Title: Game-theoretical Comparison of Information Structures in Quantum Theory

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Abstract: A family of probability distributions (i.e. a statistical model) is said to be sufficient for another, if there exists a transition matrix transforming the probability distributions in the former to the probability distributions in the latter. The so-called Blackwell-Sherman-Stein Theorem provides necessary and sufficient conditions for one statistical model to be sufficient for another, by comparing their "information values" in a game-theoretical framework. In this talk, I will extend some of these ideas to the quantum case.
I will begin by considering the comparison of ensembles of quantum states in terms of their "information value" in quantum statistical decision problems. In this case, I will prove that one ensemble is "more informative" than another if and only if there exists a suitable processing of the former into the latter.
I will then move on to the comparison of bipartite quantum states in terms of their "nonlocality value" in nonlocal games. In this case, I will prove that one bipartite state is "more nonlocal" than another if and only if the former can be transformed into the latter by local operations and shared randomness, arguing, moreover, that the framework provided by nonlocal games can be useful in understanding analogies and differences between the notions of quantum entanglement and nonlocality.

Game-Theoretical Comparison of Information Structures in Quantum Theory (arXiv:1004.3794, arXiv:1106.6095)

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Perimeter Institute, Seminar Series in Quantum Foundations

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Introduction, motivation

Resource-theoretical framework:

1. identify a *resource* $\rightsquigarrow$ allowed processes: do not use such resource
2. define resource *measures* $\rightsquigarrow$ monotonicity properties: resource-processing inequalities

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Examples: equilibrium thermodynamics (with adiabatic processes) and entanglement theory (with asymptotic SEPP operations)

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1. *Statistical decision theory* (tasks = statistical decision problems; resource = information; carriers = statistical models; allowed processes = statistical morphisms; measures = information values)
2. *Nonlocality theory* (tasks = nonlocal games; resource = nonlocality/entanglement; carriers = bipartite quantum states; allowed processes = local operations with shared randomness; measures = nonlocality values)

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Part I: statistical decision theory

A **(classical) statistical model** is given by:

1. parameter set: $\Theta = \{\theta\}$
2. state space: $\Delta = \{\delta\}$; $\mathscr{P}(\Delta)$: set of probability distributions (p.d.) over Δ
3. experiment: function $p : \Theta \to \mathscr{P}(\Delta)$; equiv. a family of p.d. $(p_\theta; \theta \in \Theta)$

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To define a **statistical game** (equiv. **statistical decision problem**) we need three more ingredients:

4. action set: $\mathcal{X} = \{x\}$
5. decision/observation: affine function $\pi : \mathscr{P}(\Delta) \to \mathscr{P}(\mathcal{X})$
6. payoff function: $\wp : \Theta \times \mathcal{X} \to \mathbb{R}$

$$\theta \xrightarrow{\text{exper.}} p_\theta(\delta) \xrightarrow{\text{decision}} \pi(x|\theta) = (\pi \circ p_\theta)(x) \xrightarrow{\text{payoff}} \wp_\theta(p,\pi) := \sum_x \pi(x|\theta)\wp(\theta,x)$$

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Def. The information value of a statistical model $E = (p_\theta; \theta \in \Theta)$ with respect to a statistical game $\mathsf{G} = (\Theta, \mathcal{X}, \wp)$ is given by $\wp^*_{\mathsf{G}}(E) := \max_{\pi:\text{decision}} \sum_\theta \wp_\theta(p, \pi)$.

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The Blackwell-Sherman-Stein theorem

For a *fixed* parameter set Θ, let us consider two statistical models $E = (p_\theta; \theta \in \Theta)$ and $F = (q_\theta; \theta \in \Theta)$, with state spaces $\Delta = \{\delta\}$ and $\Gamma = \{\gamma\}$, resp.

Def. E is said to be statistically more informative than F, written $E \supseteq_{\rm stat} F$, if and only if $\wp^*_{\mathsf{G}}(E) \geqslant \wp^*_{\mathsf{G}}(F)$, for all statistical games $\mathsf{G} = (\Theta, \mathcal{X}, \wp)$.

