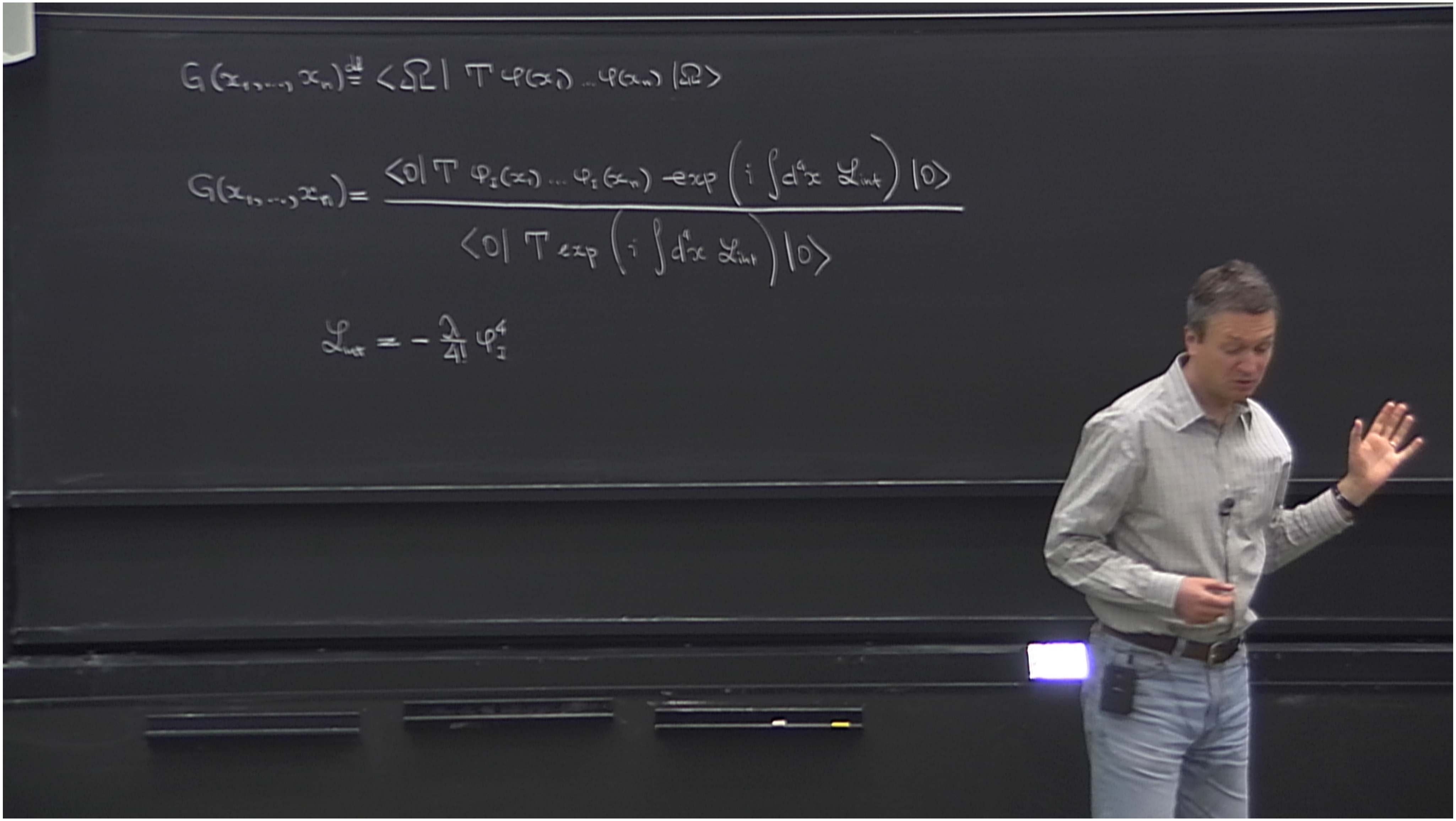
Title: Quantum Field Theory I - Lecture 11

Date: Oct 18, 2011  09:00 AM

URL: http://pirsa.org/11100021

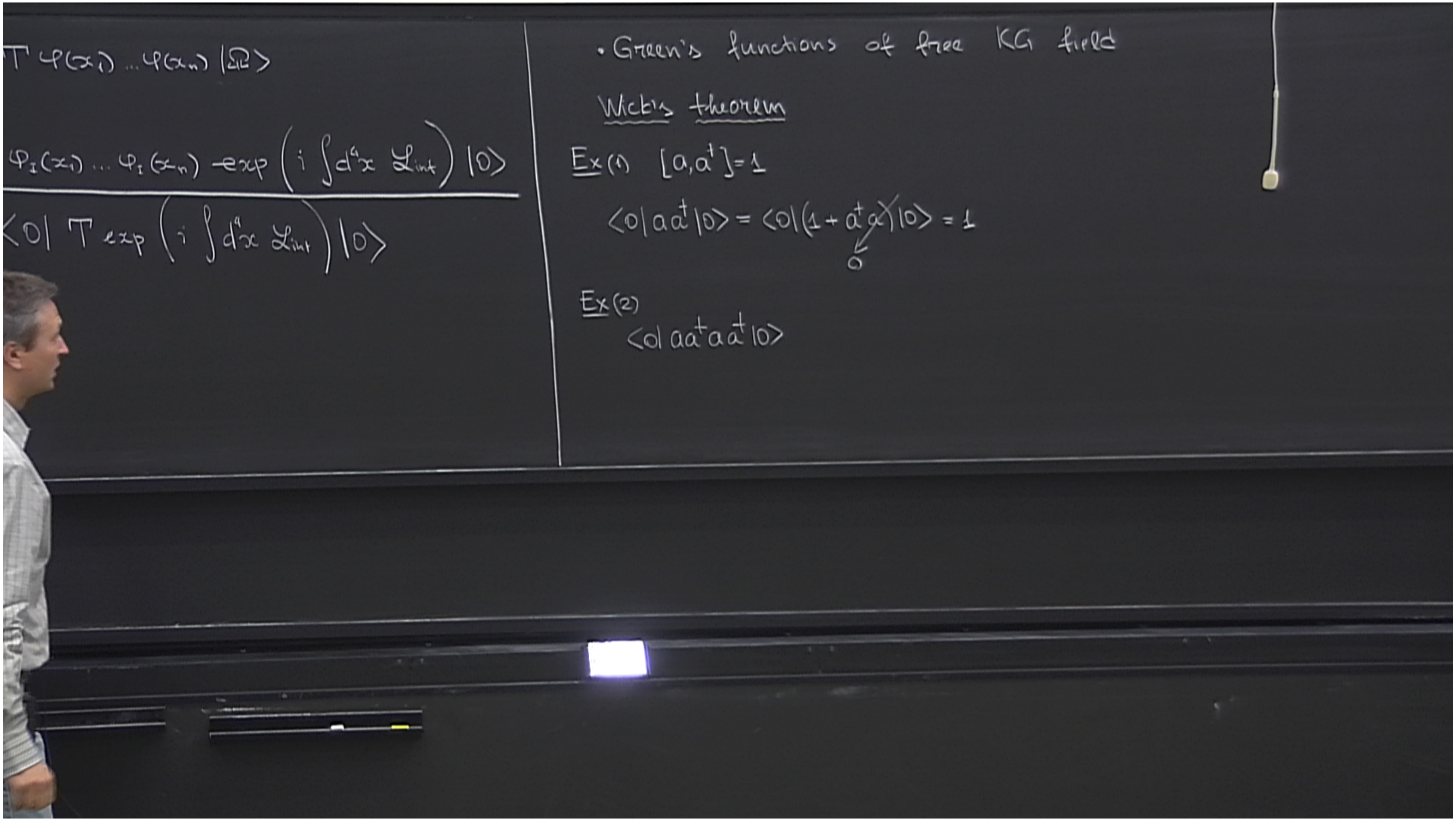
Abstract:


$G(x_1, \ldots, x_n) \stackrel{\text{def}}{=} \langle \Omega | T \varphi(x_1) \ldots \varphi(x_n) | \Omega \rangle$
$G(x_1, \ldots, x_n) = \frac{\langle 0 | T \varphi_I(x_1) \ldots \varphi_I(x_n) \exp\left(i \int d^4x \, \mathcal{L}_{int}\right) | 0 \rangle}{\langle 0 | T \exp\left(i \int d^4x \, \mathcal{L}_{int}\right) | 0 \rangle}$
$\mathcal{L}_{int} = -\frac{\lambda}{4!} \varphi_I^4$

$$G(x_1,\dots,x_n) \stackrel{\text{def}}{=} \langle\Omega|\, T\,\varphi(x_1)\dots\varphi(x_n)\,|\Omega\rangle$$

$$G(x_1,\dots,x_n) = \frac{\langle 0|\, T\,\varphi_I(x_1)\dots\varphi_I(x_n)\,\exp\left(i\int d^4x\,\mathcal{L}_{int}\right)|0\rangle}{\langle 0|\, T\exp\left(i\int d^4x\,\mathcal{L}_{int}\right)|0\rangle}$$

$$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi_I^4$$


$T\,\varphi(x_1)\ldots\varphi(x_n)\,|\Omega\rangle$
$\varphi_I(x_1)\ldots\varphi_I(x_n)\,\exp\left(i\int d^4x\,\mathcal{L}_{int}\right)|0\rangle$
$\langle 0|\,T\exp\left(i\int d^4x\,\mathcal{L}_{int}\right)|0\rangle$
• Green's functions of free KG field
Wick's theorem
Ex (1) $[a,a^\dagger]=1$
$\langle 0|aa^\dagger|0\rangle = \langle 0|(1+a^\dagger a)|0\rangle = 1$
0
Ex (2)
$\langle 0|aa^\dagger a a^\dagger|0\rangle$

$T\,\varphi(x_1)\dots\varphi(x_n)\,|\Omega\rangle$

$$\frac{\varphi_I(x_1)\dots\varphi_I(x_n)\,\exp\left(i\int d^4x\,\mathcal{L}_{int}\right)|0\rangle}{\langle 0|\,T\exp\left(i\int d^4x\,\mathcal{L}_{int}\right)|0\rangle}$$

$\varphi_I^4$

• Green's functions of free KG field

Wick's theorem

Ex (1) $[a,a^\dagger]=1$

$$\langle 0|aa^\dagger|0\rangle = \langle 0|(1+\underbrace{\cancel{a^\dagger a}}_{0})|0\rangle = 1$$

