

Title: Quantum Field Theory I - Lecture 11

Date: Oct 18, 2011 09:00 AM

URL: <http://pirsa.org/11100021>

Abstract:

$$G(x_1, \dots, x_n) \stackrel{\text{def}}{=} \langle \Omega | T \varphi(x_1) \dots \varphi(x_n) | \Omega \rangle$$

$$G(x_1, \dots, x_n) = \frac{\langle 0 | T \varphi_i(x_1) \dots \varphi_i(x_n) \exp\left(i \int d^4x \mathcal{L}_{\text{int}}\right) | 0 \rangle}{\langle 0 | T \exp\left(i \int d^4x \mathcal{L}_{\text{int}}\right) | 0 \rangle}$$

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4!} \varphi^4$$



$$G(x_1, \dots, x_n) \stackrel{\text{def}}{=} \langle \Omega | T \varphi(x_1) \dots \varphi(x_n) | \Omega \rangle$$

$$G(x_1, \dots, x_n) = \frac{\langle 0 | T \varphi_I(x_1) \dots \varphi_I(x_n) \exp\left(i \int d^4x \mathcal{L}_{\text{int}}\right) | 0 \rangle}{\langle 0 | T \exp\left(i \int d^4x \mathcal{L}_{\text{int}}\right) | 0 \rangle}$$

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4!} \varphi_I^4$$

$$T \varphi(x_1) \dots \varphi(x_n) | \Omega \rangle$$

$$\varphi_i(x_1) \dots \varphi_i(x_n) \exp \left( i \int d^4x \mathcal{L}_{int} \right) | 0 \rangle$$

$$\langle 0 | T \exp \left( i \int d^4x \mathcal{L}_{int} \right) | 0 \rangle$$

• Green's functions of free KG field

Wick's theorem

Ex (1)  $[a, a^\dagger] = 1$

$$\langle 0 | a a^\dagger | 0 \rangle = \langle 0 | (1 + \cancel{a^\dagger a}) | 0 \rangle = 1$$

Ex (2)

$$\langle 0 | a a^\dagger a a^\dagger | 0 \rangle$$

$$T \varphi(x_1) \dots \varphi(x_n) | \Omega \rangle$$

$$\varphi_i(x_1) \dots \varphi_i(x_n) \rightarrow \exp \left( i \int d^4x \mathcal{L}_{int} \right) | 0 \rangle$$

$$\langle 0 | T \exp \left( i \int d^4x \mathcal{L}_{int} \right) | 0 \rangle$$

$$\varphi_i^4$$

• Green's functions of free KG field

Wick's theorem

Ex (1)  $[a, a^\dagger] = 1$

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Ex (2)

$$\langle 0 | a a^\dagger a a^\dagger | 0 \rangle = \langle 0 | a a^\dagger | 0 \rangle = 1$$

$$A_i = c_i a + b_i a^+ ; c_i, b_i \in \mathbb{C}$$

$$\langle 0 | A_1 \dots A_n | 0 \rangle$$

$$A_i = c_i a + b_i a^\dagger ; c_i, b_i \in \mathbb{C}$$

$$\langle 0 | A_1 \dots A_n | 0 \rangle = \sum_{\text{pairings}} \underbrace{A_1 \dots A_n}_{\text{pairings}} \quad (\text{Wick's theorem})$$

$$\underbrace{A_i A_j} \equiv \langle 0 | A_i A_j | 0 \rangle$$

$$A_i A_j \equiv \langle 0 | A_i A_j | 0 \rangle$$

If  $\varphi(x_i)$  are free fields (linear in  $a_p, a_p^\dagger$ ):

$$\langle 0 | T \varphi(x_1) \dots \varphi(x_n) | 0 \rangle = \sum_{\text{pairings}} \underbrace{\varphi(x_1) \dots \varphi(x_n)}$$





$$A_i = c_i a + b_i a^\dagger ; c_i, b_i \in \mathbb{C}$$

$$\langle 0 | A_1 \dots A_n | 0 \rangle = \sum_{\text{pairings}} \underbrace{A_1 \dots A_n}_{\text{pairings}} \quad (\text{Wick's theorem})$$

