Title: Statistical Mechanics - Lecture 1

Date: Oct 03, 2011 10:30 AM

URL: http://pirsa.org/11100025

Abstract:

Where do we come from?

Undergraduate Institution:

Level of Instruction:

Different in US, Britain, Europe, Canada

Stat Mech Courses:

Thermo Course:

Concepts which specifically belong to statistical physics: P.W. Anderson "More IS Different" Physics statement, Math statement

Not in few particle quantum mechanics or in Classical Mechanics
Physics of Large systems is different from physics of small ones
19C early 20C getting few degrees of freedom straight
later looking for effects of infinities N--> infinity, t-->infinity

- Temperature
- Butterflies-Lorenz

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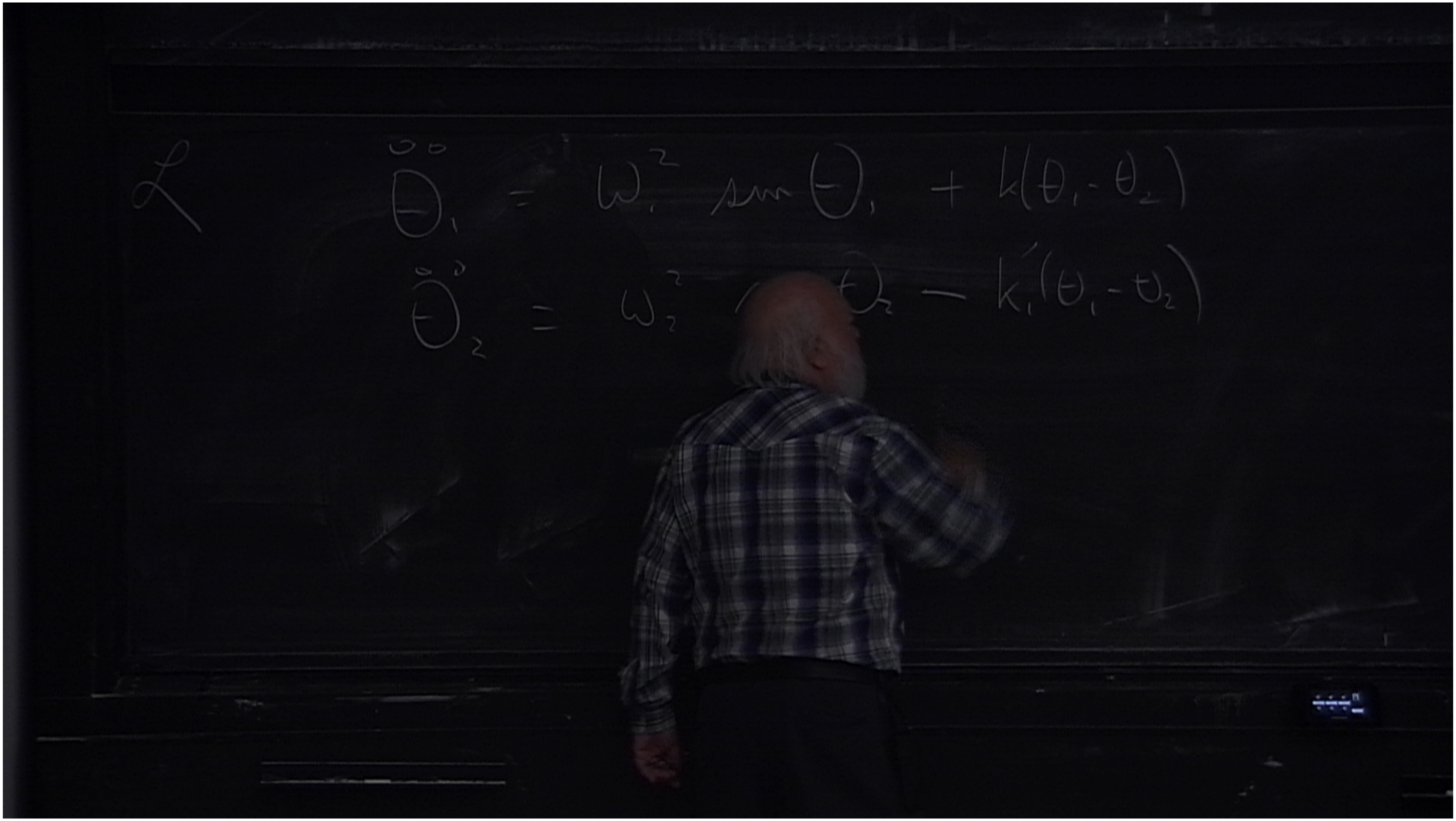
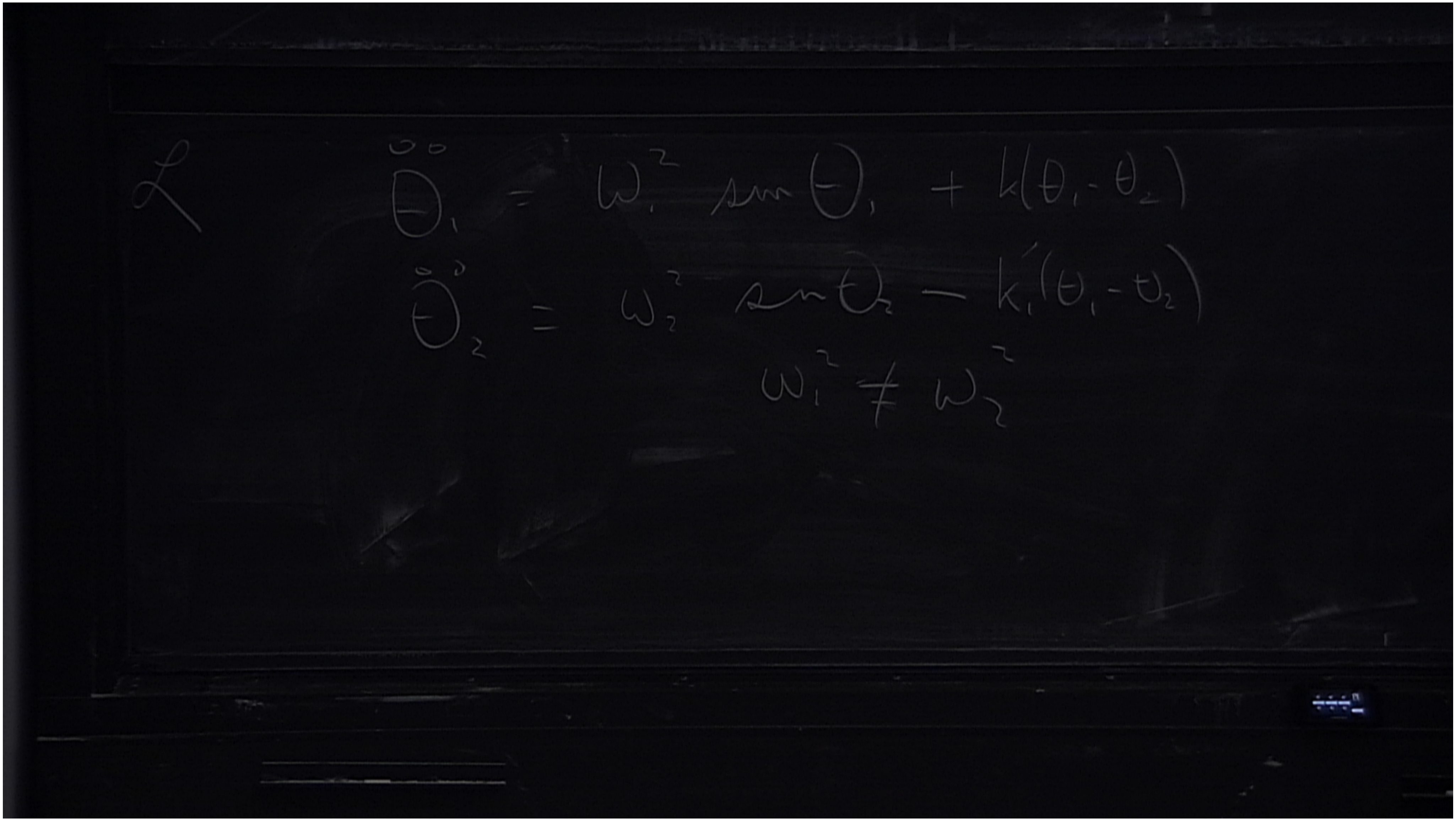
Example of Moire is Different:
Christian Huygens clock synchronization

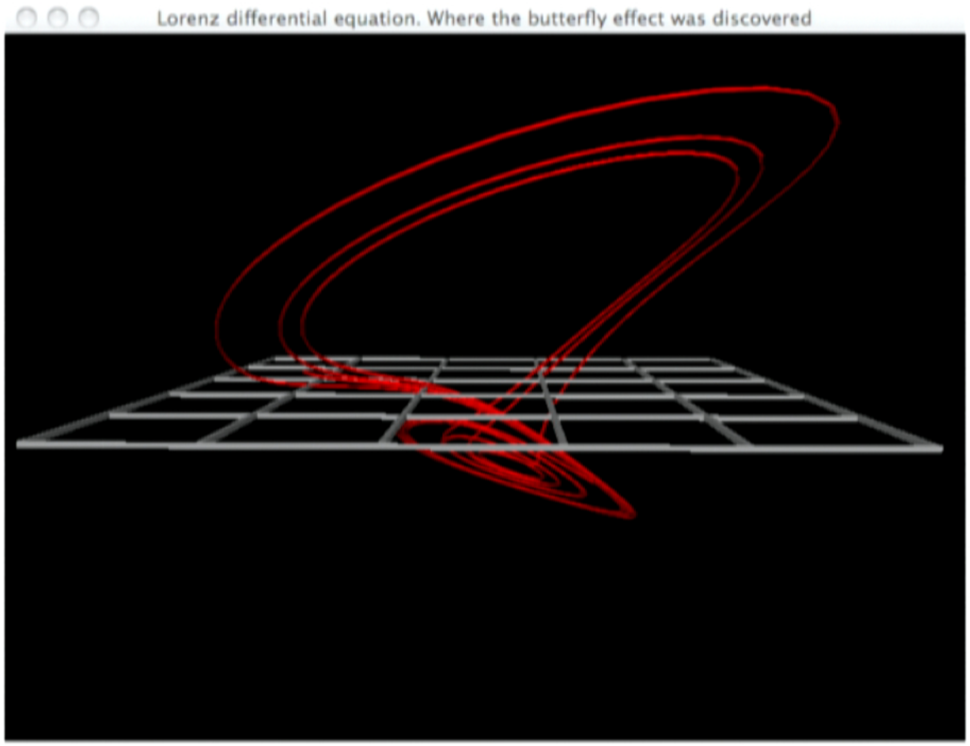



$\mathcal{L}$
$\ddot{\Theta}_1 = \omega_1^2 \sin\Theta_1 + k(\theta_1 - \theta_2)$
$\ddot{\Theta}_2 = \omega_2^2 \sin\theta_2 - k_1'(\theta_1 - \theta_2)$
$\omega_1^2 \neq \omega_2^2$

Butterfly effect--go for app

Lorenz, Edward N. (March 1963). "Deterministic Nonperiodic Flow". Journal of the Atmospheric Sciences 20 (2): 130–141. One of the most important scientific papers in the 1960s. Prior to this people believed that the behavior of physical systems was essentially predictable.

An orbit:
after Ed. Lorenz



How did we learn about the butterfly effect
Here I express things as an algorithm

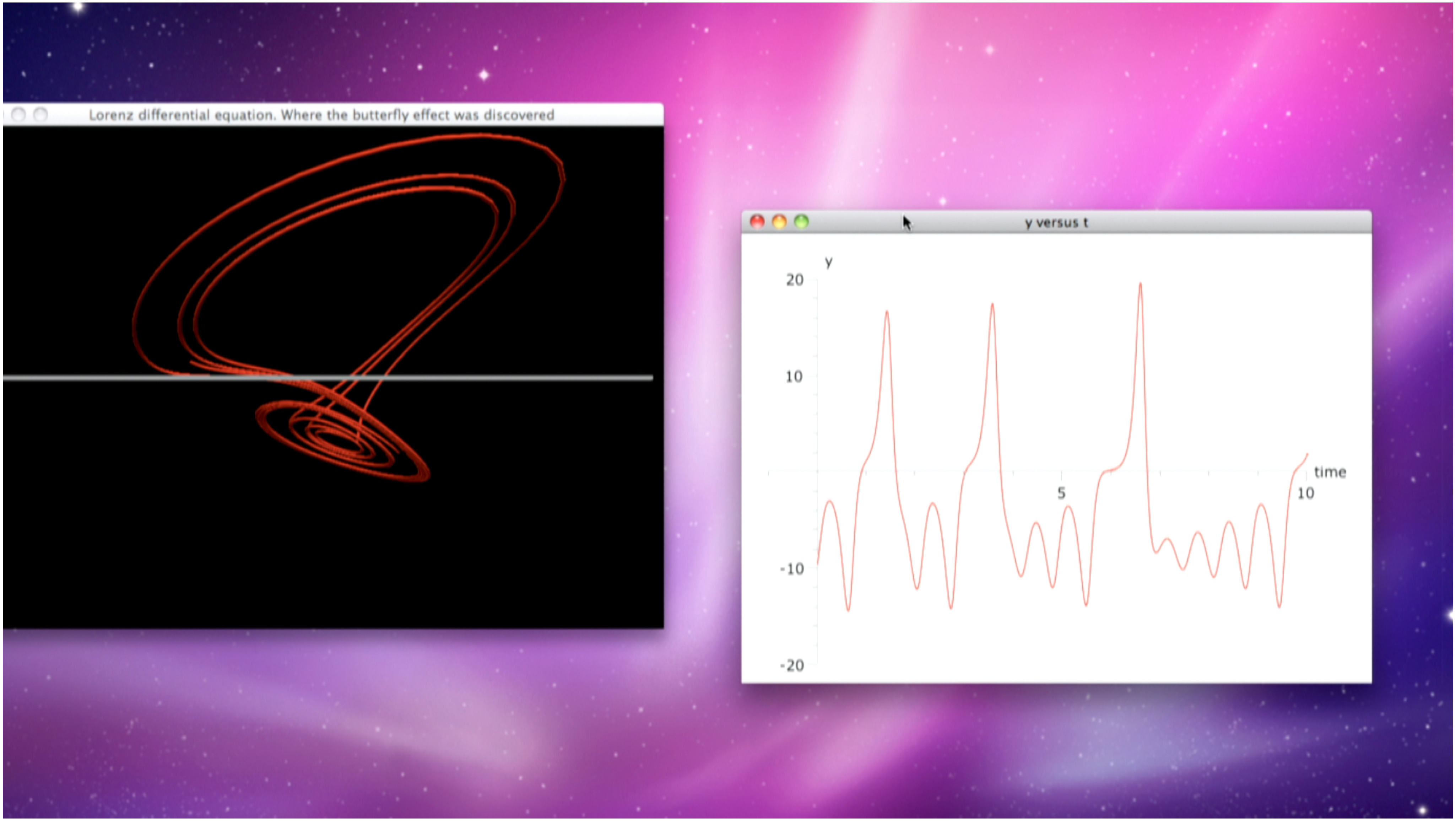
Ed Lorenz put together a very simple model of the weather: Three variables r=(r.x,r.y., r.z) a simple differential equation:

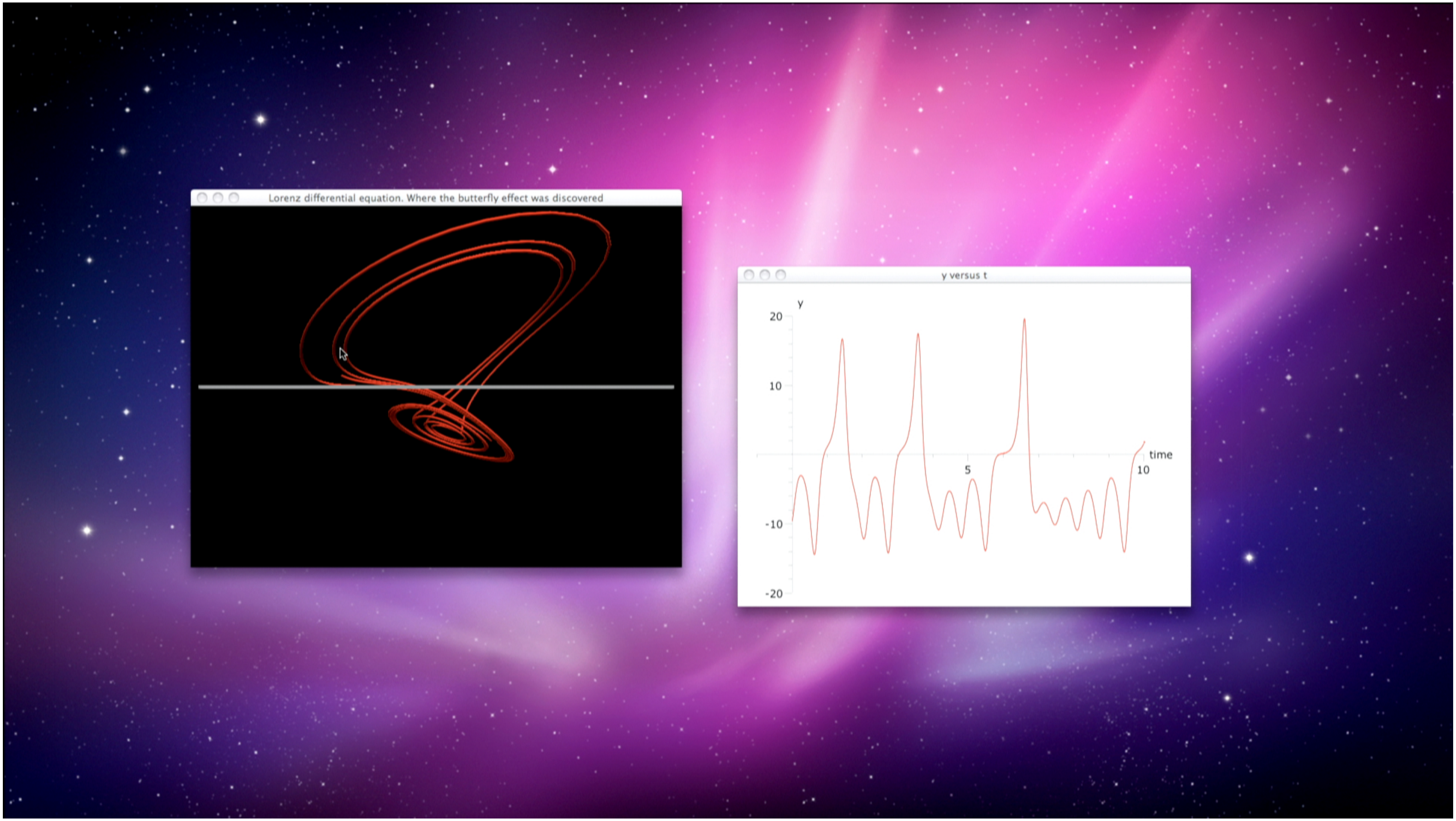
```
drdt = vector( - 8.0/3*r.x          + r.y*r.z,
                      - 10*r.y +   10*r.z,
               -  r.y*r.x + 28*r.y -     r.z );
r = r + drdt*dt
t=t+dt
```

Loop through this tons of times and you get

```
initial values:
dy=.01
maxt=20
dt = 0.01
r = vector(35, -10, -7) #red
r1=vector(35, -10+dy, -7)   #green
```

and this was called the butterfly effect


Lorenz differential equation. Where the butterfly effect was discovered
y versus t
y
20
10
-10
-20
5
10
time


