

Title: Three Perspectives on Eternal Inflation

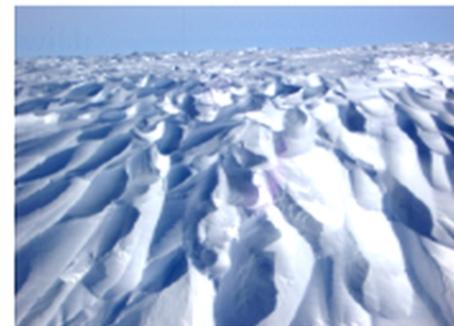
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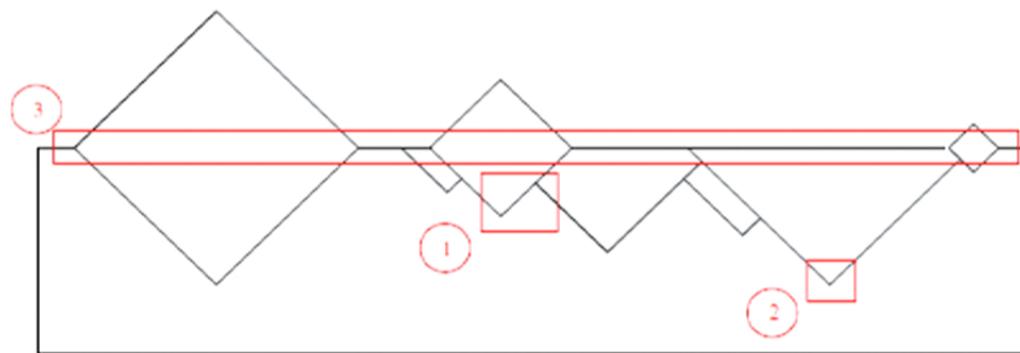
Abstract: I will discuss three ways in which (the string landscape and) eternal inflation is fun: (1) because it motivates revisiting some beautiful, classic calculations; (2) because its global description requires asking novel questions with possible broad ramifications; and (3) because it leads to experimental predictions.

## Three perspectives on eternal inflation

- String theory predicts a potential landscape with many vacua
- CDL instantons mediate nucleations of bubbles filled with lower energy vacua
- Resulting bubbles contain open FRW universes



This leads to the following picture of eternal inflation:



Plan: zoom in on this picture in 3 ways  $\leftrightarrow$  3 perspectives:

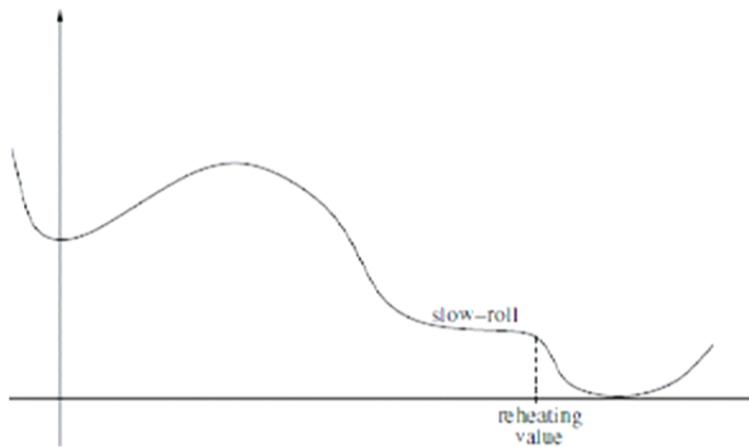
- ① On the interior of a bubble after collision  $\rightarrow$  observational prediction
- ② On the instanton mediating the nucleation  $\rightarrow$  to explore more general bubbles
- ③ On future infinity  $\rightarrow$  for theoretical insight

