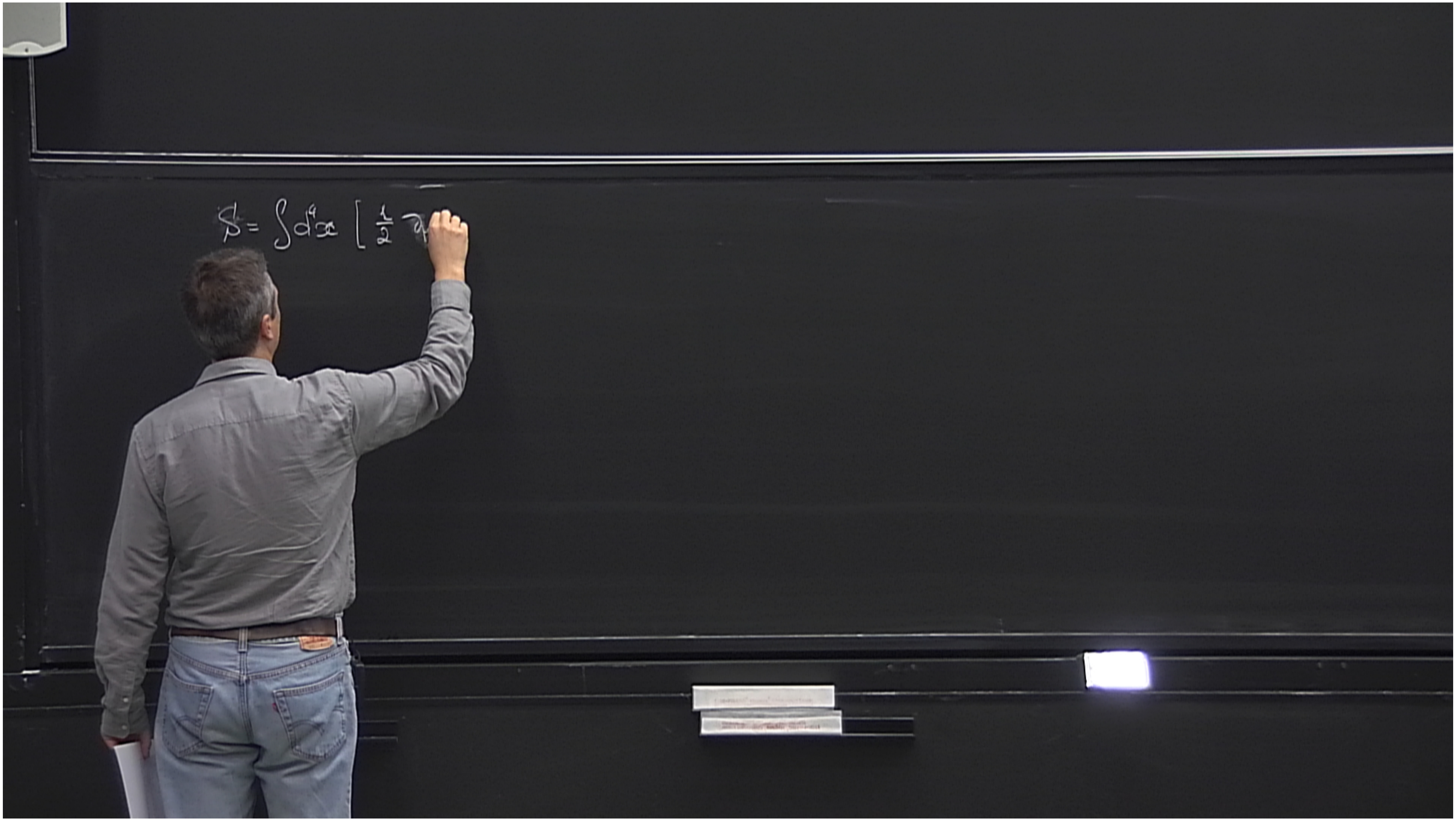
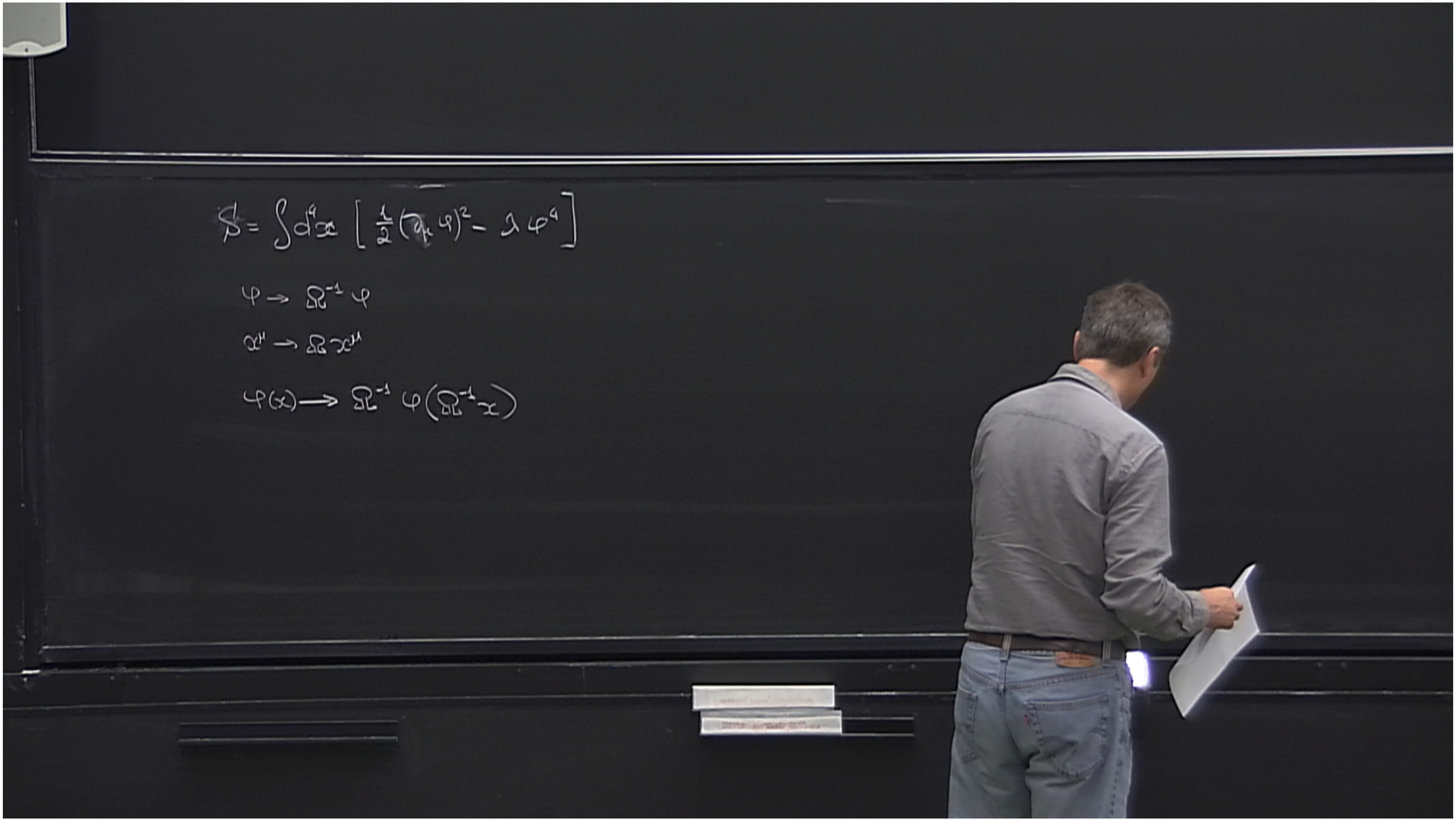
Title: Quantum Field Theory I - Lecture 9

Date: Oct 14, 2011 09:00 AM

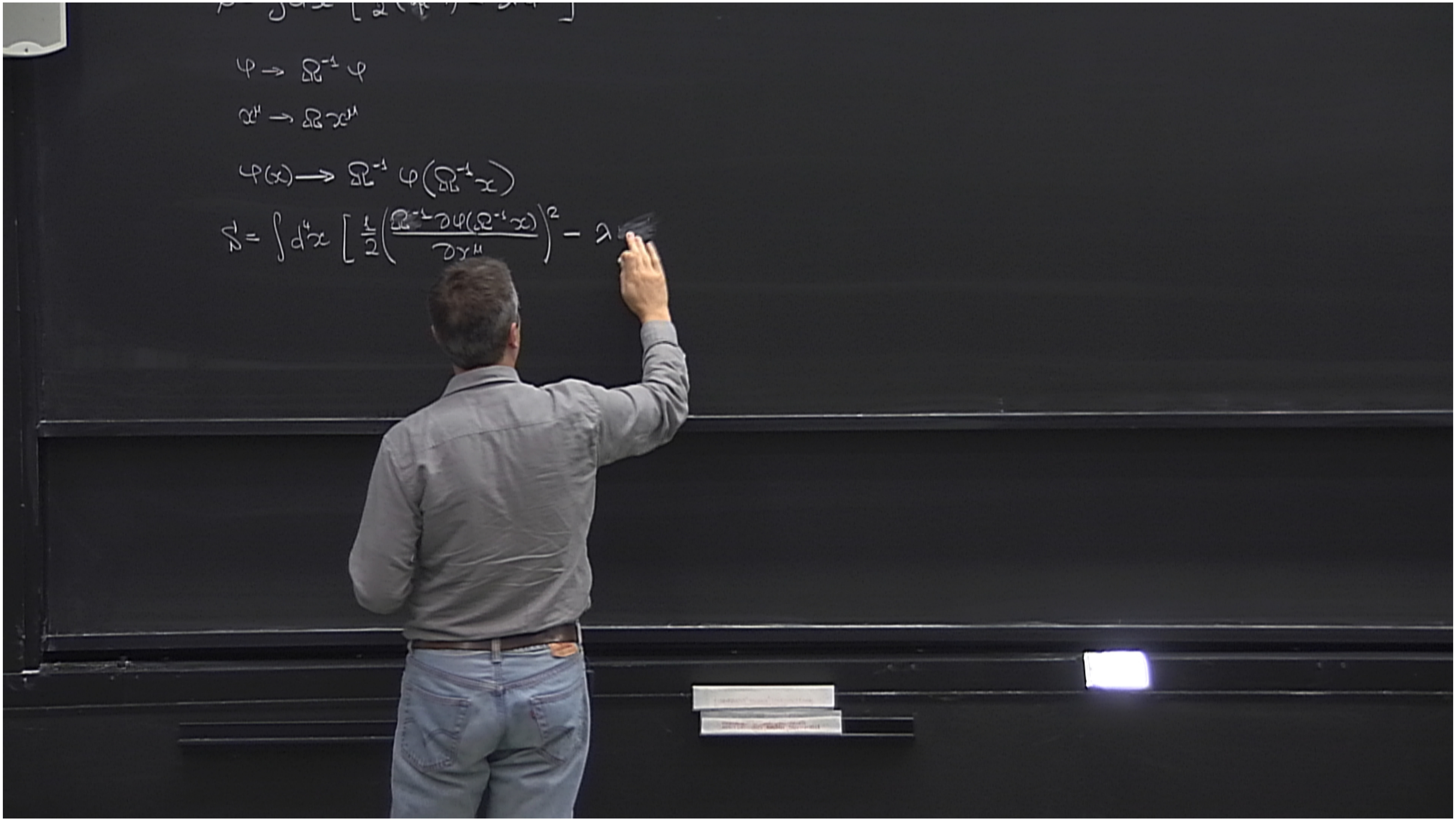
URL: http://pirsa.org/11100017

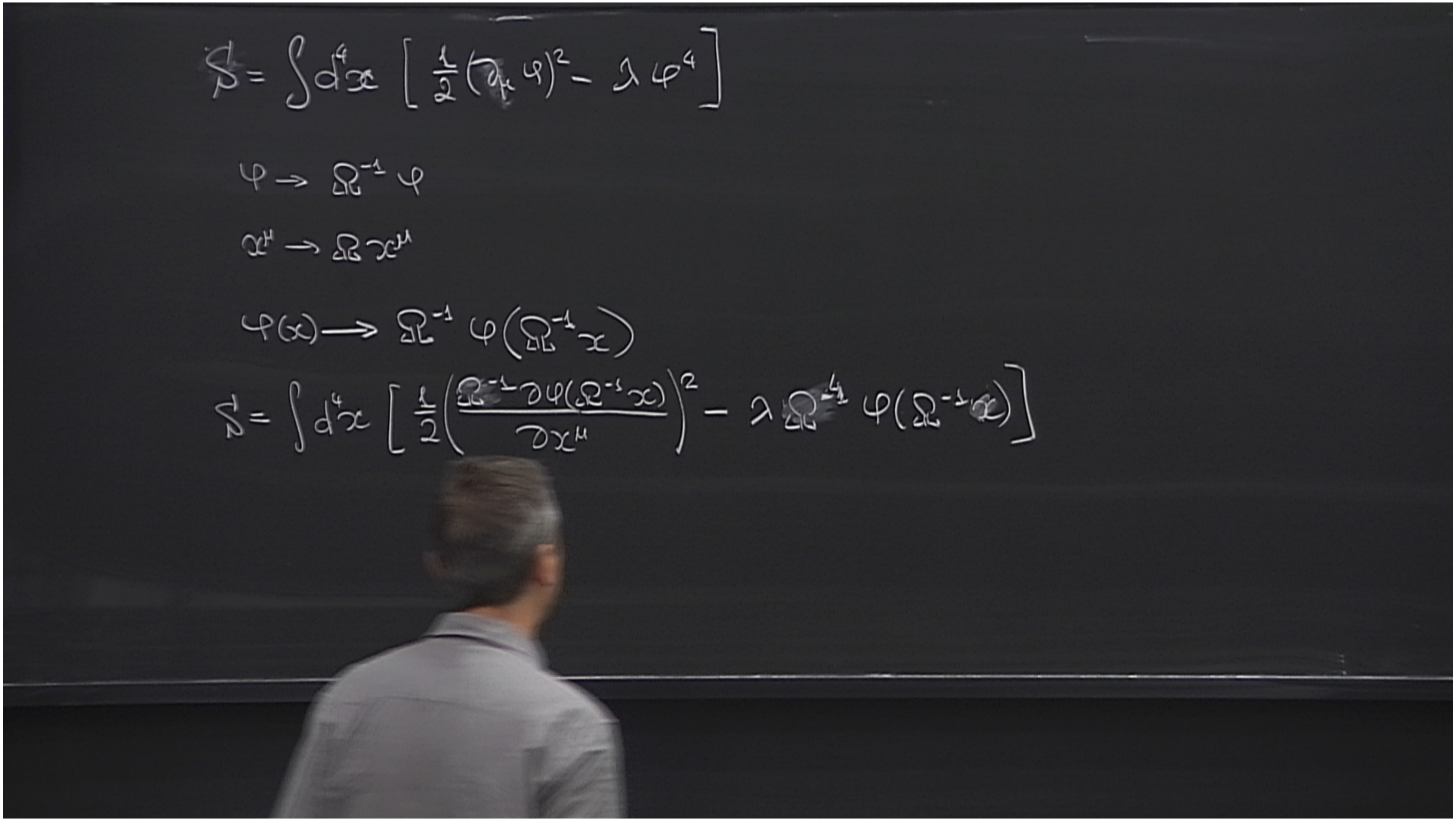
Abstract:

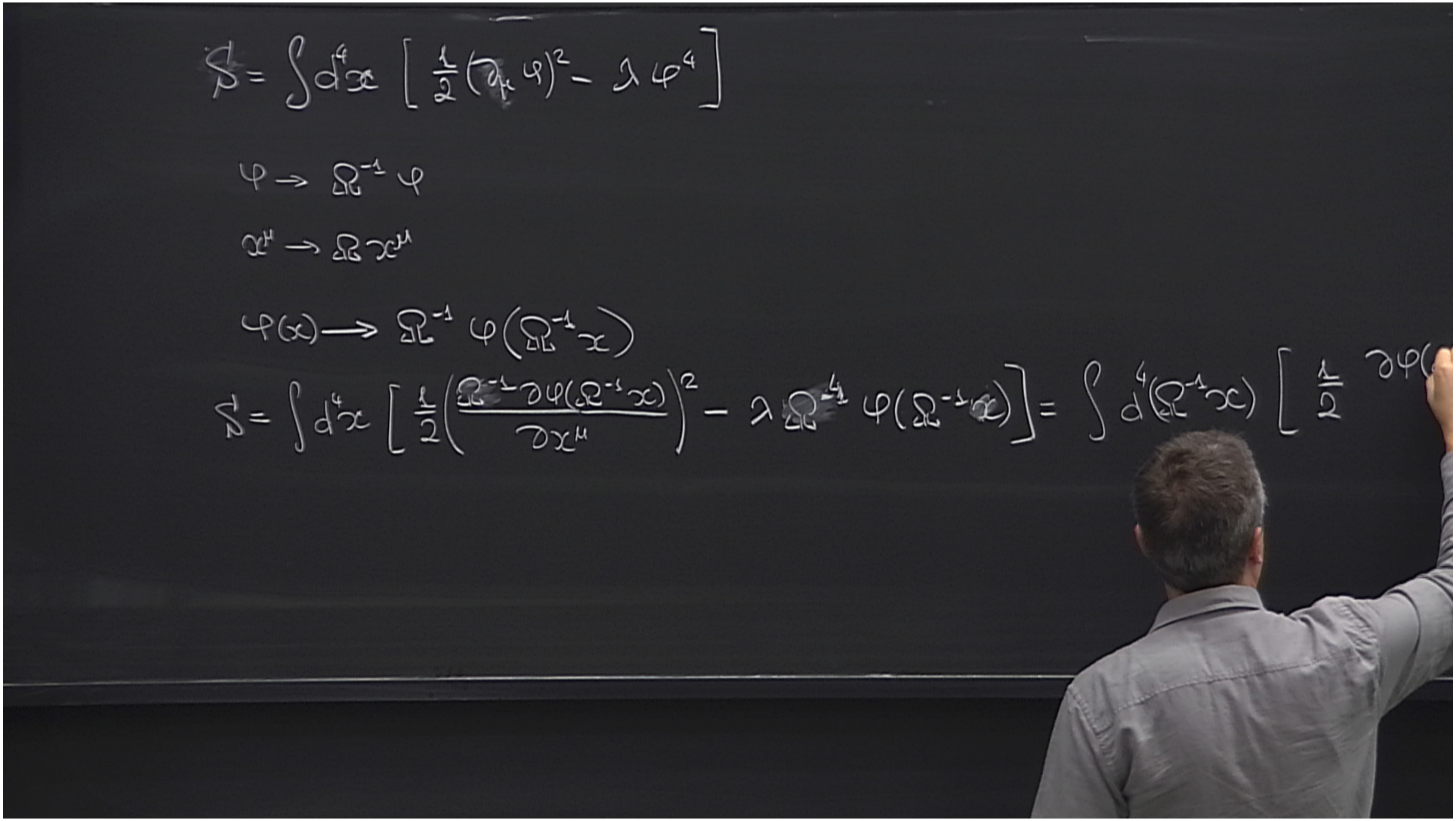

S = ∫d^4x [1/2



$S = \int d^4x \left[\frac{1}{2} (\partial_\mu \varphi)^2 - \lambda \varphi^4 \right]$
$\varphi \rightarrow \Omega^{-1} \varphi$
$x^\mu \rightarrow \Omega x^\mu$
$\varphi(x) \longrightarrow \Omega^{-1} \varphi(\Omega^{-1} x)$


$\varphi \to \Omega^{-1} \varphi$
$x^\mu \to \Omega x^\mu$
$\varphi(x) \longrightarrow \Omega^{-1} \varphi(\Omega^{-1} x)$
$S = \int d^4x \left[\frac{1}{2} \left(\frac{\Omega^{-1} \partial \varphi(\Omega^{-1} x)}{\partial x^\mu} \right)^2 - \lambda \right.$


$S = \int d^4x \left[\frac{1}{2}(\partial_\mu \varphi)^2 - \lambda \varphi^4 \right]$
$\varphi \to \Omega^{-1} \varphi$
$x^\mu \to \Omega x^\mu$
$\varphi(x) \longrightarrow \Omega^{-1} \varphi(\Omega^{-1} x)$
$S = \int d^4x \left[\frac{1}{2}\left(\frac{\Omega^{-1} \partial \varphi(\Omega^{-1} x)}{\partial x^\mu} \right)^2 - \lambda \Omega^{-4} \varphi(\Omega^{-1} x) \right]$


$S = \int d^4x \left[\frac{1}{2}(\partial_\mu \varphi)^2 - \lambda \varphi^4 \right]$
$\varphi \to \Omega^{-1} \varphi$
$x^\mu \to \Omega x^\mu$
$\varphi(x) \longrightarrow \Omega^{-1} \varphi(\Omega^{-1} x)$
$S = \int d^4x \left[\frac{1}{2}\left(\frac{\Omega^{-1} \partial\varphi(\Omega^{-1}x)}{\partial x^\mu}\right)^2 - \lambda \Omega^{-4} \varphi(\Omega^{-1}x) \right] = \int d^4(\Omega^{-1}x) \left[\frac{1}{2} \partial\varphi(\right.$

$$S = \int d^4x \left[\frac{1}{2} (\partial_\mu \varphi)^2 - \lambda \varphi^4 \right]$$

$$\varphi \to \Omega^{-1} \varphi$$

$$x^\mu \to \Omega x^\mu$$

$$\varphi(x) \longrightarrow \Omega^{-1} \varphi(\Omega^{-1} x)$$

$$S = \int d^4x \left[\frac{1}{2} \left(\frac{\Omega^{-1} \partial \varphi(\Omega^{-1} x)}{\partial x^\mu} \right)^2 - \lambda \Omega^{-4} \varphi^4(\Omega^{-1} x) \right] = \int d^4(\Omega^{-1} x) \left[\frac{1}{2} \frac{\partial \varphi(\Omega^{-1} x)}{\partial (\Omega^{-1} x)^\mu} - \lambda \ldots \right.$$

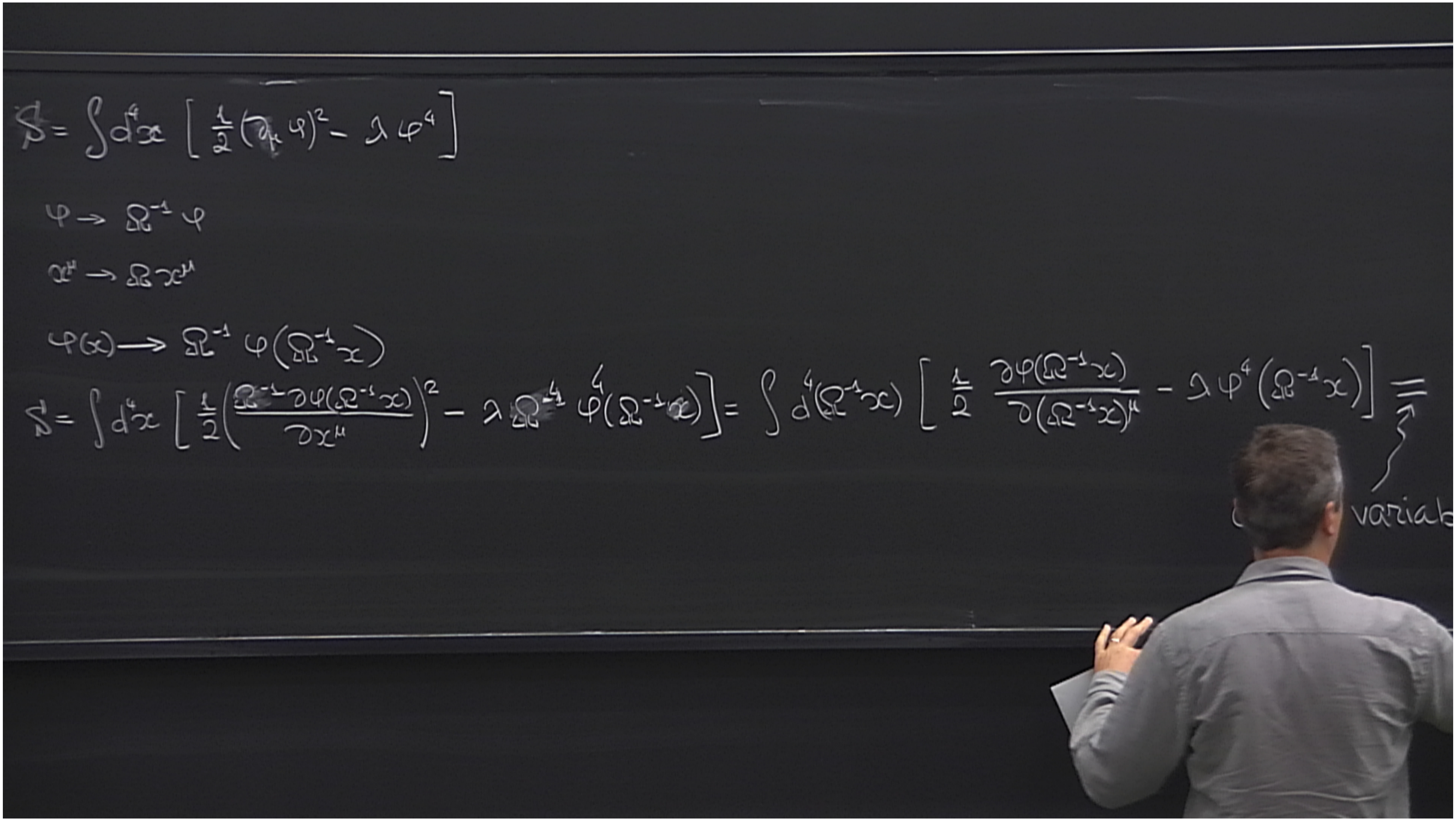
$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu \varphi)^2 - \lambda \varphi^4 \right]$$

$$\varphi \to \Omega^{-1} \varphi$$

$$x^\mu \to \Omega x^\mu$$

$$\varphi(x) \longrightarrow \Omega^{-1} \varphi(\Omega^{-1} x)$$

$$S = \int d^4x \left[\frac{1}{2}\left(\frac{\Omega^{-1} \partial \varphi(\Omega^{-1} x)}{\partial x^\mu}\right)^2 - \lambda \Omega^{-4} \varphi^4(\Omega^{-1} x) \right] = \int d^4(\Omega^{-1} x) \left[\frac{1}{2} \frac{\partial \varphi(\Omega^{-1} x)}{\partial (\Omega^{-1} x)^\mu} - \lambda \varphi^4 (\Omega^{-1} x) \right]$$


$S = \int d^4x \left[\frac{1}{2}(\partial_\mu \varphi)^2 - \lambda \varphi^4 \right]$
$\varphi \to \Omega^{-1} \varphi$
$x^\mu \to \Omega x^\mu$
$\varphi(x) \longrightarrow \Omega^{-1} \varphi(\Omega^{-1} x)$
$S' = \int d^4x \left[\frac{1}{2}\left(\frac{\Omega^{-1} \partial \varphi(\Omega^{-1}x)}{\partial x^\mu} \right)^2 - \lambda \Omega^{-4} \varphi^4(\Omega^{-1}x) \right] = \int d^4(\Omega^{-1}x) \left[\frac{1}{2} \frac{\partial \varphi(\Omega^{-1}x)}{\partial (\Omega^{-1}x)^\mu} - \lambda \varphi^4(\Omega^{-1}x) \right] =$
variab

$$S = \int d^4x \left[\tfrac{1}{2}(\partial_\mu \varphi)^2 - \lambda \varphi^4 \right]$$