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stochastic transformation: linear map $\Sigma : \mathscr{P}(\Delta) \to \mathscr{P}(\Gamma)$

Data-processing inequality. *For any statistical model* $E = (p_\theta; \theta \in \Theta)$ *and any stochastic transformation* $\Sigma : \mathscr{P}(\Delta) \to \mathscr{P}(\Gamma)$, $E \supseteq_{\mathrm{stat}} \Sigma(E)$, *where* $\Sigma(E) := (\Sigma p_\theta; \theta \in \Theta)$.

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Theorem (Blackwell 1949~1953, Sherman 1951, Stein 1951)

Fix a parameter set Θ. Let $E = (p_\theta; \theta \in \Theta)$ and $F = (q_\theta; \theta \in \Theta)$ be two statistical models on state spaces Δ and Γ, respectively. Then, $E \supseteq_{\mathrm{stat}} F$ if and only if $F = \Sigma(E)$, for some stochastic transformation $\Sigma : \mathscr{P}(\Delta) \to \mathscr{P}(\Gamma)$.

Quantum statistical models

A **quantum statistical model** is given by:

1. parameter set: $\Theta = \{\theta\}$
2. Hilbert space: $\mathcal{H}$; $\mathfrak{S}(\mathcal{H})$: set of density matrices acting on $\mathcal{H}$
3. quantum experiment: function $\varrho : \Theta \to \mathfrak{S}(\mathcal{H})$; equiv. a family of density matrices $(\varrho_\theta ; \theta \in \Theta)$

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5. decision: affine function $\pi : \mathfrak{S}(\mathcal{H}) \to \mathscr{P}(\mathcal{X})$; equivalently, POVM $\pi = (\pi^x; x \in \mathcal{X})$, with $\pi^x \geqslant 0$ and $\sum_x \pi^x = \mathbb{1}_{\mathcal{H}}$
6. payoff function: $\wp : \Theta \times \mathcal{X} \to \mathbb{R}$

$$\theta \xrightarrow{\text{exper.}} \varrho_\theta \xrightarrow{\text{POVM}} p(x|\theta) = \mathrm{Tr}[\pi^x \varrho_\theta] \xrightarrow{\text{payoff}} \wp_\theta(\varrho, \pi) = \sum_x p(x|\theta)\wp(\theta, x)$$

Comparison of quantum statistical models

For a *fixed* parameter set Θ, let us consider two quantum statistical models $R = (\varrho_\theta; \theta \in \Theta)$ and $S = (\sigma_\theta; \theta \in \Theta)$, on Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$, resp.

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Quantum data-processing inequality. *For any statistical model* $R = (\varrho_\theta; \theta \in \Theta)$ *and any CPTP map* $\Phi : \mathfrak{L}(\mathcal{H}) \to \mathfrak{L}(\mathcal{K})$, $R \supseteq_{\mathrm{stat}} \Phi(R)$, *where* $\Phi(R) := (\Phi\varrho_\theta; \theta \in \Theta)$.

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Fact. *There exist quantum statistical models $R = (\varrho_\theta; \theta \in \Theta)$ and $S = (\sigma_\theta; \theta \in \Theta)$ such that $R \supseteq_{\mathrm{stat}} S$, but $\nexists \Phi$ CPTP such that $S = \Phi(R)$.*

