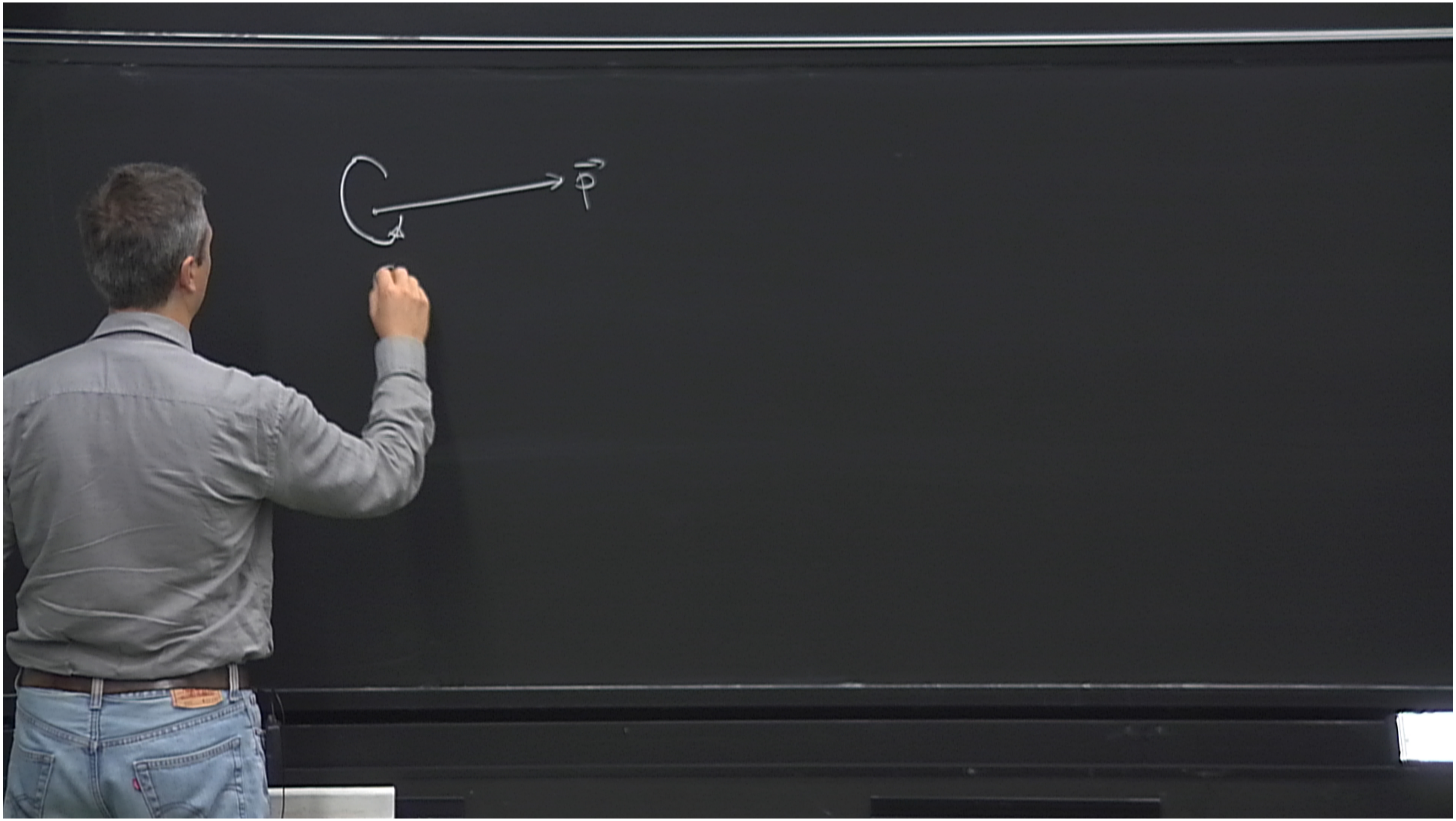
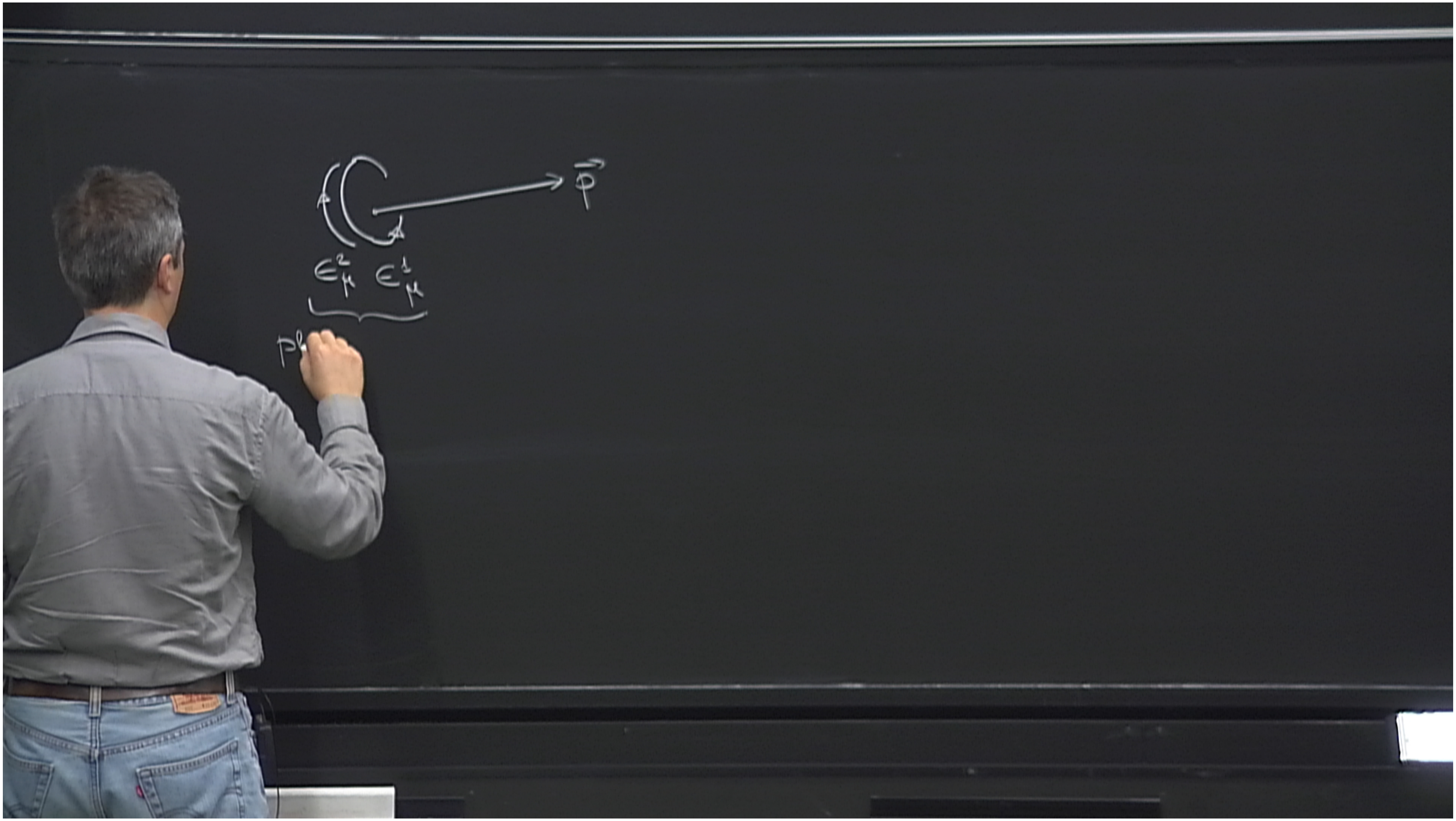
Title: Quantum Field Theory I - Lecture 8

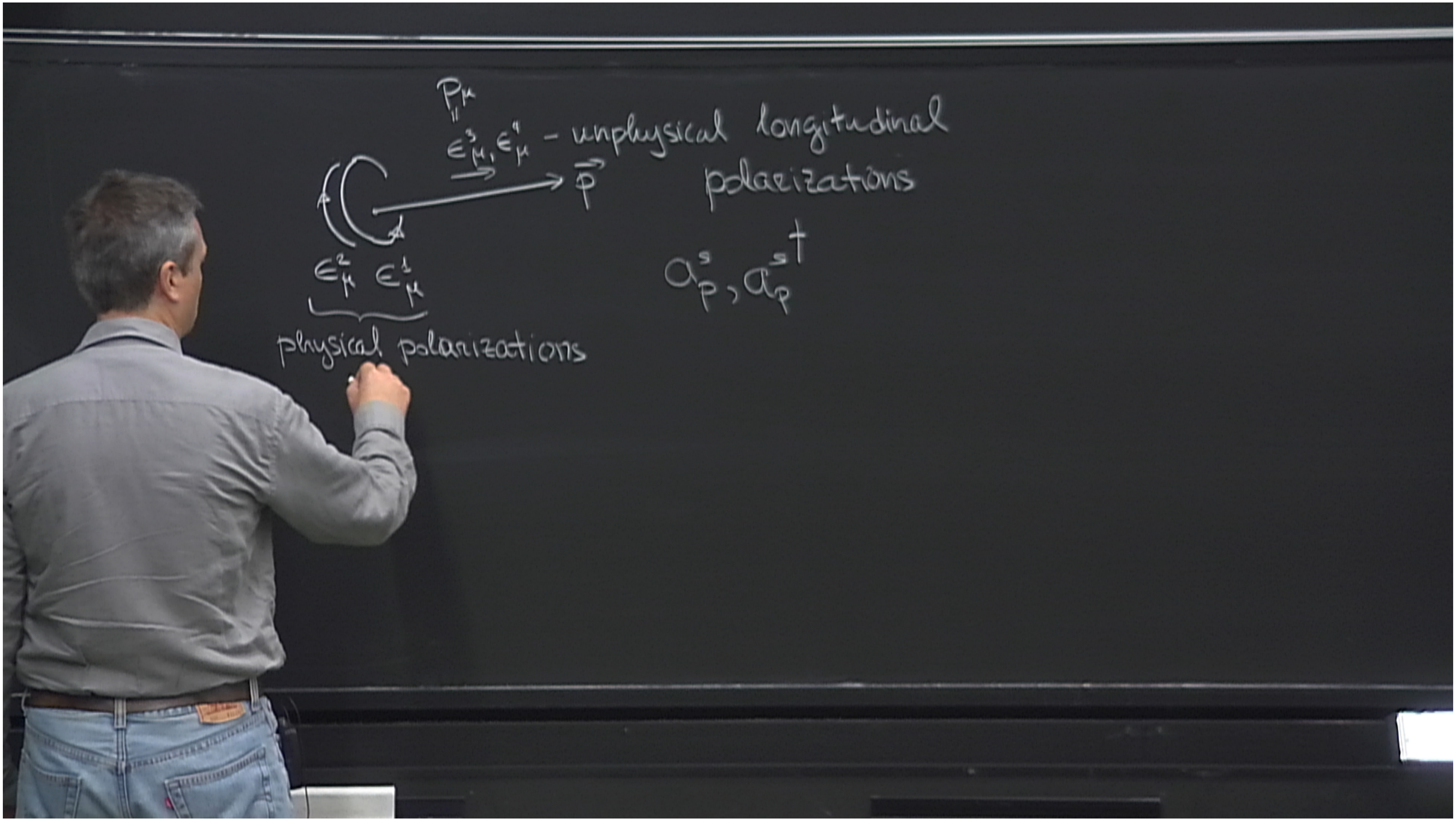
Date: Oct 13, 2011 09:00 AM

URL: http://pirsa.org/11100016

Abstract:


