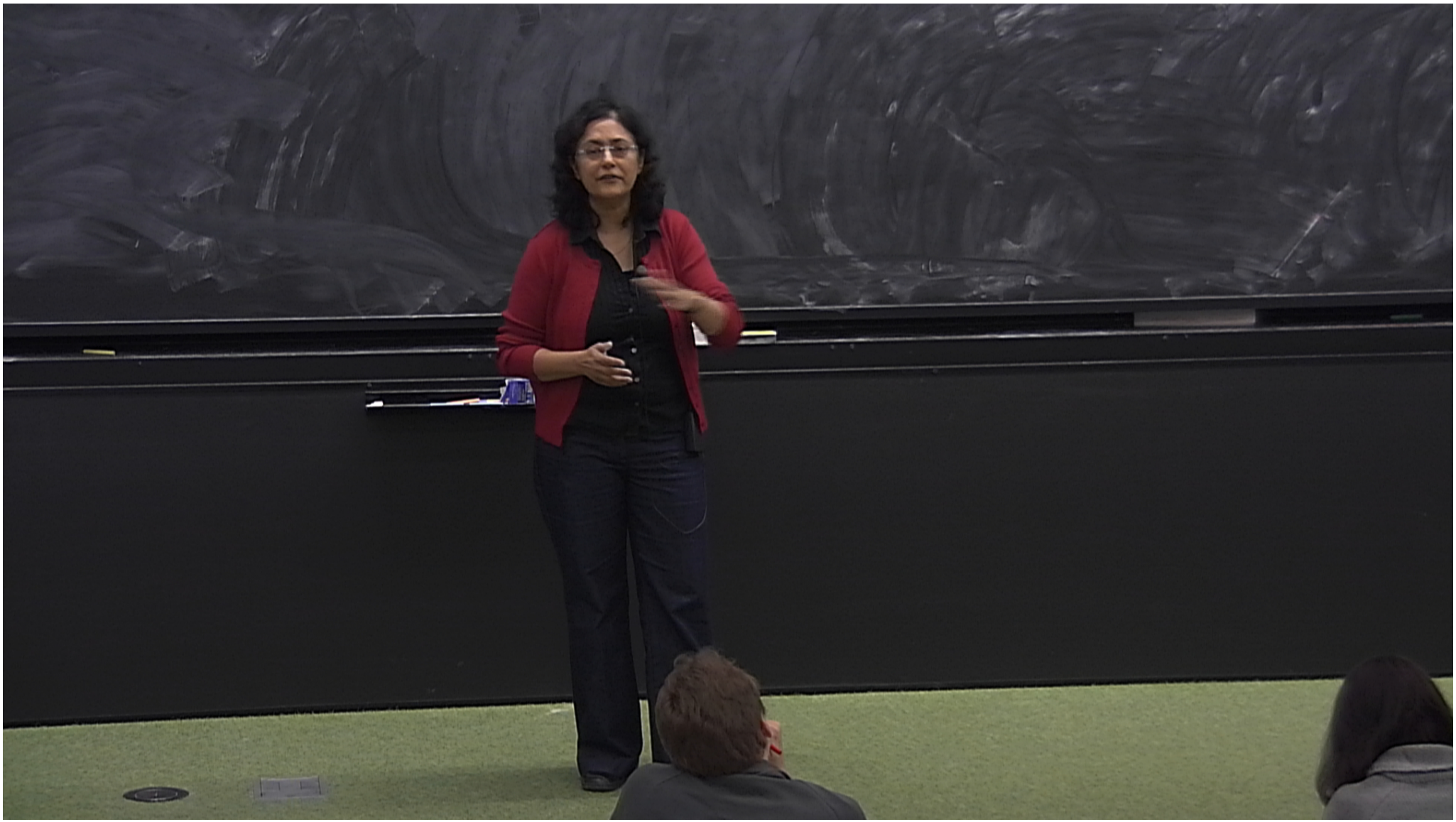
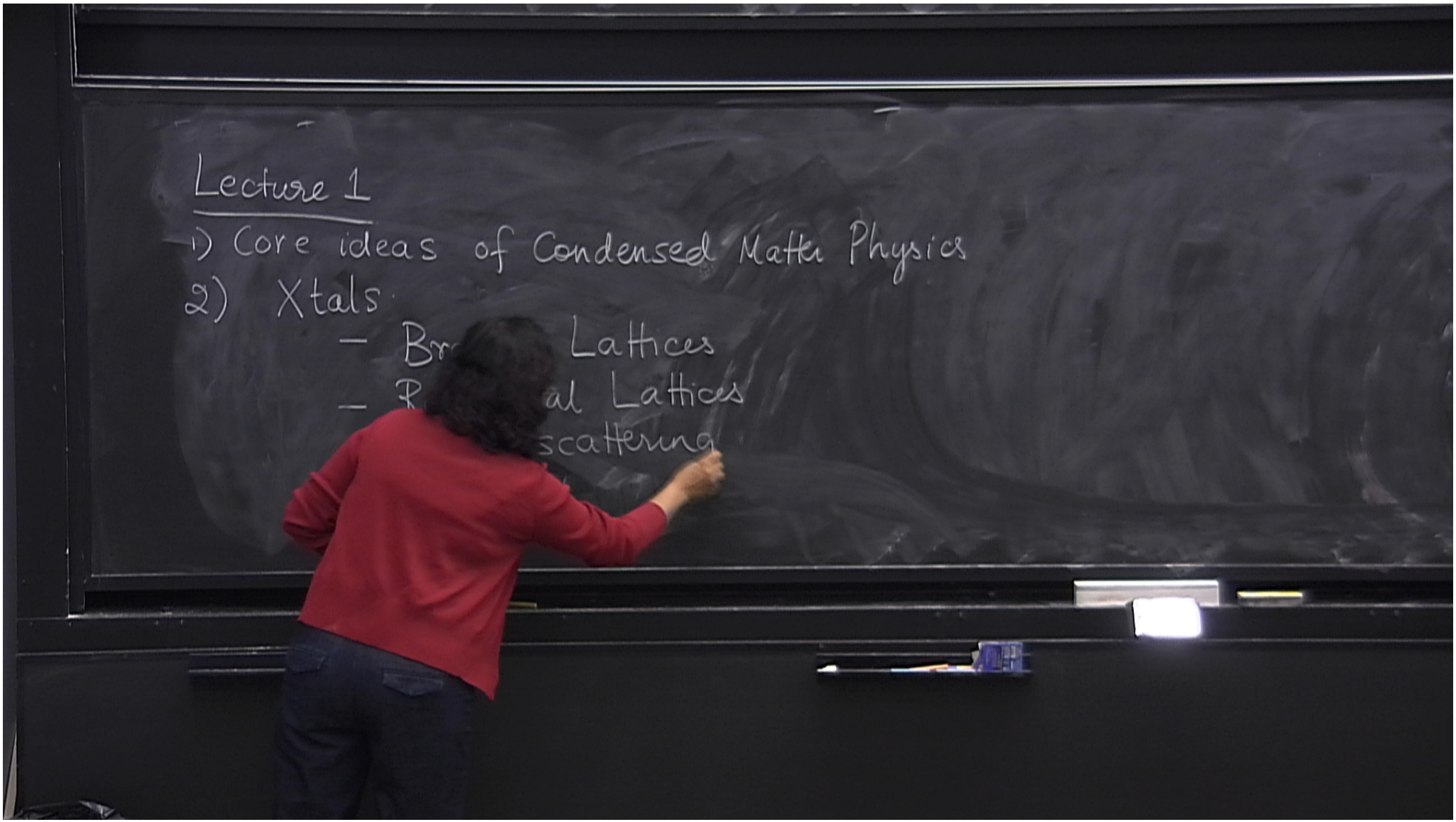
Title: Condensed Matter - Lecture 1

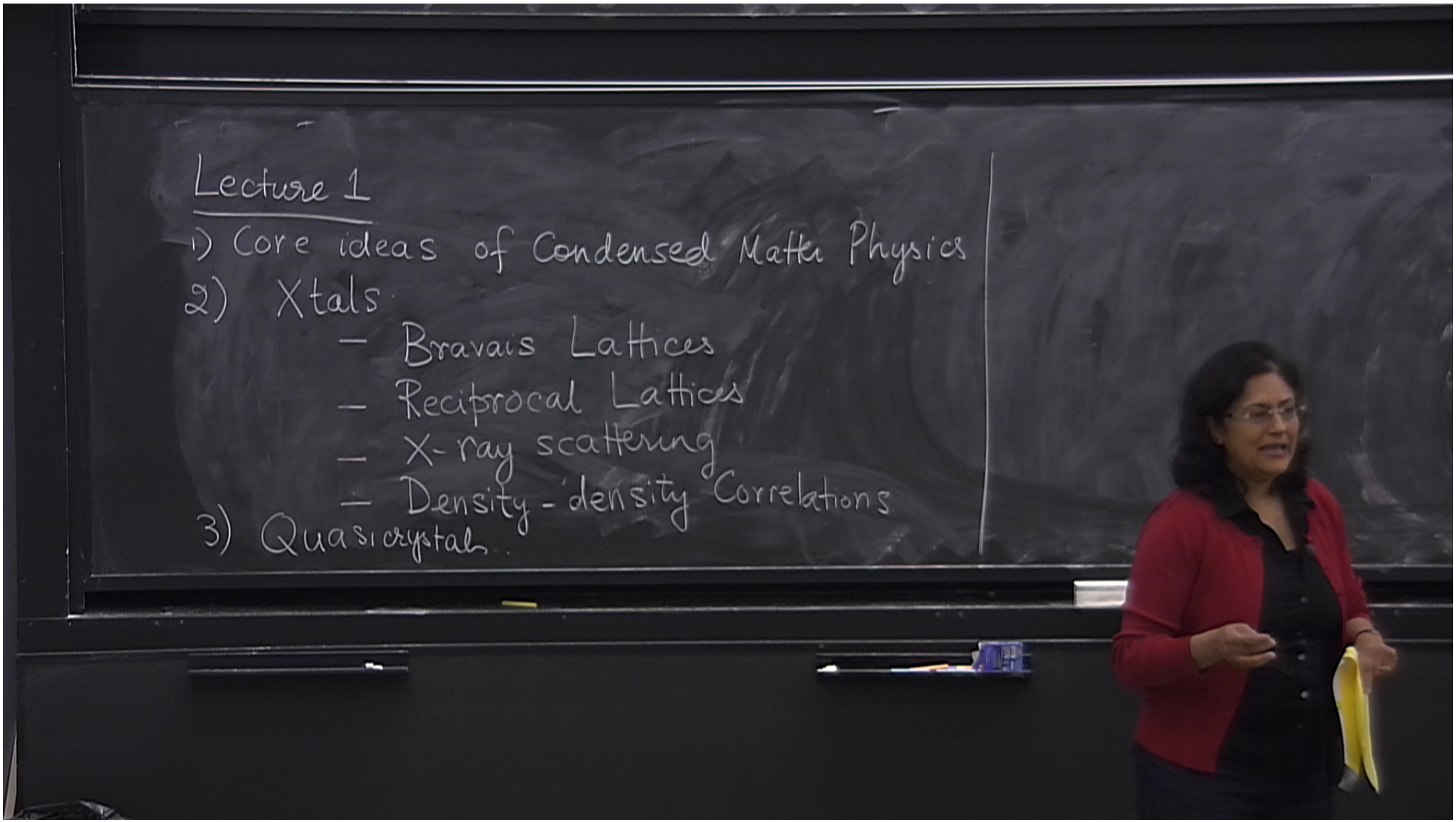
Date: Oct 31, 2011 10:30 AM

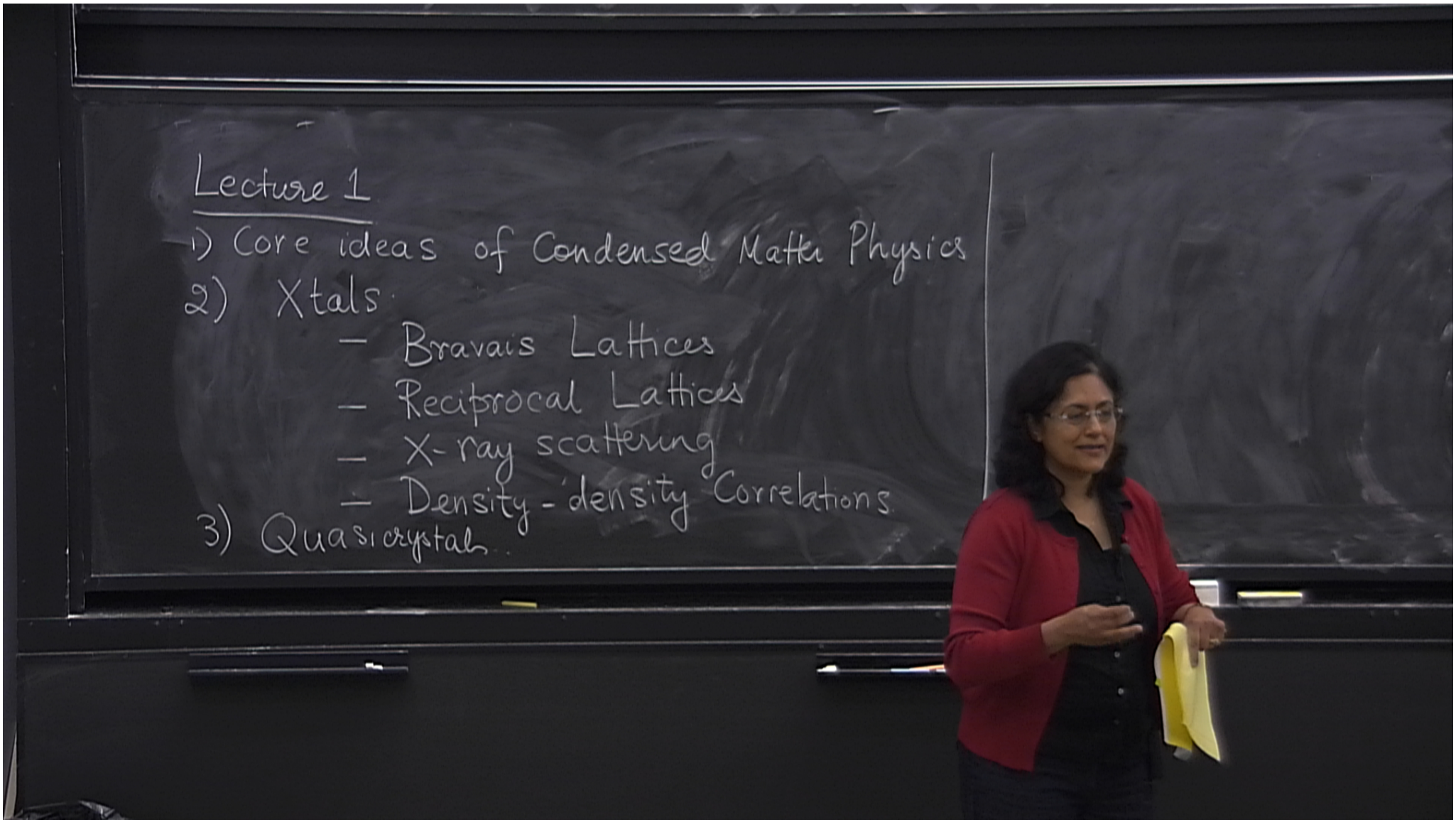
URL: http://pirsa.org/11100045

Abstract:




Lecture 1
1) Core ideas of Condensed Matter Physics
2) Xtals
– Br Lattices
– R al Lattices
scattering


Lecture 1
1) Core ideas of Condensed Matter Physics
2) Xtals
– Bravais Lattices
– Reciprocal Lattices
– X-ray scattering
– Density-density Correlations
3) Quasicrystals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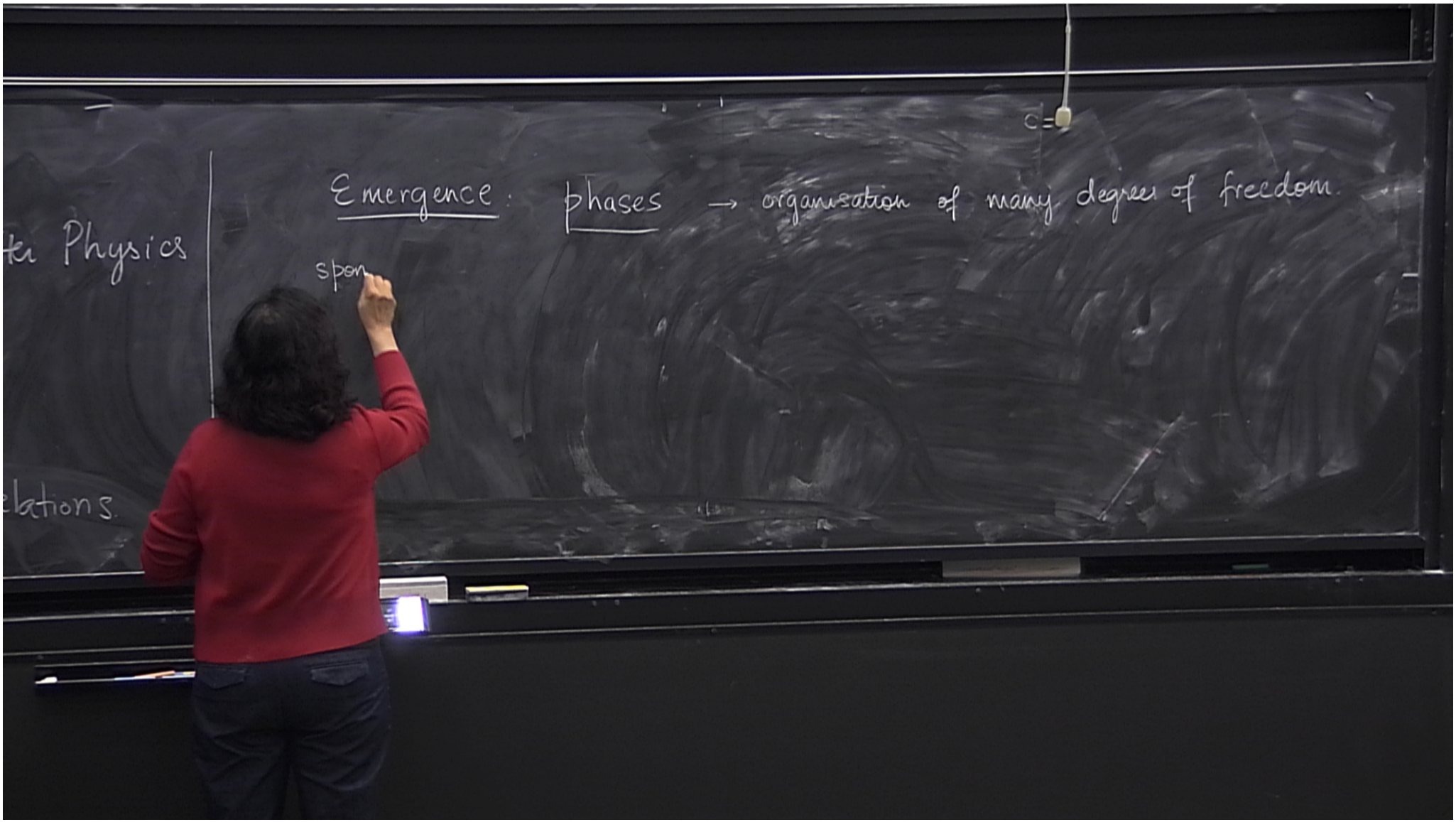
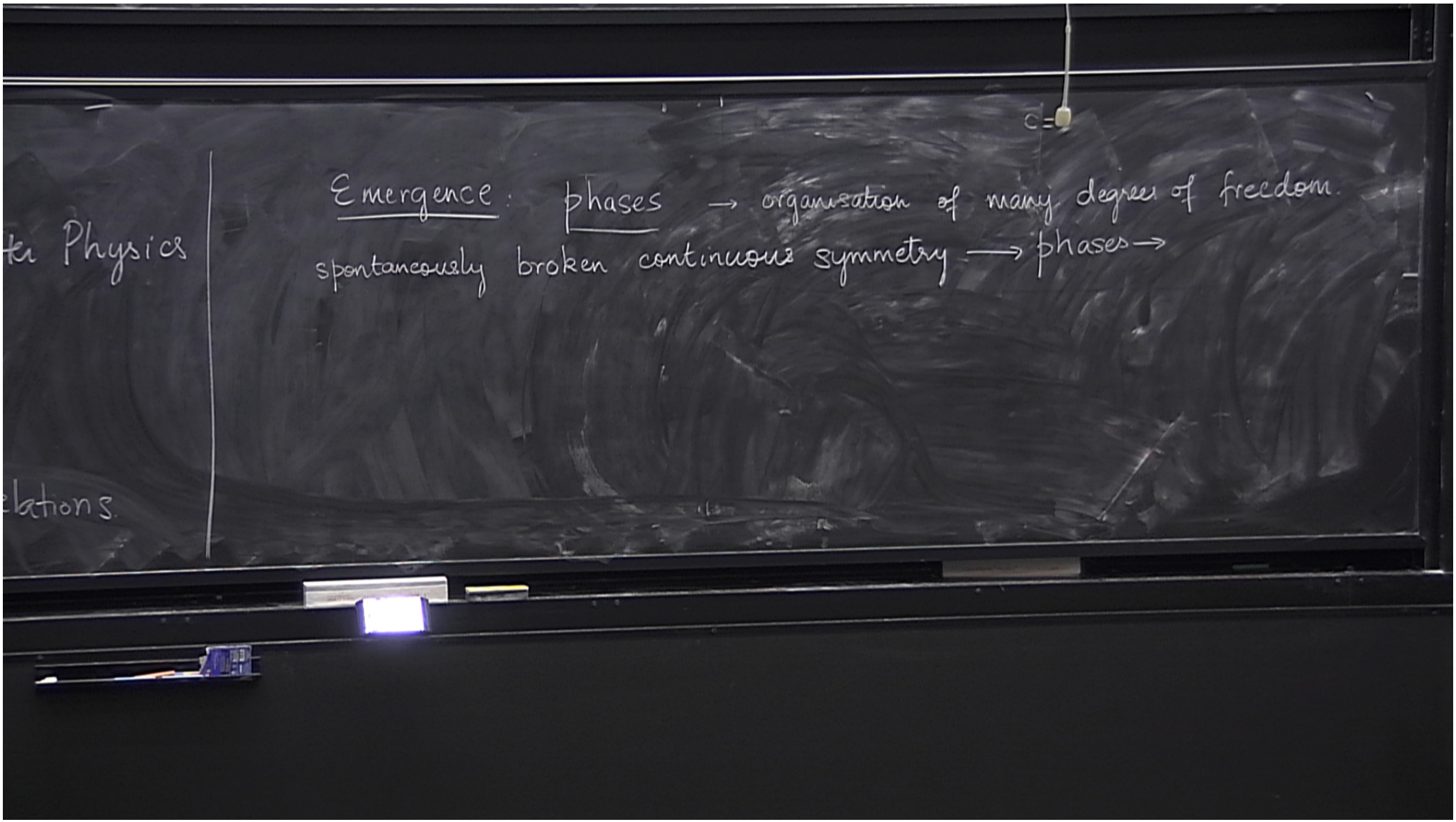
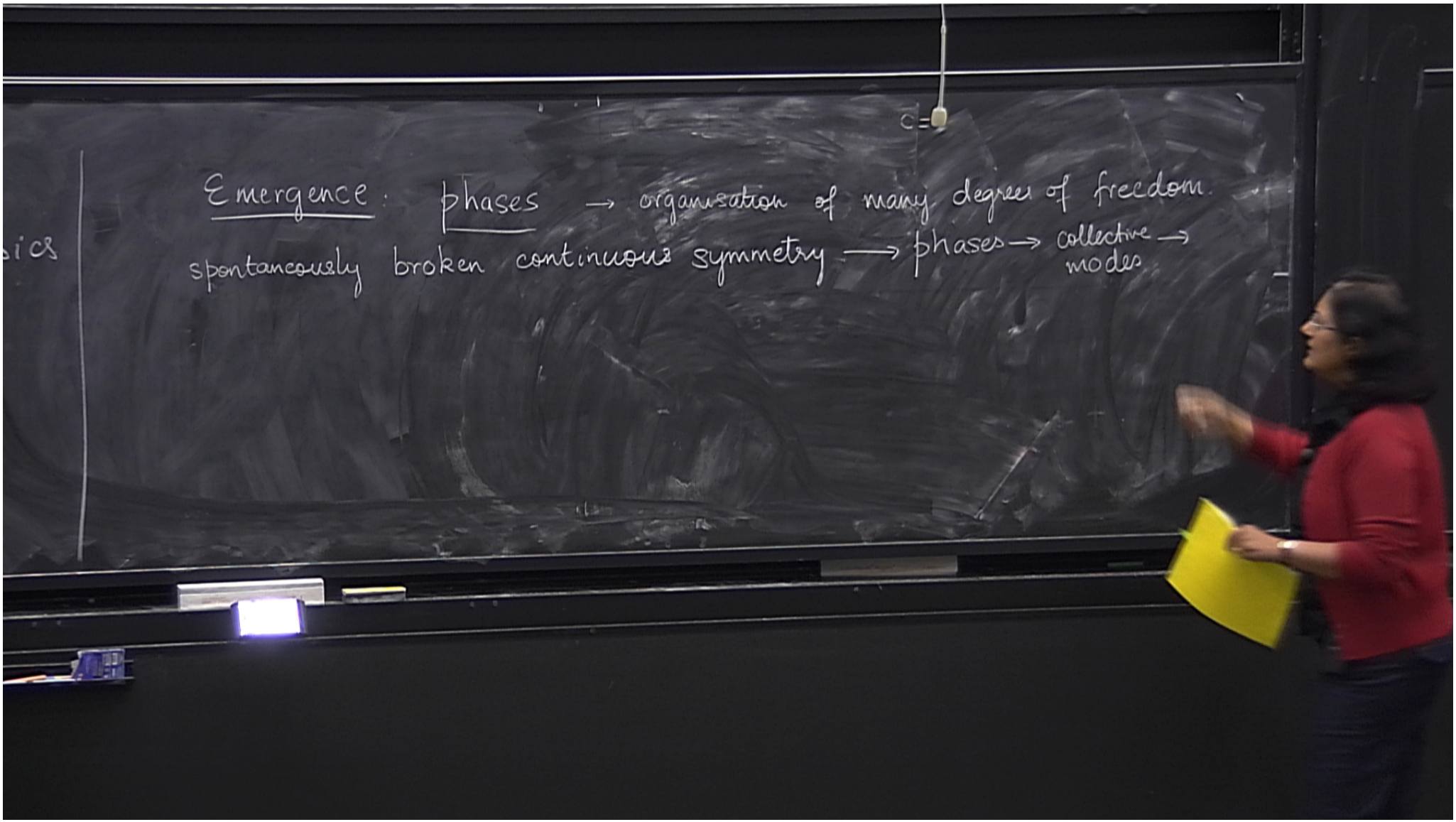
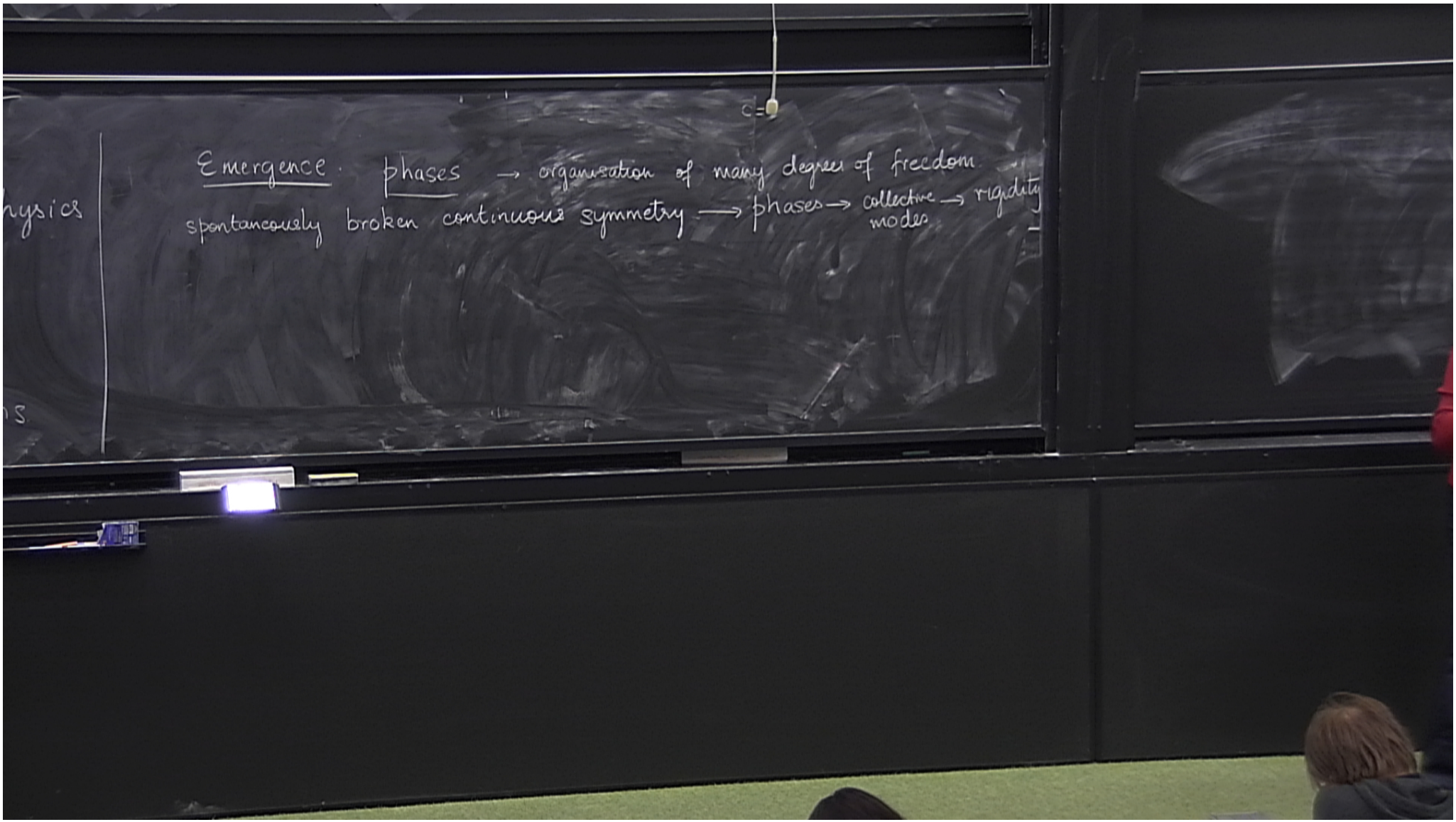

Emergence: phases → organisation of many degrees of freedom.
spon
Physics
elations


Emergence: phases → organisation of many degrees of freedom.
spontaneously broken continuous symmetry → phases →


Emergence: phases → organisation of many degrees of freedom.
spontaneously broken continuous symmetry → phases → collective modes →


Emergence · phases → organization of many degrees of freedom
spontaneously broken continuous symmetry → phases → collective modes → rigidity

Matter Physics

s

Correlations

Emergence · phases → organisation of many degrees of freedom

spontaneously broken continuous symmetry → phases → collective modes → rigidity

Xtals

Magnets

Superconductors / Superfluids

Matter Physics

s

Correlations

Emergence · phases → organisation of many degrees of freedom

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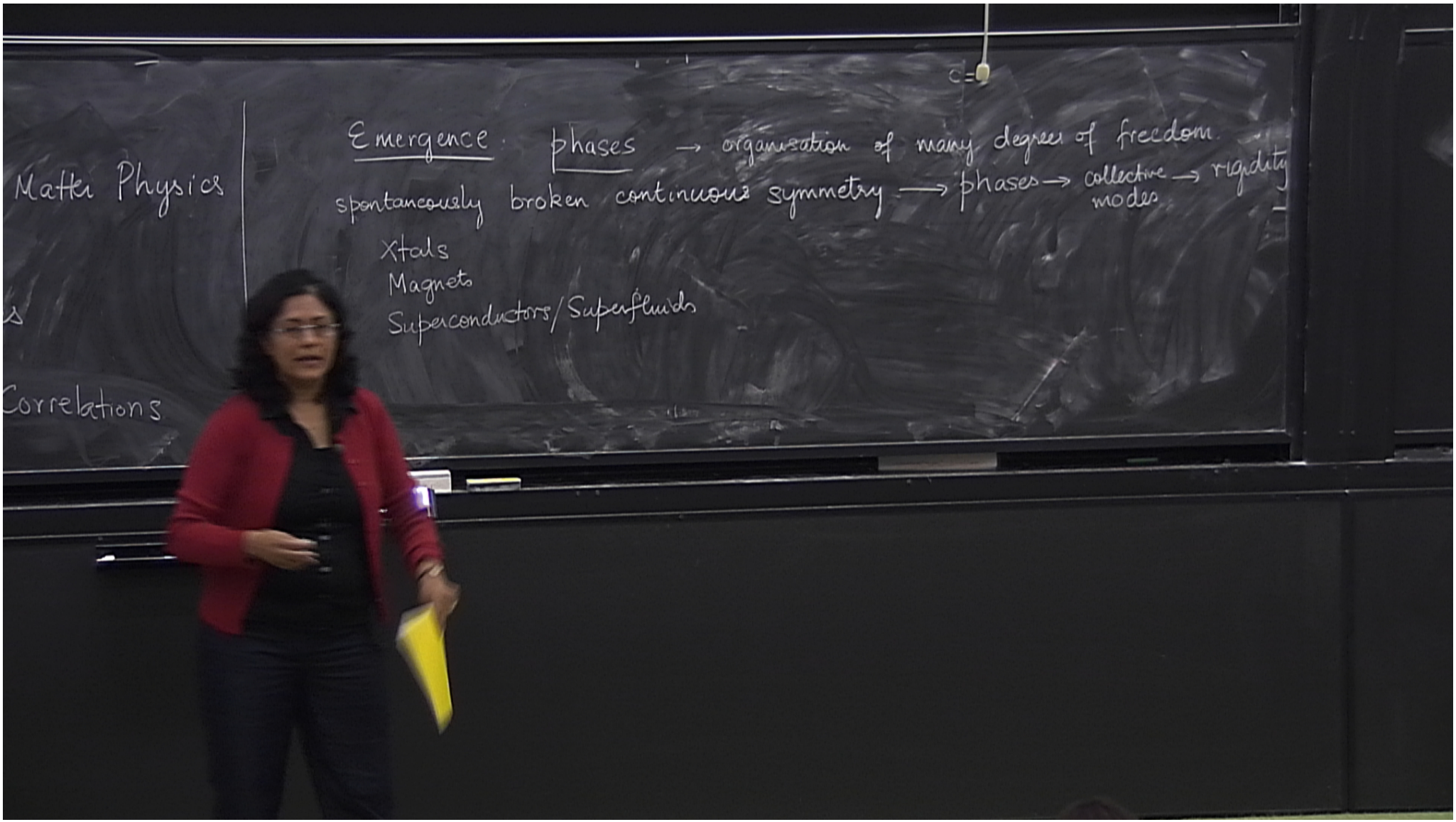
Xtals

