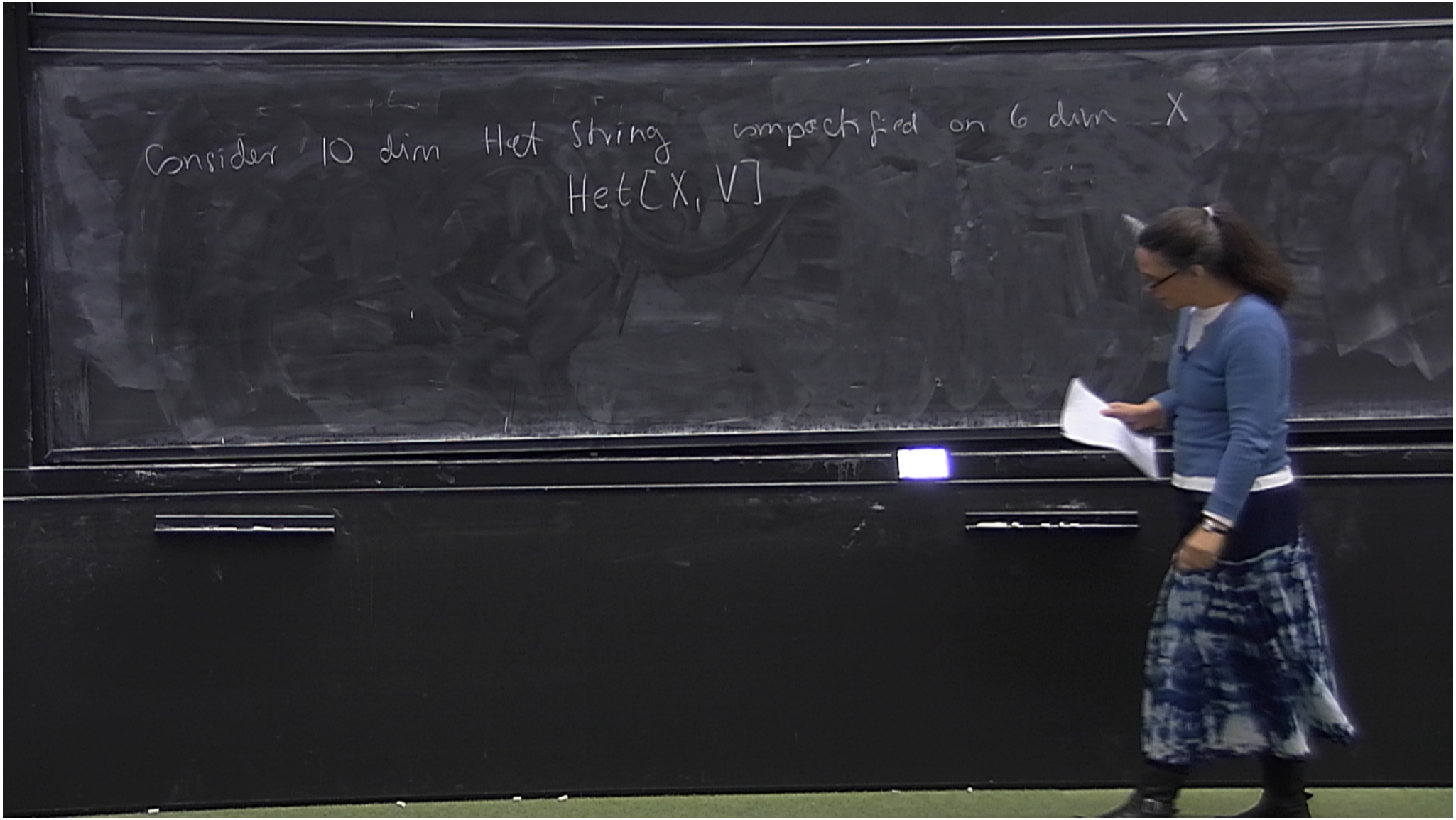
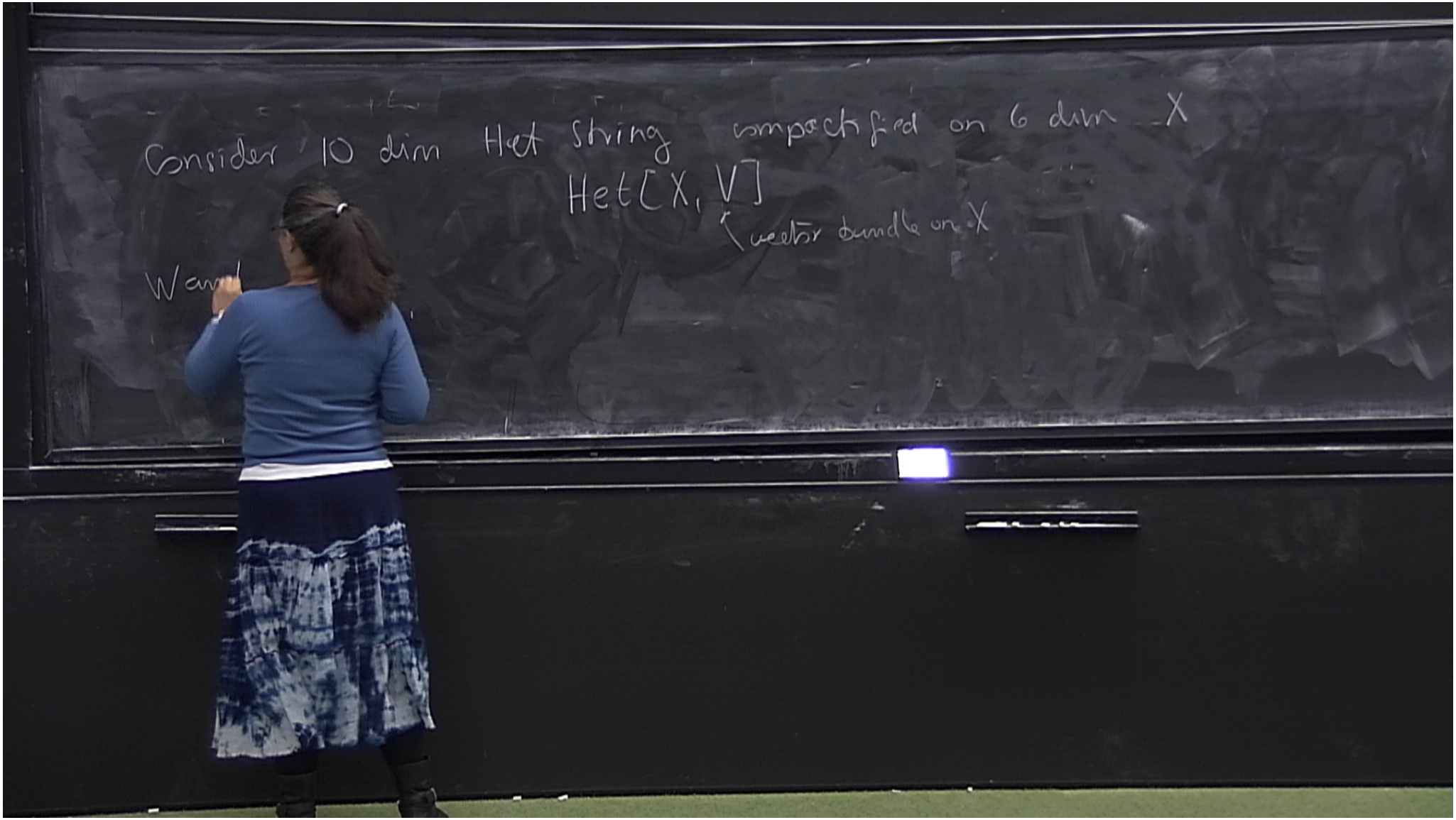
Title: Geometry and Connectedness of Heterotic String Compactifications with Fluxes

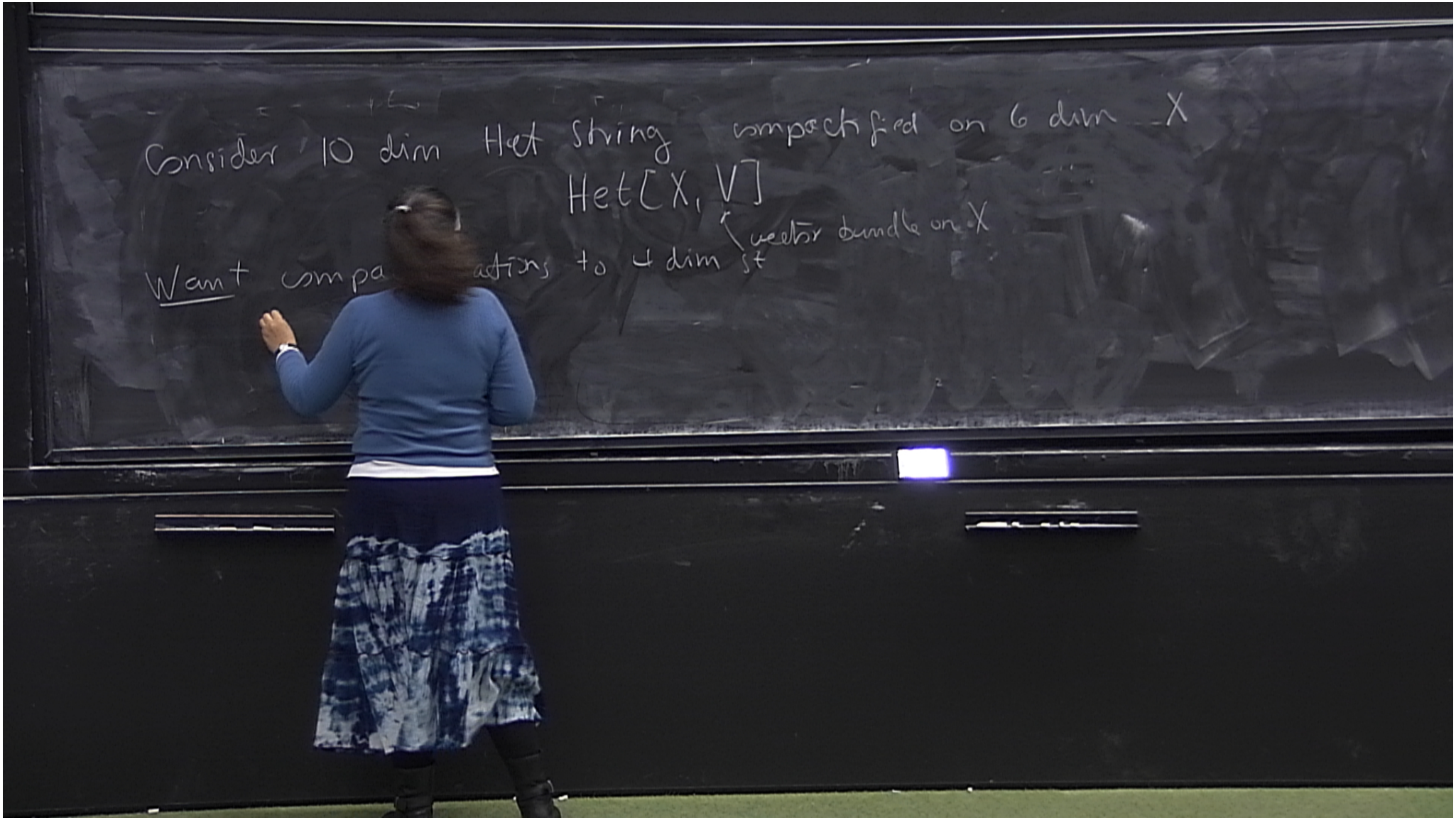
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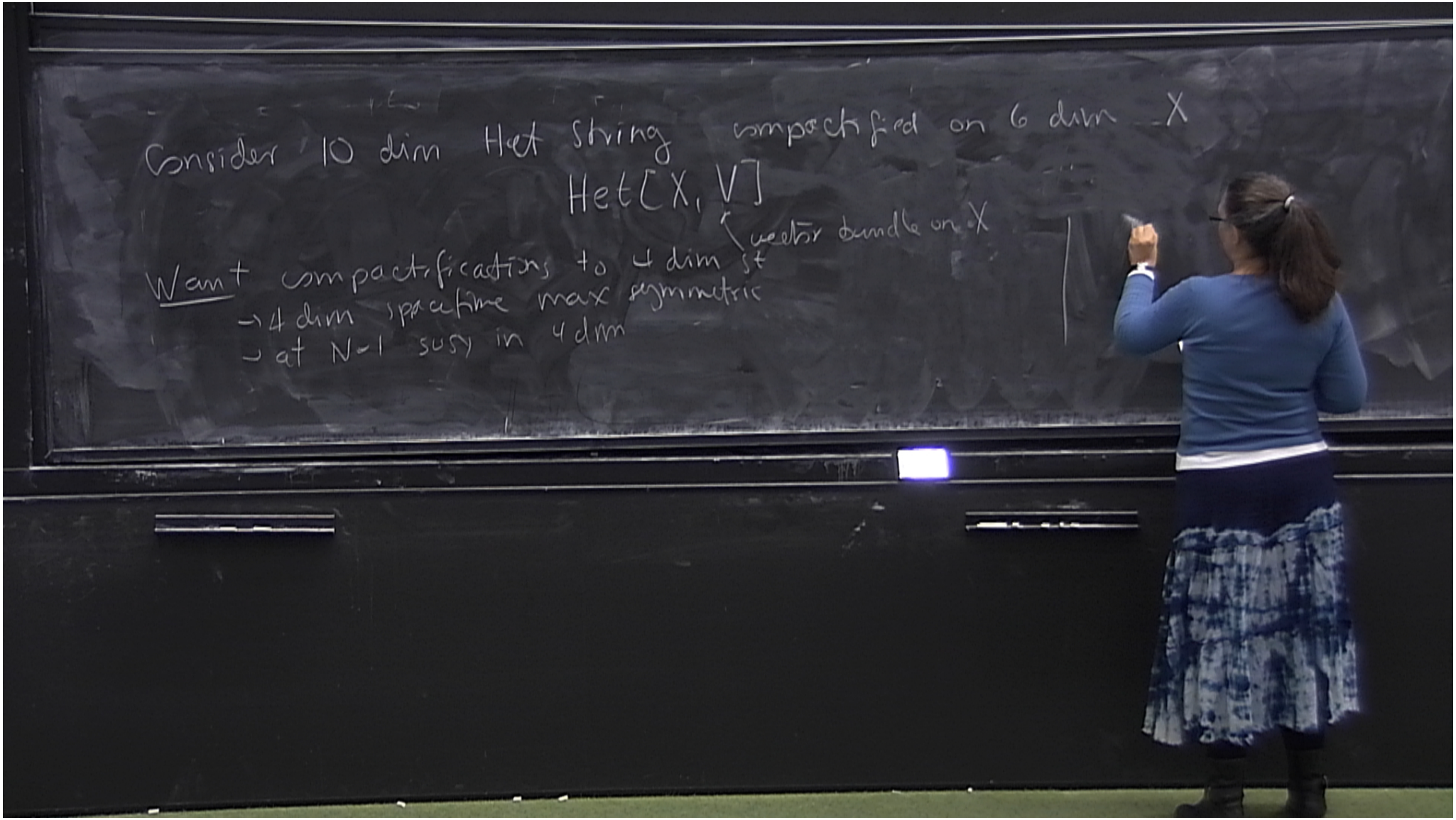
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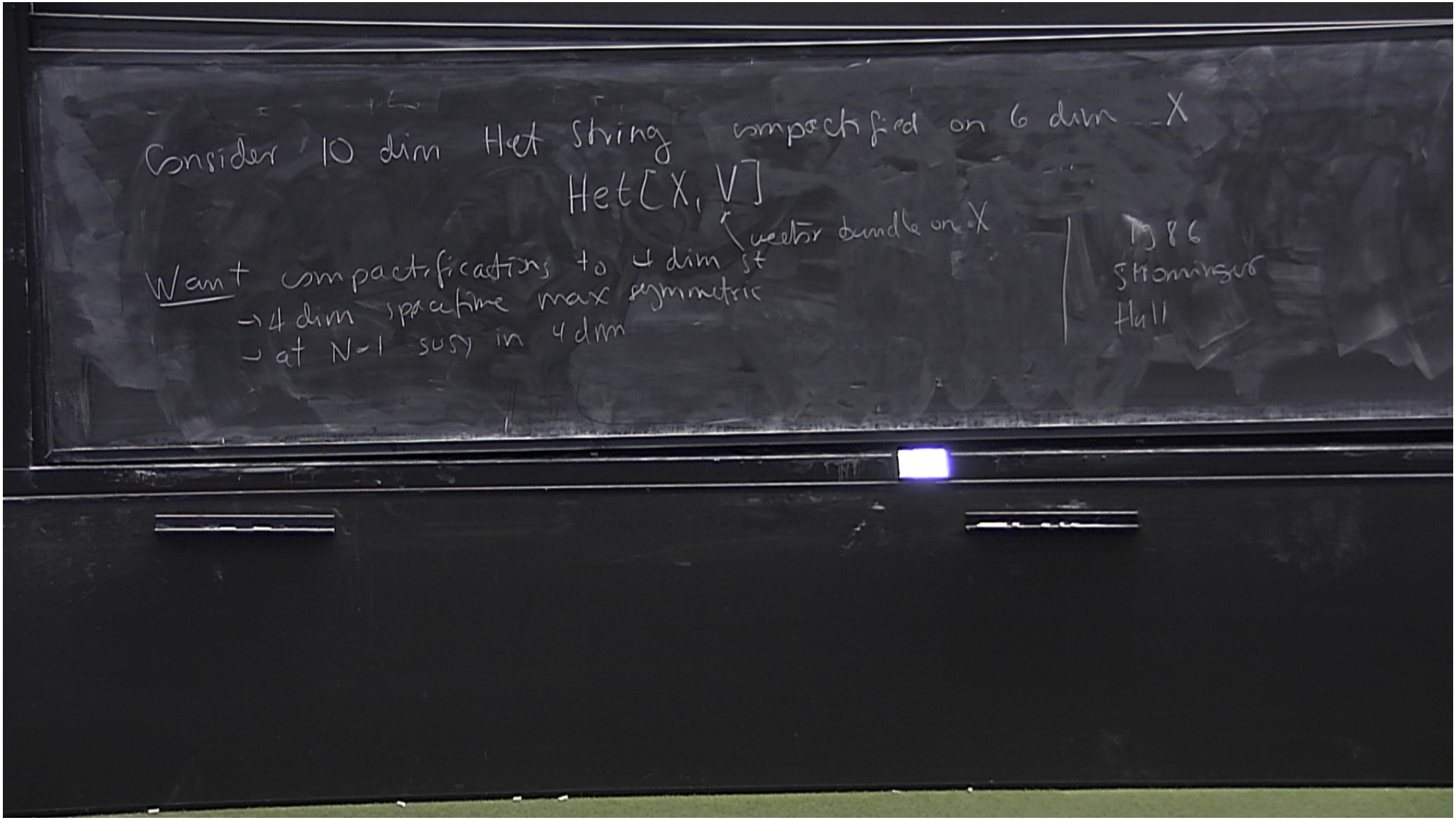
Abstract: I will discuss the geometry of heterotic string compactifications with fluxes. The compactifications on 6 dimensional manifolds which preserve N=1 supersymmetry in 4 dimensions must be complex manifolds with vanishing first Chern class, but which are not in general Kahler (and therefore not Calabi-Yau manifolds) together with a vector bundle on the manifold which must satisfy a complicated differential equation. The flux, which can be viewed as a torsion, is the obstruction to the manifold being Kahler. I will describe how these compactifications are connected to the more traditional compactifications on Calabi-Yau manifolds through geometric transitions like flops and conifold transitions. For instance, one can construct solutions by flopping rational curves in a Calabi-Yau manifold in such a way that the resulting manifold is no longer Kahler. Time permitting, I will discuss open problems, for example the understanding of the the moduli space of heterotic compactifications and the related problem of determining the massless spectrum in the effective 4 dimensional supersymmetric field theory. The study of these compactifications is interesting on its own right both in string theory, in order to understand more generally the degrees of freedom of these theories, and also in mathematics. For instance, the connectedness between the solutions is related to problems in mathematics like the conjecture by Mile Reid that complex manifolds with trivial canonical bundle are all connected through geometric transitions.

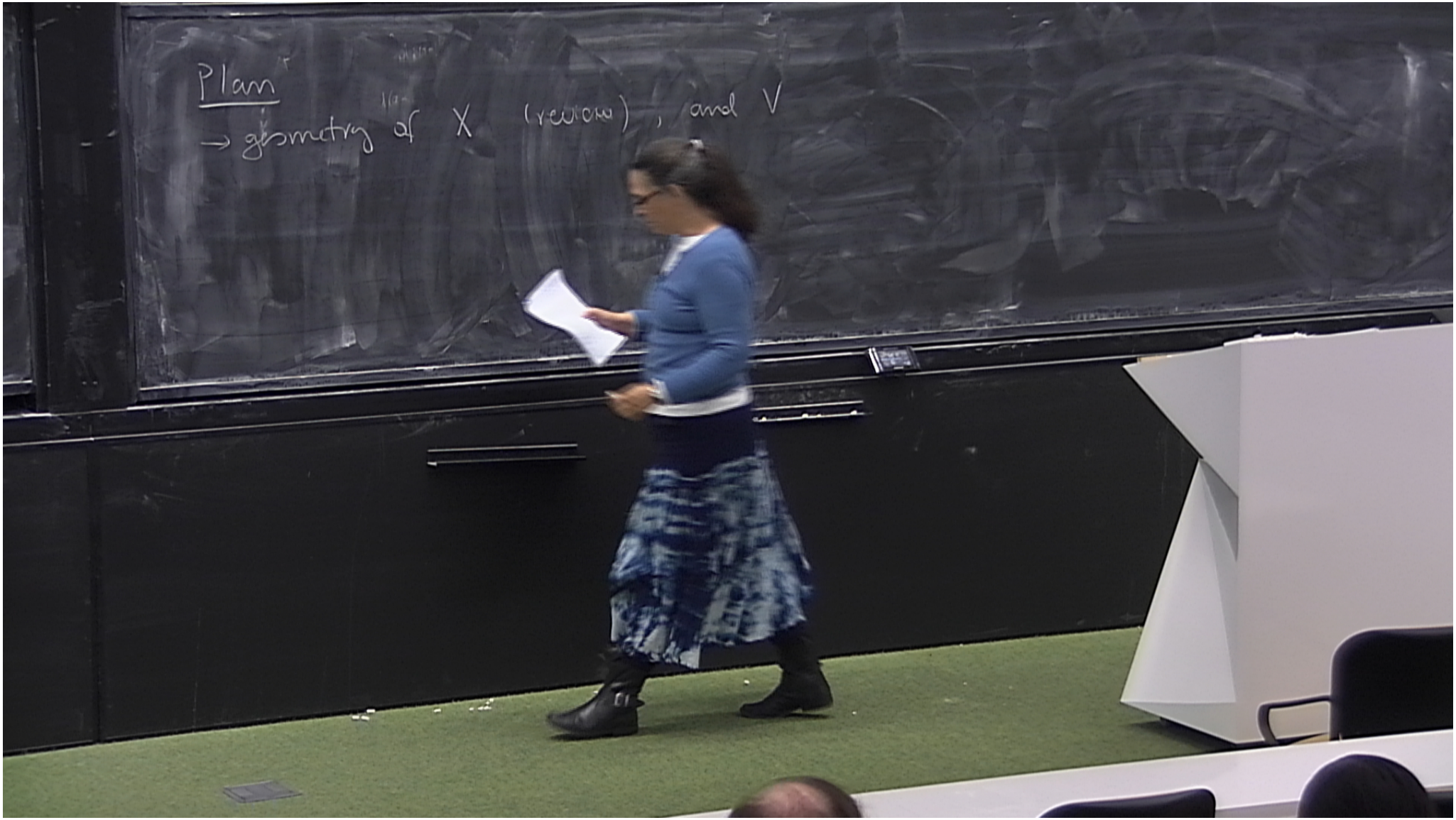

Consider 10 dim Het string compactified on 6 dim X
Het[X, V]

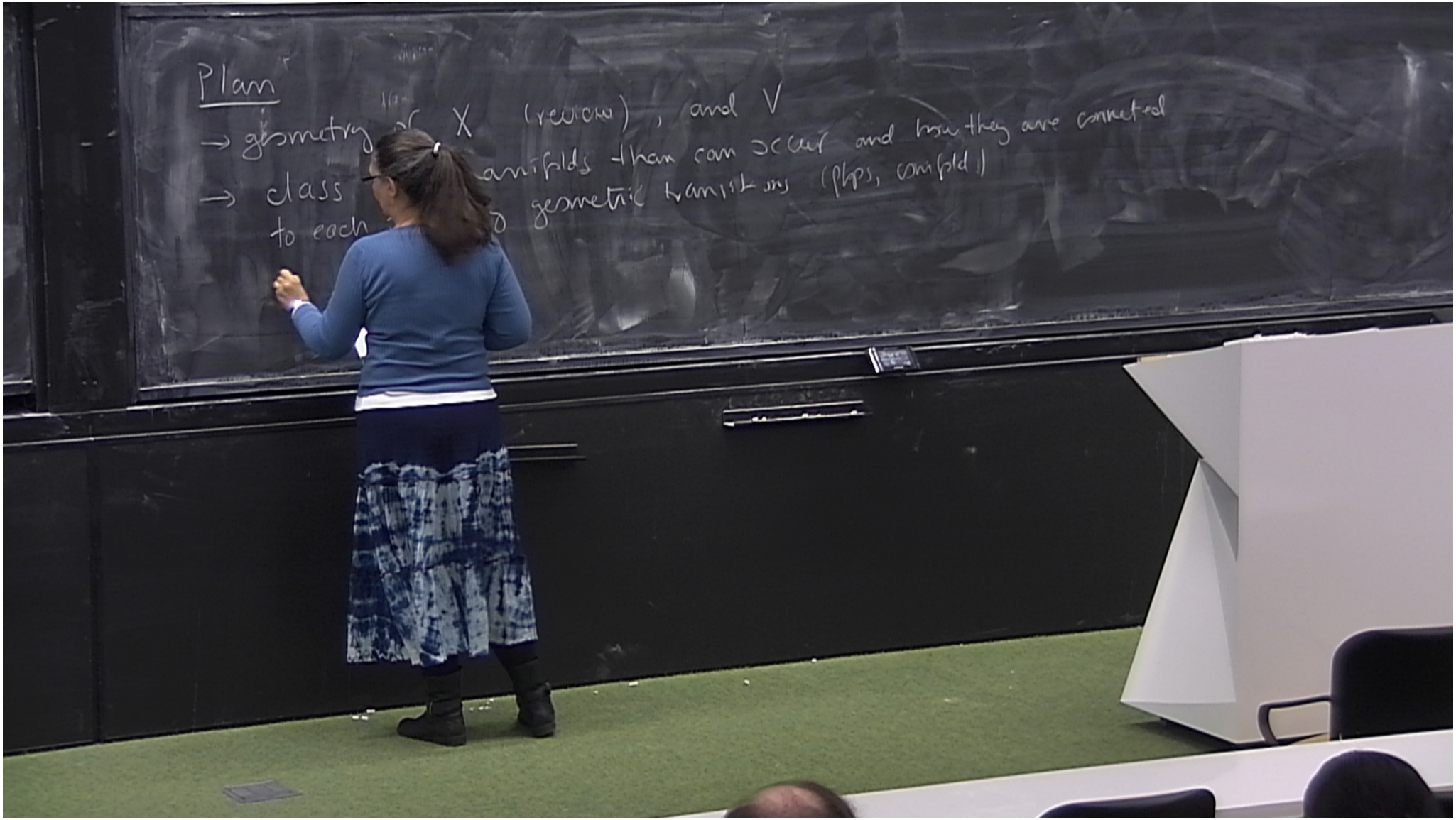

Consider 10 dim Het string compactified on 6 dim X
Het[X, V]
vector bundle on X

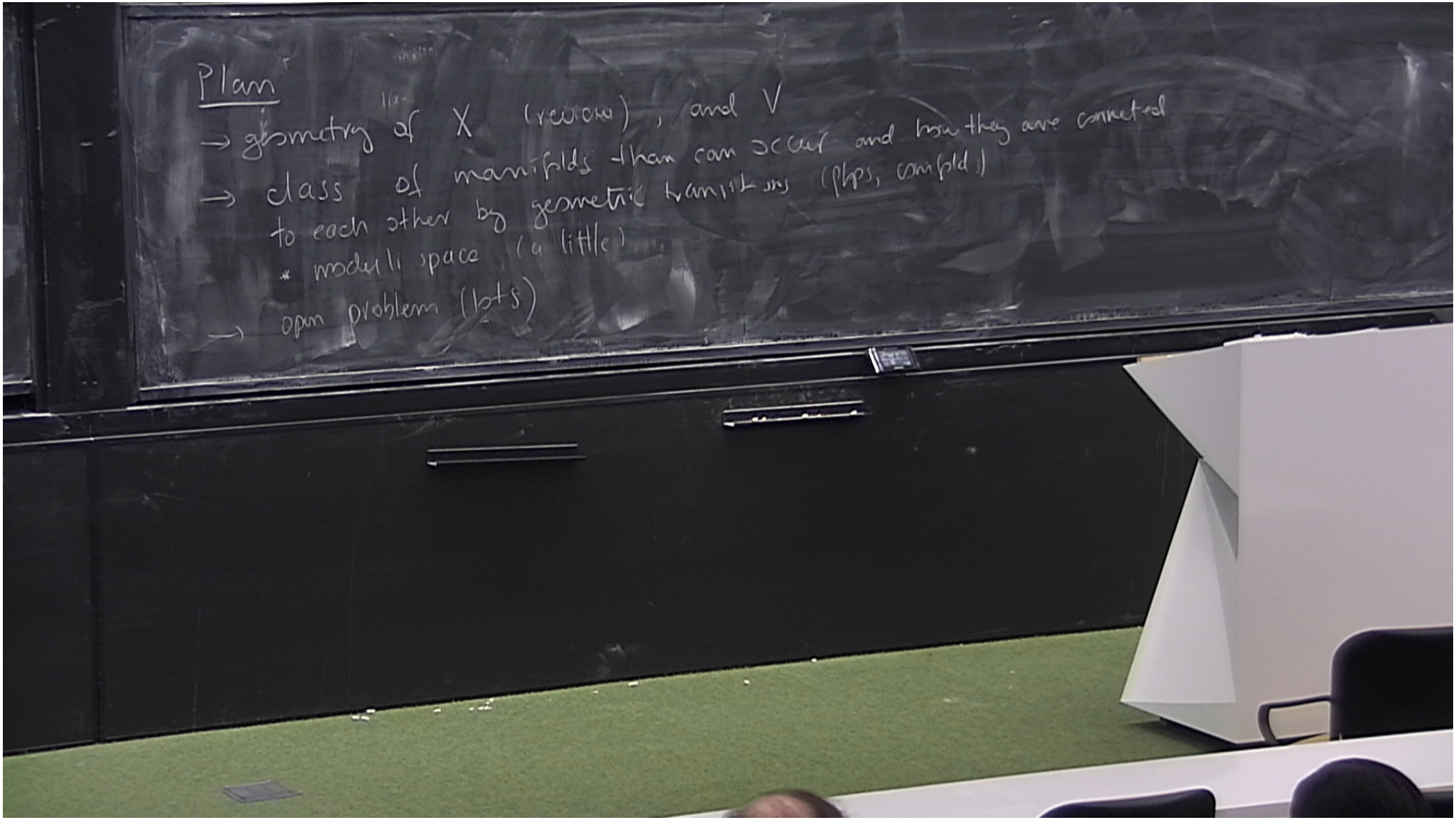

Consider 10 dim Het string compactified on 6 dim X
Het[X, V]
vector bundle on X
Want compa ations to 4 dim st

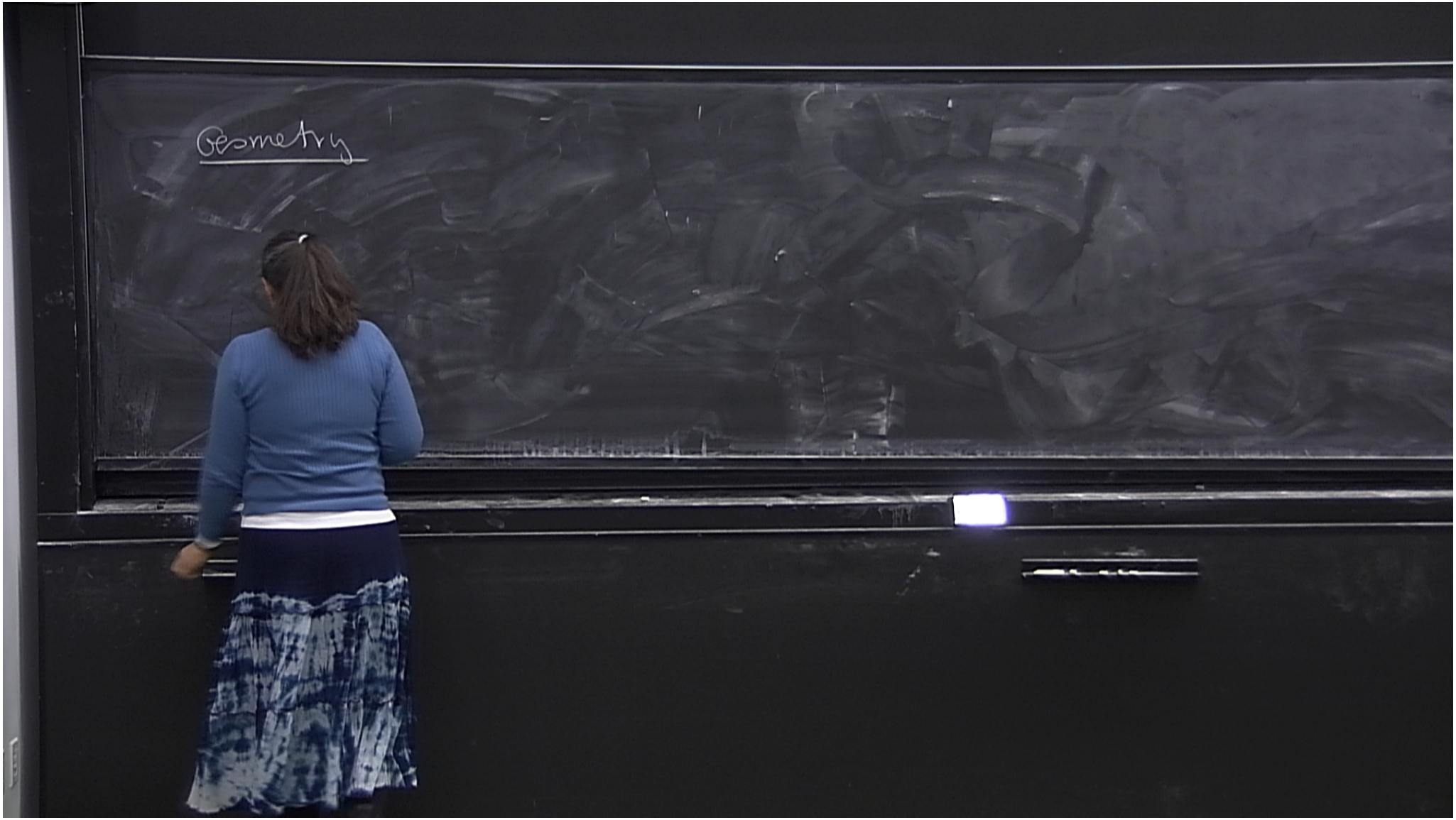

Consider 10 dim Het string compactified on 6 dim X
Het[X, V]
vector bundle on X
Want compactifications to 4 dim st
→ 4 dim spacetime max symmetric
→ at N=1 susy in 4 dim

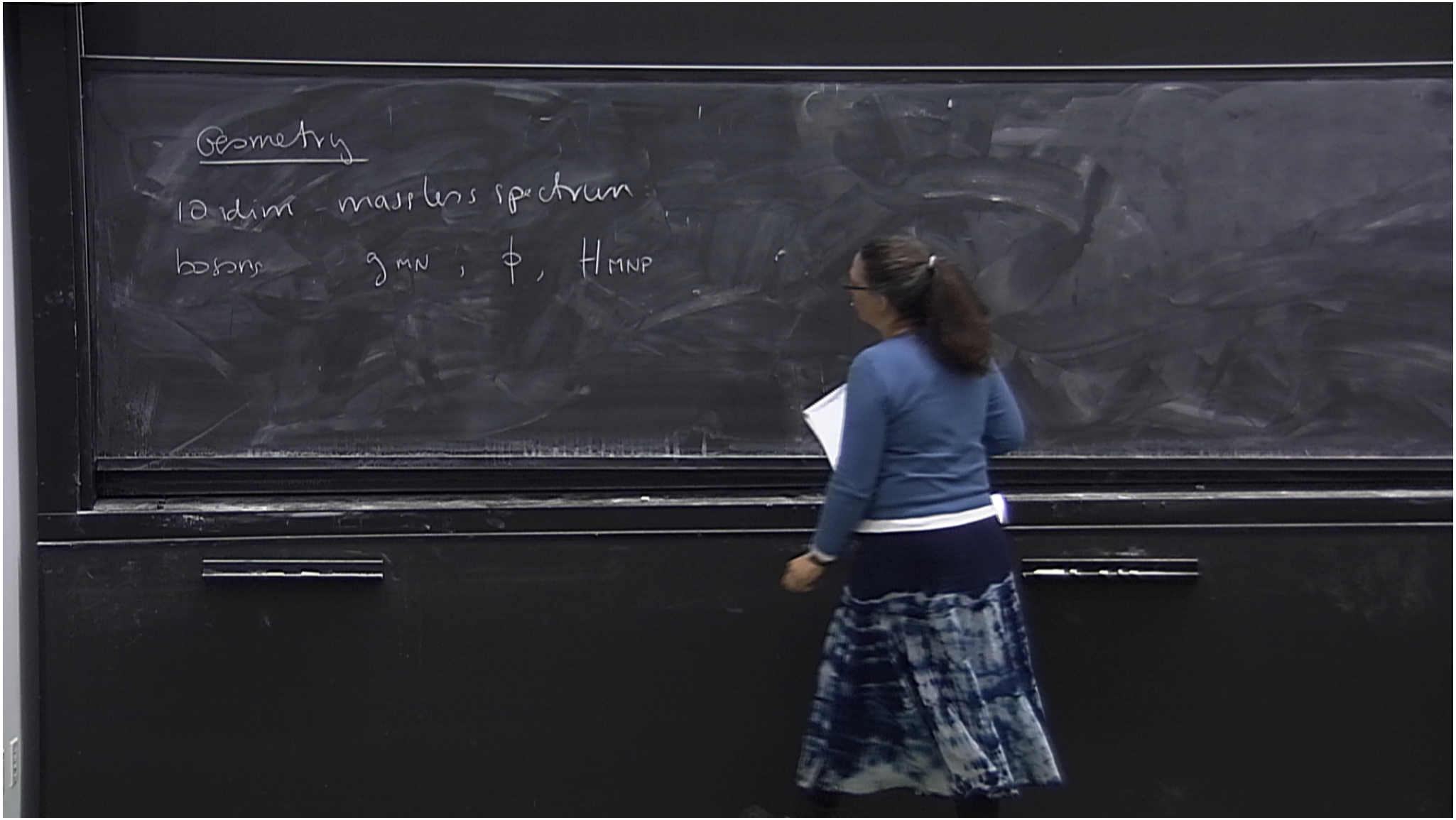

Consider 10 dim Het string compactified on 6 dim X
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Want compactifications to 4 dim st
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1986
Strominger
Hull

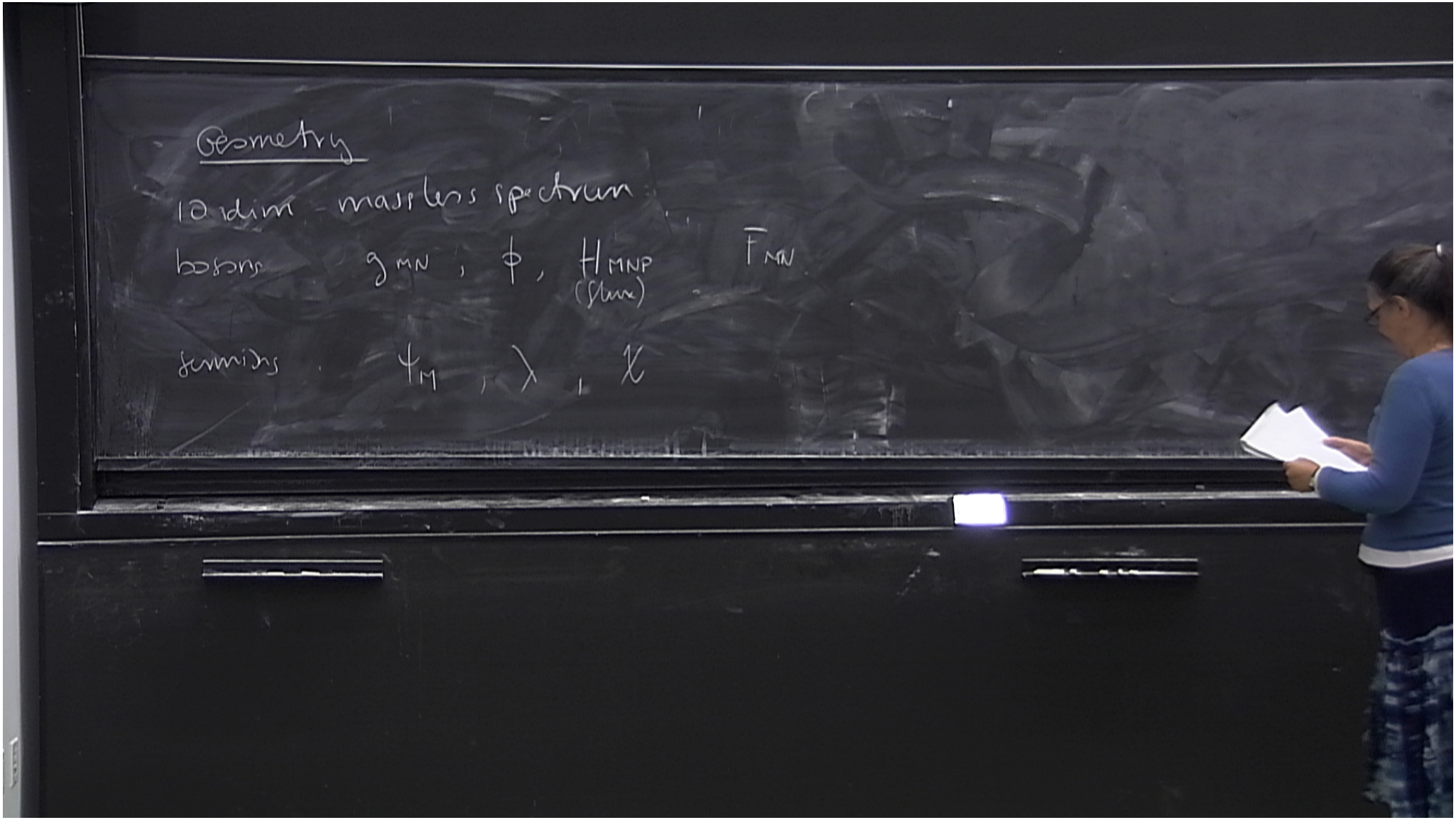

Plan
→ geometry of X (review), and V

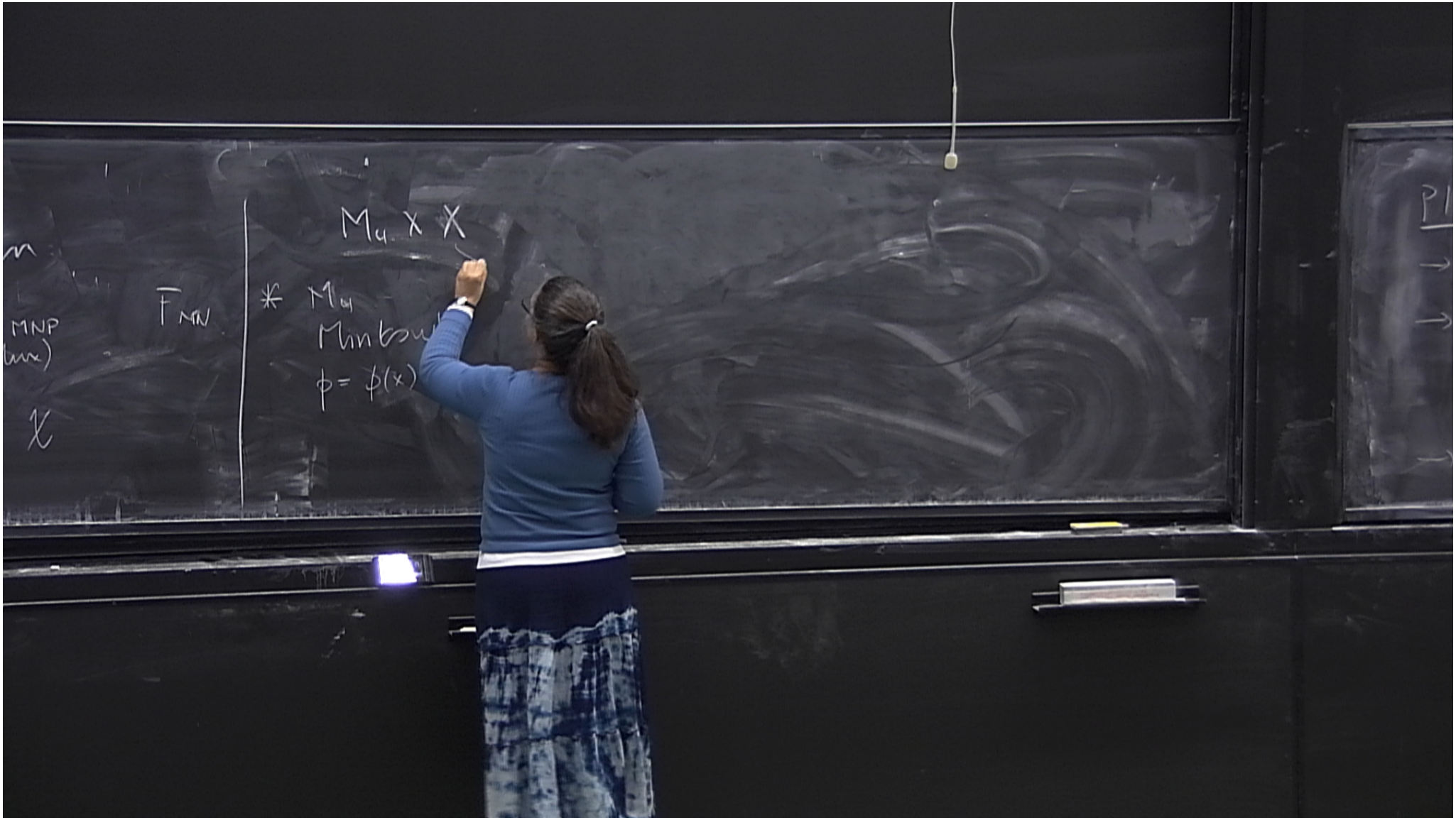

Plan
→ geometry of X (review), and V
→ class
to each
manifolds that can occur and how they are connected
geometric transitions (flips, conifold)

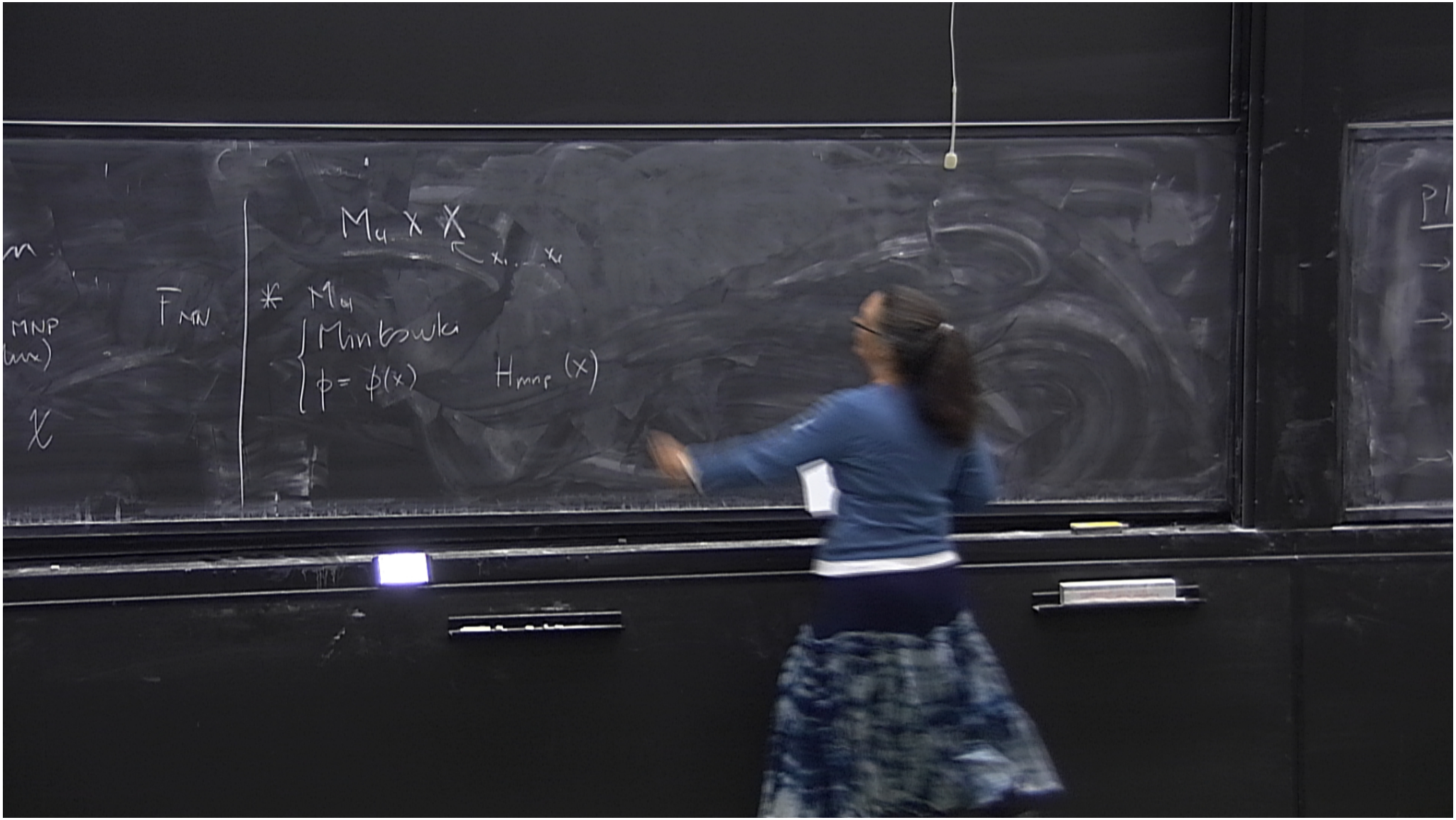

Plan
→ geometry of X (review), and V
→ class of manifolds that can occur and how they are connected to each other by geometric transitions (flips, conifold)
* moduli space (a little)
→ open problem (bts)

