Title: Quantum Field Theory I - Lecture 13

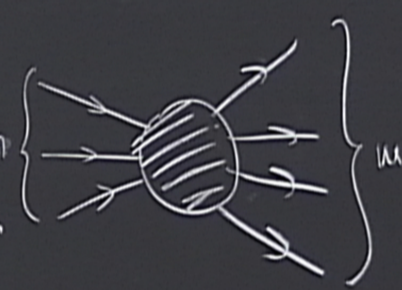
Date: Oct 20, 2011 09:00 AM

URL: http://pirsa.org/11100023

Abstract:

$$S = \mathbb{1} + iT$$

$$\langle \underset{m}{\text{out}} | \, iT \, | \underset{n}{\text{in}} \rangle = \sum_{\text{diagrams}}$$



connected
without connections
to external lines

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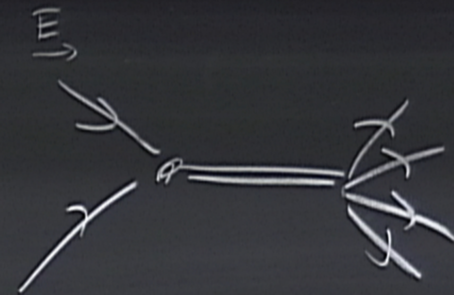
without connections
to external lines

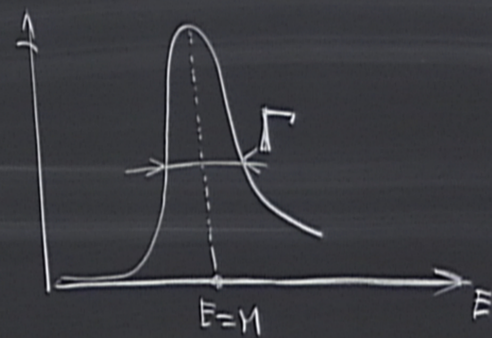
$$\langle \text{out} | iT | \text{in} \rangle = i(2\pi)^4 \delta\left(\sum_{i=1}^{m} p_i - \sum_{j=1}^{n} p_j\right) \mathcal{M}_{n\to m}$$

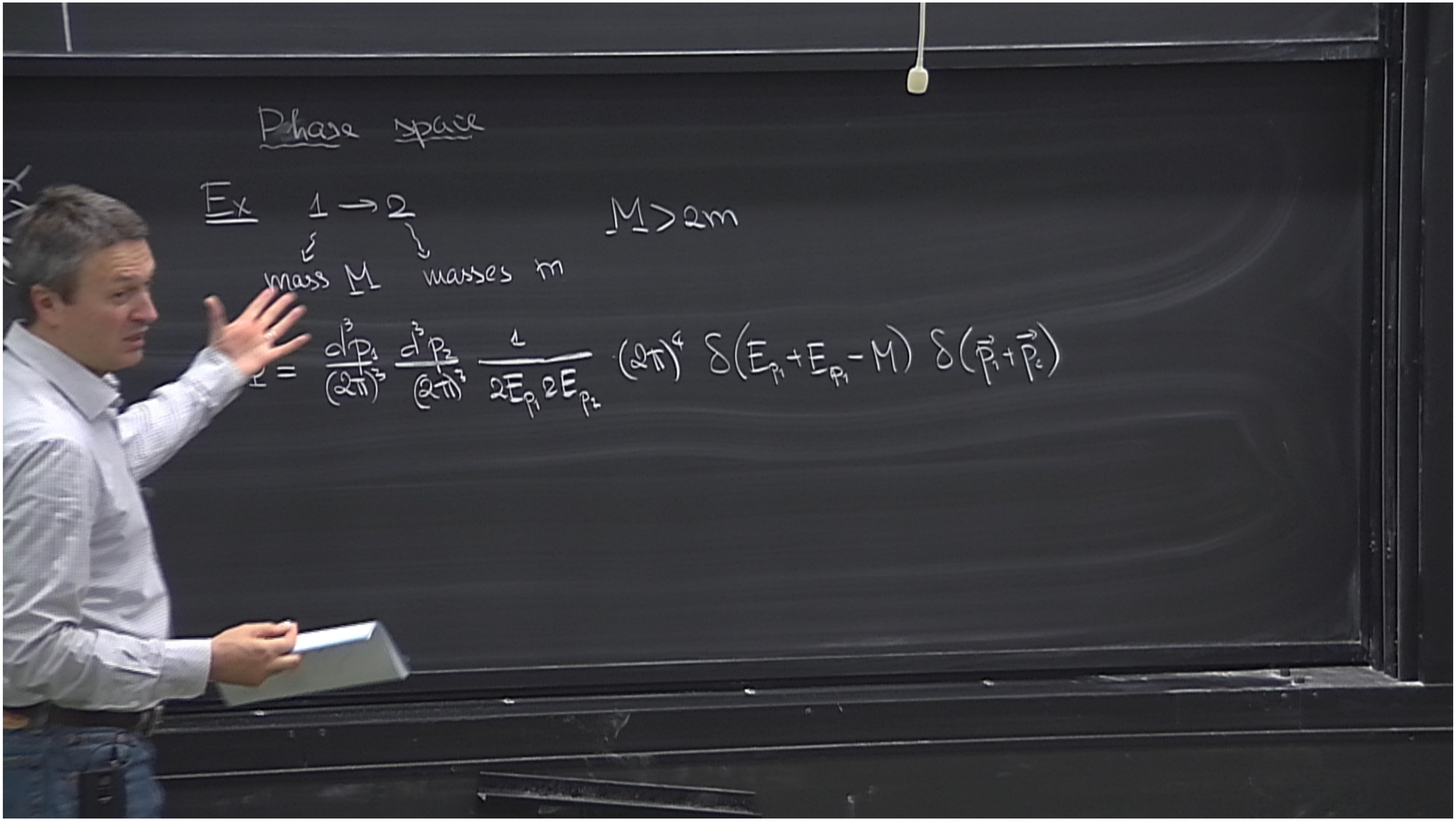
<u>Ex</u> $2\to 2$ in $\lambda\varphi^4$ theory: $\mathcal{M}_{2\to 2} = -\lambda + \mathcal{O}(\lambda^2)$

