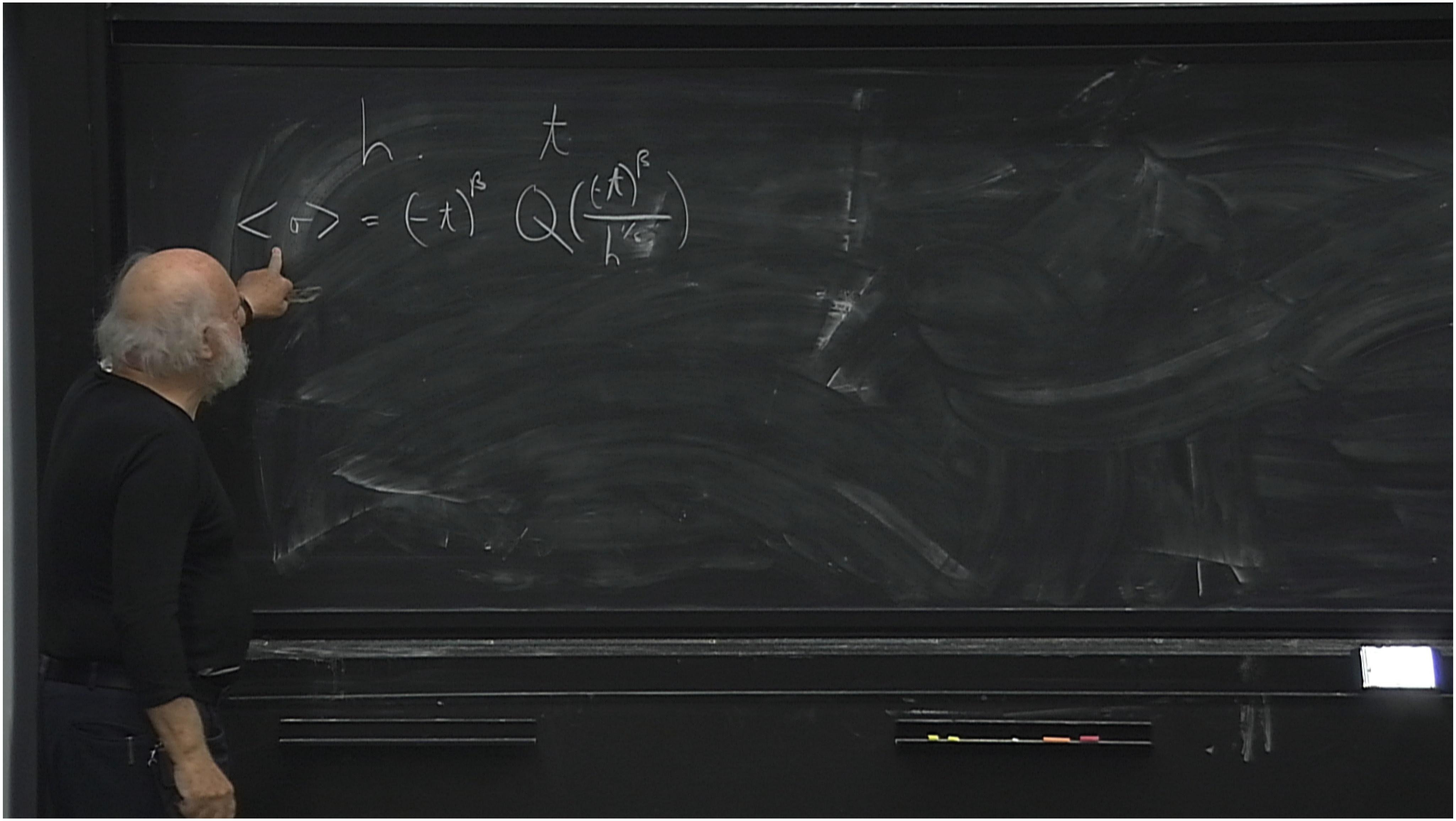
Title: Statistical Mechanics - Lecture 14

Date: Oct 21, 2011 10:30 AM

URL: http://pirsa.org/11100043

Abstract:


h
t
$\langle \sigma \rangle = (-t)^{\beta} \, Q\left(\frac{(-t)^{\beta}}{h^{1/\delta}}\right)$

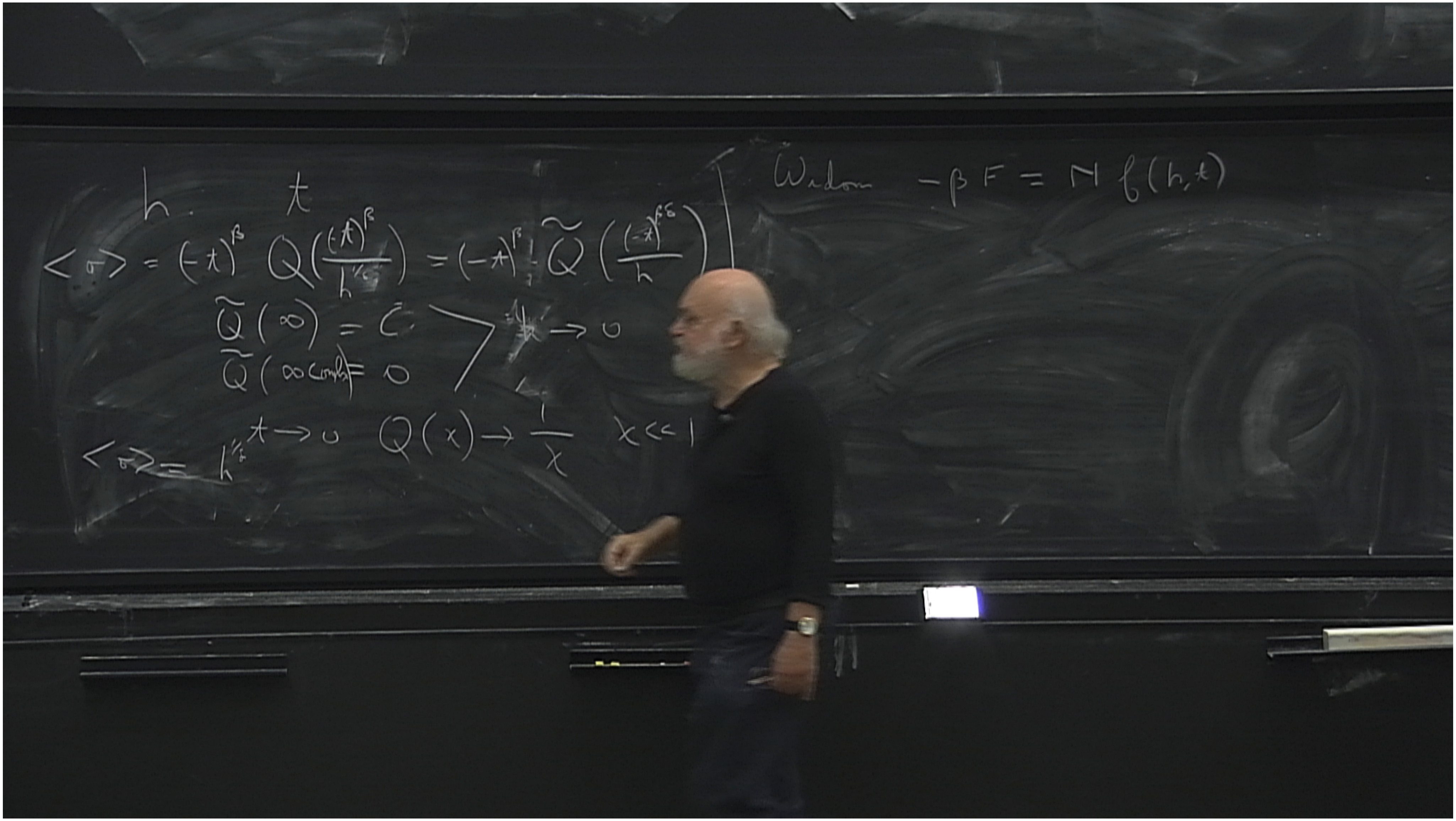
$h \quad t$

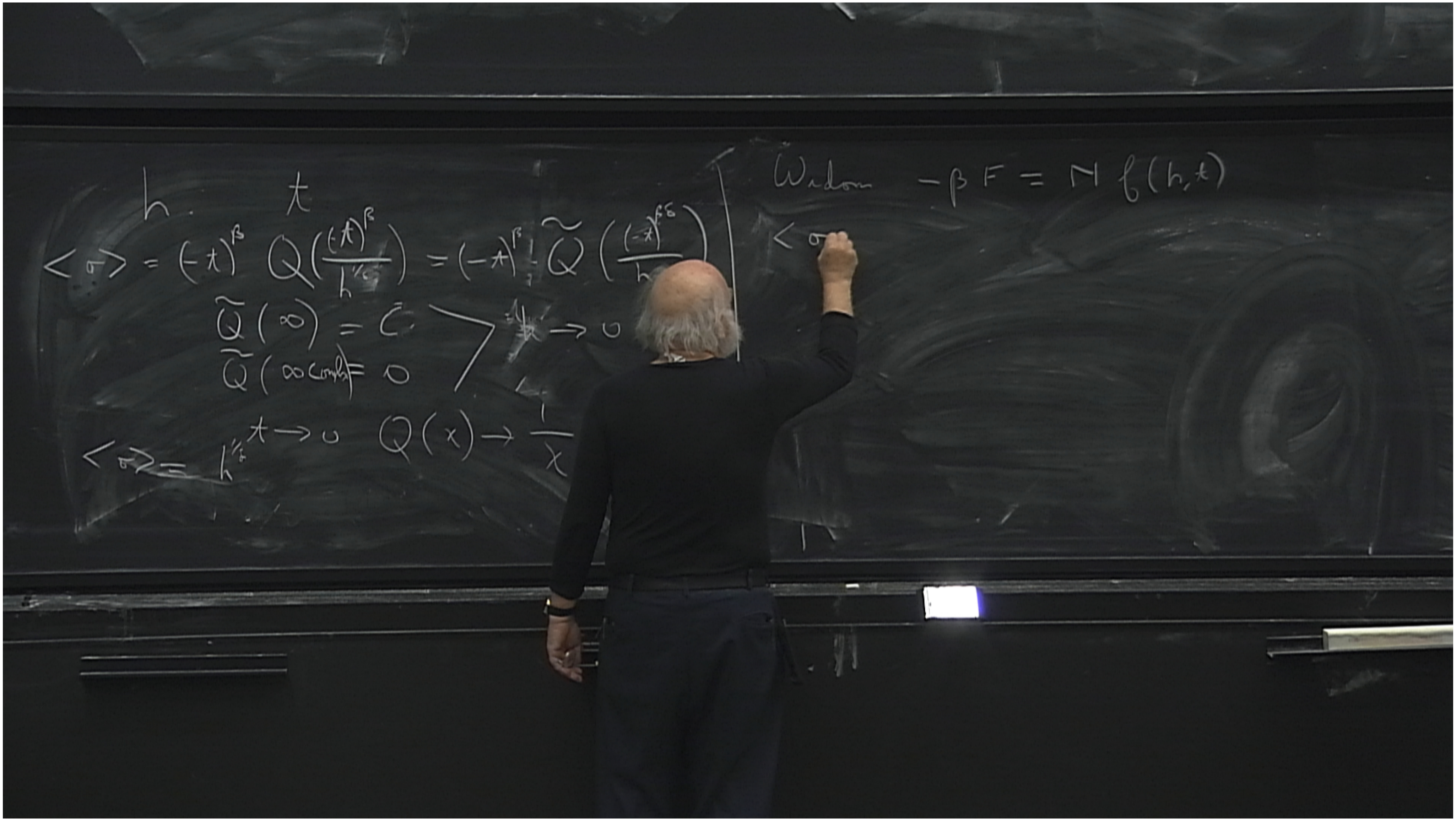
$$\langle \sigma \rangle = (-t)^{\beta}\, Q\left(\frac{(-t)^{\beta}}{h^{1/\delta}}\right) = (-t)^{\beta}\, \tilde{Q}\left(\frac{(-t)^{\beta\delta}}{h}\right)$$

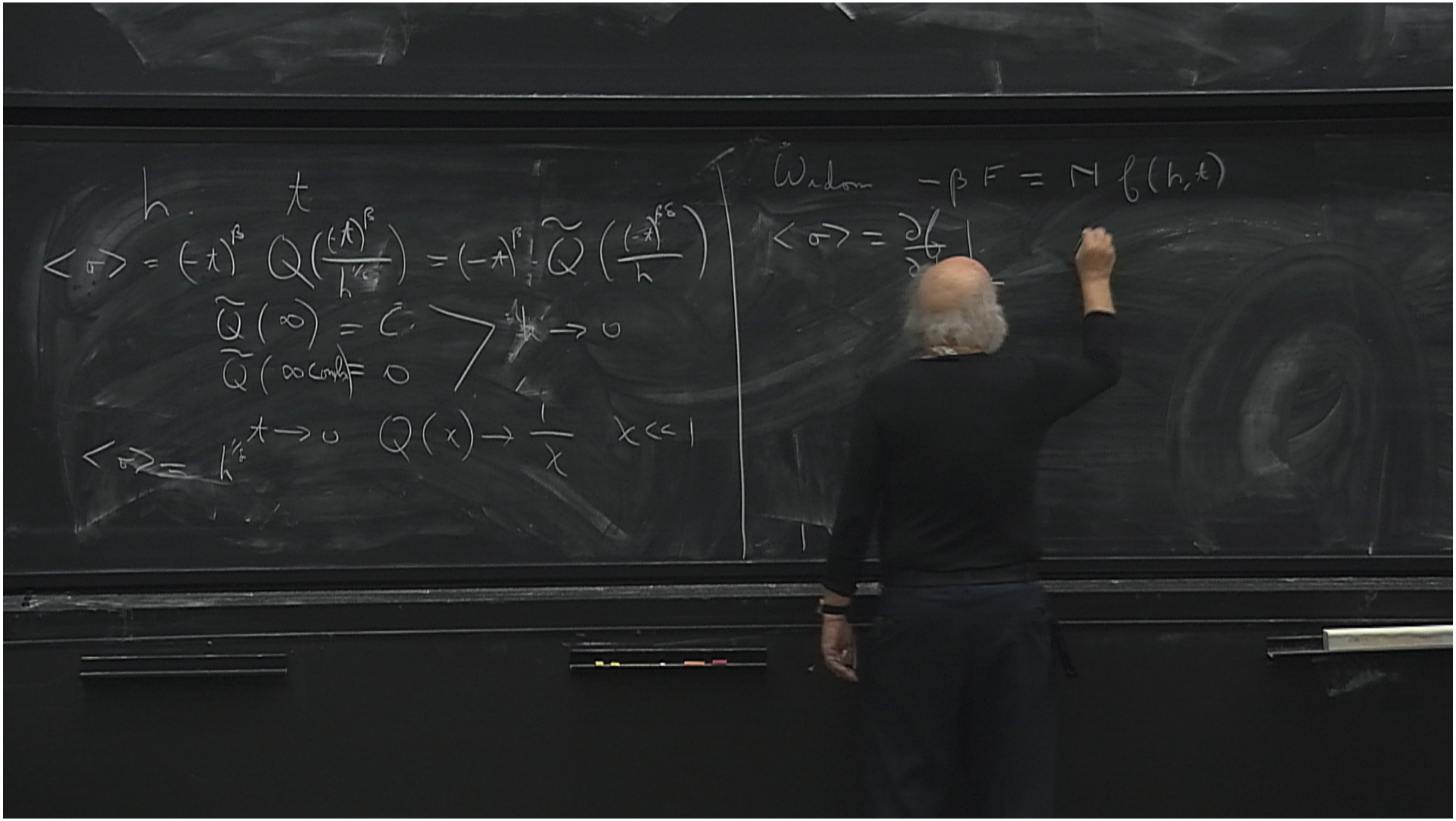
$$\left.\begin{array}{l} \tilde{Q}(\infty) = C \\ \tilde{Q}(\infty \text{ complex}) = 0 \end{array}\right\} \; t \to 0$$

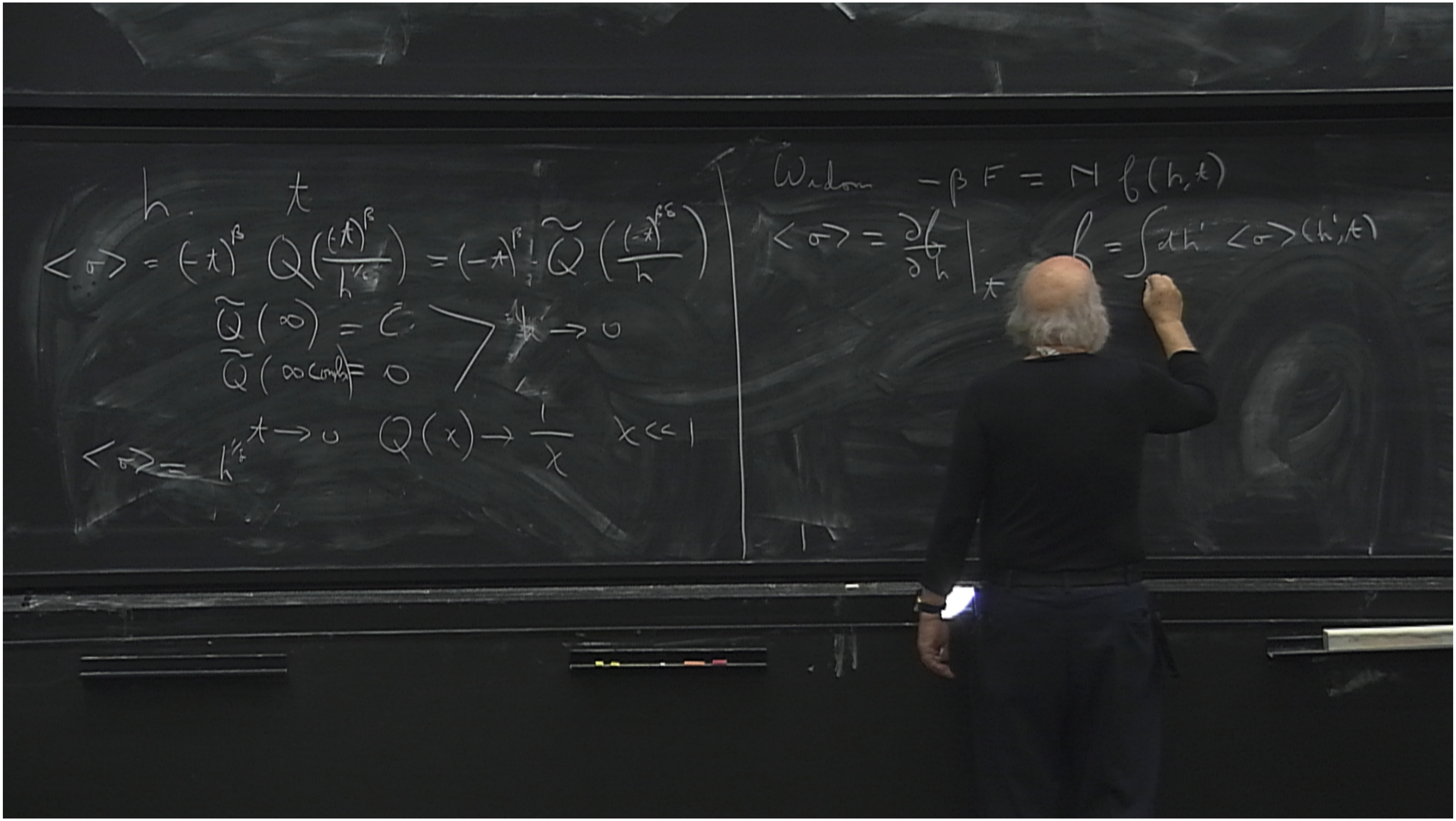
$$t \to 0 \qquad Q(x) \to \frac{1}{x} \qquad x \ll 1$$

$$\langle \sigma \rangle = h^{1/\delta}$$

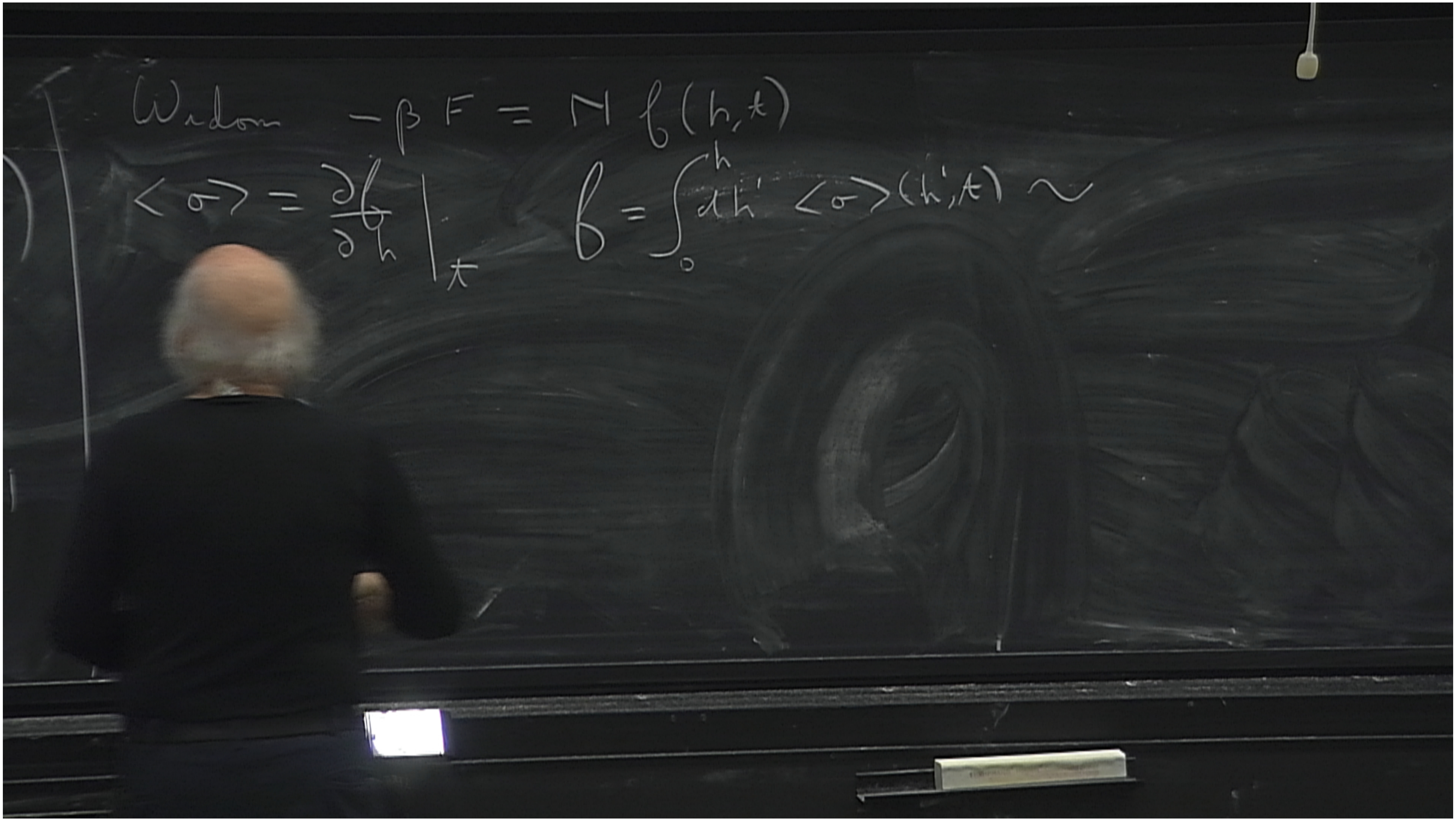

h t
$\langle\sigma\rangle = (-t)^{\beta} Q\left(\frac{(-t)^{\beta}}{h^{1/\delta}}\right) = (-t)^{\beta} \tilde{Q}\left(\frac{(-t)^{\beta\delta}}{h}\right)$
$\tilde{Q}(\infty) = C$
$\tilde{Q}(\infty \text{ complex}) = 0$
$t \to 0$
$\langle\sigma\rangle = h^{1/\delta}$ $t \to 0$ $Q(x) \to \frac{1}{x}$ $x \ll 1$
Widom $-\beta F = N f(h,t)$

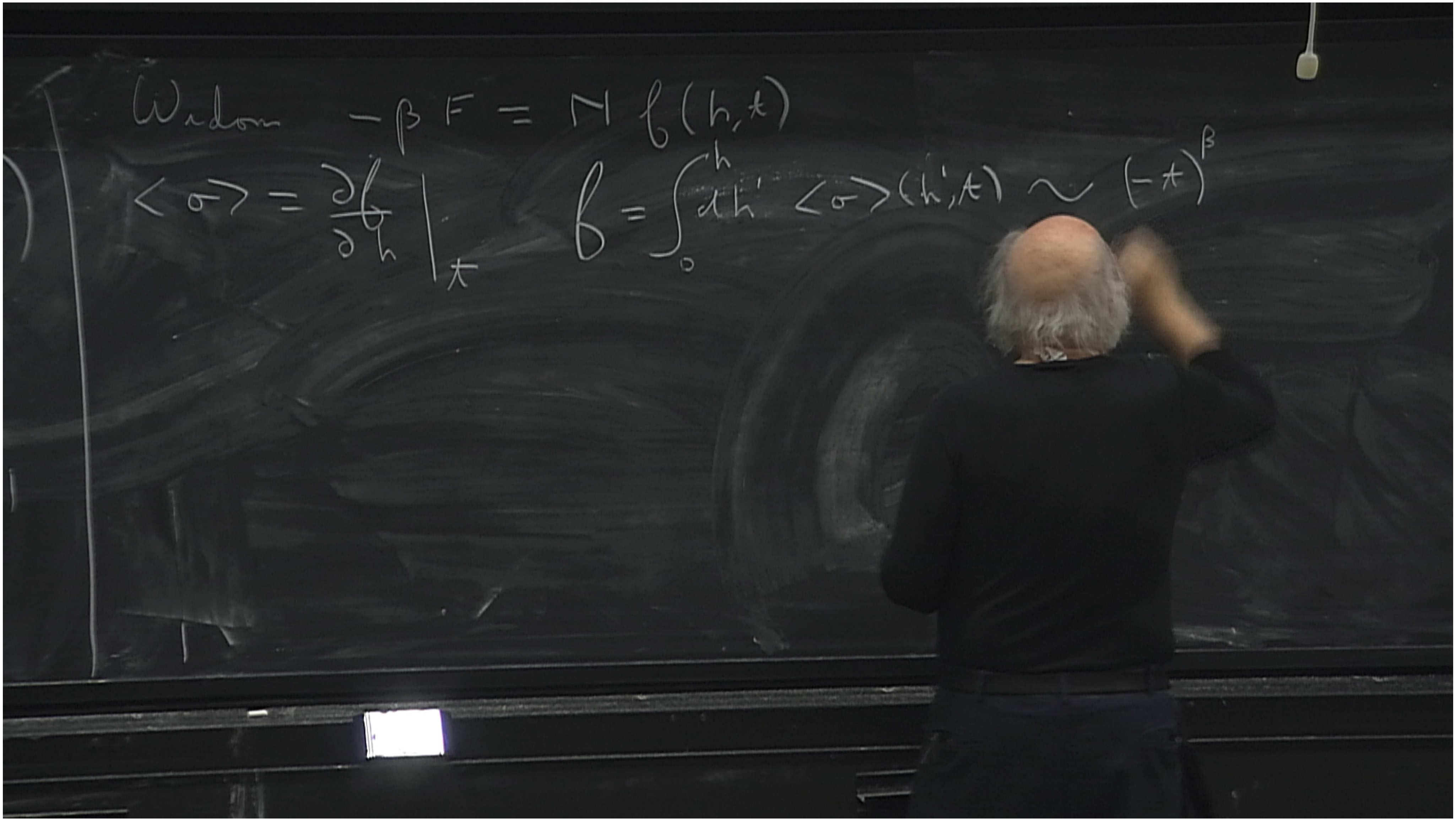

Widom −βF = N f(h,t)

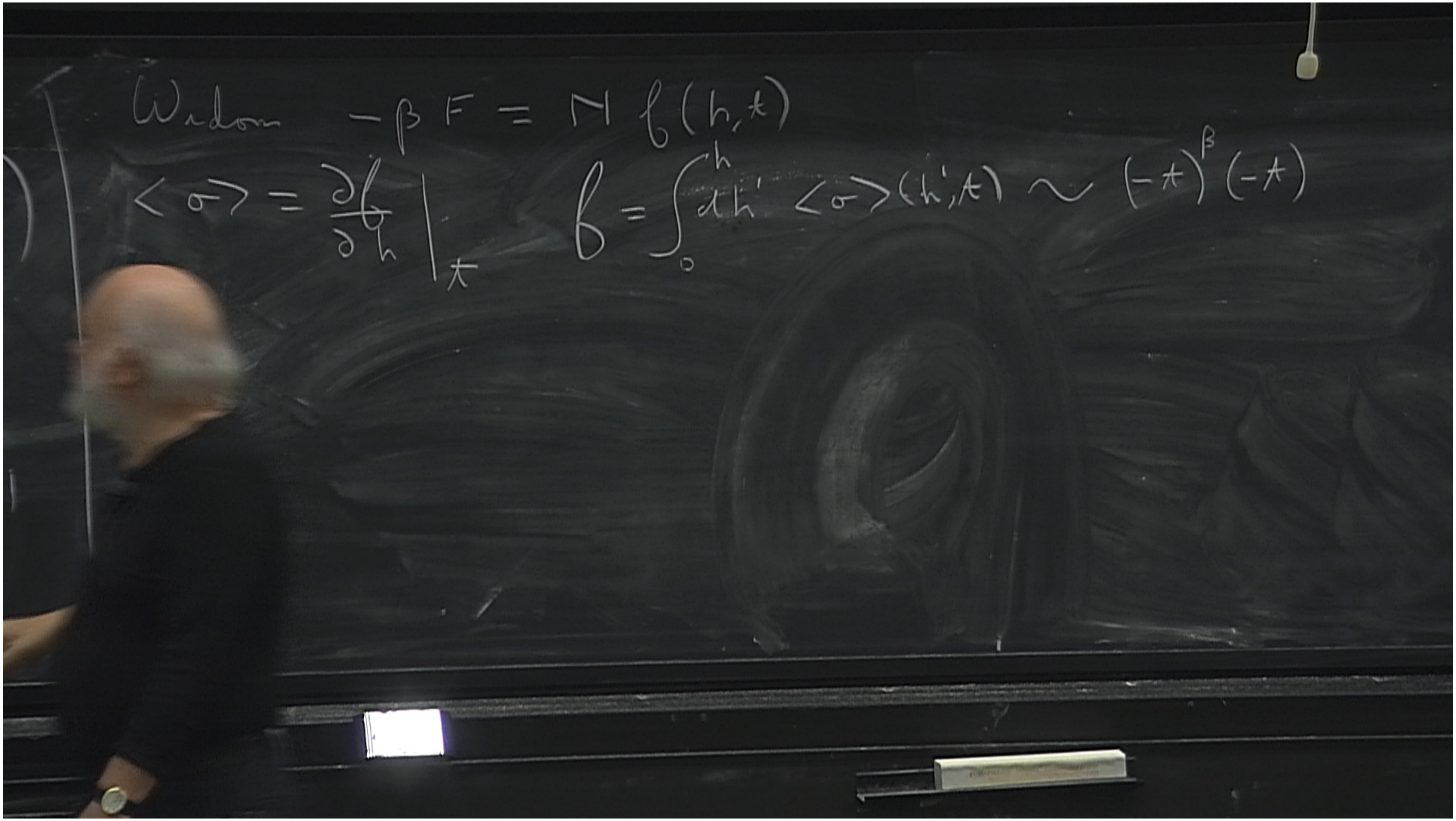

h   t
<σ> = (-t)^β Q((-t)^β / h^{1/δ}) = (-t)^β Q̃((-t)^{βδ} / h)
Q̃(∞) = C
Q̃(∞ (complex)) = 0
t → 0
<σ> = h^{1/δ}   t → 0   Q(x) → 1/x   x << 1
Widom   −βF = N f(h,t)
<σ> = ∂f/∂h

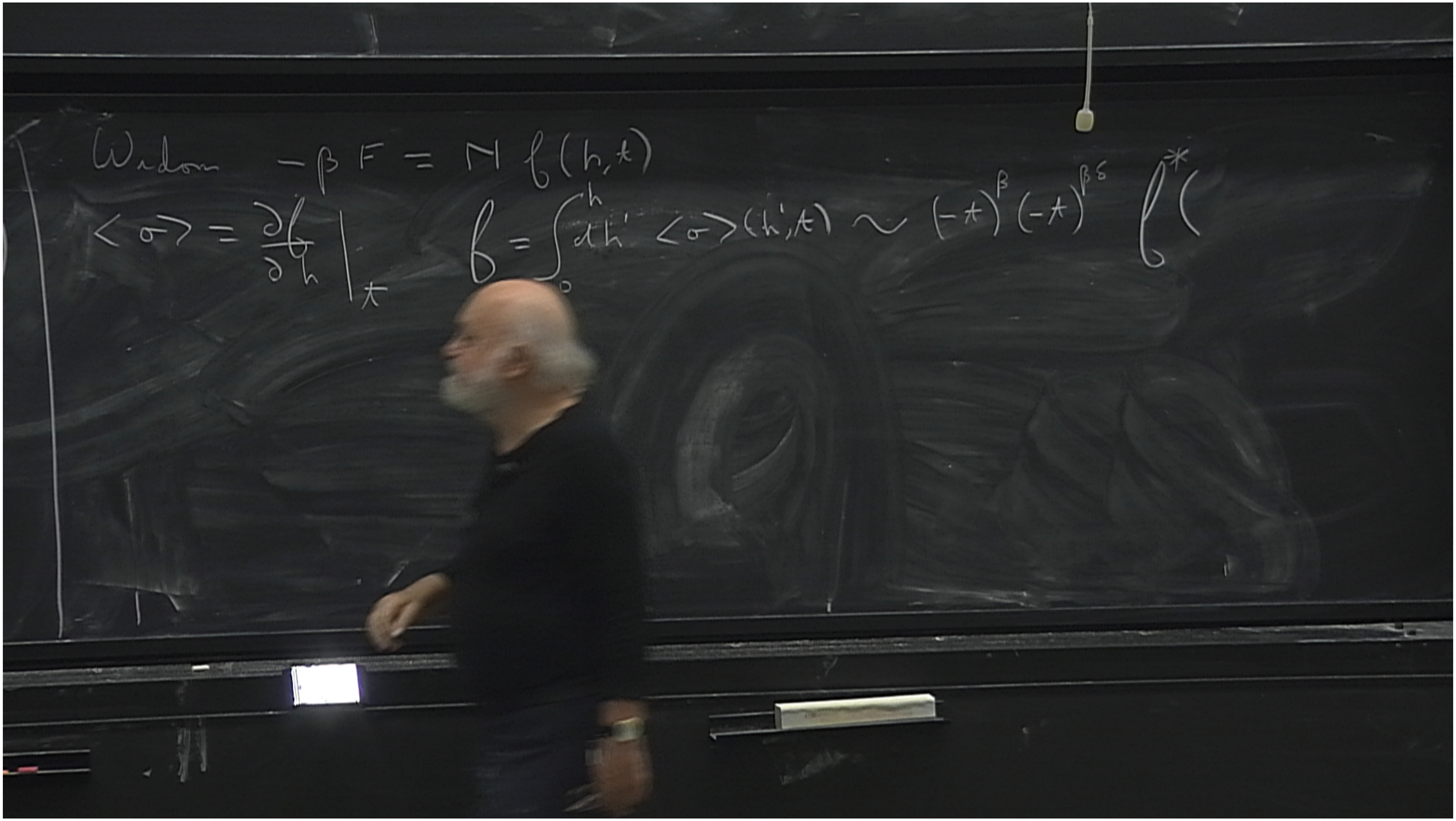


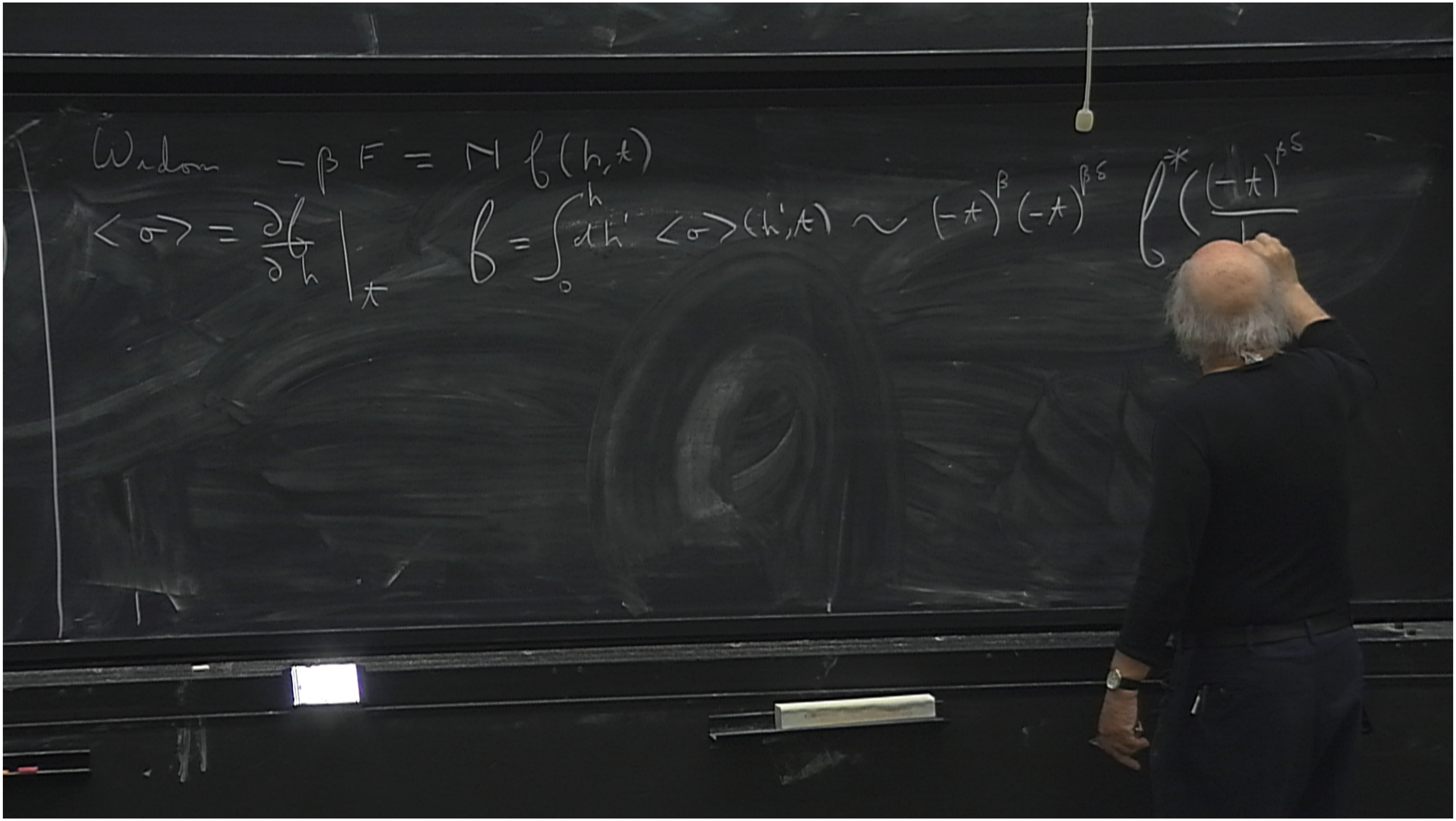
h t
<σ> = (-t)^β Q((-t)^β / h^{1/δ}) = (-t)^β Q̃((-t)^{βδ} / h)
Q̃(∞) = C
Q̃(∞) = 0
t → 0 Q(x) → 1/x x << 1
<σ> = h^{1/δ}
Widom −βF = N f(h,t)
<σ> = ∂f/∂h |t
f = ∫dh' <σ>(h';t)


Widom $-\beta F = N f(h,t)$
$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t$
$f = \int_0^h dh' \langle \sigma \rangle (h',t) \sim$


Widom $-\beta F = N f(h,t)$
$\langle\sigma\rangle = \frac{\partial f}{\partial h}\Big|_t$
$f = \int_0^h dh' \langle\sigma\rangle(h',t) \sim (-t)^\beta$

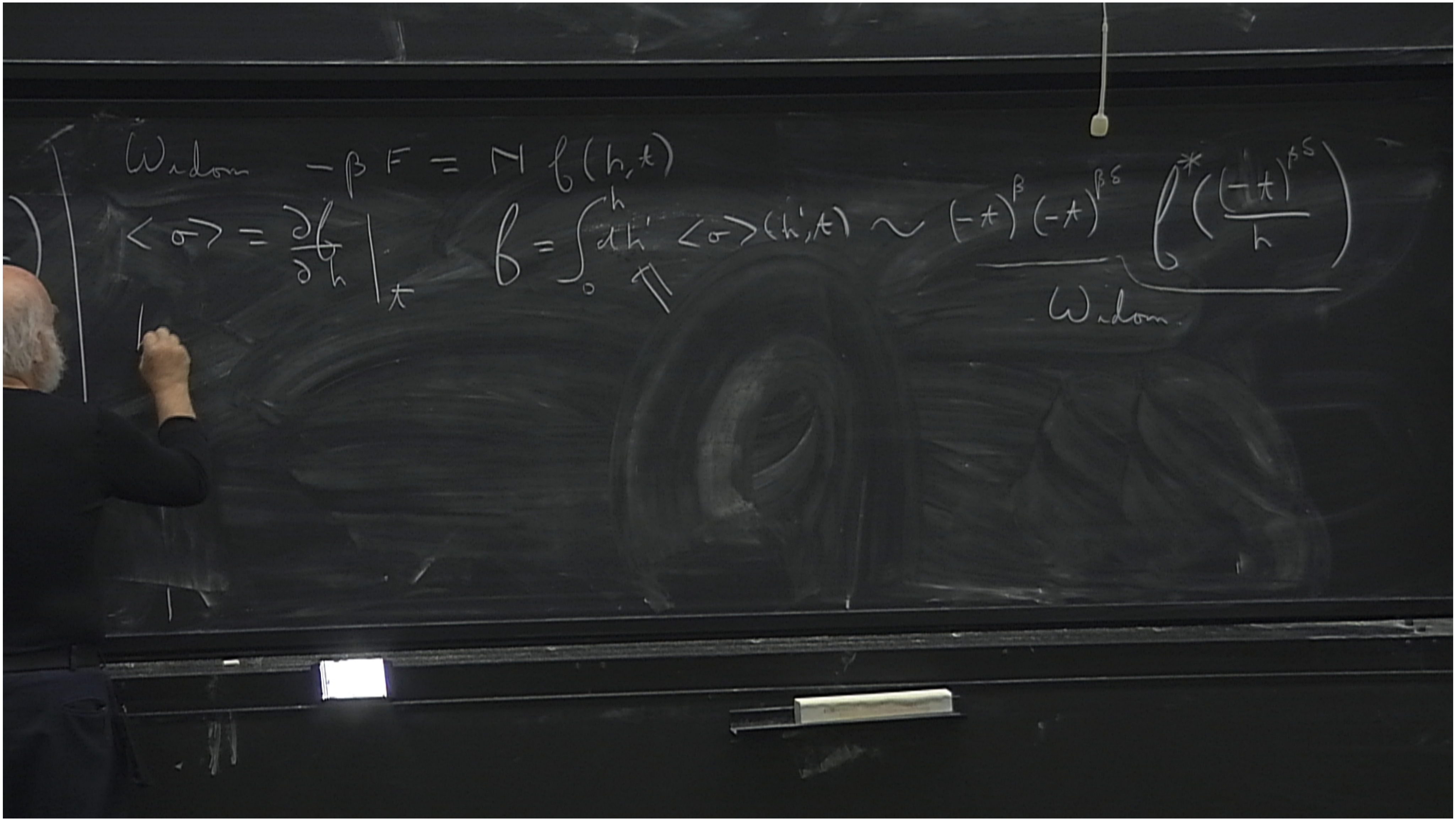

Widom $-\beta F = N f(h,t)$
$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t$
$f = \int_0^h dh' \langle \sigma \rangle (h',t) \sim (-t)^\beta (-t)$

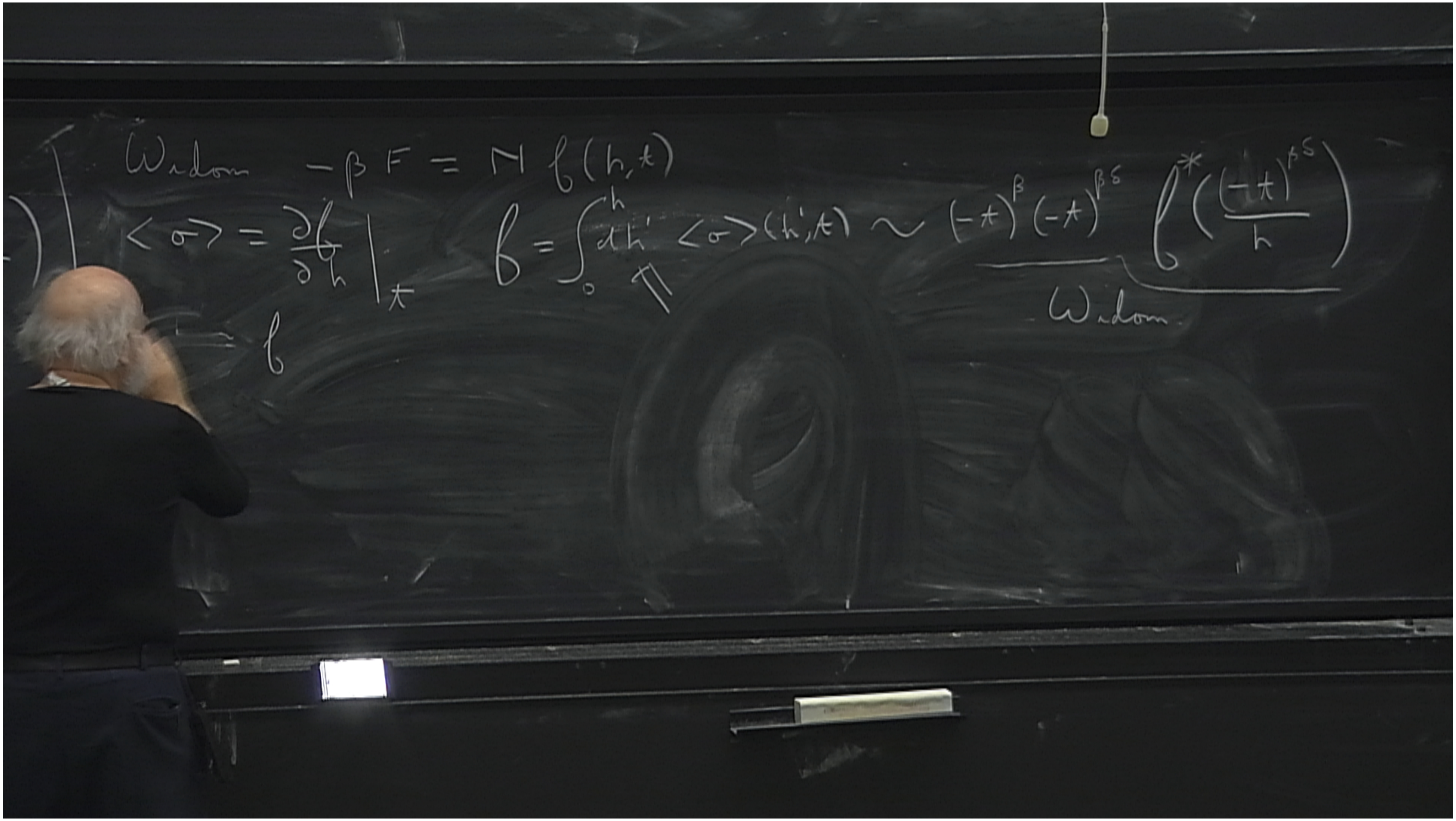

Widom $-\beta F = N f(h,t)$
$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t$ $f = \int_0^h dh' \langle \sigma \rangle (h',t) \sim (-t)^{\beta} (-t)^{\beta\delta} f^*($