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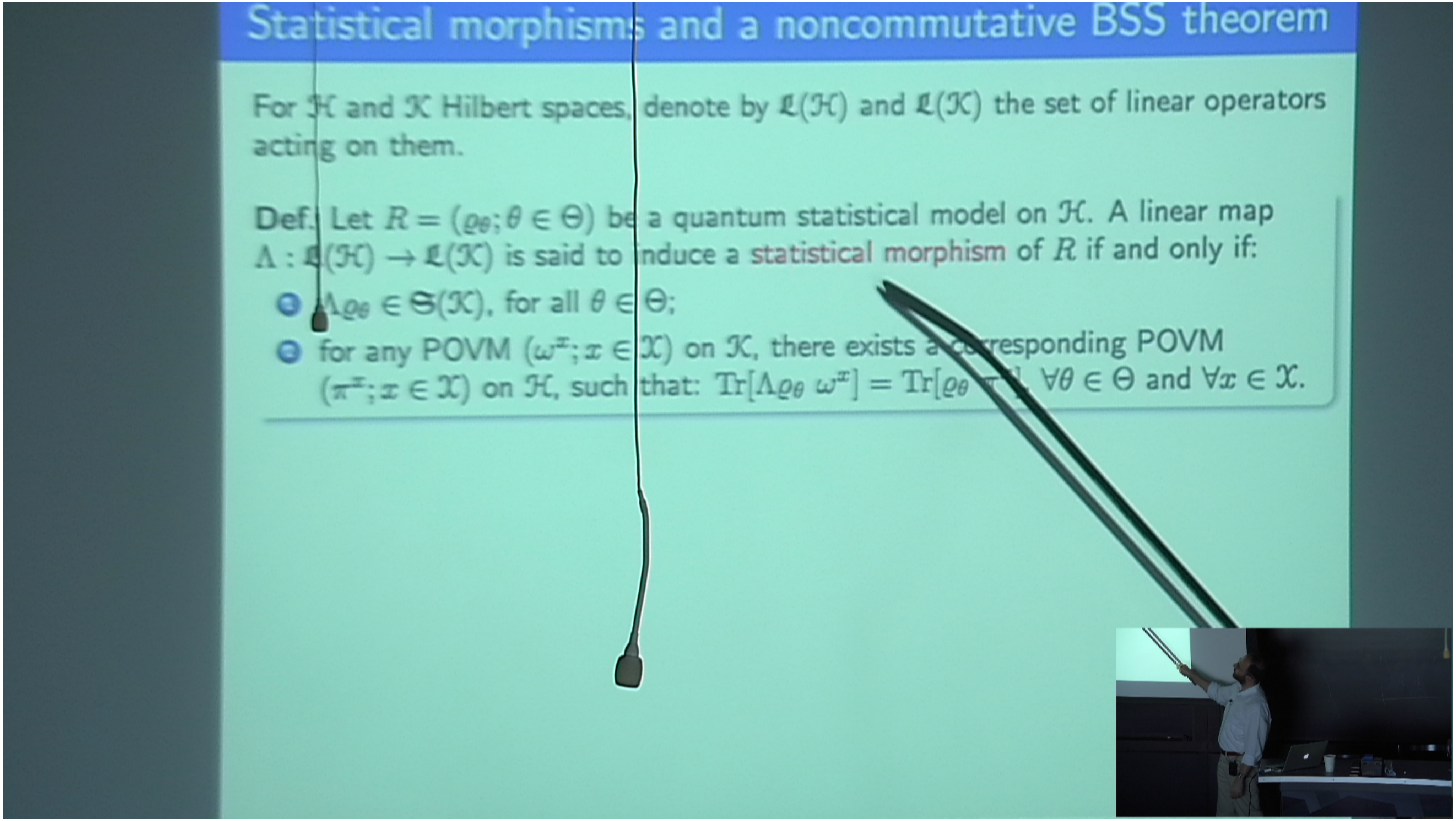
Statistical morphisms and a noncommutative BSS theorem

For $\mathcal{H}$ and $\mathcal{K}$ Hilbert spaces, denote by $\mathfrak{L}(\mathcal{H})$ and $\mathfrak{L}(\mathcal{K})$ the set of linear operators acting on them.