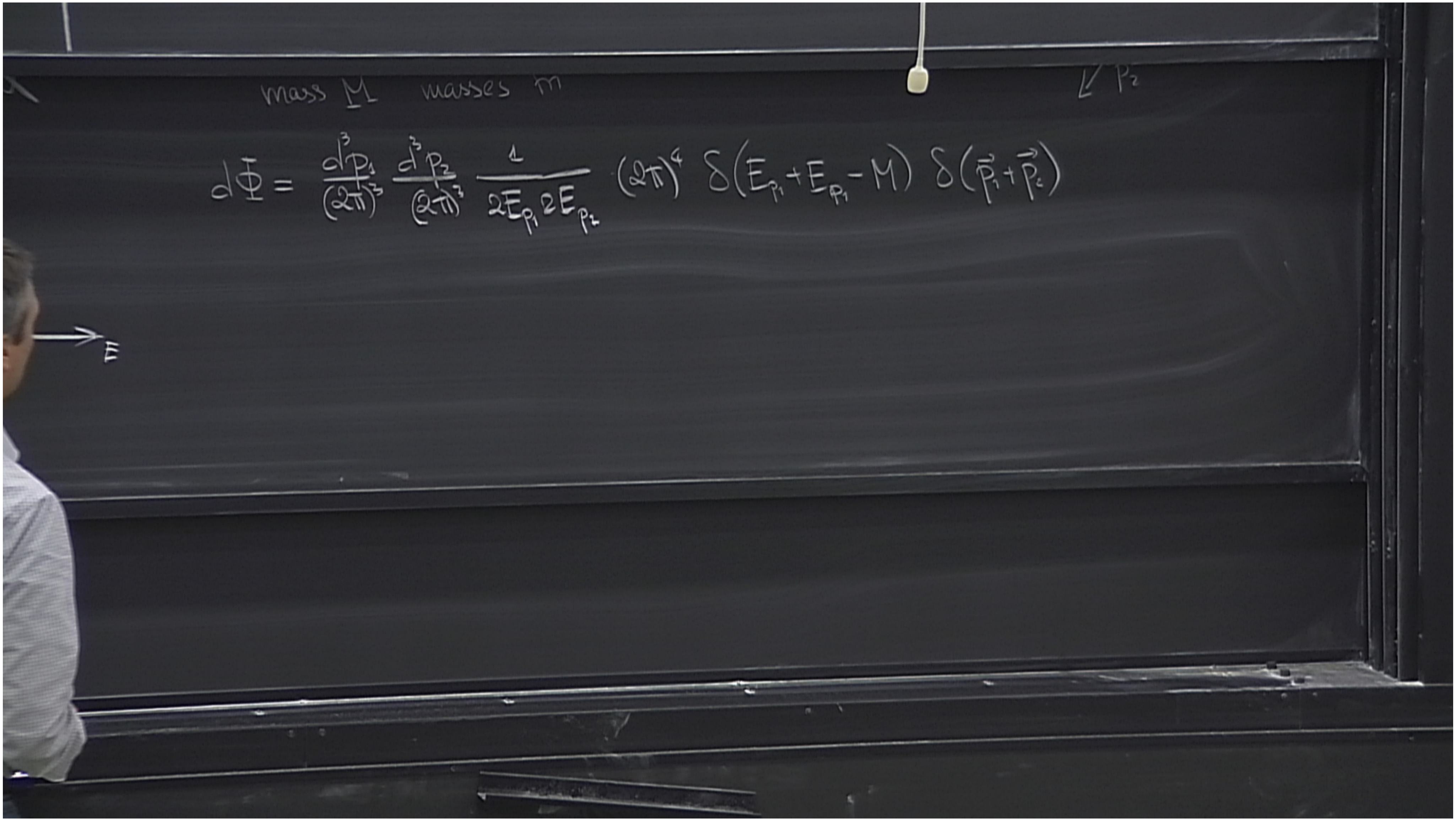
$$\Gamma_{total} = \sum_{channels} \Gamma_{partial}$$