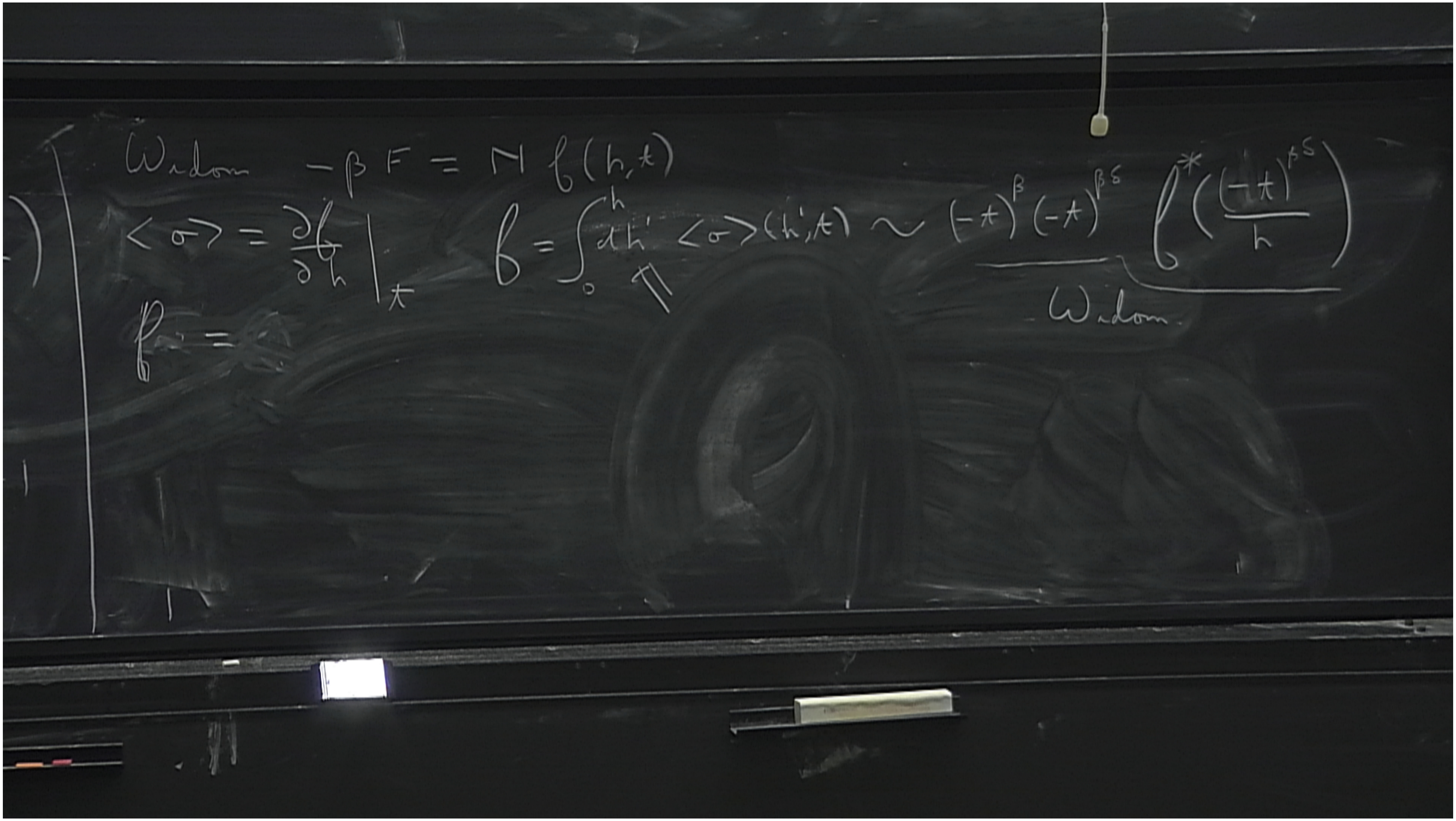

Widom $-\beta F = N f(h,t)$
$\langle\sigma\rangle = \frac{\partial f}{\partial h}\Big|_t$
$f = \int_0^h dh' \langle\sigma\rangle(h',t) \sim (-t)^{\beta}(-t)^{\beta\delta}$
$f^*\left(\frac{(-t)^{\beta\delta}}{h}\right)$

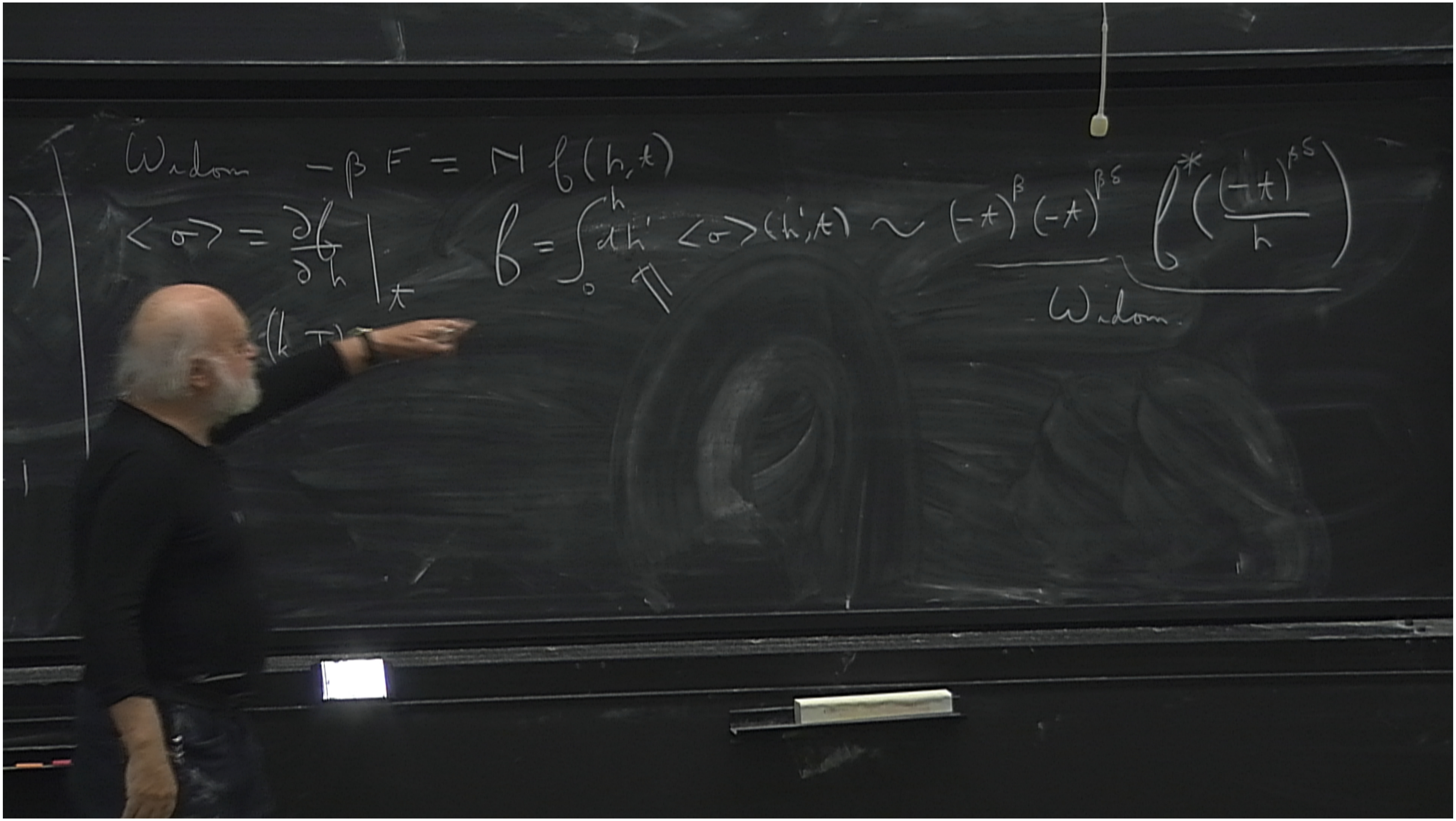
Widom $\quad -\beta F = N f(h,t)$

$$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t \qquad f = \int_0^h dh' \, \langle \sigma \rangle (h', t) \sim \underbrace{(-t)^{\beta} (-t)^{\beta\delta} \, f^*\!\left(\frac{(-t)^{\beta\delta}}{h} \right)}_{\text{Widom}}$$


Widom $-\beta F = N f(h,t)$
$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t$
$f = \int_0^h dh' \langle \sigma \rangle (h',t) \sim (-t)^\beta (-t)^{\beta\delta} f^*\left(\frac{(-t)^{\beta\delta}}{h}\right)$
Widom


Widom $-\beta F = N f(h,t)$
$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t$
$f = \int_0^h dh' \langle \sigma \rangle (h',t) \sim (-t)^{\beta} (-t)^{\beta\delta} f^*\left(\frac{(-t)^{\beta\delta}}{h}\right)$
Widom
f


Widom $-\beta F = N f(h,t)$
$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t$
$f = \int_0^h dh' \langle \sigma \rangle (h',t) \sim (-t)^{\beta} (-t)^{\beta\delta} f^*\left(\frac{(-t)^{\beta\delta}}{h}\right)$
Widom


Widom −βF = N f(h,t)
⟨σ⟩ = ∂f/∂h |t
f = ∫₀ʰ dh' ⟨σ⟩(h',t) ~ (−t)^β (−t)^βδ f*((−t)^βδ / h)
Widom

Widom $\quad -\beta F = N f(h,t)$

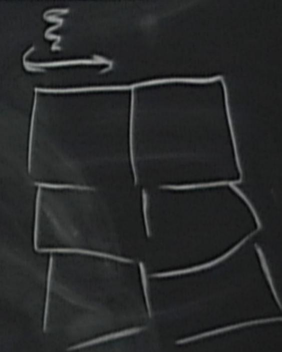
$$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t \qquad f = \int_0^h dh' \langle\sigma\rangle(h',t) \sim \underbrace{(-t)^{\beta}(-t)^{\beta\delta}\, f^*\!\left(\frac{(-t)^{\beta\delta}}{h}\right)}_{\text{Widom}}$$

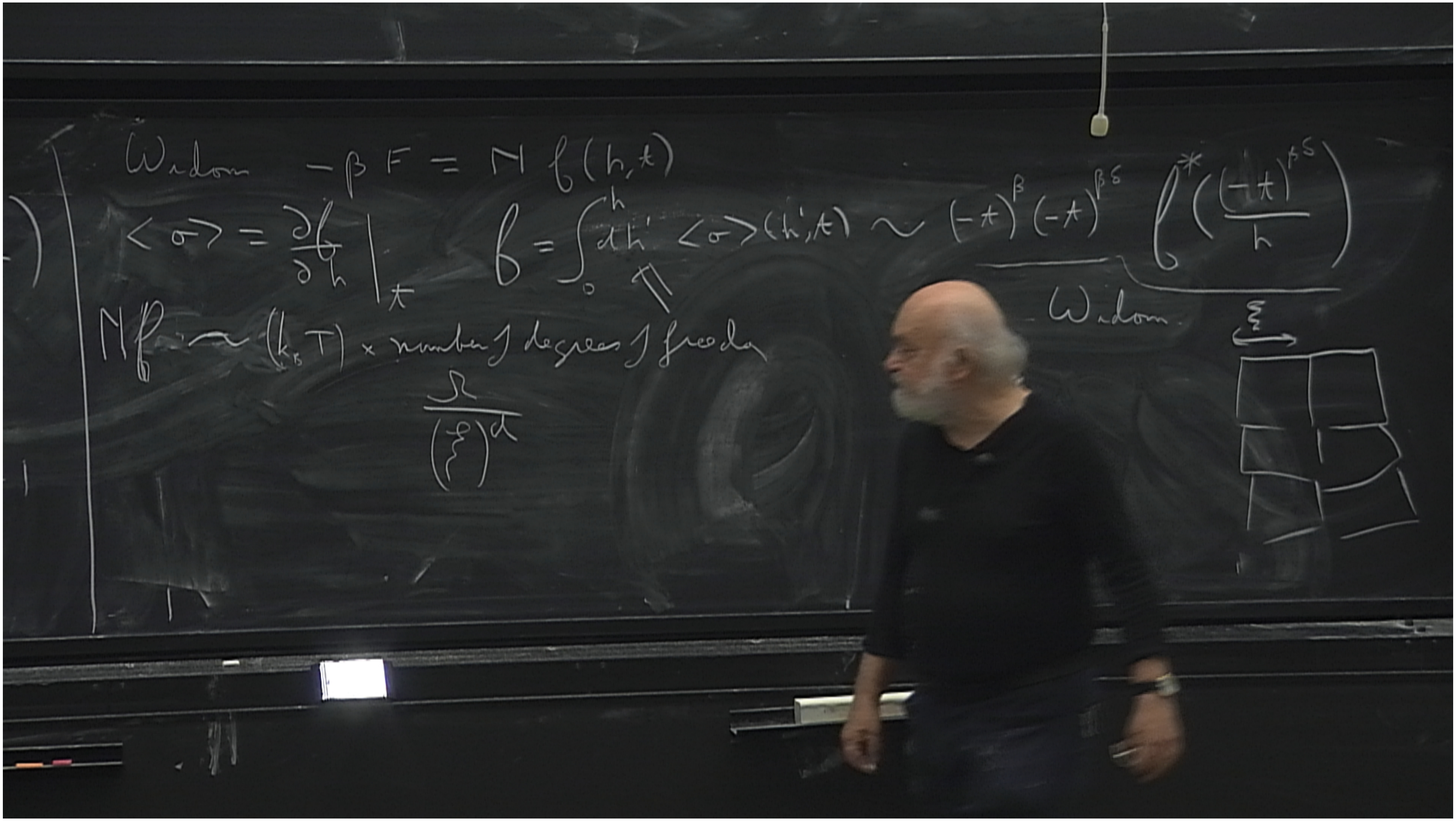
$Nf \sim (k_B T) \times$ number of degrees of freedom

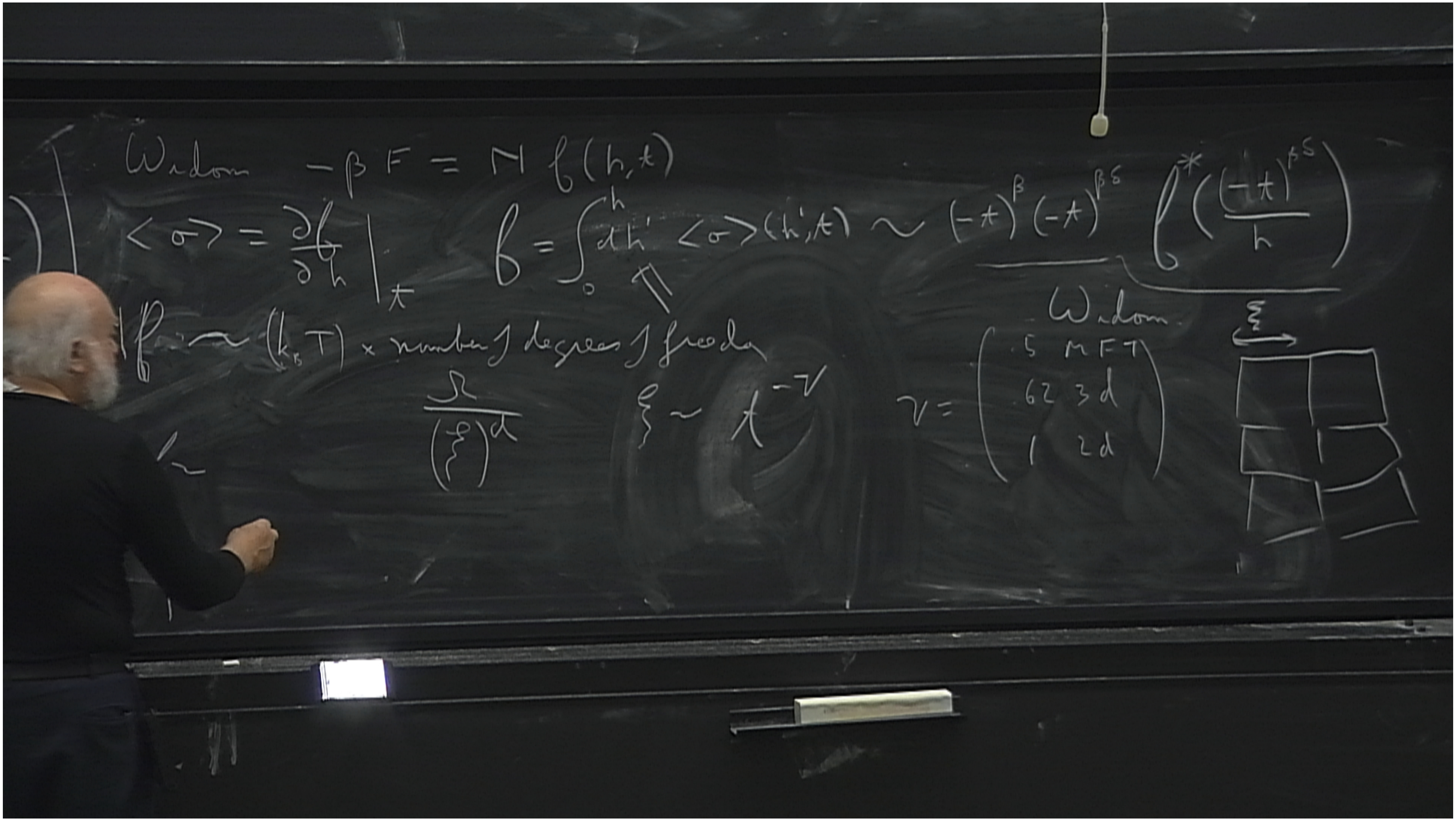
Widom $\quad -\beta F = N f(h,t)$

$$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t \qquad f = \int_0^h dh' \, \langle \sigma \rangle (h',t) \sim \underbrace{(-t)^{\beta} (-t)^{\beta\delta} f^*\left(\frac{(-t)^{\beta\delta}}{h}\right)}_{\text{Widom}}$$

$N f \sim (k_B T) \times$ number of degrees of freedom




Widom −βF = N f(h,t)
⟨σ⟩ = ∂f/∂h |t
f = ∫0h dh' ⟨σ⟩(h',t) ~ (−t)β (−t)βδ f*((−t)βδ/h)
Widom
Nf ~ (kB T) × number of degrees of freedom
Ω/(ξ)d
ξ


Widom −βF = N f(h,t)
<σ> = ∂f/∂h |_t
f = ∫_0^h dh' <σ>(h',t) ~ (−t)^β (−t)^{βδ} f*((−t)^{βδ}/h)
Widom
f ~ (k_B T) × number of degrees of freedom
Ω/(ξ)^d
ξ ~ t^{−ν}
ν =
.5 MFT
.62 3d
1 2d
ξ

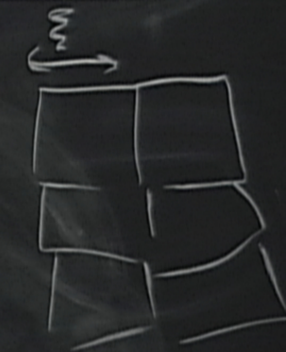
Widom $-\beta F = N f(h,t)$

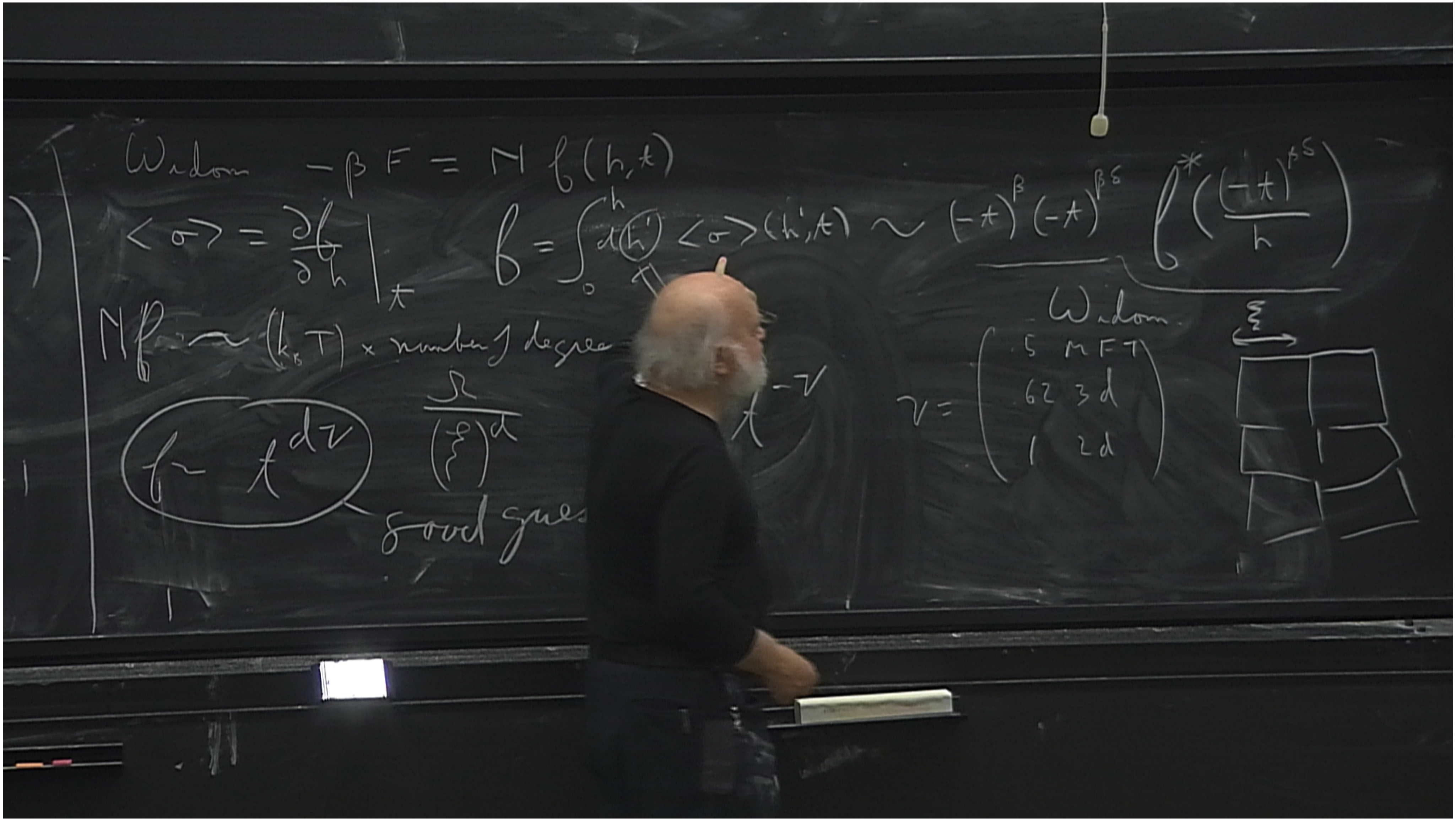
$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_t \qquad f = \int_0^h dh' \langle \sigma \rangle (h',t) \sim (-t)^{\beta} (-t)^{\beta\delta}\, f^*\left(\frac{(-t)^{\beta\delta}}{h}\right)$

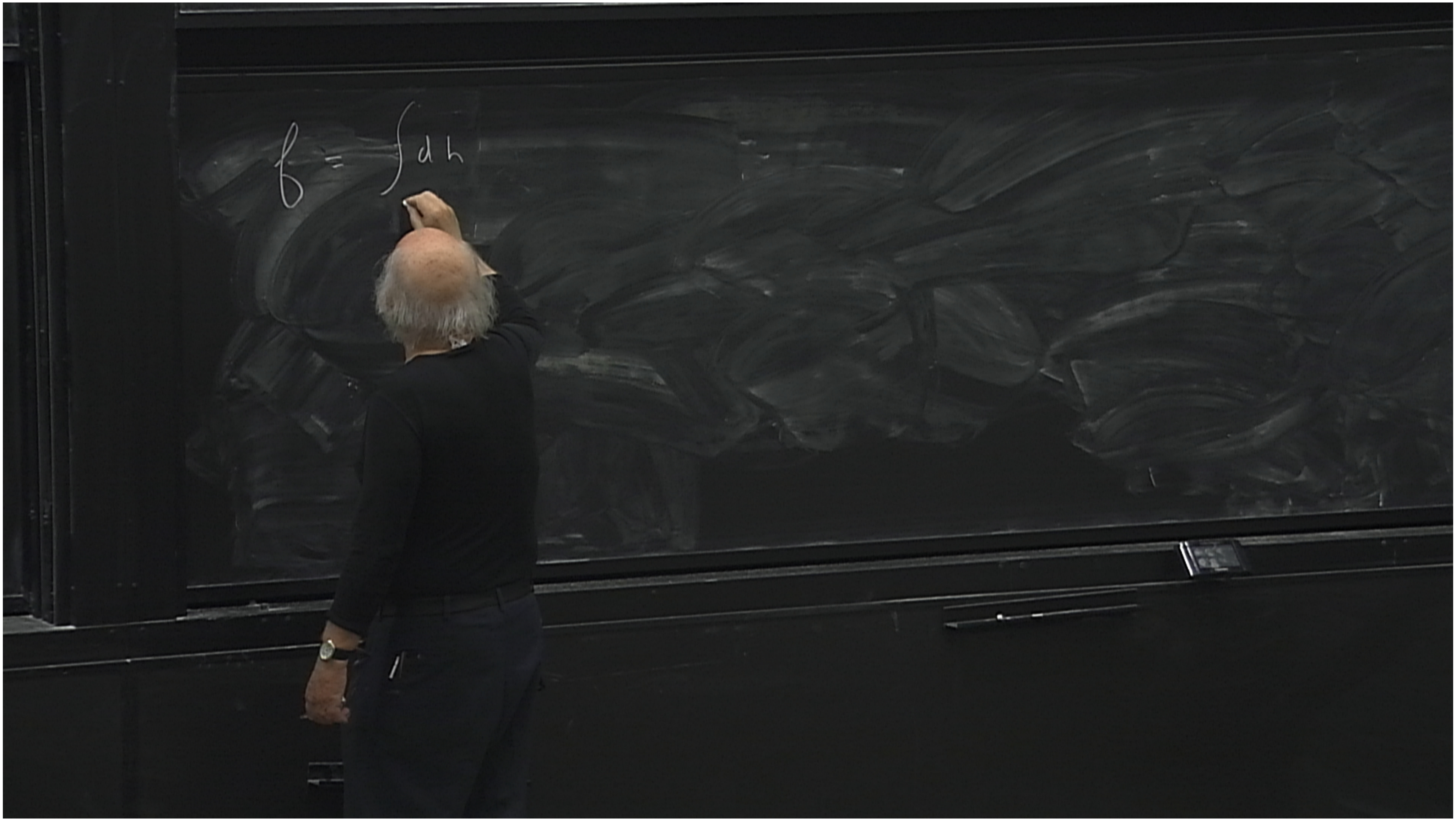
$N f \sim (k_B T) \times$ number of degrees of freedom

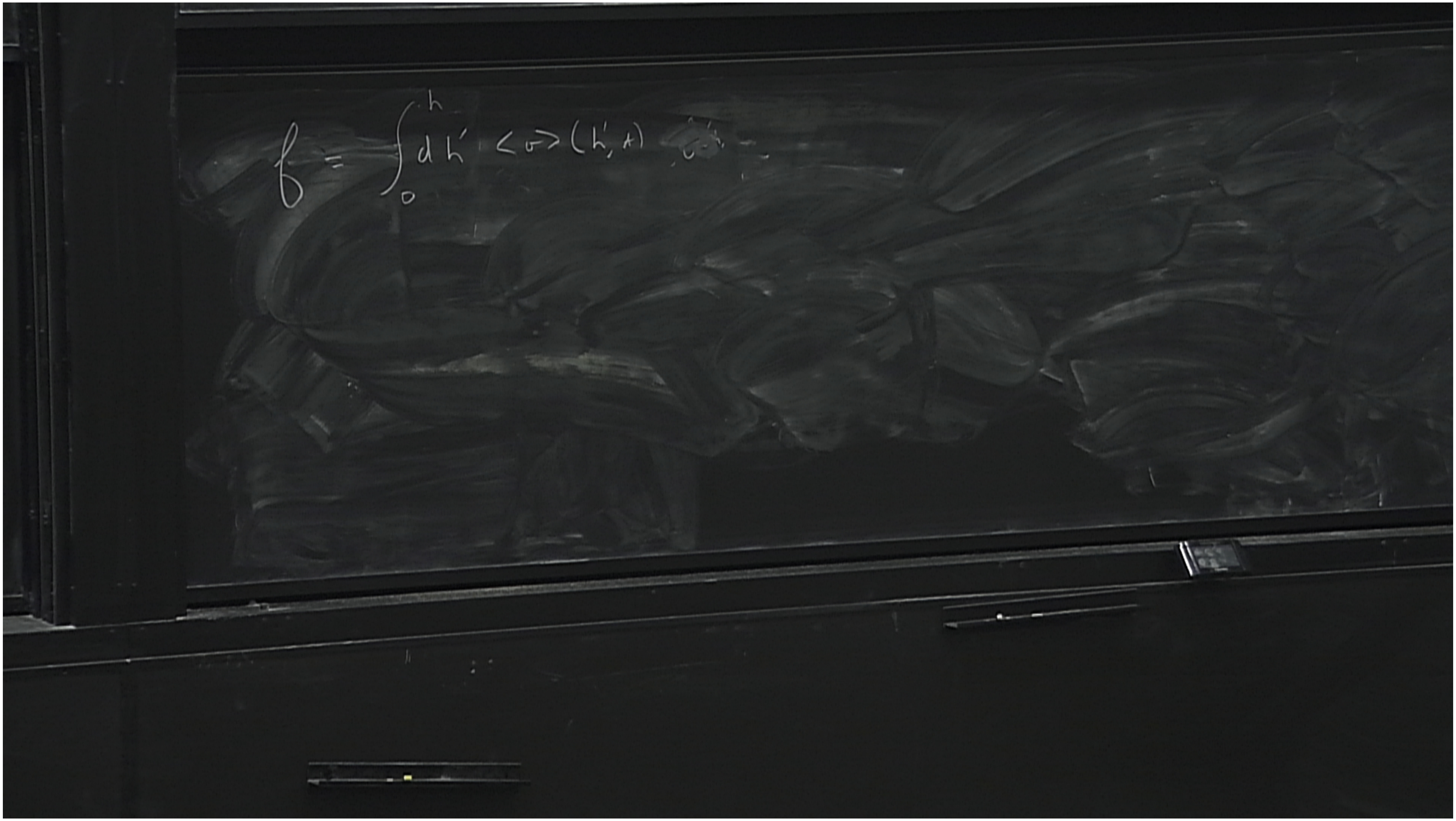
$f \sim t^{d\nu} \qquad \frac{\Omega}{(\xi)^d} \qquad \xi \sim t^{-\nu}$

$$\nu = \begin{array}{cc} \multicolumn{2}{c}{\text{Widom}} \\ \left(\begin{array}{cc} .5 & \text{MFT} \\ .62 & 3d \\ 1 & 2d \end{array}\right) \end{array}$$




Widom −βF = N f(h,t)
<σ> = ∂f/∂h |t
f = ∫ d(h') <σ>(h',t) ~ (−t)^β (−t)^βδ f*((−t)^βδ / h)
Nf ~ (kB T) × number of degree
f ~ t^dν
good gues
Widom
ν =
5 MFT
62 3d
1 2d
ξ


f = ∫ d h


$f = \int_0^h dh' \langle v \rangle (h', t)$

$h \quad t \quad = (-t)^{\beta} \tilde{\tilde{Q}}\left(\frac{h}{(-t)^{\beta\delta}}\right)$

$$\langle \sigma \rangle = (-t)^{\beta} Q\left(\frac{(-t)^{\beta}}{h^{1/\delta}}\right) = (-t)^{\beta} \tilde{Q}\left(\frac{(-t)^{\beta\delta}}{h}\right)$$

$$\tilde{Q}(\infty) = C$$

$$\tilde{Q}(\infty \text{ complex}) = 0$$

$t \to 0$

$$\langle \sigma \rangle = h^{1/\delta} \quad t \to 0 \quad Q(x) \to \frac{1}{x} \quad x \ll 1$$

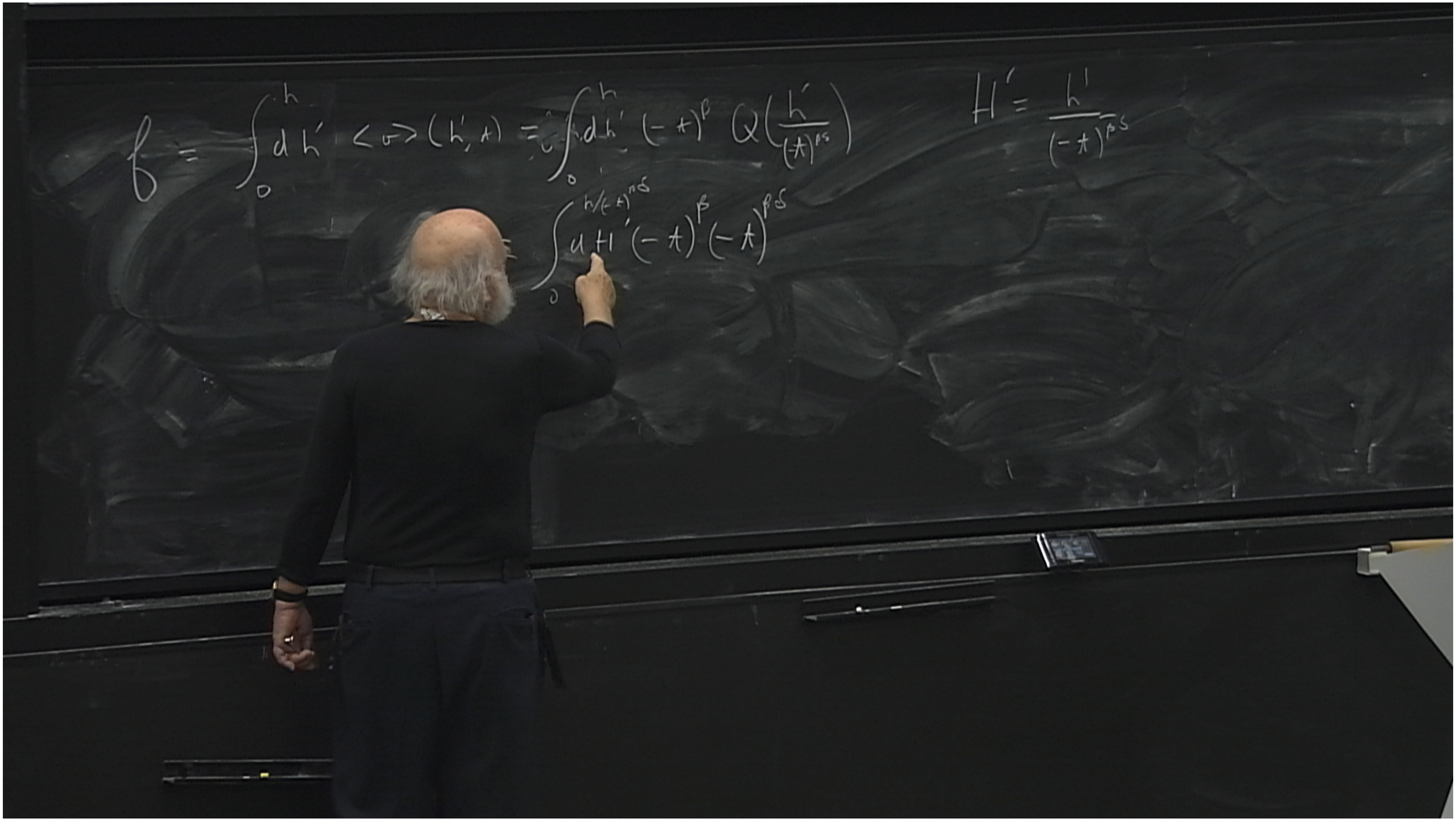
Widom $-\beta F =$

$$\langle \sigma \rangle = \frac{\partial f}{\partial h}\Big|_{t}$$

$f \sim (k_B T) \times$ numb

$f \sim t^{d\nu}$

good

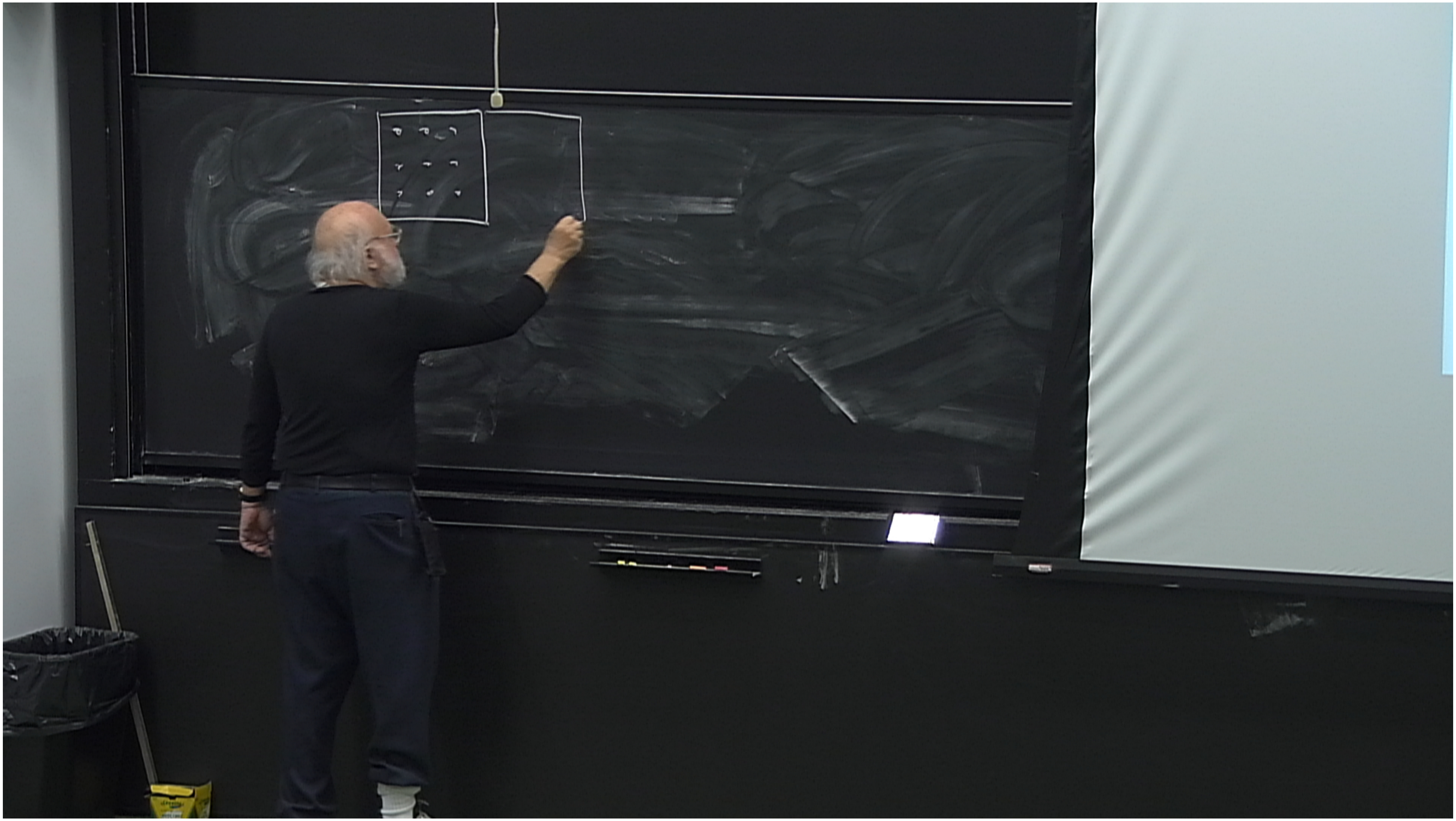

$f = \int_0^h dh' \langle\sigma\rangle(h', t) = \int_0^h dh' (-t)^\beta Q\left(\frac{h'}{(-t)^{\beta\delta}}\right)$
$H' = \frac{h'}{(-t)^{\beta\delta}}$
$= \int_0^{h/(-t)^{\beta\delta}} dH' (-t)^\beta (-t)^{\beta\delta}$

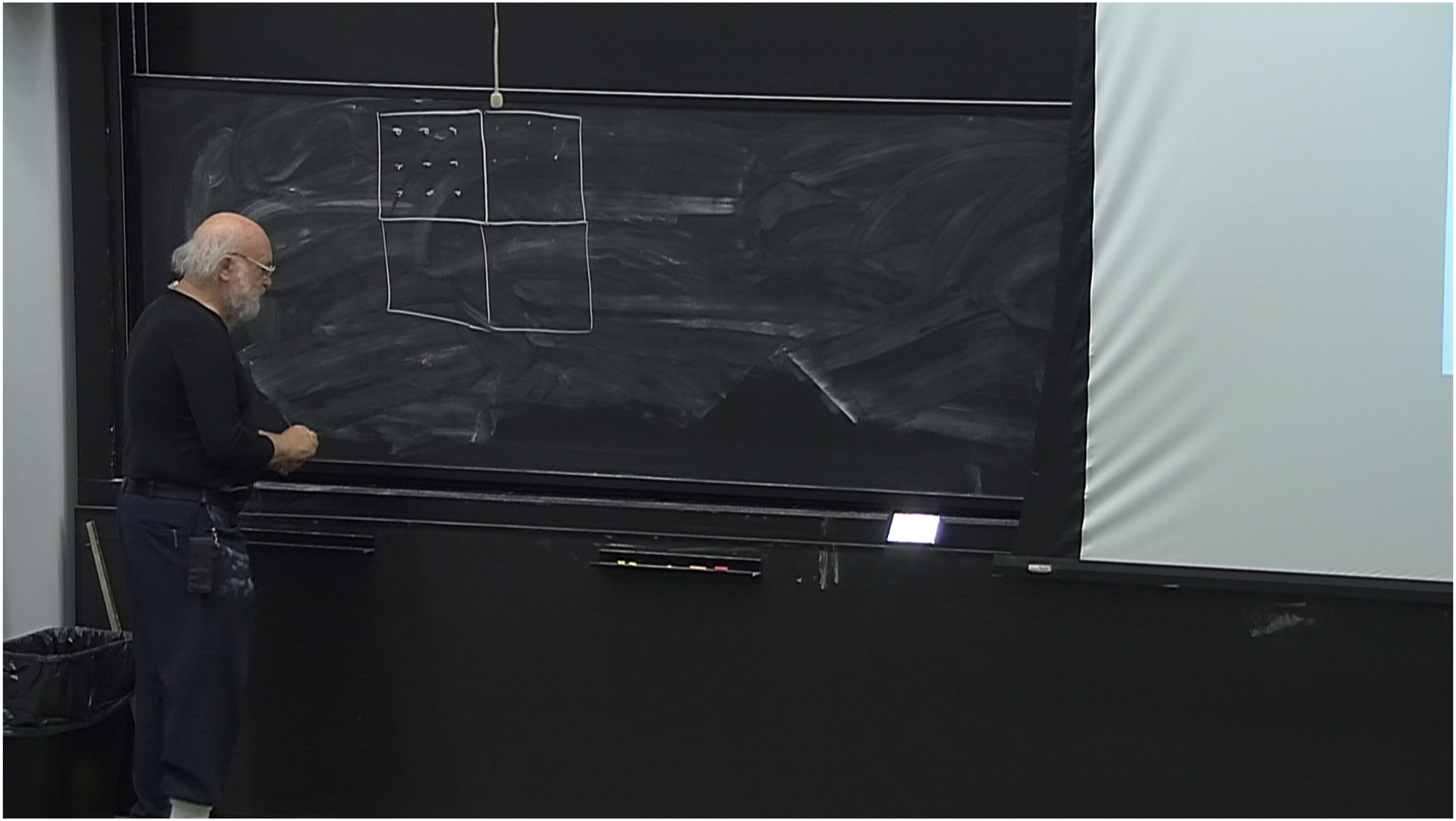
$$f = \int_0^h dh' \, \langle \sigma \rangle (h', t) = \int_0^h dh' \, (-t)^{\beta} \, \tilde{\tilde{Q}}\left(\frac{h'}{(-t)^{\beta\delta}}\right)$$

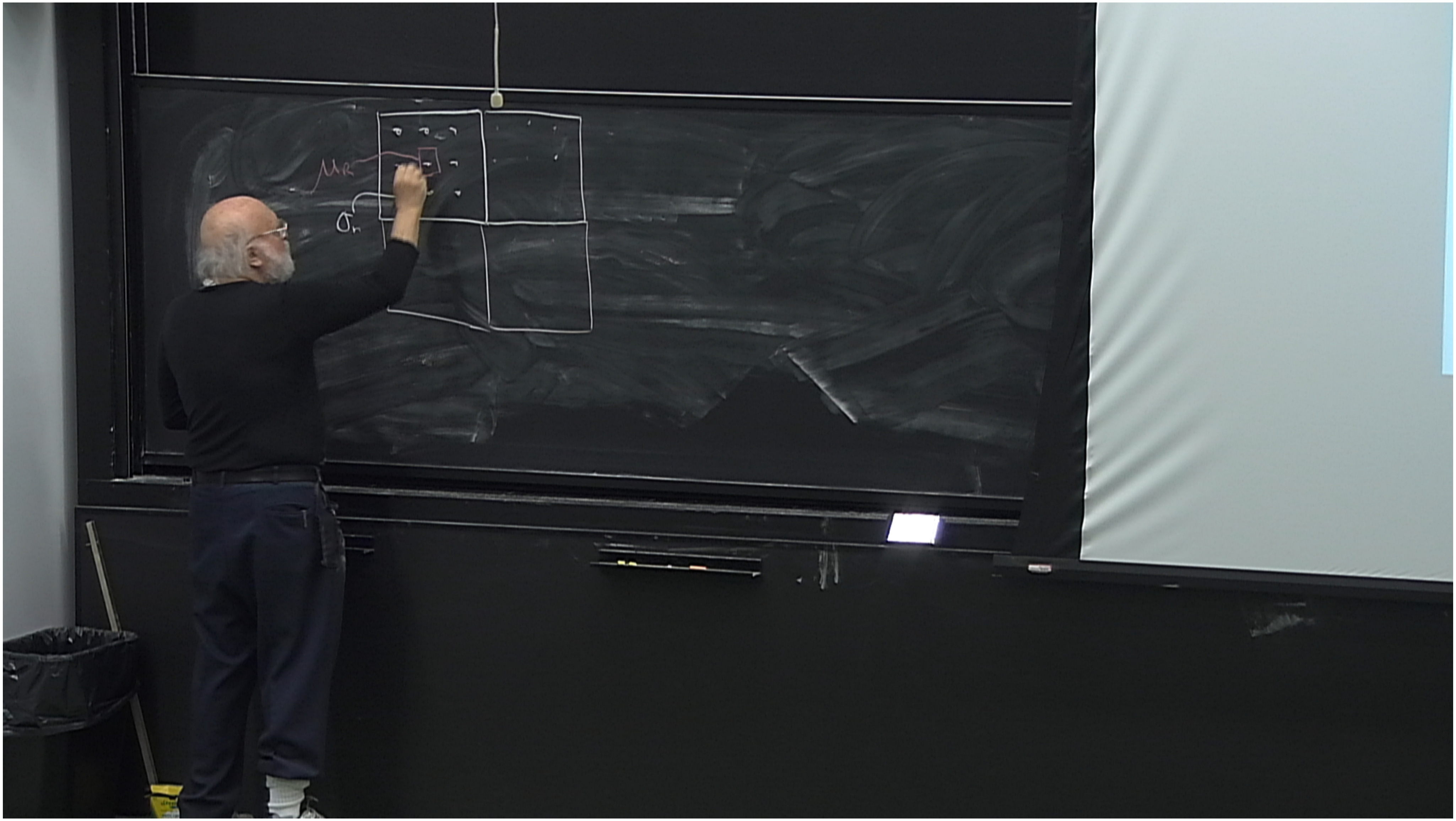
$$H' = \frac{h'}{(-t)^{\beta\delta}}$$

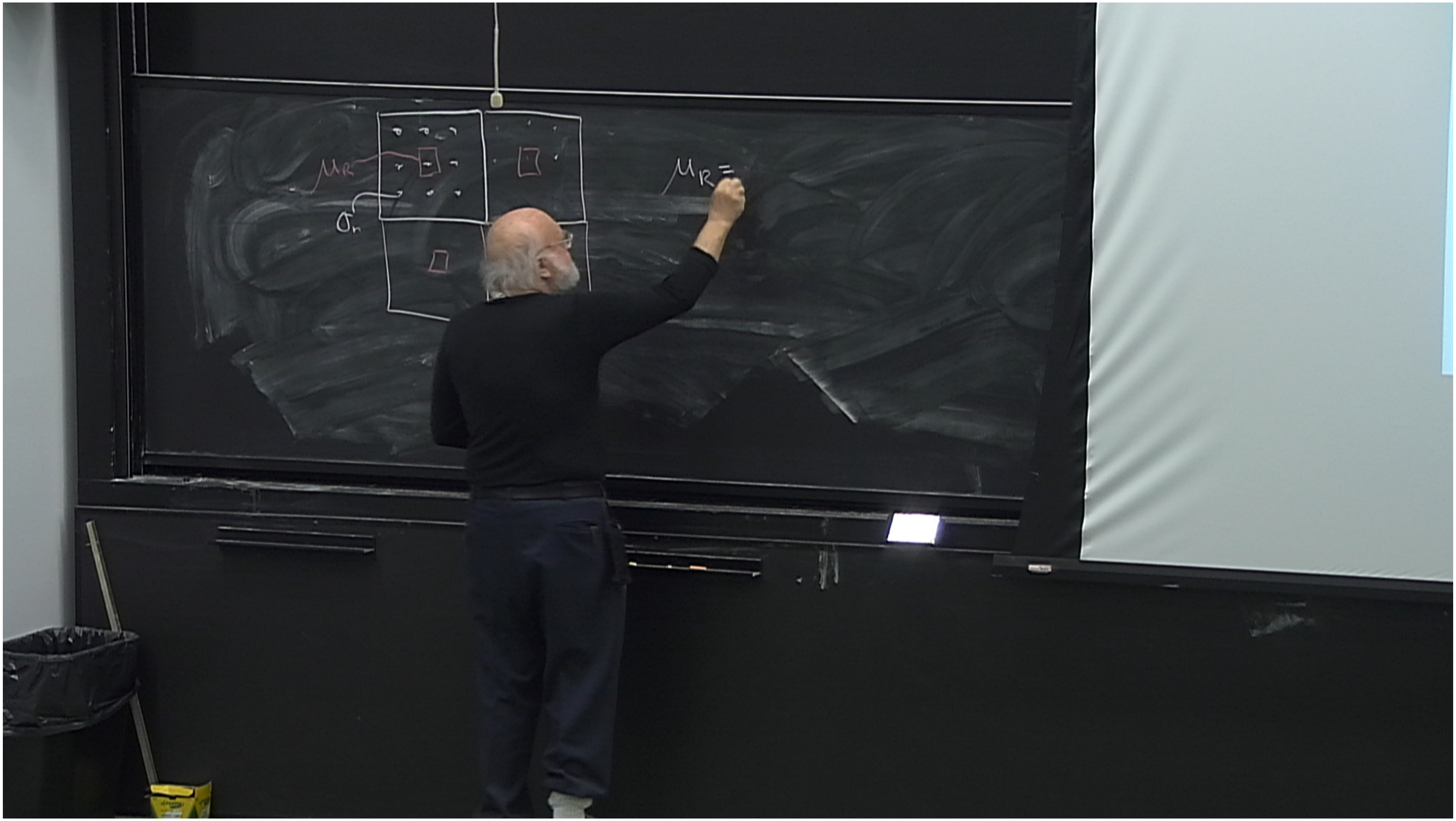
$$= \int_0^{h/(-t)^{\beta\delta}} dH' \, (-t)^{\beta} (-t)^{\beta\delta} \, \tilde{\tilde{Q}}(H')$$

