Title: Crossing Tsirelson's Bound with Super Non-localStates

Date: Oct 12, 2011 04:00 PM

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Abstract: We construct a class of entangled supersymmetric states which is used as a non-local resource in the CHSH game. This class of super entangled states is more non-local then maximally entangled states if the supersymmetric degrees of freedom are accessible to measurement.
Consequently, we show that the winning probability for the CHSH game is greater than cos2(pi/8) corresponding to an expected value greater than Tsirelson's bound.

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- Theoretical proposal of the first ‘physical’ model aspiring to provide states more non-local than usual QM
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- ‘Physical’ because supersymmetry is a beautiful but so far only theoretical concept
- Condensed matter physics is an option
- For now can only hypothesize what is actually measurable on these

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SUperSYmmetry does not supersede QM but rather extends it

To add (disperse) confusion:

1. SUSY QM is something different than the topic of this talk (it is a different supergroup)

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To add (disperse) confusion:

1. SUSY QM is something different than the topic of this talk (it is a different supergroup)
2. SUSY is usually studied on supermanifolds being the super extension of the Poincare group

What is it about?

- The key notion is a **superqubit** with the following concepts playing a key role:

Duff et al PRA 2010, Grosse et al CMP 1997, Bartocci et al JMP 1990, Balantekin et al JMP 1988

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2. Grassmann algebra
3. Superstar operation
4. Graded adjoint (superadjoint)

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- All materialize in Lie superalgebras, namely osp(1|2)

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Grassmann algebra

$\mathbb{Z}_2$ - graded vector space *W*

grade

$$W = W^{[0]} \oplus W^{[1]}$$

Grassmann algebra

Grassmann (exterior) algebra $\Lambda_N(W)$ is a $\mathbb{Z}_2$ - graded vector space *W*

grade

$$W = W^{[0]} \oplus W^{[1]} = \bigoplus_{k=0}^{N} W_k$$

$$\dim W = 2^{N-1} | 2^{N-1}$$

odd dimension

even dimension

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We define a product (the wedge product) $[,] : W \times W \mapsto W$

Let w_i be a homogeneous element of grade $i \in \{0, 1\}$

$$[w_i, w_j] = (-1)^{ij} [w_j, w_i]$$

The GA is an example of a super vector (or supercommutative) space

Grassmann algebra

Let η_i be generators of the GA $\Lambda_N(W)$

Supernumber: $\xi \in \Lambda_N(W)$ $\quad \xi = \xi_o + \xi_e$

Odd and even subspace

$$\xi_e = z_0 + \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} \frac{1}{(2k)!} z_I \eta^I$$

$$\xi_o = \sum_{k=0}^{\lfloor \frac{N+1}{2} \rfloor} \frac{1}{(2k+1)!} z_J \eta^J$$

$$z_0, z_I, z_J \in \mathbb{C}$$

$$\eta^I = \eta^{i_1} \wedge \ldots \wedge \eta^{i_{2k}} \in W_e$$

$$\eta^J = \eta^{j_1} \wedge \ldots \wedge \eta^{j_{2k+1}} \in W_o$$

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soul

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Grassmann algebra

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- Both comes from the 'reality condition'

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$$\mathrm{conj}(\eta \wedge \mathrm{conj}(\eta)) = \eta \wedge \mathrm{conj}(\eta)$$

- The usual bar operator (the star operation)

$$\overline{\eta_i \eta_j} = \overline{\eta}_j \overline{\eta}_i$$

- The unusual hash operator

$$(\eta_i \eta_j)^{\#} = \eta_i^{\#} \eta_j^{\#}$$

and

$$(\eta^{\#})^{\#} = -\eta$$

It's also called the **superstar** (**super conjugate**) operation

$\mathbb{Z}_2$- graded vector spaces (finite-dimensional)

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- Super linear algebra generalizes many concepts from ordinary LA
- Supermatrices, supertrace, supertranspose…
- A super Hilbert space is $\mathbb{Z}_2$- graded vector spaces with a positive-definite form
- We need a super Hilbert space that reduces to the qubit Hilbert space

Lie superalgebras

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- Lie superalgebra is $\mathbb{Z}_2$- graded vector space $W = W^{p|q}$
  with a product $[,] : W \times W \mapsto W$
  satisfying $[w_i, w_j] = -(-1)^{ij}[w_j, w_i]$

$$(-1)^{ik}[w_i,[w_j,w_k]] + (-1)^{ji}[w_j,[w_k,w_i]] + (-1)^{kj}[w_k,[w_i,w_j]] = 0$$

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We define the graded commutator

$$[w_i, w_j] = w_i w_j - (-1)^{ij} w_j w_i$$

Commutator for $\{ij\} = \{00, 01, 10\}$

Anticommutator for $\{ij\} = \{11\}$

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  Example: $gl(p|q) \stackrel{\mathrm{df}}{=} \mathrm{End}(W^{p|q})$

