

Title: Critical Gravity from AdS Boundary

Date: Nov 15, 2011 02:00 PM

URL: <http://pirsa.org/11100067>

Abstract: Critical theories of gravity are certain higher derivative theories in which parameters are so tuned as to eliminate massive excitations for the spin-2 field. Asymptotically AdS black hole entropy in these theories works out to be zero. We show that such theories arise naturally on the boundary of AdS in the form of counterterms. Such counterterms are derived by demanding cutoff independence of the Euclidean onshell action and black hole entropy.

The resummed higher derivative action so obtained can be shown to be critical. Connections with log-CFTs will be discussed. For a specific choice of parameters, these theories turn out to be non-dynamical.

BOUNDARY

$$I_{3d}^{ct} = \int d^3x \sqrt{\epsilon^{a_1 b_1 c_1} \epsilon^{a_2 b_2 c_2} G_{a_1 a_2} G_{b_1 b_2} G_{c_1 c_2}}$$

$$G_{ab} = R_{ab} - \frac{1}{2} R h_{ab} - \frac{1}{L^2} h_{ab}$$

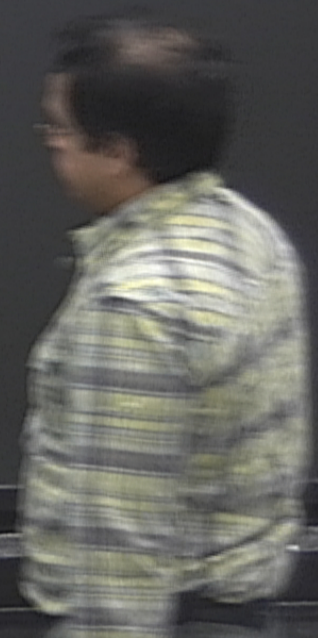
II)

(IMS_c)

$$I_{5d}^{ct} = \int d^5x \sqrt{\epsilon^{a_1 b_1 c_1 d_1 e_1} \epsilon^{a_2 b_2 c_2 d_2 e_2} G_{a_1 a_2} G_{b_1 b_2} G_{c_1 c_2} H_{d_1 d_2} H_{e_1 e_2}}$$

$$G_{ab} = R_{ab} - \frac{1}{4} R h_{ab} - \frac{1}{L^2} h_{ab}$$

$$H_{ab} = R_{ab} - \frac{1}{4} R h_{ab} + \frac{3}{2L^2} h_{ab}$$



MOTIVATION

$$\int \sqrt{-g} \left[(R - 2\Lambda) + \alpha R_{ab} R^{ab} + \beta R^2 \right] dx \rightarrow \text{Stelle} \rightarrow \text{Renormalizable} \rightarrow \text{Non-unitary}$$

$$\rightarrow \frac{1}{\beta^2 (p^2 + m^2)} \sim \frac{1}{m^2} \left(\frac{1}{p^2} - \frac{1}{p^2 + m^2} \right) \rightarrow \text{Bergshoeff, Hohm, Townsend}$$

$$\rightarrow 3\text{dim} \rightarrow \frac{D(D-3)}{2} = 0$$

New massive gravity

$$\int \sqrt{-g} (-R + \dots)$$



$\int R^2 dx \rightarrow$ Stelle \rightarrow Renormalizable \rightarrow Non-unitary

Bergshoeff, Hohm, Townsend

\rightarrow 3dim $\rightarrow \frac{D(D-3)}{2} = 0$

New massive gravity

$$\int \sqrt{g} \left[-R + \alpha \left(R_{ab} R^{ab} - \frac{3}{8} R^2 \right) \right]$$

PAIS-UHLENBECK OSCILLATOR

$$L = -\frac{1}{2} \phi \left[\prod_{i=1}^N (\square - M_i^2) \right] \phi$$

$$N=2, M_1=M_2$$

5d bulk \rightarrow 4d CFT \rightarrow cba

$$S_{\text{BH}} = 0 \rightarrow$$

0

$$\hat{g}_{ab} = \bar{g}_{ab} + \gamma_{ab} \quad \rightarrow \quad [\cancel{f(\alpha, p)} \square + g(\alpha, p)] \gamma = 0 \Rightarrow \gamma = 0$$

$$[\square + \frac{2}{L^2} - \cancel{h^2(\alpha, p)}] [\square + \frac{2}{L^2}] \gamma_{ab} = 0$$

