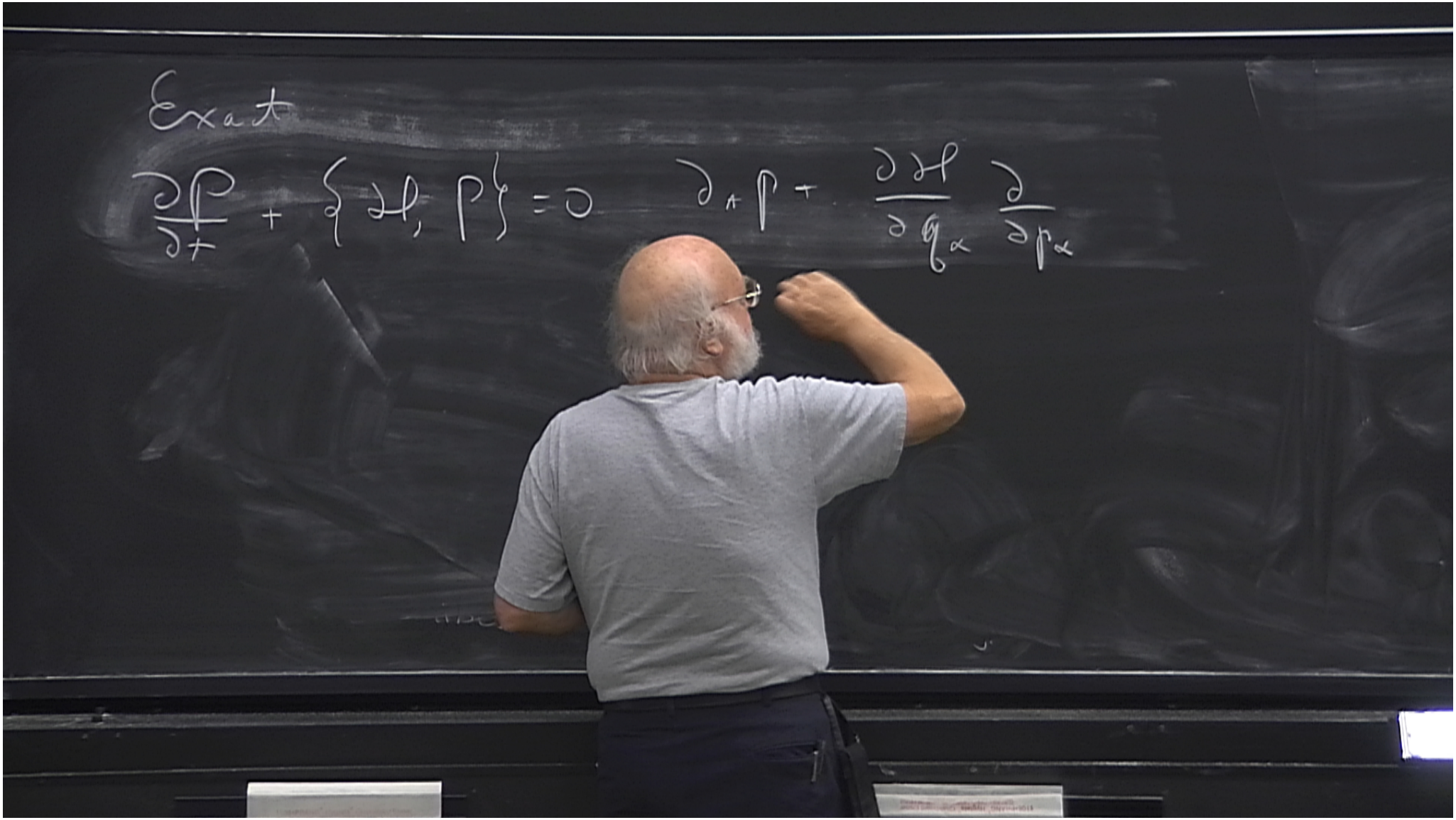
Title: Statistical Mechanics - Lecture 9

Date: Oct 14, 2011 10:30 AM

URL: http://pirsa.org/11100036

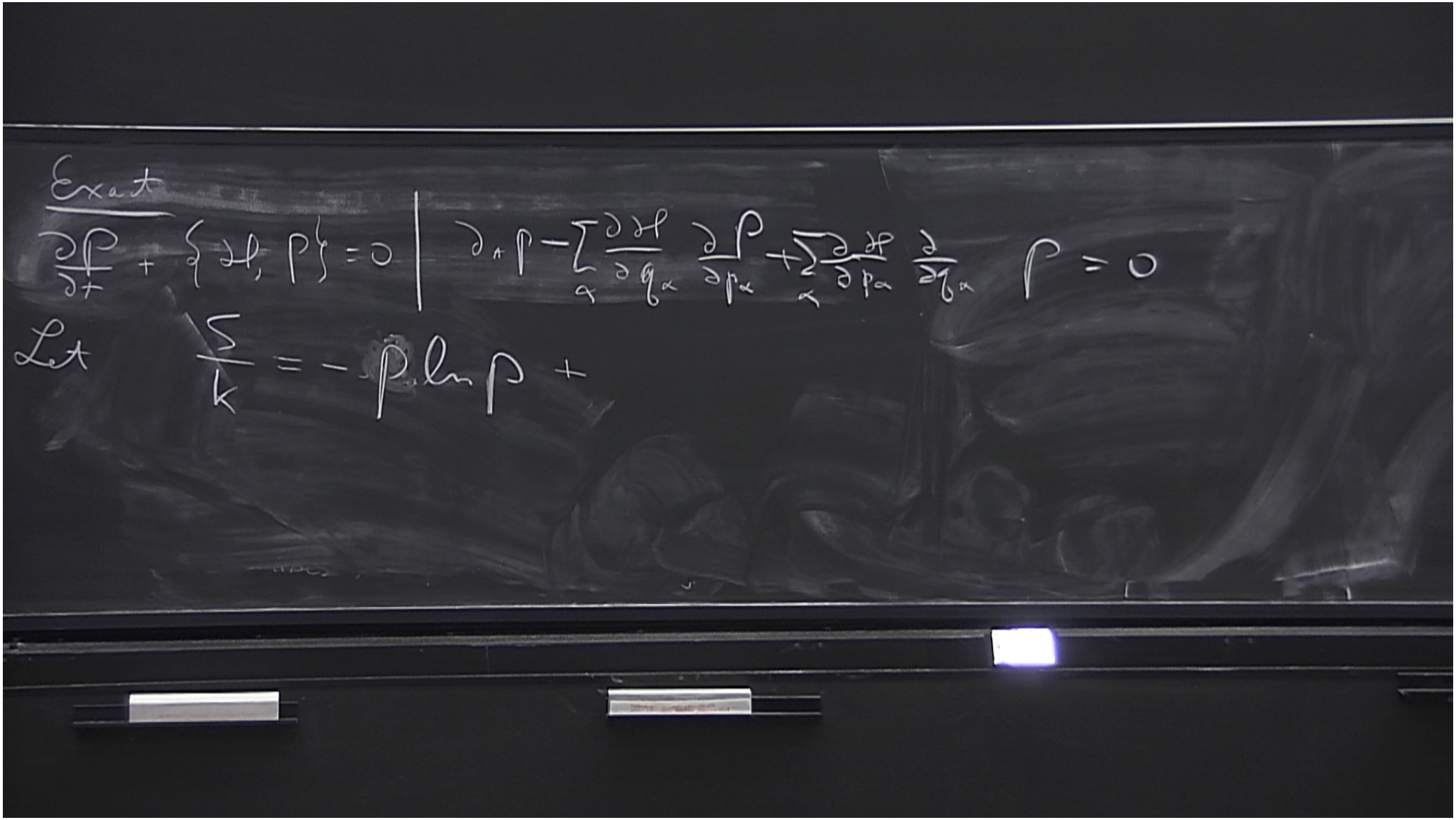
Abstract:

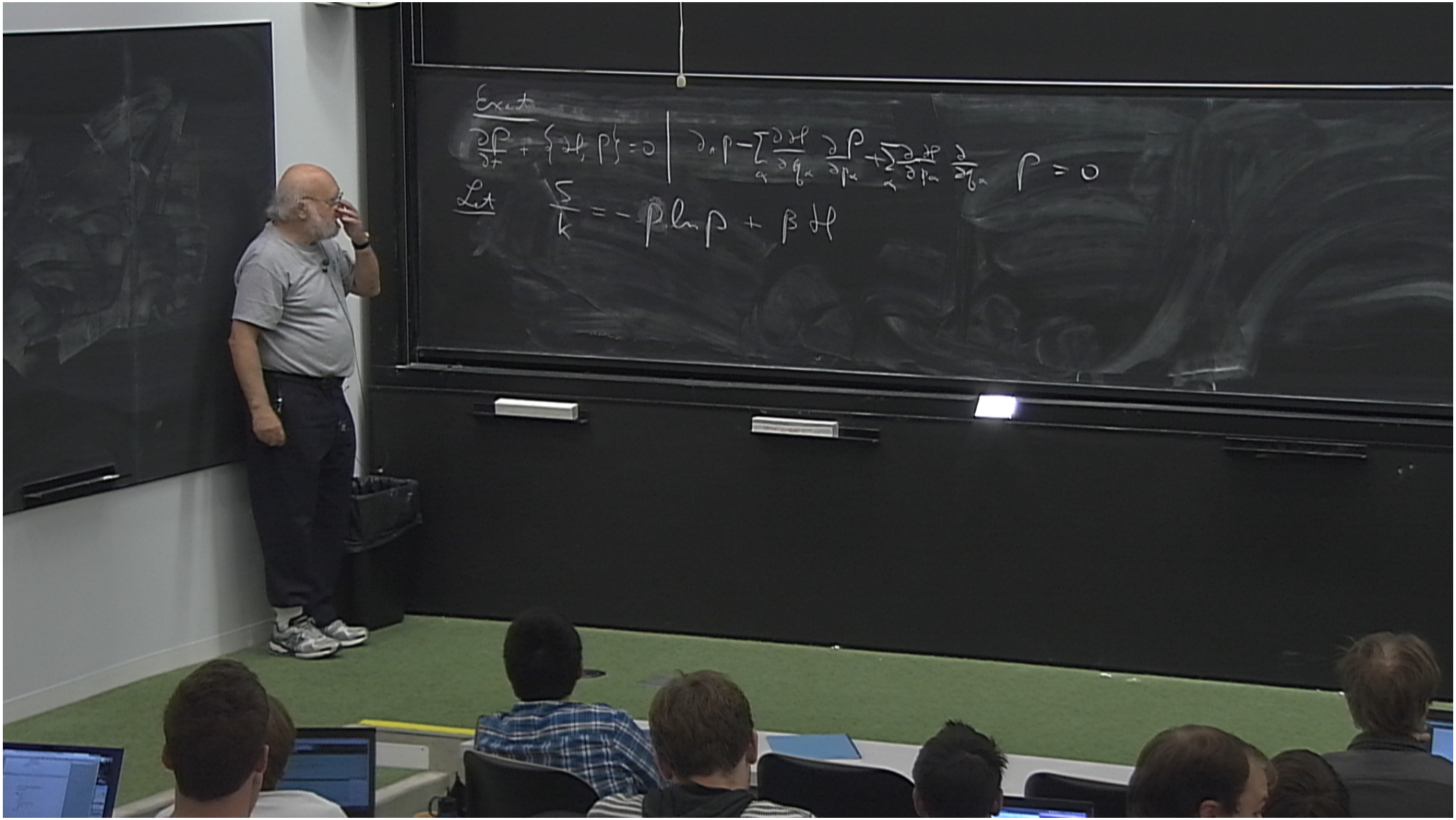


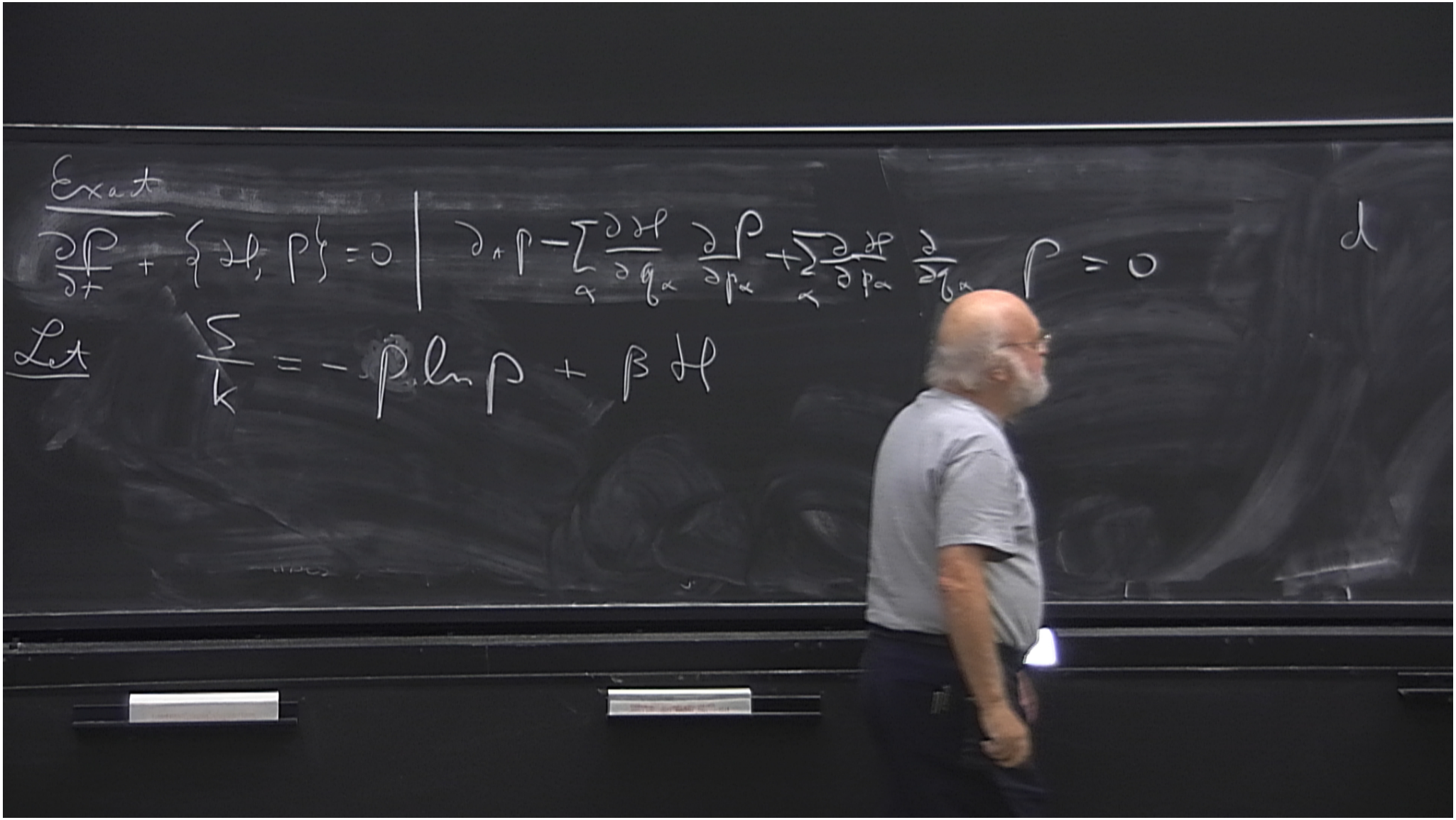

Exact
$\frac{\partial \rho}{\partial t} + \{H, \rho\} = 0$
$\partial_t \rho + \frac{\partial H}{\partial q_\alpha} \frac{\partial}{\partial p_\alpha}$

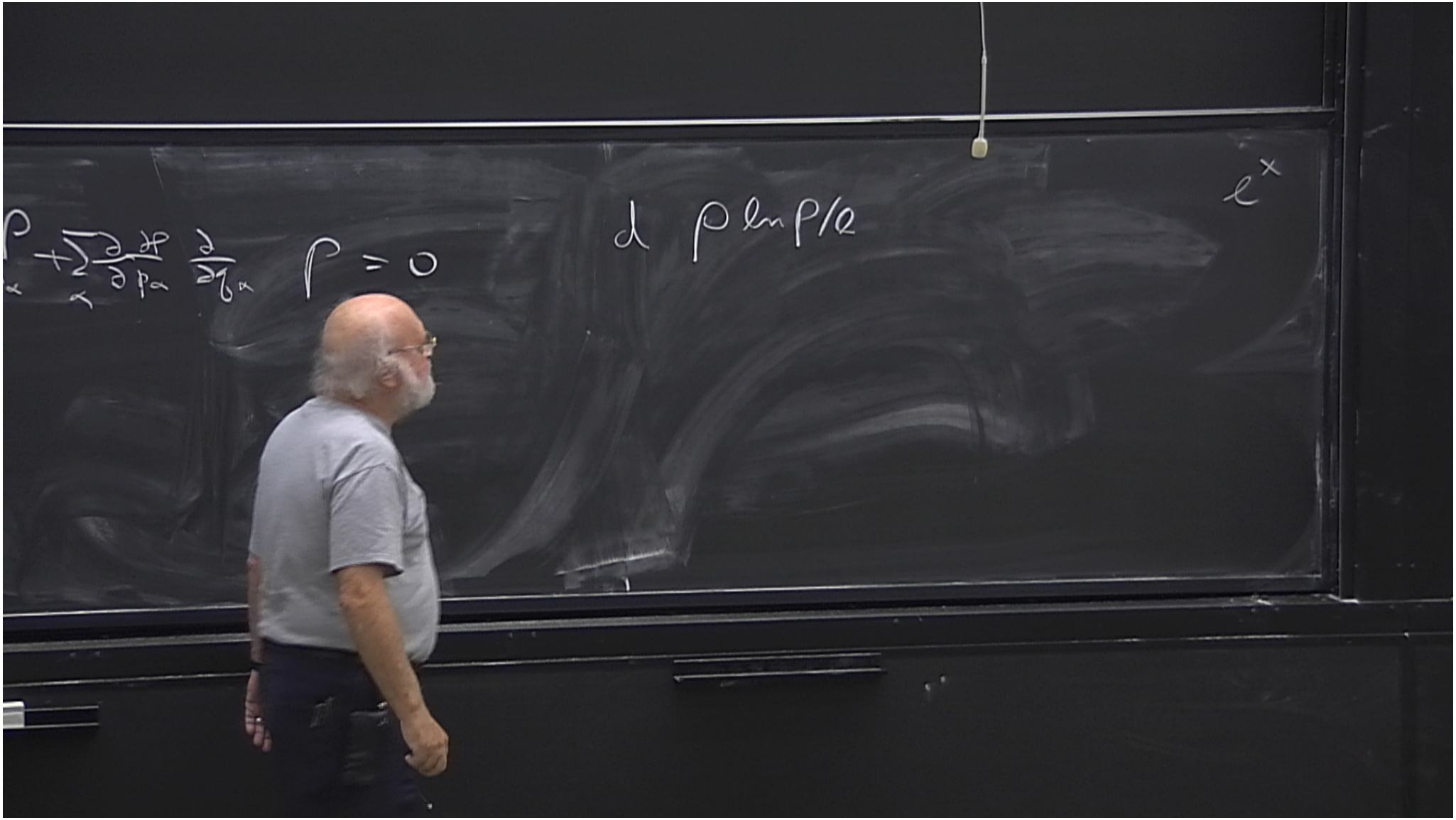
Exact

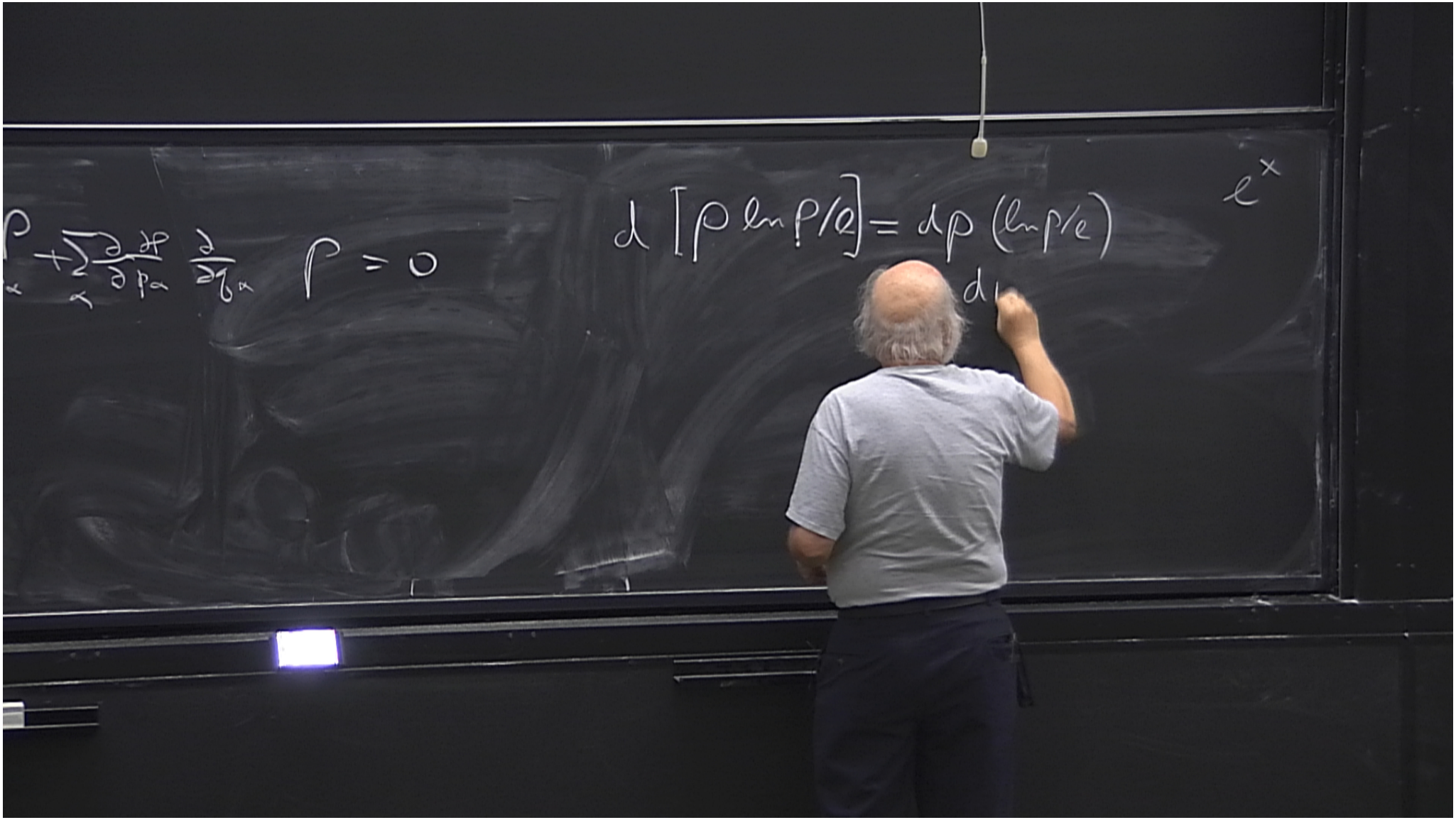
$$\frac{\partial \rho}{\partial t} + \{H, \rho\} = 0 \quad \Big| \quad \partial_t \rho - \sum_k \frac{\partial \rho}{\partial x_k}\frac{\partial H}{\partial p_k} + \sum_k \frac{\partial \rho}{\partial p_k}\frac{\partial H}{\partial x_k} = 0$$

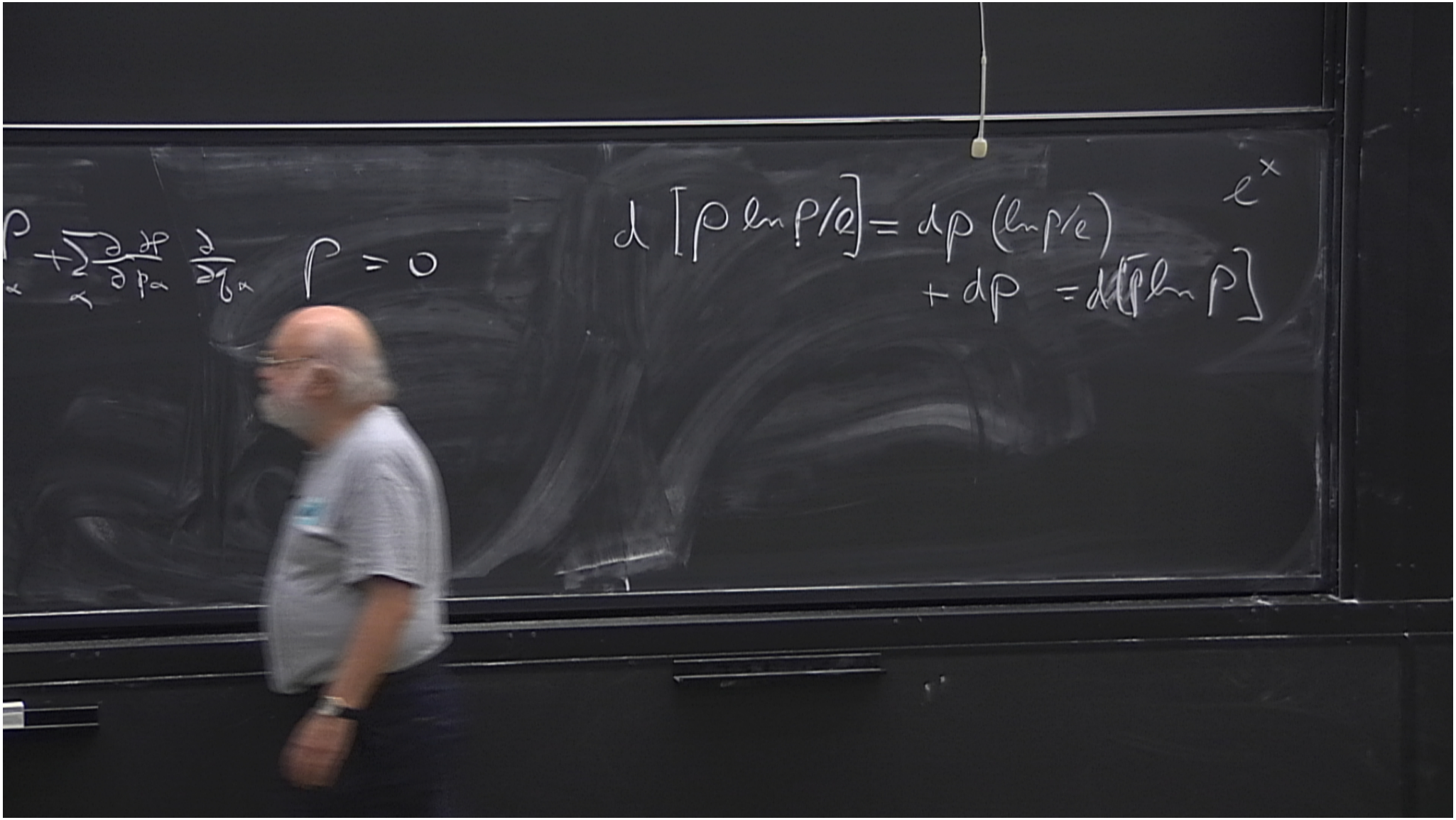

Exact
$\frac{\partial \rho}{\partial t} + \{H, \rho\} = 0$
$\partial_t \rho - \sum_\alpha \frac{\partial H}{\partial q_\alpha} \frac{\partial \rho}{\partial p_\alpha} + \sum_\alpha \frac{\partial H}{\partial p_\alpha} \frac{\partial}{\partial q_\alpha} \rho = 0$
Let $\frac{S}{k} = -\int \rho \ln \rho +$

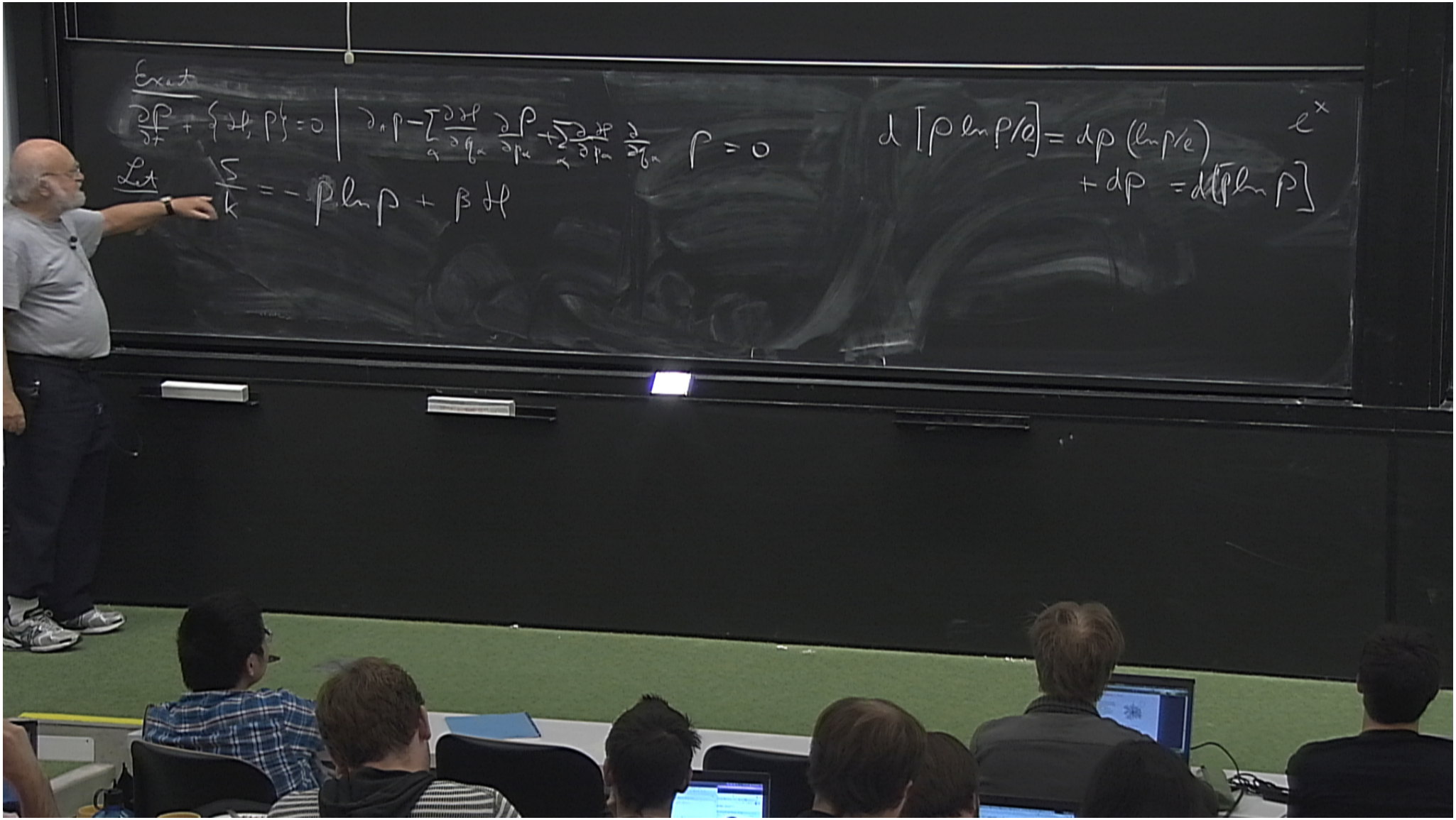

Exact
Let

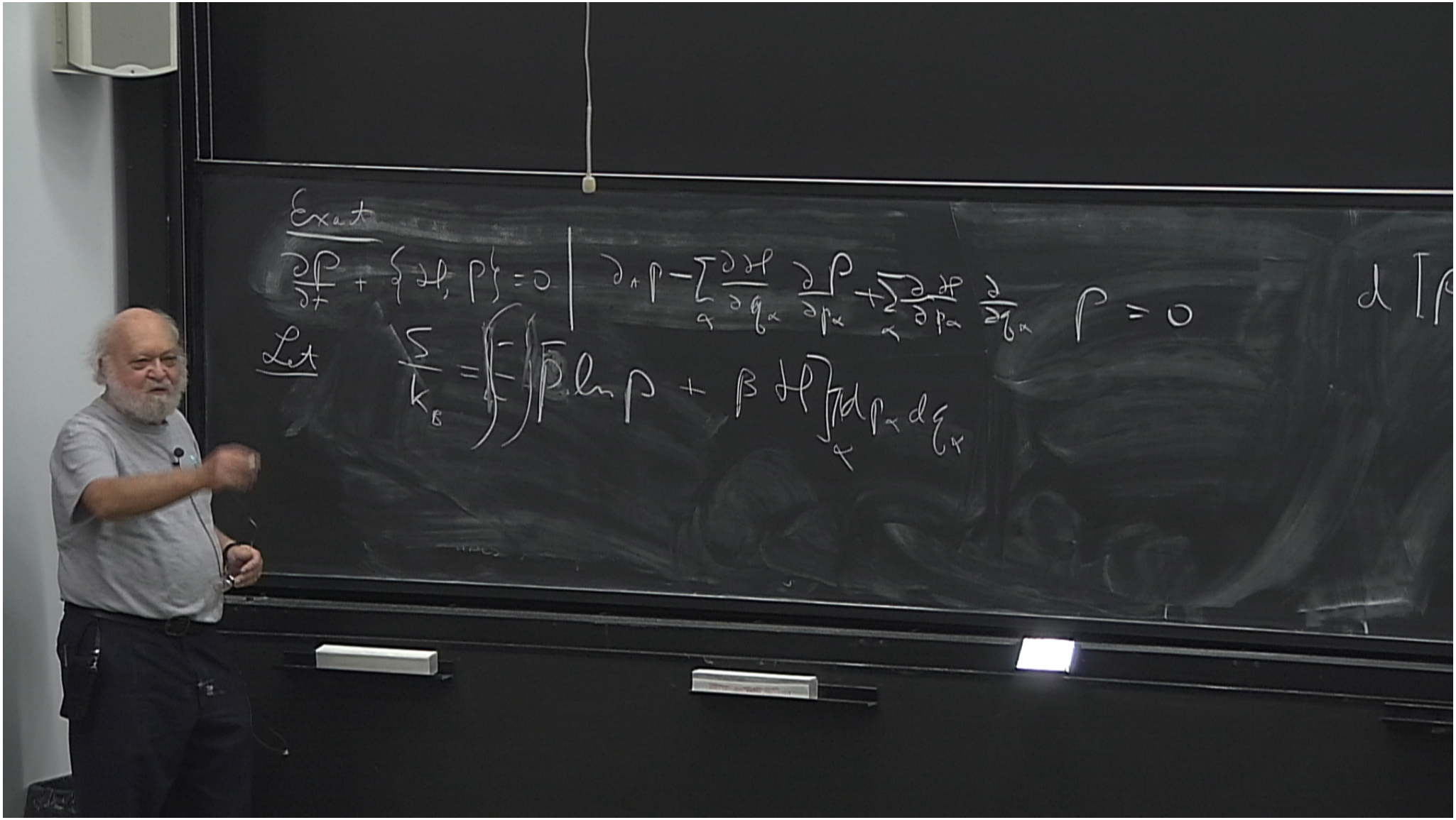

Exact
$\frac{\partial \rho}{\partial t} + \{H, \rho\} = 0$
$\partial_t \rho - \sum_\alpha \frac{\partial H}{\partial q_\alpha}\frac{\partial \rho}{\partial p_\alpha} + \sum_\alpha \frac{\partial H}{\partial p_\alpha}\frac{\partial \rho}{\partial q_\alpha} = 0$
Let
$\frac{S}{k} = -\rho \ln \rho + \beta H$

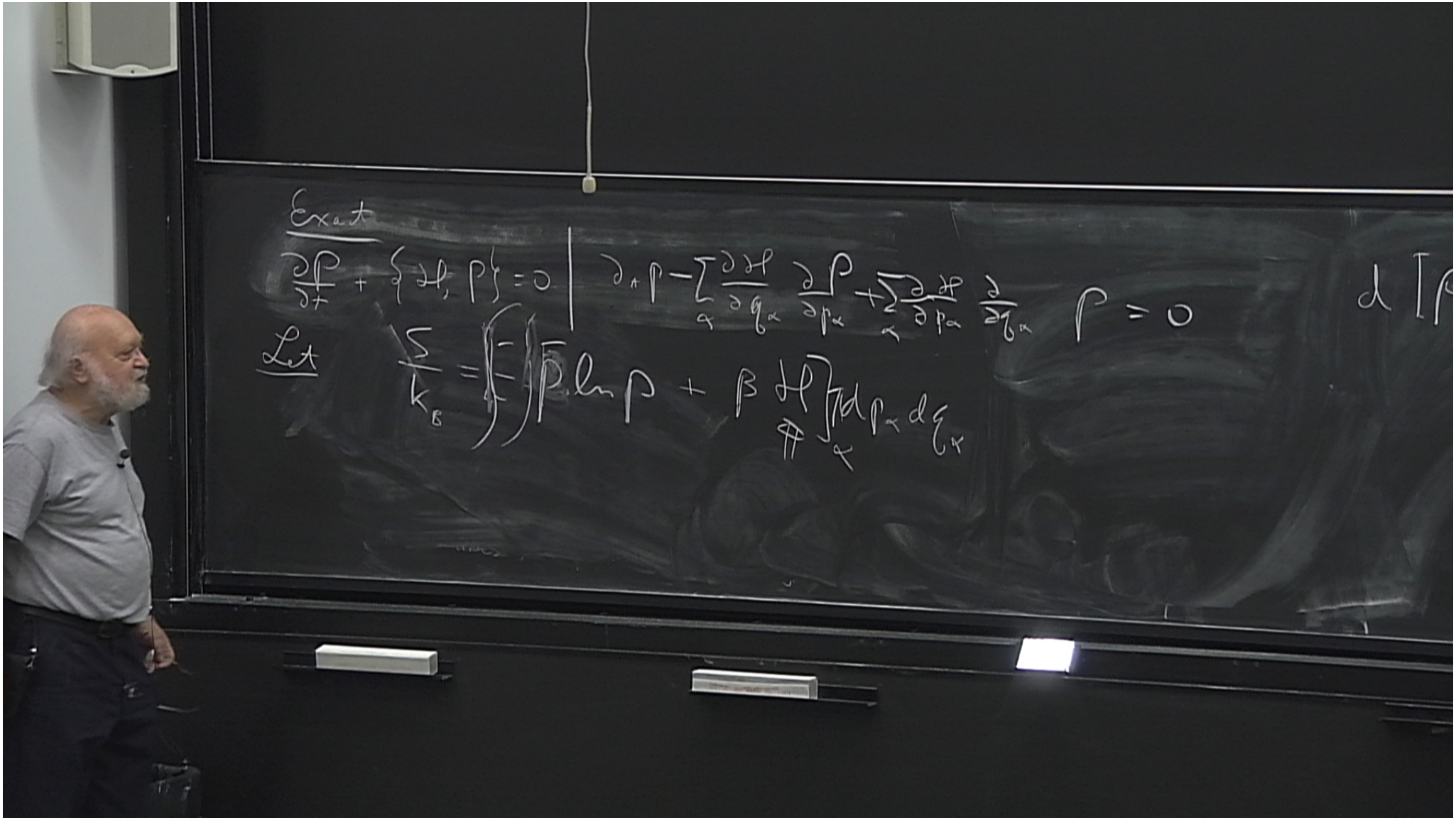

ρ
$\rho = 0$
$d\ \rho \ln \rho / a$
e^x

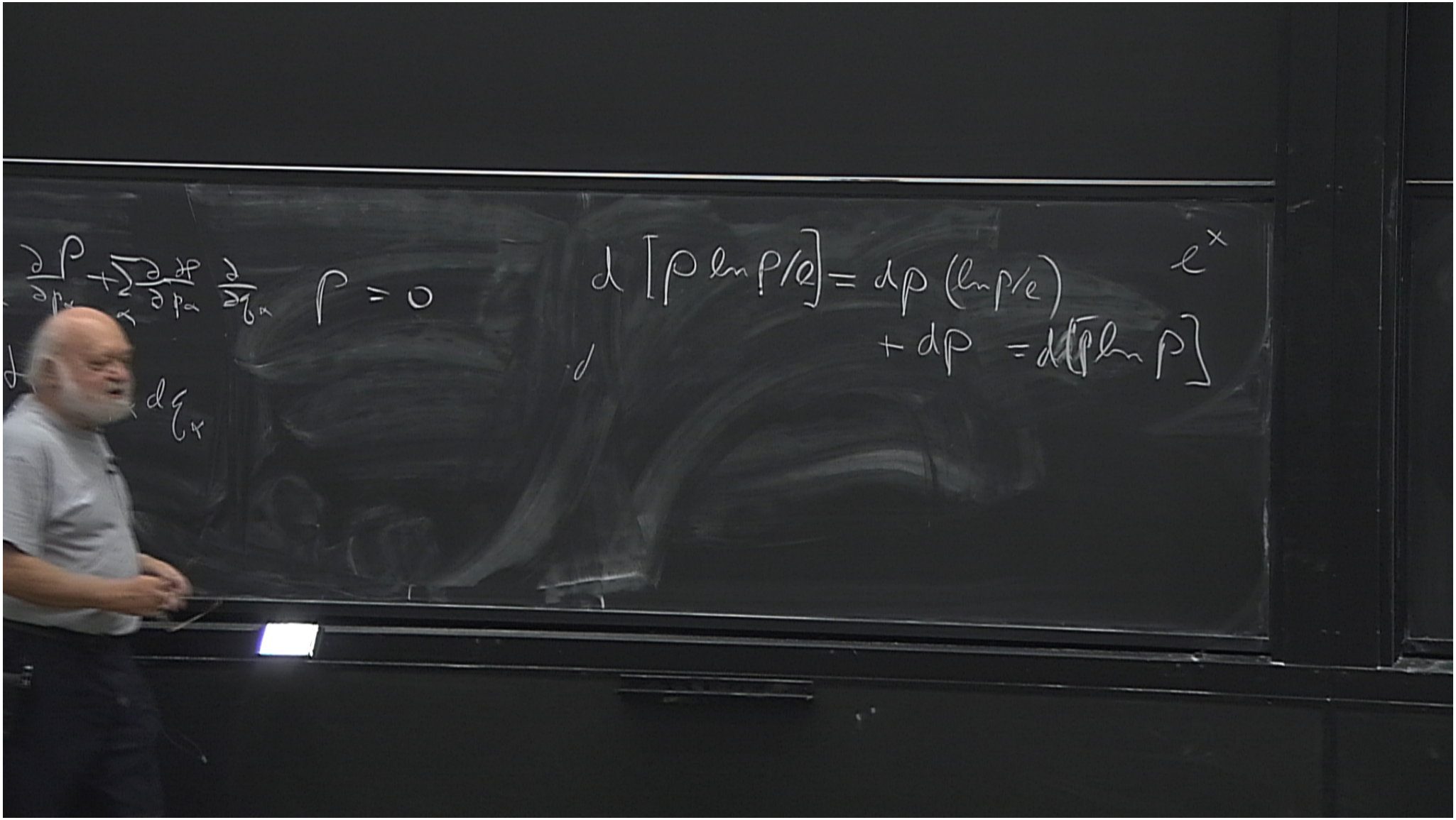

$\rho = 0$
$d[\rho \ln \rho/a] = d\rho (\ln \rho/a)$
e^x

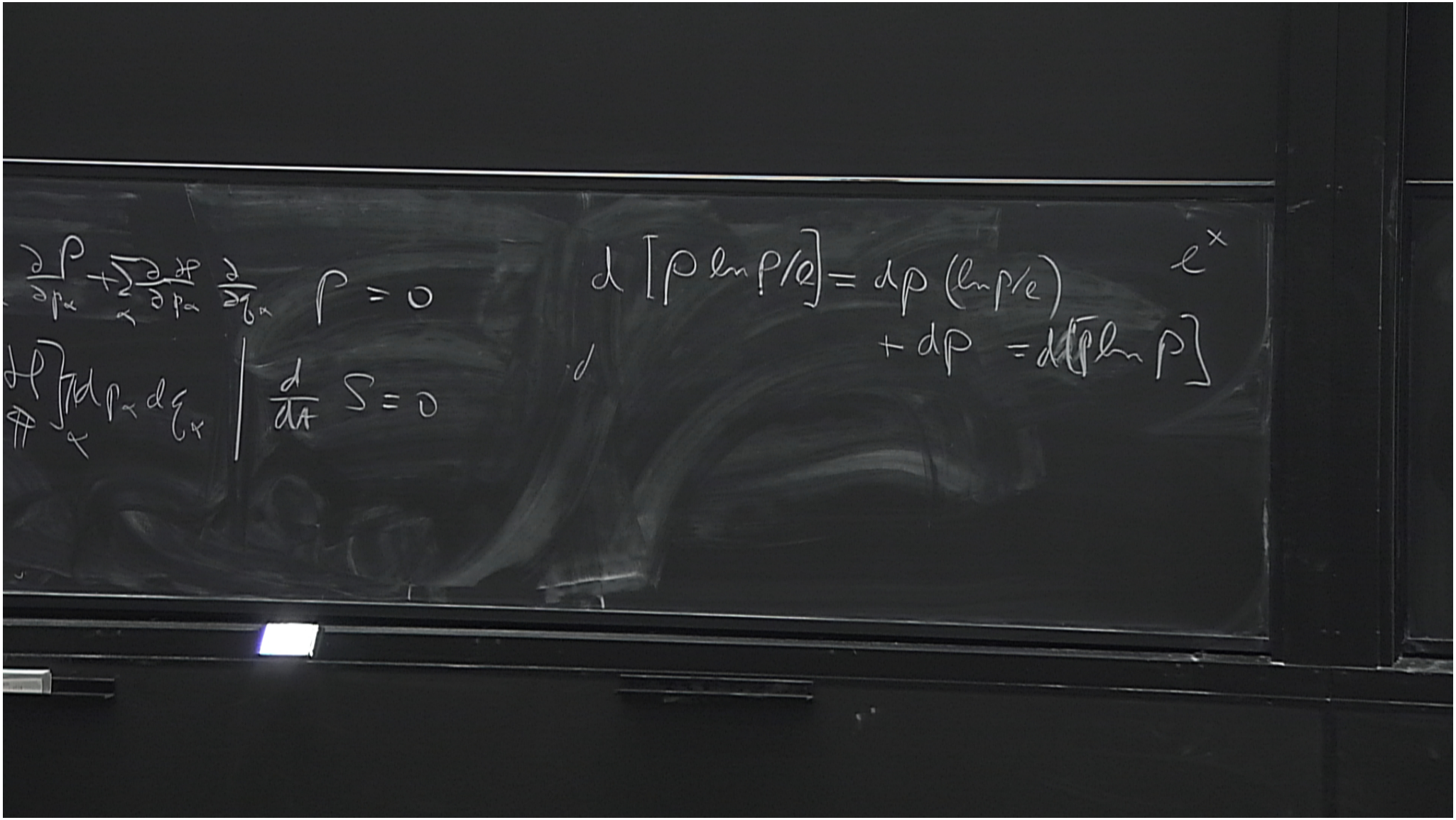

$\frac{\partial \rho}{\partial t} + \sum_\alpha \frac{\partial H}{\partial p_\alpha} \frac{\partial \rho}{\partial q_\alpha} - \frac{\partial \rho}{\partial t} = 0$
$d[\rho \ln \rho/a] = d\rho (\ln \rho/a)$
$+ d\rho = d[\rho \ln \rho]$
e^x

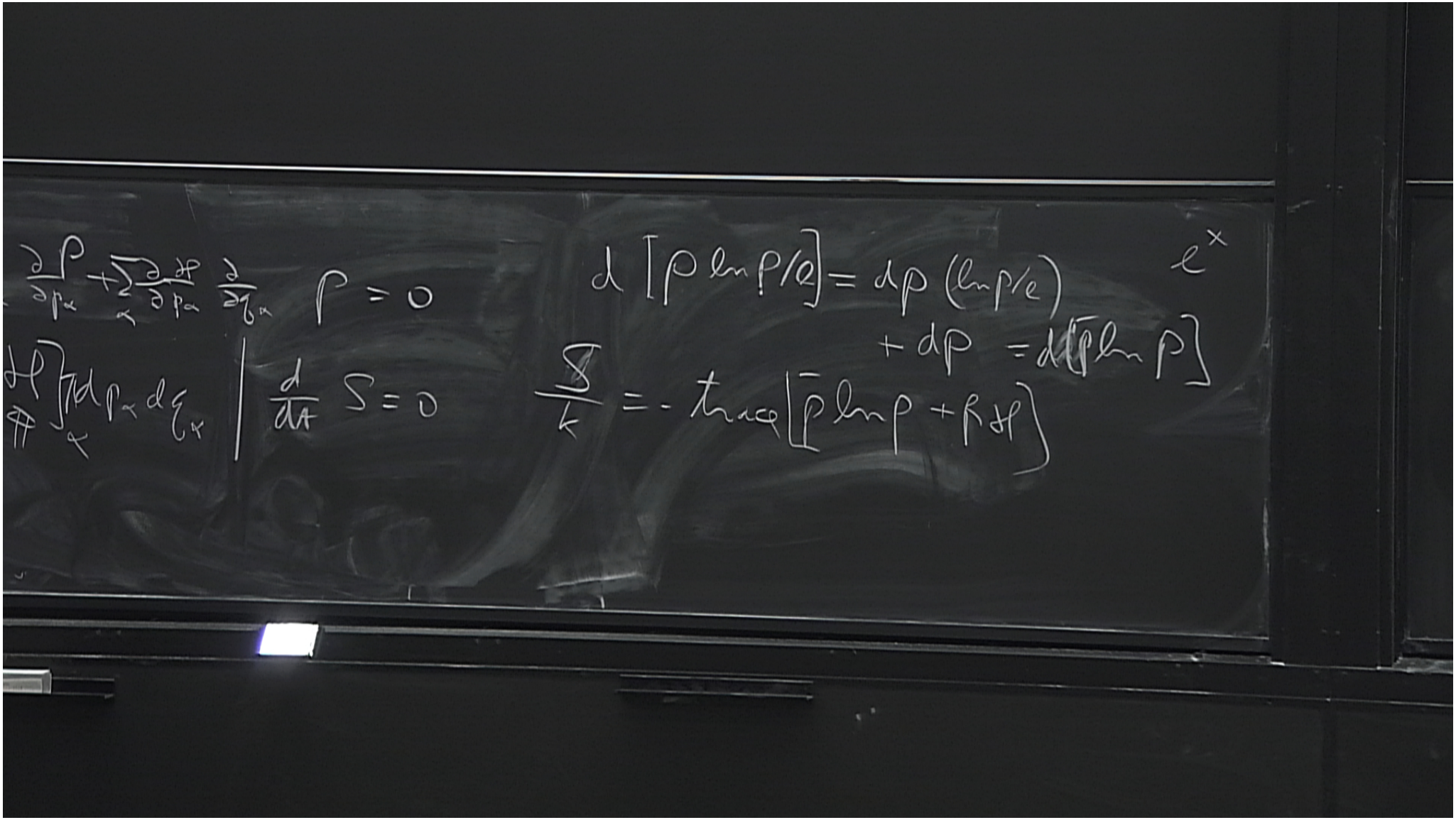

Exact
Let
e^x

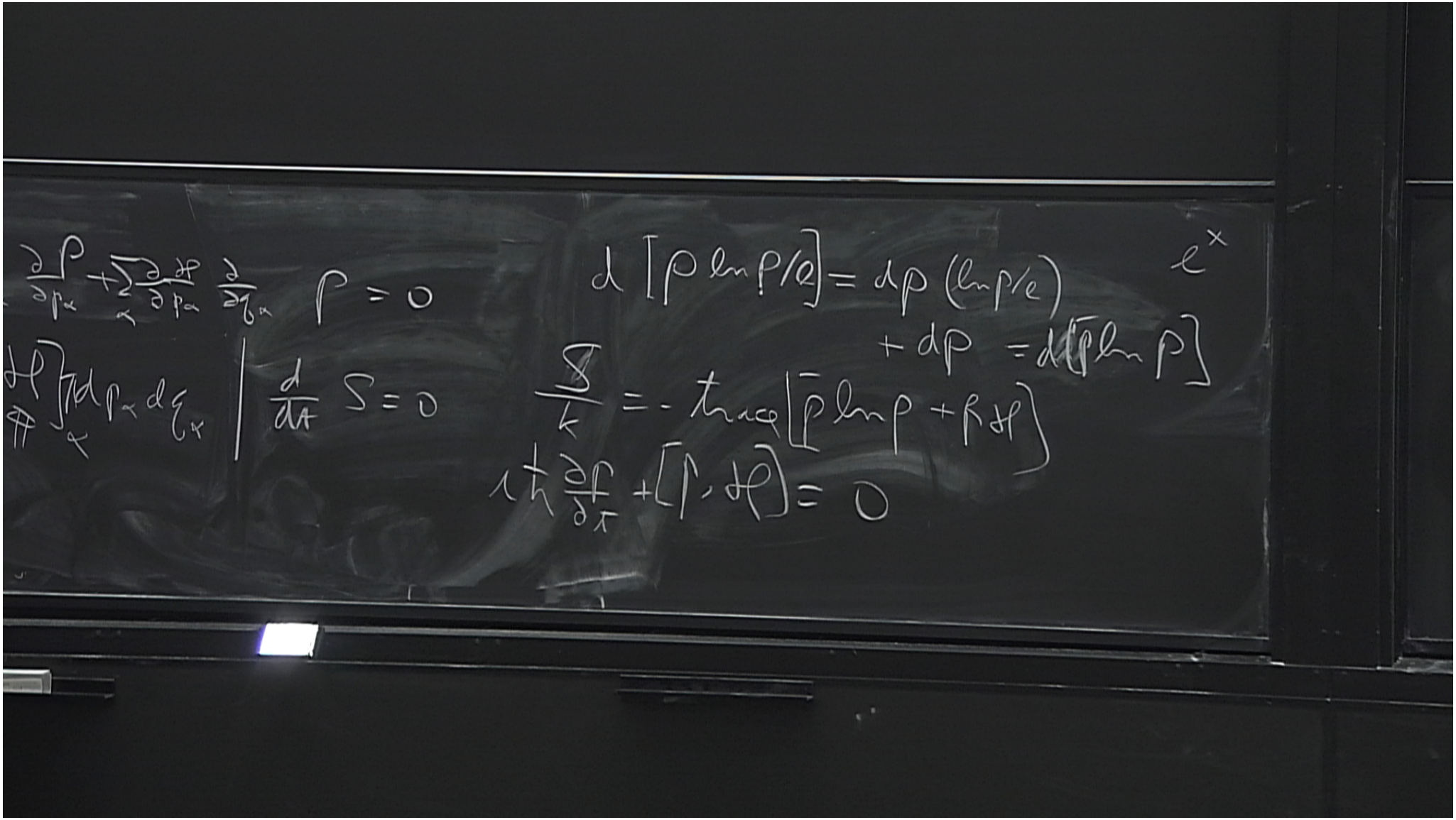

Exact
Let

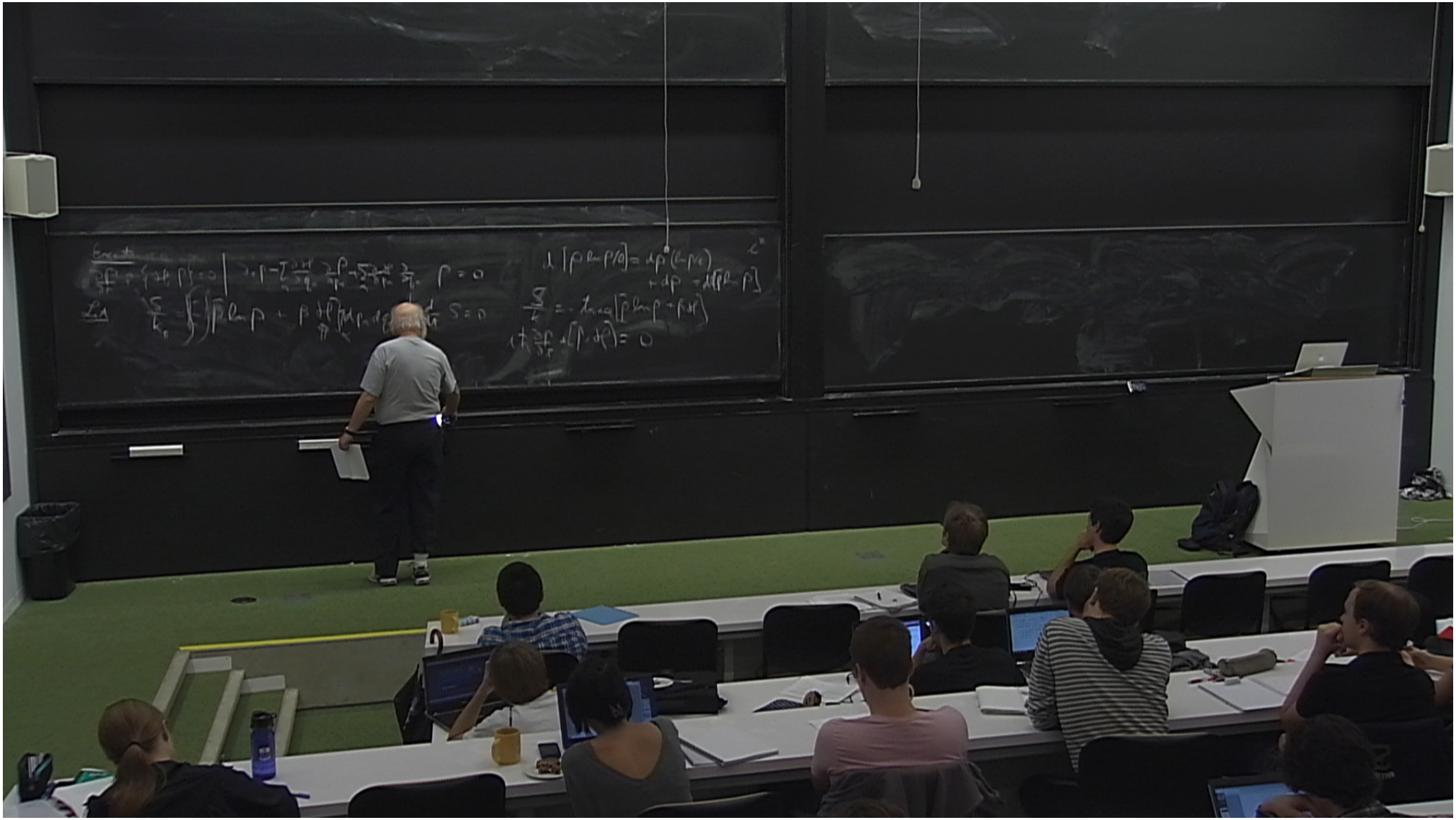

Exact
Let

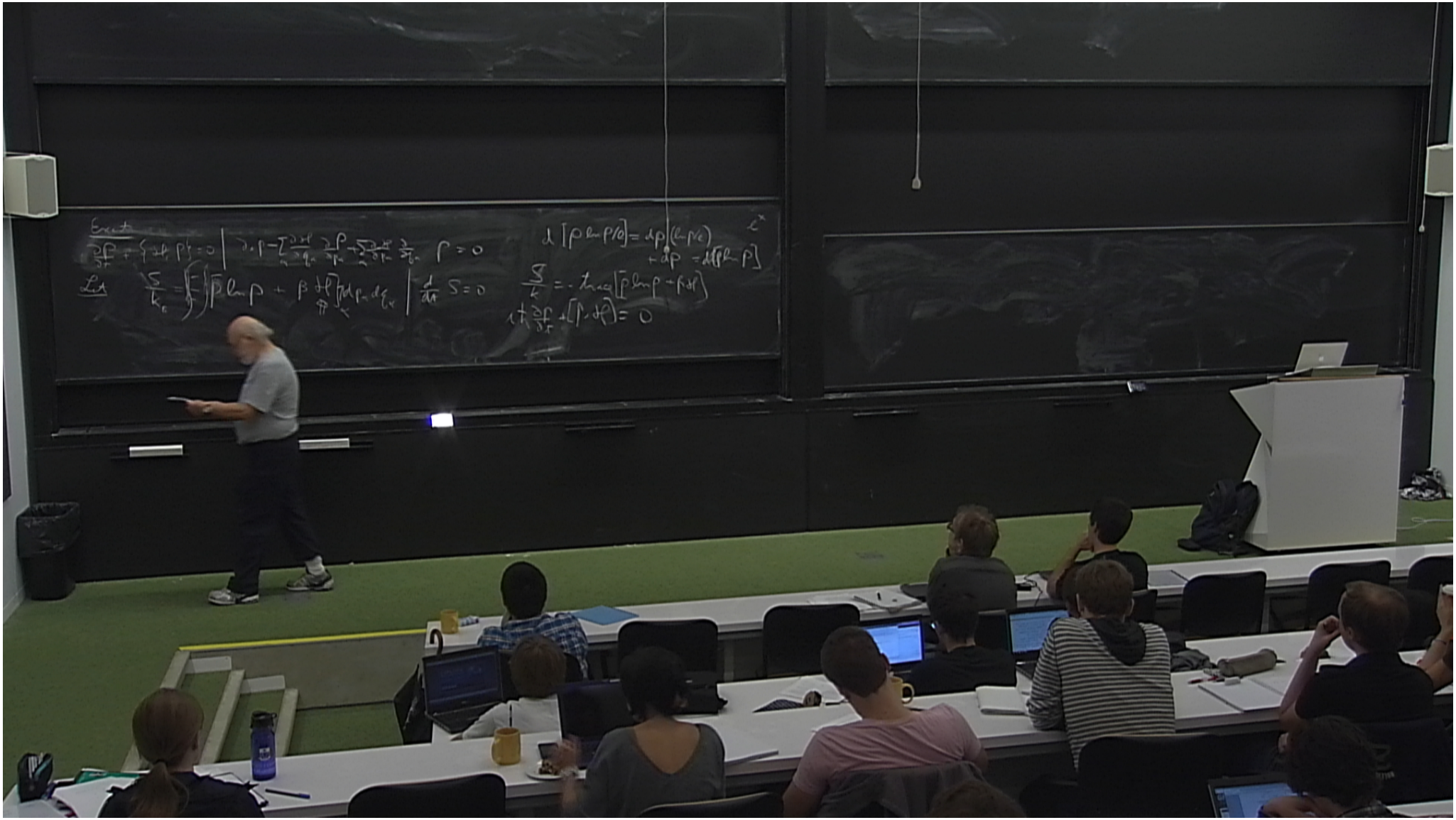

$\rho = 0$
$d[\rho \ln \rho/e] = d\rho\,(\ln \rho/e)$
$+ d\rho = d[\rho \ln \rho]$
e^x