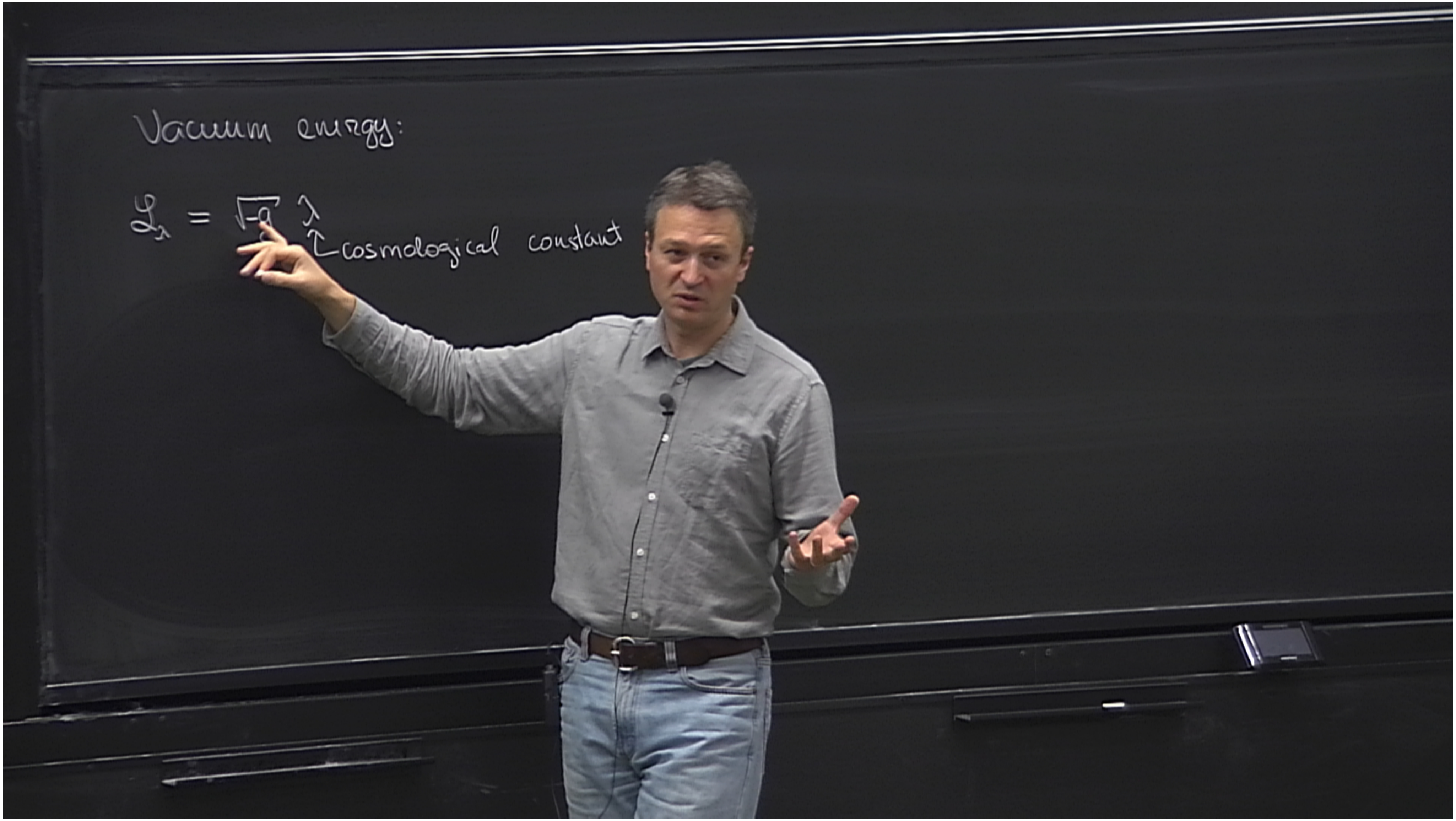
$$\varphi \to \Omega^{-1}\varphi$$

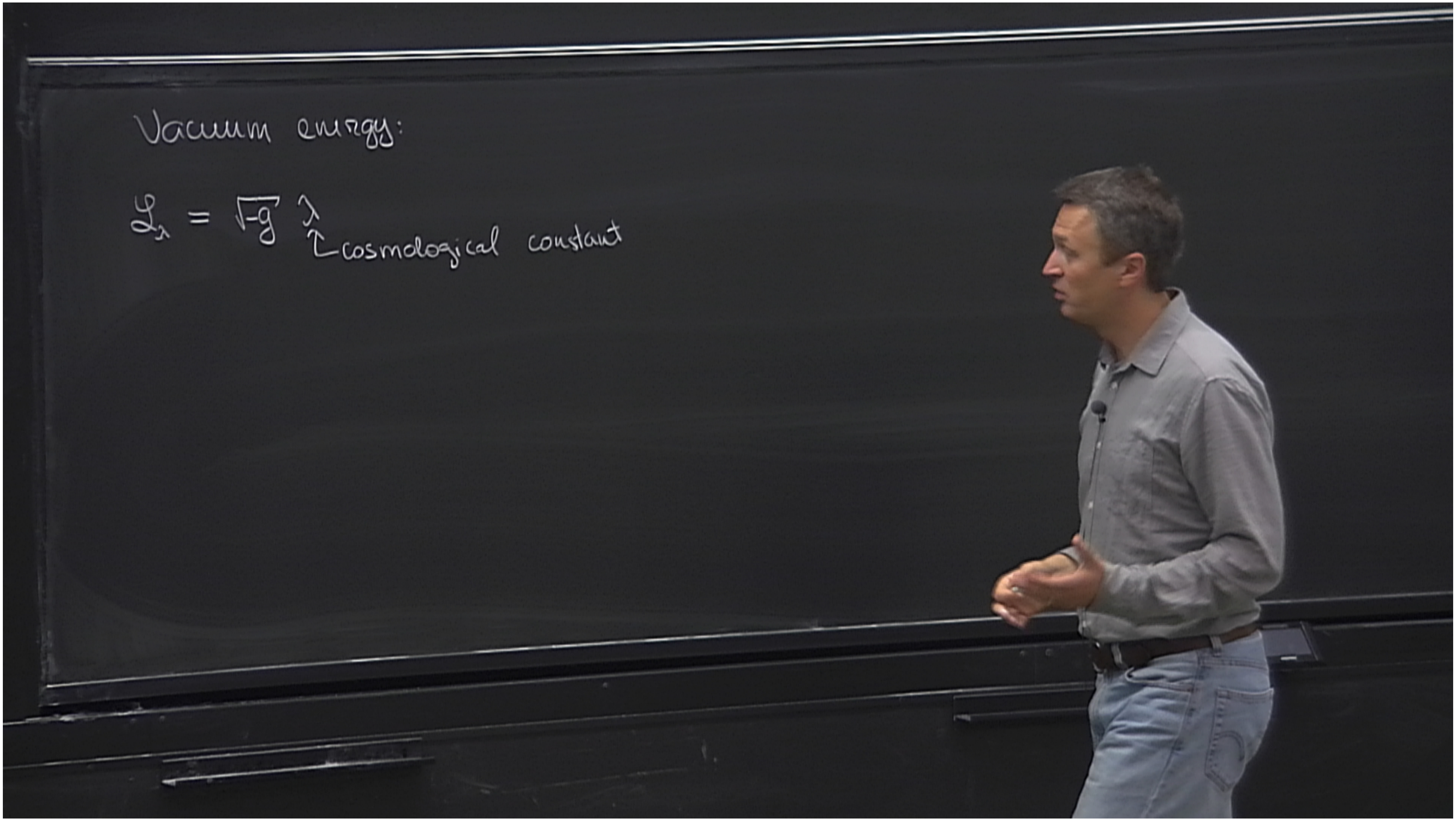
$$x^\mu \to \Omega x^\mu$$

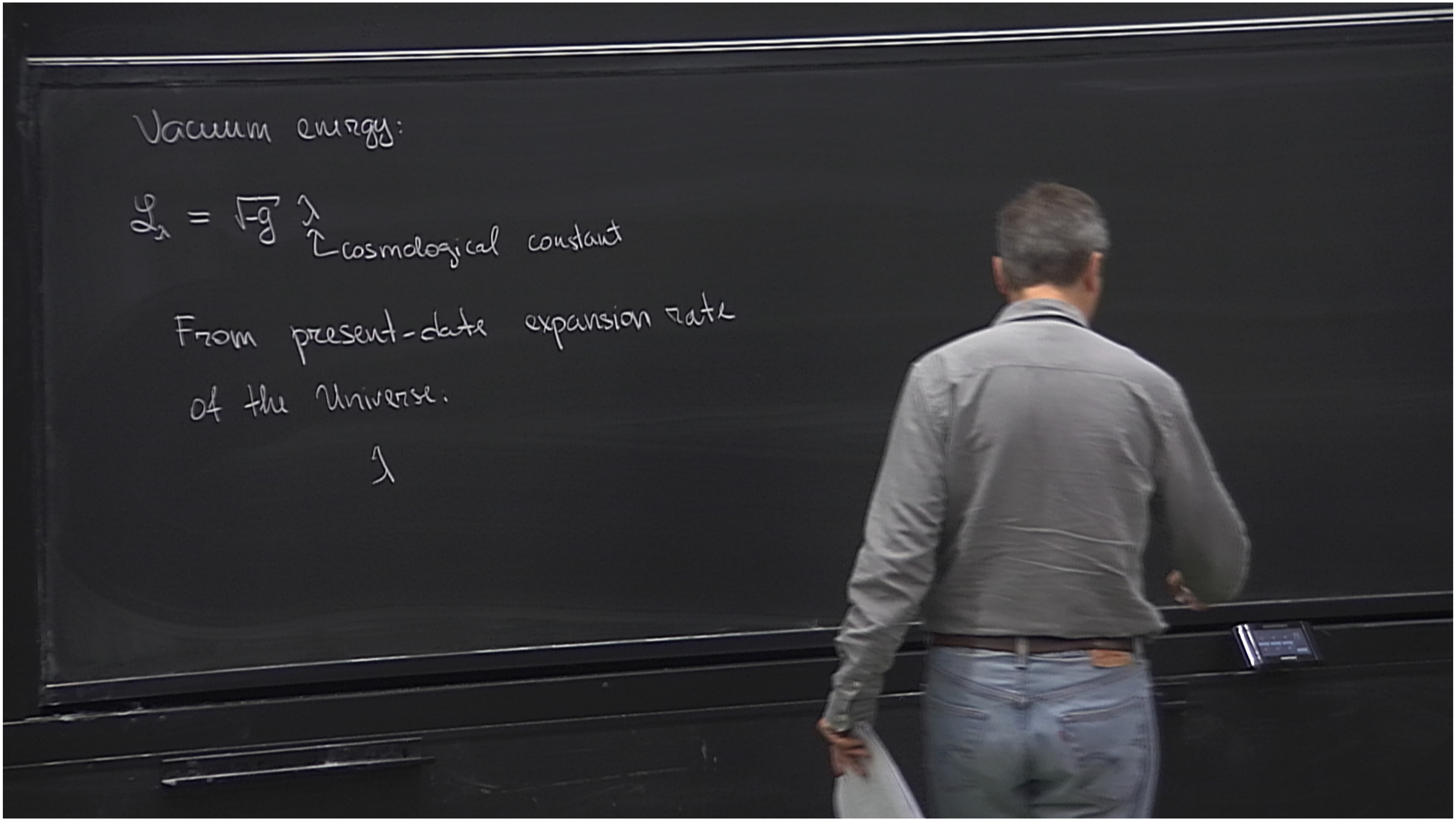
$$\varphi(x) \longrightarrow \Omega^{-1}\varphi(\Omega^{-1}x)$$

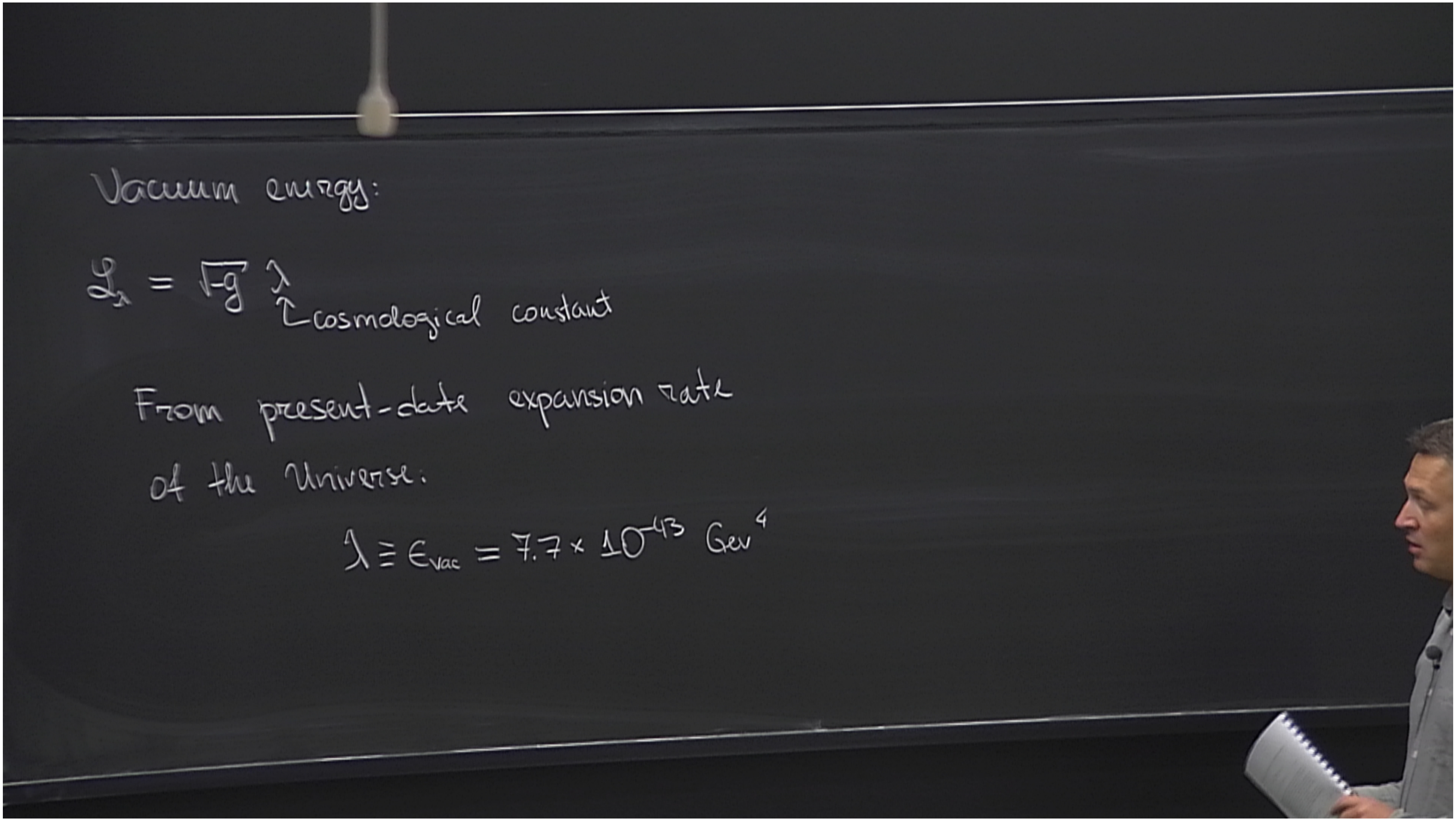
$$S = \int d^4x \left[\frac{1}{2}\left(\frac{\Omega^{-1}\partial\varphi(\Omega^{-1}x)}{\partial x^\mu}\right)^2 - \lambda\Omega^{-4}\varphi^4(\Omega^{-1}x)\right] = \int d^4(\Omega^{-1}x)\left[\frac{1}{2}\frac{\partial\varphi(\Omega^{-1}x)}{\partial(\Omega^{-1}x)^\mu} - \lambda\varphi^4(\Omega^{-1}x)\right] \equiv \int d^4y \left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial y^\mu}\right)^2 - \lambda\varphi^4\right]$$

Change variables to $y^\mu = \Omega^{-1}x^\mu$


Vacuum energy:
$\mathcal{L}_\lambda = \sqrt{-g}\,\lambda$
cosmological constant


Vacuum energy:
$\mathcal{L}_\lambda = \sqrt{-g}\,\lambda$
cosmological constant


Vacuum energy:
$\mathcal{L}_\lambda = \sqrt{-g}\,\lambda$
cosmological constant
From present-date expansion rate
of the Universe.
λ


Vacuum energy:
$\mathcal{L}_\lambda = \sqrt{-g}\,\lambda$
cosmological constant
From present-date expansion rate
of the Universe:
$\lambda \equiv \epsilon_{vac} = 7.7 \times 10^{-43}\ \mathrm{GeV}^4$

In QFT:

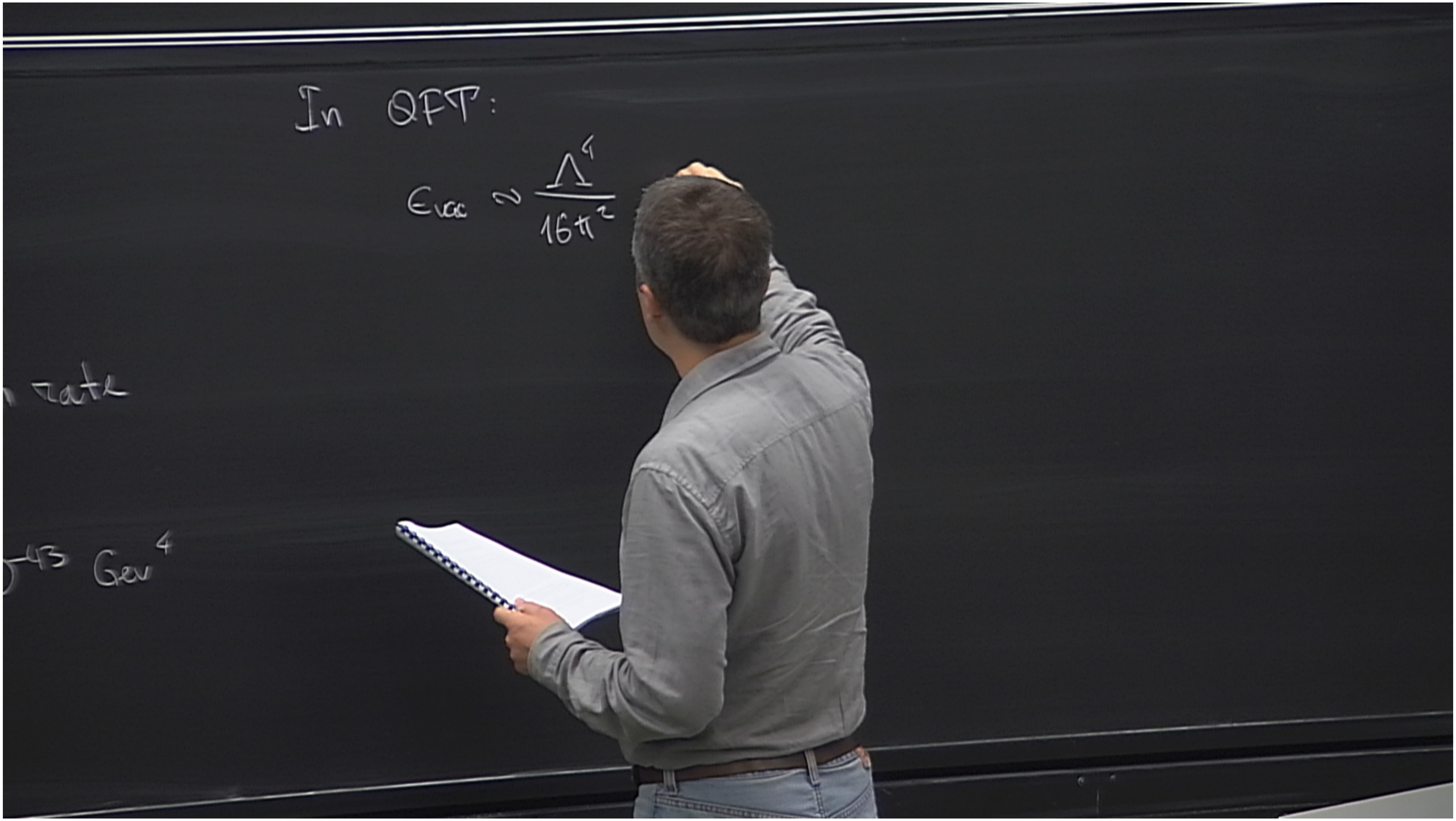
Vacuum energy:

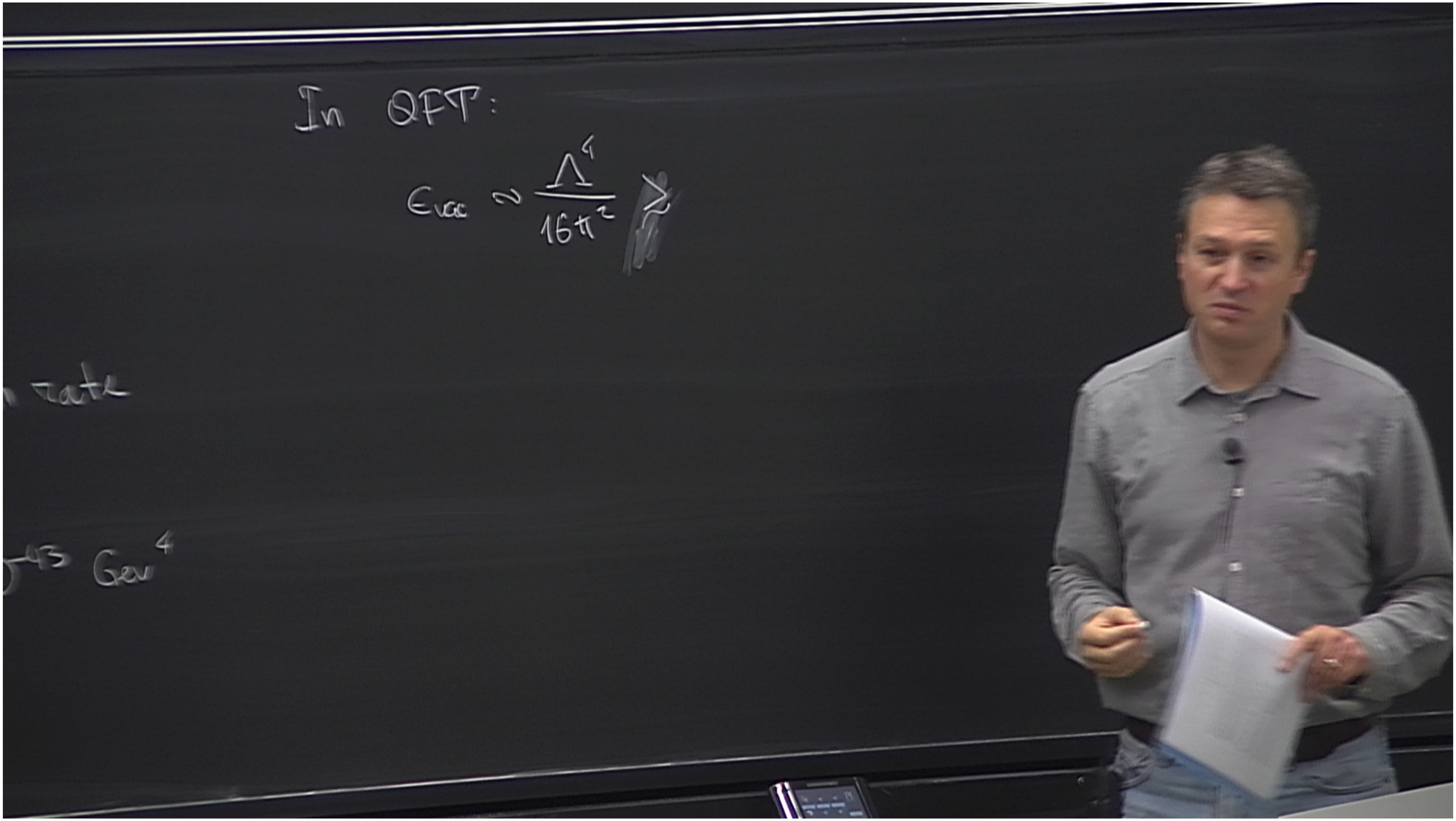
$$\mathcal{L}_\lambda = \sqrt{-g}\,\lambda$$

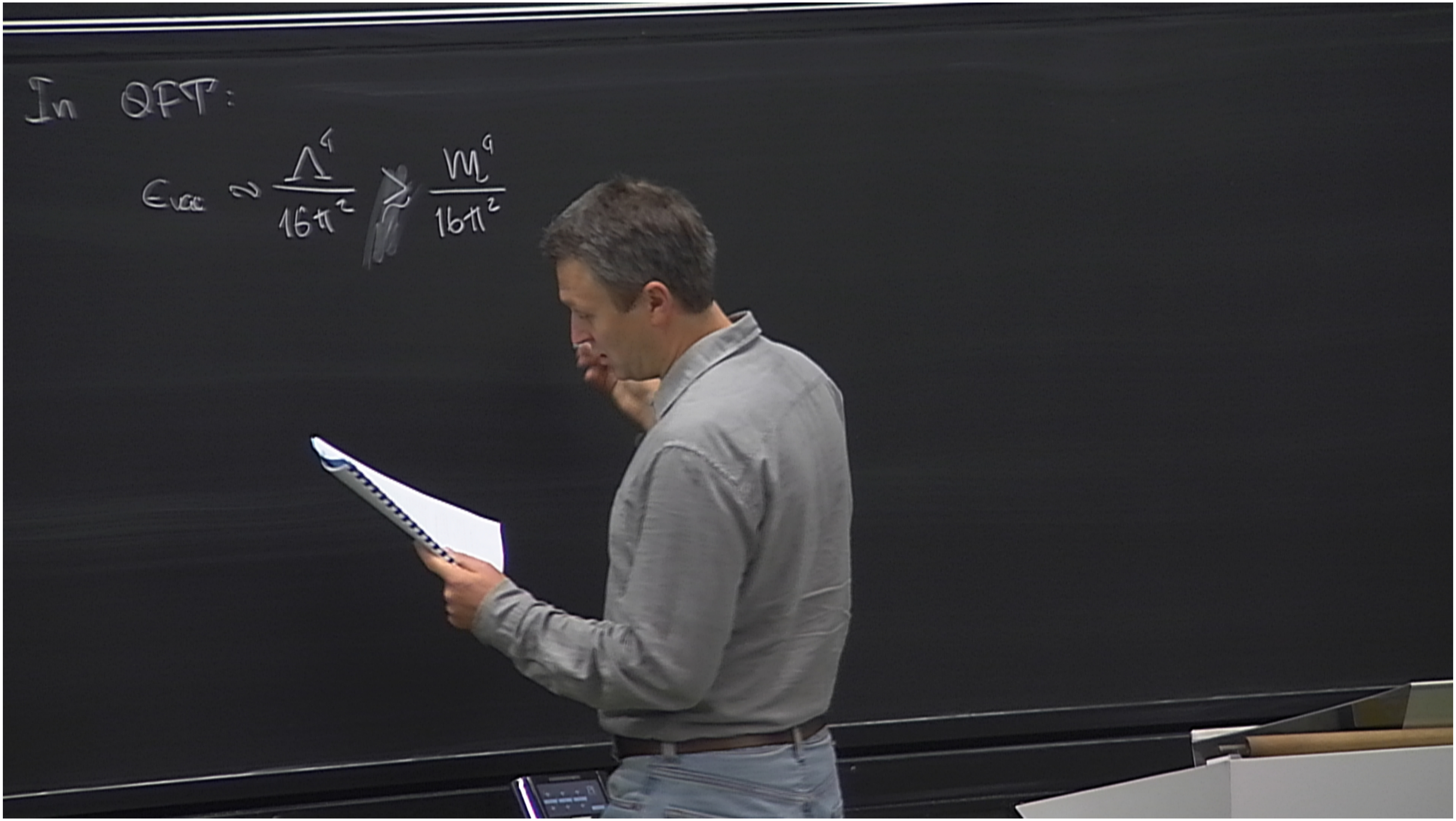
λ — cosmological constant

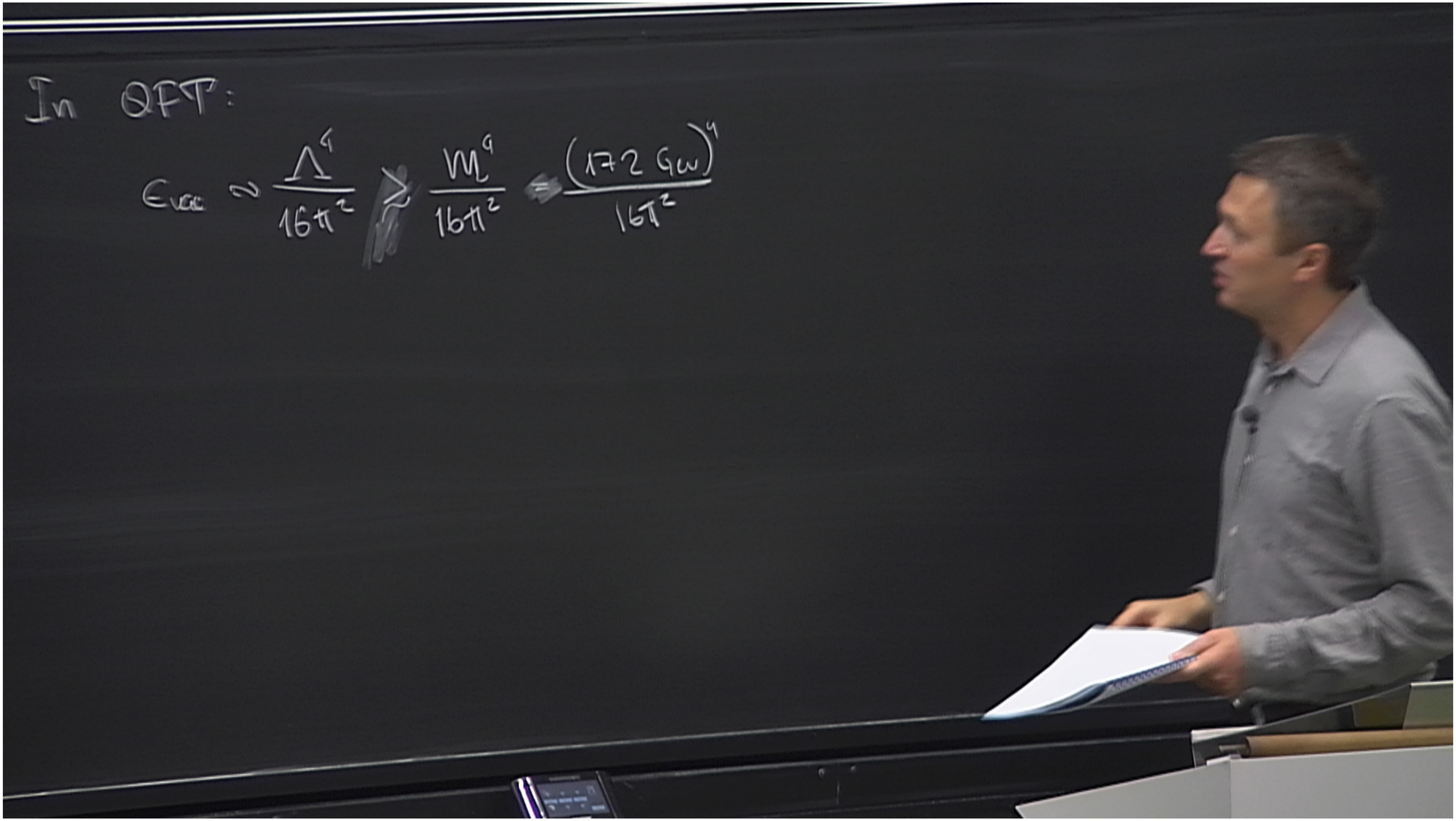
From present-date expansion rate of the Universe:

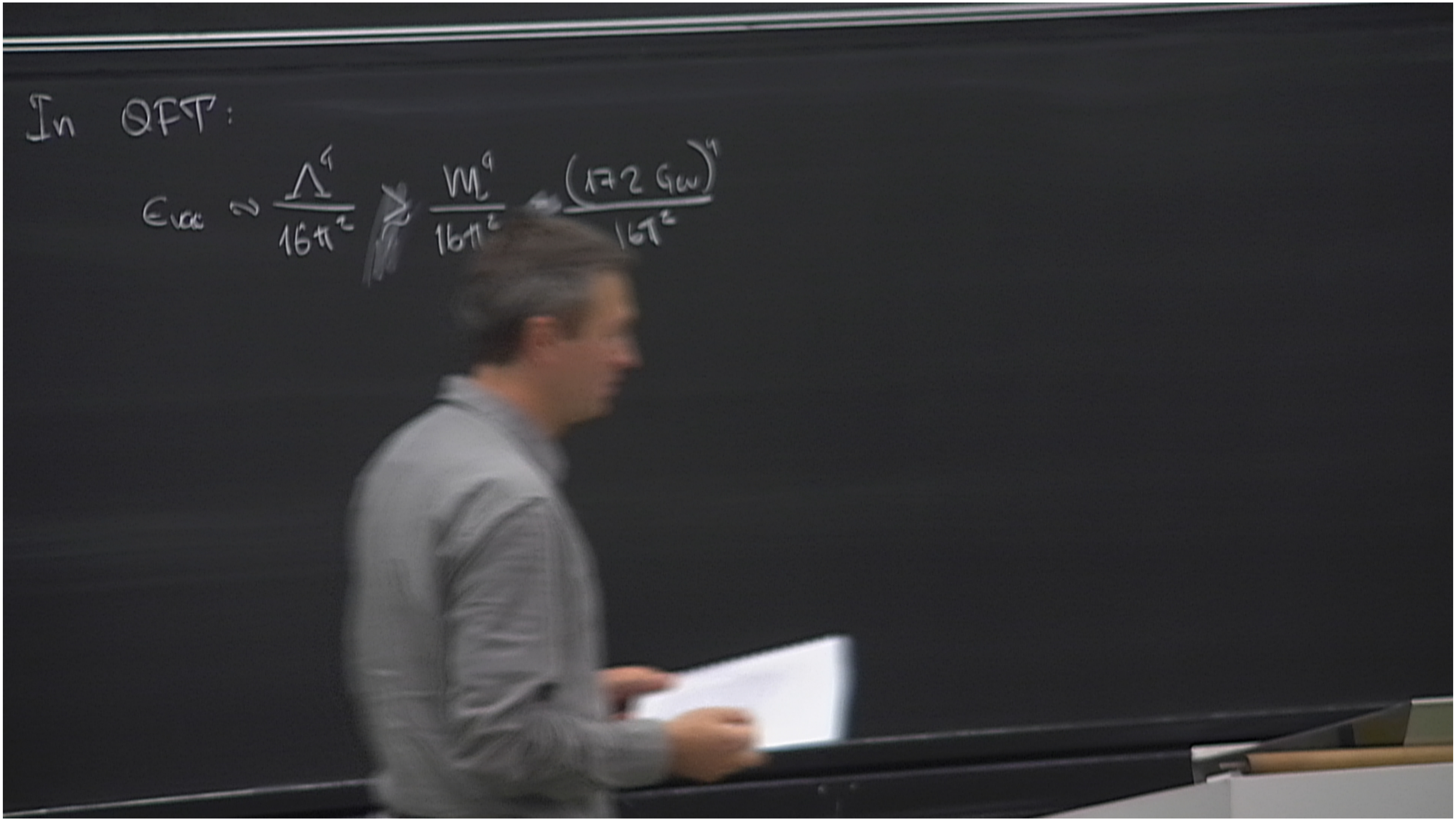
$$\lambda \cong \epsilon_{vac} = 7.7 \times 10^{-43}\ \mathrm{GeV}^4$$

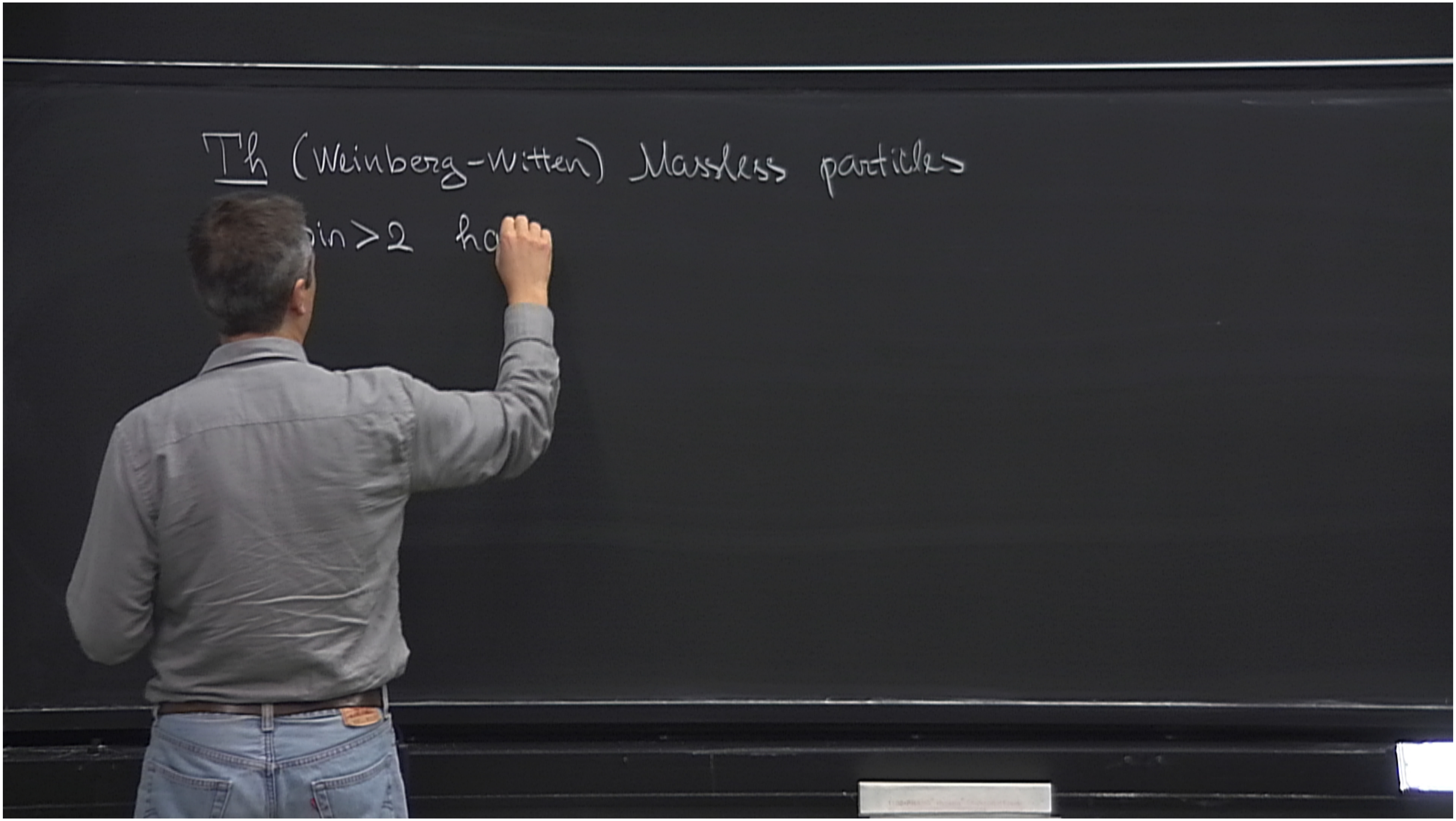

In QFT:
$\epsilon_{vac} \approx \frac{\Lambda^4}{16\pi^2}$
rate
Gev⁴


In QFT:
$\epsilon_{vac} \sim \frac{\Lambda^4}{16\pi^2}$
rate
Gev 4

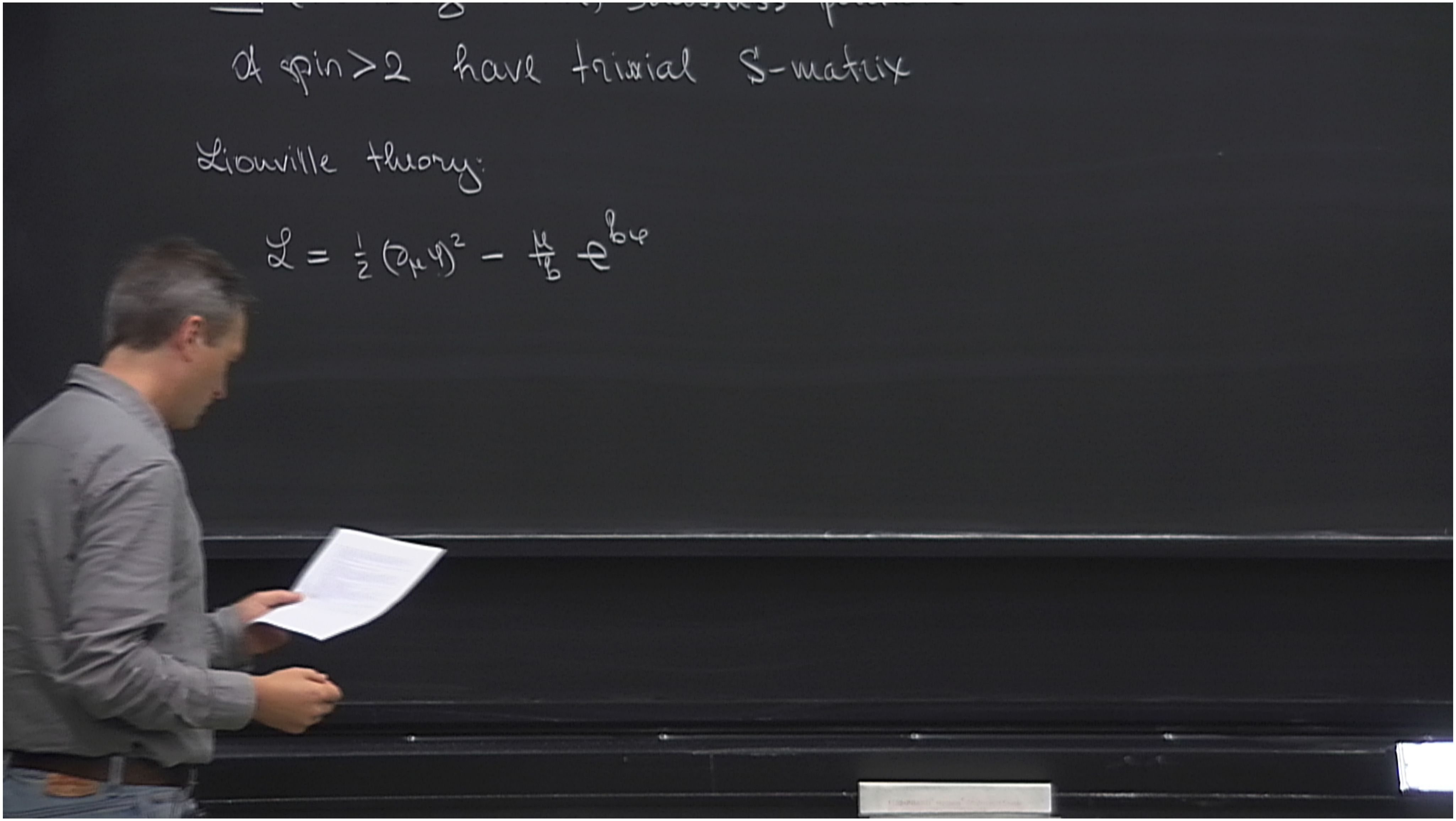

In QFT:
$\epsilon_{vac} \sim \frac{\Lambda^4}{16\pi^2} \gg \frac{m^4}{16\pi^2}$

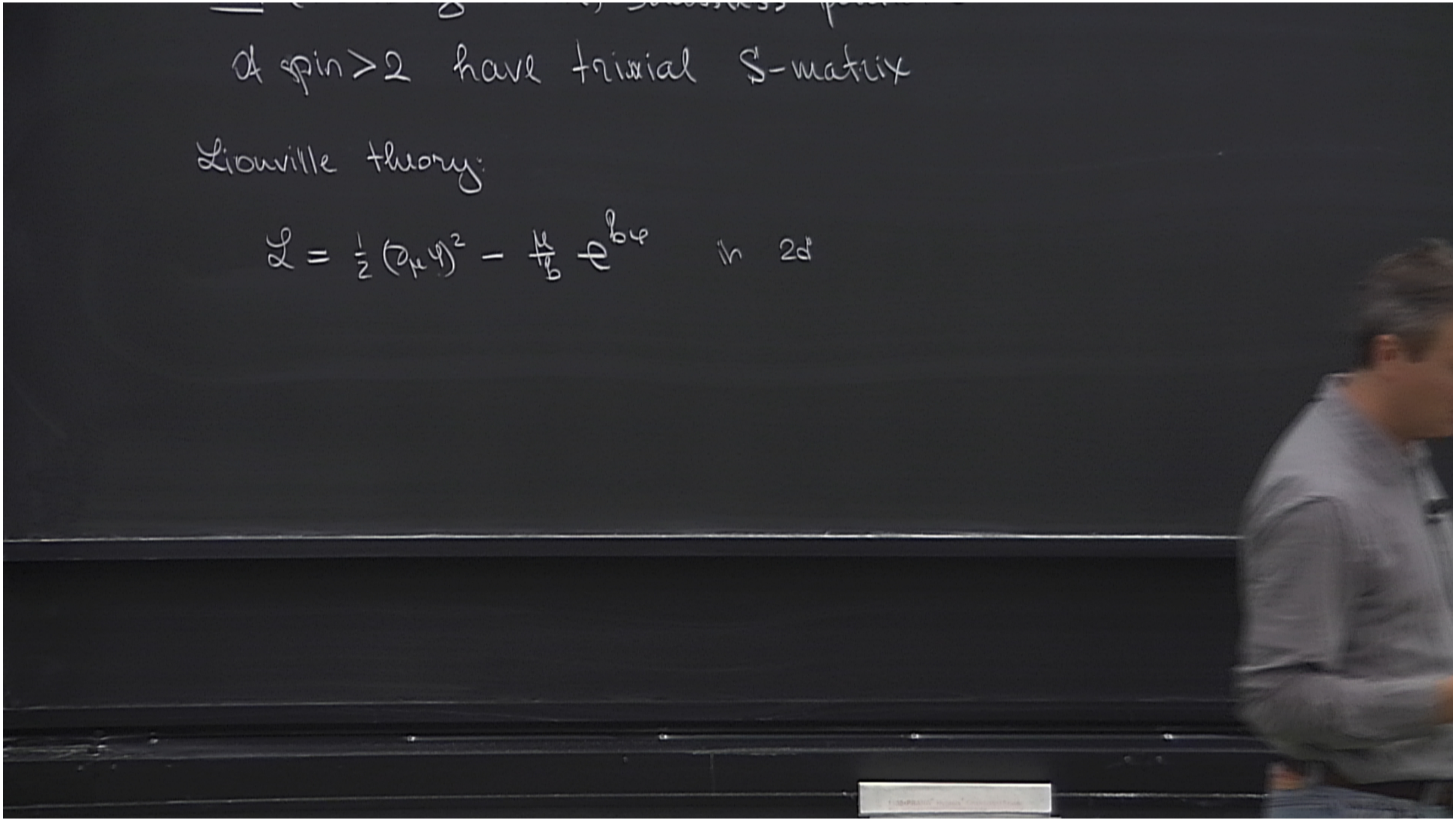

In QFT:
$\epsilon_{vac} \approx \frac{\Lambda^4}{16\pi^2} \gtrsim \frac{m_t^4}{16\pi^2} = \frac{(172\ GeV)^4}{16\pi^2}$

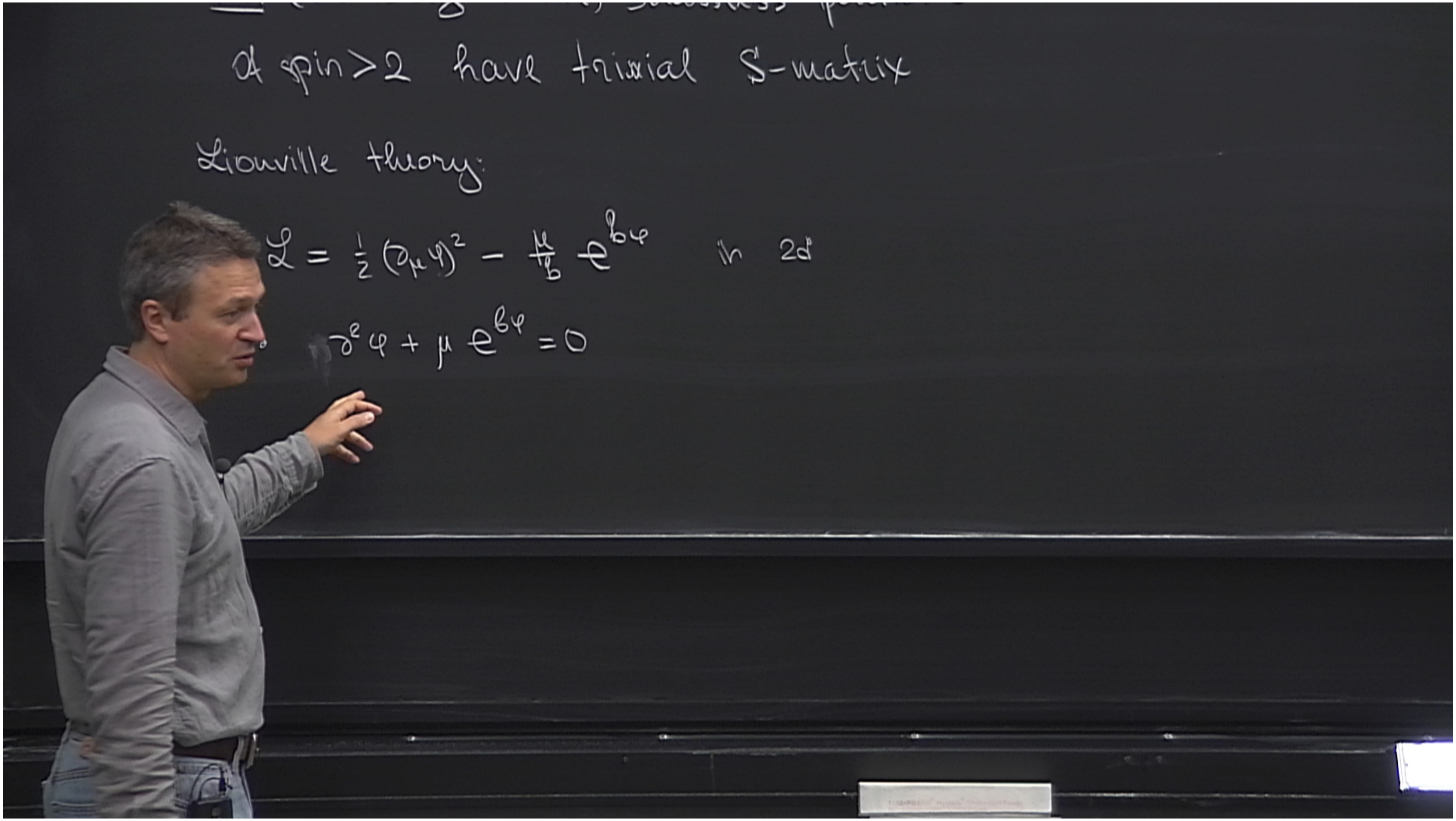

In QFT:
$\epsilon_{vac} \sim \frac{\Lambda^4}{16\pi^2} \gtrsim \frac{m_t^4}{16\pi^2} \sim \frac{(172\ GeV)^4}{16\pi^2}$

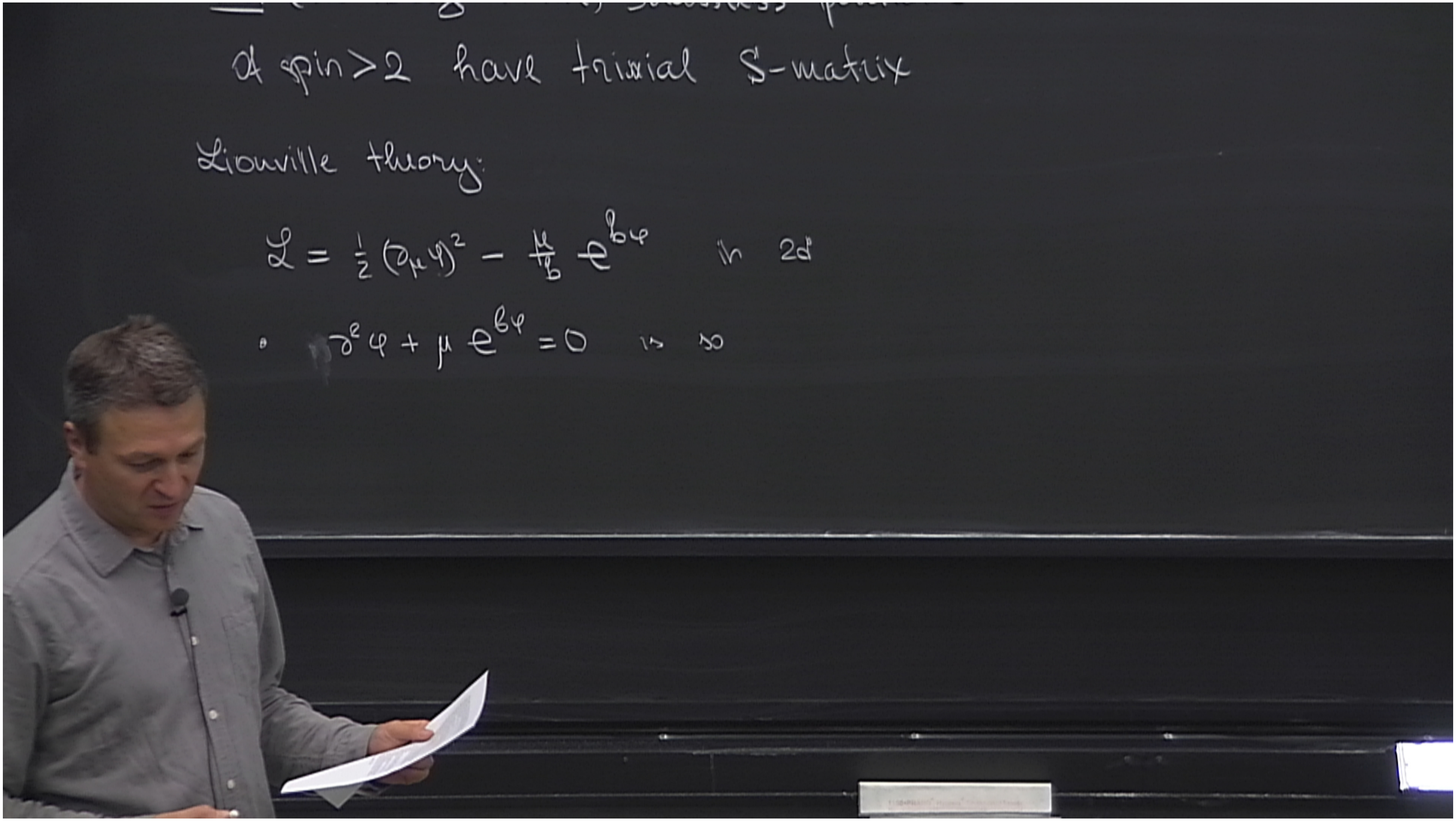

Th (Weinberg-Witten) Massless particles
in>2 ha

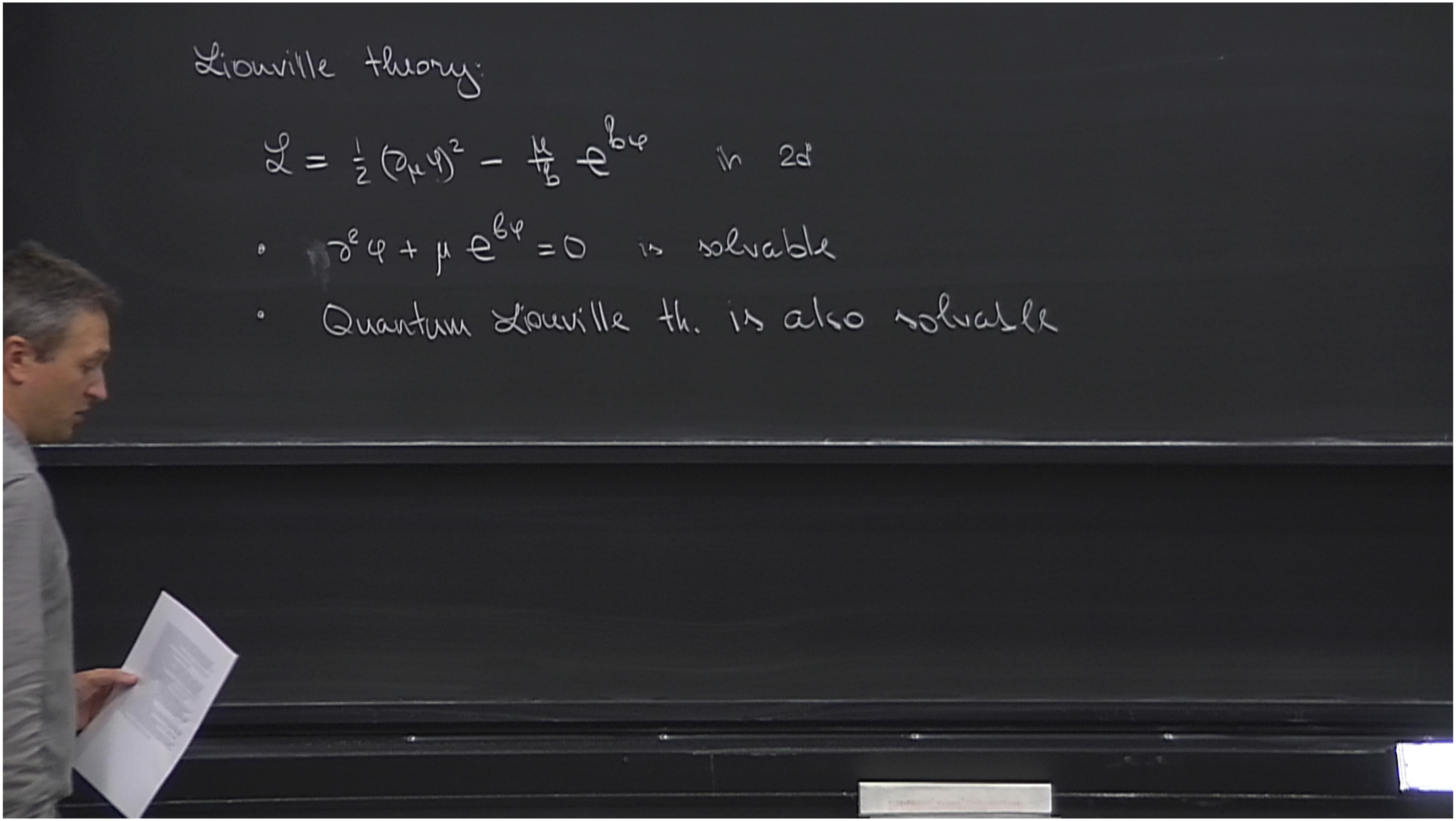
Th (Weinberg-Witten) Massless particles of spin > 2 have trivial S-matrix

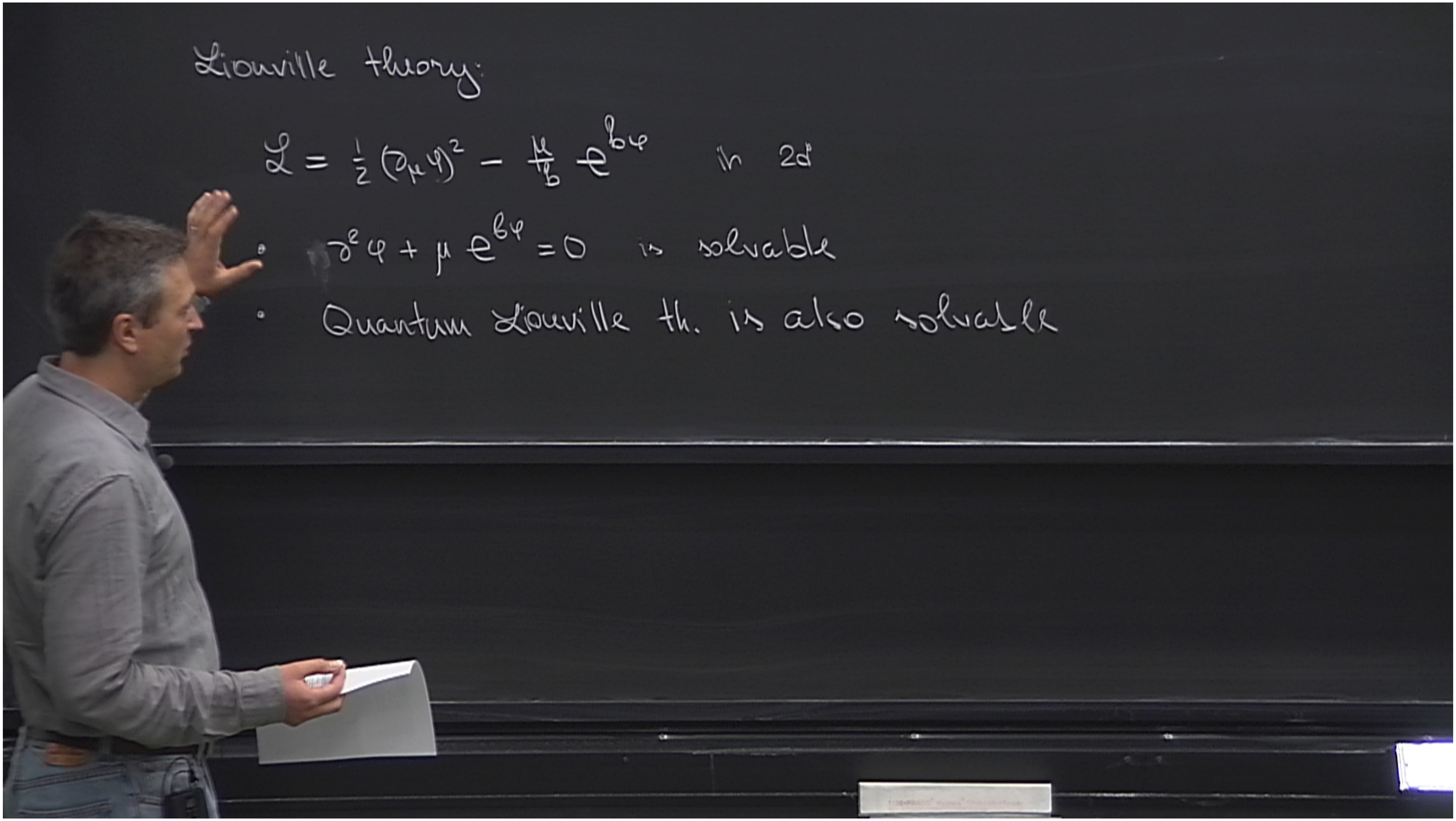

of spin > 2 have trivial S-matrix
Liouville theory:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi}$


of spin>2 have trivial S-matrix
Liouville theory:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi}$ in 2d


of spin>2 have trivial S-matrix
Liouville theory:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi}$ in 2d
$\partial^2 \varphi + \mu e^{b\varphi} = 0$


of spin>2 have trivial S-matrix
Liouville theory:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi}$ in 2d
$\partial^2 \varphi + \mu e^{b\varphi} = 0$ is so


