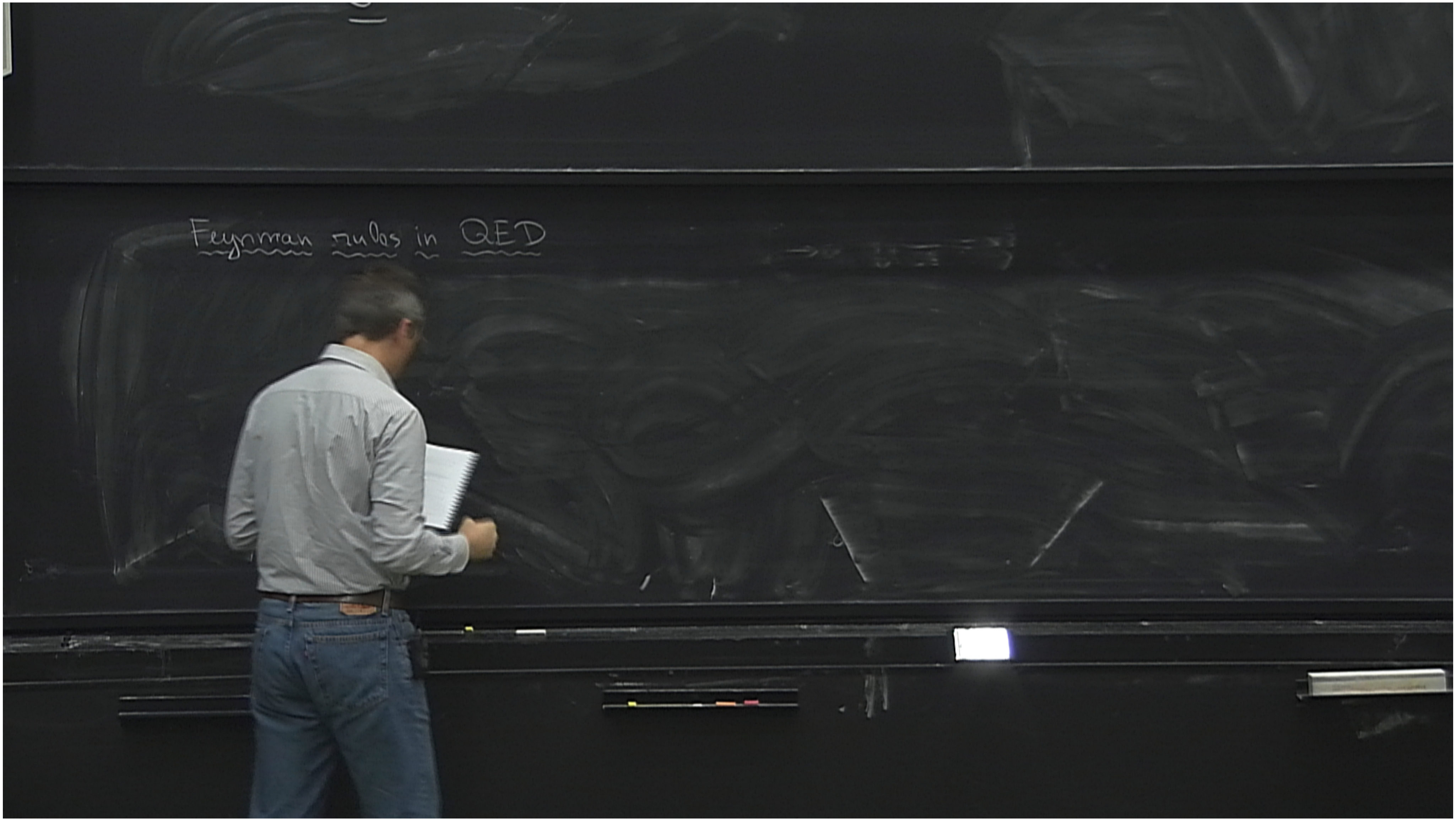
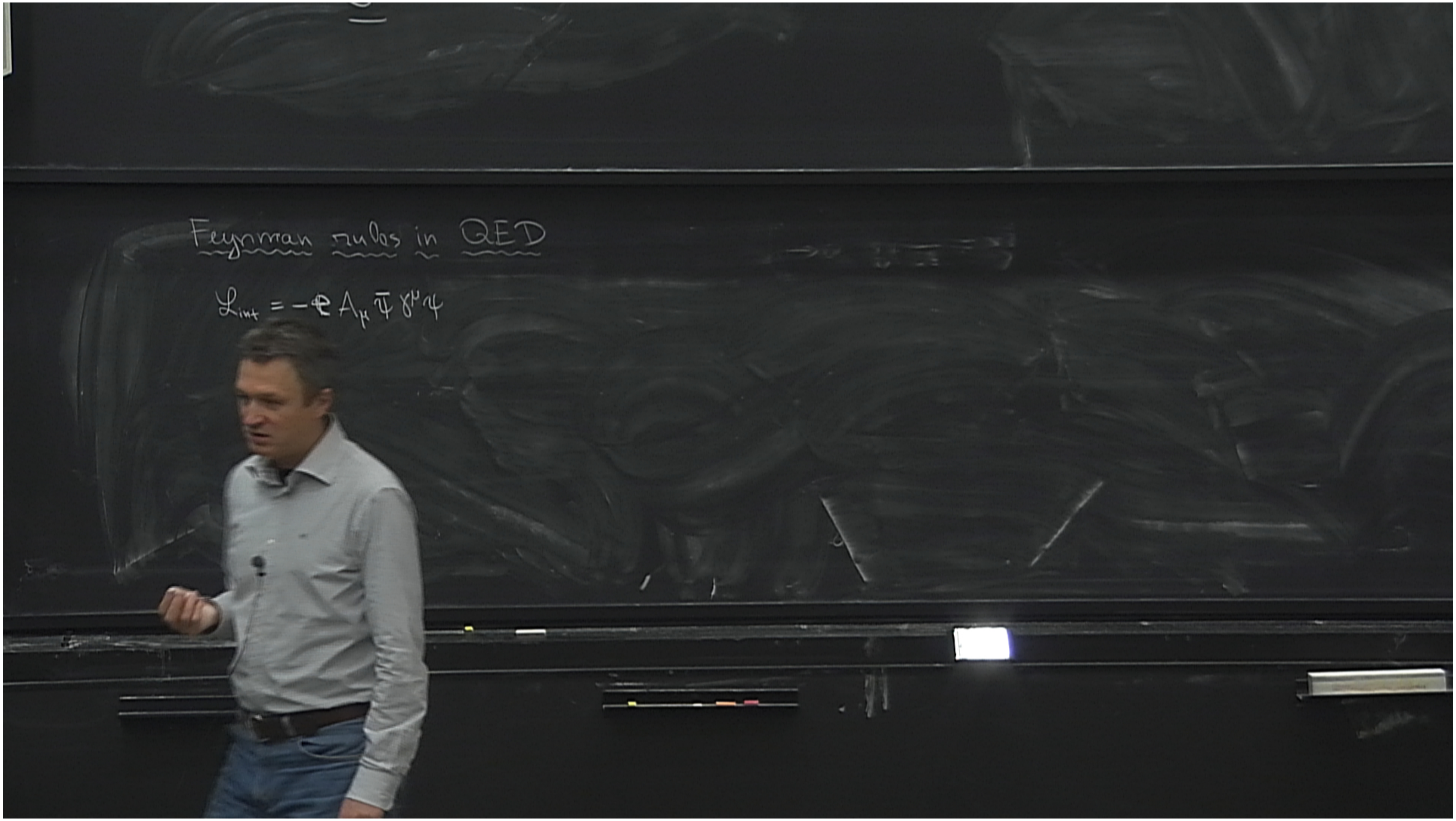
Title: Quantum Field Theory I - Lecture 14

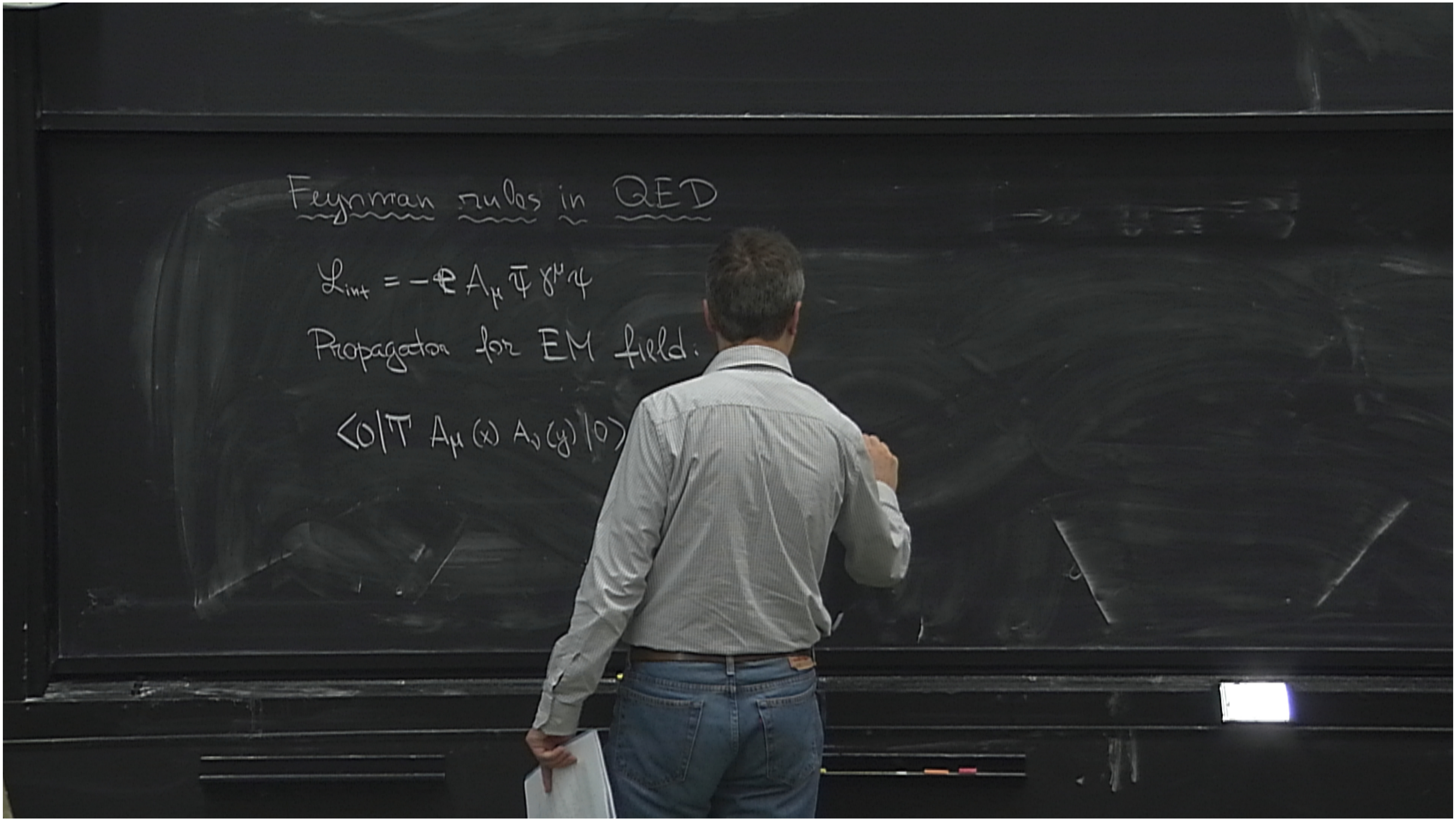
Date: Oct 21, 2011 09:00 AM

URL: http://pirsa.org/11100024

Abstract:


Feynman rules in QED


Feynman rules in QED
$\mathcal{L}_{int} = -e A_\mu \bar{\Psi} \gamma^\mu \psi$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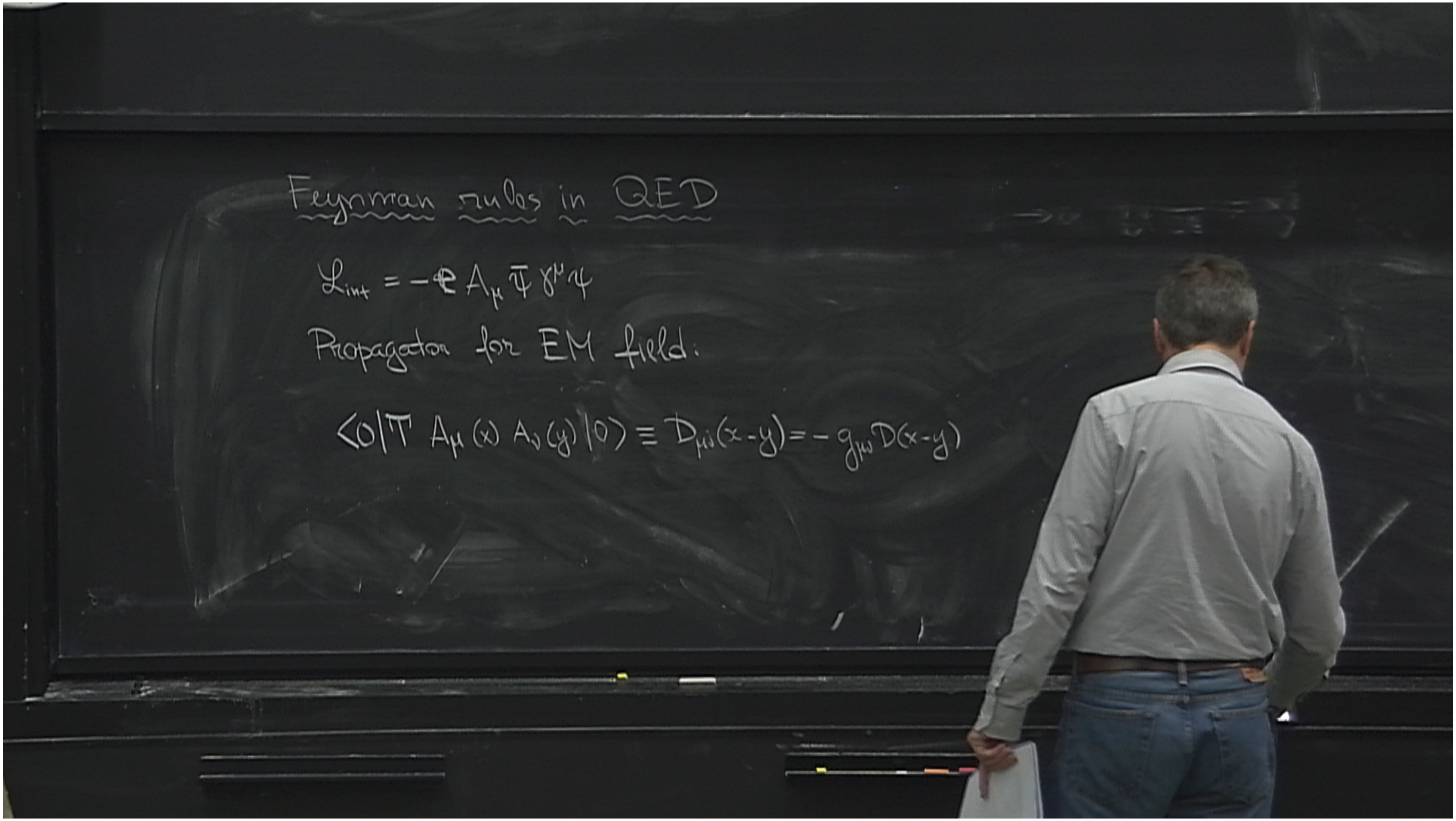
Propagator for EM field:
$\langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle$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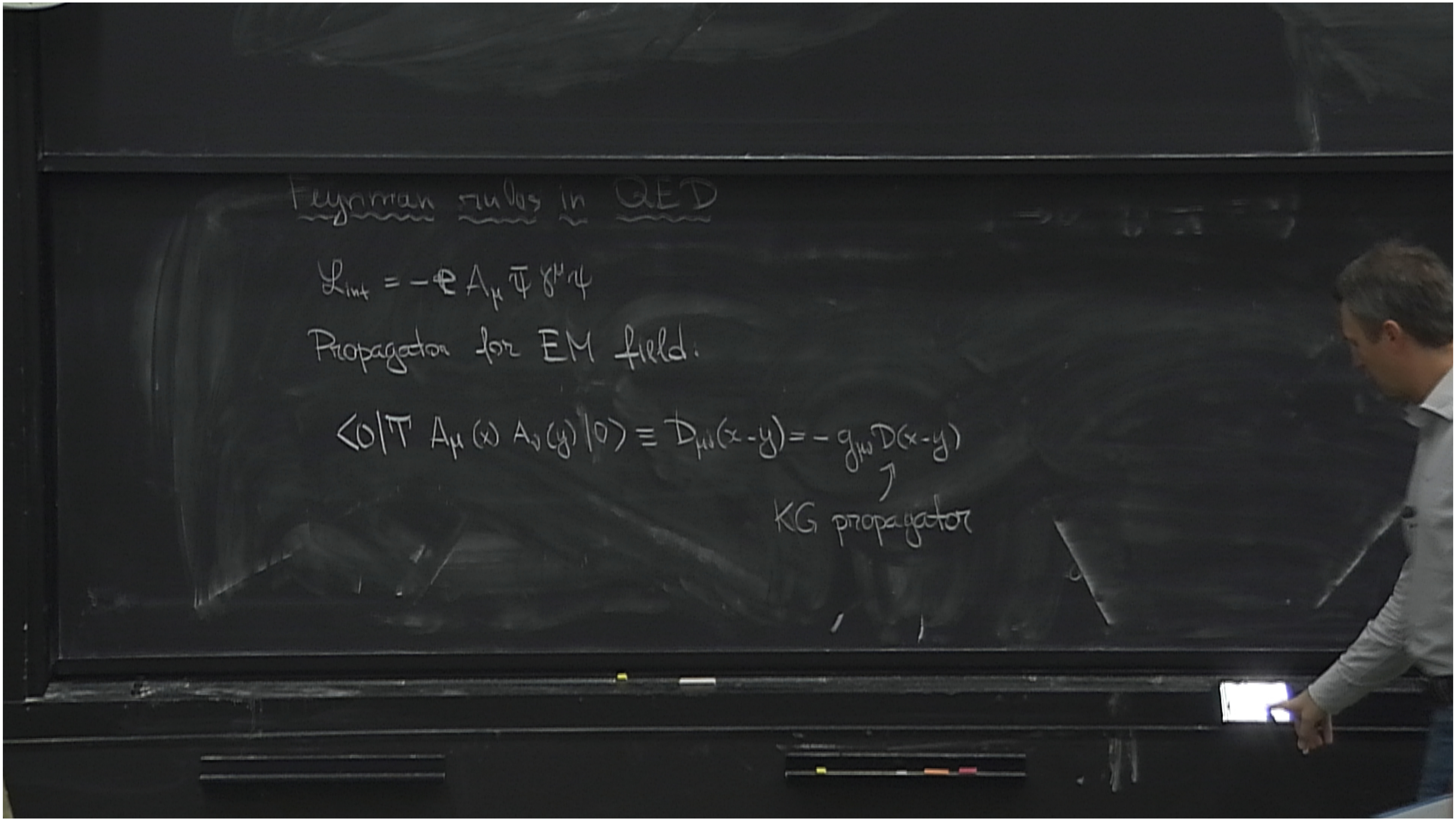
Propagator for EM field:
$\langle 0 | T\, A_\mu(x) A_\nu(y) | 0 \rangle \equiv D_{\mu\nu}(x-y) = -g_{\mu\nu} D(x-y)$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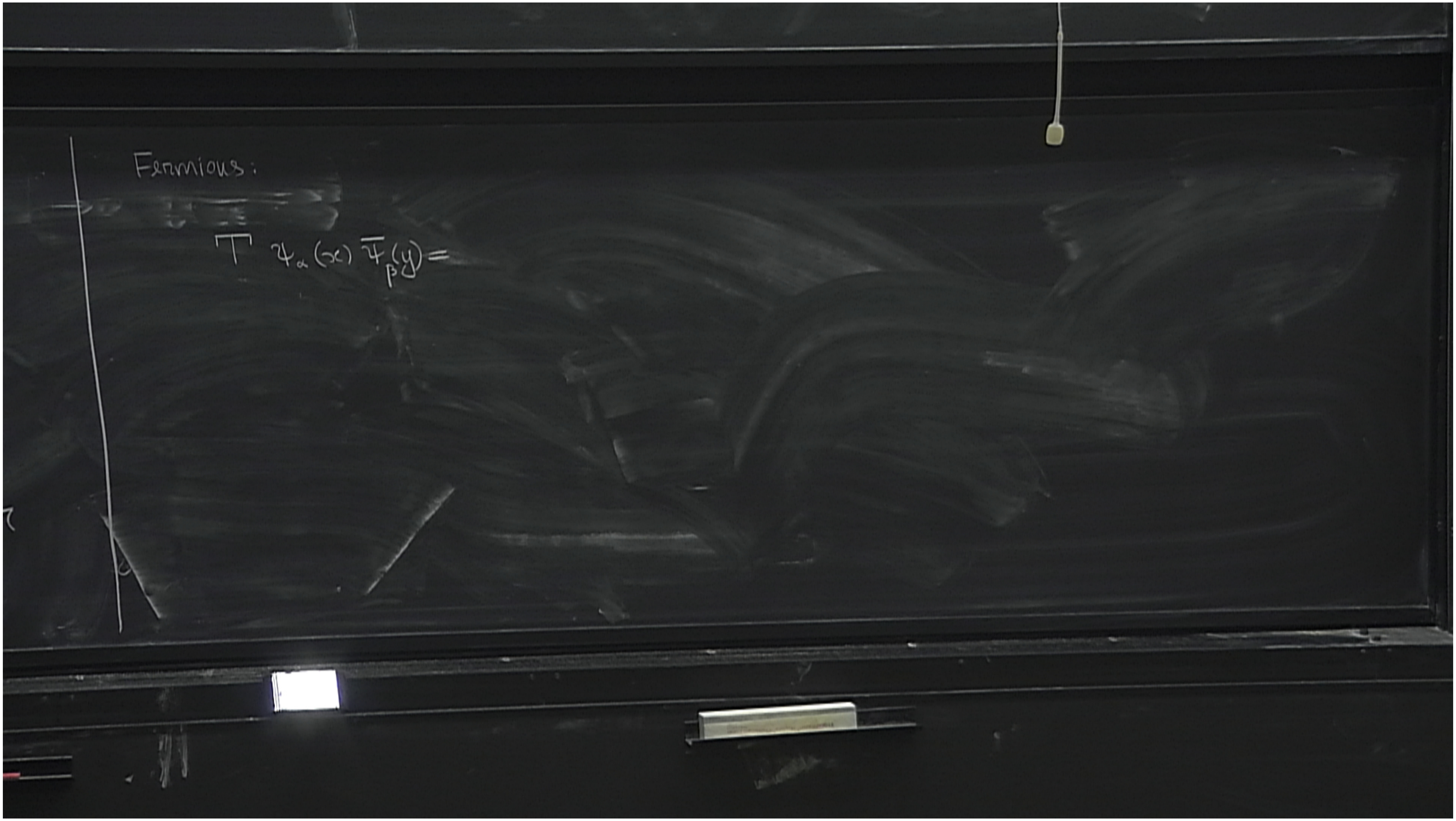
KG propagator


Fermions:
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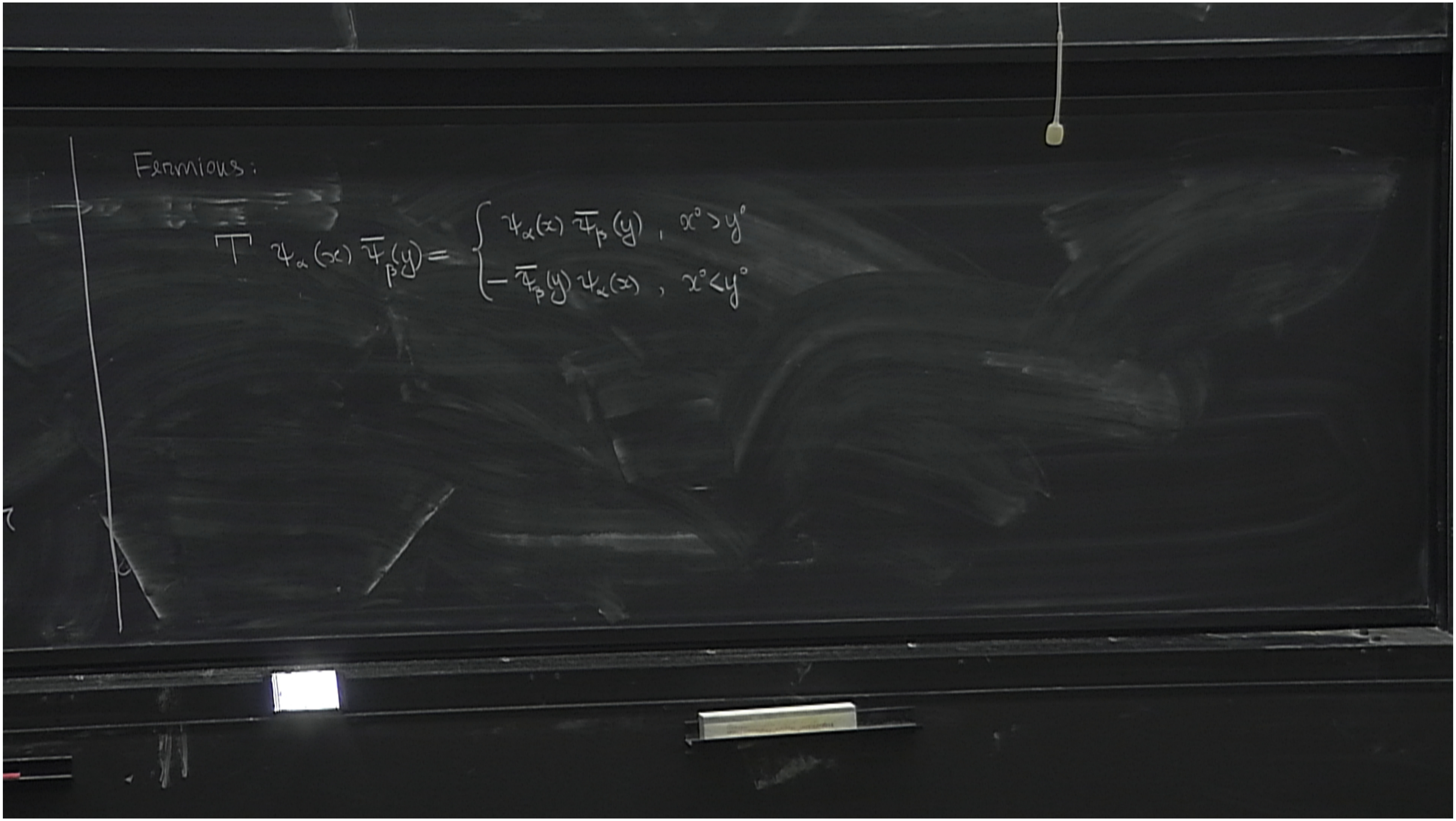
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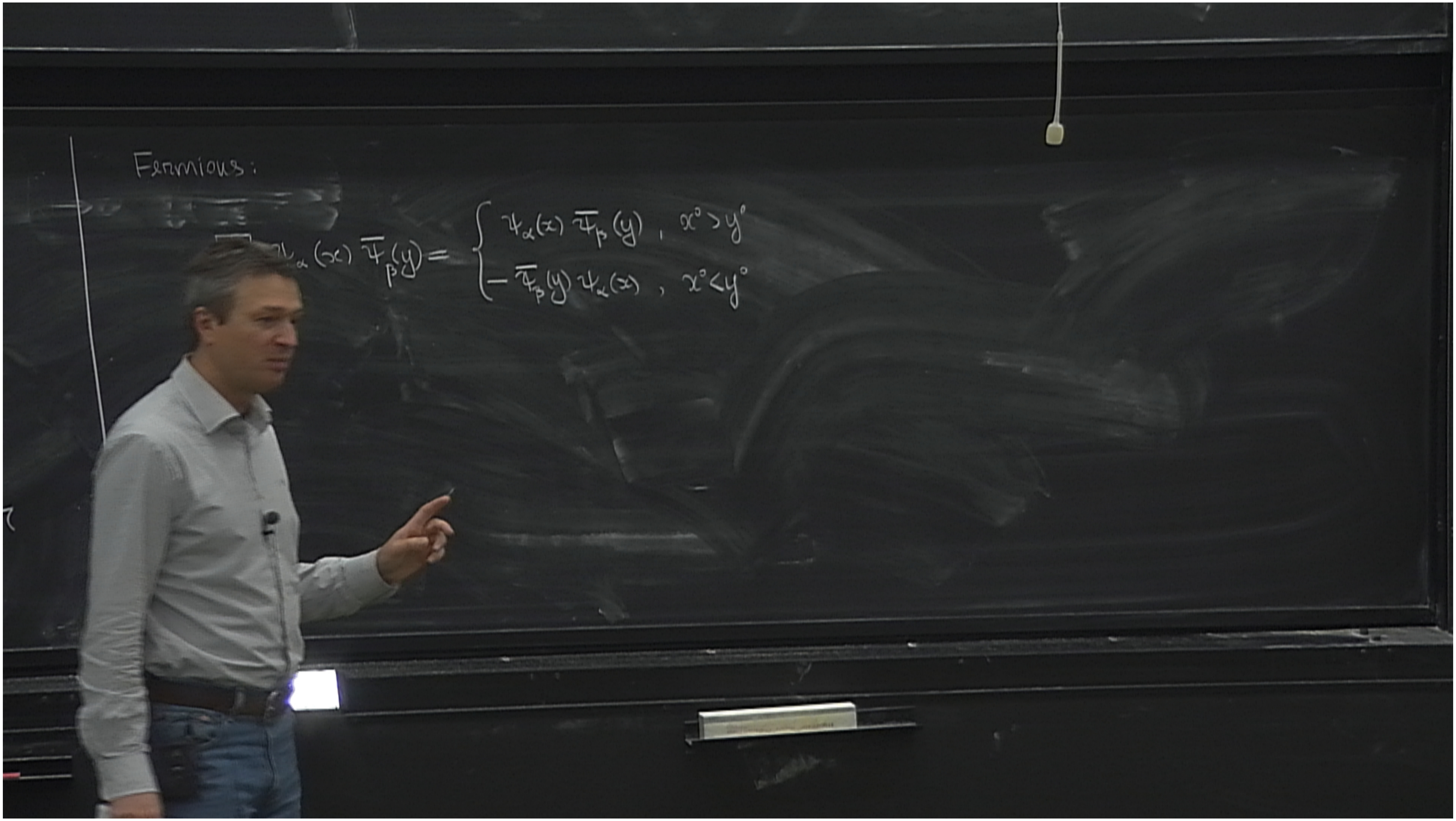
$T\,\psi_\alpha(x)\,\bar\psi_\beta(y) = \begin{cases} \psi_\alpha(x)\,\bar\psi_\beta(y), & x^0 > y^0 \\ -\bar\psi_\beta(y)\,\psi_\alpha(x), & x^0 < y^0 \end{cases}$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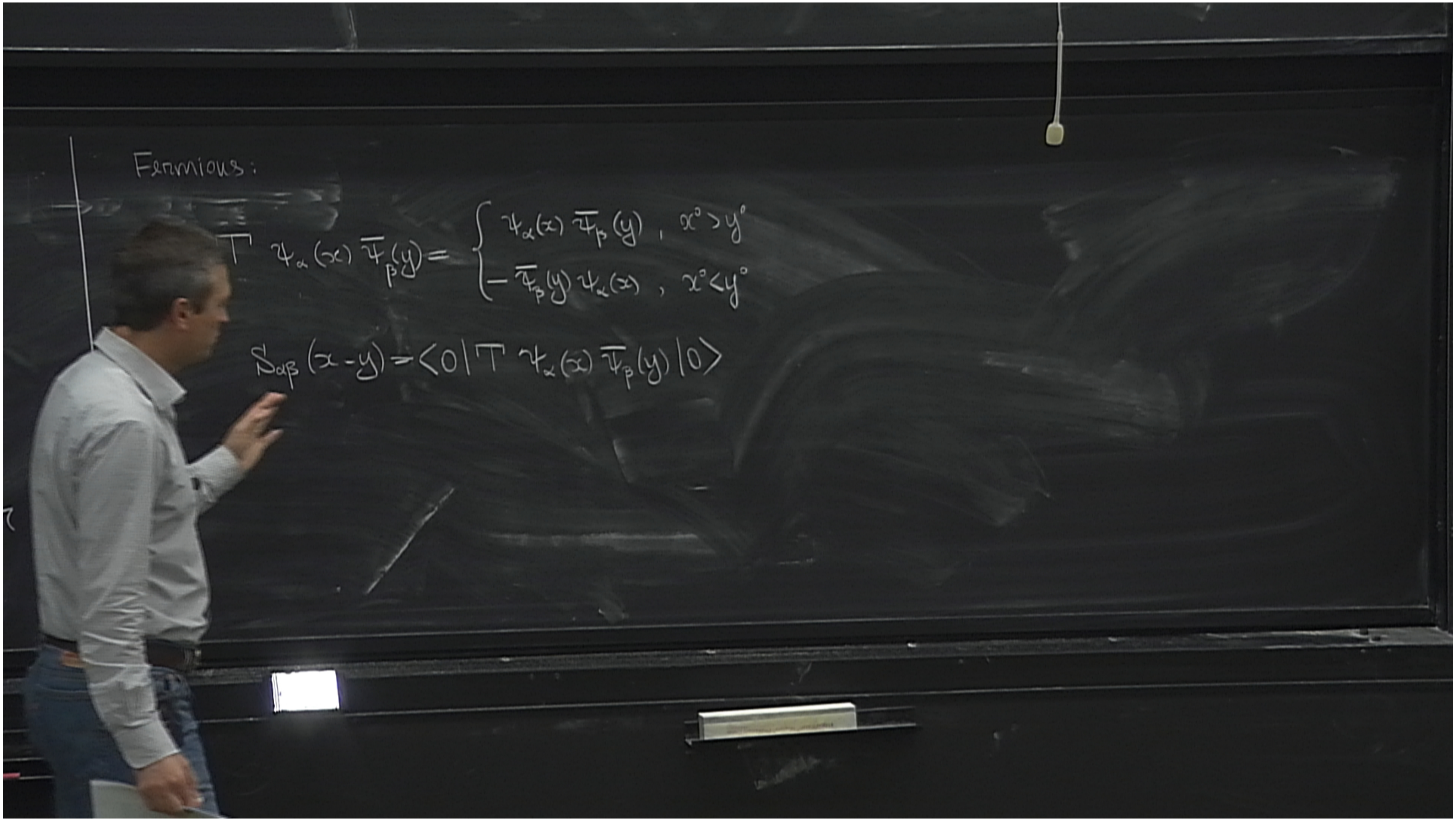
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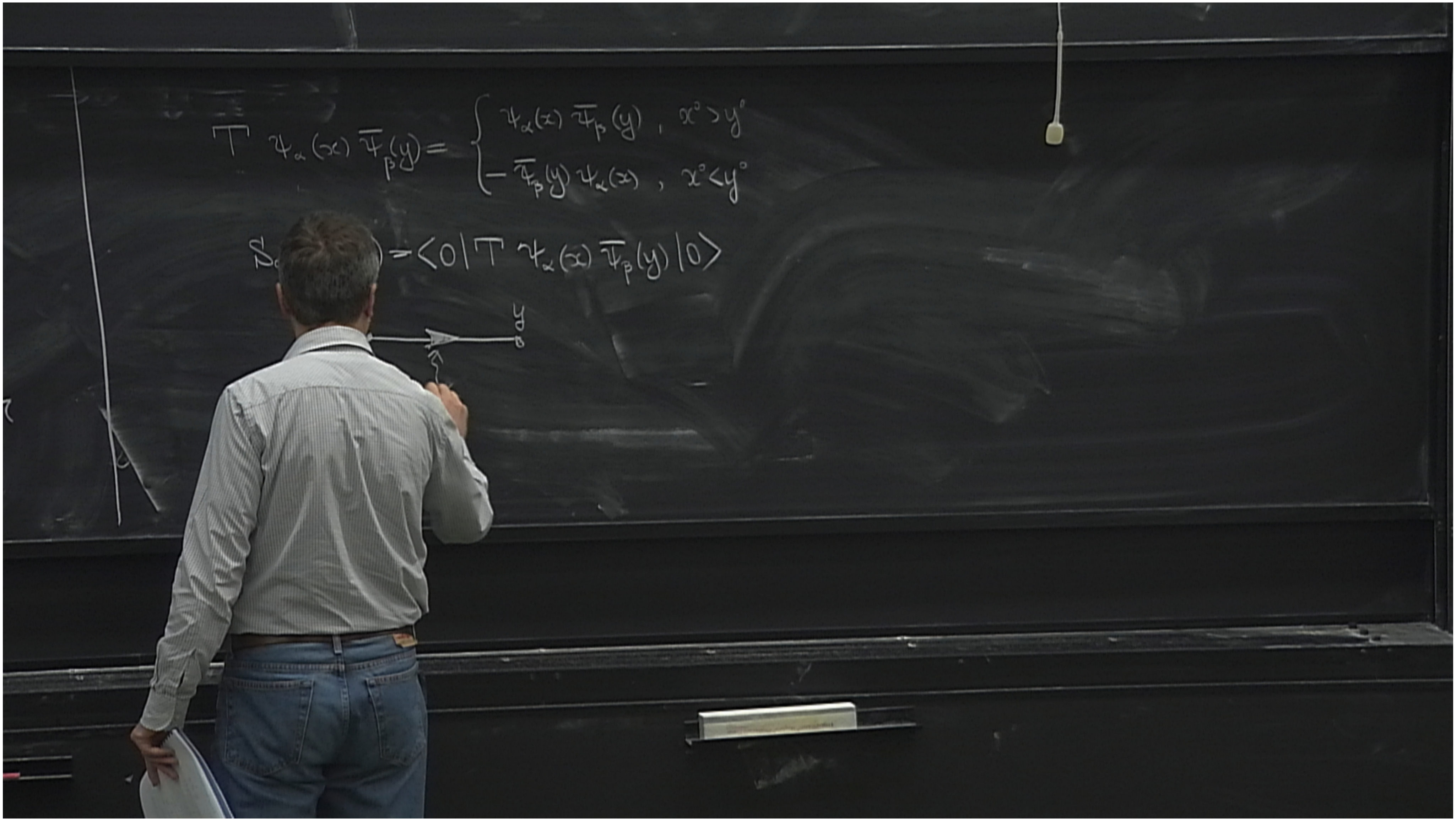
$S_{\alpha\beta}(x-y) = \langle 0|T\,\psi_\alpha(x)\,\bar{\psi}_\beta(y)|0\rangle$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$= \langle 0|T\,\psi_\alpha(x)\,\bar\psi_\beta(y)|0\rangle$
y

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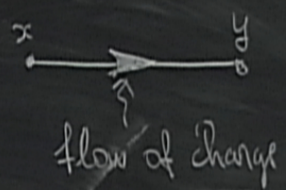

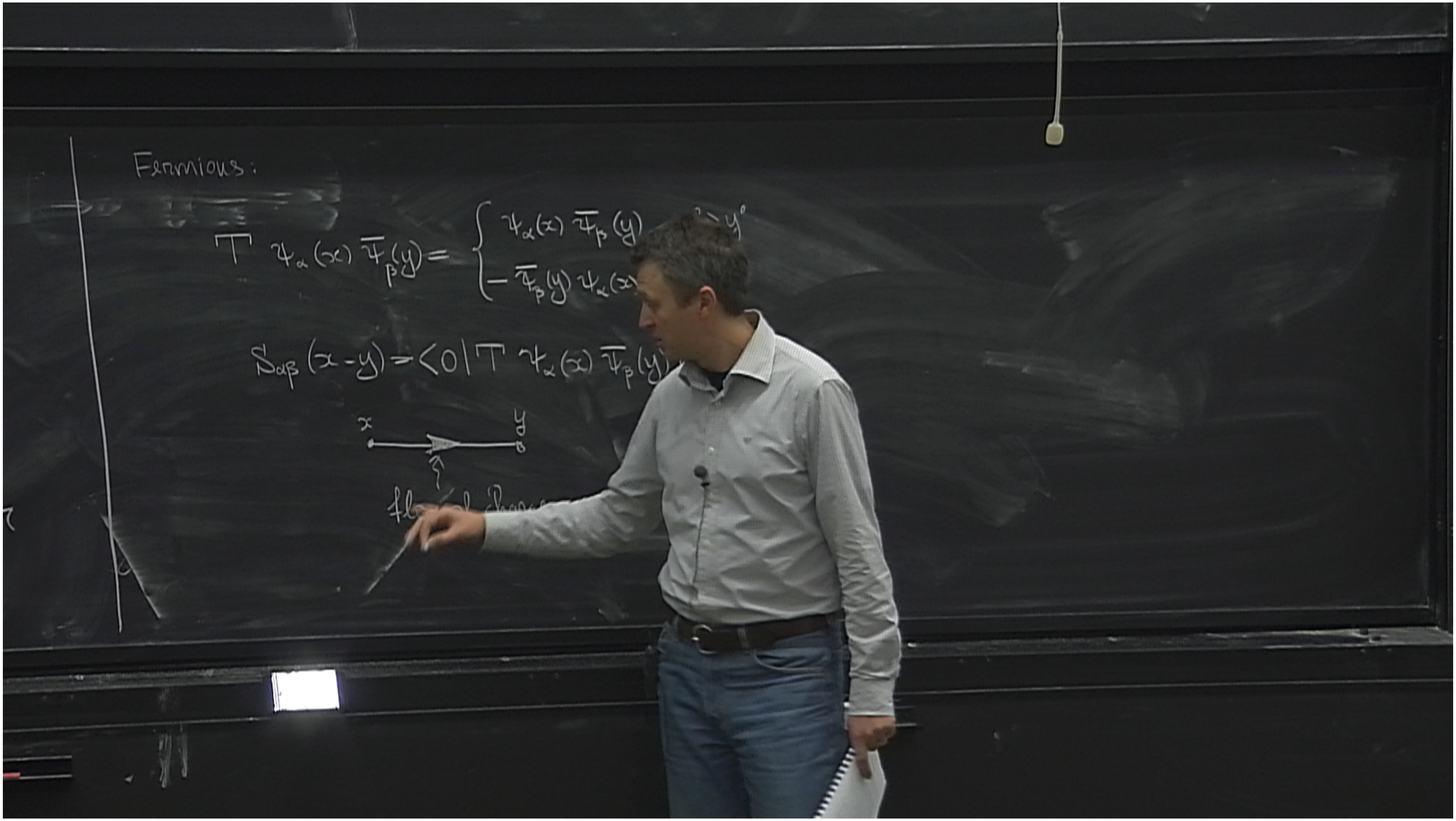
Fermions:
T ψα(x) ψ̄β(y) =
ψα(x) ψ̄β(y)
− ψ̄β(y) ψα(x
Sαβ(x − y) = <0| T ψα(x) ψ̄β(y)
x
y

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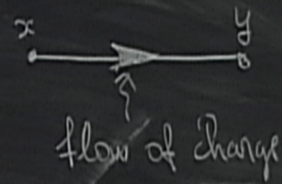