$$\tilde{\tilde{Q}} \sim 1$$

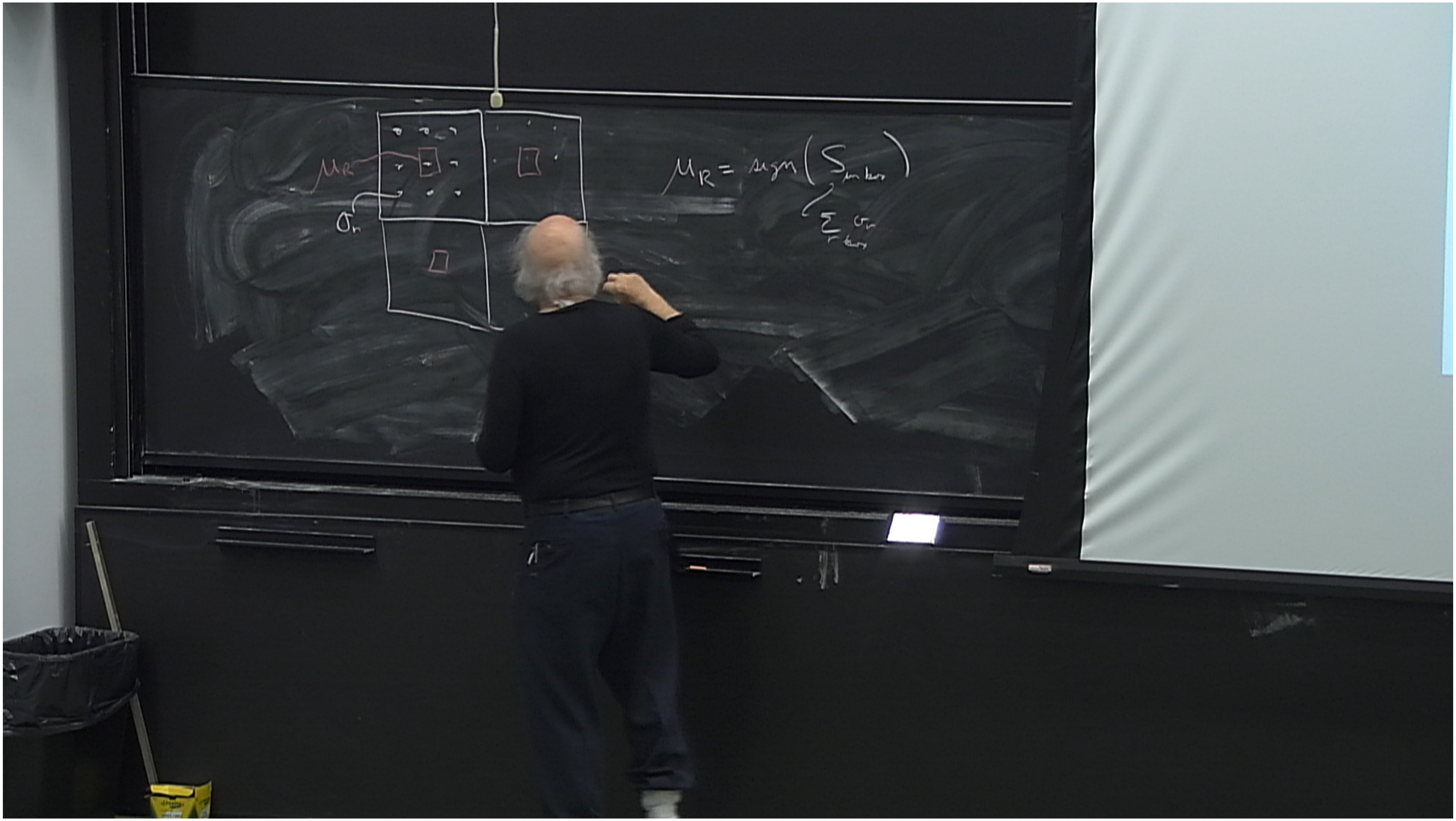


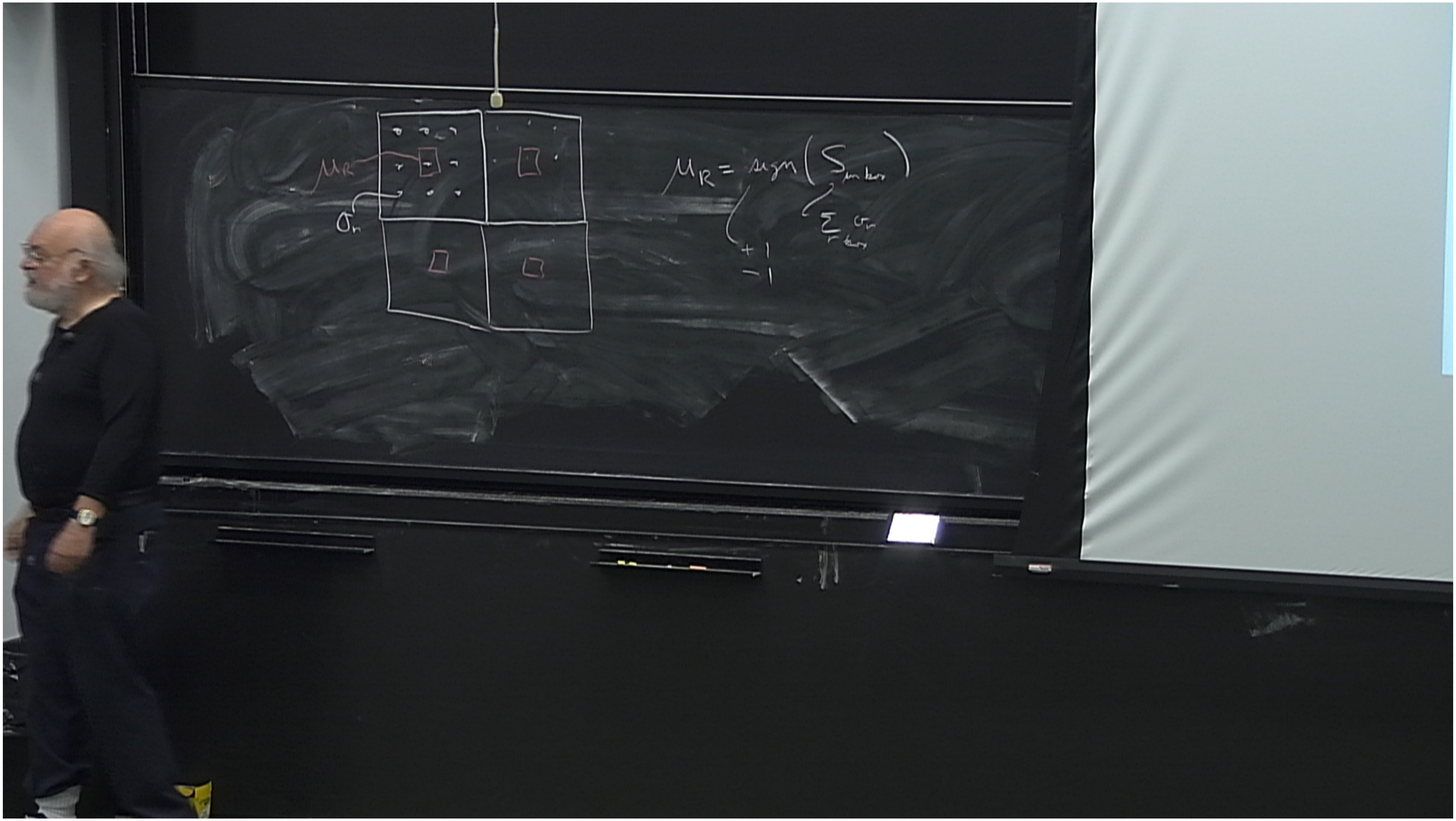




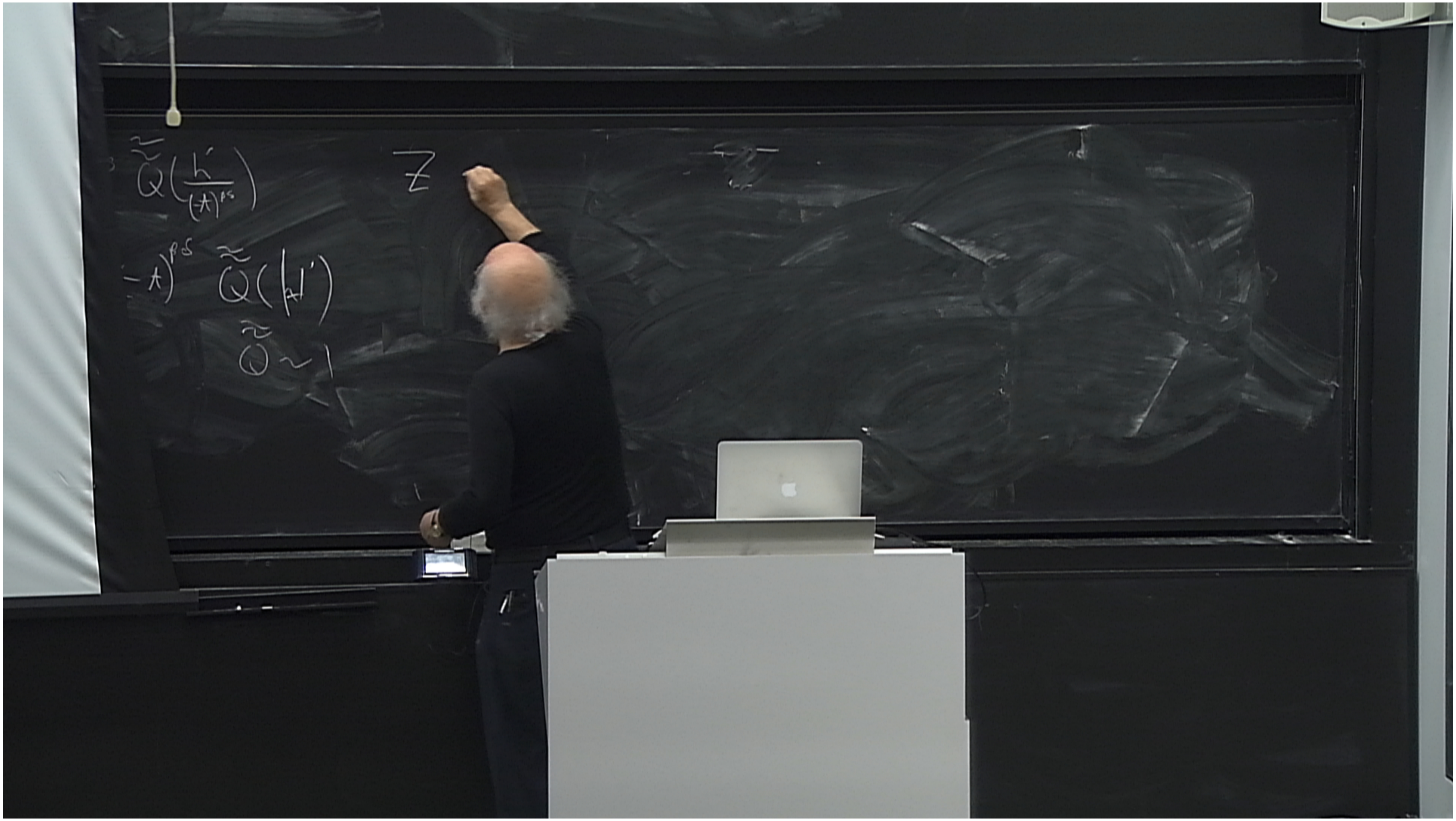


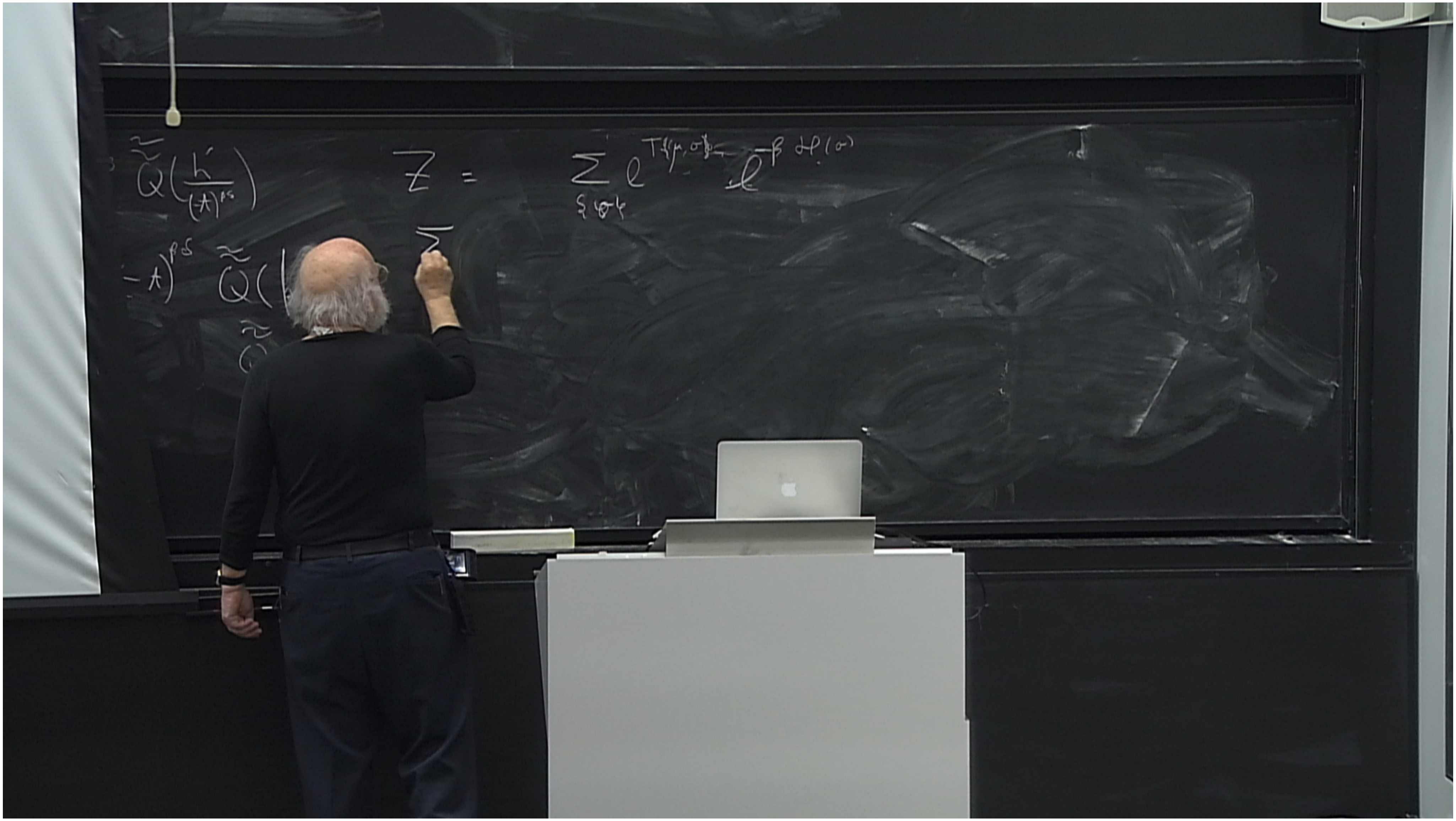
μ_R
σ_n
$\mu_R =$

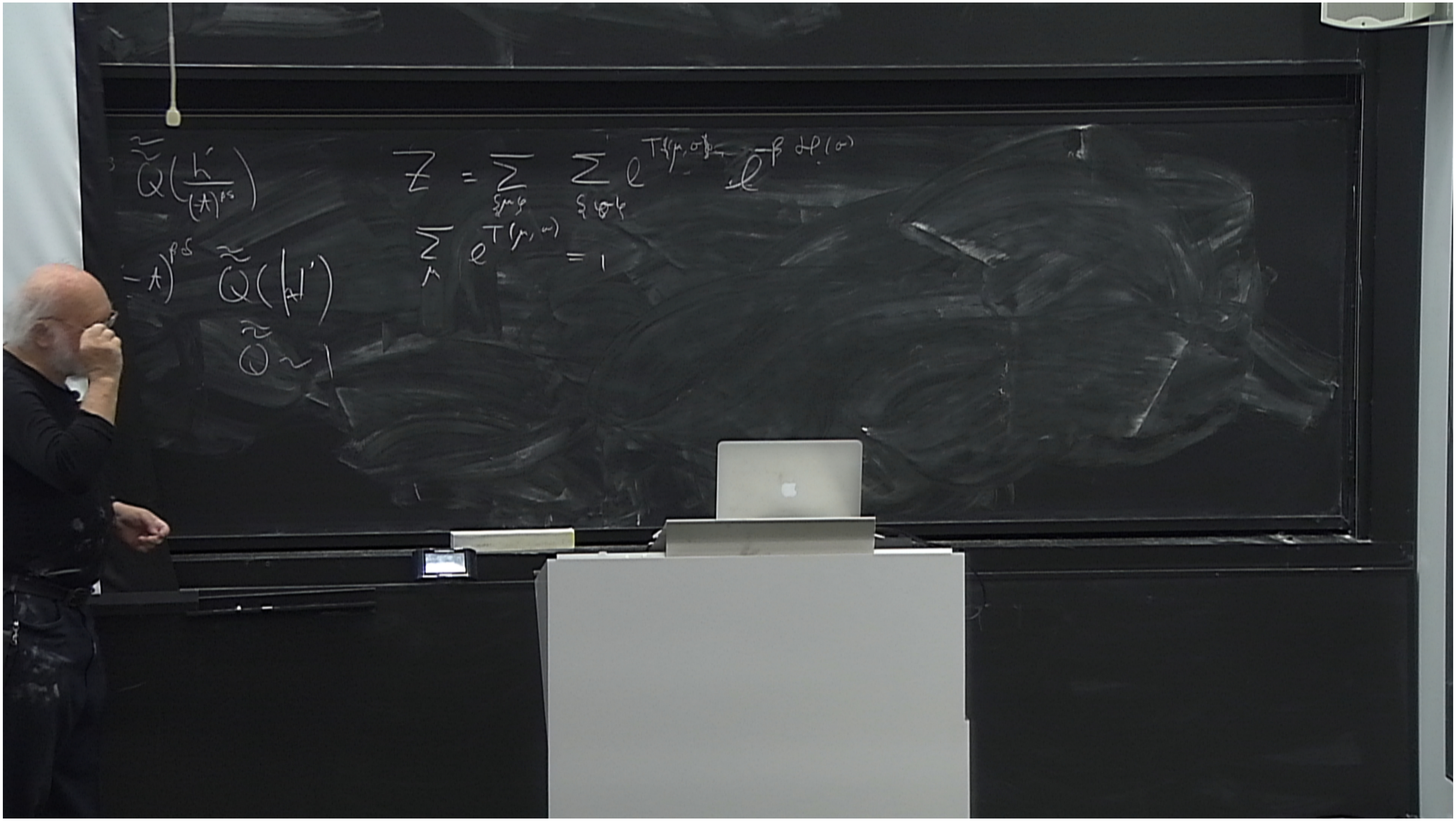


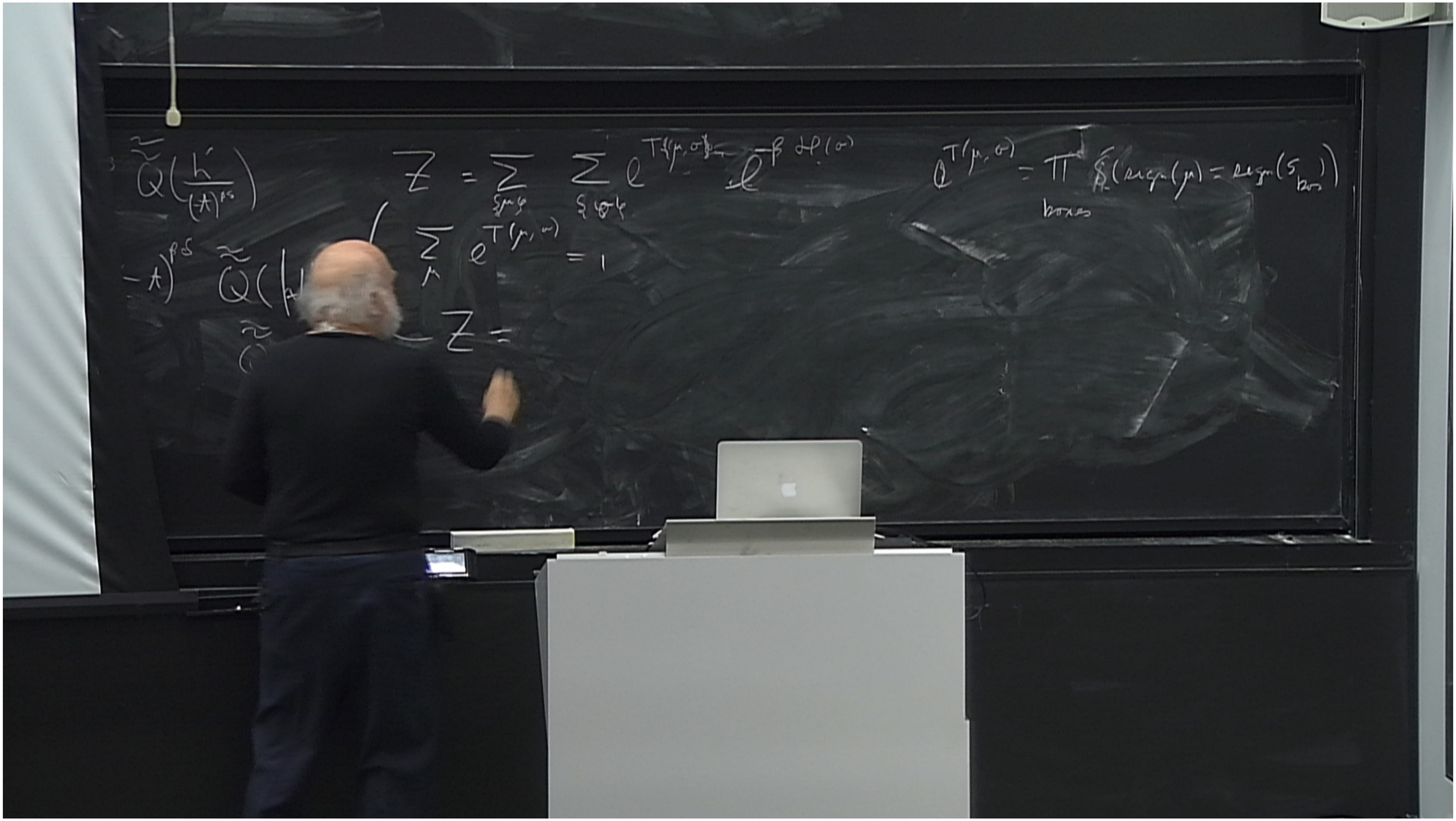


μR
σr
μR = sign (S)
+1
−1






Q̃(h'/(-A)^{ps})
(-A)^{ps} Q̃(h')
Q̃ ~ 1
Z = Σ Σ e^{T{μ,σ}} e^{-β H(σ)}
Σ_μ e^{T(μ,σ)} = 1



$$\tilde{\tilde{Q}}\left(\frac{h'}{(-\hbar)^{\beta s}}\right)$$

$$Z = \sum_{\{\mu\}} \sum_{\{\sigma\}} e^{T\{\mu,\sigma\}} e^{-\beta \mathcal{H}(\sigma)}$$

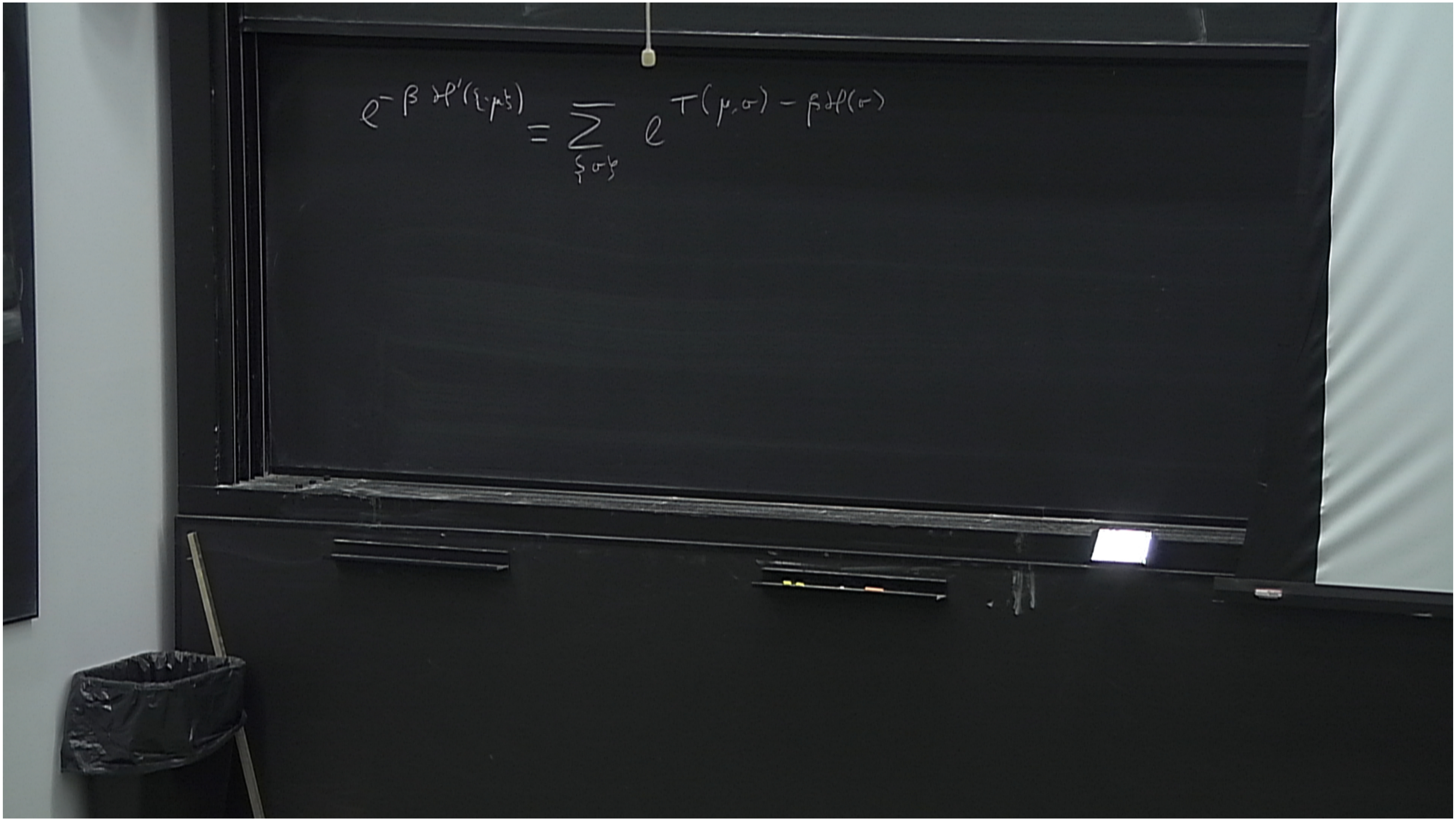
$$e^{T(\mu,\sigma)} = \prod_{\text{boxes}} \delta(\mathrm{sign}(\mu) = \mathrm{sign}(\sigma_{\text{box}}))$$

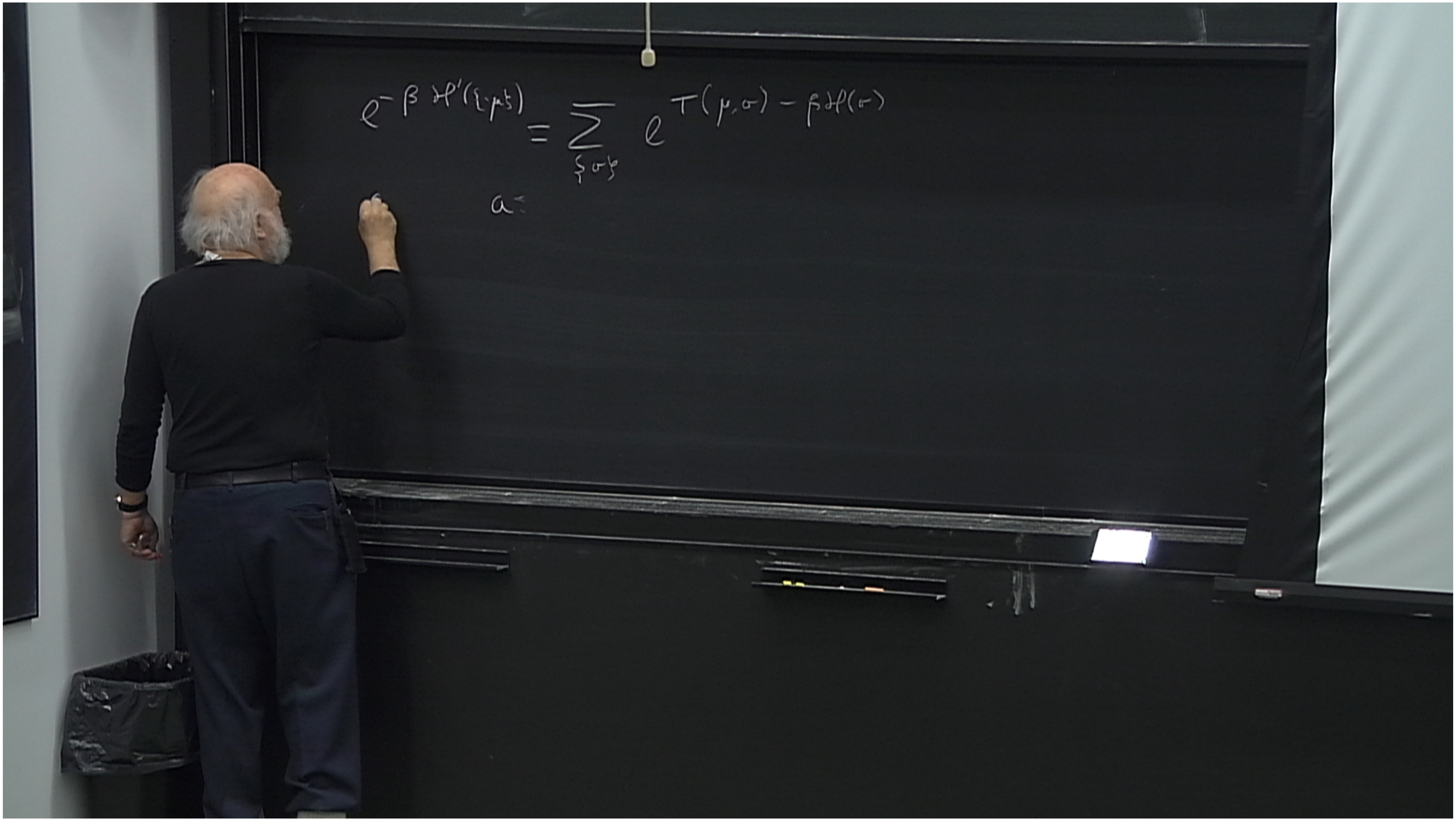
$$(-\hbar)^{\beta s} \tilde{\tilde{Q}}(\hbar')$$

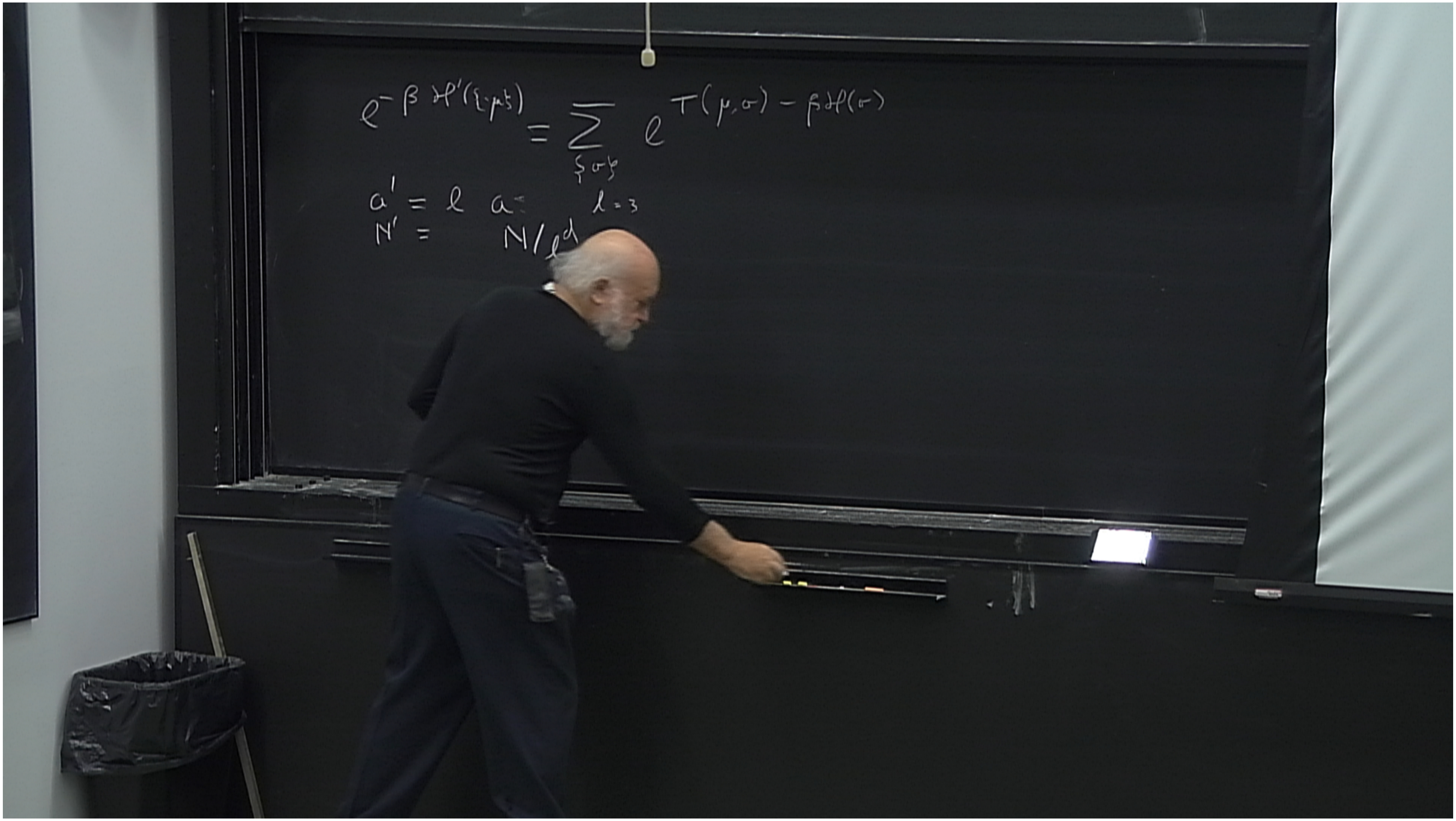
$$\sum_{\mu} e^{T(\mu,\sigma)} = 1$$

$$Z = \sum_{\{\mu\}} e^{-\beta \mathcal{H}'(\mu)}$$

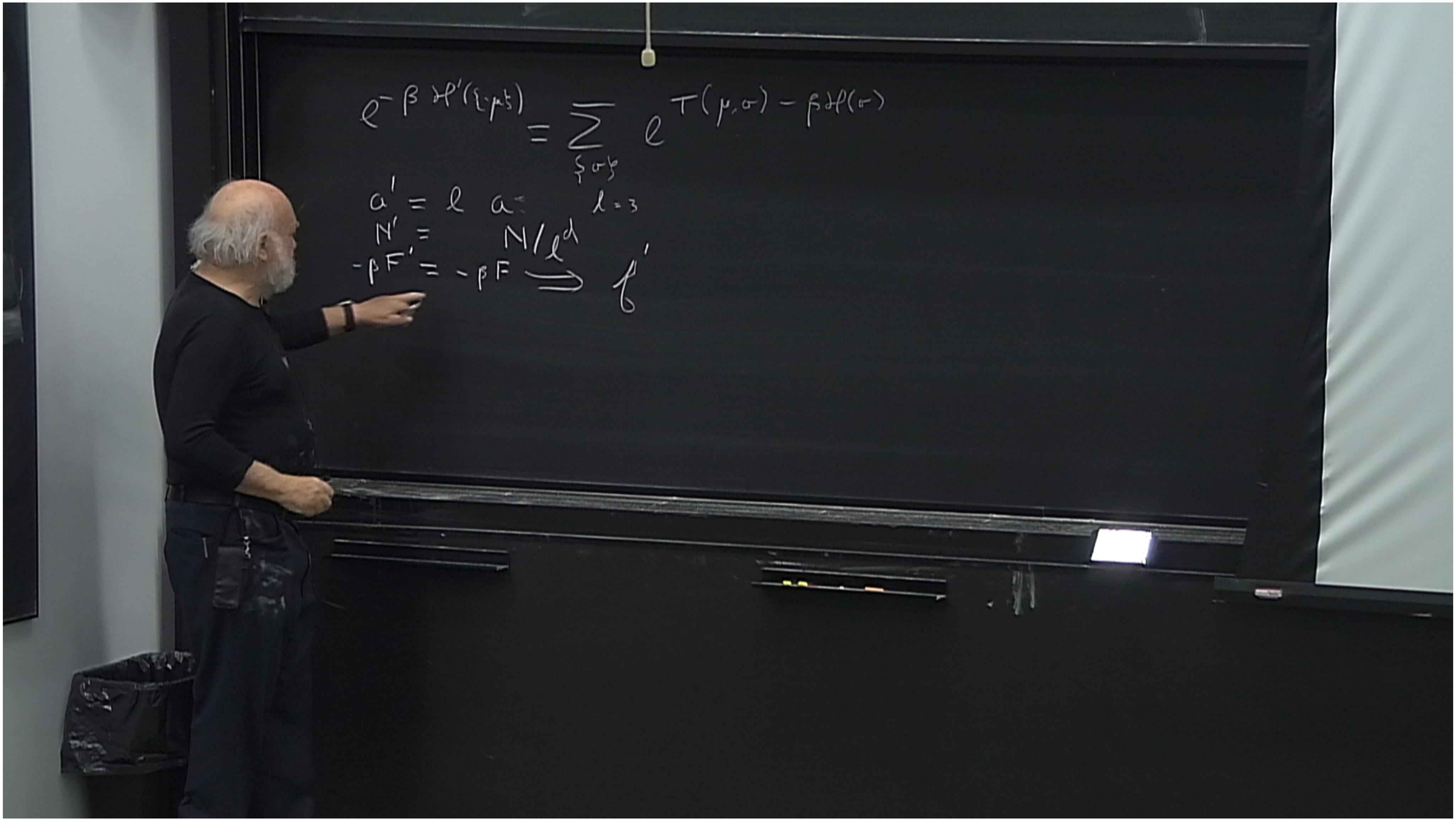
$$\tilde{\tilde{Q}} \sim 1$$

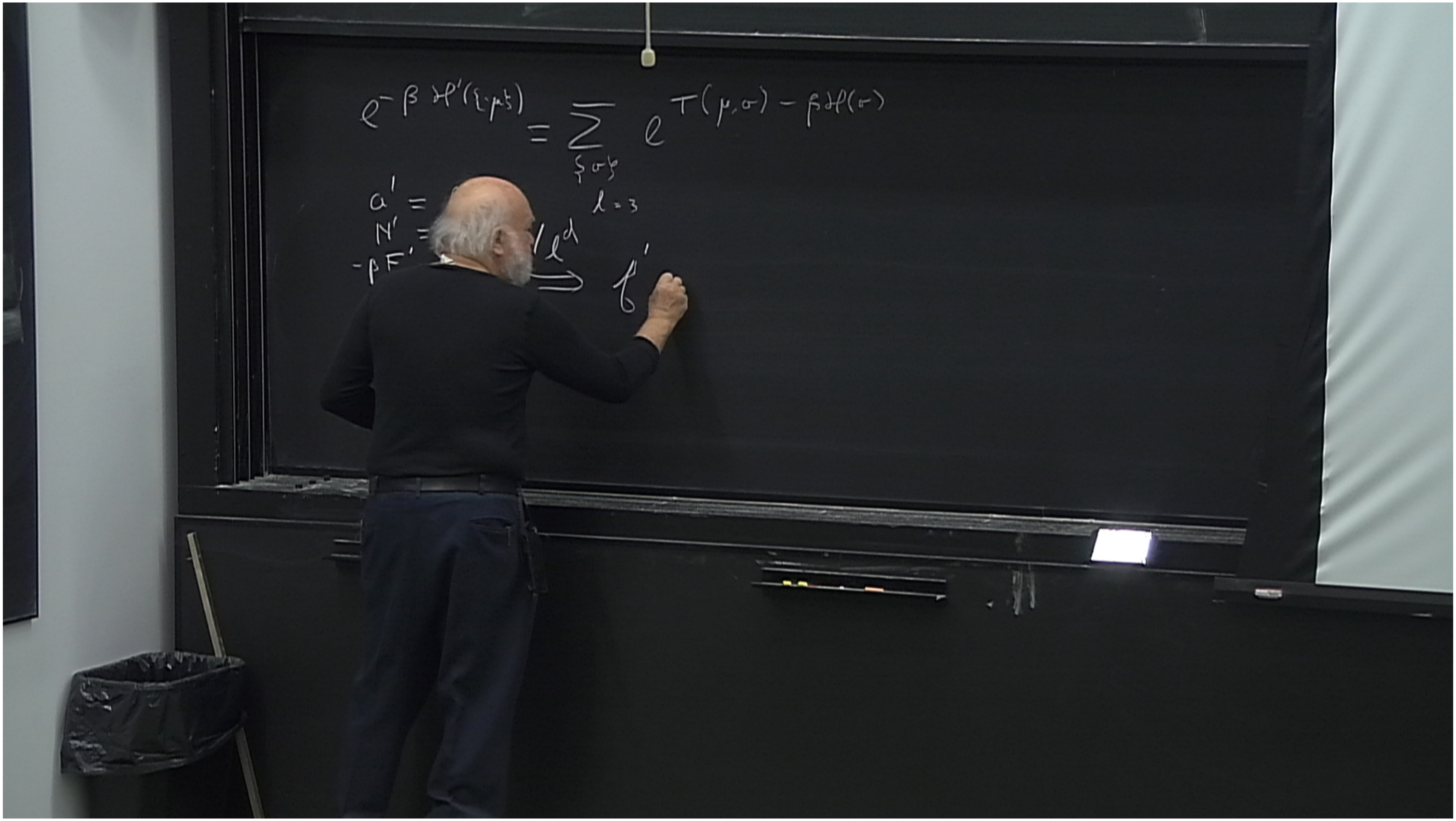

$e^{-\beta \mathcal{H}'(\{\mu\})} = \sum_{\{\sigma\}} e^{T(\mu,\sigma) - \beta \mathcal{H}(\sigma)}$

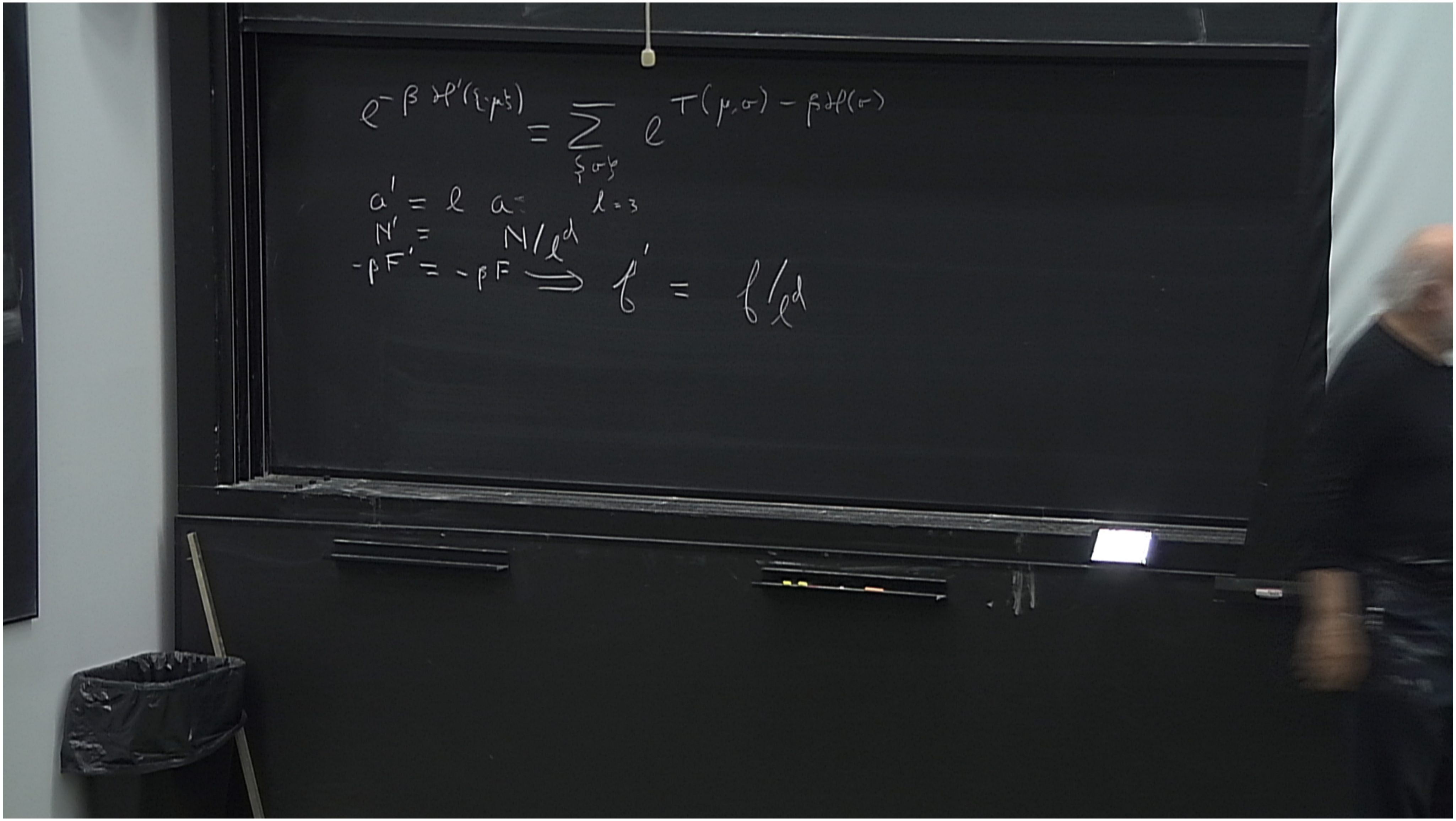

$e^{-\beta \mathcal{H}'(\{\mu\})} = \sum_{\{\sigma\}} e^{T(\mu,\sigma) - \beta \mathcal{H}(\sigma)}$
$a =$