$$\underbrace{A_i A_j} \equiv \langle 0 | A_i A_j | 0 \rangle$$

If  $\varphi(x_i)$  are free fields (linear in  $a_p, a_p^\dagger$ ):

$$\langle 0 | T \varphi(x_1) \dots \varphi(x_n) | 0 \rangle = \sum_{\text{pairings}} \underbrace{\varphi(x_1) \dots \varphi(x_n)}_{\text{pairings}}$$

$$\underbrace{\varphi(x_i) \varphi(x_j)} \equiv \langle 0 | T \varphi(x_i) \varphi(x_j) | 0 \rangle$$

$$\underline{\text{Ex (2)}} \quad G(x_1, x_2, x_3, x_4) = \langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \underbrace{\varphi_1 \varphi_2 \varphi_3 \varphi_4} + \underbrace{\varphi_1 \varphi_2 \varphi_3 \varphi_4} + \underbrace{\varphi_1 \varphi_2 \varphi_3 \varphi_4}$$

$$\varphi_i \equiv \varphi(x_i) \quad = \mathcal{D}(x_1 - x_2) \mathcal{D}(x_1 - x_4) + \mathcal{D}(x_1 - x_3) \mathcal{D}(x_2 - x_4) + \mathcal{D}(x_1 - x_4) \mathcal{D}(x_2 - x_3)$$

Feynman propagator

Ex (1)  $G(x_1, x_2) = D(x_1 - x_2)$

Ex (2)  $G(x_1, x_2, x_3, x_4) = \langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \underbrace{\varphi_1 \varphi_2 \varphi_3 \varphi_4} + \underbrace{\varphi_1 \varphi_2 \varphi_3 \varphi_4} + \underbrace{\varphi_1 \varphi_2 \varphi_3 \varphi_4}$

$\varphi_i \equiv \varphi(x_i) = D(x_1 - x_2) D(x_1 - x_4) + D(x_1 - x_3) D(x_2 - x_4) + D(x_1 - x_4) D(x_2 - x_3)$

Feynman propagator

Space-time picture:

$$\langle 0 | T \psi_1 \psi_2 | 0 \rangle = D(x_1 - x_2) =$$

$$\underbrace{\langle 0 | a_p a_k^\dagger | 0 \rangle}_W$$

$x_1$

Space-time picture:

$$\langle 0 | T \psi_1 \psi_2 | 0 \rangle = D(x_1 - x_2) = \text{---} x_1 \text{---} x_2 \text{---}$$

$$\cup$$

$$\langle 0 | \underbrace{a_i a_k^\dagger} | 0 \rangle$$

$$G(x_1, x_2, x_3, x_4) = \text{---} 1 \text{---} 2 \text{---} + \text{---} 1 \text{---} 2 \text{---} \text{---} 3 \text{---} 4 \text{---}$$

man dia

Space-time picture:

$$\langle 0 | T \psi_1 \psi_2 | 0 \rangle = D(x_1 - x_2) = \begin{array}{c} \text{---} \\ x_1 \quad x_2 \end{array}$$

U

$$\langle 0 | \underbrace{a_p a_k^\dagger} | 0 \rangle$$

$$G(x_1, x_2, x_3, x_4) = \begin{array}{c} 1 \text{---} 2 \\ 4 \text{---} 3 \end{array} + \begin{array}{c} 1 \quad 2 \\ \diagdown \quad \diagup \\ 4 \quad 3 \end{array}$$

Feynman diagrams

Space-time picture:

$$\langle 0 | T \psi_1 \psi_2 | 0 \rangle = D(x_1 - x_2) = \begin{array}{c} \text{---} \\ x_1 \quad x_2 \end{array}$$