Lorenz differential equation. Where the butterfly effect was discovered
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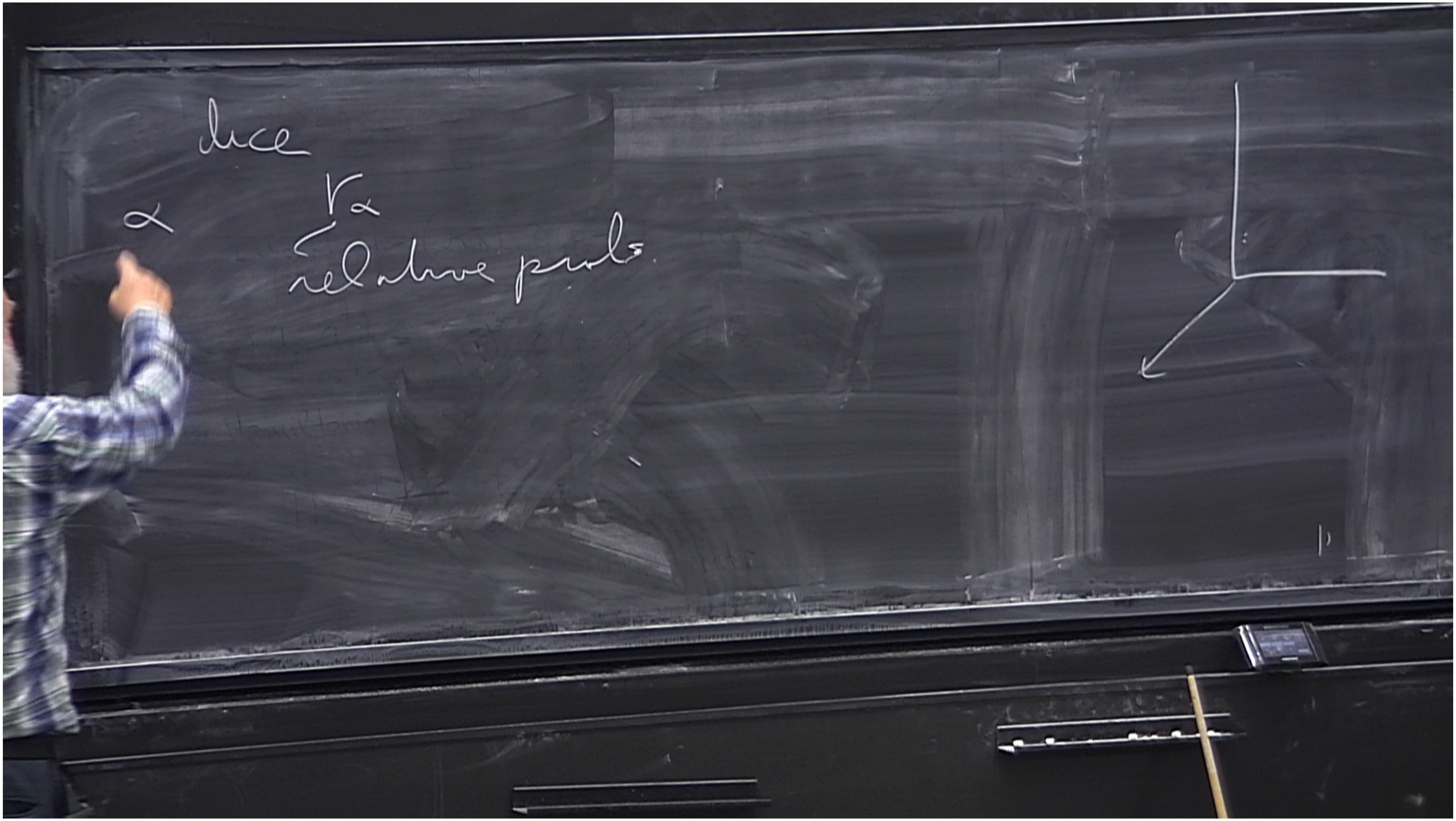
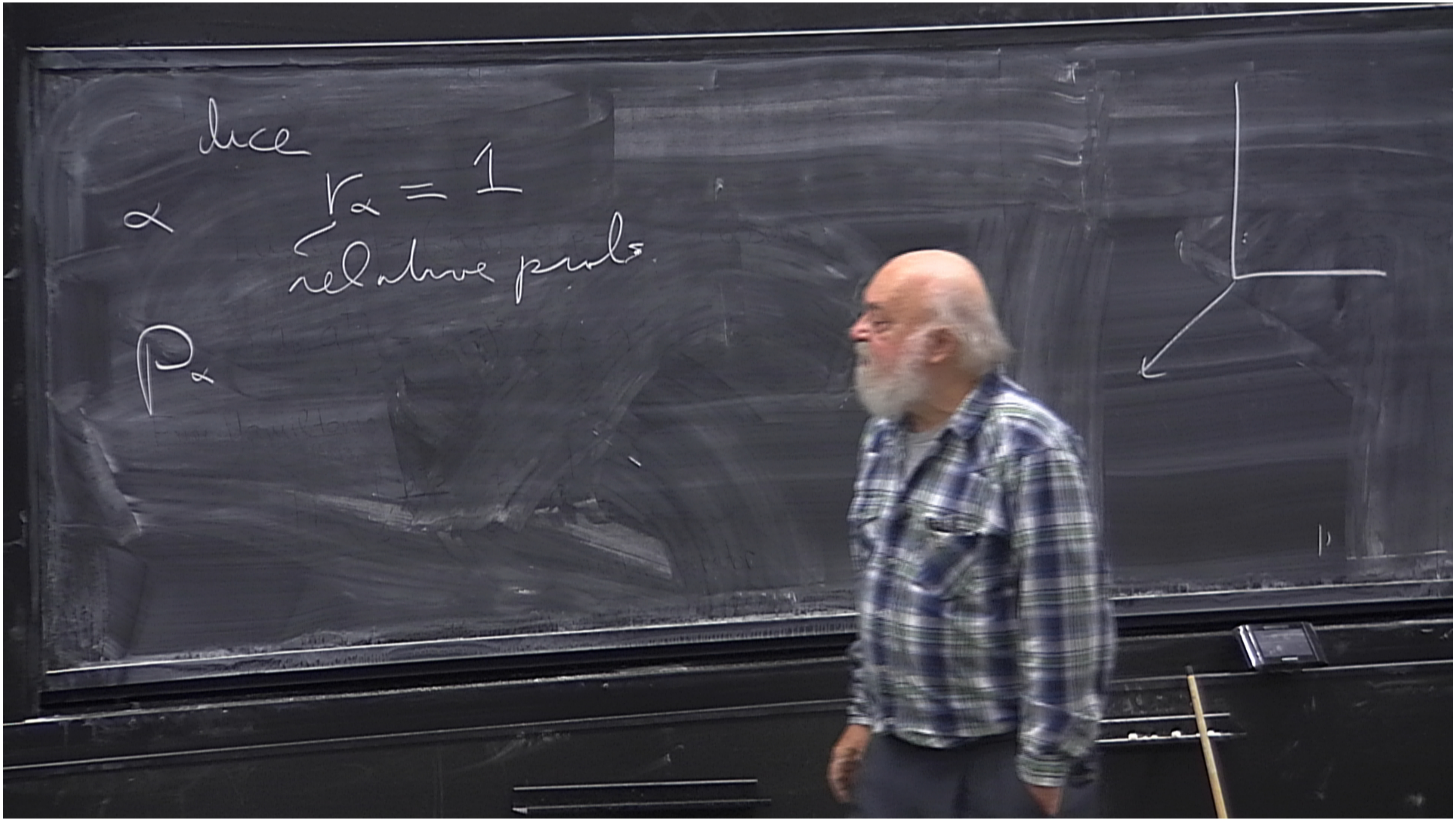
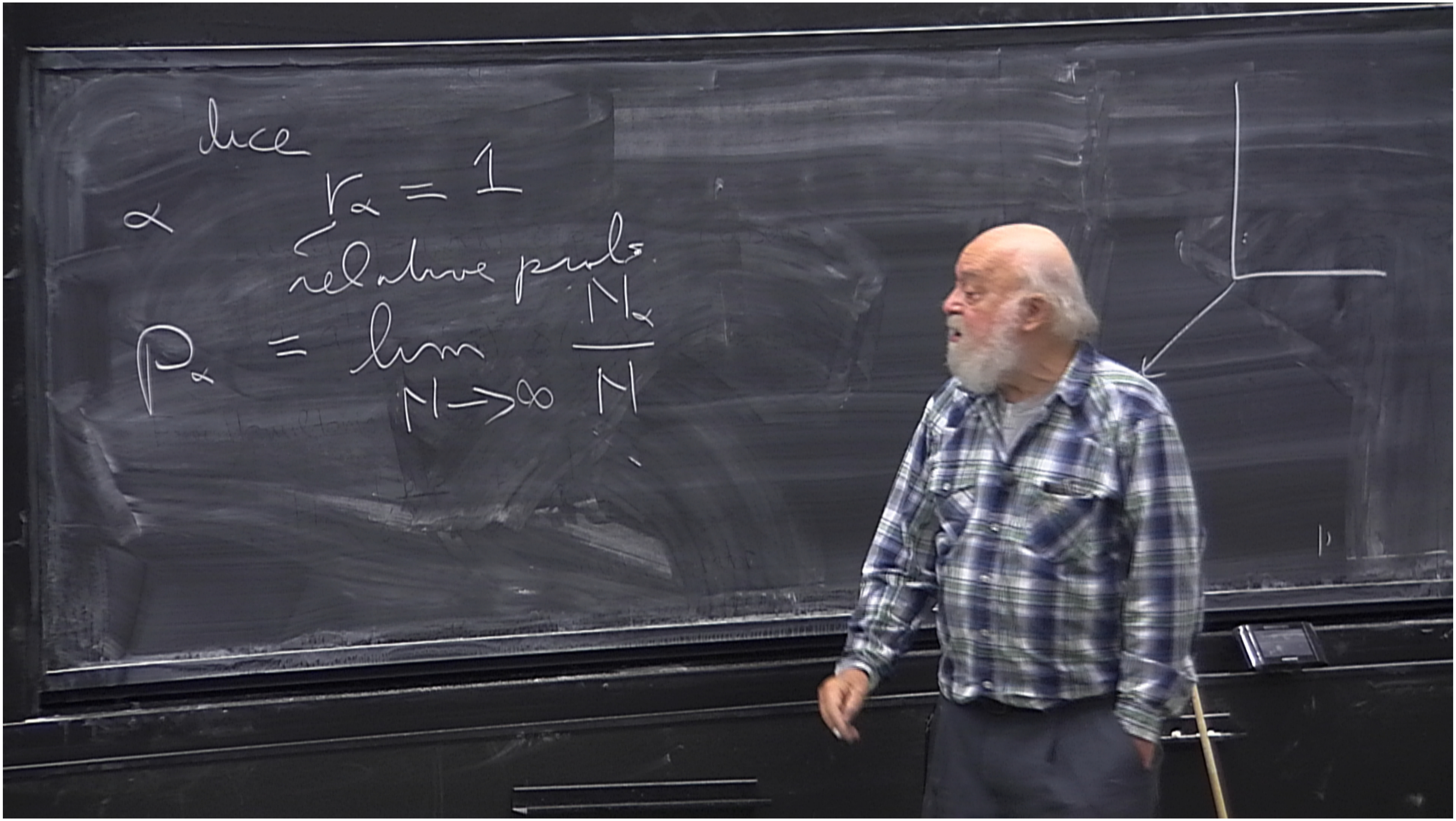
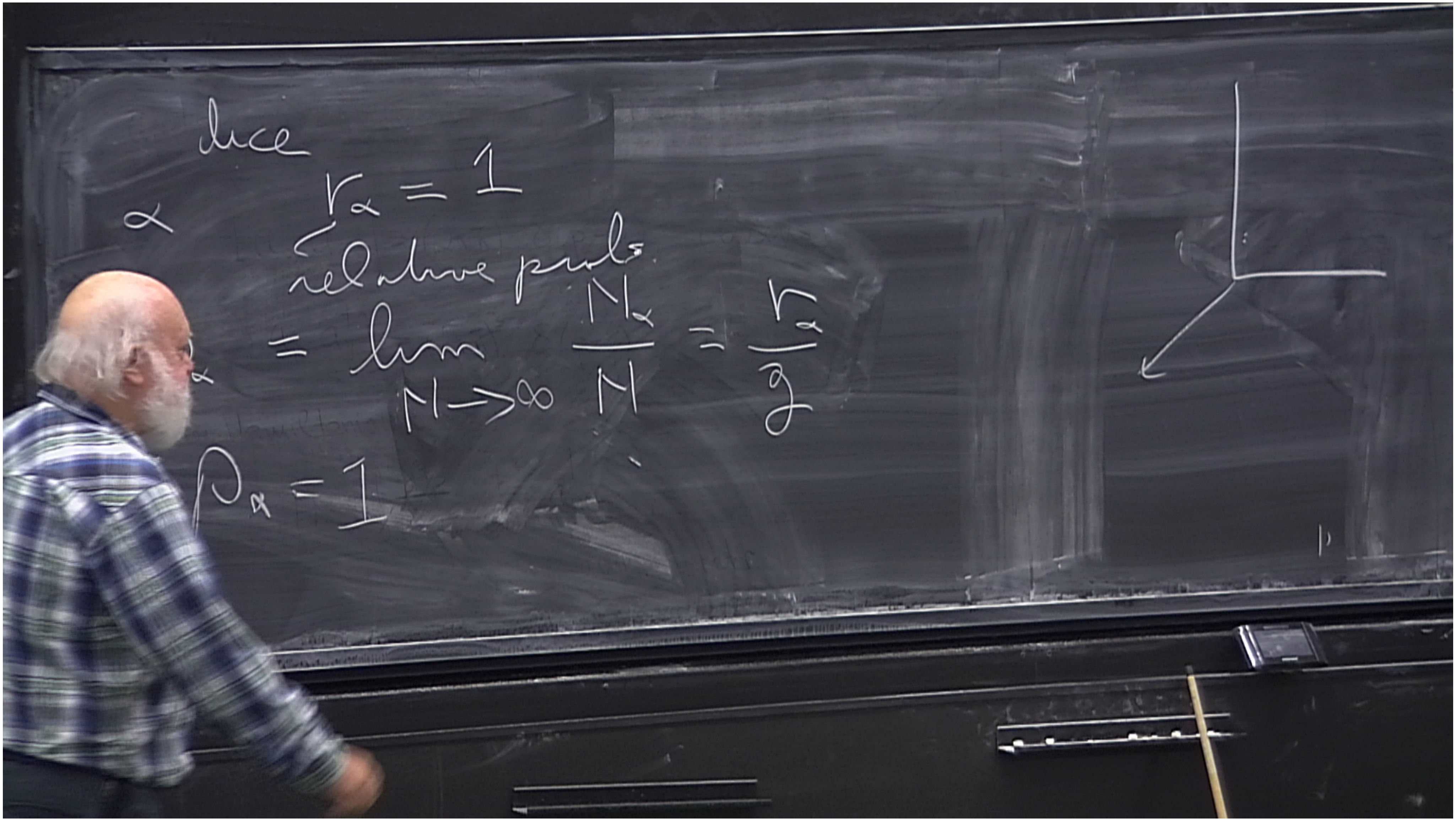
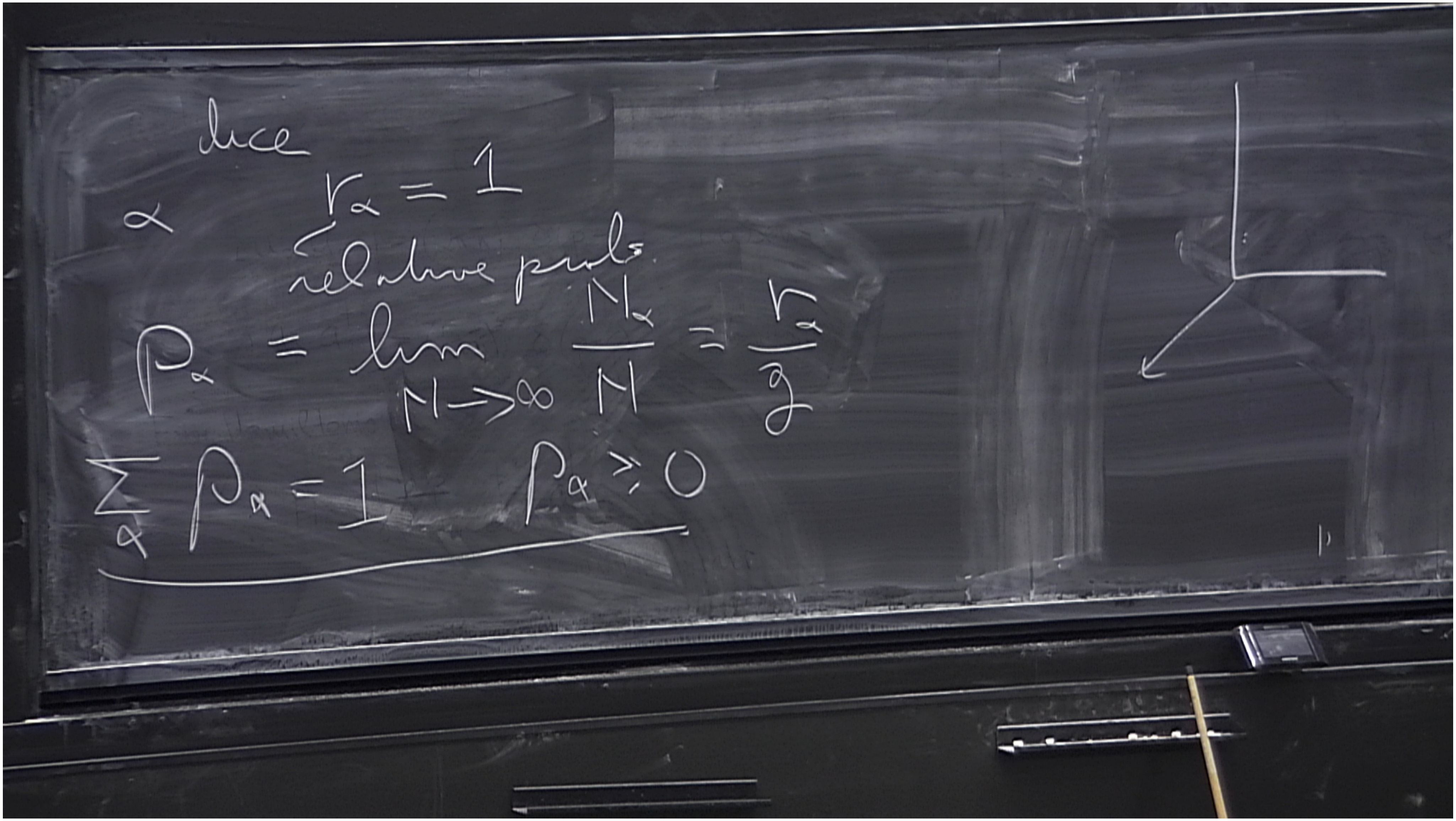
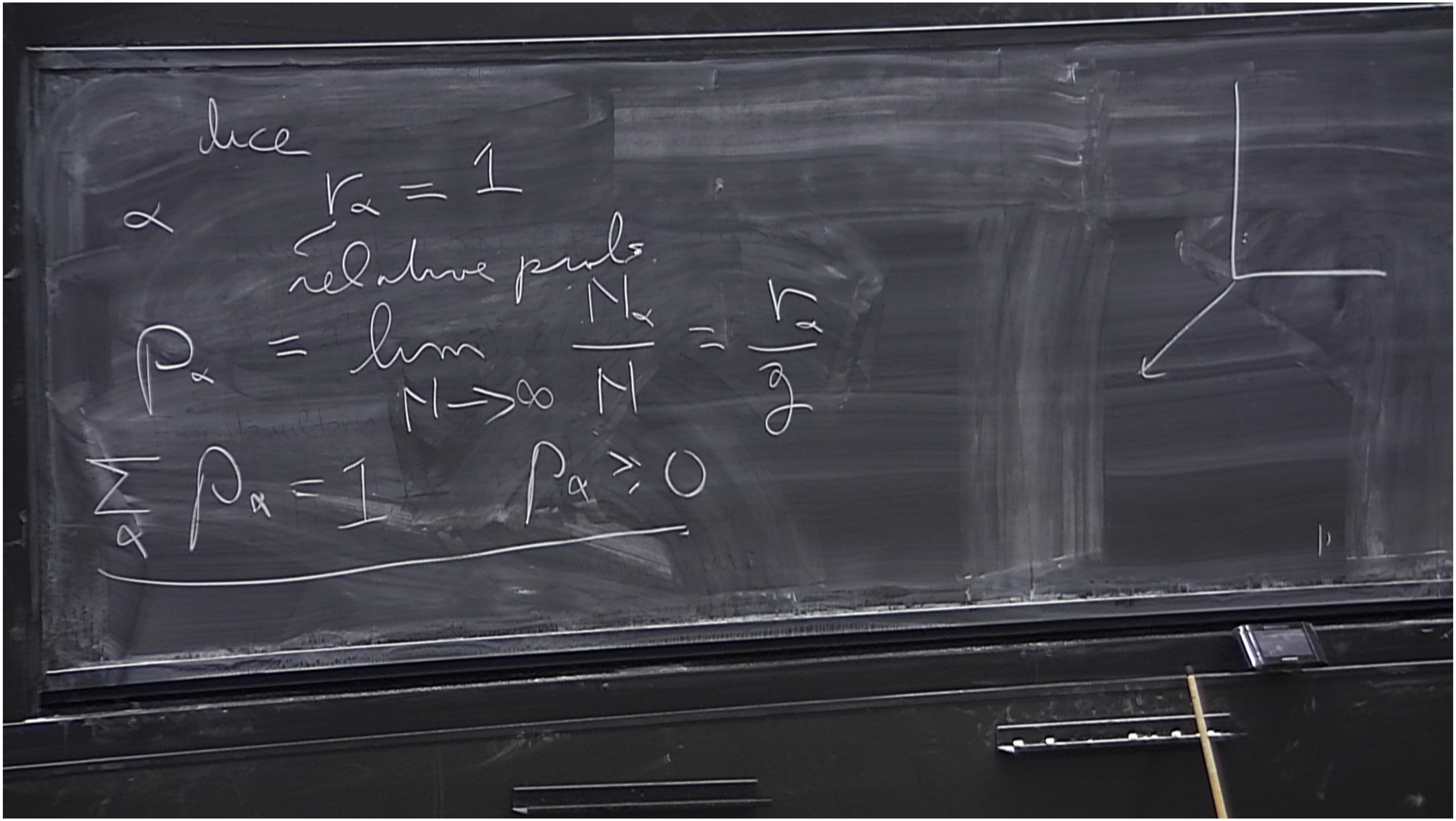

relative


$r_\alpha = 1$
relative probs


dice
α
r_α = 1
relative prob
P_α = lim N_α / N
N → ∞


rα = 1
ρα = 1


dice
α
$r_\alpha = 1$
relative prob
$\rho_\alpha = \lim_{N \to \infty} \frac{N_\alpha}{N} = \frac{r_\alpha}{g}$
$\sum_\alpha \rho_\alpha = 1$
$\rho_\alpha \geq 0$


dice
$r_\alpha = 1$
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$\sum_\alpha \rho_\alpha = 1$
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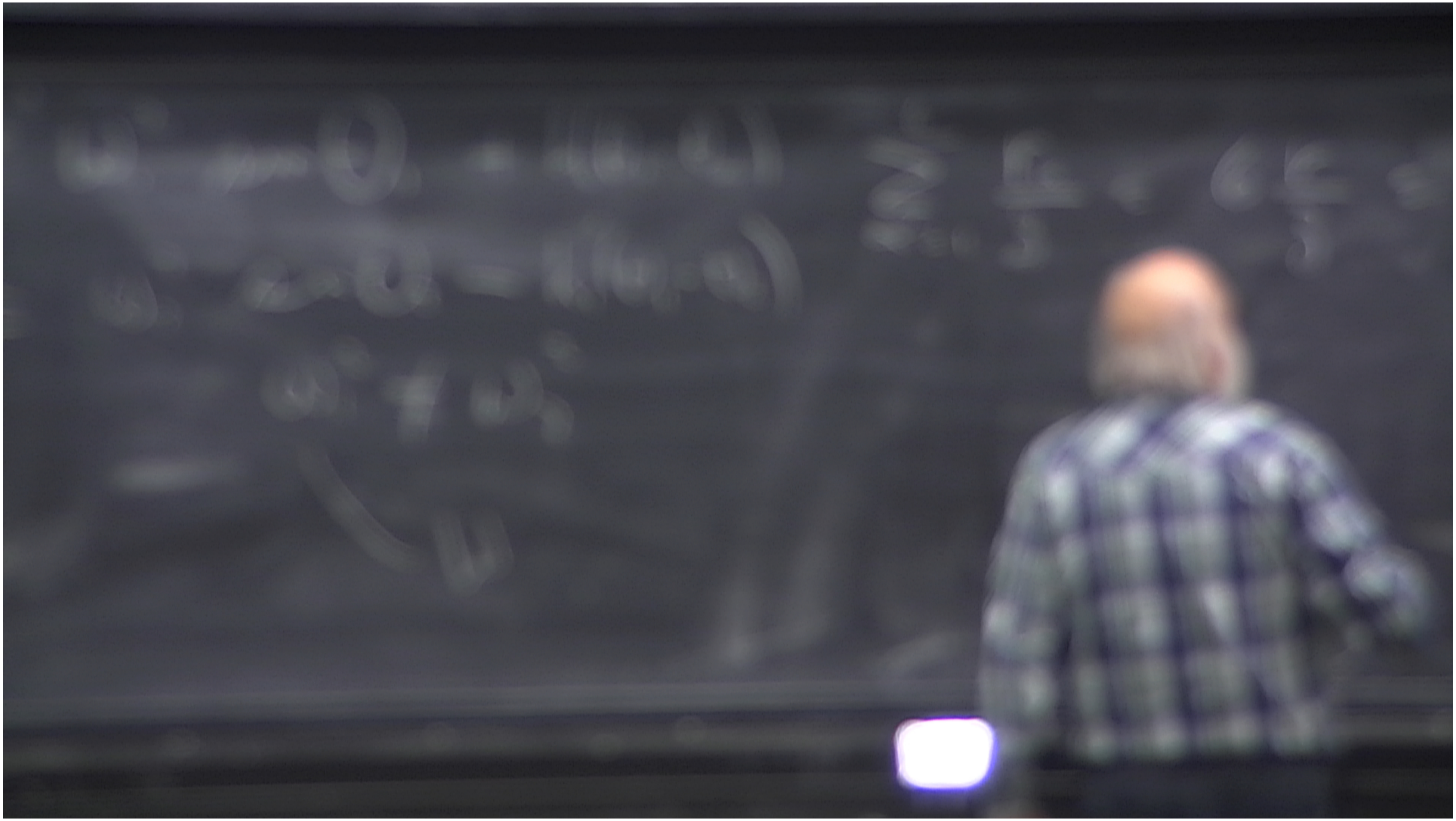
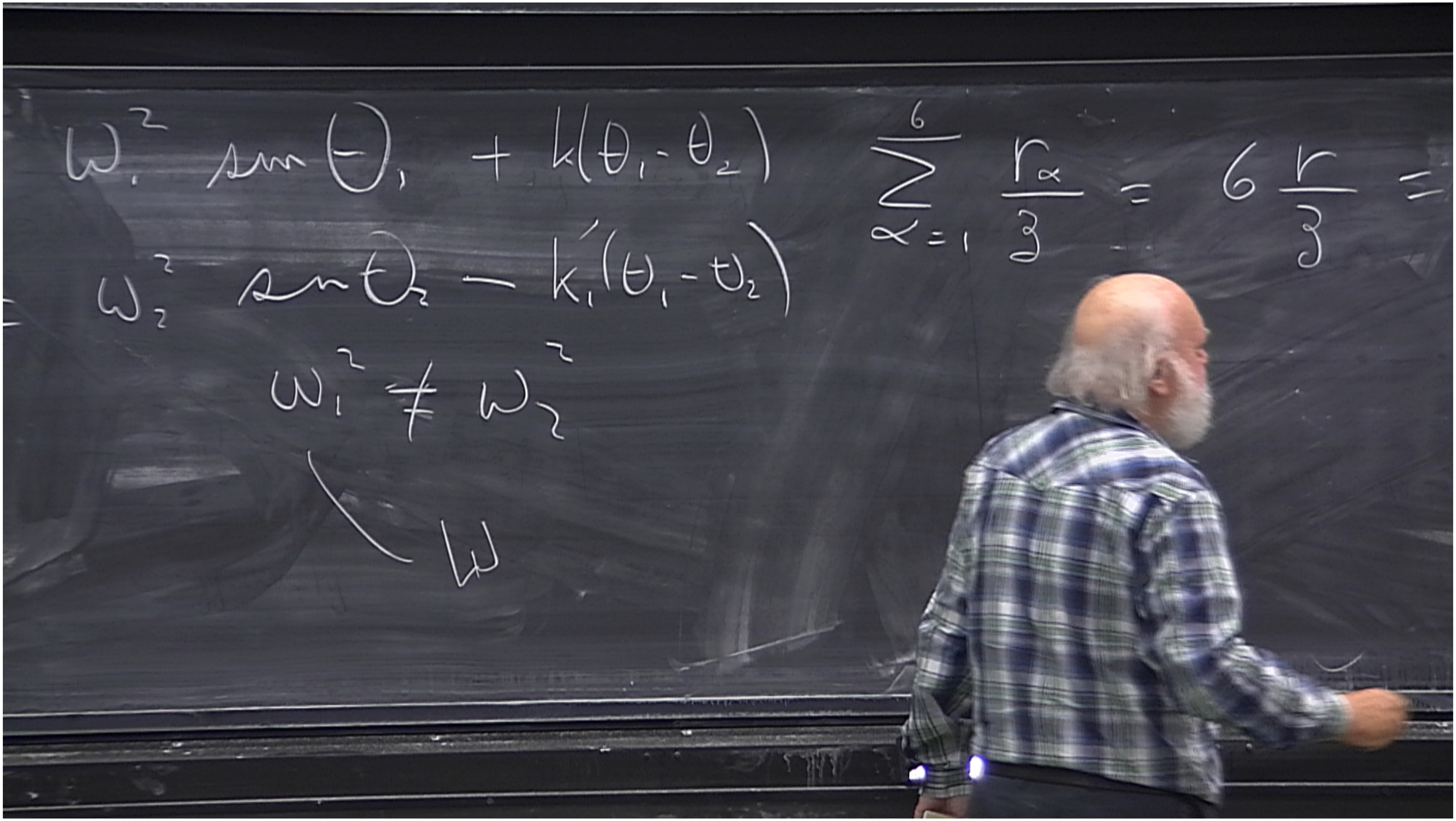
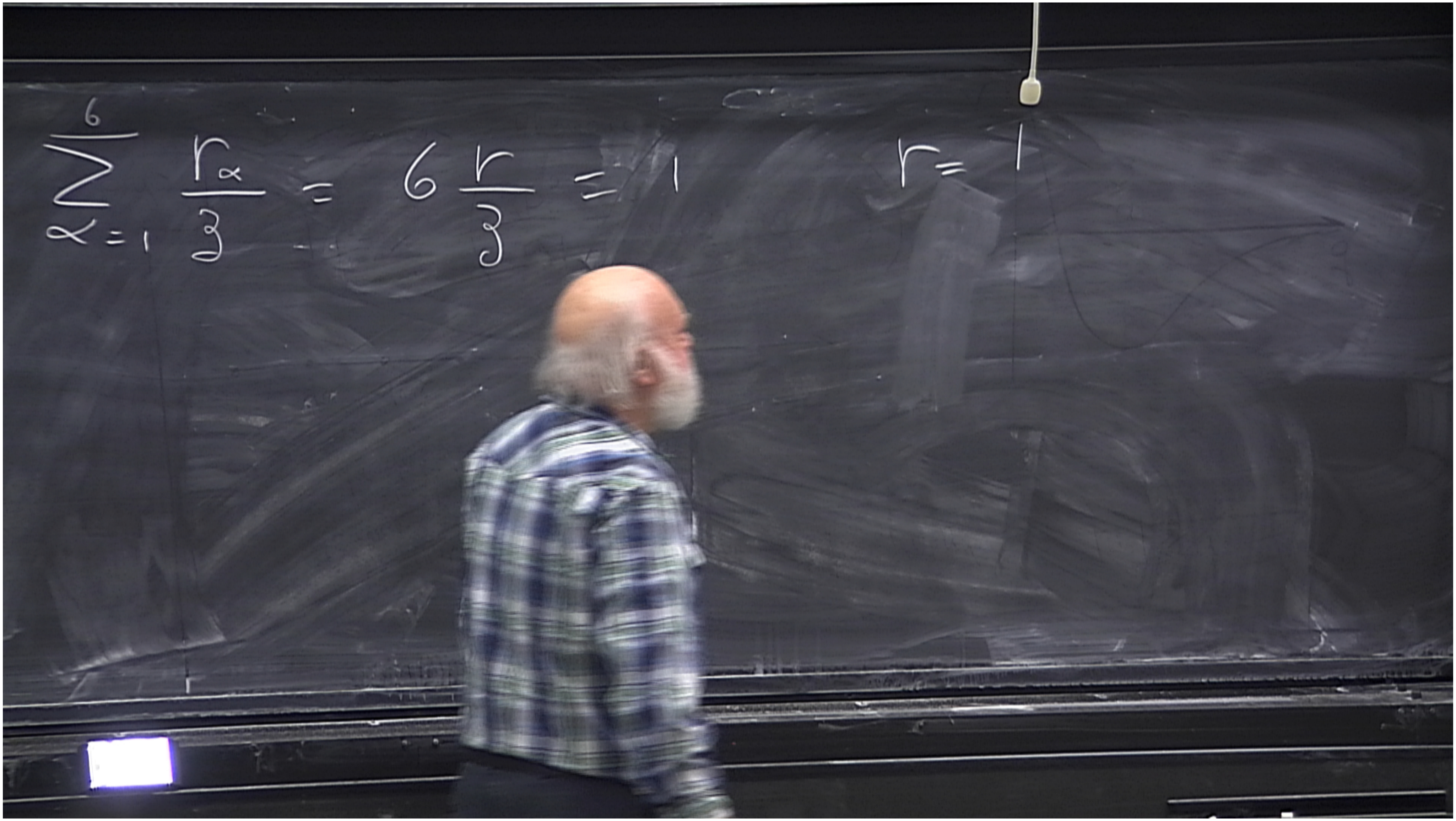
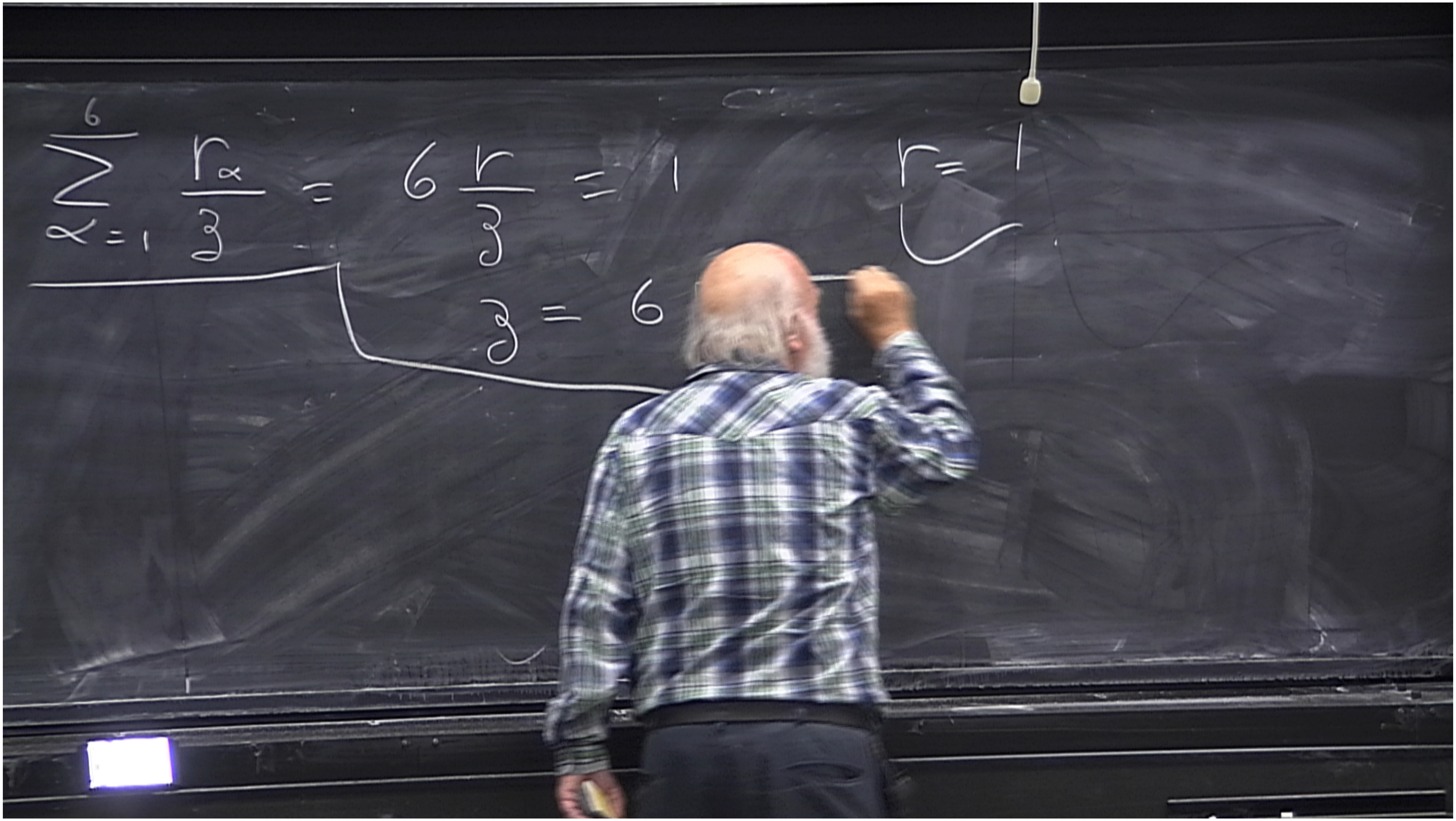
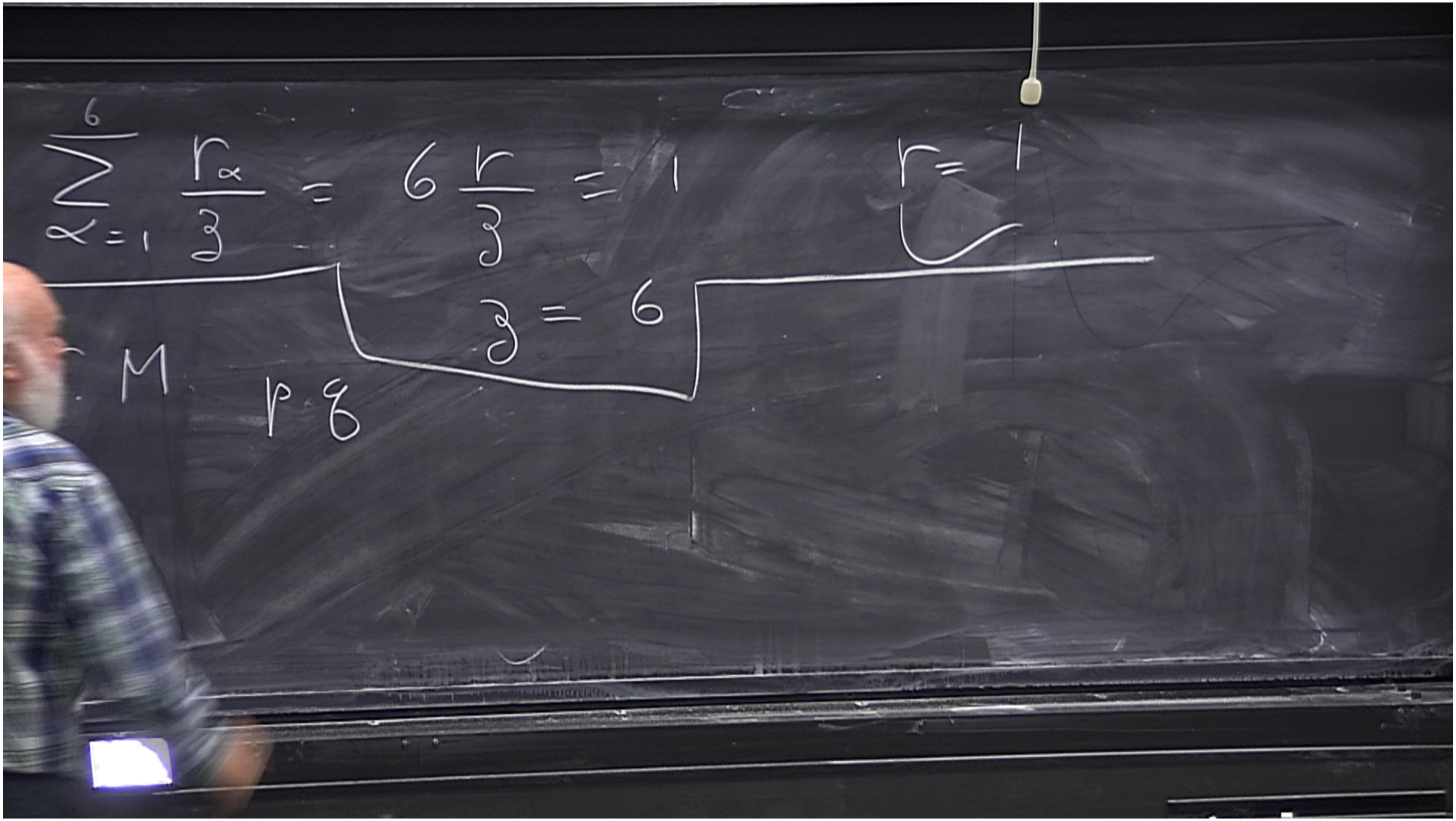


