Title: Gravitational Turbulent Instability of Anti-de Sitter Space

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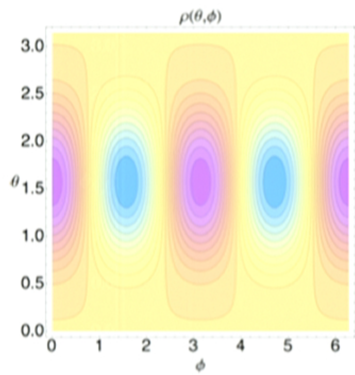
Abstract: Bizon and Rostworowski have recently suggested that anti-de Sitter spacetime might be nonlinearly unstable to transfering energy to smaller and smaller scales and eventually forming a small black hole. We consider pure gravity with a negative cosmological constant and find strong support for this idea. While one can start with a single linearized mode and add higher order corrections to construct a nonlinear geon, this is not possible starting with a linear combination of two or more modes. One is forced to add higher frequency modes with growing amplitude. The implications of this turbulent instability for the dual field theory are discussed.

Gravitational turbulent instability of AdS

Jorge E. Santos



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- In this work we want to study far from equilibrium dynamics in gravity, and try to understand its field theory interpretation.
  - A poor's man approach: break down of perturbation theory - onset of interesting dynamics.

Anti-de Sitter spacetime - 1/2

Anti-de Sitter space is a maximally symmetric solution to

$$S = \frac{1}{16\pi G}\int \mathrm{d}^d x\sqrt{-g}\left[R + \frac{(d-1)(d-2)}{L^2}\right],$$

which in global coordinates can be expressed as

$$\mathrm{d}s^2 \equiv \bar{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b = -\left(\frac{r^2}{L^2}+1\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\frac{r^2}{L^2}+1} + r^2\mathrm{d}\Omega^2_{d-2}.$$

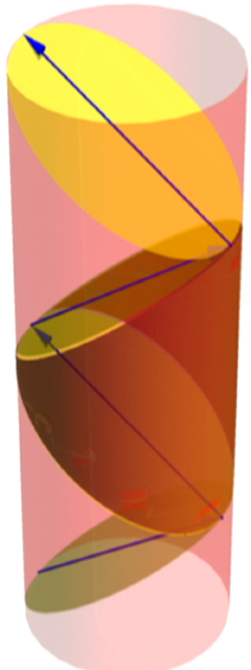
The Poincaré coordinates

$$\mathrm{d}s^2 = R^2(-\mathrm{d}\tau^2 + \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}) + \frac{L^2\mathrm{d}R^2}{R^2}$$

do not cover the entire spacetime.

Anti-de Sitter spacetime - 2/2

Conformally, AdS looks like the interior of a cylinder



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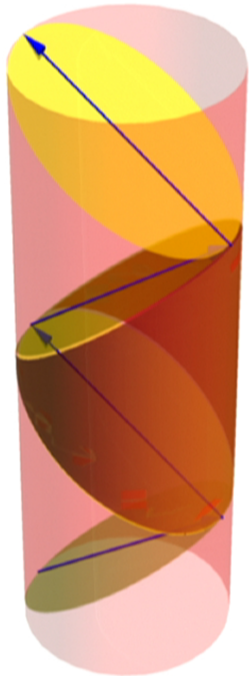

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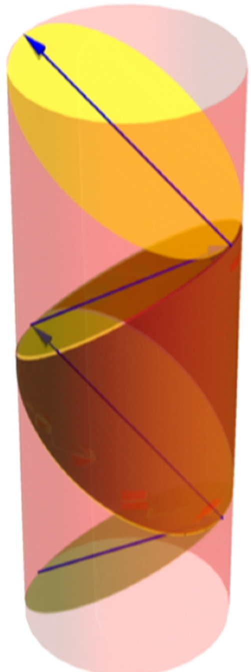

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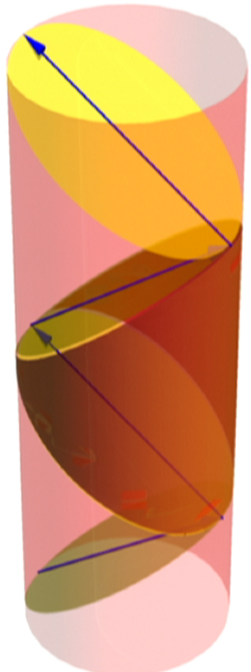

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- The instability described in this talk will occur in global AdS only.
- The dual field theory lives on $\mathbb{R}_t \times S^{d-2}$.
- With energy preserving boundary conditions, waves bounce off infinity and return in finite time.

Minkowski, dS and AdS spacetimes

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- The energy cascades from low to high frequency modes in a manner reminiscent of the onset of turbulence.