Def. Let $R = (\varrho_\theta; \theta \in \Theta)$ be a quantum statistical model on $\mathcal{H}$. A linear map $\Lambda : \mathfrak{L}(\mathcal{H}) \to \mathfrak{L}(\mathcal{K})$ is said to induce a statistical morphism of R if and only if:

1. $\Lambda\varrho_\theta \in \mathfrak{S}(\mathcal{K})$, for all $\theta \in \Theta$;
2. for any POVM $(\omega^x; x \in \mathcal{X})$ on $\mathcal{K}$, there exists a corresponding POVM $(\pi^x; x \in \mathcal{X})$ on $\mathcal{H}$, such that: $\mathrm{Tr}[\Lambda\varrho_\theta\ \omega^x] = \mathrm{Tr}[\varrho_\theta\ \pi^x]$, $\forall\theta \in \Theta$ and $\forall x \in \mathcal{X}$.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Generalized data-processing inequality. *Given a statistical model $R = (\varrho_\theta; \theta \in \Theta)$, let $\Lambda : \mathfrak{L}(\mathcal{H}) \to \mathfrak{L}(\mathcal{K})$ induce a statistical morphism of R. Then, $R \supseteq_{\mathrm{stat}} \Lambda(R)$.*

Noncommutative BSS theorem. *Fix a parameter set Θ. Let $R = (\varrho_\theta; \theta \in \Theta)$ and $S = (\sigma_\theta; \theta \in \Theta)$ be two quantum statistical models on Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$, respectively. Then, $R \supseteq_{\mathrm{stat}} S$ if and only if $S = \Lambda(R)$, for some statistical morphism $\Lambda : \mathfrak{L}(\mathcal{H}) \to \mathfrak{L}(\mathcal{K})$.*

From statistical morphisms to CPTP maps

First extension theorem. *Let* $R = (\varrho_\theta; \theta \in \Theta)$ *and* $S = (\sigma_\theta; \theta \in \Theta)$ *be two quantum statistical models on Hilbert spaces* $\mathcal{H}$ *and* $\mathcal{K}$*, respectively. Let* $[\sigma_\theta, \sigma_{\theta'}] = 0$, $\forall \theta, \theta' \in \Theta$*. Then,* $S = \Lambda(R)$*, with* Λ *statistical morphism, if and only if* $S = \Phi(R)$*, with* Φ *CPTP map.*

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Second dilation theorem. *Let* $R = (\varrho_\theta; \theta \in \Theta)$, $S = (\sigma_\theta; \theta \in \Theta)$*, and* $T = (\tau_\xi; \xi \in \Xi)$*, be three quantum statistical models on Hilbert spaces* $\mathcal{H}$, $\mathcal{K}$*, and* $\mathcal{L}$*, respectively. Let the composition* $T \otimes R$ *be defined as the statistical model* $(\tau_\xi \otimes \varrho_\theta; \xi \in \Xi, \theta \in \Theta)$ *on* $\mathcal{L} \otimes \mathcal{H}$*. Let* $T \otimes S$ *be defined analogously. Moreover, suppose that* $\mathcal{L} \cong \mathcal{K}$ *and* $\mathrm{span}\{\tau_\xi\} \cong \mathfrak{L}(\mathcal{K})$*. Then, TFAE:*

1. $T \otimes S = \Lambda(T \otimes R)$*, with* $\Lambda : \mathfrak{L}(\mathcal{L} \otimes \mathcal{H}) \to \mathfrak{L}(\mathcal{L} \otimes \mathcal{K})$ *statistical morphism;*
2. $T \otimes S = \Phi(T \otimes R)$*, with* $\Phi : \mathfrak{L}(\mathcal{L} \otimes \mathcal{H}) \to \mathfrak{L}(\mathcal{L} \otimes \mathcal{K})$ *CPTP map;*
3. $S = \tilde{\Phi}(R)$*, for some CPTP map* $\tilde{\Phi} : \mathfrak{L}(\mathcal{H}) \to \mathfrak{L}(\mathcal{K})$*.*

Quantum BSS theorems

Semi-classical BSS theorem. *Let* $R = (\varrho_\theta; \theta \in \Theta)$ *and* $S = (\sigma_\theta; \theta \in \Theta)$ *be two quantum statistical models on Hilbert spaces* $\mathcal{H}$ *and* $\mathcal{K}$*, respectively. Let moreover* $[\sigma_\theta, \sigma_{\theta'}] = 0$, $\forall \theta, \theta' \in \Theta$. *Then,* $R \supseteq_{\text{stat}} S$ *if and only if* $S = \Phi(R)$, *for some CPTP map* $\Phi : \mathfrak{L}(\mathcal{H}) \to \mathfrak{L}(\mathcal{K})$.

