Title: Statistical Mechanics - Lecture 13

Date: Oct 20, 2011 10:30 AM

URL: http://pirsa.org/11100042

Abstract:

# MFT's applications

Mean field theory has a host of different applications. In fact, the vast majority of theoretical knowledge of condensed matter systems comes from applications of MFT to such topics as mixtures of different fluids, crystal alloys composed of two kinds of molecules, liquid crystals-- that is liquids with a preferred direction of orientation and many, many other situations.

Some Forms of Liquid Crystals

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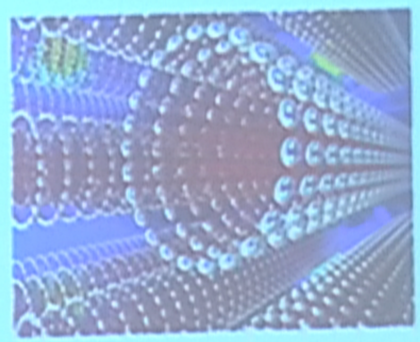

mycell

# Mean Field Theory's application to electrodynamics of continuous media

In MFT, a particle is affected by the average field produced by particles around it. A good and accurate example of MFT is the electrodynamics of continuous media, basically going back to Maxwell who described the media by **E,D,B,H** fields.

Fields produced externally to material are **D,H**

Fields **E,B** include, in addition, averaged effects of charges and currents within material.

This kind of mean field theory is usually very accurate because electrodynamics includes long-ranged forces and many charges in mutual interaction. It fails in nanoscopic materials in which a it can be true that only a small number of particles interact substantially with a given particle .

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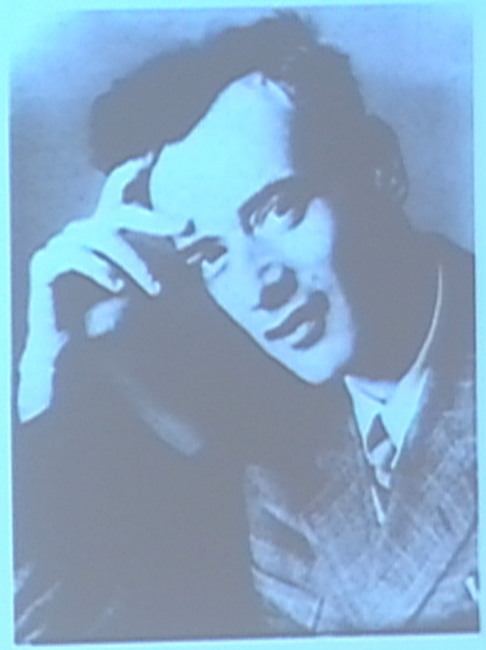
# Order Parameter, generalized

By 1937 many different phases were described by MFT's, and these were generally accepted as approximate, but essentially correct.

- In that year, Lev Landau suggested that phase transitions were manifestations of a broken symmetry, and used the order parameter to measure the extent of breaking of the symmetry.

- in ferromagnet, parameter = magnetization
- in fluid, order parameter = density
- in Ising model, order parameter = <σ>



L.D. Landau

31

# This same approach is still in extensive use.

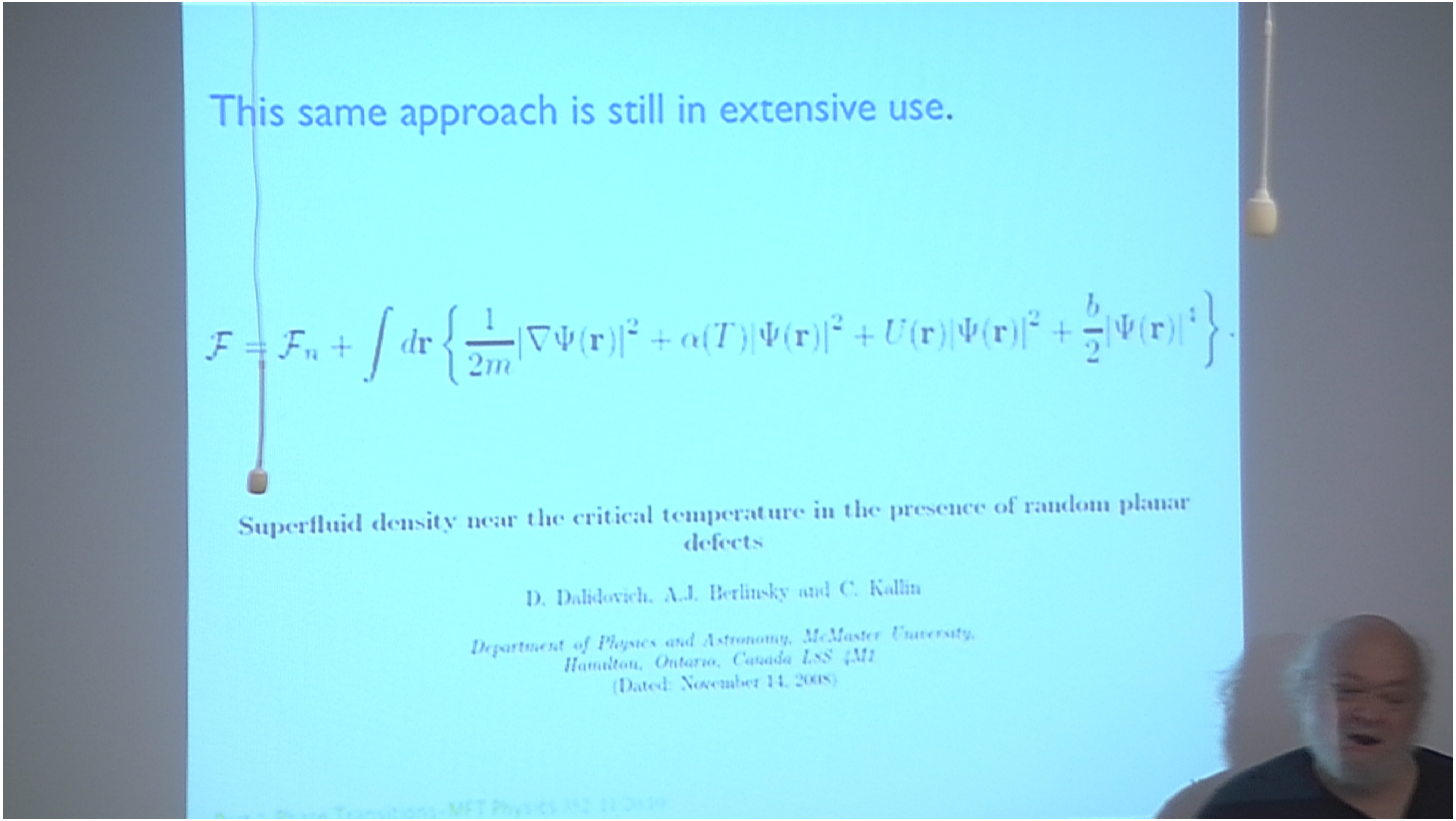
$$\mathcal{F} = \mathcal{F}_n + \int d\mathbf{r} \left\{ \frac{1}{2m} |\nabla \Psi(\mathbf{r})|^2 + \alpha(T) |\Psi(\mathbf{r})|^2 + U(\mathbf{r}) |\Psi(\mathbf{r})|^2 + \frac{b}{2} |\Psi(\mathbf{r})|^4 \right\} .$$

**Superfluid density near the critical temperature in the presence of random planar defects**

D. Dalidovich, A.J. Berlinsky and C. Kallin

*Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, Canada L8S 4M1*

(Dated: November 14, 2008)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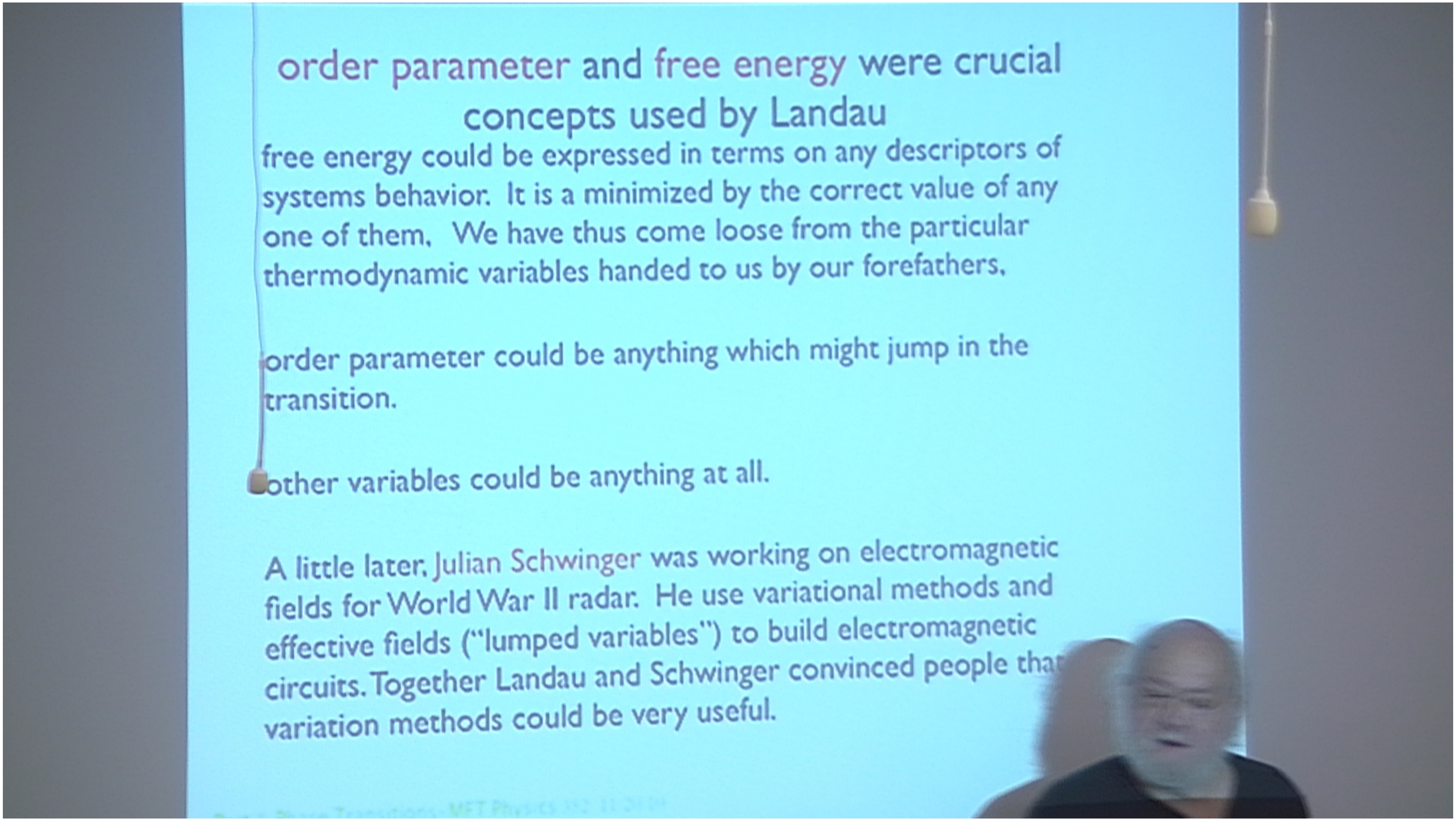
## order parameter and free energy were crucial concepts used by Landau

free energy could be expressed in terms on any descriptors of systems behavior. It is a minimized by the correct value of any one of them. We have thus come loose from the particular thermodynamic variables handed to us by our forefathers.

order parameter could be anything which might jump in the transition.

other variables could be anything at all.

A little later, Julian Schwinger was working on electromagnetic fields for World War II radar. He use variational methods and effective fields ("lumped variables") to build electromagnetic circuits. Together Landau and Schwinger convinced people that variation methods could be very useful.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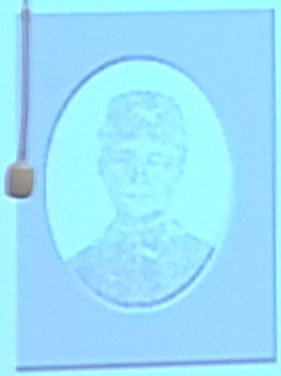
# Bardeen Family

The Bardeens are something special too...Charles Russell Bardeen, John's father, was the first person to graduate from the medical school of Johns Hopkins University. He then created a new medical school at, where else, Wisconsin and became its dean. John's mother Althea Harmer was an educator, who worked for John Dewey until Dewey had a falling out with the University of Chicago. John's grandfather, Charles William Bardeen, was also an educator and a publicist for better education. He wrote a book on rhetoric, and also2



Little Massachusetts Fifer, Diary entries and memoir o roguish 14-year-old Charles William Bardeen.

Both families are thus connected with the University of Wisconsin, founded in 1848.

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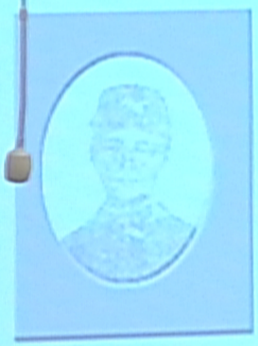

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# BCS Theory

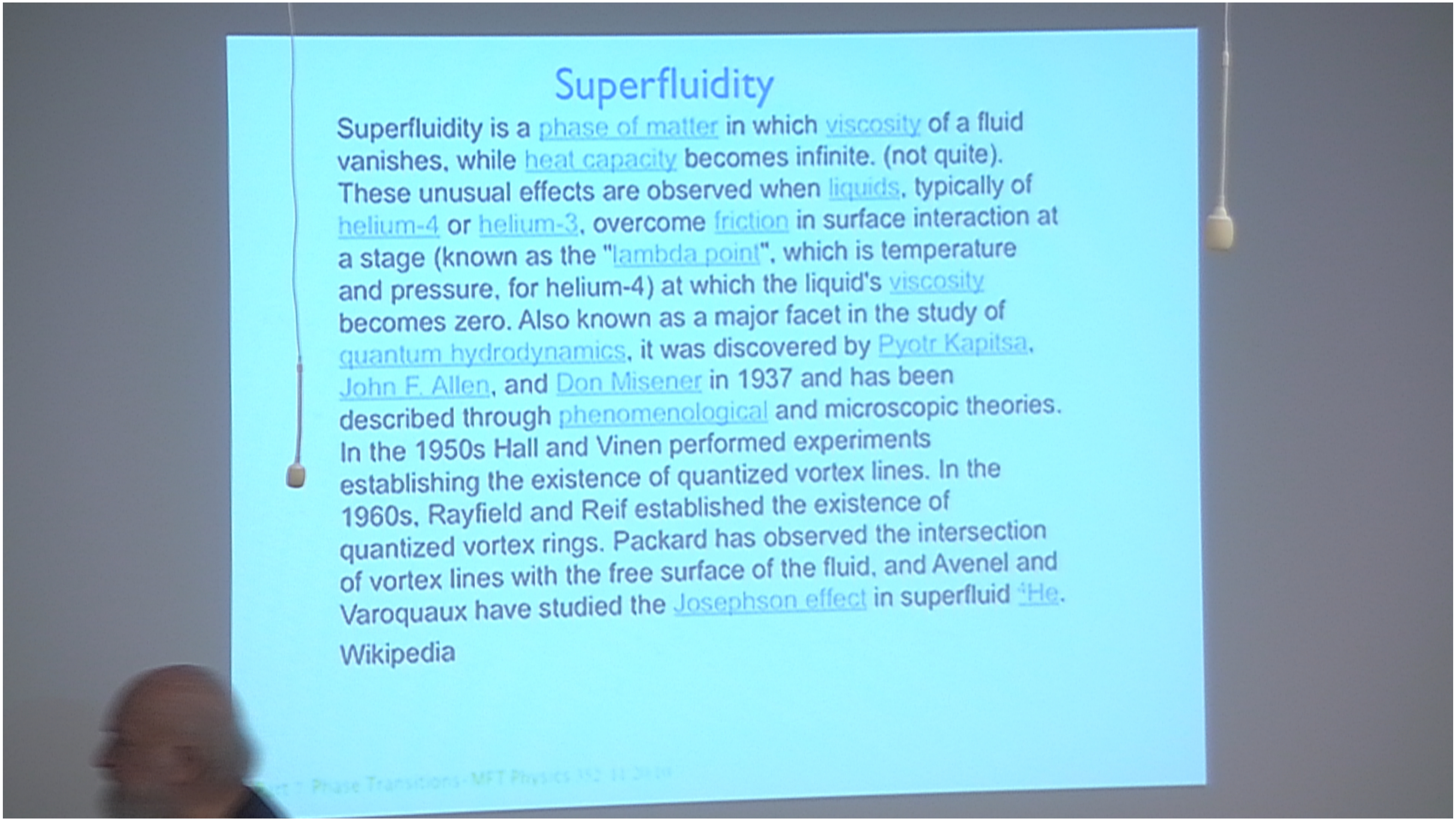
BCS put together a most impressive theory of superconductivity. Bardeen was the impressario and leader. He had questions he wanted answered about superconductivity and made sure the joint work answered everything he could think of about the subject.

Leon Cooper, a postdoc, did a theory of pairing of electrons based upon an attractive interaction between electrons produced by the frequency dependent electron phonon interaction.

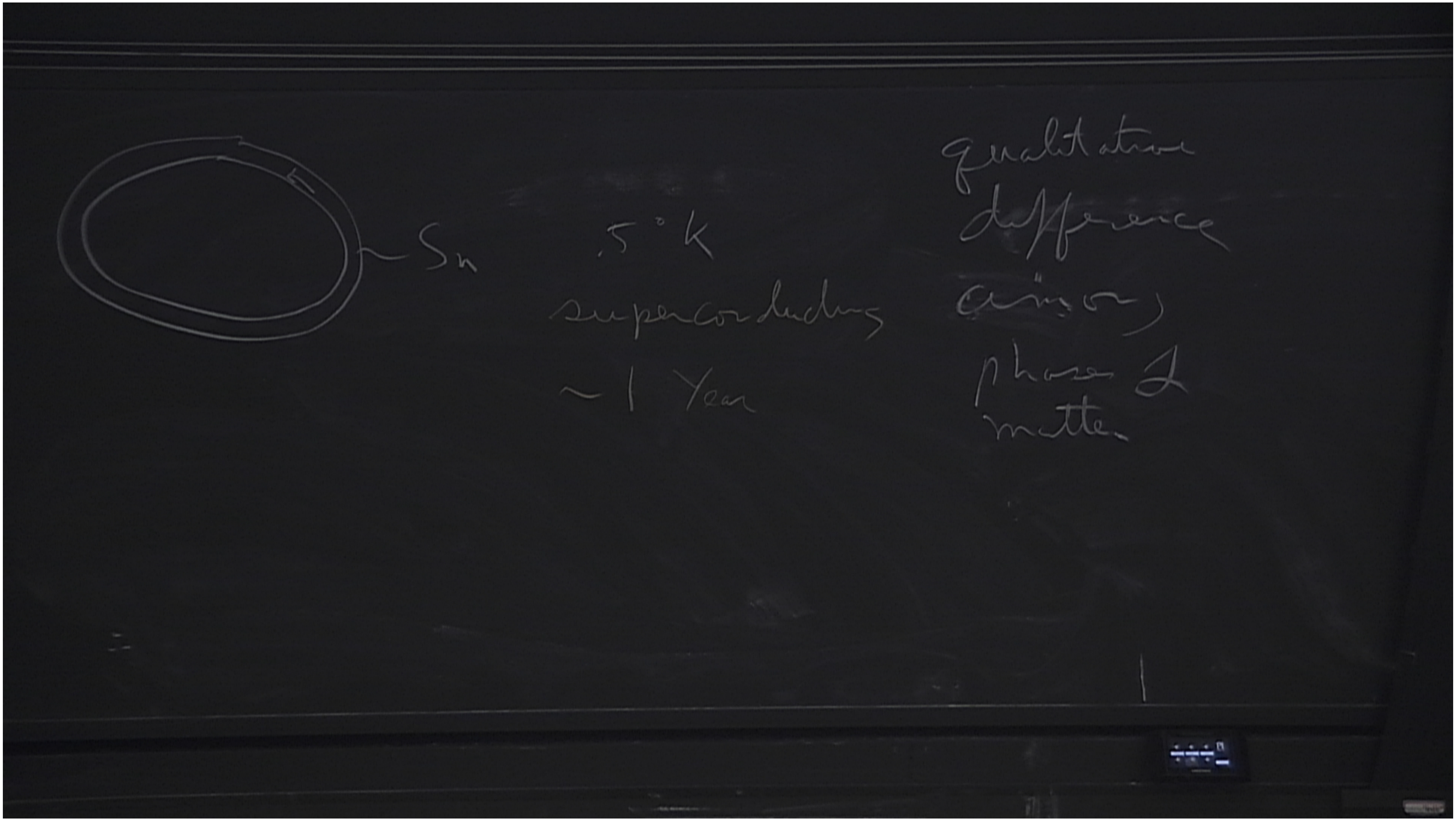
Bob Schrieffer, a brilliant grad. student produced a trial wave function (based upon the previous work of Tomonaga) to be plugged into a variational principle.

Together they explained almost all the known microscopic properties of superconductors.

But microscopics is only part of the story.