Magnets

Superconductors / Superfluids


Matter Physics
Correlations
Emergence
phases → organisation of many degrees of freedom
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Xtals
Magnets
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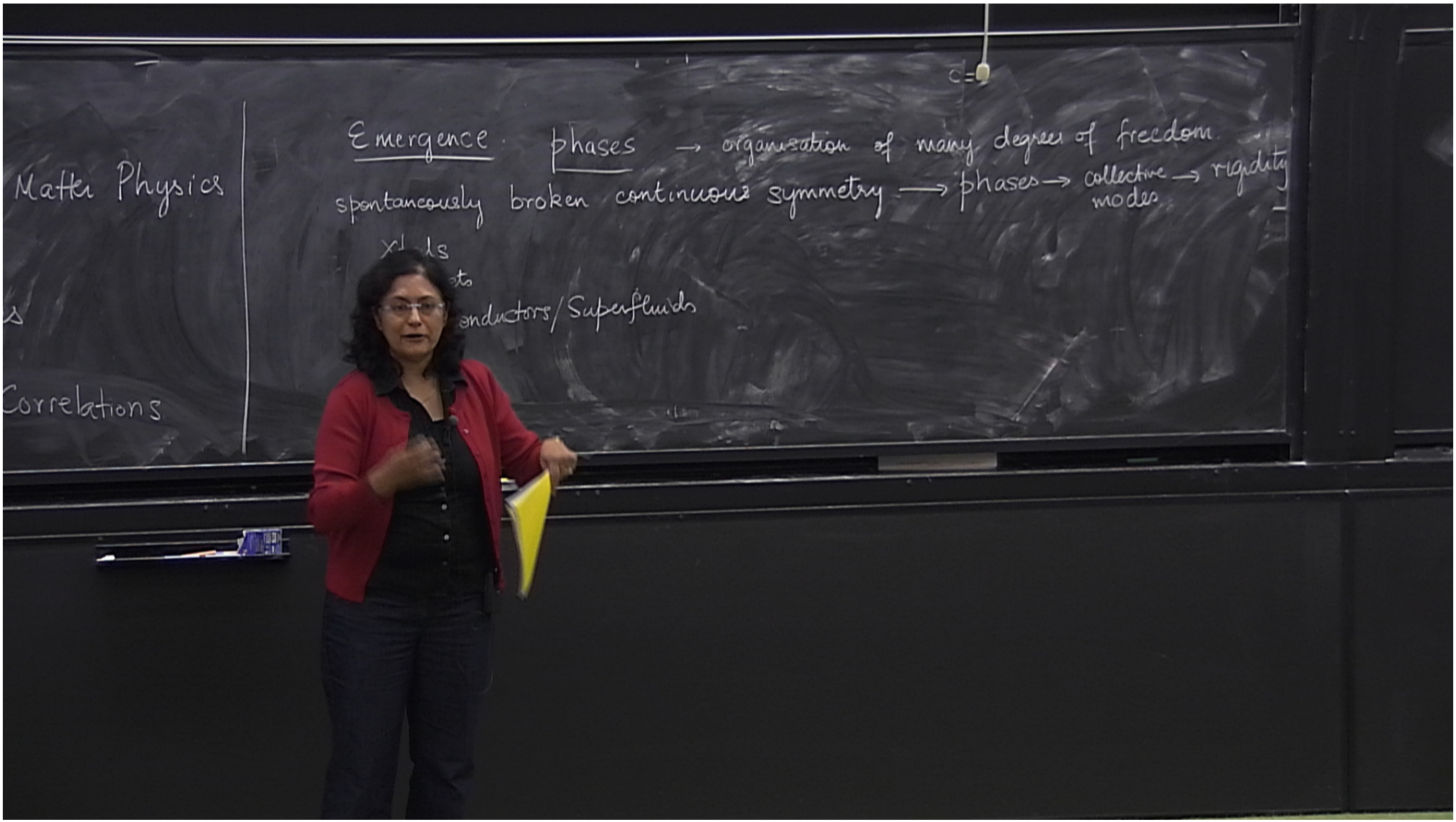
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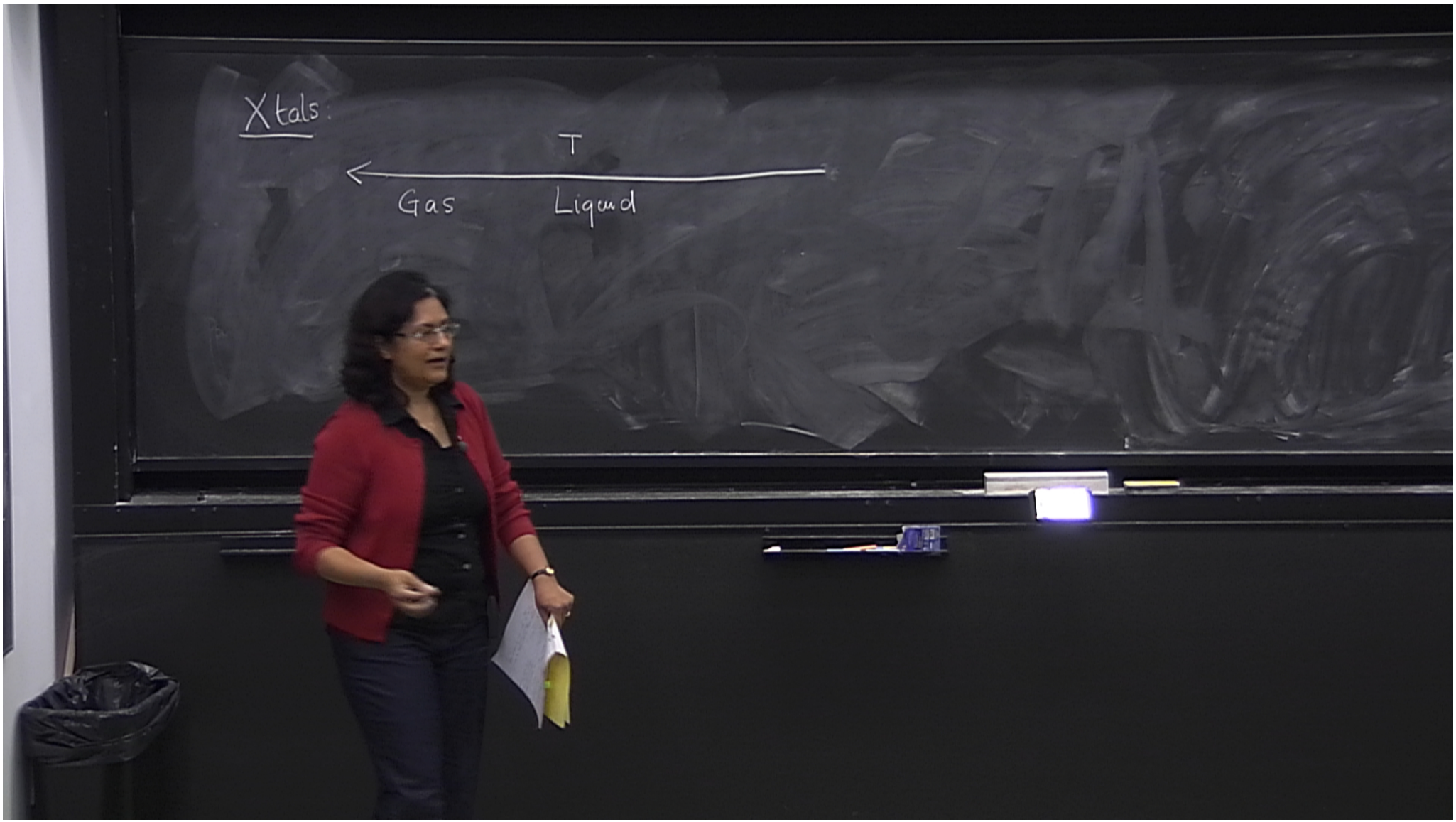
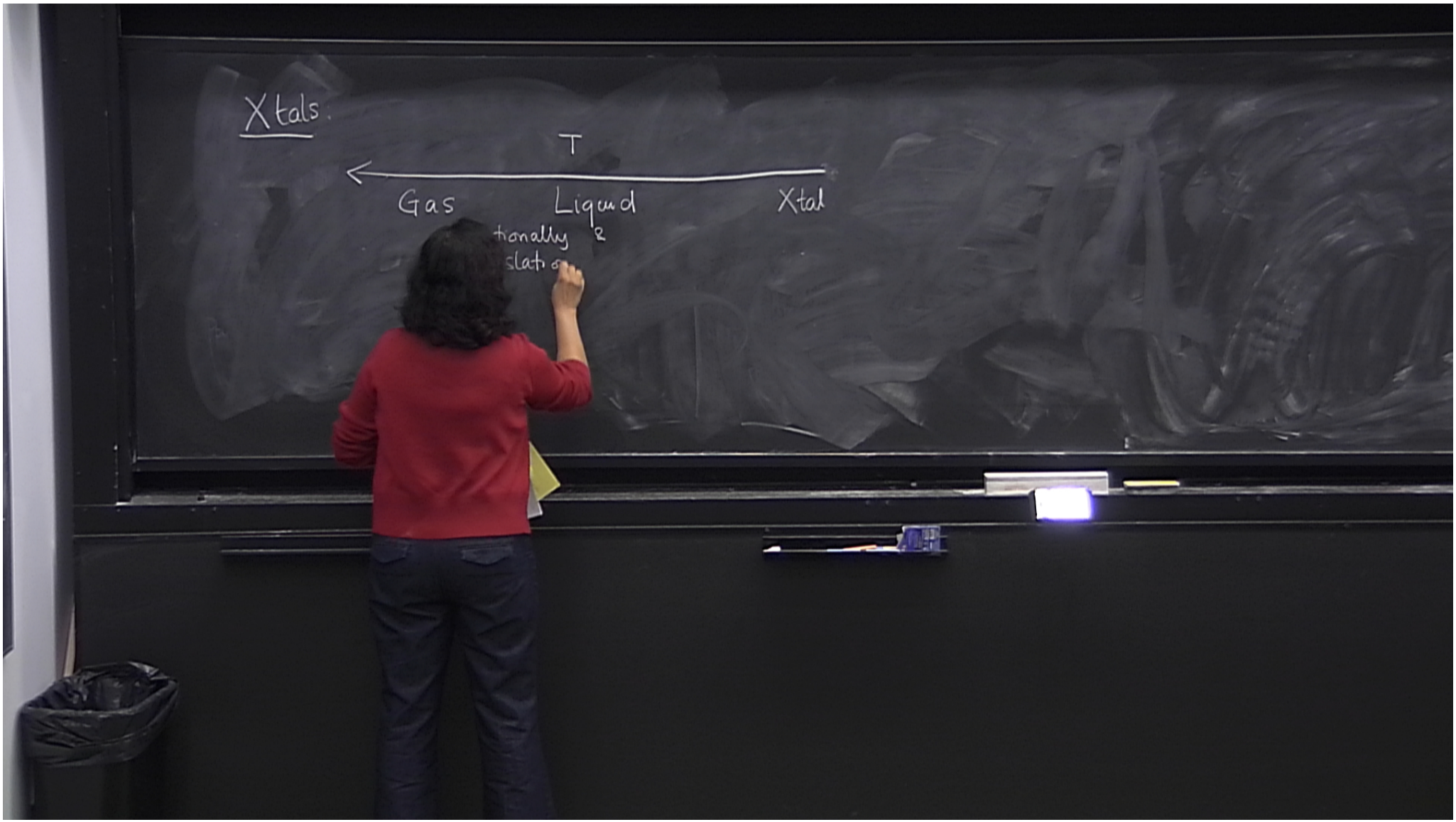

Xtals:
T
Gas
Liquid


Xtals
T
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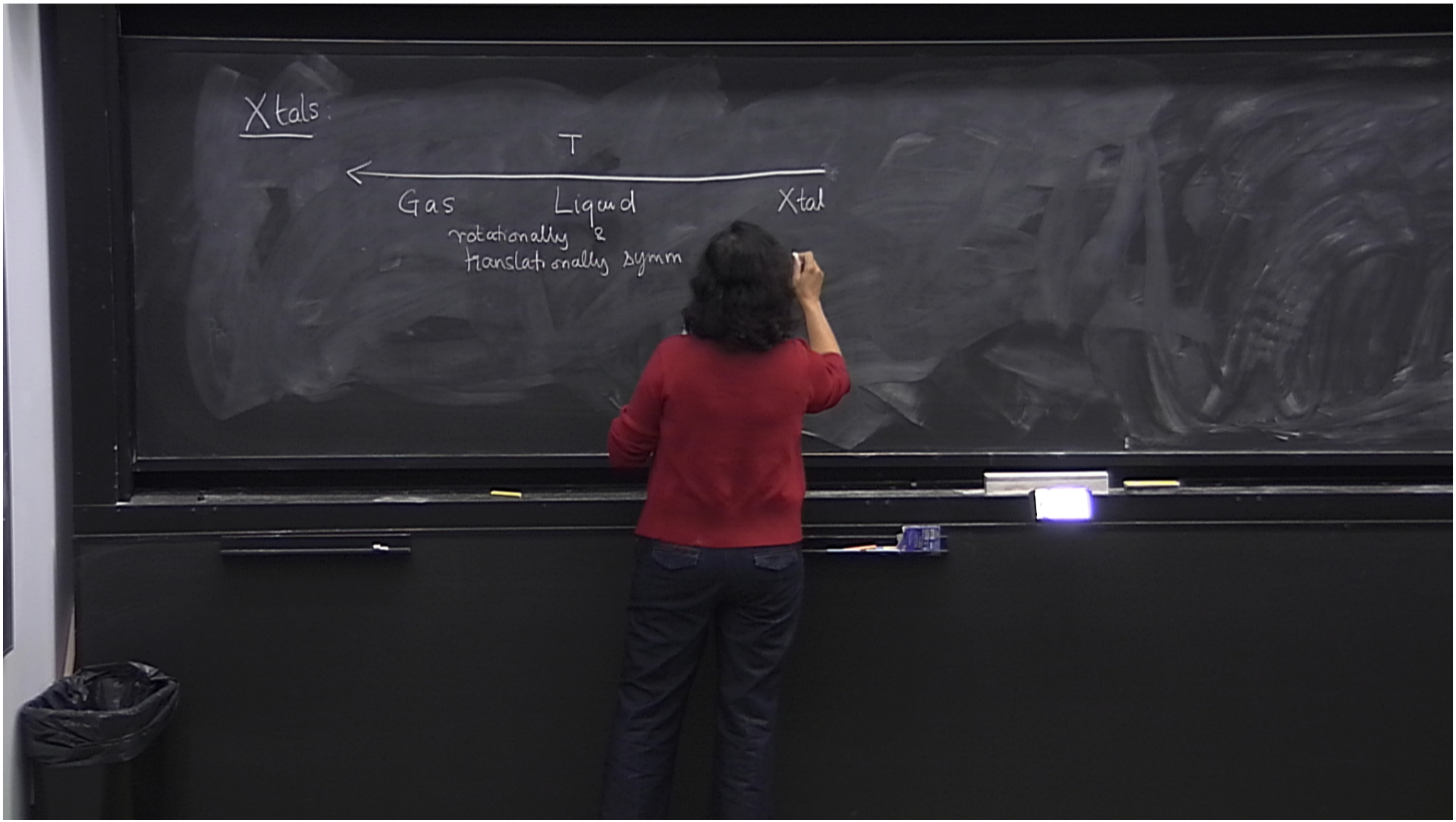
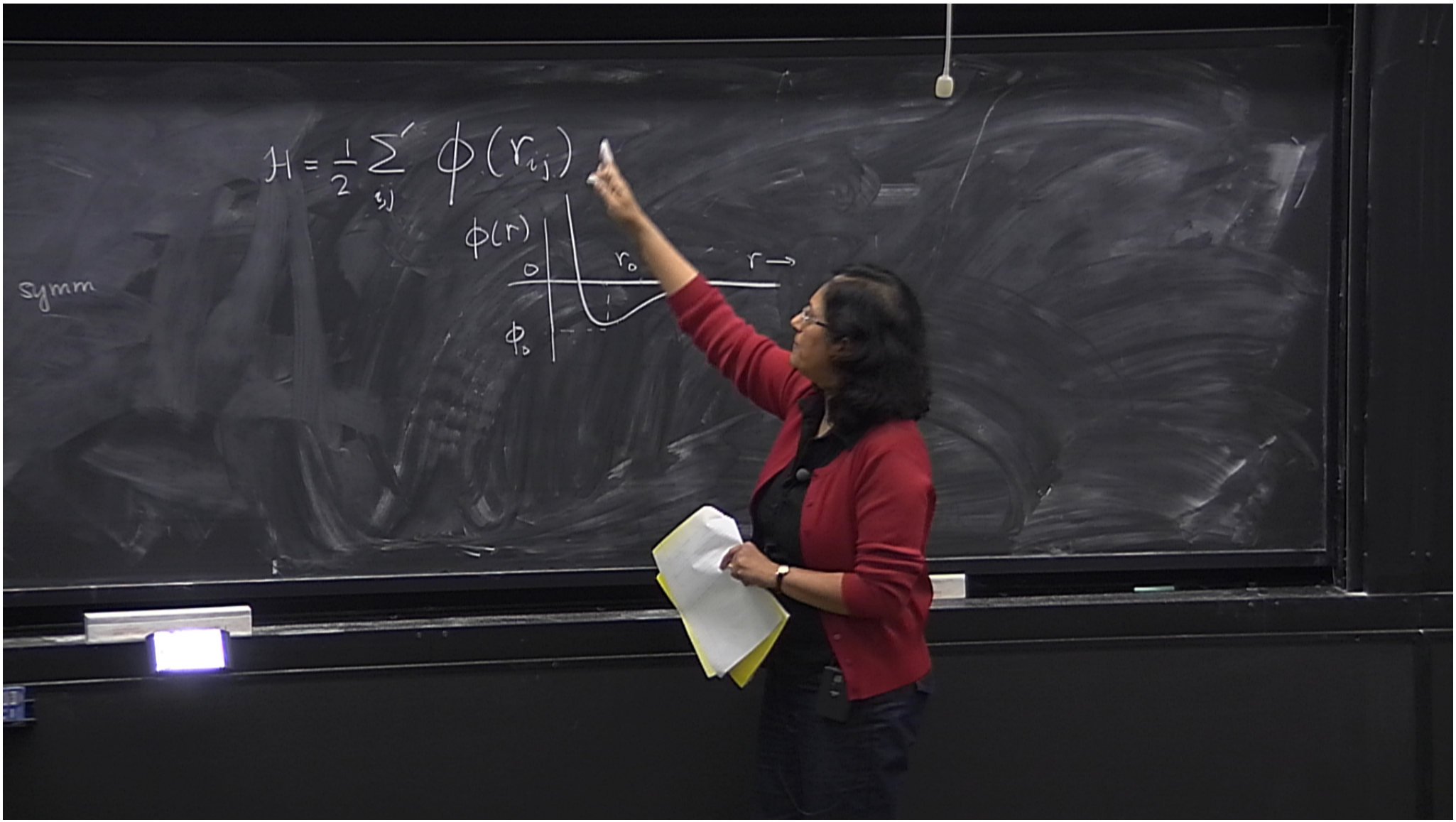
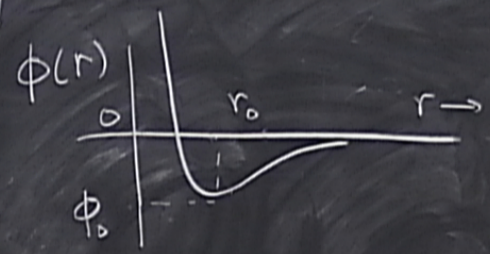
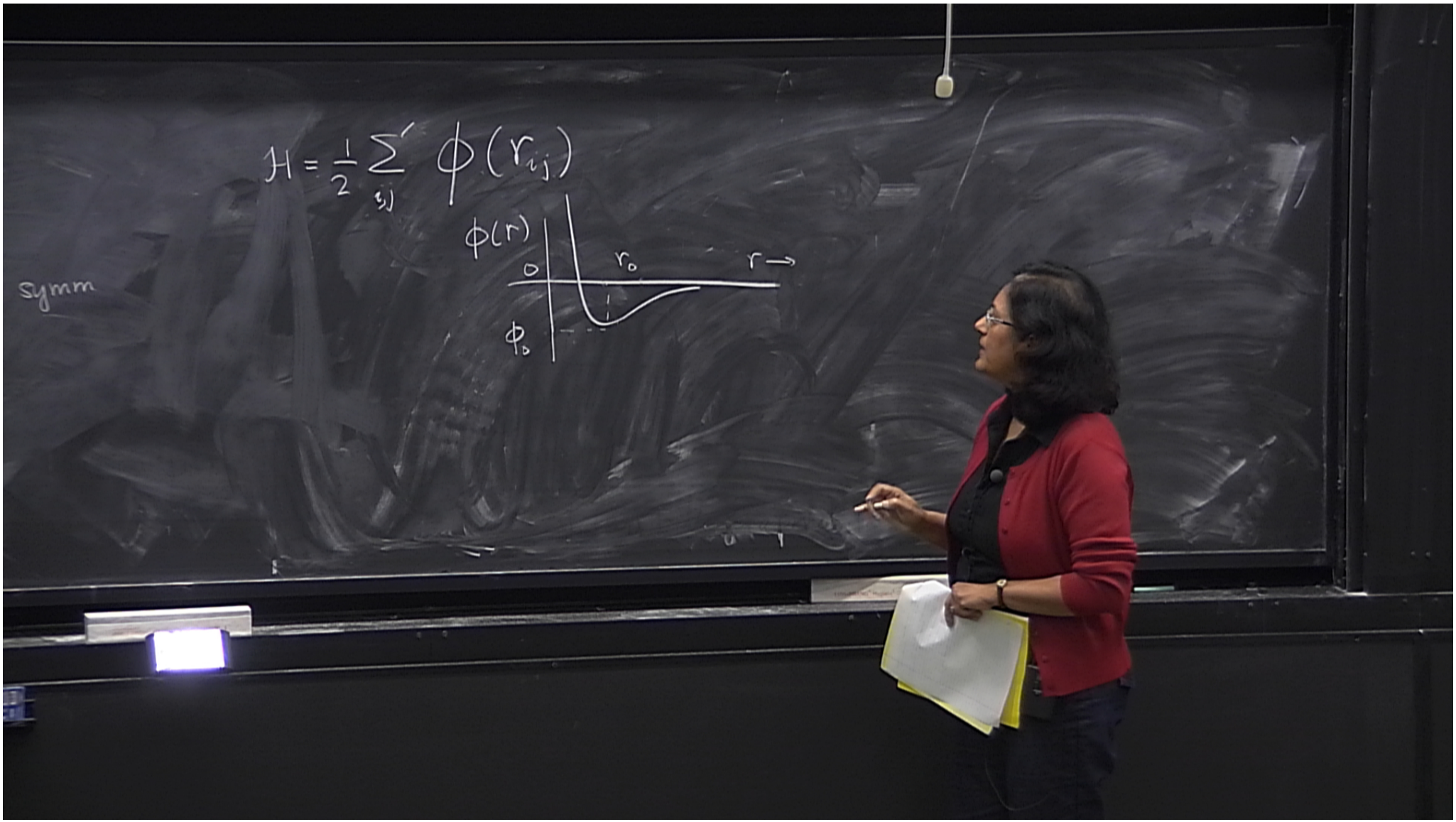

Xtals:
T
Gas
Liquid
Xtal
rotationally &
translationally symm


$\mathcal{H} = \frac{1}{2} \sum_{ij}' \phi(r_{ij})$
$\phi(r)$
0
r_0
$r \rightarrow$
ϕ_0
symm

$$\mathcal{H} = \frac{1}{2}\sum_{i,j}{}' \phi(r_{ij})$$



symm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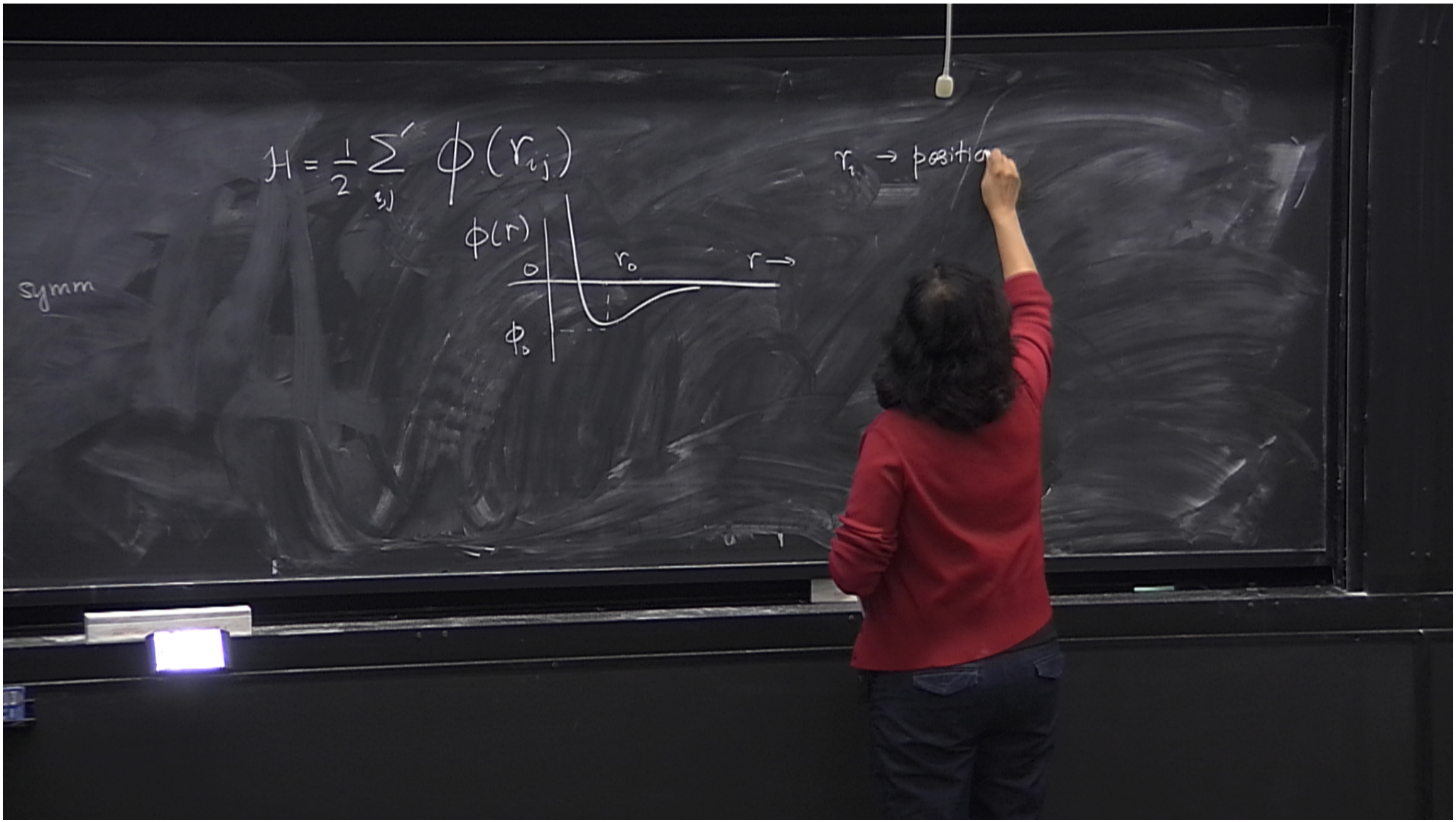
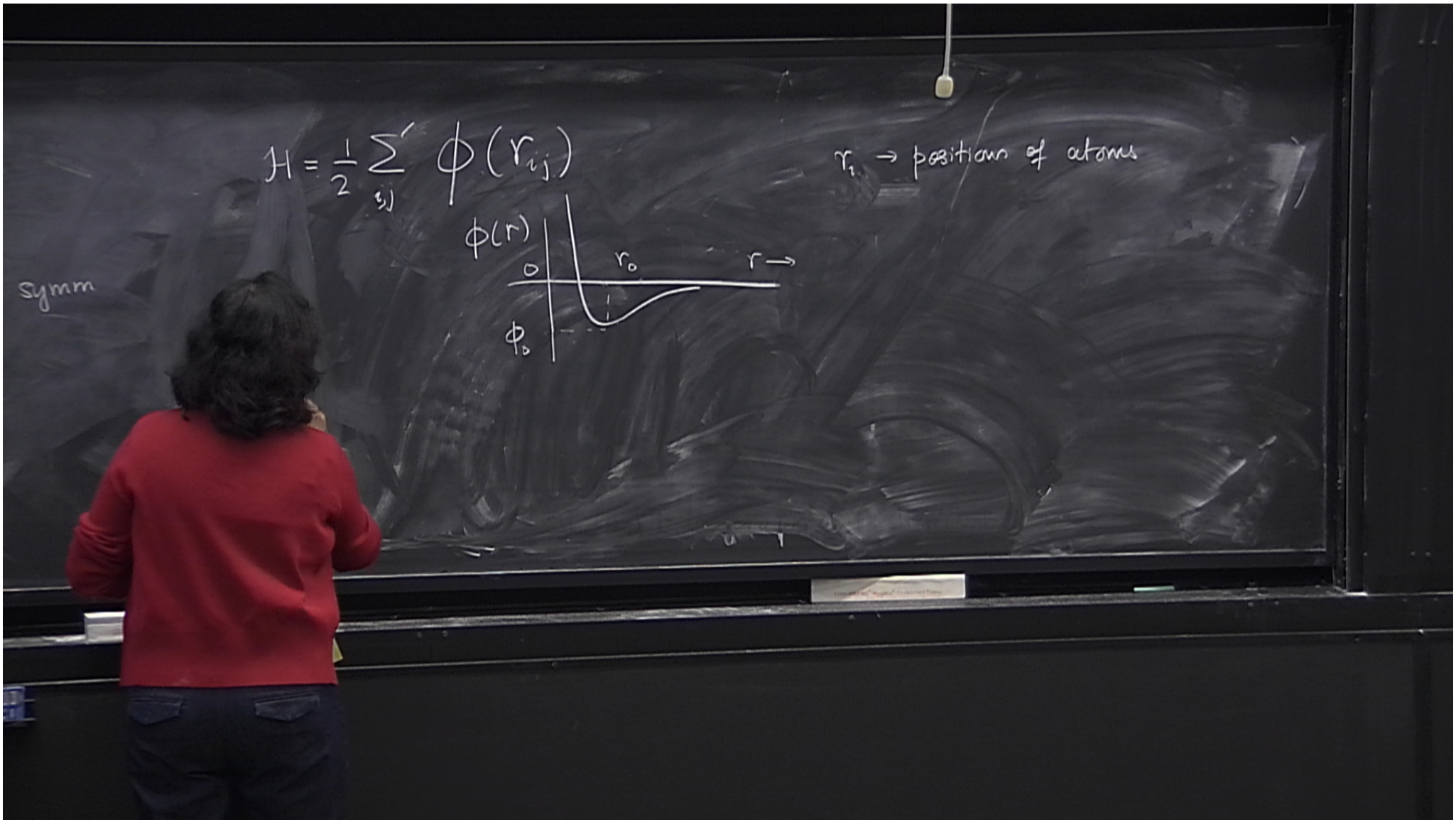
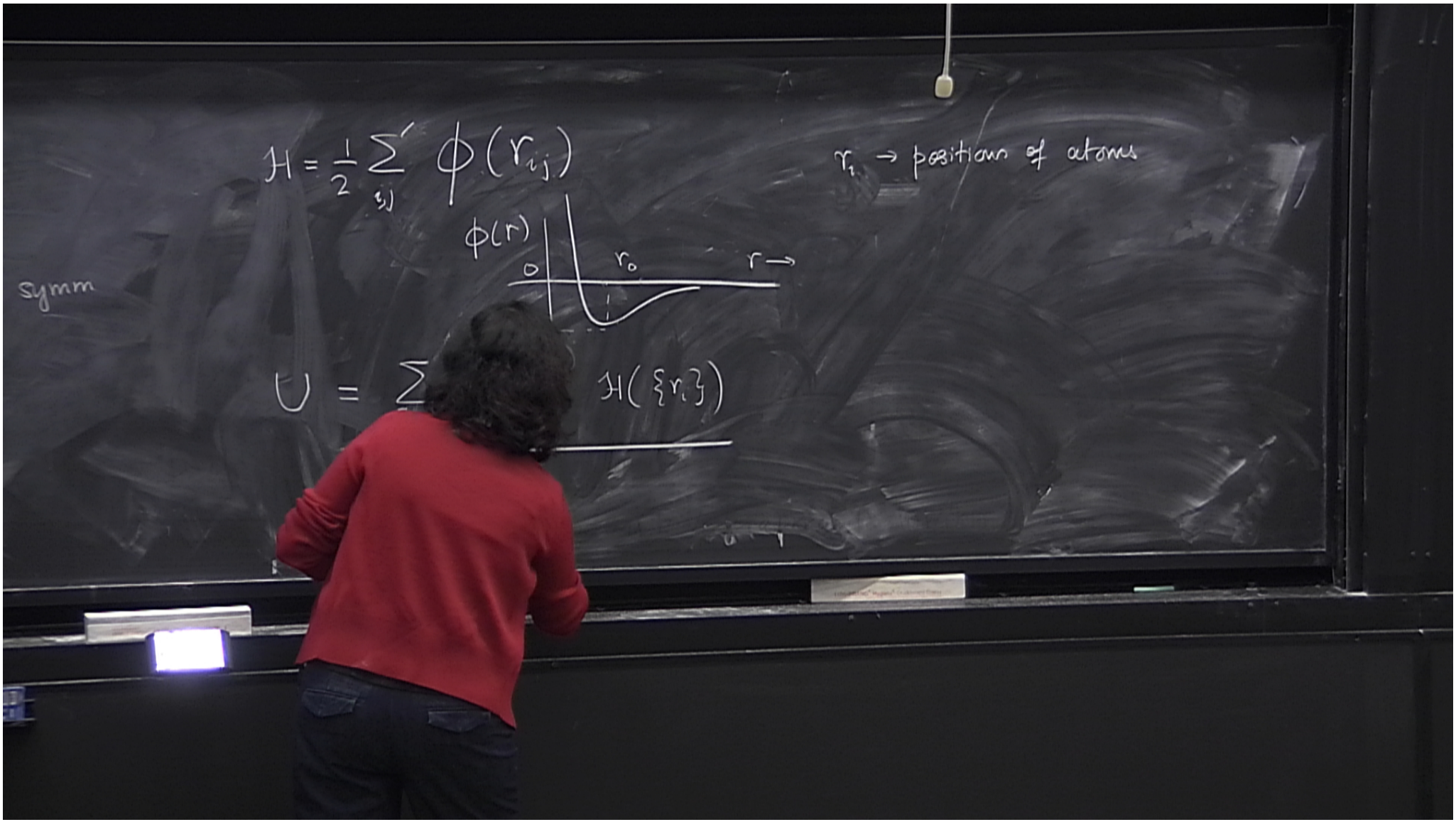
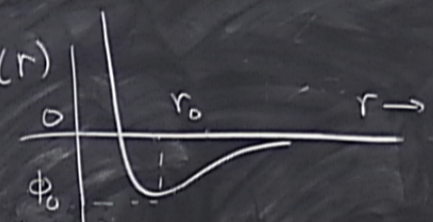

$\mathcal{H} = \frac{1}{2} \sum_{i,j}' \phi(r_{ij})$
$r_i \rightarrow$ position
$\phi(r)$
0
r_0
$r \rightarrow$
ϕ_0
symm


$H = \frac{1}{2} \sum_{i,j}' \phi(r_{ij})$
$r_i \rightarrow$ positions of atoms
$\phi(r)$
0
r_0
$r \rightarrow$
ϕ_0
symm


$\mathcal{H} = \frac{1}{2} \sum_{i,j}' \phi(r_{ij})$
$r_i \rightarrow$ positions of atoms
$\phi(r)$
0
r_0
$r \rightarrow$
symm
$U = \sum \mathcal{H}(\{r_i\})$

$$\mathcal{H} = \frac{1}{2}\sum_{i,j}{}' \phi(r_{ij})$$

$r_i \rightarrow$ positions of atoms



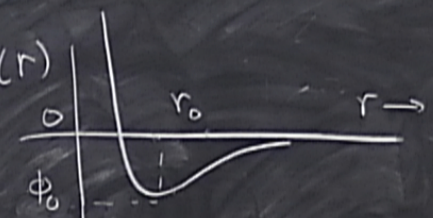
symm

$$U = \frac{\sum_{\{r_i\}} e^{-\beta \mathcal{H}(\{r_i\})} \mathcal{H}(\{r_i\})}{Z}$$

$$\{r_i\} = \{\bar{r}_1, \ldots \bar{r}_N\}$$

$$\mathcal{H} = \frac{1}{2}\sum_{i,j}{}' \phi(r_{ij})$$

$r_i \rightarrow$ positions of atoms



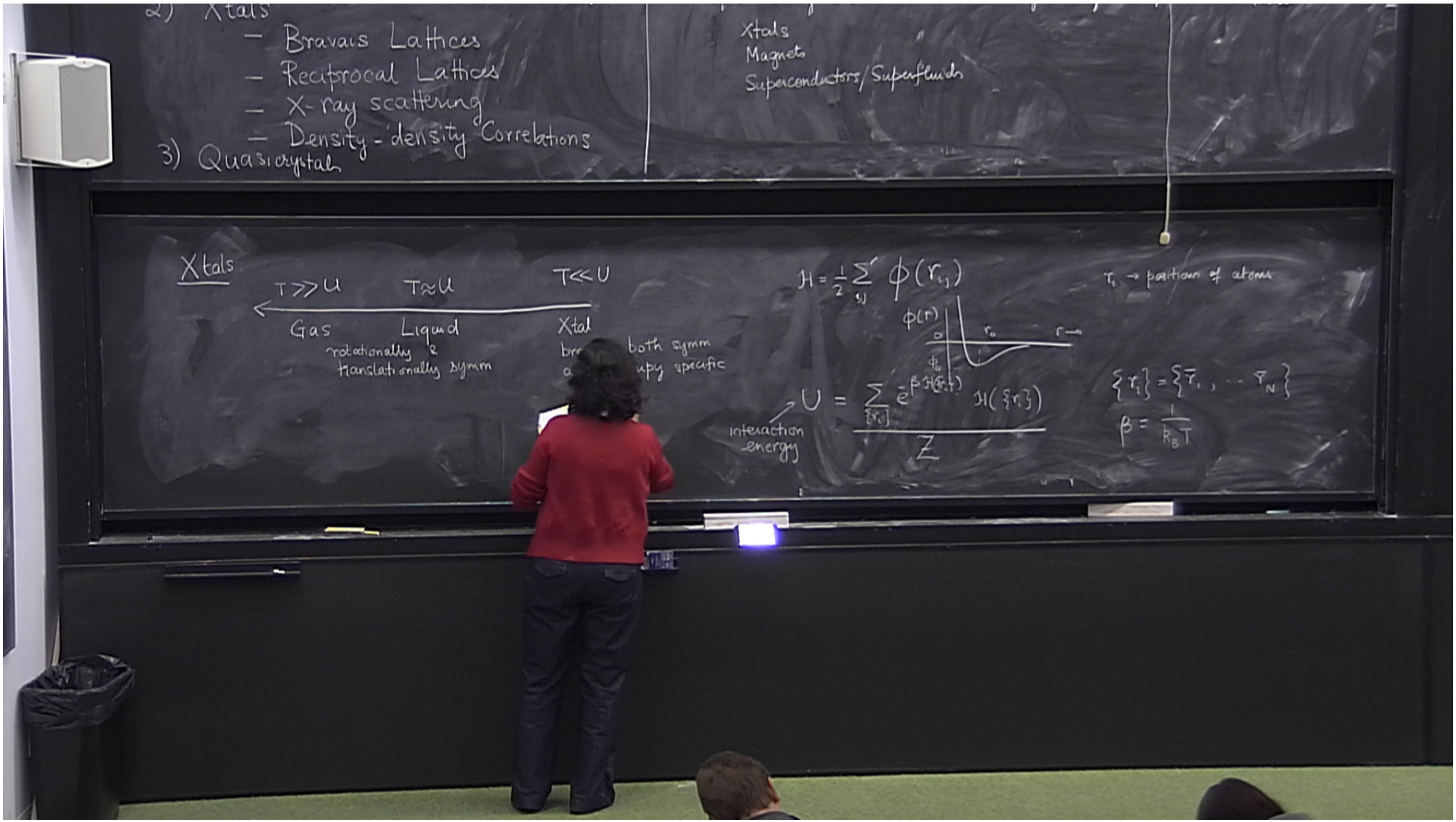
symm

$$U = \frac{\sum_{\{r_i\}} e^{-\beta \mathcal{H}(\{r_i\})} \mathcal{H}(\{r_i\})}{Z}$$