## Preliminaries to ① – a single bubble

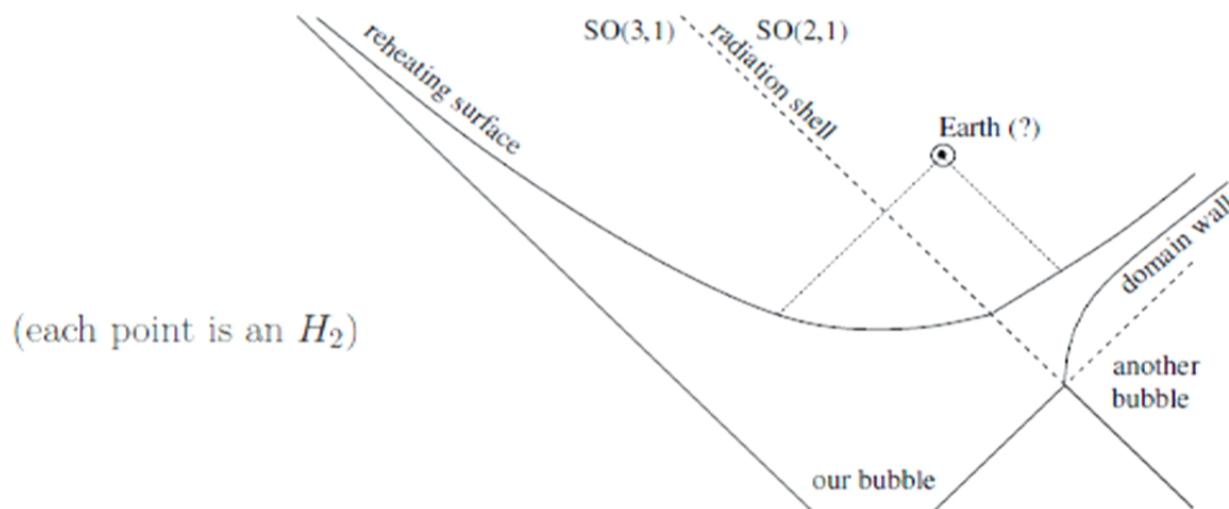
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- This is a complete FRW universe.
- If we inhabit this bubble, we need slow-roll inflation inside it.
- It is most natural to identify the inflaton with the tunneling field.
- The reheating surface is a level set of the field.



## ① A bubble collision

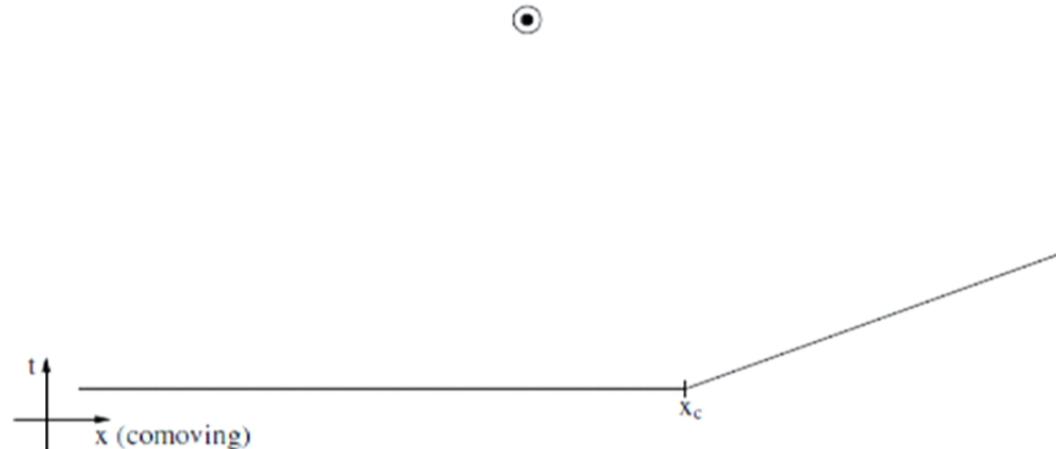


- Assume that the domain wall accelerates away from us
- Use Israel junction conditions to solve for the spacetime (Freivogel, Horowitz, Shenker, and Chang, Kleban, Levi 2007)
- Solve the scalar equation to find the reheating surface (Chang, Kleban, Levi 2008)
- Locate Earth, so Earthians see small effects of a collision
- To the future of the reheating surface, inflation has diluted curvature, so substitute  $H_2 \rightarrow \mathbb{R}^2$  and  $H_3 \rightarrow \mathbb{R}^3$

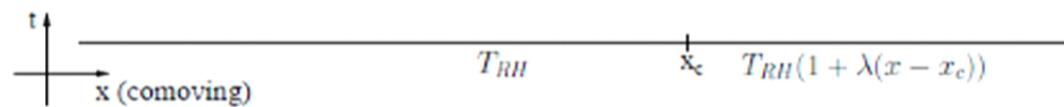
This leads to the following picture of the reheating surface:

## ① From the reheating surface to a cold / hot spot

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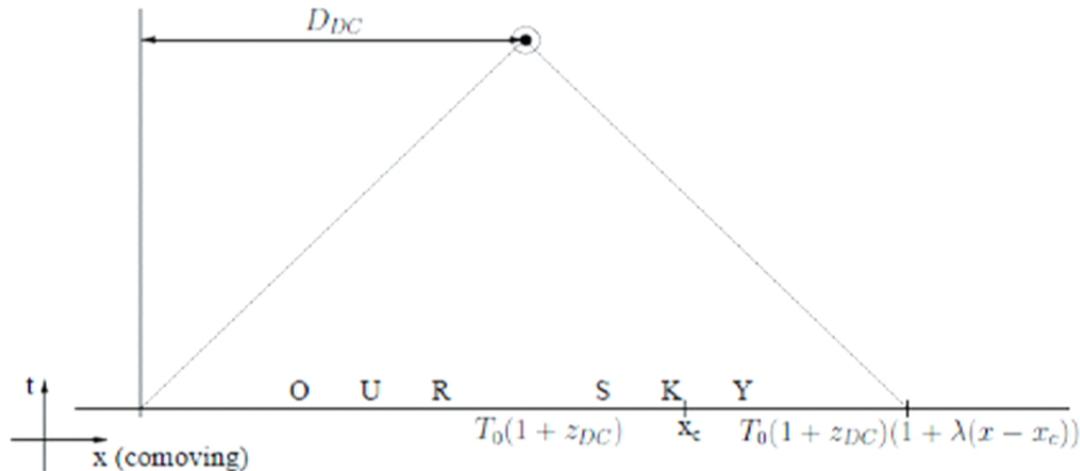


## ① From the reheating surface to a cold / hot spot



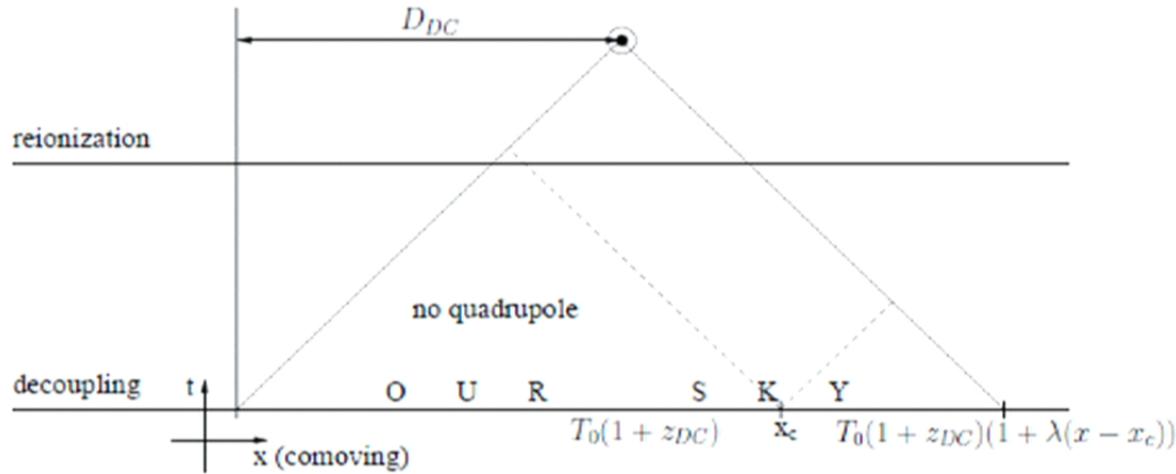
- But it is more convenient to pretend that the reheating surface is a straight line and package the effect of the collision into a temperature profile.
- N.B.  $\lambda$  determines the magnitude of the effect.

## ① From the reheating surface to a cold / hot spot



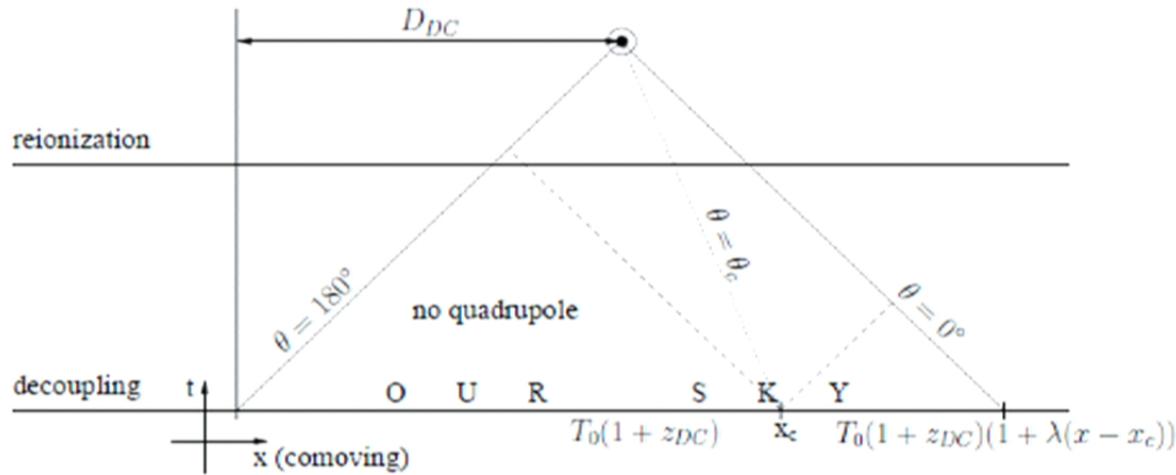
- But it is more convenient to pretend that the reheating surface is a straight line and package the effect of the collision into a temperature profile.
- N.B.  $\lambda$  determines the magnitude of the effect.
- Propagate the profile to the decoupling surface.
- Locate our Sky: each point on this segment is an azimuthal circle.  
 $\therefore$  A collision results in a cold / hot spot on our Sky.  
(There is already a candidate in the CMB.)