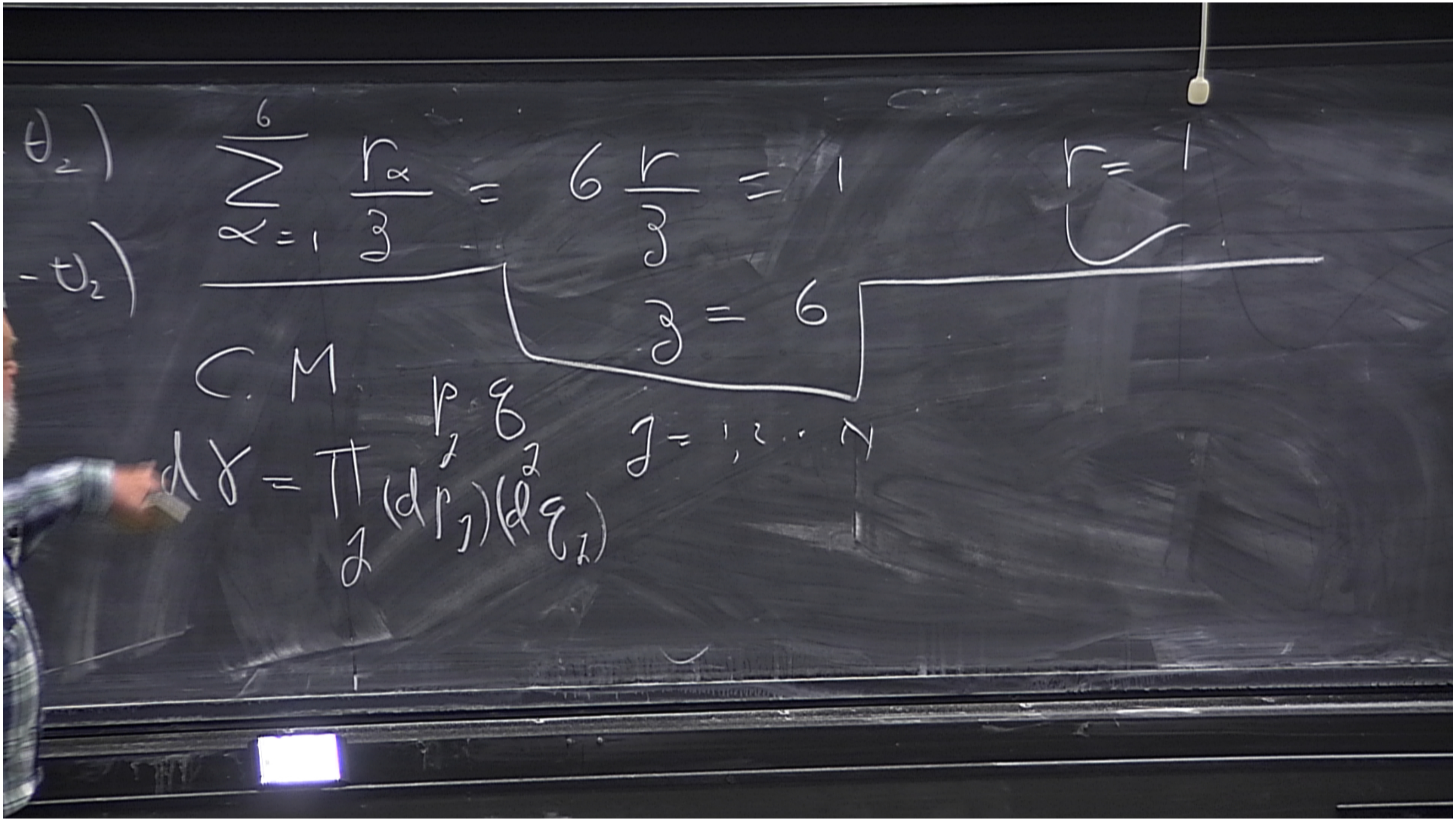

$\omega_1^2 \sin\theta_1 + k(\theta_1 - \theta_2)$
$\omega_2^2 \sin\theta_2 - k_1'(\theta_1 - \theta_2)$
$\omega_1^2 \neq \omega_2^2$
ω
$\sum_{\alpha=1}^{6} \frac{r_\alpha}{3} = 6\frac{r}{3} =$


$\sum_{\alpha=1}^{6} \frac{r_\alpha}{3} = 6\frac{r}{3} = 1$
$r = 1$

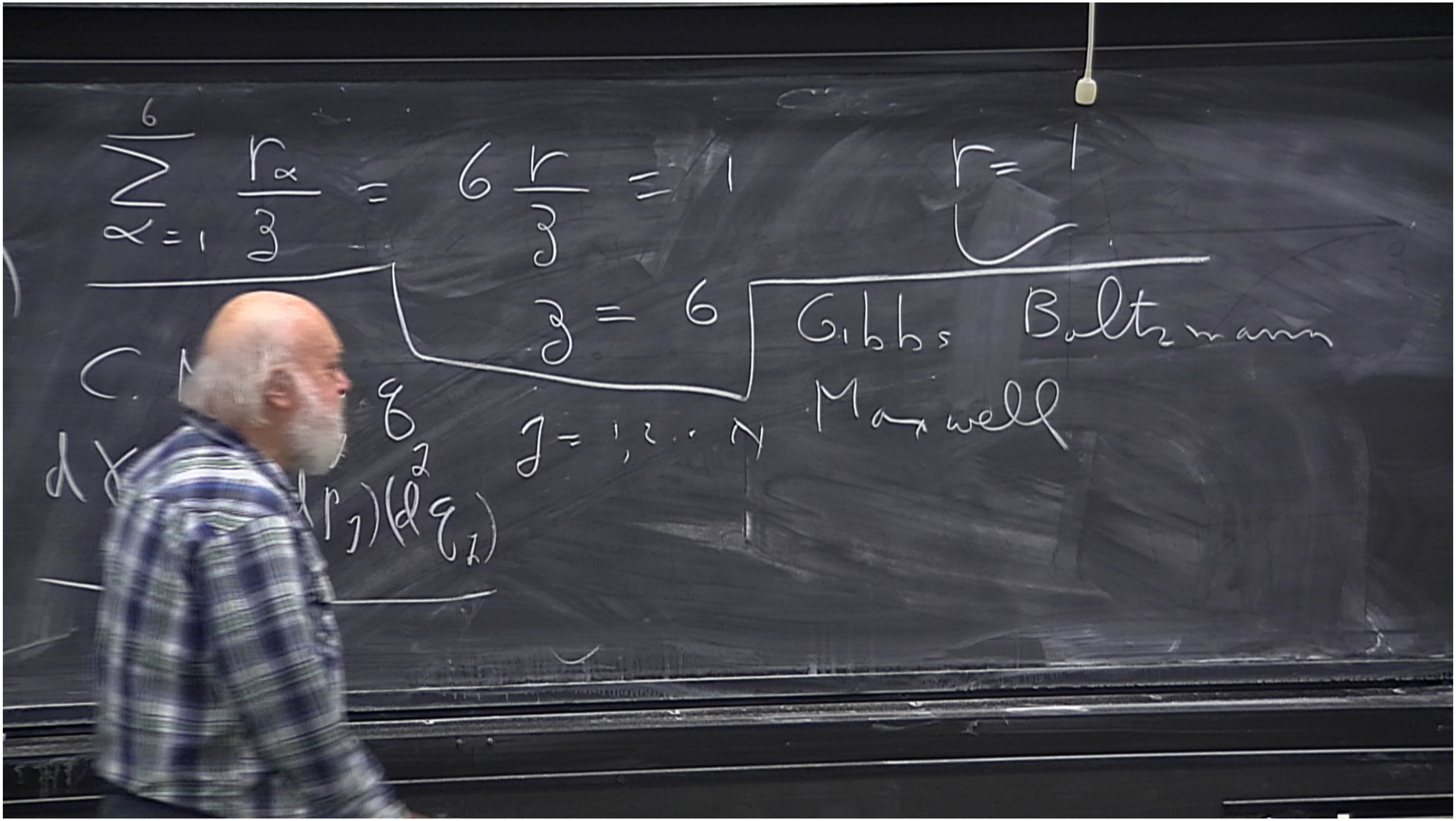


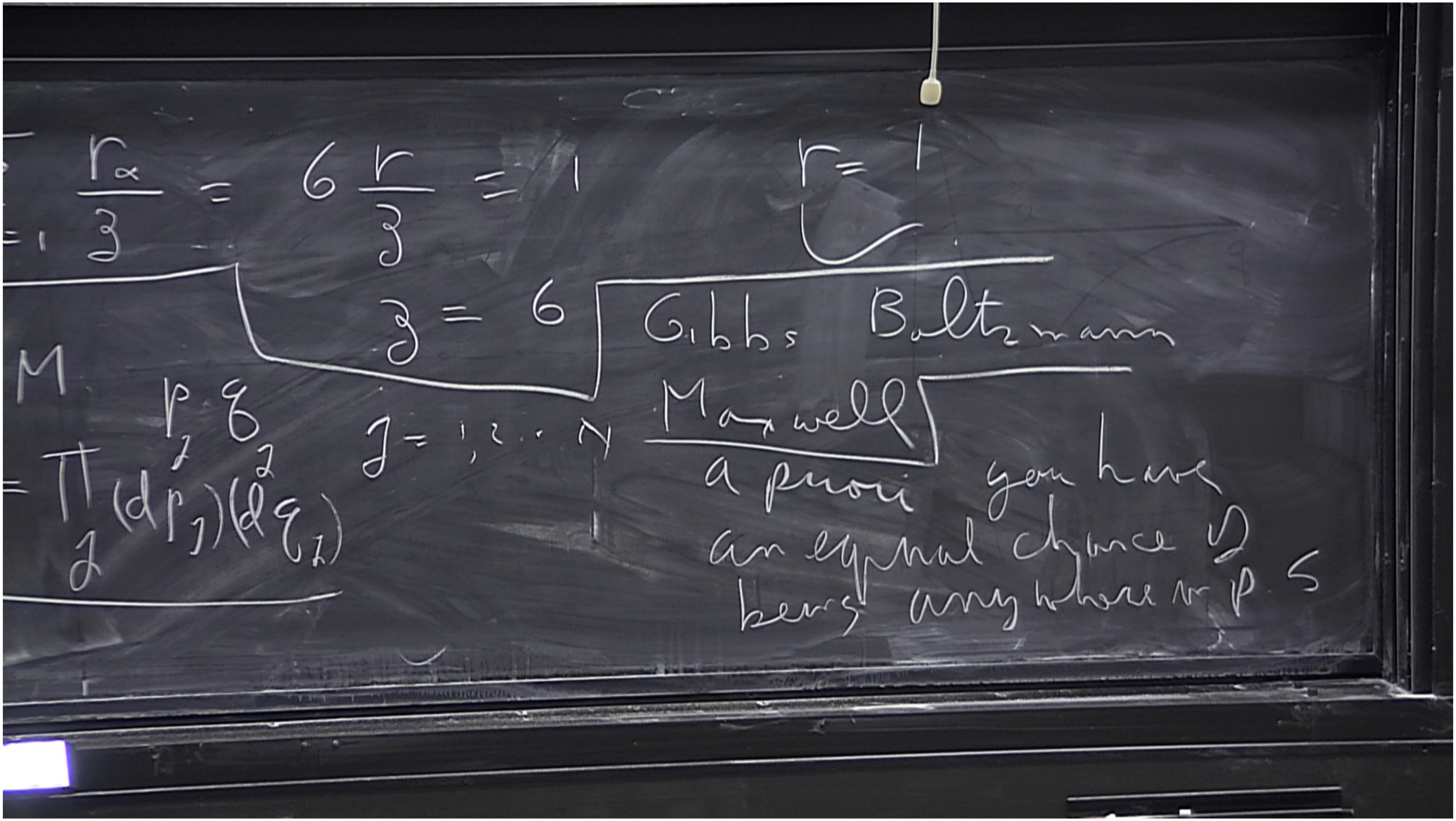
$\sum_{\alpha=1}^{6} \frac{r_\alpha}{3} = 6\frac{r}{3} = 1$
$r = 1$
$3 = 6$

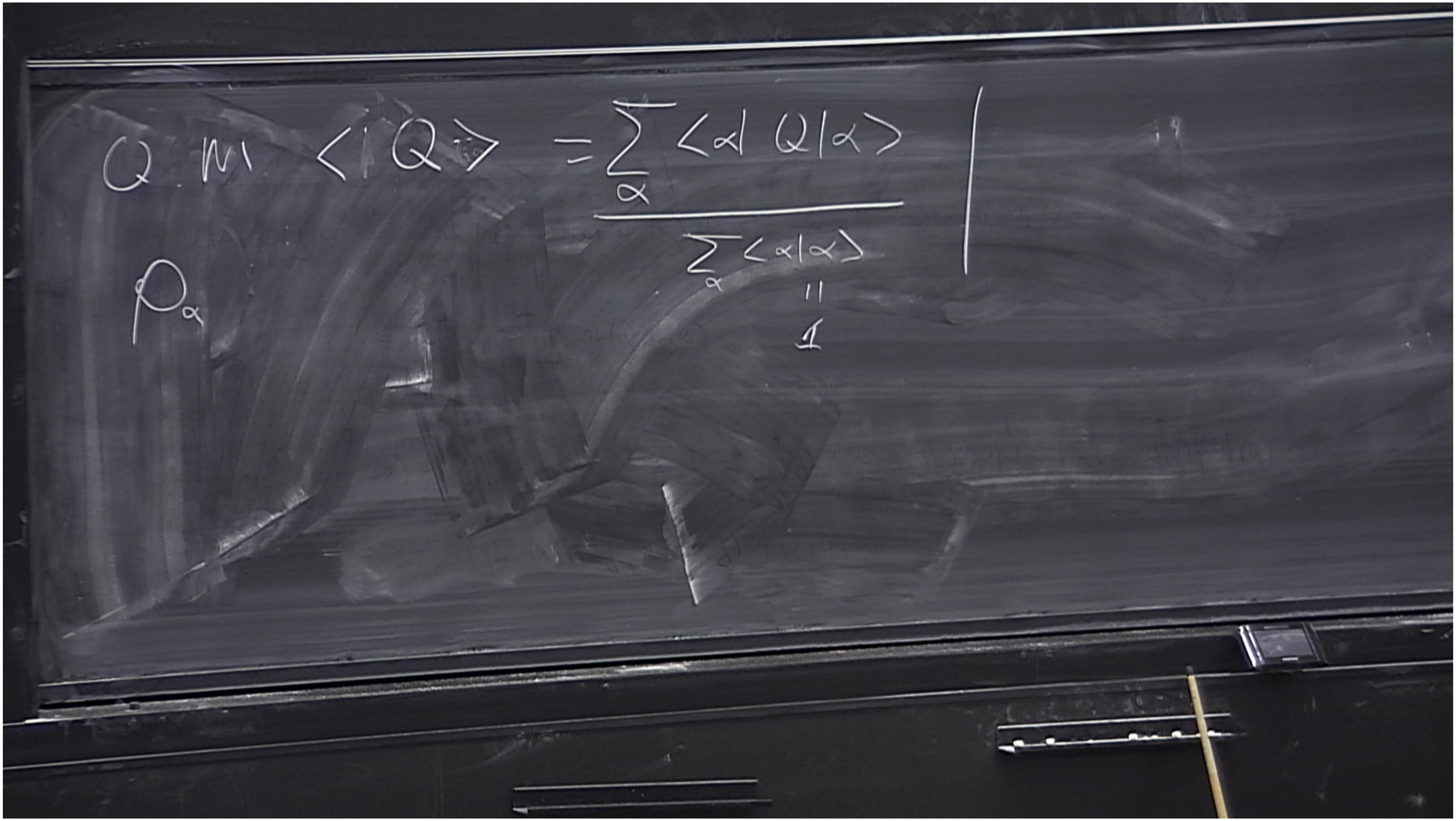

$\sum_{\alpha=1}^{6} \frac{r_\alpha}{ℨ} = 6\frac{r}{ℨ} = 1$
$r = 1$
$ℨ = 6$
M
P 8

