Title: Quantum Field Theory I - Lecture 7

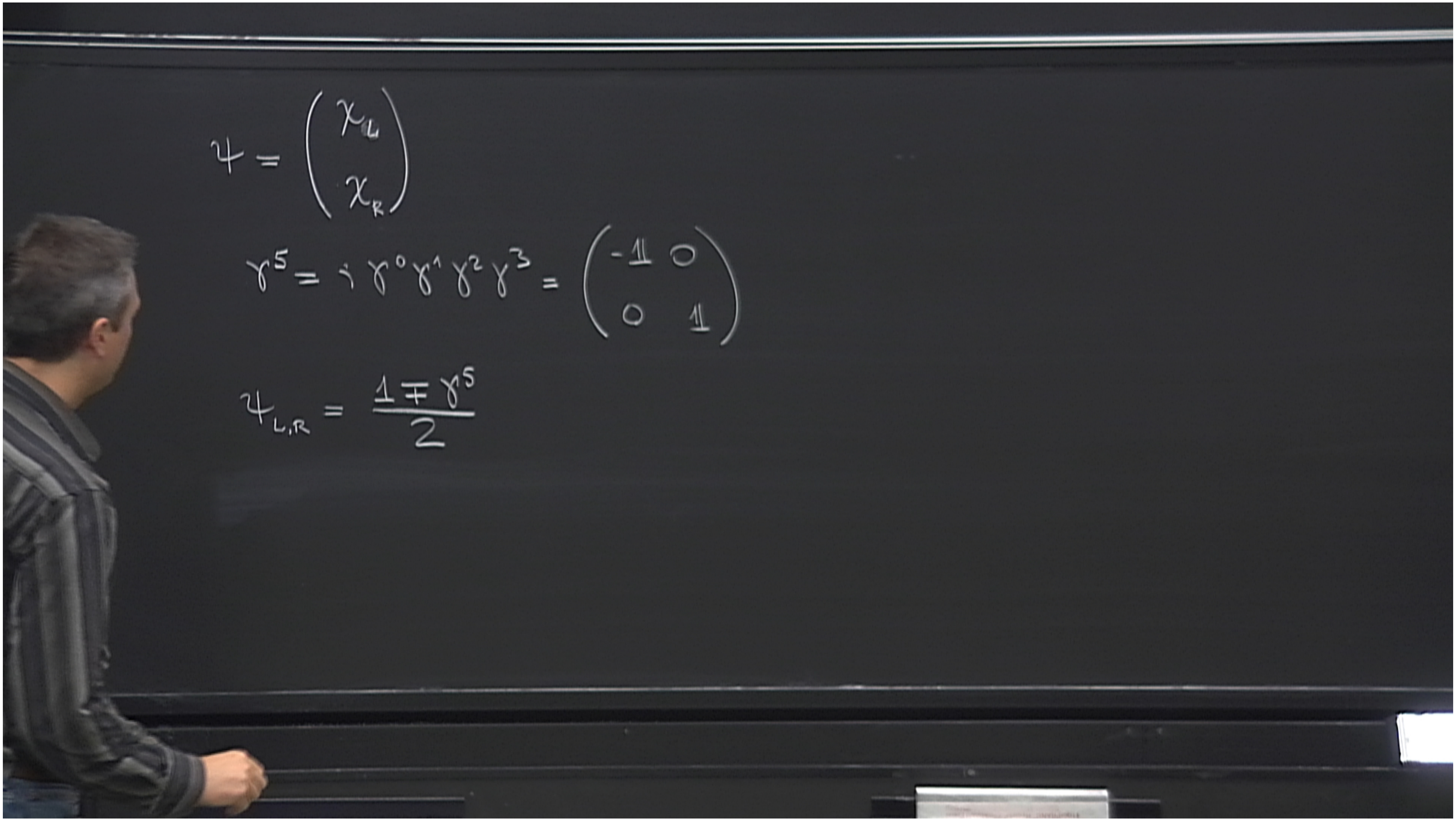
Date: Oct 12, 2011 09:00 AM

URL: http://pirsa.org/11100015

Abstract:

$$\psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}$$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$\psi_{L,R} = \frac{1 \mp \gamma^5}{2}$