Ex (2)

$$\langle 0|aa^\dagger aa^\dagger|0\rangle = \langle 0|aa^\dagger|0\rangle = 1$$

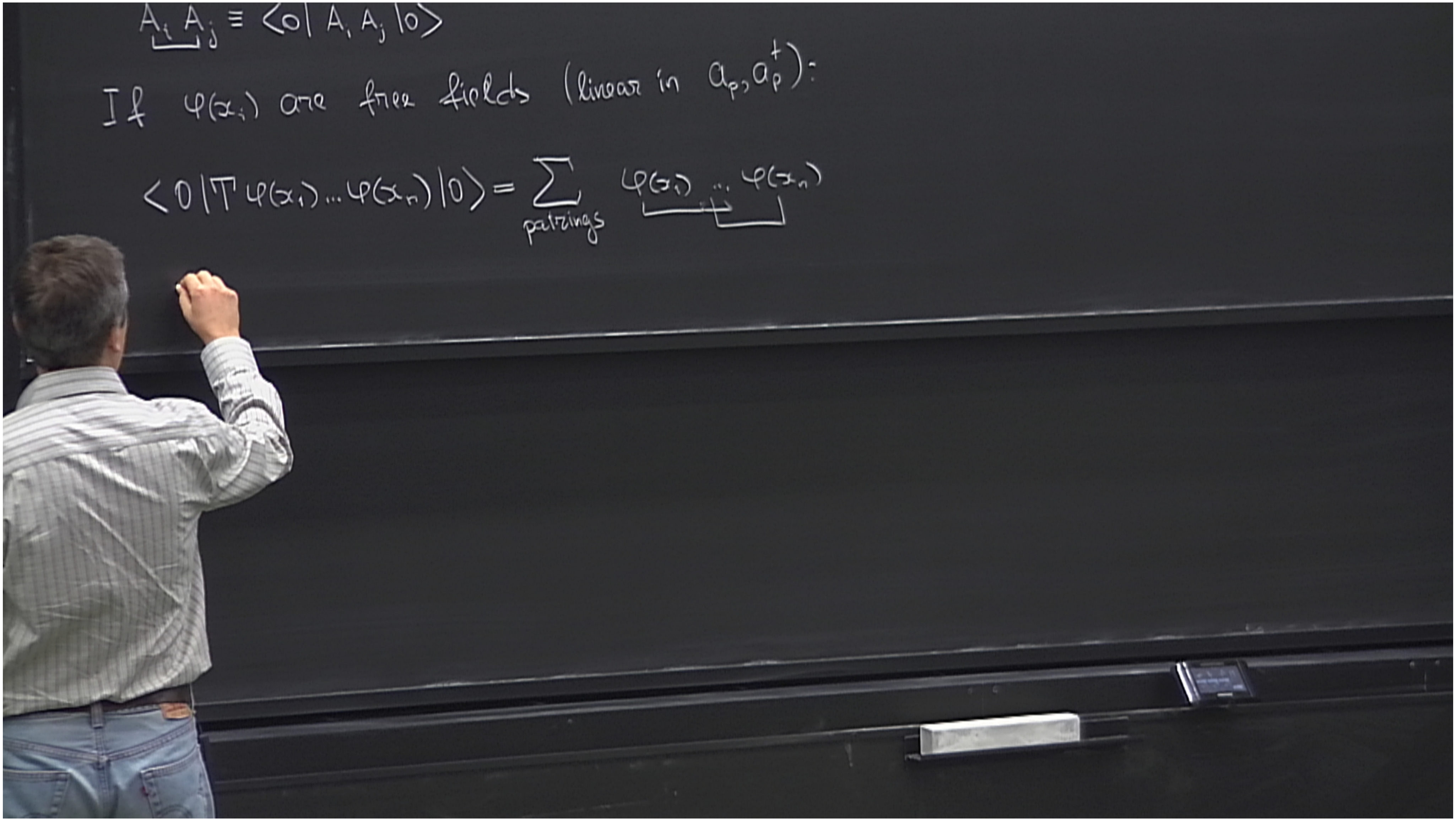
$$A_i = c_i a + b_i a^\dagger \; ; \; c_i, b_i \in \mathbb{C}$$

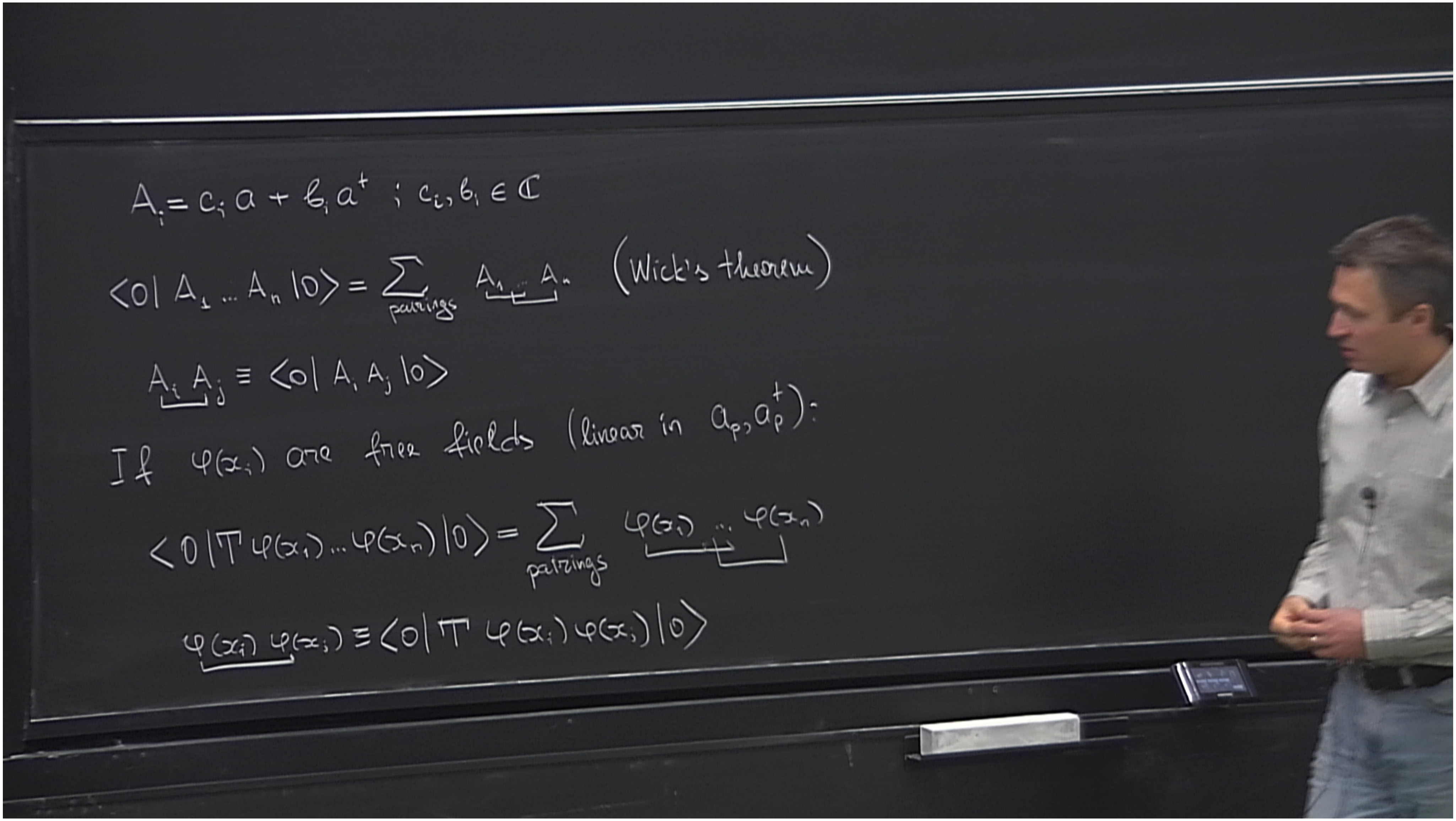
$$\langle 0 | A_1 \dots A_n | 0 \rangle$$

$$A_i = c_i\, a + b_i\, a^{+} \; ; \; c_i, b_i \in \mathbb{C}$$

$$\langle 0 | A_1 \dots A_n | 0 \rangle = \sum_{\text{pairings}} A_1 \dots A_n \qquad \text{(Wick's theorem)}$$

$$A_i A_j \equiv \langle 0 | A_i A_j | 0 \rangle$$


$\underbrace{A_i A_j} \equiv \langle 0 | A_i A_j | 0 \rangle$
If $\varphi(x_i)$ are free fields (linear in $a_p, a_p^\dagger$):
$\langle 0 | T \varphi(x_1) \ldots \varphi(x_n) | 0 \rangle = \sum_{\text{pairings}} \varphi(x_1) \ldots \varphi(x_n)$


$A_i = c_i a + b_i a^{+} \;;\; c_i, b_i \in \mathbb{C}$
$\langle 0| A_1 \dots A_n |0\rangle = \sum_{\text{pairings}} A_1 \dots A_n$ (Wick's theorem)
$A_i A_j \equiv \langle 0| A_i A_j |0\rangle$
If $\varphi(x_i)$ are free fields (linear in $a_p, a_p^{+}$):
$\langle 0| T \varphi(x_1) \dots \varphi(x_n) |0\rangle = \sum_{\text{pairings}} \varphi(x_1) \dots \varphi(x_n)$
$\varphi(x_i)\varphi(x_j) \equiv \langle 0| T \varphi(x_i)\varphi(x_j) |0\rangle$

Ex (2) $G(x_1, x_2, x_3, x_4) = \langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \varphi_1 \varphi_2 \varphi_3 \varphi_4 + \varphi_1 \varphi_2 \varphi_3 \varphi_4 + \varphi_1 \varphi_2 \varphi_3 \varphi_4$

$\varphi_i \equiv \varphi(x_i)$ $\qquad = D(x_1 - x_2) D(x_3 - x_4) + D(x_1 - x_3) D(x_2 - x_4) + D(x_1 - x_4) D(x_2 - x_3)$