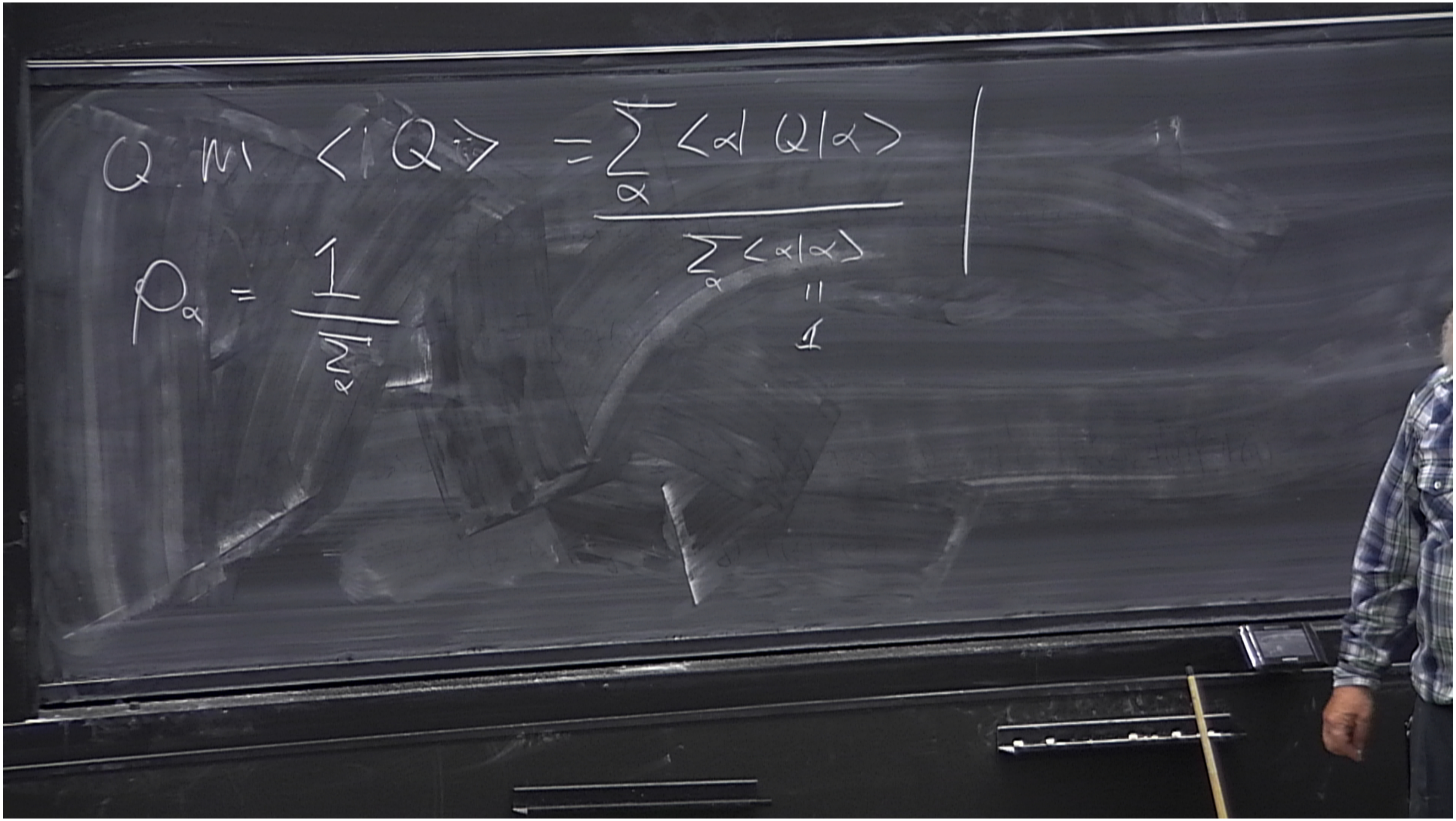


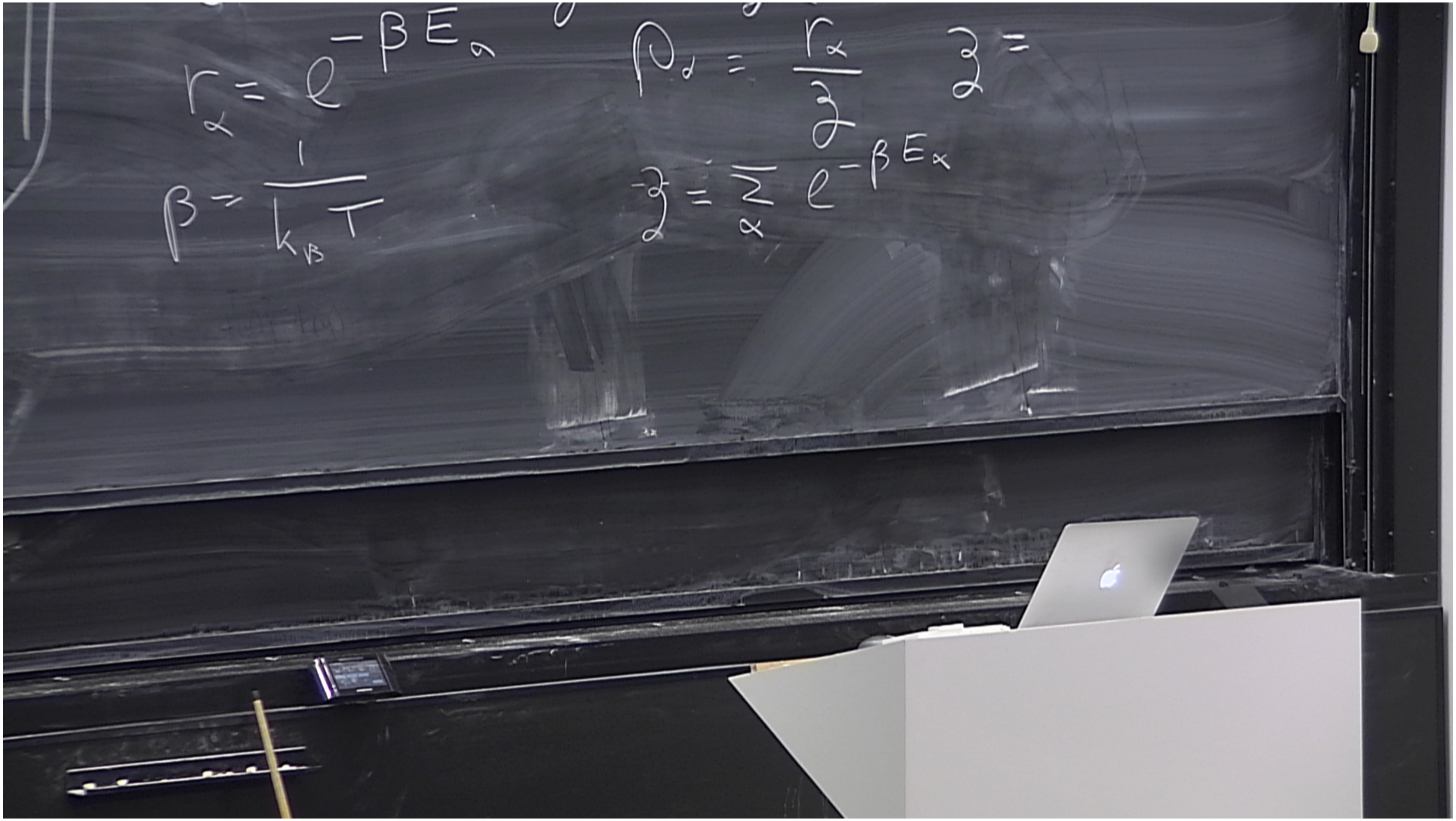
$\sum_{\alpha=1}^{6} \frac{r_\alpha}{3} = 6\frac{r}{3} = 1$
$r = 1$
$3 = 6$
C. M
$d\gamma = \prod_j (d^2 p_j)(d^2 q_j)$
$j = 1, 2, \ldots N$

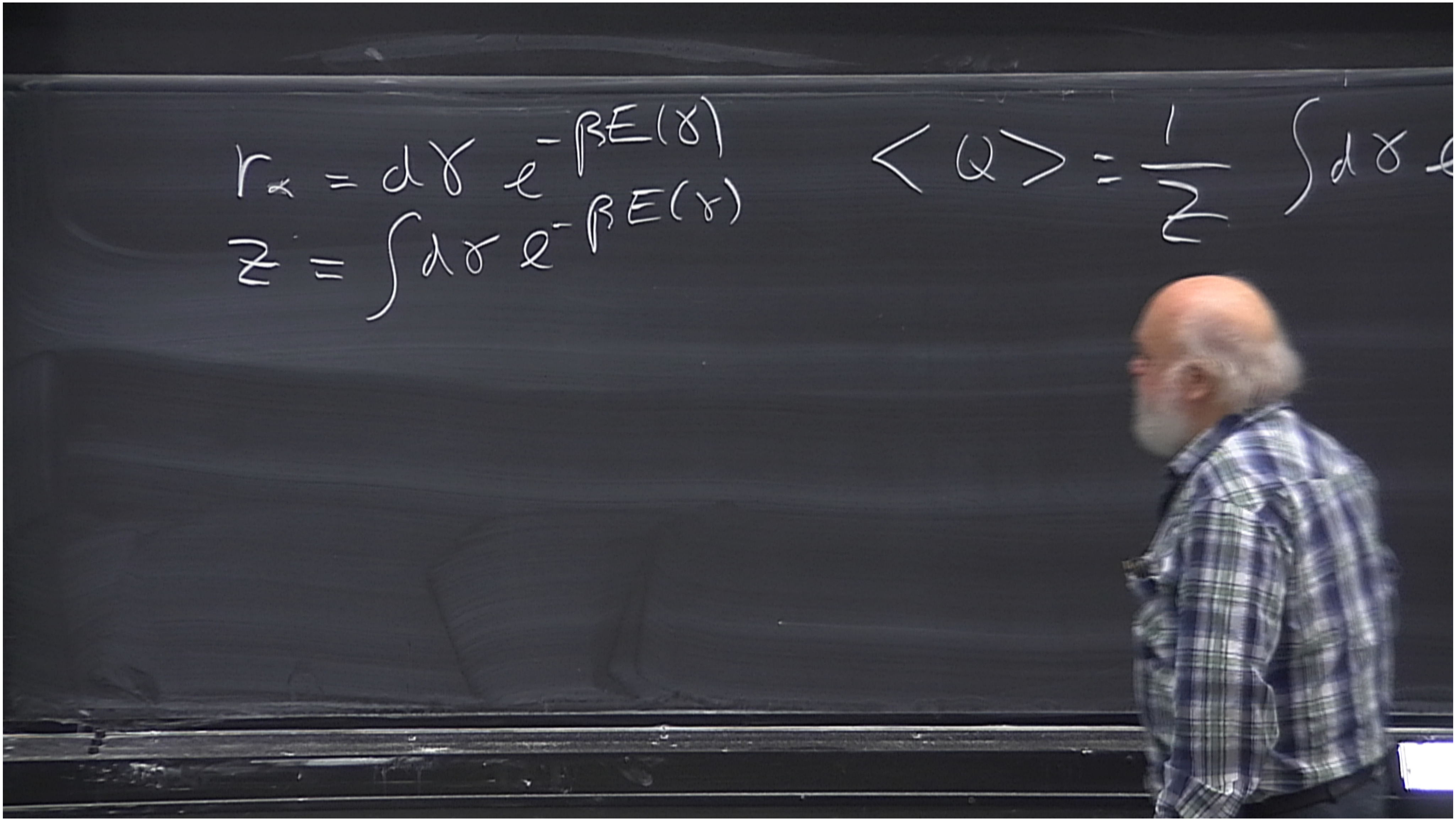

$\sum_{\alpha=1}^{6} \frac{r_\alpha}{3} = 6\frac{r}{3} = 1$
$r = 1$
$3 = 6$
Gibbs Boltzmann
Maxwell

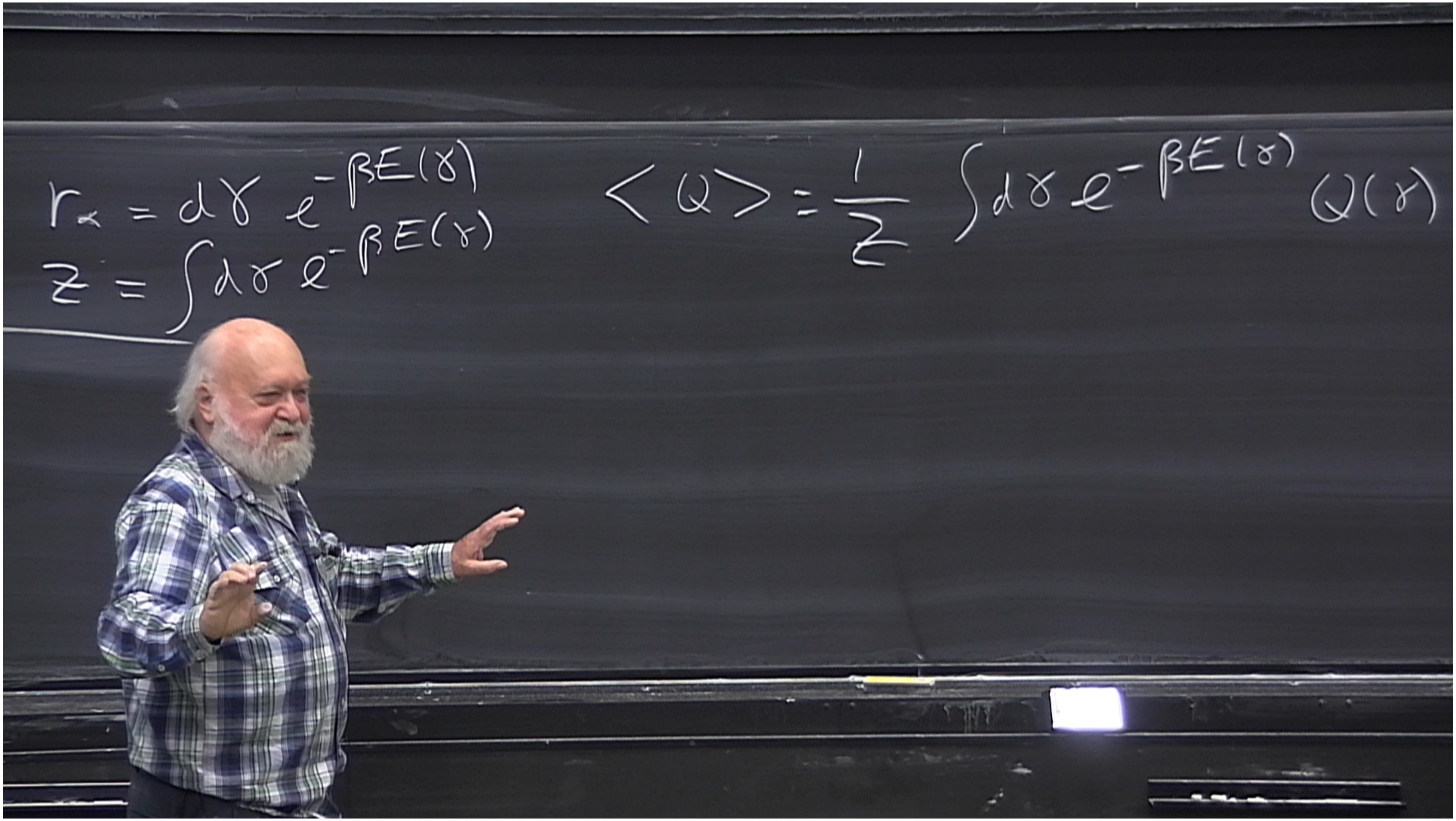

r = 1
Gibbs Boltzmann
Maxwell
a priori you have
an equal chance of
being anywhere in p.s

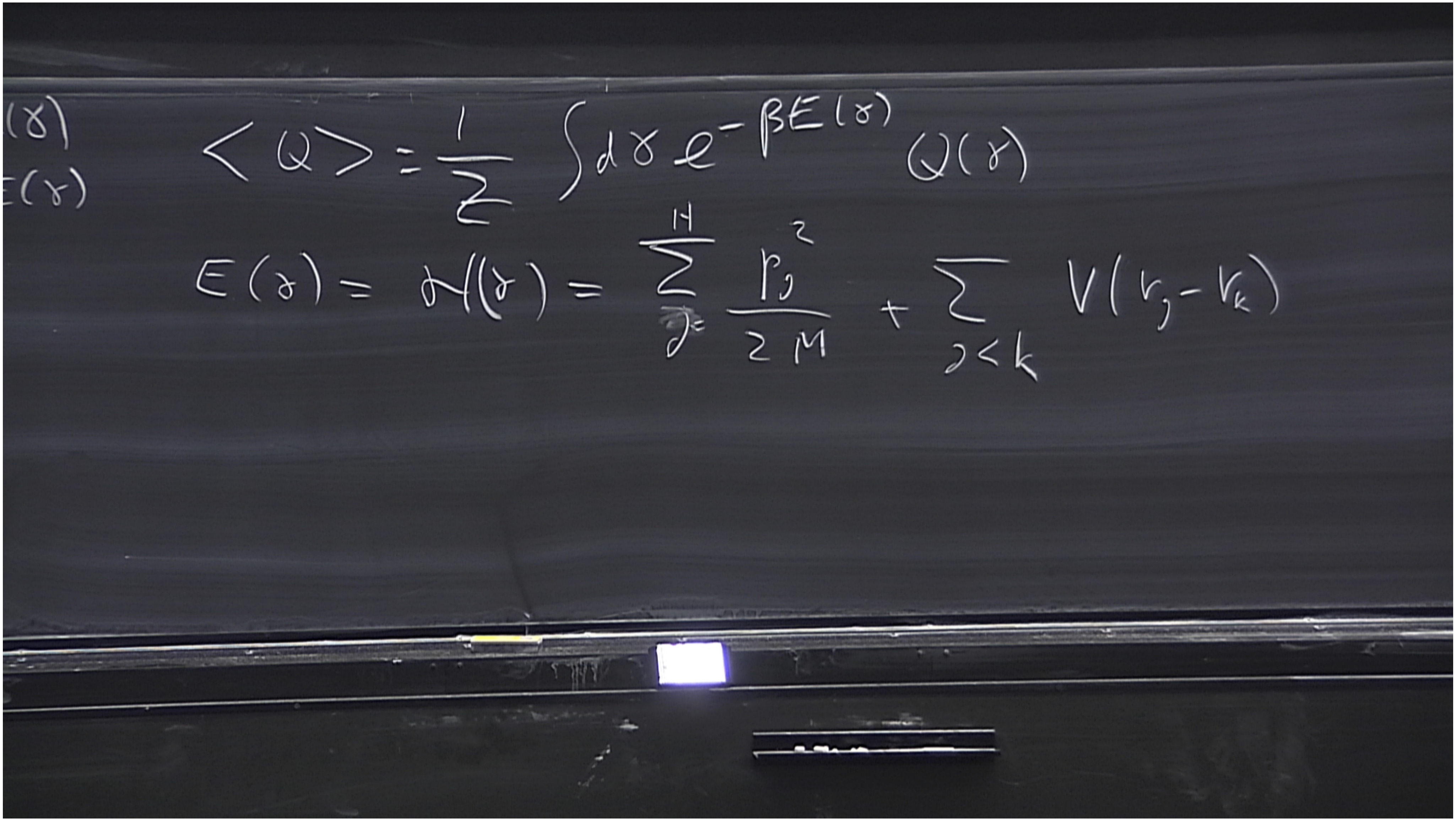

$Q.M. \quad \langle Q \rangle = \frac{\sum_\alpha \langle \alpha | Q | \alpha \rangle}{\sum_\alpha \langle \alpha | \alpha \rangle}$
$\sum_\alpha \langle \alpha | \alpha \rangle = 1$
ρ_α


QM ⟨Q⟩ = Σ_α ⟨α| Q |α⟩ / Σ_α ⟨α|α⟩
= 1
ρ_α = 1 / Σ_α


$r_\alpha = e^{-\beta E_\alpha}$
$\rho_\alpha = \frac{r_\alpha}{Z}$
$Z =$
$\beta = \frac{1}{k_B T}$
$Z = \sum_\alpha e^{-\beta E_\alpha}$


$r_\alpha = d\gamma\, e^{-\beta E(\gamma)}$
$Z = \int d\gamma\, e^{-\beta E(\gamma)}$
$\langle Q \rangle = \frac{1}{Z} \int d\gamma\, e$

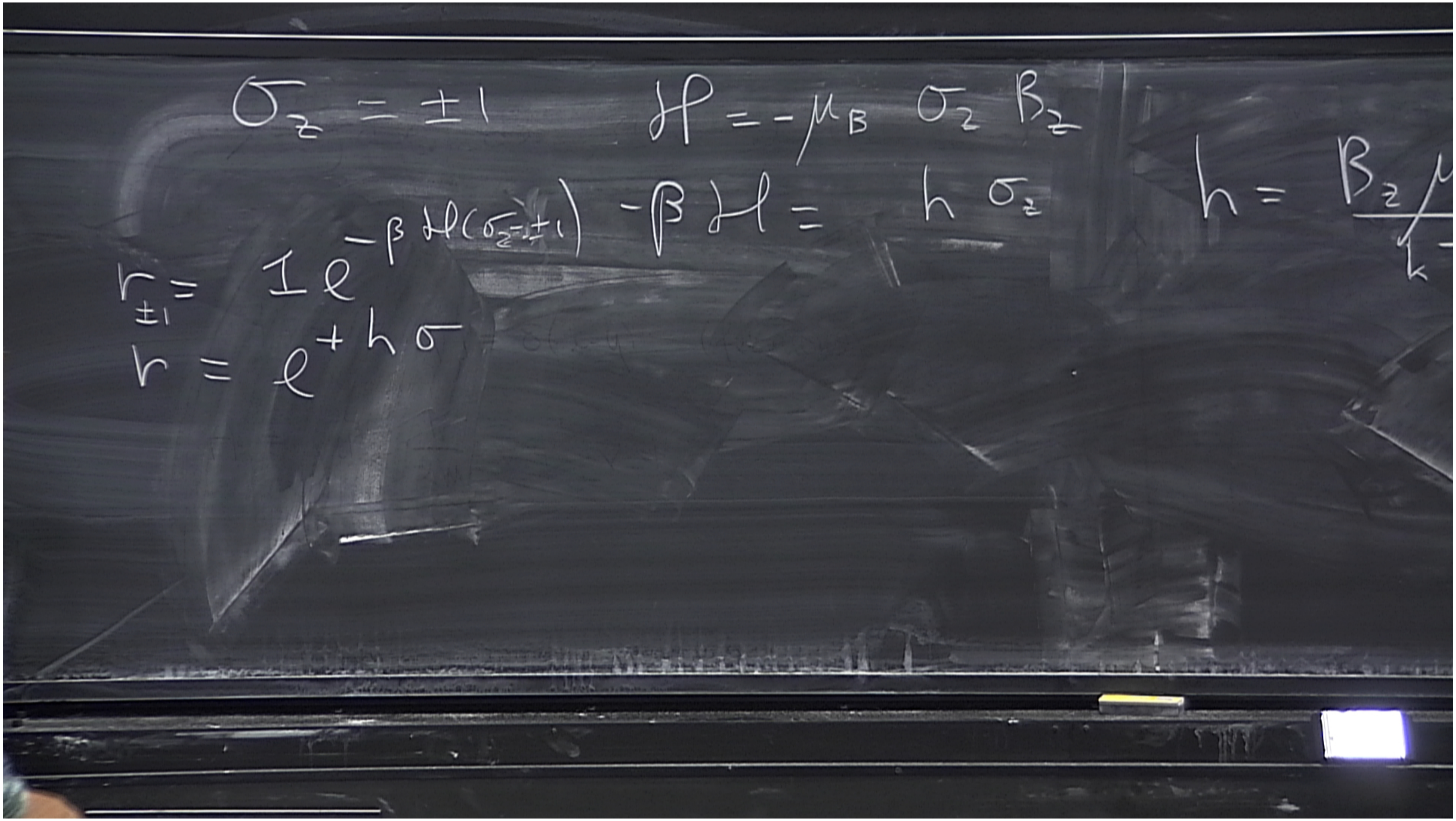

$r_\alpha = d\gamma\, e^{-\beta E(\gamma)}$
$Z = \int d\gamma\, e^{-\beta E(\gamma)}$
$\langle Q \rangle = \frac{1}{Z} \int d\gamma\, e^{-\beta E(\gamma)} Q(\gamma)$