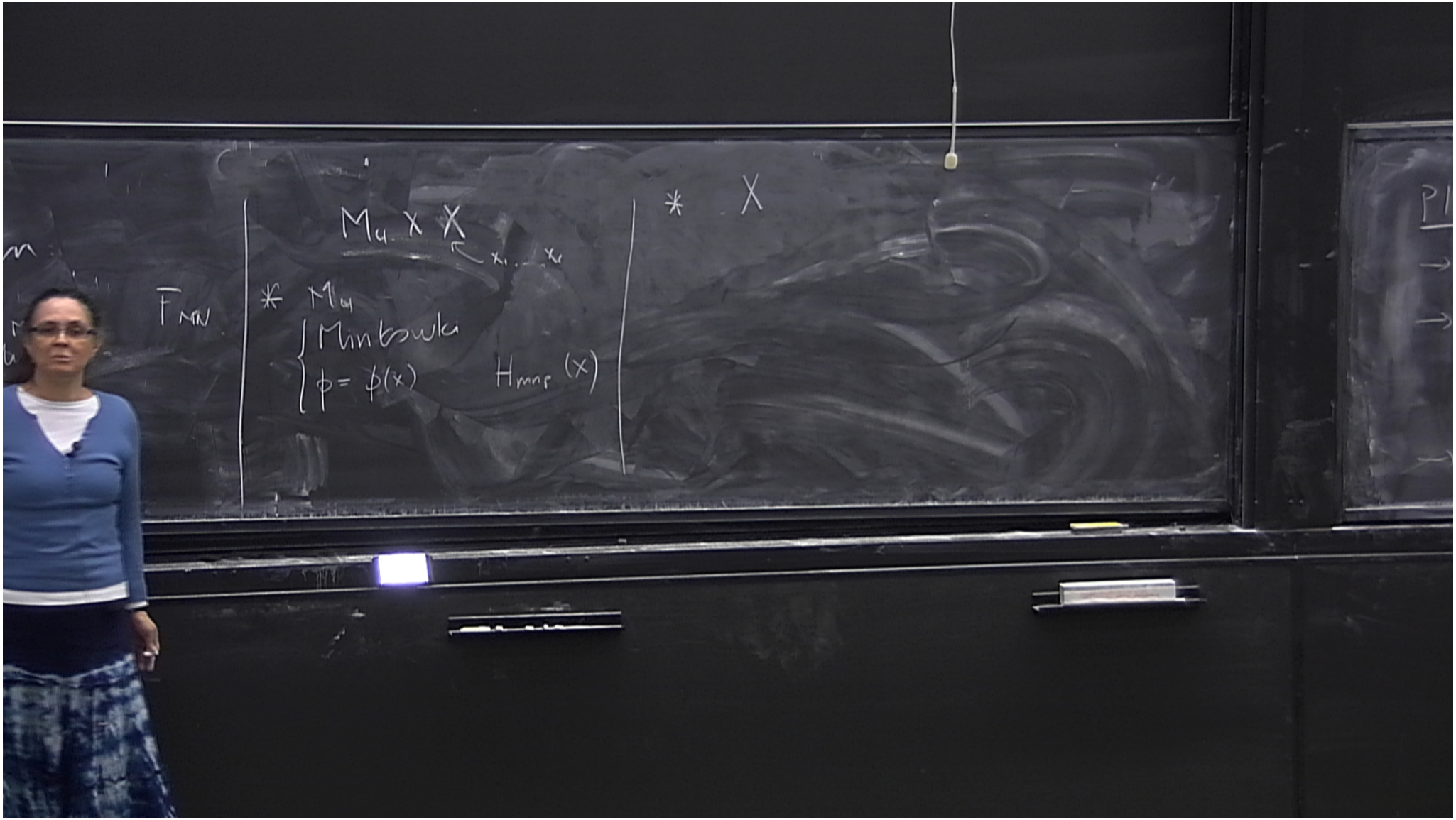

Geometry

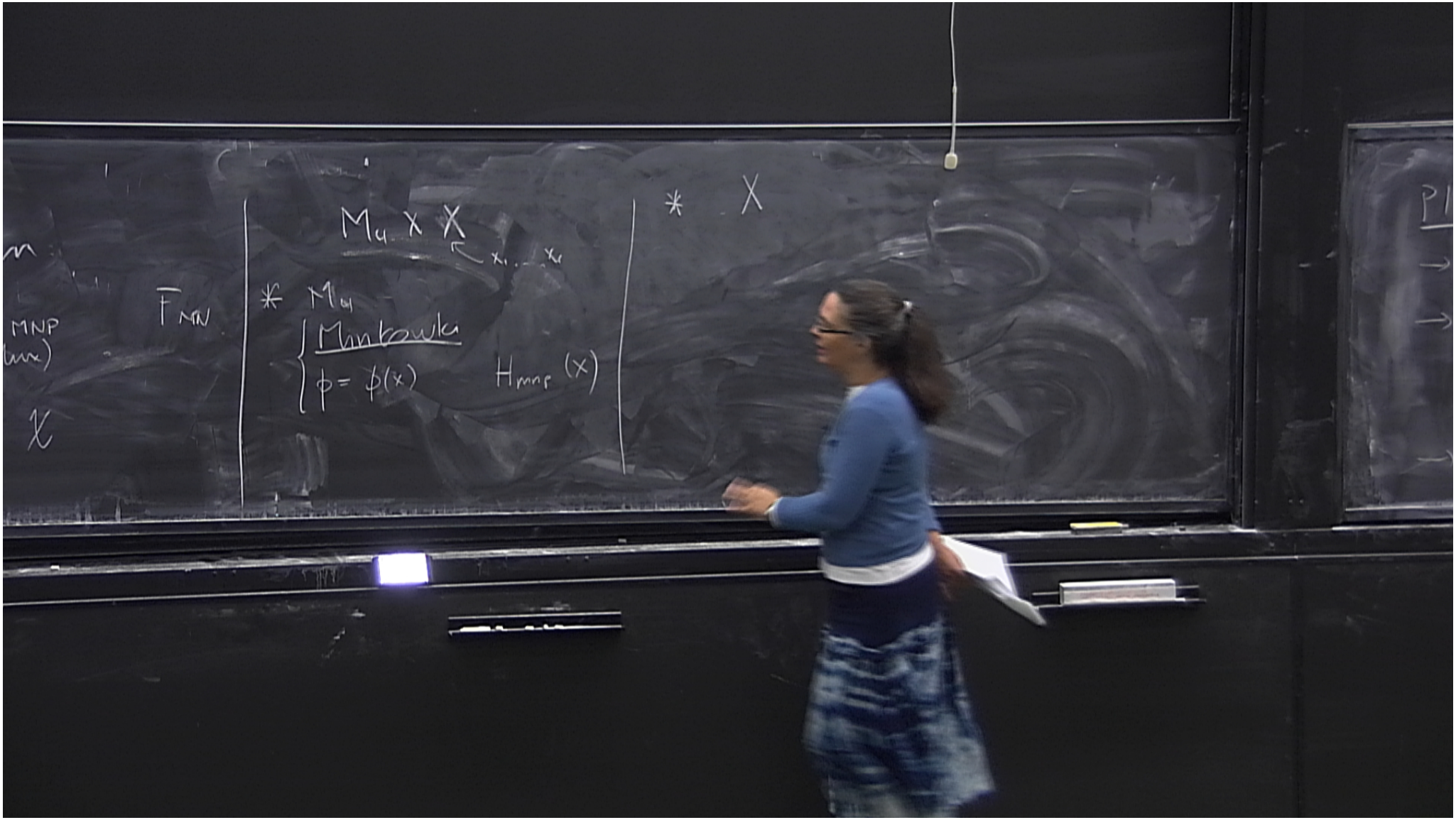

Geometry
10 dim - massless spectrum
bosons g_{MN} , ϕ , H_{MNP}

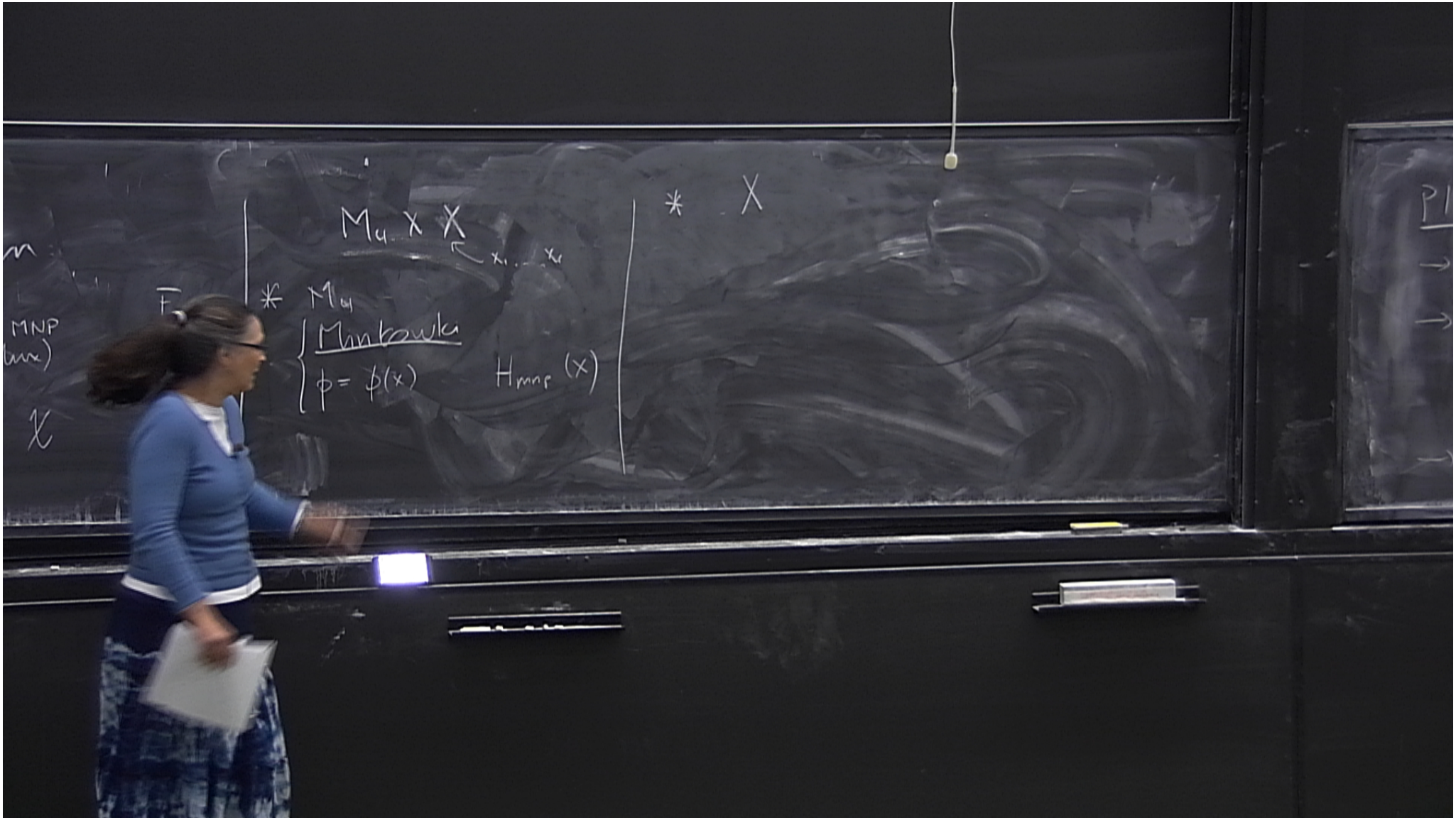

Geometry
10 dim - massless spectrum
bosons g_{MN} , ϕ , H_{MNP} (flux) F_{MN}
fermions ψ_M , λ , χ

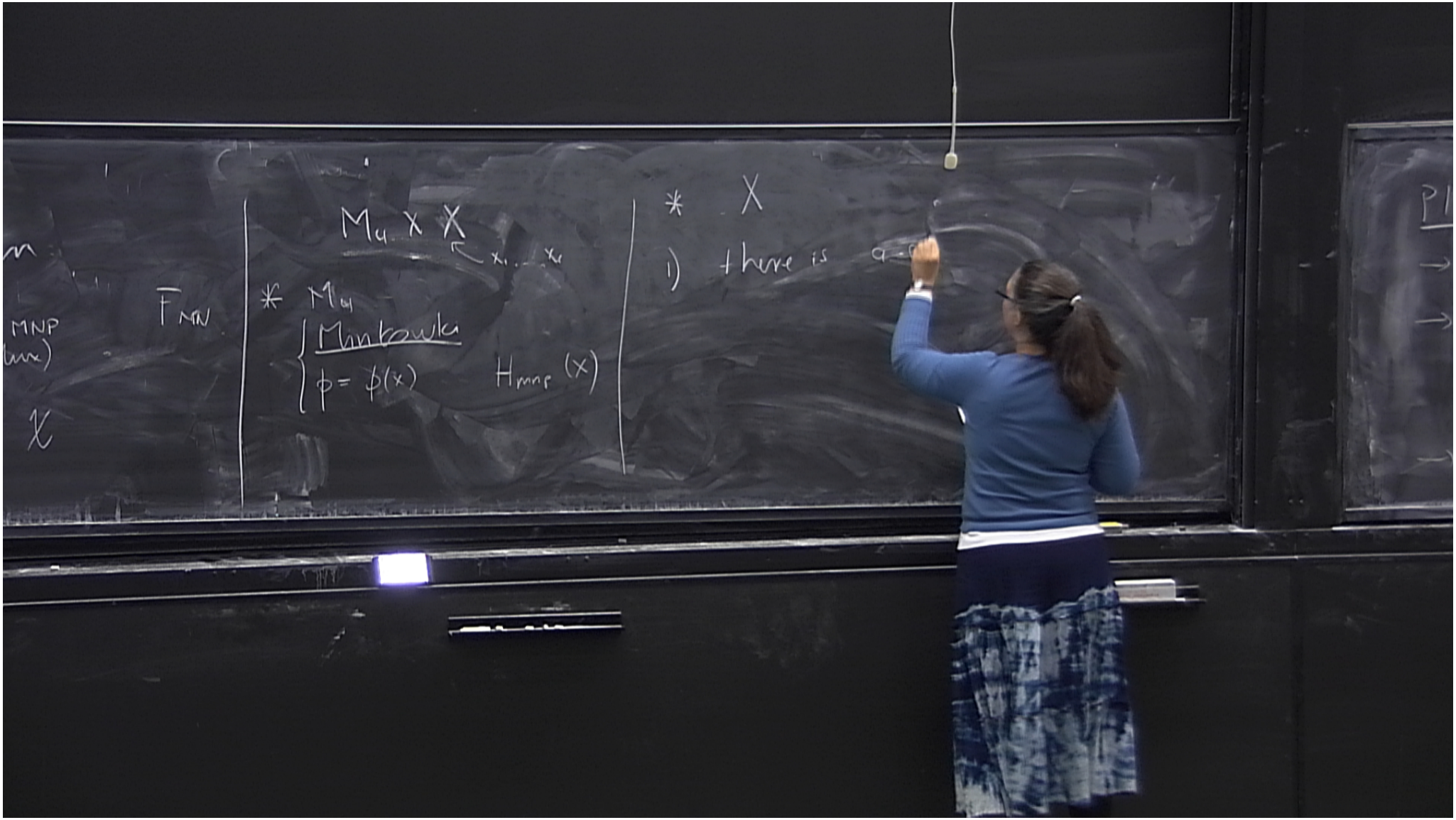

$M_4 \times X$
F_{MN}
$\phi = \phi(x)$

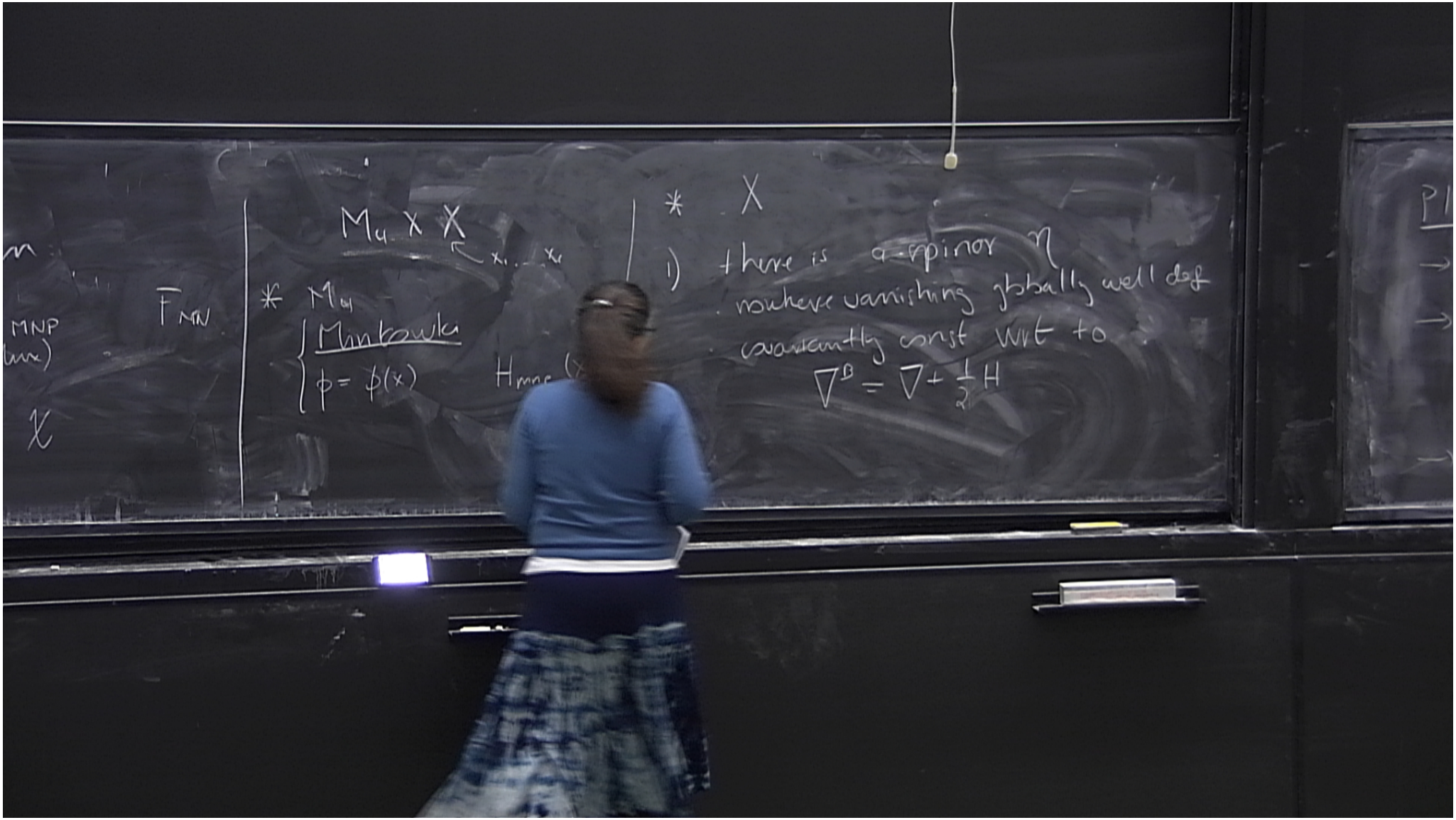

MNP
F_{MN}
$M_{4n} \times X$
$x_1 \ldots x_n$
* M_{4n}
Minkowski
$\phi = \phi(x)$
$H_{mn}(x)$

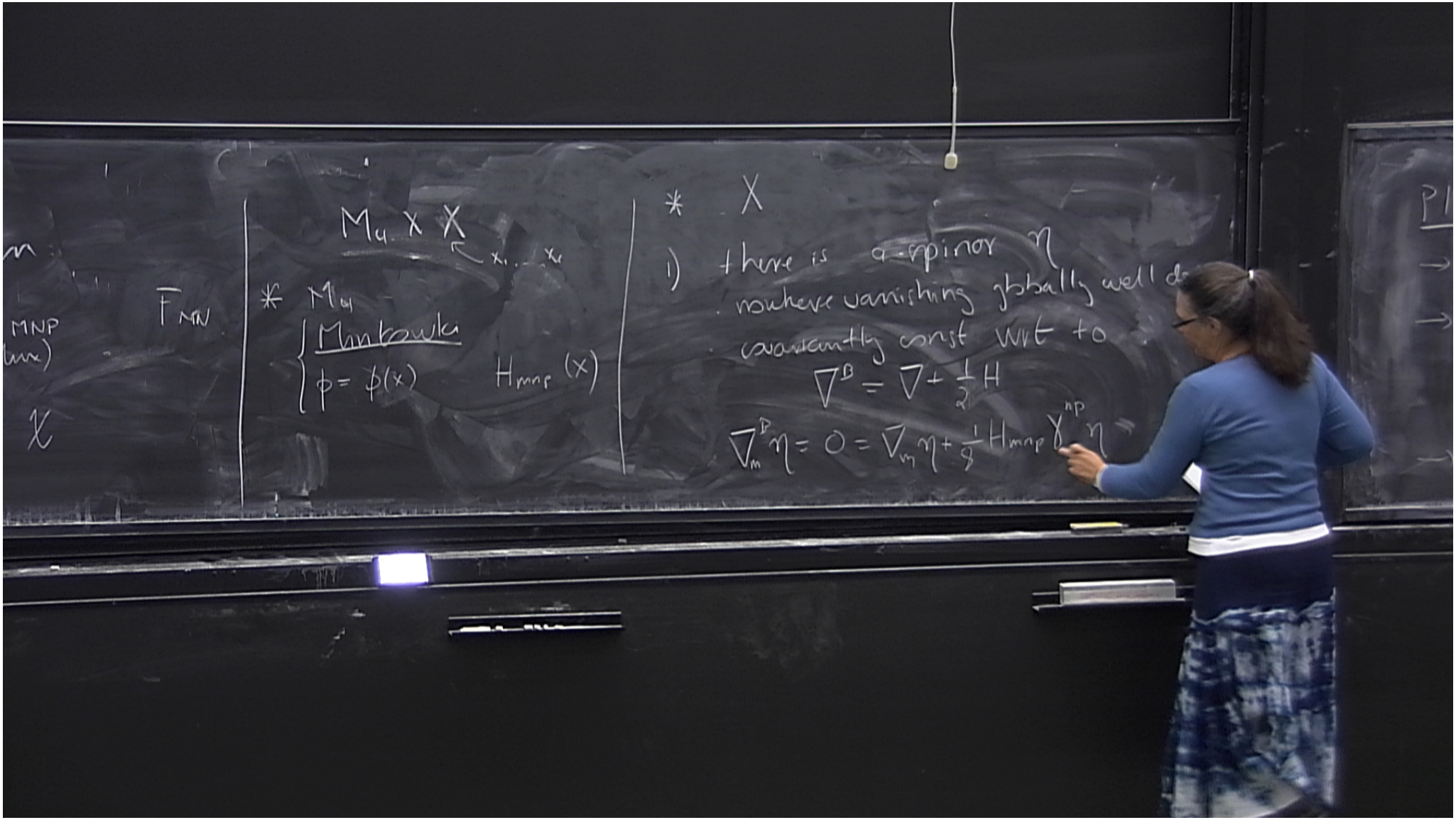

$M_4 \times X$
$x_1 \dots x_4$
F_{MN}
* M_4
Minkowski
$\phi = \phi(x)$
$H_{mn}(x)$
* X

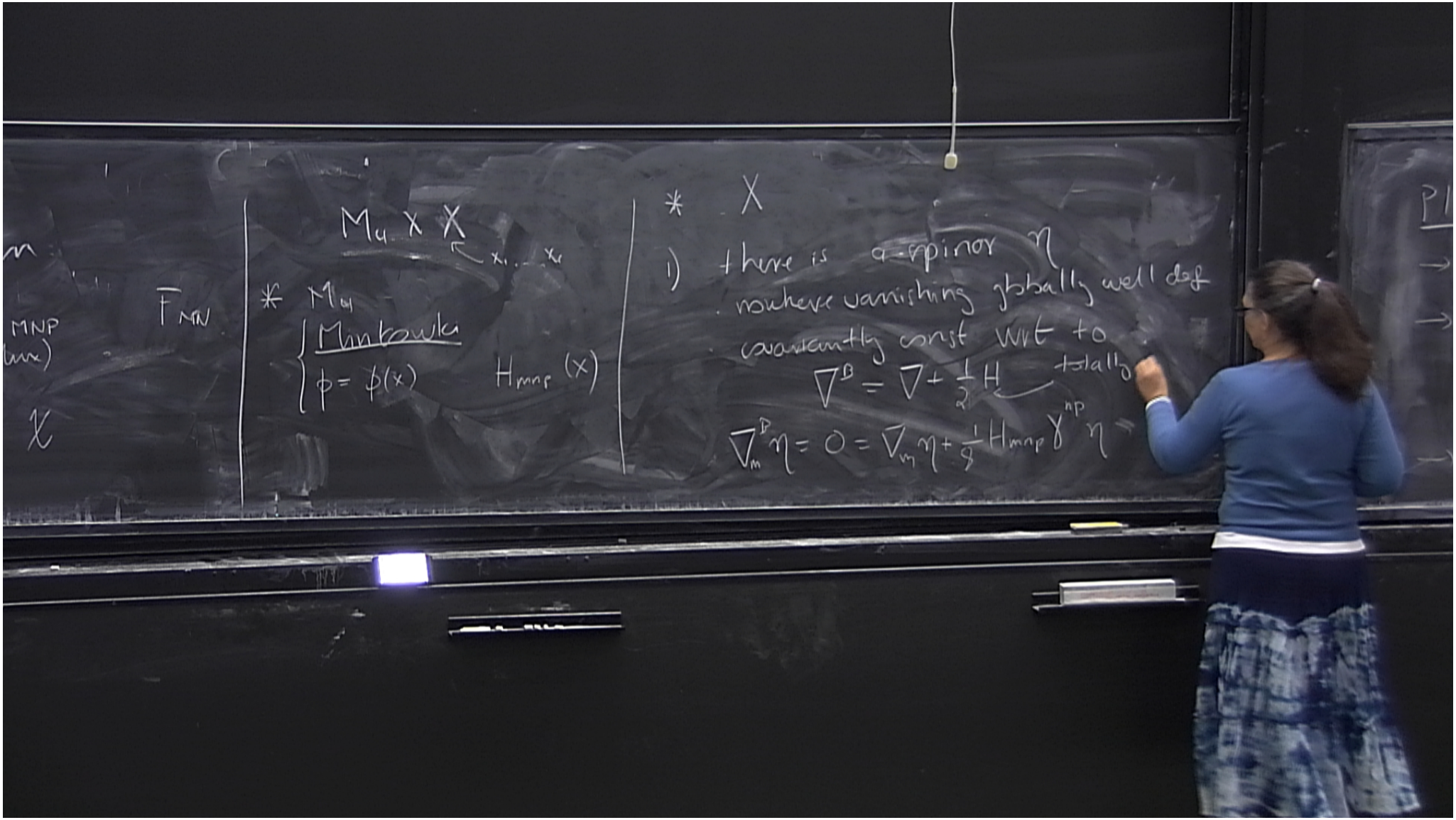

$M_4 \times X$
$x_1 \dots x_4$
F_{MN}
MNP
$*\ M_4$
Minkowski
$\phi = \phi(x)$
$H_{mnp}(x)$
$*\ X$

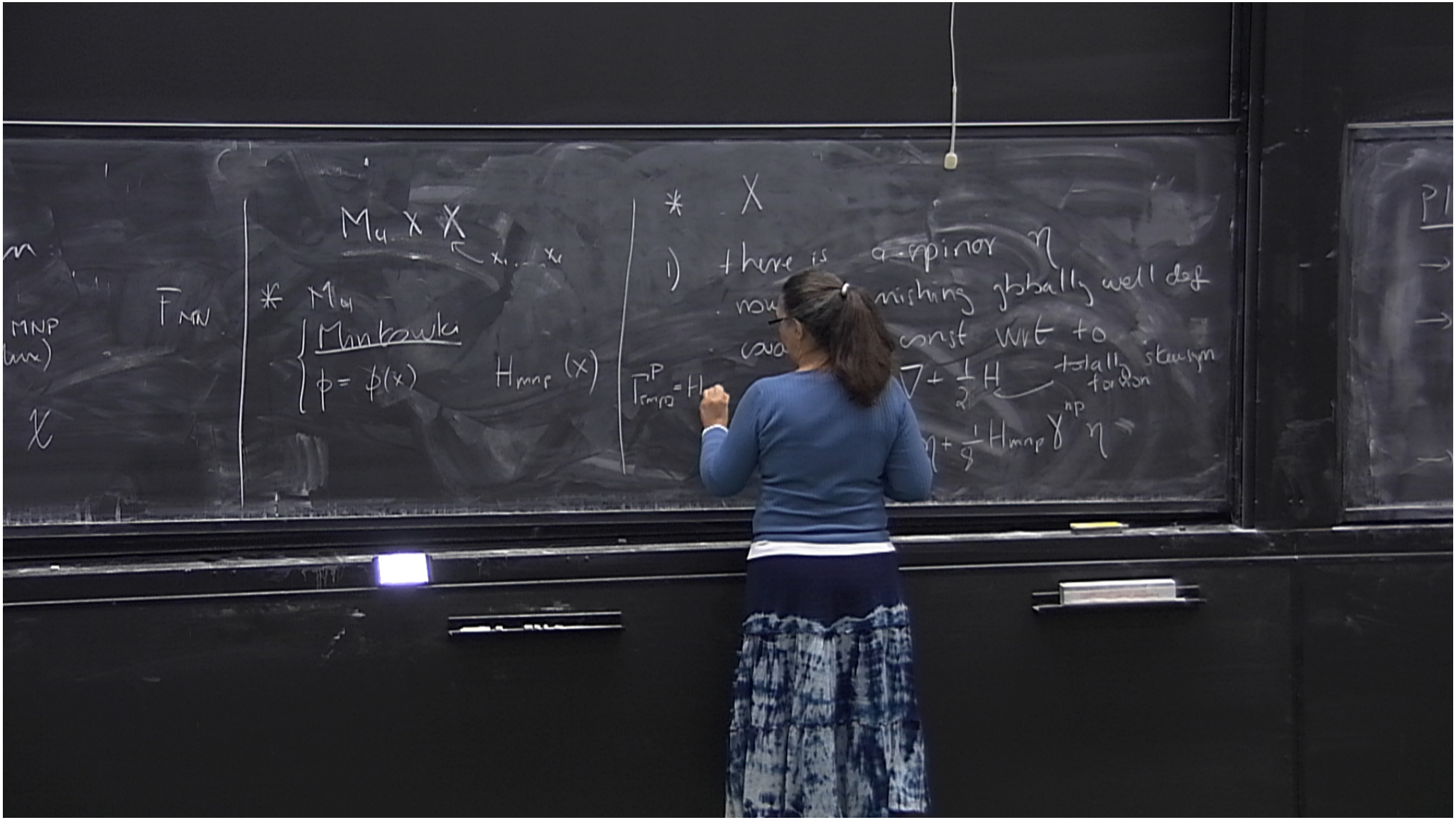

$M_4 \times X$
$x_1 \ldots x_4$
$* \; M_4$
Minkowski
$\phi = \phi(x)$
$H_{mnp}(x)$
$* \; X$

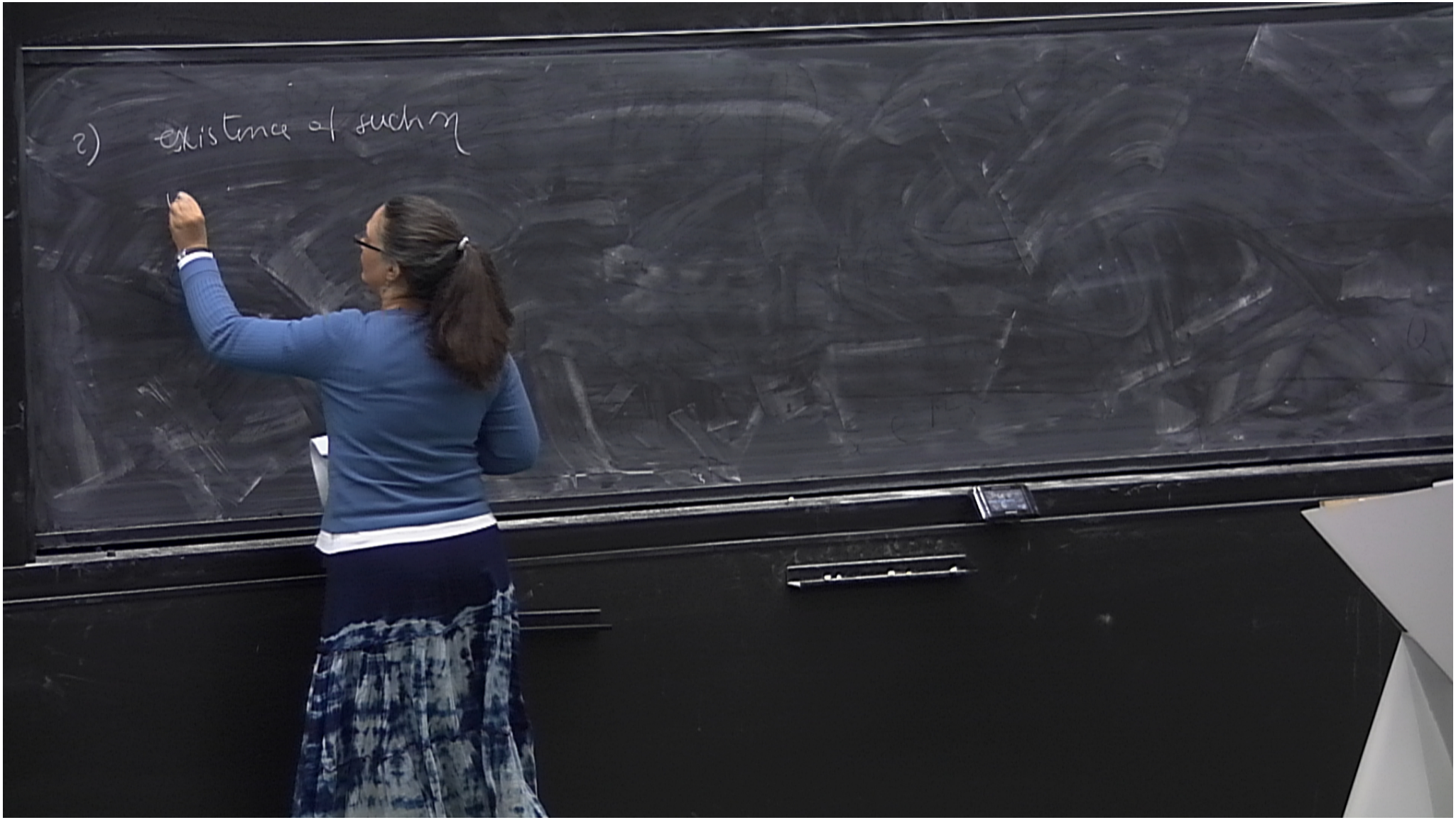

$M_4 \times X$
$x_1 \ldots x_4$
* M_4
Minkowski
$\phi = \phi(x)$
$H_{mn}(x)$
* X
1) there is
MNP
F_{MN}

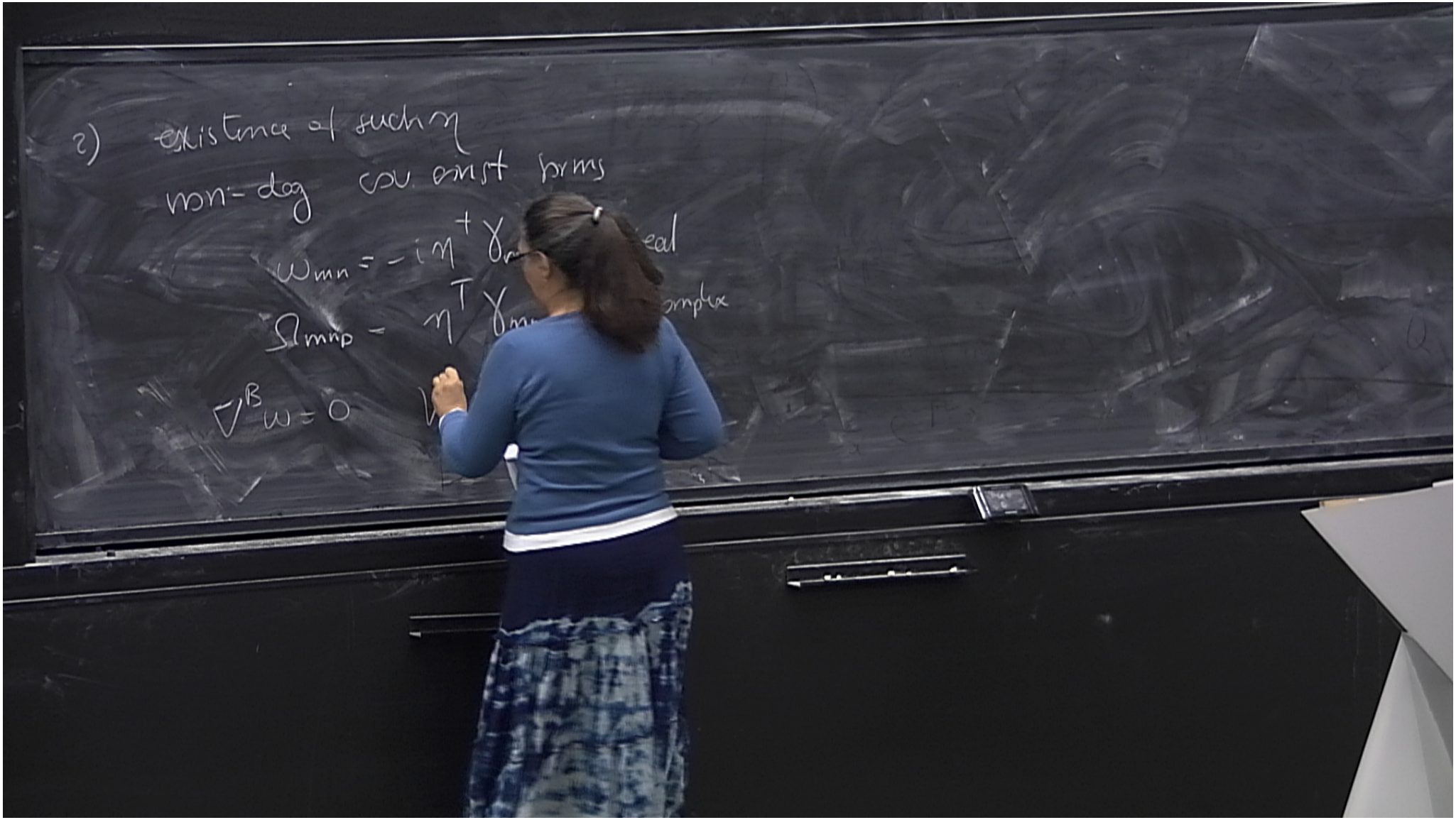

$M_4 \times X$
$x_1 \dots x_4$
F_{MN}
MNP
(MN)
χ
* M_4
Minkowski
$\phi = \phi(x)$
* X
1) there is a spinor η
nowhere vanishing globally well def
covariantly const wrt to
$\nabla^B = \nabla + \frac{1}{2} H$

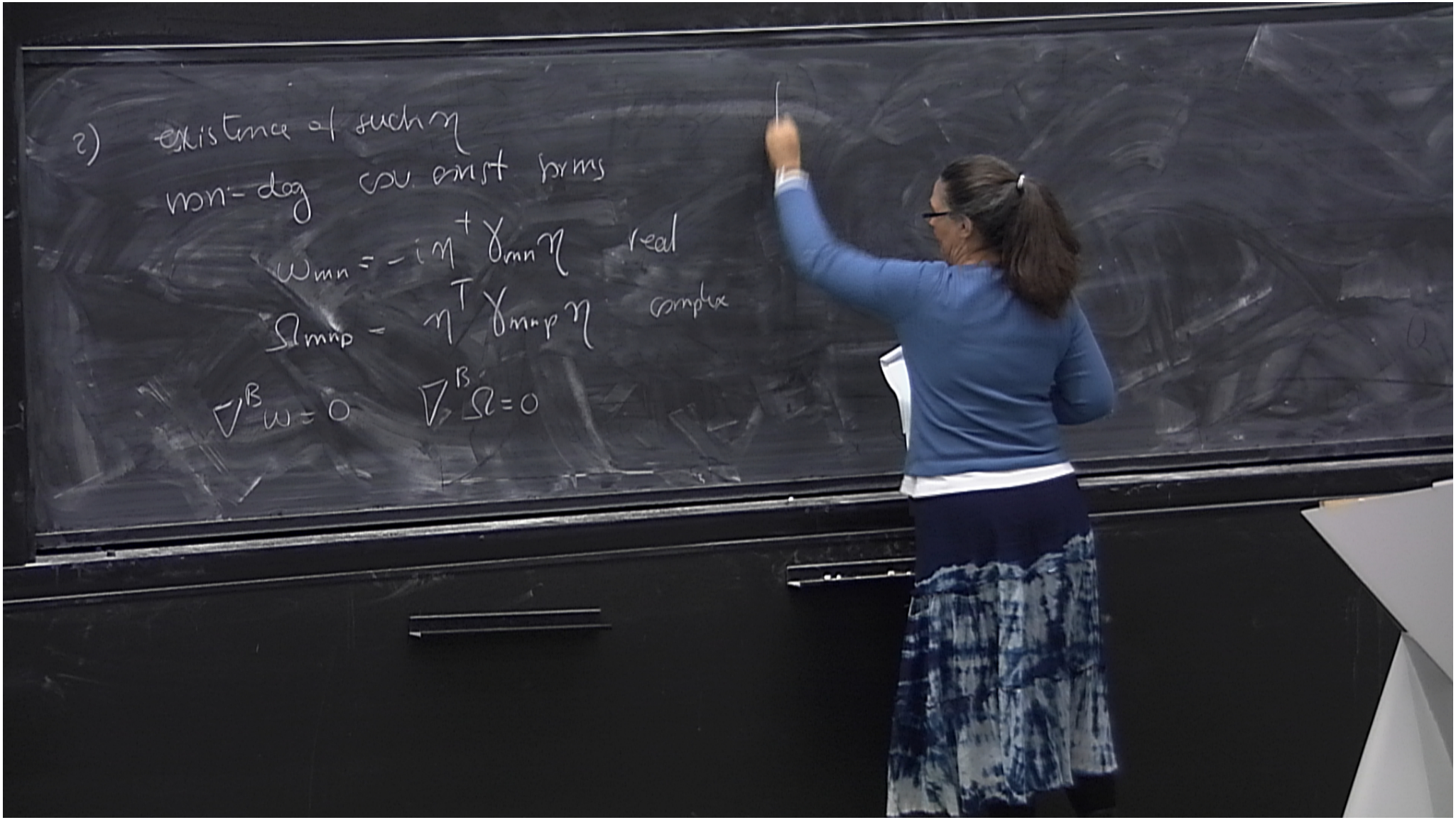

$M_4 \times X$
$x_1 \ldots x_r$
* M_4
Minkowski
$\phi = \phi(x)$
$H_{mnp}(x)$
* X
1) there is a spinor η
nowhere vanishing globally well d
covariantly const wrt to
$\nabla^B = \nabla + \frac{1}{2} H$
$\nabla^B_m \eta = 0 = \nabla_m \eta + \frac{1}{8} H_{mnp} \gamma^{np} \eta$
F_{MN}
MNP
(MN)

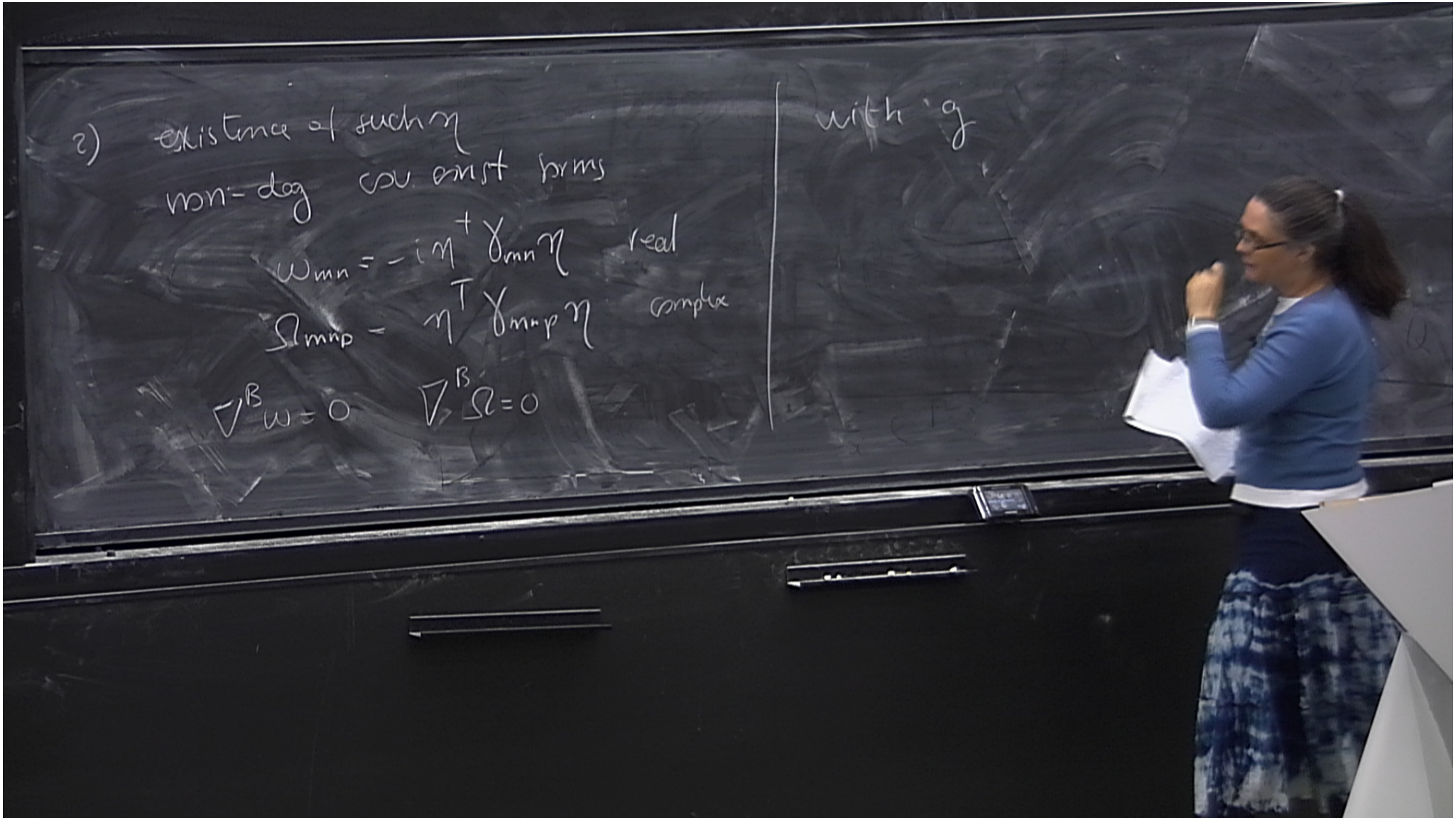

$M_4 \times X$
$x_1 \ldots x_r$
* M_4
Minkowski
$\phi = \phi(x)$
$H_{mnp}(x)$
* X
1) there is a spinor η
nowhere vanishing globally well def
covariantly const wrt to
$\nabla^B = \nabla + \frac{1}{2} H$ totally
$\nabla^B_m \eta = 0 = \nabla_m \eta + \frac{1}{8} H_{mnp} \gamma^{np} \eta$
F_{MN}

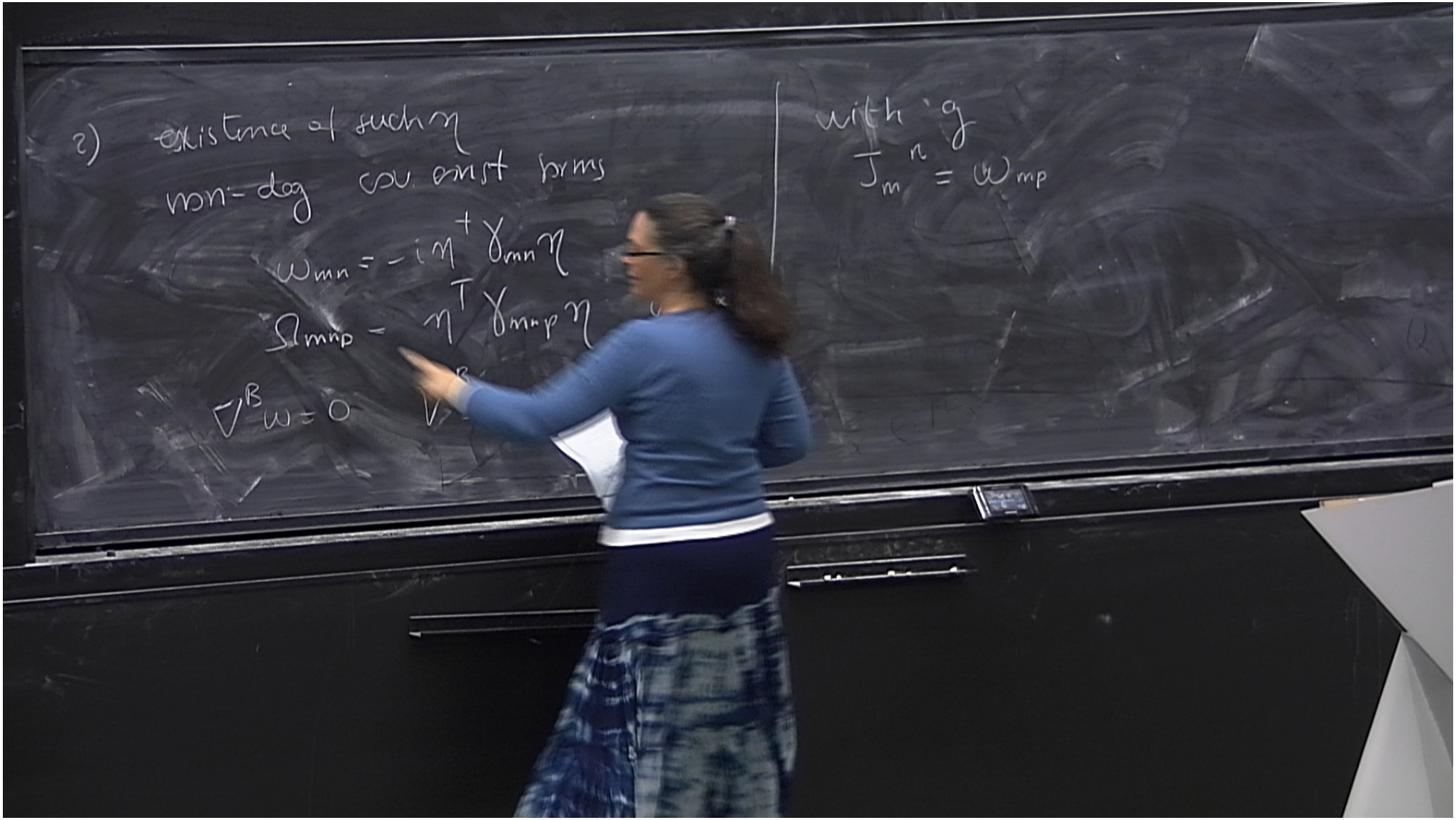

$M_4 \times X$
$x_1 \dots x_r$
* M_4
Minkowski
$\phi = \phi(x)$
$H_{mnp}(x)$
* X
1) there is a spinor η
$\nabla + \frac{1}{2} H$
total all torsion
$\eta + \frac{1}{8} H_{mnp} \gamma^{np} \eta$