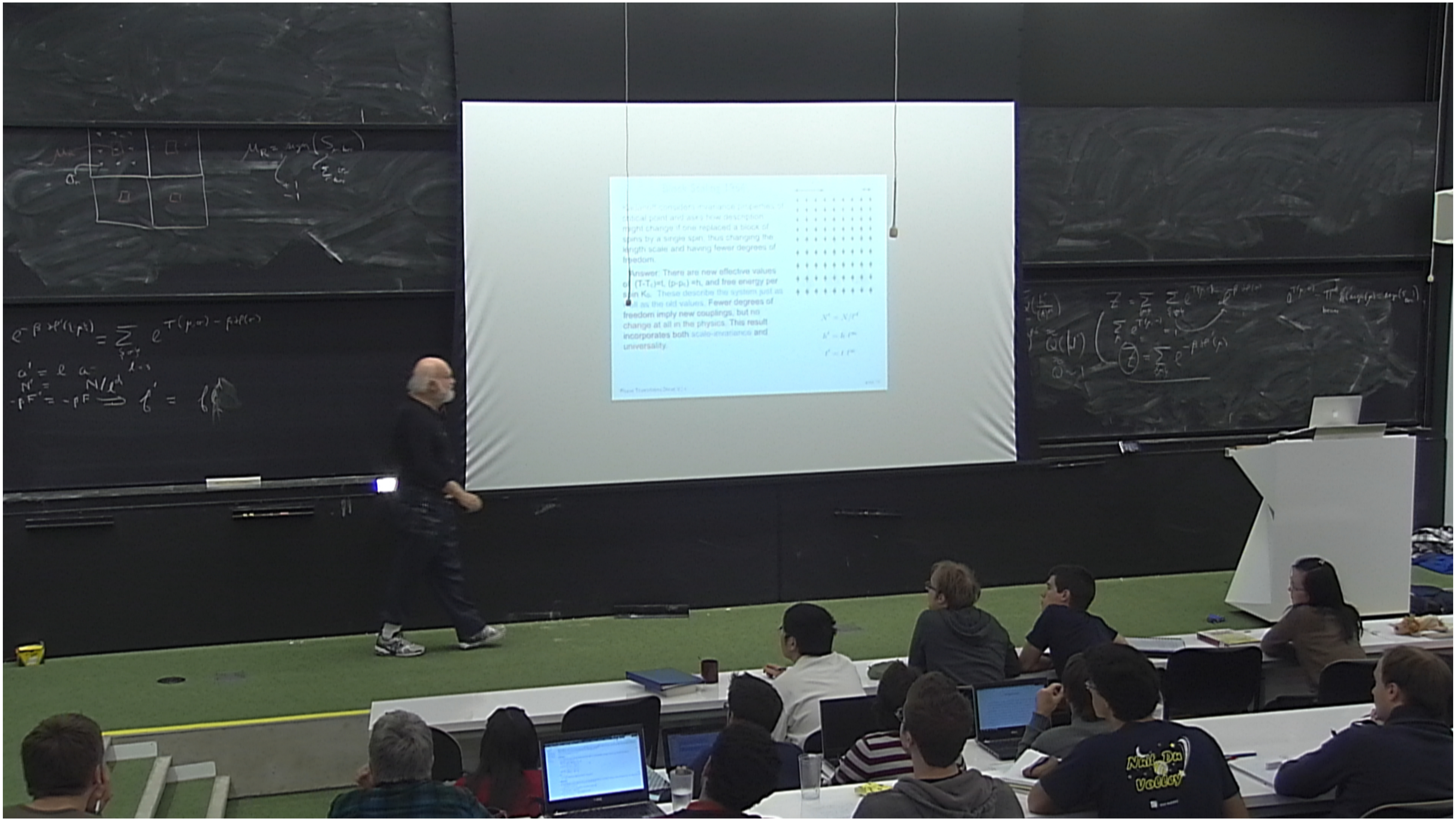


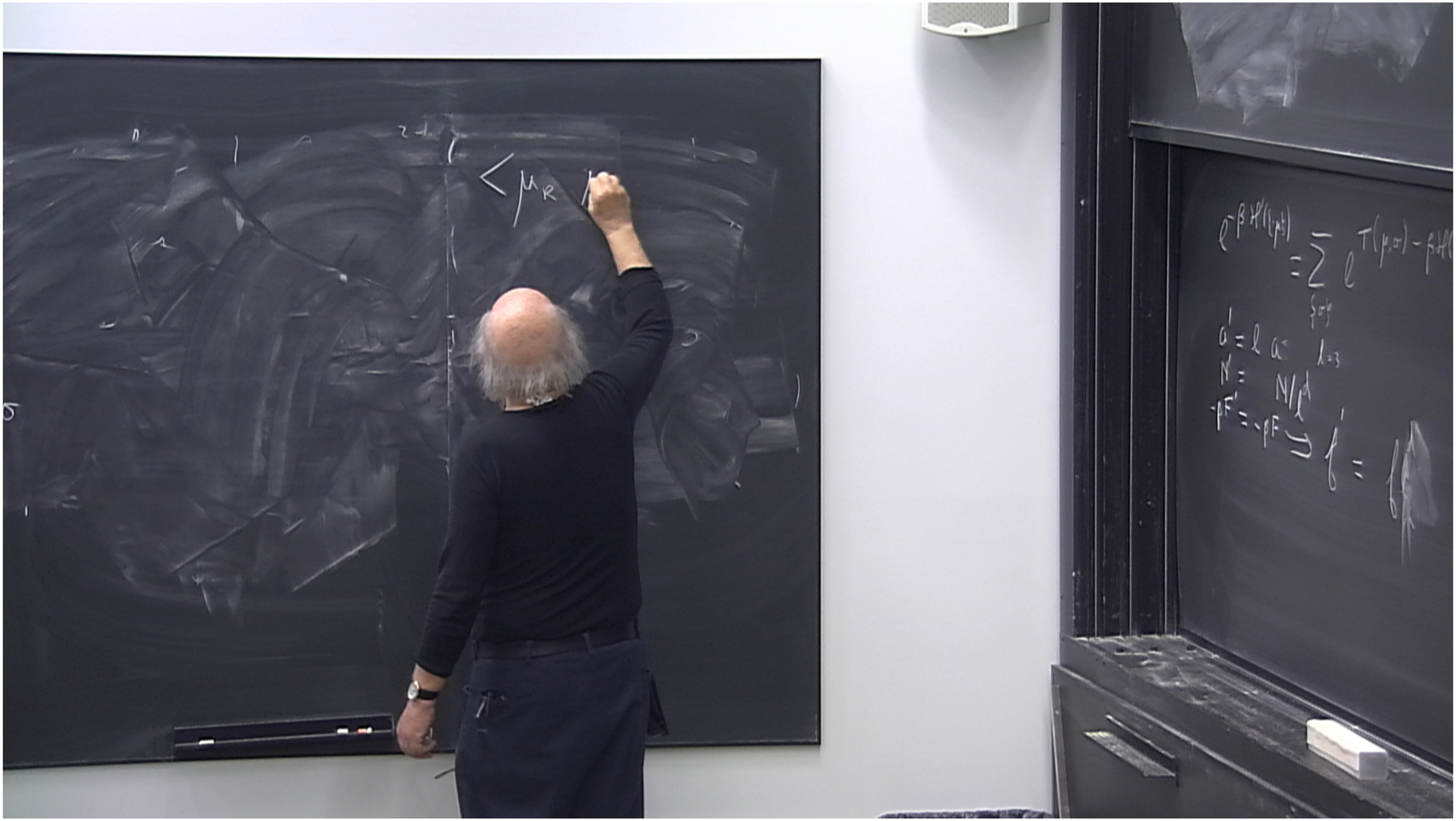
$e^{-\beta \mathcal{H}'(\{\mu\})} = \sum_{\{\sigma\}} e^{T(\mu,\sigma) - \beta \mathcal{H}(\sigma)}$
$a' = \ell\ a$
$\ell = 3$
$N' = N/\ell^d$

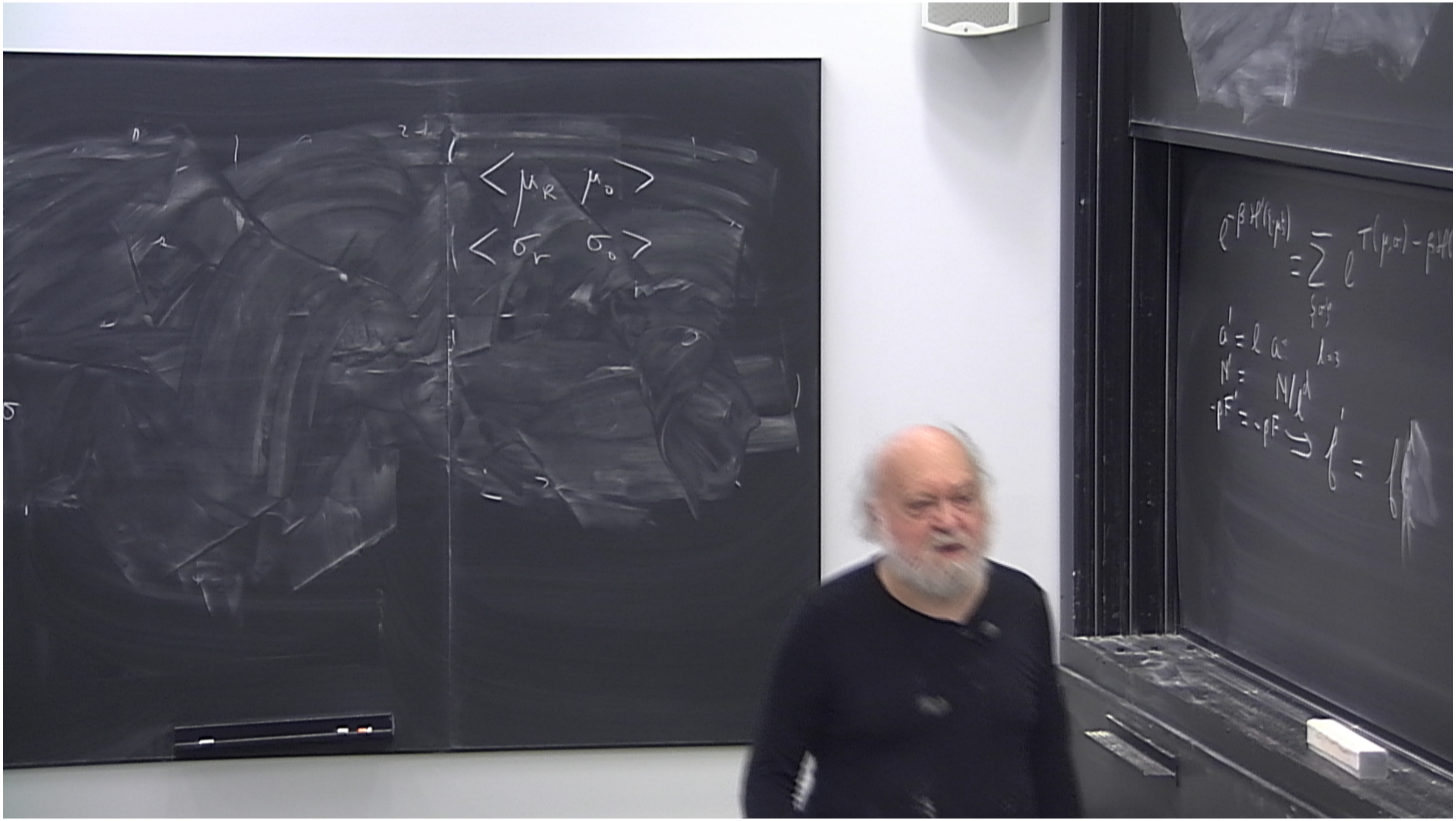

$e^{-\beta \mathcal{H}'(\{\mu\})} = \sum_{\{\sigma\}} e^{T(\mu,\sigma) - \beta \mathcal{H}(\sigma)}$
$a' = \ell\, a$
$\ell = 3$
$N' = N/\ell^d$
$-\beta F' = -\beta F \Longrightarrow f'$

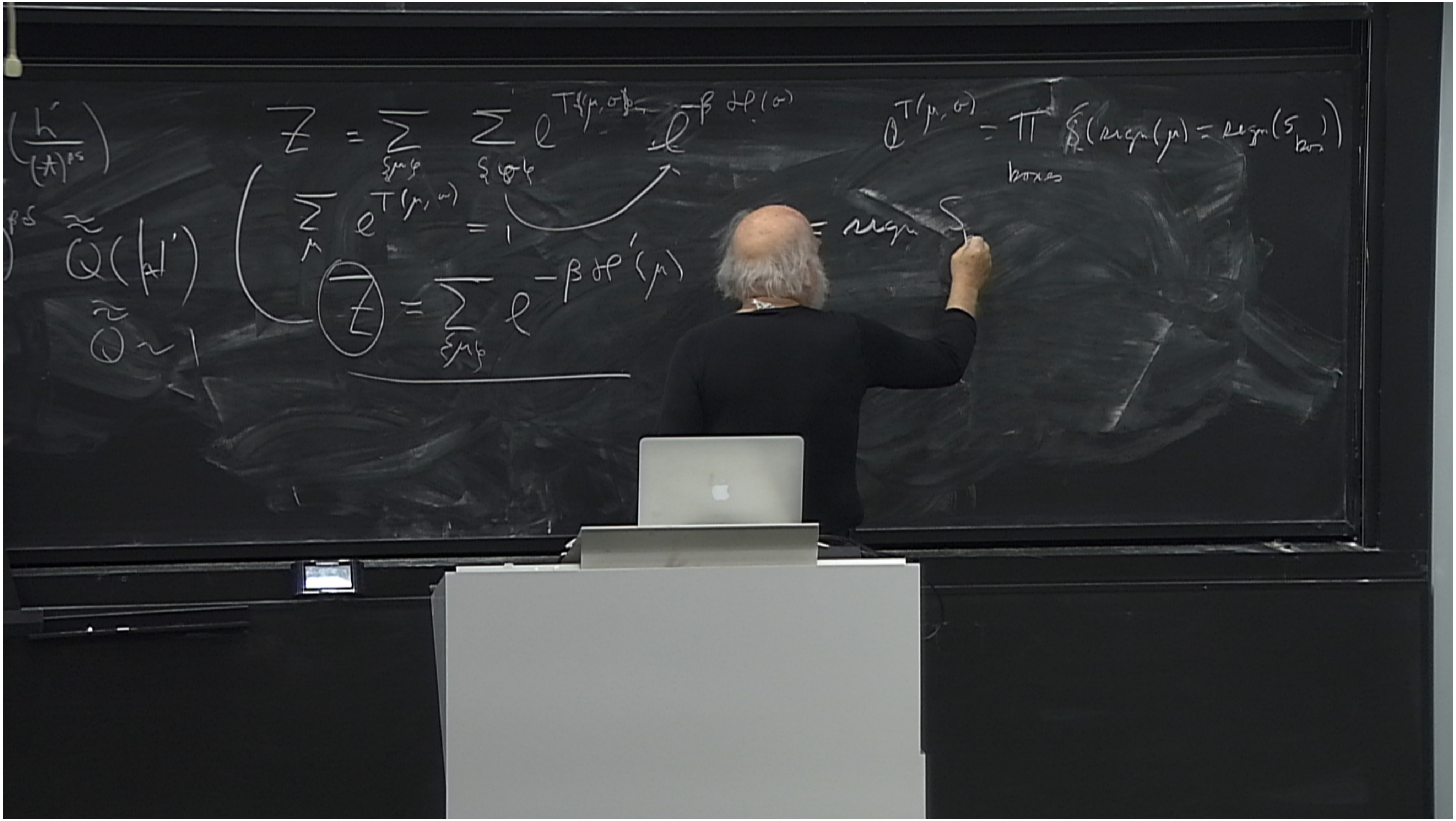

l = 3

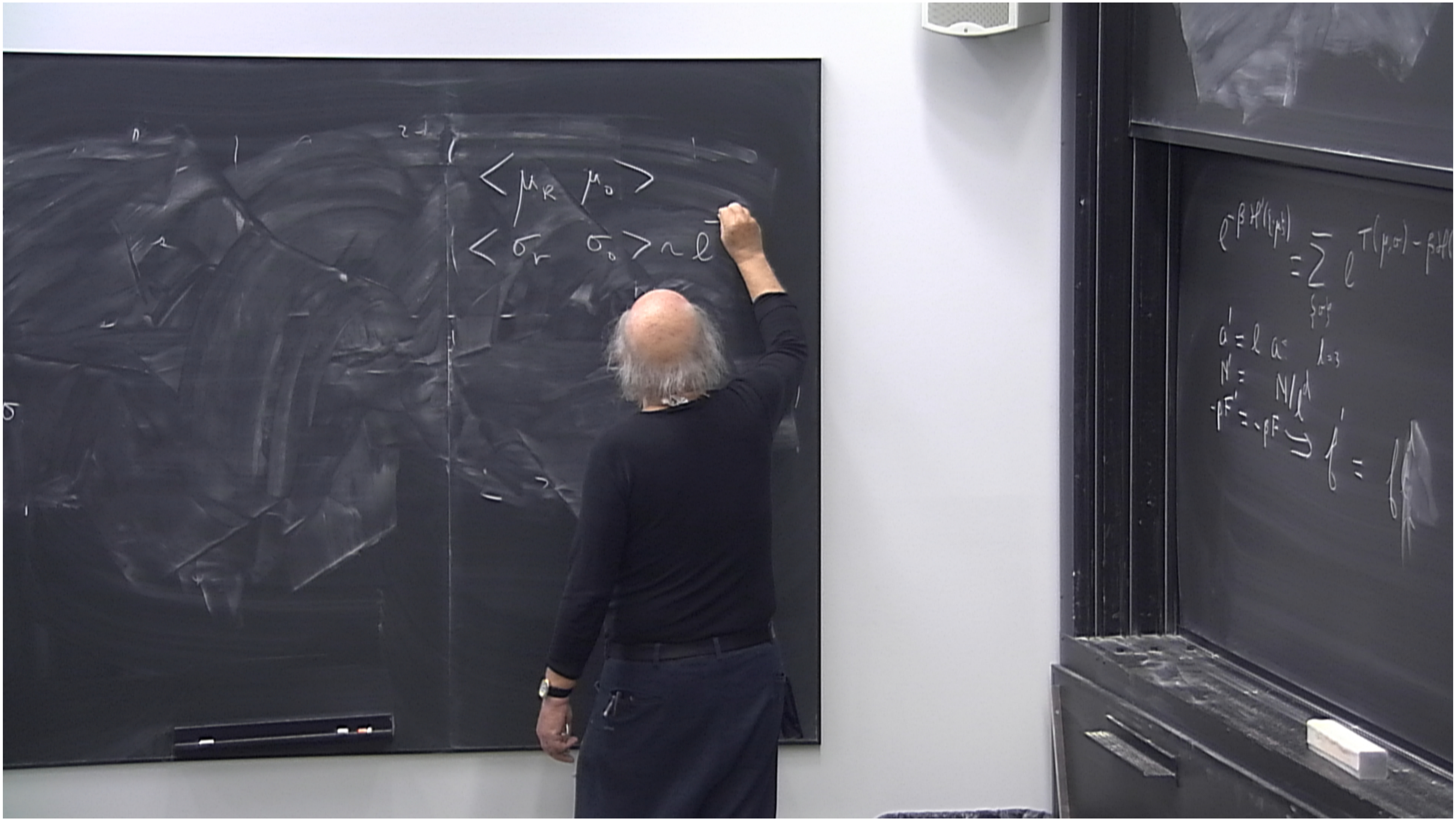

$e^{-\beta \mathcal{H}'(\{\mu\})} = \sum_{\{\sigma\}} e^{T(\mu,\sigma) - \beta \mathcal{H}(\sigma)}$
$a' = \ell a$
$\ell = 3$
$N' = N/\ell^d$
$-\beta F' = -\beta F \Longrightarrow f' = f/\ell^d$

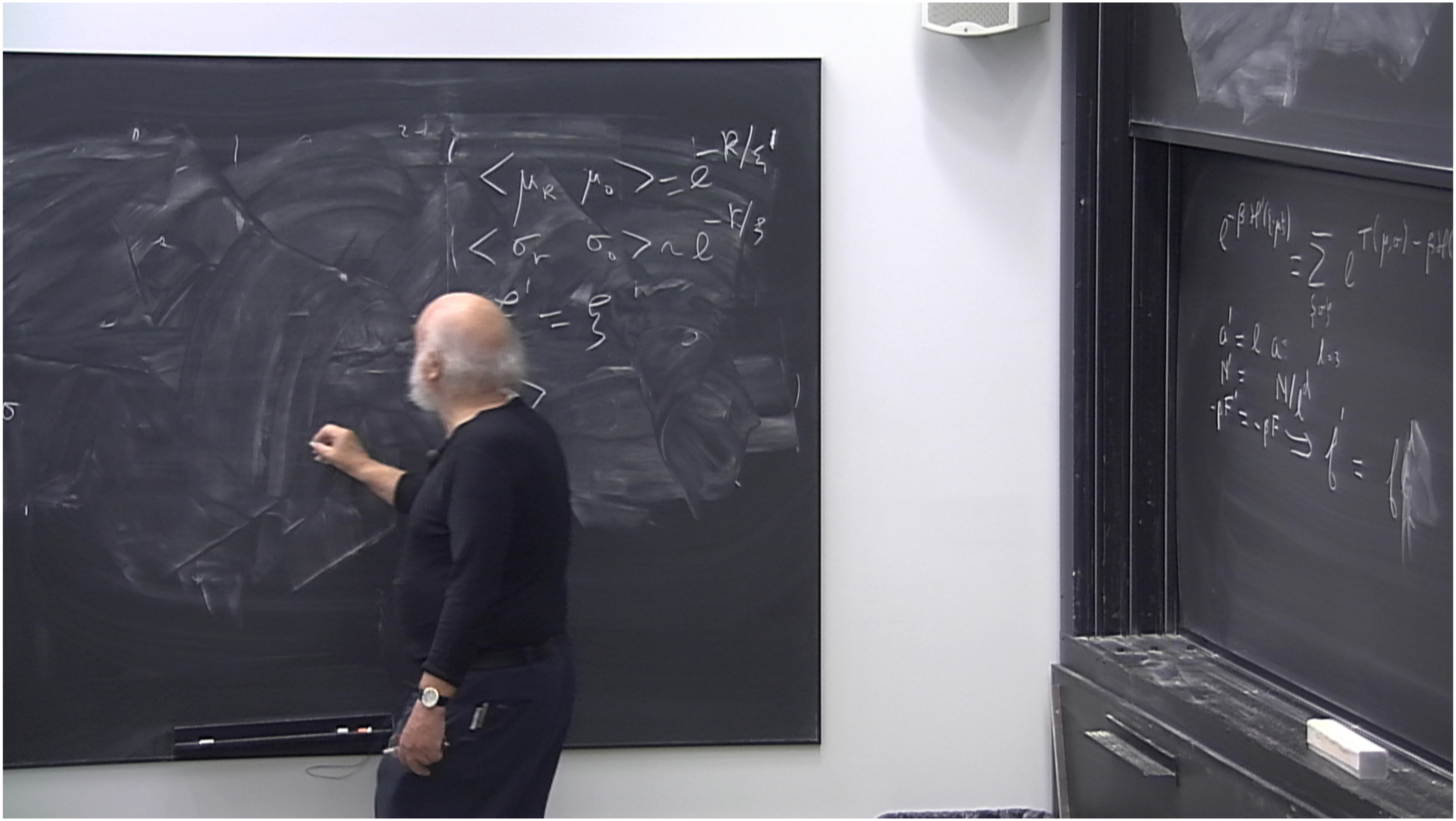

might change if one replaced a block of
spins by a single spin, thus changing the
length scale and having fewer degrees of
freedom.
Answer: There are new effective values
(T-Tc)=t, (p-pc) =h, and free energy per
These describe the system just as
Fewer degrees of
freedom imply new couplings, but no
change at all in the physics. This result
incorporates both scale-invariance and
universality.

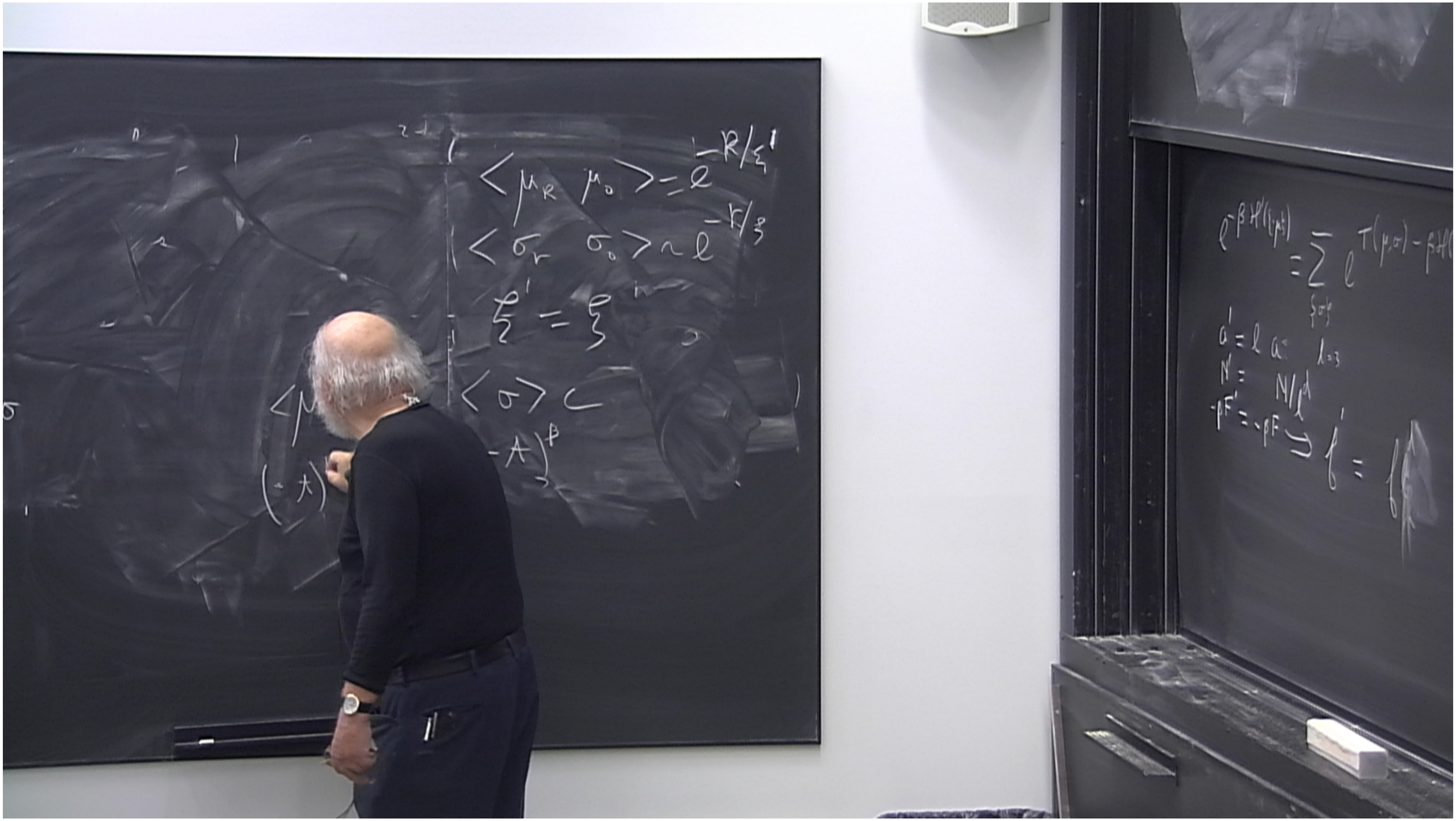


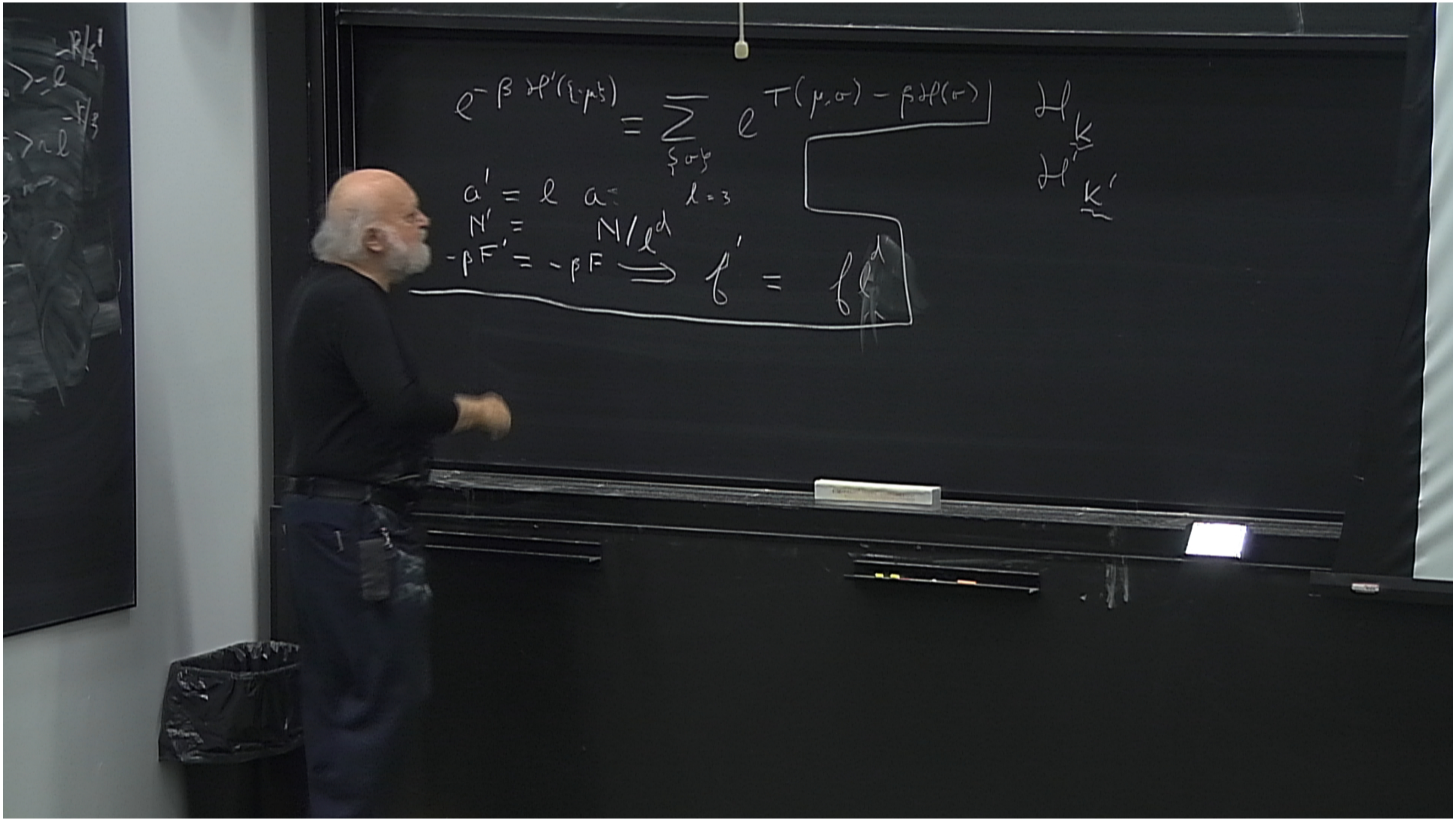

⟨μ_R μ_0⟩
⟨σ_r σ_0⟩
N' = N/l^d

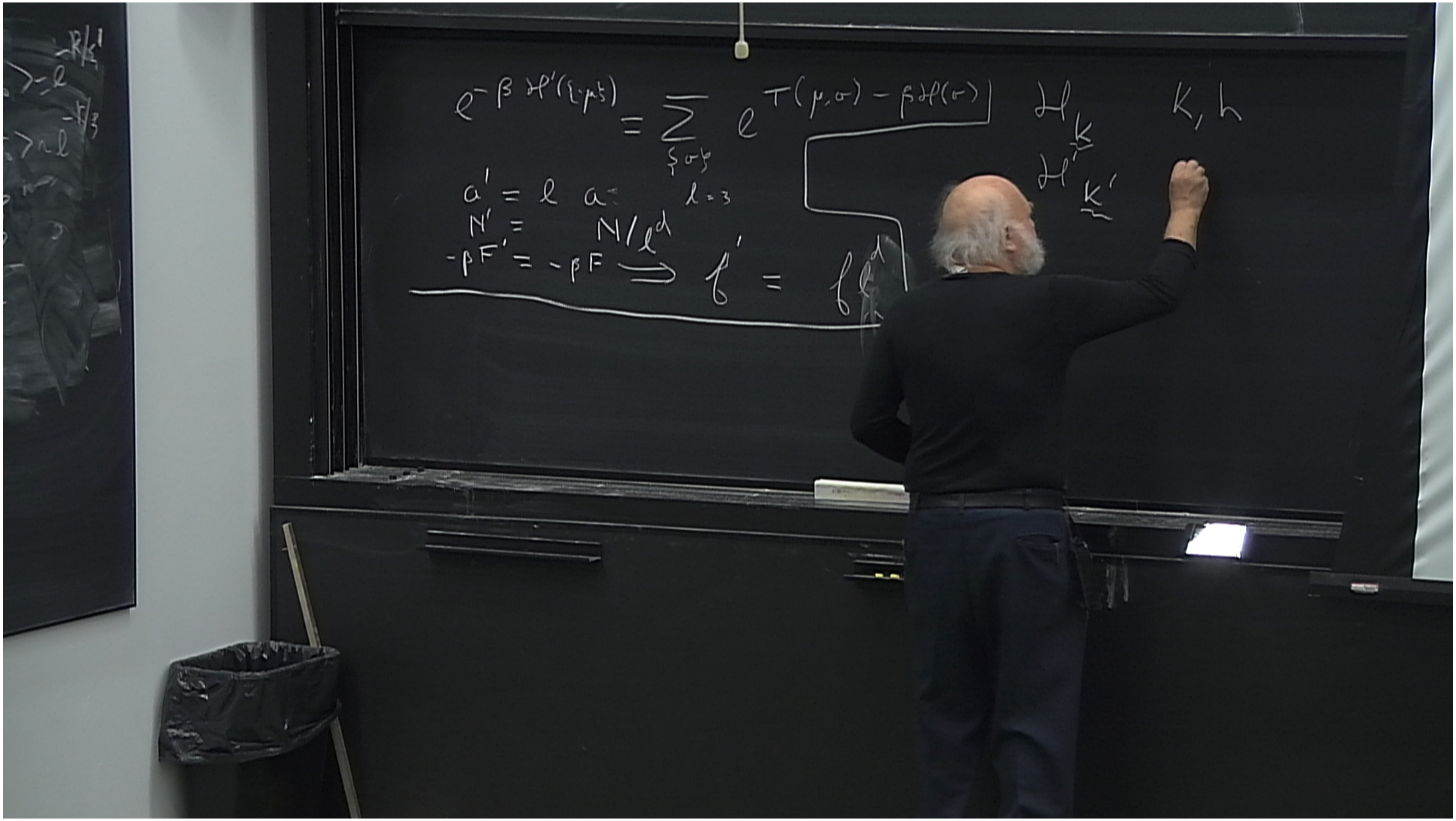

boxes

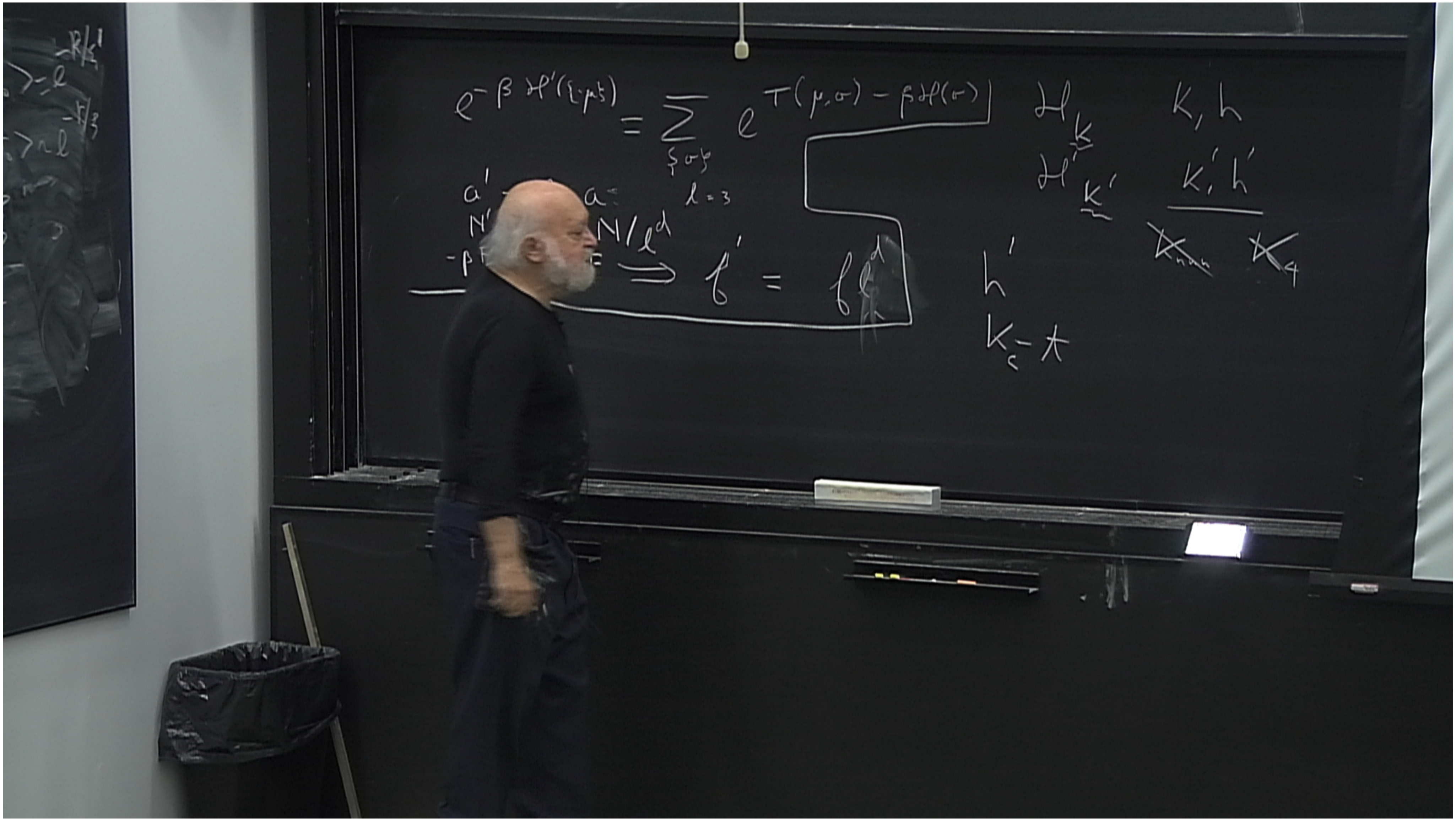

⟨μR μ0⟩
⟨σr σ0⟩ ~ e
N' = N/l^d

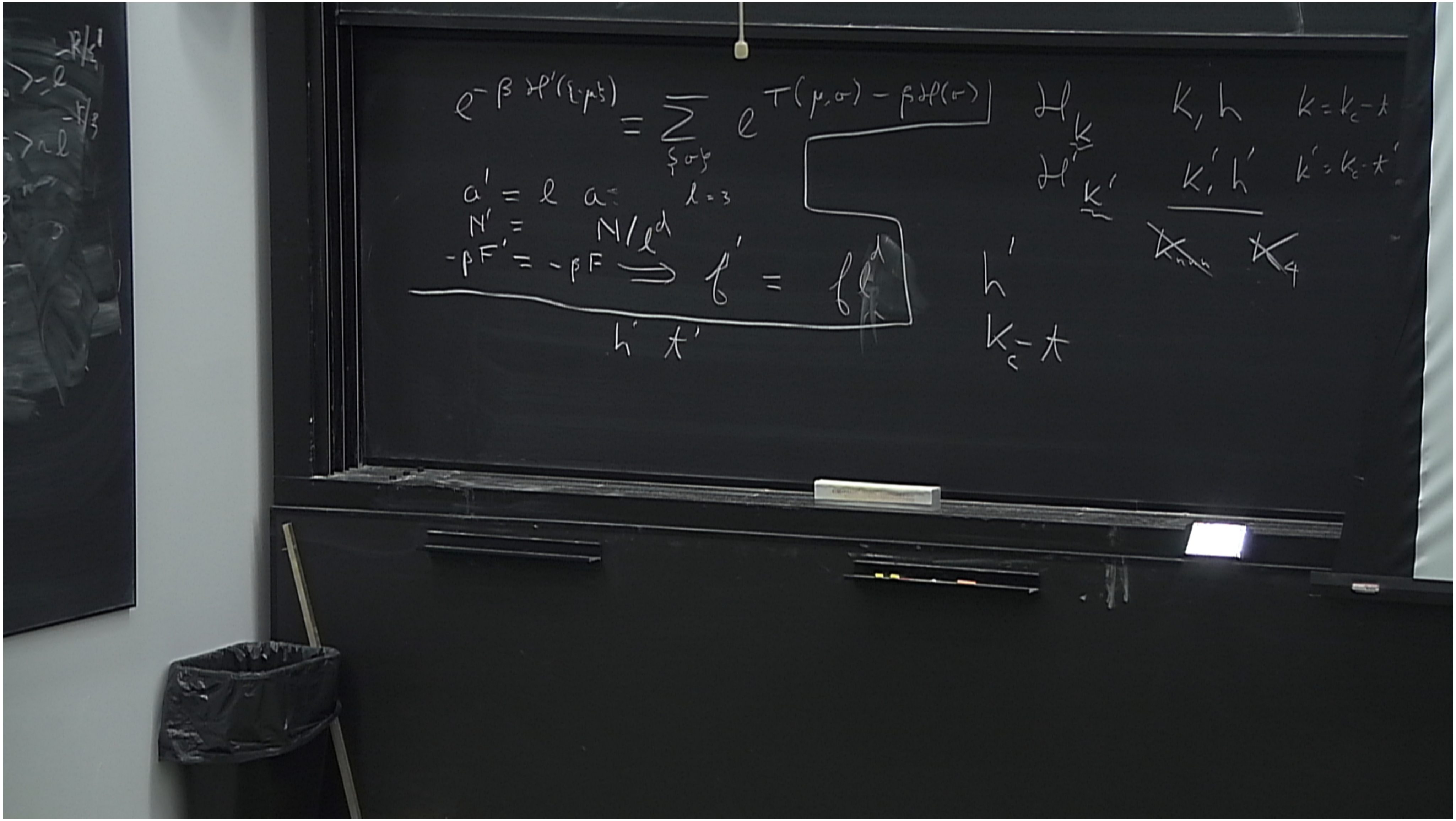


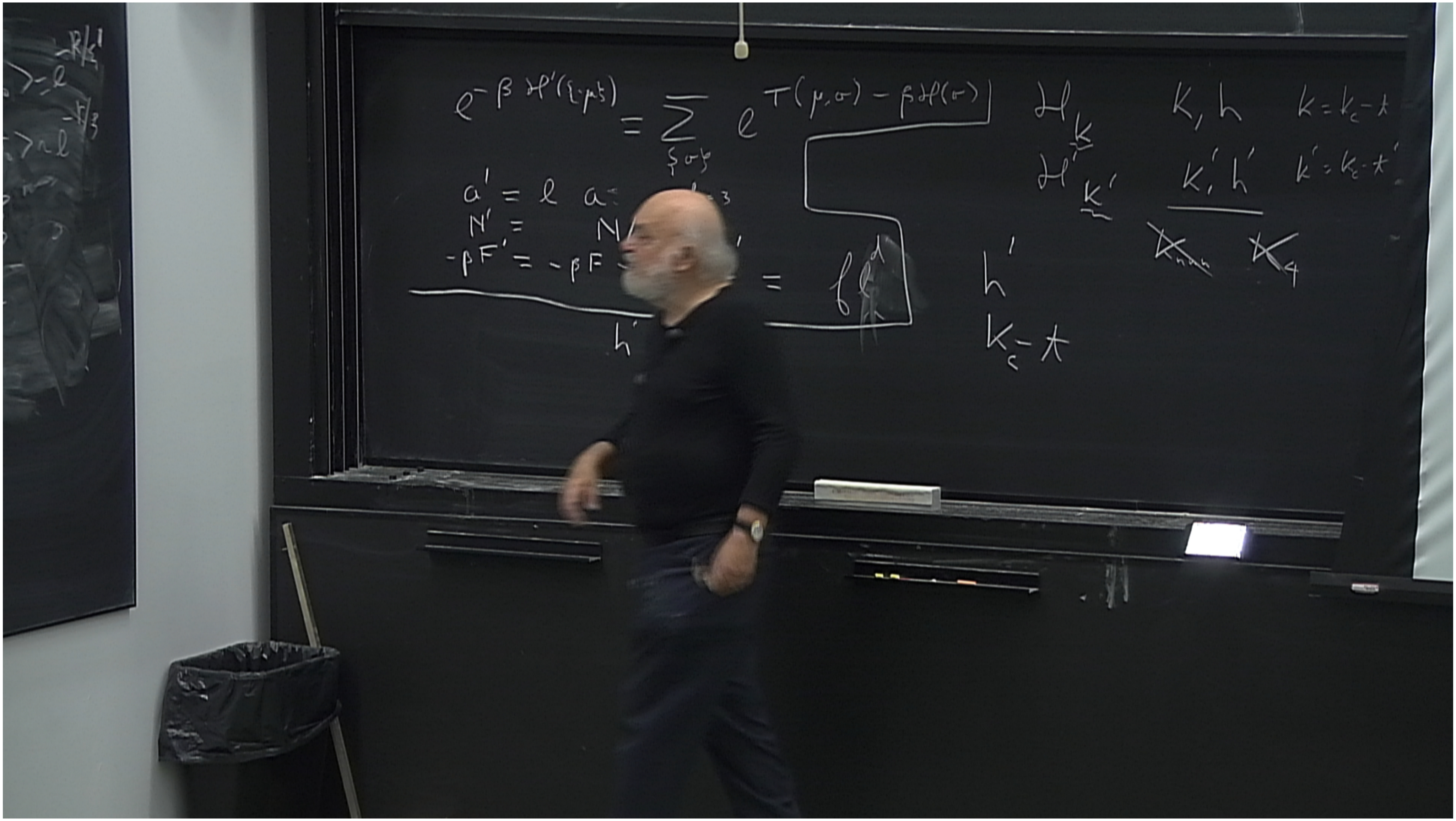


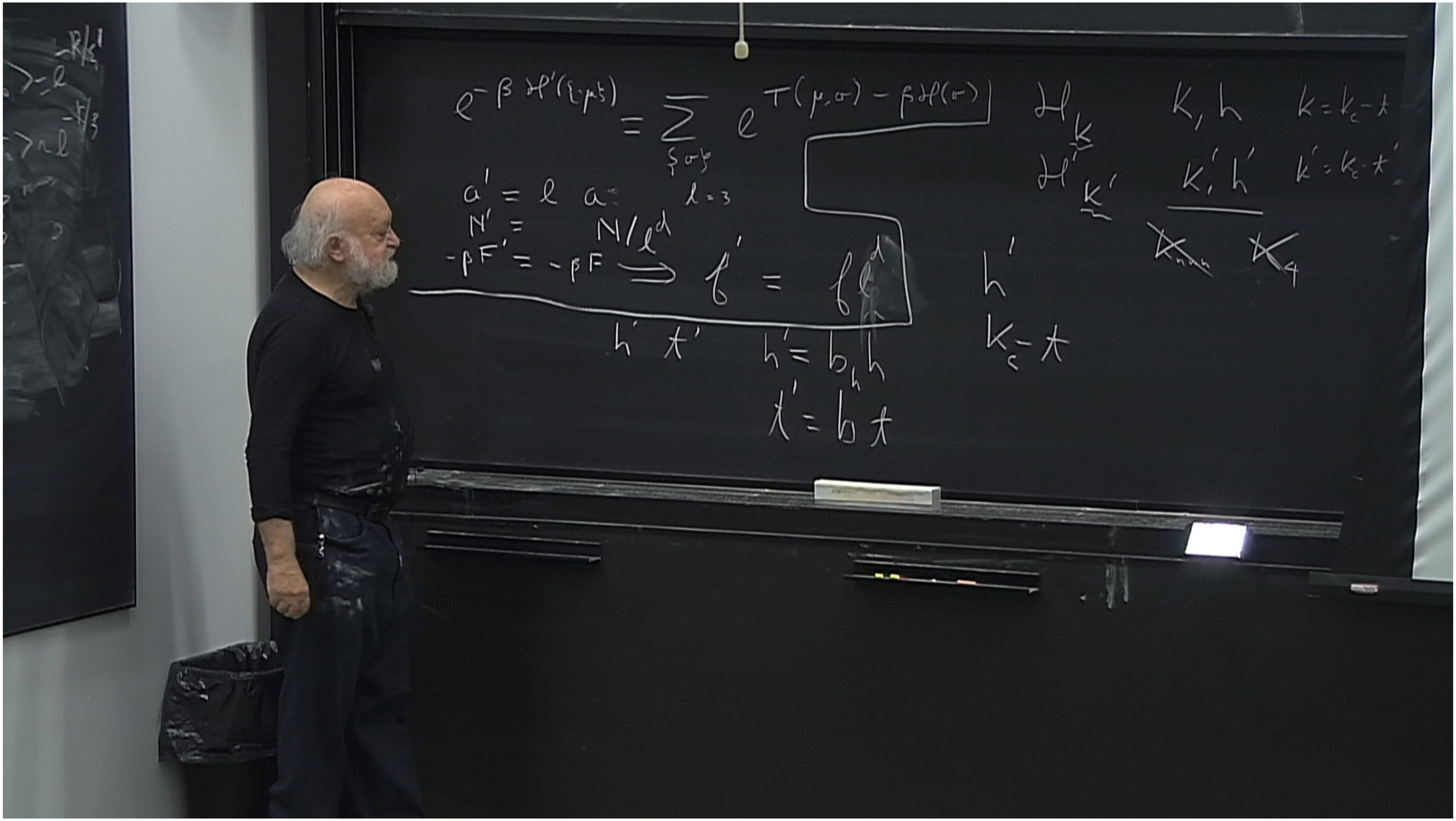

$e^{-\beta \mathcal{H}'(\{\mu\})} = \sum_{\{\sigma\}} e^{T(\mu,\sigma) - \beta\mathcal{H}(\sigma)}$
$\mathcal{H}_{\underline{k}}$
$\mathcal{H}'_{\underline{k}'}$
$a' = \ell a$
$\ell = 3$
$N' = N/\ell^d$
$-\beta F' = -\beta F \Rightarrow f' = f\ell^d$

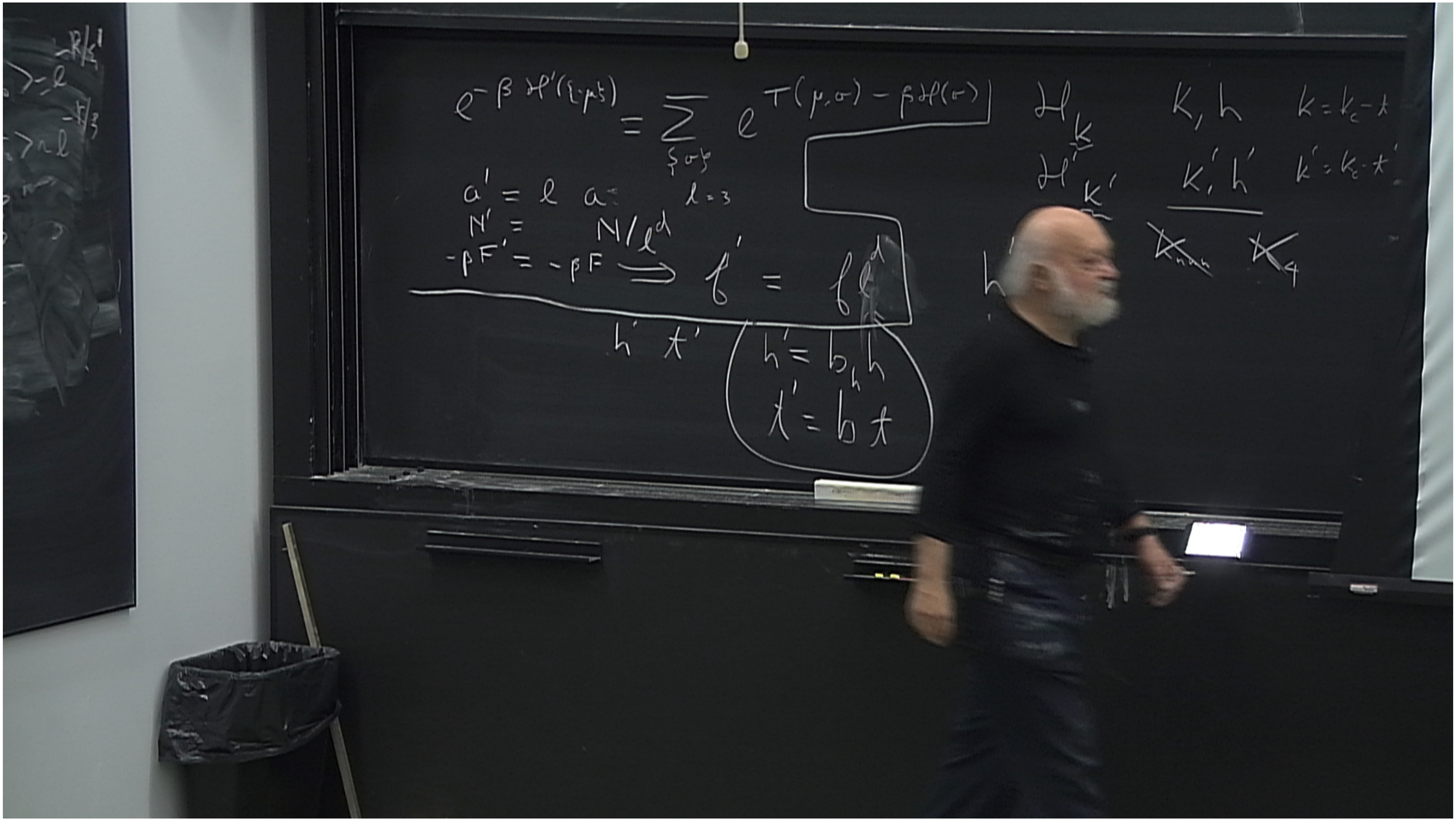

K, h
l = 3


K, h
K', h'
h'