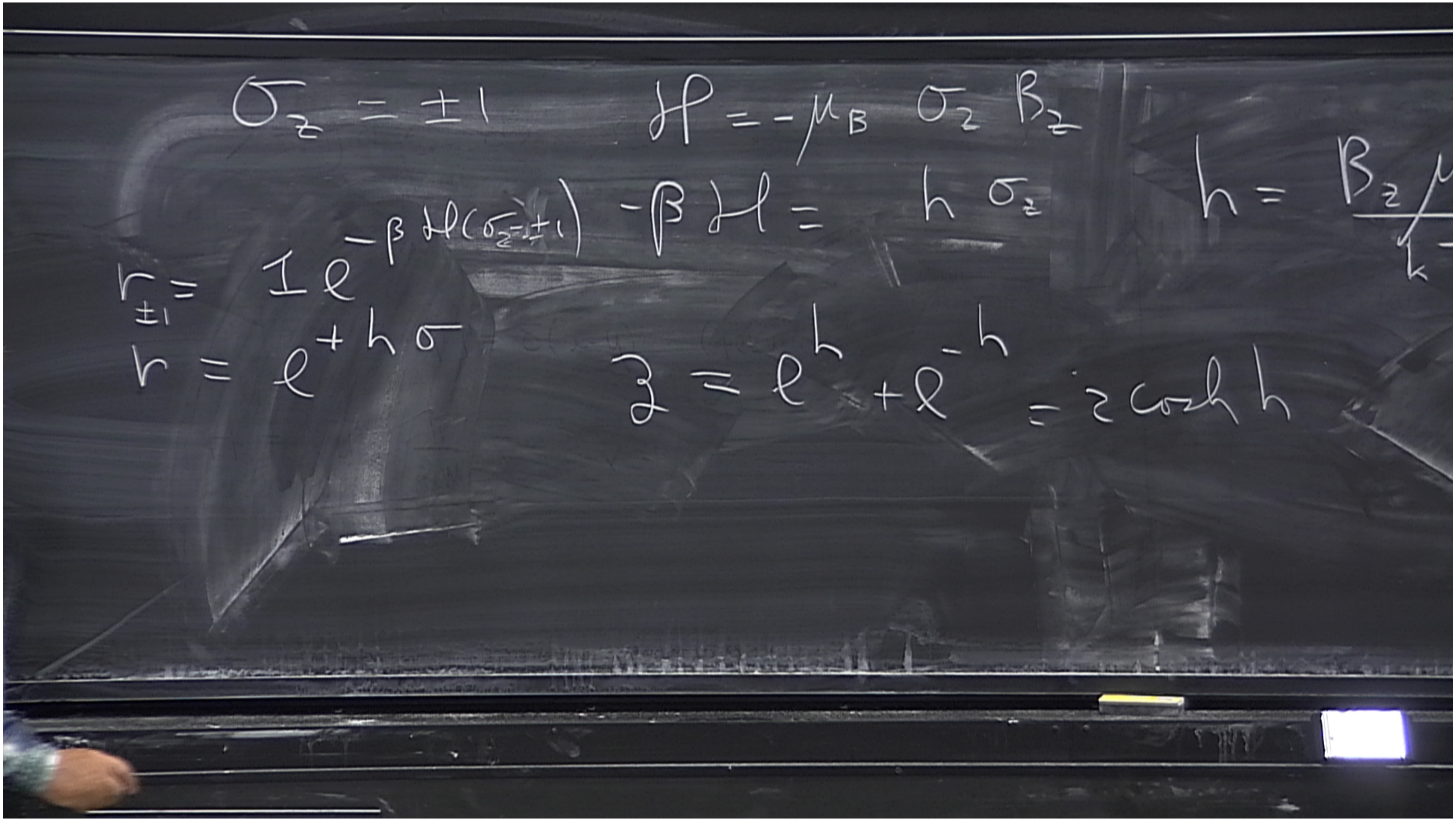


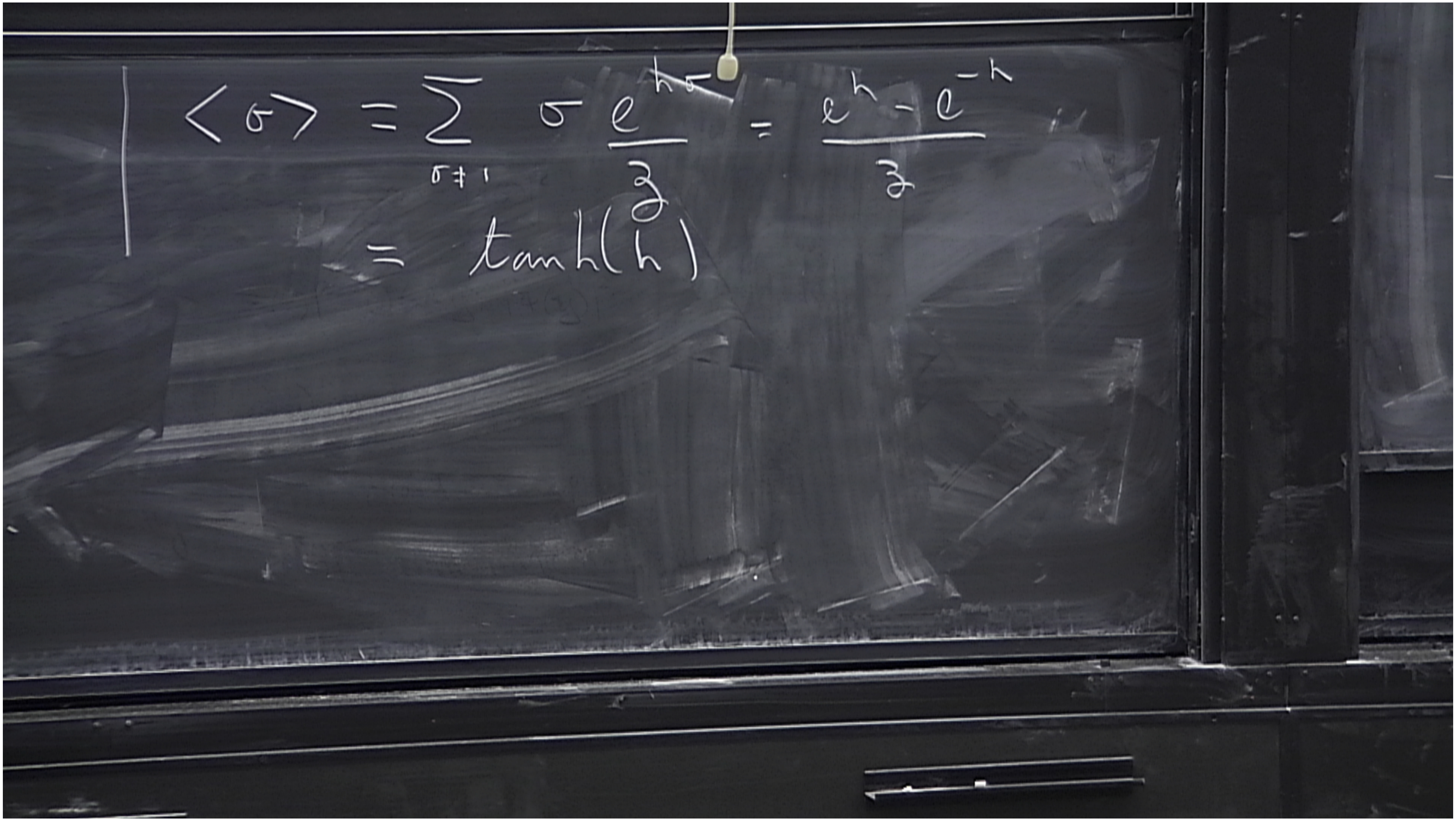
(γ)
$E(\gamma)$
$\langle Q \rangle = \frac{1}{Z} \int d\gamma \, e^{-\beta E(\gamma)} Q(\gamma)$
$E(\gamma) = \mathcal{H}(\gamma) = \sum_{j=1}^{N} \frac{p_j^2}{2M} + \sum_{j<k} V(r_j - r_k)$

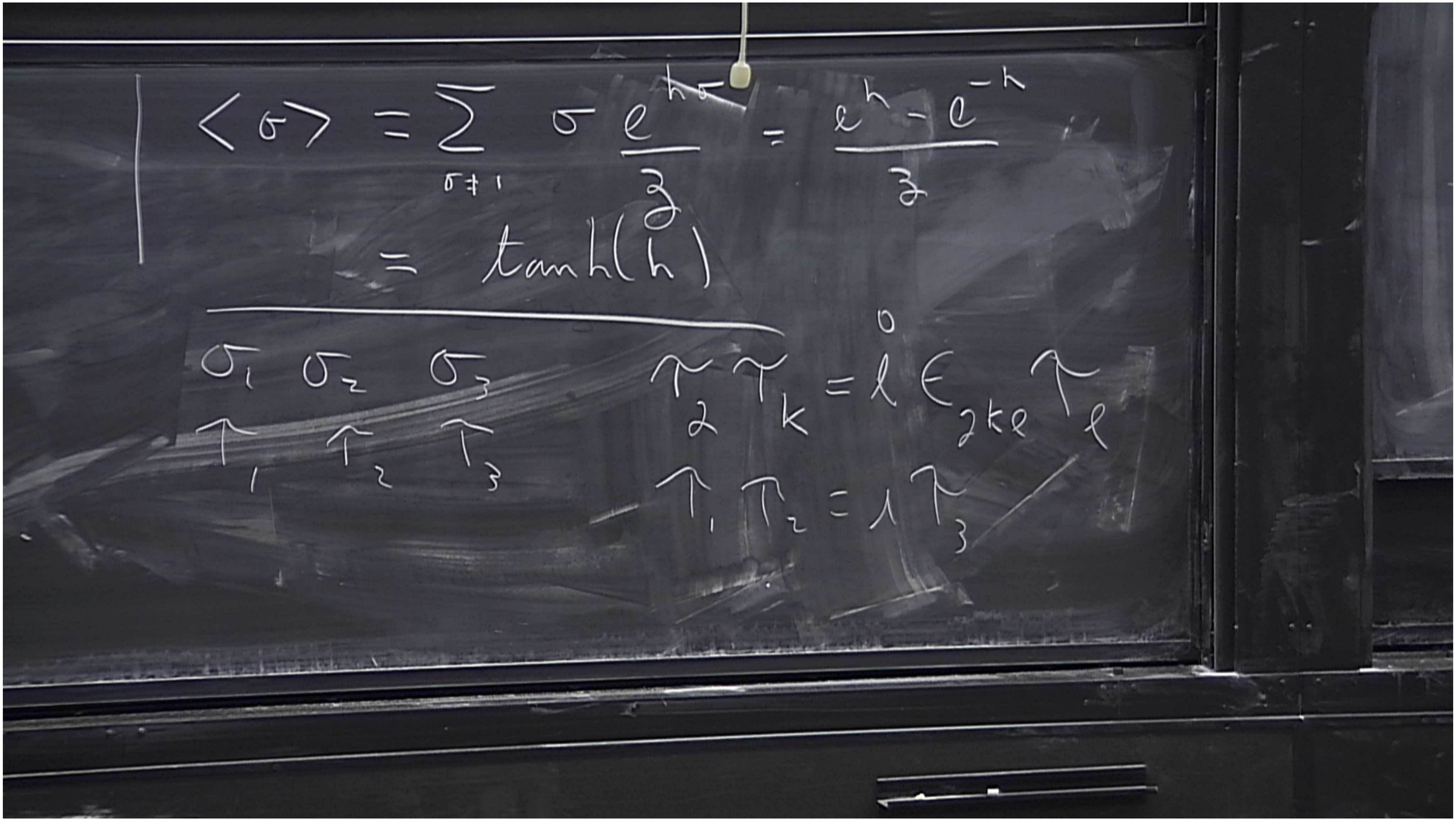
$$\sigma_z = \pm 1 \qquad \mathcal{H} = -\mu_B \, \sigma_z \, B_z$$

$$-\beta \mathcal{H} = h \, \sigma_z \qquad h = \frac{B_z \mu}{kT}$$

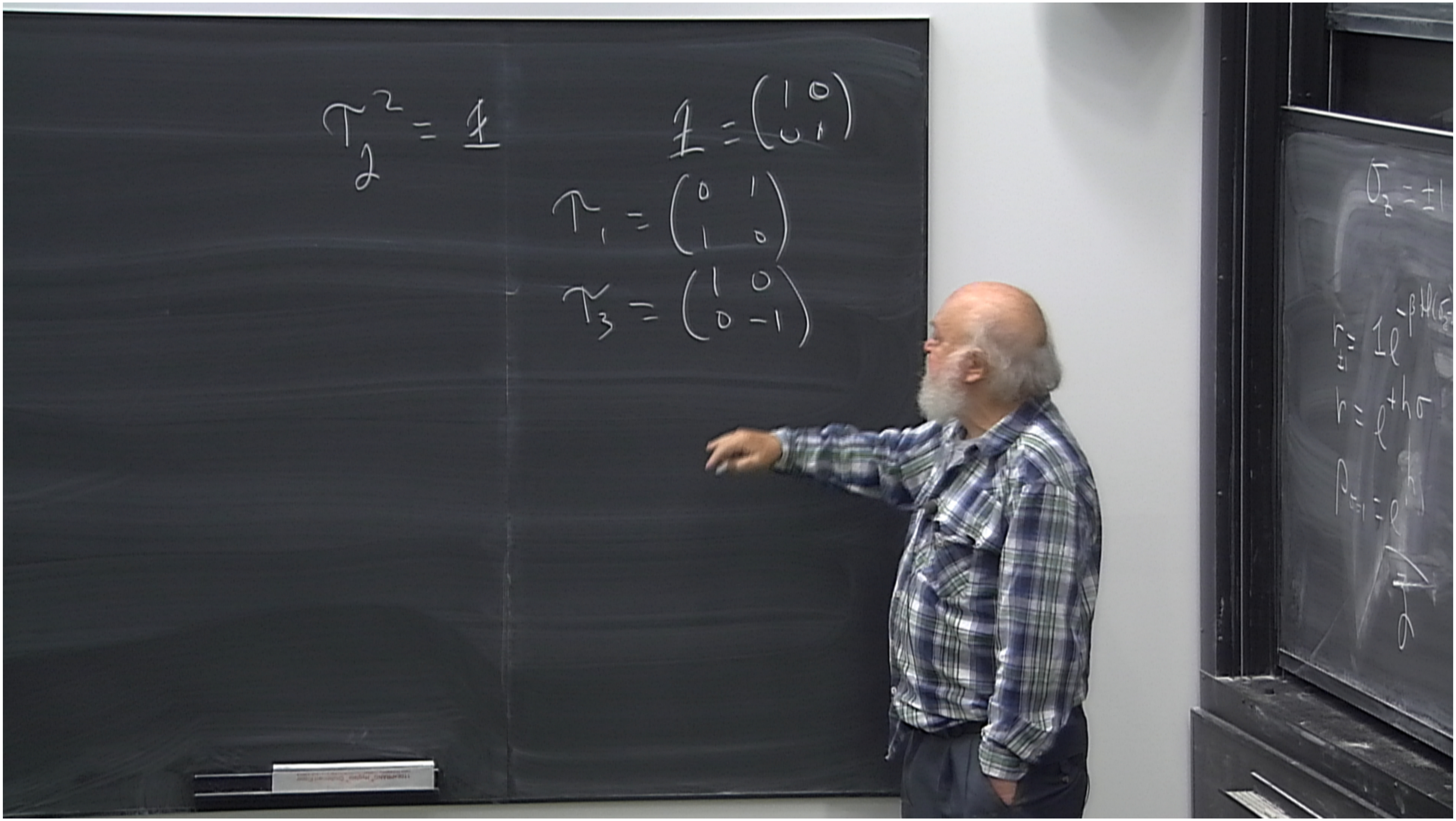

σ_z = ±1
H = -μ_B σ_z B_z
-βH = h σ_z
h = B_z μ / k
r_±1 = 1 e^{-βH(σ_z=±1)}
r = e^{+hσ}

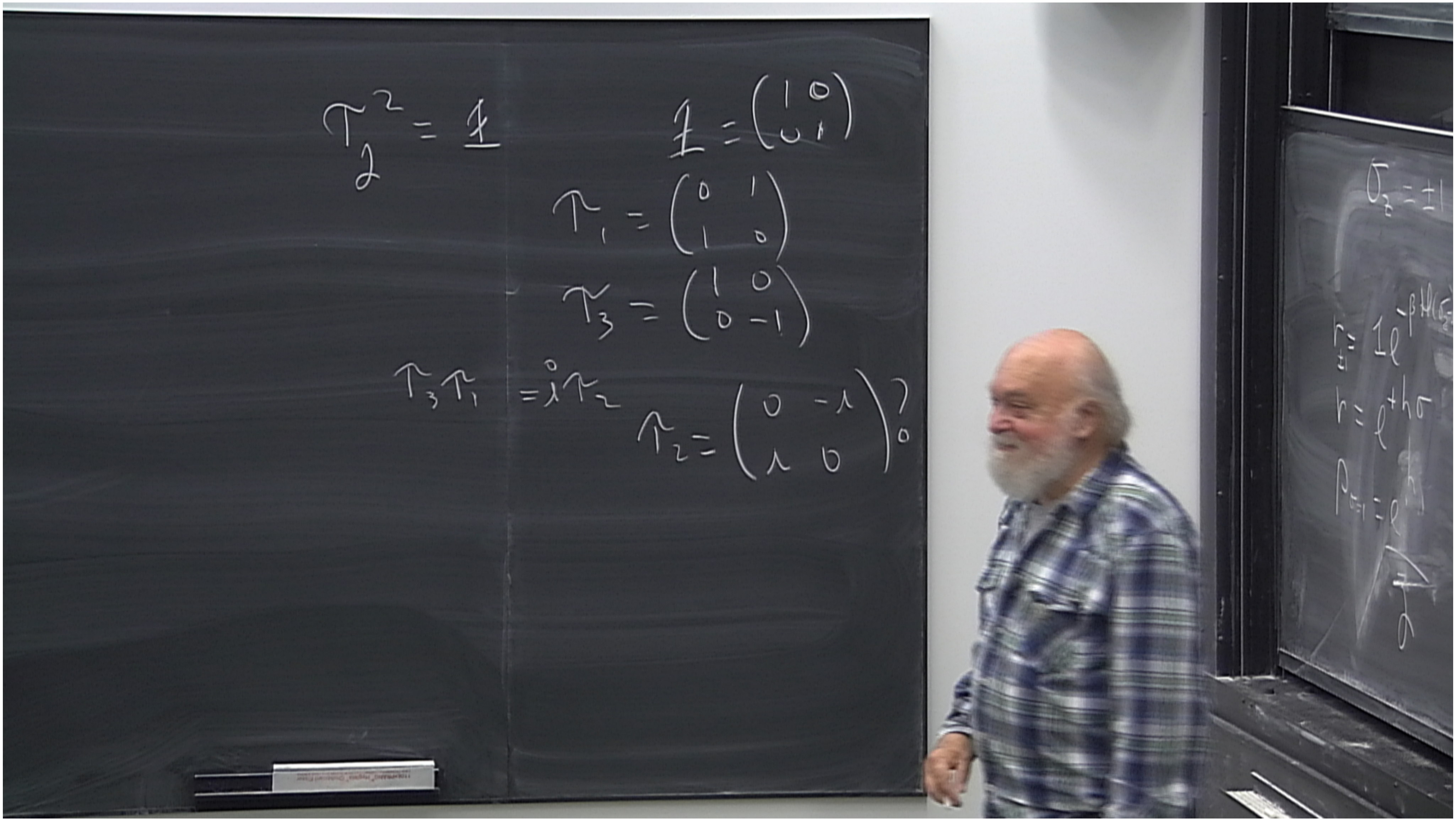

σz = ±1
H = -μB σz Bz
-βH = h σz
h = Bz μ
r±1 = 1 e^(-βH(σz=±1))
r = e^(+hσ)
Z = e^h + e^-h = 2cosh h

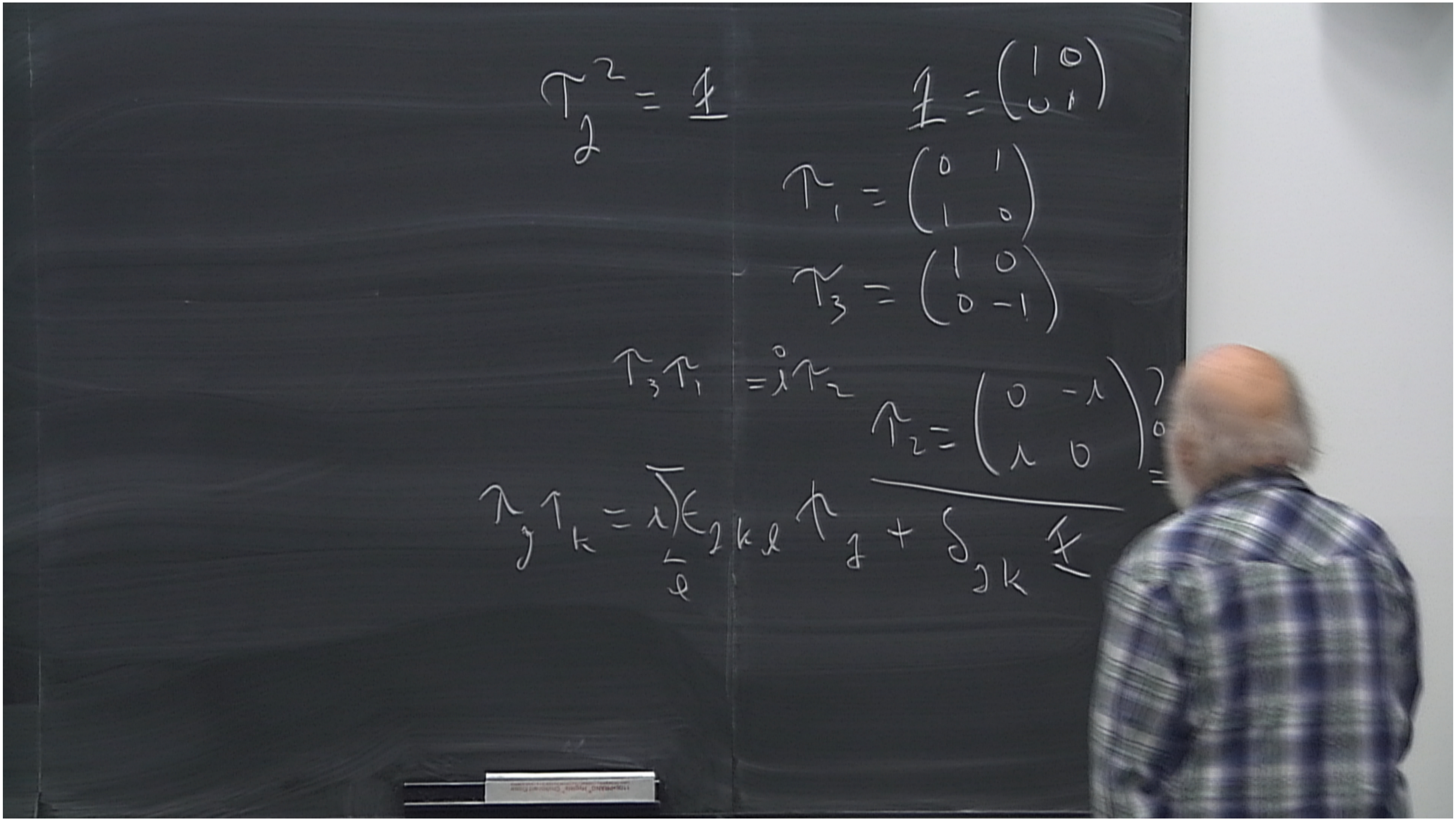

$\langle \sigma \rangle = \sum_{\sigma = \pm 1} \sigma \frac{e^{h\sigma}}{Z} = \frac{e^{h} - e^{-h}}{Z}$
$= \tanh(h)$



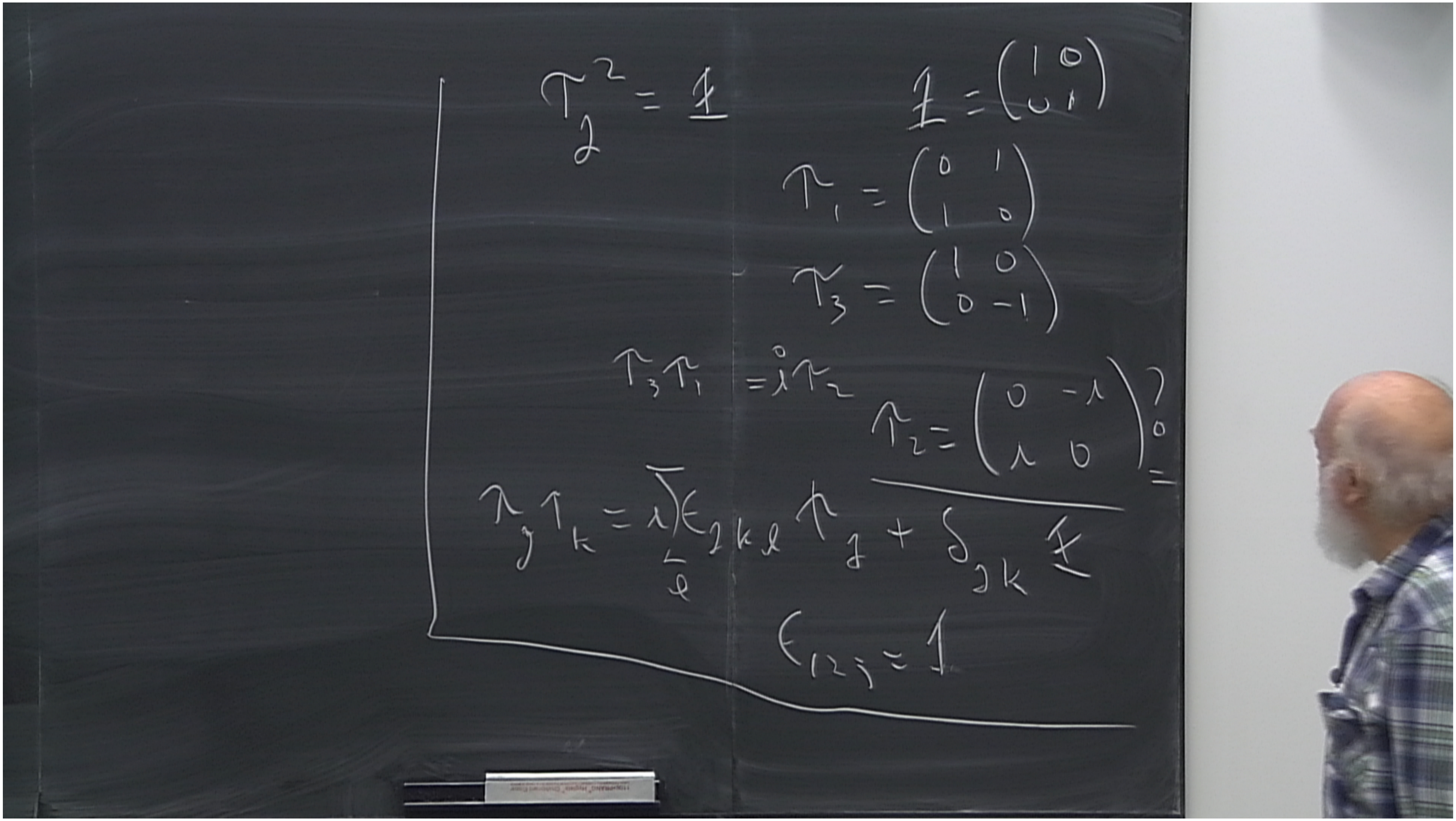
$\langle \sigma \rangle = \sum_{\sigma = \pm 1} \sigma \frac{e^{h\sigma}}{Z} = \frac{e^{h} - e^{-h}}{Z}$
$= \tanh(h)$
$\sigma_1 \quad \sigma_2 \quad \sigma_3$
$\tau_1 \quad \tau_2 \quad \tau_3$
$\tau_j \tau_k = i \epsilon_{jkl} \tau_l$
$\tau_1 \tau_2 = i \tau_3$


$\tau_2^2 = \mathbb{1}$
$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$


τ₂² = 1
1 = (1 0 / 0 1)
τ₁ = (0 1 / 1 0)
τ₃ = (1 0 / 0 −1)
τ₃τ₁ = iτ₂
τ₂ = (0 −i / i 0)



