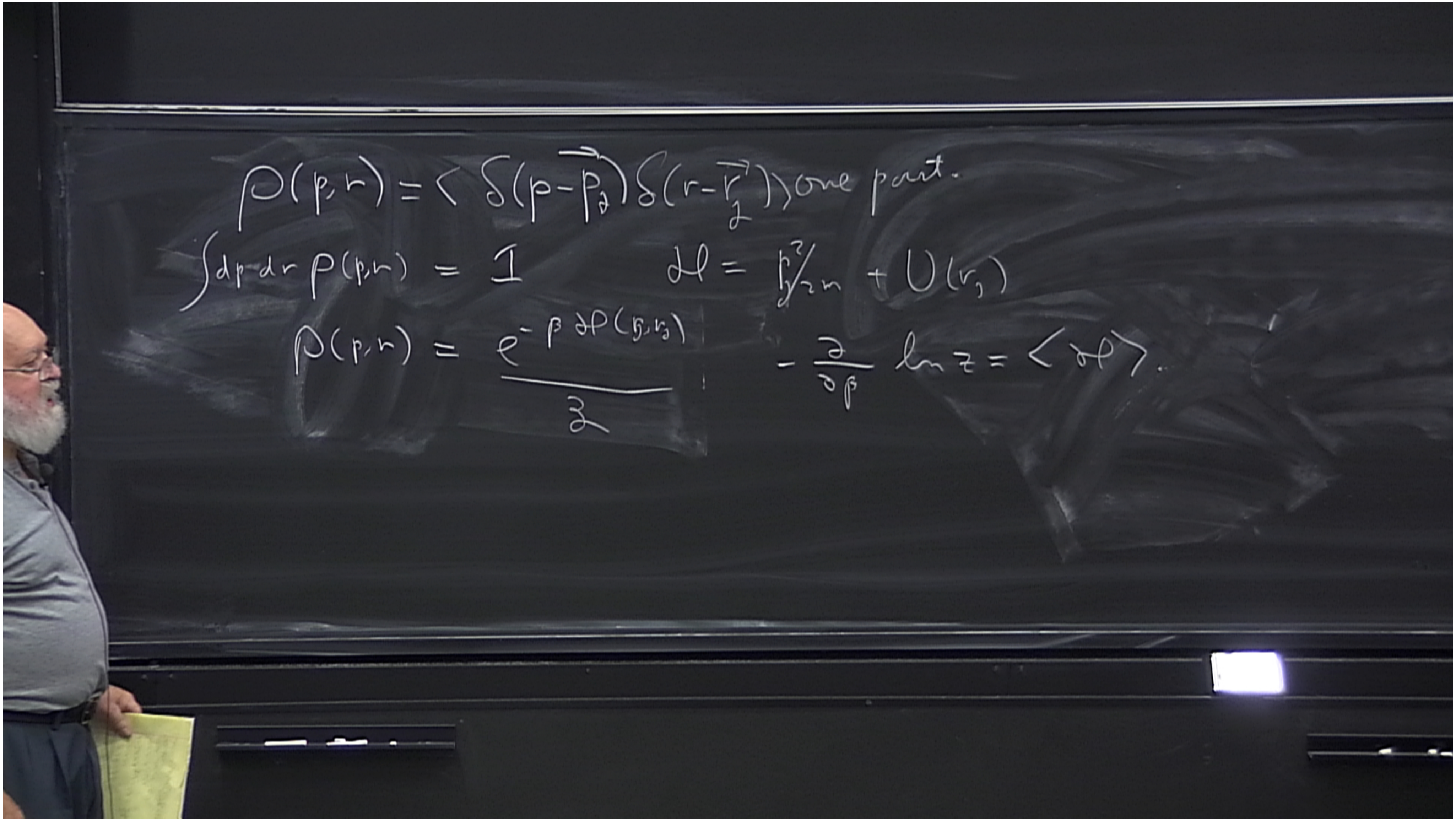
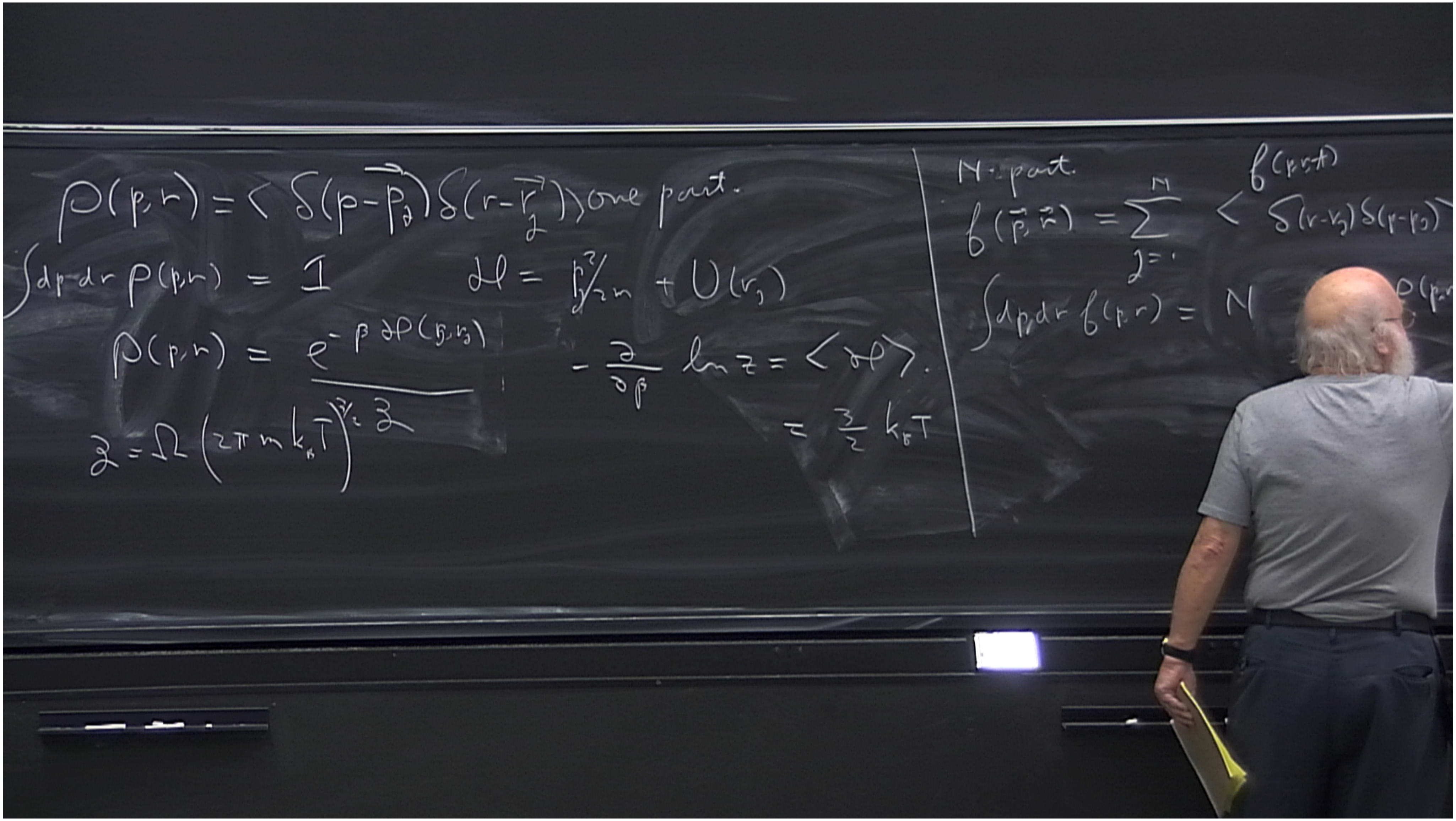
Title: Statistical Mechanics - Lecture 3

Date: Oct 05, 2011 10:30 AM

URL: http://pirsa.org/11100027

Abstract:


$\rho(p,r) = \langle \delta(p-\vec{p}_1)\,\delta(r-\vec{r}_1)\rangle$ one part.
$\int dp\,dr\,\rho(p,r) = 1$
$\mathcal{H} = p_1^2/2m + U(r_1)$
$\rho(p,r) = \frac{e^{-\beta \mathcal{H}(p_1,r_1)}}{Z}$
$-\frac{\partial}{\partial\beta}\ln Z = \langle \mathcal{H} \rangle.$


ρ(p,r) = < δ(p−p_j) δ(r−r_j) > one part.
∫dp dr ρ(p,r) = 1
H = p_j²/2m + U(r_j)
ρ(p,r) = e^{−β H(p_j,r_j)} / Z
−∂/∂β ln Z = < H >
= 3/2 k_B T
Z = Ω (2π m k_B T)^{3/2}
N part.
f(p,r) = Σ_{j=1}^{N} < δ(r−r_j) δ(p−p_j) >
∫dp dr f(p,r) = N

$(r-\vec{r}_j)\rangle$ one part.

$\mathcal{H} = p_j^2/2m + U(r_j)$

$-\frac{\partial}{\partial\beta}\ln Z = \langle \mathcal{H} \rangle.$

$= \frac{3}{2} k_B T$

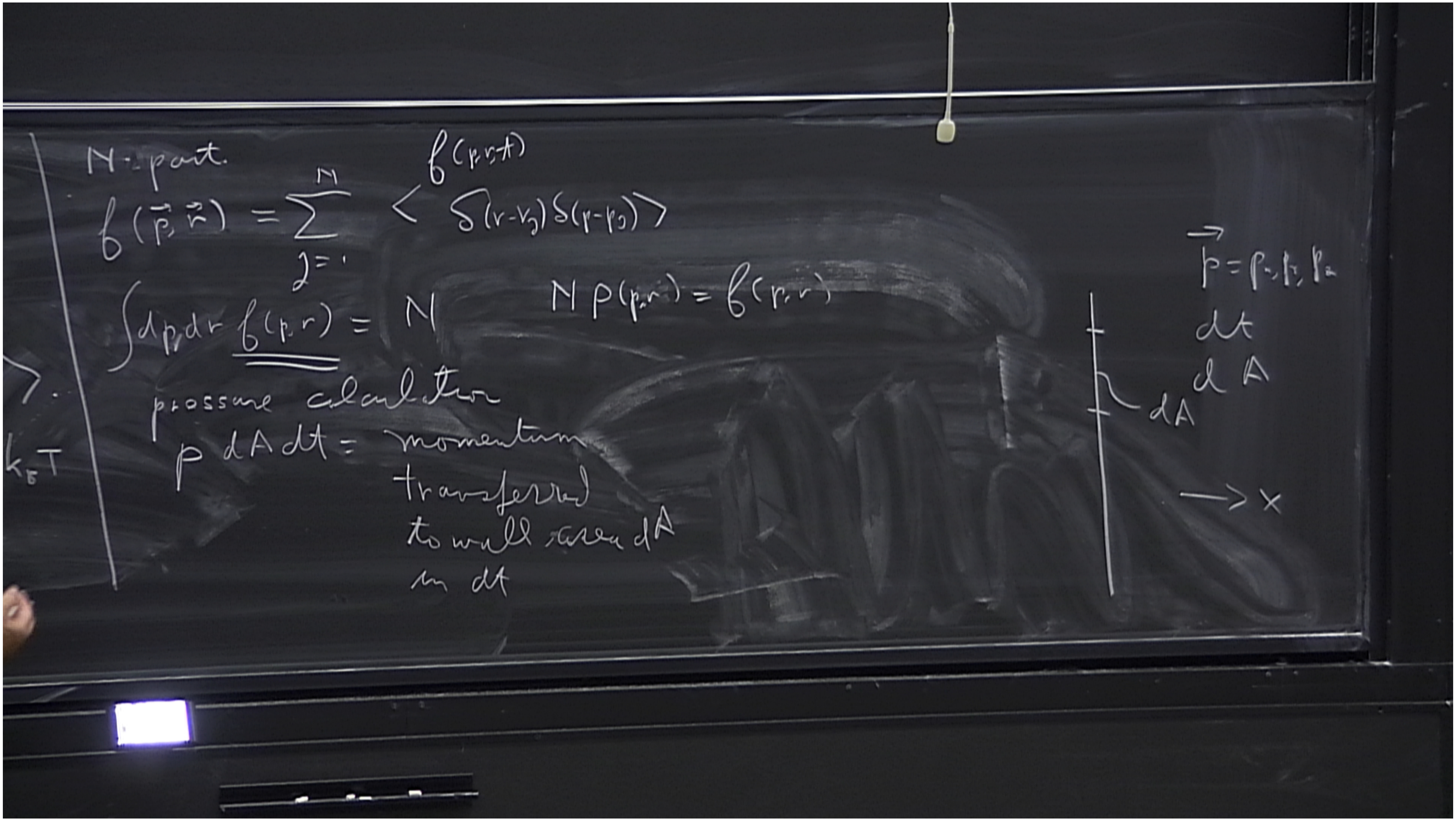
N-part.

$f(p,r,t)$

$f(\vec{p},\vec{r}) = \sum_{j=1}^{N} \langle \delta(r-r_j)\,\delta(p-p_j) \rangle$

$\int dp\,dr\, \underline{\underline{f(p,r)}} = N$

$N\rho(p,r) = f(p,r)$


N-part.
$f(p,r,t)$
$f(\vec{p},\vec{r}) = \sum_{j=1}^{N} \langle \delta(r-r_j)\,\delta(p-p_j) \rangle$
$\int dp\,dr\, f(p,r) = N$
$N\rho(p,r) = f(p,r)$
pressure calculation
$p\, dA\, dt$ = momentum transferred to wall area dA in dt
$\vec{p} = p_x, p_y, p_z$
dt
dA
dA
$\rightarrow x$
k_BT

$\rangle$.

$k_B T$

N-part.

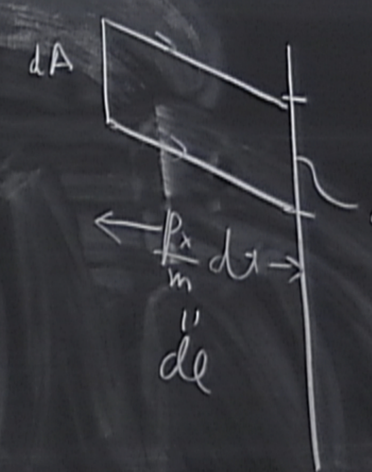
$$f(\vec{p},\vec{r}) = \sum_{j=1}^{N} \langle \delta(r-r_j)\,\delta(p-p_j) \rangle \qquad f(p,r,t)$$

$$\int dp\,dr\, f(p,r) = N \qquad N\rho(p,r) = f(p,r)$$

pressure calculation

$P\,dA\,dt$ = momentum transferred to wall, area dA in dt

elastic collision

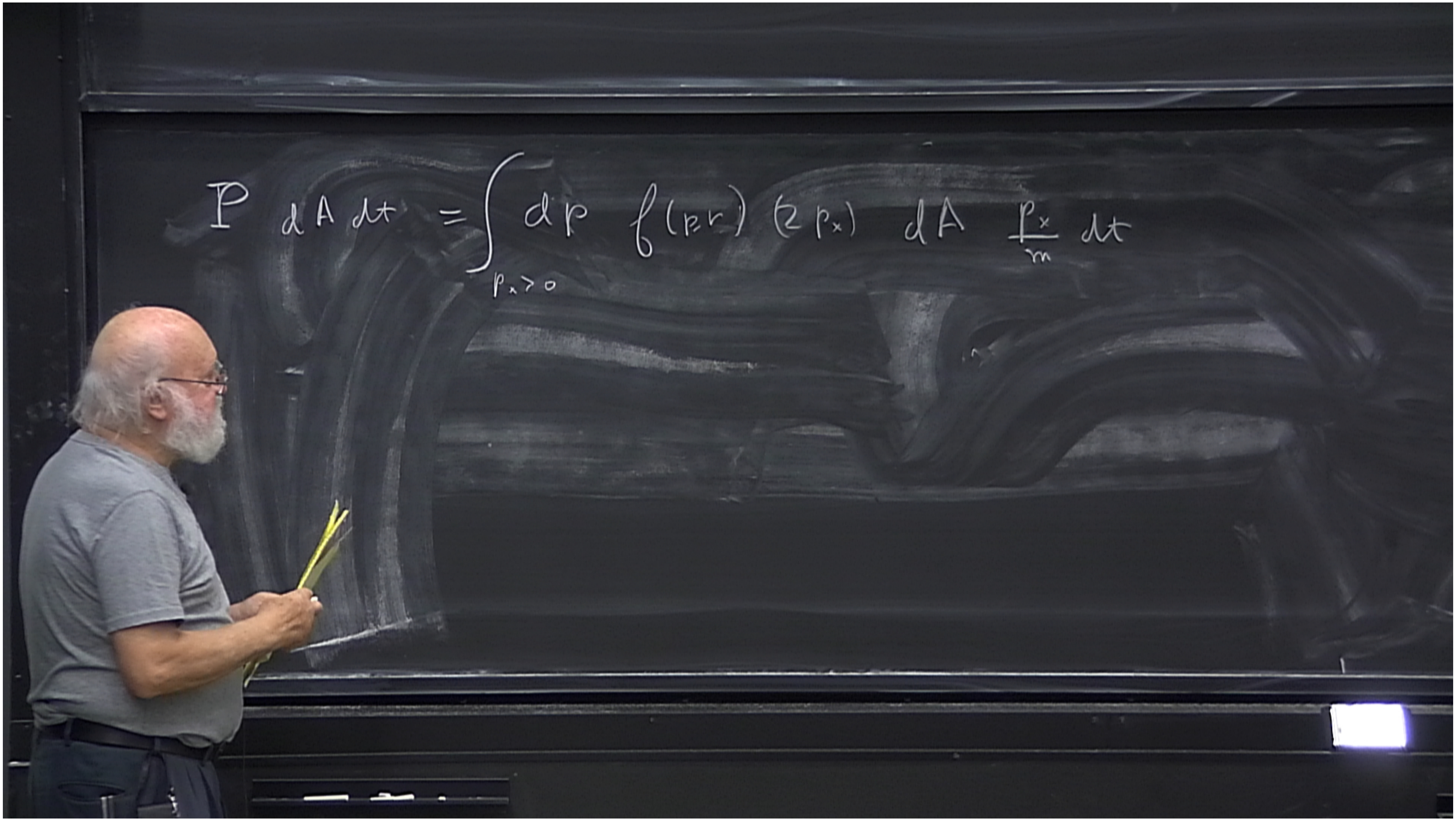


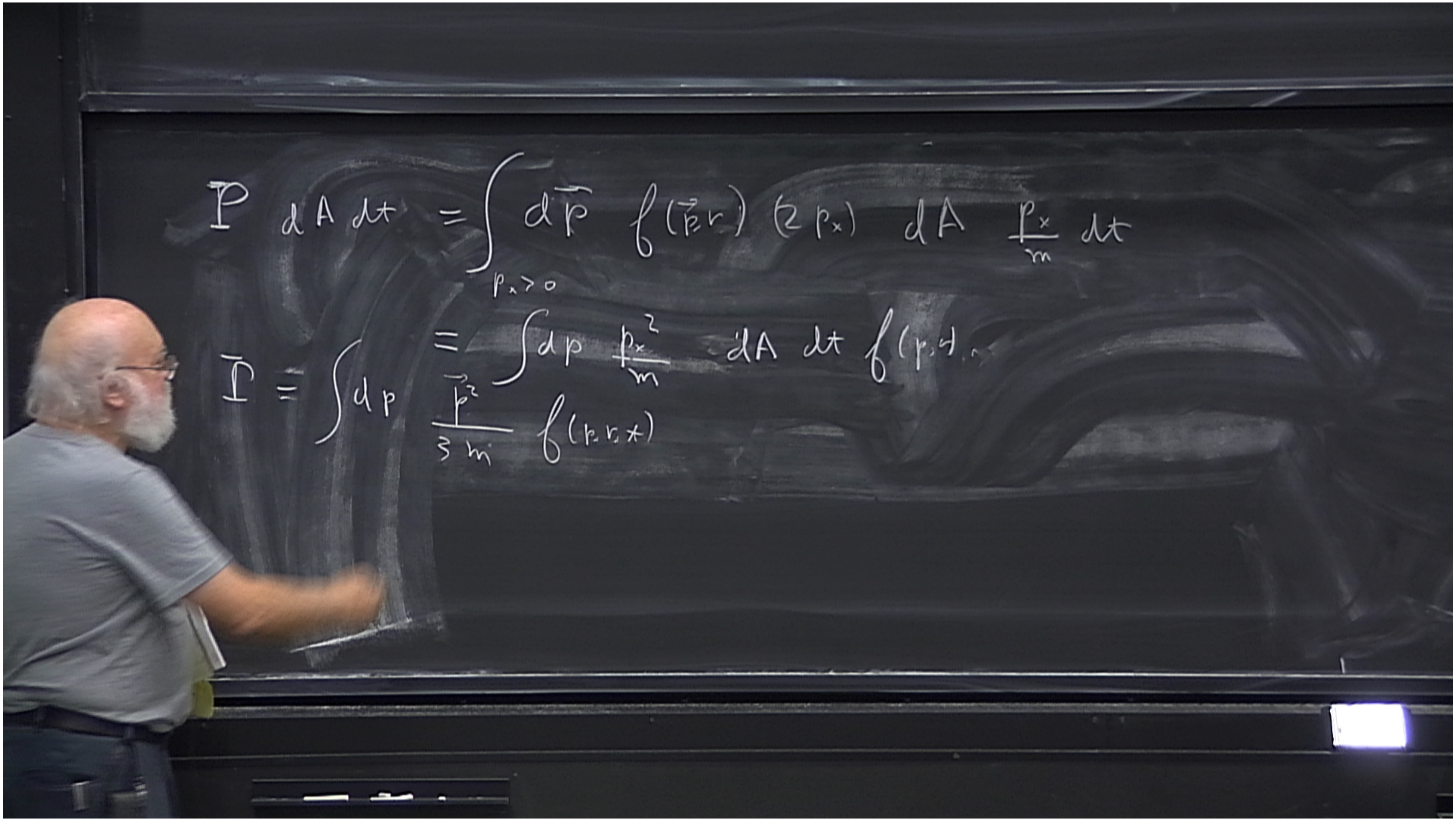
$\vec{p} = p_x, p_y, p_z$

dt

dA

$\longrightarrow x$


$P \, dA \, dt = \int_{p_x > 0} dp \; f(p, r) \, (2 p_x) \; dA \; \frac{p_x}{m} \, dt$


$P \, dA \, dt = \int_{p_x > 0} d\vec{p} \; f(\vec{p}, r) \, (2 p_x) \, dA \, \frac{p_x}{m} \, dt$
$= \int dp \, \frac{p_x^2}{m} \, dA \, dt \, f(p, r, t)$
$P = \int dp \, \frac{\vec{p}^2}{3m} \, f(p, r, t)$

$$P\, dA\, dt = \int_{p_x>0} d\vec{p}\; f(\vec{p}, r)\,(2p_x)\; dA\; \frac{p_x}{m}\, dt$$

$$= \int dp\; \frac{p_x^2}{m}\; dA\; dt\; f(p, t, \ldots$$

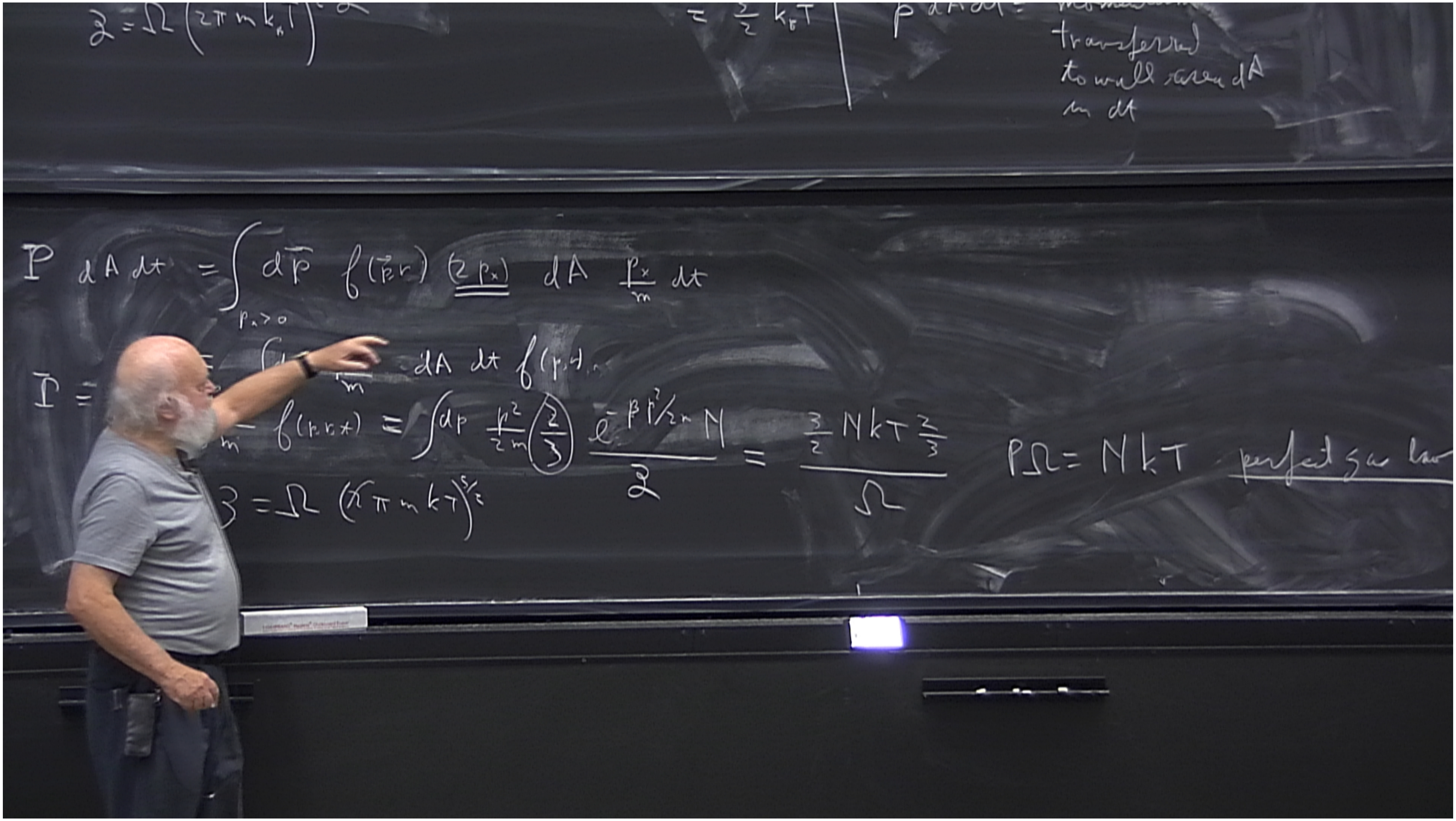
$$P = \int d\vec{p}\; \frac{\vec{p}^{\,2}}{3m}\, f(p, r, t) = \langle \epsilon \rangle \frac{2}{3} = \frac{2\epsilon}{3}$$