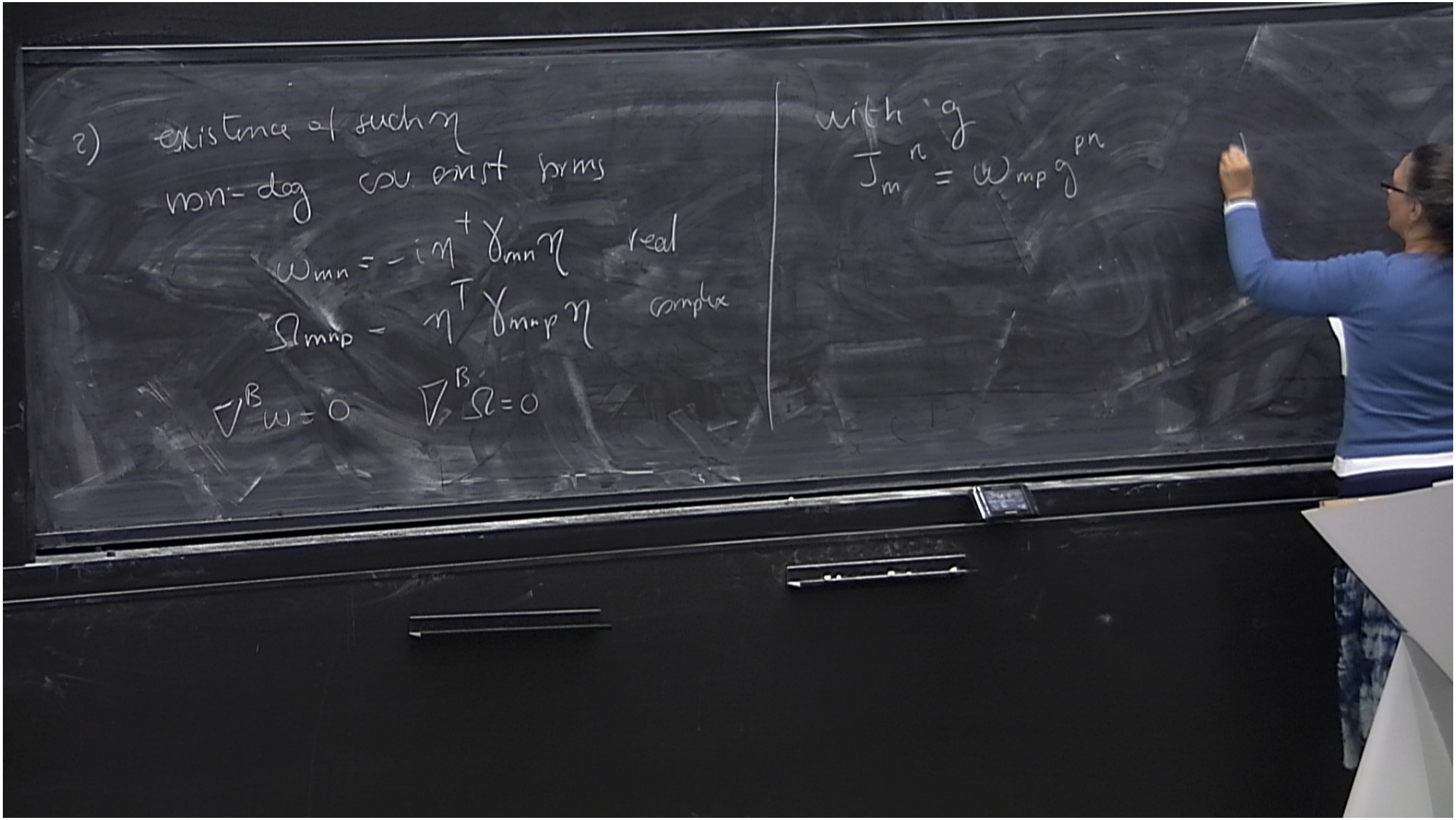

2) existence of such n

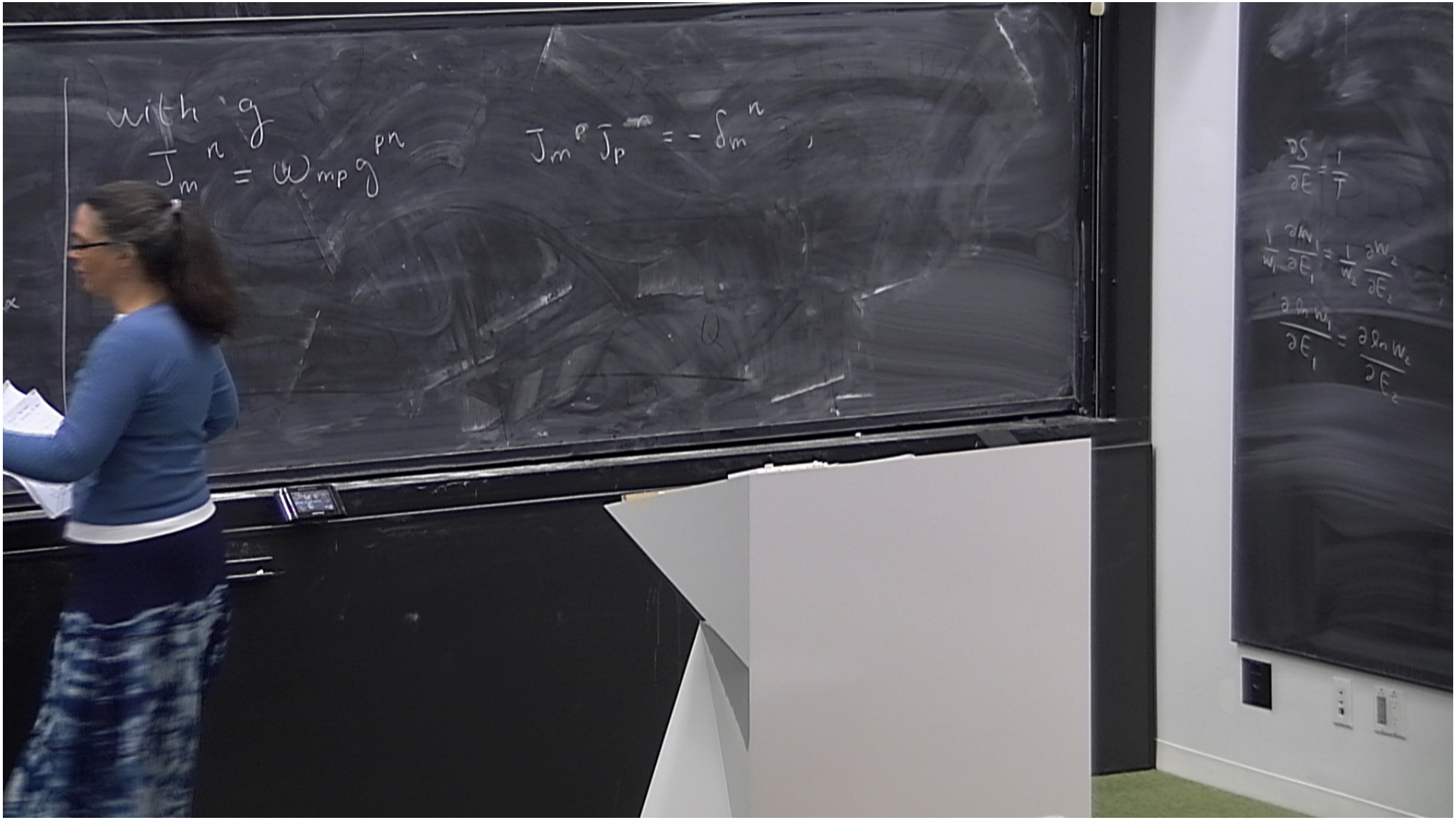

2) existence of such η
non-deg
$\omega_{mn} = -i\eta^{+}\gamma$
$\Omega_{mnp} = \eta^{T}\gamma$
$\nabla^{B}\omega = 0$

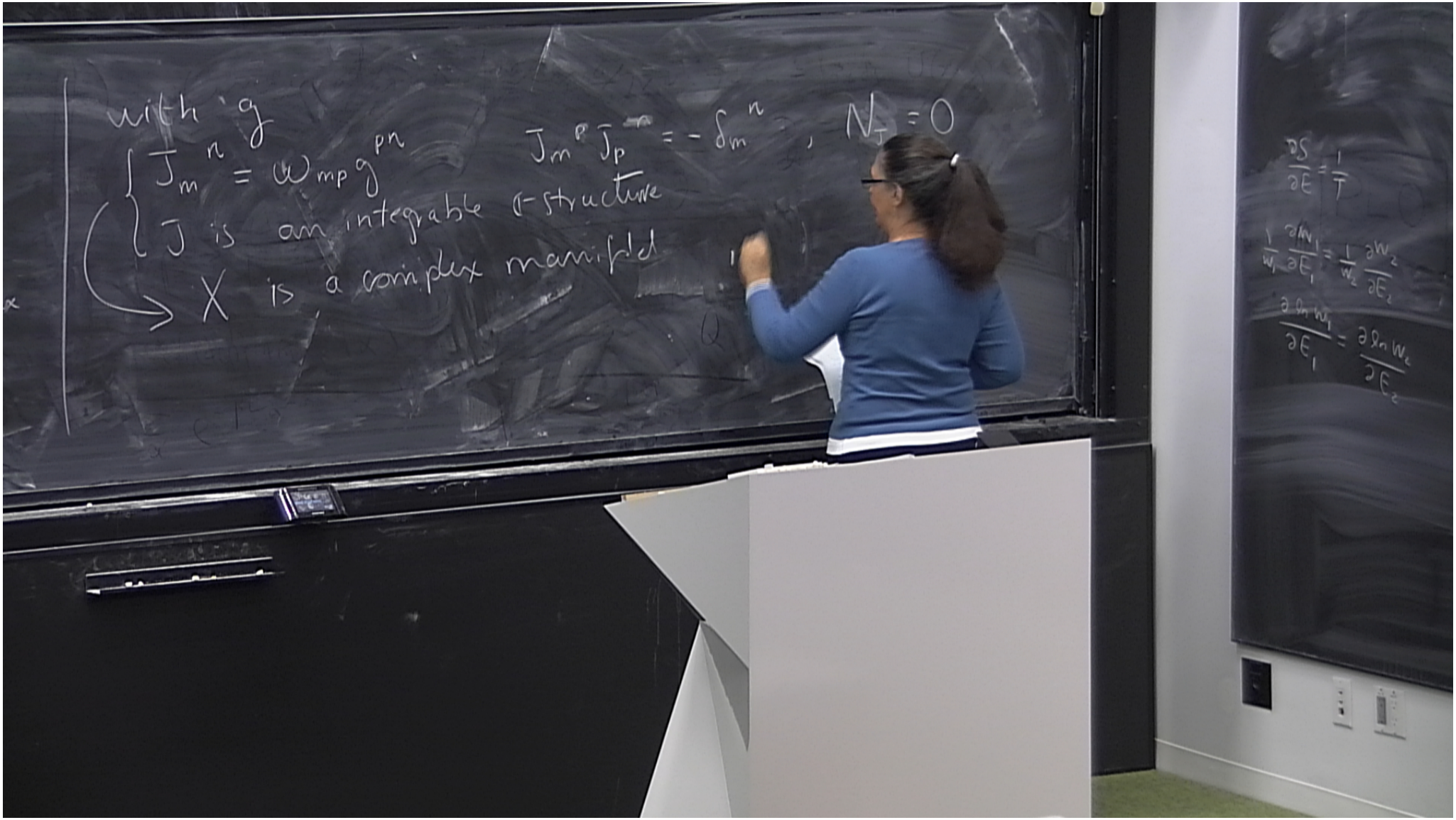

2) existence of such η
non-deg cov exist forms
$\omega_{mn} = -i\eta^{\dagger}\gamma_{mn}\eta$ real
$\Omega_{mnp} = \eta^{T}\gamma_{mnp}\eta$ complex
$\nabla^{B}\omega = 0$
$\nabla^{B}\Omega = 0$

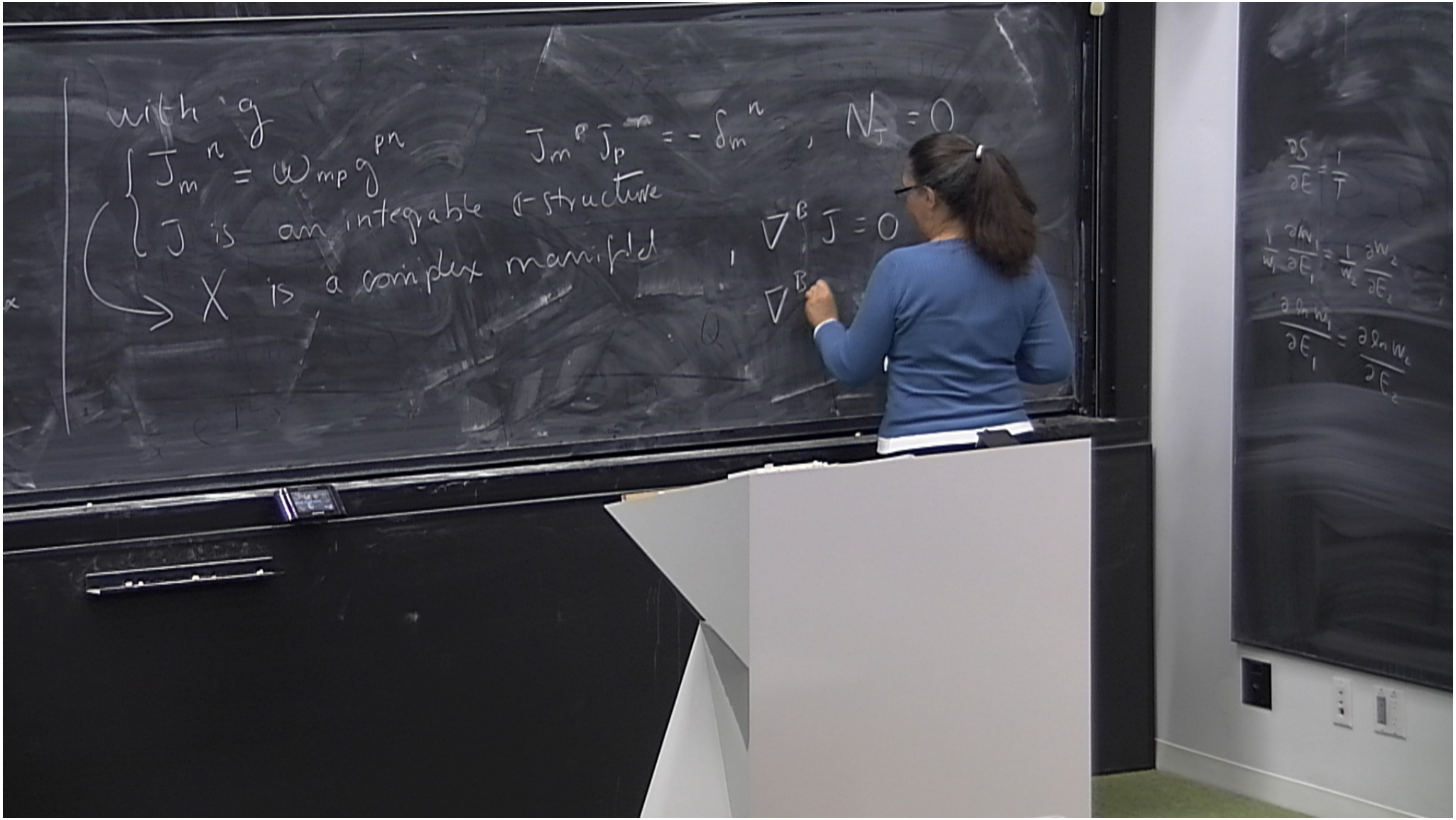

2) existence of such η
non-deg cov exist bvms
$\omega_{mn} = -i\eta^{\dagger}\gamma_{mn}\eta$ real
$\Omega_{mnp} = \eta^{T}\gamma_{mnp}\eta$ complex
$\nabla^{B}\omega = 0$ $\nabla^{B}\Omega = 0$
with g

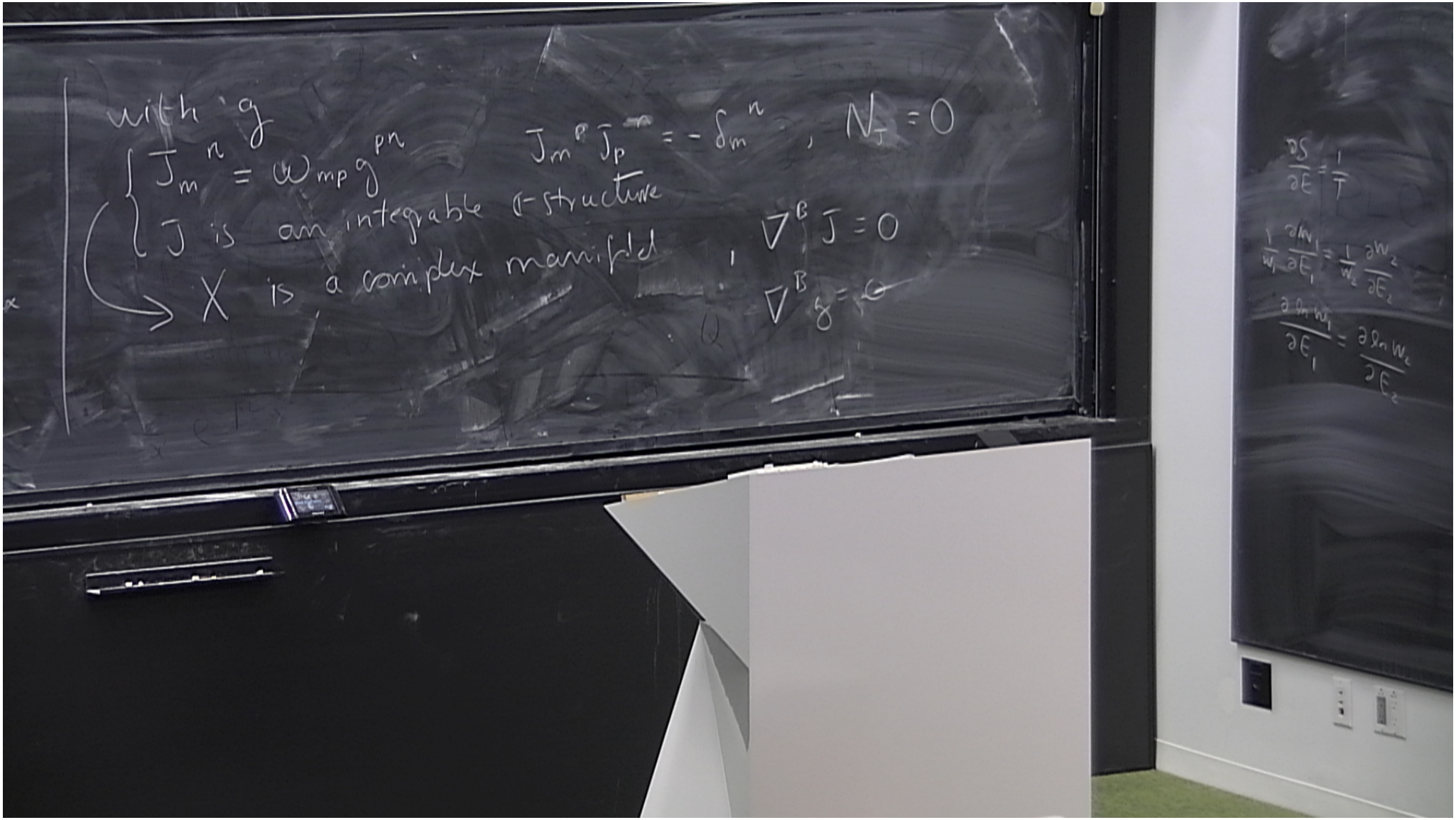

2) existence of such η
non-deg cov exist forms
$\omega_{mn} = -i\eta^{\dagger}\gamma_{mn}\eta$
$\Omega_{mnp} = \eta^{T}\gamma_{mnp}\eta$
$\nabla^{B}\omega = 0$
with g
$J_m^{\ n} = \omega_{mp}$

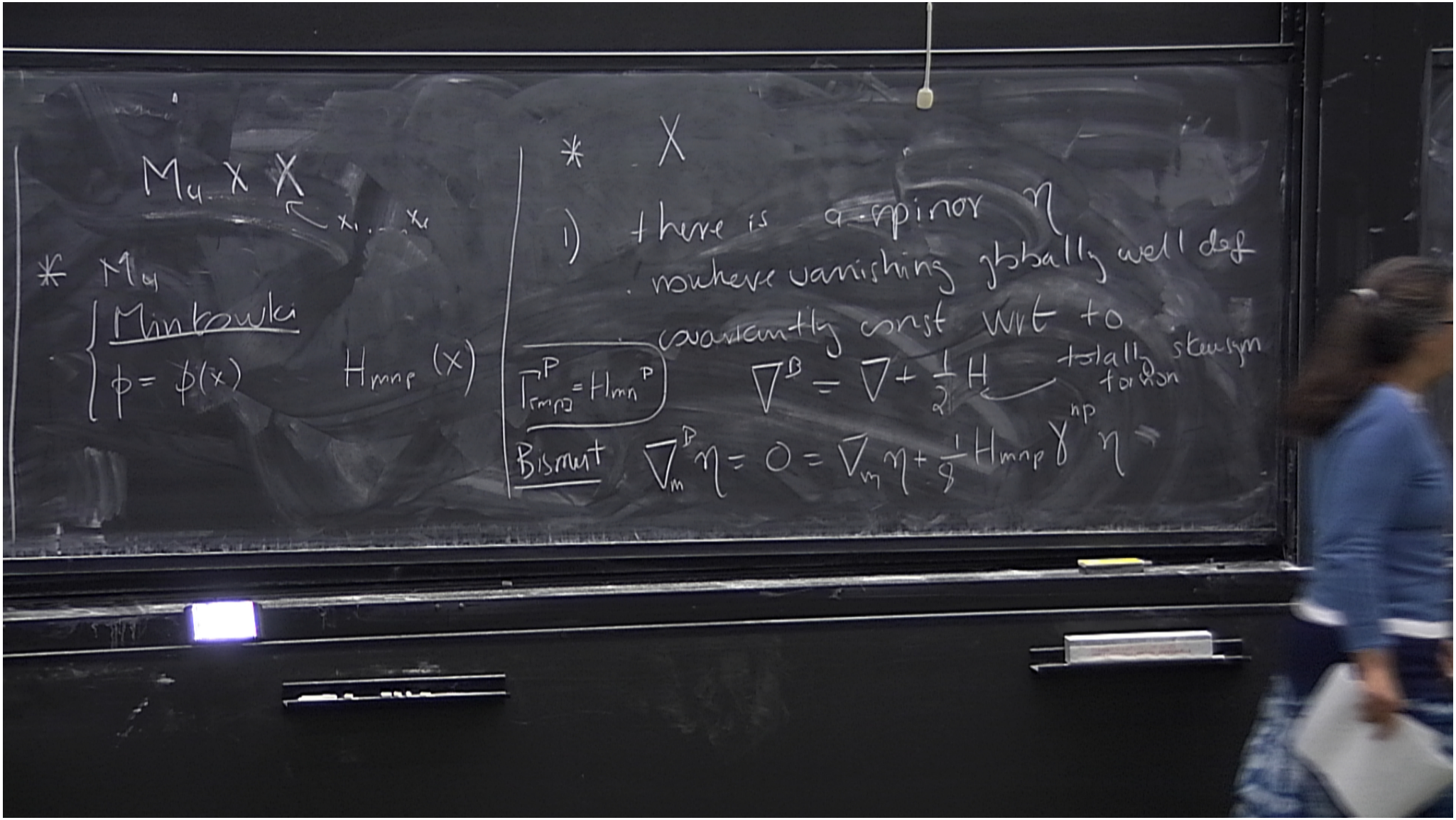

2) existence of such η
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$\omega_{mn} = -i\eta^{\dagger}\gamma_{mn}\eta$ real
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$\nabla^{B}\omega = 0$ $\nabla^{B}\Omega = 0$
with g
$J_m{}^n = \omega_{mp} g^{pn}$

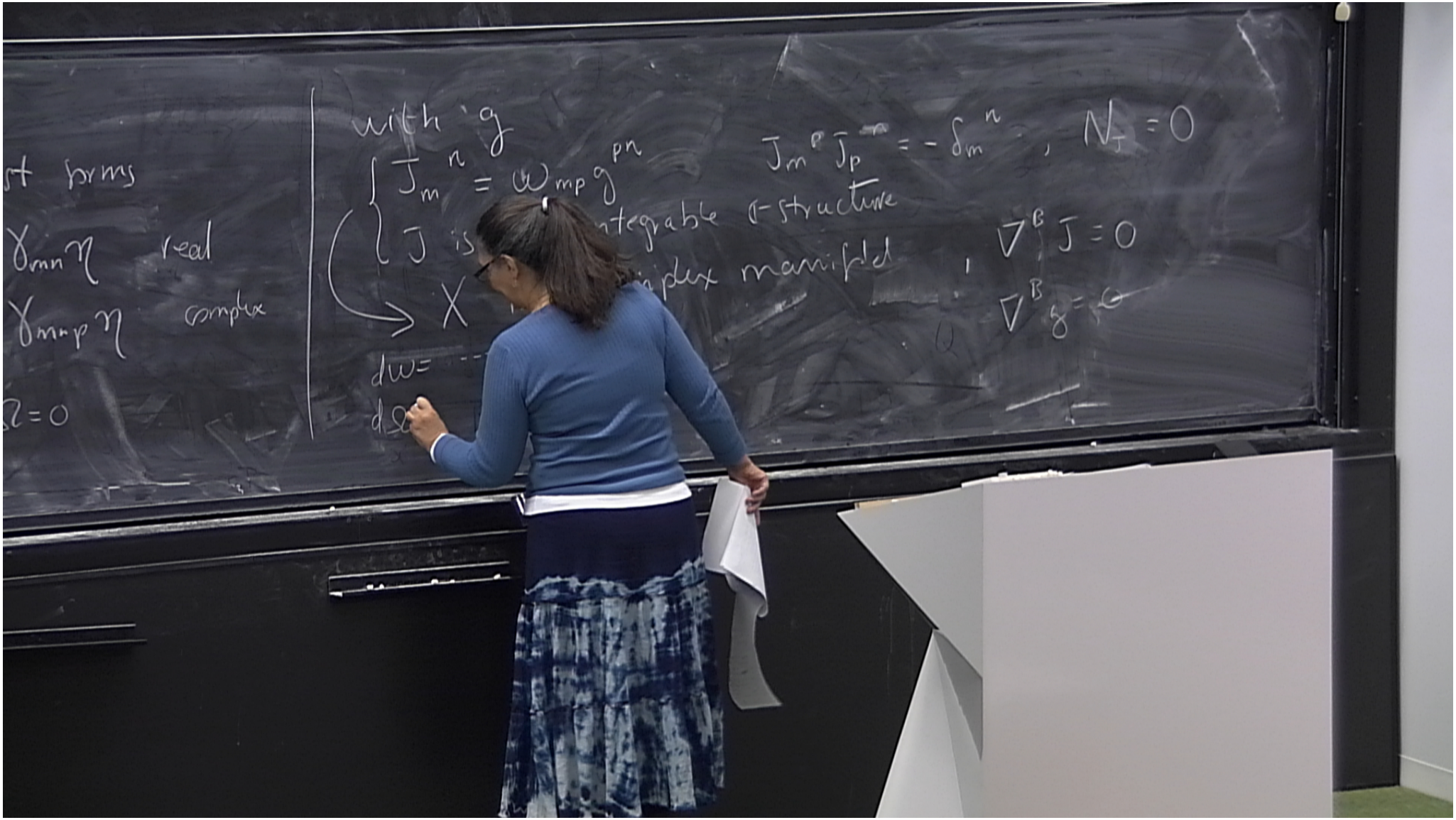

with g
$J_m{}^n = \omega_{mp} g^{pn}$
$J_m{}^p \bar{J}_p{}^n = -\delta_m{}^n$
$\frac{\partial S}{\partial E} = \frac{1}{T}$
$\frac{1}{w_1}\frac{\partial w_1}{\partial E_1} = \frac{1}{w_2}\frac{\partial w_2}{\partial E_2}$
$\frac{\partial \rho_m w_1}{\partial E_1} = \frac{\partial \rho_n w_2}{\partial E_2}$

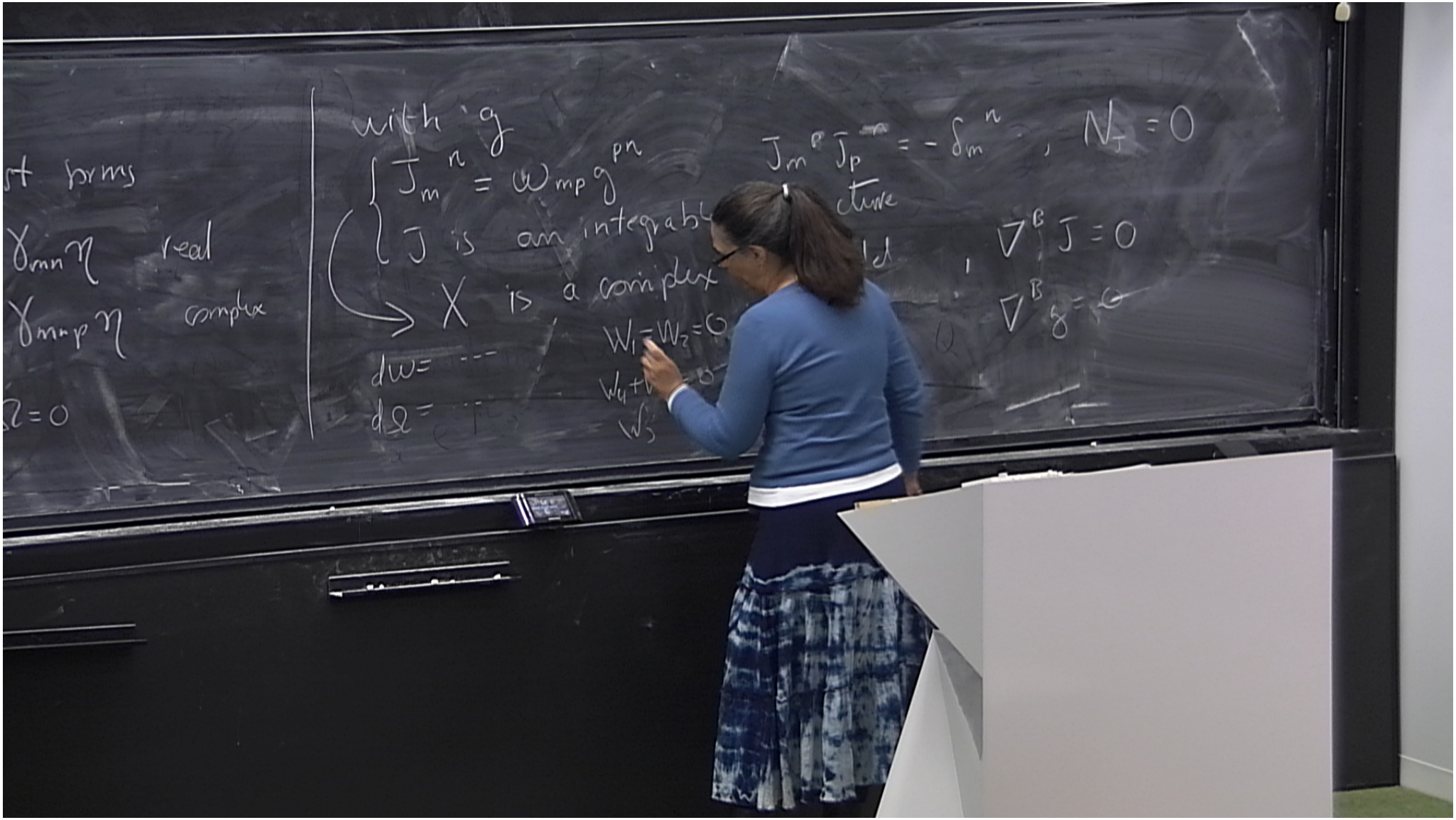

with g
$J_m^{\ n} = \omega_{mp} g^{pn}$
$J_m^{\ p} J_p^{\ n} = -\delta_m^{\ n}$, $N_J = 0$
J is an integrable C-structure
X is a complex manifold
$\frac{\partial S}{\partial E} = \frac{1}{T}$
$\frac{1}{w_1}\frac{\partial w_1}{\partial E_1} = \frac{1}{w_2}\frac{\partial w_2}{\partial E_2}$
$\frac{\partial \ln w_1}{\partial E_1} = \frac{\partial \ln w_2}{\partial E_2}$

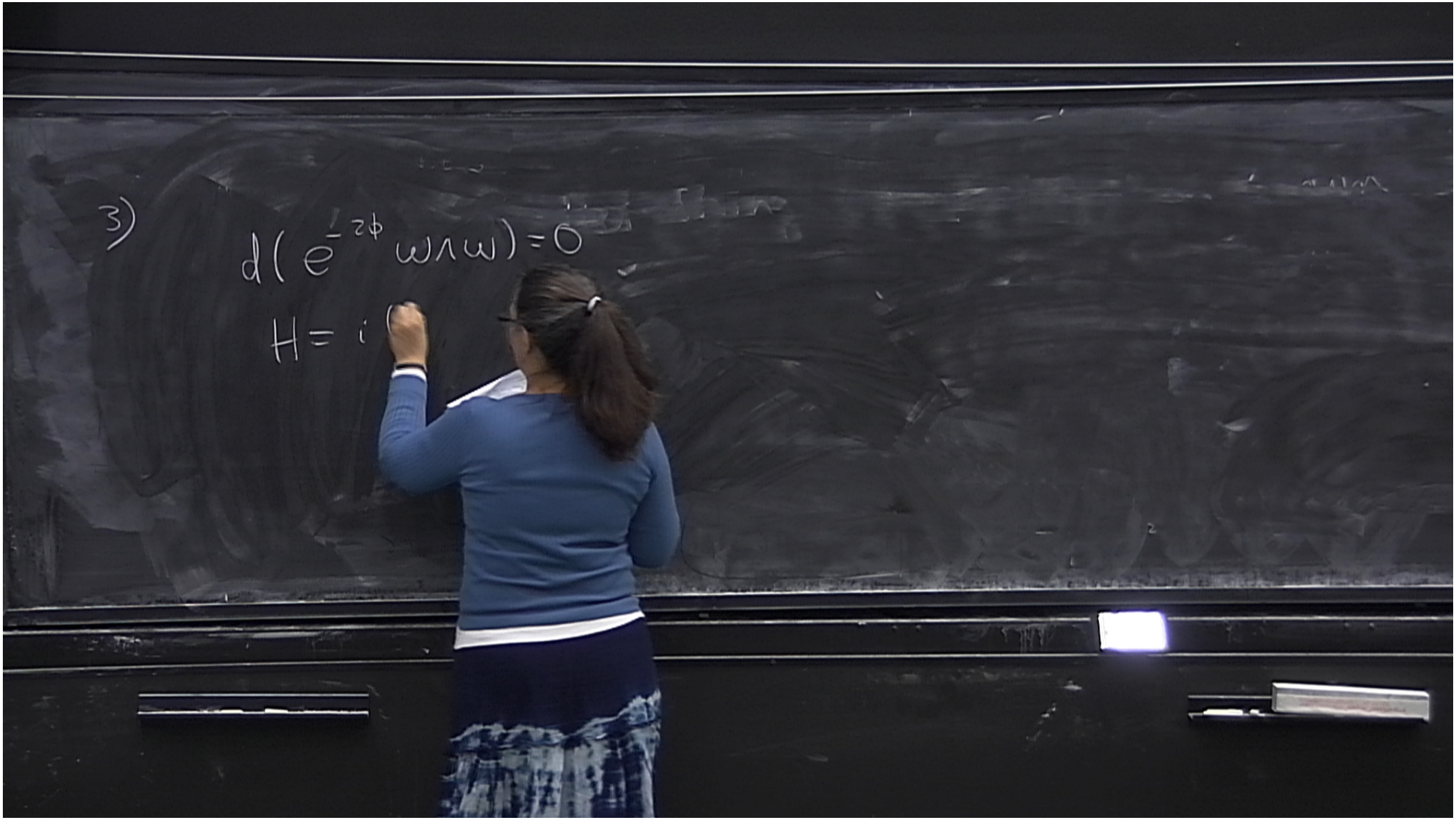

with g
$\bar{J}_m{}^n = \omega_{mp} g^{pn}$
$J_m{}^p \bar{J}_p{}^n = -\delta_m{}^n$, $N_J = 0$
J is an integrable C-structure
X is a complex manifold
, $\nabla^B \bar{J} = 0$
∇^B
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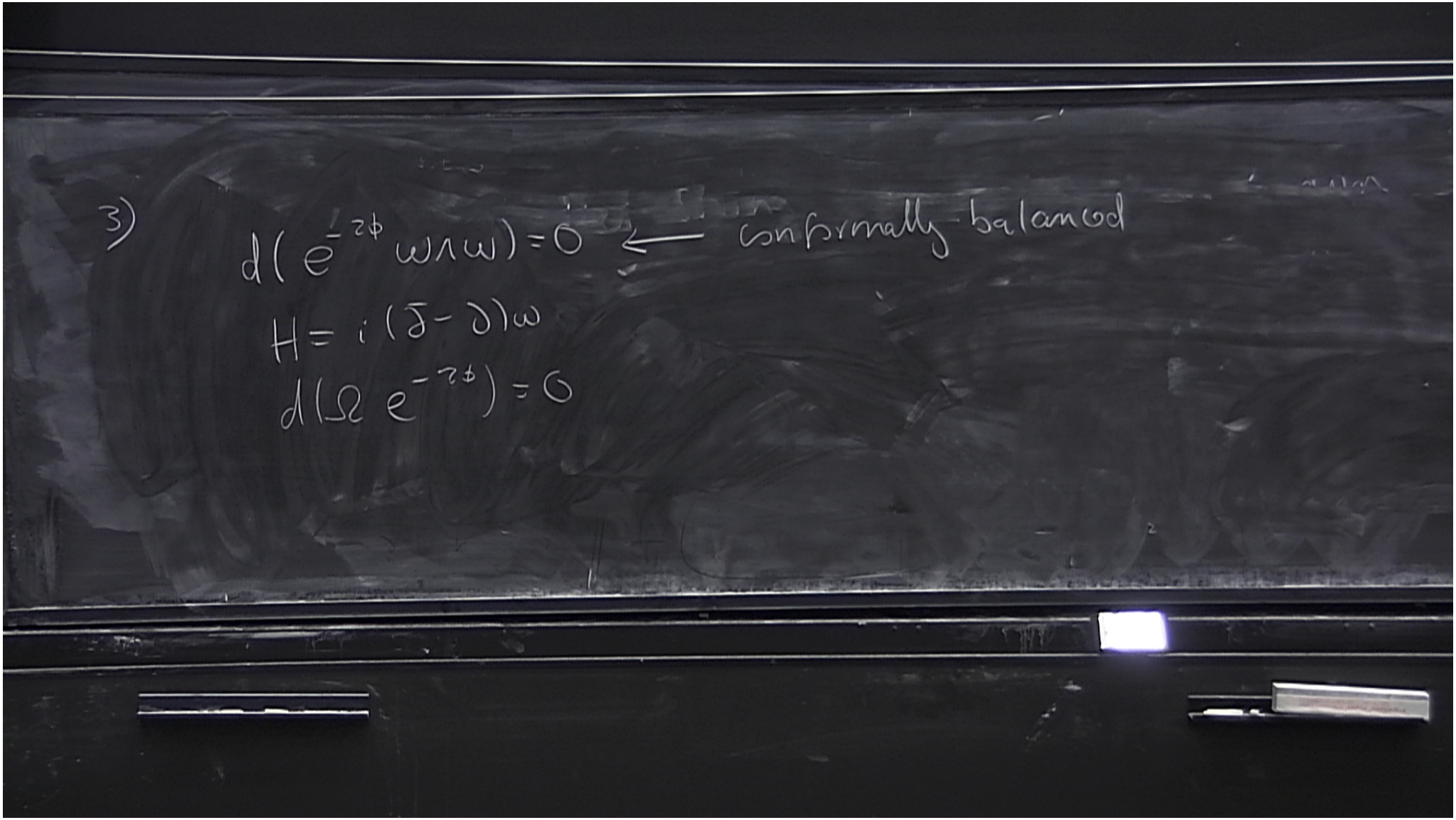

with g
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J is an integrable C-structure
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, $\nabla^B J = 0$
$\nabla^B g = 0$
$\frac{\partial S}{\partial E} = \frac{1}{T}$
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$\frac{\partial \ln w_1}{\partial E_1} = \frac{\partial \ln w_2}{\partial E_2}$

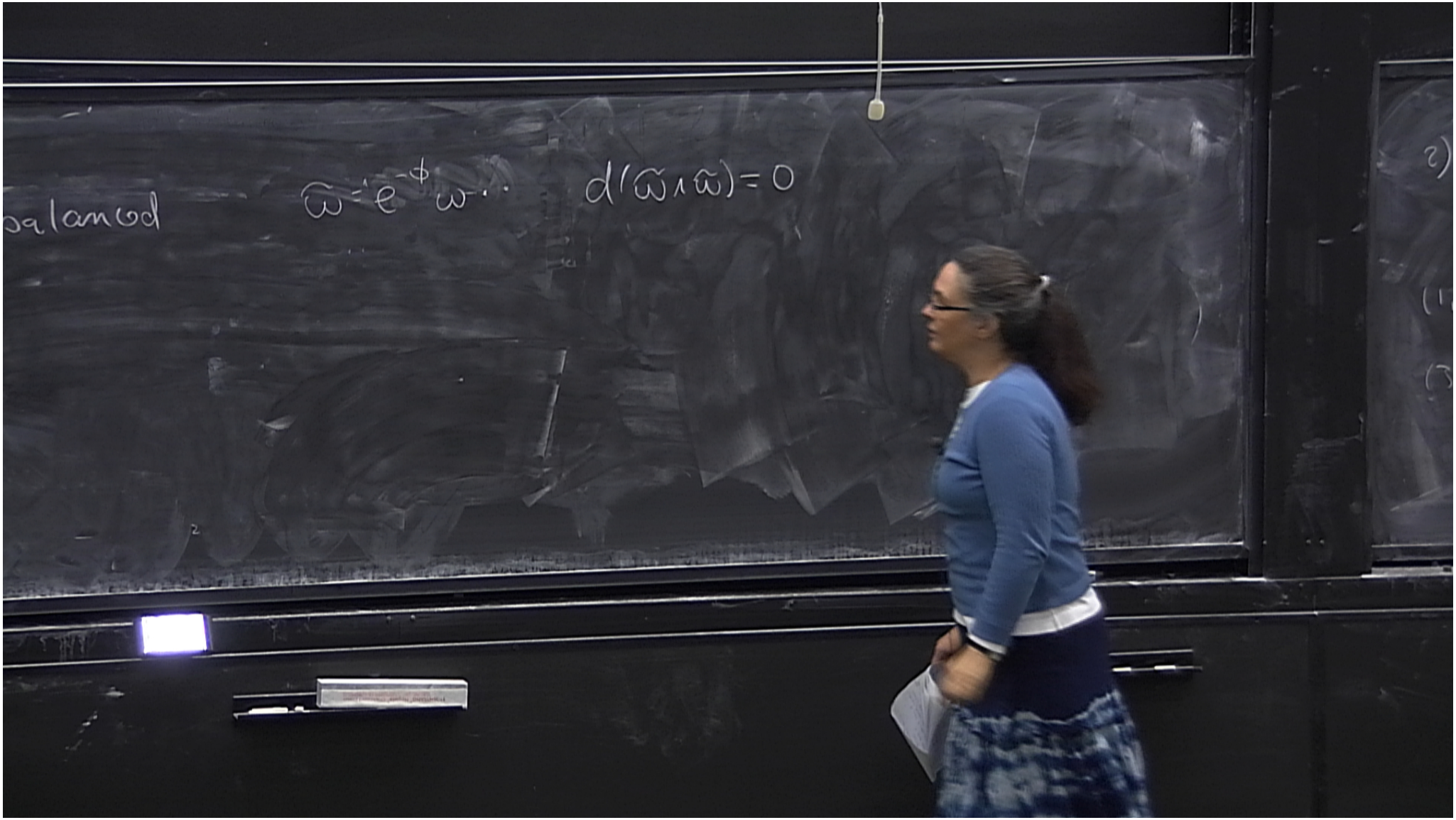

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Bismut
$\nabla^B_m \eta = 0 = \nabla_m \eta + \frac{1}{8} H_{mnp} \gamma^{np} \eta$