Liouville theory:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi}$ in 2d
• $\partial^2 \varphi + \mu e^{b\varphi} = 0$ is solvable
• Quantum Liouville th. is also solvable

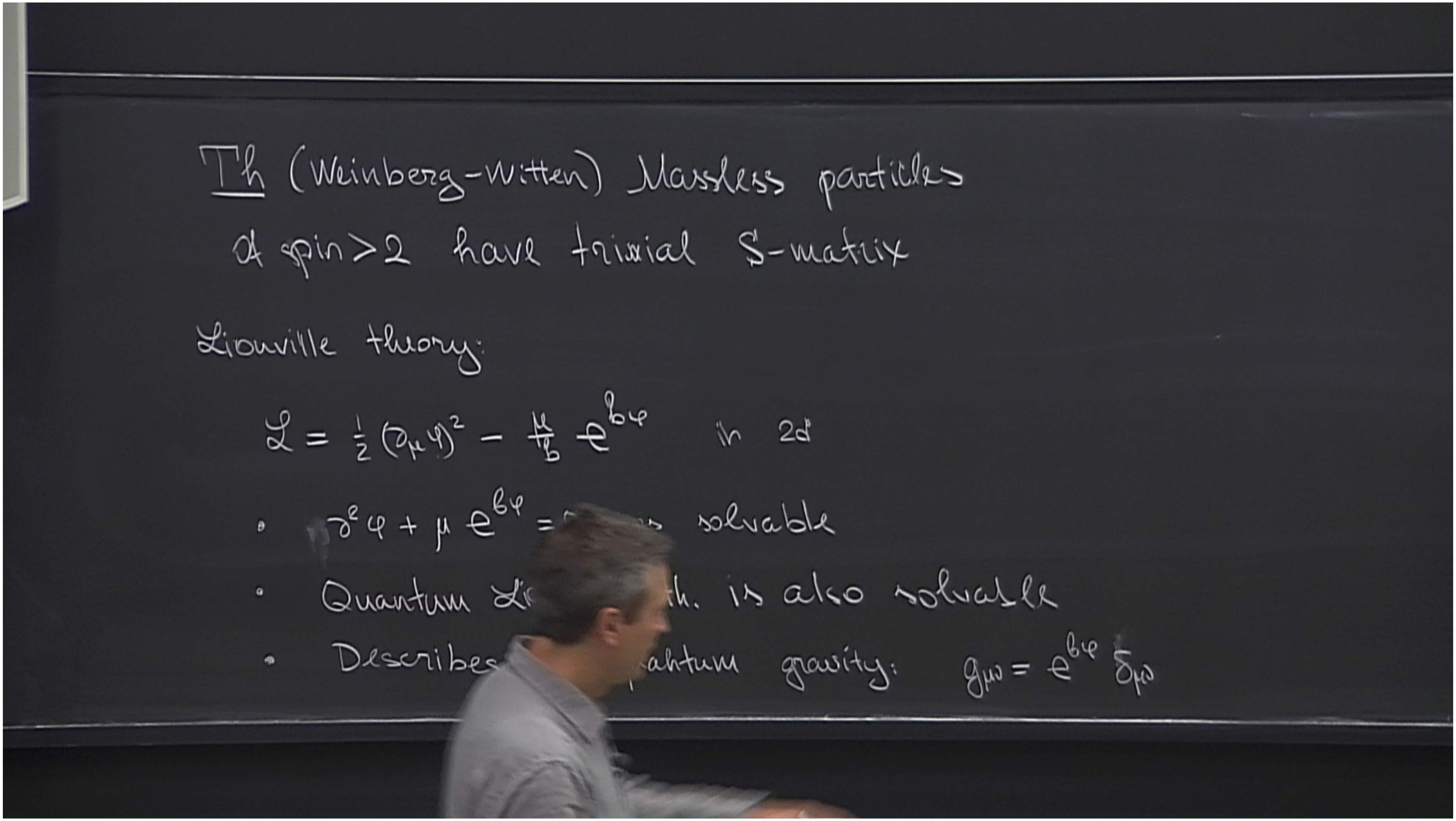

Liouville theory:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi}$ in 2d
• $\partial^2 \varphi + \mu e^{b\varphi} = 0$ is solvable
• Quantum Liouville th. is also solvable

Th (Weinberg-Witten) Massless particles of spin > 2 have trivial S-matrix

Liouville theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi} \quad \text{in } 2d$$

- $\partial^2 \varphi + \mu e^{b\varphi} = 0$ … solvable
- Quantum Liou… is also solvable
- Describes … quantum gravity: $g_{\mu\nu} =$

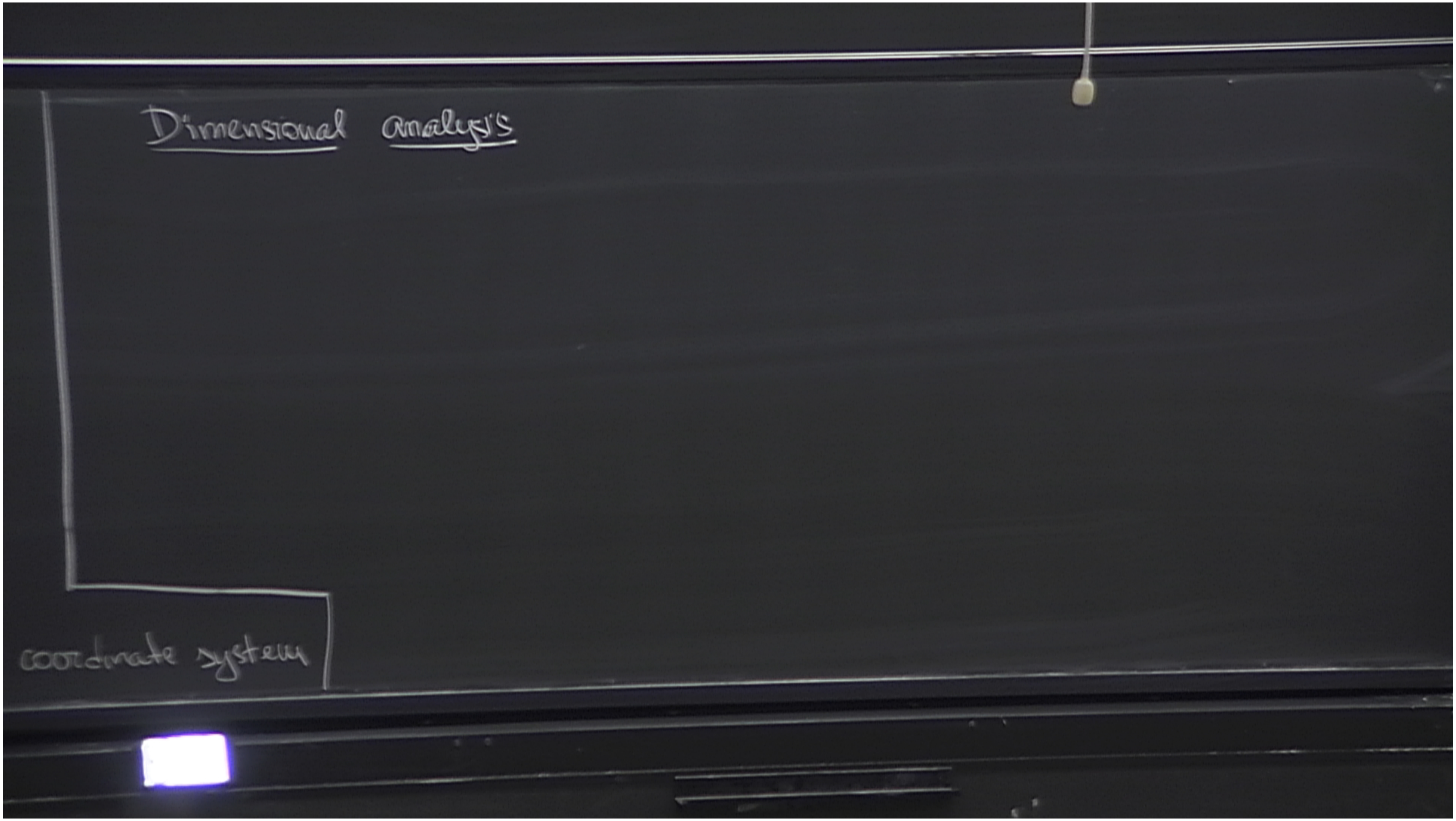

Th (Weinberg-Witten) Massless particles
of spin>2 have trivial S-matrix
Liouville theory:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi}$ in 2d
$\partial^2 \varphi + \mu e^{b\varphi} =$
solvable
Quantum
h. is also solvable
Describes
antum gravity: $g_{\mu\nu} = e^{b\varphi} \delta_{\mu\nu}$

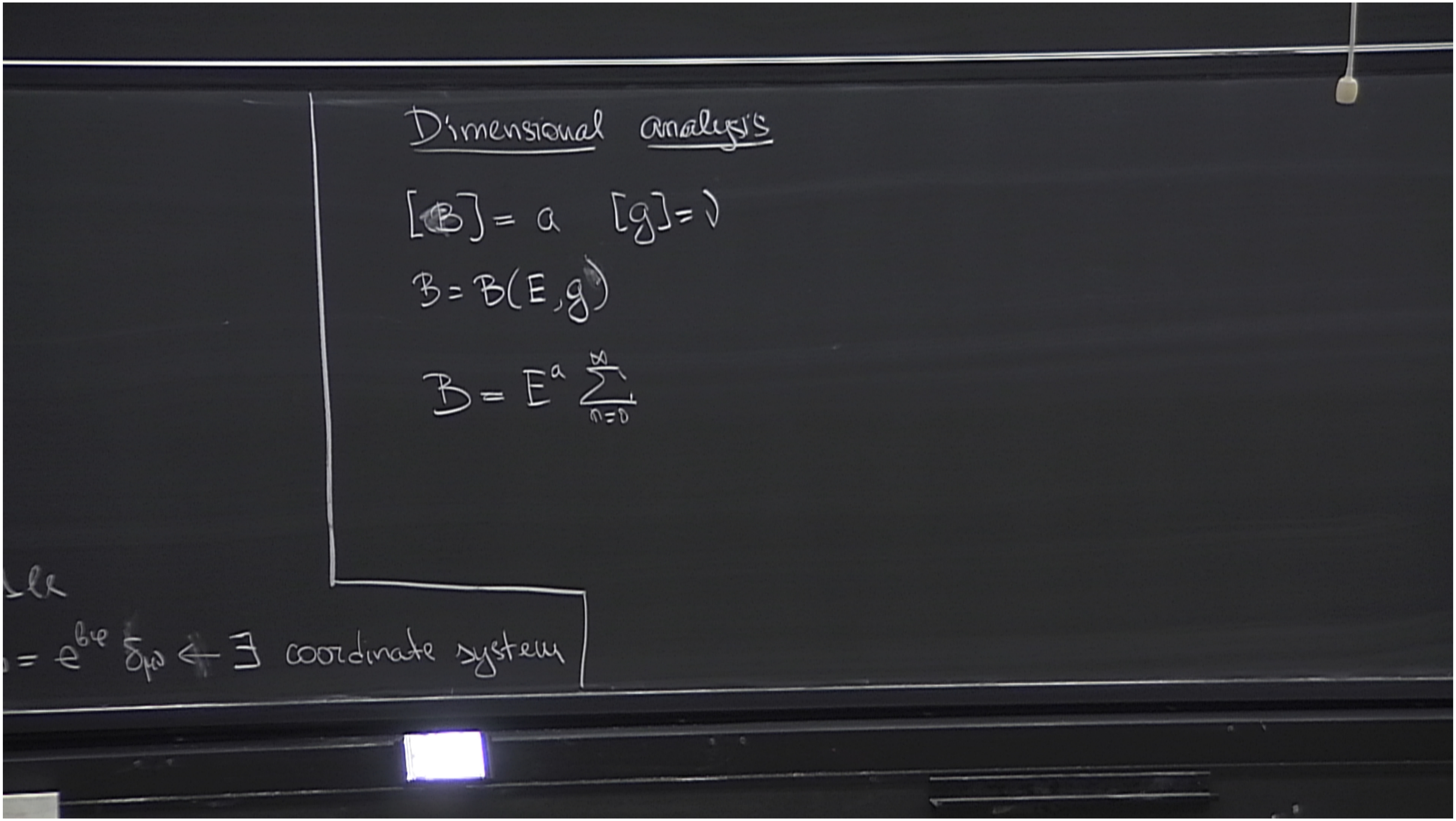
Th (Weinberg-Witten) Massless particles of spin > 2 have trivial S-matrix

Liouville theory:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\mu}{b} e^{b\varphi} \quad \text{in } 2d$$

- $\partial^2 \varphi + \mu e^{b\varphi} = 0$ is solvable
- Quantum Liouville th. is also solvable
- Describes 2d quantum gravity: $g_{\mu\nu} = e^{b\varphi} \delta_{\mu\nu} \leftarrow \exists$ coordinate system


Dimensional analysis
coordinate system


Dimensional analysis
[B] = a [g] = 1
B = B(E, g)
B = E^a Σ_{n=0}^∞
= e^{bφ} δ_{μν} ← ∃ coordinate system

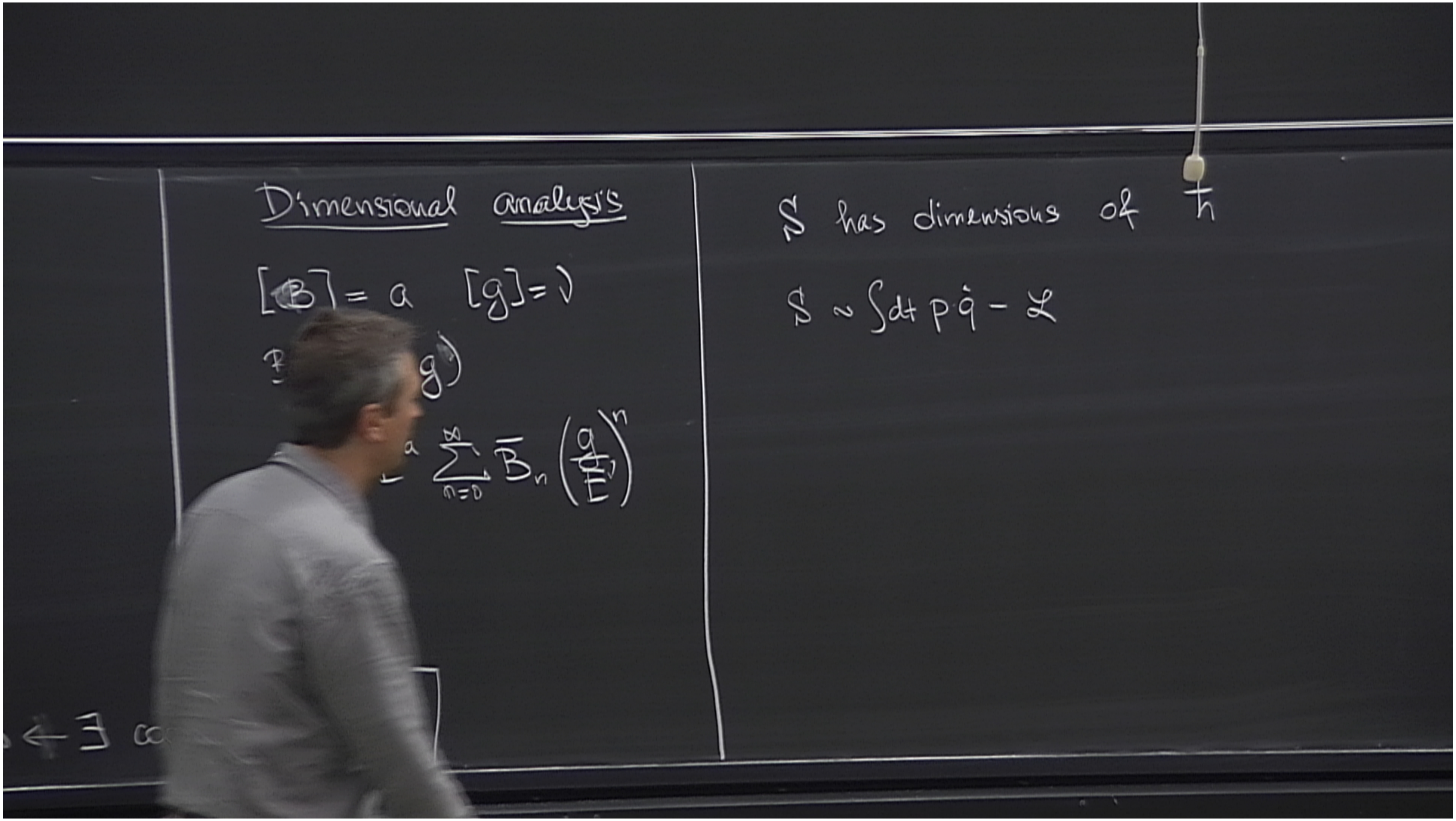
Dimensional analysis

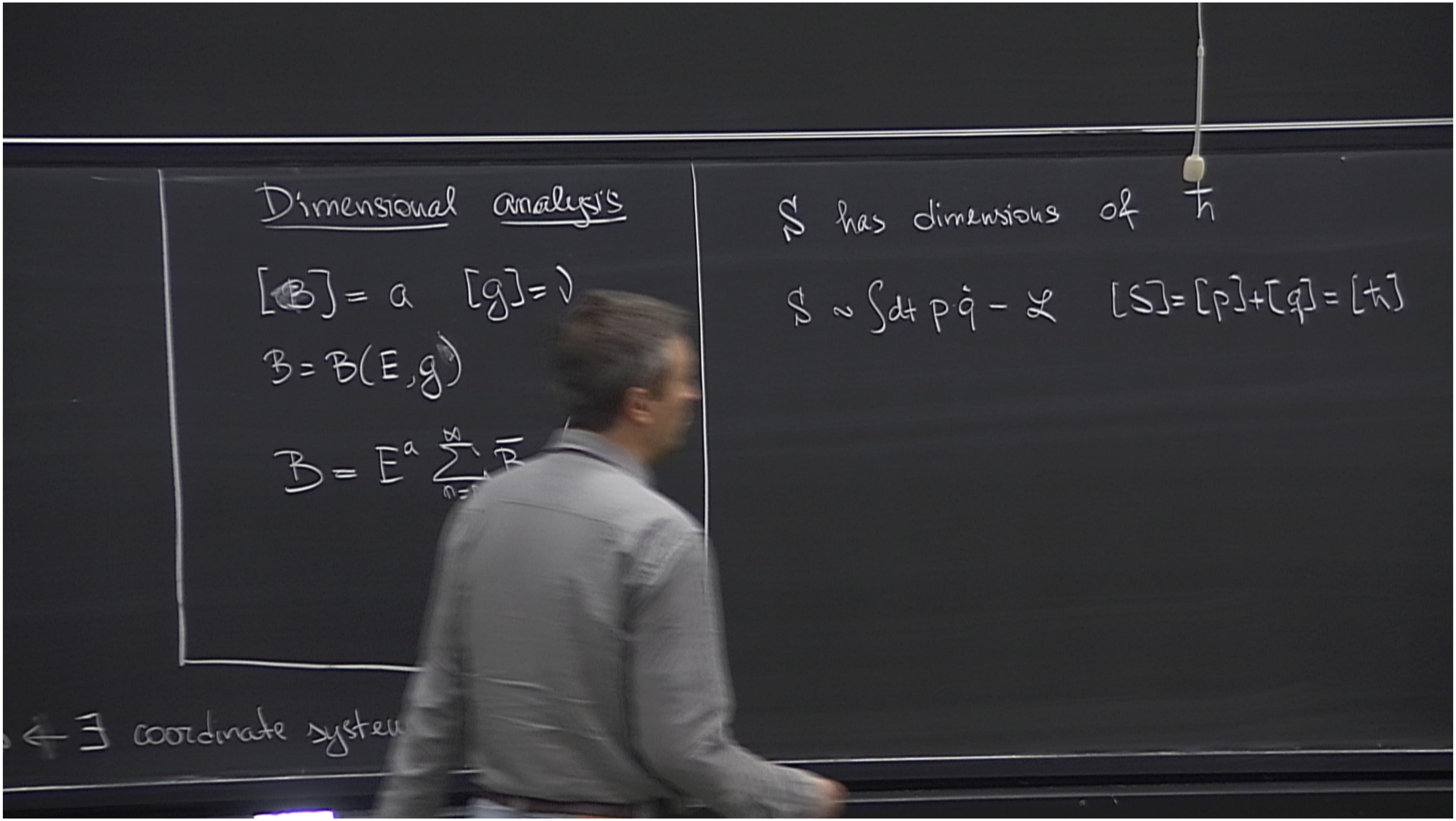
$[B] = a \qquad [g] = 1$

$B = B(E, g)$

$$B = E^a \sum_{n=0}^{\infty} \bar{B}_n \left(\frac{g}{E}\right)^n$$

$\leftarrow \exists$ coordinate system


Dimensional analysis
[B] = a   [g] = ν
S has dimensions of ħ
S ~ ∫dt p·q̇ − ℒ


Dimensional analysis
[B] = a   [g] = ν
B = B(E, g)
S has dimensions of ħ
S ~ ∫dt p·q̇ − L   [S] = [p] + [q] = [ħ]
∃ coordinate

nsional analysis

$= a \quad [g] = 1$

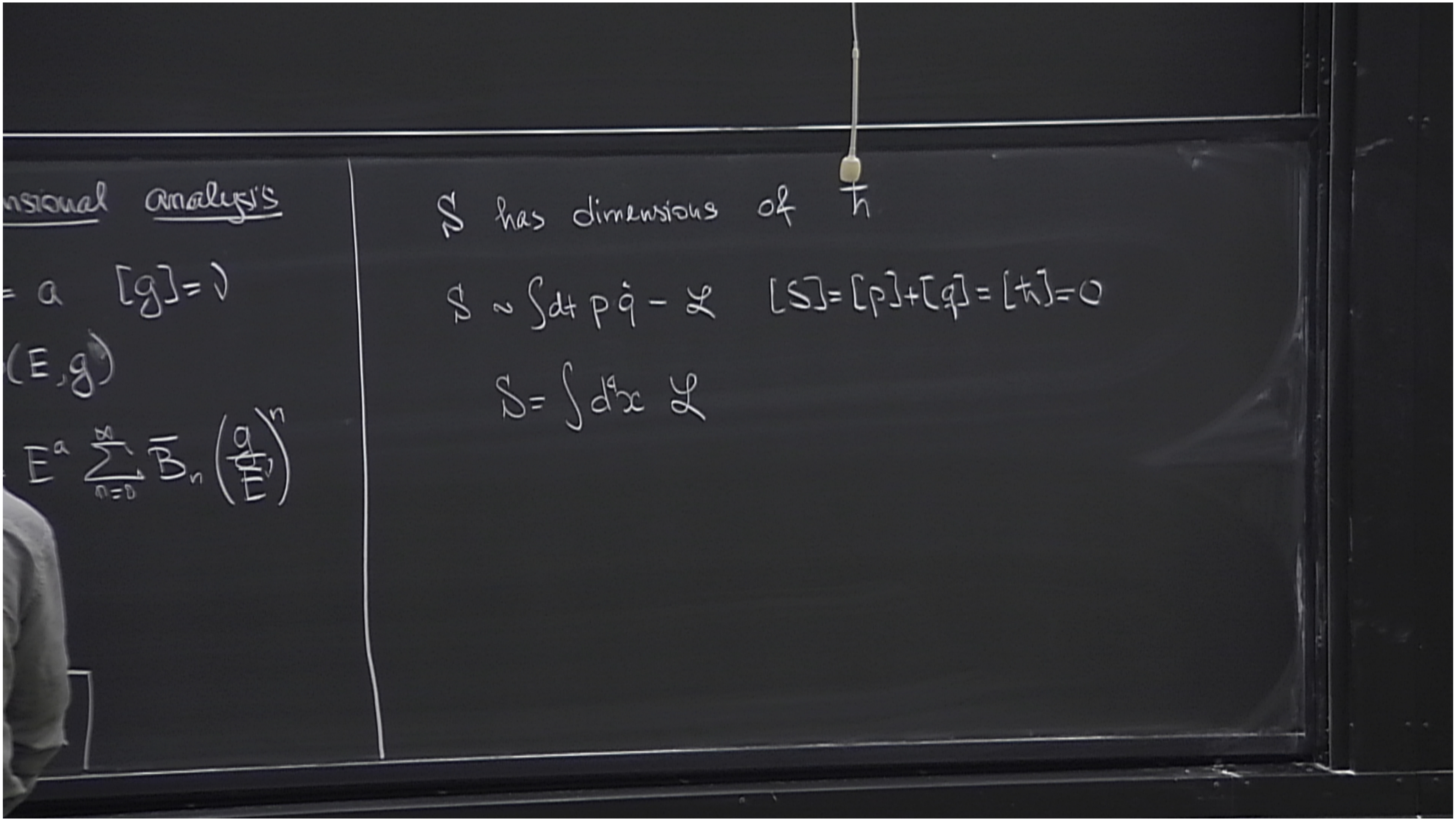
(E, g)

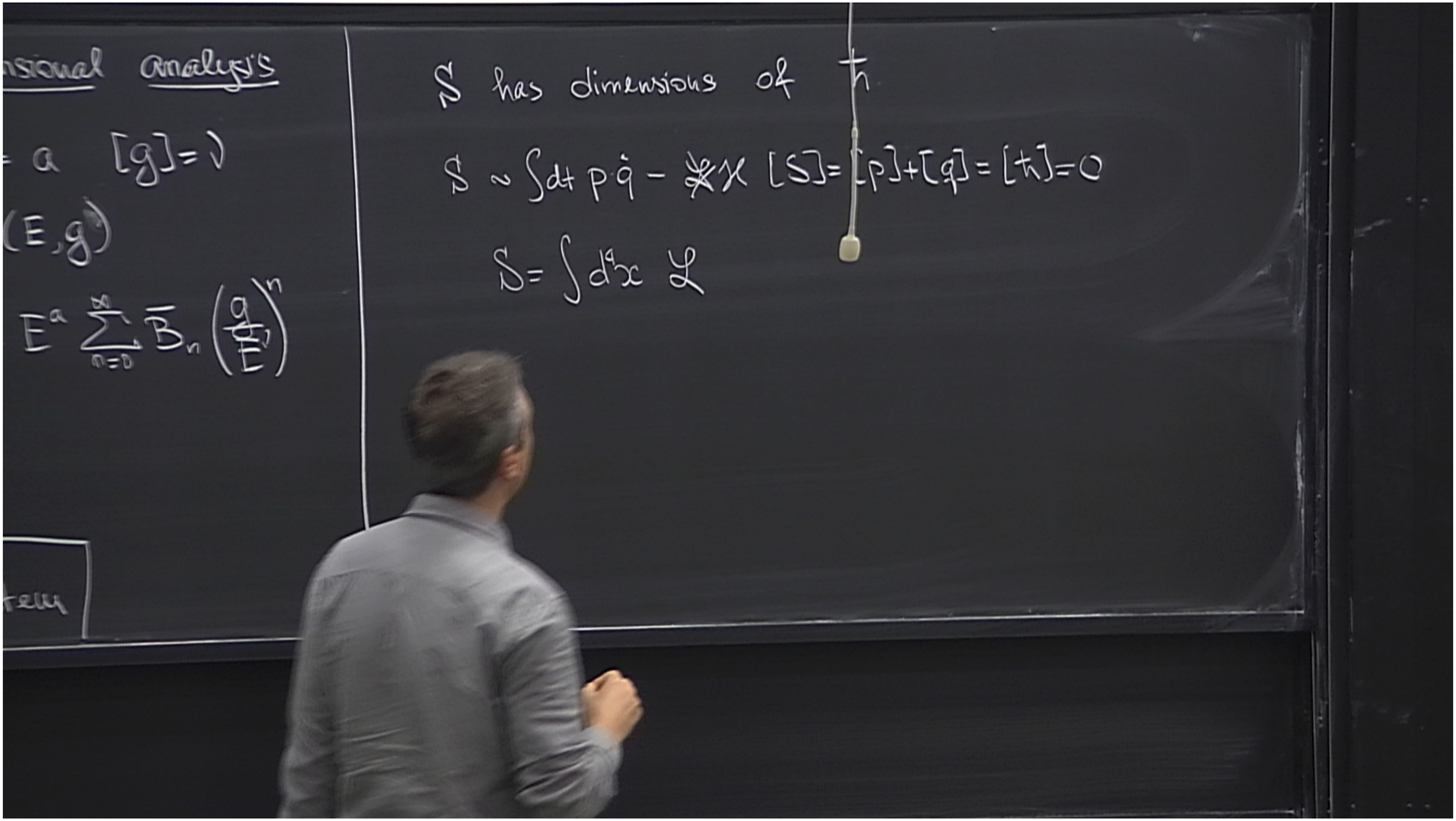
$E^a \sum_{n=0}^{\infty} \bar{B}_n \left(\frac{g}{E}\right)^n$