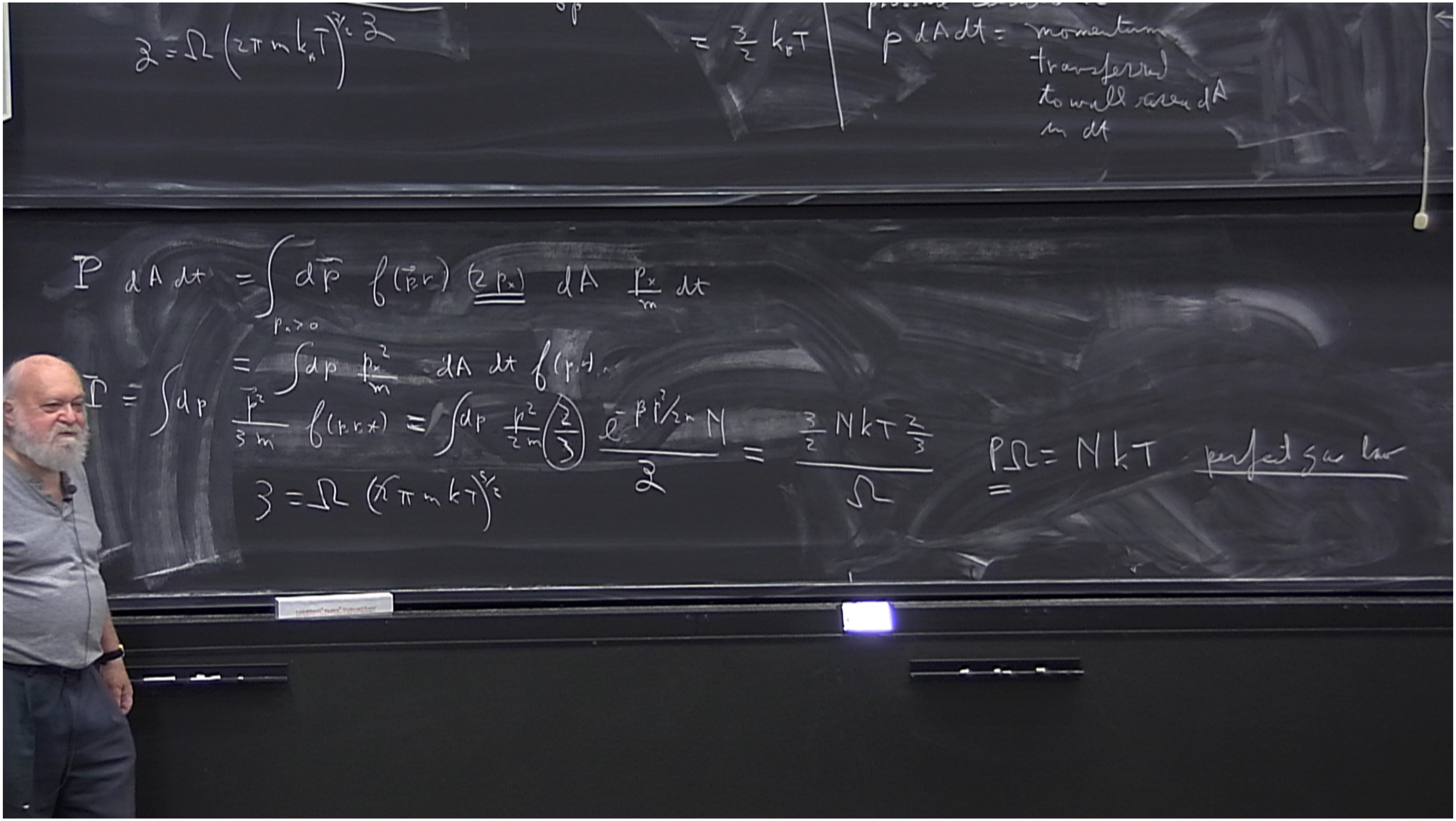
$$P\, dA\, dt = \int_{p_x > 0} d\vec{p}\; f(\vec{p}, r)\,(2 p_x)\; dA\; \frac{p_x}{m}\, dt$$

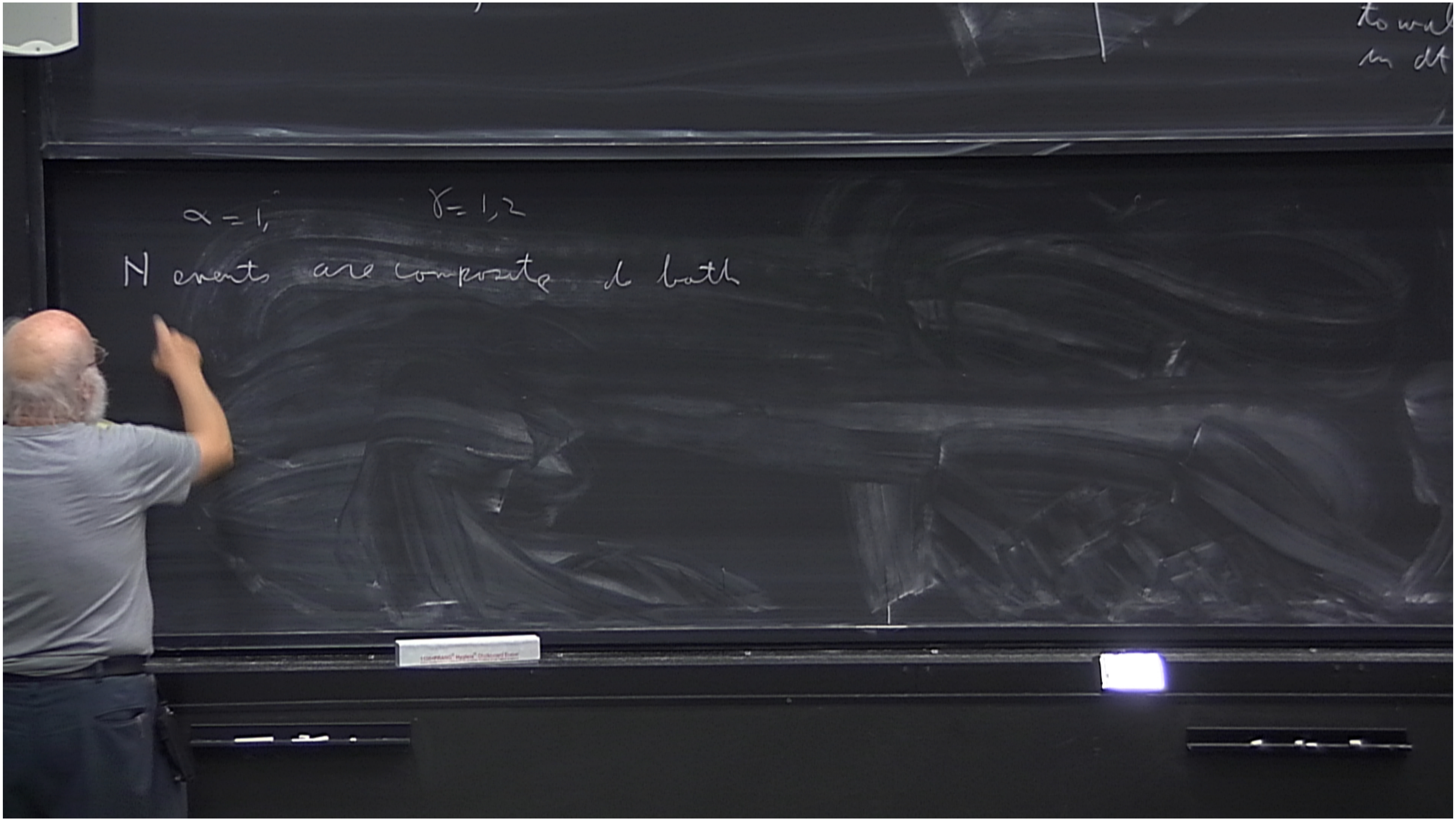
$$= \int dp\; \frac{p_x^2}{m}\; dA\, dt\; f(p, r \ldots$$

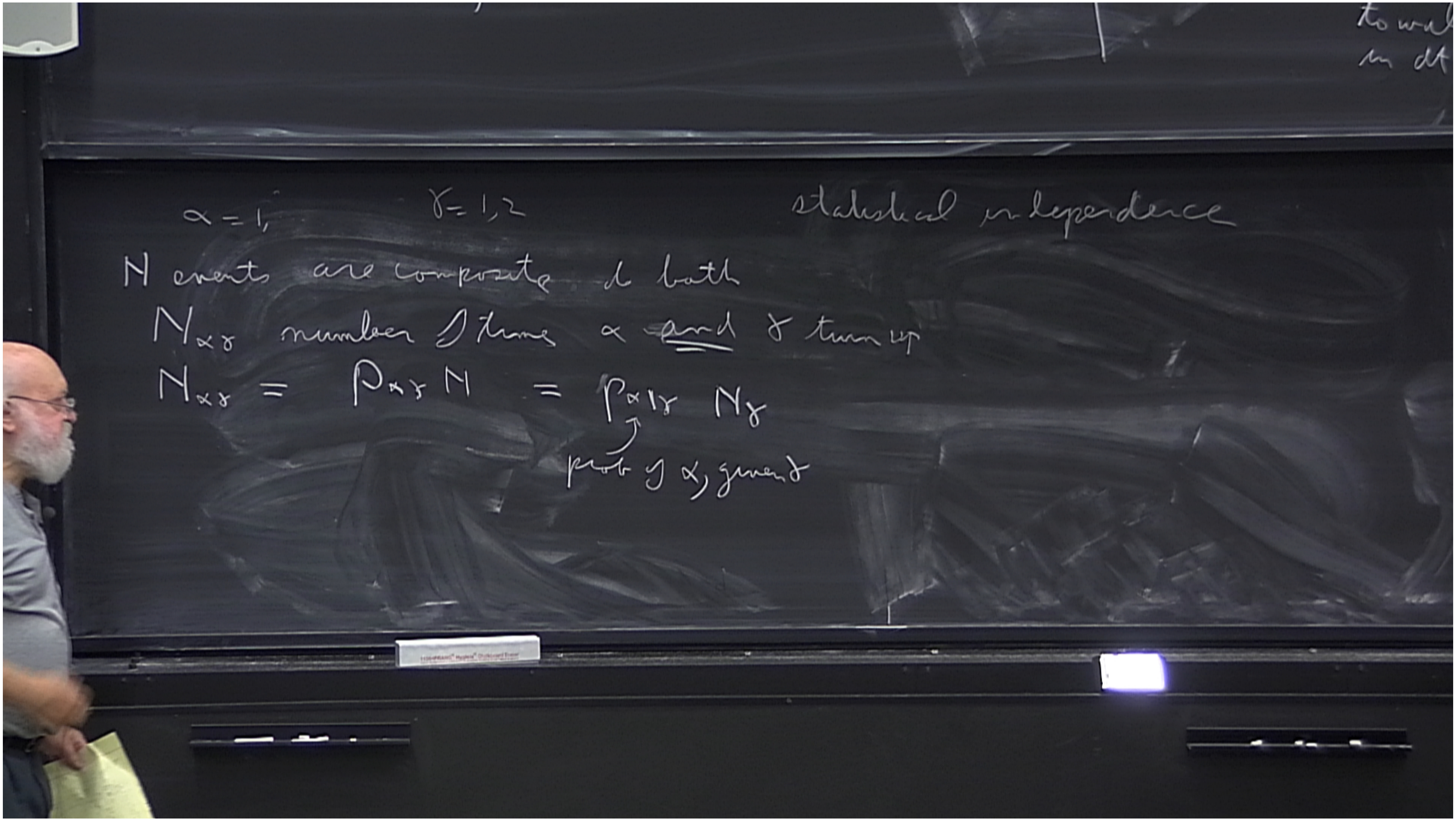
$$P = \int dp\; \frac{\vec{p}^{\,2}}{3m}\, f(p, r, t) = \int dp\; \frac{p^2}{2m} \frac{2}{3}\; \frac{e^{-\beta p^2/2m} N}{Z} = \frac{\frac{3}{2} N k T \frac{2}{3}}{\Omega}$$

$$Z = \Omega\, (2\pi m k T)^{3/2}$$


transferred
to wall area dA
in dt
$P \, dA \, dt = \int_{p_x > 0} d\vec{p} \; f(\vec{p}, r) \, (2 p_x) \, dA \, \frac{p_x}{m} \, dt$
$P \Omega = N k T$
perfect gas law


z = Ω (2π m k_B T)^{3/2} Z
= 3/2 k_B T
p dA dt = momentum transferred to wall area dA in dt
P dA dt = ∫_{p_x>0} dp f(p,r) (2p_x) dA p_x/m dt
= ∫dp p_x²/m dA dt f(p,r)
P = ∫dp p²/3m f(p,r,t) = ∫dp p²/2m (2/3) e^{-βp²/2m} N / Z = (3/2 NkT 2/3) / Ω
Z = Ω (2π m kT)^{3/2}
PΩ = NkT perfect gas law


α = 1,
γ = 1,2
N events are composite & both


α = 1,   γ = 1,2
statistical independence
N events are composite of both
N_αγ number of times α and γ turn up
N_αγ = p_αγ N = p_α|γ N_γ
prob of α, given γ

to wall area d
in dt

$\alpha = 1,$ $\qquad \gamma = 1, 2$ $\qquad$ statistical independence $\qquad e^{-\beta H} =$

N events are composite of both

$N_{\alpha\gamma}$ number of times α <u>and</u> γ turn up

$N_{\alpha\gamma} = P_{\alpha\gamma} N = P_{\alpha|\gamma} N_\gamma$ $\qquad P_{\alpha|\gamma} = P_\alpha$ independent of γ

prob of α, given γ

transferred
to wall area dA
in dt

$d\ell$

$\longrightarrow x$

statistical independence

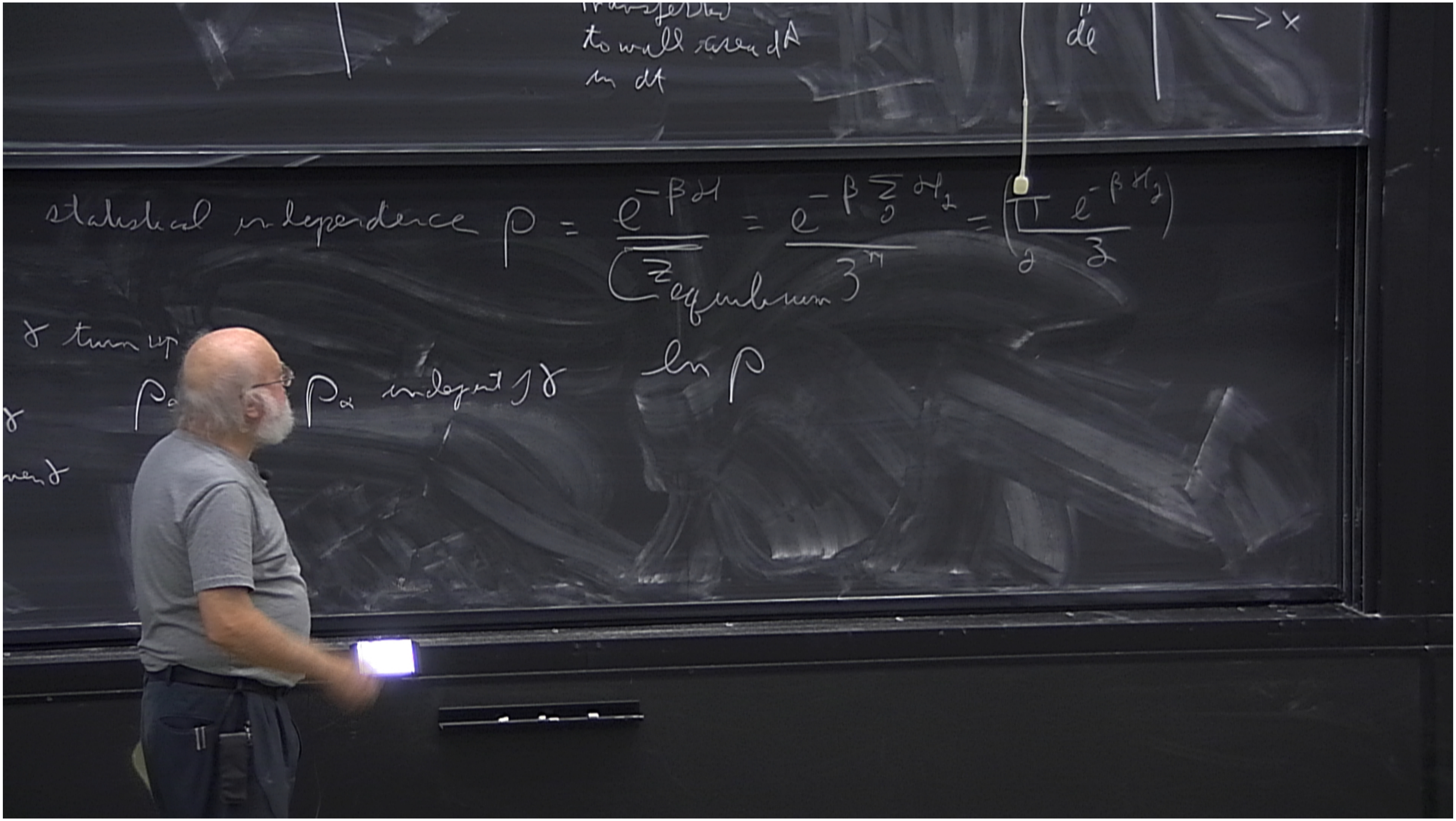
$$\underline{\underline{e^{-\beta H}}} = e^{-\beta \sum_j H_j} = \left(\prod_j e^{-\beta H_j}\right)$$

γ turn up

$\rho_{\alpha|\gamma} = \rho_\alpha$ independent of γ

γ

ment γ


to wall area dA
in dt
statistical independence
γ turn up
ln ρ

transferred
to wall area dA
in dt

$d\ell$

$\longrightarrow x$

statistical independence $\rho = \dfrac{e^{-\beta H}}{Z_{\text{equilibrium}}} = \dfrac{e^{-\beta \sum_j H_j}}{Z^N} = \left(\prod_j \dfrac{e^{-\beta H_j}}{Z} \right)$

γ turn up

$\rho_{\alpha|\gamma} = \rho_\alpha$ indep't of γ

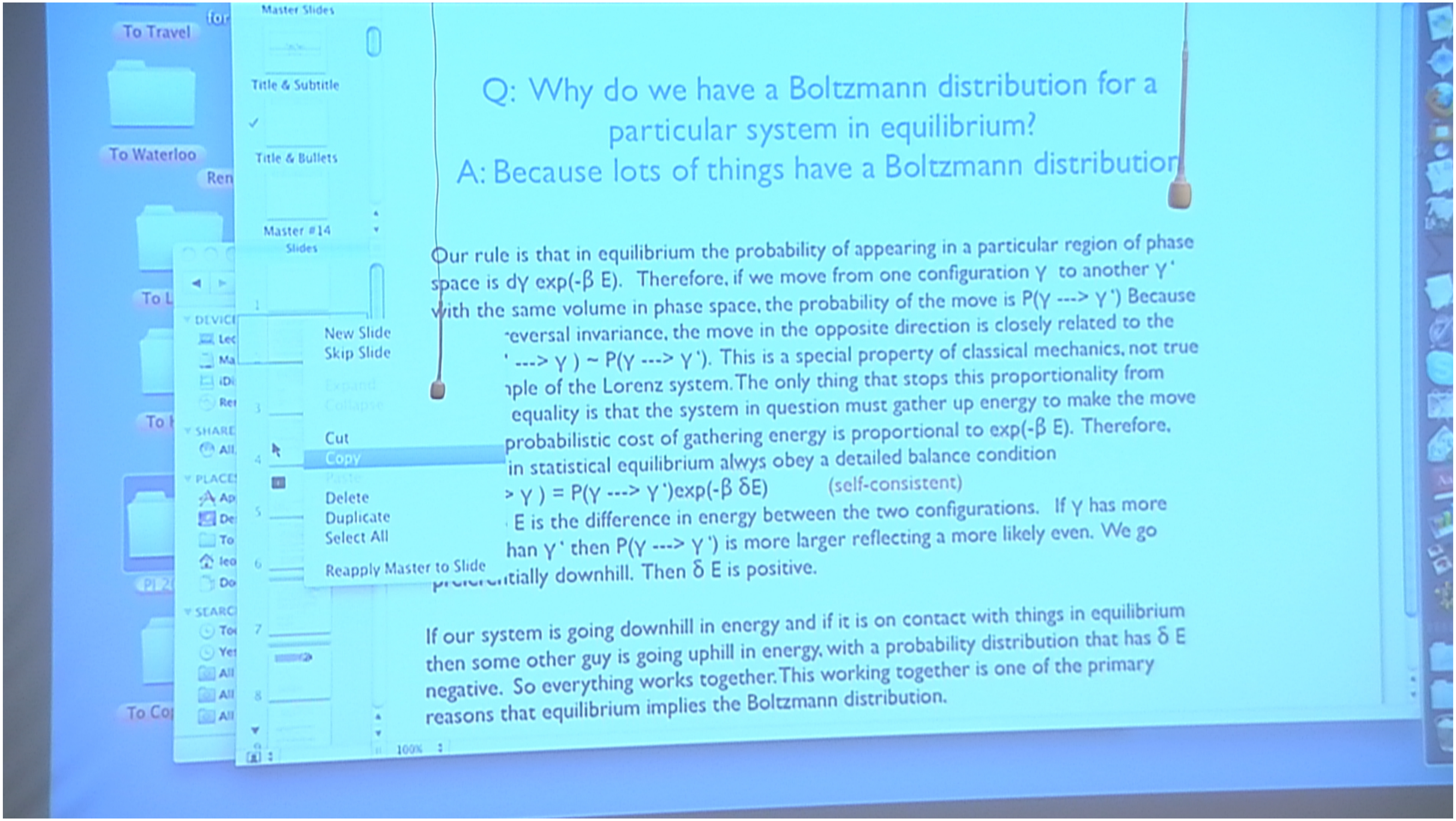
$\delta [\ln \rho_A] \sim -\beta \delta H_A$

A B

detailed balance

γ

…ment γ


To Travel
To Waterloo
Master Slides
Title & Subtitle
Title & Bullets
Master #14
Slides
New Slide
Skip Slide
Cut
Copy
Delete
Duplicate
Select All
Reapply Master to Slide
Q: Why do we have a Boltzmann distribution for a particular system in equilibrium?
A: Because lots of things have a Boltzmann distributio
Our rule is that in equilibrium the probability of appearing in a particular region of phase
space is dγ exp(-β E). Therefore, if we move from one configuration γ to another γ'
with the same volume in phase space, the probability of the move is P(γ ---> γ') Because
eversal invariance, the move in the opposite direction is closely related to the
' ---> γ) ~ P(γ ---> γ'). This is a special property of classical mechanics, not true
nple of the Lorenz system. The only thing that stops this proportionality from
equality is that the system in question must gather up energy to make the move
probabilistic cost of gathering energy is proportional to exp(-β E). Therefore,
in statistical equilibrium alwys obey a detailed balance condition
> γ) = P(γ ---> γ')exp(-β δE) (self-consistent)
E is the difference in energy between the two configurations. If γ has more
han γ' then P(γ ---> γ') is more larger reflecting a more likely even. We go
tially downhill. Then δ E is positive.
If our system is going downhill in energy and if it is on contact with things in equilibrium
then some other guy is going uphill in energy, with a probability distribution that has δ E
negative. So everything works together. This working together is one of the primary
reasons that equilibrium implies the Boltzmann distribution.

Q: Why do we have a Boltzmann distribution for a particular system in equilibrium?

A: Because lots of things have a Boltzmann distribution

Our rule is that in equilibrium the probability of appearing in a particular region of phase space is $d\gamma \exp(-\beta E)$. Therefore, if we move from one configuration γ to another γ' with the same volume in phase space, the probability of the move is $P(\gamma \dashrightarrow \gamma')$ Because of time reversal invariance, the move in the opposite direction is closely related to the first $P(\gamma' \dashrightarrow \gamma) \sim P(\gamma \dashrightarrow \gamma')$. This is a special property of classical mechanics, not true for example of the Lorenz system. The only thing that stops this proportionality from being an equality is that the system in question must gather up energy to make the move and the probabilistic cost of gathering energy is proportional to $\exp(-\beta E)$. Therefore, systems in statistical equilibrium alwys obey a detailed balance condition

$P(\gamma' \dashrightarrow \gamma) = P(\gamma \dashrightarrow \gamma')\exp(-\beta\, \delta E)$ (self-consistent)

where δE is the difference in energy between the two configurations. If γ has more energy than γ' then $P(\gamma \dashrightarrow \gamma')$ is more larger reflecting a more likely even. We go preferentially downhill. Then δE is positive.

If our system is going downhill in energy and if it is on contact with things in equilibrium then some other guy is going uphill in energy, with a probability distribution that has δE negative. So everything works together. This working together is one of the primary reasons that equilibrium implies the Boltzmann distribution.