interaction energy

$$\{r_i\} = \{\bar{r}_1, \ldots \bar{r}_N\}$$

$$\beta = \frac{1}{k_B T}$$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Xtals
T >> U
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$\mathcal{H} = \frac{1}{2}\sum_{ij}' \phi(r_{ij})$
$r_i \to$ positions of atoms
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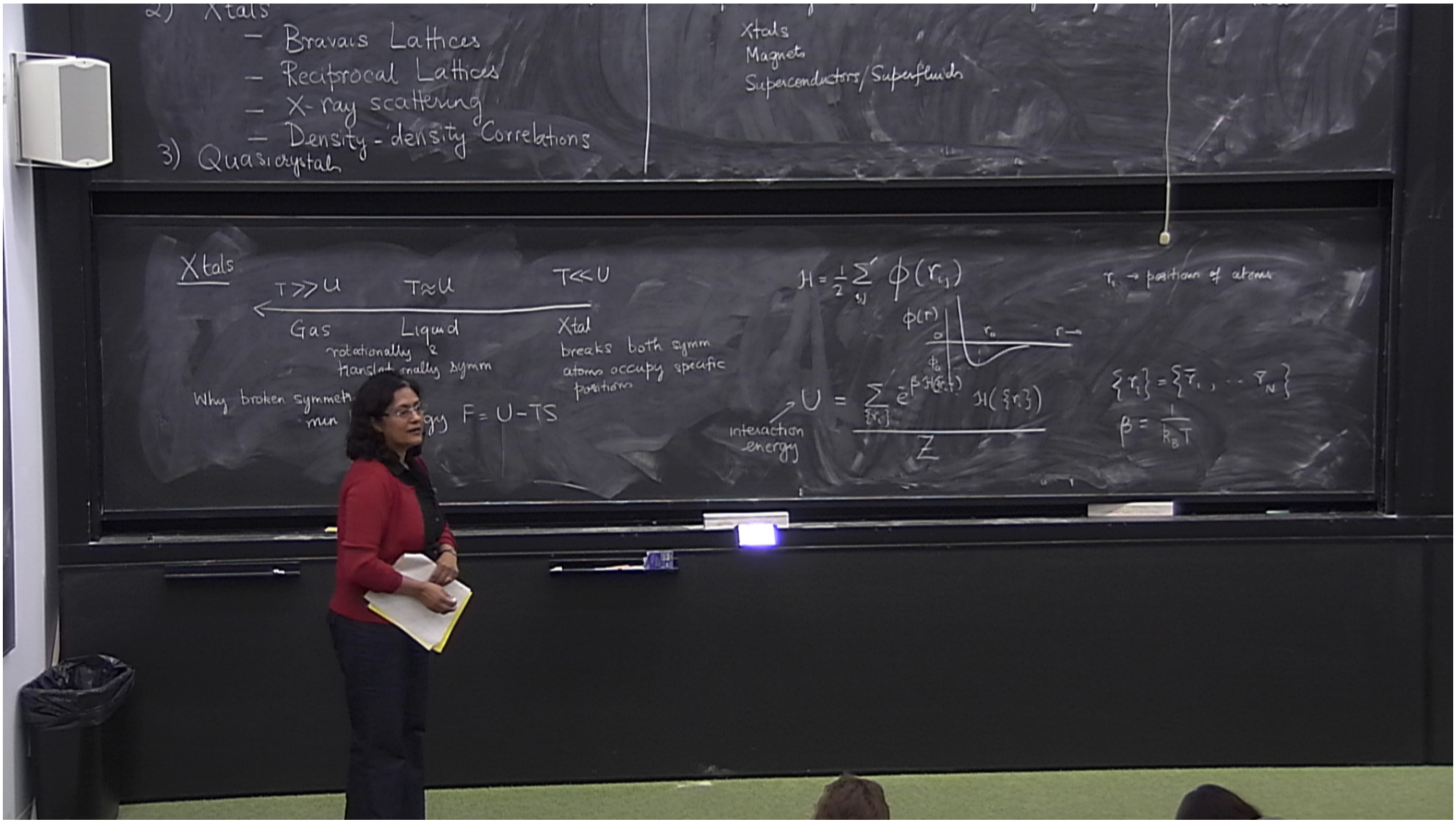
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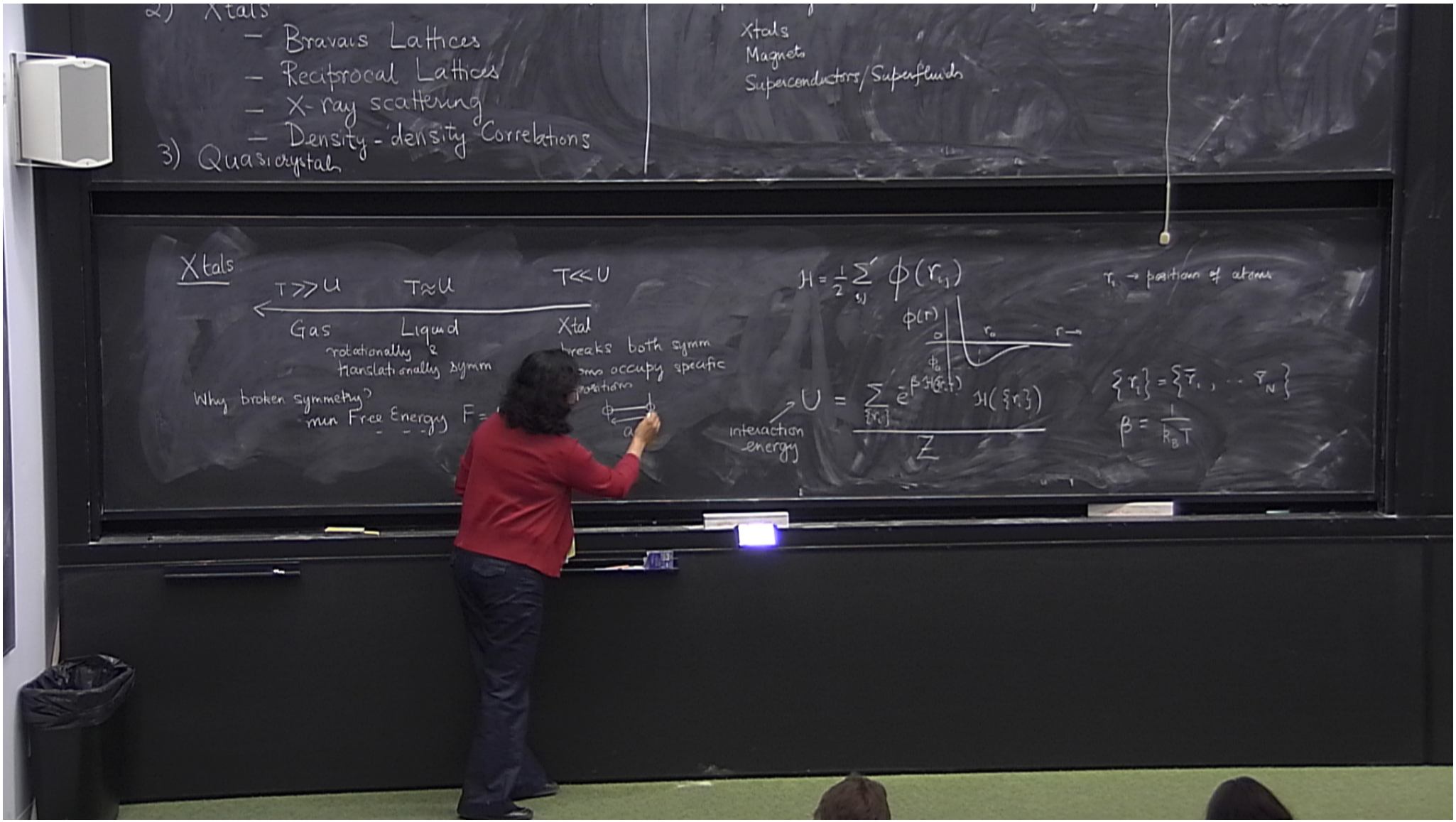
Magnets
Superconductors/Superfluids
Xtals
T>>U
T≈U
T<<U
Gas
Liquid
Xtal
rotationally & translationally symm
breaks both symm
atoms occupy specific positions
Why broken symmetry
F = U − TS
interaction energy
r_i → positions of atoms
β = 1/k_B T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Xtals
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Superconductors/Superfluids
Xtals
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Gas
Liquid
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rotationally &
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Why broken symmetry?
min Free Energy
$\mathcal{H} = \frac{1}{2}\sum_{ij}' \phi(r_{ij})$
$r_i \to$ positions of atoms
$\phi(r)$
r_0
$r \to$
ϕ_0
$U = \frac{\sum_{\{r_i\}} e^{-\beta \mathcal{H}(\{r_i\})} \mathcal{H}(\{r_i\})}{Z}$
interaction energy
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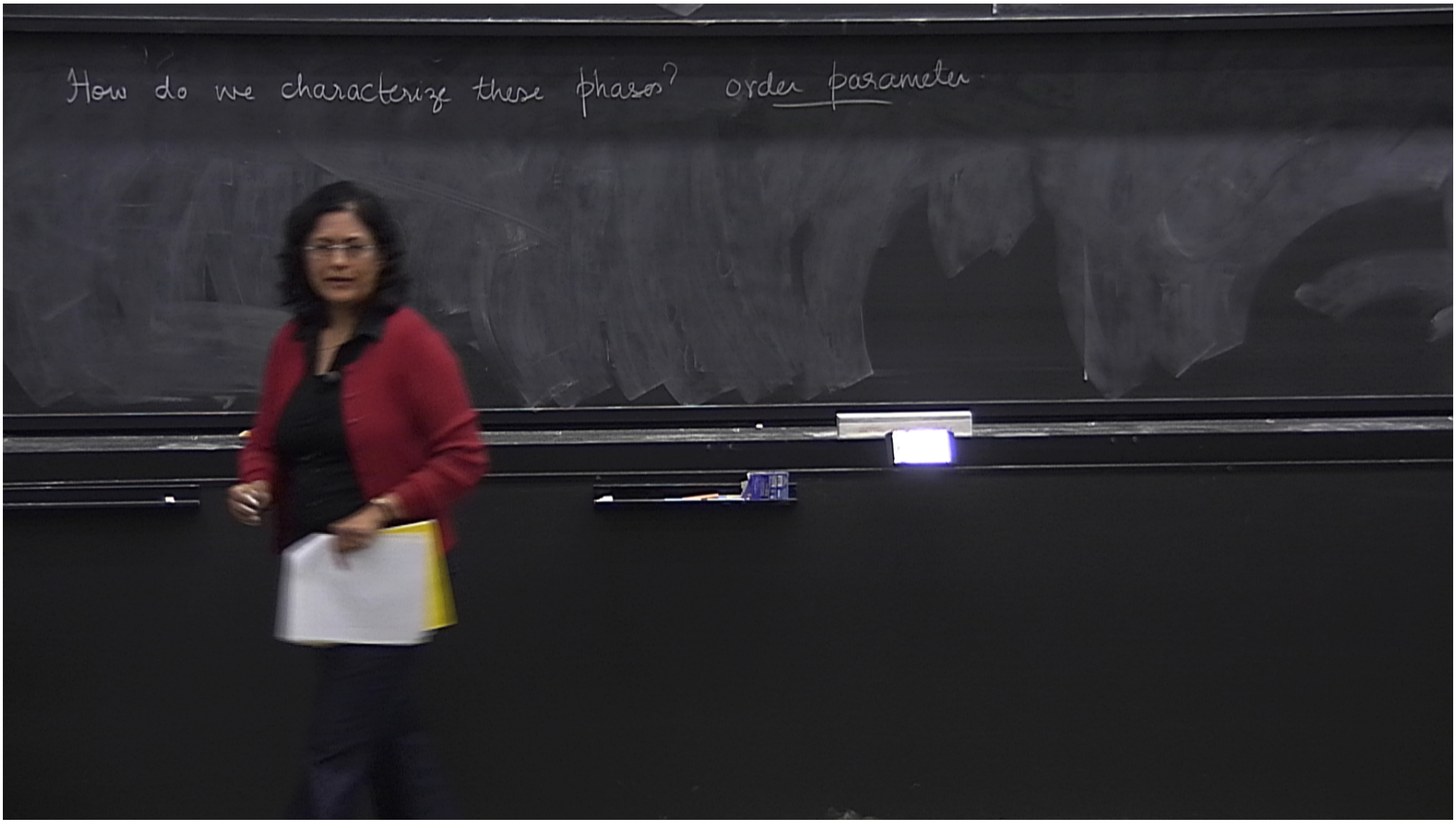
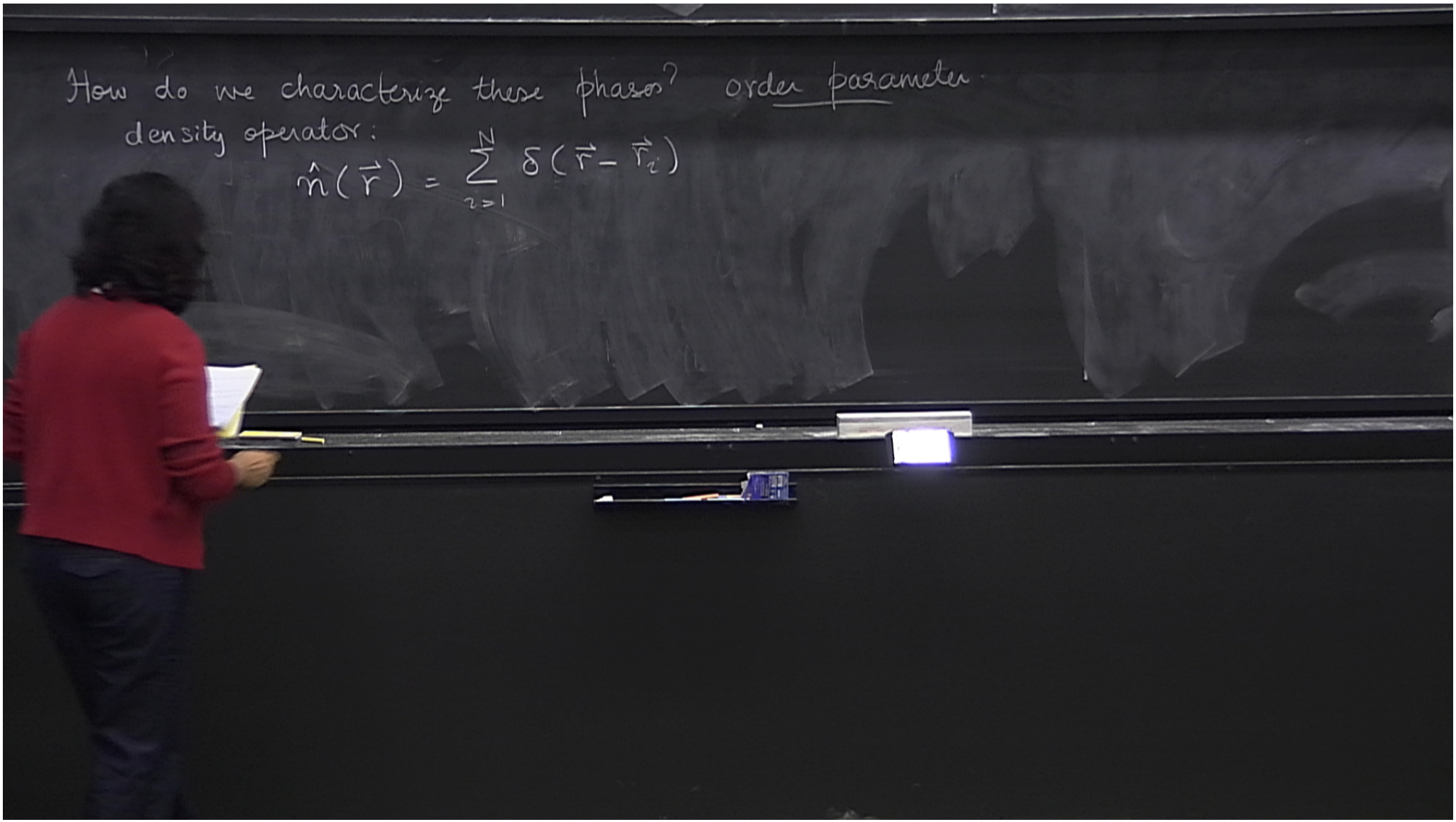
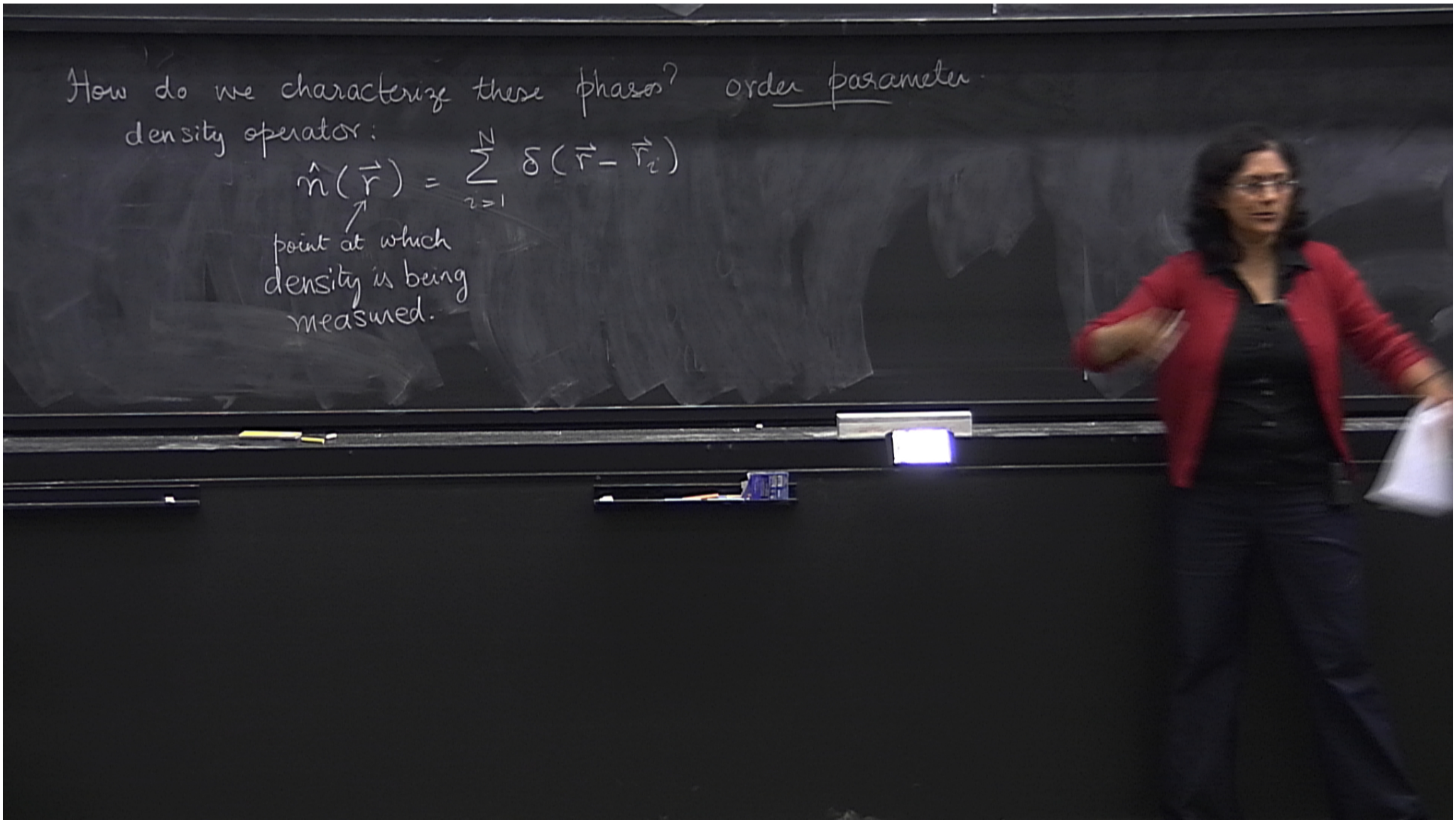

How do we characterize these phases? order parameter


How do we characterize these phases? order parameter.
density operator:
$\hat{n}(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i)$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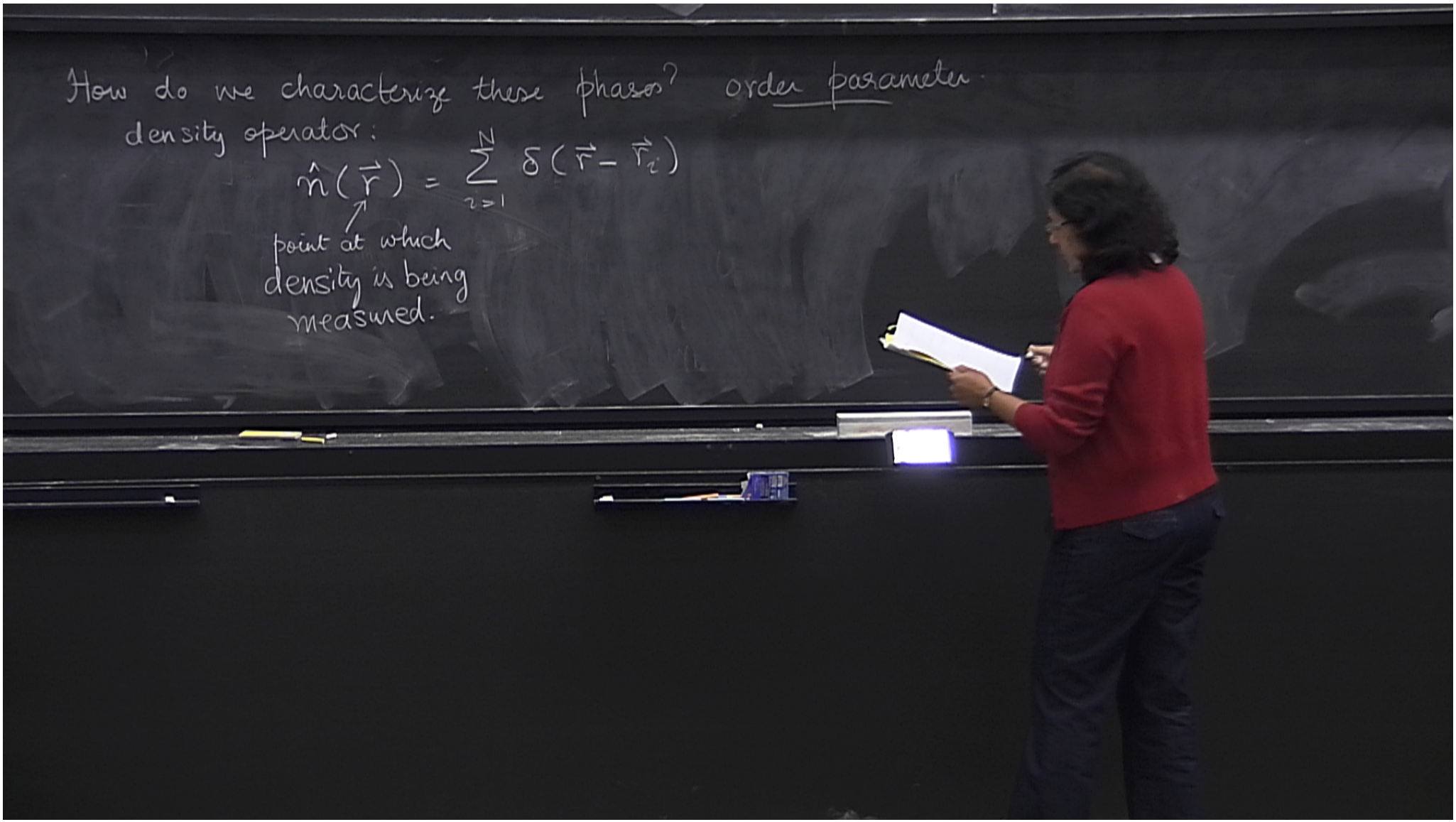
$\hat{n}(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i)$
point at which density is being measured.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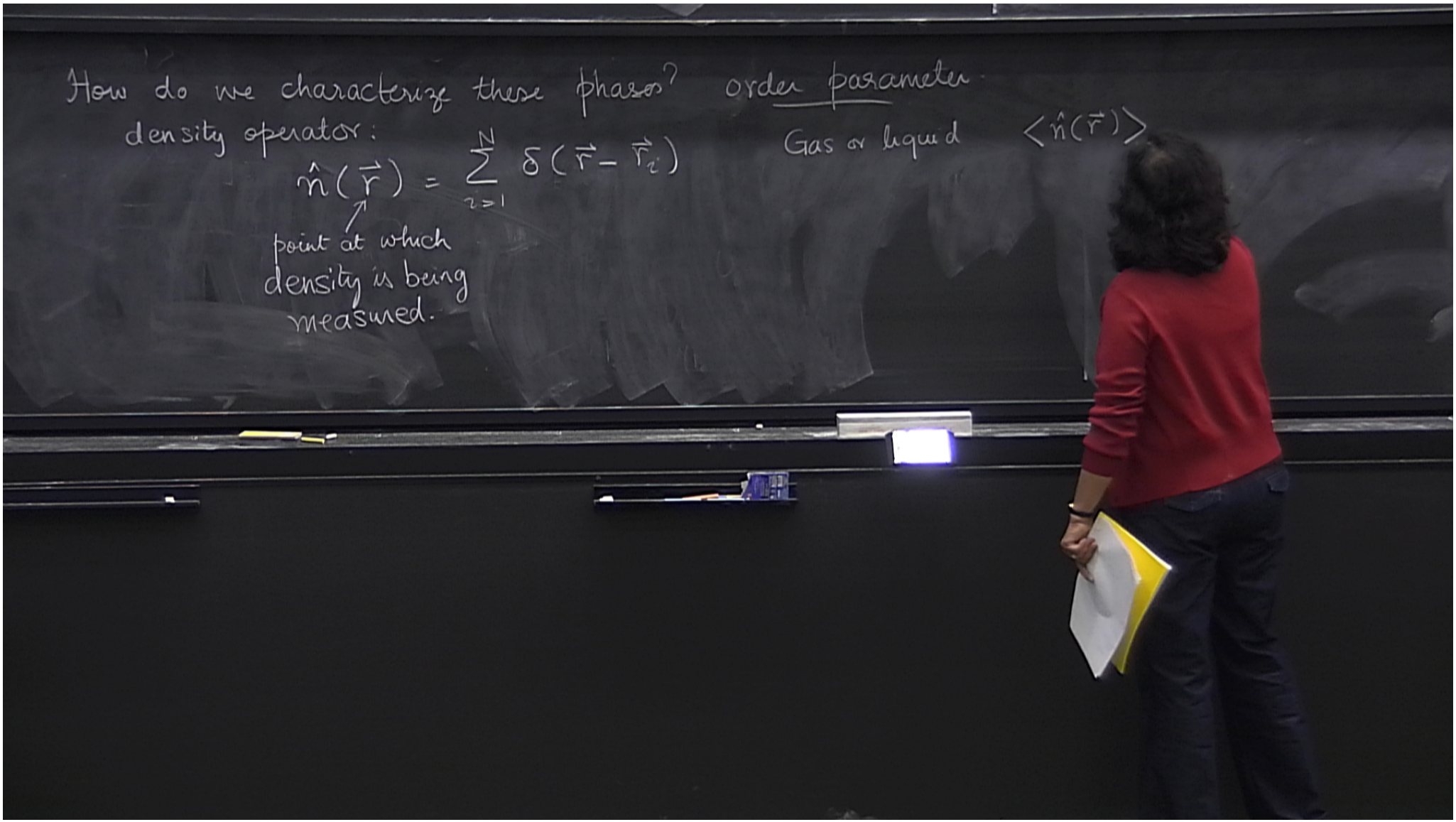
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density operator:
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point at which density is being measured.
Gas or liquid $\langle \hat{n}(\vec{r}) \rangle$

How do we characterize these phases? order parameter.

density operator:

$$\hat{n}(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i)$$

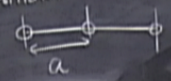
point at which density is being measured.

Gas or liquid $\langle \hat{n}(\vec{r}) \rangle$ statistical average

Why broken symmetry?

min Free Energy $F = U - TS$

positions



$$U = \frac{\sum_{\{r_i\}} e^{-\beta \mathcal{H}(\{r_i\})} \mathcal{H}(\{r_i\})}{Z}$$

interaction energy

$\{r_i\} = \{r_1, \ldots r_N\}$

$\beta = \frac{1}{k_B T}$

How do we characterize these phases? order parameter

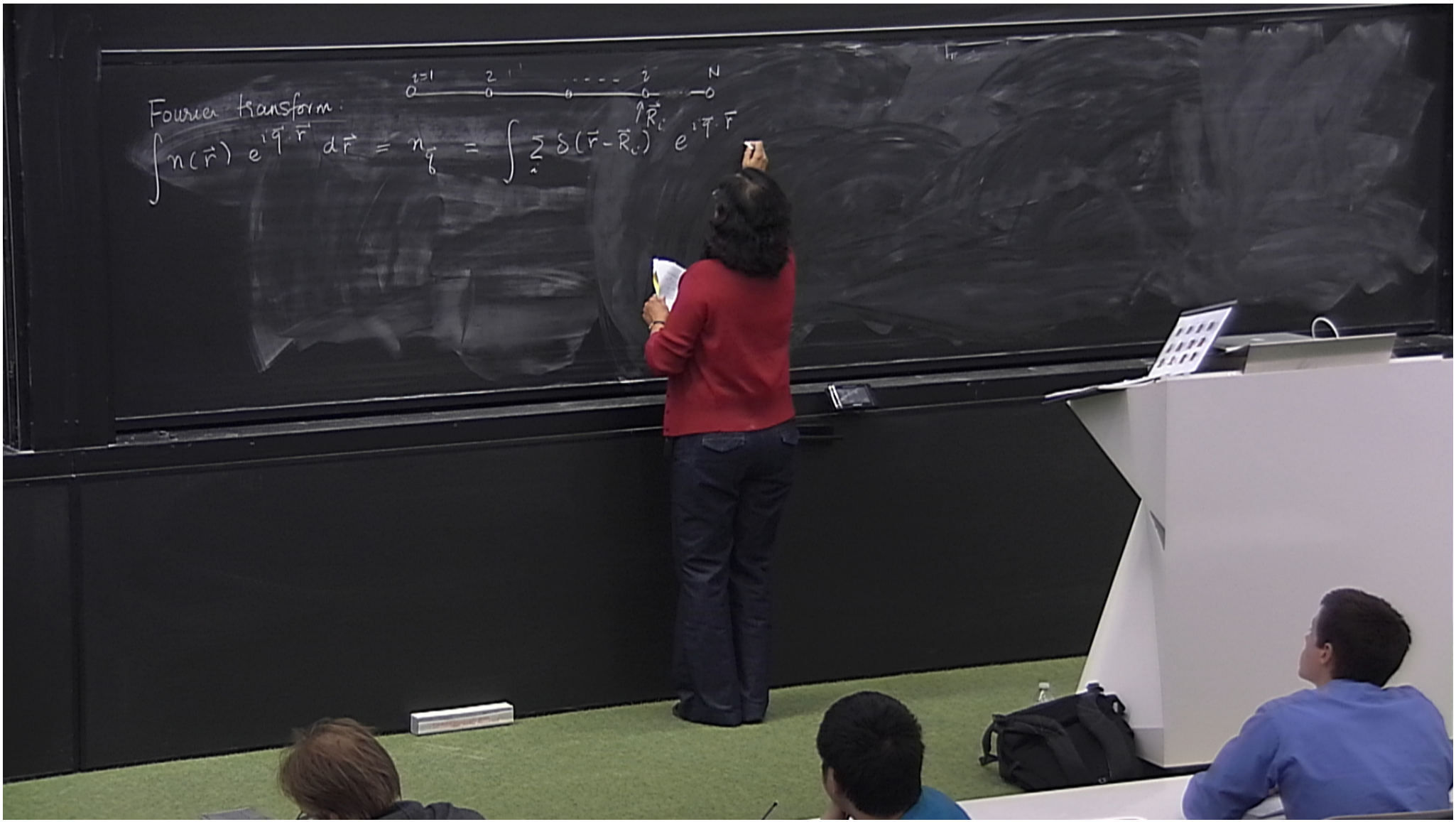
density operator:

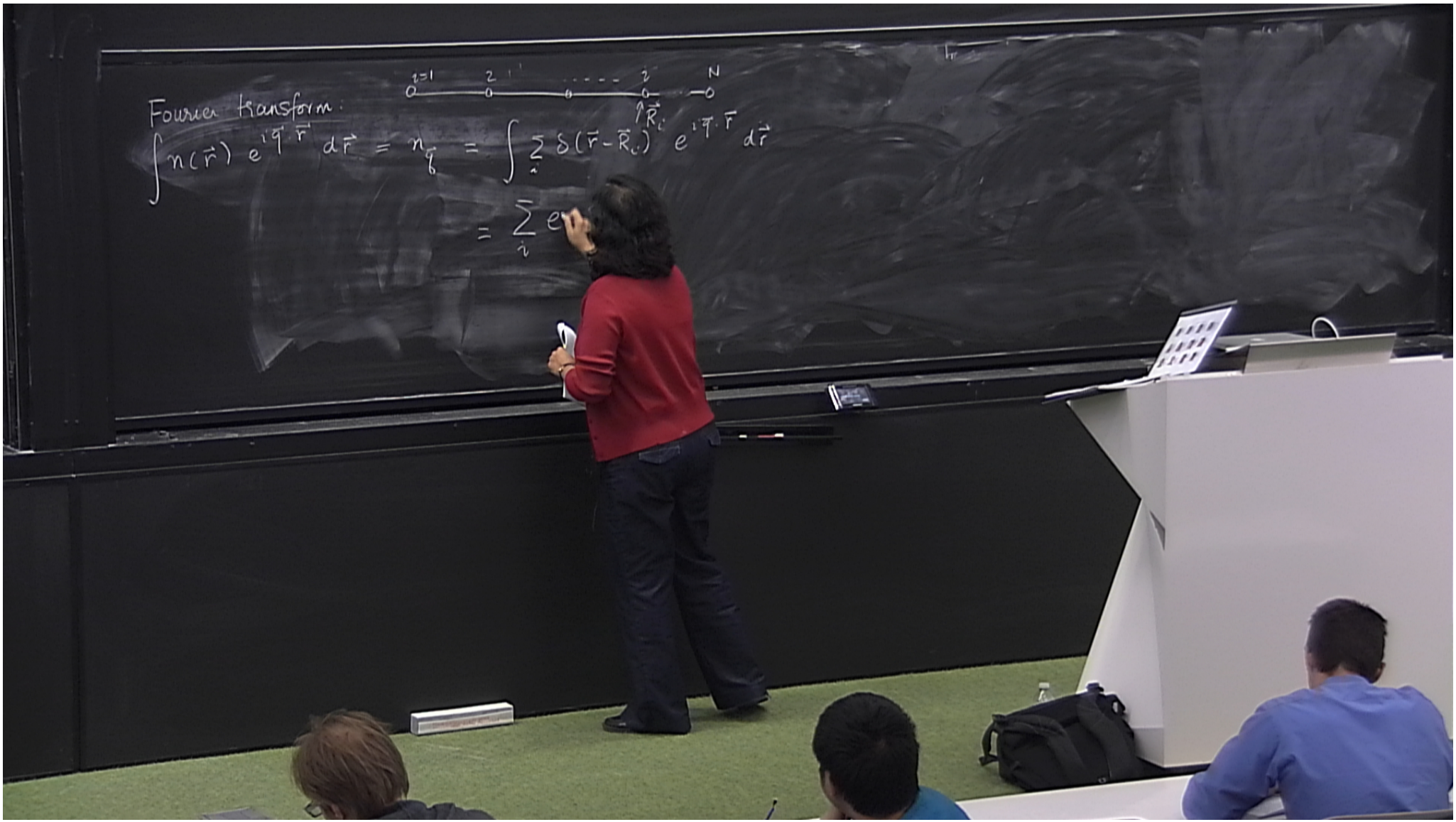
$$\hat{n}(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i)$$

point at which density is being measured.

Gas or liquid $\langle \hat{n}(\vec{r}) \rangle$ statistical average $= n_0 = \frac{N}{V}$ constant

Xtal: $n(\vec{r}) = \sum_i \delta(\vec{r} - \vec{R}_i)$; R_i are called Bravais Lattice positions