## ① Toward CMB Polarization



- Polarization comes from Thomson scattering off electrons that see a quadrupole temperature anisotropy.
- It only depends on  $\theta$ , so it is fully E-mode (Stokes parameter  $Q$ ):
$$Q(\theta) = \frac{\sqrt{6}}{10} \sum_{m=-2}^2 \pm 2 Y_{2m} \int_{D_{DC}}^0 dD g(D) T_{2m}(D \hat{n}_\theta)$$
  - Integrate over  $\theta$ -rays
  - Measure is the “visibility function” – peaked at decoupling and reionization

## ① CMB Polarization



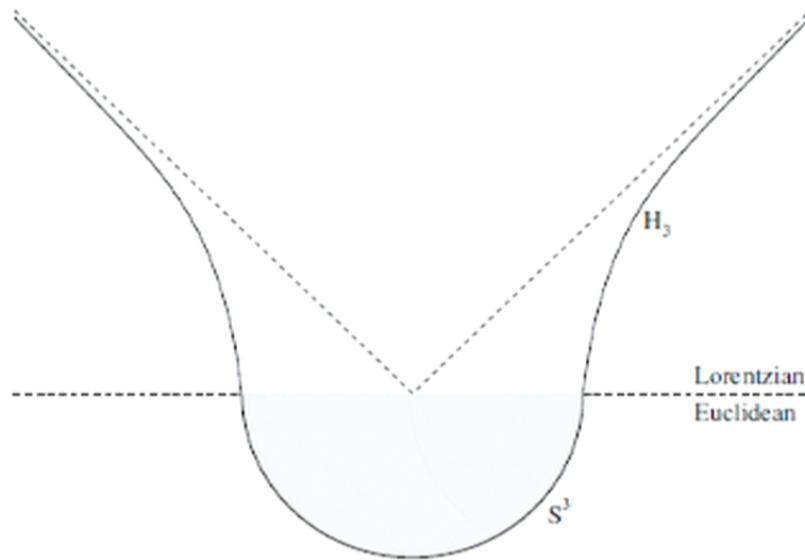
∴ There are two azimuthal peaks:

- narrow, cold / hot spot-sized, from decoupling
  - broad from reionization (this one spills over the whole Sky for small spots)

This will be measured by Planck in the near future.

## ② Are spherical bubbles the whole story?

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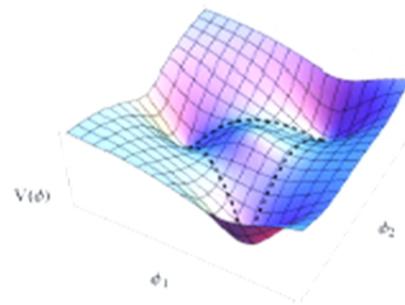


SAGREDO: Yes! Coleman, Glaser, and Martin told us so.

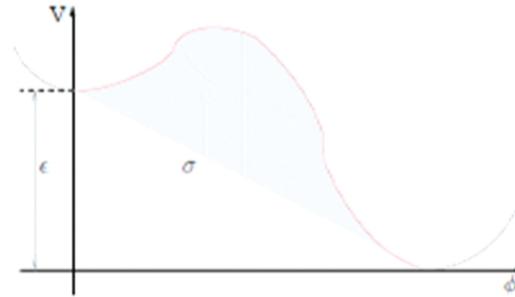
SALVIATI: But their proof only applies when the field space is one-dimensional.  
This is very different from the string landscape.

- More general instantons could significantly alter our picture of eternal inflation.
- From ①, their effects might even be observable.

## ② Setup



$$\{\epsilon_{AB}, \epsilon_{AC}, \epsilon_{BC}, \sigma_{AB}, \sigma_{AC}, \sigma_{BC}\}$$

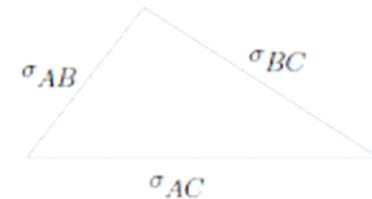


$$\{\epsilon, \sigma\}$$

- As a first step, just do field theory.
- Work in the thin wall approximation.
- The thin wall parameters are subject to relations:

$$\epsilon_{AC} = \epsilon_{AB} + \epsilon_{BC}$$

$$\sigma_{AC} = \min_{A \rightarrow C} \int_A^C dl \sqrt{V(l)} \quad \Rightarrow \quad \text{triangle inequality:}$$