Monte Carlo

This rule for relative probabilities is the basis for a tremendously useful calculational method called a Monte Carlo simulation. The basic idea is to divide the phase space up into equal sized volumes defined by an index γ. Then we imagine setting up a process in which a system cycles through the states, γ, γ', γ'' etc. Pick a probability for hopping from γ to γ' as $P(\gamma \dashrightarrow \gamma')$ obeying the detailed balance condition. Then imagine that we go through a process specifically $\gamma \dashrightarrow \gamma'$ with $P(\gamma \dashrightarrow \gamma') >= 0$

$P(\gamma' \dashrightarrow \gamma) = P(\gamma \dashrightarrow \gamma')\exp(-\beta\,\delta E)$ with $\delta E=E(\gamma')-E(\gamma)$
being the energy difference between γ and γ' -detailed balance condition

e.g. use $P(\gamma \dashrightarrow \gamma') = 1$ if $\delta E < 0$ and $= \exp(-\beta\,\delta E)$ if $\delta E >= 0$

```
loop:
start with state= γ
pick a γ' at random
let del=-β [E(γ')-E(γ)]
if del <0 :
      state = γ'
      go to loop
Else:
     generate a random number, r, between 0 and 1
     if r<= exp(-del):
          state = γ'
     else no change
go to loop
```

This process generates the correct probability distribution

Let $\rho(\gamma,t)$ be the probability for being in a state γ at time t.

or $\rho(\gamma, t+1)-\rho(\gamma, t)= \Sigma_{\gamma'} [\rho(\gamma', t) P(\gamma'\text{-->}\gamma)- \rho(\gamma, t)(P(\gamma\text{-->}\gamma'))]$

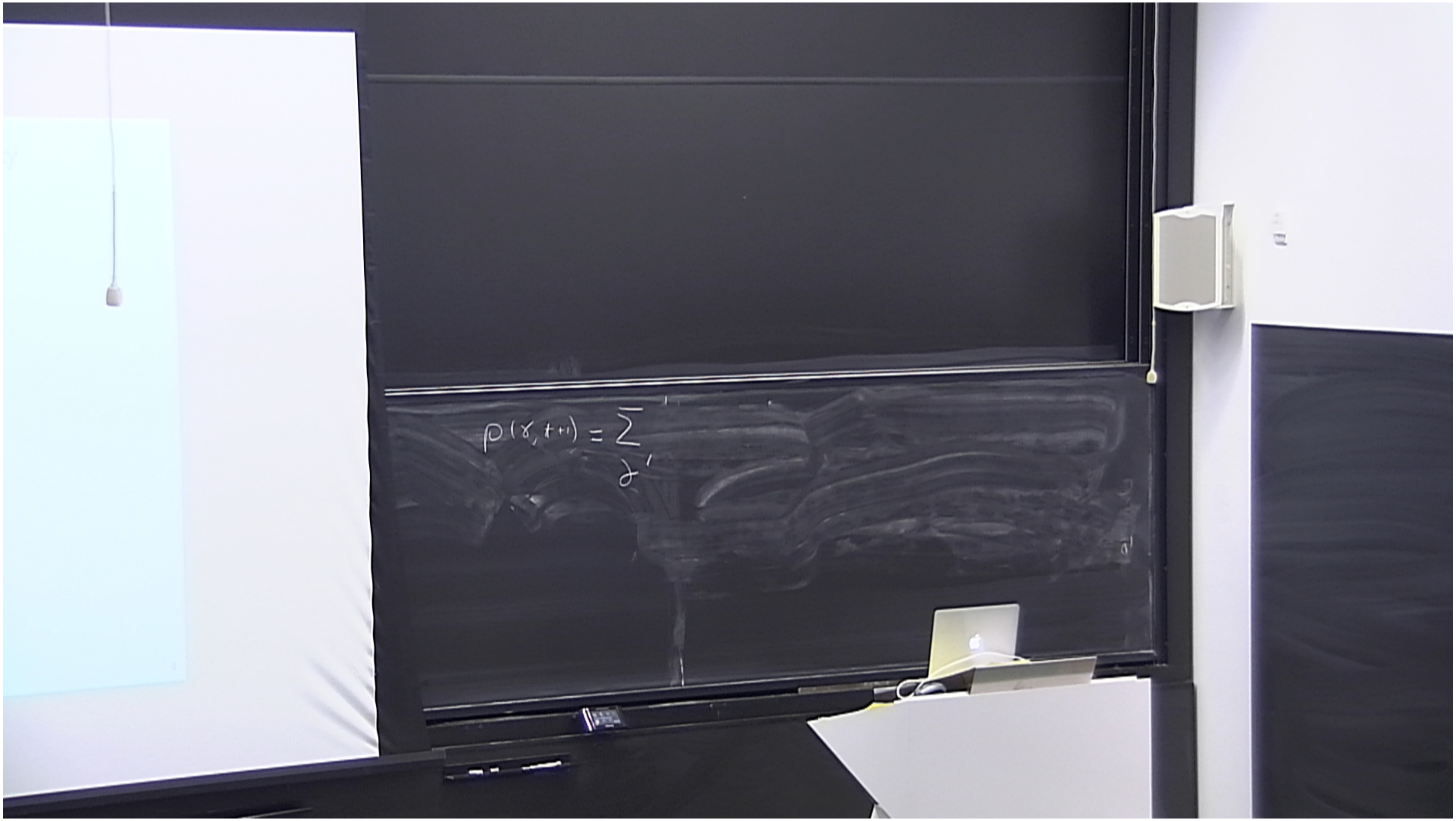
see whether this has the correct equilibrium distribution

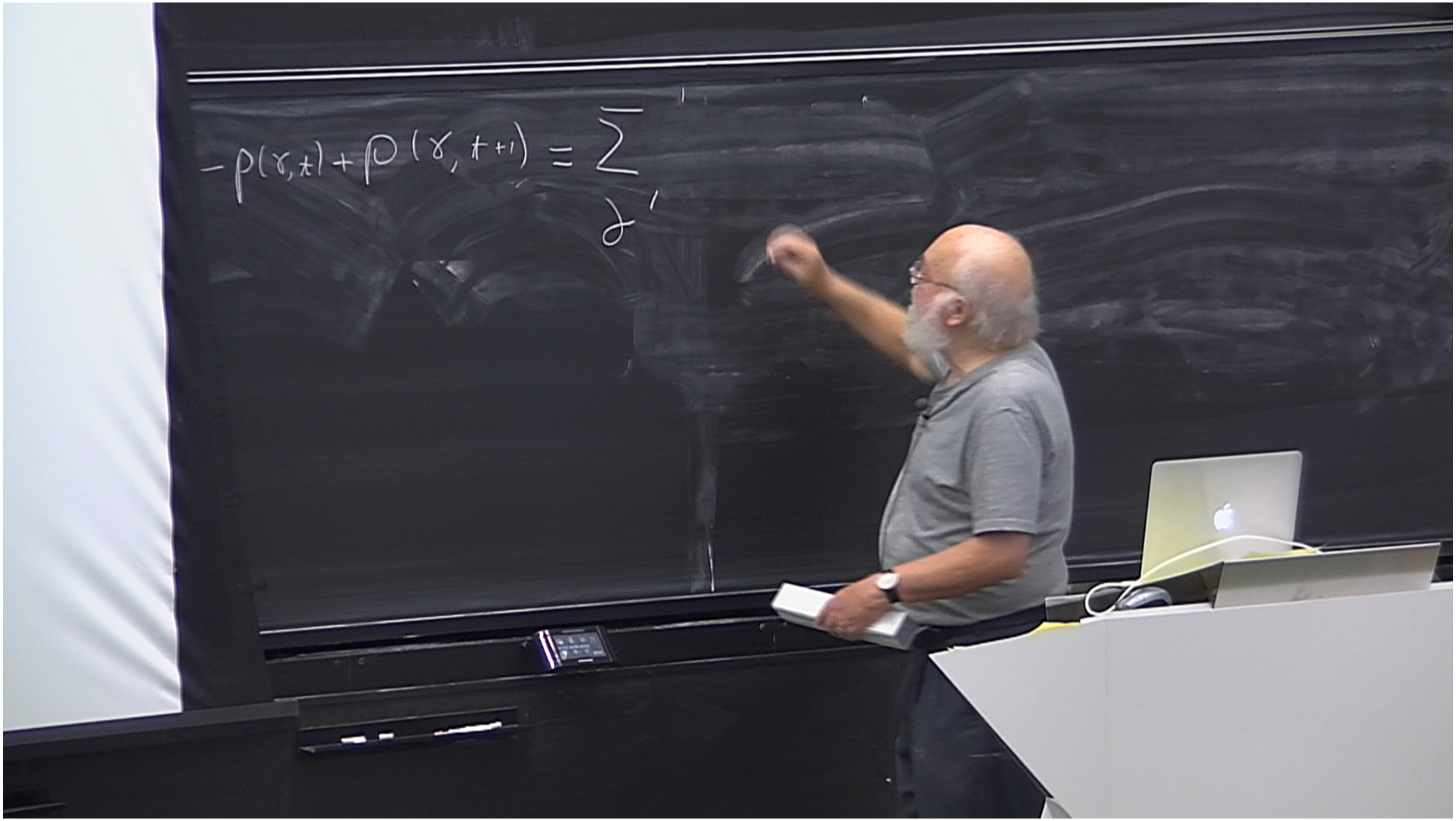
$\rho(\gamma ,t) = 1/z \exp(-\beta H(\gamma))$ independent of t

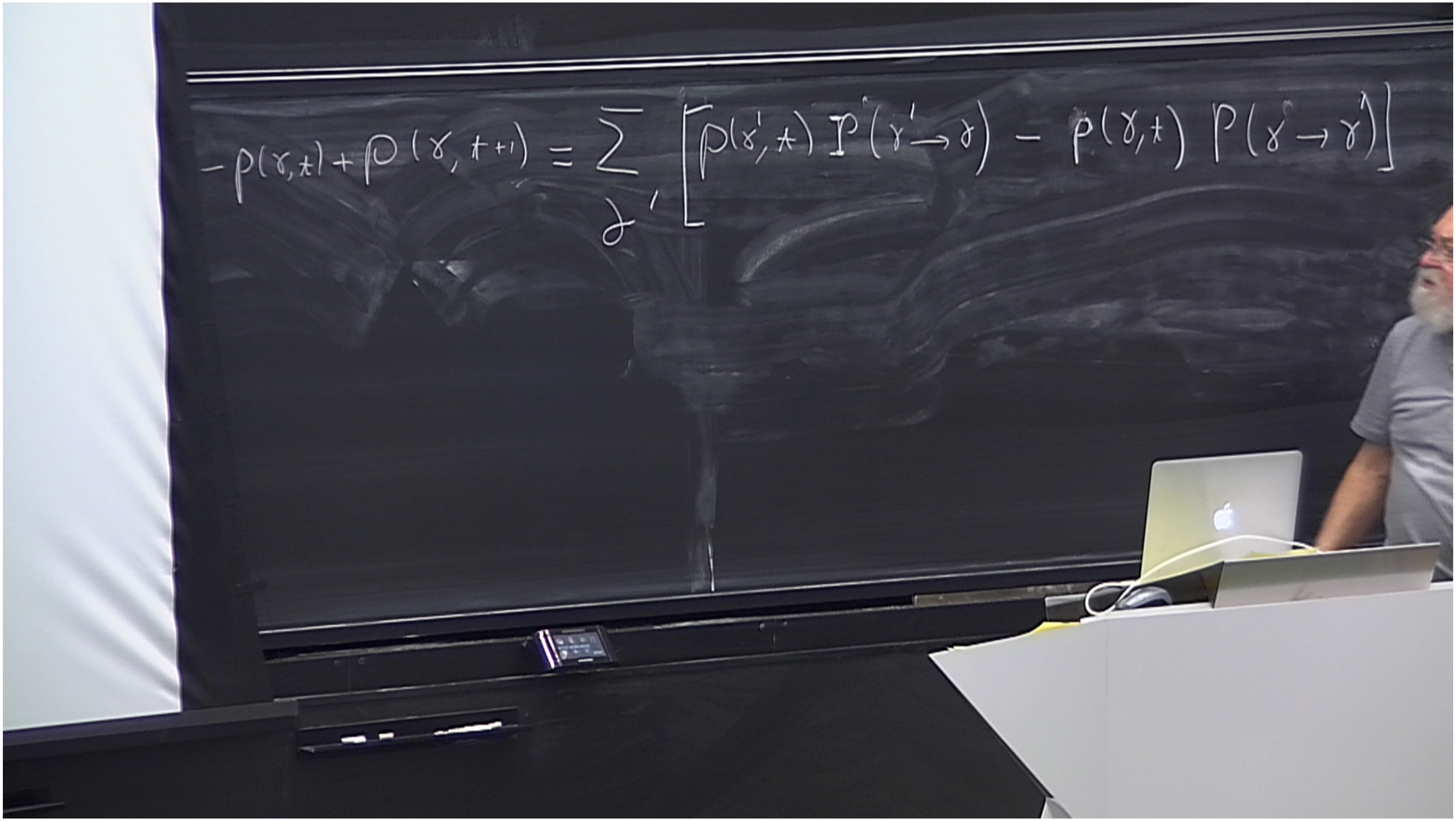
RHS = $\Sigma_{\gamma'} [\exp(-\beta H(\gamma')) P(\gamma'\text{-->}\gamma)-\exp(-\beta H(\gamma)) P(\gamma\text{-->}\gamma')$

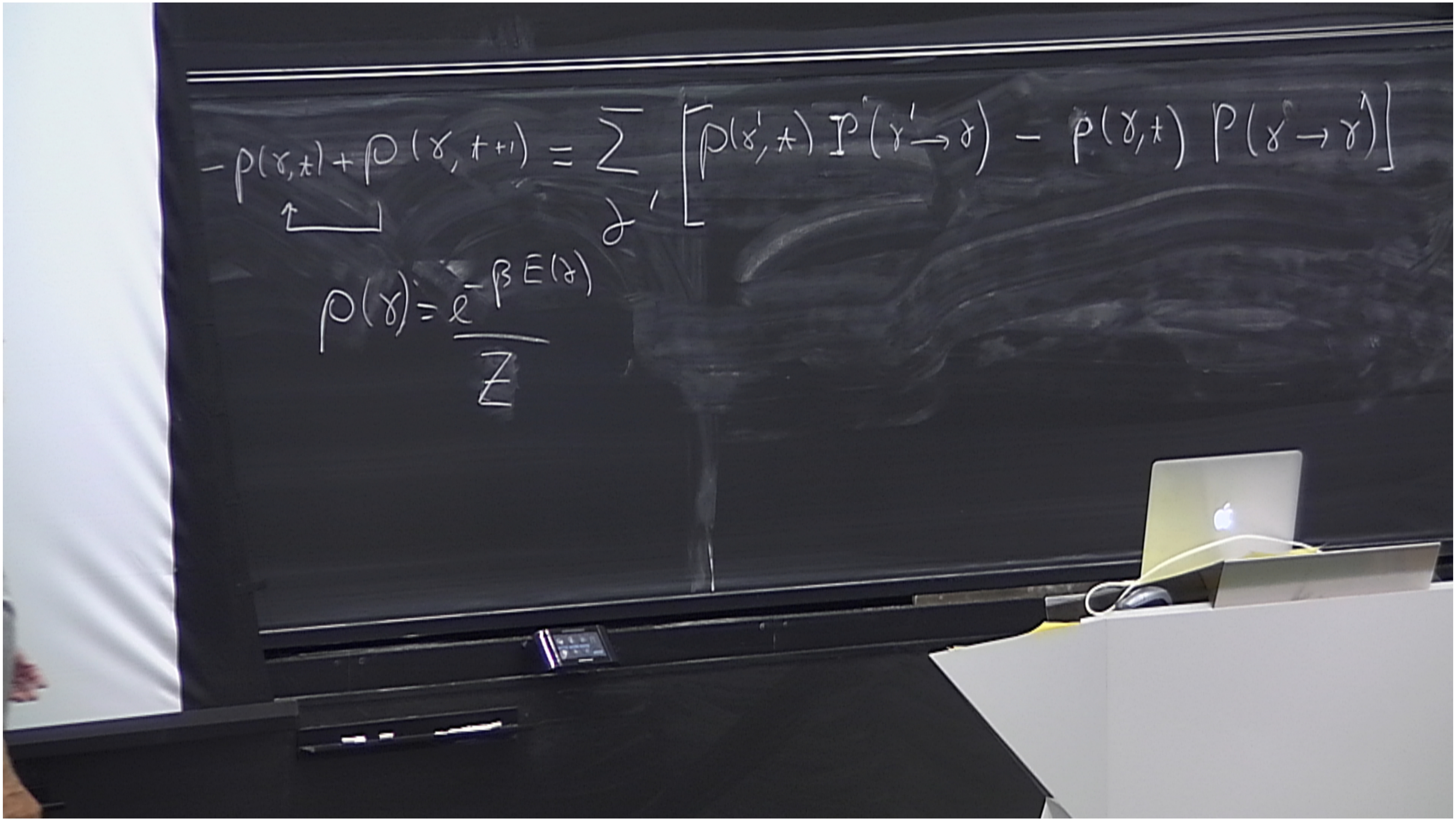
$\exp(-\beta H(\gamma'))/\exp(-\beta H(\gamma)) = \exp(-\beta \delta E)$ QED

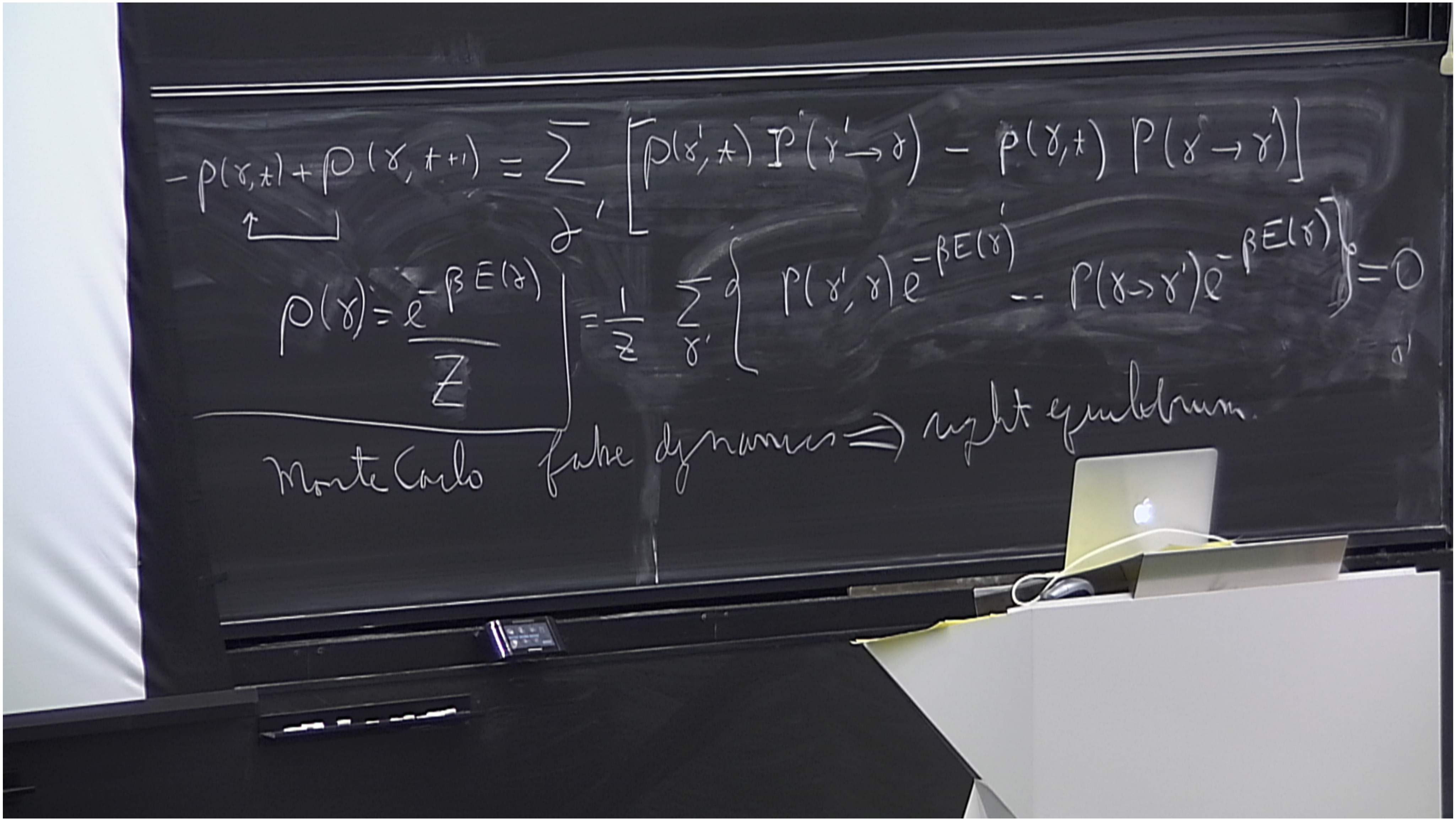
I think that you can prove that this will converge to the right answer if the number of states is finite and all P's are nonzero

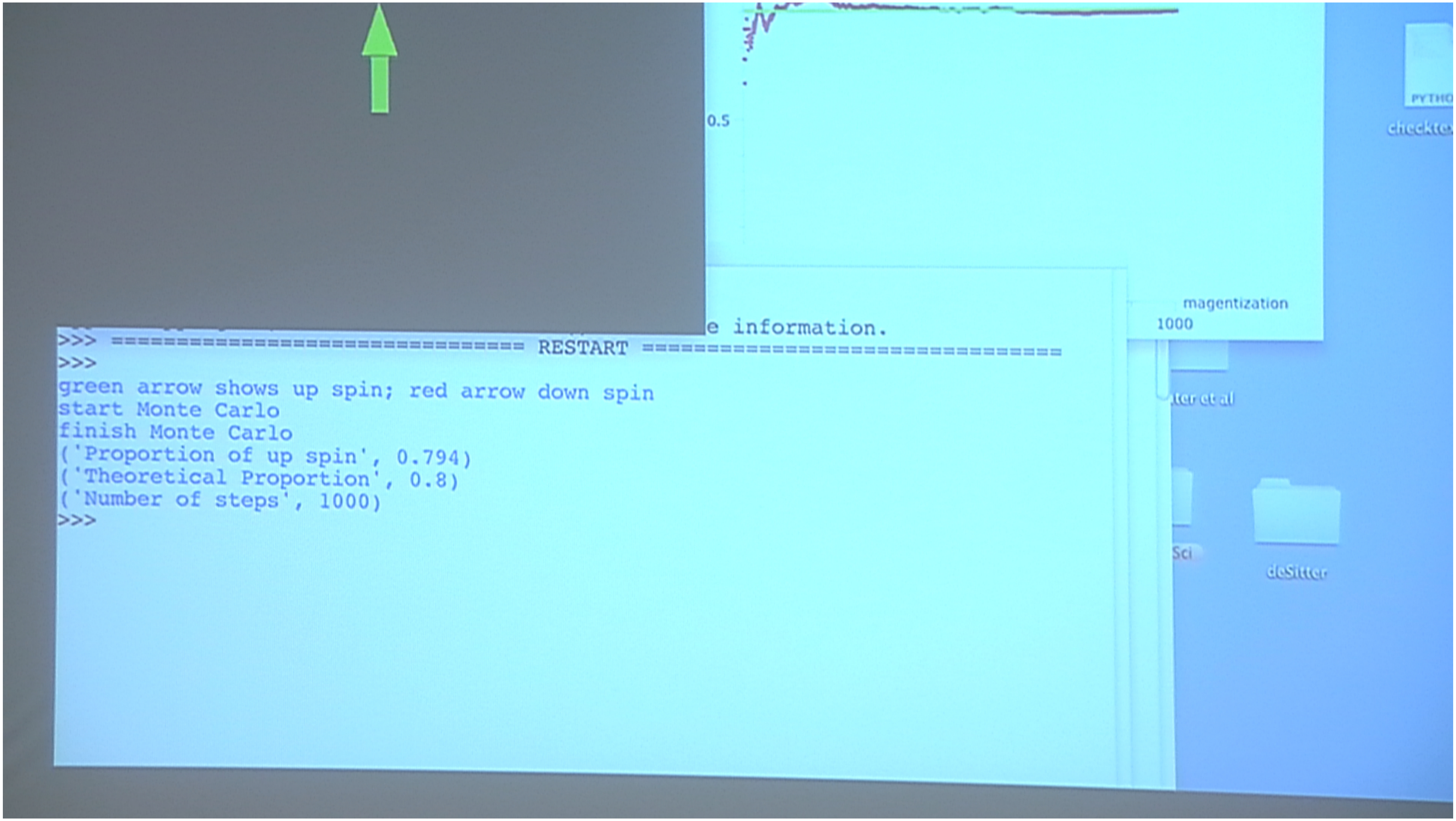

ρ(γ, t+1) = Σ γ'

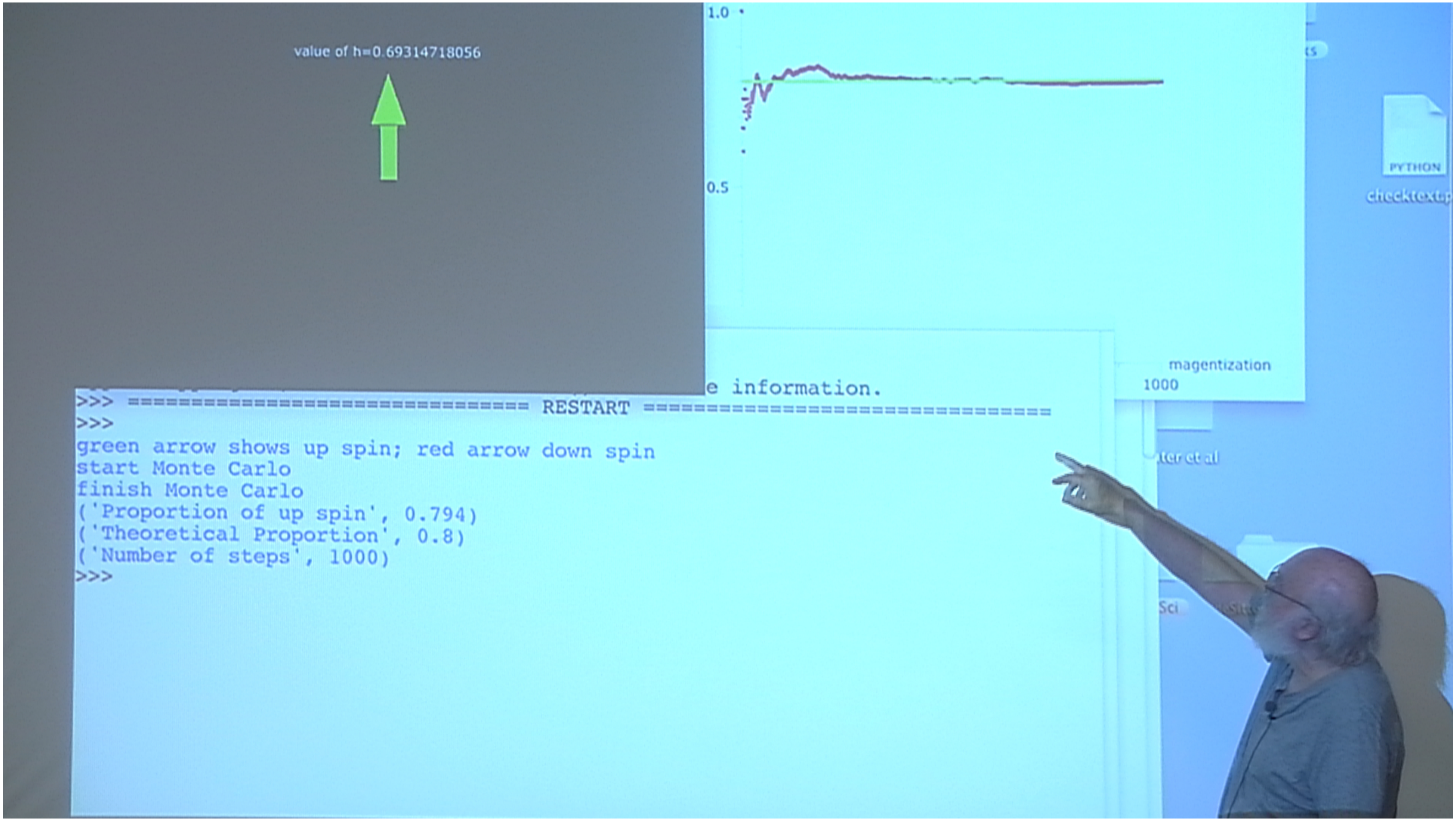

$-\rho(r,t)+\rho(r,t+1)=\sum_{r'}$

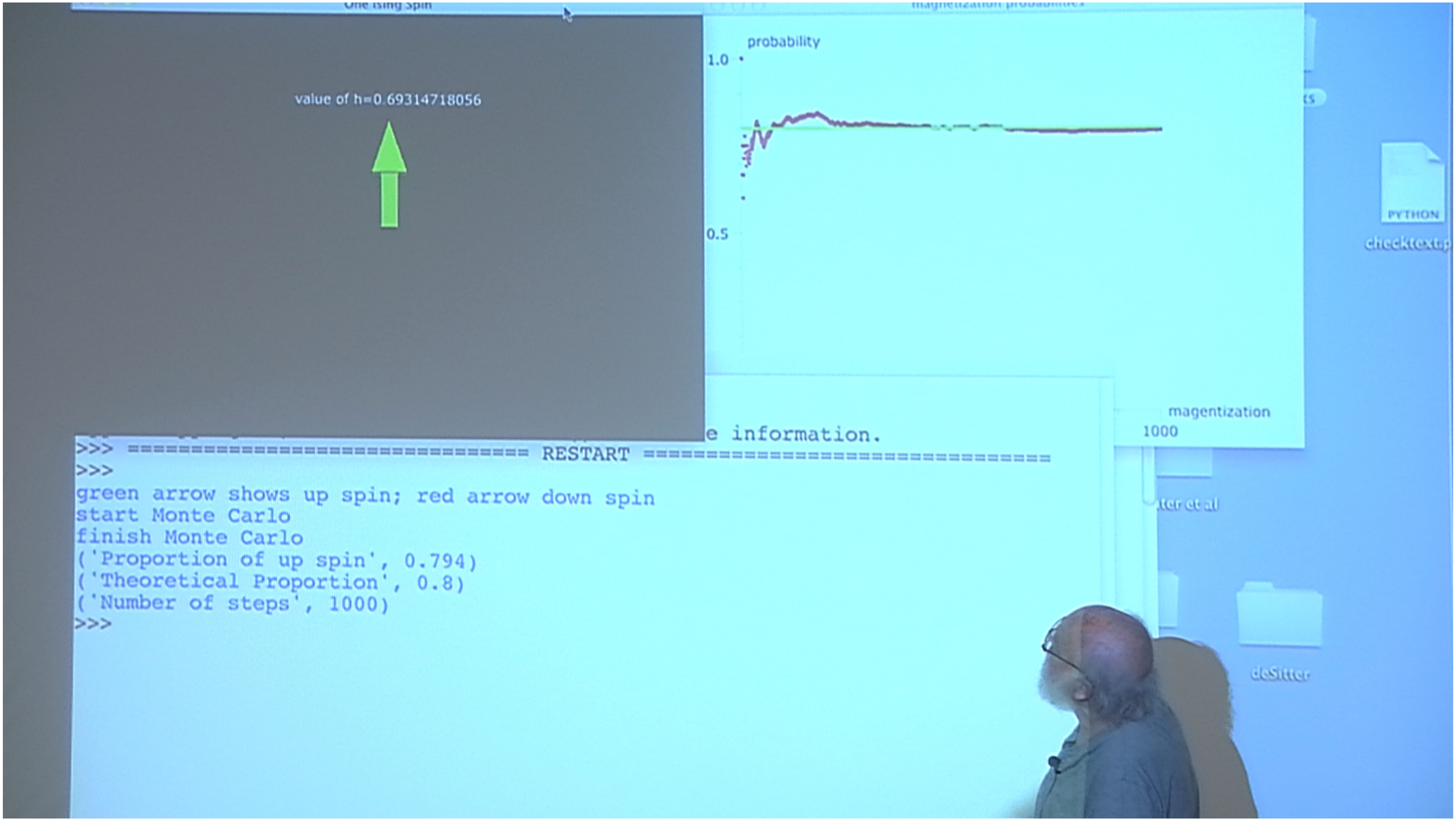

$-\rho(\gamma,t)+\rho(\gamma,t+1)=\sum_{\gamma'}\left[\rho(\gamma',t)\,P(\gamma'\to\gamma)-\rho(\gamma,t)\,P(\gamma\to\gamma')\right]$


$-\rho(\gamma,t)+\rho(\gamma,t+1)=\sum_{\gamma'}\left[\rho(\gamma',t)\,P(\gamma'\to\gamma)-\rho(\gamma,t)\,P(\gamma\to\gamma')\right]$
$\rho(\gamma)=\frac{e^{-\beta E(\gamma)}}{Z}$


$-\rho(\gamma,t)+\rho(\gamma,t+1)=\sum_{\gamma'}\left[\rho(\gamma',t)\,P(\gamma'\to\gamma)-\rho(\gamma,t)\,P(\gamma\to\gamma')\right]$
$\rho(\gamma)=\frac{e^{-\beta E(\gamma)}}{Z}$
$=\frac{1}{Z}\sum_{\gamma'}\left\{P(\gamma'\to\gamma)e^{-\beta E(\gamma')}-P(\gamma\to\gamma')e^{-\beta E(\gamma)}\right\}=0$
Monte Carlo
fake dynamics ⇒ right equilibrium.