$$(\square + \frac{2}{L^2})^2 \gamma_{ab} = 0$$

$$(\square + \frac{2}{L^2})$$

$$[\square + g(\alpha, \rho)] \gamma = 0 \Rightarrow \gamma = 0$$

$$\left[\square - \frac{1}{L^2} g(\alpha, \rho) \right] \gamma_{nl} = 0$$

$$\left(\square + \frac{z}{L} \right)^2 \gamma_{ab} = 0$$

$$\left(\square + \frac{z}{L} \right) \gamma_{ab} \neq 0$$

Poincaré + Roberts

$$H \psi = E_0 \psi \leftarrow$$

$$H \phi = E_0 \phi + \psi$$

$$H \begin{pmatrix} \psi \\ \phi \end{pmatrix} = \begin{pmatrix} z & 1 \\ 0 & z \end{pmatrix} \begin{pmatrix} \psi \\ \phi \end{pmatrix}$$

Jordan

$$(H - E_0)^2 \phi = (H - E_0) \psi = 0$$

$$\langle \phi, (H - E_0)^2 \phi \rangle = \langle (H - E_0) \psi, (H - E_0) \psi \rangle = \langle \psi, \psi \rangle = 0$$

$$\langle S, \psi \rangle \neq 0$$

$$\langle S, S \rangle > 0$$

$$|S + z\psi| < 0$$

LOG-CFTS \rightarrow Gurarie

Disorder

Mean fields

Large Disorder \rightarrow Random critical point

$2+1$ d turbulence

Polyakov \rightarrow Minimal model

$E(\omega) \sim \frac{1}{\omega^2}$

Kraichnan
scaling

LOG-CFTS \rightarrow Gurarie

Disorder

Mean fields

Large Disorder \rightarrow Random critical point

$2+1$ d turbulence

Polyakov \rightarrow Minimal model

$E(k) \sim \frac{1}{k^\alpha}$ Kraichnan scaling

$\alpha \rightarrow 3-4$

$$\left(R_{ab} R^{ab} - \frac{n}{4(n-1)} R^2 \right)$$

Comparison + Johnson + Myers 9903238

$$I = -\frac{1}{2l^{d-1}} \int_{\Sigma} \sqrt{g} \left(R + \frac{d(d-1)}{L^2} \right) - \frac{1}{l_P^{d-1}} \int_{\Sigma} d^d x \sqrt{h} K + I_{ct}$$

$$I_{\text{eff}} = \frac{1}{L^{\frac{d-1}{2}}} \int_{\mathcal{M}} d^d x \sqrt{h} \left[\frac{d-1}{L} + \frac{L}{2(d-2)} R + \frac{L^2}{2(d-4)(d-2)} (R_{\mu\nu} R^{\mu\nu} - \frac{d}{4} R^2) \right]$$

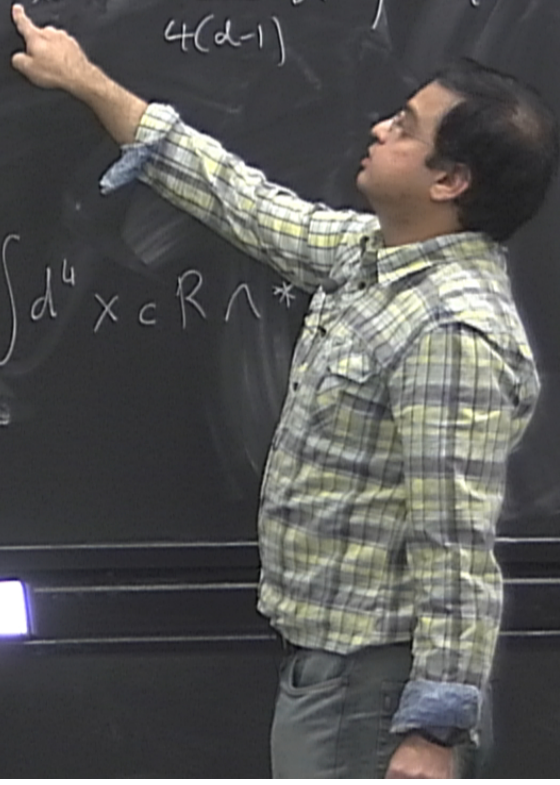
$$\frac{1}{2(d-2)} R + \frac{L^2}{2(d-4)(d-2)} \left(R_{\mu\nu} R^{\mu\nu} - \frac{d}{4(d-1)} R^2 \right) + \dots$$

$$\frac{1}{L^d} \int d^d x \sqrt{h} \left[\frac{d-1}{L} + \frac{L}{2(d-2)} R + \frac{L^2}{2(d-4)(d-2)} \left(R_{\mu\nu} R^{\mu\nu} - \frac{d}{4(d-1)} R^2 \right) + \dots \right]$$

$$\int d^3 x \sqrt{g} (R - 2\Lambda) + \epsilon^{\mu\nu\rho} \left(\Gamma^\sigma \Gamma^\rho + \frac{2}{3} \Gamma^\sigma \Gamma^\rho \right)$$

