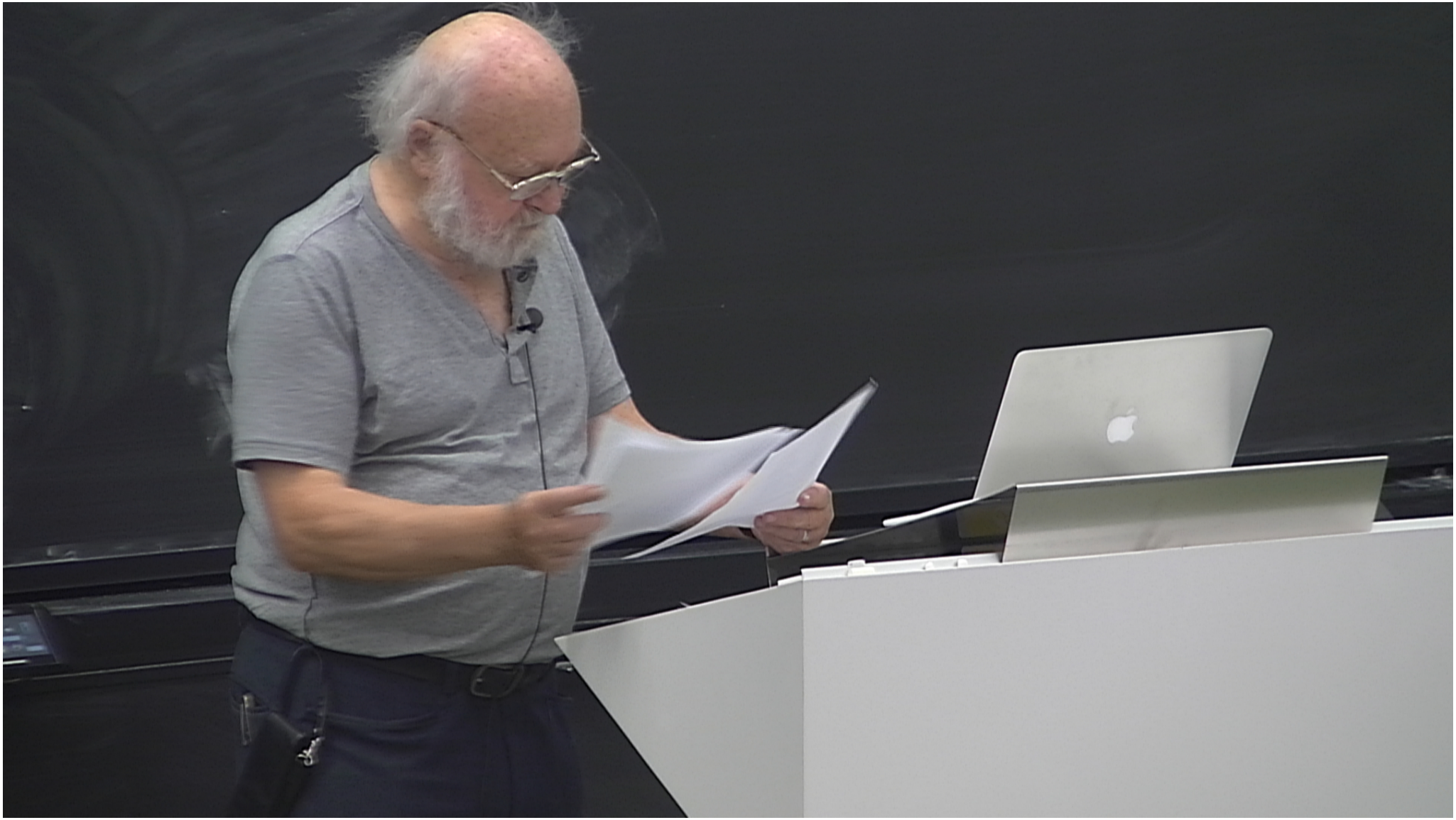
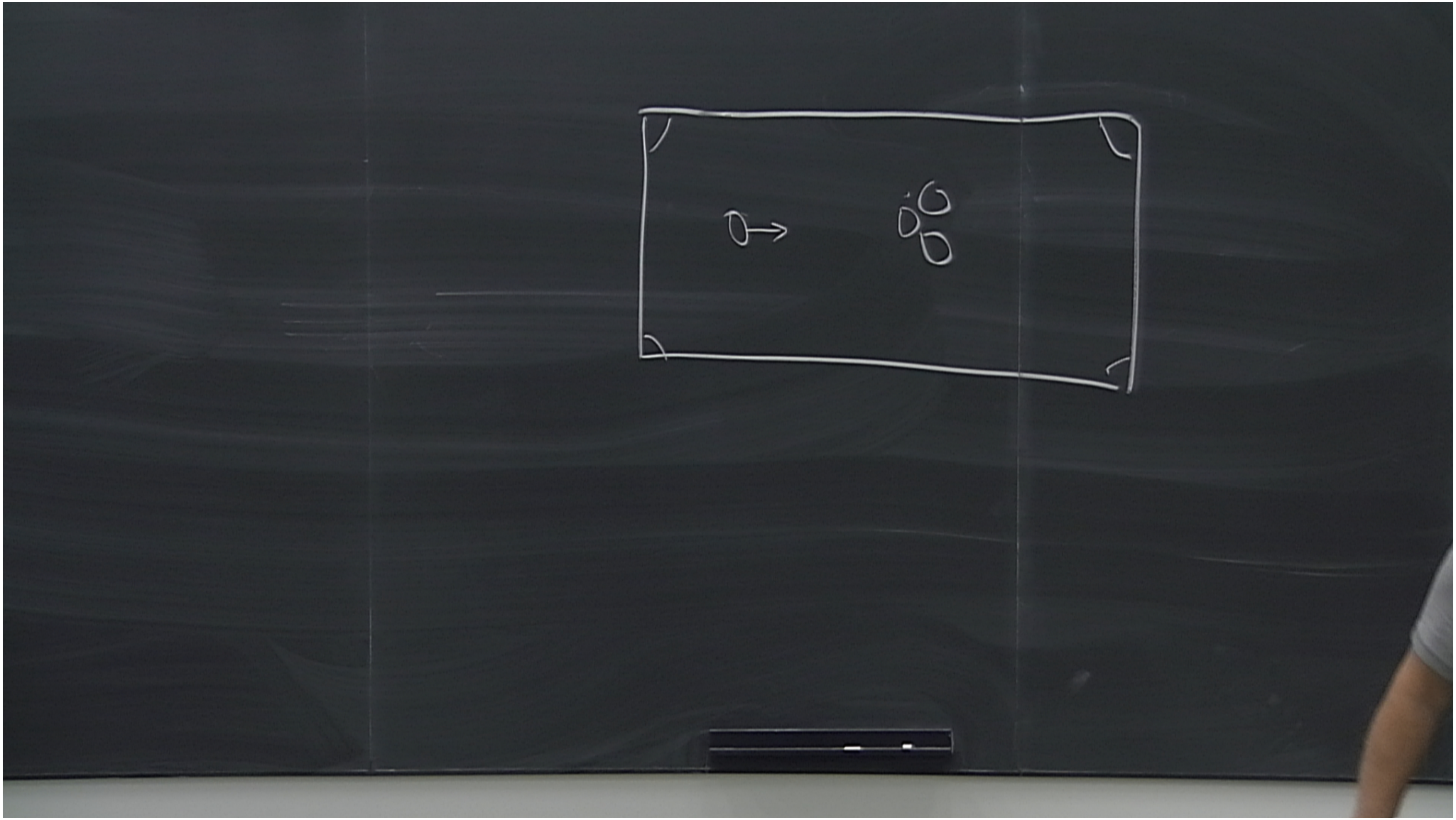
Title: Statistical Mechanics - Lecture 10

Date: Oct 17, 2011 10:30 AM

URL: http://pirsa.org/11100039

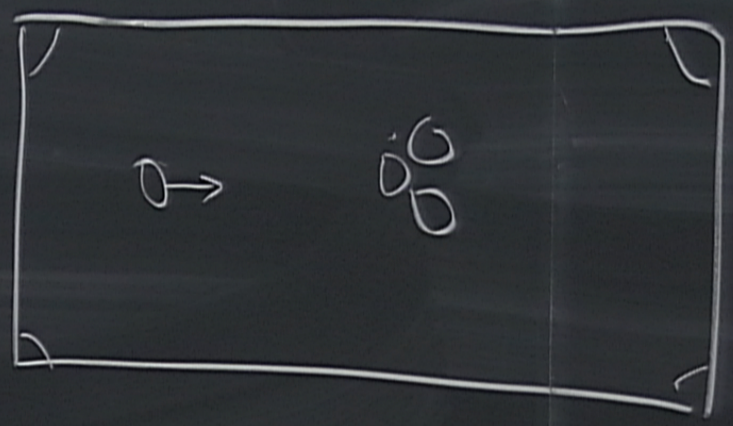
Abstract:





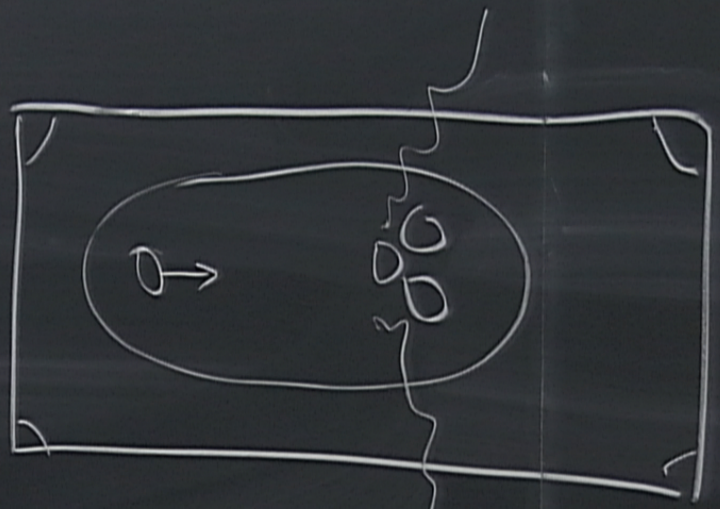
entropy is defined exactly for eq^n systems

$$Z = e^{\pm S/k \cdot \beta E}$$



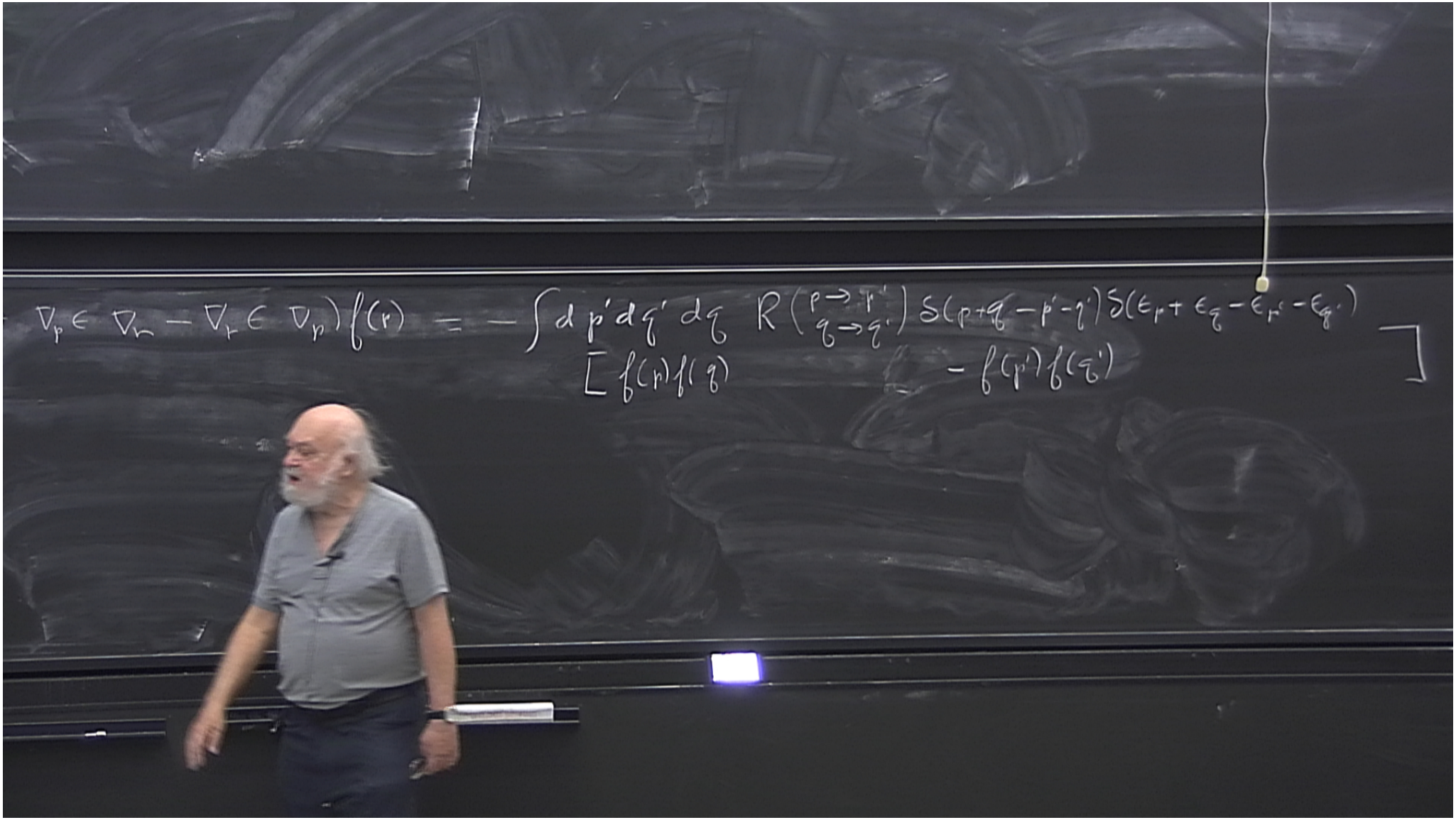
✓ entropy is defined exactly for eq^m systems