Superfluidity
Superfluidity is a phase of matter in which viscosity of a fluid vanishes, while heat capacity becomes infinite. (not quite). These unusual effects are observed when liquids, typically of helium-4 or helium-3, overcome friction in surface interaction at a stage (known as the "lambda point", which is temperature and pressure, for helium-4) at which the liquid's viscosity becomes zero. Also known as a major facet in the study of quantum hydrodynamics, it was discovered by Pyotr Kapitsa, John F. Allen, and Don Misener in 1937 and has been described through phenomenological and microscopic theories. In the 1950s Hall and Vinen performed experiments establishing the existence of quantized vortex lines. In the 1960s, Rayfield and Reif established the existence of quantized vortex rings. Packard has observed the intersection of vortex lines with the free surface of the fluid, and Avenel and Varoquaux have studied the Josephson effect in superfluid 4He.
Wikipedia


Sn
.5°K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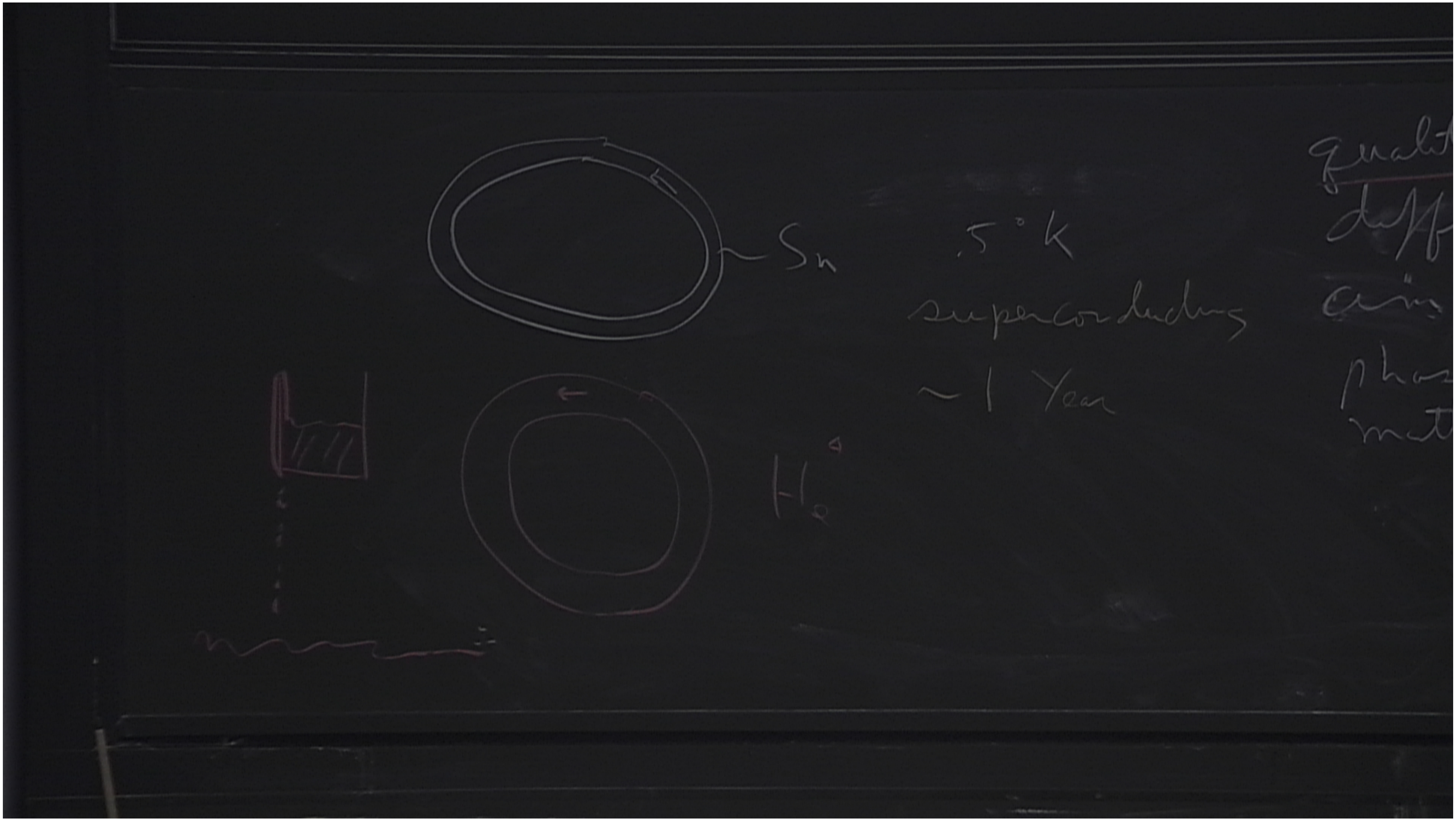
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He

# "Hydrodynamics" and transport in different phase of matter

Different phases of matter are qualitatively different. The only exception mentioned here is the liquid/vapor system in which both phases are qualitatively similar.

Each kind of phase has its own kind of long-wave-length or transport process. For example, a liquid has no rigidity so it cannot transfer a torque over a long distance, but a solid can. At low frequencies momentum transfer in liquids involve one kind of sound wave and two components of diffusion limited by viscosity. The latter is described by the Navier Stokes equations, extensively used in fluid research.

$$\partial_t \mathbf{v}(r,t)+(\mathbf{v}(r,t)\cdot\nabla)\, \mathbf{v}(r,t)= \eta\nabla^2\mathbf{v}(r,t) + \nabla p(r,t) \qquad \nabla\cdot\mathbf{v}(r,t) =0$$

In contrast, solids have three modes of sound propagation: one longitudinal and two transverse. Each of these modes obeys an equation of the form

$$\partial_t^2\mathbf{v}(r,t) - c^2\nabla^2\mathbf{v}(r,t)=0$$

# For superconductor

$$F=\int dr\{t|\psi|^2+c|\psi\}^4+|[p-2eA(r,t)]\psi)|^2\}$$

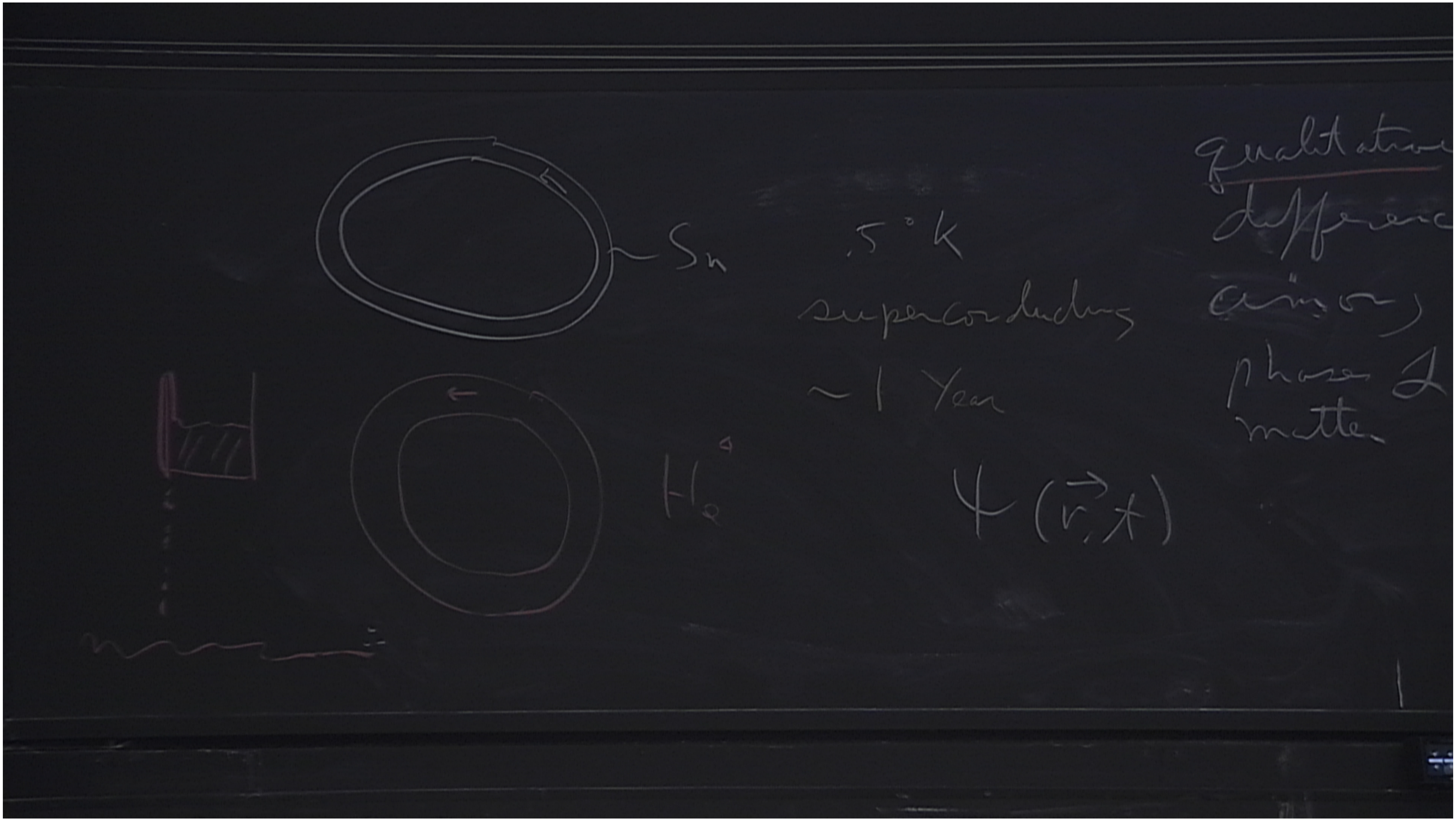
$$p=\hbar\nabla/i$$

Minimize with respect to the condensate wave function. $\psi$. Current is the derivative of F with respect to A(r,t)

$$\alpha\psi+\beta|\psi|^2\psi+\frac{1}{2m}\left(-i\hbar\nabla-2e\mathbf{A}\right)^2\psi=0$$

$$\mathrm{j}=\frac{2e}{m}\mathrm{Re}\left\{\psi^*\left(-i\hbar\nabla-2e\mathbf{A}\right)\psi\right\}$$

Note that the theory includes F, so when there are several solutions, we can pick the right one.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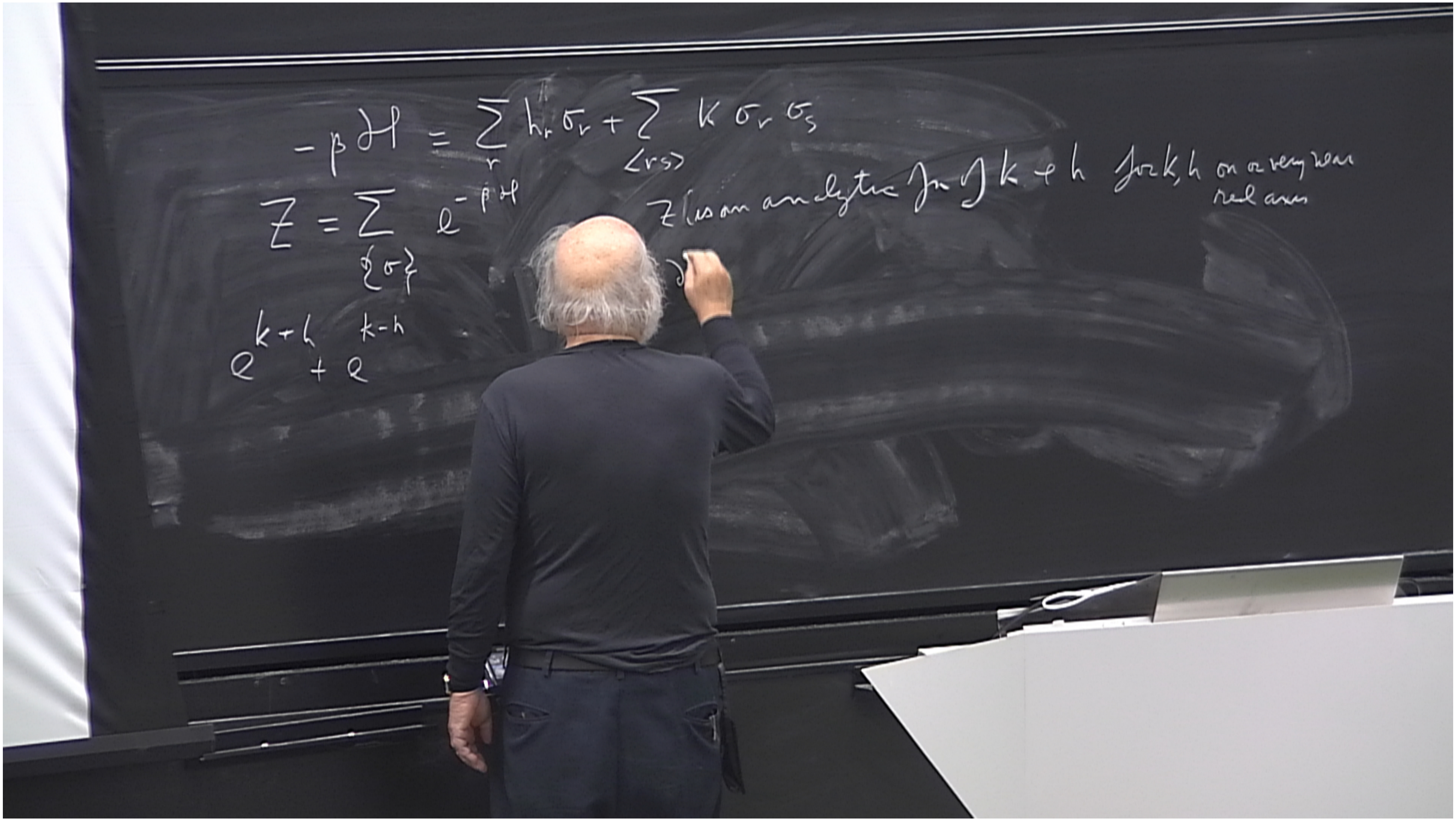
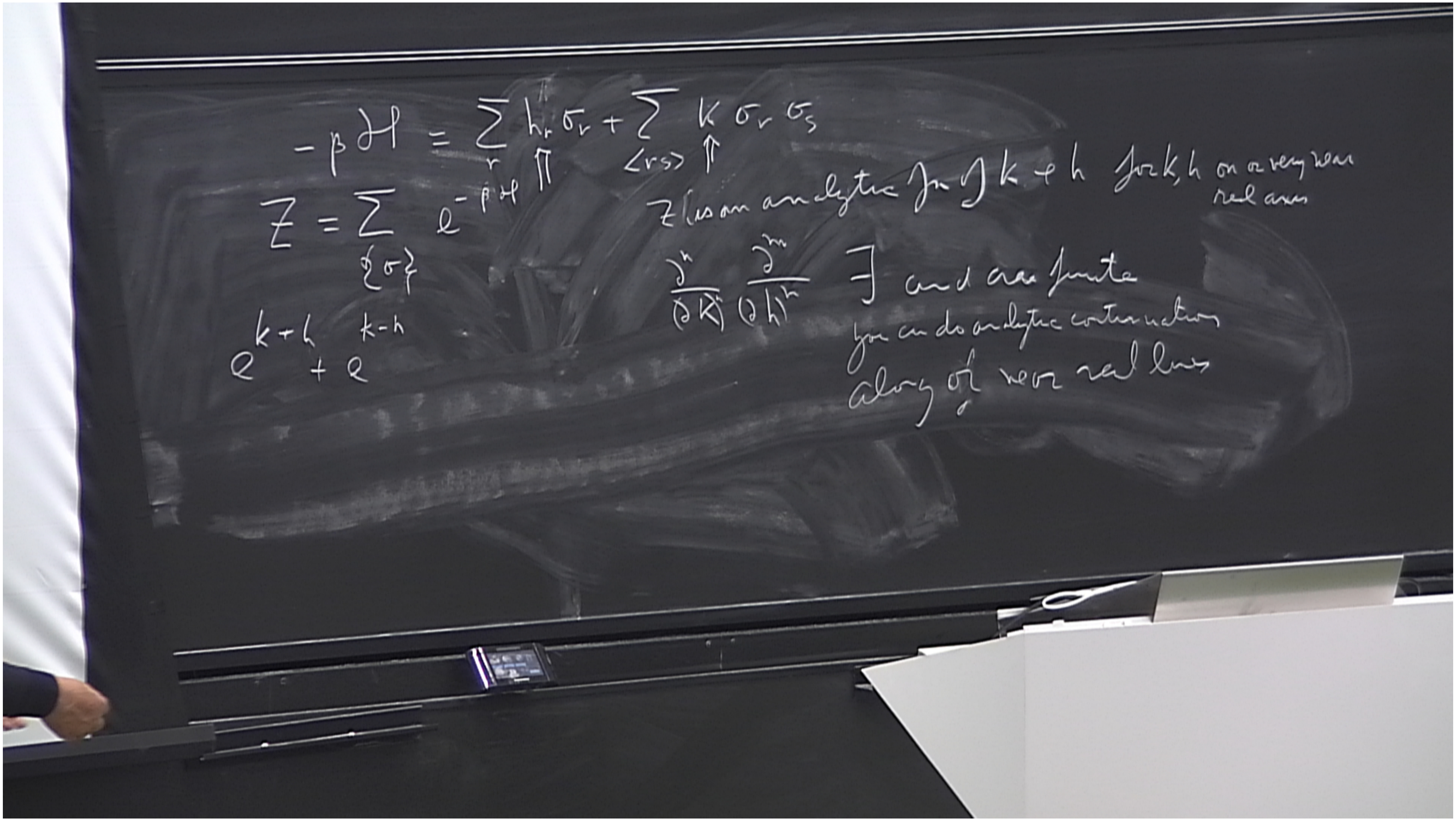