$\vec{p}$


$\vec{p}$
ϵ^2_μ ϵ^1_μ


p_μ
$\epsilon^3_\mu, \epsilon^4_\mu$ - unphysical longitudinal polarizations
$\vec{p}$
ϵ^2_μ ϵ^1_μ
physical polarizations
$a^s_p, a^{s\dagger}_p$

$\epsilon^2_\mu \quad \epsilon^1_\mu \qquad\qquad a^s_p, a^{s\dagger}_p$

$\underbrace{}$

physical polarizations

$\hookrightarrow$ physical states

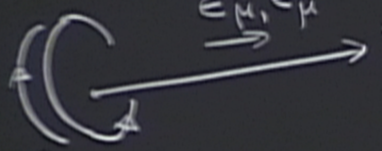
In intermediate states, all polarizations appear:

$$\sum_s \epsilon^{s*}_\mu(p)\,\epsilon^s_\nu(p) = g_{\mu\nu}$$

p_μ

$\parallel$

$\epsilon^3_\mu, \epsilon^4_\mu$ – unphysical longitudinal polarizations



$\vec{p}$

$a^s_p, a^{s\dagger}_p$

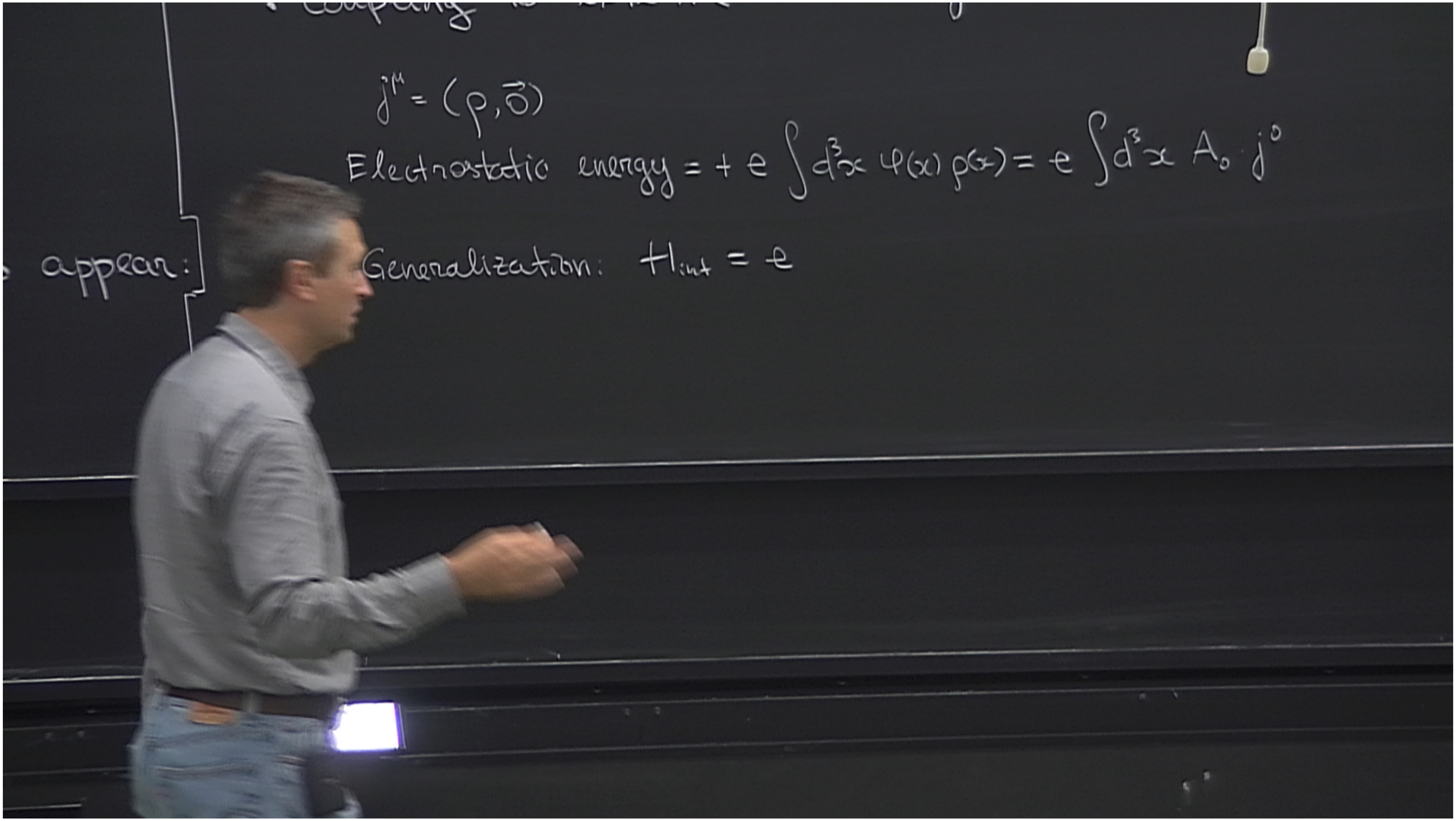
$\underbrace{\epsilon^2_\mu \ \epsilon^1_\mu}_{\text{physical polarizations}}$

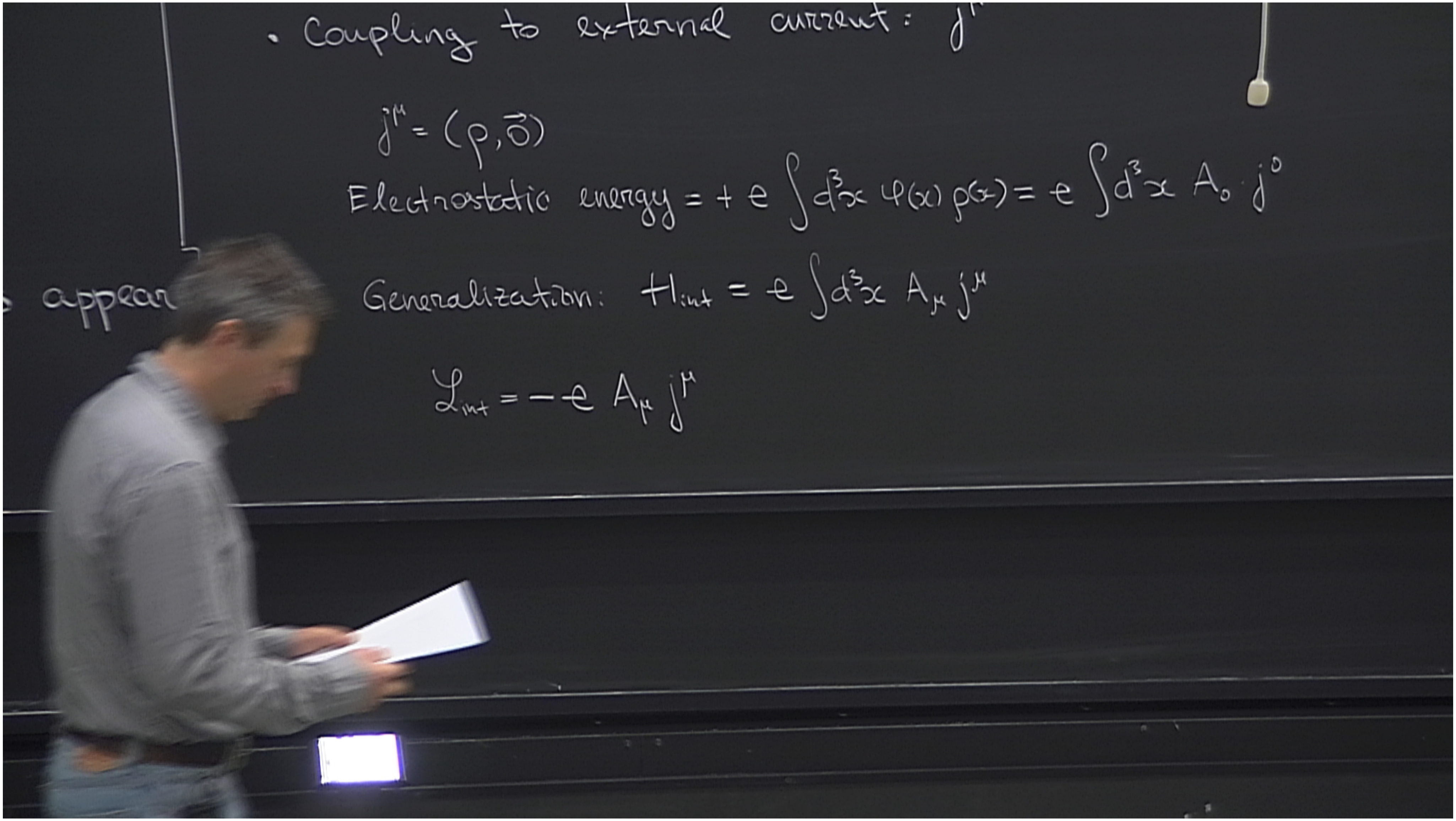
physical polarizations

$\hookrightarrow$ physical states

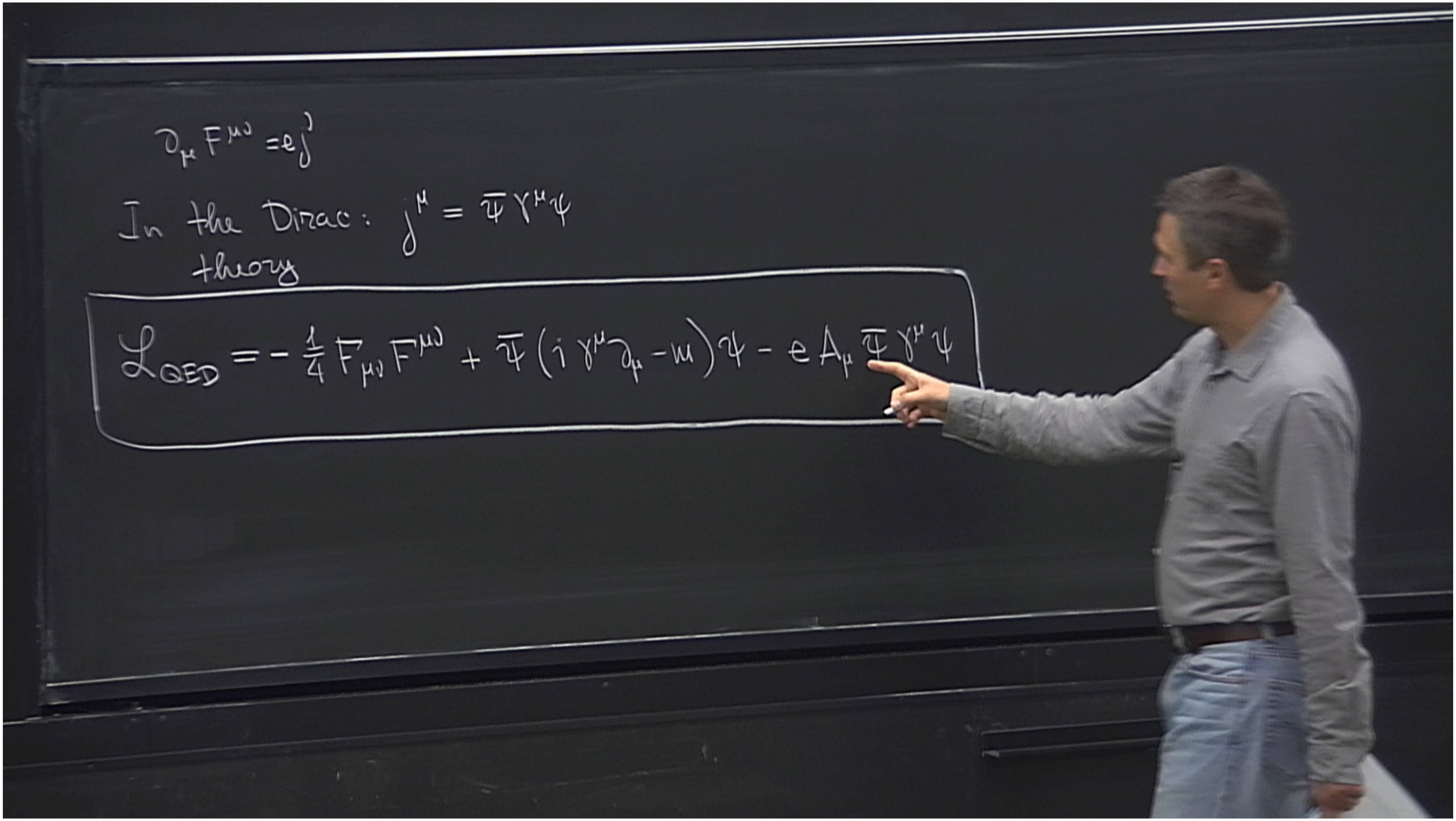
In intermediate states, all polarizations appear:

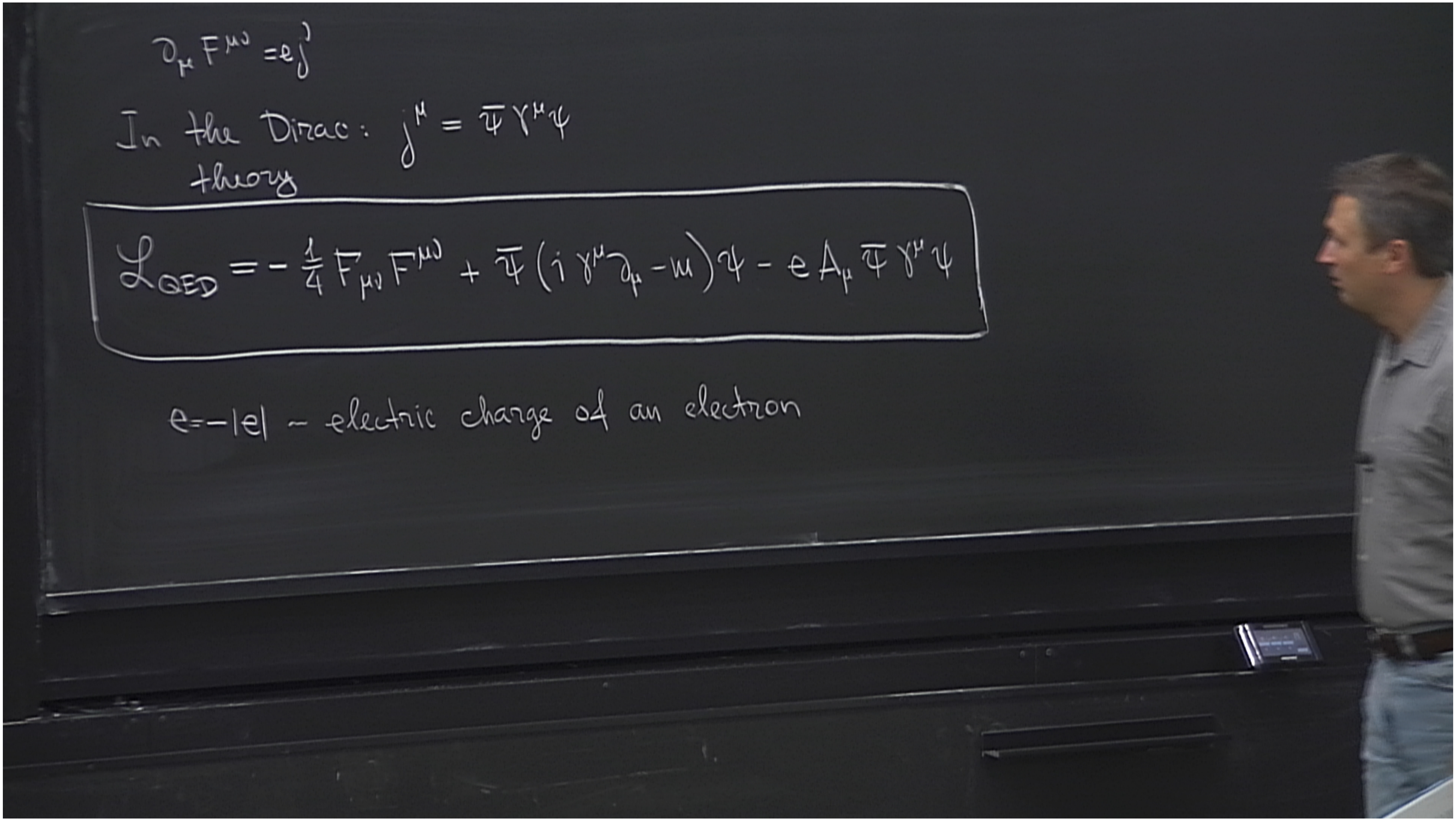
$$\sum_s \epsilon^{s*}_\mu(p)\, \epsilon^s_\nu(p) = g_{\mu\nu}$$


$j^\mu = (\rho, \vec{0})$
Electrostatic energy $= + e \int d^3x \, \varphi(x) \rho(x) = e \int d^3x \, A_0 \cdot j^0$
appear:
Generalization: $H_{int} = e$



• Coupling to external current: j^μ
$j^\mu = (\rho, \vec{0})$
Electrostatic energy $= + e \int d^3x \, \varphi(x) \rho(x) = e \int d^3x \, A_0 \cdot j^0$
Generalization: $H_{int} = e \int d^3x \, A_\mu j^\mu$
$\mathcal{L}_{int} = - e A_\mu j^\mu$
appear


$\partial_\mu F^{\mu\nu} = e j^\nu$
In the Dirac theory: $j^\mu = \bar\psi \gamma^\mu \psi$
$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar\psi (i \gamma^\mu \partial_\mu - m)\psi - e A_\mu \bar\psi \gamma^\mu \psi$


$\partial_\mu F^{\mu\nu} = e j^\nu$
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$e = -|e|$ ~ electric charge of an electron