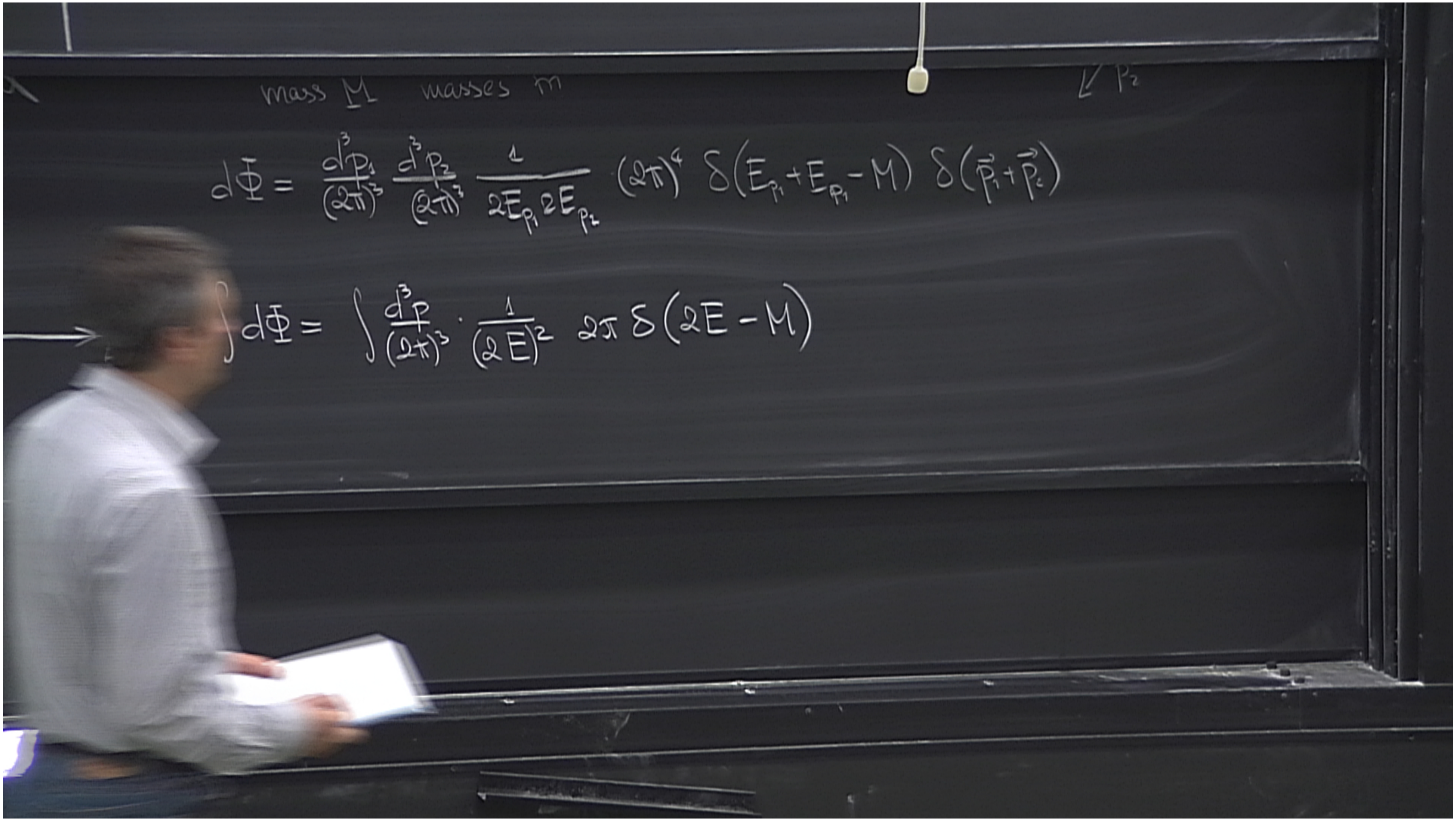
$$\tau = \frac{1}{\Gamma_{total}}$$ ~ mean lifetime of an unstable particle