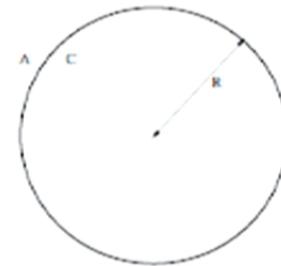
## ② Ansatz

2-vacuum problem

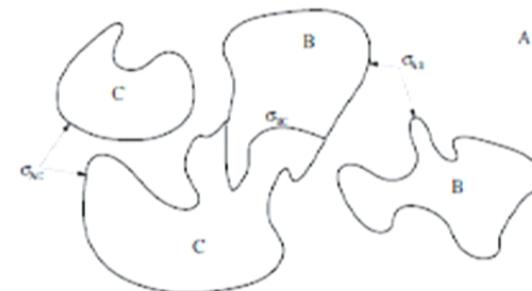


- regions of 2 / 3 vacua separated by walls

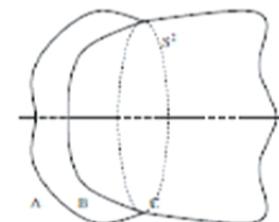
- take a single region
- form a maximally (spherically / cylindrically) symmetric object



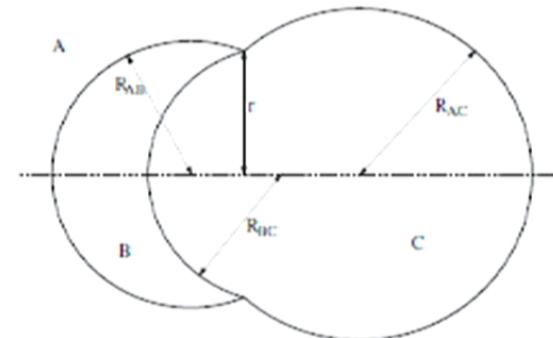
3-vacuum problem



... with a single BC-interface



- find optimal surfaces with an  $S^2$  boundary (junction)



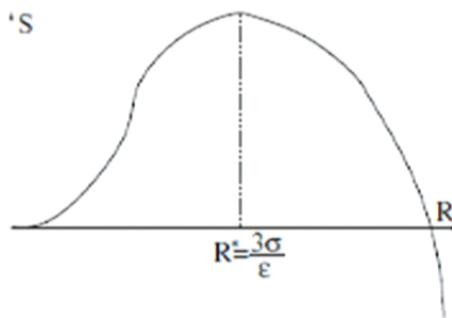
## ② Calculation

2-vacuum problem

parameters:  $R$

action:  $-\epsilon R^4 \text{vol}(B^4) + \sigma R^3 \text{area}(S^3)$

extremize:



negative modes: one -  $R$

$\therefore$

3-vacuum problem

$R_{AB}, R_{AC}, R_{BC}, r$  (junction radius)

$$\begin{aligned} & -\epsilon_{AB} \text{vol}(AB) + \sigma_{AB} \text{area}(AB) \\ & -\epsilon_{AC} \text{vol}(AC) + \sigma_{AC} \text{area}(AC) \\ & -\epsilon_{BC} \text{vol}(BC) + \sigma_{BC} \text{area}(BC) \end{aligned}$$

$$R_X^* = \frac{3\sigma_X}{\epsilon_X}$$

(same as in the 2-vacuum case)

$$\begin{aligned} & r = 0 \text{ (spherical bubble)} \\ & \text{and} \\ & r = r^* \text{ (new)} \end{aligned}$$

Hessian is diagonal:

$$\begin{aligned} \frac{\partial^2 S}{\partial R_X \partial R_Y} &= 0 \\ \frac{\partial^2 S}{\partial r \partial R_X} &\propto R_X - \frac{3\sigma_X}{\epsilon_X} = 0 \text{ (by E.O.M.)} \end{aligned}$$

count negative modes:

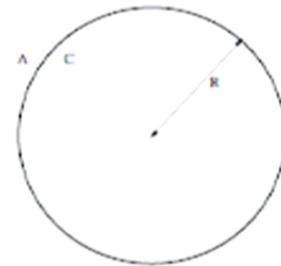
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2-vacuum problem