The osp(1|2) algebra

- It is called the orthosymplectic algebra of dimension 1|2

$$osp(1|2) = \{X \in gl(1|2) | X^{ST} g + gX = 0\}$$

$$g = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

ST – supertranspose

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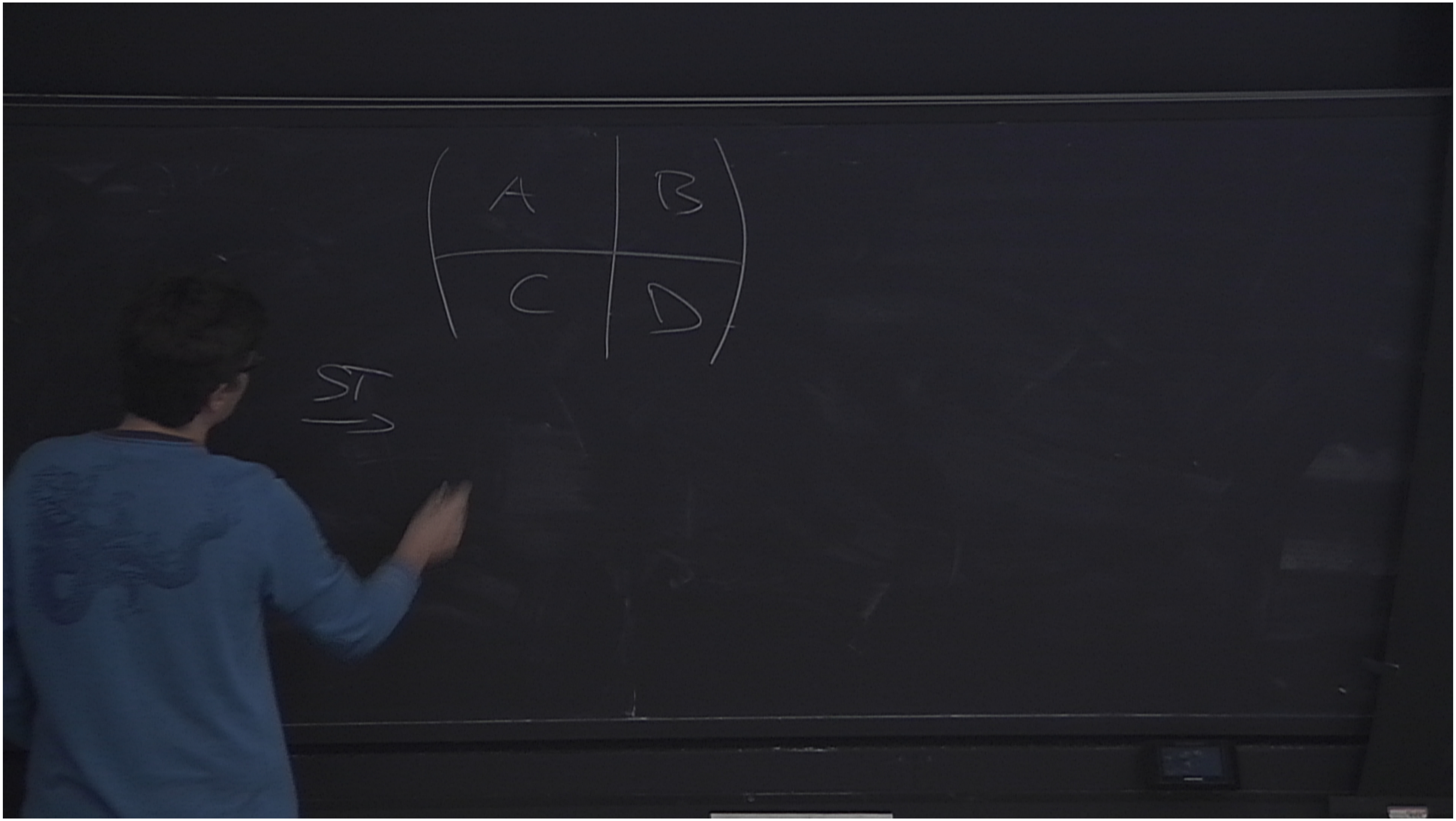
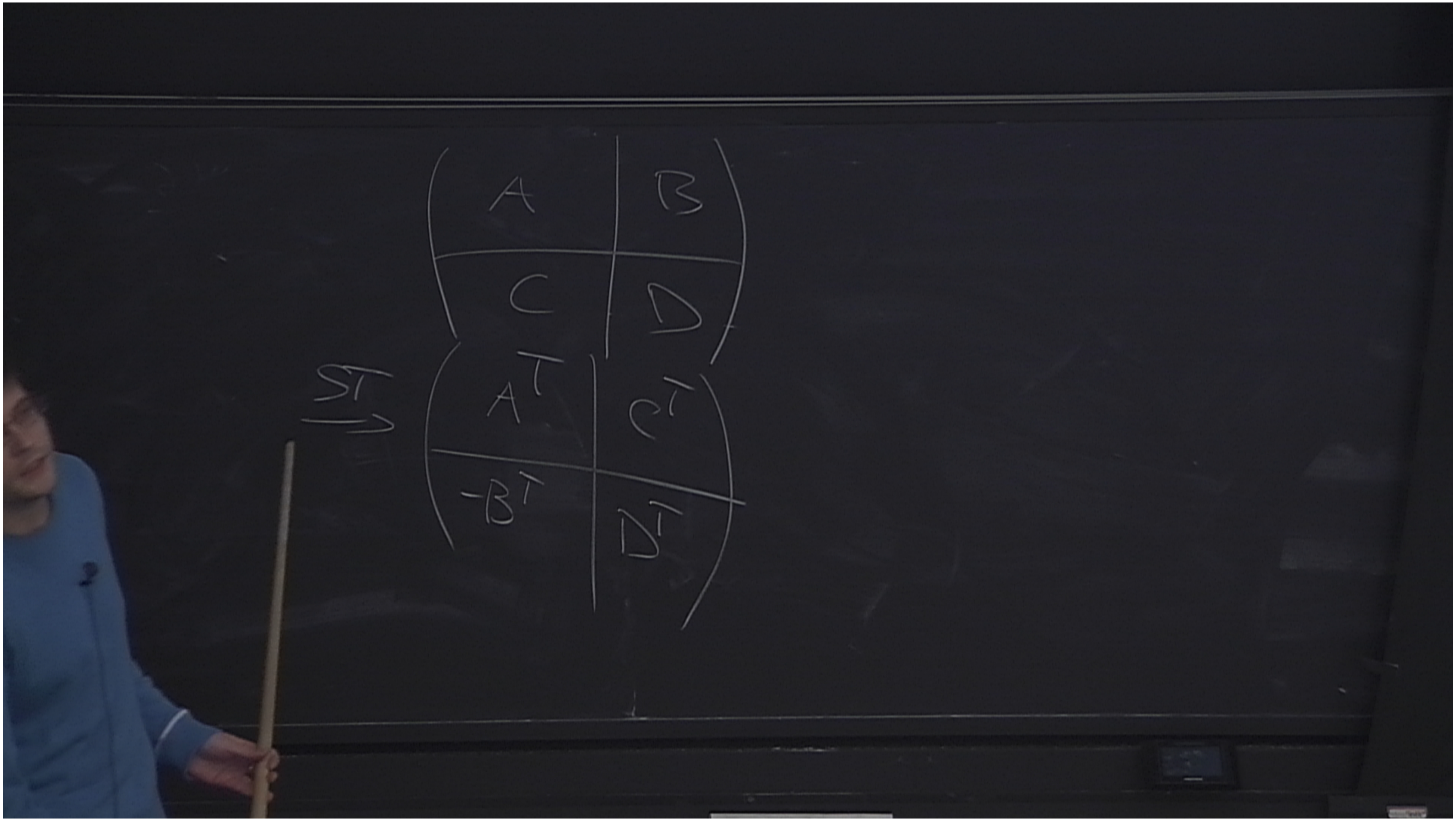
- We define a compact real form (unitary orthosymplectic algebra)

$$uosp(1|2) = \{X \in osp(1|2) | X^{\ddagger} = -X\}$$

$$X^{\ddagger} = (X^{ST})^{\#}$$

The usual adjoint 'trivializes' the osp(1|2) algebra


A
B
C
D
ST


A B
C D
ST
A^T C^T
$-B^T$ D^T

The uosp(1|2) algebra

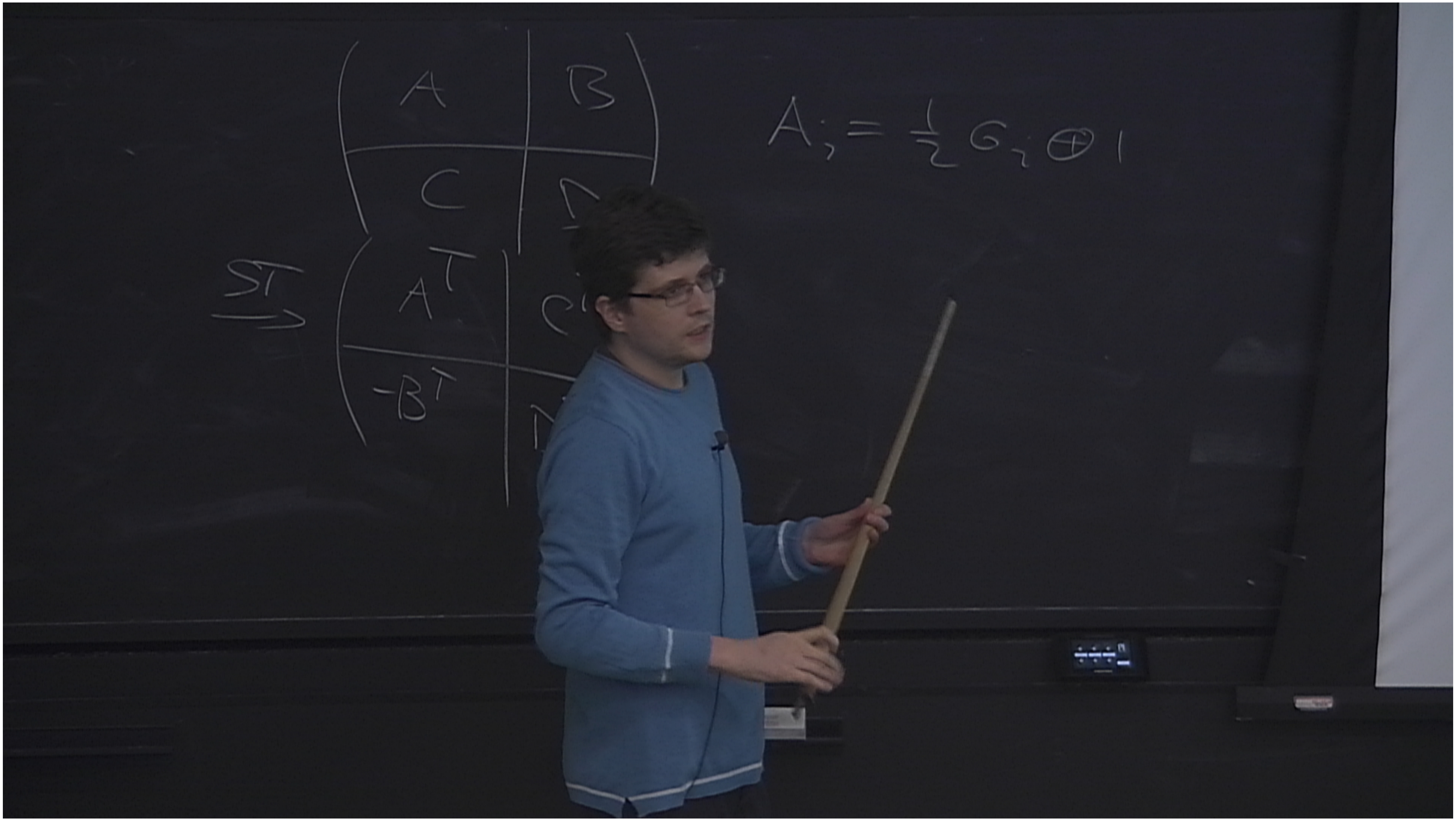
- An arbitrary element:

$$X = \underbrace{a_1 A_1 + a_2 A_2 + a_3 A_3}_{su(2)} + \eta^{\#} Q_- + \eta Q_+$$