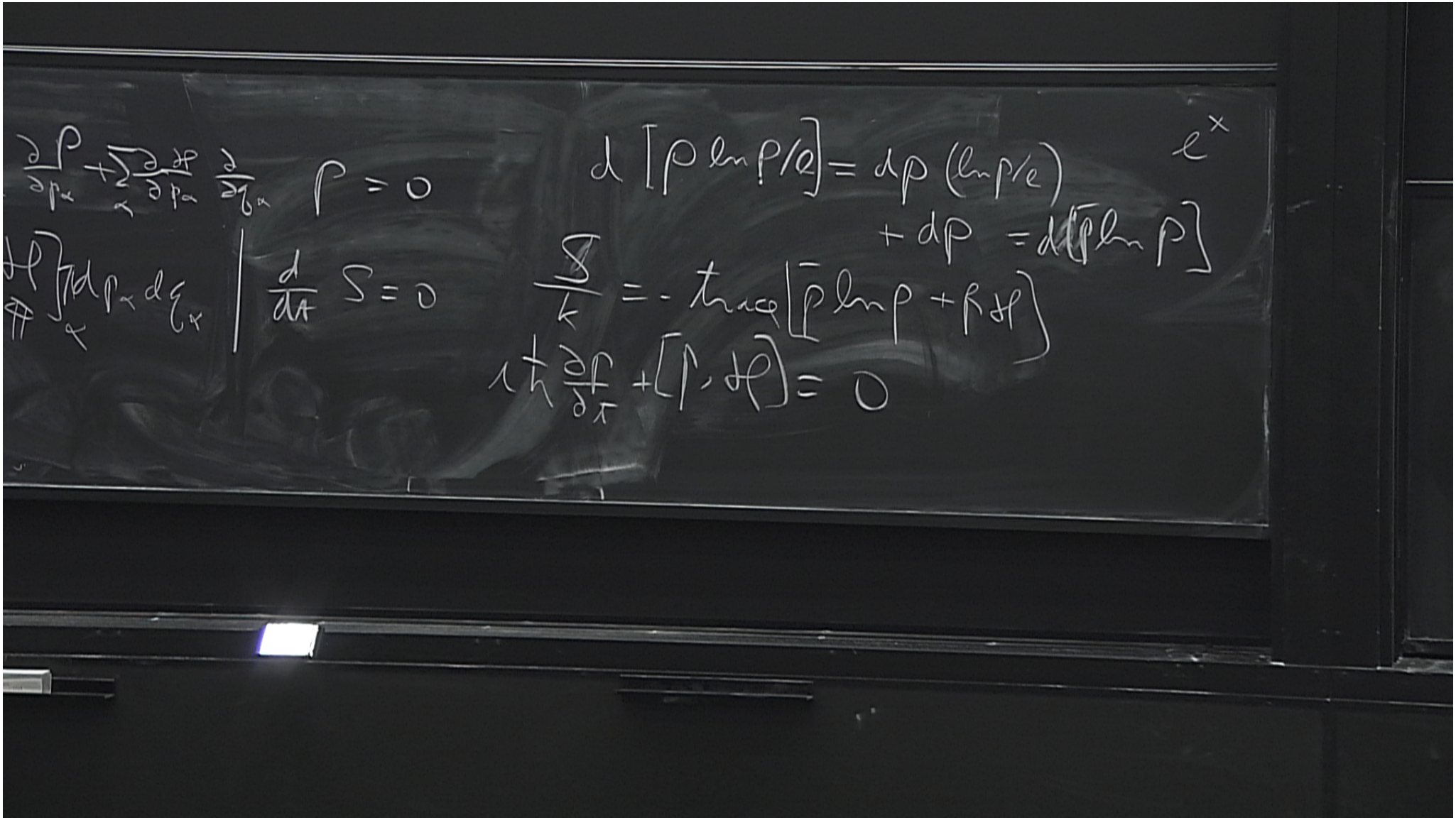

$\frac{\partial \rho}{\partial t} + \sum_\alpha \frac{\partial \rho}{\partial p_\alpha} \frac{\partial}{\partial q_\alpha} \rho = 0$
$d[\rho \ln \rho/e] = dp\,(\ln \rho/e)$
$+ dp = d[\rho \ln \rho]$
e^x
$\frac{d}{dt} S = 0$

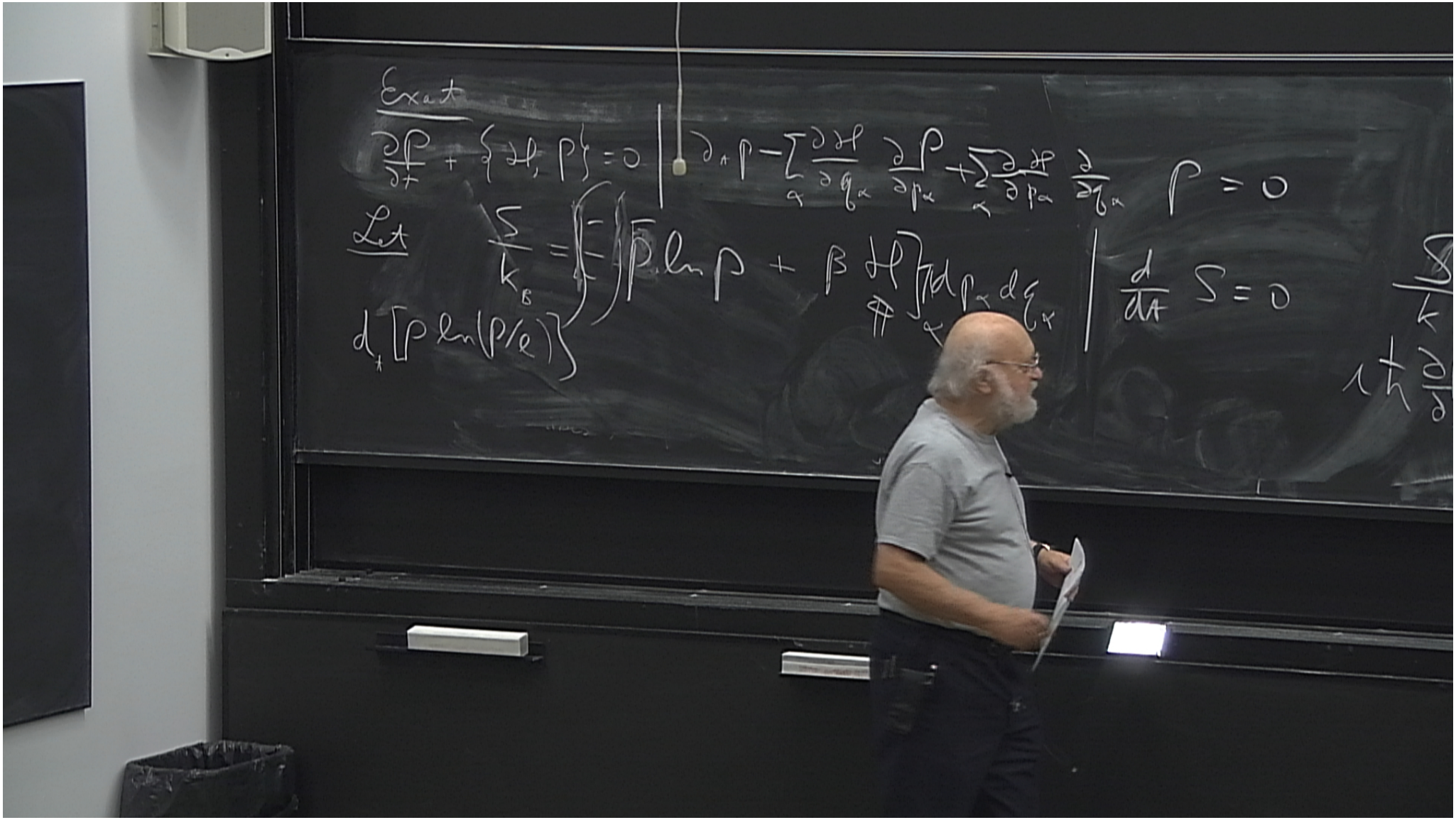

$\frac{\partial \rho}{\partial t} + \sum_\alpha \frac{\partial H}{\partial p_\alpha} \frac{\partial}{\partial q_\alpha} \rho = 0$
$d[\rho \ln \rho/\rho] = d\rho (\ln \rho/\rho)$
$+ d\rho = d[\rho \ln \rho]$
e^x
$\frac{d}{dt} S = 0$
$\frac{S}{k} = -\mathrm{tr}_{\mathrm{aq}}[\bar{\rho} \ln \rho + \rho H]$

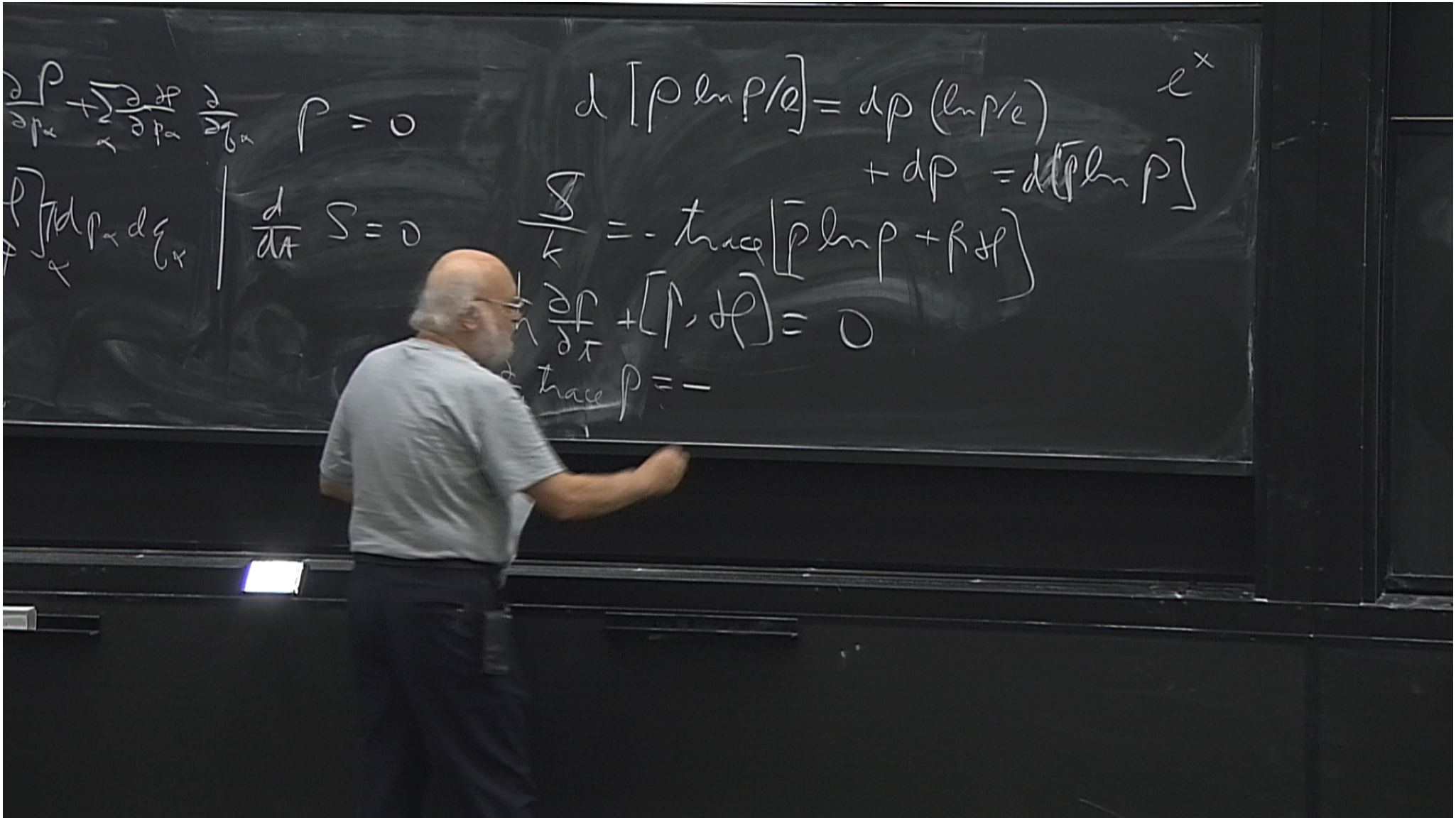

$\frac{\partial \rho}{\partial t} + \sum_\alpha \frac{\partial H}{\partial p_\alpha} \frac{\partial}{\partial q_\alpha} \rho = 0$
$d[\rho \ln \rho/e] = d\rho (\ln \rho/e)$
$+ d\rho = d[\rho \ln \rho]$
e^x
$\frac{d}{dt} S = 0$
$\frac{S}{k} = -\,\mathrm{tr}_{\mathrm{aux}}[\rho \ln \rho + \rho H]$
$i\hbar \frac{\partial \rho}{\partial t} + [\rho, H] = 0$

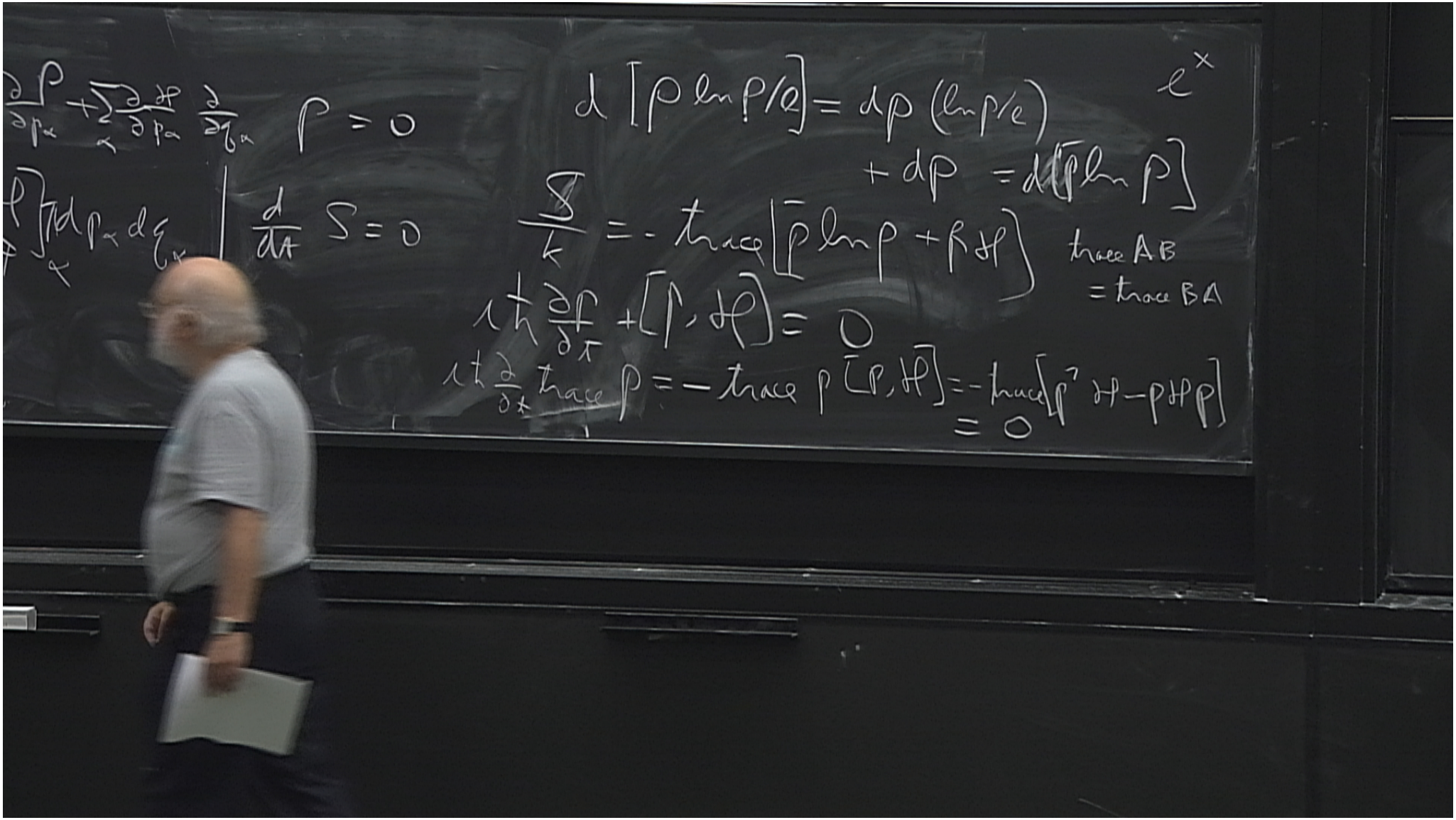


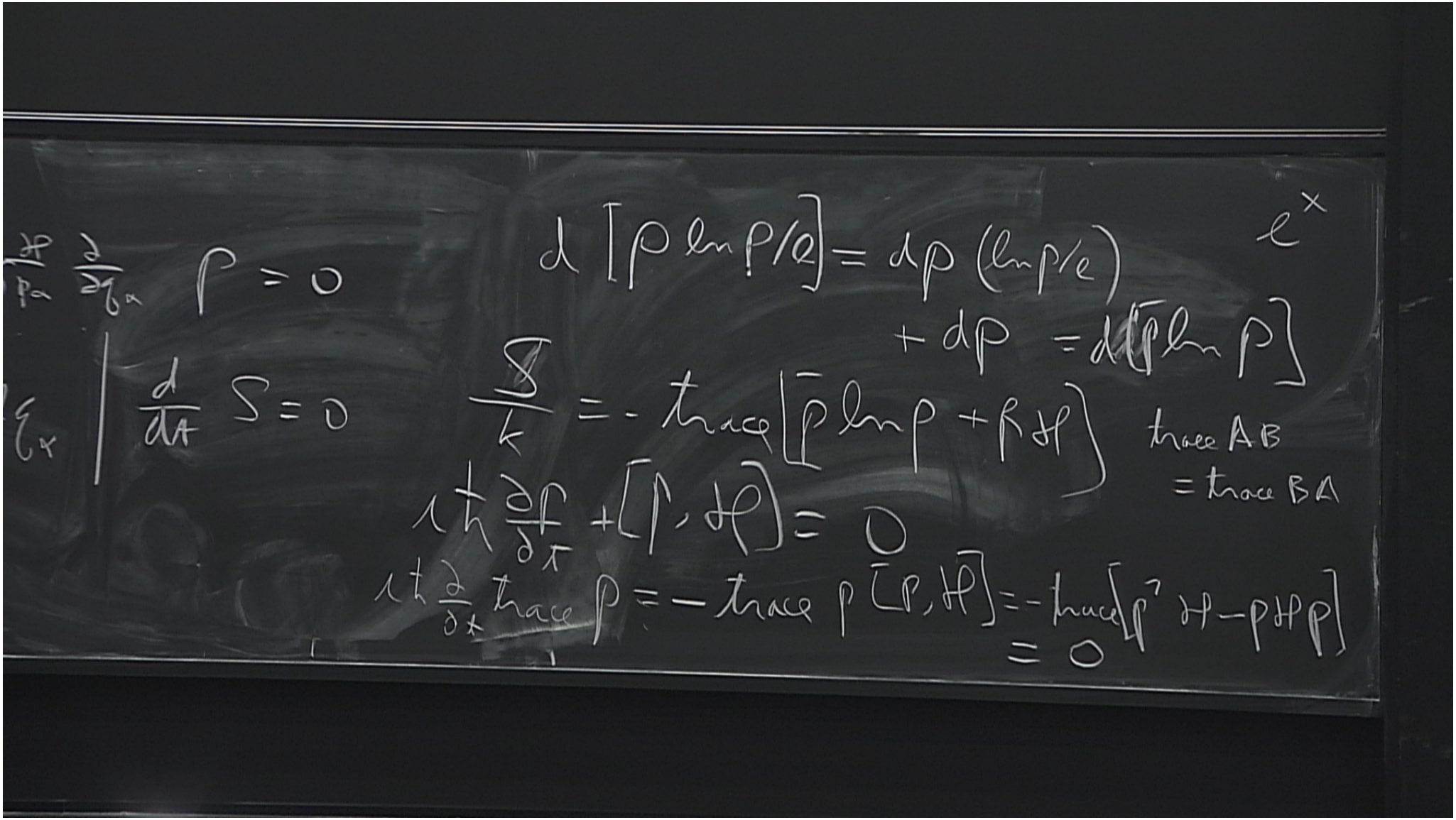


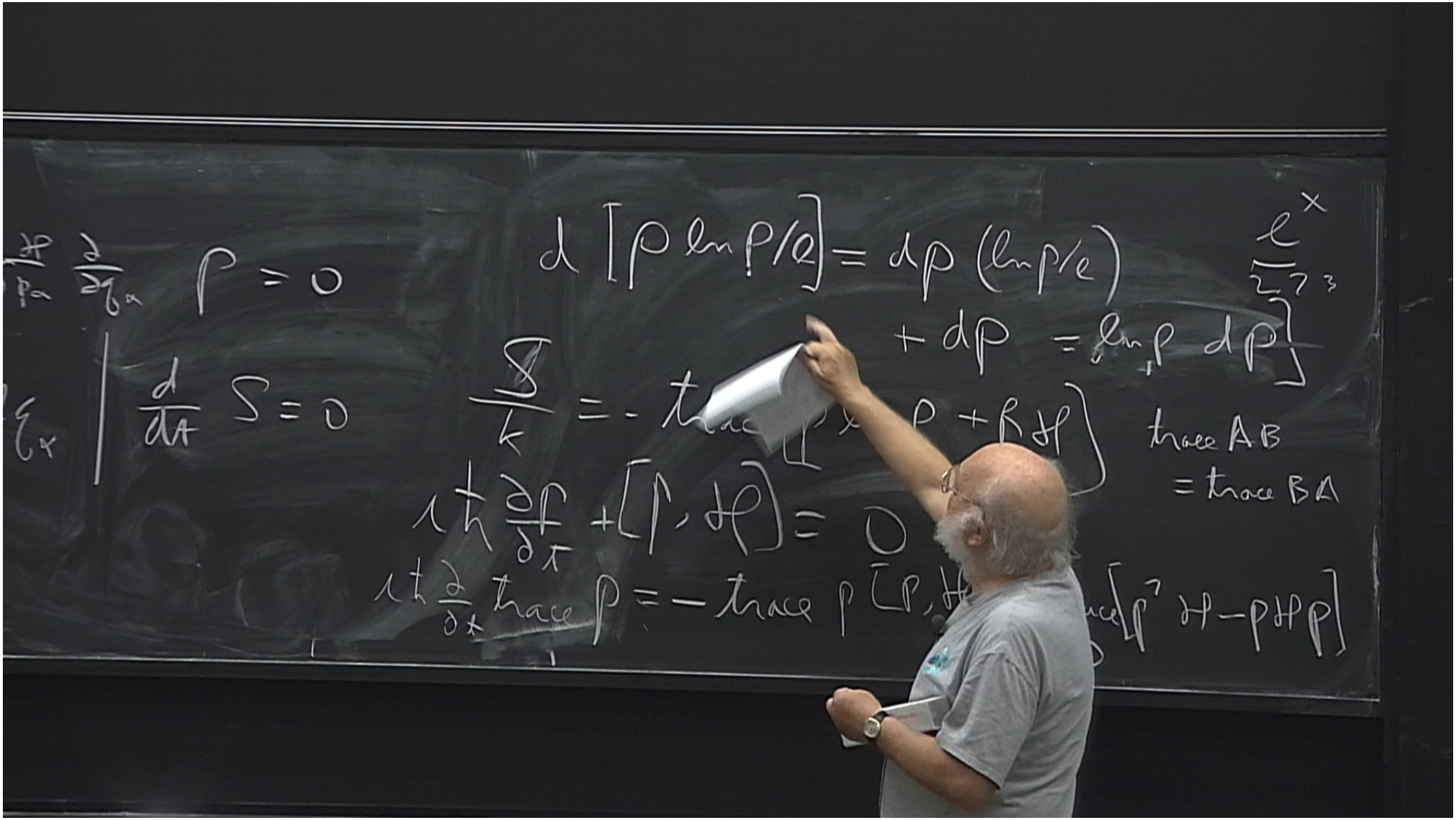

$\frac{\partial \rho}{\partial t} + \sum_\alpha \frac{\partial H}{\partial p_\alpha} \frac{\partial}{\partial q_\alpha} \rho = 0$
$d[\rho \ln \rho/e] = d\rho (\ln \rho/e)$
$+ d\rho = d[\rho \ln \rho]$
e^x
$\frac{d}{dt} S = 0$
$\frac{S}{k} = -\mathrm{tr}_{\mathrm{aq}}[\bar{\rho} \ln \rho + \rho H]$
$i\hbar \frac{\partial \rho}{\partial t} + [\rho, H] = 0$

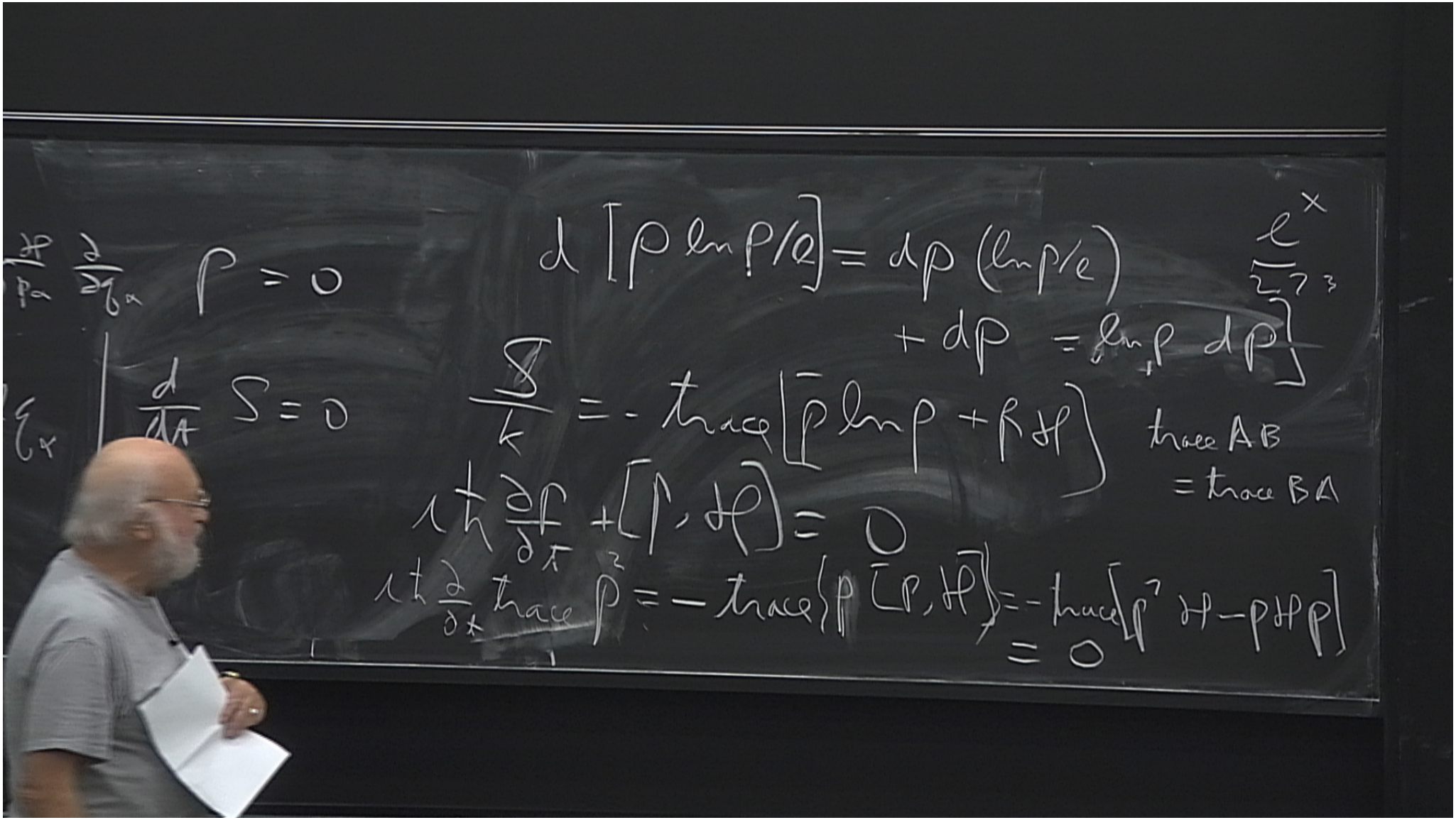

Exact
Let

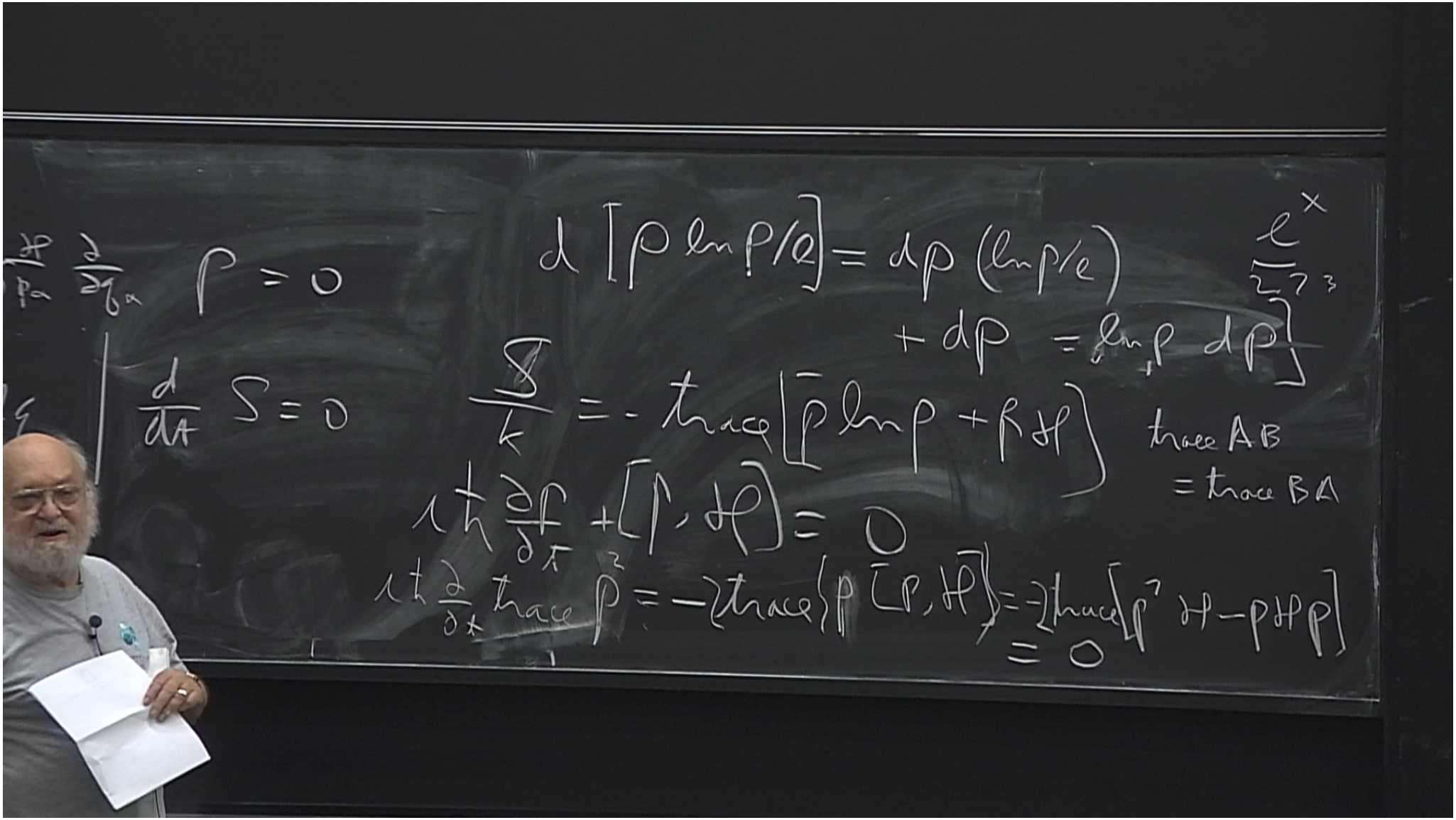

$\frac{\partial \rho}{\partial t} + \sum_\alpha \frac{\partial H}{\partial p_\alpha} \frac{\partial}{\partial q_\alpha} \rho = 0$
$\frac{d}{dt} S = 0$
$d[\rho \ln \rho / e] = d\rho (\ln \rho / e)$
$+ d\rho = d[\rho \ln \rho]$
e^x
$\frac{S}{k} = -\mathrm{trace}[\bar{\rho} \ln \rho + \beta H]$
$\frac{\partial f}{\partial t} + [f, H] = 0$
$\mathrm{trace}\, \rho = -$

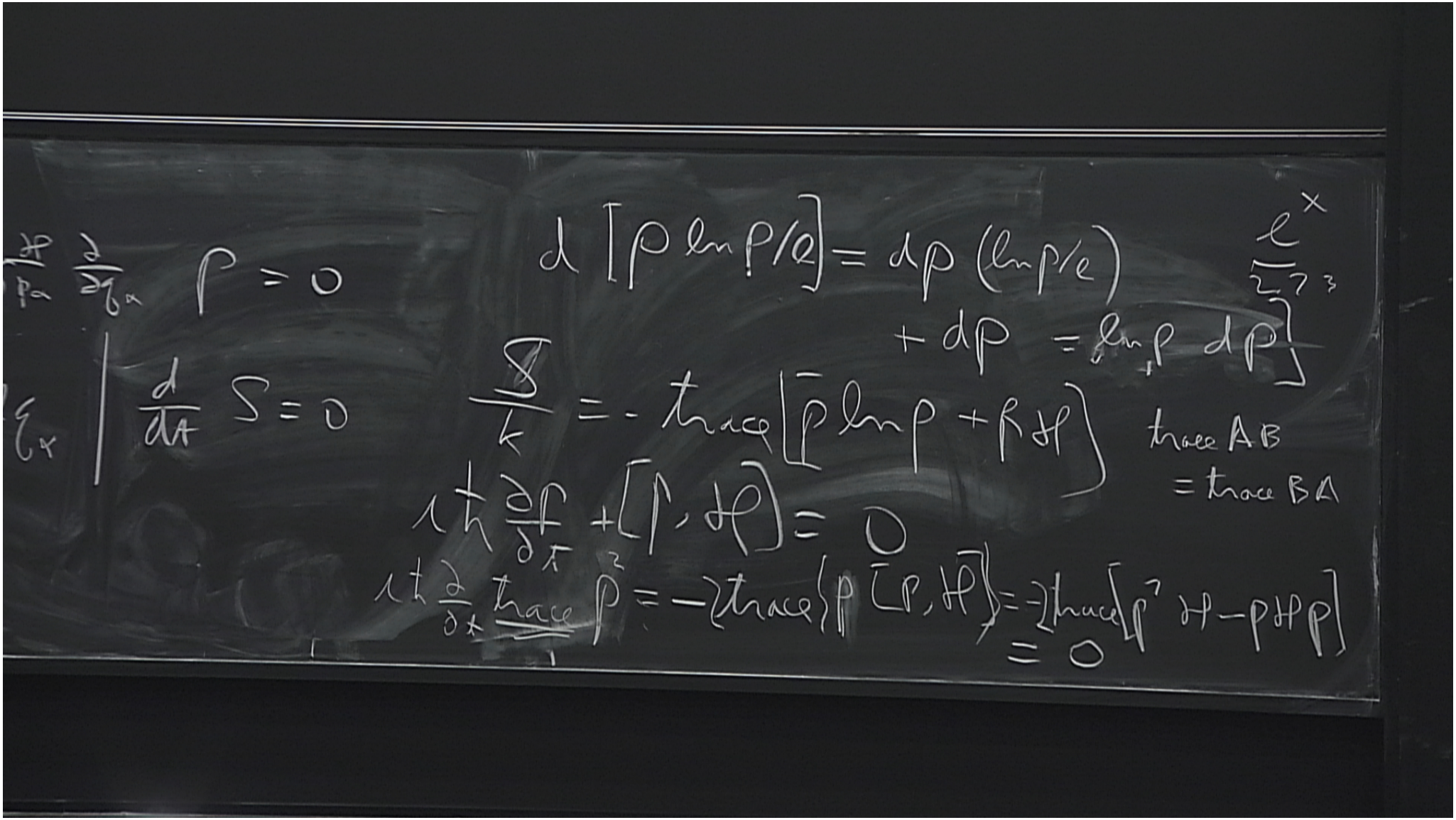

$\frac{\partial \rho}{\partial t} + \sum_\alpha \frac{\partial H}{\partial p_\alpha}\frac{\partial}{\partial q_\alpha}\rho = 0$
$\frac{d}{dt} S = 0$
$d[\rho \ln \rho/e] = d\rho\,(\ln \rho/e) + d\rho = d[\rho \ln \rho]$
e^x
$\frac{S}{k} = -\,\mathrm{trace}[\bar{\rho}\ln\rho + \beta H]$
trace AB = trace BA
$i\hbar \frac{\partial \rho}{\partial t} + [\rho, H] = 0$
$i\hbar \frac{\partial}{\partial t}\mathrm{trace}\,\rho = -\,\mathrm{trace}\,\rho\,[\rho, H] = -\,\mathrm{trace}[\rho^2 H - \rho H \rho]$
$= 0$

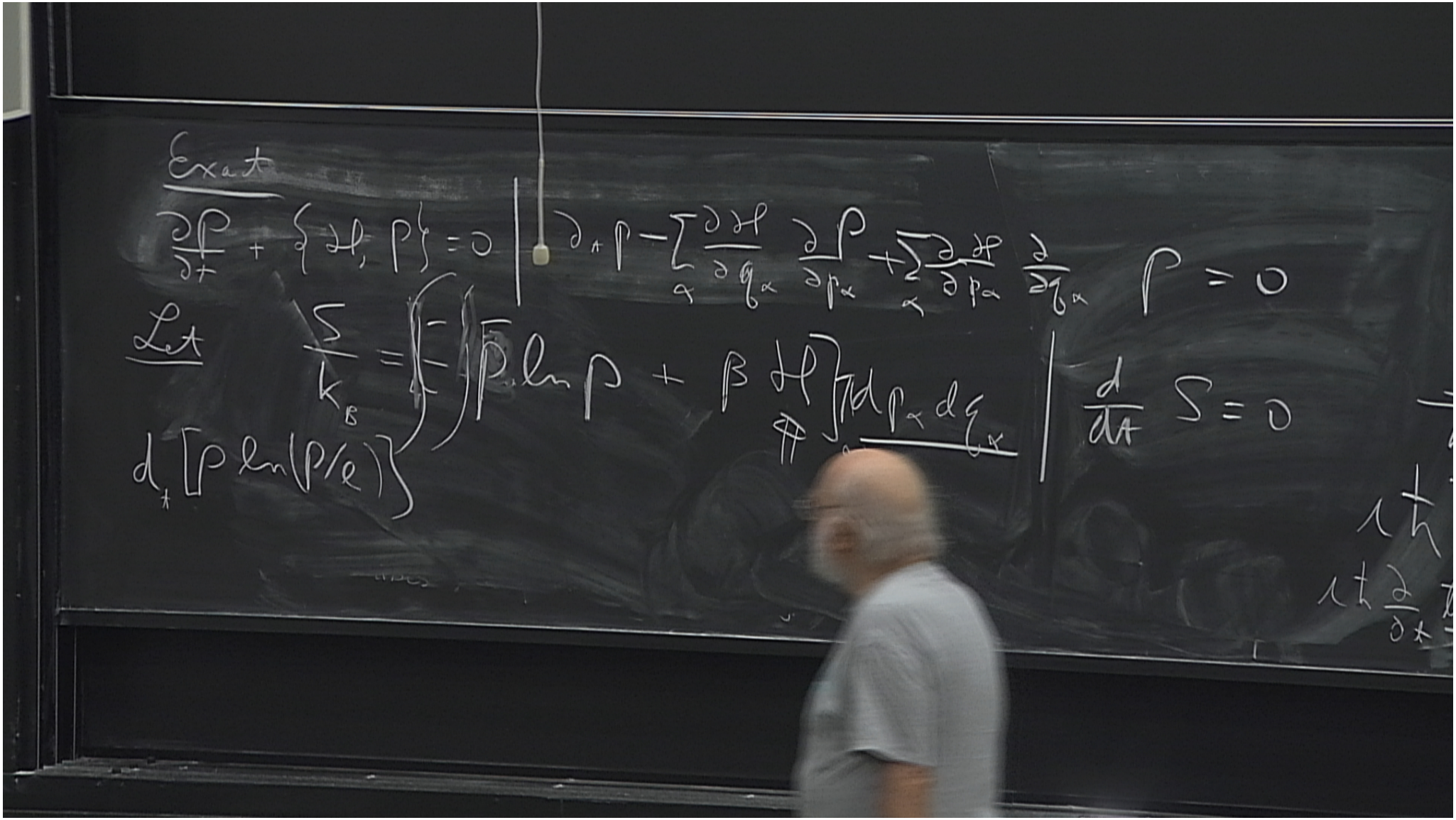

$\frac{\partial \rho}{\partial \alpha} \frac{\partial}{\partial \xi_\alpha} \rho = 0$
$\xi_\alpha \mid \frac{d}{dt} S = 0$
$d[\rho \ln \rho/e] = d\rho (\ln \rho/e)$
$+ d\rho = d[\rho \ln \rho]$
e^x
$\frac{S}{k} = -\text{trace}[\bar{\rho} \ln \rho + \beta H]$
trace AB
= trace BA
$i\hbar \frac{\partial f}{\partial t} + [f, H] = 0$
$i\hbar \frac{\partial}{\partial t} \text{trace}\, \rho = -\text{trace}\, \rho [\rho, H] = -\text{trace}[\rho^2 H - \rho H \rho]$
$= 0$

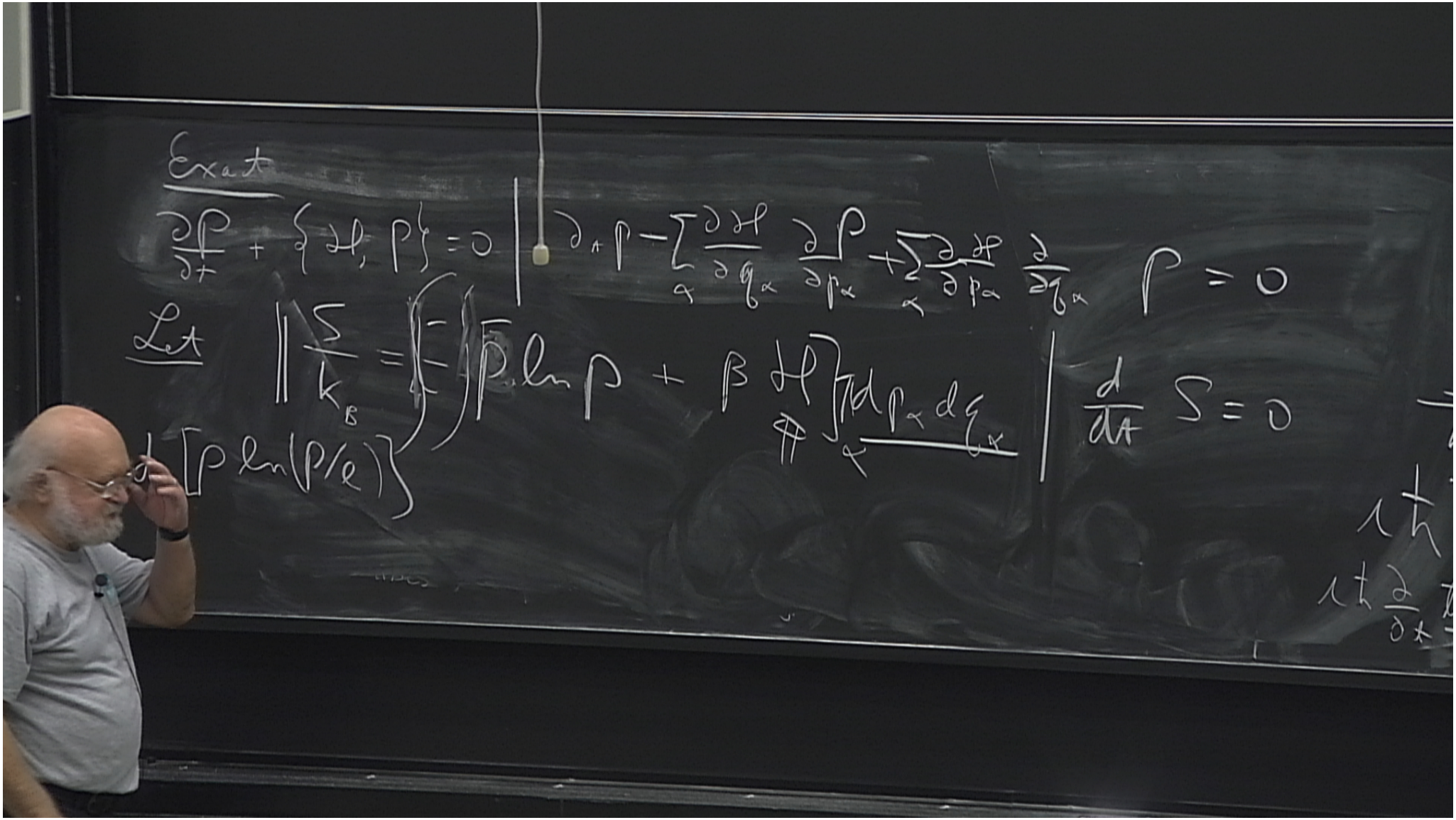

$\frac{\partial}{\partial p_a}\frac{\partial}{\partial \eta_a}\rho = 0$
$\frac{d}{dt}S = 0$
$d[\rho \ln \rho/e] = d\rho\,(\ln \rho/e) + d\rho = \ln \rho\, d\rho]$
$\frac{S}{k} = -\mathrm{trace}[\rho \ln \rho + \beta H]$
$\mathrm{trace}\,AB = \mathrm{trace}\,BA$
$i\hbar \frac{\partial f}{\partial t} + [f, H] = 0$
$i\hbar \frac{\partial}{\partial t}\mathrm{trace}\,\rho = -\mathrm{trace}\,\rho[\rho, H]$
$[\rho^2 H - \rho H \rho]$

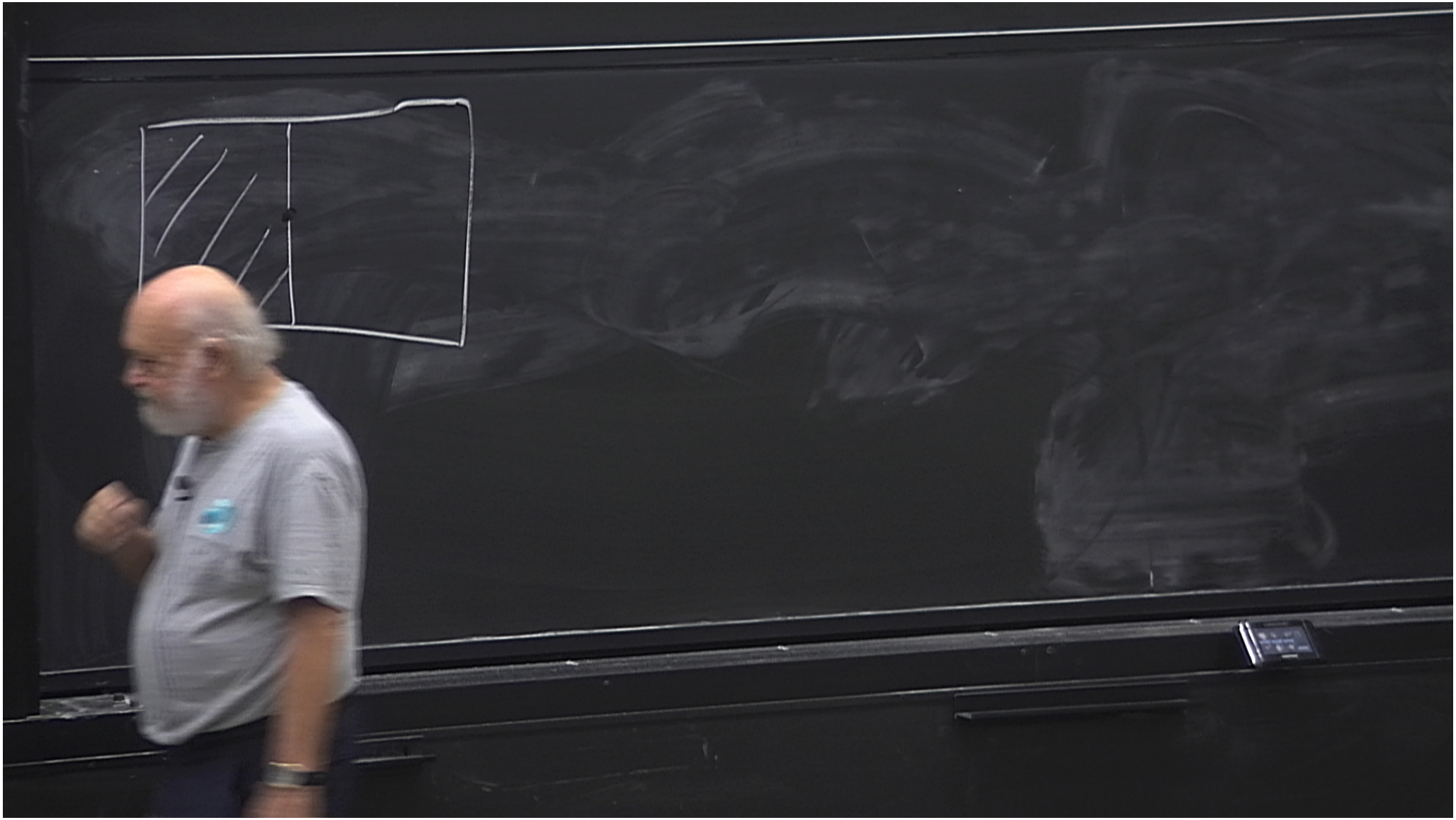

$\frac{\partial}{\partial p_\alpha}\frac{\partial}{\partial \eta_\alpha}\rho = 0$
$\frac{d}{dt} S = 0$
$d[\rho \ln \rho/e] = d\rho\,(\ln \rho/e) + d\rho = \ln \rho\, d\rho]$
e^x
$\frac{S}{k} = -\mathrm{trace}[\bar{\rho} \ln \rho + \beta H]$
trace AB = trace BA
$i\hbar \frac{\partial f}{\partial t} + [f, H] = 0$
$i\hbar \frac{\partial}{\partial t} \mathrm{trace}\, \rho^2 = -\mathrm{trace}\{\rho\,[\rho, H]\} = -\mathrm{trace}[\rho^2 H - \rho H \rho]$
$= 0$

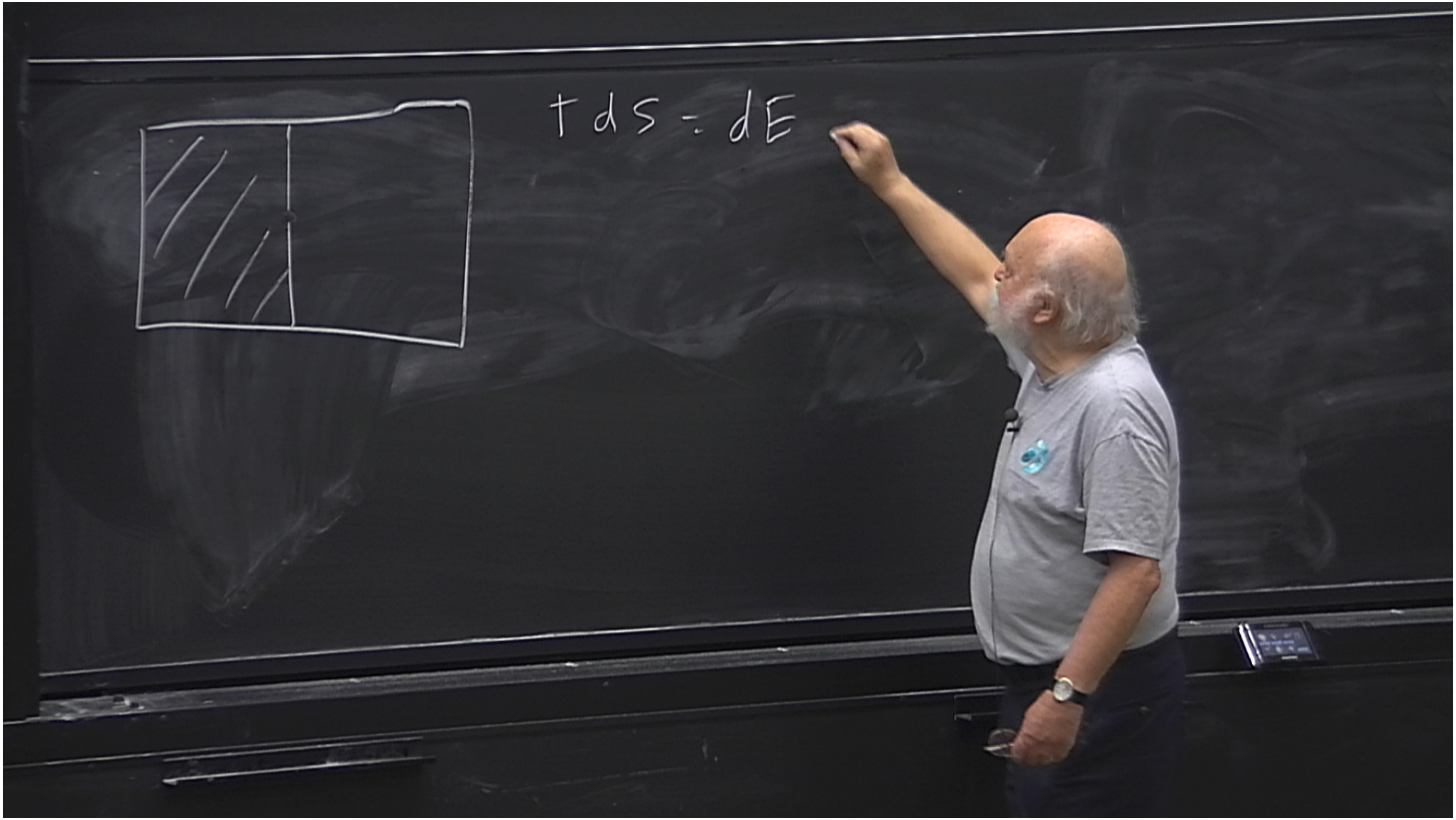

$\frac{\partial}{\partial p_a}\frac{\partial}{\partial \eta_a}\rho = 0$
$\frac{d}{dt} S = 0$
$d[\rho \ln \rho/e] = d\rho\,(\ln \rho/e) + d\rho = \ln \rho\, d\rho]$
$\frac{S}{k} = -\,\mathrm{trace}[\bar{\rho} \ln \rho + \rho H]$
e^x
$\mathrm{trace}\,AB = \mathrm{trace}\,BA$
$i\hbar \frac{\partial f}{\partial t} + [f, H] = 0$
$i\hbar \frac{\partial}{\partial t} \mathrm{trace}\,\rho^2 = -2\,\mathrm{trace}\{\rho\,[\rho, H]\} = -2\,\mathrm{trace}[\rho^2 H - \rho H \rho]$
$= 0$

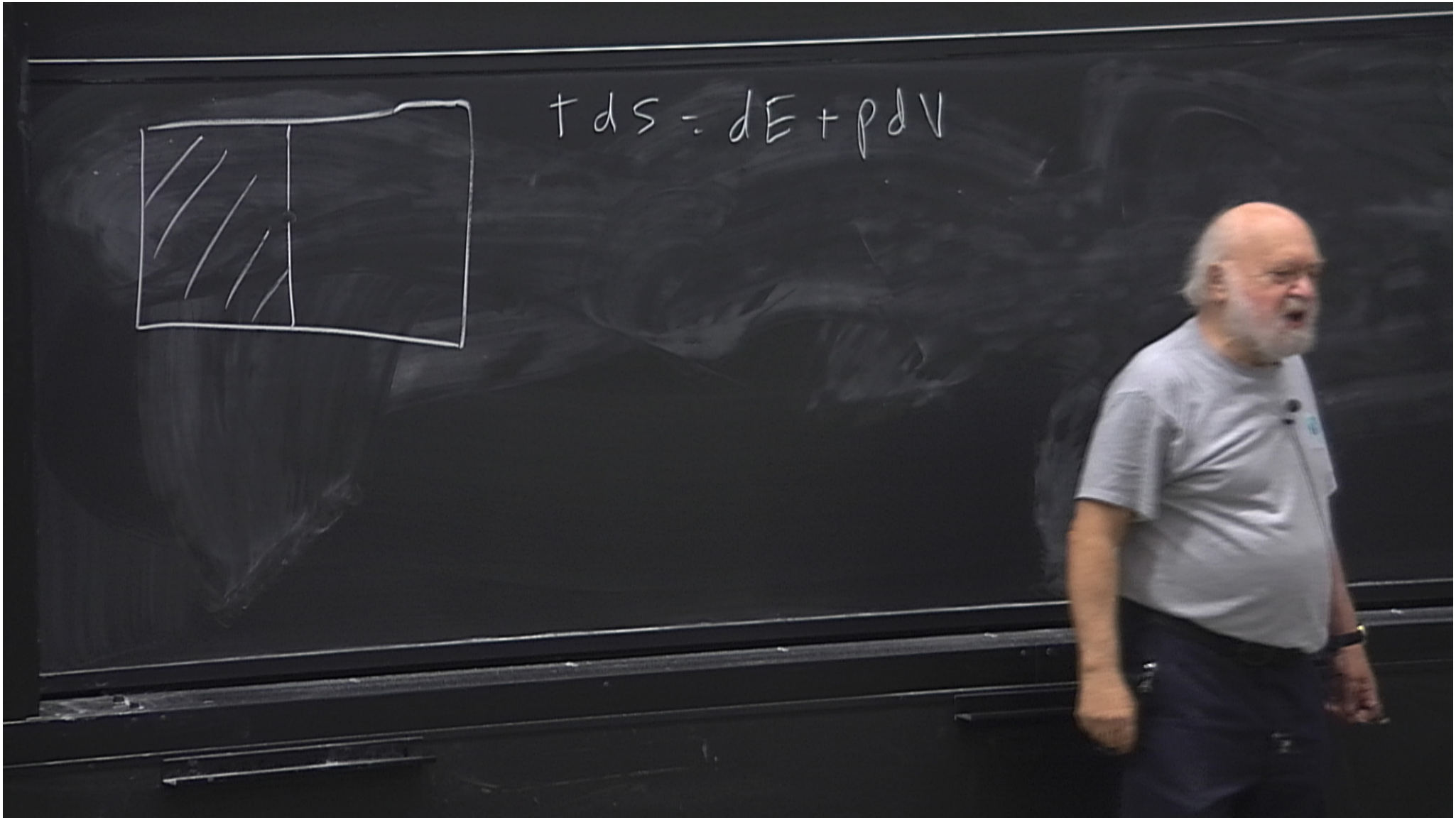

$\frac{\partial \rho}{\partial \alpha} \frac{\partial}{\partial \xi_\alpha} \rho = 0$
$\xi_\alpha \mid \frac{d}{dt} S = 0$
$d[\rho \ln \rho/e] = d\rho (\ln \rho/e) + d\rho = \ln \rho \, d\rho]$
e^x
$\frac{S}{k} = -\text{trace}[\bar{\rho} \ln \rho + \beta H]$
trace AB = trace BA
$i\hbar \frac{\partial f}{\partial t} + [f, H] = 0$
$i\hbar \frac{\partial}{\partial t} \text{trace} \rho^2 = -\text{trace}\{\rho [\rho, H]\} = -\text{trace}[\rho^2 H - \rho H \rho]$
$= 0$

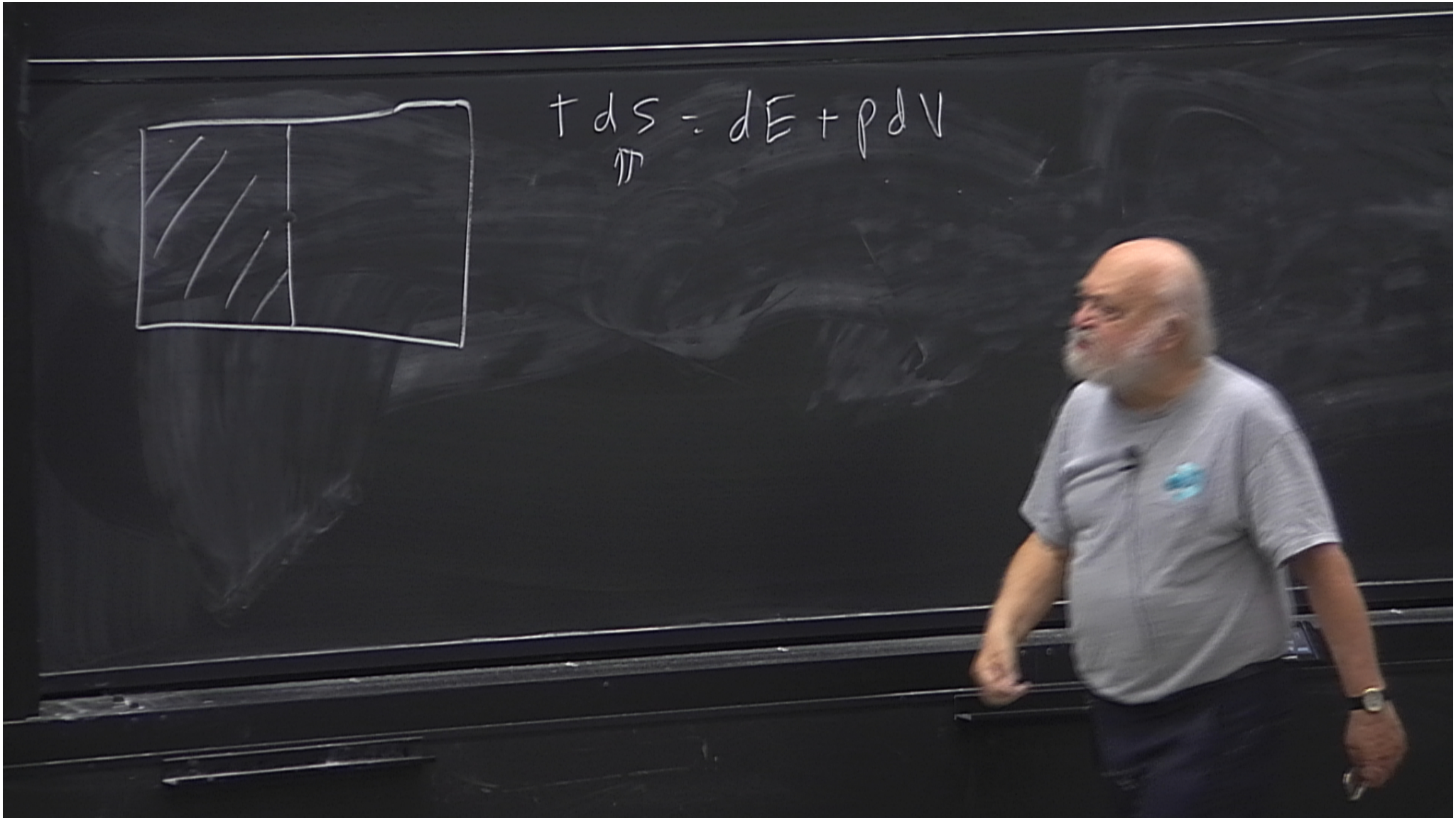

Exact
Let
$\frac{\partial \rho}{\partial t} + \{H, \rho\} = 0$
$\partial_t \rho - \sum_\alpha \frac{\partial H}{\partial q_\alpha}\frac{\partial \rho}{\partial p_\alpha} + \sum_\alpha \frac{\partial H}{\partial p_\alpha}\frac{\partial \rho}{\partial q_\alpha} = 0$
$\frac{S}{k_B} = -\int \rho \ln \rho + \beta H \int d p_\alpha d q_\alpha$
$\frac{d}{dt} S = 0$
$\partial_t [\rho \ln(\rho/e)]$