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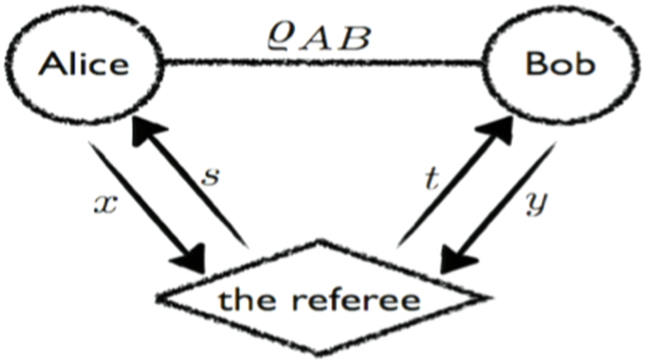
1. $T \otimes R \supseteq_{\rm stat} T \otimes S$*, for any auxiliary quantum statistical model* T*;*
2. $T \otimes R \supseteq_{\rm stat} T \otimes S$*, for some auxiliary quantum statistical model* $T = (\tau_\xi; \xi \in \Xi)$ *on* $\mathcal{K}$*, such that* $\mathrm{span}\{\tau_\xi\} = \mathfrak{L}(\mathcal{K})$*;*
3. $S = \Phi(R)$*, for some CPTP map* $\Phi : \mathfrak{L}(\mathcal{H}) \to \mathfrak{L}(\mathcal{K})$.

Complete positivity is always related with the possibility of *extending* quantum systems by tensoring.

here beginneth the second part: nonlocality theory



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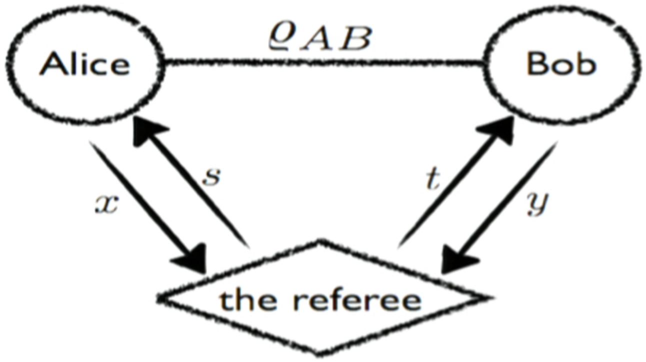

The rules of a **nonlocal game** G_{nl} consist of:

1. four (finite) index sets $\mathcal{S} = \{s\}$, $\mathcal{T} = \{t\}$, $\mathcal{X} = \{x\}$, and $\mathcal{Y} = \{y\}$;
2. a payoff function $\wp : \mathcal{S} \times \mathcal{T} \times \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$.

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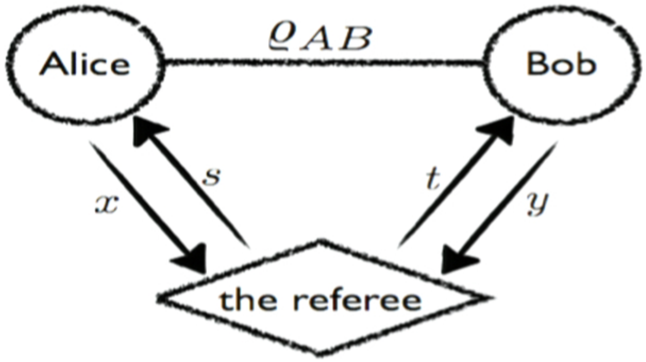

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Notation: questions are encoded on orthonormal quantum states $|s\rangle\langle s| \in \mathfrak{S}(\mathcal{H}_{A_0})$ and $|t\rangle\langle t| \in \mathfrak{S}(\mathcal{H}_{B_0})$.