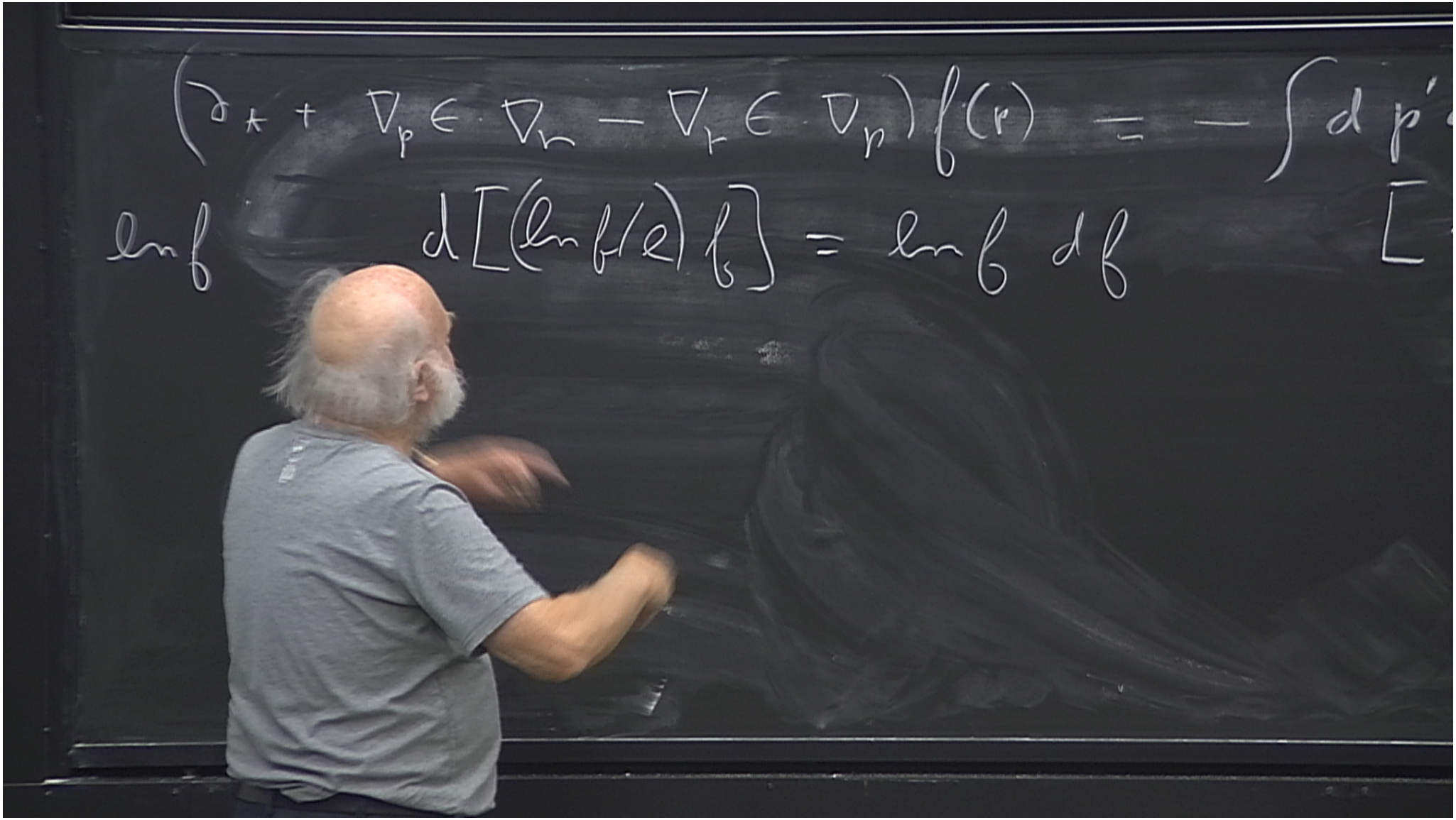
$Z = e^{\pm S/k \cdot \beta E}$

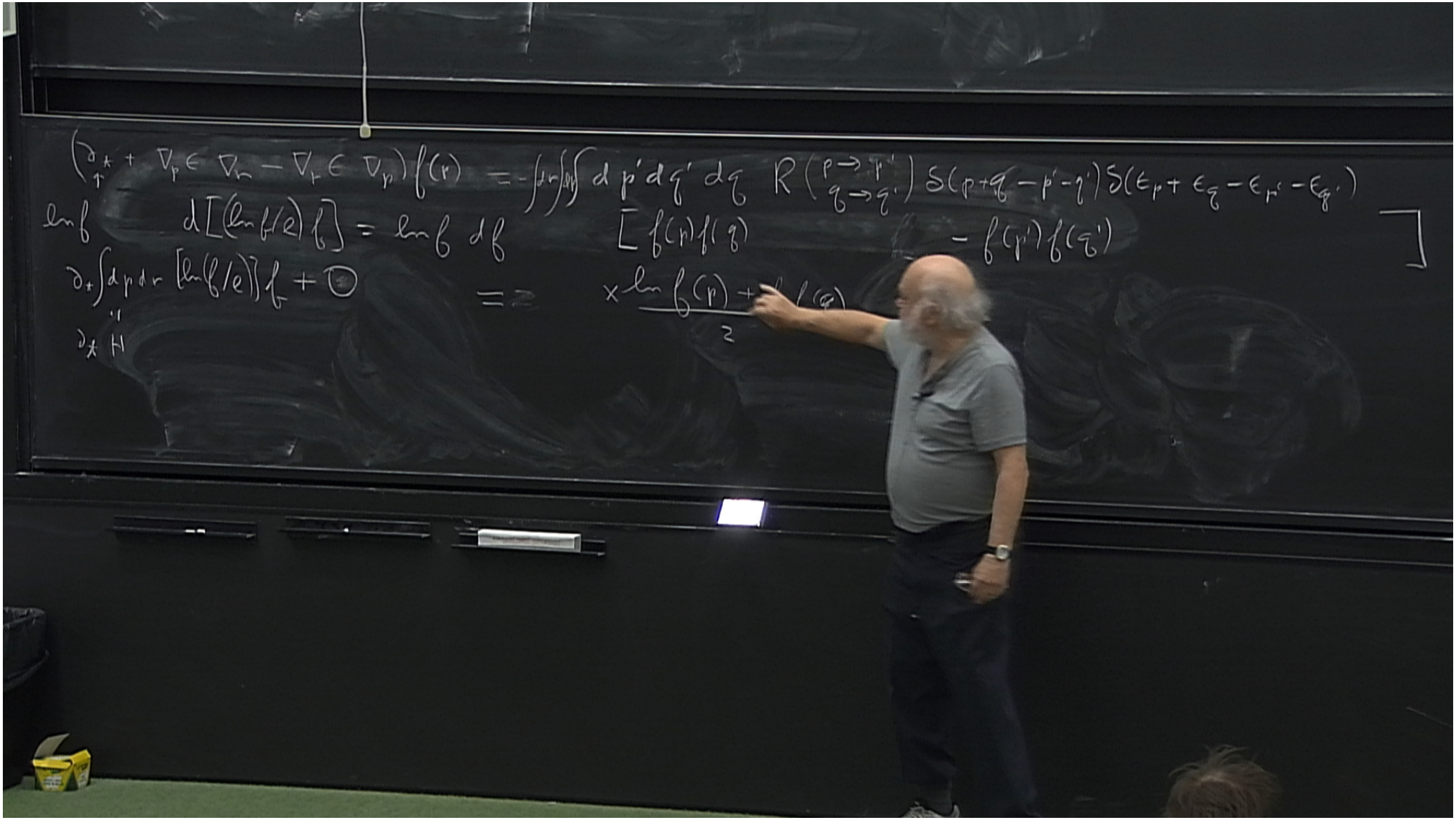


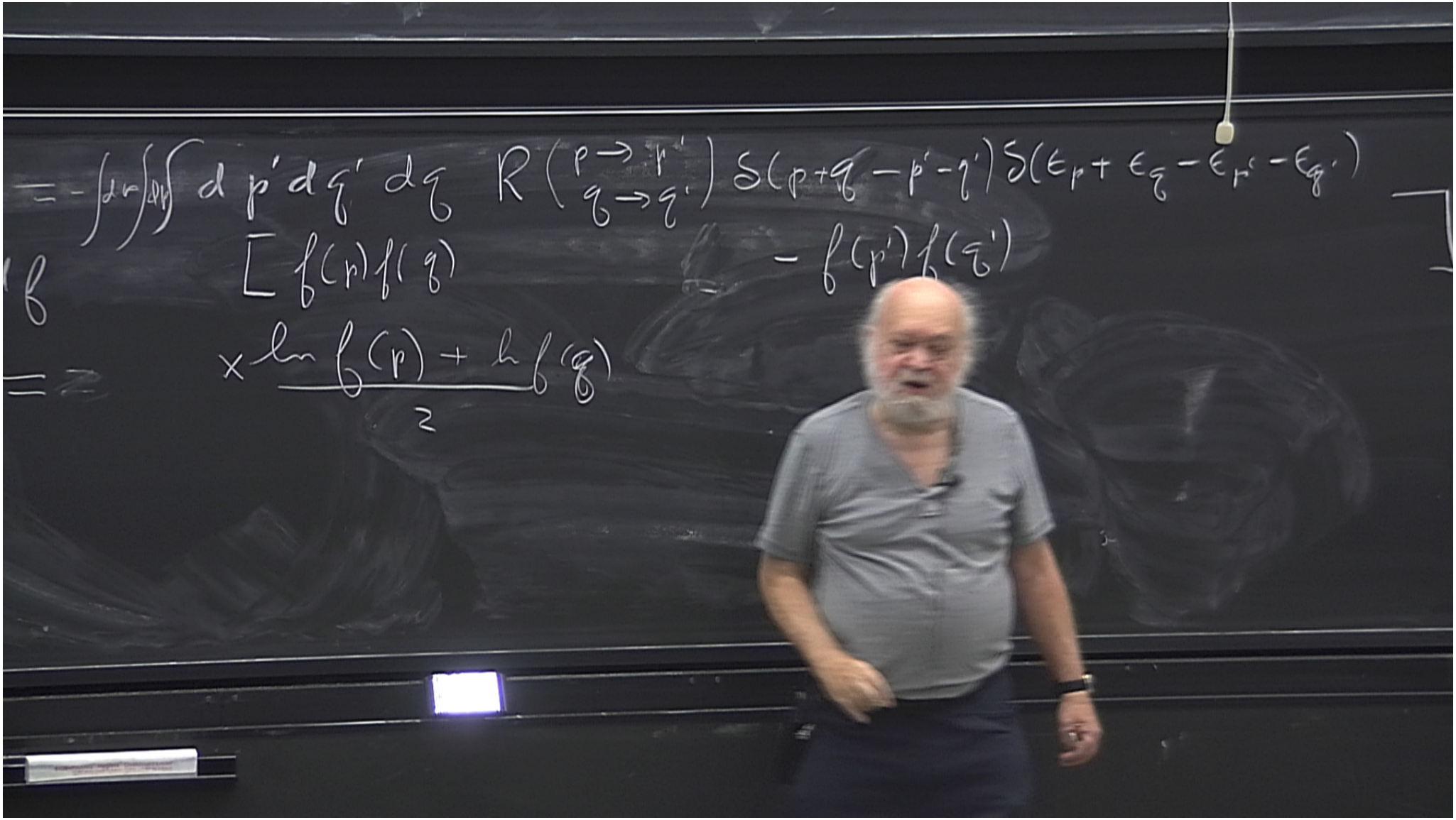
approx for non-eqm

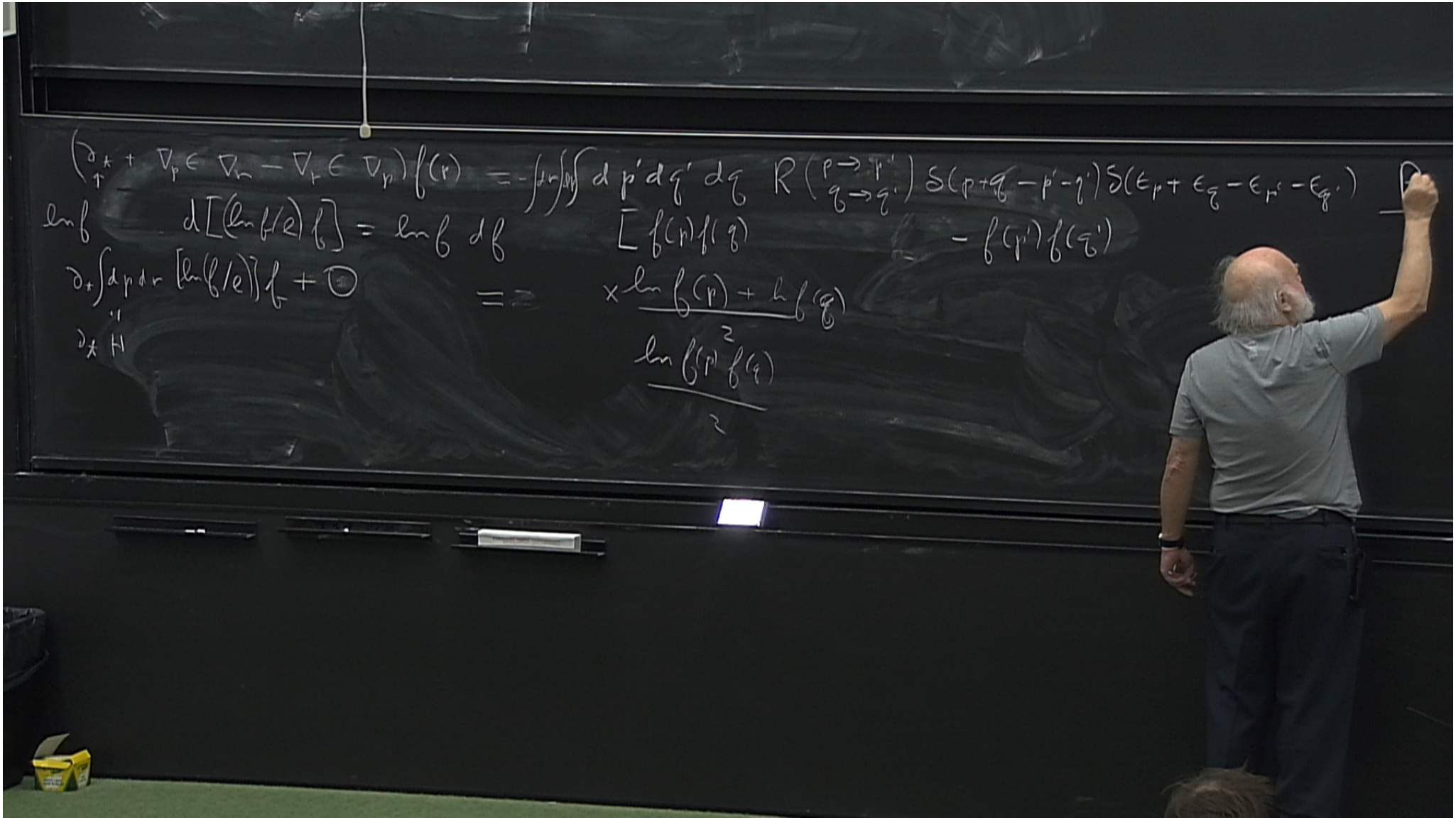
$\frac{\pm S}{k} = H = \int d\rho\, \rho(r) \ln \rho(r)$

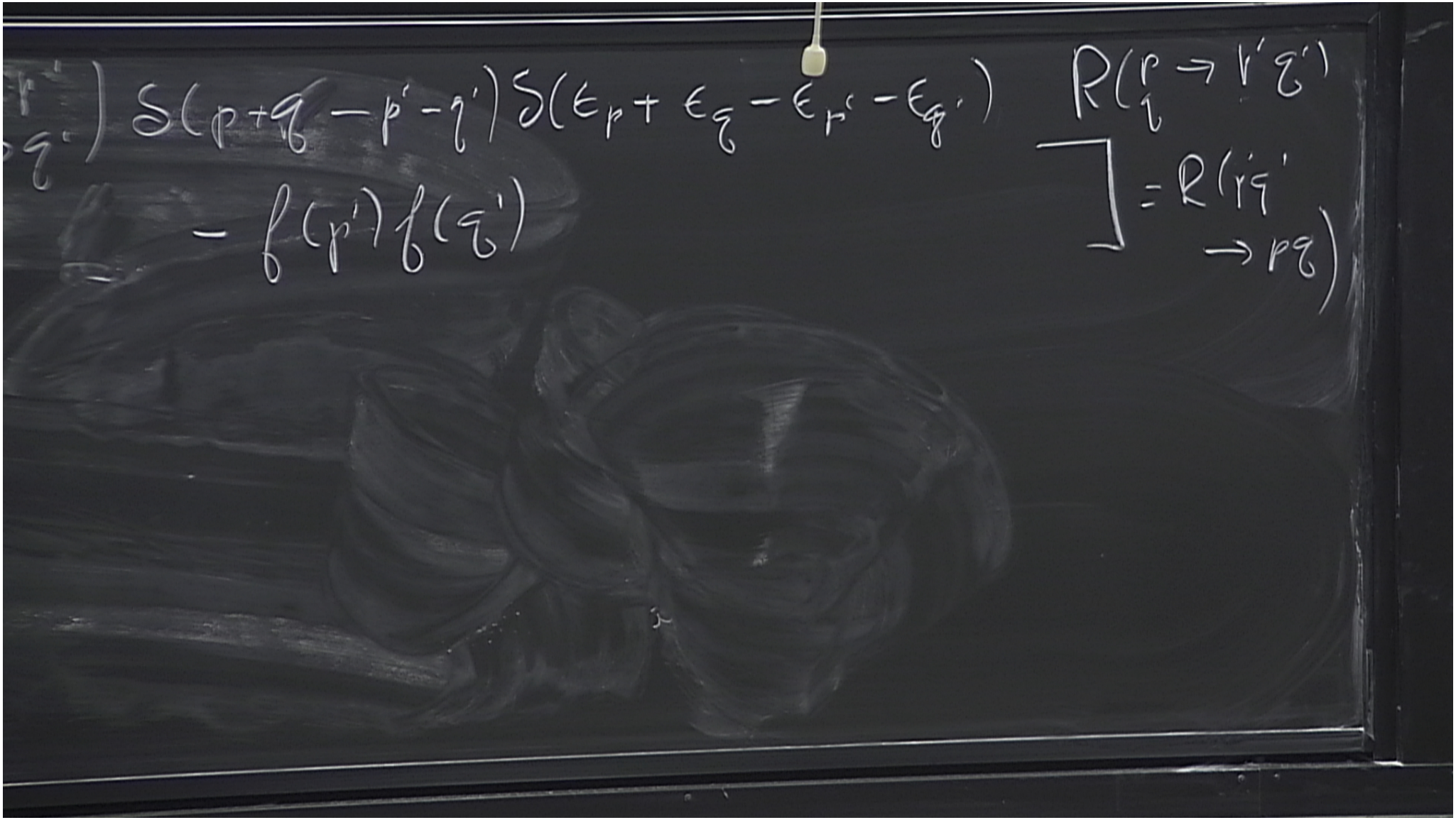

$\nabla_p \in \nabla_r - \nabla_r \in \nabla_p) f(r) = -\int dp' dq' dq \; R\begin{pmatrix} p \to p' \\ q \to q' \end{pmatrix} \delta(p+q-p'-q') \delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$
$[f(r) f(q)$
$- f(p') f(q')$
]

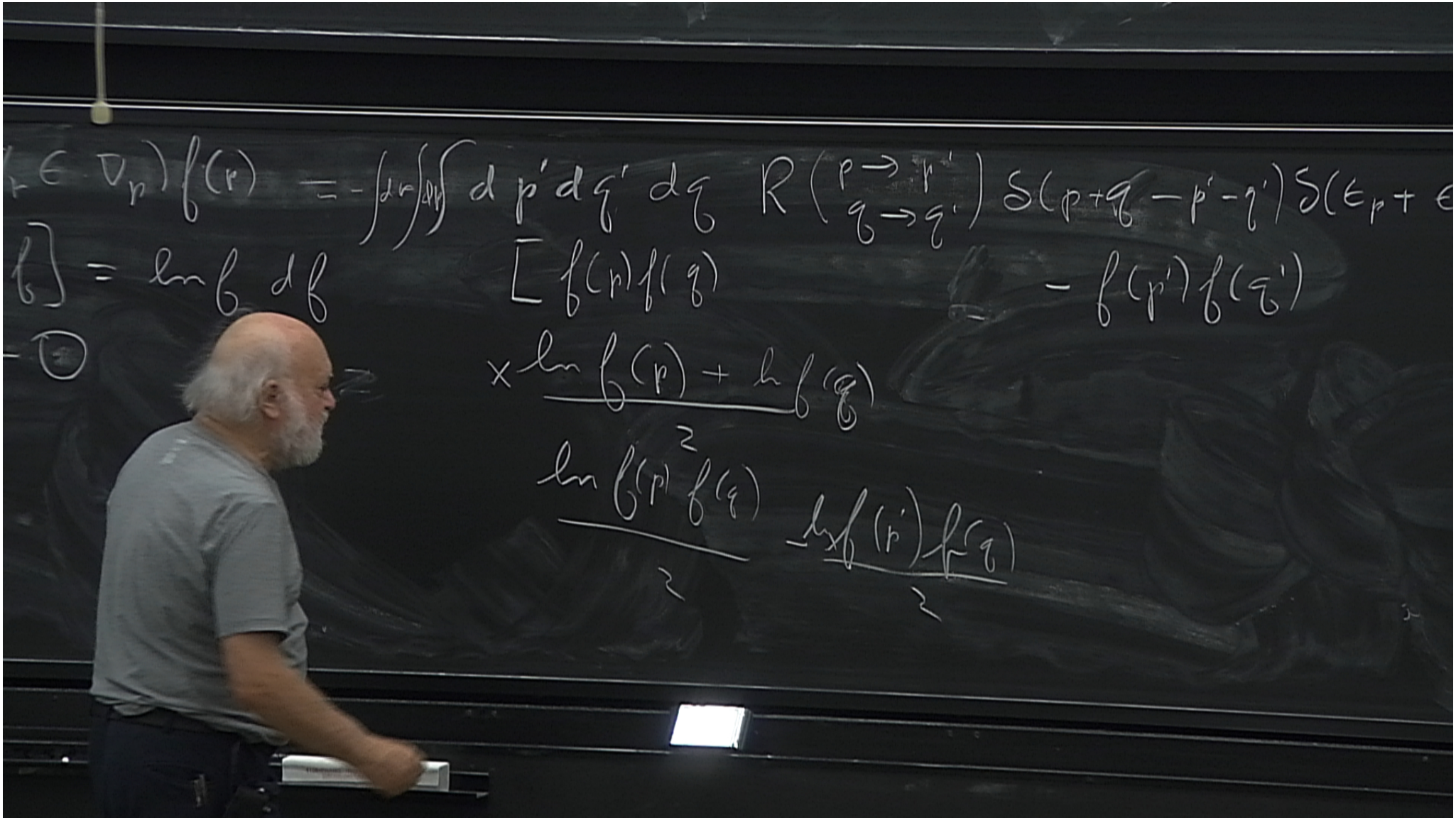

$\left(\partial_t + \nabla_p \in \cdot \nabla_r - \nabla_r \in \cdot \nabla_p\right) f(r) = -\int d p'$
$\ln f \quad d[(\ln f/z) f] = \ln f \, df$

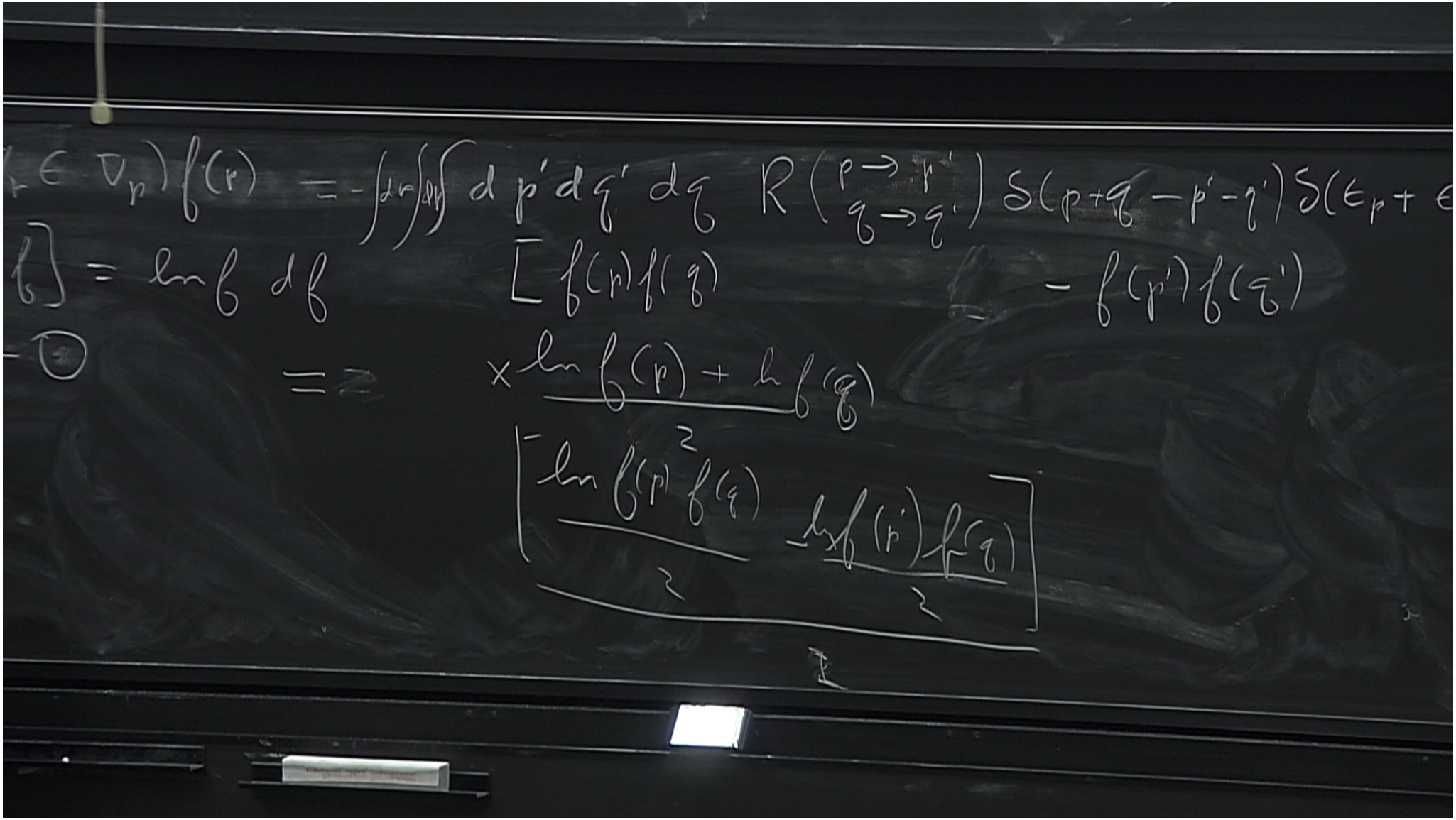


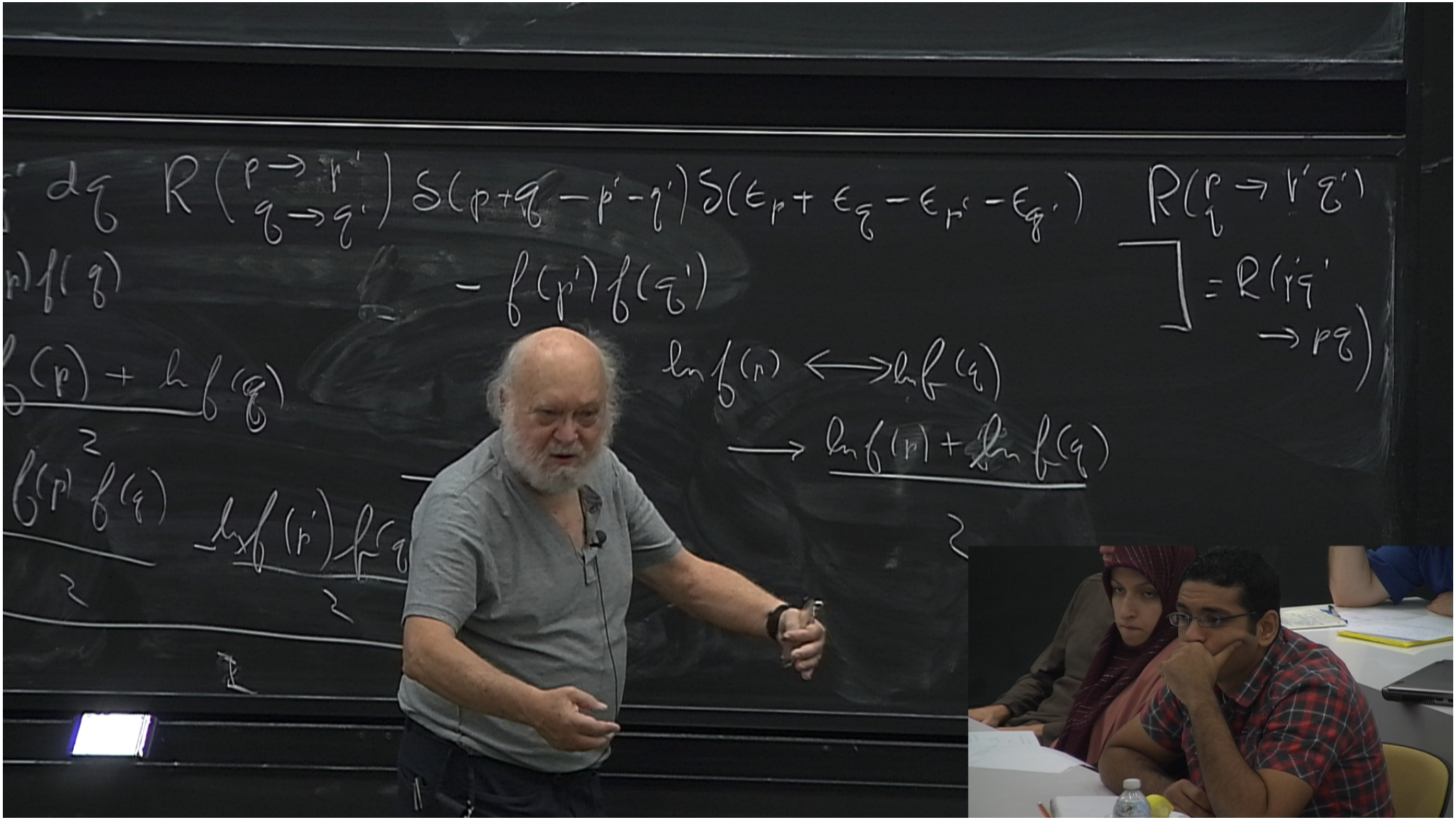

$= -\int d\nu \int dp \int dp' dq' dq \; R\left(\begin{matrix} p \to p' \\ q \to q' \end{matrix}\right) \delta(p+q-p'-q') \delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$
$[f(p) f(q)$
$- f(p') f(q')$
$\times \frac{\ln f(p) + \ln f(q)}{2}$