$a_k \in \Lambda_N(W^{[0]})$

even supernumbers

- The even part (called bosonic) is a subalgebra



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$\eta, \eta^{\#} \in \Lambda_N(W^{[1]})$

odd supernumbers

$$Q_- = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad Q_+ = \frac{1}{2}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$[A_i, Q_\alpha]_- = \frac{1}{2}(\sigma_i)_{MN} Q_\beta$$

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- The odd part (fermionic) is not closed wrt the graded commutator

The UOSP(1|2) group

- We define

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Berezin, CMP 1981

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$$UOSP(1|2) = \{U = \exp X | X \in uosp(1|2)\}$$

- The anti self-superadjoint condition becomes the superunitary condition

$$U^{\ddagger}U = U^{\ddagger}U = 1$$

Berezin, CMP 1981

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Superqubits

- Superqubits carry a (projective) rep of UOSP(1|2)
- The orthogonal basis is $\{|0\rangle, |1\rangle, |\bullet\rangle\}$

$$|\psi\rangle = \left(1 - \tfrac{1}{8}\eta\eta^{\#}\right)\underbrace{(\alpha|0\rangle + \beta|1\rangle)}_{\text{normalized qubit}} - \tfrac{1}{2}\eta|\bullet\rangle \qquad \alpha, \beta, \eta\eta^{\#} \in \Lambda_N(W^{[0]})$$

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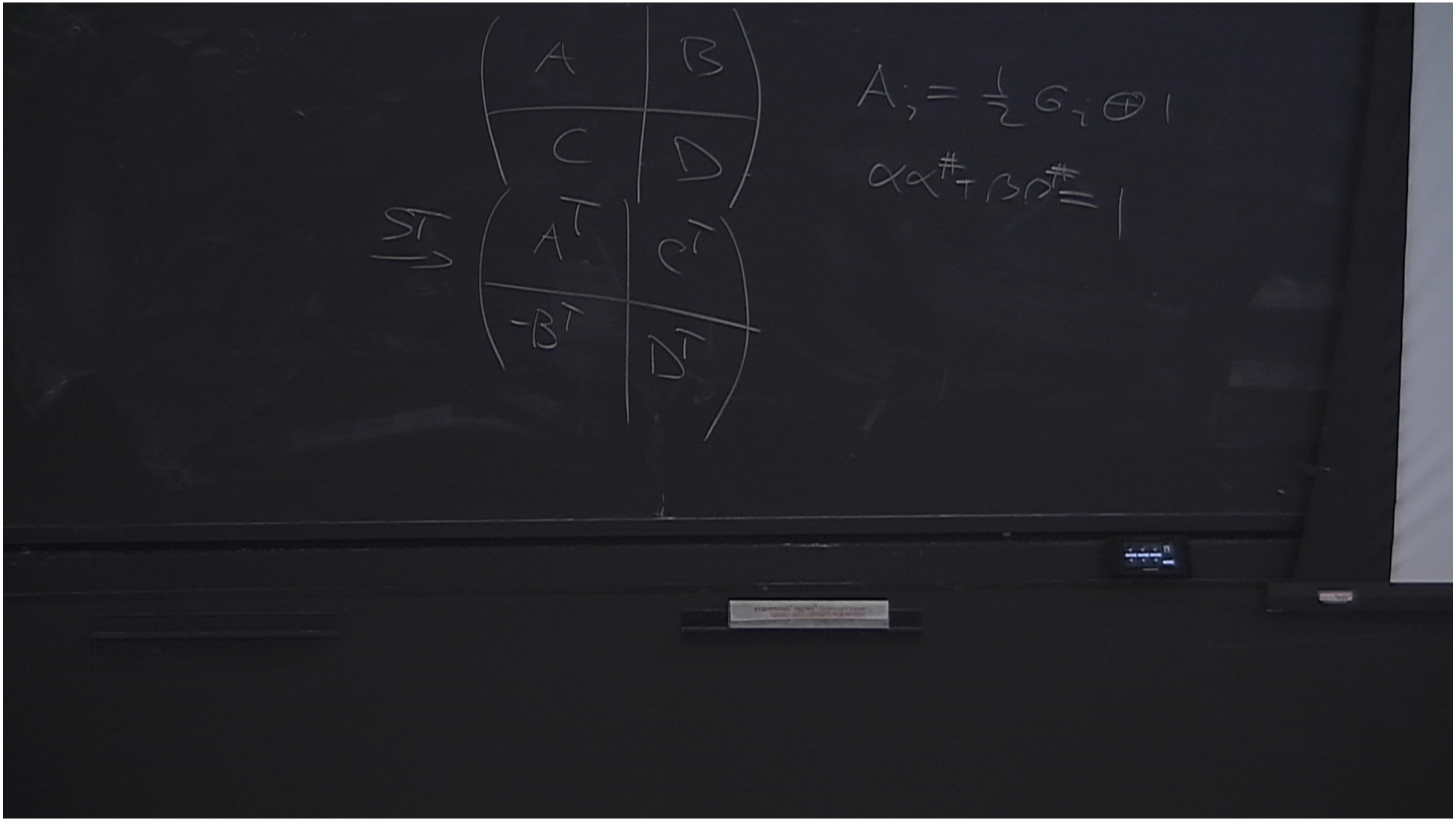
This is how one can mix fermions and bosons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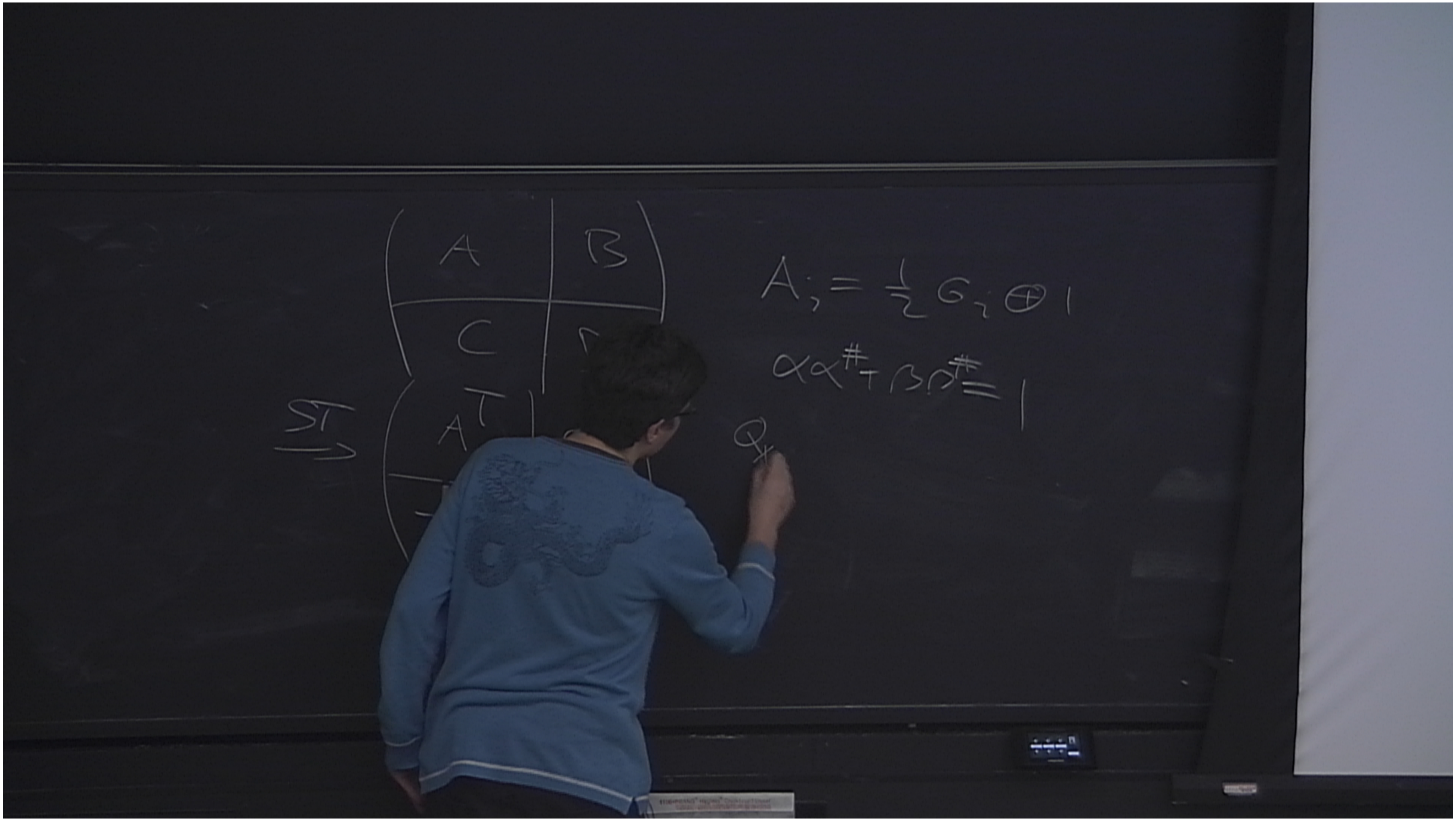
$$|\bullet\rangle = f^{\ddagger}|vac\rangle$$