0.5
magentization
1000
e information.
>>> ================================ RESTART ================================
>>>
green arrow shows up spin; red arrow down spin
start Monte Carlo
finish Monte Carlo
('Proportion of up spin', 0.794)
('Theoretical Proportion', 0.8)
('Number of steps', 1000)
>>>
deSitter


value of h=0.69314718056
1.0
0.5
magentization
1000
e information.
>>> ================================ RESTART ================================
>>>
green arrow shows up spin; red arrow down spin
start Monte Carlo
finish Monte Carlo
('Proportion of up spin', 0.794)
('Theoretical Proportion', 0.8)
('Number of steps', 1000)
>>>
PYTHON
checktext.p



value of h=0.69314718056
probability
1.0
0.5
magentization
1000
e information.
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>>>
green arrow shows up spin; red arrow down spin
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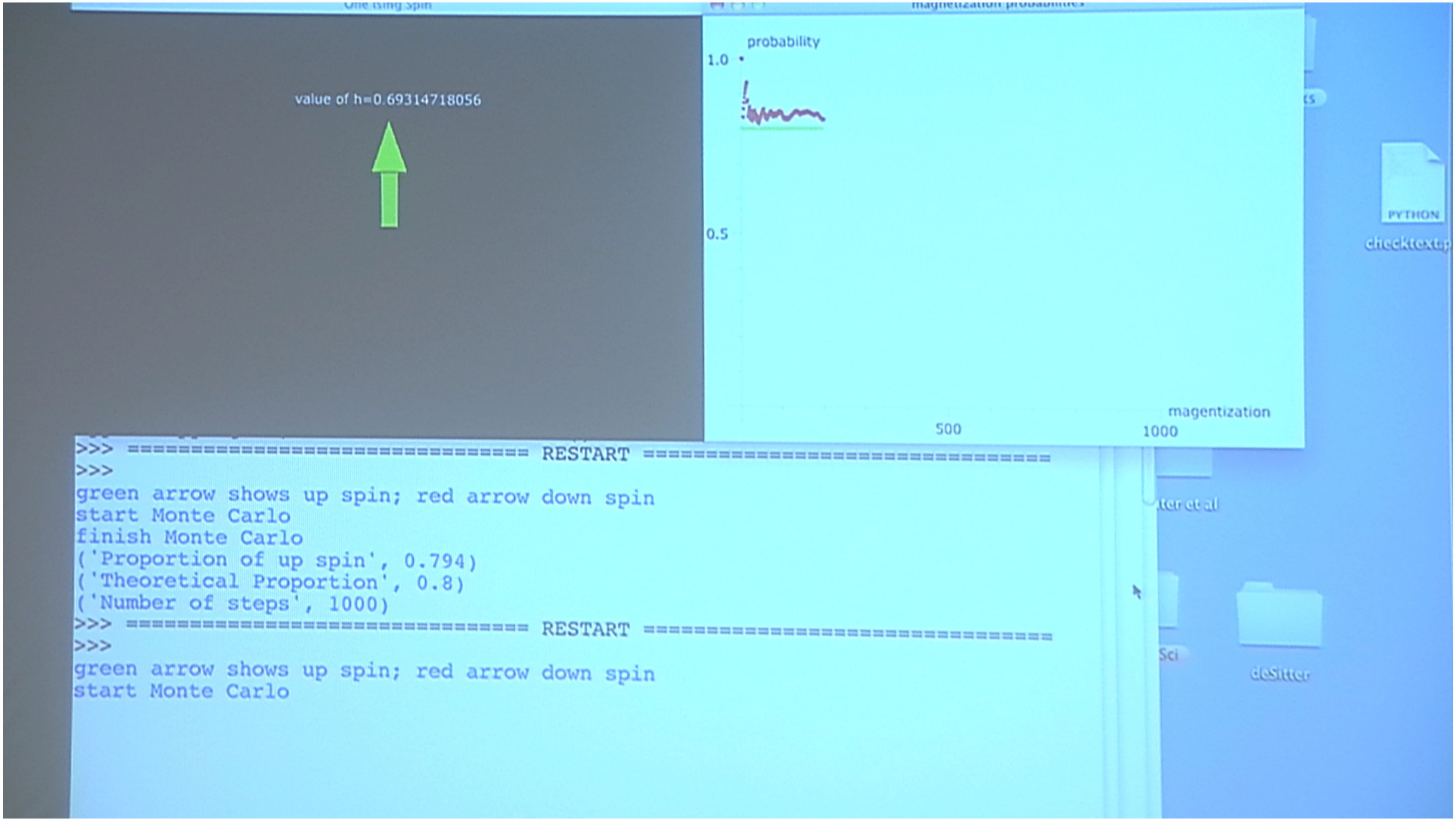
('Theoretical Proportion', 0.8)
('Number of steps', 1000)
>>>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1000
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PYTHON
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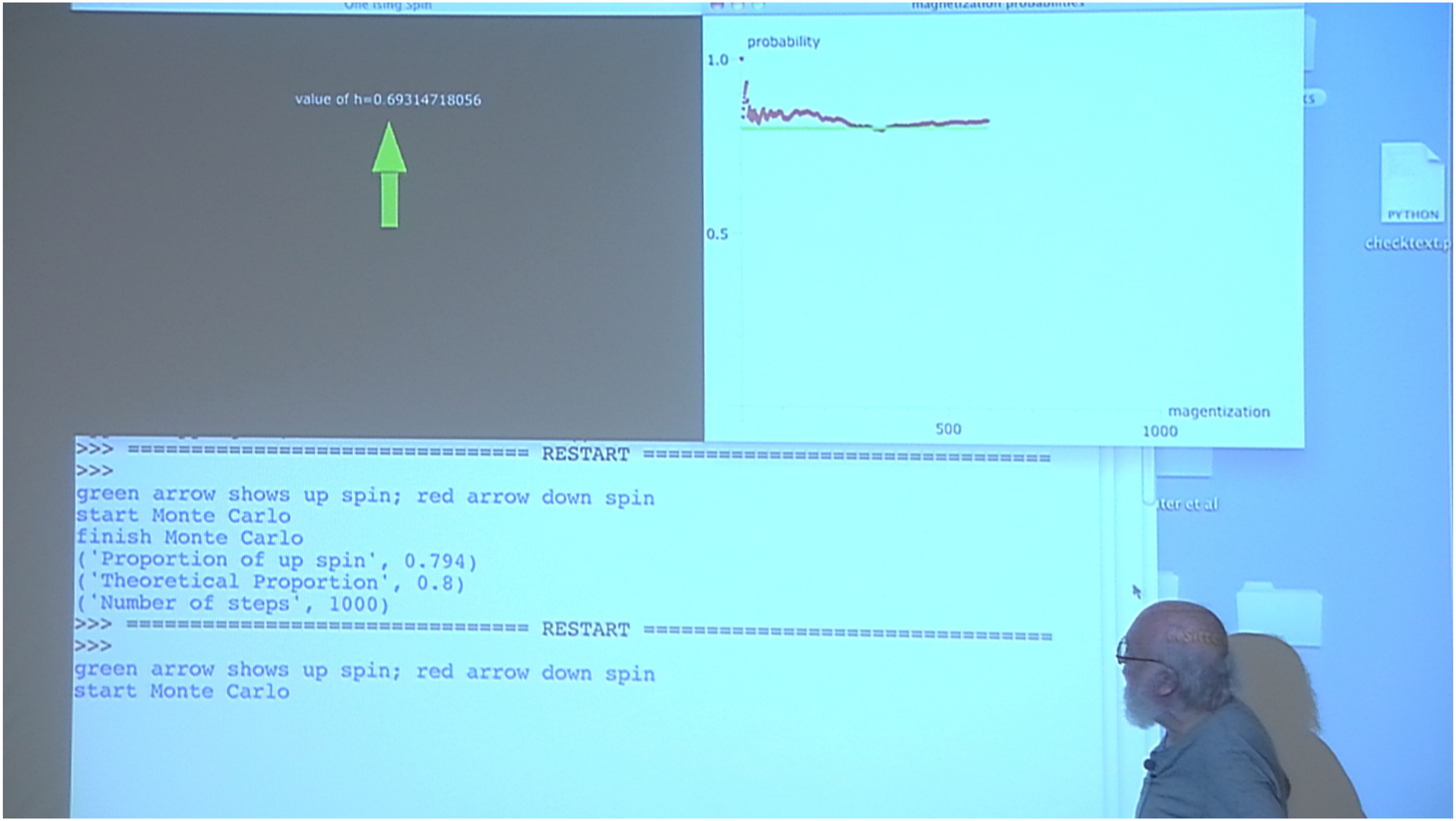
value of h=0.69314718056
probability
1.0
0.5
500
1000
magentization
>>> ================================ RESTART ================================
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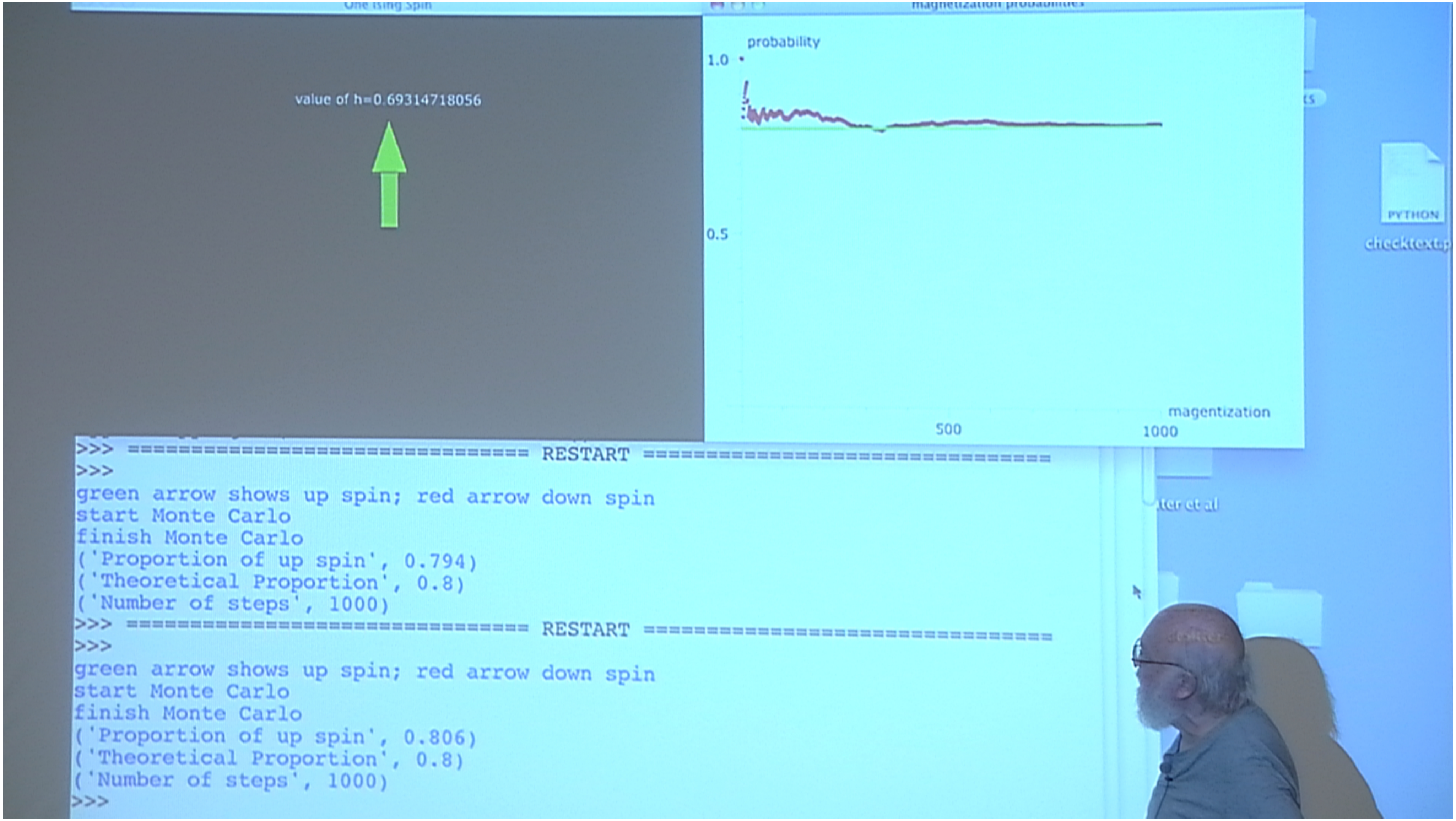
green arrow shows up spin; red arrow down spin
start Monte Carlo
finish Monte Carlo
('Proportion of up spin', 0.794)
('Theoretical Proportion', 0.8)
('Number of steps', 1000)
>>> ================================ RESTART ================================
>>>
green arrow shows up spin; red arrow down spin
start Monte Carlo
PYTHON


value of h=0.69314718056
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0.5
magentization
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1000
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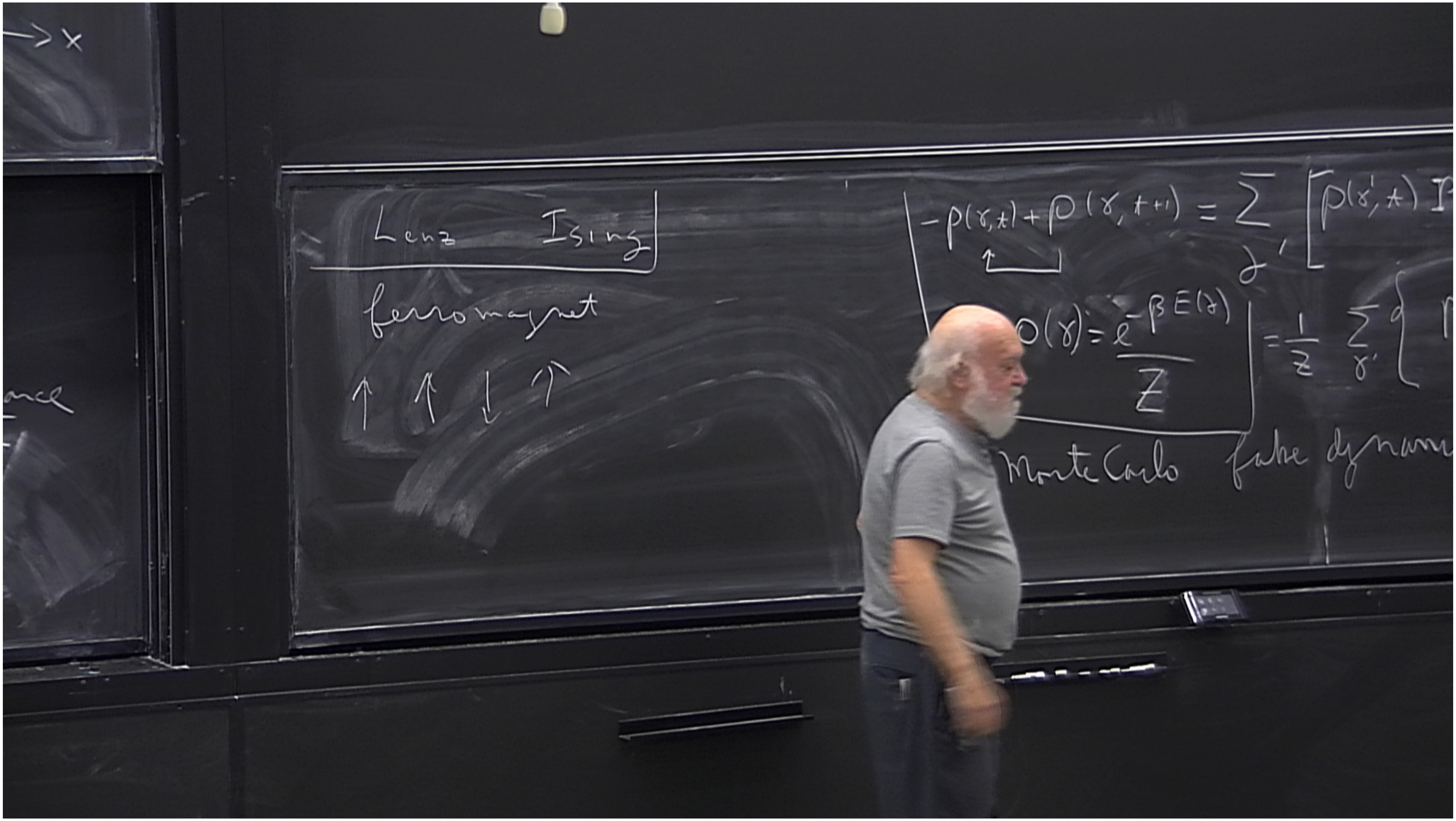
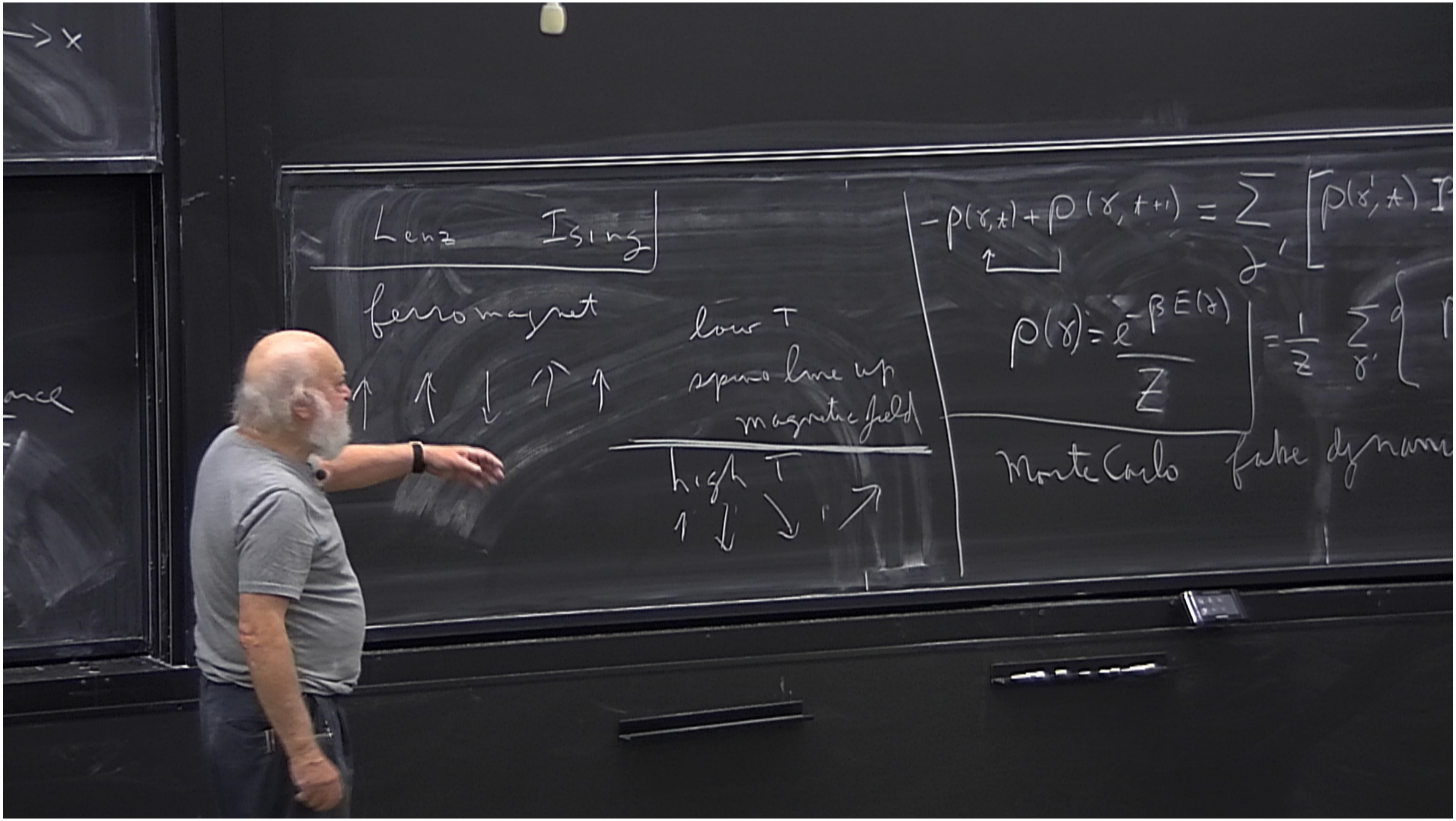
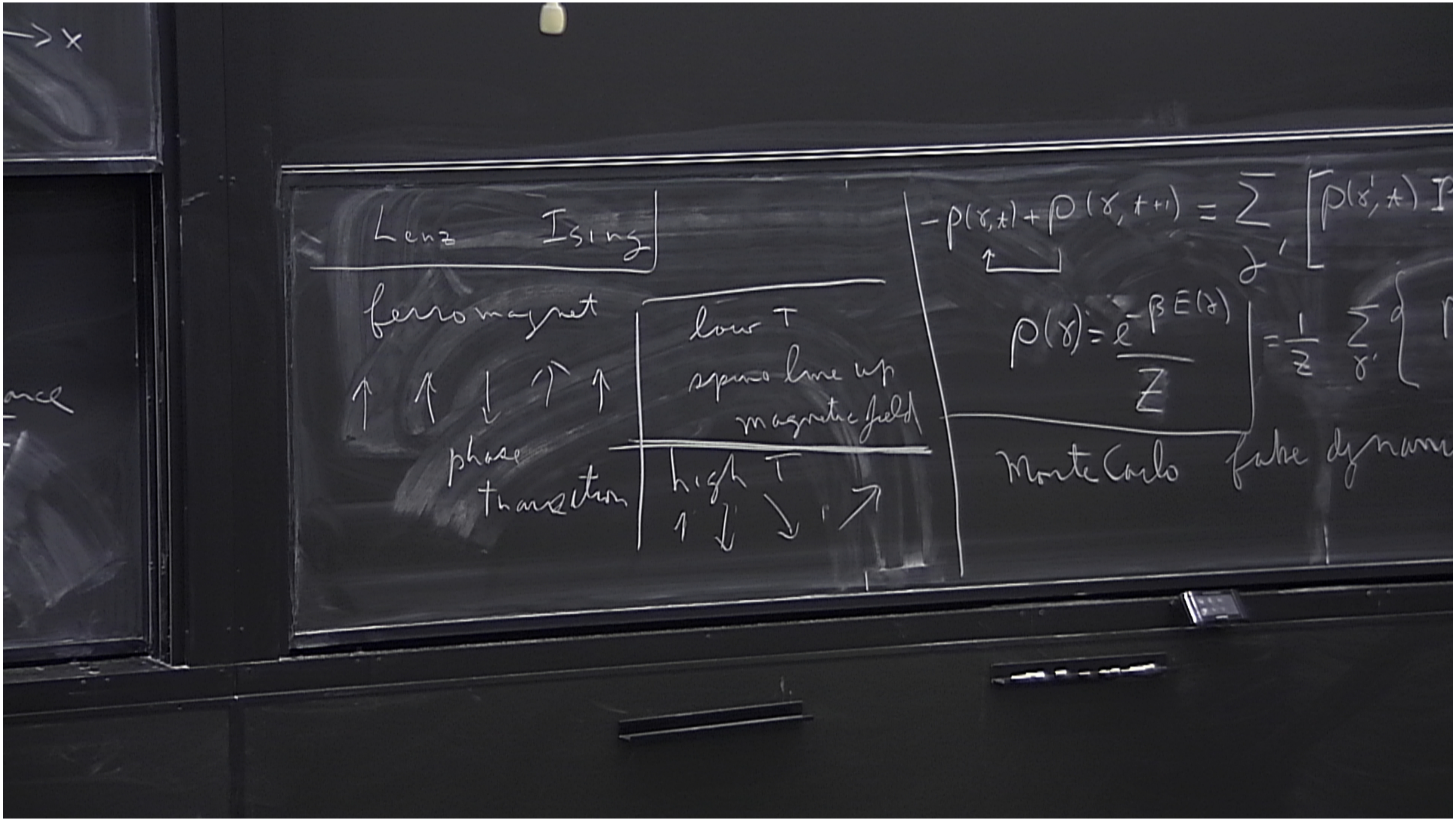
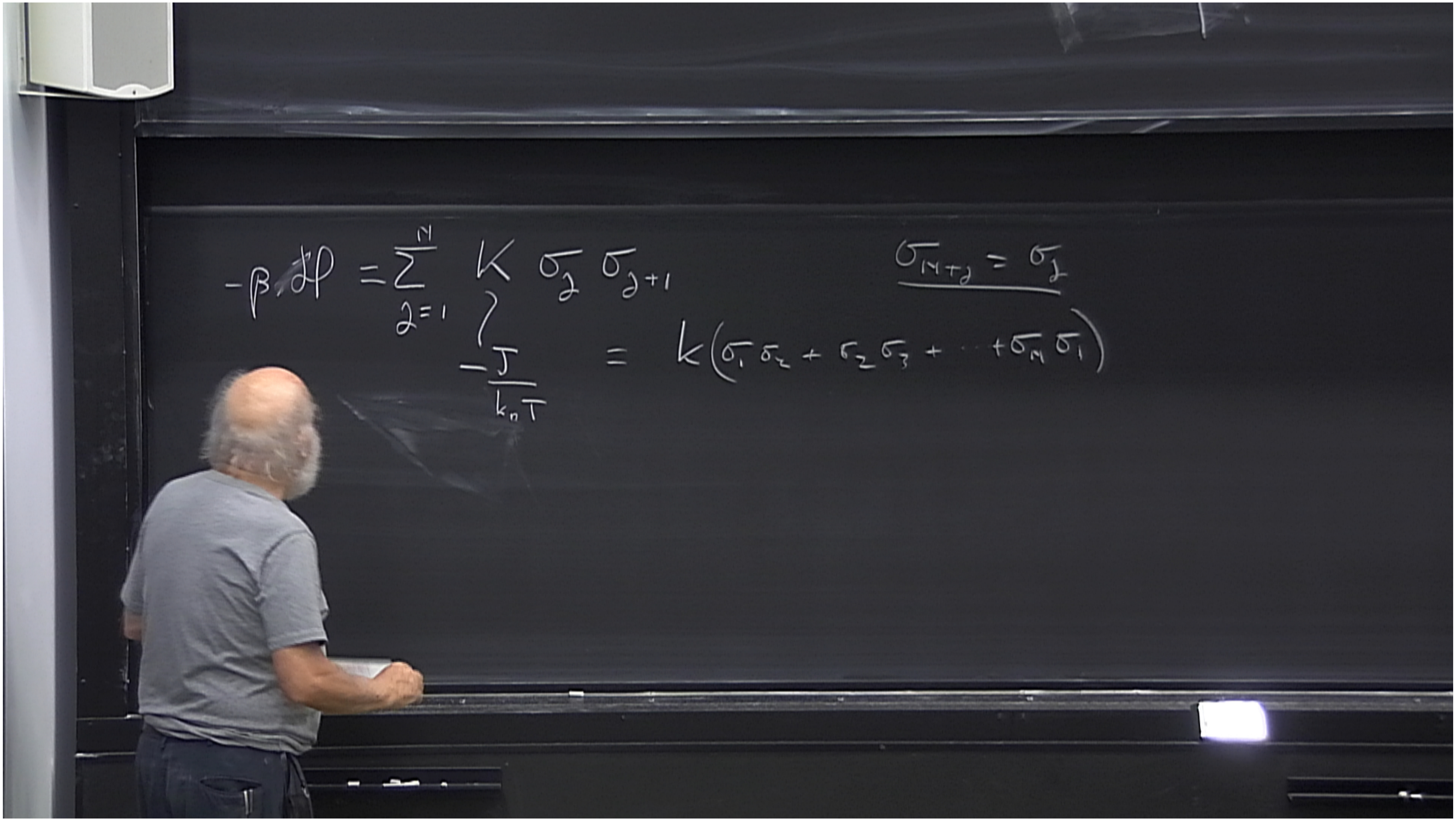
green arrow shows up spin; red arrow down spin
start Monte Carlo
finish Monte Carlo
('Proportion of up spin', 0.794)
('Theoretical Proportion', 0.8)
('Number of steps', 1000)
>>> ================================ RESTART ================================
>>>
green arrow shows up spin; red arrow down spin
start Monte Carlo
finish Monte Carlo
('Proportion of up spin', 0.806)
('Theoretical Proportion', 0.8)
('Number of steps', 1000)
>>>
PYTHON
checktext.p