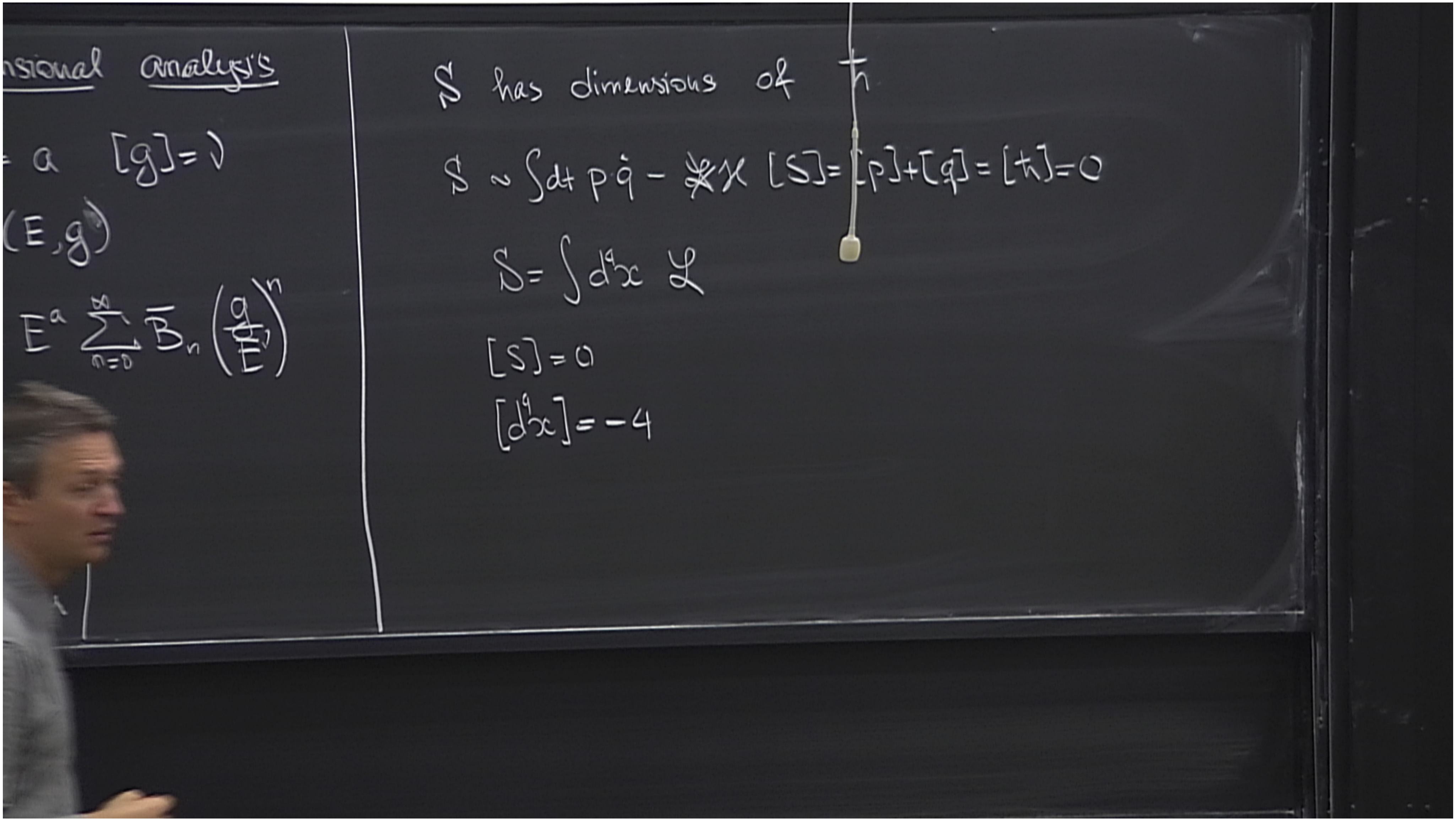
tem

S has dimensions of $\hbar$

$S \sim \int dt\, p\dot{q} - \mathcal{L} \quad [S] = [p] + [q] = [\hbar] = 0$


nsional analysis
= a [g]=1
(E,g)
E^a Σ_{n=0}^∞ B̄_n (g/E)^n
S has dimensions of ħ
S ~ ∫dt p q̇ − L
[S]=[p]+[q]=[ħ]=0
S=∫d^4x L


S has dimensions of ħ
S ~ ∫dt p q̇ − H [S] = [p] + [q] = [ħ] = 0
S = ∫d^4x ℒ
analysis
[g] = ?
(E, g)
E^a Σ_{n=0}^∞ B̄_n (g/E)^n


analysis
(E, g)
$E^a \sum_{n=0}^{\infty} \bar{B}_n \left(\frac{g}{E}\right)^n$
S has dimensions of $\hbar$
$S \sim \int dt\, p\dot{q} - H \quad [S] = [p] + [q] = [\hbar] = 0$
$S = \int d^4x\, \mathcal{L}$
$[S] = 0$
$[d^4x] = -4$

nsional analysis

$= a \quad [g] = ?$

(E, g)

$E^a \sum_{n=0}^{\infty} \bar{B}_n \left(\frac{g}{E^?}\right)^n$

tem

S has dimensions of $\hbar$

$S \sim \int dt \, p\dot{q} - \cancel{*}\mathcal{H} \quad [S] = [p] + [q] = [\hbar] = 0$

$S = \int d^4x \, \mathcal{L}$

$[S] = 0$

$[d^4x] = -4 \quad \Rightarrow [\mathcal{L}] = 4$

analysis

$[g] = 0$

$\bar{B}_n \left(\frac{g}{E}\right)^n$

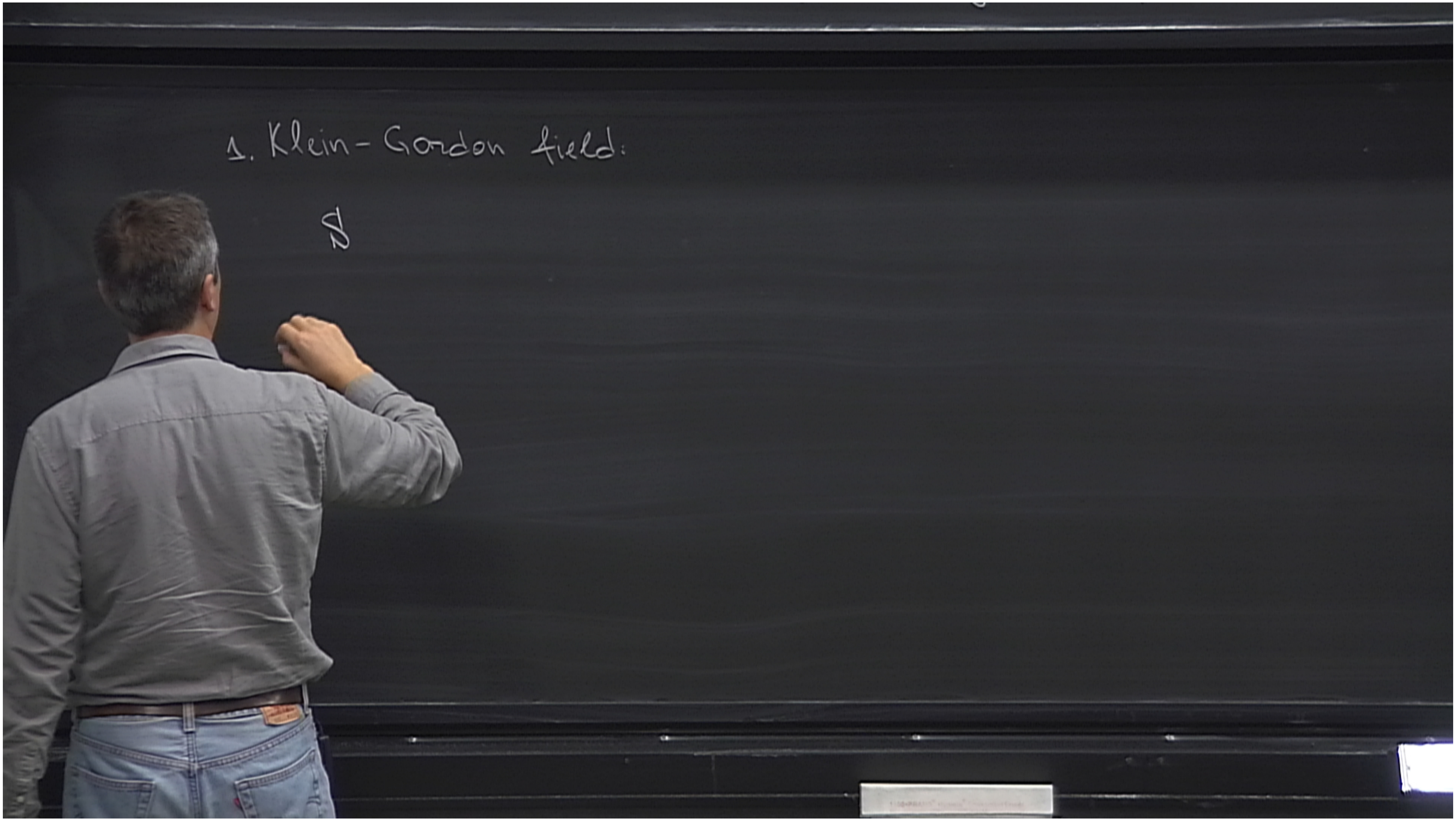
S has dimensions of $\hbar$

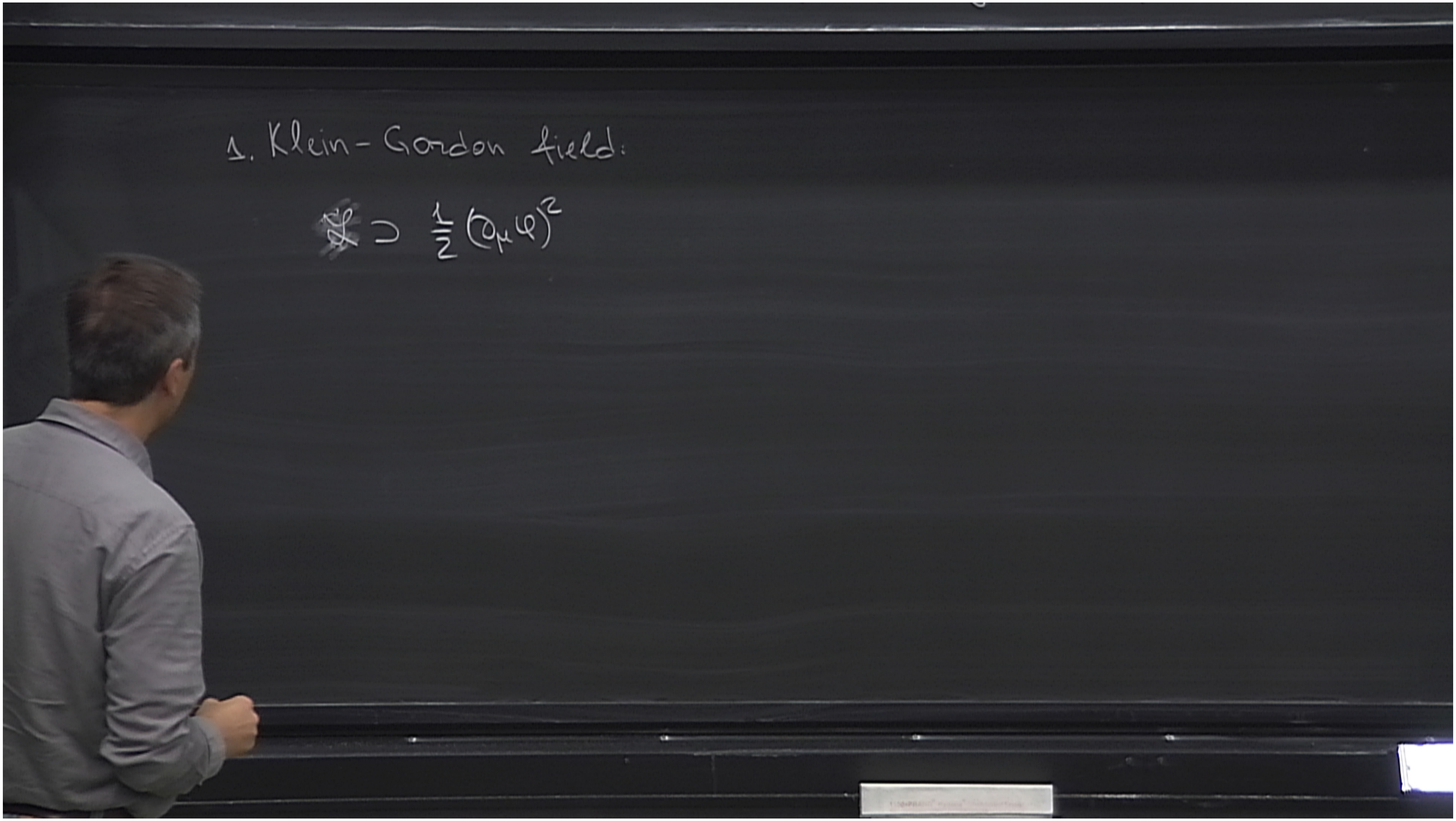
$$S \sim \int dt \; p \cdot \dot{q} - \cancel{H} \mathcal{H} \qquad [S] = [p] + [q] = [\hbar] = 0$$

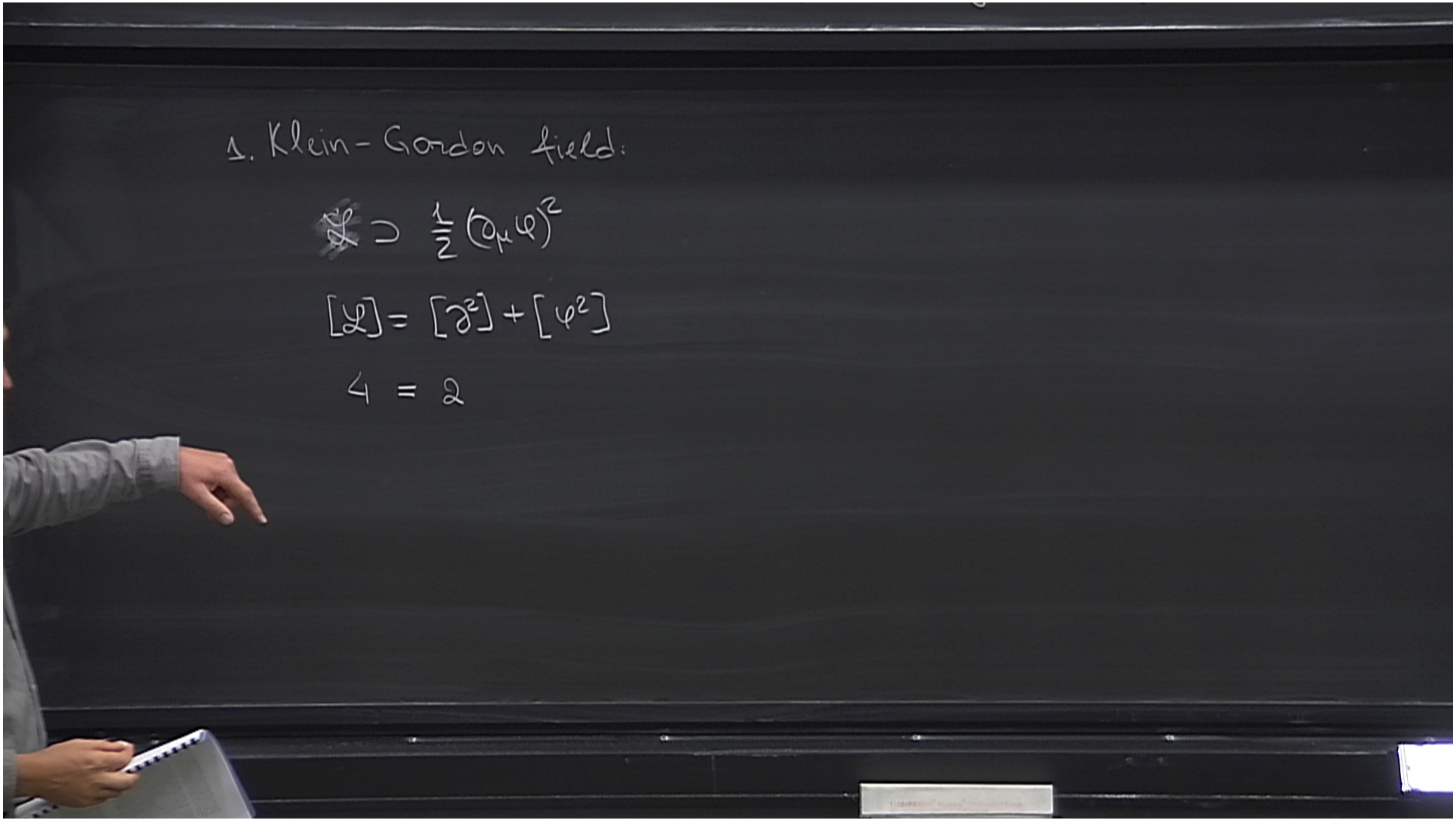
$$S = \int d^4x \; \mathcal{L}$$

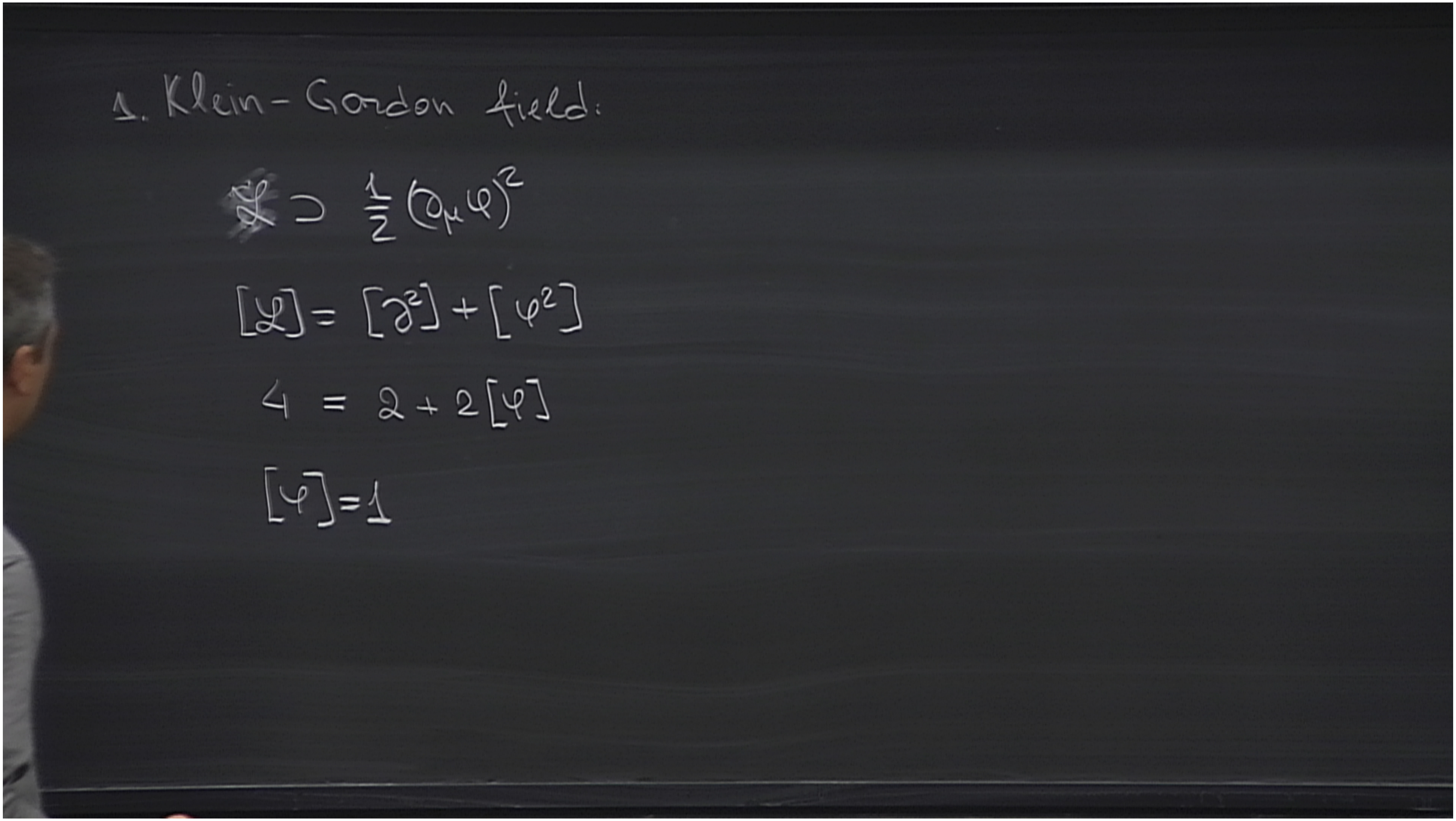
$$[S] = 0$$

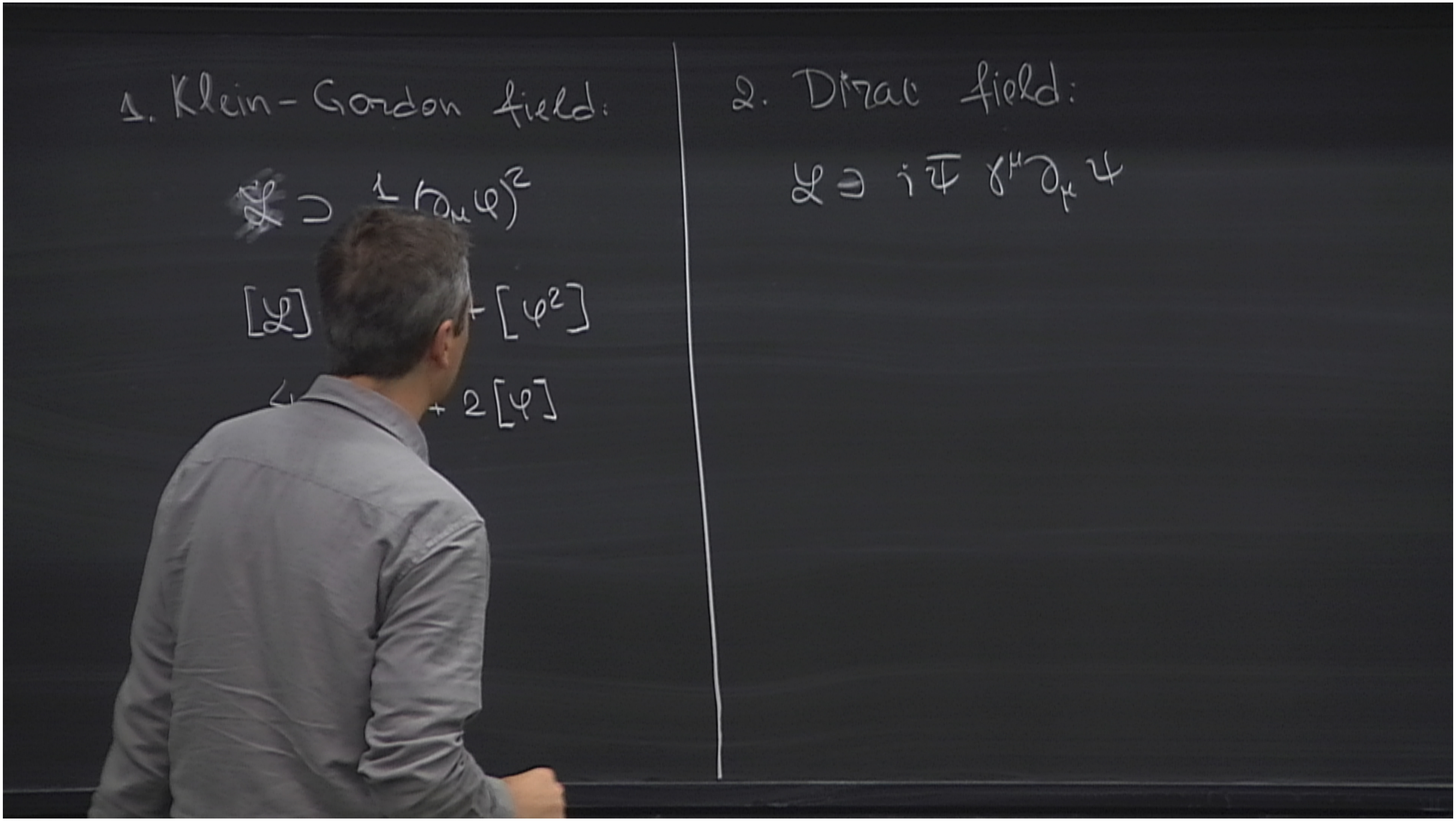
$$[d^4x] = -4 \qquad \Rightarrow \qquad [\mathcal{L}] = 4$$