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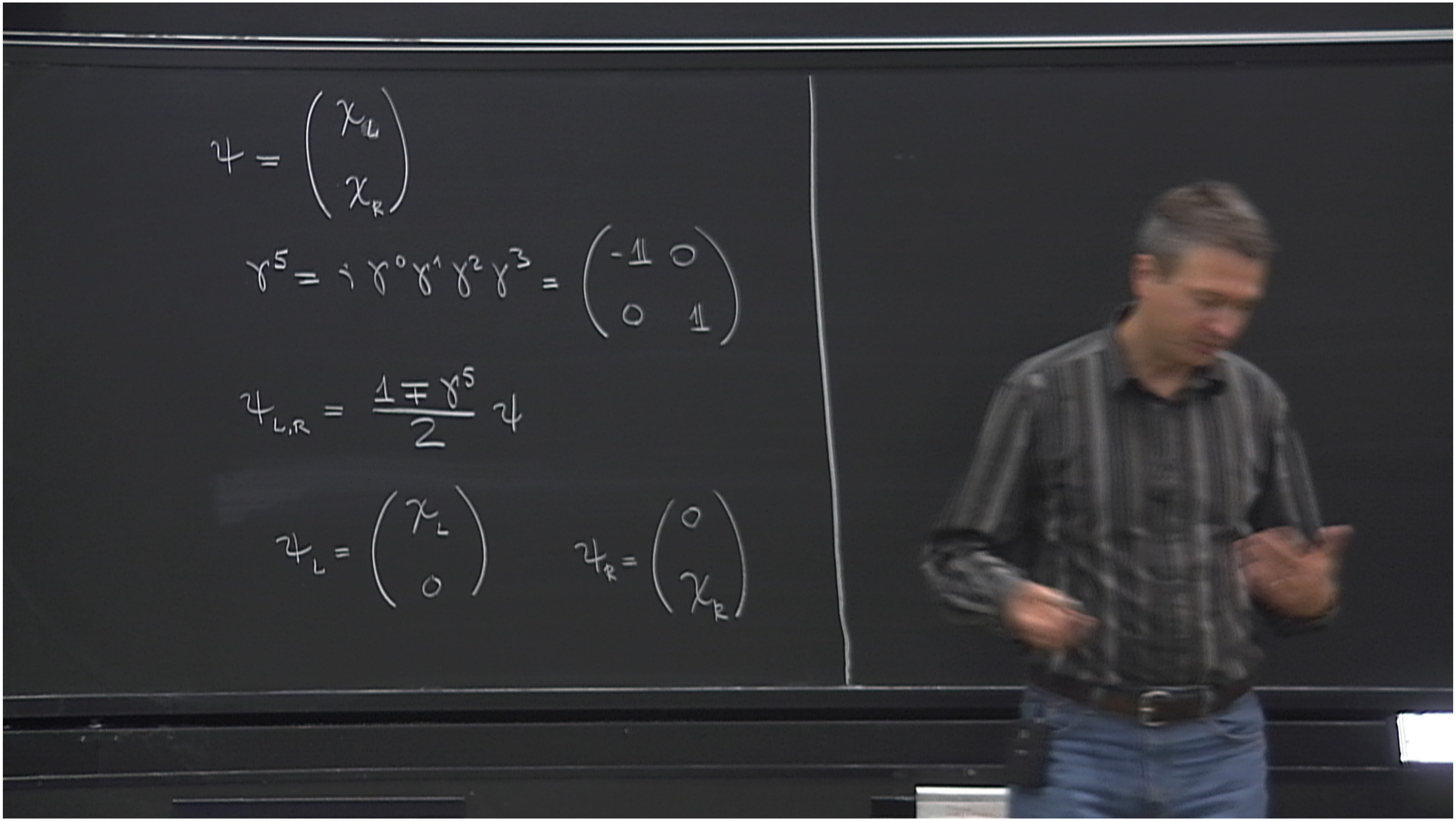
$$\psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix} \qquad \psi_R = \begin{pmatrix} 0 \\ \chi_R \end{pmatrix}$$


$\psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$