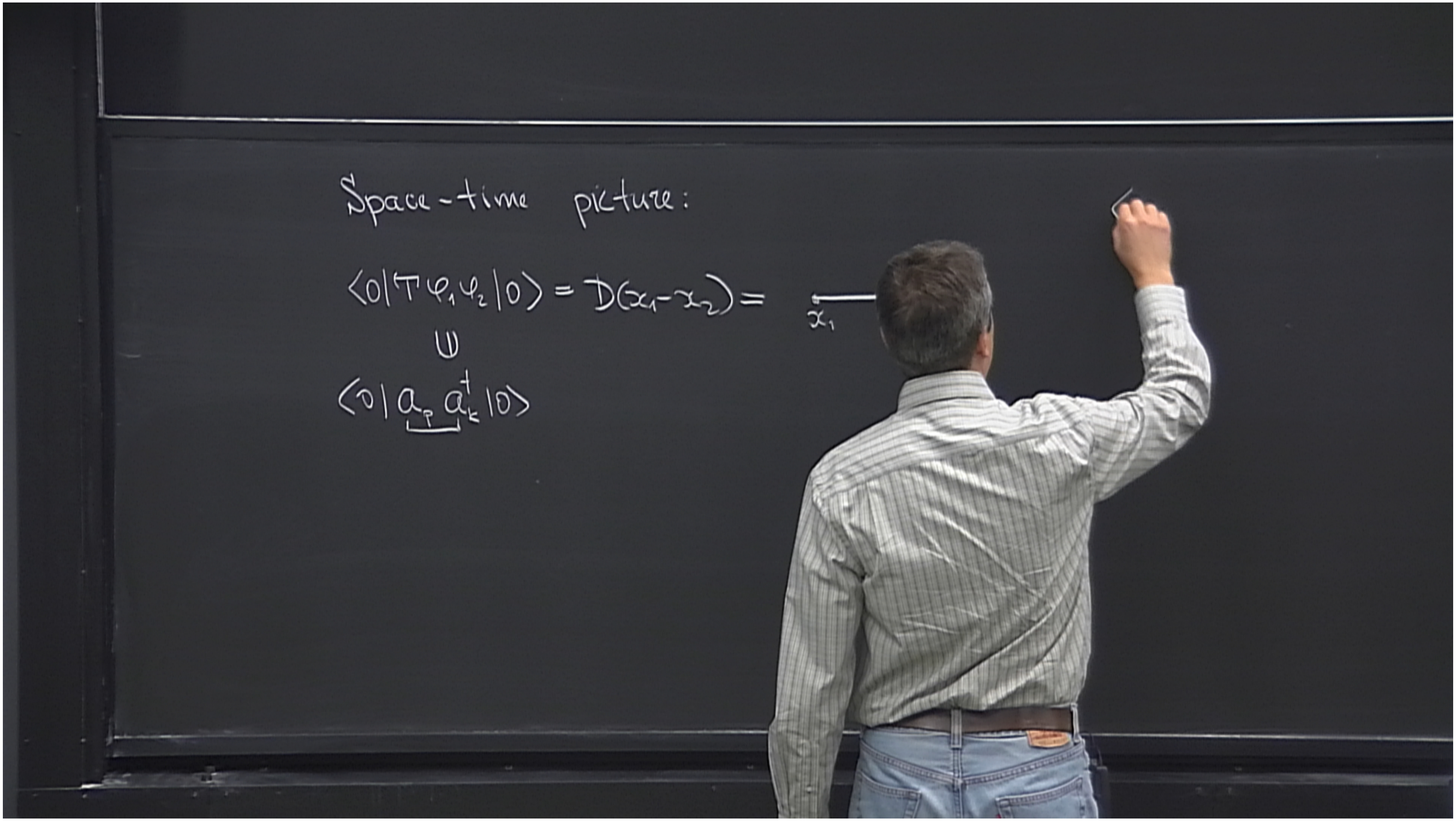
Feynman propagator

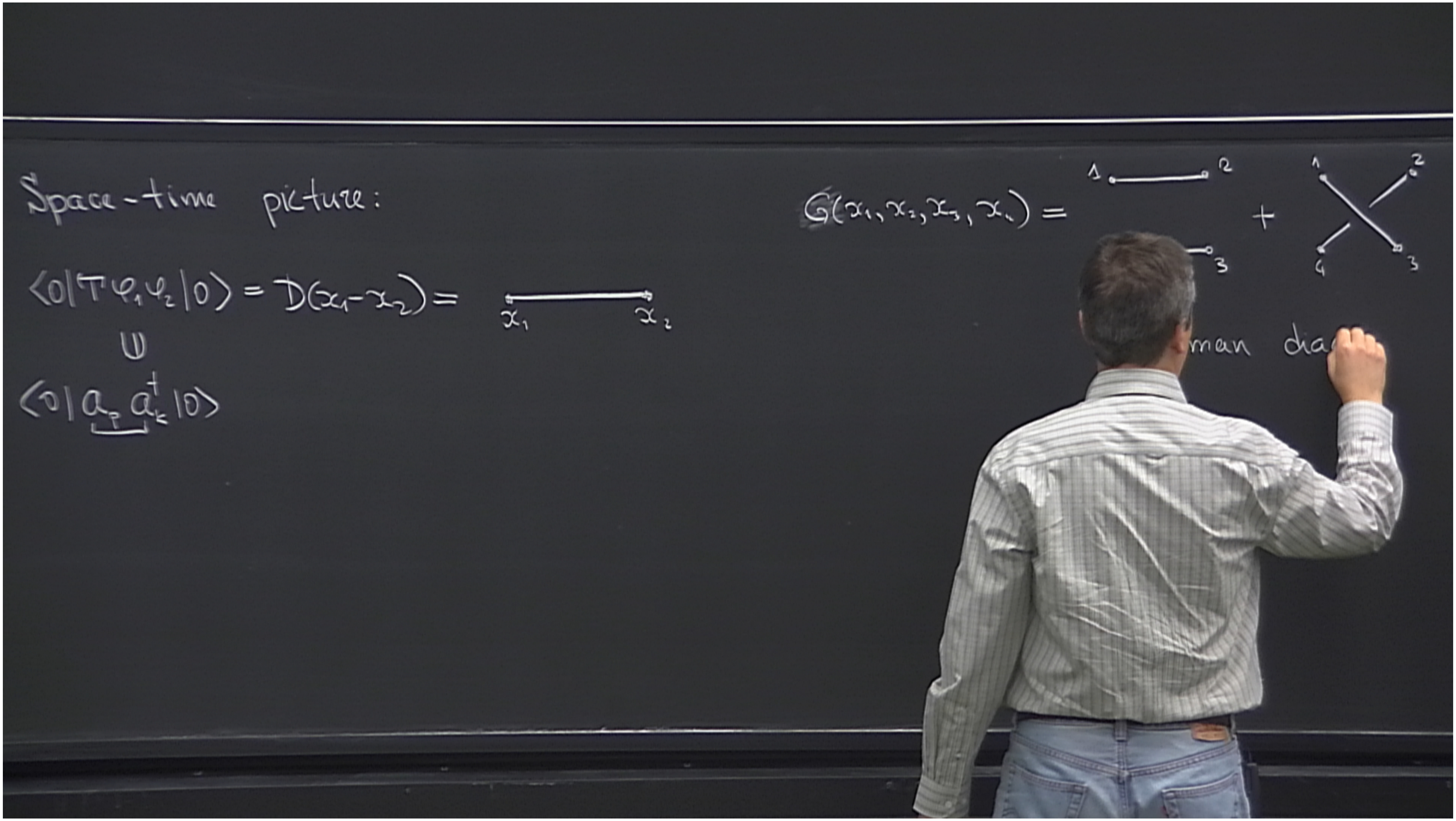
Ex. (1) $G(x_1, x_2) = D(x_1 - x_2)$

Ex. (2) $G(x_1, x_2, x_3, x_4) = \langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \varphi_1 \varphi_2 \varphi_3 \varphi_4 + \varphi_1 \varphi_2 \varphi_3 \varphi_4 + \varphi_1 \varphi_2 \varphi_3 \varphi_4$

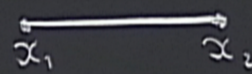
$\varphi_i \equiv \varphi(x_i)$ $\qquad = D(x_1 - x_2) D(x_3 - x_4) + D(x_1 - x_3) D(x_2 - x_4) + D(x_1 - x_4) D(x_2 - x_3)$

Feynman propagator


Space-time picture:
⟨0|T φ1 φ2|0⟩ = D(x1 − x2) =
x1
⟨0| ap ak† |0⟩

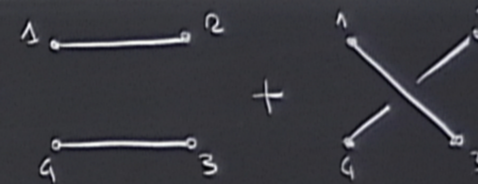

Space-time picture:
$\langle 0|T\varphi_1\varphi_2|0\rangle = D(x_1-x_2) =$
$x_1$
$x_2$
$\langle 0|a_p a_k^\dagger|0\rangle$
$G(x_1,x_2,x_3,x_4) =$
1
2
3
+
1
2
4
3
man dia

Space-time picture:

$\langle 0|T\varphi_1\varphi_2|0\rangle = D(x_1-x_2) =$ 


$\Downarrow$

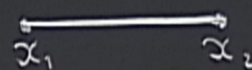
$\langle 0| a_p a_k^\dagger |0\rangle$

$G(x_1, x_2, x_3, x_4) =$ 


Feynman diagrams

Space-time picture:

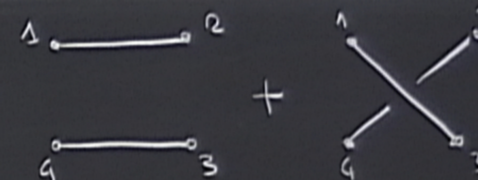
$$\langle 0|T\varphi_1\varphi_2|0\rangle = D(x_1-x_2) =$$