The bullet state is a fermion but not the usual one

This is how one can mix fermions and bosons

- The state is normalized
- The body part of the norm is positive (and normalized too)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C
ST
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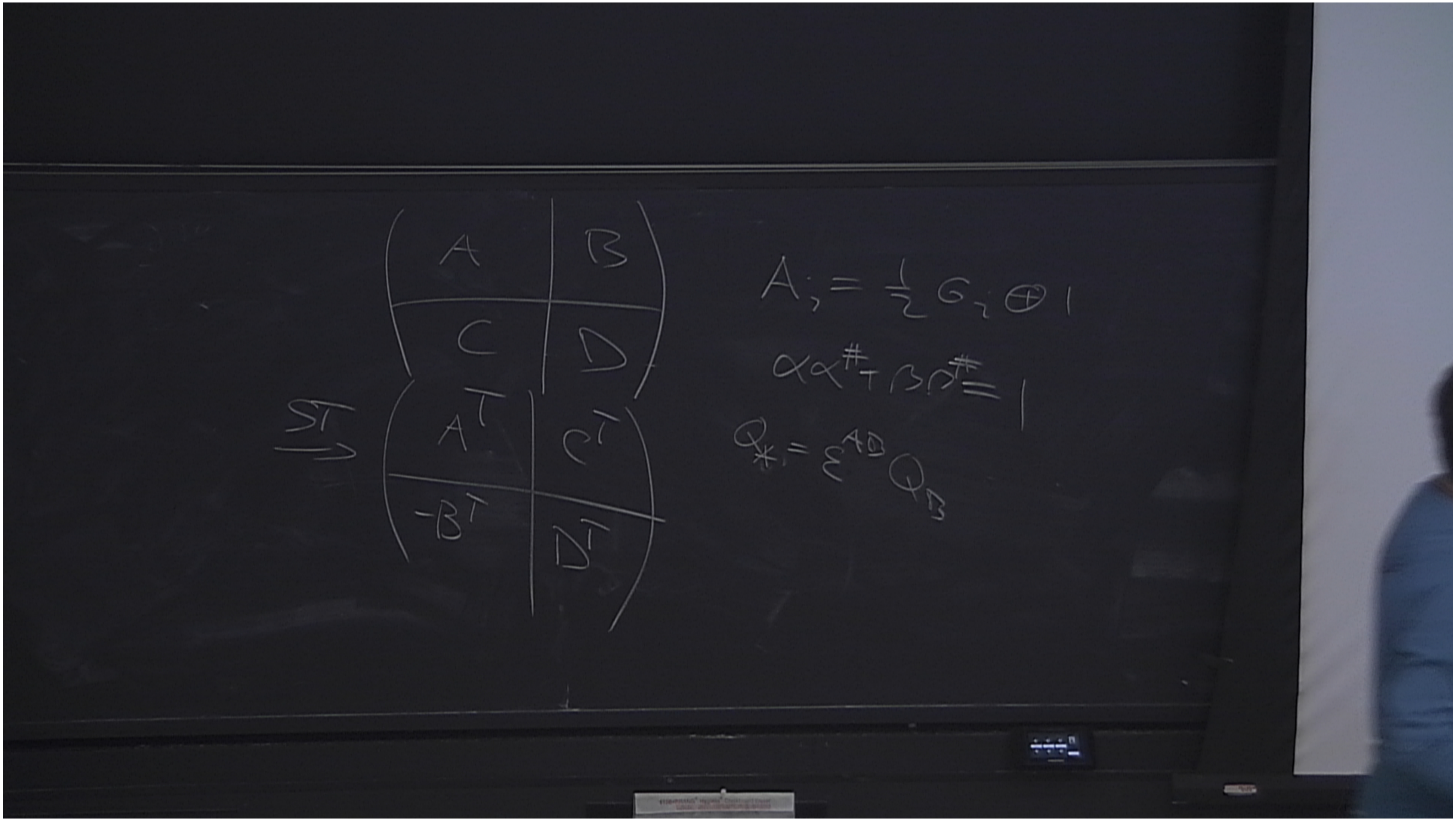
A
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- The superadjoint action:

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$$\downarrow \ddagger$$

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- It forms a density matrix whose eigenvalues are 'real'
- It is a superhermitian (self-superadjoint) matrix
- Its supertrace is one

$$|\psi\rangle\langle\psi| = \begin{pmatrix} -\frac{1}{4}\eta\eta^{\#} & -\frac{1}{2}\eta \\ \frac{1}{2}\eta^{\#} & 1 - \frac{1}{4}\eta\eta^{\#} \end{pmatrix}$$

Superqubits

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The problem of measurement

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The problem of measurement

- The CHSH game is better

Superqubits

- It does not seem to be that straightforward as in ordinary QM

  (metric – norms – probabilities)
- We cannot interpret or order Grassmann numbers

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- We define the order of products of **even** Grassmann numbers and give a physical meaning to their coefficients
- Only even Grassmanns are given by the Grassmann-valued probability function

$$p_{Grass}(\psi, n) = \langle\psi|n\rangle(\langle\psi|n\rangle)^{\#}$$

Superqubits

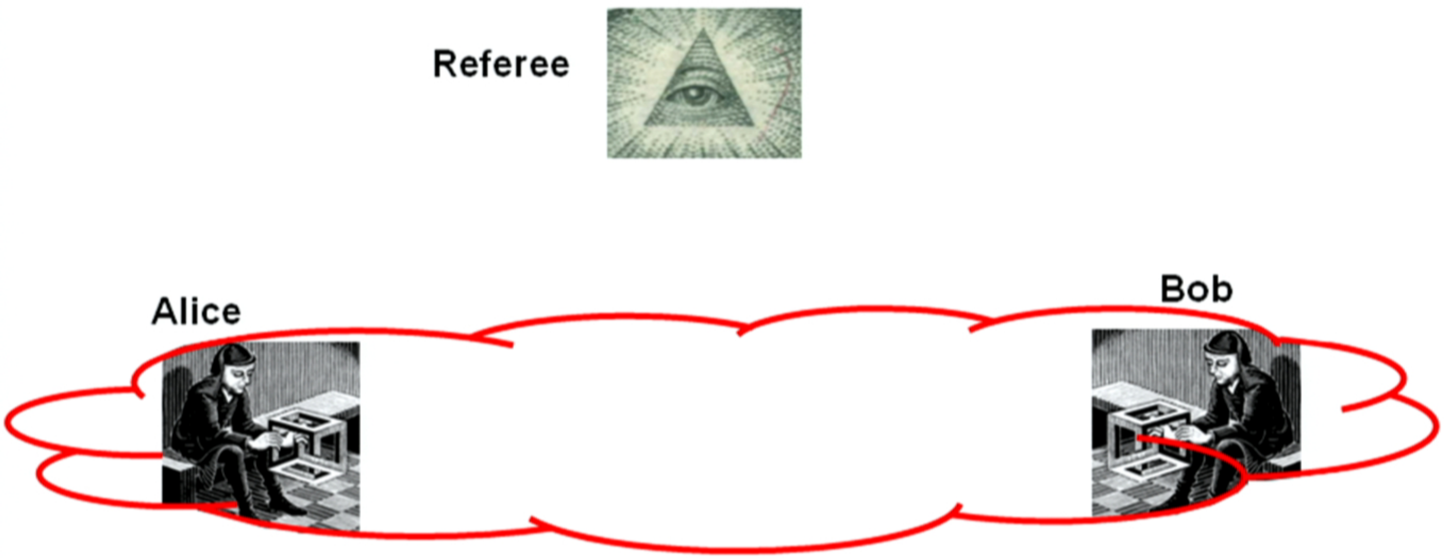
- Norms on Grassmann numbers do something similar, for example, the Rogers norm