-βH = Σ_r h_r σ_r + Σ_⟨rs⟩ K σ_r σ_s
Z = Σ_{σ} e^{-βH}
e^{k+h} + e^{k-h}
Z is an analytic fn of K & h for K, h on or very near real axes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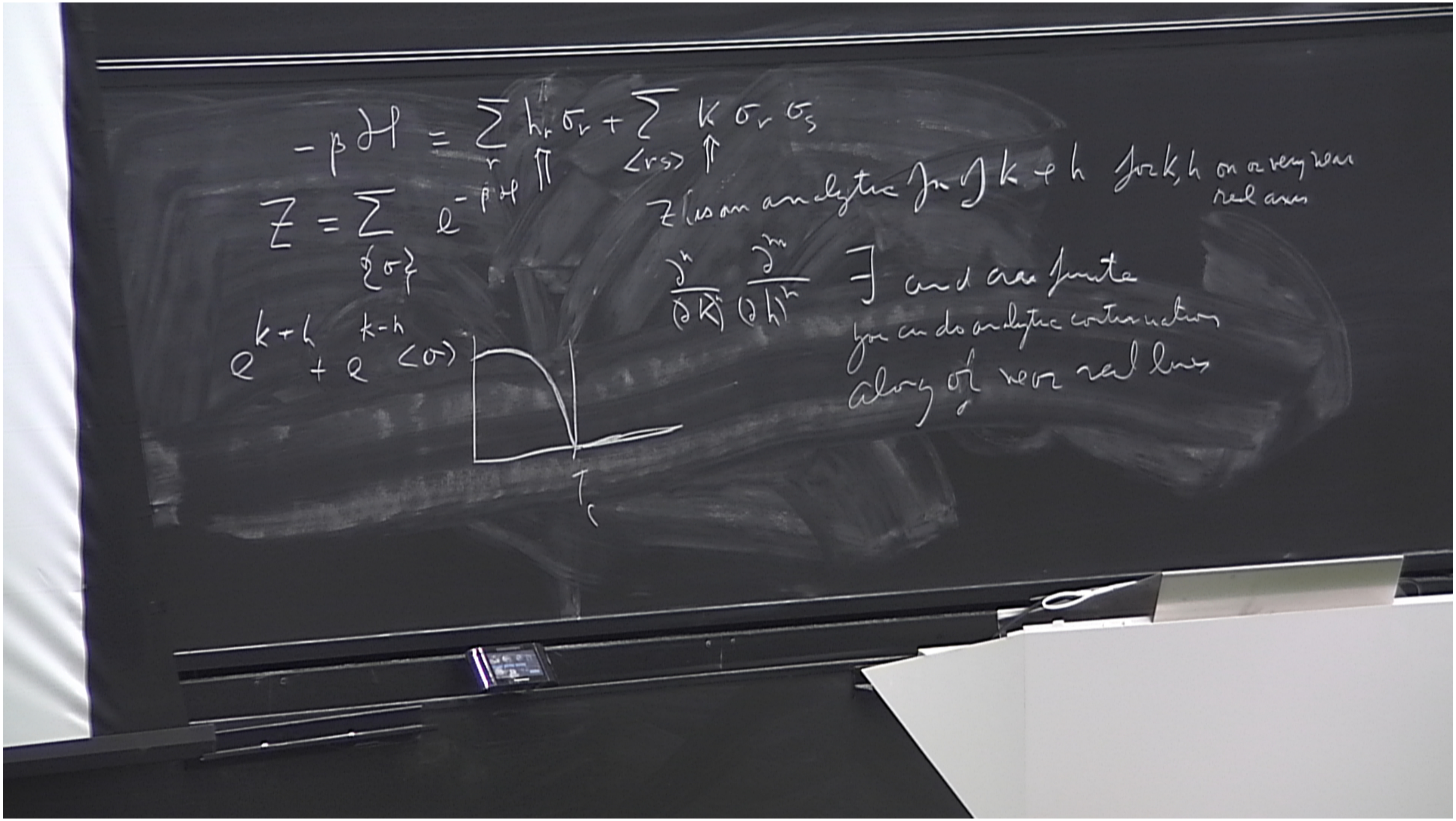
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$\langle \sigma \rangle$
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$\frac{\partial^n}{(\partial K)^n} \frac{\partial^m}{(\partial h)^m}$ ∃ and are finite
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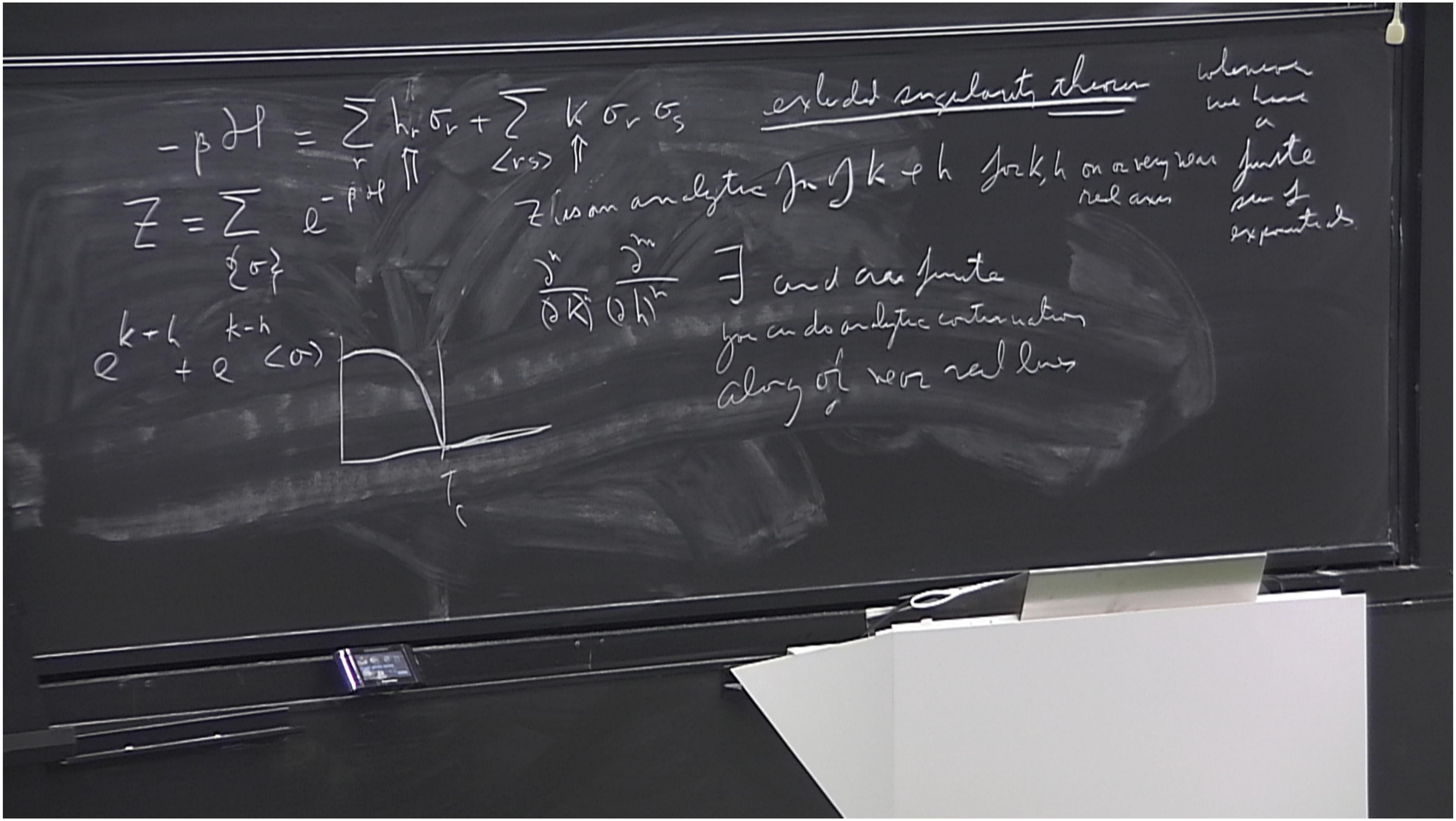
$T_c$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-βH = Σ_r h_r σ_r + Σ_<rs> K σ_r σ_s
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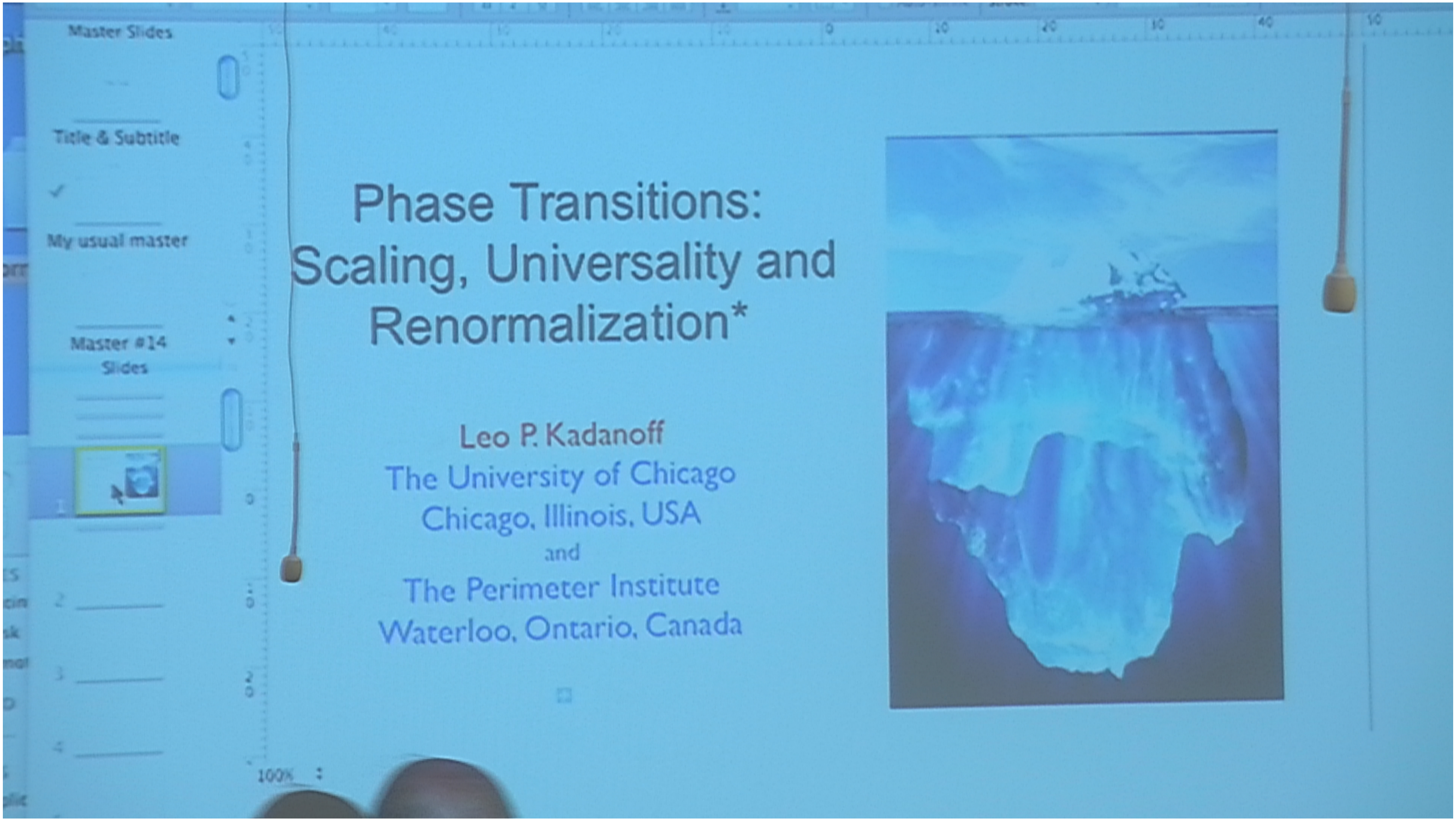
Z has an analytic fn of K & h for K, h on or very near real axis
∂^n/(∂K)^n ∂^m/(∂h)^m ∃ and are finite
you can do analytic continuation along or near real lines
whenever we have a finite sum of exponentials
e^{K+h} + e^{K-h}
<σ>
T_c


Master Slides
Title & Subtitle
My usual master
Master #14
Slides
Phase Transitions:
Scaling, Universality and
Renormalization*
Leo P. Kadanoff
The University of Chicago
Chicago, Illinois, USA
and
The Perimeter Institute
Waterloo, Ontario, Canada
100%

abstract

Historically, the renormalization group method, as developed by Kenneth G. Wilson, is rooted in the theory of phase transitions. Theoretical treatments of phase transitions are based upon mean field theories like the ones developed by van der Waals and Landau.

Sharp phase transitions are necessarily connected with singularities in statistical mechanics, which in turn require infinite systems for their realization. (extended singularity theorem) A discussion of this point apparently marked a 1937 meeting in Amsterdam on van der Waals theory.

Mean field theories, like van der Waals', neither demand nor employ spatial infinities in their descriptions of phase transitions. Another theory is required that weds a breaking of internal symmetries with a proper description of spatial infinities. The renormalization (semi-) group provides such a wedding. Its nature is described. The major ideas surrounding this point of view are described including especially scaling and universality.

# Who am I?

A condensed matter theorist, with an interest in the history of science, who intends to talk about a subject closely related to condensed matter, but also to the philosophy of science and particle physics. I am not an expert in either of the latter subjects.

condensed matter physics: formulations clear (stat mech, Schrodinger equation, etc.) goal: explain amazing variety of nature. Nature = an Onion, exposed layer after layer. We hope to see mathematical and conceptual beauty arise from the mundane.

particle physics: simple results=masses, cross-sections goal: seek clear and final (!!) theoretic formulations based upon experiment and observations. Hope to see the mundane arise from the mathematical beauty of a single truth.

# Connections in Condensed Matter Physics

Condensed matter physics relates the observable, often macroscopic, properties of liquids, gases, solids and all everyday materials to more microscopic theories, often the quantum theory of atoms and molecules. Since the macroscopic theories are themselves non-trivial, e.g. elasticity, hydrodynamics, the electrodynamics of materials, it follows that condensed matter physics is largely an exercise in connecting different kinds of theories.

Typically this connection involves different length scales

Size of molecule = $10^{-9}$ meter. Size of laboratory= 5 meter

One of the deepest aspects of this area of science is the existence of different thermodynamic phases, each with qualitatively different properties. E.g., freezing is a sudden qualitative change in which the material

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Particle physics often wishes to relate its present, phenomenological, theory to a deeper (?!) theory at a much shorter or longer length scale. e.g. Connect the standard model to physics at a LHC, unification, or Planck scale.

Previously the search for a final theory has been impeded by ugliness or singularities arising at scales far from observation. These singularities show up directly as infinities in perturbation theory and indirectly as algebraic behavior ($1/|x-y|^p$) in a correlation function

I am going to follow condensed matter physics for the next parts of this talk, but particle physics and condensed matter physics are essentially similar.

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# Further Connections    Dirac's ideas

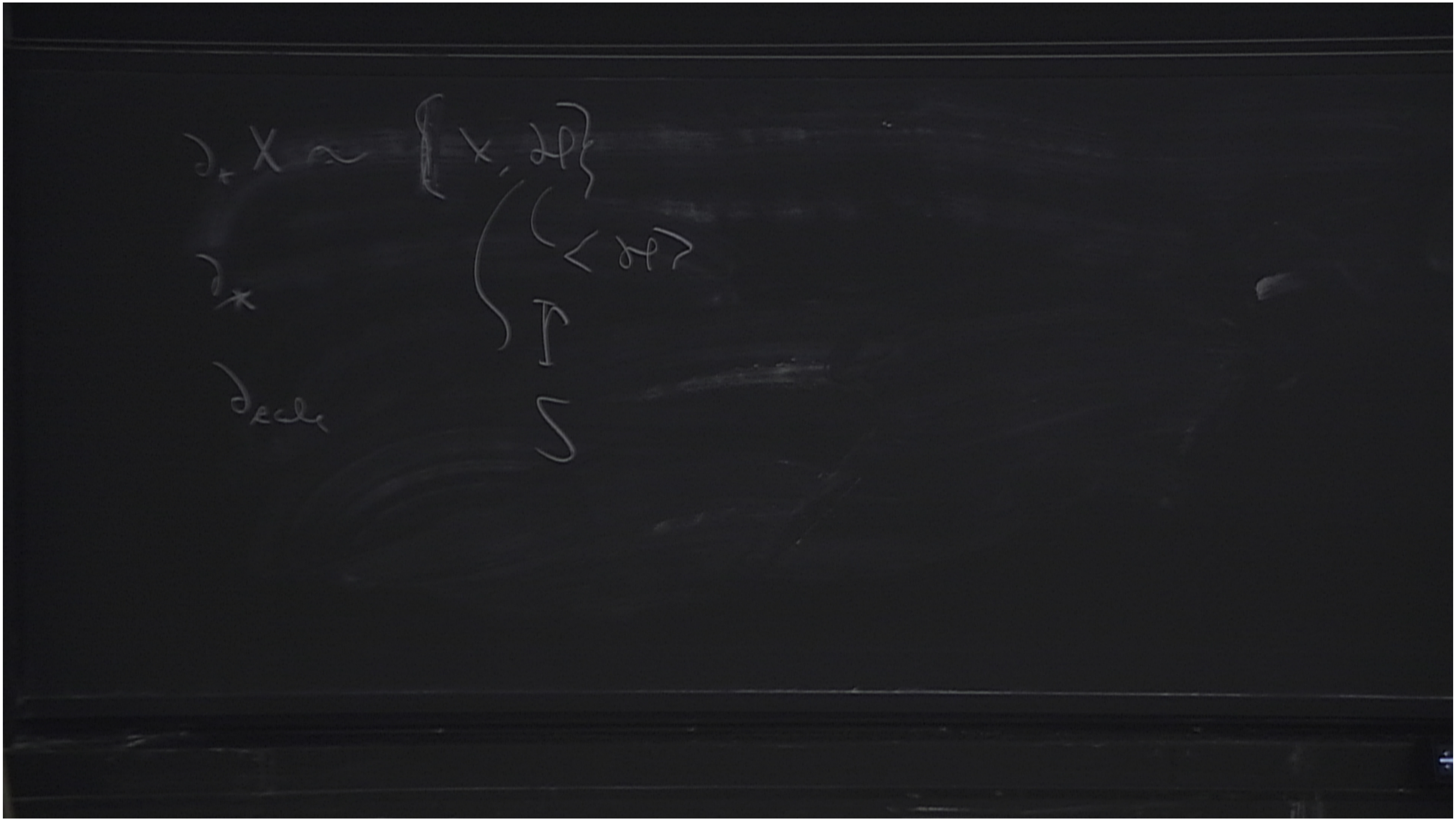
Field Theory and Statistical Mechanics are closely connected. A Wick rotation $t \to i/(kT)$ will take you from one to the other.

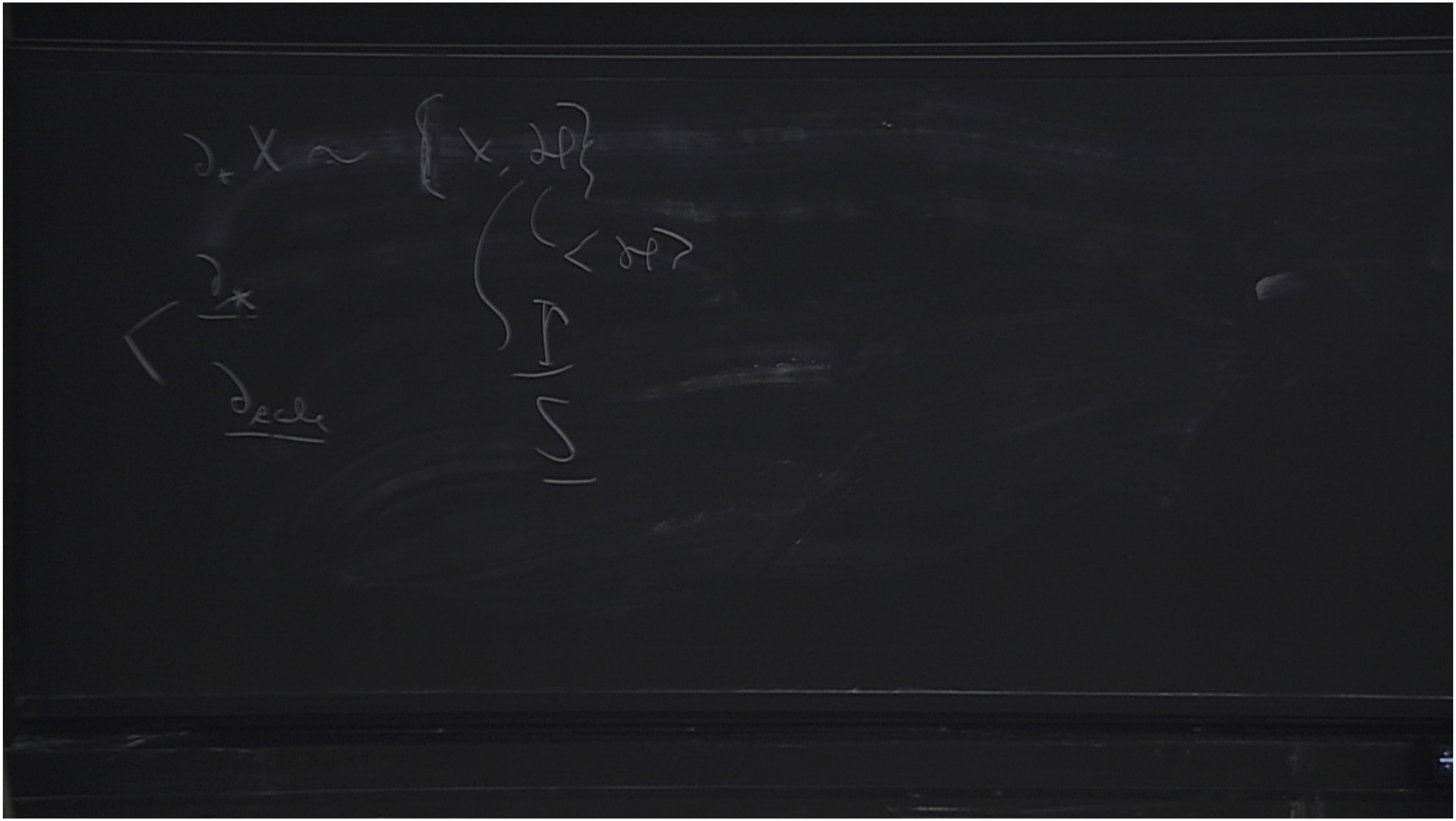
Quantum Mechanics and Classical Mechanics are closely connected. Both employ Hamiltonians as basic generators of time development as do Field Theory and Statistical Mechanics.

All four have a dual structure in which terms in the Hamiltonian both describe measurable quantities and equally generate changes in development.

All four have the same structure: Poisson Bracket and Commutator, conjugate variables = p's and q's.

I shall talk mostly about statistical mechanics.


∂t X ~ {X, ∂H}
< ∂H >



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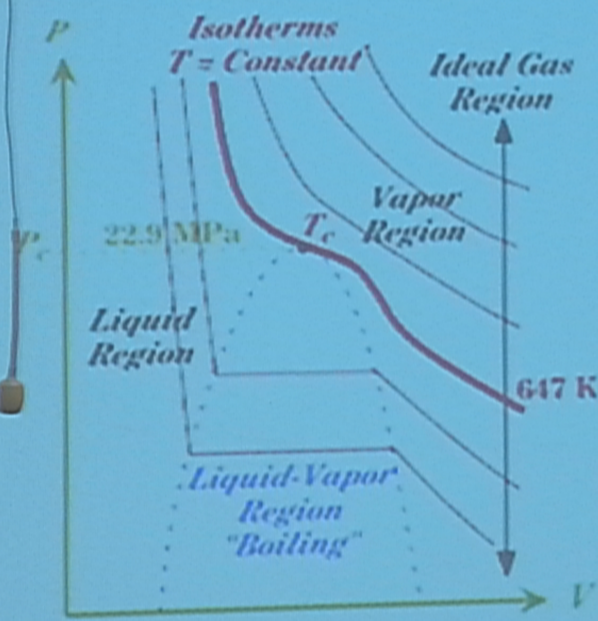
# History:

(1869 ) Thomas Andrews, experimentally studied the P-V diagram of $CO_2$. He discovered the critical point. His data look roughly like:

Phil. Trans. Roy. Soc. **159 p.** 575 (1869 )




Cartoon is PVT plot for water, but $CO_2$ is similar, with a more accessible critical point.

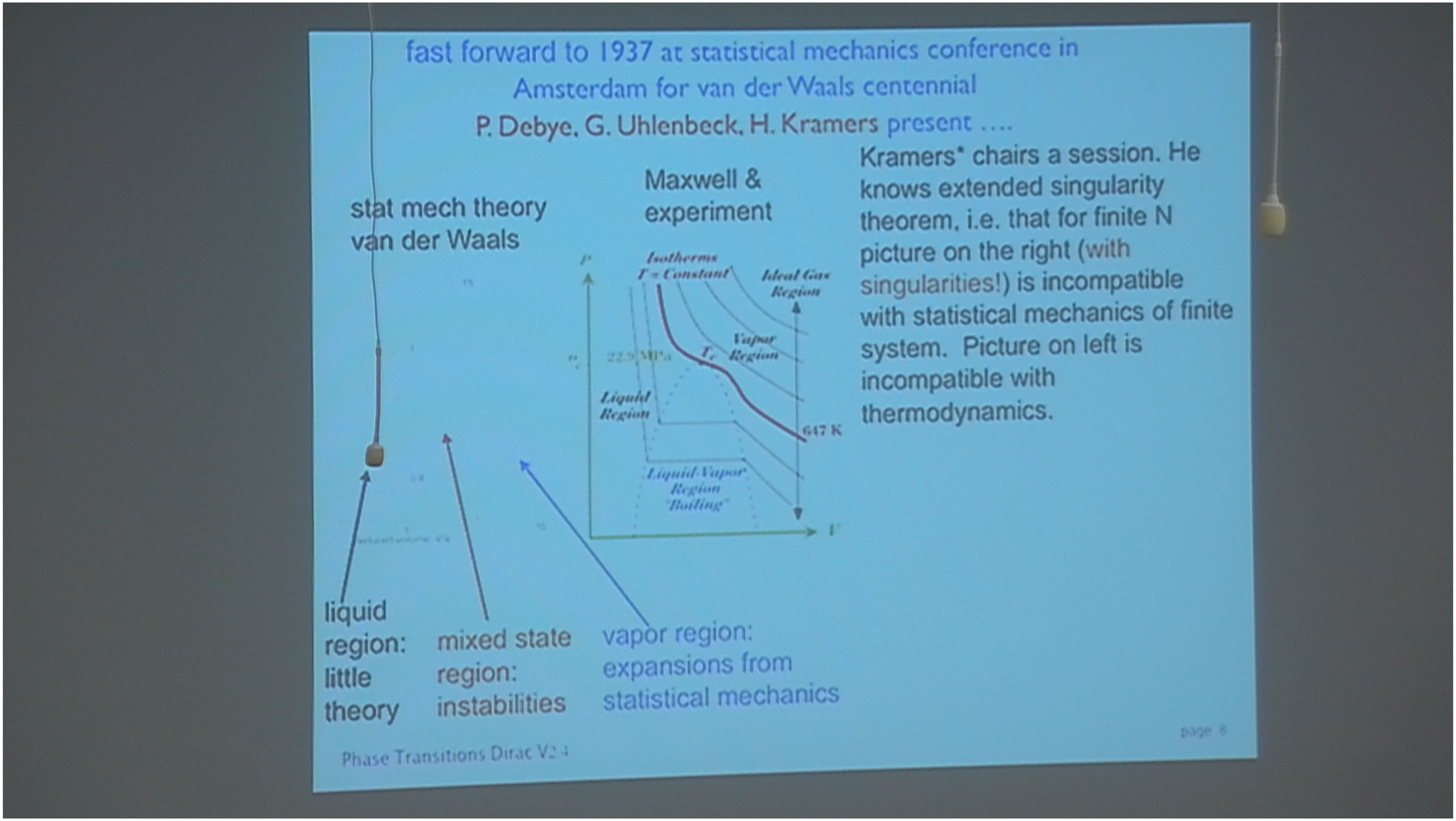
Note qualitative changes.