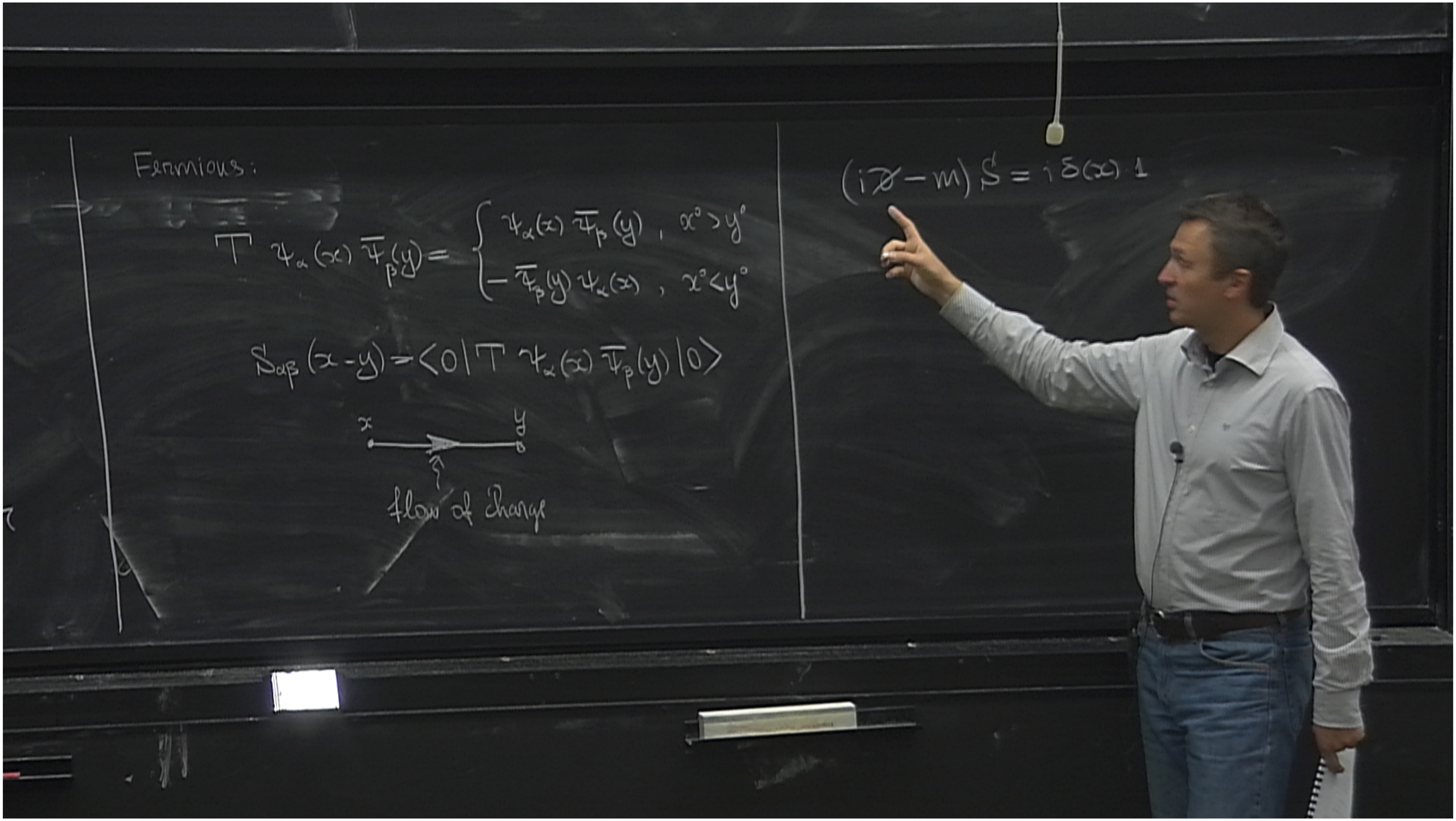
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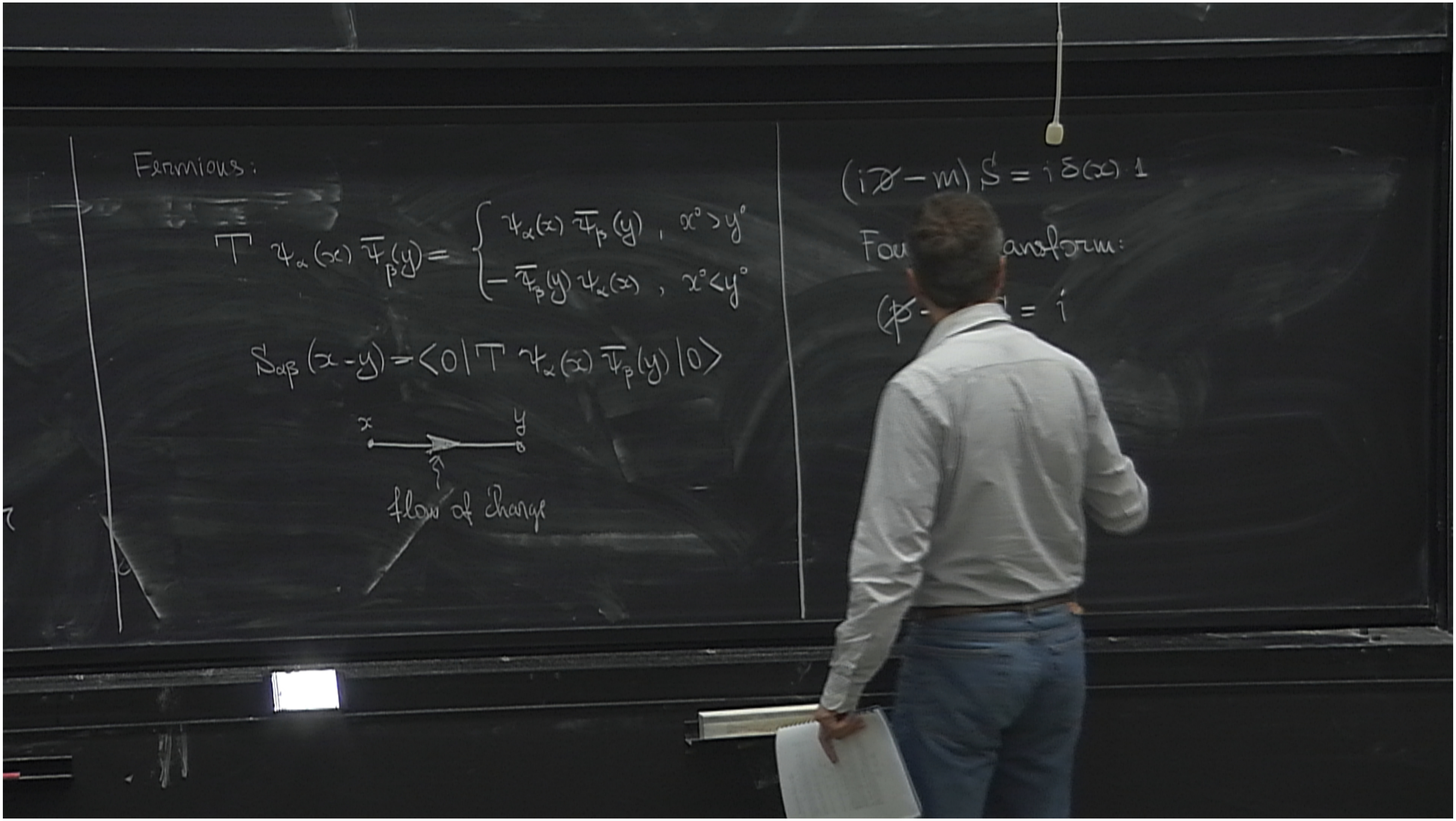
$S_{\alpha\beta}(x-y) = \langle 0 | T\,\psi_\alpha(x)\bar{\psi}_\beta(y) | 0 \rangle$
x
y
flow of charge
$(i\partial\!\!\!/ - m) S = i\delta(x)\cdot 1$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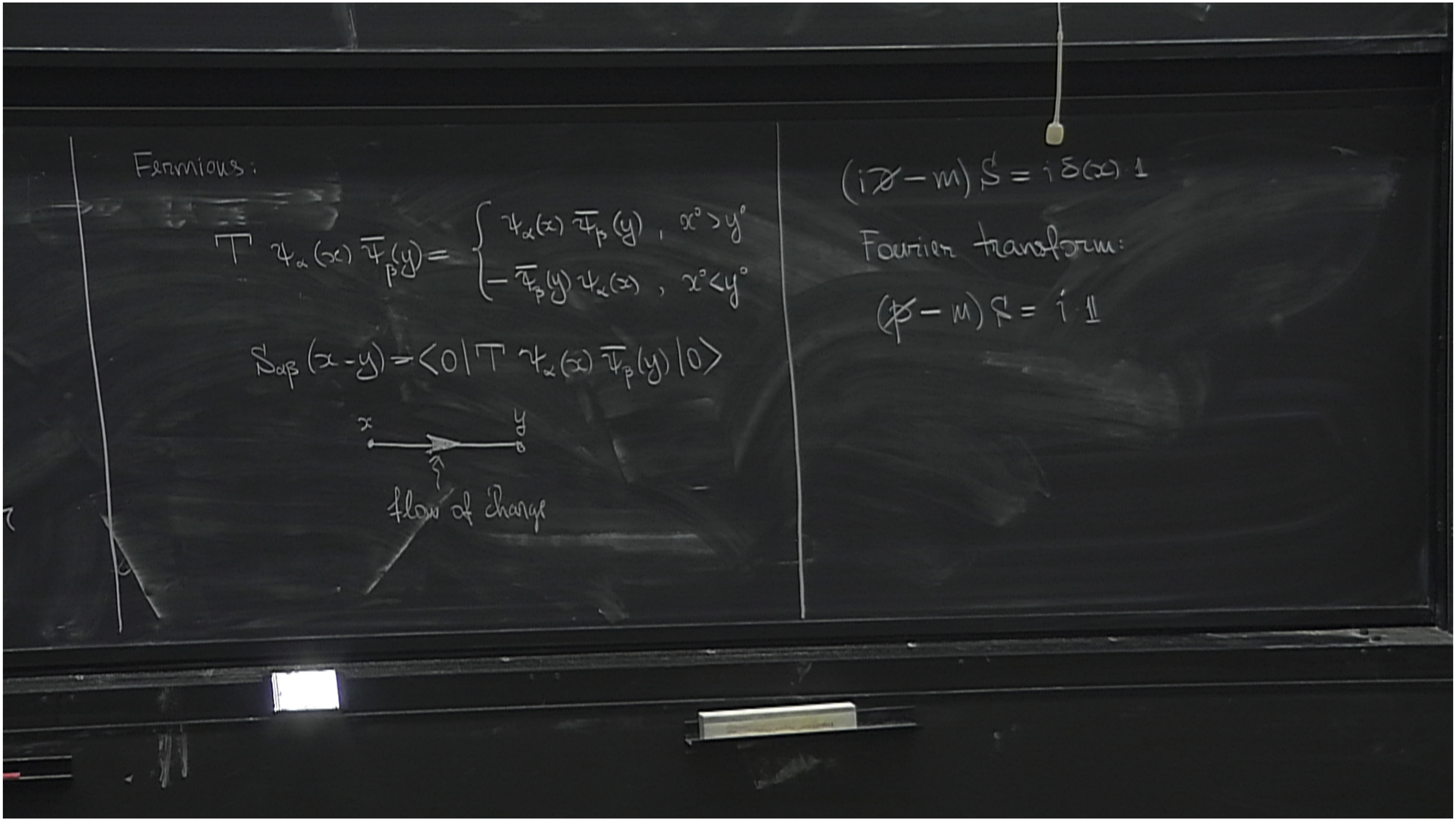
x
y
flow of charge
$(i\partial\!\!\!/ - m)\,S = i\,\delta(x)\,1$
Fou
ansform:
= i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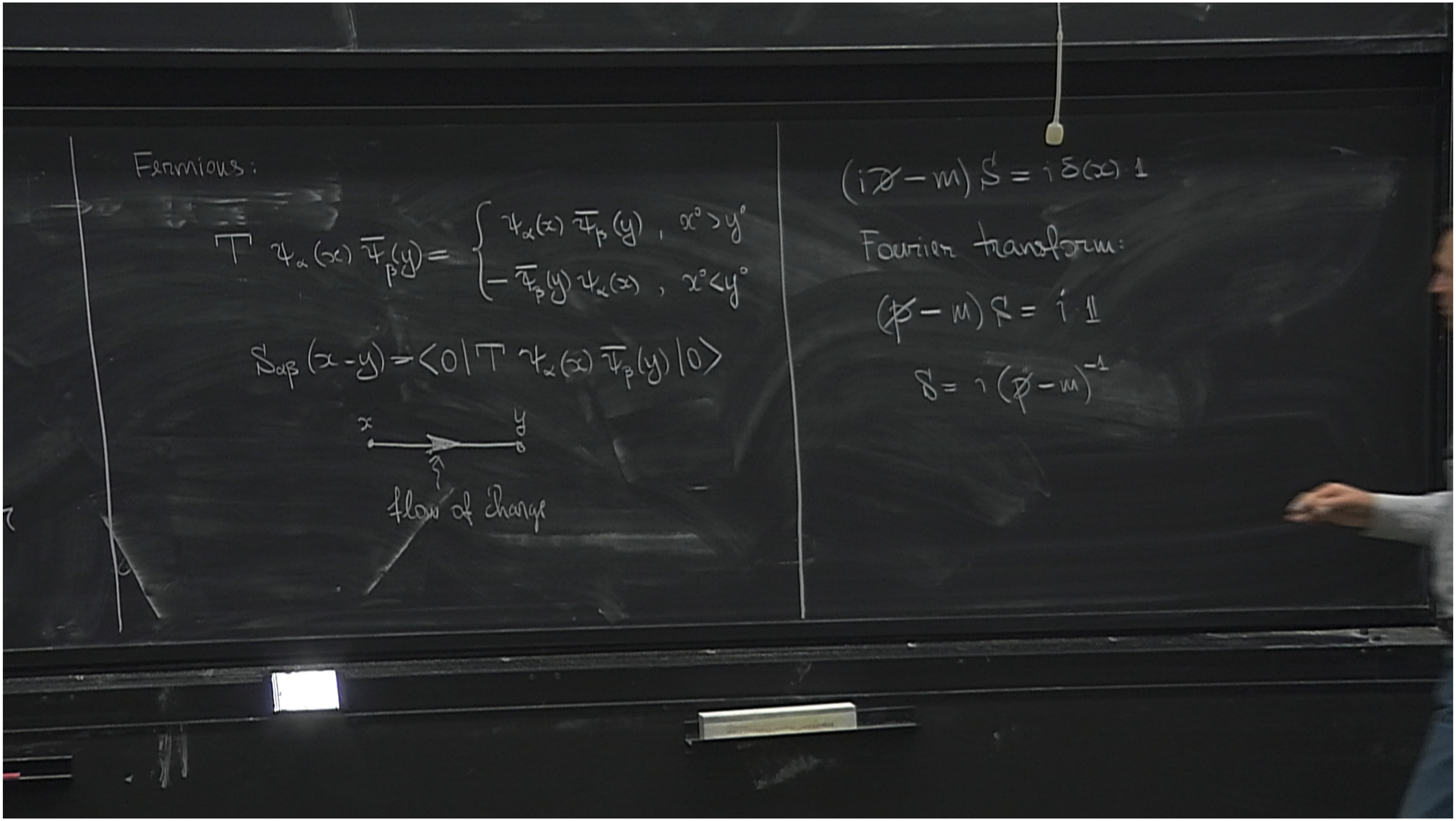
x
y
flow of charge
$(i\partial\!\!\!/ - m)S = i\delta(x)\cdot 1$
Fourier transform:
$(p\!\!\!/ - m)S = i\cdot 1\!\!1$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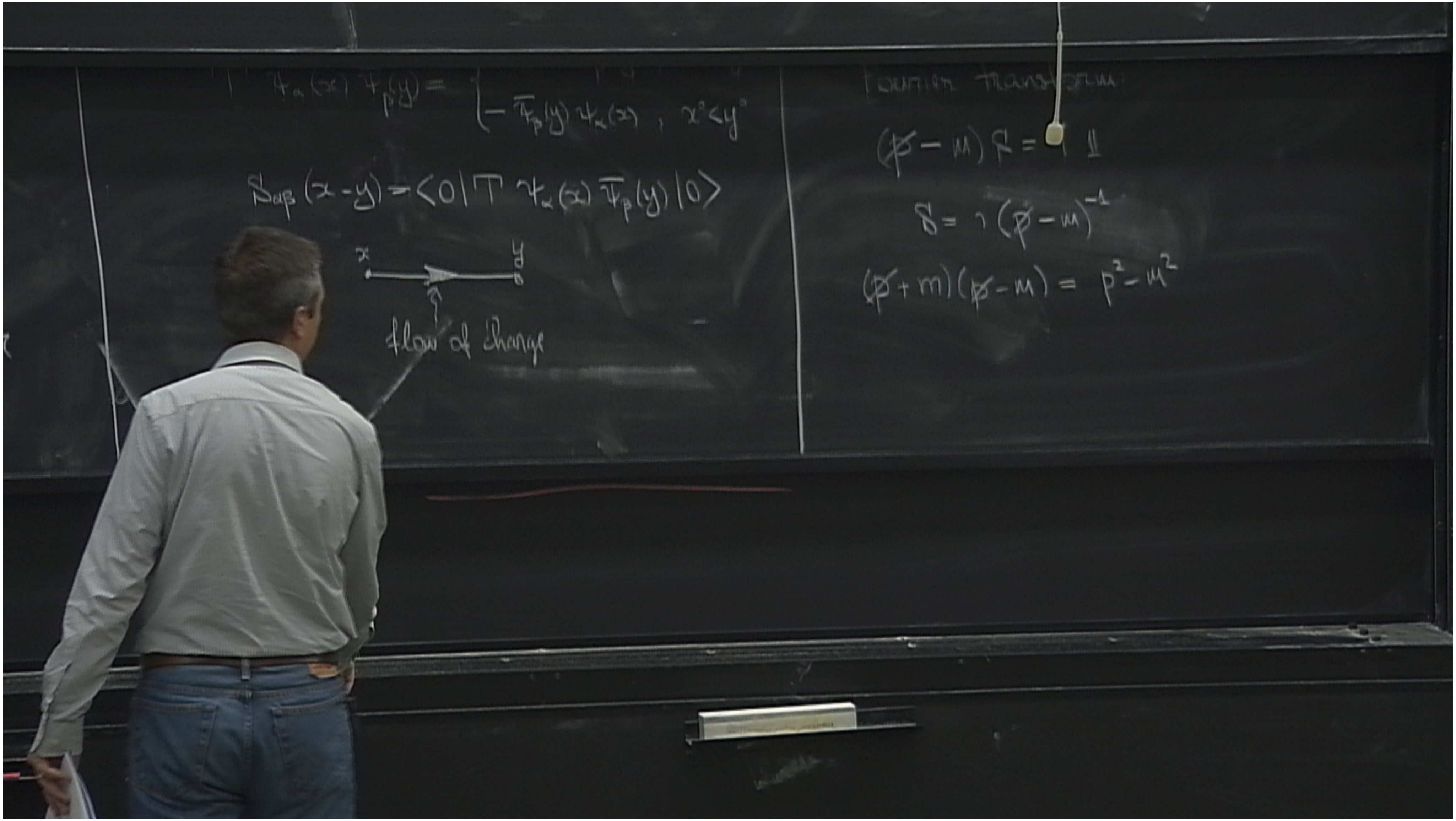
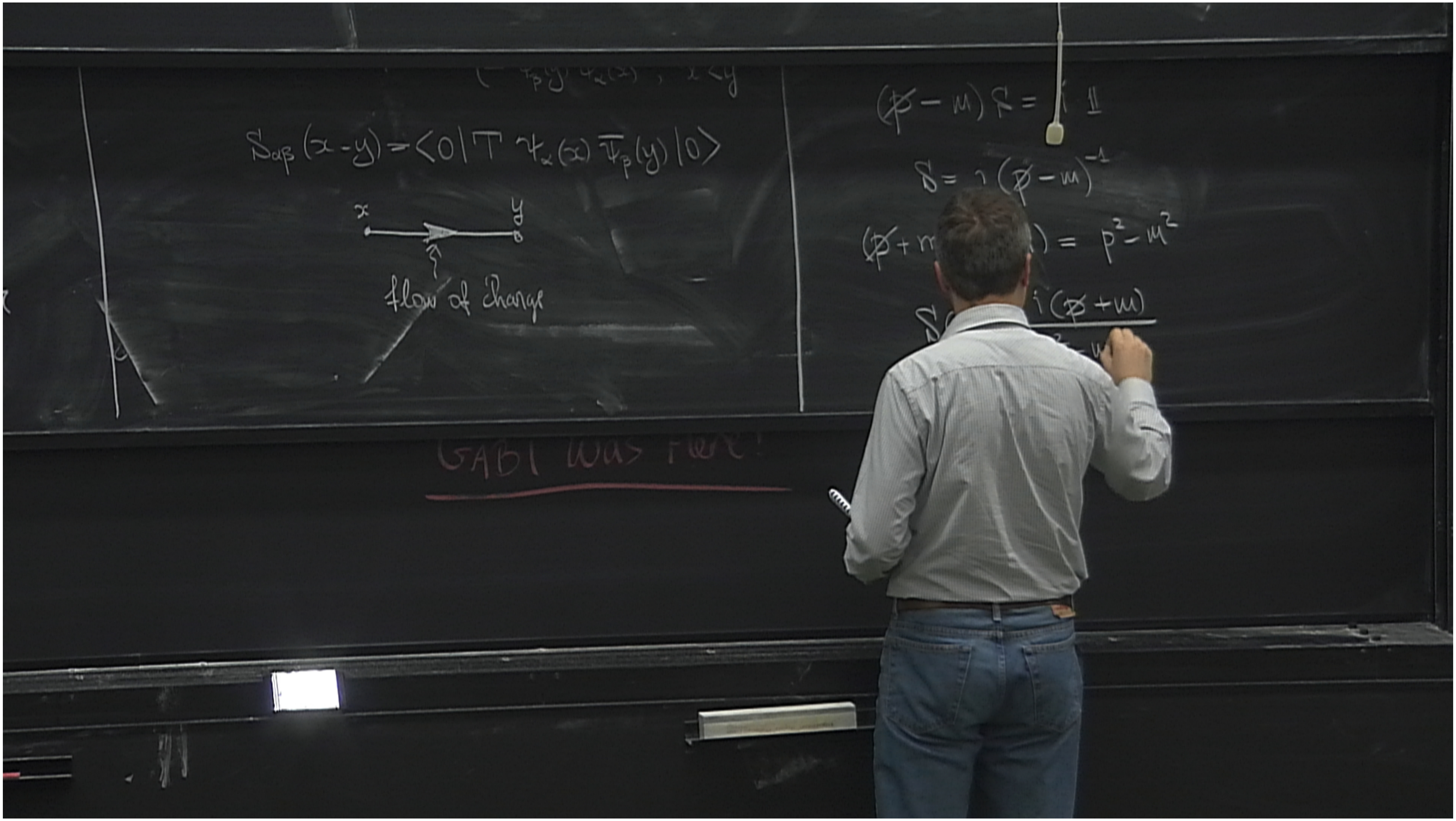
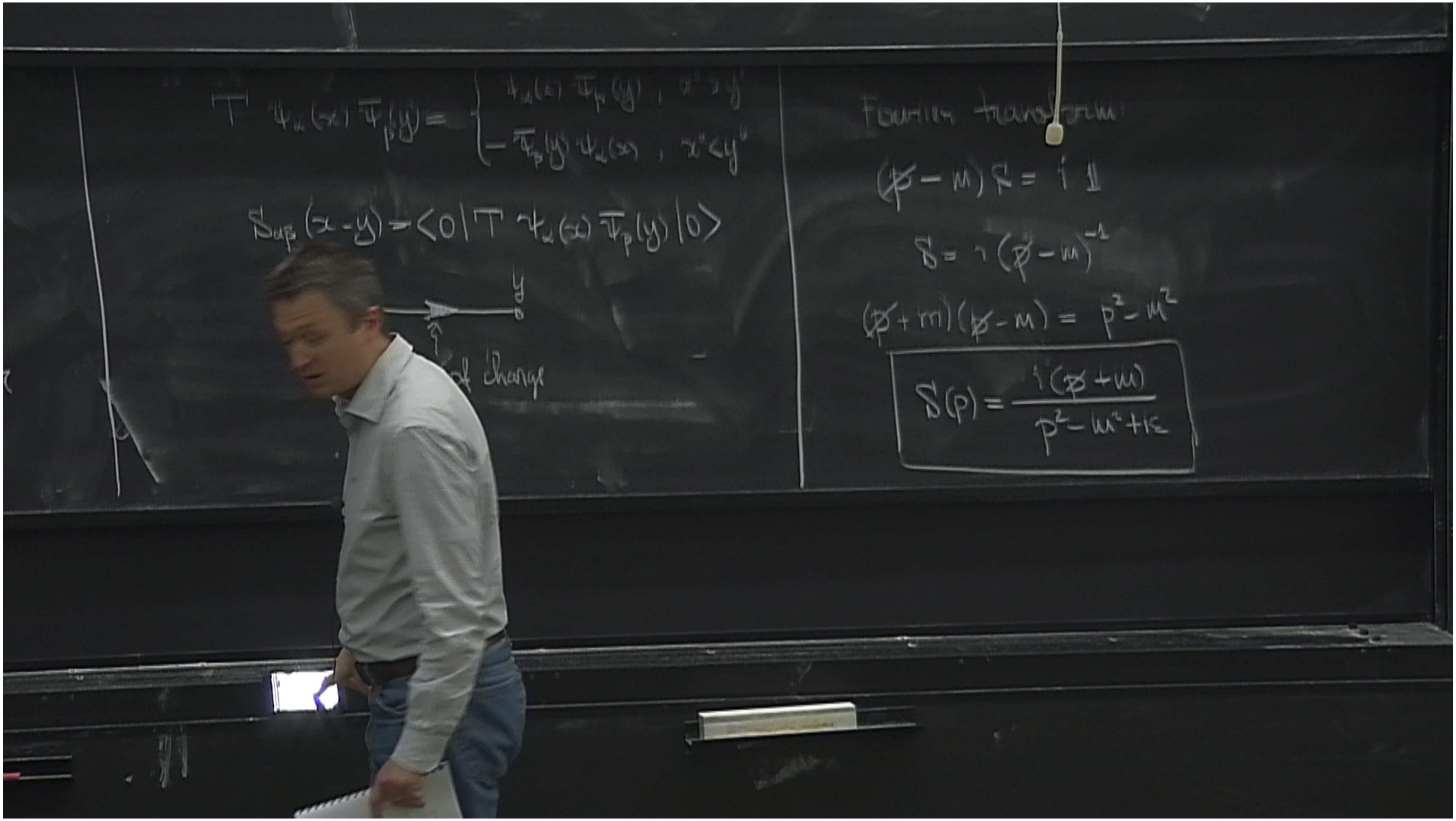
$S_{\alpha\beta}(x-y) = \langle 0|\, T\, \psi_\alpha(x)\, \bar\psi_\beta(y)\, |0\rangle$
x
y
flow of charge
$(i\not\partial - m)\, S = i\,\delta(x)\cdot 1$
Fourier transform:
$(\not p - m)\, S = i \cdot 1\!\!1$
$S = i\,(\not p - m)^{-1}$


flow of charge

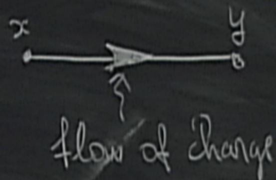

Sαβ (x - y) = <0| T ψα(x) ψ̄β(y) |0>
x
y
flow of charge
(p̸ − m) S = i 𝟙
S = i (p̸ − m)^-1
= p² − m²
i (p̸ + m)
GABI was here!


Fourier transform
(p̸ − m) S = i 𝟙
S = i (p̸ − m)^-1
(p̸ + m)(p̸ − m) = p² − m²
S(p) = i(p̸ + m) / (p² − m² + iε)
S_αβ (x − y) = <0| T ψ_α(x) ψ̄_β(y) |0>

Fermions:

$$T\,\psi_\alpha(x)\bar{\psi}_\beta(y) = \begin{cases} \psi_\alpha(x)\bar{\psi}_\beta(y), & x^0 > y^0 \\ -\bar{\psi}_\beta(y)\psi_\alpha(x), & x^0 < y^0 \end{cases}$$

$$S_{\alpha\beta}(x-y) = \langle 0|T\,\psi_\alpha(x)\bar{\psi}_\beta(y)|0\rangle$$



$$(i\partial\!\!\!/ - m)S = i\delta(x)\cdot 1$$

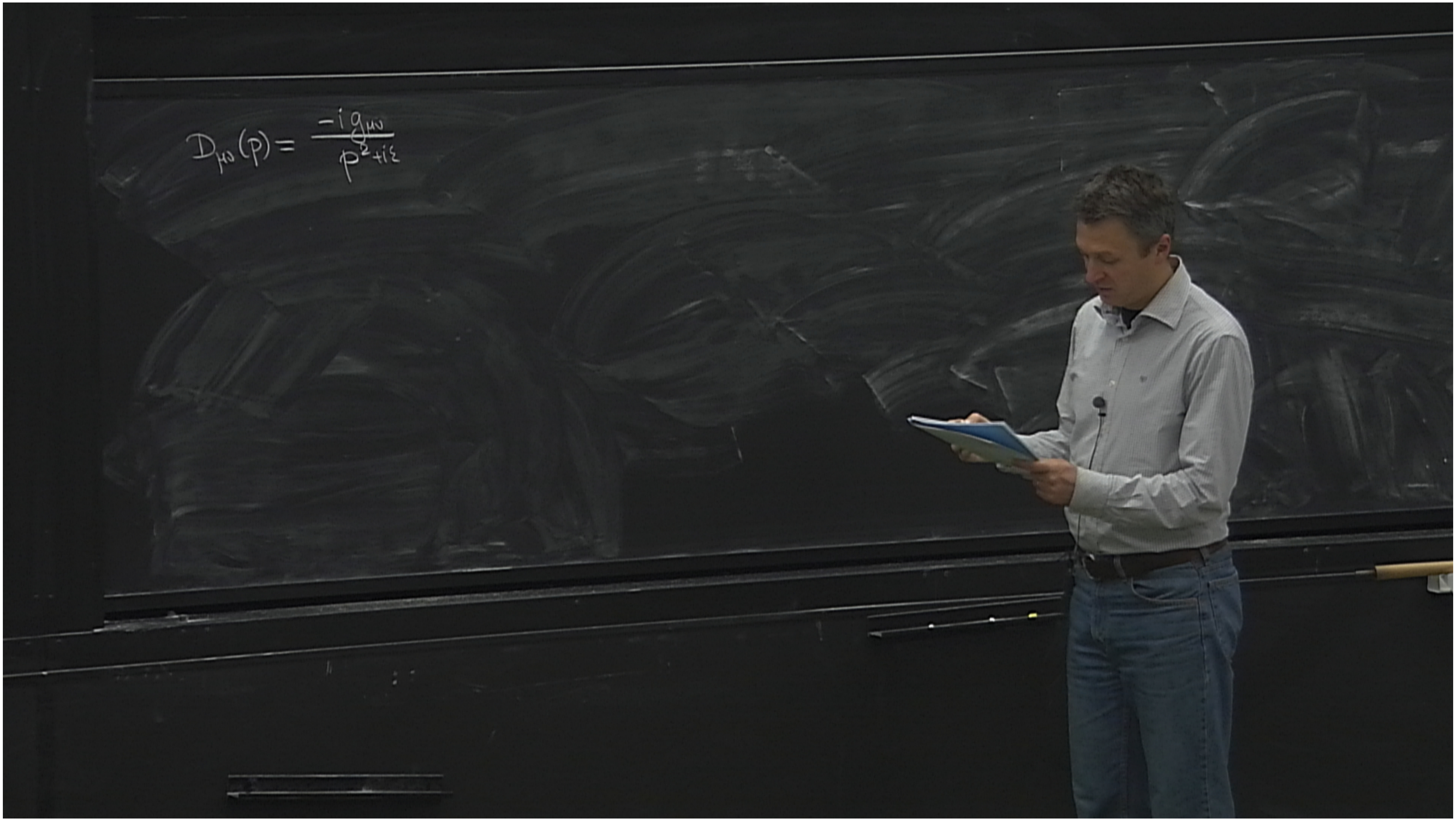
Fourier transform:

$$(p\!\!\!/ - m)S = i\cdot 1$$

$$S = i(p\!\!\!/ - m)^{-1}$$

$$(p\!\!\!/ + m)(p\!\!\!/ - m) = p^2 - m^2$$

$$\boxed{S(p) = \frac{i(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon}}$$


$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$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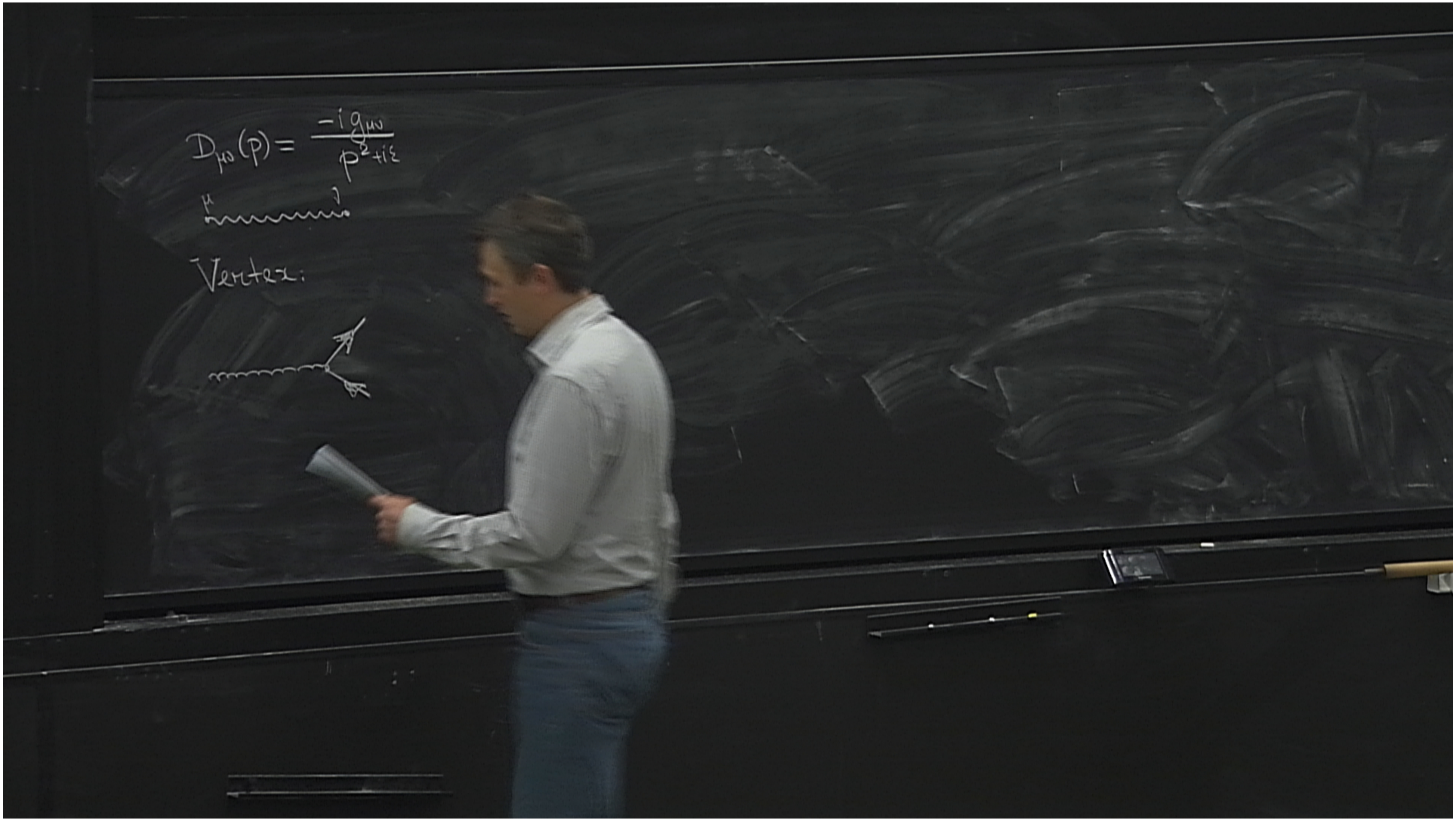
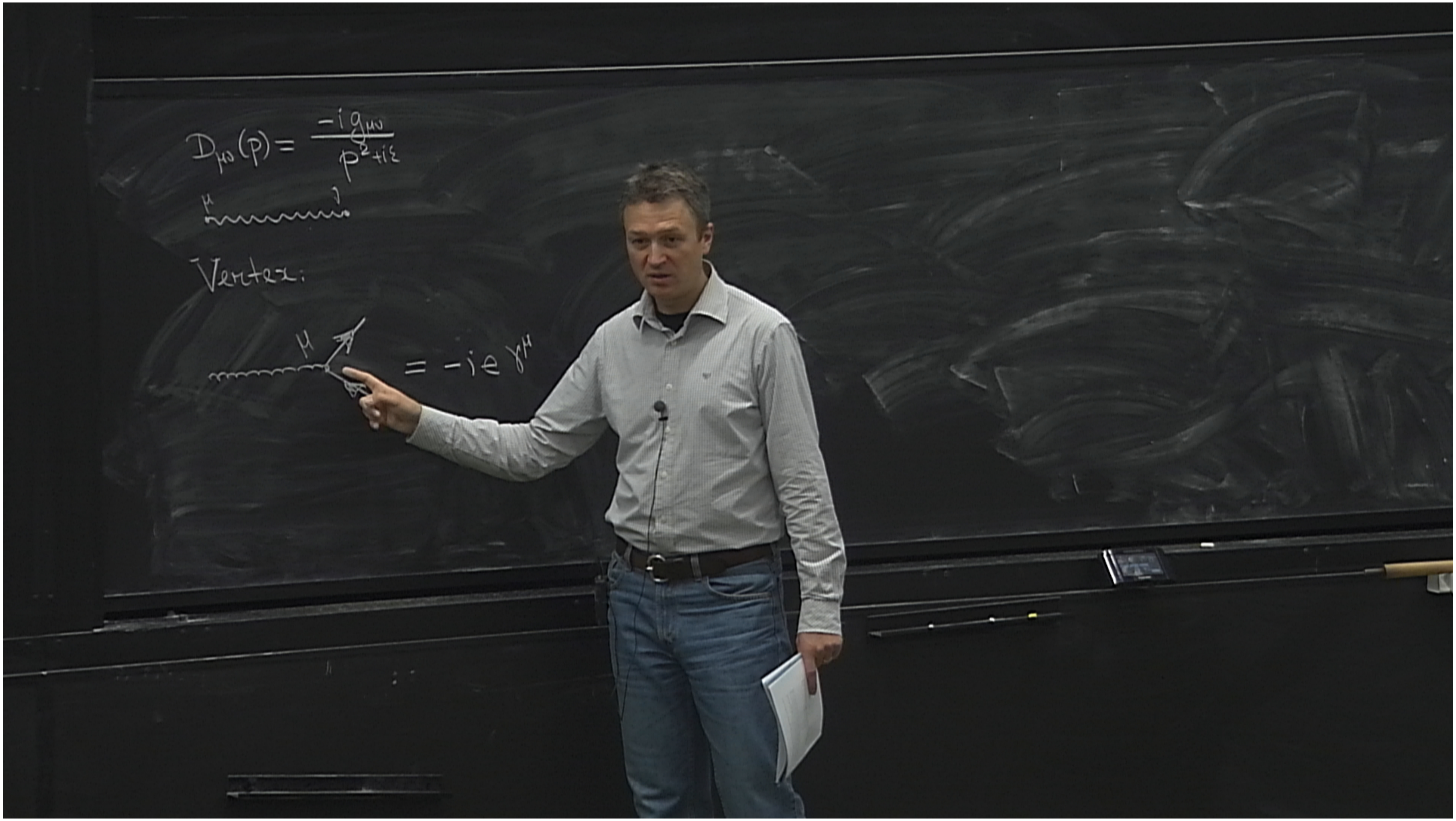
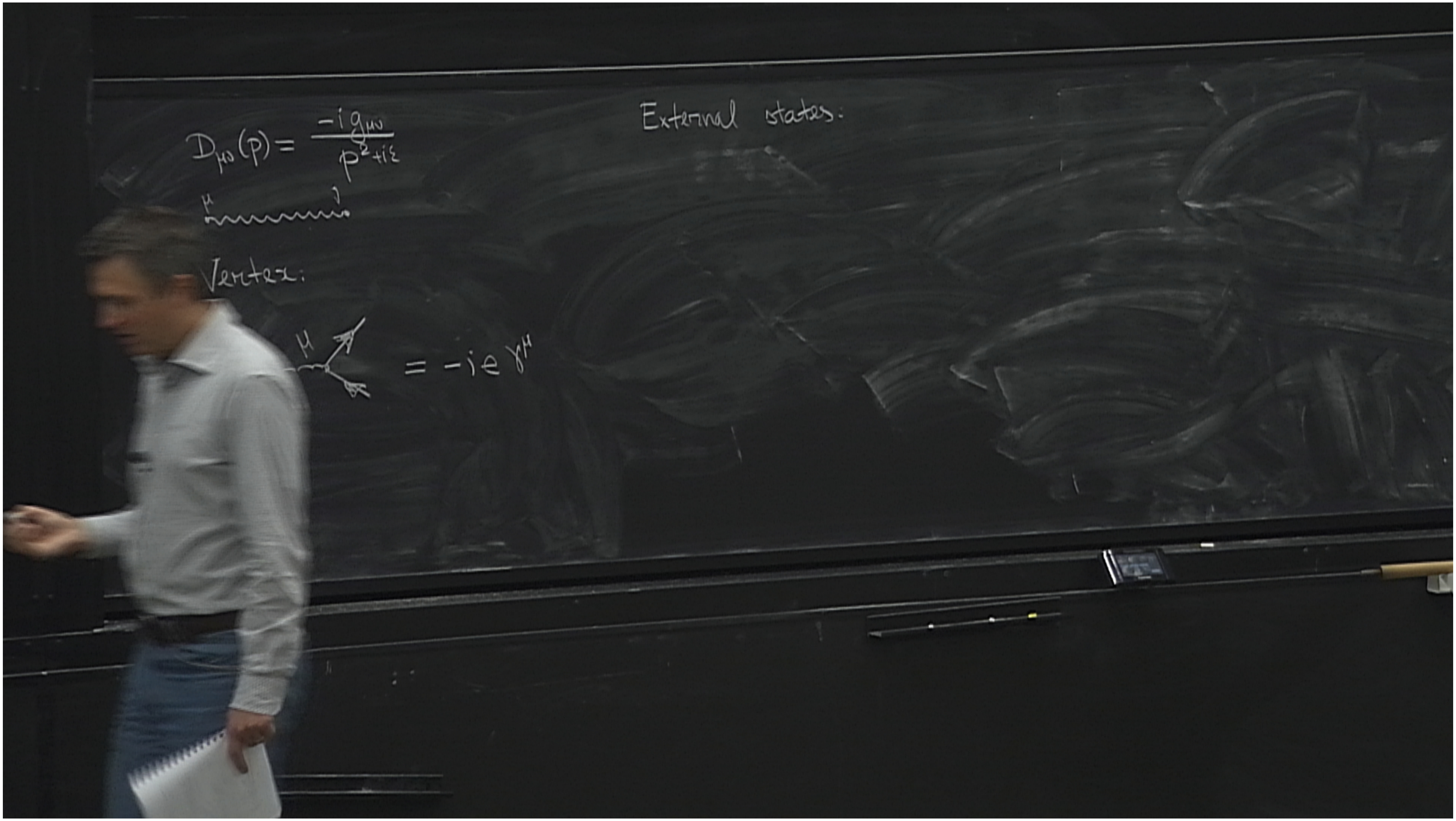
$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
μ
ν
Vertex:



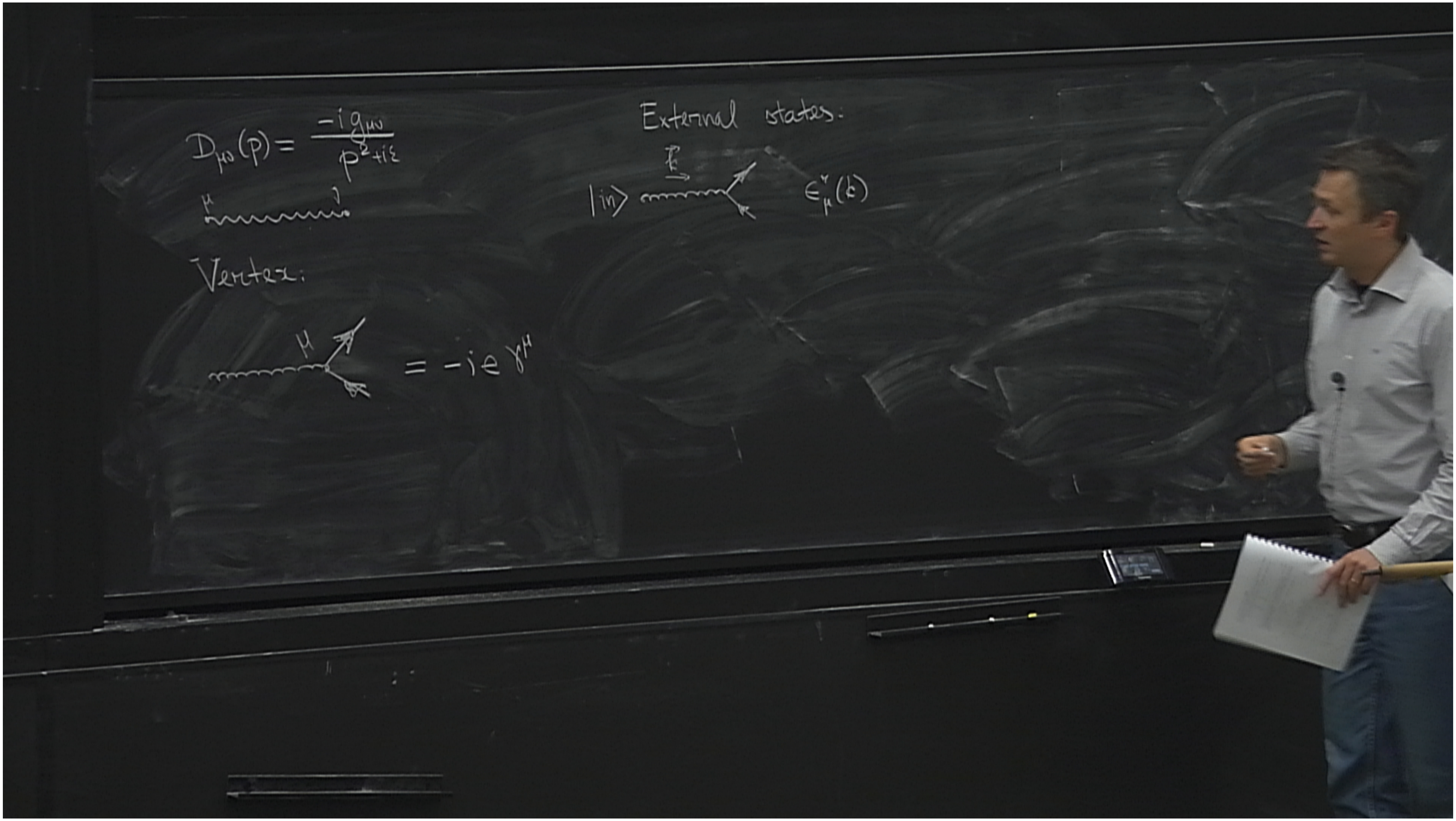
$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
μ
ν
Vertex:

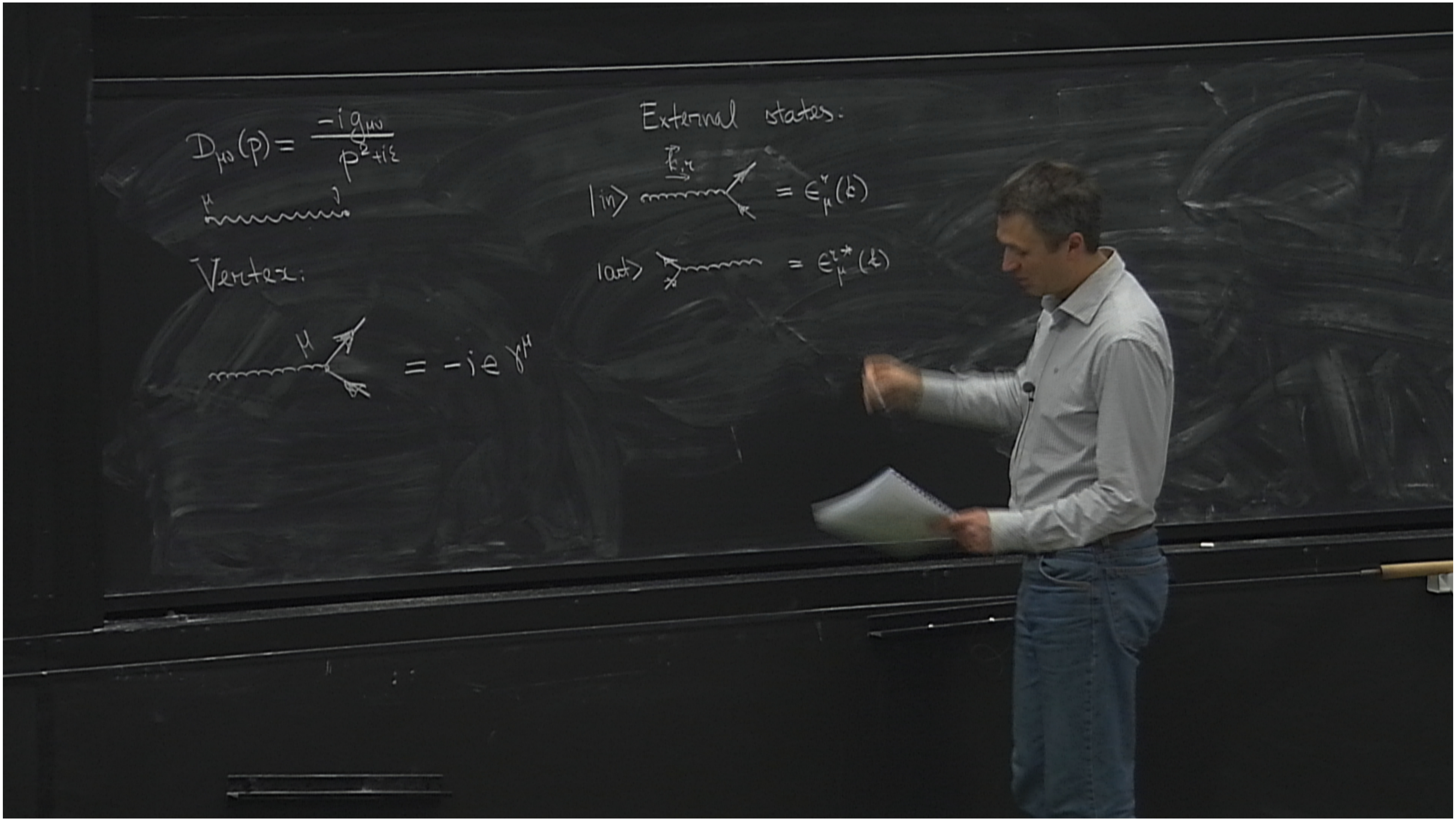


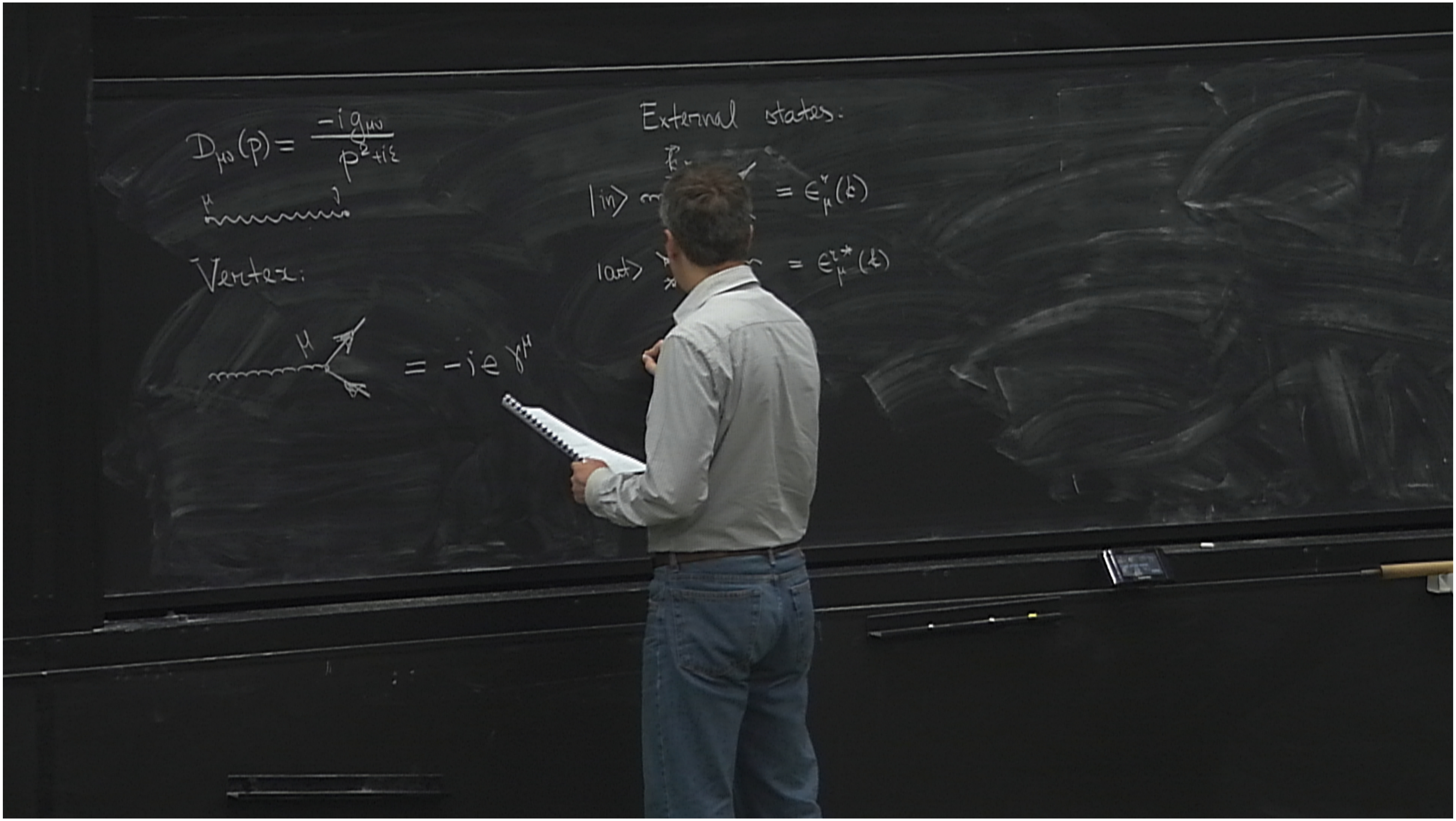
$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
$= -ie\gamma^\mu$