$$\xi_e = z_0 + \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} \frac{1}{(2k)!} z_I \eta^I \qquad \xi_0 = \sum_{k=0}^{\lfloor \frac{N+1}{2} \rfloor} \frac{1}{(2k+1)!} z_J \eta^J$$

$$|\xi|_R \stackrel{\mathrm{df}}{=} |z_0| + \sum_K |z_K|$$

Rogers JMP 1980, Rudolph CMP 2001

CHSH game



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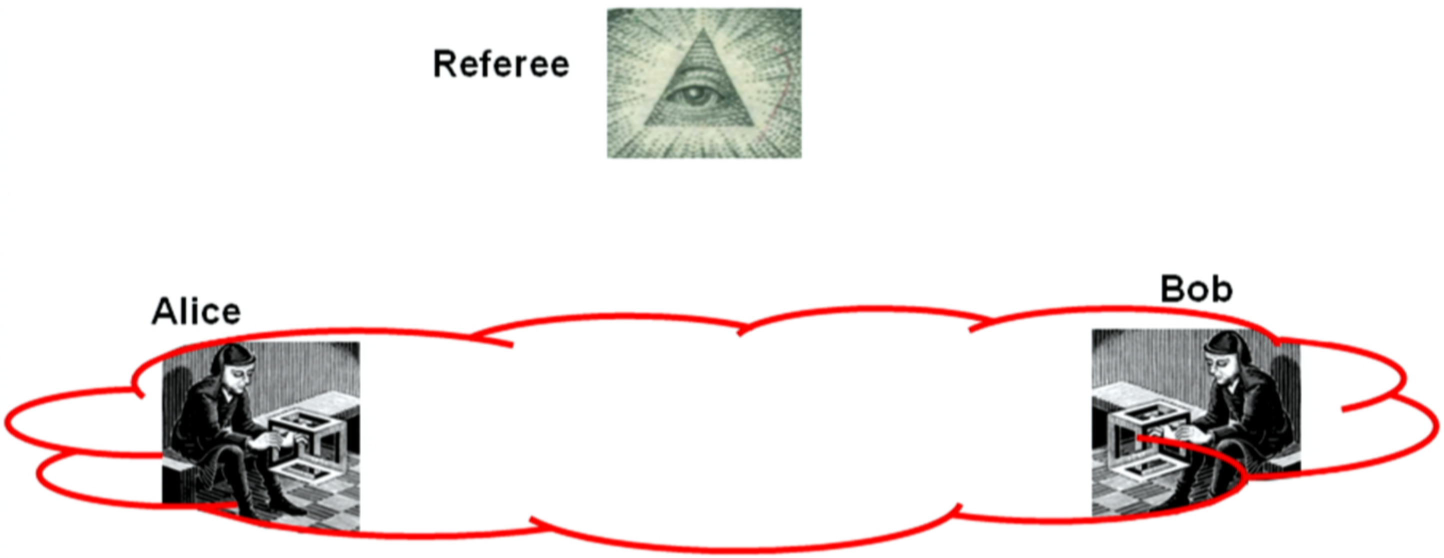
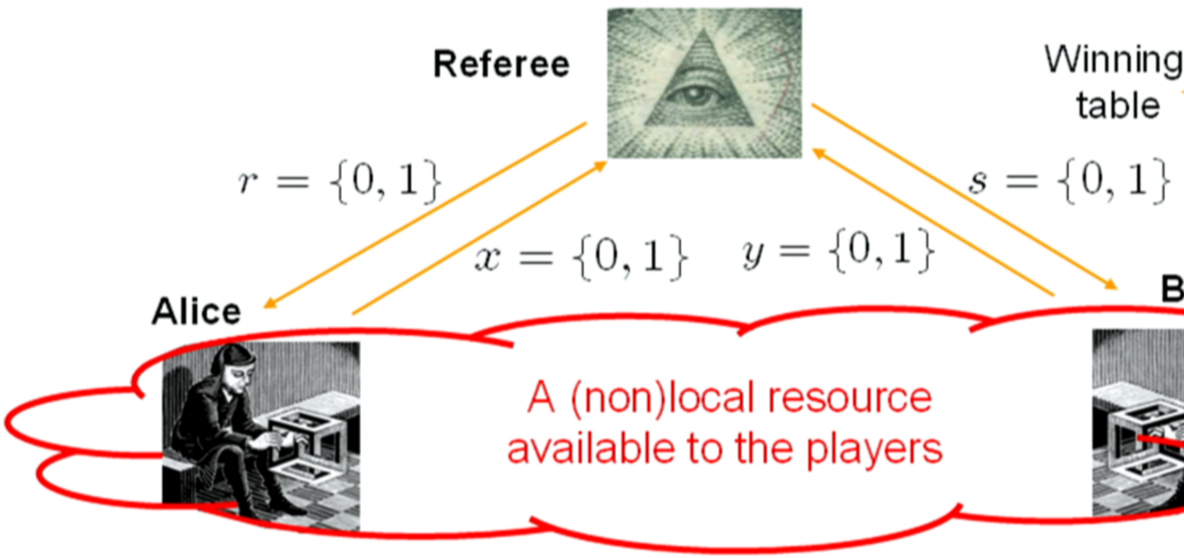
CHSH game