Fourier transform:
$\int n(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r} = n_{\vec{q}} = \int \sum_i \delta(\vec{r}-\vec{R}_i)\, e^{i\vec{q}\cdot\vec{r}}$
i=1
2
i
N
$\vec{R}_i$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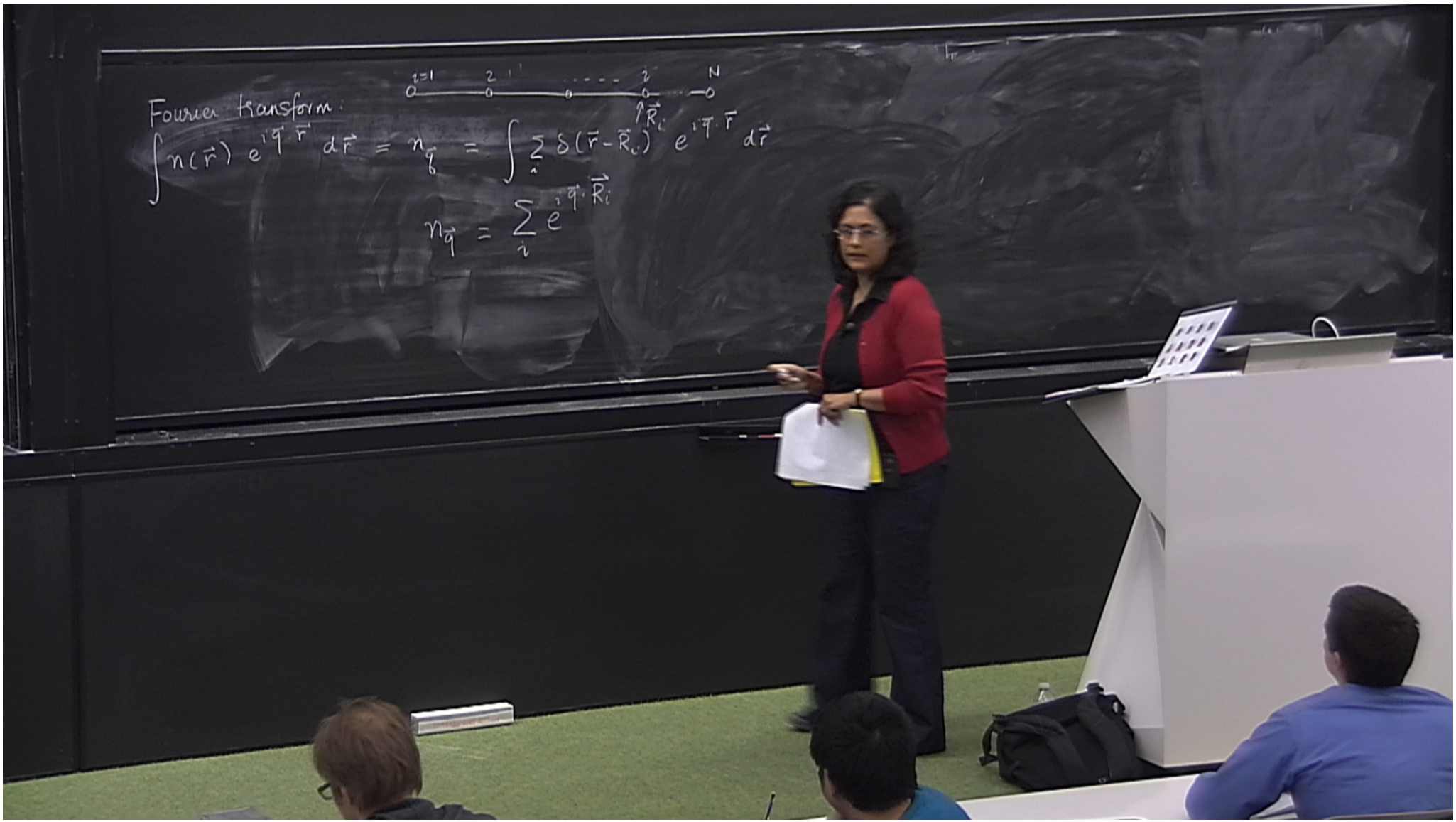
$= \sum_i e$
i=1
2
i
N
$\vec{R}_i$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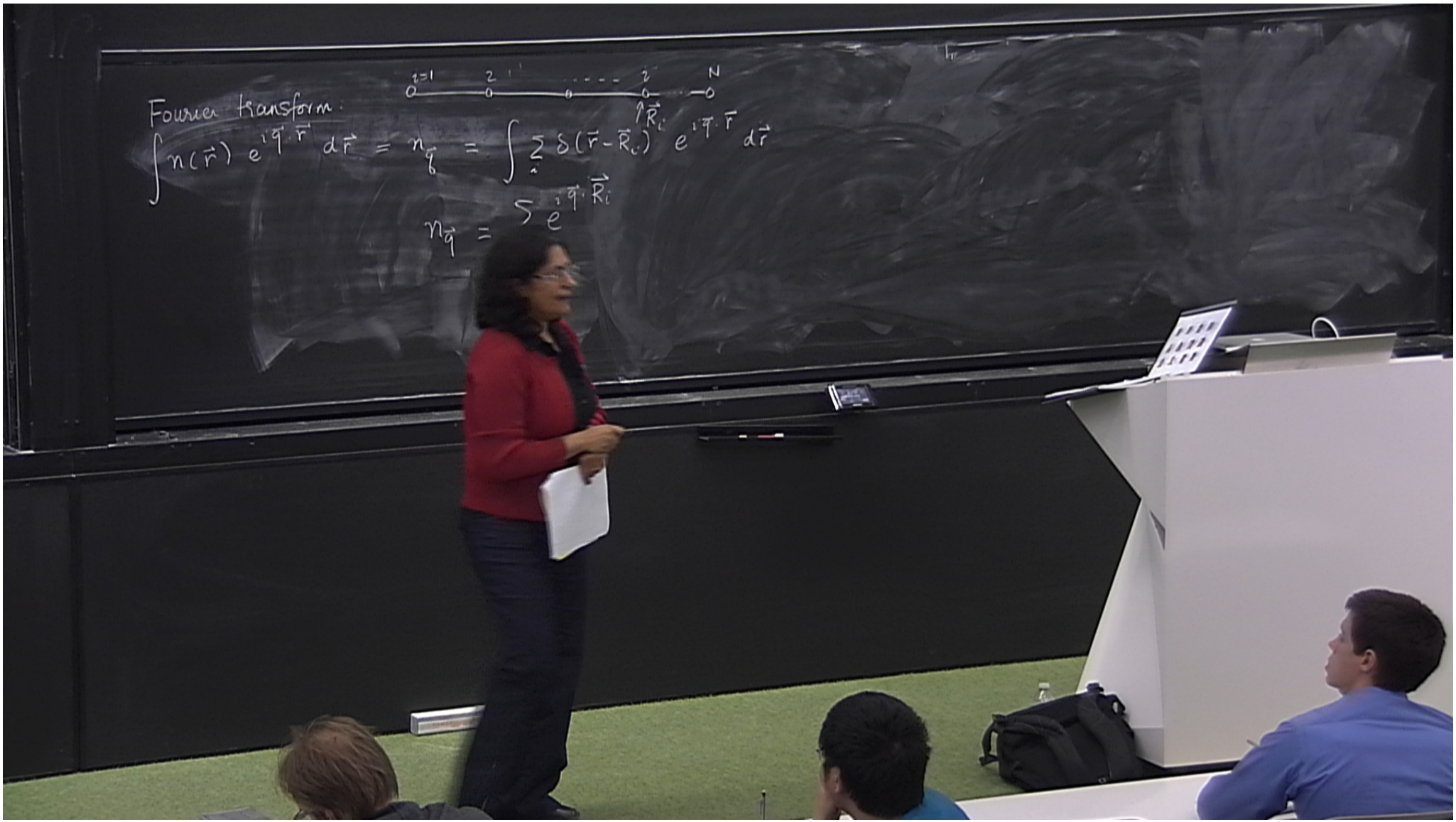
$n_{\vec{q}} = \sum_i e^{i\vec{q}\cdot\vec{R}_i}$
i=1
2
i
N
$\vec{R}_i$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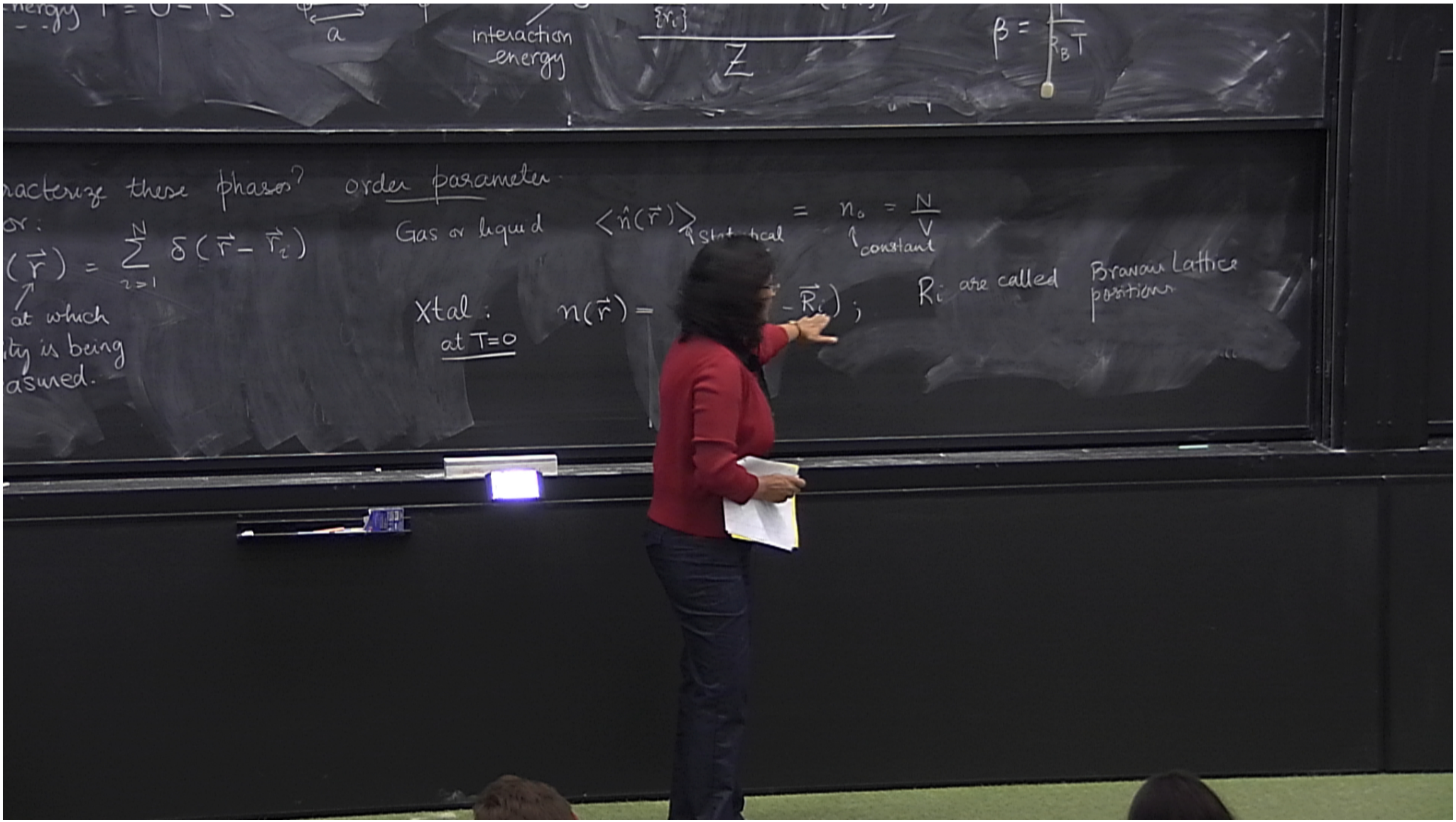
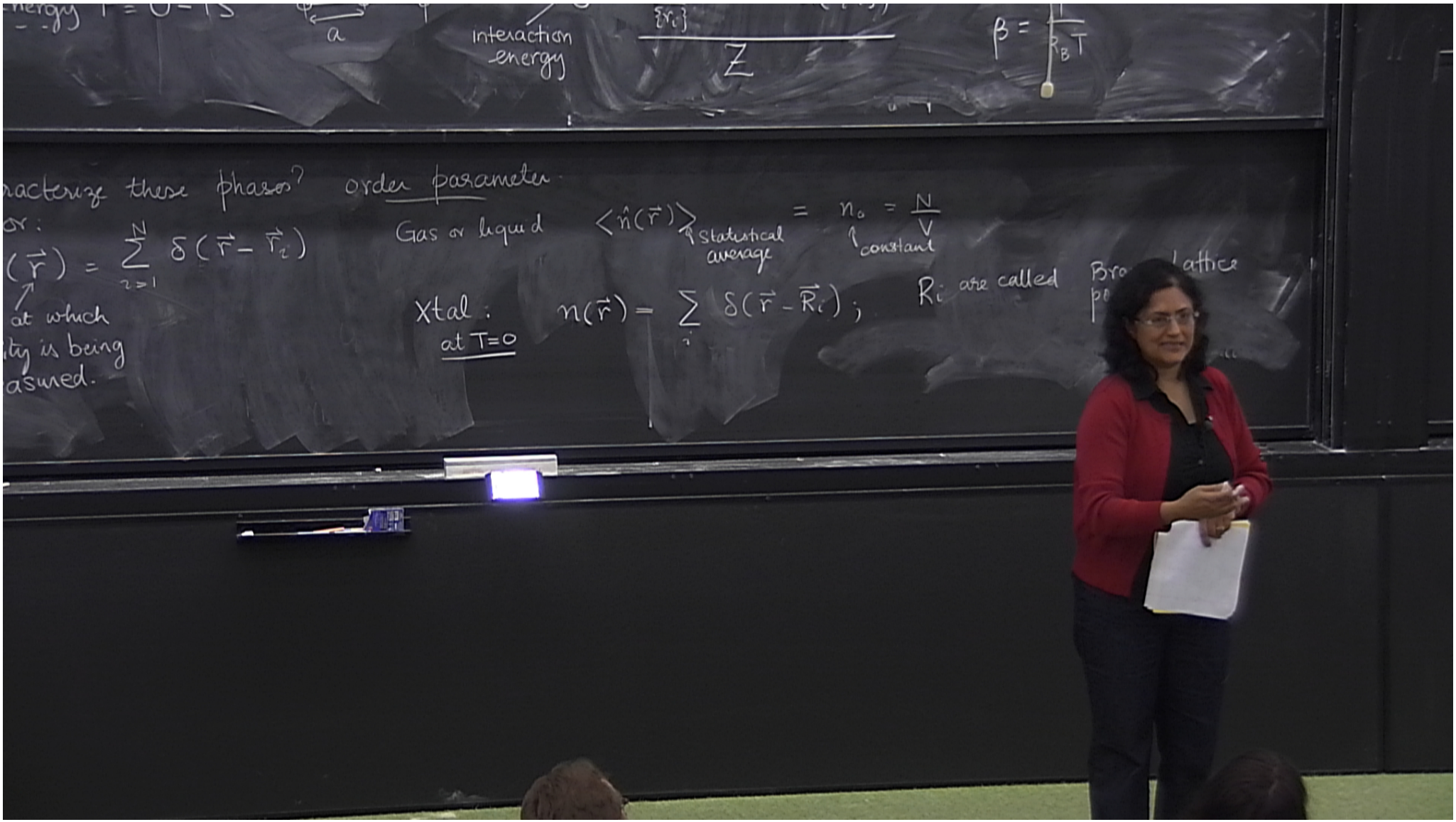
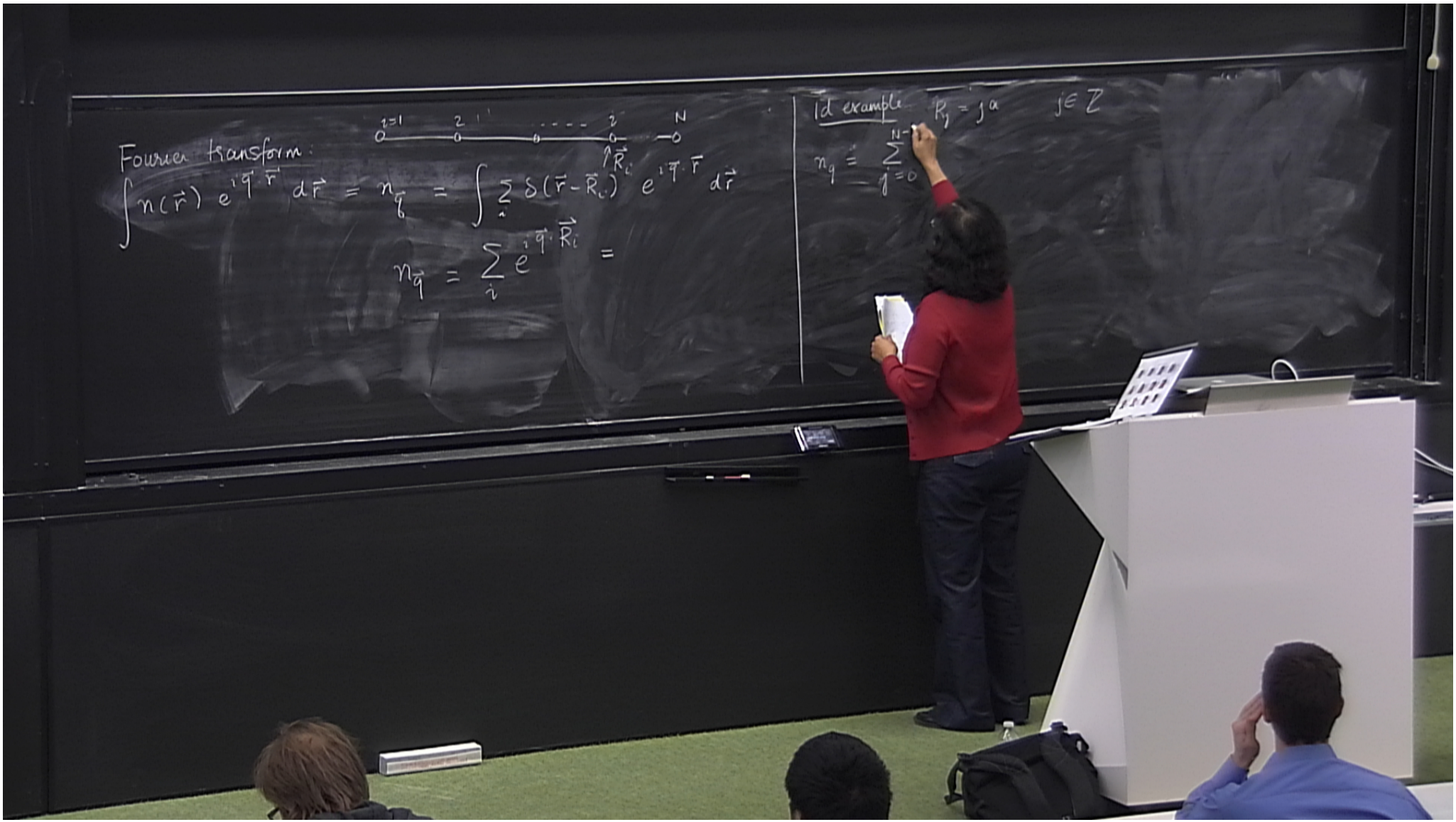
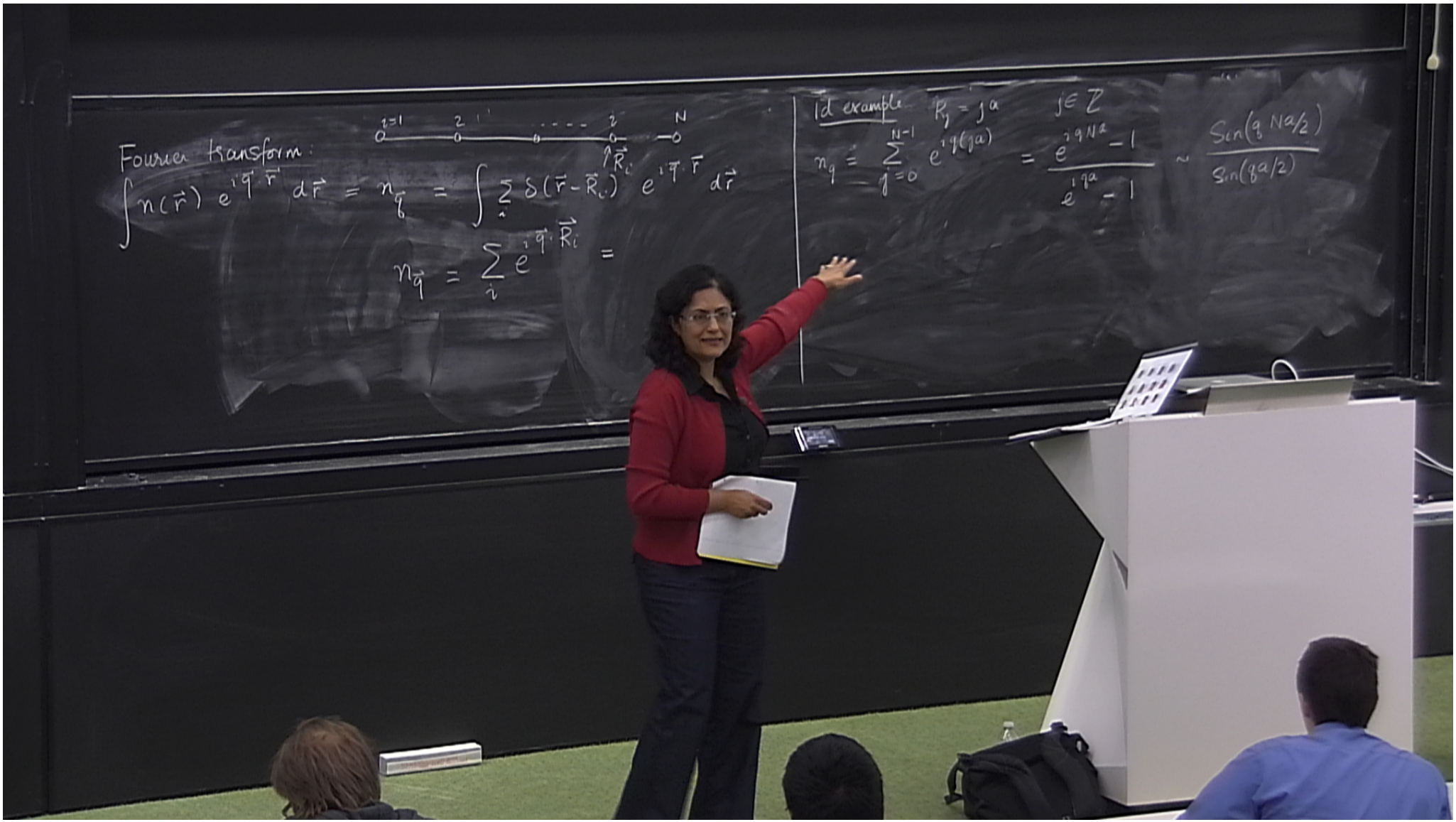
$\int n(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r} = n_{\vec{q}} = \int \sum_i \delta(\vec{r}-\vec{R}_i)\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r}$
$n_{\vec{q}} = \sum e^{i\vec{q}\cdot\vec{R}_i}$
i=1
2
i
N
$\vec{R}_i$


interaction energy
Z
β =
racterize these phases? order parameter
or:
(r) = Σ δ(r − r_i)
at which
ity is being
asured.
Gas or liquid
⟨n̂(r)⟩ = n_0 = N/V
constant
Xtal:
at T=0
n(r) =
R_i are called Bravais Lattice positions


interaction energy
Z
β =
racterize these phases? order parameter
or:
(r) = Σ δ(r − r_i)
at which
ity is being
asured.
Gas or liquid
⟨n̂(r)⟩ statistical average = n_0 = N/V
constant
Xtal: at T=0
n(r) = Σ δ(r − R_i);
R_i are called
lattice


Fourier transform:
$\int n(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r} = n_{\vec{q}} = \int \sum_i \delta(\vec{r}-\vec{R}_i)\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r}$
$n_{\vec{q}} = \sum_i e^{i\vec{q}\cdot\vec{R}_i} =$
1=1
2
N
$\vec{R}_i$
1d example $R_j = ja$ $j \in \mathbb{Z}$
$n_q = \sum_{j=0}^{N-1}$


Fourier transform:
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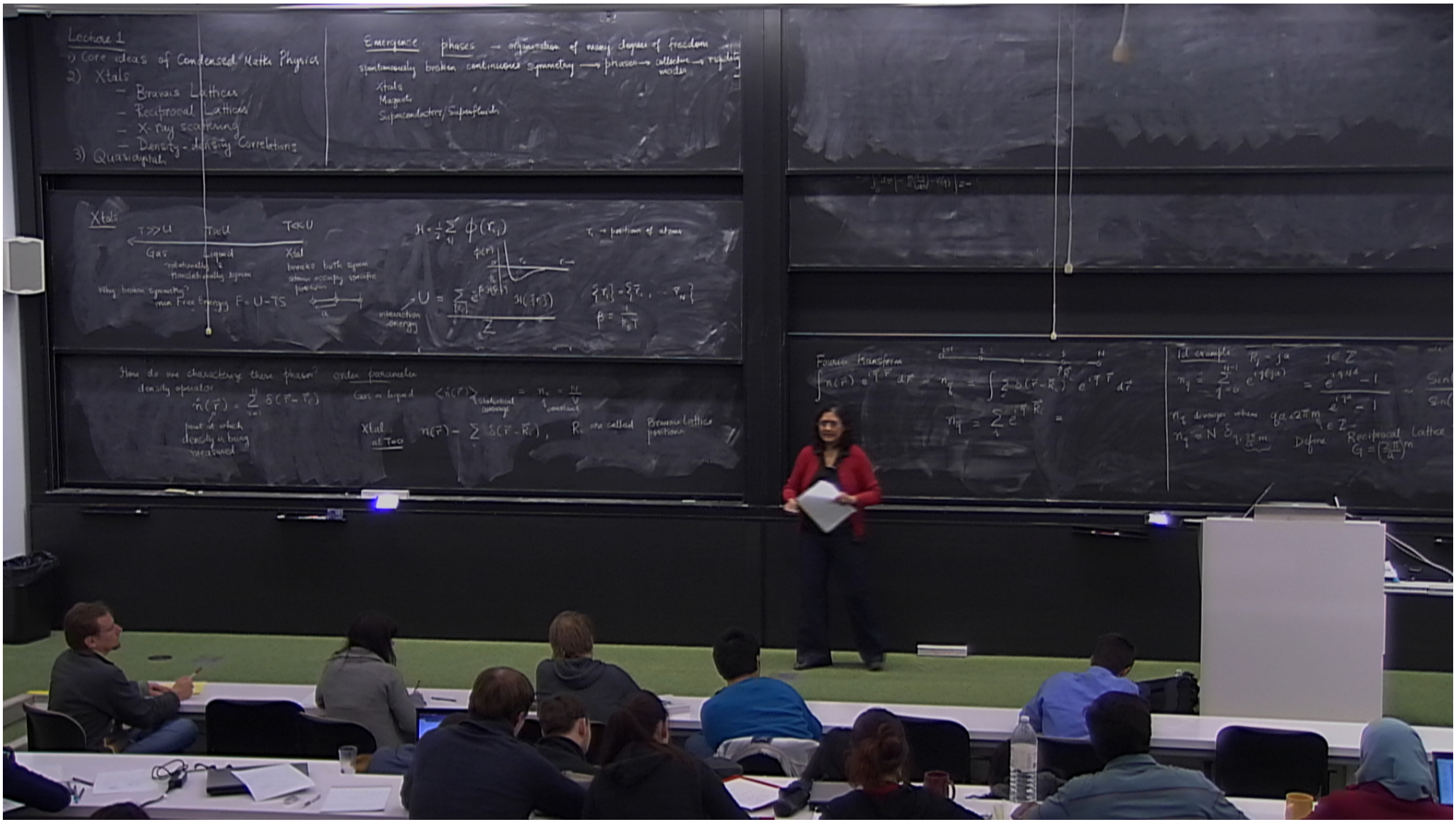
i=1
2
i
N
$\vec{R}_i$
1d example
$R_j = ja$
$j \in \mathbb{Z}$
$n_q = \sum_{j=0}^{N-1} e^{iq(ja)} = \frac{e^{iqNa}-1}{e^{iqa}-1} \sim \frac{\sin(qNa/2)}{\sin(qa/2)}$

1d example $R_j = ja \qquad j \in \mathbb{Z}$

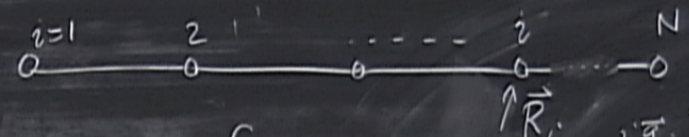
$$n_q \equiv \sum_{j=0}^{N-1} e^{iq(ja)} = \frac{e^{iqNa} - 1}{e^{iqa} - 1} \sim \frac{\sin(qNa/2)}{\sin(qa/2)}$$

n_q diverges when $qa = 2\pi m \quad m \in \mathbb{Z}$

$$n_q = N\,\delta_{q,\frac{2\pi}{a}m}$$


Lecture 1
1) Core ideas of Condensed Matter Physics
2) Xtals
– Bravais Lattices
– Reciprocal Lattices
– X-ray scattering
– Density-density Correlations
3) Quasicrystals
Emergence phases
Xtals
Magnets
Superconductors/Superfluids
Xtals
Gas
Liquid
Xtal
How do we characterize these phases?
order parameter
density operator
Xtal at T=0
Fourier transform
1d example
Define Reciprocal Lattice

Fourier transform:



$$\int n(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r} = n_{\vec{q}} = \int \sum_i \delta(\vec{r}-\vec{R}_i)\, e^{i\vec{q}\cdot\vec{r}}\, d\vec{r}$$

$$n_{\vec{q}} = \sum_i e^{i\vec{q}\cdot\vec{R}_i} = N\,\delta_{\vec{q},\vec{G}}$$

$\vec{G}$ defined by $\vec{G}\cdot\vec{R} = 2\pi m, \quad m \in \mathbb{Z}$

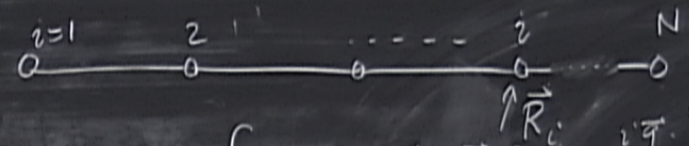
1d example

$$n_q = \sum_{j=0}^{N-1}$$

n_q diverges wh

$n_q = N\,\delta_{q,}$

Fourier transform:



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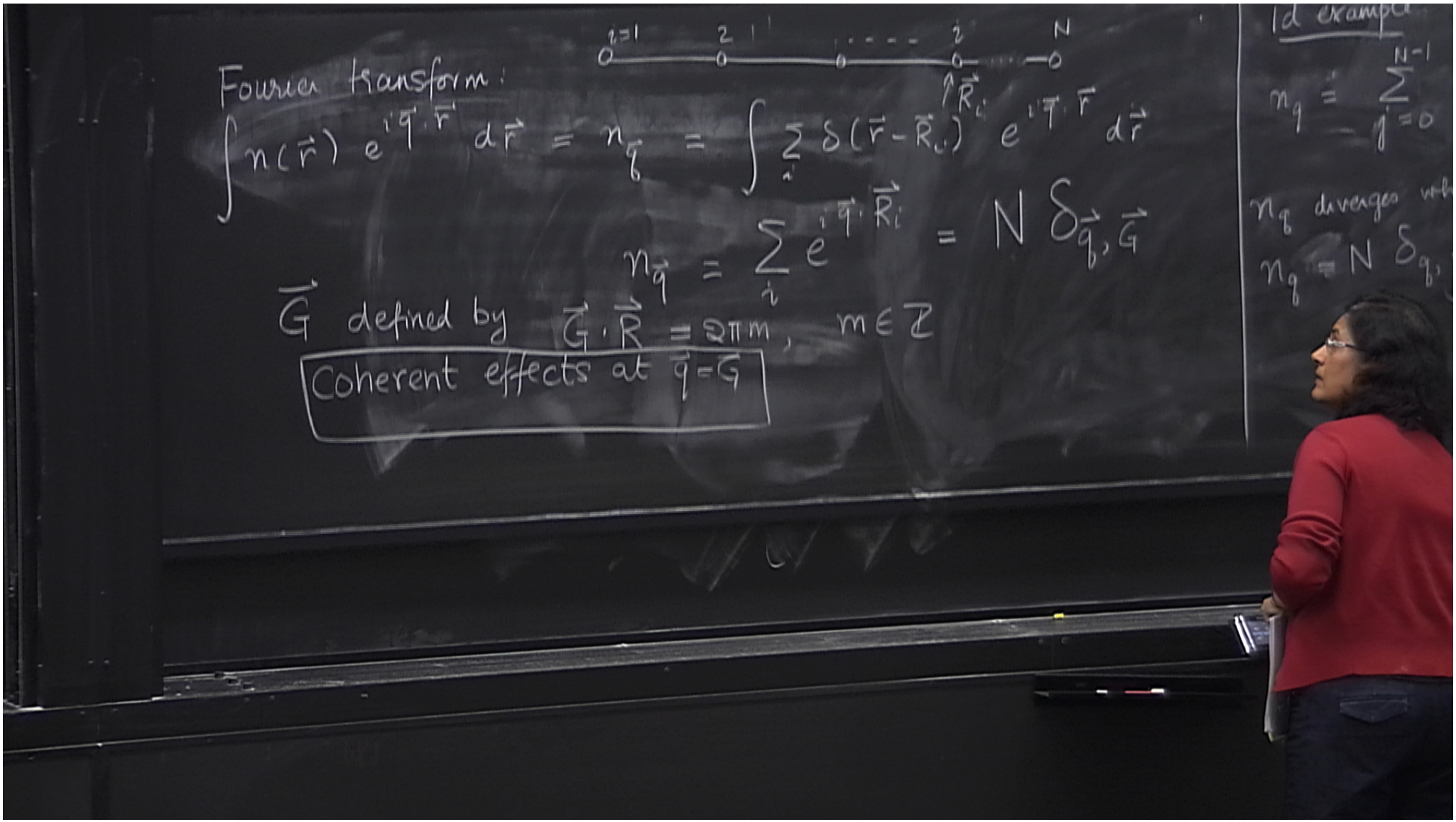
$\boxed{\text{Coherent effects at } \vec{q}=\vec{G}}$

1d example

$$n_q = \sum_{j=0}^{N-1}$$

n_q diverges wh

$n_q = N\,\delta_{q,}$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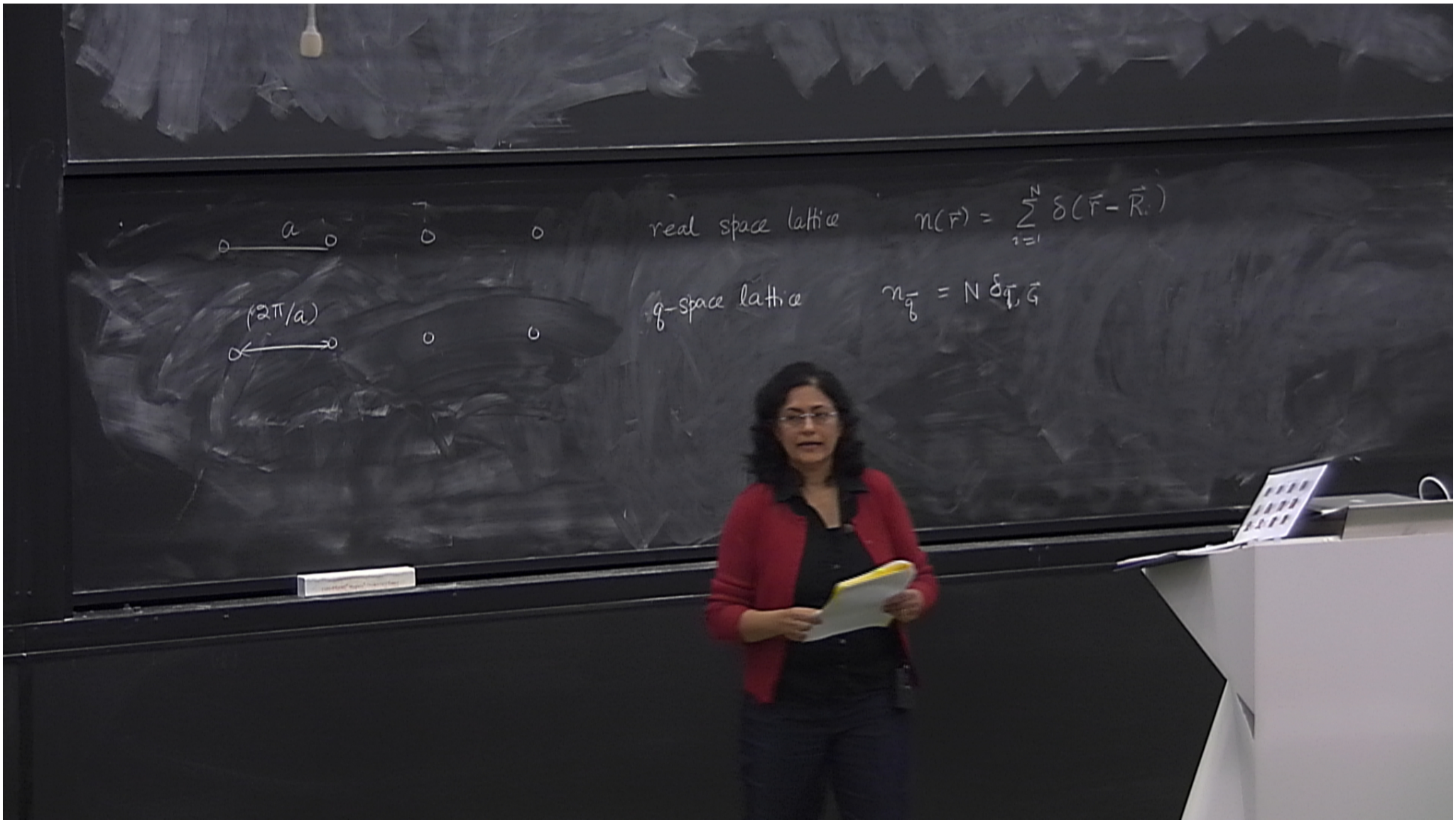
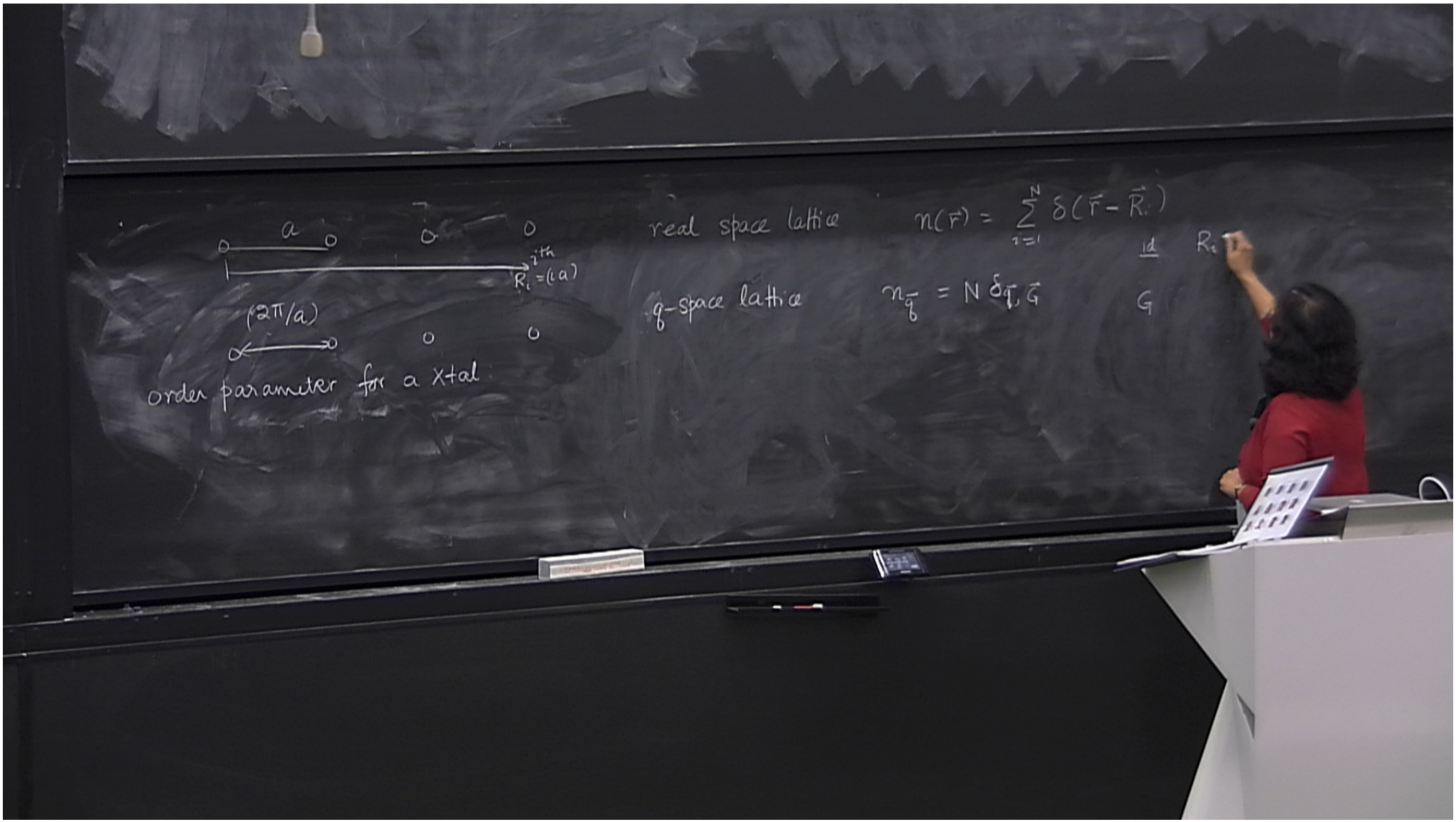
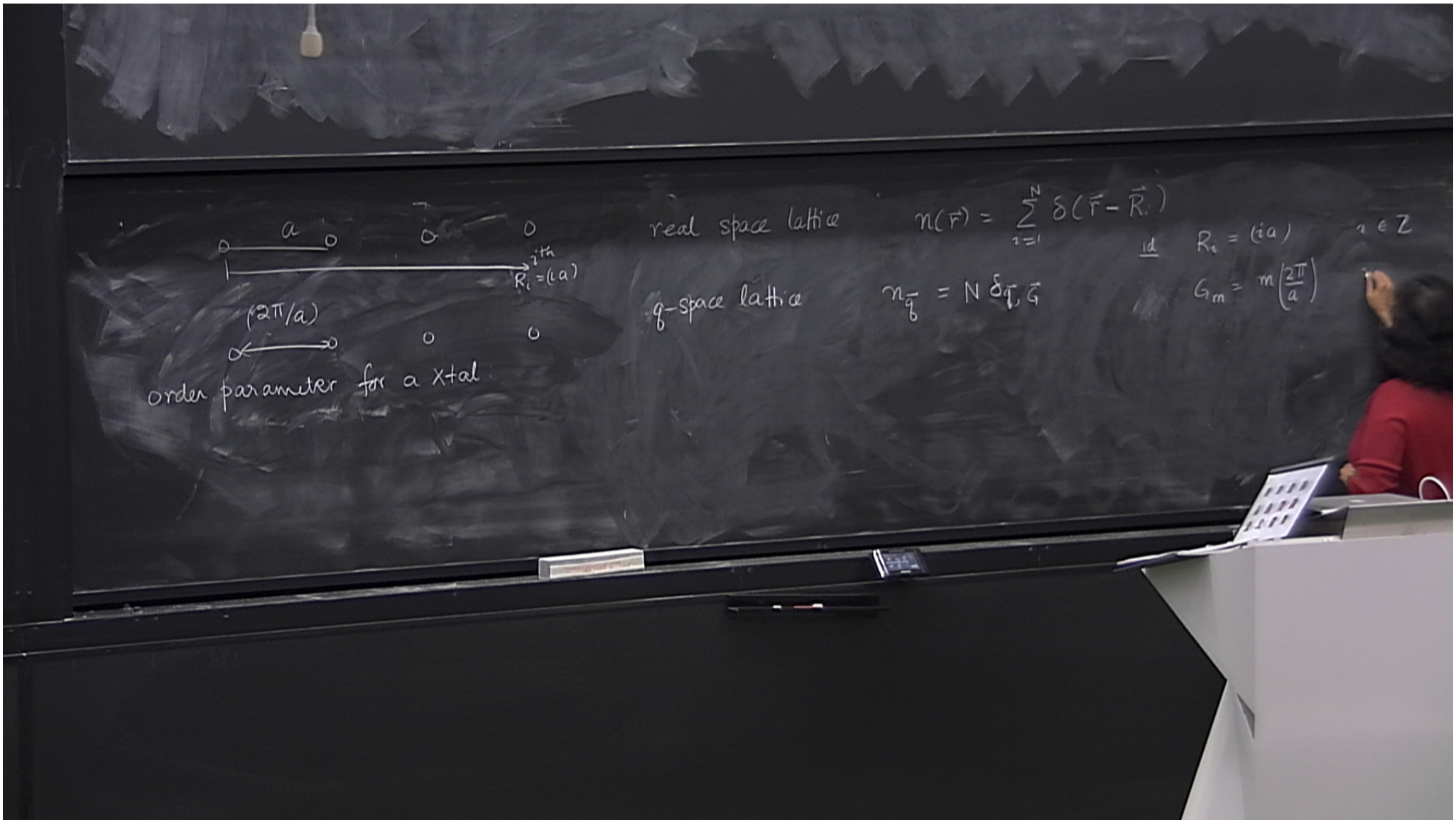
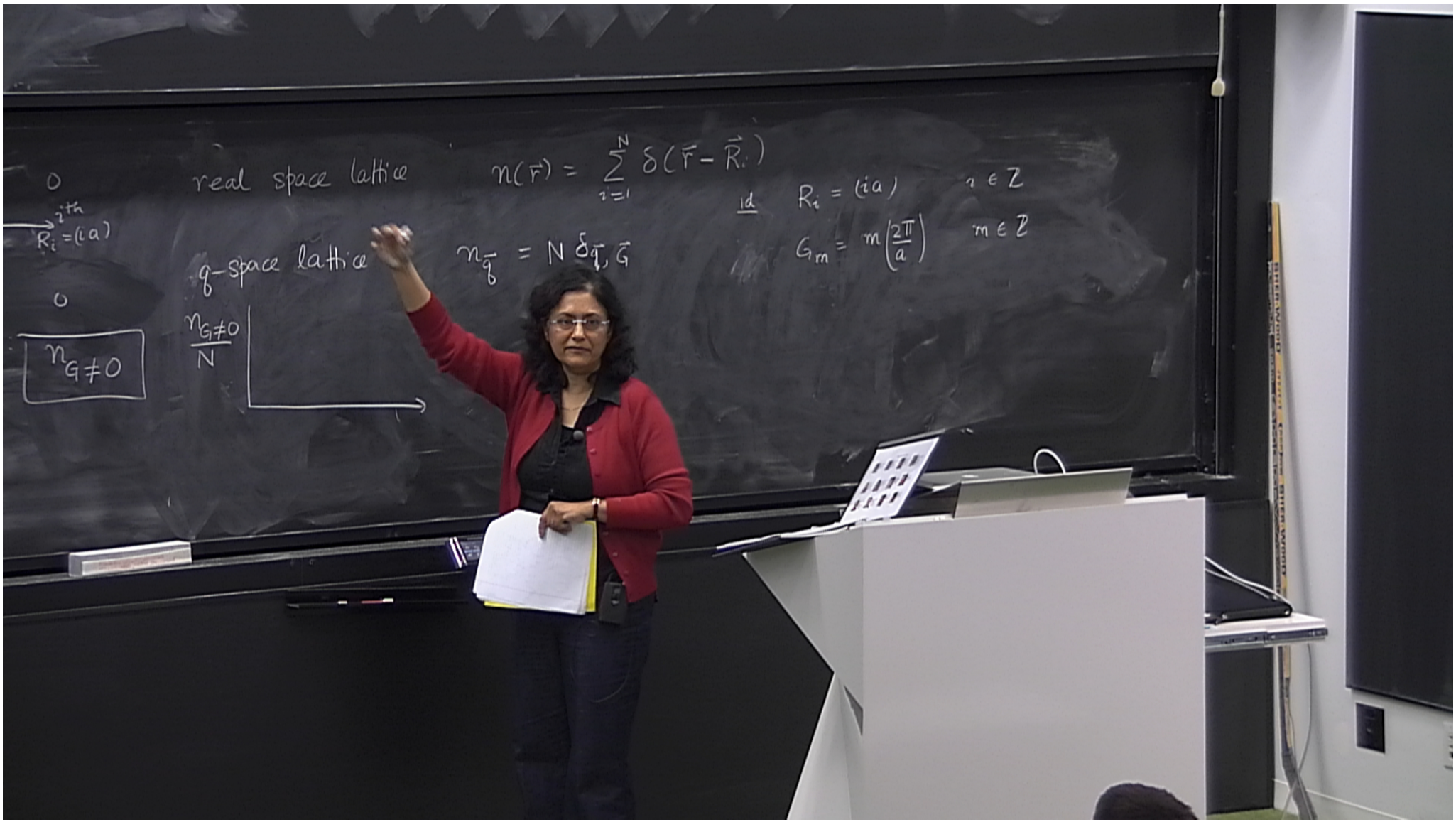
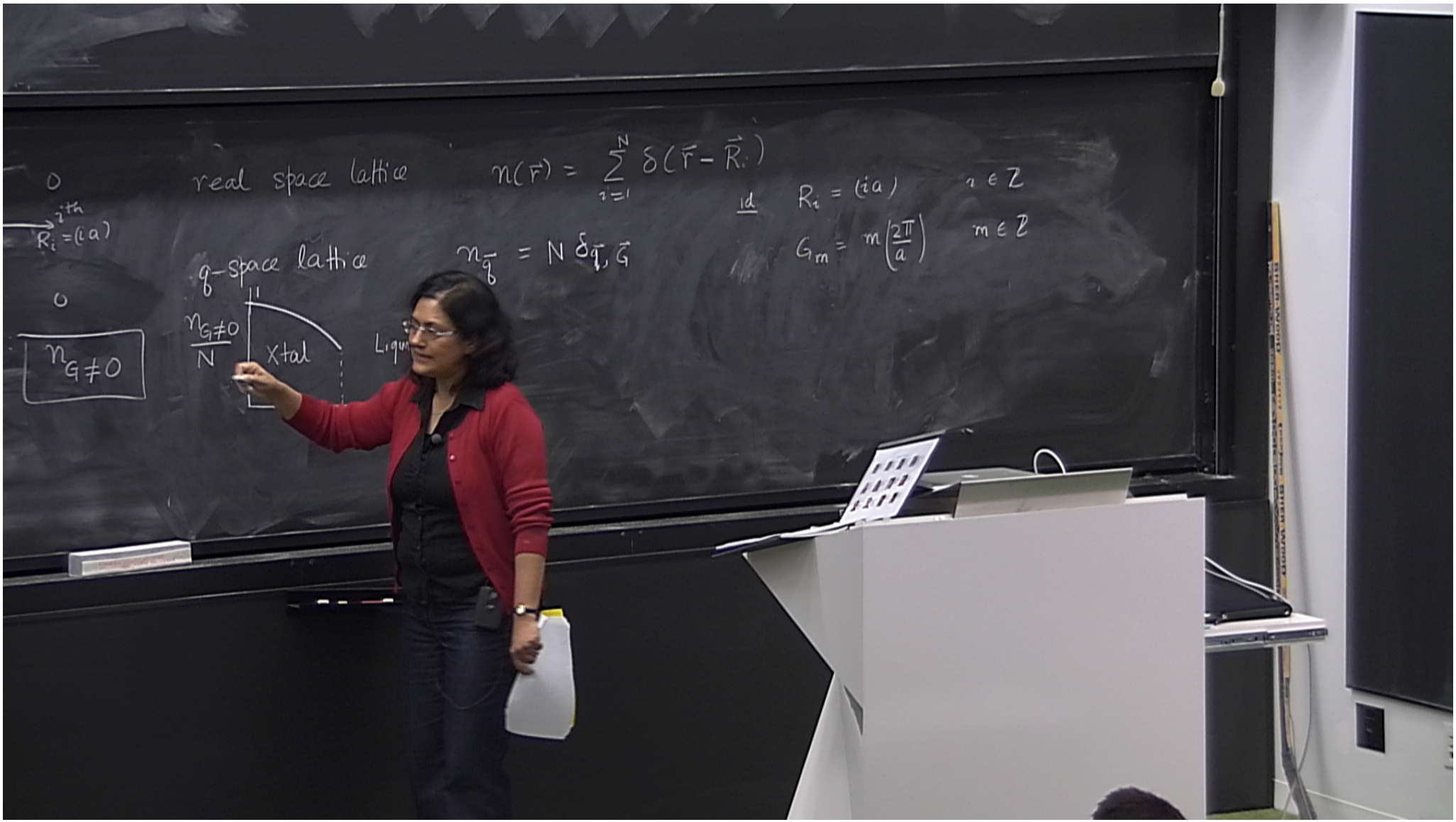
Coherent effects at $\vec{q}=\vec{G}$
i=1
2
i
N
$\vec{R}_i$
1d example
$n_q = \sum_{j=0}^{N-1}$
n_q diverges
$n_q = N\,\delta_{q,}$