Surface

$$\int d^4 x \epsilon R \wedge *$$



BOUNDARY

ii)

(IMS_c)

$$I_{3d}^{ct} = \int d^3x \sqrt{\epsilon^{a_1 b_1 c_1} \epsilon^{a_2 b_2 c_2} G_{a_1 a_2} G_{b_1 b_2} G_{c_1 c_2}}$$

$$G_{ab} = R_{ab} - \frac{1}{2} R h_{ab} - \frac{1}{L^2} h_{ab}$$

$$I_{5d}^{ct} = \int d^5x \sqrt{\epsilon^{a_1 b_1 c_1 d_1 e_1} \epsilon^{a_2 b_2 c_2 d_2 e_2} G_{a_1 a_2} G_{b_1 b_2} G_{c_1 c_2} H_{d_1 d_2} H_{e_1 e_2}}$$

$$G_{ab} = R_{ab} - \frac{1}{4} R h_{ab} - \frac{1}{L^2} h_{ab}$$

$$H_{ab} = R_{ab} - \frac{1}{4} R h_{ab} + \frac{3}{2L^2} h_{ab}$$

det - action \rightarrow DBI gravity

Eddington

$$\int \sqrt{-g} R$$

$$- \frac{1}{\Lambda} \int \sqrt{\det R_{\mu\nu}(\Gamma)}$$

$$\int \sqrt{-g} (R(\Gamma) g_{ab} - 2\Lambda)$$

$$g_{ab} \rightarrow g_{ab} + \nabla_a v_b + \nabla_b v_a \rightarrow \nabla_a Y^a_b = \nabla_b Y$$

$$h_{ab} \rightarrow h_{ab} + \nabla_a w_b + \nabla_b w_a \rightarrow \text{Restricted gauge}$$

$$\therefore \nabla \cdot W = 0$$

$$\sum_{a_1 b_1 c_1} G_{a_1 a_2} G_{b_1 b_2} G_{c_1 c_2}$$

$$S = \left(C_5 \right) T^\alpha$$

$$\frac{1}{2} R_{hab} - \frac{1}{L^2} h_{ab}$$

$$\sum_{a_1 b_1 c_1 d_1 e_1} G_{a_1 a_2} G_{b_1 b_2} G_{c_1 c_2} H_{d_1 d_2} H_{e_1 e_2}$$