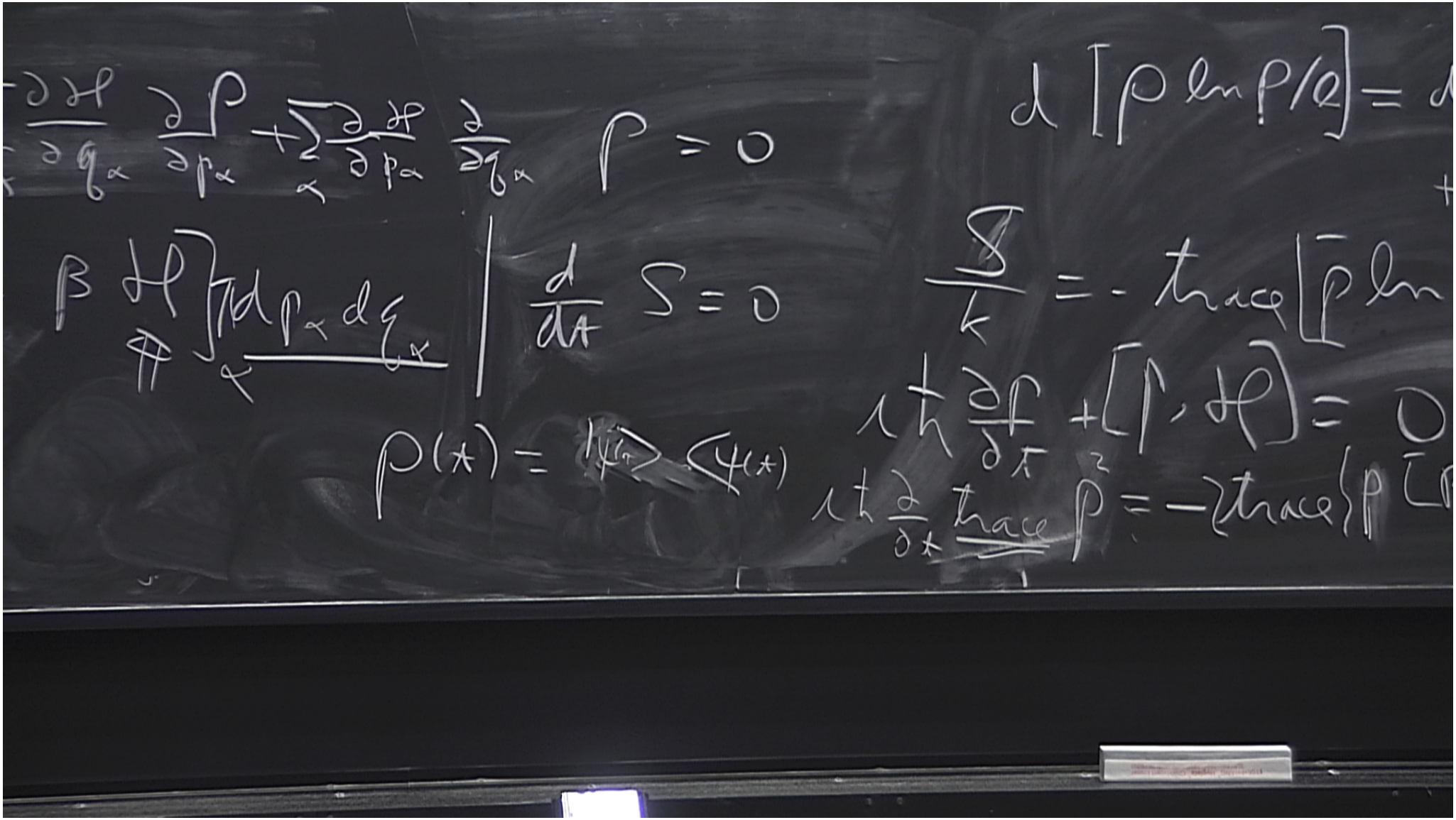

Exact
Let
$\frac{\partial \rho}{\partial t} + \{\mathcal{H}, \rho\} = 0$
$\frac{d}{dt} S = 0$

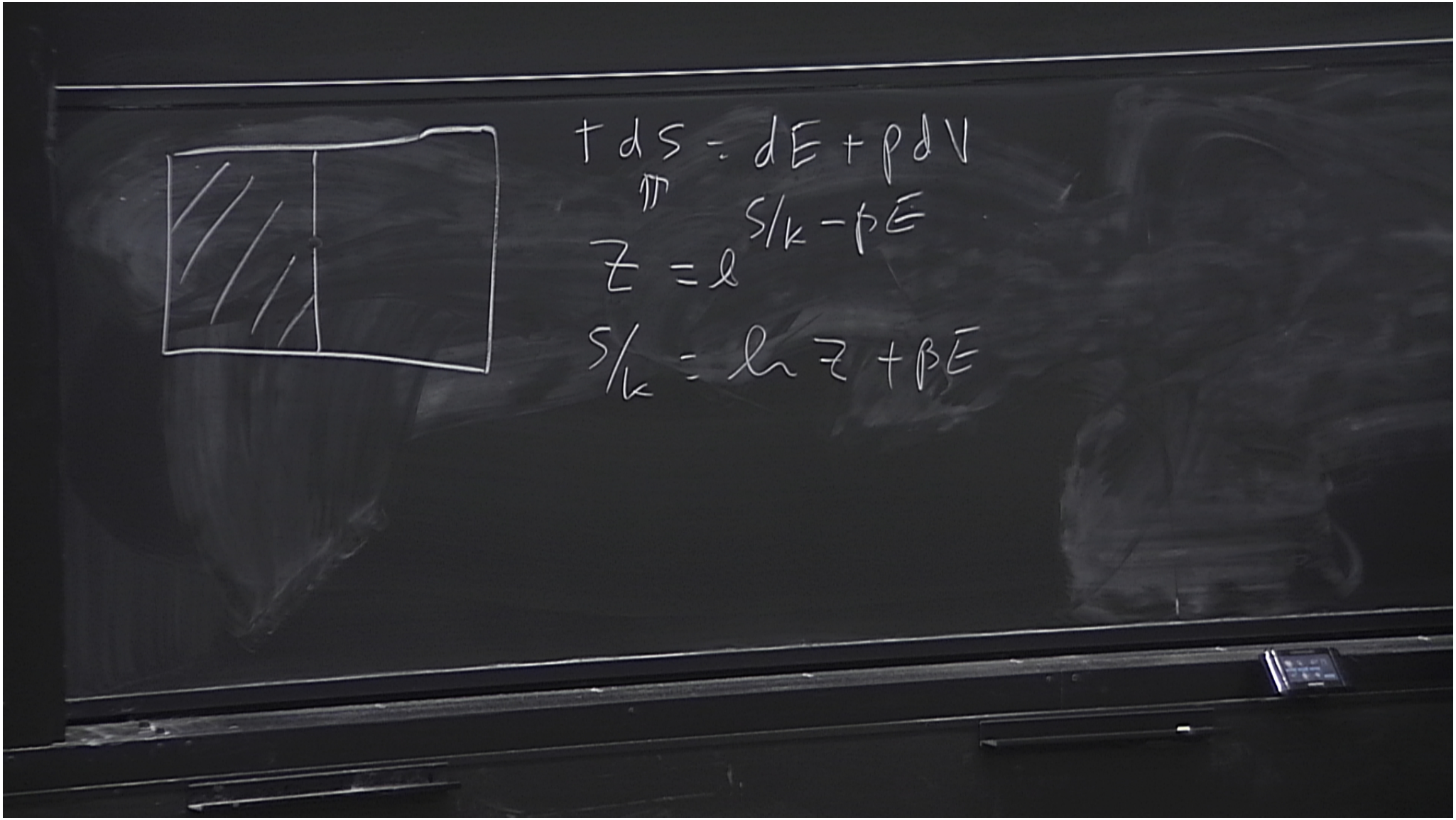


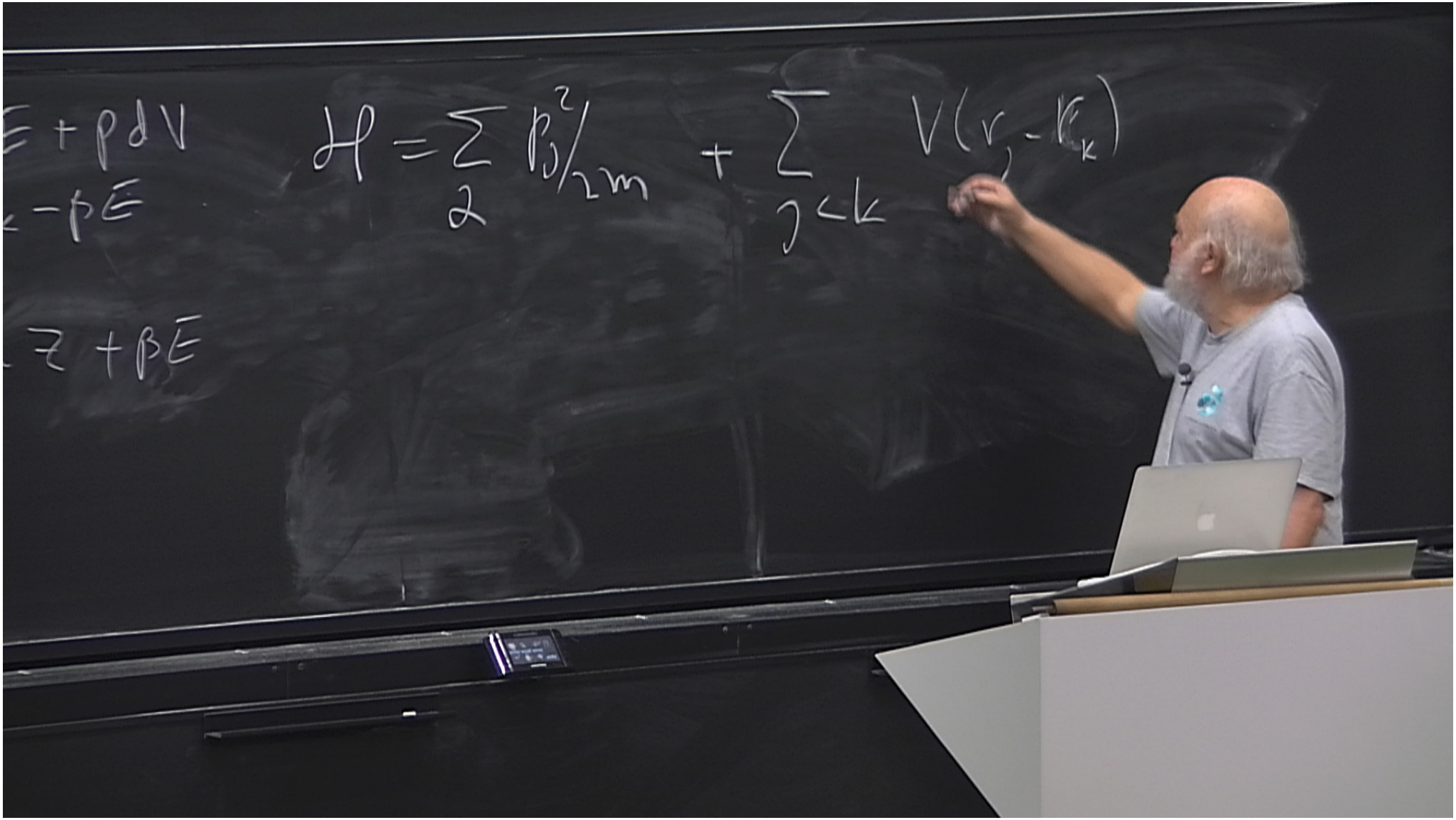

T dS = dE

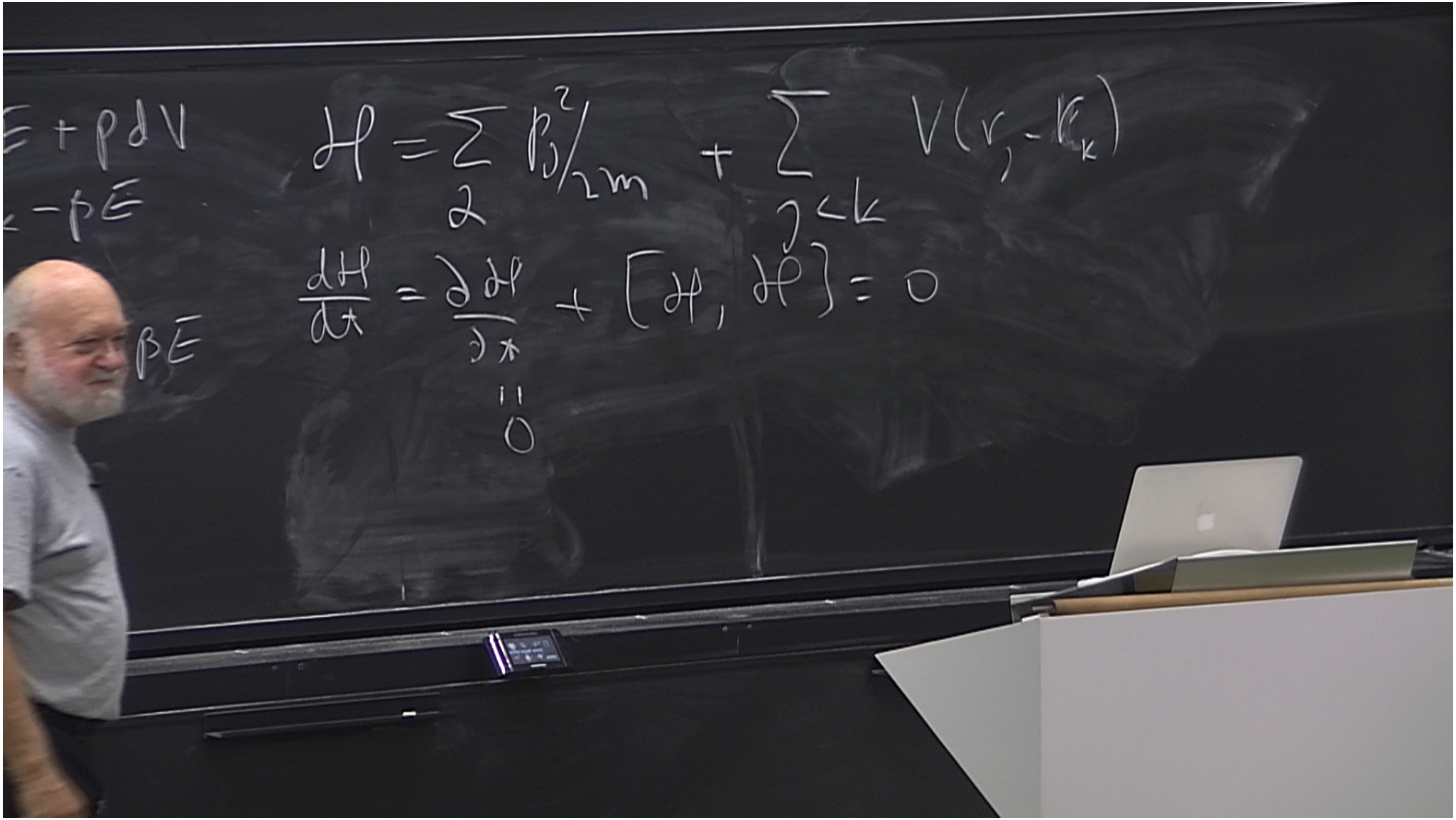

T dS = dE + pdV

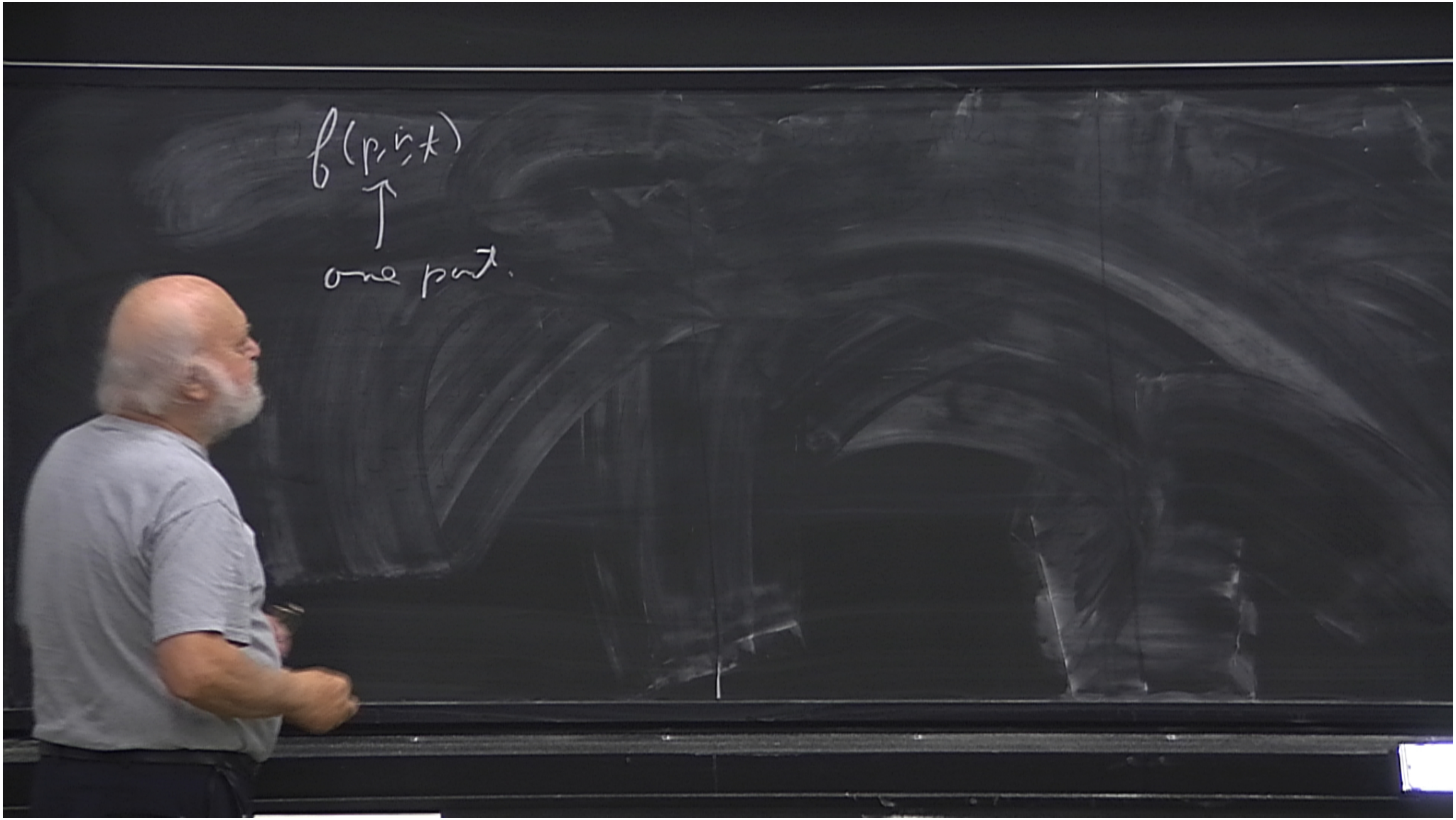

T dS = dE + pdV

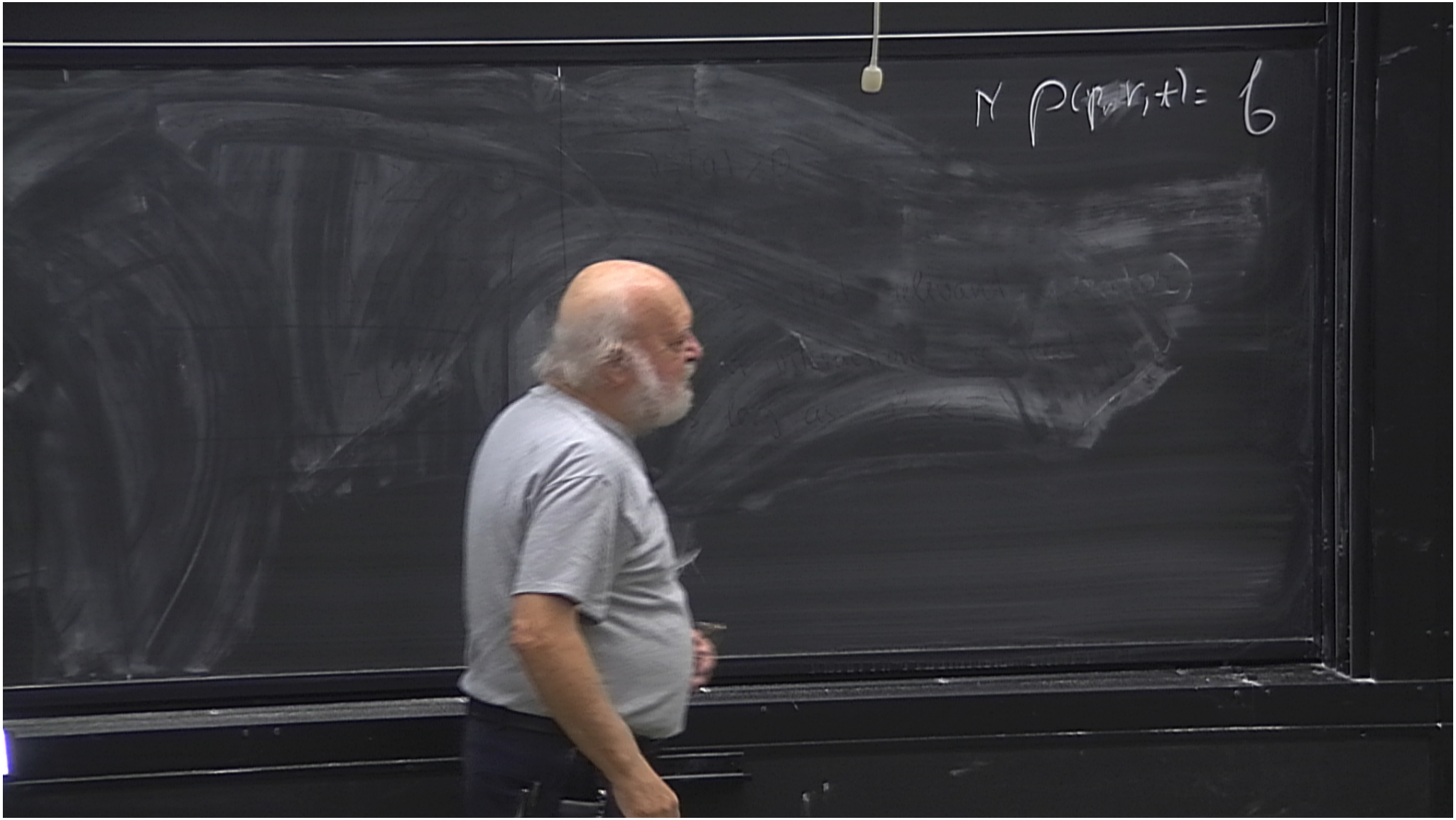

$\frac{\partial H}{\partial q_\alpha}\frac{\partial \rho}{\partial p_\alpha} + \sum_\alpha \frac{\partial H}{\partial p_\alpha}\frac{\partial}{\partial q_\alpha}\rho = 0$
$\frac{d}{dt} S = 0$
$\rho(t) = |\psi(t)\rangle\langle\psi(t)|$
$\frac{S}{k} = -\mathrm{trace}[\bar{\rho} \ln$
$i\hbar \frac{\partial \rho}{\partial t} + [\rho, H] = 0$
$i\hbar \frac{\partial}{\partial t} \mathrm{trace}\,\rho^2 = -2\,\mathrm{trace}\{\rho$

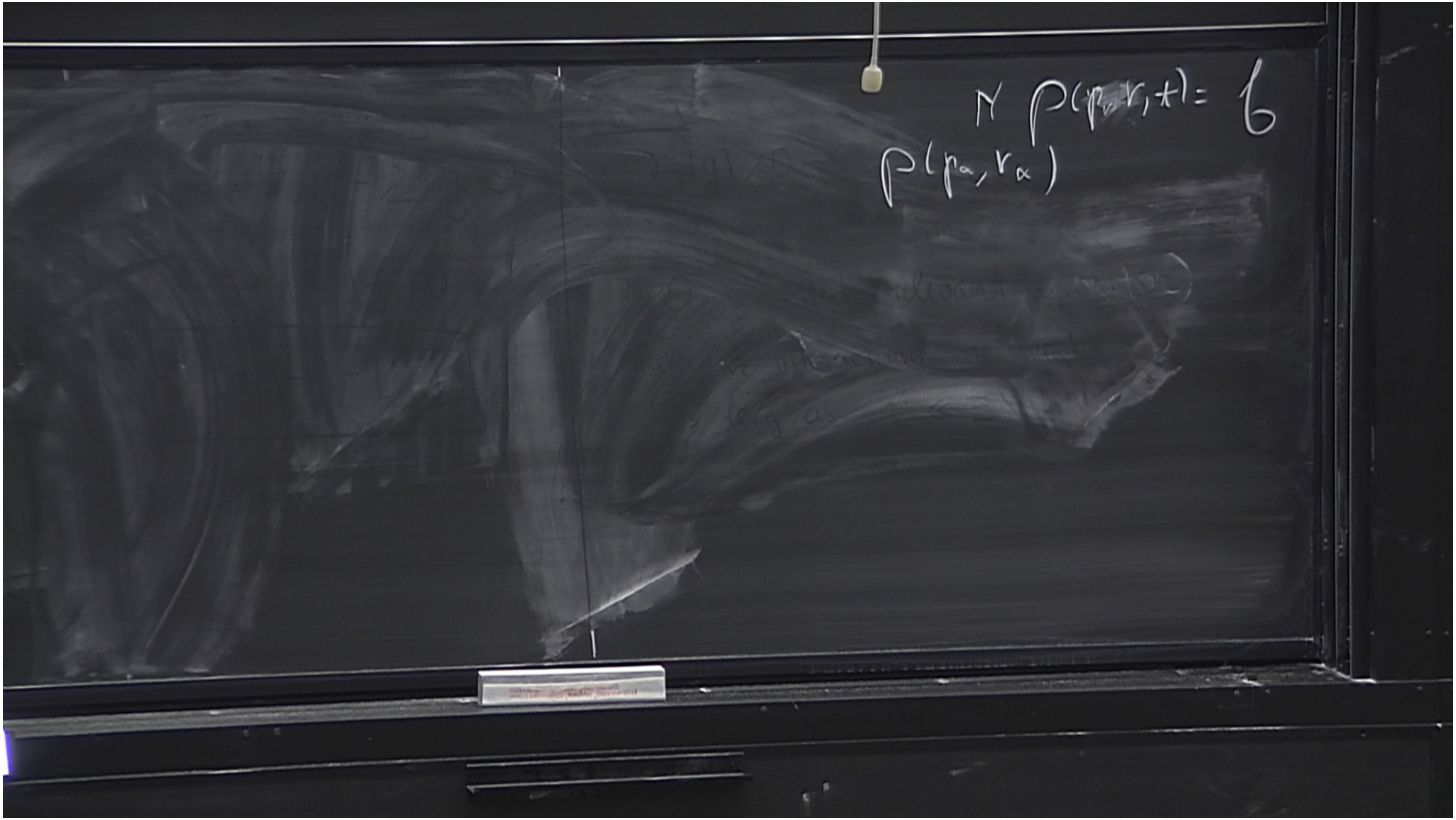

T dS = dE + p dV
Z = e^{S/k - βE}
S/k = ln Z + βE

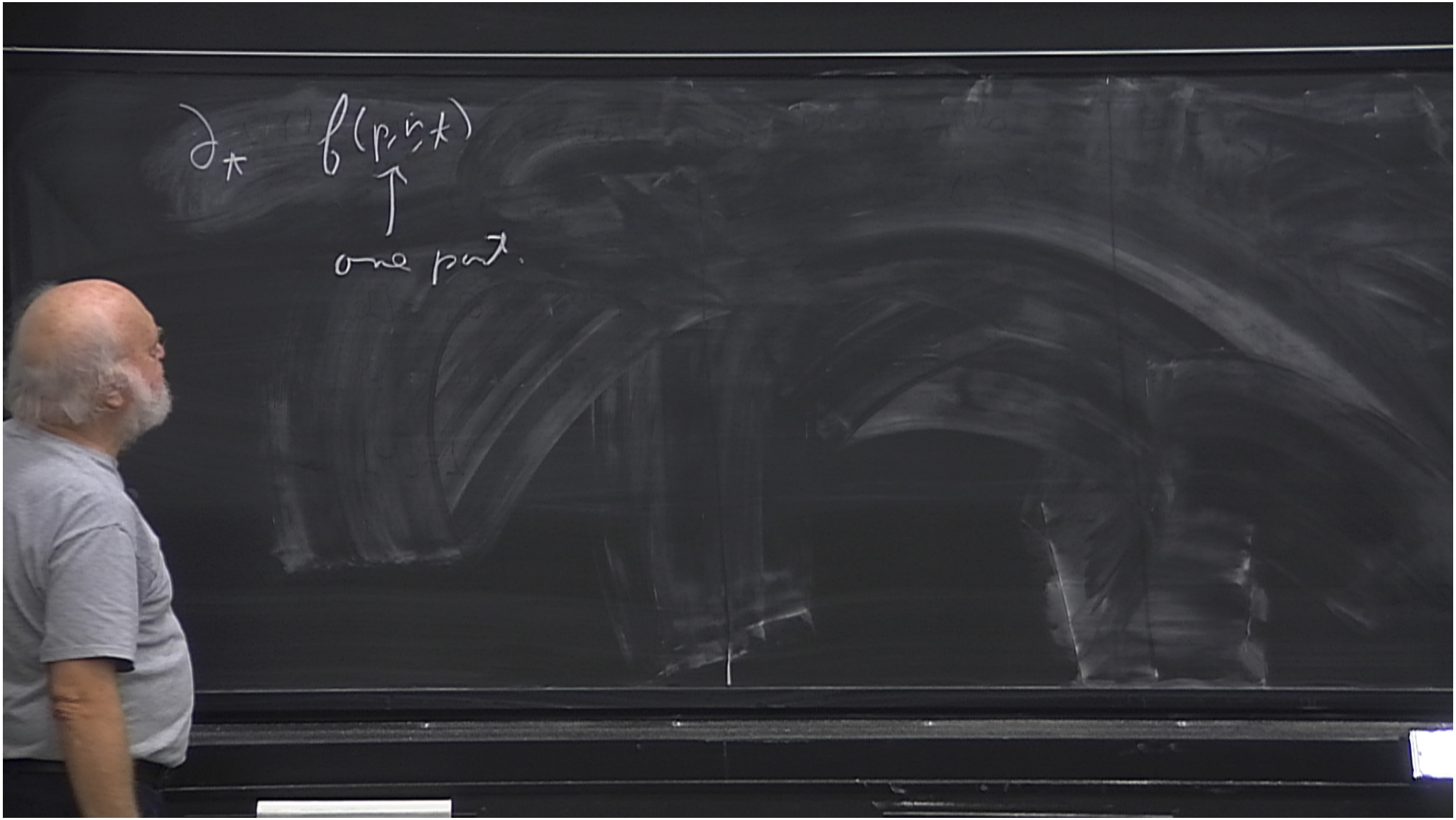

E + pdV
x - pE
z + βE
$\mathcal{H} = \sum_i p_i^2/2m + \sum_{j<k} V(r_j - r_k)$

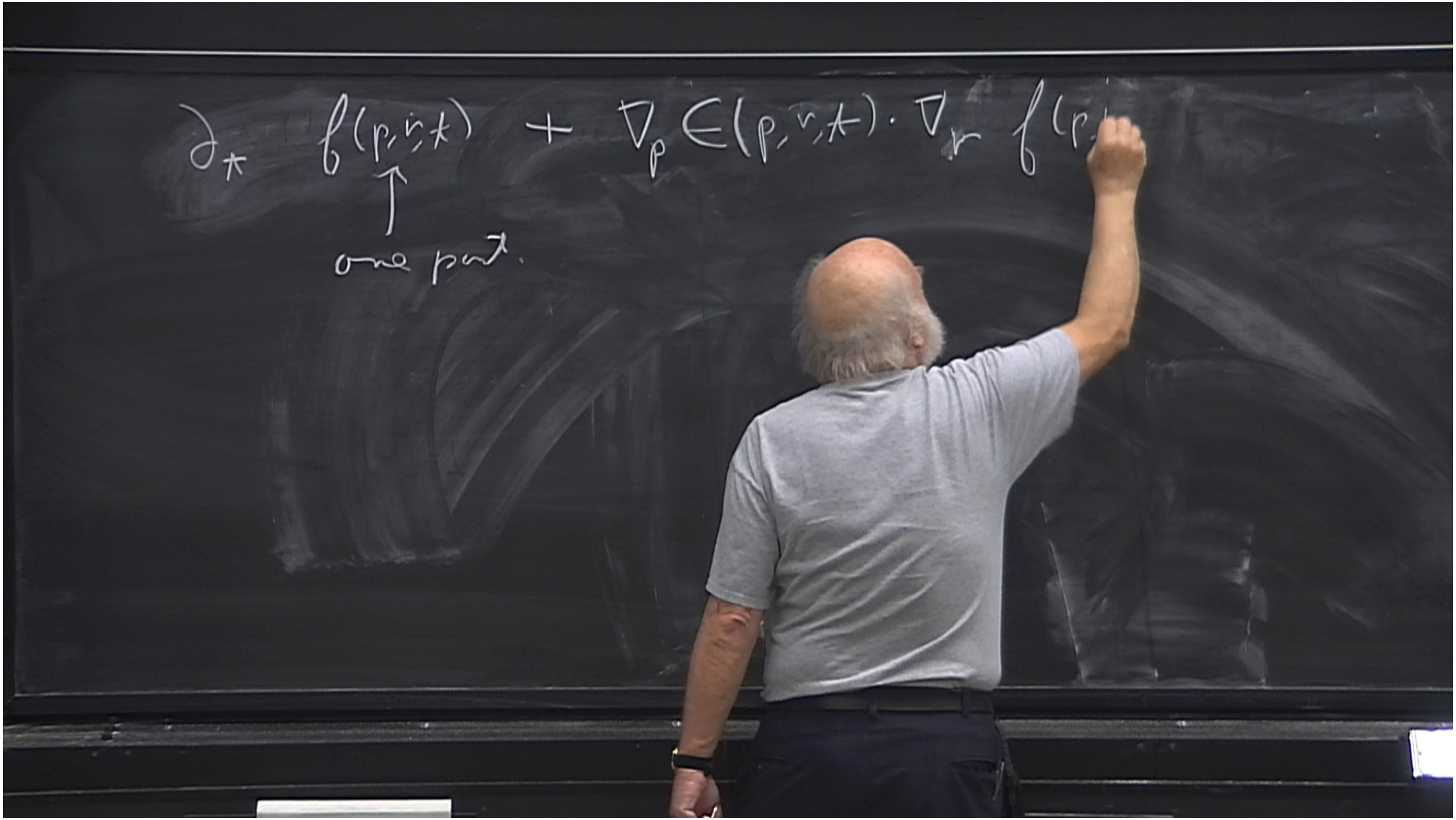

E + pdV
x - pE
βE
$\mathcal{H} = \sum_i p_i^2/2m + \sum_{j<k} V(r_j - r_k)$
$\frac{dH}{dt} = \frac{\partial \mathcal{H}}{\partial t} + [\mathcal{H}, \mathcal{H}] = 0$
$\frac{\partial \mathcal{H}}{\partial t} = 0$

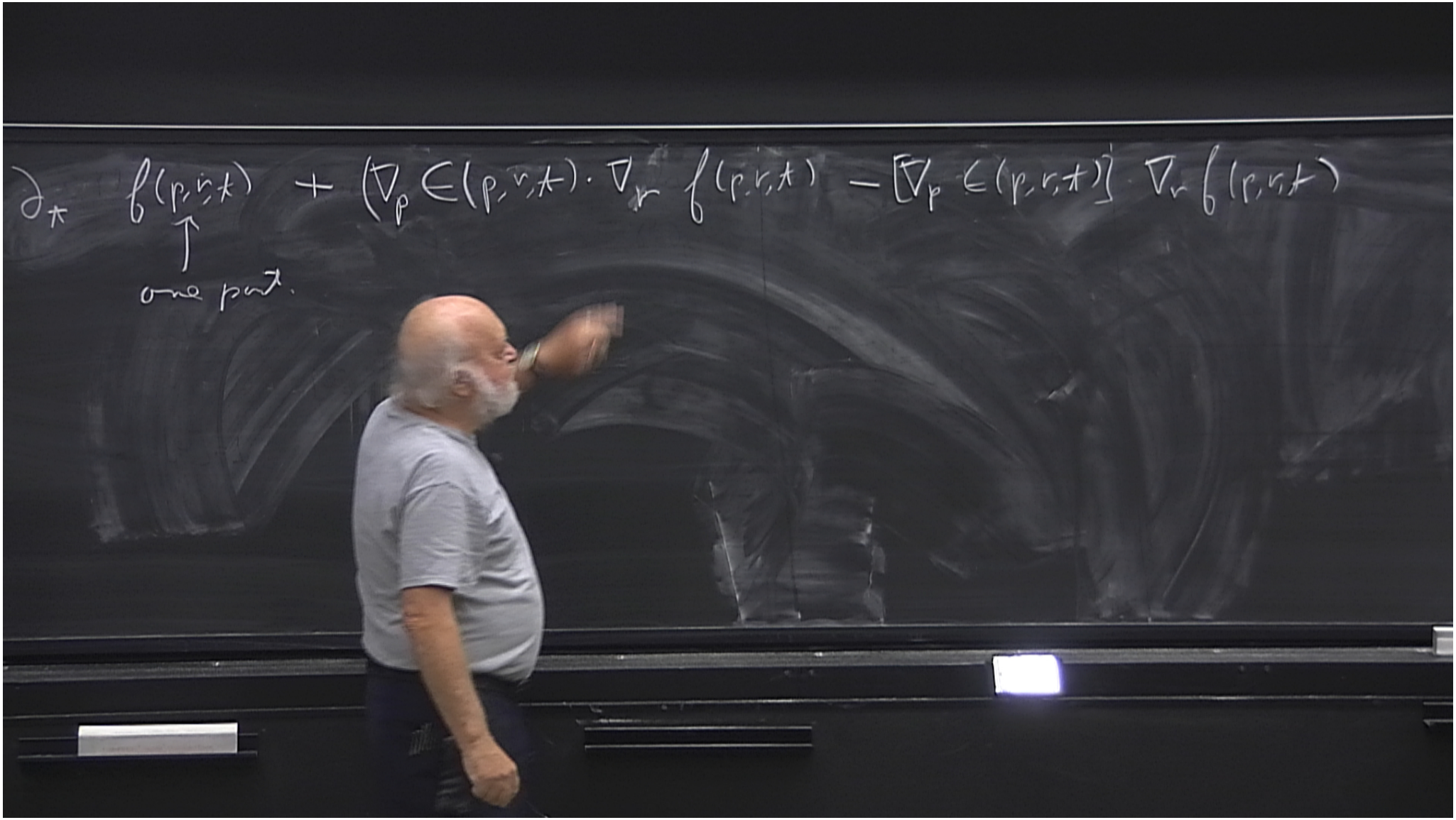

f(p, i, t)
one part.

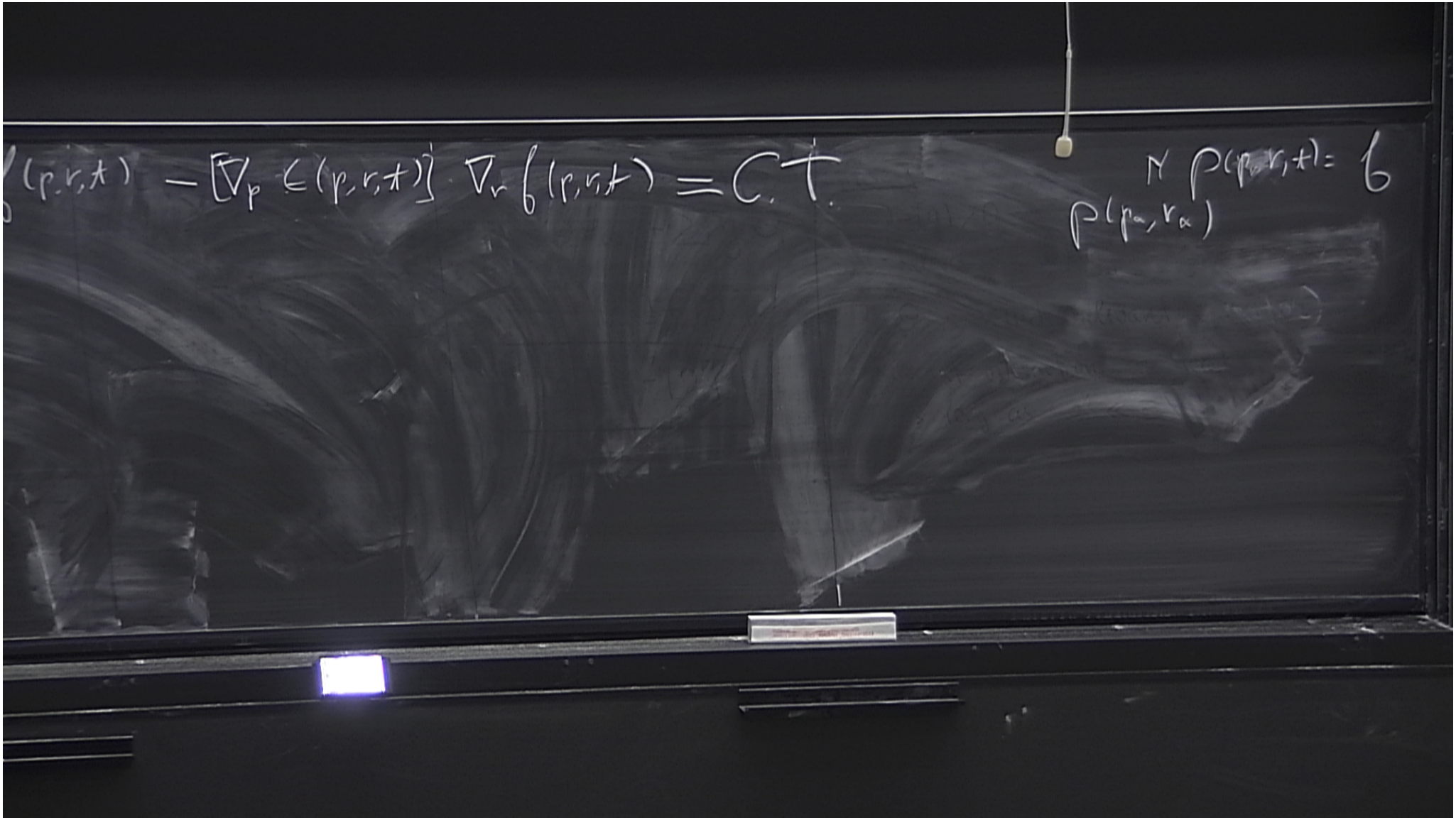

N ρ(p, r, t) = 6

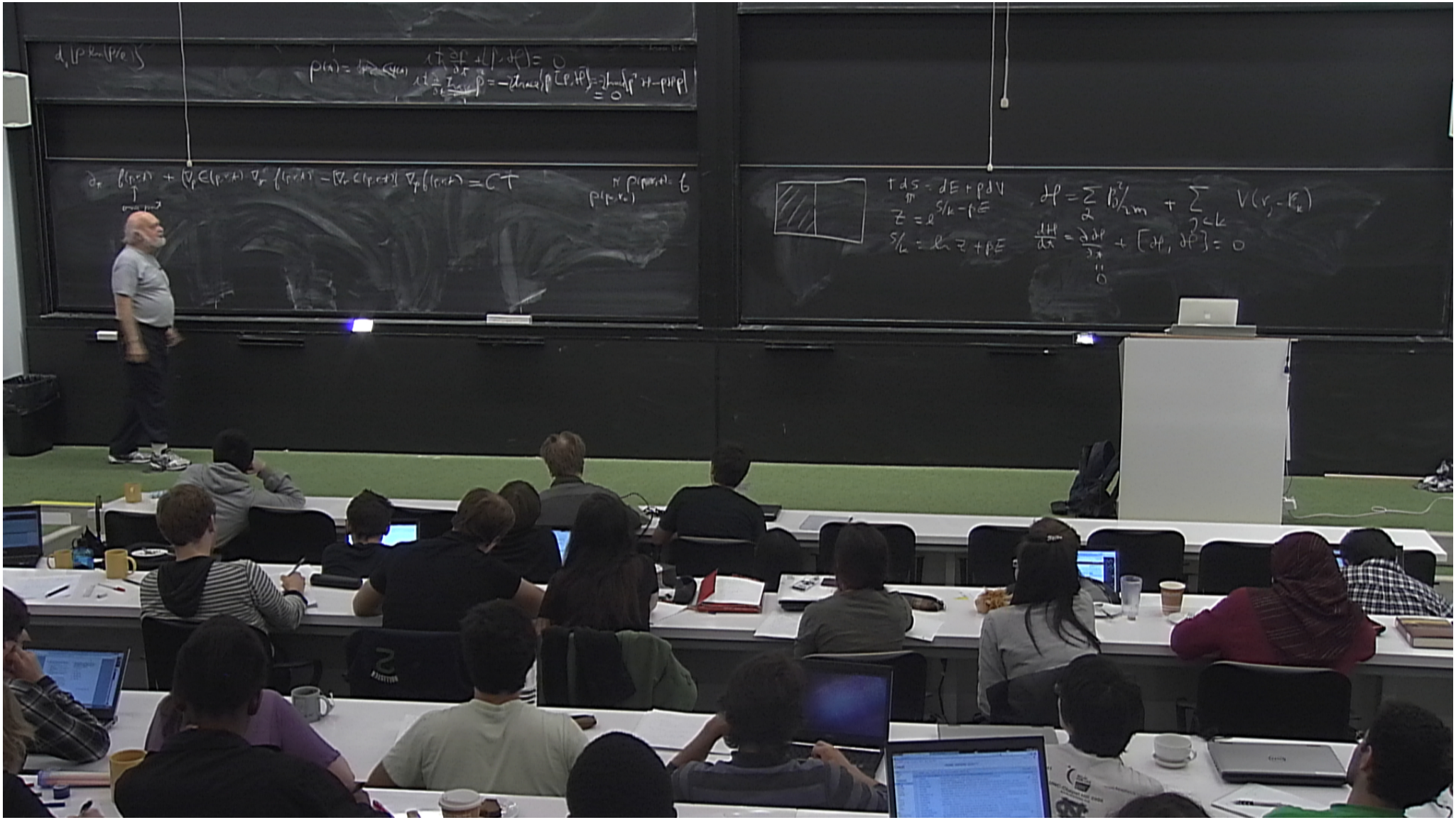

N ρ(p, r, t) = 6
ρ(p_α, r_α)

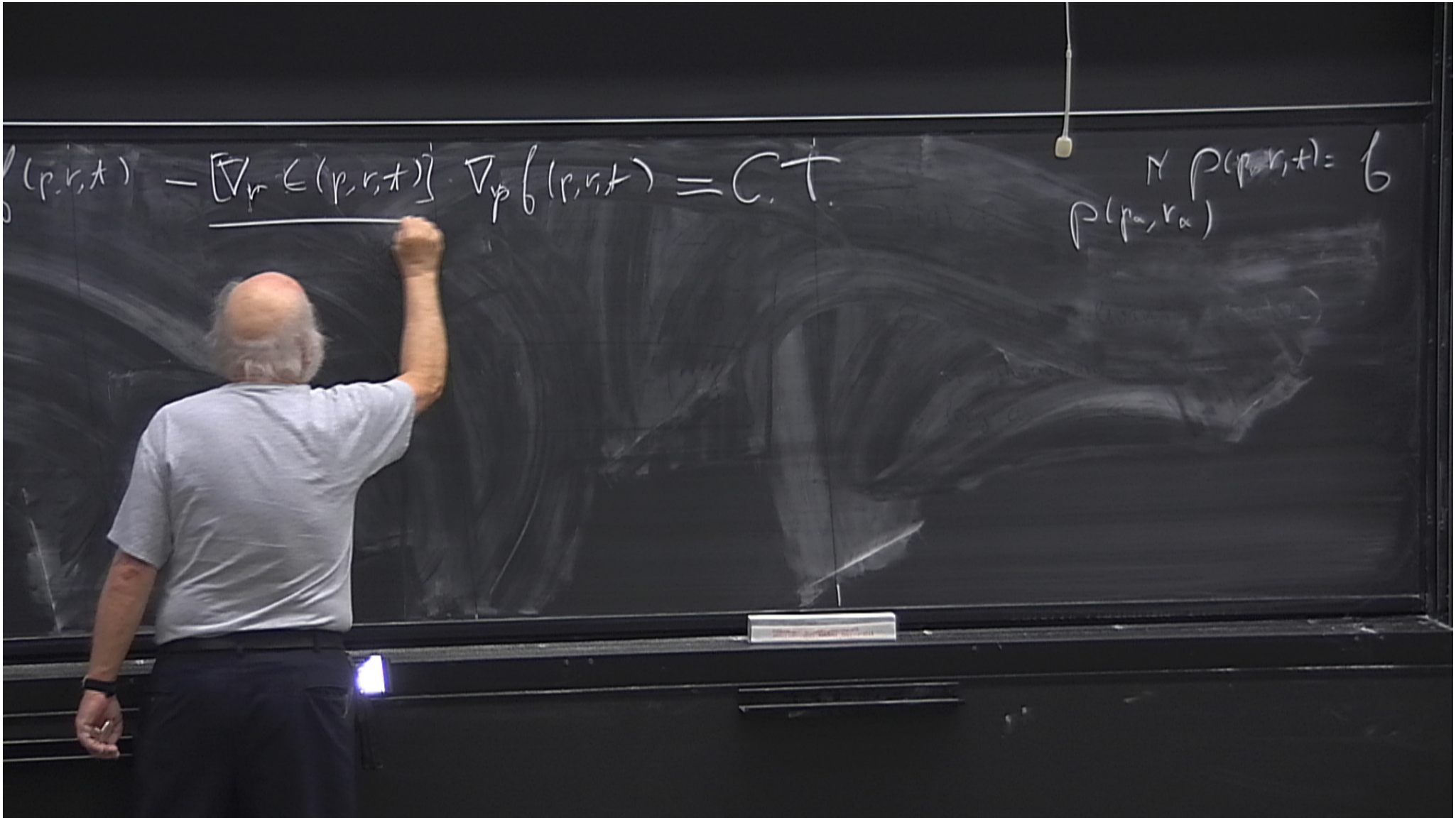

∂_* $f(p,\vec{p},*)$
one part.

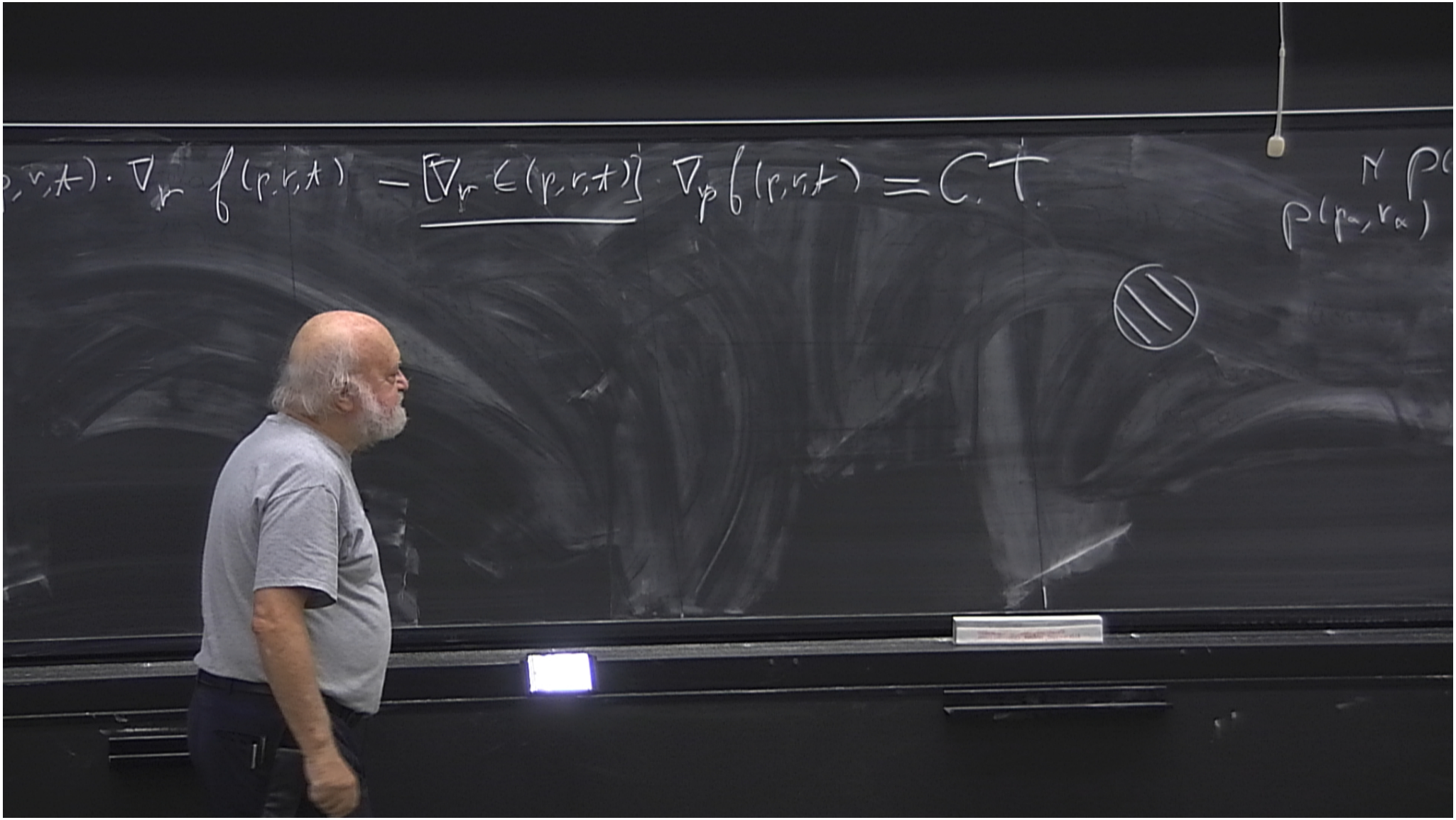

$\partial_t f(p,r,t) + \nabla_p \in (p,r,t) \cdot \nabla_r f(r,$
one part.

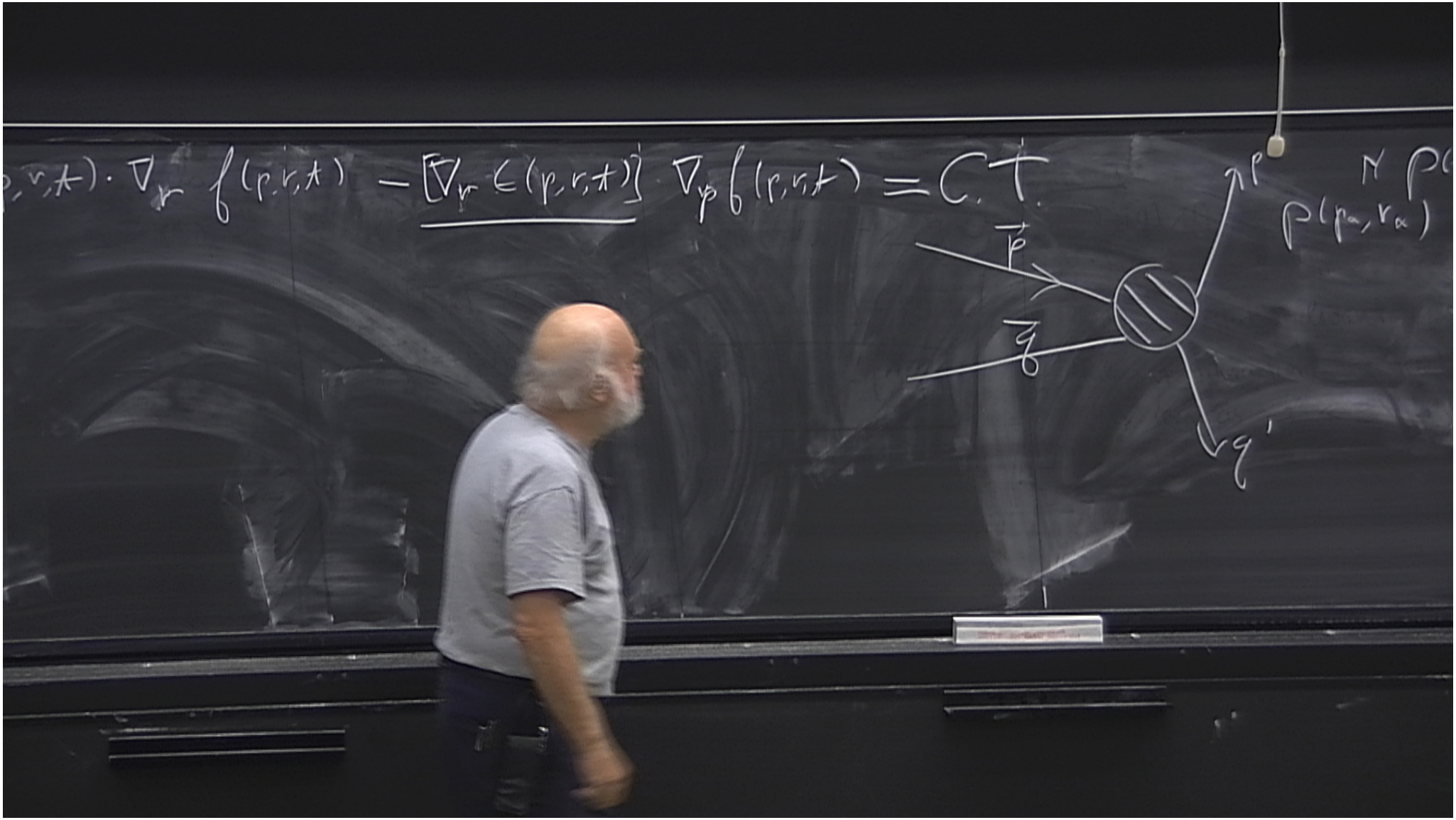

$\partial_t f(p,r,t) + (\nabla_p \in(p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_p \in(p,r,t)] \nabla_r f(p,r,t)$
one part.