- as boiling takes one from liquid to vapor
- as one passes from isotherm to isotherm through critical point

These qualitative changes are mathematical singularities.

page 7

Phase Transitions Dirac V2.4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stat mech theory
van der Waals
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experiment
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T = Constant
Ideal Gas
Region
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Region
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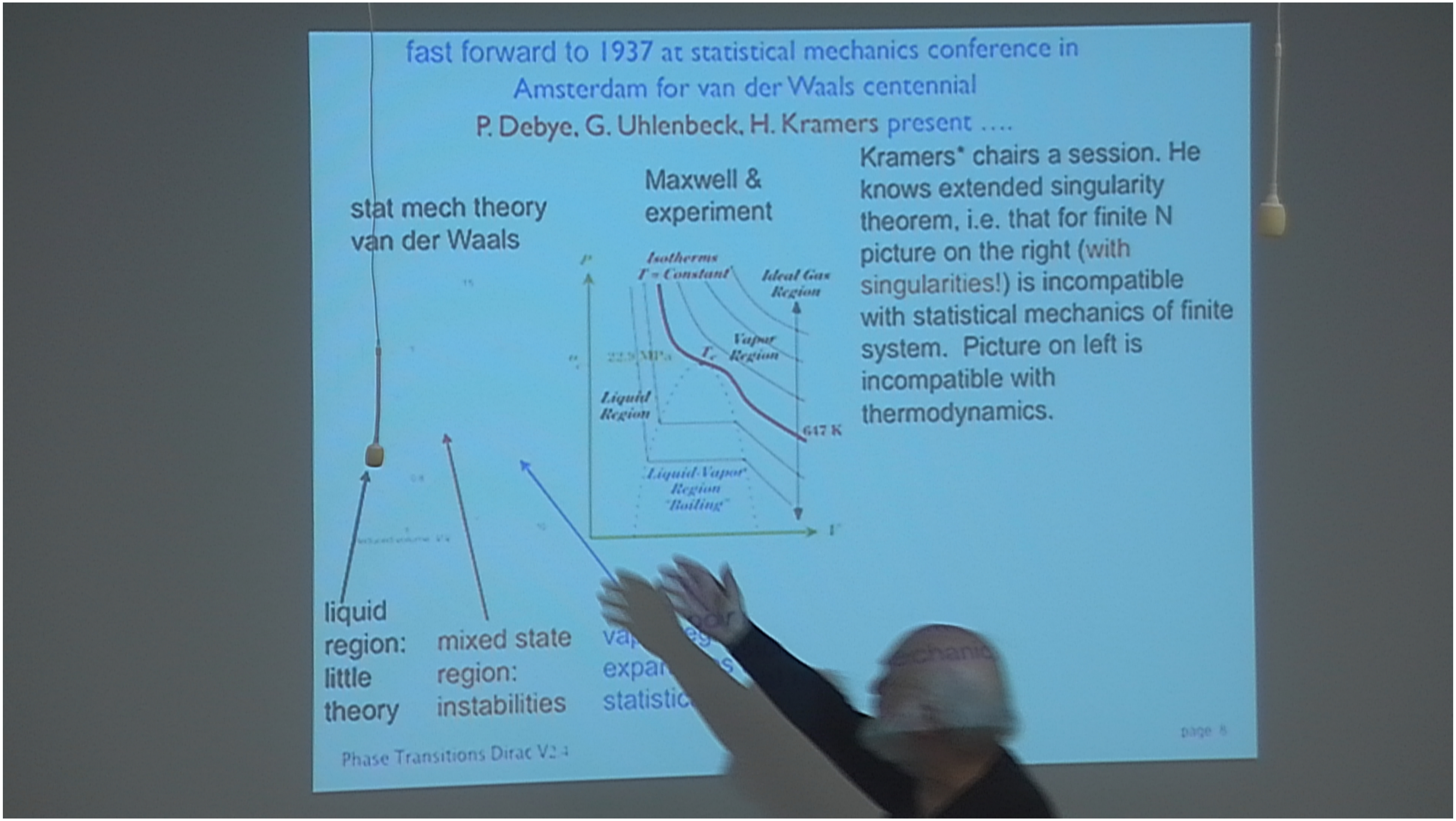
Region
Liquid-Vapor
Region
"Boiling"
647 K
Kramers* chairs a session. He knows extended singularity theorem, i.e. that for finite N picture on the right (with singularities!) is incompatible with statistical mechanics of finite system. Picture on left is incompatible with thermodynamics.
liquid region: little theory
mixed state region: instabilities
vapor region: expansions from statistical mechanics
Phase Transitions Dirac V2.4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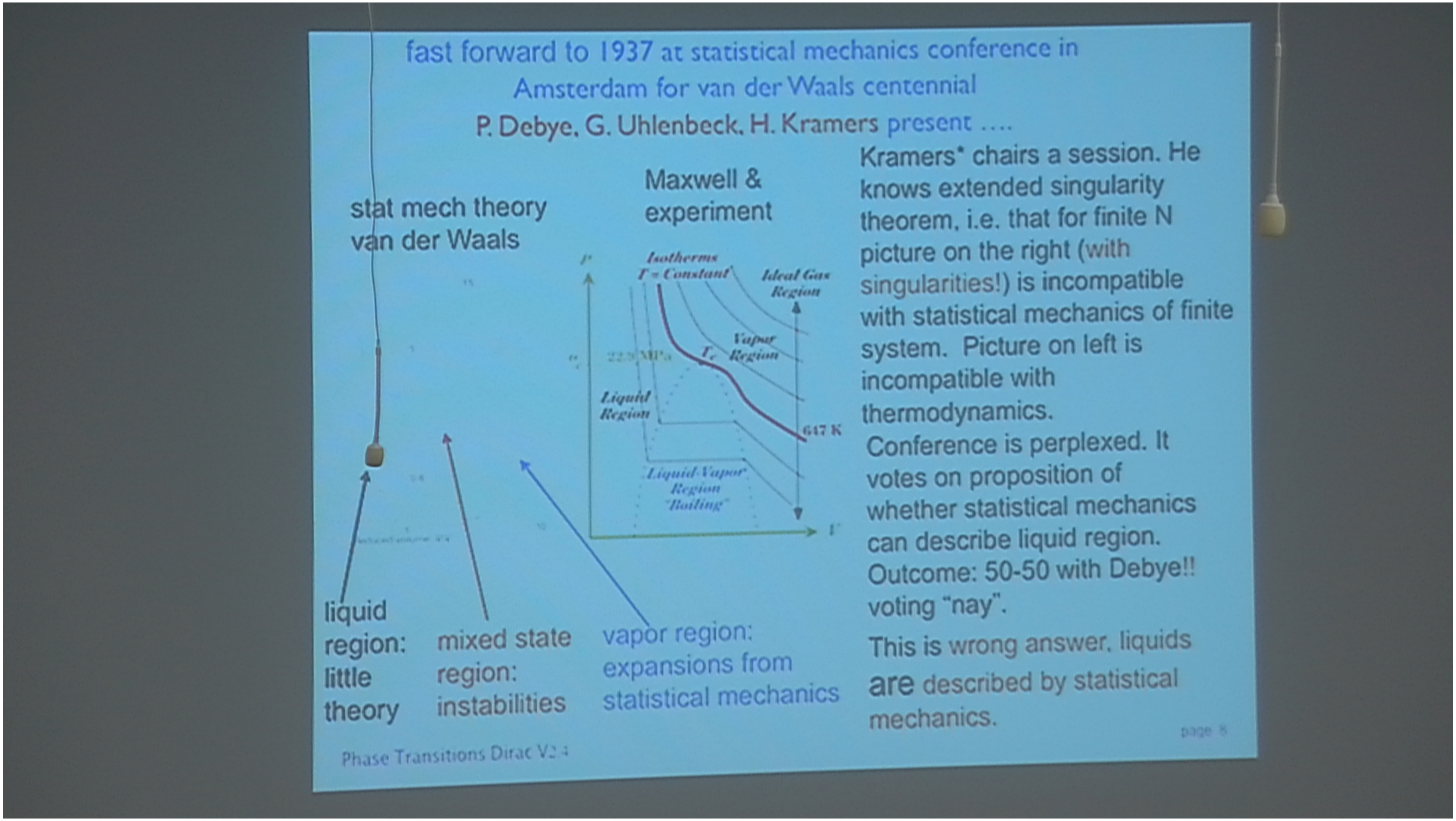
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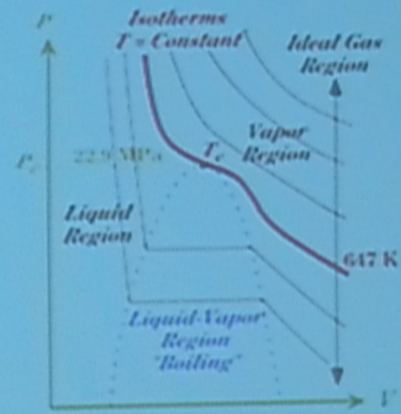
Conference is perplexed. It votes on proposition of whether statistical mechanics can describe liquid region. Outcome: 50-50 with Debye!! voting "nay".
This is wrong answer, liquids are described by statistical mechanics.
liquid region: little theory
mixed state region: instabilities
vapor region: expansions from statistical mechanics
Phase Transitions Dirac V2.4

# Application to Phase Transitions: today's view

- thermodynamic phase transitions involve singularities, and infinities arising (almost always) from unbounded numbers of particles
- these infinities appear in thermodynamic derivatives which is caused by a coherence length (correlation length) that diverges*
- in practice coherence length describes spatial extent of fluctuations that look like regions of two phases intermixed. e.g. drops of vapor in liquid or drops of liquid in vapor.

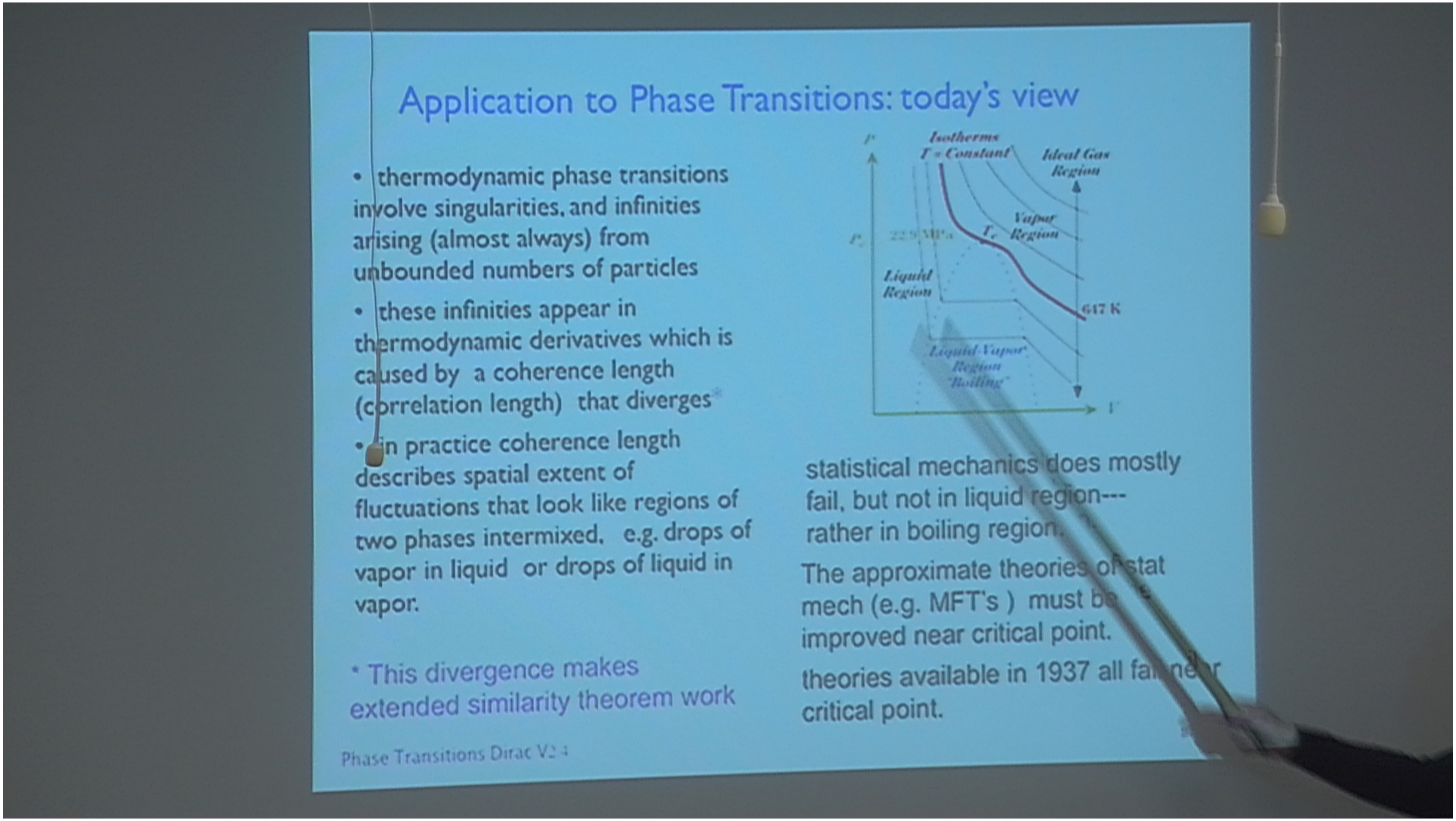
* This divergence makes extended similarity theorem work




statistical mechanics does mostly fail, but not in liquid region--- rather in boiling region.

The approximate theories of stat mech (e.g. MFT's ) must be improved near critical point.

theories available in 1937 all fail near critical point.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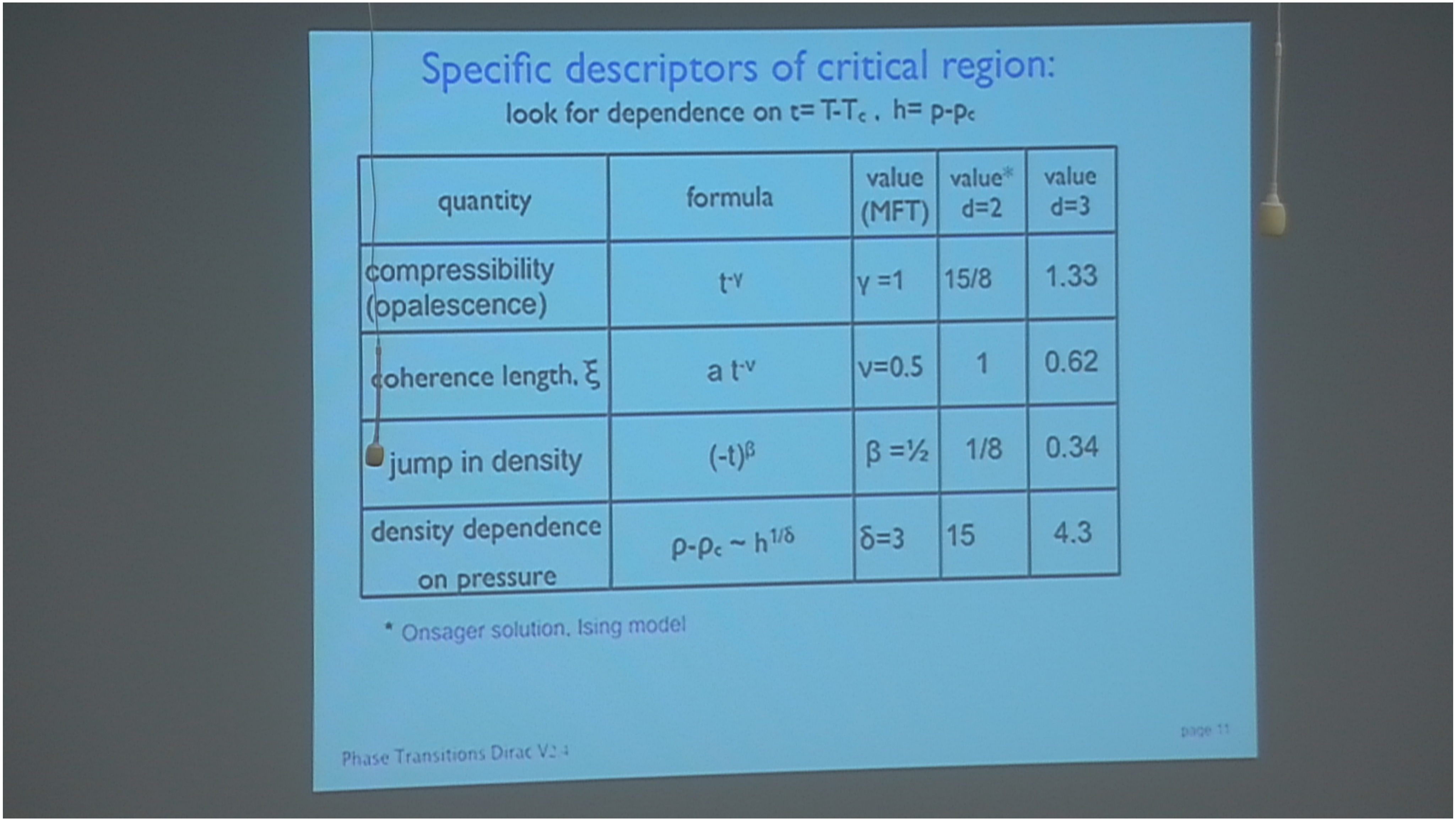
"Boiling"
statistical mechanics does mostly fail, but not in liquid region--- rather in boiling region.
The approximate theories of stat mech (e.g. MFT's ) must be improved near critical point.
theories available in 1937 all fail near critical point.
Phase Transitions Dirac V2.4


Specific descriptors of critical region:
look for dependence on t= T-Tc , h= p-pc
quantity | formula | value (MFT) | value* d=2 | value d=3
compressibility (opalescence) | t-γ | γ =1 | 15/8 | 1.33
coherence length, ξ | a t-ν | ν=0.5 | 1 | 0.62
jump in density | (-t)β | β =½ | 1/8 | 0.34
density dependence on pressure | ρ-ρc ~ h1/δ | δ=3 | 15 | 4.3
* Onsager solution, Ising model
Phase Transitions Dirac V2.4
page 11

# Specific descriptors of critical region:

look for dependence on $t = T - T_c$ , $h = p - p_c$

| quantity | formula | value (MFT) | value* d=2 | value d=3 |
|---|---|---|---|---|
| compressibility (opalescence) | $t^{-\gamma}$ | $\gamma = 1$ | 15/8 | 1.33 |
| coherence length, $\xi$ | $a\, t^{-\nu}$ | $\nu = 0.5$ | 1 | 0.62 |
| jump in density | $(-t)^{\beta}$ | $\beta = ½$ | 1/8 | 0.34 |
| density dependence on pressure | $\rho - \rho_c \sim h^{1/\delta}$ | $\delta = 3$ | 15 | 4.3 |

* Onsager solution, Ising model

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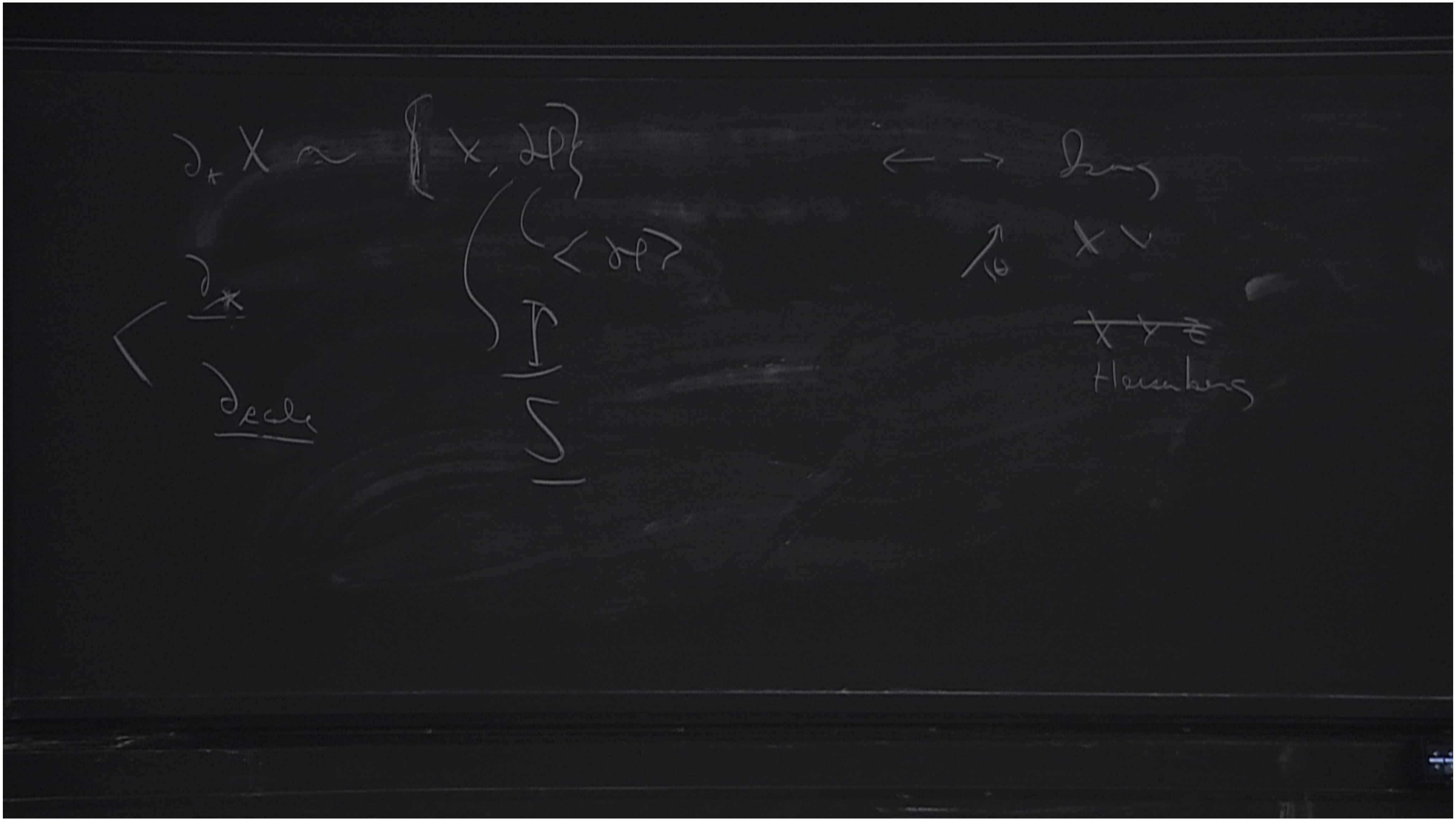
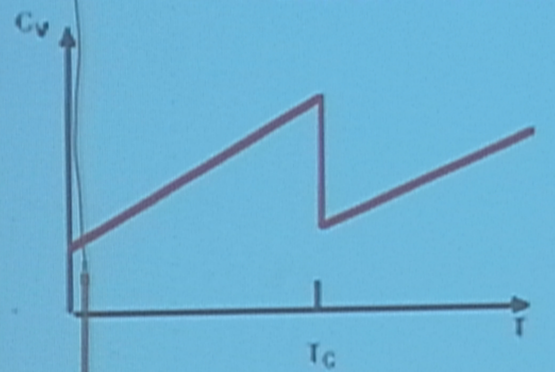
# Mean Field Theory is useless in predicting phase transitions and ordering over long distances

It predicts transitions in one-dimensional systems with finite-range interactions at non-zero temperatures.. (In fact, these transitions never occur. )

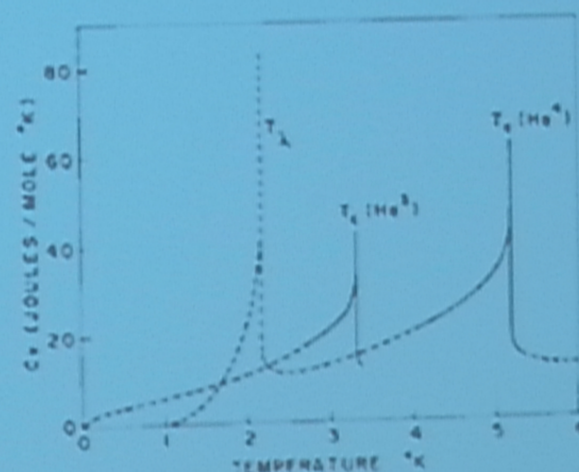
It predicts average order and transitions in two dimensions for Ising models, XY models, and Heisenberg models at non-zero temperatures. In these cases. there are respectively transitions plus order ($<\sigma_r> \neq 0$), transitions but no average order ($<\sigma_r>=0$), and no transitions or ordering.