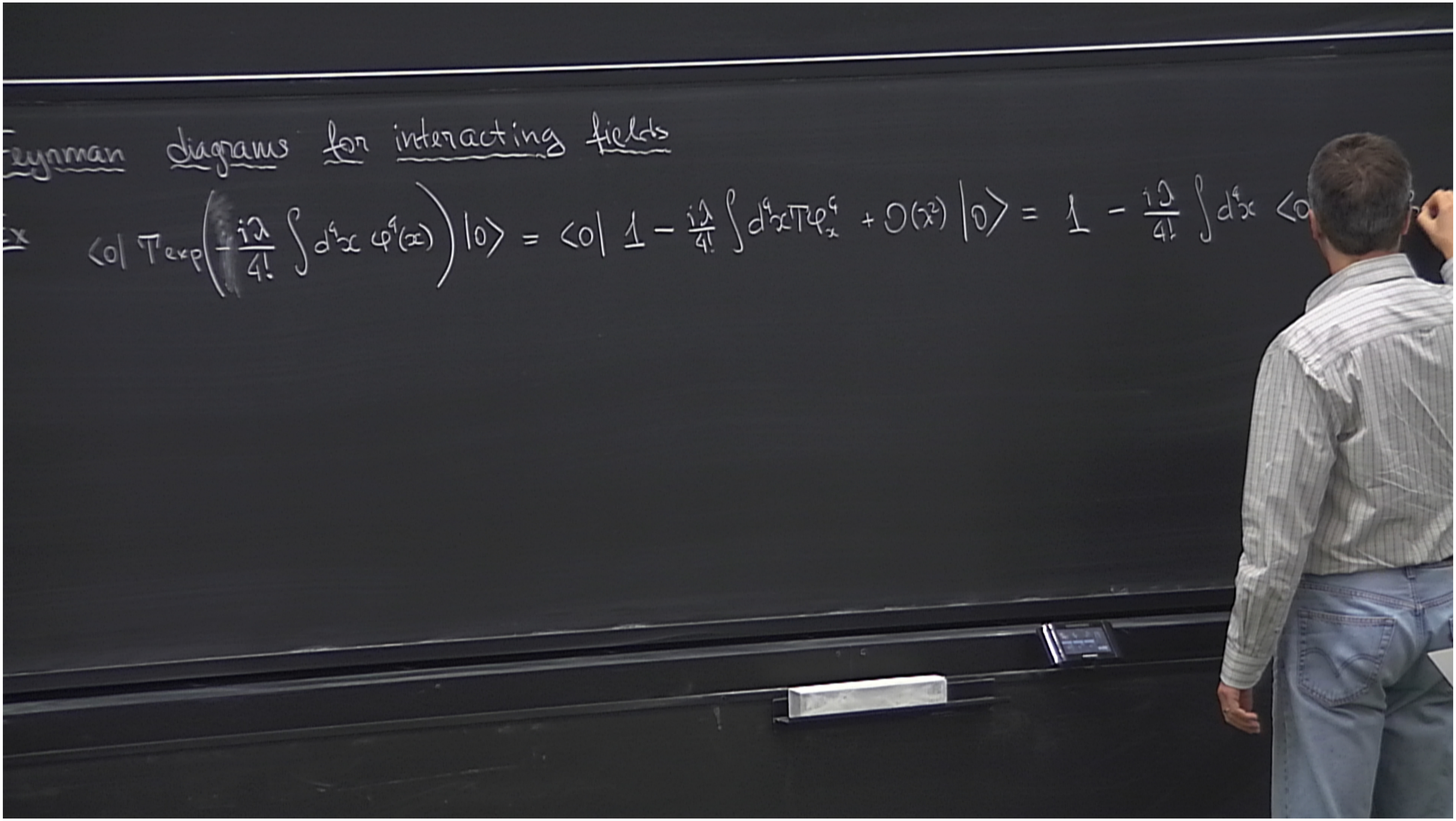

$$\cup$$

$$\langle 0|a_p a_k^\dagger|0\rangle$$

$$G(x_1,x_2,x_3,x_4) =$$




Feynman diagrams



eynman diagrams for interacting fields
Ex $\langle 0|\, T\exp\left(-\frac{i\lambda}{4!}\int d^4x\, \varphi^4(x)\right)|0\rangle = \langle 0|\, 1 - \frac{i\lambda}{4!}\int d^4x\, T\varphi_x^4 + O(\lambda^2)\,|0\rangle = 1 - \frac{i\lambda}{4!}\int d^4x\, \langle 0$

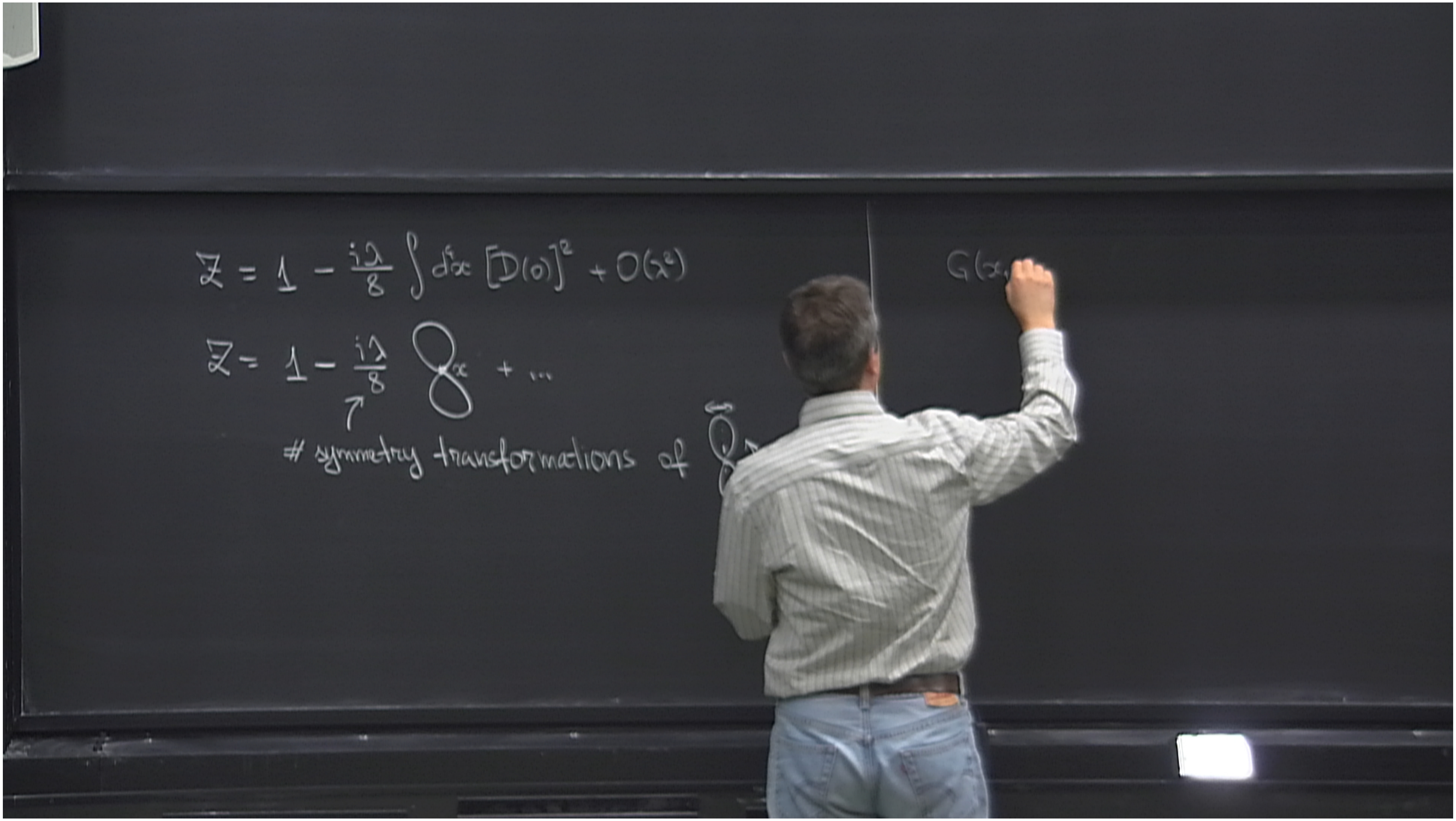
Feynman diagrams for interacting fields

Ex $\langle 0|\, T\exp\left(-\frac{i\lambda}{4!}\int d^4x\, \varphi^4(x)\right)|0\rangle = \langle 0|\, 1 - \frac{i\lambda}{4!}\int d^4x\, T\varphi_x^4 + O(\lambda^2)\, |0\rangle = 1 - \frac{i\lambda}{4!}\int d^4x\, \langle 0|\, T\,\varphi_x\varphi_x\varphi_x\varphi_x\, |0\rangle$

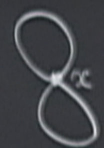
## Feynman diagrams for interacting fields

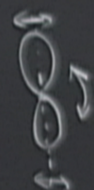
Ex
$$\langle 0|\, T \exp\left(-\frac{i\lambda}{4!}\int d^4x\, \varphi^4(x)\right)|0\rangle = \langle 0|\, 1 - \frac{i\lambda}{4!}\int d^4x\, T\varphi_x^4 + O(\lambda^2)\,|0\rangle = 1 - \frac{i\lambda}{4!}\int d^4x\, \langle 0|\, T\, \varphi_x\varphi_x\varphi_x\varphi_x\,|0\rangle + \dots$$

1) Wick contractions: $\varphi_x\varphi_x\varphi_x\varphi_x + \varphi_x\varphi_x\varphi_x\varphi_x + \varphi_x\varphi_x\varphi_x\varphi_x = 3\left[D(0)\right]^2$


$Z = 1 - \frac{i\lambda}{8} \int d^4x \, [D(0)]^2 + O(\lambda^2)$
$Z = 1 - \frac{i\lambda}{8}$ x $+ \dots$
# symmetry transformations of
$G(x,$

$$Z = 1 - \frac{i\lambda}{8} \int d^4x \, [D(0)]^2 + O(\lambda^2)$$

$$Z = 1 - \frac{i\lambda}{8}$$  $+ \dots$

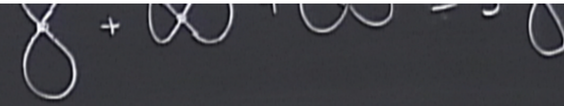
\# symmetry transformations of 

$$G(x_1, x_2) = \langle 0 | T \varphi_1 \varphi_2 | 0 \rangle - \frac{i\lambda}{4!} \int d^4x$$

$$G(x_1,x_2) = \langle 0|T\,\varphi_1\varphi_2|0\rangle - \frac{i\lambda}{4!}\int d^4x\, \langle 0|T\,\varphi_1\varphi_2\,\varphi_x\varphi_x\varphi_x\varphi_x|0\rangle$$