U

$$\langle 0 | \underbrace{a_p a_k^\dagger} | 0 \rangle$$

$$G(x_1, x_2, x_3, x_4) = \begin{array}{c} 1 \text{---} 2 \\ 4 \text{---} 3 \end{array} + \begin{array}{c} 1 \quad 2 \\ \diagdown \quad \diagup \\ 4 \quad 3 \end{array}$$

Feynman diagrams

Feynman diagrams for interacting fields

$$\langle 0 | T \exp\left(-\frac{i\lambda}{4!} \int d^4x \varphi^4(x)\right) | 0 \rangle = \langle 0 | 1 - \frac{i\lambda}{4!} \int d^4x T \varphi^4 + \mathcal{O}(\lambda^2) | 0 \rangle = 1 - \frac{i\lambda}{4!} \int d^4x \langle 0 | T \varphi^4(x) | 0 \rangle + \dots$$



Feynman diagrams for interacting fields

Ex  $\langle 0 | T \exp\left(-\frac{i\lambda}{4!} \int d^4x \varphi^4(x)\right) | 0 \rangle = \langle 0 | 1 - \frac{i\lambda}{4!} \int d^4x \varphi^4 + O(\lambda^2) | 0 \rangle = 1 - \frac{i\lambda}{4!} \int d^4x \langle 0 | T \varphi_x \varphi_x \varphi_x \varphi_x | 0 \rangle$

## Feynman diagrams for interacting fields

$$\underline{\text{Ex}} \quad \langle 0 | T \exp\left(-\frac{i\lambda}{4!} \int d^4x \varphi^4(x)\right) | 0 \rangle = \langle 0 | 1 - \frac{i\lambda}{4!} \int d^4x T \varphi^4 + \mathcal{O}(\lambda^2) | 0 \rangle = 1 - \frac{i\lambda}{4!} \int d^4x \langle 0 | T \varphi_x \varphi_x \varphi_x \varphi_x | 0 \rangle +$$

1) Wick contractions:  $\underbrace{\varphi_x \varphi_x}_{\text{1}} \underbrace{\varphi_x \varphi_x}_{\text{1}} + \underbrace{\varphi_x \varphi_x}_{\text{1}} \underbrace{\varphi_x \varphi_x}_{\text{1}} + \underbrace{\varphi_x \varphi_x}_{\text{1}} \underbrace{\varphi_x \varphi_x}_{\text{1}} = 3 [D(0)]^2$

$$\mathcal{Z} = 1 - \frac{i\lambda}{8} \int d^4x [D(x)]^2 + O(\lambda^2)$$

$$\mathcal{Z} = 1 - \frac{i\lambda}{8} \mathcal{I} + \dots$$

# symmetry transformations of  $\mathcal{I}$

$G(x)$

$$\mathcal{Z} = 1 - \frac{i\lambda}{8} \int d^4x [D(0)]^2 + O(\lambda^2)$$

$$\mathcal{Z} = 1 - \frac{i\lambda}{8} \int d^4x + \dots$$

# symmetry transformations of  $\int d^4x$

$$G(x_1, x_2) = \langle 0 | T \phi(x_1) \phi(x_2) | 0 \rangle - \frac{i\lambda}{4!} \int d^4x$$

$$G(x_1, x_2) = \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle - \frac{i\lambda}{4!} \int d^4x \langle 0 | T \varphi_1 \varphi_2 \varphi_x \varphi_x \varphi_x | 0 \rangle$$

$$+ \frac{i\lambda}{4!} \int d^4x \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle \langle 0 | T \varphi_x \varphi_x \varphi_x \varphi_x | 0 \rangle + \mathcal{O}(\lambda^2)$$