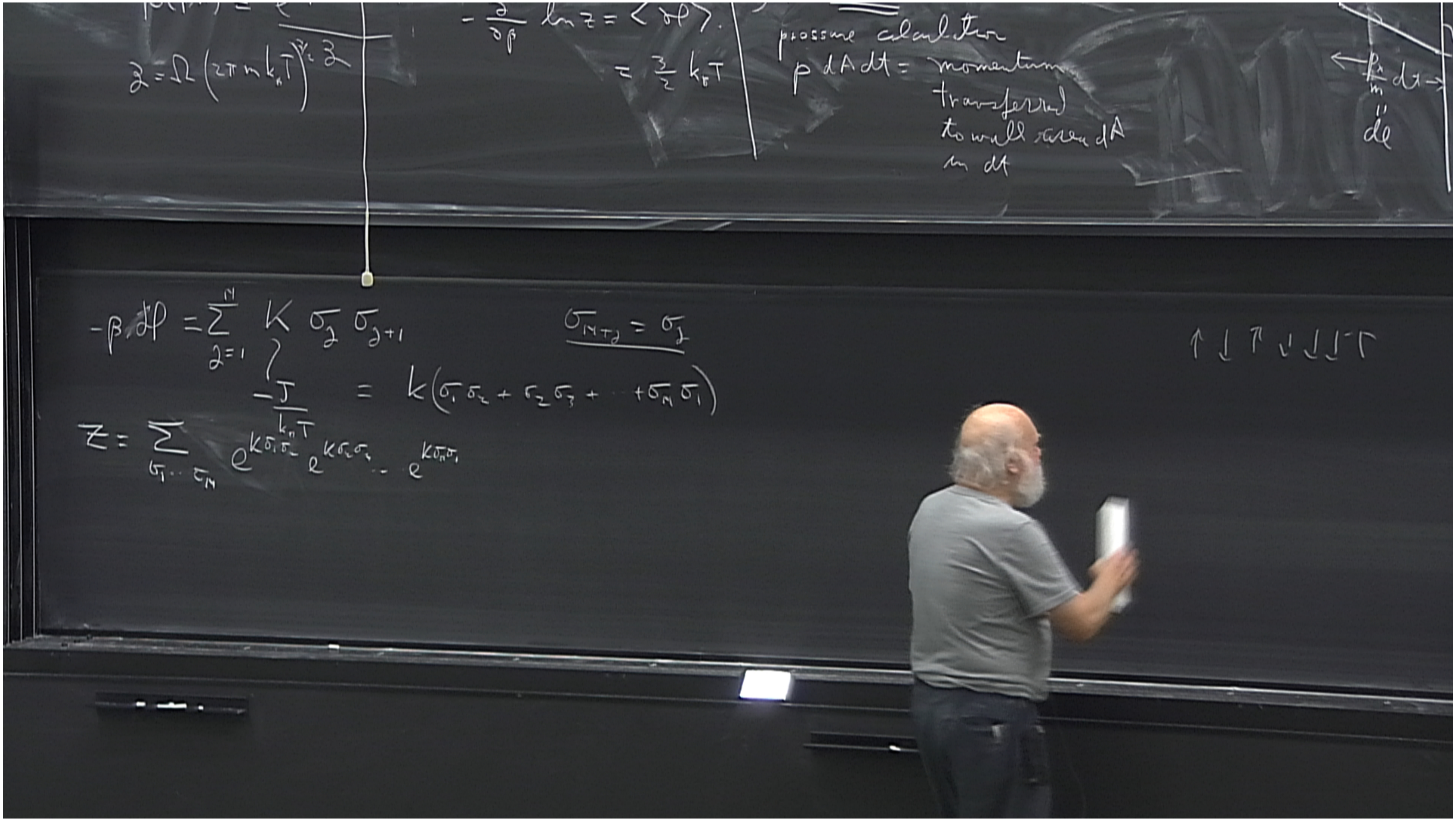

Lenz Ising
ferromagnet
Monte Carlo

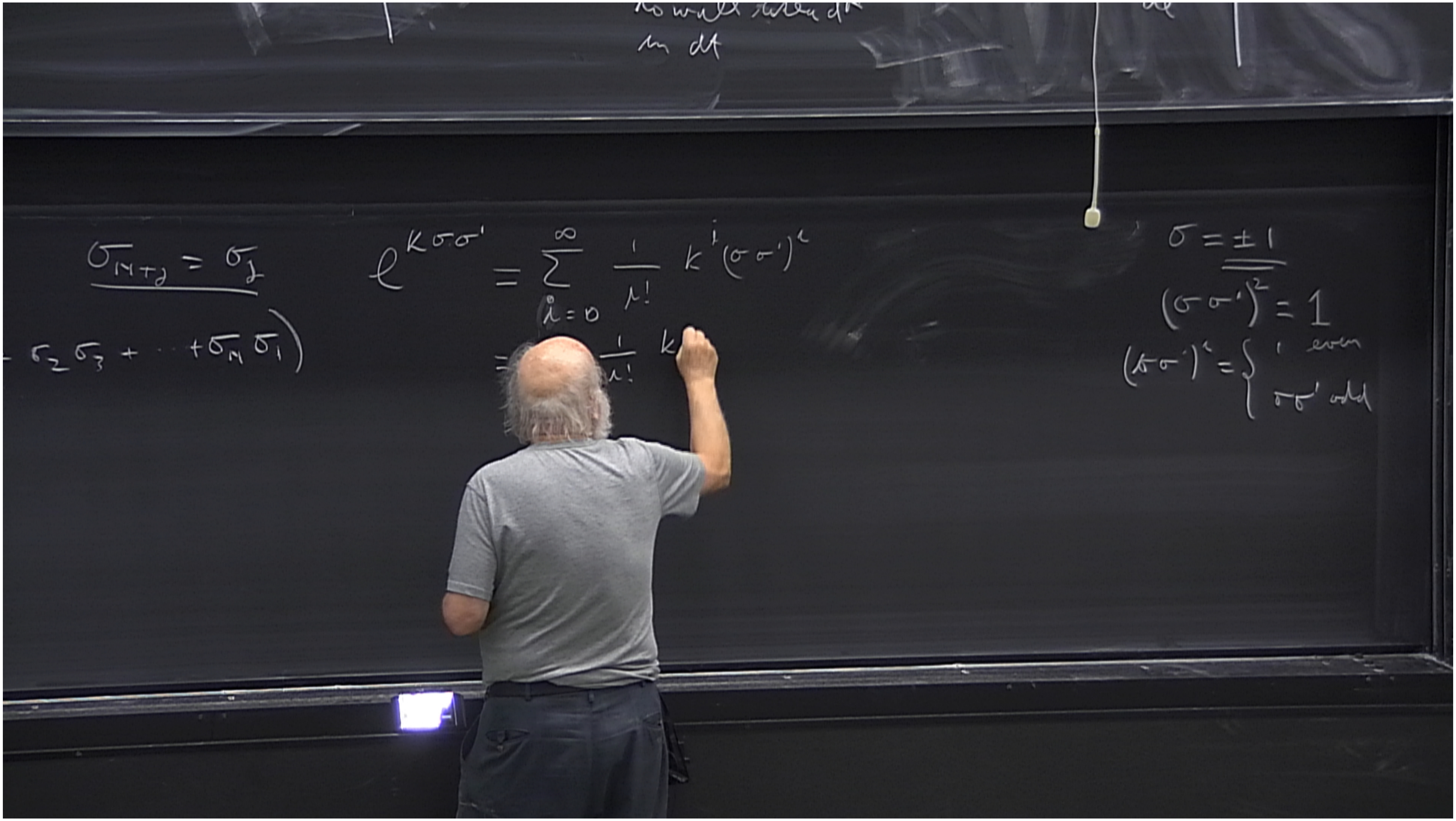

Lenz Ising
ferromagnet
low T
spins line up
magnetic field
high T
$-\rho(\gamma,t)+\rho(\gamma,t+1)=\sum_{\gamma'}$
$\rho(\gamma)=\frac{e^{-\beta E(\gamma)}}{Z}$
Monte Carlo

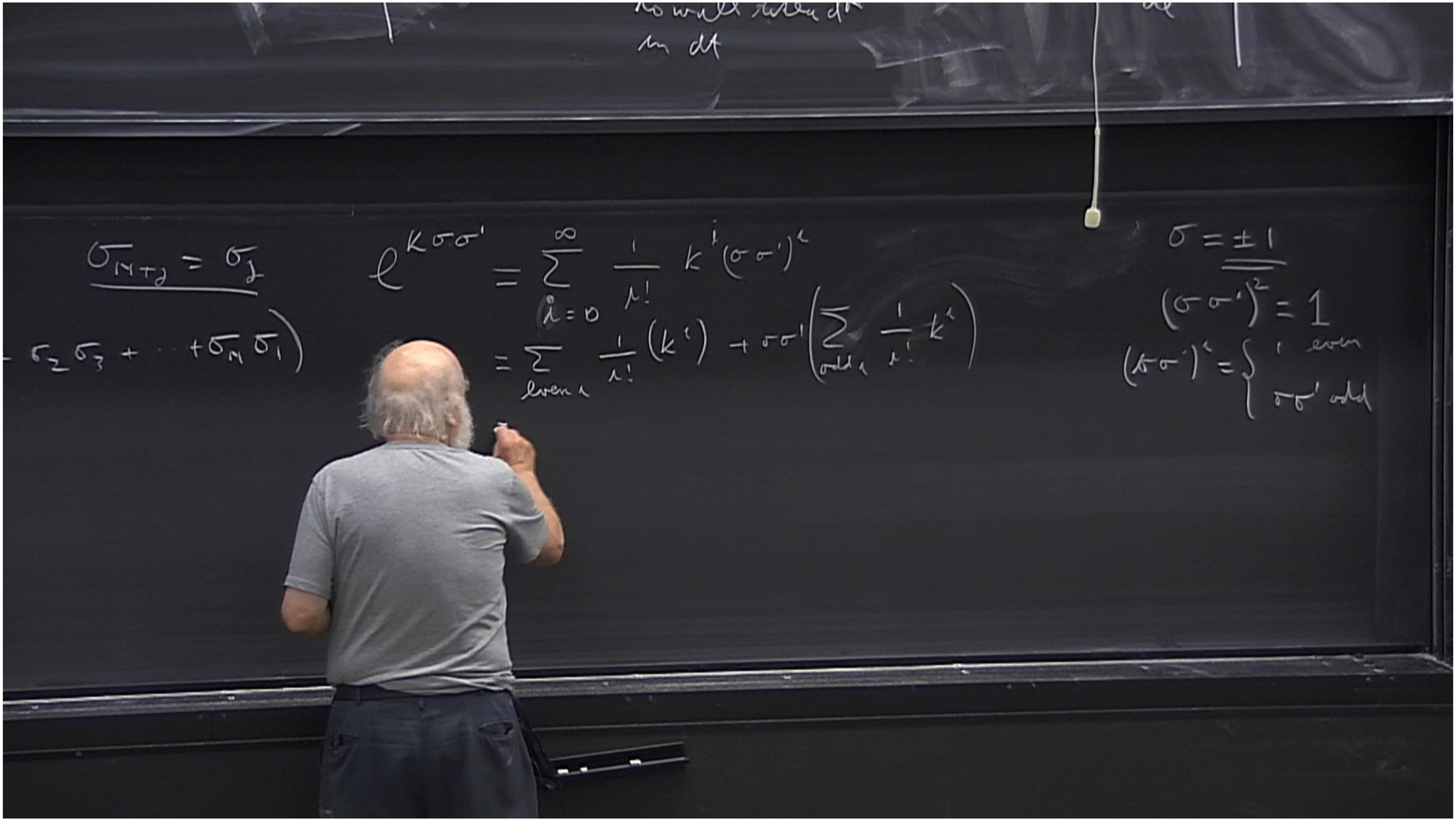

Lenz Ising
ferromagnet
phase transition
low T
spins line up
magnetic field
high T
ρ(γ) = e^{-βE(γ)} / Z
Monte Carlo

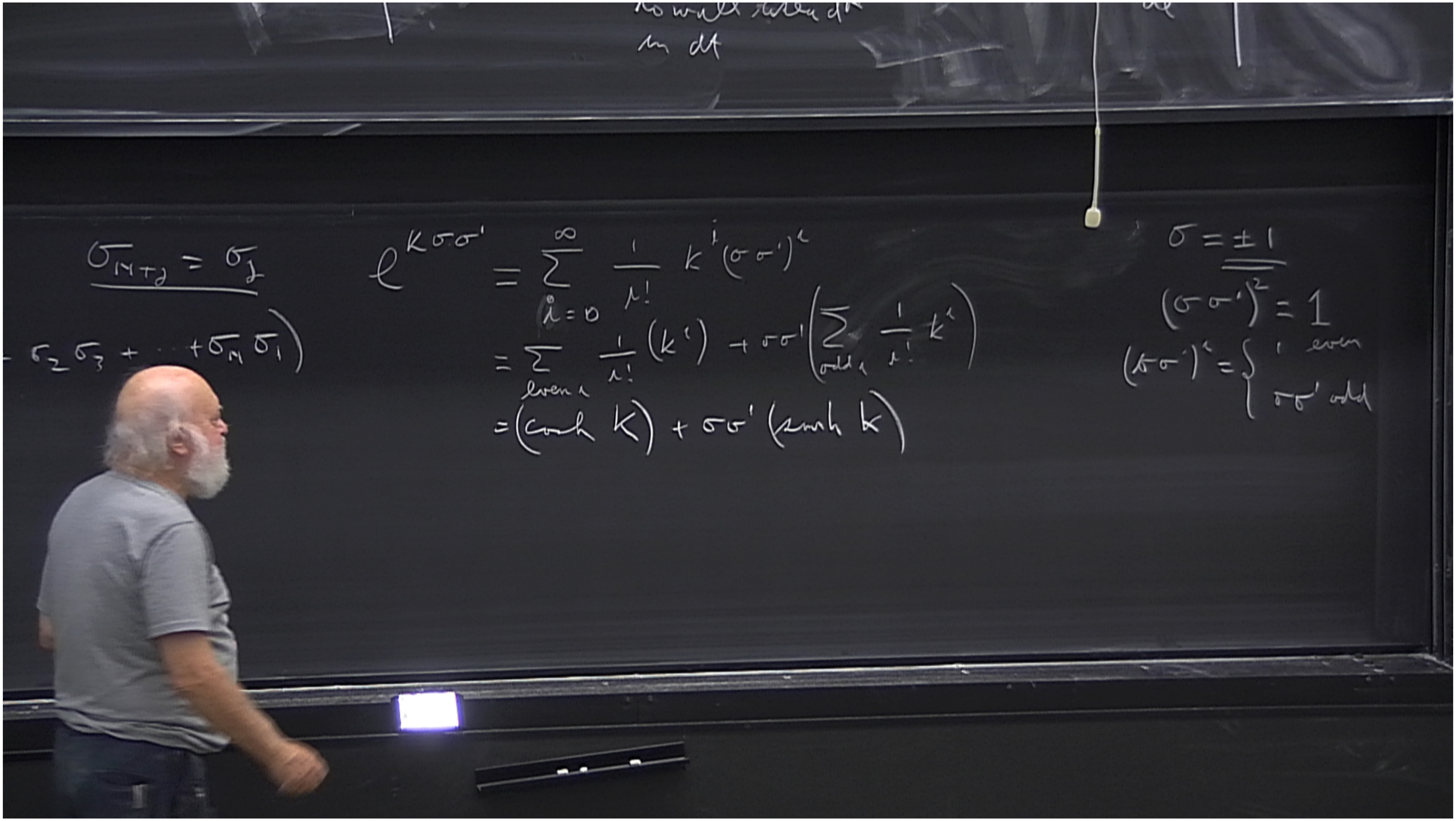
$$-\beta \cdot \partial \rho = \Sigma$$

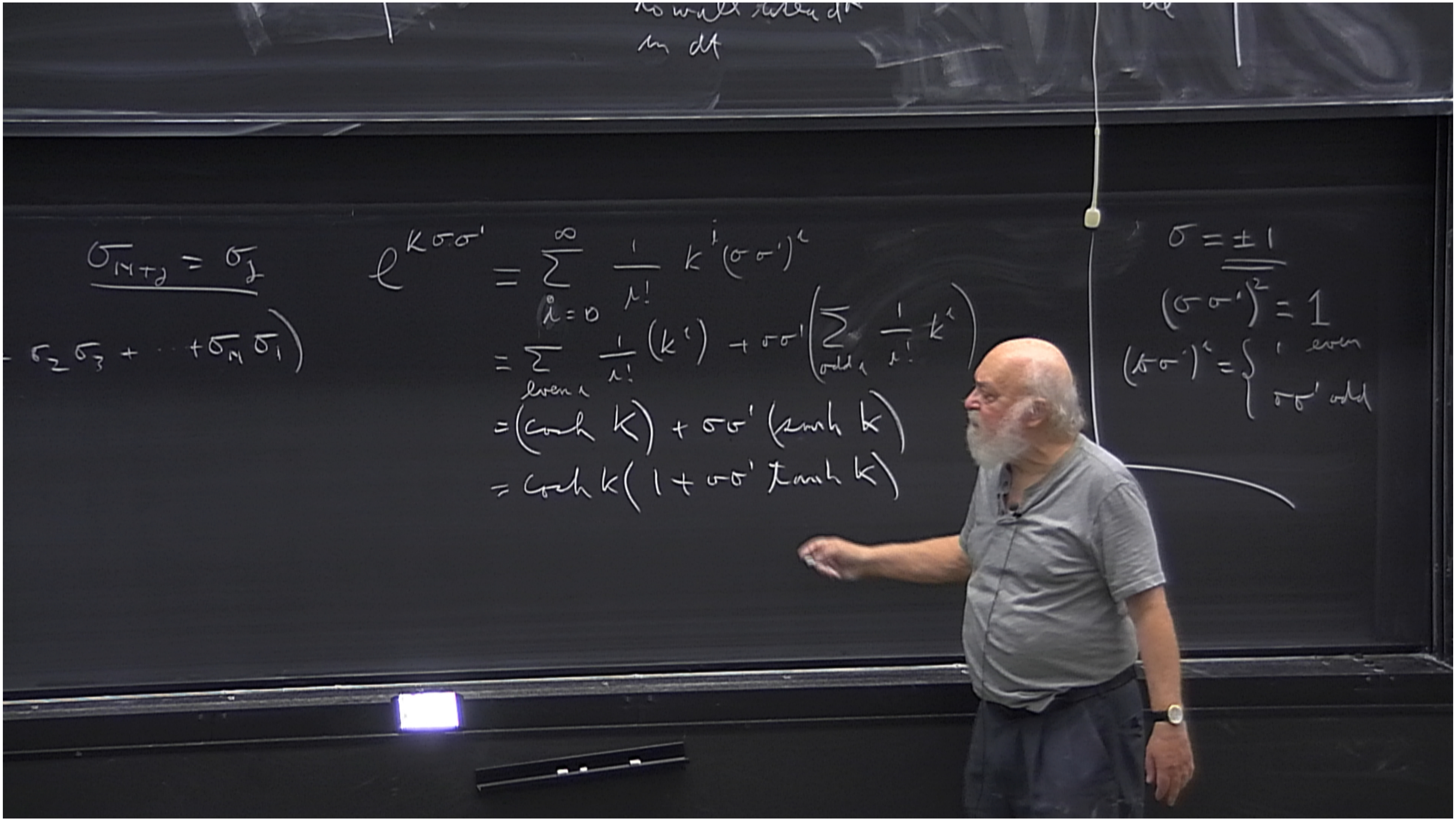

$-\beta \mathcal{H} = \sum_{j=1}^{N} K \sigma_j \sigma_{j+1}$
$\sigma_{N+j} = \sigma_j$
$= K(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \cdots + \sigma_N\sigma_1)$
$-\frac{J}{k_B T}$

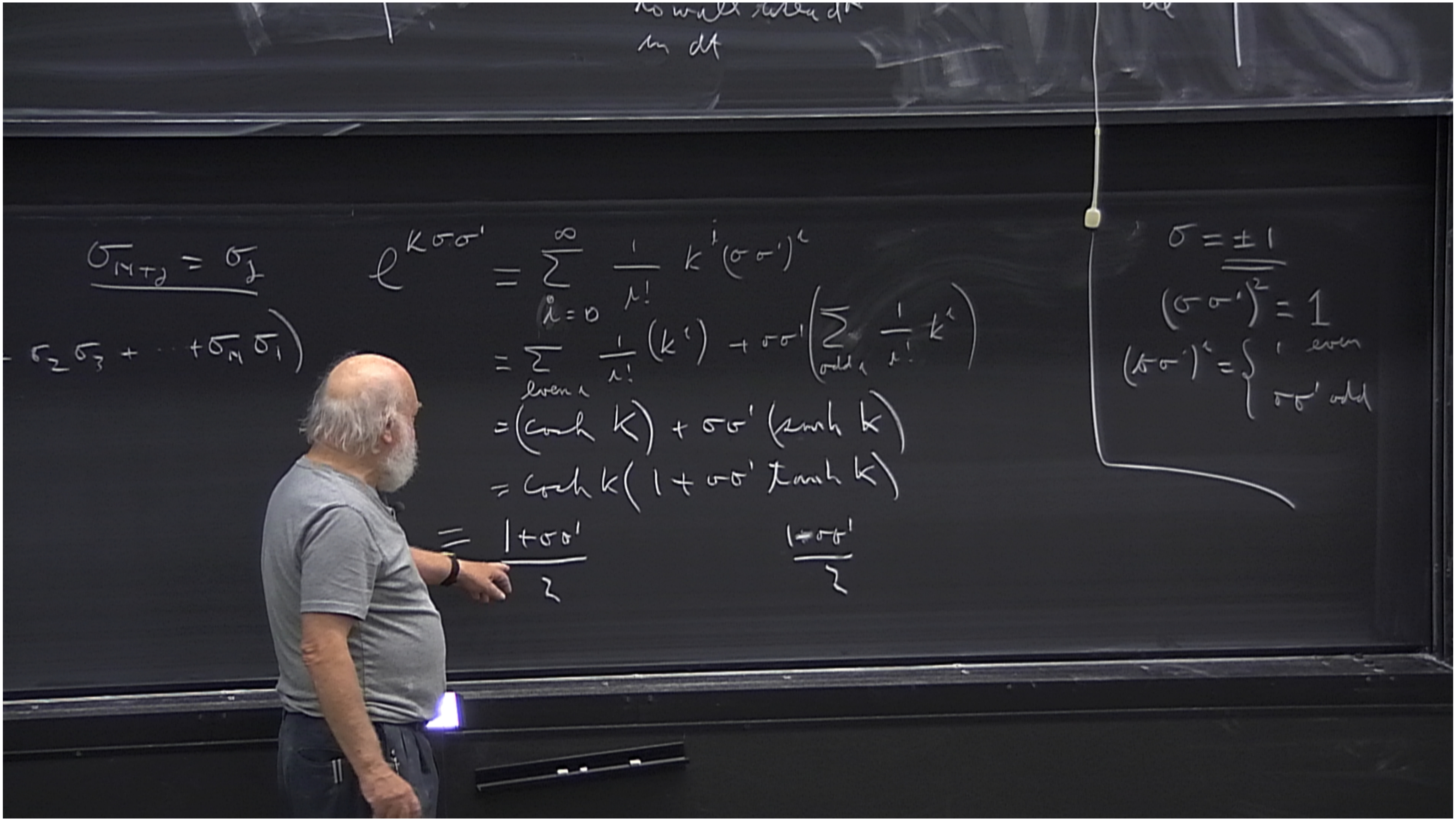

pressure calculation
p dA dt = momentum transferred to wall area dA in dt
= 3/2 k_B T
-βH = Σ_{j=1}^{N} K σ_j σ_{j+1}
σ_{N+j} = σ_j
-J/k_B T
= K(σ_1σ_2 + σ_2σ_3 + ... + σ_Nσ_1)
Z = Σ_{σ_1...σ_N} e^{Kσ_1σ_2} e^{Kσ_2σ_3} ... e^{Kσ_Nσ_1}