Def. The nonlocality value of a bipartite state $\varrho_{AB} \in \mathfrak{S}(\mathcal{H}_A \otimes \mathcal{H}_B)$ with respect to a nonlocal game $\mathsf{G}_{\mathrm{nl}} = (\mathcal{S}, \mathcal{T}, \mathcal{X}, \mathcal{Y}, \wp)$ is given by

$$\wp^*(\varrho_{AB}; \mathsf{G}_{\mathrm{nl}}) :=$$

$$\max_{P,Q:\text{POVM's}} \sum_{s,t,x,y} \wp(s,t,x,y) \underbrace{\mathrm{Tr}\left[\left(P^x_{A_0A} \otimes Q^y_{BB_0}\right) \ \left(|s\rangle\langle s|_{A_0} \otimes \varrho_{AB} \otimes |t\rangle\langle t|_{B_0}\right)\right]}_{p(x,y|s,t)} .$$

Comparison of bipartite states (1/2)

Def. A bipartite state $\varrho_{AB} \in \mathfrak{S}(\mathcal{H}_A \otimes \mathcal{H}_B)$ is said to be more nonlocal than another $\sigma_{A'B'} \in \mathfrak{S}(\mathcal{H}_{A'} \otimes \mathcal{H}_{B'})$, written $\varrho_{AB} \supseteq_{\mathrm{nl}} \sigma_{A'B'}$, if and only if $\wp^*(\varrho_{AB}; \mathsf{G}_{\mathrm{nl}}) \geqslant \wp^*(\sigma_{A'B'}; \mathsf{G}_{\mathrm{nl}})$, for all nonlocal games G_{nl}.

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Local operations with shared randomness (LOSR): bipartite CPTP maps $\Phi : \mathfrak{L}(\mathcal{H}_A \otimes \mathcal{H}_B) \to \mathfrak{L}(\mathcal{H}_{A'} \otimes \mathcal{H}_{B'})$ that can be written as $\Phi = \sum_i \mu(i)\mathcal{E}^i \otimes \mathcal{F}^i$, for some μ probability distribution, and some CPTP maps $\mathcal{E}^i : \mathfrak{L}(\mathcal{H}_A) \to \mathfrak{L}(\mathcal{H}_{A'})$ and $\mathcal{F}^i : \mathfrak{L}(\mathcal{H}_B) \to \mathfrak{L}(\mathcal{H}_{B'})$.

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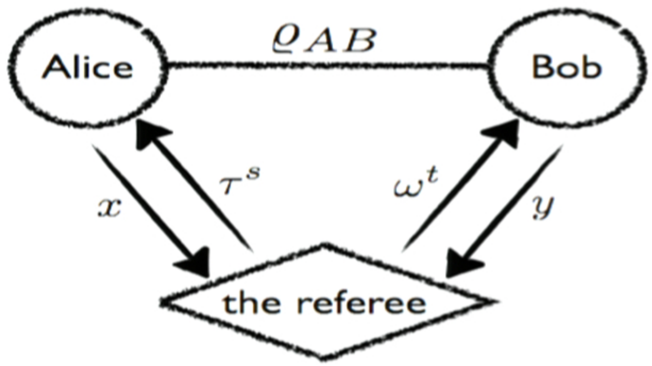
Semi-quantum nonlocal games



The rules of a **semi-quantum nonlocal game** G_{nl} consist of:

1. four (finite) index sets $\mathcal{S} = \{s\}$, $\mathcal{T} = \{t\}$, $\mathcal{X} = \{x\}$, and $\mathcal{Y} = \{y\}$;
2. two families of density matrices $\tau := (\tau^s; s \in \mathcal{S})$ and $\omega := (\omega^t; t \in \mathcal{T})$ on A_0 and B_0, respectively;
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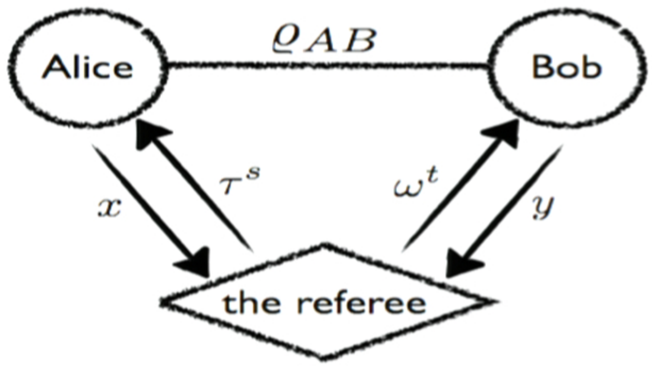