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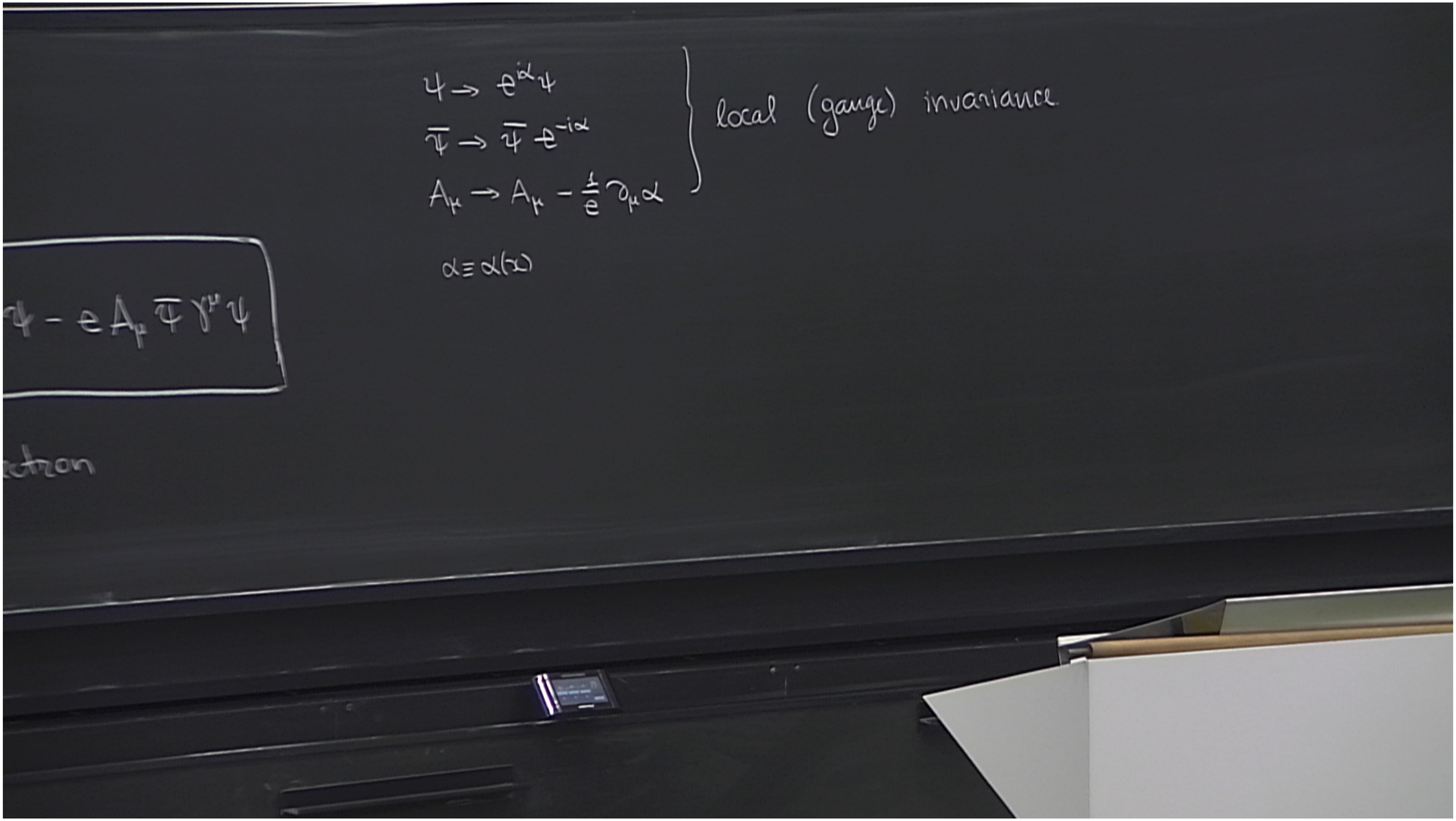
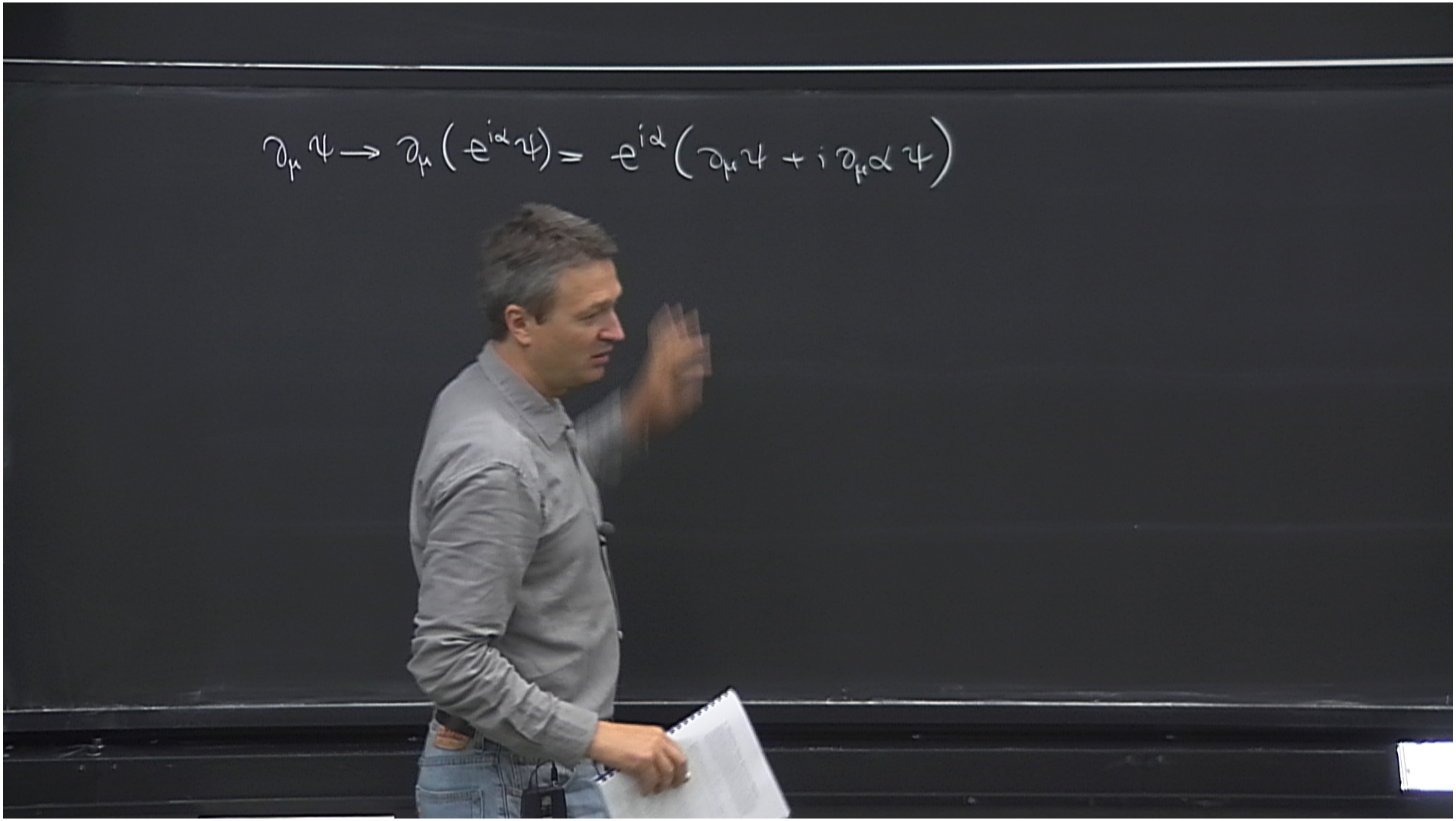
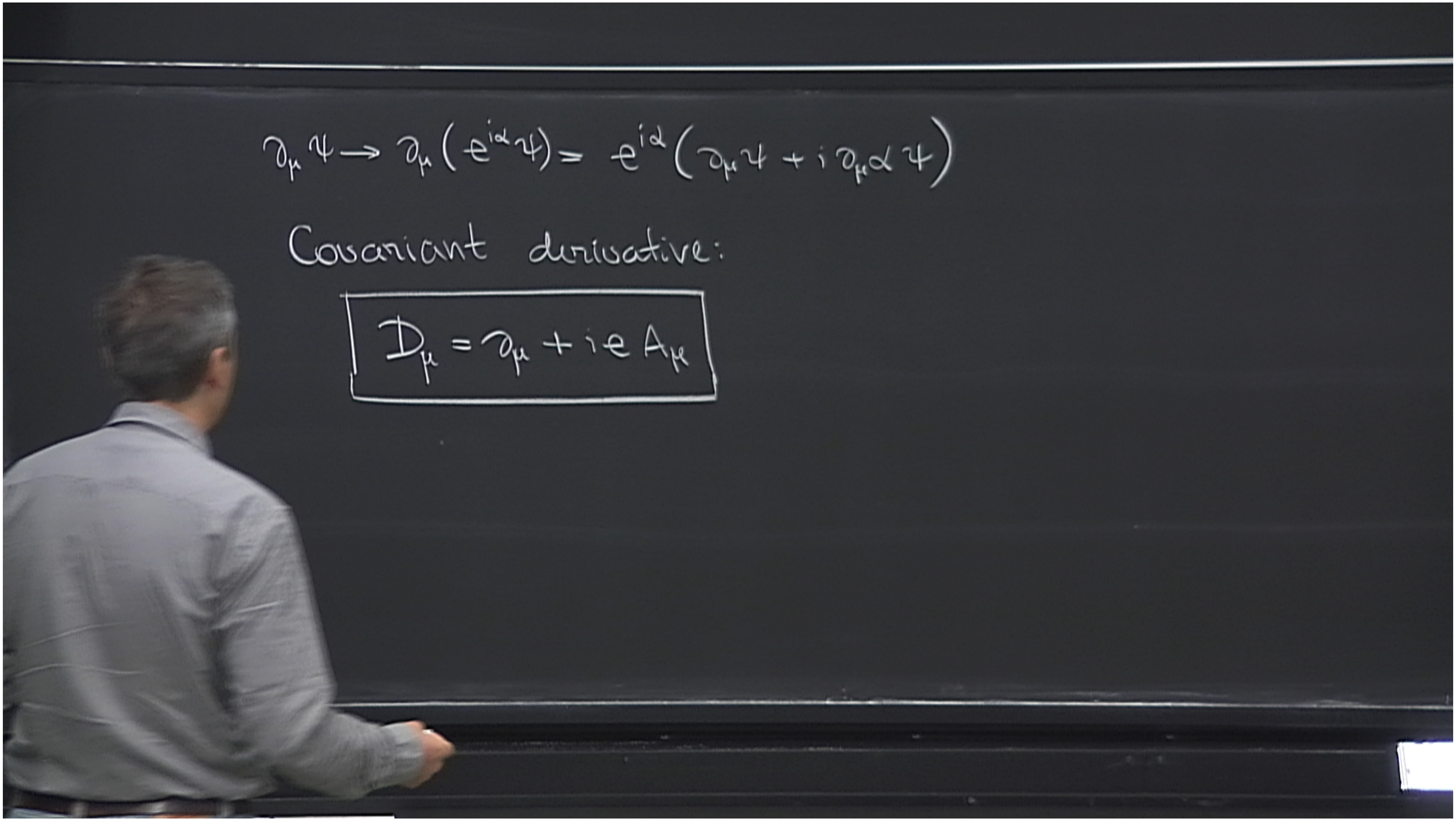
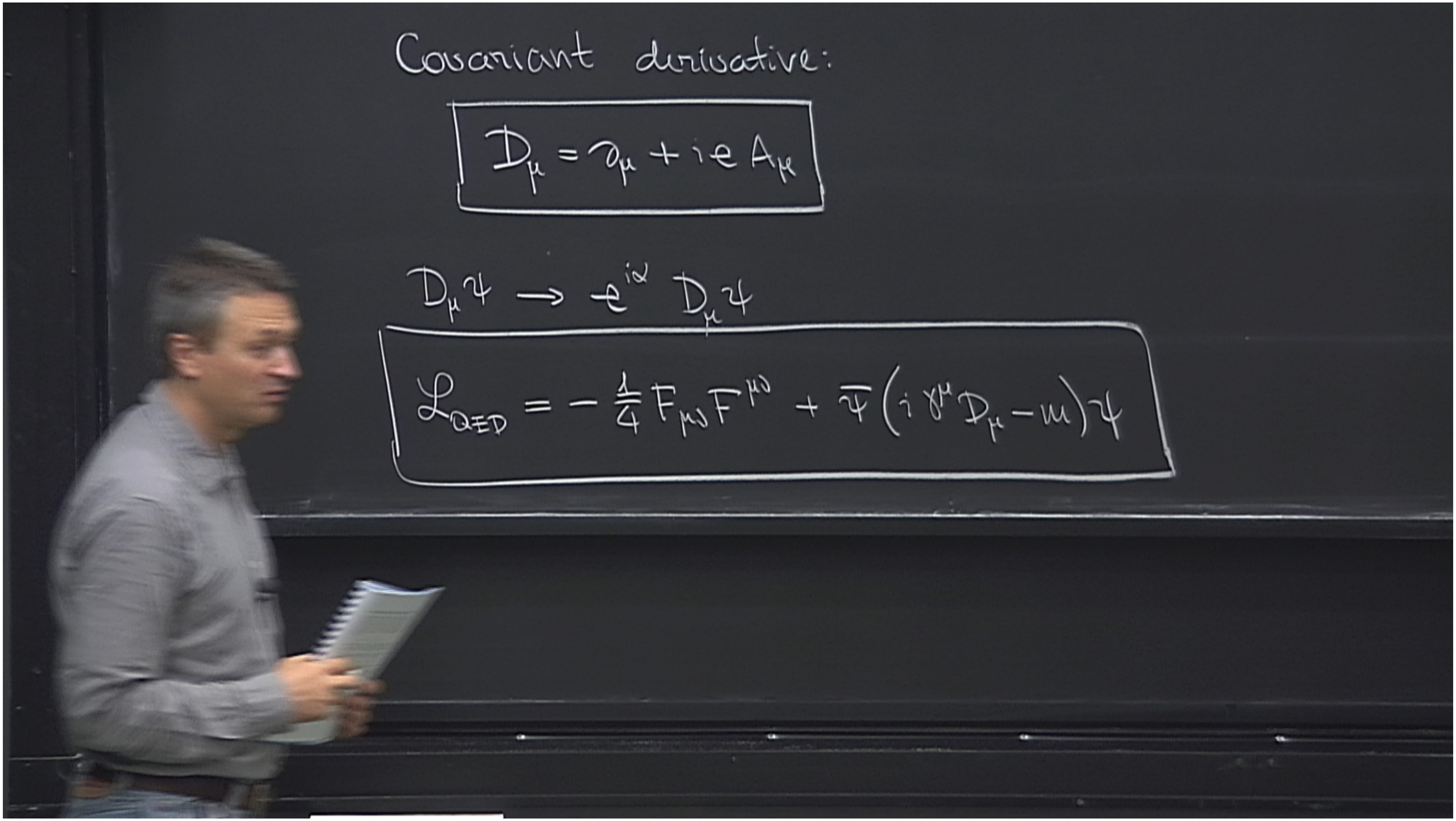

$\psi \to e^{i\alpha}\psi$
$\bar{\psi} \to \bar{\psi}\, e^{-i\alpha}$
$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \alpha$
local (gauge) invariance
$\alpha \equiv \alpha(x)$
$\psi - e A_\mu \bar{\psi}\gamma^\mu \psi$
ctron


$\partial_\mu \psi \to \partial_\mu (e^{i\alpha}\psi) = e^{i\alpha}(\partial_\mu \psi + i\,\partial_\mu \alpha\, \psi)$


$\partial_\mu \psi \to \partial_\mu (e^{i\alpha}\psi) = e^{i\alpha}(\partial_\mu \psi + i\,\partial_\mu \alpha\, \psi)$
Covariant derivative:
$\boxed{D_\mu = \partial_\mu + ieA_\mu}$