$\tau_j^2 = \mathbb{1}$
$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\tau_3 \tau_1 = i \tau_2$
$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\tau_j \tau_k = i \sum_l \epsilon_{jkl} \tau_l + \delta_{jk} \mathbb{1}$


$\tau_j^2 = 1$
$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\tau_3 \tau_1 = i \tau_2$
$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$?
$\tau_j \tau_k = i \sum_l \epsilon_{jkl} \tau_l + \delta_{jk} 1$
$\epsilon_{123} = 1$

$$\tau_j^2 = \mathbb{1} \qquad \mathbb{1} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\tau_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

Pauli spin matrix

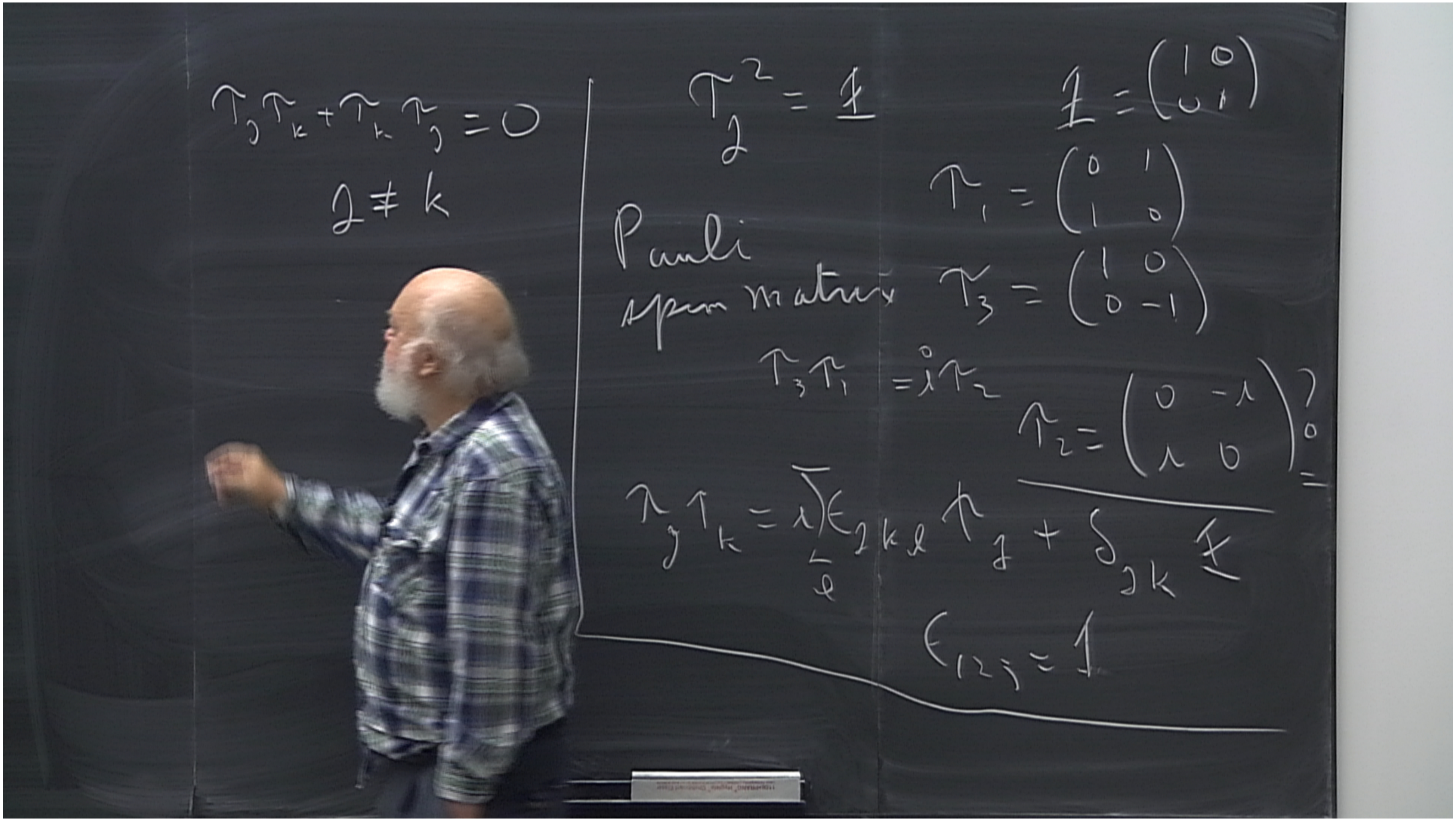
$$\tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

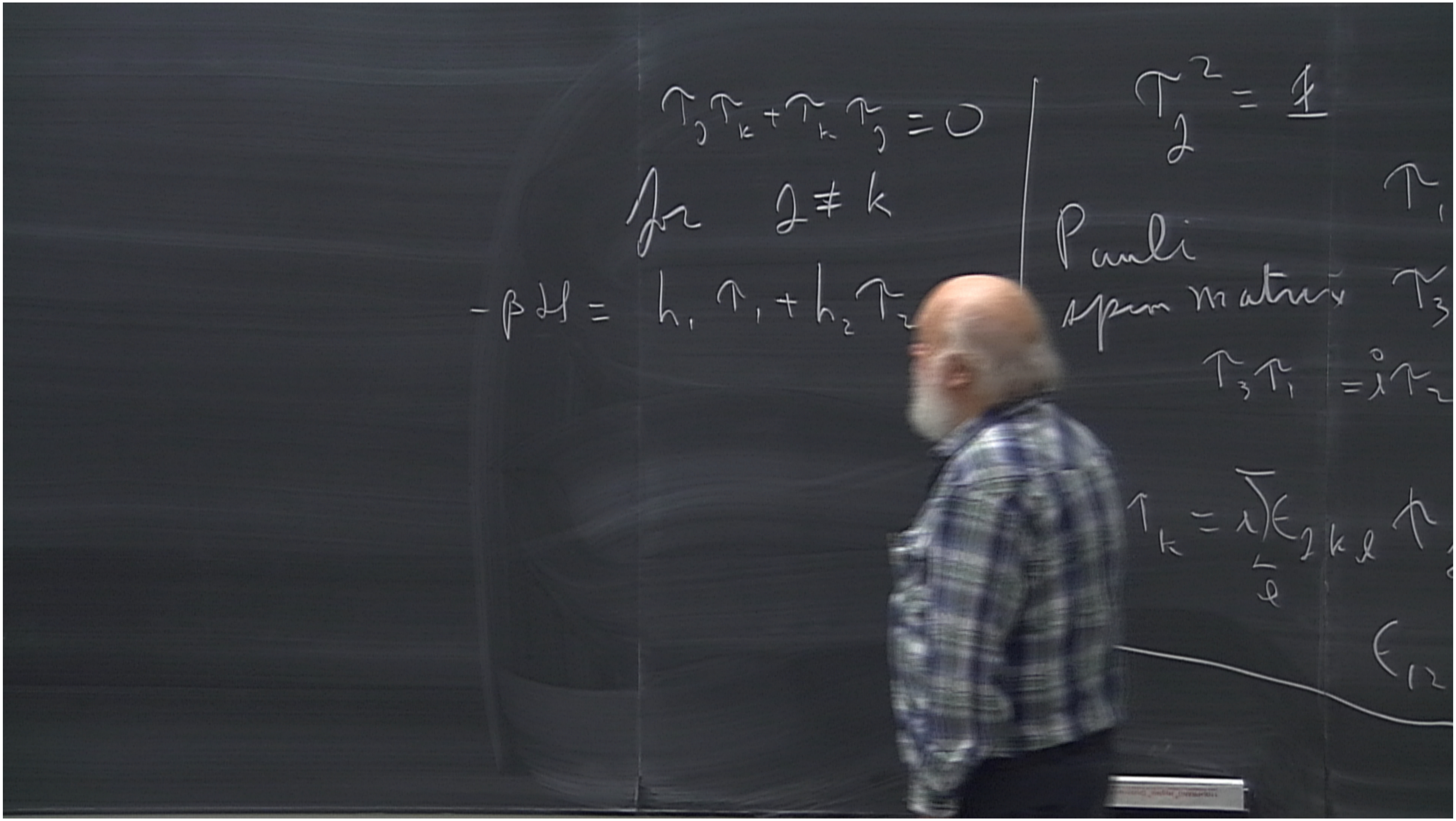
$$\tau_3 \tau_1 = i\tau_2$$

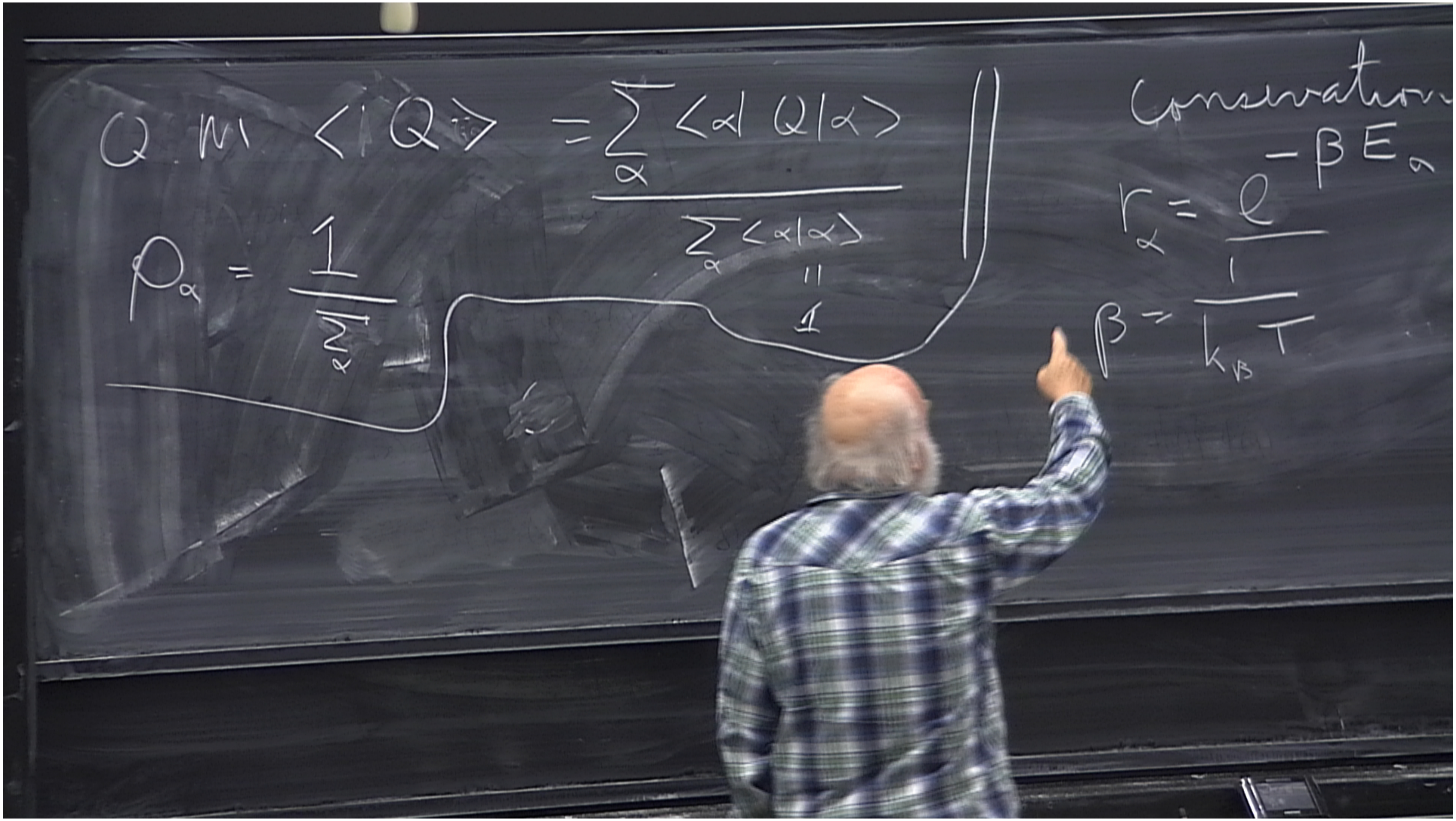
$$\tau_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \stackrel{?}{=}$$

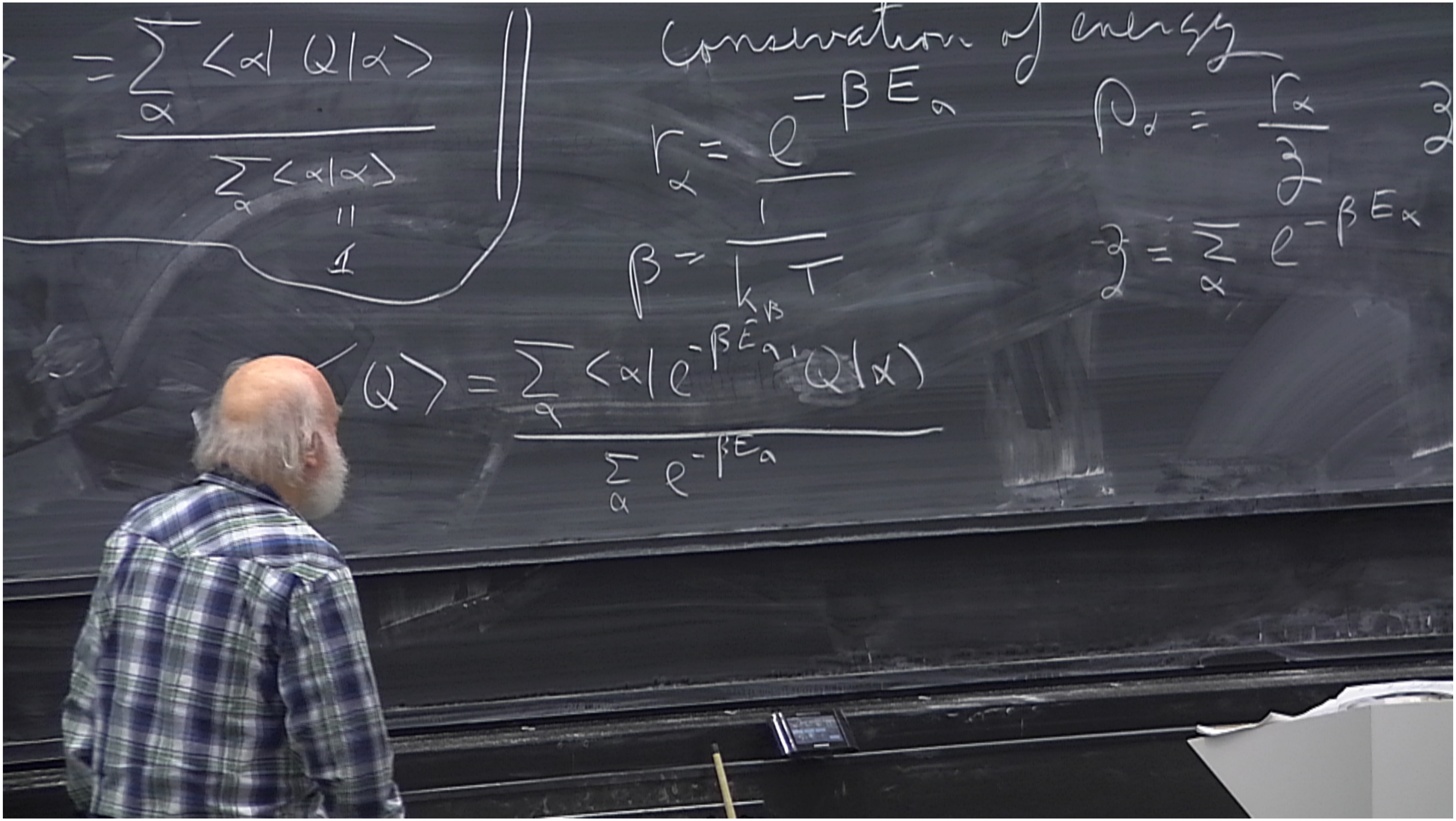
$$\tau_j \tau_k = i\sum_l \epsilon_{jkl}\tau_l + \delta_{jk}\mathbb{1}$$

$$\epsilon_{123} = 1$$

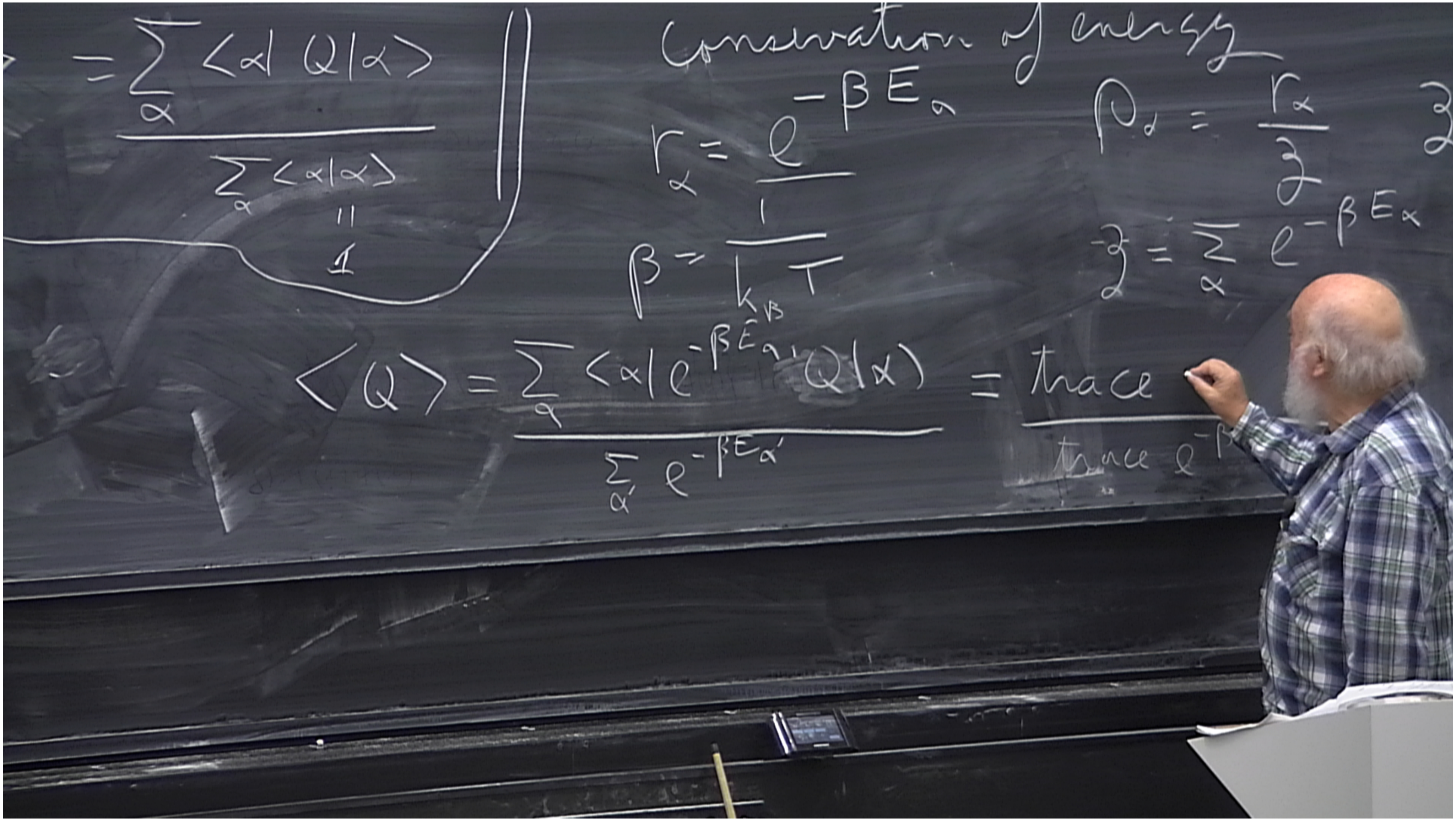

$\tau_j \tau_k + \tau_k \tau_j = 0$
$j \neq k$
$\tau_j^2 = \mathbb{1}$
$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Pauli
spin matrix
$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\tau_3 \tau_1 = i \tau_2$
$\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\tau_j \tau_k = i \sum_l \epsilon_{jkl} \tau_l + \delta_{jk} \mathbb{1}$
$\epsilon_{123} = 1$

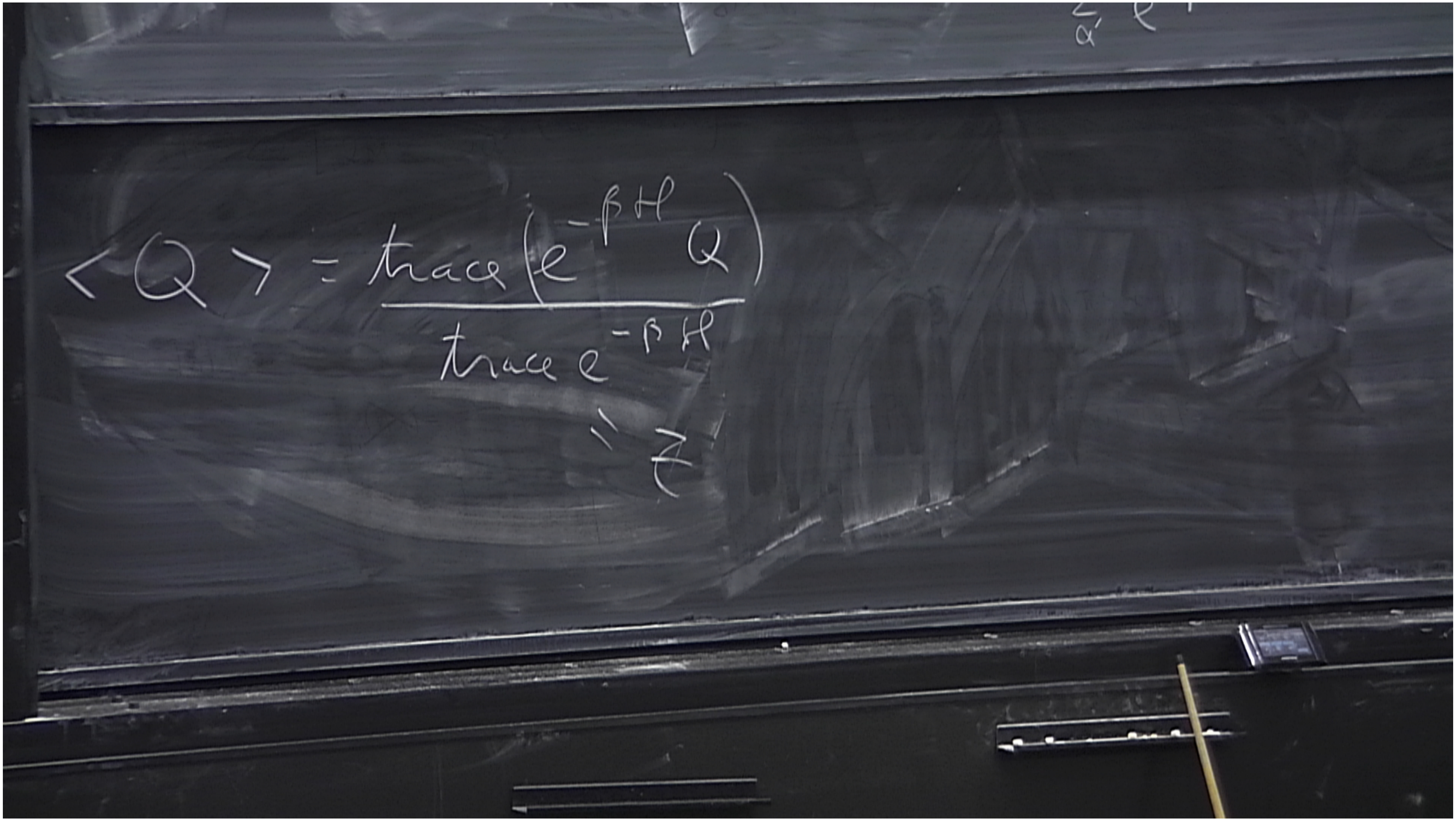

τj τk + τk τj = 0
for j ≠ k
−βH = h1 τ1 + h2 τ2
τj² = 1
Pauli
spin matrix
τ3 τ1 = i τ2