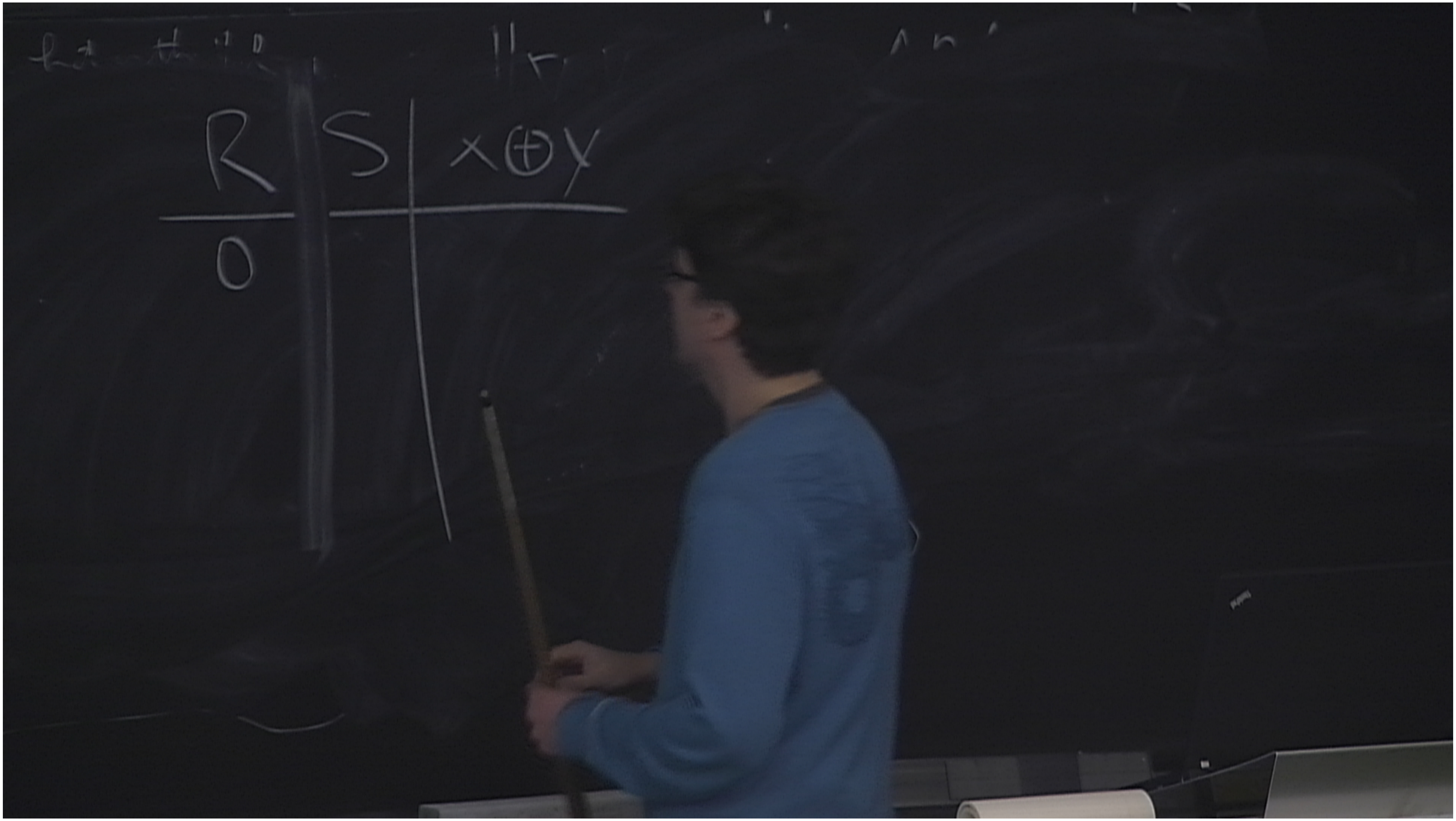
CHSH game

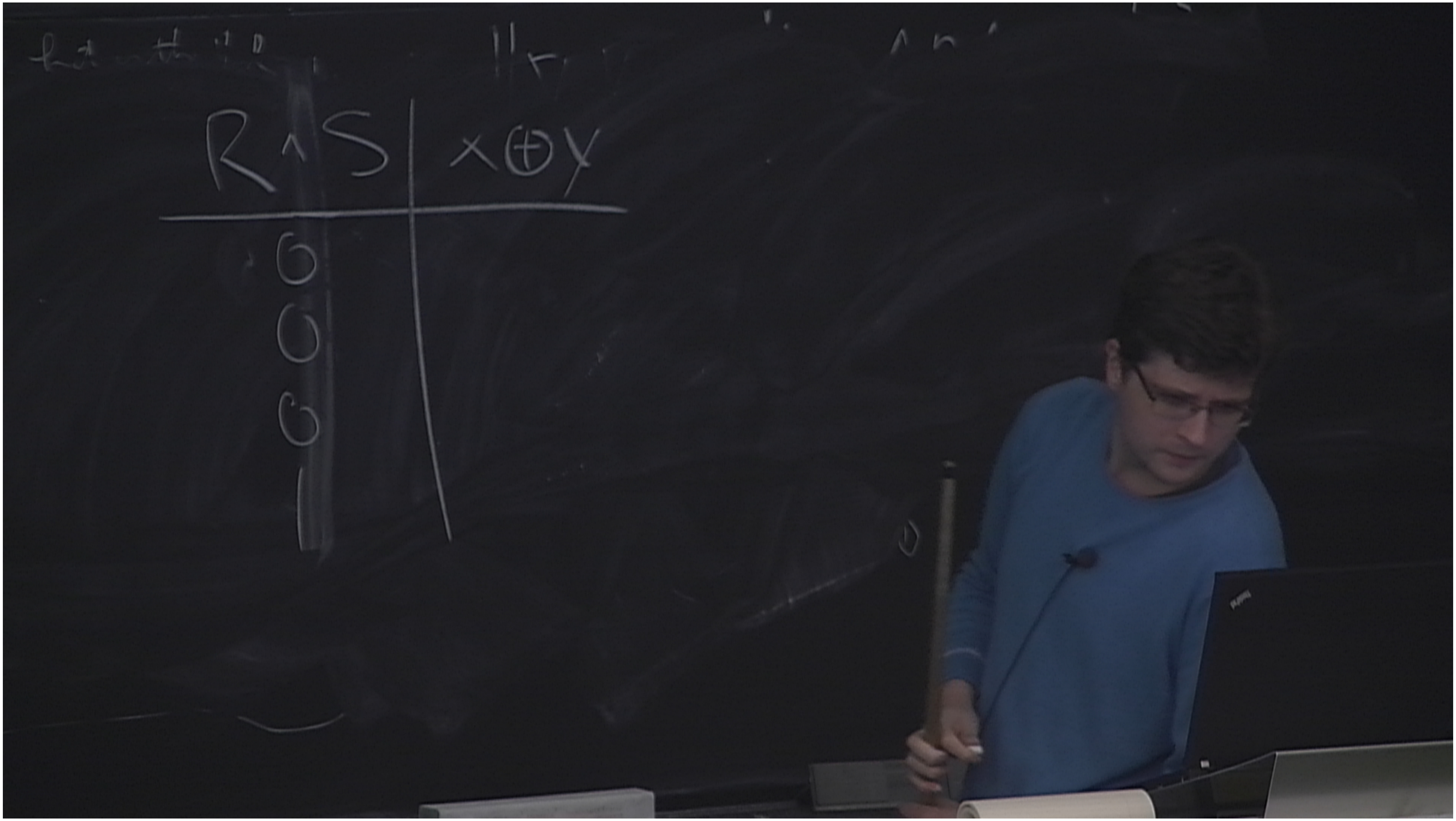


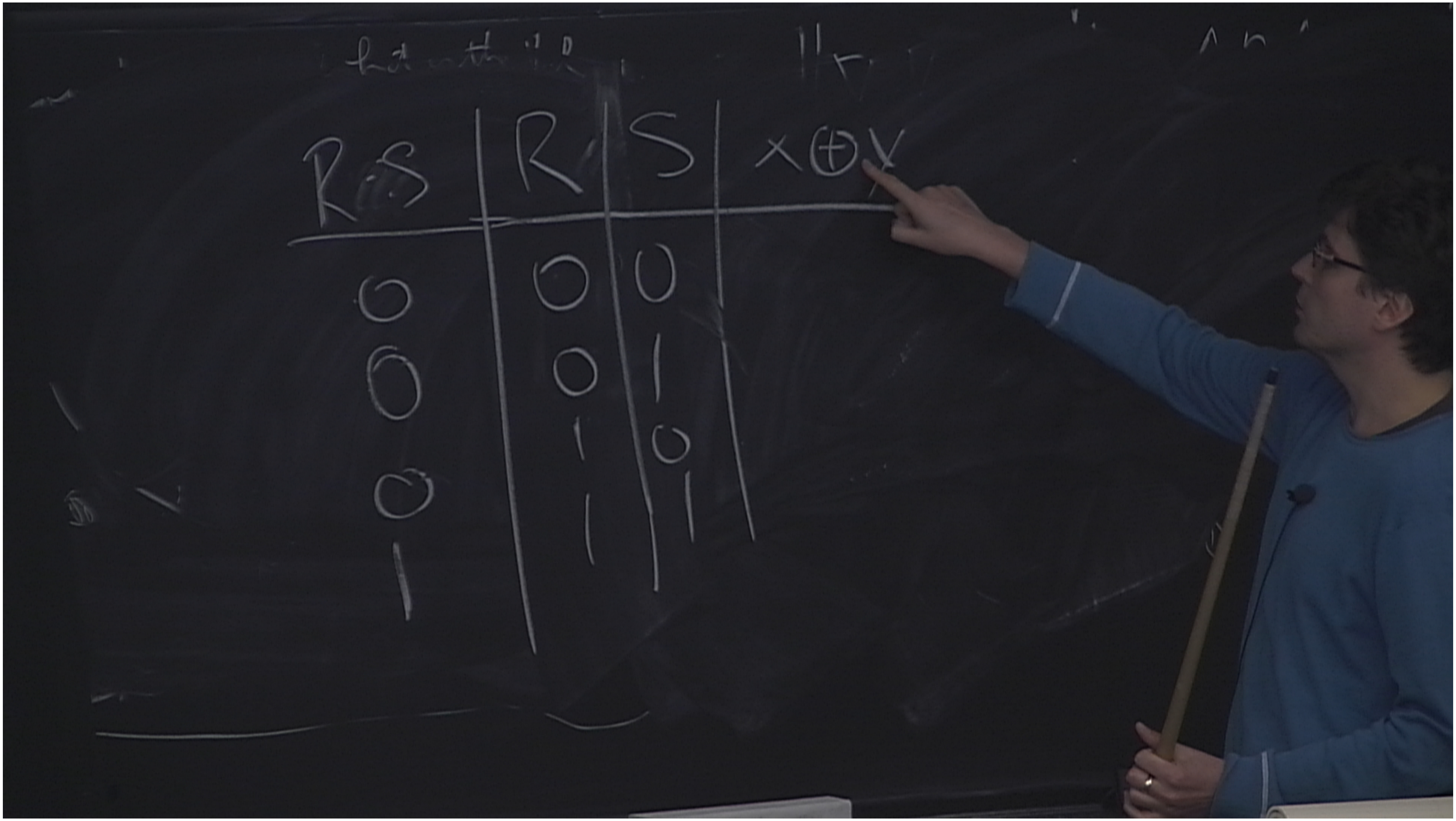
Winning table

rs	x⊕y
00	0
01	0
10	0
11	1

No classical communication is allowed after the game starts