a
(2π/a)
real space lattice
$n(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{R}_i)$
q-space lattice
$n_{\vec{q}} = N \delta_{\vec{q}, \vec{G}}$


a
i^th
R_i = (i a)
(2π/a)
order parameter for a Xtal
real space lattice
n(r) = Σ_{i=1}^{N} δ(r − R_i)
q-space lattice
n_q = N δ_{q,G}
G


a
i^th
R_i = (ia)
(2π/a)
order parameter for a xtal
real space lattice
n(r) = Σ_{i=1}^{N} δ(r − R_i)
q-space lattice
n_q = N δ_{q,G}
1d
R_i = (ia)
i ∈ Z
G_m = m(2π/a)


real space lattice
$n(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{R}_i)$
1d
$R_i = (ia)$
$i \in \mathbb{Z}$
$G_m = m\left(\frac{2\pi}{a}\right)$
$m \in \mathbb{Z}$
i^{th}
$R_i = (ia)$
q-space lattice
$n_{\vec{q}} = N \delta_{\vec{q}, \vec{G}}$
$n_{G \neq 0}$
$\frac{n_{G \neq 0}}{N}$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$n(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{R}_i)$
1d
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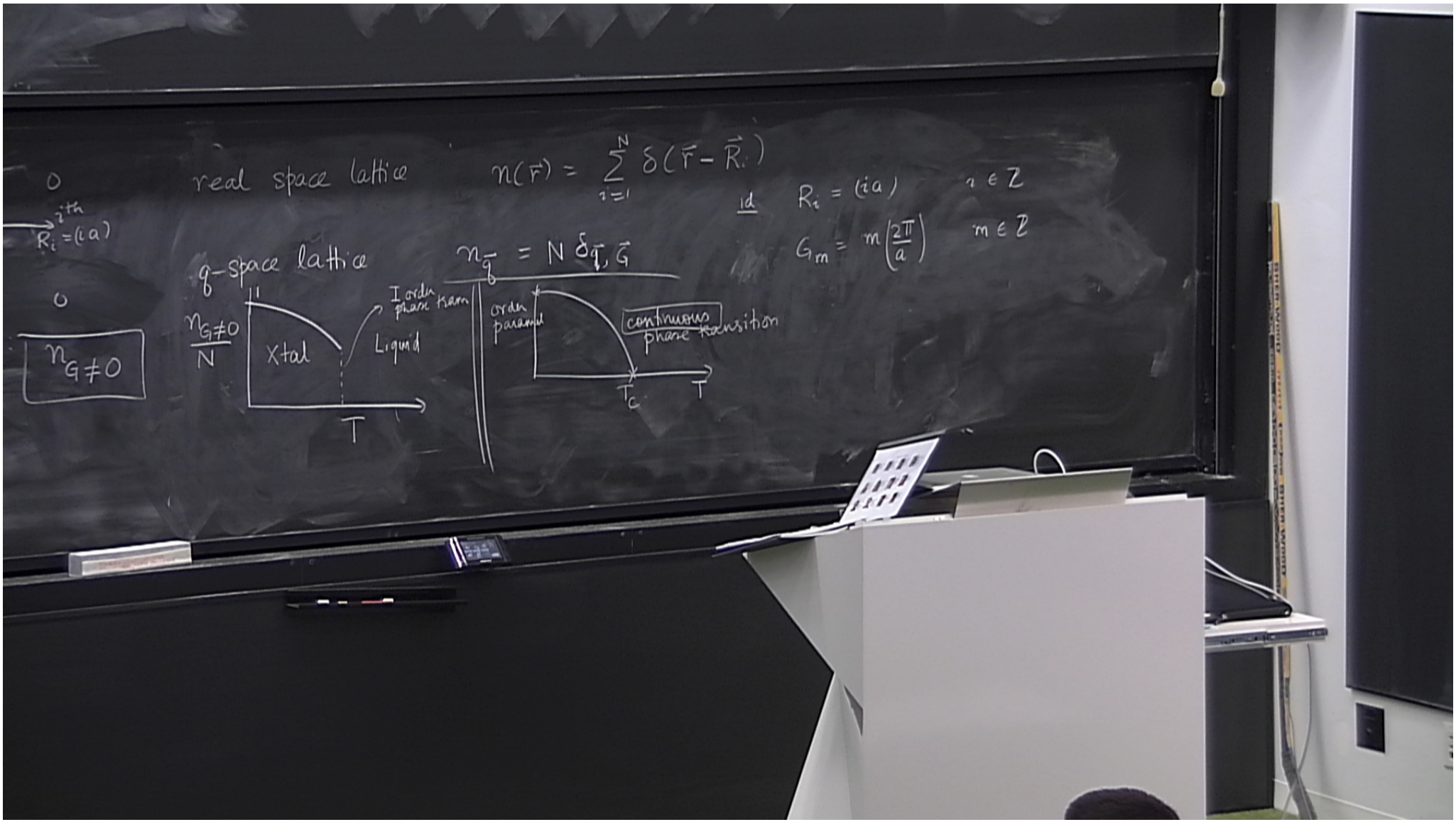
$i \in \mathbb{Z}$
$G_m = m\left(\frac{2\pi}{a}\right)$
$m \in \mathbb{Z}$
$R_i = (ia)$
$n_{G \neq 0}$
q-space lattice
$n_{\vec{q}} = N \delta_{\vec{q}, \vec{G}}$
$\frac{n_{G \neq 0}}{N}$
Xtal


real space lattice
$n(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{R}_i)$
1d
$R_i = (ia)$
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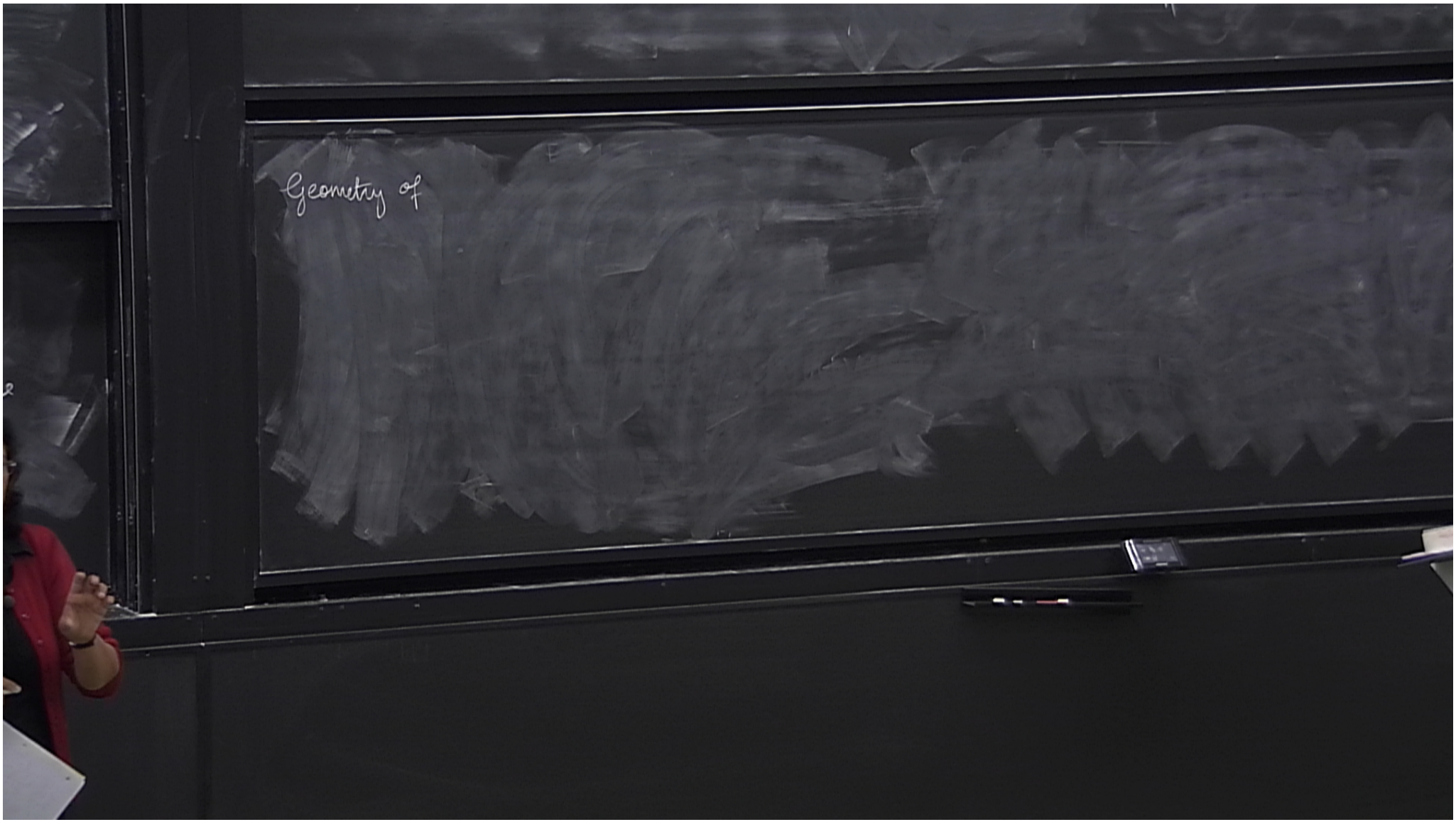
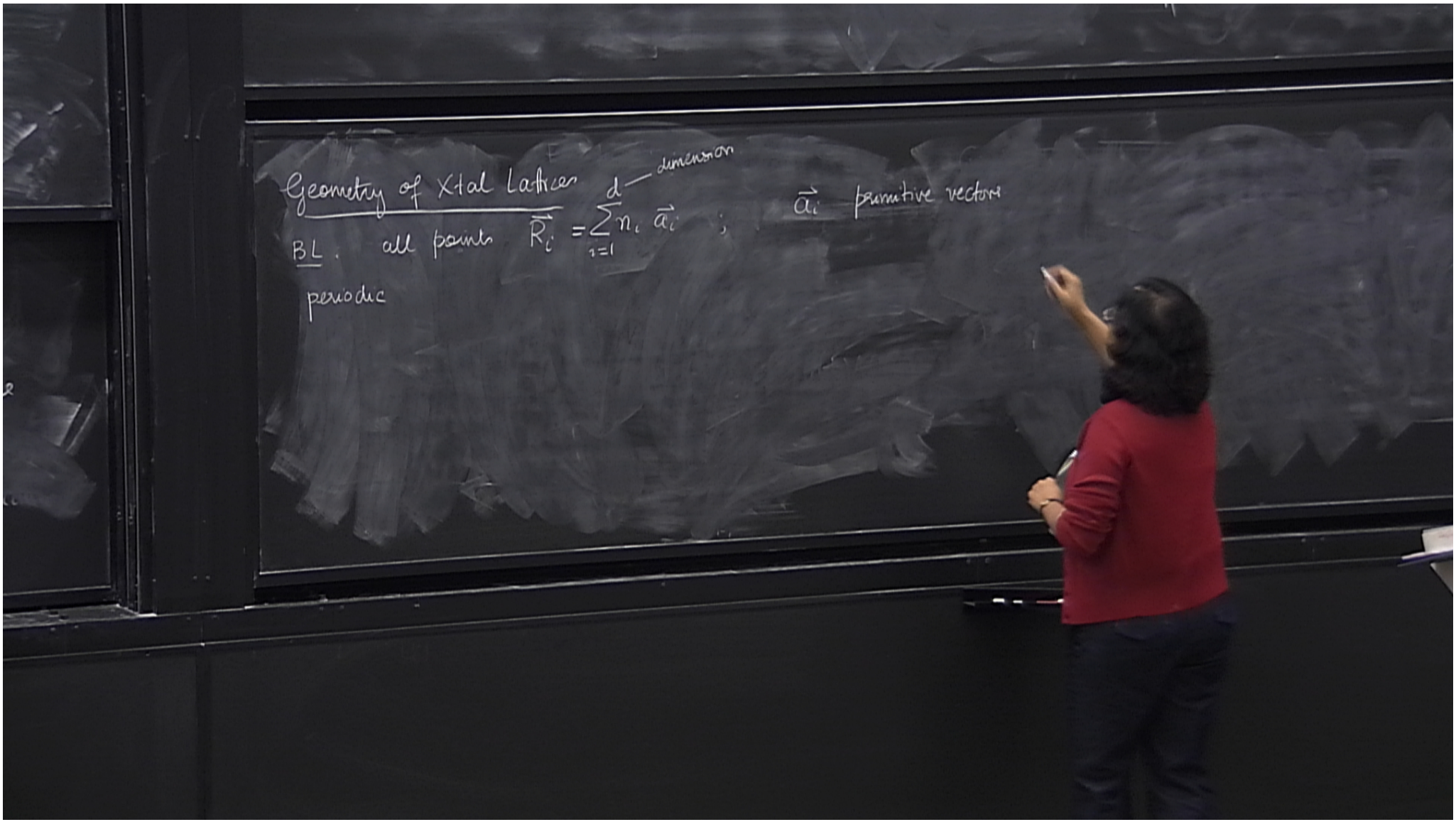
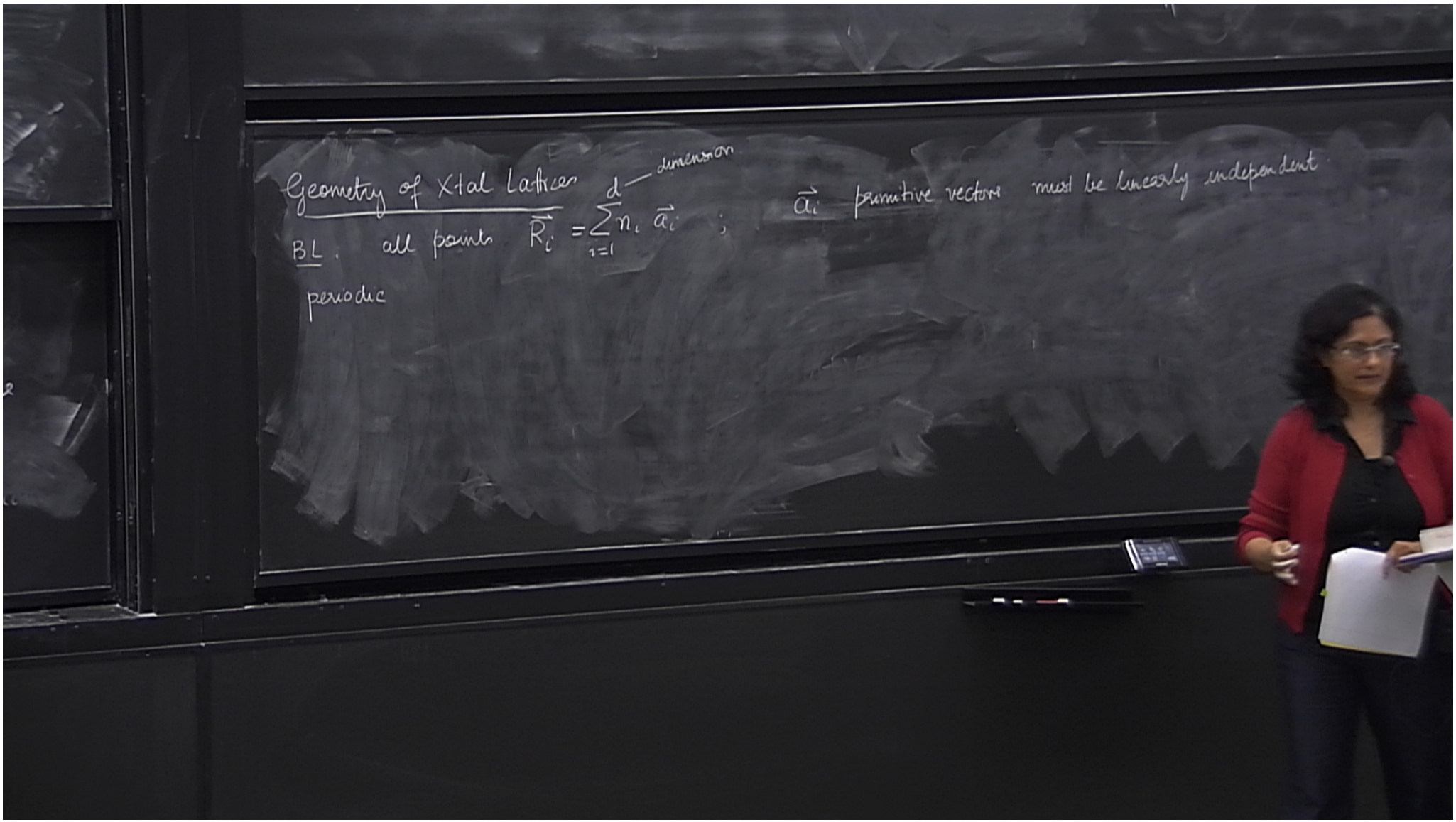
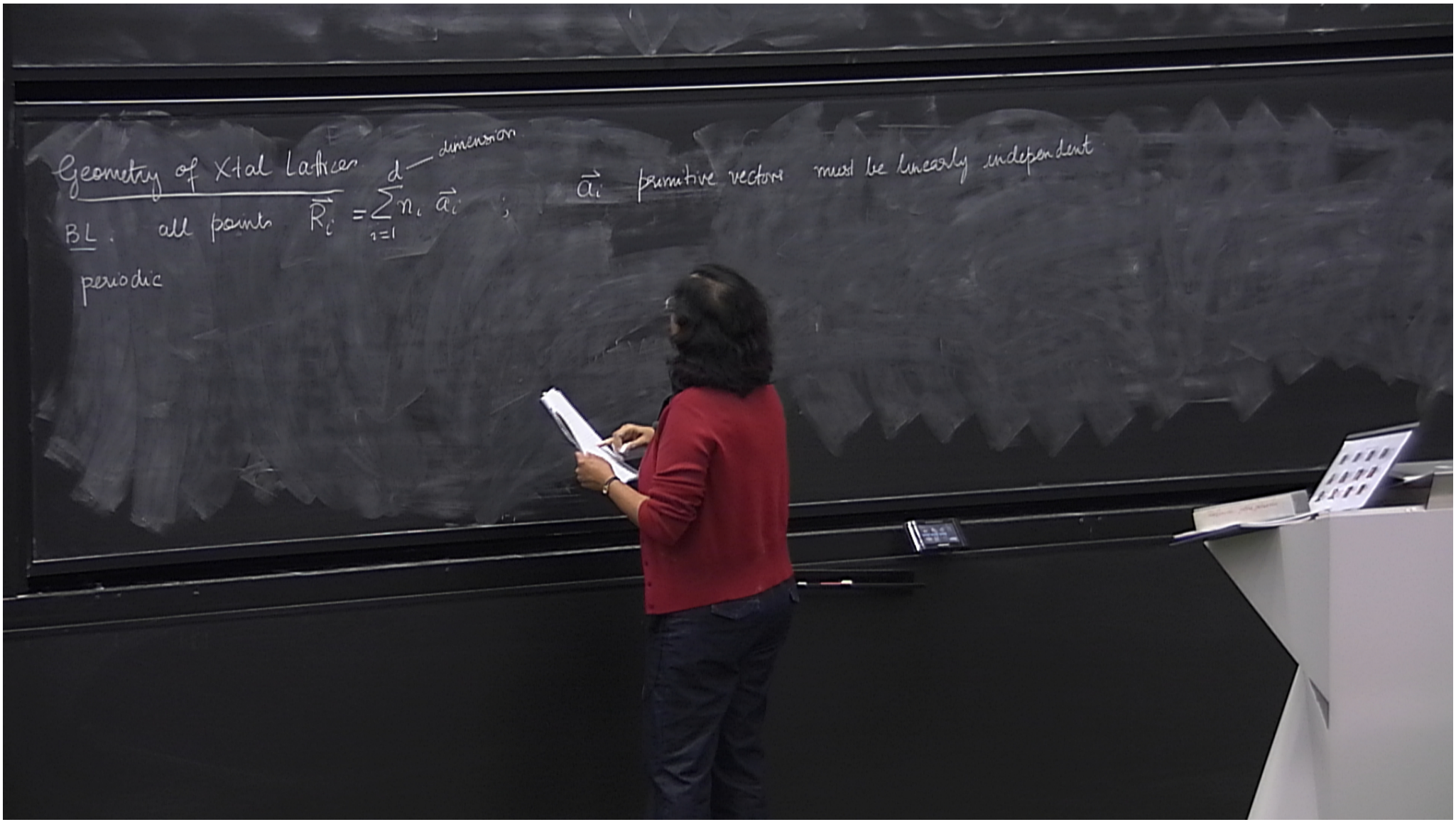
$m \in \mathbb{Z}$
$R_i = (ia)$
q-space lattice
$n_{\vec{q}} = N \delta_{\vec{q}, \vec{G}}$
$n_{G \neq 0}$
$\frac{n_{G \neq 0}}{N}$
Xtal
Liquid
T
I order phase trans
order param
continuous phase transition
T_c
T


Geometry of


Geometry of Xtal Lattices
d — dimension
BL. all points $\vec{R}_i = \sum_{i=1}^{d} n_i \vec{a}_i$; $\vec{a}_i$ primitive vectors
periodic


Geometry of Xtal Lattices
d — dimension
BL. all points $\vec{R}_i = \sum_{i=1}^{d} n_i \vec{a}_i$; $\vec{a}_i$ primitive vectors must be linearly independent
periodic


Geometry of Xtal Lattices
d — dimension
BL. all points $\vec{R}_i = \sum_{i=1} n_i \vec{a}_i$;
$\vec{a}_i$ primitive vectors must be linearly independent
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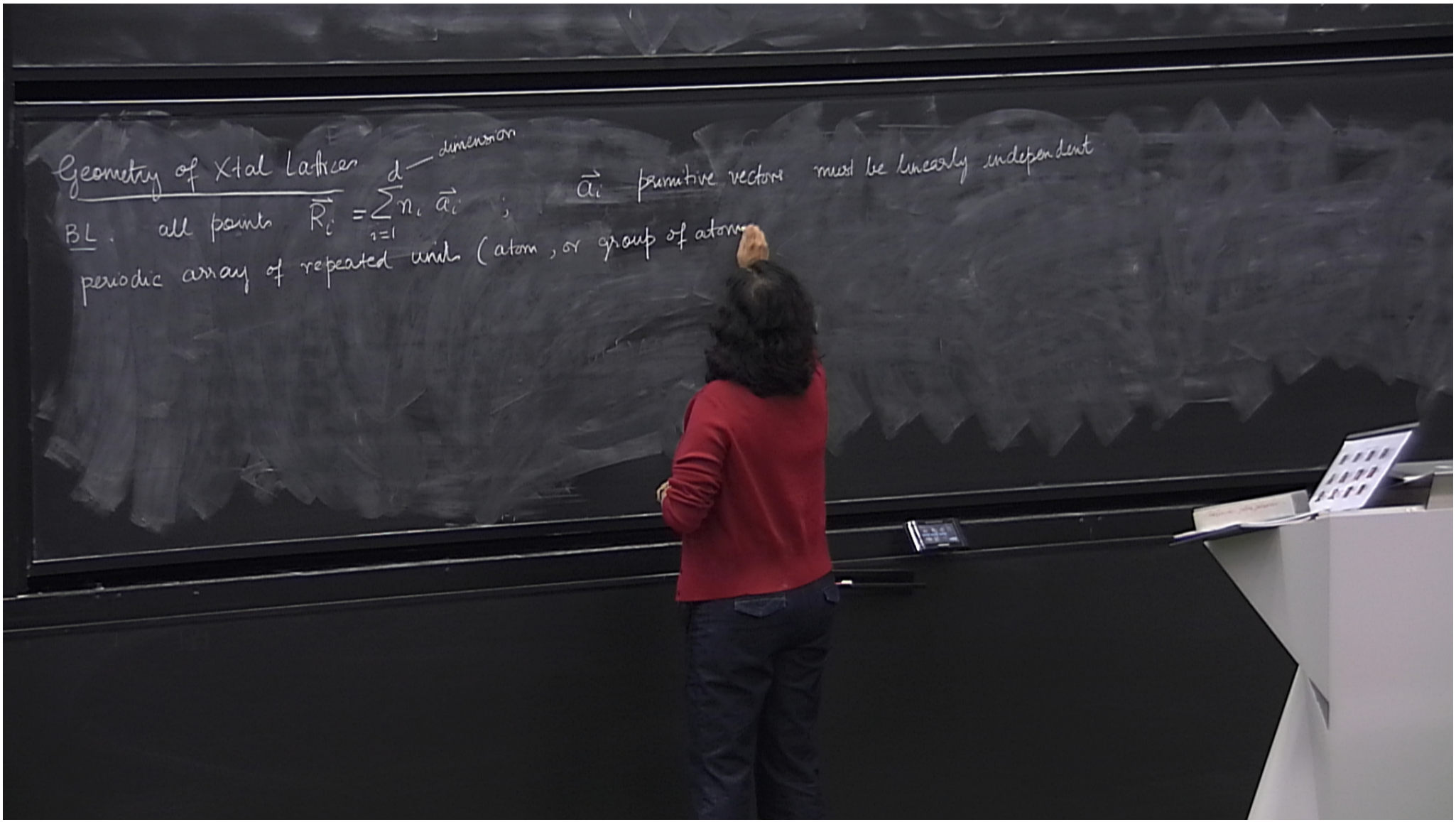
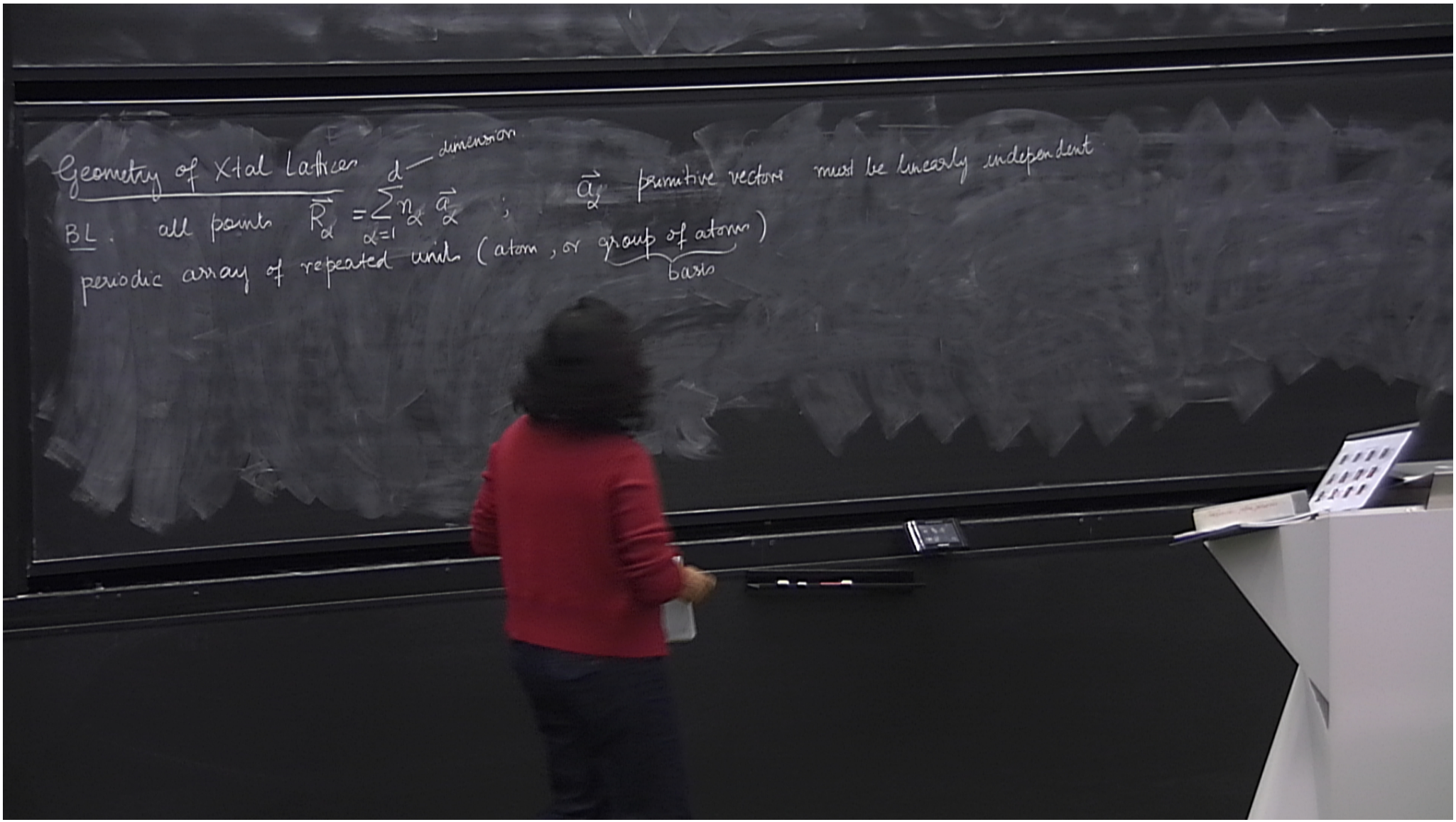

Geometry of Xtal Lattices
d — dimension
$\vec{a}_i$ primitive vectors must be linearly independent
BL. all points $\vec{R}_i = \sum_{i=1}^{d} n_i \vec{a}_i$;
periodic array of repeated units (atom, or group of atoms


Geometry of Xtal Lattices
d — dimension
BL. all points $\vec{R}_\alpha = \sum_{\alpha=1}^{d} n_\alpha \vec{a}_\alpha$; $\vec{a}_\alpha$ primitive vectors must be linearly independent
periodic array of repeated units (atom, or group of atoms)
basis

Geometry of Xtal Lattices

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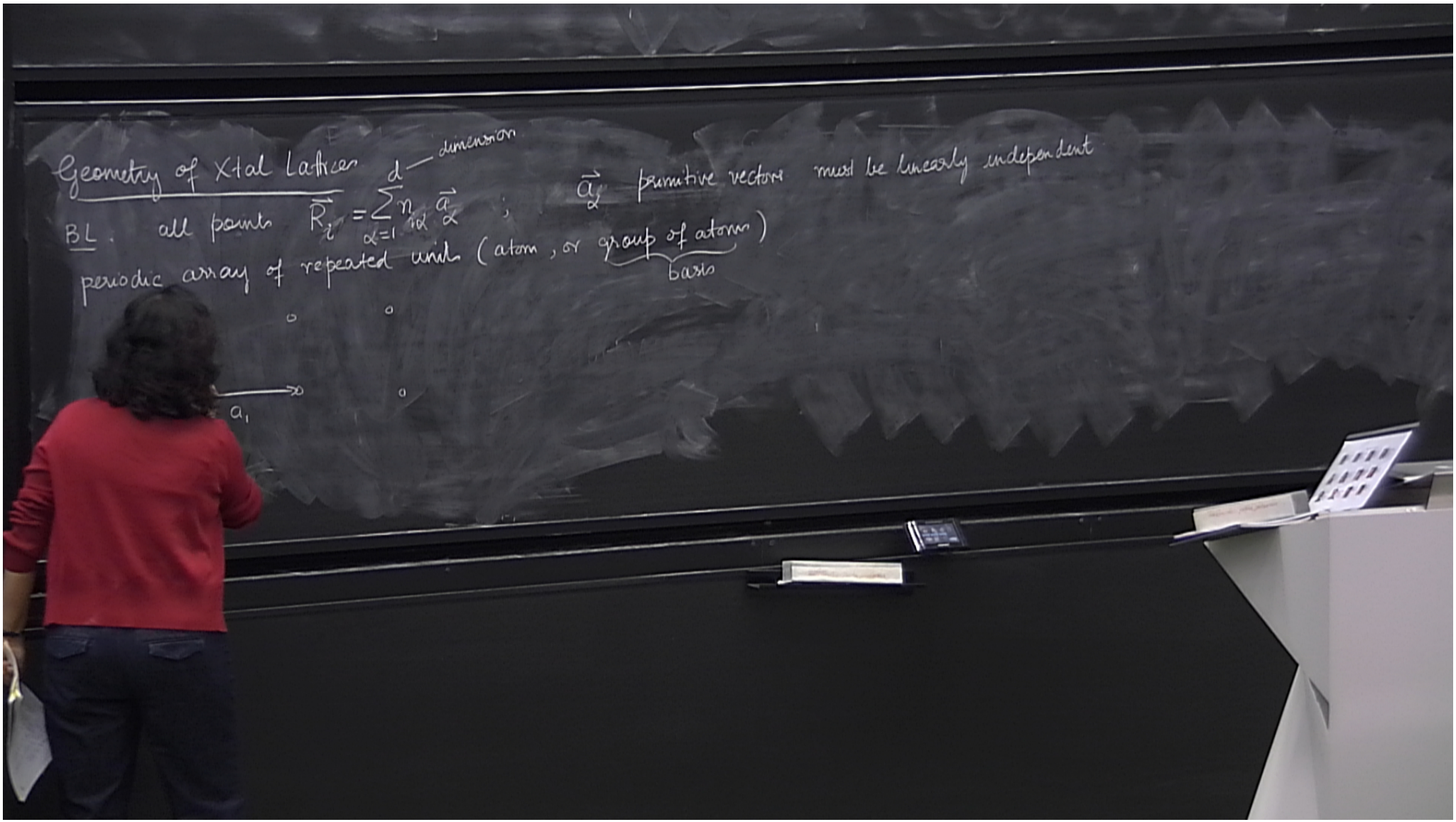
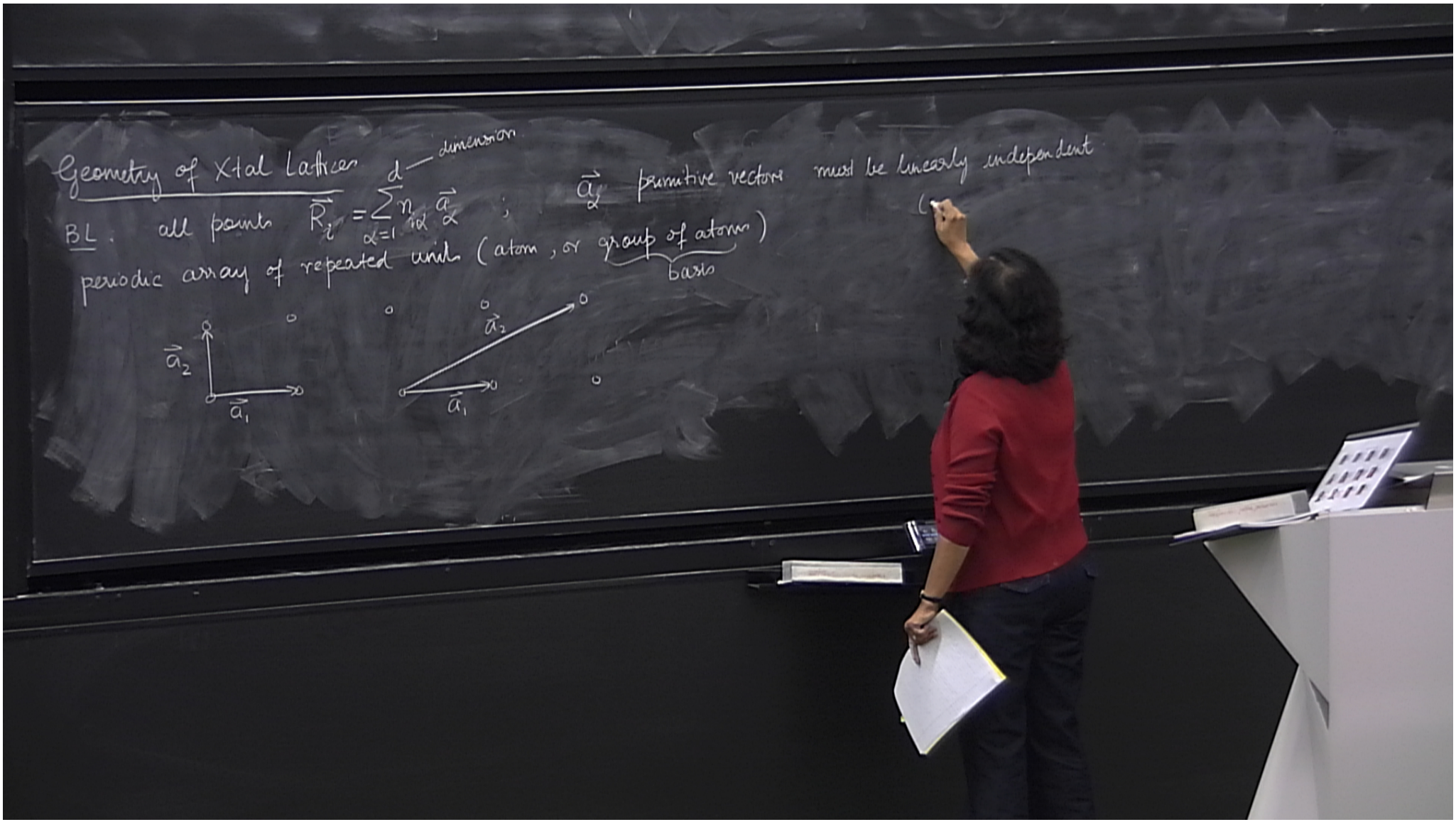
d — dimension

periodic array of repeated units (atom, or $\underbrace{\text{group of atoms}}_{\text{basis}}$)