$$R_{ab} - \frac{1}{4} R_{hab} - \frac{1}{L^2} h_{ab}$$

$$= R_{ab} - \frac{1}{4} R_{hab} + \frac{3}{2L^2} h_{ab}$$

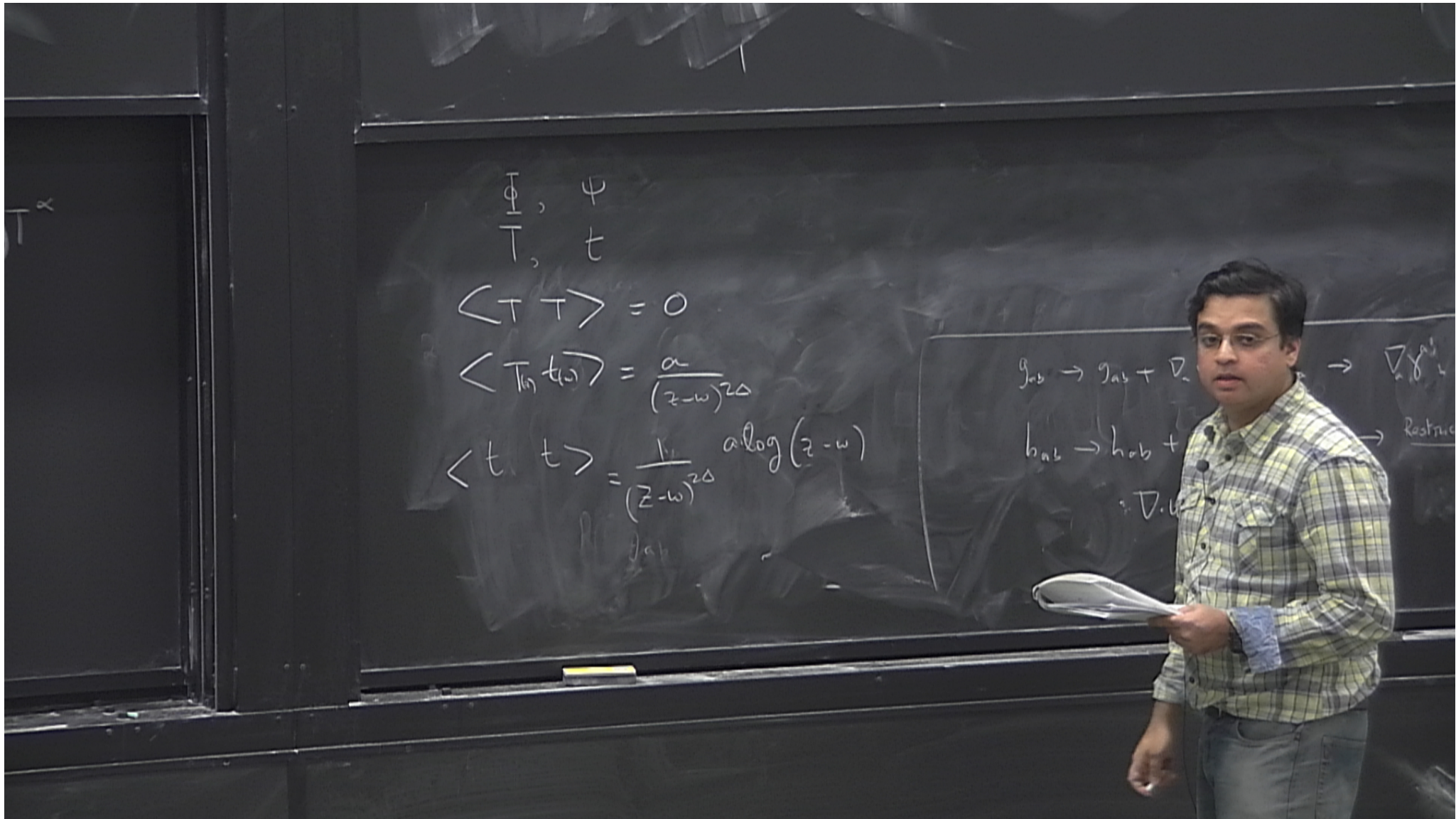
det - action \rightarrow DBI

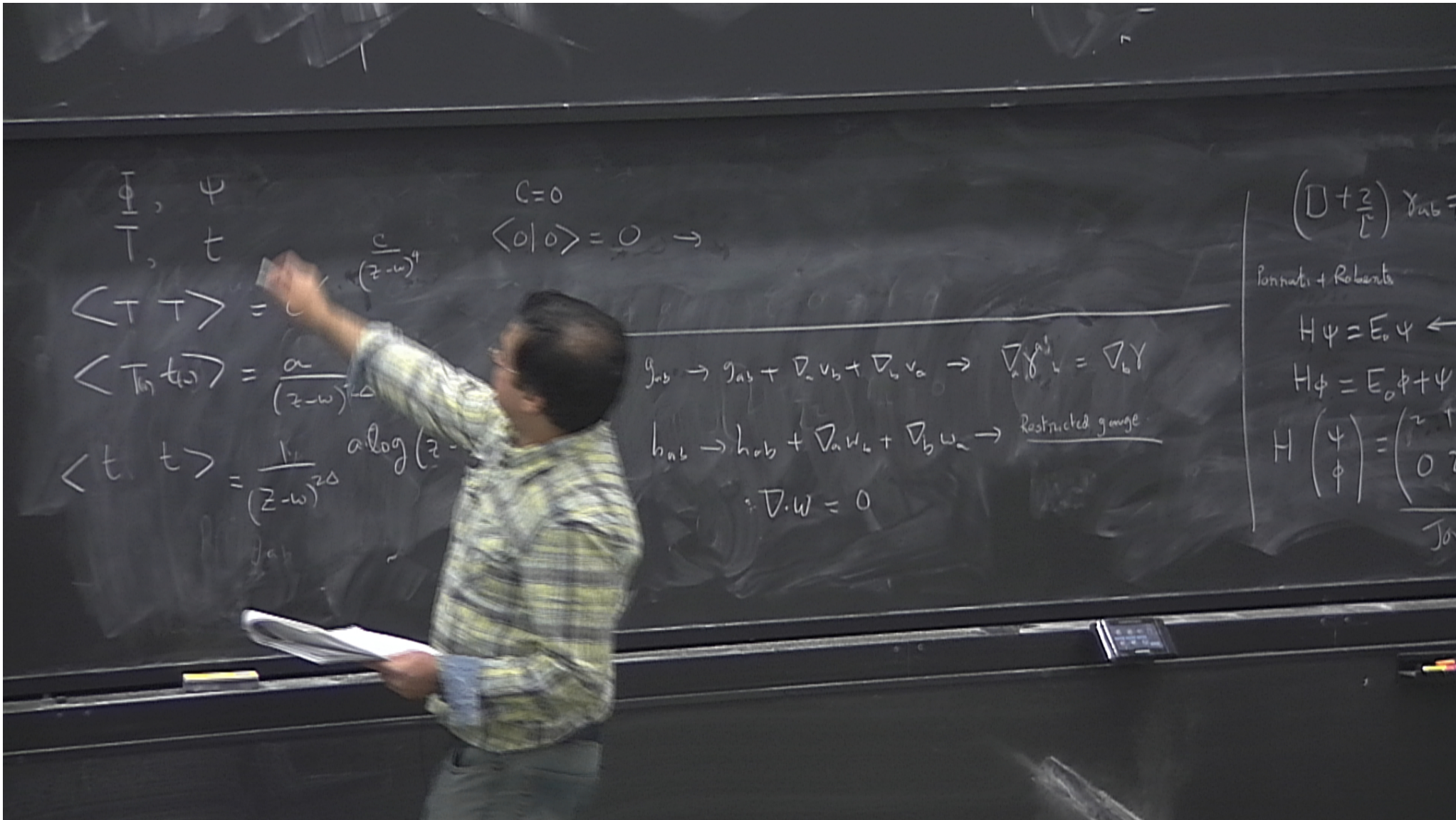
Eddington

$$\int \sqrt{-g} R$$

$$= \alpha \frac{1}{\lambda} \int \sqrt{\det R_{\mu\nu}(\Gamma)}$$

$$\int \sqrt{-g} \left(R^{ab}(\Gamma) g_{ab} - 2\Lambda \right)$$





$$\begin{aligned}
 & \Phi, \psi \\
 & \bar{T}, t \\
 & \langle T T \rangle = \frac{c}{(z-w)^4} \\
 & \langle \bar{T}_n t_m \rangle = \frac{a}{(z-w)^{2\Delta}} \\
 & \langle t_n t_m \rangle = \frac{1}{(z-w)^{2\Delta}} \alpha \log(z-w)
 \end{aligned}$$

$$c=0 \\
 \langle 0|0 \rangle = 0 \rightarrow$$

$$\begin{aligned}
 g_{ab} & \rightarrow g_{ab} + \nabla_a v_b + \nabla_b v_a \rightarrow \nabla_a \gamma^a_b = \nabla_b \gamma \\
 h_{ab} & \rightarrow h_{ab} + \nabla_a w_b + \nabla_b w_a \rightarrow \text{Restricted gauge} \\
 & \nabla \cdot w = 0
 \end{aligned}$$