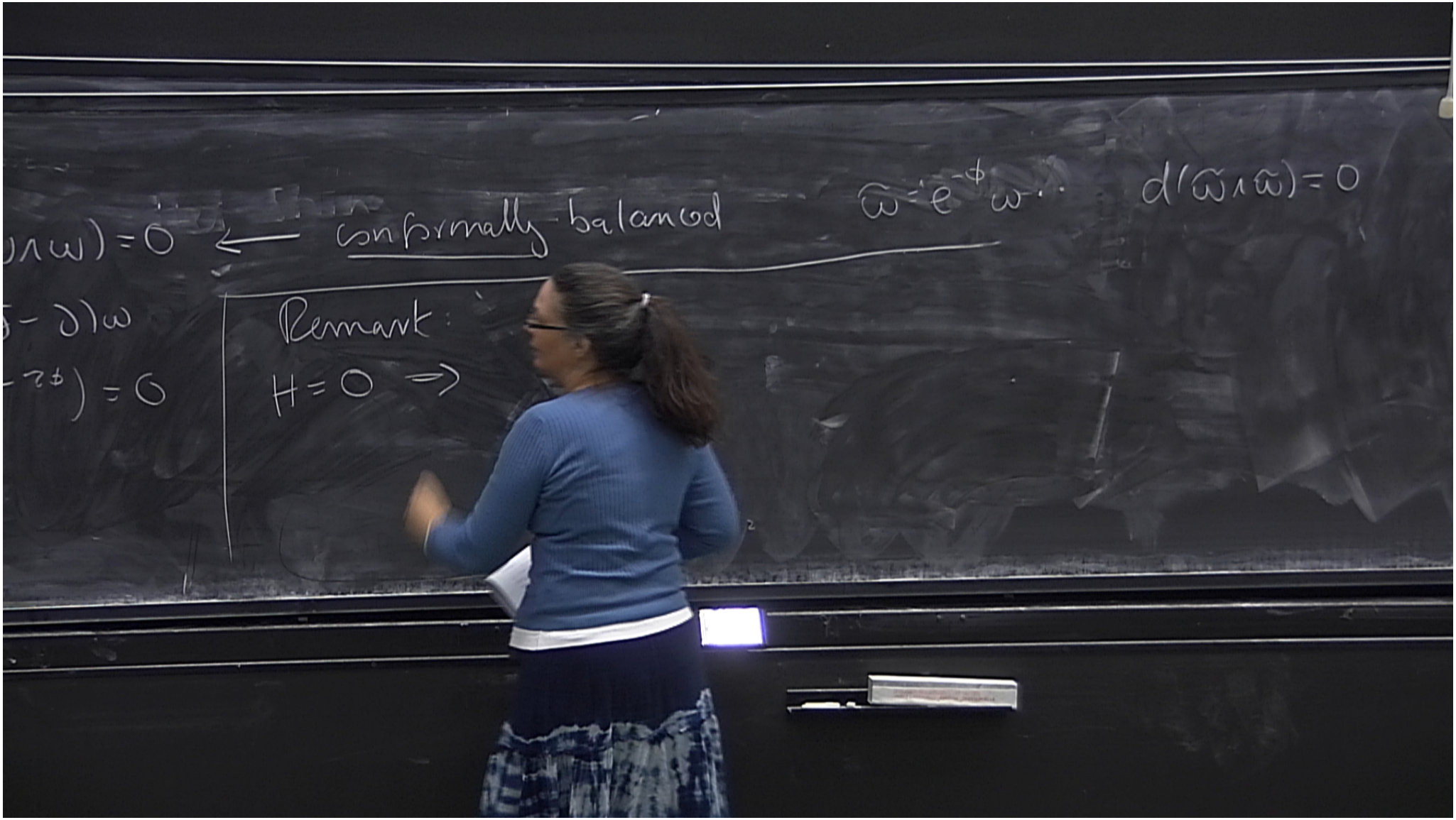

real
complex
with g
integrable
manifold

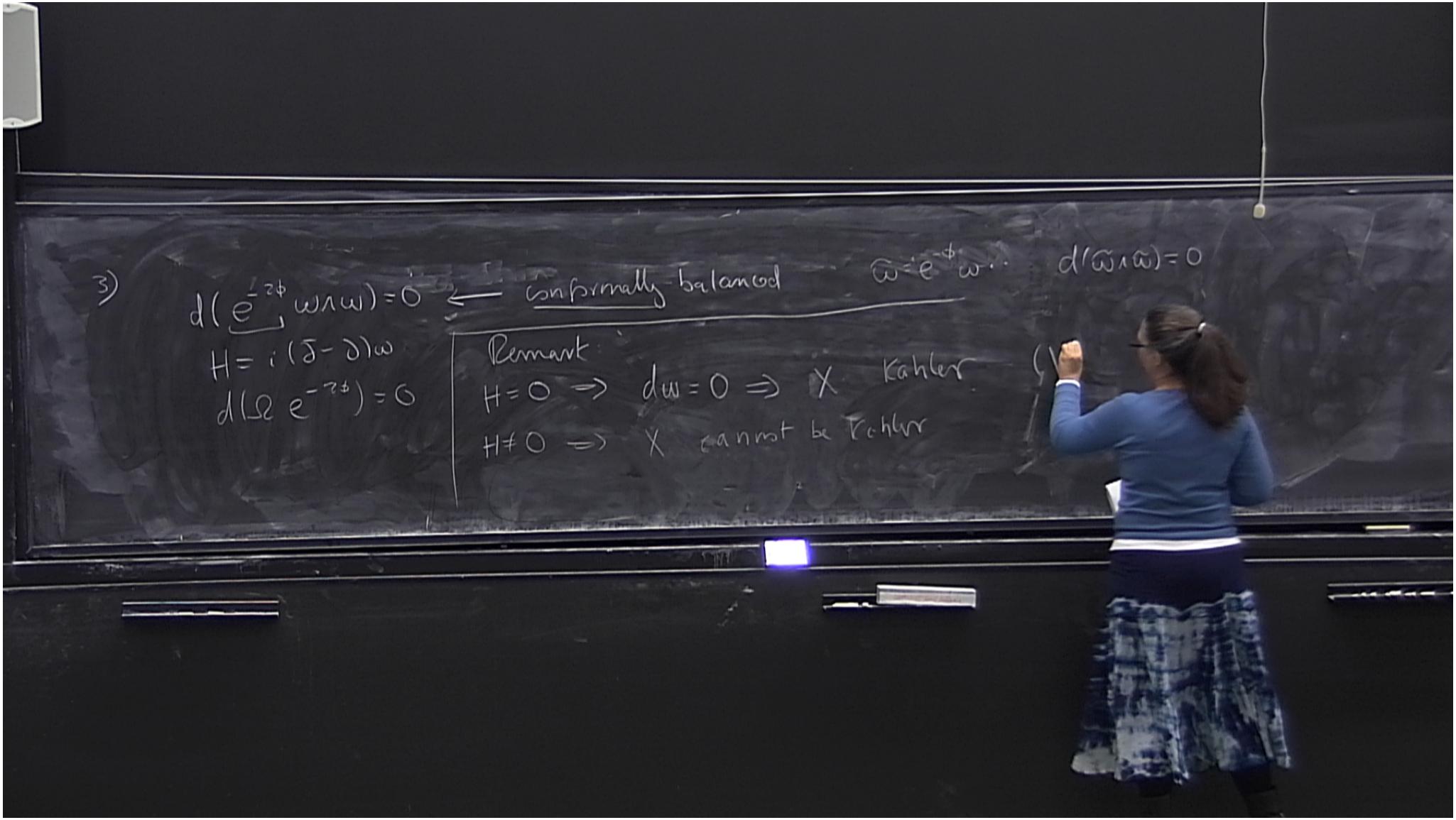

with g
real
complex
is a complex
integrable
dω=
dΩ=
W1=W2=0
Ω=0
∇B J=0
∇B g=0

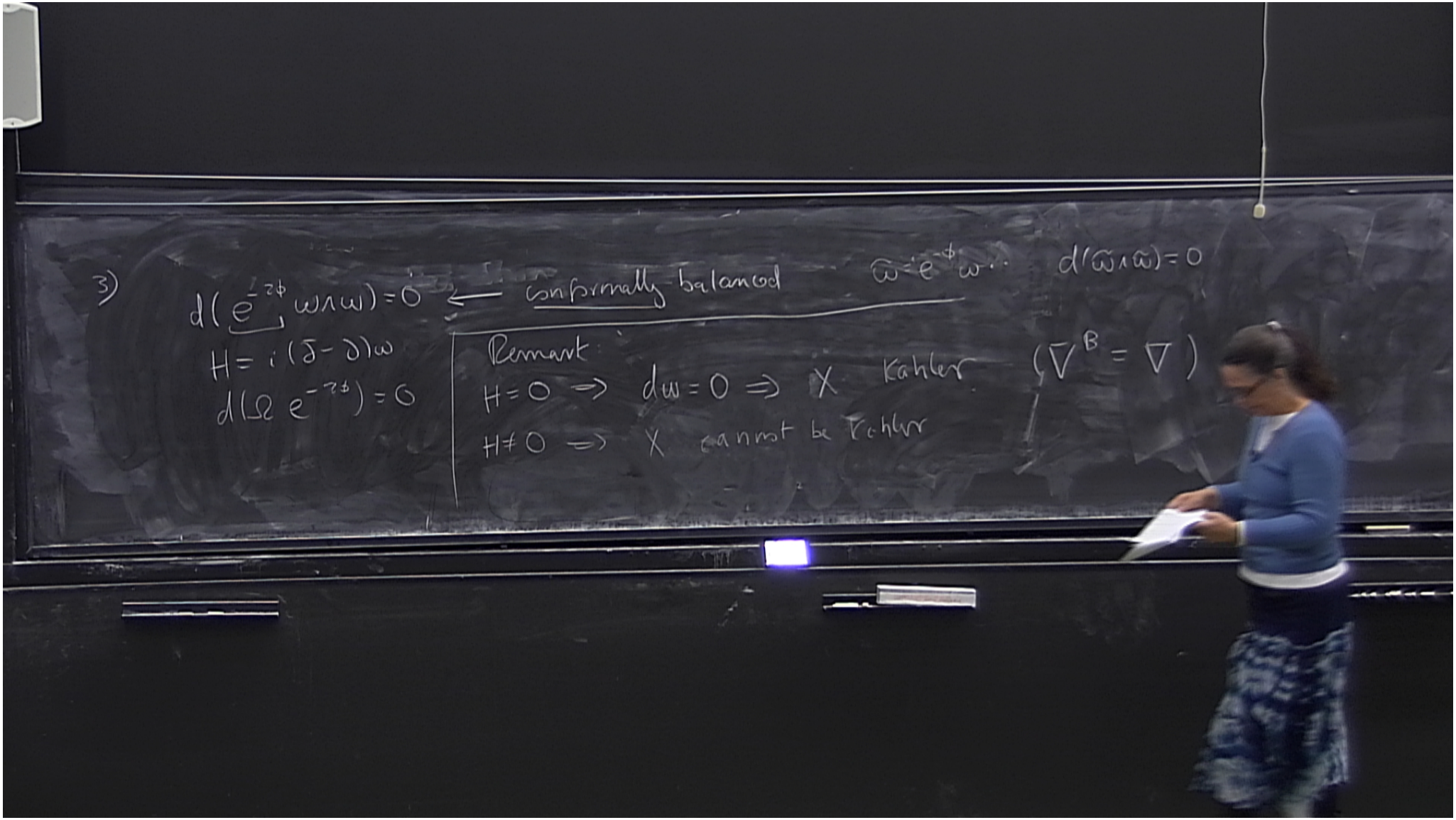

3)
$d(e^{-2\phi}\omega\wedge\omega)=0$
$H=i$

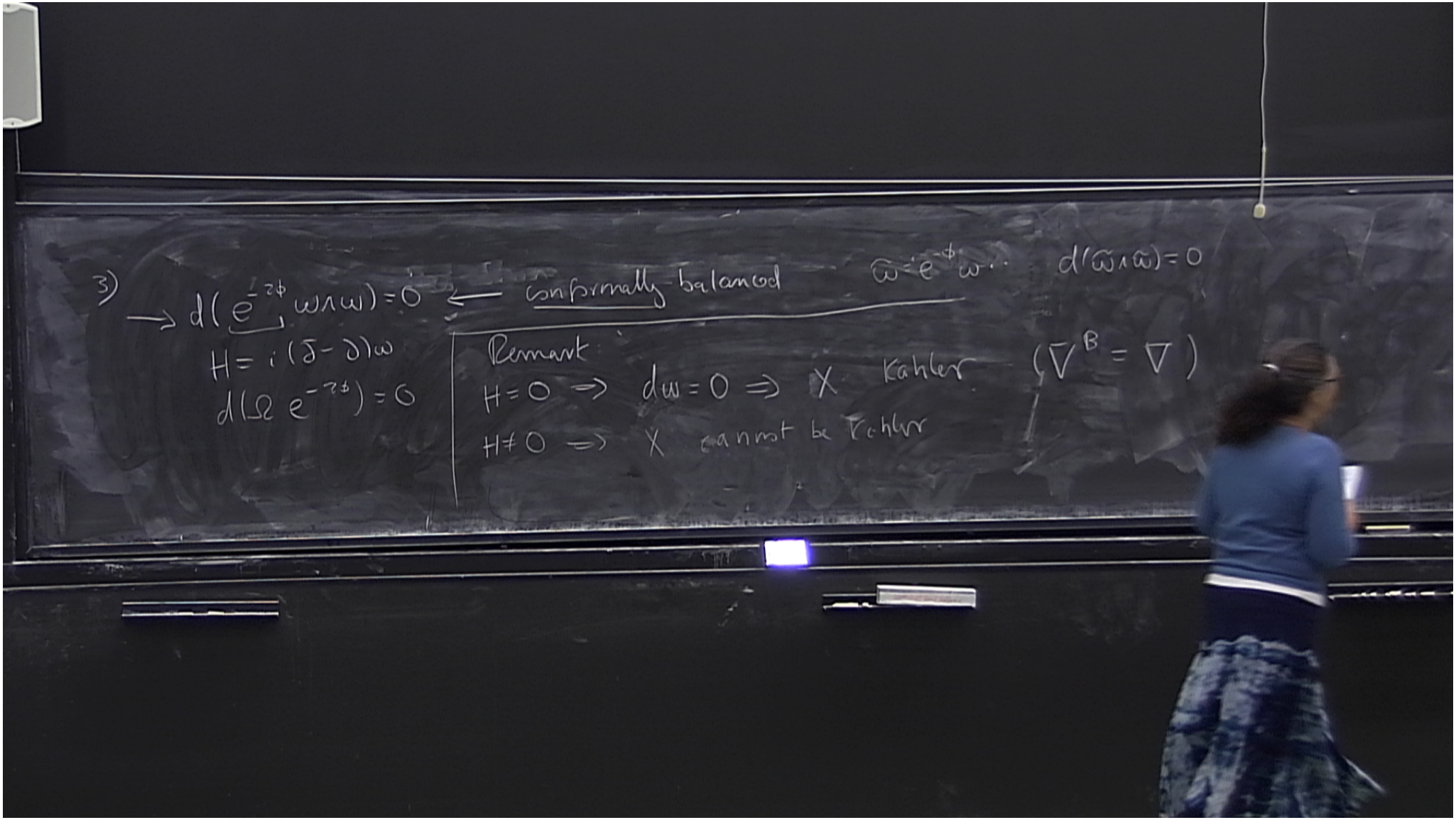

3)
$d(e^{-2\phi}\omega\wedge\omega)=0$ ⟵ conformally balanced
$H=i(\bar\partial-\partial)\omega$
$d(\Omega e^{-2\phi})=0$

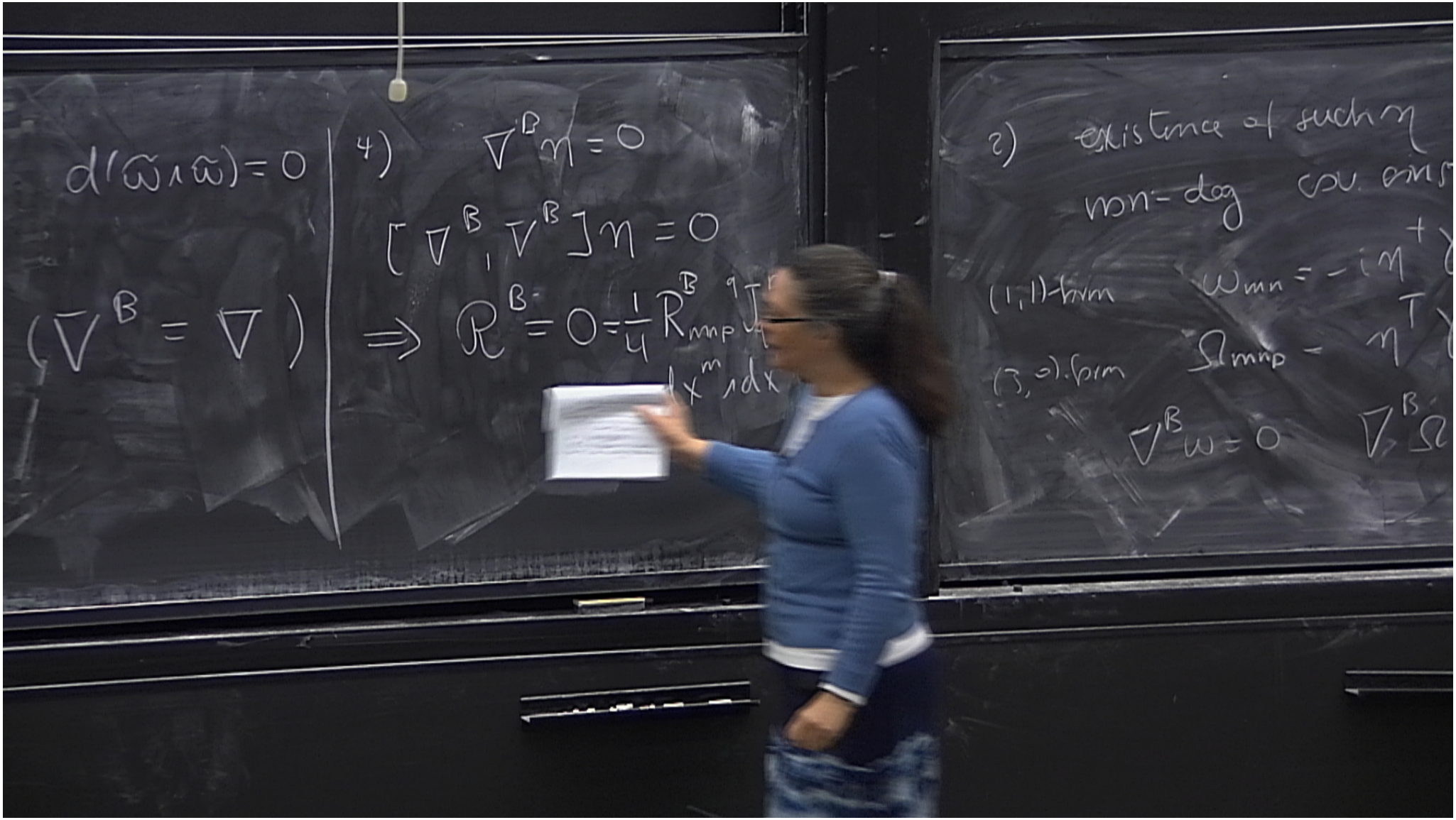

balanced
$\tilde{\omega} = e^{-\phi}\omega$
$d(\tilde{\omega}\wedge\tilde{\omega}) = 0$

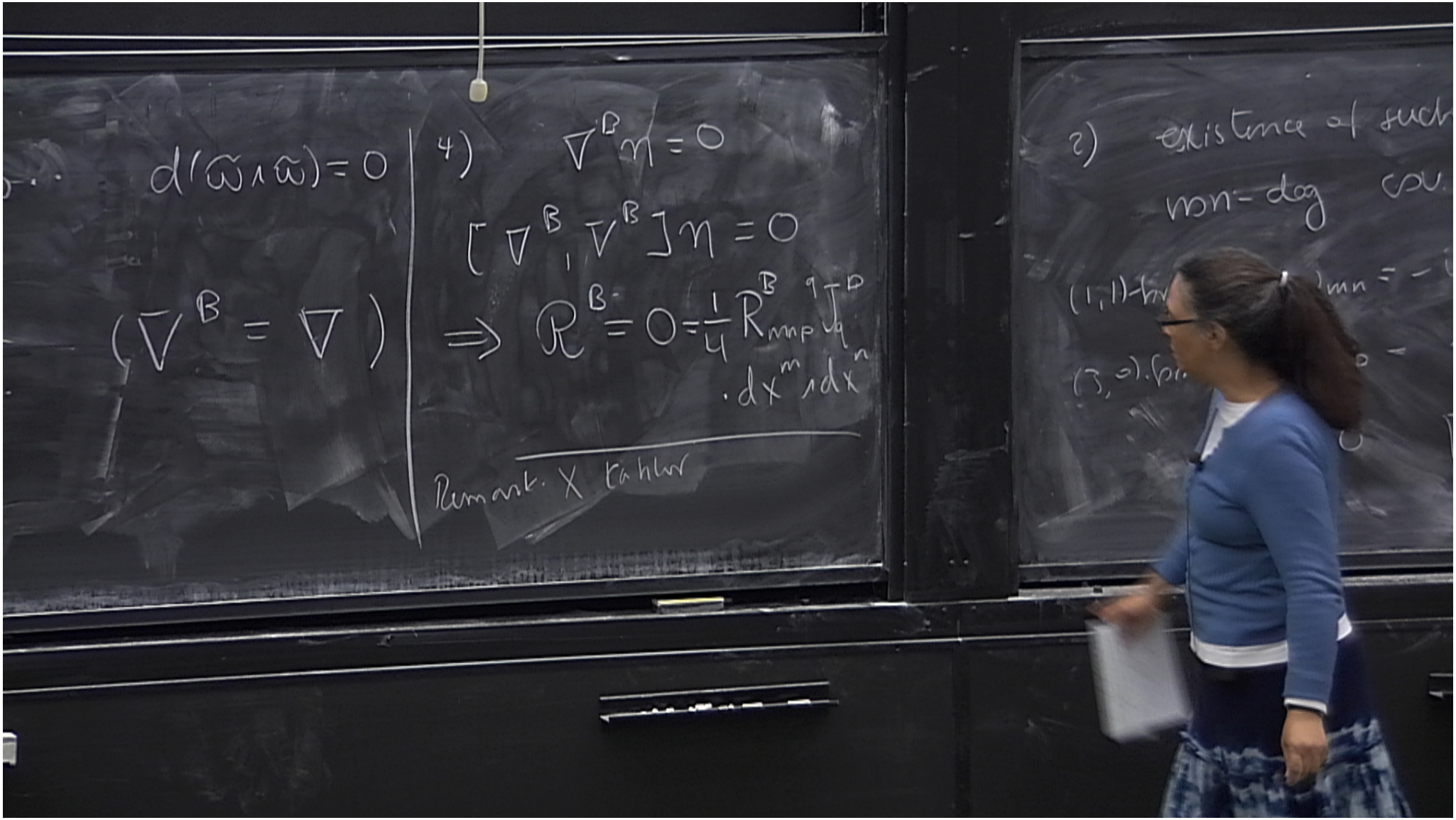

conformally balanced
Remark:
H = 0 ⇒

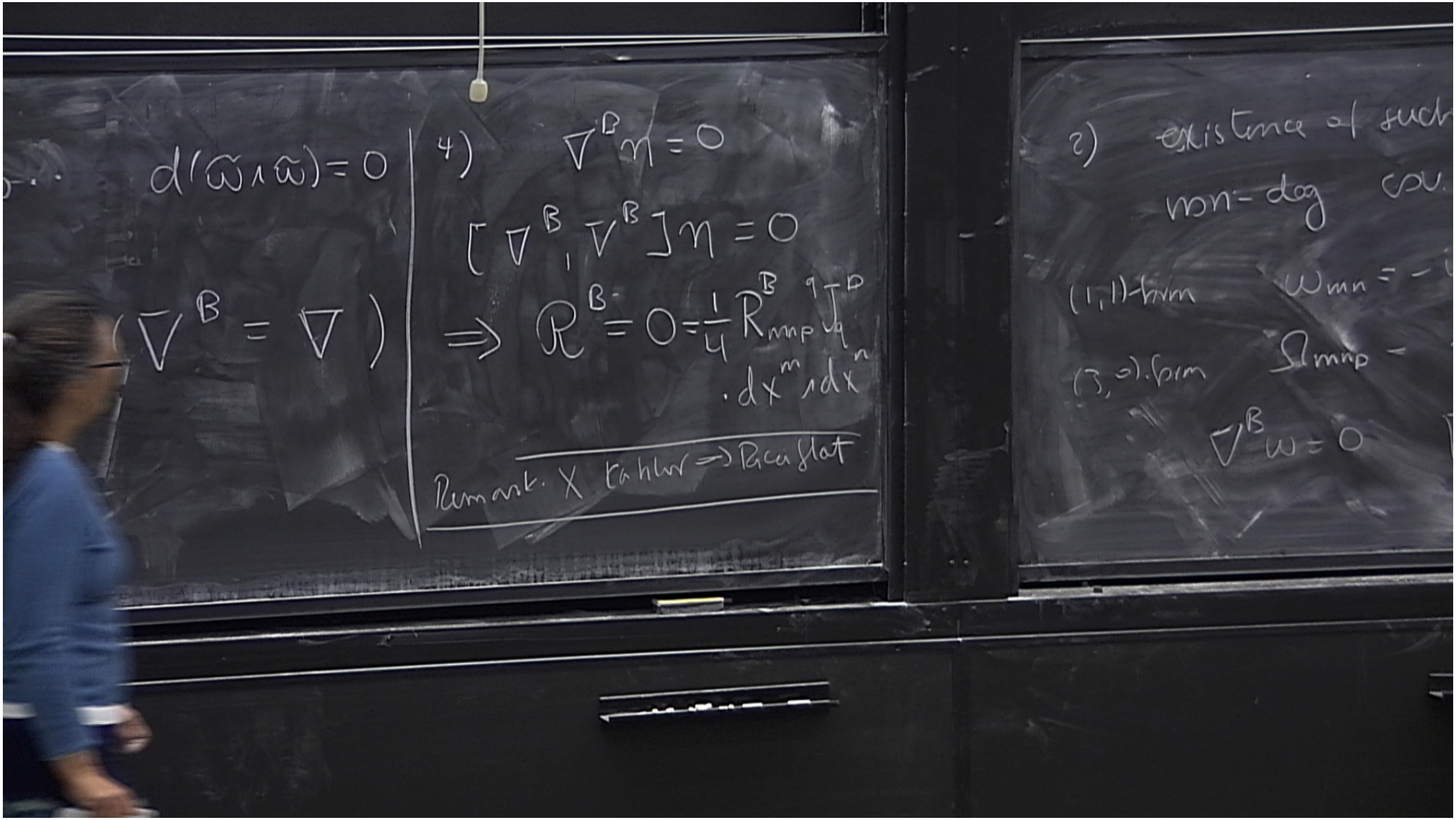

3)
$d(e^{-2\phi}\omega\wedge\omega)=0$ ⟸ conformally balanced
$\tilde{\omega}=e^{-\phi}\omega$: $d(\tilde{\omega}\wedge\omega)=0$
$H=i(\partial-\bar{\partial})\omega$
$d(\Omega\, e^{-2\phi})=0$
Remark
$H=0 \Rightarrow d\omega=0 \Rightarrow X$ Kahler
$H\neq 0 \Rightarrow X$ cannot be Kahler

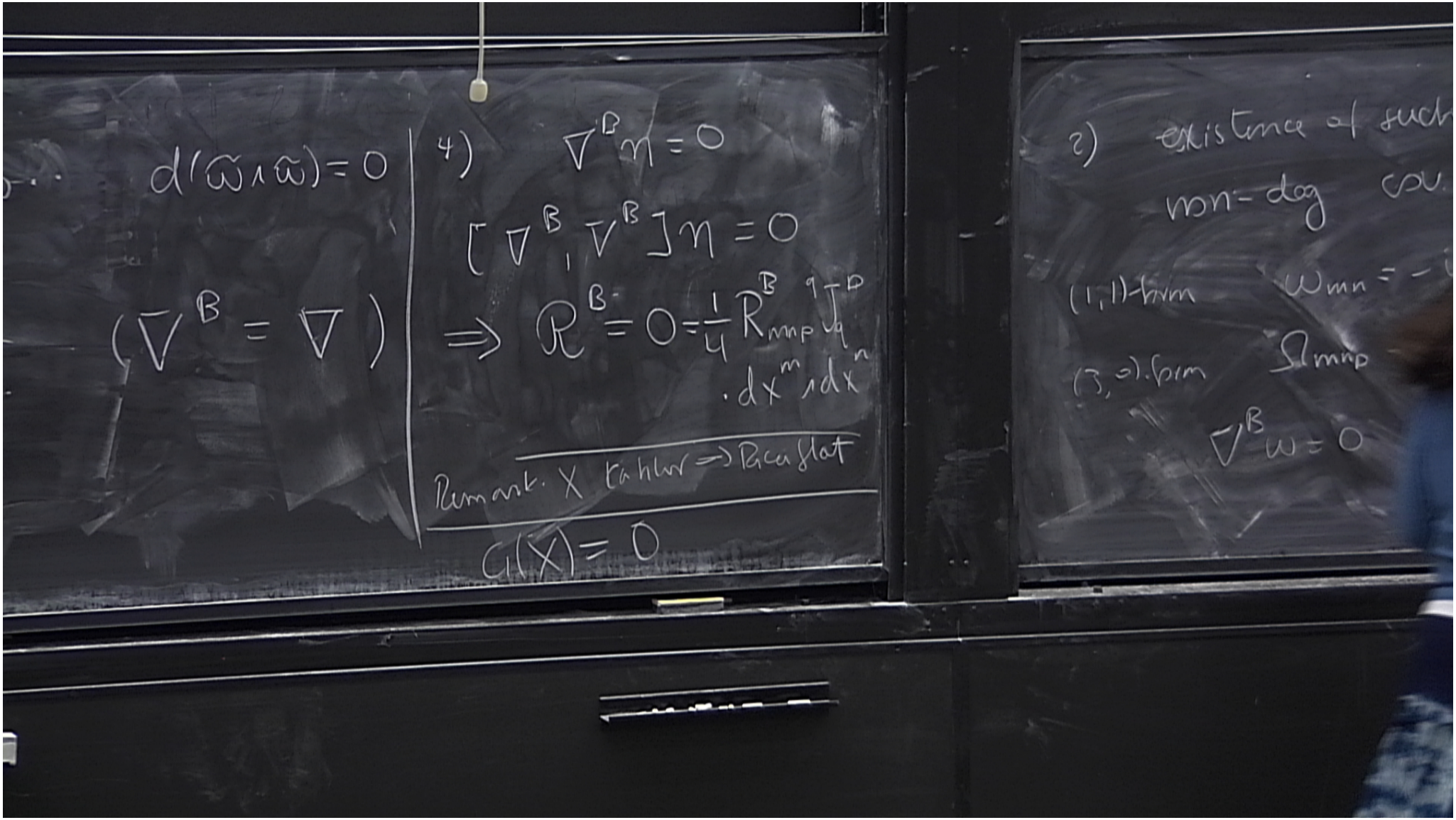

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Remark
$H=0 \Rightarrow d\omega=0 \Rightarrow X$ Kahler $(\nabla^B=\nabla)$
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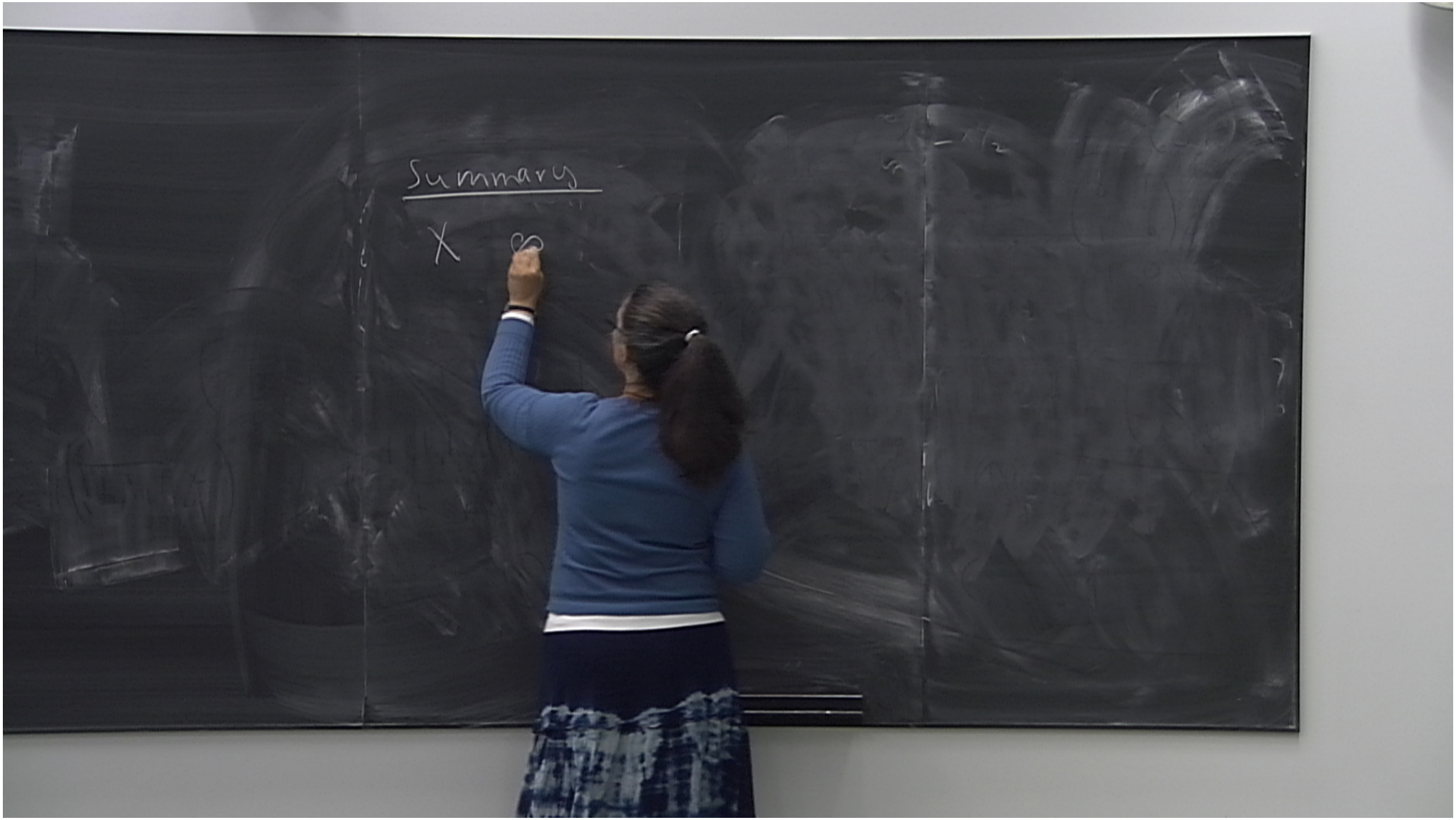

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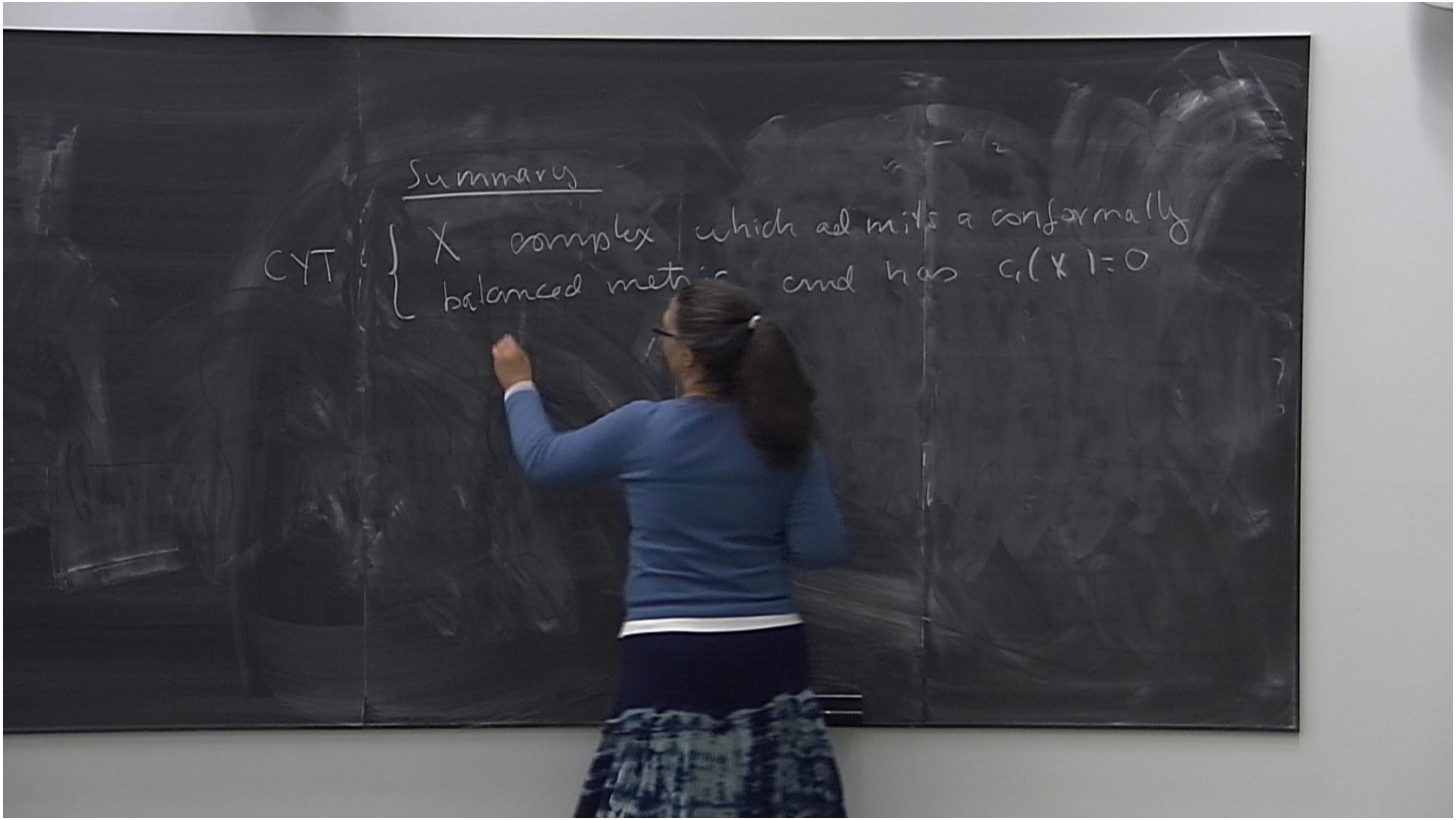

$d(\tilde{\omega} \wedge \tilde{\omega}) = 0$
$(\nabla^B = \nabla)$
4) $\nabla^B \eta = 0$
$[\nabla^B, \nabla^B]\eta = 0$
$\Rightarrow \mathcal{R}^B = 0 = \frac{1}{4} R^B_{mnp}$
$dx^m \wedge dx$
2) existence of such η
non-deg
(1,1)-form $\omega_{mn} = -i\eta^+$
(3,0)-form $\Omega_{mnp} = \eta^T$
$\nabla^B \omega = 0$
$\nabla^B \Omega$

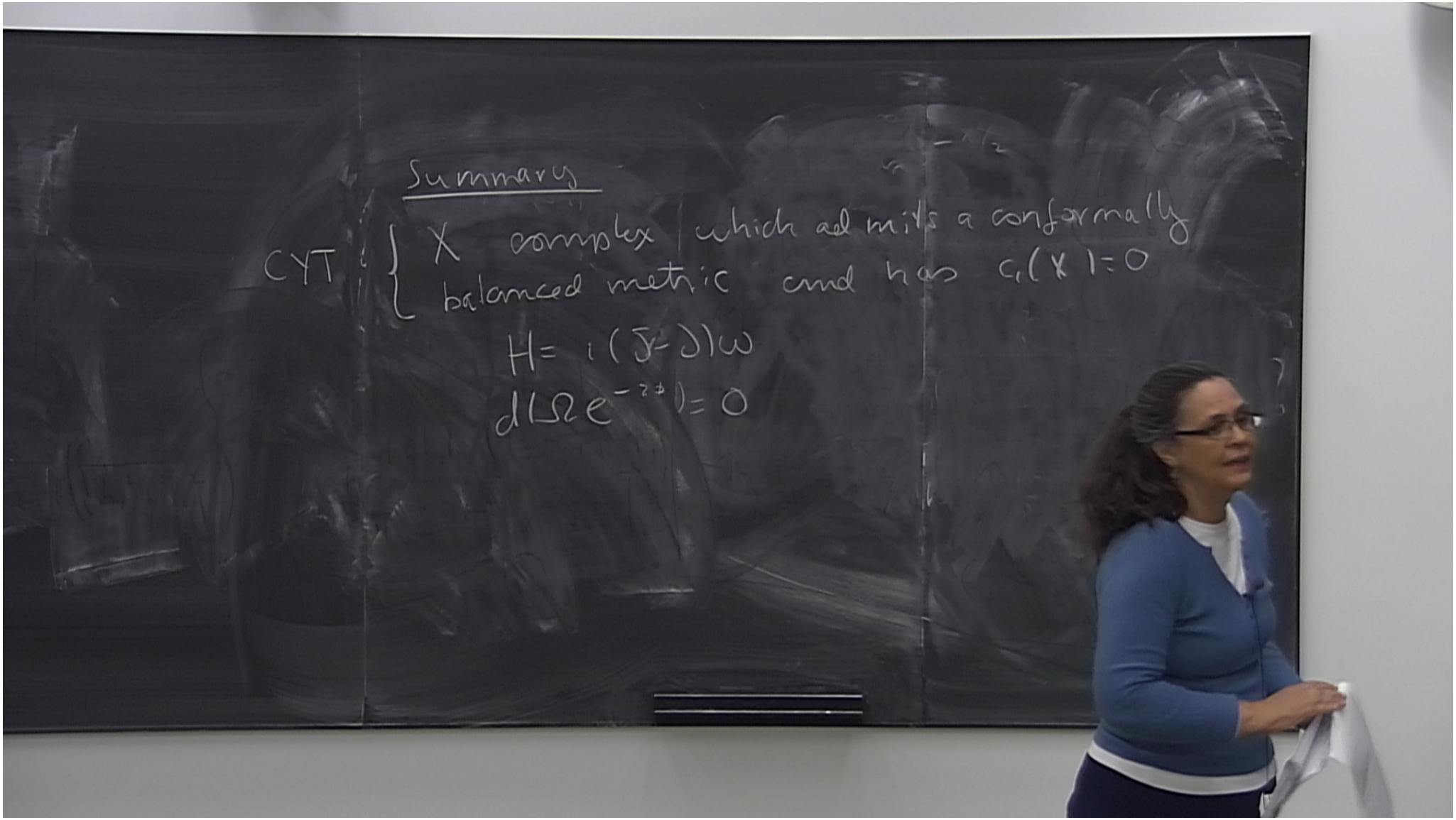

$d(\omega \wedge \omega) = 0$
$(\nabla^B = \nabla)$
4) $\nabla^B \eta = 0$
$[\nabla^B, \nabla^B]\eta = 0$
$\Rightarrow \mathcal{R}^B = 0 = \frac{1}{4} R^B_{mnpq} J^D \cdot dx^m \wedge dx^n$
Remark. X Kahler
2) existence of such
non-deg
(1,1)-form
(3,0)-form

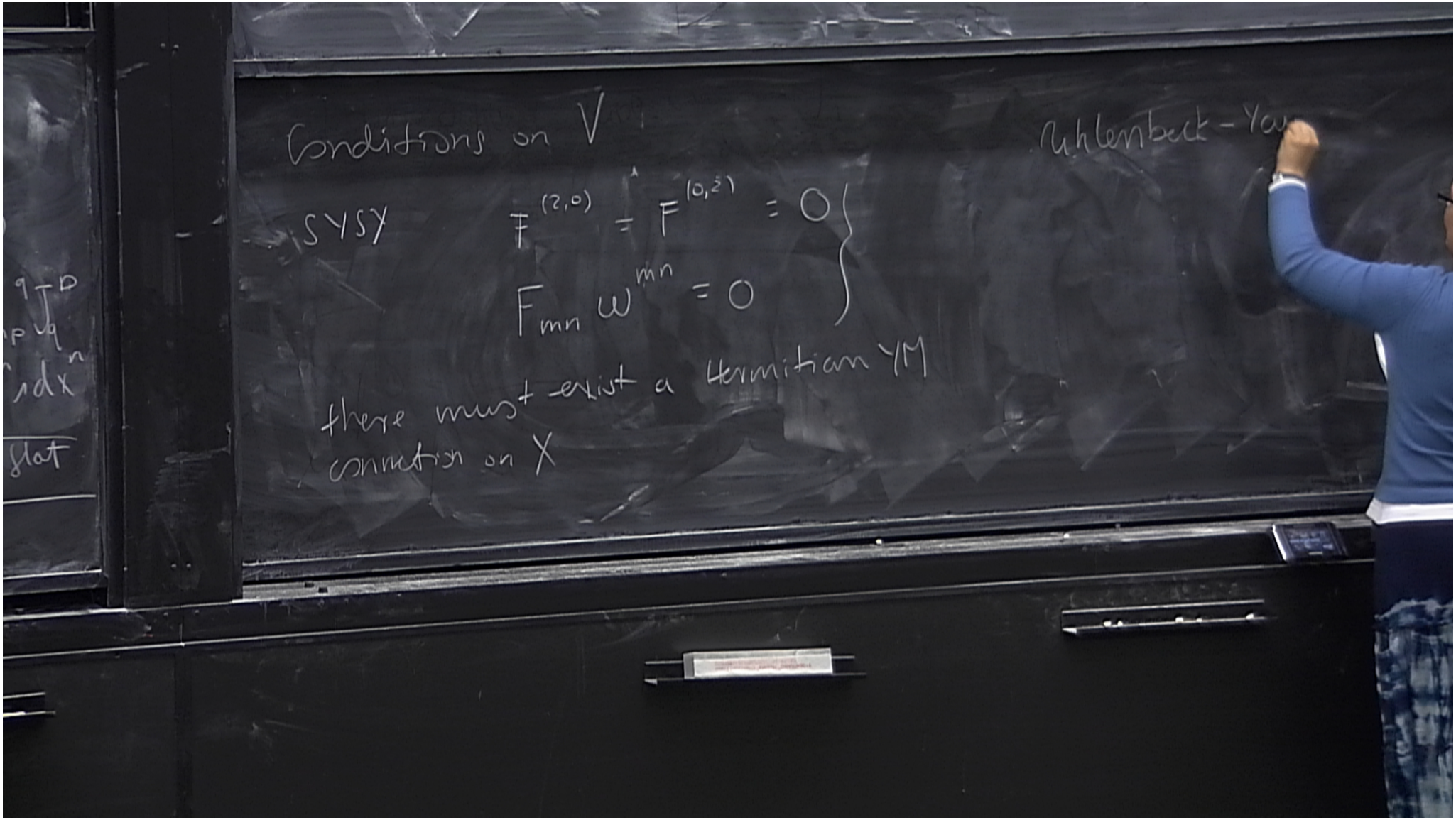

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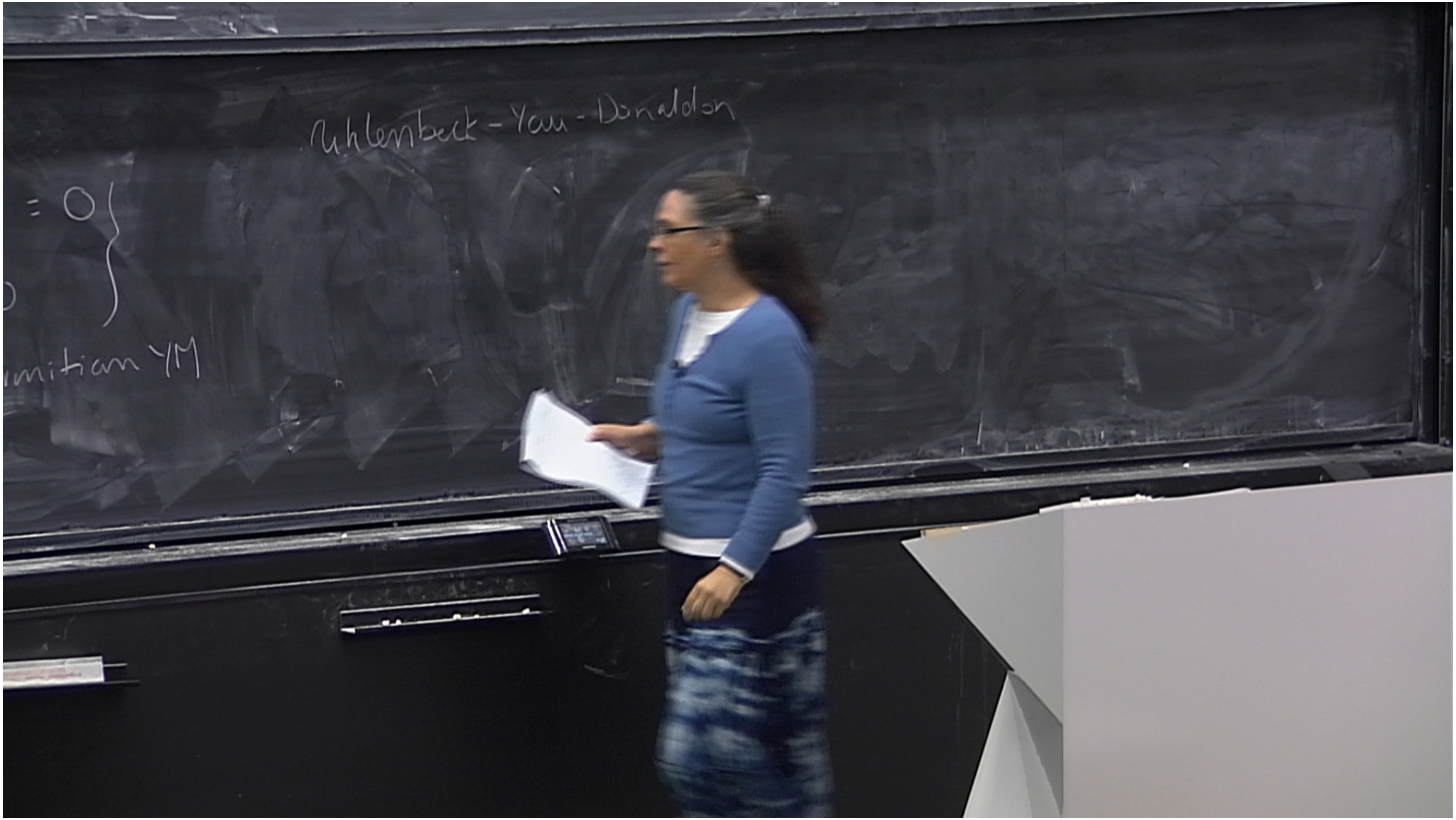

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$c_1(X) = 0$
2) existence of such
non-deg
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(3,0)-form Ω_{mnp}
$\nabla^B \omega = 0$

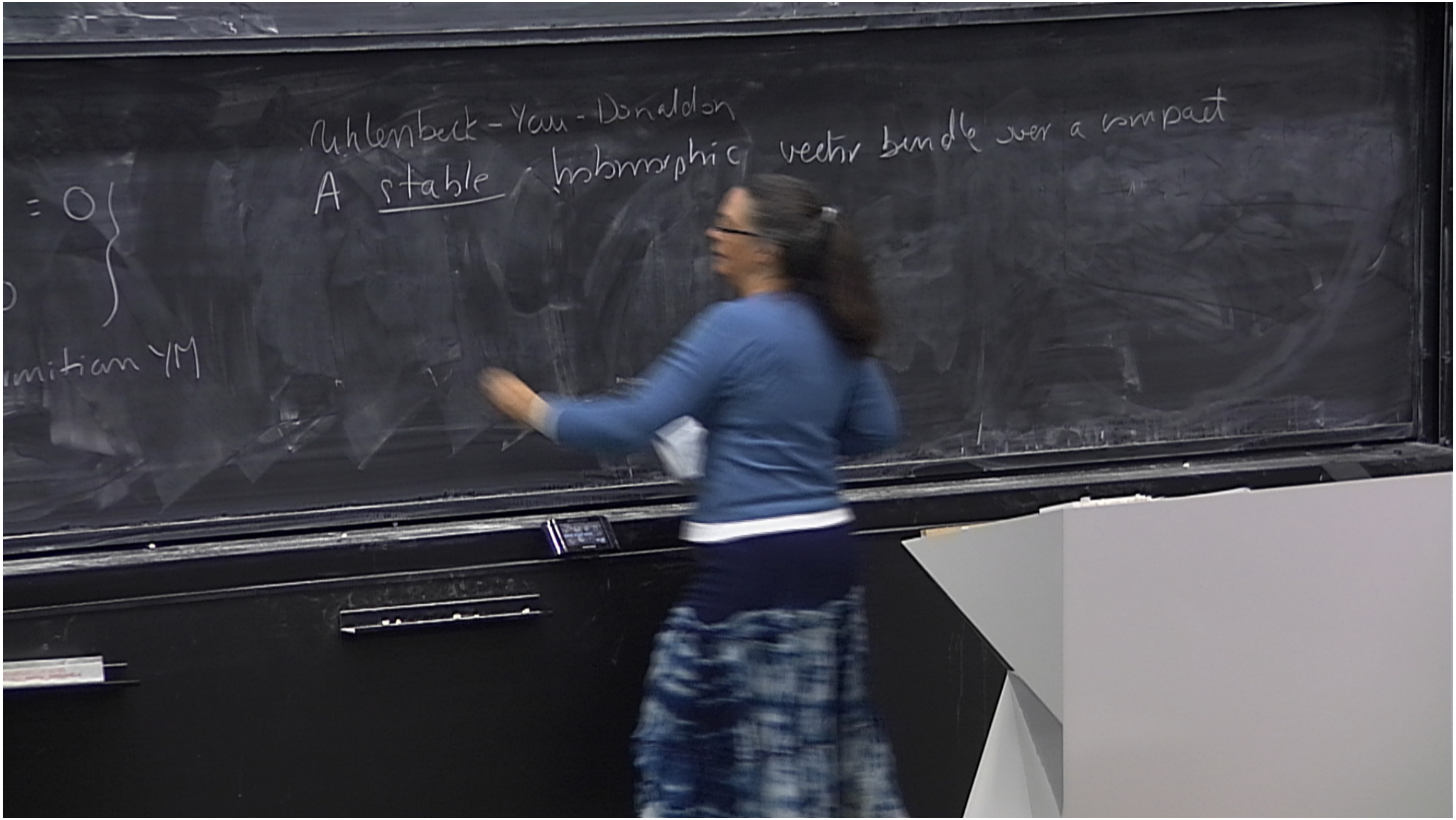

Summary
X

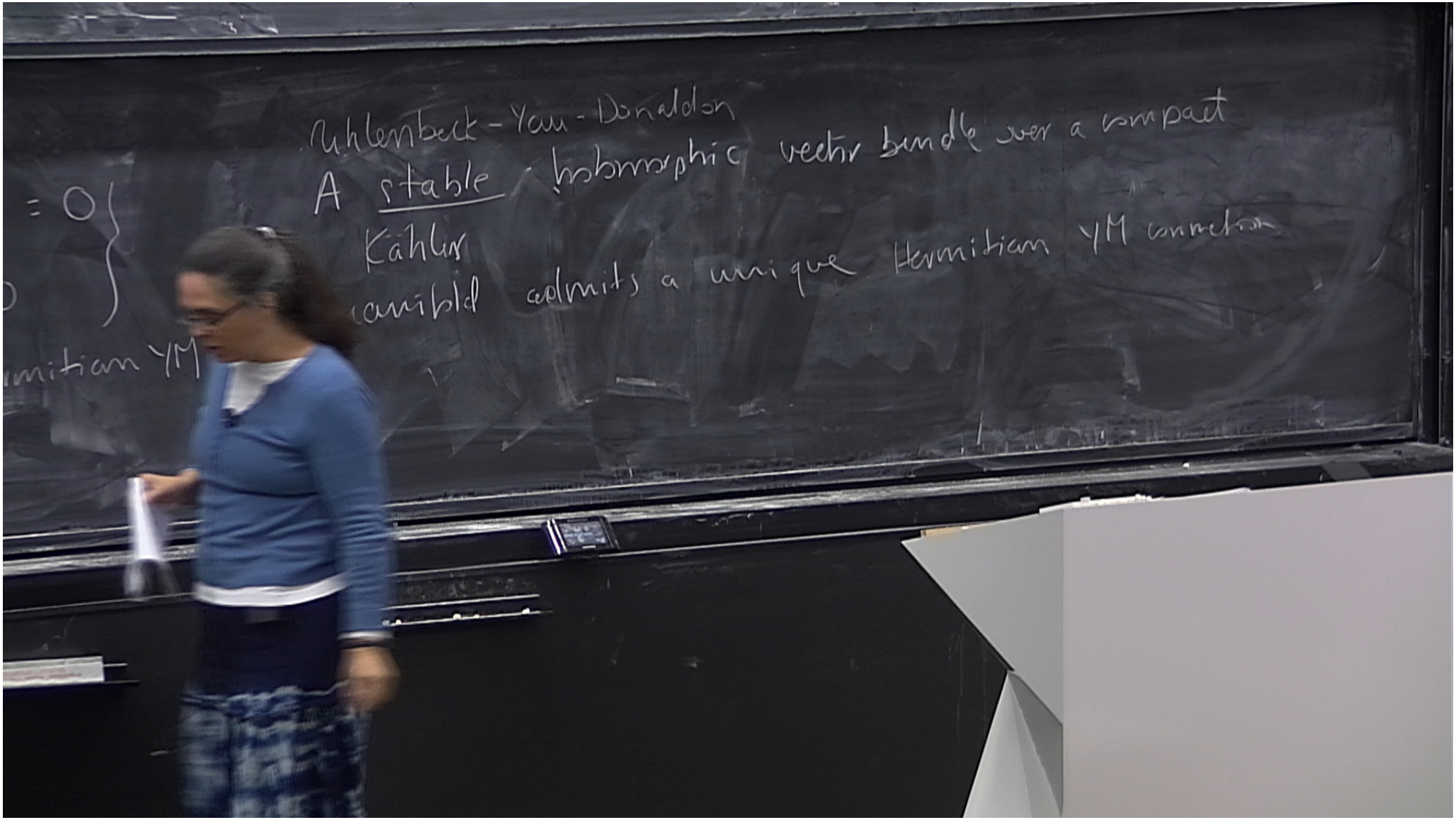

Summary
CYT
X complex which admits a conformally
balanced metric and has $c_1(X)=0$