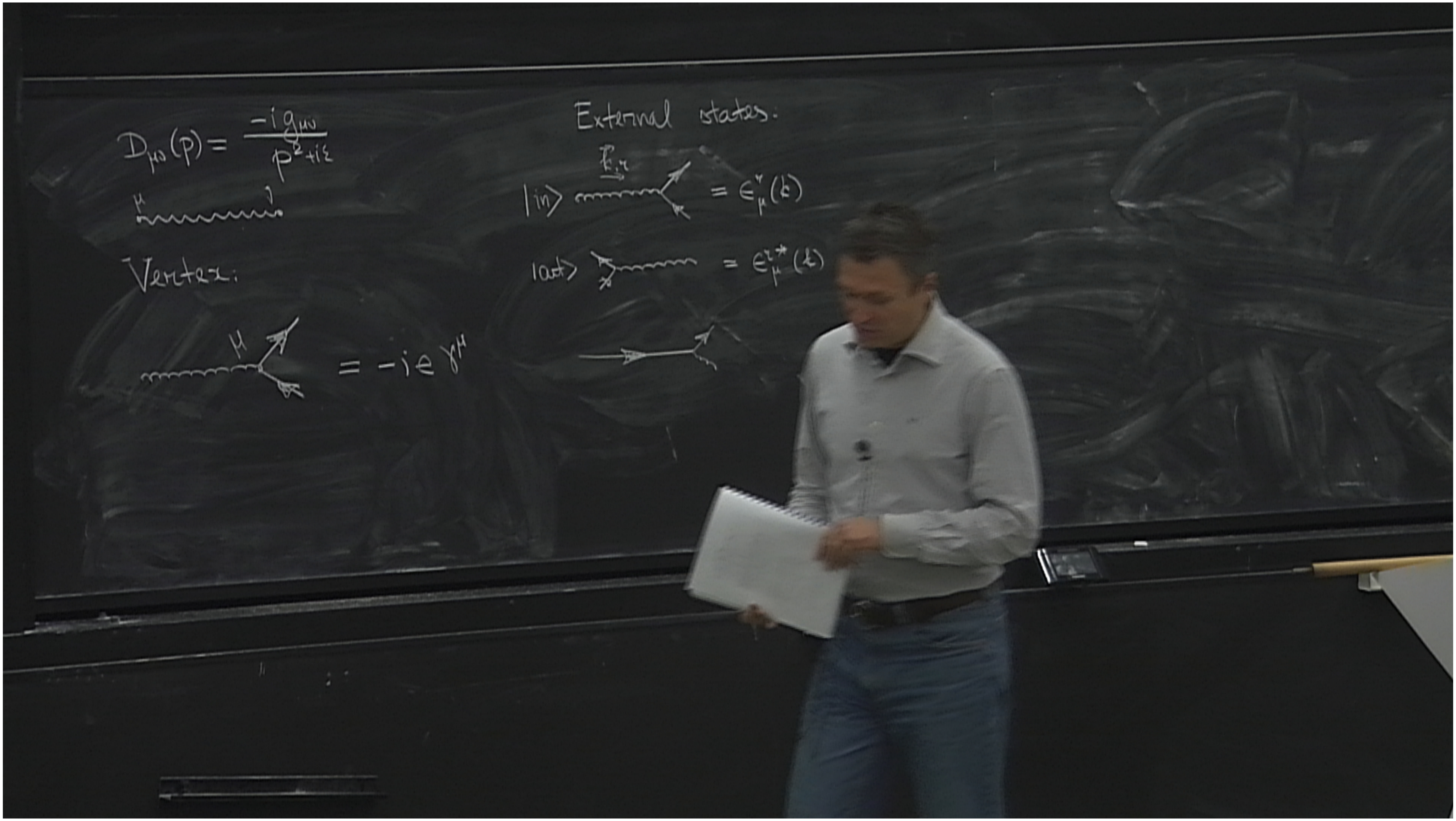
$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
$= -ie\gamma^\mu$
External states:

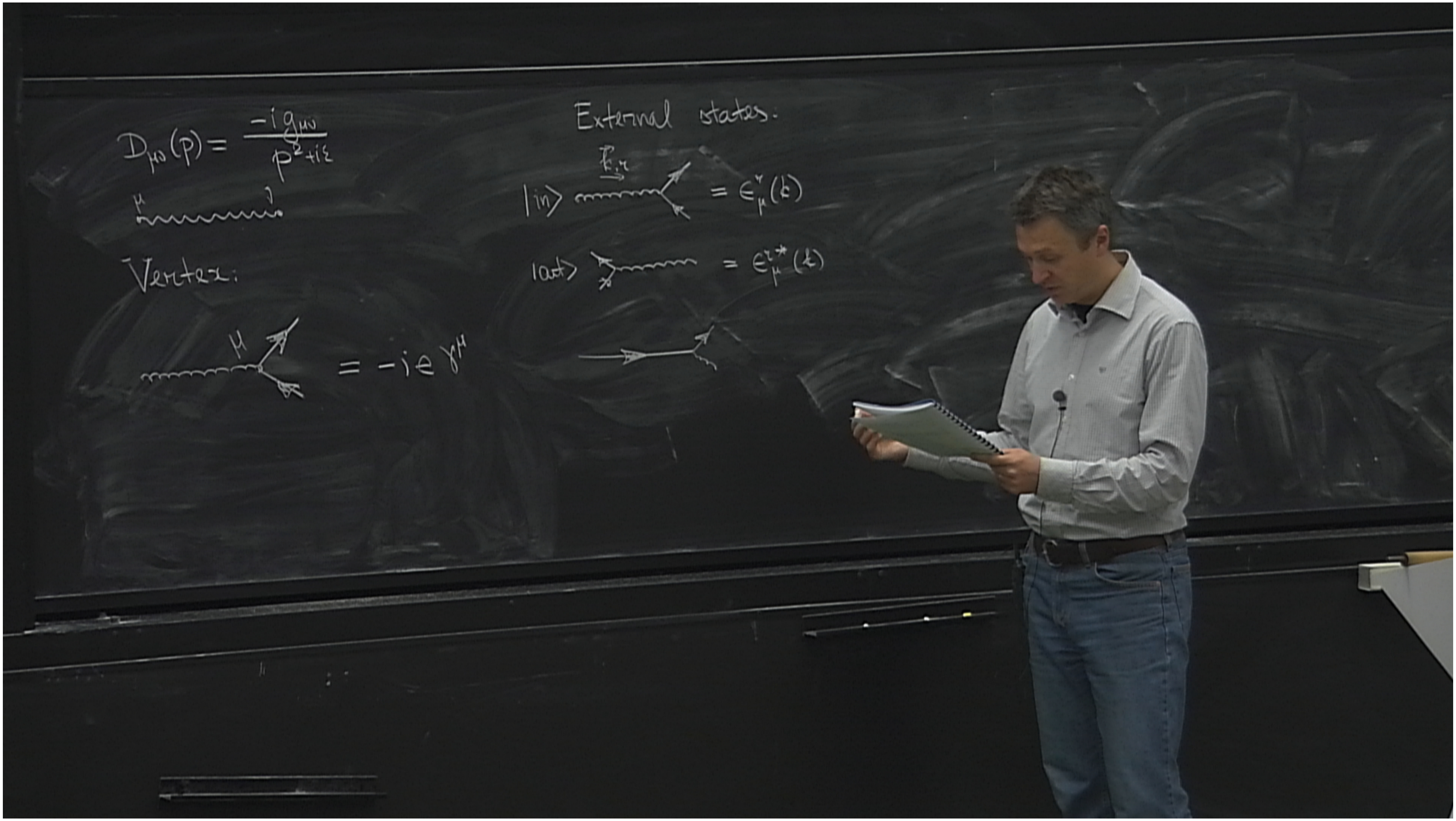

$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
μ ν
Vertex:
μ
$= -ie\gamma^\mu$
External states:
$|in\rangle$
k
$\epsilon^*_\mu(k)$


$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
$= -ie\gamma^\mu$
External states:
$|in\rangle = \epsilon^r_\mu(k)$
$|out\rangle = \epsilon^{r*}_\mu(k)$



$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
$= -ie\gamma^\mu$
External states:
$|in\rangle$
$= \epsilon^r_\mu(k)$
$|out\rangle$
$= \epsilon^{r*}_\mu(k)$


D_μν(p) = -i g_μν / (p² + iε)
Vertex:
= -ie γ^μ
External states.
|in⟩
|out⟩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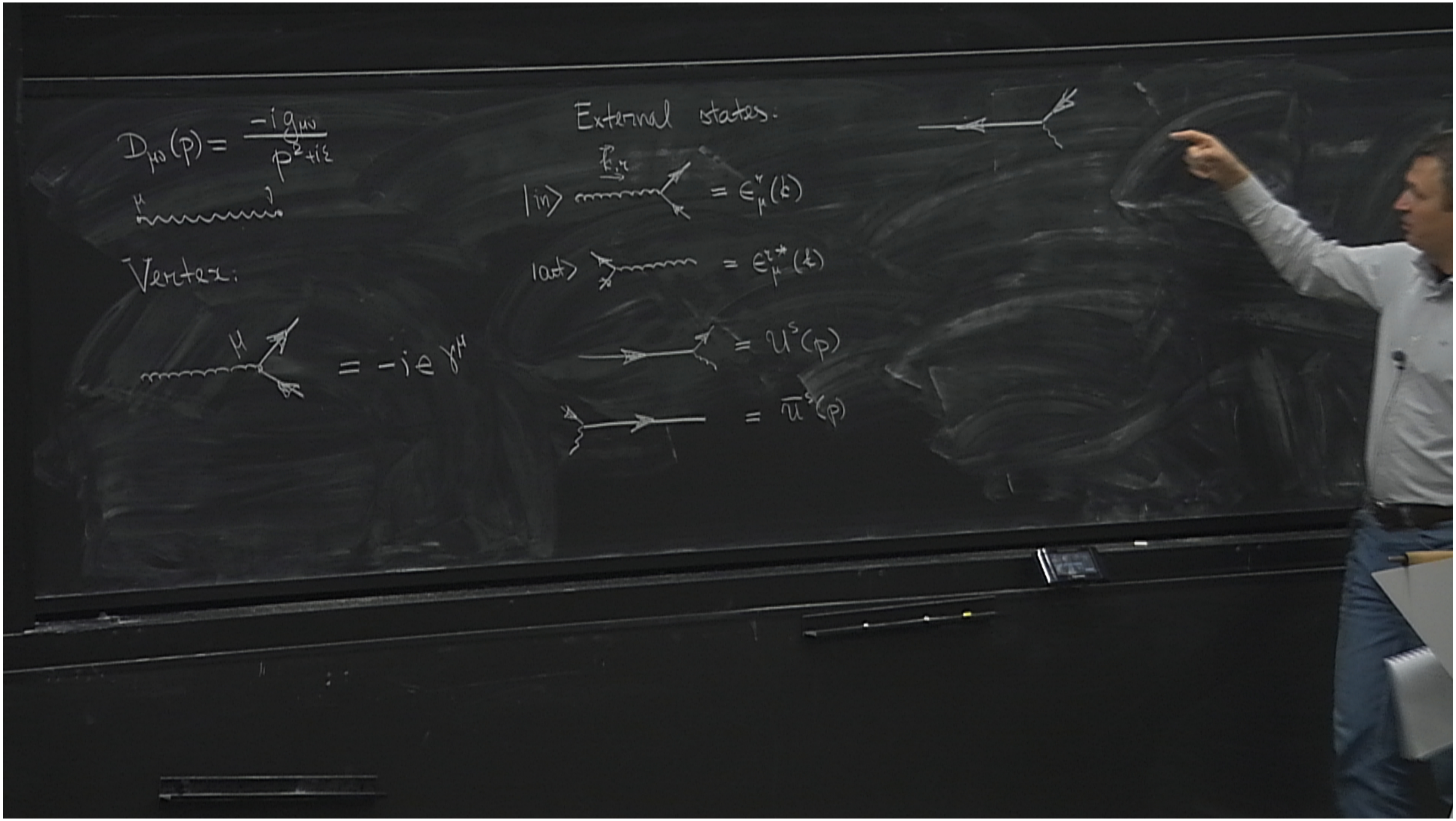
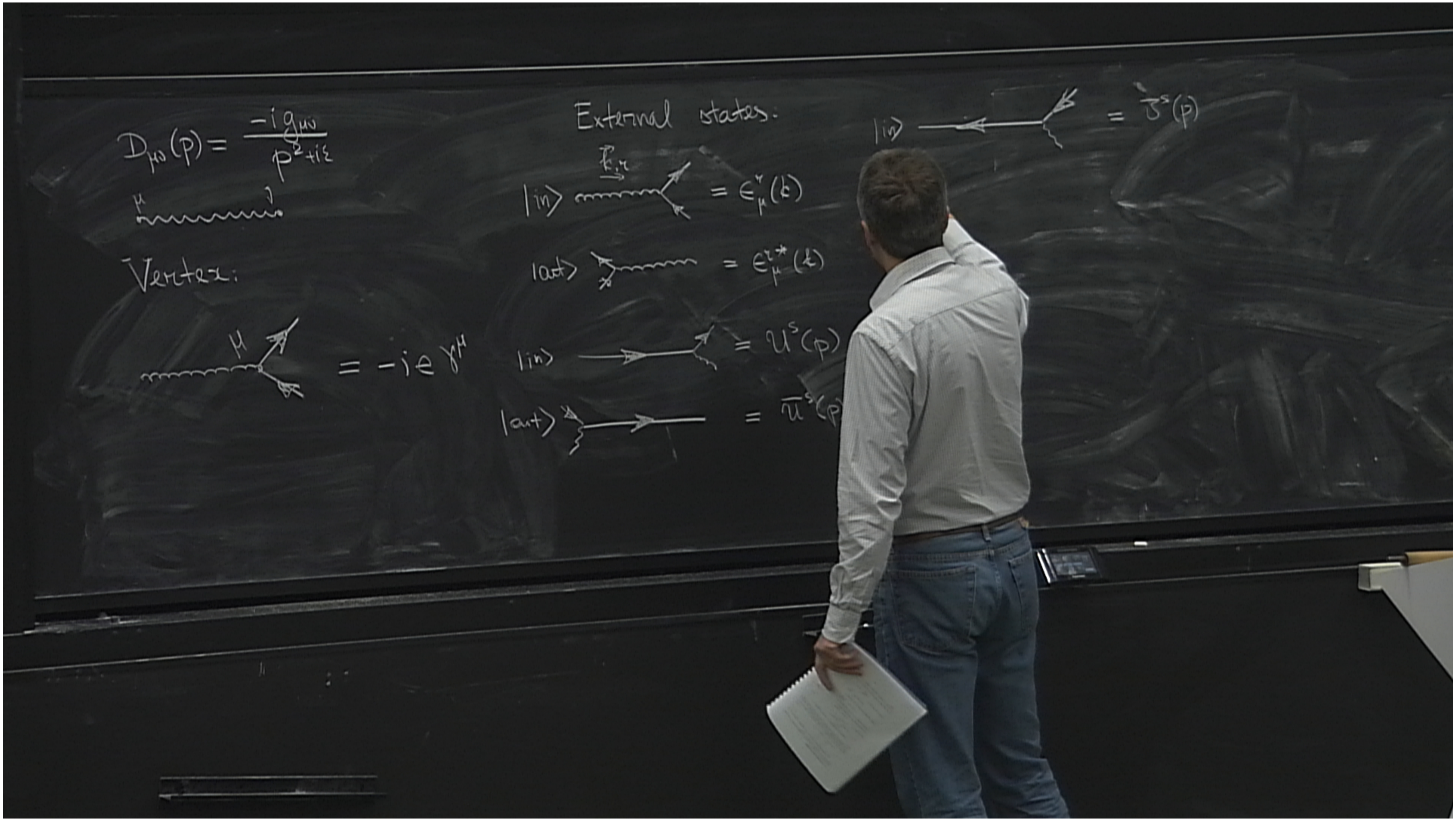
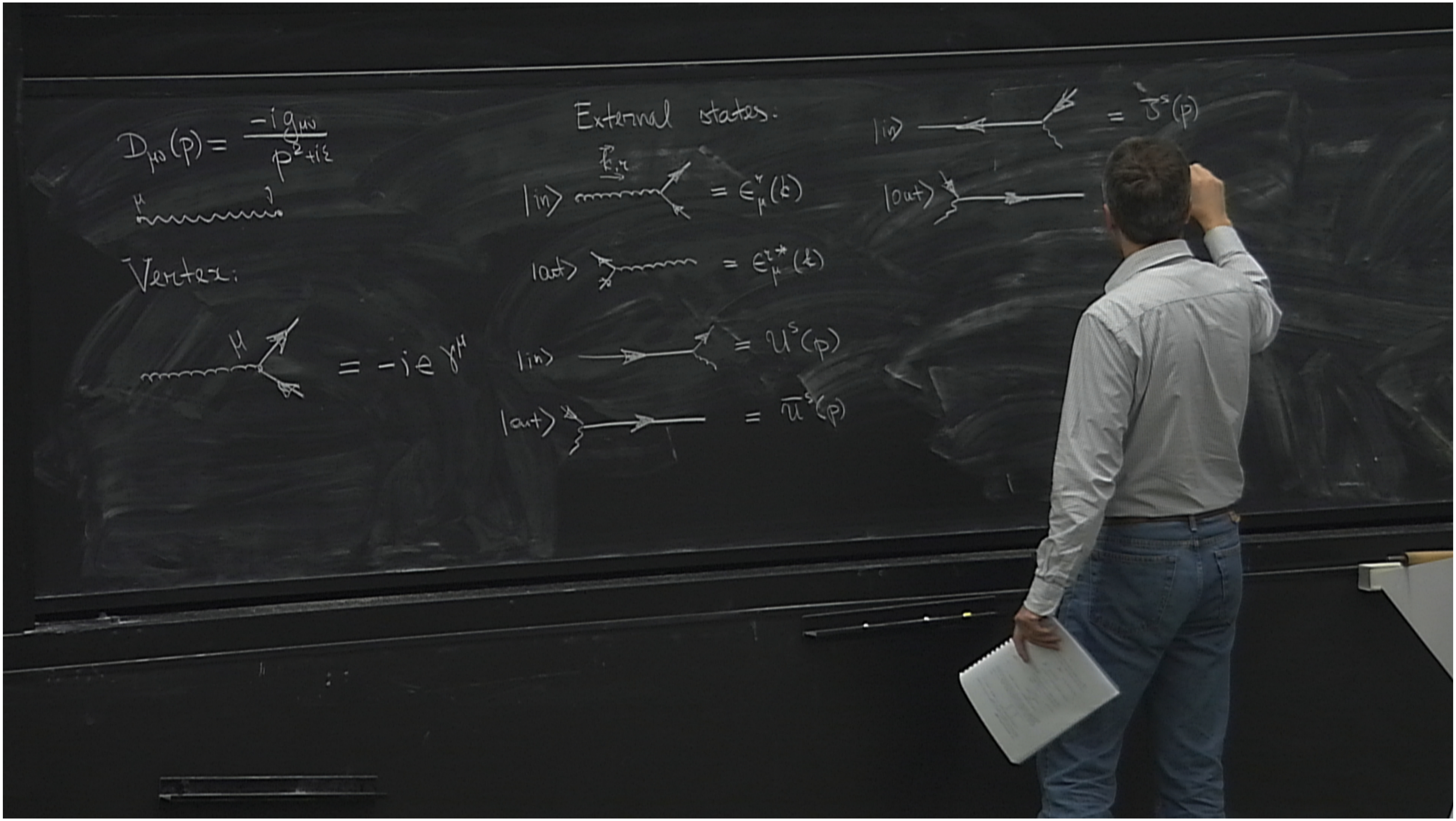
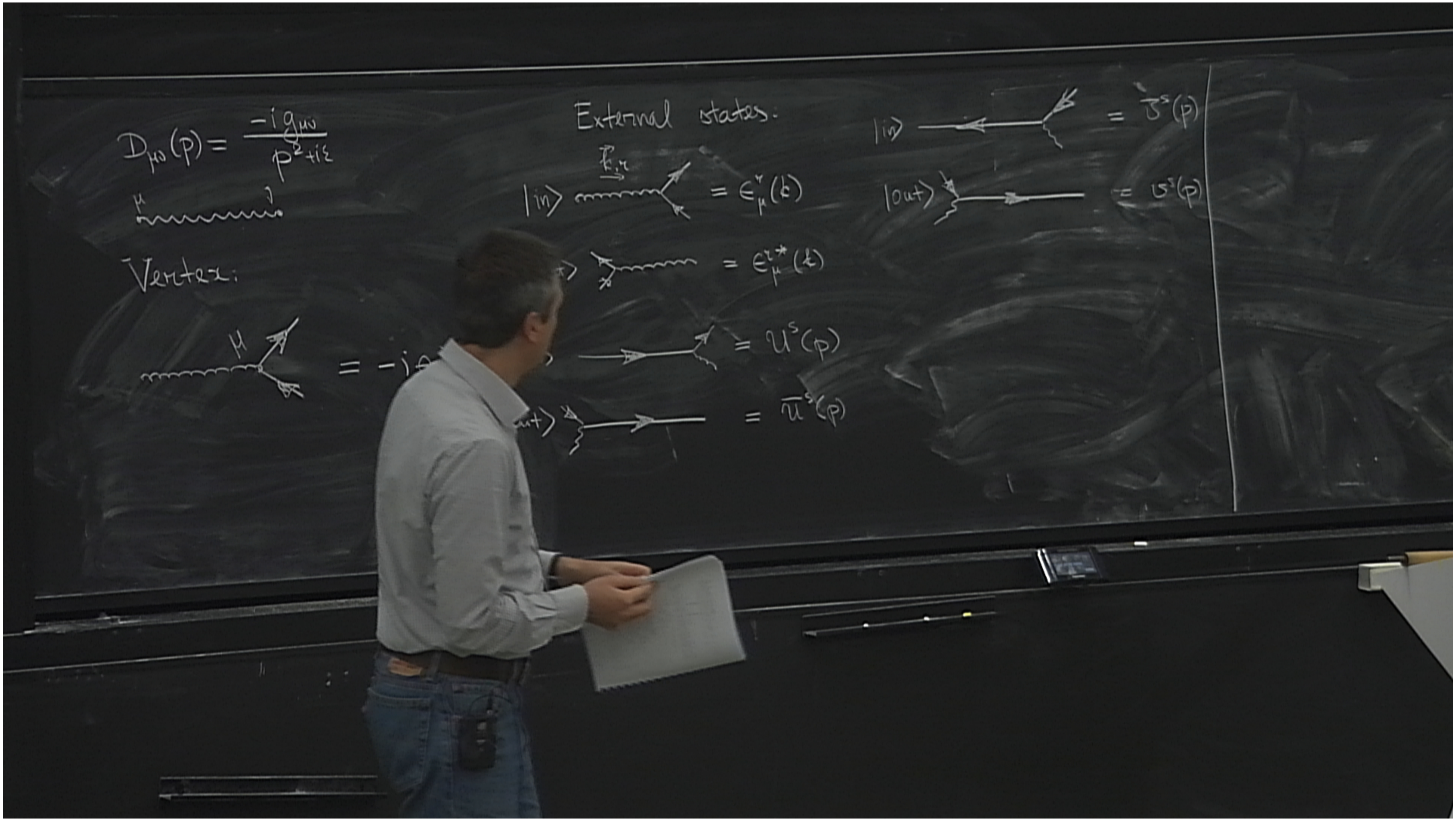
Vertex:
$= -ie\gamma^\mu$
External states.
$|in\rangle$
$= \epsilon^r_\mu(k)$
$|out\rangle$
$= \epsilon^{r*}_\mu(k)$


D_μν(p) = -i g_μν / (p^2 + iε)
μ ν
Vertex:
μ
= -ie γ^μ
External states:
k, r
|in⟩ = ε^r_μ(k)
|out⟩ = ε^r*_μ(k)
= u^s(p)
= ū^s(p)

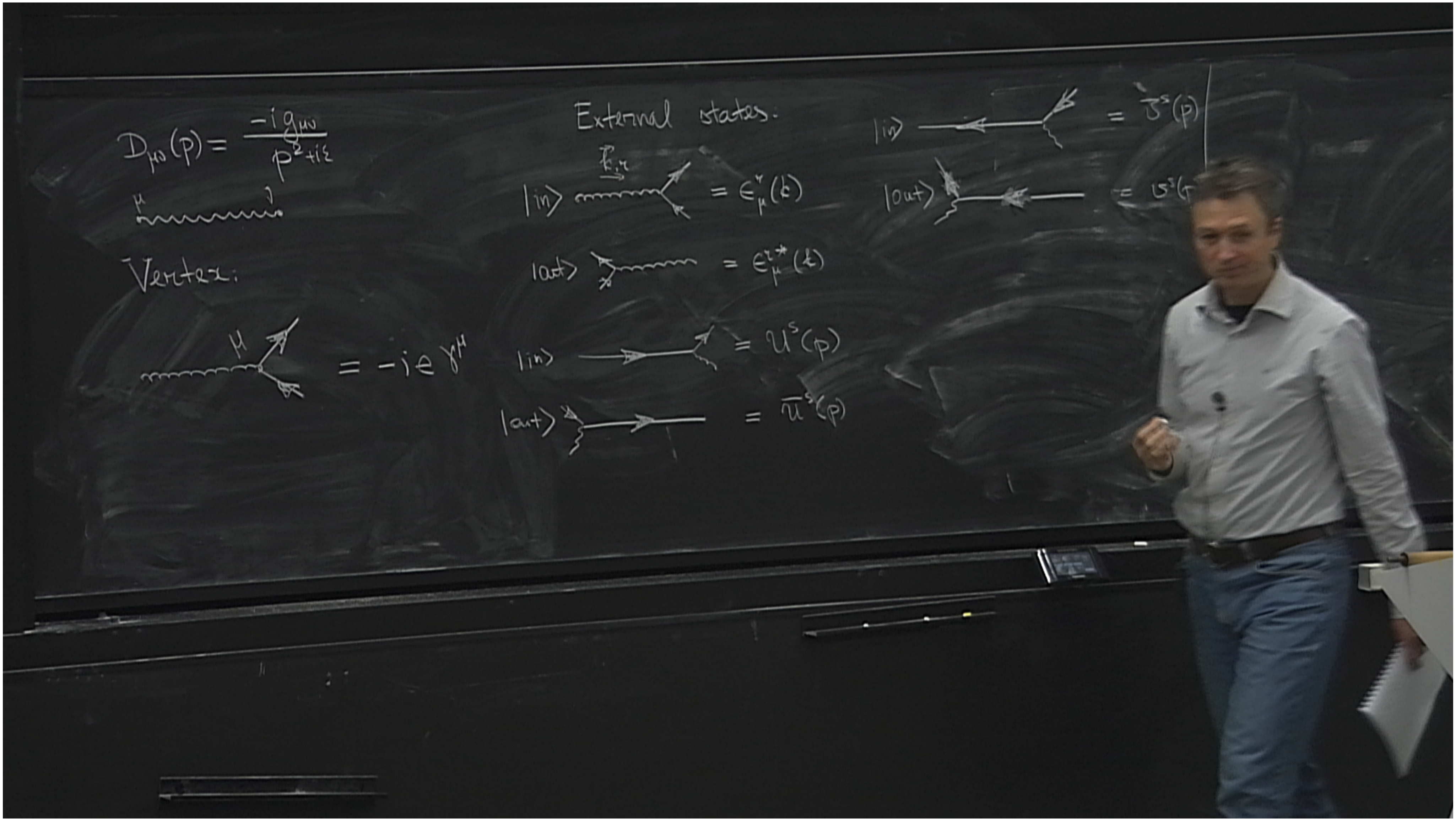


External states:
$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
$= -ie\gamma^\mu$
$|in\rangle$ $= \epsilon^r_\mu(k)$
$|out\rangle$ $= \epsilon^{r*}_\mu(k)$
$|in\rangle$ $= u^s(p)$
$|out\rangle$ $= \bar{u}^s(p)$
$|in\rangle$ $= \bar{v}^s(p)$


$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
$= -ie\gamma^\mu$
External states:
|in⟩ = $\epsilon^r_\mu(k)$
|out⟩ = $\epsilon^{r*}_\mu(k)$
|in⟩ = $u^s(p)$
|out⟩ = $\bar{u}^s(p)$
|in⟩ = $\bar{v}^s(p)$
|out⟩



$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
External states.
|in⟩ $= \epsilon^r_\mu(k)$
$= \epsilon^{r*}_\mu(k)$
$= u^s(p)$
$= \bar{u}^s(p)$
|in⟩ $= \bar{v}^s(p)$
|out⟩ $= v^s(p)$


$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
μ ν
Vertex:
μ
$= -ie\gamma^\mu$
External states:
k, r
$|in\rangle$ $= \epsilon^r_\mu(k)$
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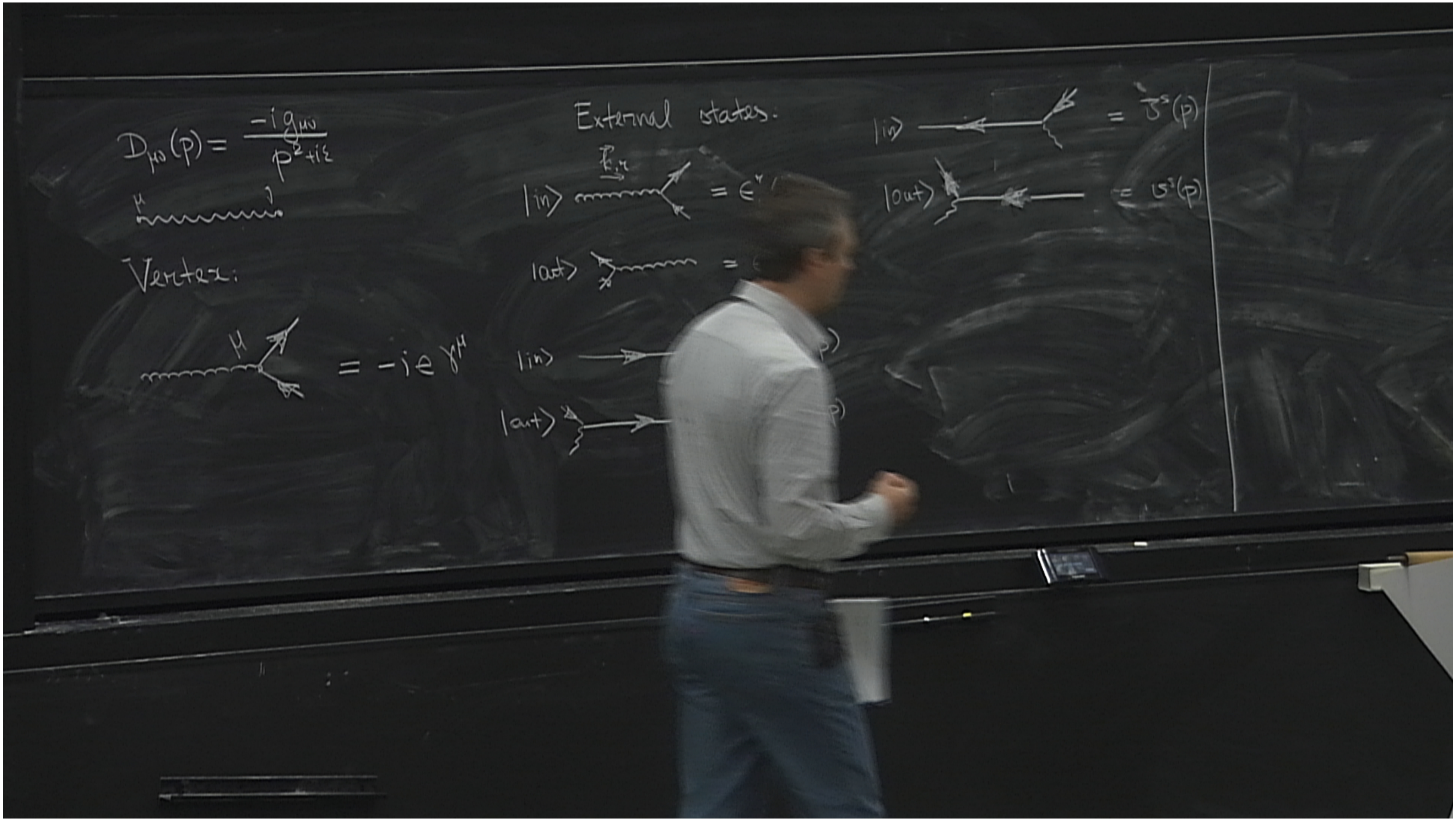
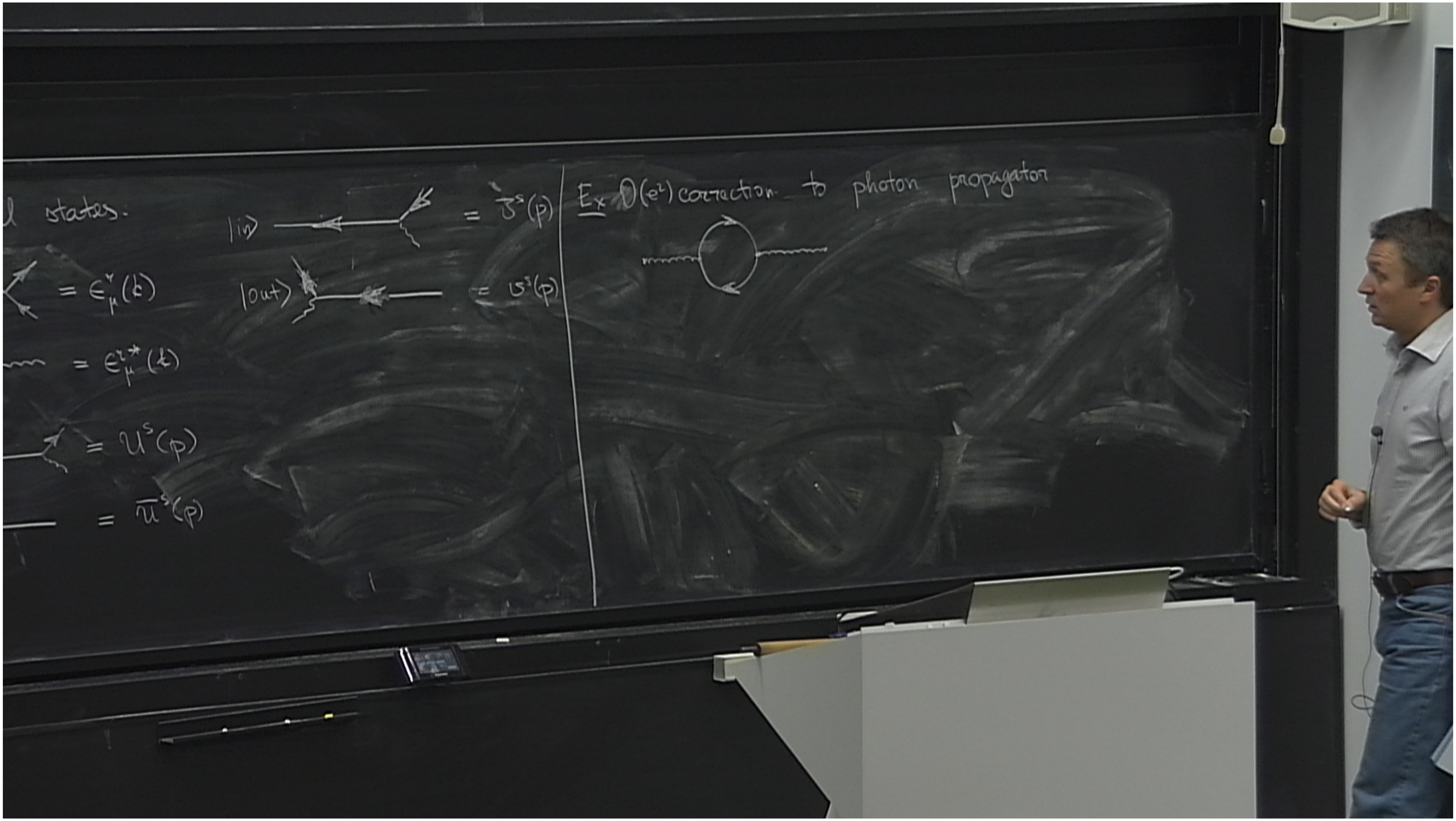
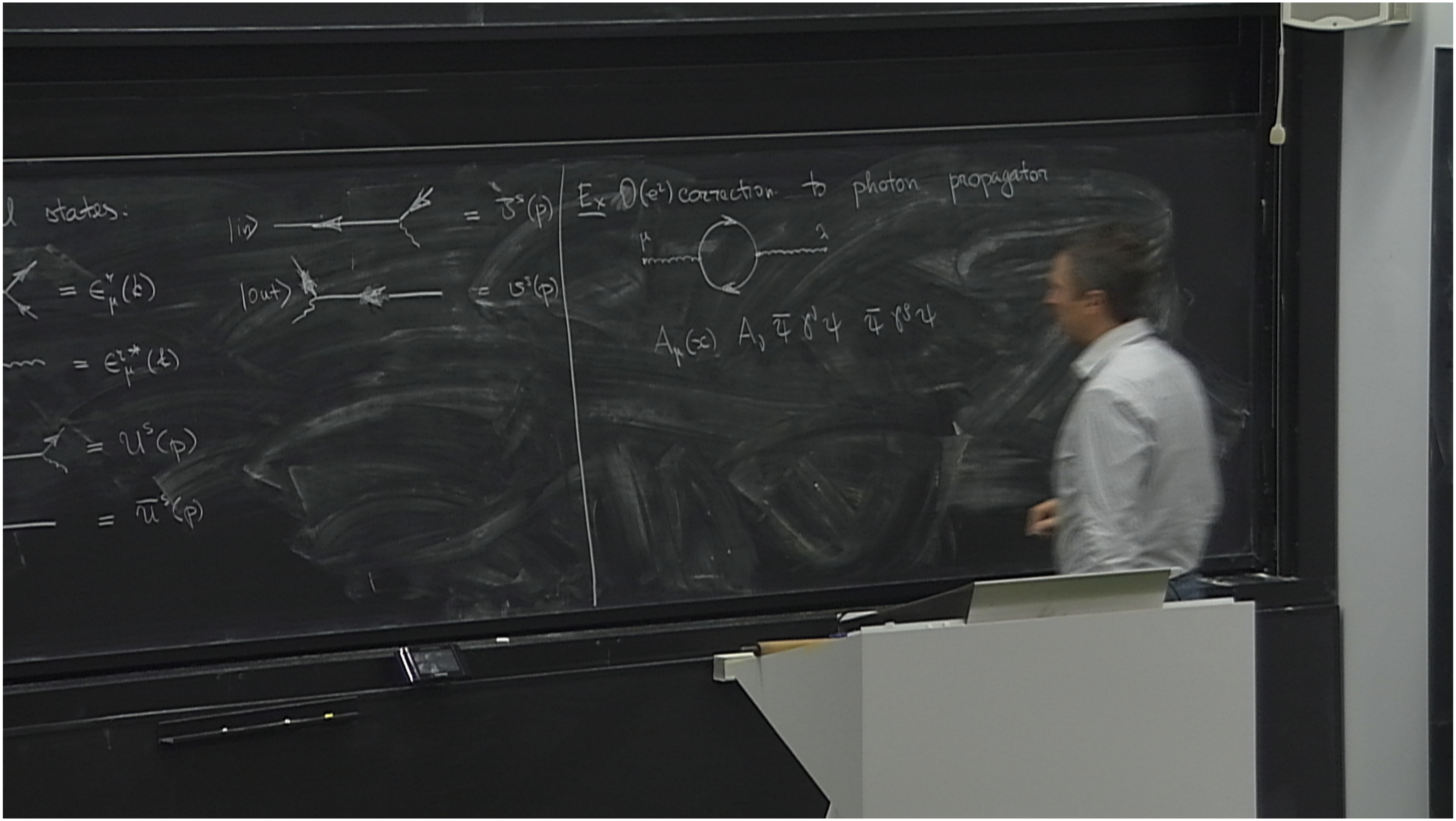
$|in\rangle$ $= u^s(p)$
$|out\rangle$ $= \bar{u}^s(p)$
$|in\rangle$ $= \bar{v}^s(p)$
$|out\rangle$

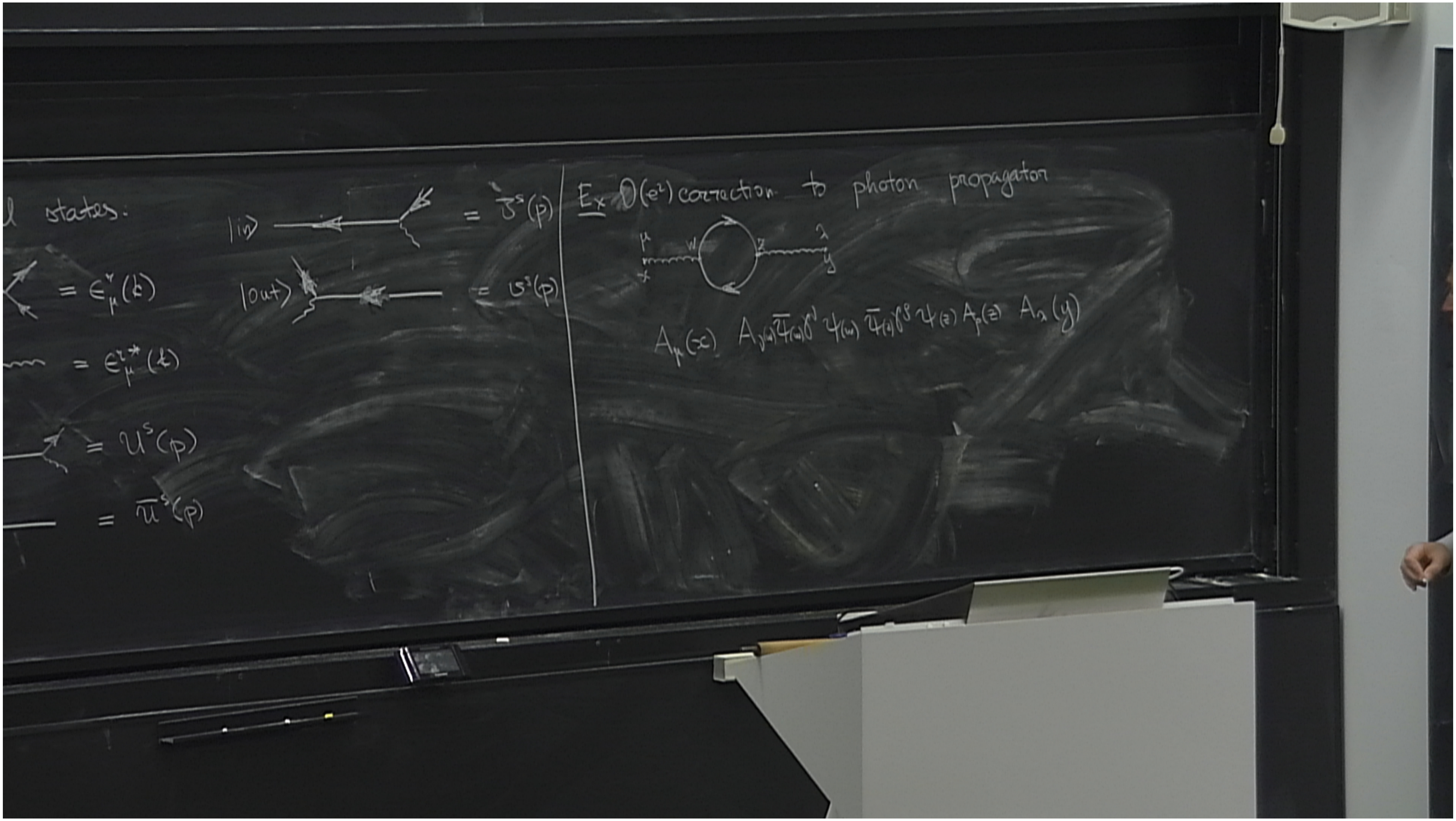


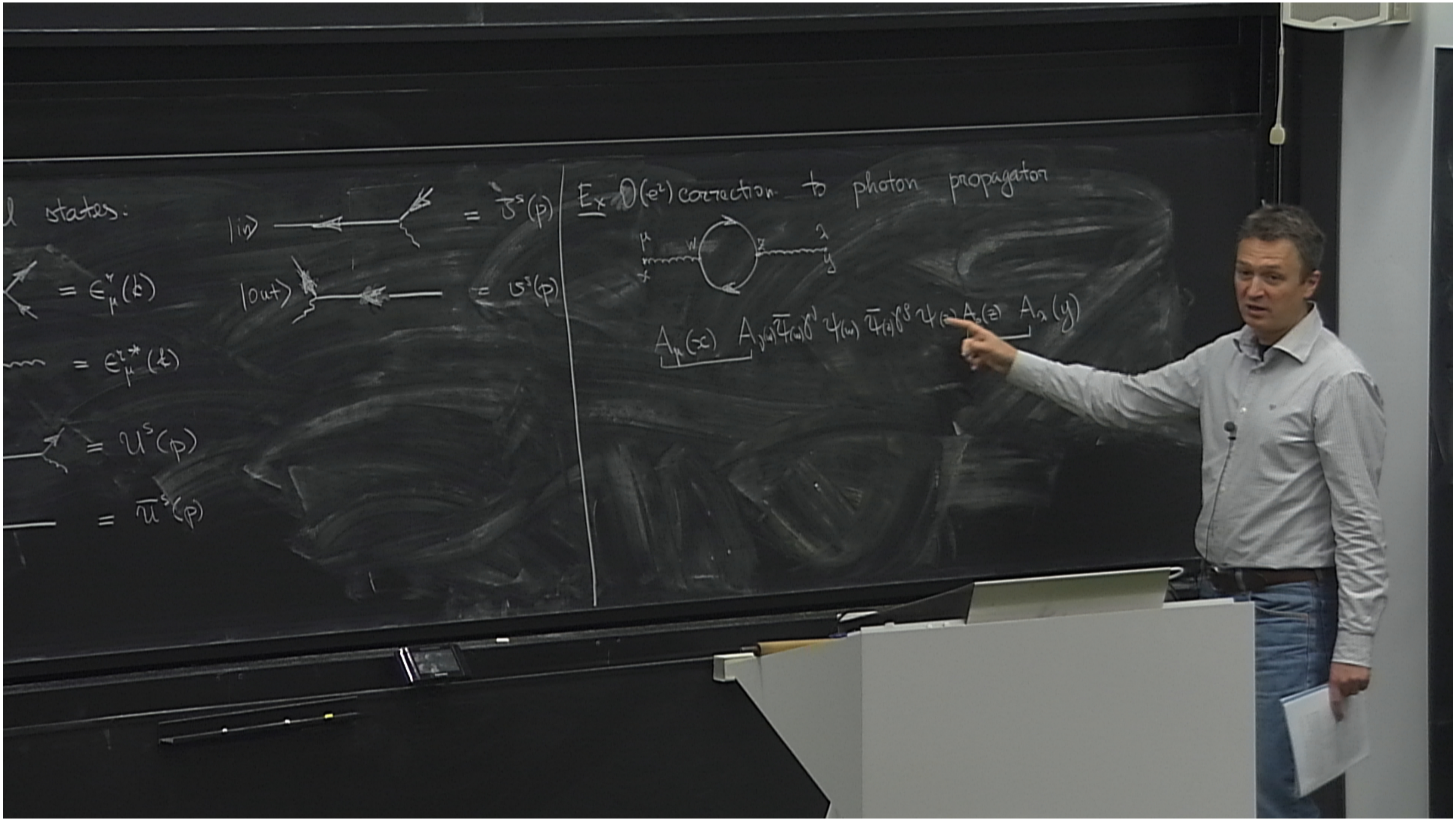
$D_{\mu\nu}(p) = \frac{-i g_{\mu\nu}}{p^2 + i\varepsilon}$
Vertex:
$= -ie\gamma^\mu$
External states.
|in⟩
|out⟩