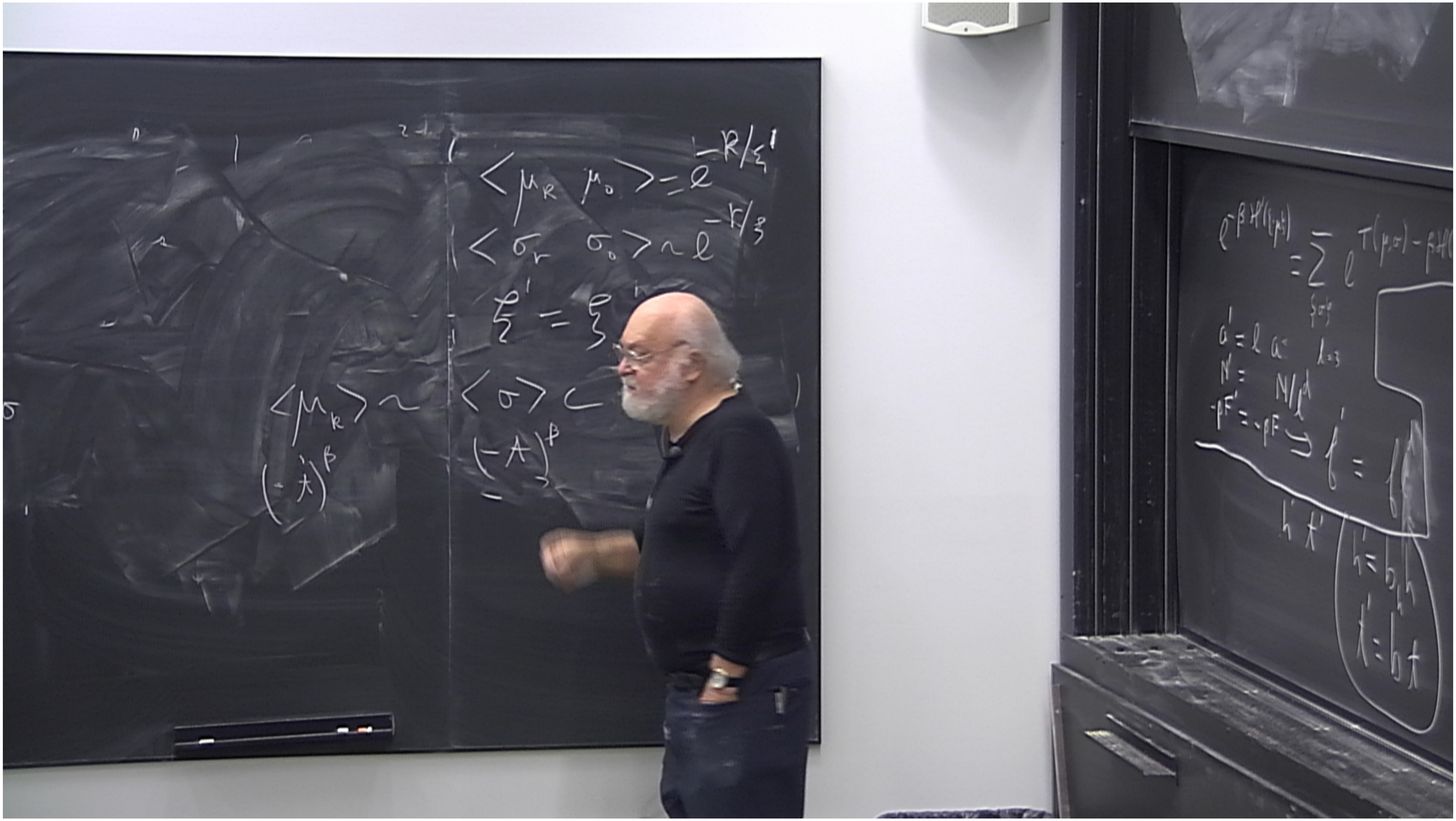

e^{-β H'(μ)} = Σ_{σ} e^{T(μ,σ) - β H(σ)}
a' = l a
l = 3
N' = N/l^d
-βF' = -βF ⟹ f' = f l^d
h' t'
H K
H' K'
K, h
K', h'
k = k_c - t
k' = k_c - t'
h'
k_c - t

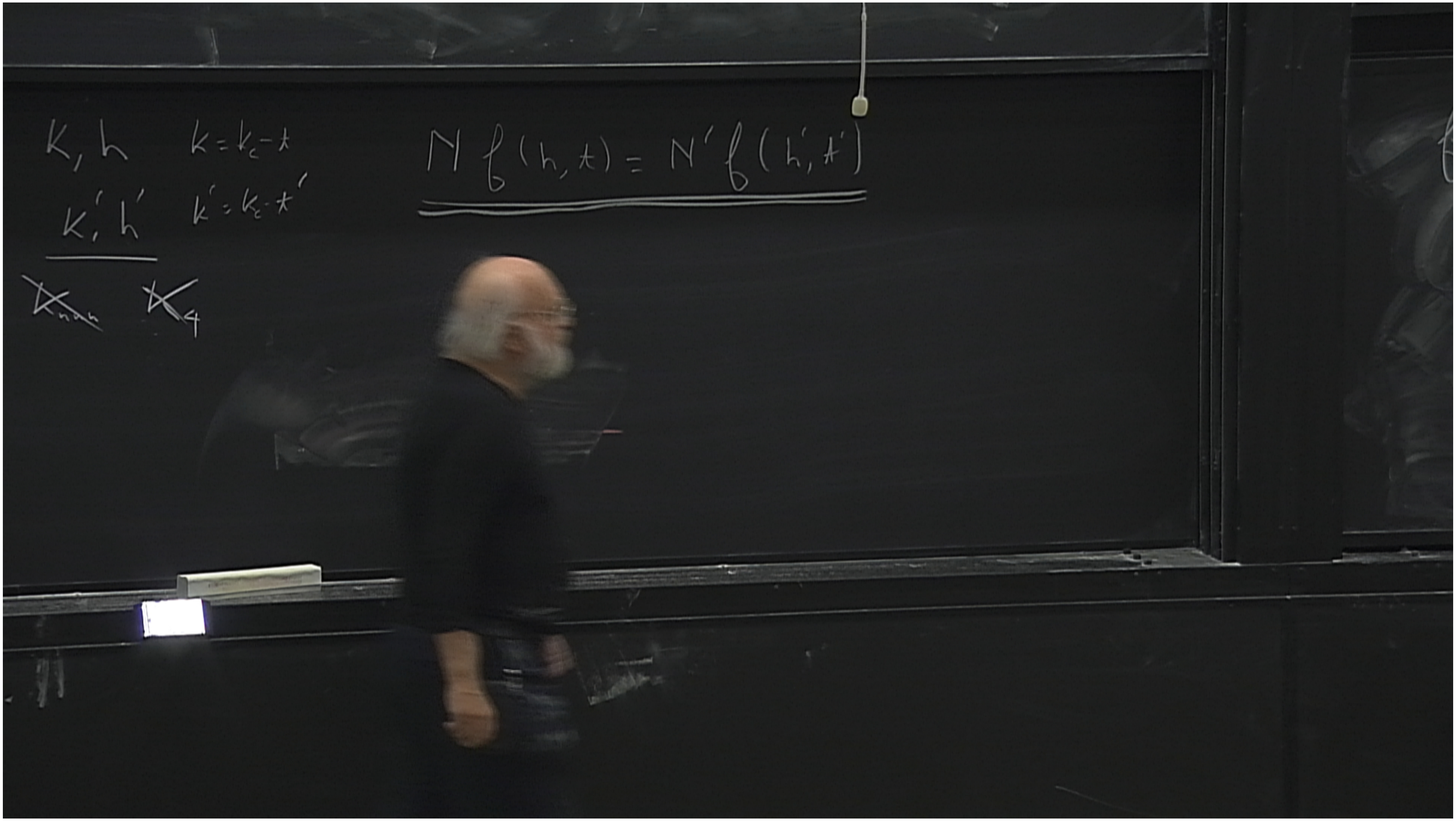

K, h
K', h'
h'

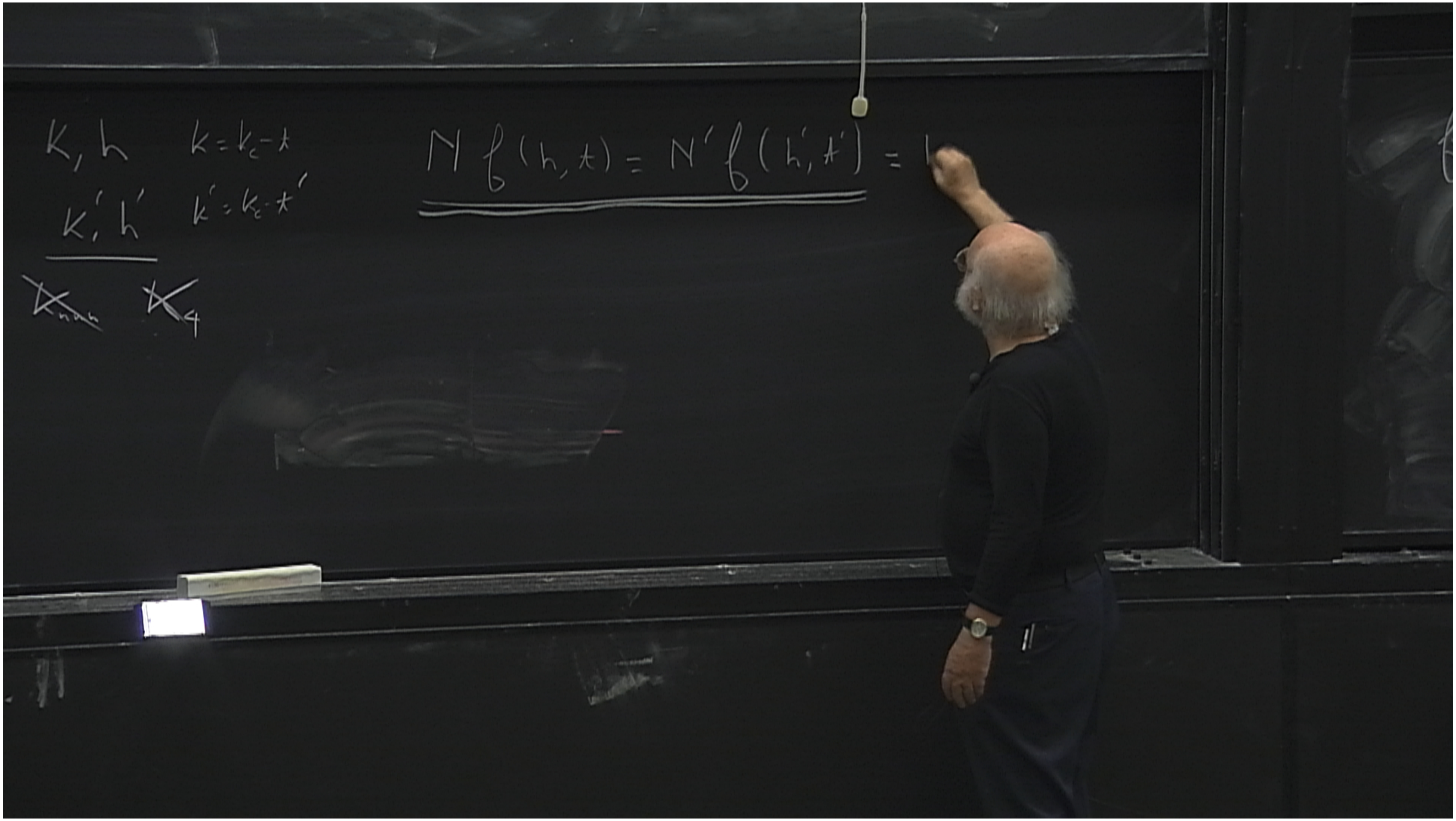

$e^{-\beta \mathcal{H}'(\{\mu\})} = \sum_{\{\sigma\}} e^{T(\mu,\sigma) - \beta\mathcal{H}(\sigma)}$
$a' = l\, a$
$l = 3$
$N' = N/l^d$
$-\beta F' = -\beta F \Longrightarrow f' = f l^d$
$h'\ t'$
$h' = b_h h$
$t' = b t$
$\mathcal{H}_K$
$\mathcal{H}'_{K'}$
K, h
K', h'
$k = k_c - t$
$k' = k_c - t'$
h'
$K_c - t$

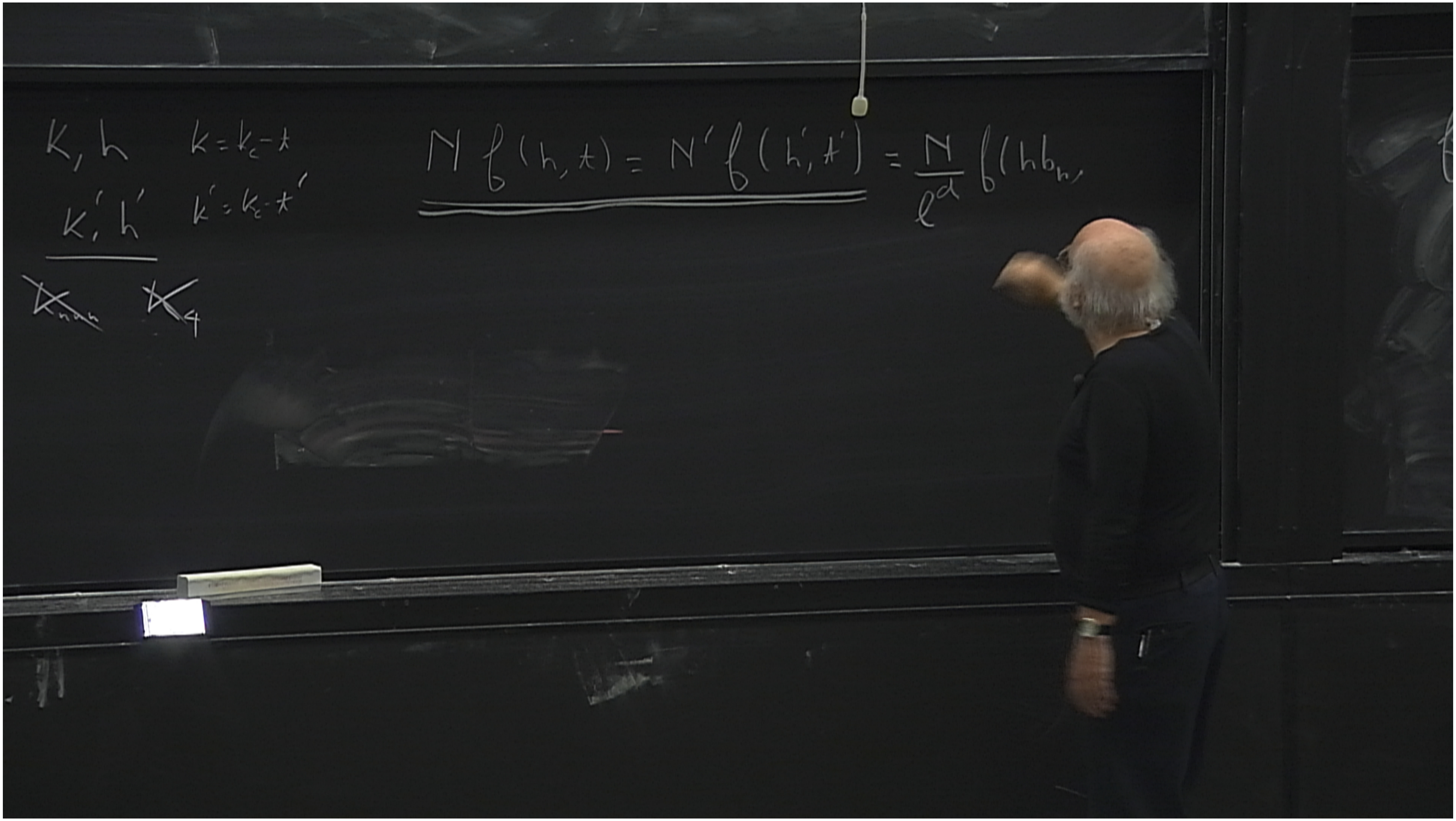


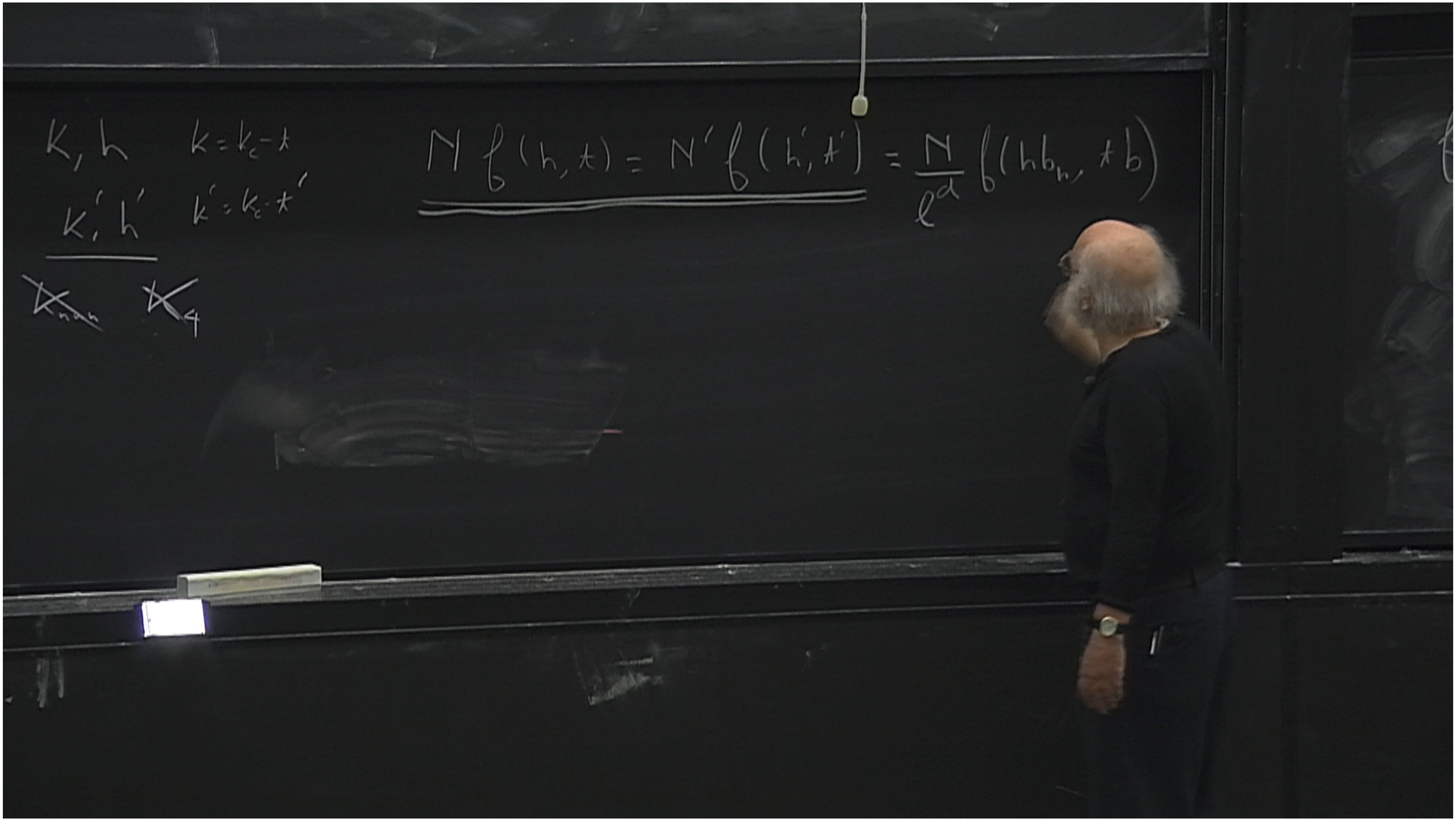
e^{-βH'(...)} = Σ_{σ} e^{T(μ,σ) - βH(σ)}
a' = l a   l = 3
N' = N/l^d
-βF' = -βF ⟹ f' = f l^d
h' t'
h' = b_h h
t' = b t
H_K   K, h   k = k_c - t
H'_K'   K', h'   k' = k_c - t'

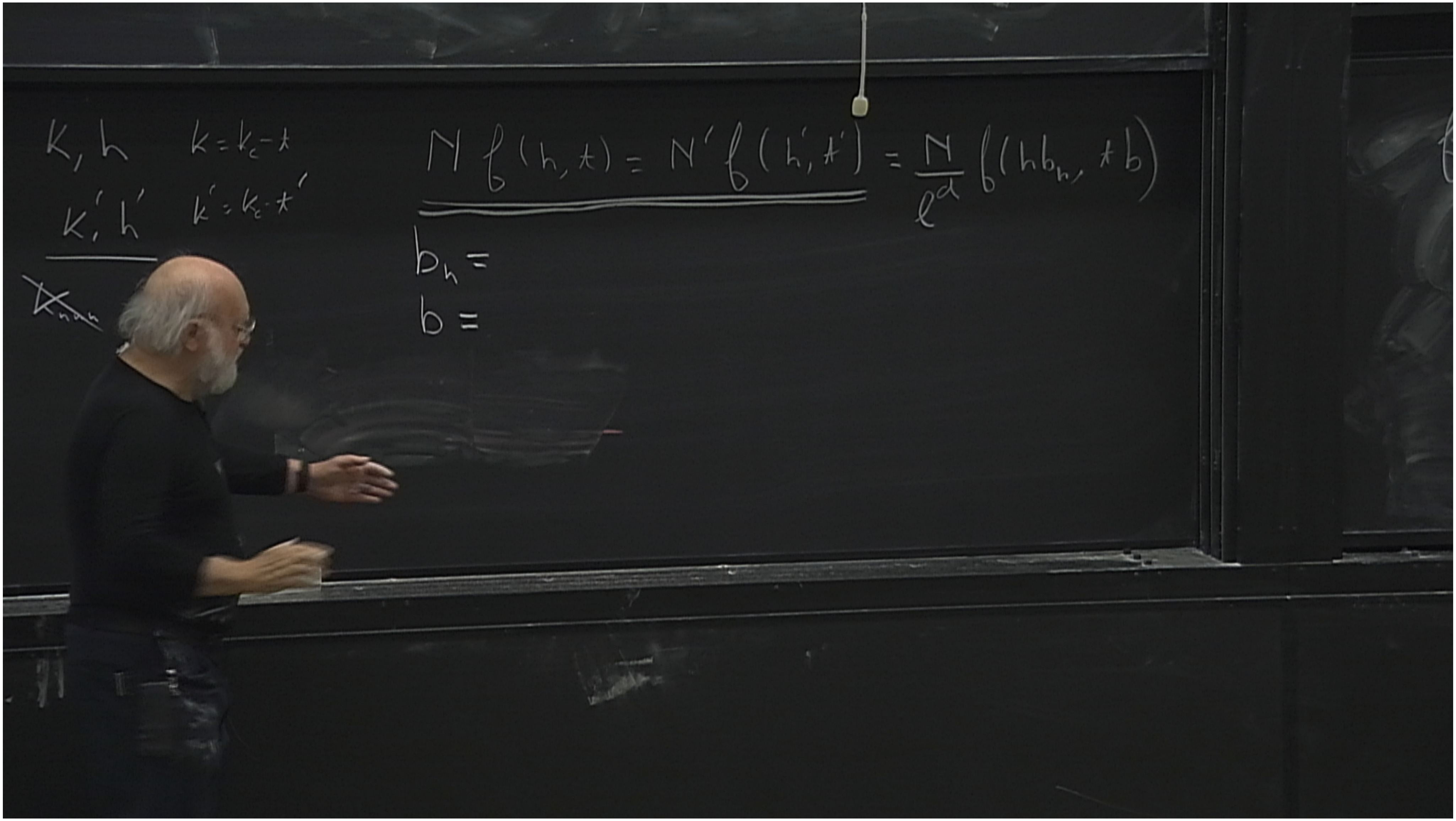



K, h   k = k_c − t
K', h'   k' = k_c − t'
N f(h, t) = N' f(h', t')

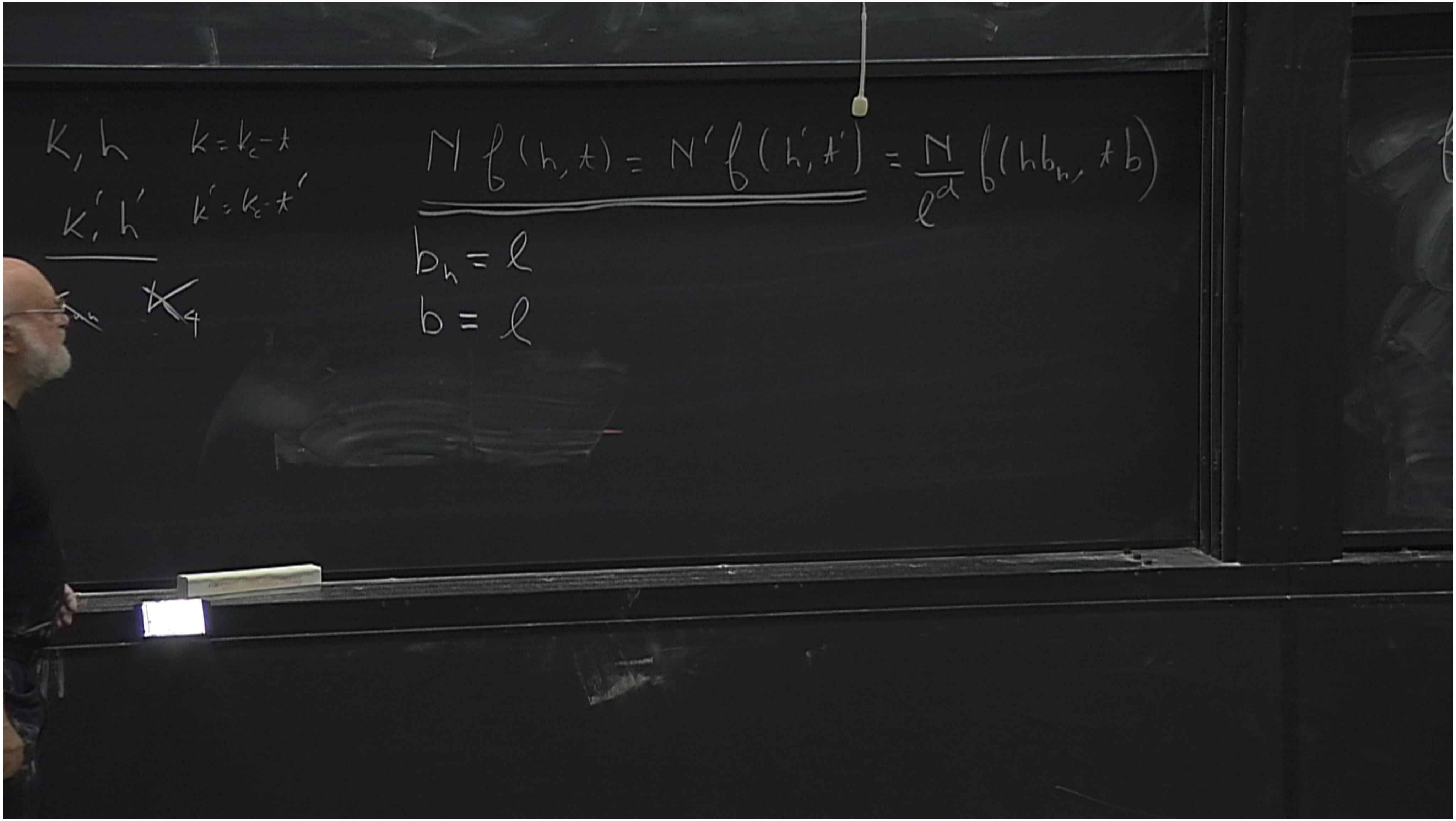

K, h   k = k_c - t
K', h'   k' = k_c - t'
~~K_nnn~~   ~~K_4~~
N f(h, t) = N' f(h', t') =

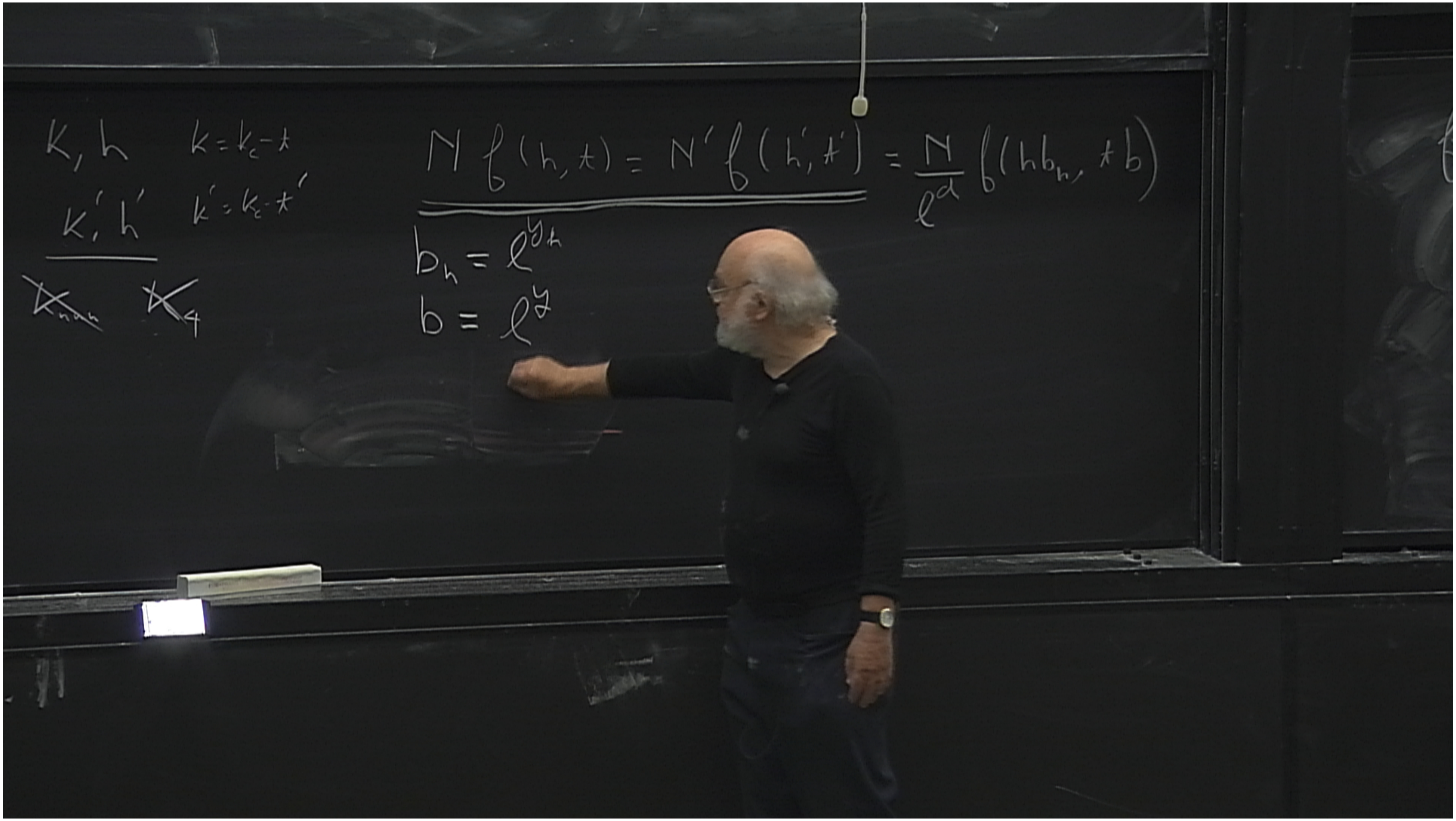

K, h   k = k_c − t
K', h'   k' = k_c − t'
K_nnn   K_4
N f(h, t) = N' f(h', t') = N/ℓ^d f(h b_h,


K, h   k = k_c − t
K', h'   k' = k_c − t'
K_nnn   K_4
N f(h, t) = N' f(h', t') = N/ℓ^d f(h b_h, t b)

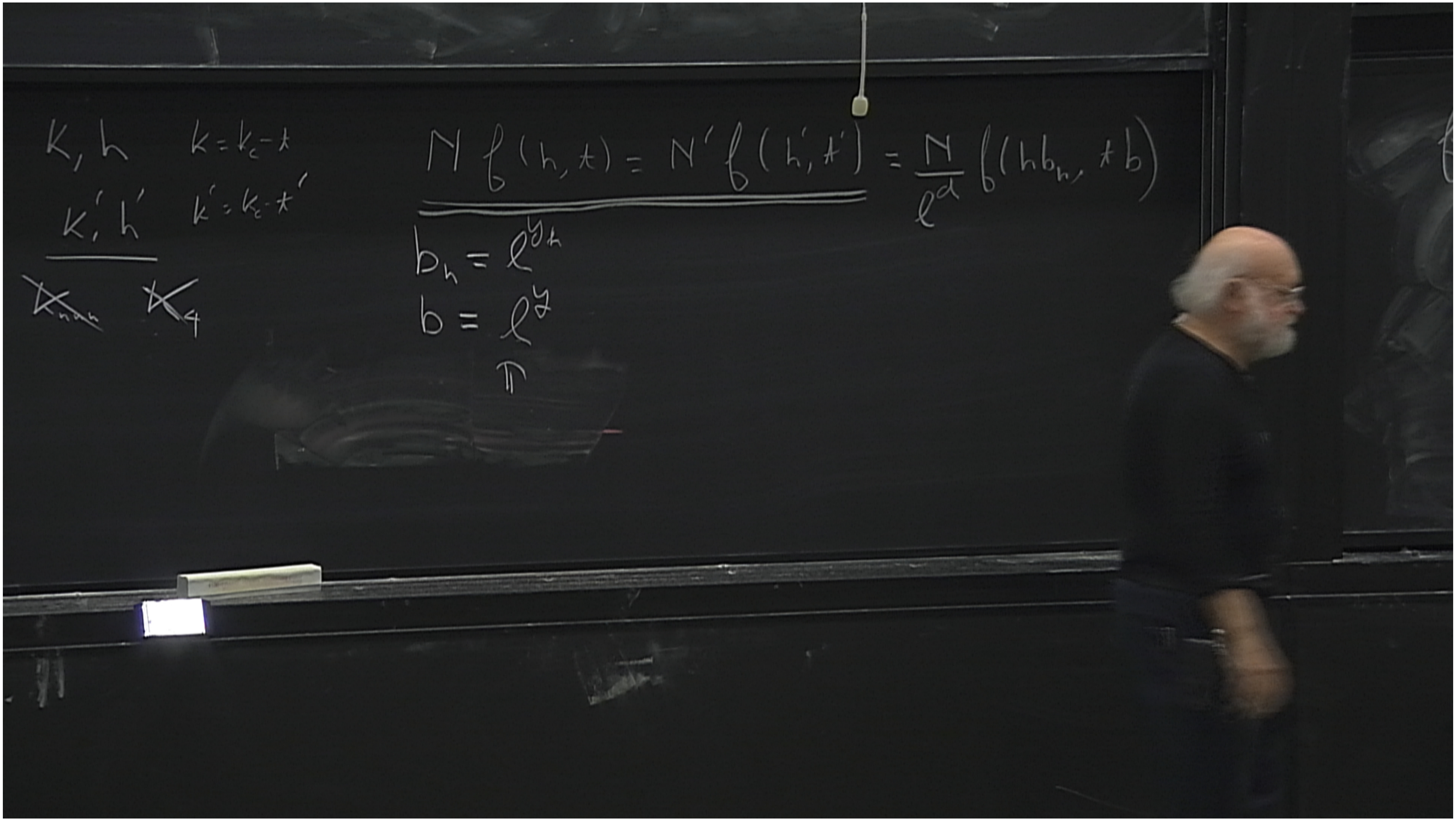


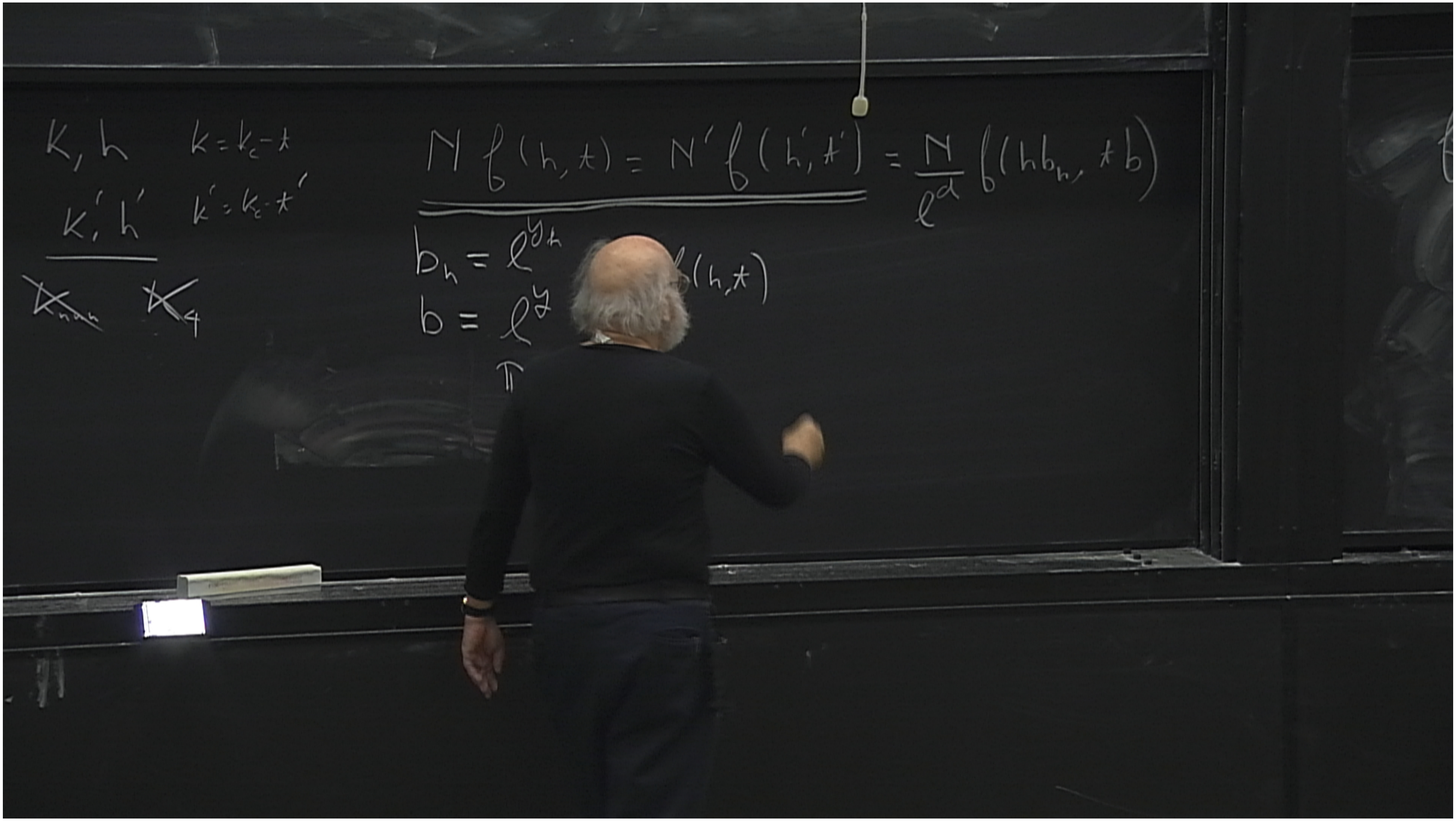
$K, h \quad k = k_c - t$
$K', h' \quad k' = k_c - t'$
$\cancel{K_{nnn}}$
$N f(h,t) = N' f(h', t') = \frac{N}{\ell^d} f(h b_h, t b)$
$b_h =$
$b =$

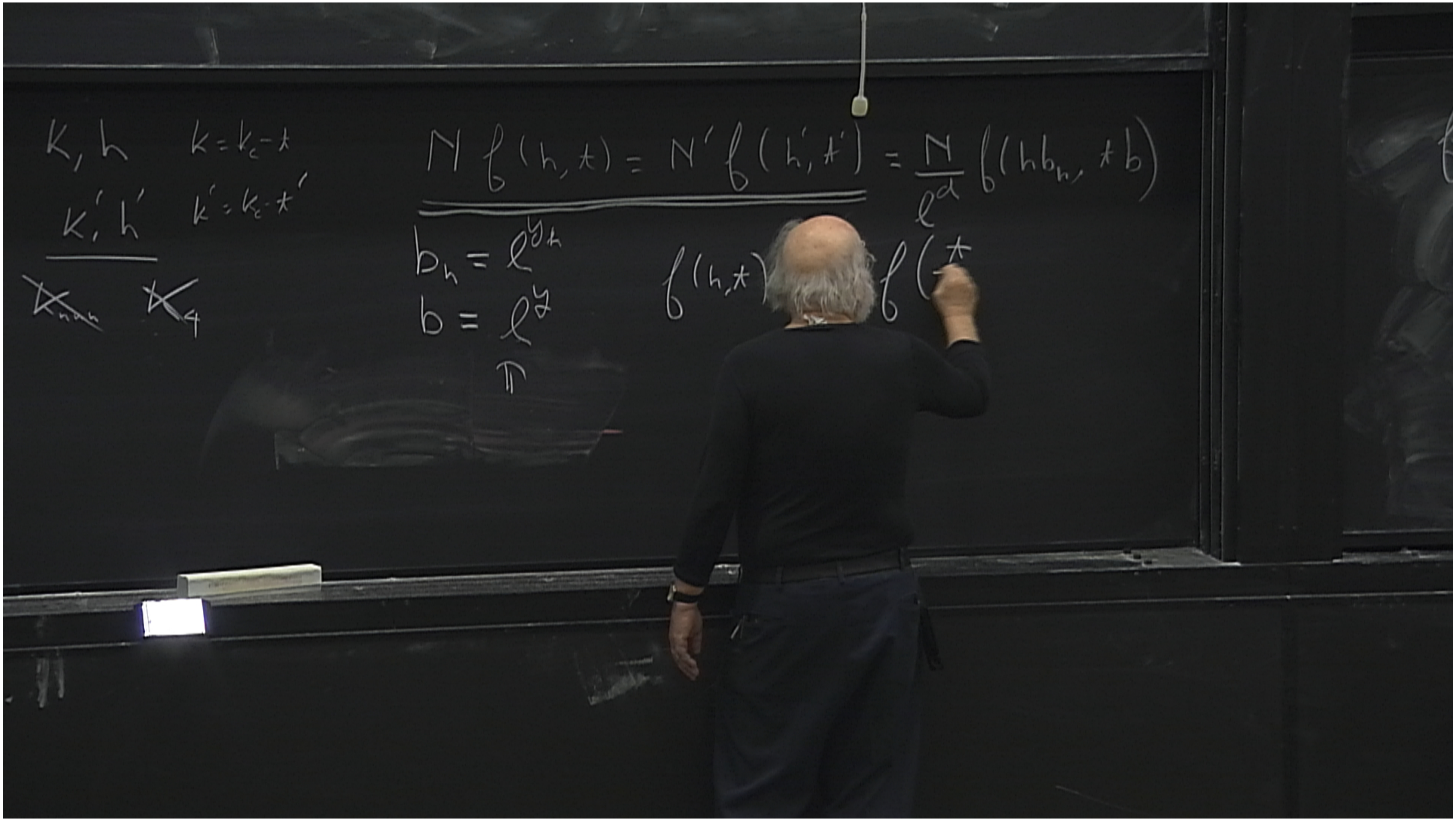

K, h   k = kc − t
K', h'   k' = kc − t'
N f(h, t) = N' f(h', t') = N/ℓ^d f(h bh, t b)
bh = ℓ
b = ℓ

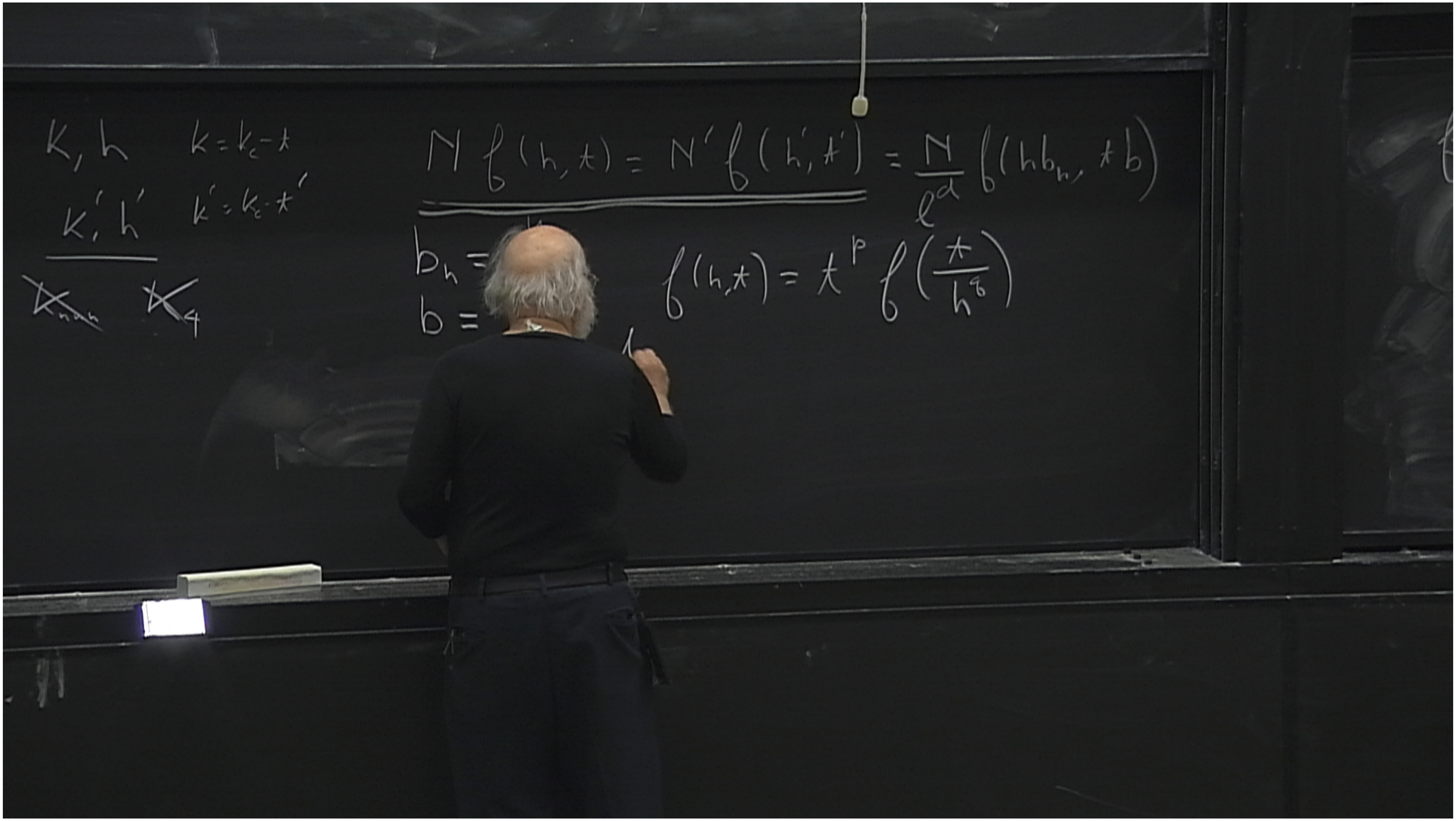


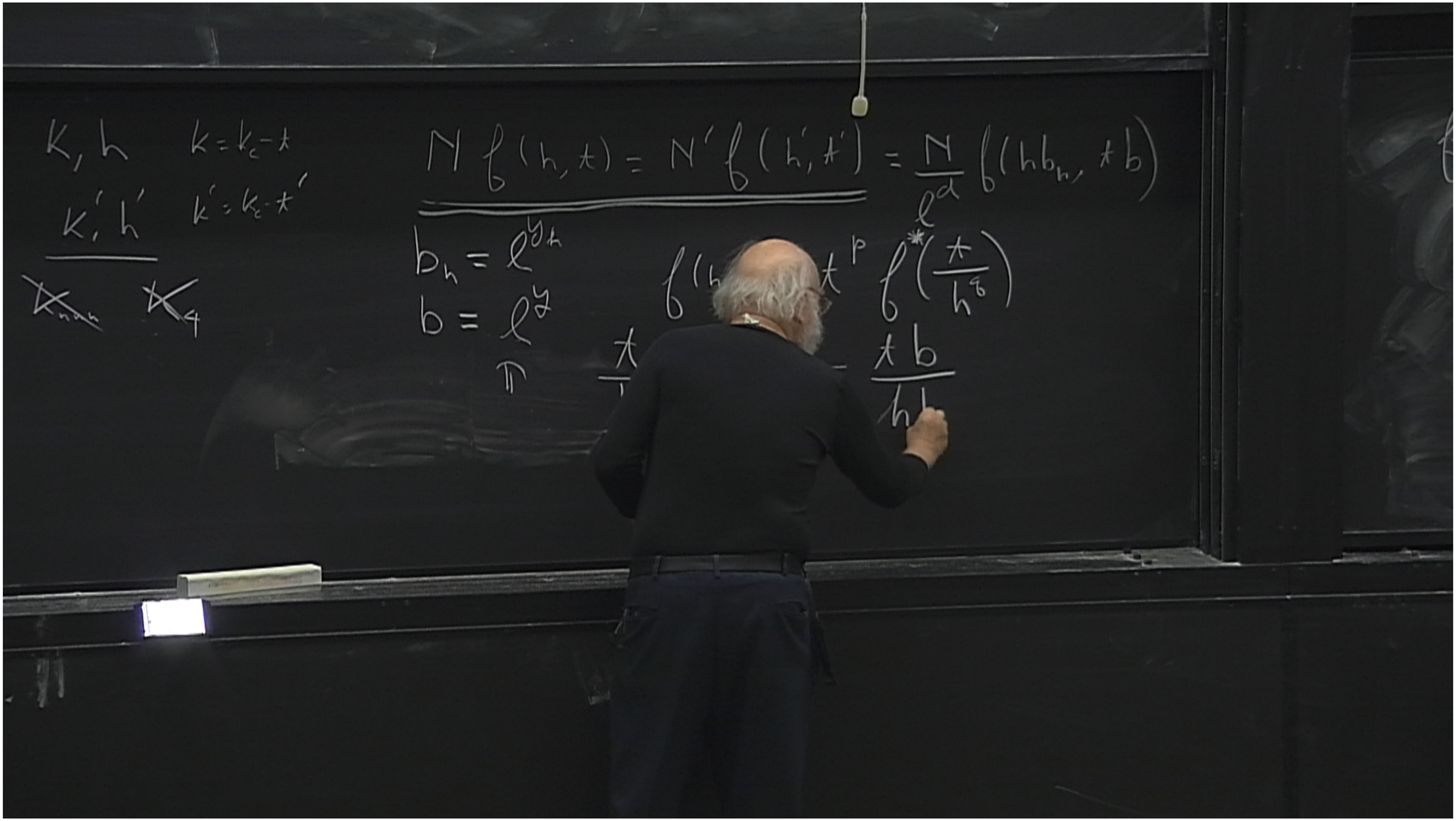
K, h
k = k_c - t
K', h'
k' = k_c - t'
~~K_nn~~ ~~K_4~~
N f(h, t) = N' f(h', t') = N/l^d f(h b_h, t b)
b_h = l^{y_h}
b = l^y