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The short answer, is *NO* :

- Positivity energy theorem: if matter satisfies the dominant energy condition, then $E \geq 0$ for all nonsingular, asymptotically AdS initial data, being zero for AdS only.

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- Positivity energy theorem: if matter satisfies the dominant energy condition, then $E \geq 0$ for all nonsingular, asymptotically AdS initial data, being zero for AdS only.
  - This ensures that AdS cannot decay.
  - It does not ensure that a small amount of energy added to AdS will not generically form a small black hole.

Folklore - 2/2

Example (Dafermos):

Consider the following action:

$$S = \int \mathrm{d}^d x \sqrt{-g}\left[R - (\nabla\phi)^2\right].$$

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Consider now the same action, but with the *wrong* sign for the scalar kinetic term:

$$S = \int \mathrm{d}^d x \sqrt{-g}\,\left[R + (\nabla\phi)^2\right].$$

There is no positivity energy theorem, but Minkowski space is still nonlinearly stable.

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- Special (fine tuned) solutions need not lead to the formation of black holes.
  - We will see that for each linearized gravitational *mode* there will be a corresponding nonlinear solution - geon.
  - These solutions are special since they are exactly periodic in time and invariant under a single continuous symmetry.

Perturbative construction - 1/3

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where $T^{(i)}$ depends on $\{h^{(j \leq i-1)}\}$ and their derivatives and

$$2\tilde{\Delta}_L h_{ab}^{(i)} \equiv -\bar{\nabla}^2 h_{ab}^{(i)} - 2\bar{R}_a{}^c{}_b{}^d h_{cd}^{(i)} - \bar{\nabla}_a \bar{\nabla}_b h^{(i)} + 2\bar{\nabla}_{(a} \bar{\nabla}^c h_{b)c}^{(i)}.$$

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- Any smooth symmetric two-tensor can be expressed as a sum of fundamental building blocks, $\mathcal{T}^{\ell m}_{ab}$, that have definite transformation properties under the $SO(d-1)$ subgroup of AdS.

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- We go beyond linear order: need real representation for $Y_{\ell m}$ - $Y^c_{\ell m} = \cos\phi \, \mathrm{L}^m_\ell(\theta)$ and $Y^s_{\ell m} = \sin\phi \, \mathrm{L}^m_\ell(\theta)$.

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- At each order, we can reduce the metric perturbations to 4 gauge invariant functions satisfying (Kodama and Ishibashi '03 for $i = 1$):

$$\Box_2 \Phi^{\alpha,(i)}_{\ell m}(t,r) + V^{(i)}_\ell(r) \Phi^{\alpha,(i)}_{\ell m}(t,r) = \tilde{T}^{\alpha,(i)}_{\ell m}(t,r),$$

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  where $\alpha \in \{c, s\}$ and $\Box_2$ is the w. op. in the (t,r) orbit space.
- Choice of initial data relates $\Phi^{c,(i)}_{\ell m}$ and $\Phi^{s,(i)}_{\ell m}$: 2 PDEs to solve.

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- This is also the choice that conforms with finite energy for the standard definition of "*gravitational energy*".

Linear Perturbations

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- Because AdS acts like a confining box, only certain frequencies are allowed to propagate

$$\omega_\ell^2 L^2 = (1 + \ell + 2p)^2,$$

where p is the radial overtone. These are the so-called normal modes of AdS. The fact that $\omega^2 L^2 > 0$ means that AdS is linearly stable.

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4. If $\tilde{T}_{\ell m}^{\alpha,(i+1)}(t,r)$ has an harmonic time dependence $\cos(\omega\, t)$, then $\Phi_{\ell m}^{\alpha,(i+1)}(t,r)$ will exhibit the same dependence, EXCEPT when ω agrees with one of the normal frequencies of AdS:

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5. If for a given perturbation one can construct $\Phi^{\alpha,(i)}_{\ell m}$ to any order, without ever introducing a term growing linearly in time, the solution is said to be stable and is unstable otherwise.

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- The structure of the equations indicate that there is only one resonant term at each odd order, and that the amplitude of the growing mode can be set to zero by correcting the frequency.
- One can compute the asymptotic charges to fourth order, and they readily obey to the first order of thermodynamics:

$$E_g = \frac{3J_g}{2L}\left(1 - \frac{4901\,J_g}{7560\pi L^2}\right), \quad \omega_2 = \frac{3}{L}\left(1 - \frac{4901\,J_g}{3780\pi L^2}\right),$$

where we defined ϵ by $J_g = \frac{27}{128}\pi L^2 \epsilon^2$.

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- At the nonlinear level, we have the same type of symmetry, but ω changes.
- Resonances occur because normal modes of AdS take integer values:
    - Geons are likely to be "*more*" stable than AdS because the normal modes of the Geons correspond to continuous deformations of the AdS normal modes.

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- Spherical scalar field collapse in AdS - Bizon and Rostworowski, '11.