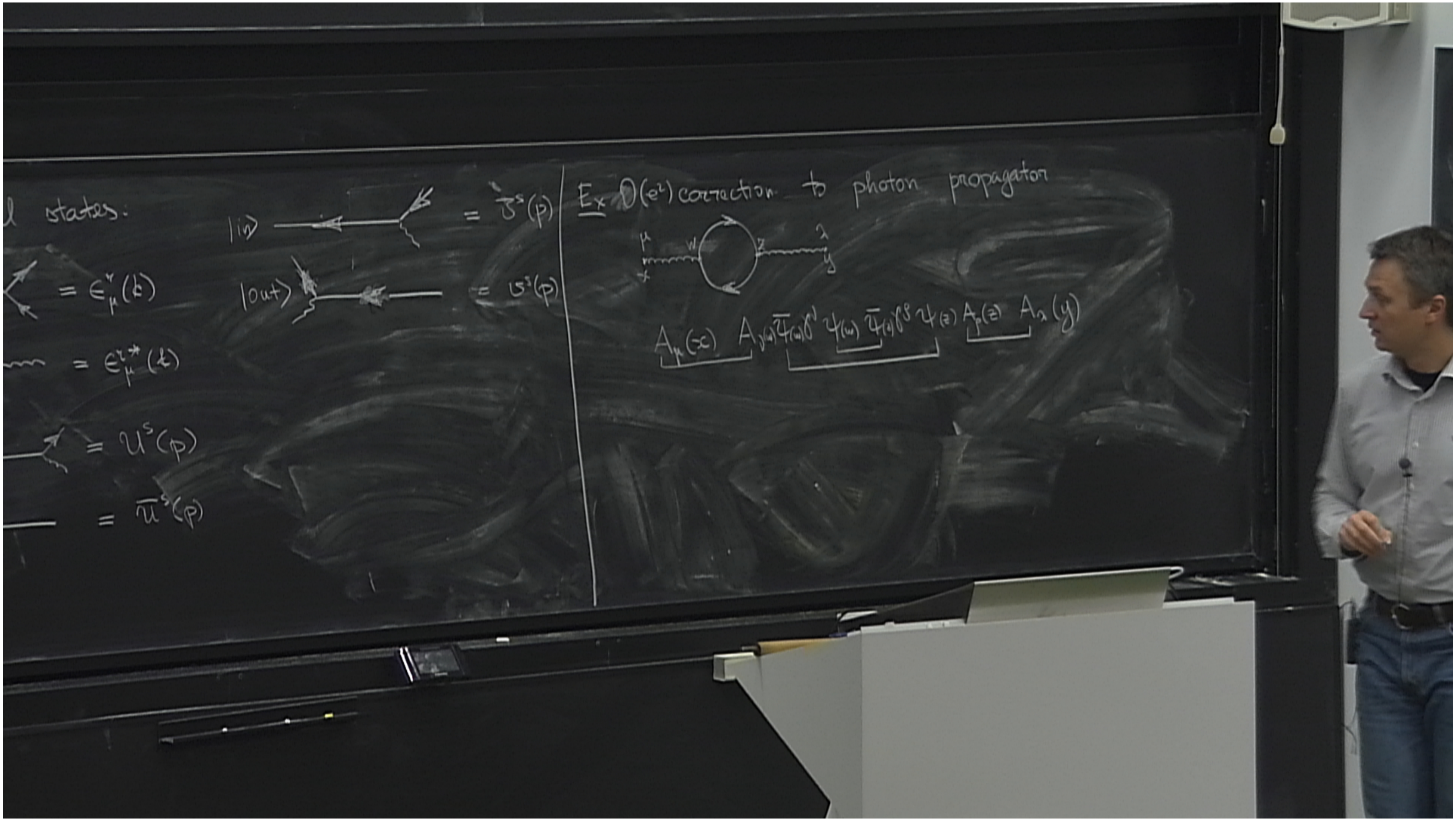


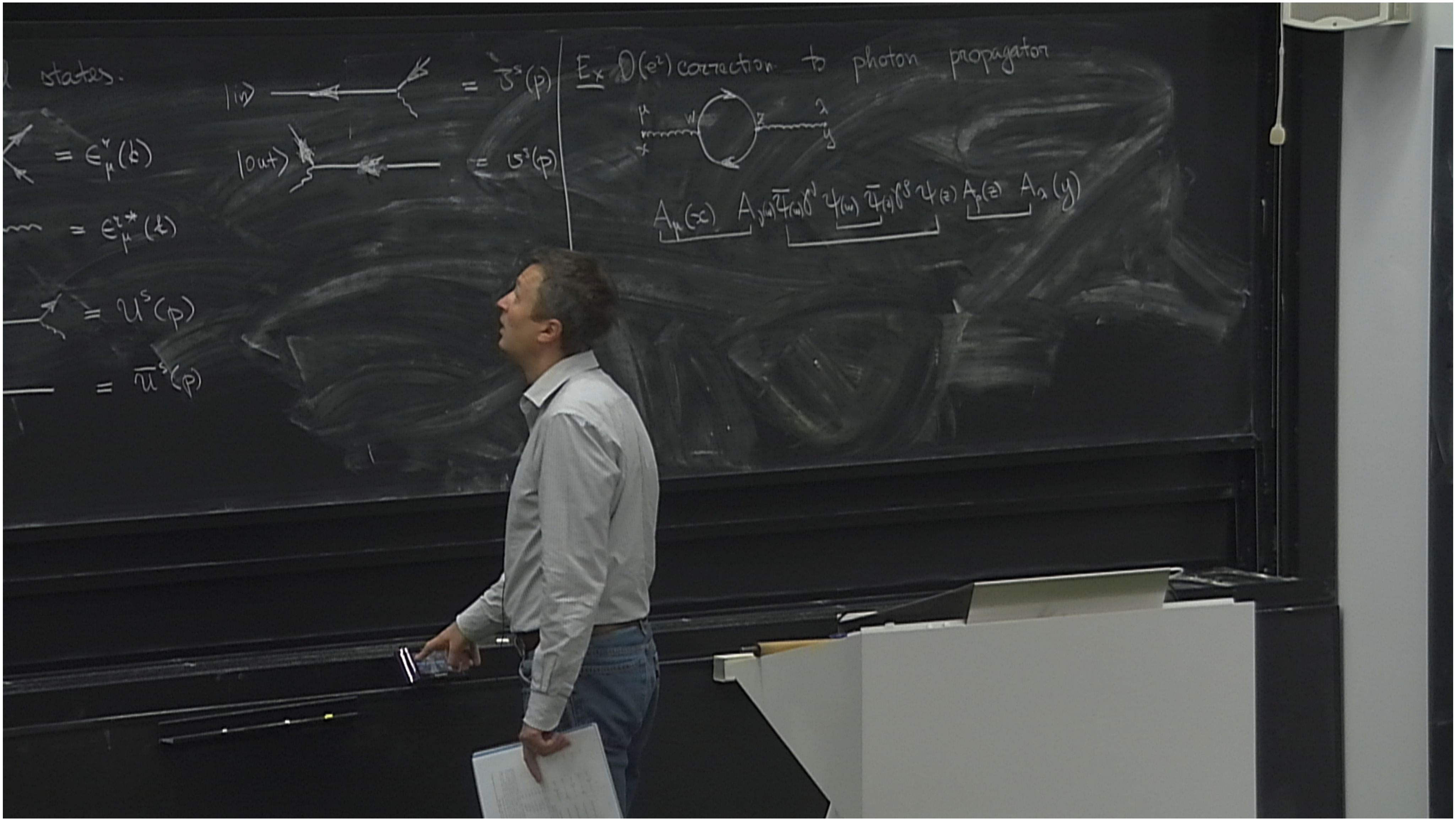
states.
|in⟩
|out⟩
Ex O(e²) correction to photon propagator

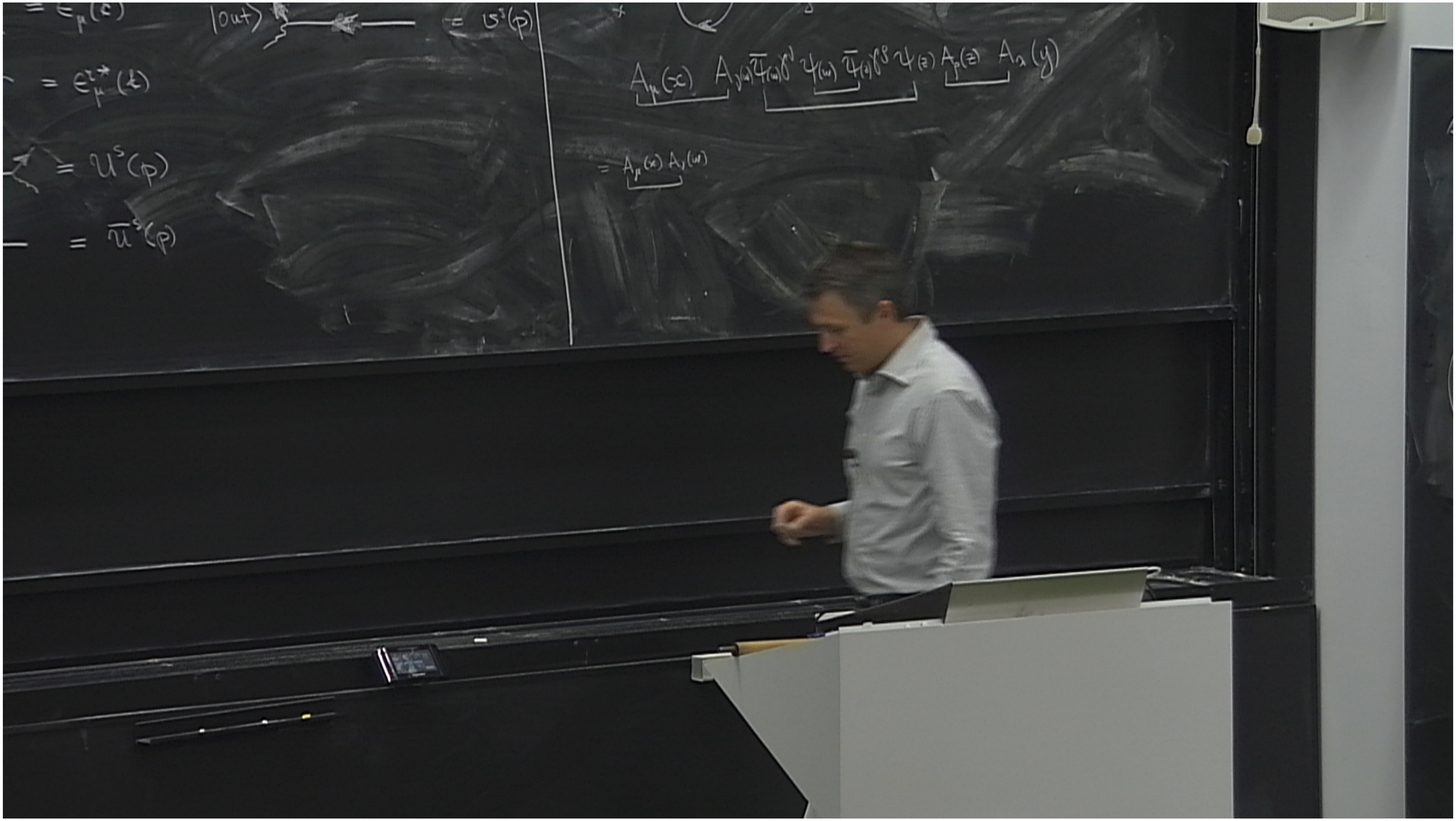

states:
|in⟩
|out⟩
Ex O(e²) correction to photon propagator
A_μ(x) A_ν ψ̄ γ^ν ψ ψ̄ γ^ρ ψ


states:
|in⟩
|out⟩
Ex O(e²) correction to photon propagator
$A_\mu(x)\ A_{\nu(w)}\bar\psi_{(w)}\gamma^\nu\psi_{(w)}\ \bar\psi_{(z)}\gamma^\rho\psi_{(z)}\ A_{\rho}(z)\ A_\lambda(y)$

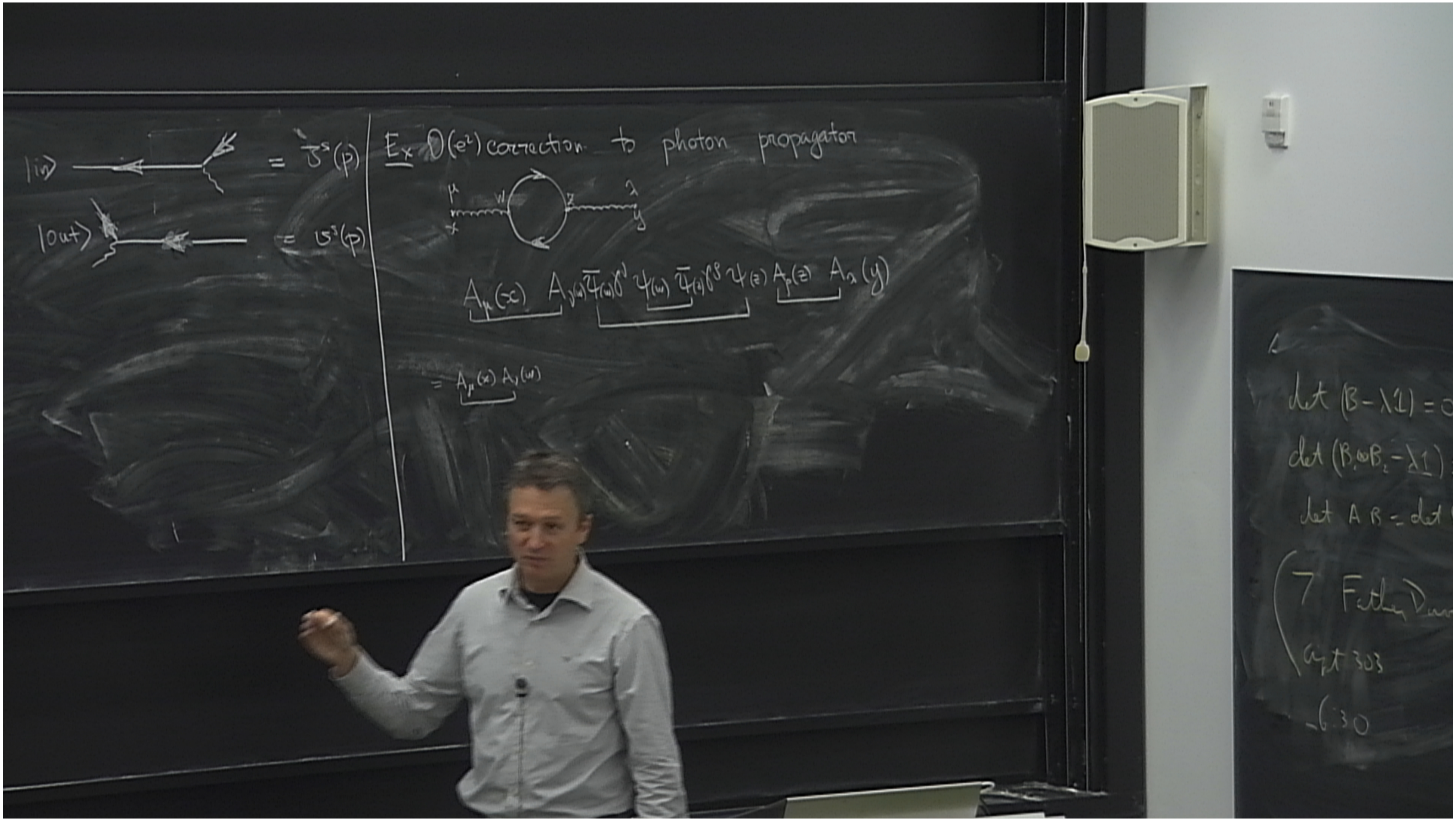

Ex O(e²) correction to photon propagator
states:
|in⟩
|out⟩

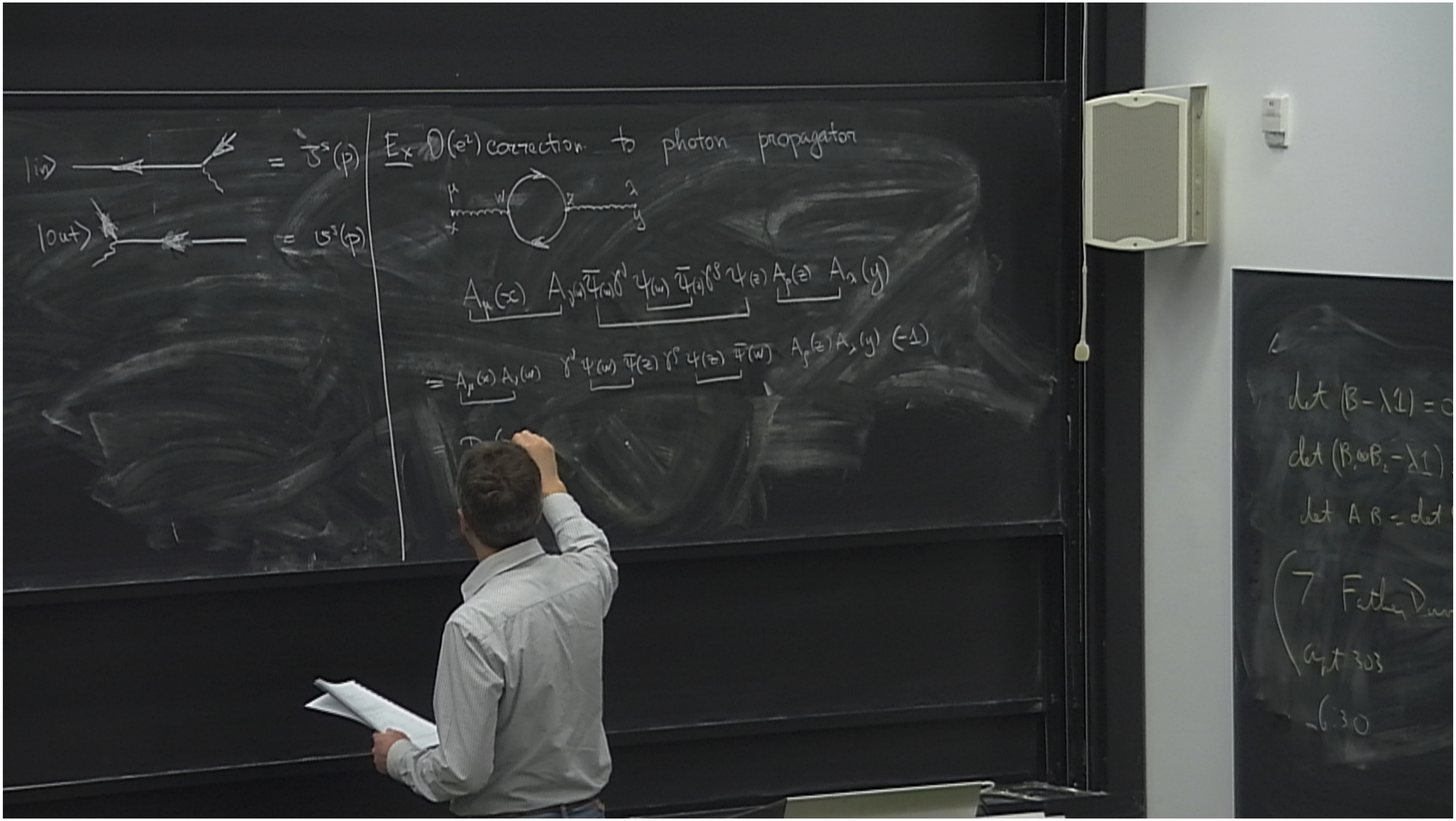

states:
|in⟩
|out⟩
Ex O(e²) correction to photon propagator
$A_\mu(x)\ A_\nu(w)\bar{\psi}(w)\gamma^\nu\psi(w)\ \bar{\psi}(z)\gamma^\rho\psi(z)\ A_\rho(z)\ A_\lambda(y)$



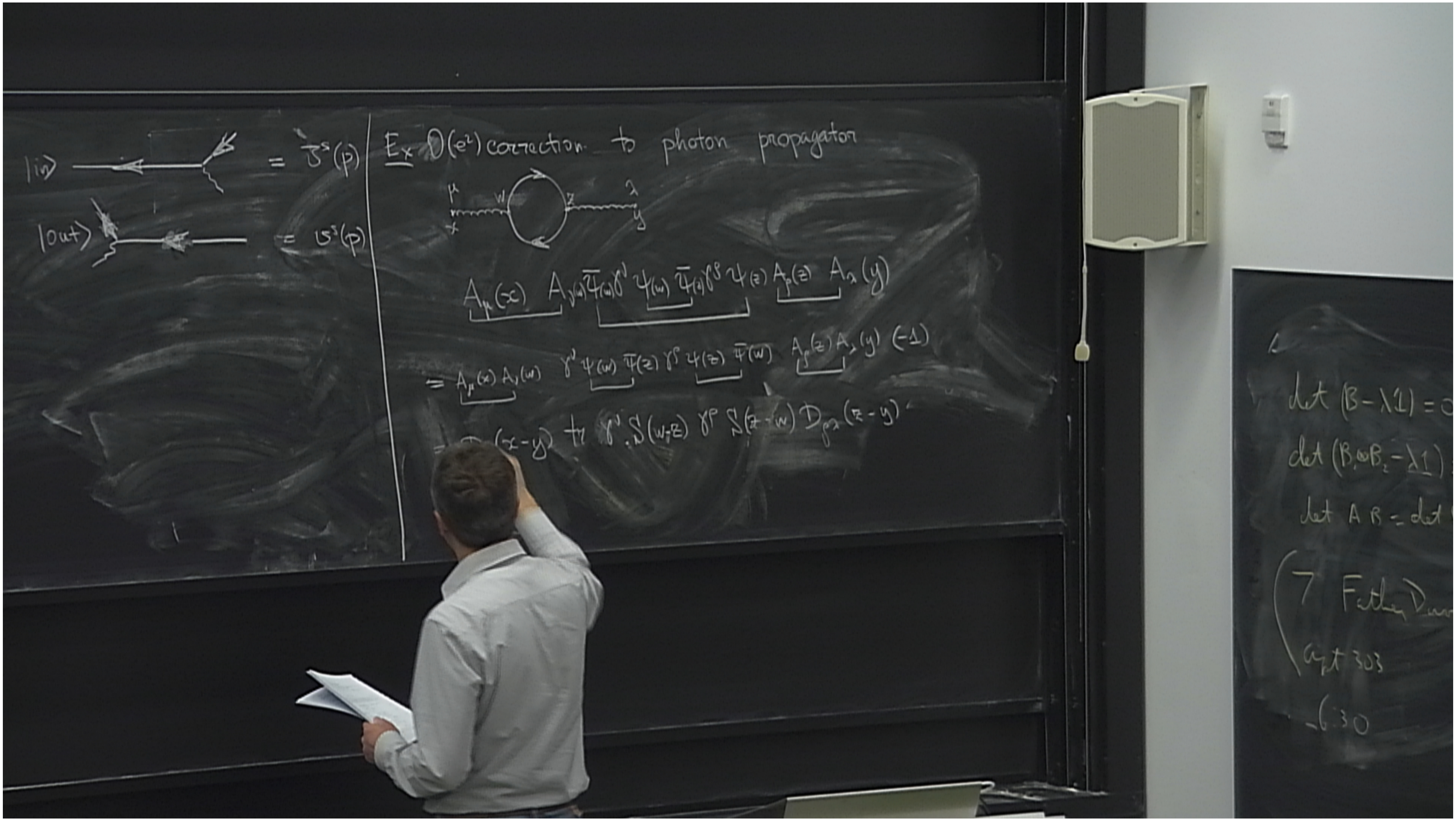


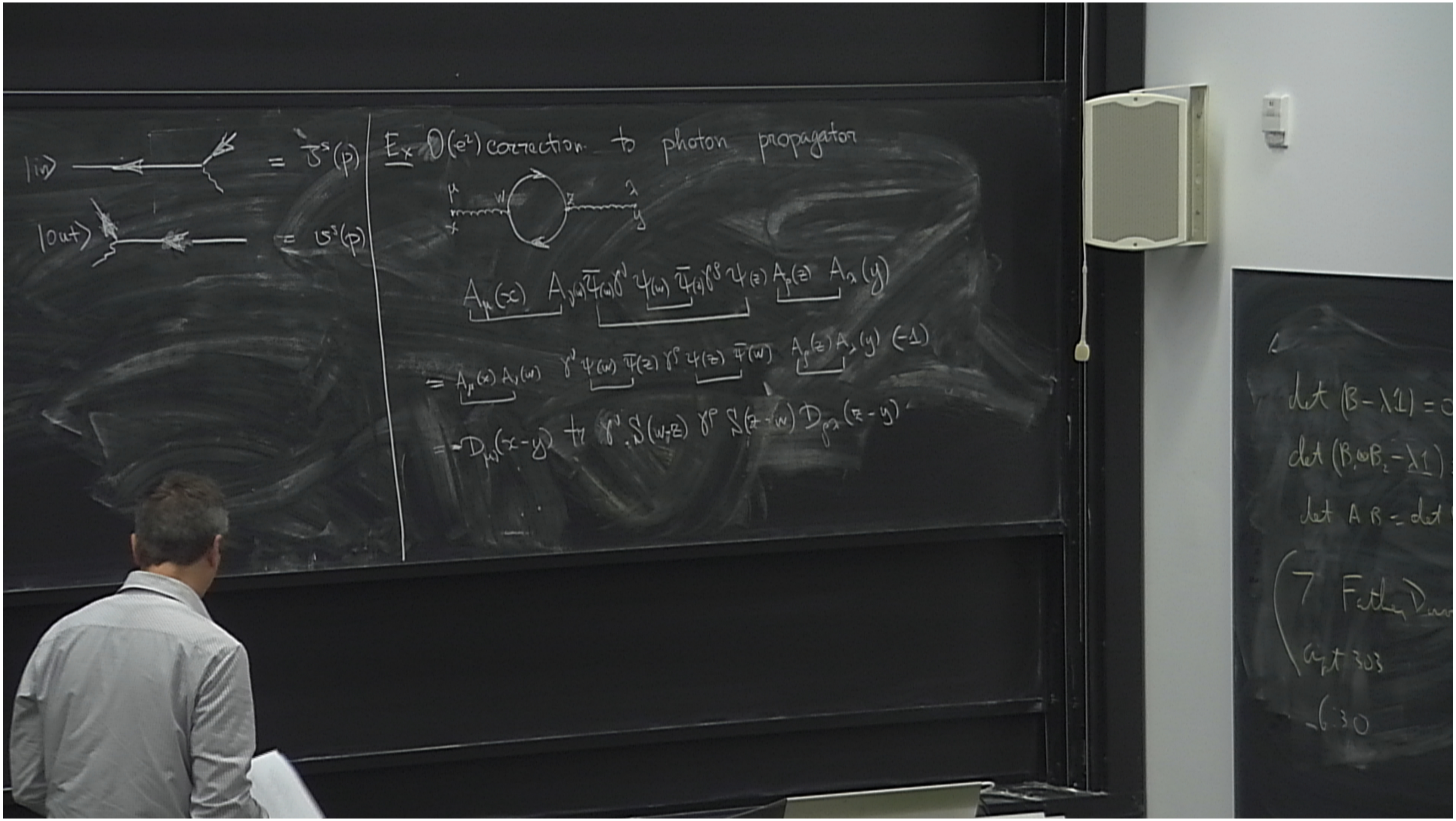
$= \epsilon_\mu(k)$
$|out\rangle$
$= v^s(p)$
$= \epsilon^{r*}_\mu(k)$
$= u^s(p)$
$= \bar{u}^s(p)$
$A_\mu(x)\ A_\nu(w)\bar{\psi}(w)\gamma^\nu\psi(w)\ \bar{\psi}(z)\gamma^\rho\psi(z)\ A_\rho(z)\ A_\alpha(y)$
$= A_\mu(x)\ A_\nu(w)$

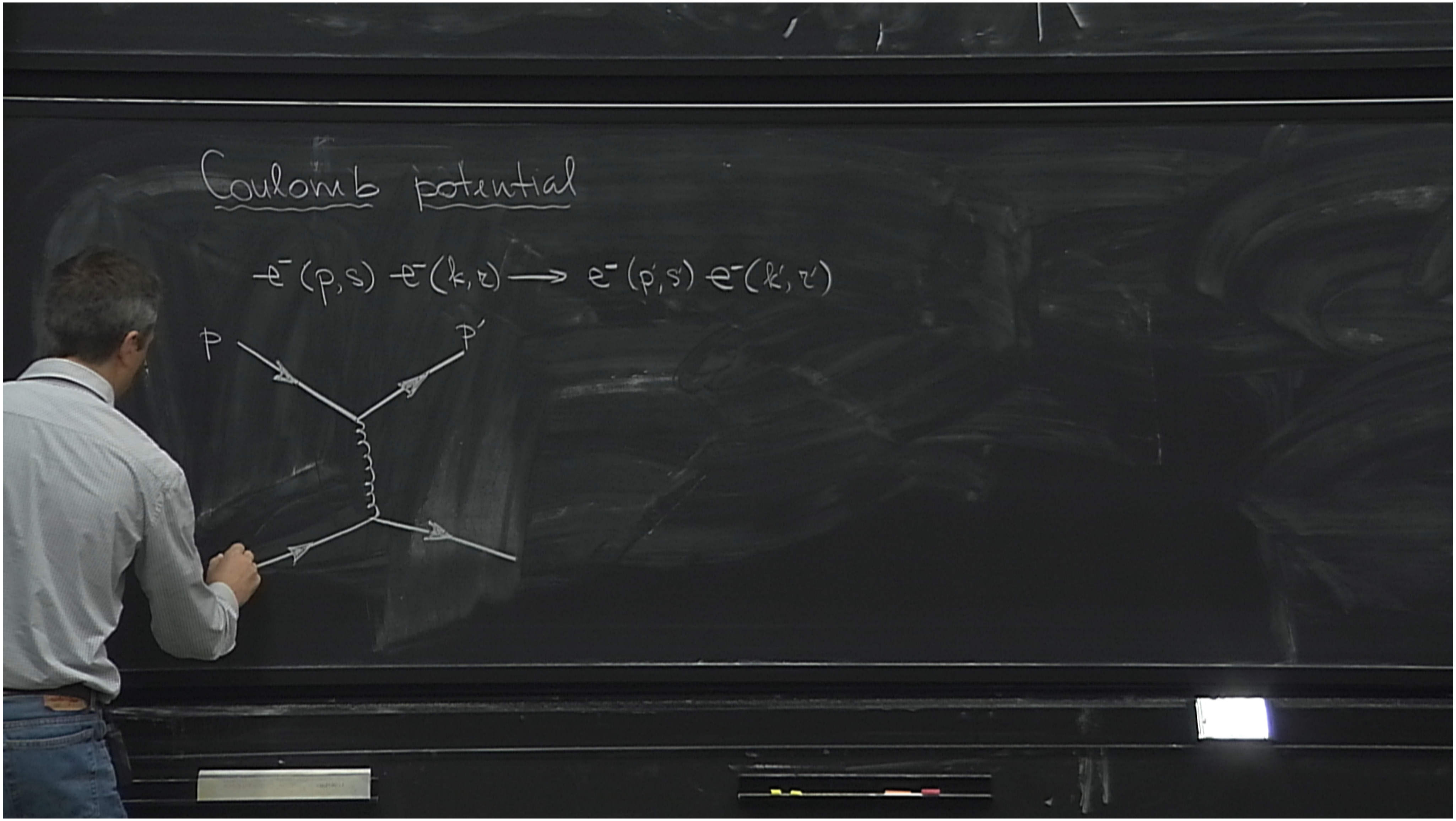

Ex O(e²) correction to photon propagator
|in⟩
|out⟩
μ
w
z
λ
x
y
= A_μ(x) A_ν(w)
7 Father Dun...
6:30

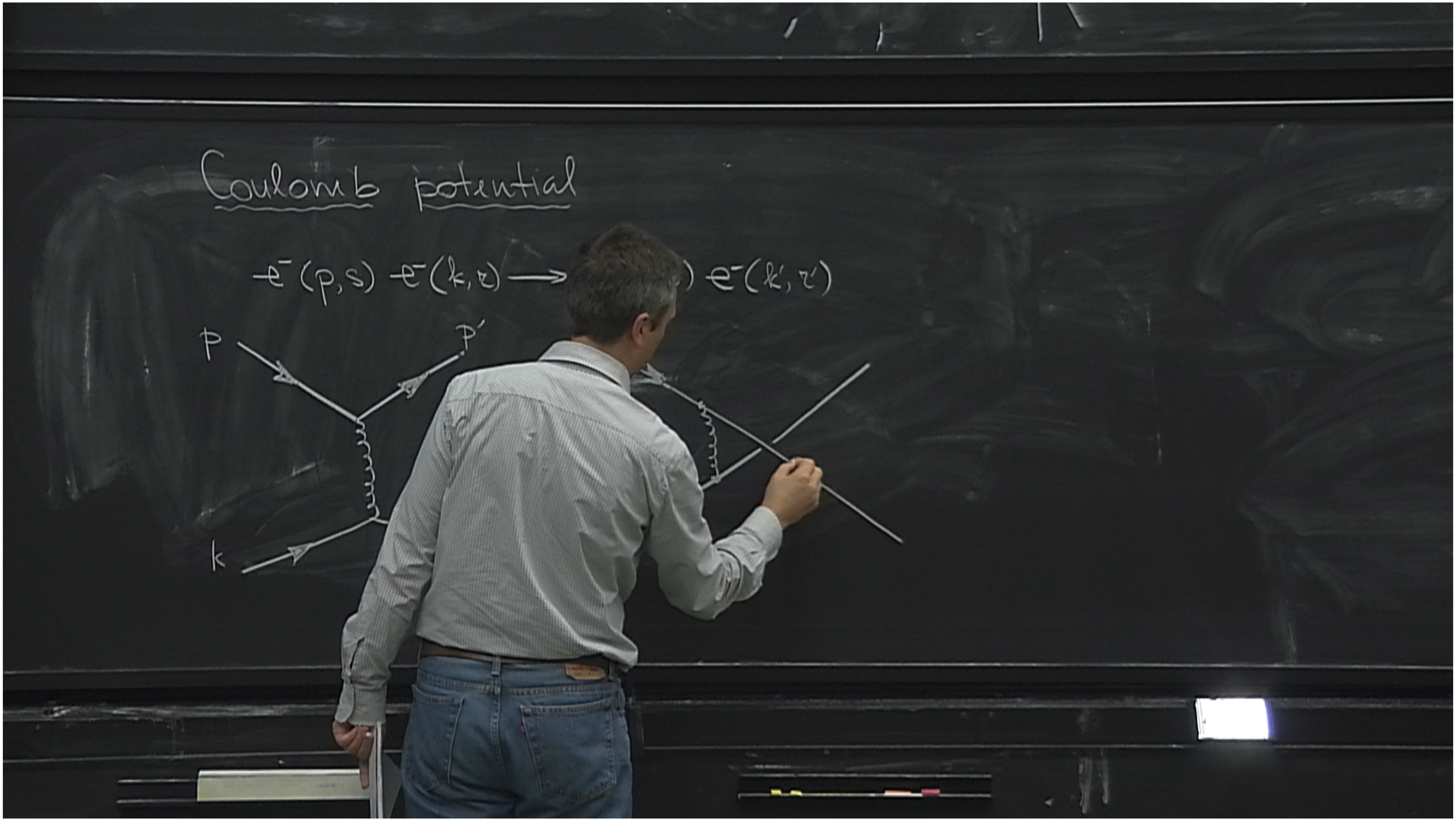


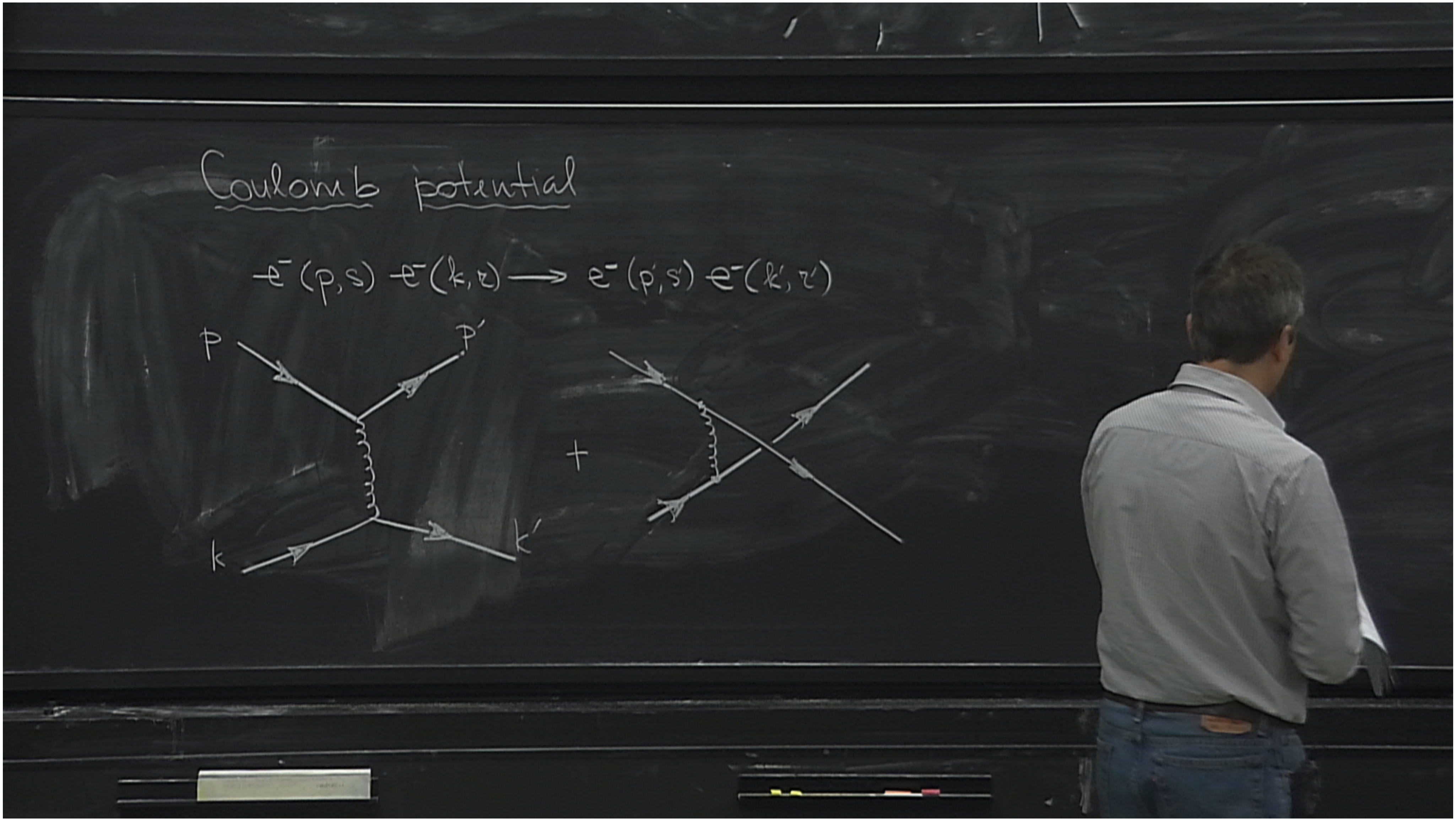
|in⟩
= $\bar{v}^s(p)$
|out⟩
= $v^s(p)$
Ex $\mathcal{O}(e^2)$ correction to photon propagator
μ w z λ
x y
$A_\mu(x)\ A_\nu(w)\bar\Psi(w)\gamma^\nu\ \psi(w)\ \bar\Psi(z)\gamma^\rho\psi(z)\ A_\rho(z)\ A_\lambda(y)$
$= A_\mu(x)\ A_\nu(w)\ \gamma^\nu\ \psi(w)\ \bar\Psi(z)\ \gamma^\rho\ \psi(z)\ \bar\Psi(w)\ A_\rho(z)\ A_\lambda(y)\ (-1)$
det (B − λ1) =
det (B₁⊗B₂ − λ1)
det A B = det
7 Father Dunn
Cap't 303
6:30

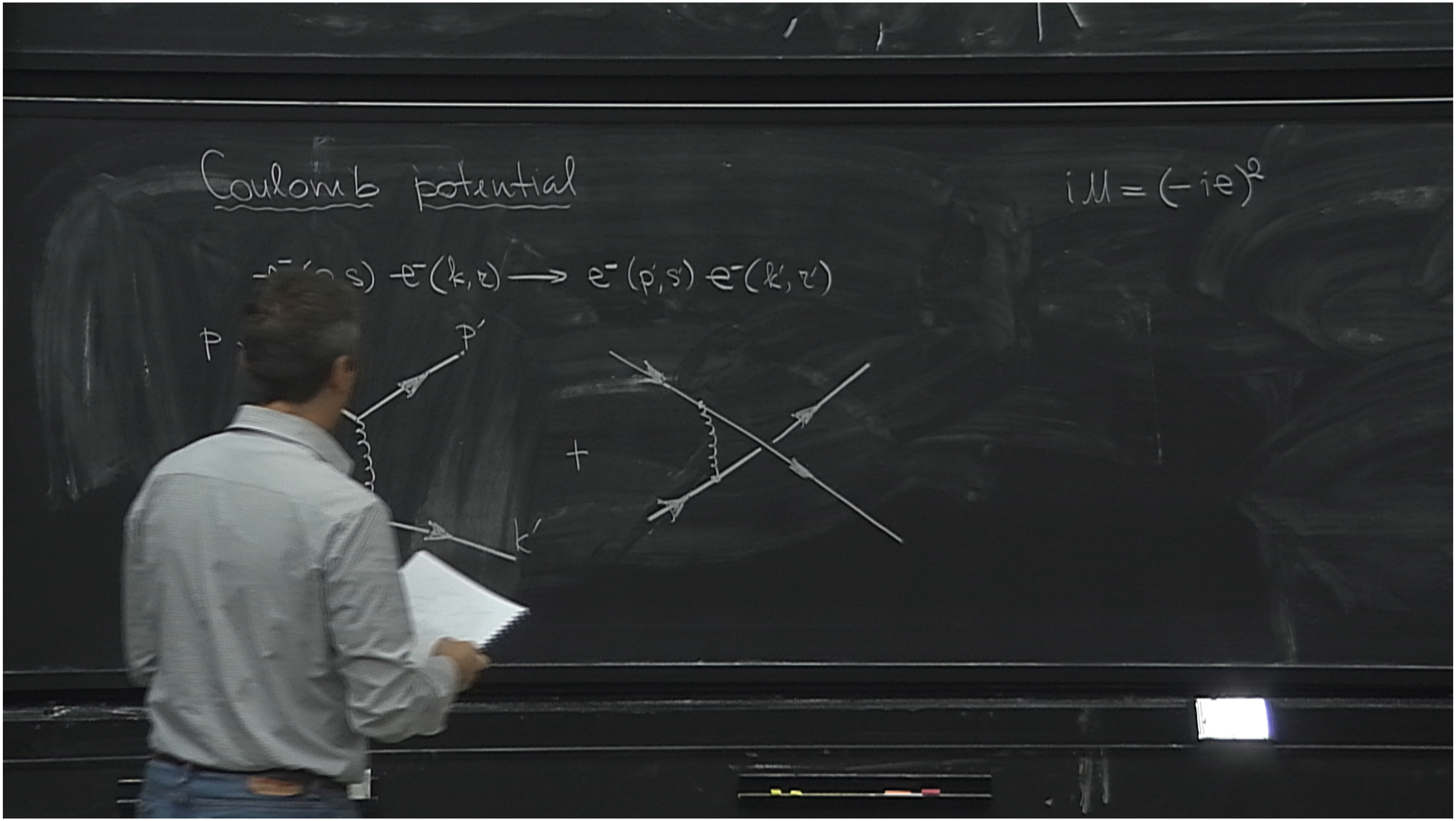

|in⟩
|out⟩
Ex O(e²) correction to photon propagator
7 Feynman Diagrams

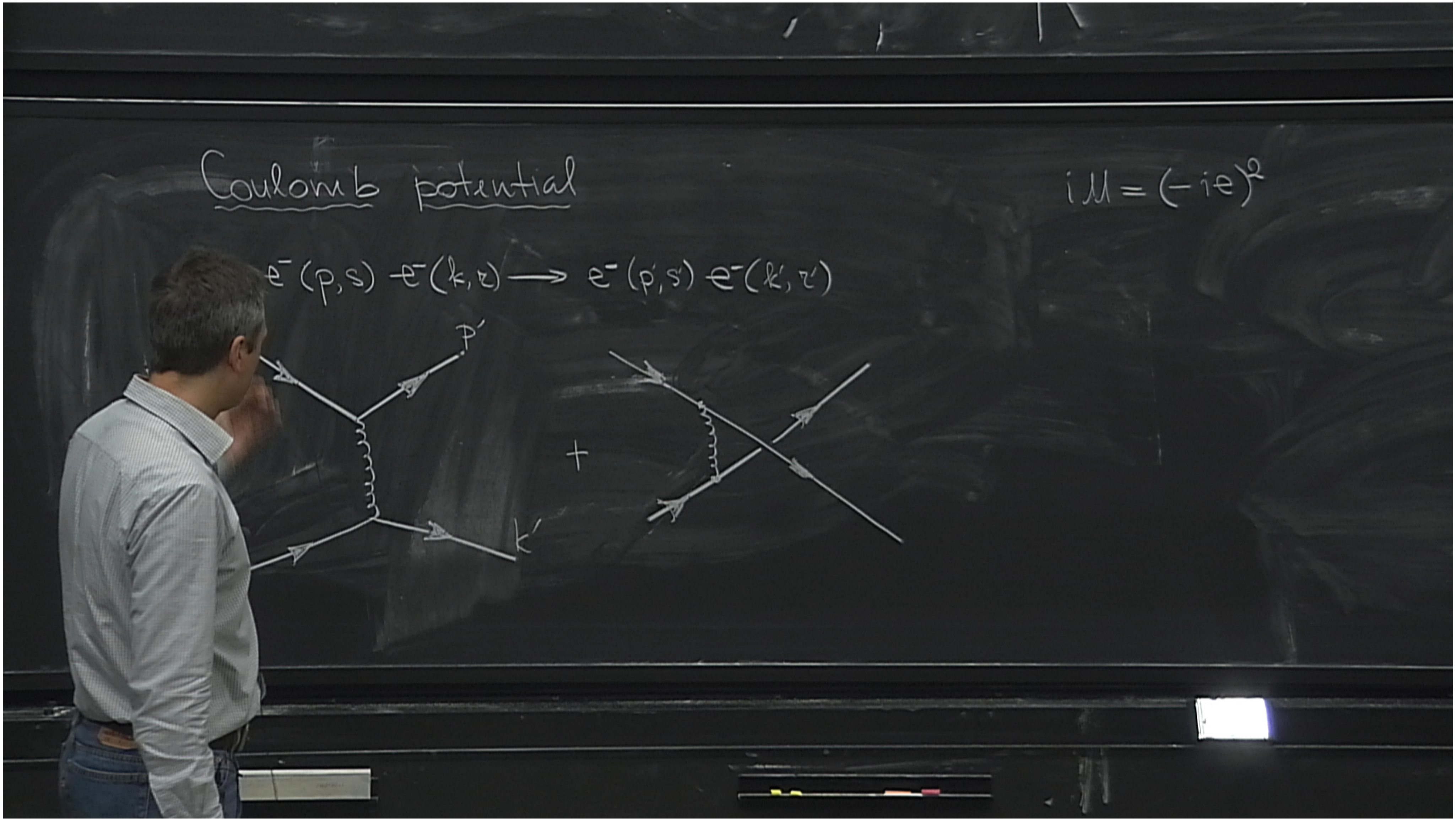

Ex O(e²) correction to photon propagator

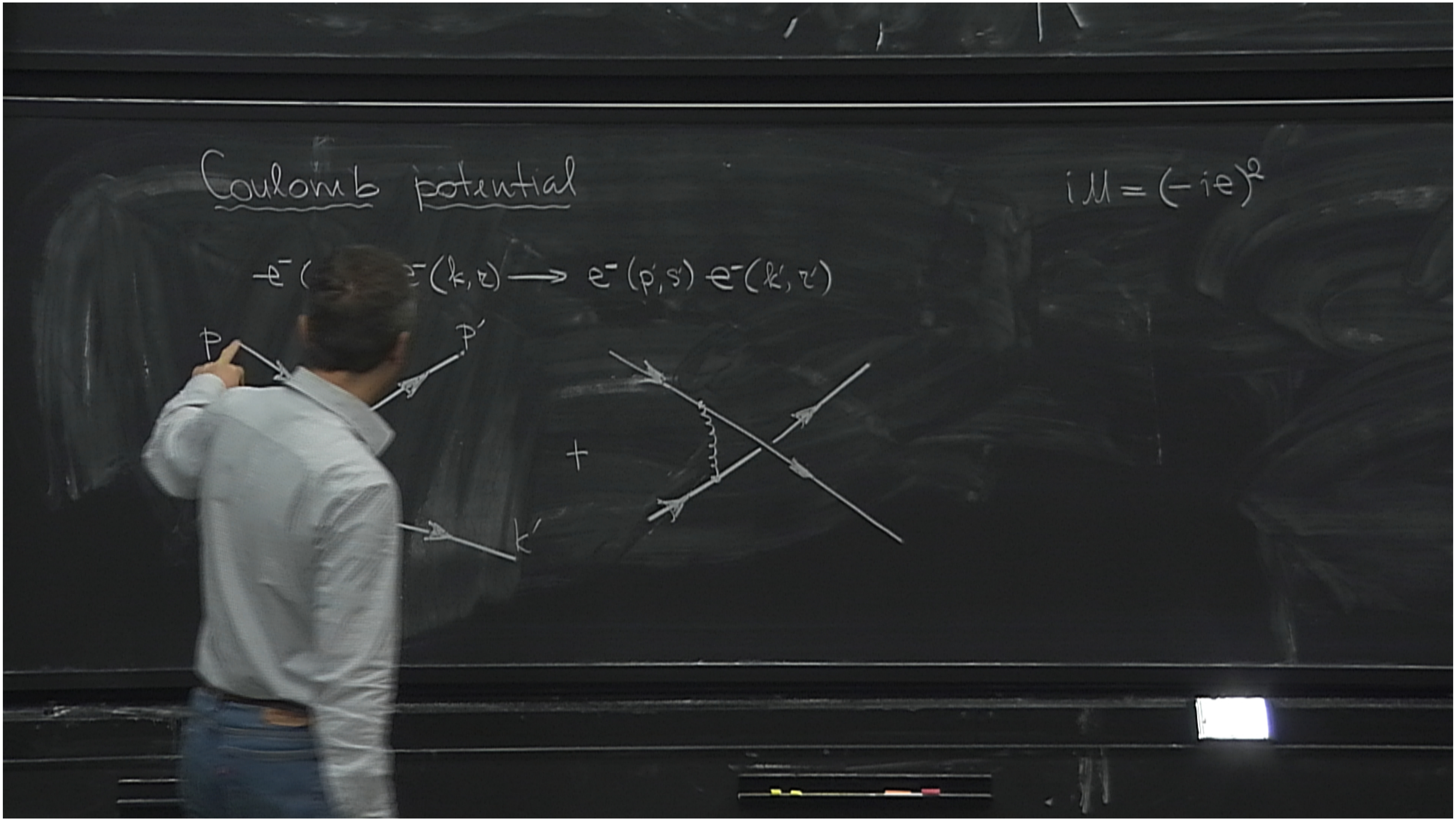

Coulomb potential
e-(p,s) e-(k,r) → e-(p',s') e-(k',r')
p
p'