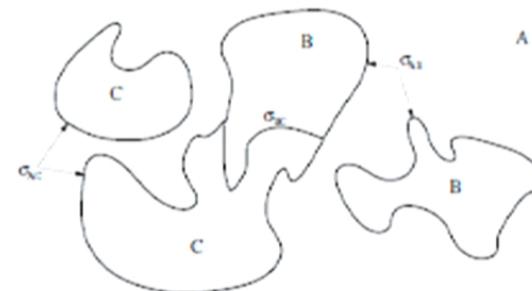


- regions of 2 / 3 vacua separated by walls

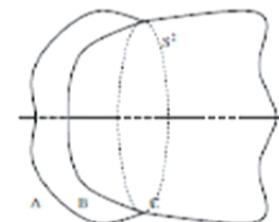
- take a single region
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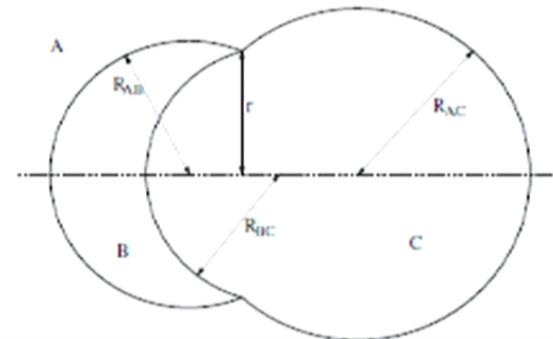
3-vacuum problem



... with a single BC-interface



- find optimal surfaces with an  $S^2$  boundary (junction)



## ② Negative modes

$$\frac{\partial^2 S}{\partial R_X^2} \begin{cases} < 0 & \text{if } X \text{ is bigger than a hemisphere} \\ > 0 & \text{if } X \text{ is smaller than a hemisphere} \end{cases}$$
$$\frac{\partial^2 S}{\partial r^2} \quad \text{- obtain by analyzing } S(r):$$

- Because  $S(r)$  has two extrema at 0 and  $r^*$ :

$$\frac{\partial^2 S}{\partial r^2}|_{r=r^*} > 0 (< 0) \Leftrightarrow r=0 \text{ is a local max (min) of } S(r)$$

- But  $\frac{\partial^2 S}{\partial r^2}|_{r=0} = 0 \Rightarrow$  this requires explanation

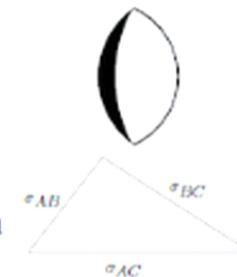
$\Rightarrow$  we must go to cubic order:

- $\frac{1}{8\pi} \frac{\partial^3 S}{\partial r^3}|_{r=0} = \pm \sigma_{AB} \pm \sigma_{AC} \pm \sigma_{BC} \gtrless 0 \Leftrightarrow \frac{\partial^2 S}{\partial r^2}|_{r=r^*} \lessgtr 0$   
+ (-) sign for regions smaller (bigger) than a hemisphere

- We want exactly 1 negative mode:

case (1):  $S_{rr} < 0 \Rightarrow$  all three  $S_{RR} > 0 \Rightarrow$   
three smaller-than-hemisphere regions

case (2): one  $S_{RR} < 0 \Rightarrow$  exactly two  $S_{RR} > 0 \Rightarrow$   
two smaller-, one bigger-than-hemisphere region



$\therefore$  All non-trivial saddle points have 2 or more negative modes.

## ② Loose end

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$$\frac{\partial^2 S}{\partial r^2} \Big|_{r=0} = 0$$

SAGREDO:  $r = 0$  is the good, old spherical instanton.

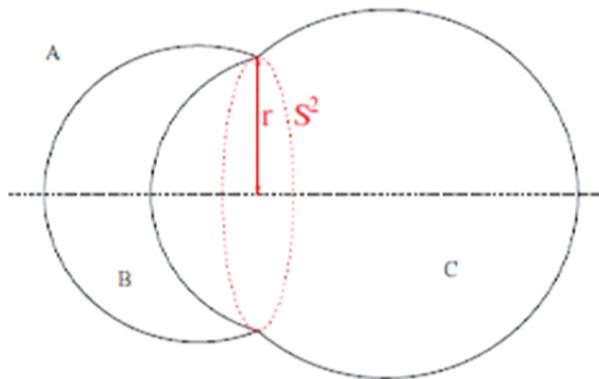
Does this mean that it has a non-translational zero mode?

Does it enhance the nucleation rate?

SALVIATI: No, because we neglected a quadratic piece of the action.