Covariant derivative:
$\boxed{D_\mu = \partial_\mu + ieA_\mu}$
$D_\mu \psi \longrightarrow e^{i\alpha} D_\mu \psi$
$\boxed{\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi}$

$\psi)$

A_μ - compensating field that makes Dirac Lagrangian gauge invariant

$\boxed{{}_\mu - m)\psi}$

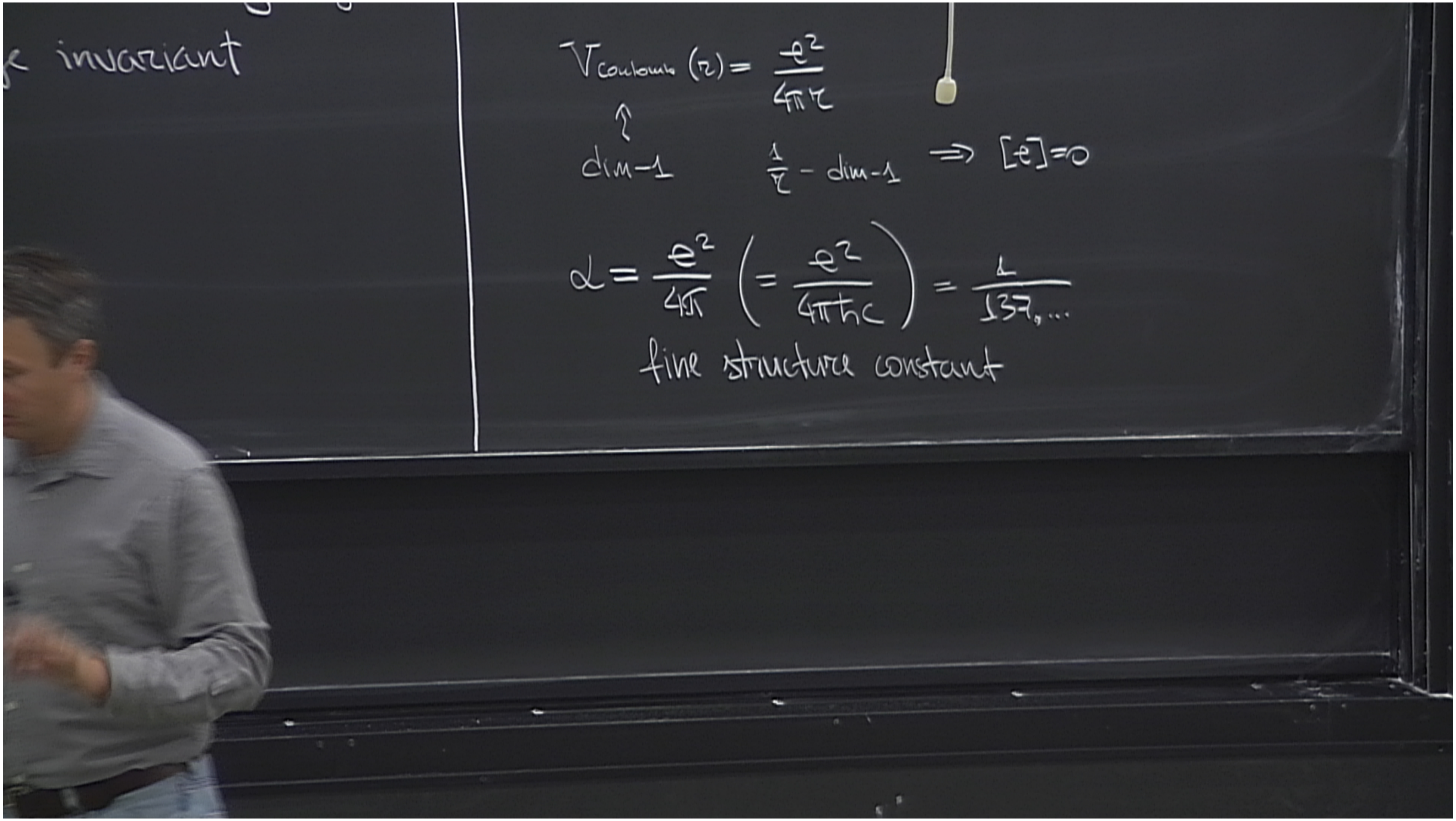
ompensating field that
s Dirac Lagrangian
ge invariant

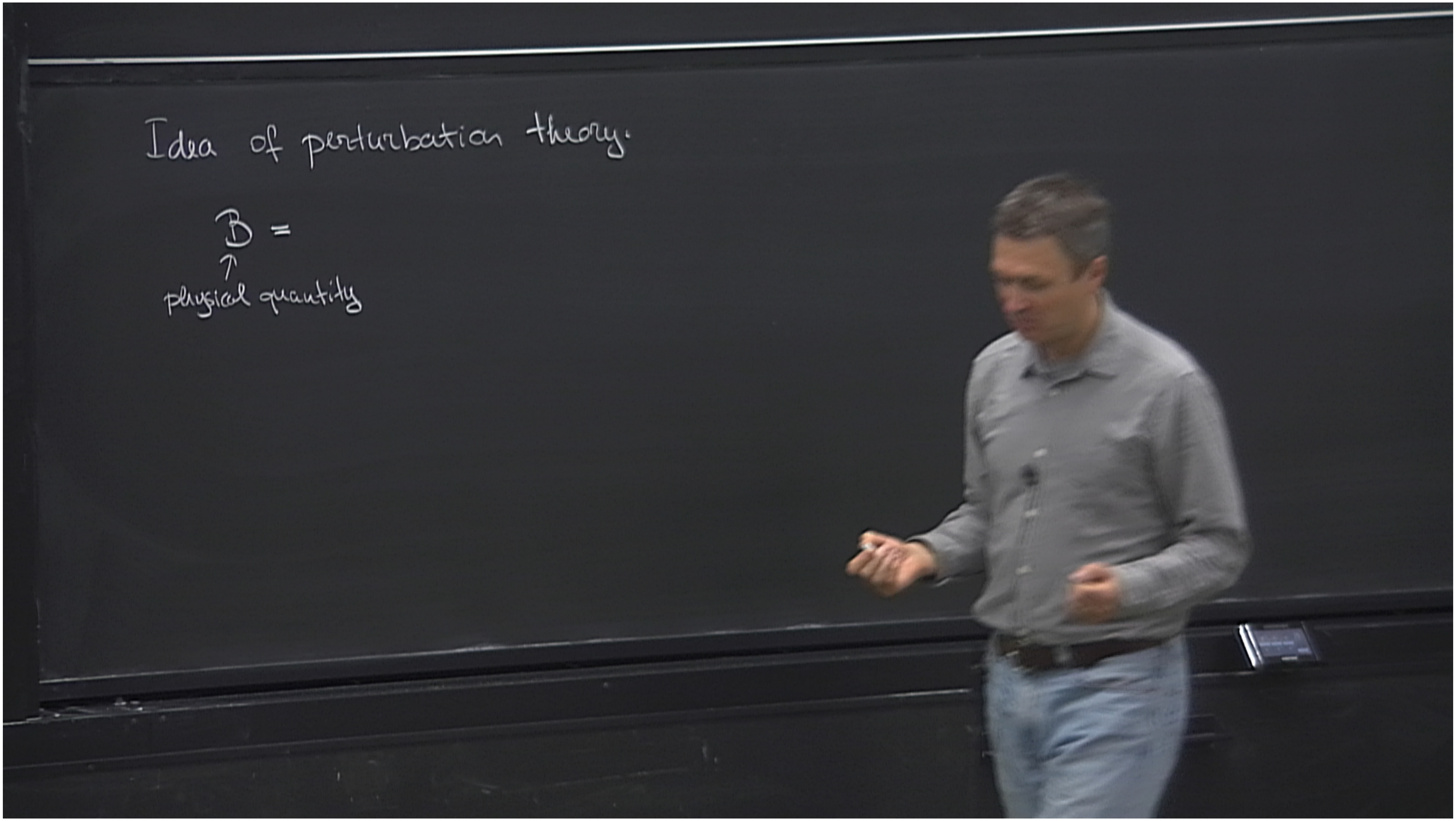
Perturbation Theory

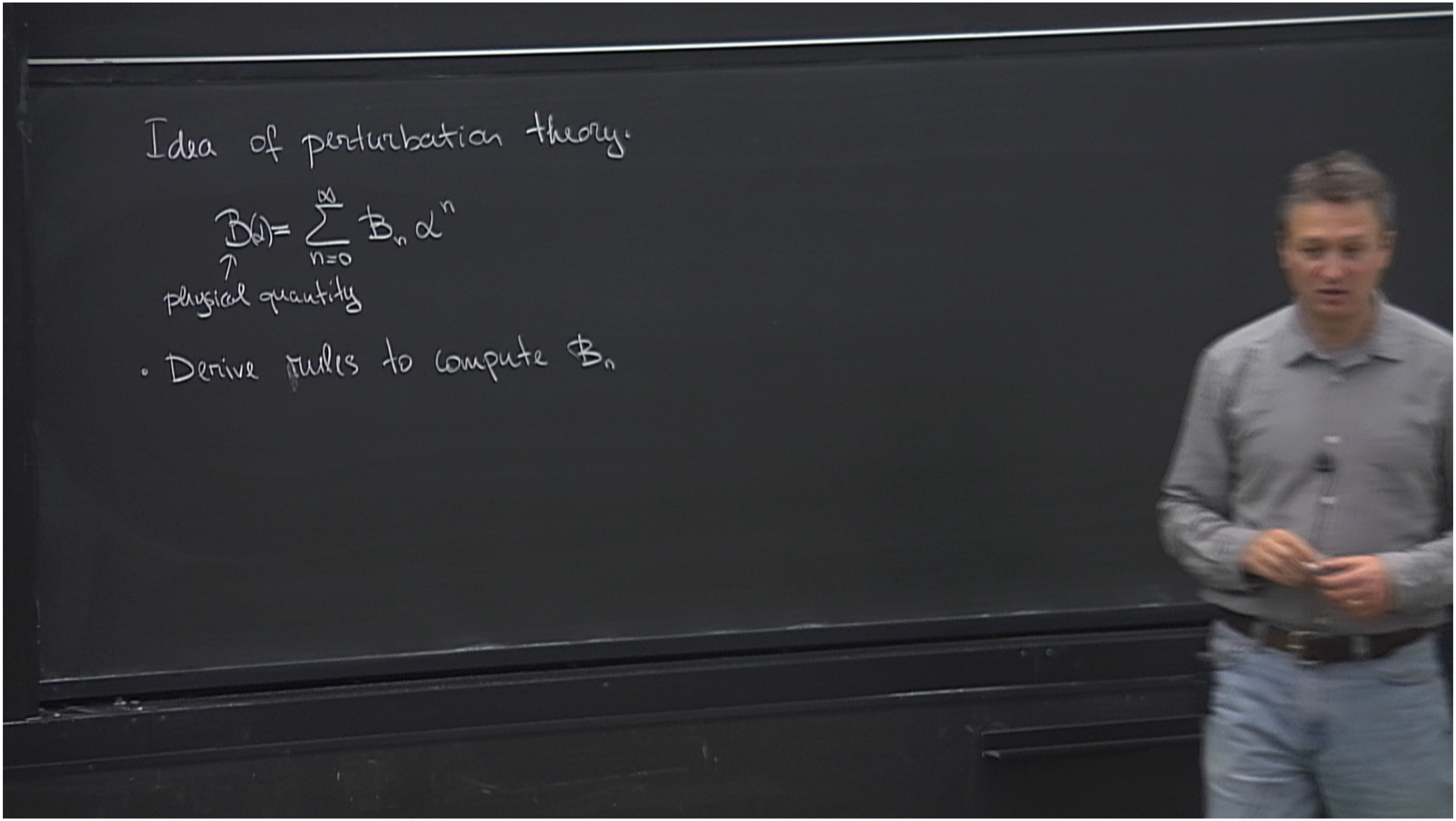
- e is dimensionless.