Geometry of Xtal Lattices
d — dimension
$\vec{a}_\alpha$ primitive vectors must be linearly independent
BL. all points $\vec{R}_i = \sum_{\alpha=1}^{d} n_{i\alpha} \vec{a}_\alpha$;
periodic array of repeated units (atom, or group of atoms)
basis
a_1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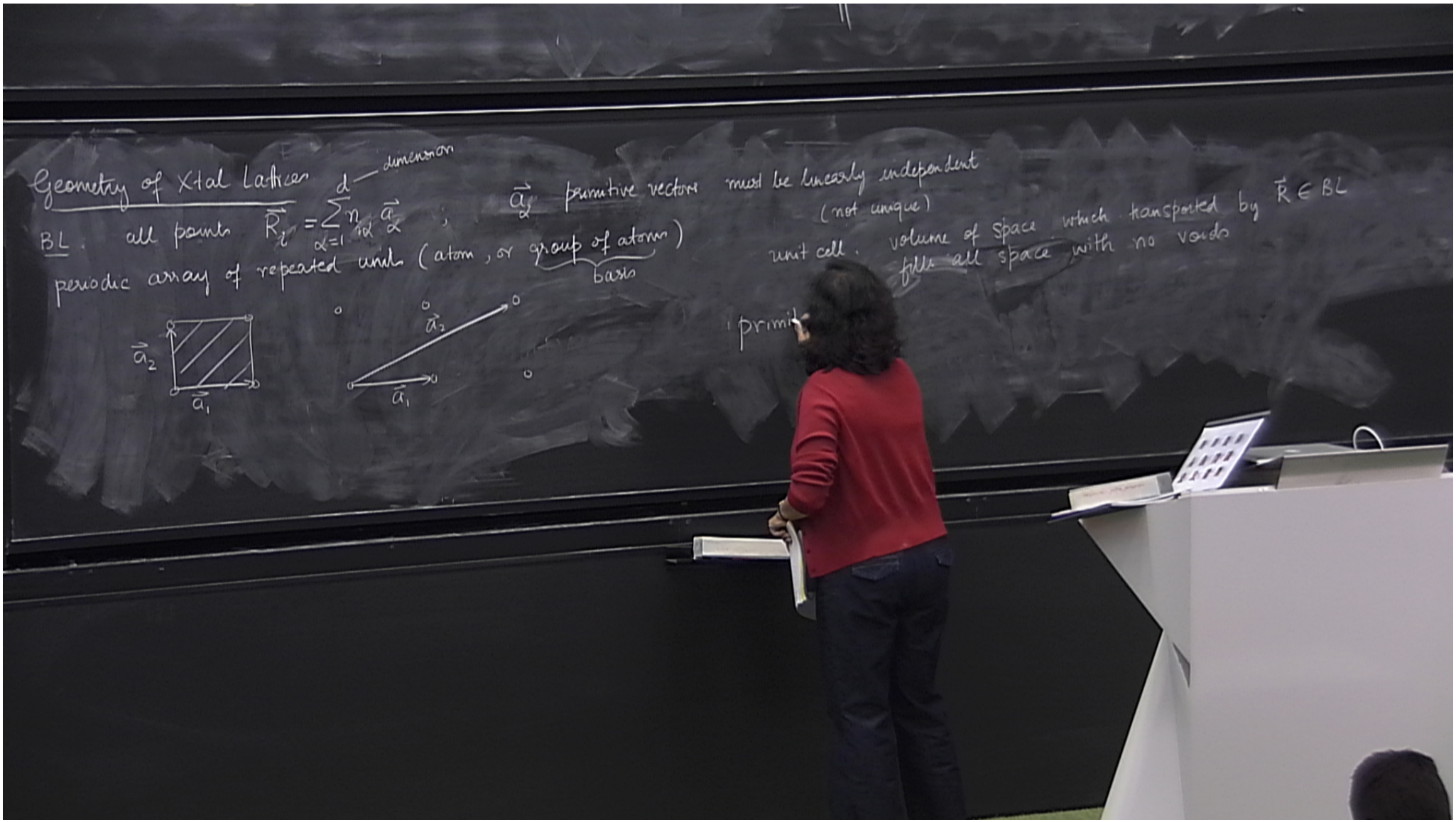
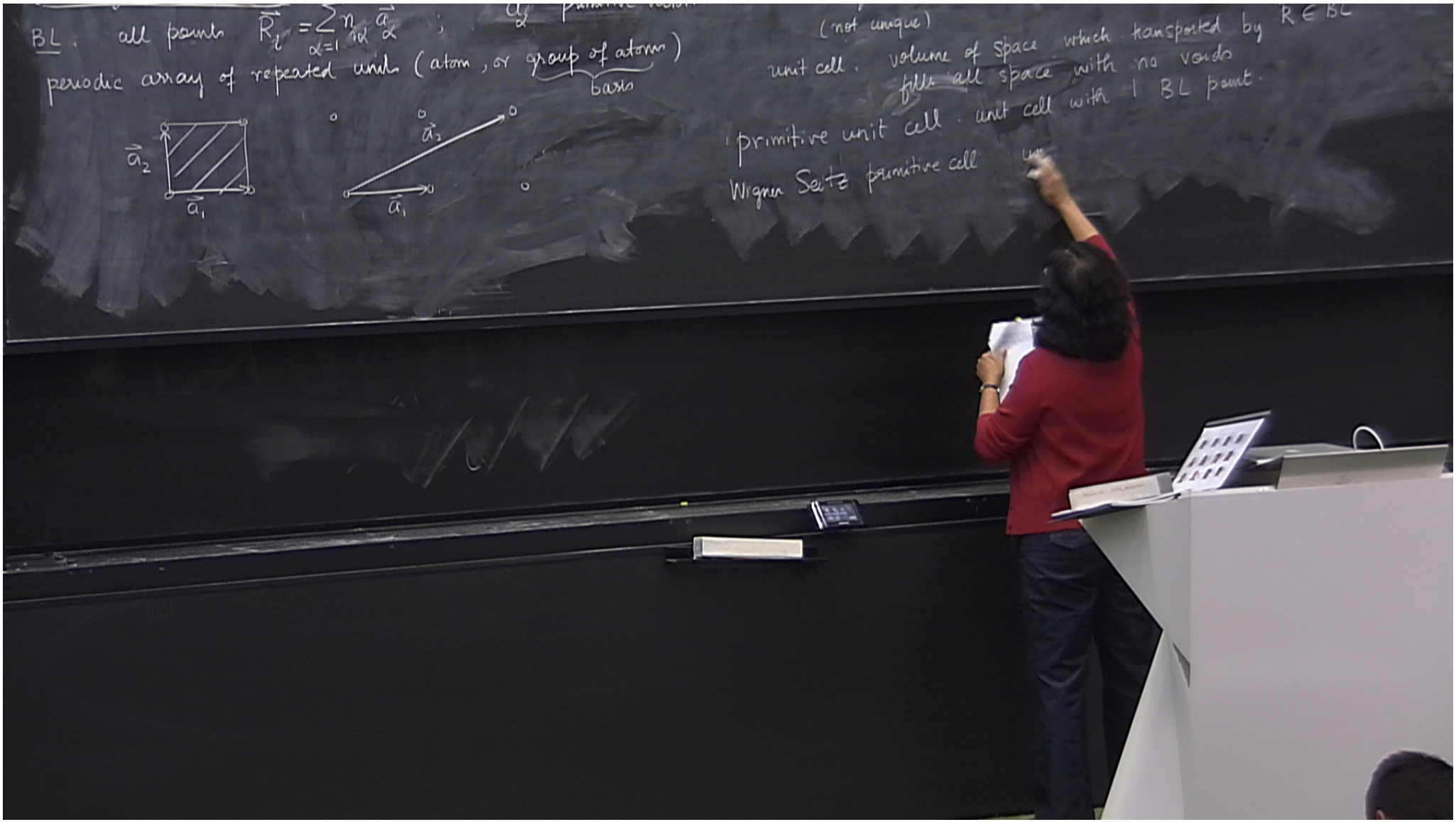
BL. all points $\vec{R}_i = \sum_{\alpha=1}^{d} n_{i\alpha} \vec{a}_\alpha$;
periodic array of repeated units (atom, or group of atoms)
basis
$\vec{a}_2$
$\vec{a}_1$
$\vec{a}_2$
$\vec{a}_1$


Geometry of Xtal Lattices
d — dimension
BL. all points $\vec{R}_i = \sum_{\alpha=1}^{d} n_{i\alpha} \vec{a}_\alpha$; $\vec{a}_\alpha$ primitive vectors must be linearly independent
(not unique)
periodic array of repeated units (atom, or group of atoms)
basis
unit cell. volume of space which transported by $\vec{R} \in BL$
fills all space with no voids
$\vec{a}_2$
$\vec{a}_1$
$\vec{a}_2$
$\vec{a}_1$
primi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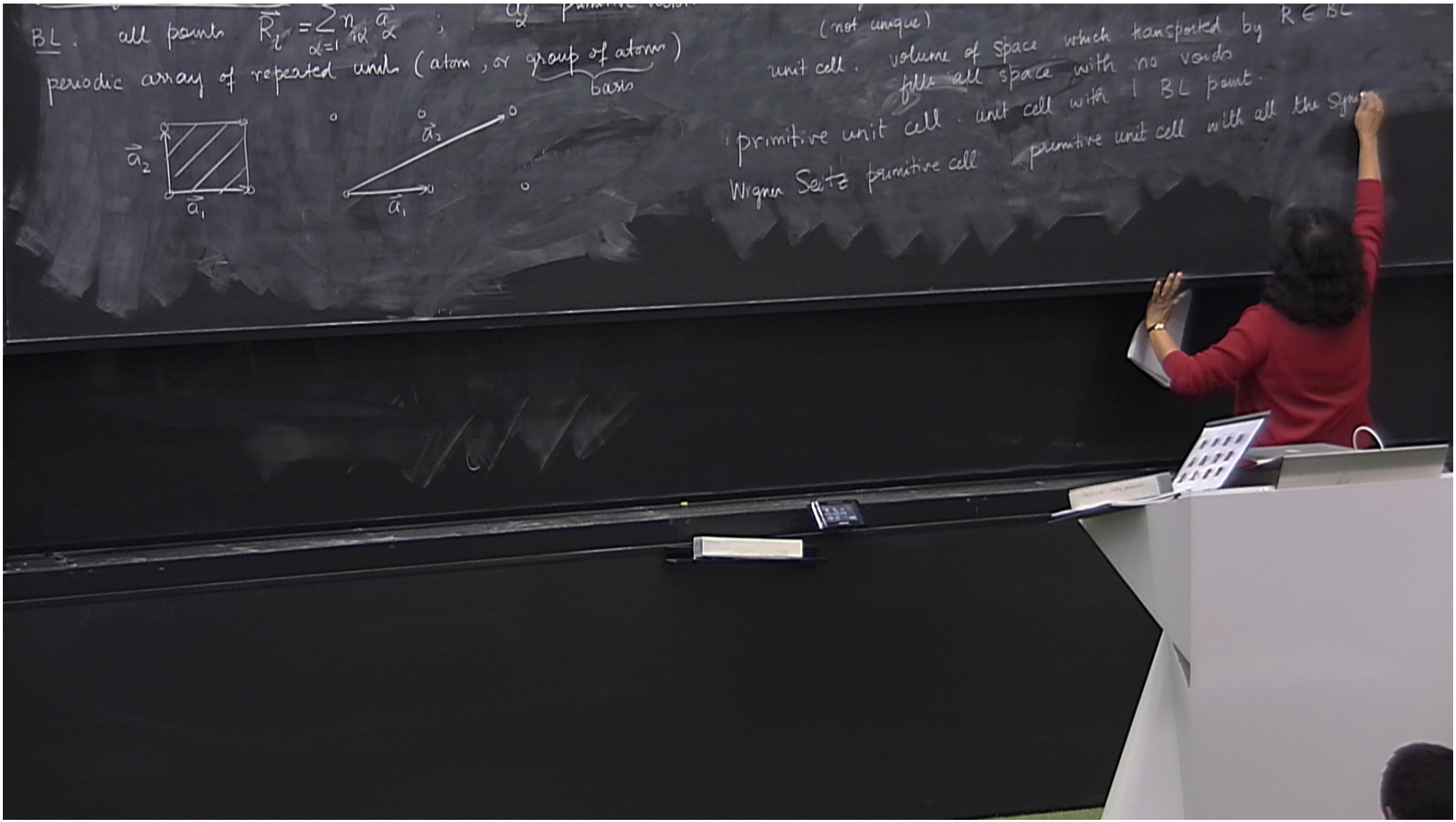
periodic array of repeated units (atom, or group of atoms)
basis
(not unique)
unit cell. volume of space which transported by $R \in BL$
fills all space with no voids
primitive unit cell. unit cell with 1 BL point.
Wigner Seitz primitive cell
$\vec{a}_2$
$\vec{a}_1$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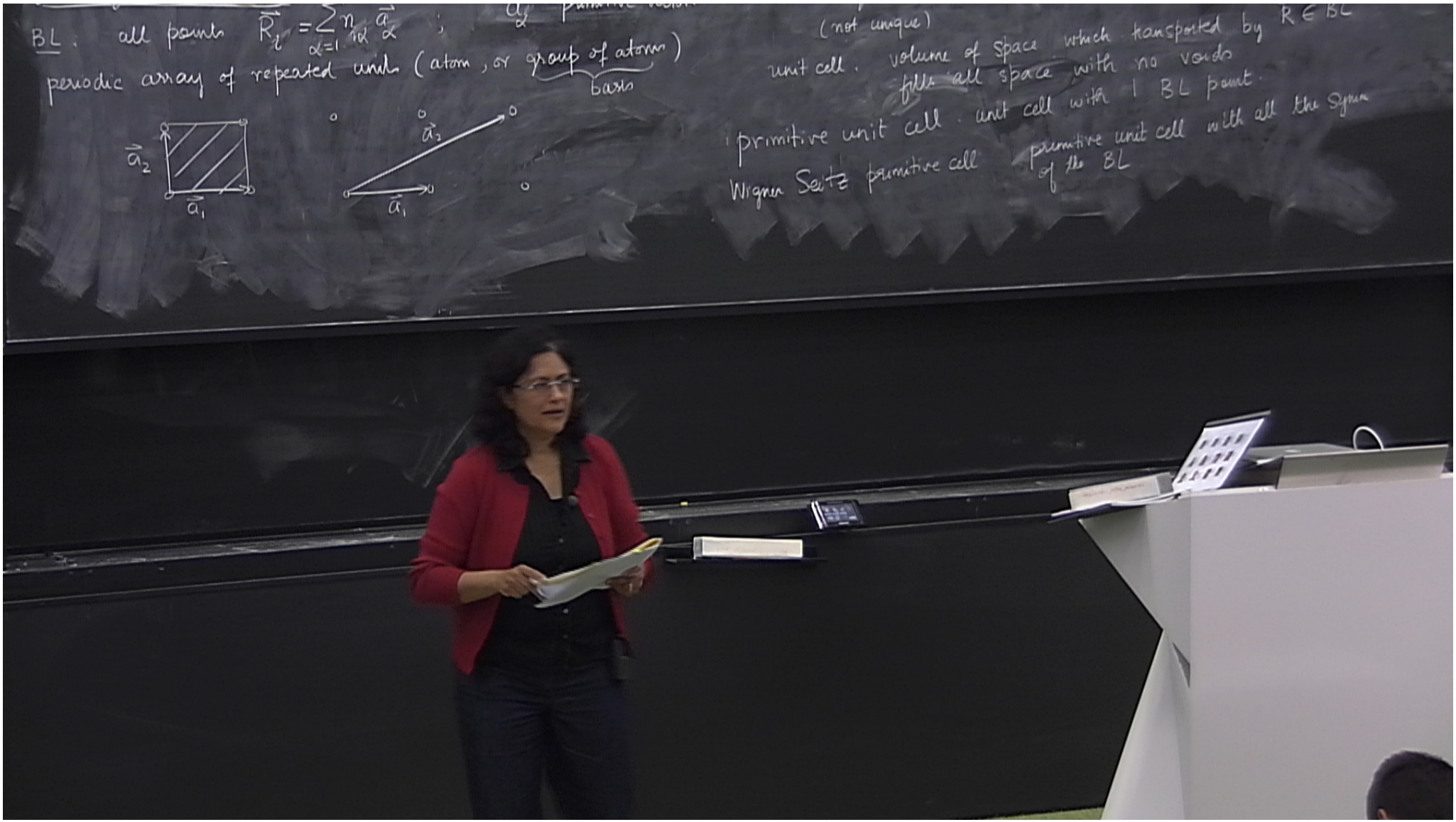
Wigner Seitz primitive cell primitive unit cell with all the sym
$\vec{a}_2$
$\vec{a}_1$
$\vec{a}_2$
$\vec{a}_1$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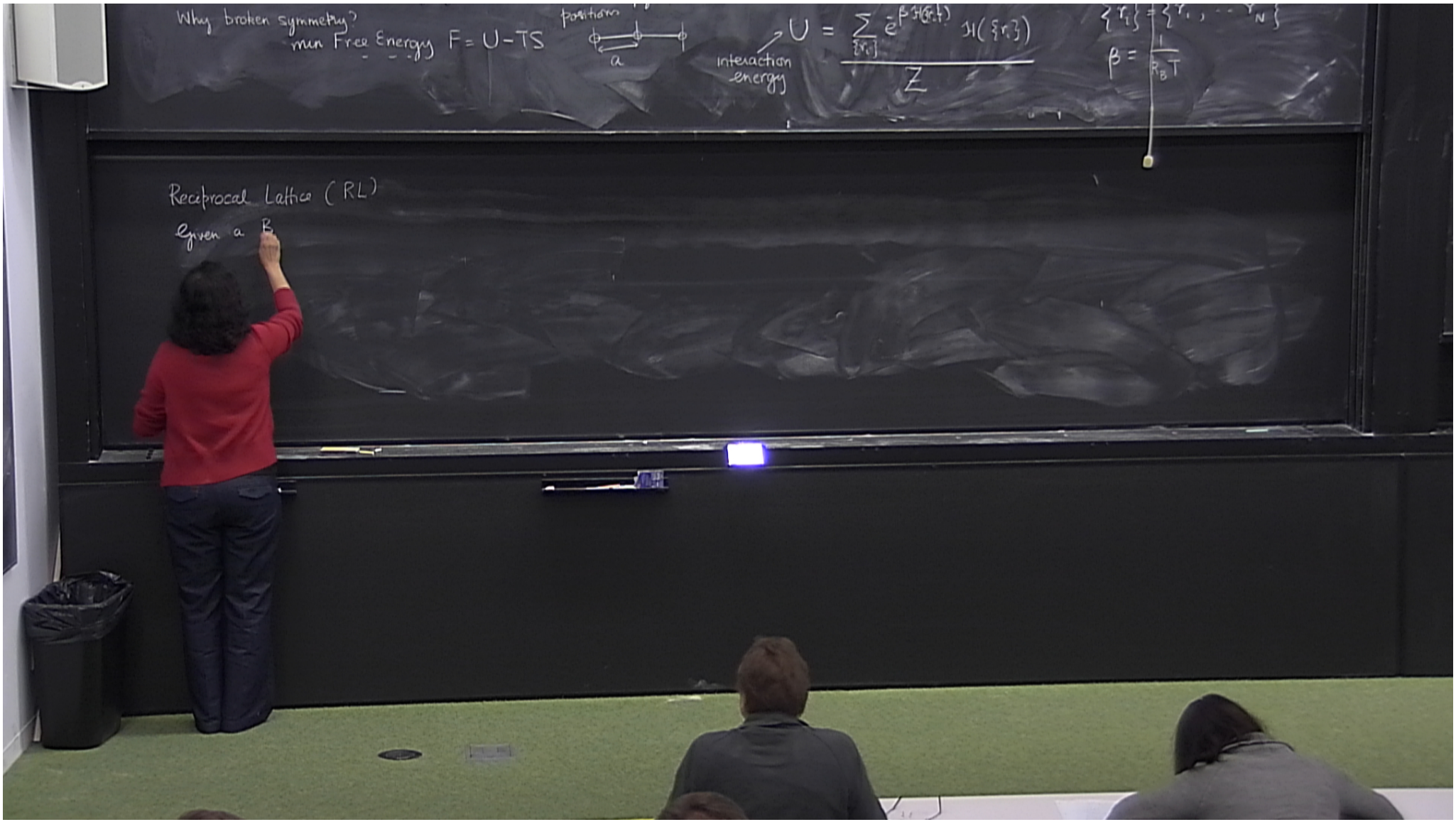
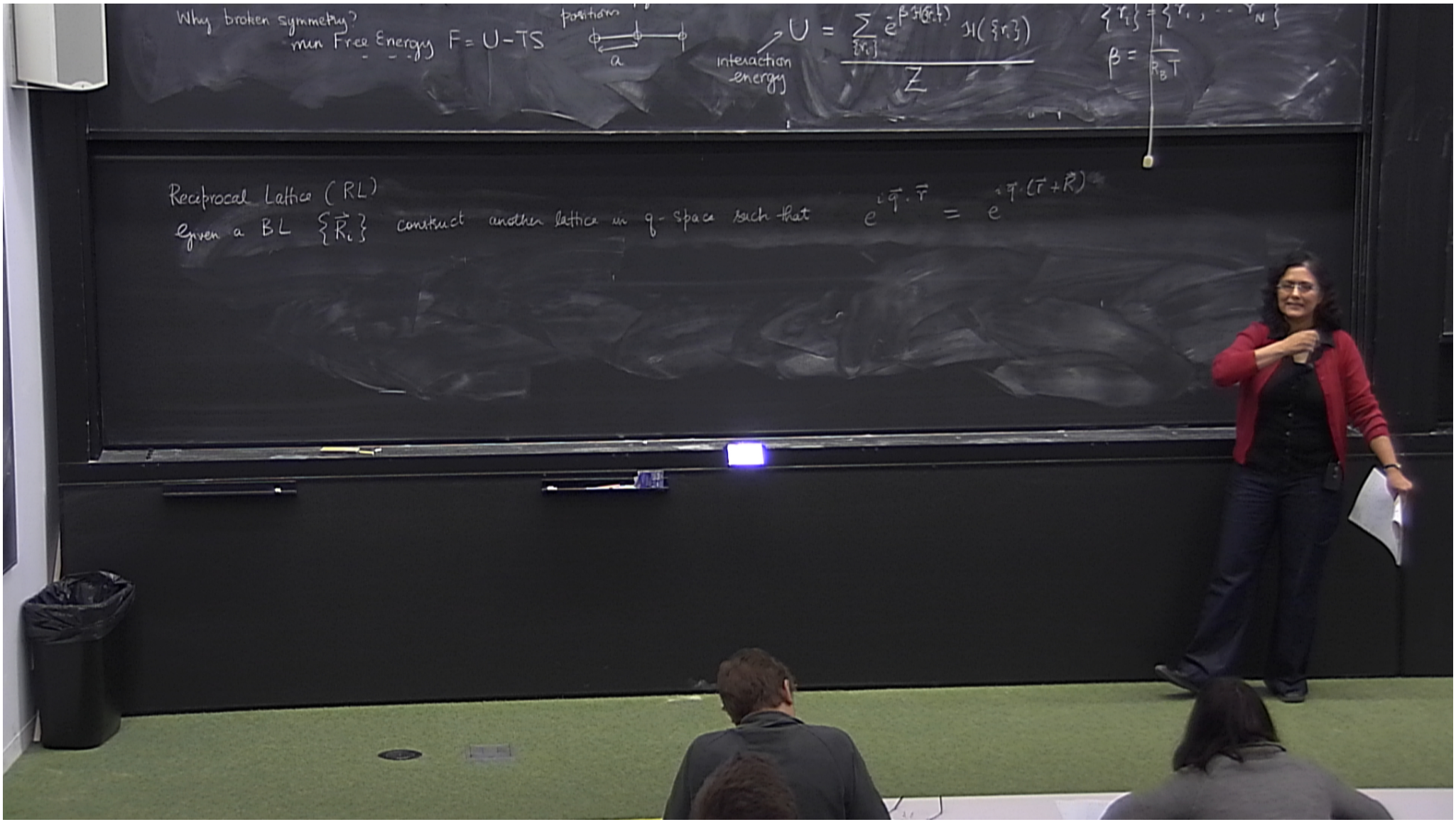
Wigner Seitz primitive cell primitive unit cell with all the symm of the BL
$\vec{a}_2$
$\vec{a}_1$


Why broken symmetry?
min Free Energy F = U - TS
positions
a
interaction energy
U = Σ e^{-β H({r_i})} H({r_i}) / Z
{r_i} = {r_1, ... r_N}
β = 1/(k_B T)
Reciprocal Lattice (RL)
Given a B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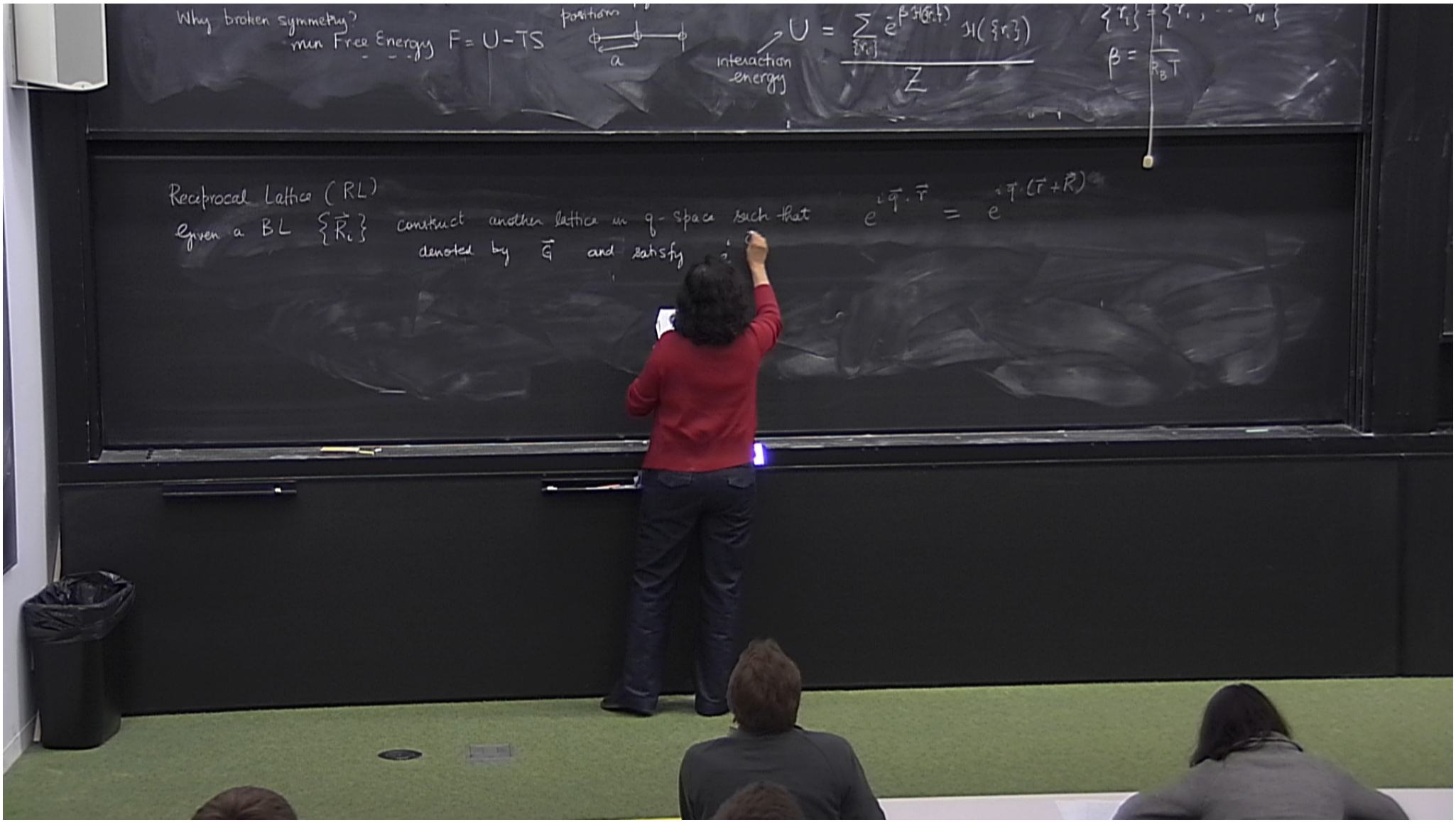
{r_i} = {r_1, ... r_N}
β = 1/(k_B T)
Reciprocal Lattice (RL)
given a BL {R_i} construct another lattice in q-space such that
e^{i q·r} = e^{i q·(r + R)}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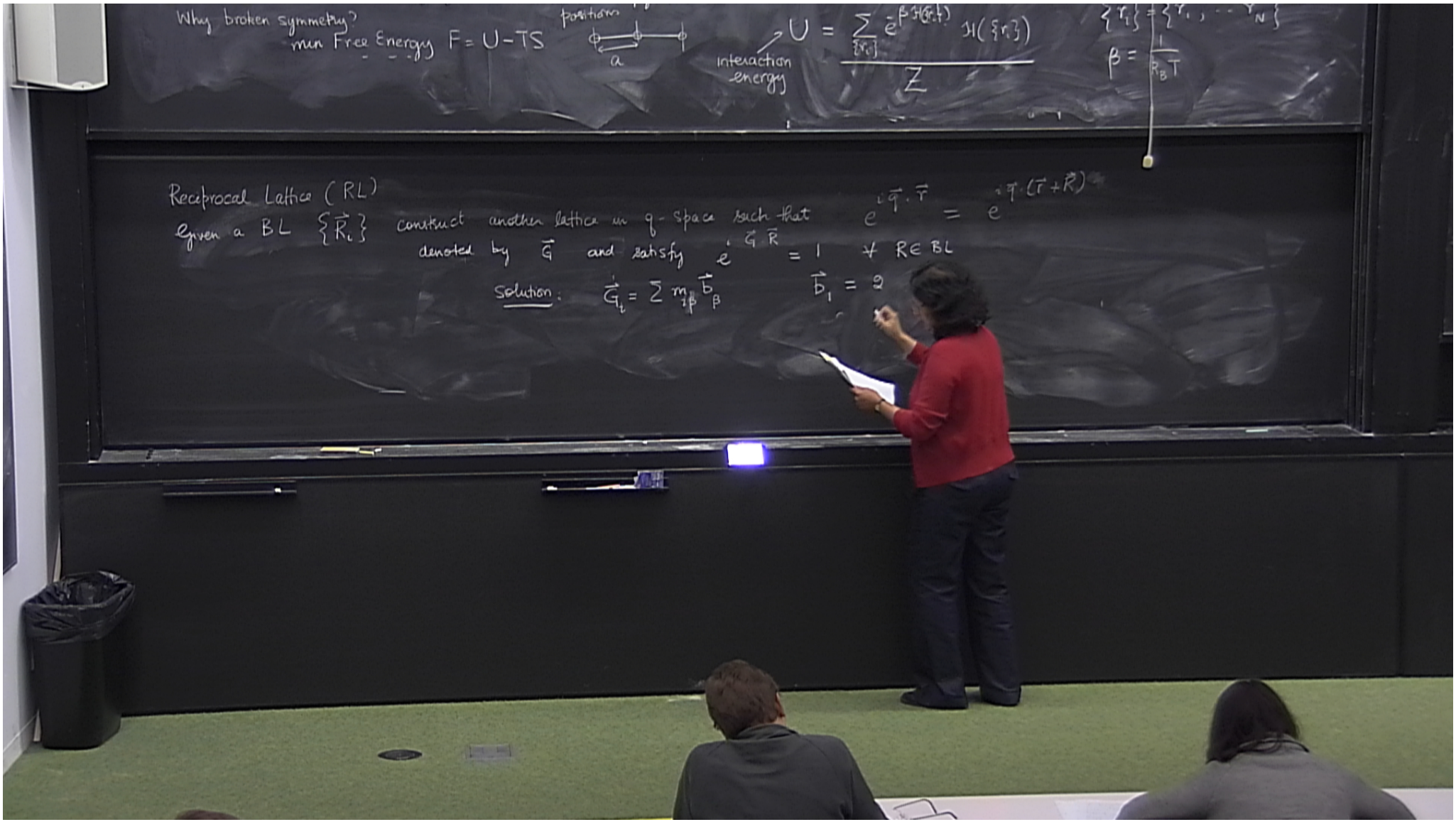
e^{iq·r} = e^{iq·(r+R)}
denoted by G and satisfy


Why broken symmetry?
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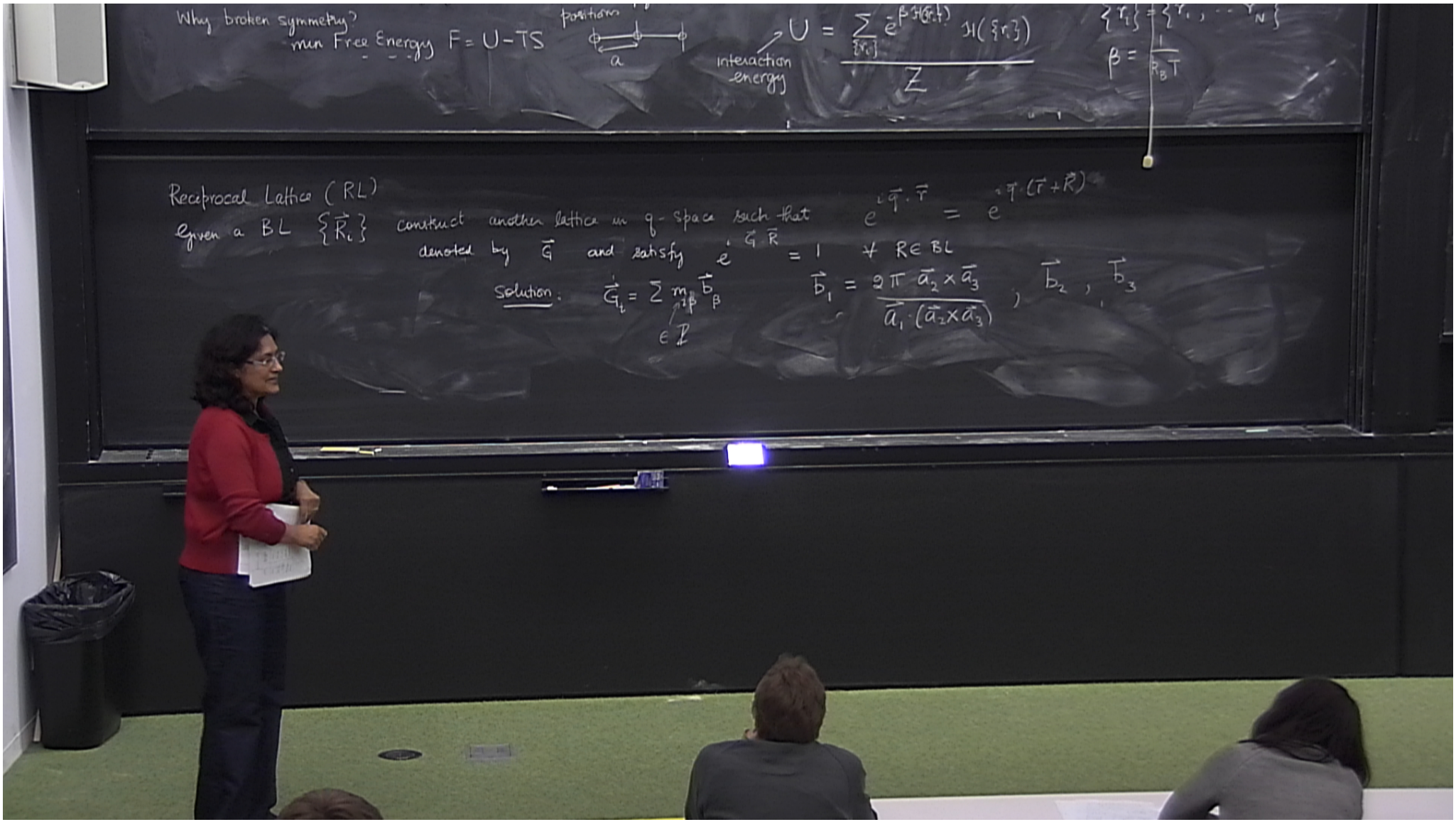
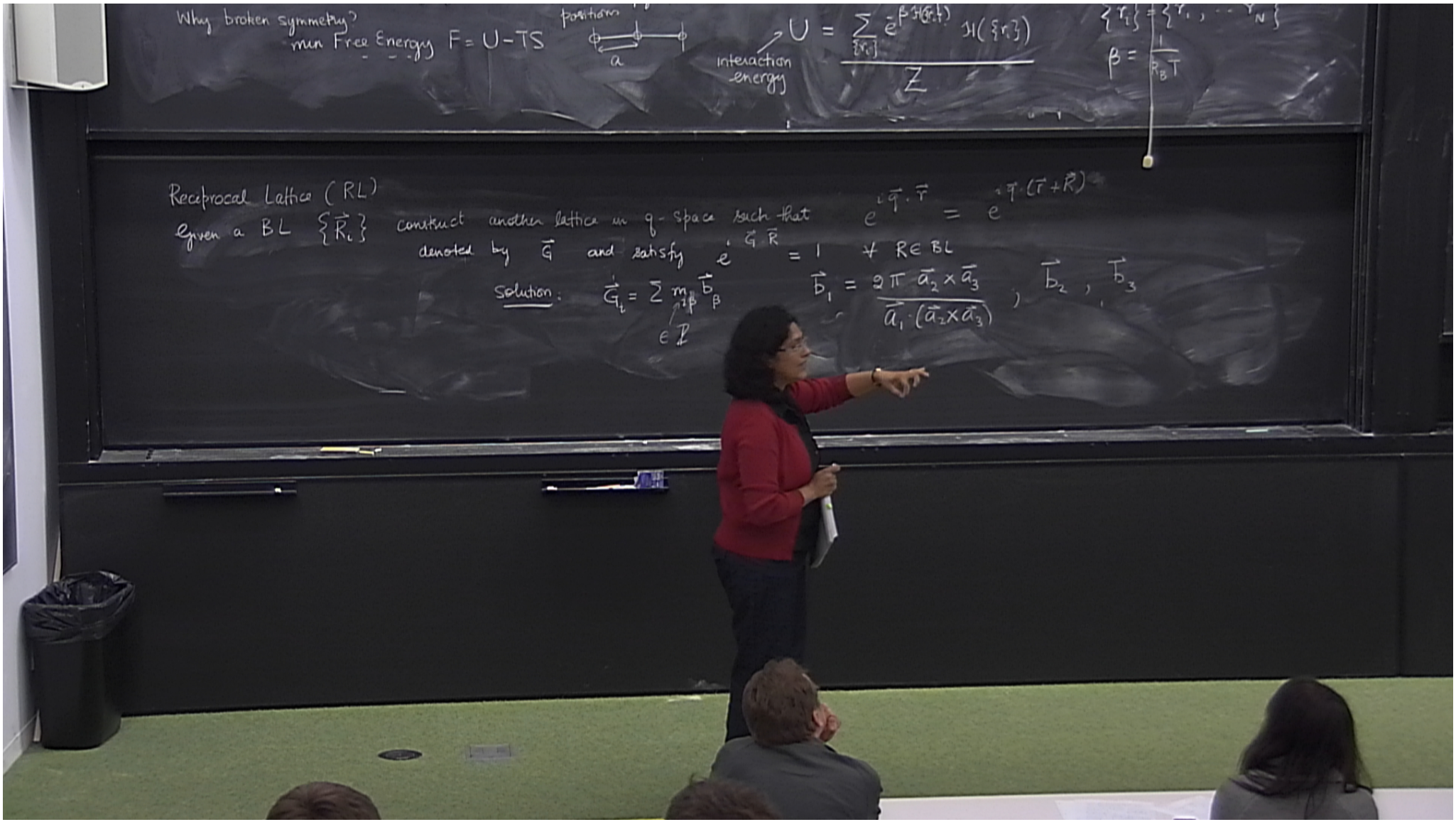
U = Σ_{{r_i}} e^{-β H({r_i})} H({r_i}) / Z
{r_i} = {r_1, ... r_N}
β = 1/(k_B T)
Reciprocal Lattice (RL)
Given a BL {R_i} construct another lattice in q-space such that e^{iq·r} = e^{iq·(r+R)}
denoted by G and satisfy e^{iGR} = 1 ∀ R ∈ BL
Solution: G_i = Σ m_{iβ} b_β
b_1 = 2