Heisenberg

# Mean Field Theory is Useless near Critical Point: Look at heat capacity, $C_v$




Mean Field Theory= discontinuous but finite jump at $T_c$

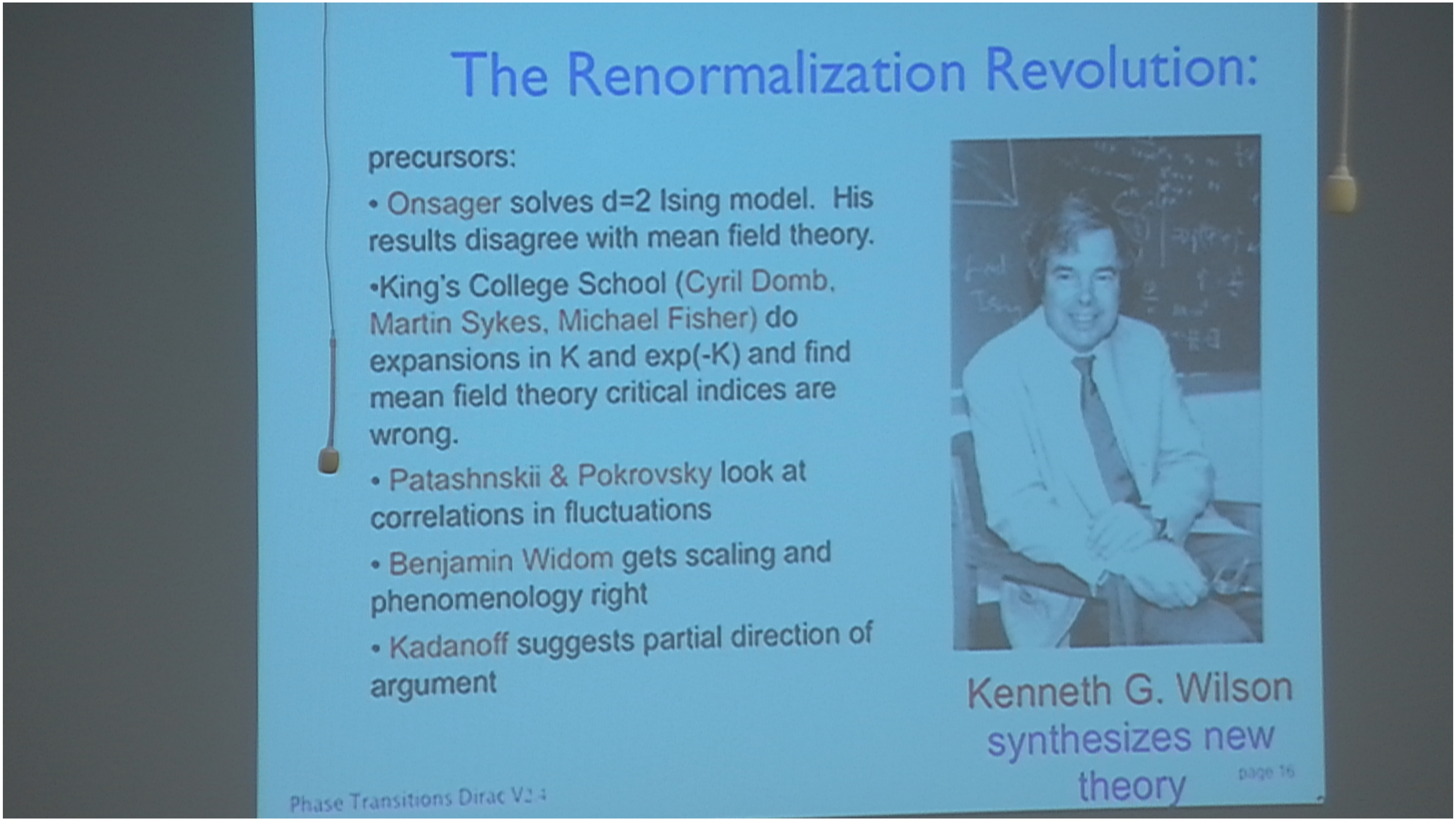



Moldover and Little see singular result, probably going to infinity

# The physics is in fluctuations

which extend over an indefinite range at critical point. t and h limit range of fluctuations to finite value, called the correlation length, $\xi$. How can we convert this fact into a theory?

At the singularities these fluctuations are droplets of fluid which have all different scales from the microscopic to as large as you want. Away from singularity correlation length serves to cut off the largest-scale fluctuations. These droplets are regions of density different from that of the surrounding fluid.

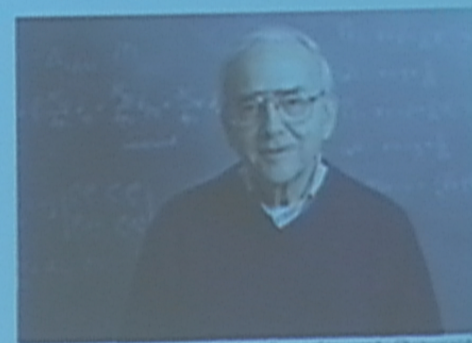

The Renormalization Revolution:
precursors:
• Onsager solves d=2 Ising model. His results disagree with mean field theory.
•King's College School (Cyril Domb, Martin Sykes, Michael Fisher) do expansions in K and exp(-K) and find mean field theory critical indices are wrong.
• Patashnskii & Pokrovsky look at correlations in fluctuations
• Benjamin Widom gets scaling and phenomenology right
• Kadanoff suggests partial direction of argument
Kenneth G. Wilson
synthesizes new theory
page 16
Phase Transitions Dirac V2.4

# Toward the revolution

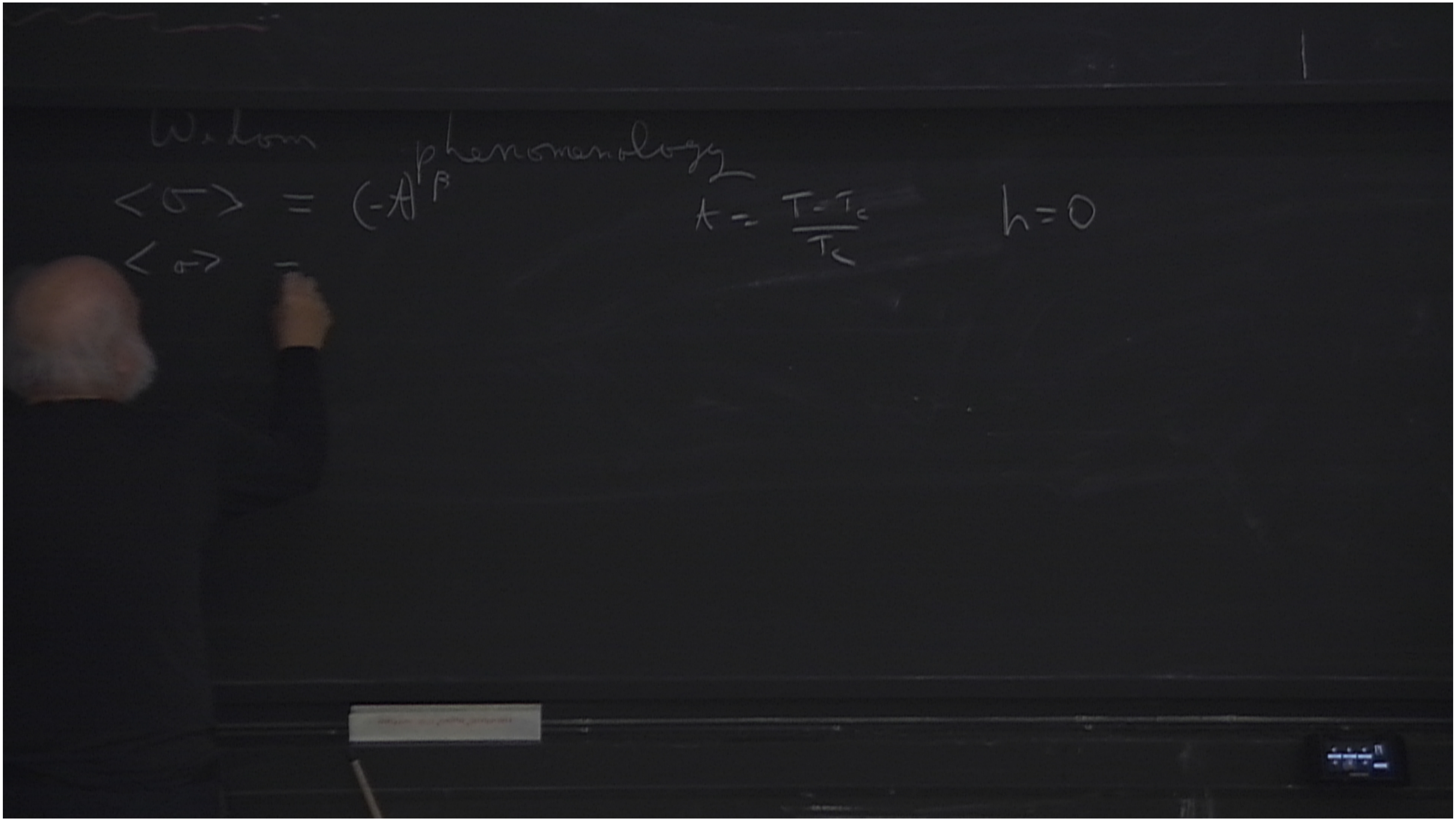
## The phenomenology

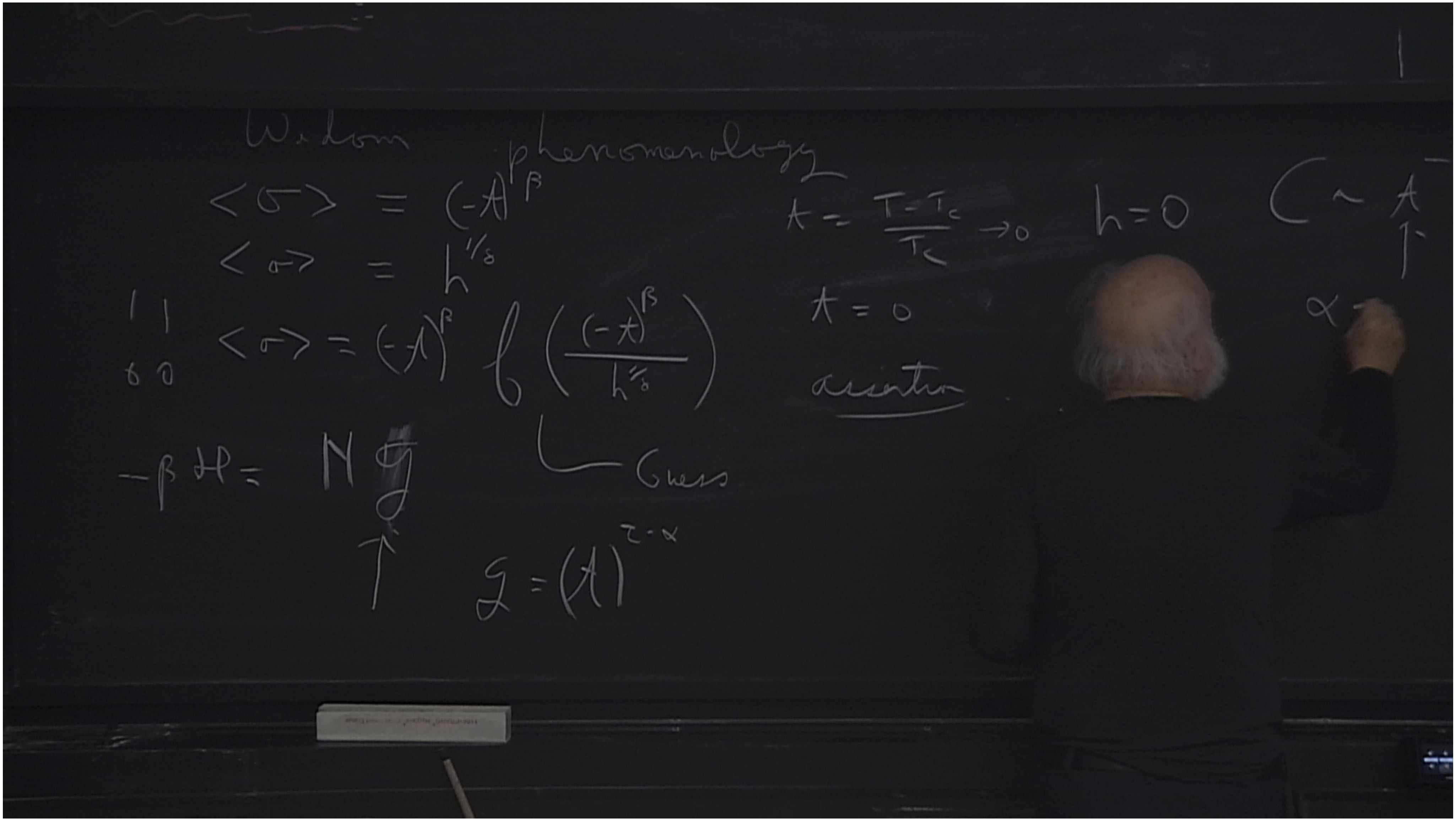
Ben Widom noticed the most significant scaling properties of critical phenomena, but did not detail where they might have come from.

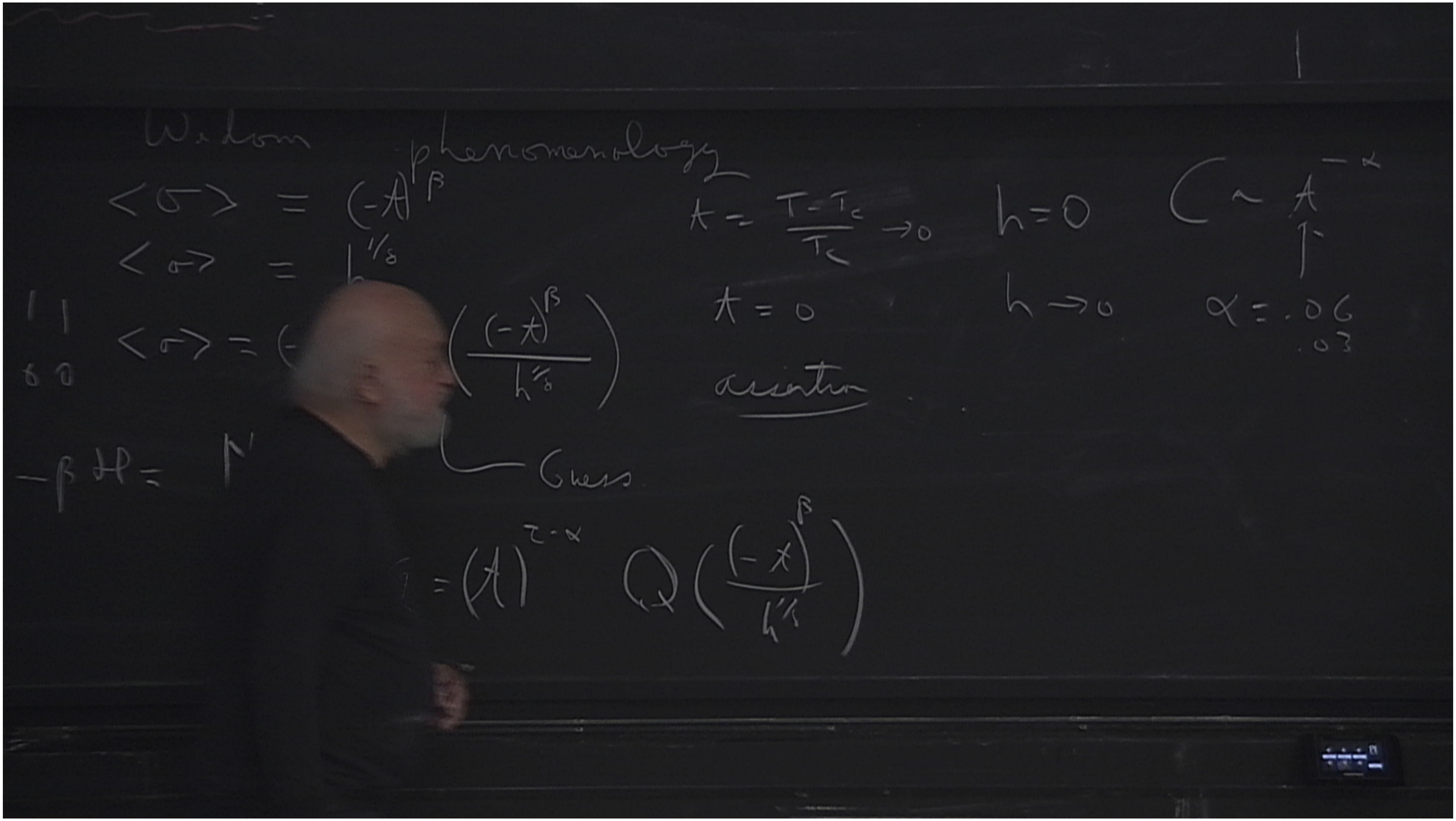
B. Widom. J. Chem. Phys. **43** 3892 and 3896 (1965).