It arises from the cost of creating a junction:

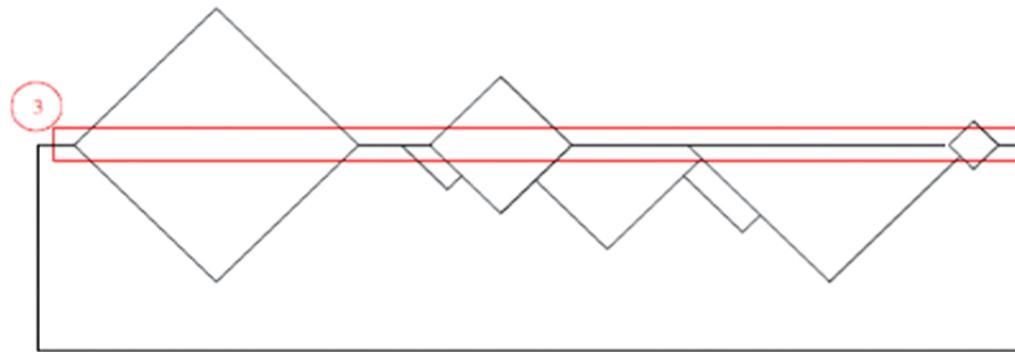
$$S = S_{\text{before}} + \kappa r^2$$



- In the thin-wall approximation, codimension-2 junctions generalize objects of codimension-1 (walls) and codimension-0 (vacua).
- Microscopically, junction tensions depend on hills in the landscape.
- They are necessary to resolve the apparent zero modes.

### ③ Topology at future infinity

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Motivation:

- well-defined (independent of slicing)
- independent of the measure problem
- theoretical significance (e.g. for FRW / CFT)
- mathematically fun

### ③ Discretization

- Re-draw diagram in comoving coordinates:
- Bubbles attain a fixed comoving size:

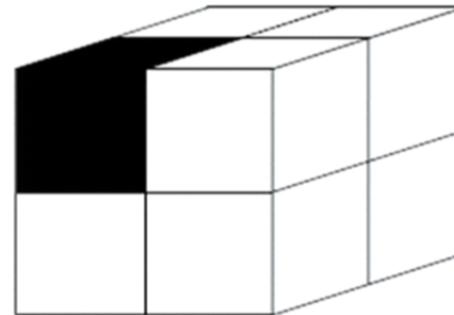
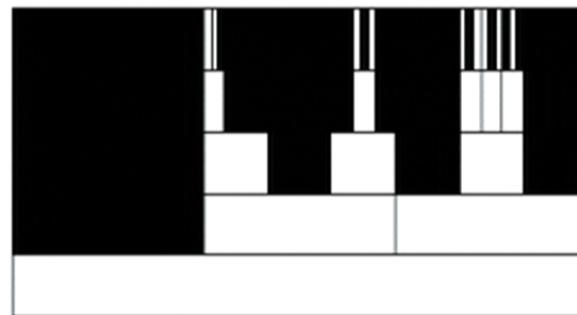
$$c = 1 = Ha\Delta x = \dot{a}\Delta x$$

(Hubble radius in comoving coordinates)

$$\Delta x = (\dot{a})^{-1} \propto e^{-t} \text{ in de Sitter}$$

- Re-draw diagram with discrete cells:
- Set  $\Delta x = (\dot{a})^{-1} \propto e^{-t}$
- After time  $\Delta t$ , the spatial cell size decreases by a factor  $\frac{\dot{a}(t)^{-1}}{\dot{a}(t+\Delta t)^{-1}}$
- Set  $\Delta t$  so this ratio is a natural number.
- Here  $N = 3$ .

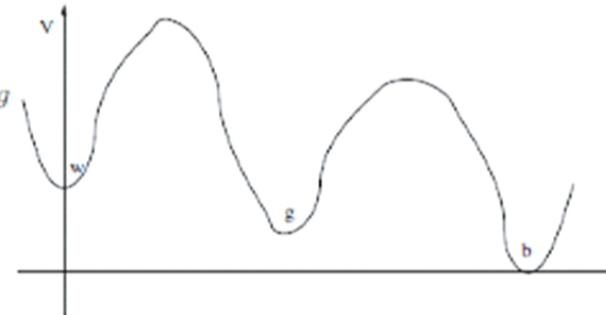
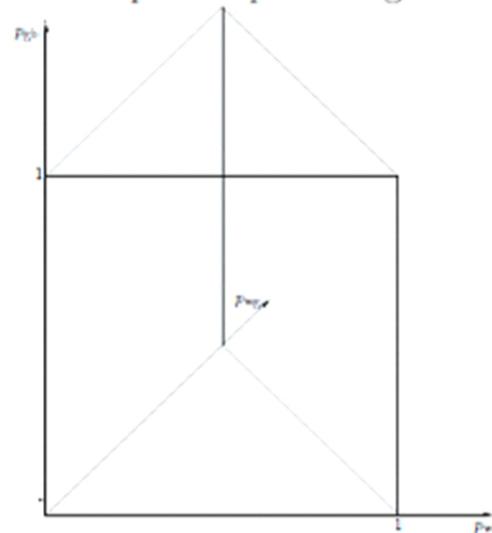
- This defines the Mandelbrot model (in 3 dimensions)
- 2 colors  $\leftrightarrow$  vacua; 2 parameters:  
 $N^3 = \# \text{ of daughter cells} \sim e^{3H\Delta t}$   
 $p = \text{prob. of coloring / nucleation} \sim \Gamma(\Delta x)^3 \Delta t$