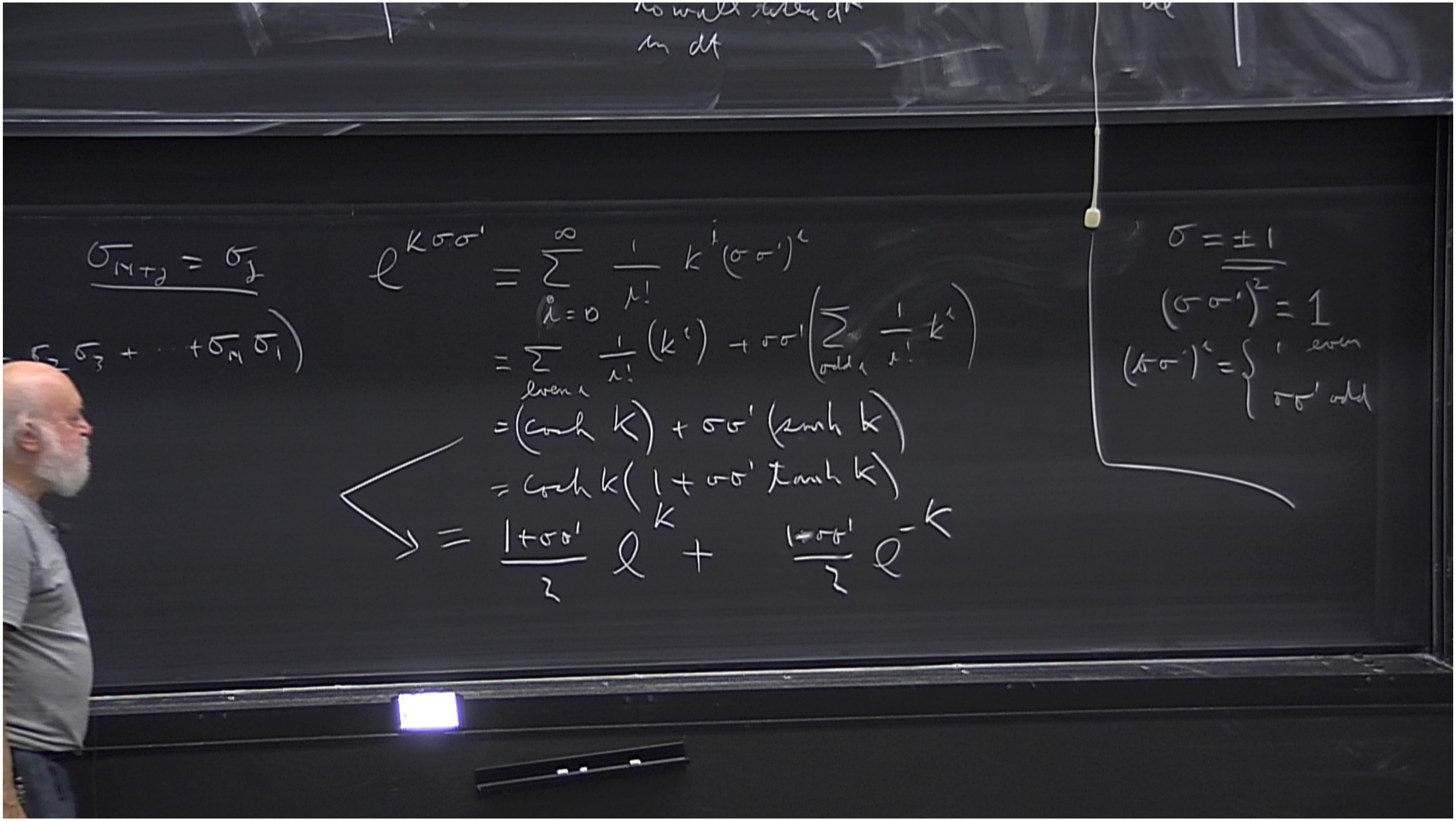

σ_{N+j} = σ_j
σ_2 σ_3 + ... + σ_N σ_1)
e^{Kσσ'} = Σ_{i=0}^∞ 1/i! K^i (σσ')^i
= 1/i! K
σ = ±1
(σσ')^2 = 1
(σσ')^i = { 1 even
σσ' odd


σ_{N+j} = σ_j
e^{Kσσ'} = Σ_{i=0}^{∞} 1/i! K^i (σσ')^i
= Σ_{even i} 1/i! (K^i) + σσ' (Σ_{odd i} 1/i! K^i)
σ = ±1
(σσ')^2 = 1
(σσ')^i = { 1 even; σσ' odd
σ_2 σ_3 + ⋯ + σ_N σ_1)
in dt


σ_{N+j} = σ_j
σ_2 σ_3 + ⋯ + σ_N σ_1)
e^{Kσσ'} = Σ_{i=0}^{∞} (1/i!) K^i (σσ')^i
= Σ_{even i} (1/i!) (K^i) + σσ' (Σ_{odd i} (1/i!) K^i)
= (cosh K) + σσ' (sinh K)
σ = ±1
(σσ')^2 = 1
(σσ')^i = { 1 even ; σσ' odd


σN+j = σj
σ2 σ3 + ··· + σN σ1)
e^{Kσσ'} = Σ_{i=0}^{∞} 1/i! K^i (σσ')^i
= Σ_{even i} 1/i! (K^i) + σσ' (Σ_{odd i} 1/i! K^i)
= (cosh K) + σσ' (sinh K)
= cosh K (1 + σσ' tanh K)
σ = ±1
(σσ')^2 = 1
(σσ')^i = { 1 even, σσ' odd


$\sigma_{14+j} = \sigma_j$
$\ldots - \sigma_2\sigma_3 + \cdots + \sigma_{14}\sigma_1)$
$e^{K\sigma\sigma'} = \sum_{i=0}^{\infty} \frac{1}{i!} K^i (\sigma\sigma')^i$
$= \sum_{\text{even } i} \frac{1}{i!} (K^i) + \sigma\sigma' \left(\sum_{\text{odd } i} \frac{1}{i!} K^i \right)$
$= (\cosh K) + \sigma\sigma' (\sinh K)$
$= \cosh K (1 + \sigma\sigma' \tanh K)$
$= \frac{1+\sigma\sigma'}{2}$
$\frac{1-\sigma\sigma'}{2}$
$\sigma = \pm 1$
$(\sigma\sigma')^2 = 1$
$(\sigma\sigma')^i = \begin{cases} 1 & \text{even} \\ \sigma\sigma' & \text{odd} \end{cases}$


$\sigma_{N+j} = \sigma_j$
$\sigma_2 \sigma_3 + \cdots + \sigma_N \sigma_1)$
$e^{K\sigma\sigma'} = \sum_{i=0}^{\infty} \frac{1}{i!} K^i (\sigma\sigma')^i$
$= \sum_{\text{even } i} \frac{1}{i!} (K^i) + \sigma\sigma' \left(\sum_{\text{odd } i} \frac{1}{i!} K^i \right)$
$= (\cosh K) + \sigma\sigma' (\sinh K)$
$= \cosh K (1 + \sigma\sigma' \tanh K)$
$= \frac{1+\sigma\sigma'}{2} e^{K} + \frac{1-\sigma\sigma'}{2} e^{-K}$
$\sigma = \pm 1$
$(\sigma\sigma')^2 = 1$
$(\sigma\sigma')^i = \begin{cases} 1 & \text{even} \\ \sigma\sigma' & \text{odd} \end{cases}$

$$\sigma_{N+j} = \sigma_j$$

$$\ldots \sigma_2 \sigma_3 + \cdots + \sigma_N \sigma_1)$$

$$e^{K\sigma\sigma'} = \sum_{i=0}^{\infty} \frac{1}{i!} K^i (\sigma\sigma')^i$$

$$= \sum_{\text{even } i} \frac{1}{i!} (K^i) + \sigma\sigma' \left(\sum_{\text{odd } i} \frac{1}{i!} K^i \right)$$

$$= (\cosh K) + \sigma\sigma' (\sinh K)$$

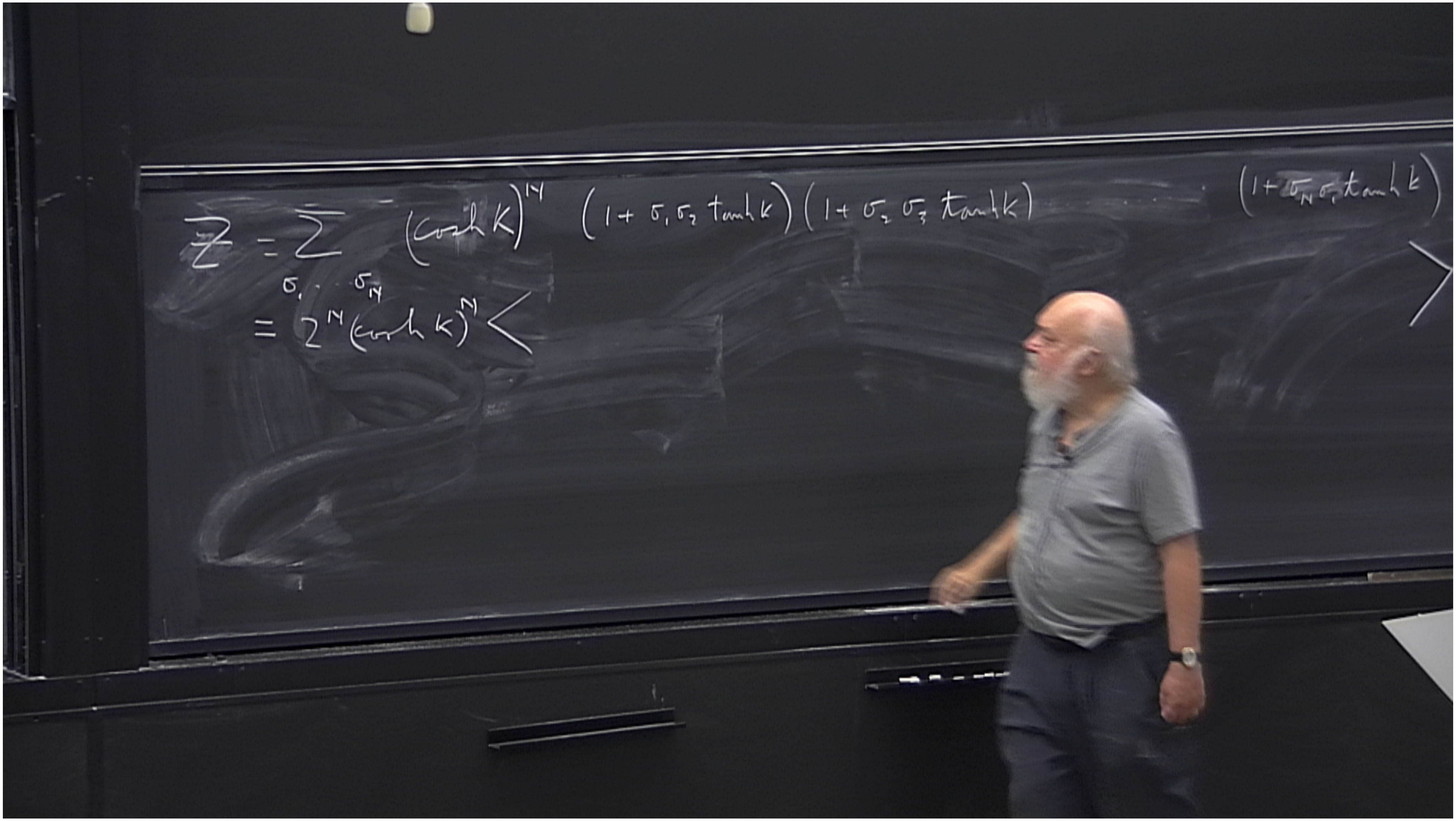
$$= \cosh K (1 + \sigma\sigma' \tanh K)$$

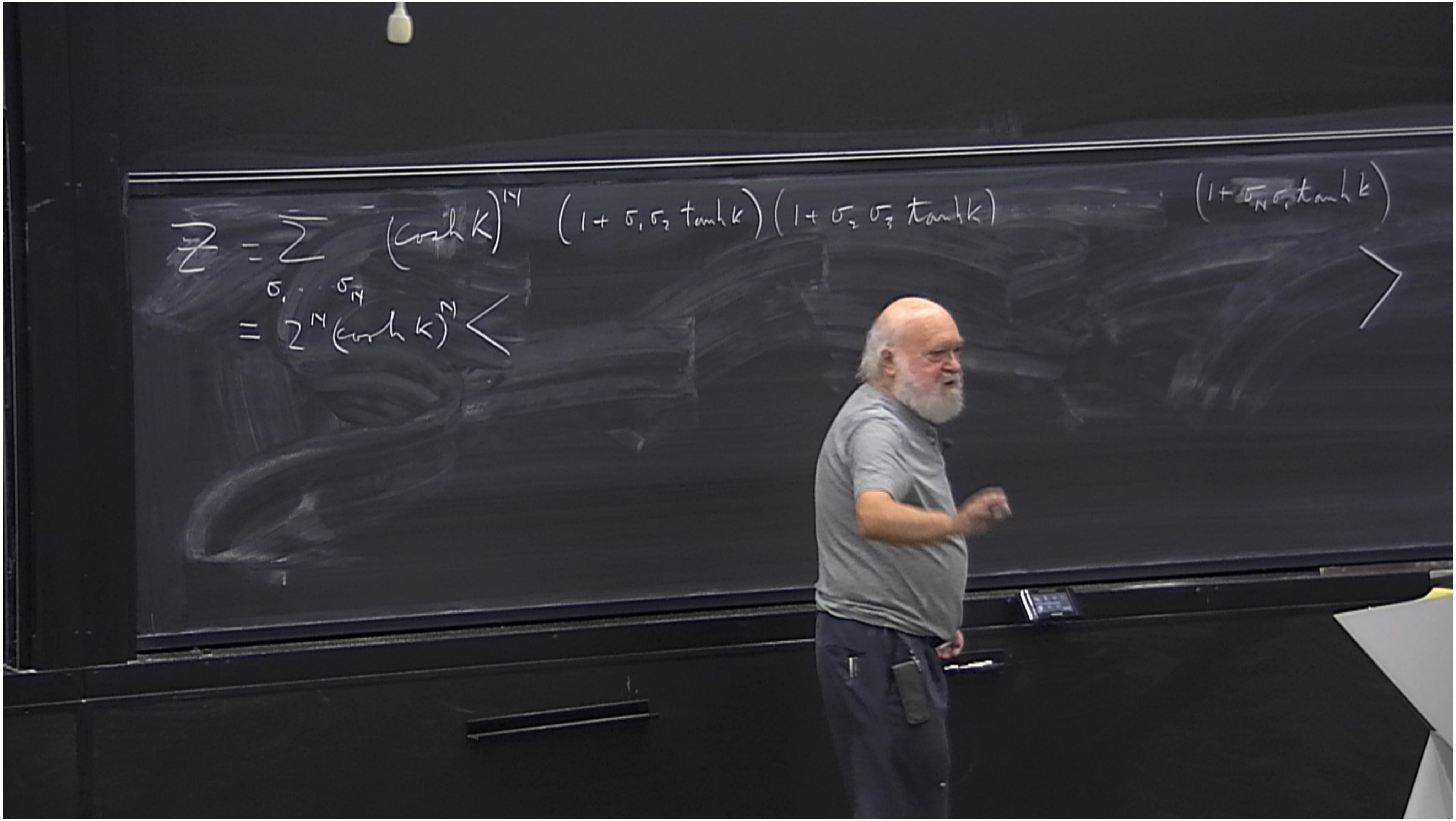
$$= \frac{1+\sigma\sigma'}{2} e^{K} + \frac{1-\sigma\sigma'}{2} e^{-K} = e^{K} \left(\frac{1+\sigma\sigma'}{2} + \frac{1-\sigma\sigma'}{2} e^{-2K} \right)$$

$$\sigma = \pm 1$$

$$(\sigma\sigma')^2 = 1$$

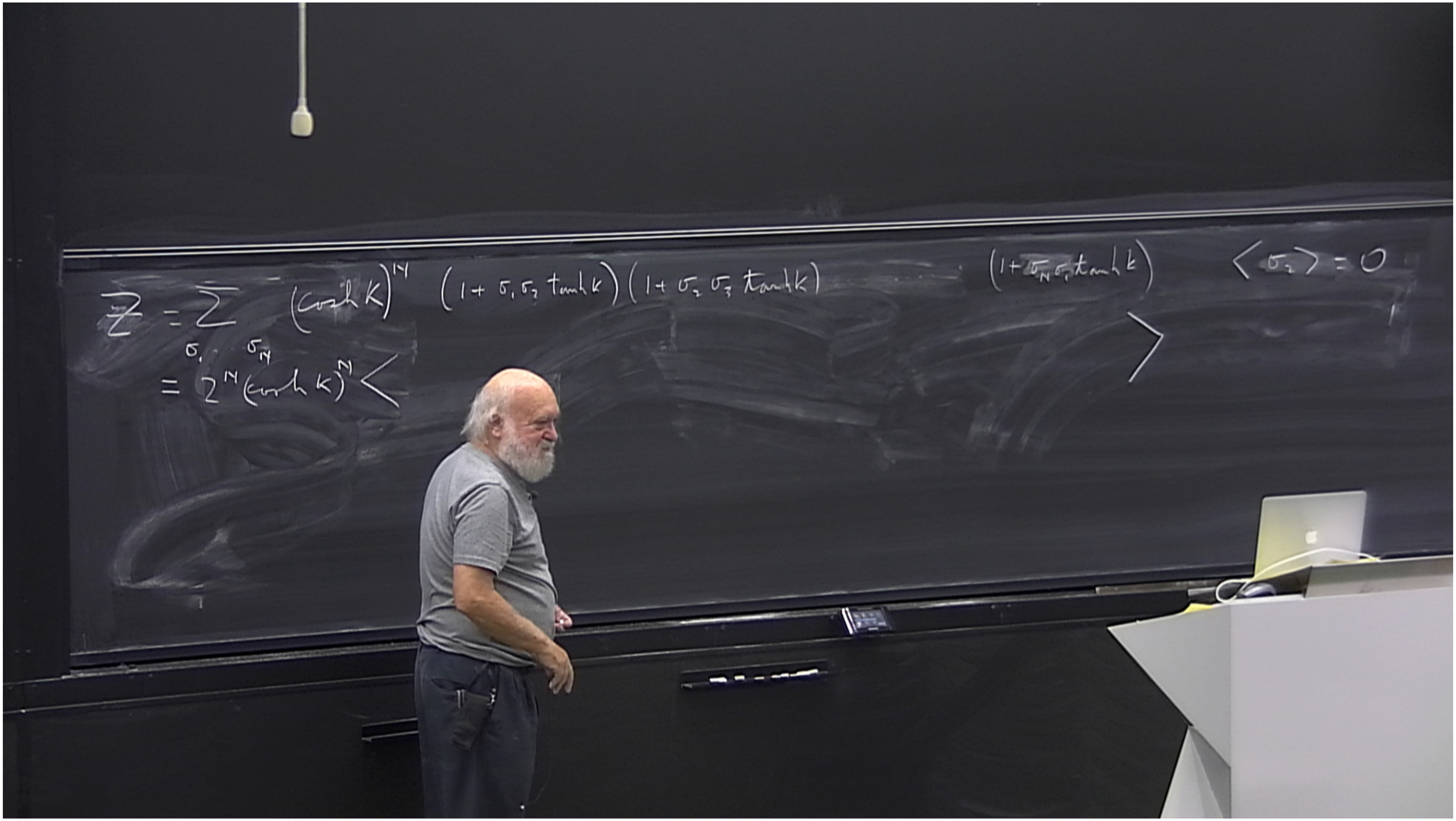
$$(\sigma\sigma')^i = \begin{cases} 1 & \text{even} \\ \sigma\sigma' & \text{odd} \end{cases}$$

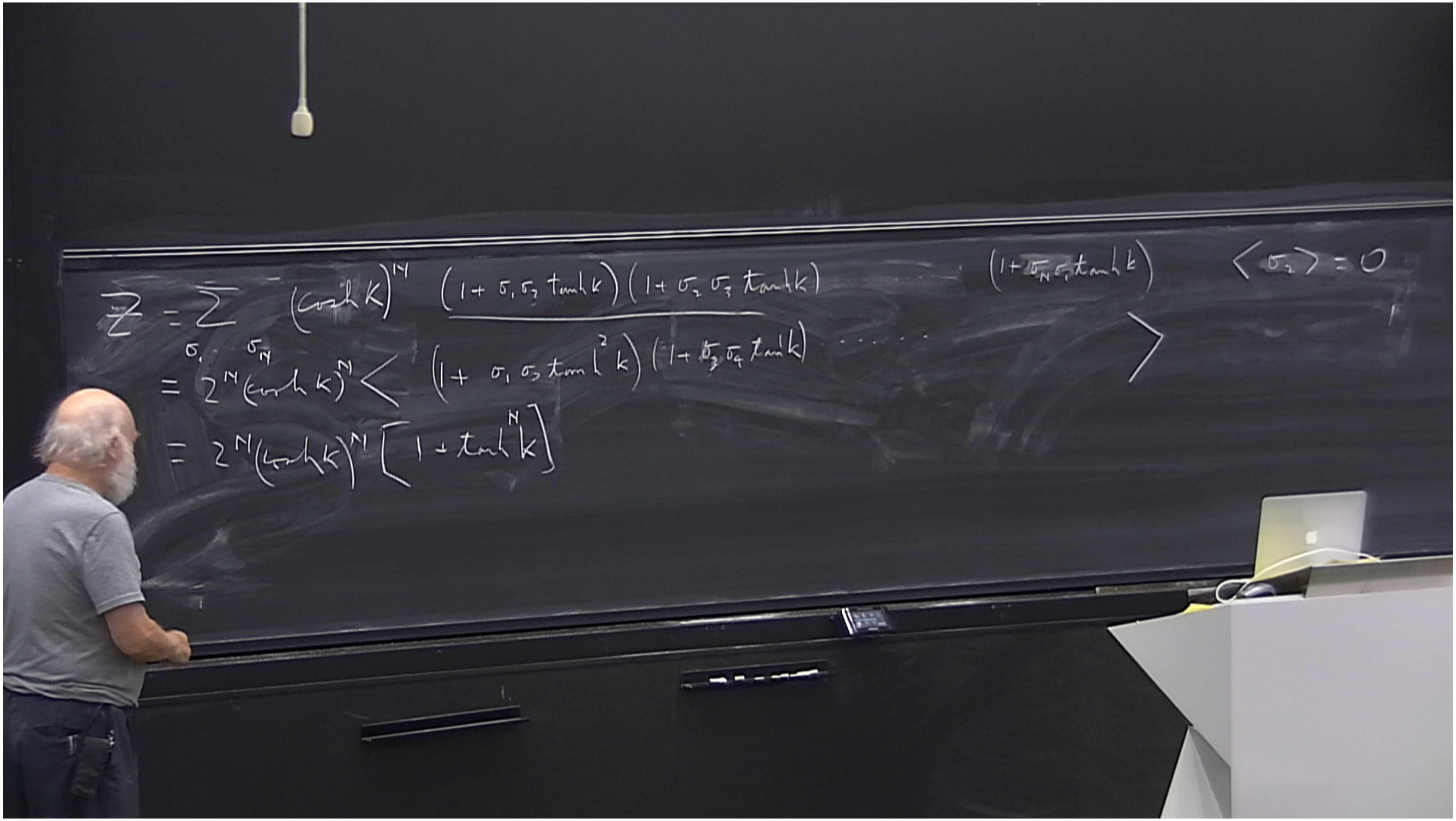

Z = Σ (cosh K)^N (1+σ1σ2 tanh K)(1+σ2σ3 tanh K) ... (1+σNσ1 tanh K)
σ1 ... σN
= 2^N (cosh K)^N < >


$Z = \sum_{\sigma_1 \ldots \sigma_N} (\cosh K)^N (1+\sigma_1\sigma_2 \tanh K)(1+\sigma_2\sigma_3 \tanh K) \ldots (1+\sigma_N\sigma_1 \tanh K)$
$= 2^N (\cosh K)^N \langle \quad \rangle$