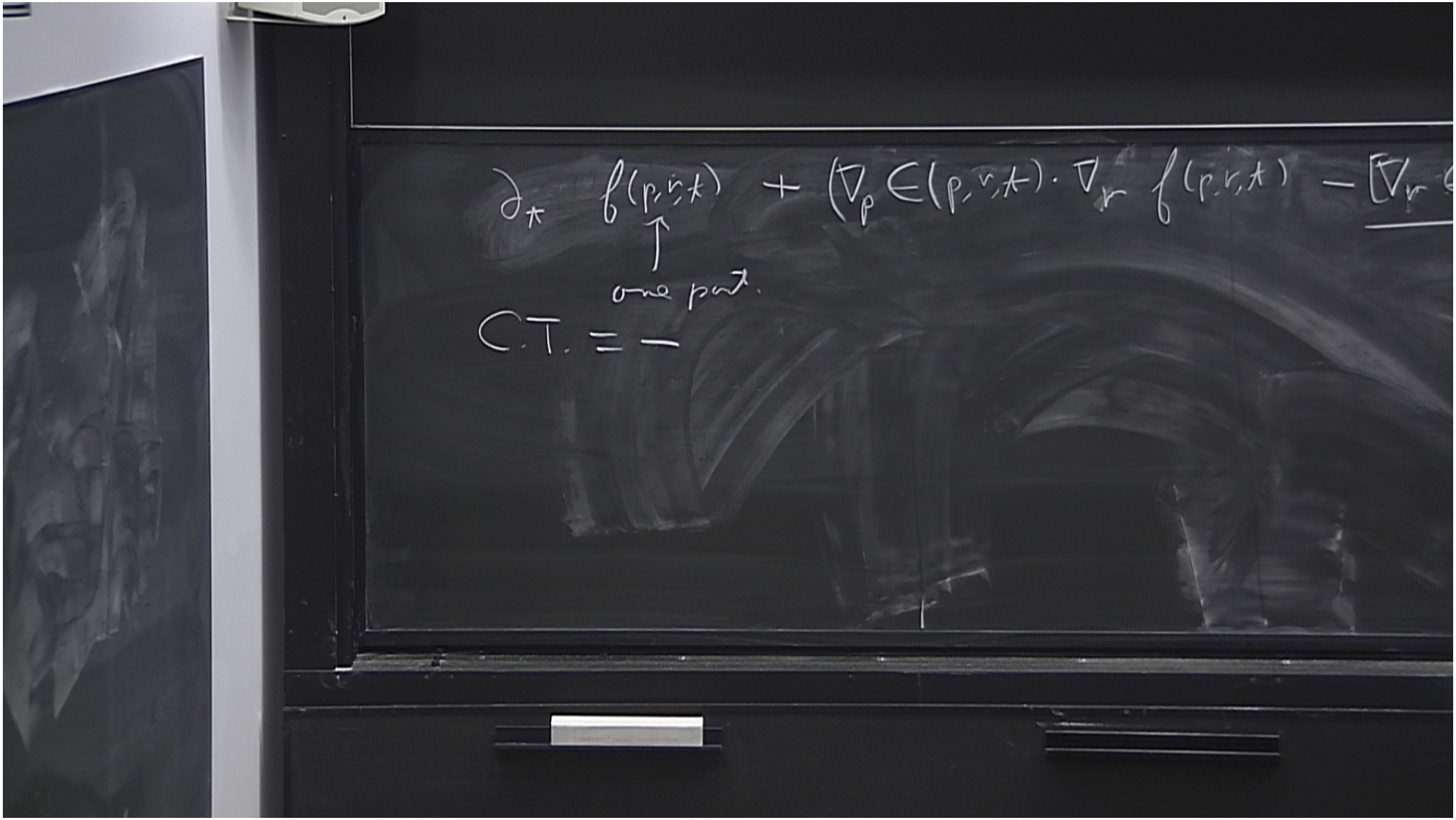

$(p,r,t) - [\nabla_p \epsilon(p,r,t)] \cdot \nabla_r f(p,r,t) = C.T.$
$N \rho(p,r,t) = 6$
$\rho(p_\alpha, r_\alpha)$

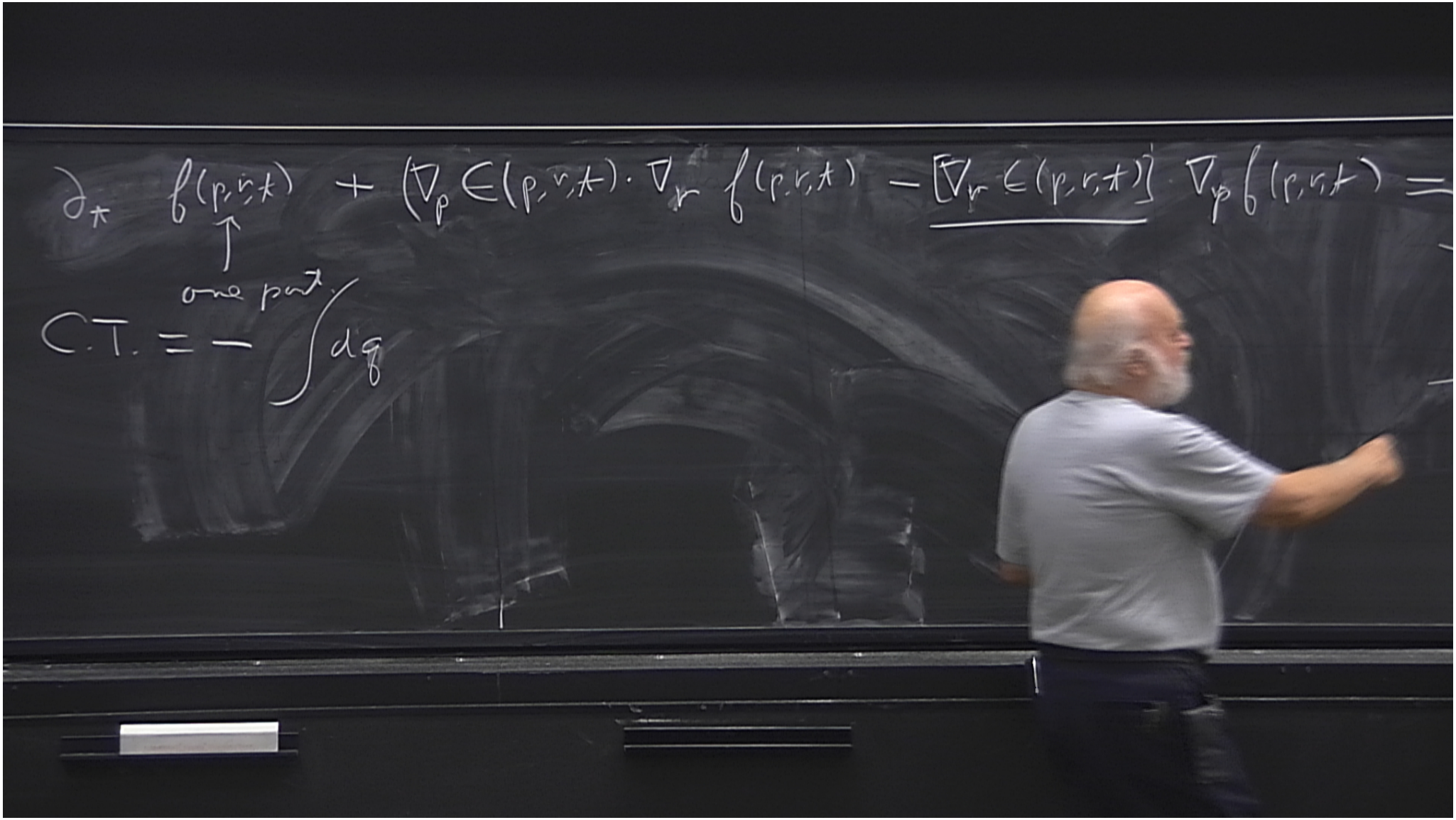


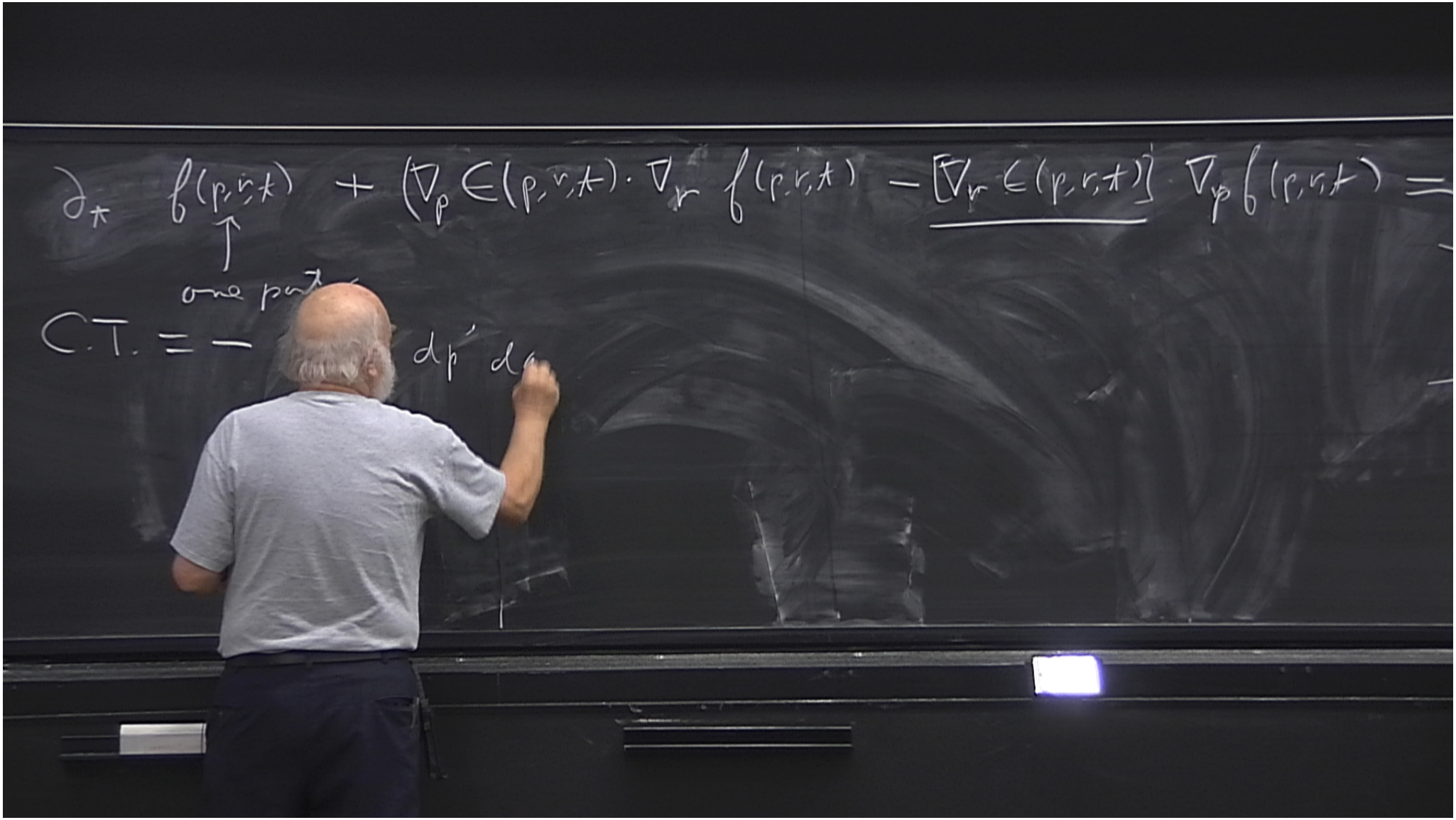

$\rho(p_\alpha, r_\alpha)$

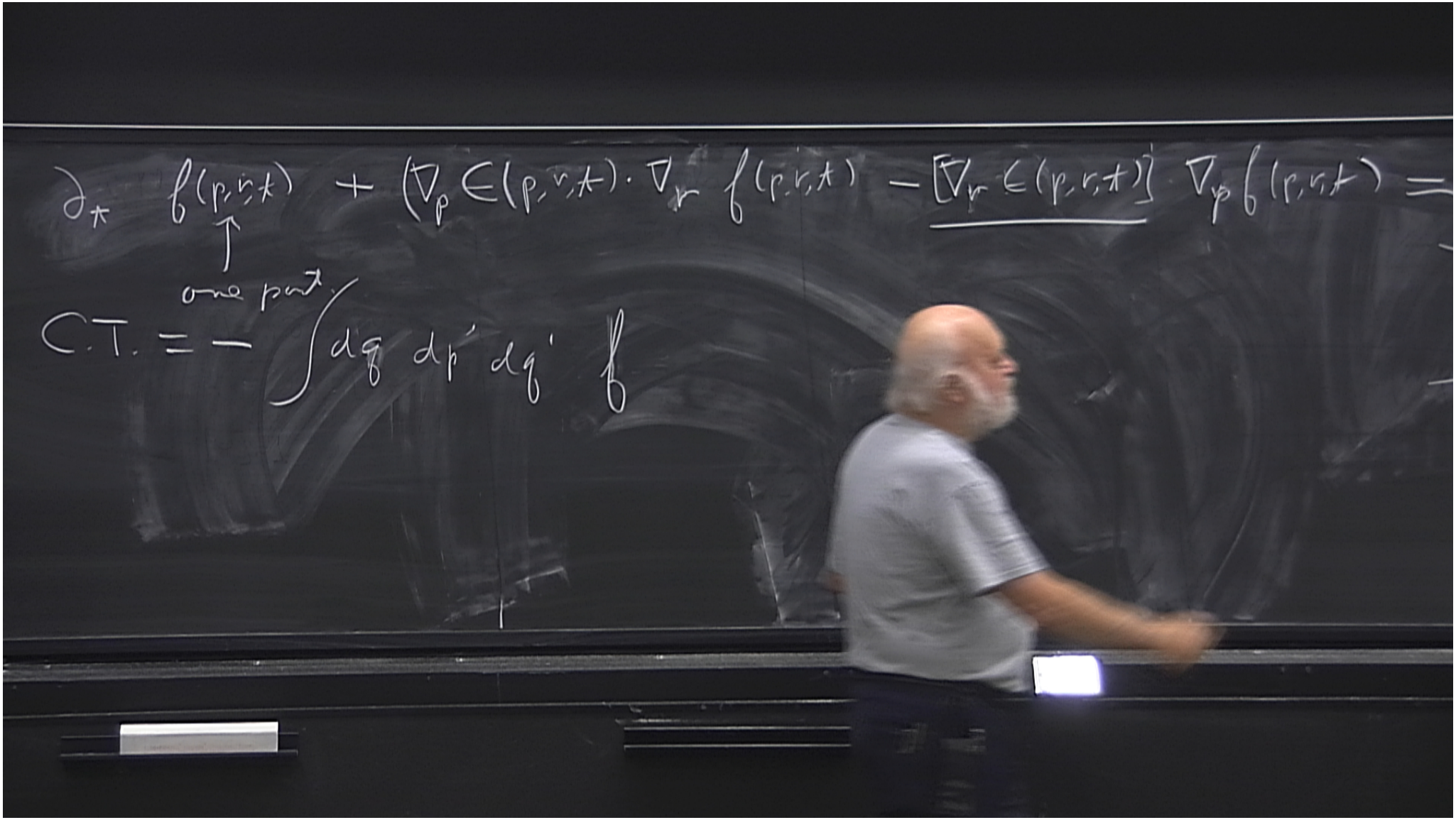

$\cdot \nabla_r f(p,r,t) - [\nabla_r \in (p,r,t)] \nabla_p f(p,r,t) = C.T.$
$\rho(p_\alpha, r_\alpha)$

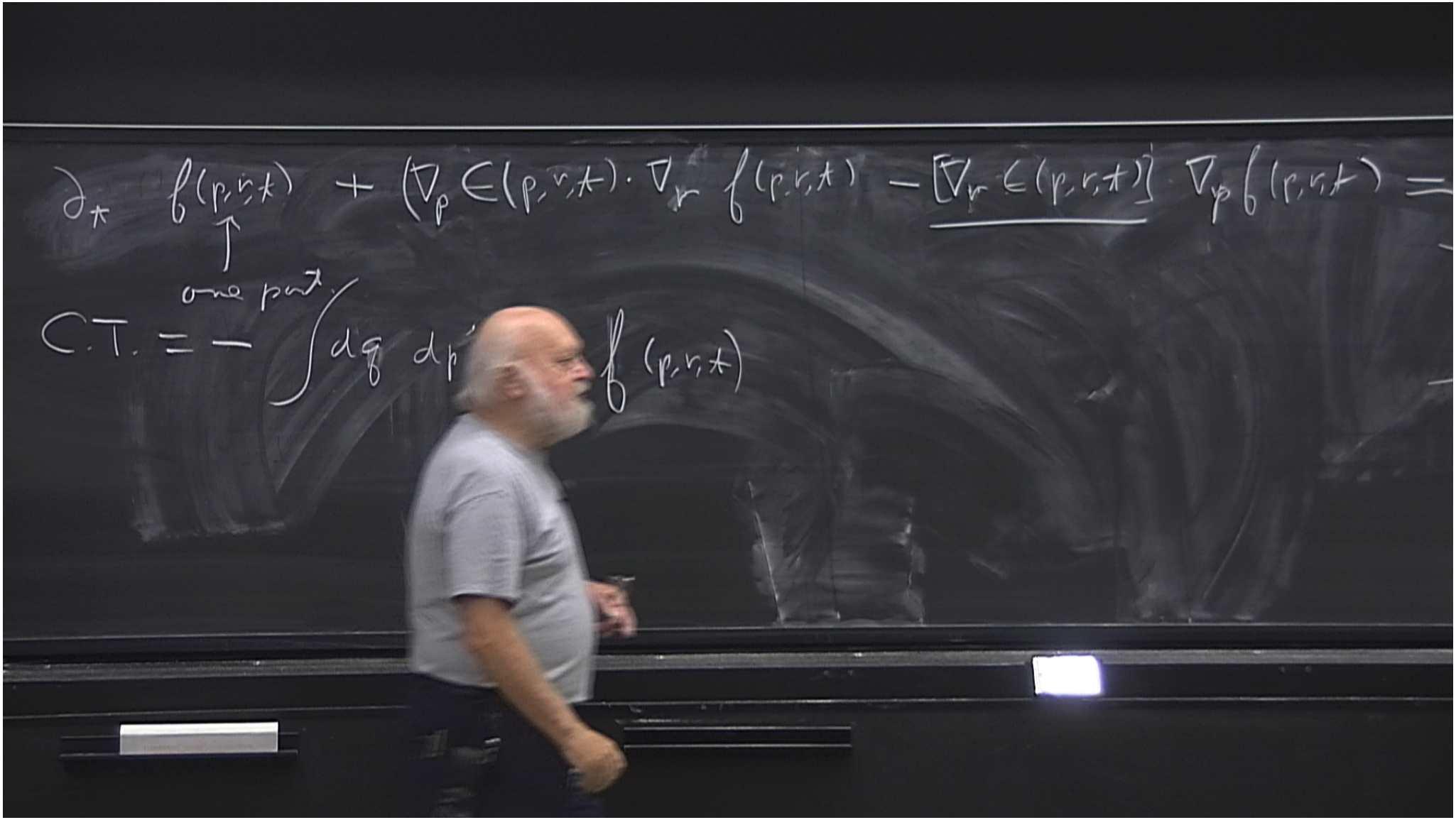

$\nabla_r f(p,r,t) - [\nabla_r \in (p,r,t)] \nabla_p f(p,r,t) = C.\dot{T}.$
$\rho(p_\alpha, r_\alpha)$
$\vec{p}$
$\vec{q}$
$\vec{q}'$

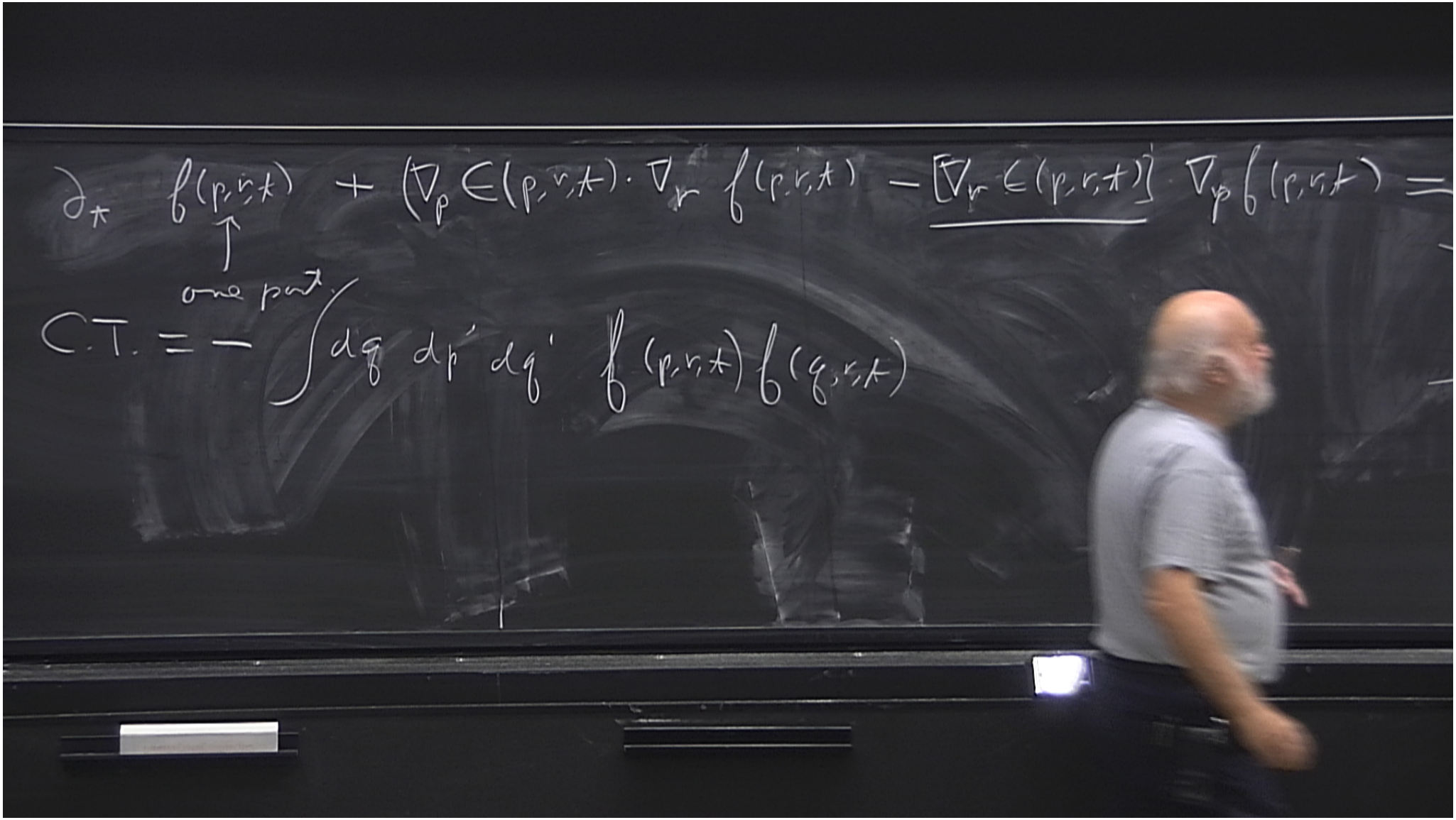

$\partial_t f(p,r,t) + (\nabla_p \in (p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r$
one part.
C.T. = —

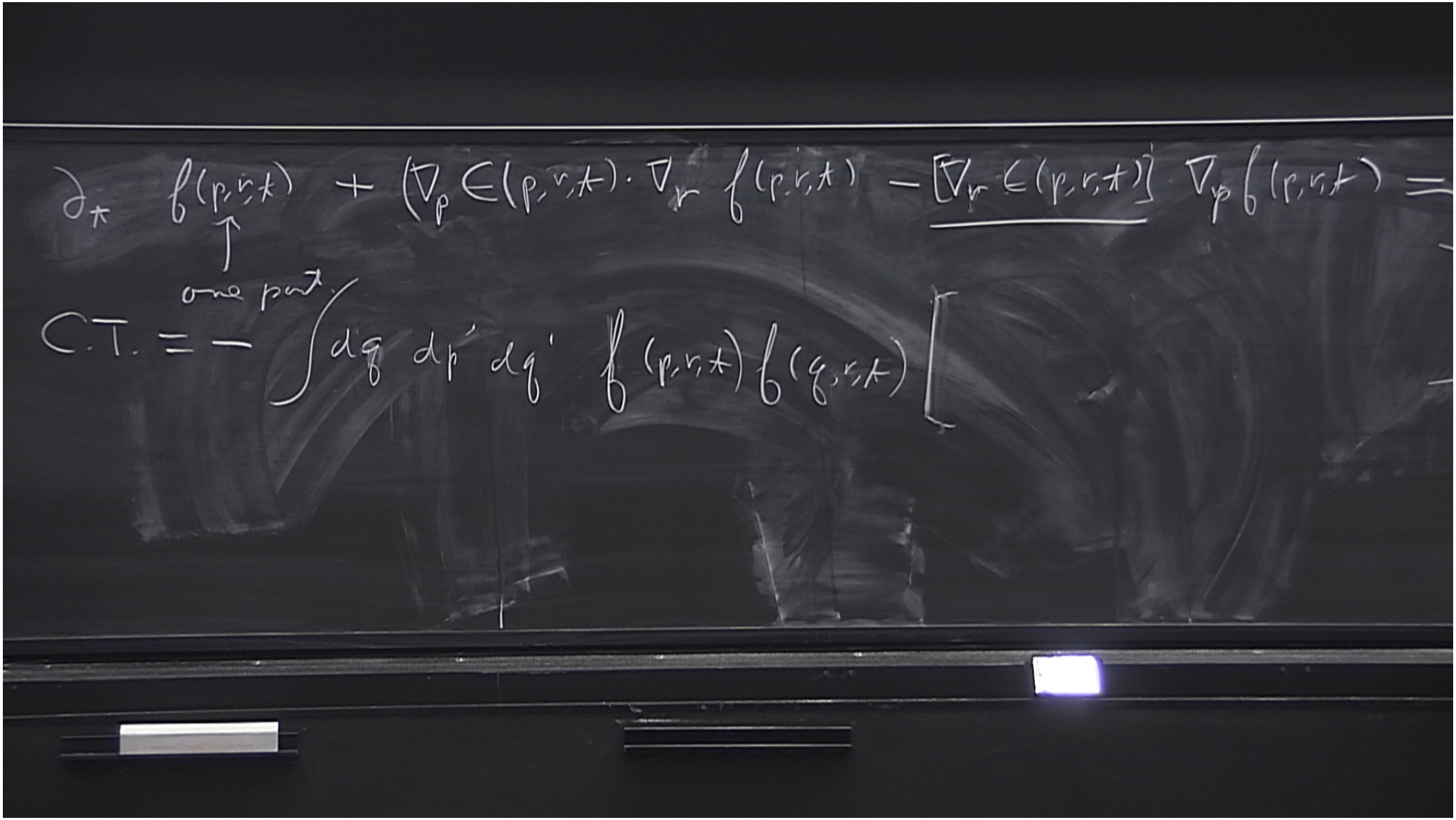

one part.

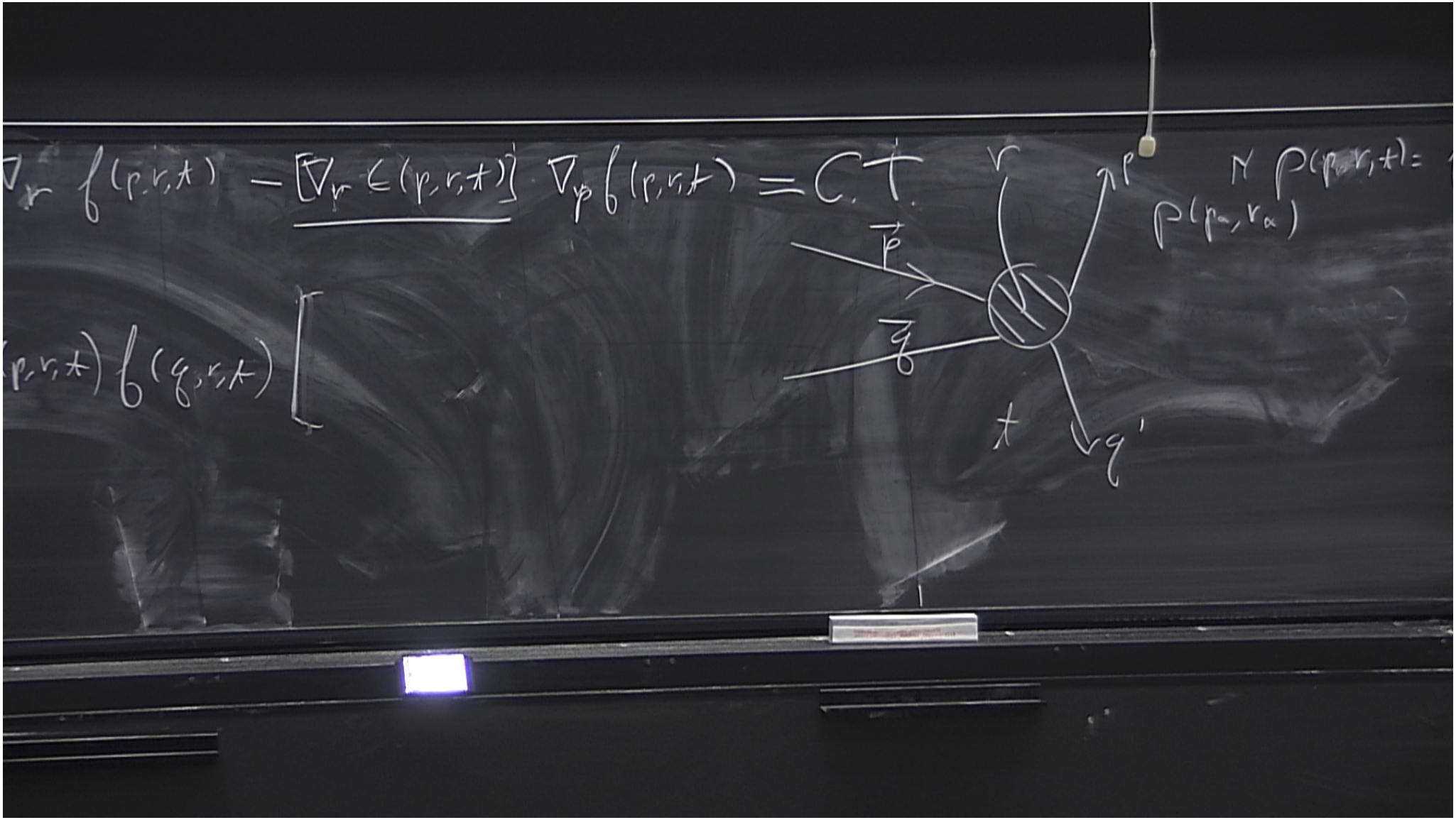

$\partial_t f(p,r,t) + (\nabla_p \in(p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r \in(p,r,t)] \nabla_p f(p,r,t) =$
one
C.T. = —
$dp' dq$

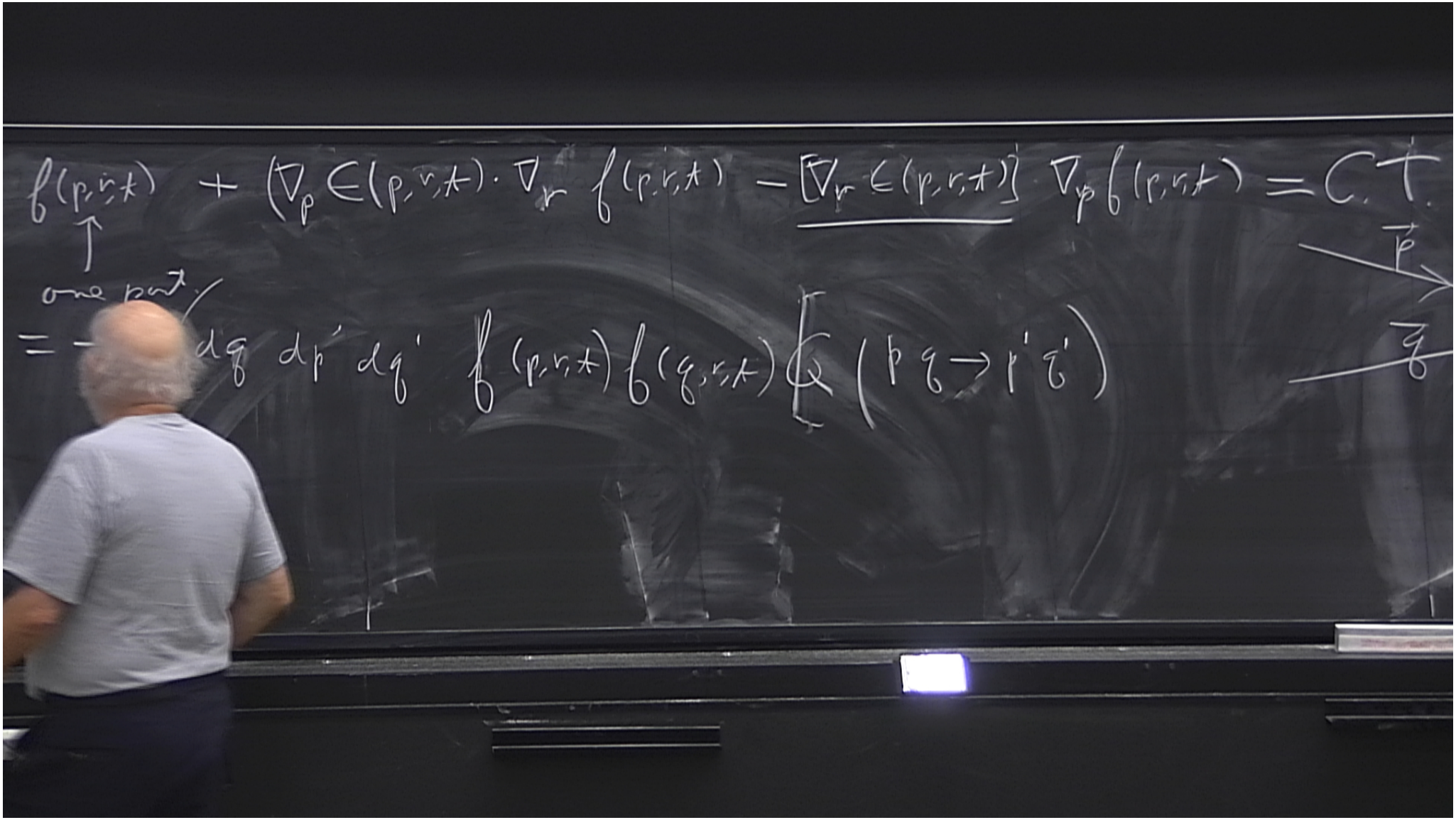

$\partial_t f(p,r,t) + (\nabla_p \in(p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r \in(p,r,t)] \nabla_p f(p,r,t) =$
one part.
$C.T. = -\int dq\, dp'\, dq'\, f$

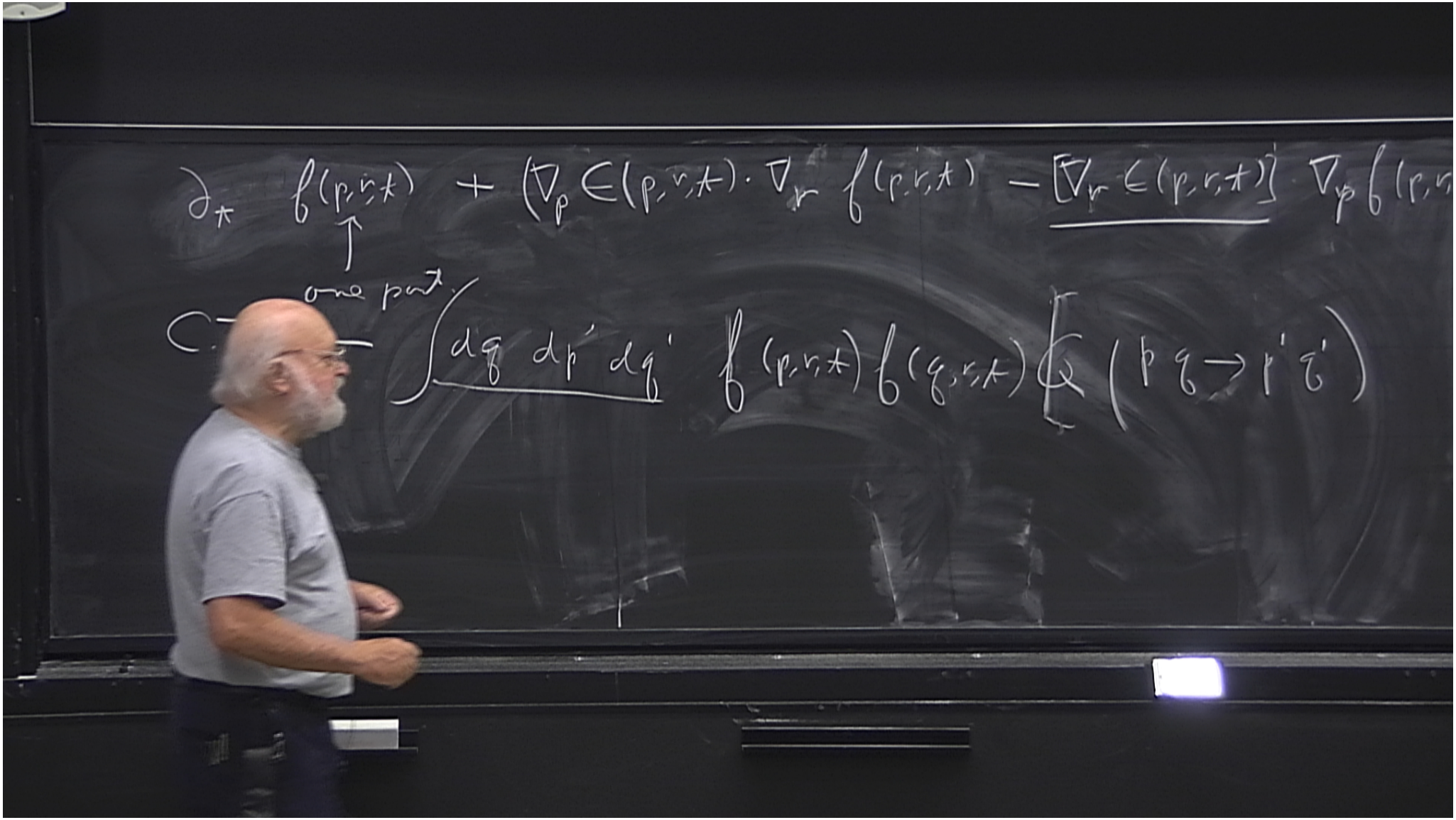

one part.

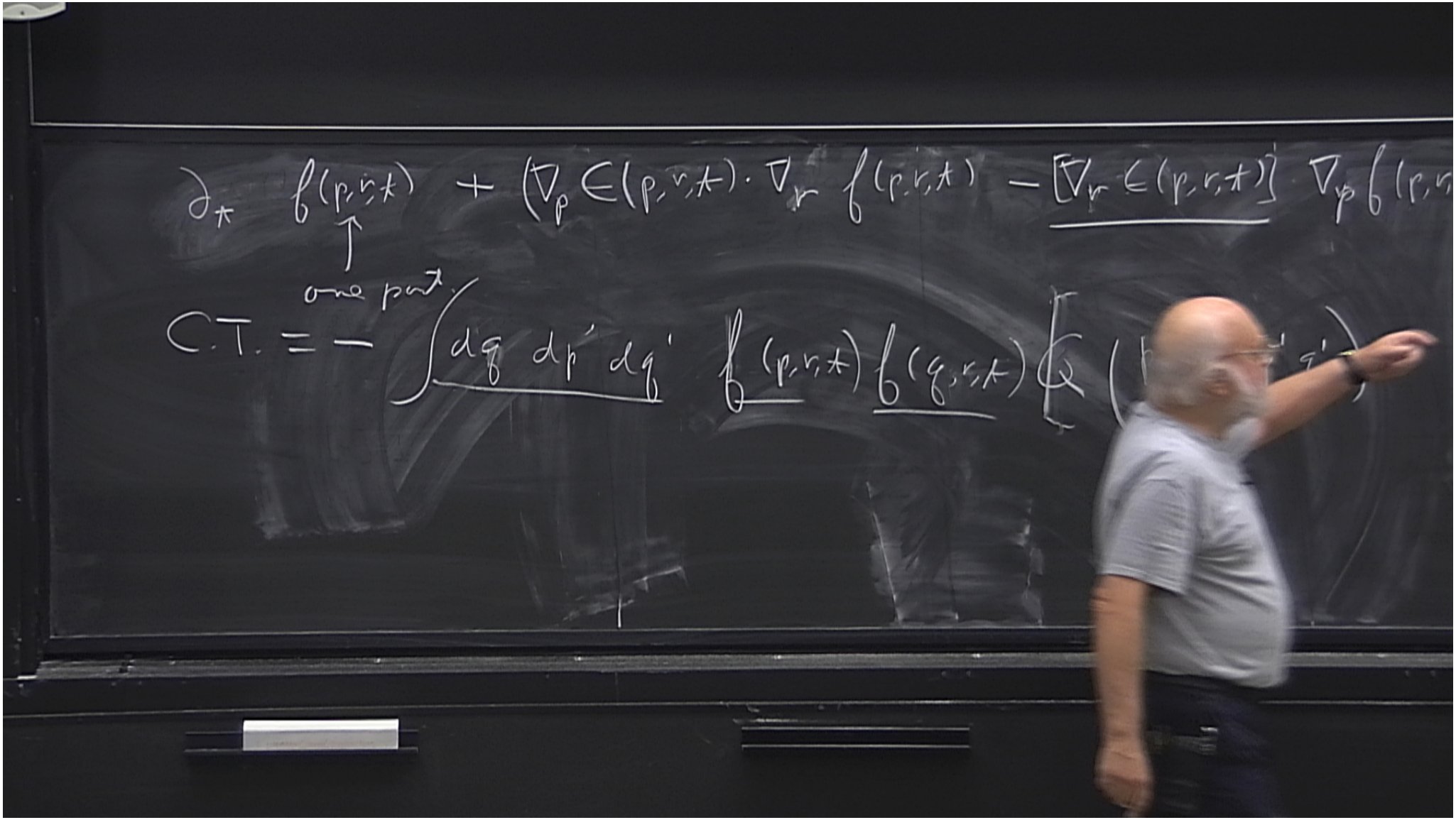

$\partial_t f(p,r,t) + (\nabla_p \in(p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r \in(p,r,t)] \cdot \nabla_p f(p,r,t) =$
one part.
$C.T. = -\int dq\, dp'\, dq'\, f(p,r,t) f(q,r,t)$

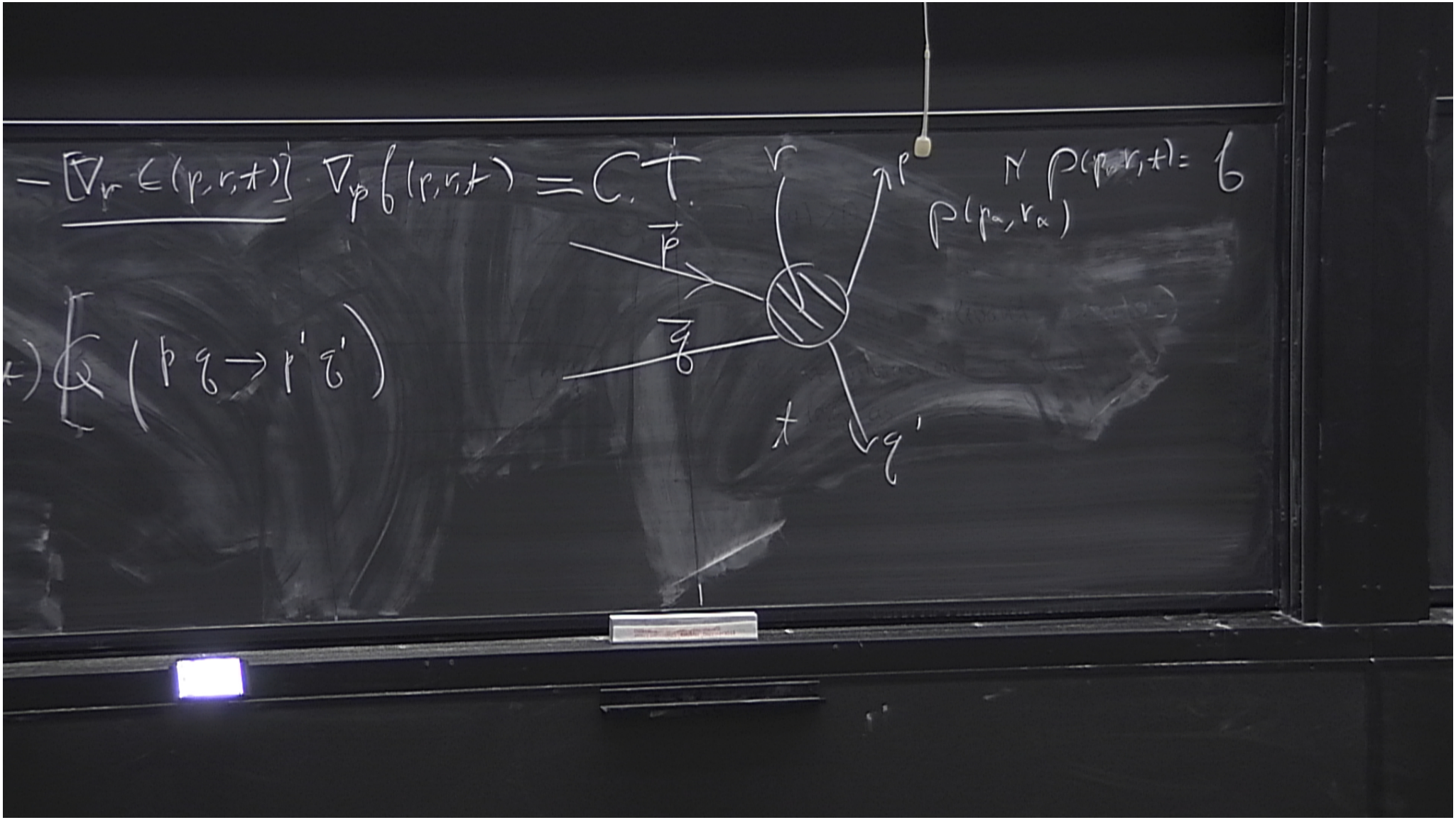

$\partial_t f(p,r,t) + (\nabla_p \in(p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r \in(p,r,t)] \nabla_p f(p,r,t) =$
one part.
$C.T. = -\int dq\, dp'\, dq'\, f(p,r,t) f(q,r,t) [$

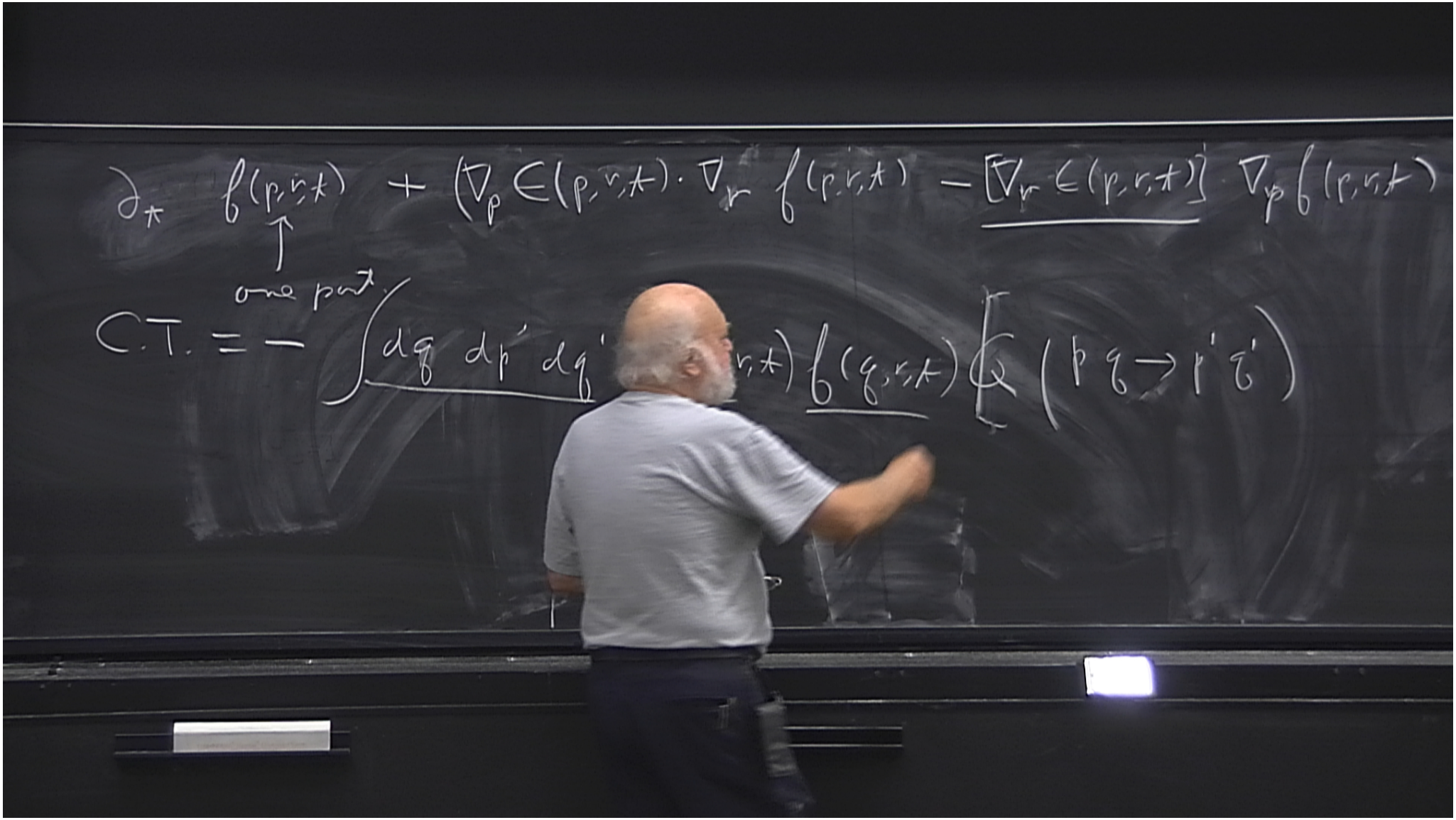

r
p
p
q
q'
t

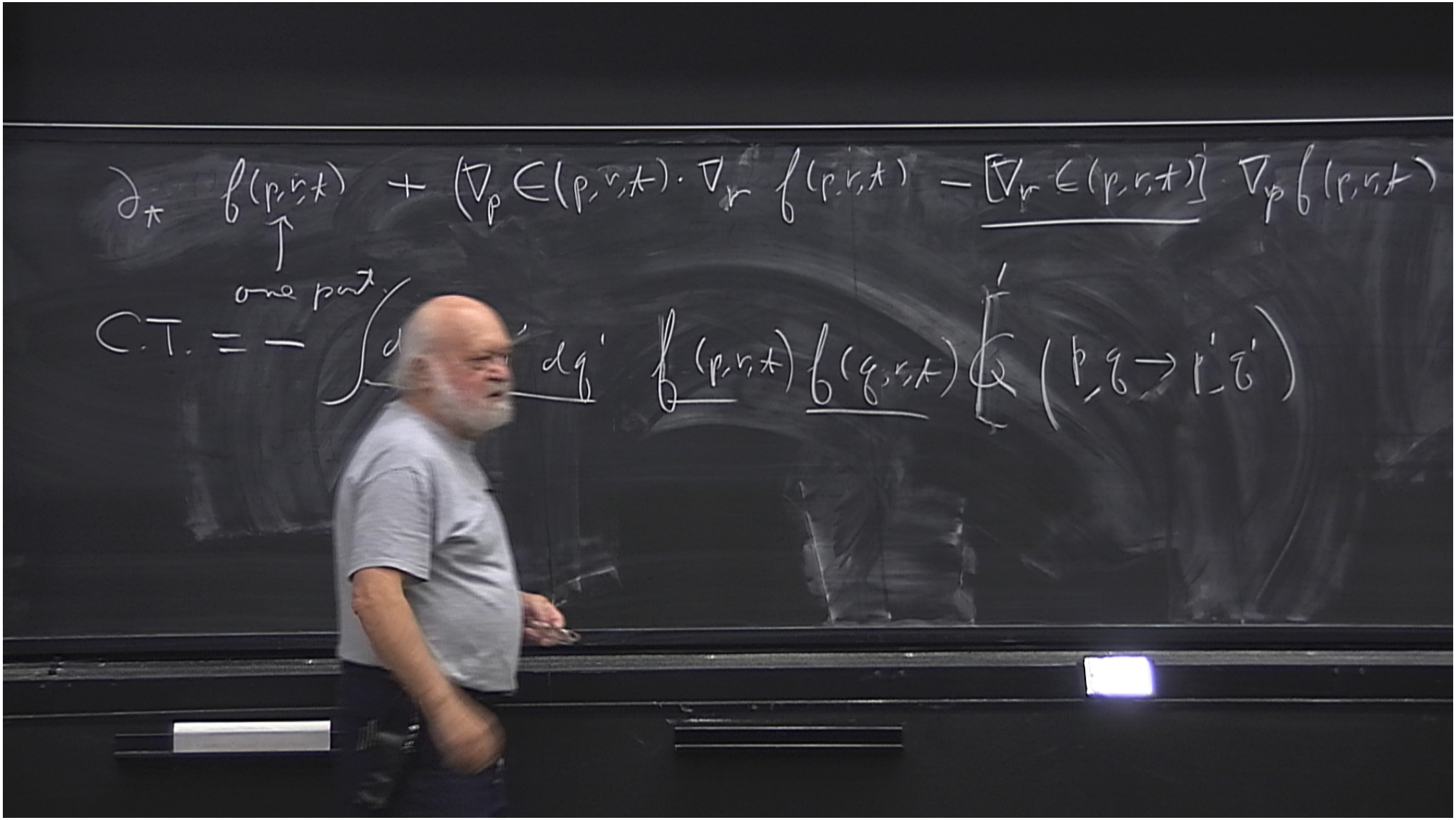

$f(p,r,t) + (\nabla_p \in (p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r \in (p,r,t)] \nabla_p f(p,r,t) = C.\dot{T}.$
one part
$= - \int dq\, dp'\, dq'\, f(p,r,t) f(q,r,t) \left(pq \to p'q' \right)$
$\vec{p}$
$\vec{q}$

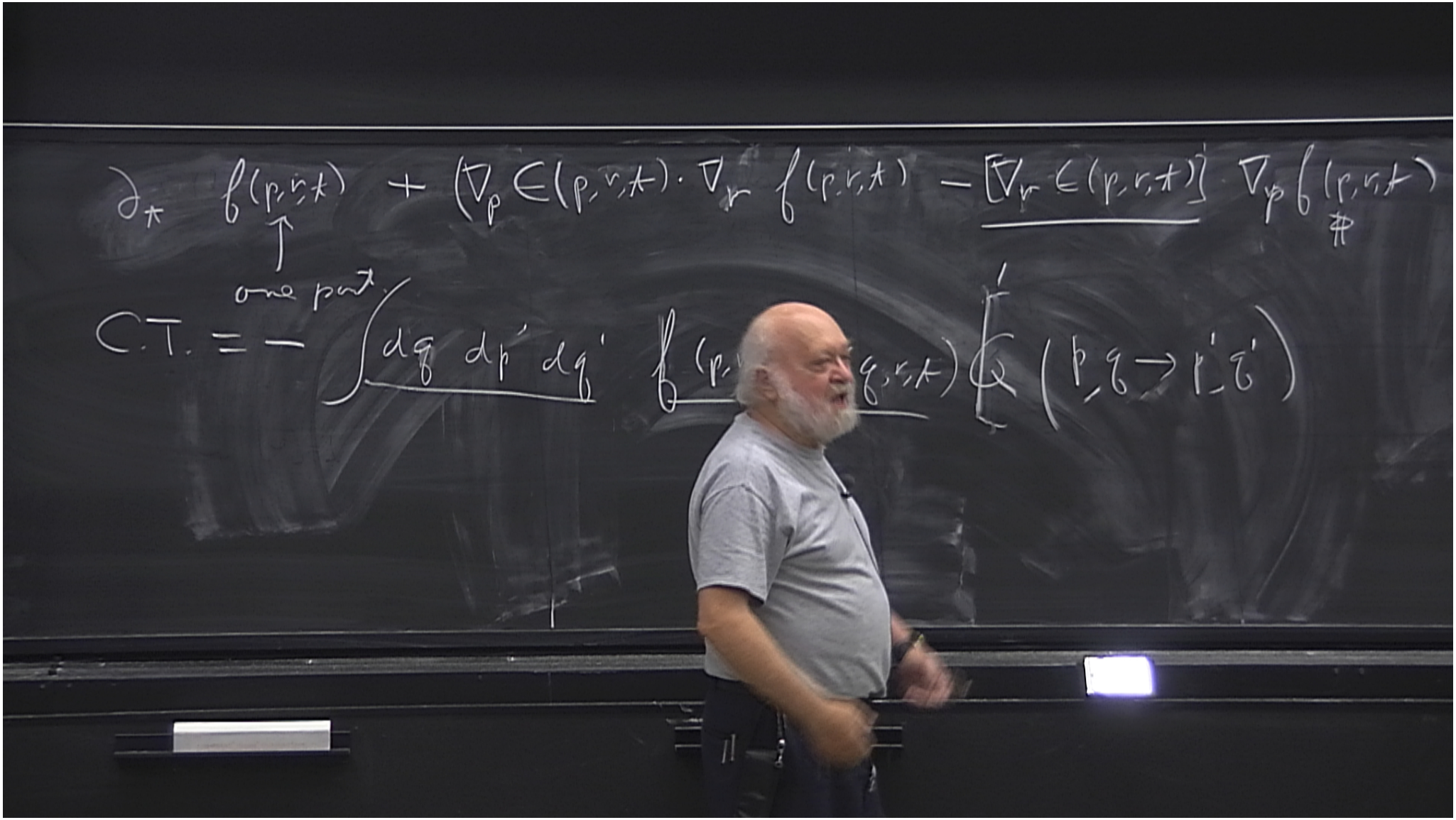

one part.

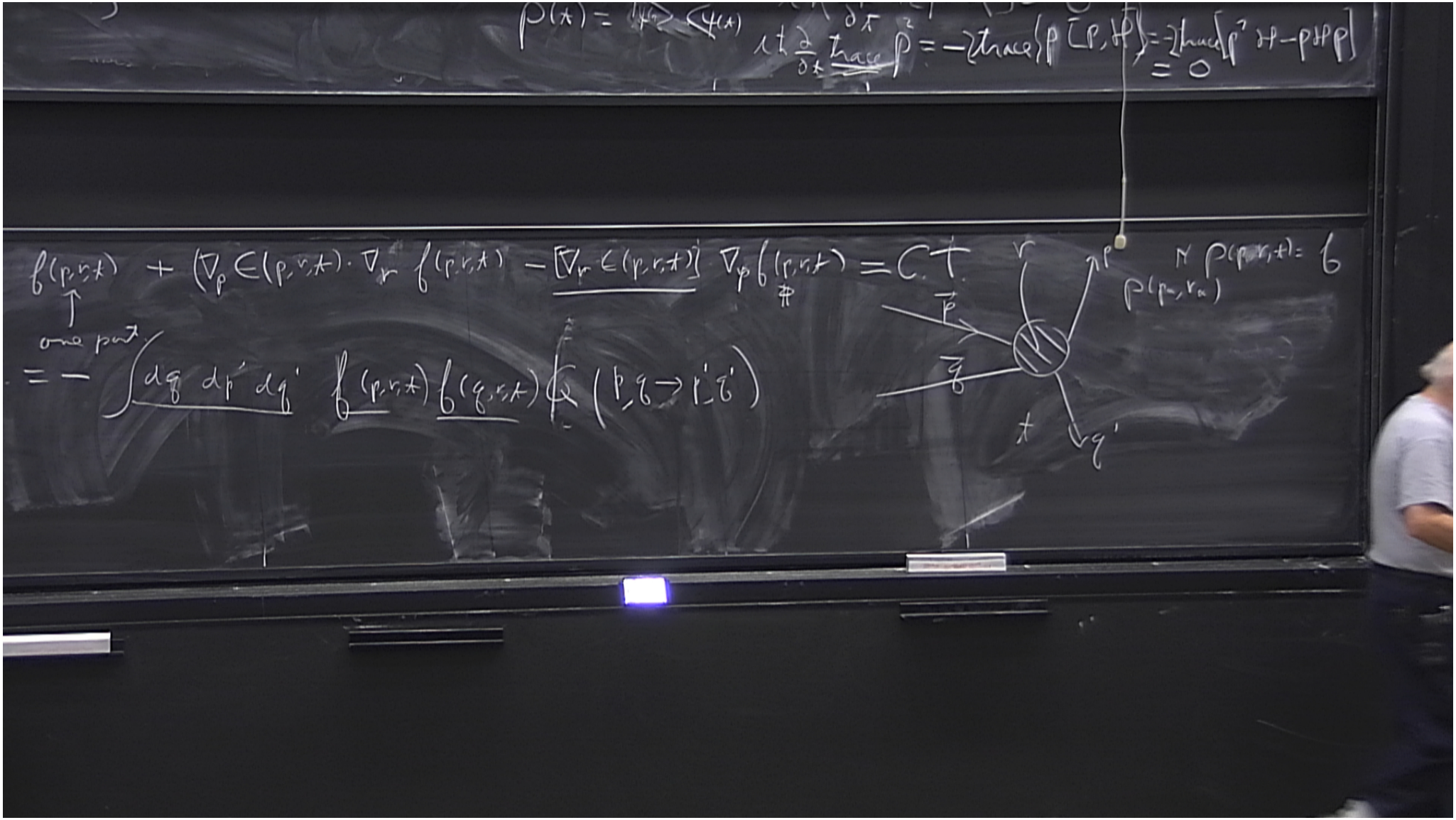

one part.

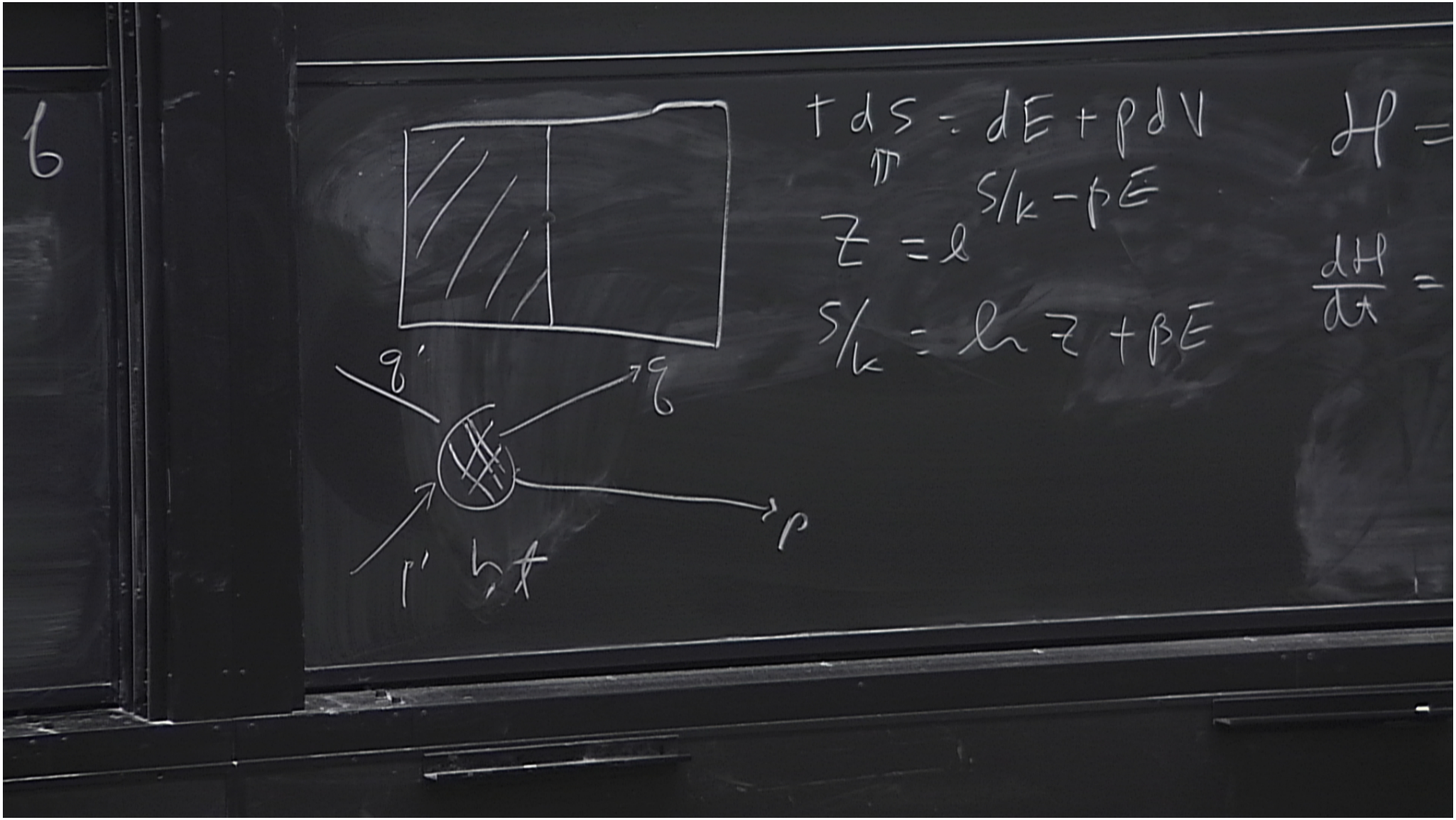

$-[\nabla_r \in (r,v,t)]\ \nabla_p f(r,v,t) = C.\dot{T}.$
$N\ \rho(r,v,t) = 6$
$\rho(r_\alpha, v_\alpha)$
$\vec{p}$
$\vec{q}$
r
p
t
q'
$+)\left[Q\left(pq \to p'q'\right)\right.$

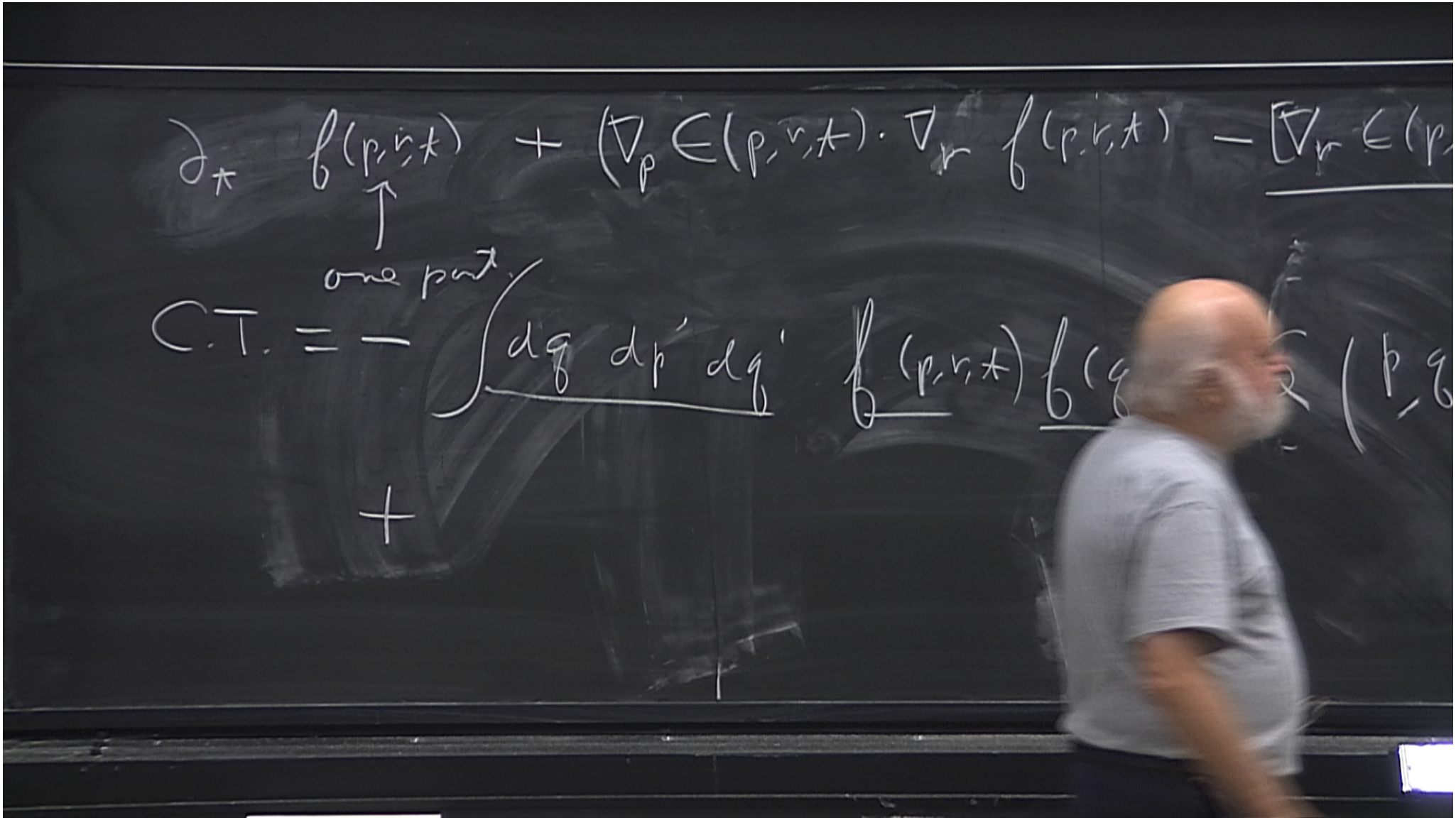

one part.