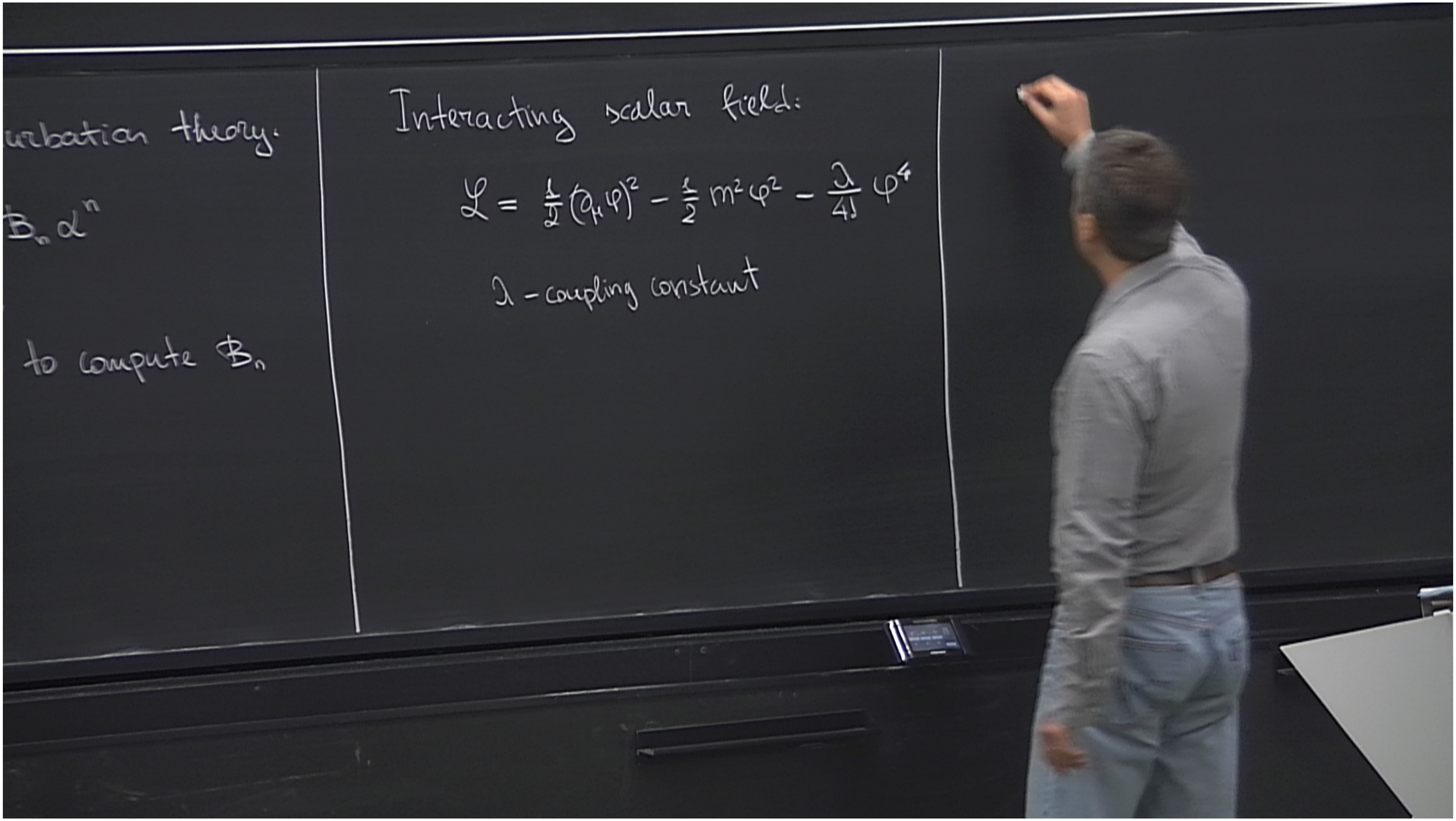
$$V_{\text{Coulomb}}(r) = \frac{-e^2}{4\pi r}$$

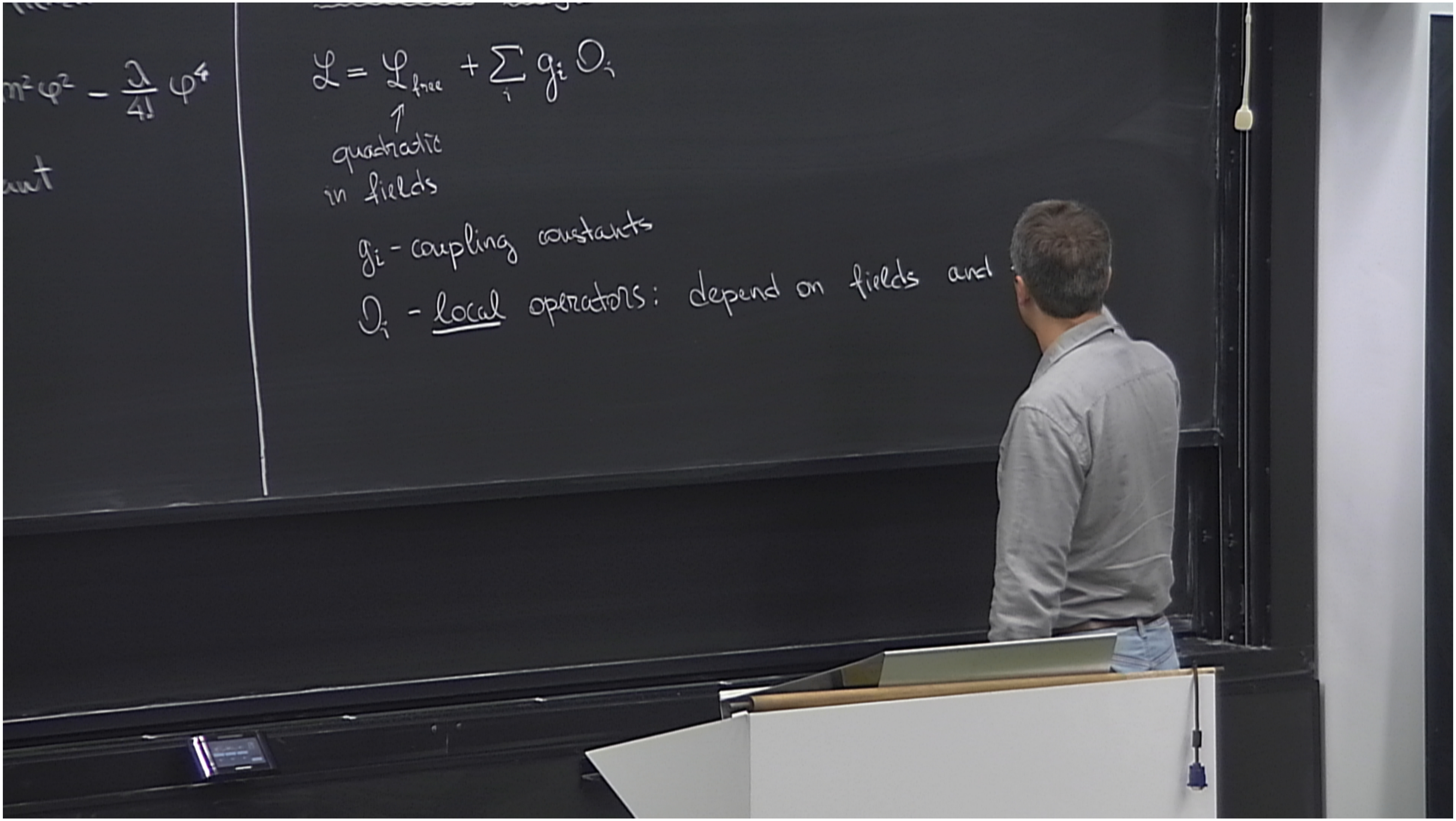
dim-1 $\qquad \frac{1}{r}$ - dim-1 $\Rightarrow [e]=0$

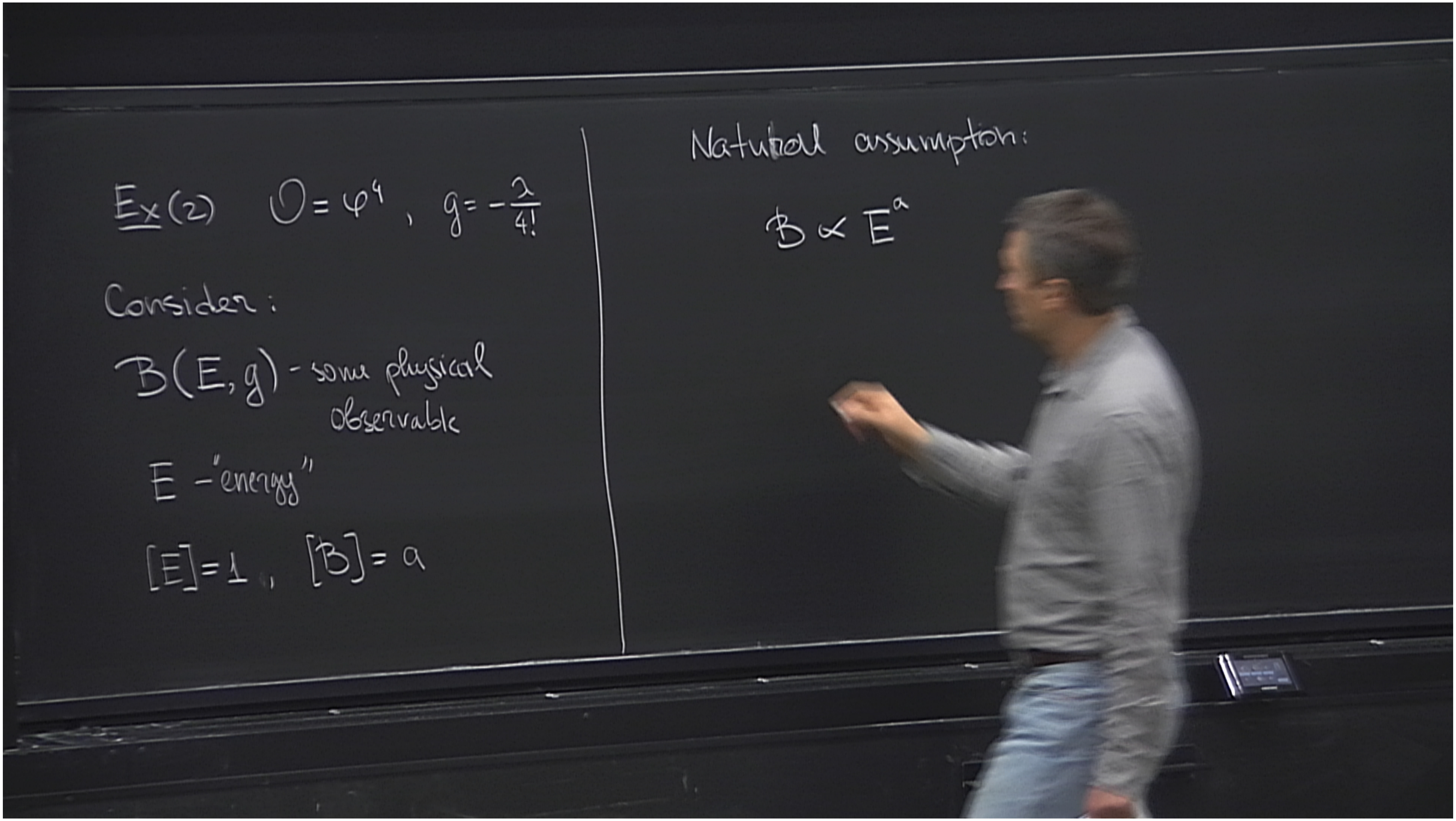

invariant
$V_{\text{Coulomb}}(r) = \frac{e^2}{4\pi r}$
dim-1
$\frac{1}{r}$ - dim-1 $\Rightarrow [e]=0$
$\alpha = \frac{e^2}{4\pi} \left(= \frac{e^2}{4\pi\hbar c} \right) = \frac{1}{137,\ldots}$
fine structure constant