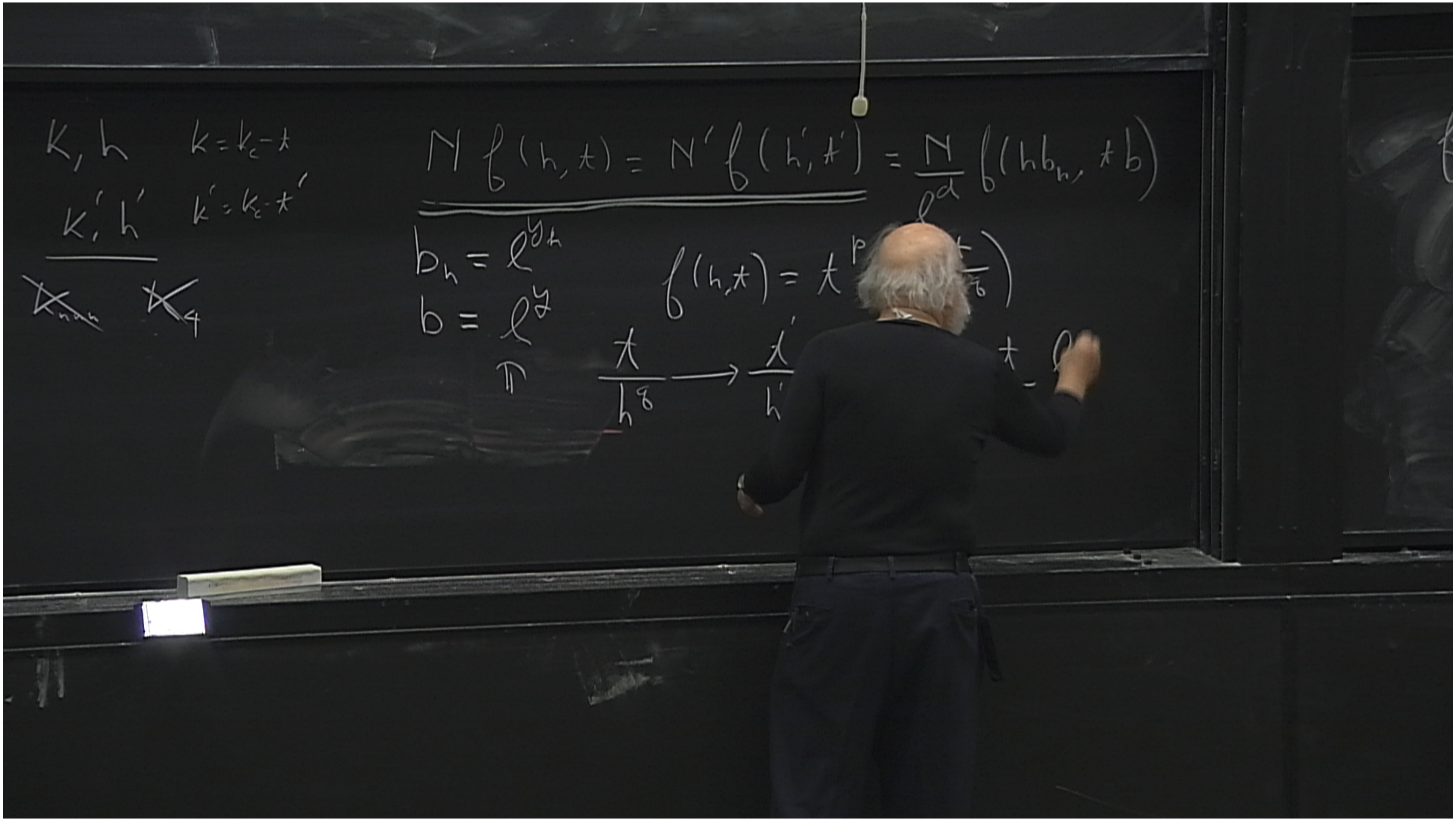

K, h   k = k_c - t
K', h'   k' = k_c - t'
K_nn   K_4
N f(h, t) = N' f(h', t') = N/l^d f(h b_h, t b)
b_h = l^{y_h}
b = l^y

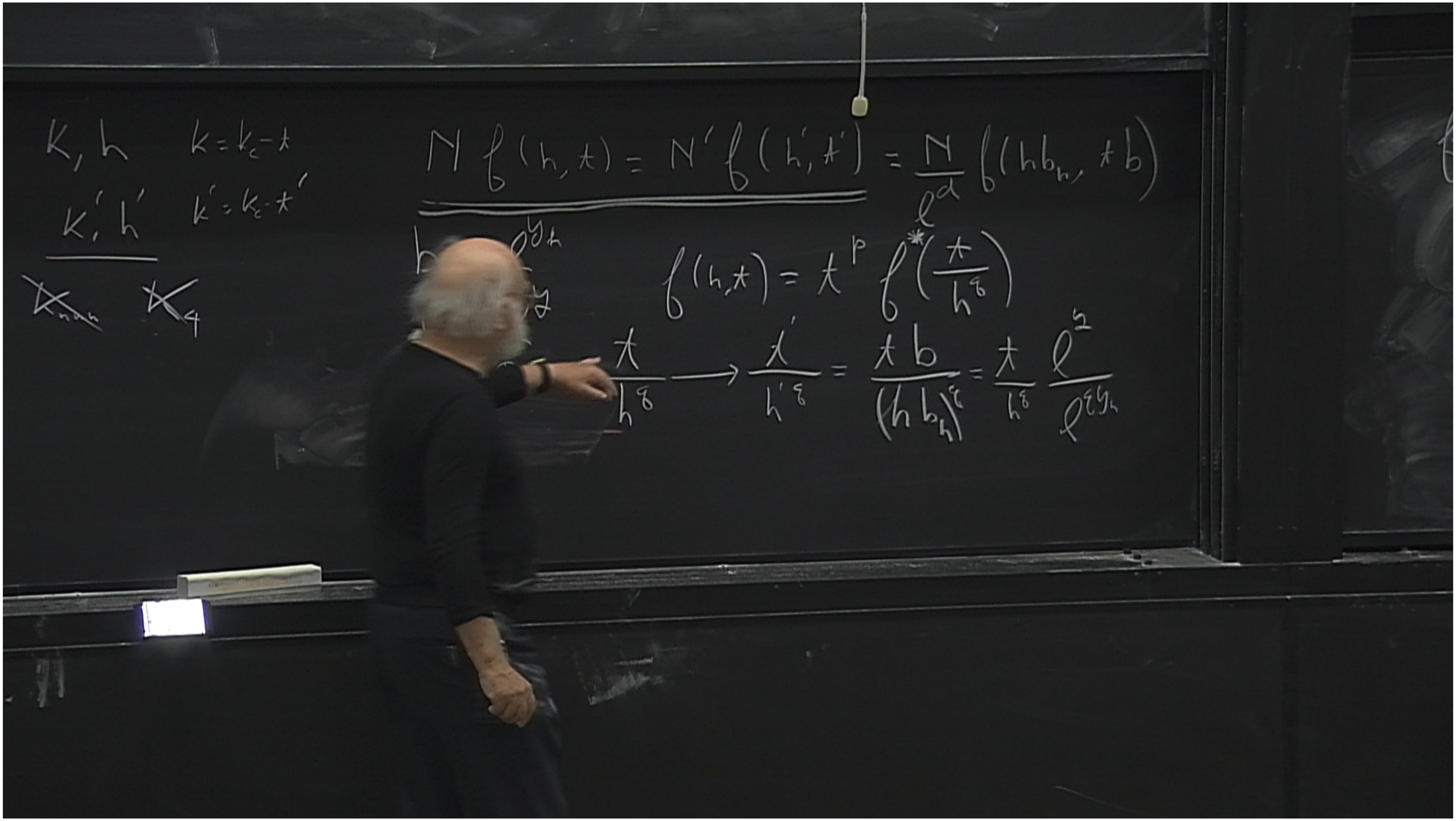

K, h   k = k_c − t
K', h'   k' = k_c − t'
N f(h, t) = N' f(h', t') = N/ℓ^d f(h b_h, t b)
b_h = ℓ^{y_h}
b = ℓ^y


K, h k = kc − t
K′, h′ k′ = kc − t′
N f(h, t) = N′ f(h′, t′) = N/ℓ^d f(h b_h, t b)
b_h = ℓ^(y_h)
b = ℓ^y


K, h   k = k_c − t
K', h'   k' = k_c − t'
N f(h, t) = N' f(h', t') = N/ℓ^d f(h b_h, t b)
b_h =
b =
f(h, t) = t^p f(t/h^q)


K, h   k = k_c − t
k', h'   k' = k_c − t'
K_nn   K_4
N f(h, t) = N' f(h', t') = N / ℓ^d f(h b_h, t b)
b_h = ℓ^{y_h}
b = ℓ^y
t^p f^*(t / h^δ)
t b / h b


K, h
k = k_c − t
K', h'
k' = k_c − t'
N f(h, t) = N' f(h', t') = N/ℓ^d f(h b_h, t b)
b_h = ℓ^{y_h}
b = ℓ^y
f(h, t) = t^p


K, h   k = k_c − t
K', h'   k' = k_c − t'
N f(h, t) = N' f(h', t') = N/ℓ^d f(h b_h, t b)
f(h, t) = t^p f*(t/h^ε)
t/h^ε → t'/h'^ε = t b/(h b_h)^ε = t/h^ε ℓ^2/ℓ^{ε y_h}

$K, h \qquad k = k_c - t$

$K', h' \qquad k' = k_c - t'$

$\underline{K', h'}$

$\cancel{K_{nnn}} \qquad \cancel{K_4}$

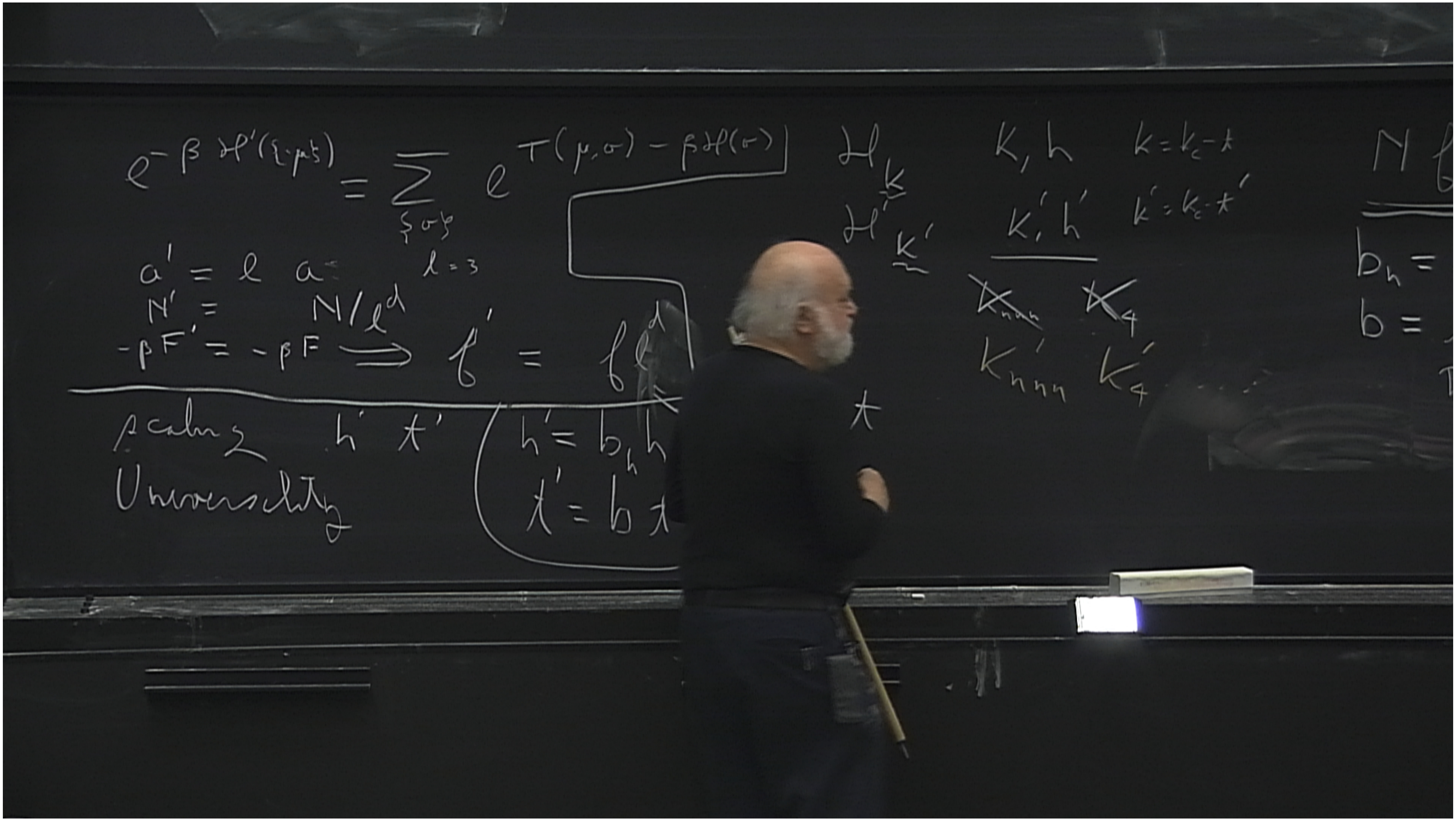
$$\underline{\underline{N f(h,t) = N' f(h',t')}} = \frac{N}{\ell^d} f(h b_h, t b)$$

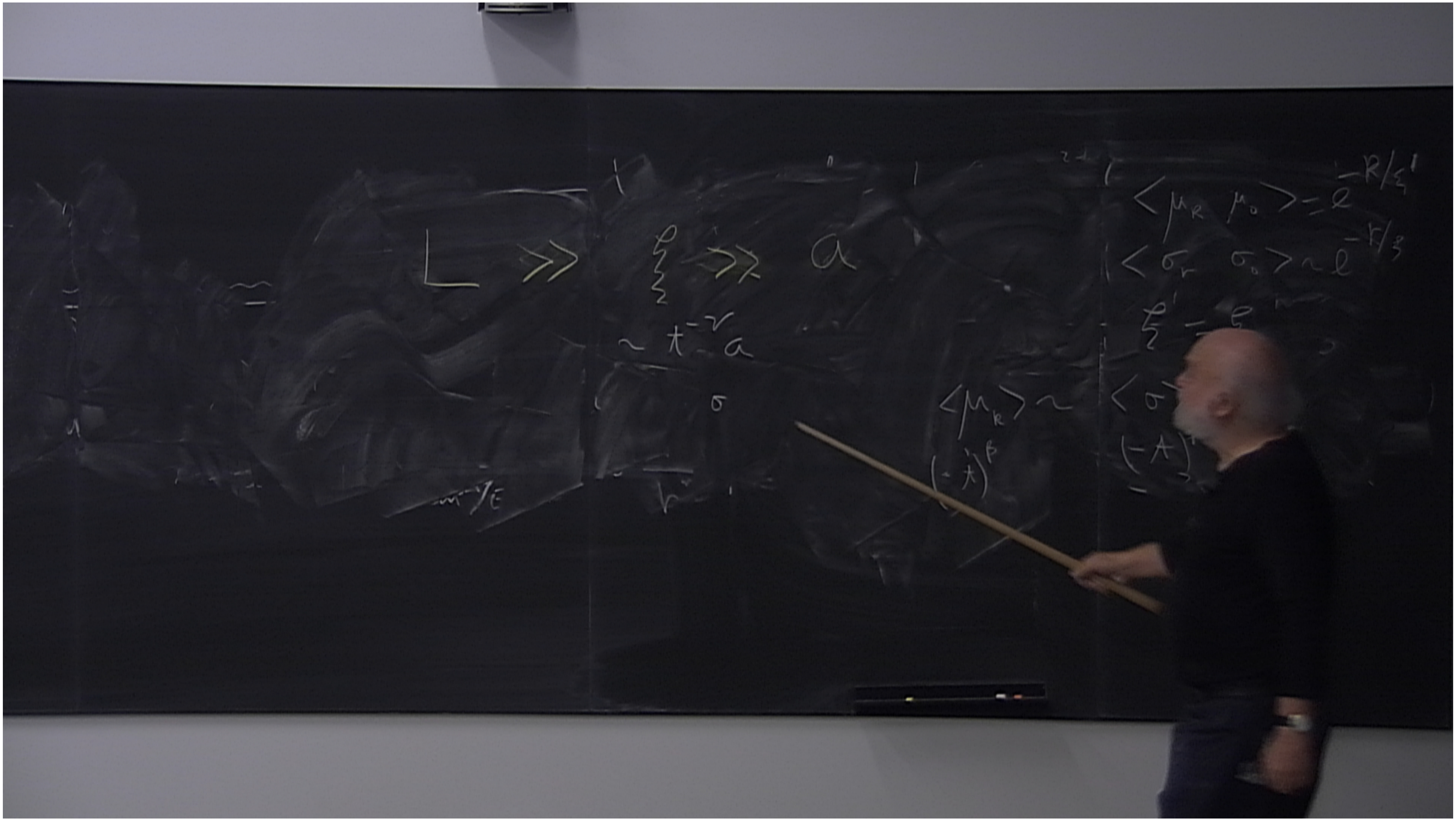
$$b_h = \ell^{y_h}$$

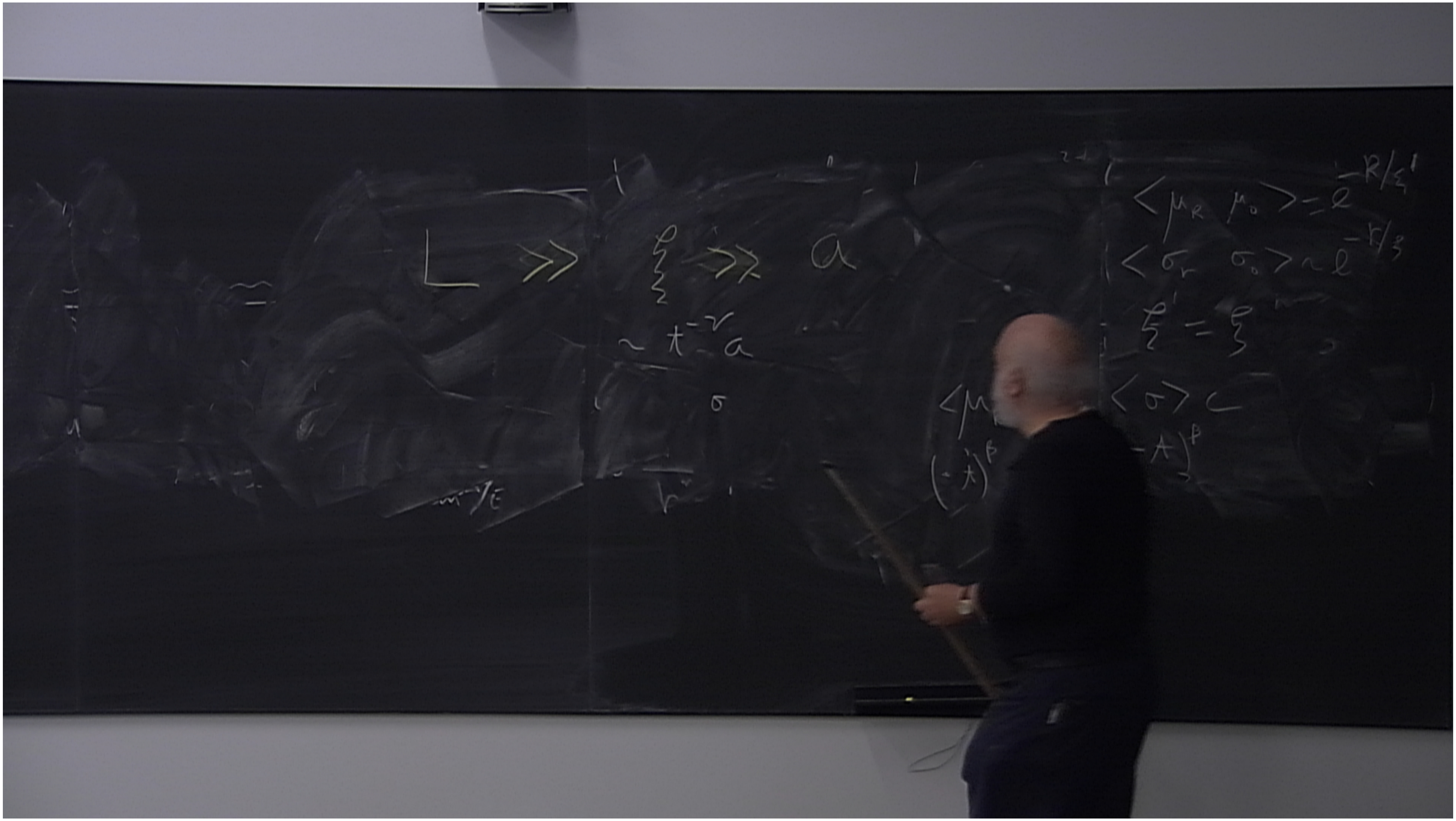
$$b = \ell^{y}$$

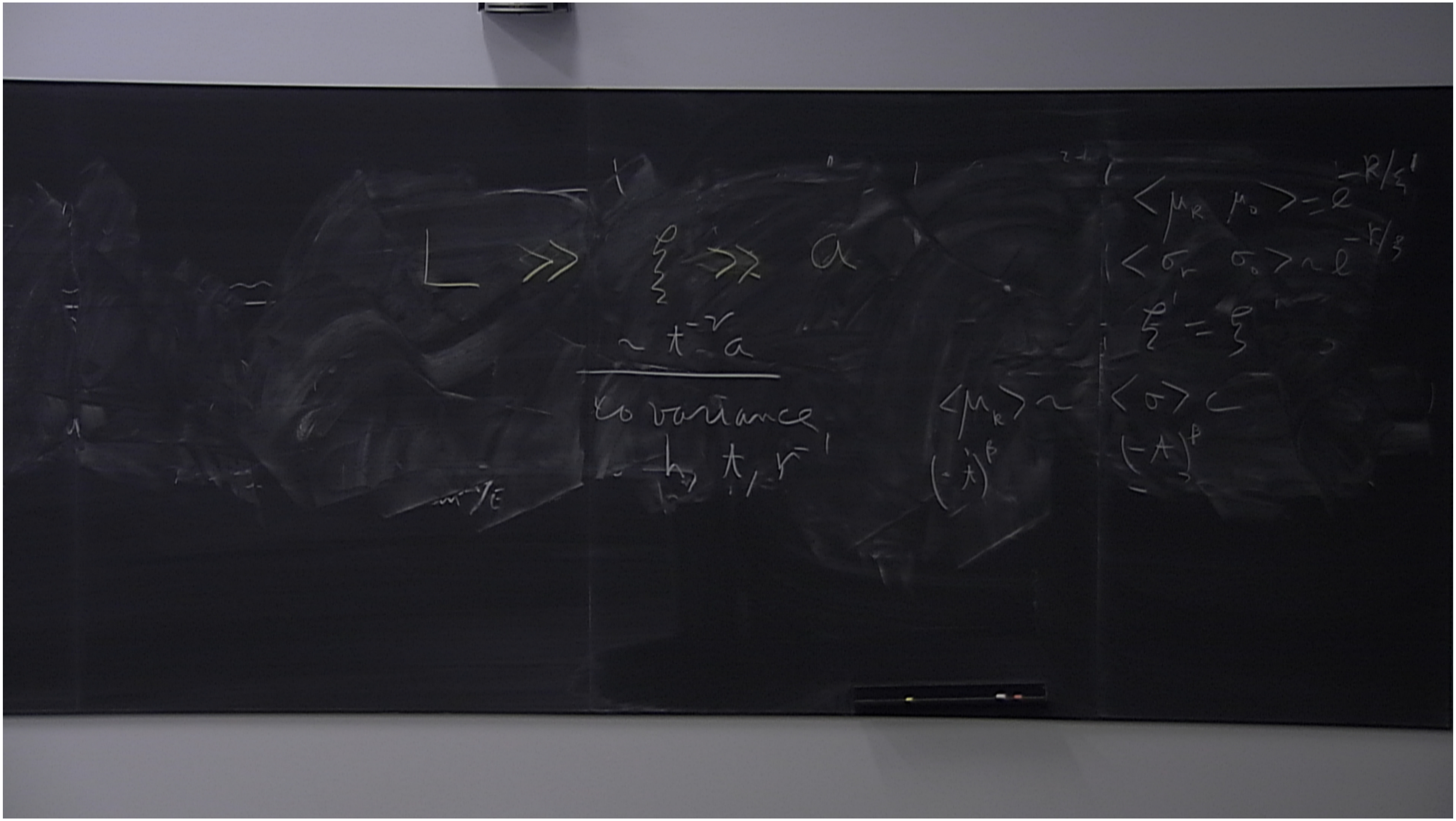
$$f(h,t) = t^p f^*\left(\frac{t}{h^\varepsilon}\right)$$

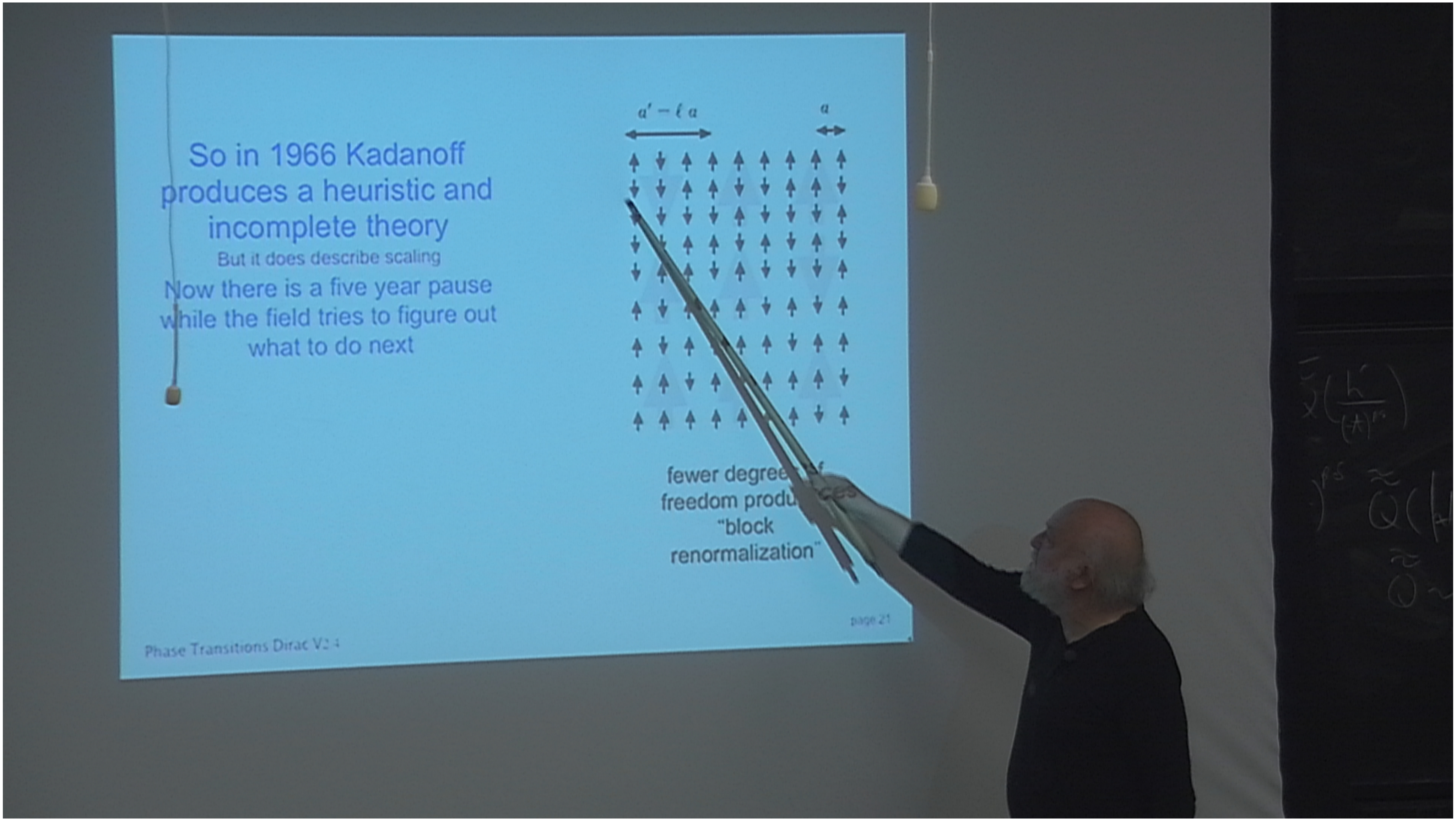
$$\frac{t}{h^\varepsilon} \longrightarrow \frac{t'}{h'^\varepsilon} = \frac{t b}{(h b_h)^\varepsilon} = \frac{t}{h^\varepsilon} \frac{\ell^y}{\ell^{\varepsilon y_h}}$$

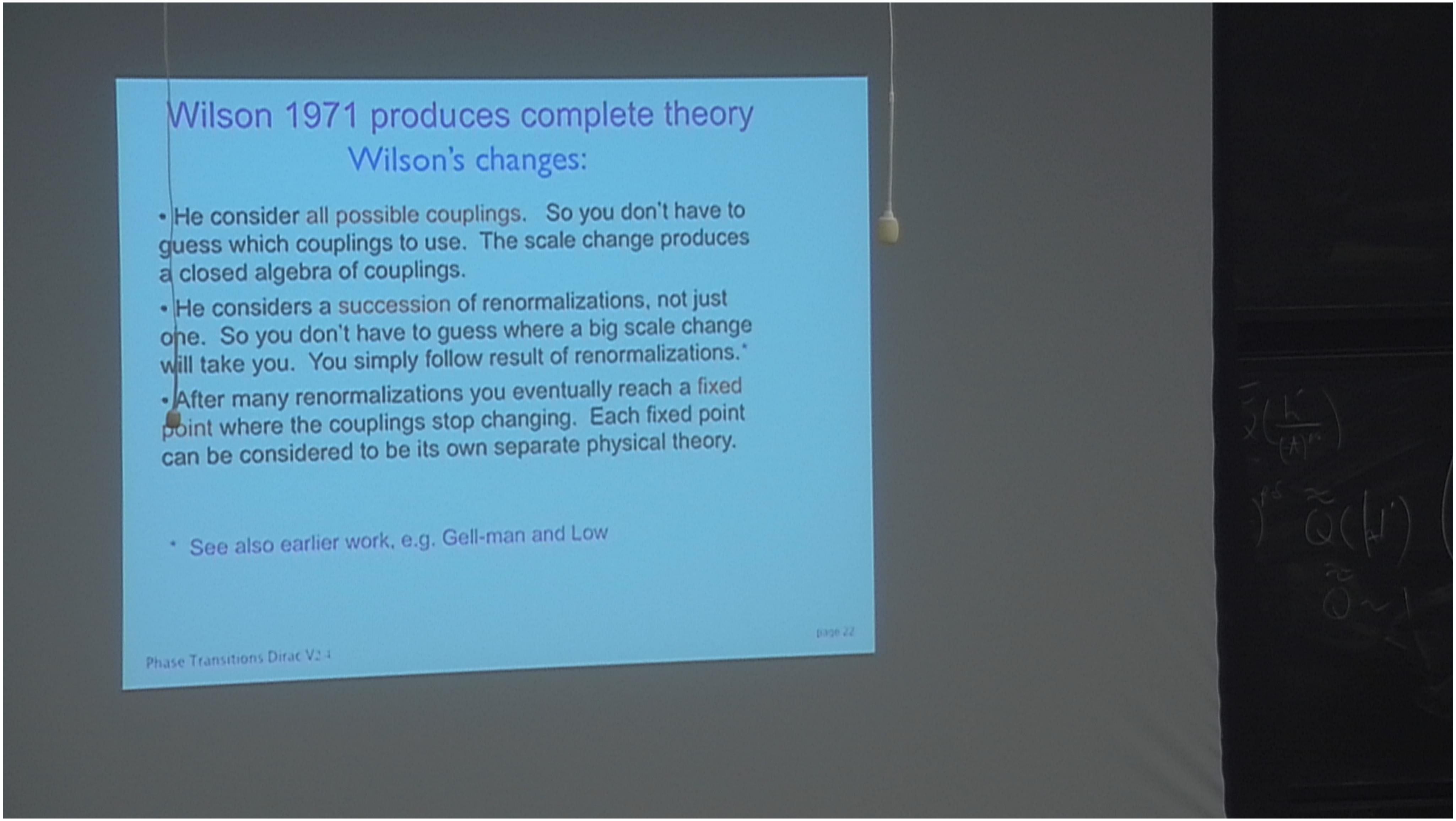

a' = l a
l = 3
N' = N/l^d
-βF' = -βF
K, h
K', h'
k = k_c - t
k' = k_c - t'
K_nnn
K_4
scaling h' t'
h' = b_h h
t' = b t
Universality
b_h =
b =

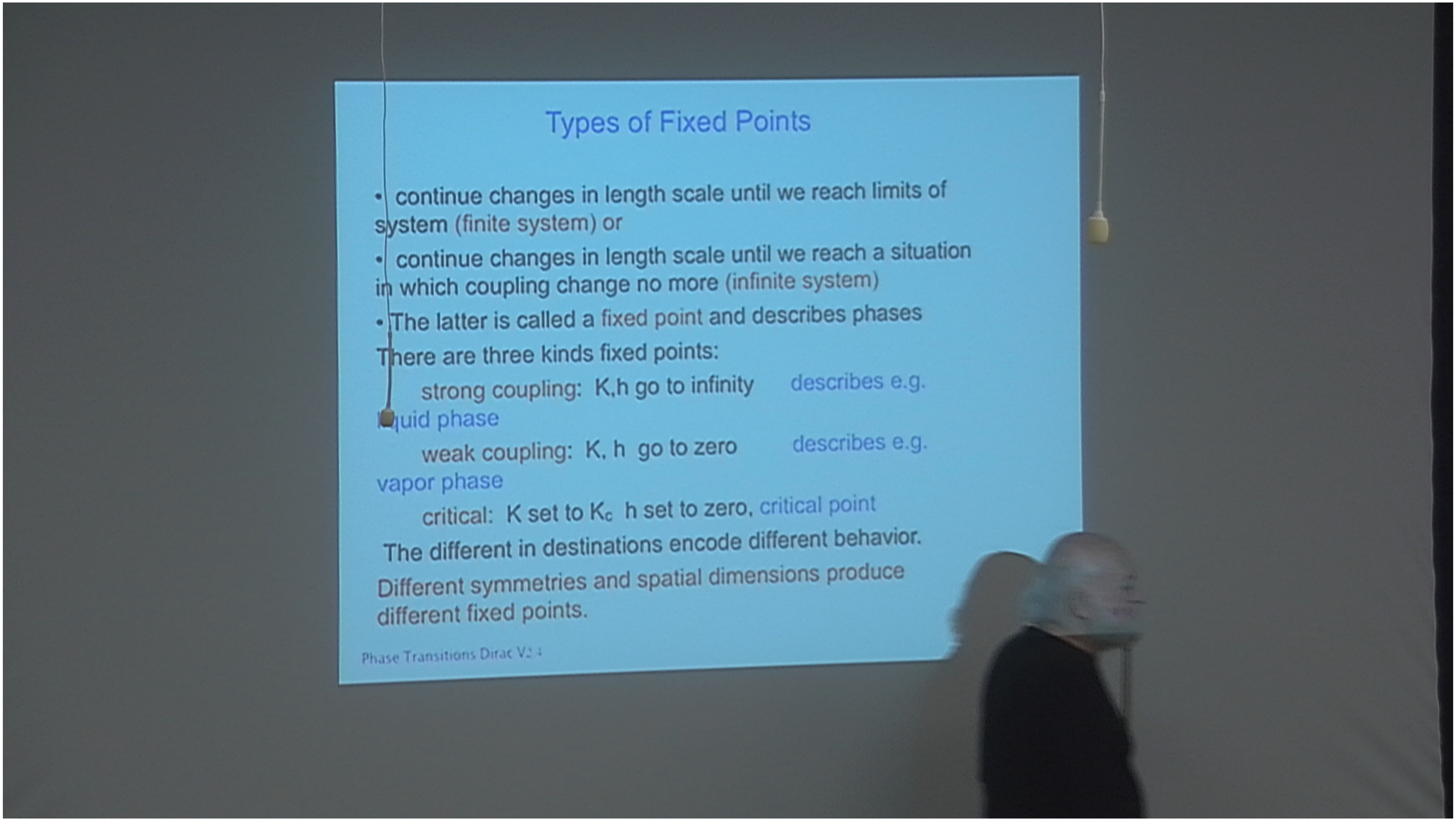

L >> ξ >> a
⟨μ_R μ_0⟩ = e^{-R/ξ'}
⟨σ_r σ_0⟩ ~ e^{-r/ξ}

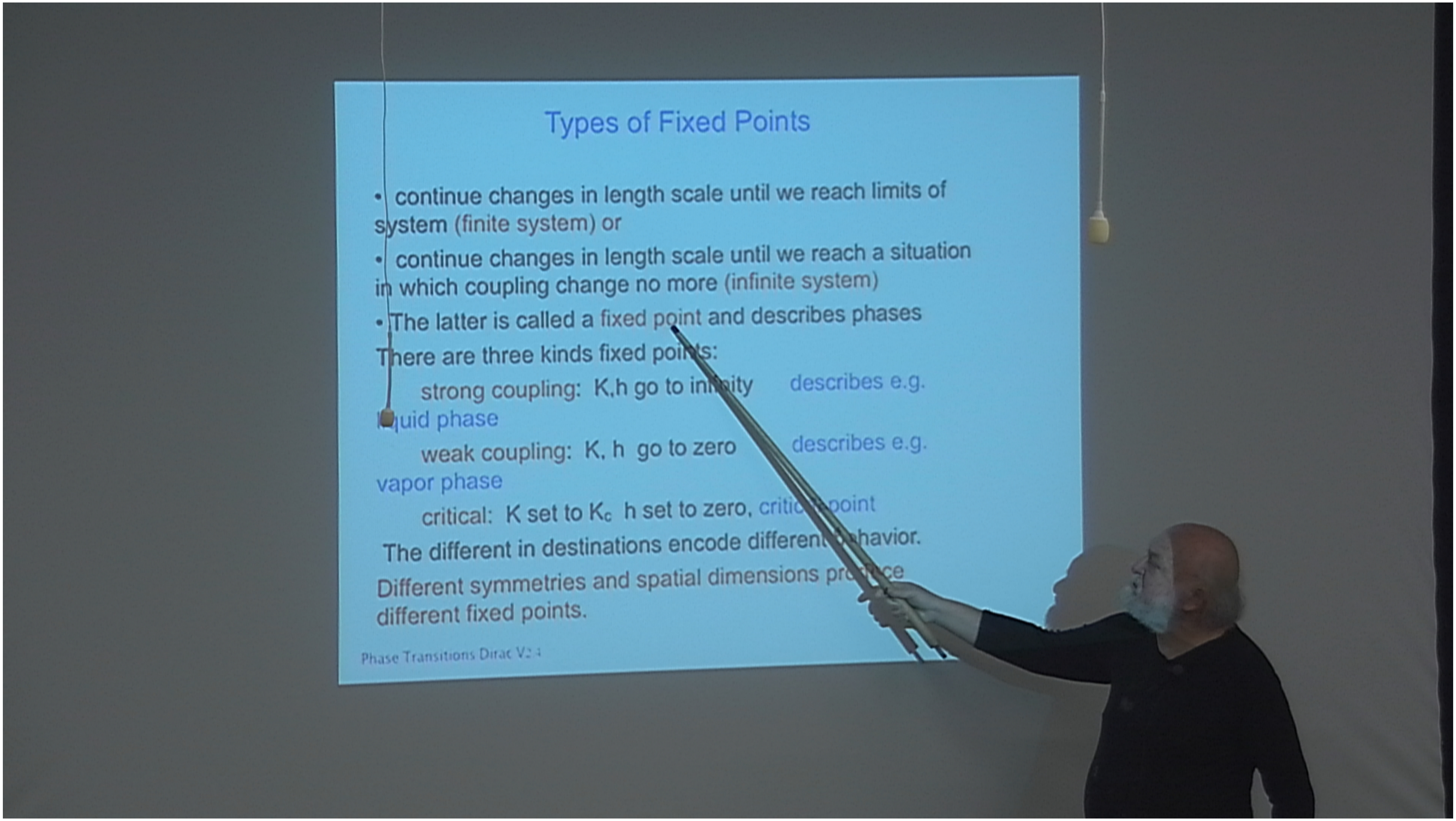

L >> ξ >> a
~ t^{-ν} a
⟨μ_R μ_0⟩ = e^{-R/ξ'}
⟨σ_r σ_0⟩ ~ e^{-r/ξ}
ξ' = ξ
(-t)^β


$L \gg \xi \gg a$
$\sim t^{-\nu} a$
Covariance
$\sim h, t, r^{-1}$
$\langle \mu_R \; \mu_0 \rangle = e^{-R/\xi'}$
$\langle \sigma_r \; \sigma_0 \rangle \sim e^{-r/\xi}$
$\xi' = \xi$
$\langle \mu_R \rangle \sim (-t)^{\beta}$
$\langle \sigma \rangle \sim (-t)^{\beta}$


So in 1966 Kadanoff produces a heuristic and incomplete theory
But it does describe scaling
Now there is a five year pause while the field tries to figure out what to do next
$a' - \ell a$
a
fewer degrees of freedom produces "block renormalization"
page 21
Phase Transitions Dirac V2.4


Wilson 1971 produces complete theory
Wilson's changes:
• He consider all possible couplings. So you don't have to guess which couplings to use. The scale change produces a closed algebra of couplings.
• He considers a succession of renormalizations, not just one. So you don't have to guess where a big scale change will take you. You simply follow result of renormalizations.*
• After many renormalizations you eventually reach a fixed point where the couplings stop changing. Each fixed point can be considered to be its own separate physical theory.
* See also earlier work, e.g. Gell-man and Low
page 22
Phase Transitions Dirac V2.4