↪ Wick contractions:  $\varphi_1 \varphi_2 \varphi_x \varphi_x \varphi_x \varphi_x$   
 3 possible contractions

$$g + \dots$$

$$\exp\left(i \int d^4x \mathcal{L}_int\right) = 1 + i \int d^4x \mathcal{L}_int$$

$$\frac{\langle \dots \rangle}{1 + i \int d^4x \langle \mathcal{L}_int \rangle} = \langle \dots \rangle \left( 1 - i \int d^4x \langle \mathcal{L}_int \rangle + \dots \right) = \langle \dots \rangle - i \int d^4x \langle \dots \rangle \langle \mathcal{L}_int \rangle$$

$$G(x_1, x_2) = \langle 0 | T \phi_1 \phi_2 | 0 \rangle - \frac{i\lambda}{4!} \int d^4x \langle 0 | T \phi_1 \phi_2 \phi_x \phi_x \phi_x \phi_x | 0 \rangle$$

$$+ \frac{i\lambda}{4!} \int d^4x \langle 0 | T \phi_1 \phi_2 | 0 \rangle \langle 0 | T \phi_x \phi_x \phi_x \phi_x | 0 \rangle + \mathcal{O}(\lambda^2)$$

1) Wick contraction

$$= \underbrace{\phi_1 \phi_2}_{\text{3 possible contractions}} \underbrace{\phi_x \phi_x \phi_x \phi_x}_{\text{3 possible contractions}} + \underbrace{\phi_1 \phi_2}_{\text{3 possible contractions}} \underbrace{\phi_x \phi_x \phi_x \phi_x}_{\text{3 possible contractions}} = 0 \text{?}$$

$$\underbrace{\phi_1 \phi_2}_{\text{3}} \underbrace{\phi_x \phi_x \phi_x \phi_x}_{\text{3}} = 4 \cdot 3 \cdot D(x_1 - x) D(x_2 - x) D(0)$$

$$G(x_1, x_2) = \langle 0 | T \psi_1 \psi_2 | 0 \rangle - \frac{i\lambda}{4!} \int d^4x \langle 0 | T \psi_1 \psi_2 \psi_x \psi_x \psi_x \psi_x | 0 \rangle + \frac{i\lambda}{4!} \int d^4x \langle 0 | T \psi_1 \psi_2 | 0 \rangle \langle 0 | T \psi_x \psi_x \psi_x \psi_x | 0 \rangle + \mathcal{O}(\lambda^2)$$

1) Wick contractions:  $-\underbrace{\psi_1 \psi_2 \psi_x \psi_x \psi_x \psi_x}_{3 \text{ possible contractions}} + \underbrace{\psi_1 \psi_2 \psi_x \psi_x \psi_x \psi_x}_{3 \text{ possible contractions}} = 0$

$$\underbrace{\psi_1 \psi_2 \psi_x \psi_x \psi_x \psi_x}_{4 \text{ } \underbrace{\psi_x \psi_x \psi_x \psi_x}_{3}} = 4 \cdot 3 D(x_1 - x) D(x_2 - x) D(0)$$

$$G(x_1, x_2) = D(x_1 - x_2) - \frac{i\lambda}{2} \int d^4x D(x_1 - x) D(x_2 - x) D(0) + \mathcal{O}(\lambda^2)$$

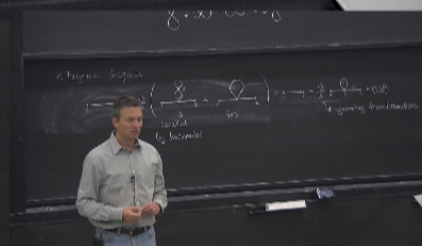
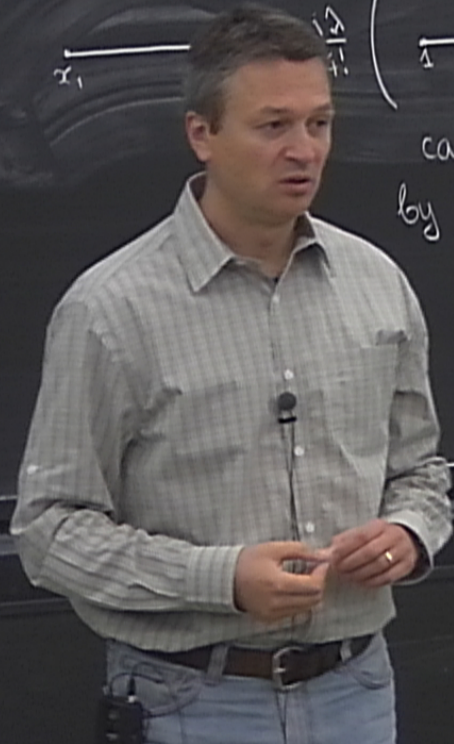