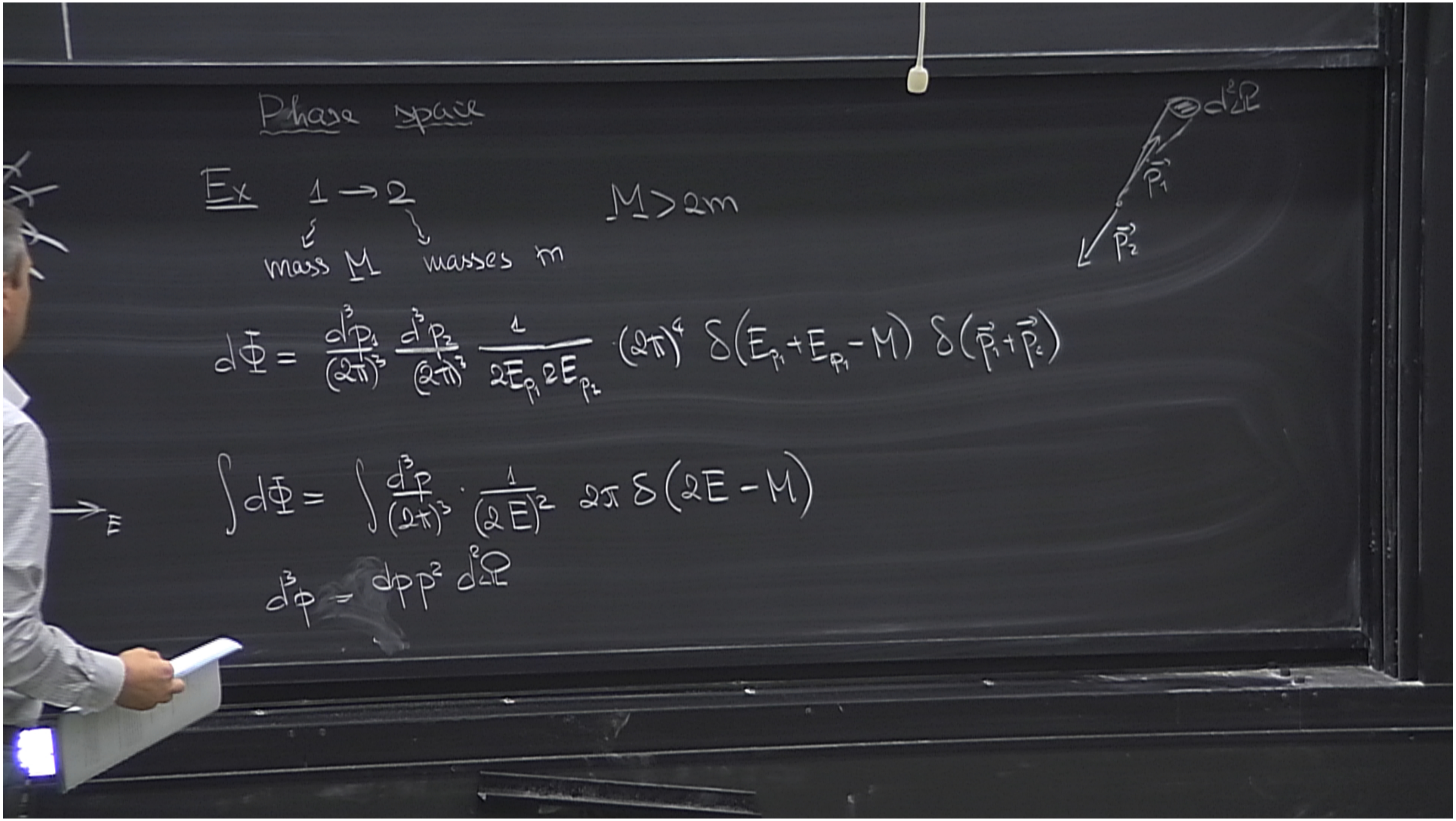


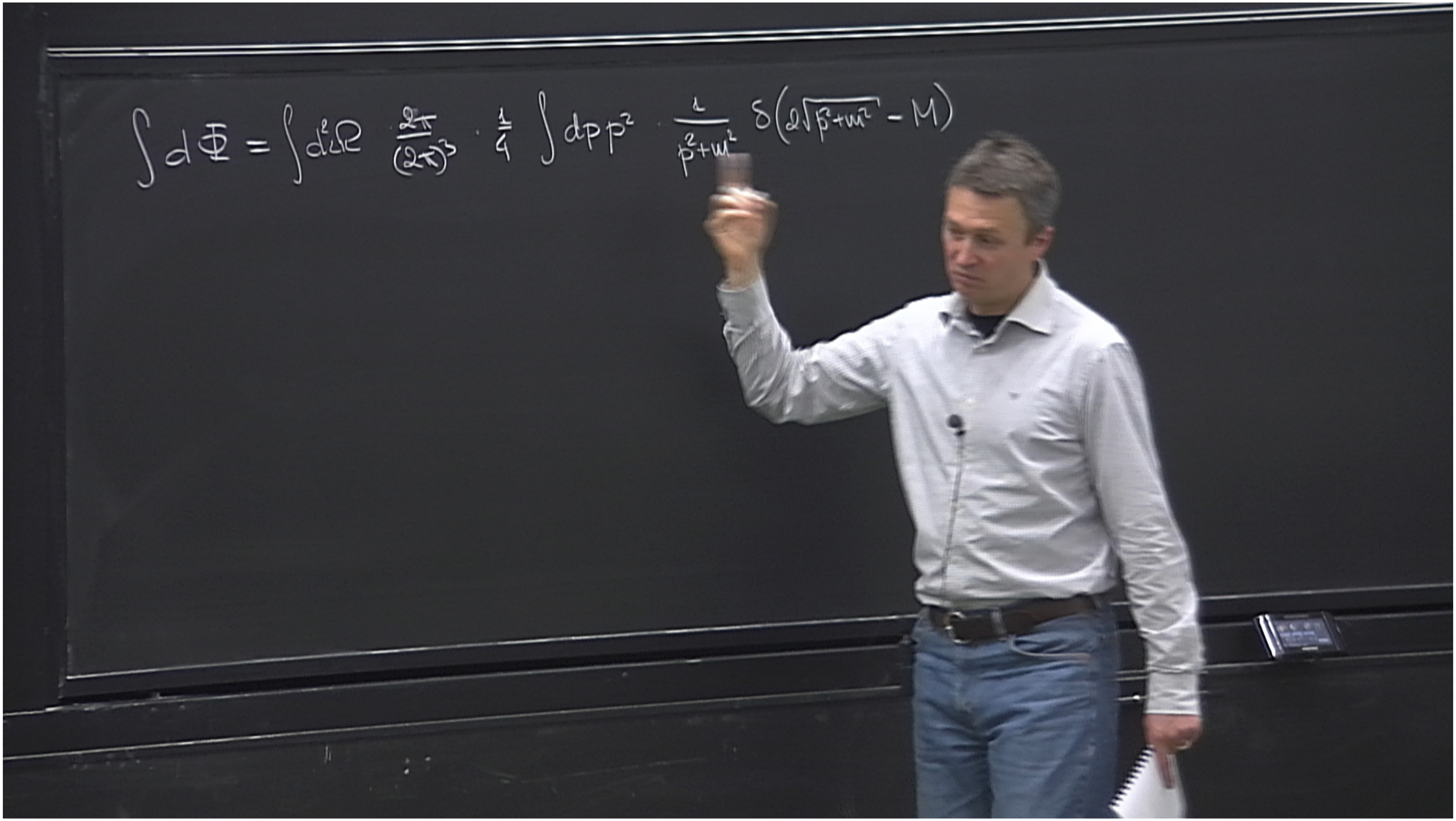



Phase space
Ex $1 \to 2$
$M > 2m$
mass M
masses m
$= \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{1}{2E_{p_1} 2E_{p_2}} (2\pi)^4 \delta(E_{p_1} + E_{p_1} - M) \delta(\vec{p}_1 + \vec{p}_2)$


mass M masses m
p_2
$d\Phi = \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_{p_1}2E_{p_2}}(2\pi)^4\,\delta(E_{p_1}+E_{p_1}-M)\,\delta(\vec{p}_1+\vec{p}_2)$
E


mass M masses m
$d\Phi = \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \frac{1}{2E_{p_1} 2E_{p_2}} (2\pi)^4 \delta(E_{p_1}+E_{p_1}-M)\, \delta(\vec{p}_1+\vec{p}_2)$
$\int d\Phi = \int \frac{d^3p}{(2\pi)^3} \cdot \frac{1}{(2E)^2}\, 2\pi\, \delta(2E-M)$
p_2


Phase space
Ex 1 → 2
mass M
masses m
$M > 2m$
$d\Phi = \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_{p_1}2E_{p_2}}(2\pi)^4\,\delta(E_{p_1}+E_{p_1}-M)\,\delta(\vec{p}_1+\vec{p}_2)$
$\int d\Phi = \int \frac{d^3p}{(2\pi)^3}\cdot\frac{1}{(2E)^2}\,2\pi\,\delta(2E-M)$
$d^3p = dp\,p^2\,d^2\Omega$
$d^2\Omega$
$\vec{p}_1$
$\vec{p}_2$
E