$\left(\partial_t + \nabla_p \epsilon \nabla_r - \nabla_r \epsilon \nabla_p\right) f(r) = -\int\int\int dp'dq'dq\, R\left(\begin{smallmatrix} p \to p' \\ q \to q' \end{smallmatrix}\right) \delta(p+q-p'-q') \delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$
$\ln f \quad d[(\ln f/e) f] = \ln f\, df$
$[f(r) f(q)$
$- f(p') f(q')$
$\partial_t \int dp\, dr\, [\ln f/e] f + 0$
$\Rightarrow$
$\times \frac{\ln f(p) + \ln f(q)}{\ln f(p') f(q')}$
$\partial_t H$


$\delta(p+q-p'-q')\,\delta(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'})$
$R(pq \to p'q')$
$- f(p')f(q')$
$] = R(p'q' \to pq)$

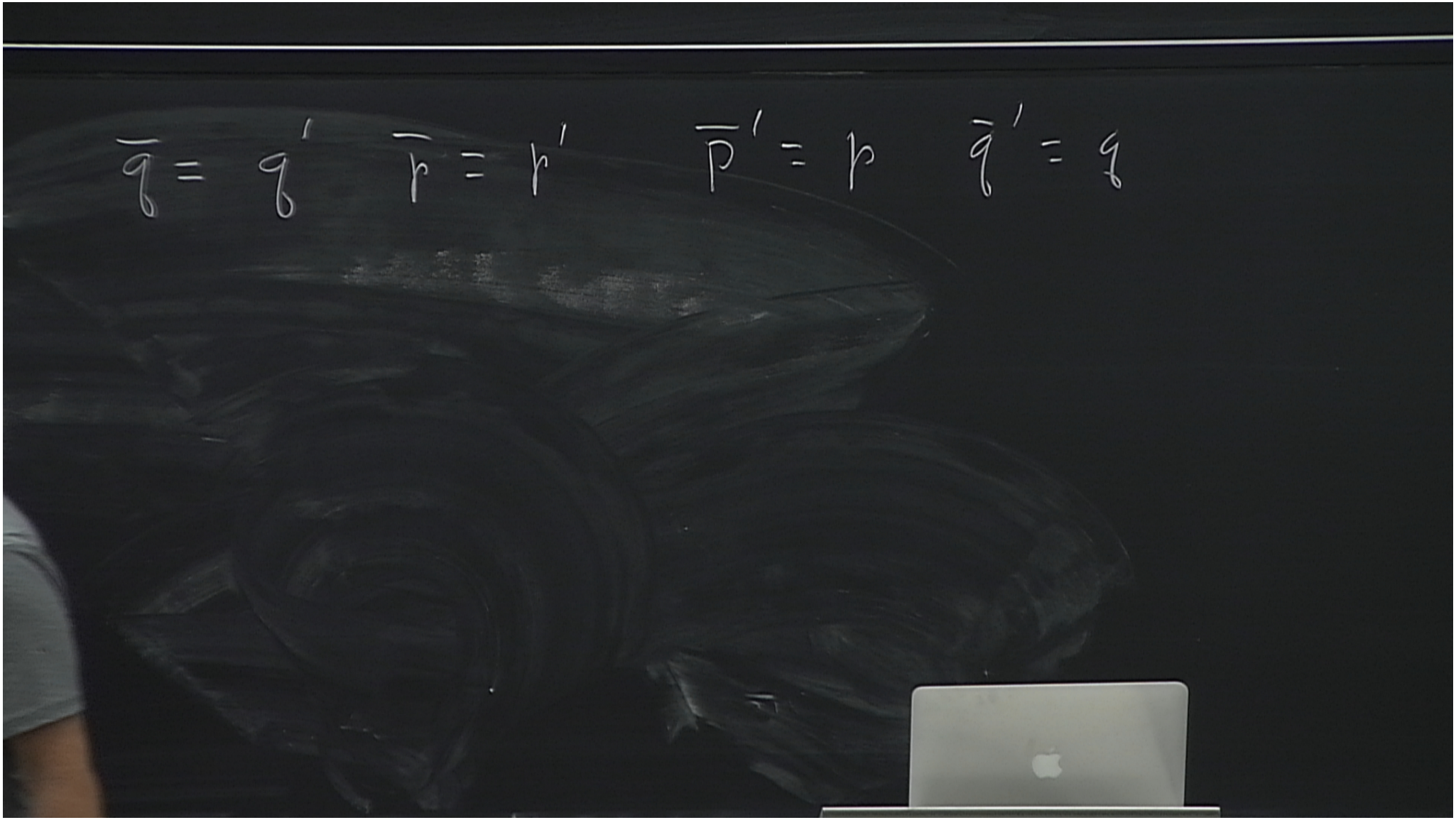

$\in \nabla_p) f(r) = -\int dr \int dp \int dp' dq' dq \; R\left(\begin{smallmatrix} p \to p' \\ q \to q' \end{smallmatrix}\right) \delta(p+q-p'-q') \delta(\epsilon_p + \epsilon$
$f] = \ln f \, df$
$[f(r) f(q)$
$- f(p') f(q')$
$- \textcircled{1}$
$\times \ln \frac{f(r) + \ln f(q)}{2}$
$\frac{\ln f(r)^2 f(q)}{2} - \frac{\ln f(r') f(q)}{2}$

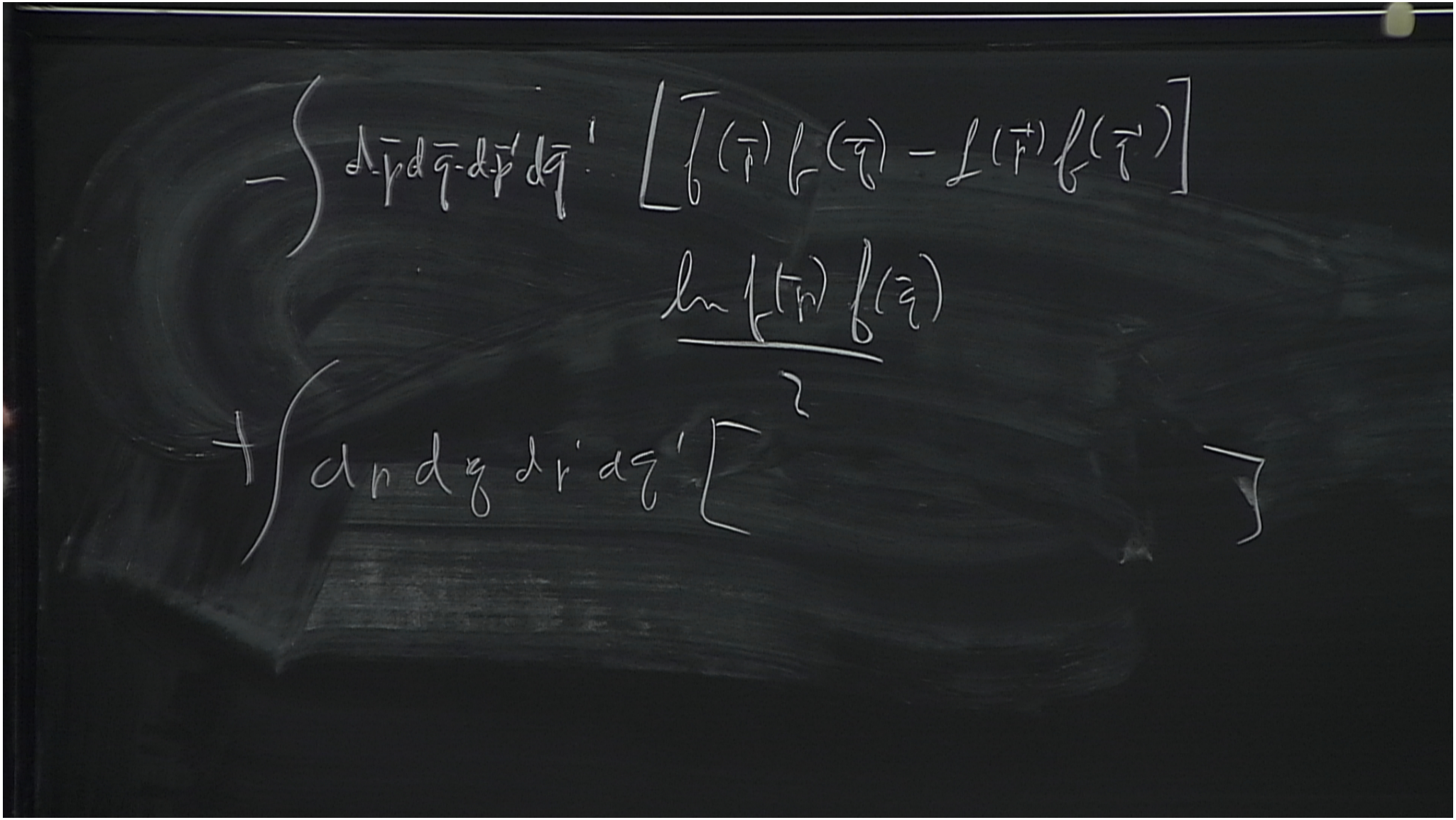

$\in \nabla_p) f(p) = -\int \int \int dp' dq' dq \; R\left(\begin{smallmatrix} p \to p' \\ q \to q' \end{smallmatrix}\right) \delta(p+q-p'-q') \delta(\epsilon_p + \epsilon$
$f] = \ln f \, df$
$[f(p) f(q) - f(p') f(q')$
$\times \ln \frac{f(p) + \ln f(q)}{2}$
$\left[\frac{\ln f(p') f(q)}{2} - \frac{\ln f(p') f(q)}{2} \right]$

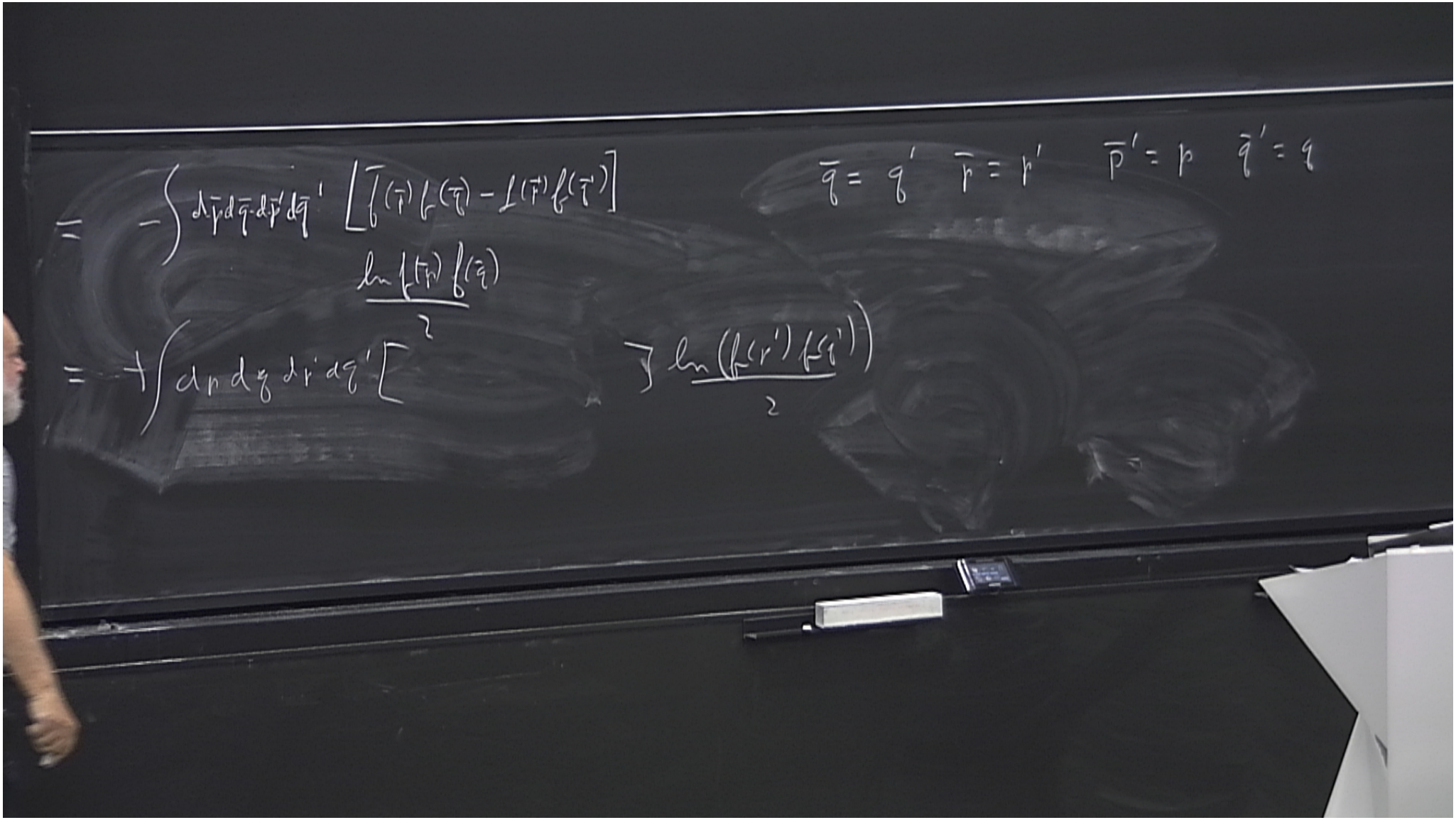

R(p → p' q → q') δ(p+q−p'−q') δ(ε_p + ε_q − ε_p' − ε_q')
− f(p') f(q')
ln f(n) ⟷ ln f(q)
→ ln f(n) + ln f(q)
R(p → p' q → q')
= R(pq → p'q')

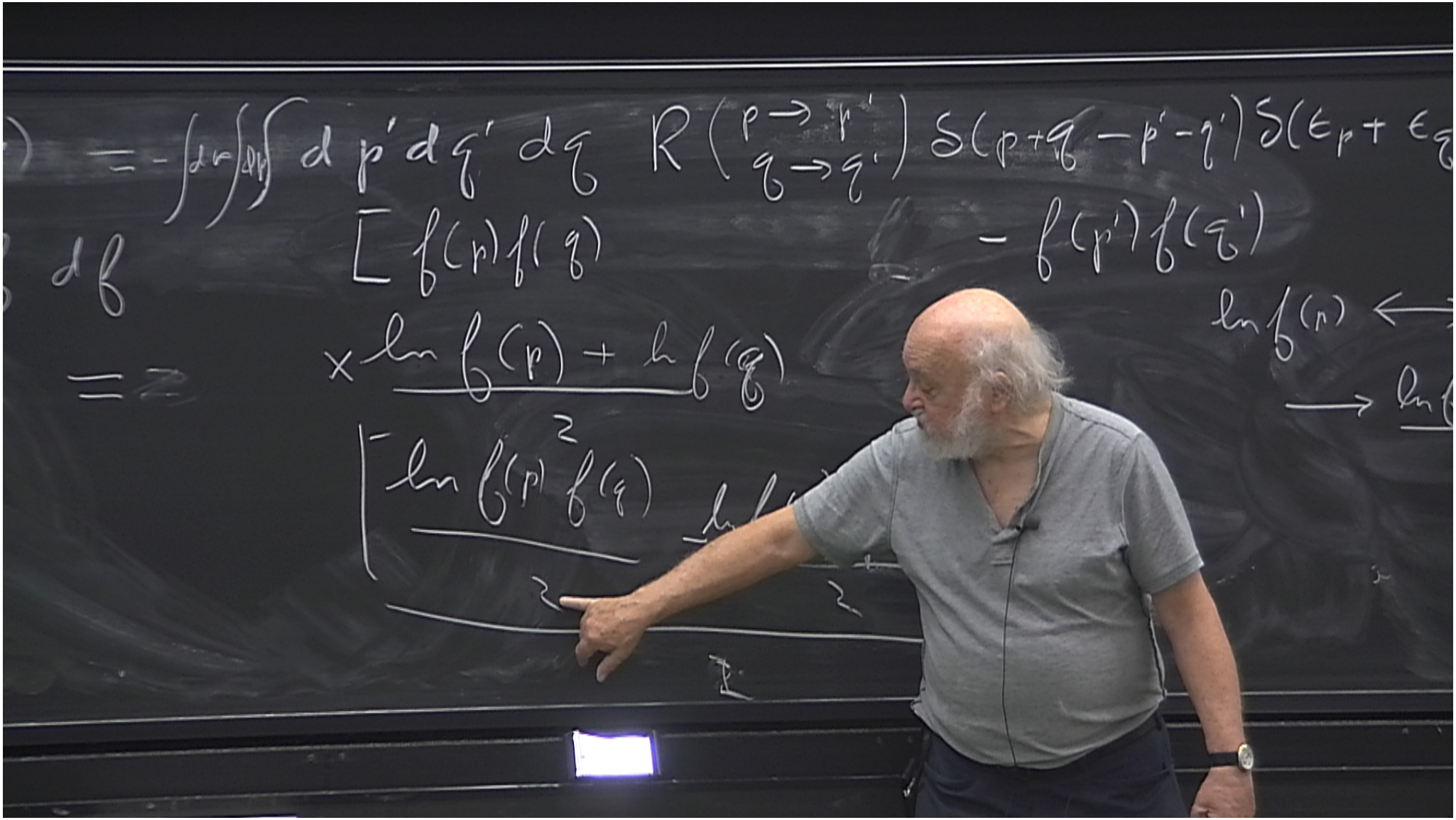
$$-\int d_1 p\, dq\, dp'\, dq' \quad f(p) f(q) - f(p') f(q')$$