Why broken symmetry?
min Free Energy F = U − TS
positions
a
interaction energy
U = Σ_{{r_i}} e^{−β H({r_i})} H({r_i}) / Z
{r_i} = {r_1, ... r_N}
β = 1/(k_B T)
Reciprocal Lattice (RL)
Given a BL {R_i} construct another lattice in q-space such that e^{iq·r} = e^{iq·(r+R)}
denoted by G and satisfy e^{iG·R} = 1 ∀ R ∈ BL
Solution: G_i = Σ m_{iβ} b_β
∈ I
b_1 = 2π a_2 × a_3 / a_1·(a_2 × a_3) , b_2 , b_3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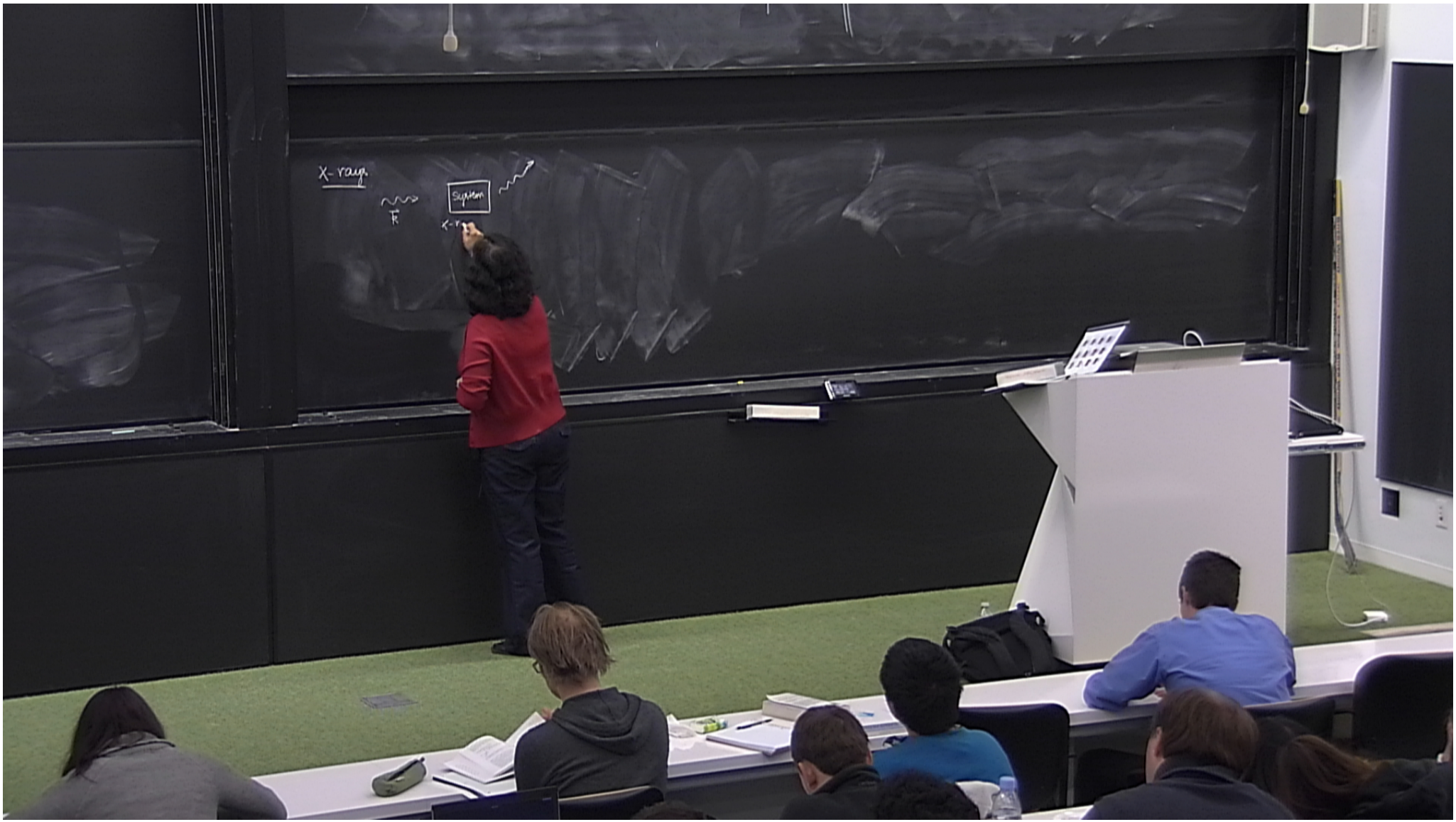
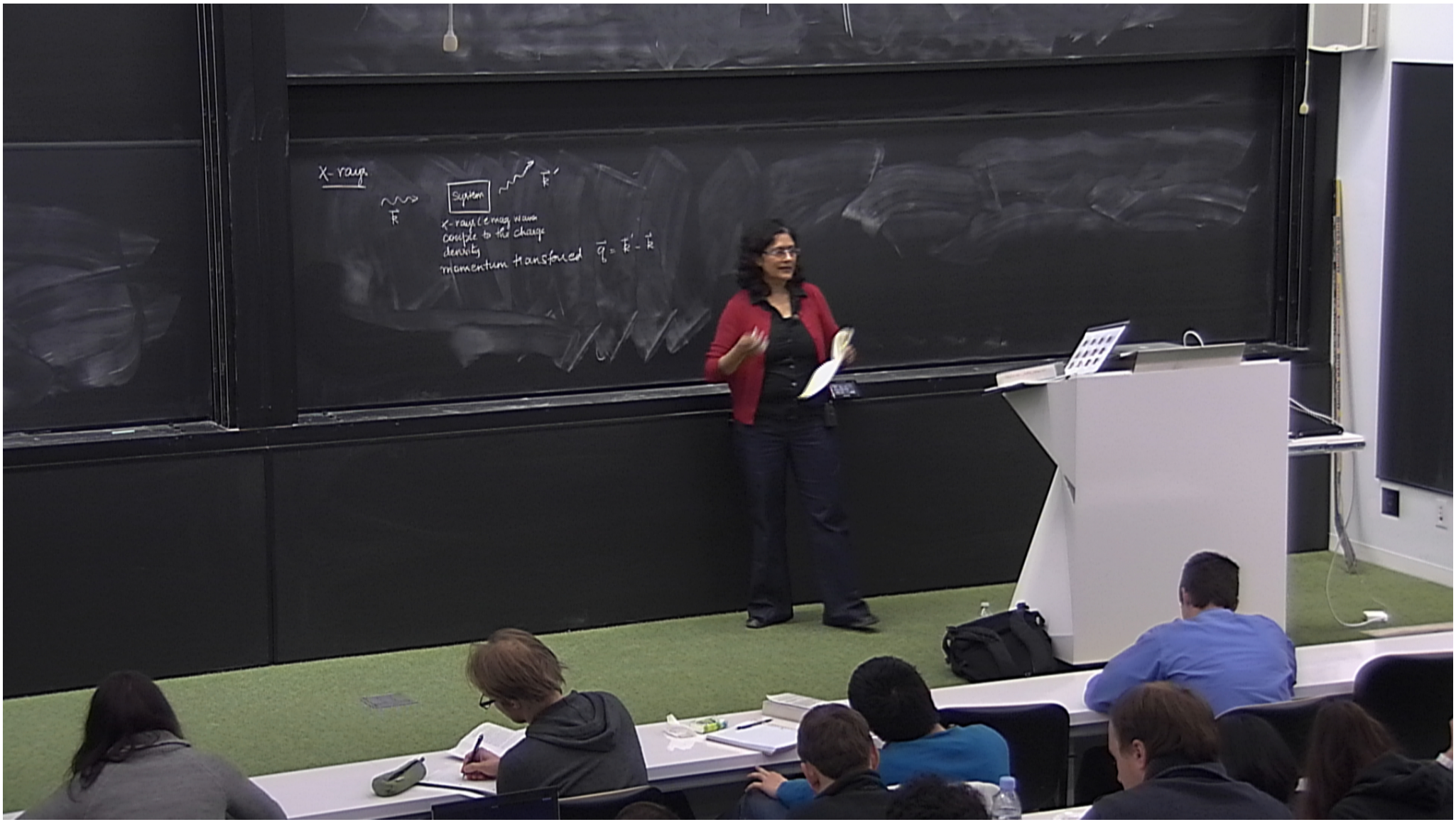
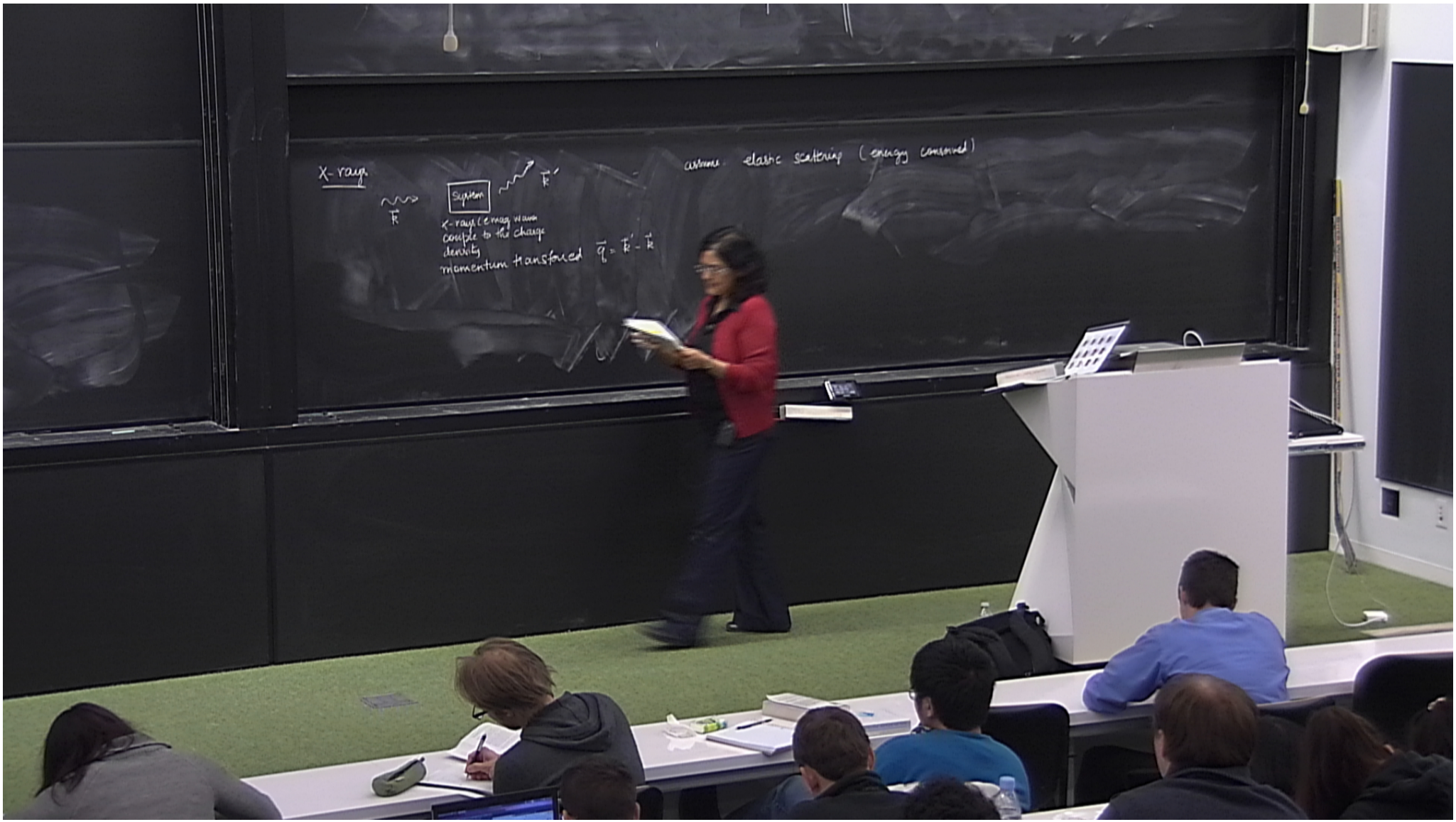
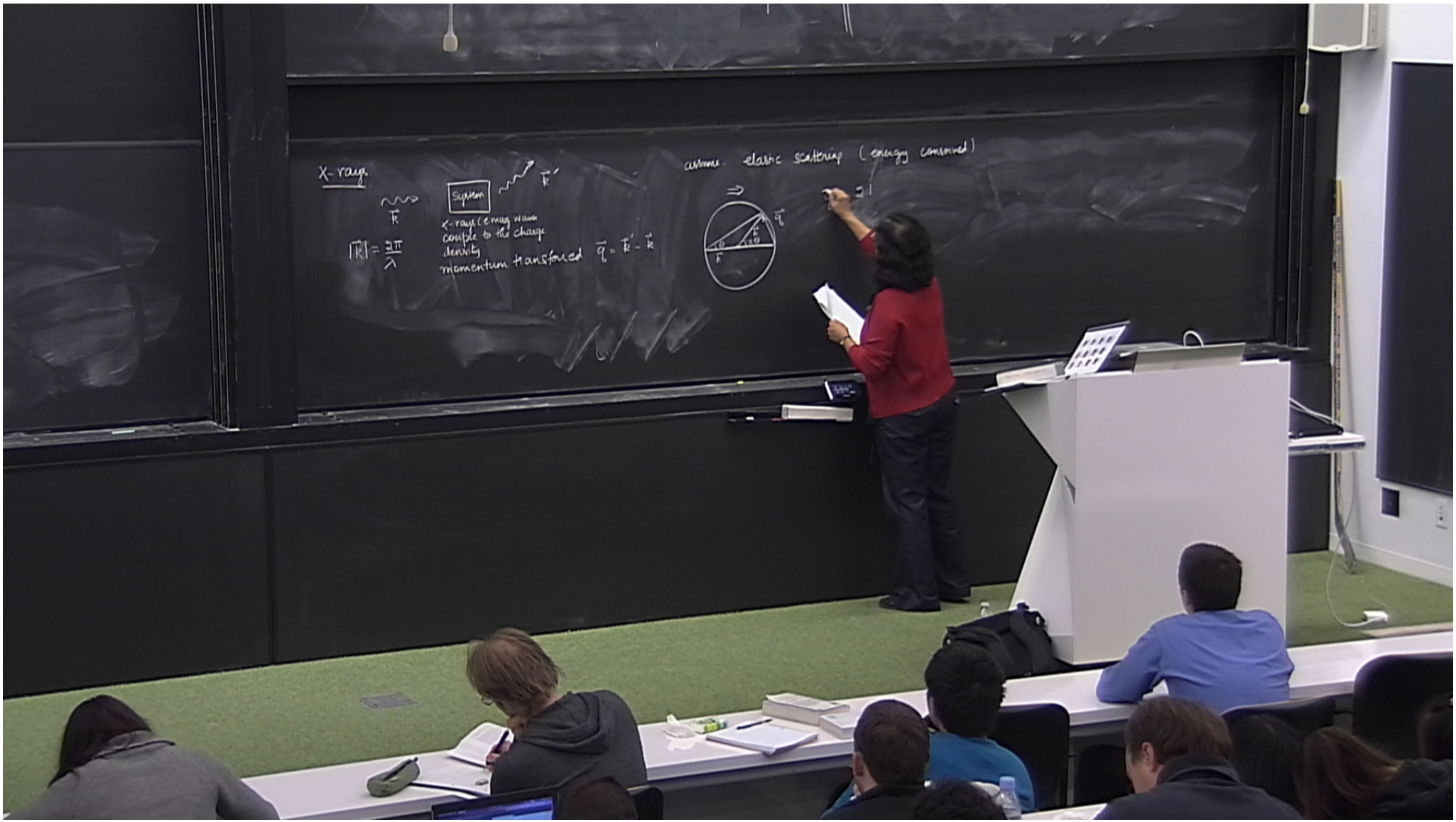
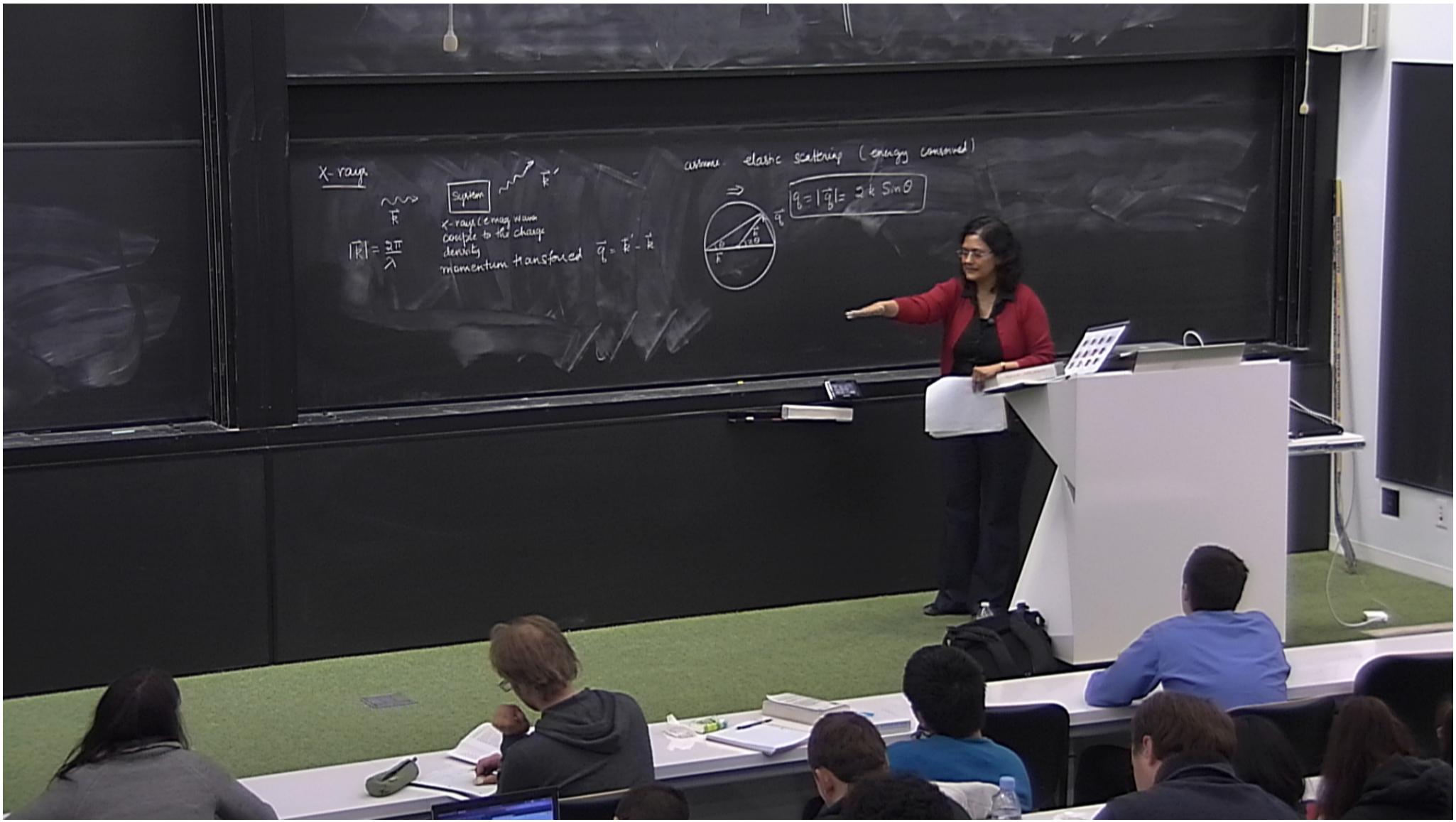

X-rays
System

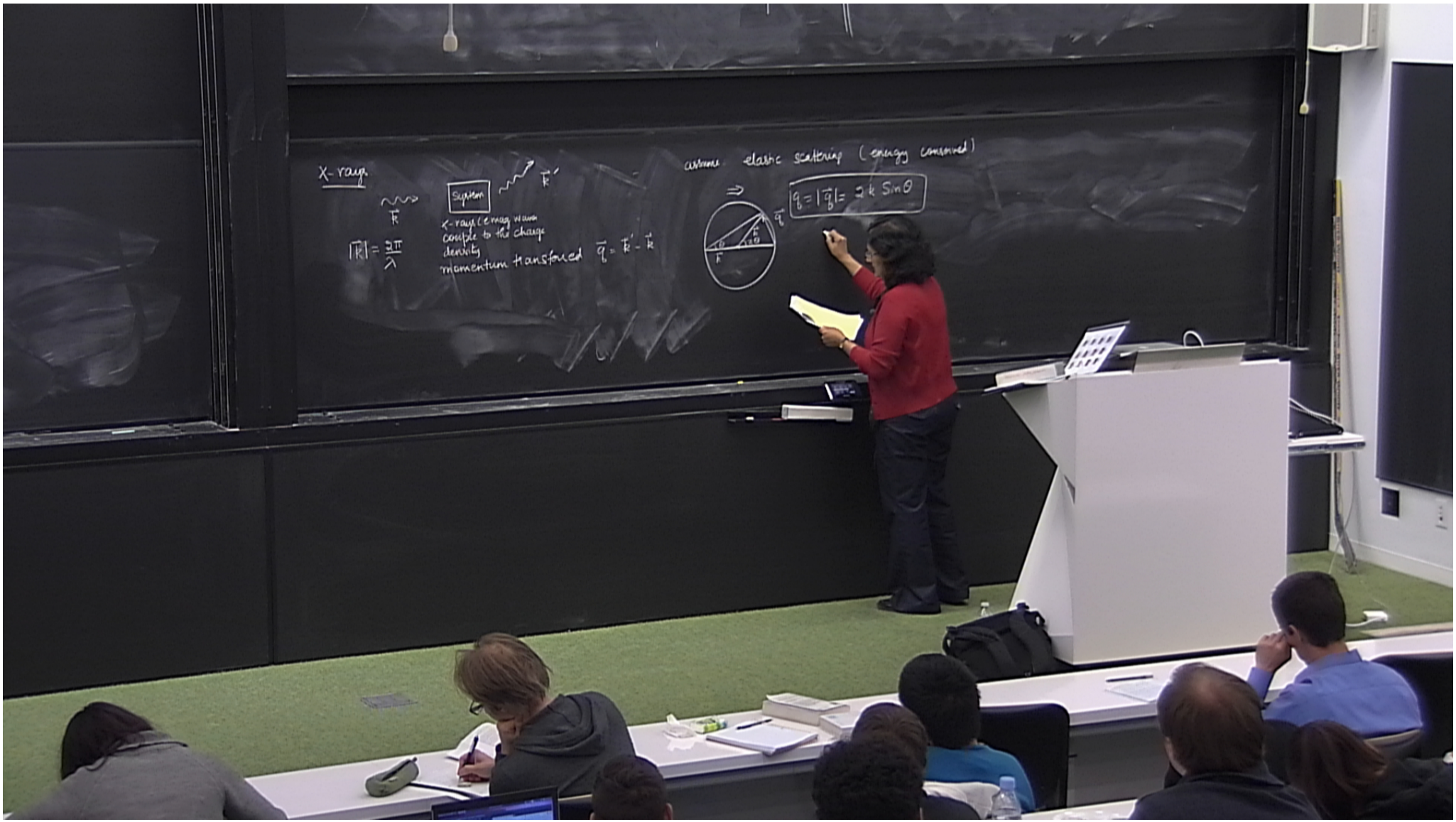
X-rays
System
X-rays (e mag wave
couple to the charge
density

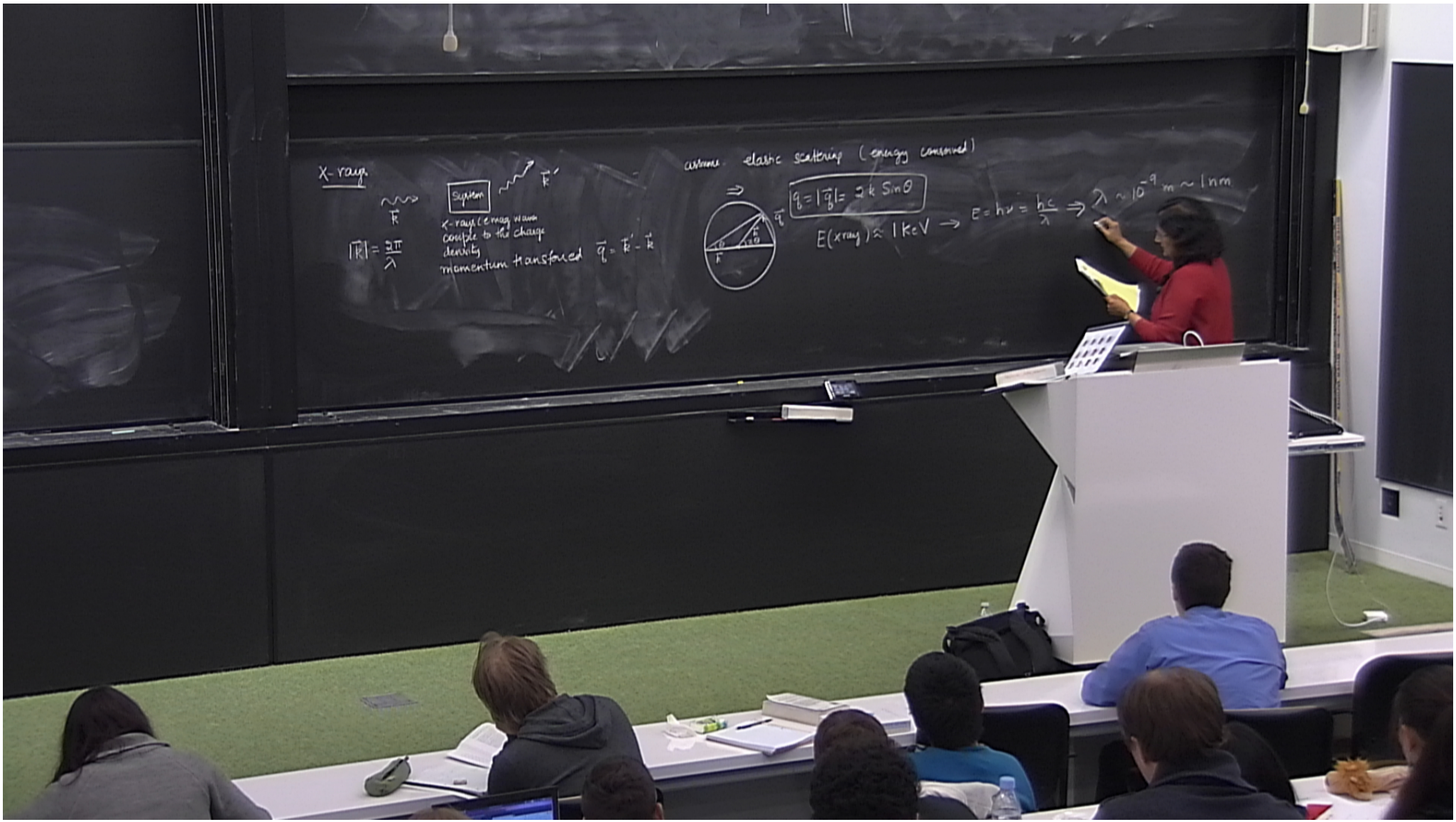

X-rays
System
k
k'
X-rays (e mag wave
couple to the charge
density
momentum transferred q = k' - k


X-rays
System
elastic scattering (energy conserved)

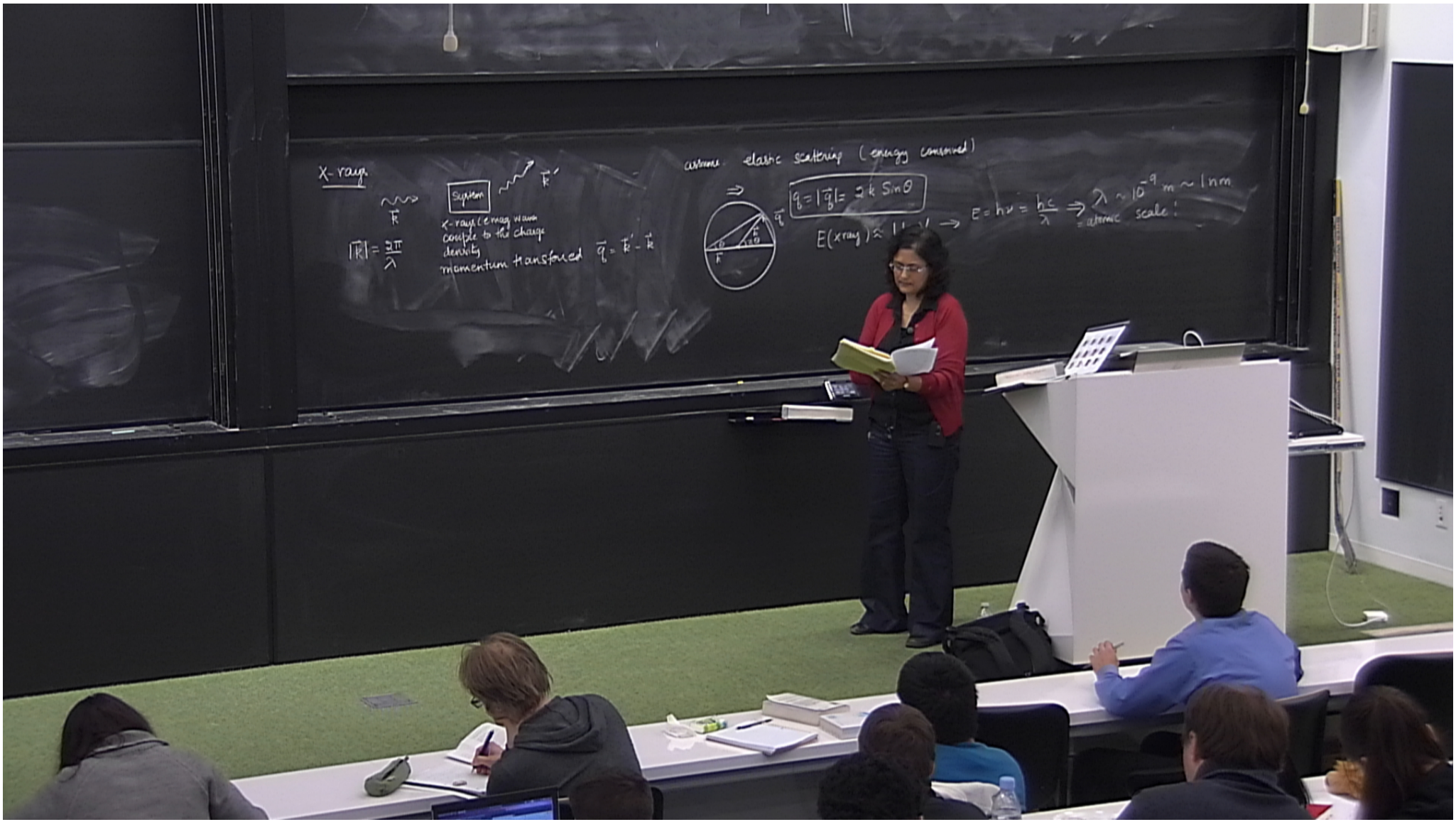

X-rays
System
X-rays (e mag wave
couple to the charge
density
momentum transferred
assume elastic scattering (energy conserved)

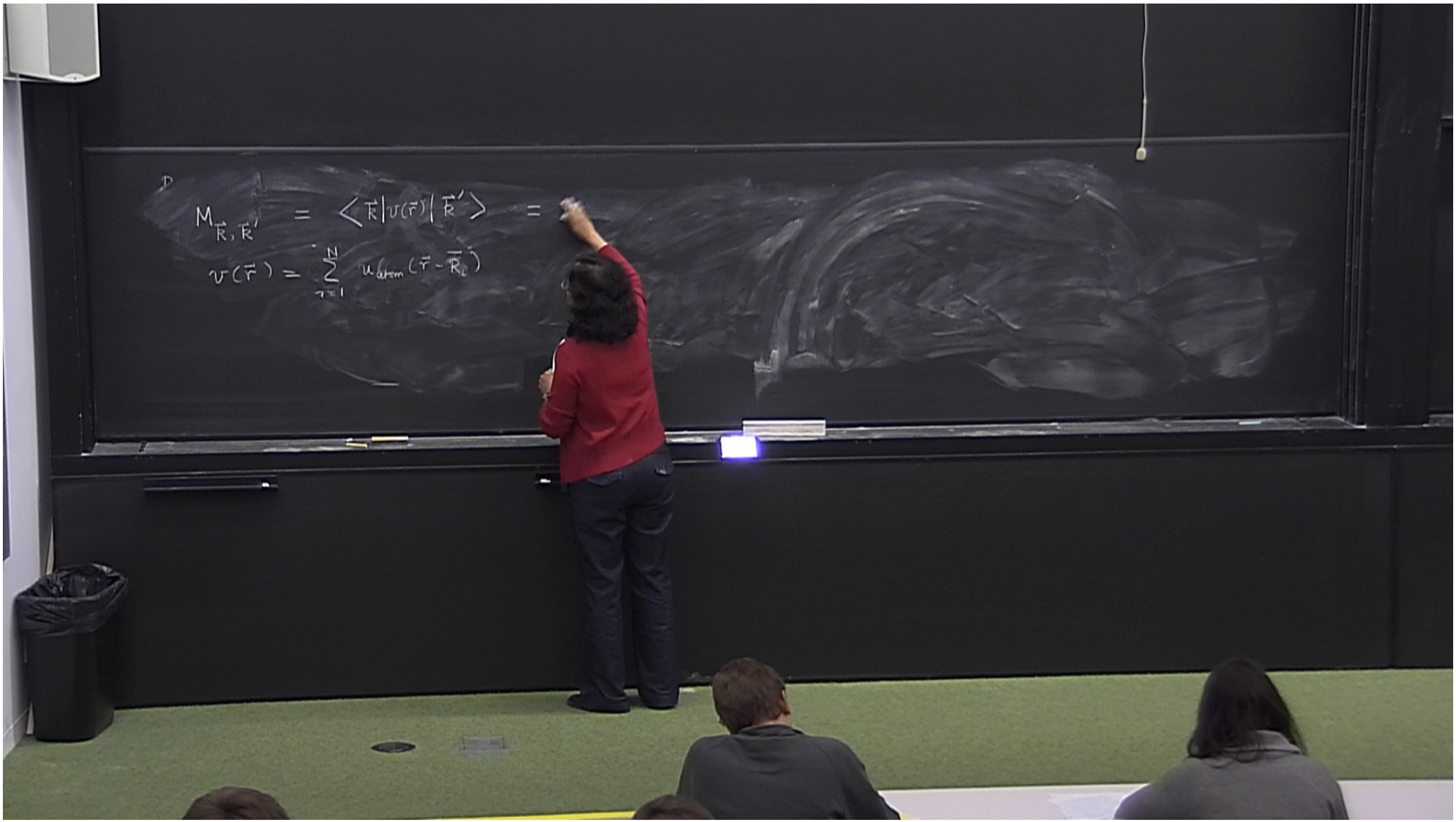

X-rays
System
X-rays (e mag wave
couple to the charge
density
momentum transferred
elastic scattering (energy conserved)

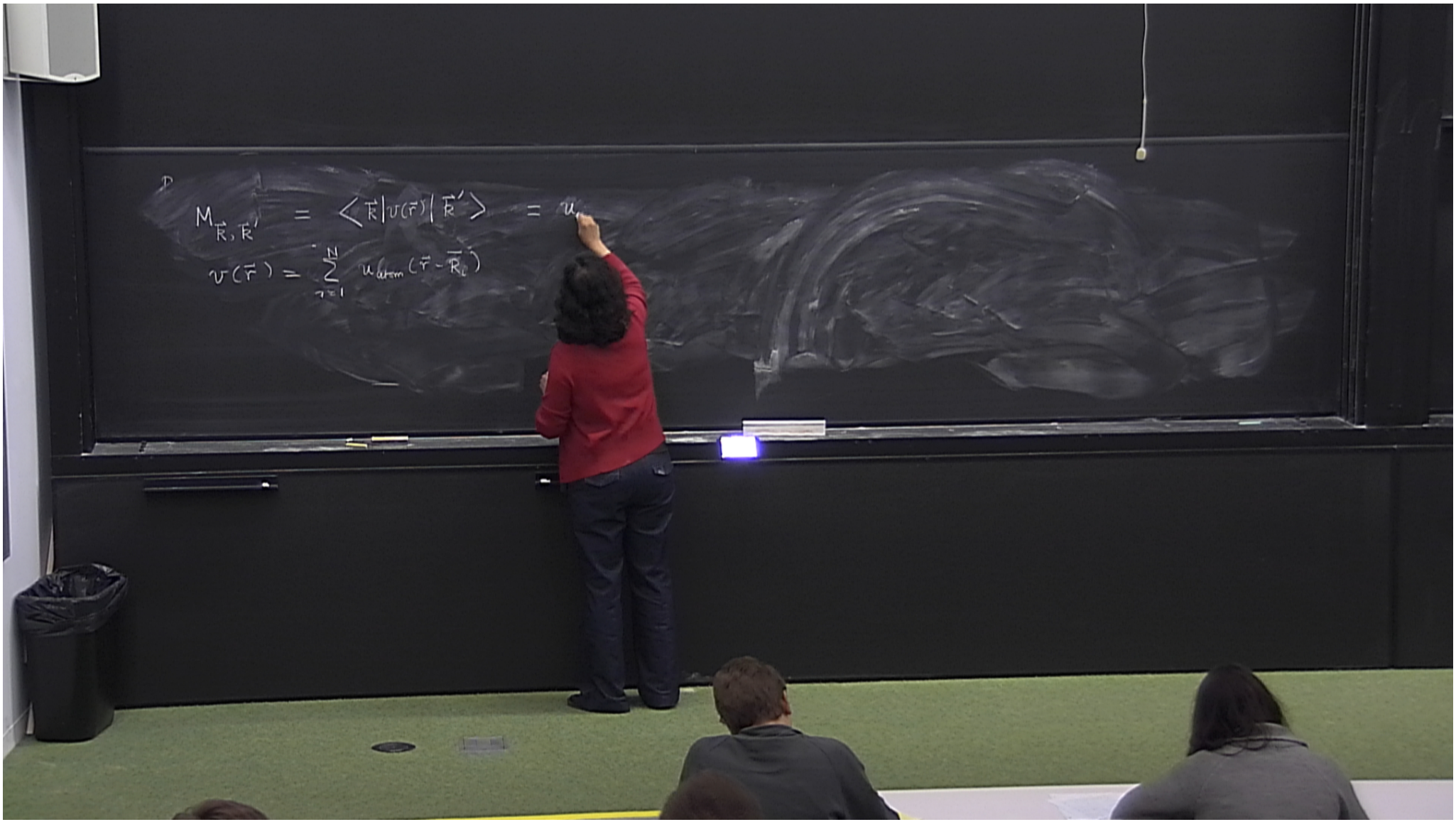

X-rays
System
X-rays (e mag wave
couple to the charge
density
momentum transferred
assume elastic scattering (energy conserved)


X-rays
System
X-rays (e mag wave couple to the charge density
momentum transferred
assume elastic scattering (energy conserved)
E(xray) ≈ 1 keV

X-rays
System
momentum transferred
elastic scattering (energy conserved)
E(xray) ≈ 1 keV
scale!