### ③ Generalize to three vacua

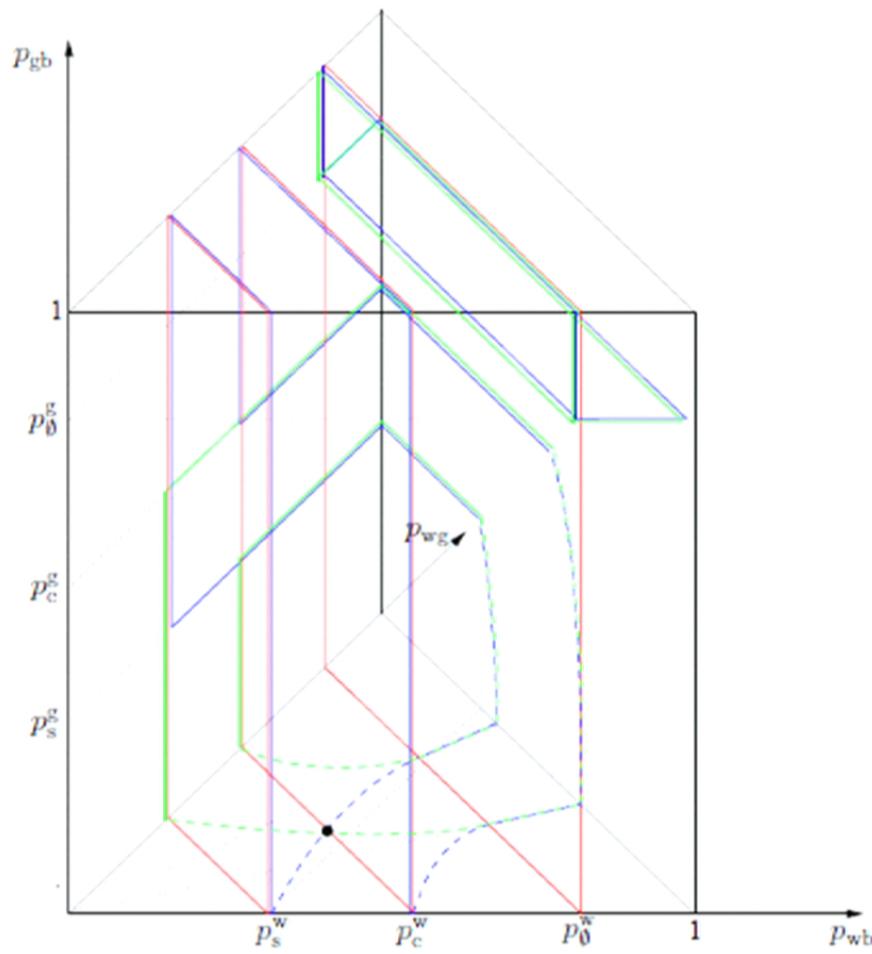
- Consider the 3-vacuum system.
- There are 5 parameters:  $p_{wg}$ ,  $p_{wb}$ ,  $p_{gb}$ ,  $N_w$ ,  $N_g$
- But a shift in  $N_w$ ,  $N_g$  can always be undone by a compensating shift in the probabilities

∴ The phase space diagram will look like this:



- Proceed by bootstrapping results from the 2-vacuum system.
- Example: white islands:  
2-vacuum:  $p_c \leq p_{wb} \leq p_\emptyset$   
3-vacuum:  $p_c \leq p_{wb} + p_{wg} \leq p_\emptyset$

### ③ The 3-vacuum phase diagram



### ③ Lessons

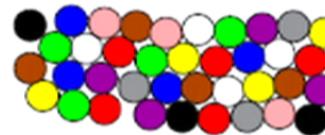
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- In the 2-vacuum case, we had crossing surfaces or two colors of crossing curves.
- In the 3-vacuum case, much of the phase diagram is occupied by phases:

1) white crossing curves	gray islands	black islands
2) white islands	gray crossing curves	black islands
3) white islands	gray islands	black crossing curves

∴ In the many-vacuum case, all colors will be generically present in island form.

∴ The “grainy phase” is generic.



∴ This leads to the following picture of eternal inflation:

## Summary

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- ① We predicted CMB polarization patterns, which could corroborate the string landscape.
- ② We excluded previously unconsidered, putative instantons, which would combine regions of two true(-r) vacua.
- ② We appreciated the role of “junctions” for regulating zero modes in thin-wall calculations of nucleation rates.
- ③ We saw that interesting topology may arise in eternal inflation, but mostly in the later generations and on the intra-bubble scale.

THANK YOU!