2) Feynman diagrams

$$\frac{i\lambda}{4!} \left( \text{diagram 1} + \text{diagram 2} \right) = \text{diagram 3} - \frac{i\lambda}{2} \text{diagram 4} + O(\lambda^2)$$

The first diagram is a self-energy loop on a propagator between points 1 and 2, with a factor of 3 below it. The second diagram is a tadpole loop on a propagator between points 1 and 2, with a factor of  $4 \times 3$  below it. The third diagram is a simple propagator between points 1 and 2. The fourth diagram is a tadpole loop on a propagator between points 1 and 2, with a factor of  $\frac{1}{2}$  below it.

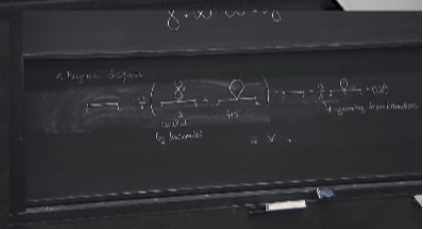
Annotations: "cancelled by denominator" points to the first two diagrams. "# symmetry transformations" points to the fourth diagram.



$$g + \text{loop} + \dots = g$$

2) Feynman diagrams

$$\begin{aligned}
 & \text{Diagram } x_1 \text{---} x_2 \quad - \frac{i\lambda}{4!} \left( \text{Diagram 1} + \text{Diagram 2} \right) = \text{Diagram 1} - \frac{i\lambda}{2} \text{Diagram 2} + O(\lambda^2) \\
 & \text{Diagram 1: } \begin{array}{c} \text{Loop} \\ \text{---} \\ 1 \quad 2 \end{array} \quad \text{Diagram 2: } \begin{array}{c} \text{Loop} \\ \text{---} \\ 1 \quad 2 \end{array} \\
 & \text{Diagram 1: } 3 \text{ cancelled by denominator} \quad \text{Diagram 2: } 4 \times 3 \\
 & \text{Diagram 2: } \# \text{ symmetry transformations}
 \end{aligned}$$



Feynman rules:

$$G(x_1, \dots, x_n)$$

- draw  $n$  external lines  $x_i$
- add any number of vertices  $x$
- connect them w. propagators


Feynman rules:

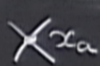
$G(x_1, \dots, x_n)$

- draw  $n$  external lines  $x_i$
- add any number of vertices  $x$
- connect them w. propagators  
in all possible ways\*
- \* no vacuum bubbles!

$$G(x_1, x_2) = D(x_1 - x_2) - \frac{i\lambda}{2} \int d^d x D(x_1 - x) D(x_2 - x) D(0) + O(\lambda^2)$$

res:

- Assign  $D(y_i - y_j)$  to 

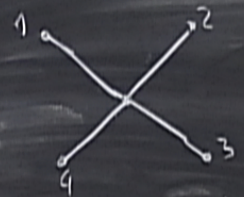
- Assign  $-i\lambda \int d^d x_a$  to 

$$G(x_1, \dots, x_n) = \sum_{\text{all diagrams}} \frac{\text{\# vertices} \times \text{\# propagators}}{\text{\# symmetries of the diagram}}$$

ten  
nes  $x_i$   
of vertices  $x$   
propagators  
ays \*  
es ?