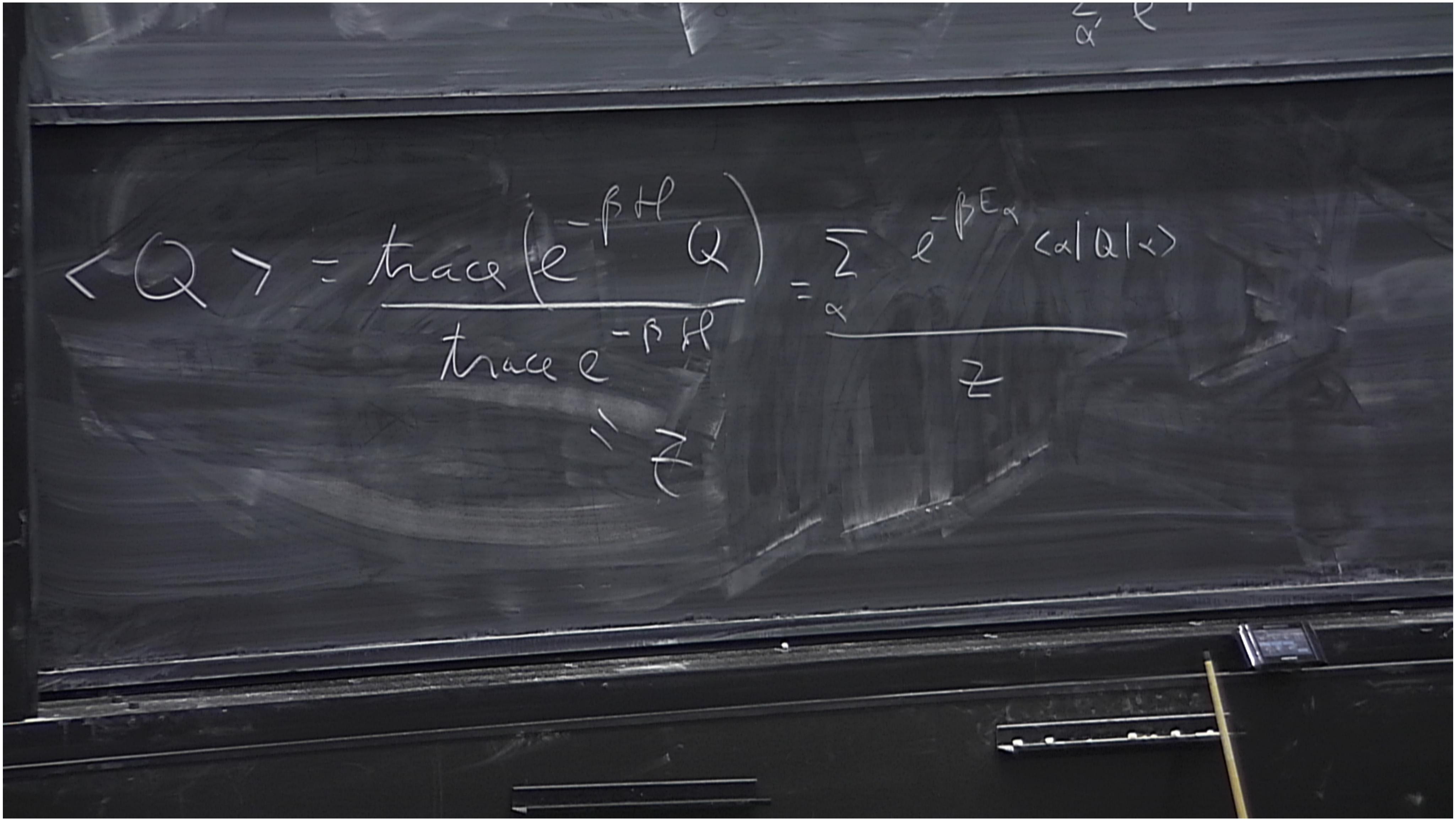


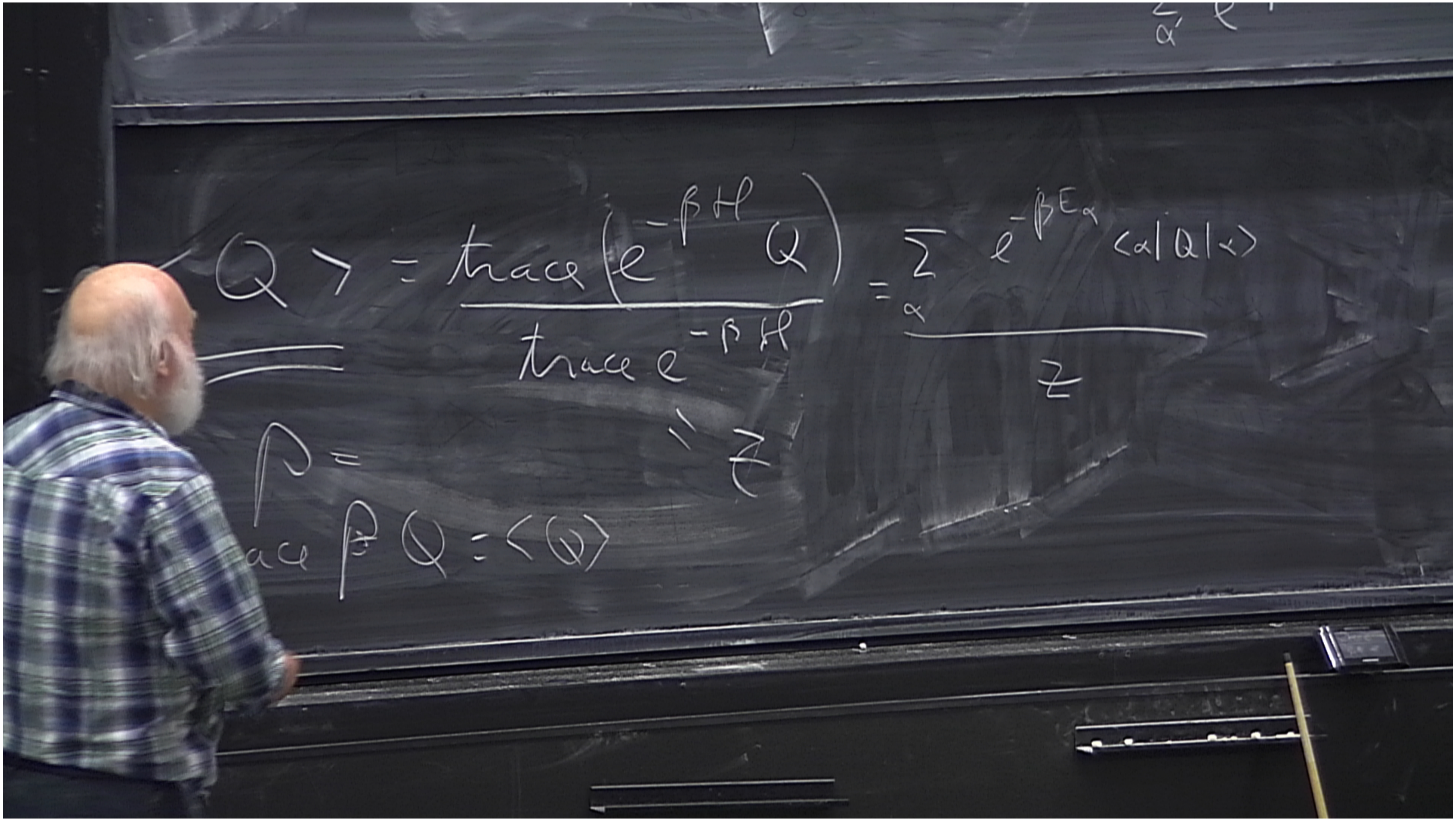
Conservation of energy
$= \frac{\sum_\alpha \langle \alpha | Q | \alpha \rangle}{\sum_\alpha \langle \alpha | \alpha \rangle}$
$\sum_\alpha \langle \alpha | \alpha \rangle = 1$
$r_\alpha = e^{-\beta E_\alpha}$
$\beta = \frac{1}{k_B T}$
$p_\alpha = \frac{r_\alpha}{Z}$
$Z = \sum_\alpha e^{-\beta E_\alpha}$
$\langle Q \rangle = \frac{\sum_\alpha \langle \alpha | e^{-\beta E_\alpha} Q | \alpha \rangle}{\sum_\alpha e^{-\beta E_\alpha}}$

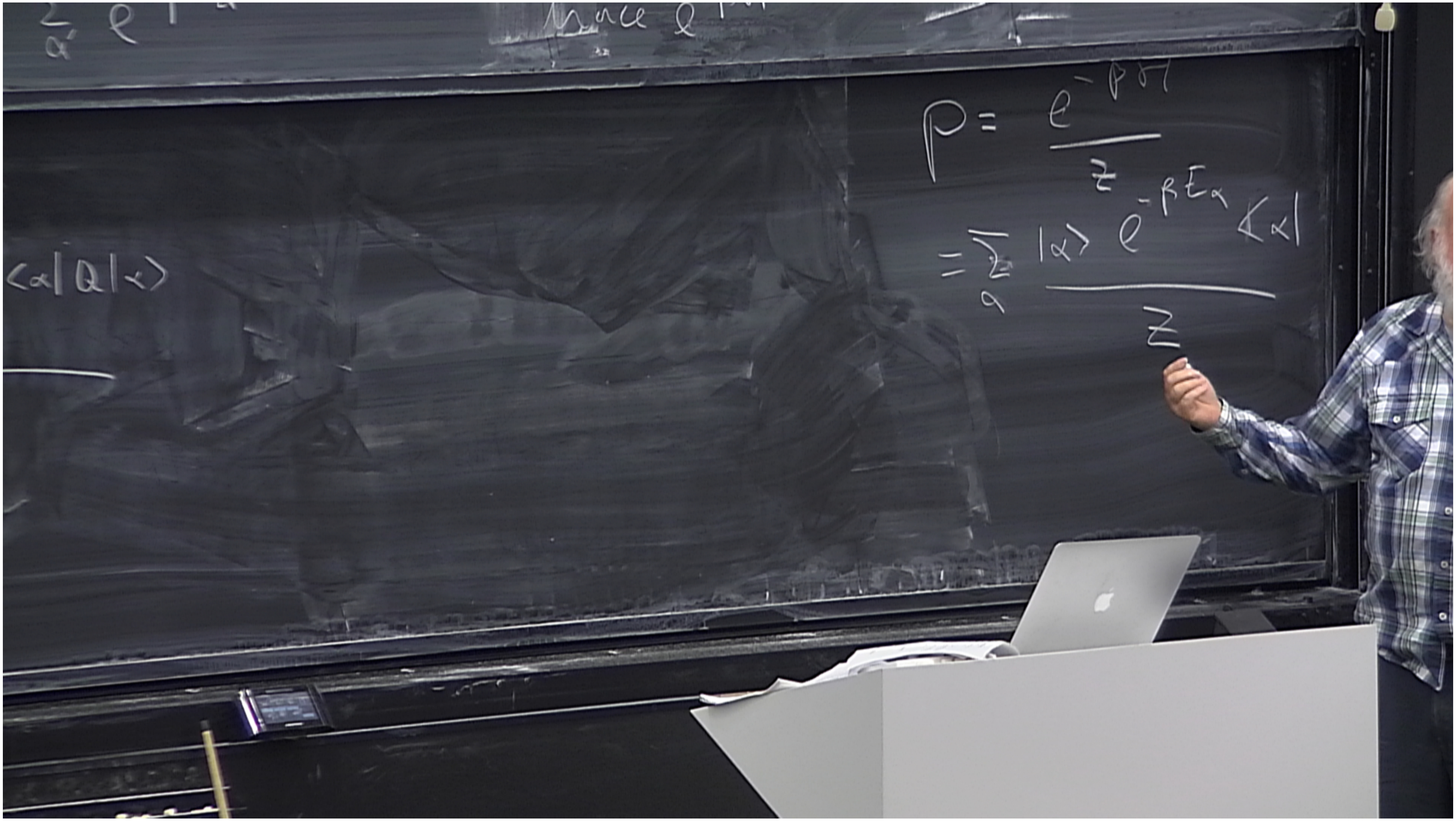

Conservation of energy
$= \frac{\sum_\alpha \langle\alpha| Q |\alpha\rangle}{\sum_\alpha \langle\alpha|\alpha\rangle}$
$r_\alpha = \frac{e^{-\beta E_\alpha}}{1}$
$\beta = \frac{1}{k_B T}$
$\rho_\alpha = \frac{r_\alpha}{Z}$
$Z = \sum_\alpha e^{-\beta E_\alpha}$
$\langle Q \rangle = \frac{\sum_\alpha \langle\alpha| e^{-\beta E_\alpha} Q |\alpha\rangle}{\sum_{\alpha'} e^{-\beta E_{\alpha'}}} = \frac{\text{trace}}{\text{trace } e^{-\beta}}$

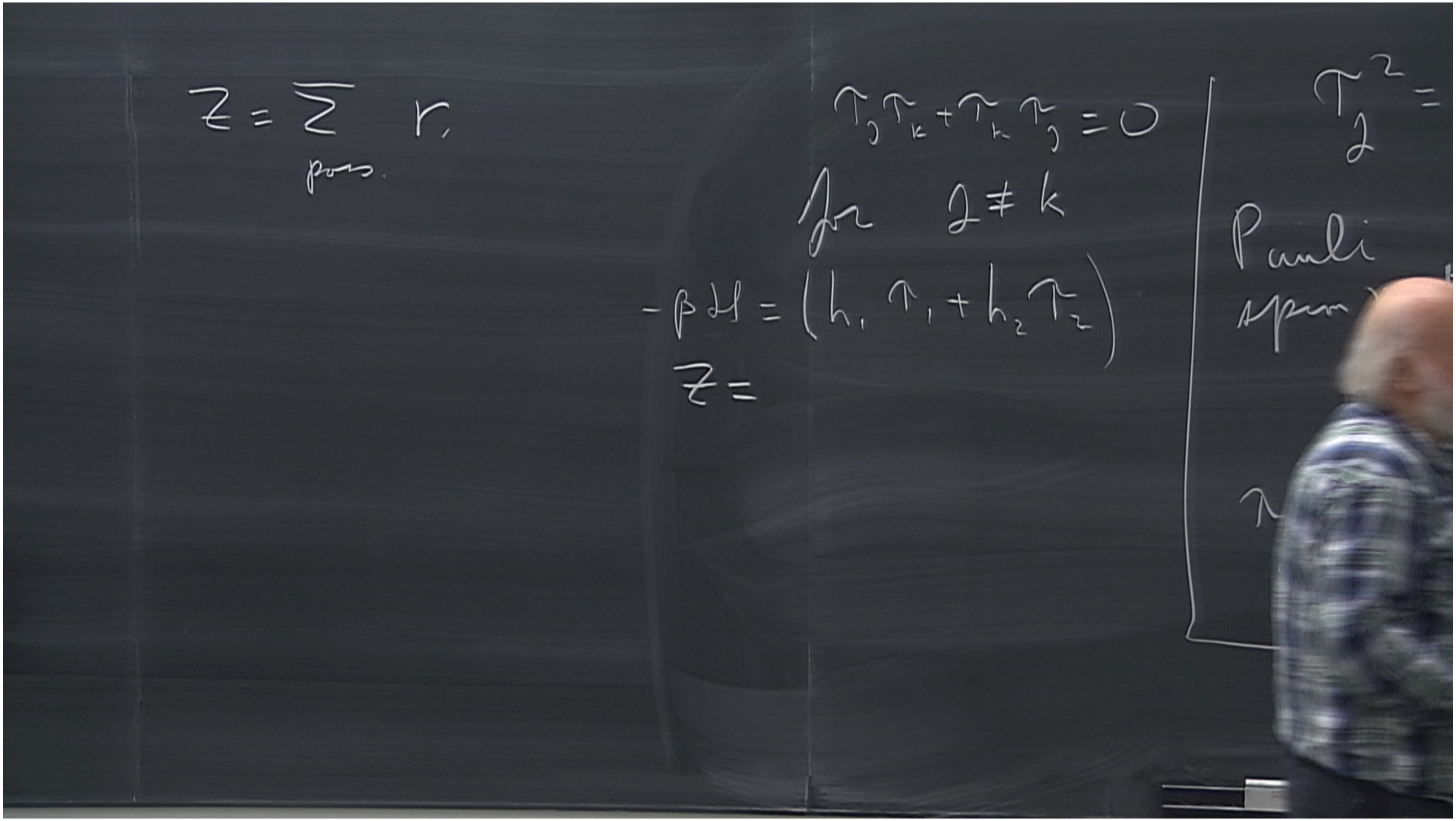


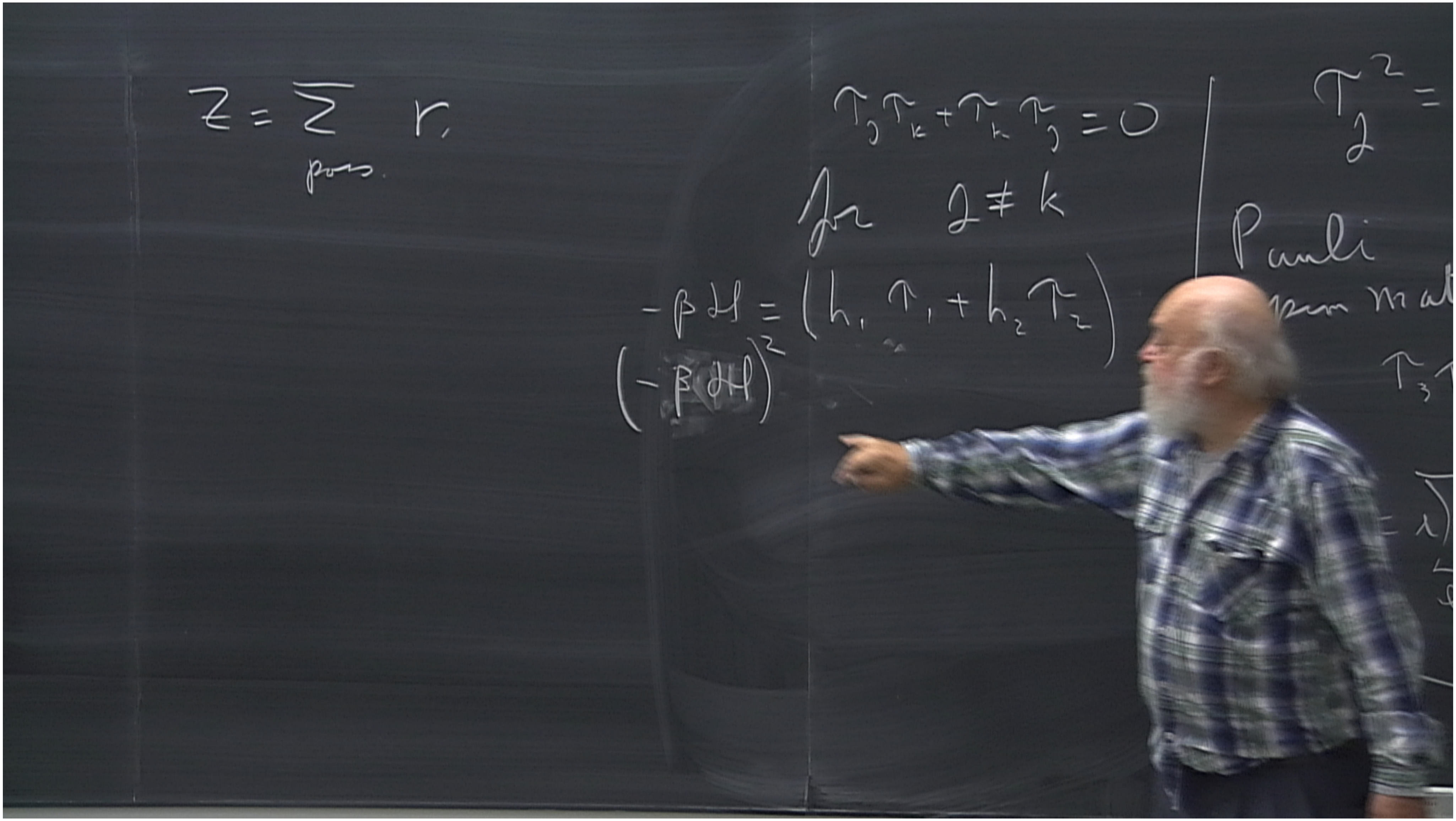
<Q> = trace(e^-βH Q) / trace e^-βH
= Z


$\langle Q \rangle = \frac{\text{trace}\left(e^{-\beta H} Q\right)}{\text{trace}\, e^{-\beta H}} = \frac{\sum_\alpha e^{-\beta E_\alpha} \langle \alpha | Q | \alpha \rangle}{Z}$
$" = Z"$


⟨Q⟩ = trace(e^{-βH} Q) / trace e^{-βH} = Σ_α e^{-βE_α} ⟨α|Q|α⟩ / Z
trace e^{-βH} = Z
ρ =
trace ρ Q = ⟨Q⟩


$\rho = \frac{e^{-\beta H}}{Z}$
$= \sum_\alpha \frac{|\alpha\rangle e^{-\beta E_\alpha} \langle\alpha|}{Z}$
$\langle\alpha|Q|\alpha\rangle$


Z = Σ r_i
poss.
τ_j τ_k + τ_k τ_j = 0
for j ≠ k
-βH = (h_1 τ_1 + h_2 τ_2)
Z =
τ_j^2 =
Pauli
spin


Pauli

$$Z = \sum_{\text{poss.}} r_i$$

$$\tau_j \tau_k + \tau_k \tau_j = 0$$

for $j \neq k$

$$-\beta \mathcal{H} = (h_1 \tau_1 + h_2 \tau_2)$$

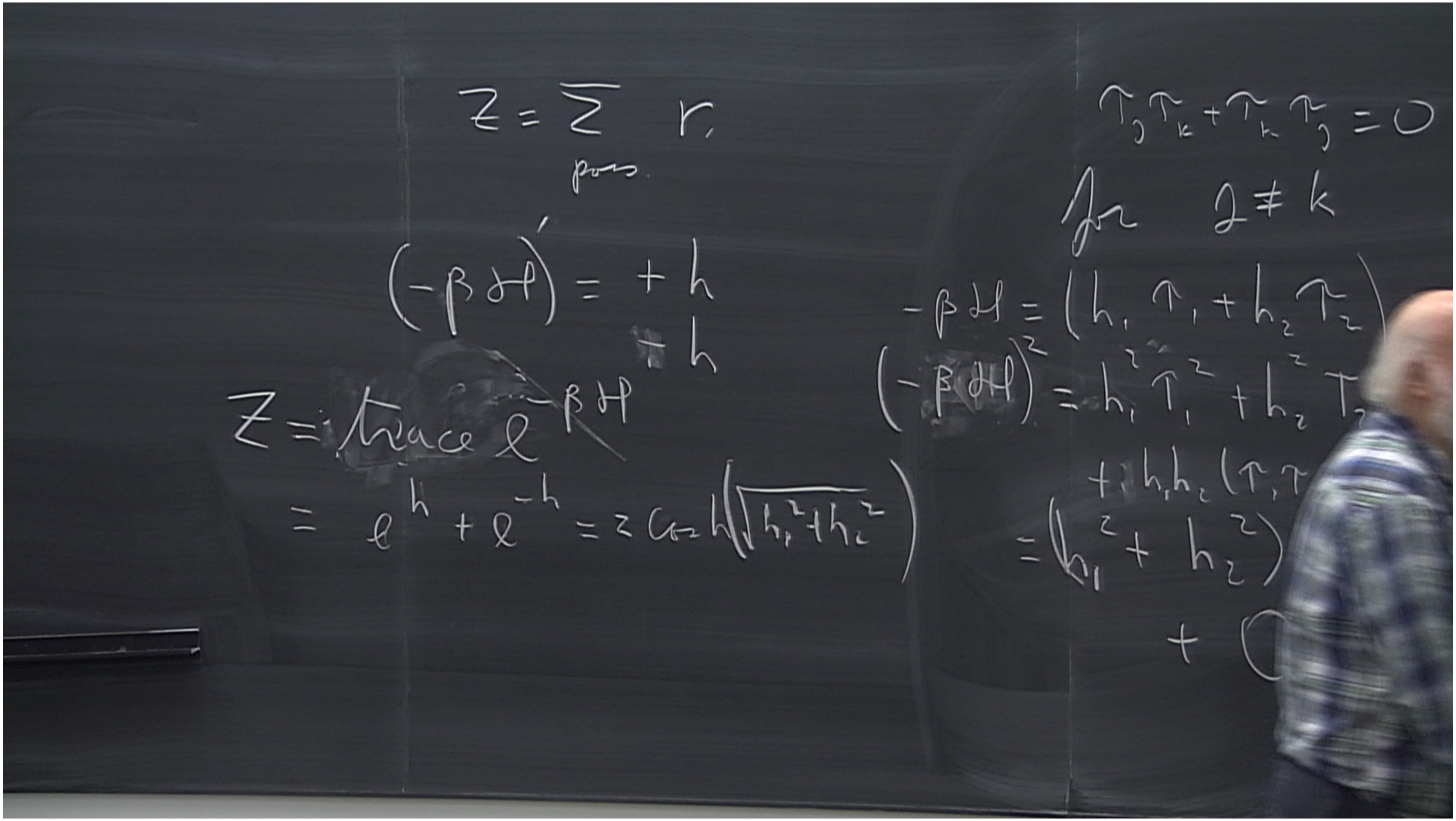
$$(-\beta \mathcal{H})^2 = h_1^2 \tau_1^2 + h_2^2 \tau_2^2 + h_1 h_2 (\tau_1 \tau_2 + \tau_2 \tau_1)$$