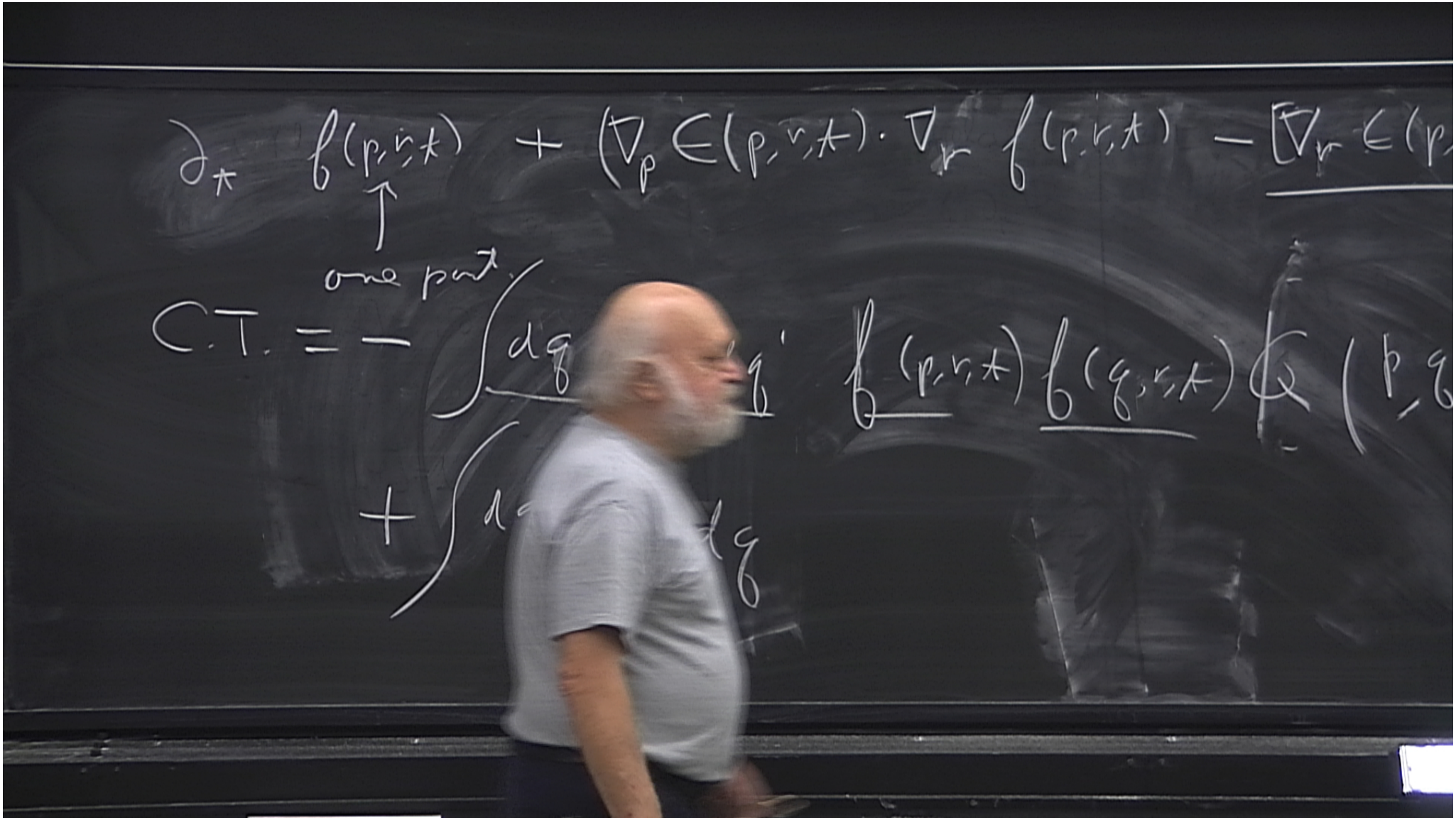

one part.

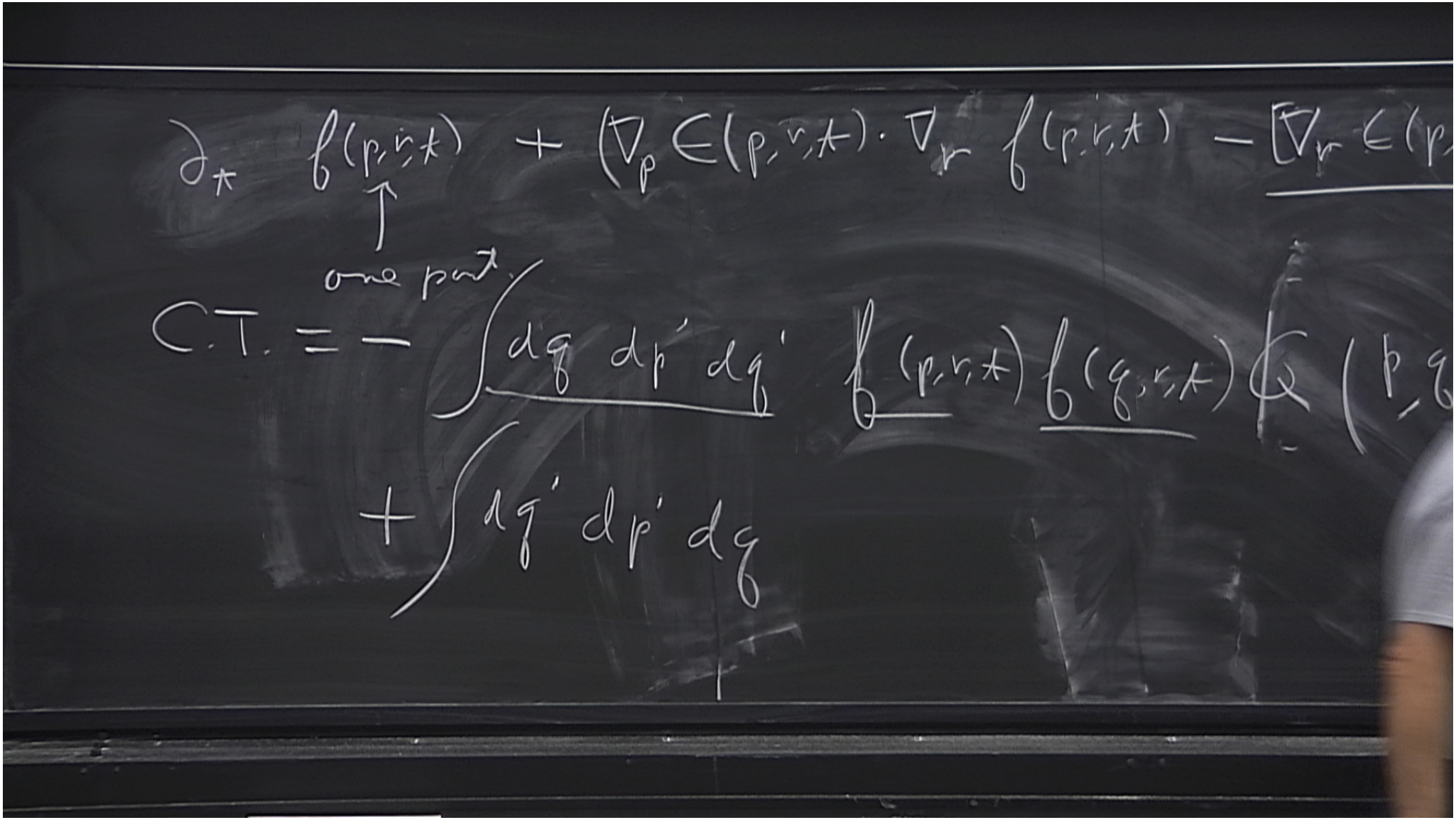

one part.

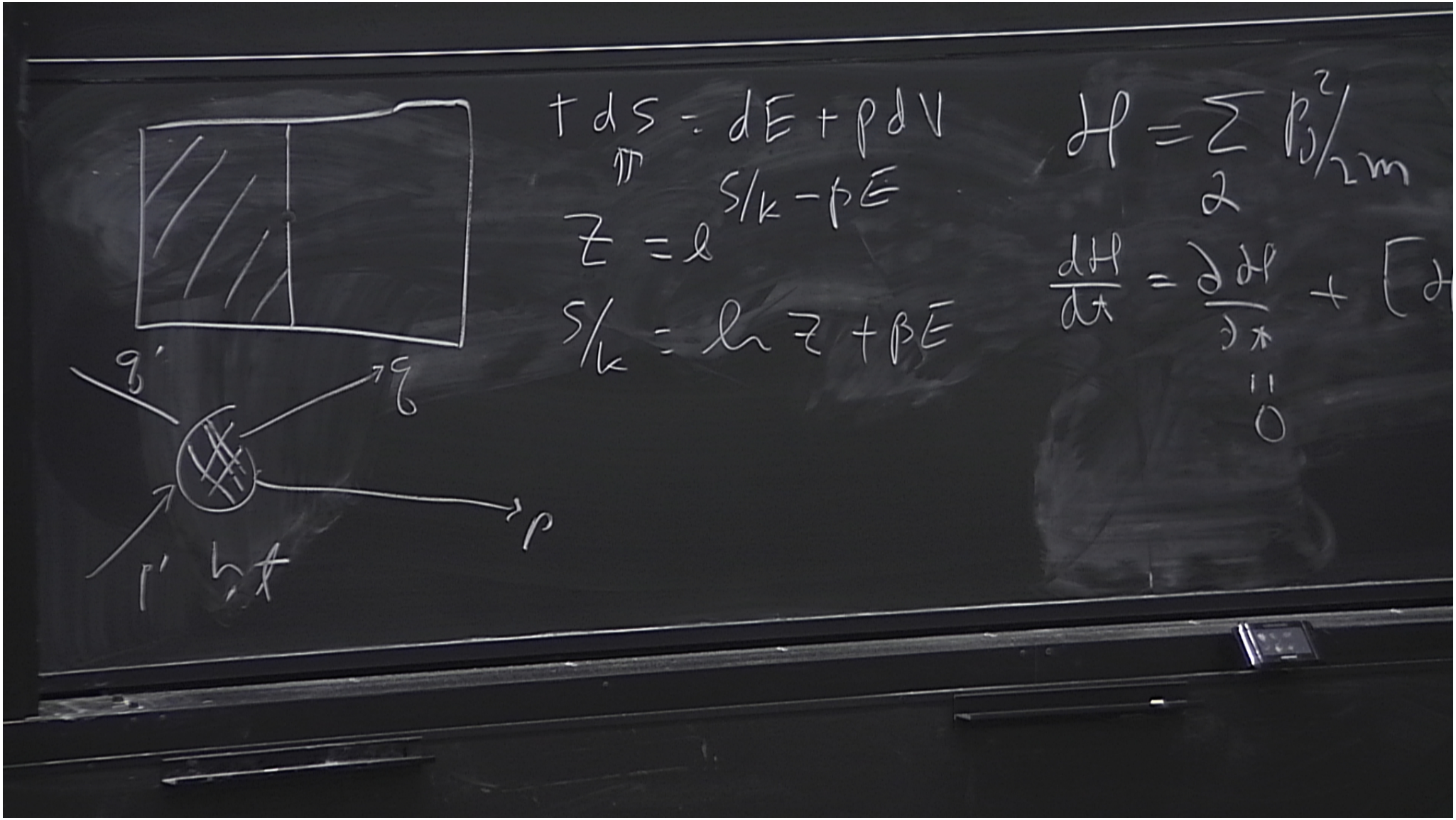

$\rho(t) = |\psi\rangle\langle\psi(t)|$
$i\hbar \frac{\partial}{\partial t} \text{Trace}\,\rho^2 = -\text{Trace}\{\rho\,[\rho, H]\} = -\text{Trace}[\rho^2 H - \rho H \rho] = 0$
$f(p,r,t) + (\nabla_p \in(p,r,t)\cdot\nabla_r f(p,r,t) - [\nabla_r \in(p,r,t)]\,\nabla_p f(p,r,t) = C.T.$
one part
$= -\int dq\,dp'\,dq'\; f(p,r,t)\,f(q,r,t)\,\Phi(p,q \to p',q')$
$N\,\rho(p,r,t) = 6$
$\rho(p_n, r_n)$
p
q
r
p'
q'

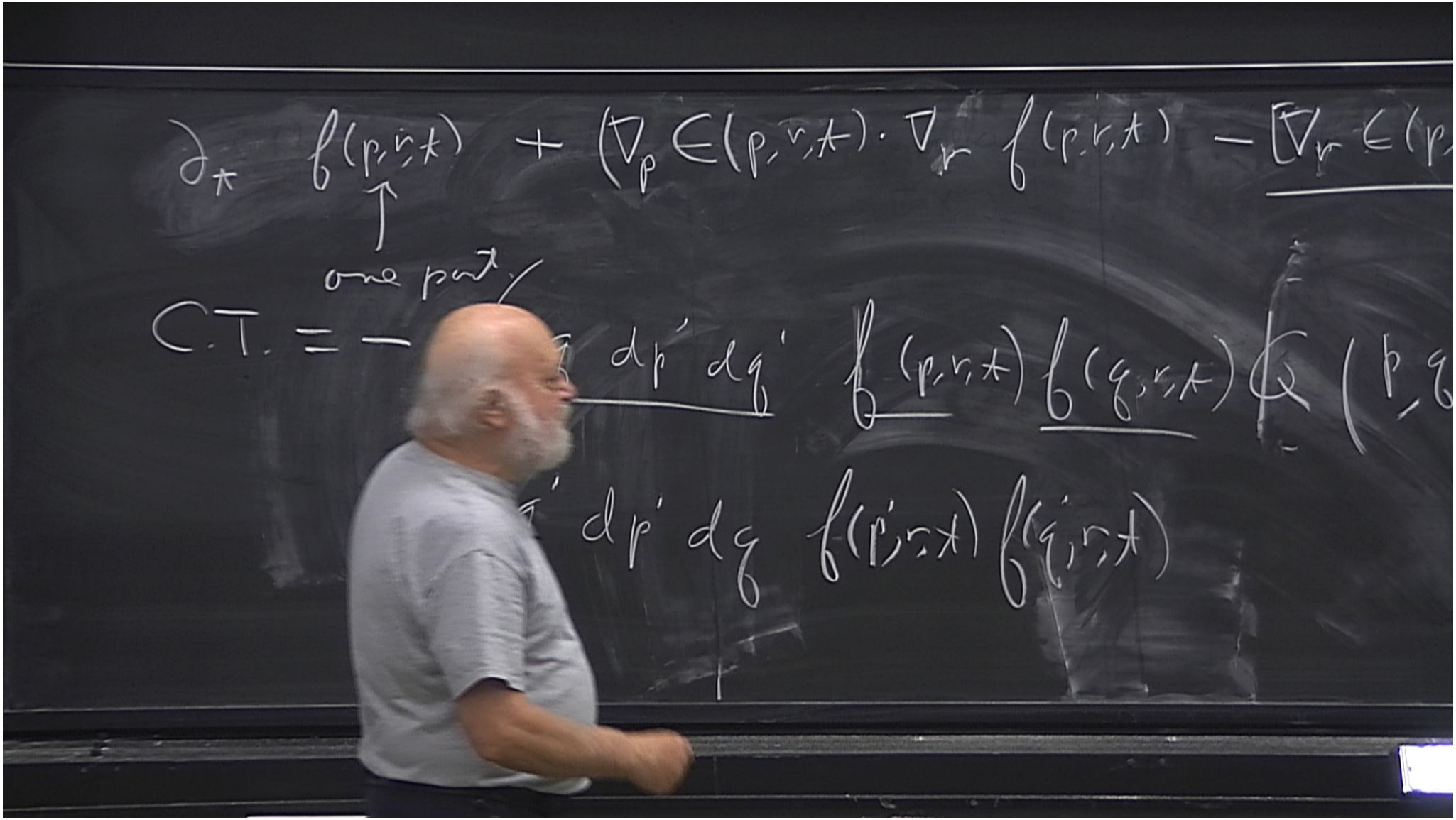

6
$TdS = dE + pdV$
Π
$Z = e^{S/k - \beta E}$
$S/k = \ln Z + \beta E$
$d\rho =$
$\frac{dH}{dt} =$
q'
q
p
p' h, t


one part.


one part


$\partial_t f(p,r,t) + (\nabla_p \in (p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r \in (p,$
one part.
$C.T. = -\int dq\, dp'\, dq'\, f(p,r,t) f(q,r,t) \Phi(p,q$
$+ \int dq'\, dp'\, dq$


$TdS = dE + pdV$
$\Rightarrow$
$Z = e^{S/k - \beta E}$
$S/k = \ln Z + \beta E$
$\mathcal{H} = \sum_2 p^2/2m$
$\frac{dH}{dt} = \frac{\partial \mathcal{H}}{\partial t} + [\mathcal{H}$
$\| 0$
q'
q
p
p' h,t

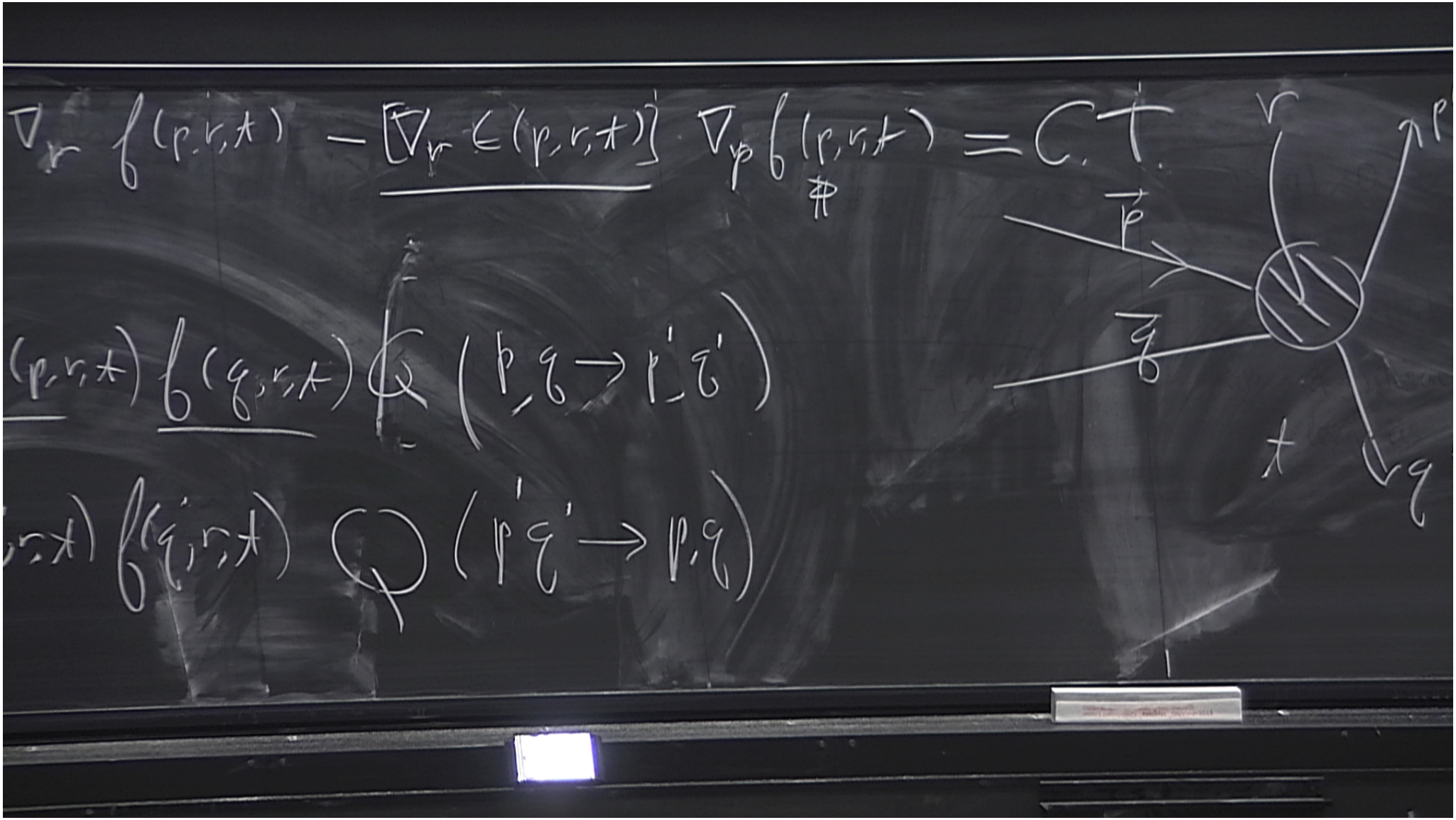

one part

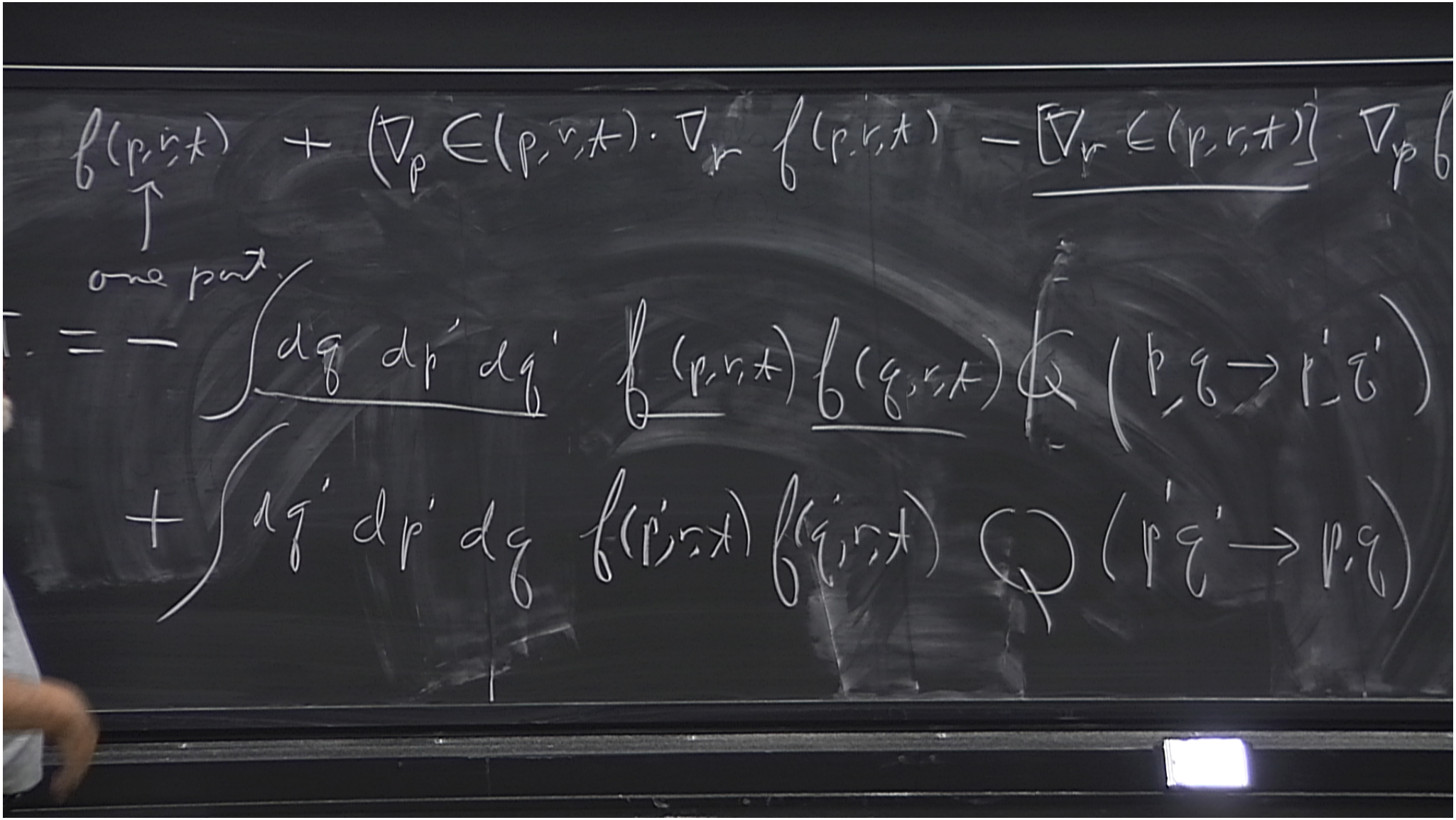
$$\partial_t f(p,r,t) + (\nabla_p \epsilon(p,r,t) \cdot \nabla_r f(p,r,t) - [\nabla_r \epsilon(p,$$

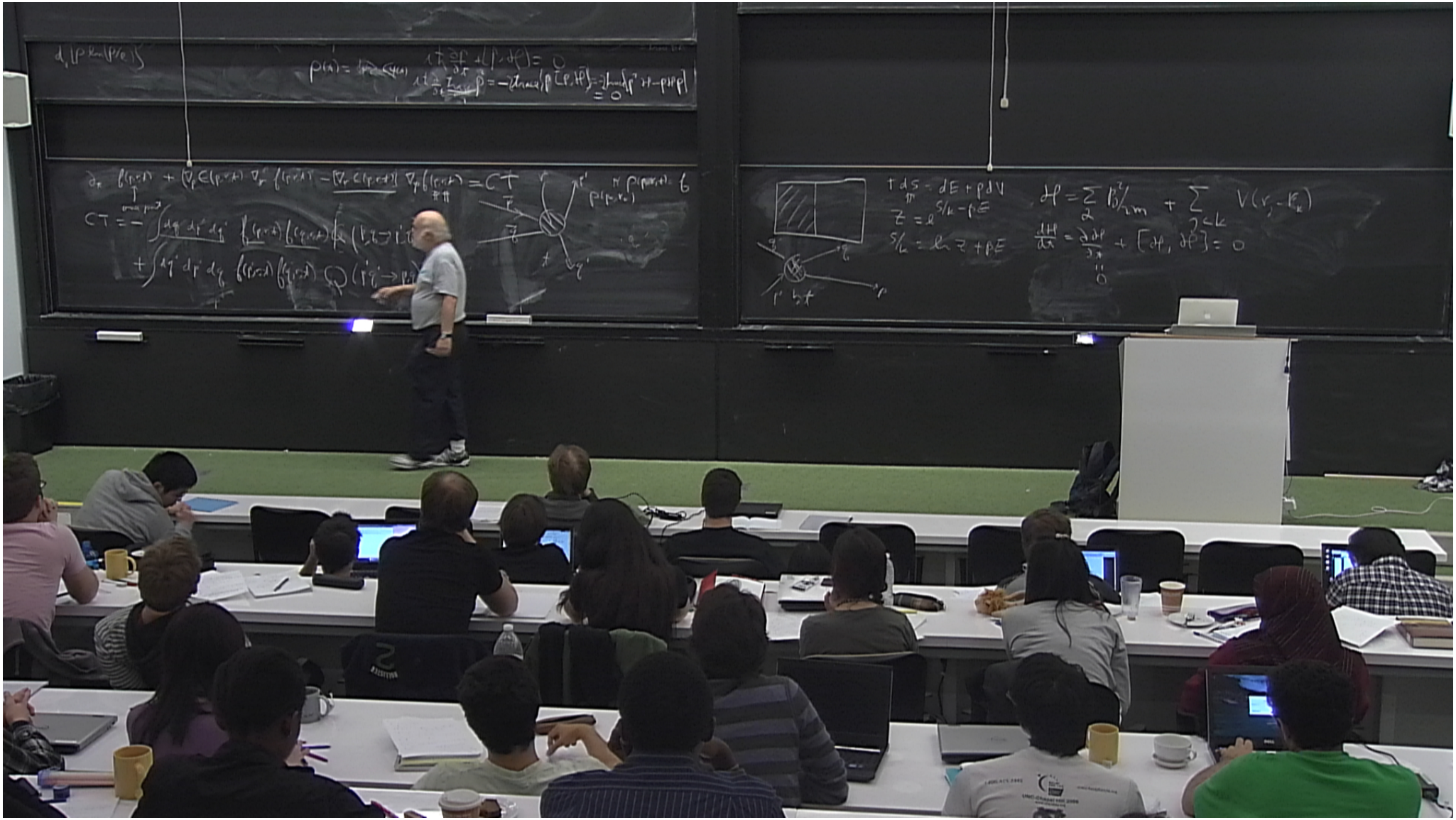
one part

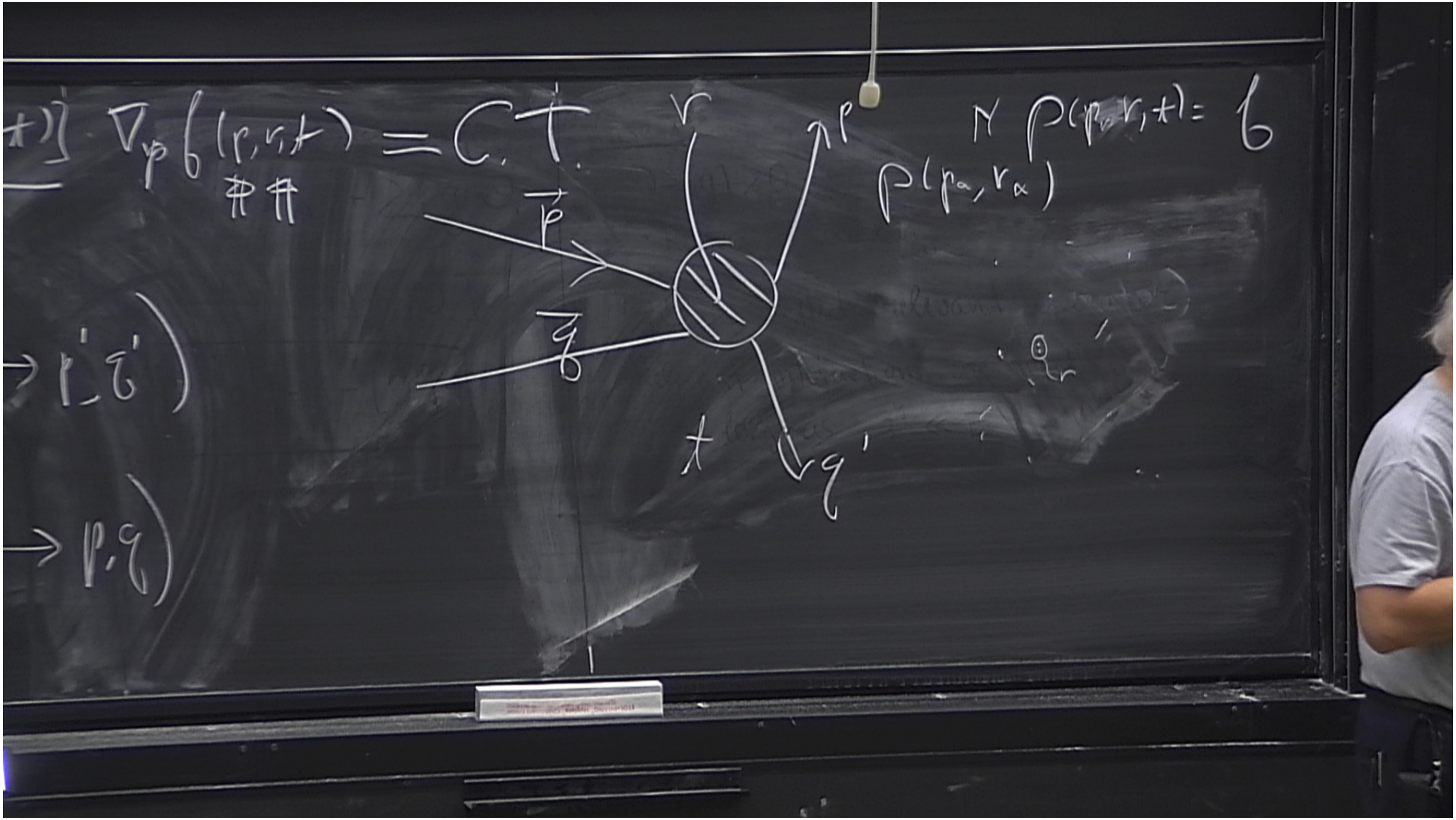
$$C.T. = -\int dq\, dp'\, dq'\, f(p,r,t)\, f(q,r,t)\, Q(p,q$$

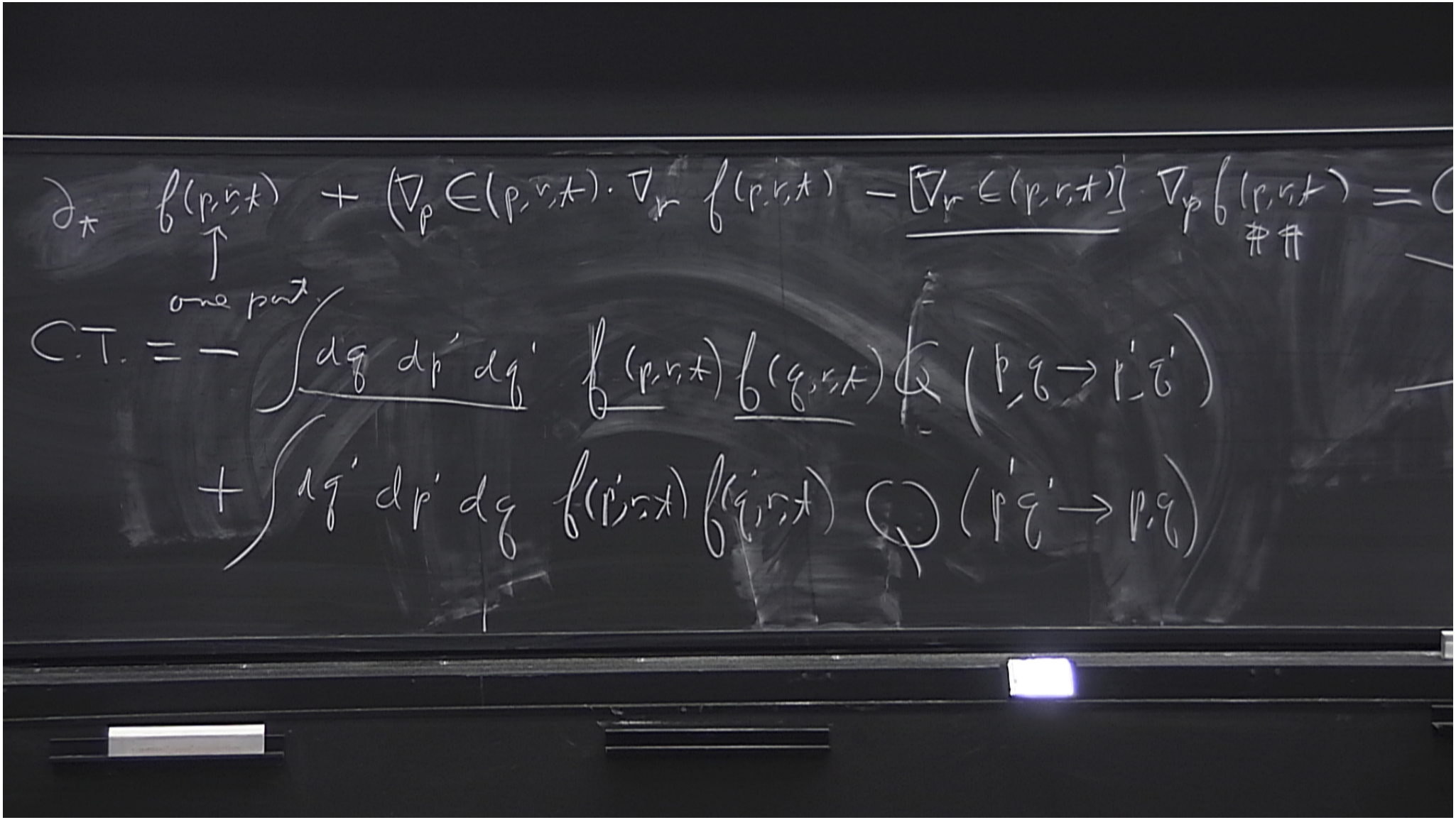
$$+ \int dq'\, dp'\, dq\, f(p',r,t)\, f(q,r,t)\, Q$$

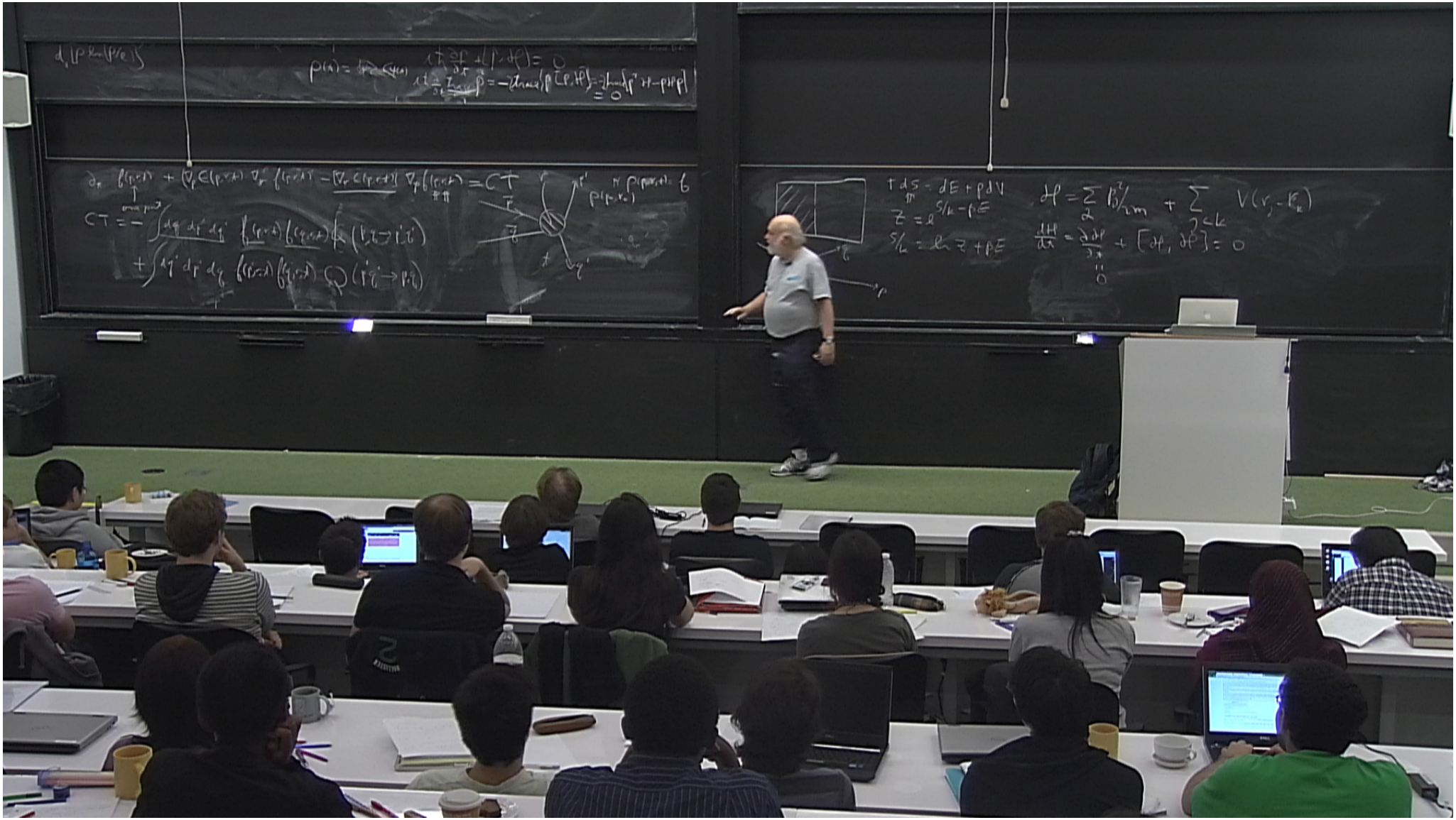

$\nabla_r f(r,v,t) - [\nabla_r \in (r,v,t)] \nabla_p f(r,v,t) = C.\dot{t}$
$(r,v,t) f(q,v,t) Q(p,q \to p',q')$
$f(q',v,t) Q(p'q' \to p,q)$
$\vec{p}$
$\vec{q}$
t

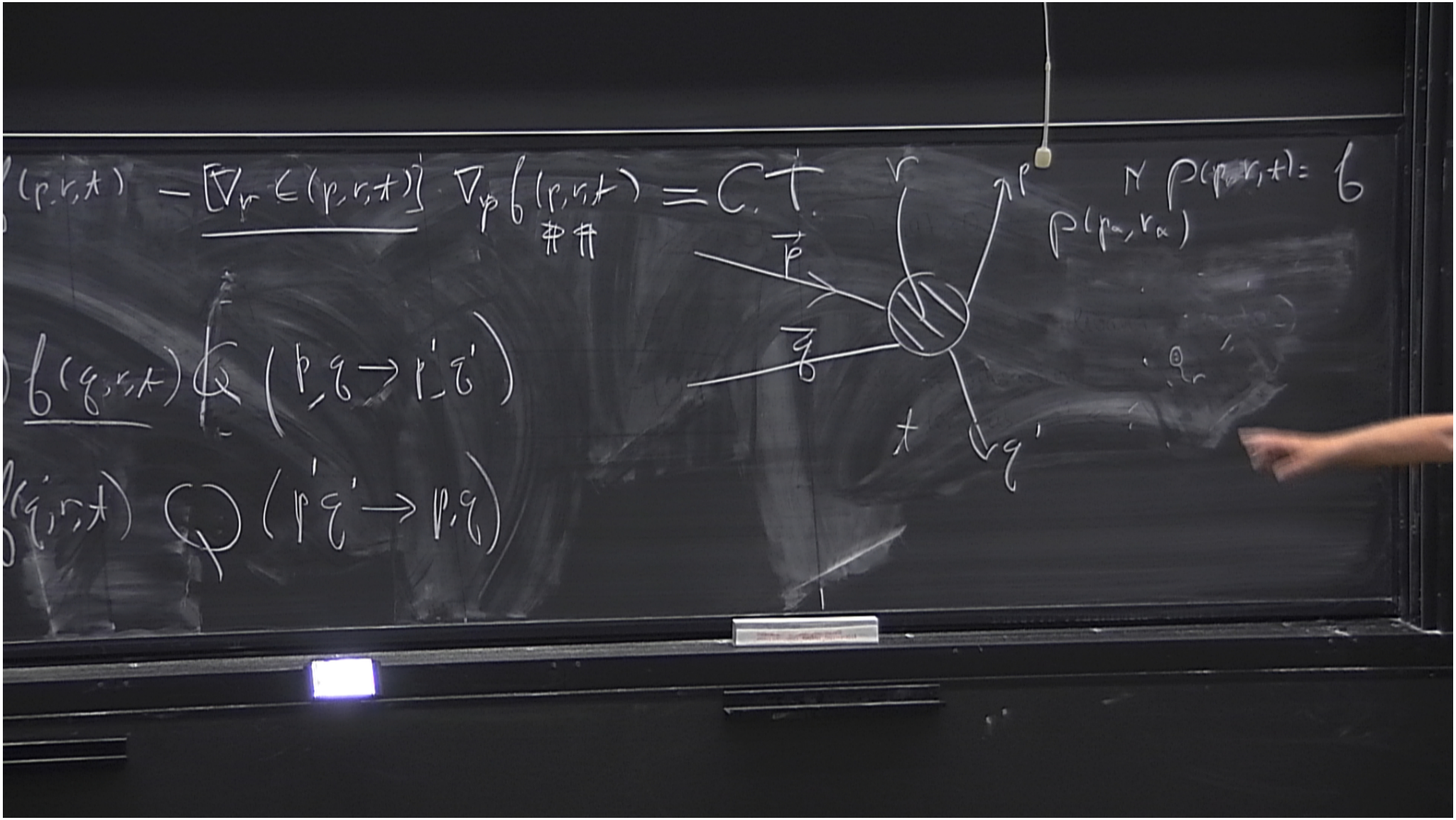

f(p,r,t) + (∇_p ∈(p,r,t) · ∇_r f(p,r,t) − [∇_r ∈(p,r,t)] · ∇_p f
one part.
= − ∫ dq dp' dq' f(p,r,t) f(q,r,t) Q(p,q → p',q')
+ ∫ dq' dp' dq f(p',r,t) f(q',r,t) Q(p'q' → p,q)

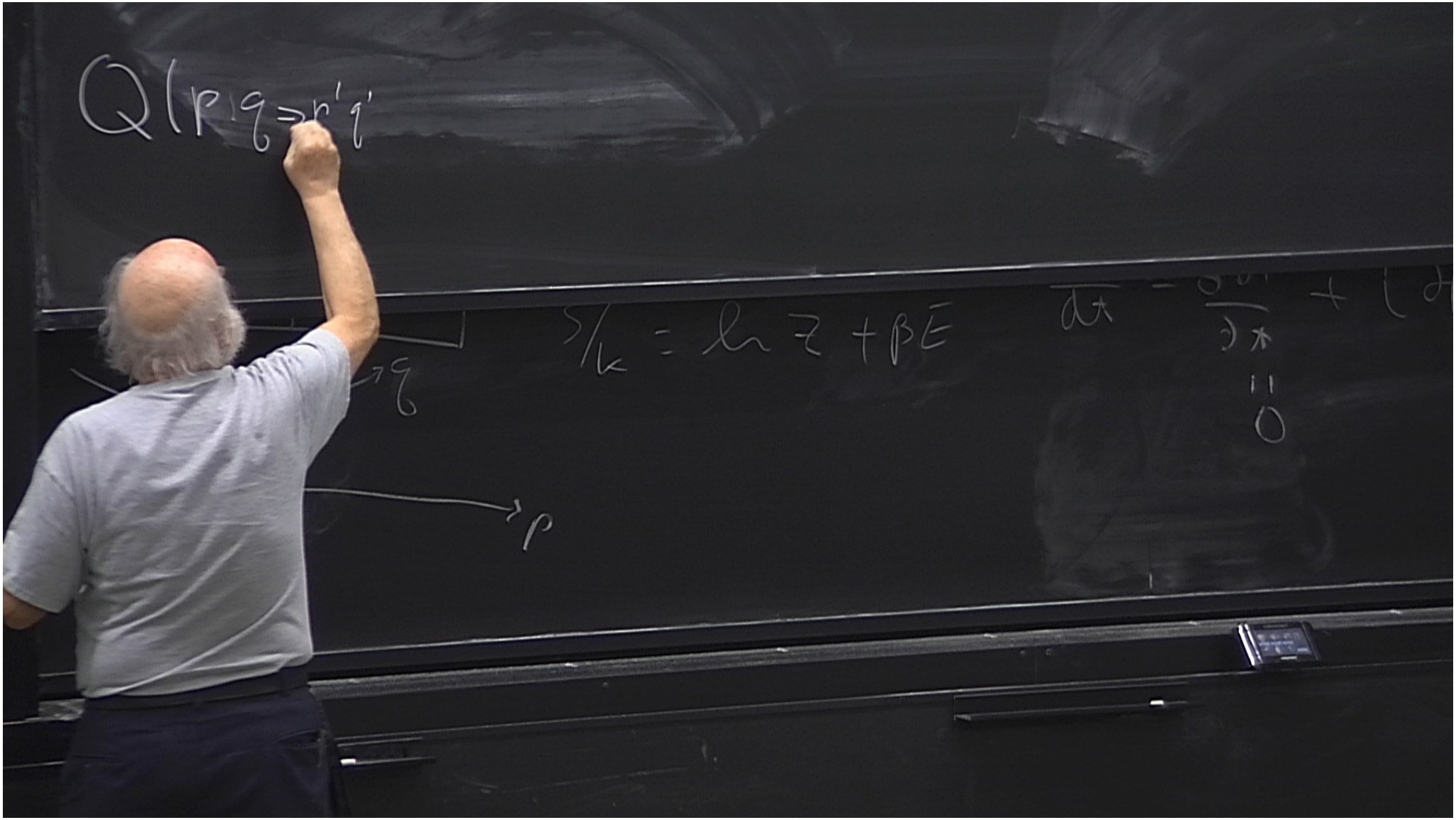



$\nabla_p f(r,v,t) = C \cdot \dot{t}$
$\rho(p_\alpha, r_\alpha)$
$N \rho(p_\alpha, r_\alpha, t) = 1$
$\vec{p}$
$\vec{q}$
q'

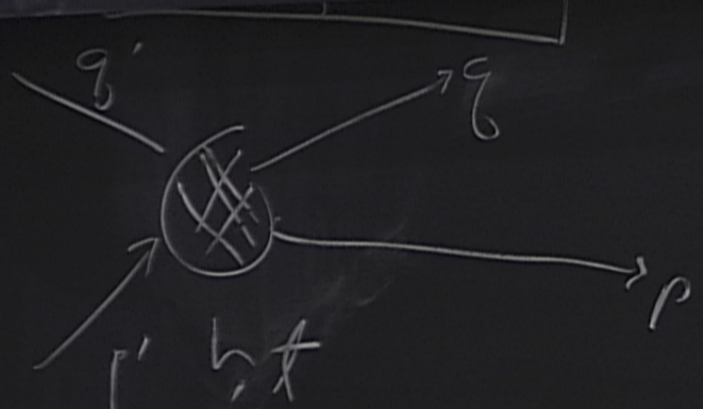

$\partial_t f(p,r,t) + (\nabla_p \in(p,r,t)\cdot\nabla_r f(p,r,t) - [\nabla_r \in(p,r,t)]\cdot\nabla_p f(p,r,t) = C$
one part
$C.T. = -\int dq\, dp'\, dq'\, f(p,r,t) f(q,r,t) Q(p,q \to p',q')$
$+\int dq'\, dp'\, dq\, f(p',r,t) f(q',r,t) Q(p'q' \to p,q)$




$\vec{p}$
$\vec{q}$
$\vec{p}'$
$\vec{q}'$


Q|p,q⟩=|p',q'⟩
q
p

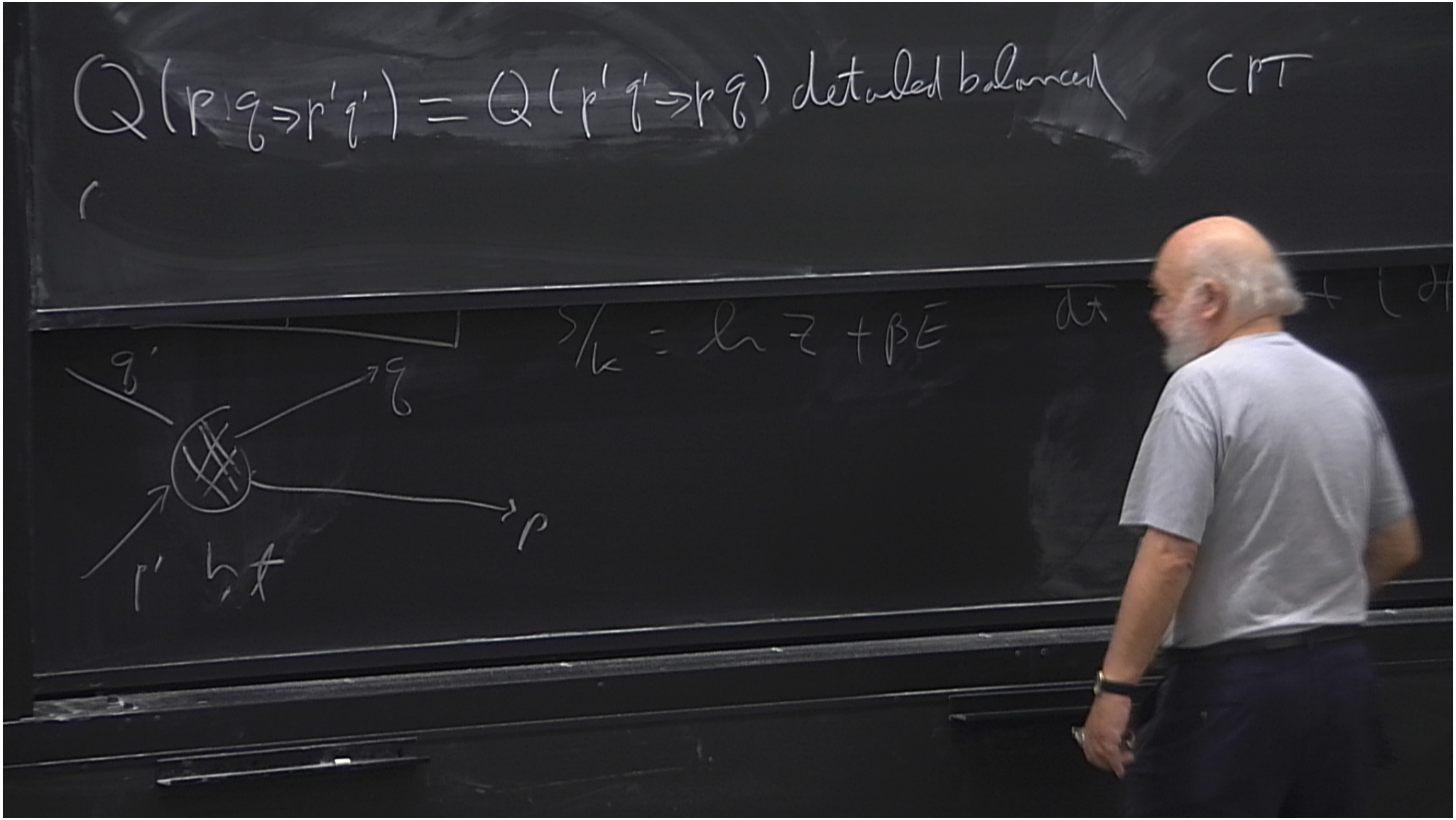
$Q(pq \to p'q') = Q(p'q' \to pq)$ detailed balance



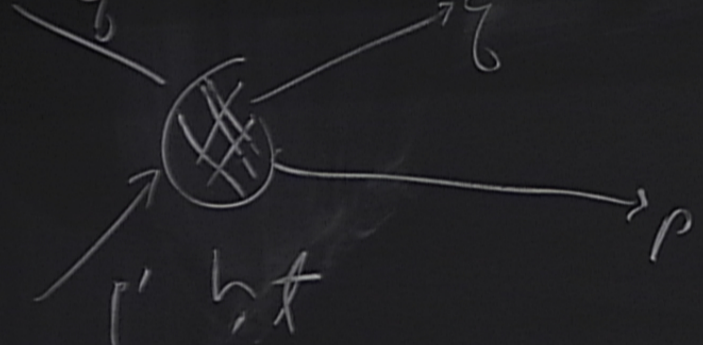
$S_k = \ln z + \beta E$

$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \dots$

$\frac{\partial S}{\partial t} = 0$


Q(pq⇒p'q') = Q(p'q'⇒pq) detailed balance
CPT
S/k = ln Z + βE
q'
q
p
p'

$Q(p,q \Rightarrow r',q') = Q(p',q' \Rightarrow r,q)$ detailed balance



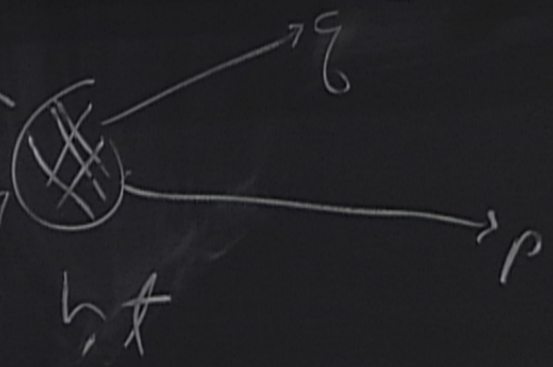
$S/k = \ln Z + \beta \bar{E}$

$\frac{d\bar{A}}{\partial \lambda} = \frac{\partial \bar{A}}{\partial \lambda} + \left\langle \frac{\partial A}{\partial \lambda} \right\rangle$

$= 0$

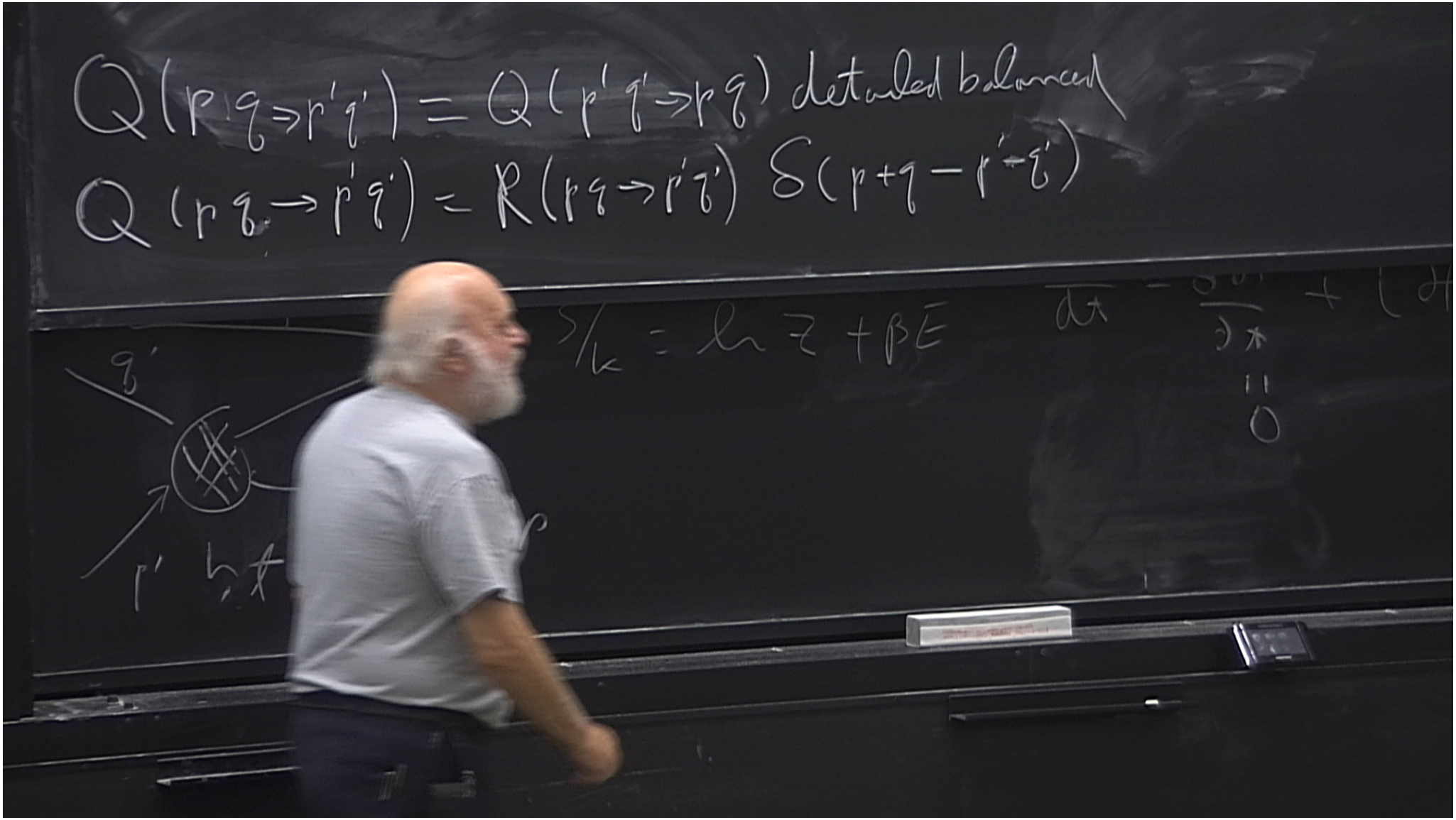
$Q(pq \to p'q') = Q(p'q' \to pq)$ detailed balance

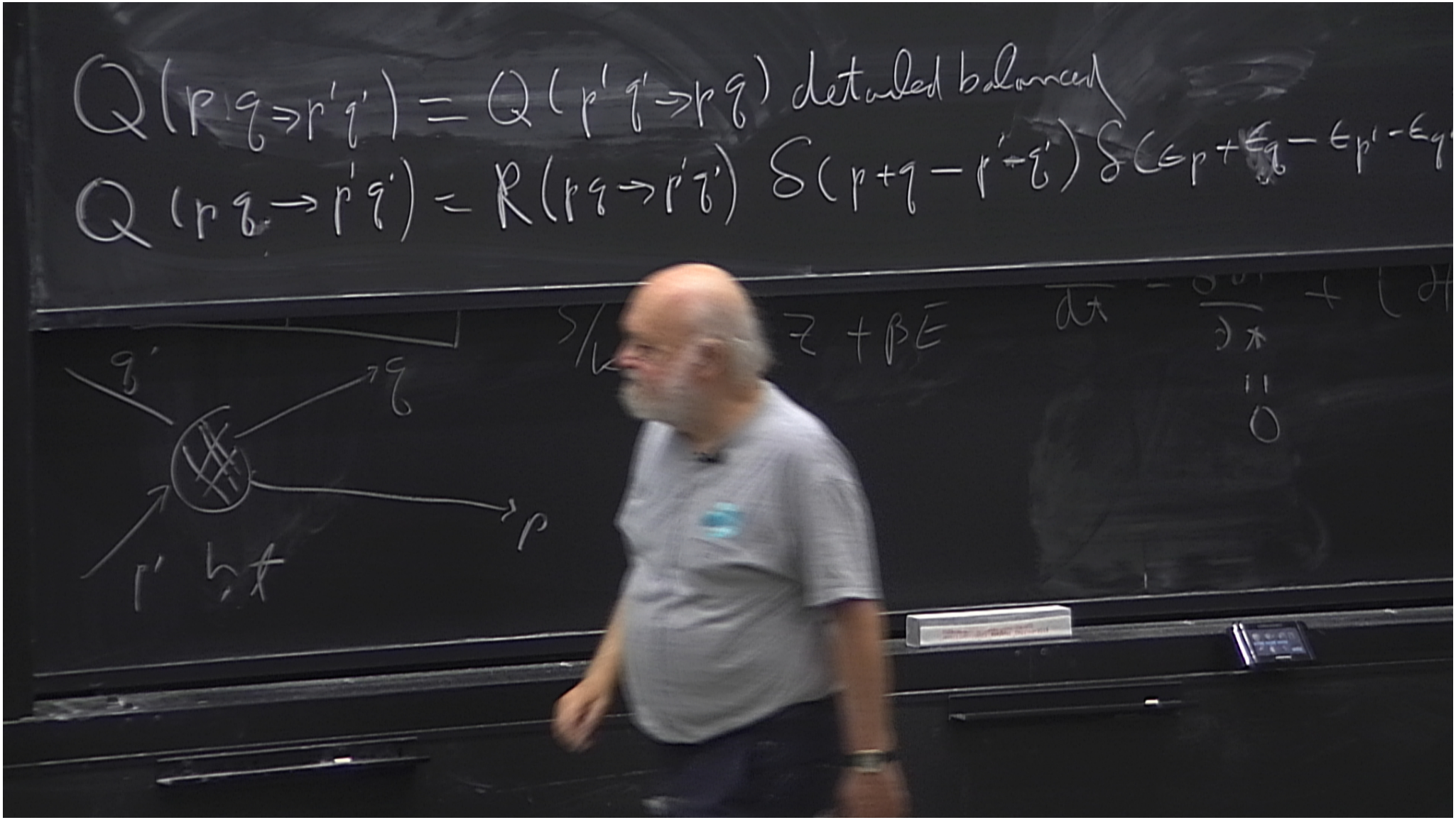
$Q(pq \to p'q')$



$S/k = \ln Z + \beta E$

$\frac{\partial}{\partial} = 0$


$Q(pq \to p'q') = Q(p'q' \to pq)$ detailed balance
$Q(pq \to p'q') = R(pq \to p'q')\,\delta(p+q-p'-q')$
$S/k = \ln Z + \beta E$
$\frac{dS}{dt} = \frac{\partial S}{\partial t} + \{S, H\}$
$\frac{\partial S}{\partial t} = 0$
q'
p'


Q(pq→p'q') = Q(p'q'→pq) detailed balance
Q(pq→p'q') = R(pq→p'q') δ(p+q−p'−q') δ(ε_p+ε_q−ε_p'−ε_q
q'
q
p
p'

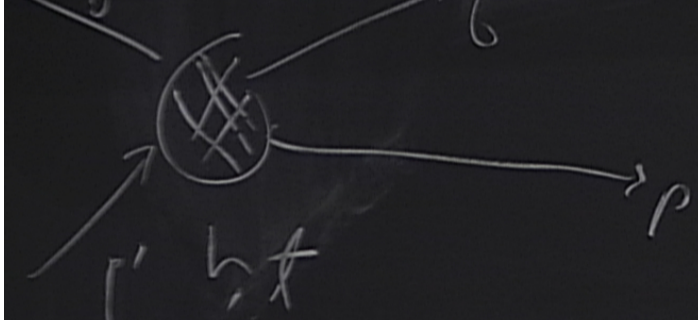
$Q(pq \to p'q') = Q(p'q' \to pq)$ detailed balanced