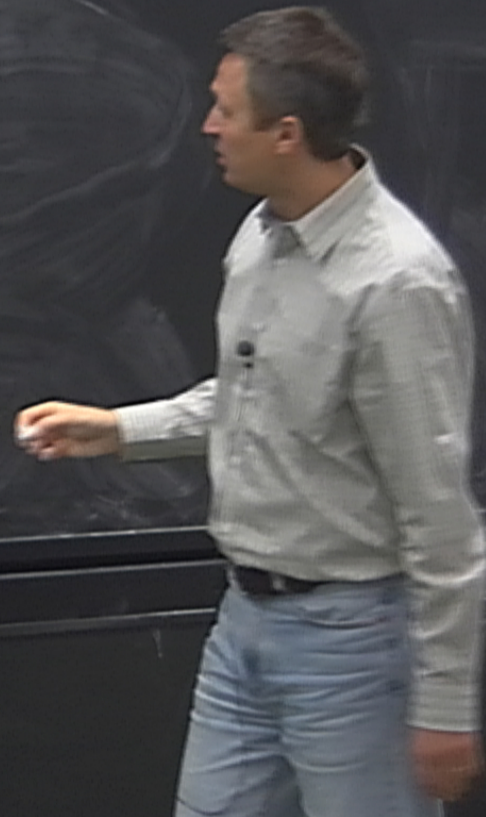
$$g + \infty + \infty = g$$

Ex  $G(x_1, x_2, x_3, x_4)$  at  $O(2)$



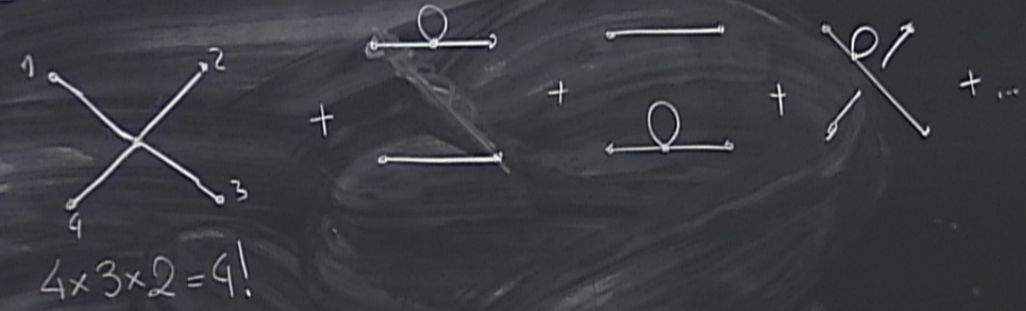
+

$$4 \times 3 \times 2 = 4!$$

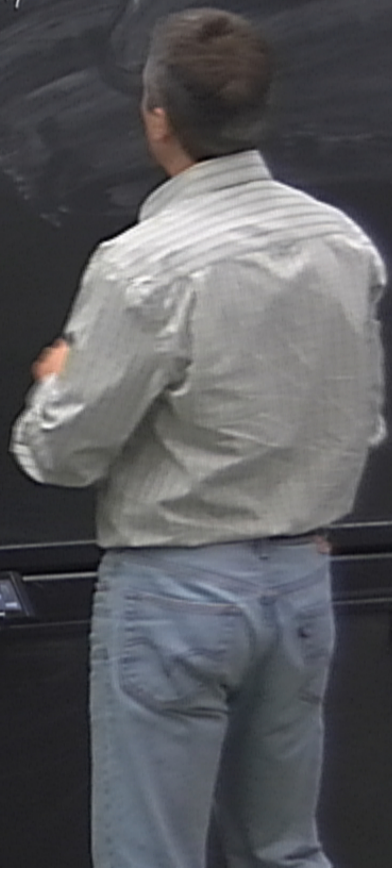