Types of Fixed Points
• continue changes in length scale until we reach limits of system (finite system) or
• continue changes in length scale until we reach a situation in which coupling change no more (infinite system)
• The latter is called a fixed point and describes phases
There are three kinds fixed points:
strong coupling: K,h go to infinity describes e.g. liquid phase
weak coupling: K, h go to zero describes e.g. vapor phase
critical: K set to Kc h set to zero, critical point
The different in destinations encode different behavior.
Different symmetries and spatial dimensions produce different fixed points.
Phase Transitions Dirac V2.4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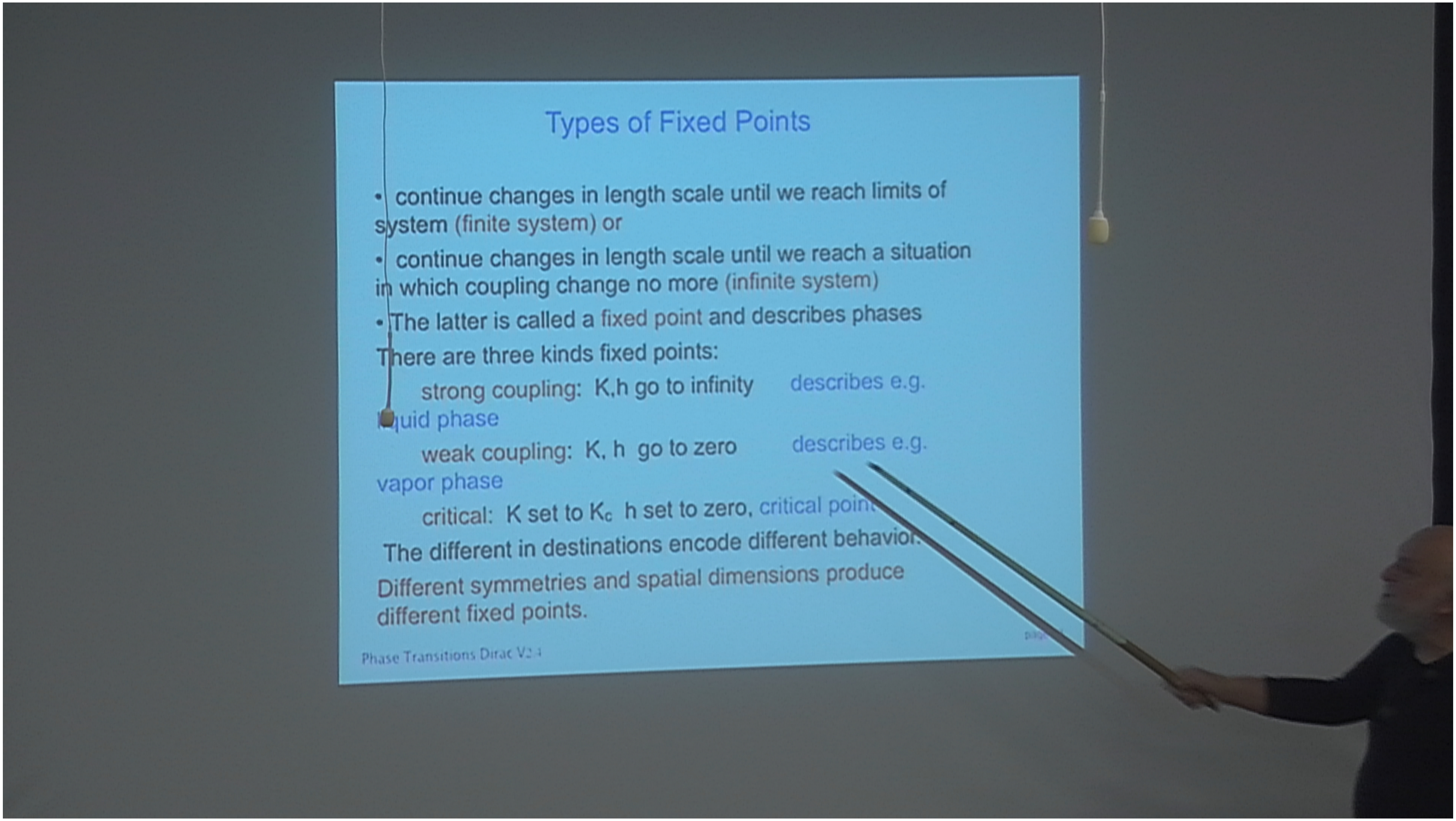
Phase Transitions Dirac V2.4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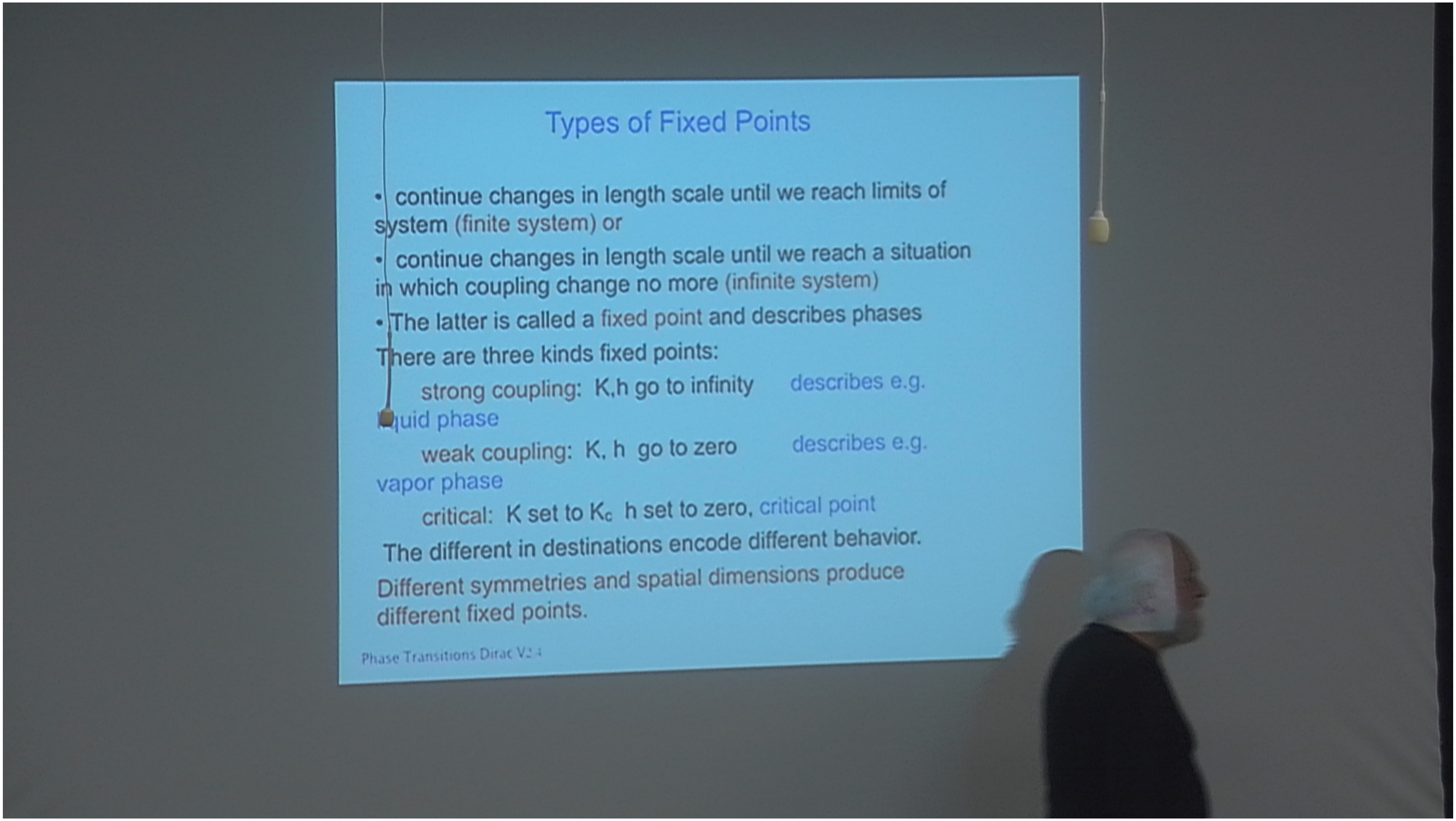
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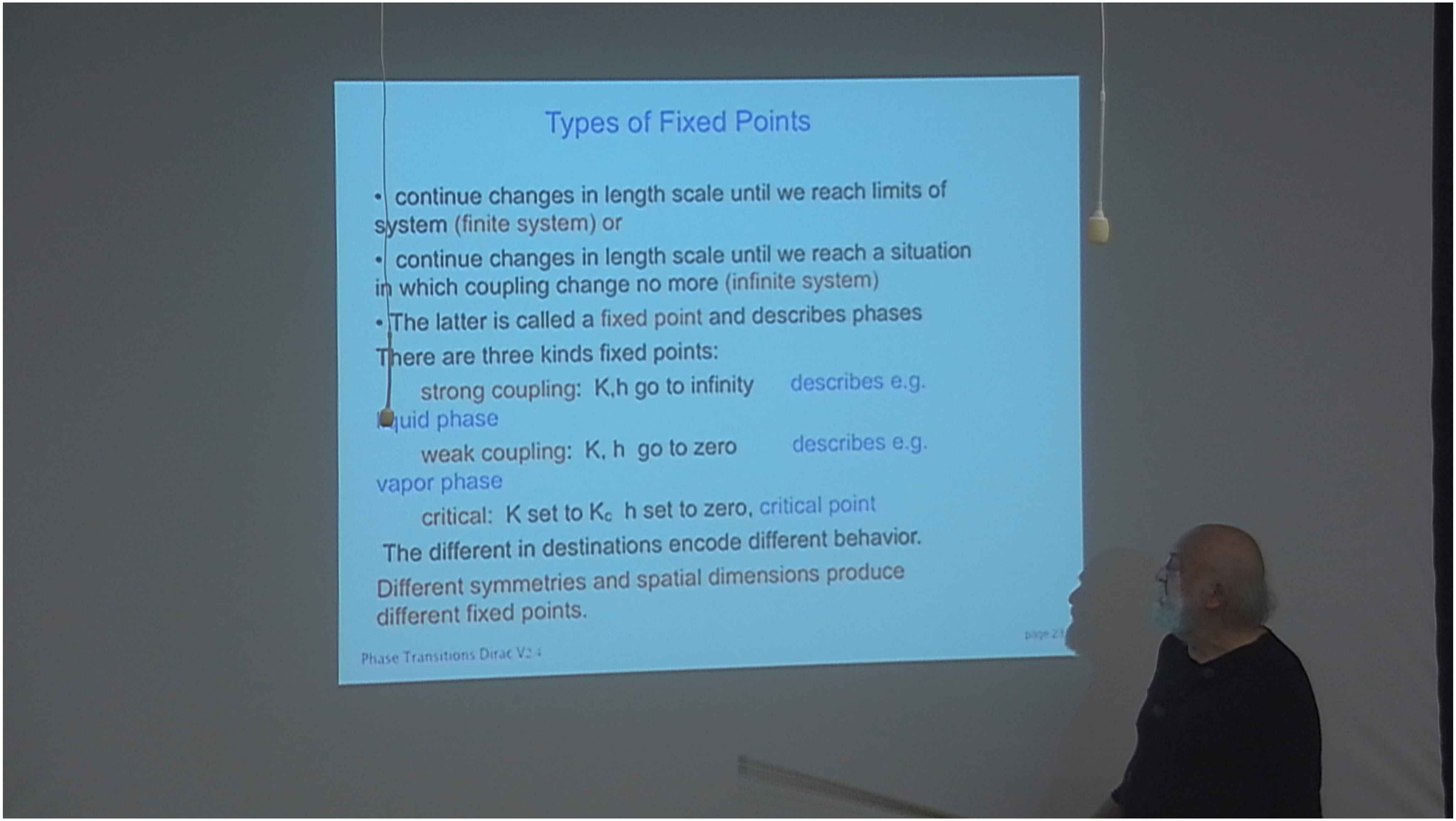
Types of Fixed Points
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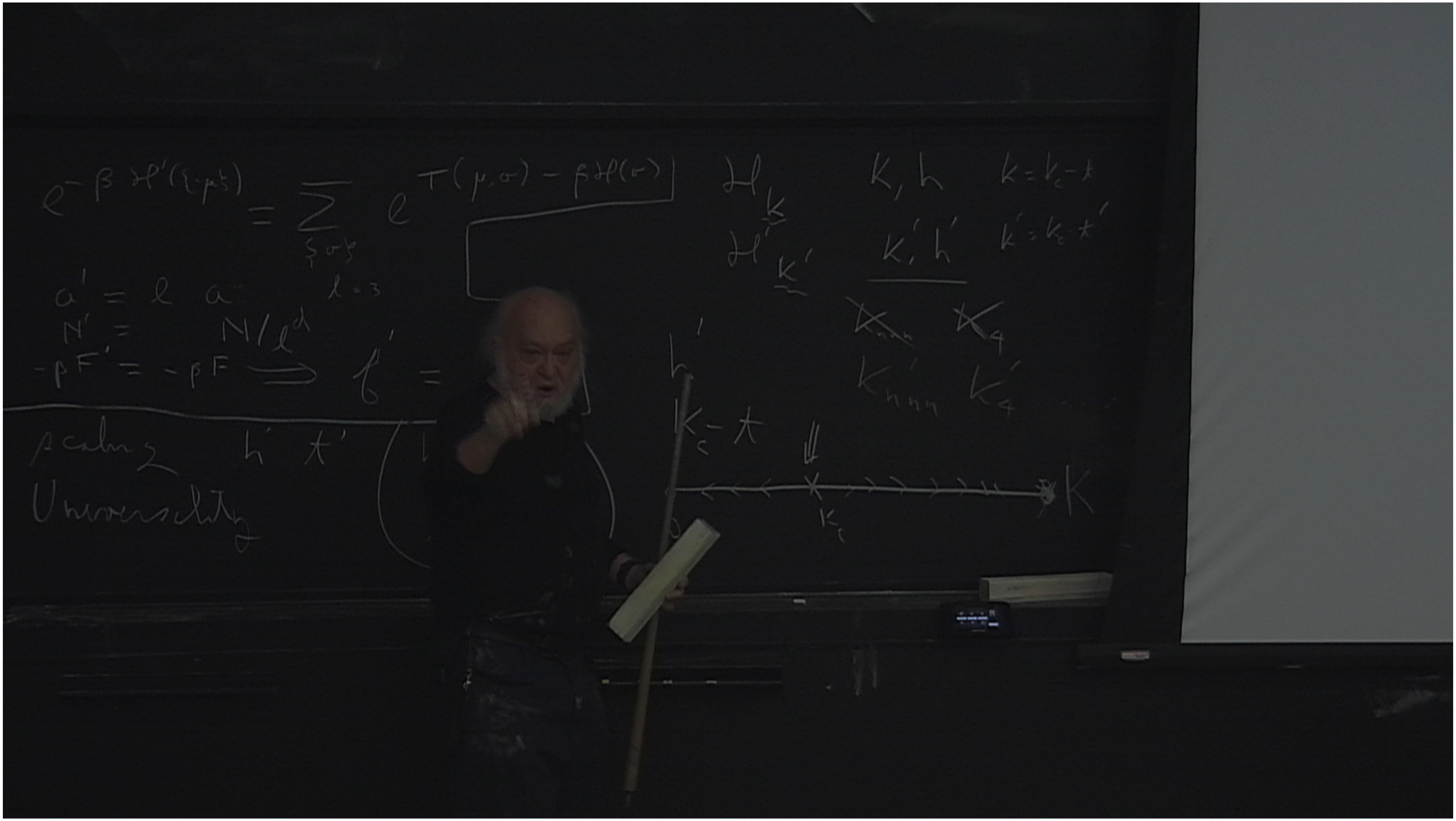
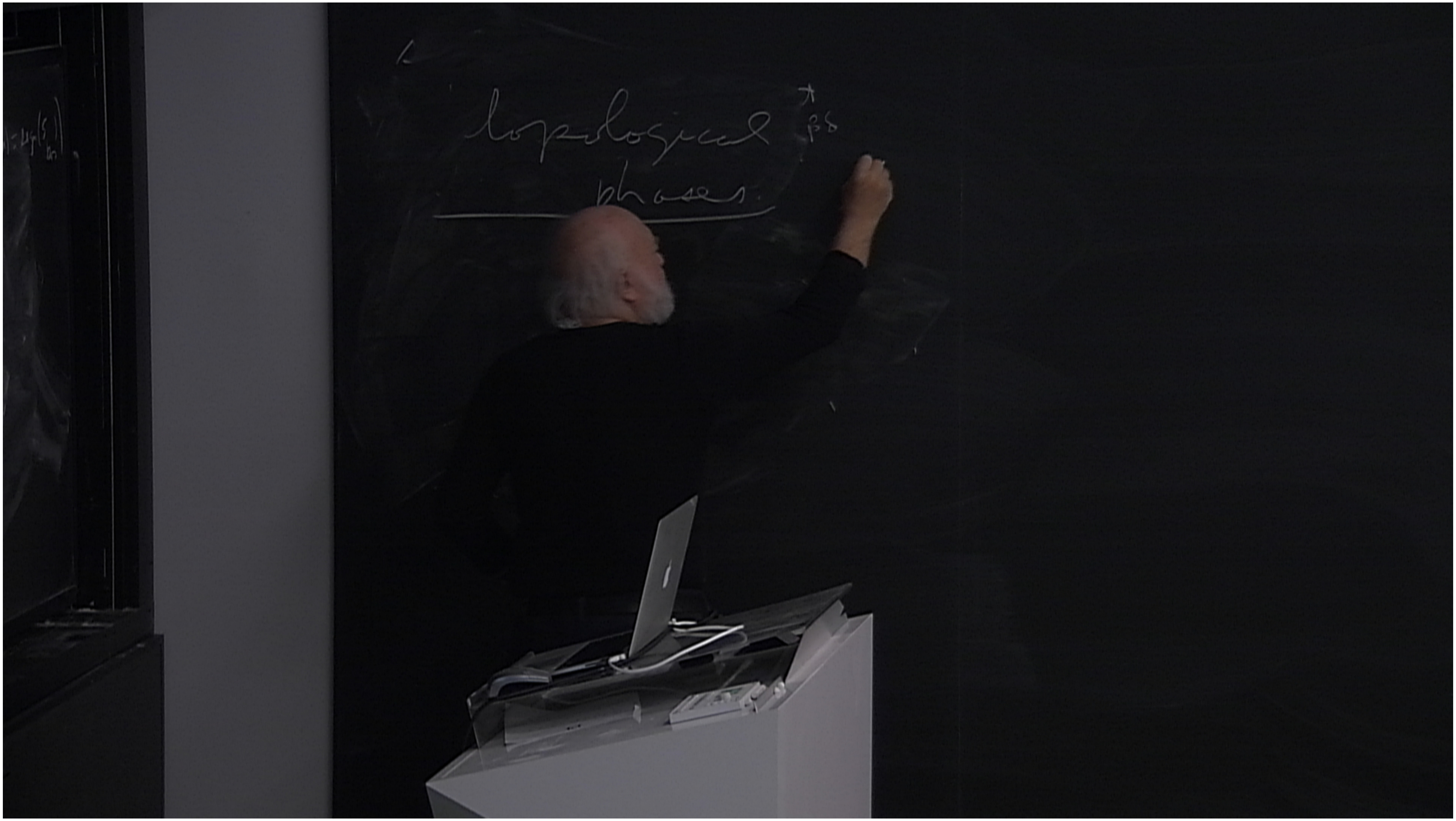
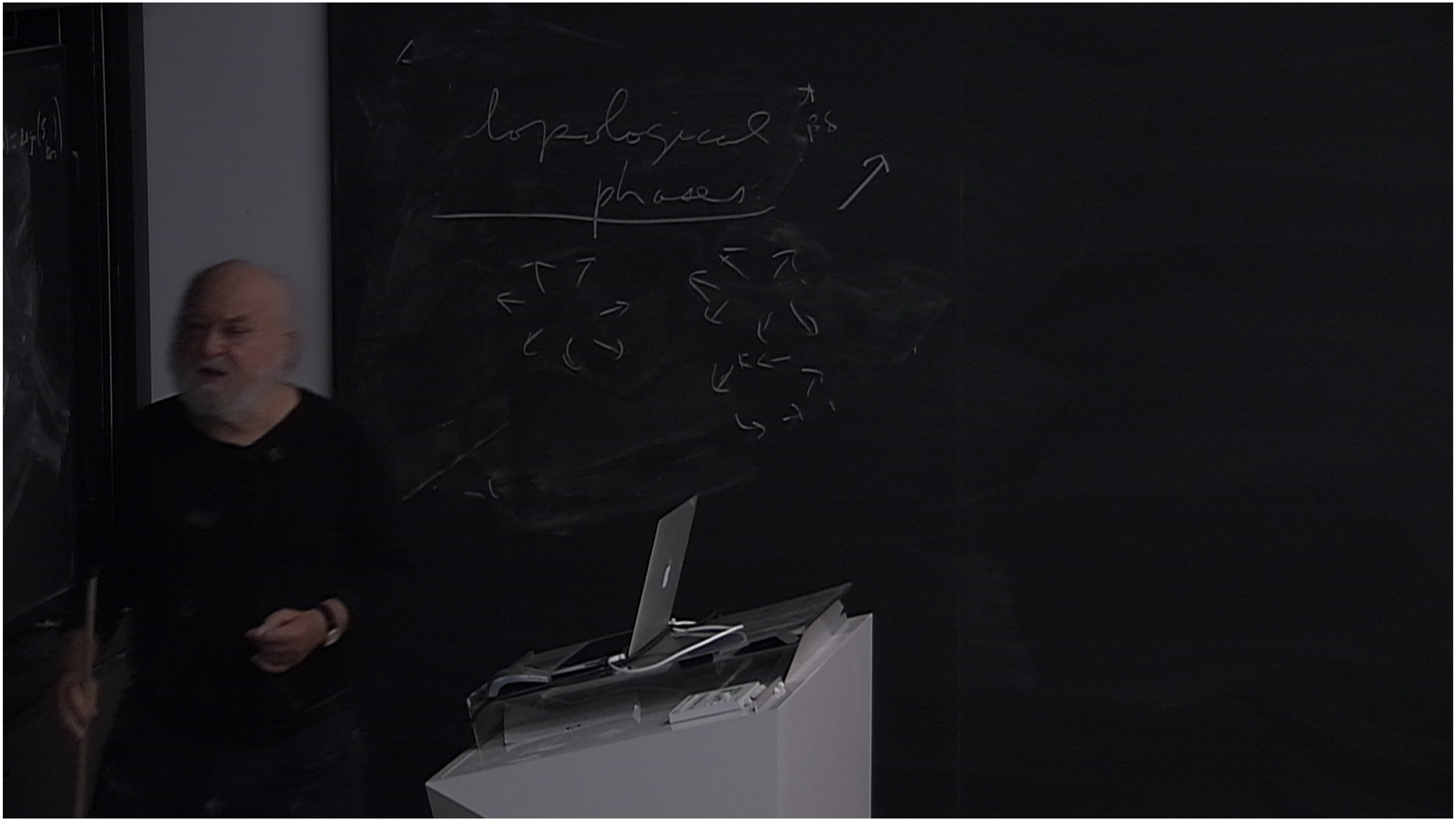
Phase Transitions Dirac V2.4




topological
phases


topological
phases

Franz Wegner: At a particular fixed point there is a list of couplings.

We use eigenvalue analysis to pick out the linear combinations of couplings which have a simple change under the renormalization analysis:

$$K'_\mu = K_\mu \, \ell^{y_\mu}$$

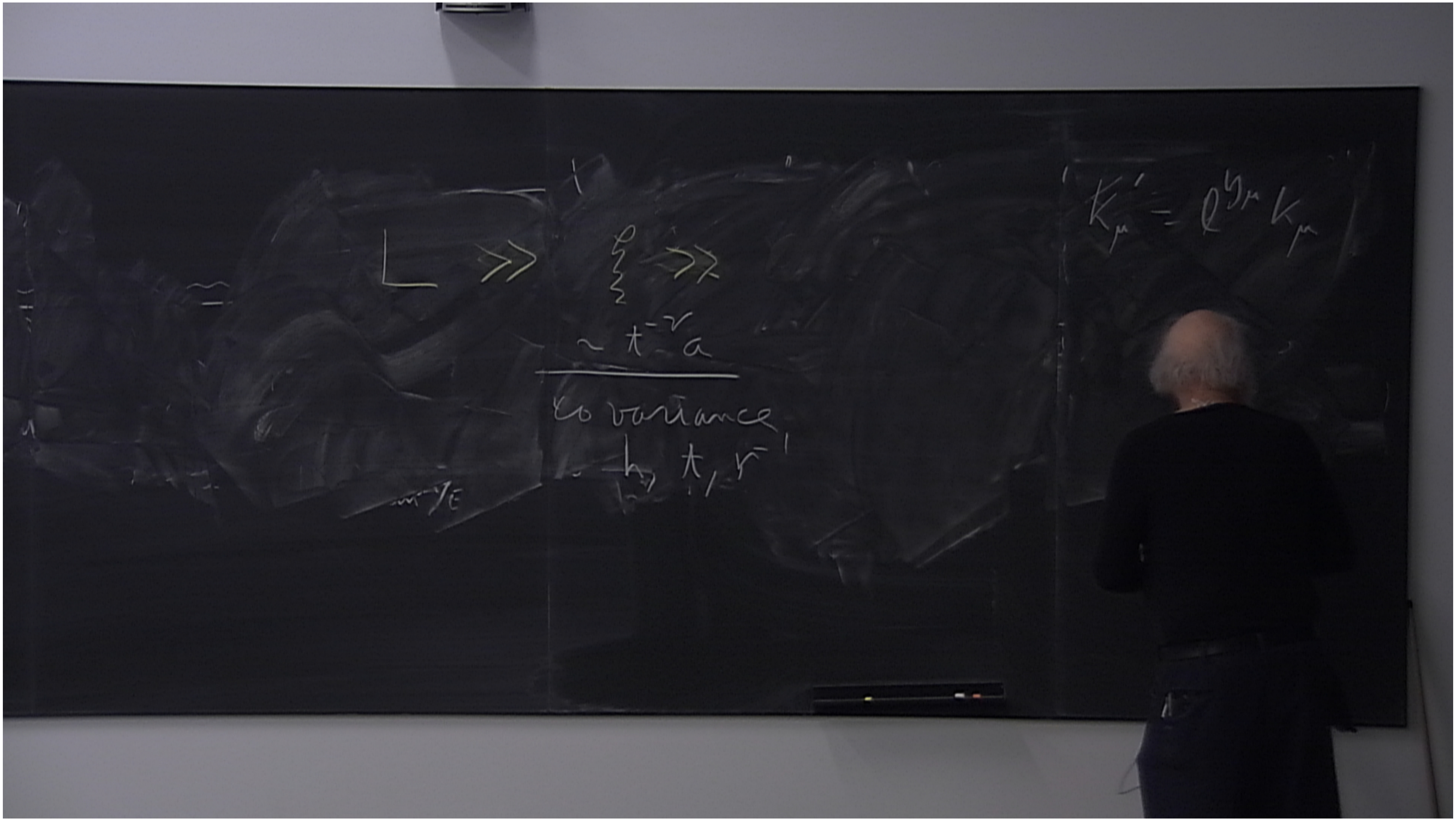
These couplings appear in the near-critical Hamiltonian in the form of a linear variation about the fixed point Hamiltonian. $\mathcal{H}^*$

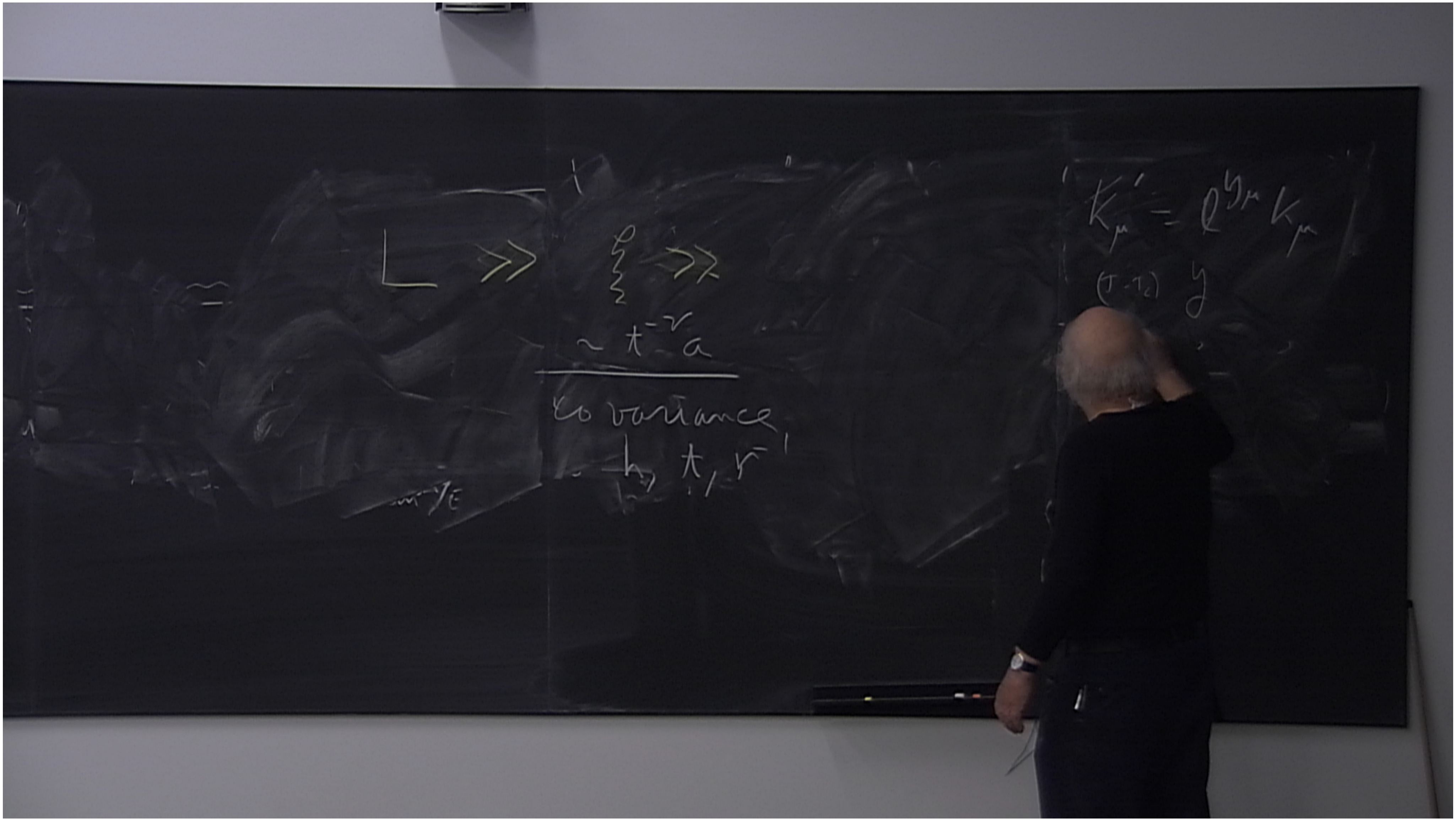
$$\mathcal{H}' = \mathcal{H}^* + \sum_\mu \int d\mathbf{r} \; K_\mu O_\mu(\mathbf{r})$$

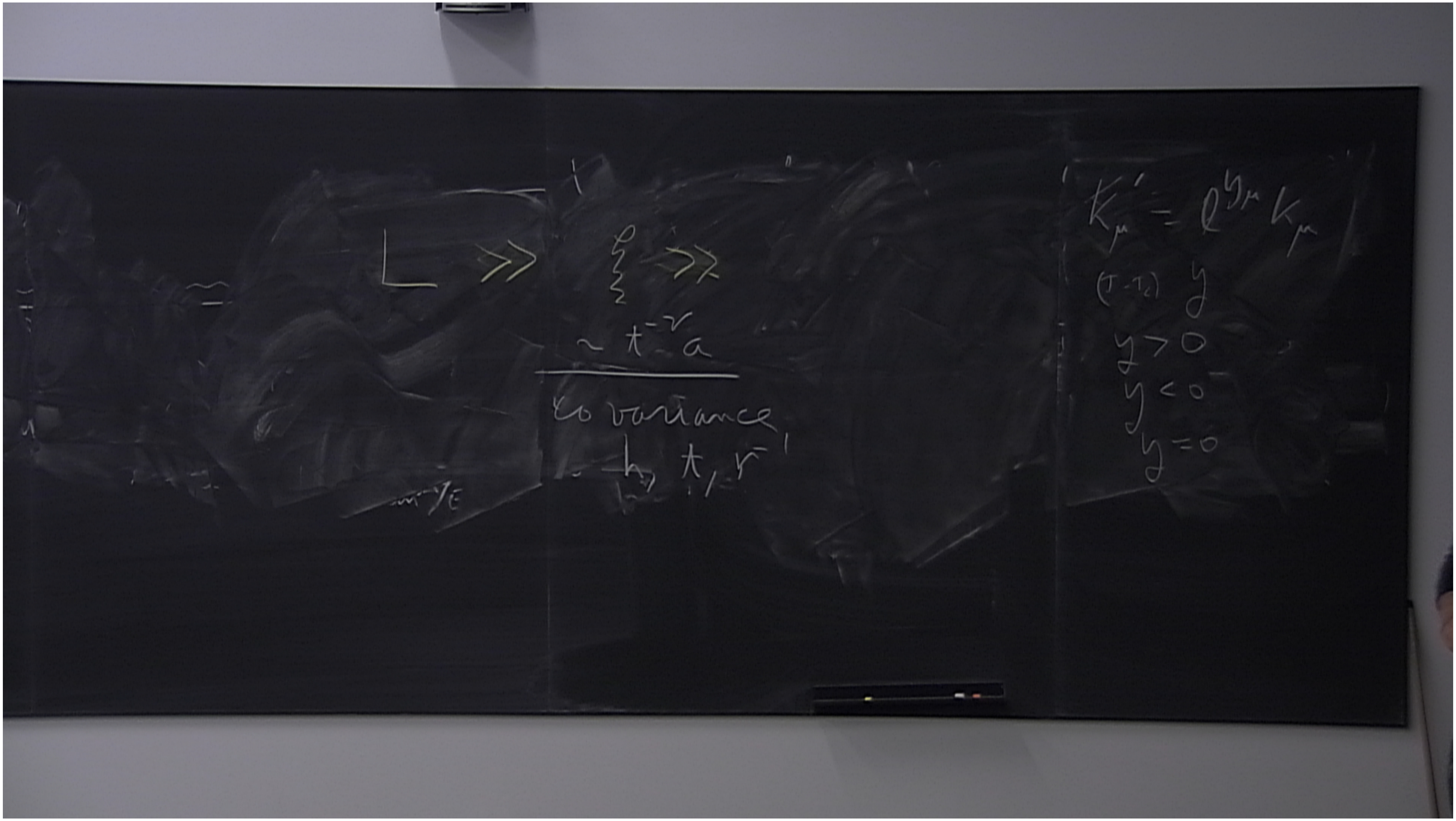
Here the $O_\mu(\mathbf{r})$ describes the local density of some fluctuating quantity, like σ_r. This particular one is conjugate to the coupling K_μ

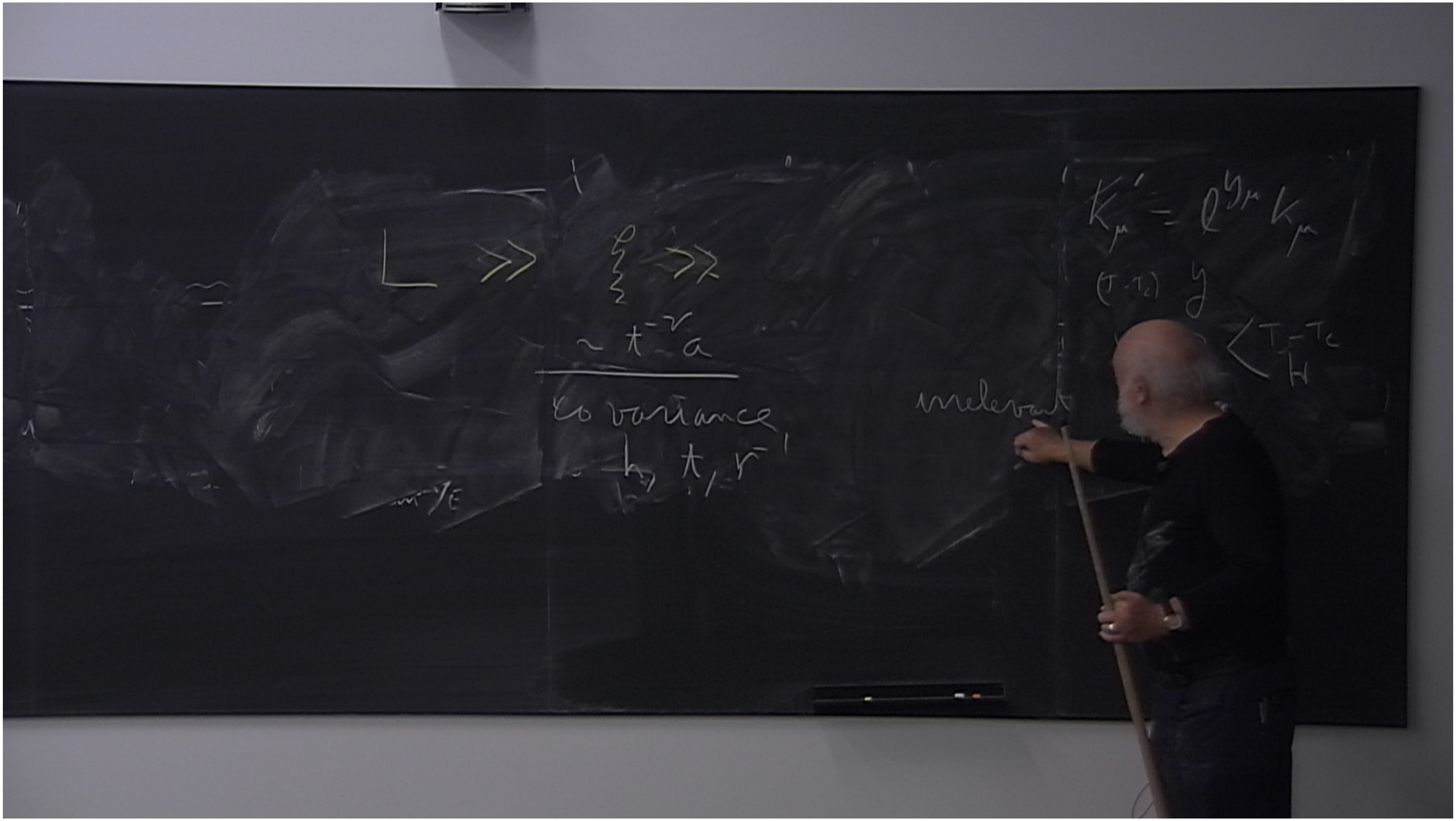
If the coupling scales with an index y_μ, then the local operator, O, scales with an index $x_\mu = d - y_\mu$.

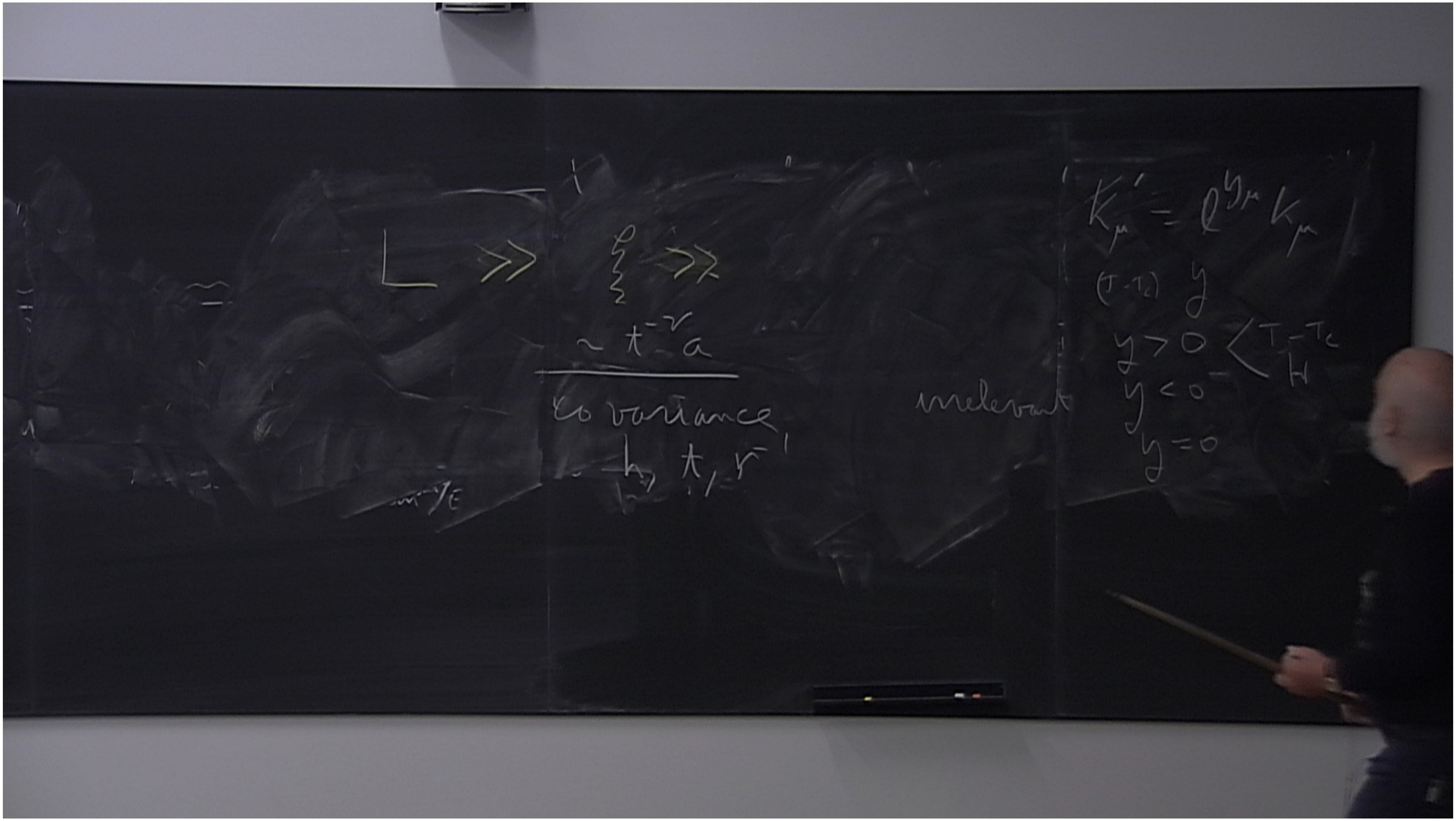
Phase Transitions Dirac V2.4

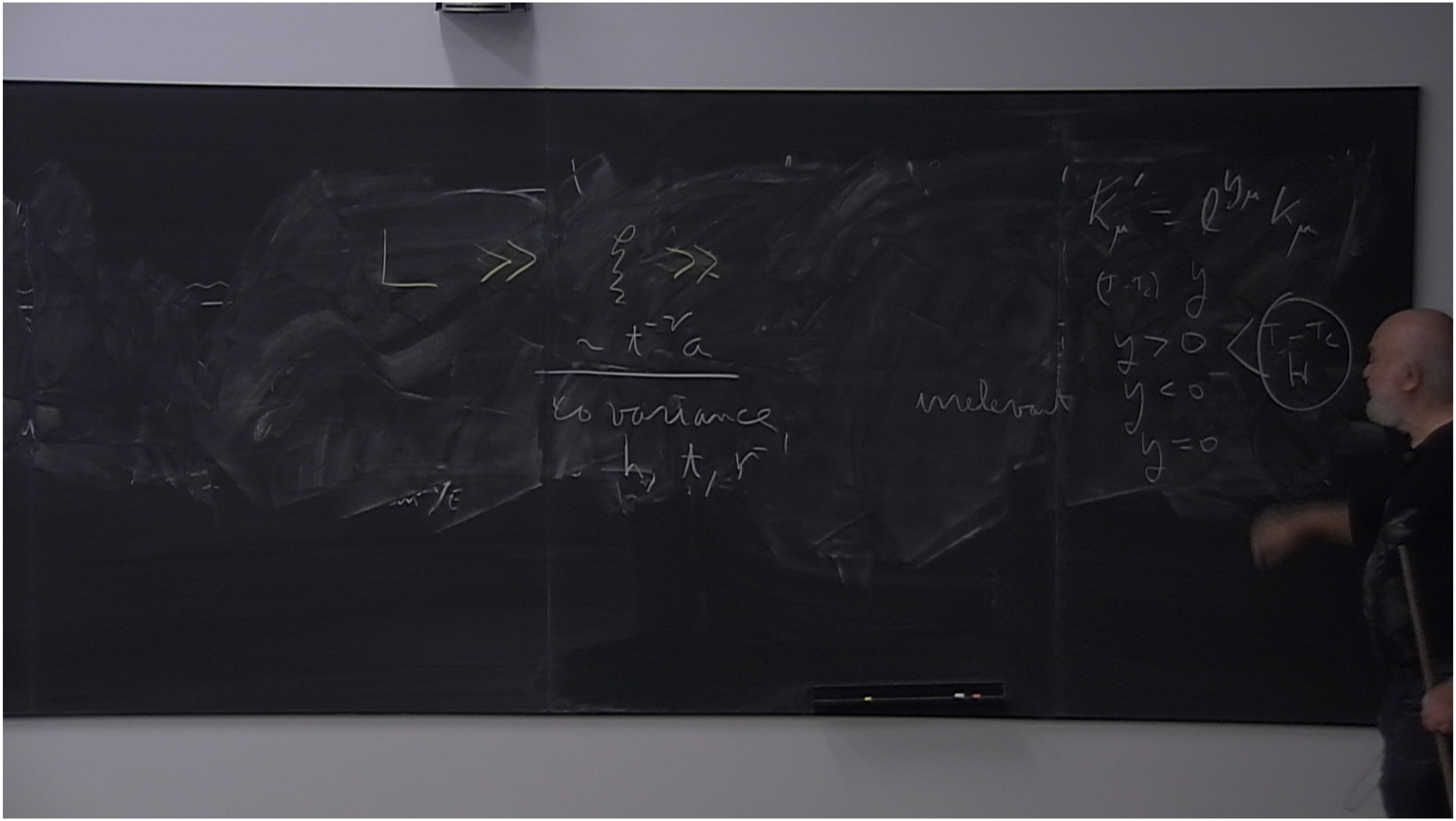

L >> ℓ >>
~ t^-ν a
co variance
h, t, r^-1
K_μ' = ℓ^{y_μ} K_μ


co variance


L >> ξ >> a
~ t^-ν a
covariance
h, t, r^-1
K_μ' = l^yμ K_μ
(T - T_c) y
y > 0
y < 0
y = 0


$K_\mu' = \ell^{y_\mu} K_\mu$
$\sim t^{-\nu} a$
covariance


L >> ξ >> a
~ t^-ν a
co variance
h, t, r^-1
irrelevant
K'_μ = l^yμ K_μ
(T-Tc) y
y > 0 < T - Tc, H
y < 0
y = 0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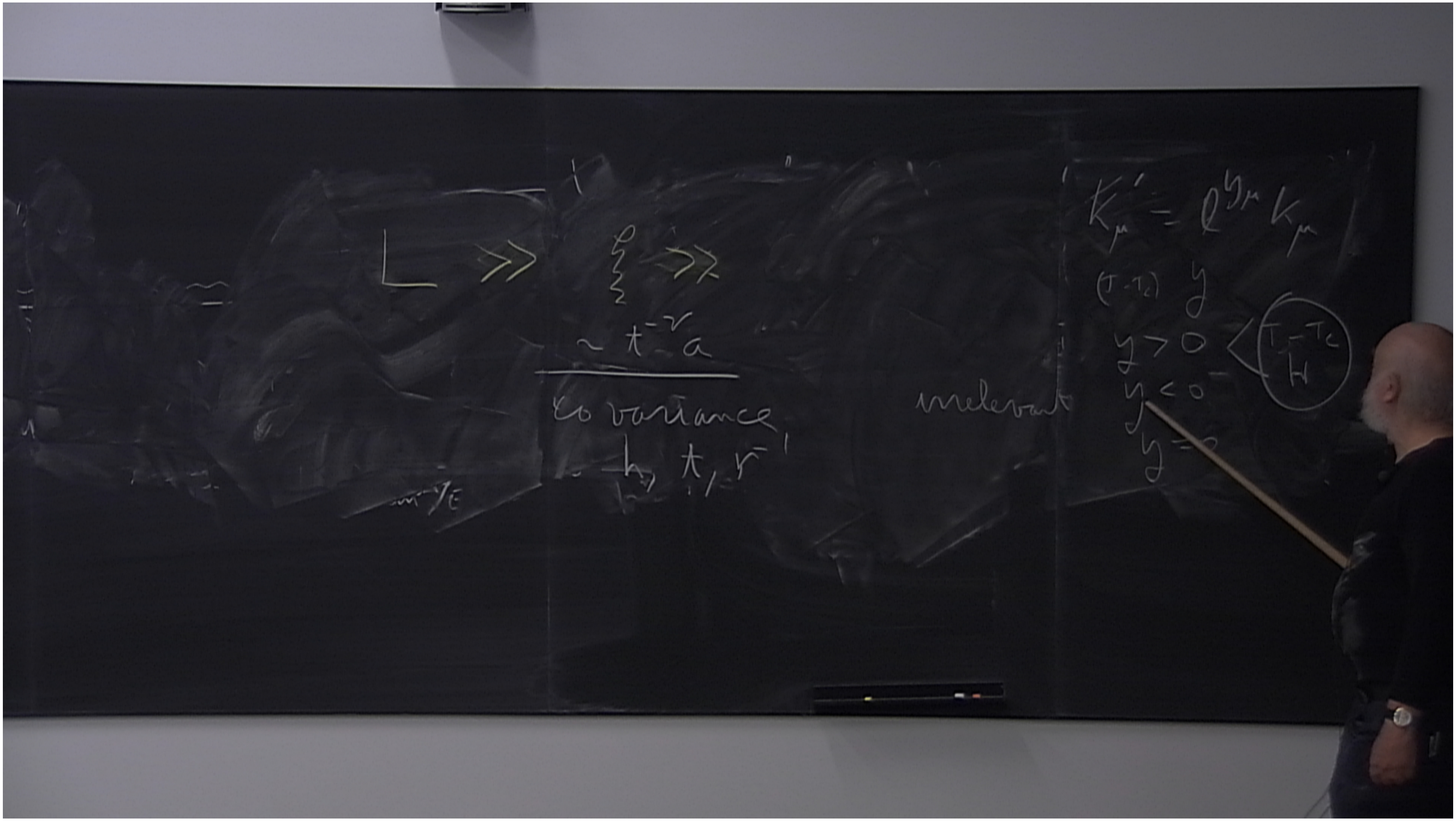
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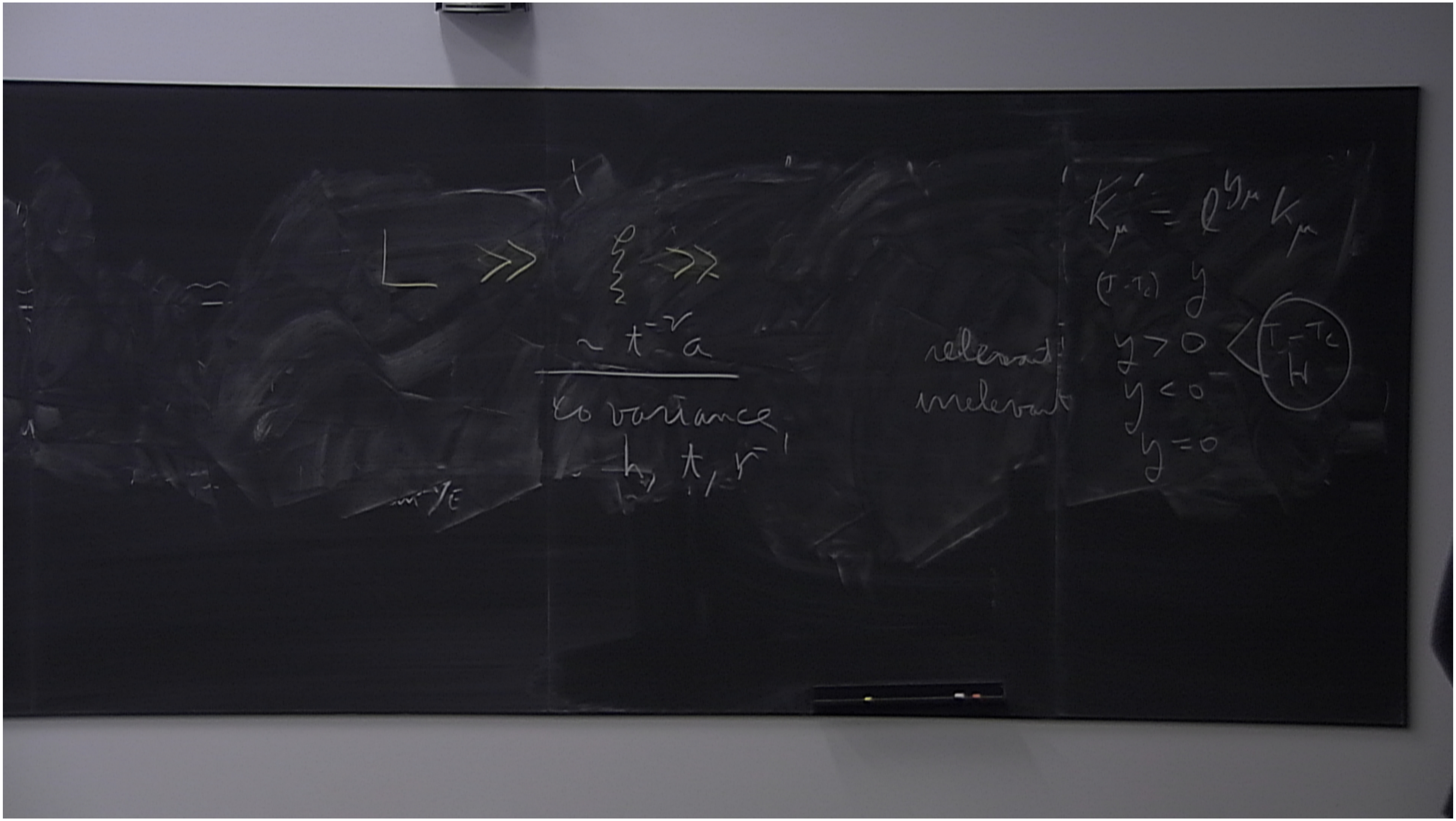
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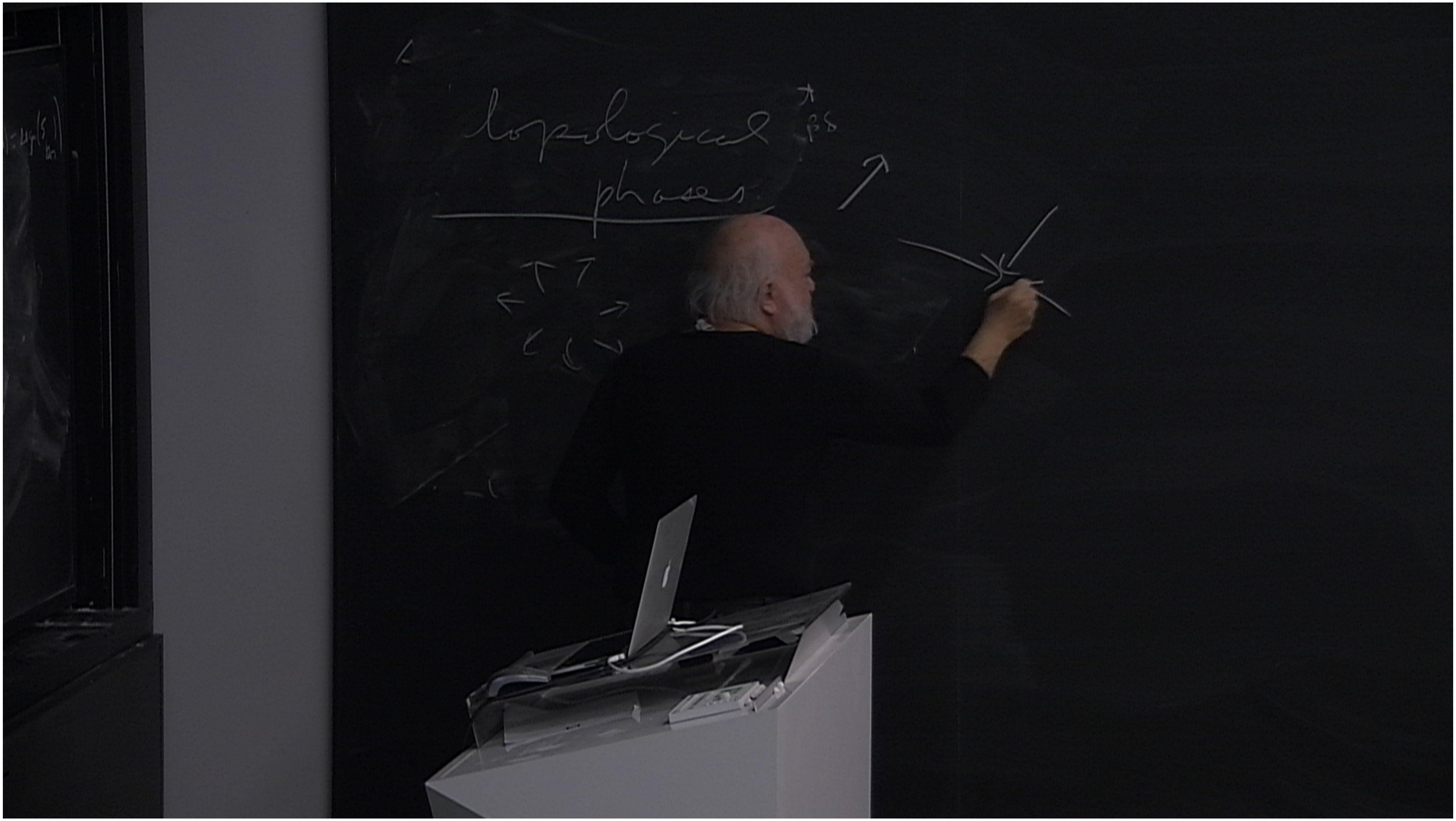
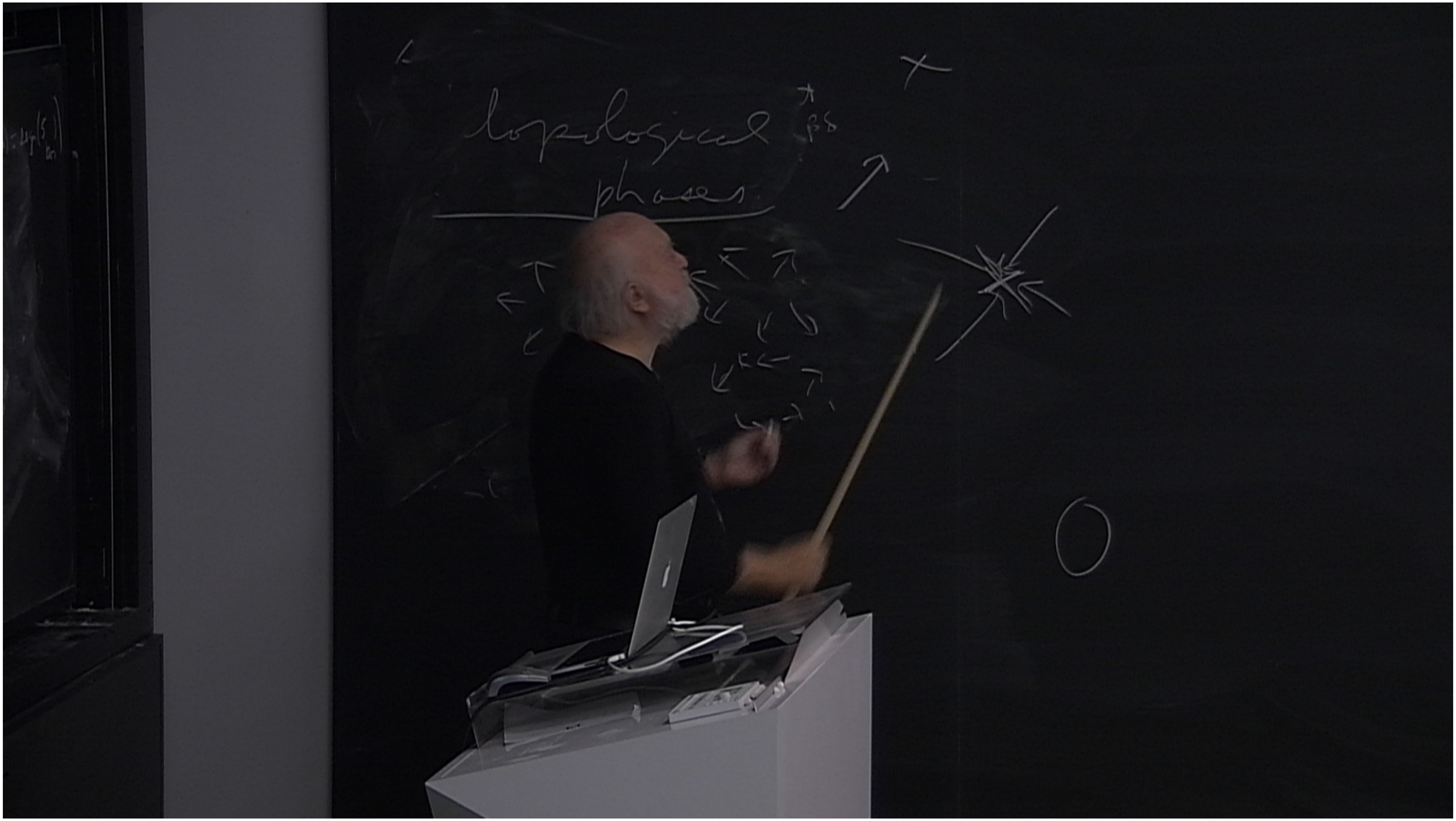