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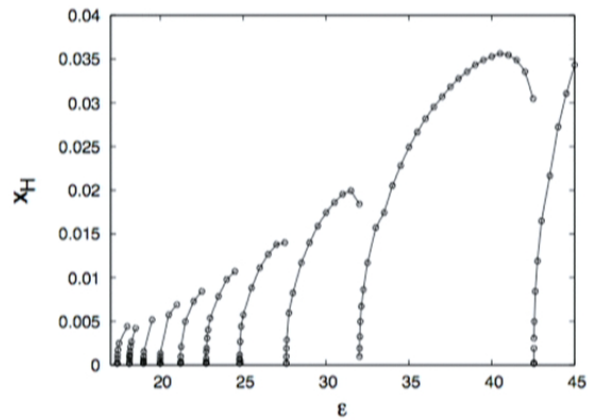
- Spherical scalar field collapse in AdS - Bizon and Rostworowski, '11.
- No matter how small you make the initial amplitude, the curvature at the origin grows and you eventually form a small black hole.



What happens in String Theory?

Consider IIB string theory on $\mathrm{AdS}_5 \times S^5$, with AdS length scale L.

There are two energy scales: the Planck energy and the string energy $E_s < E_p$.

Example II - 2/2

- The frequency of the growing mode is higher than any of the frequencies we started with!
- The "energy" is transferred to modes of higher frequency.
- Expect this to continue. When the $\omega L = 7$, $\ell = m = 6$ mode grows, it will source even higher frequency modes with growing amplitude.

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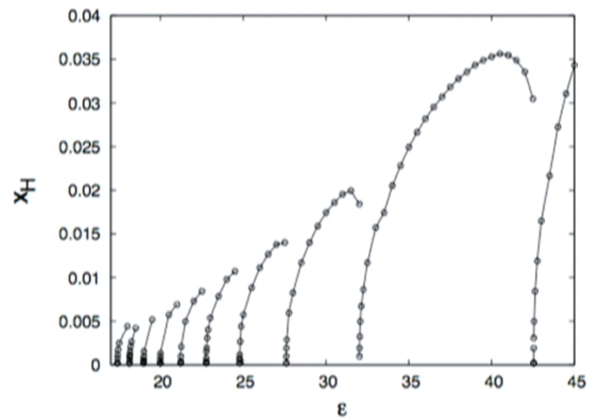

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- If the initial energy is $E_s < E < E_{corr}$, one forms an excited string.
- If $E < E_s$, the cascades stops at frequencies $\omega = E$, and one gets a gas of particles in AdS.

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In 2+1 dimensions, classical turbulence has an inverse energy cascade due to an extra conserved quantity - the enstrophy. Our results indicate that in a strongly coupled quantum theory, there is a standard energy cascade.

Caveat: This intuition comes from solving the Navier Stokes equations in 2+1 dimensions. Because our regime is non-hydro, we don't know how to define enstrophy.

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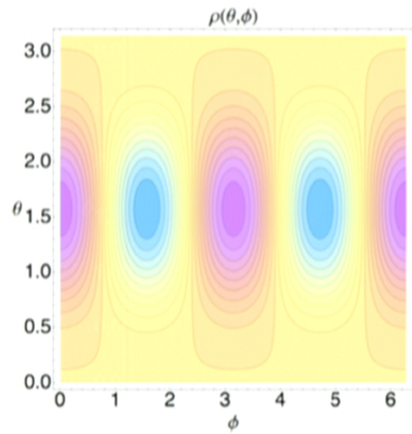

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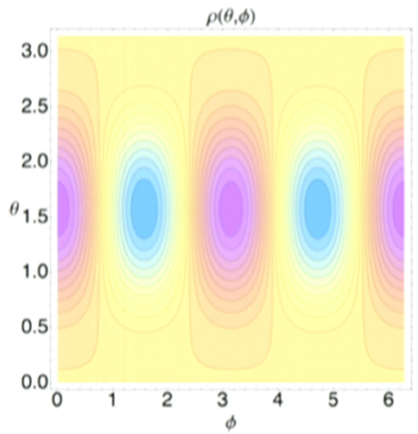
$$K = \frac{\partial}{\partial t} + \frac{\omega}{m}\frac{\partial}{\partial \phi},$$

which is timelike near the poles but spacelike near the equator.



Gravitational hairy black holes with a single $U(1)$ - 1/2.

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Two possibilities:

1. If the black hole absorbs the higher frequency modes faster than they can be created, might stabilize with gravitational waves sloshing around outside the black hole .

Conclusions & Open questions

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Open questions:

- Understand why the energy cascade in 2+1 quantum theory is different from the classical theory.
- Prove a singularity theorem for anti-de Sitter.
- Understand the late time behavior of the superradiance instability.
- Understand the space of CFT states that do not thermalize.