$\int d\Phi = \int d^2\Omega \, \frac{2\pi}{(2\pi)^3} \cdot \frac{1}{4} \int dp \, p^2 \cdot \frac{1}{p^2+m^2} \, \delta\left(2\sqrt{p^2+m^2} - M\right)$

$$\Phi = \int d^2\Omega \, \frac{2\pi}{(2\pi)^3} \cdot \frac{1}{4} \int_0^\infty dp\, p^2 \cdot \frac{1}{p^2+m^2}\, \delta\left(2\sqrt{p^2+m^2} - M\right) \qquad \sqrt{p^2+m^2} = \frac{M}{2}$$

$$p = \sqrt{\frac{M^2}{4} - m^2} = \frac{M}{2}\sqrt{1 - \frac{4m^2}{M^2}}$$

$$= \int \frac{d^2\Omega}{4(2\pi)^2} \cdot \frac{p^2}{p^2+m^2} \cdot \frac{\sqrt{p^2+m^2}}{2p} \Bigg|_{\sqrt{p^2+m^2} = \frac{M}{2}}$$

$$\frac{d\Phi}{d\Omega} = \frac{1}{32\pi^2}\left(\frac{p}{\sqrt{p^2+m^2}}\right)$$

v - velocity of a produced particles

$$\boxed{\frac{d\Phi}{d\Omega} = \frac{1}{32\pi^2}\sqrt{1 - \frac{4m^2}{M^2}}}$$

Scattering cro

$$\int d\Phi = \int d^2\Omega \, \frac{2\pi}{(2\pi)^3} \cdot \frac{1}{4} \int_0^\infty dp\, p^2 \cdot \frac{1}{p^2+m^2}\, \delta\left(2\sqrt{p^2+m^2} - M\right)$$

$$\sqrt{p^2+m^2} = \frac{M}{2}$$

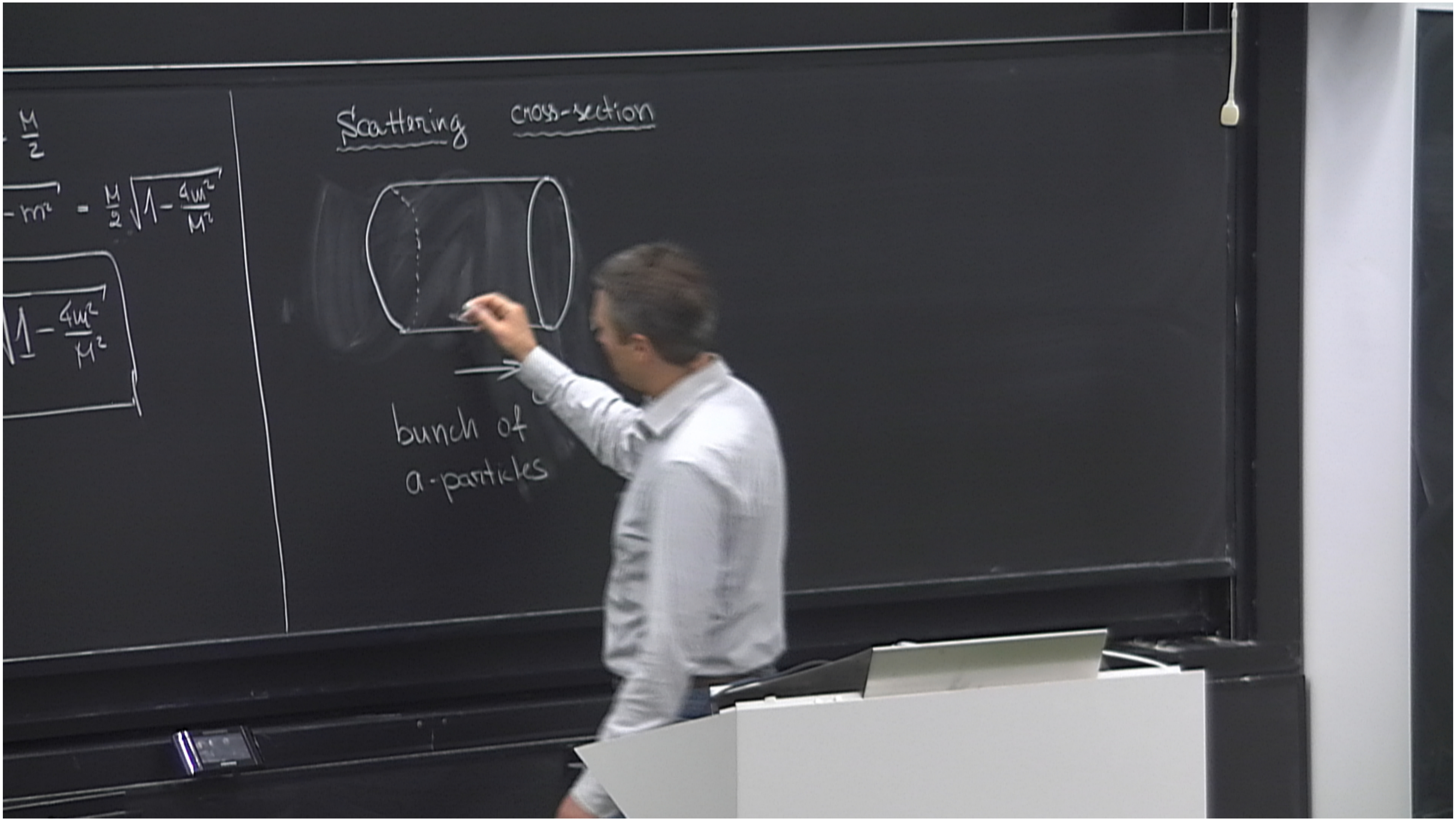
$$p = \sqrt{\frac{M^2}{4} - m^2} = \frac{M}{2}\sqrt{1 - \frac{4m^2}{M^2}}$$