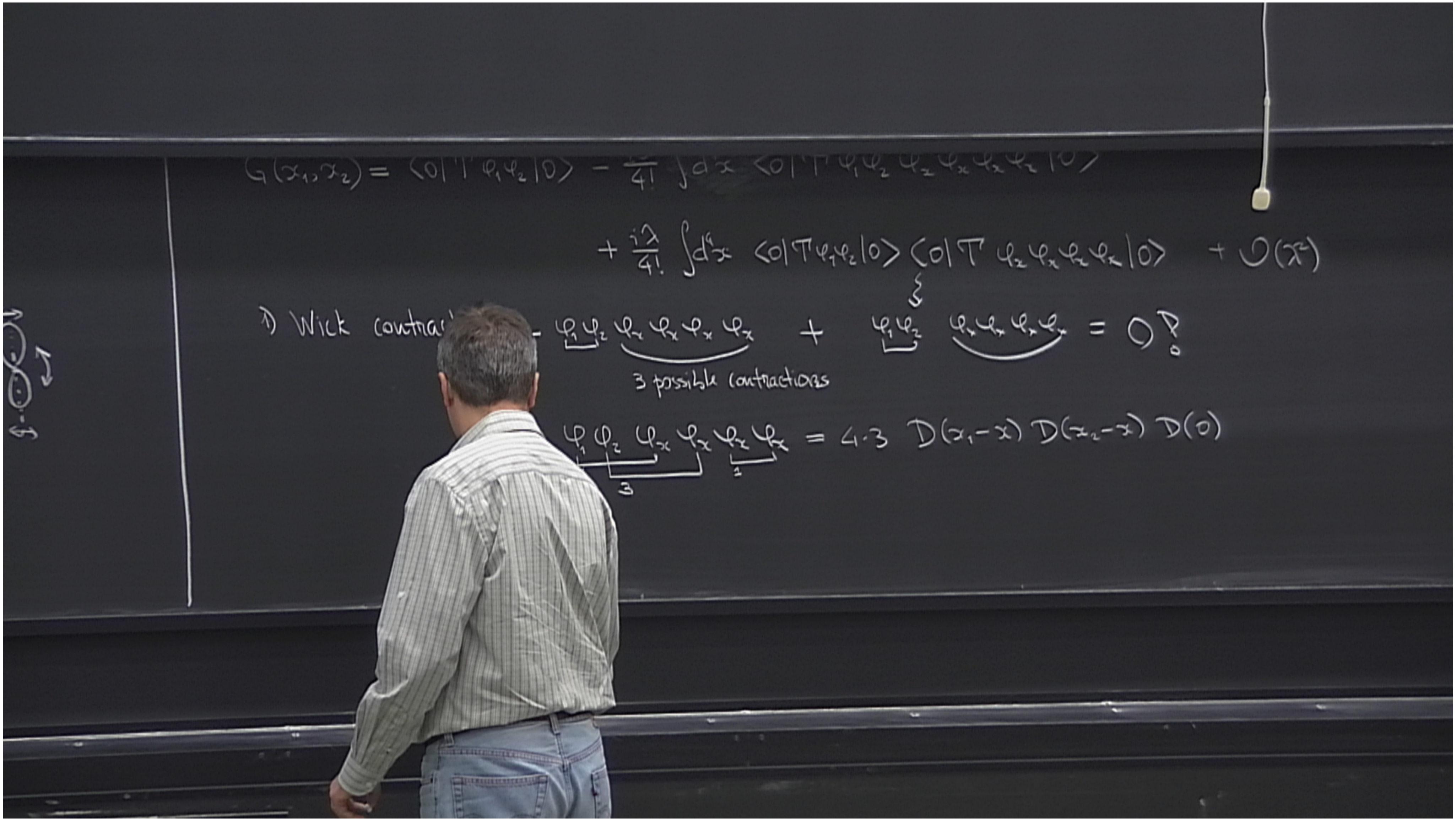
$$+ \frac{i\lambda}{4!}\int d^4x\, \langle 0|T\varphi_1\varphi_2|0\rangle\langle 0|T\,\varphi_x\varphi_x\varphi_x\varphi_x|0\rangle + \mathcal{O}(\lambda^2)$$

1) Wick contractions: $\varphi_1\varphi_2\,\underbrace{\varphi_x\varphi_x\varphi_x\varphi_x}_{\text{3 possible contractions}}$



$$\exp\left(i\int d^4x\, \mathcal{L}_{int}\right) = 1 + i\int d^4x\, \mathcal{L}_{int}$$

$$\frac{\langle \ldots \rangle}{1 + i\int d^4x\, \langle 0|\mathcal{L}_{int}|0\rangle} = \langle \ldots \rangle \left(1 - i\int d^4x \langle 0|\mathcal{L}_{int}|0\rangle + \ldots\right) = \langle \ldots \rangle - i\int d^4x\, \langle \ldots \rangle \langle 0|\mathcal{L}_{int}|0\rangle$$


$G(x_1,x_2) = \langle 0|T\varphi_1\varphi_2|0\rangle - \frac{i\lambda}{4!}\int d^4x\, \langle 0|T\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x|0\rangle$
$+ \frac{i\lambda}{4!}\int d^4x\, \langle 0|T\varphi_1\varphi_2|0\rangle\langle 0|T\varphi_x\varphi_x\varphi_x\varphi_x|0\rangle + \mathcal{O}(\lambda^2)$
1) Wick contrac
$\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x + \varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x = 0?$
3 possible contractions
$\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x = 4\cdot 3\, D(x_1-x)\,D(x_2-x)\,D(0)$
3
1

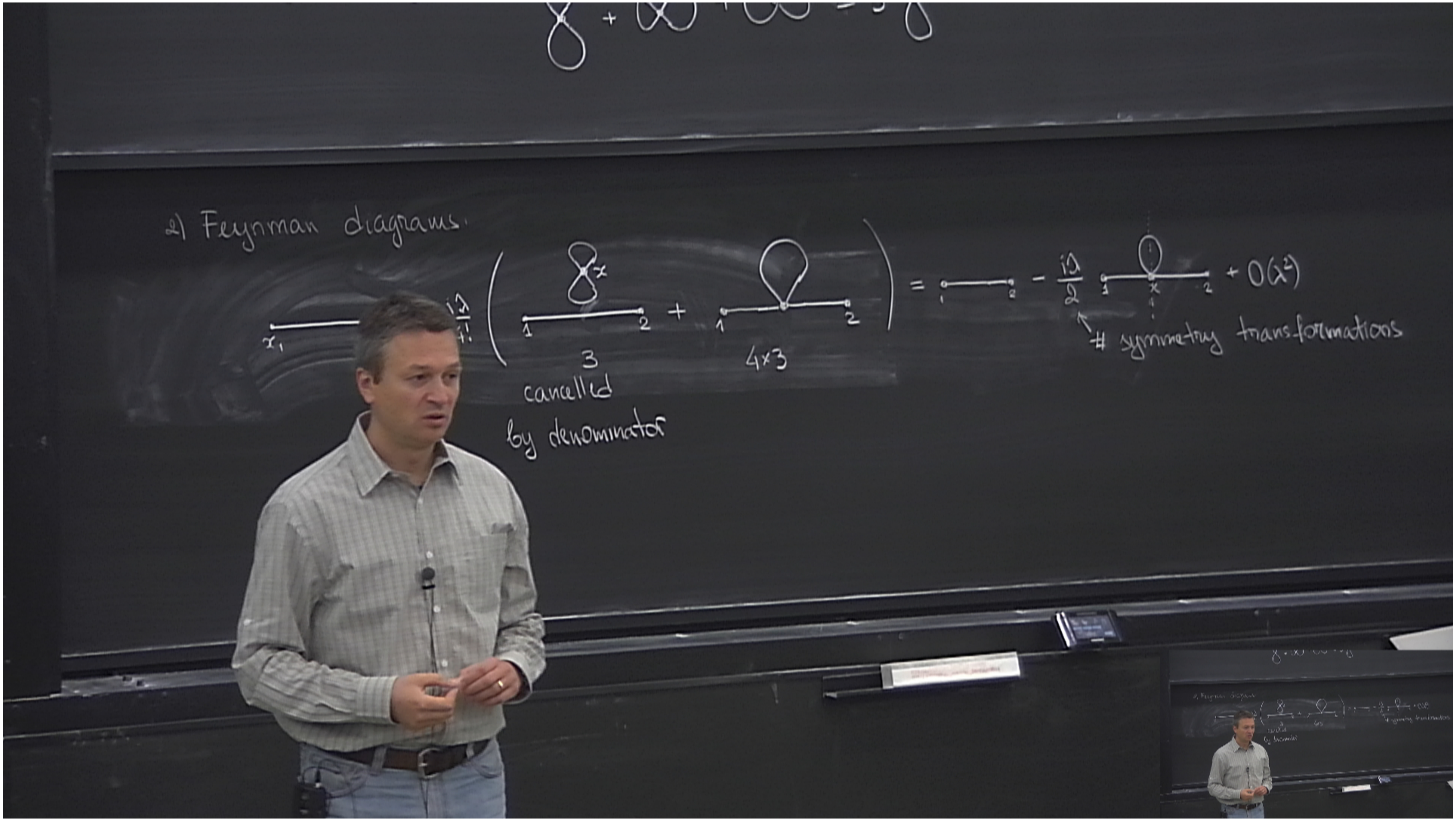
$$G(x_1,x_2) = \langle 0|T\varphi_1\varphi_2|0\rangle - \frac{i\lambda}{4!}\int d^4x\, \langle 0|T\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x|0\rangle$$