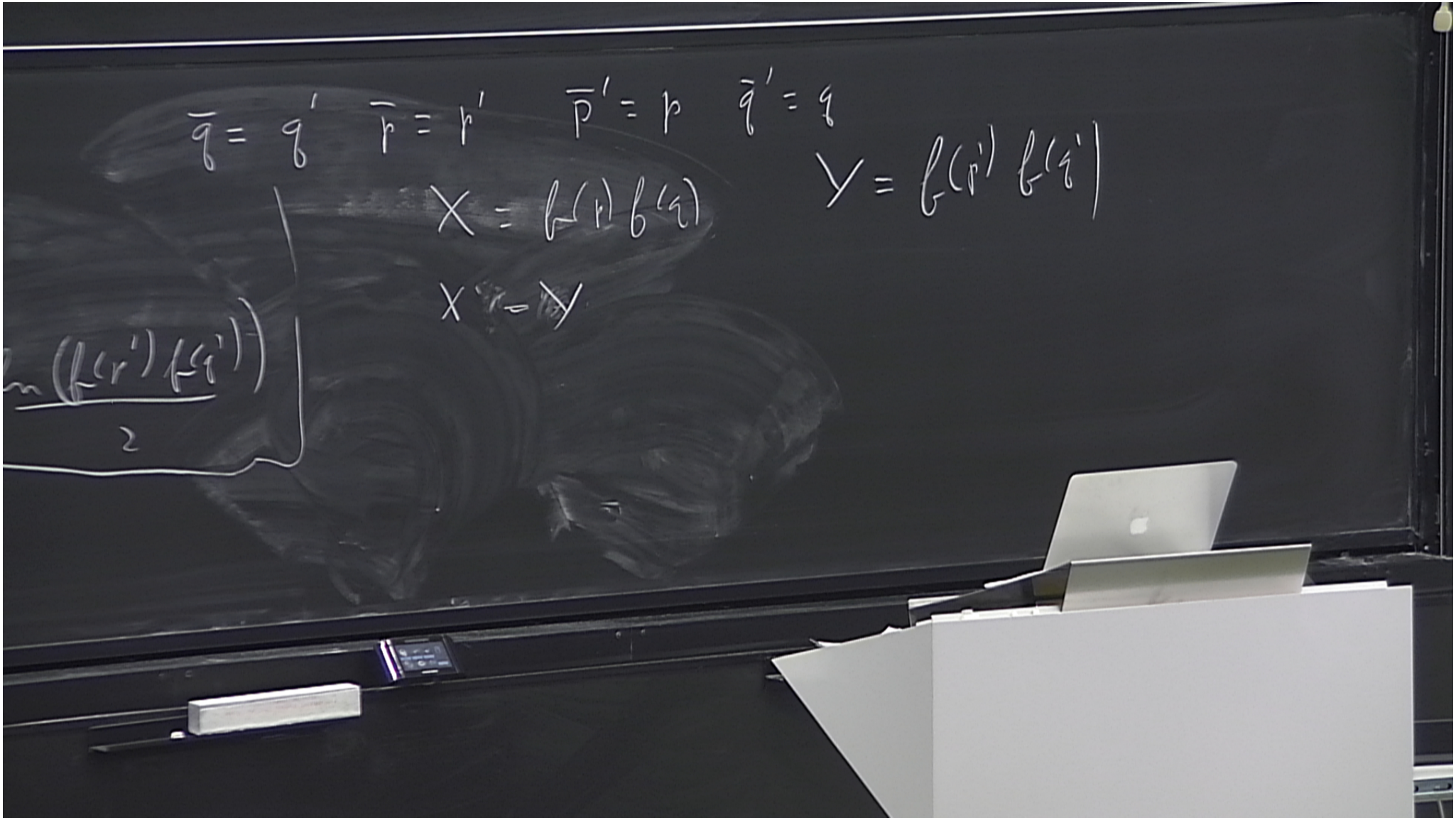
$$\frac{\ln f(p) f(q)}{2}$$

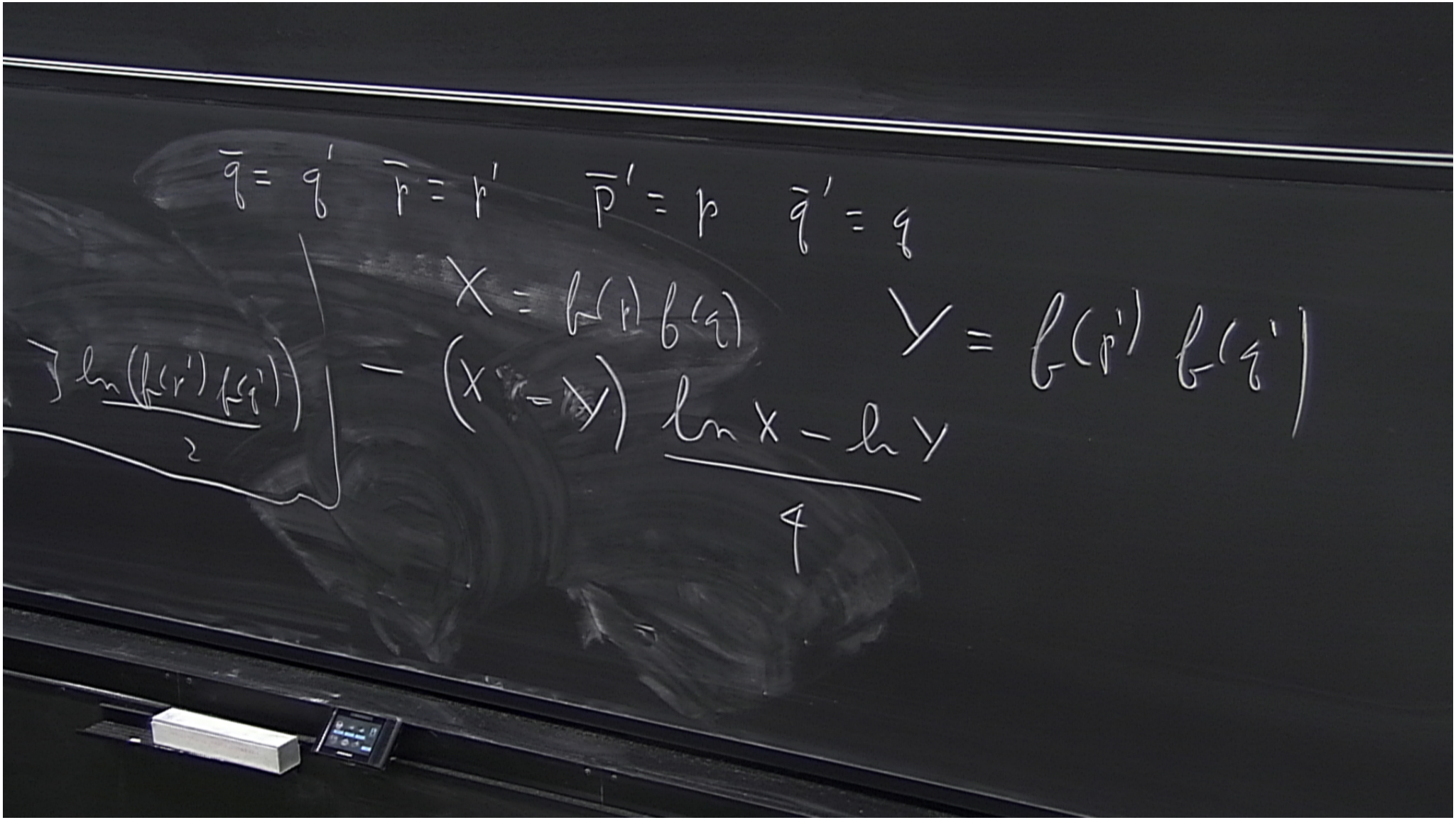

$\bar{q} = q'$ $\bar{r} = r'$ $\bar{p}' = p$ $\bar{q}' = q$

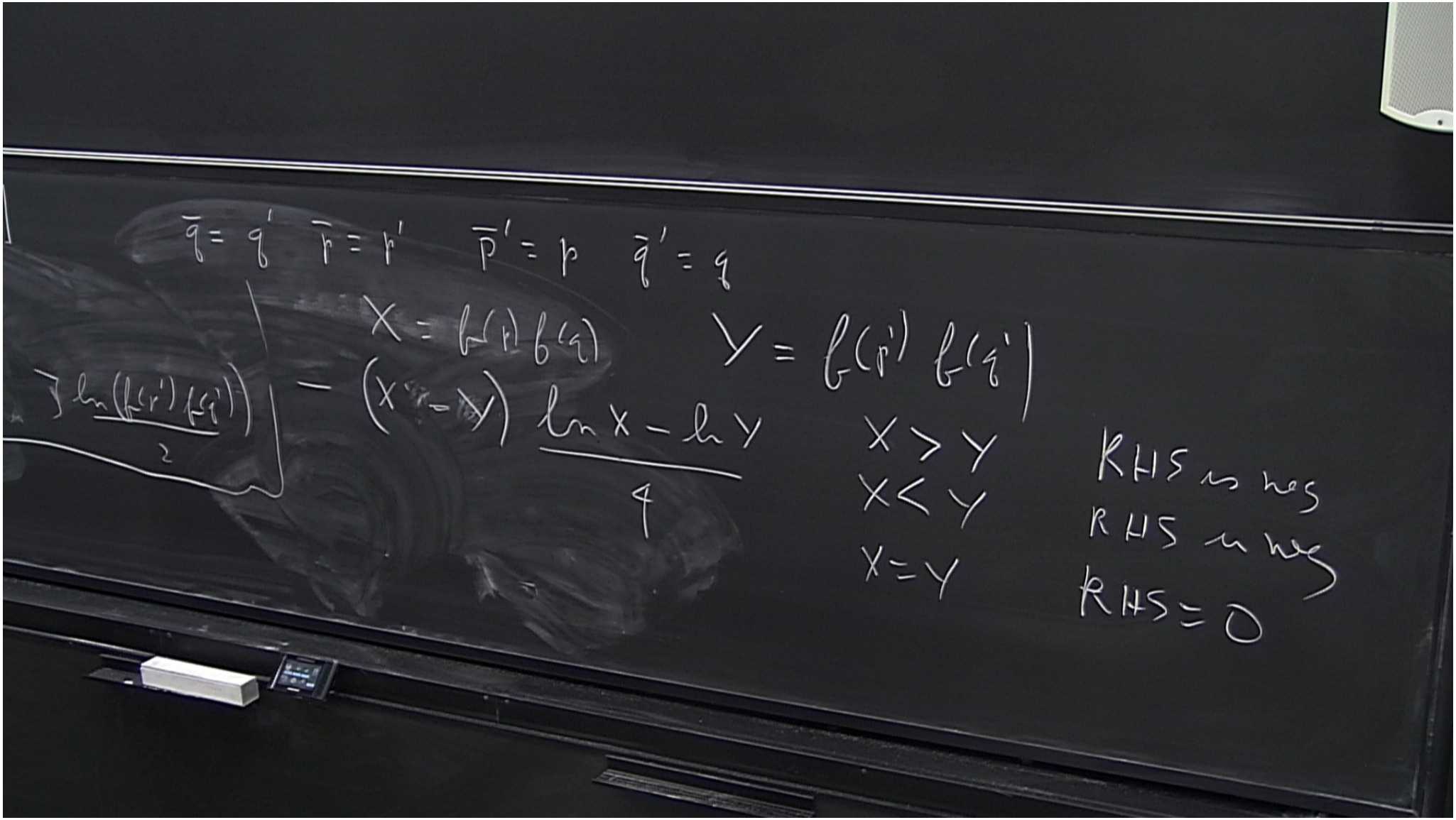


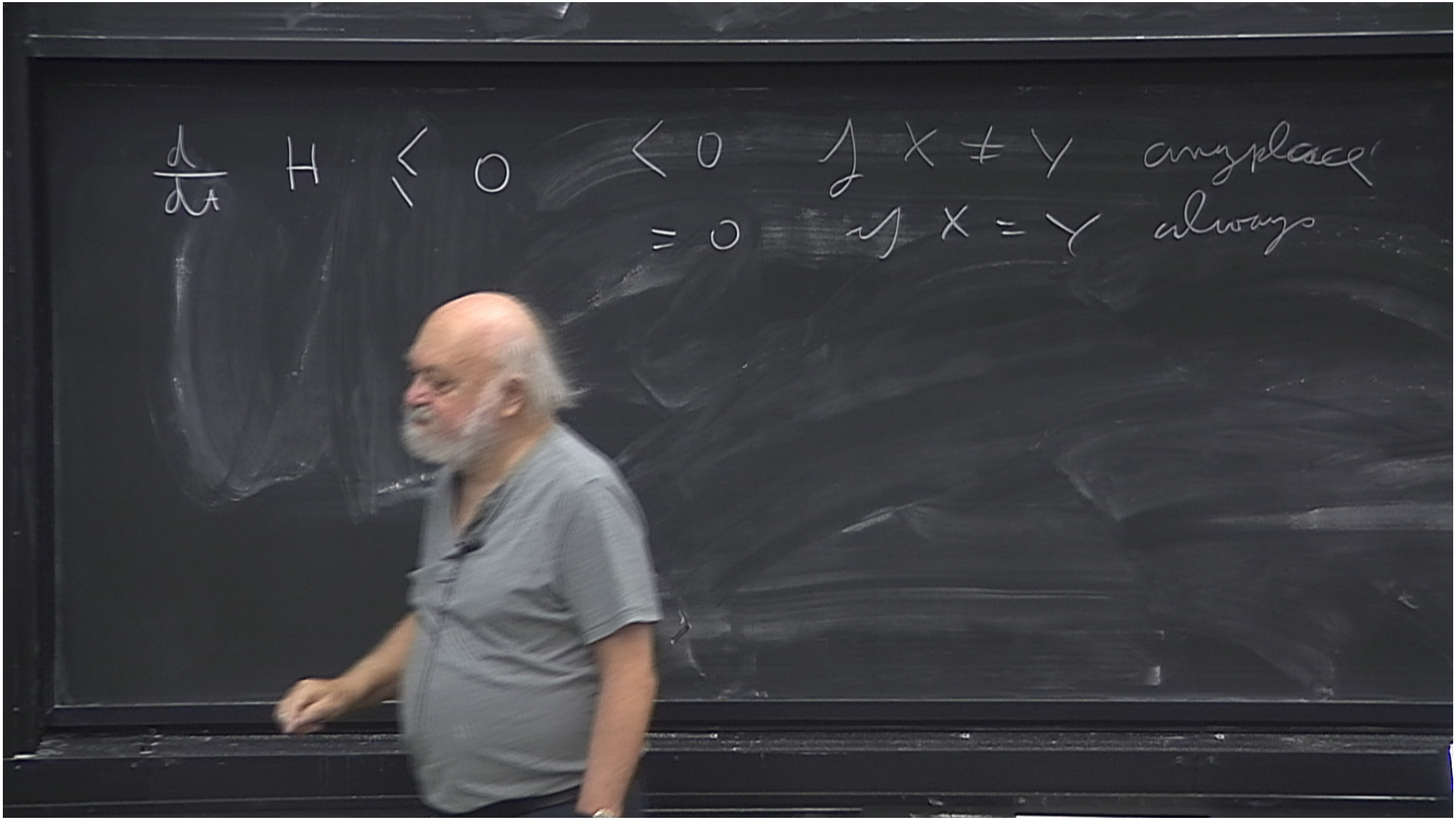

$\bar{q} = q' \quad \bar{r} = r' \quad \bar{p}' = p \quad \bar{q}' = q$

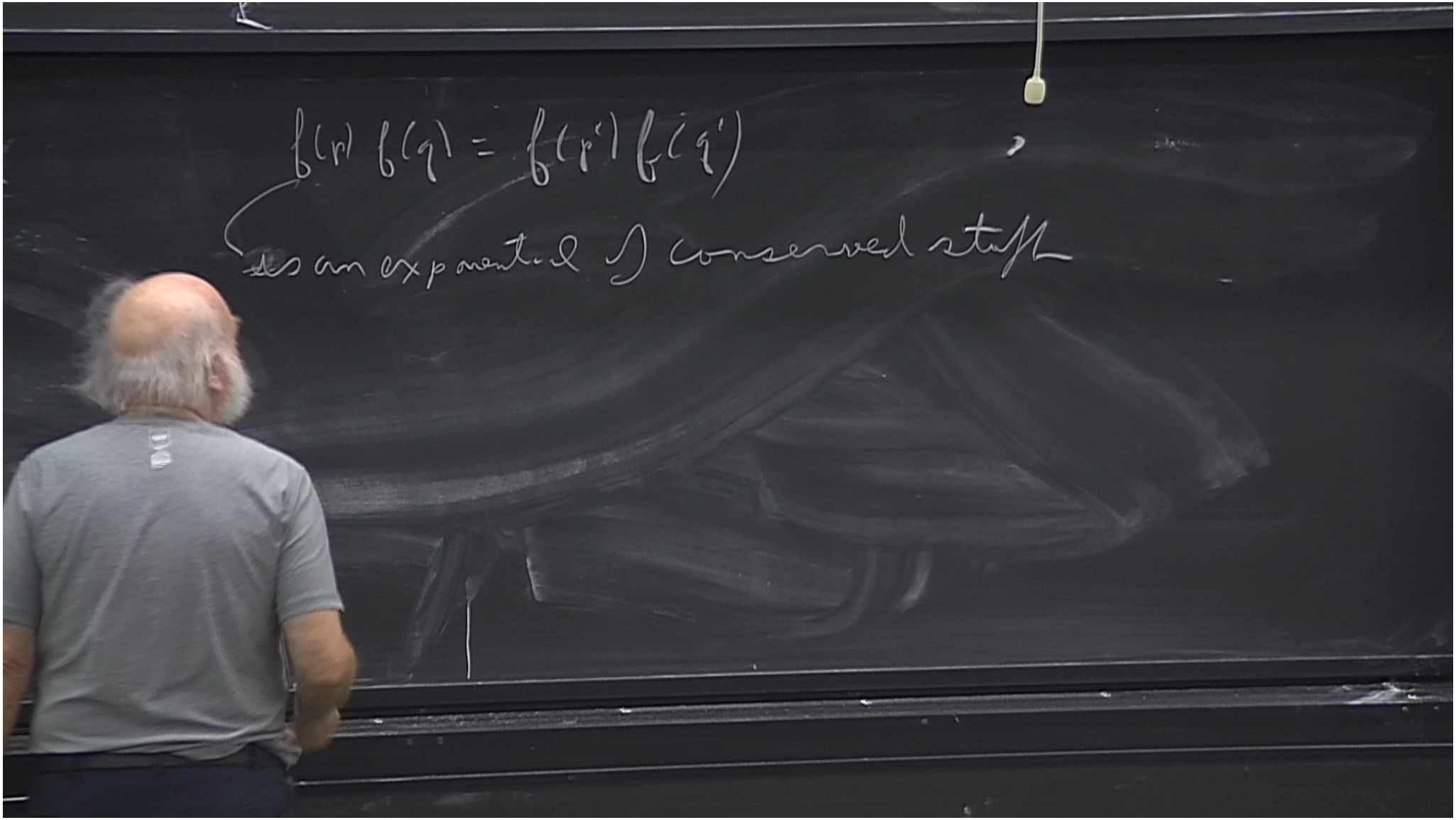

df
= z
[f(p) f(q)
ln f(p) + ln f(q)
ln f(p)² f(q)
ln f(p)

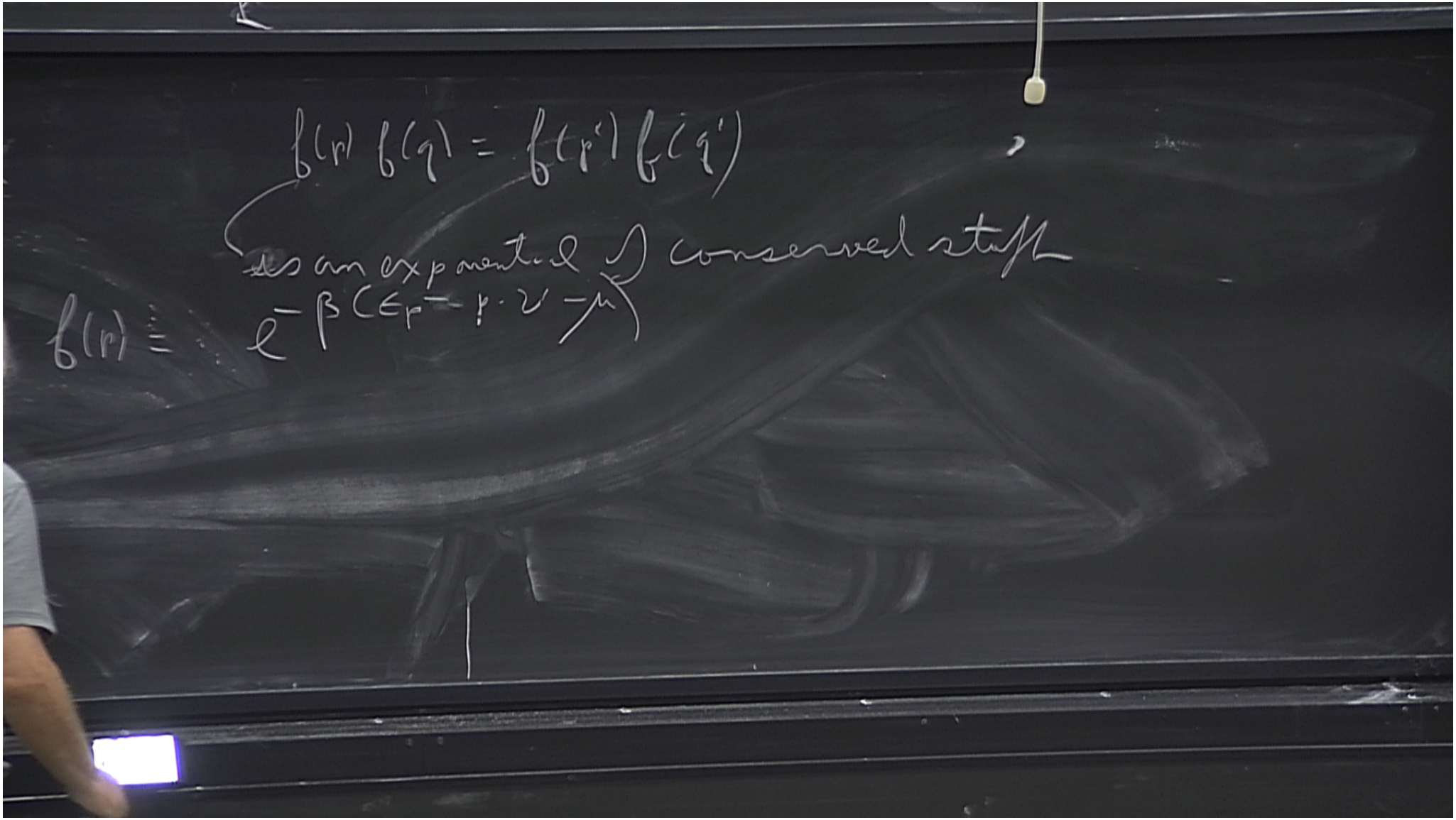

$\bar{q} = q'$ $\bar{r} = r'$ $\bar{p}' = p$ $\bar{q}' = q$
$X = f(r) f(q)$
$Y = f(r') f(q')$
$X \sim Y$

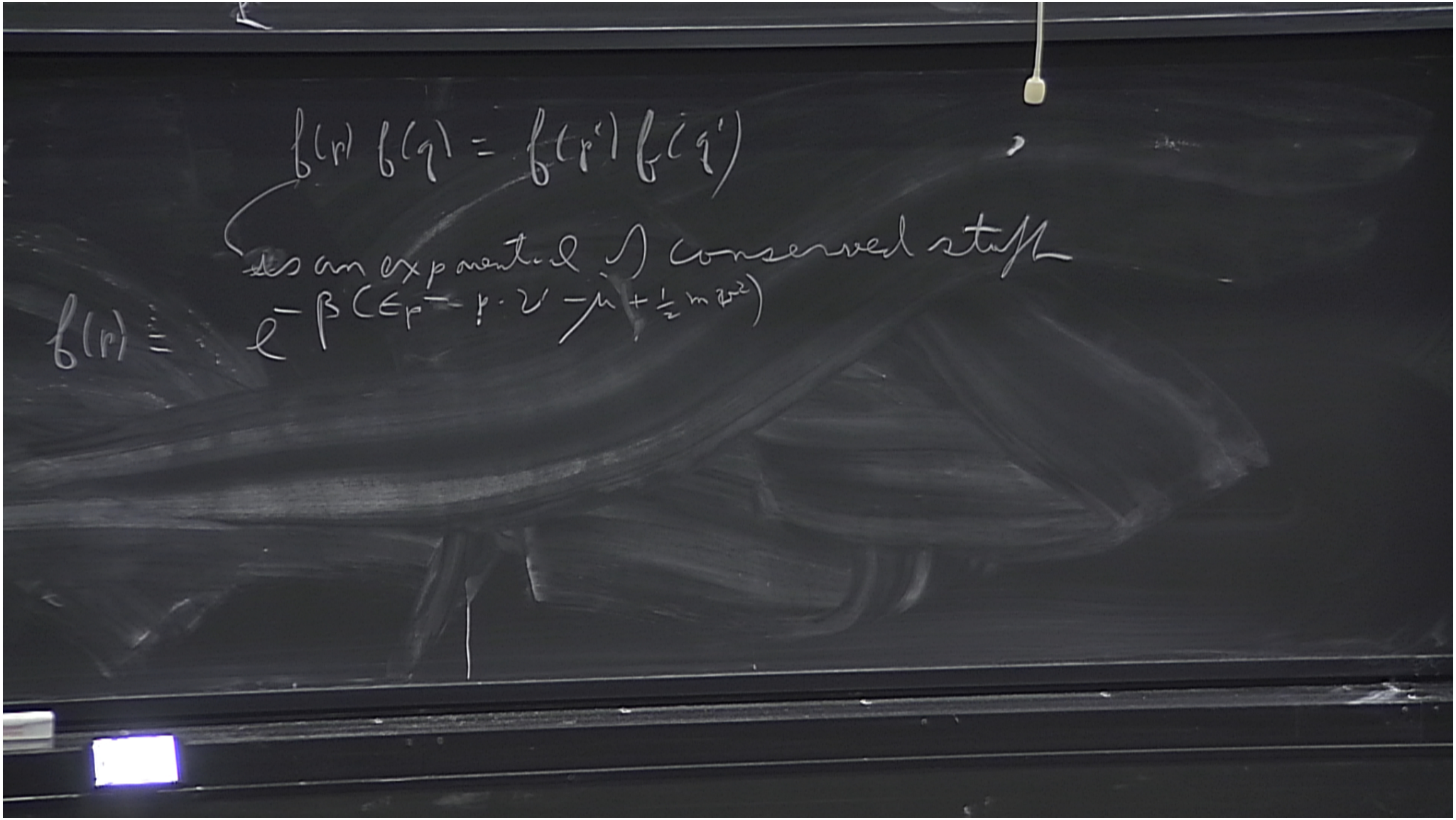

$\bar{q} = q' \quad \bar{r} = r' \quad \bar{p}' = p \quad \bar{q}' = q$
$X = f(r) f(q) \qquad Y = f(p') f(q')$
$\ln\frac{(f(r')f(q'))}{2}$
$-(X-Y)\frac{\ln X - \ln Y}{4}$