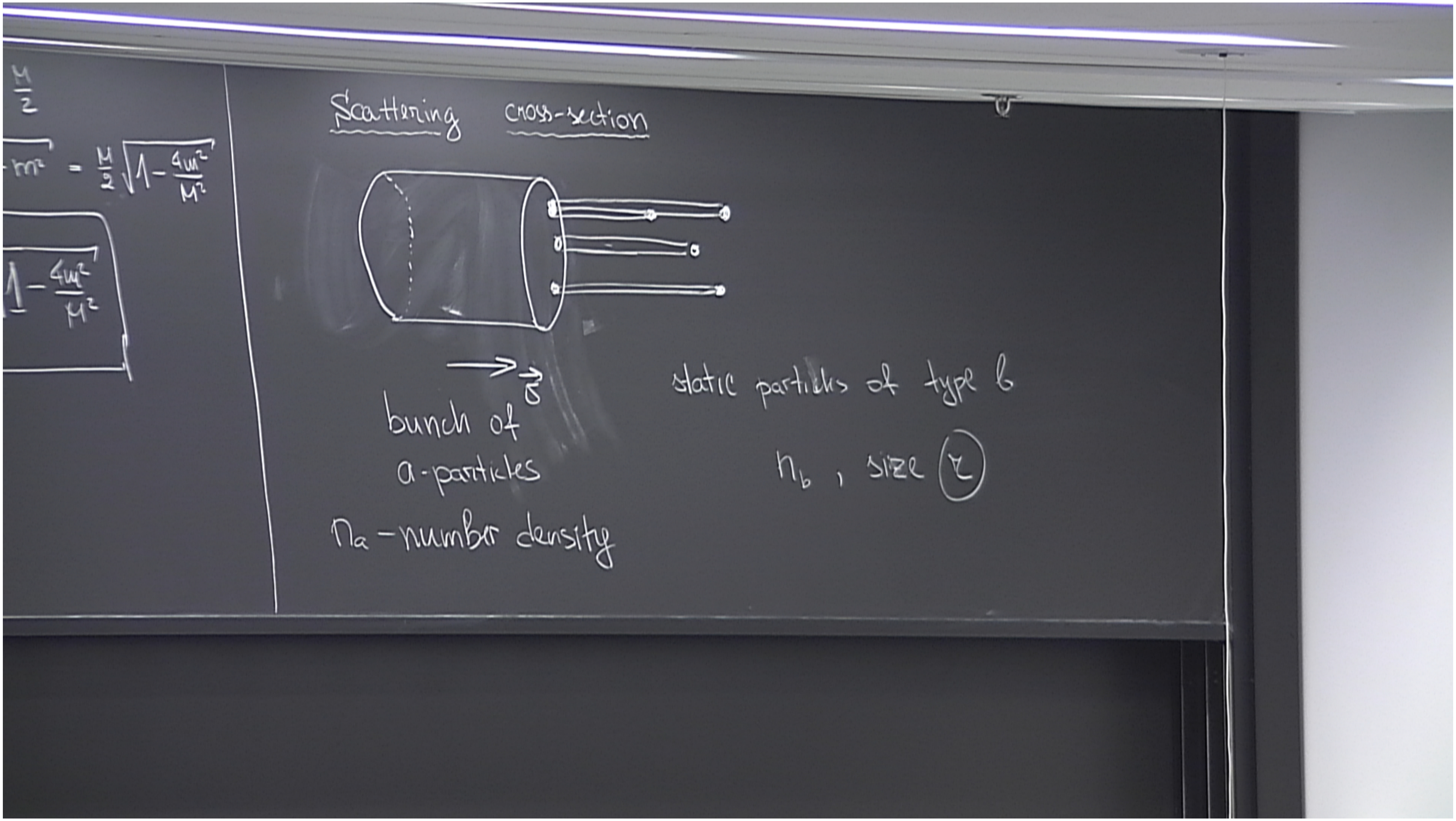
$$= \int \frac{d^2\Omega}{4(2\pi)^2} \cdot \frac{p^2}{p^2+m^2} \cdot \frac{\sqrt{p^2+m^2}}{2p} \Bigg|_{\sqrt{p^2+m^2} = \frac{M}{2}}$$

$$\boxed{\frac{d\Phi}{d\Omega} = \frac{1}{32\pi^2}\sqrt{1 - \frac{4m^2}{M^2}}}$$

$$\frac{d\Phi}{d^2\Omega} = \frac{1}{32\pi^2}\left(\frac{p}{\sqrt{p^2+m^2}}\right)$$

v - velocity of a produced particles


Scattering cross-section
bunch of
a-particles


Scattering cross-section
$\vec{v}$
bunch of
a-particles
n_a - number density
static particles of type b
n_b , size r
$= \frac{M}{2}\sqrt{1-\frac{4m^2}{M^2}}$

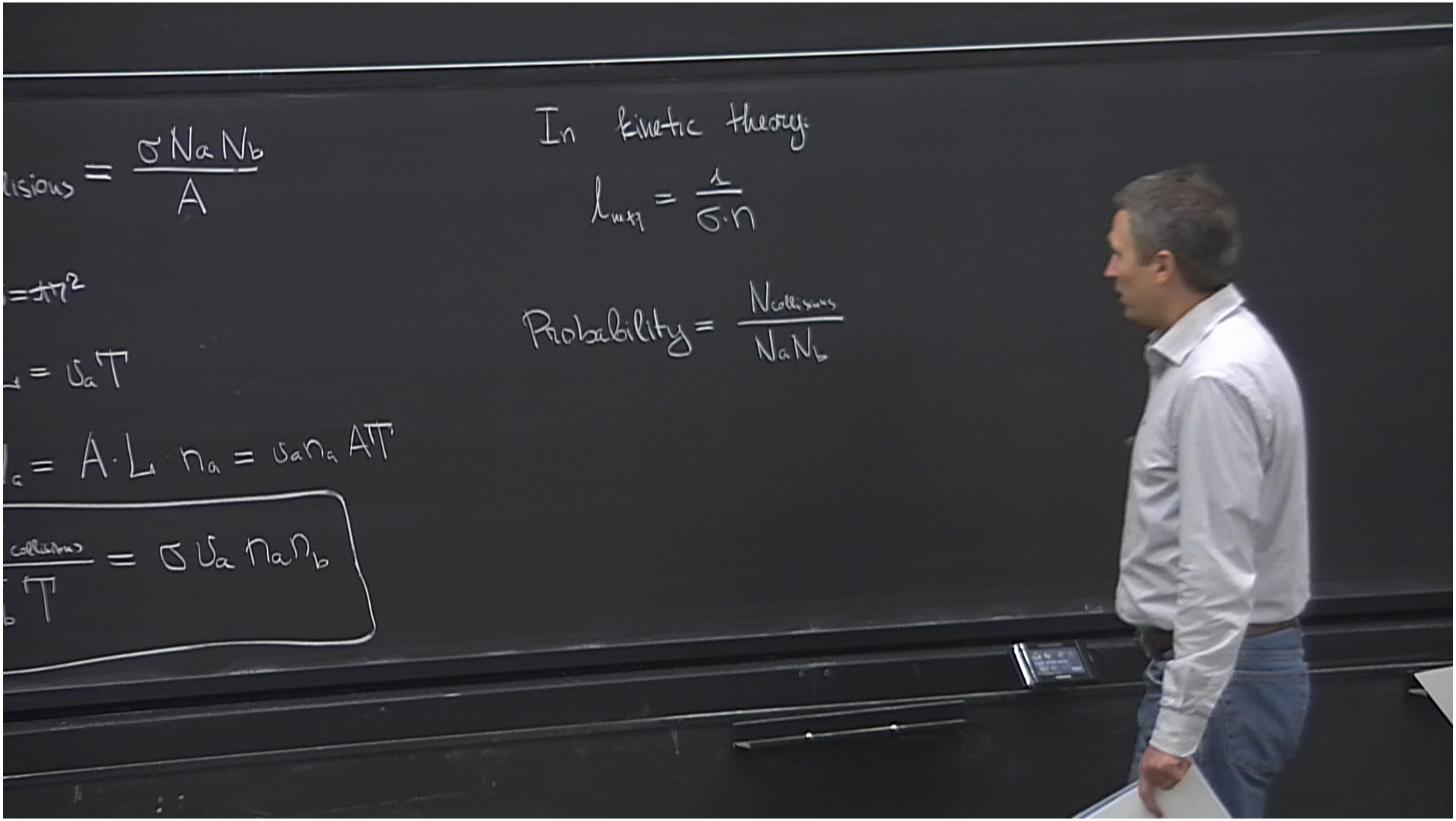
$$N_{collisions} = \frac{\sigma N_a N_b}{A}$$