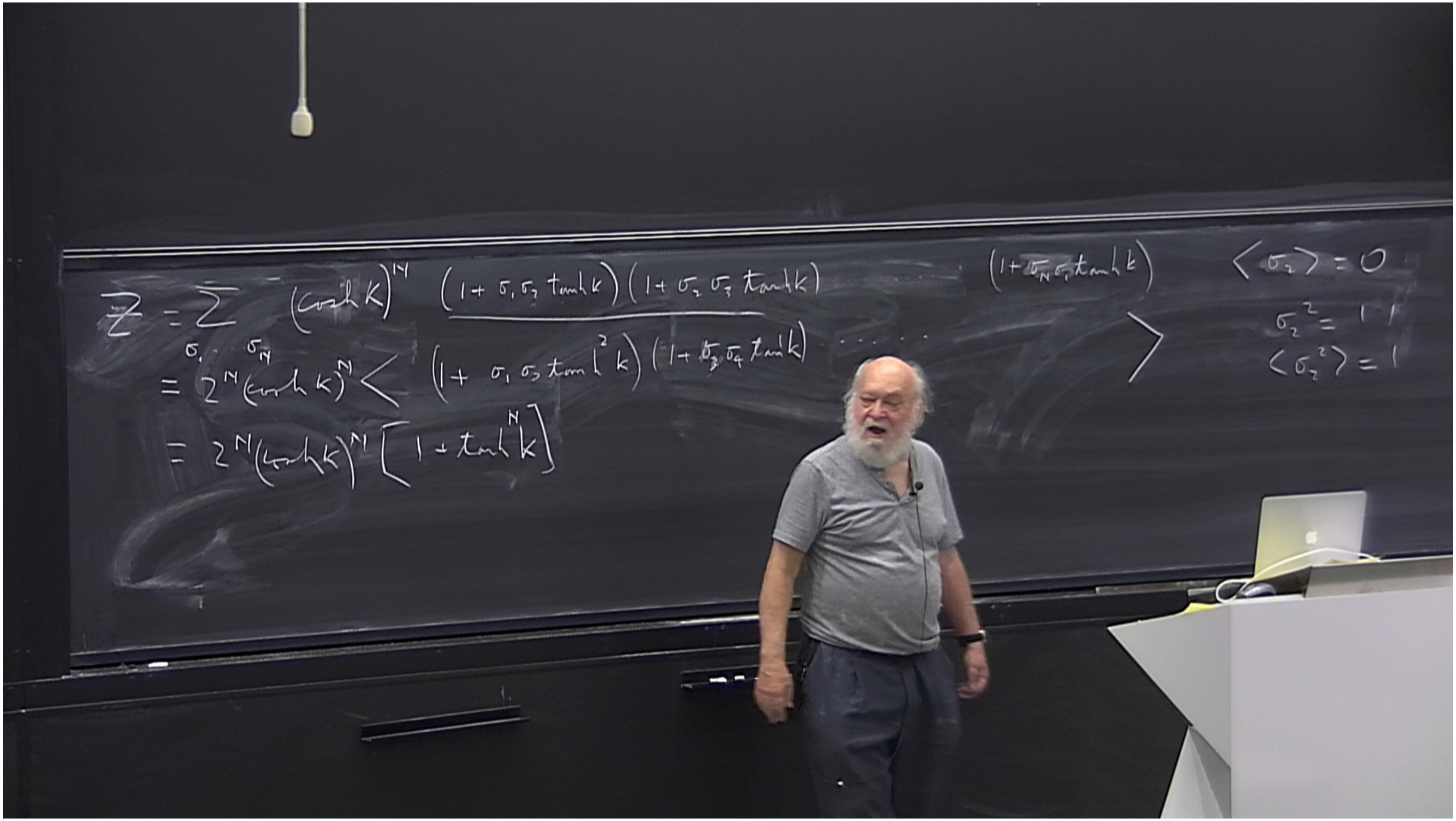
$$Z = \sum_{\sigma_1 \cdots \sigma_N} (\cosh K)^N \, (1+\sigma_1\sigma_2 \tanh K)(1+\sigma_2\sigma_3 \tanh K) \cdots (1+\sigma_N\sigma_1 \tanh K)$$

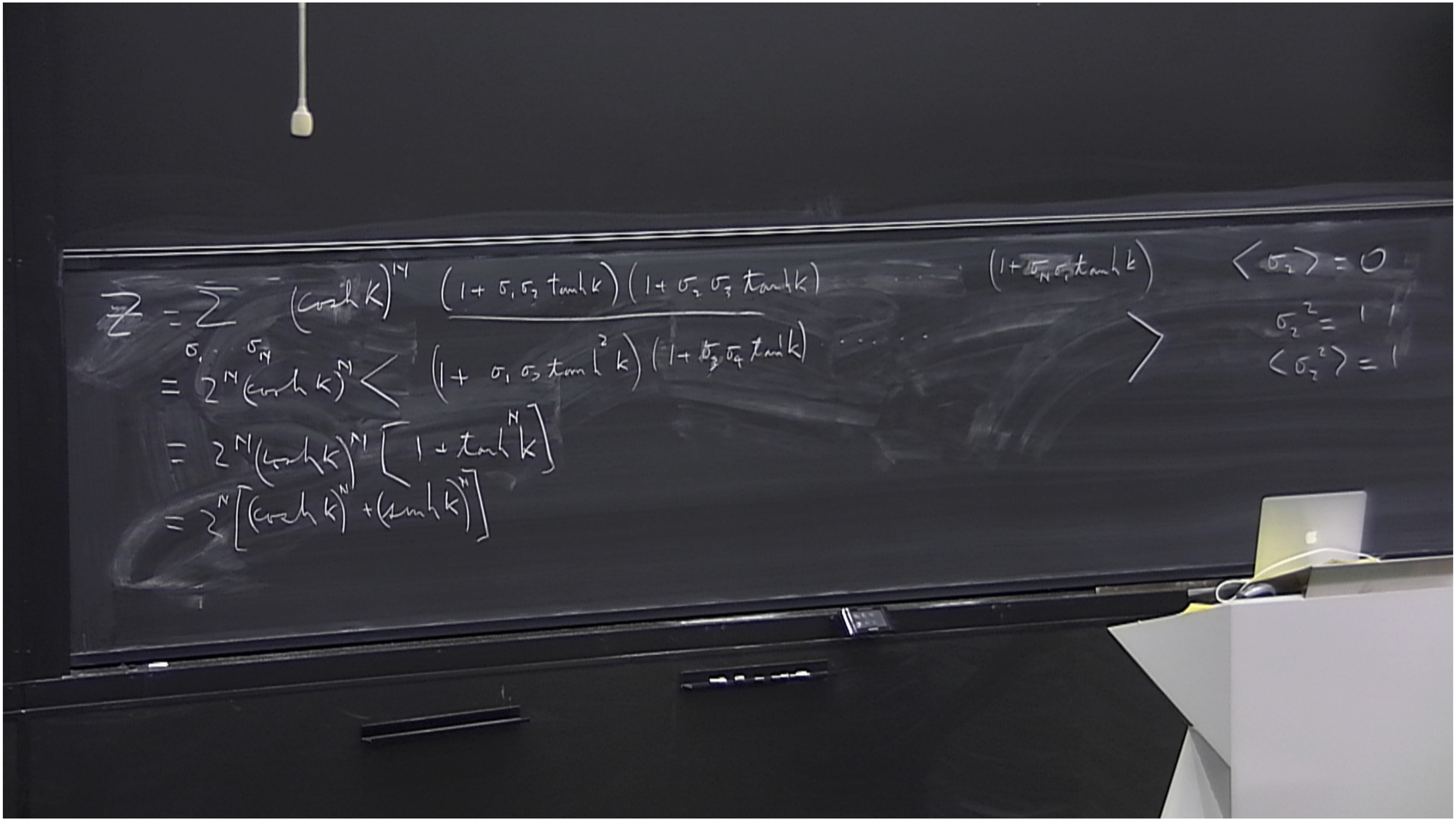
$$= 2^N (\cosh K)^N \langle \qquad \rangle$$

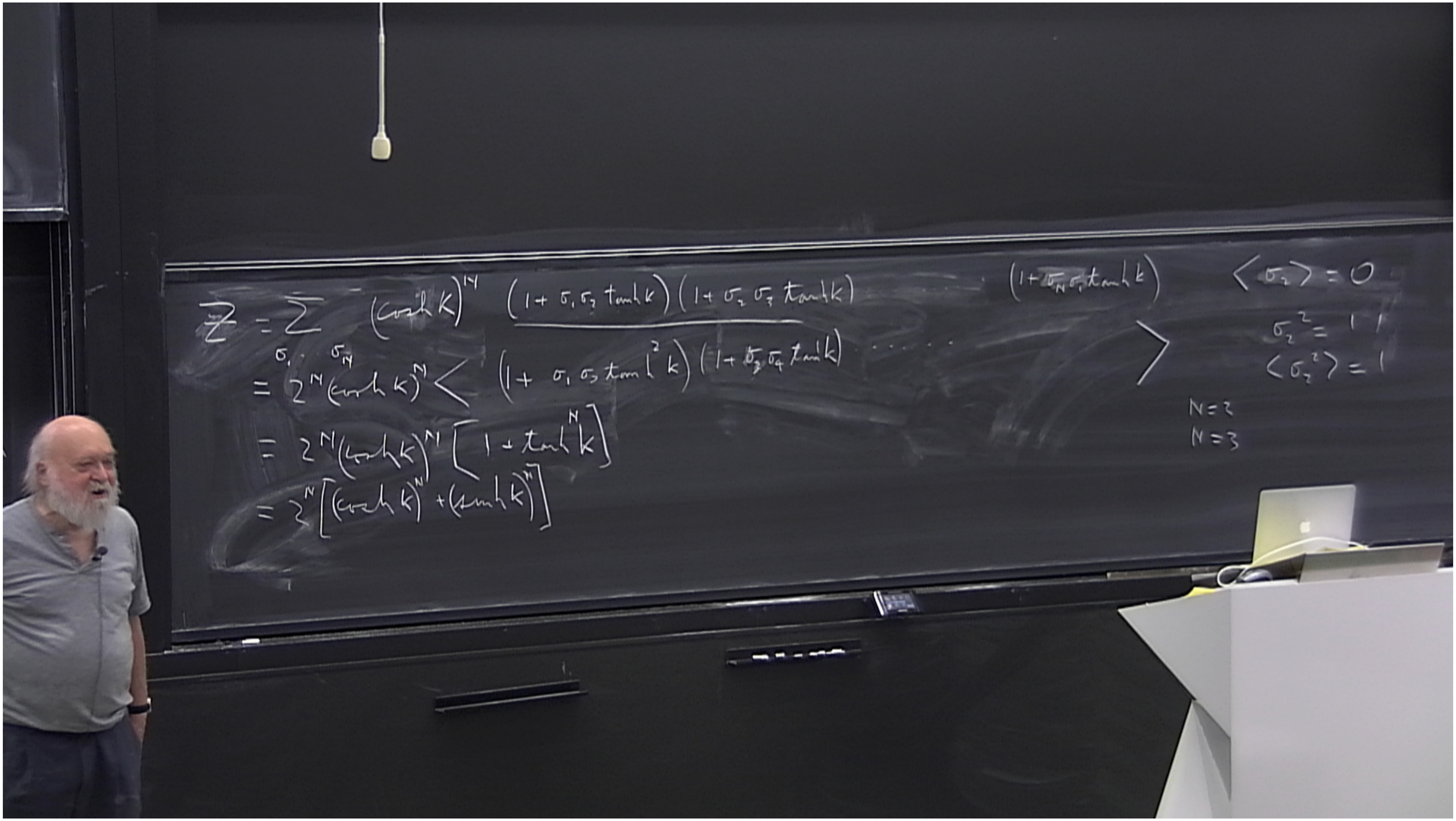

$Z = \sum_{\sigma_1 \cdots \sigma_N} (\cosh K)^N (1 + \sigma_1\sigma_2 \tanh K)(1 + \sigma_2\sigma_3 \tanh K) \cdots (1 + \sigma_N\sigma_1 \tanh K)$
$= 2^N (\cosh K)^N \langle \quad \rangle$
$\langle \sigma_2 \rangle = 0$



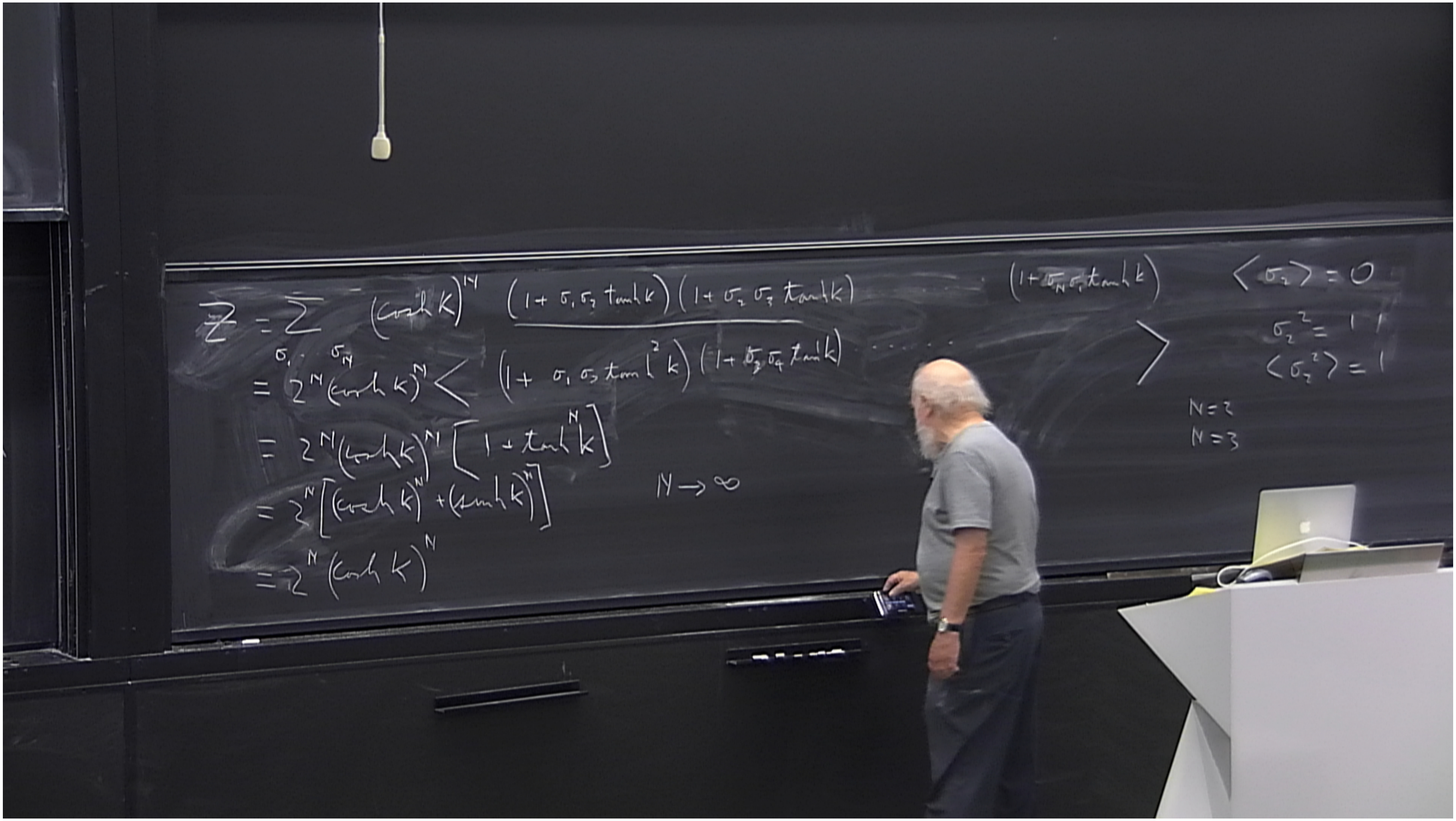
$Z = \sum_{\sigma_1 \cdots \sigma_N} (\cosh K)^N (1+\sigma_1\sigma_2 \tanh K)(1+\sigma_2\sigma_3 \tanh K) \cdots (1+\sigma_N\sigma_1 \tanh K)$
$\langle \sigma_i \rangle = 0$
$= 2^N (\cosh K)^N \langle (1+\sigma_1\sigma_3 \tanh^2 K)(1+\sigma_3\sigma_4 \tanh K) \cdots \rangle$
$= 2^N (\cosh K)^N [1 + \tanh^N K]$

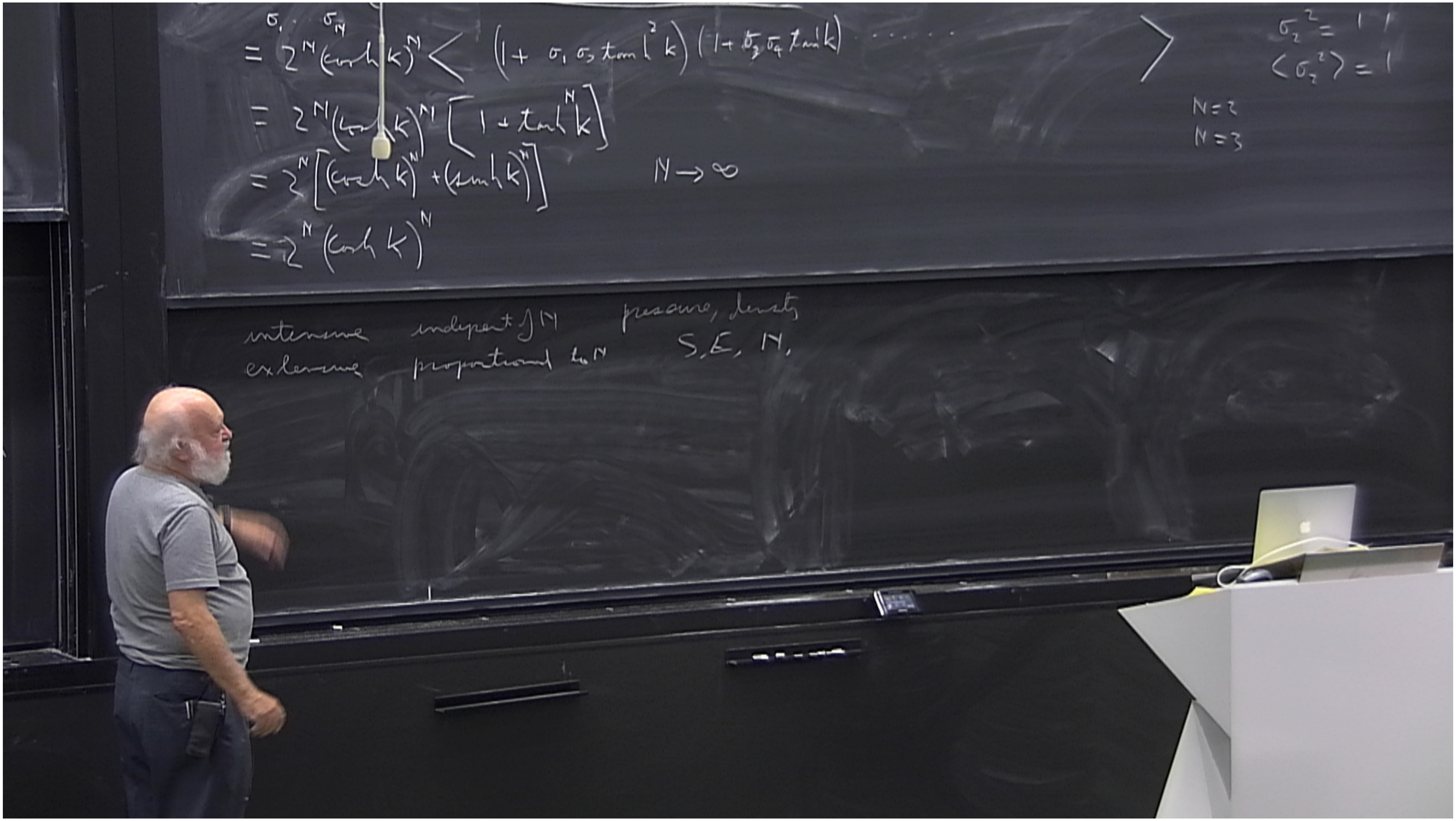

$Z = \sum_{\sigma_1 \cdots \sigma_N} (\cosh K)^N (1 + \sigma_1 \sigma_2 \tanh K)(1 + \sigma_2 \sigma_3 \tanh K) \cdots (1 + \sigma_N \sigma_1 \tanh K)$
$= 2^N (\cosh K)^N \langle (1 + \sigma_1 \sigma_3 \tanh^2 K)(1 + \sigma_3 \sigma_4 \tanh K) \cdots \rangle$
$= 2^N (\cosh K)^N [1 + \tanh^N K]$
$\langle \sigma_2 \rangle = 0$
$\sigma_2^2 = 1$
$\langle \sigma_2^2 \rangle = 1$

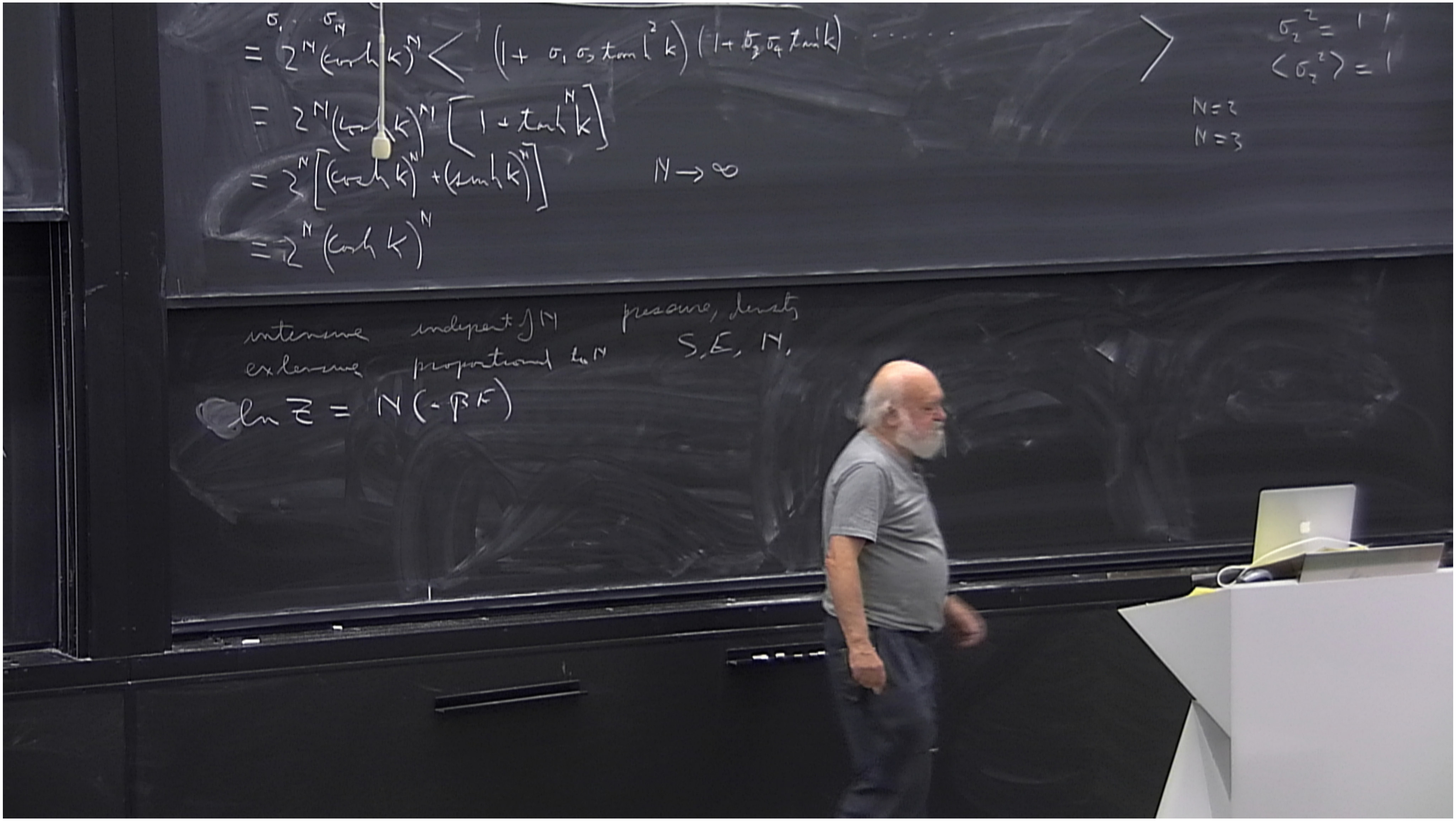

$Z = \sum_{\sigma_1 \cdots \sigma_N} (\cosh K)^N (1+\sigma_1\sigma_2 \tanh K)(1+\sigma_2\sigma_3 \tanh K) \cdots (1+\sigma_N\sigma_1 \tanh K)$
$= 2^N (\cosh K)^N \langle (1+\sigma_1\sigma_3 \tanh^2 K)(1+\sigma_3\sigma_4 \tanh K) \cdots \rangle$
$= 2^N (\cosh K)^N [1+\tanh^N K]$
$= 2^N [(\cosh K)^N + (\sinh K)^N]$
$\langle \sigma_2 \rangle = 0$
$\sigma_2^2 = 1$
$\langle \sigma_2^2 \rangle = 1$