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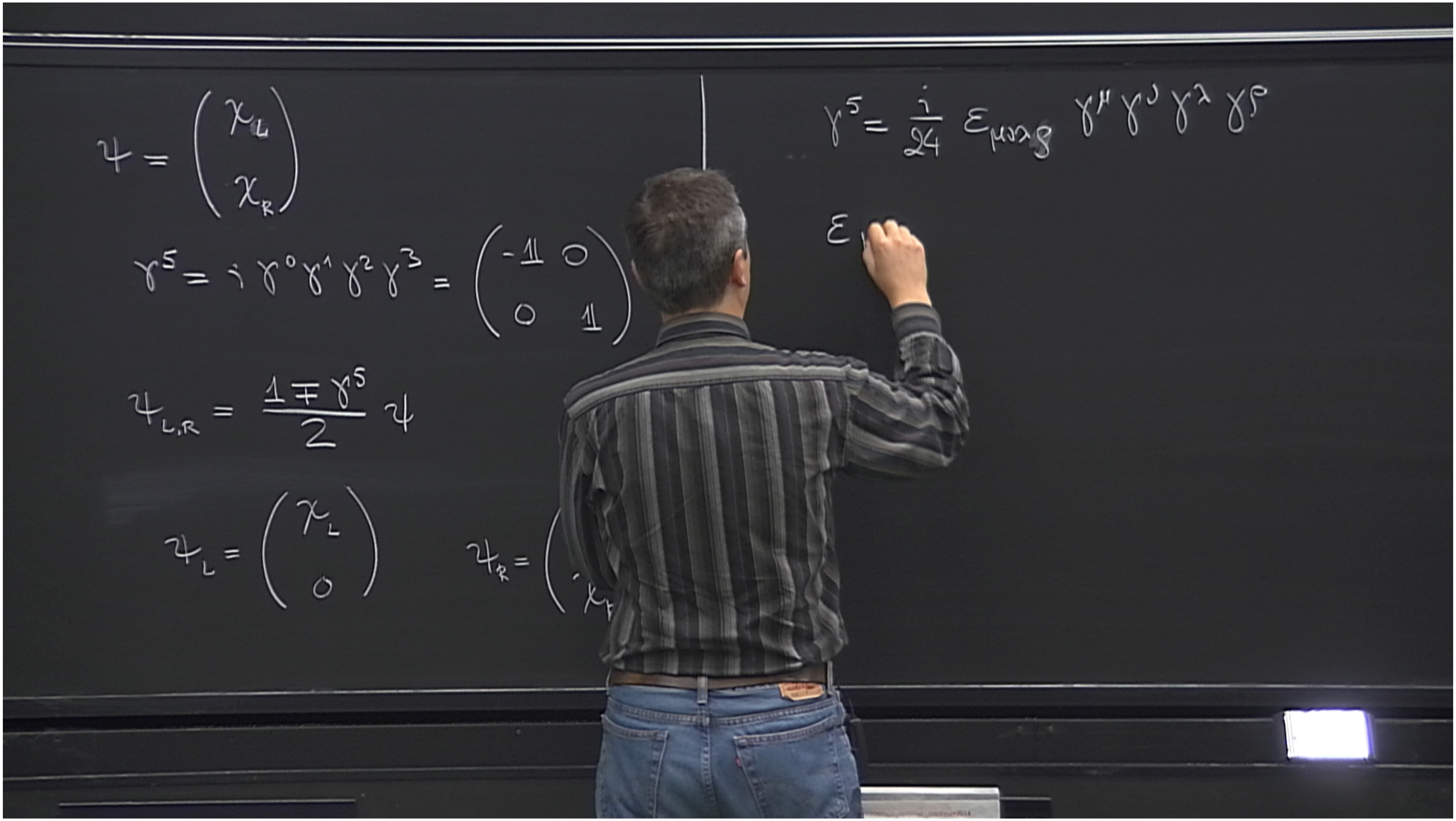
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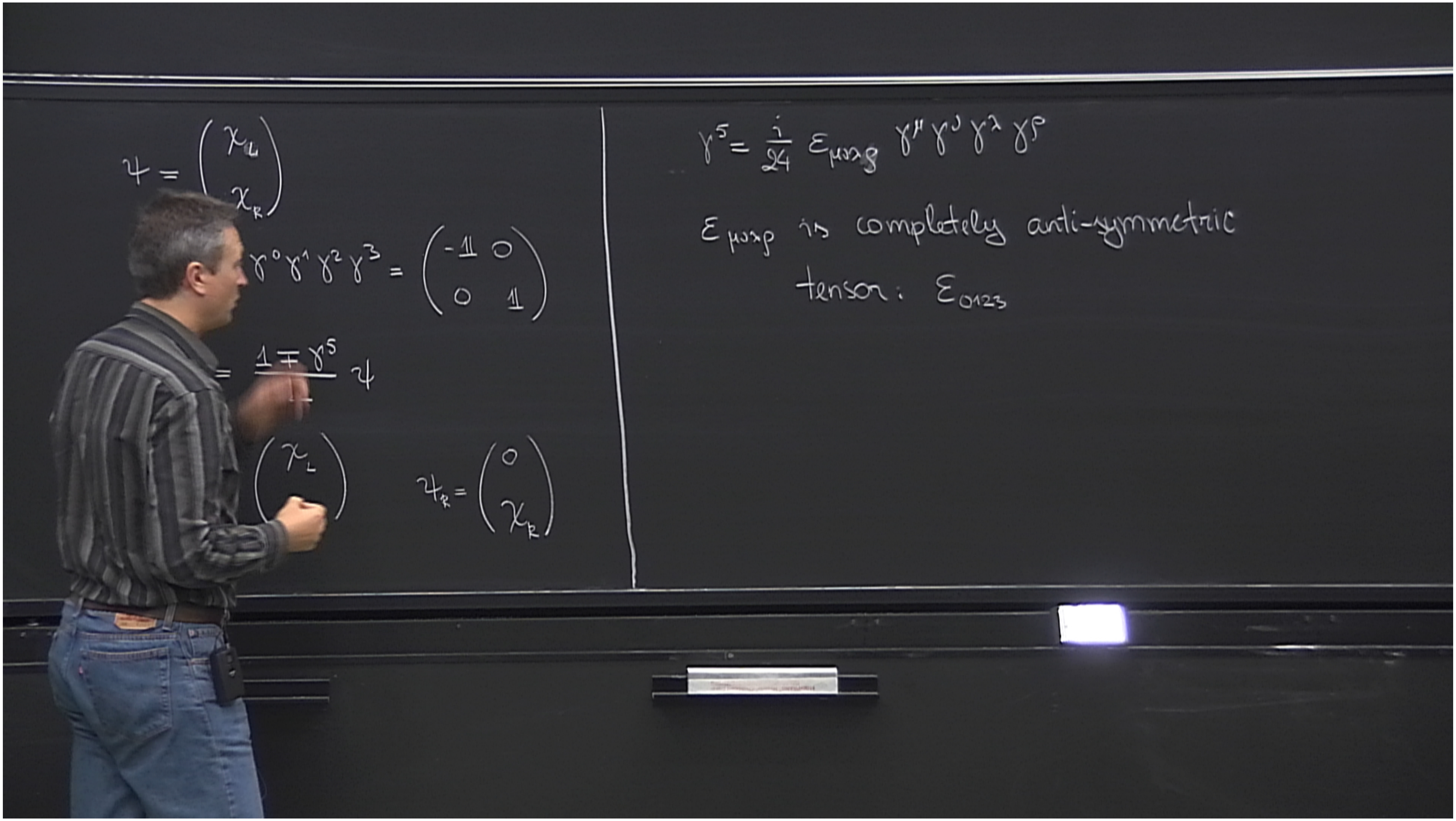
$\psi_{L,R} = \frac{1 \mp \gamma^5}{2}\psi$