The rules of a **semi-quantum nonlocal game** $\mathsf{G}_{\rm nl}$ consist of:

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3. a payoff function $\wp : \mathcal{S} \times \mathcal{T} \times \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$.

Def. The nonlocality value of a bipartite state $\varrho_{AB} \in \mathfrak{S}(\mathcal{H}_A \otimes \mathcal{H}_B)$ with respect to a semi-quantum nonlocal game $\mathsf{G}_{\rm nl}^{\rm s\text{-}q} = (\mathcal{S}, \mathcal{T}, \mathcal{X}, \mathcal{Y}, \tau, \omega, \wp)$ is given by

$$\wp^*(\varrho_{AB}; \mathsf{G}_{\rm nl}^{\rm s\text{-}q}) := \max_{P,Q:\text{POVM's}} \sum_{s,t,x,y} \wp(s,t,x,y) \operatorname{Tr}\left[\left(P^x_{A_0A} \otimes Q^y_{BB_0}\right) \left(\tau^s_{A_0} \otimes \varrho_{AB} \otimes \omega^t_{B_0}\right)\right].$$

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Def. Given two bipartite states $\varrho_{AB} \in \mathfrak{S}(\mathcal{H}_A \otimes \mathcal{H}_B)$ and $\sigma_{A'B'} \in \mathfrak{S}(\mathcal{H}_{A'} \otimes \mathcal{H}_{B'})$, we write $\varrho_{AB} \supseteq_{\text{s-q}} \sigma_{A'B'}$ if and only if $\wp^*(\varrho_{AB}; \mathsf{G}_{\text{nl}}^{\text{s-q}}) \geqslant \wp^*(\sigma_{A'B'}; \mathsf{G}_{\text{nl}}^{\text{s-q}})$, for all semi-quantum nonlocal games $\mathsf{G}_{\text{nl}}^{\text{s-q}}$.

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Consequences. *Separable states are the endpoints of the relation $\supseteq_{\text{s-q}}$. All separable states have the same nonlocality values, i.e. $\wp^*(\varrho_{AB}; \mathsf{G}_{\text{nl}}^{\text{s-q}}) = \wp^*_{\text{sep}}(\mathsf{G}_{\text{nl}}^{\text{s-q}})$, for all separable states ϱ_{AB} and all semi-quantum nonlocal games $\mathsf{G}_{\text{nl}}^{\text{s-q}}$.*

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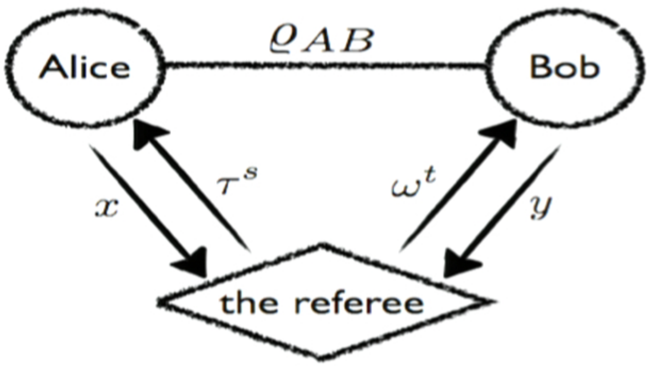
Semi-quantum nonlocal games



The rules of a **semi-quantum nonlocal game** G_{nl} consist of:

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Concluding remarks

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Summary:

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