R | S | x ⊕ y
0


R∧S | x⊕y
0
0
0
1


R·S
R
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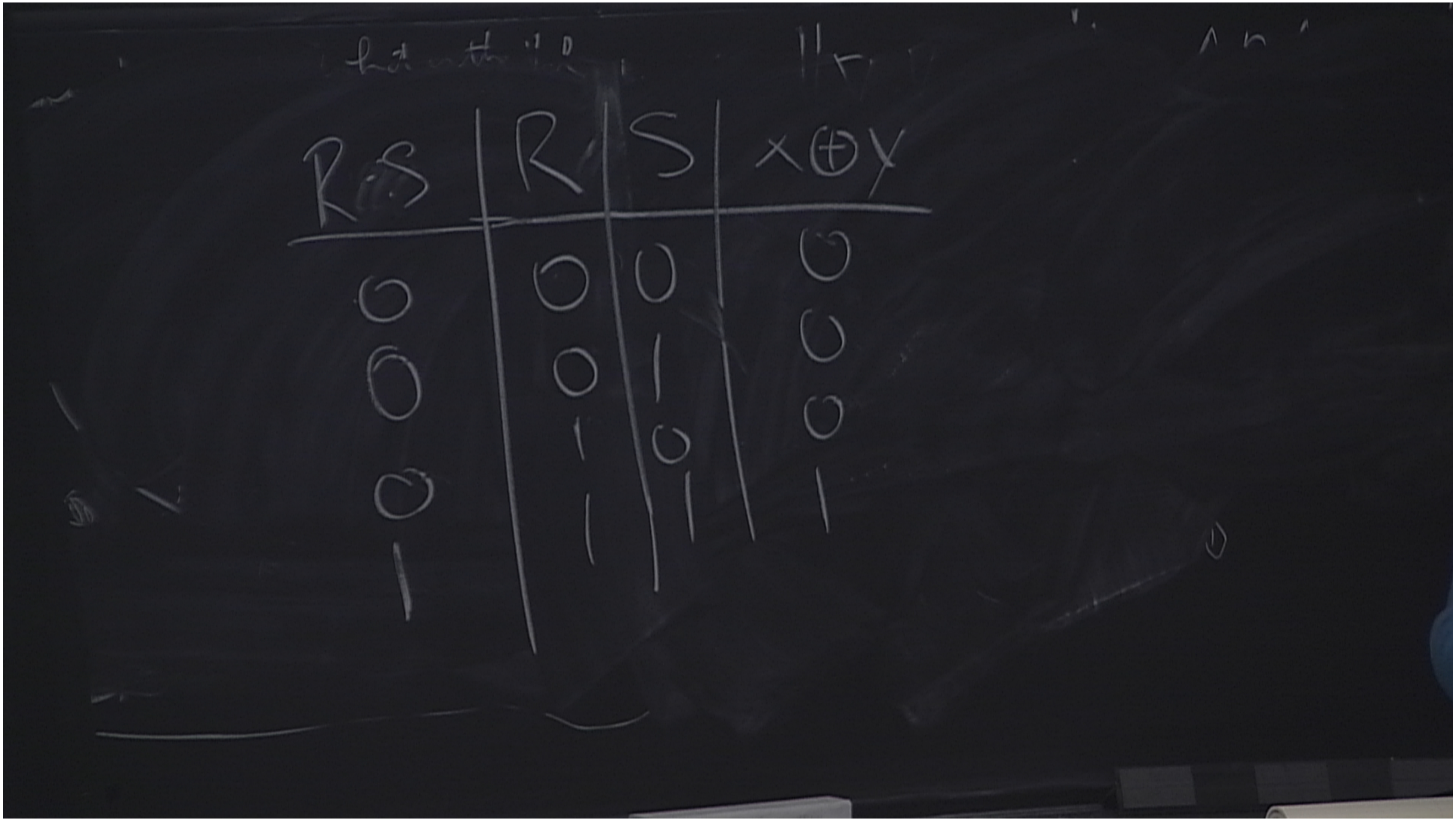
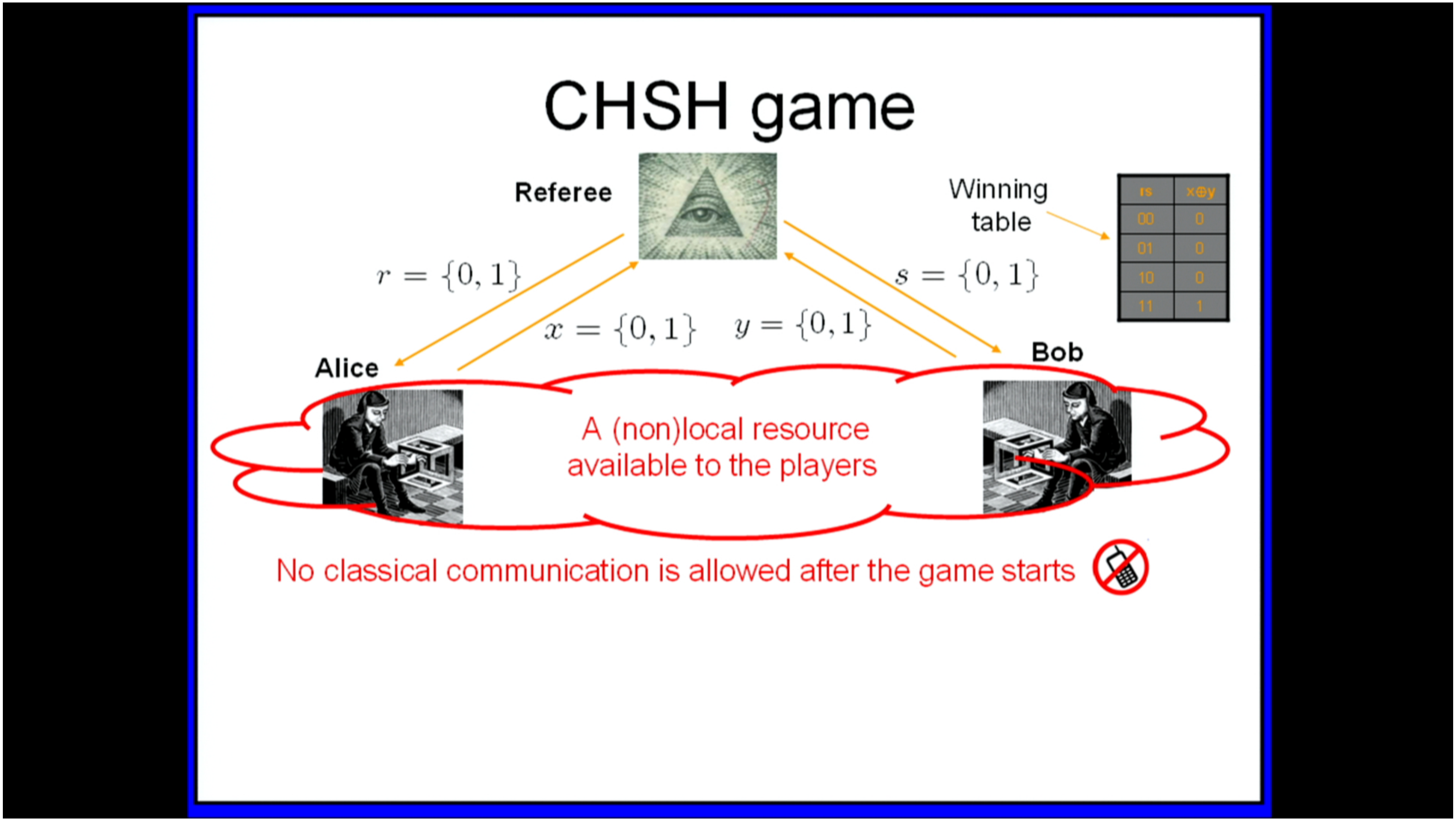
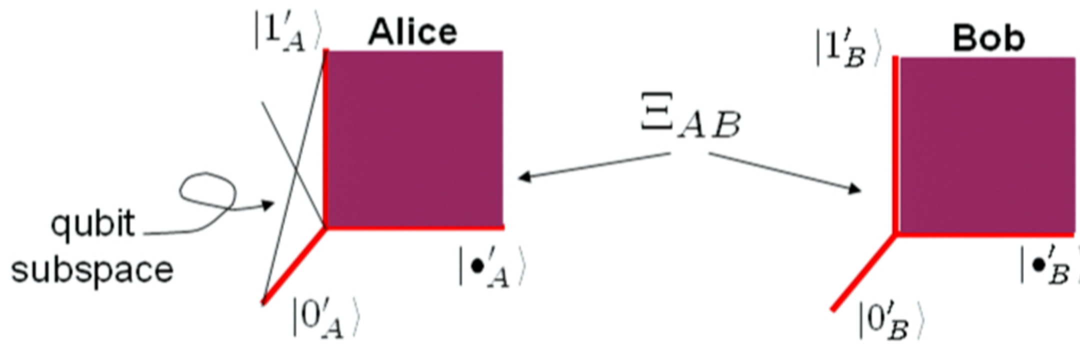
X⊕Y