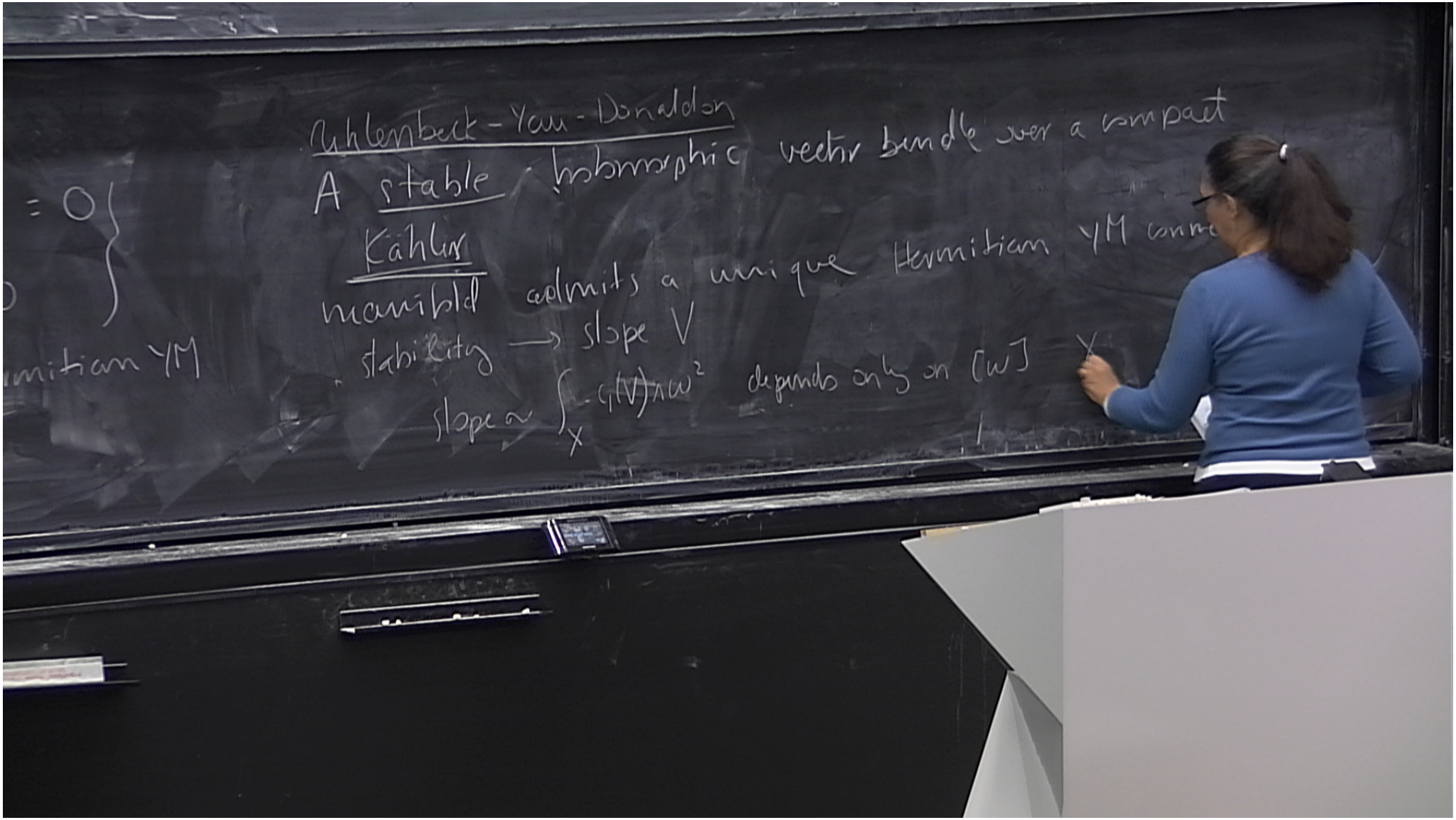

Summary
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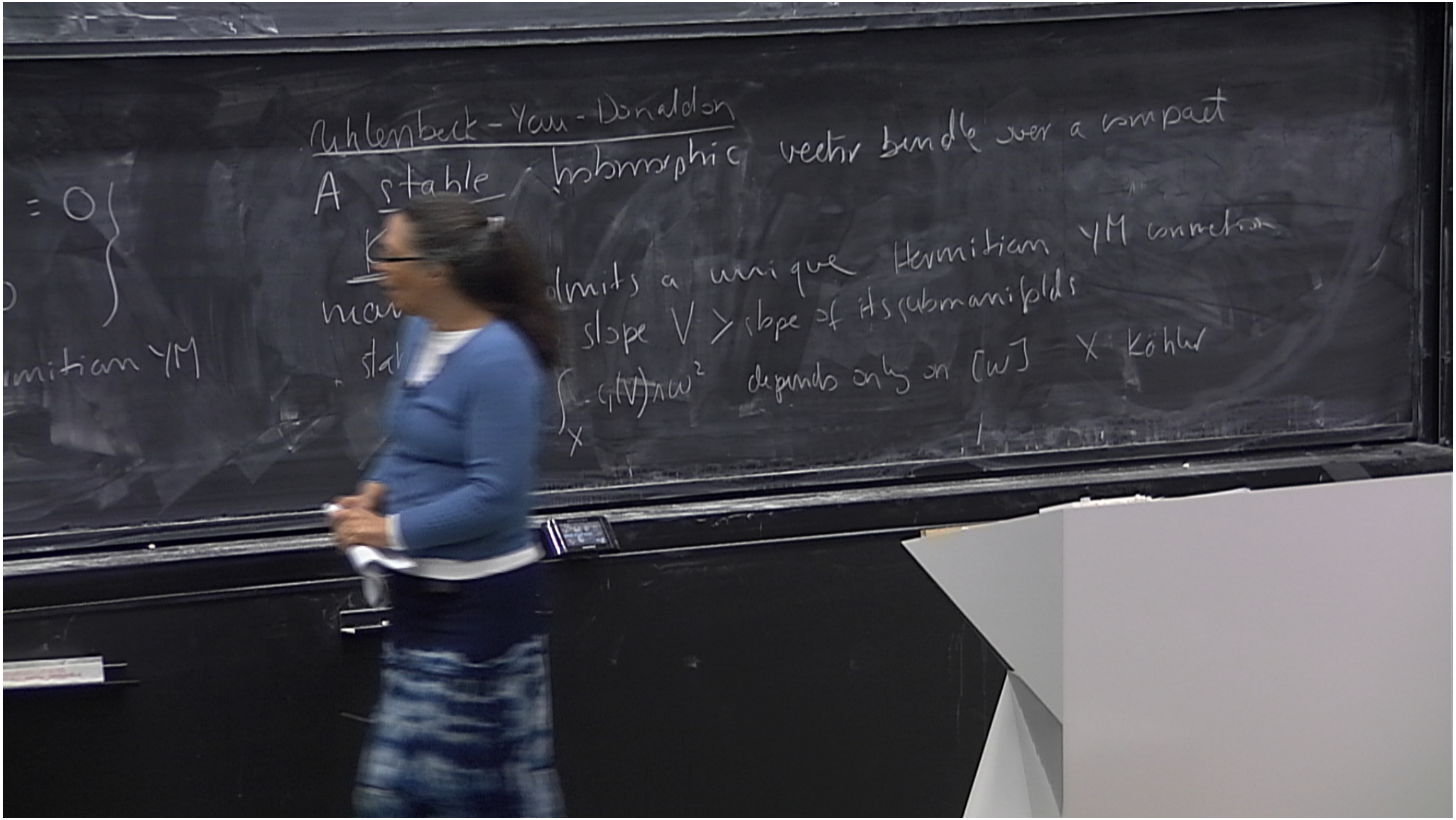

Conditions on V
SYSY
$F^{(2,0)} = F^{(0,2)} = 0$
$F_{mn}\, \omega^{mn} = 0$
there must exist a Hermitian YM
connection on X
Uhlenbeck - Ya

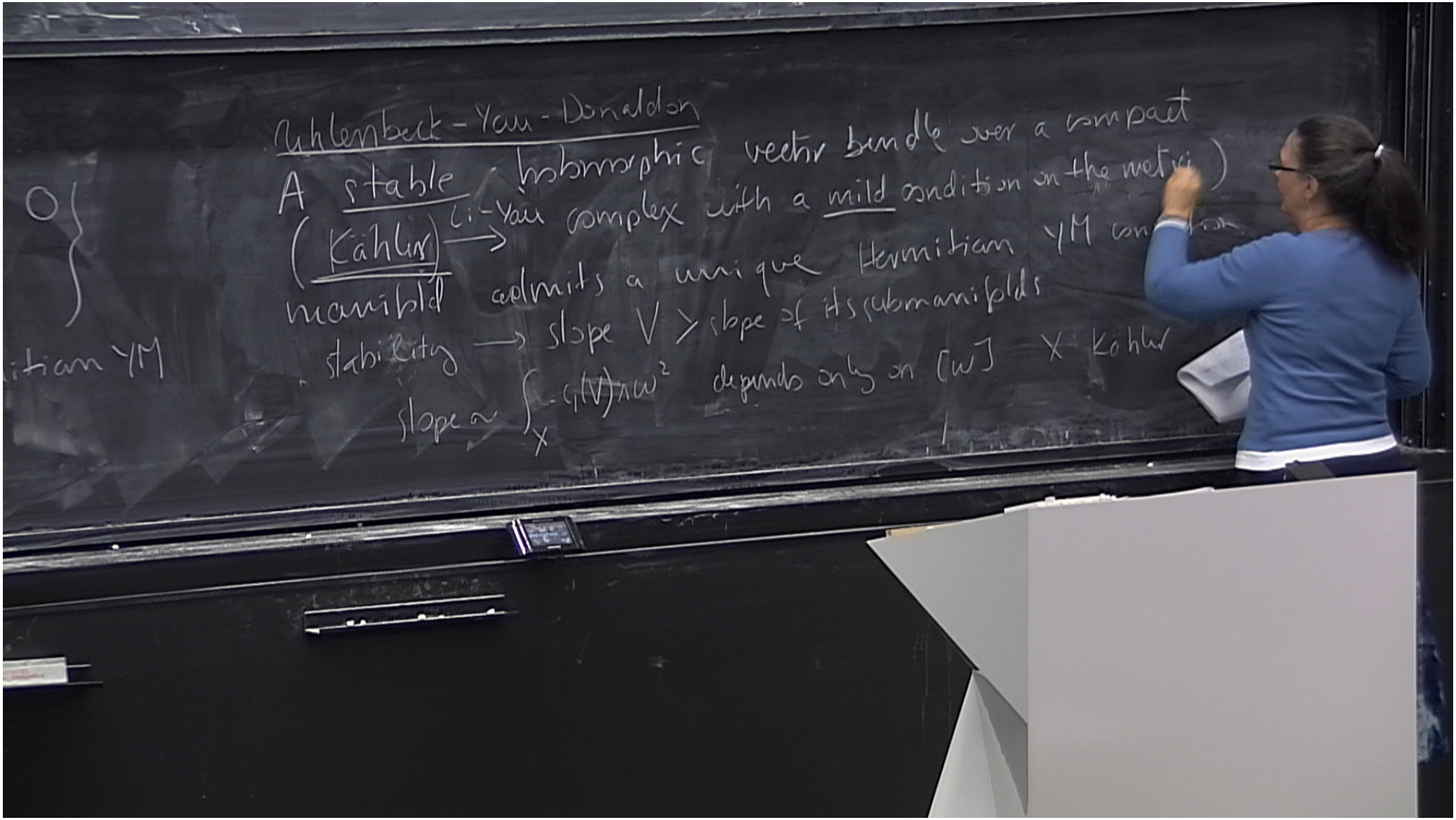

Uhlenbeck - Yau - Donaldson
= 0
mitian YM

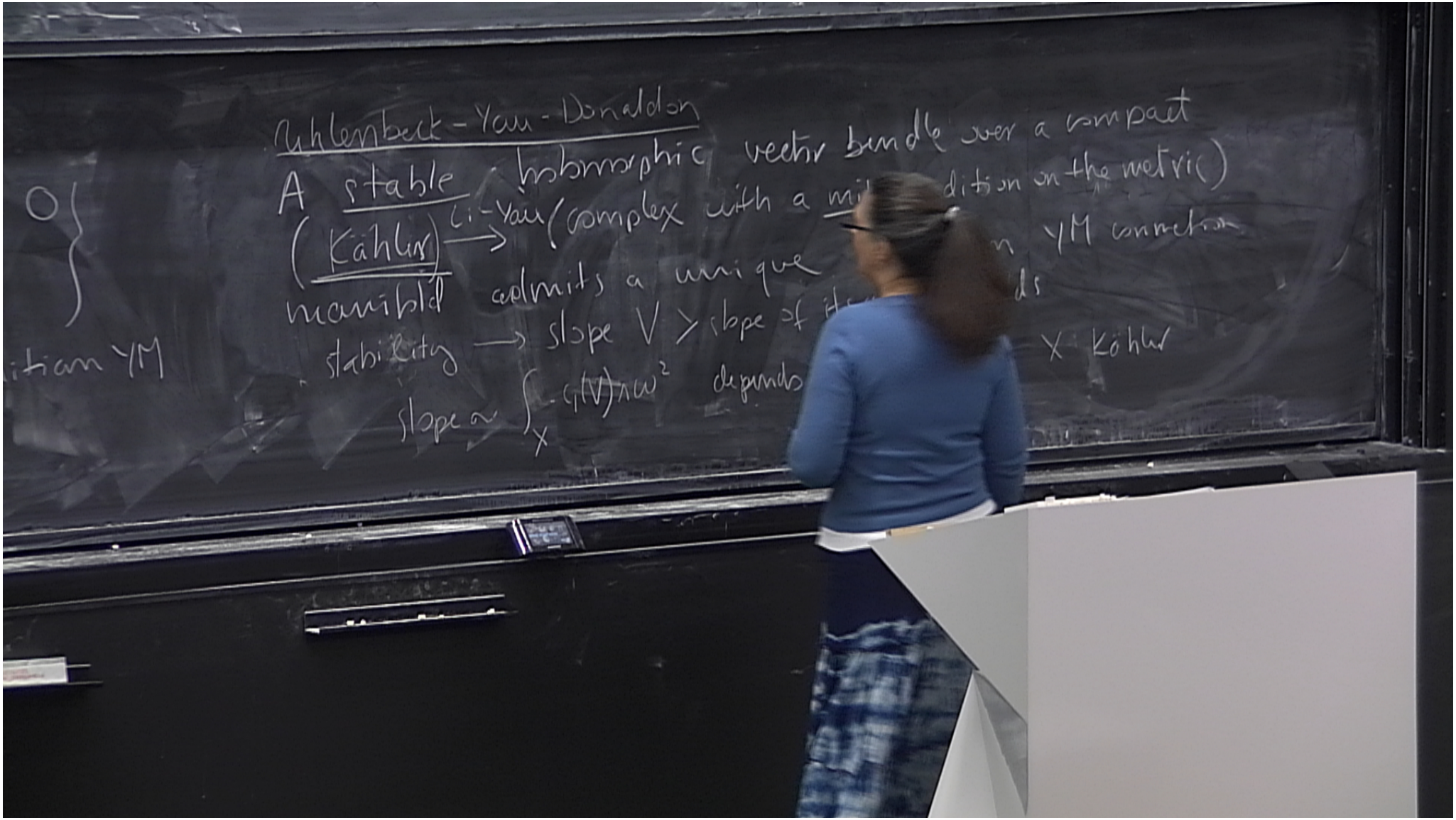

Uhlenbeck - Yau - Donaldson
A stable holomorphic vector bundle over a compact
= 0
mitian YM

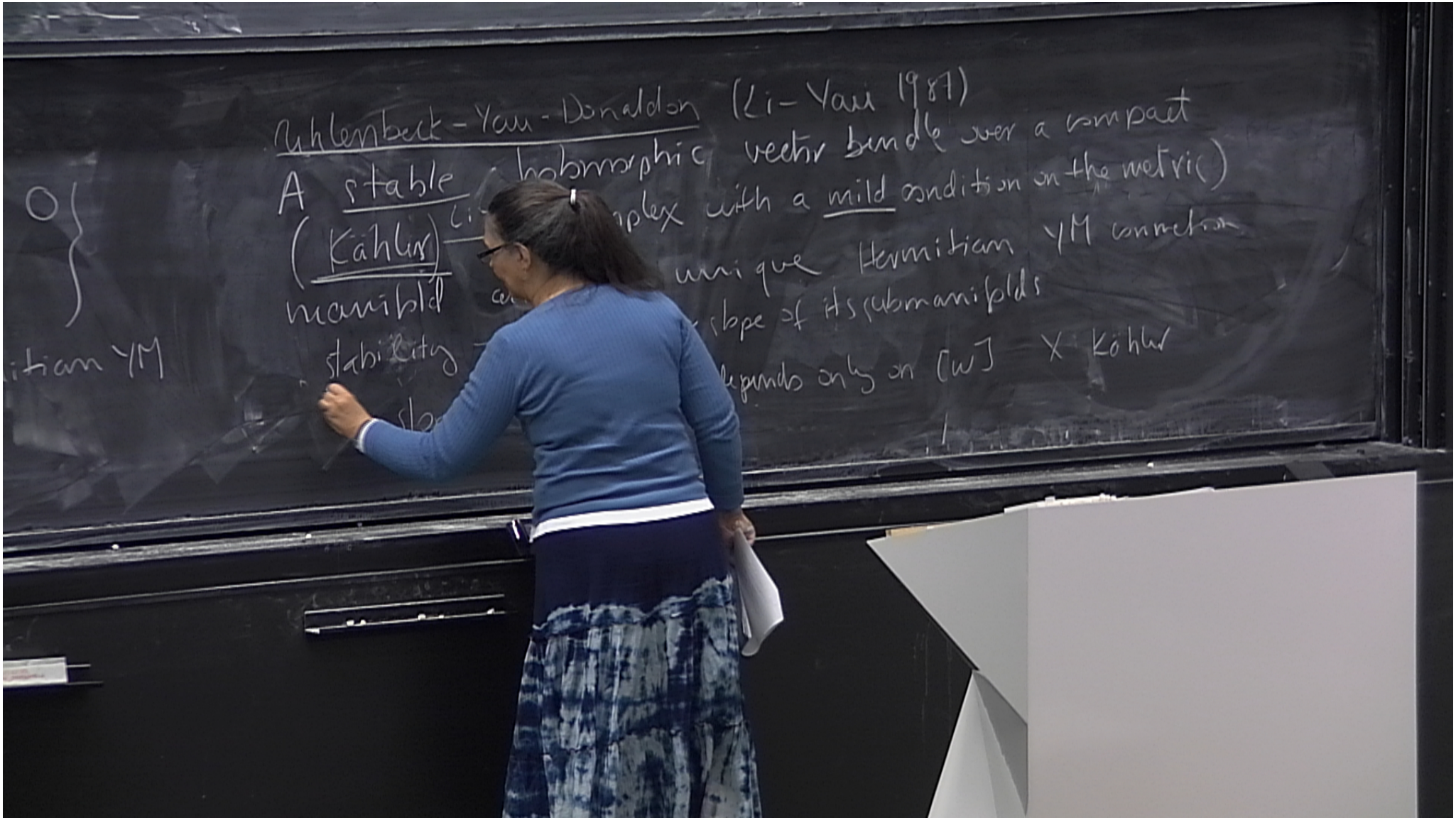

Uhlenbeck - Yau - Donaldson
A stable holomorphic vector bundle over a compact
Kähler
admits a unique Hermitian YM connection

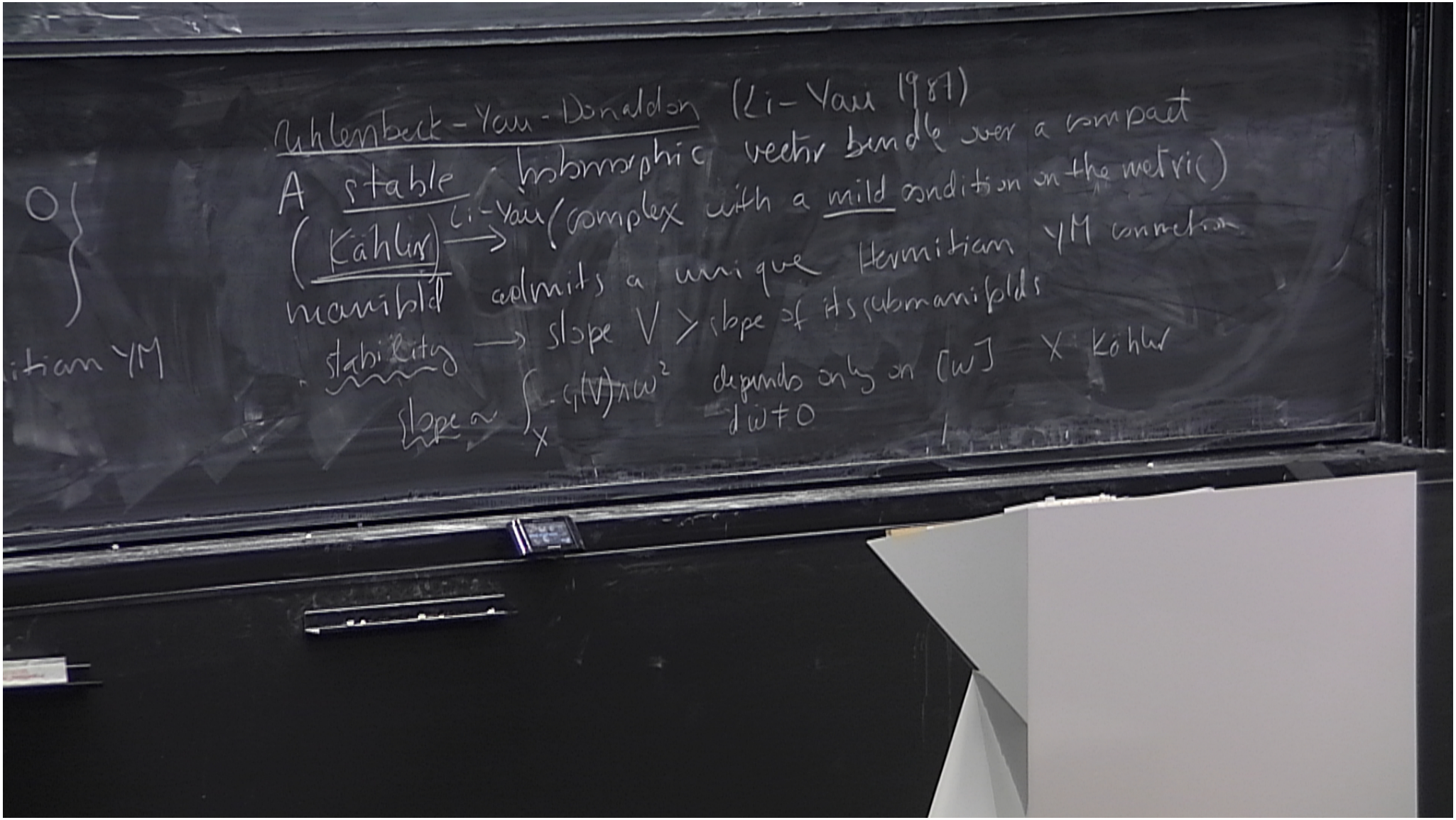

Uhlenbeck - Yau - Donaldson
A stable holomorphic vector bundle over a compact
Kähler
manifold admits a unique Hermitian YM conn
stability → slope V
slope ~ $\int_X c_1(V) \wedge \omega^2$ depends only on [ω]
mitian YM

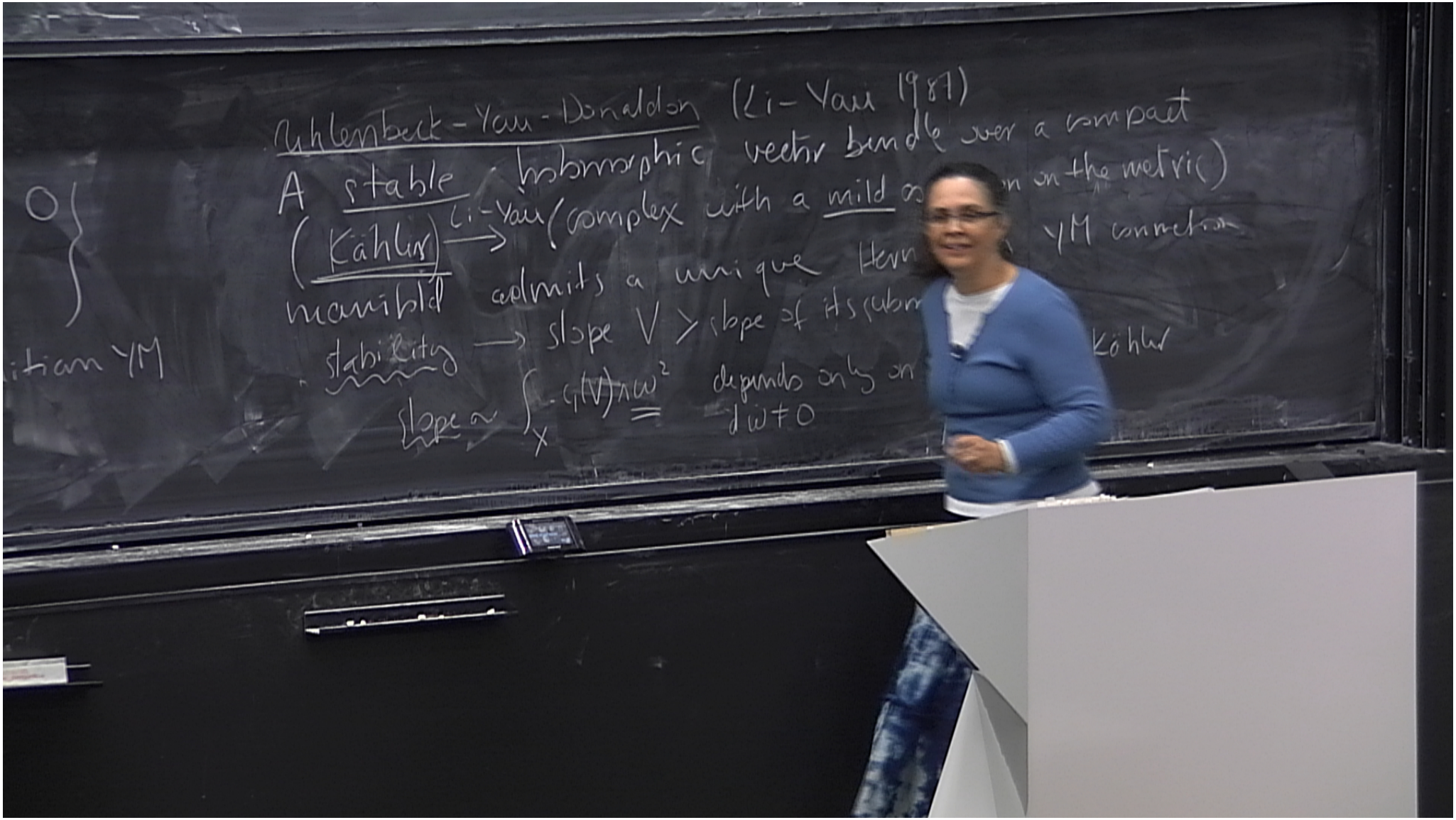

Uhlenbeck - Yau - Donaldson
A stable holomorphic vector bundle over a compact
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slope V > slope of its submanifolds
depends only on [ω]
X Kähler

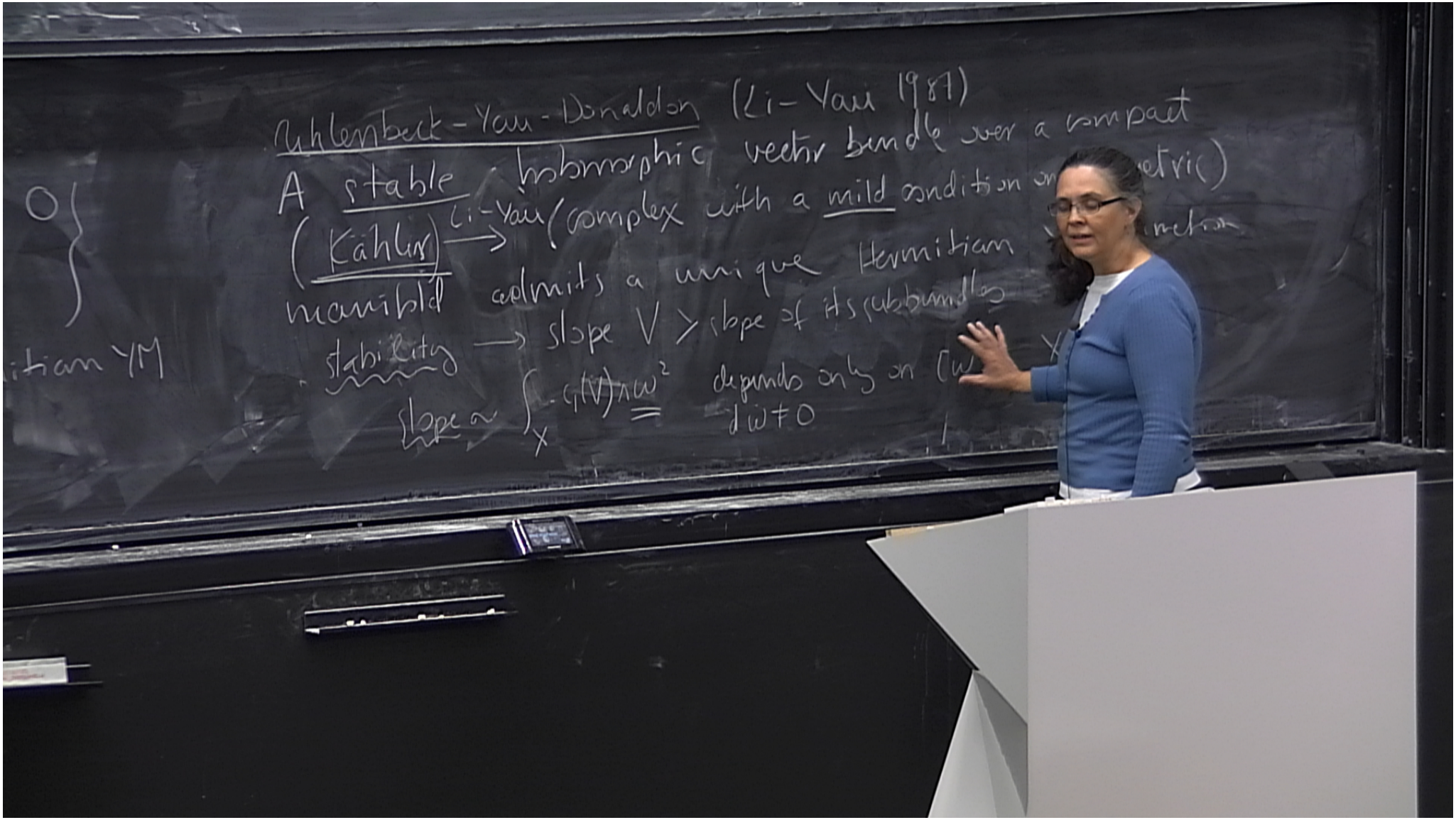

Uhlenbeck - Yau - Donaldson
A stable holomorphic vector bundle over a compact
(Kähler) Li-Yau complex with a mild condition on the metric)
manifold admits a unique Hermitian YM connection
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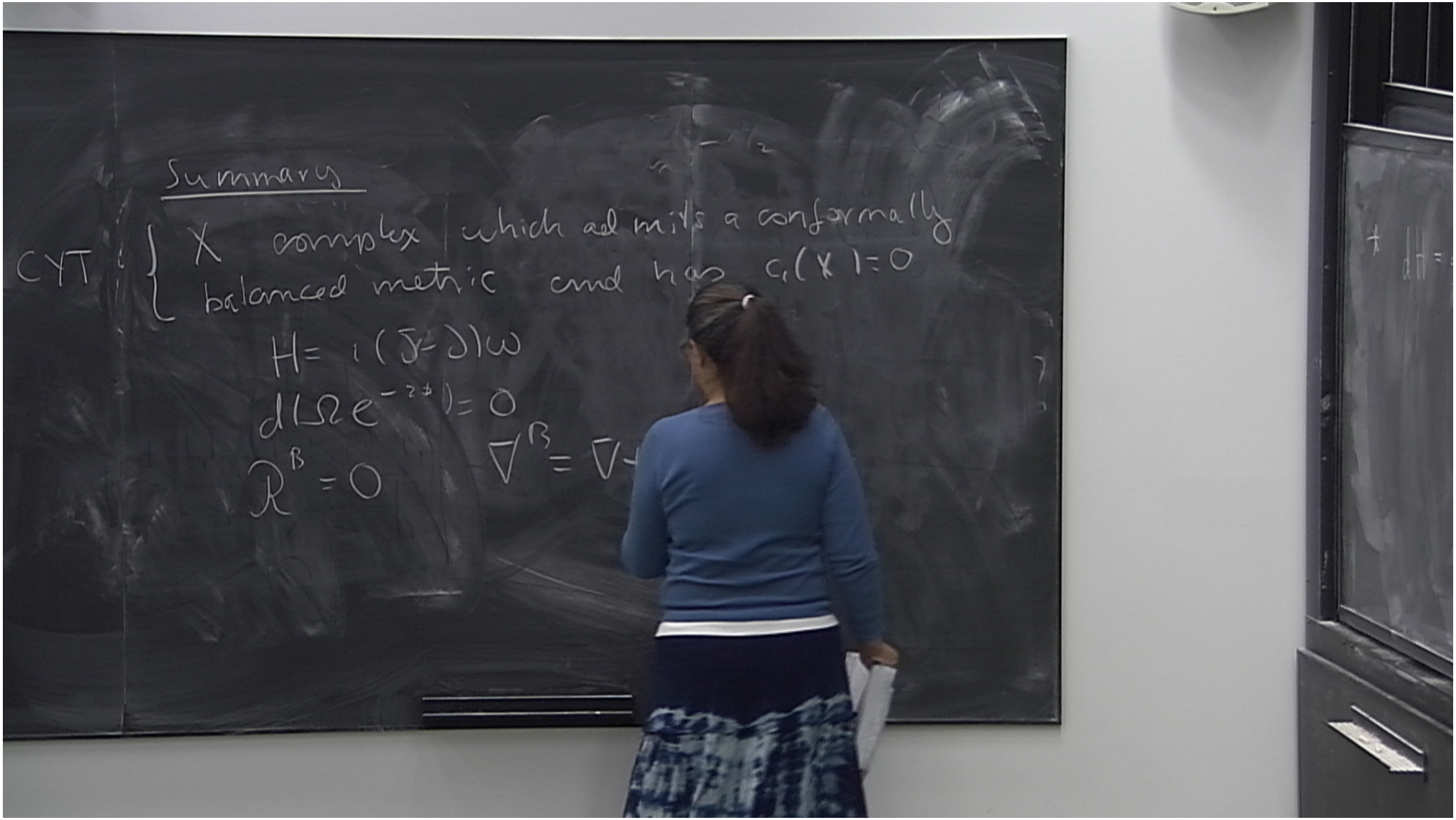

Uhlenbeck - Yau - Donaldson
A stable holomorphic vector bundle over a compact
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X Kähler
ition YM


Uhlenbeck - Yau - Donaldson (Li - Yau 1987)
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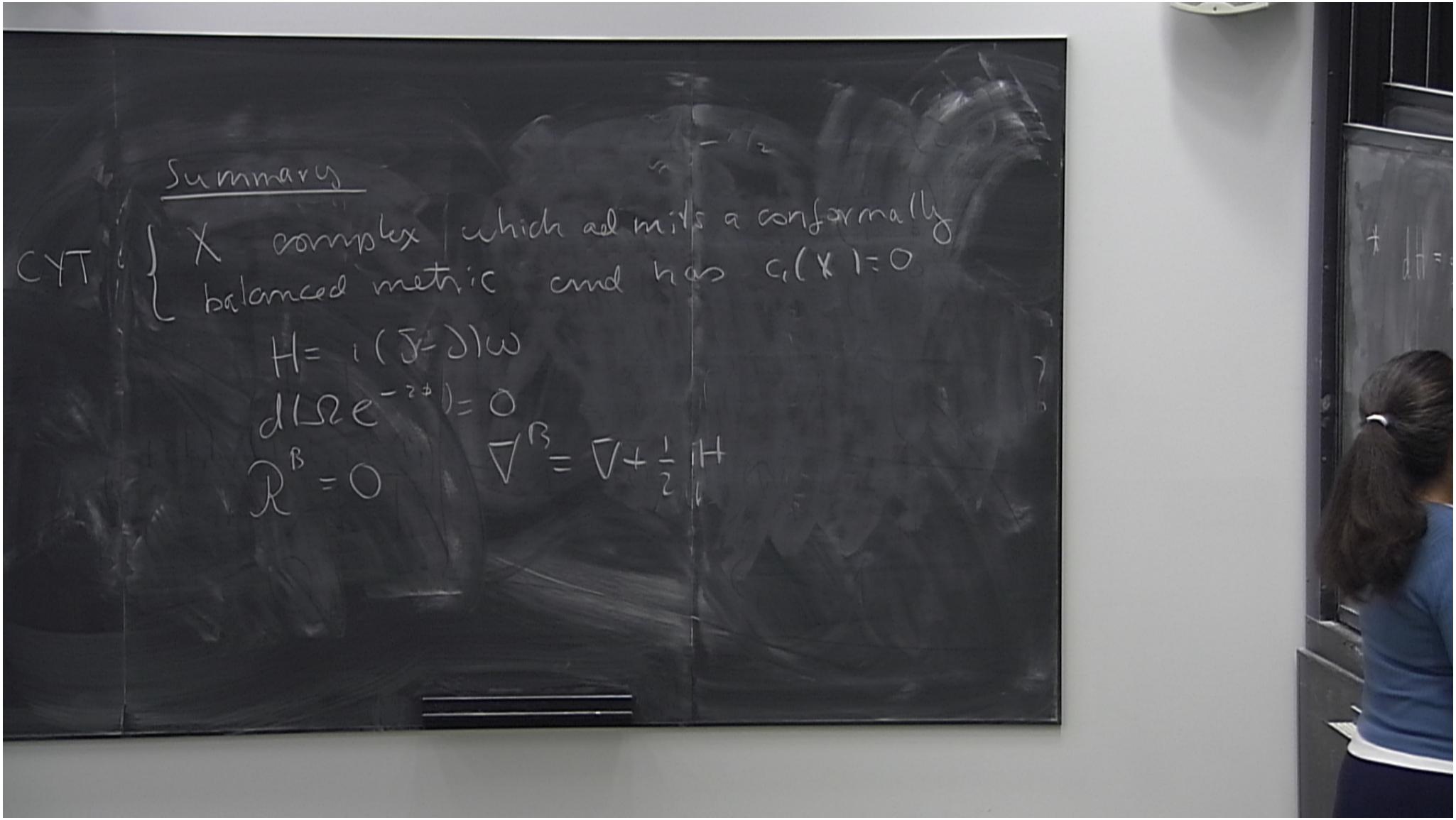

Uhlenbeck-Yau-Donaldson (Li-Yau 1987)
A stable holomorphic vector bundle over a compact
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dω≠0
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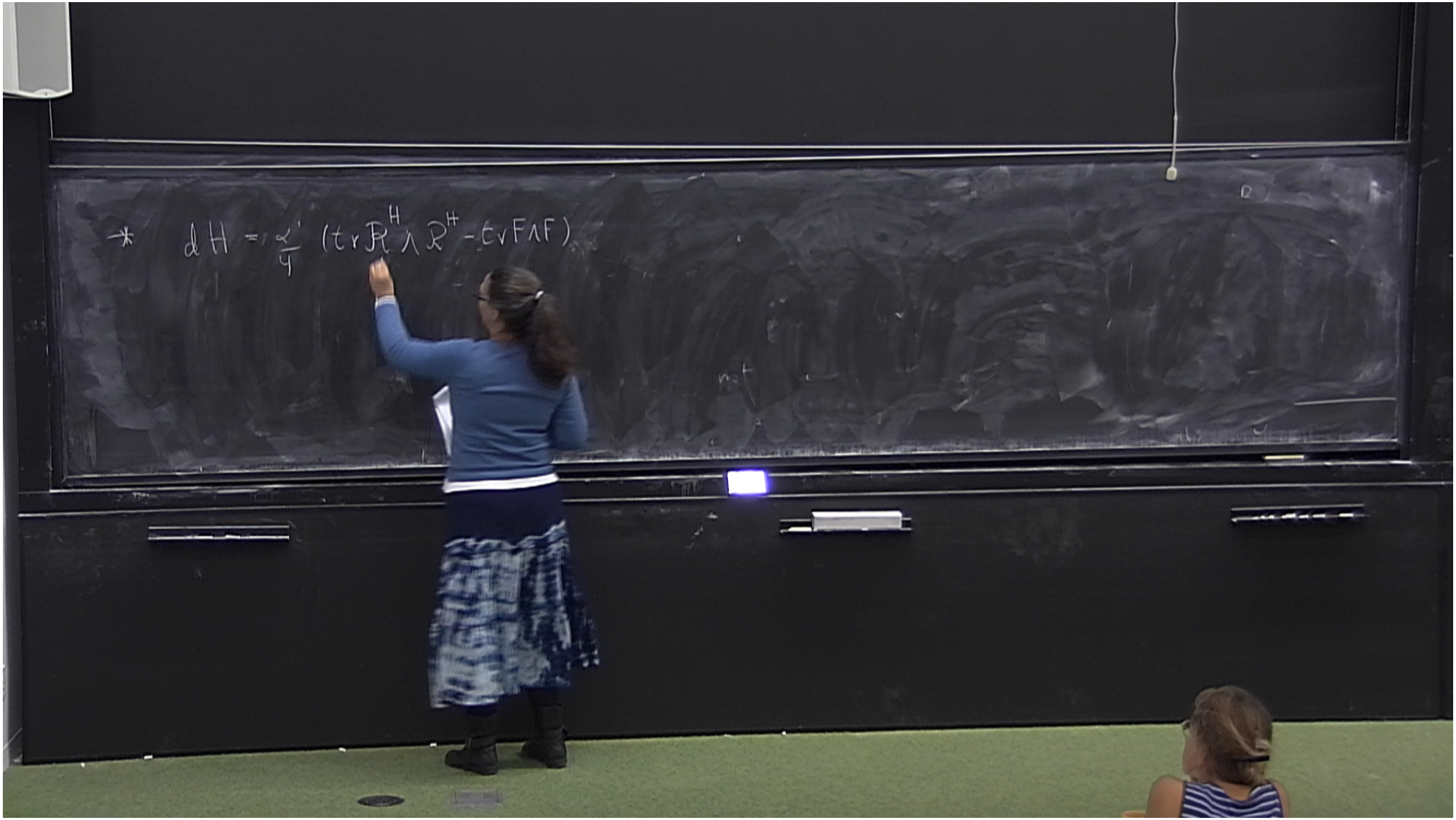

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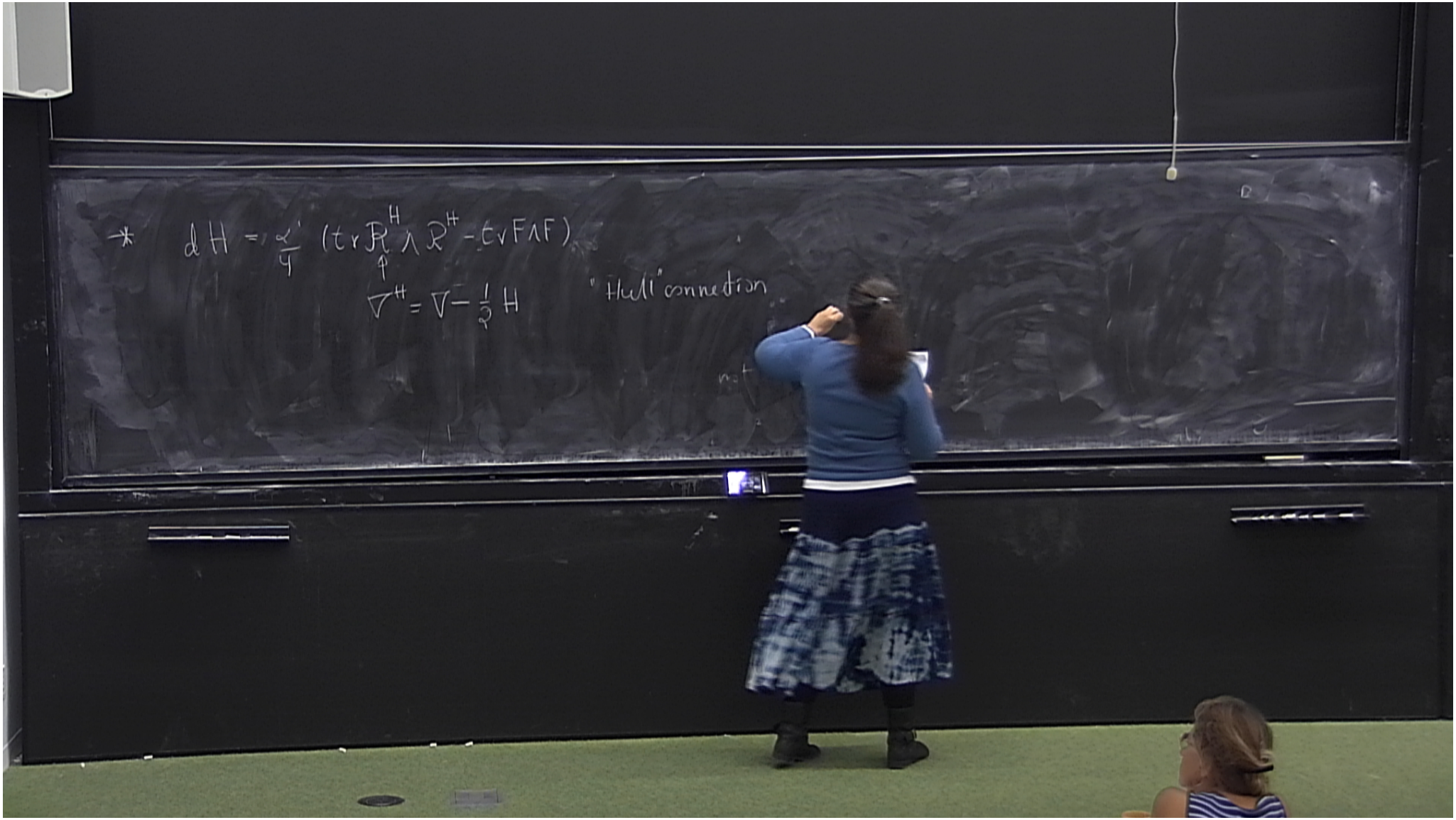