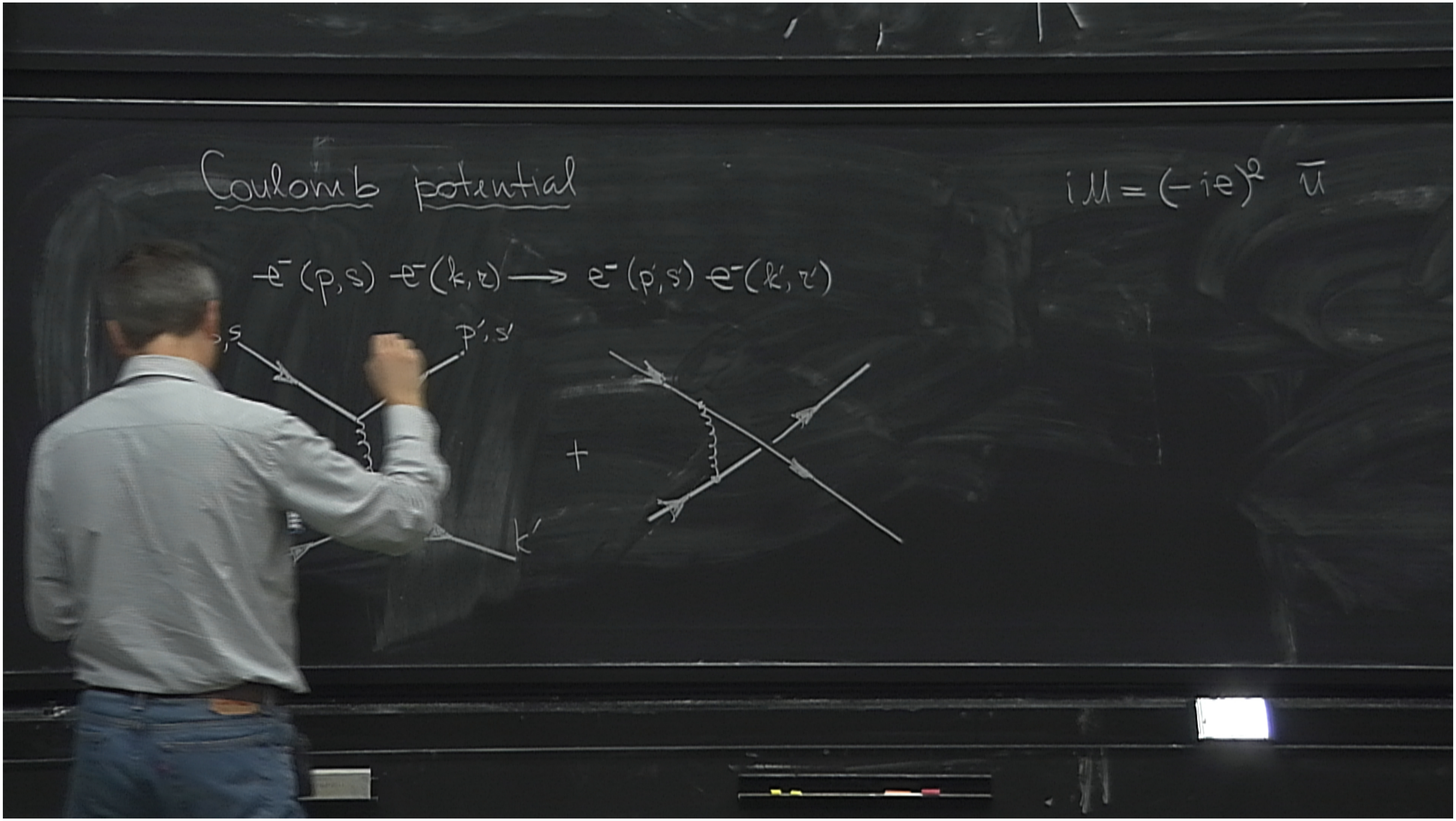

Coulomb potential
e⁻(p,s) e⁻(k,r) ⟶) e⁻(k′,r′)
p
p′
k

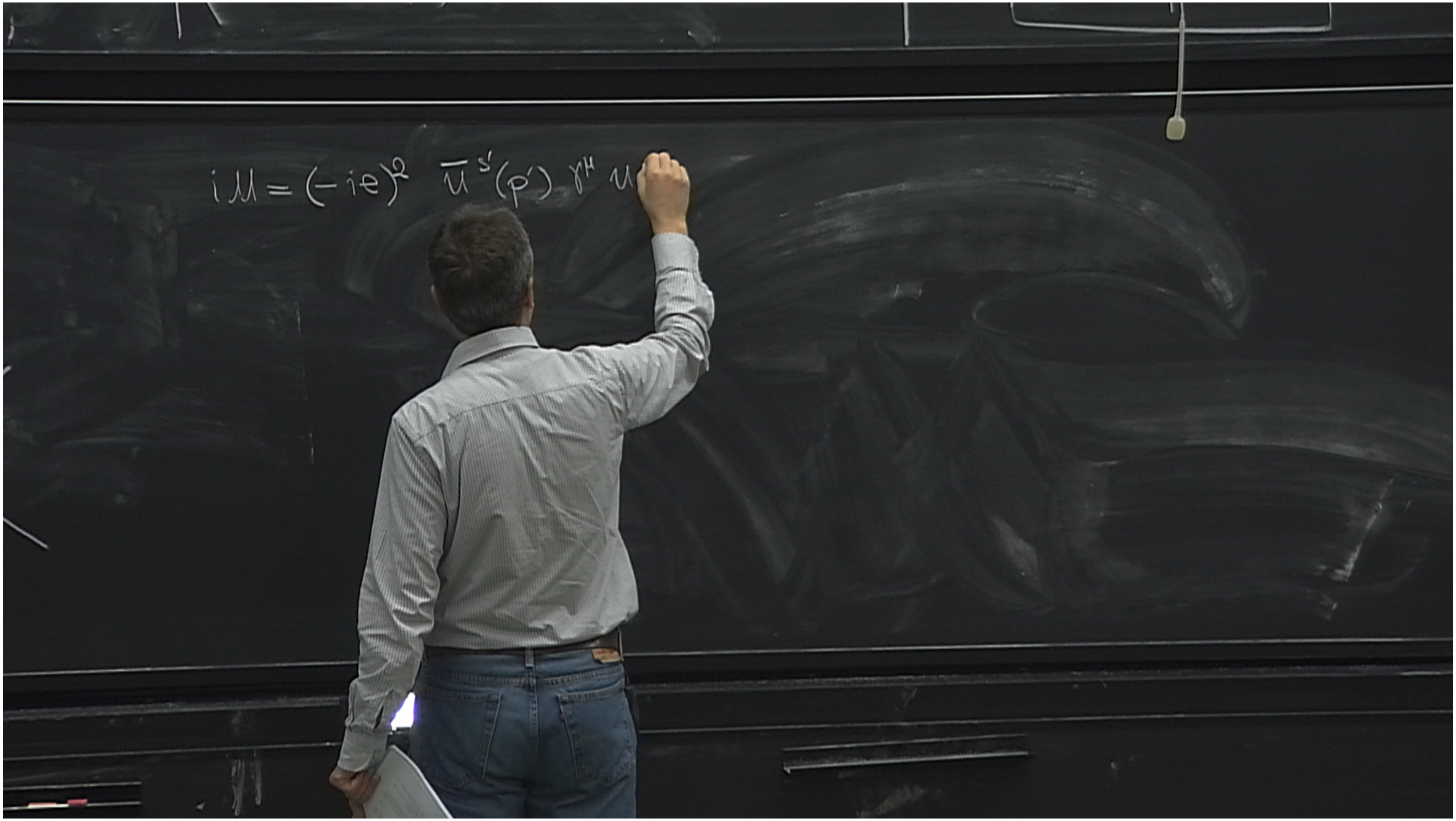

Coulomb potential
e⁻(p,s) e⁻(k,r) ⟶ e⁻(p′,s′) e⁻(k′,r′)
p
p′
k
k′
+


Coulomb potential
s) e⁻(k,r) ⟶ e⁻(p',s') e⁻(k',r')
p
p'
k'
+
iM = (−ie)²

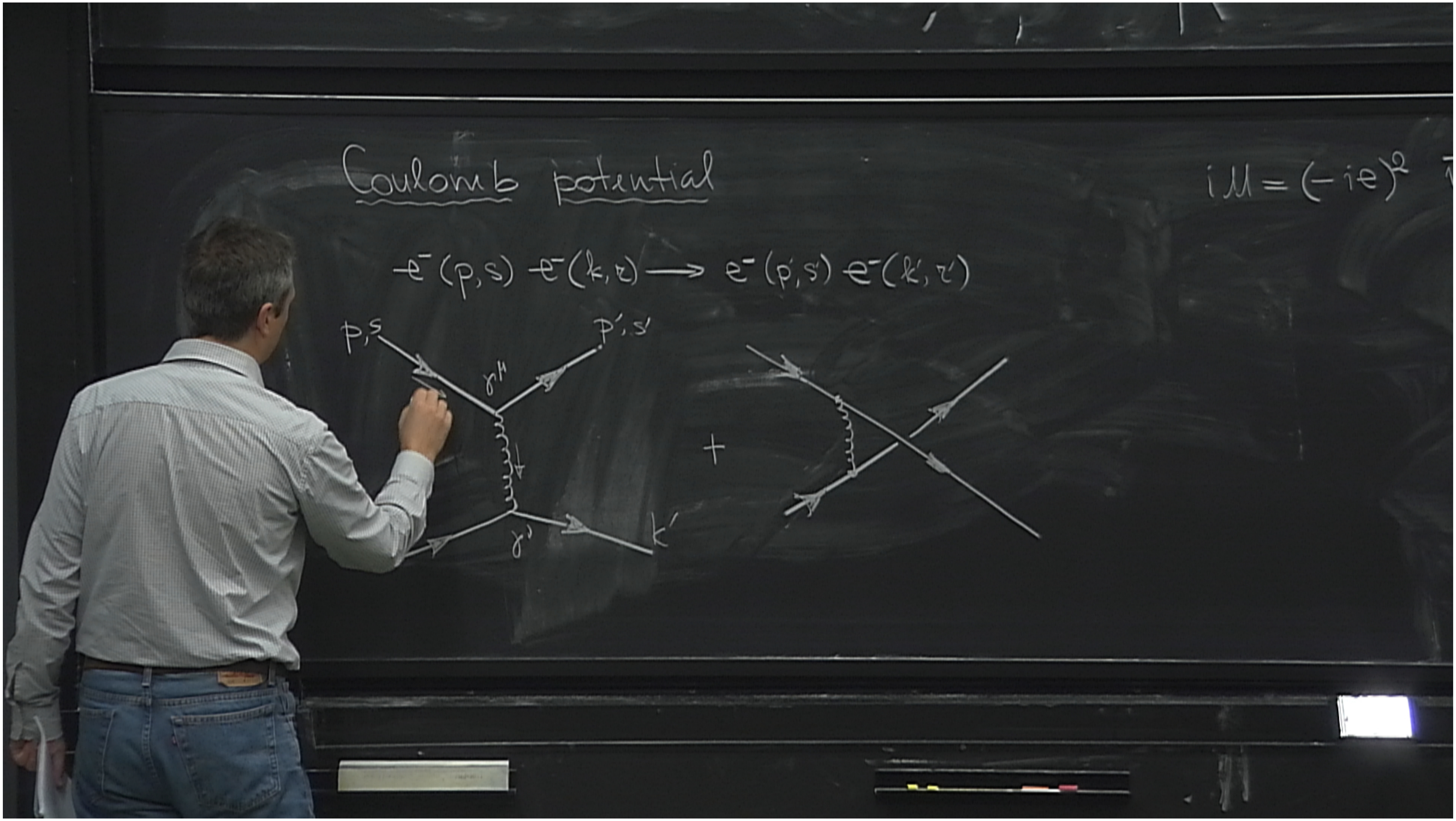

Coulomb potential
e⁻(p,s) e⁻(k,r) → e⁻(p′,s′) e⁻(k′,r′)
p′
k′
+
iM = (−ie)²

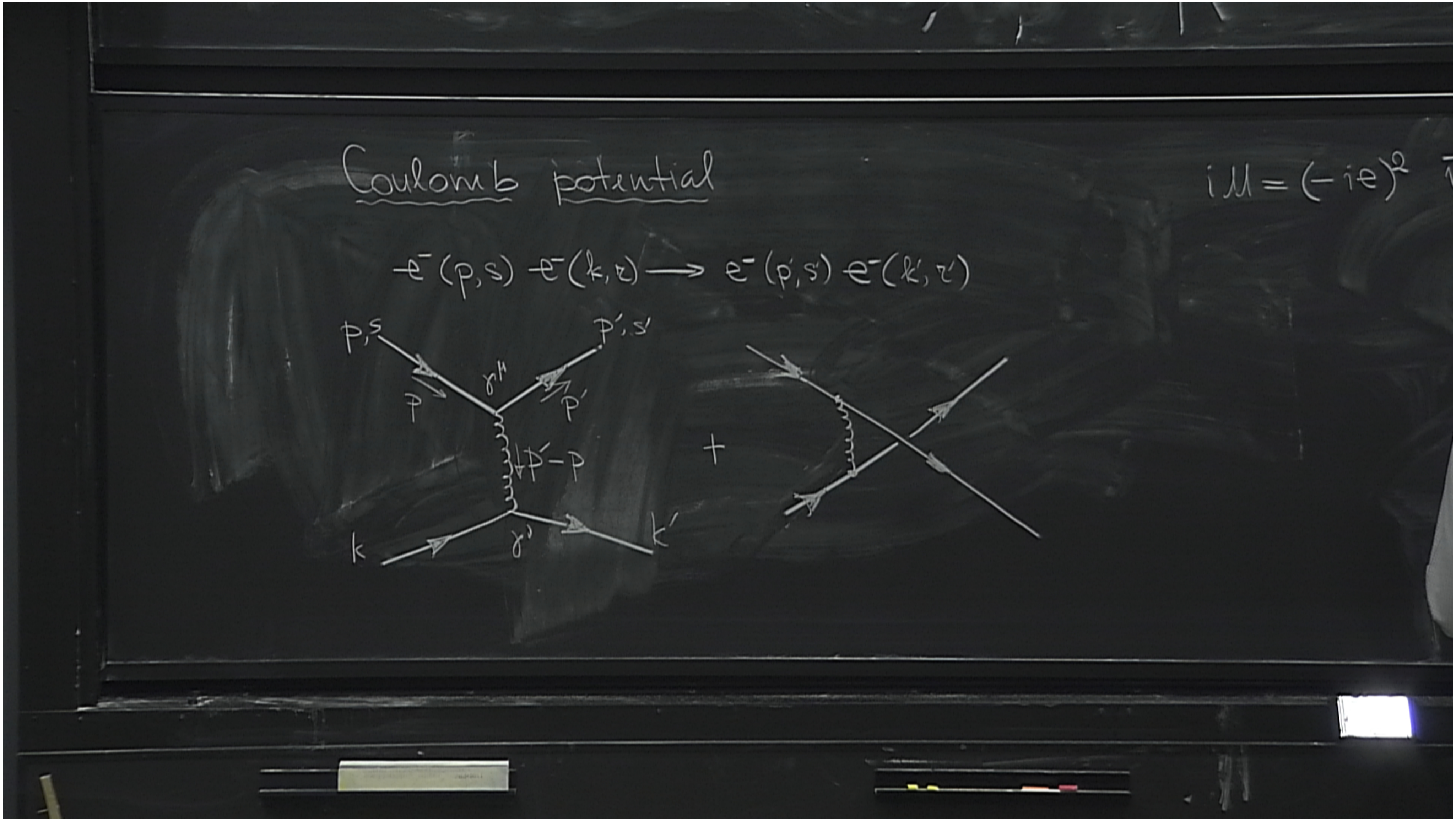

Coulomb potential
e⁻(k,r) ⟶ e⁻(p′,s′) e⁻(k′,r′)
p
p′
k′
+
iM = (−ie)²


Coulomb potential
e⁻(p,s) e⁻(k,r) ⟶ e⁻(p′,s′) e⁻(k′,r′)
p′,s′
k′
+
iM = (−ie)² ū

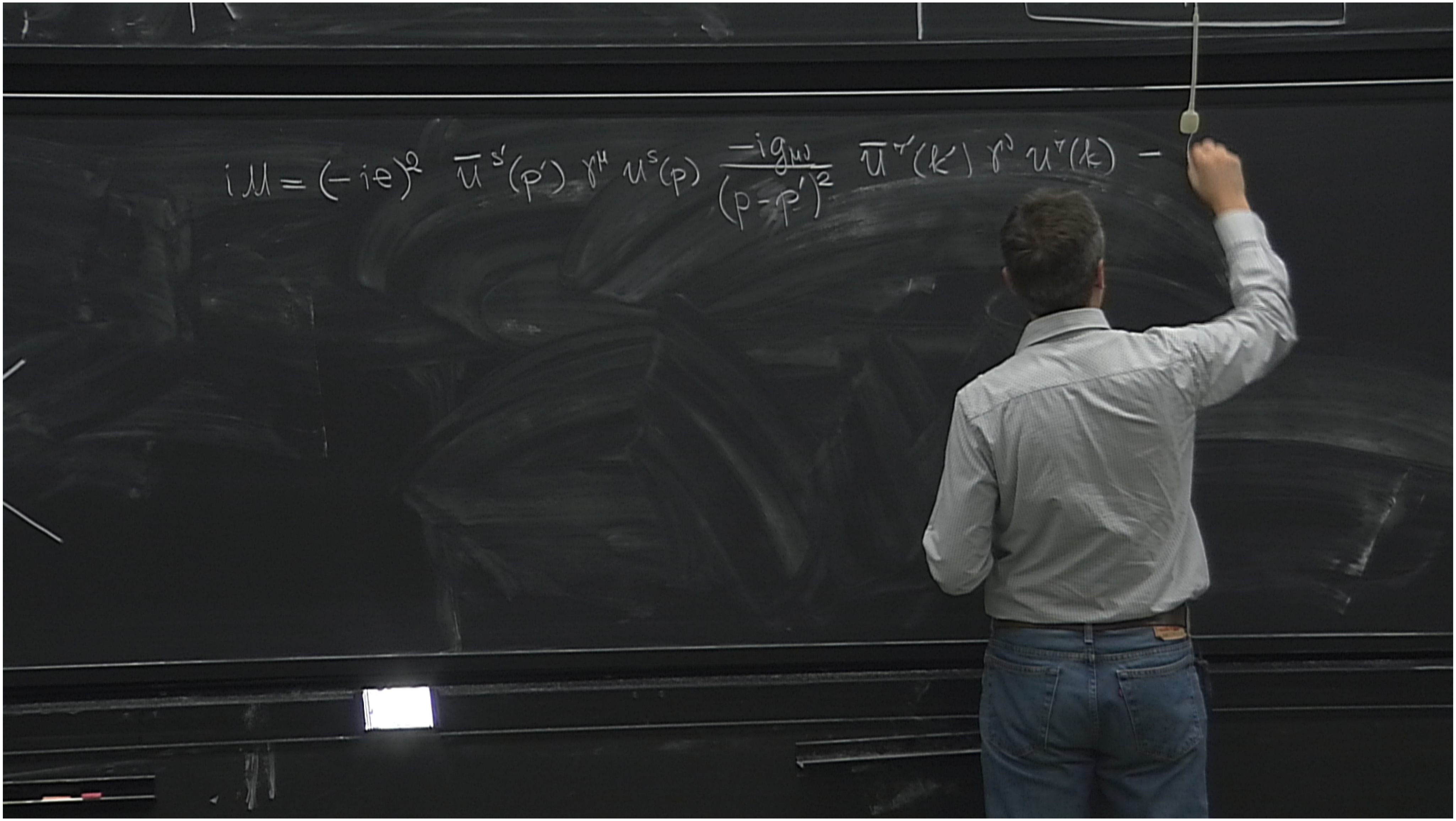

$i\mathcal{M} = (-ie)^2 \ \bar{u}^{s'}(p') \gamma^\mu u$

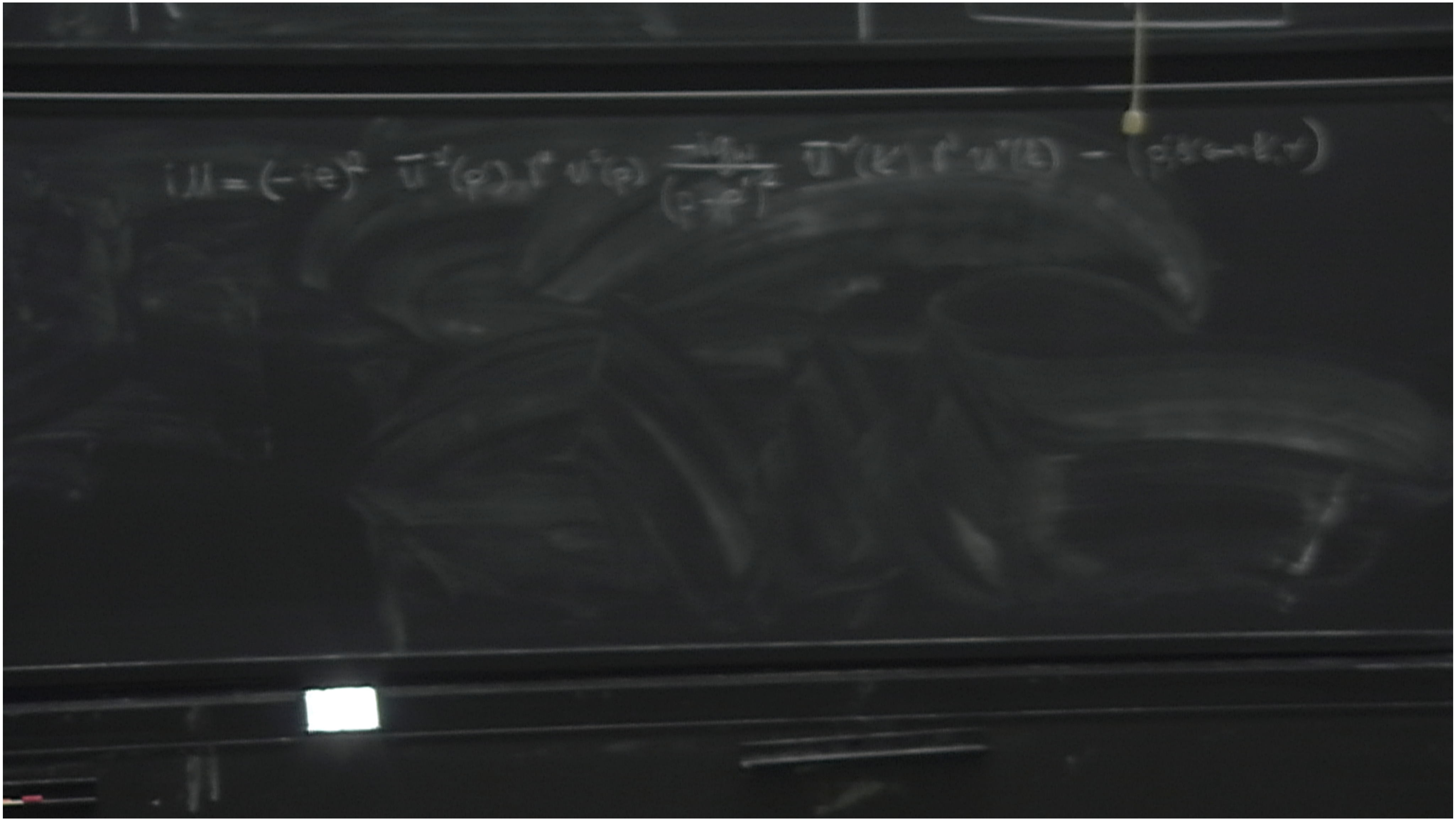
$$i\mathcal{M} = (-ie)^2\, \bar{u}^{s'}(p')\, \gamma^\mu\, u^s(p)\, \underline{-ig_{\mu\nu}}$$


Coulomb potential
e⁻(p,s) e⁻(k,r) ⟶ e⁻(p′,s′) e⁻(k′,r′)
iM = (−ie)²
p,s
p′,s′
γ^μ
γ^ν
k′
+

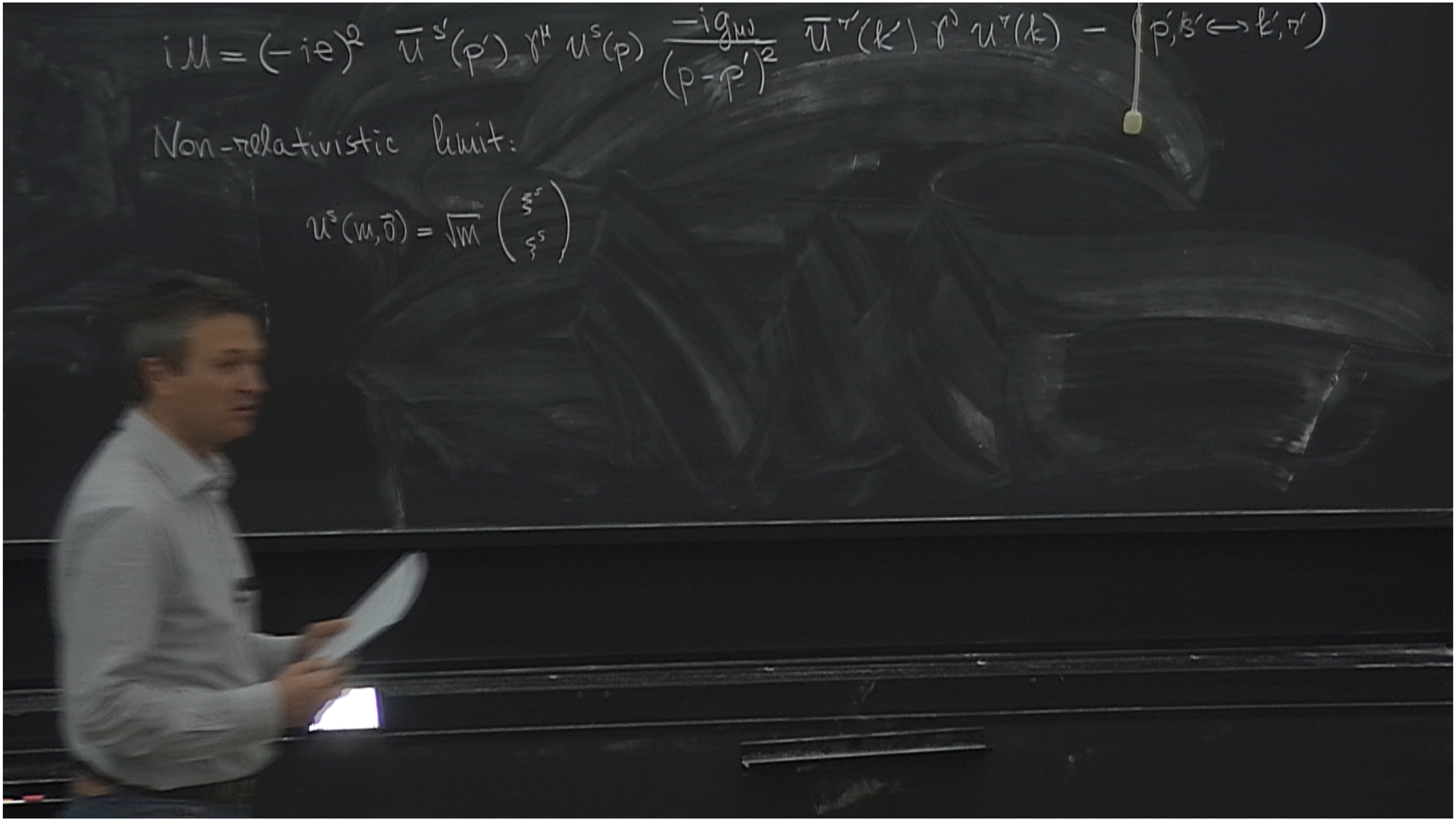

Coulomb potential
e⁻(p,s) e⁻(k,r) ⟶ e⁻(p′,s′) e⁻(k′,r′)
iM = (−ie)²
p,s
p′,s′
p
p′
γ^μ
p′−p
γ^ν
k
k′
+

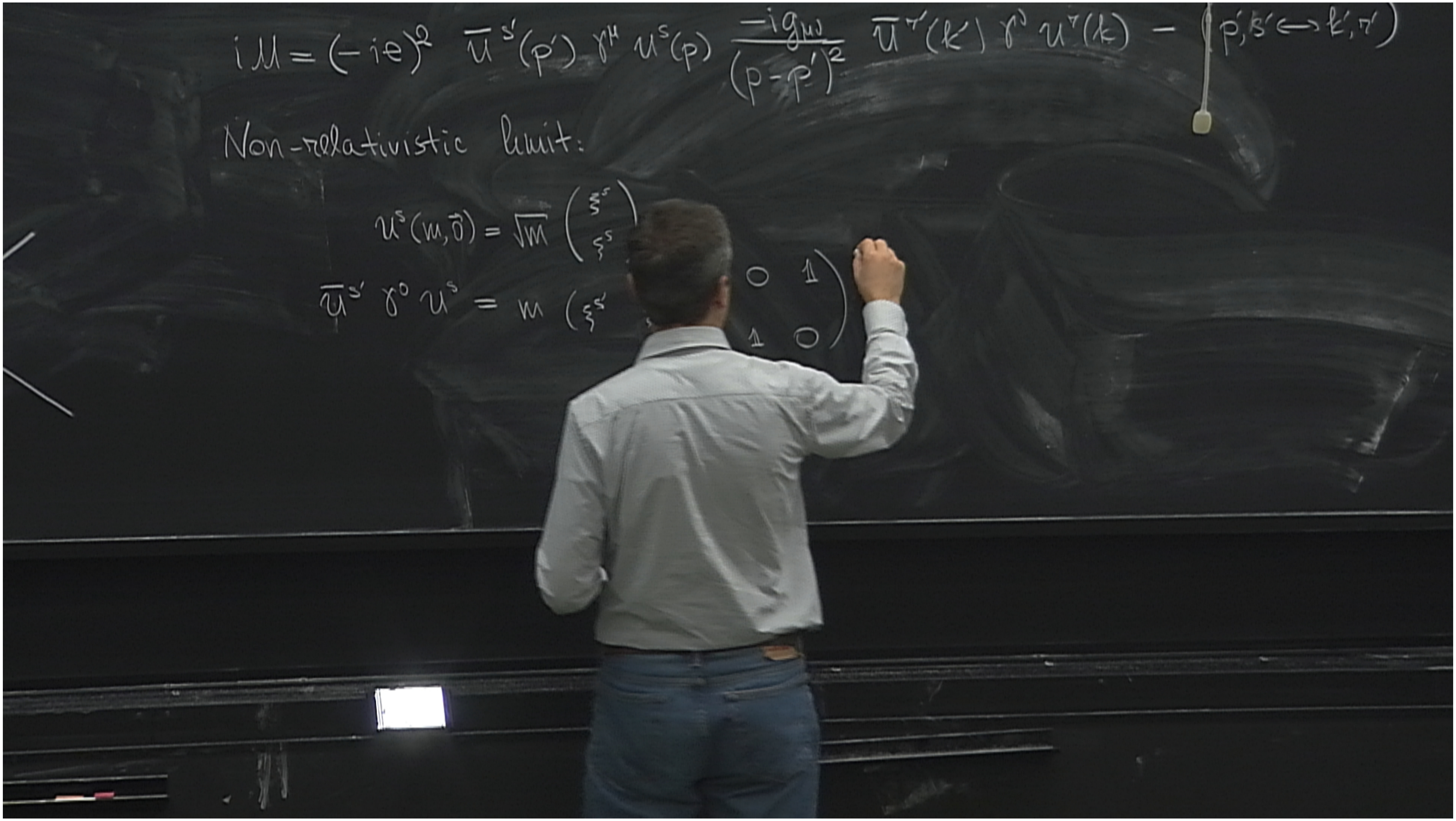
$$i\mathcal{M} = (-ie)^2\, \bar{u}^{s'}(p')\,\gamma^\mu\, u^s(p)\, \frac{-ig_{\mu\nu}}{(p-p')^2}\, \bar{u}^{r'}(k')\,\gamma^\nu\, u^r(k)$$

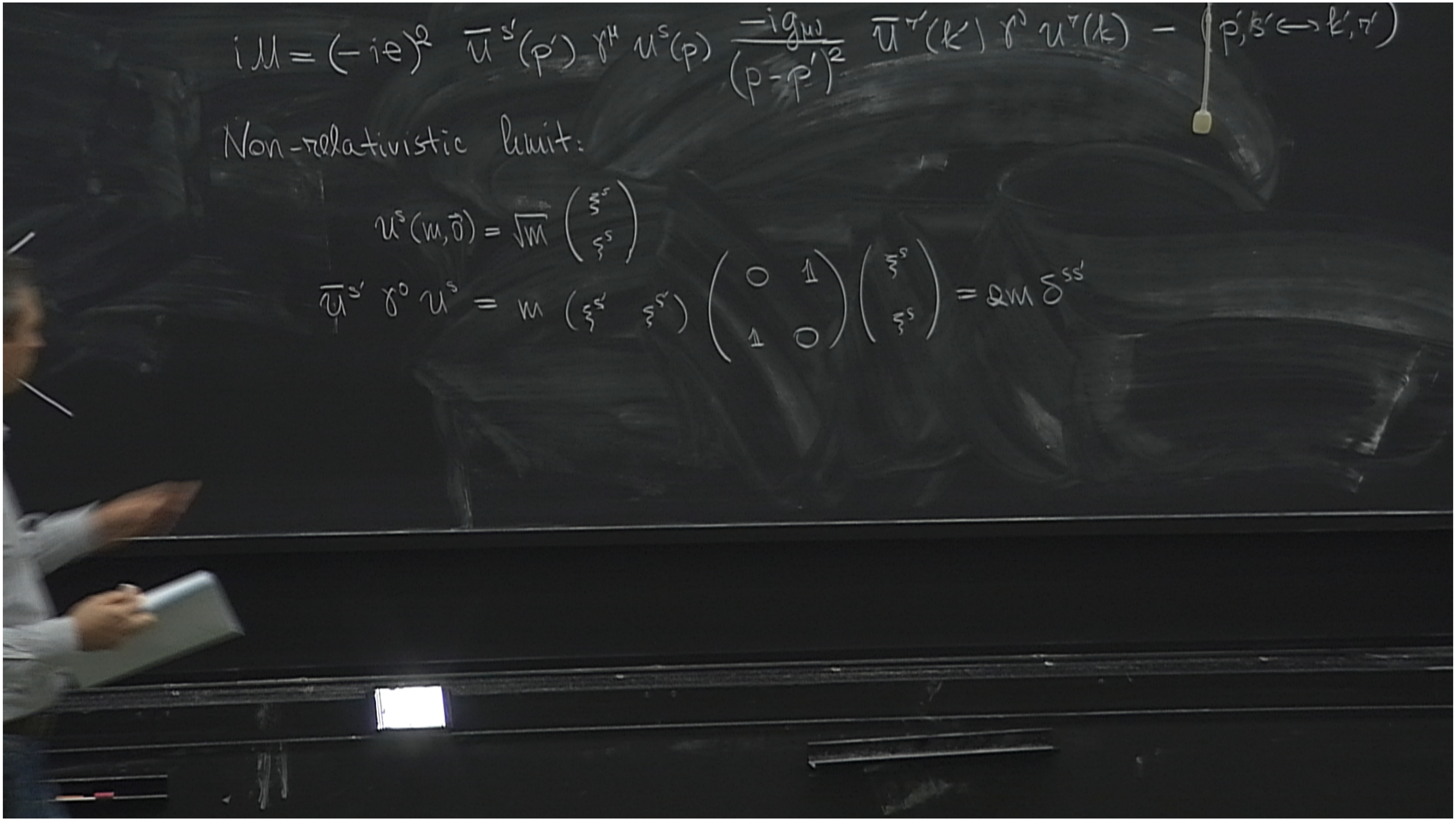

$i\mathcal{M} = (-ie)^2 \, \bar{u}^{s'}(p') \gamma^\mu u^s(p) \frac{-ig_{\mu\nu}}{(p-p')^2} \bar{u}^{r'}(k') \gamma^\nu u^r(k) -$

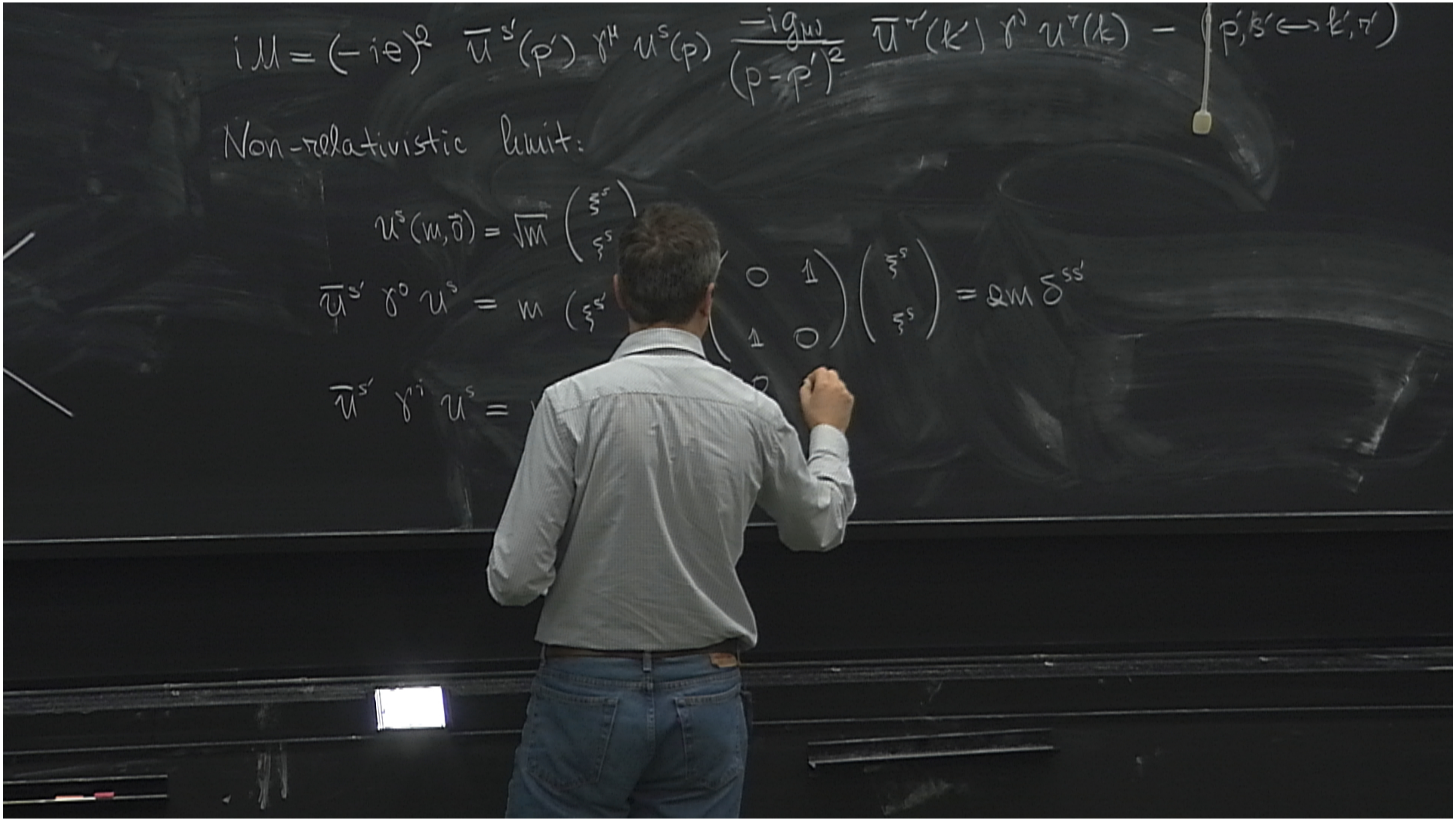


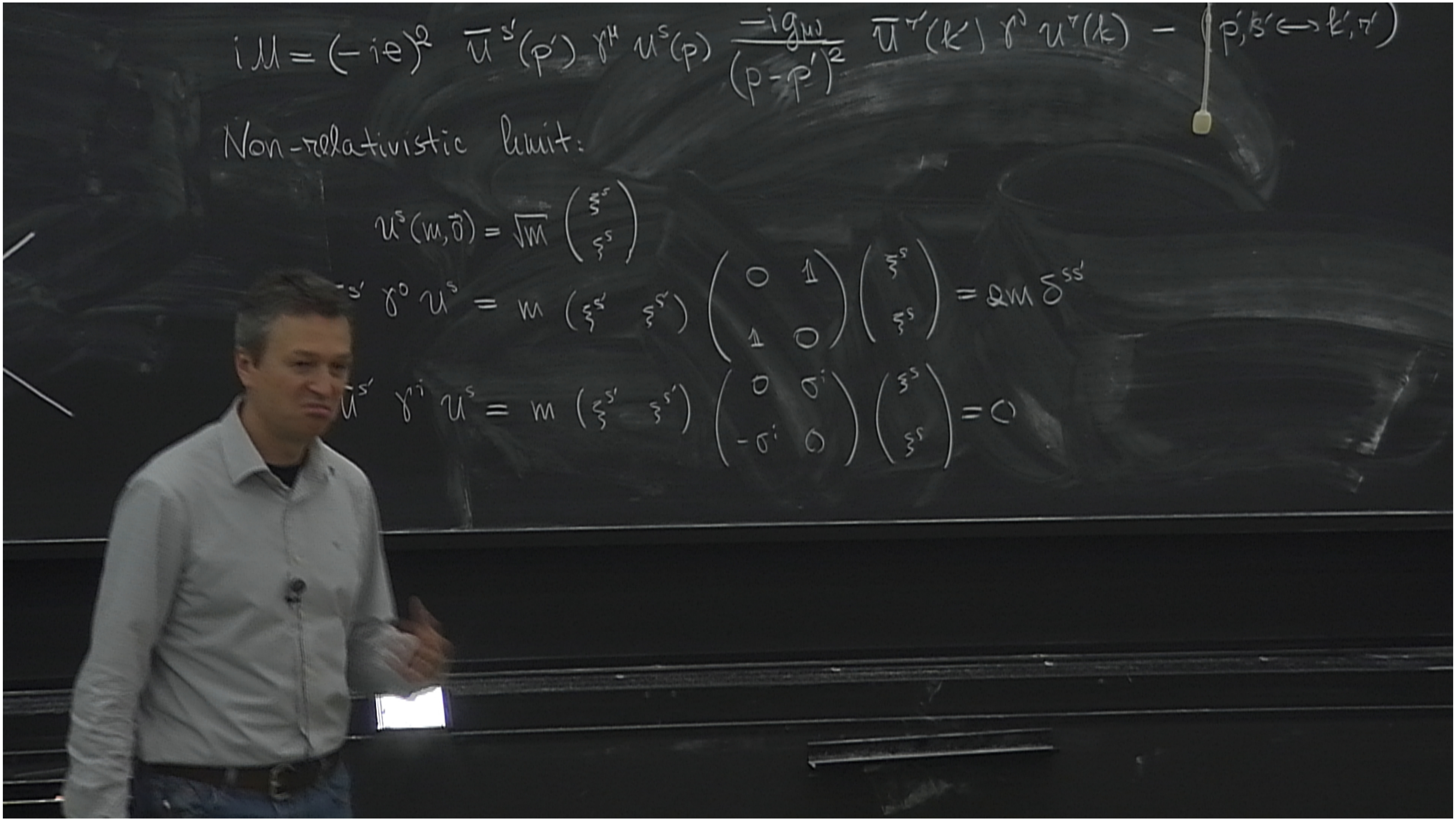
$$i\mathcal{M} = (-ie)^2 \; \bar{u}^{s'}(p')\, \gamma^\mu\, u^s(p)\, \frac{-i g_{\mu\nu}}{(p-p')^2}\, \bar{u}^{r'}(k')\, \gamma^\nu\, u^r(k) - (p', s' \longleftrightarrow k', r')$$