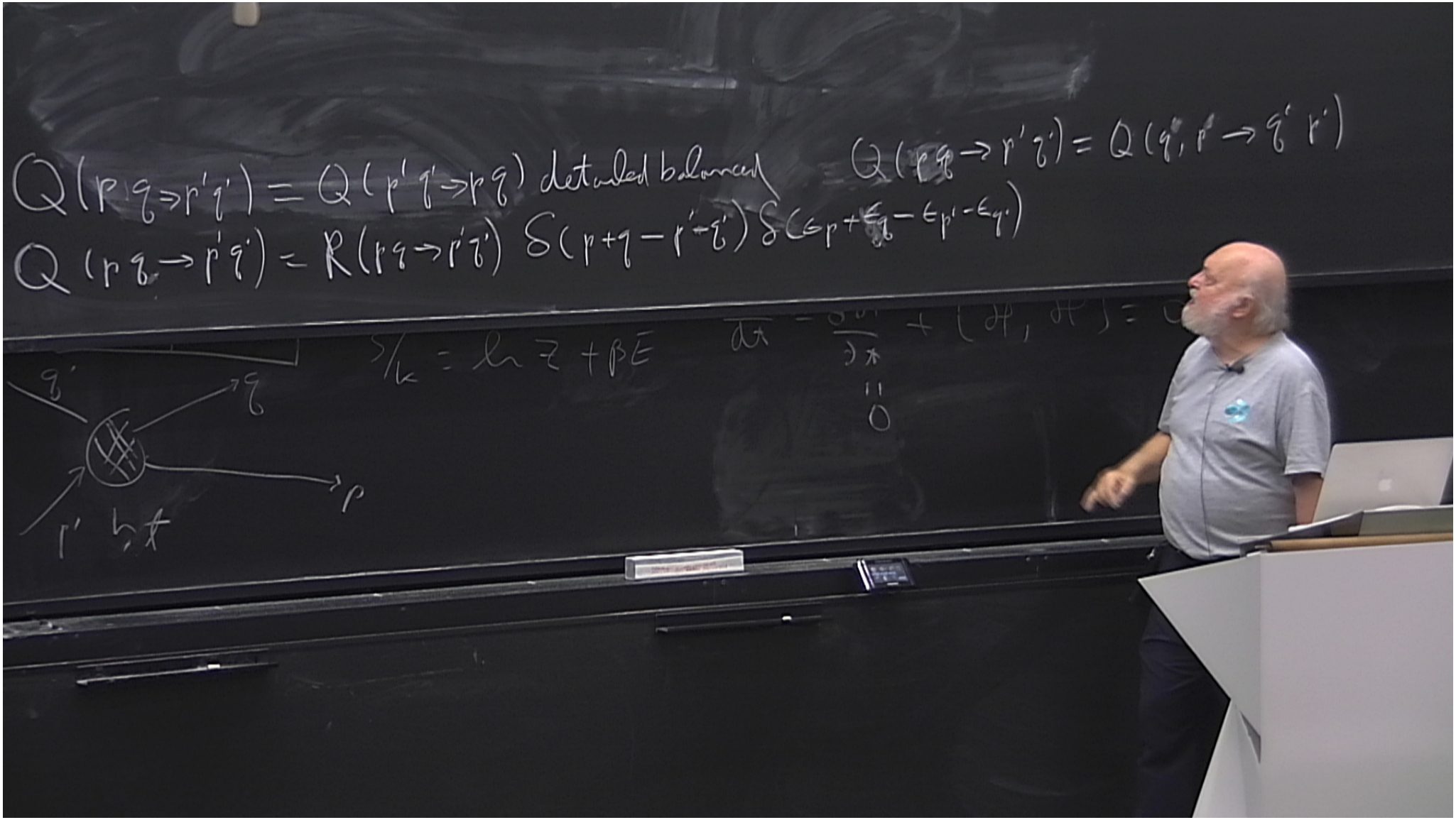
$Q(pq \to p'q') = R(pq \to p'q')\, \delta(p+q-p'-q')\, \delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$

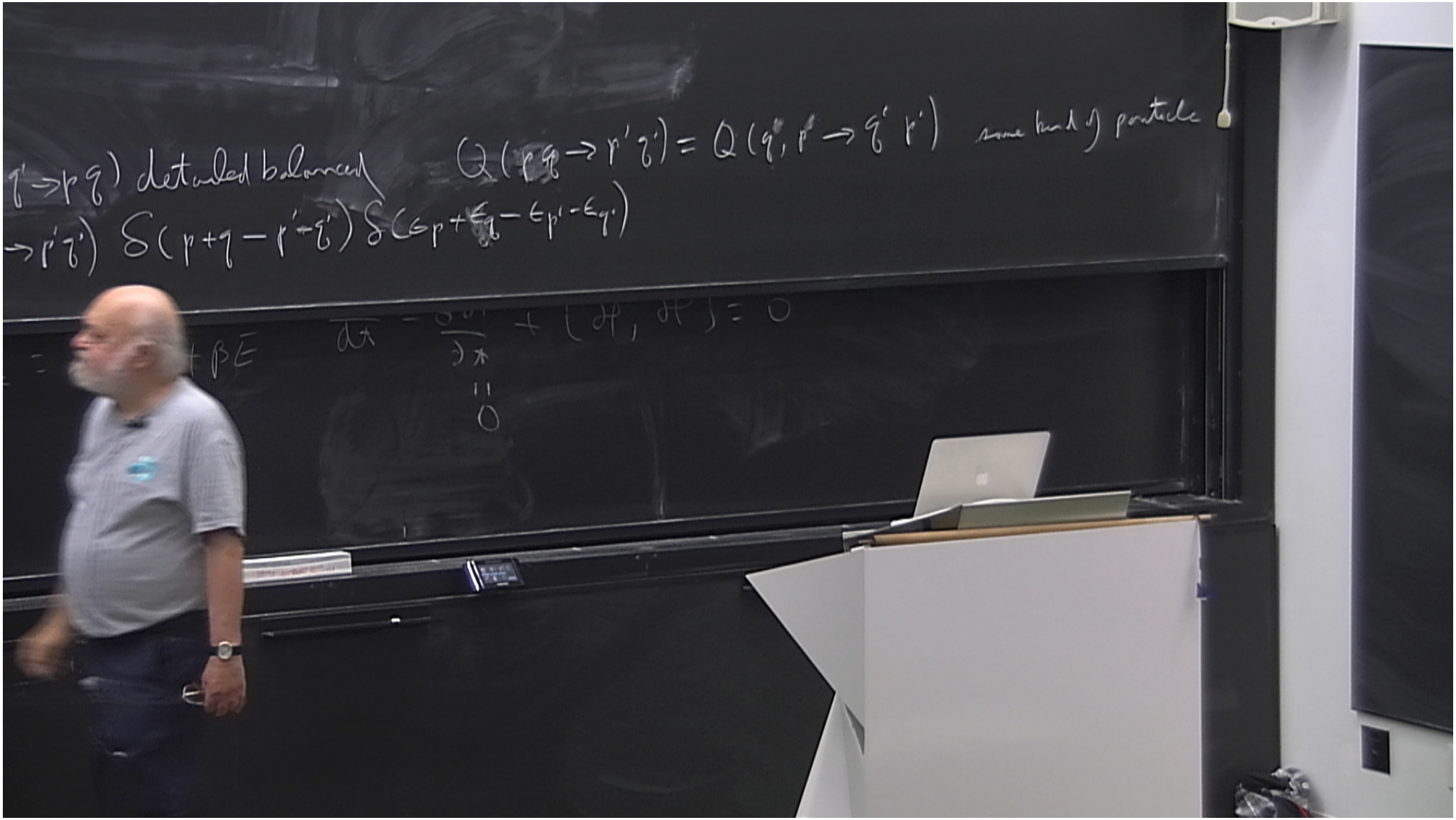
$S/k = \ln z + \beta E$

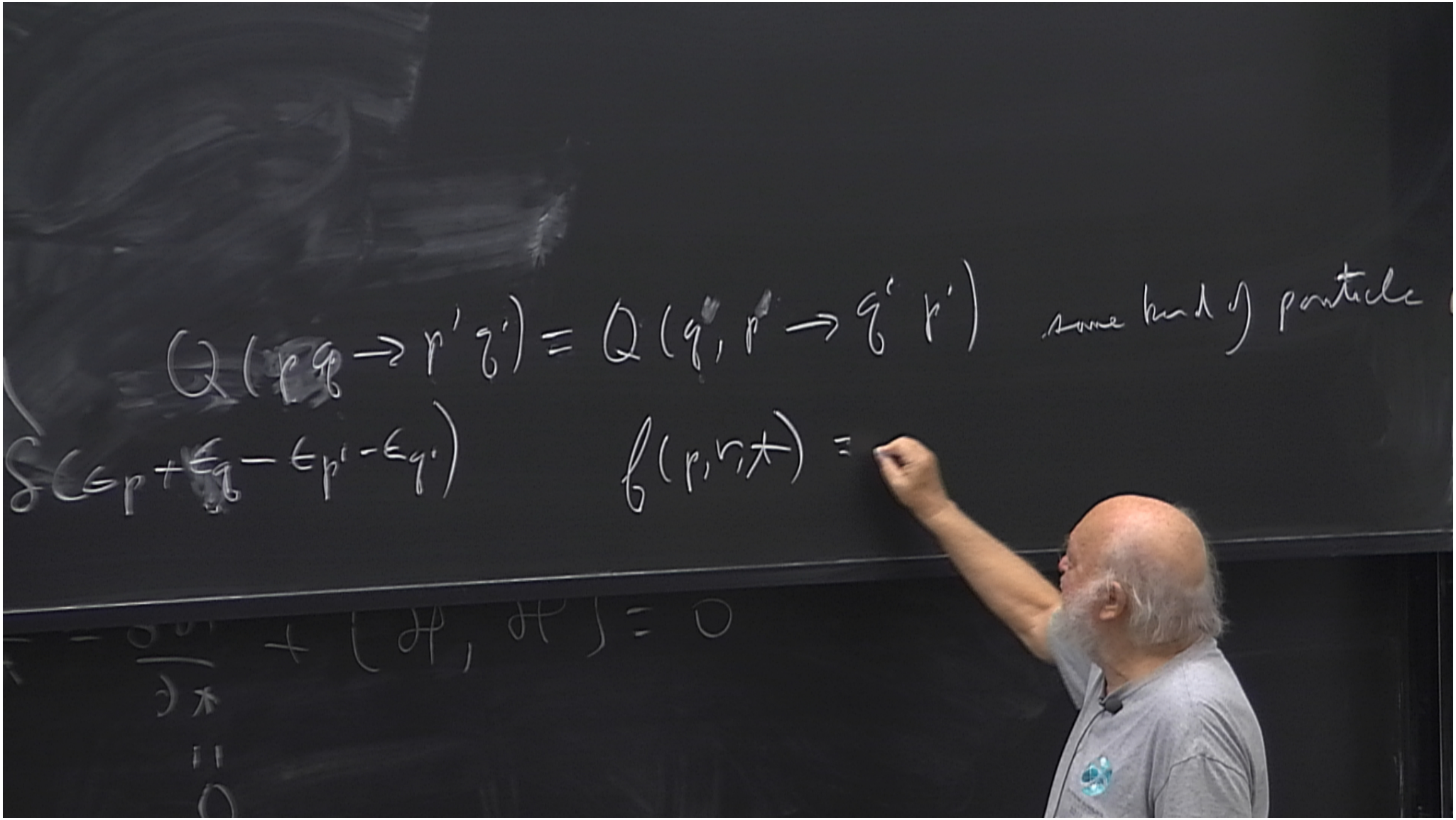
$\frac{dA}{d\lambda} = \frac{\partial A}{\partial \lambda} + \{A, \}$

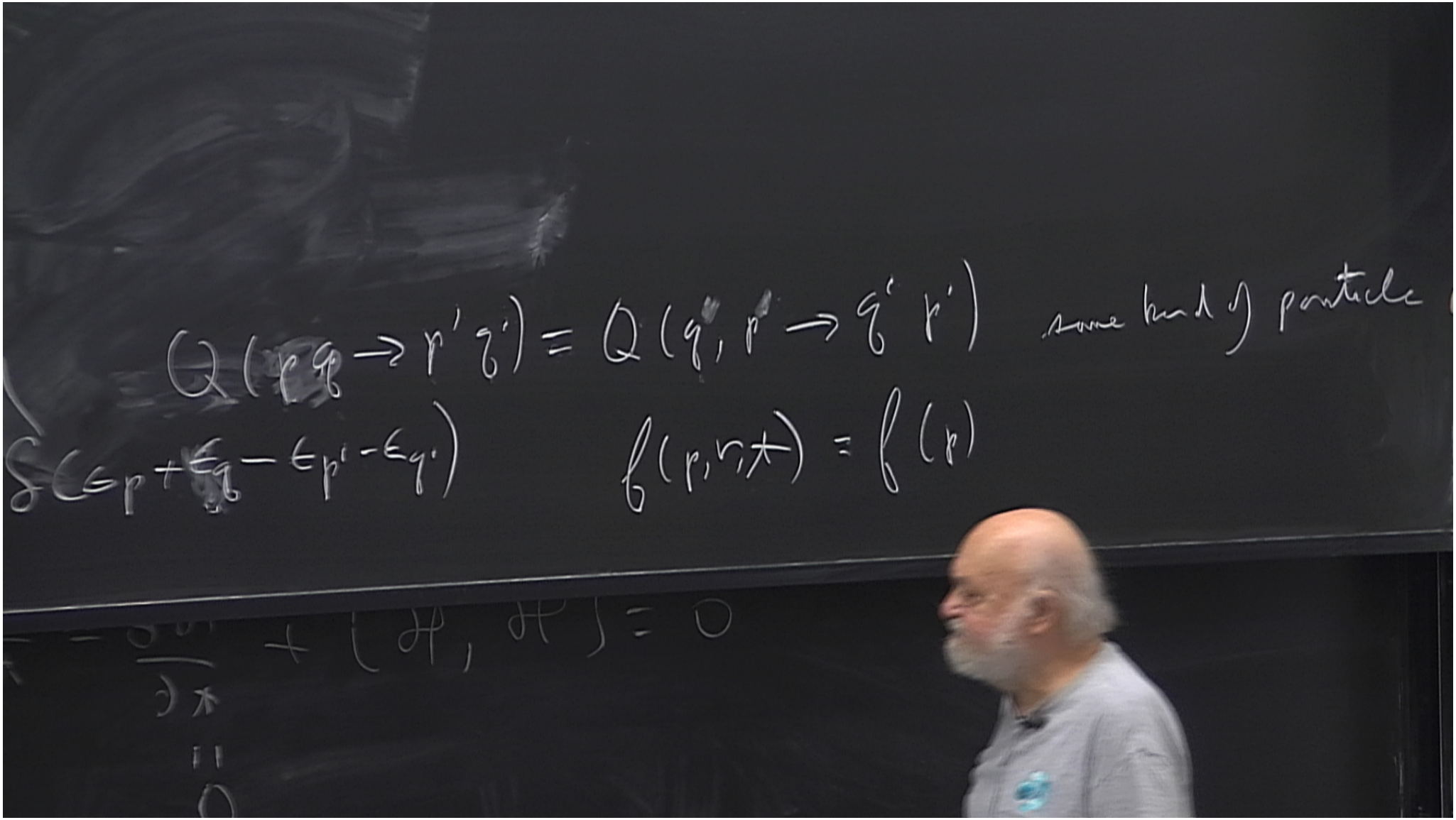
$\frac{\partial A}{\partial \lambda} = 0$




$Q(pq \to p'q') = Q(p'q' \to pq)$ detailed balance
$Q(pq \to p'q') = Q(q, p \to q'p')$
$Q(pq \to p'q') = R(pq \to p'q')\,\delta(p+q-p'-q')\,\delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$
$S/k = \ln z + \beta E$
$\frac{dx}{dt} = \frac{\partial x}{\partial t} + \{x, H\} = 0$
q'
q
p
p'


detailed balanced
same kind of particle
$\delta(p+q-p'-q')\,\delta(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'})$
$+\{H, H\} = 0$
$= 0$


Q(pq → p'q') = Q(q, p → q' p')
same kind of particle
δ(ε_p + ε_q − ε_p' − ε_q')
f(p, ↑)
+ [H, H] = 0

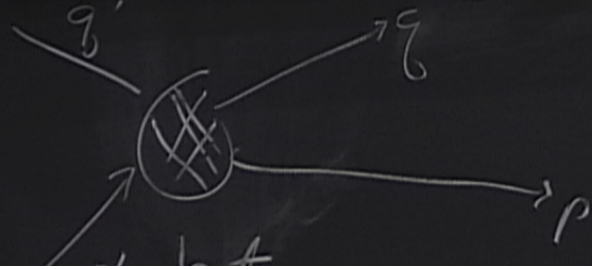

$Q(pq \to p'q') = Q(q, p \to q'p')$ some kind of particle
$\delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$
$f(p, \tau) = f(p)$
$+ [\mathcal{H}, \mathcal{H}] = 0$

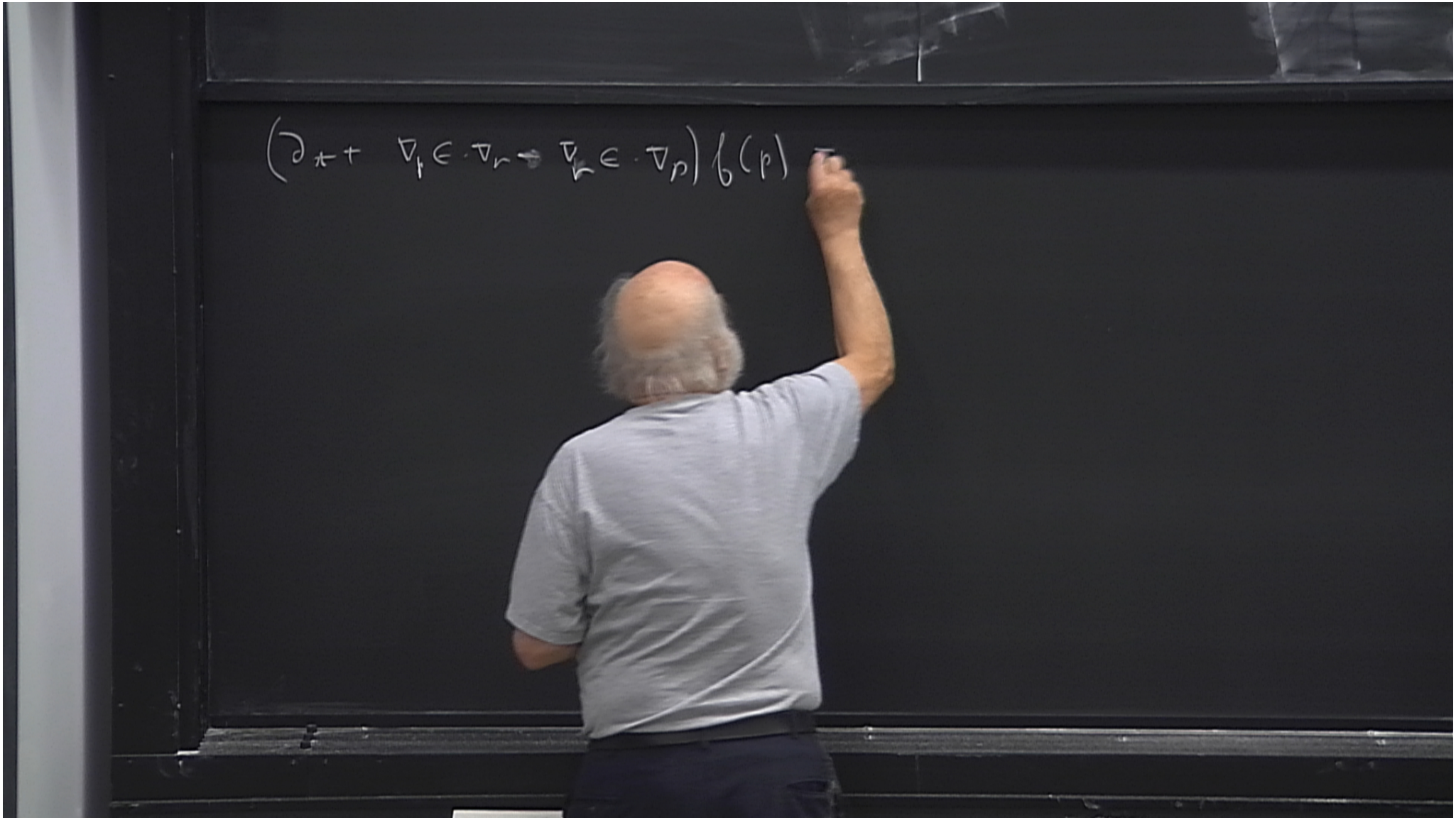
$$Q(pq \to p'q') = Q(p'q' \to pq) \quad \text{detailed balance}$$

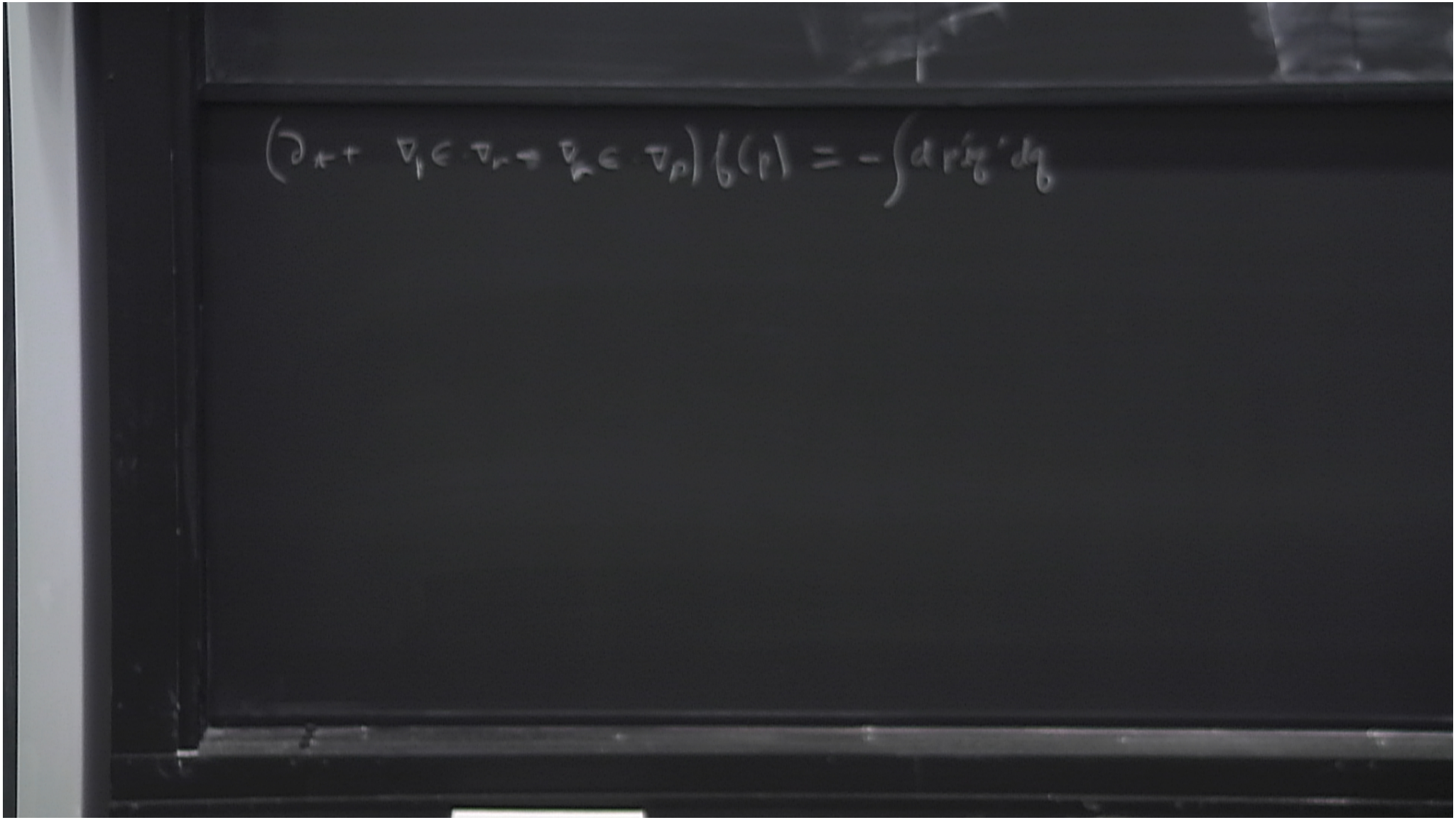
$$Q(pq \to p'q') = R(pq \to p'q')\, \delta(p+q-p'-q')\, \delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})$$

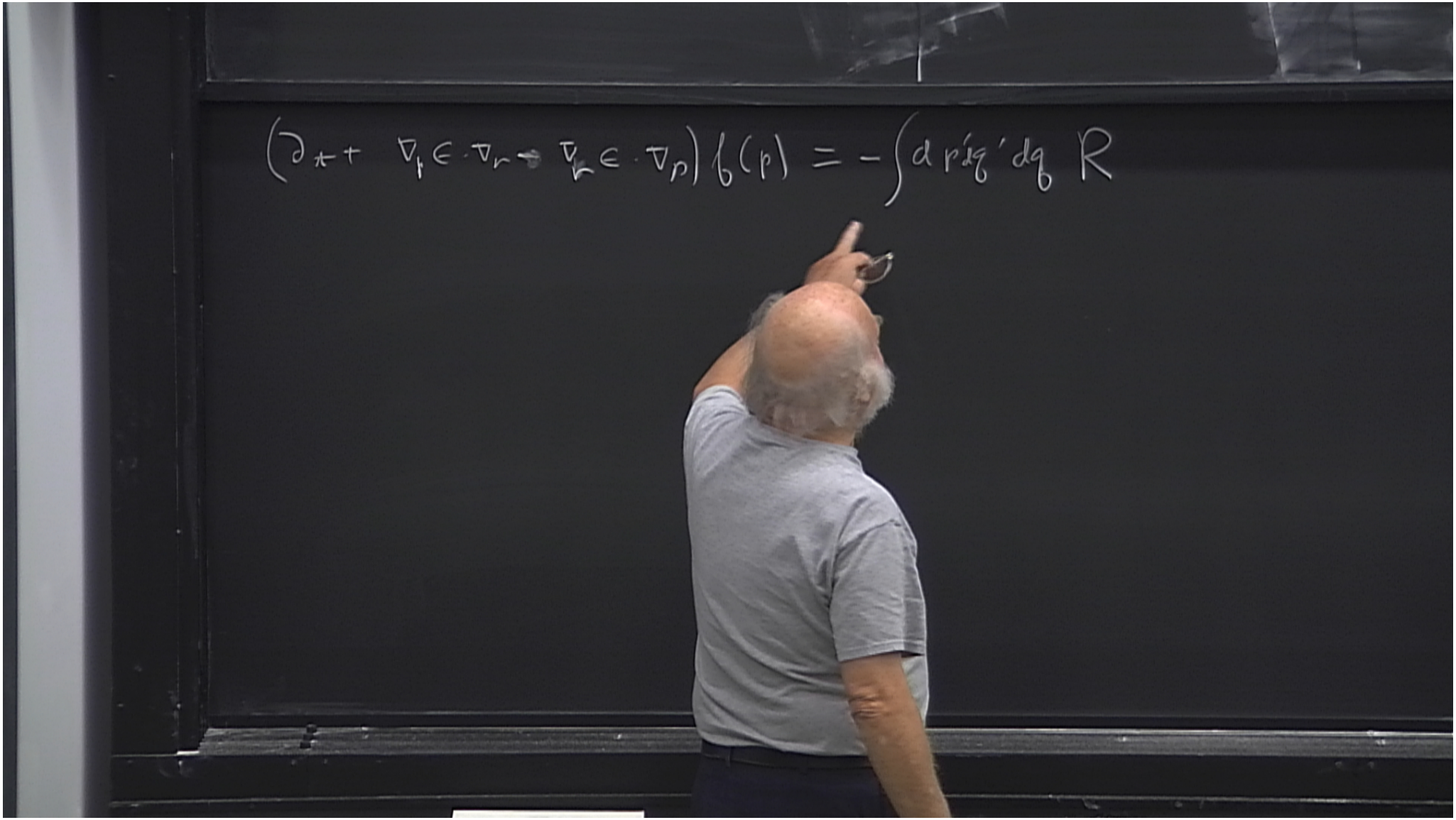
$$S/k = \ln Z + \beta E$$

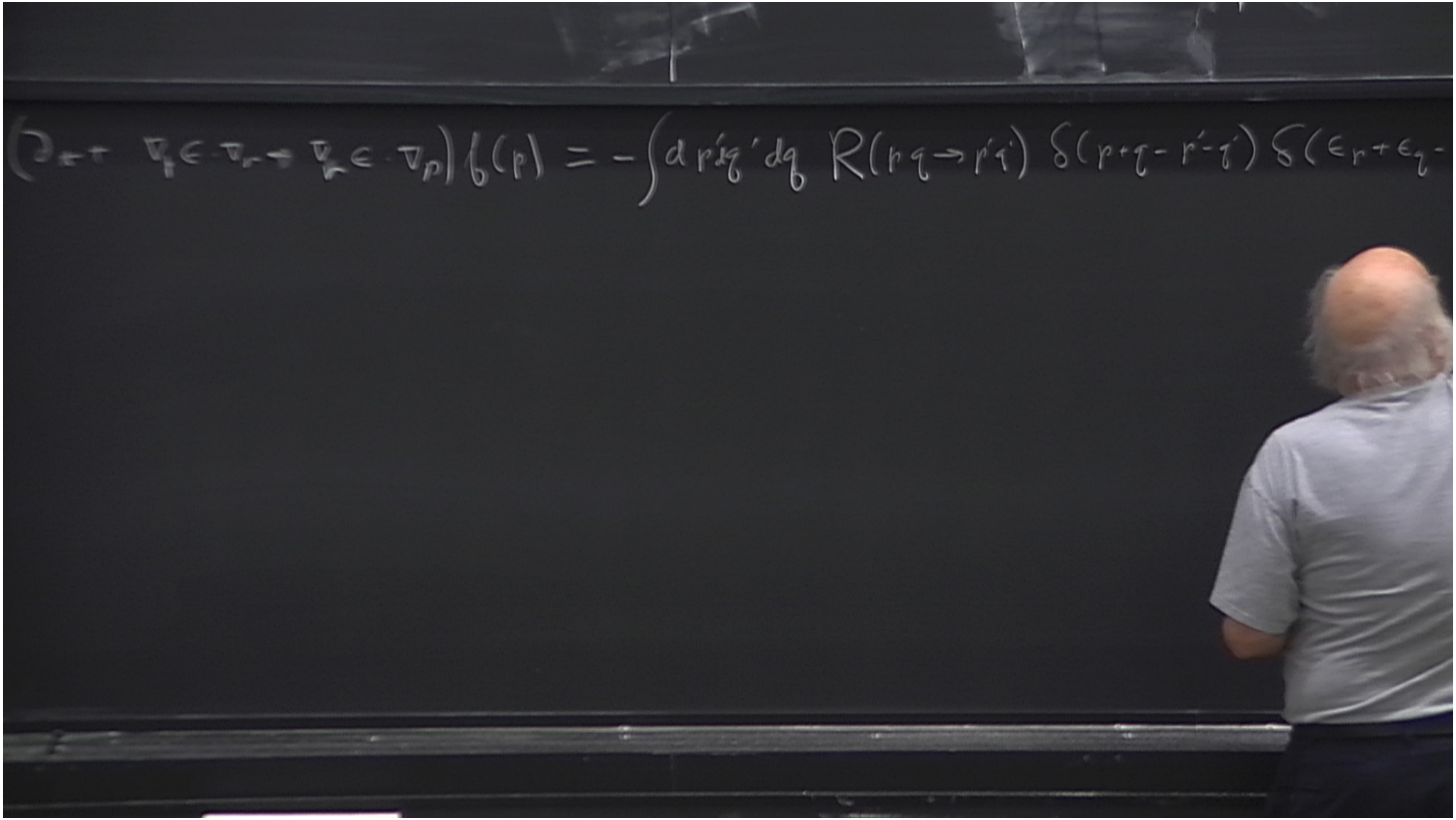
$$\frac{dS}{d\lambda} = \frac{\partial S}{\partial \lambda} + \{H, S\} = 0$$

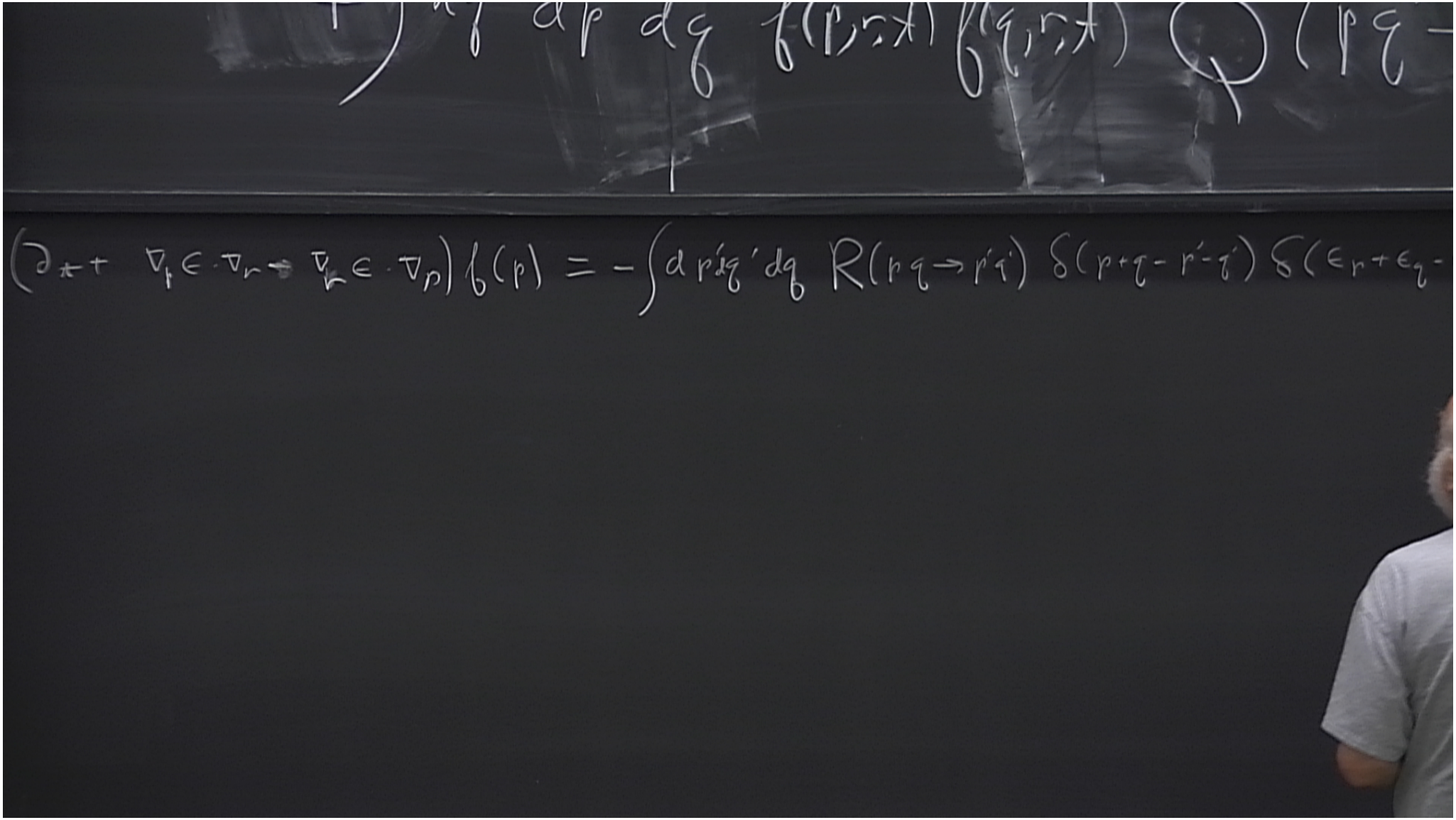


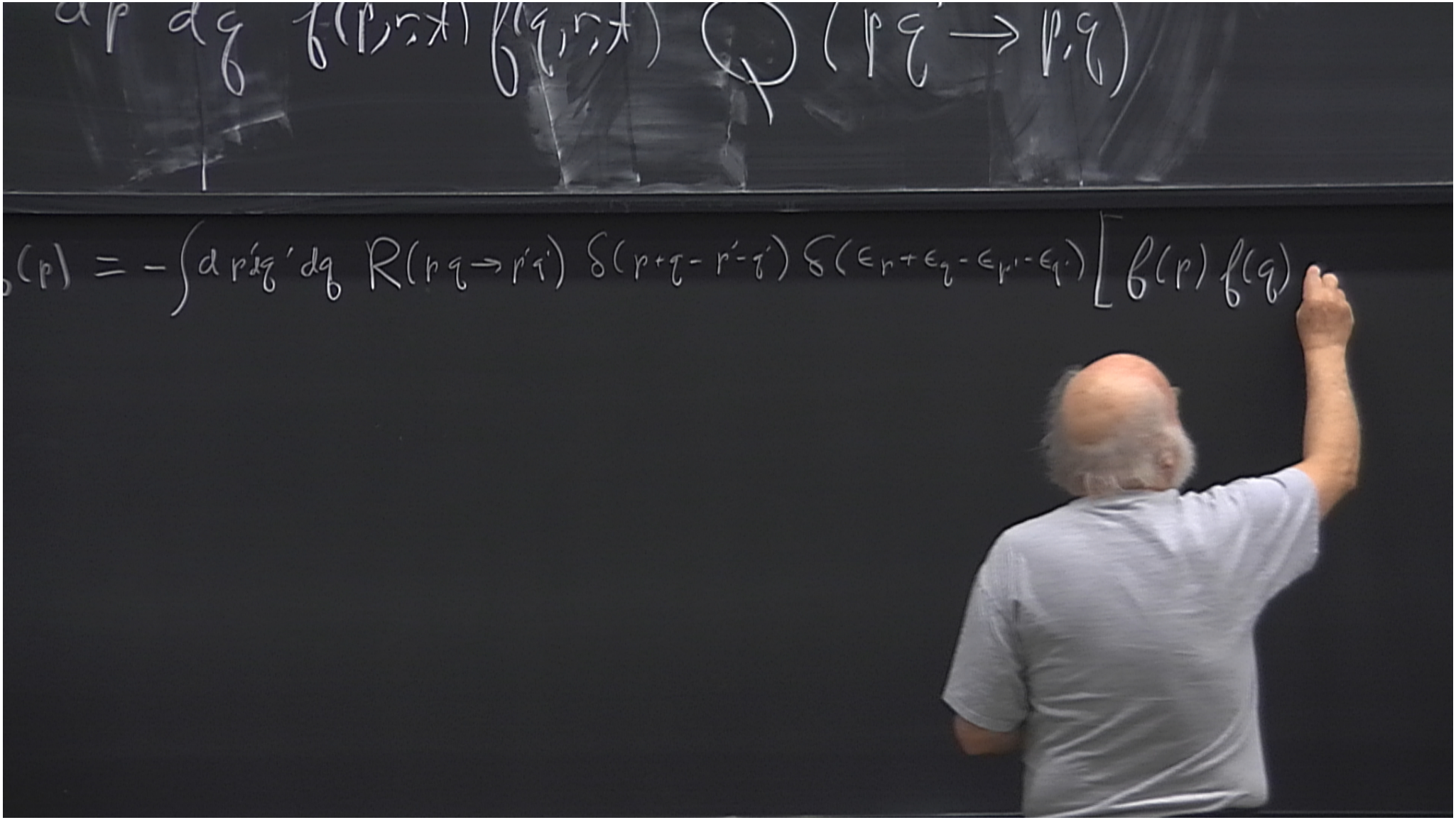

$\left(\partial_{x^+} + \nabla_{i_1} \in \cdot \nabla_{r} \rightarrow \nabla_{i_2} \in \cdot \nabla_{n}\right) f(p)$


$(\partial_{x+} \nabla_1 \in \nabla_{x-} \nabla_n \in \nabla_n) f(p) = -\int d^n p \hat{\eta}_0' d\eta_0$

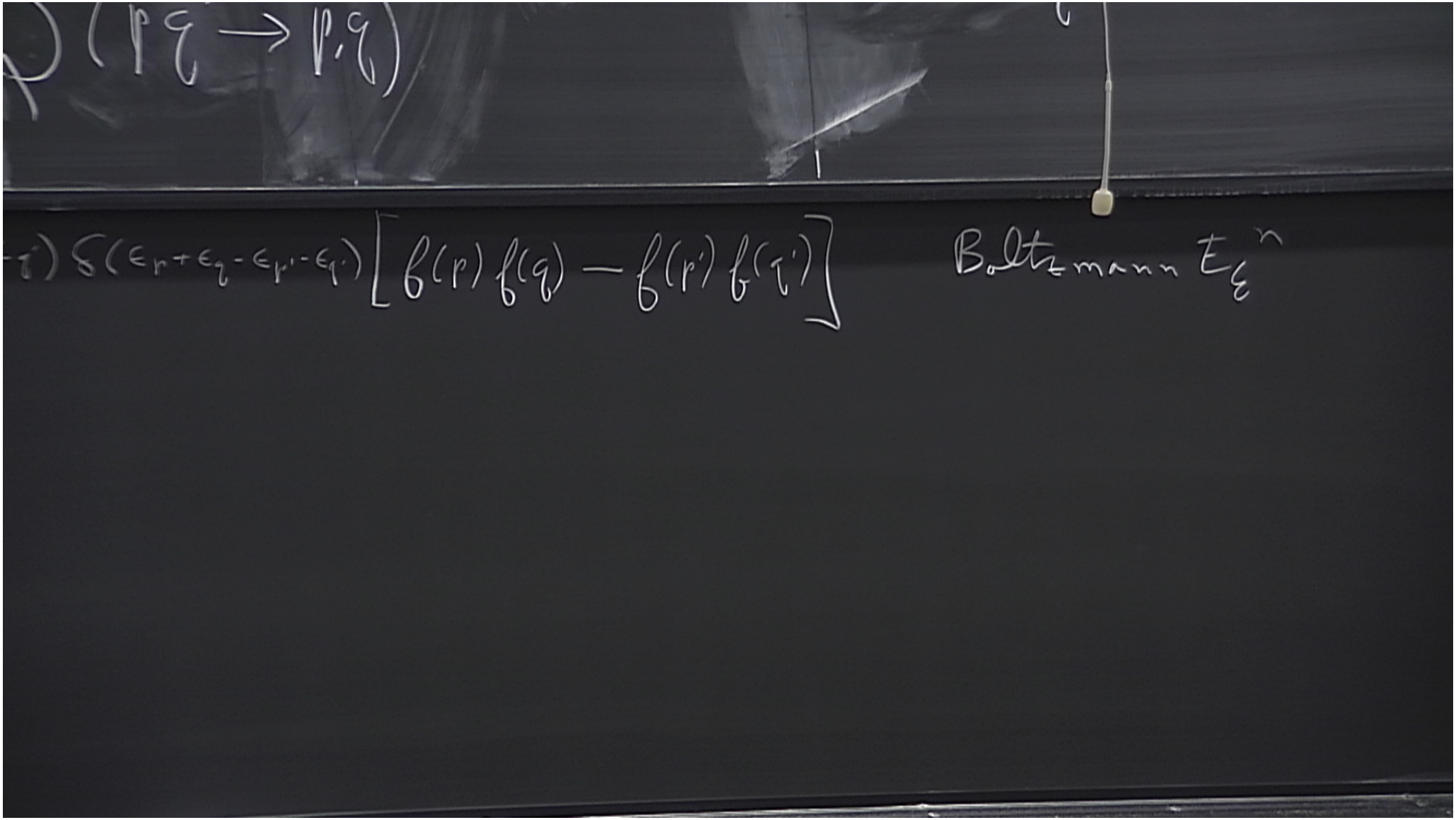

$\left(\partial_{t} + \nabla_{1} \in \cdot \nabla_{r} + \nabla_{k} \in \cdot \nabla_{n}\right) f(p) = -\int d\,r\,dq'\,dq_0\,R$

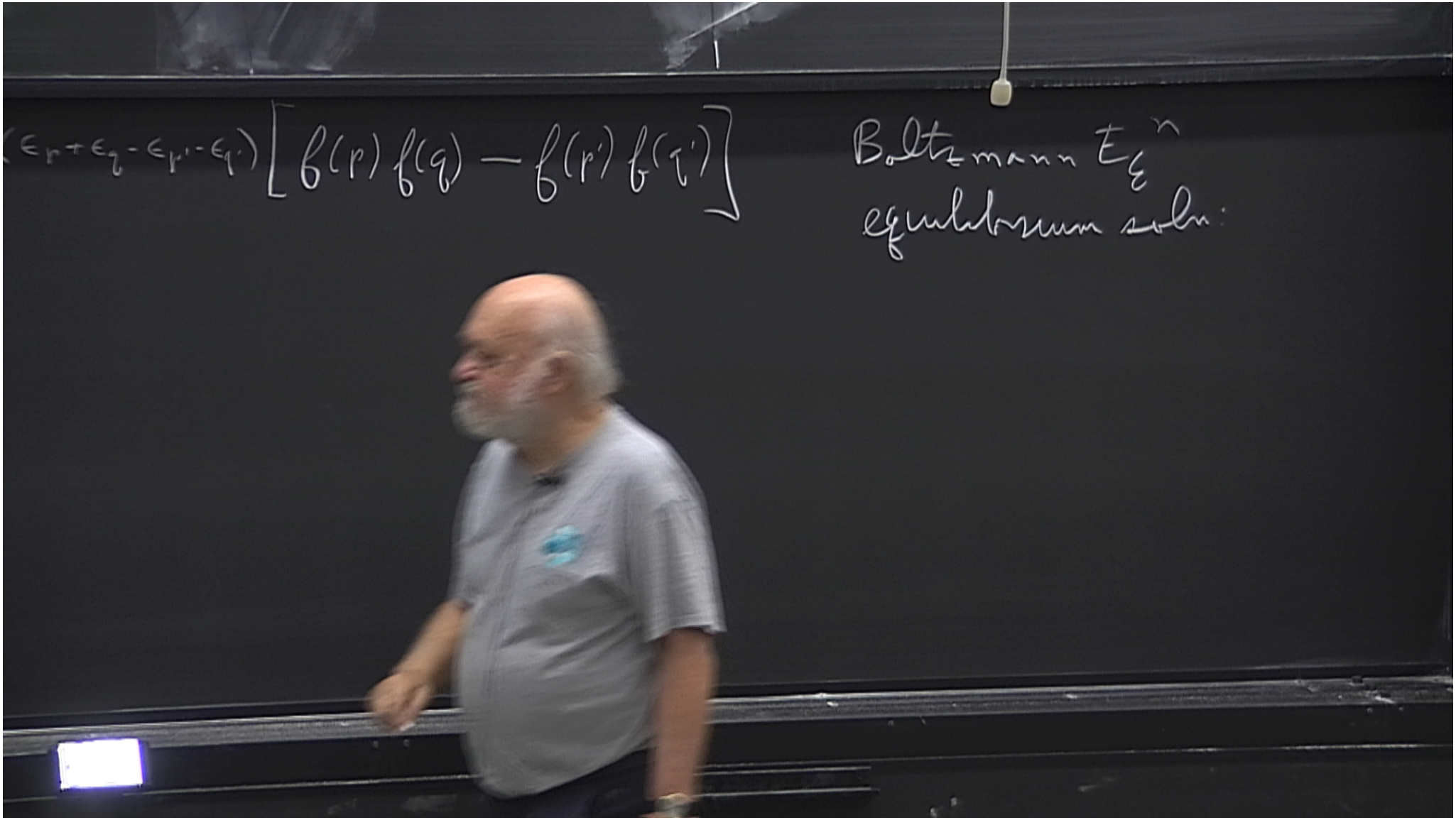

$\left(\partial_{t-t'} - \nabla_{k_1} \in \nabla_{r-r'} - \nabla_{k_2} \in \nabla_{p}\right) f_0(p) = -\int dr' dq' dq\, R(rq \to r'q')\, \delta(r+q-r'-q')\, \delta(\epsilon_r + \epsilon_q -$

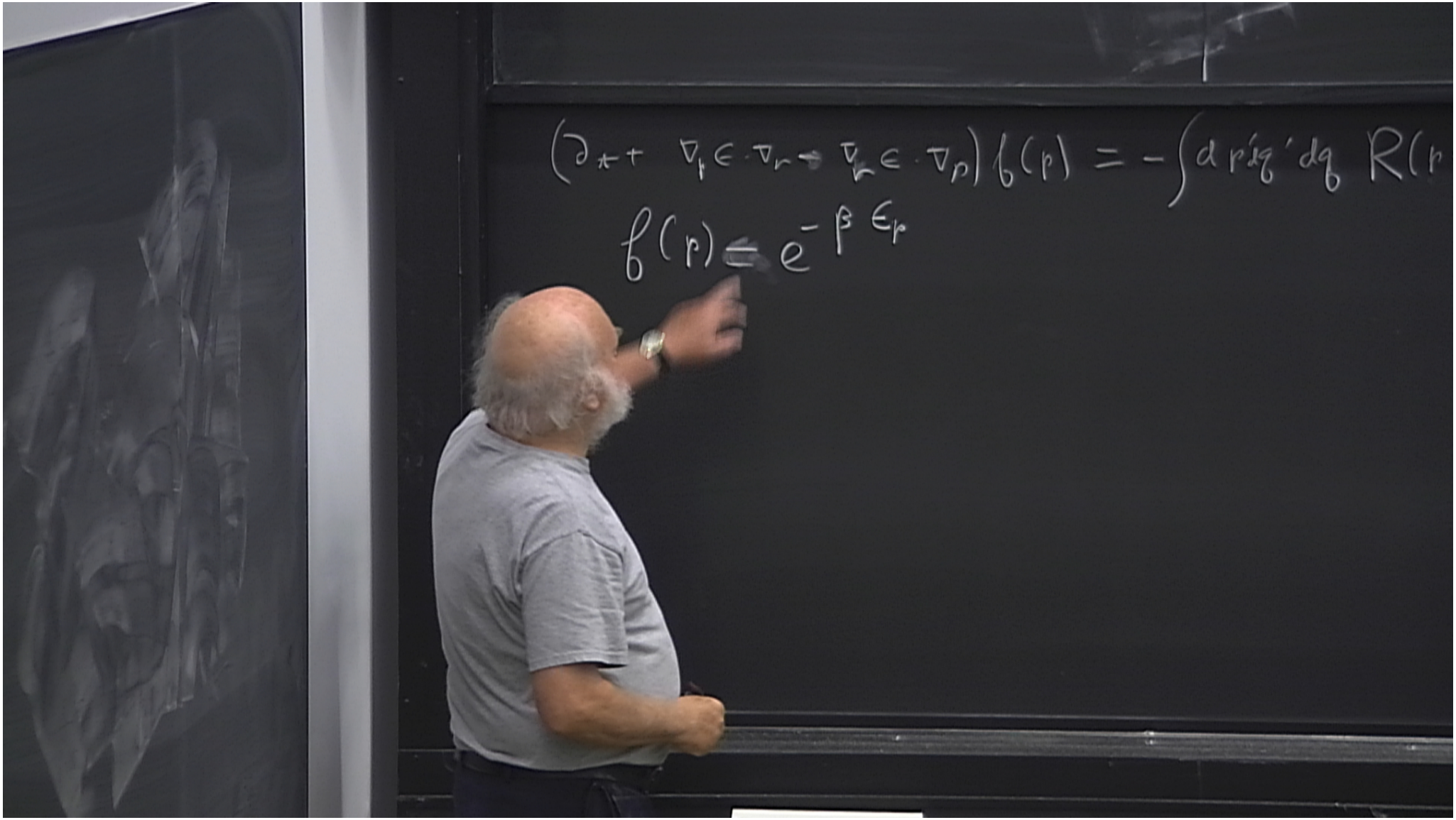

$\int dp\, dq\, f(p,x,t) f(q,x,t)$
$\left(\partial_t + \nabla_p \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p\right) f(p) = -\int dr' dq' dq\, R(pq \to p'q')\, \delta(p+q-p'-q')\, \delta(\epsilon_p + \epsilon_q -$

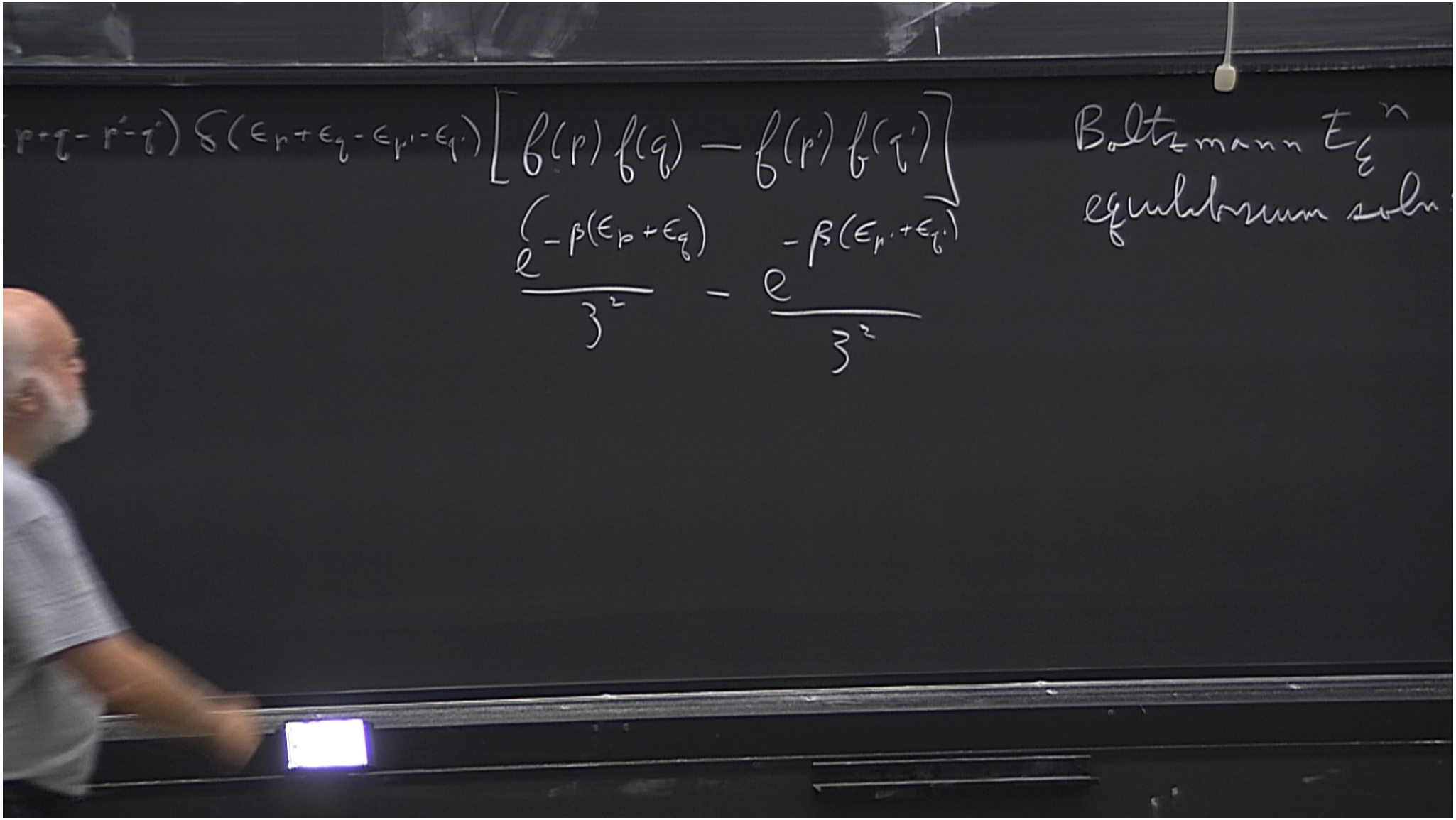


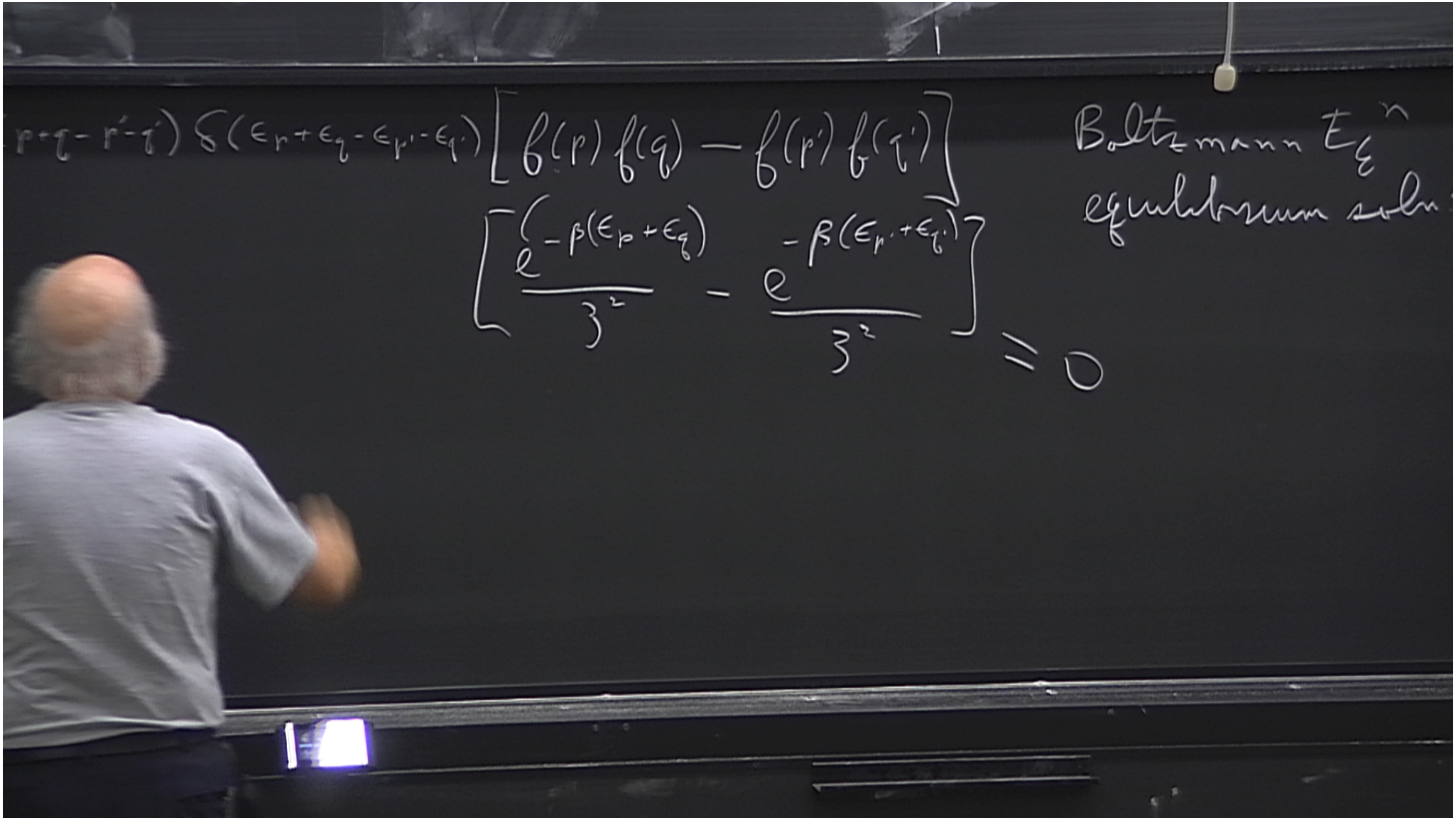
$$\int dq\, R(pq \to p'q')\, \delta(p+q-p'-q')\, \delta(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'}) \left[f(p) f(q) - f(p') f(q') \right]$$


$(p\xi \to p'q')$
$\delta(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})\left[f(p)f(q) - f(p')f(\tau')\right]$
Boltzmann E_ξ


$(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'})\left[f(p)f(q)-f(p')f(q')\right]$
Boltzmann E_ξ
equilibrium soln:

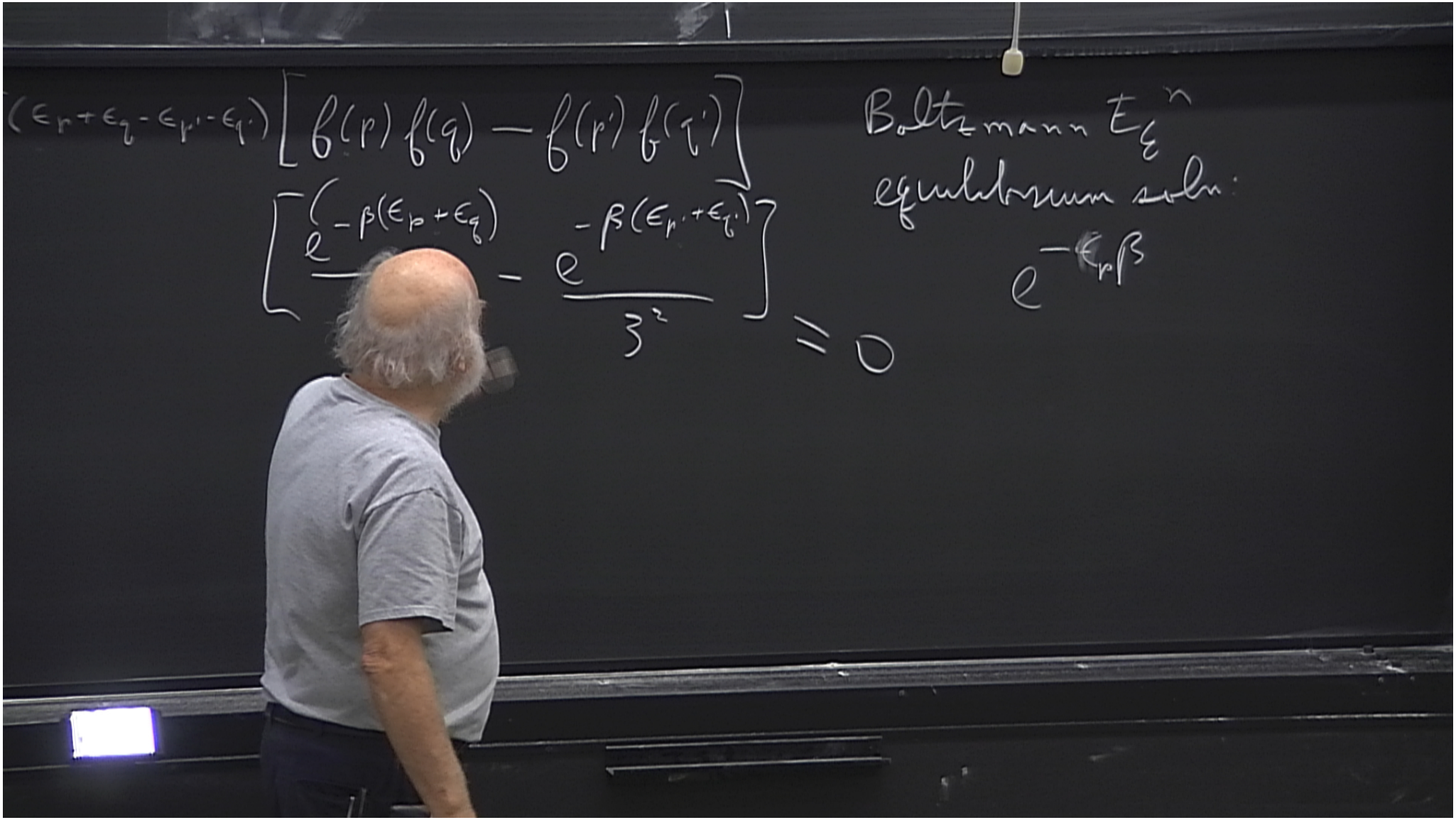

$\left(\partial_{t} + \nabla_{1} \in \cdot \nabla_{r} - \nabla_{r} \in \cdot \nabla_{p}\right) f(r) = -\int d\,r' dq' dq\, R(r$
$f(r) = e^{-\beta \epsilon_{r}}$