$\bar{q} = q' \quad \bar{r} = r' \quad \bar{p}' = p \quad \bar{q}' = q$
$X = f(r) f(q) \qquad Y = f(p') f(q')$
$\frac{\ln(f(r') f(q'))}{2}$
$= (X - Y) \frac{\ln X - \ln Y}{4}$
$X > Y$
$X < Y$
$X = Y$
RHS is neg
RHS is neg
RHS = 0


$\frac{d}{dt} H \leq 0$
< 0 if $x \neq y$ anyplace
$= 0$ if $x = y$ always


f(r) f(q) = f(r') f(q')
is an exponential of conserved stuff


f(r) f(q) = f(r') f(q')
f(r) = e^{-β(ε_p - p·v - μ)}


f(r) f(q) = f(r') f(q')
es an exponential of conserved stuff
f(r) ≡ e^{-β(ε_p - p·v - μ + ½ m v²)}

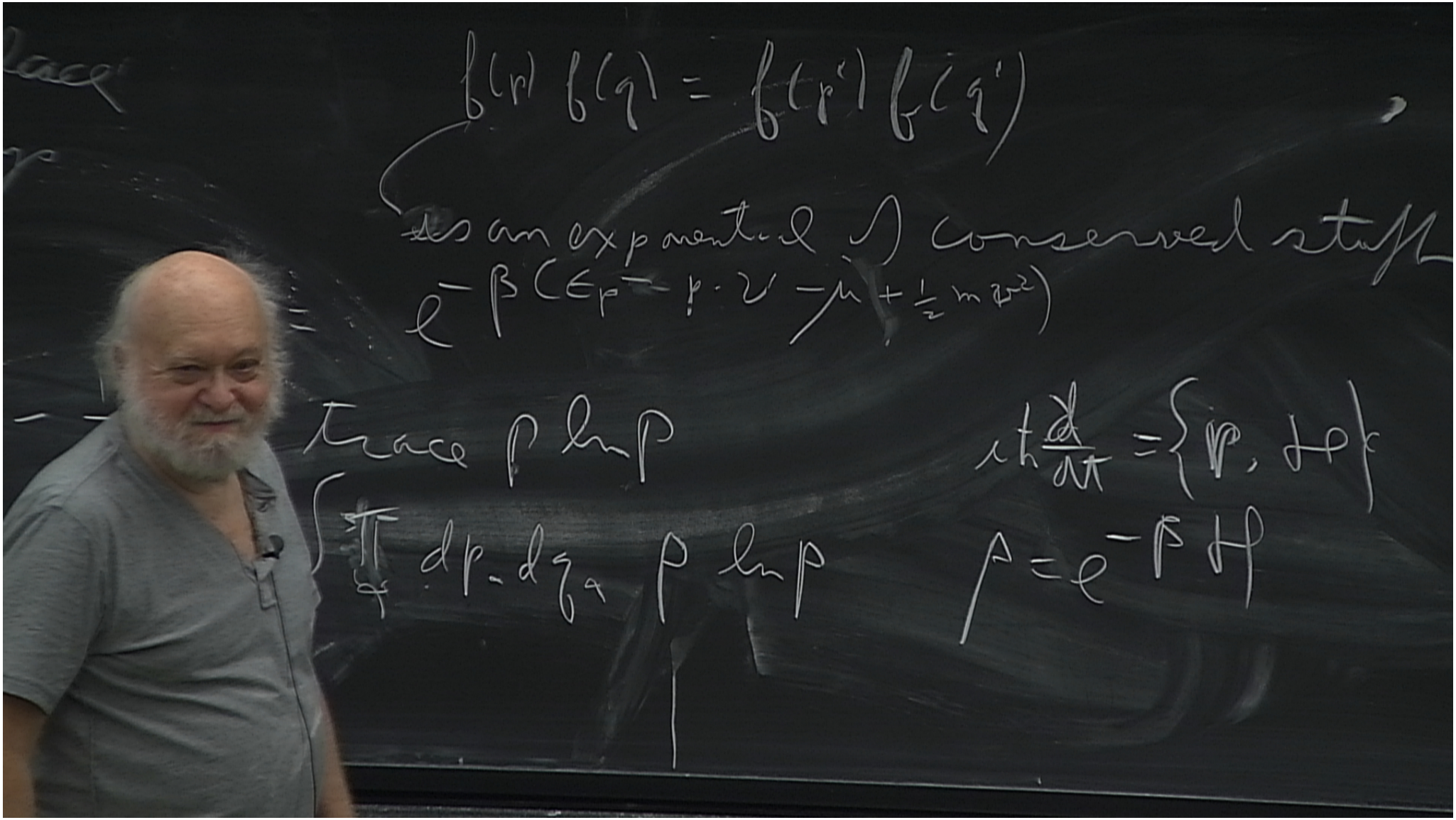
$f(r)\, f(q) = f(r')\, f(q')$

$f(r) = e^{-\beta(\epsilon_p - p\cdot v - \mu + \frac{1}{2}mv^2)}$

$-\frac{1}{k}S = \text{trace}\, \rho \ln \rho$

$= \int \prod_q dp_q\, dq_q\, \rho \ln \rho$

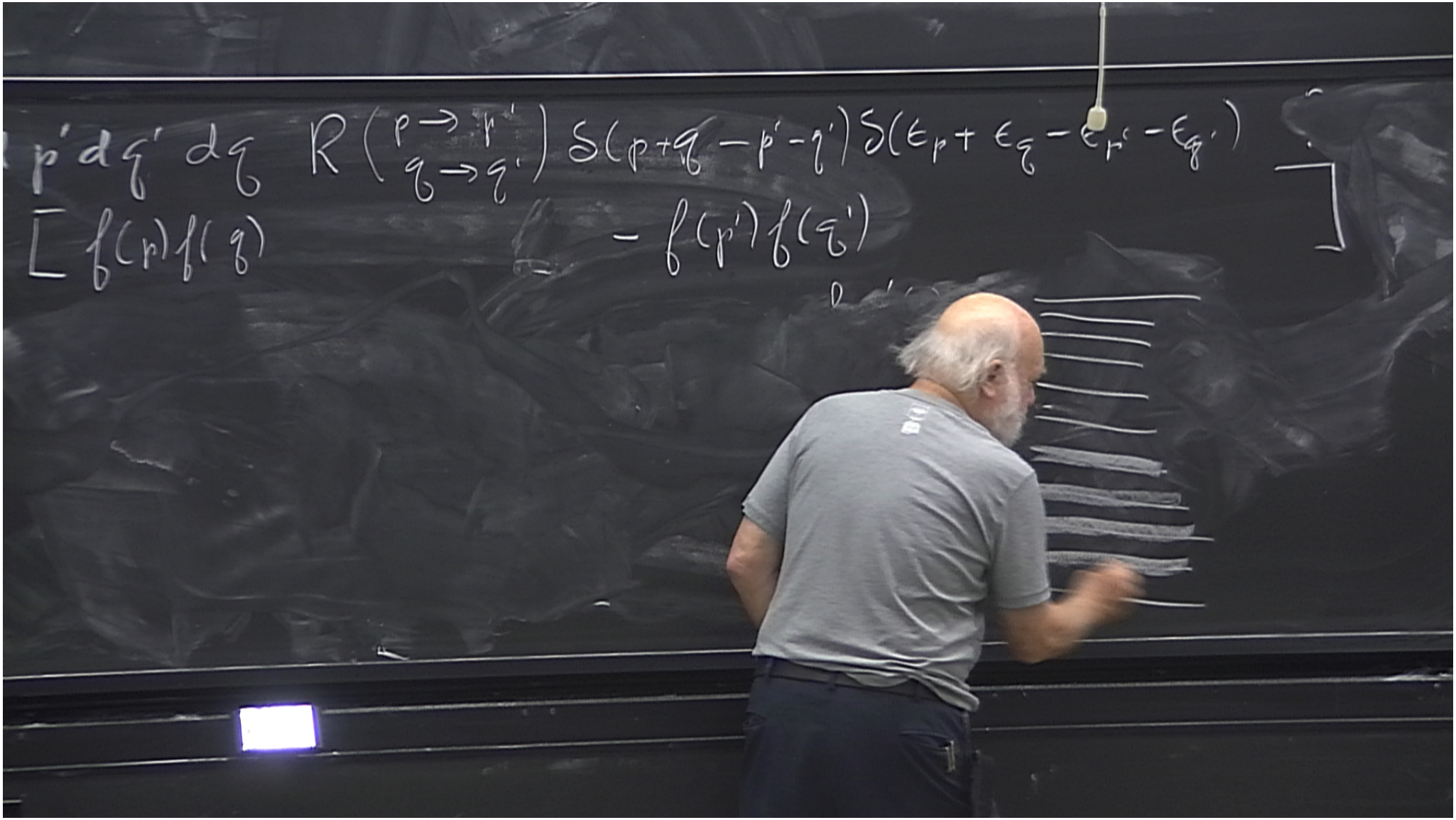
$i\hbar \frac{\partial}{\partial t} = \{H, \cdot\}$


f(r) f(q) = f(r') f(q')
is an exponential of conserved stuff
e^{-β(ε_p - p·v - μ + ½ m v²)}
Trace ρ ln ρ
∫ Π_q dp_q dq_q ρ ln ρ
ih ∂/∂t = {ρ, H}
ρ = e^{-βH}

$$\left(\partial_t + \nabla_p \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p\right) f(r) = -\int dp' dq' dq \left[f(r) f(q) \right.$$

Fermi liquid

^3He


p' dq' dq R(p → p', q → q') δ(p+q−p'−q') δ(ε_p + ε_q − ε_p' − ε_q')
[f(p) f(q)
− f(p') f(q')

$$\left(\partial_t + \nabla_p \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p\right) f(r) = -\int dp' dq' dq \left[f(r) f(q) \right.$$

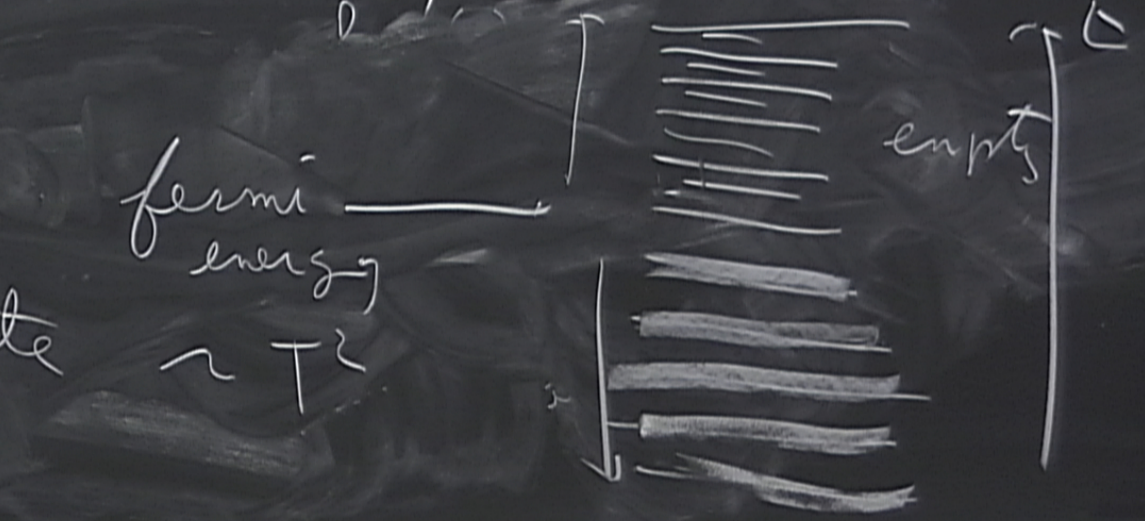
Fermi liquid – low T

He^3 10^{-3} – 1°K

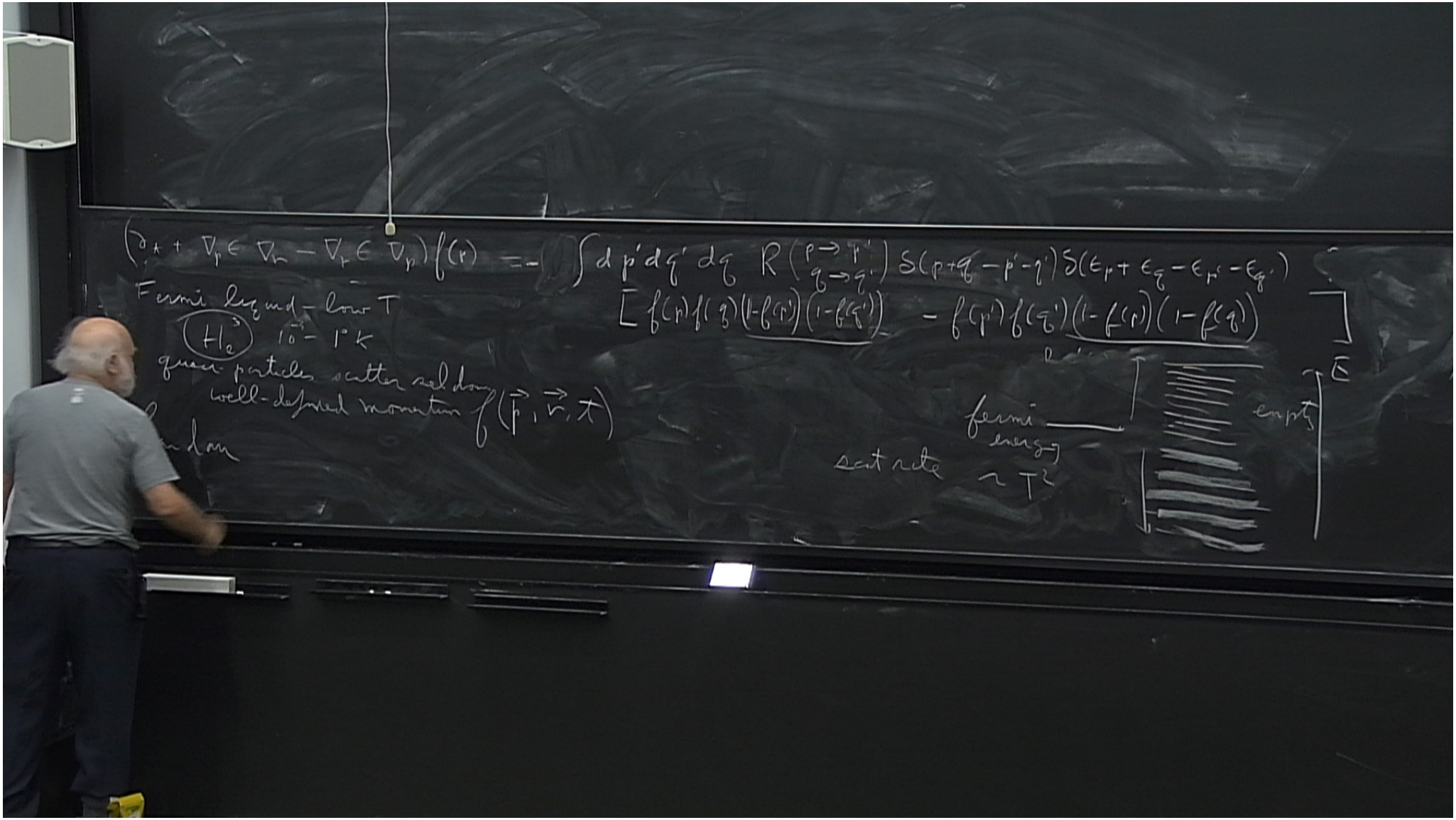
quasi-particles scatter seldom

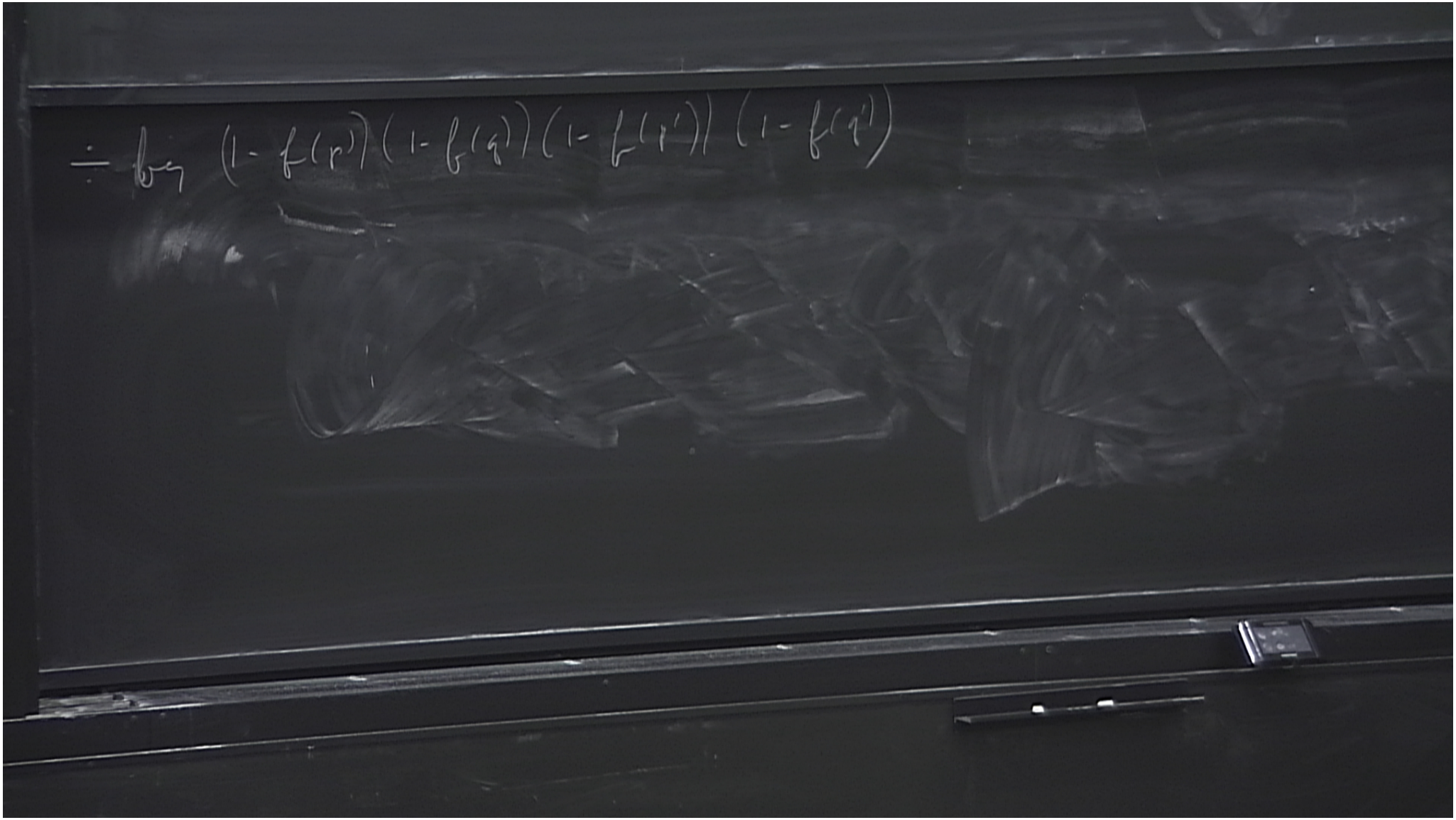
$$\int dq' \, dq \; R\left(\begin{matrix} p \to p' \\ q \to q' \end{matrix}\right) \delta(p+q-p'-q') \, \delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$$