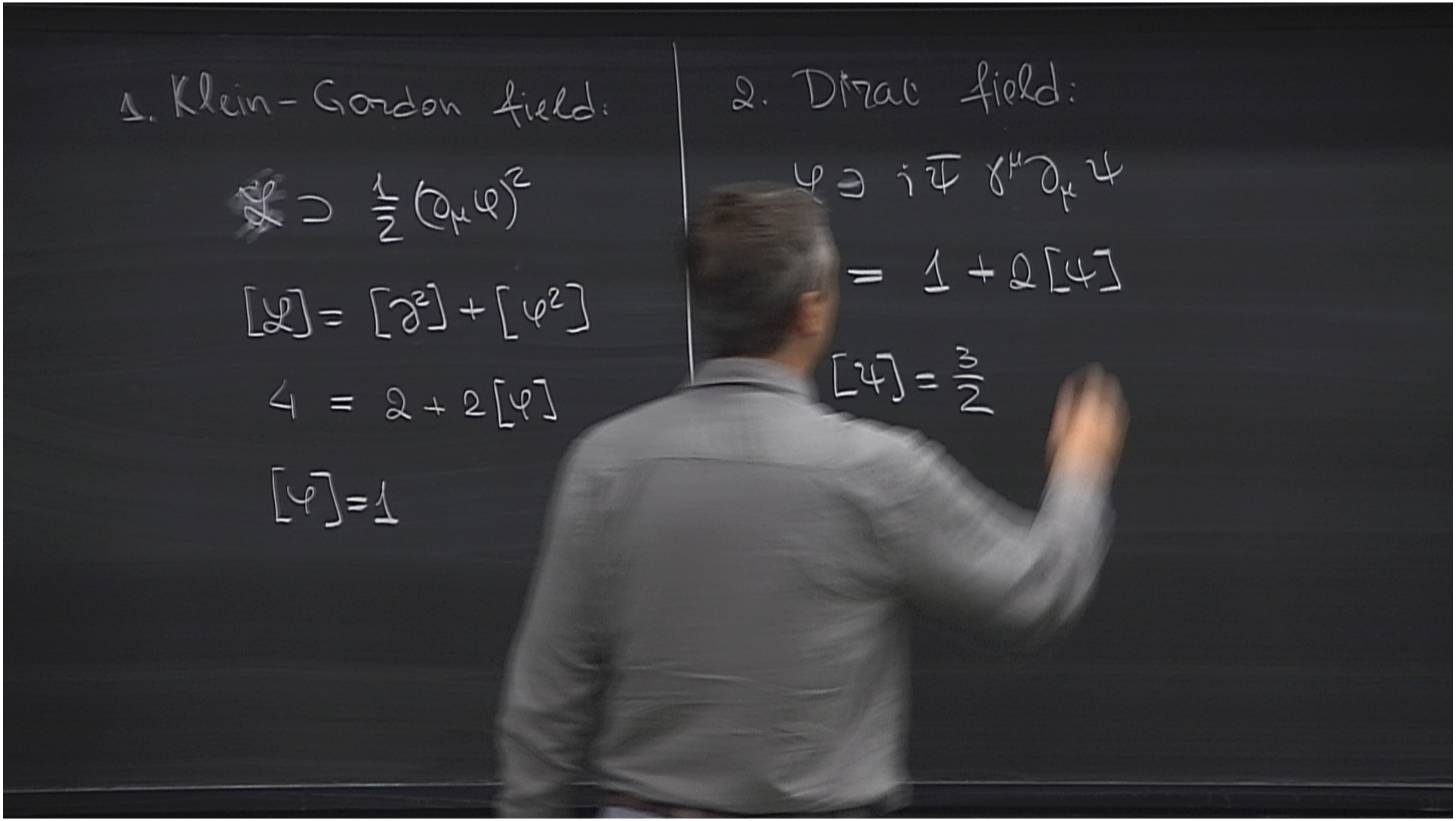

1. Klein-Gordon field:
S

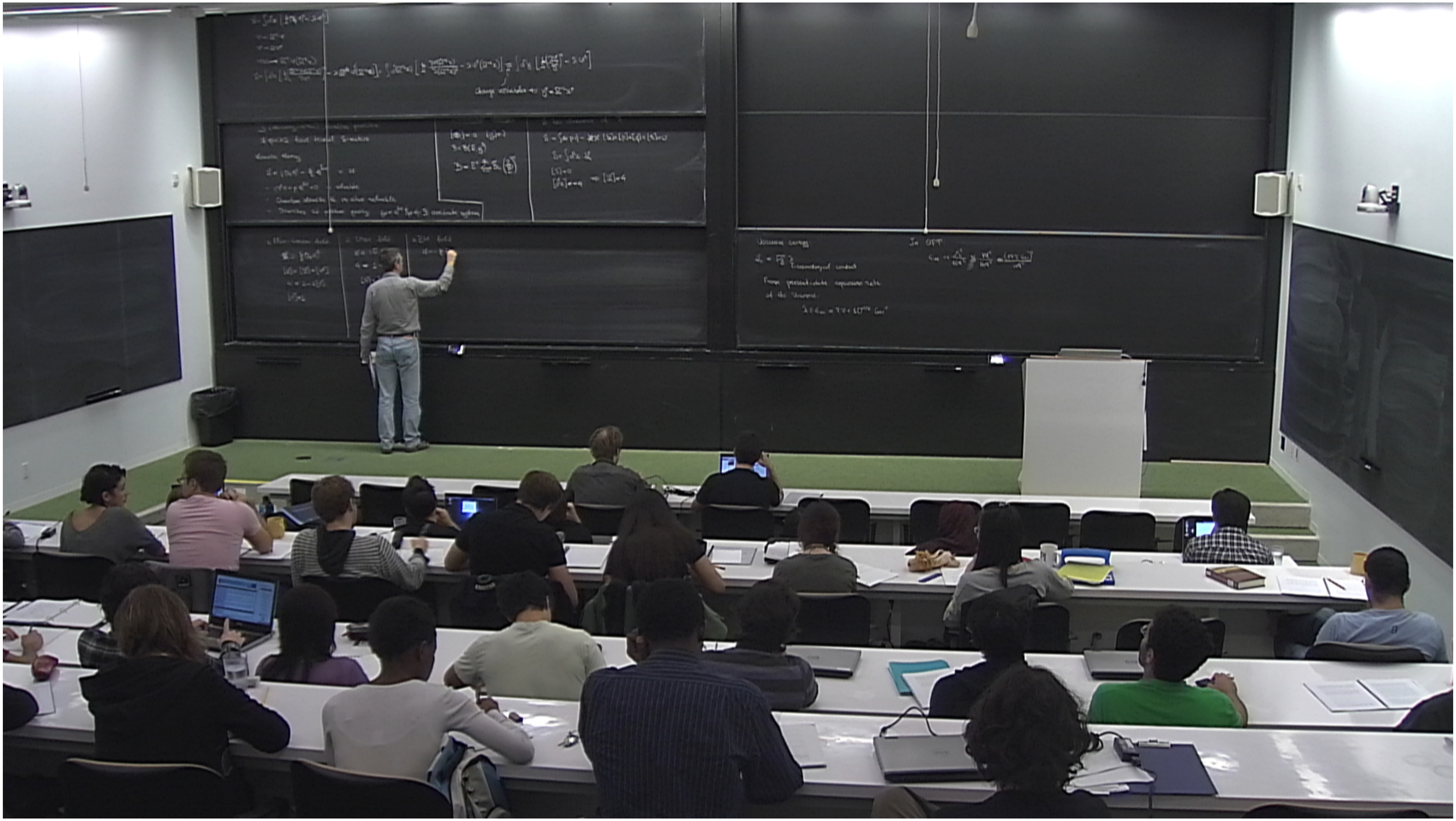

1. Klein-Gordon field:
$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \varphi)^2$

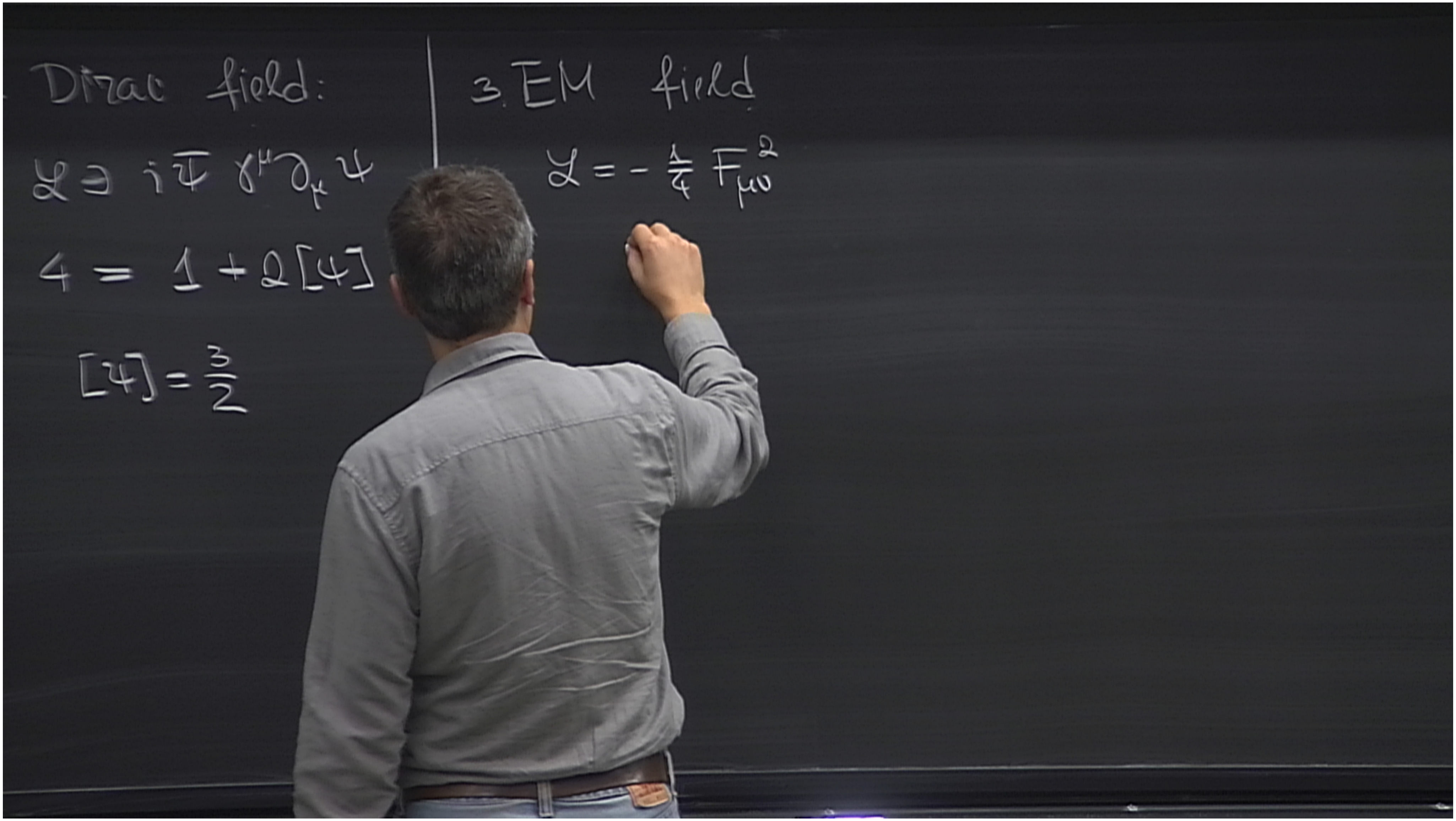

1. Klein-Gordon field:
$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \varphi)^2$
$[\mathcal{L}] = [\partial^2] + [\varphi^2]$
$4 = 2$


1. Klein-Gordon field:
$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \varphi)^2$
$[\mathcal{L}] = [\partial^2] + [\varphi^2]$
$4 = 2 + 2[\varphi]$
$[\varphi] = 1$


1. Klein-Gordon field:
2. Dirac field:
$\mathcal{L} \ni i\bar{\Psi}\gamma^\mu\partial_\mu\Psi$


1. Klein-Gordon field:
$\mathcal{L} \supset \frac{1}{2}(\partial_\mu \varphi)^2$
$[\mathcal{L}] = [\partial^2] + [\varphi^2]$
$4 = 2 + 2[\varphi]$
$[\varphi] = 1$
2. Dirac field:
$\mathcal{L} \ni i\bar{\Psi}\gamma^\mu \partial_\mu \Psi$
$= 1 + 2[\Psi]$
$[\Psi] = \frac{3}{2}$




Dirac field:
$\mathcal{L} \ni i\bar{\Psi} \gamma^\mu \partial_\mu \psi$
$4 = 1 + 2[\psi]$
$[\psi] = \frac{3}{2}$
3. EM field
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$

Dirac field:

$$\mathcal{L} \ni i\bar{\Psi}\gamma^\mu \partial_\mu \Psi$$

$$4 = 1 + 2[\Psi]$$

$$[\Psi] = \frac{3}{2}$$

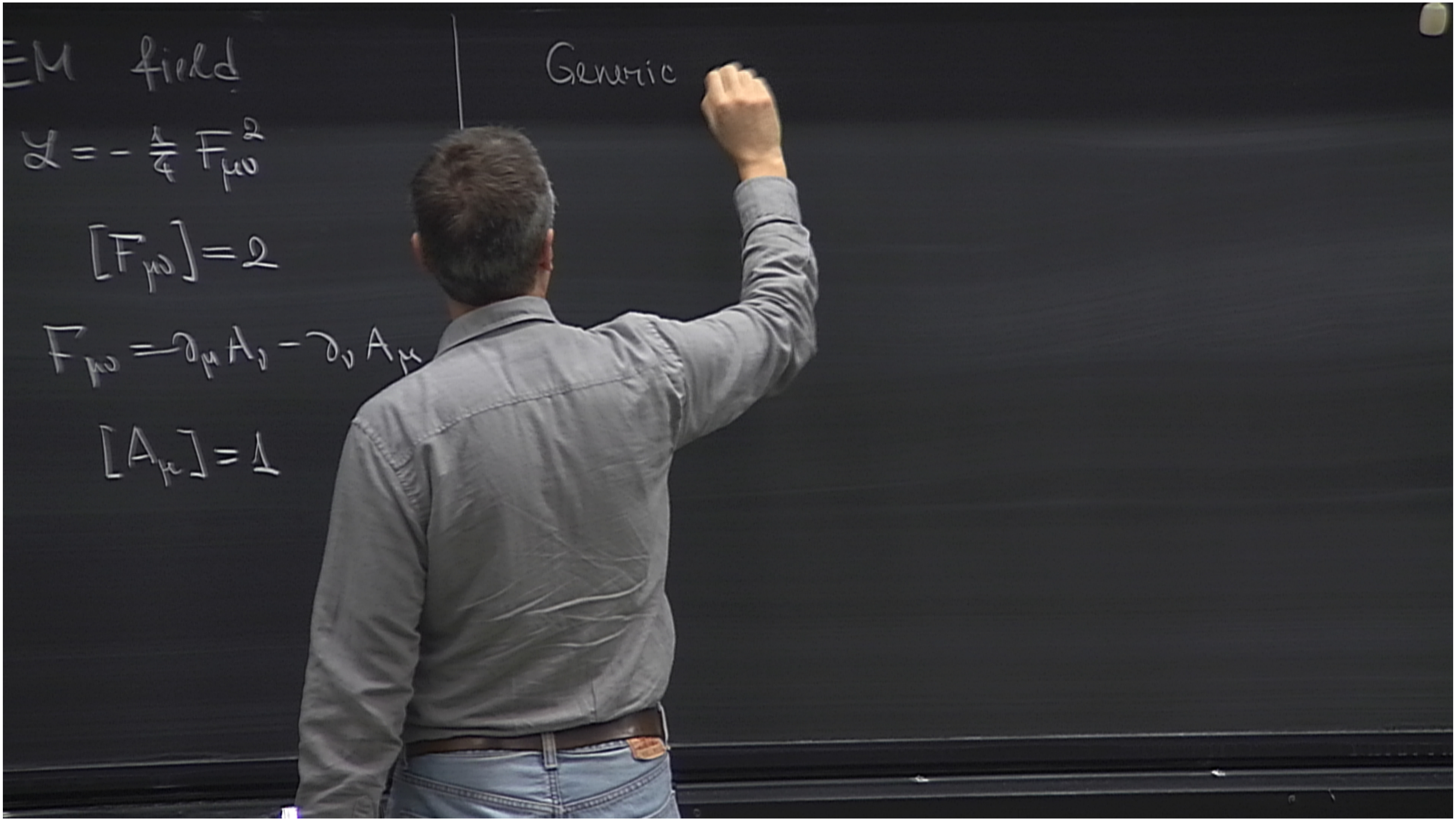
3. EM field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$$

$$[F_{\mu\nu}] = 2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$[A_\mu] = 1$$


EM field
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$
$[F_{\mu\nu}] = 2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$[A_\mu] = 1$
Generic

EM field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$$

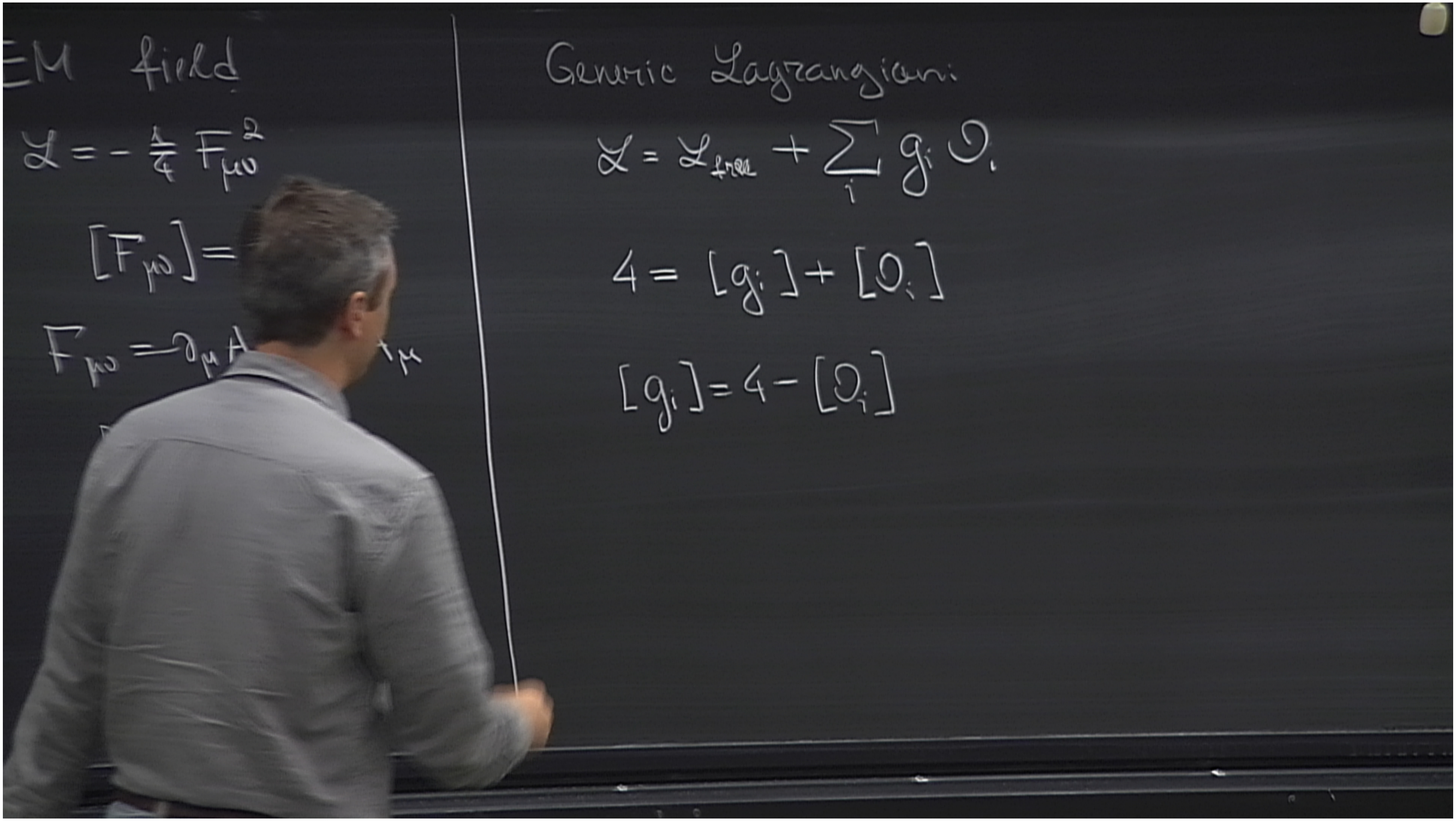
$$[F_{\mu\nu}] = 2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$[A_\mu] = 1$$

Generic Lagrangian:

$$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i \mathcal{O}_i$$


EM field
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$
$[F_{\mu\nu}] =$
$F_{\mu\nu} = \partial_\mu A$
Generic Lagrangian:
$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i \mathcal{O}_i$
$4 = [g_i] + [\mathcal{O}_i]$
$[g_i] = 4 - [\mathcal{O}_i]$

EM field

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2$$

$$[F_{\mu\nu}] = 2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$[A_\mu] = 1$$

Generic Lagrangian:

$$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i \mathcal{O}_i$$

$$4 = [g_i] + [\mathcal{O}_i]$$

$$\boxed{[g_i] = 4 - [\mathcal{O}_i]}$$

Generic Lagrangian:

$$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i \mathcal{O}_i$$

$$4 = [g_i] + [\mathcal{O}_i]$$

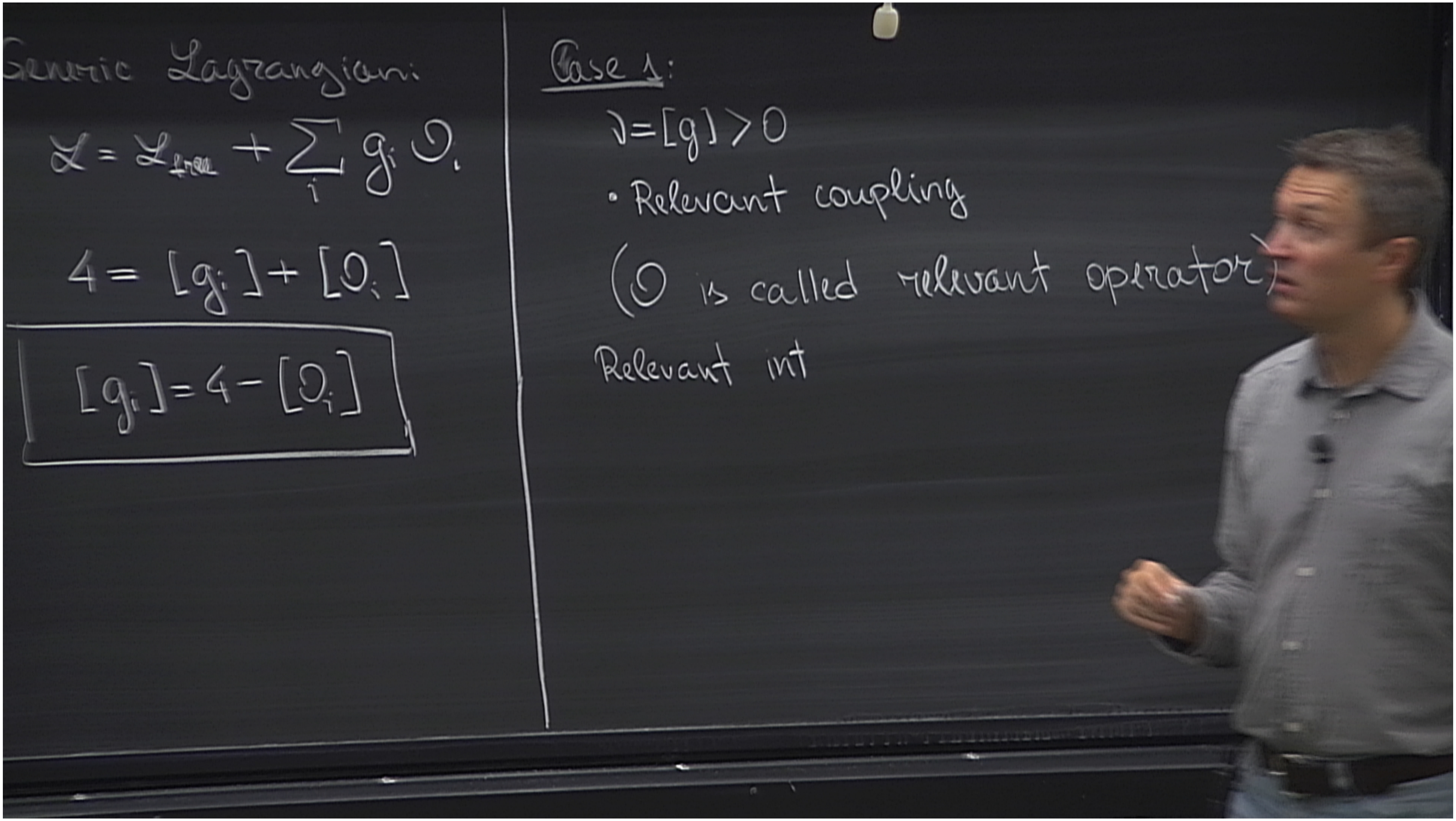
$$\boxed{[g_i] = 4 - [\mathcal{O}_i]}$$

Case 1:

$\nu = [g] > 0$

- Relevant coupling

($\mathcal{O}$ is called relevant operator)


Generic Lagrangian:
$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i \mathcal{O}_i$
$4 = [g_i] + [\mathcal{O}_i]$
$\boxed{[g_i] = 4 - [\mathcal{O}_i]}$
Case 1:
$\nu = [g] > 0$
• Relevant coupling
($\mathcal{O}$ is called relevant operator)
Relevant int

Generic Lagrangian:

$$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i \mathcal{O}_i$$

$$4 = [g_i] + [\mathcal{O}_i]$$

$$\boxed{[g_i] = 4 - [\mathcal{O}_i]}$$

<u>Case 1</u>:

$\Delta = [g] > 0$

- Relevant coupling

($\mathcal{O}$ is called relevant operator)

Relevant interactions are weak as long as $g^{1/\Delta} \ll E$

Generic Lagrangian:

$$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i \mathcal{O}_i$$

$$4 = [g_i] + [\mathcal{O}_i]$$

$$\boxed{[g_i] = 4 - [\mathcal{O}_i]}$$

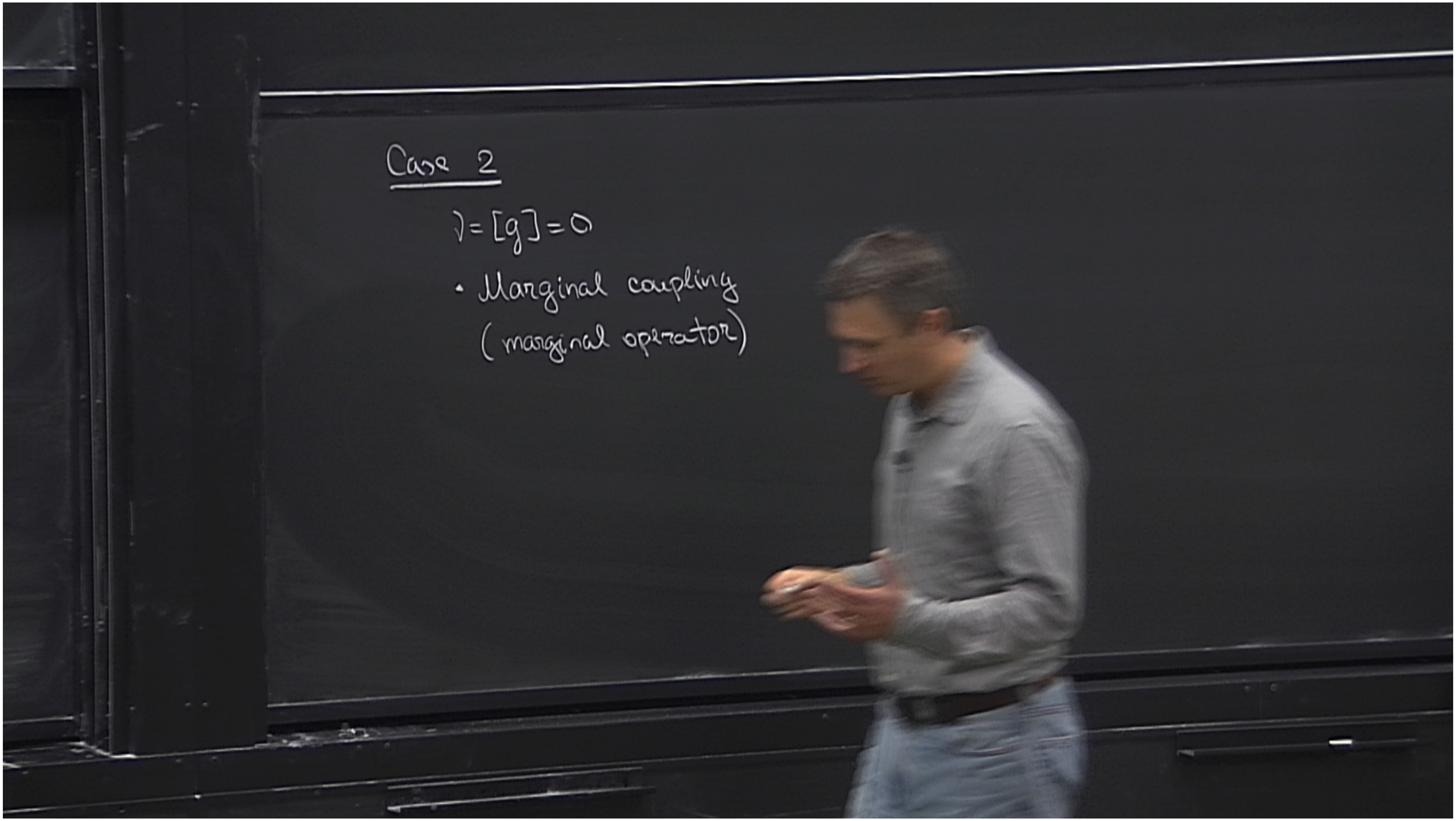
Case 1:

$\partial = [g] > 0$

- Relevant coupling

($\mathcal{O}$ is called relevant operator)

Relevant interactions are weak as long as $g^{1/\partial} \ll E$



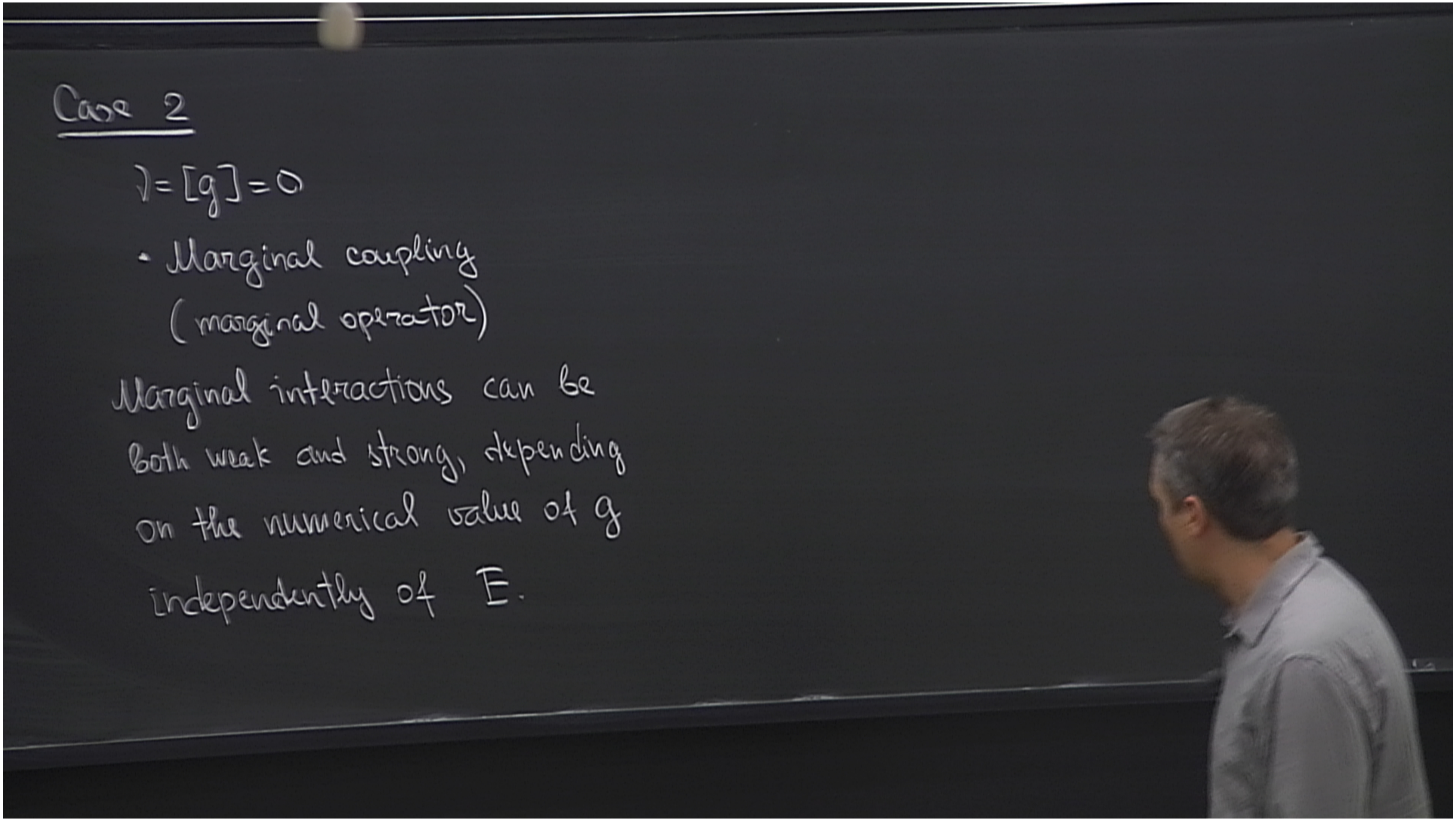
Case 2
$\lambda = [g] = 0$
• Marginal coupling
(marginal operator)

Case 2

$\lambda = [g] = 0$

- Marginal coupling
  (marginal operator)

Marginal interactions can be both weak and strong, depen


Case 2
$\lambda = [g] = 0$
• Marginal coupling
(marginal operator)
Marginal interactions can be
both weak and strong, depending
on the numerical value of g
independently of E.