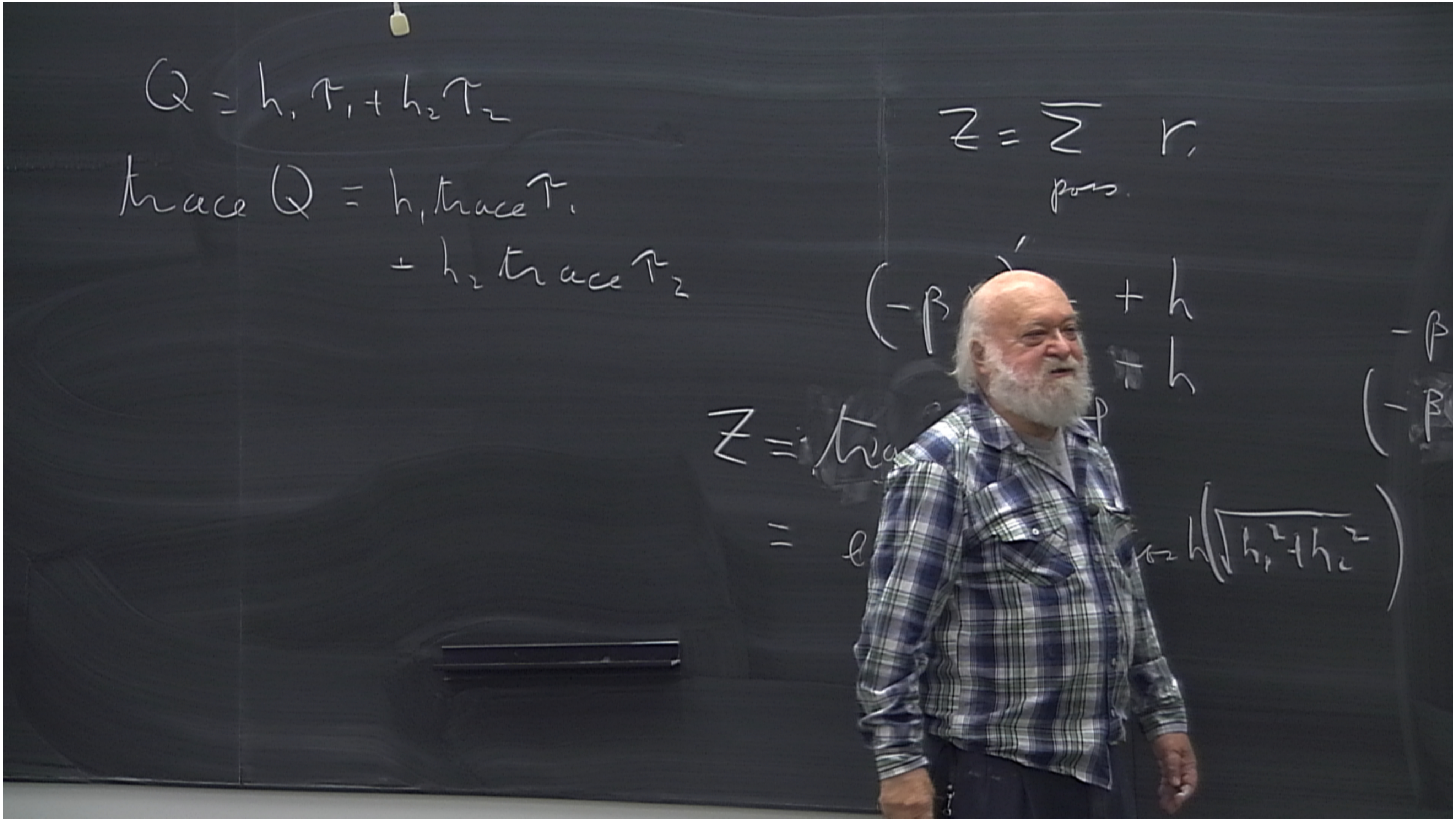
$$= (h_1^2 + h_2^2)\, \mathbb{1} + 0$$

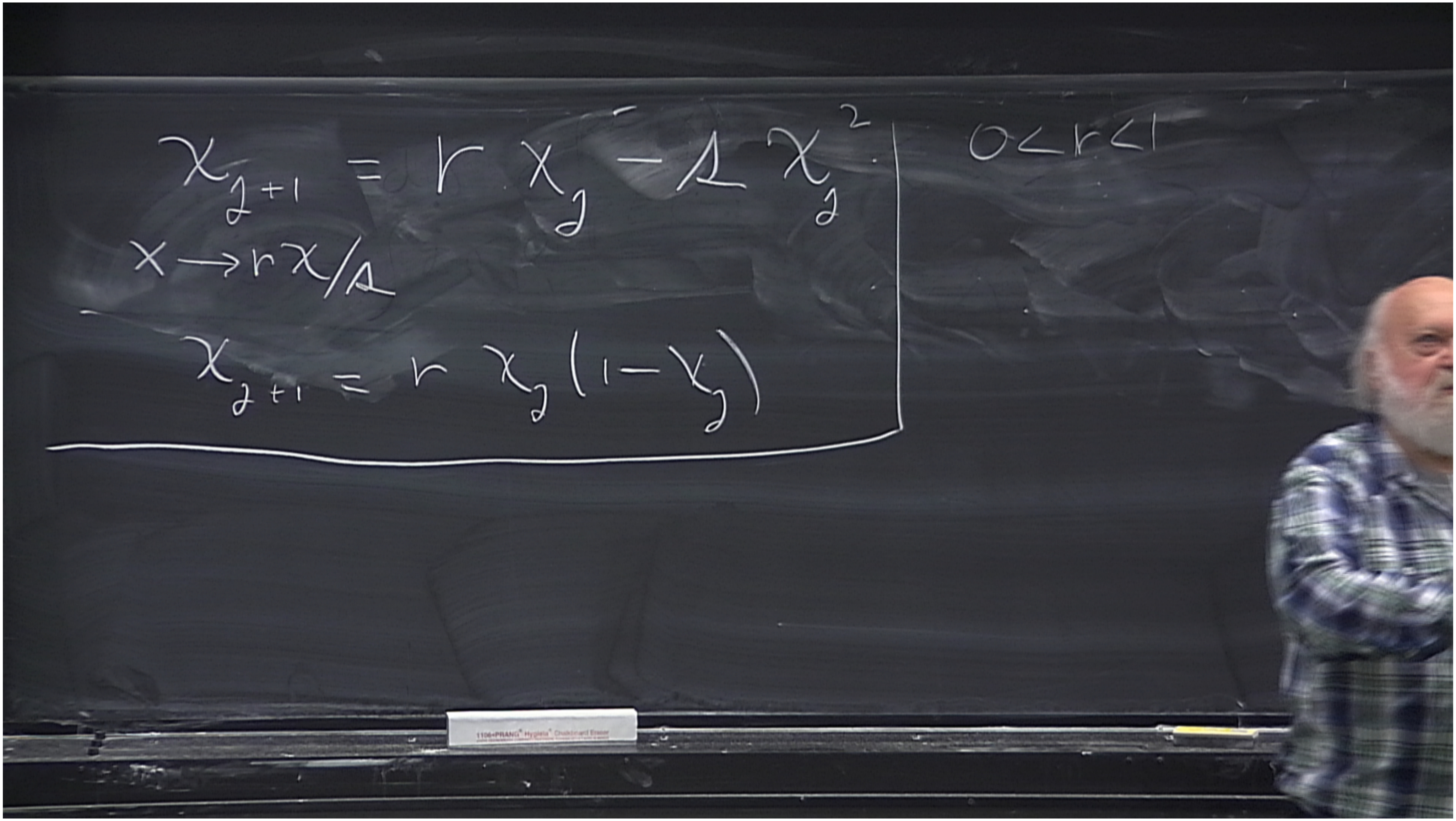
$$\tau_j^2 =$$

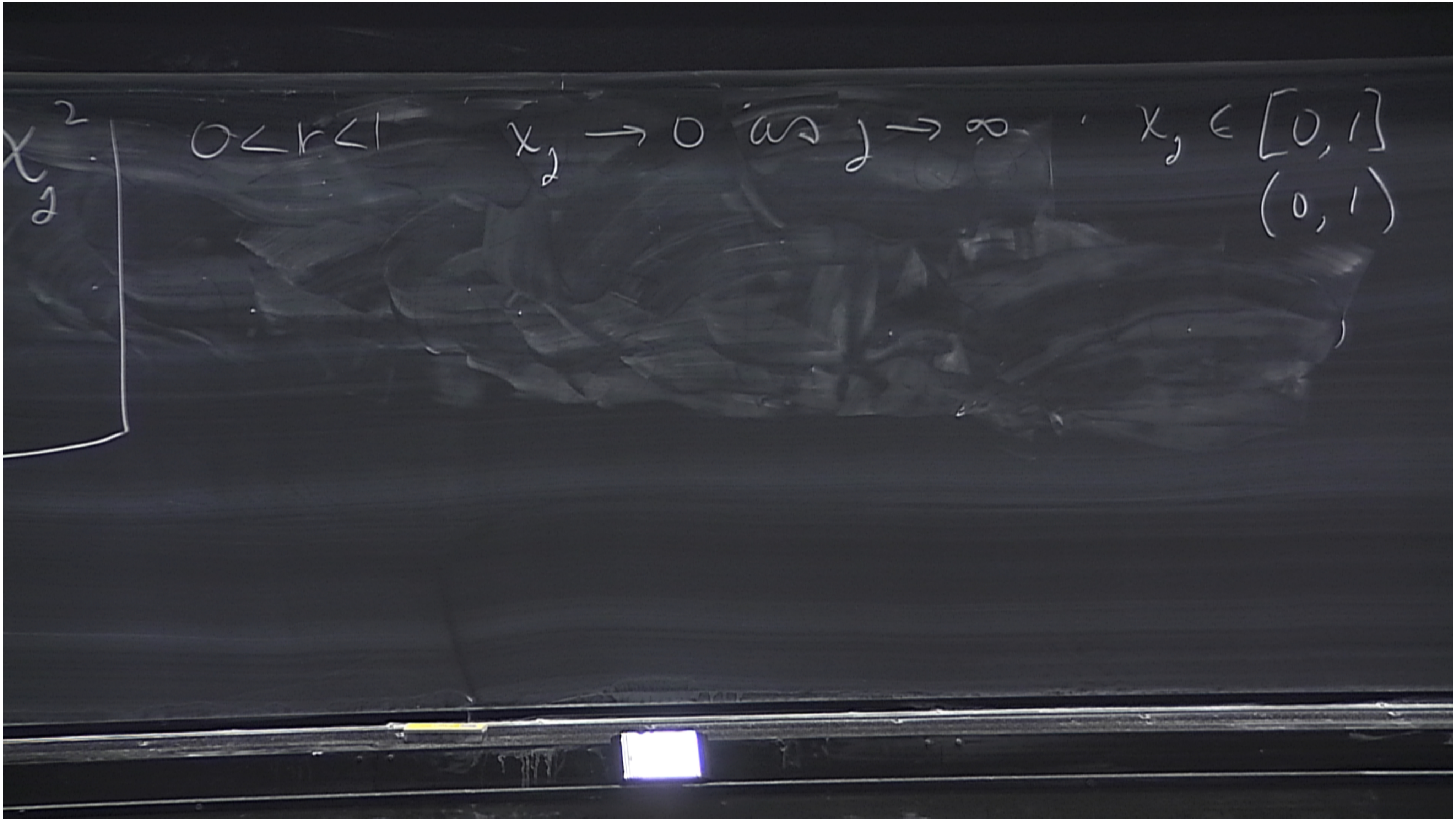
Pauli spin mat

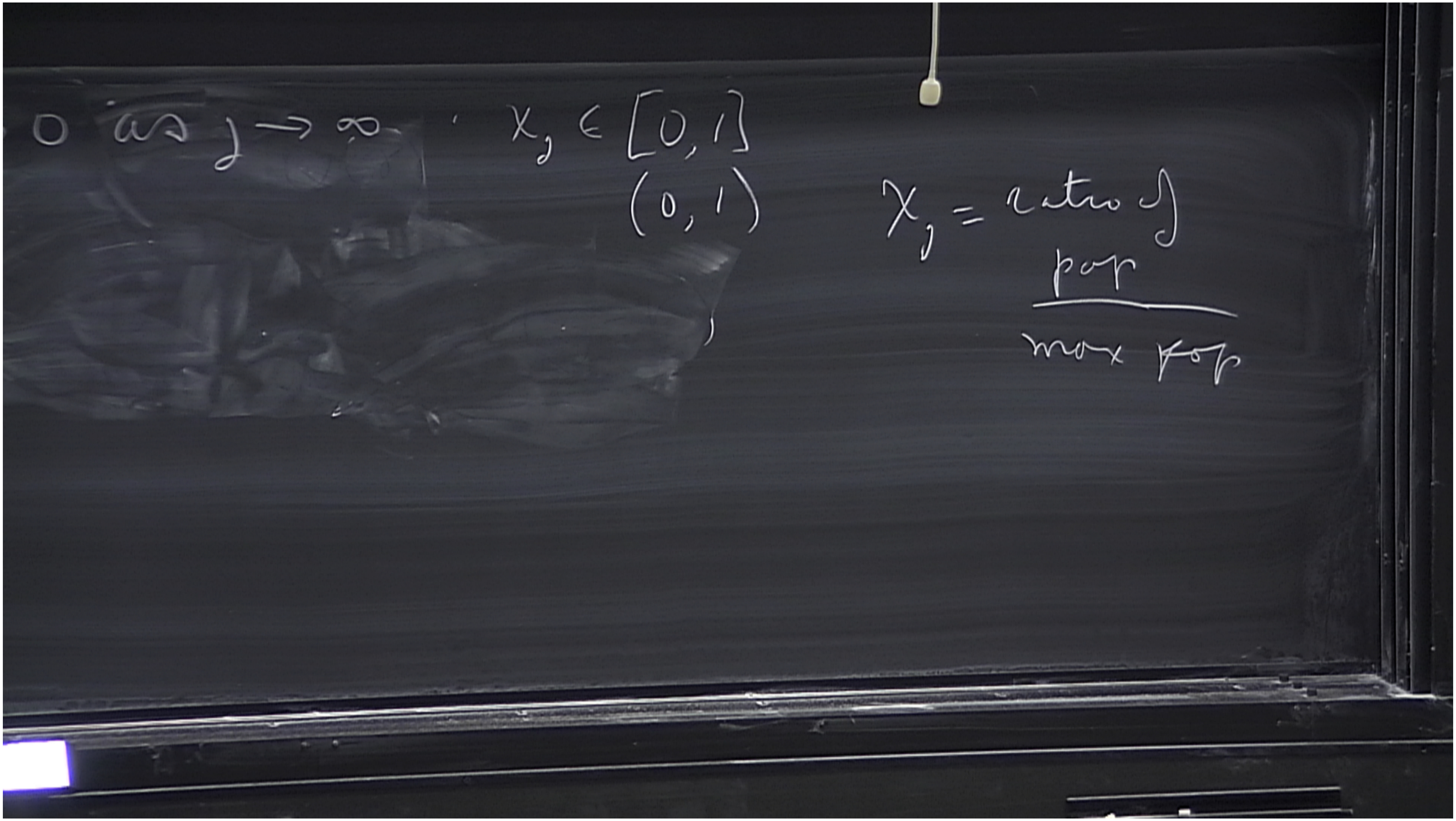
$$\tau_j \tau_k = i$$

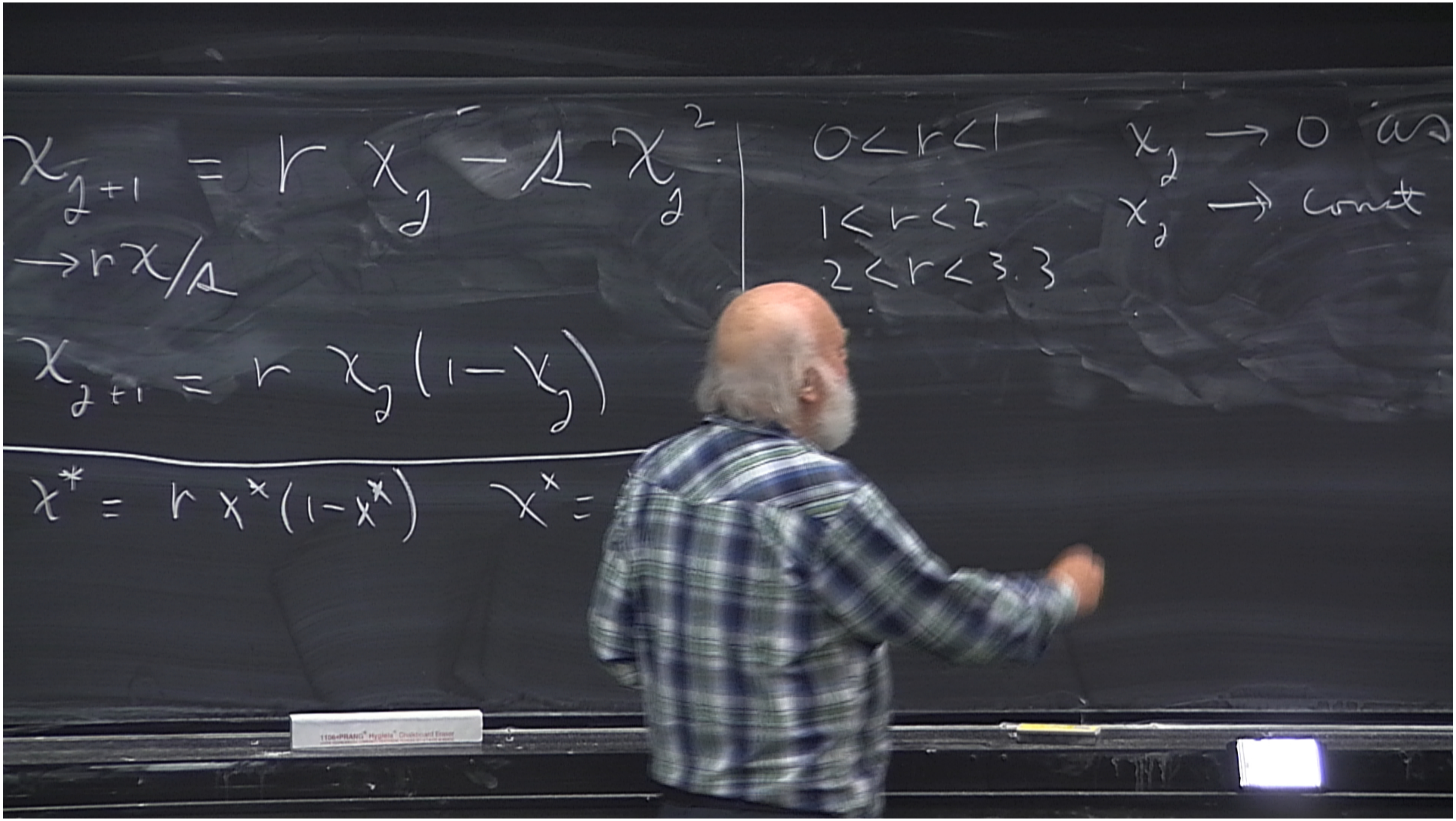

Z = Σ r
pos.
(−βℋ)′ = +h
−h
Z = trace e^−βℋ
= e^h + e^−h = 2 cosh(√(h1²+h2²))
σj σk + σk σj = 0
for j ≠ k
−βℋ = (h1 σ1 + h2 σ2
(−βℋ)² = h1² σ1² + h2² σ2²
+ h1h2 (σ1σ
= (h1² + h2²)
+

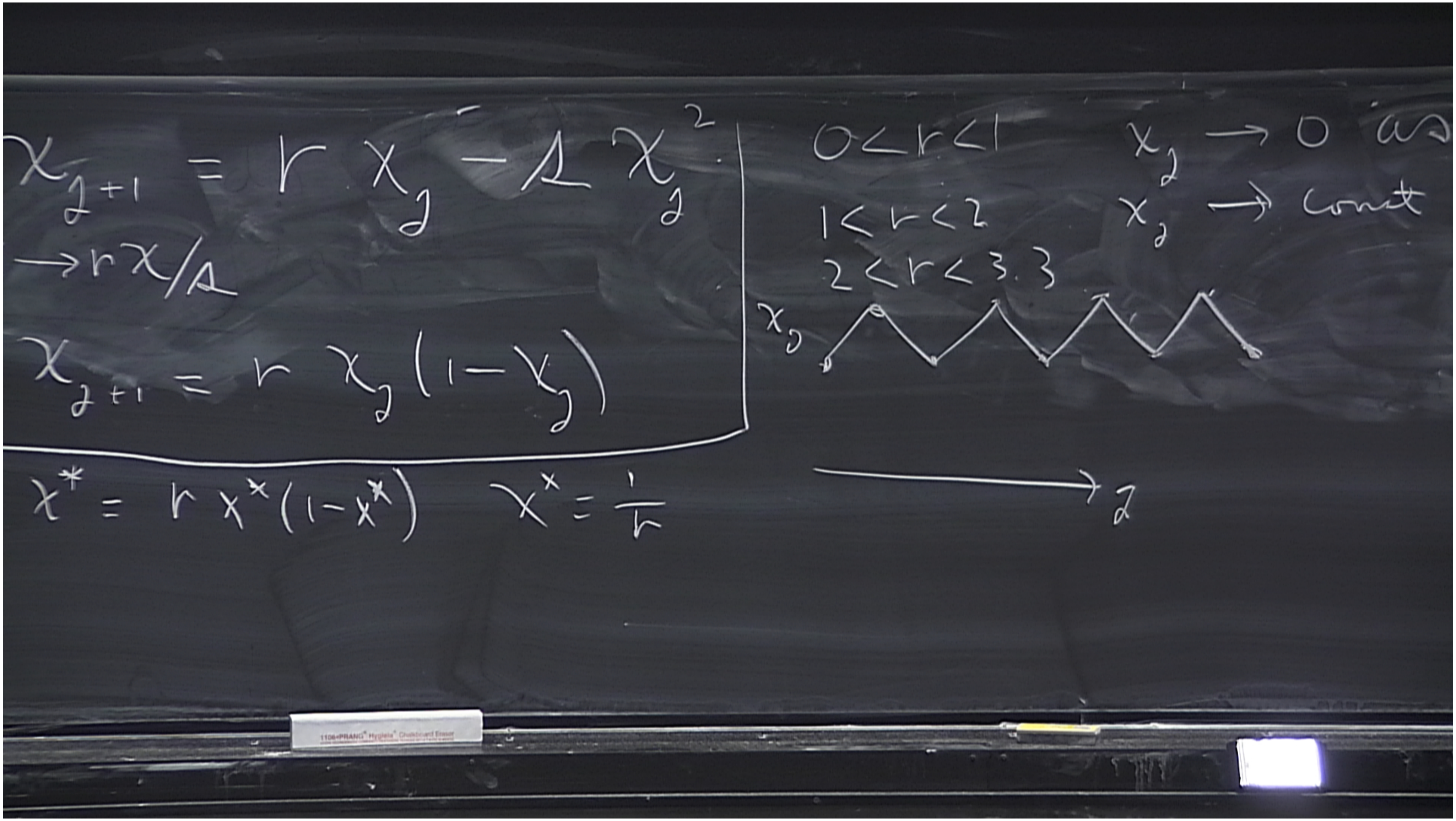

$Q = h_1 \tau_1 + h_2 \tau_2$
trace $Q = h_1$ trace τ_1
$+ h_2$ trace τ_2
$Z = \sum r_i$
pos.

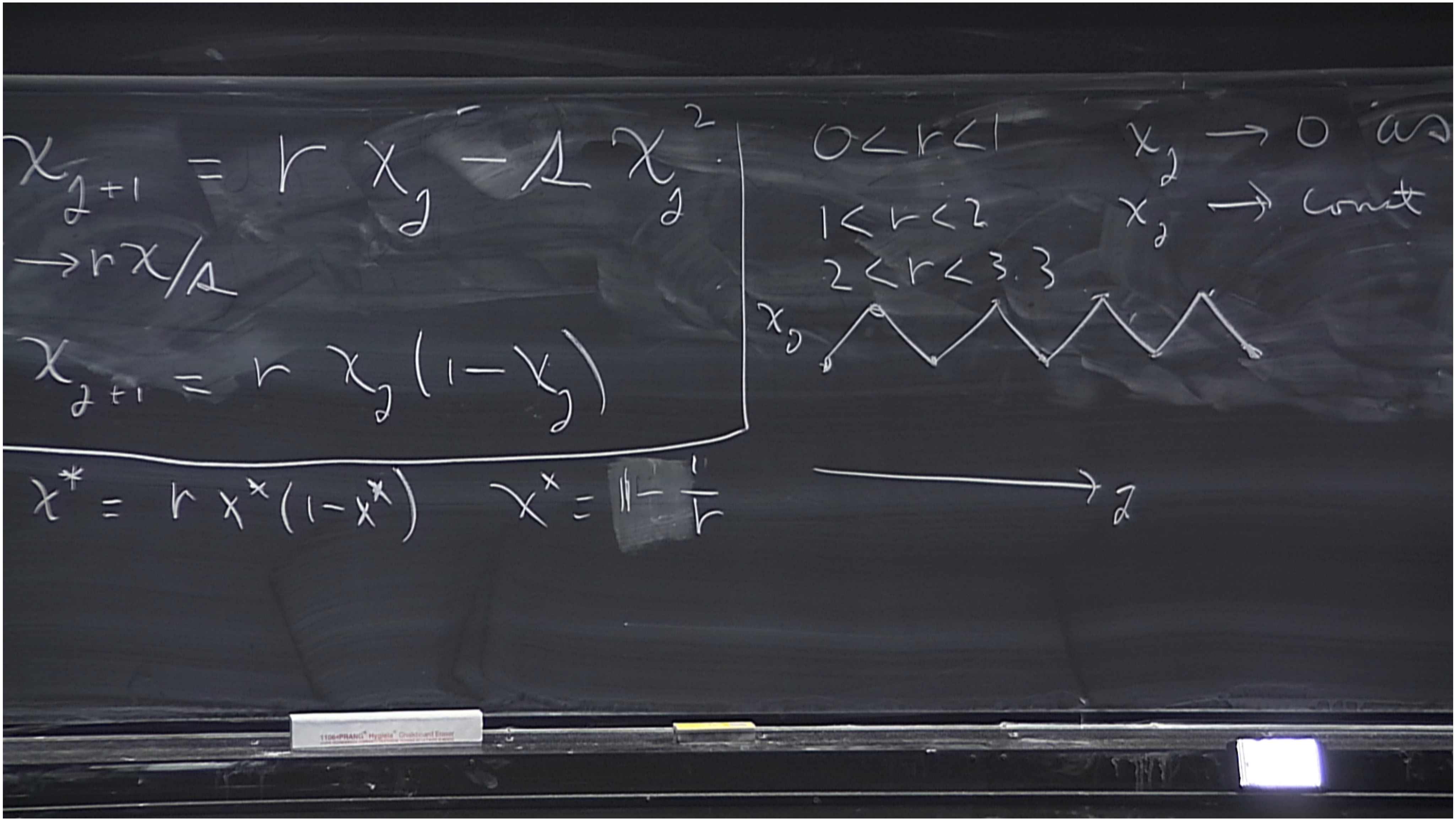

$x_{j+1} = r\,x_j - s\,x_j^2$
$0<r<1$
$x \to rx/s$
$x_{j+1} = r\,x_j(1-x_j)$


x_j^2
$0<r<1$
$x_j \to 0$ as $j \to \infty$
$x_j \in [0,1]$
$(0,1)$


0 as j → ∞
x_j ∈ [0, 1]
(0, 1)
x_j = ratio of pop
max pop


$x_{g+1} = r x_g - s x_g^2$
$\to r x / s$
$x_{g+1} = r x_g (1 - x_g)$
$x^* = r x^* (1 - x^*)$
$x^* =$
$0 < r < 1$ $x_g \to 0$ as
$1 < r < 2$ $x_g \to$ const
$2 < r < 3.3$


$x_{j+1} = r x_j - s x_j^2$
$\rightarrow r x / s$
$x_{j+1} = r x_j (1 - x_j)$
$x^* = r x^* (1 - x^*)$ $x^* = \frac{1}{r}$
$0 < r < 1$ $x_j \rightarrow 0$ as
$1 < r < 2$ $x_j \rightarrow$ const
$2 < r < 3.3$
x_j
j


$x_{j+1} = r x_j - s x_j^2$
$\to r x / s$
$x_{j+1} = r x_j (1 - x_j)$
$x^* = r x^* (1 - x^*)$
$x^* = 1 - \frac{1}{r}$
$0 < r < 1$ $x_j \to 0$ as
$1 < r < 2$ $x_j \to$ Const
$2 < r < 3.3$
x_j
j