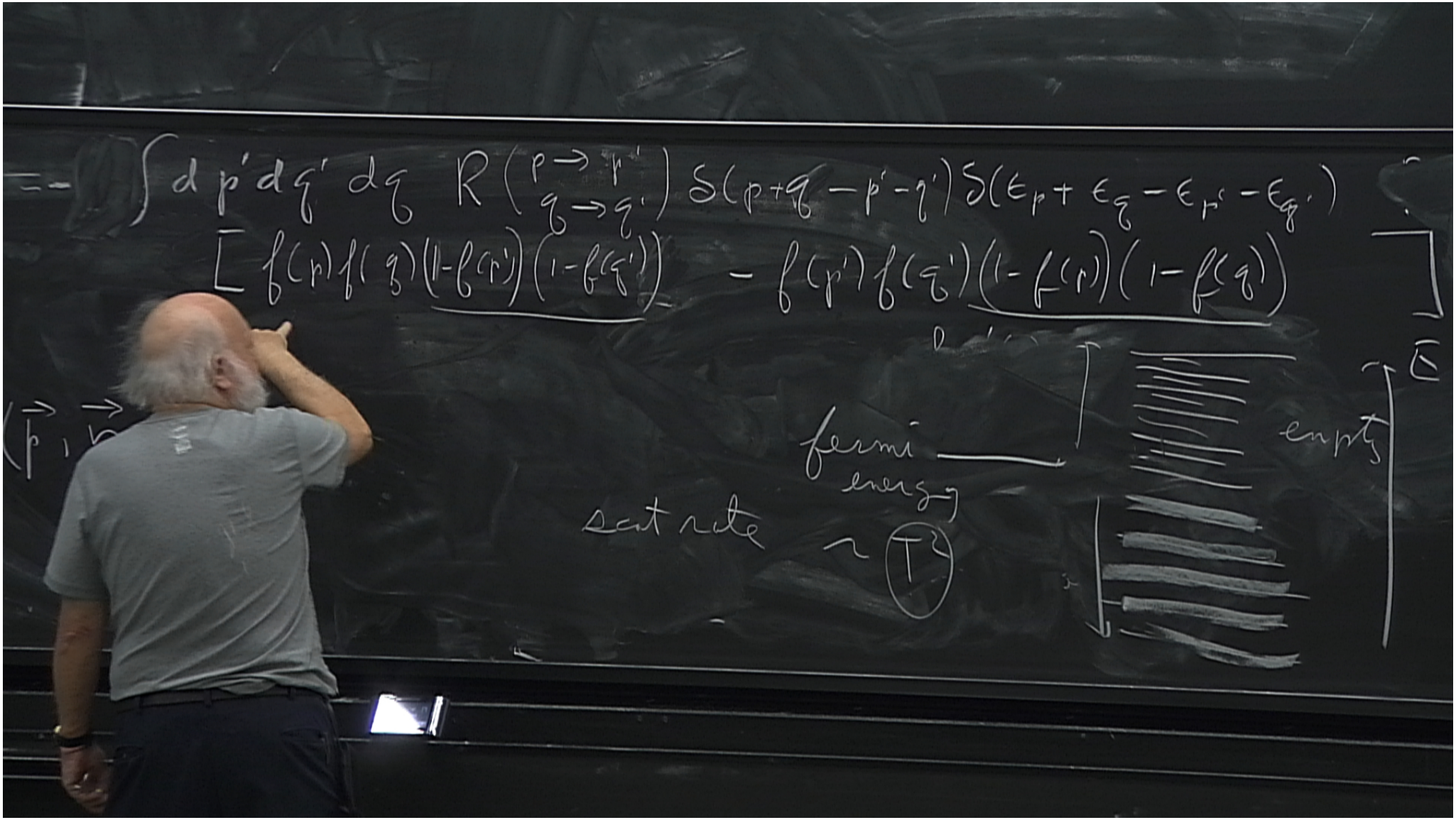
$$f(p) f(q) \qquad - f(p') f(q')$$

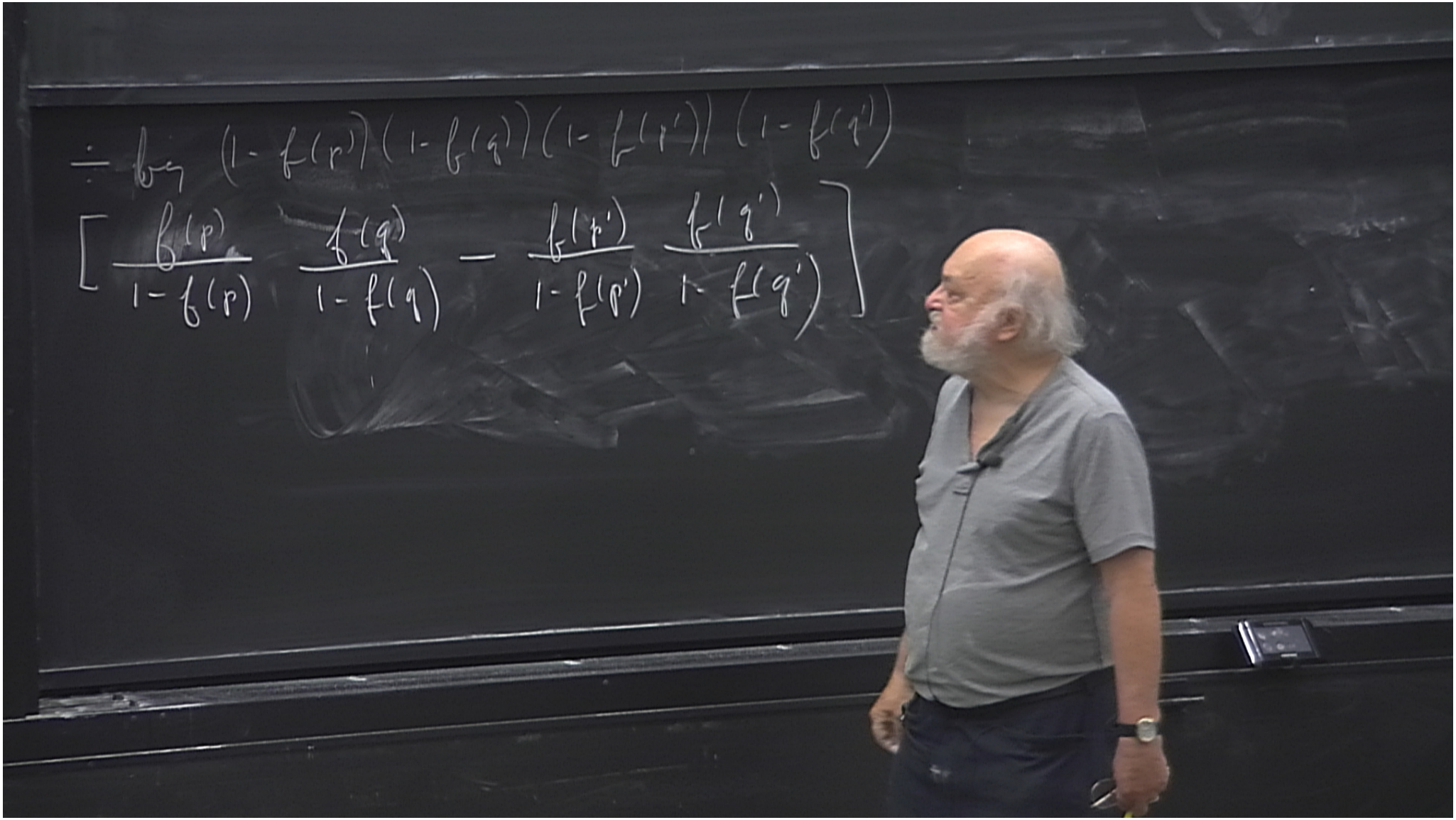


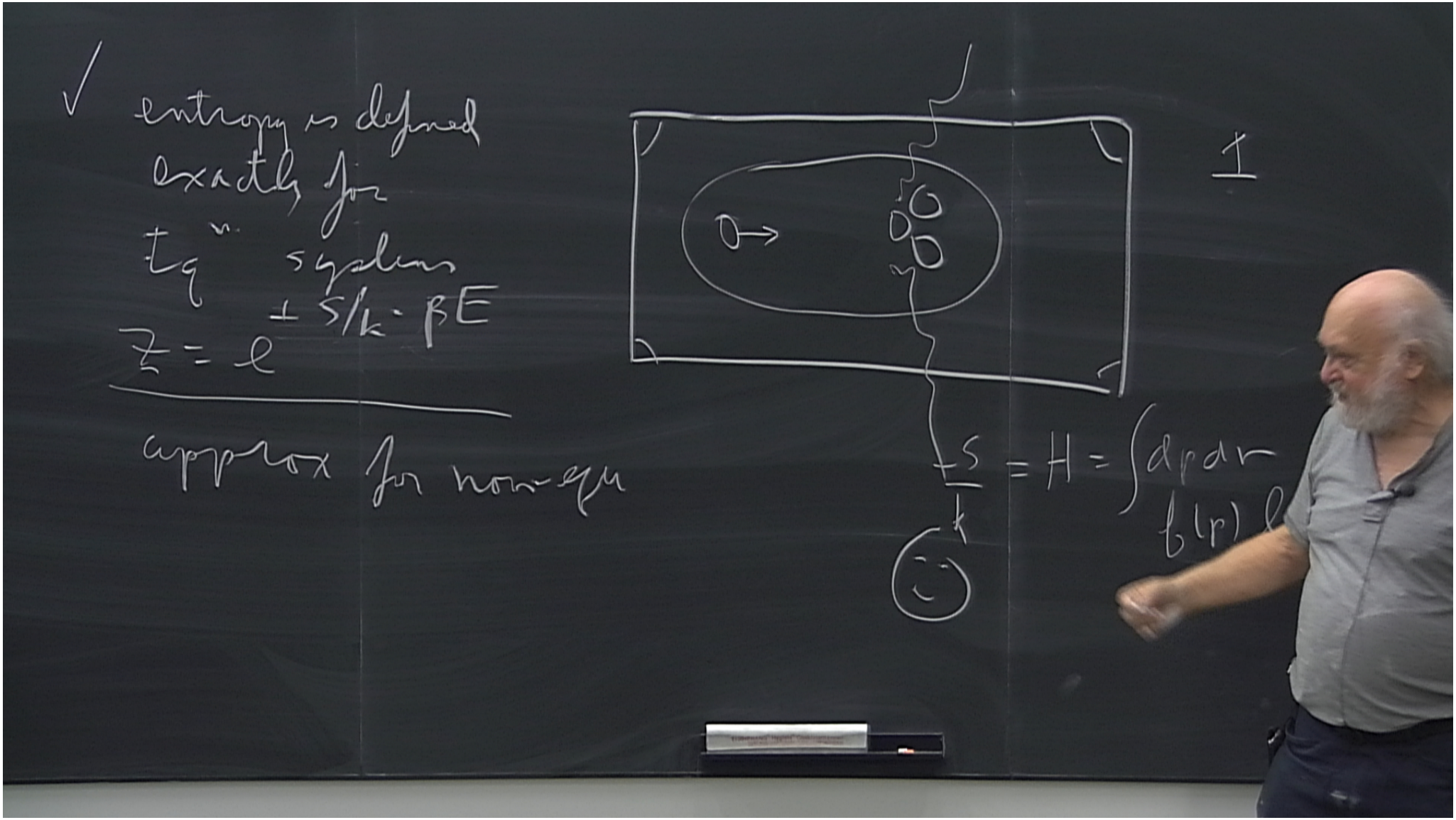
scat rate $\sim T^2$

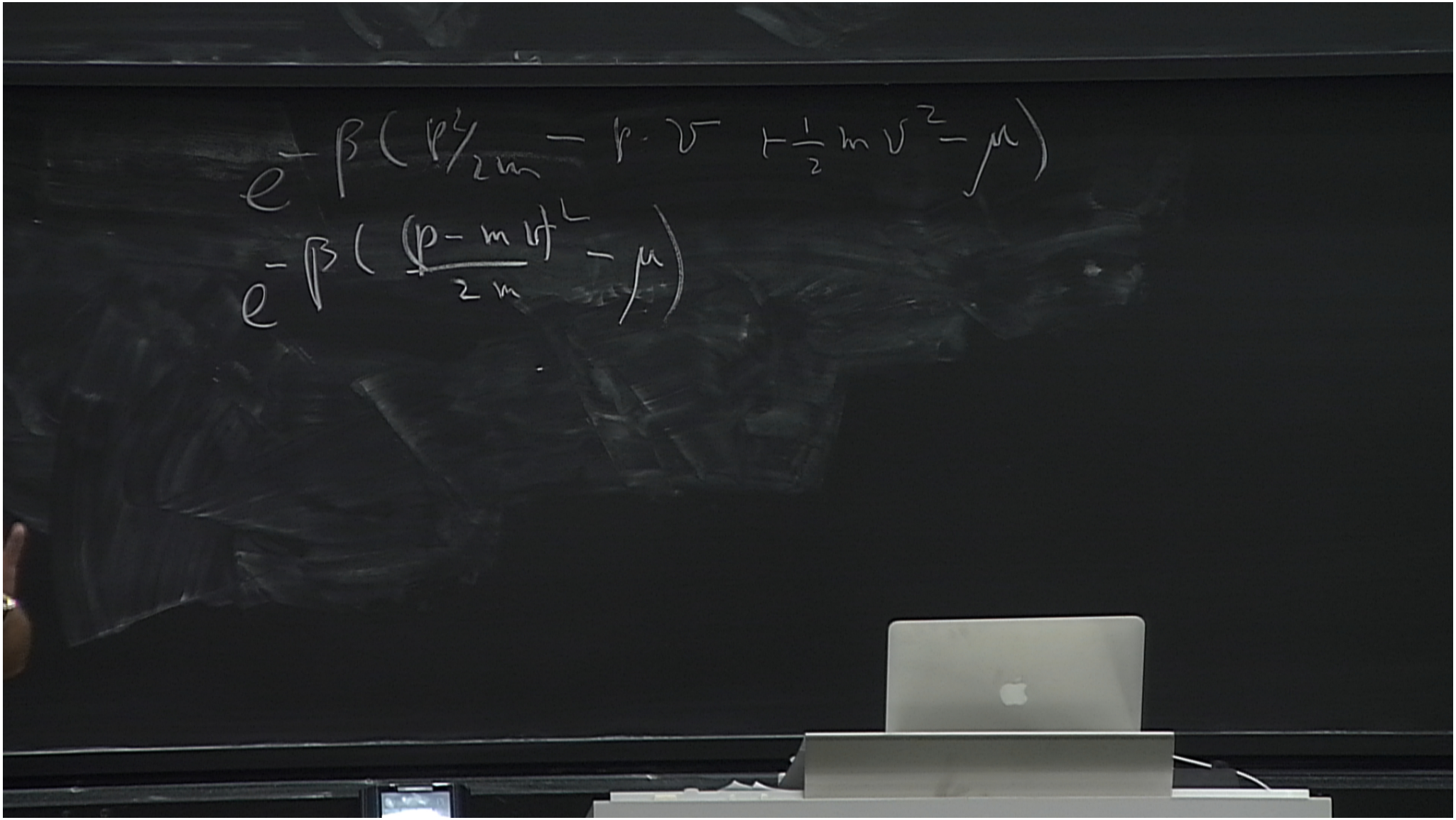

Fermi liquid – low T
quasi-particles scatter rarely
well-defined momentum
fermi energy
empty
scat rate ~ T²

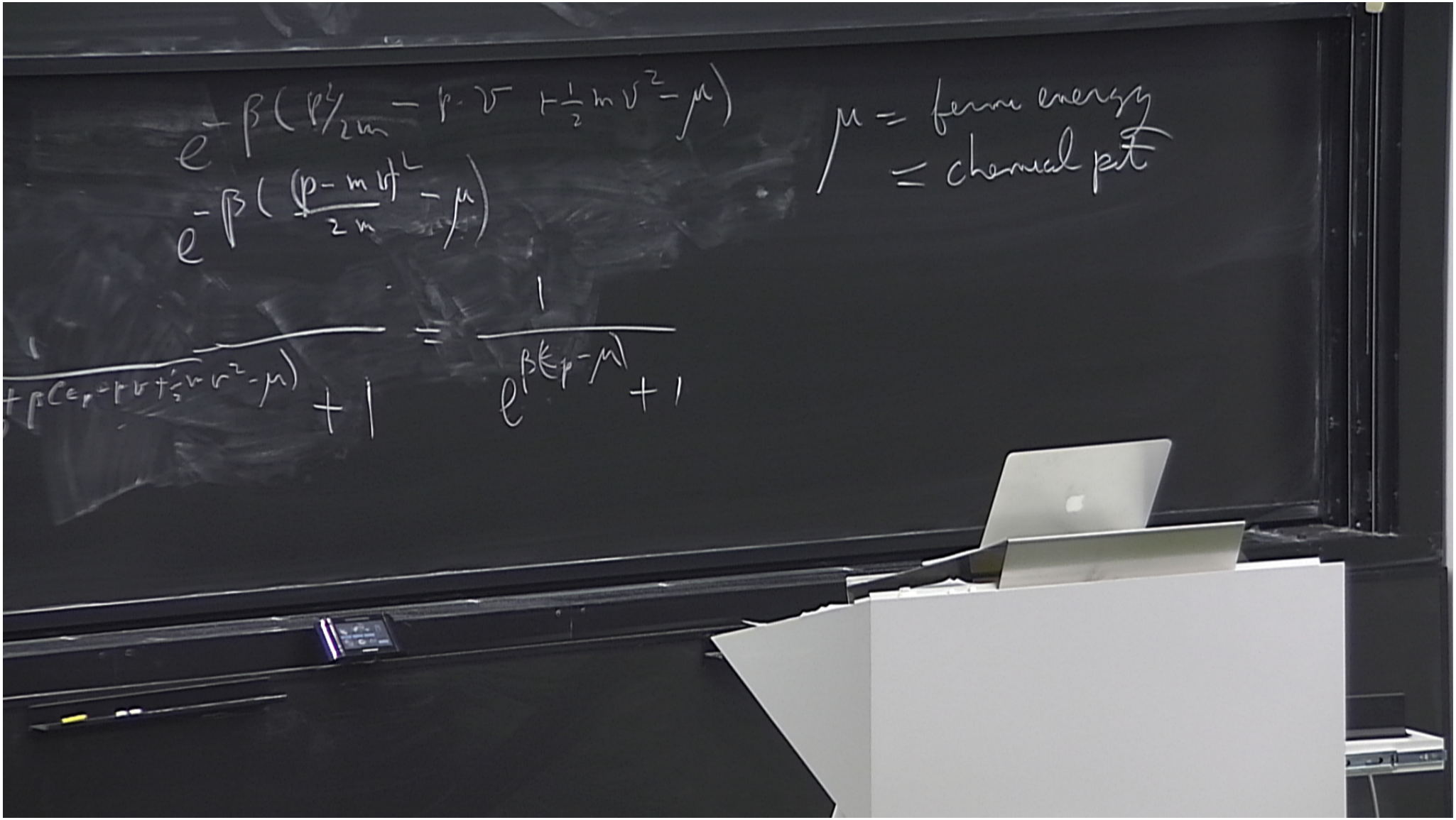

÷ by (1 - f(p')) (1 - f(q')) (1 - f(r')) (1 - f(s'))

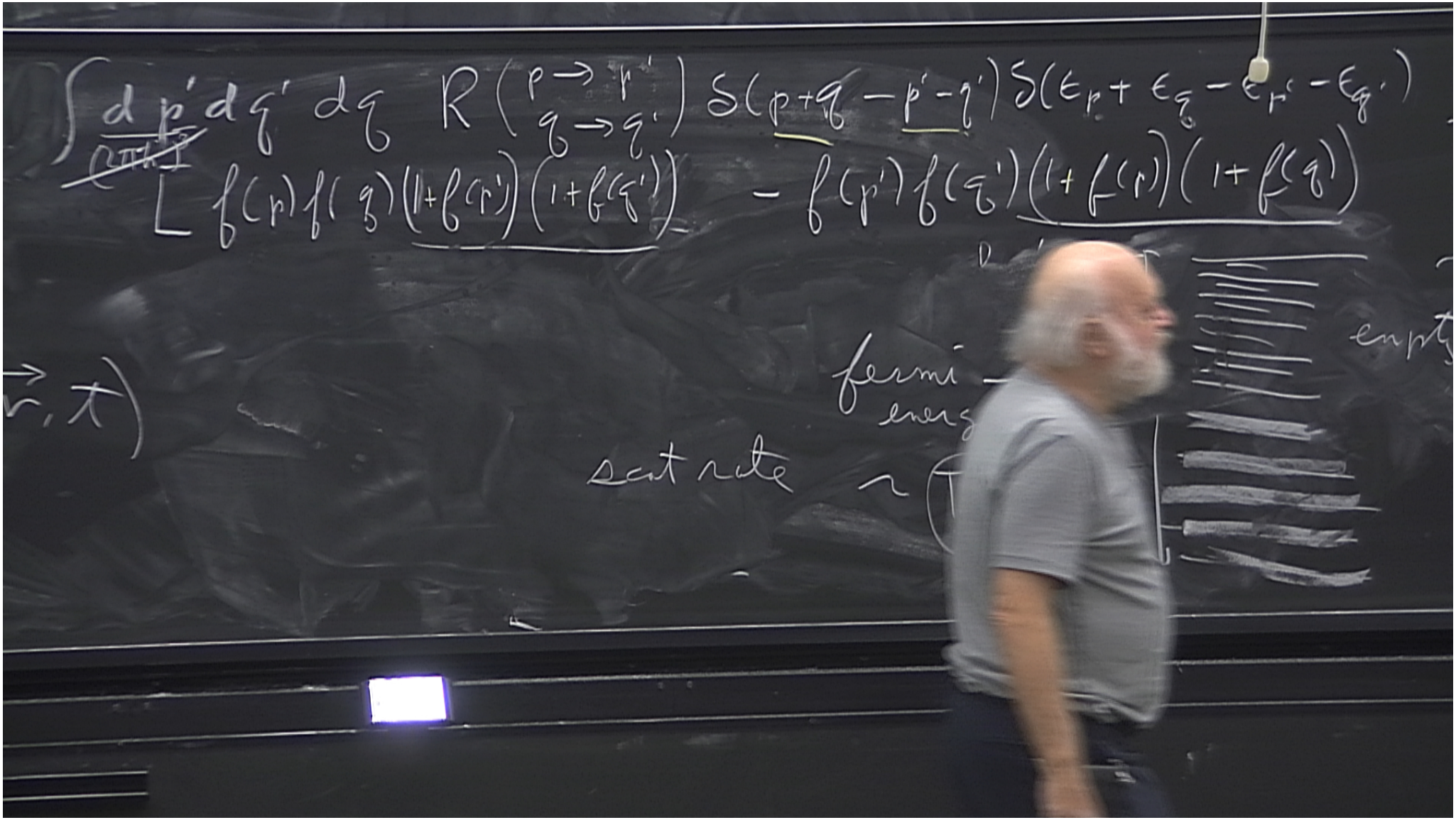

fermi energy
empty
scat rate

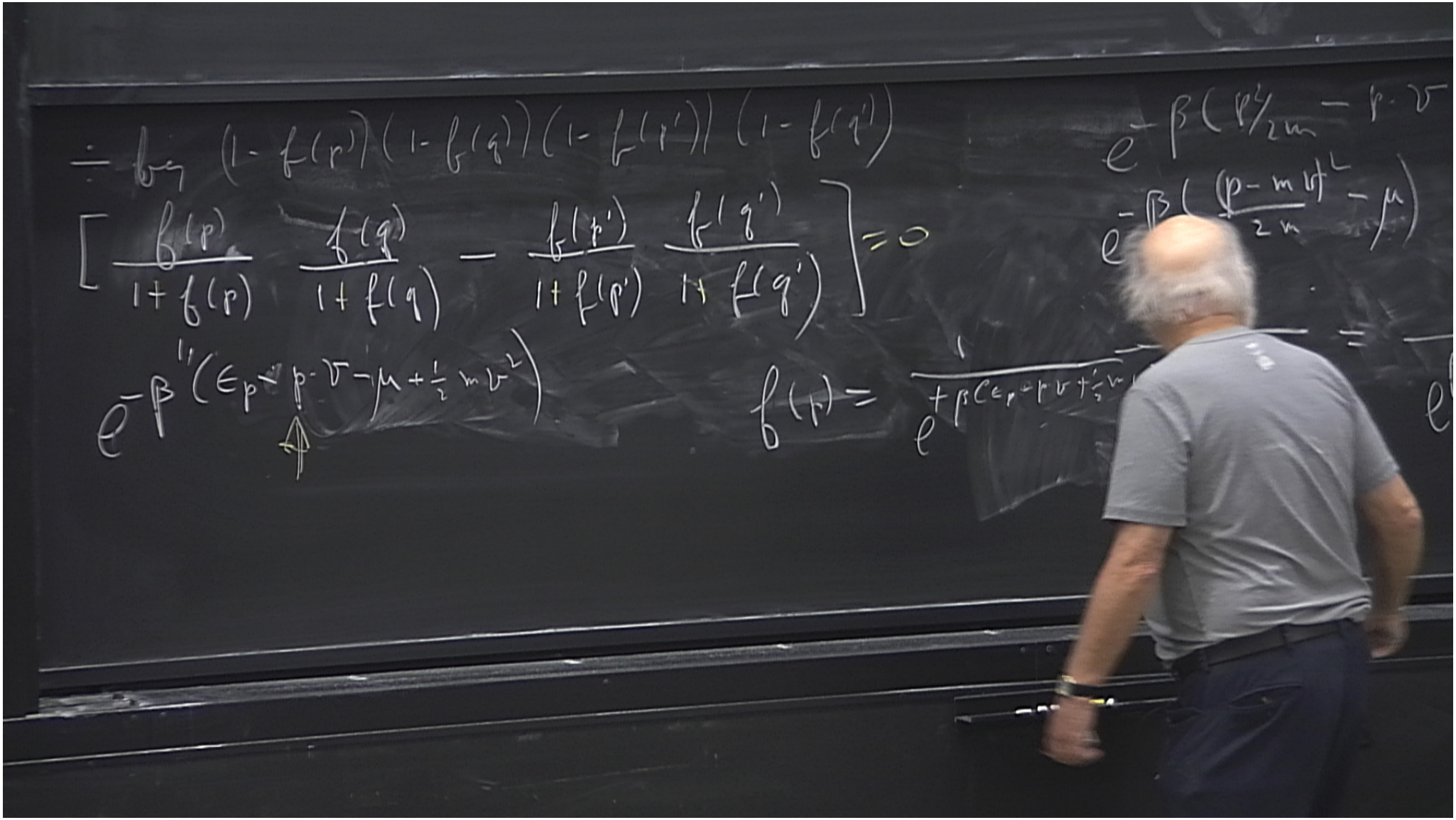

$\div \log (1-f(p))(1-f(q))(1-f(p'))(1-f(q'))$
$\left[\frac{f(p)}{1-f(p)}\frac{f(q)}{1-f(q)} - \frac{f(p')}{1-f(p')}\frac{f(q')}{1-f(q')}\right]$