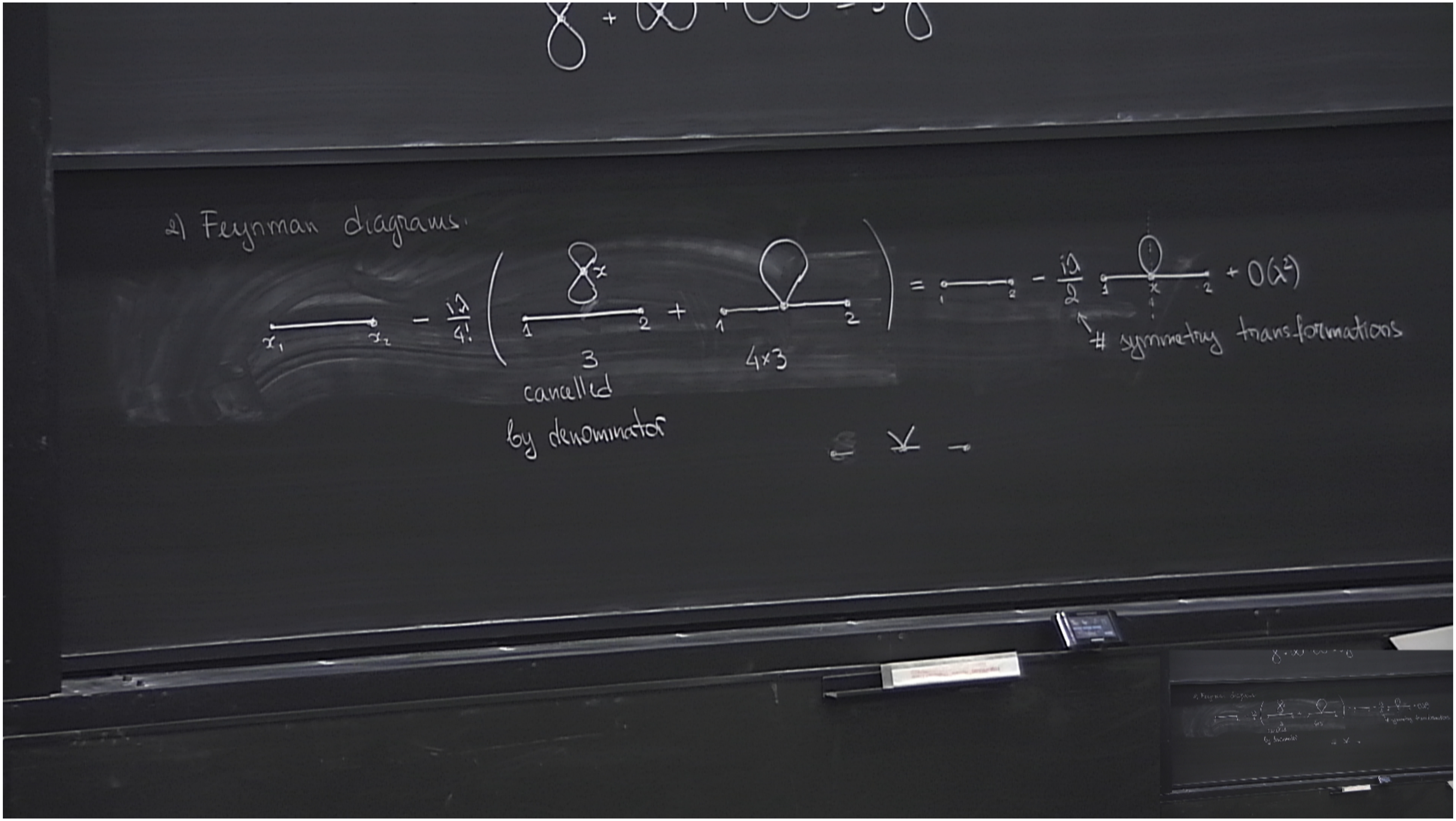
$$+ \frac{i\lambda}{4!}\int d^4x\, \langle 0|T\varphi_1\varphi_2|0\rangle\langle 0|T\varphi_x\varphi_x\varphi_x\varphi_x|0\rangle + \mathcal{O}(\lambda^2)$$

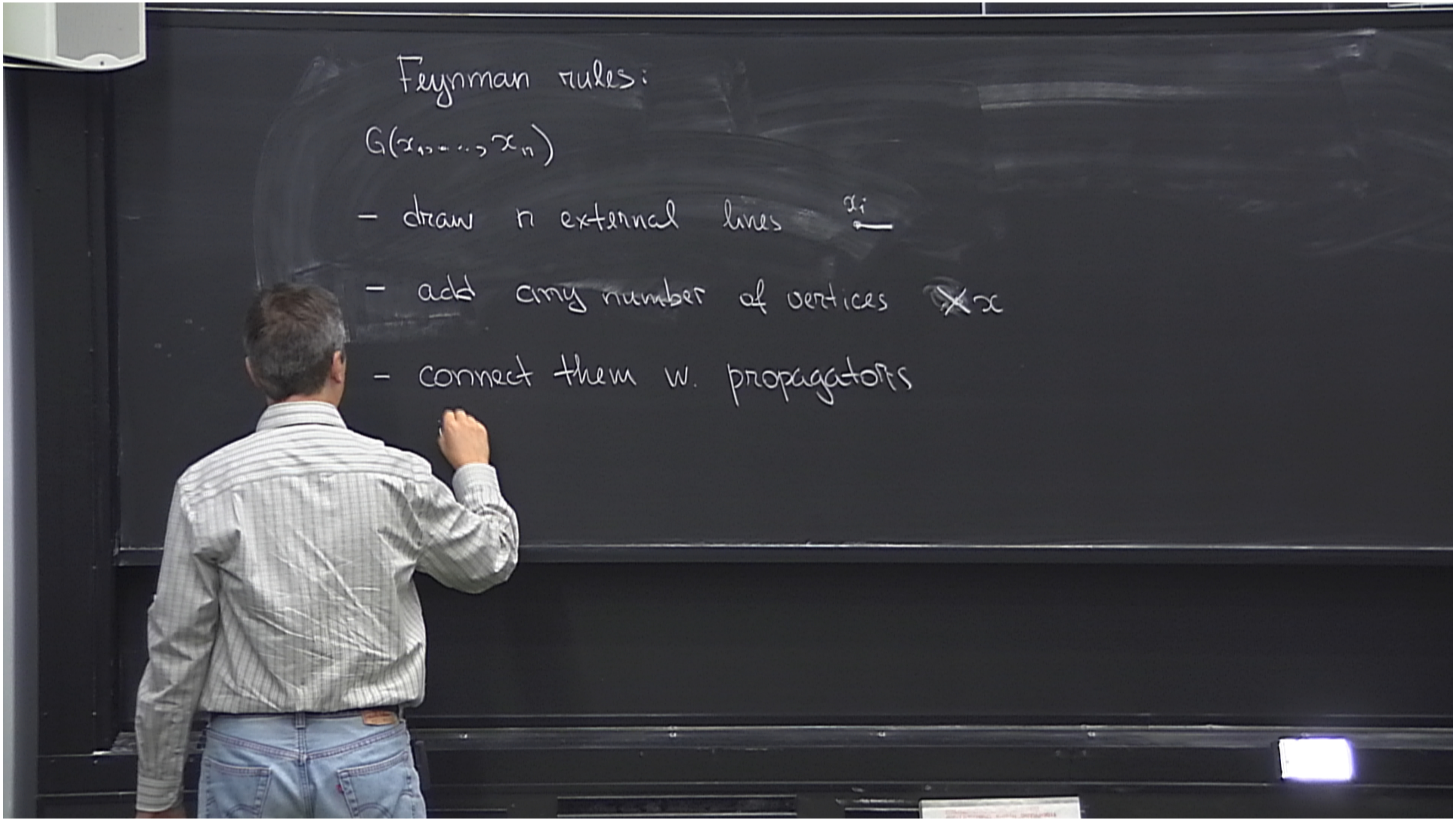
1) Wick contractions: $-\,\varphi_1\varphi_2\,\underbrace{\varphi_x\varphi_x\varphi_x\varphi_x}_{\text{3 possible contractions}} + \varphi_1\varphi_2\,\varphi_x\varphi_x\varphi_x\varphi_x = 0\,?$

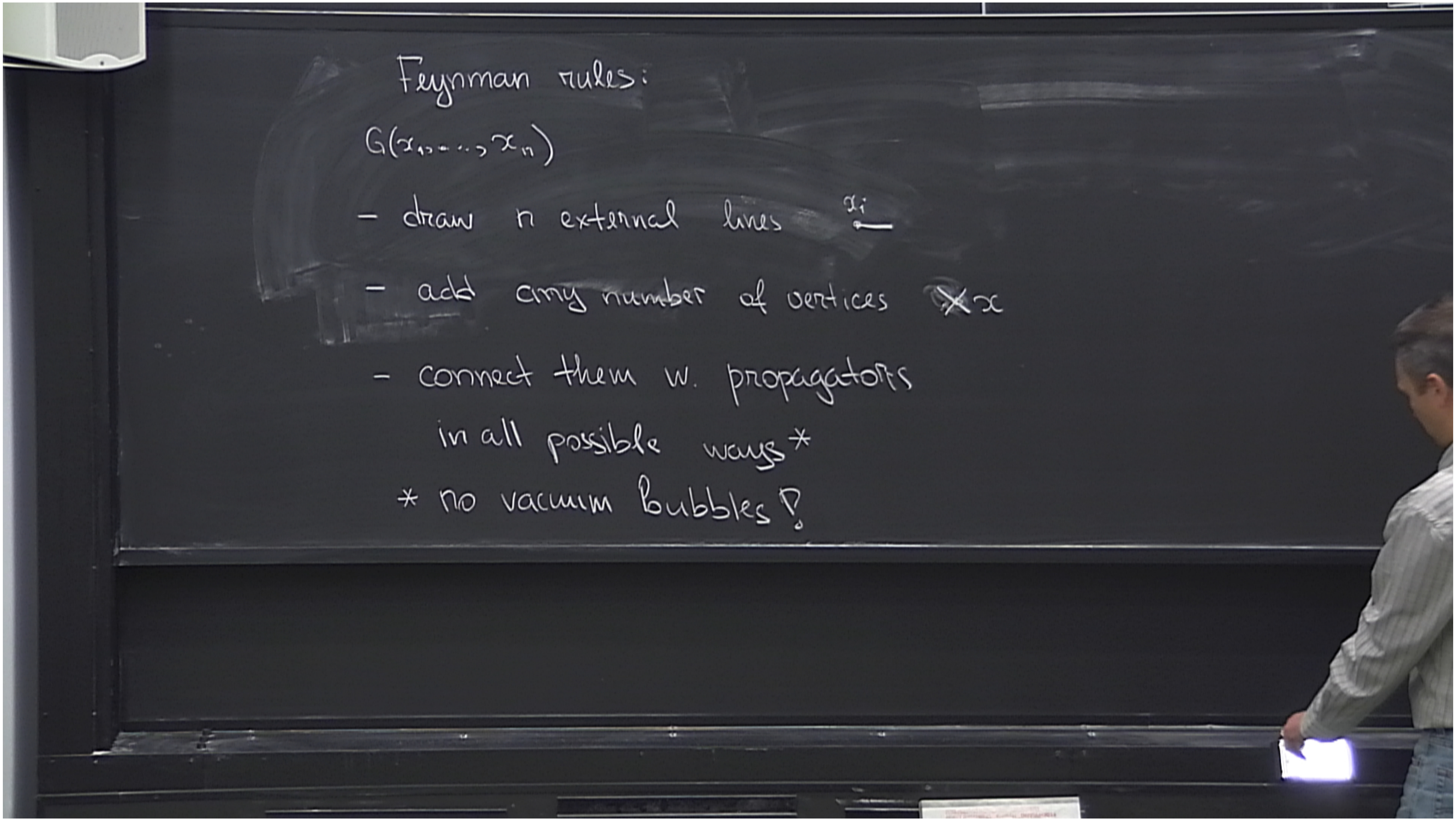
$$\varphi_1\varphi_2\varphi_x\varphi_x\varphi_x\varphi_x = 4\cdot 3\; D(x_1-x)\,D(x_2-x)\,D(0)$$