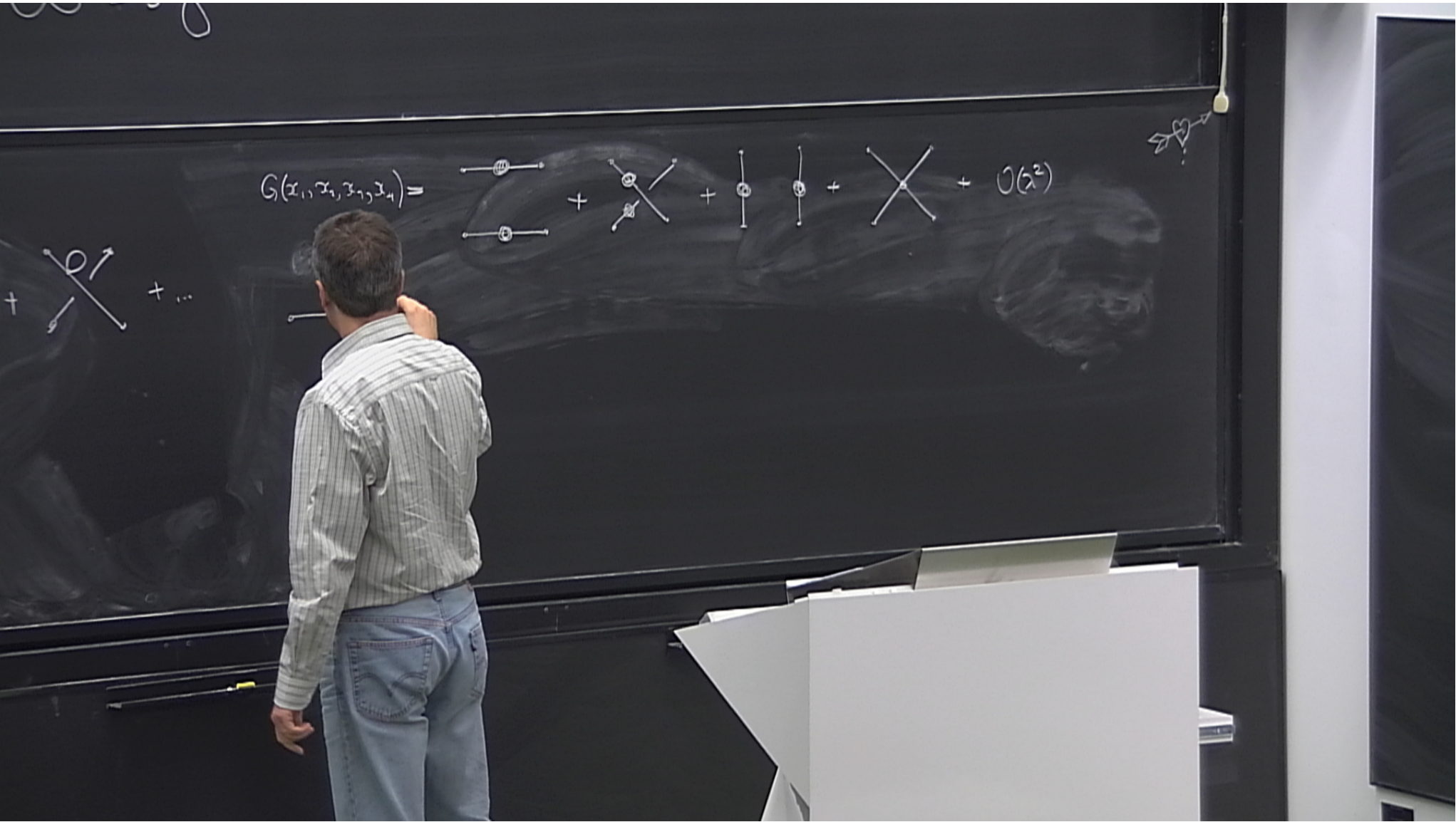
$$g + \infty + \infty = g$$

Ex  $G(x_1, x_2, x_3, x_4)$  at  $\mathcal{O}(2)$

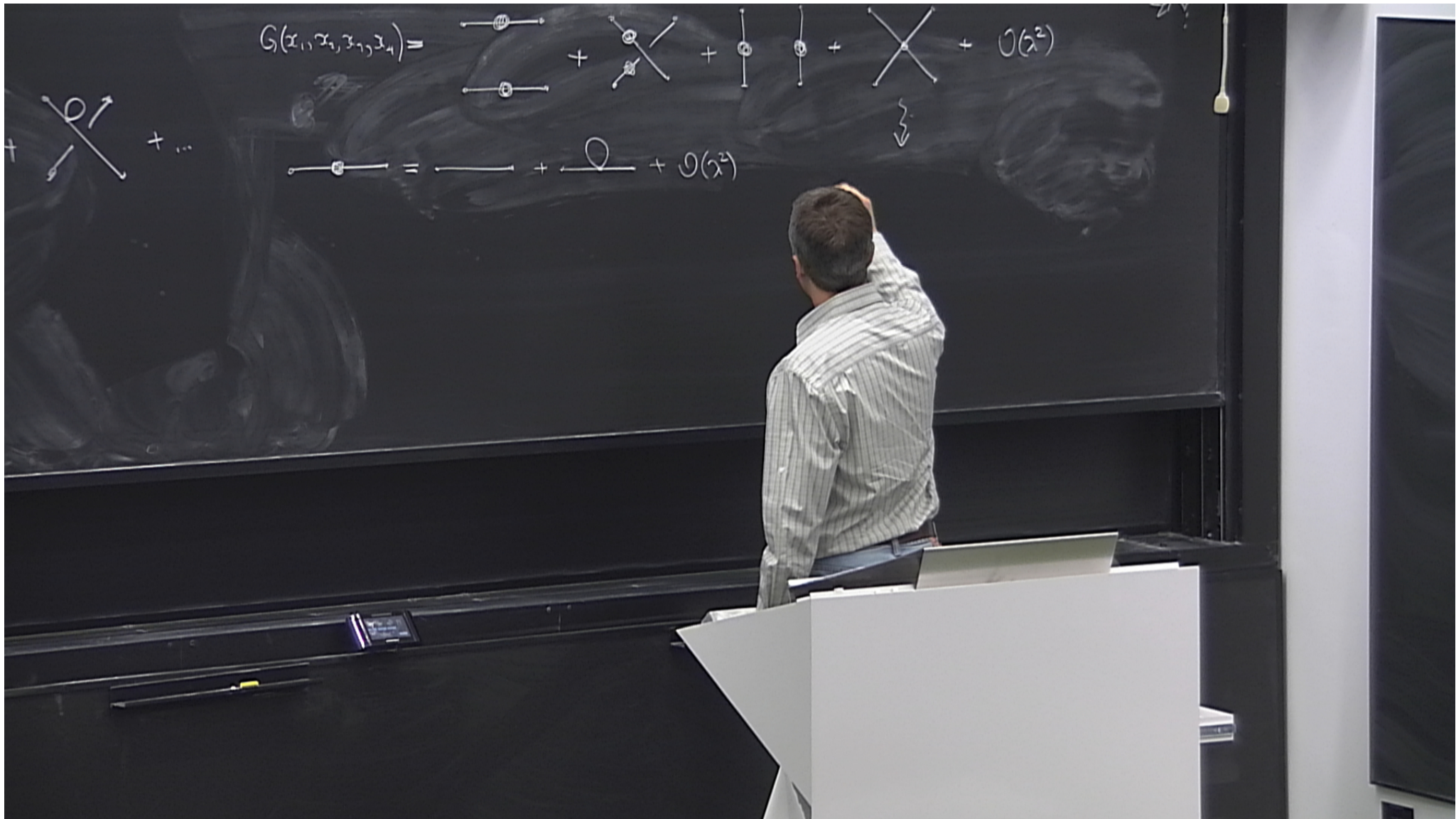


$$G(x_1, x_2, x_3, x_4) =$$









$$G(x_1, x_2, x_3, x_4) =$$



$$+ \mathcal{O}(\lambda^2)$$



$$\text{[Diagram with loop on internal line]} = \text{[Diagram with loop on vertex]} + \mathcal{O}(\lambda^2)$$

$$-i\lambda \int d^4x D(x_1-x) D(x_2-x) D(x_3-x) D(x_4-x)$$