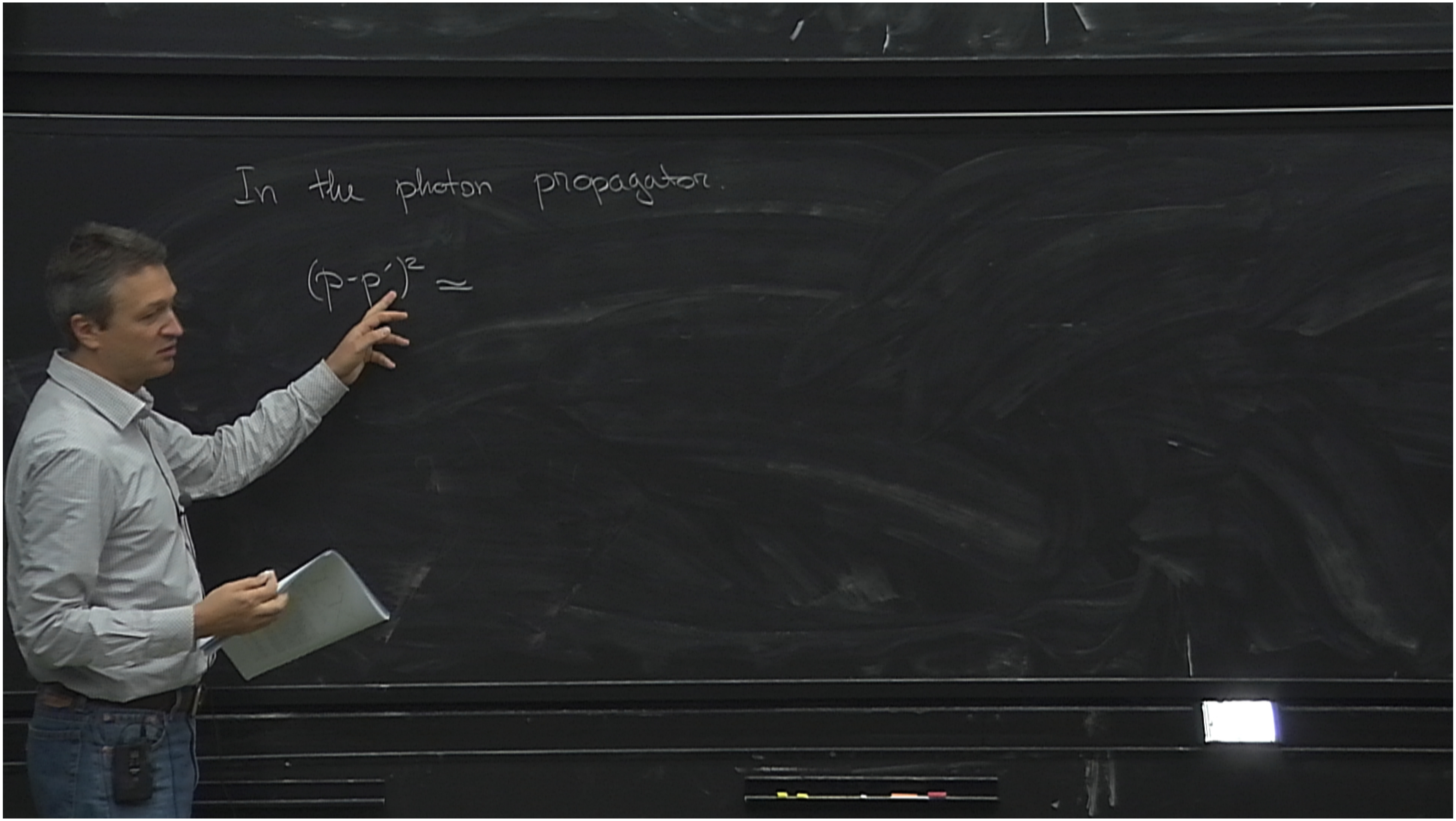

$i\mathcal{M} = (-ie)^2 \; \bar{u}^{s'}(p') \gamma^\mu u^s(p) \frac{-ig_{\mu\nu}}{(p-p')^2} \; \bar{u}^{r'}(k') \gamma^\nu u^r(k) - (p', s' \longleftrightarrow k', r')$
Non-relativistic limit:
$u^s(m, \vec{0}) = \sqrt{m} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix}$

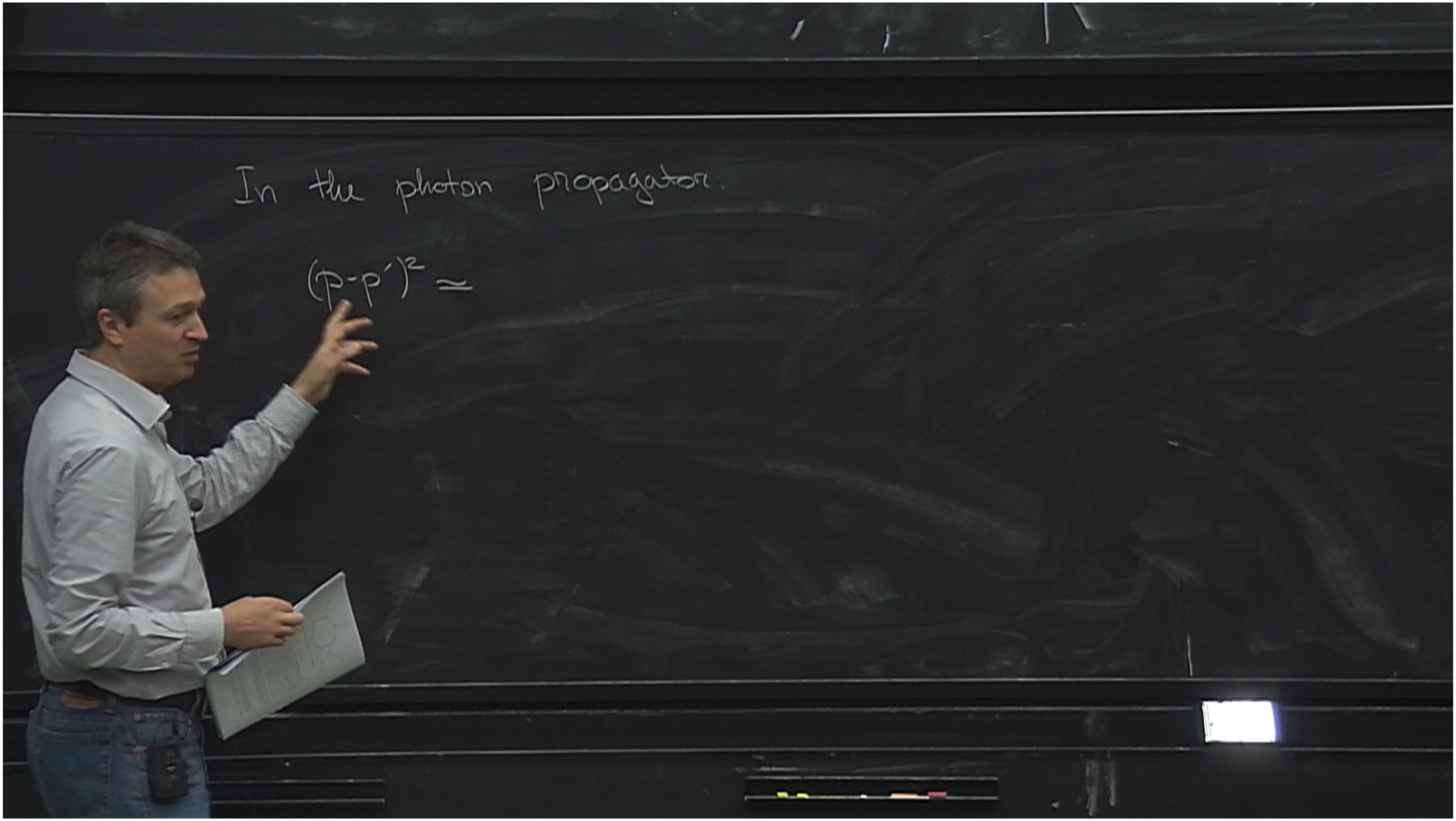

$i\mathcal{M} = (-ie)^2 \, \bar{u}^{s'}(p') \gamma^\mu u^s(p) \frac{-ig_{\mu\nu}}{(p-p')^2} \bar{u}^{r'}(k') \gamma^\nu u^r(k) - (p', s' \leftrightarrow k', r')$
Non-relativistic limit:
$u^s(m, \vec{0}) = \sqrt{m} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix}$
$\bar{u}^{s'} \gamma^0 u^s = m \, (\xi^{s'}$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

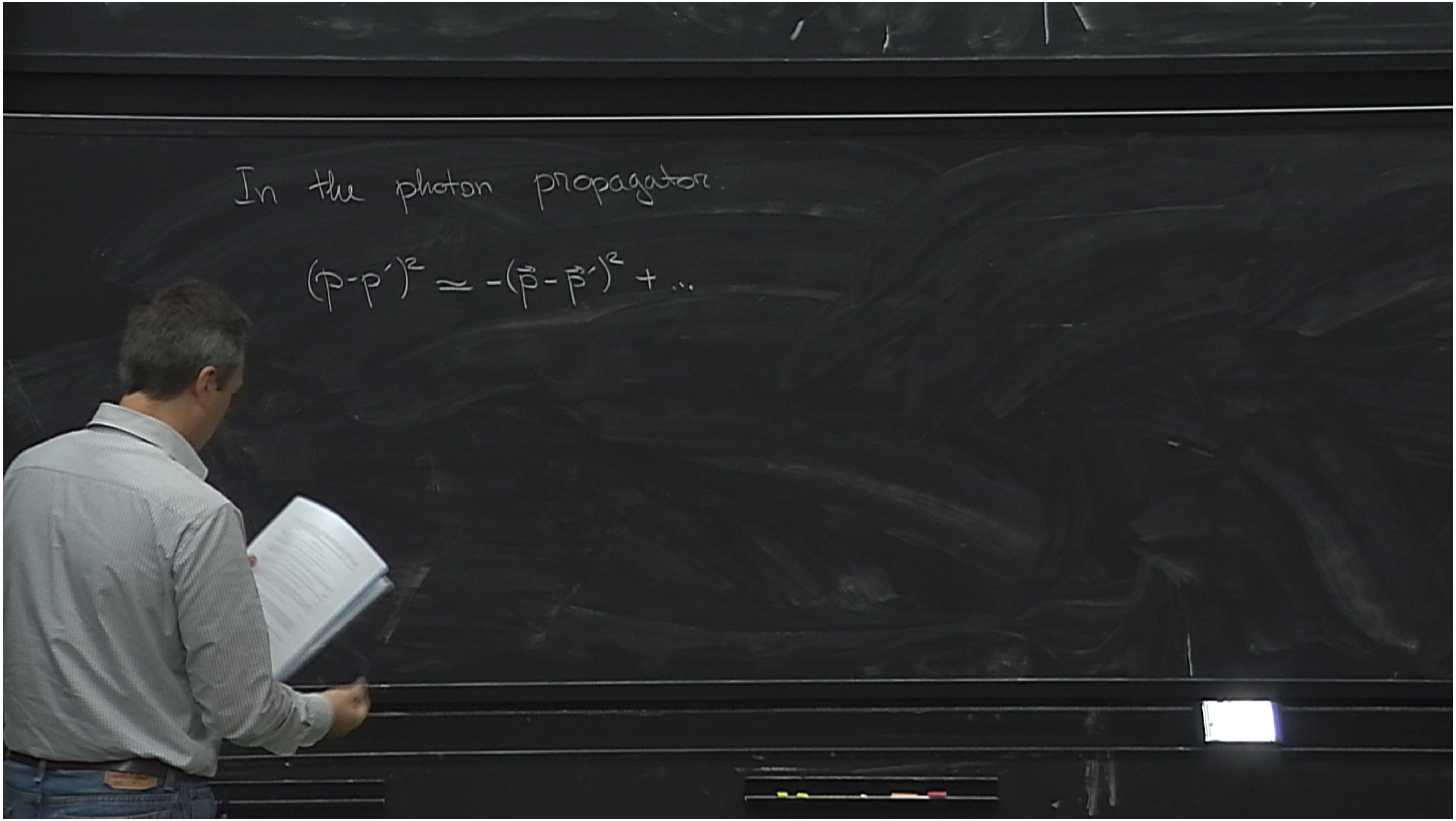

$i\mathcal{M} = (-ie)^2 \, \bar{u}^{s'}(p') \gamma^\mu u^s(p) \frac{-i g_{\mu\nu}}{(p-p')^2} \bar{u}^{r'}(k') \gamma^\nu u^r(k) - (p, s' \leftrightarrow k', r')$
Non-relativistic limit:
$u^s(m,\vec{0}) = \sqrt{m} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix}$
$\bar{u}^{s'} \gamma^0 u^s = m \, (\xi^{s'} \;\; \xi^{s'}) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix} = 2m \, \delta^{ss'}$

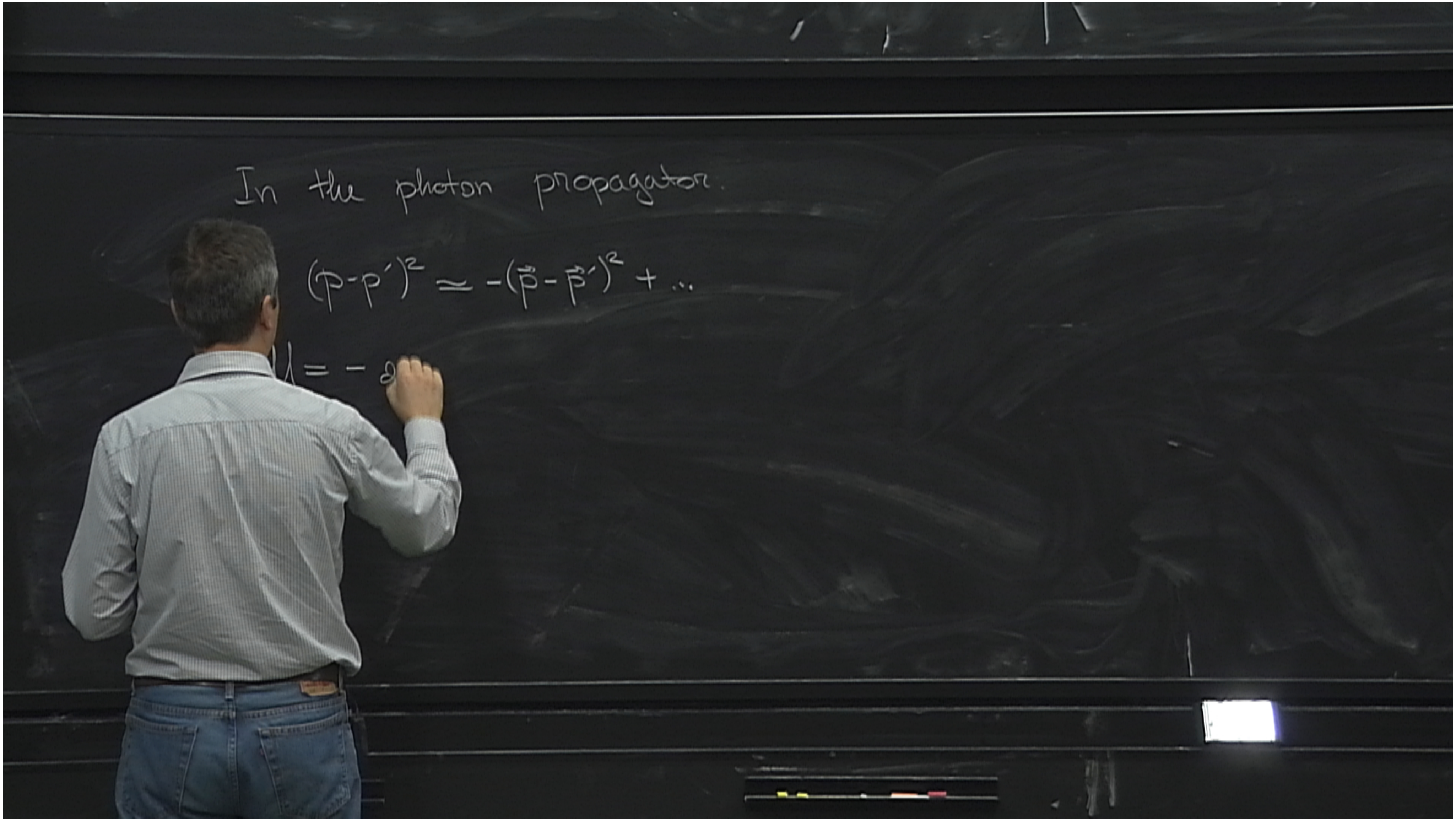

Non-relativistic limit:

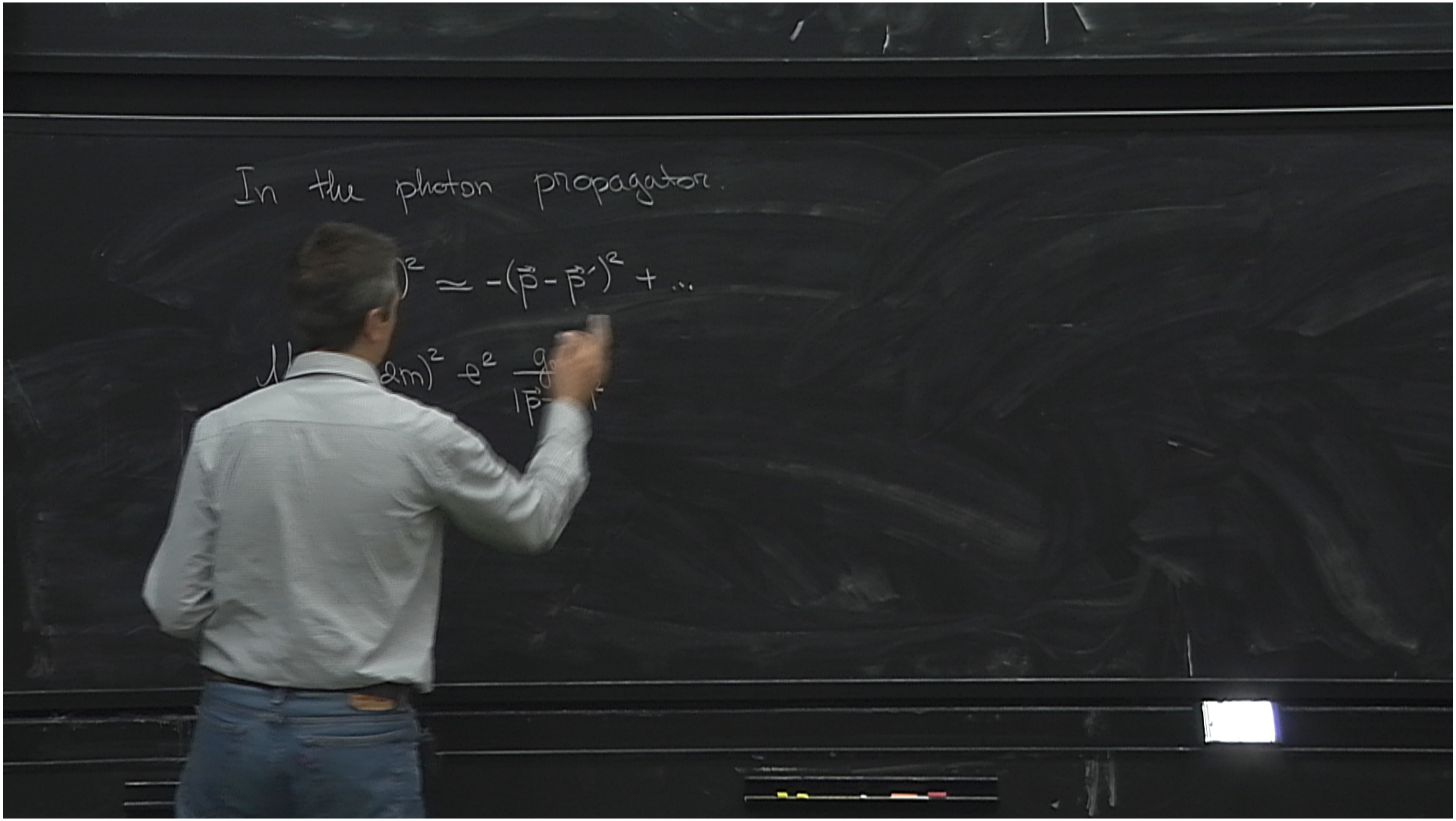

$i\mathcal{M} = (-ie)^2 \, \bar{u}^{s'}(p') \gamma^\mu u^s(p) \frac{-ig_{\mu\nu}}{(p-p')^2} \bar{u}^{r'}(k') \gamma^\nu u^r(k) - (p', s' \leftrightarrow k', r')$
Non-relativistic limit:
$u^s(m,\vec{0}) = \sqrt{m} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix}$
$\bar{u}^{s'} \gamma^0 u^s = m \begin{pmatrix} \xi^{s'} & \xi^{s'} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix} = 2m\delta^{ss'}$
$\bar{u}^{s'} \gamma^i u^s = m \begin{pmatrix} \xi^{s'} & \xi^{s'} \end{pmatrix} \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \begin{pmatrix} \xi^s \\ \xi^s \end{pmatrix} = 0$

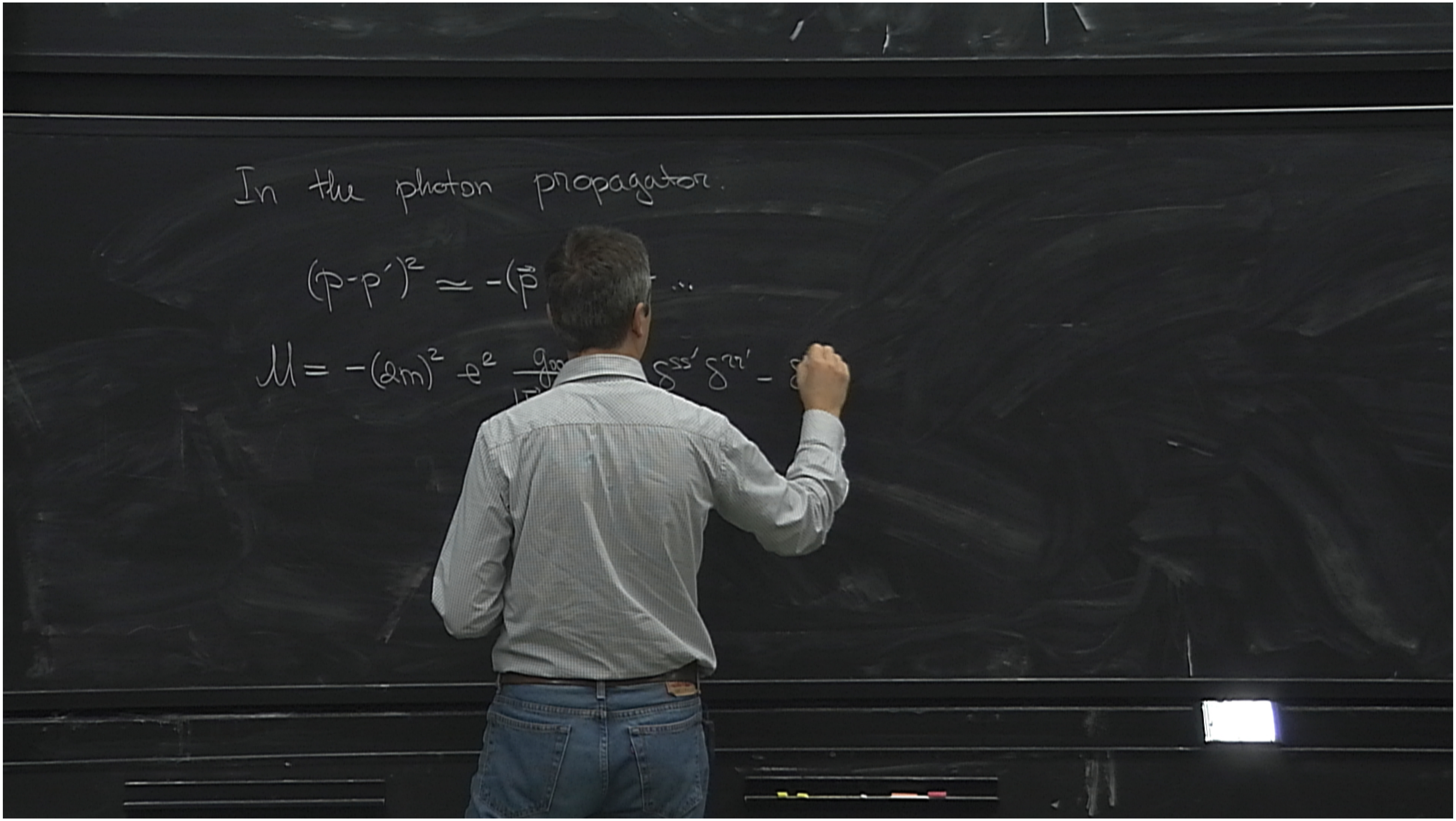

In the photon propagator.
(p-p')² ≃


In the photon propagator.
(p-p')² ≃


In the photon propagator:
$(p-p')^2 \simeq -(\vec{p}-\vec{p}\,')^2 + \dots$


In the photon propagator.
(p-p')² ≃ -(p⃗ - p⃗')² + ...

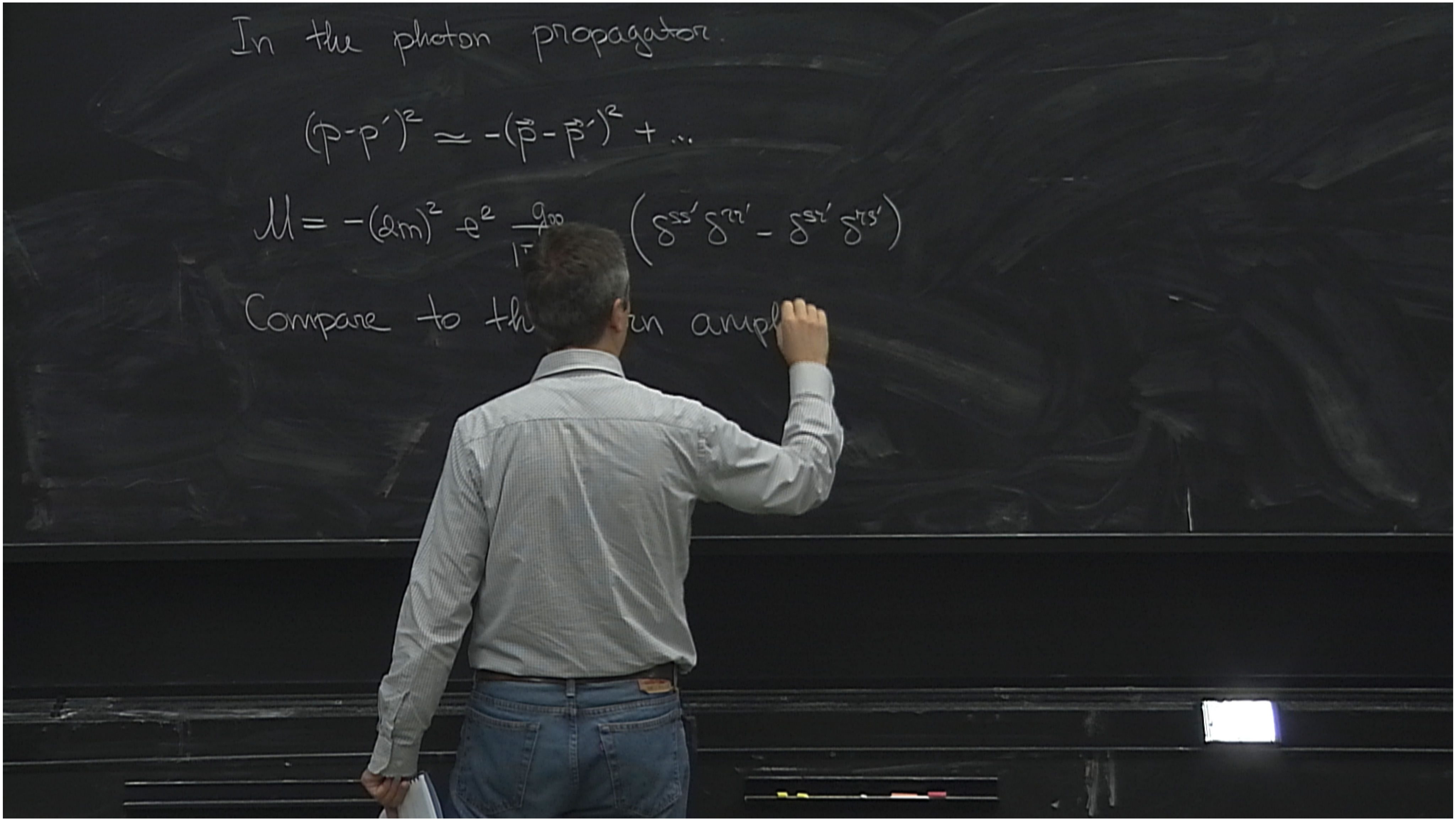

In the photon propagator.

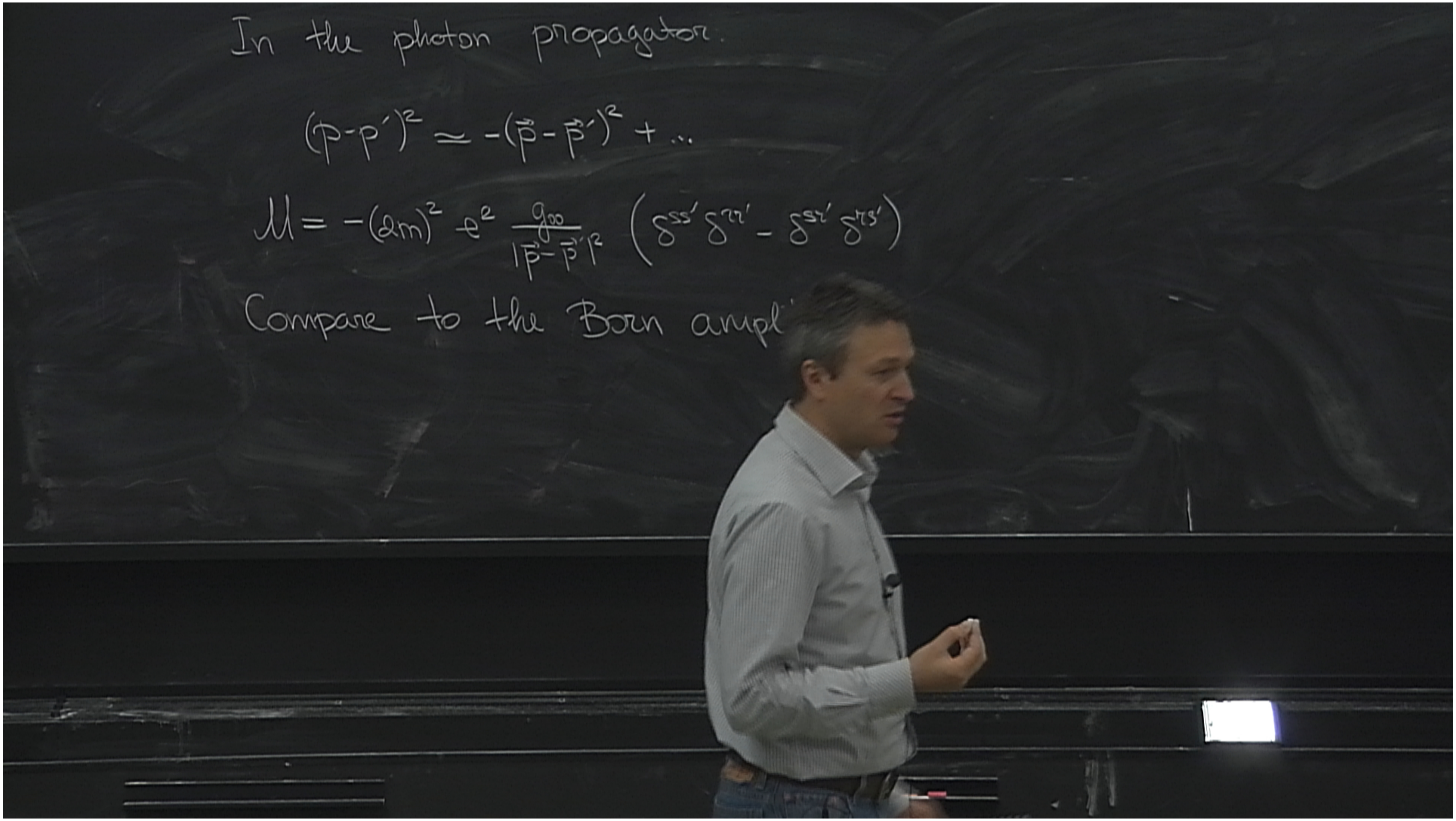

In the photon propagator.

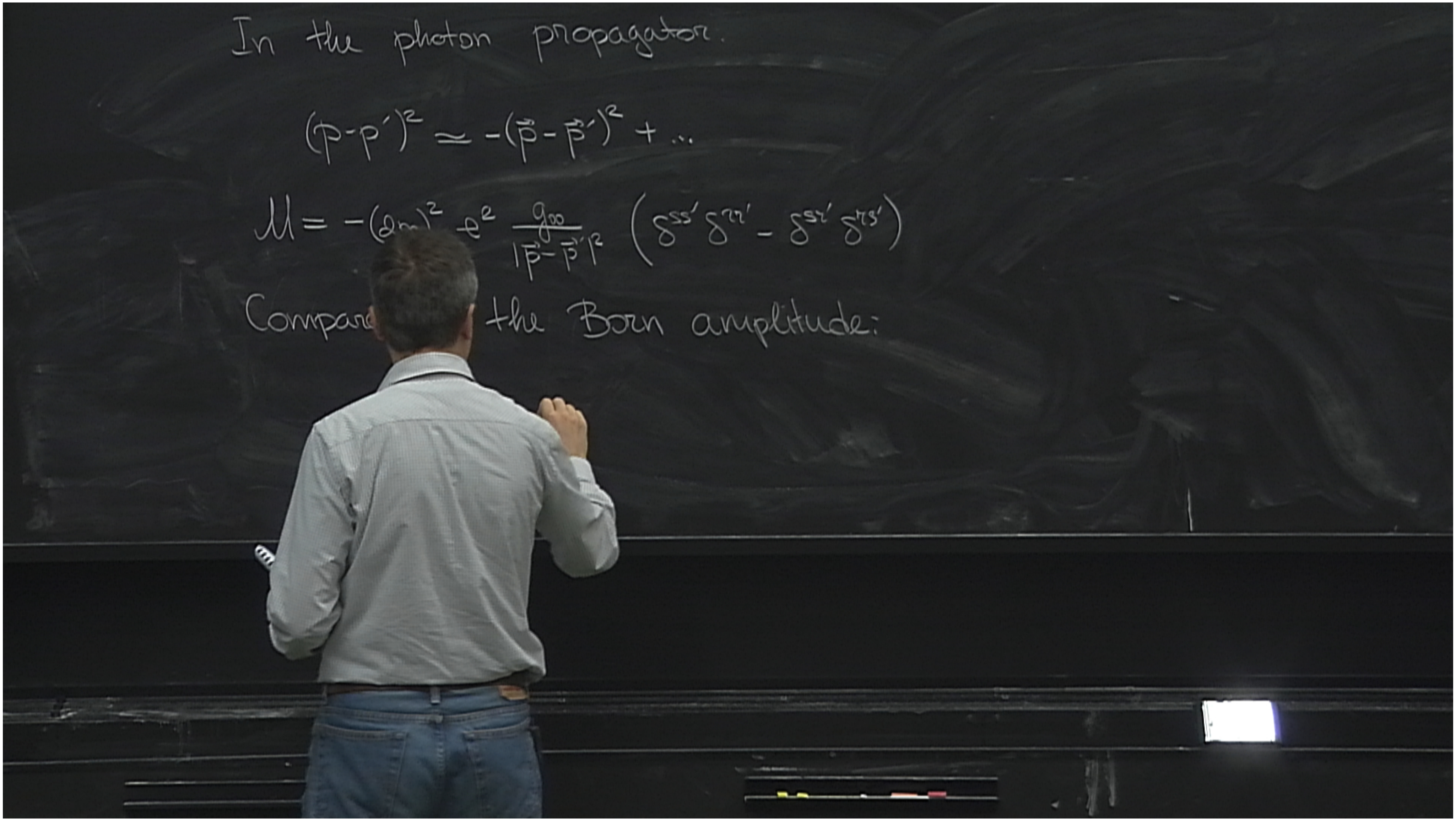
In the photon propagator:

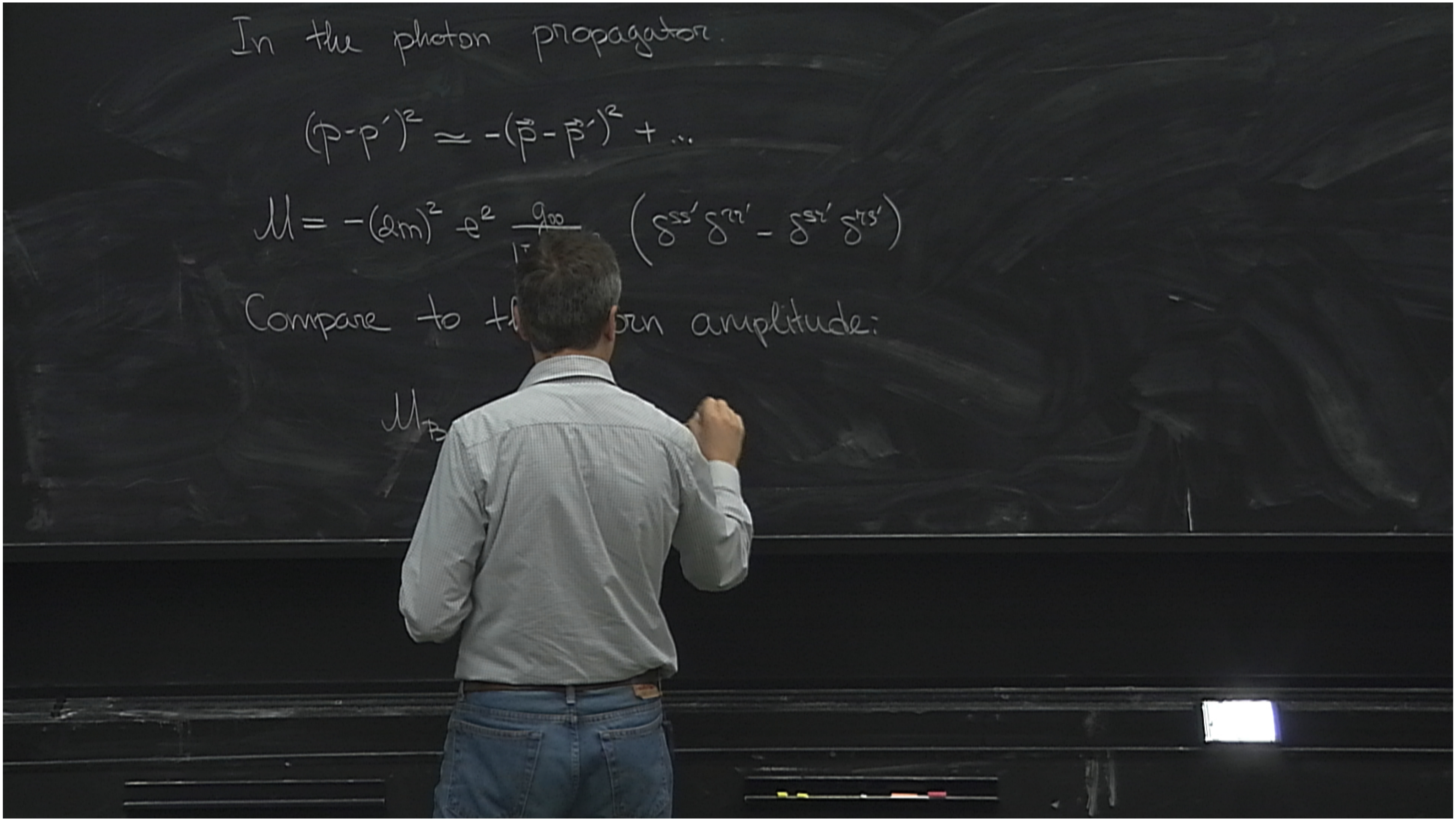
$$(p-p')^2 \simeq -(\vec{p}-\vec{p}\,')^2 + \dots$$

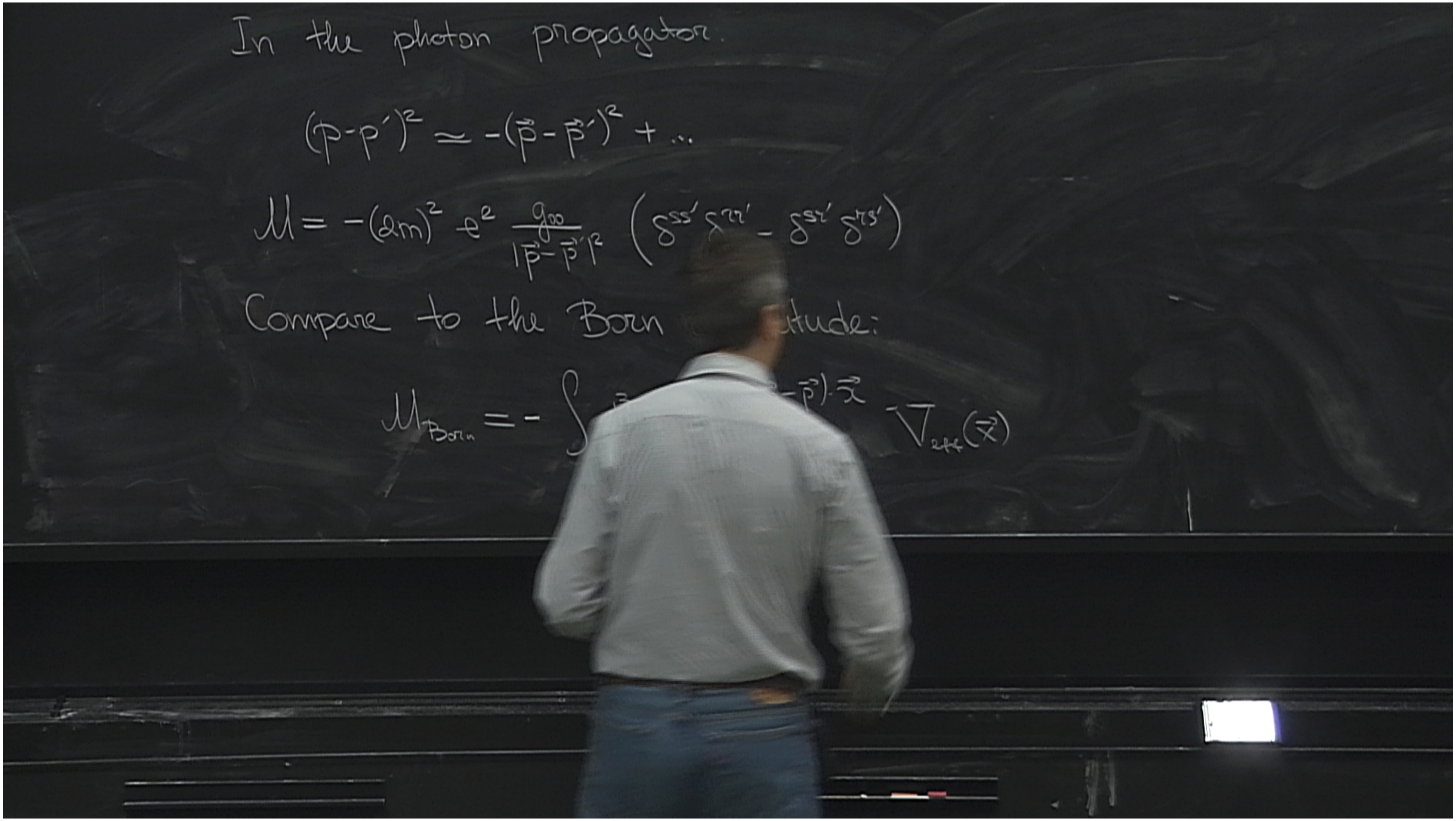
$$\mathcal{M} = -(2m)^2 e^2 \frac{g_{00}}{|\vec{p}-\vec{p}\,'|^2} \left(\delta^{ss'} \delta^{rr'} - \delta^{sr'} \delta^{rs'} \right)$$


In the photon propagator
$(p-p')^2 \simeq -(\vec{p}-\vec{p}')^2 + \ldots$
$\mathcal{M} = -(2m)^2 e^2 \frac{g_{00}}{|} \left(\delta^{ss'}\delta^{rr'} - \delta^{sr'}\delta^{rs'}\right)$
Compare to th
rn ampl


In the photon propagator:
$(p-p')^2 \simeq -(\vec{p}-\vec{p}')^2 + \dots$
$\mathcal{M} = -(2m)^2 e^2 \frac{g_{00}}{|\vec{p}-\vec{p}'|^2} \left(\delta^{ss'}\delta^{rr'} - \delta^{sr'}\delta^{rs'}\right)$
Compare to the Born ampl


In the photon propagator.
$(p-p')^2 \simeq -(\vec{p}-\vec{p}')^2 + \ldots$
$\mathcal{M} = -(2m)^2 e^2 \frac{g_{00}}{|\vec{p}-\vec{p}'|^2} \left(\delta^{ss'}\delta^{rr'} - \delta^{sr'}\delta^{rs'}\right)$
the Born amplitude:


In the photon propagator.
$(p-p')^2 \simeq -(\vec{p}-\vec{p}')^2 + \ldots$
$\mathcal{M} = -(2m)^2 e^2 \frac{g_{00}}{|\ldots} \left(\delta^{ss'}\delta^{rr'} - \delta^{sr'}\delta^{rs'}\right)$
Compare to th orn amplitude:
$\mathcal{M}_B$


In the photon propagator.
$(p-p')^2 \simeq -(\vec{p}-\vec{p}')^2 + \ldots$
$\mathcal{M} = -(2m)^2 e^2 \frac{g_{00}}{|\vec{p}-\vec{p}'|^2} \left(\delta^{ss'} \delta^{rr'} - \delta^{sr'} \delta^{rs'} \right)$
Compare to the Born

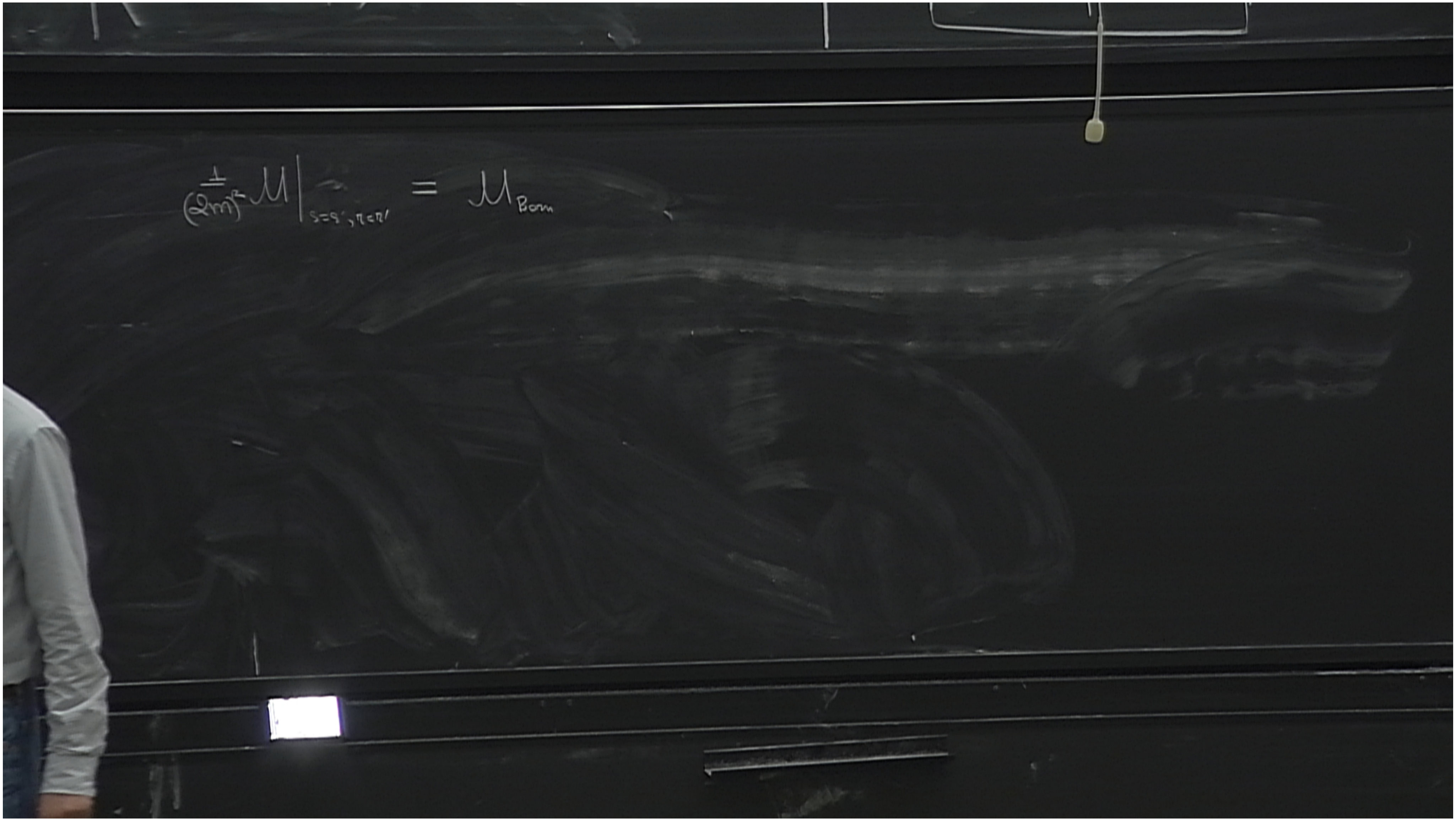
In the photon propagator:

$$(p-p')^2 \simeq -(\vec{p}-\vec{p}\,')^2 + \ldots$$

$$\mathcal{M} = -(2m)^2 e^2 \frac{g_{00}}{|\vec{p}-\vec{p}\,'|^2} \left(\delta^{ss'} \delta^{rr'} - \delta^{sr'} \delta^{rs'} \right)$$