$p+q-p'-q')\,\delta(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'})\left[f(p)f(q)-f(p')f(q')\right]$
Boltzmann
equilibrium
$\frac{e^{-\beta(\epsilon_p+\epsilon_q)}}{Z^2}-\frac{e^{-\beta(\epsilon_{p'}+\epsilon_{q'})}}{Z^2}$


$p+q-p'-q')\,\delta(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'})\left[f(p)f(q)-f(p')f(q')\right]$
$\left[\frac{e^{-\beta(\epsilon_p+\epsilon_q)}}{Z^2}-\frac{e^{-\beta(\epsilon_{p'}+\epsilon_{q'})}}{Z^2}\right]=0$
Boltzmann
equilibrium

$$\left(\partial_{t} + \nabla_{p}\epsilon\cdot\nabla_{r} - \nabla_{r}\epsilon\cdot\nabla_{p}\right) f(p) = -\int dp'dq'dq\; R(pq\to p'q')\,\delta(p+q-p'-q')\,\delta$$

$$f(p) = \frac{e^{-\beta\epsilon_{p}}}{Z}$$

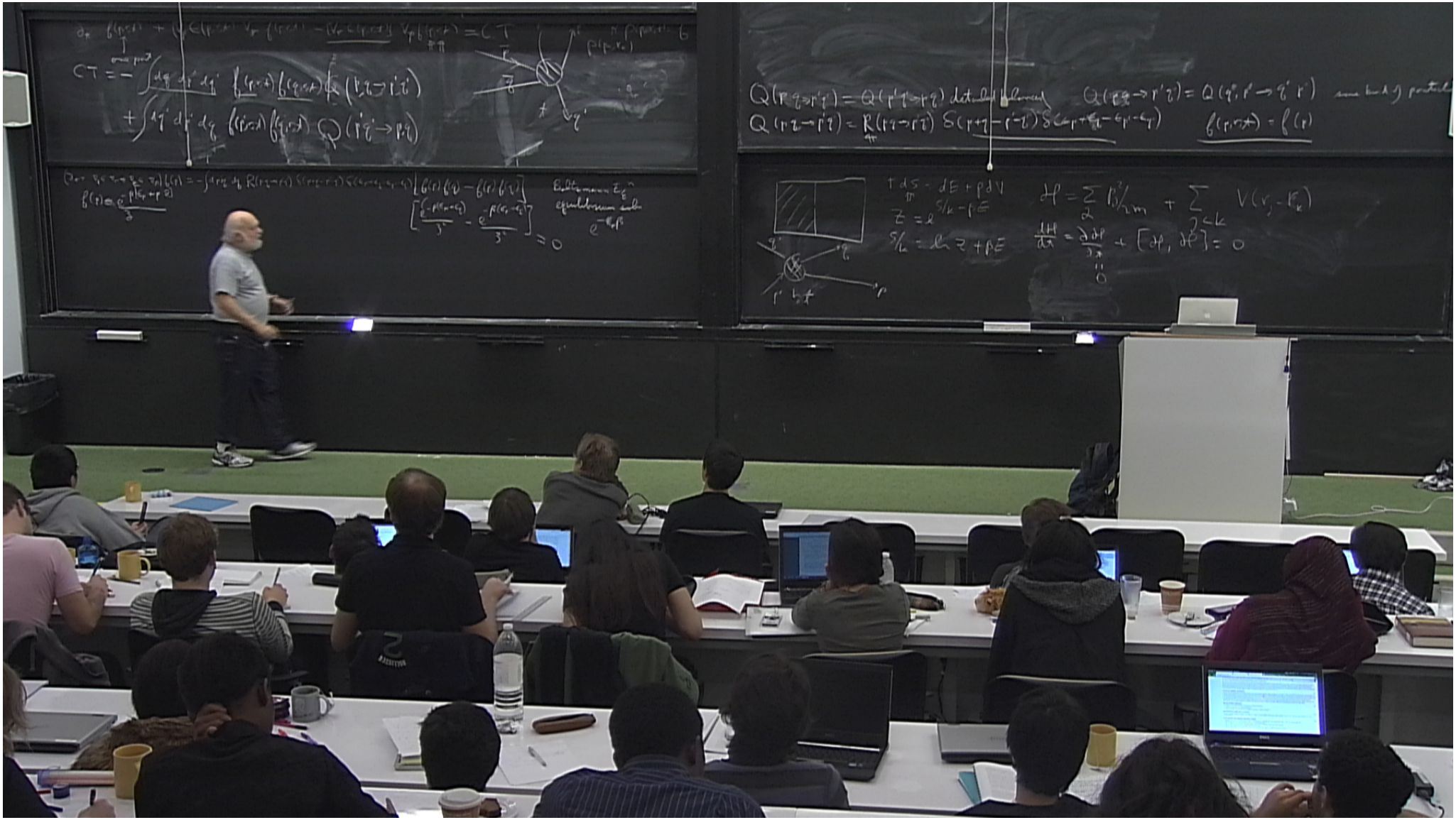

$(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})\left[f(p)f(q) - f(p')f(q')\right]$
$\left[\frac{e^{-\beta(\epsilon_p+\epsilon_q)}}{} - \frac{e^{-\beta(\epsilon_{p'}+\epsilon_{q'})}}{}\right] = 0$
Boltzmann E_ϵ
equilibrium soln:
$e^{-\epsilon_p \beta}$

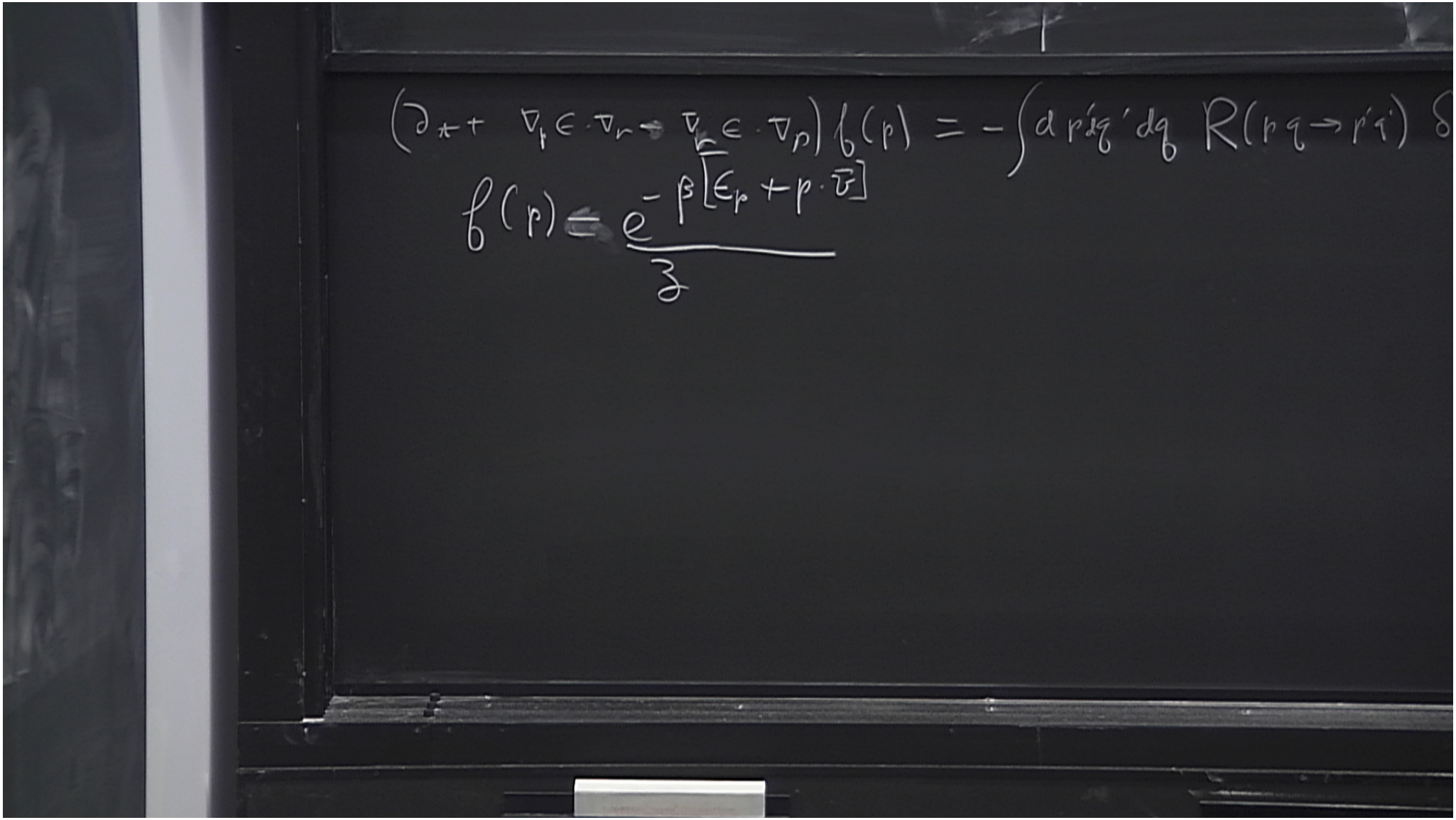
$$(\epsilon_p + \epsilon_q - \epsilon_{p'} - \epsilon_{q'})\left[f(p)f(q) - f(p')f(q')\right]$$

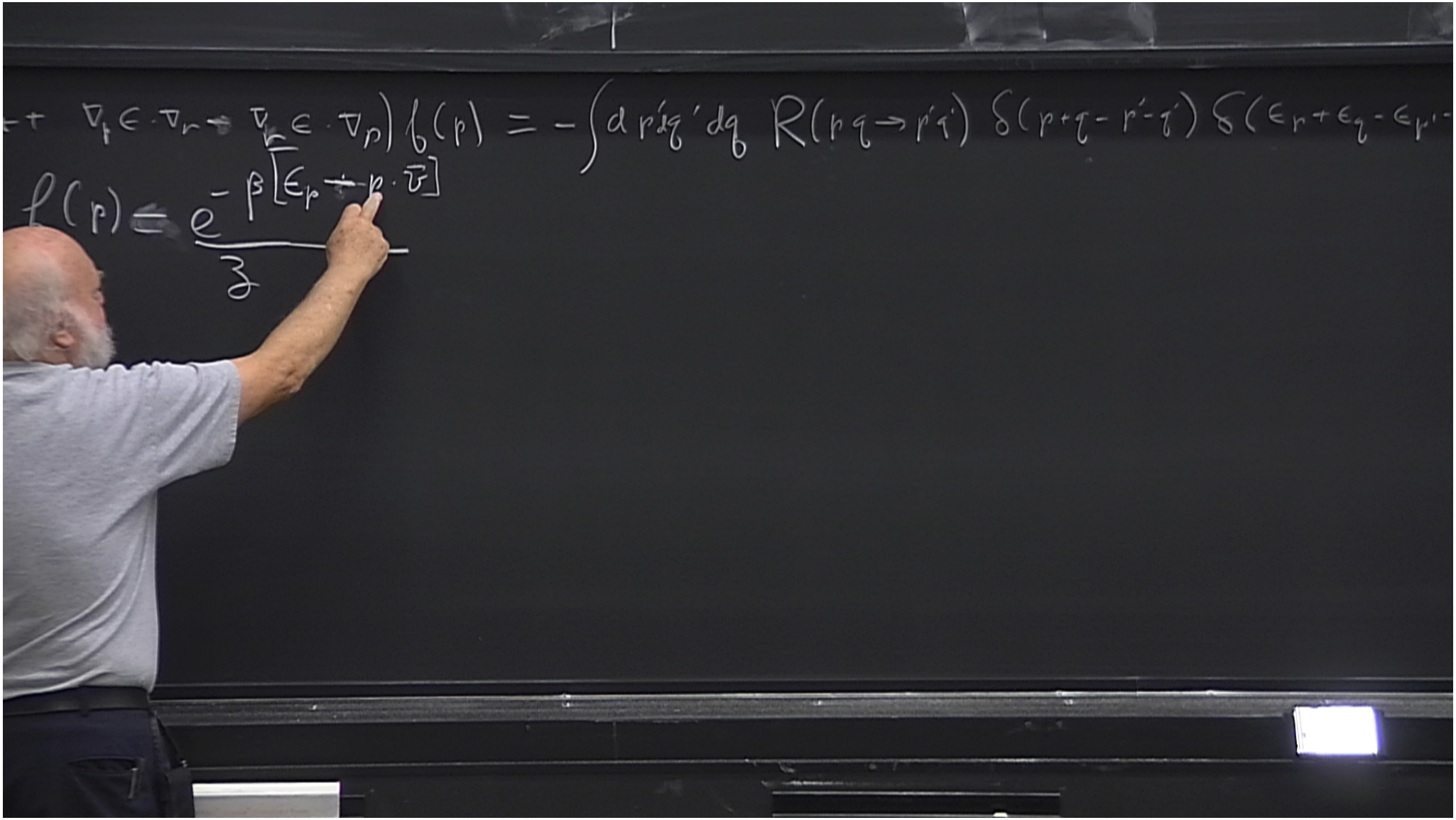
$$\left[\frac{e^{-\beta(\epsilon_p+\epsilon_q)}}{Z^2} - \frac{e^{-\beta(\epsilon_{p'}+\epsilon_{q'})}}{Z^2}\right] = 0$$

Boltzmann E_ϵ equilibrium soln:

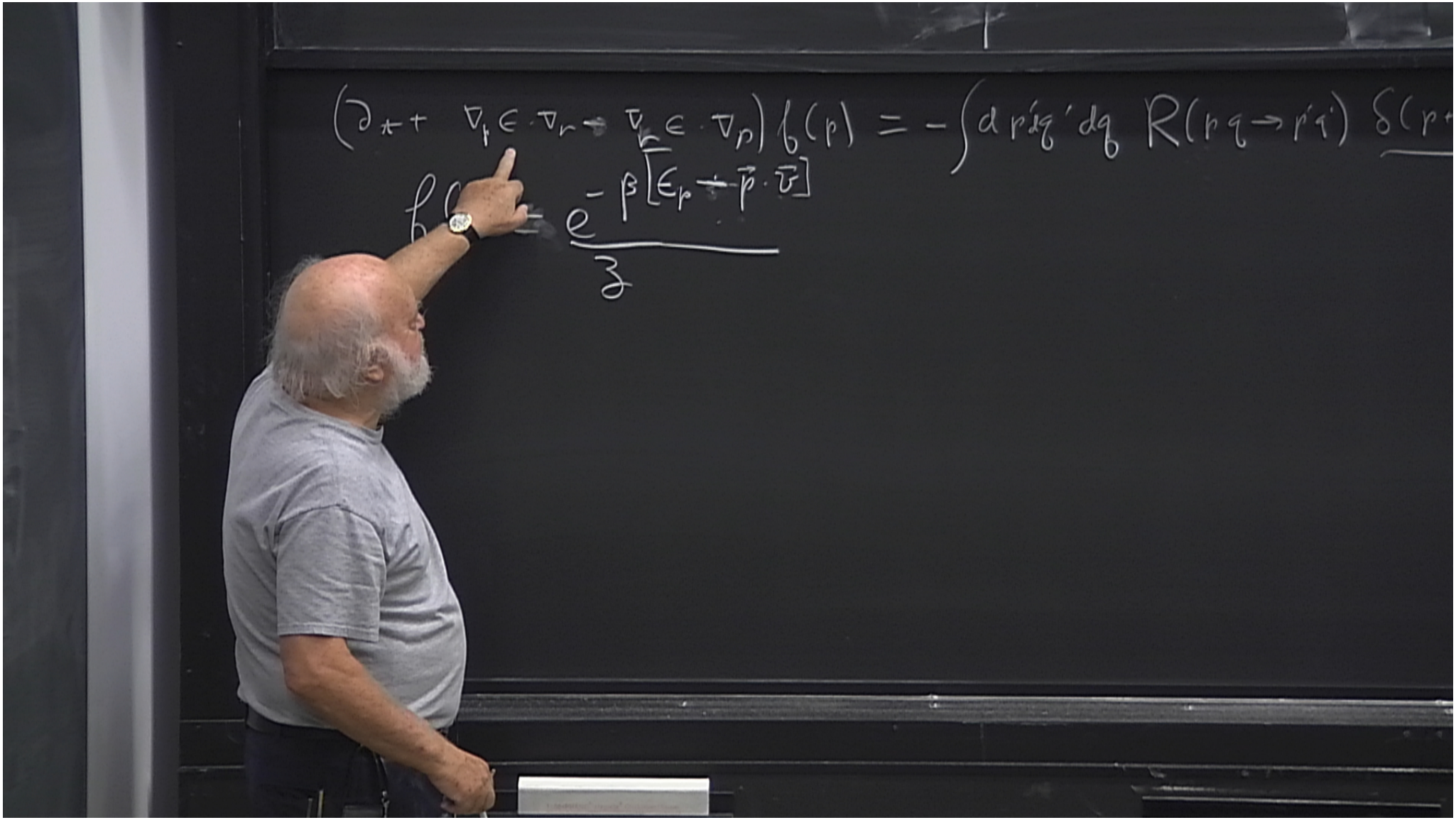
$$e^{-\epsilon_p \beta}$$

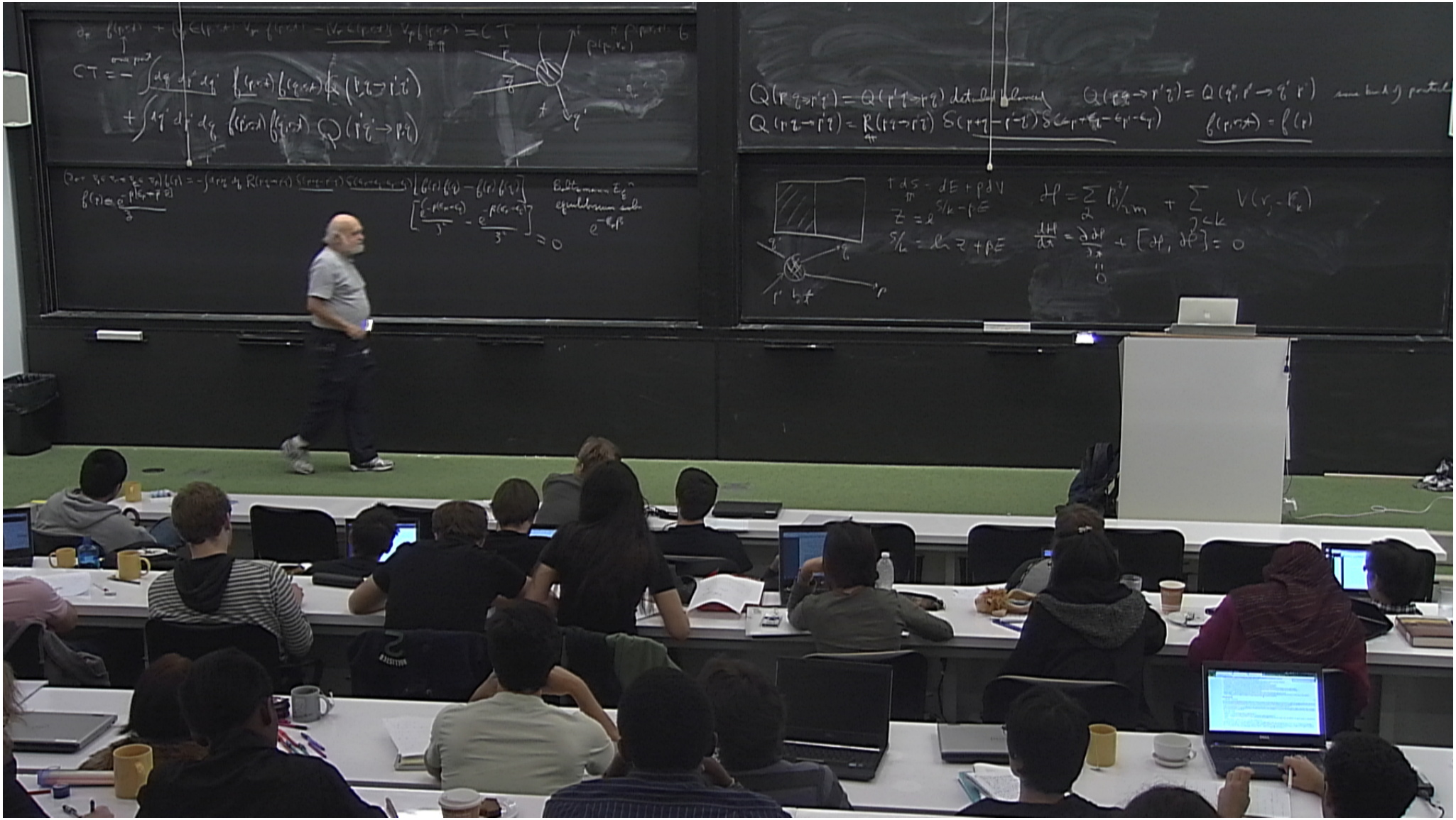



$\left(\partial_t + \nabla_r \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p\right) f(p) = -\int dr' dq' dq\, R(rq \to r'q')$
$f(p) = \frac{e^{-\beta[\epsilon_p + p \cdot \vec{v}]}}{Z}$


$\nabla_p \epsilon \cdot \nabla_r + \nabla_k \epsilon \cdot \nabla_p) f(p) = -\int dr' dq' dq\, R(rq \to r'q')\, \delta(p+q-r'-q')\, \delta(\epsilon_r + \epsilon_q - \epsilon_{r'}$
$f(p) = \frac{e^{-\beta[\epsilon_p + p \cdot \vec{v}]}}{Z}$



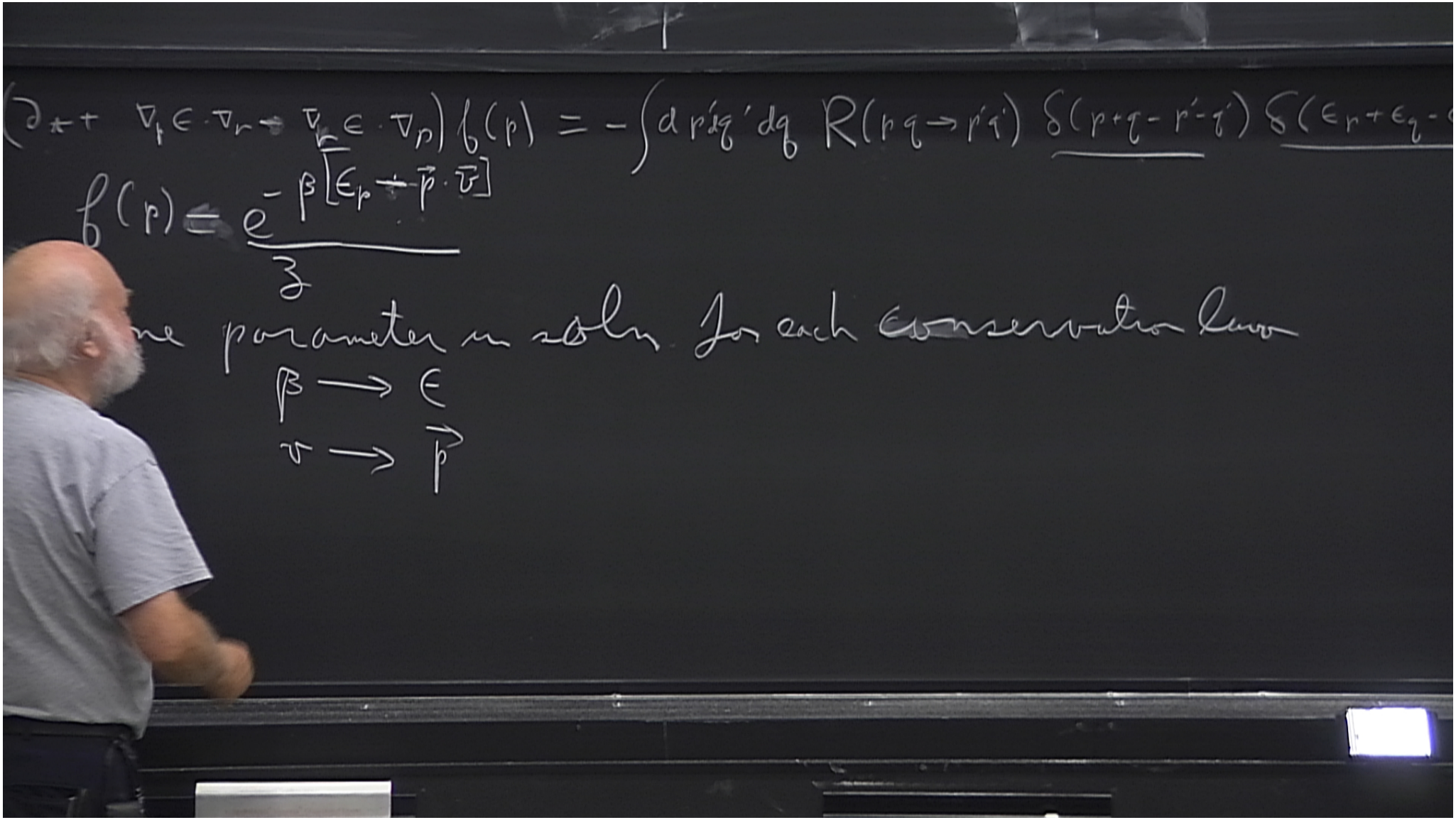

$(\partial_t + \nabla_p \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p) f(p) = -\int dr' dq' dq\, R(rq \to r'q')$
$f = \frac{e^{-\beta[\epsilon_p + \vec{p}\cdot\vec{v}]}}{Z}$



$$\left(\partial_t + \nabla_p \epsilon \cdot \nabla_r + \nabla_r \epsilon \cdot \nabla_p\right) f(p) = -\int dr' dq' dq\, R(rq \to r'q')\, \delta(p+q-r'-q')\, \delta(\epsilon_r + \epsilon_q -$$

$$f(p) = \frac{e^{-\beta\left[\epsilon_p - \vec{p}\cdot\vec{v}\right]}}{Z}$$

one parameter n solns for each conservation law


$\left(\partial_{t} + \nabla_{p}\epsilon\cdot\nabla_{r} - \nabla_{r}\epsilon\cdot\nabla_{p}\right) f(p) = -\int dr'dq'dq\, R(rq\to r'q')\,\delta(p+q-r'-q')\,\delta(\epsilon_r+\epsilon_q-$
$f(p) = \frac{e^{-\beta\left[\epsilon_p - \vec{p}\cdot\vec{v}\right]}}{Z}$
parameter solve for each conservation law
$\beta \longrightarrow \epsilon$
$v \longrightarrow \vec{p}$

$$\left(\partial_{t} + \nabla_{p}\epsilon\cdot\nabla_{r} - \nabla_{r}\epsilon\cdot\nabla_{p}\right) f(p) = -\int dr'dq'dq\; R(rq\to r'q')\;\delta(p+q-r'-q')\;\delta(\epsilon_{r}+\epsilon_{q}-$$

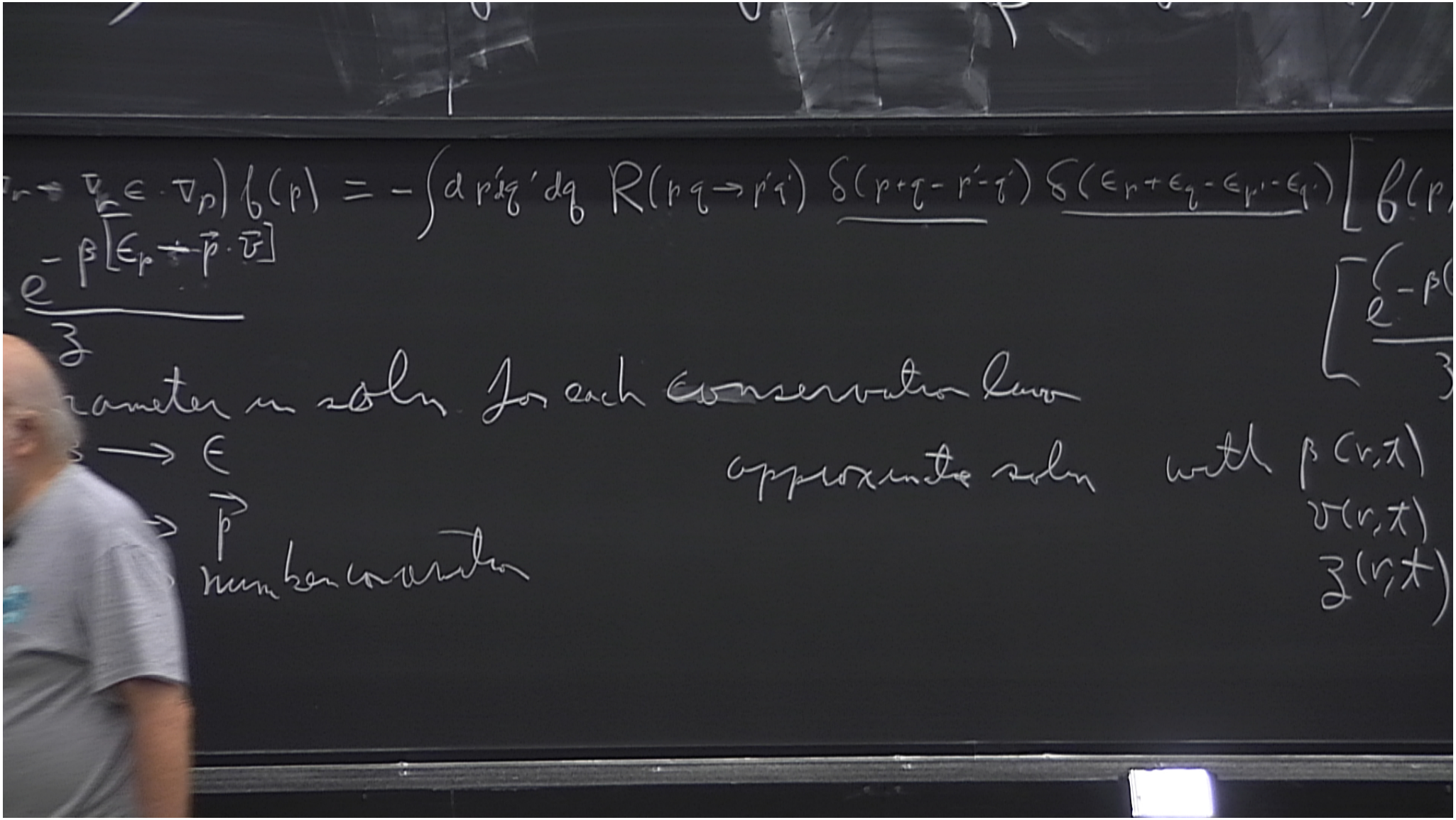
$$f(p) = \frac{e^{-\beta\left[\epsilon_{p} - \vec{p}\cdot\vec{v}\right]}}{Z}$$

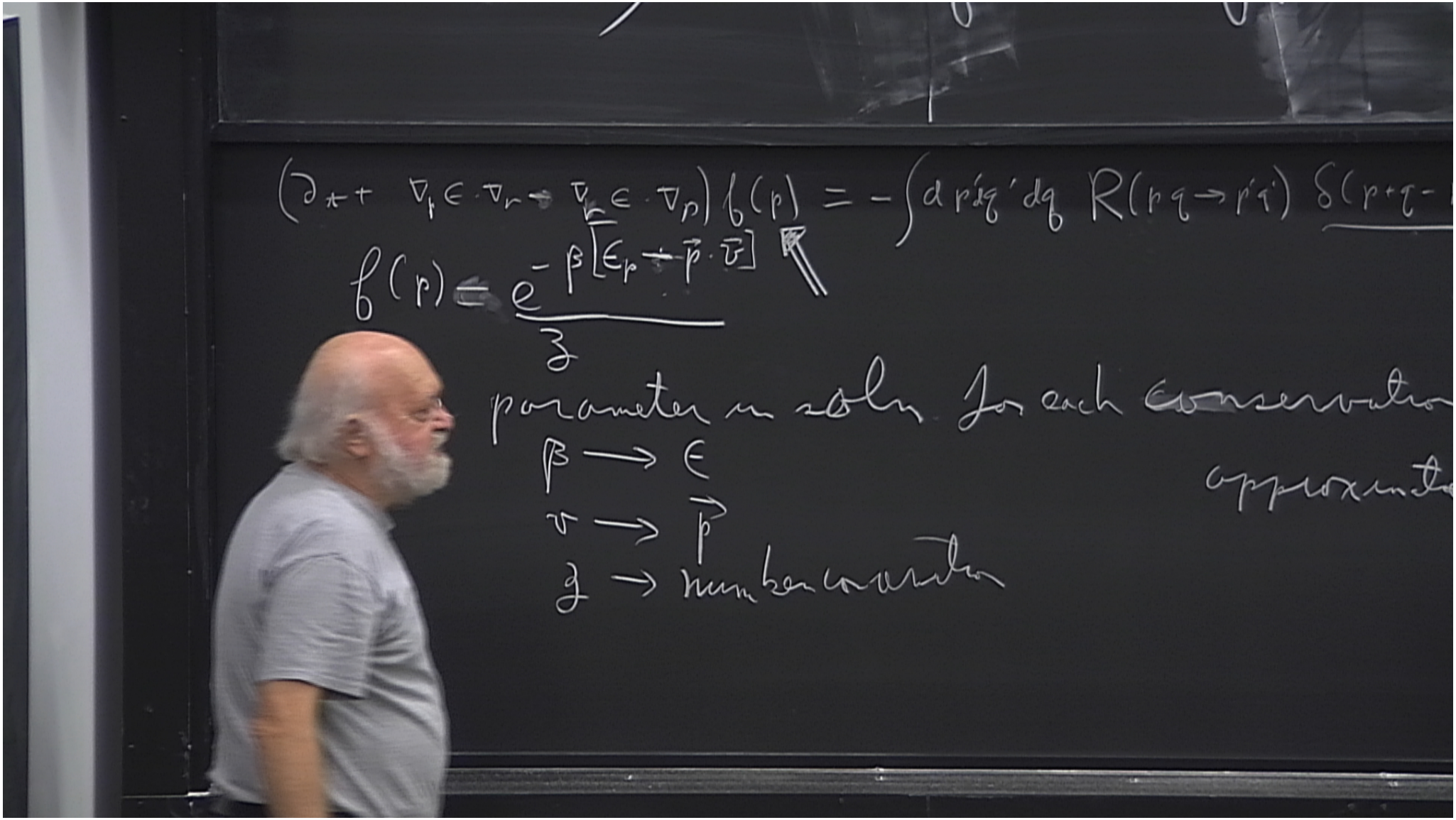
one parameter in solns for each conservation

$\beta \longrightarrow \epsilon$

$v \longrightarrow \vec{p}$

$z \to$ number conservation


$\nabla_r \to \nabla_r \in \cdot \nabla_p) f_0(p) = -\int d\,r\,dq\,dq\,dq\; R(rq \to r'q')\; \delta(p+q-r'-q')\; \delta(\epsilon_r+\epsilon_q-\epsilon_{r'}-\epsilon_{q'})$
$-\beta[\epsilon_p - \vec{p}\cdot\vec{v}]$
$\frac{e}{Z}$
rameter n solns for each conservation laws
$\to \epsilon$
$\to \vec{p}$
number conservation
approximate solns with $\beta(r,t)$
$v(r,t)$
$Z(r,t)$


$\left(\partial_t + \nabla_p \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p\right) f(p) = -\int dr' dq' dq\, R(rq \to r'q')\, \delta(p+q-$
$f(p) = \frac{e^{-\beta[\epsilon_p - \vec{p}\cdot\vec{v}]}}{Z}$
parameters solve for each conservation
approximate
$\beta \longrightarrow \epsilon$
$v \longrightarrow \vec{p}$
$Z \to$ number conservation

$$\left(\partial_t + \nabla_p \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p\right) f(p) = -\int dr' dq' dq\, R(rq \to r'q')\, \delta(r+q-$$

$$f(p) = \frac{e^{-\beta\left[\epsilon_p - \vec{p}\cdot\vec{v}\right]}}{Z}$$

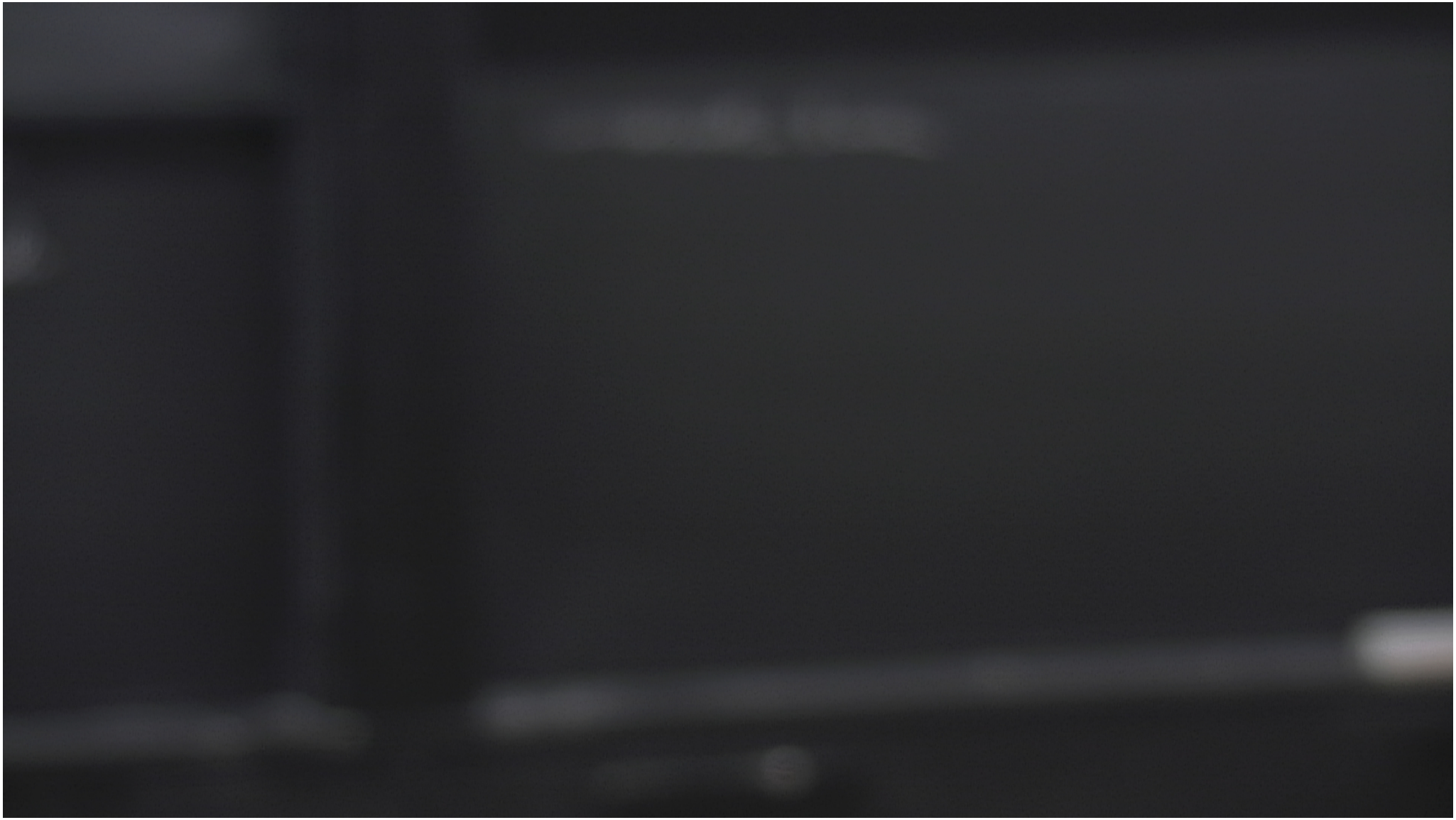
one parameter in soln for each conservation

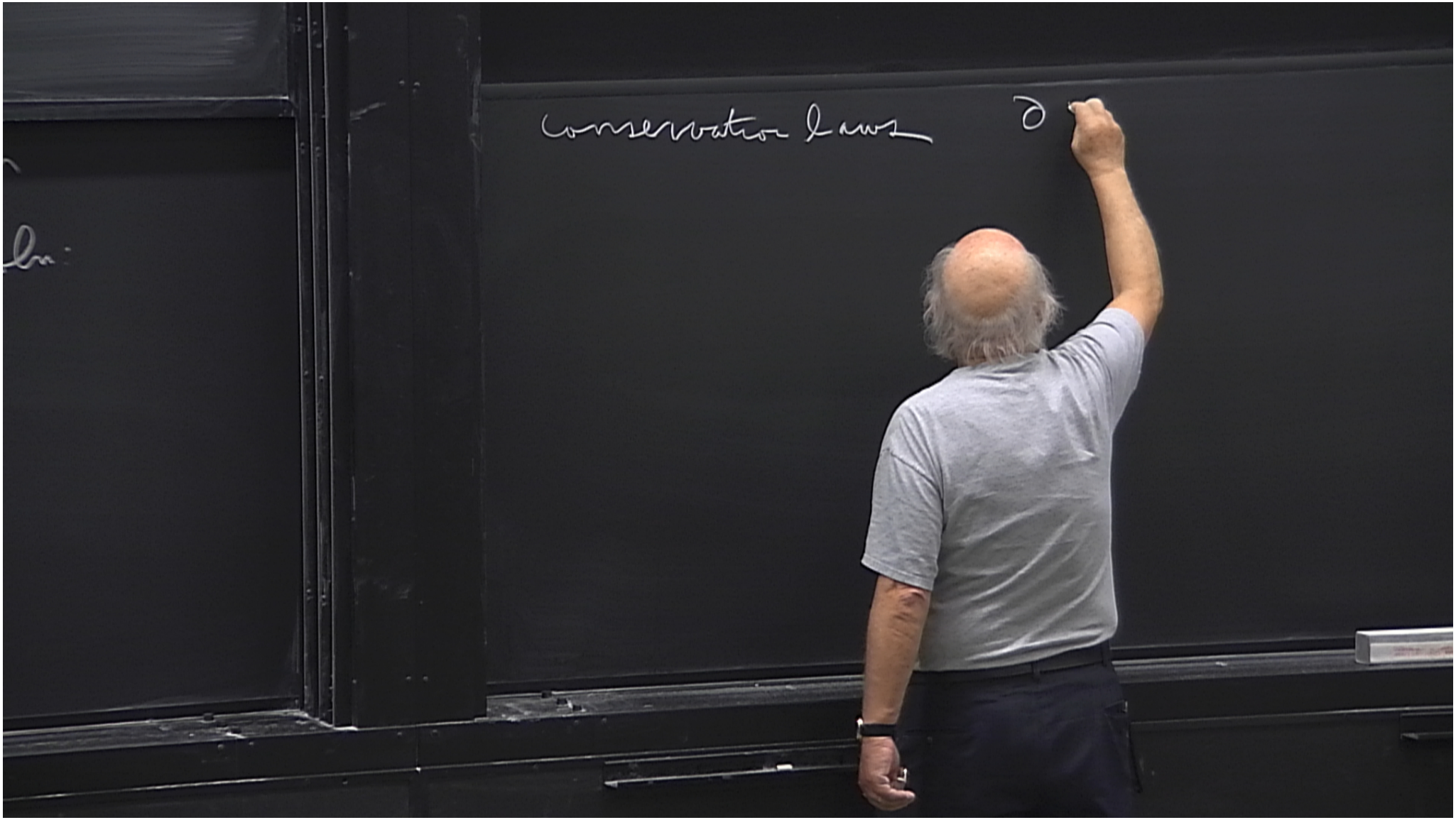
$\beta \longrightarrow \epsilon$

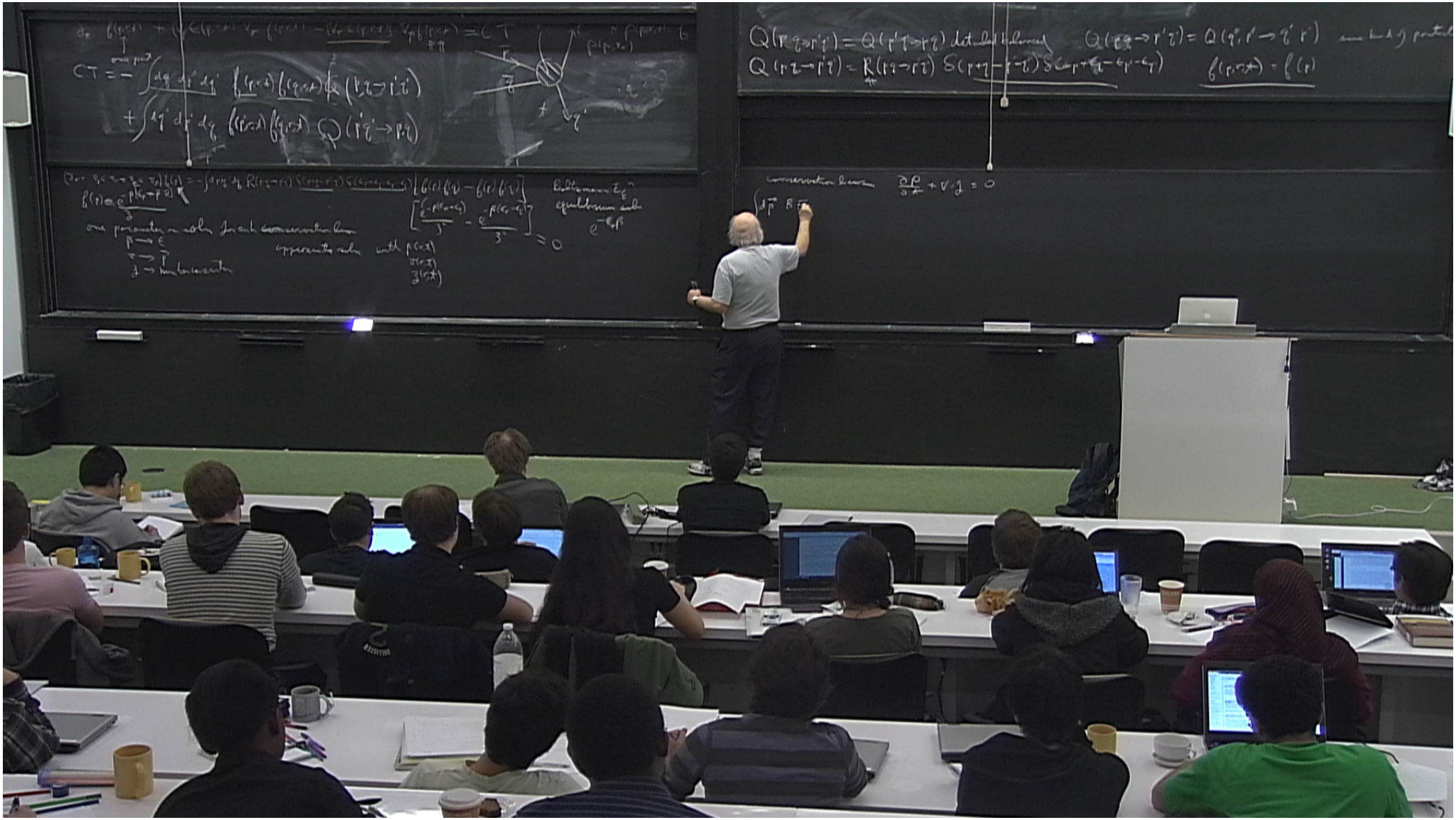
$v \longrightarrow \vec{p}$

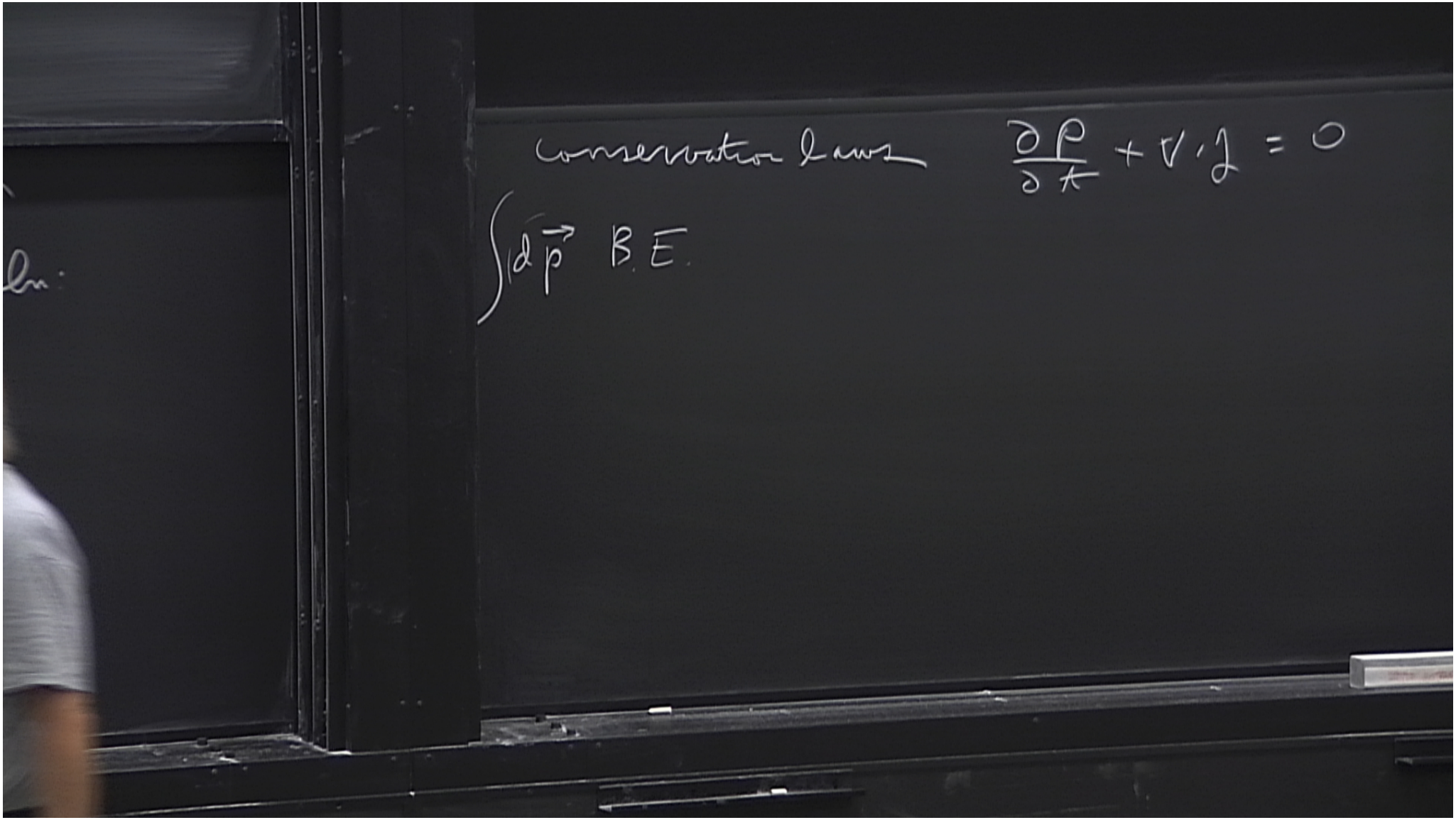
$Z \to$ number conservation

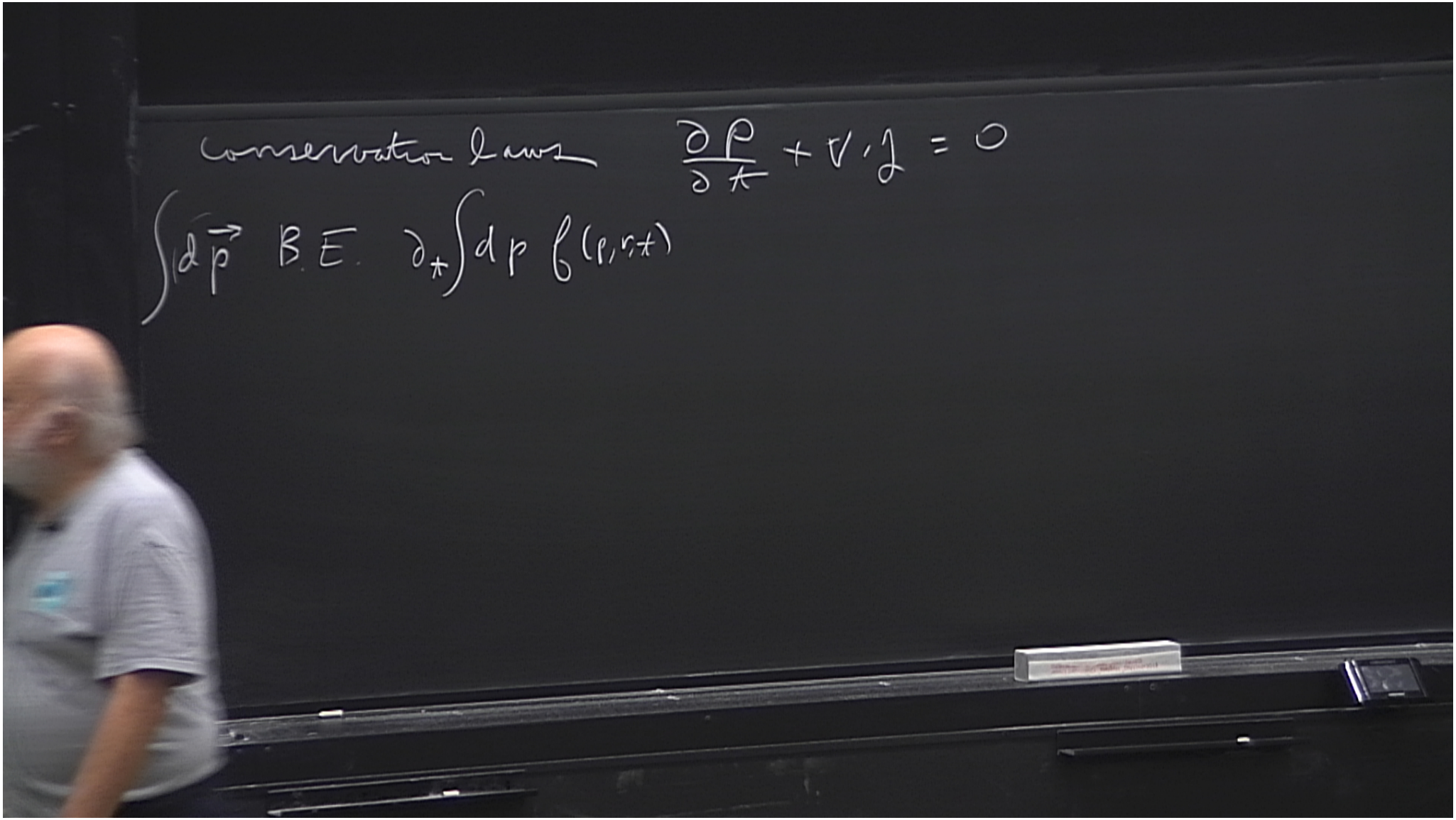
approximate

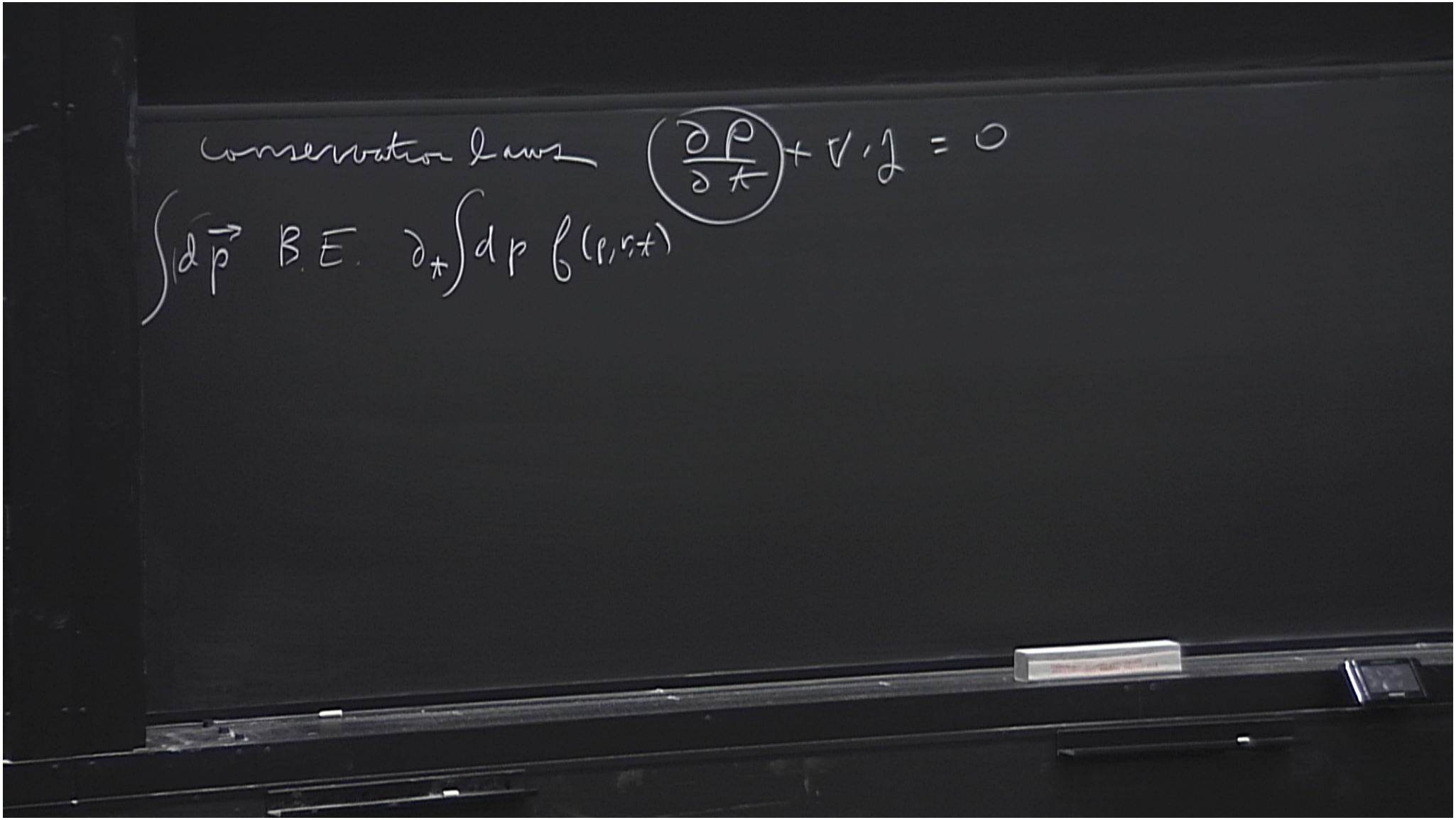



conservation laws




conservation laws
$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
$\int d\vec{p}$ B.E.