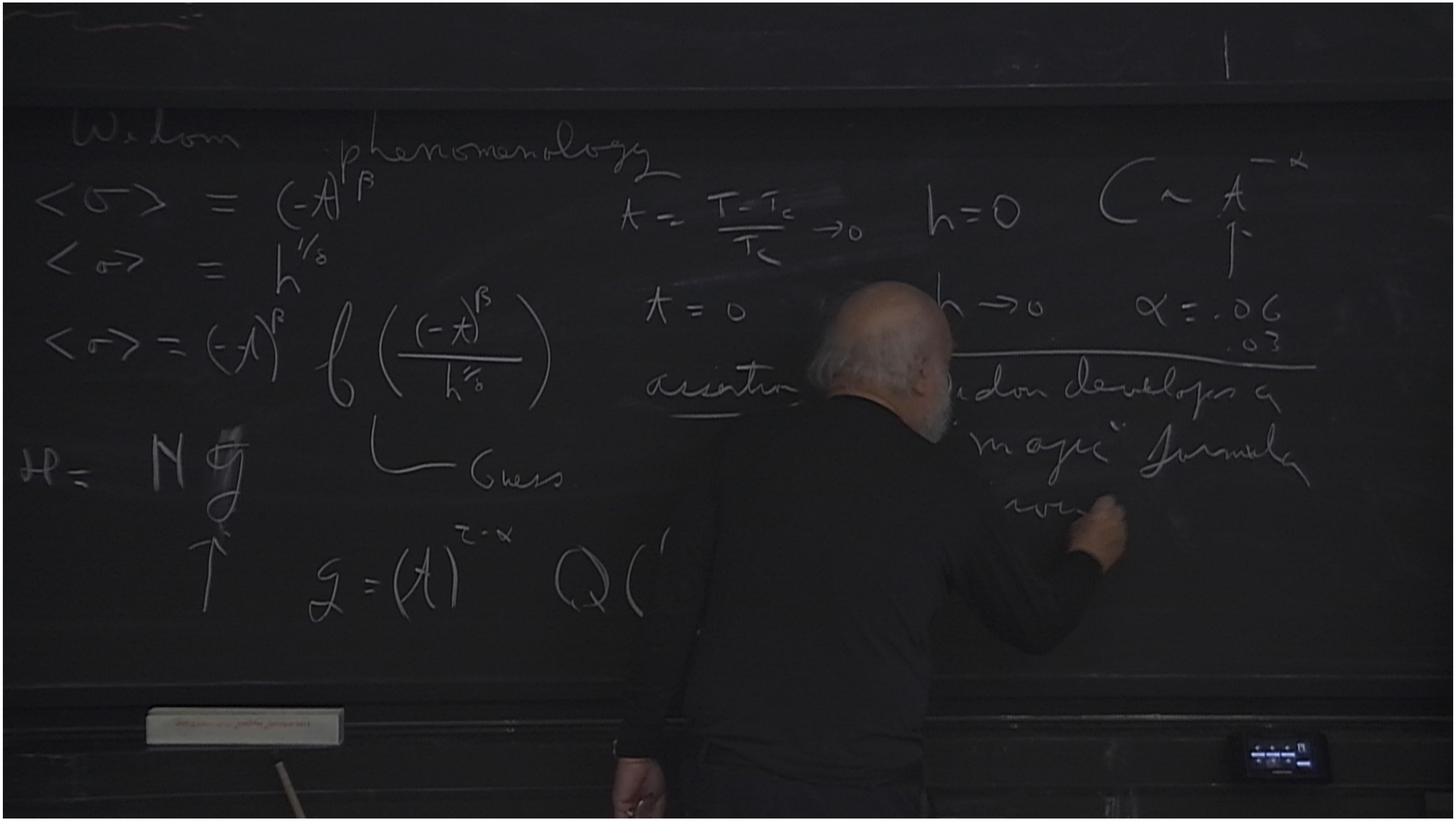


Professor Benjamin Widom in his office in Baker Lab


Widom phenomenology
<σ> = (-t)^β
t = (T - Tc)/Tc
h = 0
<σ> =


Widom phenomenology
h=0
Guess


Widom
phenomenology
<σ> = (-t)^β
<σ> = h^{1/δ}
t = (T - Tc)/Tc → 0
h = 0
C ~ t^{-α}
t = 0
h → 0
α = .06
.03
Guess


Widom phenomenology
<σ> = (-t)^β
t = (T - Tc)/Tc → 0
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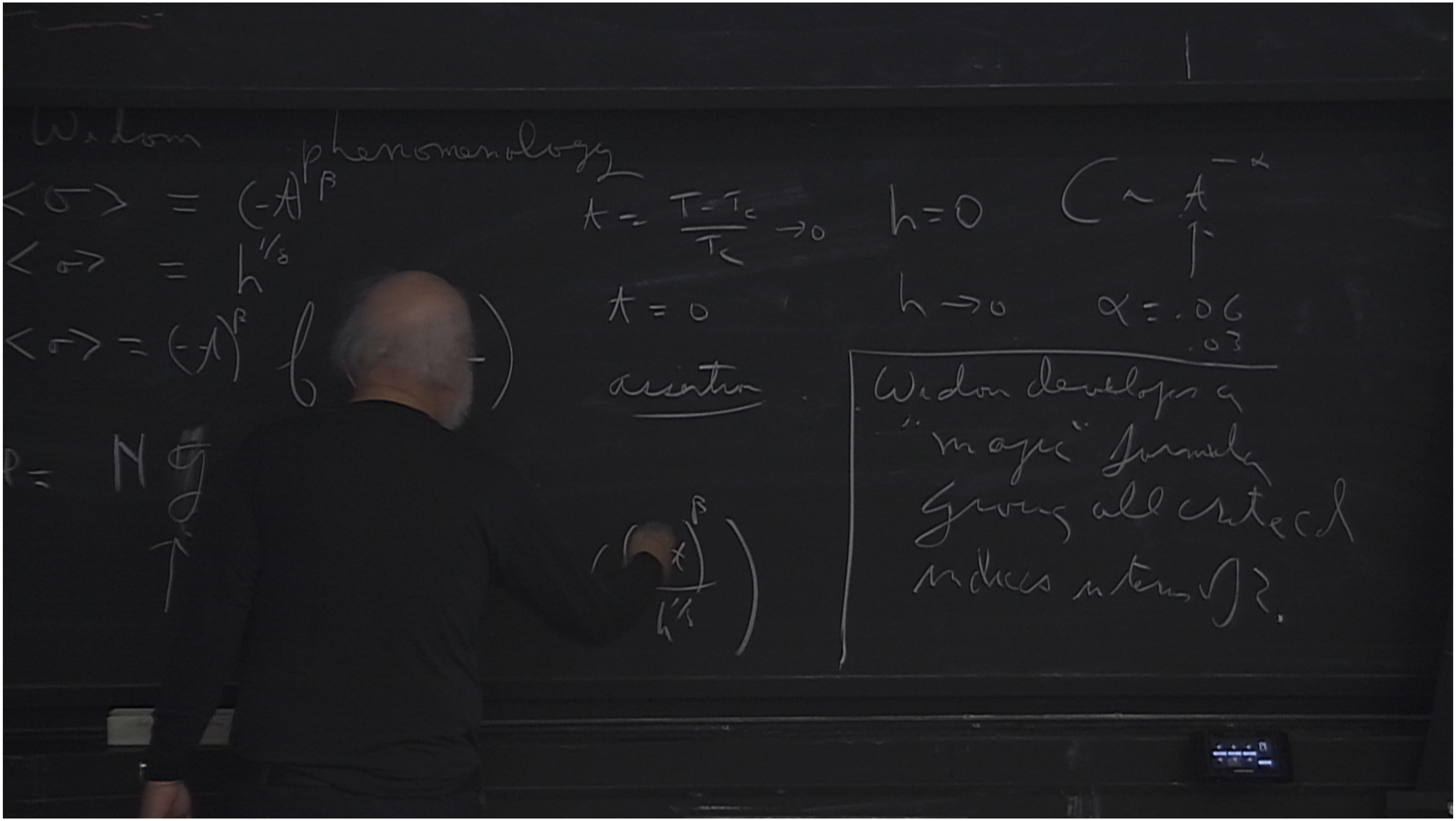
C ~ t^-α
<σ> = h^1/δ
t = 0
h → 0
α = .06
.03
<σ> = (-t)^β f((-t)^β / h^1/δ)
Guess
"magic" formula


Widom
phenomenology
h=0
h→0
α=.06
.03
Widom develops a "magic" formula giving all critical indices in terms of ?

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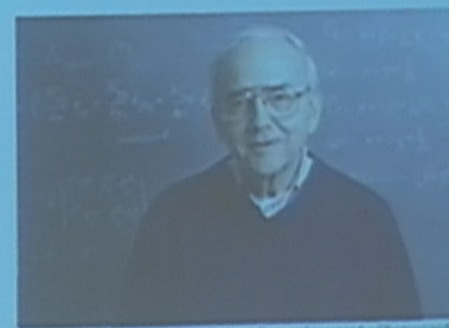

Professor Benjamin Widom in his office in Baker Lab

# Widom's results

in terms of $t=T-T_c$  $h=p-p_c$

Widom 1965: scaling result  He focuses attention on scaling near critical point.  In this region, averages and fluctuations have a characteristic size, for example   density jump ~ $(-t)^{\beta}$ when h=0

density minus critical density ~ $(h)^{1/\delta}$ when t=0

Therefore, Widom argues there is a characteristic size for h, which is

$h^* \sim (-t)^{\beta\,\delta} = (-t)^{\Delta}$ with $\Delta = \beta\,\delta$

so that density minus critical density = $(-t)^{\beta}\, g(h/t^{\Delta})$

therefore, using a little thermodynamics,  scaling for free energy is

$F(t,h) = V\, t^{\beta+\Delta} f^*(h/t^{\Delta}) + F_{non\text{-}singular}$:  (V is volume of system)

Further he says singular term in free energy given by excitations of size of coherence length with kT per excitation.  They fill all space, giving

$F - F_{non\text{-}singular} \sim (\text{Volume of system})/\,\xi^d \sim V t^{d\nu}$

Therefore "magic" relations, e.g. $\beta + \Delta = d\,\nu$

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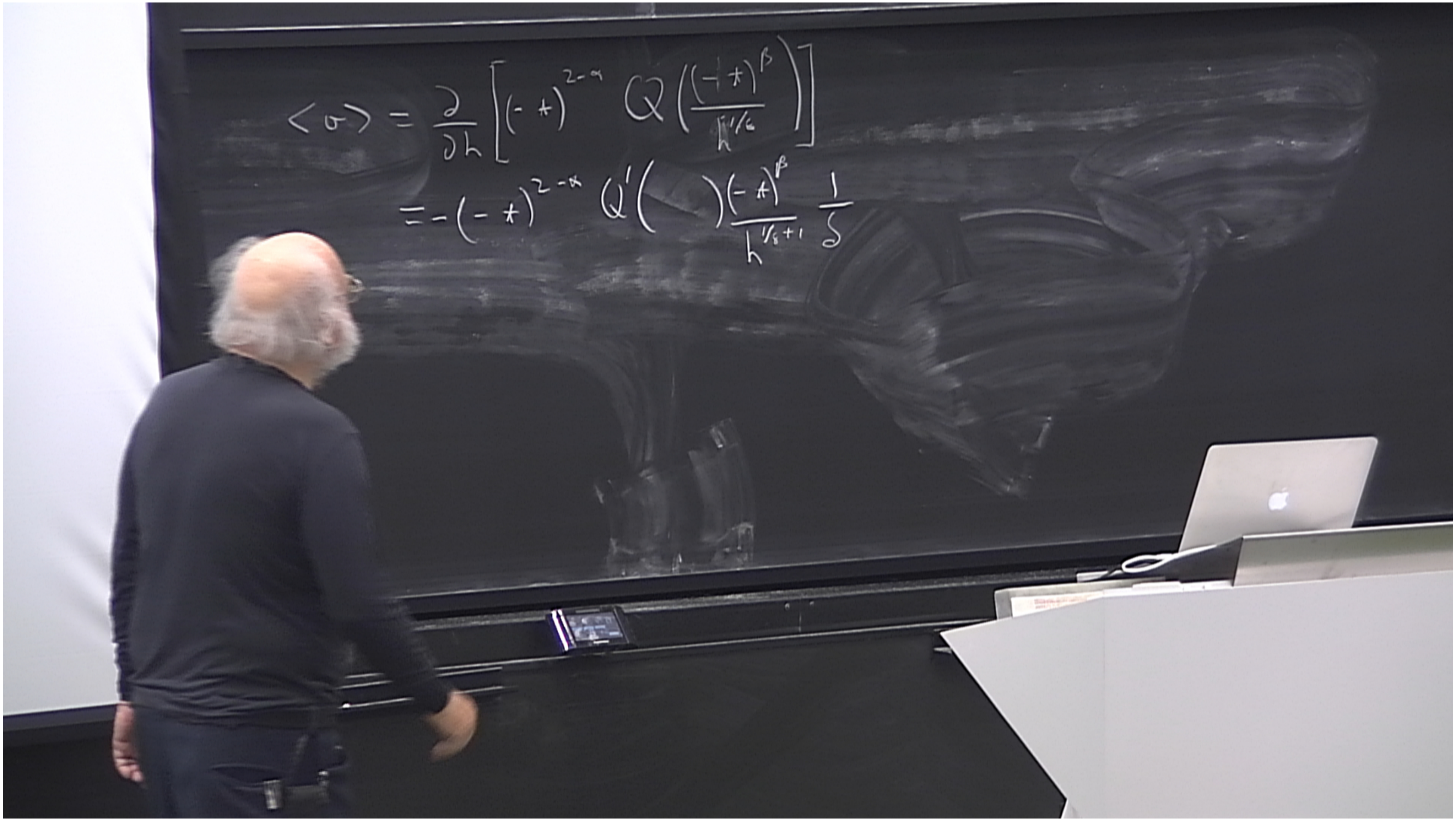
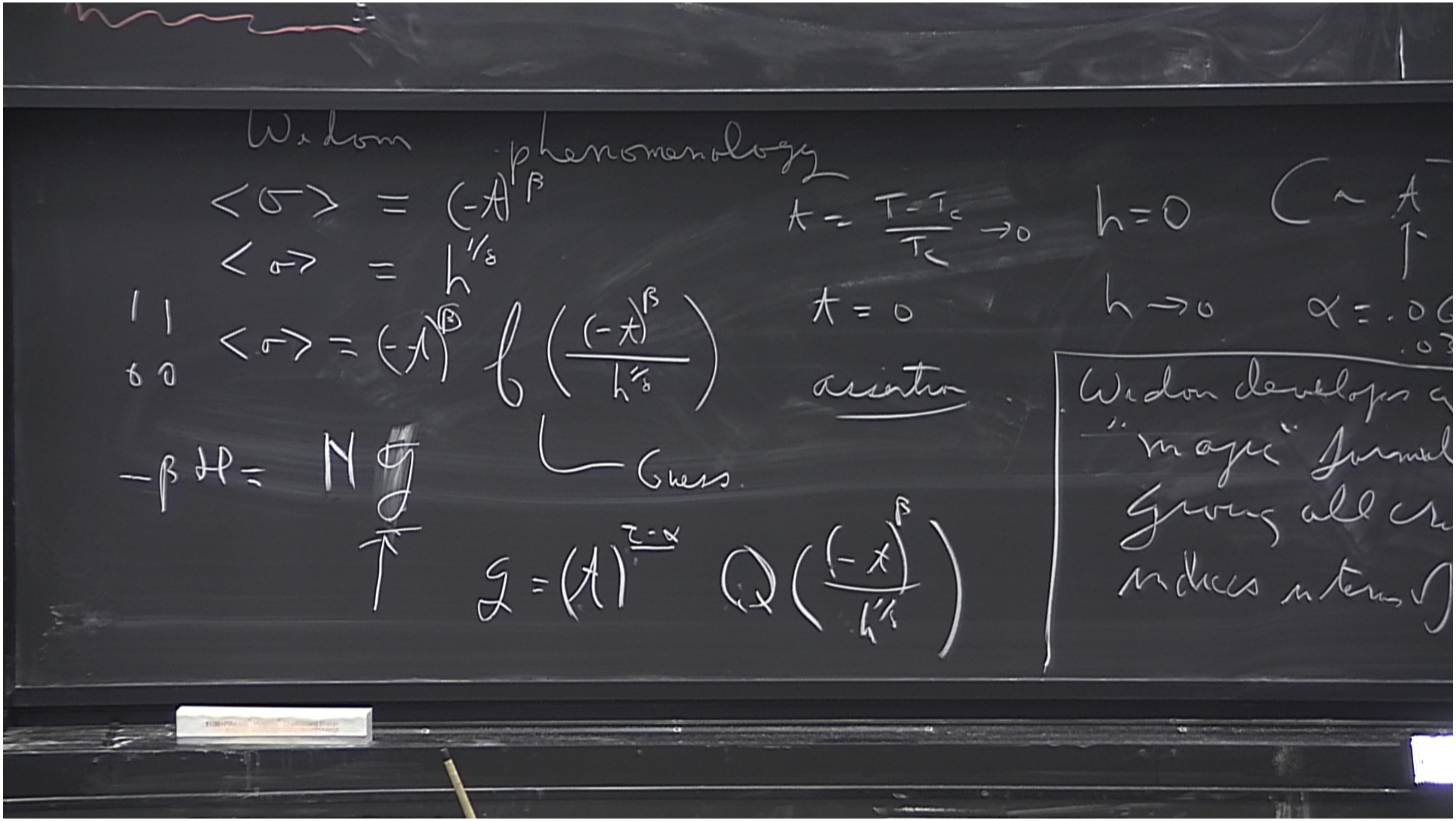

⟨σ⟩ = ∂/∂h [ (-t)^{2-α} Q( (-t)^β / h^{1/δ} ) ]
= -(-t)^{2-α} Q'( ) (-t)^β / h^{1/δ+1} 1/δ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<σ> = (-t)^β
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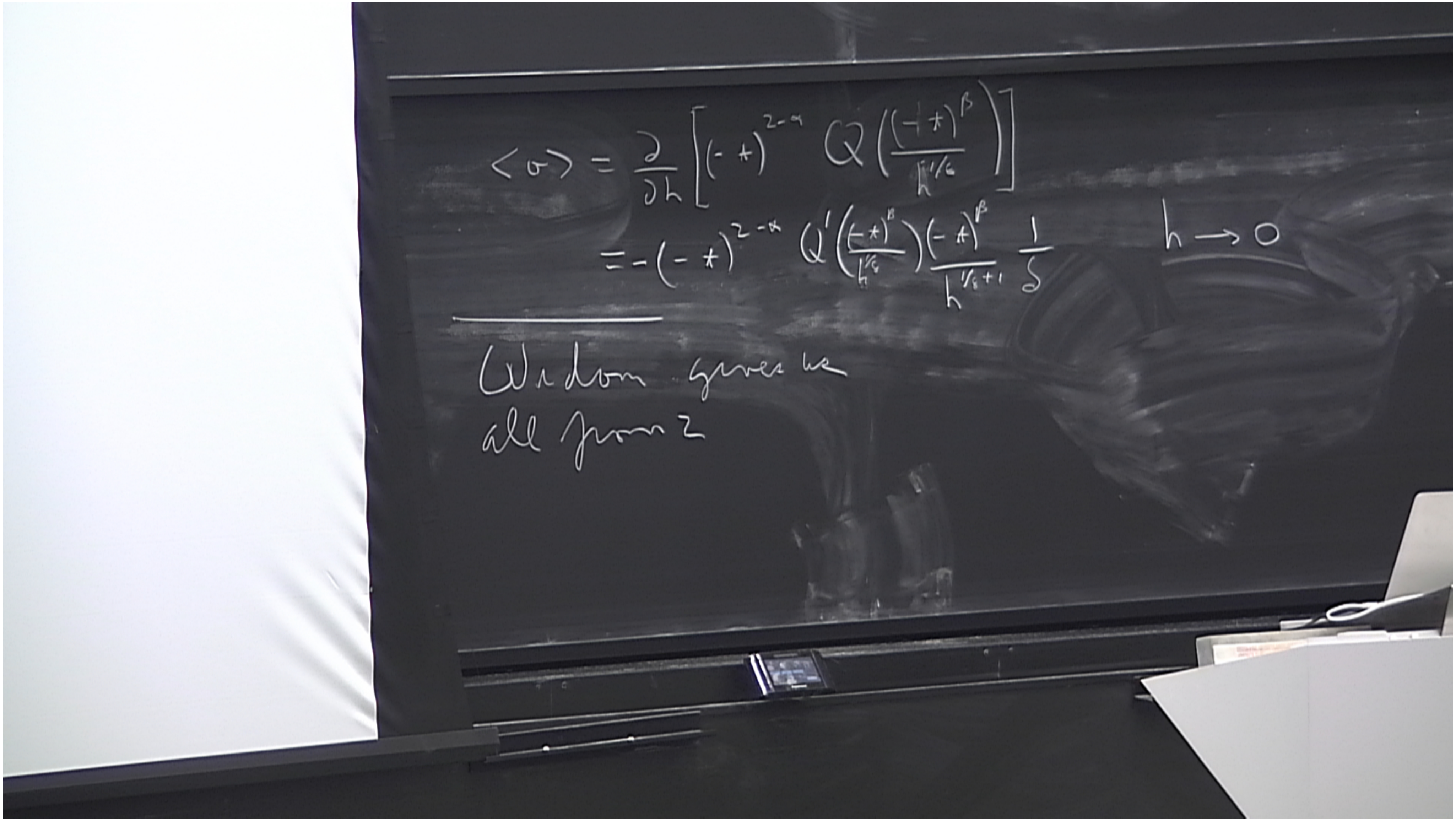
<σ> = (-t)^β f( (-t)^β / h^{1/δ} )
Guess
-βH = N g
g = (t)^{2-α} Q( (-t)^β / h^{1/δ} )
t = (T - Tc)/Tc → 0
h = 0
t = 0
h → 0
assertion
Widom develops a
"magic" formul
Giving all cr
indices in terms of



<σ> = ∂/∂h [(-t)^(2-α) Q((-t)^β / h^(1/δ))]
= -(-t)^(2-α) Q'((-t)^β / h^(1/δ)) (-t)^β / h^(1/δ+1) 1/δ
h → 0
Widom gives us
all from 2

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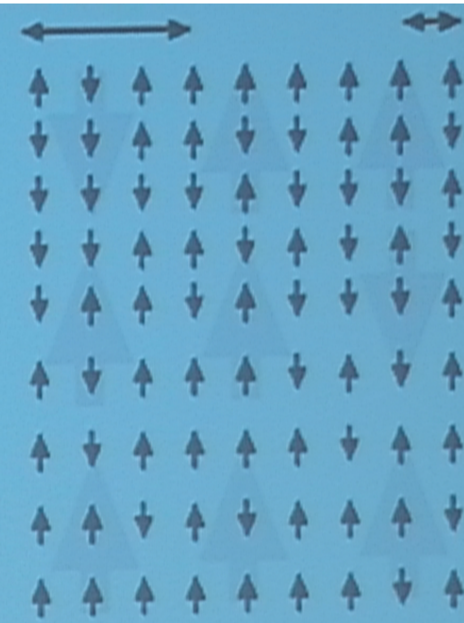
Therefore "magic" relations, e.g. $\beta + \Delta = d\,\nu$

page 18

# Block Scaling 1966

Kadanoff considers invariance properties of critical point and asks how description might change if one replaced a block of spins by a single spin, thus changing the length scale and having fewer degrees of freedom.