topological
phases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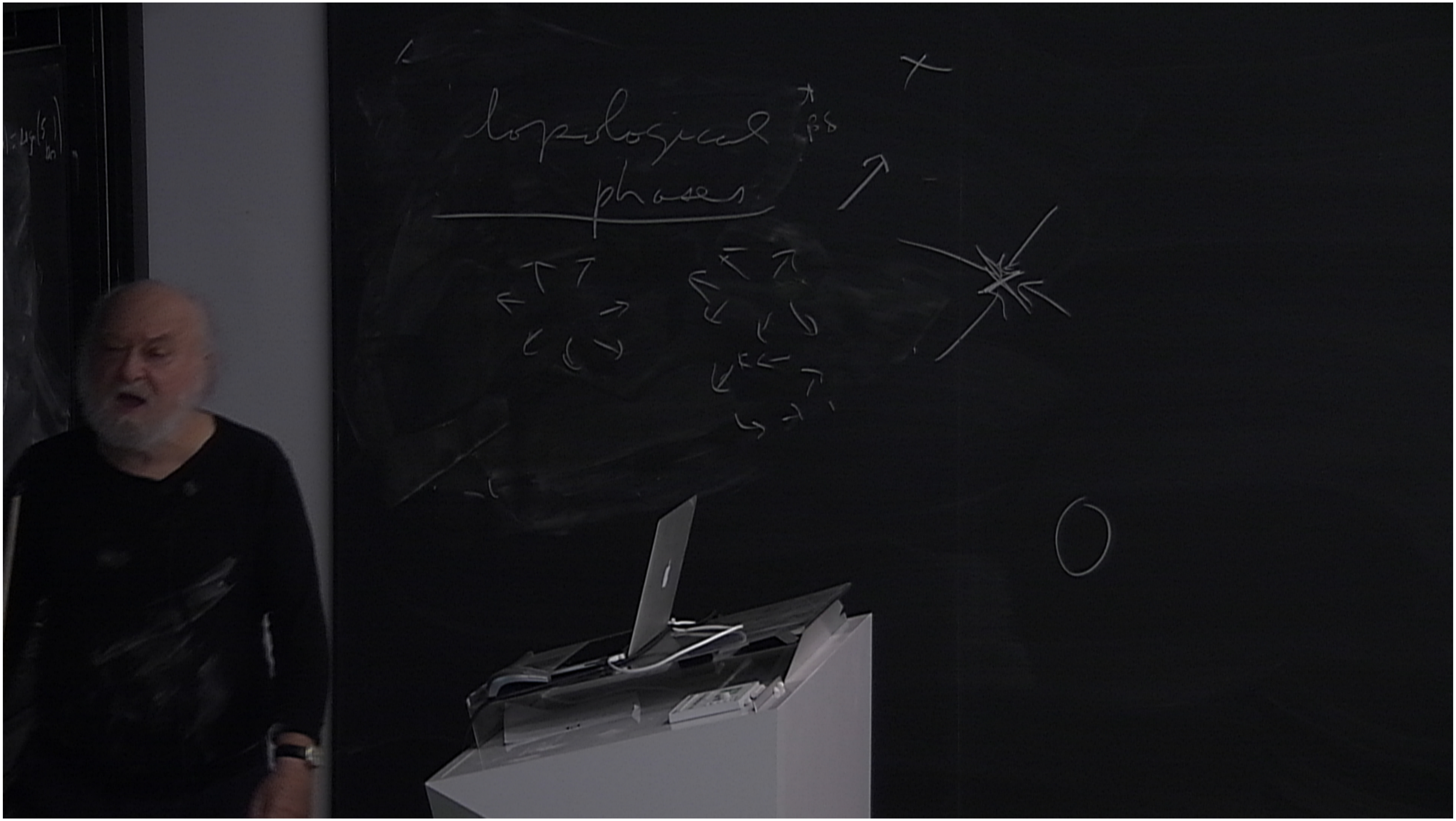
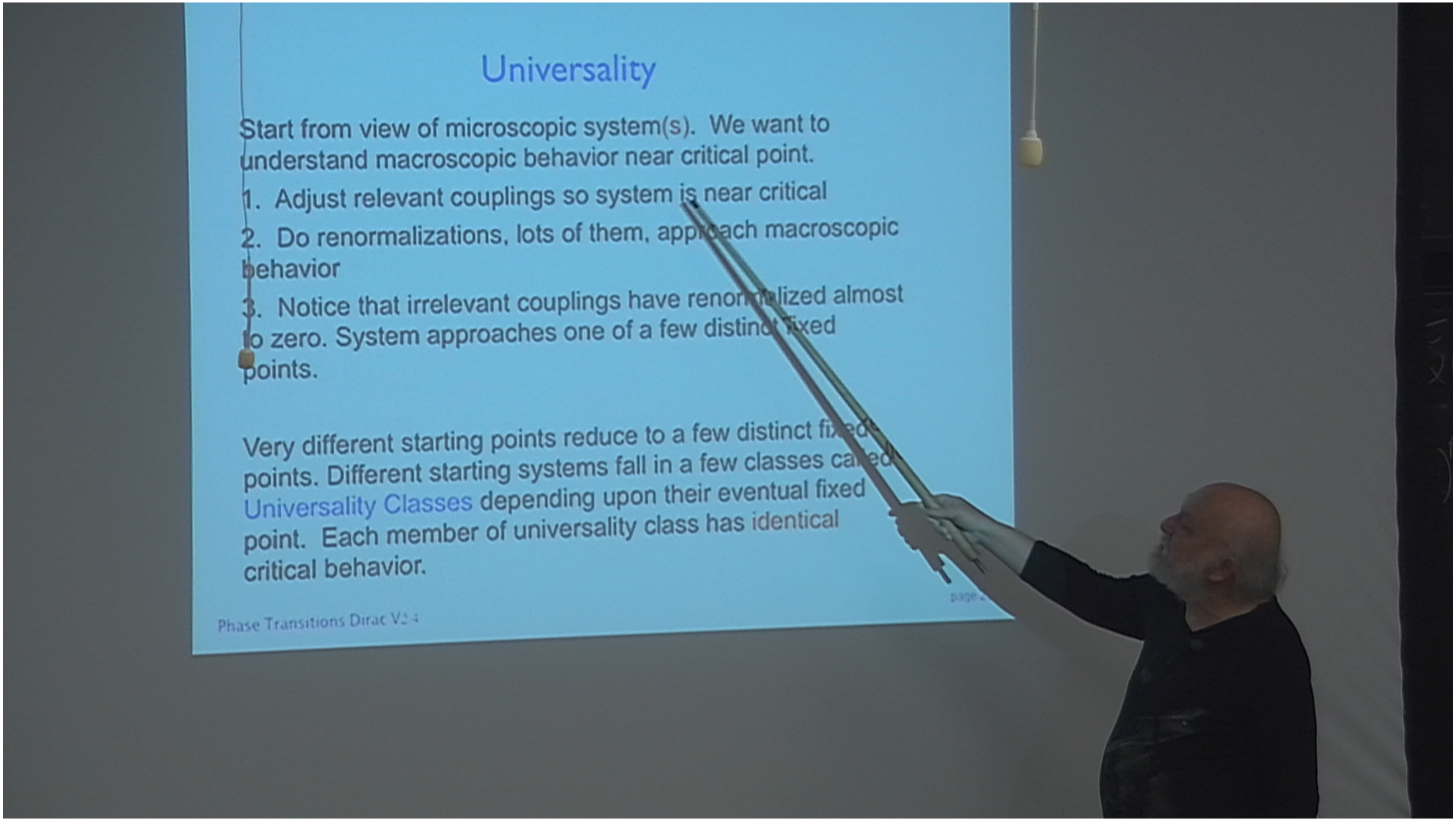
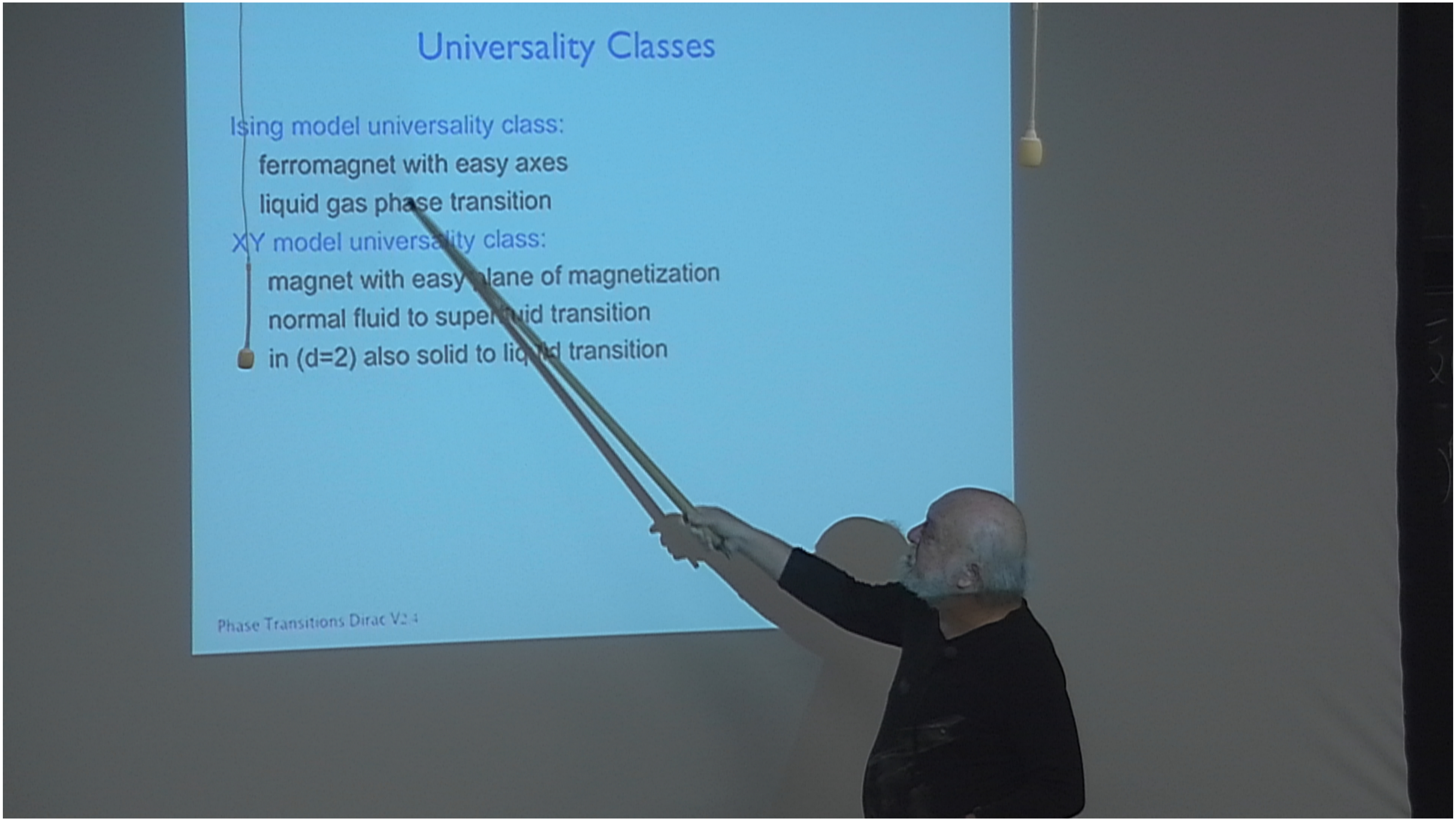
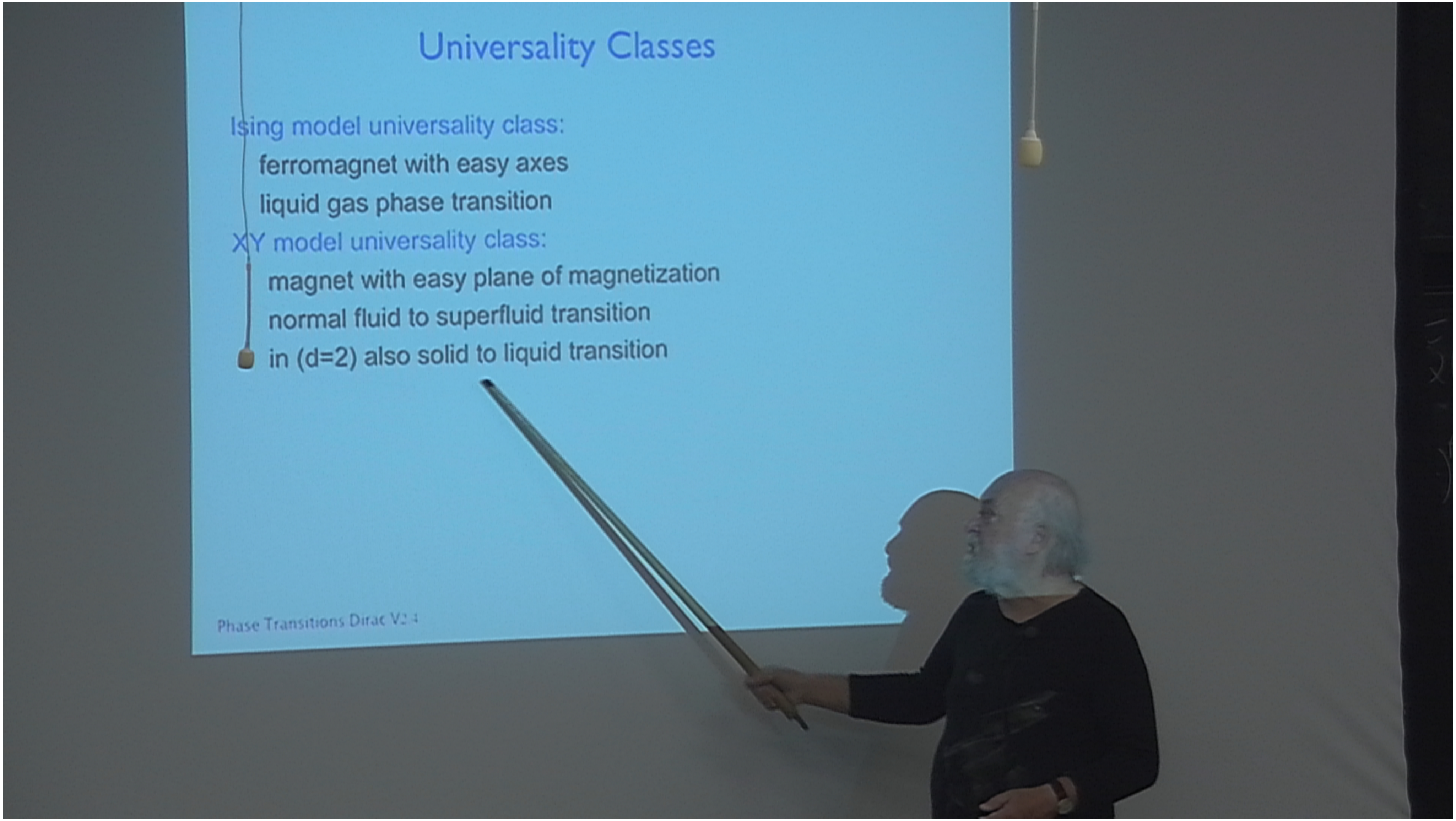

topological phases


Universality
Start from view of microscopic system(s). We want to understand macroscopic behavior near critical point.
1. Adjust relevant couplings so system is near critical
2. Do renormalizations, lots of them, approach macroscopic behavior
3. Notice that irrelevant couplings have renormalized almost to zero. System approaches one of a few distinct fixed points.
Very different starting points reduce to a few distinct fixed points. Different starting systems fall in a few classes called Universality Classes depending upon their eventual fixed point. Each member of universality class has identical critical behavior.
Phase Transitions Dirac V2.4


Universality Classes
Ising model universality class:
ferromagnet with easy axes
liquid gas phase transition
XY model universality class:
magnet with easy plane of magnetization
normal fluid to superfluid transition
in (d=2) also solid to liquid transition
Phase Transitions Dirac V2.4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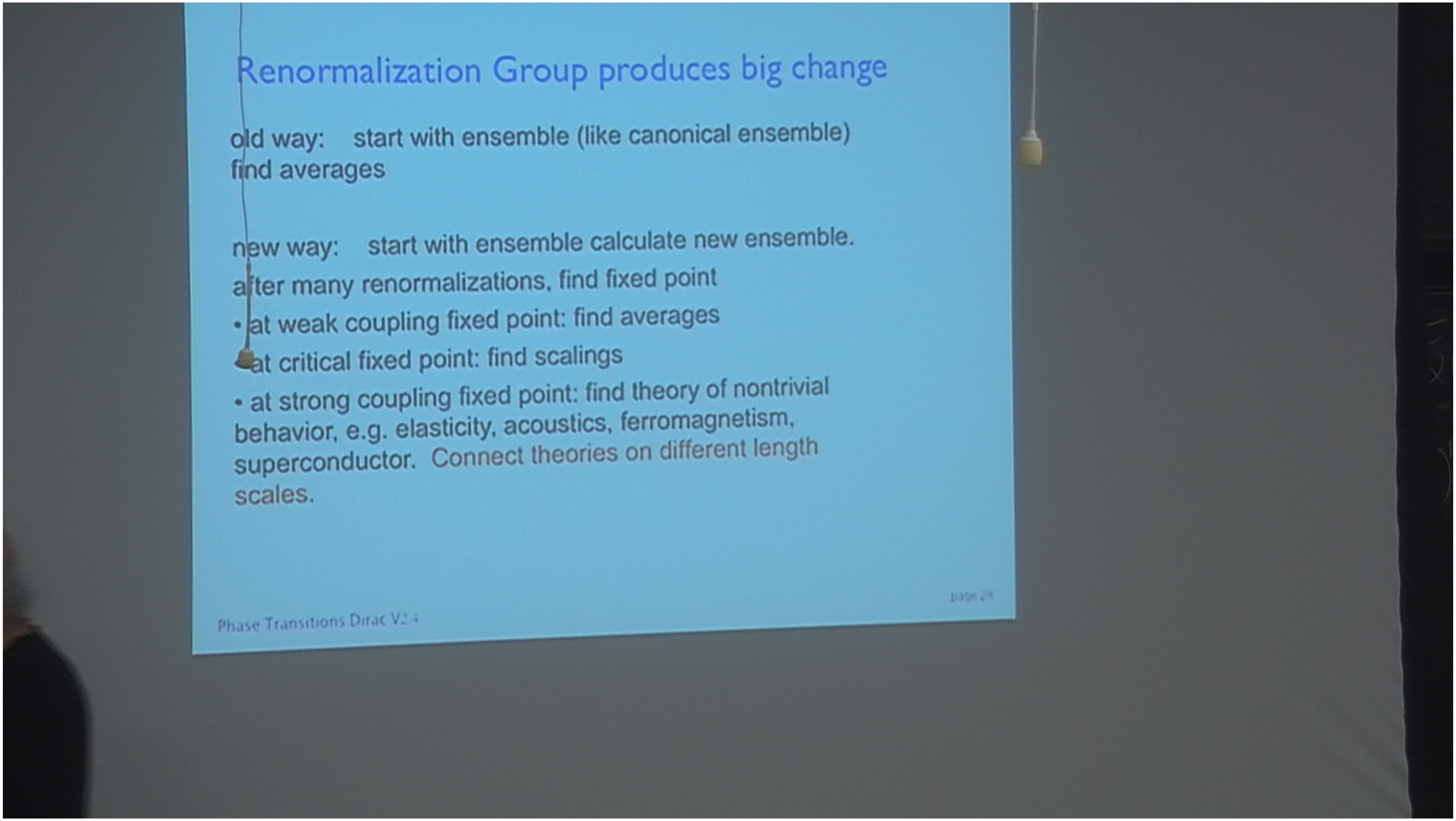
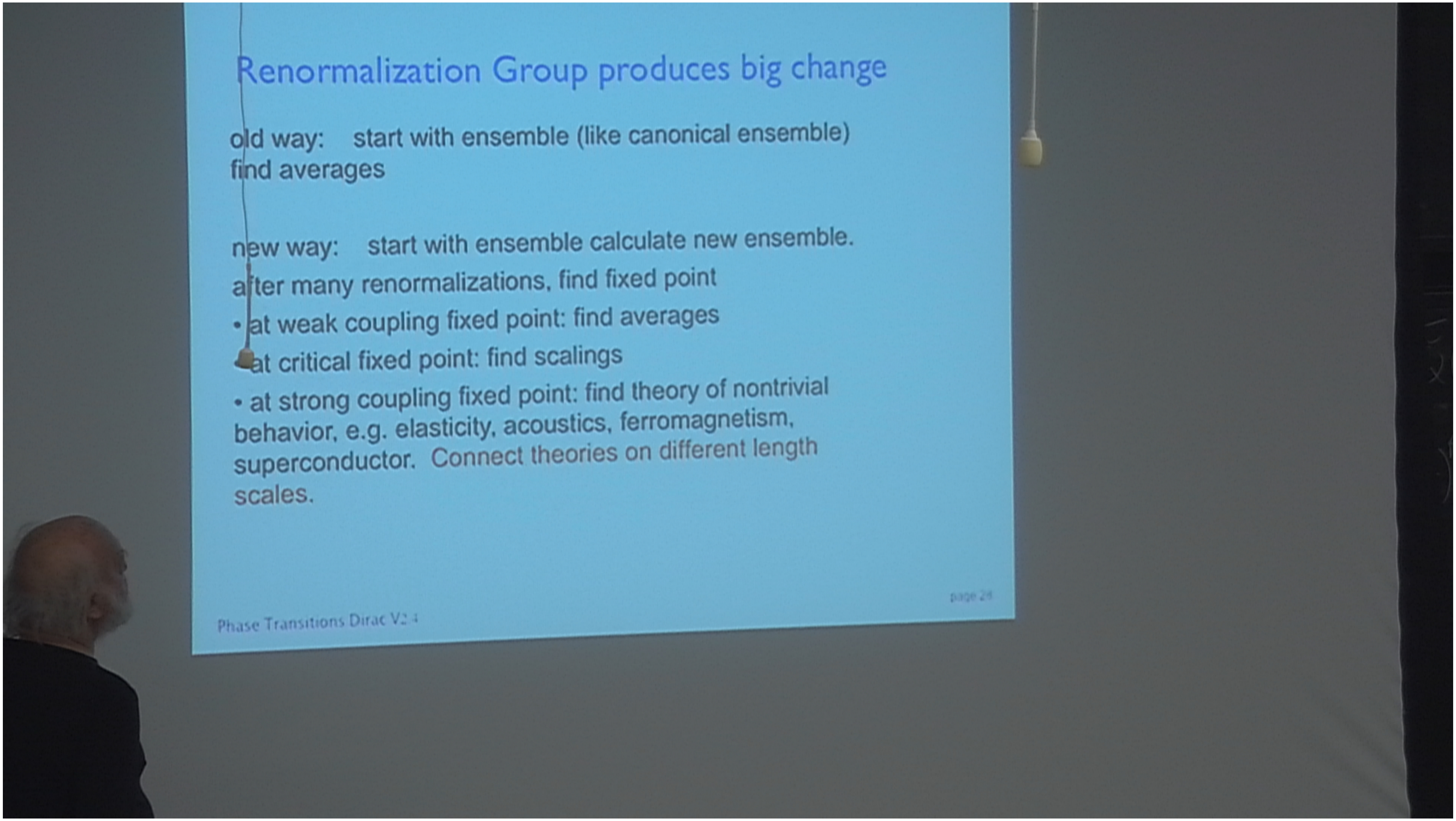

Renormalization Group produces big change
old way: start with ensemble (like canonical ensemble)
find averages
new way: start with ensemble calculate new ensemble.
after many renormalizations, find fixed point
• at weak coupling fixed point: find averages
• at critical fixed point: find scalings
• at strong coupling fixed point: find theory of nontrivial behavior, e.g. elasticity, acoustics, ferromagnetism, superconductor. Connect theories on different length scales.
Phase Transitions Dirac V2.4
page 28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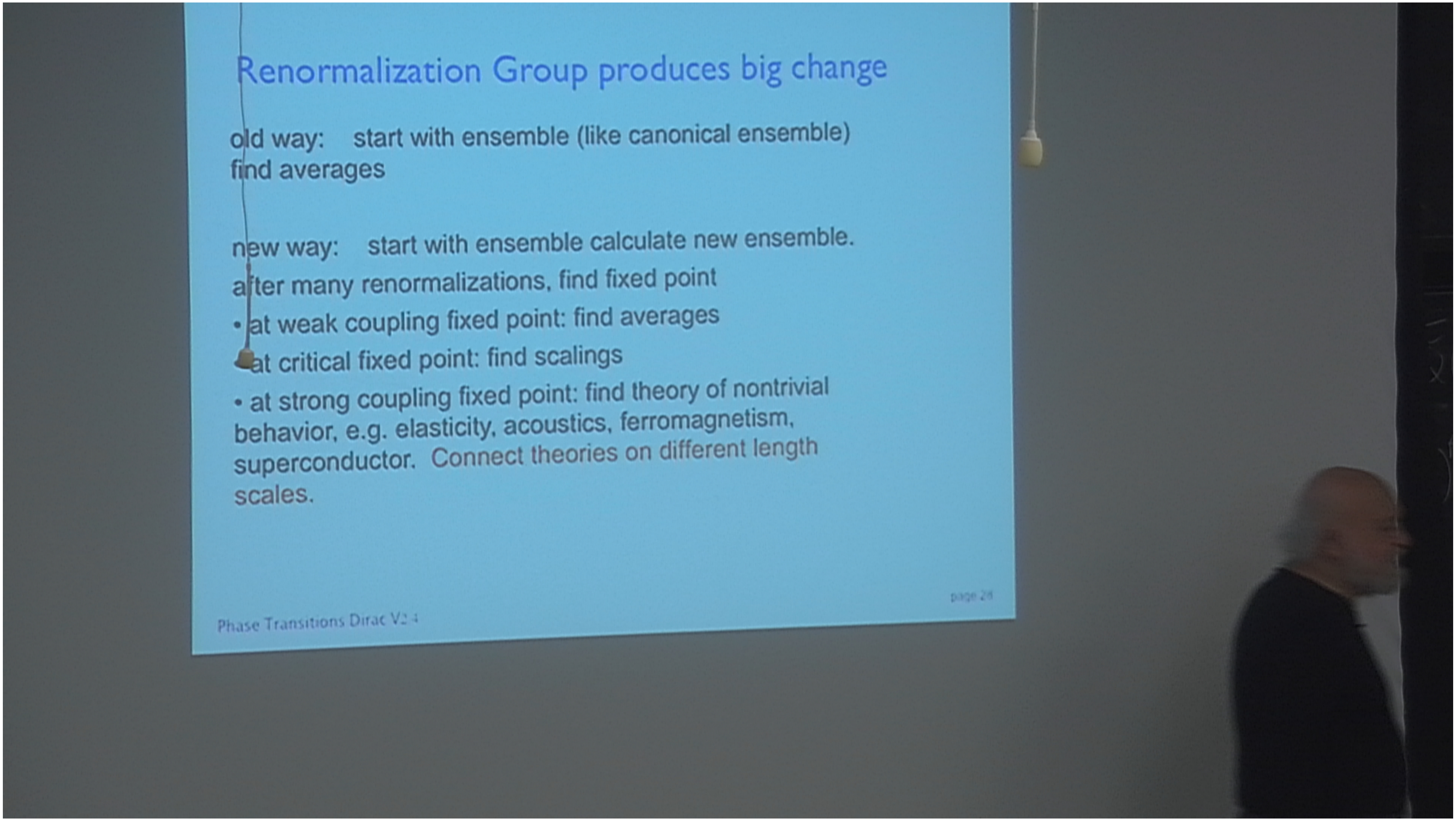
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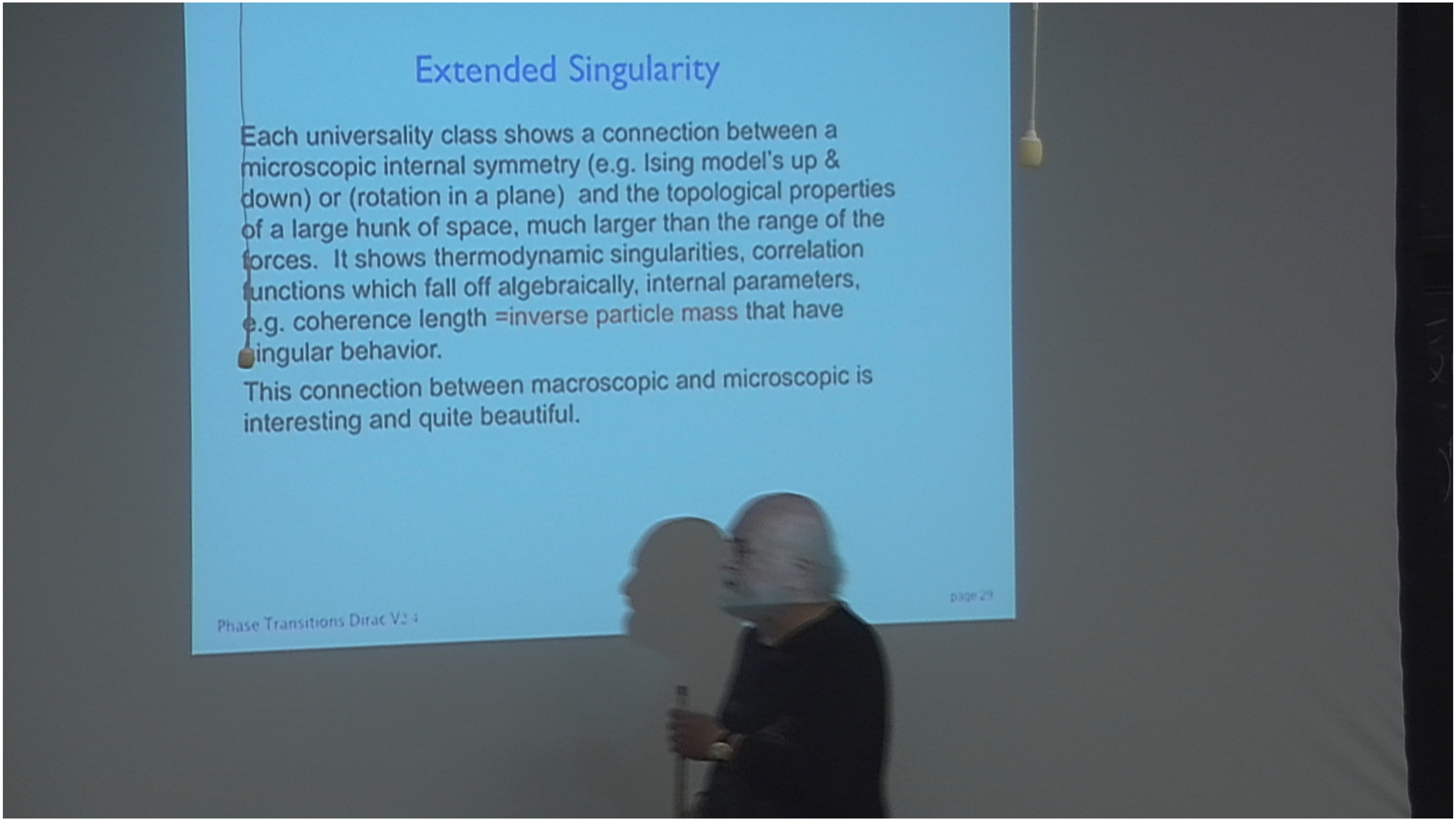
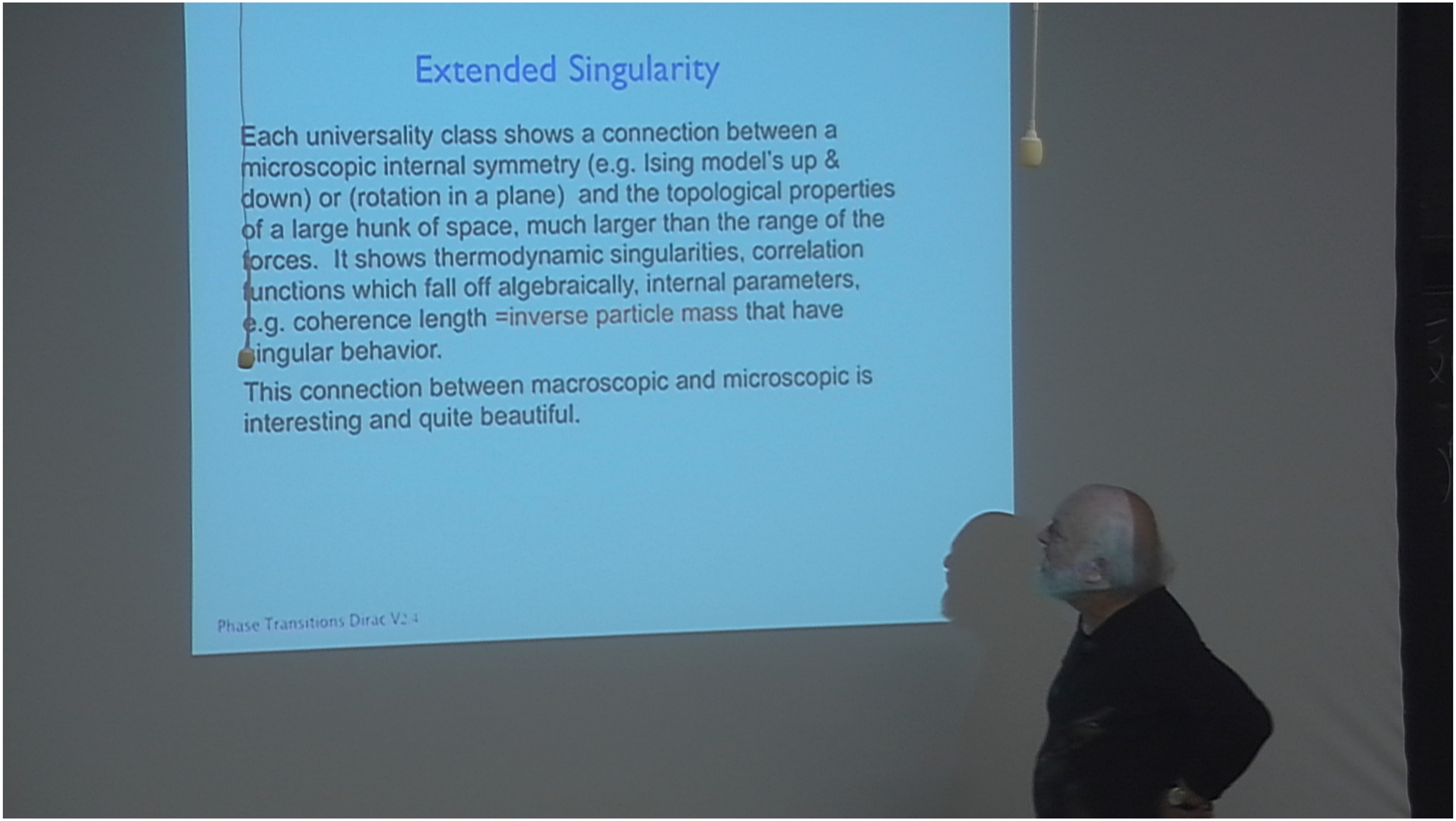

Extended Singularity
Each universality class shows a connection between a microscopic internal symmetry (e.g. Ising model's up & down) or (rotation in a plane) and the topological properties of a large hunk of space, much larger than the range of the forces. It shows thermodynamic singularities, correlation functions which fall off algebraically, internal parameters, e.g. coherence length =inverse particle mass that have singular behavior.
This connection between macroscopic and microscopic is interesting and quite beautiful.
Phase Transitions Dirac
page 29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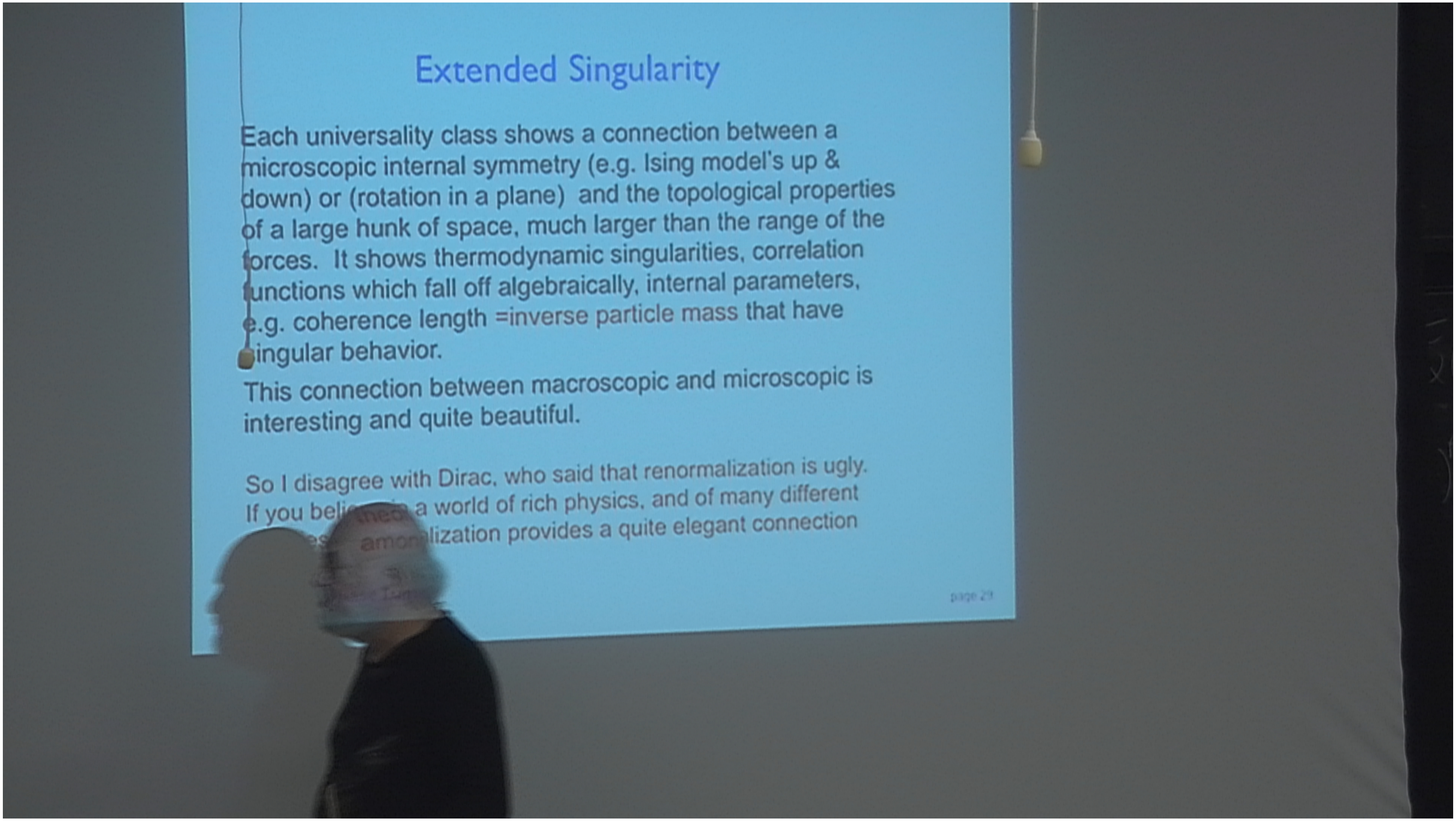
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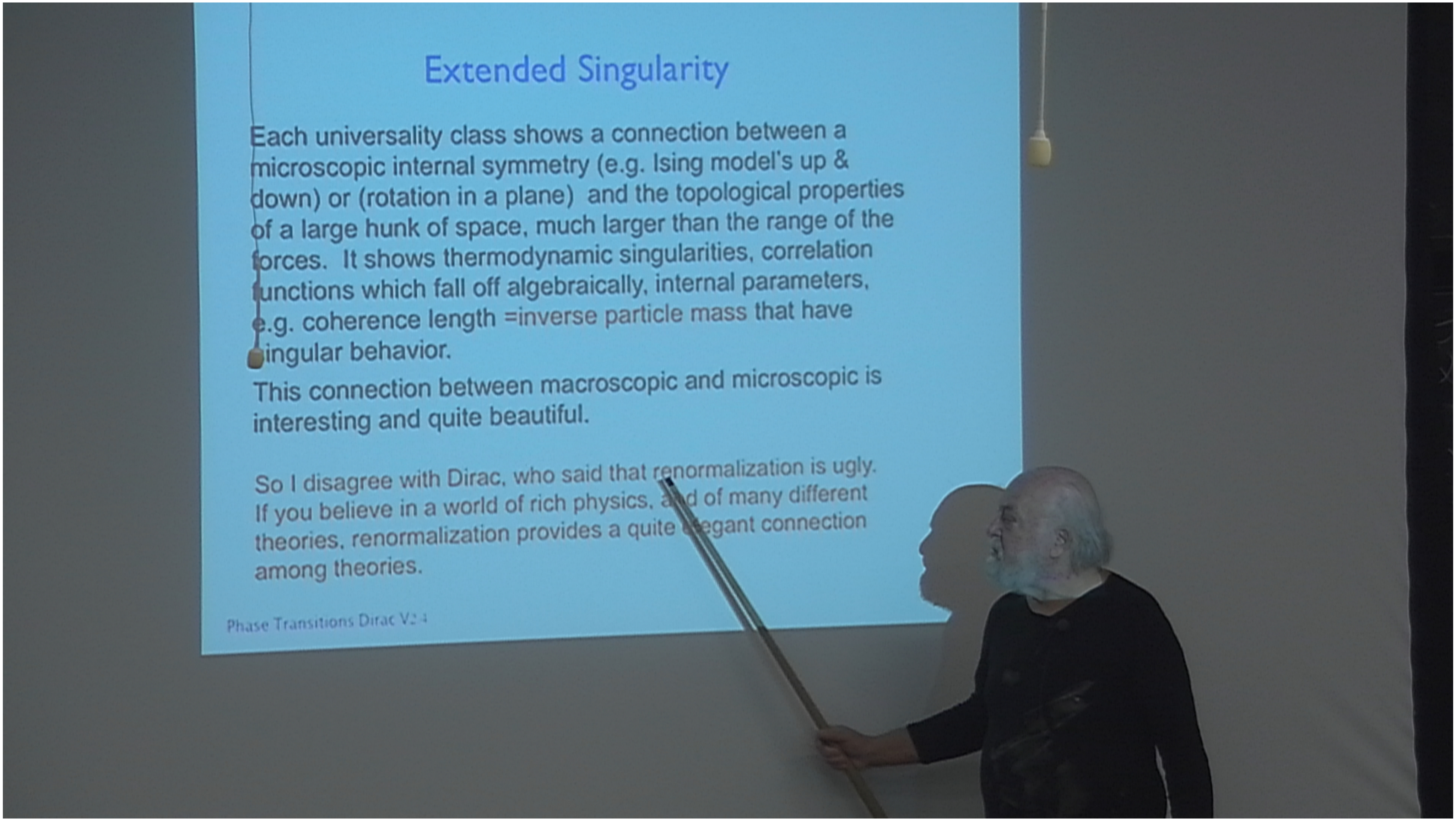

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This connection between macroscopic and microscopic is interesting and quite beautiful.
So I disagree with Dirac, who said that renormalization is ugly. If you believe in a world of rich physics, and of many different theories, renormalization provides a quite elegant connection among theories.
Phase Transitions Dirac V2.4