$$\sigma = \pi r^2$$

$$L = v_a T$$

$$N_a = A \cdot L \cdot n_a = v_a n_a A T$$

$$N_{collisions}$$


lisions = $\frac{\sigma N_a N_b}{A}$
$= \pi r^2$
$= v_a T$
$= A \cdot L \cdot n_a = v_a n_a A T$
collisions
$= \sigma v_a n_a n_b$
T
In kinetic theory:
$l_{mfp} = \frac{1}{\sigma \cdot n}$
Probability = $\frac{N_{collisions}}{N_a N_b}$

$$\text{...lisions} = \frac{\sigma N_a N_b}{A}$$

$$\ldots = \pi r^2$$

$$\ldots = v_a T$$

$$\ldots = A \cdot L \cdot n_a = v_a n_a A T$$

$$\boxed{\frac{\text{collisions}}{\ldots T} = \sigma v_a n_a n_b}$$

In kinetic theory:

$$l_{mfp} = \frac{1}{\sigma \cdot n}$$

$$\text{Probability} = \frac{N_{collisions}}{N_a N_b} = \frac{\sigma}{A} = \sigma v_{rel} \frac{T}{V}$$

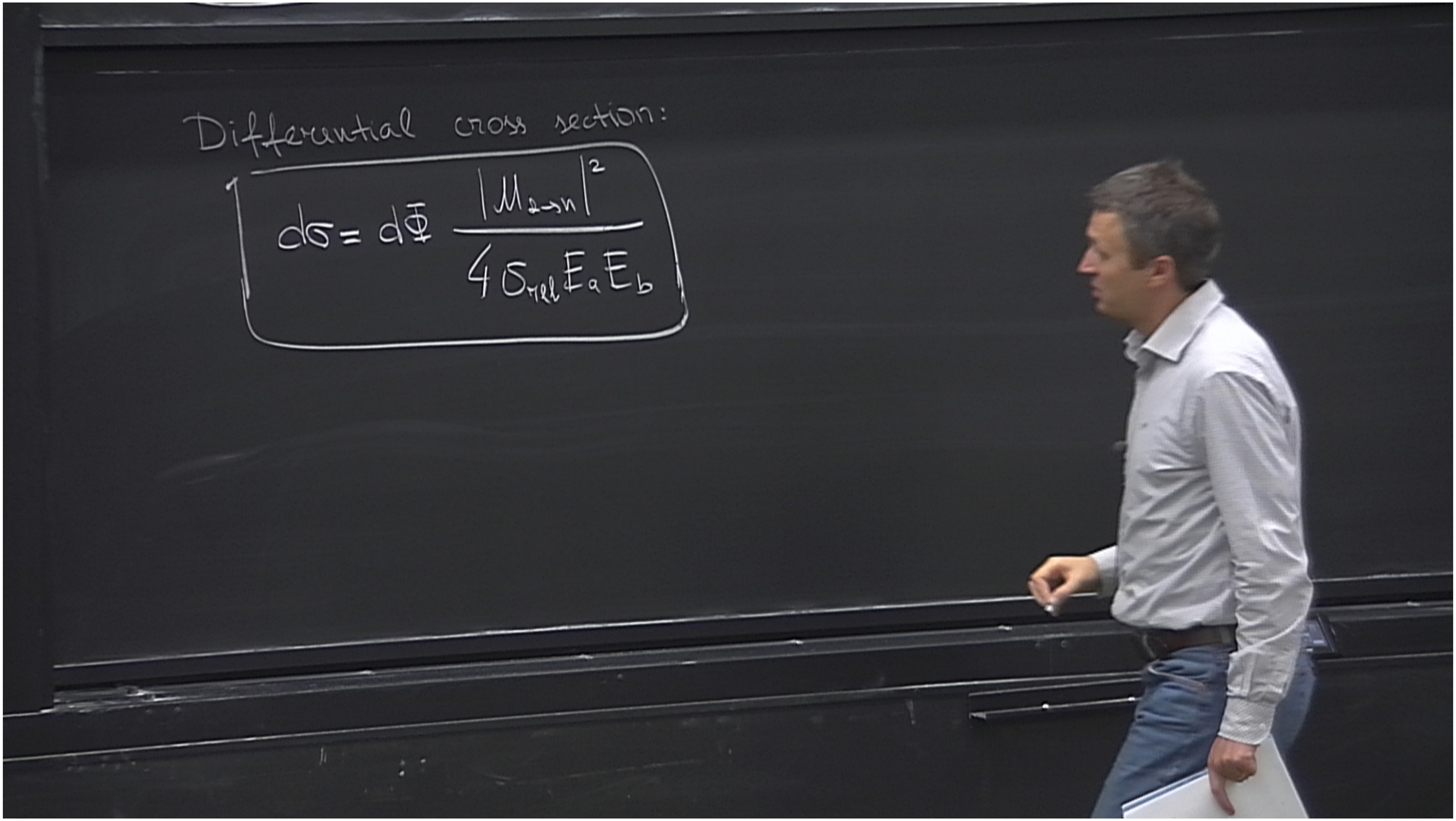
theory:

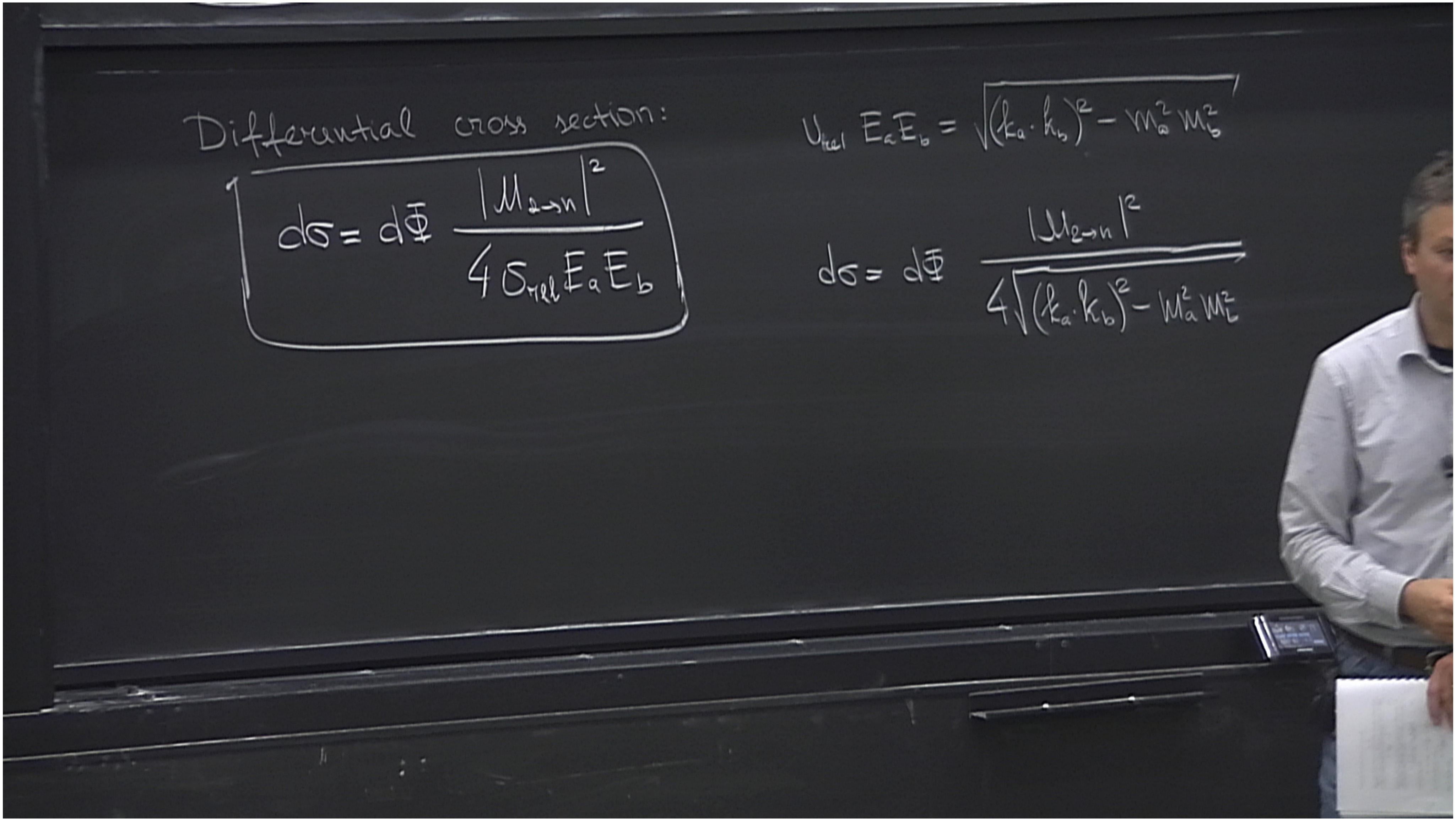
$\cdot n$

$$\frac{N_{\text{collisions}}}{N_a N_b} = \frac{\sigma}{A} = \sigma\, v_{\text{rel}}\, \frac{T}{V}$$

In QFT:

$$dP = \left[\prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3}\, \frac{1}{2E_{p_i}}\right] \frac{|\langle p_1,\ldots,p_n | T | k_a, k_b\rangle|^2}{\langle k_a|k_a\rangle \langle k_b|k_b\rangle}$$


Differential cross section:
$$d\sigma = d\Phi \frac{|\mathcal{M}_{2\to n}|^2}{4 v_{rel} E_a E_b}$$


Differential cross section:
$\boxed{d\sigma = d\Phi \frac{|\mathcal{M}_{2\to n}|^2}{4 v_{rel} E_a E_b}}$
$v_{rel} E_a E_b = \sqrt{(k_a \cdot k_b)^2 - m_a^2 m_b^2}$
$d\sigma = d\Phi \frac{|\mathcal{M}_{2\to n}|^2}{4\sqrt{(k_a \cdot k_b)^2 - m_a^2 m_b^2}}$

Differential cross section:

$$\boxed{d\sigma = d\Phi \, \frac{|\mathcal{M}_{2\to n}|^2}{4 v_{rel} E_a E_b}}$$

$$v_{rel} E_a E_b = \sqrt{(k_a \cdot k_b)^2 - m_a^2 m_b^2}$$

$$d\sigma = d\Phi \, \frac{|\mathcal{M}_{2\to n}|^2}{4\sqrt{(k_a \cdot k_b)^2 - m_a^2 m_b^2}}$$