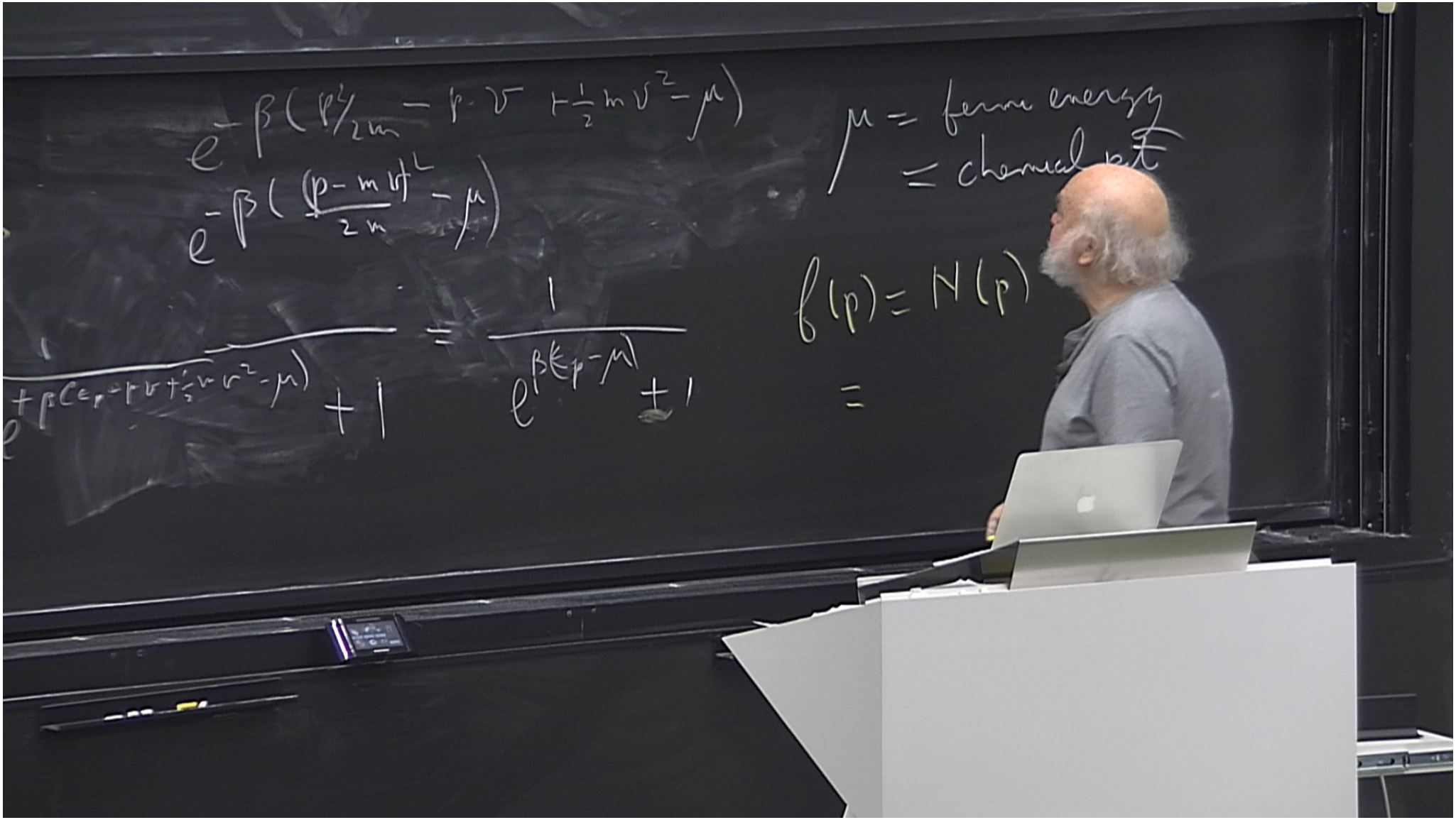

entropy is defined
exactly for
eq systems
$Z = e^{\pm S/k \cdot \beta E}$
approx for non-eq
$\frac{\pm S}{k} = H = \int dp\, dr$
$f(p)$

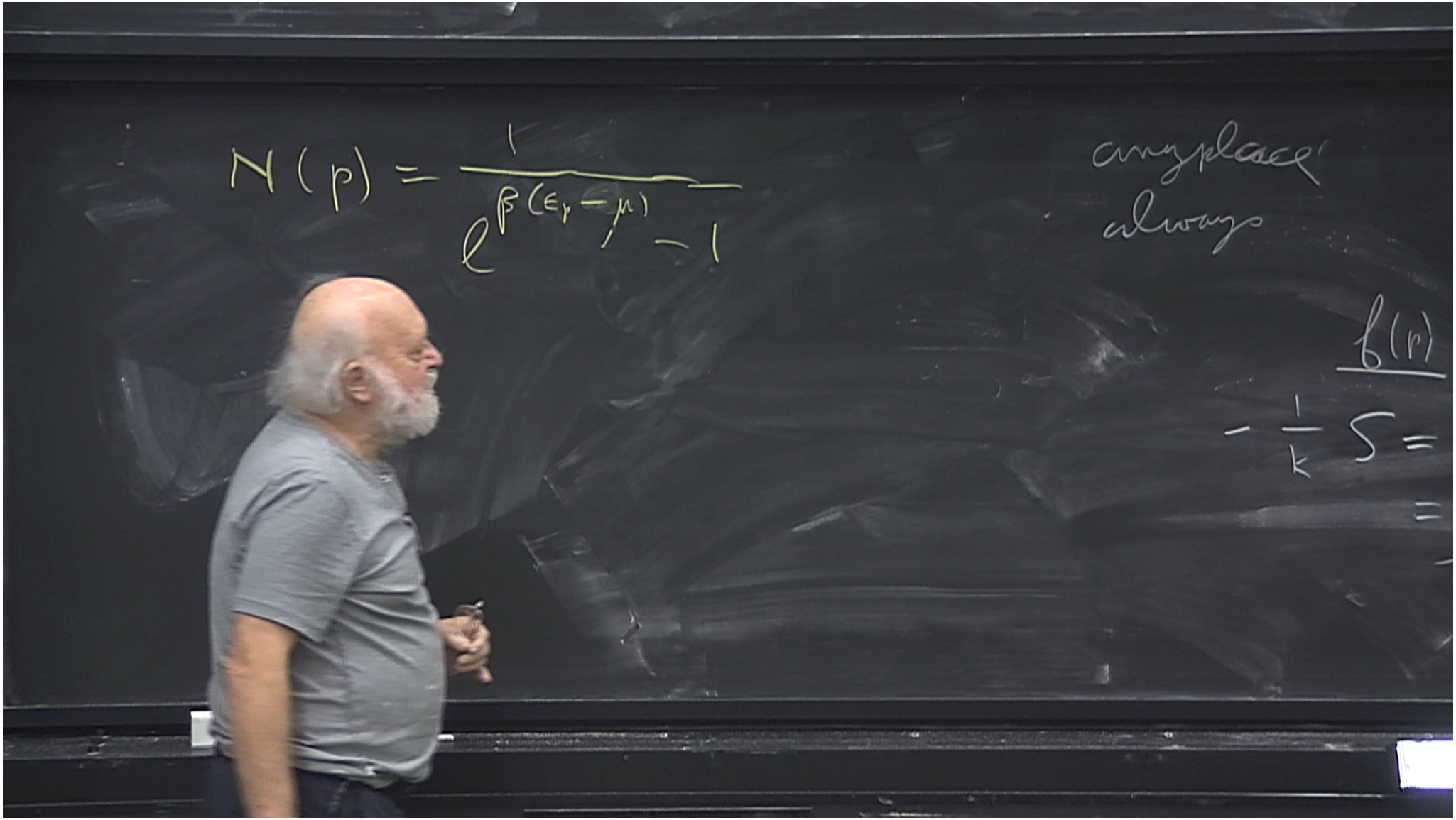

$e^{-\beta\left(p^2/2m - p\cdot v + \frac{1}{2}mv^2 - \mu\right)}$
$e^{-\beta\left(\frac{(p-mv)^2}{2m} - \mu\right)}$

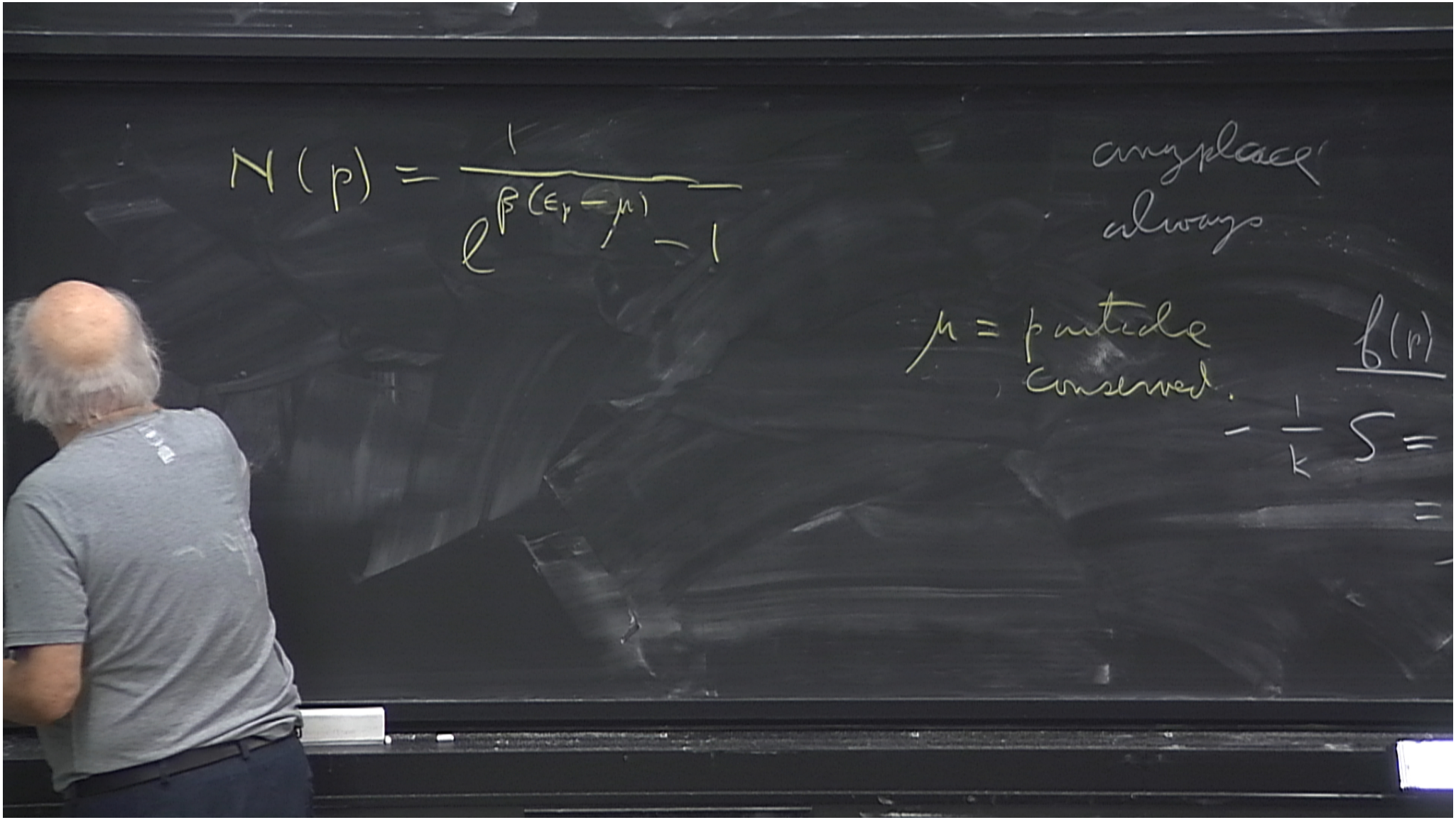

$e^{-\beta(p^2/2m - p\cdot v + \frac{1}{2}mv^2 - \mu)}$
$e^{-\beta(\frac{(p-mv)^2}{2m} - \mu)}$
μ = fermi energy
= chemical pot
$\frac{1}{e^{\beta(\epsilon_p - \mu)} + 1}$

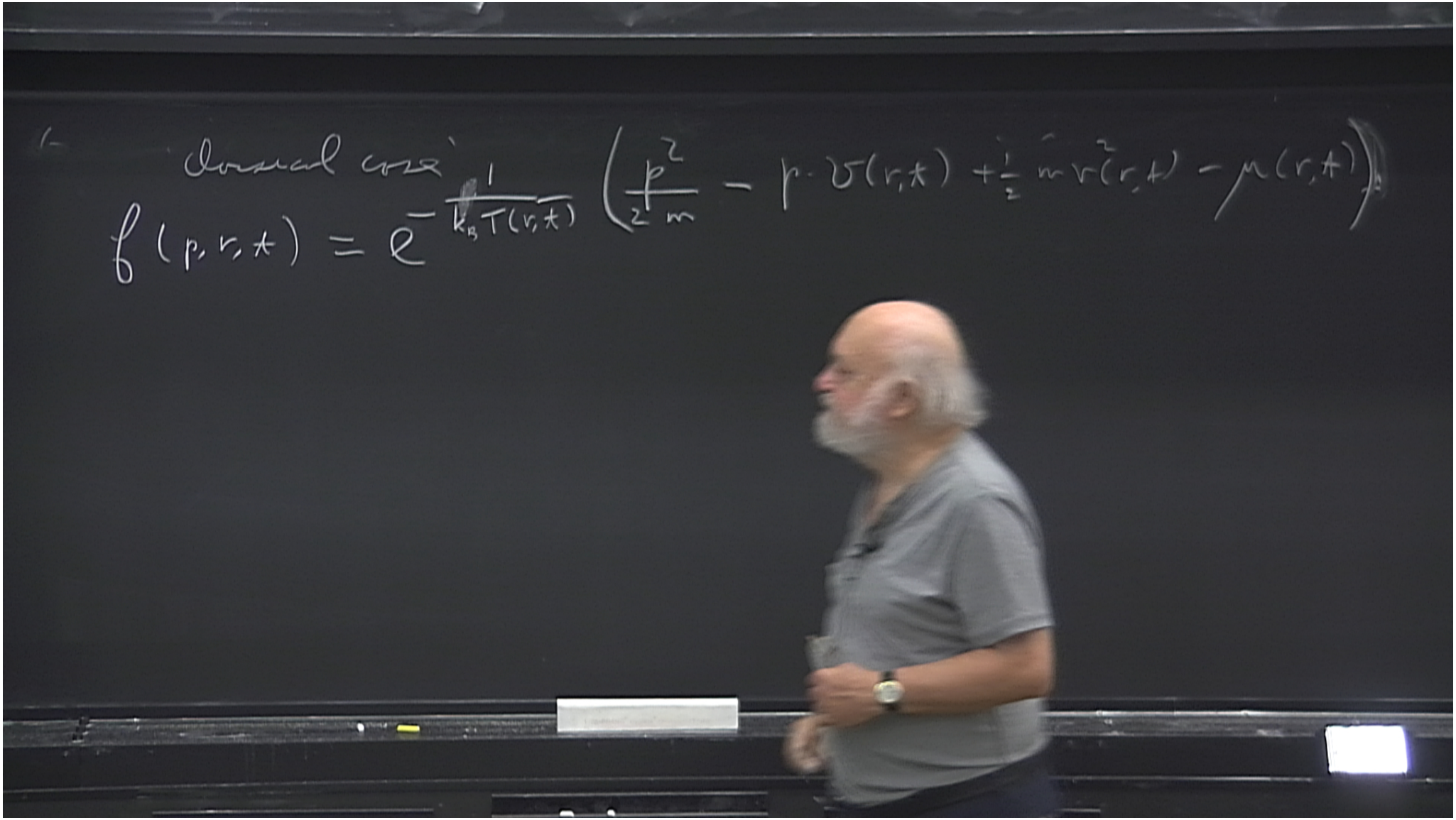

fermi
energy
empty

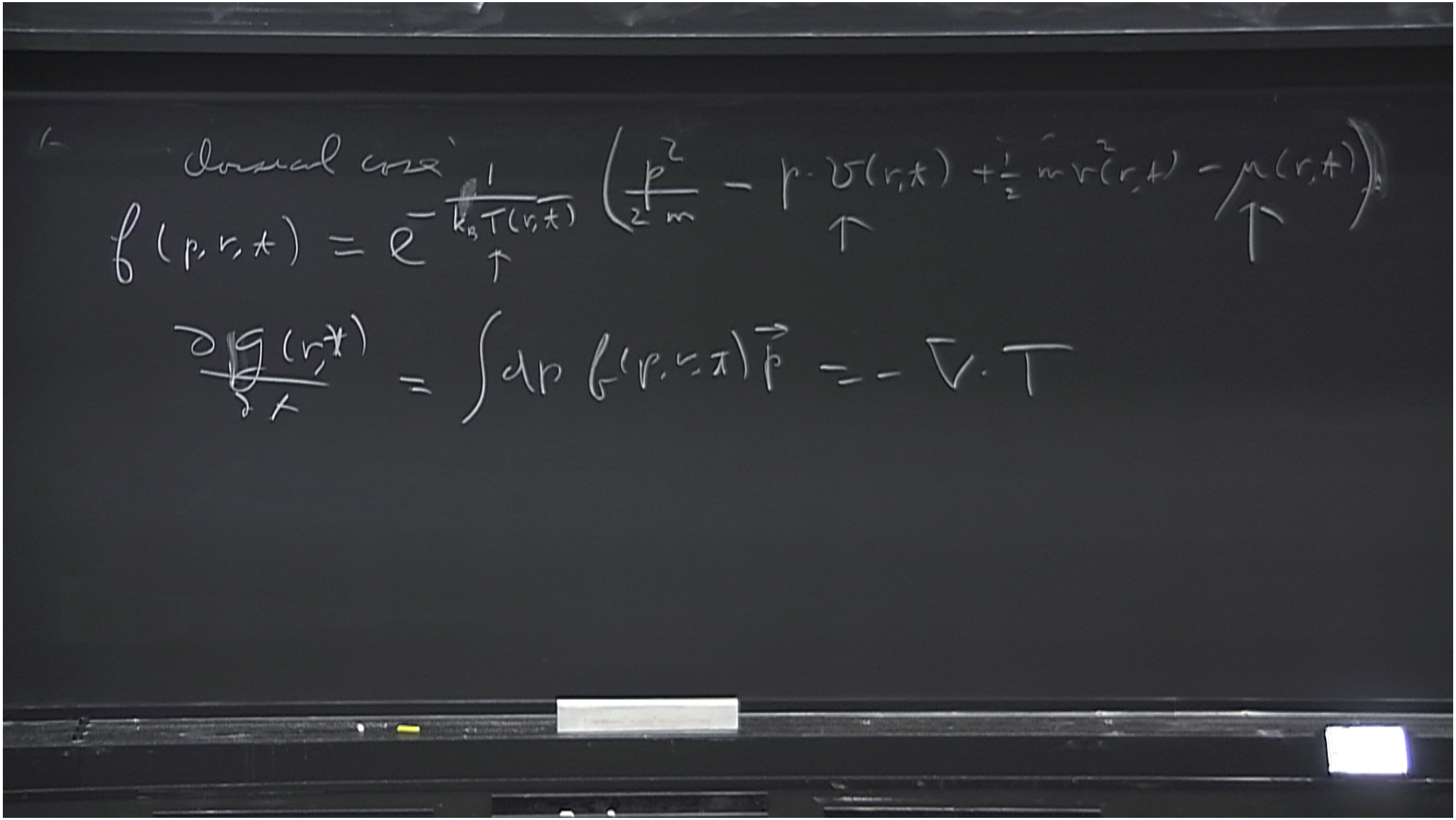

$\div \int d\Gamma\,(1-f(p))(1-f(q))(1-f(p'))(1-f(q'))$
$\left[\frac{f(p)}{1+f(p)}\frac{f(q)}{1+f(q)} - \frac{f(p')}{1+f(p')}\frac{f(q')}{1+f(q')}\right] = 0$
$e^{-\beta(\epsilon_p - p\cdot v - \mu + \frac{1}{2}mv^2)}$
$f(p) =$
$e^{-\beta(p^2/2m - p\cdot v$
$e^{-\beta($