Answer: There are new effective values of $(T-T_c)=t$, $(p-p_c)=h$, and free energy per spin $K_0$. These describe the system just as well as the old values. Fewer degrees of freedom imply new couplings, but no change at all in the physics. This result incorporates both scale-invariance and universality.



$$N' = N/\ell^d$$

$$h' = h\,\ell^{y_h}$$

$$t' = t\,\ell^{y_t}$$

page 19

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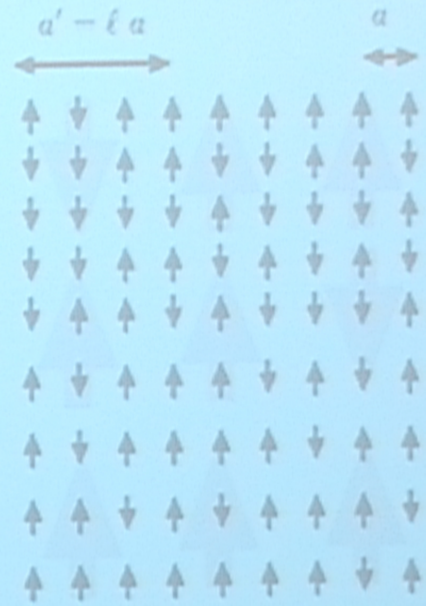


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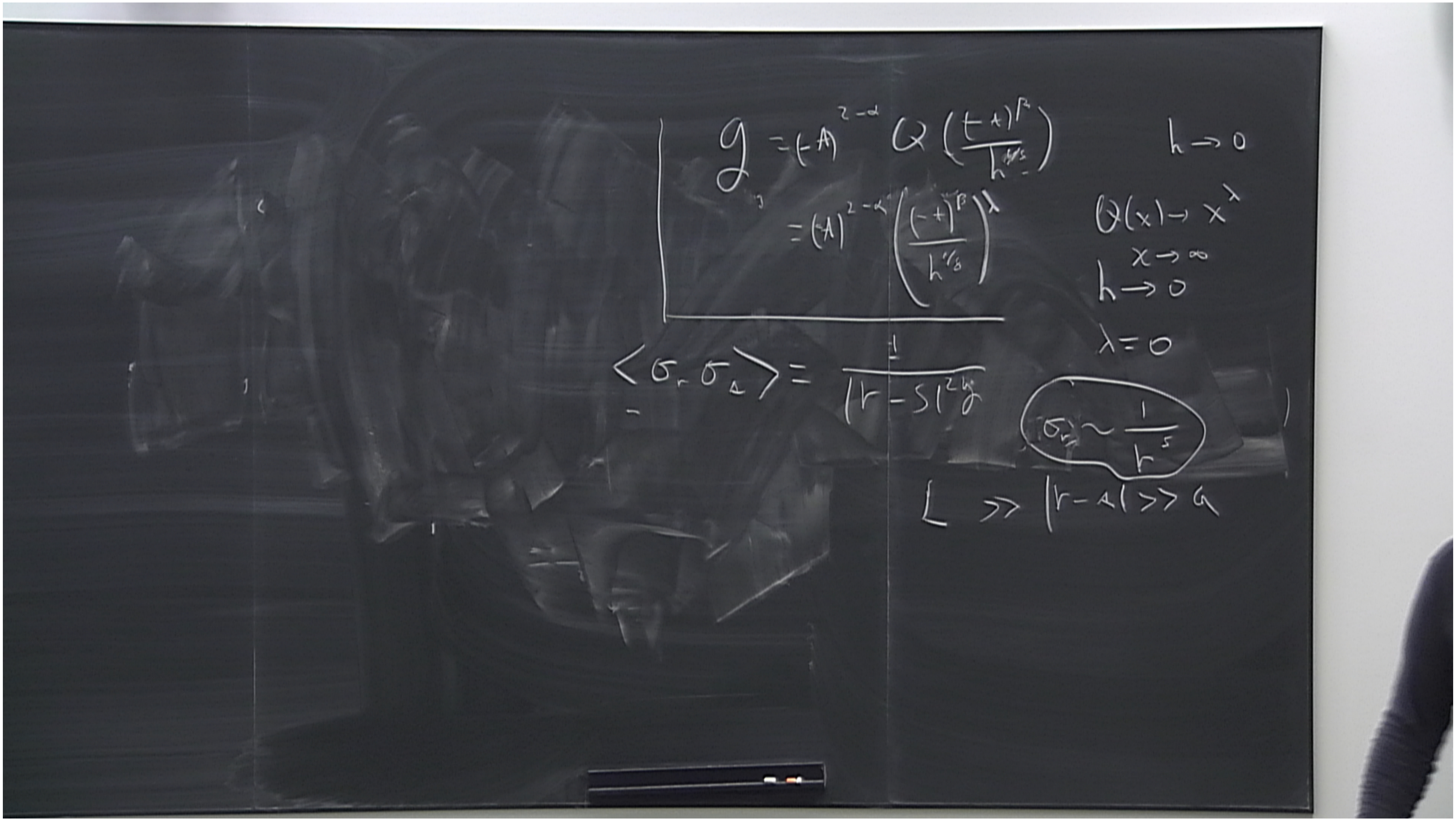
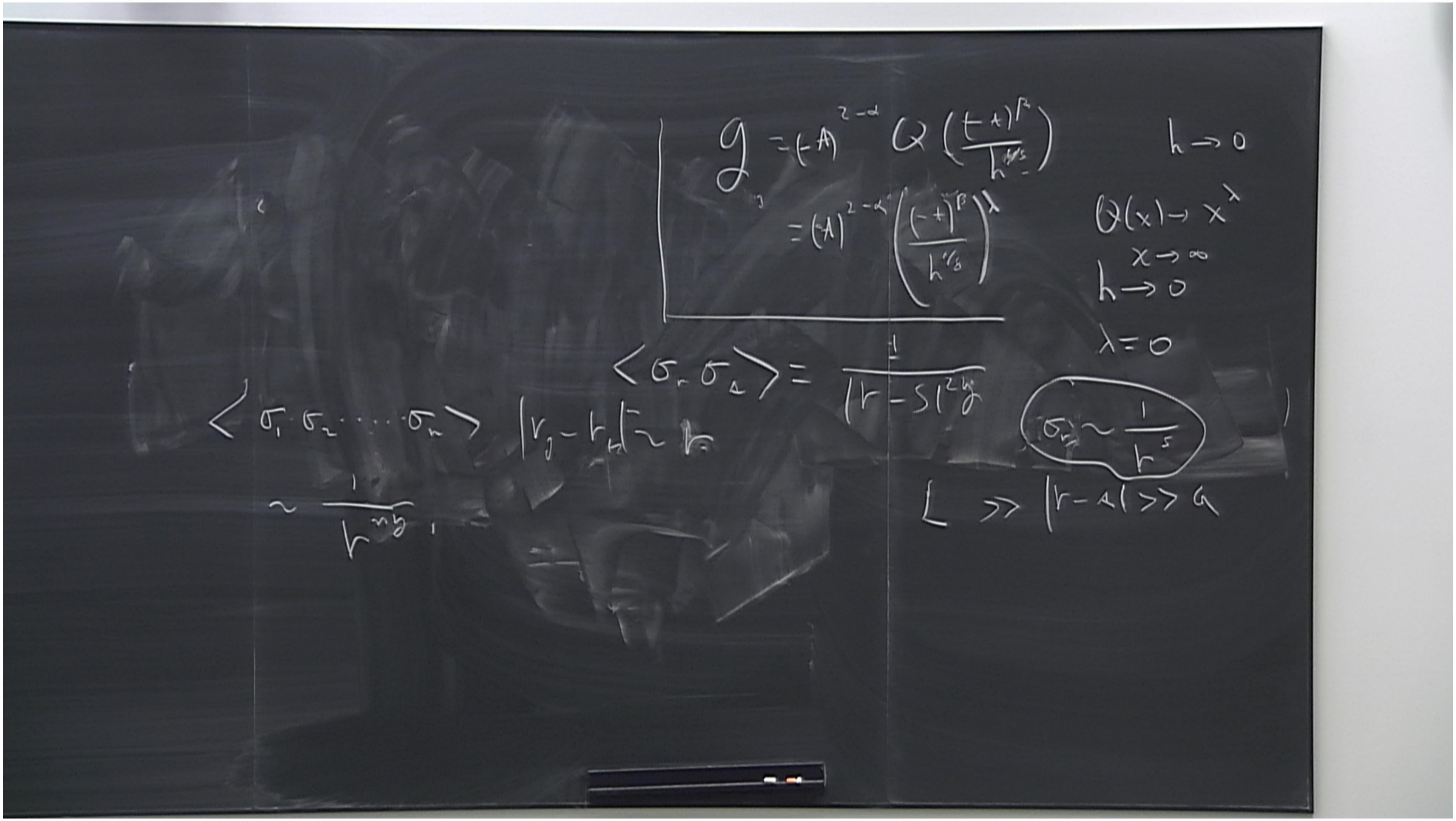
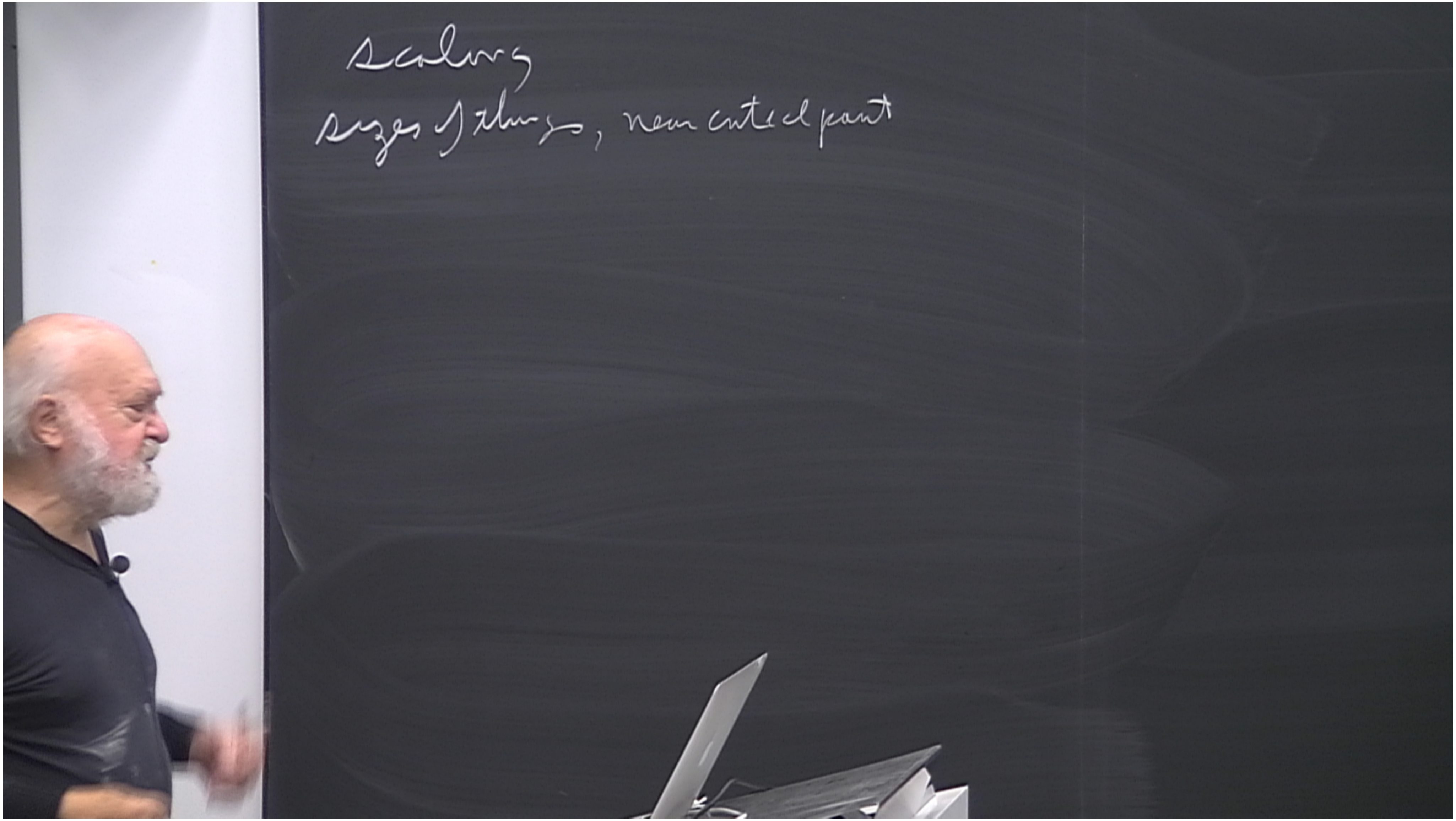
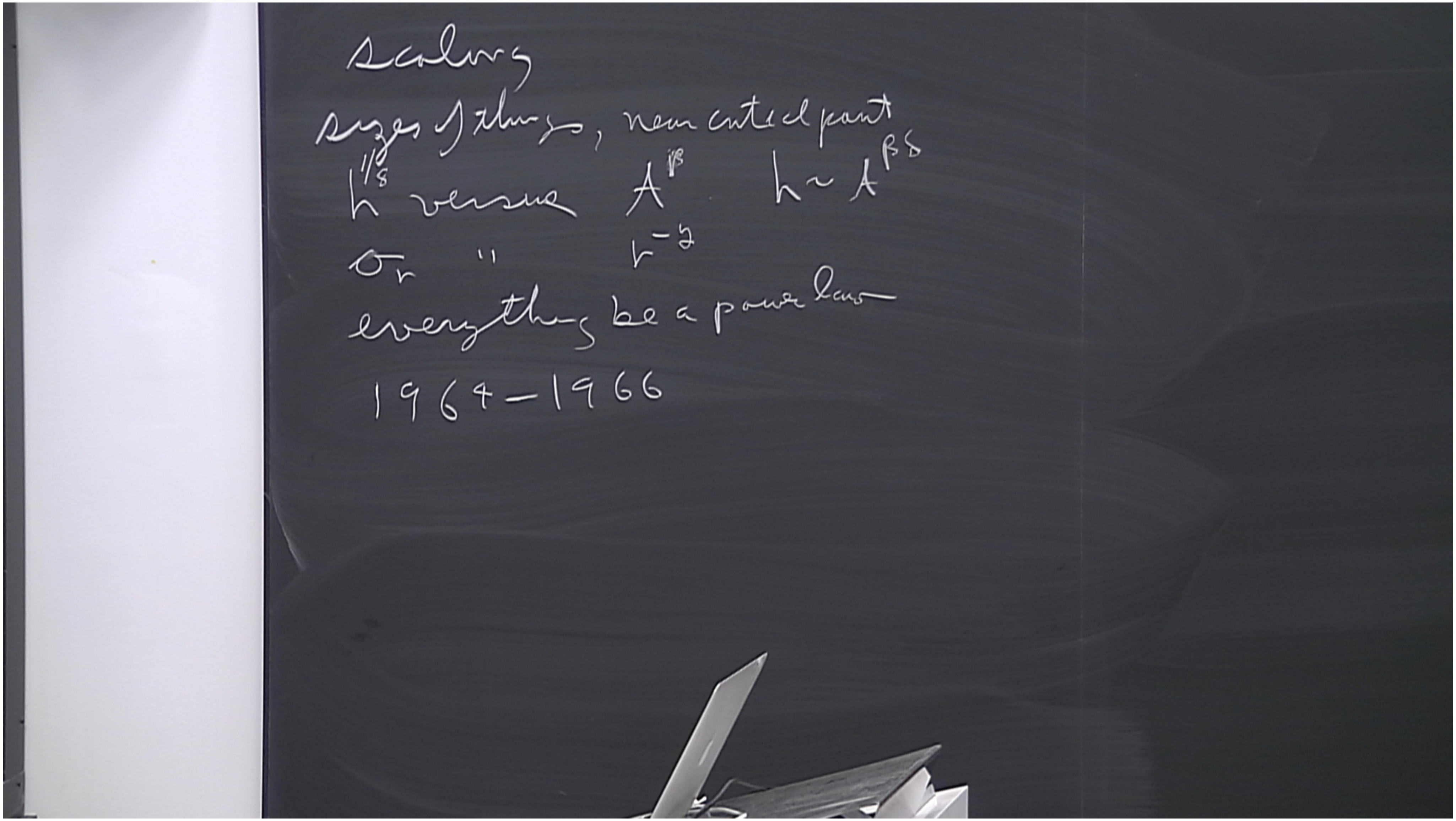
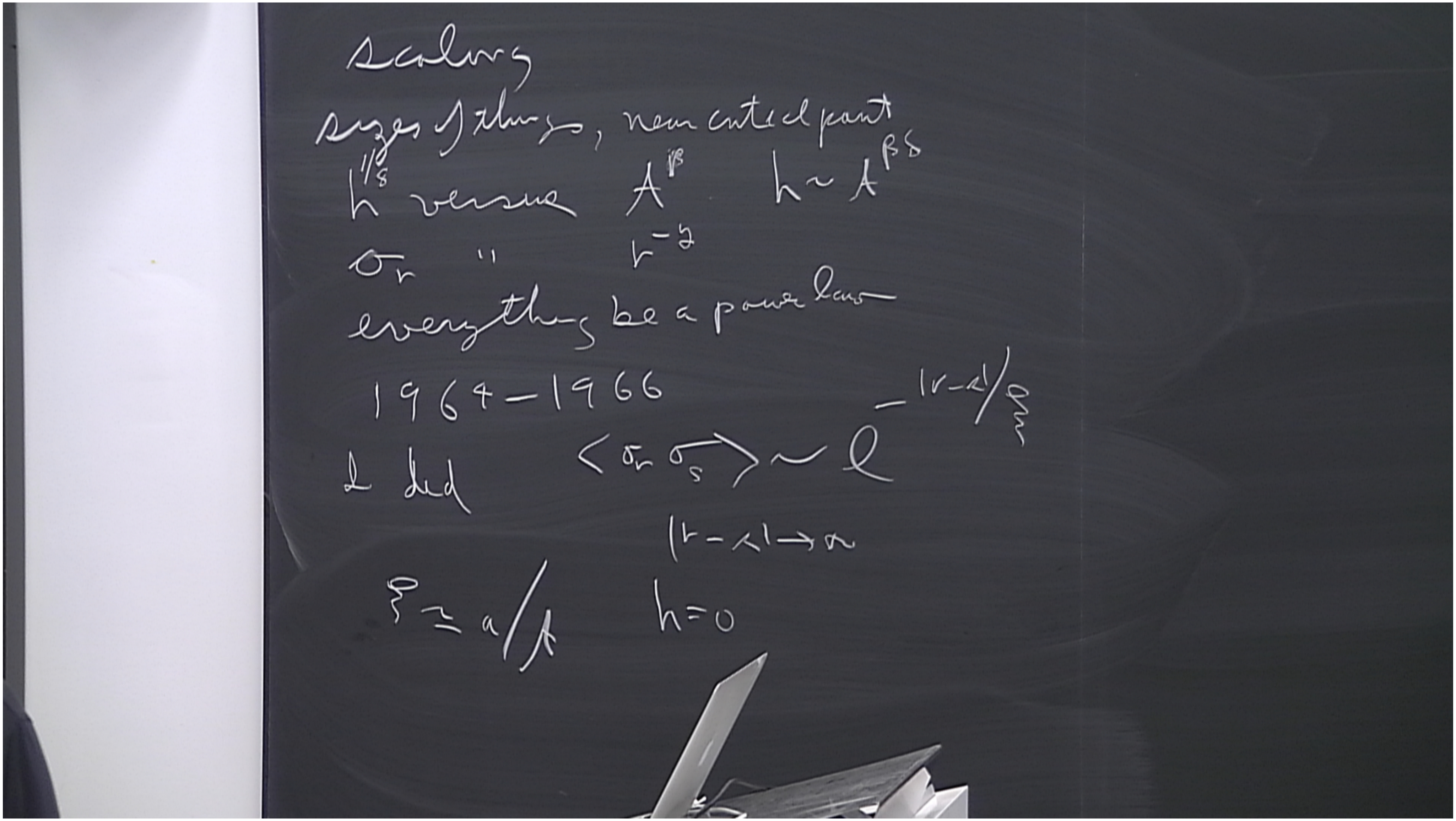
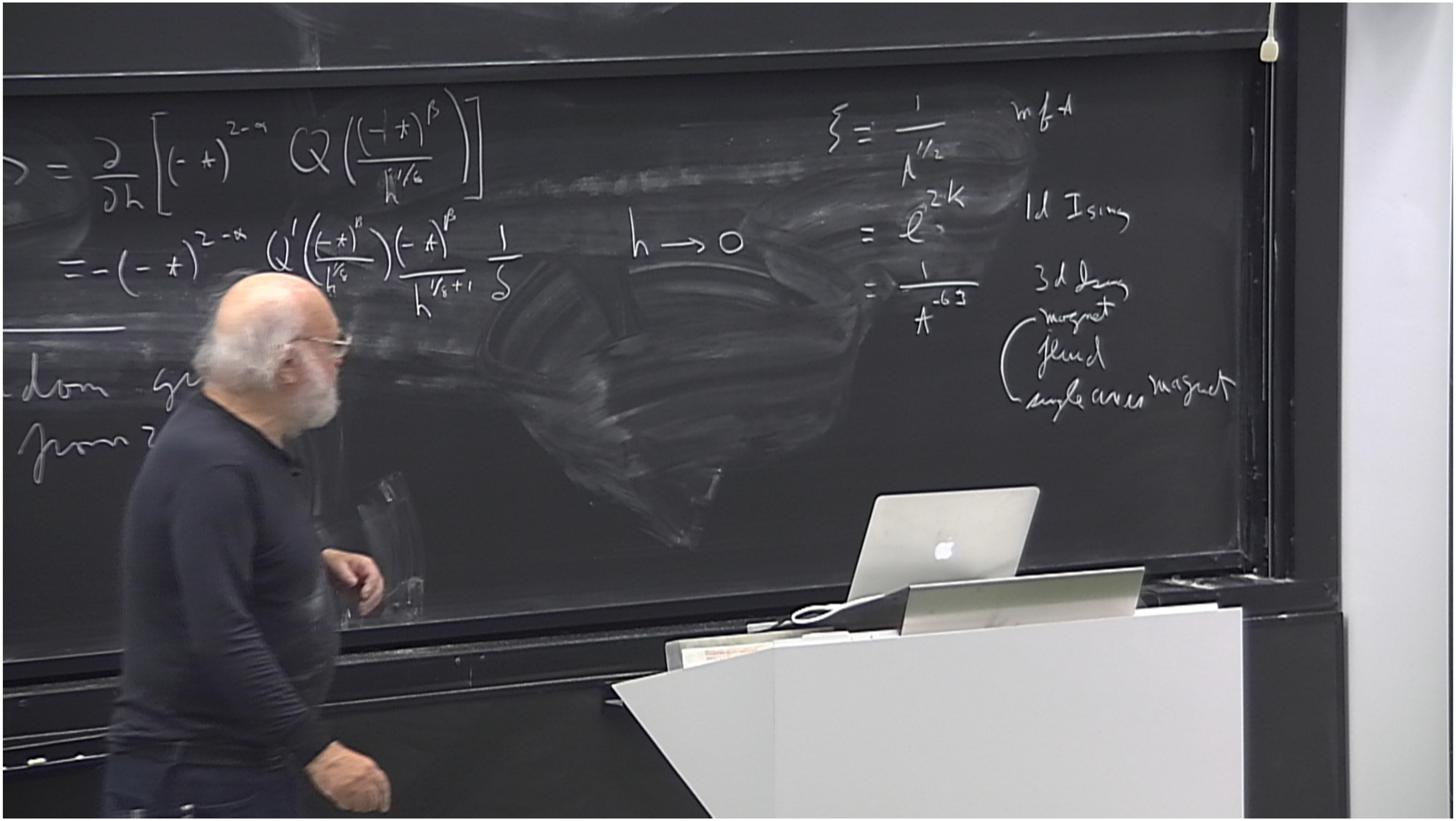

h → 0
Q(x) → x^λ
x → ∞
h → 0
λ = 0
<σ_r σ_s> = 1 / |r - s|^{2y}
σ_r ~ 1 / r^s
L >> |r - s| >> a