X-rays
System
X-rays (e mag wave
couple to the charge
density
momentum transferred
elastic scattering (energy conserved)
= atomic scale!


$M_{\vec{k},\vec{k}'} = \langle \vec{k} | v(\vec{r}) | \vec{k}' \rangle =$
$v(\vec{r}) = \sum_{l=1}^{N} u_{atom}(\vec{r} - \vec{R}_l)$


$M_{\vec{k},\vec{k}'} = \langle \vec{k} | v(\vec{r}) | \vec{k}' \rangle = u$
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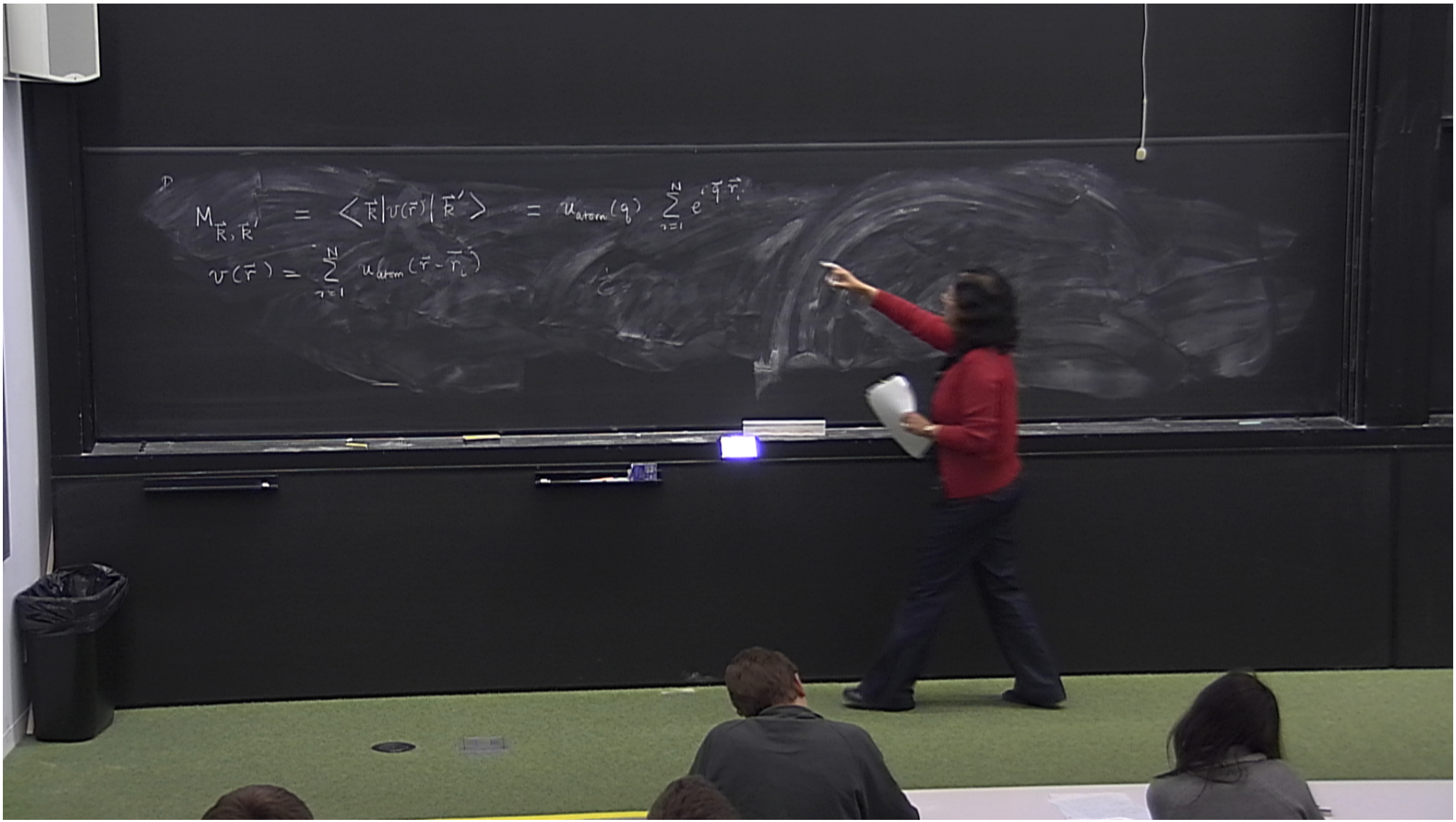
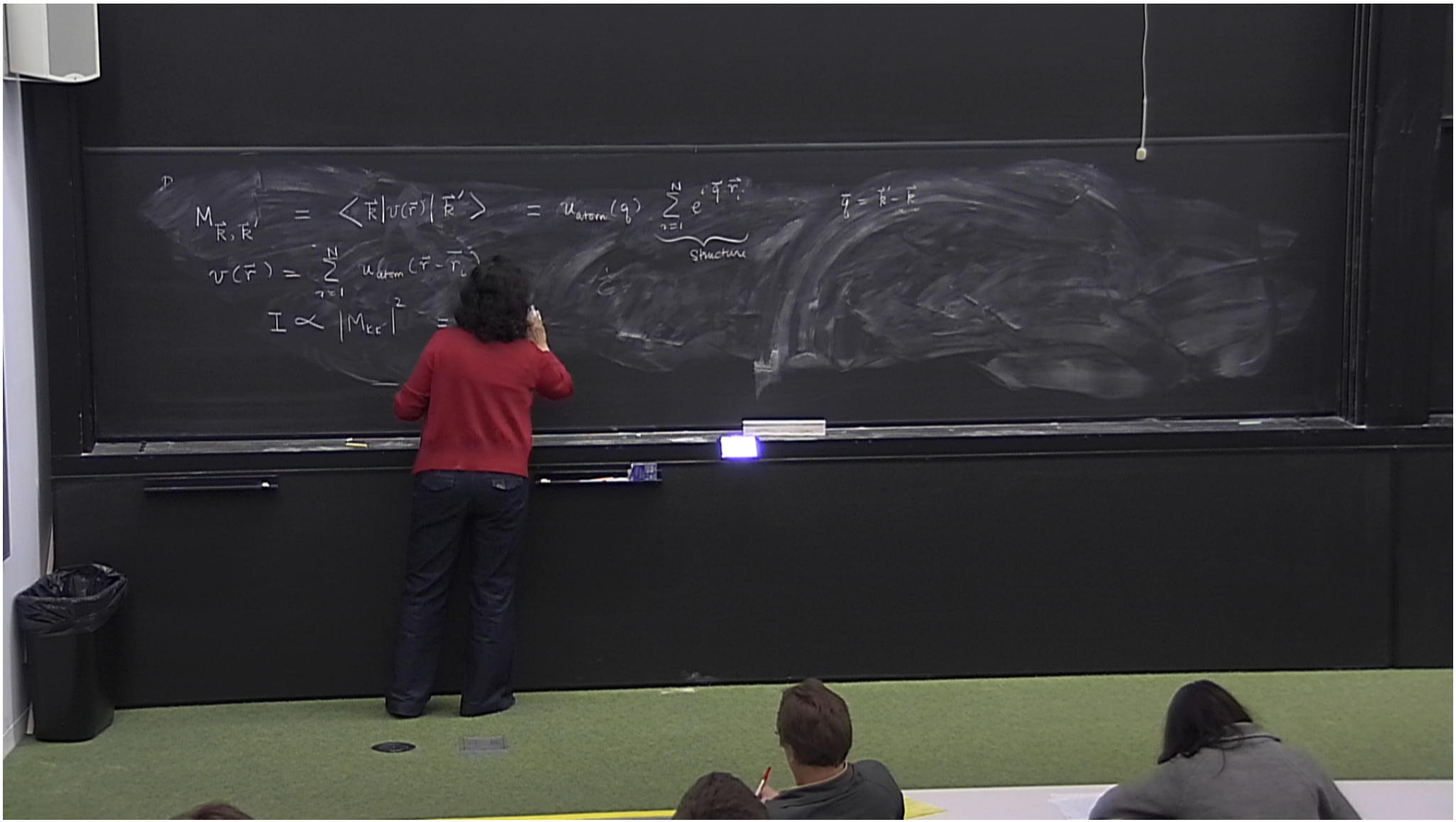

$M_{\vec{k},\vec{k}'} = \langle \vec{k} | v(\vec{r}) | \vec{k}' \rangle = u_{atom}(q) \sum_{i=1}^{N} e^{i\vec{q}\cdot\vec{r}_i}$
$v(\vec{r}) = \sum_{i=1}^{N} u_{atom}(\vec{r} - \vec{r}_i)$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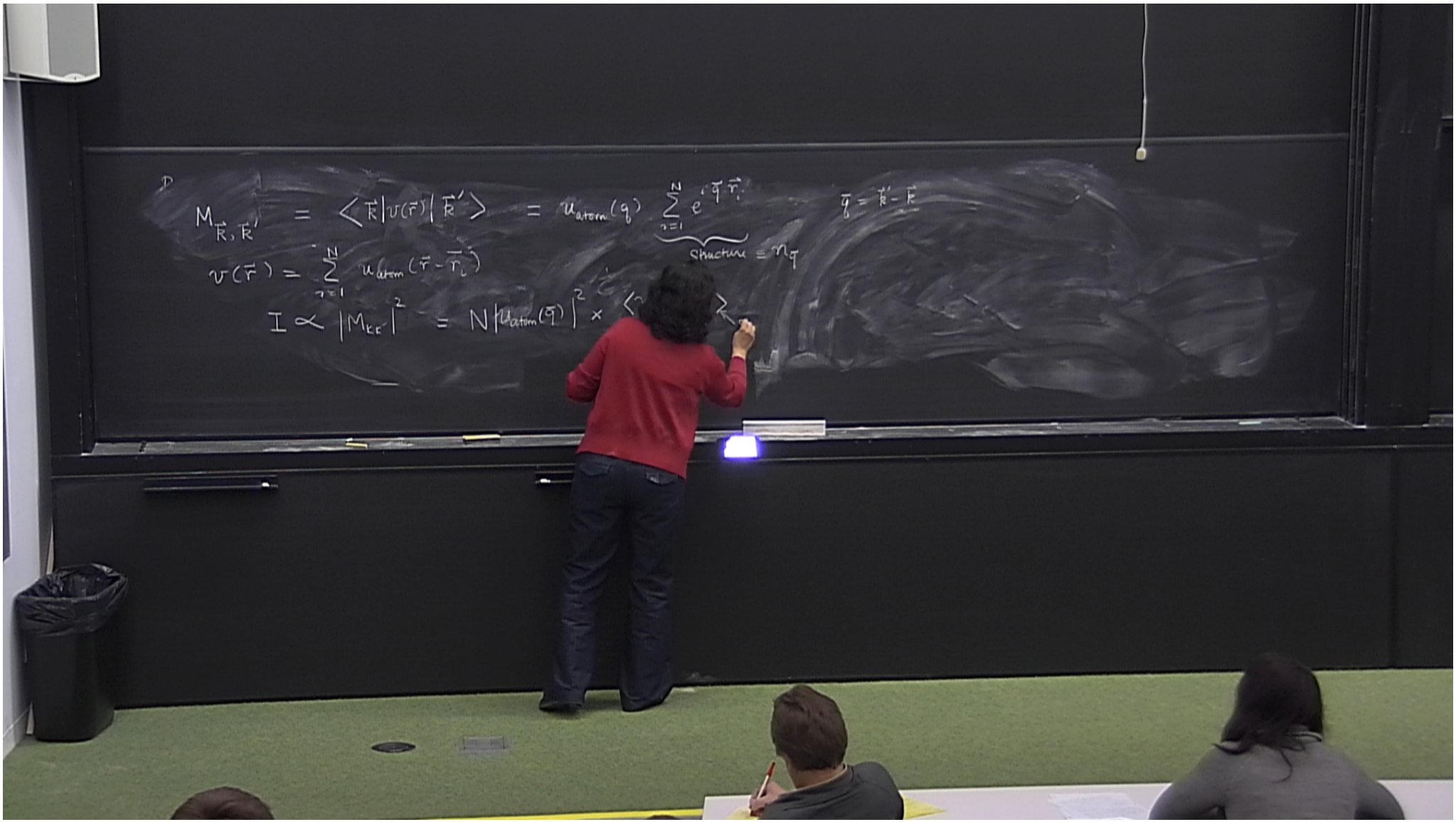
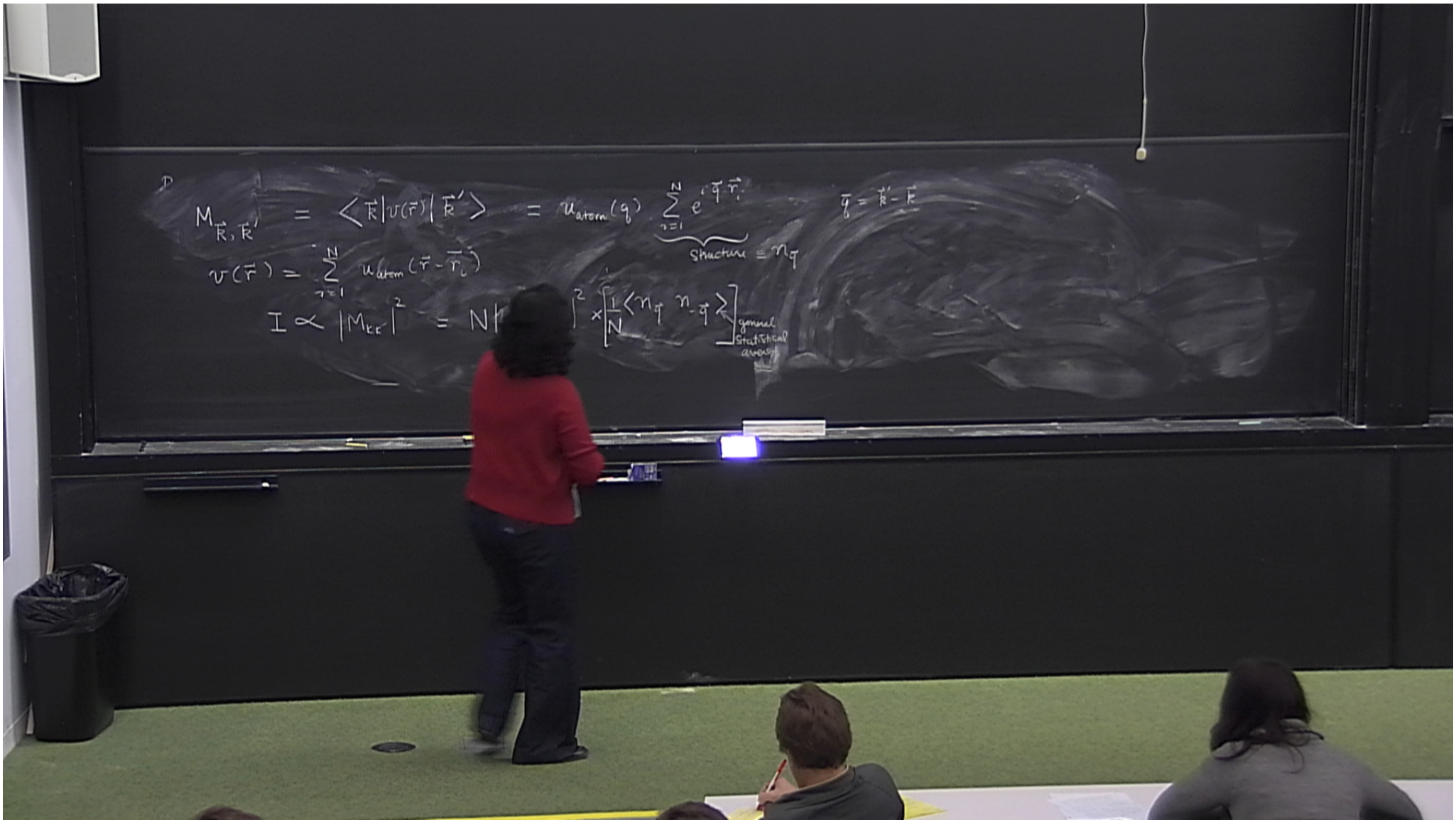
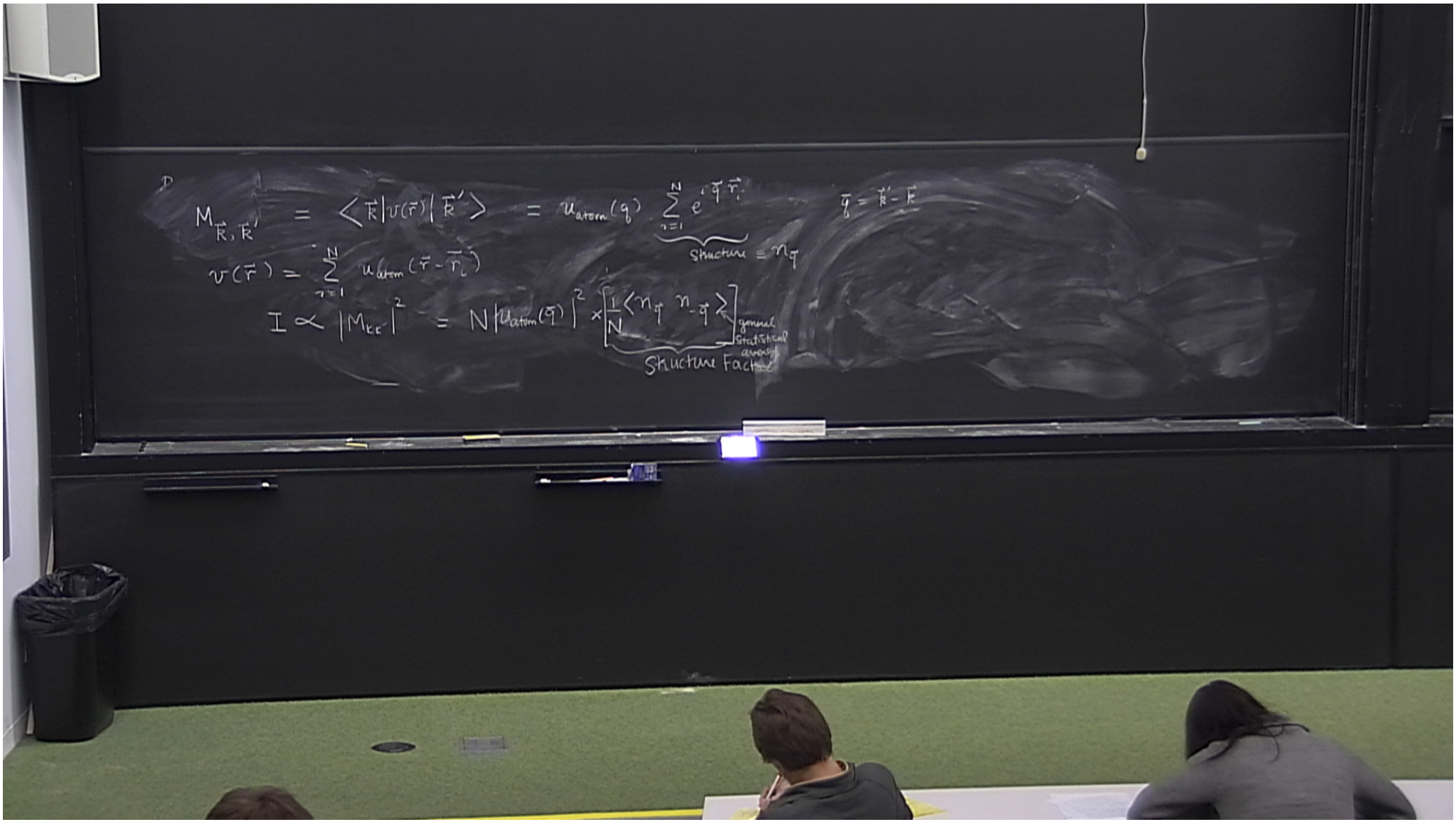
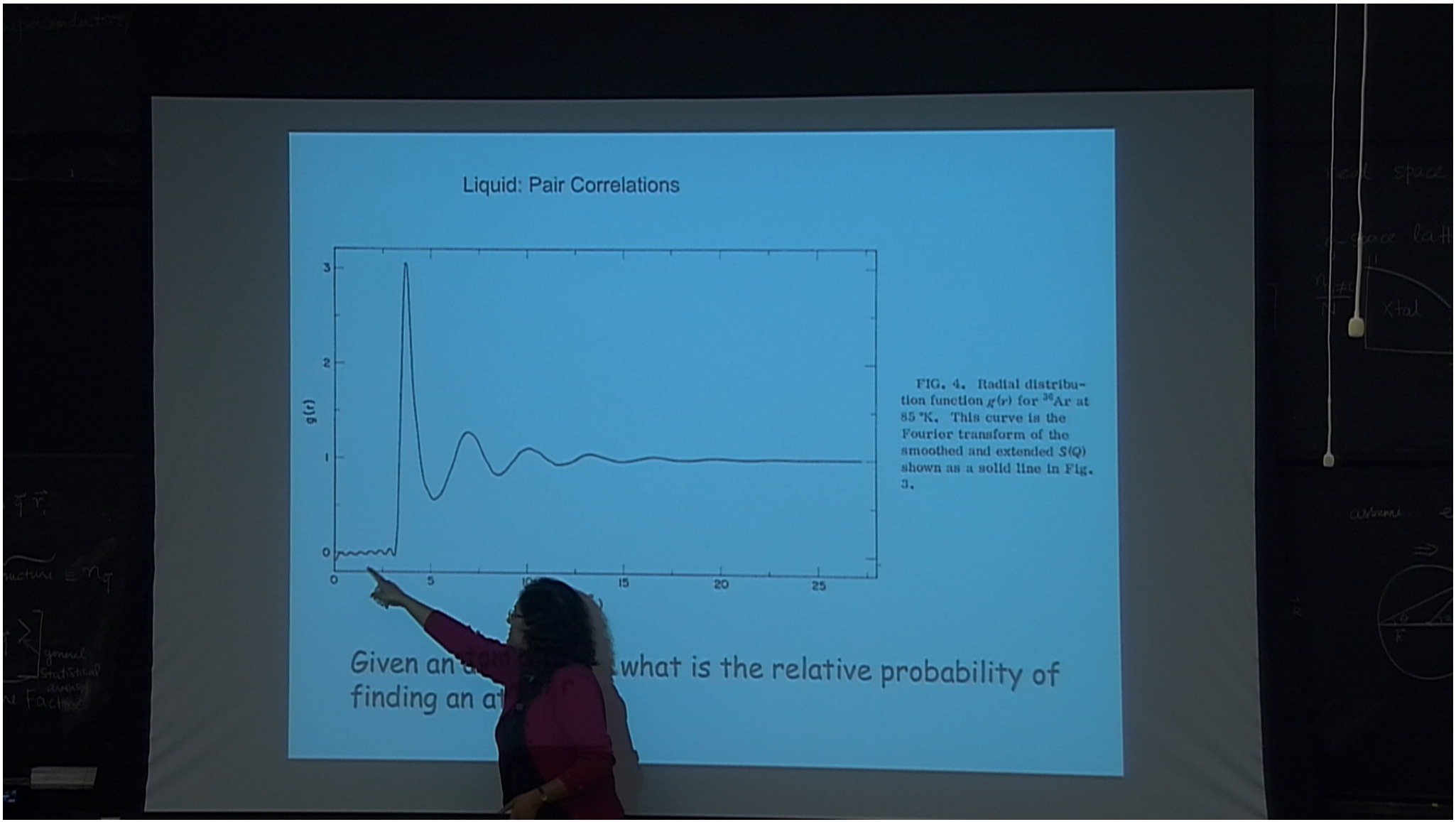
$\vec{q} = \vec{k}' - \vec{k}$
$v(\vec{r}) = \sum_{i=1}^{N} u_{atom}(\vec{r} - \vec{r}_i)$
$I \propto |M_{kk'}|^2 =$


$M_{\vec{k},\vec{k}'} = \langle \vec{k} | v(\vec{r}) | \vec{k}' \rangle = u_{atom}(q) \underbrace{\sum_{i=1}^{N} e^{i\vec{q}\cdot\vec{r}_i}}_{\text{structure} \equiv n_q}$
$\vec{q} = \vec{k}' - \vec{k}$
$v(\vec{r}) = \sum_{i=1}^{N} u_{atom}(\vec{r} - \vec{r}_i)$
$I \propto |M_{kk'}|^2 = N |u_{atom}(\vec{q})|^2 \times$


$M_{\vec{k},\vec{k}'} = \langle \vec{k} | v(\vec{r}) | \vec{k}' \rangle = u_{atom}(q) \sum_{i=1}^{N} e^{i\vec{q}\cdot\vec{r}_i}$
$\vec{q} = \vec{k}' - \vec{k}$
structure $\equiv n_{\vec{q}}$
$v(\vec{r}) = \sum_{i=1}^{N} u_{atom}(\vec{r} - \vec{r}_i)$
$I \propto |M_{kk'}|^2 = N|$
$|^2 \times \left[\frac{1}{N}\langle n_{\vec{q}}\, n_{-\vec{q}} \rangle\right]$
general statistical average


$M_{\vec{k},\vec{k}'} = \langle \vec{k} | v(\vec{r}) | \vec{k}' \rangle = u_{atom}(q) \underbrace{\sum_{i=1}^{N} e^{i\vec{q}\cdot\vec{r}_i}}_{\text{structure} \equiv n_{\vec{q}}}$
$\vec{q} = \vec{k}' - \vec{k}$
$v(\vec{r}) = \sum_{i=1}^{N} u_{atom}(\vec{r} - \vec{r}_i)$
$I \propto |M_{kk'}|^2 = N |u_{atom}(\vec{q})|^2 \times \underbrace{\left[\frac{1}{N}\langle n_{\vec{q}}\, n_{-\vec{q}} \rangle\right]}_{\text{Structure Factor}}$
general statistical average


Liquid: Pair Correlations
g(r)
0
1
2
3
0
5
10
15
20
25
FIG. 4. Radial distribution function g(r) for 36Ar at 85 °K. This curve is the Fourier transform of the smoothed and extended S(Q) shown as a solid line in Fig. 3.
Given an
what is the relative probability of
finding an

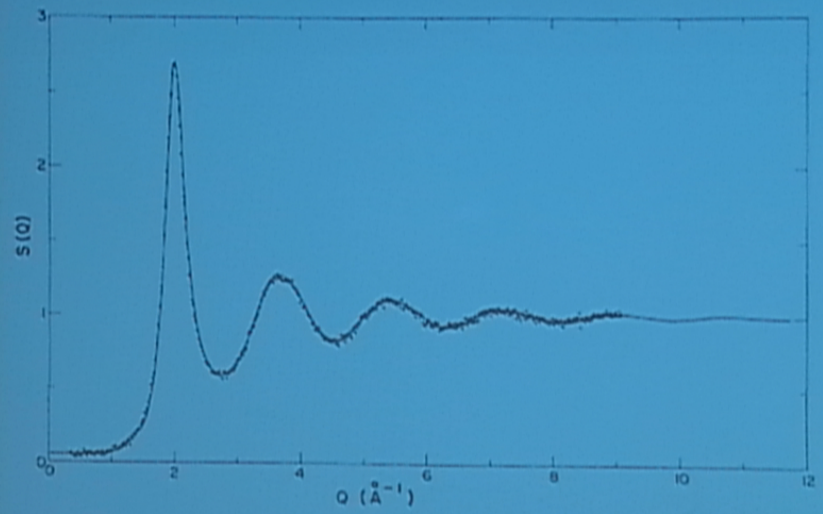


FIG. 3. Fully corrected experimental values of the liquid structure factor $S(Q)$ for ^{36}Ar at 85 °K. The solid line is a smoothed and extended $S(Q)$ obtained from the experimental data by an iterative procedure described in the text. The small dip in the experimental data near the top of the second peak ($Q = 3.75$ Å^{-1}) is apparently statistical in origin.

$$n(\vec{r}) = \sum_i \delta(\vec{r} - \vec{r}_i)$$

$$n(\vec{q}) = \sum_i e^{i\vec{q}\cdot\vec{r}_i}$$

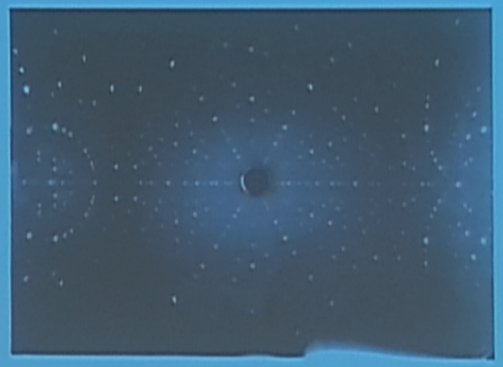
$$\langle n(\vec{r}) \rangle = n_0$$

$$S(q) = \frac{1}{N} \left\langle \sum_{i,j} e^{i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} \right\rangle$$

$$S(q) = \frac{1}{N} \langle n(q) n(-q) \rangle$$

Density-density correlatio

WOW!!!

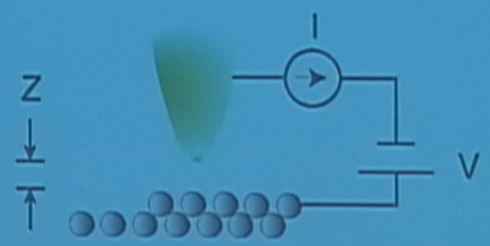


X-rays: Laue diffraction pattern for a single Si crystal.
The film wraps around the sample, so that it covers nearly 360 degrees.
There is a (111) direction right in the center.

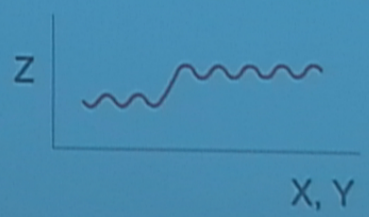
http://www.union.edu/PUBLIC/PHYDEPT/jonesc/scientific_photos.htm

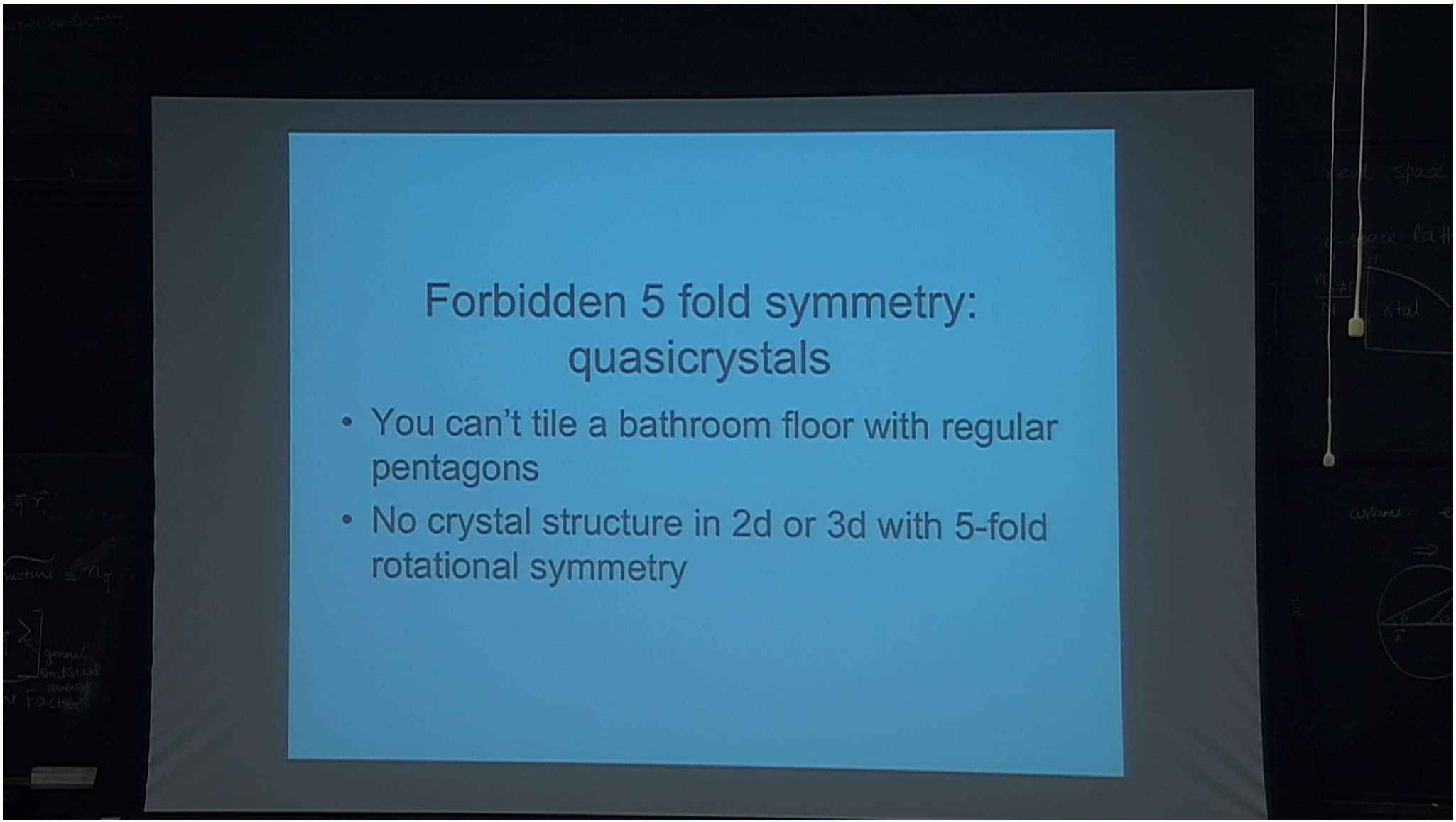
Tunneling Microscopy: Topograp

Constant Current STM



Topography




Forbidden 5 fold symmetry: quasicrystals
• You can't tile a bathroom floor with regular pentagons
• No crystal structure in 2d or 3d with 5-fold rotational symmetry

VOLUME 53, NUMBER 20 PHYSICAL REVIEW LETTERS 12 NOVEMBER 1984

Metallic Phase with Long-Range Orientational Order and No Translational Symmetry

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and

D. Gratias
Centre d'Etudes de Chimie Métallurgique, Centre National de la Recherche Scientifique, F-94400 Vitry, France

and

J. W. Cahn
Center for Materials Science, National Bureau of Standards, Gaithersburg, Maryland 20760
(Received 9 October 1984)

We have observed a metallic solid (Al–14-at.%-Mn) with long-range orientational order, but with icosahedral point group symmetry, which is inconsistent with lattice translations. Its diffraction spots are as sharp as those of crystals but cannot be indexed to any Bravais lattice. The solid is metastable and forms from the melt by a first-order transition.

discovered a 5-fold symmetric point diffraction pattern (very sharp spots) in a rapidly cooled metallic alloy Al6Mn

VOLUME 53, NUMBER 26 PHYSICAL REVIEW LETTERS 24 DECEMBER 1984

Quasicrystals: A New Class of Ordered Structures

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and

Paul Joseph Steinhardt
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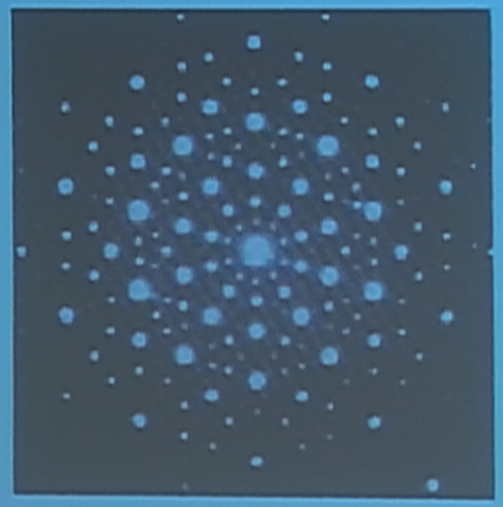
(Received 2 November 1984)

A quasicrystal is the natural extension of the notion of a crystal to structures with *quasi-periodic*, rather than periodic, translational order. We classify two- and three-dimensional quasicrystals by their symmetry under rotation and show that many disallowed crystal symmetries are allowed quasicrystal symmetries. We analytically compute the diffraction pattern of an ideal quasicrystal and show that the recently observed electron-diffraction pattern of an Al-Mn alloy is closely related to that of an icosahedral quasicrystal.

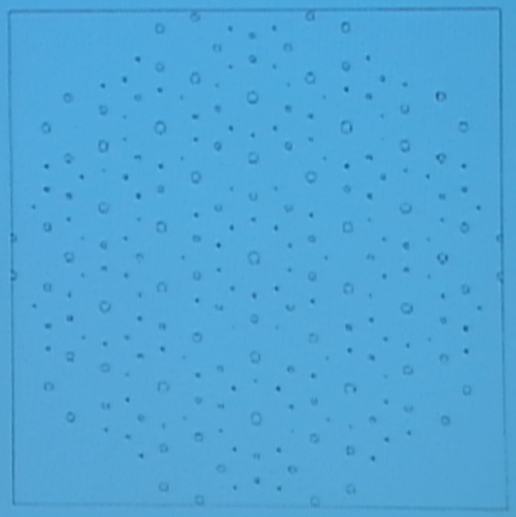
The diffraction pattern has the rotational symmetry of an icosahedron.
The icosahedron is a forbidden symmetry of classical crystallography, however in defiance of the conventional wisdom among crystallographers the Fourier transform of these lattices, despite their lack of translational periodicity turns out to have delta function Bragg peaks.

Bond orientational order maintained over micron distances



EXPT

Dan Shechtman, I. Blech, D. Gratias, and J.W. Cahn PRL 53, 1951 (1984)



THEORY

Dov Levine and Paul Steinhardt, PRL 53, 2477 (1984)

- ***OLD PARADIGM:*** SHARP SPOTS IN A DIFFRACTION PATTERN ARE A CONSEQUENCE OF TRANSLATIONAL SYMMETRY

$$sharpspots \Leftrightarrow trans.symmetry$$

- ***NEW PARADIGM:***

  WHILE 5 FOLD ROTATIONAL SYMMETRY IS NOT CONSISTENT WITH TRANSLATIONAL SYMMETRY

  IT CAN STILL GIVE A SHARP 5-FOLD DIFFRACTION PATTERN

$$sharpspots \Leftarrow trans.symmetry$$