Case 2

$\nu = [g] = 0$

- Marginal coupling
  (marginal operator)

Marginal interactions can be both weak and strong, depending on the numerical value of g independently of E.

Case 3

$\nu = [g] < 0$

- Irrelevant couplings
  (irrelevant operators)

Case 2

$\nu = [g] = 0$

- Marginal coupling (marginal operator)

Marginal interactions can be both weak and strong, depending on the numerical value of g independently of E.

Case 3

$\nu = [g] < 0$

- Irrelevant couplings (irrelevant operators)

Irrelevant interactions are weak at low energies

Case 2

$\nu = [g] = 0$

- Marginal coupling
  (marginal operator)

Marginal interactions can be both weak and strong, depending on the numerical value of g independently of E.

Case 3

$\nu = [g] < 0$

- Irrelevant couplings
  (irrelevant operators)

Irrelevant interactions are weak at low energies

Case 2

$\nu = [g] = 0$

- Marginal coupling
  (marginal operator)

Marginal interactions can be both weak and strong, depending on the numerical value of g independently of E.

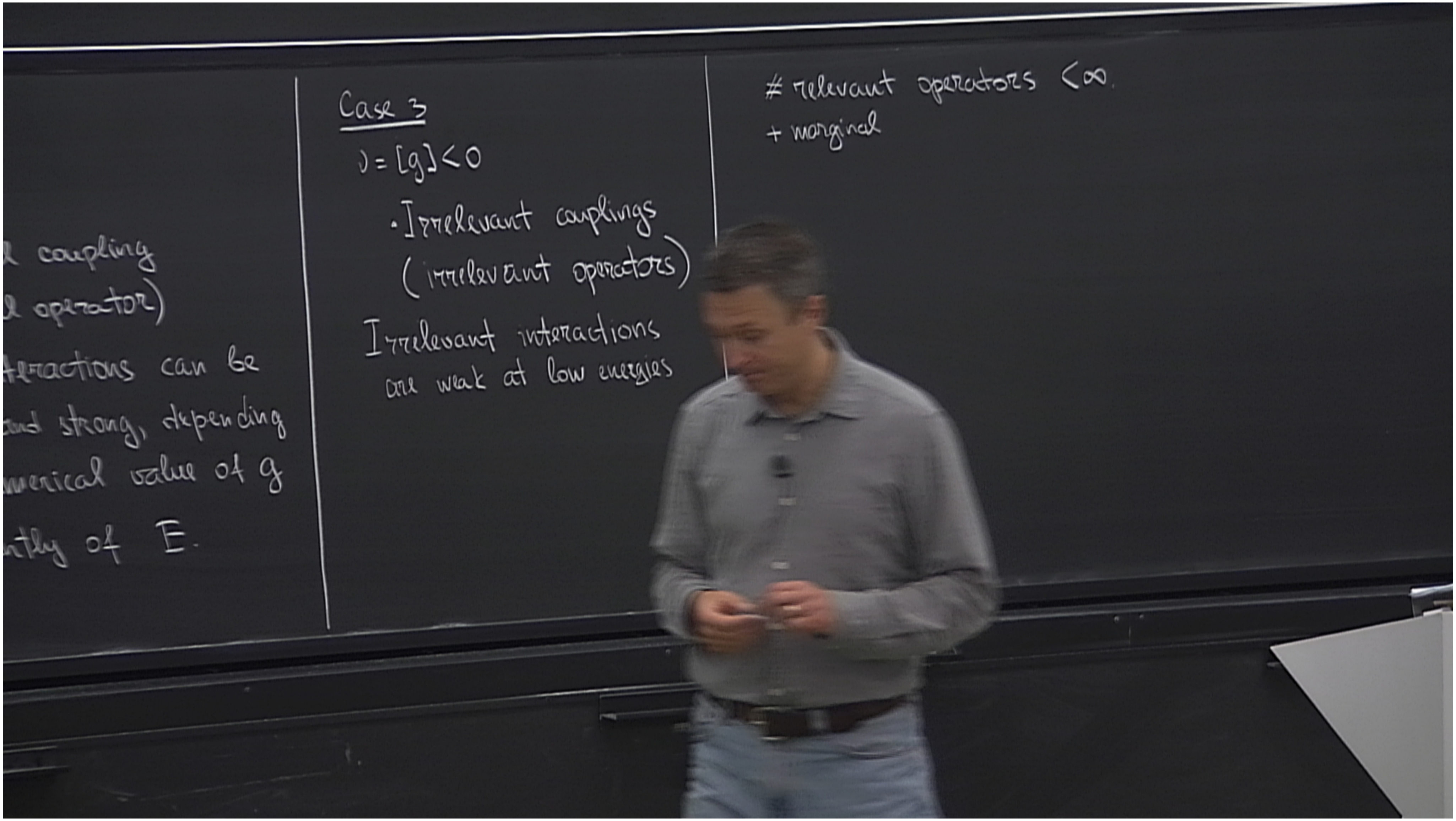
Case 3

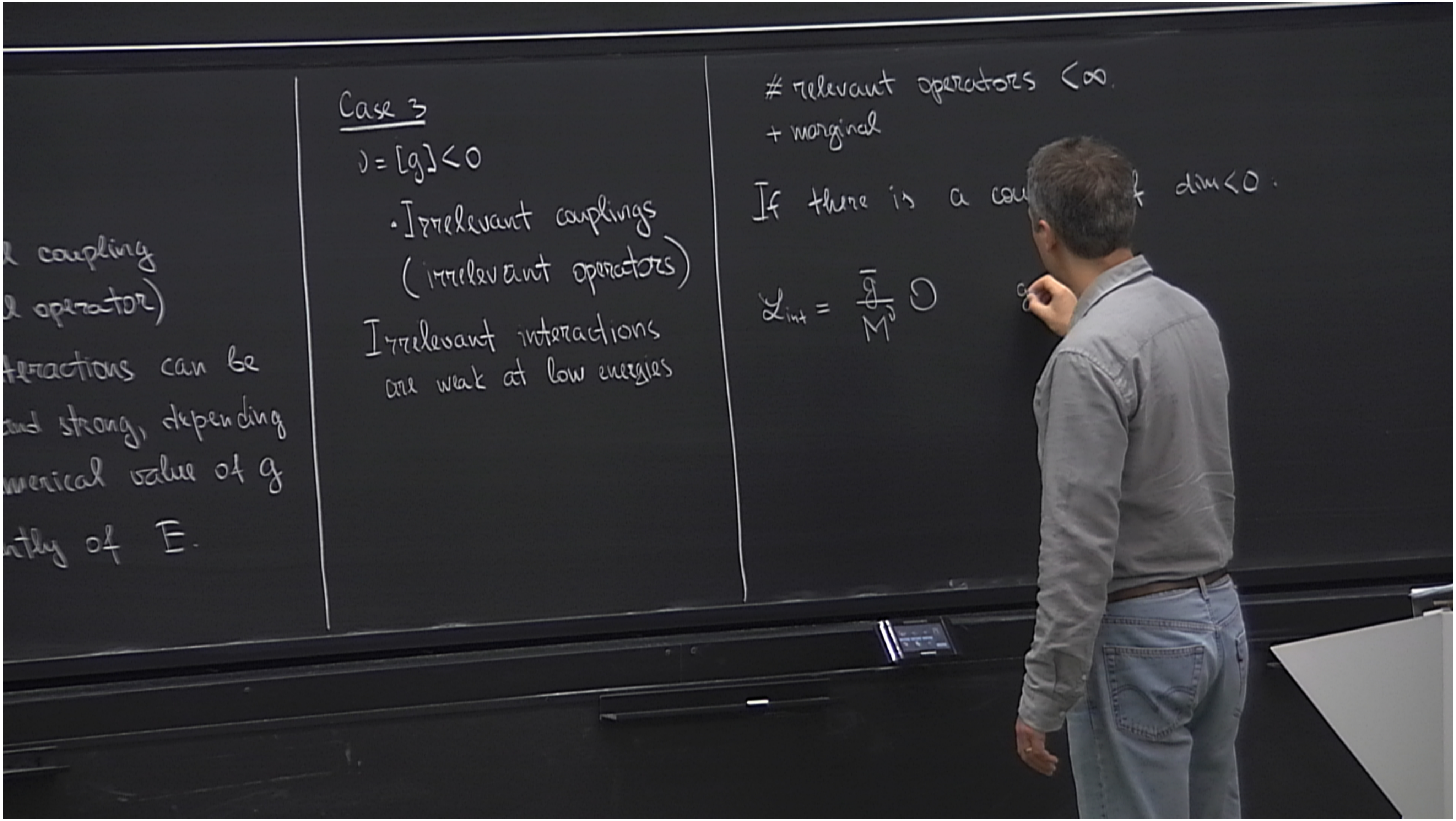
$\nu = [g] < 0$

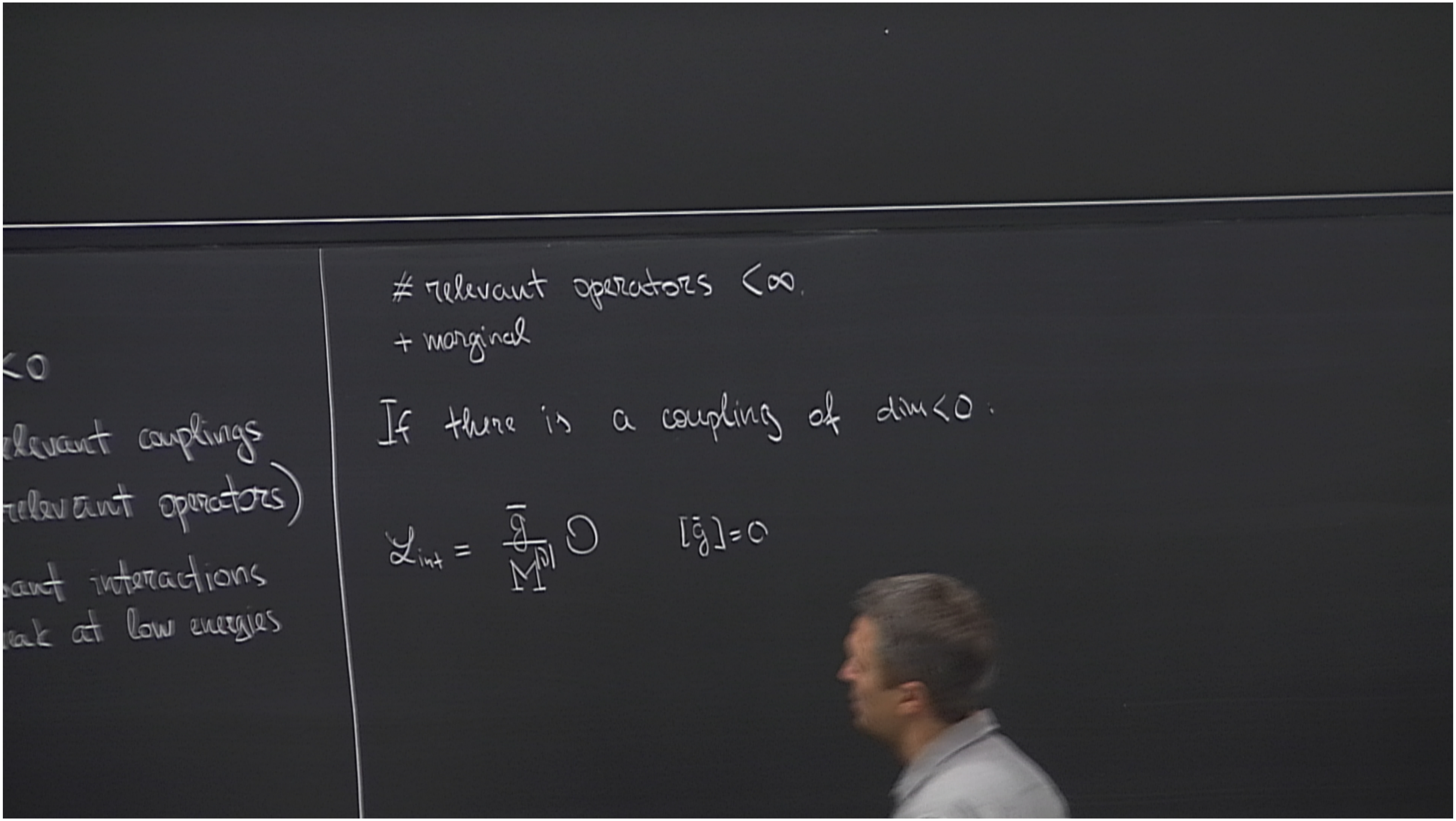
- Irrelevant couplings
  (irrelevant operators)

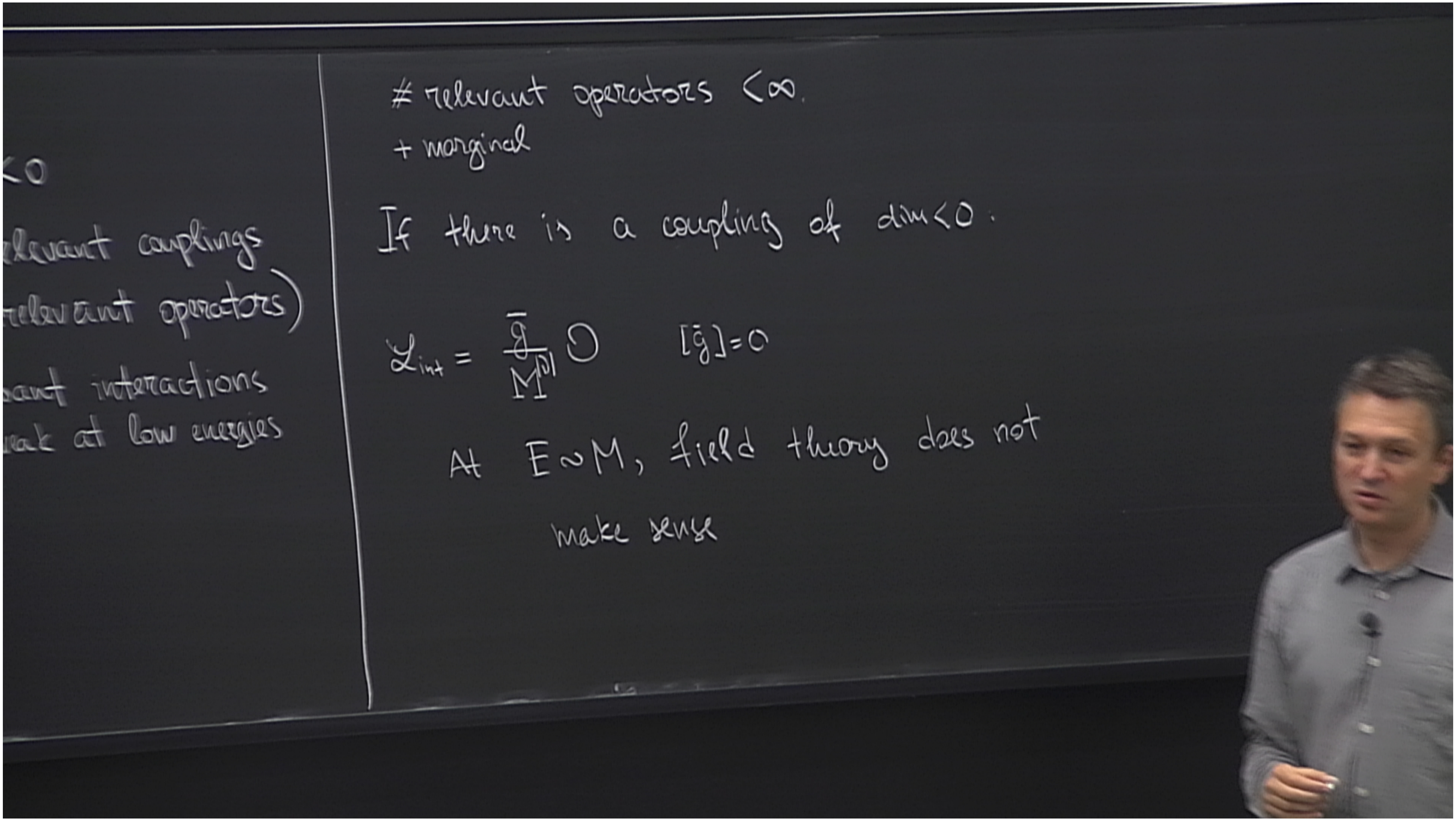
Irrelevant interactions are weak at low energies

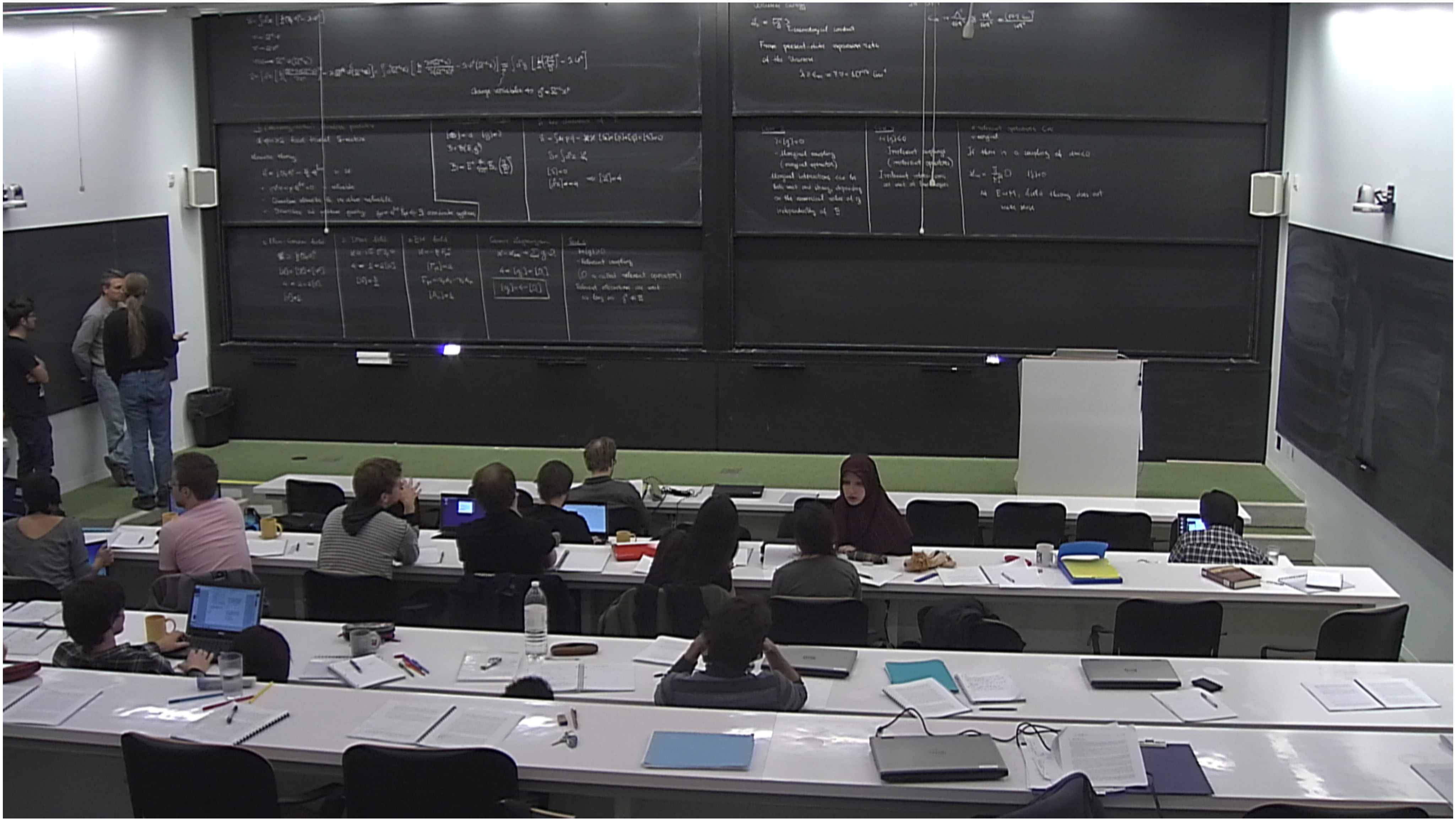
$\#$ relevant operators $< \infty$


Case 3
ν = [g] < 0
• Irrelevant couplings
(irrelevant operators)
Irrelevant interactions
are weak at low energies
relevant operators < ∞
+ marginal
coupling
operator)
teractions can be
and strong, depending
merical value of g
ntly of E.

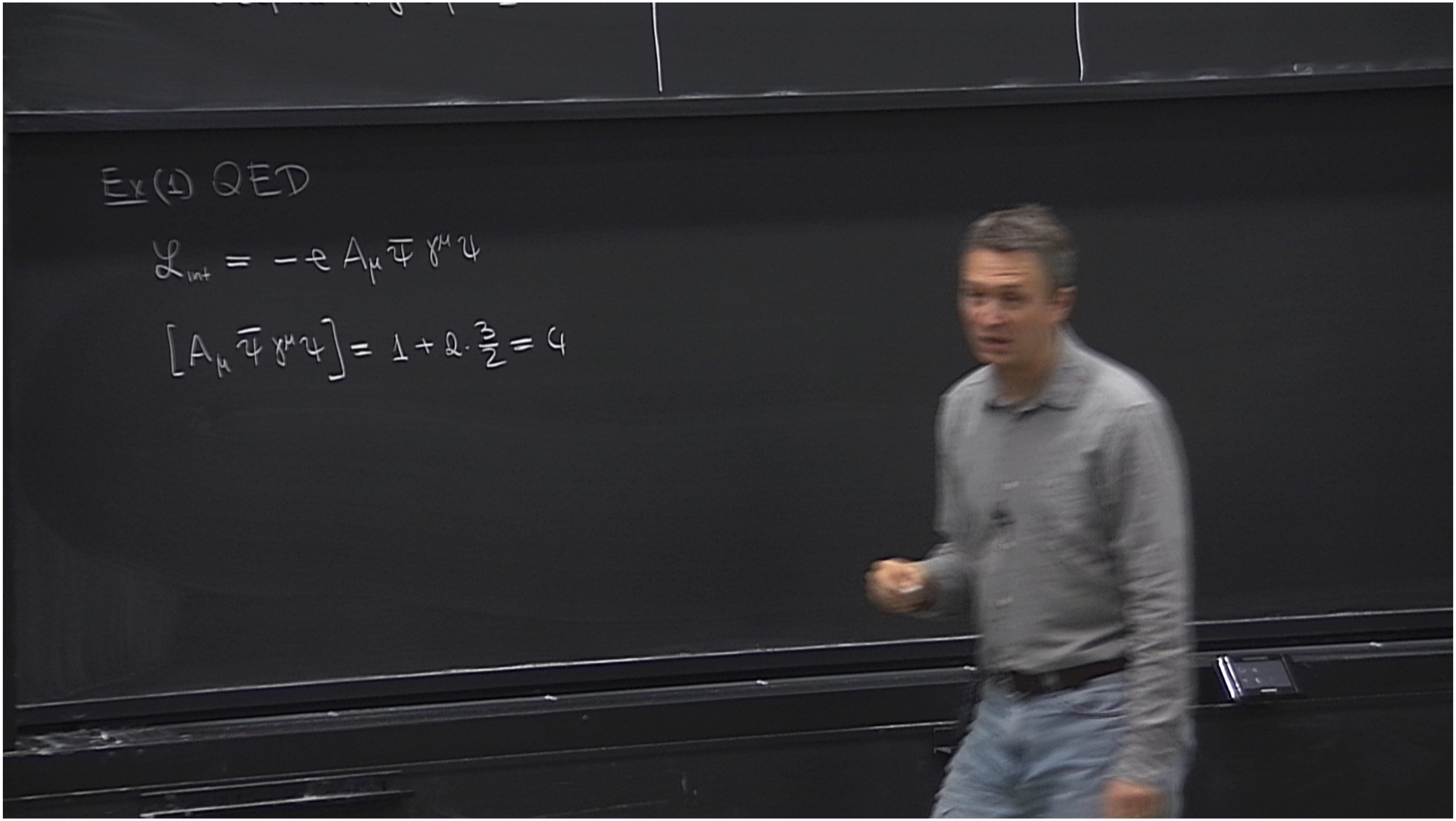

l coupling
l operator)
teractions can be
and strong, depending
merical value of g
ntly of E.
Case 3
ν = [g] < 0
· Irrelevant couplings
(irrelevant operators)
Irrelevant interactions
are weak at low energies
relevant operators < ∞.
+ marginal
If there is a cou
f dim < 0.
$\mathcal{L}_{int} = \frac{\bar{g}}{M^{\nu}} \mathcal{O}$


<0
elevant couplings
relevant operators)
ant interactions
ak at low energies
relevant operators < ∞.
+ marginal
If there is a coupling of dim<0:
$\mathcal{L}_{int} = \frac{\bar{g}}{M^{[g]}} \mathcal{O}$
$[\bar{g}]=0$

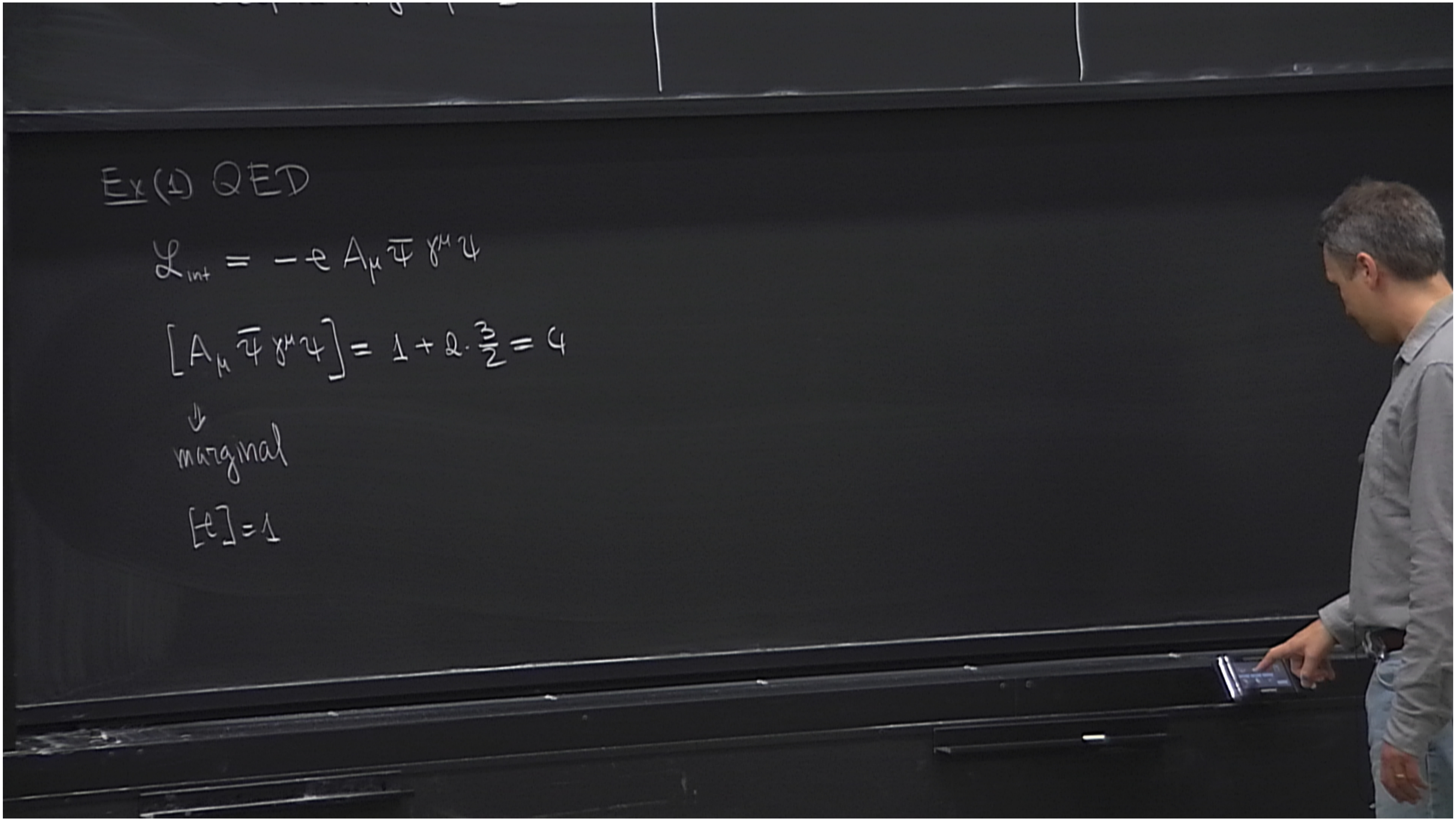

<0
elevant couplings
relevant operators)
ant interactions
eak at low energies
relevant operators < ∞.
+ marginal
If there is a coupling of dim<0.
L_int = ḡ / M^[g] O
[ḡ]=0
At E∼M, field theory does not
make sense




Ex (1) QED
$\mathcal{L}_{int} =$
A_μ



Ex (1) QED
$\mathcal{L}_{int} = -e A_\mu \bar{\psi} \gamma^\mu \psi$
$[A_\mu \bar{\psi} \gamma^\mu \psi] = 1 + 2 \cdot \frac{3}{2} = 4$



Ex (1) QED
$\mathcal{L}_{int} = -e A_\mu \bar{\Psi} \gamma^\mu \Psi$
$[A_\mu \bar{\Psi} \gamma^\mu \Psi] = 1 + 2 \cdot \frac{3}{2} = 4$
⇓
marginal
$[t] = 1$