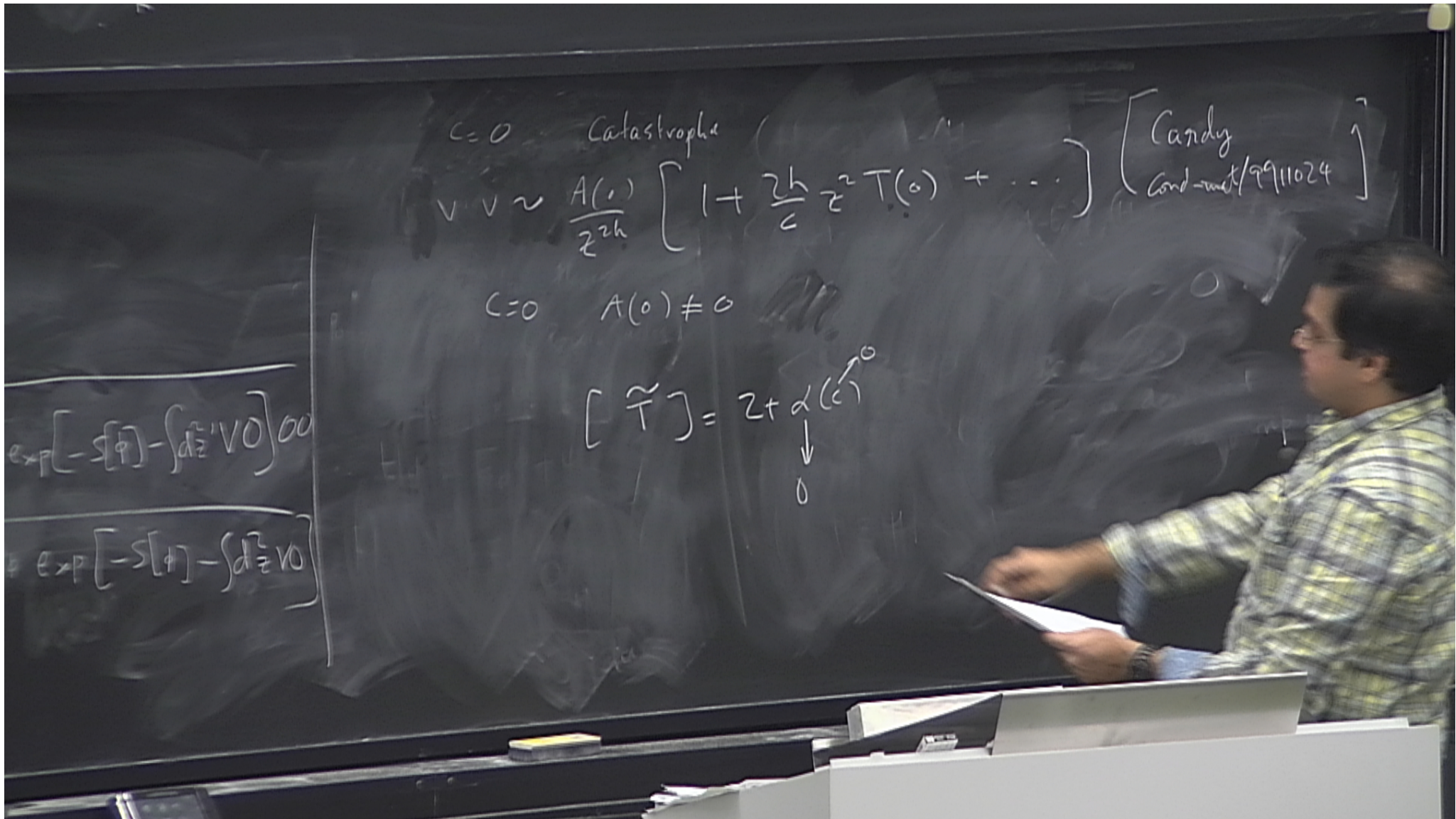
$$\left(\nabla + \frac{z}{l} \right) \gamma_{ab} =$$

Poincaré + Roberts

$$\begin{aligned}
 H\psi & = E_0 \psi \leftarrow \\
 H\phi & = E_0 \phi + \psi \\
 H \begin{pmatrix} \psi \\ \phi \end{pmatrix} & = \begin{pmatrix} z & \psi \\ 0 & \phi \end{pmatrix}
 \end{aligned}$$

Φ, Ψ ← Rank- n Jordan cell
 \bar{T}, t $C=0$
 $\langle 0|0\rangle = 0 \rightarrow$
 $\langle T T \rangle = 0 \leftarrow \frac{c}{(z-w)^4}$
 $\langle \bar{T}_h t_w \rangle = \frac{a}{(z-w)^{2\Delta}}$
 $\langle t_w t_w \rangle = \frac{1}{(z-w)^{2\Delta}} a (\log(z-w))^{h-1}$

Quenched disorder
 $\langle 0|0\rangle = \frac{\int \mathcal{D}V P[V] \int \mathcal{D}\phi \exp[-S[\phi] - \int d^2z V O]}{\int \mathcal{D}\phi \exp[-S[\phi] - \int d^2z V O]}$



$c=0$ Catastrophe

$$v \sim \frac{A(\epsilon)}{z^{2h}} \left[1 + \frac{2h}{c} z^2 T(\epsilon) + \dots \right]$$

[Candy
Cond-mat/9911024]

$c=0$ $A(\epsilon) \neq 0$

$$[\tilde{T}] = z + \alpha(\epsilon) \begin{matrix} \nearrow 0 \\ \downarrow 0 \end{matrix}$$

$$\exp[-S(\phi) - \int d^2z V(\phi)]_{00}$$
$$\exp[-S(\phi) - \int d^2z V(\phi)]$$