R.S | R | S | X⊕Y
0 | 0 | 0 | 0
0 | 0 | 1 | 0
0 | 1 | 0 | 0
1 | 1 | 1 | 1


CHSH game
Referee
Winning table
rs | x⊕y
00 | 0
01 | 0
10 | 0
11 | 1
$r = \{0, 1\}$
$s = \{0, 1\}$
$x = \{0, 1\}$
$y = \{0, 1\}$
Alice
Bob
A (non)local resource available to the players
No classical communication is allowed after the game starts

CHSH game with superqubits



Winning measurements

$rs = \{00, 01, 10\}$ $\langle\Xi|0'_A 0'_B\rangle, \langle\Xi|1'_A 1'_B\rangle, \langle\Xi|\bullet'_A 1'_B\rangle, \langle\Xi|1'_A \bullet'_B\rangle, \langle\Xi|\bullet'_A \bullet'_B\rangle$

$rs = \{11\}$ $\langle\Xi|0'_A 1'_B\rangle, \langle\Xi|1'_A 0'_B\rangle, \langle\Xi|\bullet'_A 0'_B\rangle, \langle\Xi|0'_A \bullet'_B\rangle$

CHSH game with superqubits

Consider the state

$$\tilde{\Xi}_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB} + p\eta_A \left|\bullet 1\right\rangle_{AB} + q\eta_B \left|1\bullet\right\rangle_{AB} \quad p, q \in \mathbb{R}$$

It must be normalized

$$\Xi_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) - \frac{1}{2}\left(p^2\eta_A\eta_A^{\#} + q^2\eta_B\eta_B^{\#}\right)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) + \frac{3}{4}p^2q^2\eta_A\eta_A^{\#}\eta_B\eta_B^{\#}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$+ p\eta_A\left(1 - \frac{q^2}{2}\eta_B\eta_B^{\#}\right)|\bullet 1\rangle + q\eta_B\left(1 - \frac{p^2}{2}\eta_A\eta_A^{\#}\right)|1\bullet\rangle$$

We calculate $\langle \Xi | 0'_A 0'_B \rangle \rightarrow p_{\mathbf{0}'\mathbf{0}'} = \langle \Xi | 0'_A 0'_B \rangle (\langle \Xi | 0'_A 0'_B \rangle)^{\#}$
and similarly other contributions to the winning probability

CHSH game with superqubits

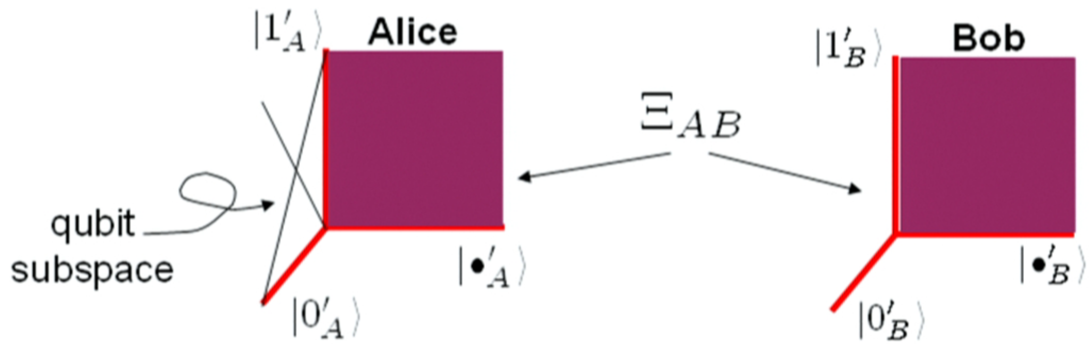
$$\tilde{p}_{win} = \cos^2\frac{\pi}{8} + \frac{1}{4}[P_A\eta_A\eta_A^{\#} + P_B\eta_B\eta_B^{\#} + P_{AB}\eta_A\eta_A^{\#}\eta_B\eta_B^{\#}]$$

$$P_A = \frac{1}{8}\left(p^2(1 - 2\cos^2\frac{\pi}{8})\right)$$
$$P_B = \frac{1}{8}\left(q^2(3 - 4\cos^2\frac{\pi}{8})\right)$$
$$P_{AB} = \frac{1}{64}\left(p^2q^2(-4 + 6\cos^2\frac{\pi}{8})\right)$$

The missing piece is the true probability function

$$f_{true} : \tilde{p}_{win} \mapsto p_{win} = \cos^2\frac{\pi}{8} + g(P_A, P_B, P_{AB}) \in [0,1]$$

CHSH game with superqubits



Winning measurements

$rs = \{00, 01, 10\}$ $\langle\Xi|0'_A 0'_B\rangle, \langle\Xi|1'_A 1'_B\rangle, \langle\Xi|\bullet'_A 1'_B\rangle, \langle\Xi|1'_A \bullet'_B\rangle, \langle\Xi|\bullet'_A \bullet'_B\rangle$

$rs = \{11\}$ $\langle\Xi|0'_A 1'_B\rangle, \langle\Xi|1'_A 0'_B\rangle, \langle\Xi|\bullet'_A 0'_B\rangle, \langle\Xi|0'_A \bullet'_B\rangle$

CHSH game with superqubits

An intuitive way to see why it might work

$$\Psi = 1/\sqrt{2}(|00\rangle + |11\rangle)$$

$$\Psi \rightarrow \tilde{\Phi} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) + \epsilon\,|\chi\rangle$$

normalized and orthogonal

$$\Phi = \frac{1}{\sqrt{1+\epsilon^2}}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) + \frac{\epsilon}{\sqrt{1+\epsilon^2}}\,|\chi\rangle$$

$$\Phi \approx \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) - \frac{1}{\sqrt{2}}\frac{\epsilon-1}{\epsilon}(|00\rangle + |11\rangle) + |\chi\rangle \qquad \text{for } \epsilon \gg 0$$

Compare it with

$$\Xi_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) - \frac{1}{2}\left(p^2\eta_A\eta_A^{\#} + q^2\eta_B\eta_B^{\#}\right)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) + \frac{3}{4}p^2q^2\eta_A\eta_A^{\#}\eta_B\eta_B^{\#}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$+\, p\eta_A\left(1 - \frac{q^2}{2}\eta_B\eta_B^{\#}\right)|\bullet 1\rangle + q\eta_B\left(1 - \frac{p^2}{2}\eta_A\eta_A^{\#}\right)|1\bullet\rangle\,.$$

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- Is the measurement model adequate?
- Is there a norm confirming the heuristic arguments?
- Superentangled states surely are different from ordinary entangled states
- Are they also super-nonlocal?
- If superqubits are more powerful resources, the rest of QIT protocols and tools is waiting to be modified and explored

Thank you for your attention