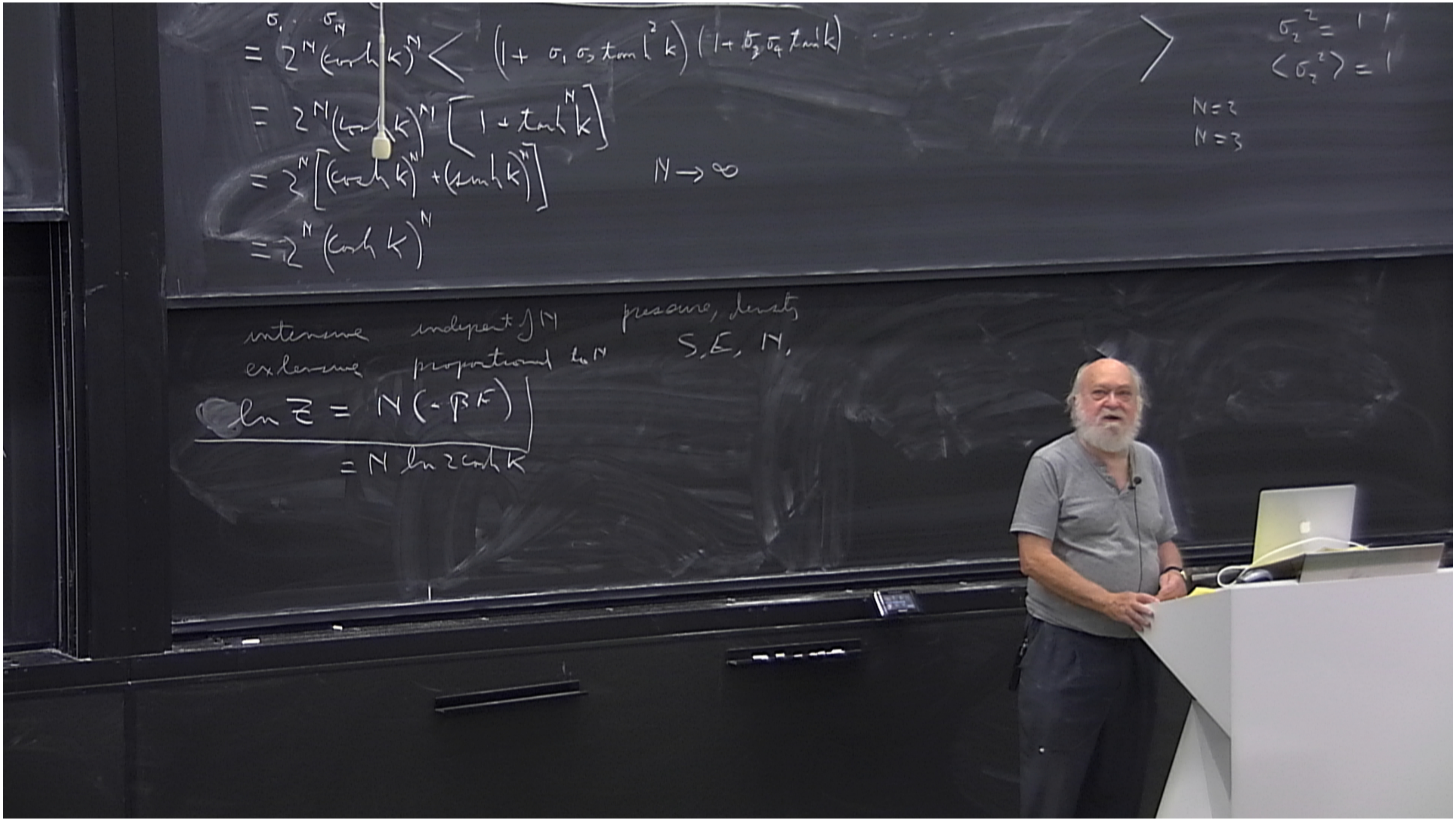


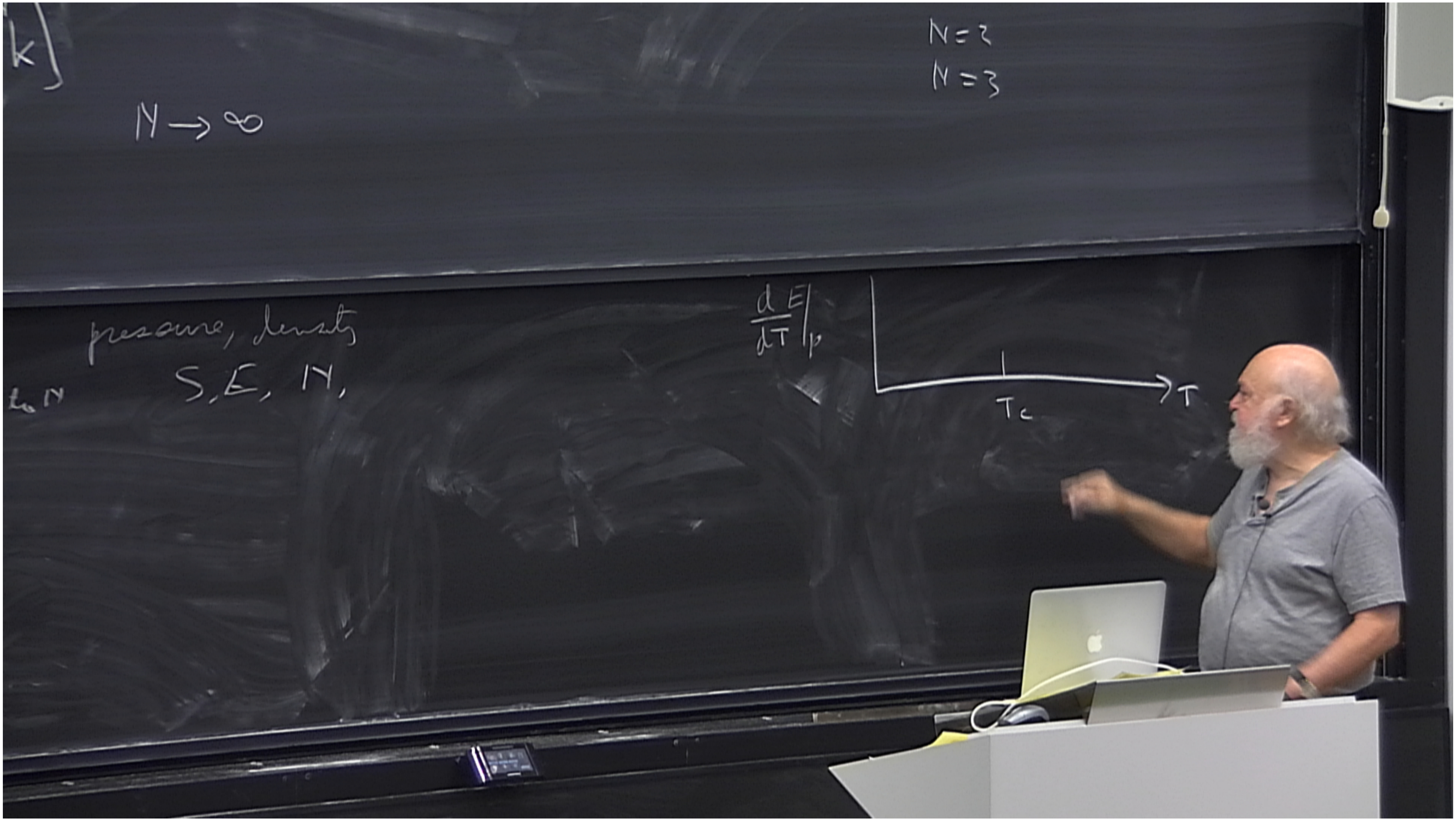
$Z = \sum_{\sigma_1 \ldots \sigma_N} (\cosh K)^N (1+\sigma_1\sigma_2 \tanh K)(1+\sigma_2\sigma_3 \tanh K) \cdots (1+\sigma_N\sigma_1 \tanh K)$
$= 2^N (\cosh K)^N \langle (1+\sigma_1\sigma_3 \tanh^2 K)(1+\sigma_3\sigma_4 \tanh K) \cdots \rangle$
$= 2^N (\cosh K)^N [1 + \tanh^N K]$
$= 2^N [(\cosh K)^N + (\sinh K)^N]$
$\langle \sigma_2 \rangle = 0$
$\sigma_2^2 = 1$
$\langle \sigma_2^2 \rangle = 1$
$N=2$
$N=3$

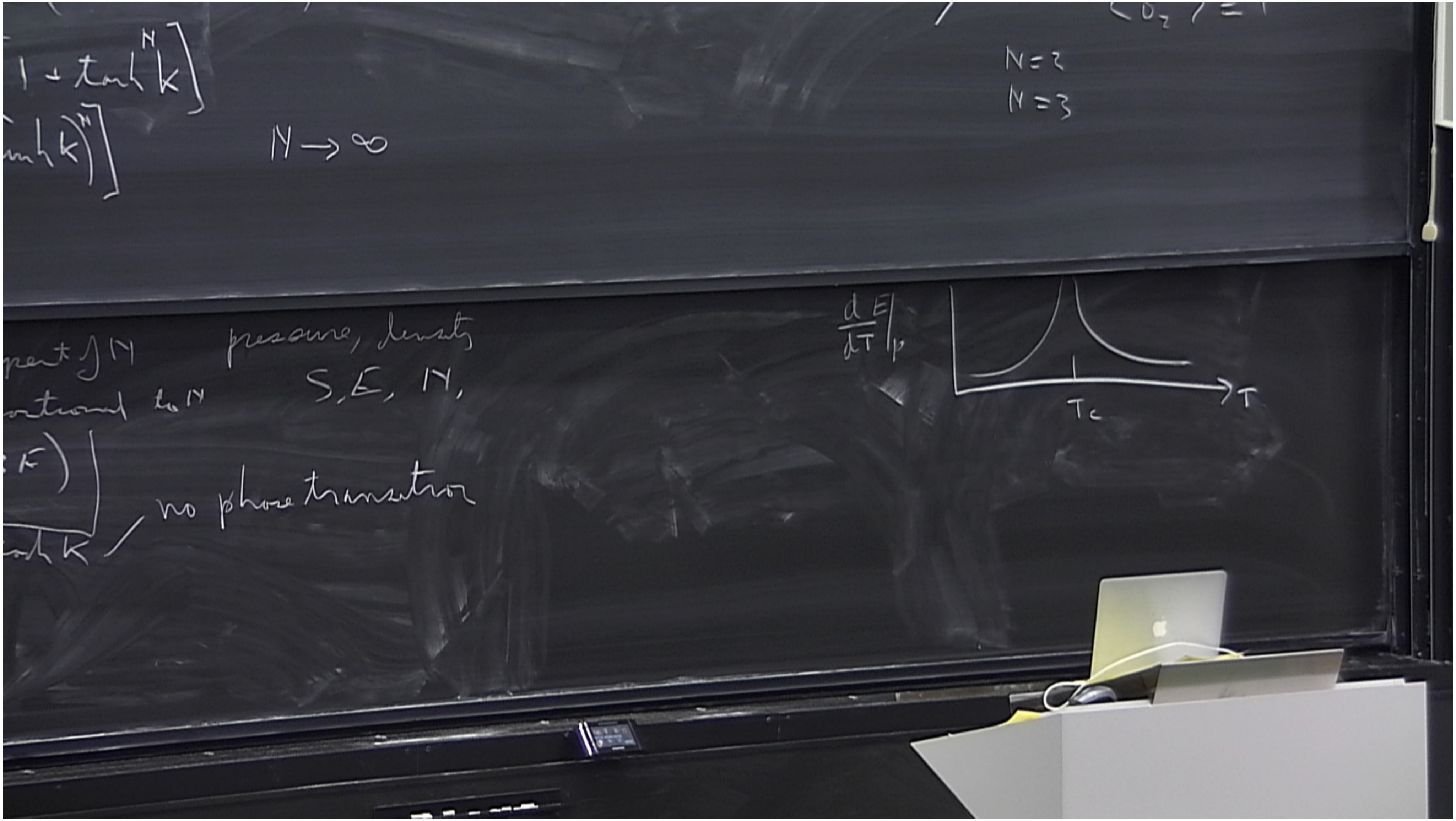

$Z = \sum_{\sigma_1 \cdots \sigma_N} (\cosh K)^N (1+\sigma_1\sigma_2 \tanh K)(1+\sigma_2\sigma_3 \tanh K) \cdots (1+\sigma_N\sigma_1 \tanh K)$
$= 2^N (\cosh K)^N \langle (1+\sigma_1\sigma_3 \tanh^2 K)(1+\sigma_3\sigma_4 \tanh K) \cdots \rangle$
$= 2^N (\cosh K)^N [1+\tanh^N K]$
$= 2^N [(\cosh K)^N + (\sinh K)^N]$
$= 2^N (\cosh K)^N$
$N \to \infty$
$\langle \sigma_2 \rangle = 0$
$\sigma_2^2 = 1$
$\langle \sigma_2^2 \rangle = 1$
$N = 2$
$N = 3$

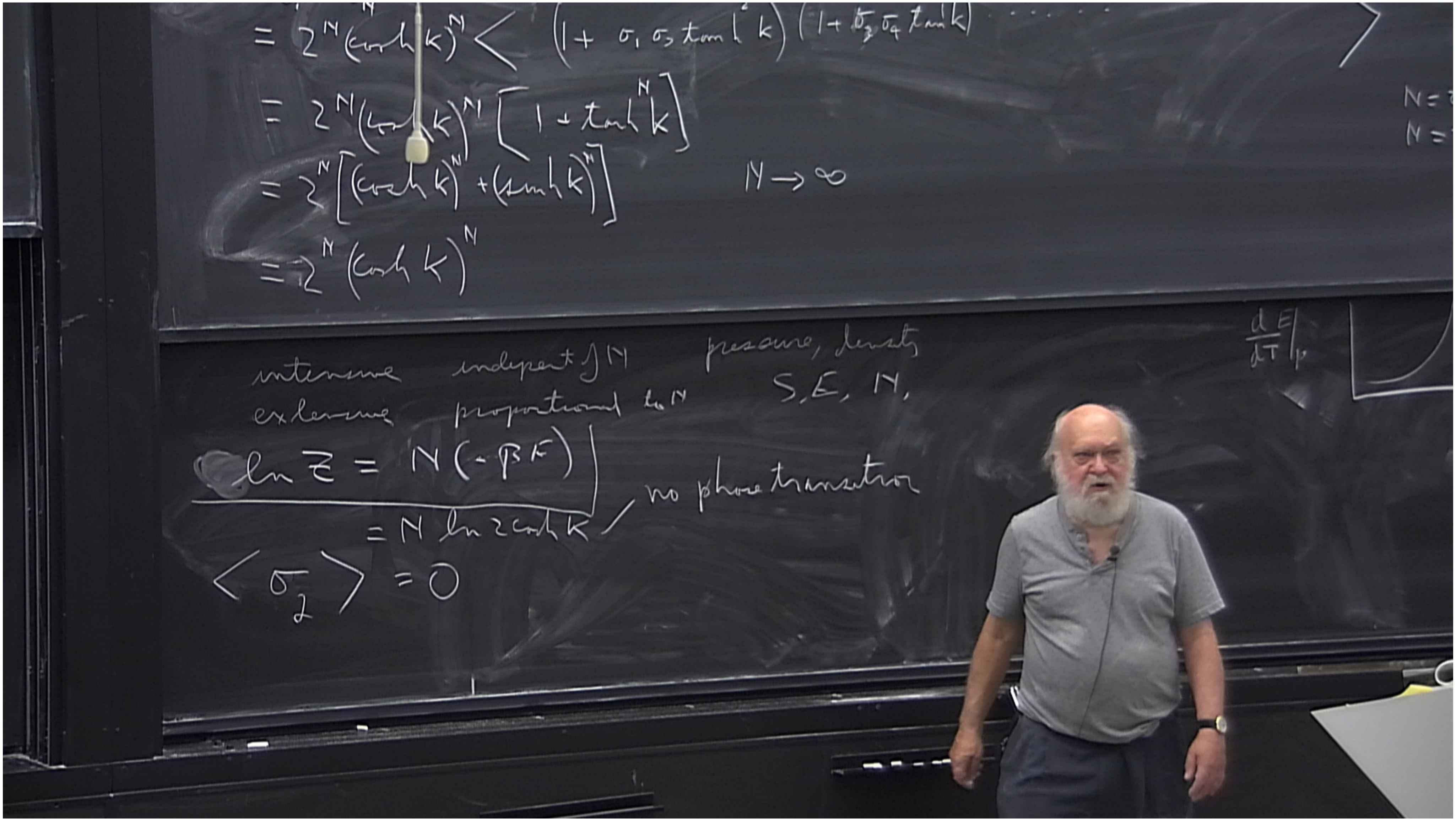

N = 2
N = 3
N → ∞
pressure, density
S, E, N,

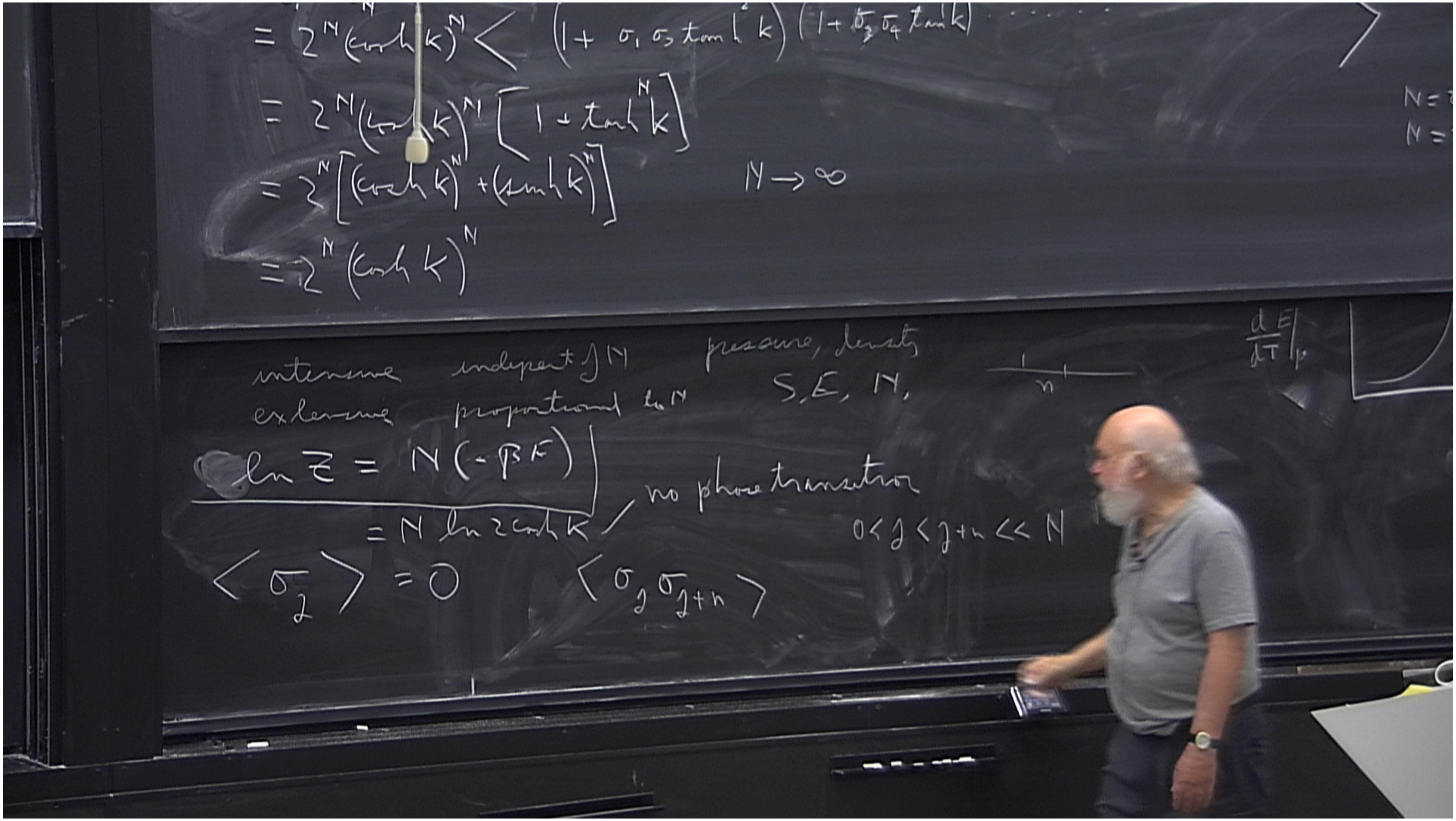

= 2^N (cosh k)^N [1 + tanh^N k]
= 2^N [(cosh k)^N + (sinh k)^N]
N → ∞
= 2^N (cosh k)^N
N = 2
N = 3
⟨σ_2^2⟩ = 1
intensive independent of N pressure, density
extensive proportional to N S, E, N,
ln Z = N(-βF)

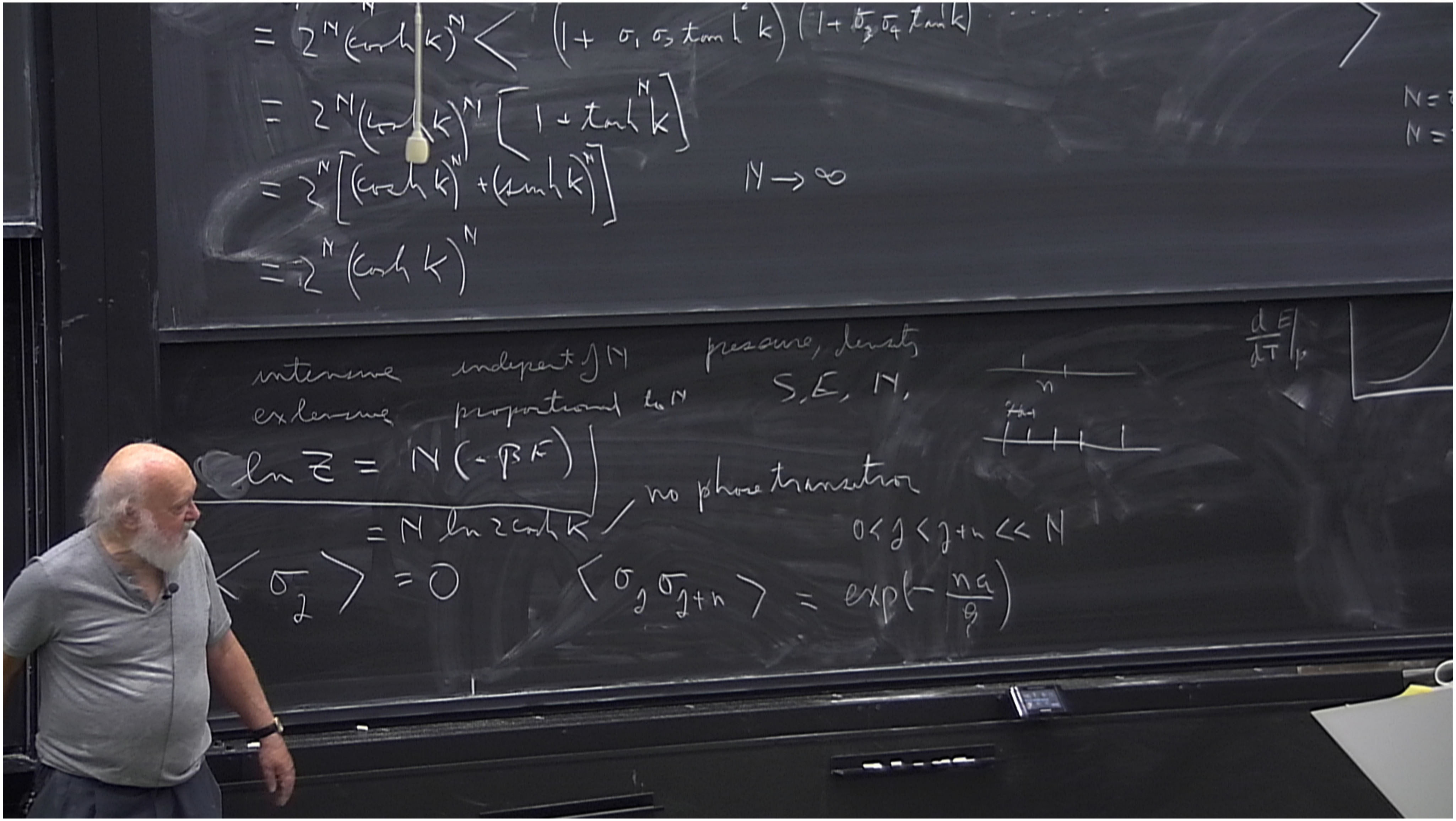

$= 2^N (\cosh k)^N \langle (1 + \sigma_1 \sigma_3 \tanh^2 k)(1 + \sigma_2 \sigma_4 \tanh k) \cdots \rangle$
$\sigma_2^2 = 1$
$\langle \sigma_2^2 \rangle = 1$
$= 2^N (\cosh k)^N [1 + \tanh^N k]$
$N = 2$
$N = 3$
$= 2^N [(\cosh k)^N + (\sinh k)^N]$
$N \to \infty$
$= 2^N (\cosh k)^N$
intensive independ of N
pressure, density
extensive proportional to N
S, E, N,
$\ln Z = N(-\beta F)$
$= N \ln 2 \cosh K$

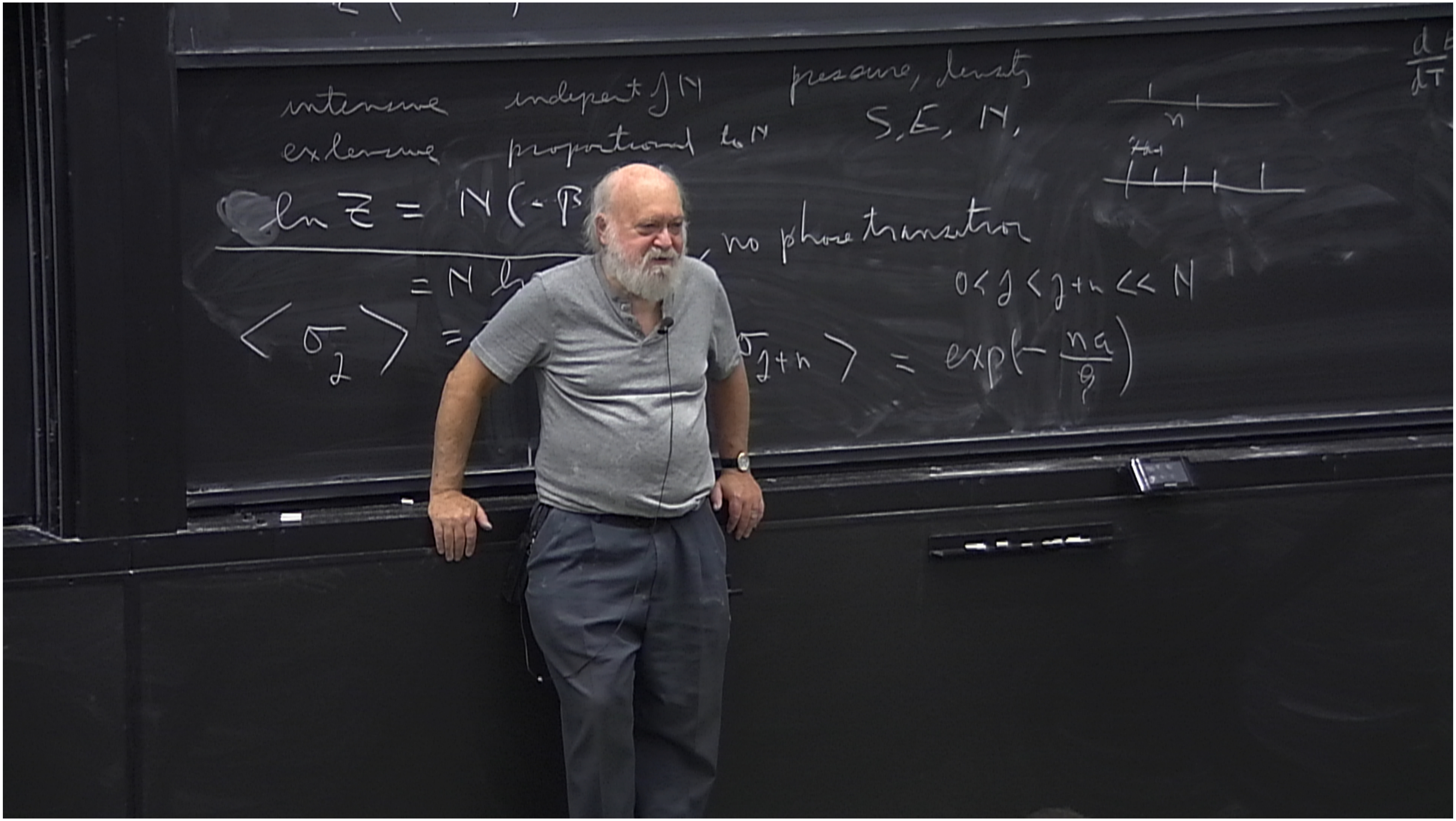

N=2
N=3
N → ∞
pressure, density
S, E, N,
dE/dT |p
Tc
T


N = 2
N = 3
N → ∞
pressure, density
S, E, N,
no phase transition
dE/dT |p
Tc
T


= 2^N (cosh k)^N < (1 + σ1 σ3 tanh^2 k)(1 + σ3 σ4 tanh k) ...... >
= 2^N (cosh k)^N [1 + tanh^N k]
= 2^N [(cosh k)^N + (sinh k)^N]
N → ∞
= 2^N (cosh k)^N
intensive independent of N
pressure, density
extensive proportional to N
S, E, N,
ln Z = N(-βF)
= N ln 2cosh k
no phase transition
< σ_2 > = 0
dE/dT |_p


= 2^N (cosh k)^N < (1 + σ1 σ3 tanh² k)(1 + σ2 σ4 tanh k) ...... >
= 2^N (cosh k)^N [1 + tanh^N k]
= 2^N [(cosh k)^N + (sinh k)^N]
N → ∞
= 2^N (cosh k)^N
intensive independent of N pressure, density
extensive proportional to N S, E, N,
ln Z = N(-βF)
= N ln 2cosh k
no phase transition
0 < j < j+n << N
< σj > = 0
< σj σj+n >


= 2^N (cosh k)^N < (1 + σ1 σ3 tanh² k)(1 + σ3 σ4 tanh k) ...... >
= 2^N (cosh k)^N [1 + tanh^N k]
= 2^N [(cosh k)^N + (sinh k)^N]
N → ∞
= 2^N (cosh k)^N
intensive independent of N pressure, density
extensive proportional to N S, E, N,
ln Z = N(-βF)
= N ln 2cosh k
no phase transition
0 < j < j+n << N
< σj > = 0
< σj σj+n > = exp(- na/θ)


intensive independent of N pressure, density
extensive proportional to N S, E, N,
ln Z = N(-β
no phase transition
= N ln
0 < j < j+n << N
⟨σ_j⟩ =
σ_{j+n}⟩ = exp(− na/ξ)