Ex (1) QED

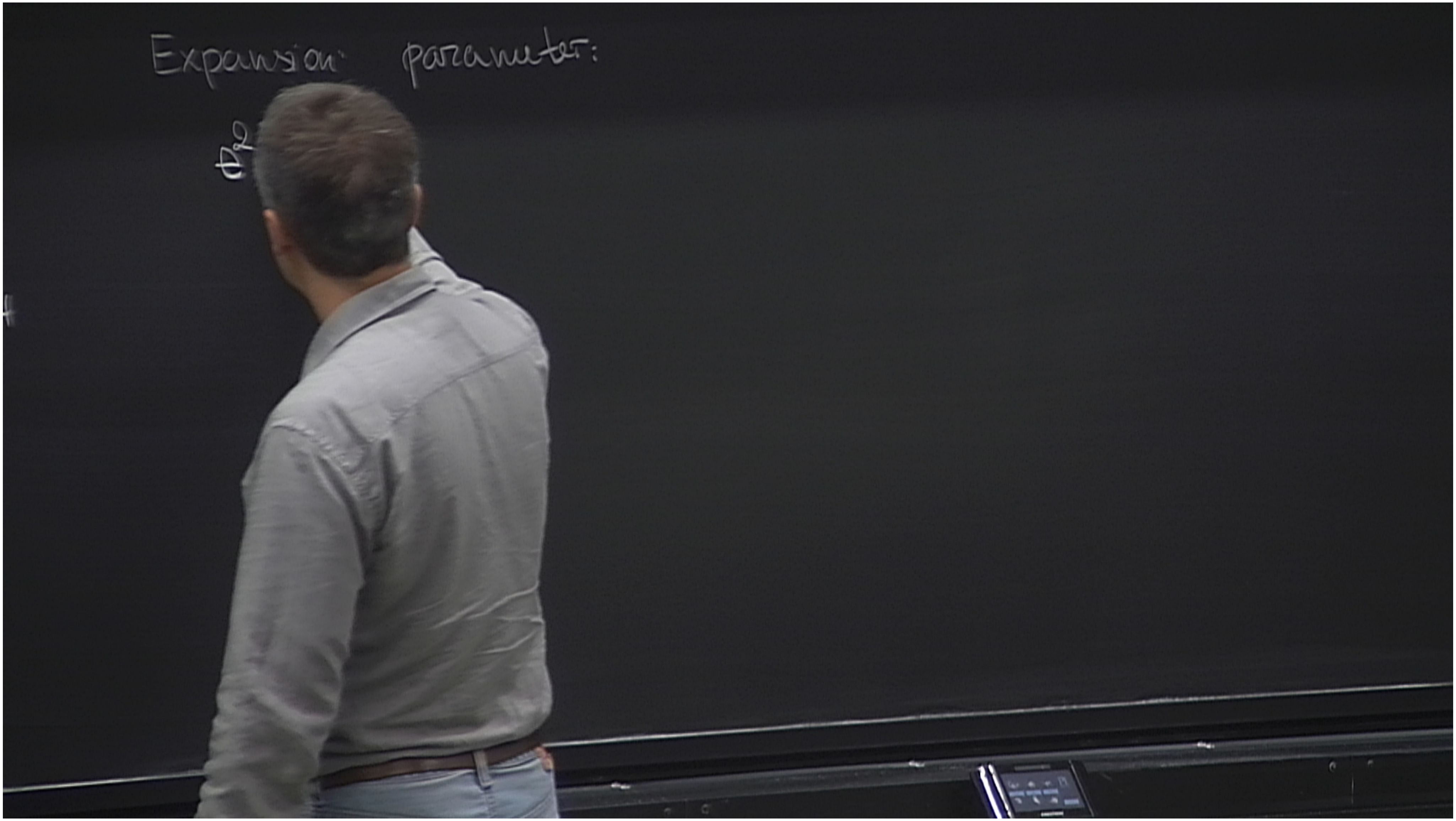
$$\mathcal{L}_{int} = -e A_\mu \bar{\psi} \gamma^\mu \psi$$

$$[A_\mu \bar{\psi} \gamma^\mu \psi] = 1 + 2 \cdot \frac{3}{2} = 4$$

$\Downarrow$

marginal

$$[e] = 0$$

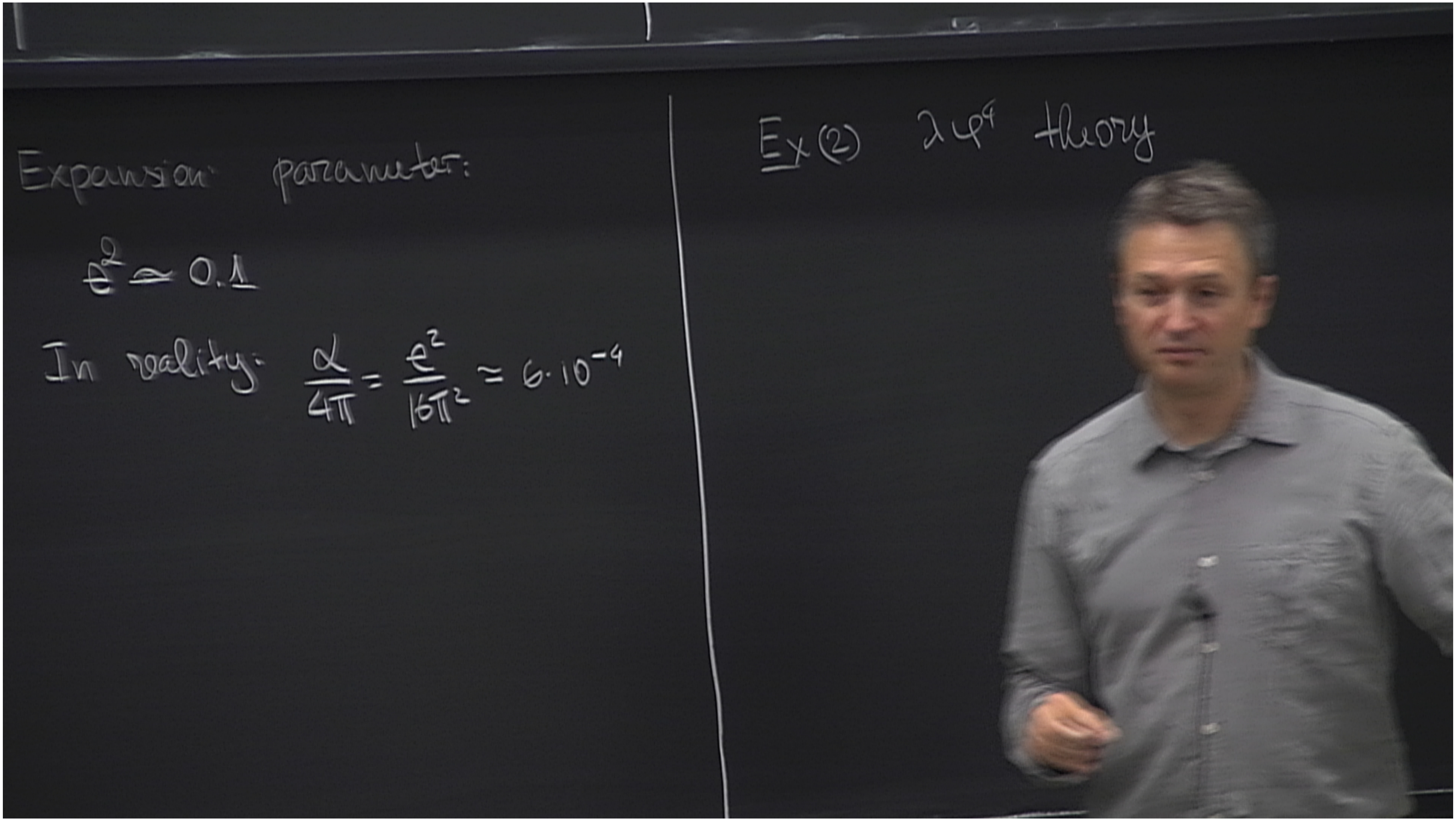

Expansion parameter:

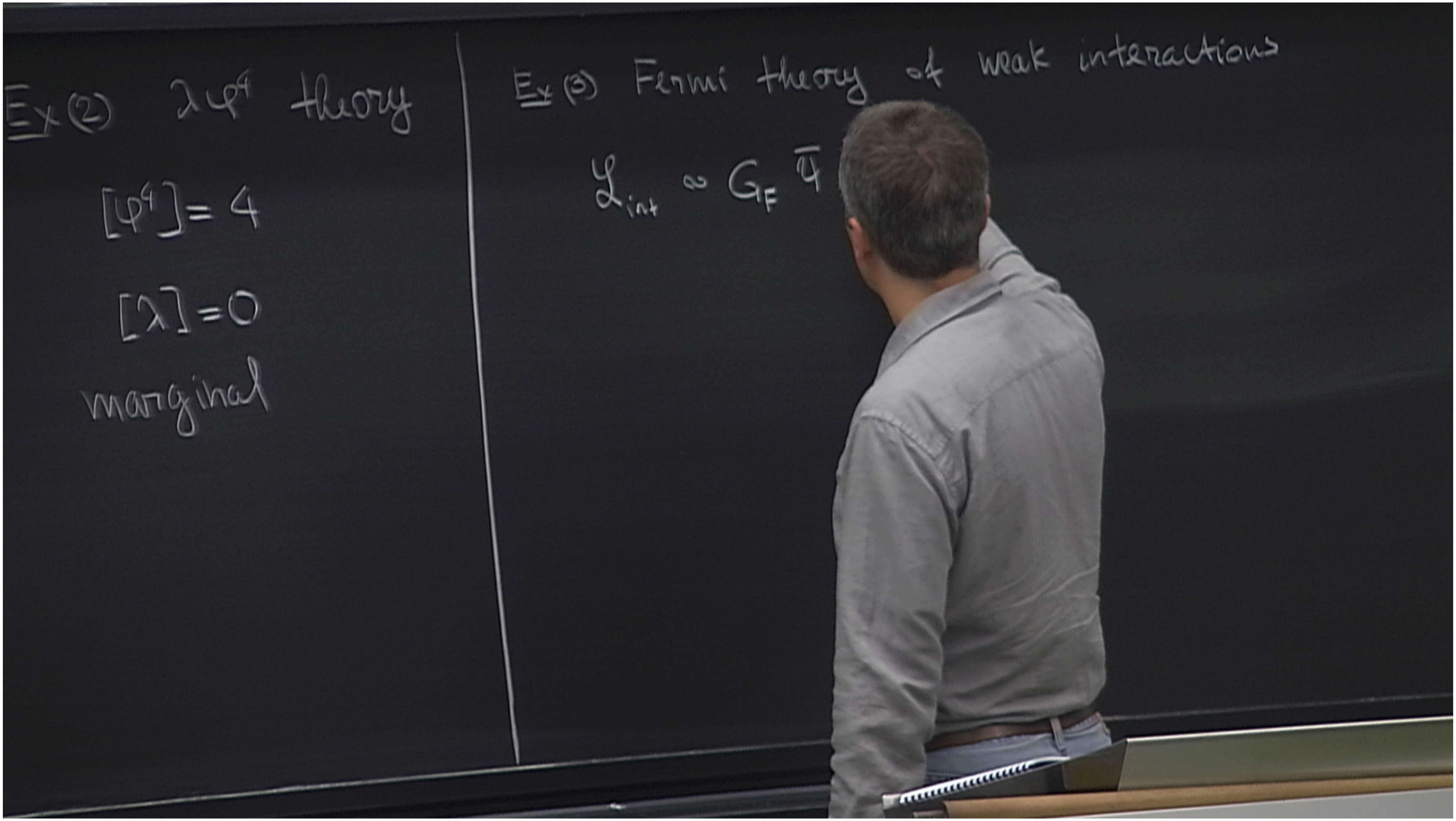
Expansion parameter:

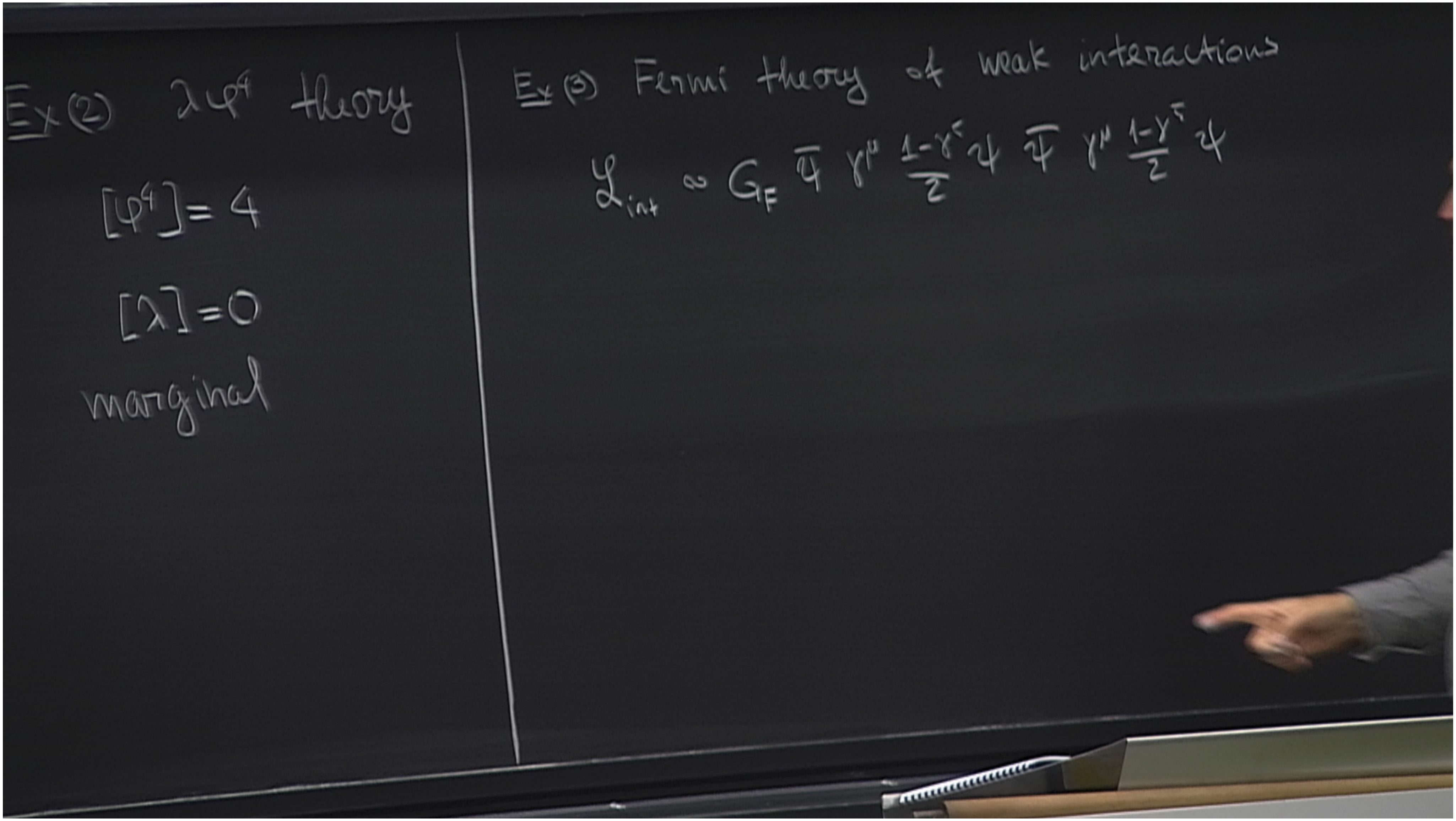
$$e^2 \simeq 0.1$$

In reality: $\dfrac{\alpha}{4\pi} = \dfrac{e^2}{16\pi^2} \simeq 6\cdot 10^{-4}$

$= 4$


Expansion parameter:
$e^2 \simeq 0.1$
In reality: $\frac{\alpha}{4\pi} = \frac{e^2}{16\pi^2} \simeq 6 \cdot 10^{-4}$
Ex (2) $\lambda \varphi^4$ theory


Ex (2) $\lambda\varphi^4$ theory
$[\varphi^4] = 4$
$[\lambda] = 0$
marginal
Ex (3) Fermi theory of weak interactions
$\mathcal{L}_{int} \sim G_F \bar{\psi}$


Ex (2) $\lambda\varphi^4$ theory
$[\varphi^4] = 4$
$[\lambda] = 0$
marginal
Ex (3) Fermi theory of weak interactions
$\mathcal{L}_{int} \propto G_F \bar{\psi} \gamma^\mu \frac{1-\gamma^5}{2} \psi \bar{\psi} \gamma^\mu \frac{1-\gamma^5}{2} \psi$

Ex (2) $\lambda\varphi^4$ theory

$$[\varphi^4] = 4$$

$$[\lambda] = 0$$

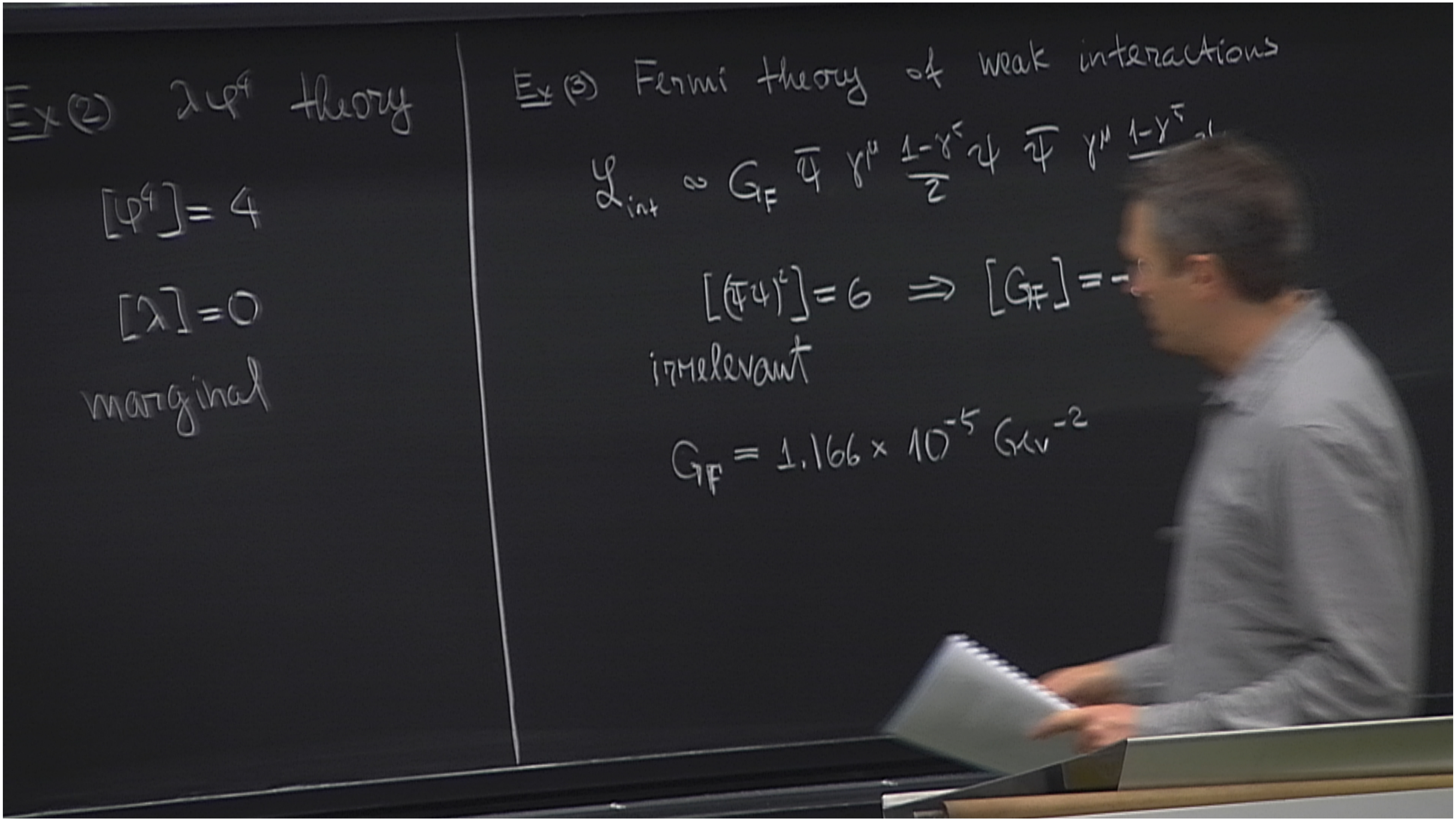
marginal

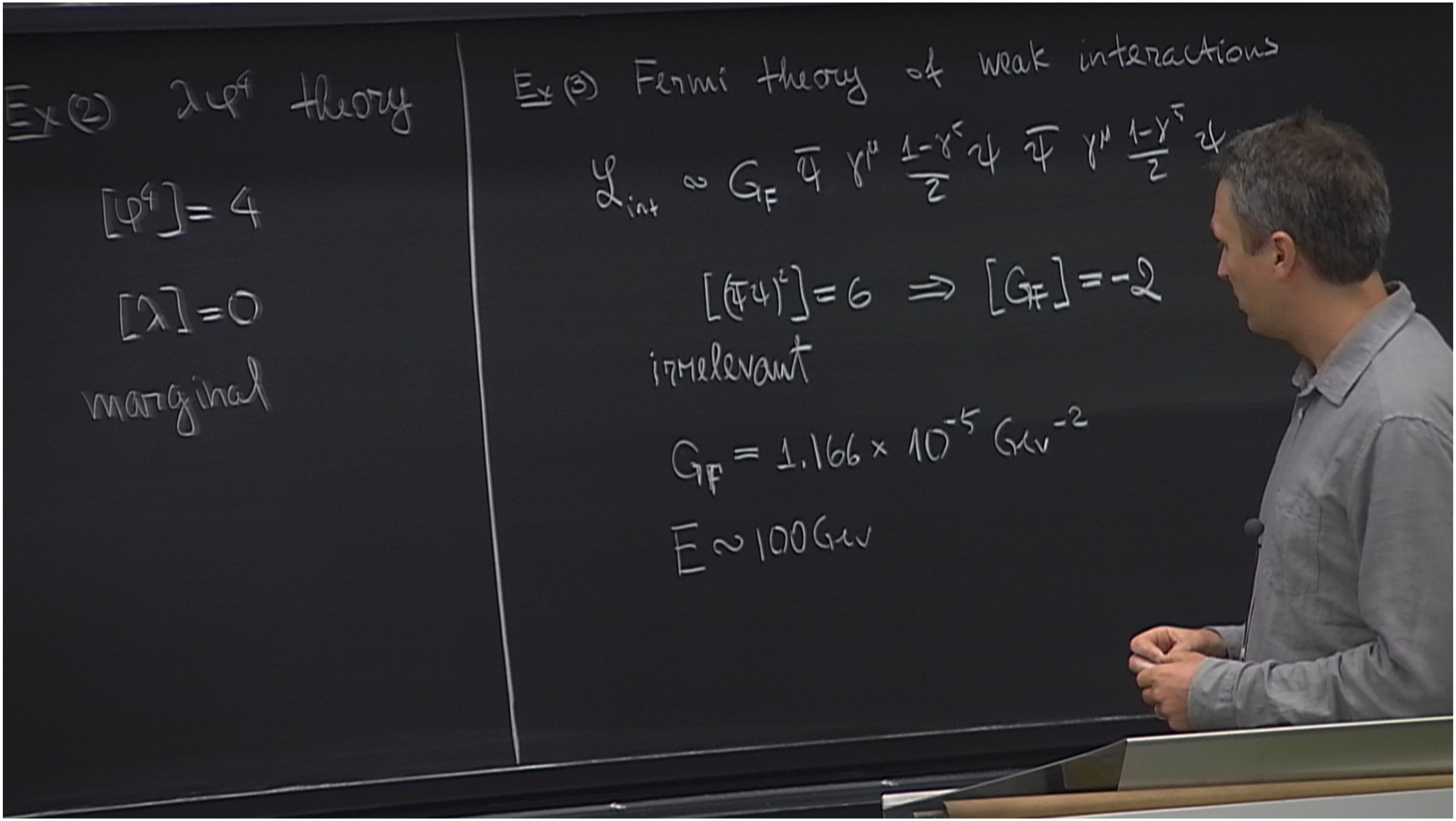
Ex (3) Fermi theory of weak interactions

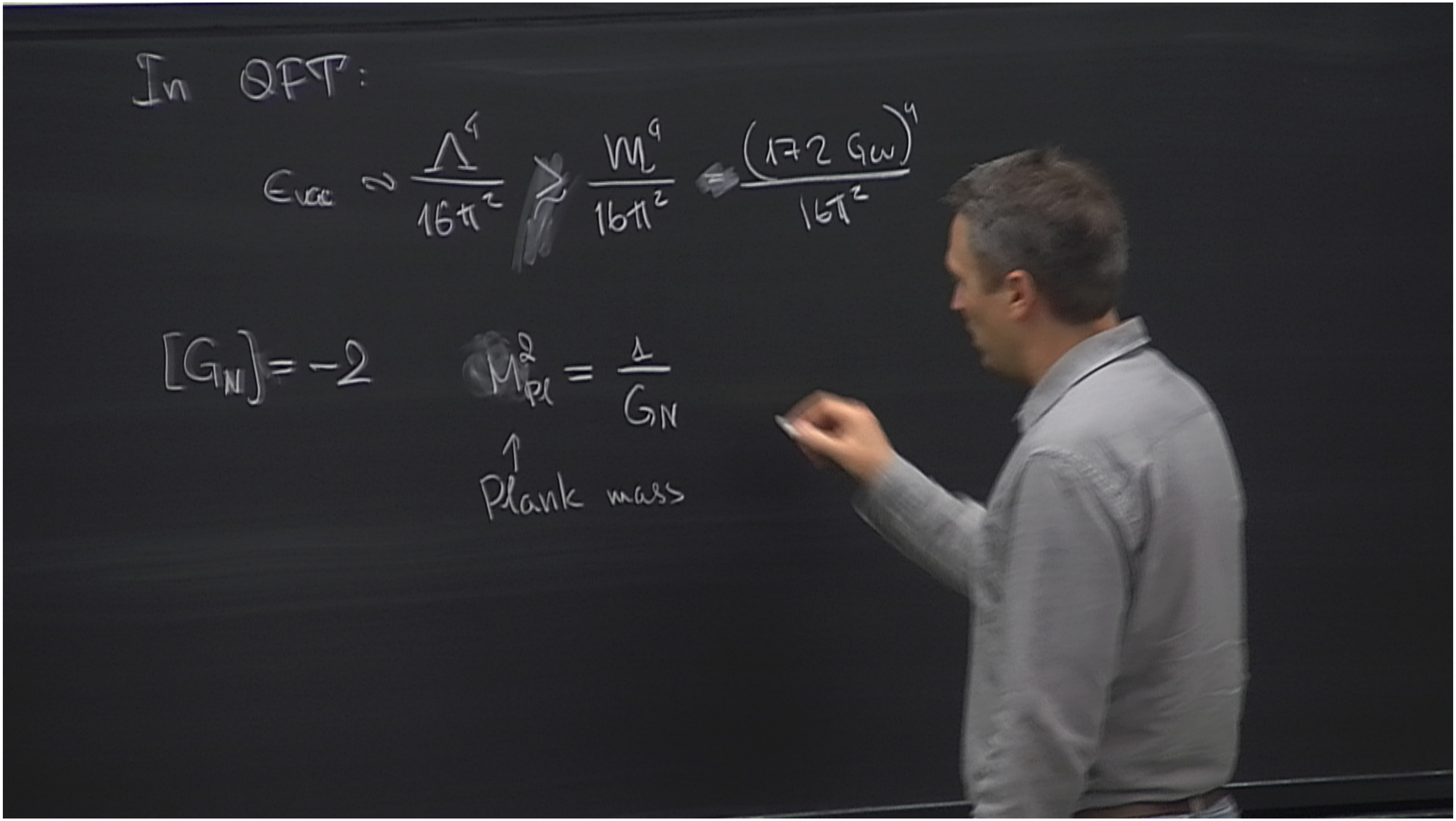
$$\mathcal{L}_{int} \propto G_F\, \bar\psi \gamma^\mu \frac{1-\gamma^5}{2} \psi\, \bar\psi \gamma^\mu \frac{1-\gamma^5}{2} \psi$$

$$[(\bar\psi\psi)^2] = 6 \Rightarrow [G_F] = -2$$

irrelevant


Ex (2) $\lambda\varphi^4$ theory
$[\varphi^4] = 4$
$[\lambda] = 0$
marginal
Ex (3) Fermi theory of weak interactions
$\mathcal{L}_{int} \propto G_F \bar{\Psi} \gamma^\mu \frac{1-\gamma^5}{2} \psi \bar{\Psi} \gamma^\mu \frac{1-\gamma^5}{2}$
$[(\bar{\Psi}\psi)^2] = 6 \Rightarrow [G_F] =$
irrelevant
$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$


Ex (2) $\lambda\varphi^4$ theory
$[\varphi^4] = 4$
$[\lambda] = 0$
marginal
Ex (3) Fermi theory of weak interactions
$\mathcal{L}_{int} \propto G_F \bar{\psi} \gamma^\mu \frac{1-\gamma^5}{2} \psi \bar{\psi} \gamma^\mu \frac{1-\gamma^5}{2} \psi$
$[(\bar{\psi}\psi)^2] = 6 \Rightarrow [G_F] = -2$
irrelevant
$G_F = 1.166 \times 10^{-5}$ GeV^{-2}
$E \sim 100$ GeV


In QFT:
$\epsilon_{vac} \sim \frac{\Lambda^4}{16\pi^2} \gtrsim \frac{m_t^4}{16\pi^2} = \frac{(172\ GeV)^4}{16\pi^2}$
$[G_N] = -2$
$M_{Pl}^2 = \frac{1}{G_N}$
Plank mass

In QFT:

$$\epsilon_{vac} \sim \frac{\Lambda^4}{16\pi^2} \gtrsim \frac{m_t^4}{16\pi^2} = \frac{(172\ \mathrm{GeV})^4}{16\pi^2}$$

$$[G_N] = -2 \qquad M_{Pl}^2 = \frac{1}{G_N} \qquad M_{Pl} \sim 10^{18}\ \mathrm{GeV}$$

Plank mass