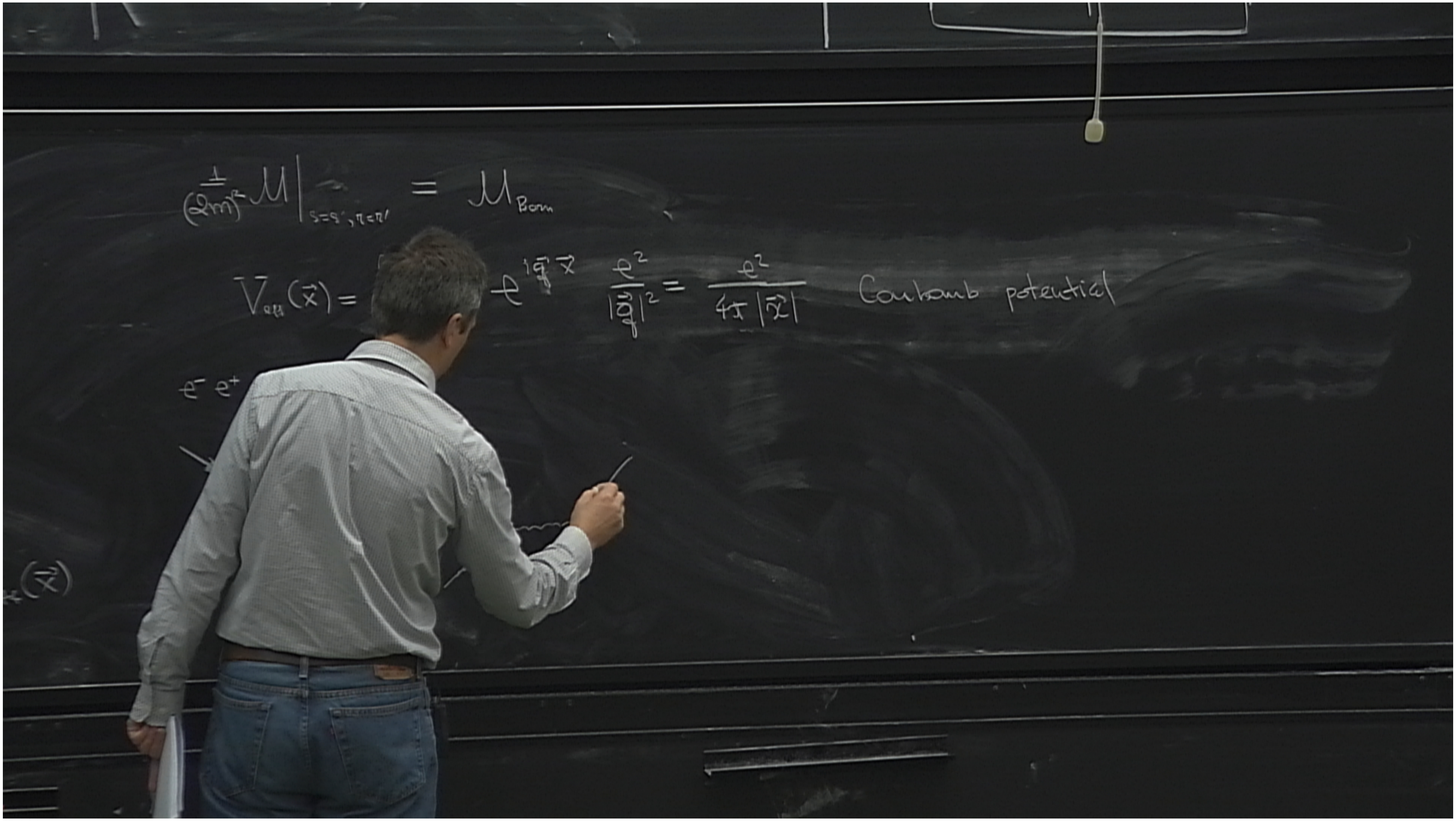
Compare to the Born amplitude:

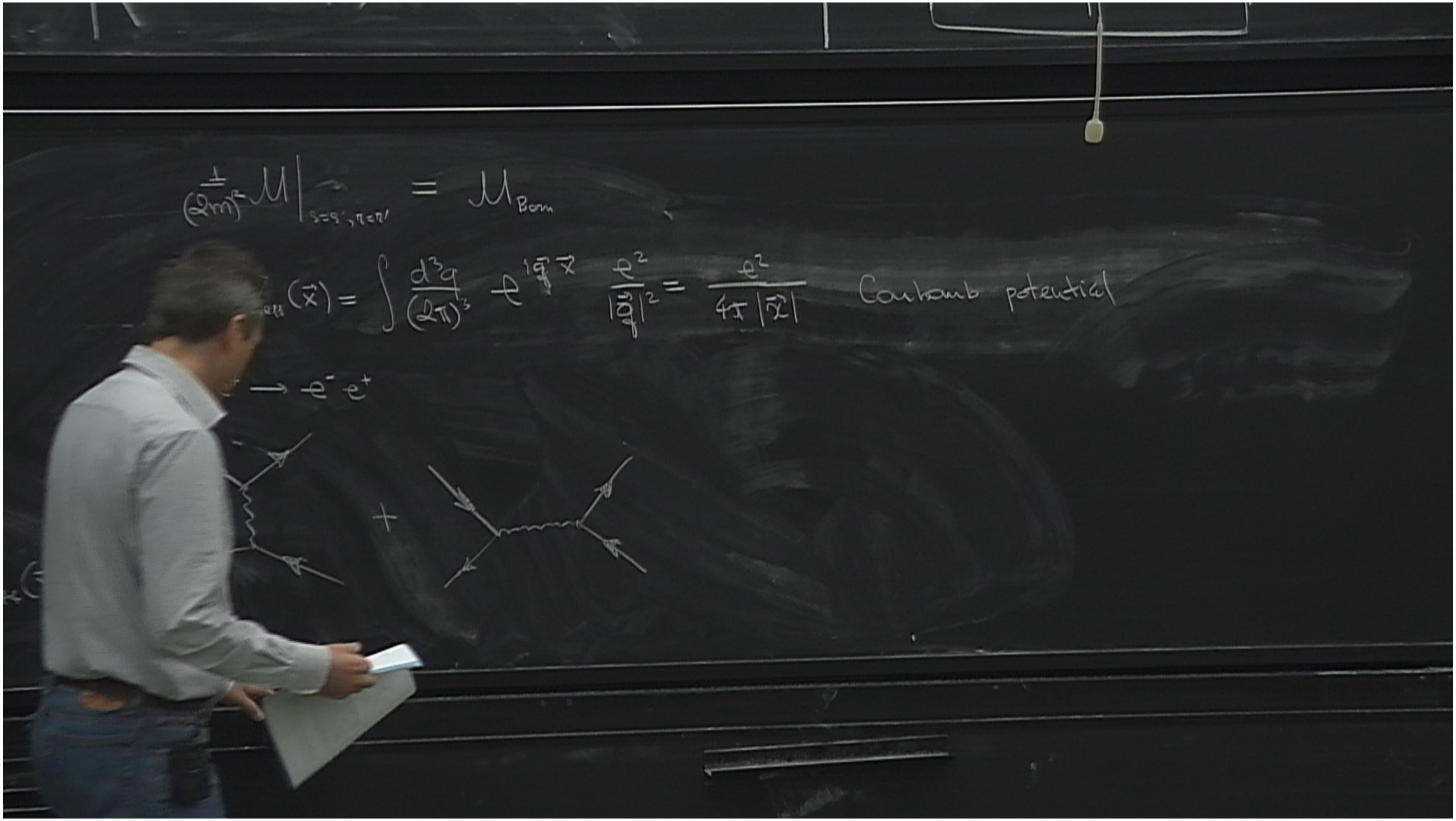
$$\mathcal{M}_{Born} = - \int d^3x \, e^{-i(\vec{p}\,'-\vec{p})\cdot\vec{x}} \, V_{eff}(\vec{x})$$

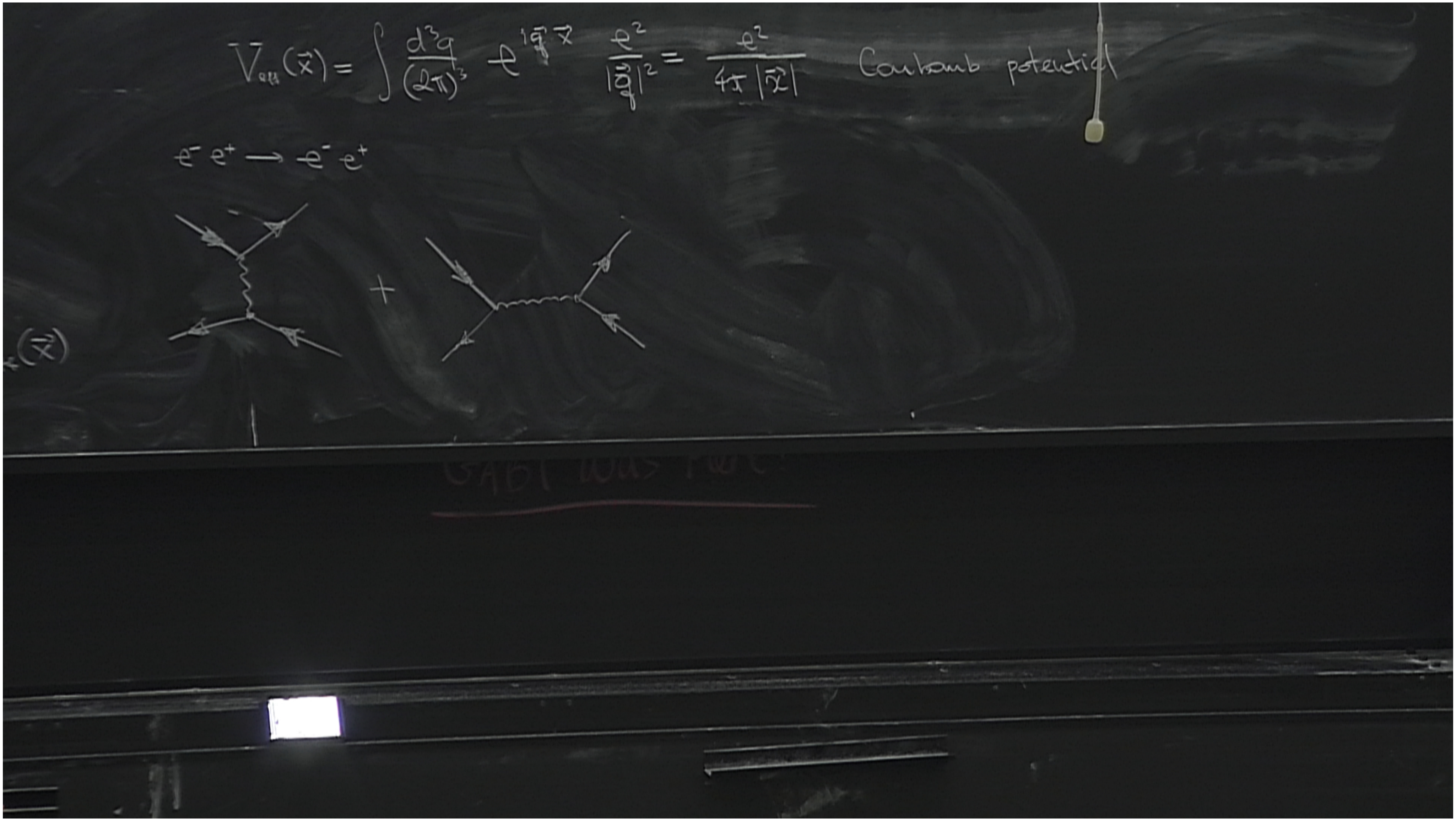

$\frac{1}{(2m)^2} \mathcal{M} \Big|_{s=q^2, r=r'} = \mathcal{M}_{Born}$

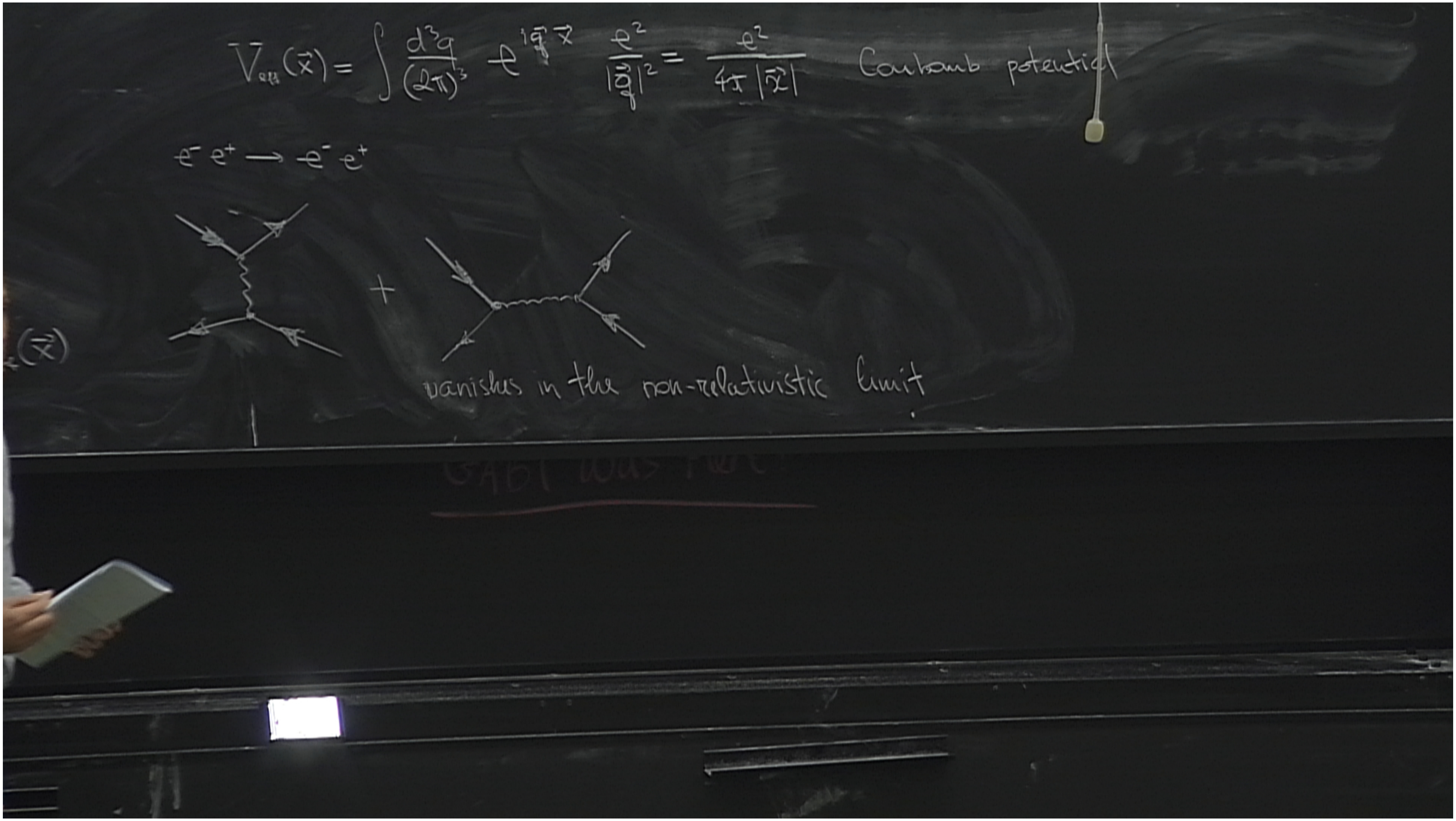
$$\frac{1}{(2m)^2}\mathcal{M}\Big|_{s=q',\, r=r'} = \mathcal{M}_{Born}$$

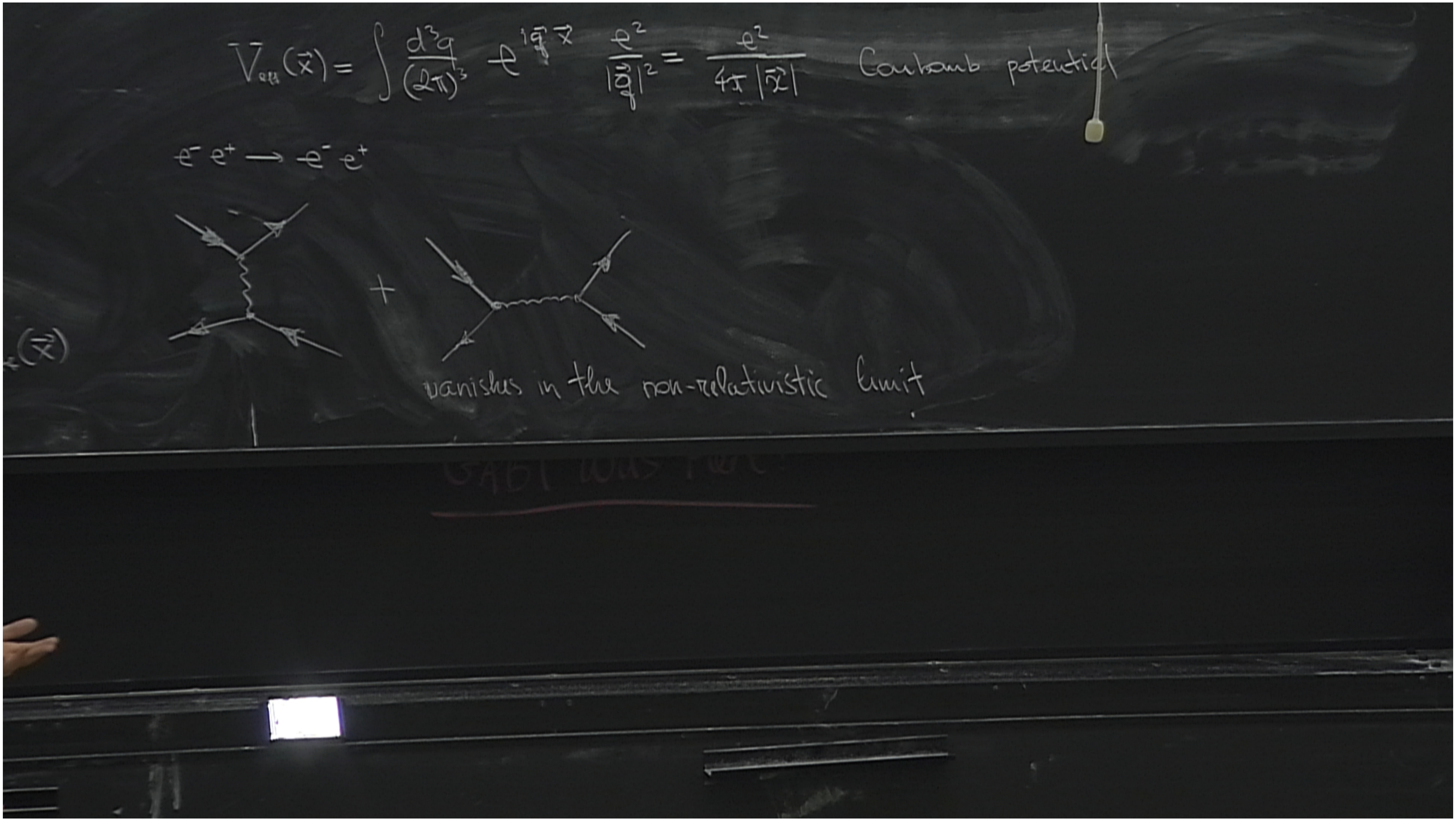
$$V_{eff}(\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\, e^{i\vec{q}\cdot\vec{x}}\, \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi|\vec{x}|} \quad \text{Coulomb potential}$$

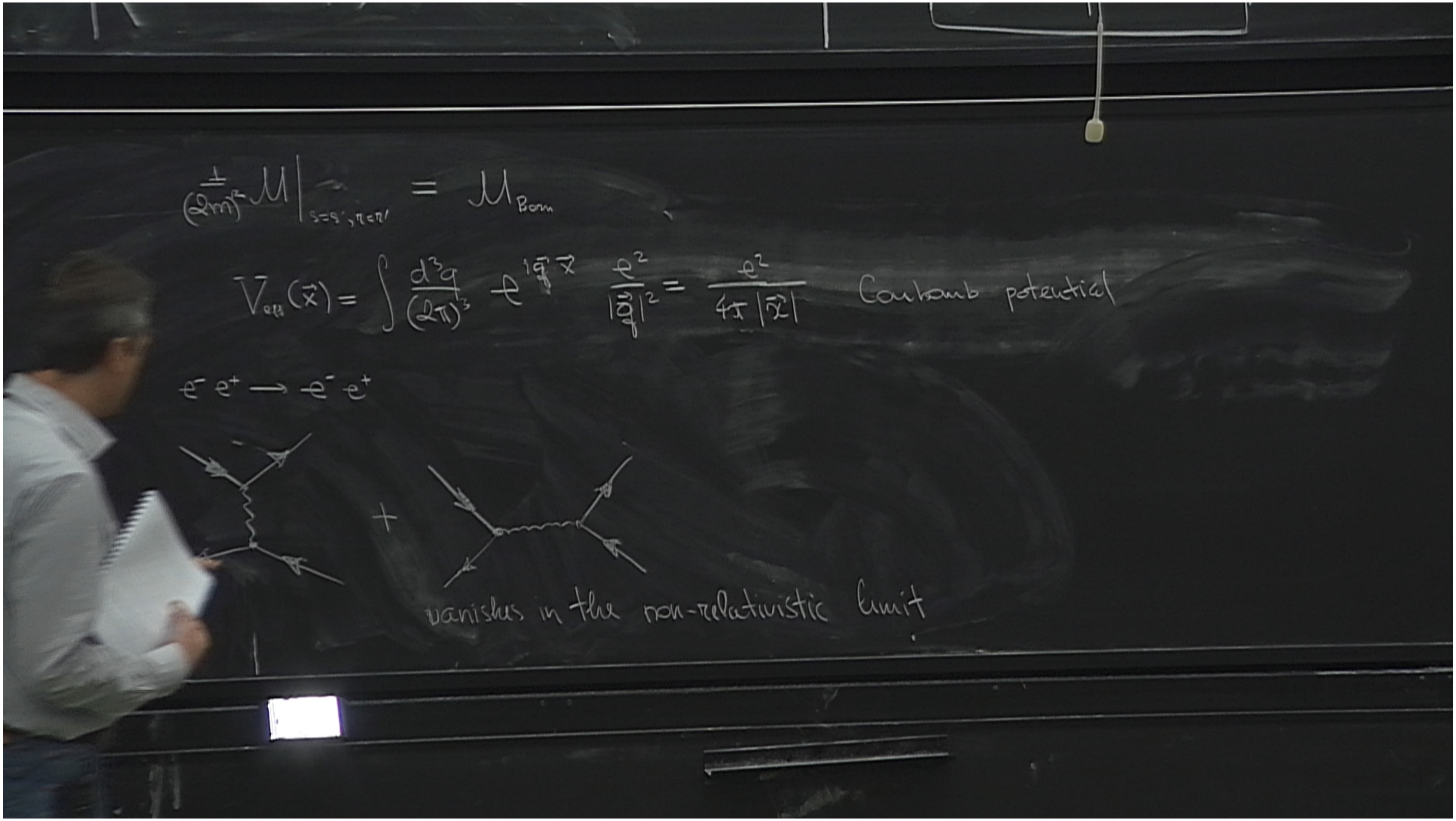

Coulomb potential

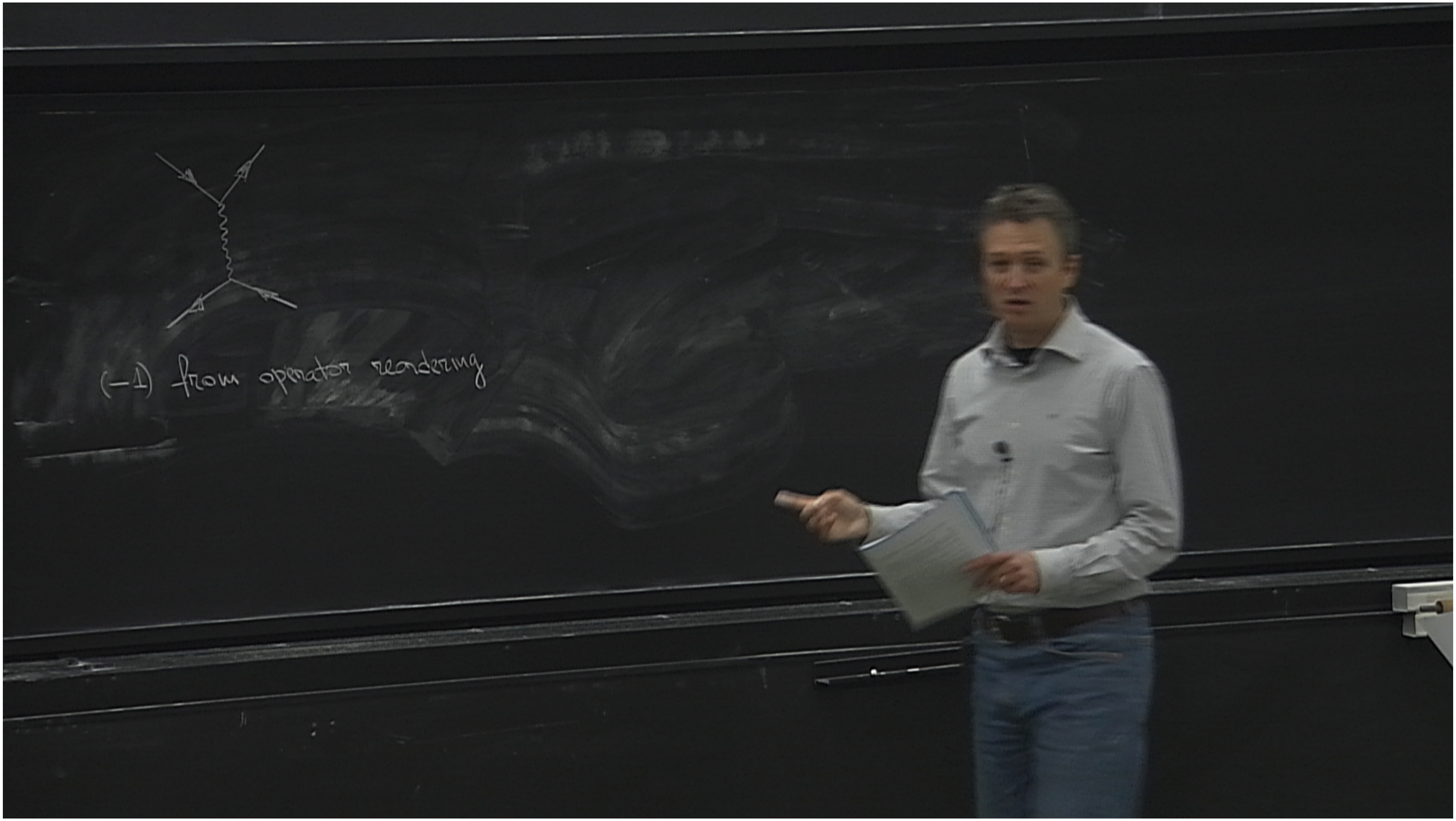

$\frac{1}{(2m)^2}\mathcal{M}\Big|_{s=q', r=r'} = \mathcal{M}_{Born}$
$(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi|\vec{x}|}$
Coulomb potential
$\to e^- e^+$

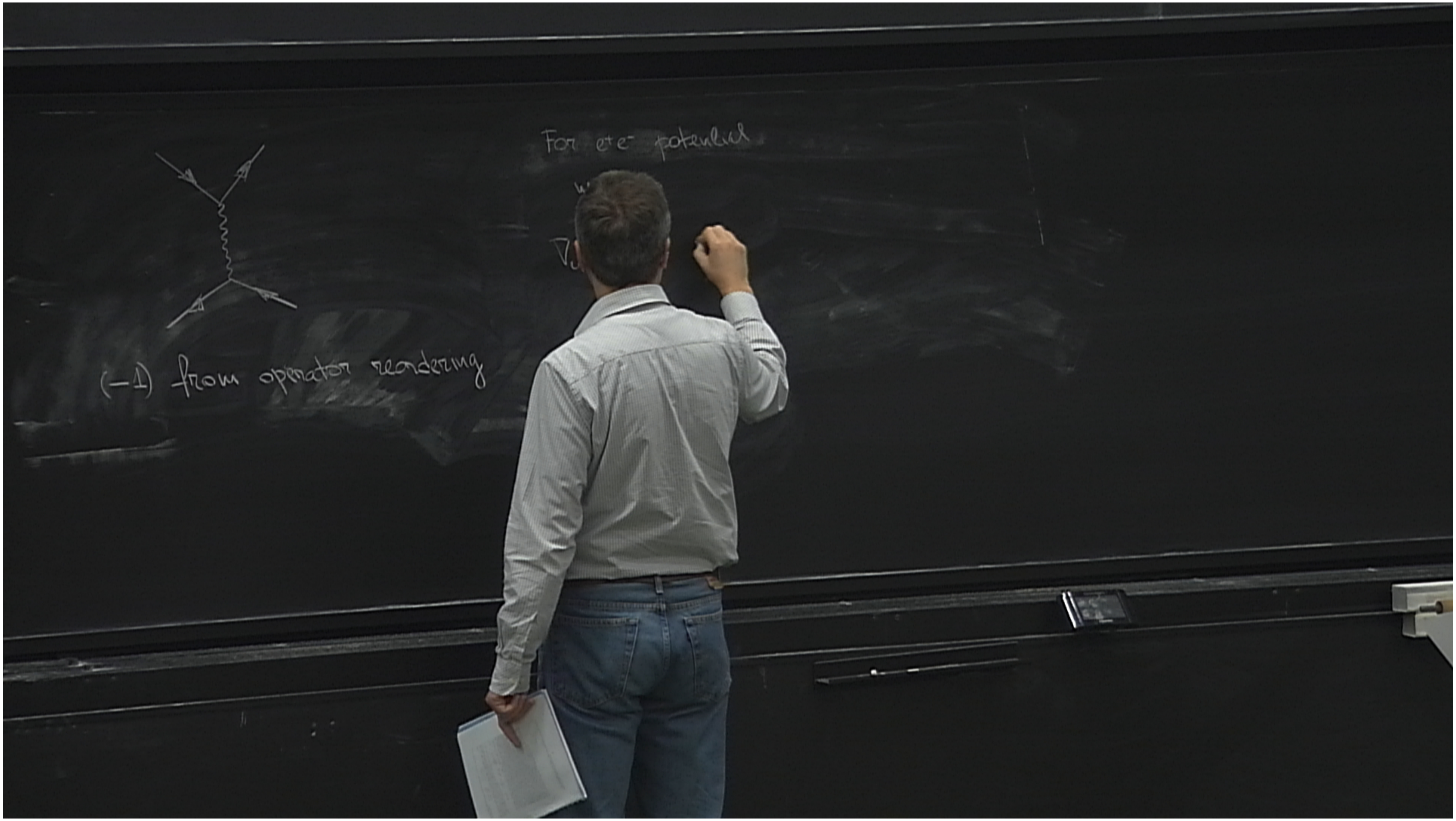

$V_{eff}(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi|\vec{x}|}$
Coulomb potential
$e^- e^+ \longrightarrow e^- e^+$
+

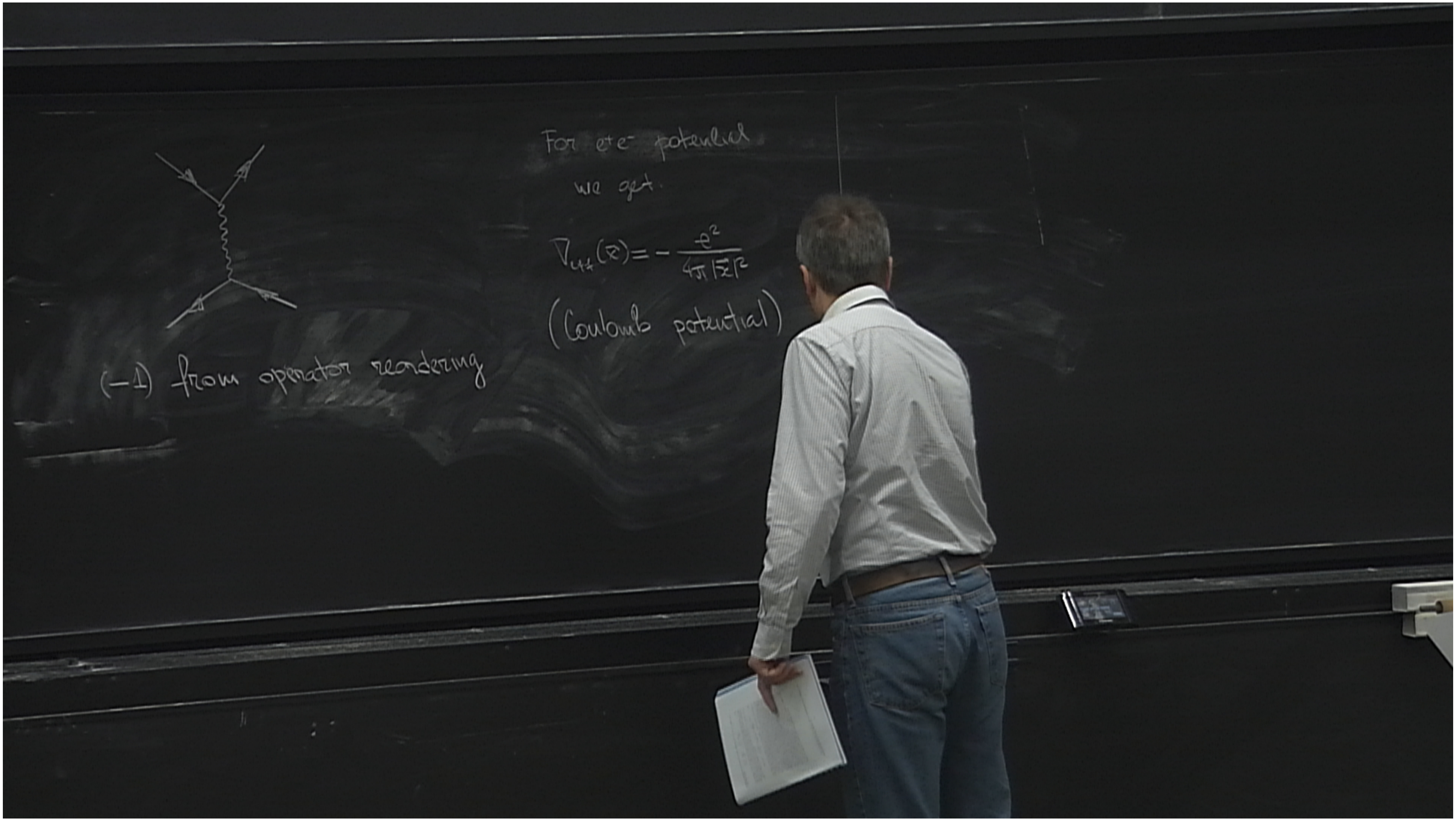

$\bar{V}_{eff}(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi|\vec{x}|}$ Coulomb potential
$e^- e^+ \longrightarrow e^- e^+$
+
vanishes in the non-relativistic limit

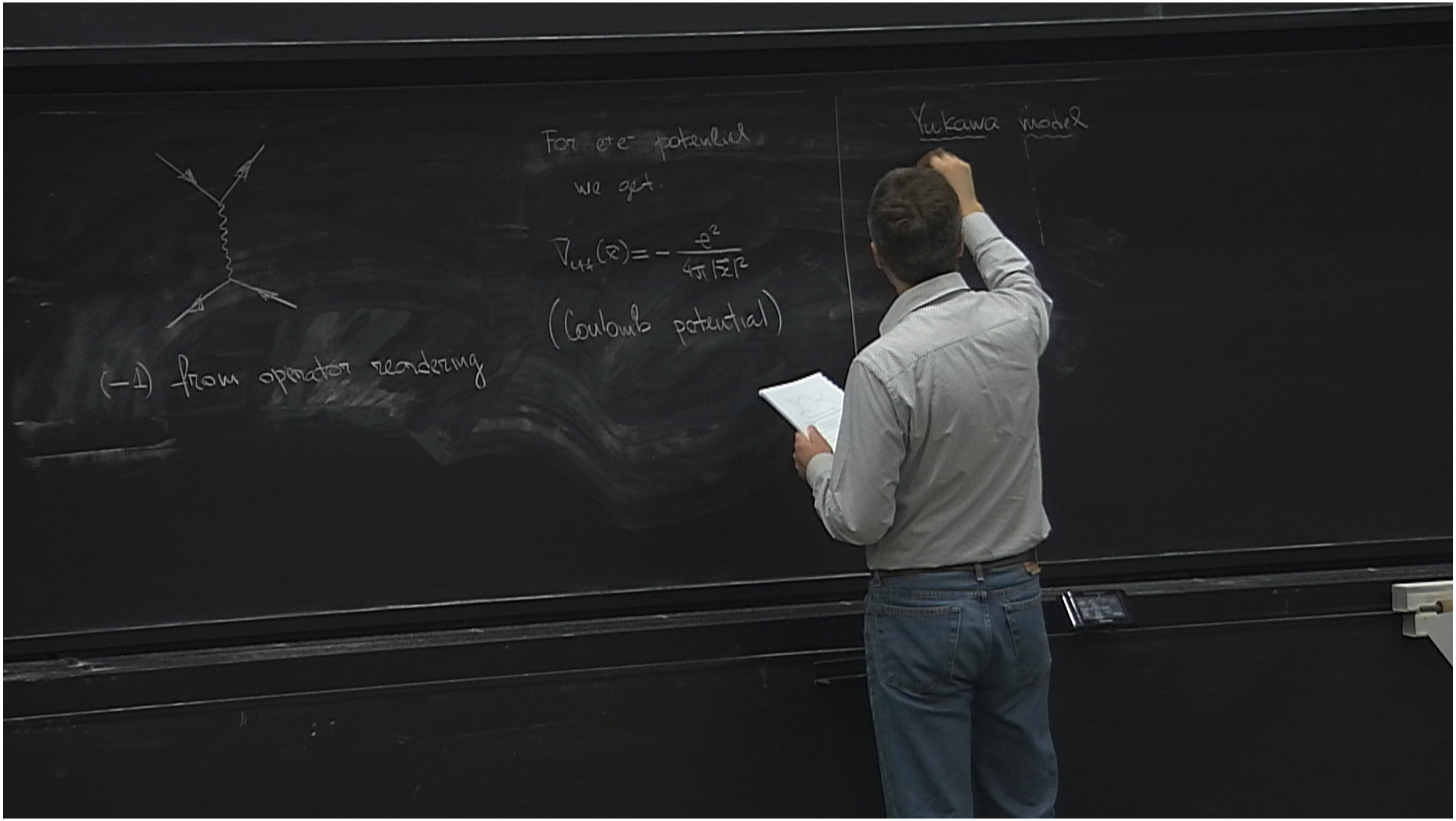

$\bar{V}_{eff}(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2} = \frac{e^2}{4\pi|\vec{x}|}$ Coulomb potential
$e^- e^+ \longrightarrow e^- e^+$
+
vanishes in the non-relativistic limit

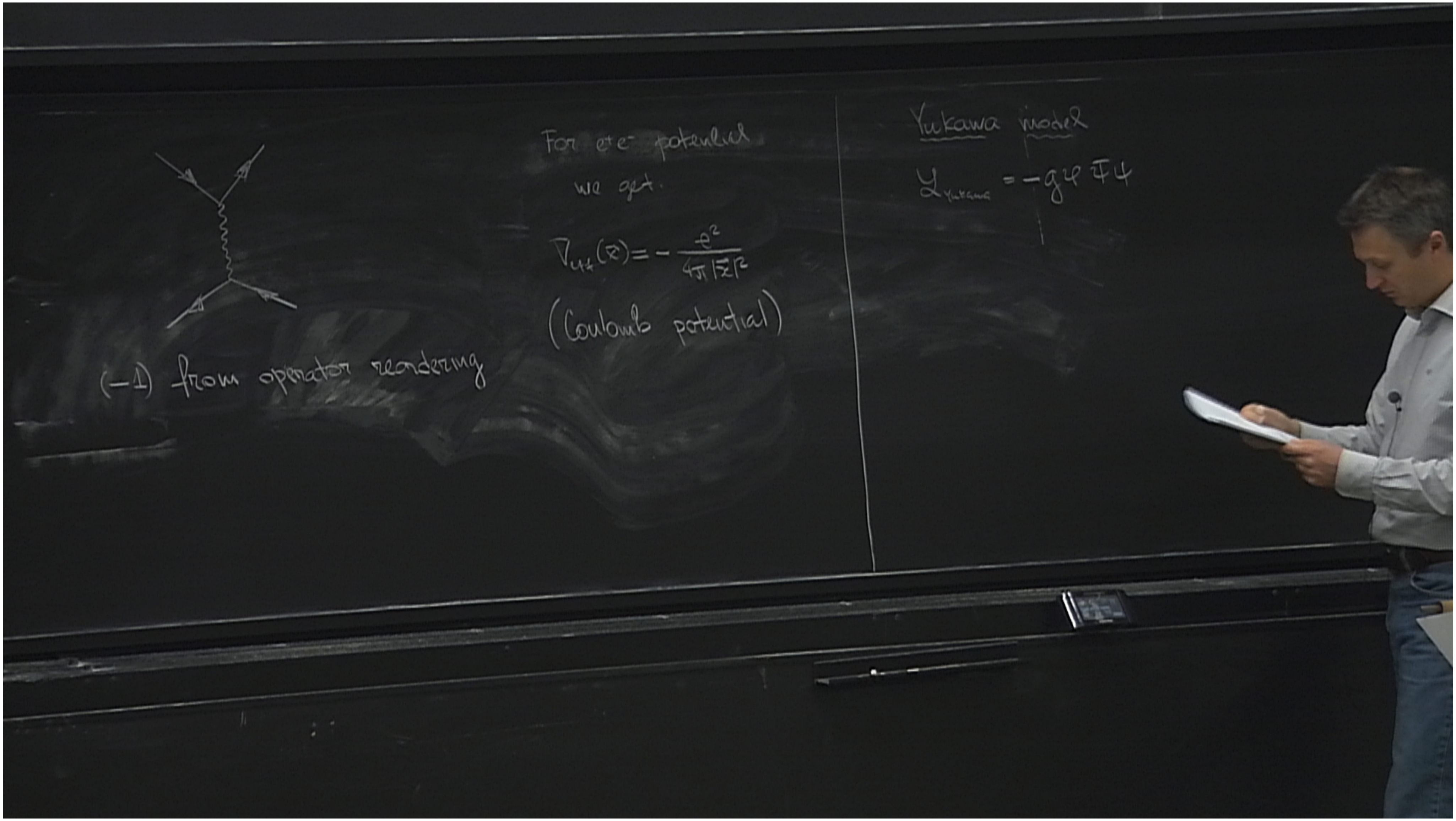

1/(2m)² M|_{s=s′, r=r′} = M_Born
V_eff(x⃗) = ∫ d³q/(2π)³ e^{i q⃗·x⃗} e²/|q⃗|² = e²/(4π|x⃗|)
Coulomb potential
e⁻ e⁺ → e⁻ e⁺
+
vanishes in the non-relativistic limit