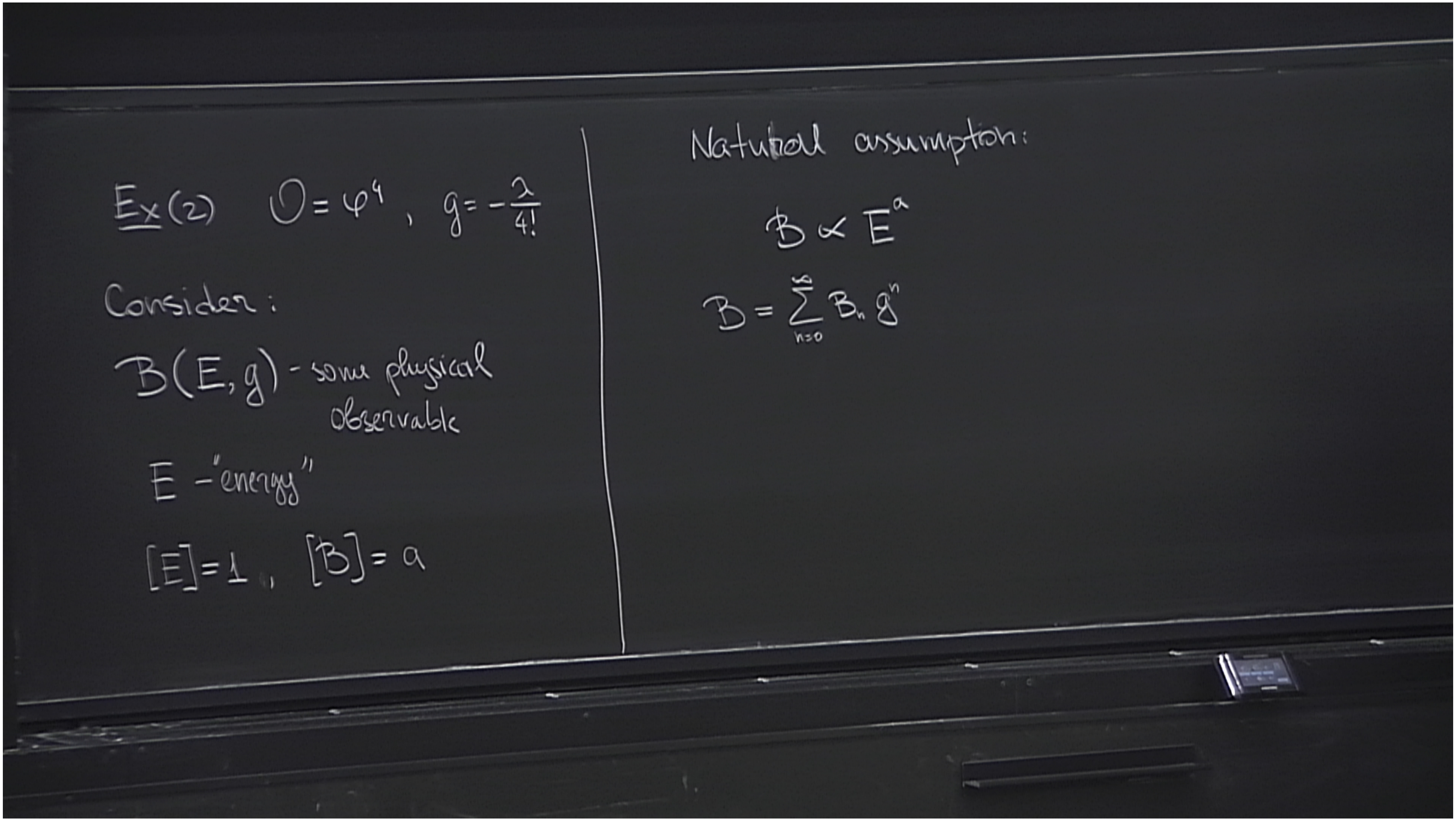

Idea of perturbation theory.
B =
physical quantity


Idea of perturbation theory.
$B(\alpha) = \sum_{n=0}^{\infty} B_n \alpha^n$
physical quantity
• Derive rules to compute B_n


urbation theory.
$B_n \alpha^n$
to compute B_n
Interacting scalar field:
$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!}\varphi^4$
λ - coupling constant


$m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4$
$\mathcal{L} = \mathcal{L}_{free} + \sum_i g_i O_i$
quadratic in fields
g_i - coupling constants
O_i - local operators: depend on fields and


Ex (2) $\mathcal{O} = \varphi^4$, $g = -\frac{\lambda}{4!}$
Consider:
$B(E, g)$ - some physical observable
E - "energy"
$[E] = 1$, $[B] = a$
Natural assumption:
$B \propto E^a$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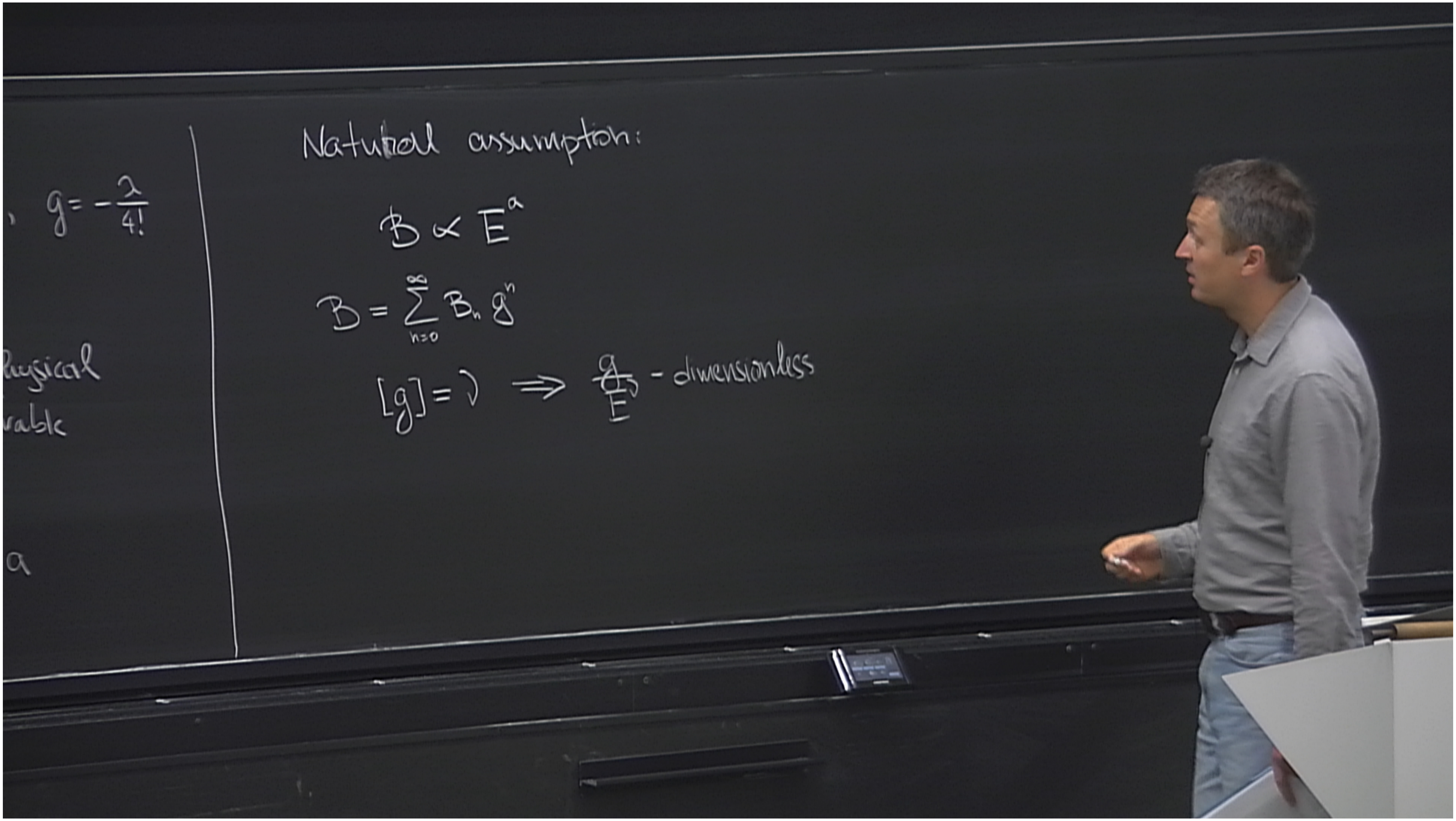
$B = \sum_{n=0}^{\infty} B_n g^n$


Natural assumption:
$B \propto E^{a}$
$B = \sum_{n=0}^{\infty} B_n g^n$
$[g] = \nu \Rightarrow \frac{g}{E^{\nu}}$ - dimensionless
$g = -\frac{\lambda}{4!}$
hysical
rable
a

Natural assumption:

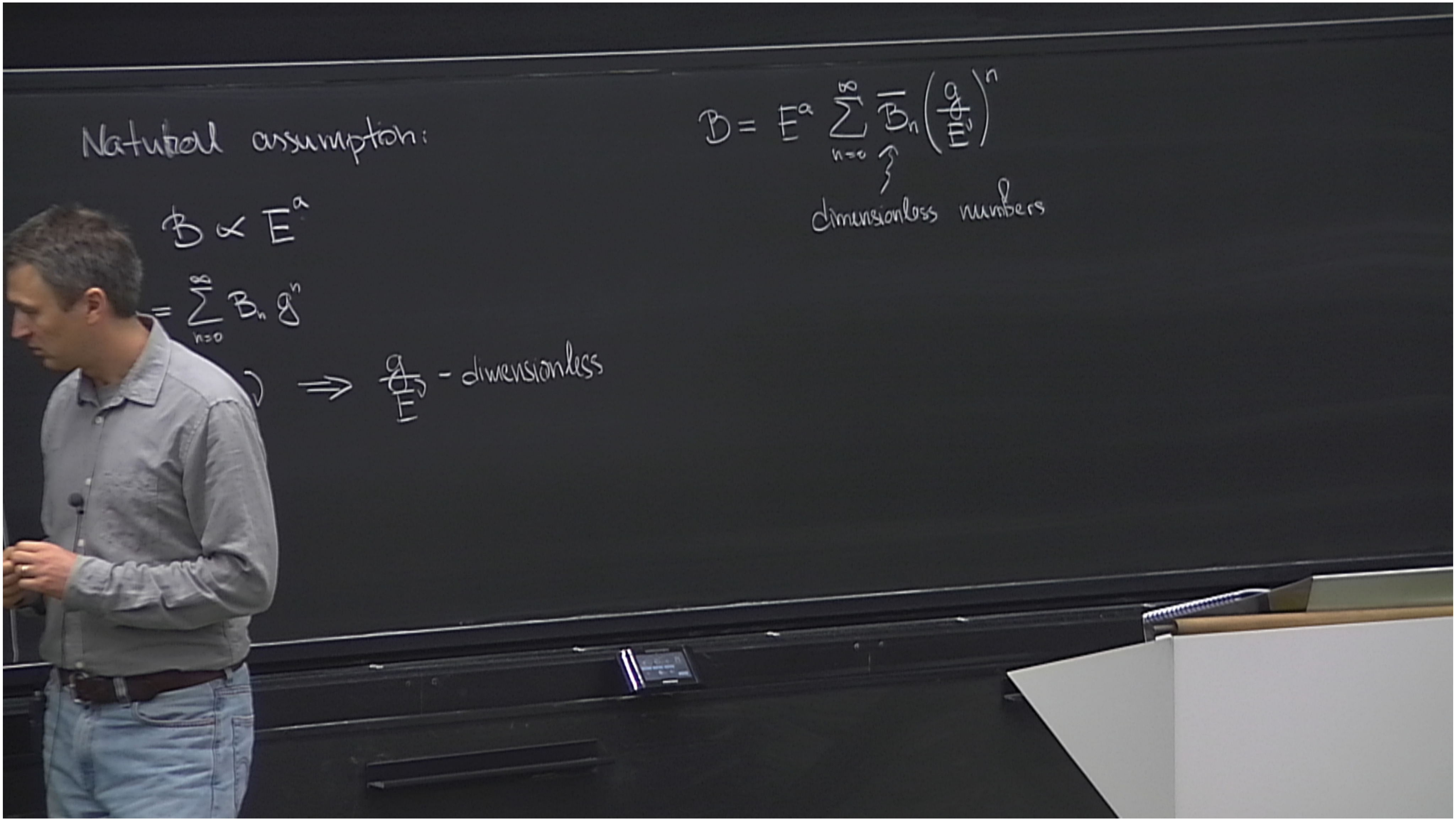
$$B \propto E^a$$

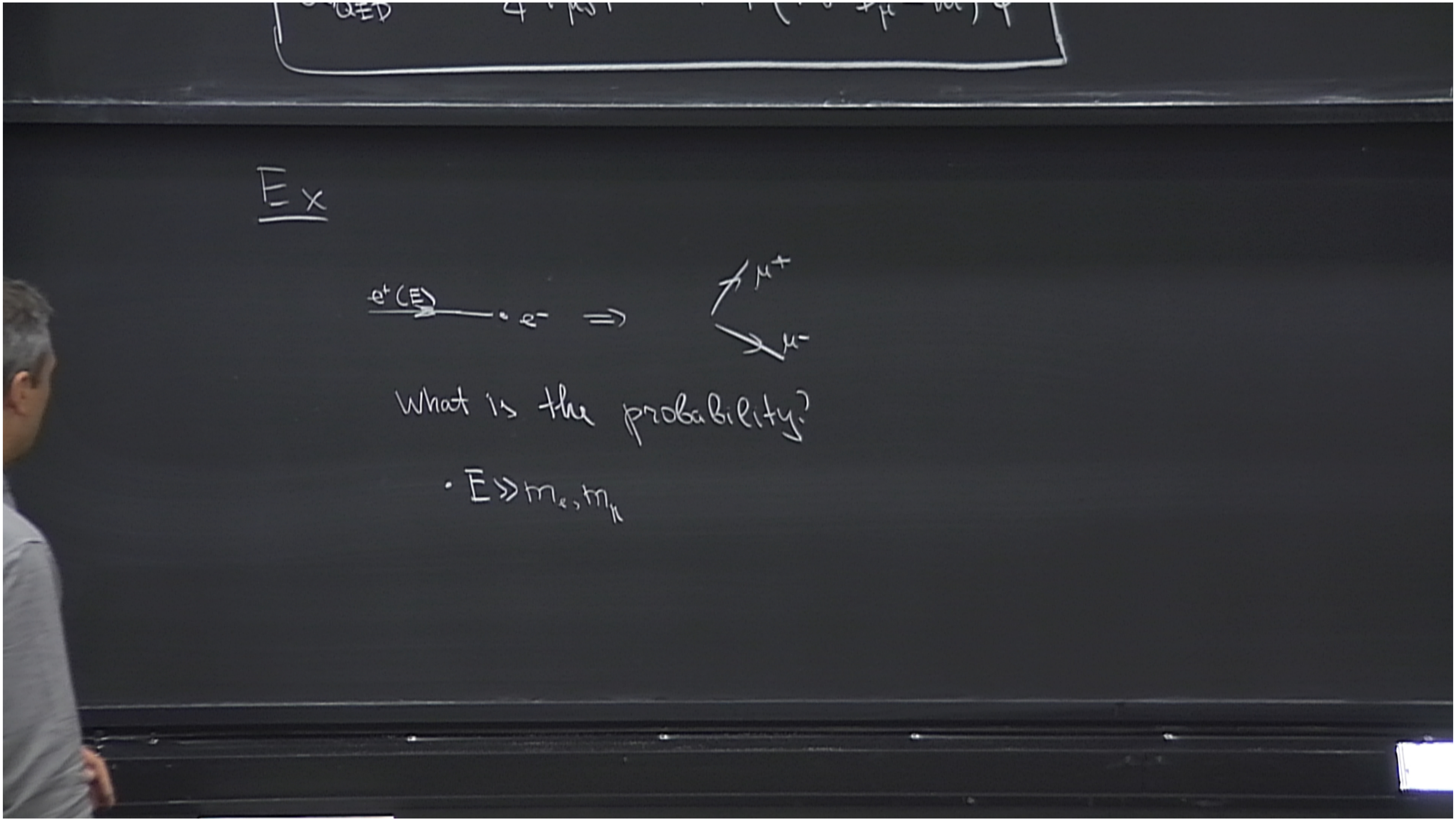
$$B = \sum_{n=0}^{\infty} B_n g^n$$

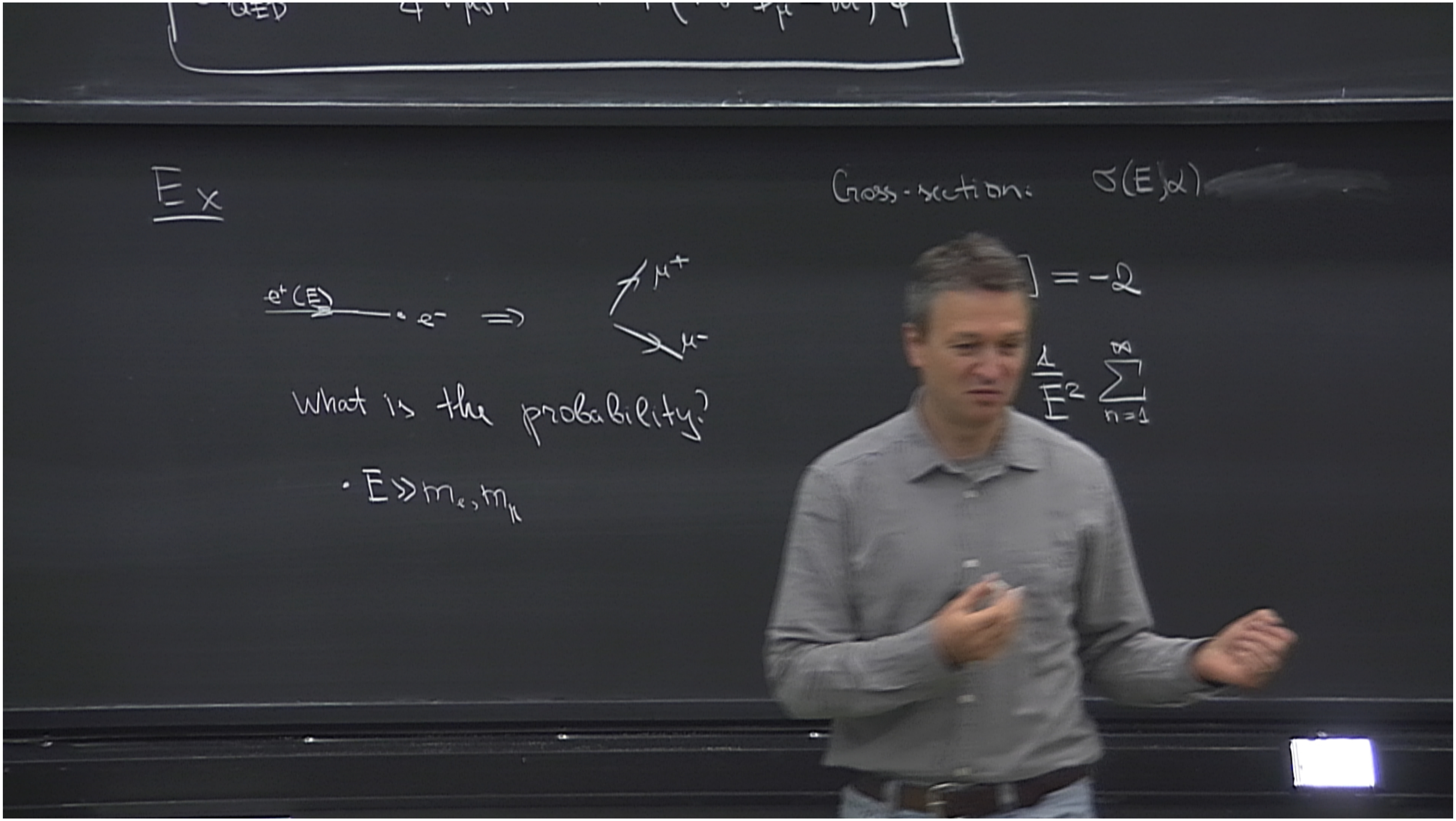
$$[g] = \nu \Rightarrow \frac{g}{E^\nu} \text{ - dimensionless}$$

$$B = E^a \sum_{n=0}^{\infty} \bar{B}_n \left(\frac{g}{E^\nu}\right)^n$$

dimensionless numbers


Natural assumption:
$B \propto E^a$
$= \sum_{n=0}^{\infty} B_n g^n$
$\Rightarrow \frac{g}{E}$ - dimensionless
$B = E^a \sum_{n=0}^{\infty} \bar{B}_n \left(\frac{g}{E^\nu}\right)^n$
dimensionless numbers


Ex
$e^+(E)$
e^-
μ^+
μ^-
What is the probability?
$\cdot\ E \gg m_e, m_\mu$


Ex
e+(E)
e-
μ+
μ-
What is the probability?
· E ≫ m_e, m_μ
Cross-section: σ(E, α)
] = -2
1/E² ∑ n=1 ∞

Ex

$e^+(E) \longrightarrow \cdot\, e^- \implies \mu^+, \mu^-$

What is the probability?

- $E \gg m_e, m_\mu$

Cross-section: $\sigma(E,\alpha)$

$$[\sigma] = -2$$

$$\sigma = \frac{1}{E^2} \sum_{n=1}^{\infty} \bar{\sigma}_n \alpha^n$$