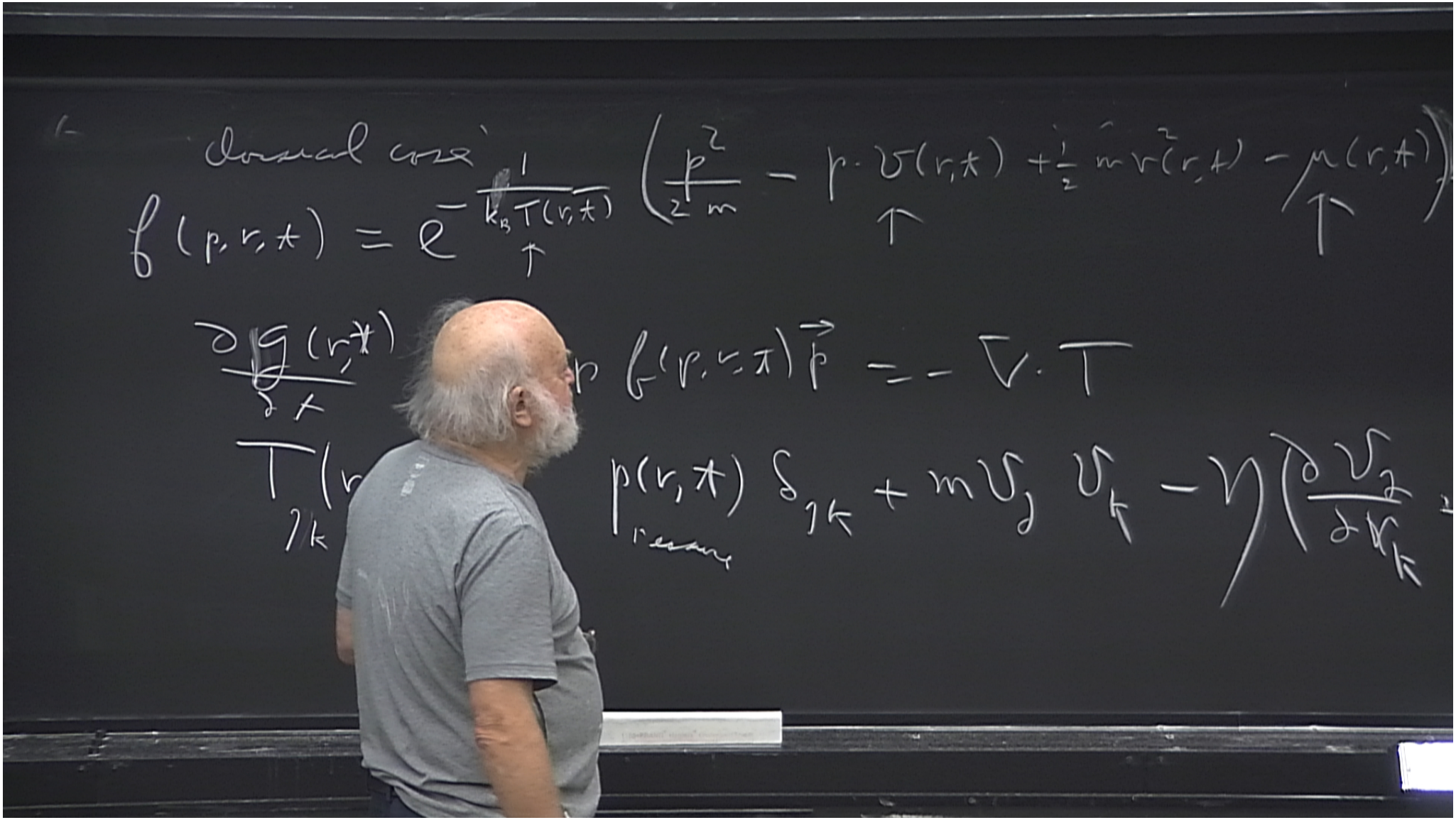

$e^{-\beta\left(\frac{p^2}{2m} - p\cdot v + \frac{1}{2}mv^2 - \mu\right)}$
$e^{-\beta\left(\frac{(p-mv)^2}{2m} - \mu\right)}$
μ = fermi energy
= chemical pot
$f(p) = N(p)$
=
$\frac{1}{e^{\beta(\epsilon_p - \mu)} \pm 1}$

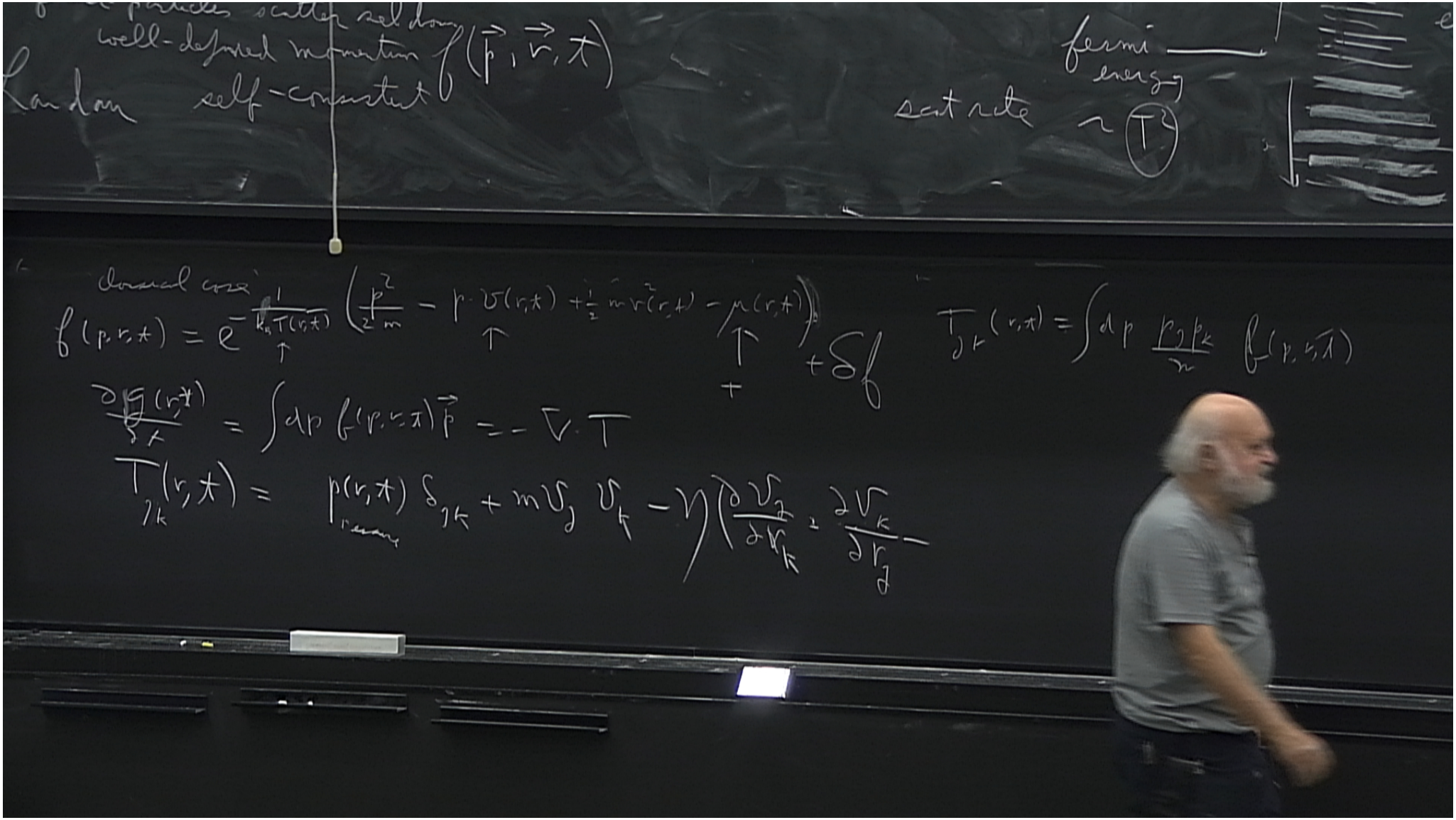

$N(p) = \frac{1}{e^{\beta(\epsilon_p - \mu)} - 1}$
anyplace
always
$\frac{b(p)}{-\frac{1}{k}S =}$

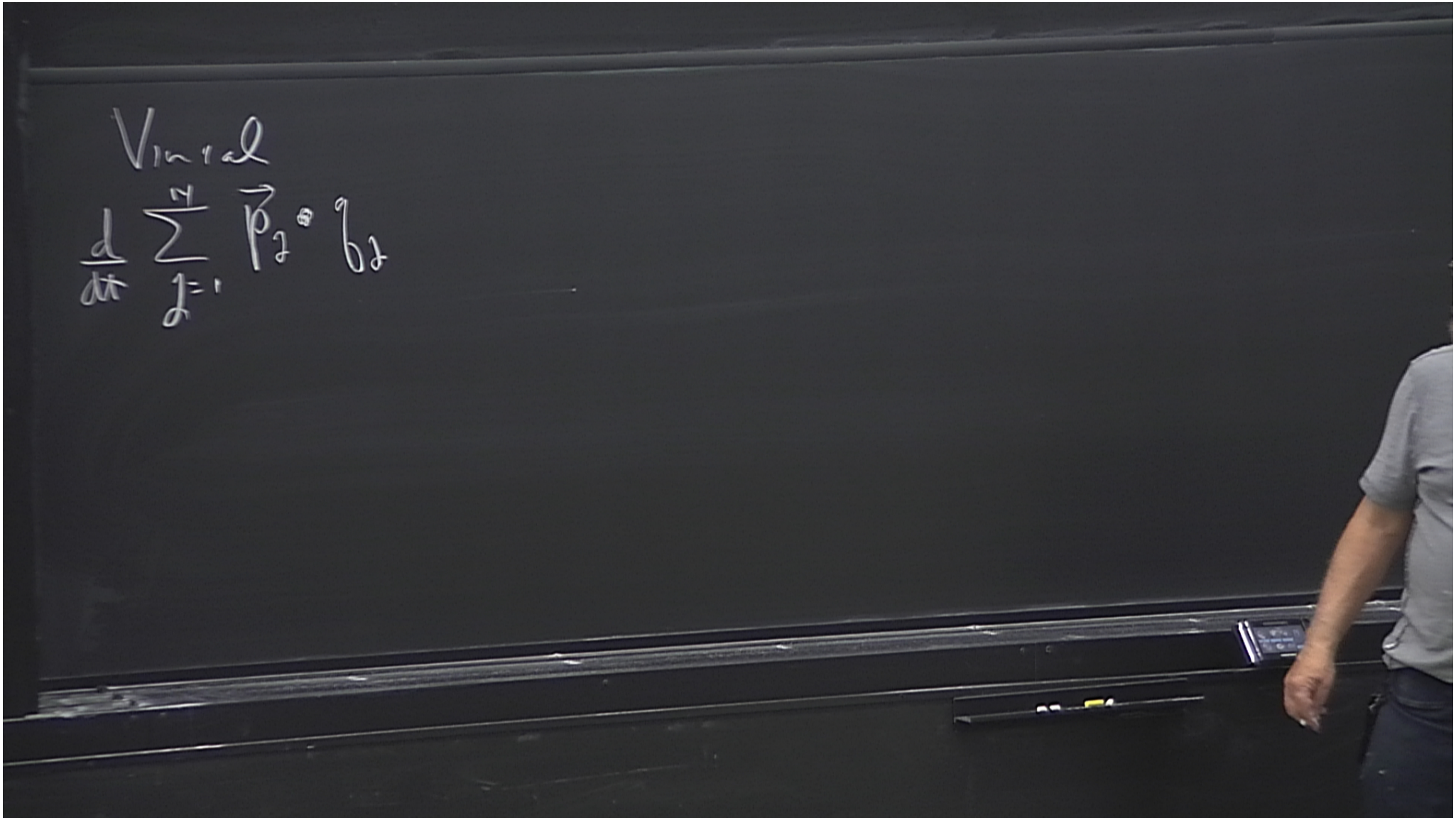

$N(p) = \frac{1}{e^{\beta(\epsilon_p - \mu)} - 1}$
anyplace
always
μ = particle
conserved.
$f(p)$
$-\frac{1}{k} S =$

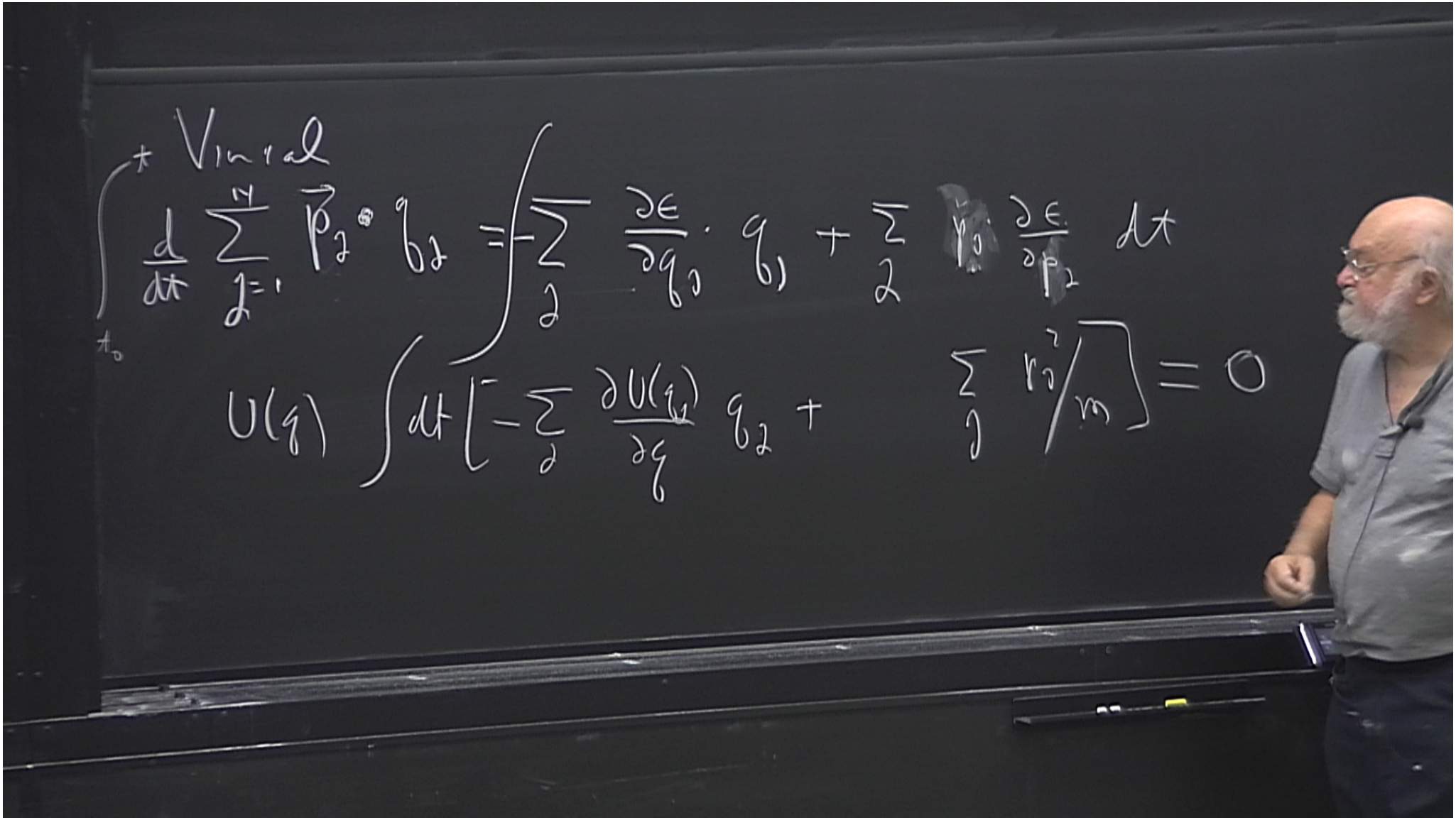

local eq
$f(p,r,t) = e^{-\frac{1}{k_B T(r,t)}\left(\frac{p^2}{2m} - p\cdot v(r,t) + \frac{1}{2} m v^2(r,t) - \mu(r,t)\right)}$

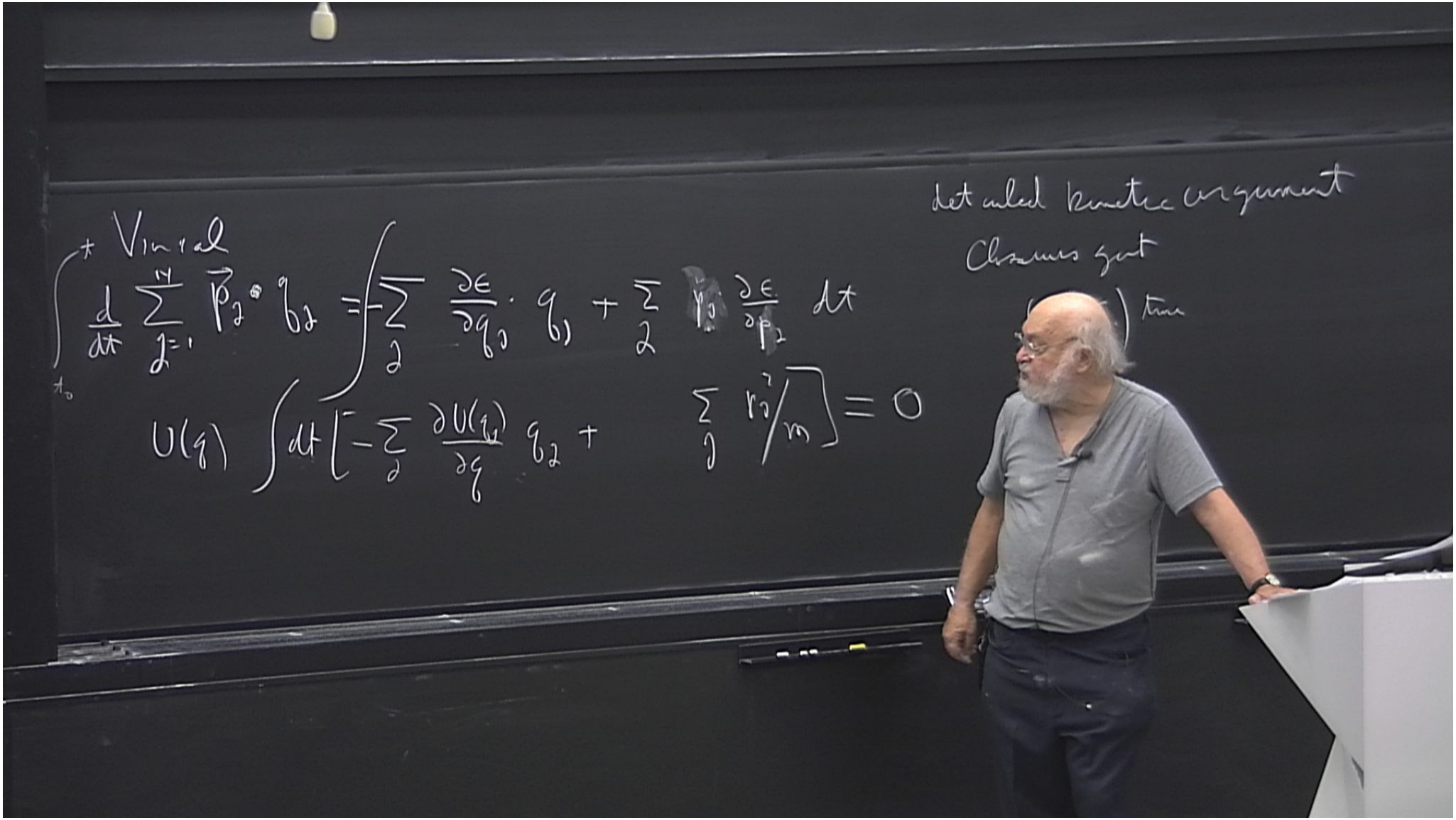

dinamical case
$f(p,r,t) = e^{-\frac{1}{k_B T(r,t)}\left(\frac{p^2}{2m} - p\cdot v(r,t) + \frac{1}{2} m v^2(r,t) - \mu(r,t)\right)}$
$\frac{\partial g(r,t)}{\partial t} = \int dp\, f(p,r,t)\vec{p} = -\nabla \cdot T$


classical case
$f(p,r,t) = e^{-\frac{1}{k_B T(r,t)}\left(\frac{p^2}{2m} - p\cdot v(r,t) + \frac{1}{2} m v^2(r,t) - \mu(r,t)\right)}$
$\frac{\partial g(r,t)}{\partial t}$
$\int f(p,r,t)\,\vec{p} = -\nabla\cdot T$
T_{jk}
$p(r,t)\,\delta_{jk} + m v_j v_k - \eta\left(\frac{\partial v_j}{\partial r_k}\right.$


well-defined momentum
fermi
energy
scat rate
classical case


Virial
$\frac{d}{dt}\sum_{j=1}^{N} \vec{p}_j \cdot \vec{q}_j$


$\int_{t_0}^{t} V_{\text{virial}}$
$\frac{d}{dt}\sum_{j=1}^{N} \vec{p}_j \cdot q_j = -\sum_j \frac{\partial \epsilon}{\partial q_j} \cdot q_j + \sum_j \dot{p}_j \frac{\partial \epsilon}{\partial p_j}\, dt$
$U(q) \int dt \left[-\sum_j \frac{\partial U(q)}{\partial q} q_j + \sum_j r_j^2 / m \right] = 0$


detailed kinetic argument
Clausius gave