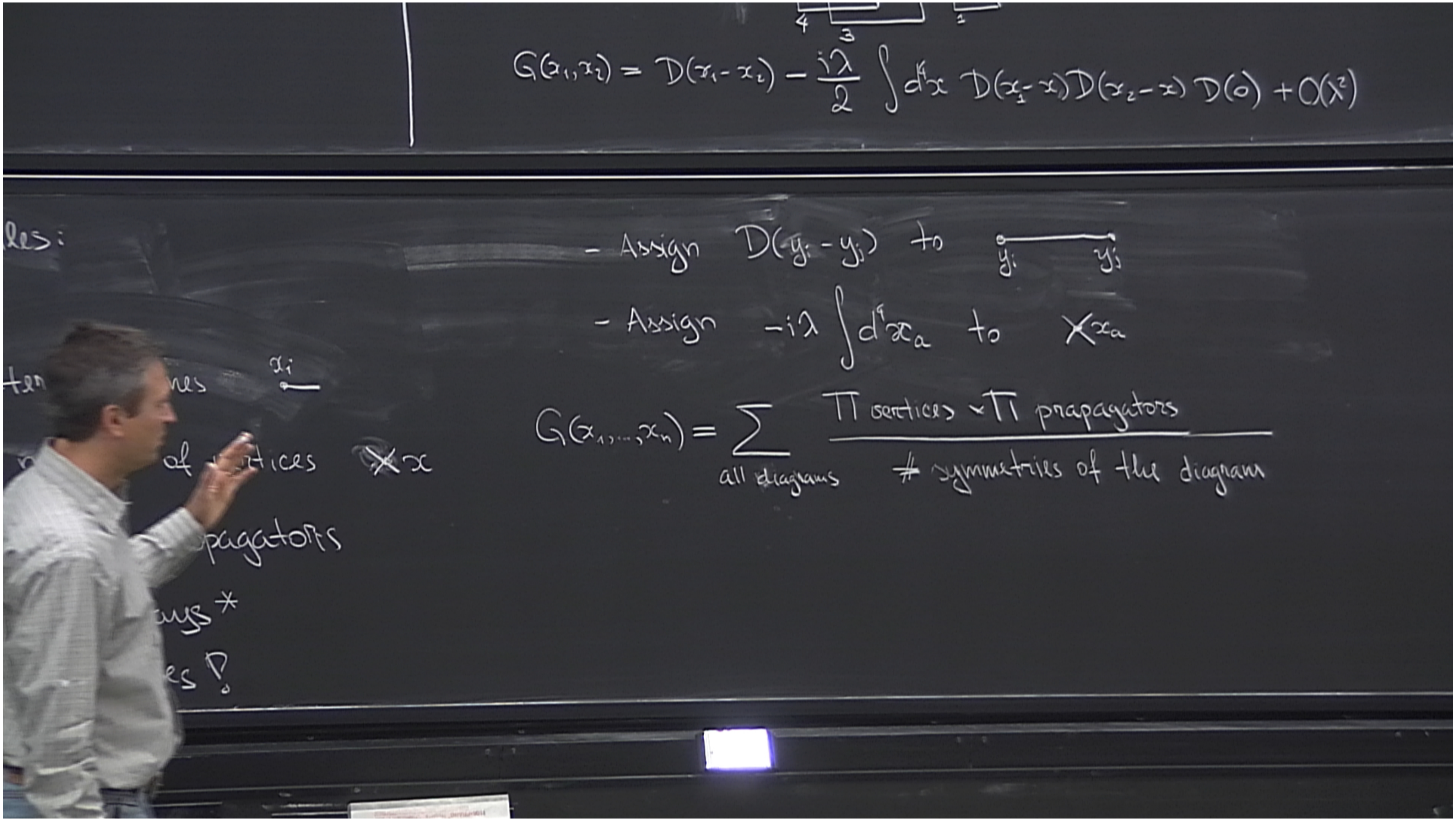
$$G(x_1,x_2) = D(x_1-x_2) - \frac{i\lambda}{2}\int d^4x\, D(x_1-x)D(x_2-x)\,D(0) + \mathcal{O}(\lambda^2)$$


2) Feynman diagrams.
3
4×3
cancelled
by denominator
# symmetry transformations

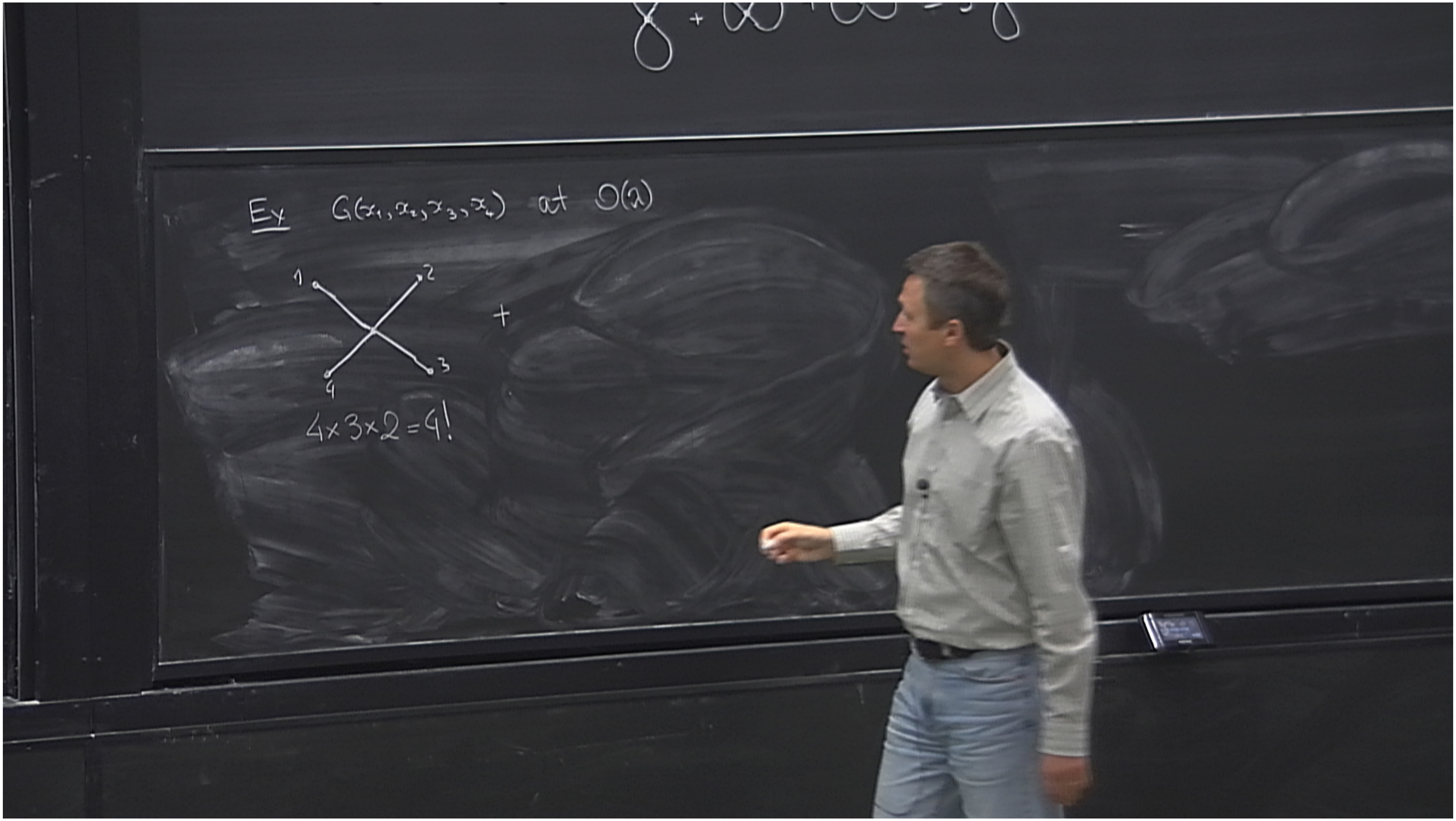

2) Feynman diagrams.
$- \frac{i\lambda}{4!}$
3
4×3
cancelled
by denominator
$- \frac{i\lambda}{2}$
$+ O(\lambda^2)$
# symmetry transformations

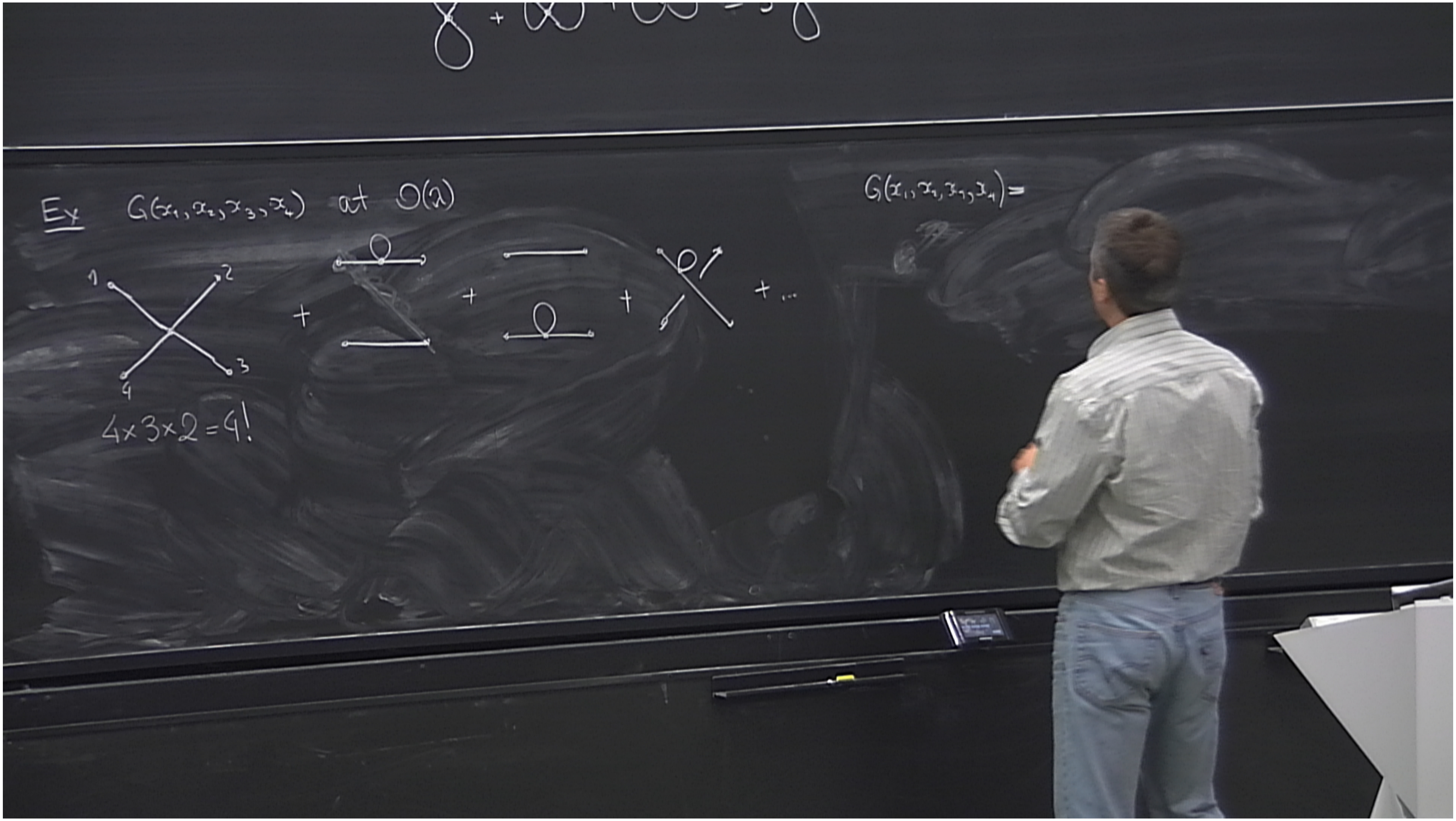

Feynman rules:
G(x1, ..., xn)
- draw n external lines xi
- add any number of vertices x
- connect them w. propagators