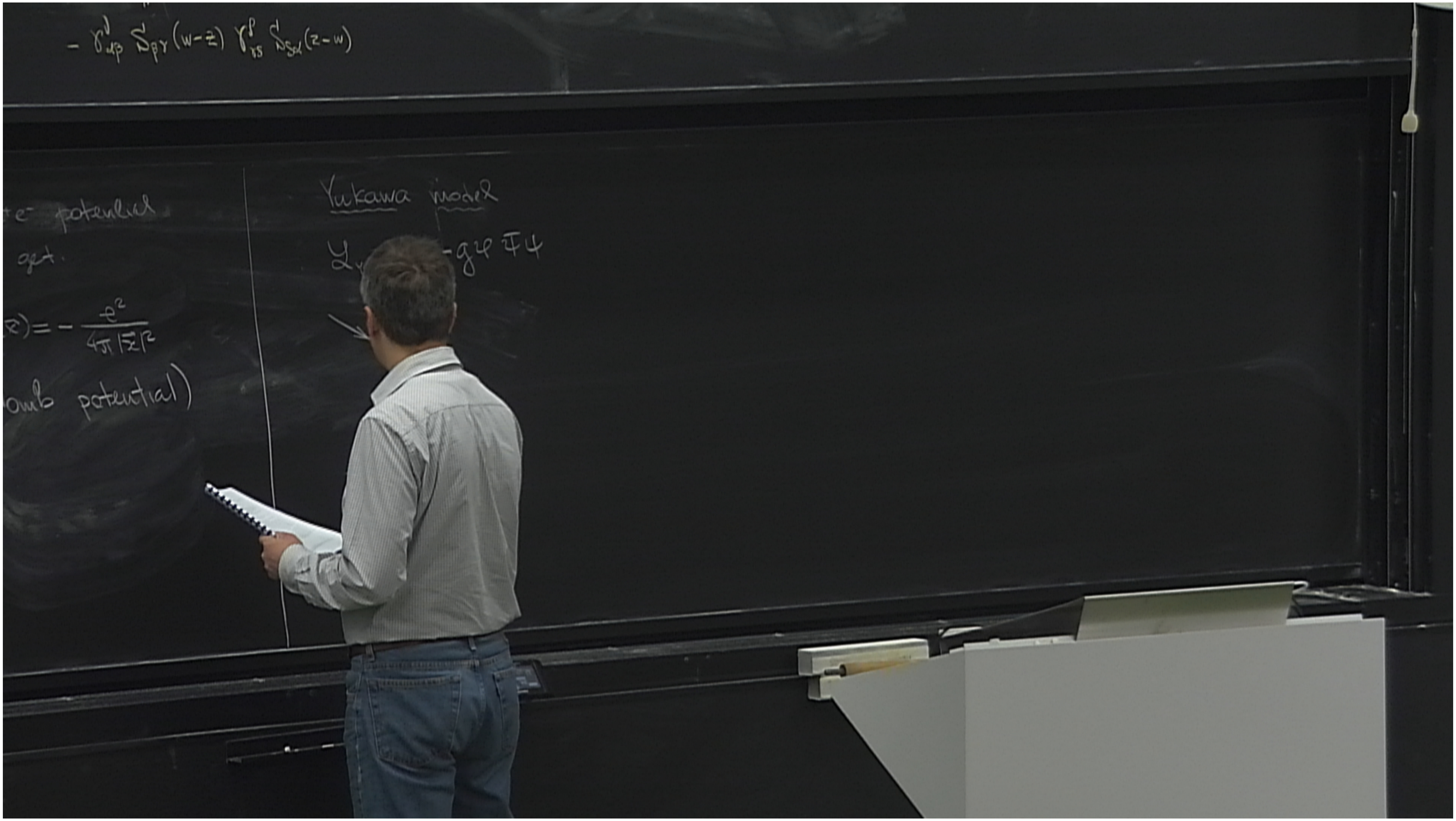

(−1) from operator reordering

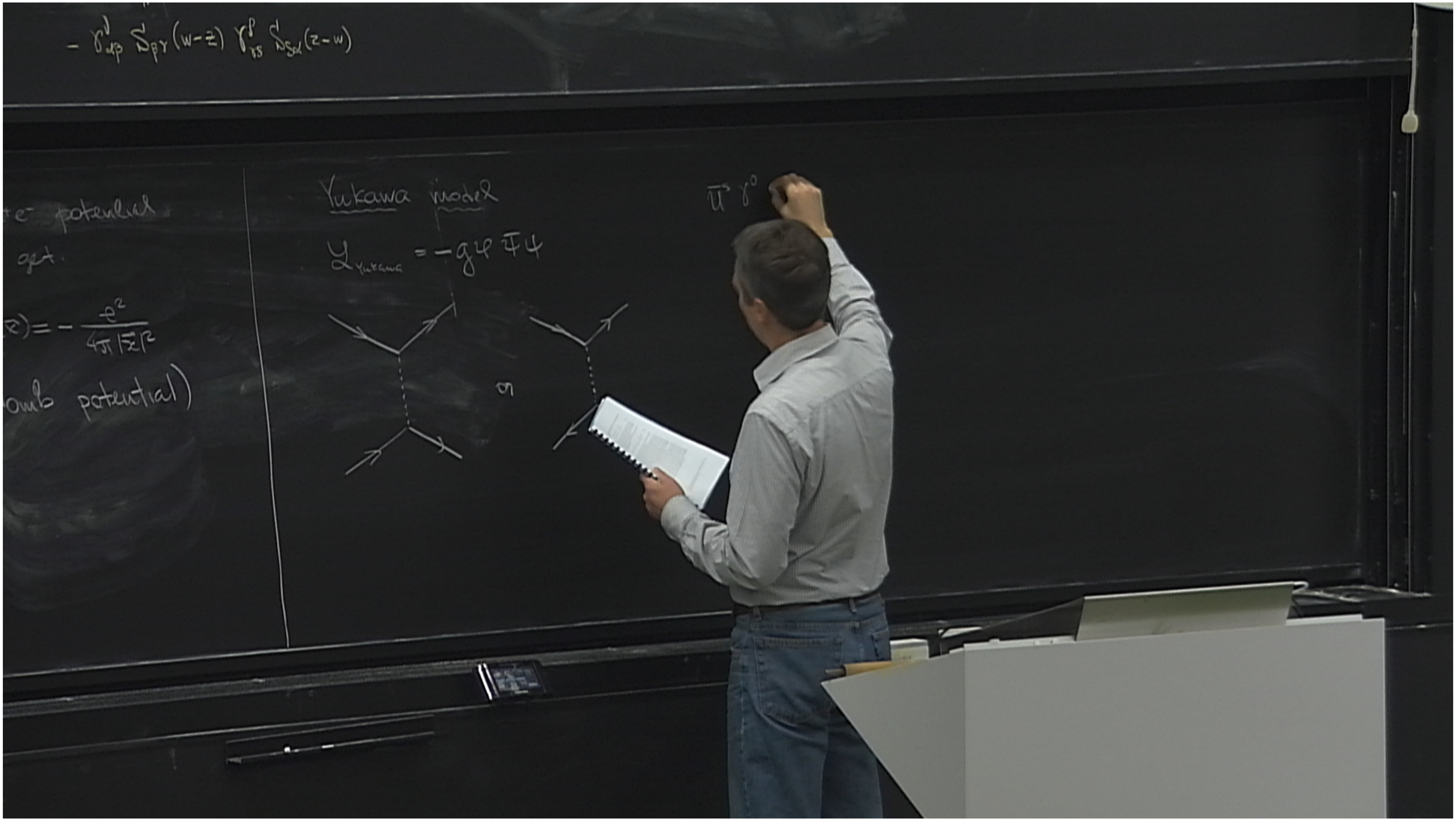

For e+e- potential
(−1) from operator reordering


For e+e- potential
we get.
(Coulomb potential)
(−1) from operator reordering

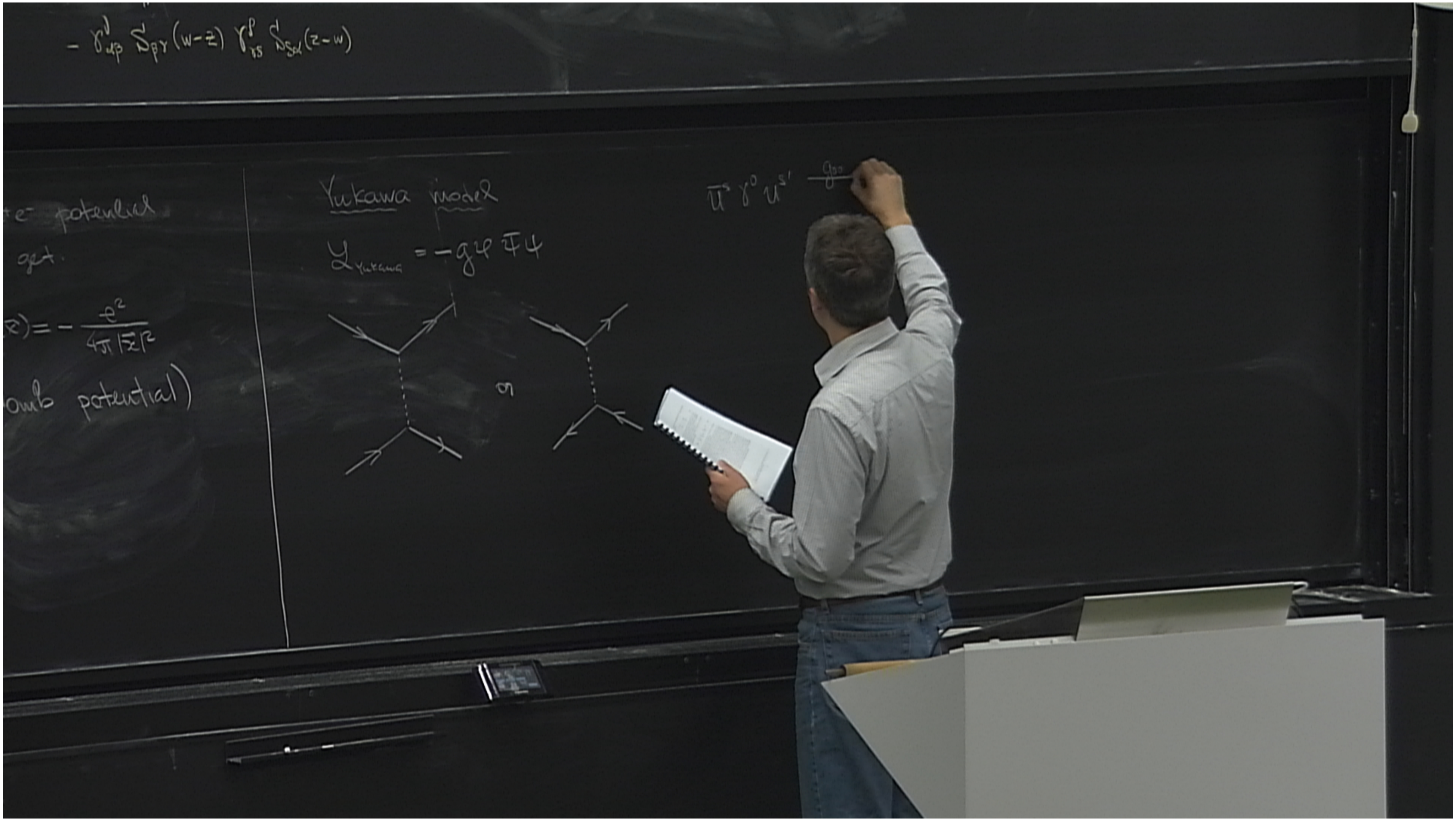

For e+e- potential
we get.
(Coulomb potential)
(-1) from operator reordering
Yukawa model

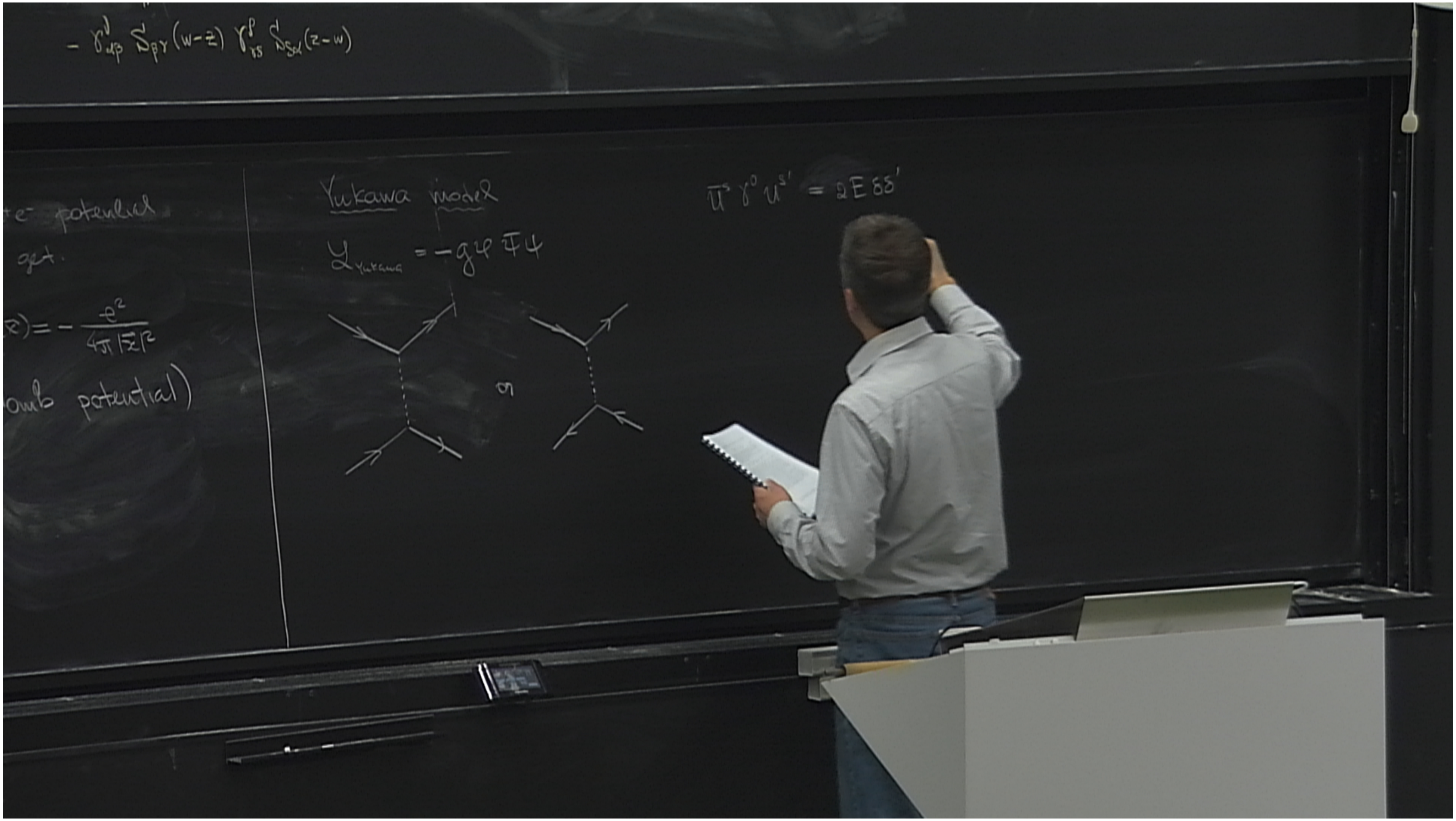

For e+e- potential
we get.
$V_{eff}(\vec{x}) = -\frac{e^2}{4\pi|\vec{x}|^2}$
(Coulomb potential)
(-1) from operator reordering
Yukawa model
$\mathcal{L}_{Yukawa} = -g\varphi\bar{\psi}\psi$

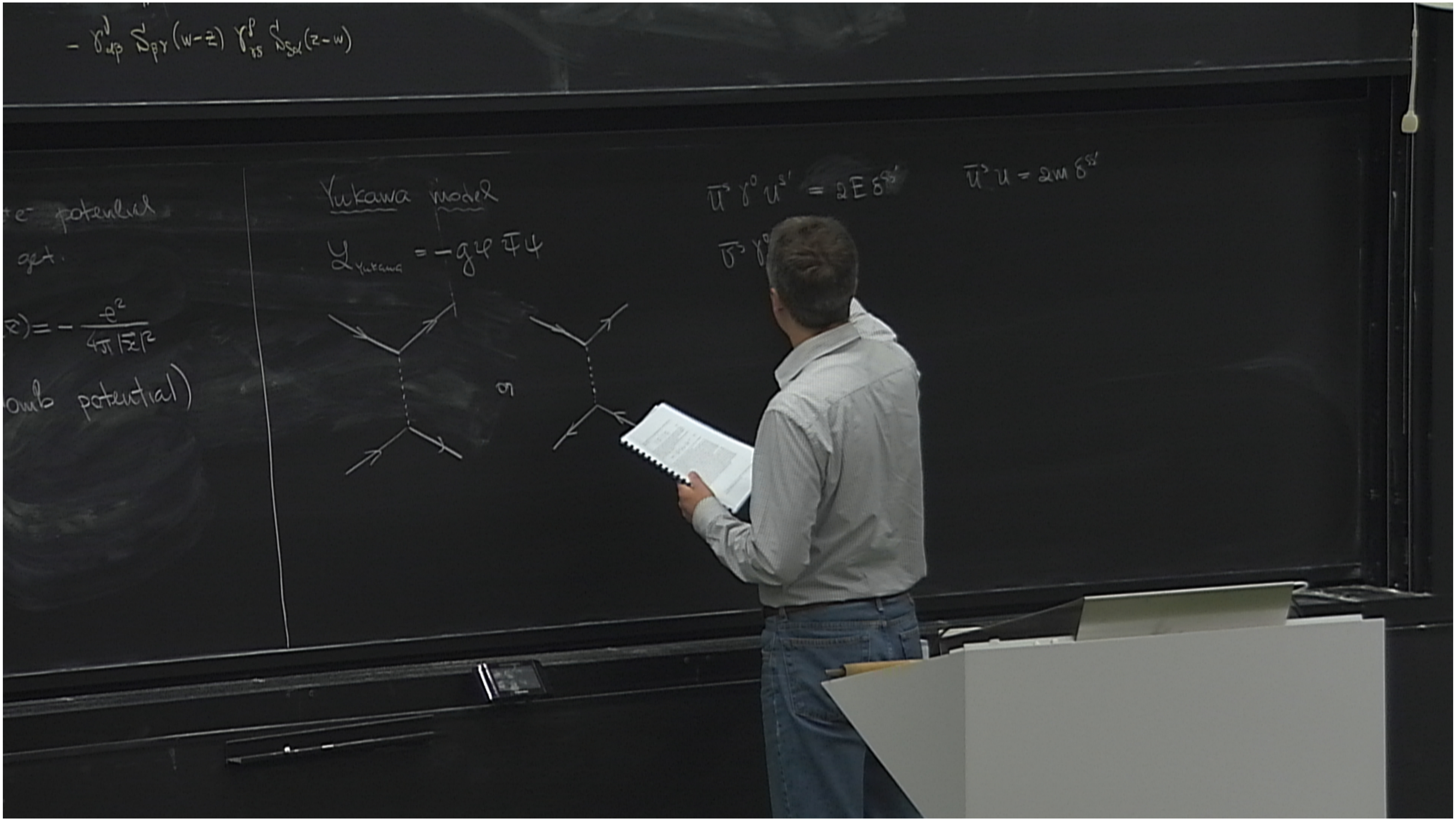

Yukawa model

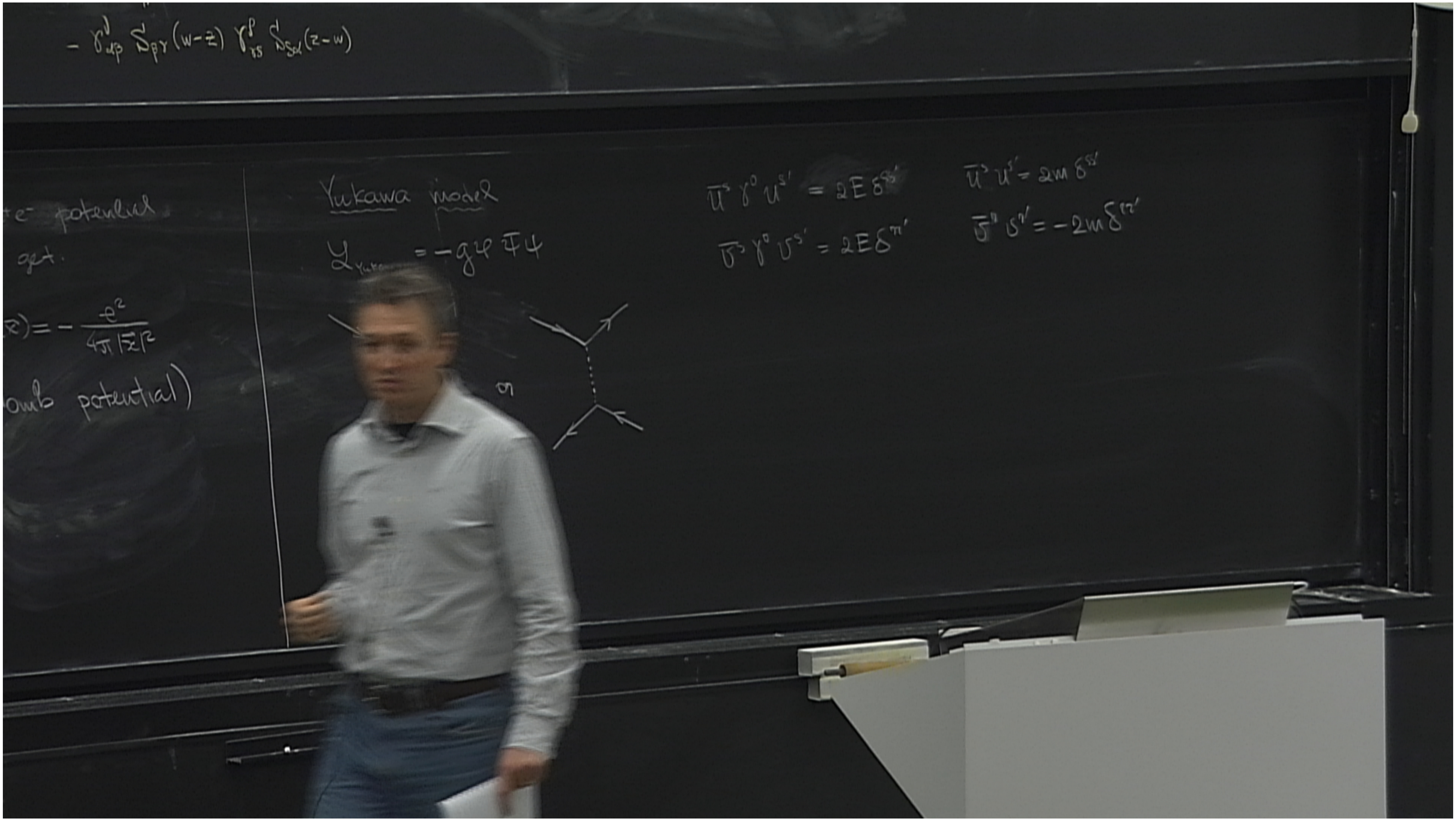


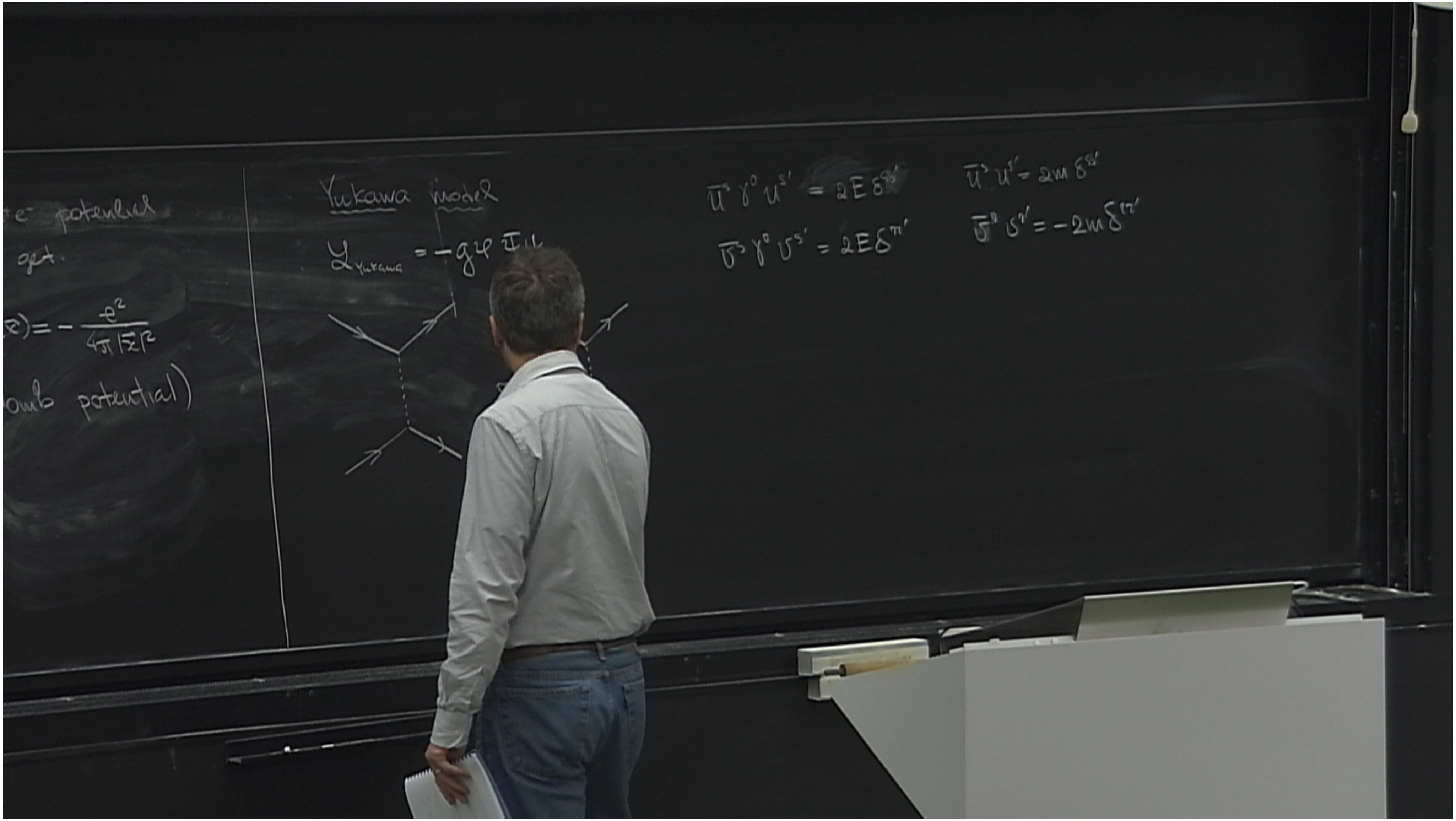
Yukawa model
potential
get.
omb potential)
or

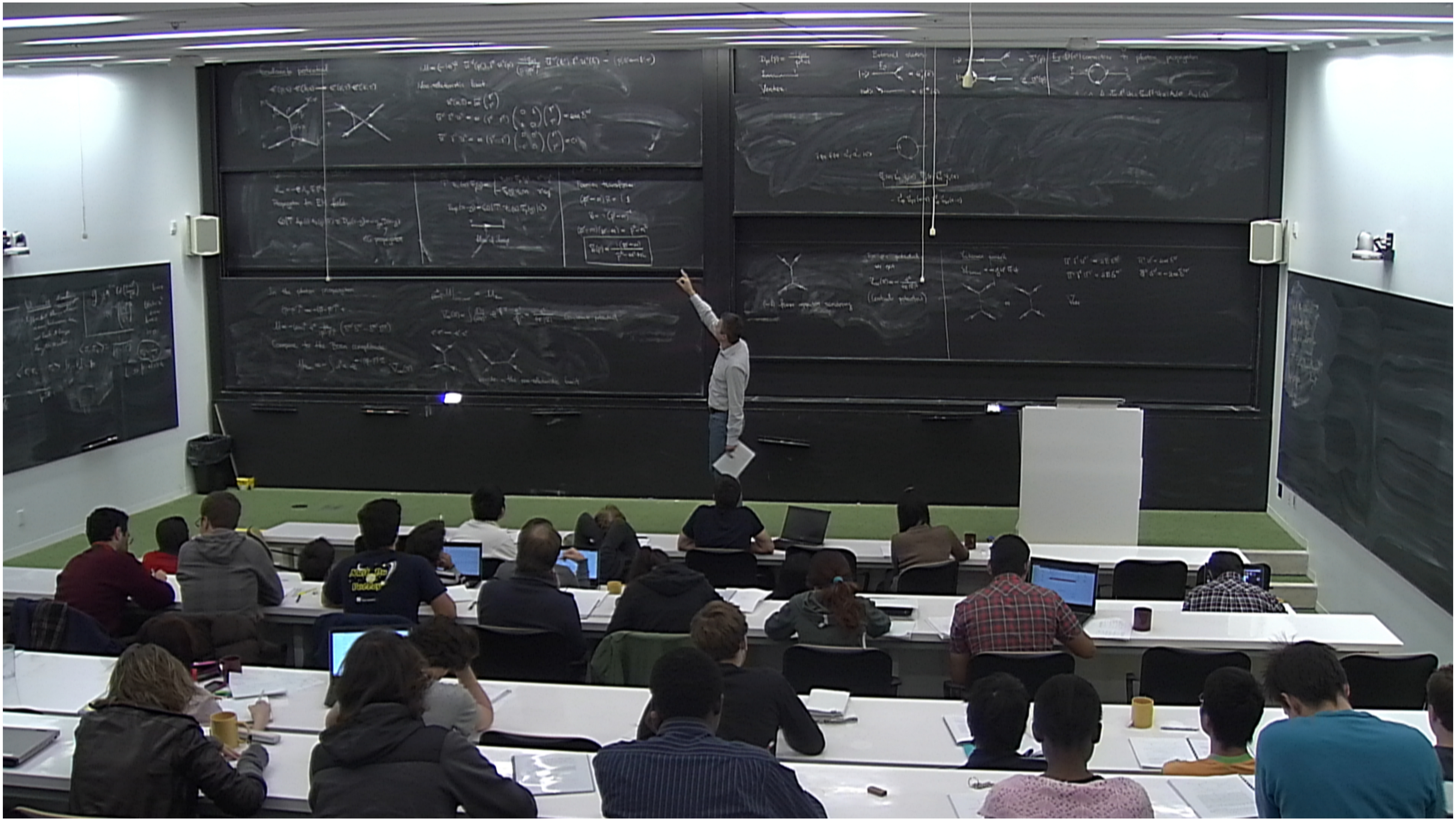

Yukawa model
or
potential

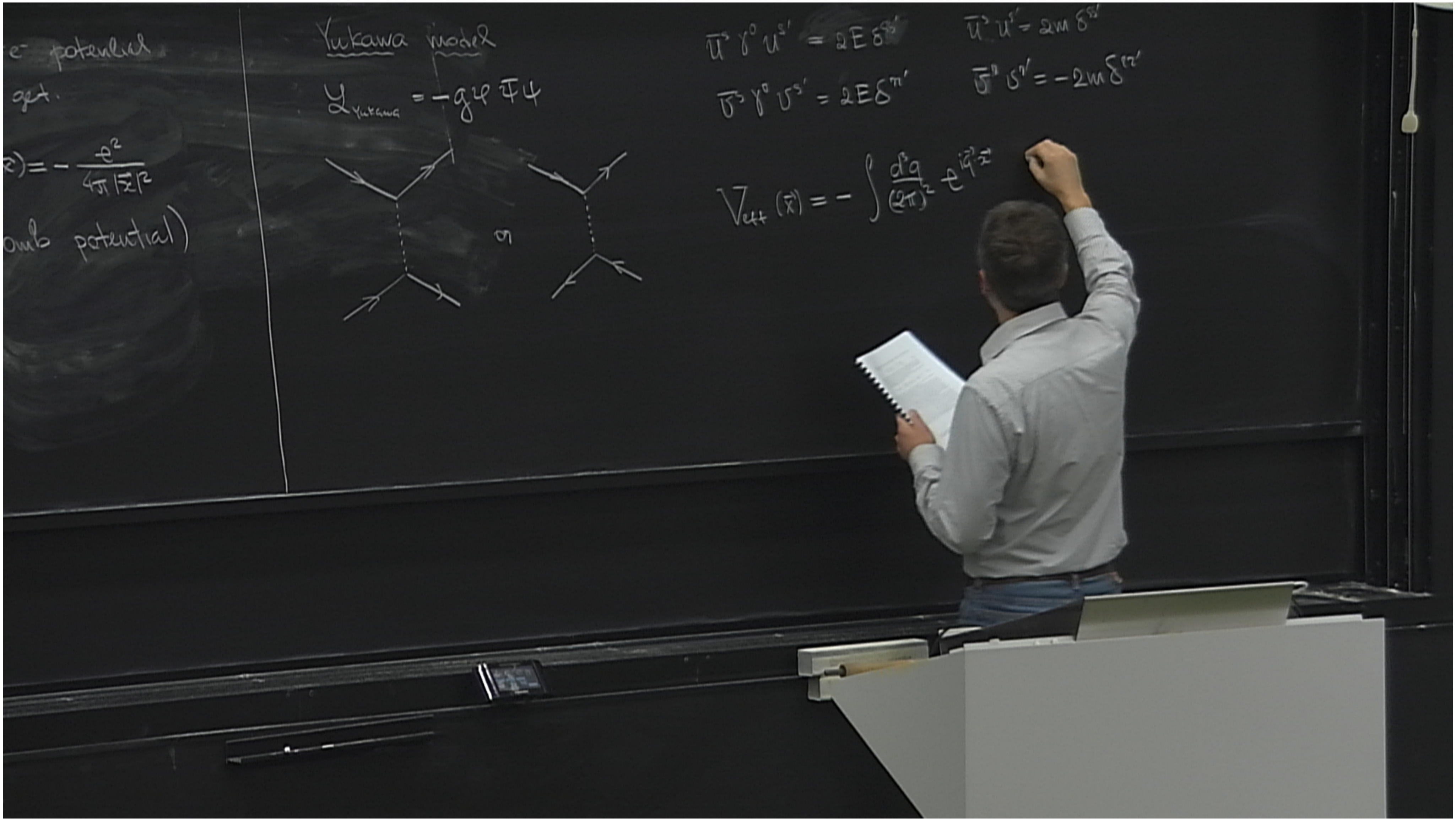

Yukawa model
or
potential

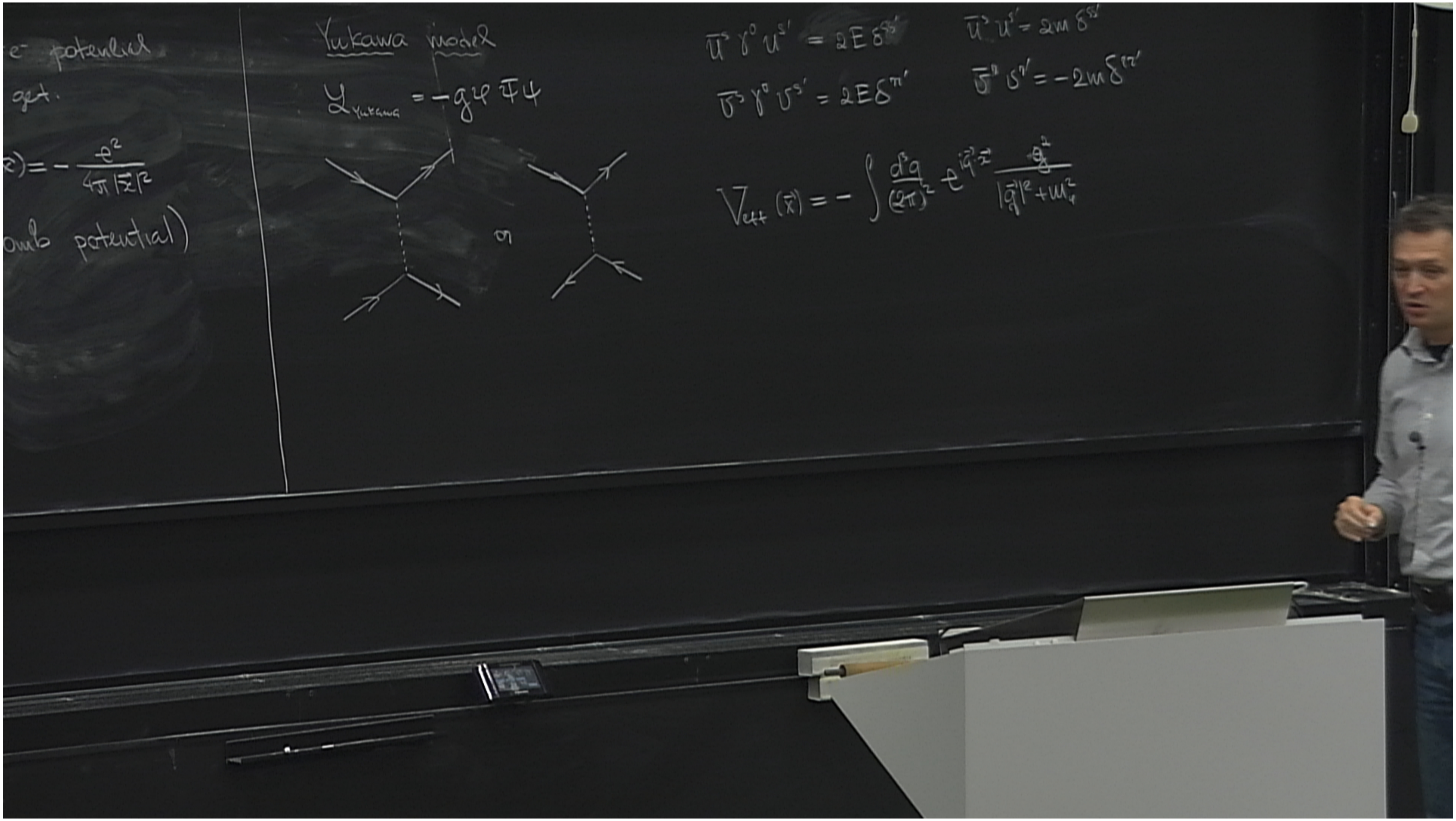

Yukawa model
potential
get.
or
omb potential)

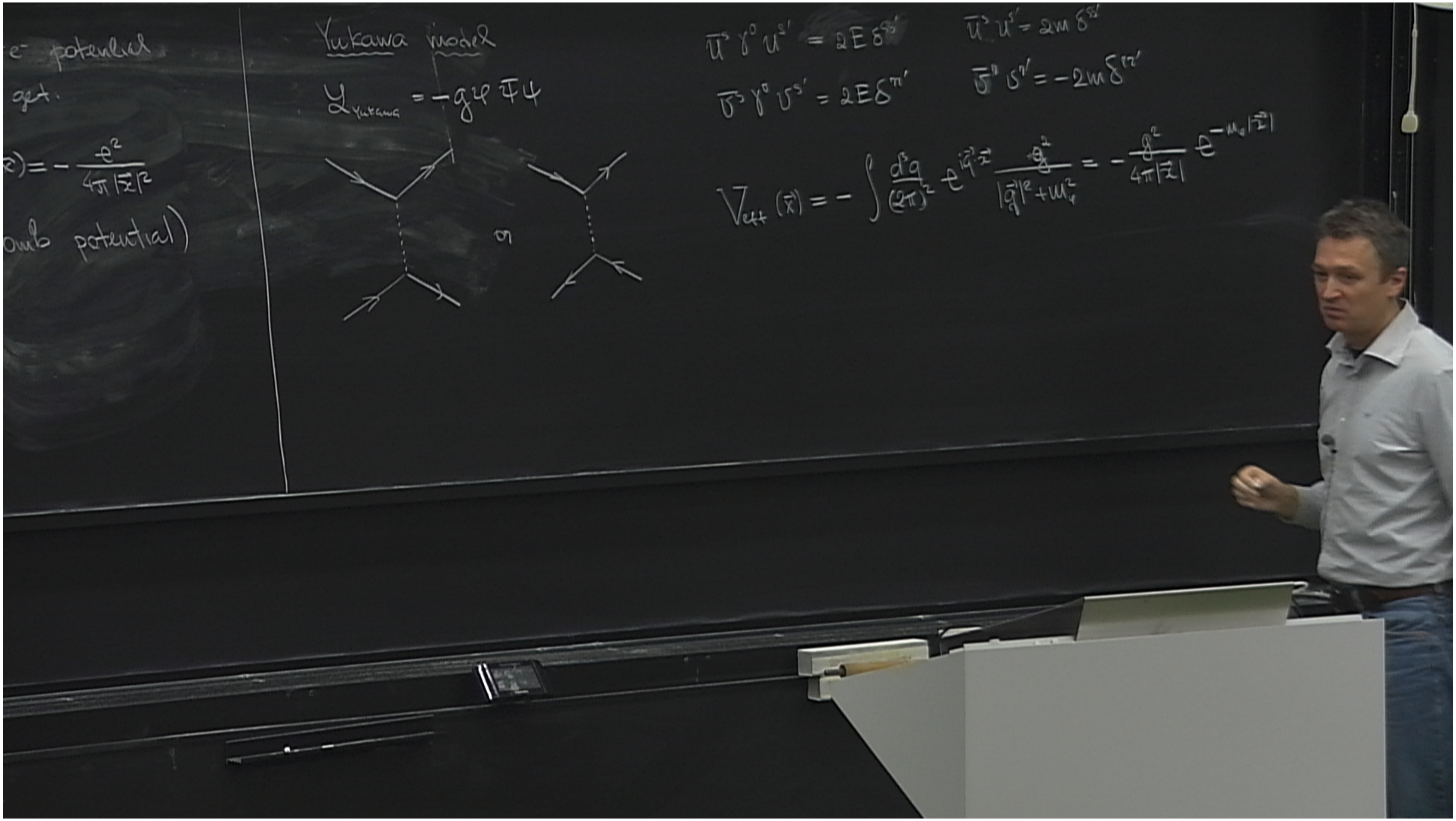

Yukawa model
potential
get.
potential)


e⁻ potential
get.
(Coulomb potential)
Yukawa model
$\mathcal{L}_{Yukawa} = -g\varphi\bar{\Psi}\Psi$
$\bar{u}^s\gamma^0 u^{s'} = 2E\delta^{ss'}$
$\bar{v}^s\gamma^0 v^{s'} = 2E\delta^{rr'}$
$\bar{u}^s u^{s'} = 2m\delta^{ss'}$
$\bar{v}^r v^{r'} = -2m\delta^{rr'}$

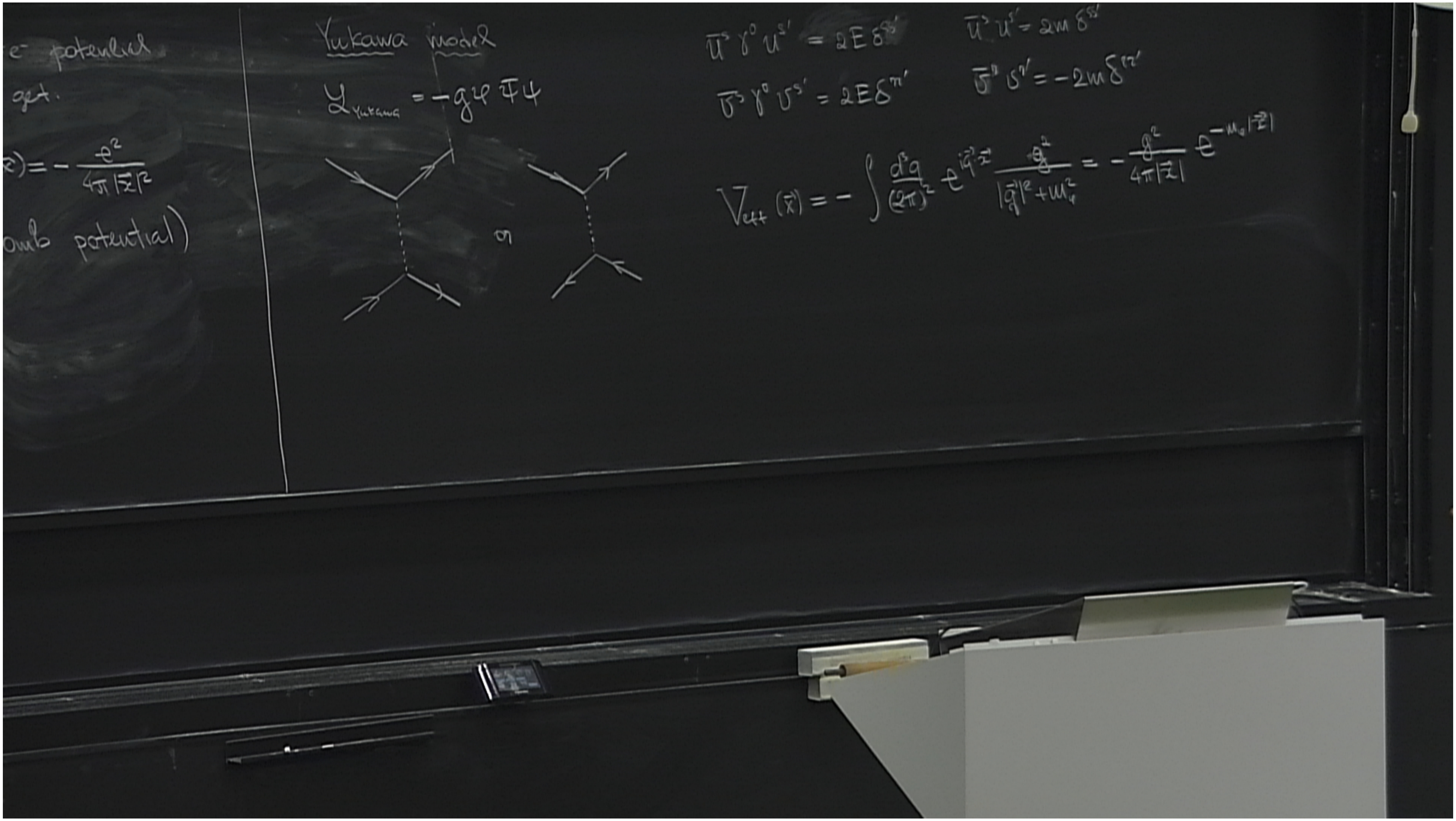


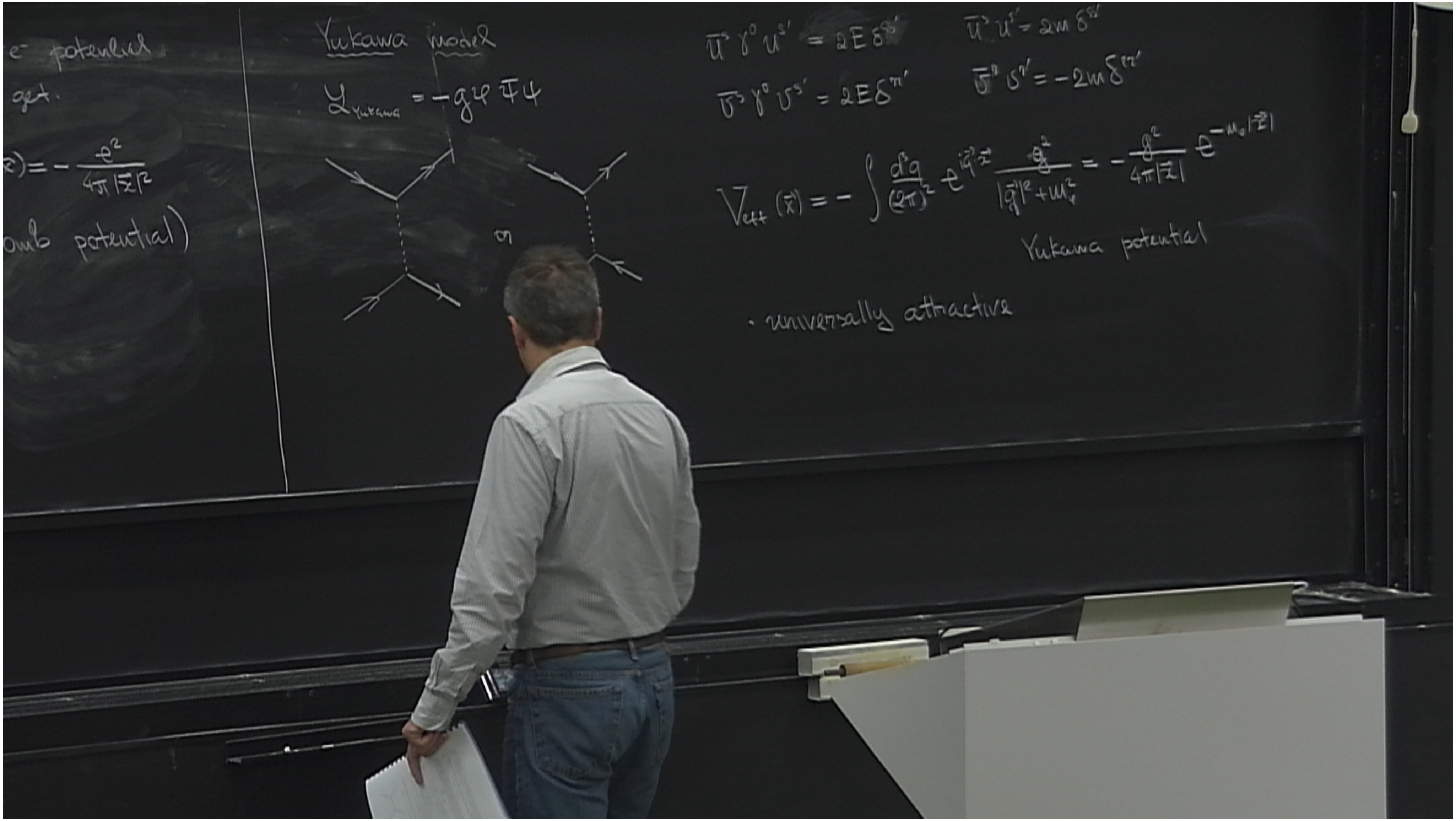

Yukawa model
or
omb potential)


Yukawa model
potential
get.
Coulomb potential)
$\mathcal{L}_{Yukawa} = -g\varphi\bar{\Psi}\Psi$
or
$\bar{u}^s \gamma^0 u^{s'} = 2E\delta^{ss'}$
$\bar{u}^s u^{s'} = 2m\delta^{ss'}$
$\bar{v}^s \gamma^0 v^{s'} = 2E\delta^{rr'}$
$\bar{v}^r v^{r'} = -2m\delta^{rr'}$
$V_{eff}(\vec{x}) = -\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{g^2}{|\vec{q}|^2 + m_\varphi^2}$



e⁻ potential
get.
Coulomb potential)
Yukawa model
$\mathcal{L}_{Yukawa} = -g\varphi\bar{\psi}\psi$
or
$\bar{u}^s\gamma^0 u^{s'} = 2E\delta^{ss'}$
$\bar{u}^s u^{s'} = 2m\delta^{ss'}$
$\bar{v}^s\gamma^0 v^{s'} = 2E\delta^{rr'}$
$\bar{v}^r v^{r'} = -2m\delta^{rr'}$
$V_{eff}(\vec{x}) = -\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{g^2}{|\vec{q}|^2+m_\varphi^2} = -\frac{g^2}{4\pi|\vec{x}|} e^{-m_\varphi|\vec{x}|}$


e⁻ potential
get.
$= -\frac{e^2}{4\pi|\vec{x}|^2}$
omb potential)
Yukawa model
$\mathcal{L}_{Yukawa} = -g\varphi\bar{\Psi}\Psi$
or
$\bar{u}^s\gamma^0 u^{s'} = 2E\delta^{ss'}$
$\bar{u}^s u^{s'} = 2m\delta^{ss'}$
$\bar{v}^s\gamma^0 v^{s'} = 2E\delta^{rr'}$
$\bar{v}^s v^{s'} = -2m\delta^{rr'}$
$V_{eff}(\vec{x}) = -\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} \frac{g^2}{|\vec{q}|^2+m_\varphi^2} = -\frac{g^2}{4\pi|\vec{x}|} e^{-m_\varphi|\vec{x}|}$


Yukawa model
Yukawa potential
universally attractive
potential)