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$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
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$$f(p) = \frac{e^{-\beta[\epsilon_p - \vec{p}\cdot\vec{v}]}}{Z}$$

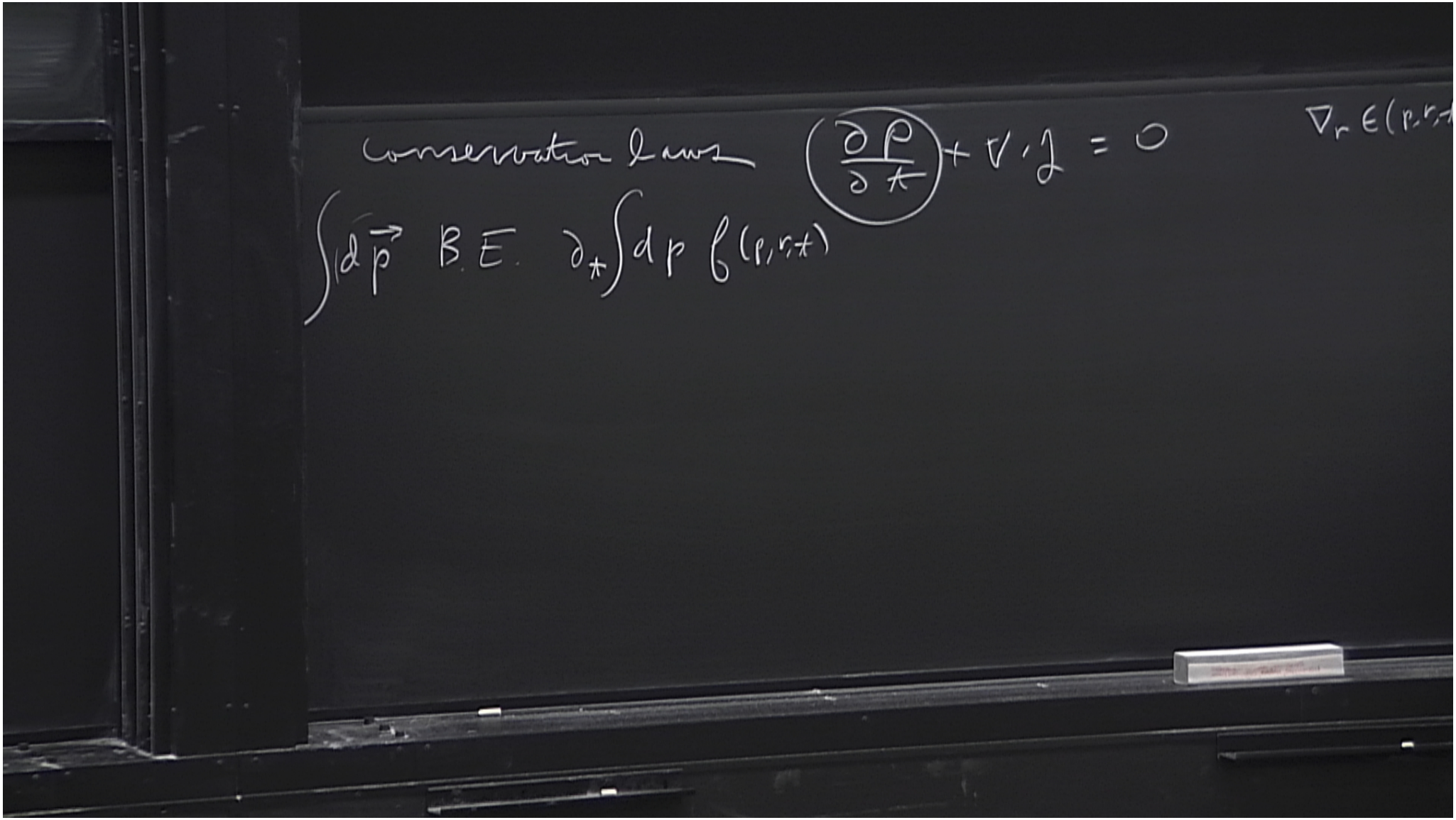
one parameter solns for each conservation

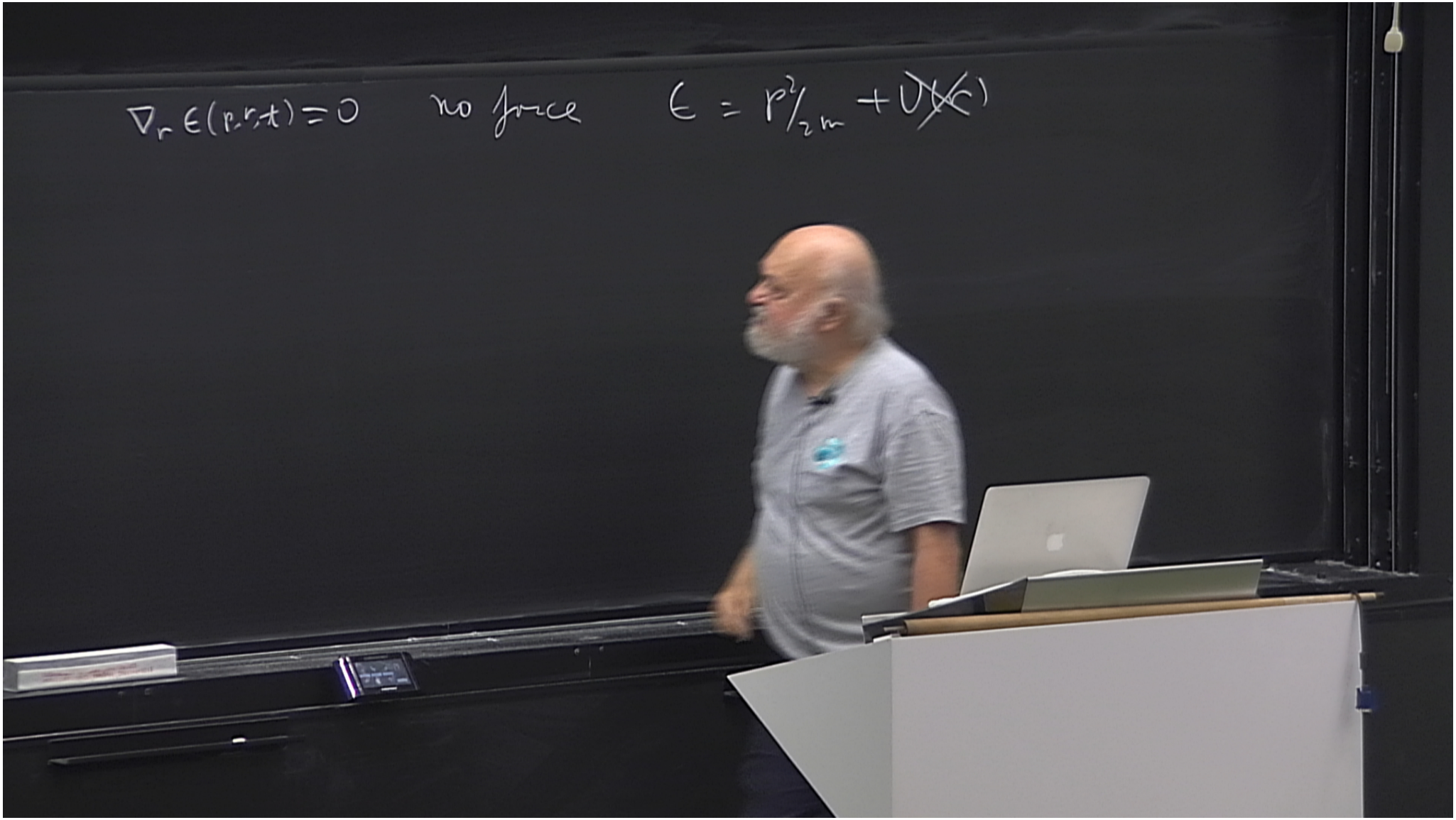
$\beta \longrightarrow \epsilon$

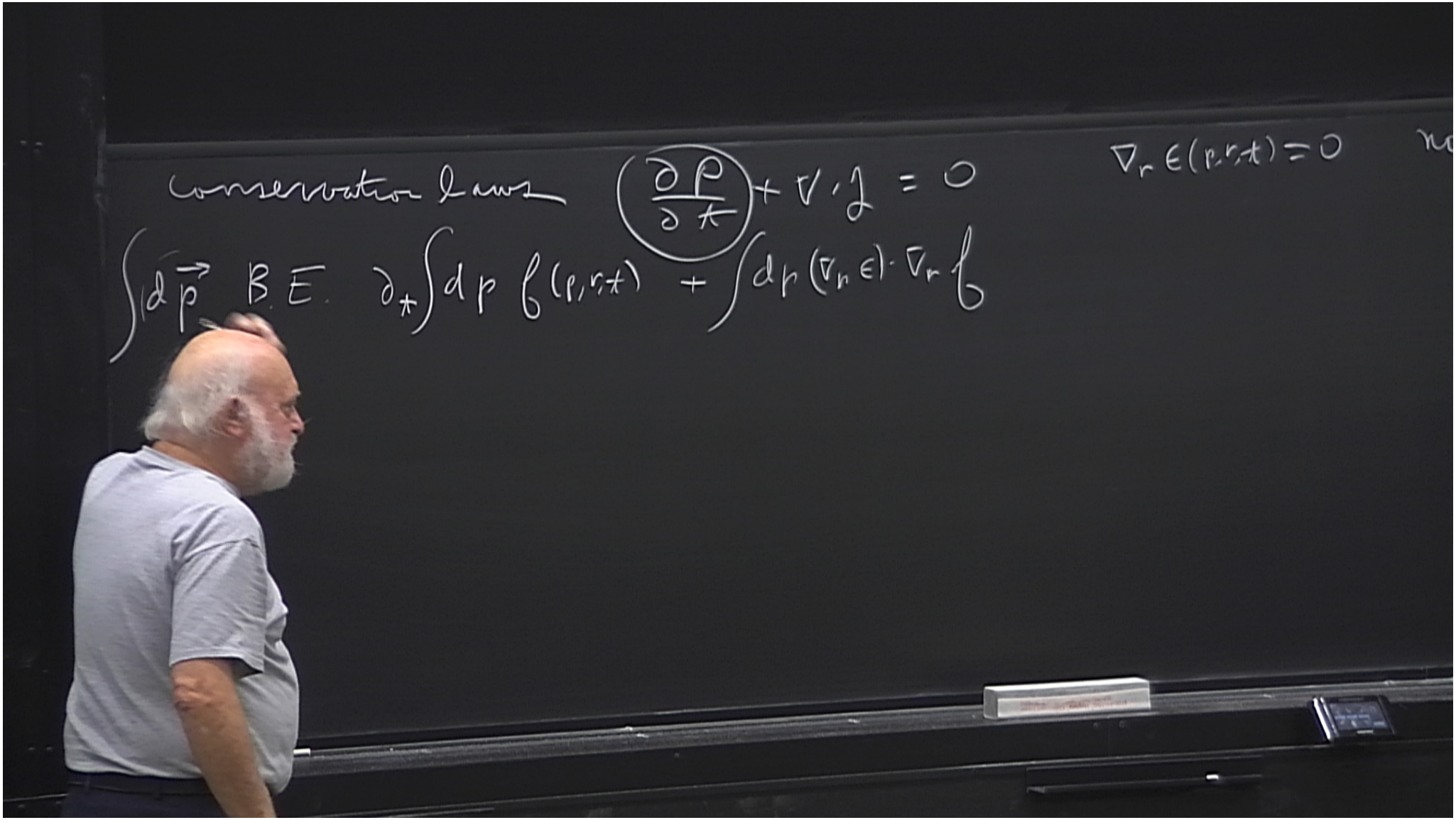
$v \longrightarrow \vec{p}$

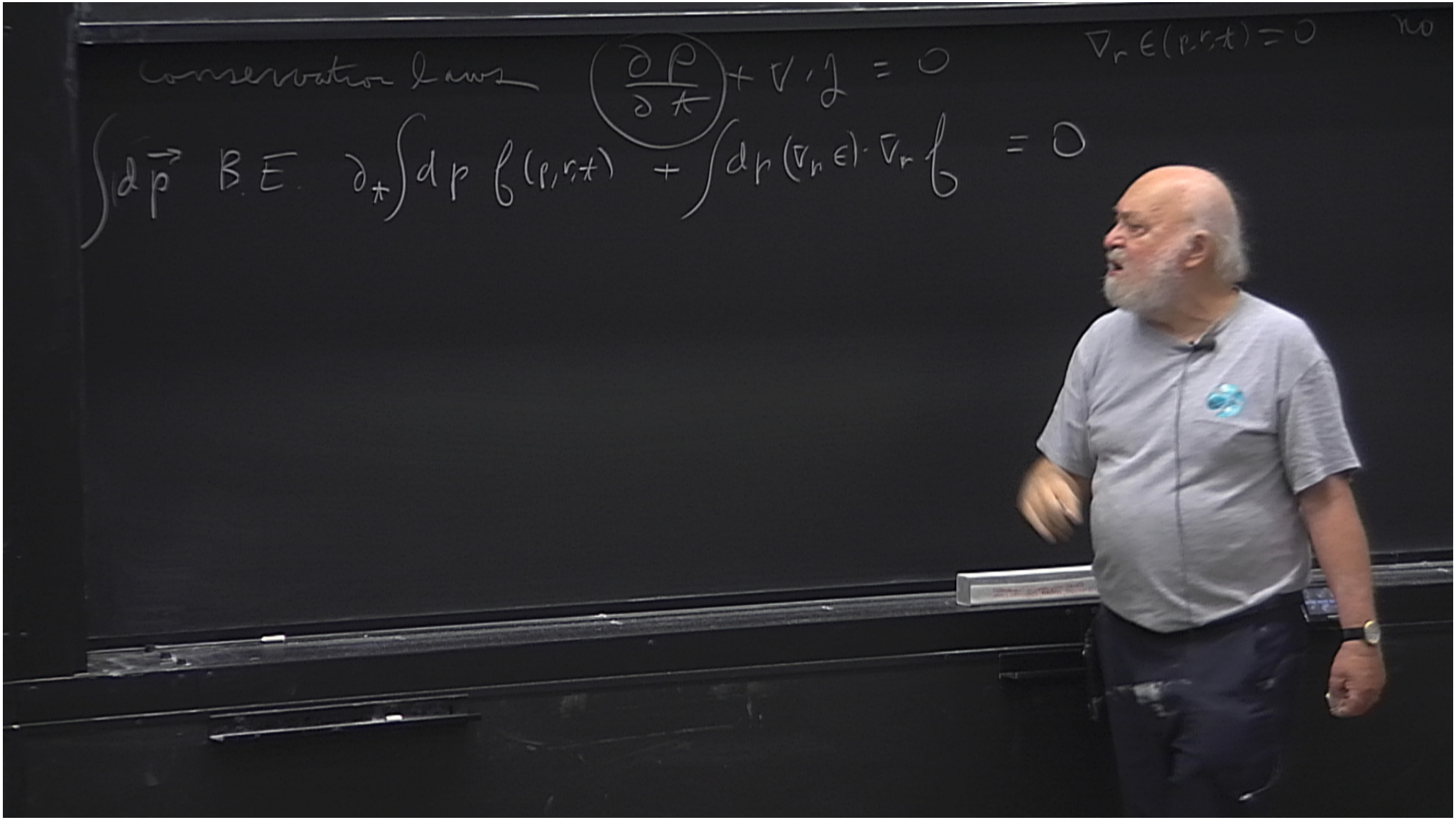
$z \to$ number conservation

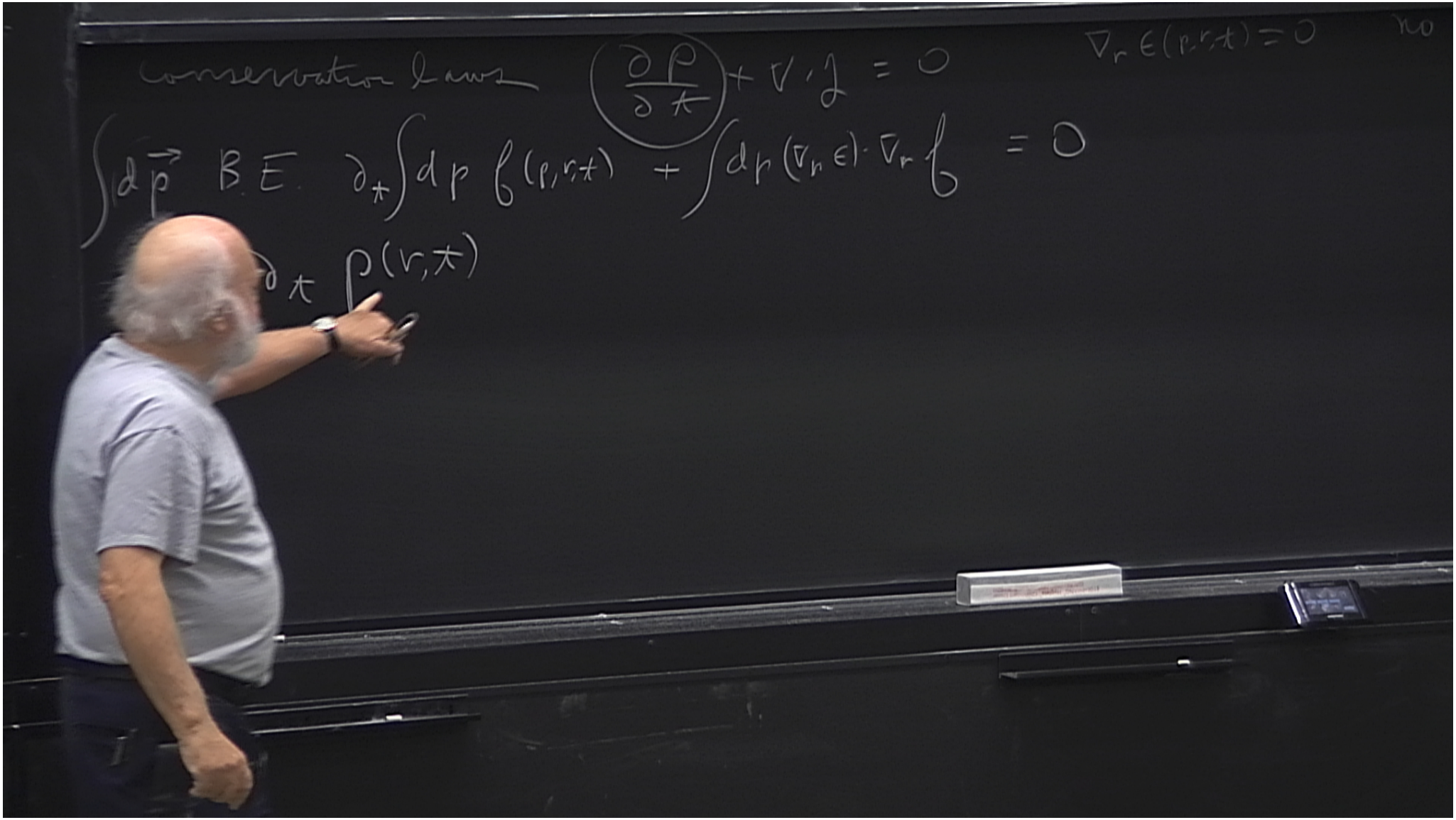
approximate

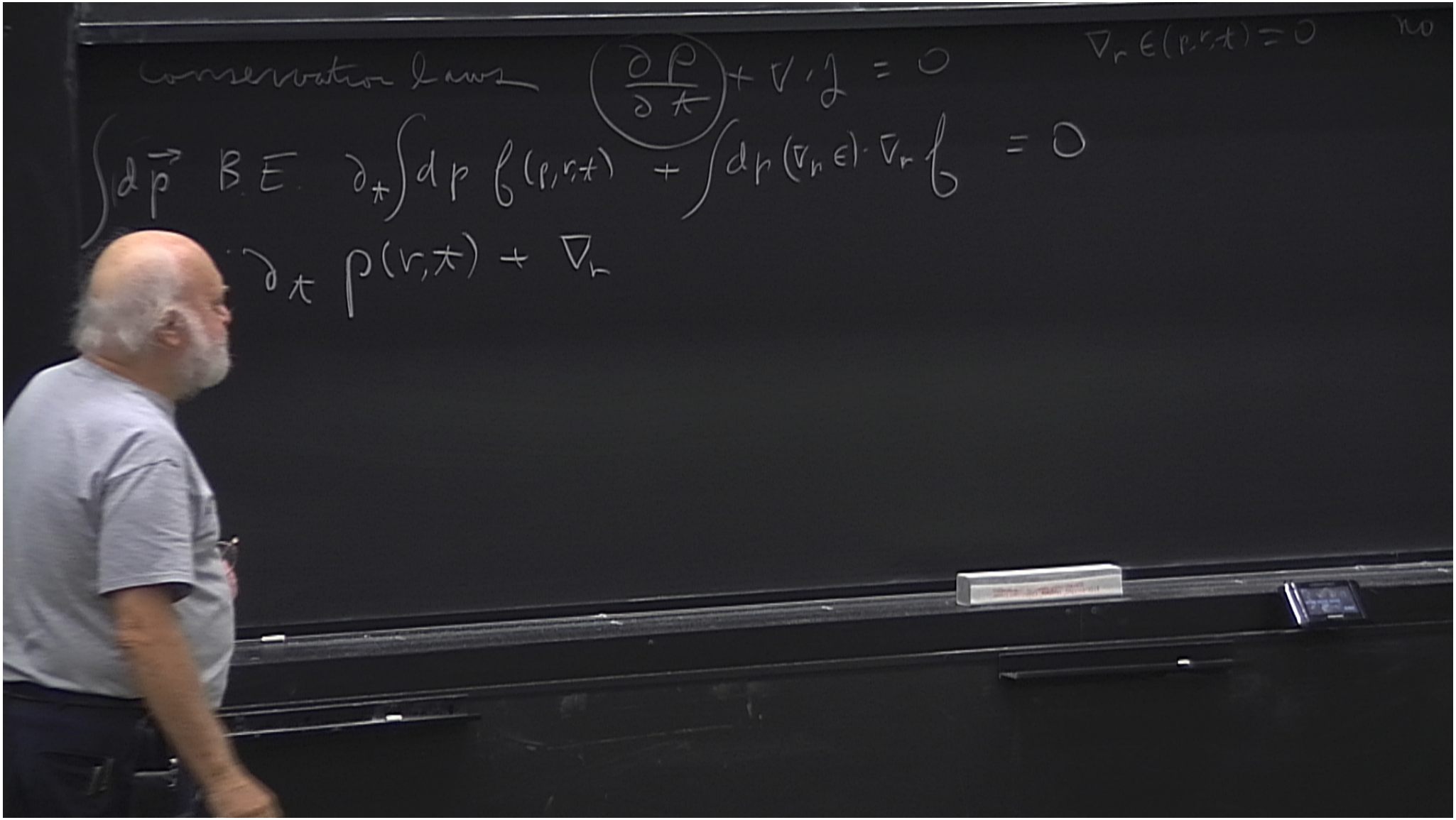

conservation laws
$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
$\int d\vec{p}$ B.E. $\partial_t \int dp\, f(p,r,t)$
$\nabla_r \epsilon(p,r,t$

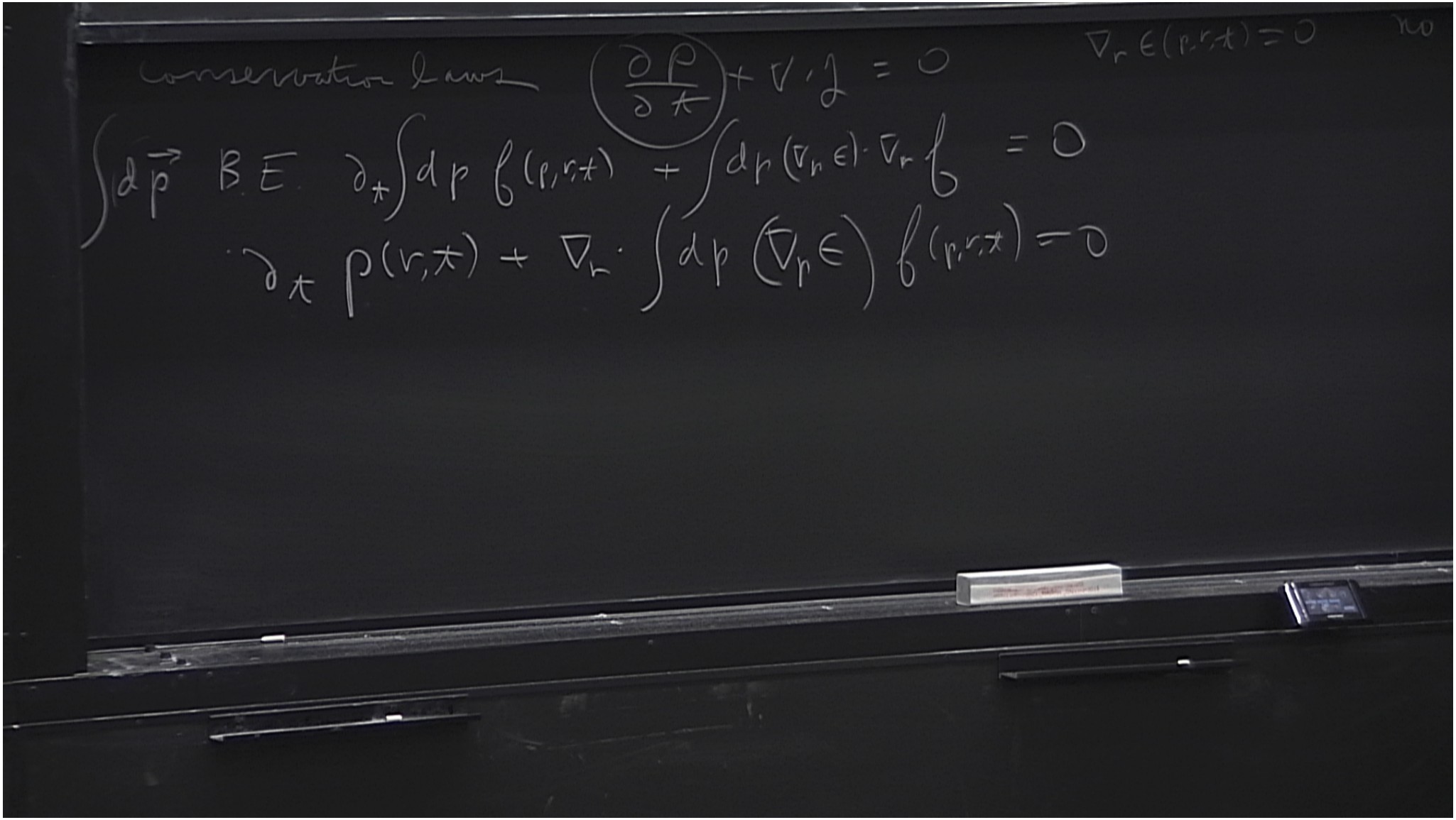

$\nabla_r \epsilon(p,r,t) = 0$ no force $\epsilon = p^2/2m + U(r)$

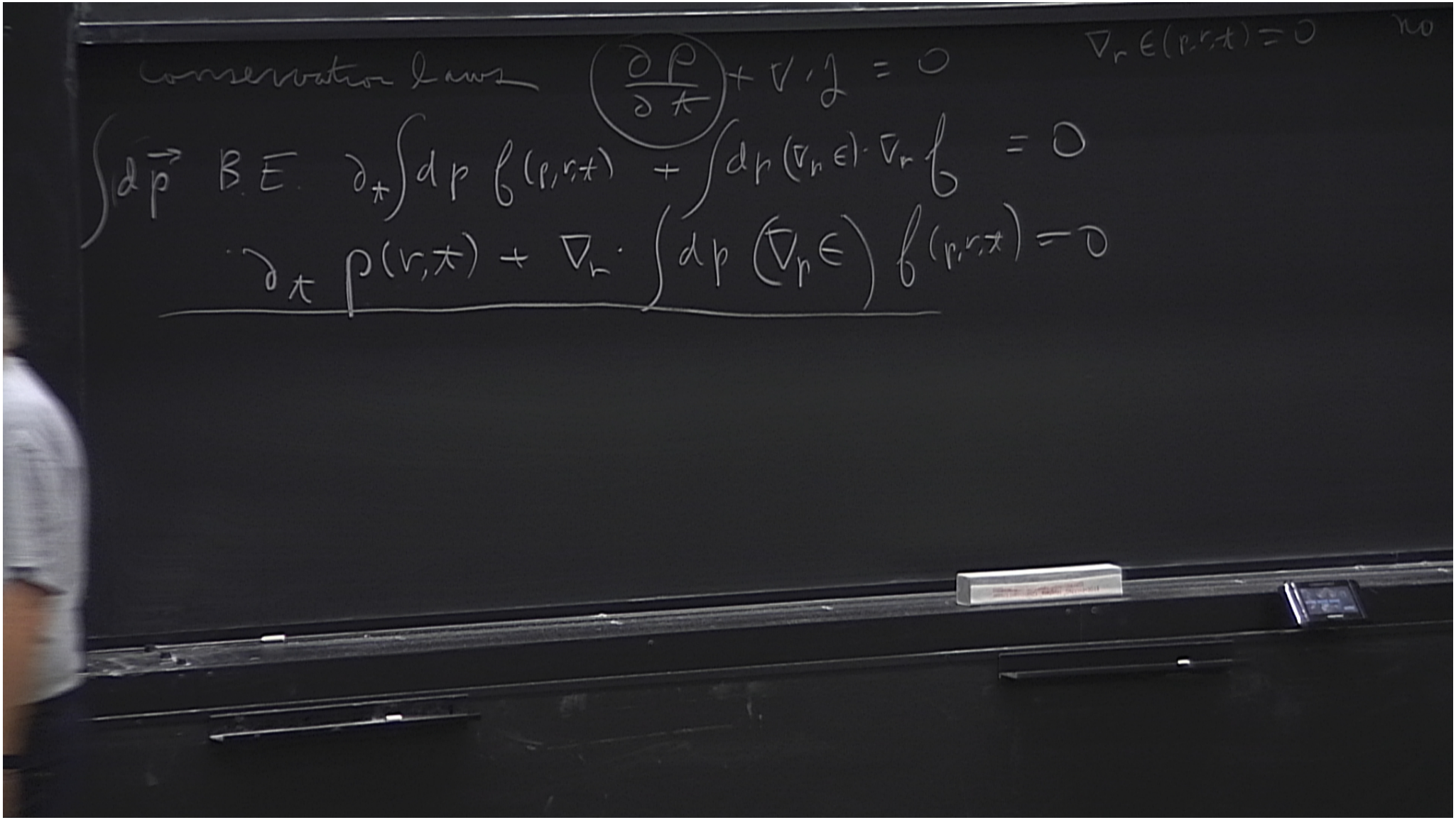

conservation laws
$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
$\nabla_r \epsilon(p,r,t) = 0$
$\int d\vec{p}$ B.E. $\partial_t \int dp\, f(p,r,t) + \int dp\, (\nabla_r \epsilon) \cdot \nabla_r f$

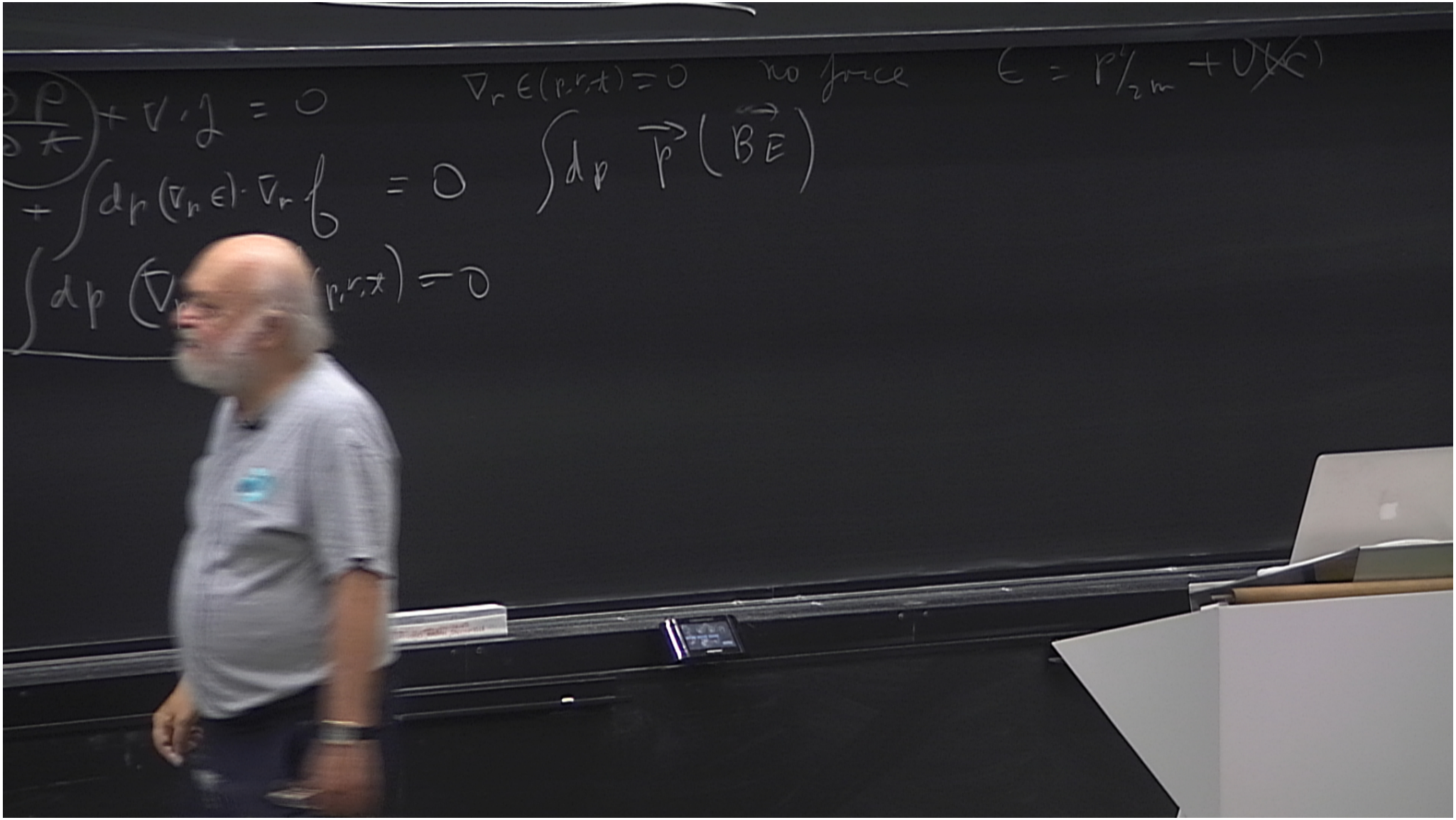

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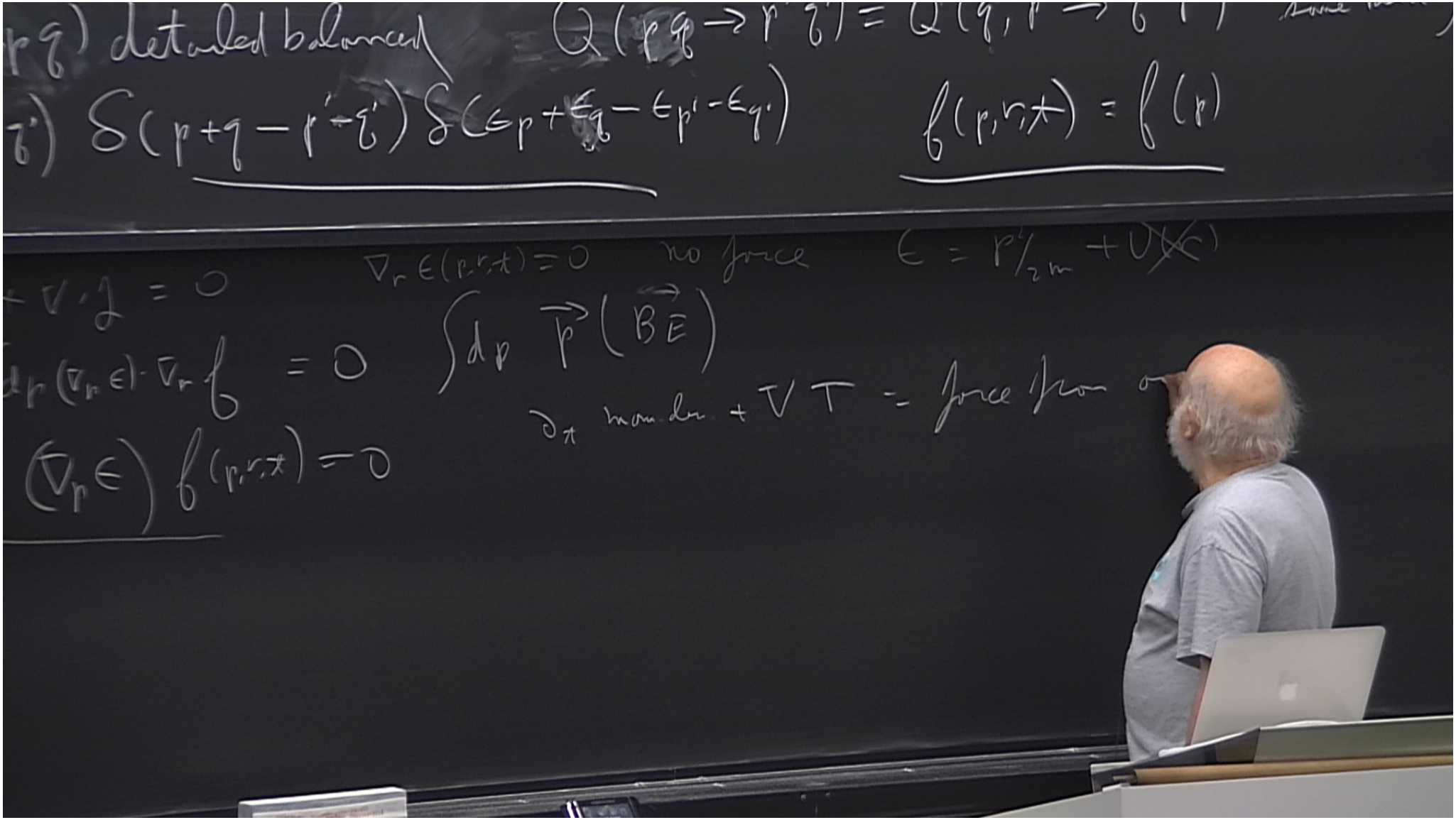

conservation laws
$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
$\nabla_r \epsilon(p,r,t) = 0$
no
$\int d\vec{p}$ B.E. $\partial_t \int dp\, f(p,r,t) + \int dp\, (\nabla_r \epsilon) \cdot \nabla_r f = 0$
$\partial_t \rho(r,t)$

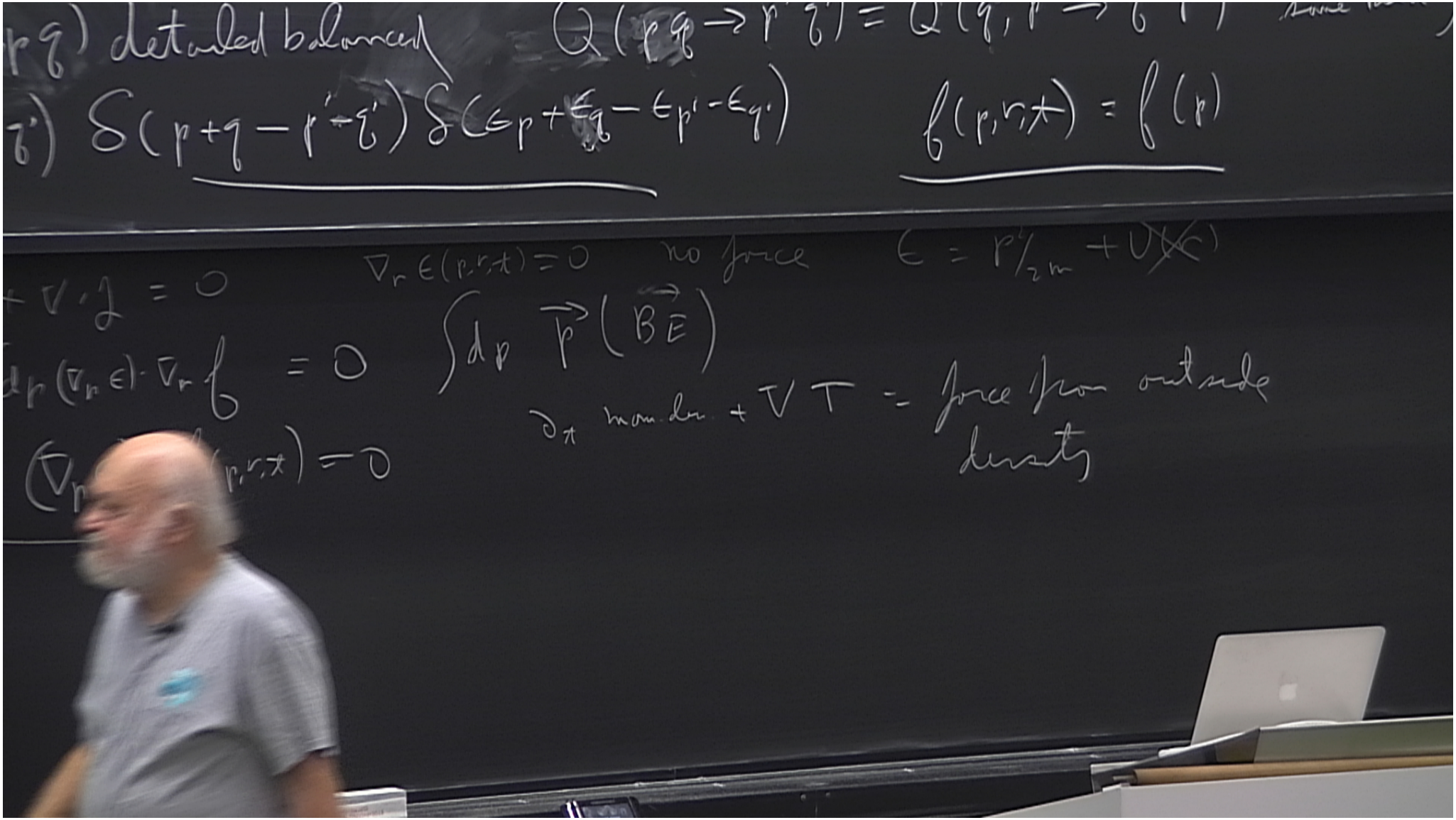

conservation laws
$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
$\nabla_r \epsilon(p,r,t) = 0$
$\int d\vec{p}$ BE $\partial_t \int dp\, f(p,r,t) + \int dp\, (\nabla_r \epsilon) \cdot \nabla_r f = 0$
$\partial_t \rho(r,t) + \nabla_r$

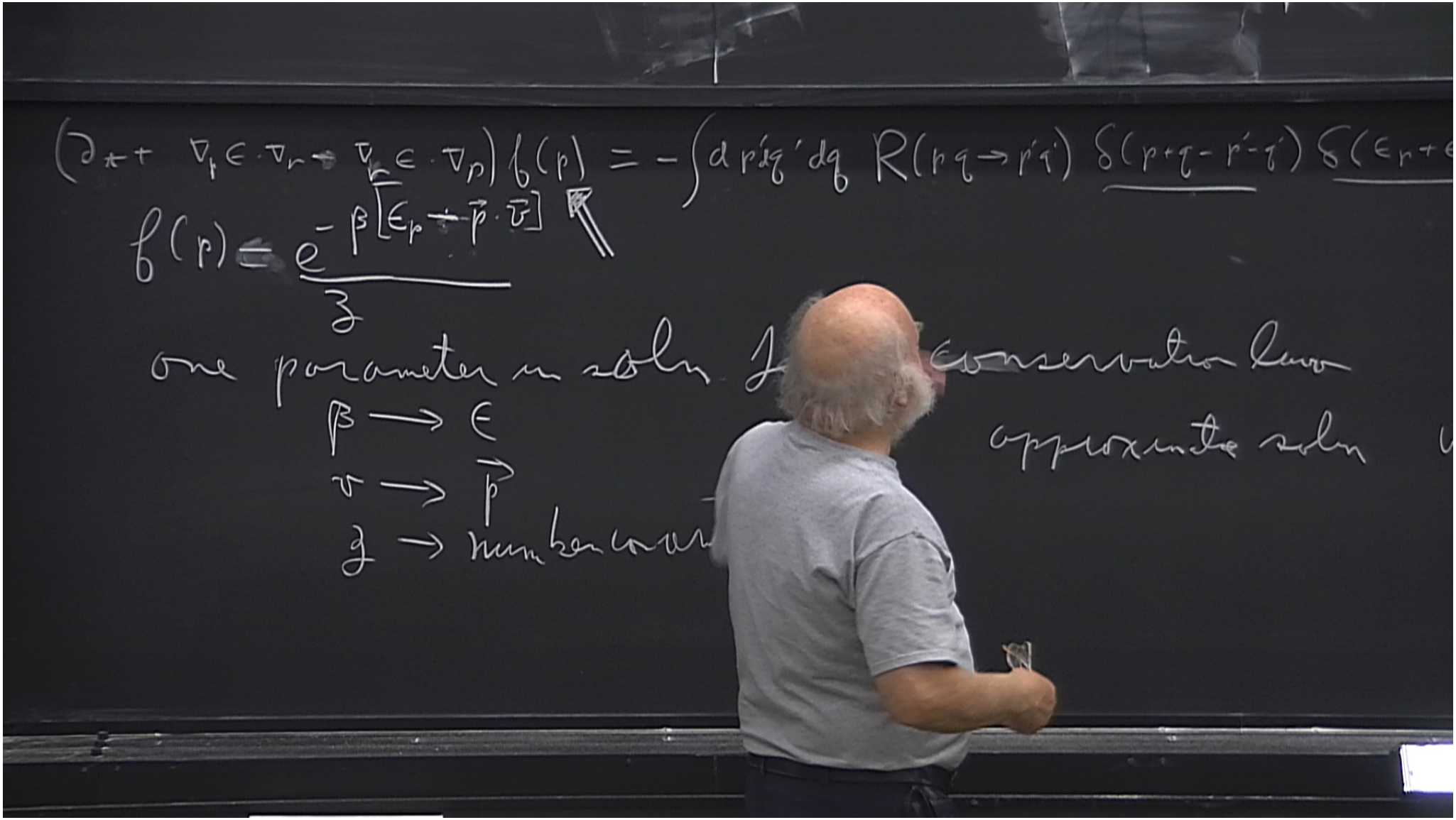

conservation laws
$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
$\nabla_r \epsilon(p,r,t) = 0$
$\int d\vec{p}$ BE $\partial_t \int dp\, f(p,r,t) + \int dp\, (\nabla_p \epsilon) \cdot \nabla_r f = 0$
$\partial_t \rho(r,t) + \nabla_r \cdot \int dp\, (\nabla_p \epsilon)\, f(p,r,t) = 0$

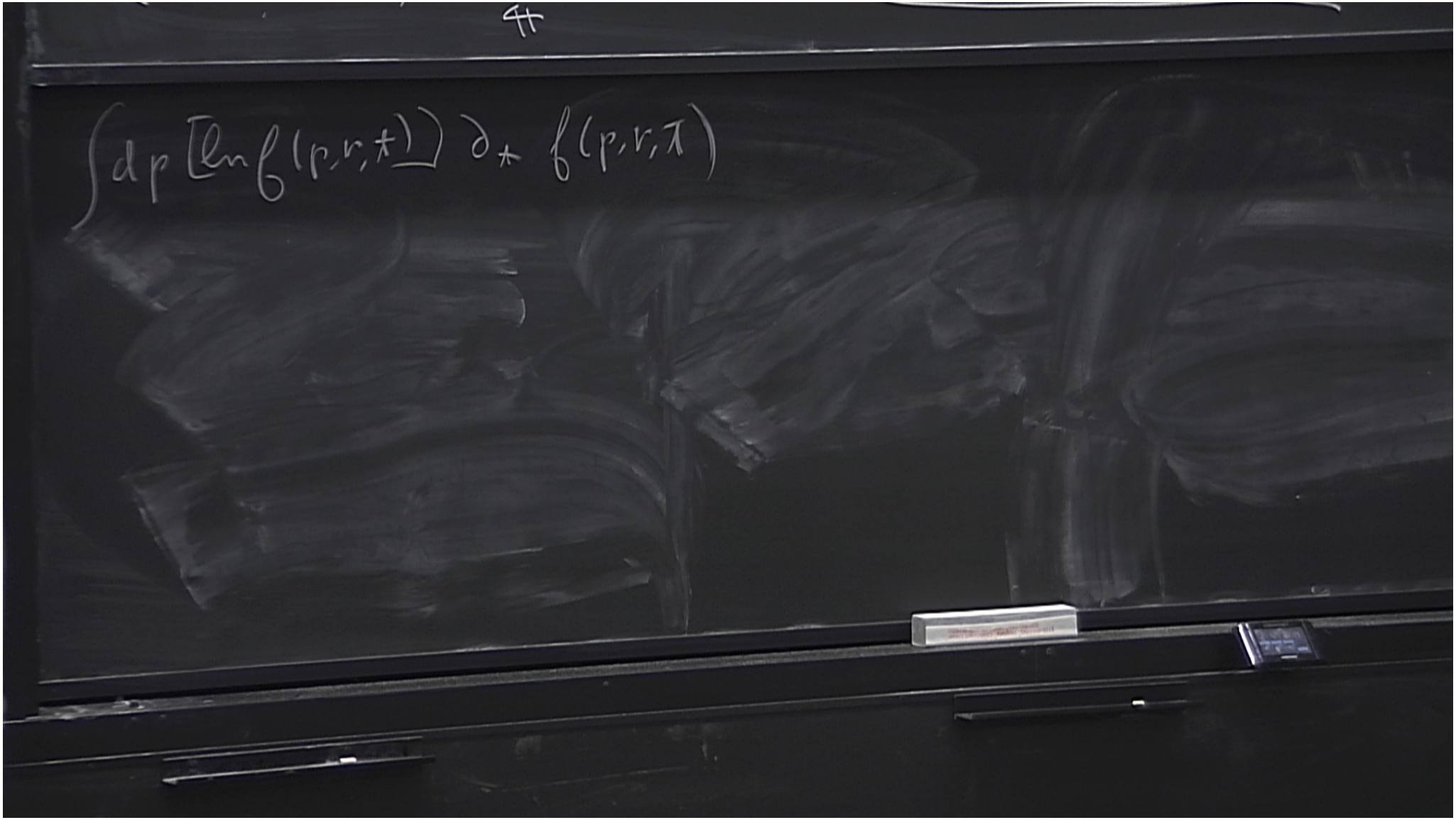

conservation laws
$\frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$
$\nabla_r \epsilon(p,r,t) = 0$ no
$\int d\vec{p}$ B.E. $\partial_t \int dp\, f(p,r,t) + \int dp\, (\nabla_p \epsilon) \cdot \nabla_r f = 0$
$\partial_t \rho(r,t) + \nabla_r \cdot \int dp\, (\nabla_p \epsilon)\, f(p,r,t) = 0$

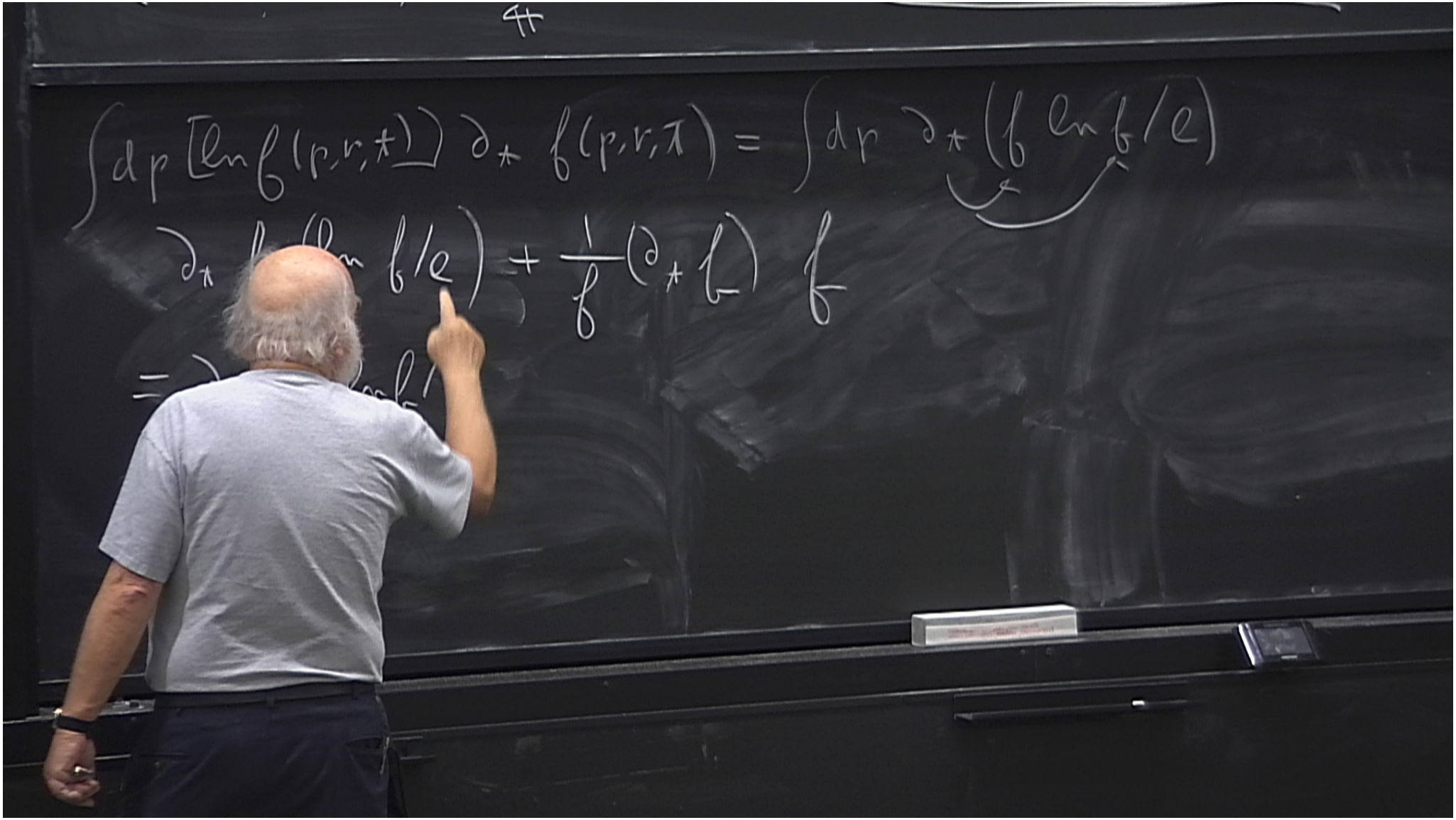

$\nabla_r \epsilon(r,v,t) = 0$
no force
$\epsilon = p^2/2m + U(x)$
$+ \nabla \cdot j = 0$
$\int dp\, \vec{p}\,(\vec{B}\vec{E})$
$+ \int dp\, (\nabla_r \epsilon) \cdot \nabla_r f = 0$
$\int dp\, (\nabla$
$(r,v,t) = 0$

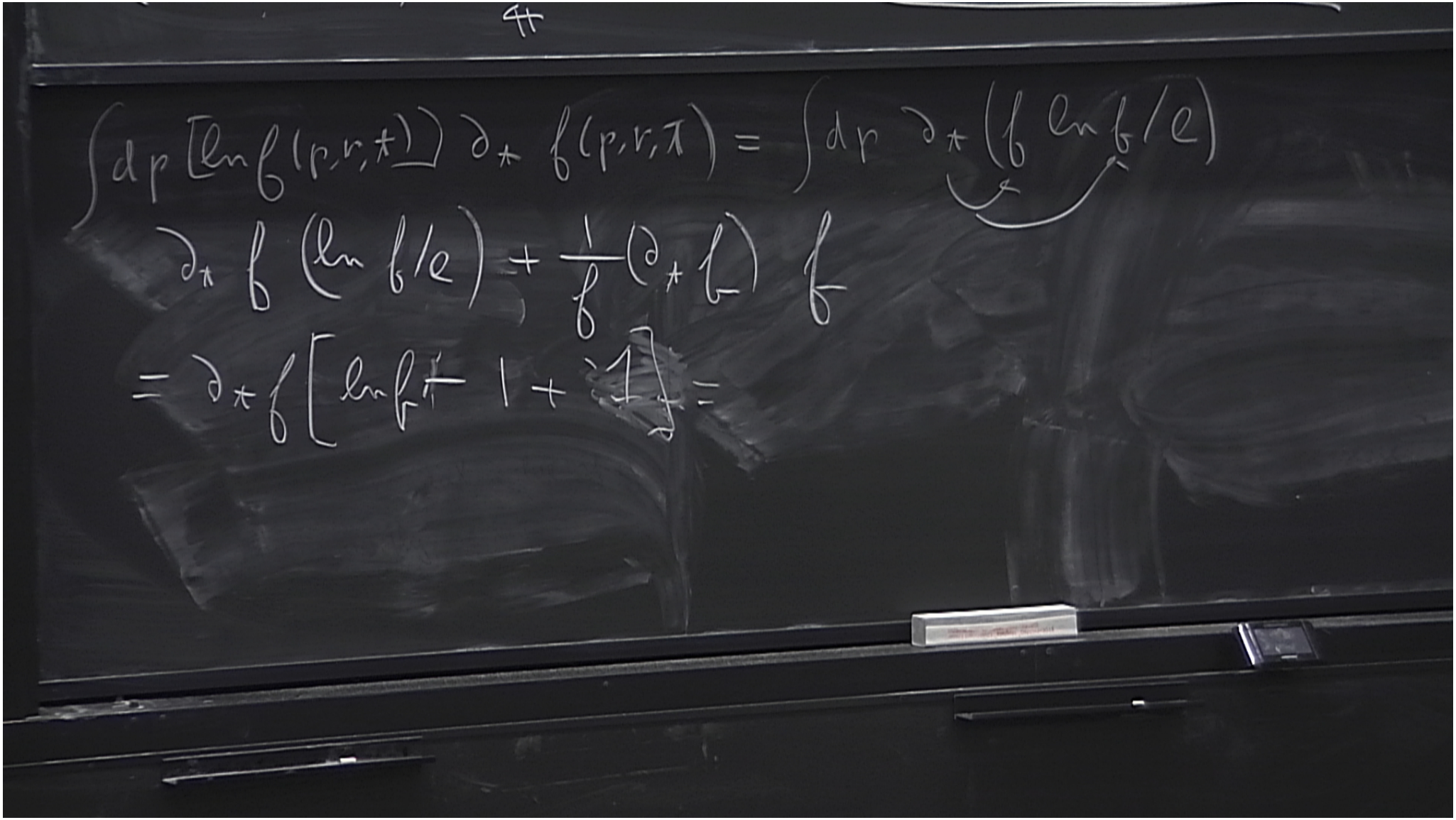

detailed balance
$\delta(p+q-p'-q')\,\delta(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'})$
$f(p,r,t) = f(p)$
$\nabla_r \epsilon(p,r,t) = 0$ no force
$\epsilon = p^2/2m + U(r)$
$\nabla \cdot j = 0$
$\int dp\, \vec{p}\,(B\vec{E})$
∂_t momentum $+ \nabla T =$ force

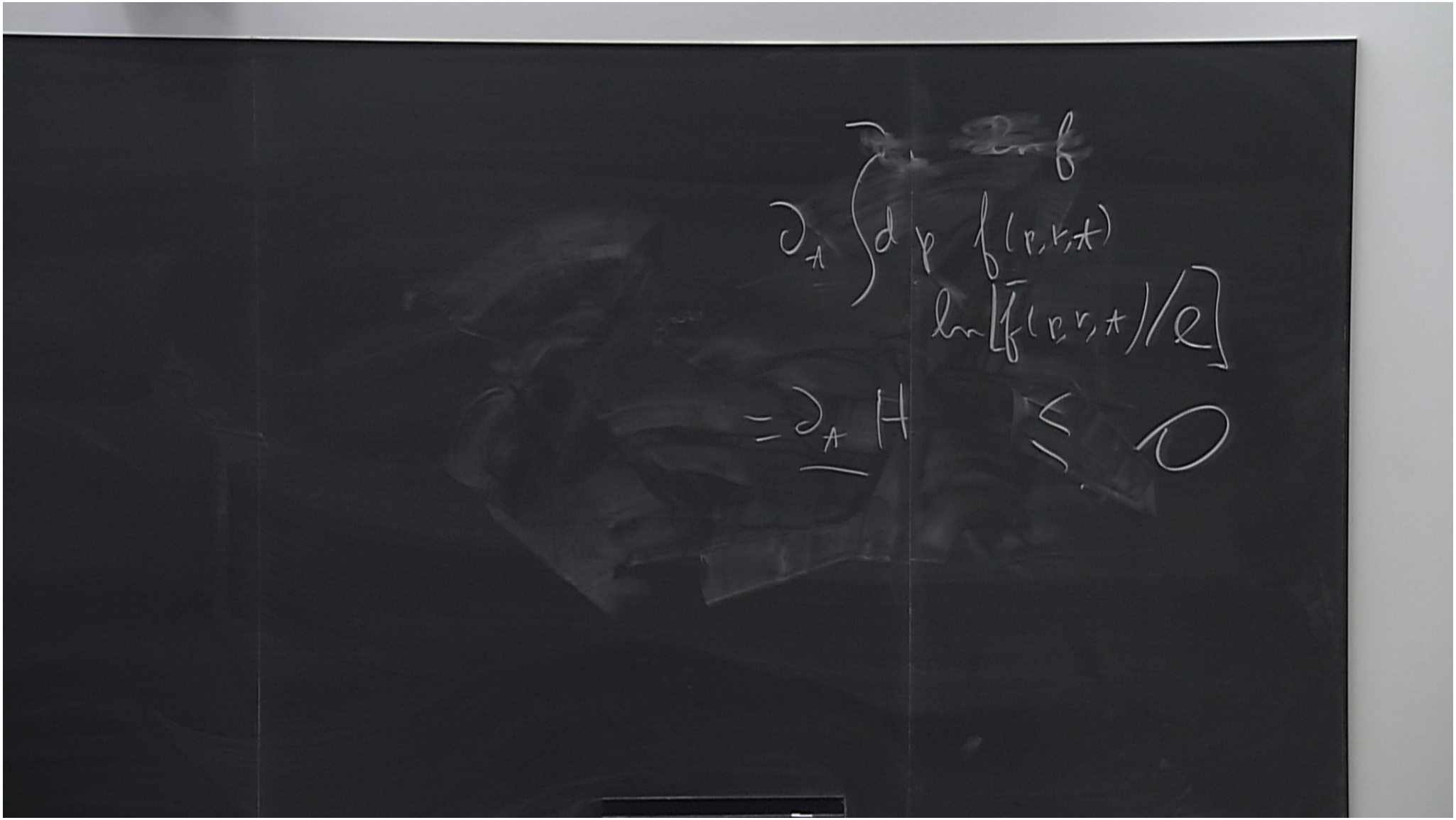

detailed balance
$Q(pq \to p'q') = Q(q,p \to q'p')$
$\delta(p+q-p'-q')\,\delta(\epsilon_p+\epsilon_q-\epsilon_{p'}-\epsilon_{q'})$
$f(p,r,t) = f(p)$
$\nabla_r \epsilon(p,r,t) = 0$ no force
$\epsilon = p^2/2m + U(r)$
$\int dp\, \vec{p}\,(\vec{B}\vec{E})$
∂_t mom. den. $+ \nabla T =$ force from outside
density

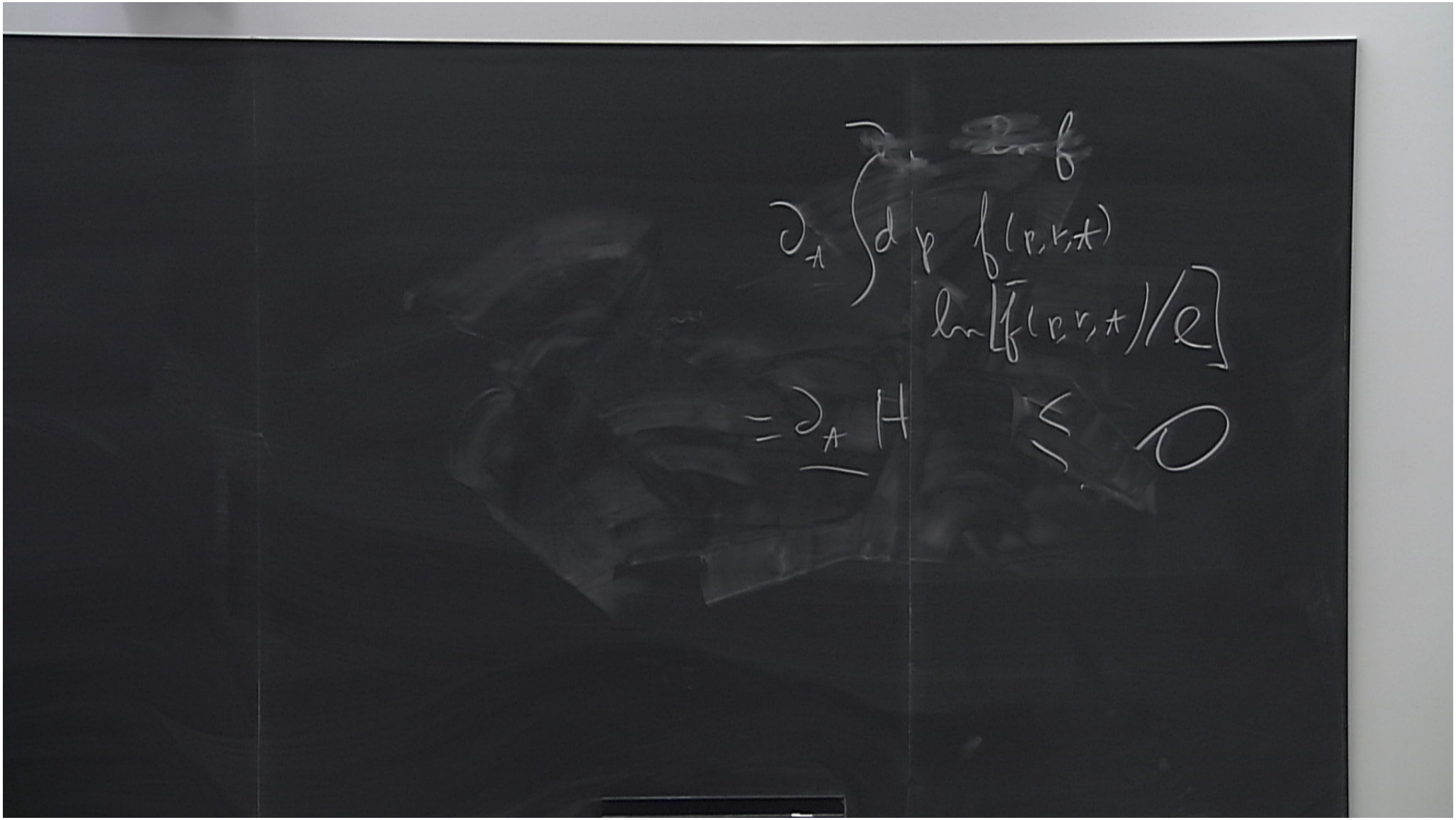

$\left(\partial_t + \nabla_r \epsilon \cdot \nabla_r - \nabla_r \epsilon \cdot \nabla_p\right) f(p) = -\int dp'dq'dq\, R(rq \to r'q')\, \delta(p+q-r'-q')\, \delta(\epsilon_r +$
$f(p) = \frac{e^{-\beta[\epsilon_p - \vec{p}\cdot\vec{v}]}}{Z}$
one parameter
$\beta \longrightarrow \epsilon$
$v \longrightarrow \vec{p}$
$Z \rightarrow$


$\int dp\, [\ln f(p,r,t)]\, \partial_t f(p,r,t)$




$\int dp\,[\ln f(p,r,t)]\,\partial_t f(p,r,t) = \int dp\,\partial_t (f \ln f/e)$
$\partial_t f\,(\ln f/e) + \frac{1}{f}(\partial_t f)\,f$
$= \partial_t f\,[\ln f + 1 + \ldots] =$


$\partial_t \int d^3p\, f(r,v,t)$
$\ln[f(r,v,t)/1]$
$= \partial_t H \leq 0$


$\partial_t \int d^3p\, f(r,v,t)$
$\ln[f(r,v,t)/e]$
$= \partial_t H \leq 0$