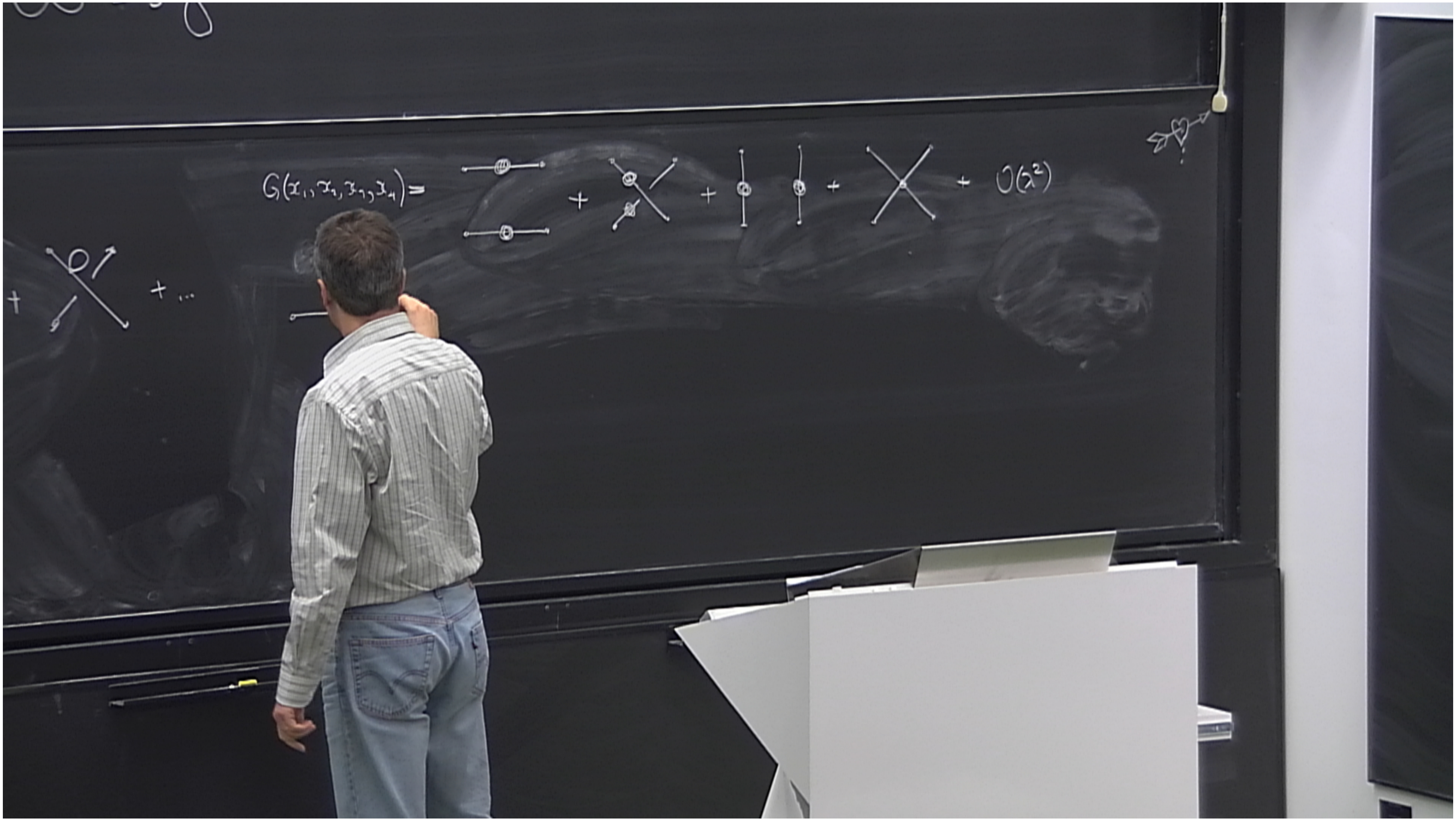

Feynman rules:
$G(x_1, \ldots, x_n)$
- draw n external lines $x_i$
- add any number of vertices $x$
- connect them w. propagators
in all possible ways *
* no vacuum bubbles!



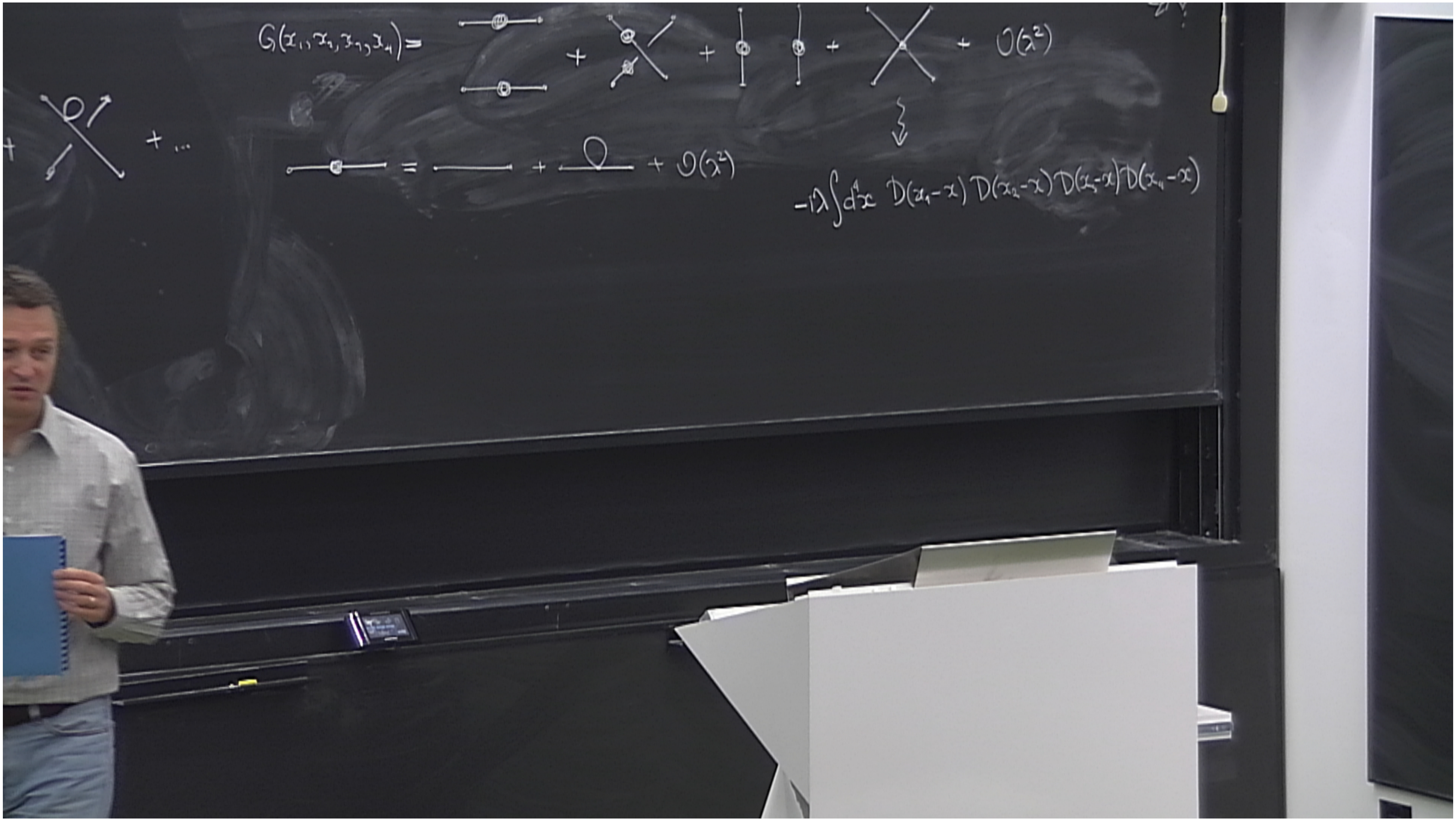
$G(x_1,x_2) = D(x_1-x_2) - \frac{i\lambda}{2}\int d^4x\, D(x_1-x)D(x_2-x)D(0) + O(\lambda^2)$
- Assign $D(y_i - y_j)$ to
$y_i$ $y_j$
- Assign $-i\lambda \int d^4x_a$ to $x_a$
$G(x_1,\dots,x_n) = \sum_{\text{all diagrams}} \frac{\prod \text{vertices} \times \prod \text{propagators}}{\#\text{ symmetries of the diagram}}$


Ex G(x1, x2, x3, x4) at O(λ)
1
2
3
4
+
4×3×2=4!


Ex G(x1, x2, x3, x4) at O(λ)
4×3×2=4!
G(x1, x2, x3, x4)=


G(x1, x2, x3, x4) =
+ O(λ²)
+ ...

G(x1, x2, x3, x4) =
+ O(λ²)
+ O(λ²)



$G(x_1,x_2,x_3,x_4) =$
$+ O(\lambda^2)$
$= + + O(\lambda^2)$
$-i\lambda \int d^4x \, D(x_1-x) D(x_2-x) D(x_3-x) D(x_4-x)$