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dω = 0

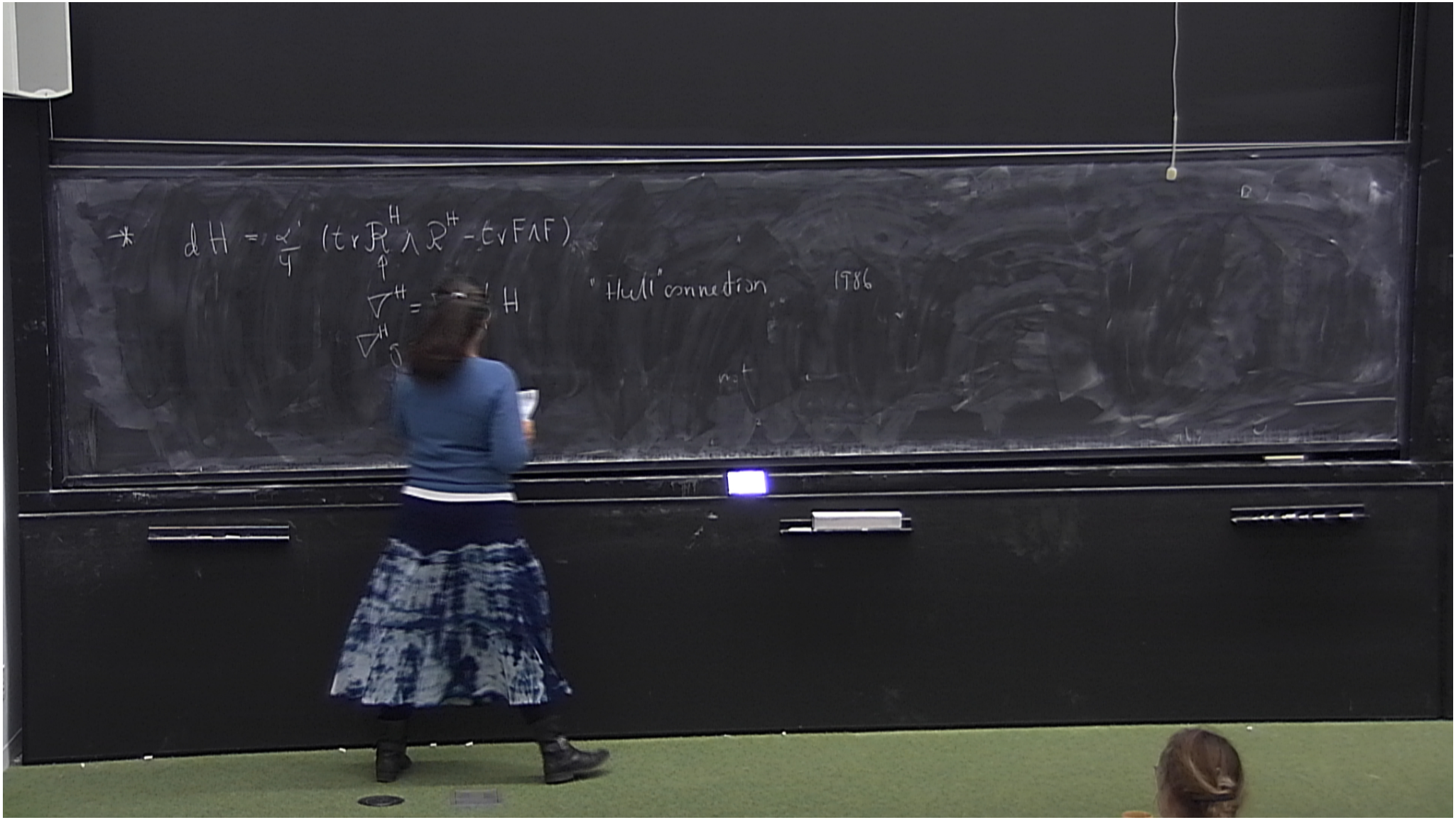

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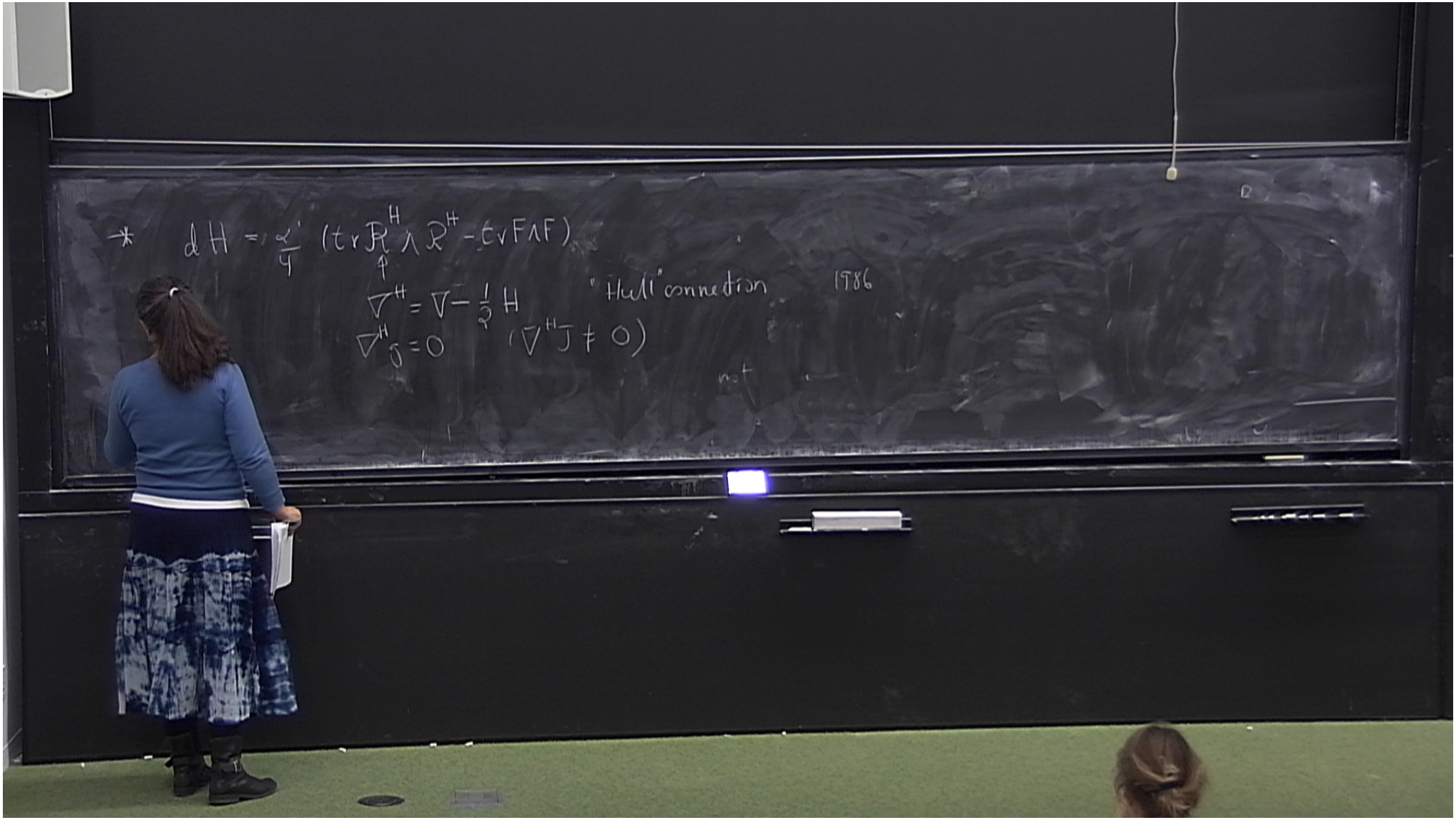


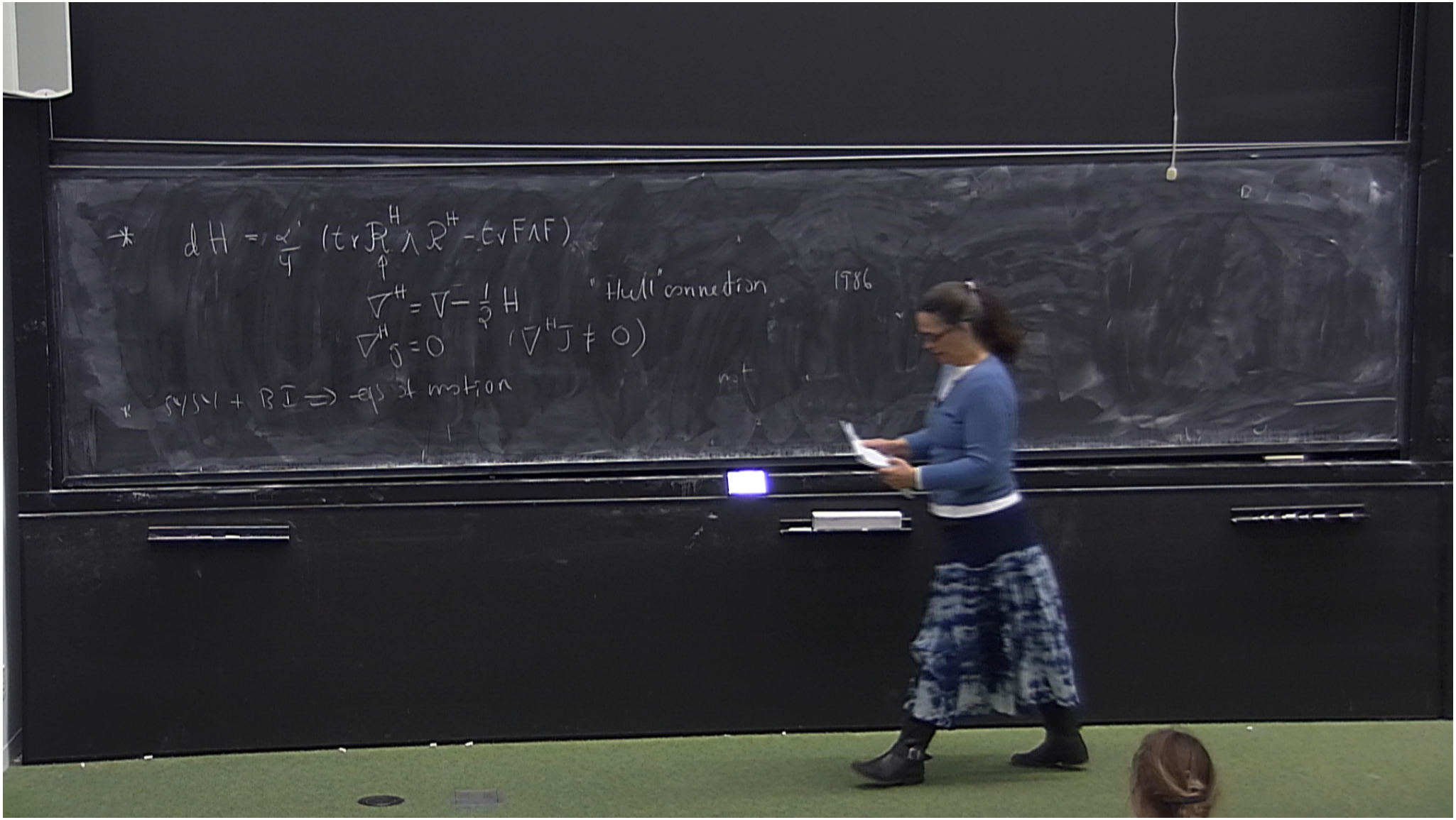

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$\mathcal{R}^{\beta} = 0$
$\nabla^{\beta} = \nabla + \frac{1}{2} H$

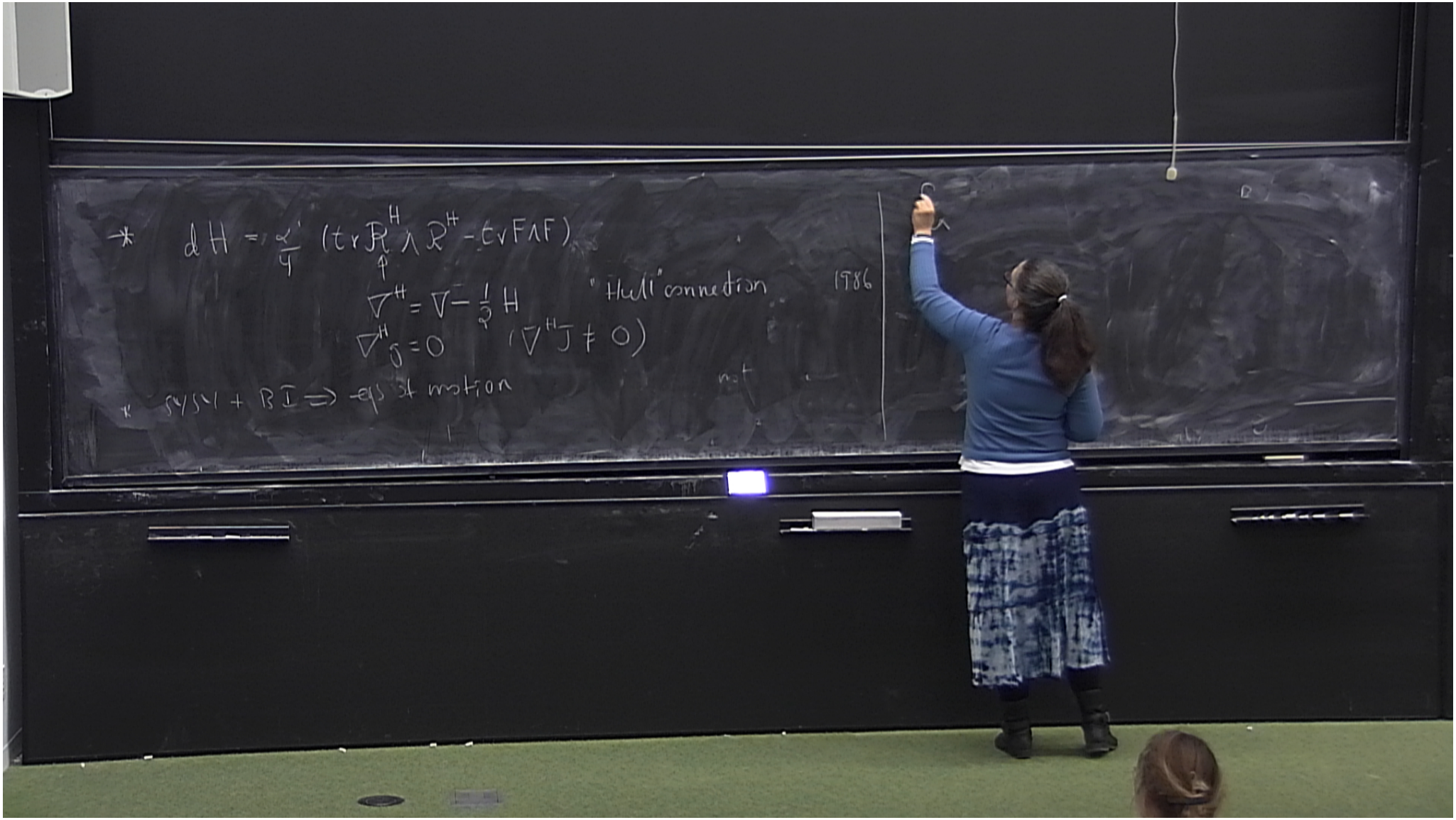

$* \quad d H = \frac{\alpha'}{4} (\mathrm{tr} R^H \wedge R^H - \mathrm{tr} F \wedge F)$

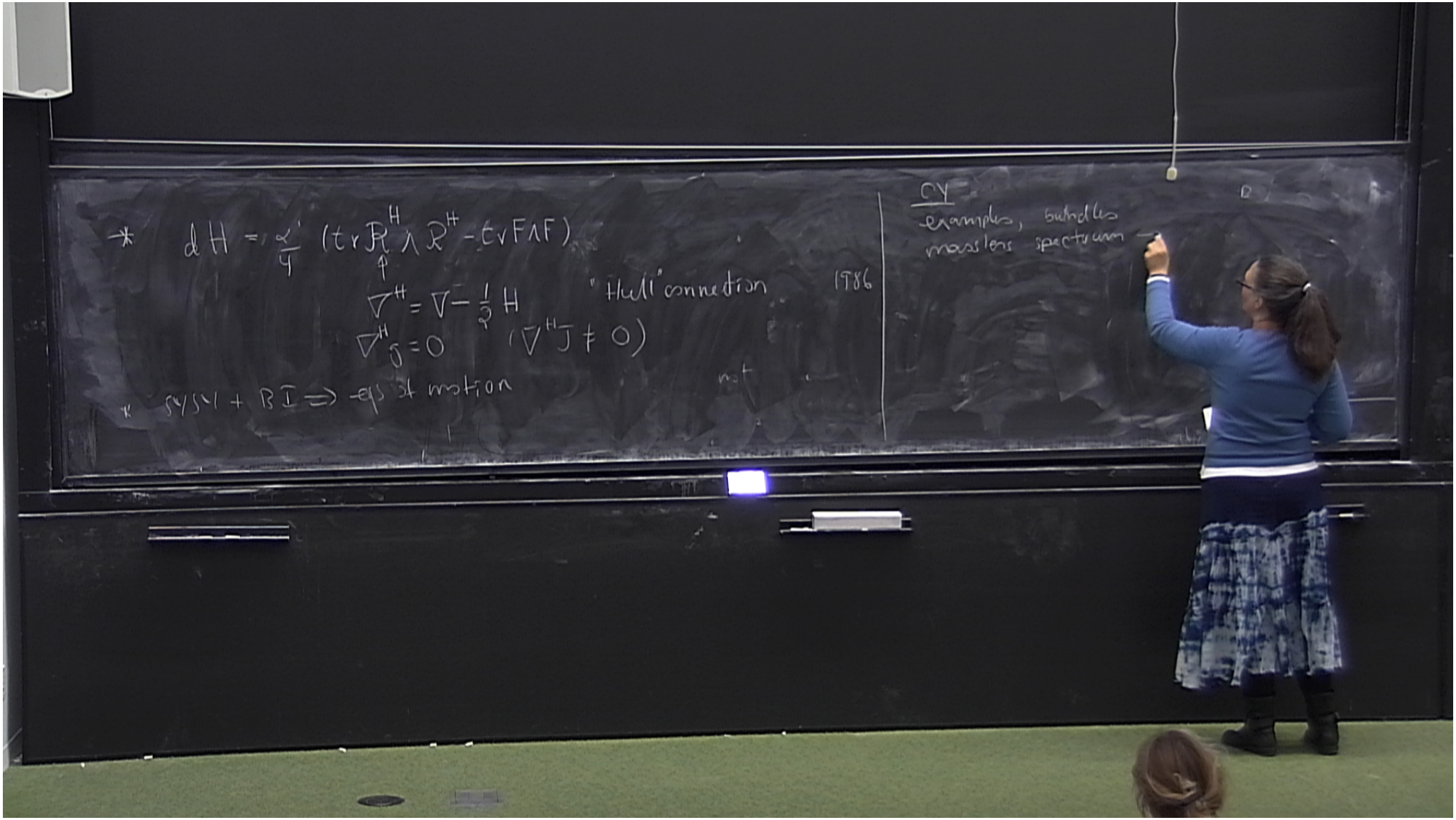

$* \quad dH = \frac{\alpha'}{4}(\mathrm{tr}\, R^H \wedge R^H - \mathrm{tr}\, F \wedge F)$
$\nabla^H = \nabla - \frac{1}{2}H$
"Hull" connection

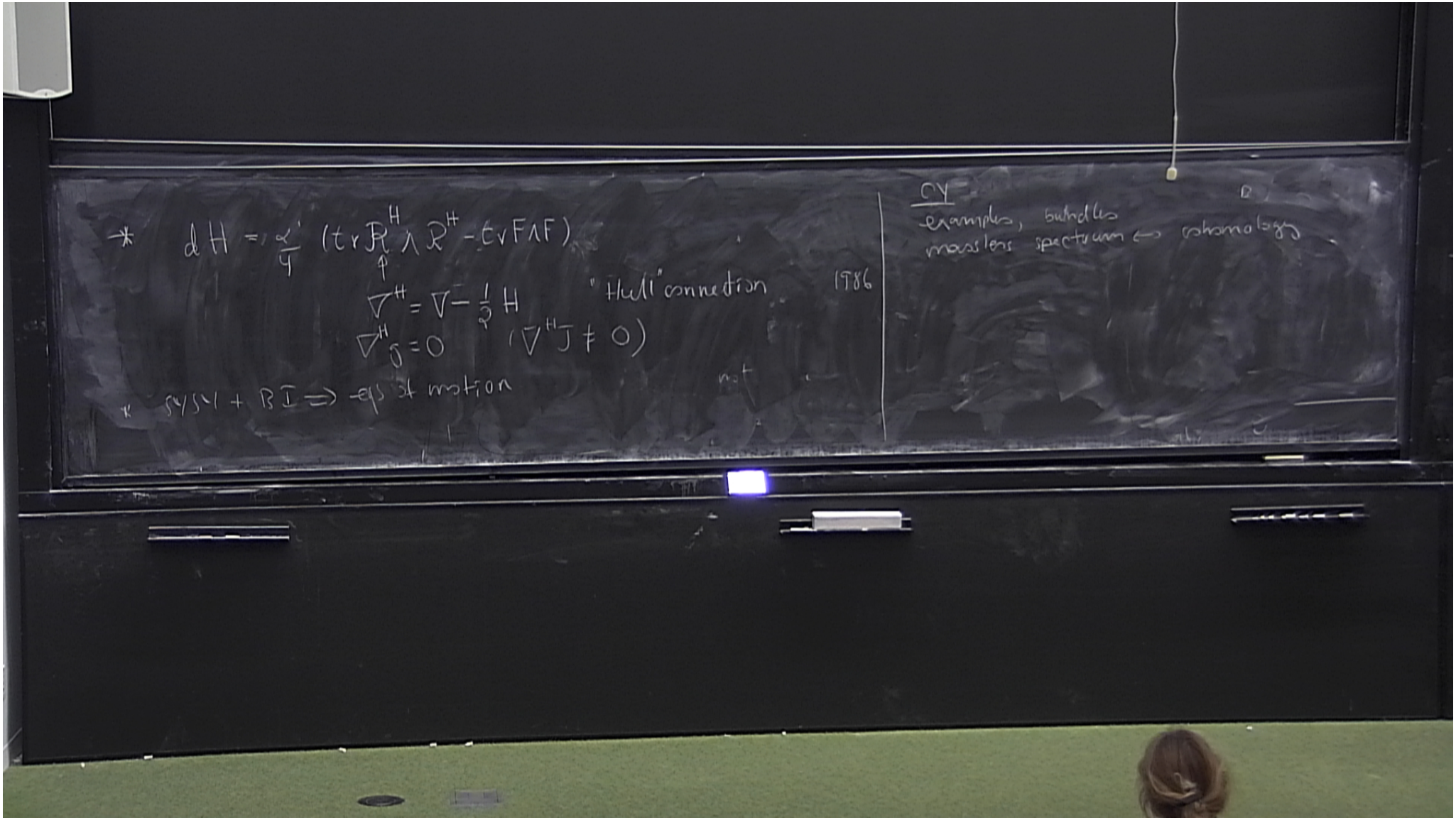

"Hull" connection
1986

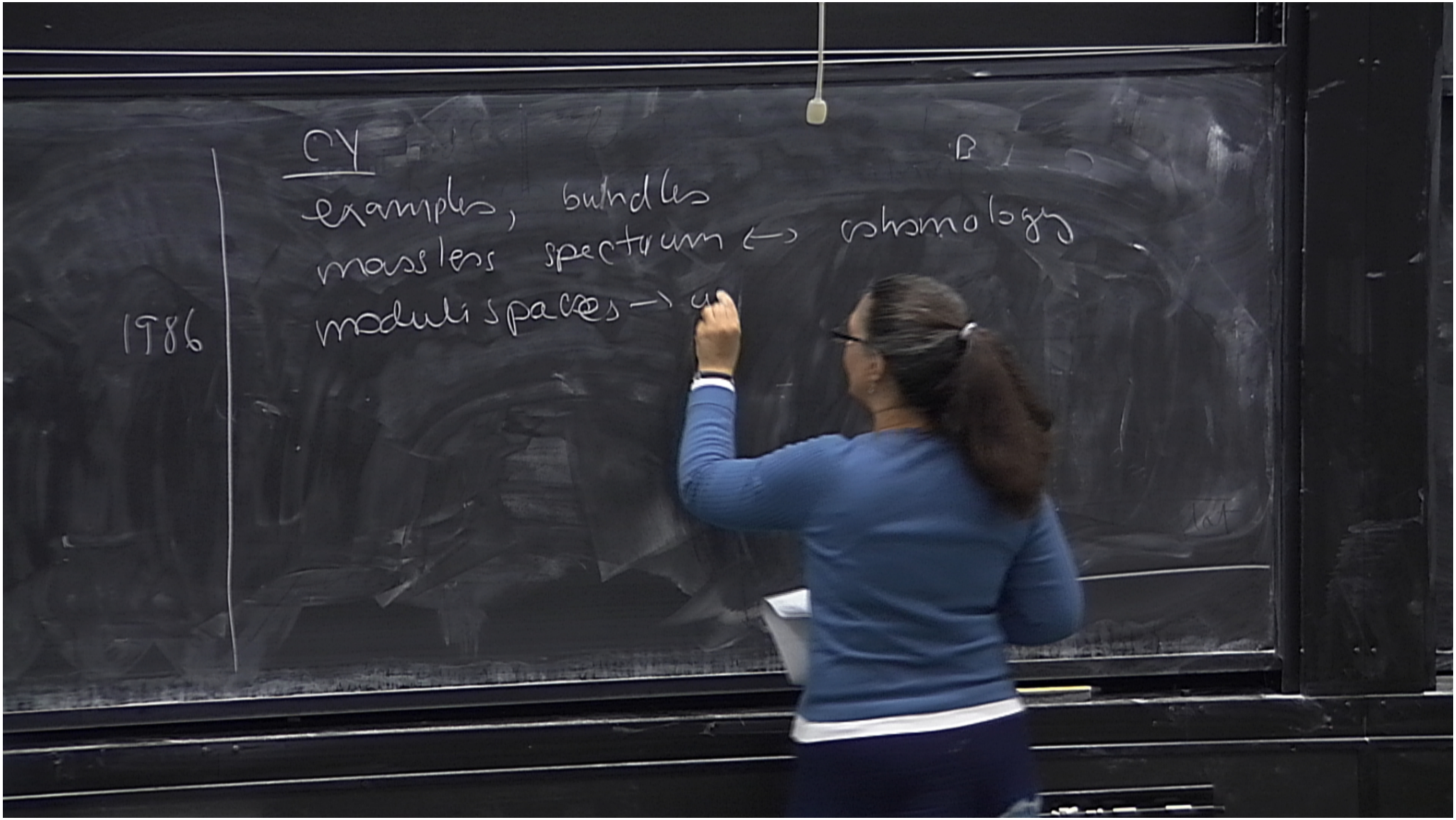

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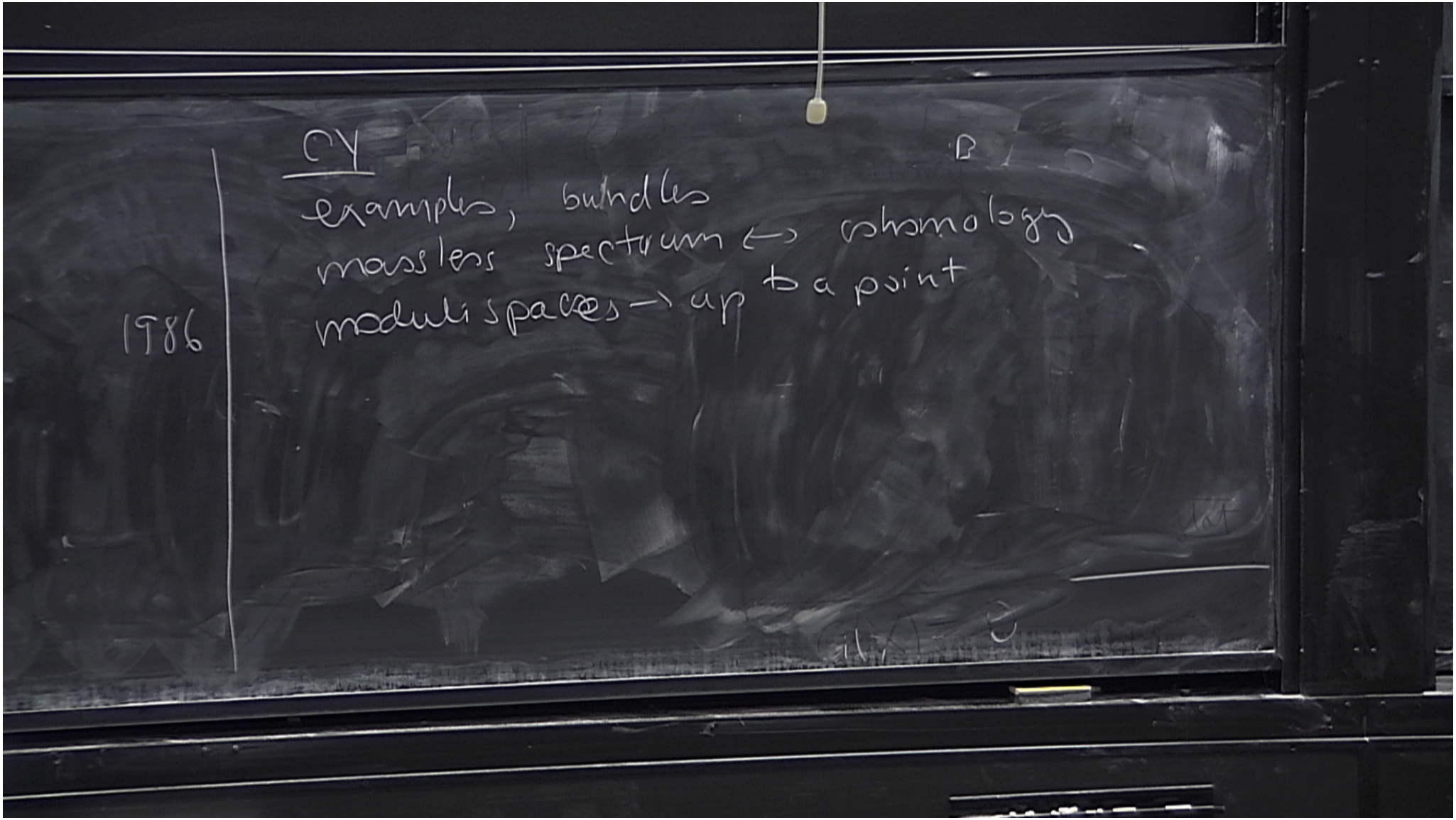

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* SUSY + BI $\Rightarrow$ eqs of motion

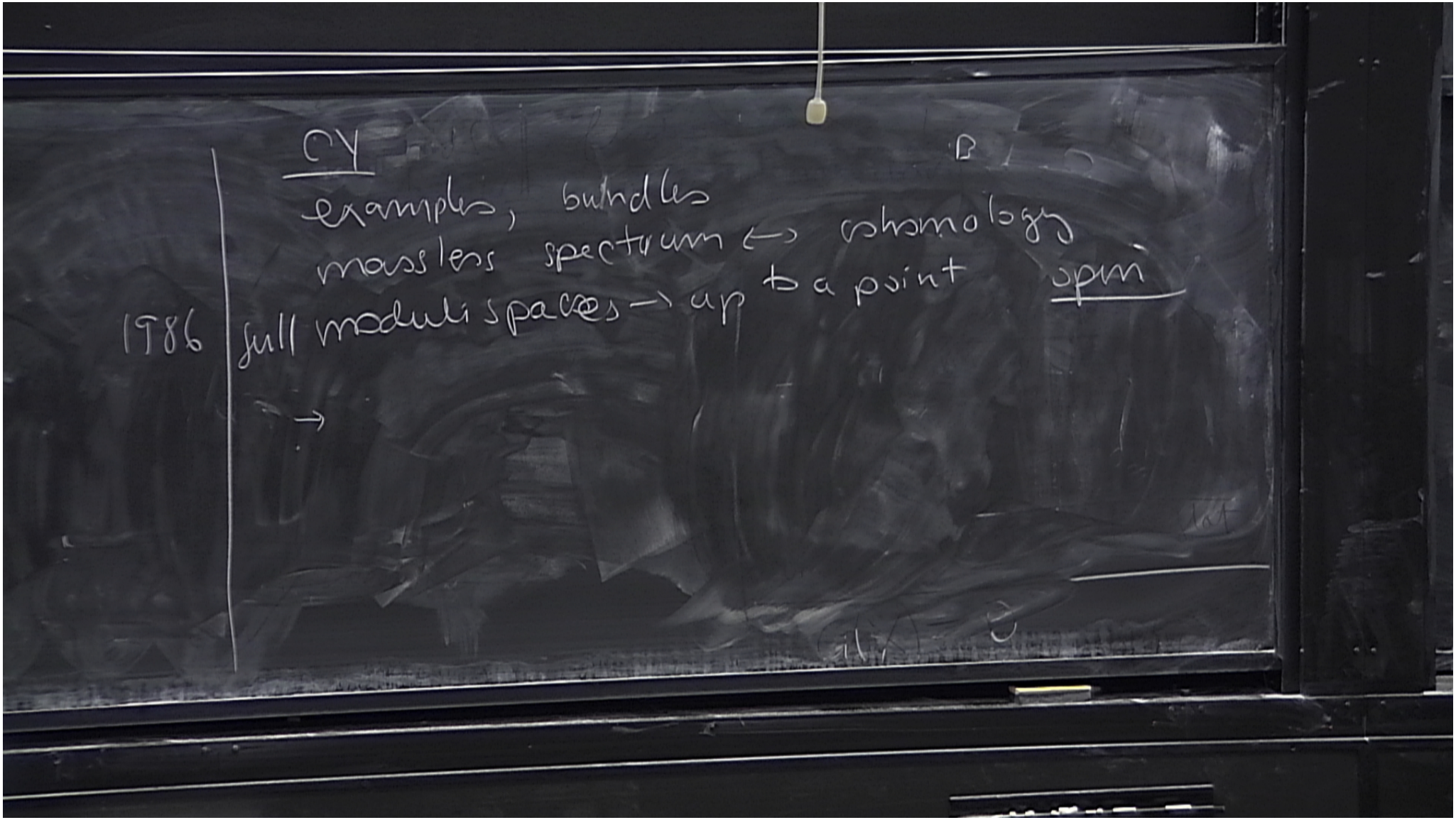

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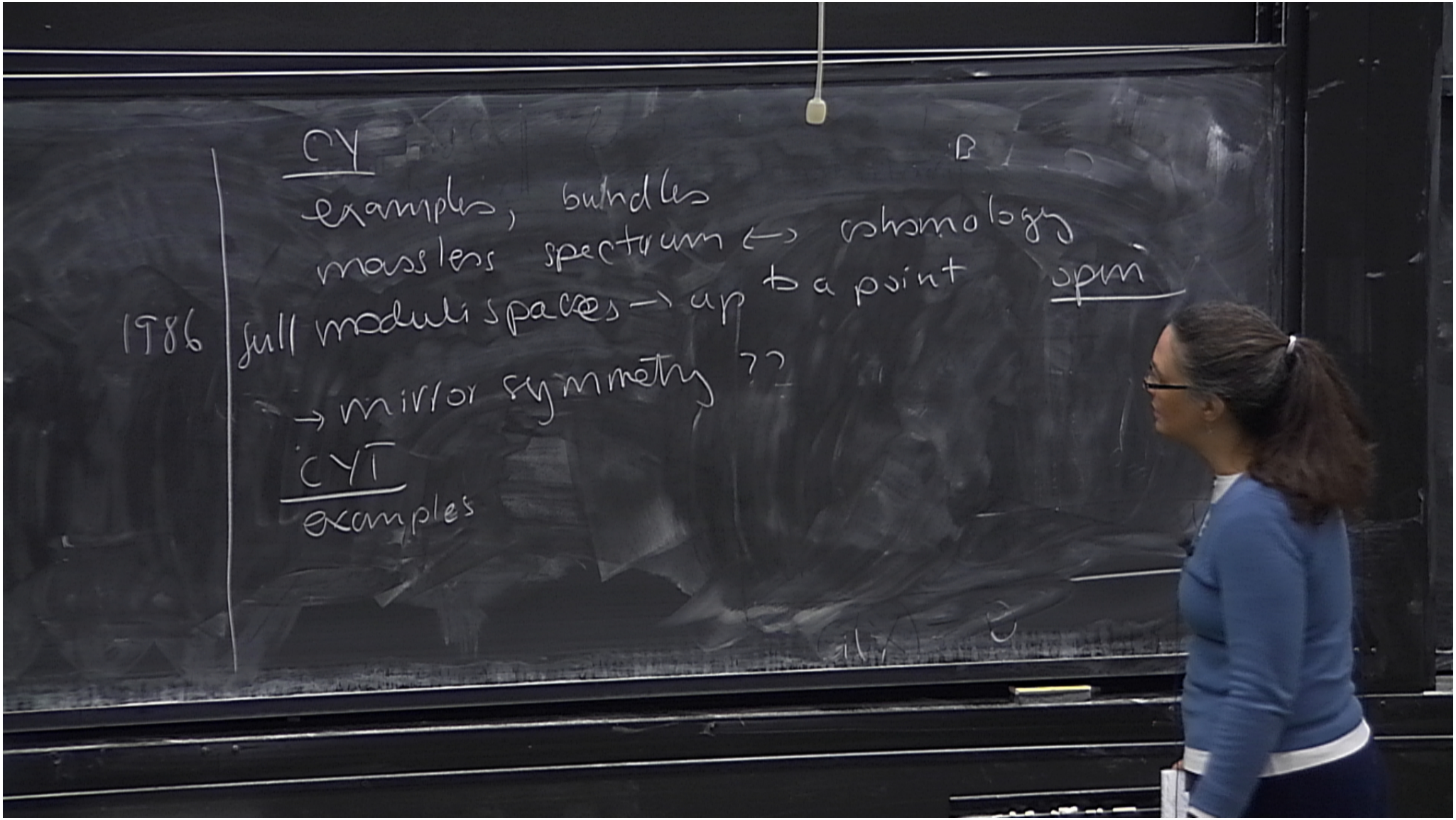

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$*$ SUSY + BI $\Rightarrow$ eqs of motion
CY
examples, bundles
massless spectrum

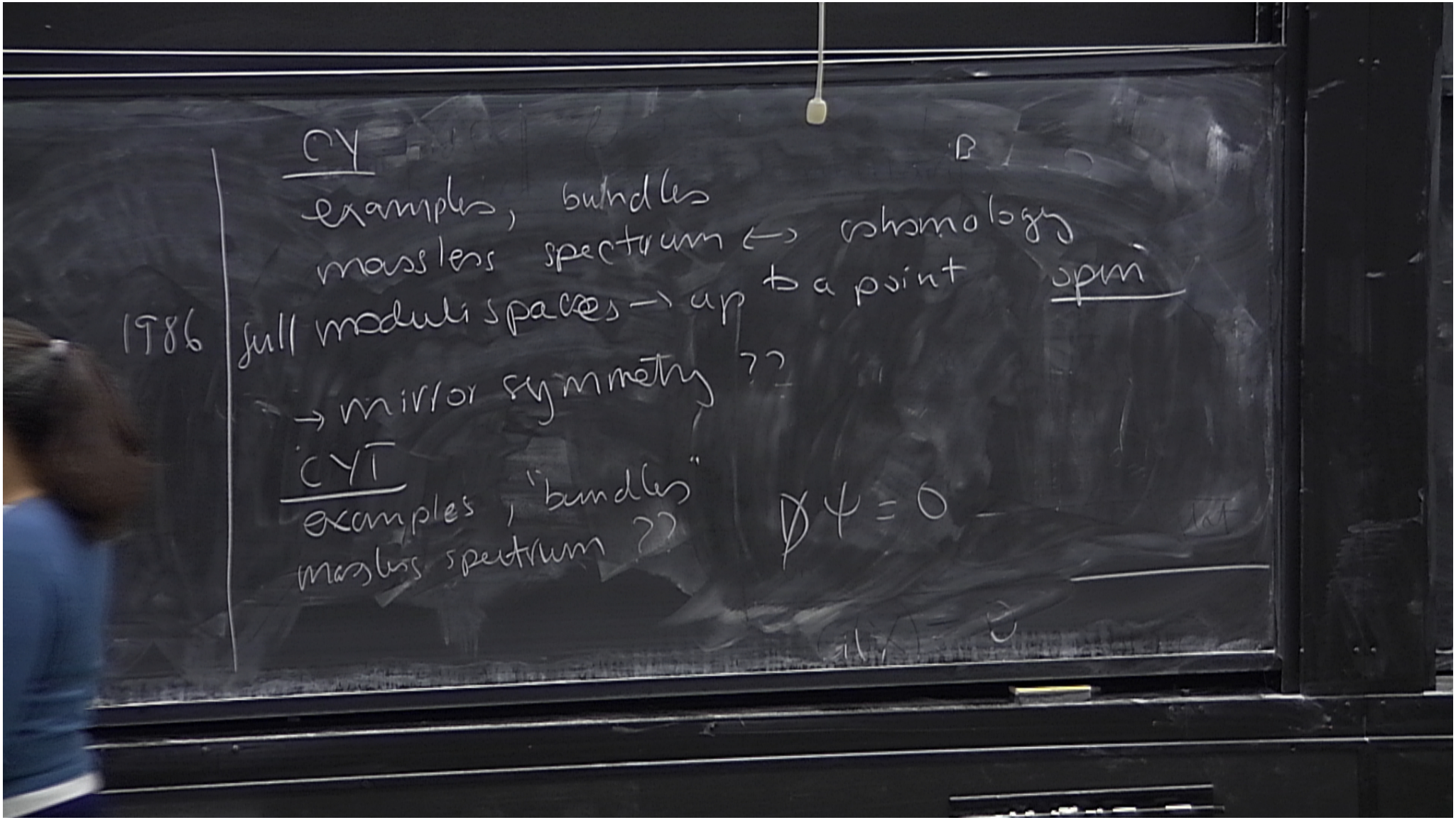

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* SUSY + BI ⇒ eqs of motion
not
CY
examples, butterflies
massless spectrum ↔ cohomology

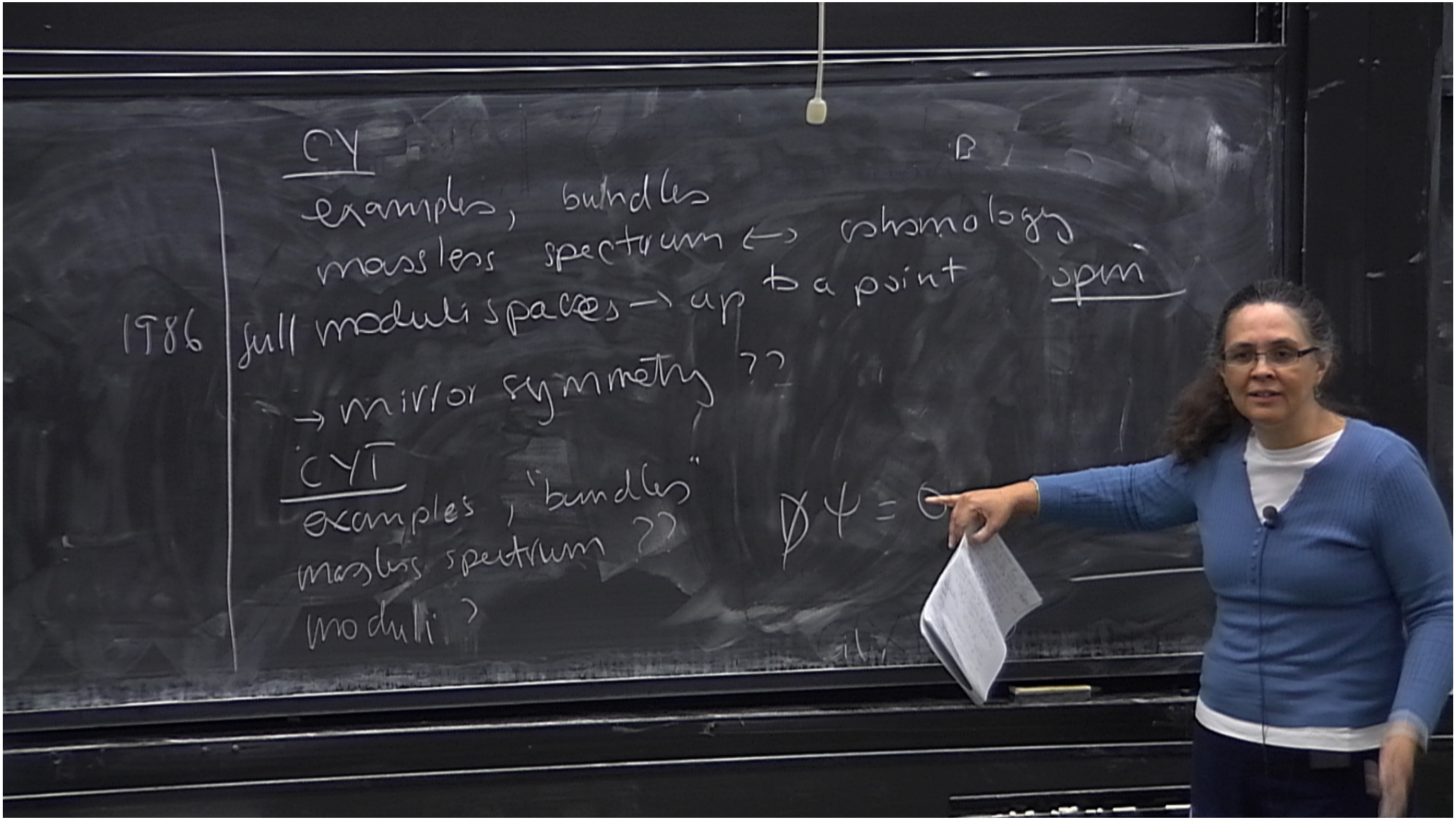

CY
examples, bundles
massless spectrum ↔ cohomology
moduli spaces →
1986

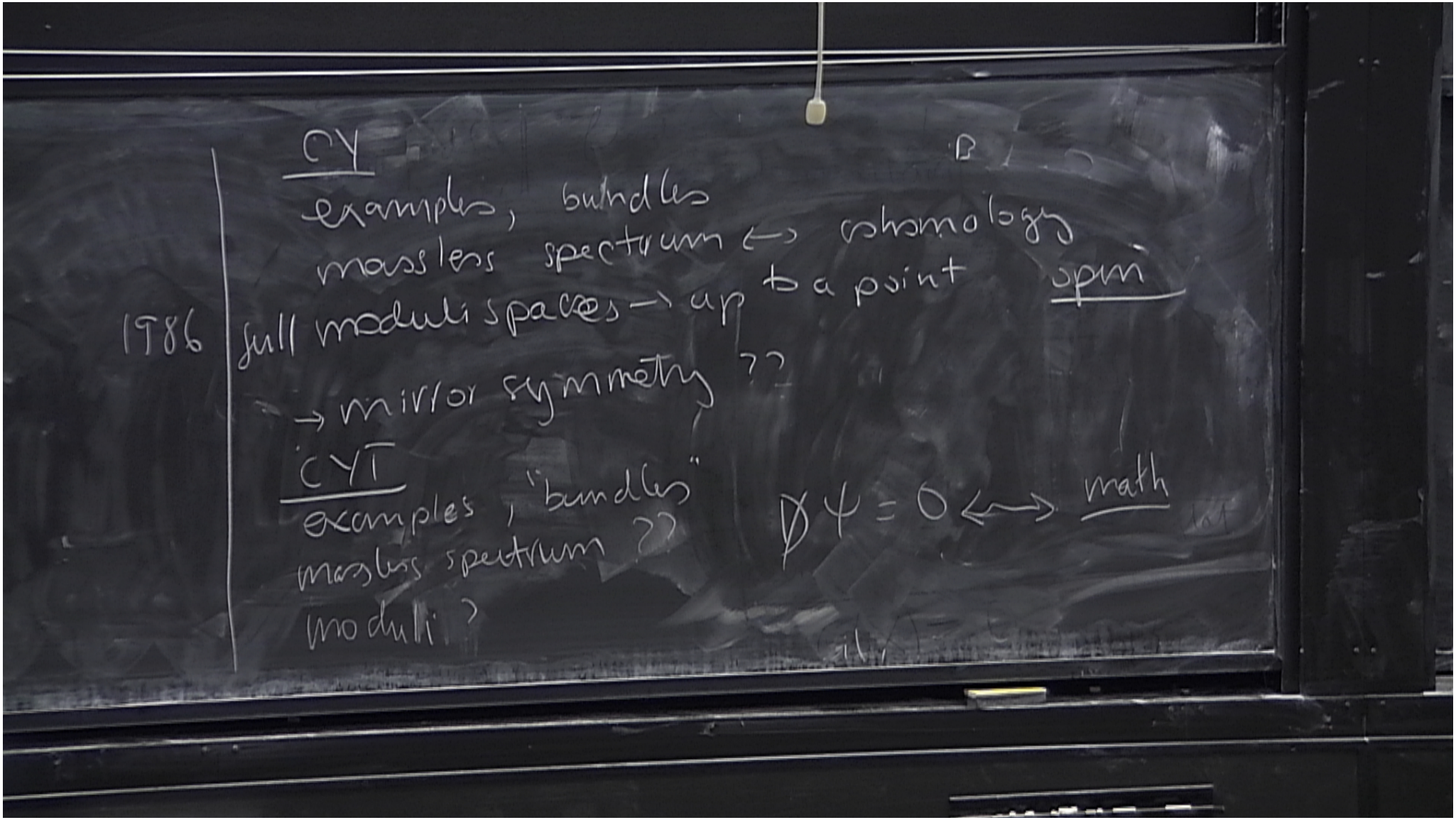

CY
examples, bundles
massless spectrum ↔ cohomology
moduli spaces → up to a point
1986

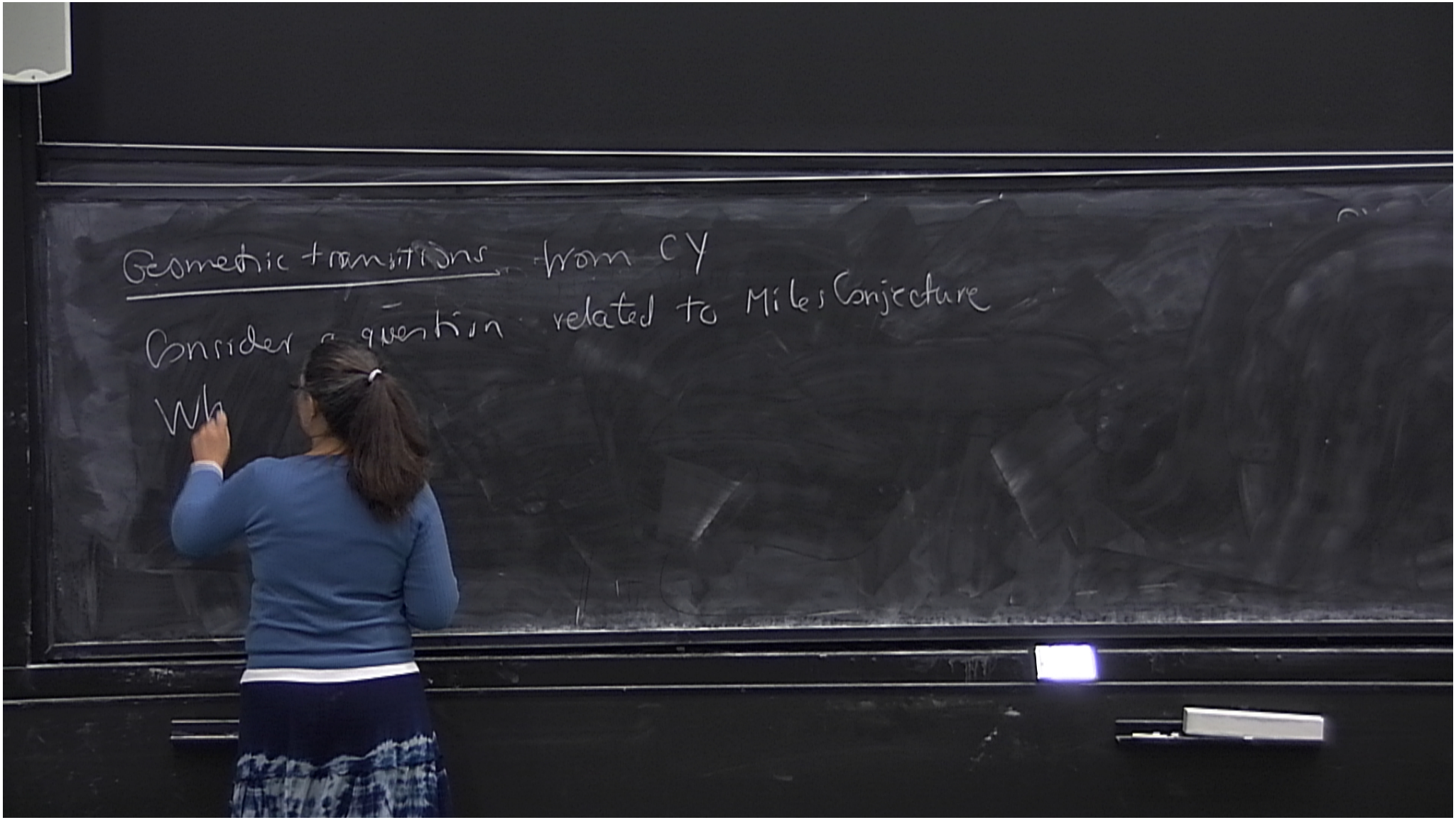

CY
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massless spectrum ↔ cohomology
1986 full moduli spaces → up to a point
susy