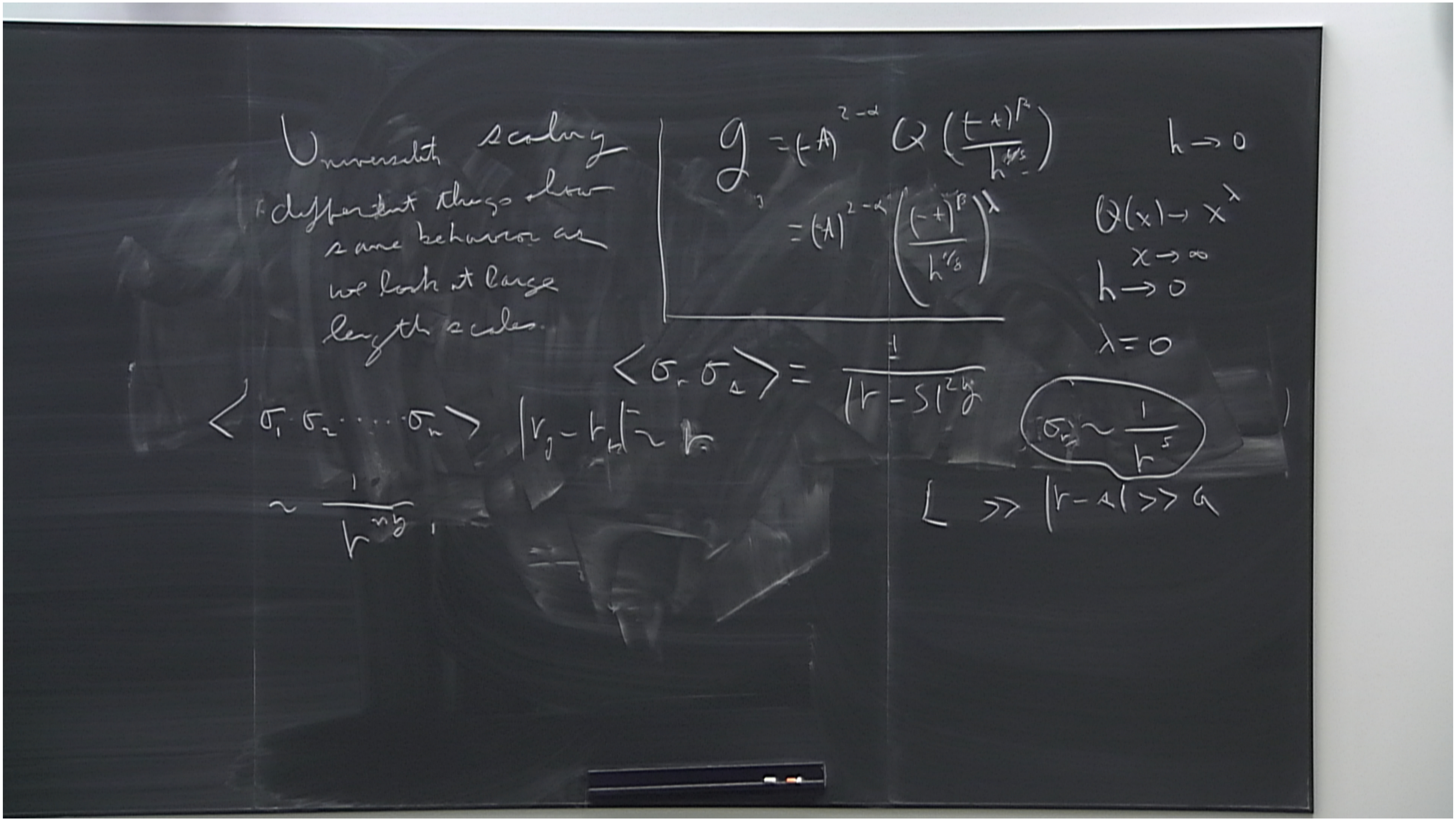

h → 0
Q(x) → x^λ
x → ∞
h → 0
λ = 0
⟨σ_r σ_s⟩ = 1 / |r − s|^{2y}
⟨σ_1 σ_2 · · · σ_n⟩ |r_j − r_k| ~ h
L ≫ |r − s| ≫ a


scaling
sizes of things, near critical point


scaling
sizes of things, near critical point
h^1/δ versus A^β   h ~ A^βδ
σ_r   "   r^-2
everything be a power law
1964–1966


scaling
sizes of things, near critical point
everything be a power law
1964–1966
I did
|r−s|→∞
h=0


h → 0
1d Ising
3d Ising
magnet
fluid
single axis magnet


Universality scaling
different things show
same behavior as
we look at large
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Q(x) → x^λ
x → ∞
h → 0
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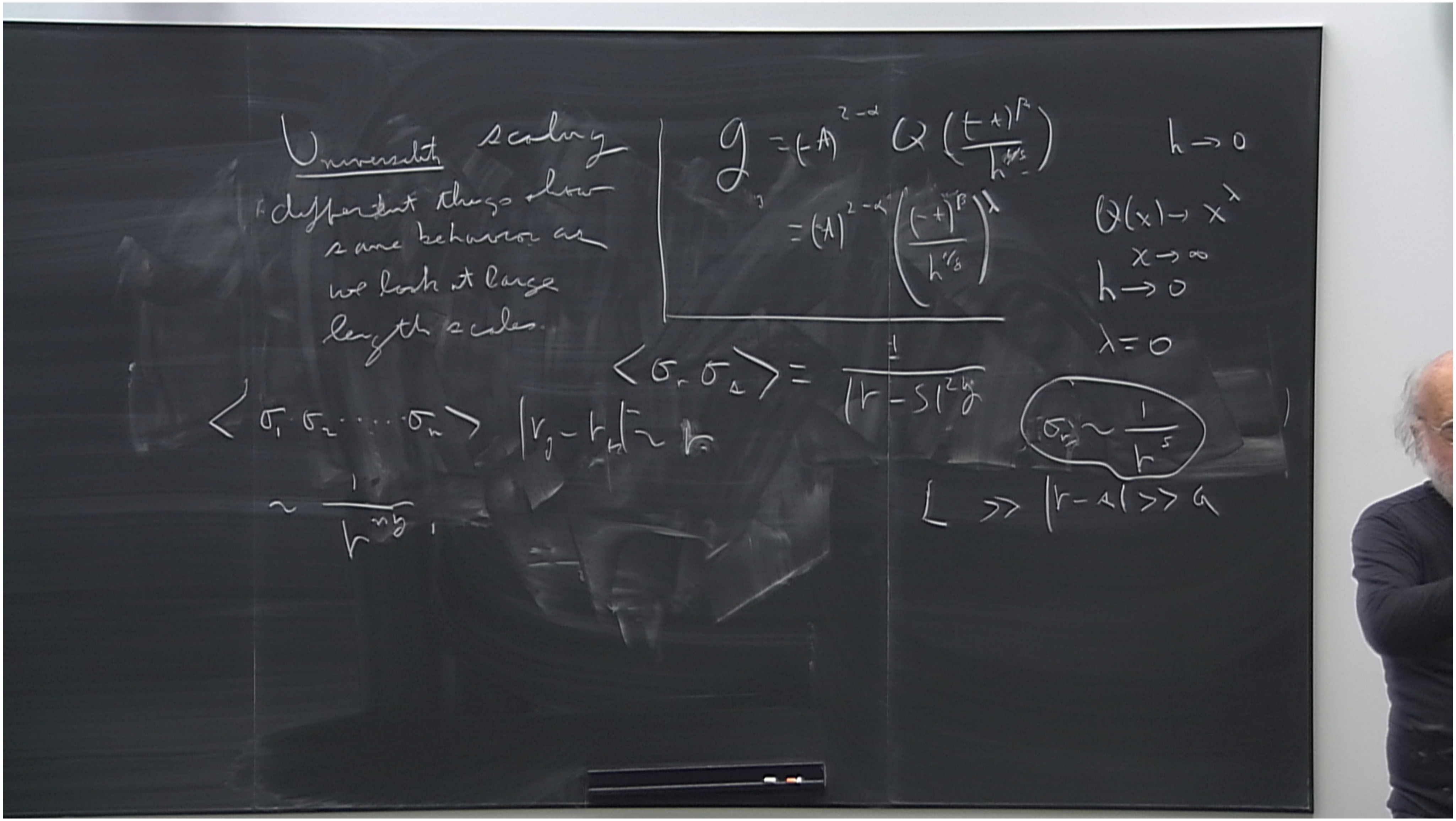
⟨σ_r σ_s⟩ = 1 / |r − s|^{2y}
L >> |r − s| >> a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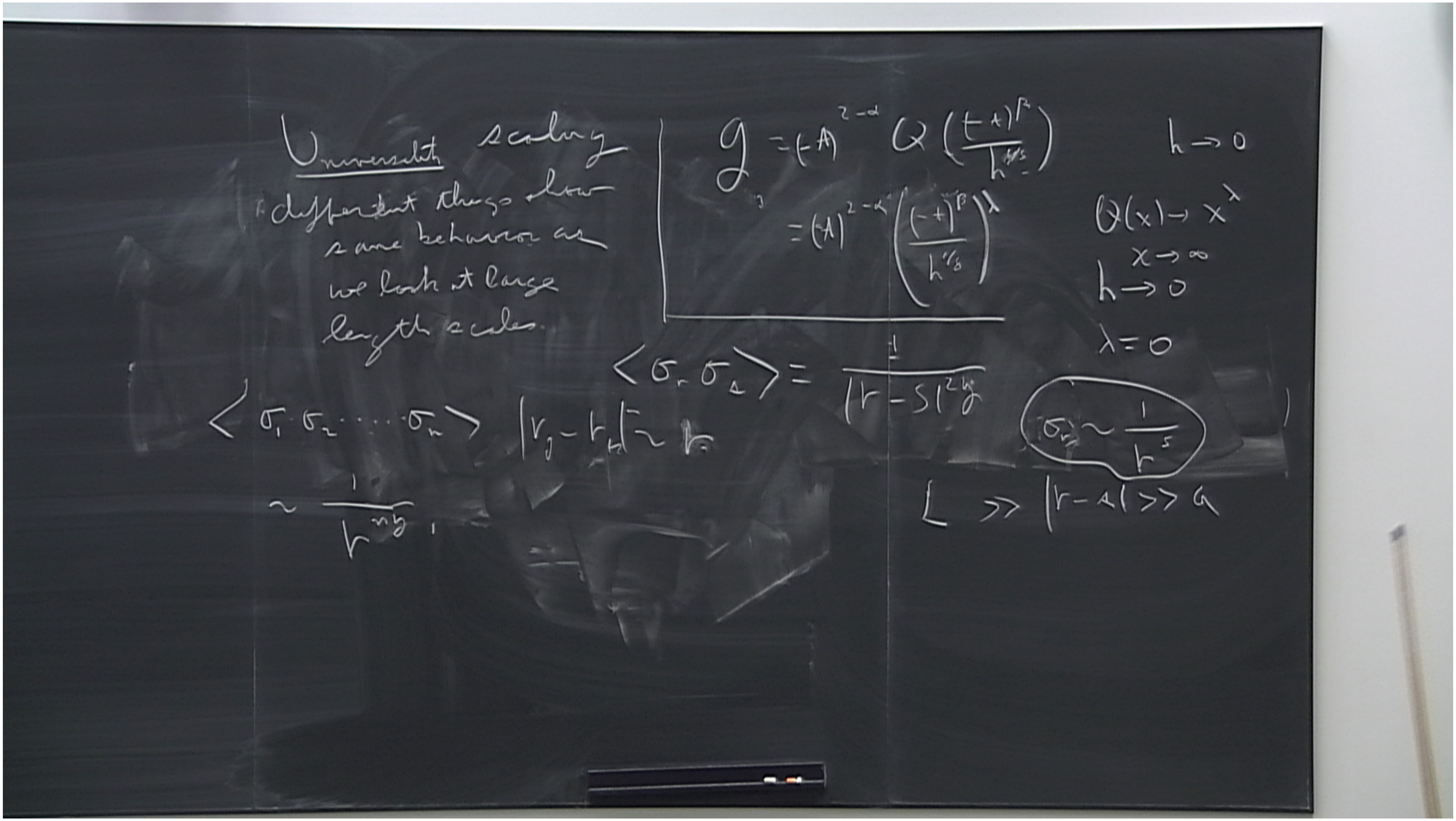
Universality scaling
different things show same behavior as we look at large length scales.
h → 0
λ = 0
L >> |r − s| >> a

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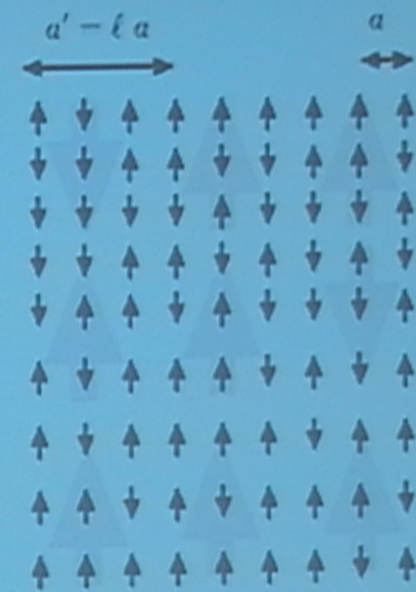


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$$h' = h\,\ell^{y_h}$$

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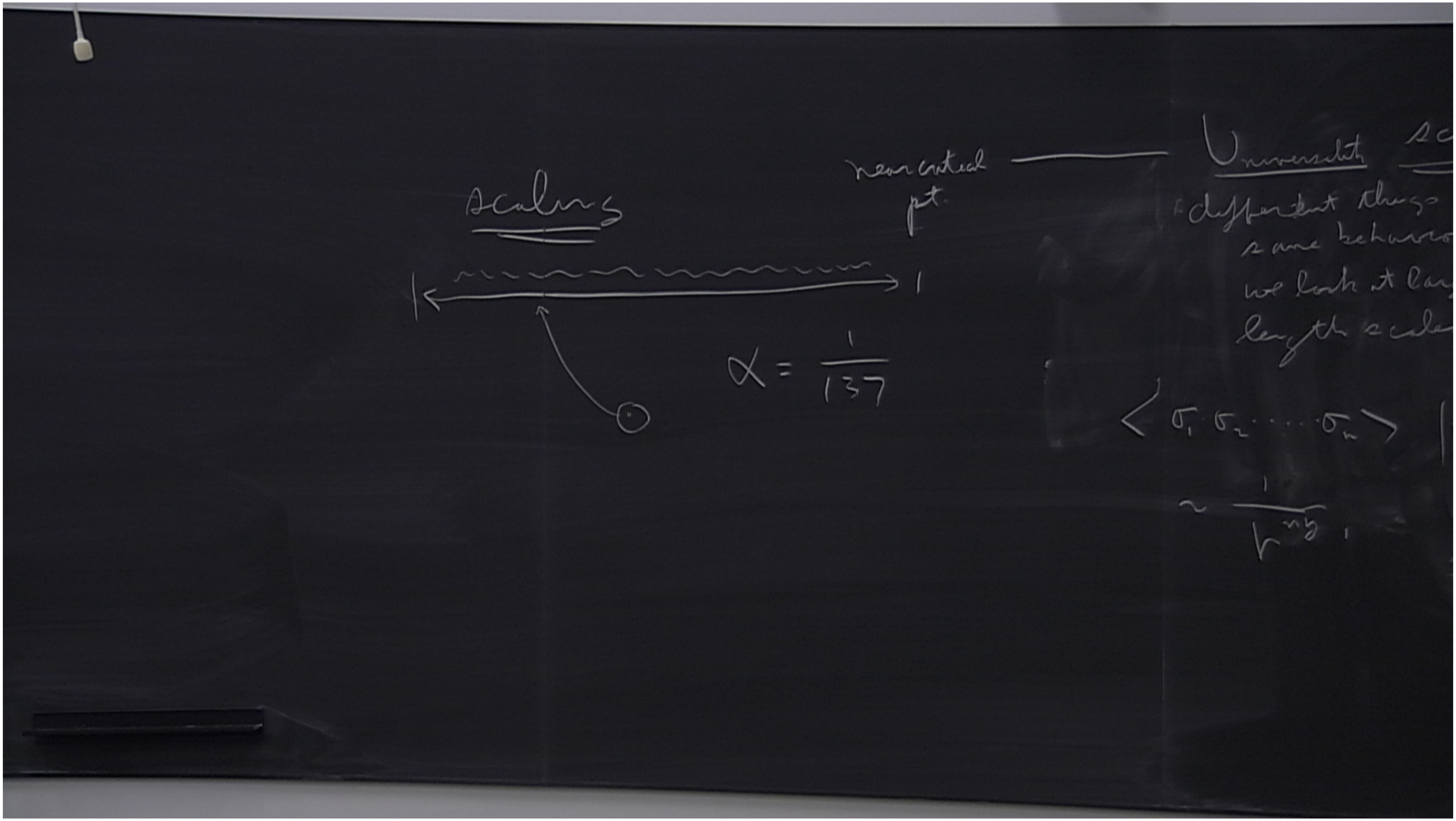
scaling
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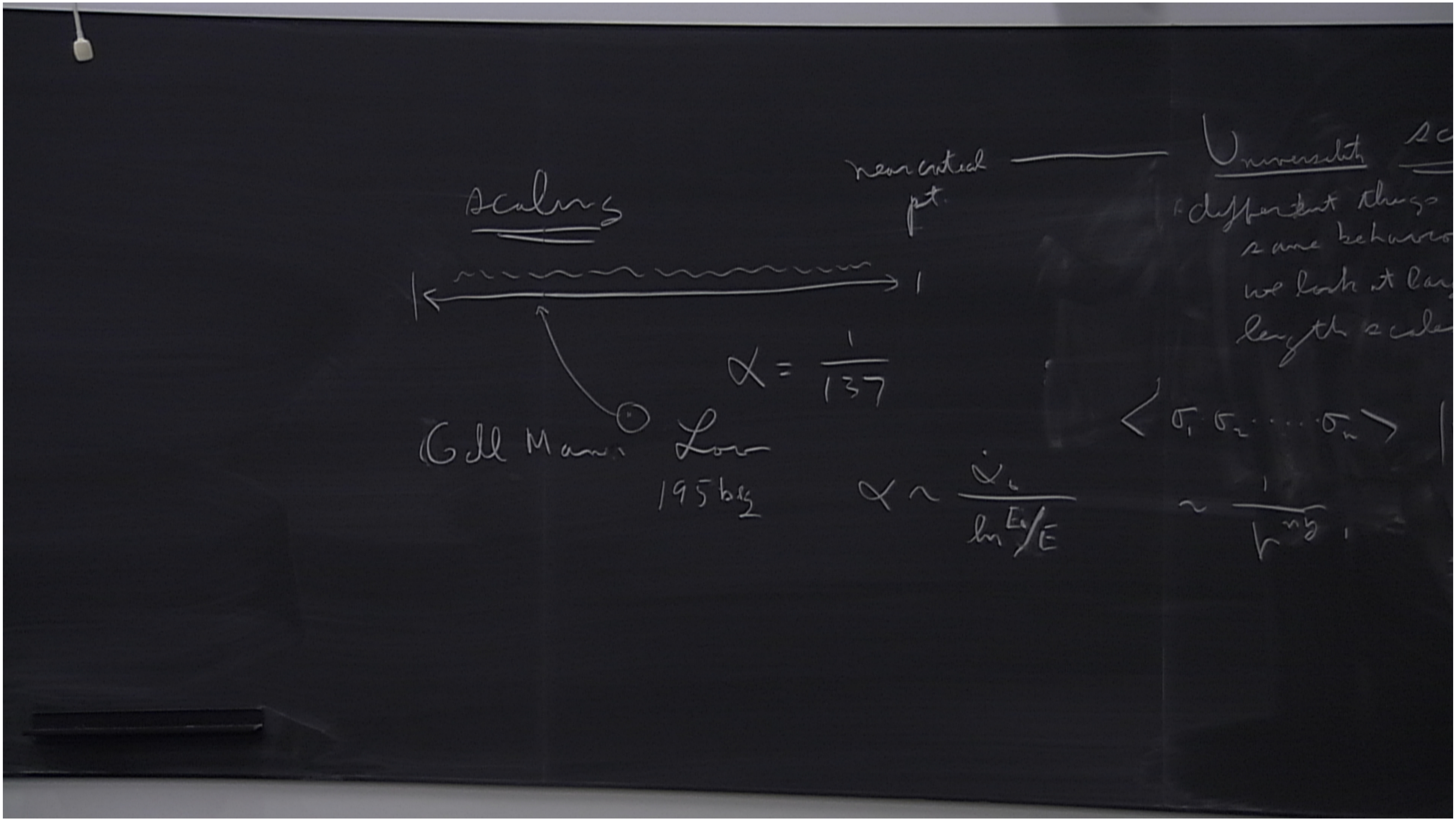
scaling
near critical pt.
Universality
different things
same behavior
α = 1/137
Gell Mann Low
1950s
α ~ α₀ / ln E₀/E
<σ₁ σ₂ …… σₙ>
~ 1/r^{nη}