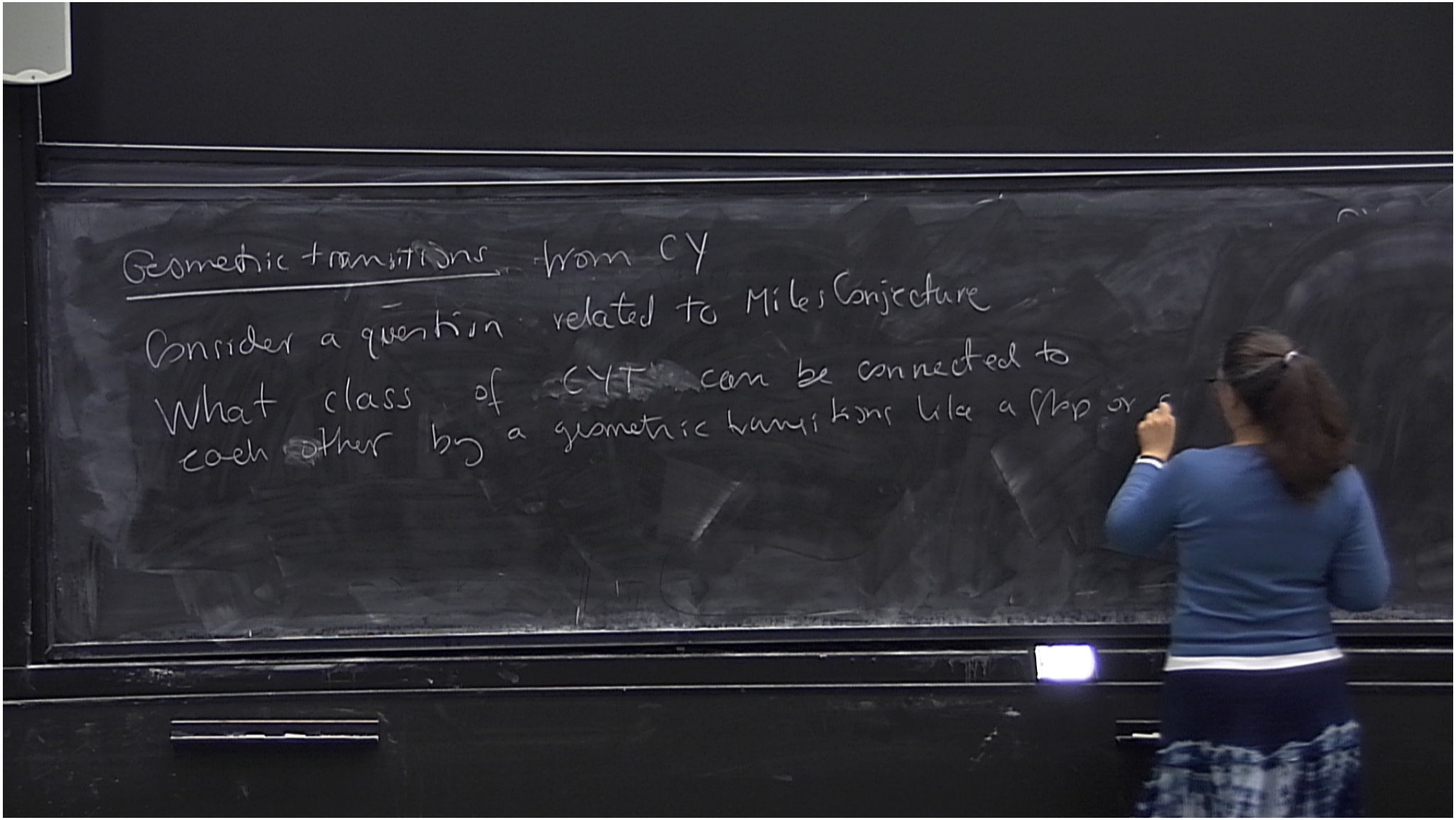

CY
examples, bundles
massless spectrum ↔ cohomology
1986 full moduli spaces → up to a point
spin
→ mirror symmetry ??
CYT
examples

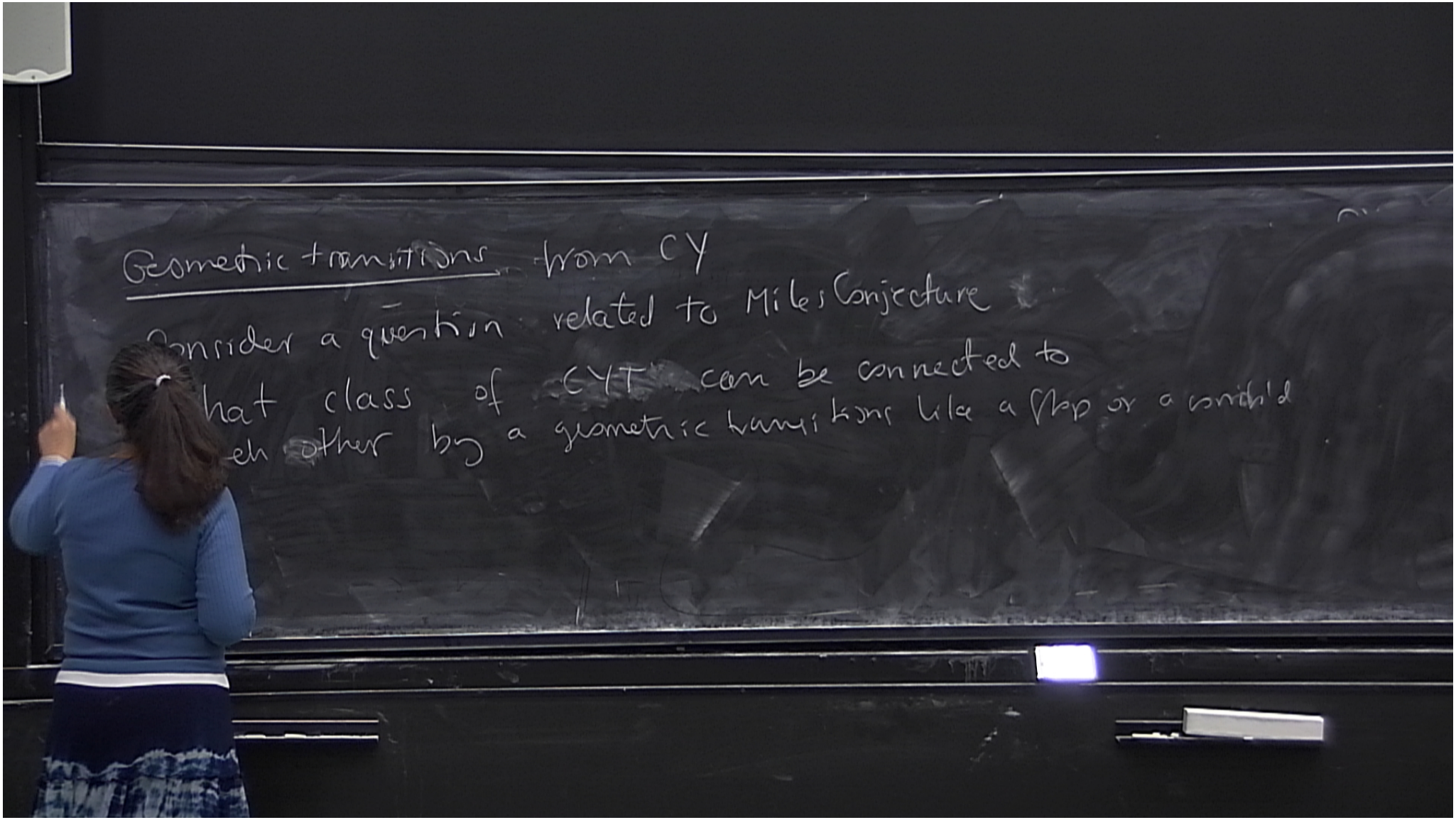

CY
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CYT
examples, "bundles"
massless spectrum ??
∂̸ψ = 0

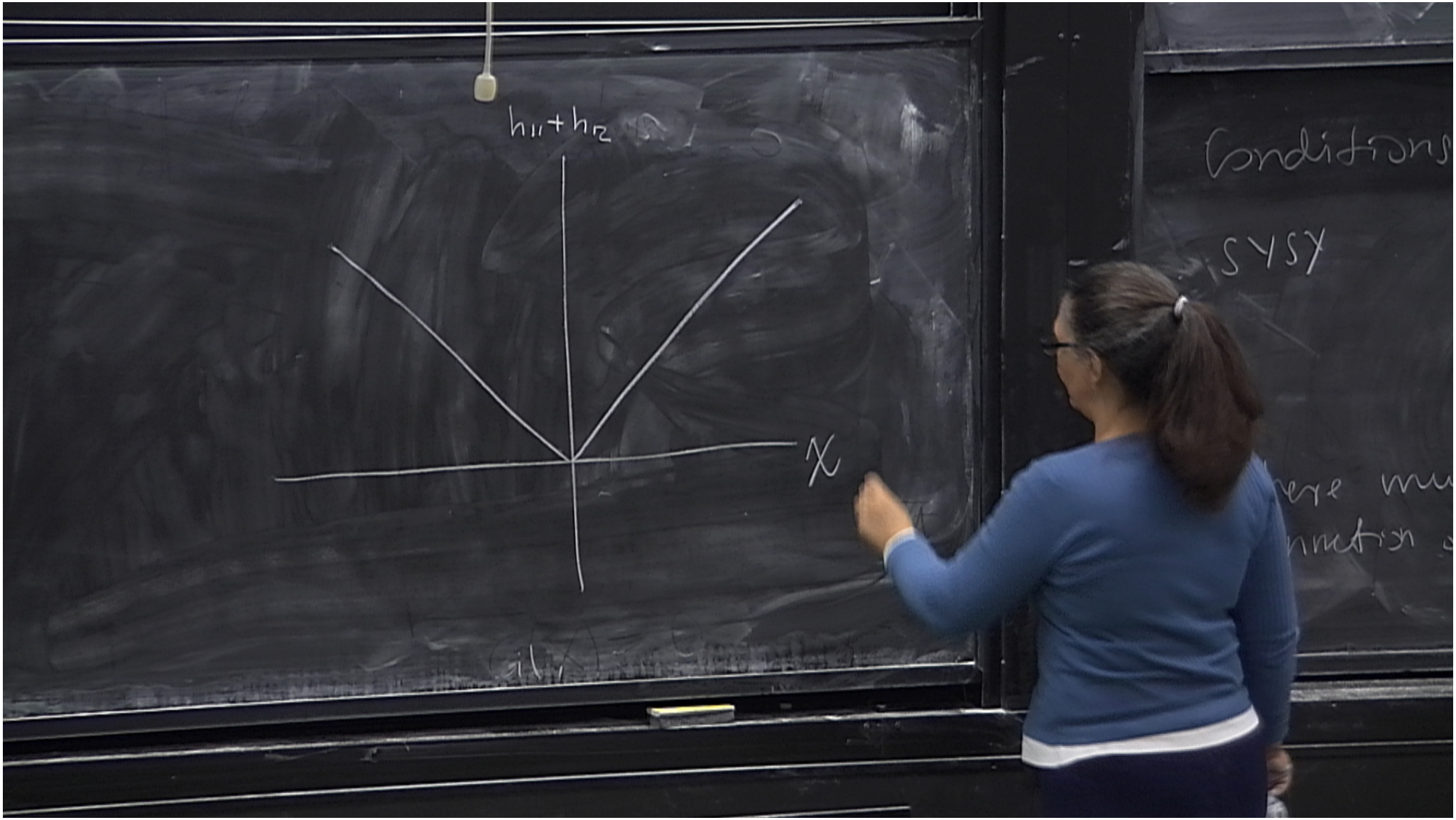

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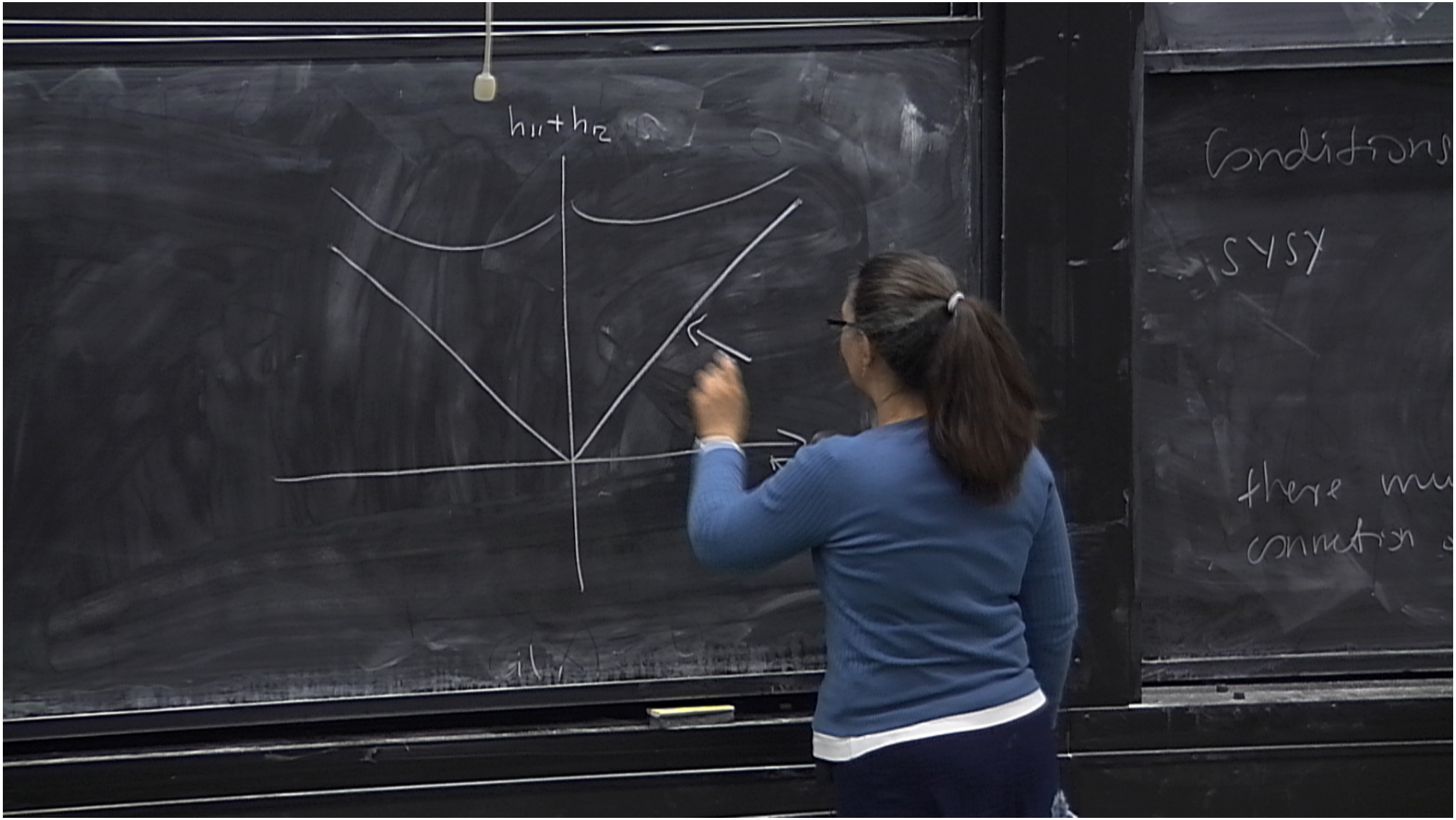

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CYT
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massless spectrum ??
moduli ?
$\not{D}\psi = 0$ ↔ math

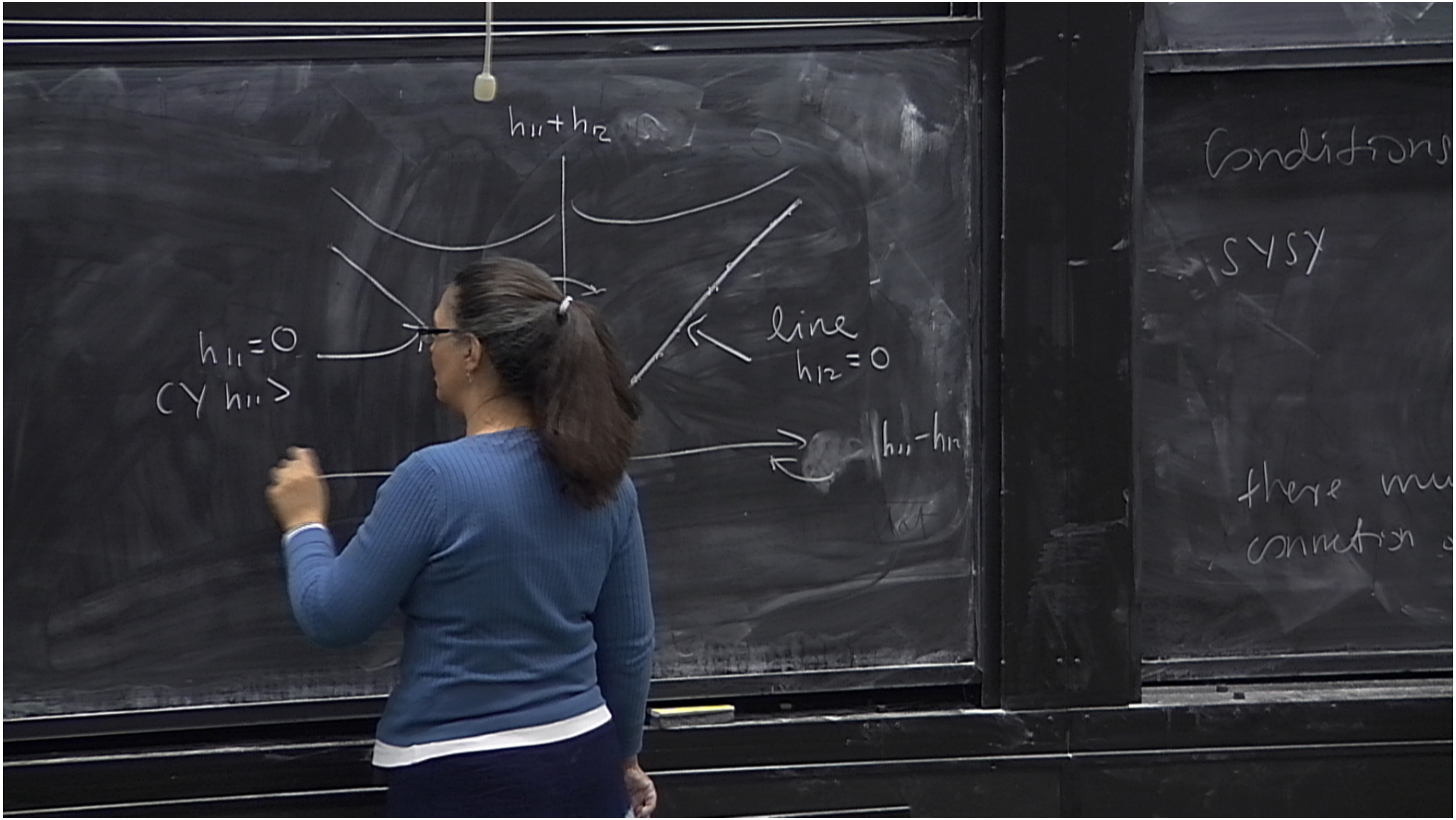

Geometric transitions from CY
related to Miles Conjecture

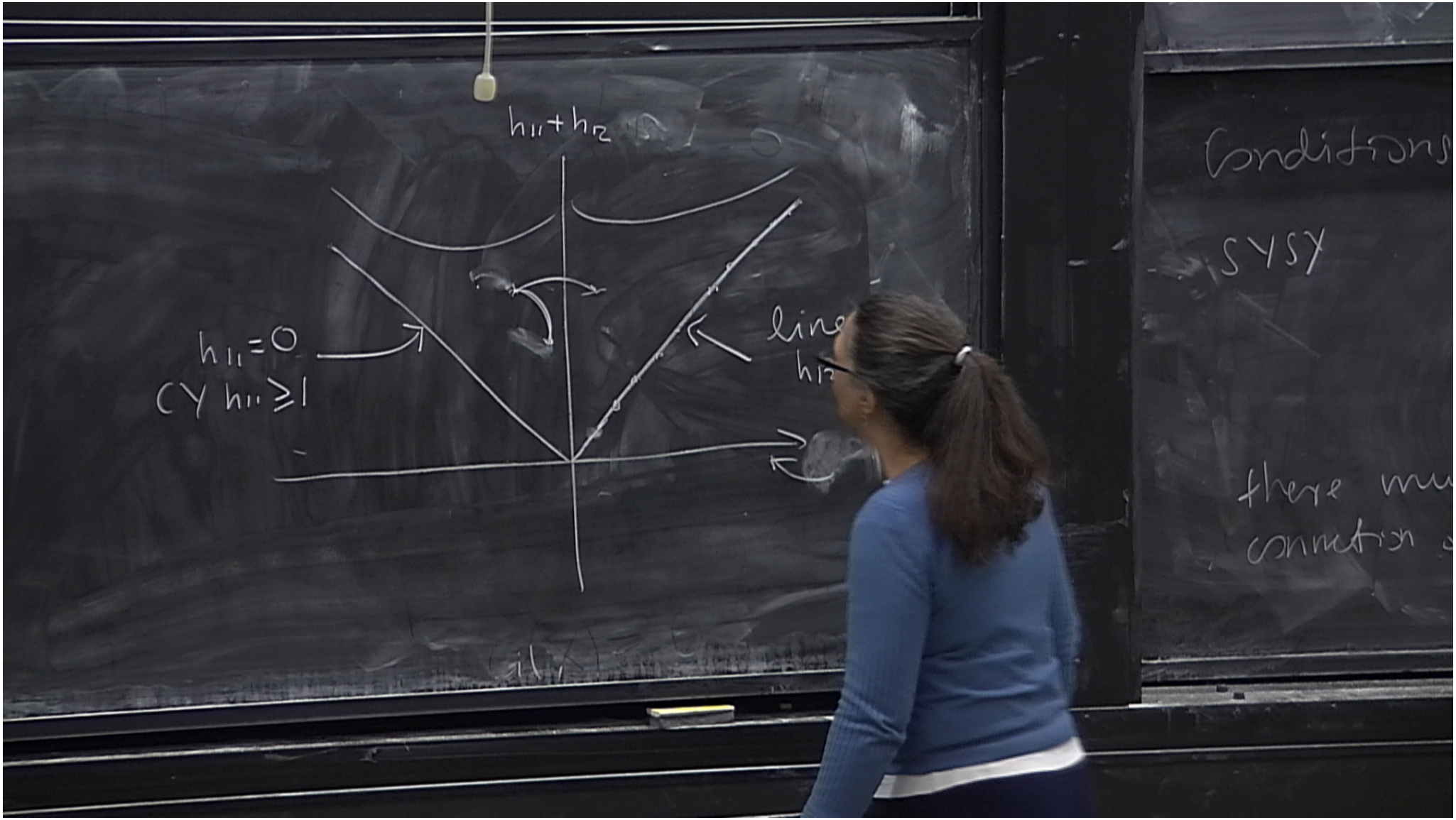

Geometric transitions from CY
Consider a question related to Miles Conjecture
What class of CYT can be connected to
each other by a geometric transition like a flop or

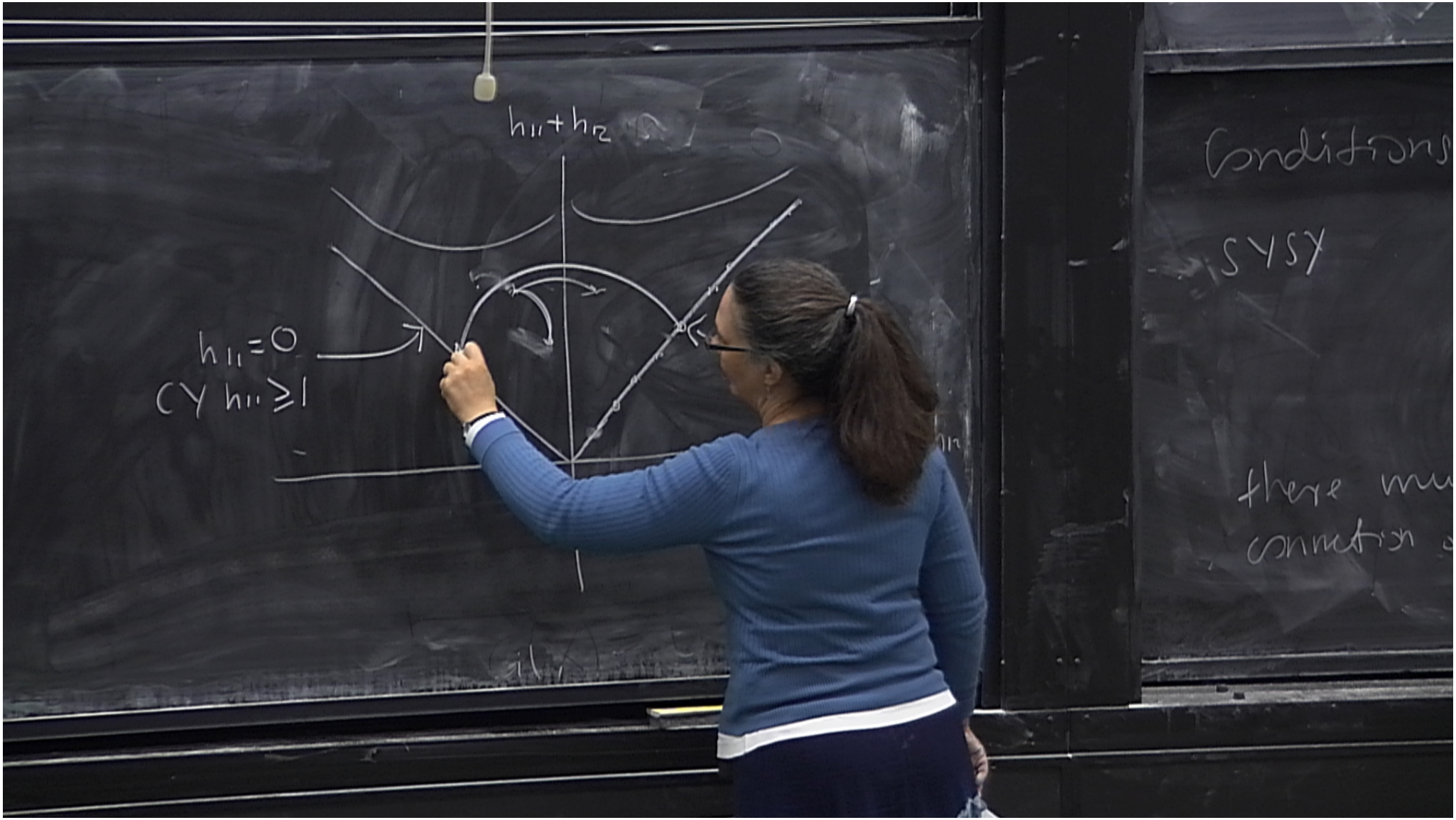

Geometric transitions from CY
Consider a question related to Miles Conjecture
What class of CYT can be connected to
each other by a geometric transition like a flop or a conifold

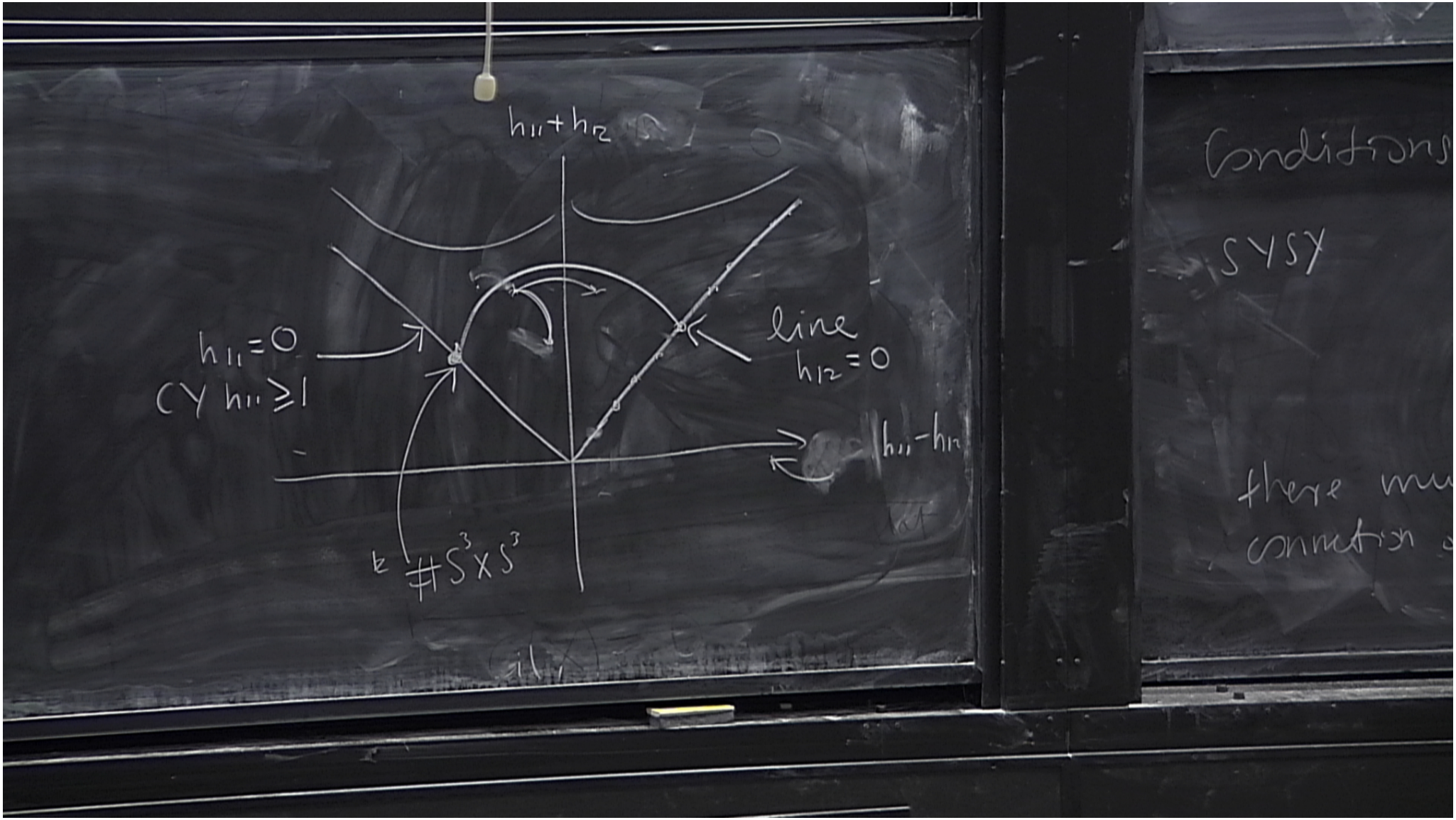

$h_1 + h_2$
χ
Conditions
SYSY

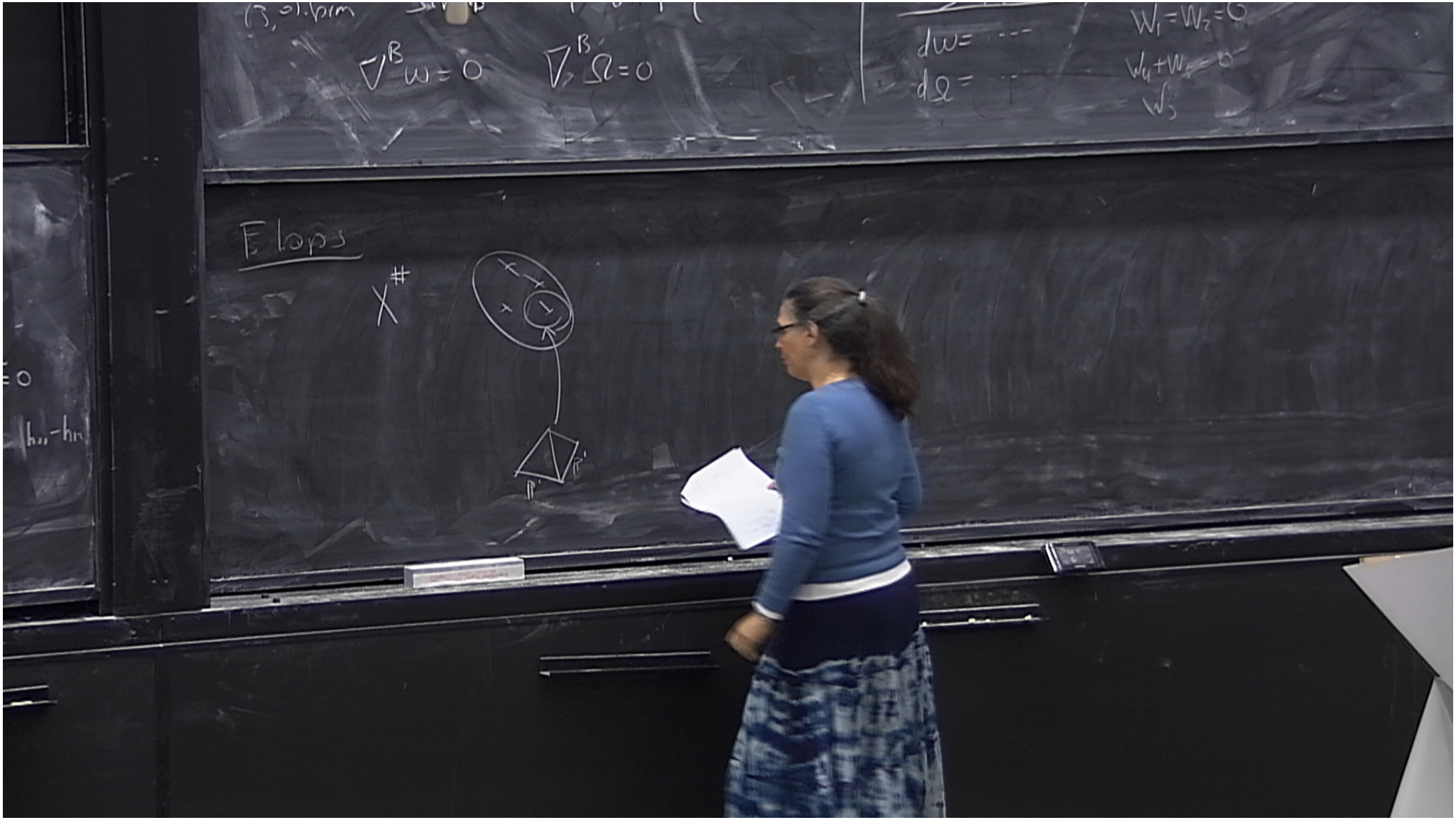

$h_1 + h_2$
conditions
SYSY
there mu
connection

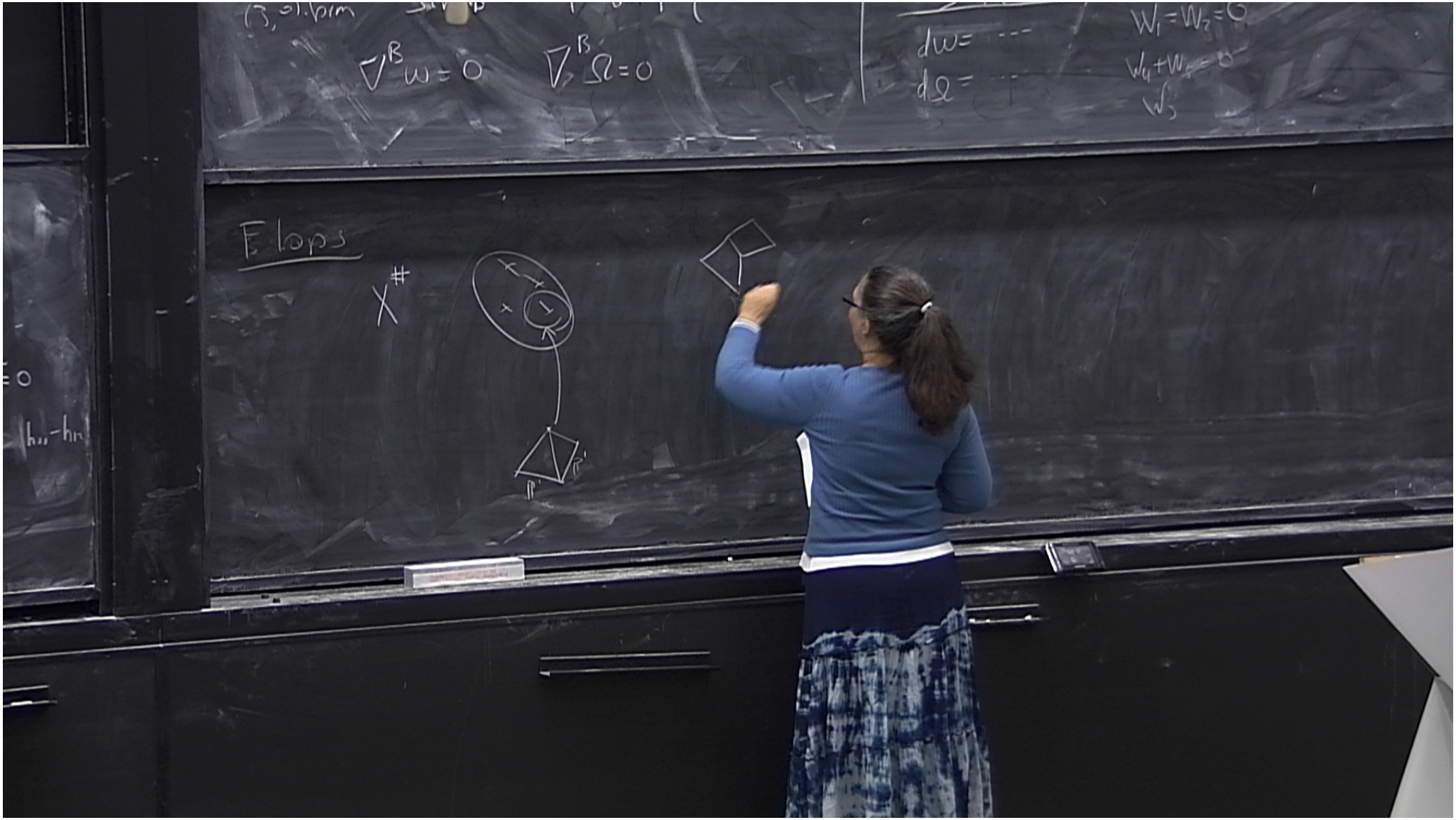

$h_{11}+h_{12}$
$h_{11}=0$
$CY\ h_{11}>$
line
$h_{12}=0$
$h_{11}-h_{12}$
Conditions
SYSY
there mu
connection

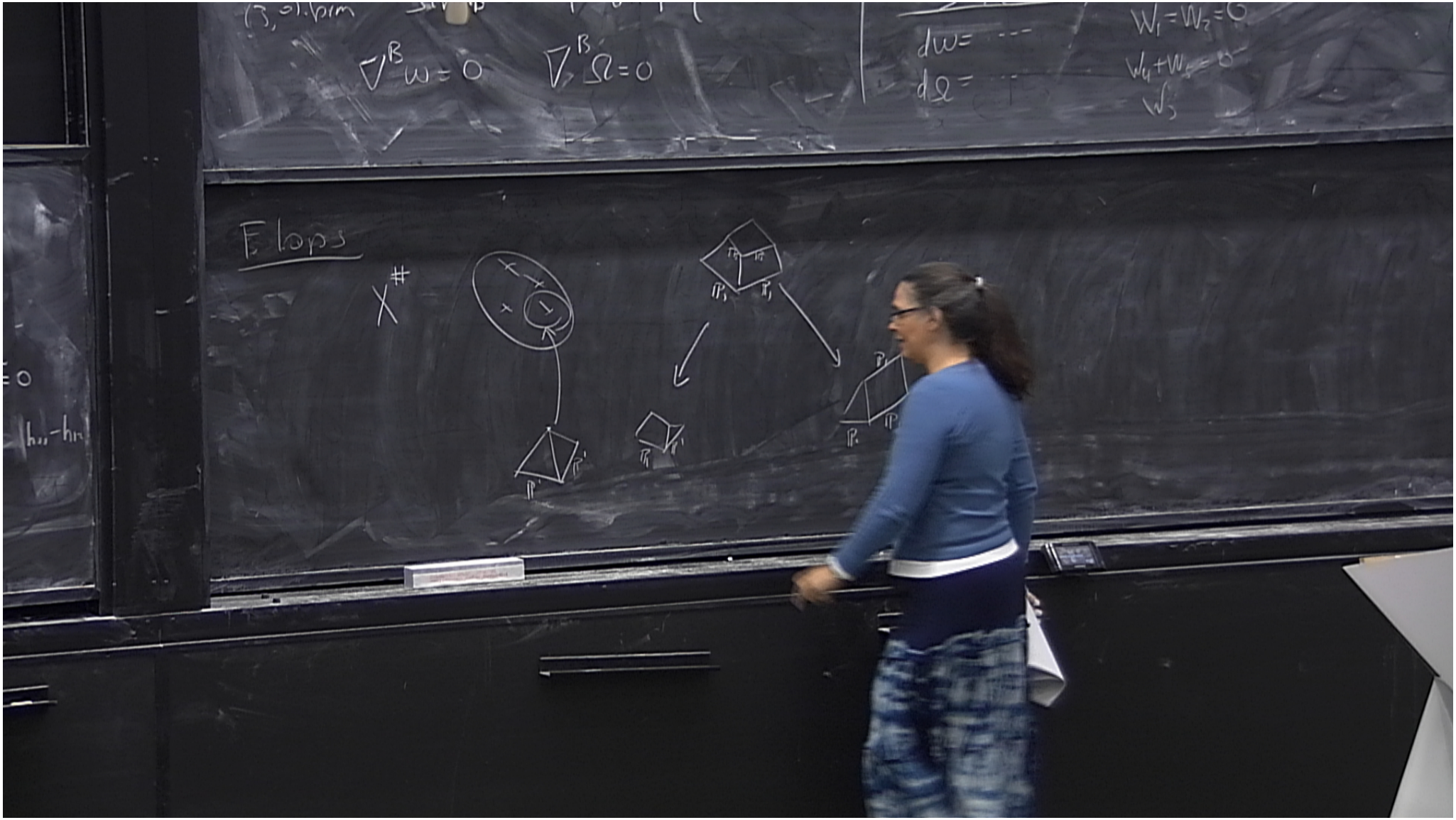

$h_{11}+h_2$
$h_{11}=0$
CY $h_{11}\geq 1$
line
conditions
SYSY
there mu
connection

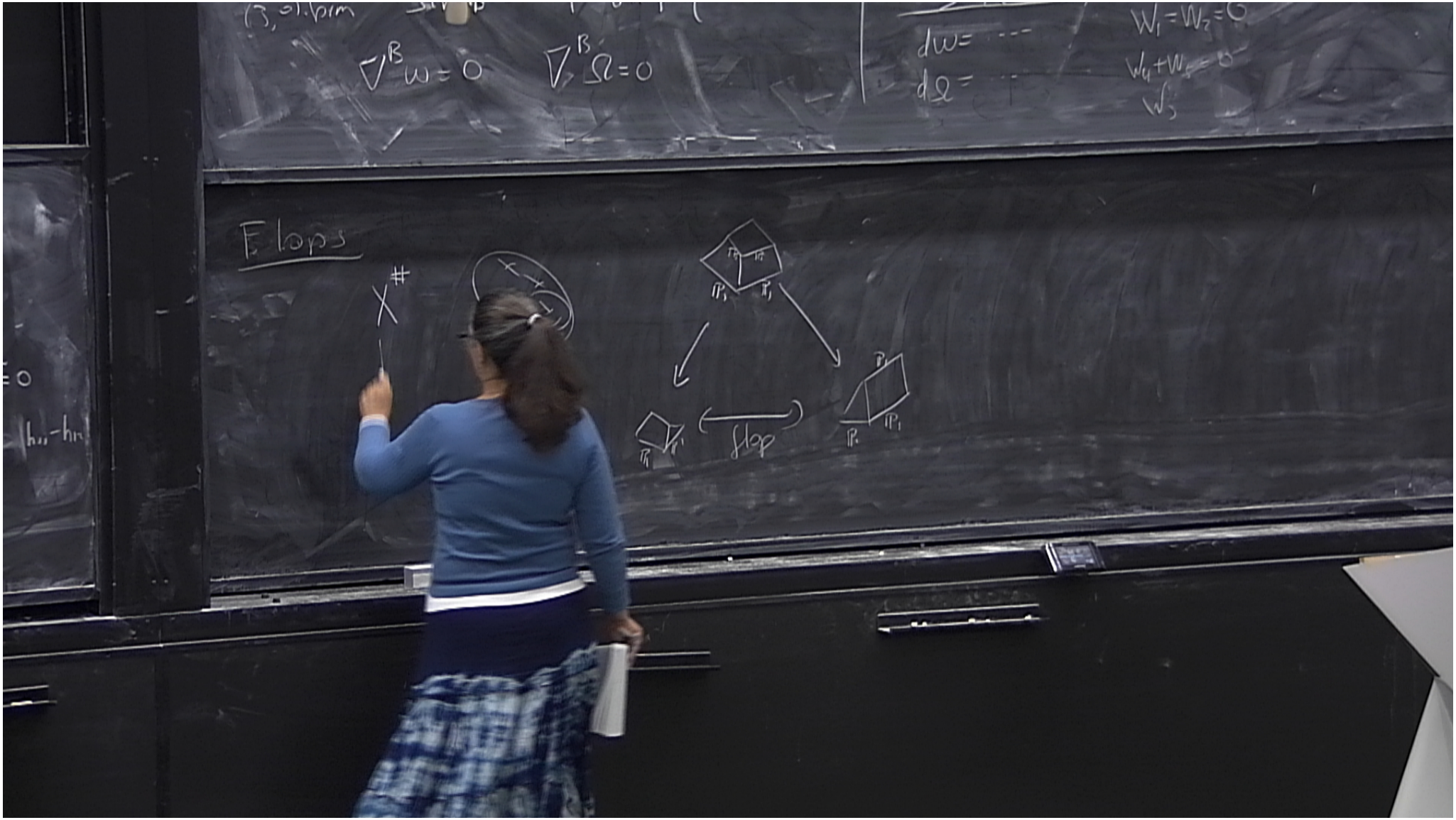

$h_{11}+h_2$
$h_{11}=0$
CY $h_{11}\geq 1$
Conditions
SYSY
there mu
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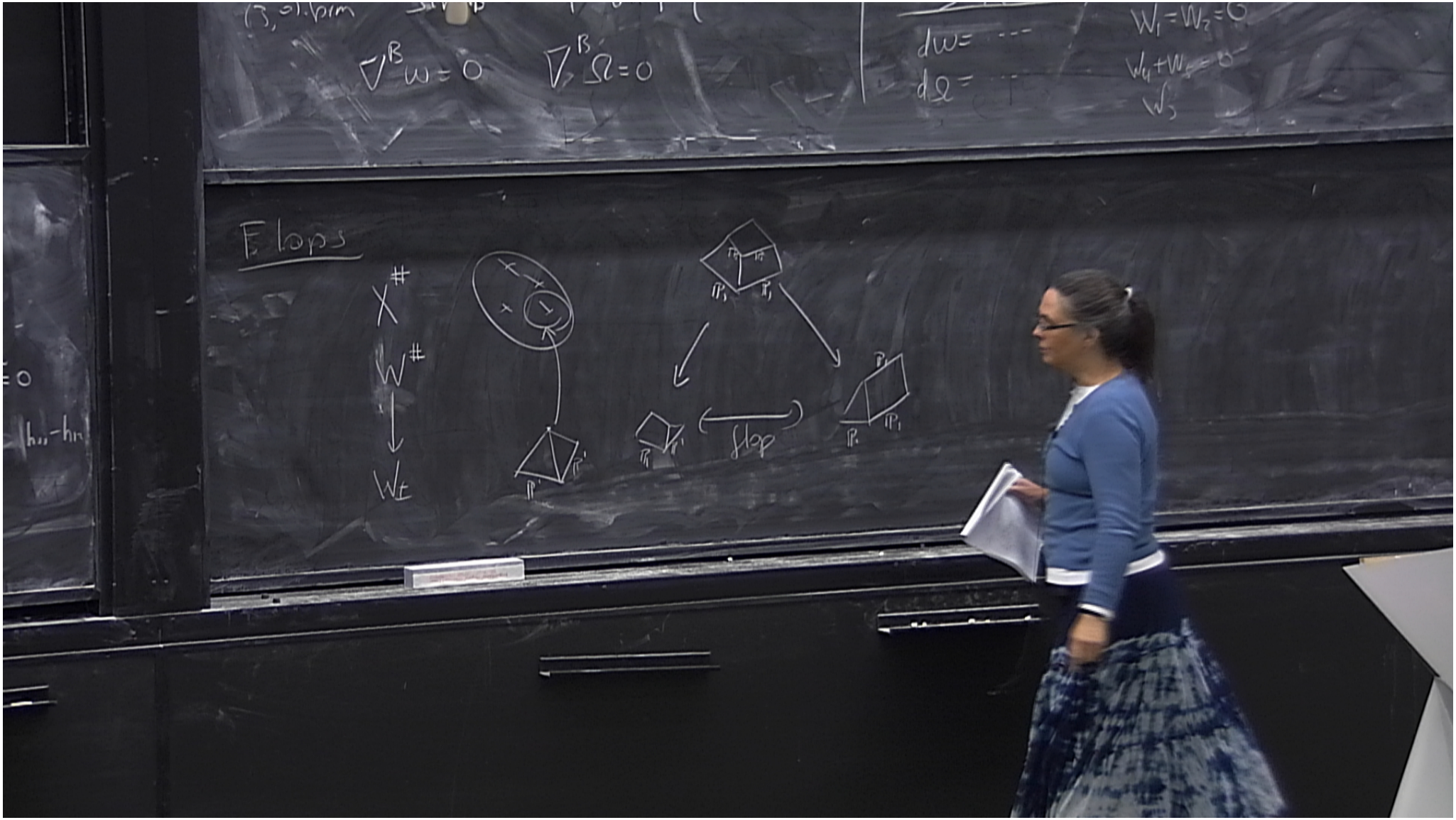

$h_{11}+h_{12}$
$h_{11}=0$
CY $h_{11}\geq 1$
line
$h_{12}=0$
$h_{11}-h_{12}$
$\#S^3\times S^3$
Conditions
SYSY
there mu
connection

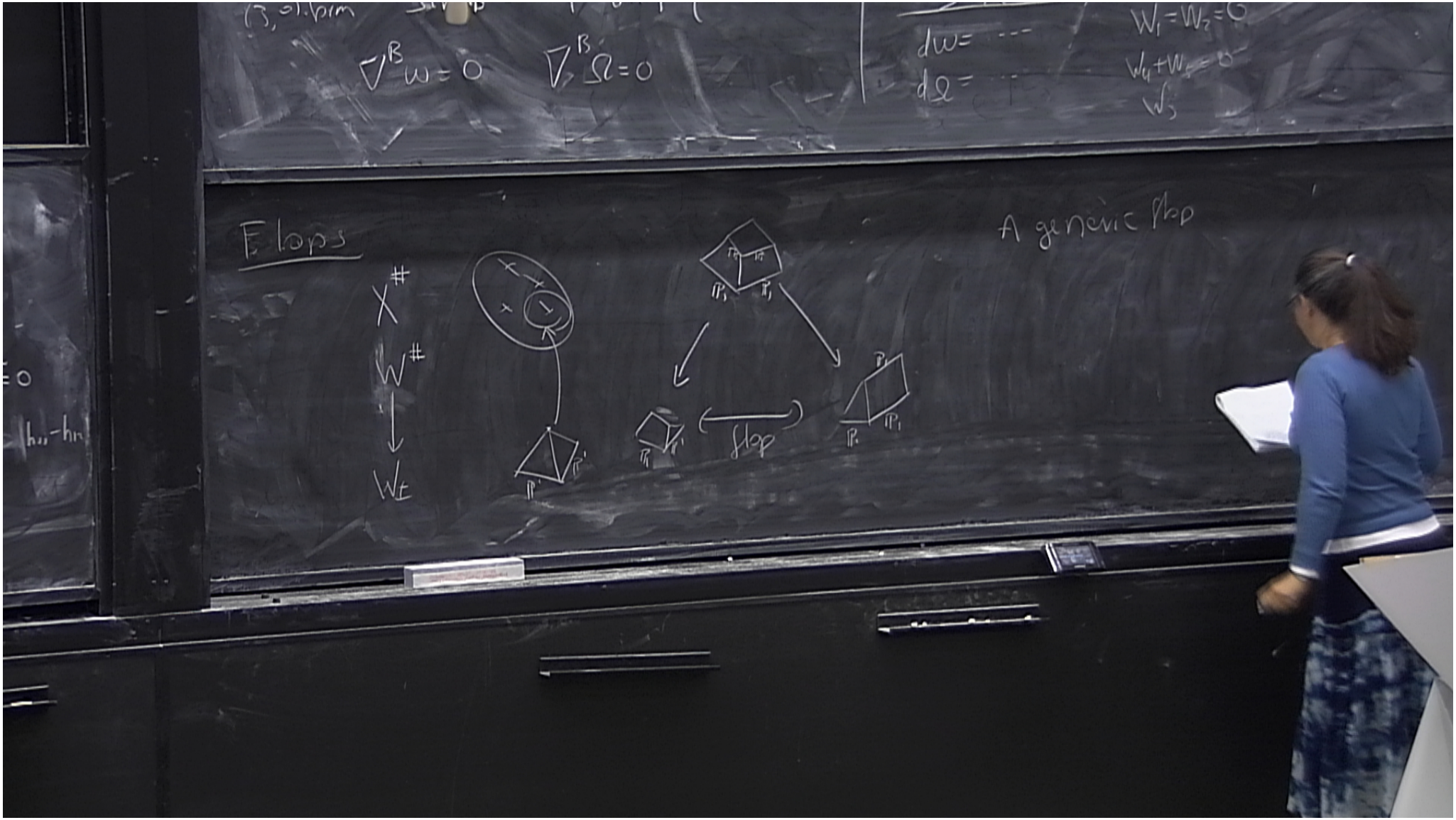

Flops
$X^{\#}$

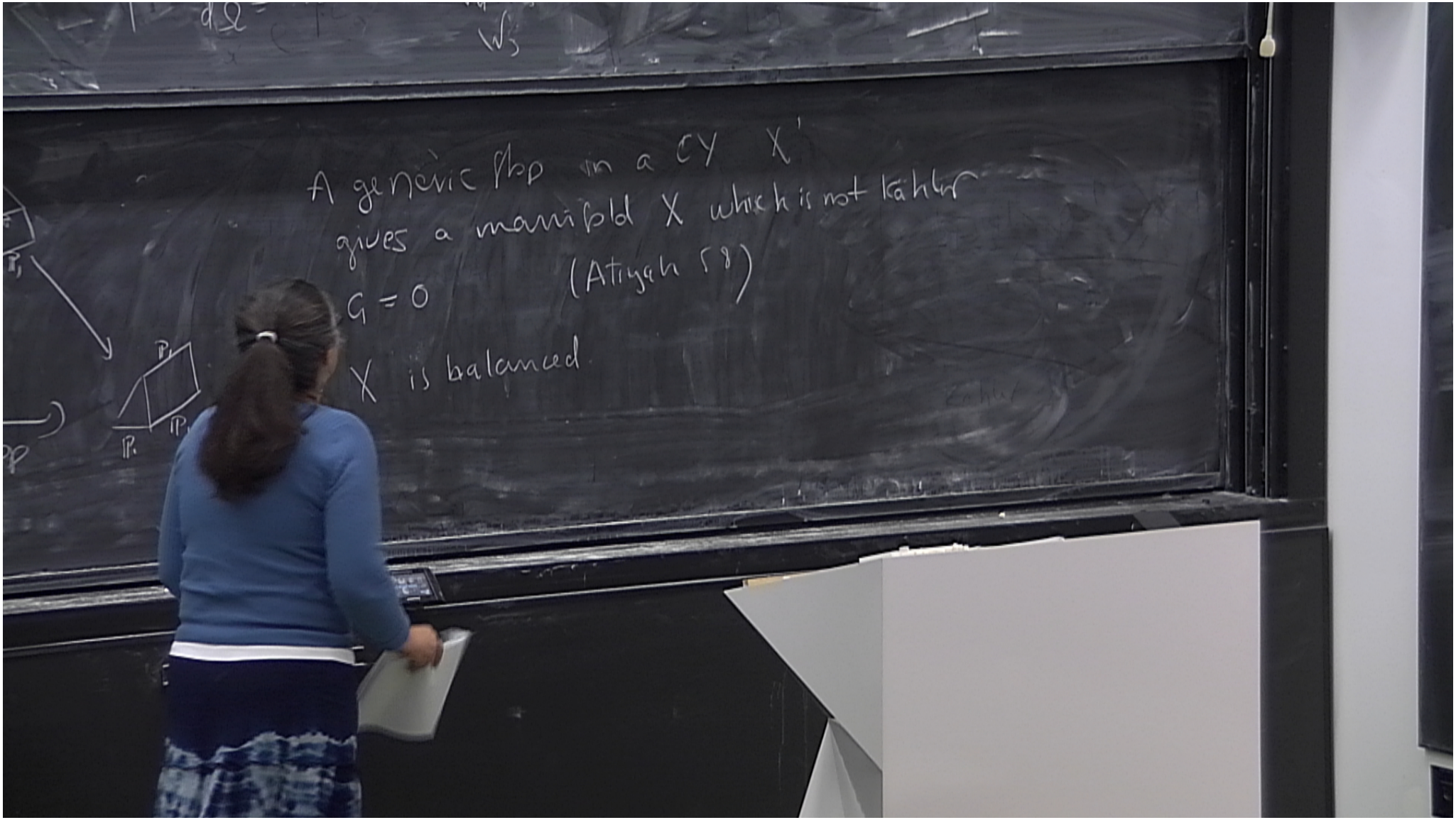

Flops

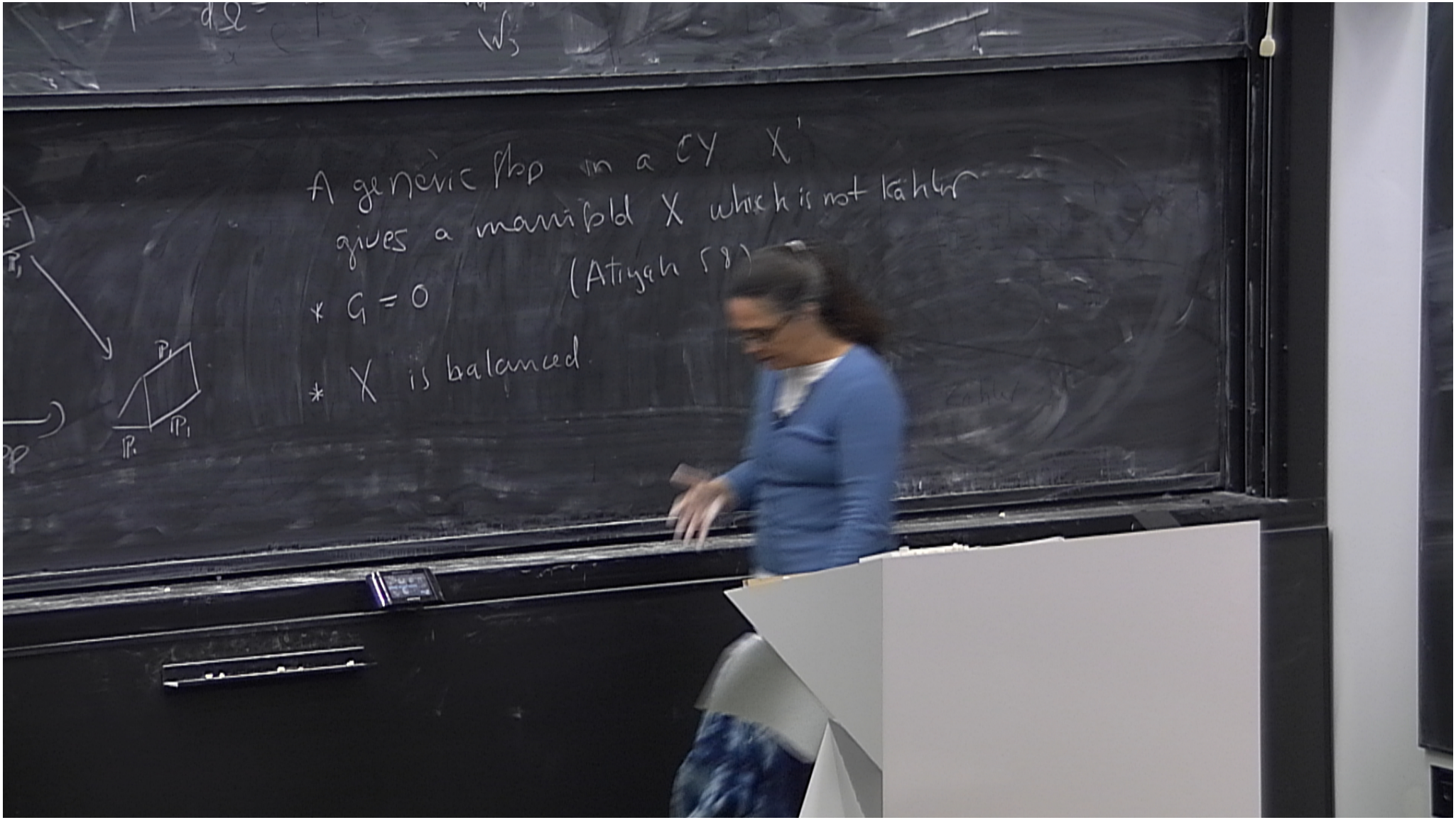

Flops

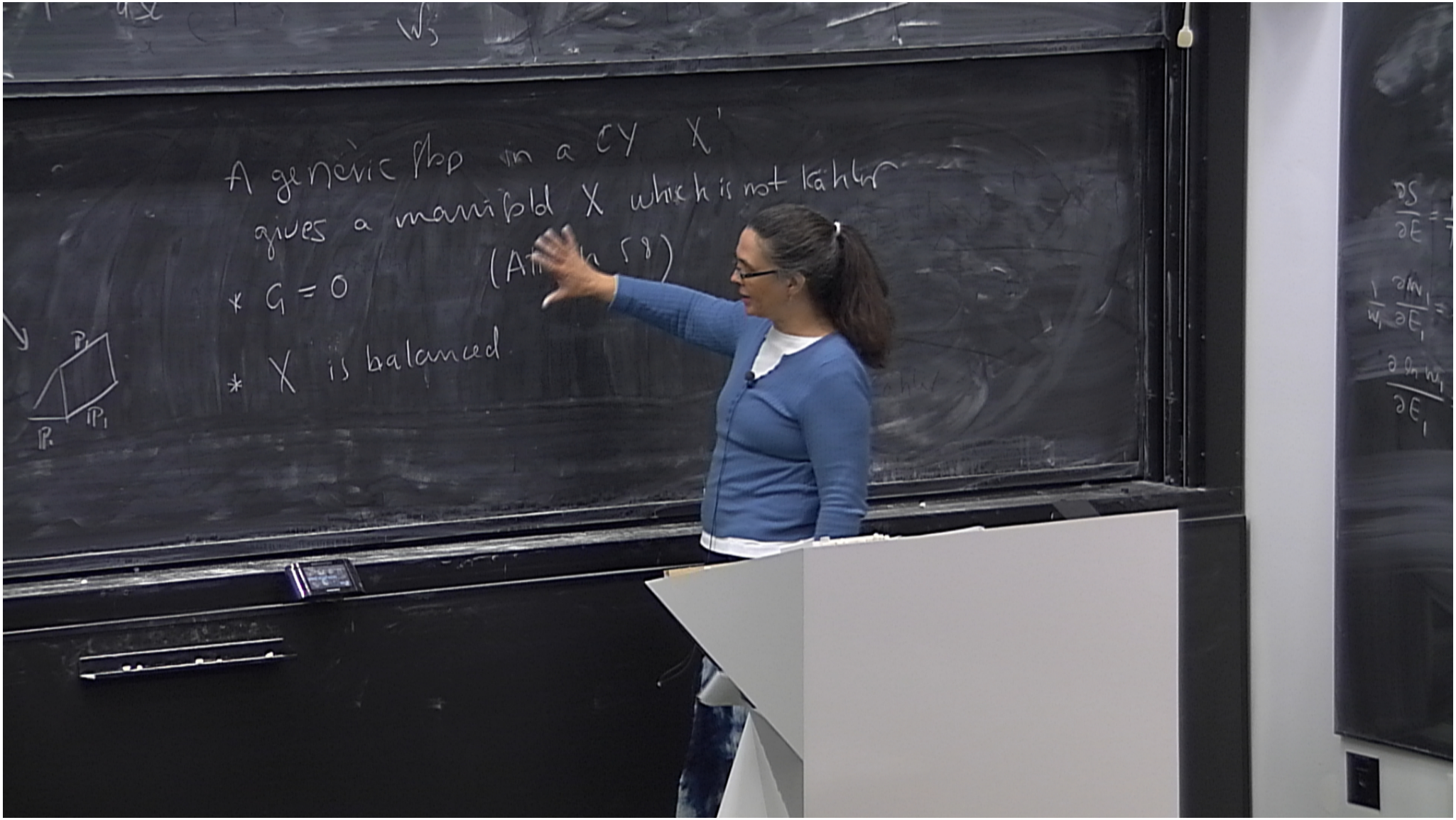

Flops
flop

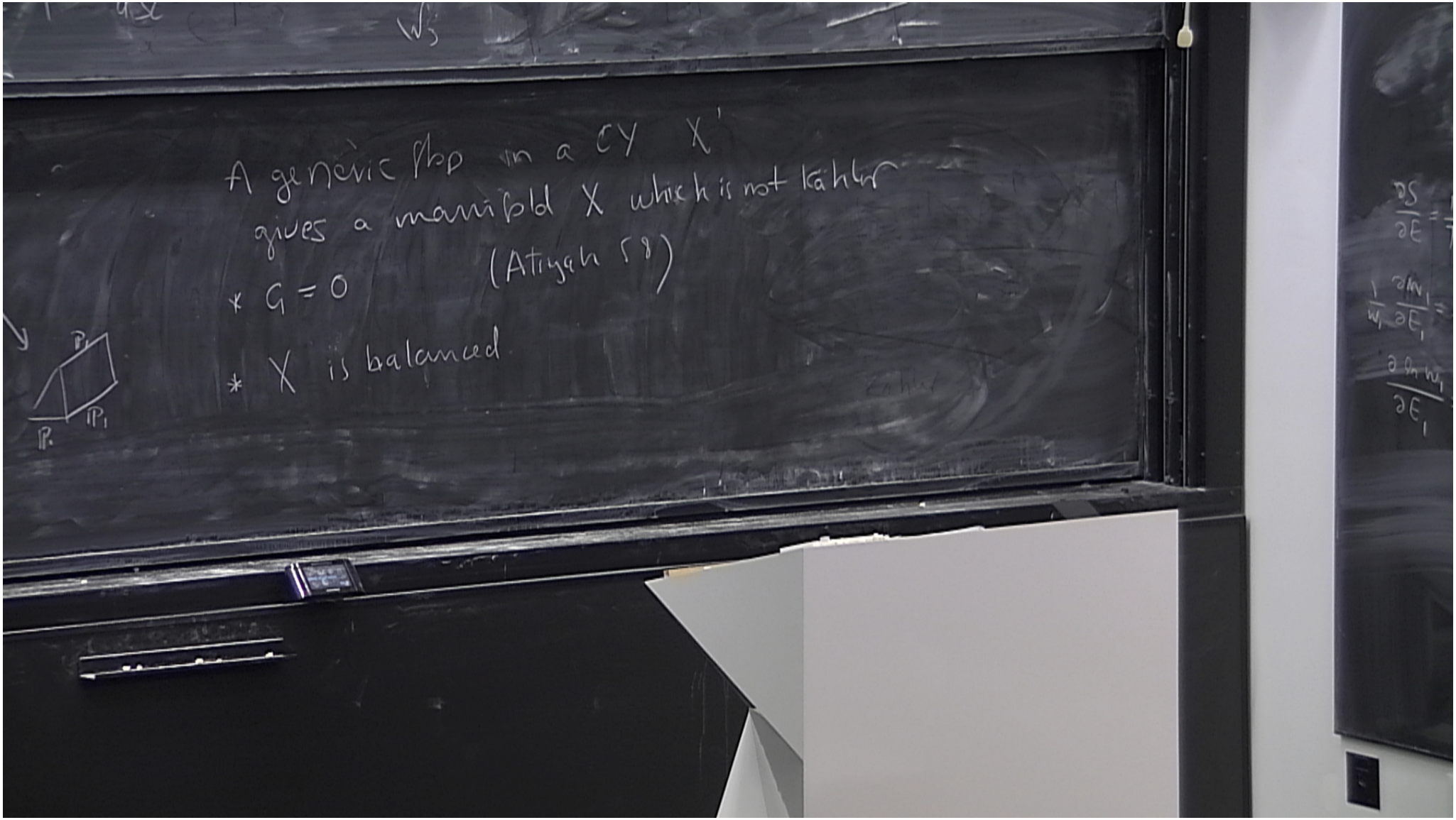

Flops
flop

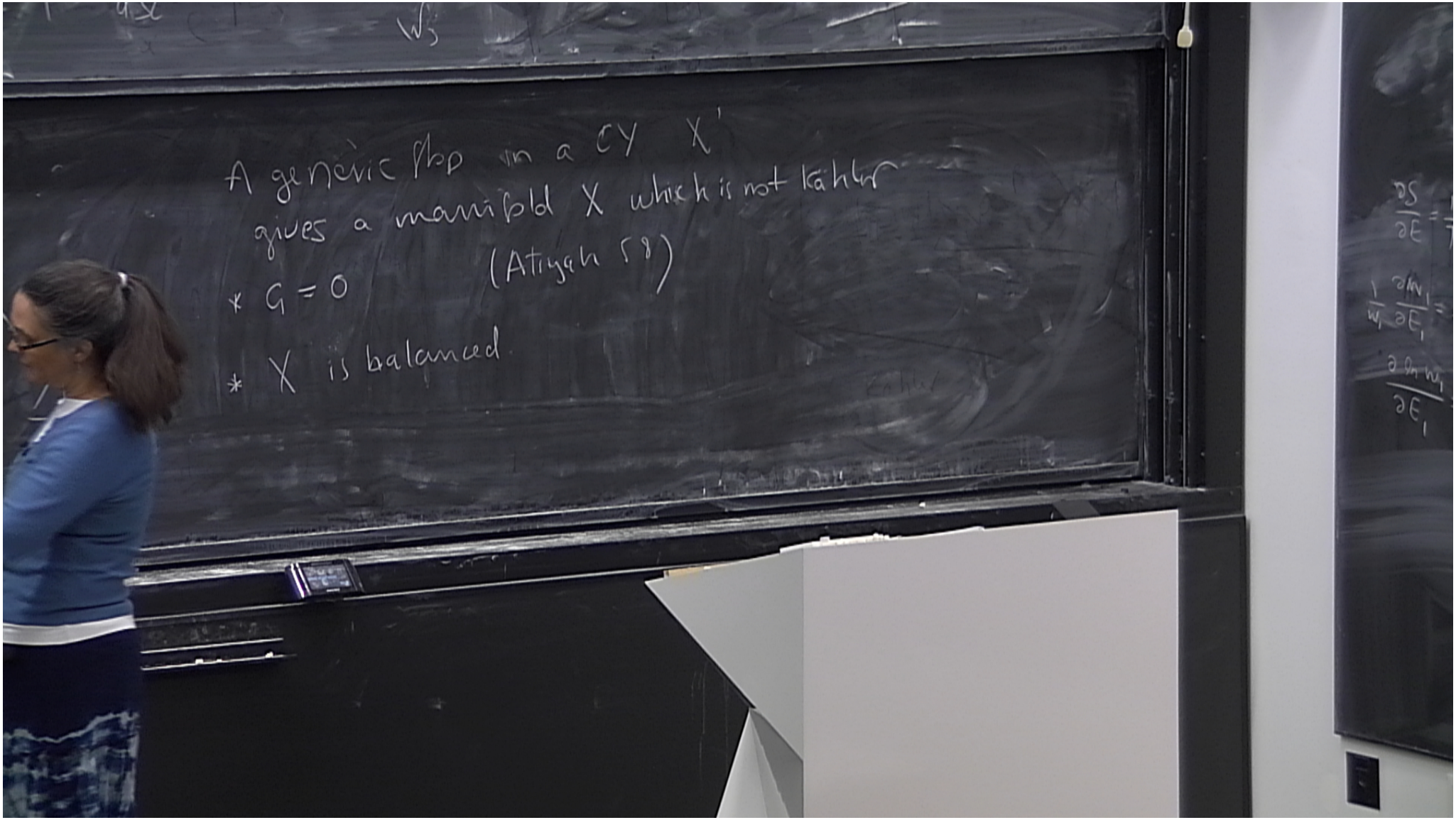

Flops
A generic flop
flop

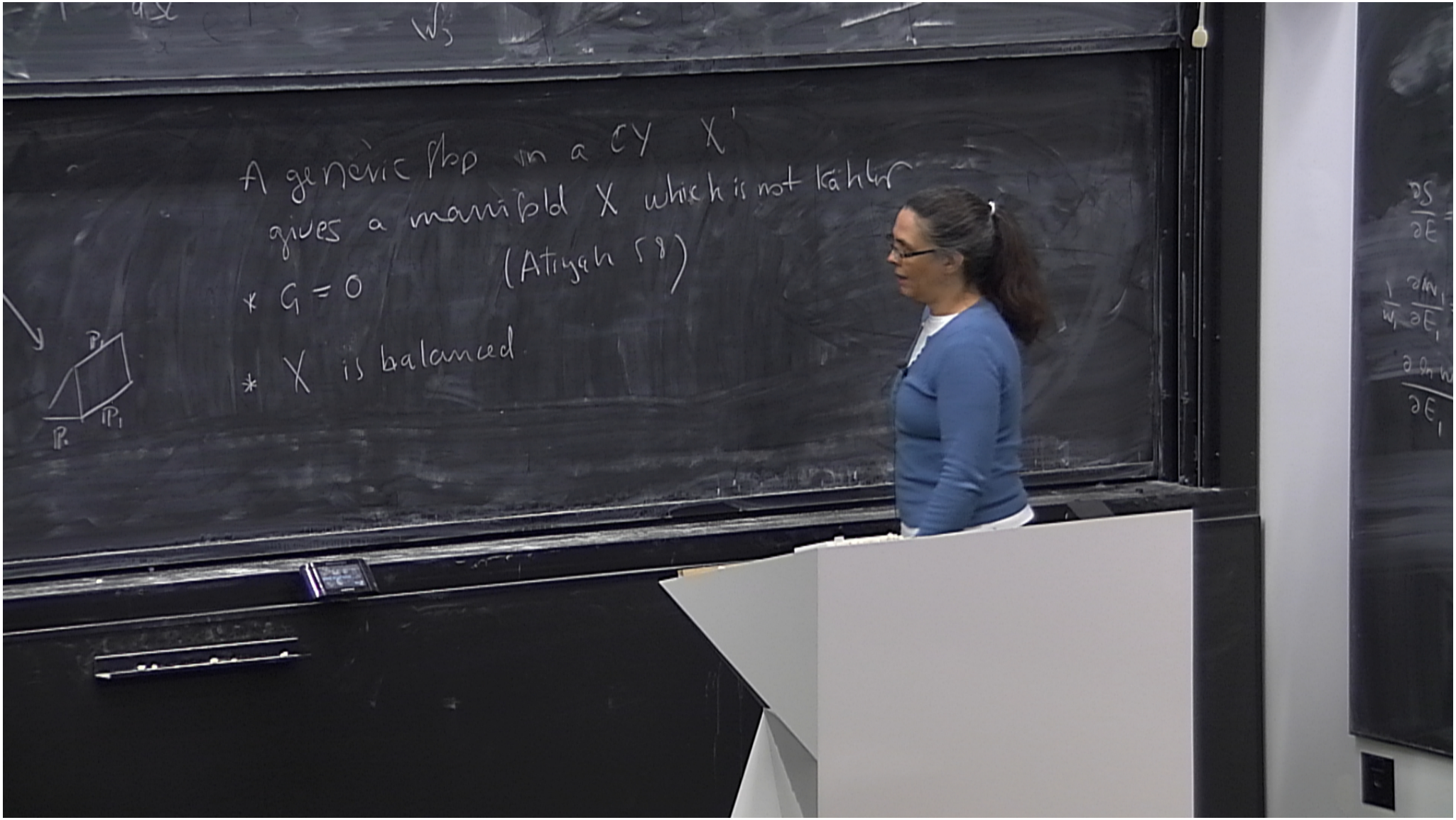

A generic flop in a CY X'
gives a manifold X which is not kähler
(Atiyah 58)
$G = 0$
X is balanced

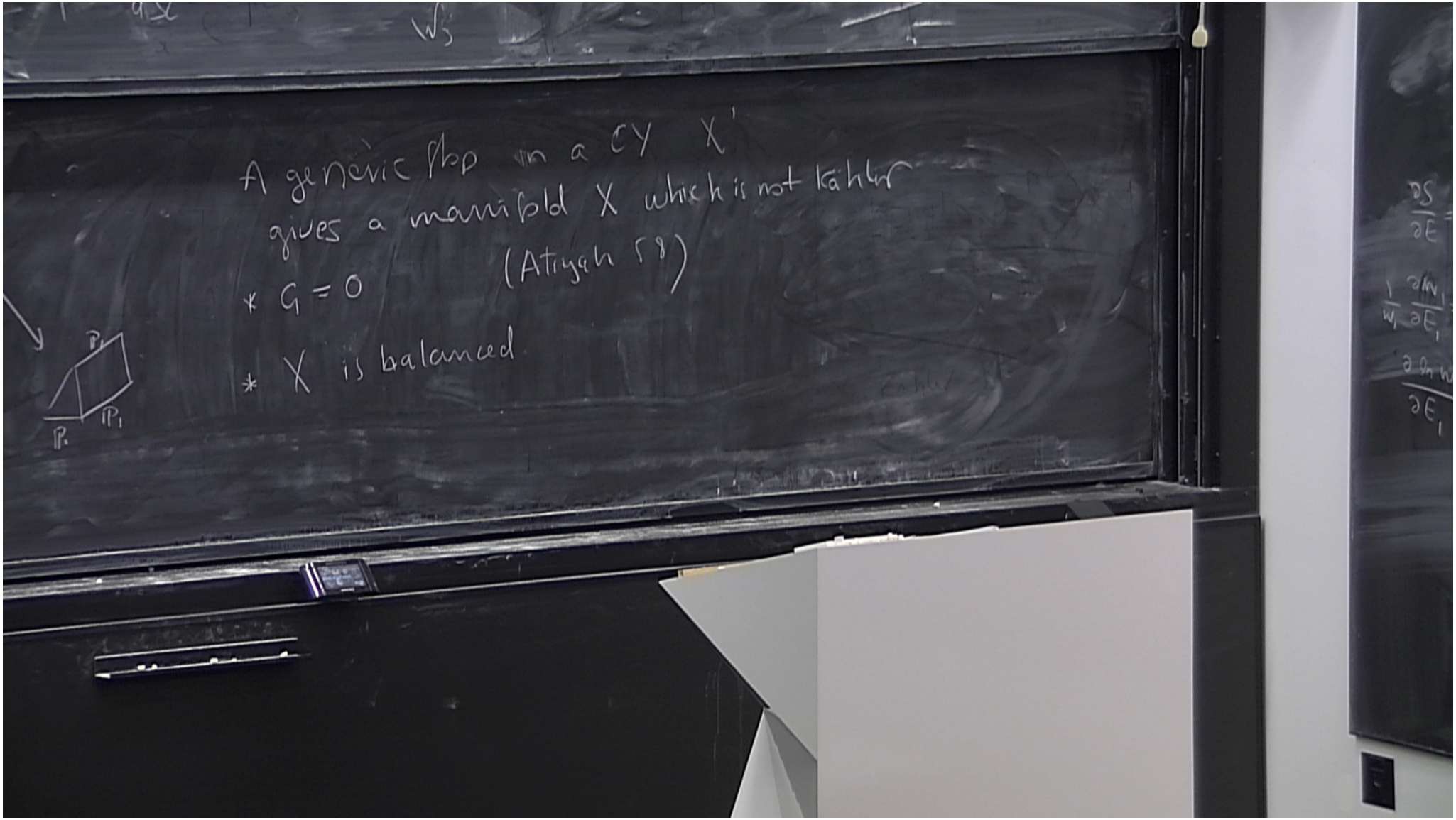

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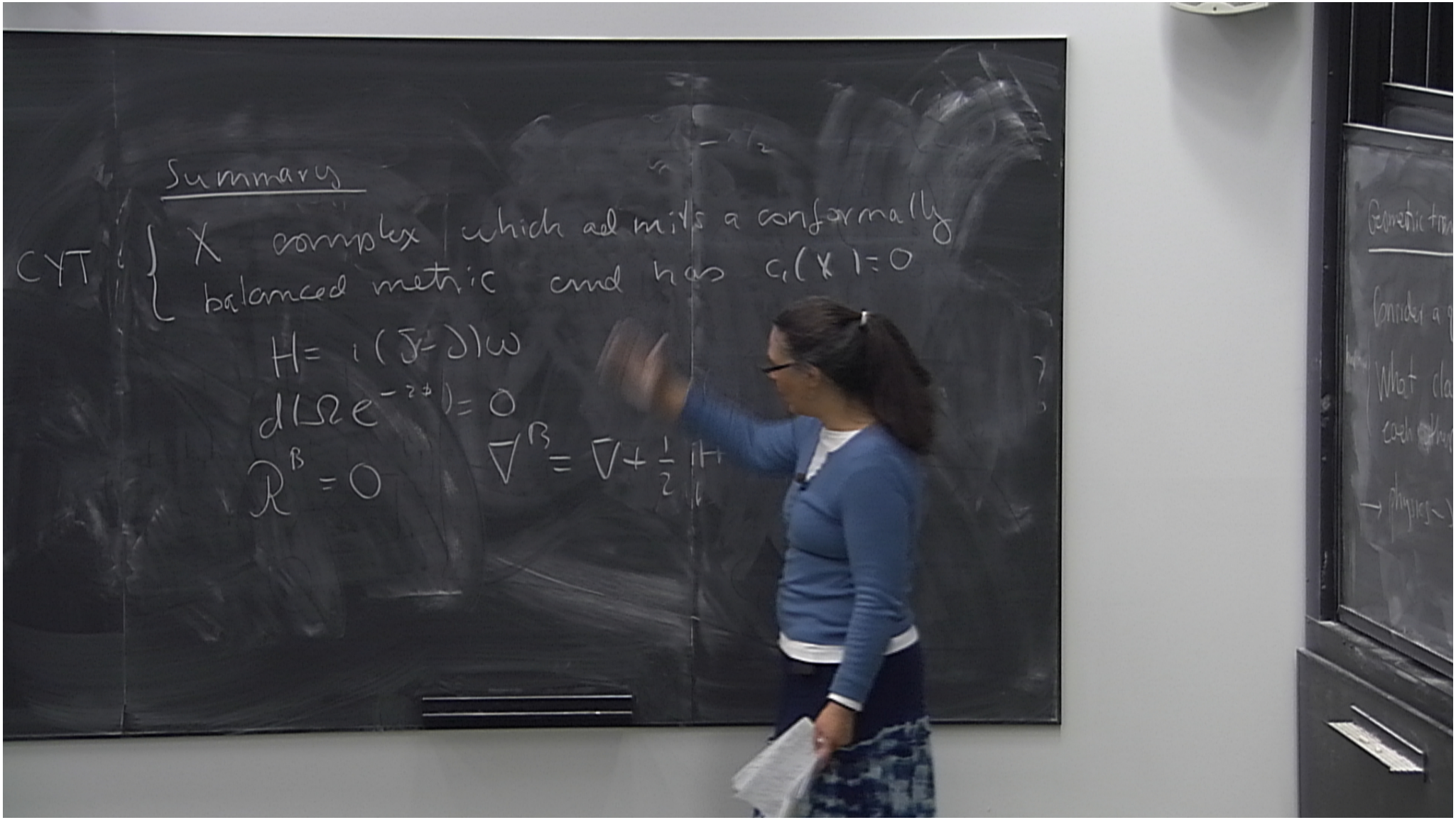

A generic fibre in a CY X'
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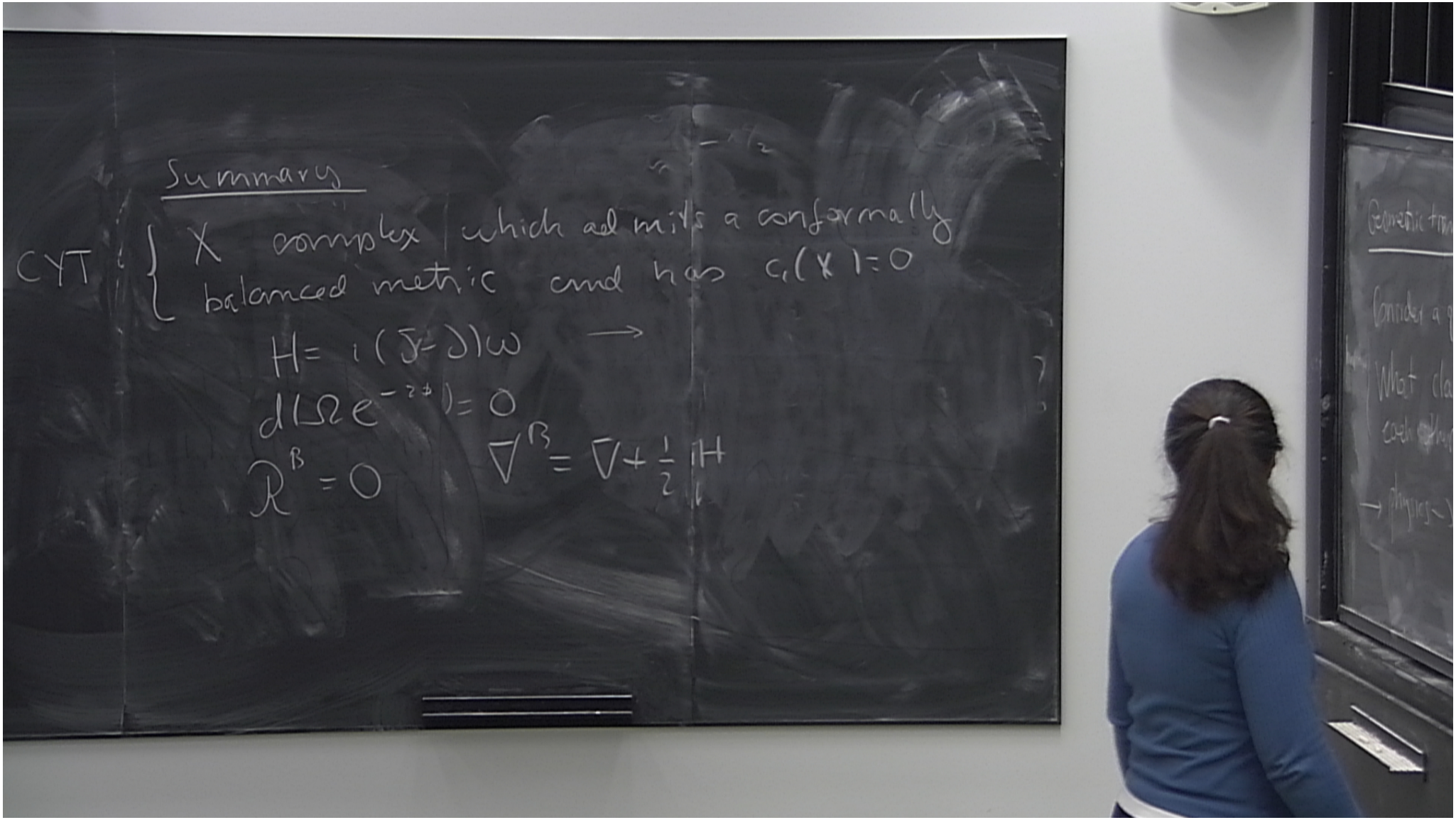

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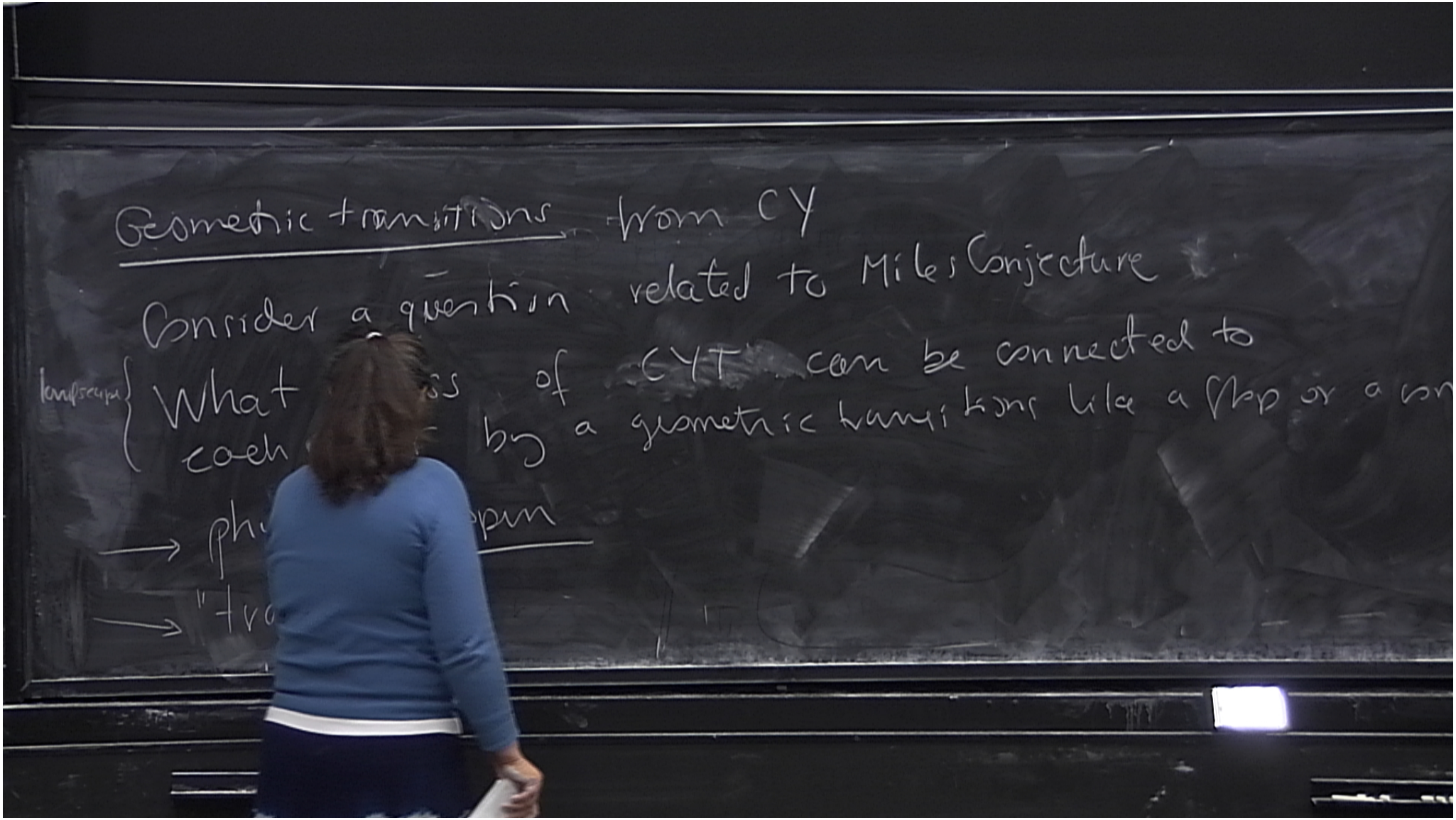

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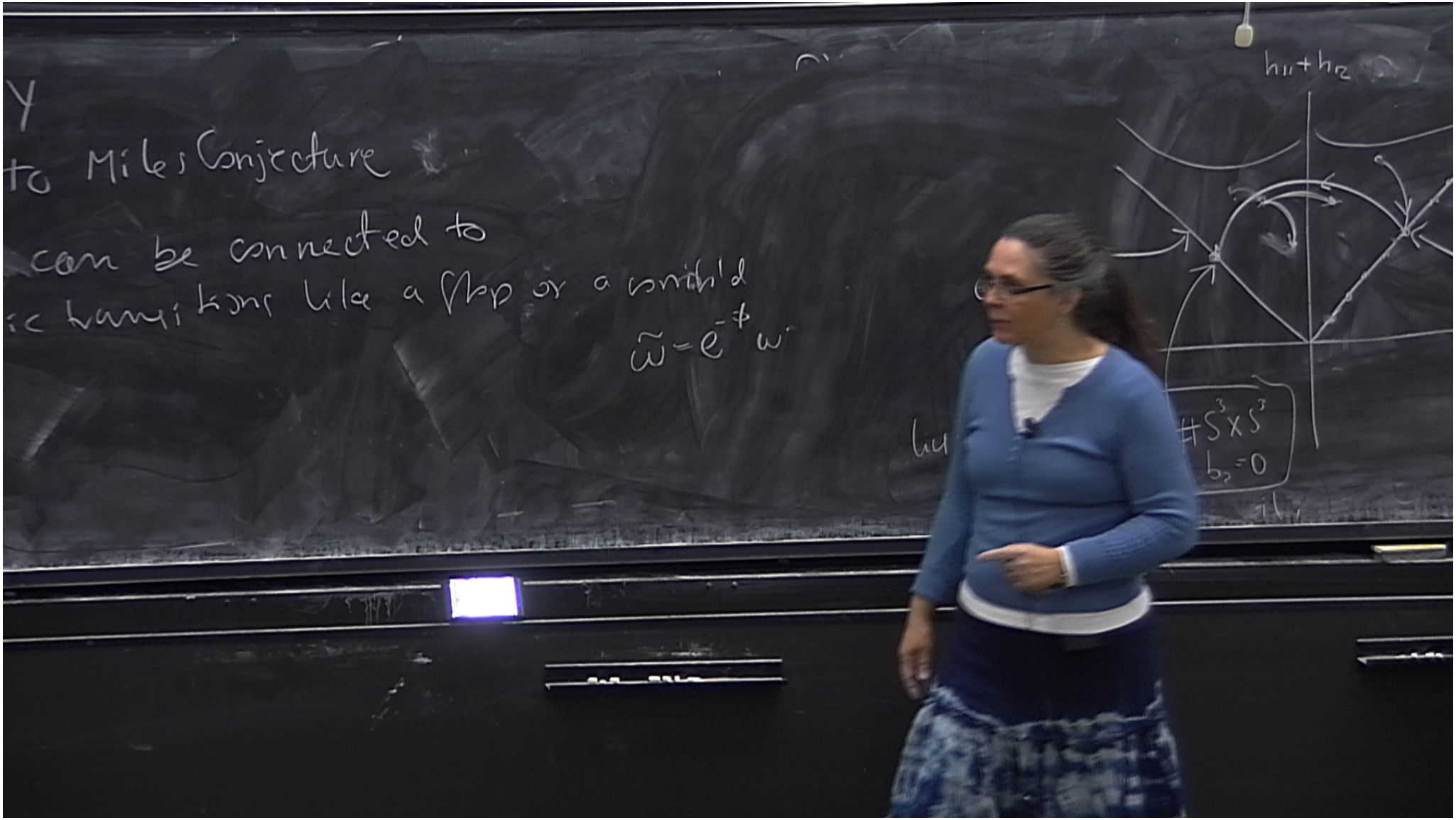

A generic flop in a CY X'
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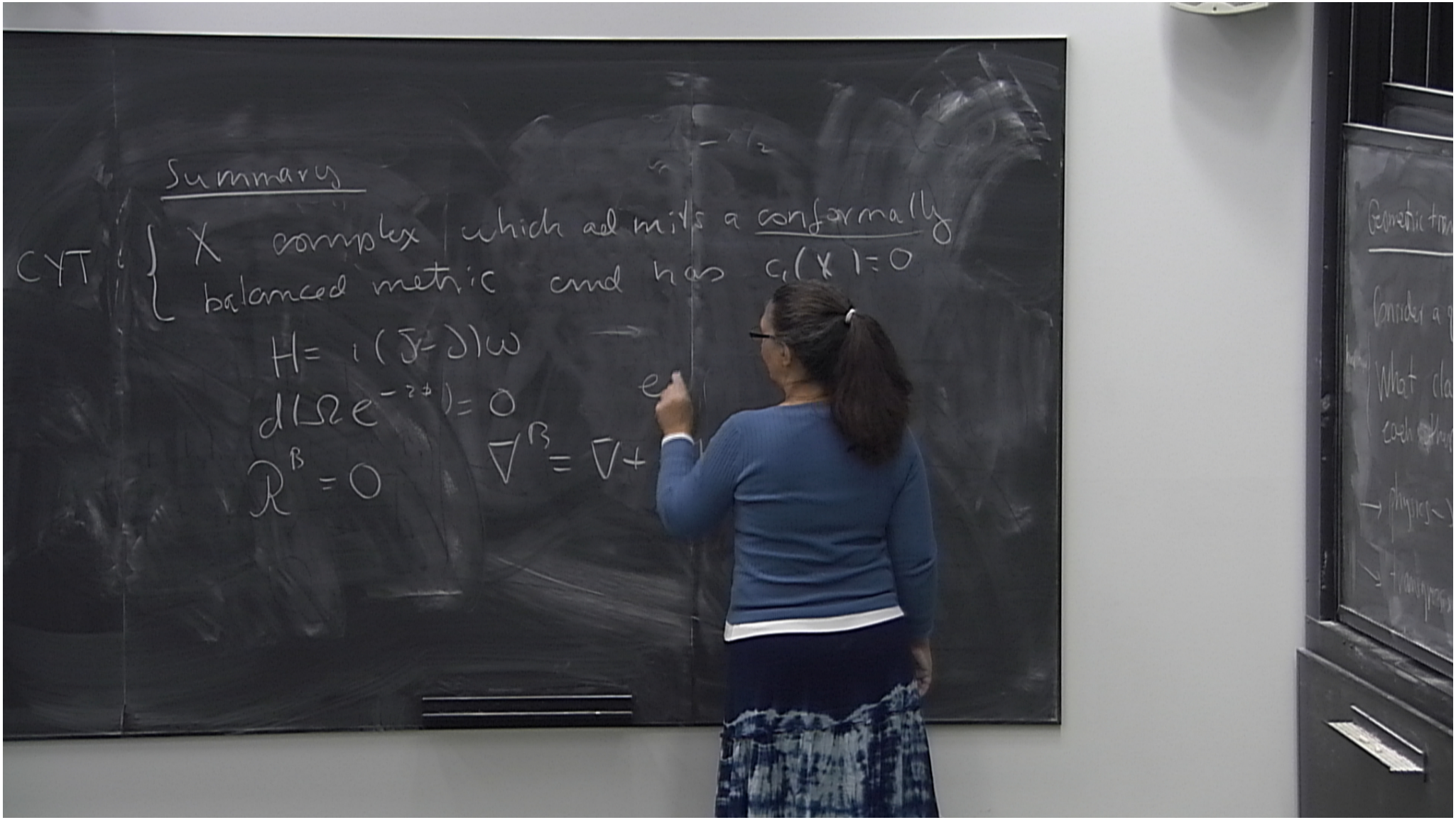

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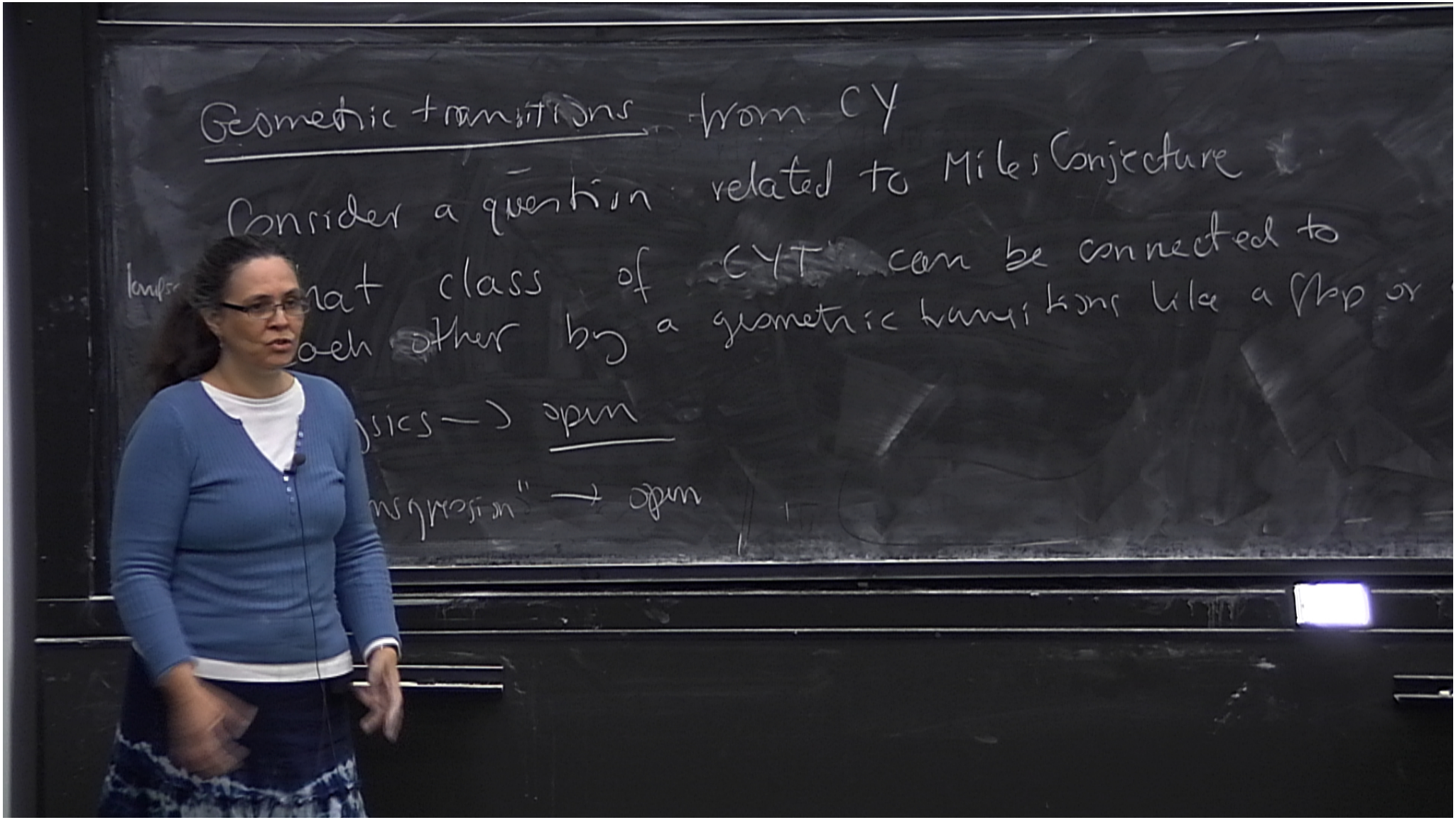

Summary
CYT
X complex which admits a conformally balanced metric and has $c_1(X)=0$
$H = i(\bar{\partial}-\partial)\omega$
$d(\Omega e^{-2\phi}) = 0$
$R^{B} = 0$
$\nabla^{B} = \nabla + \frac{1}{2}H$


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Geometric transitions from CY
Consider a question related to Miles Conjecture
What
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Y
to Miles Conjecture
can be connected to
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$\tilde{\omega} = e^{-t} \omega$
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$\#S^3 \times S^3$
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sics → open
nsgression" → open