$\psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$
$\psi_R =$
$\gamma^5 = \frac{i}{24}\varepsilon_{\mu\nu\lambda\rho}\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho$
ε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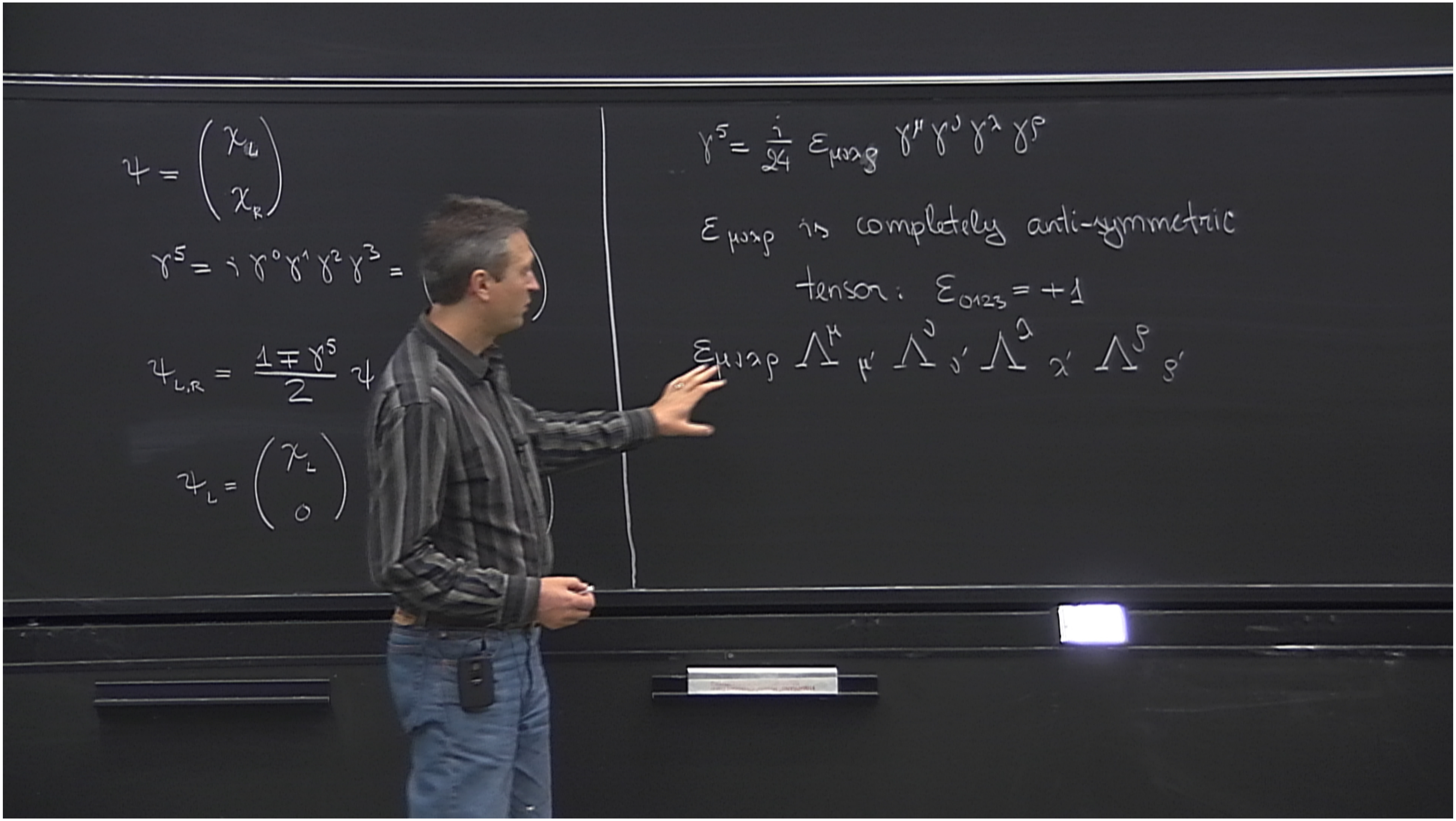
$= \frac{1 \mp \gamma^5}{2} \psi$
$\begin{pmatrix} \chi_L \\ \end{pmatrix}$
$\psi_R = \begin{pmatrix} 0 \\ \chi_R \end{pmatrix}$
$\gamma^5 = \frac{i}{24} \varepsilon_{\mu\nu\lambda\rho} \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\rho$
$\varepsilon_{\mu\nu\lambda\rho}$ is completely anti-symmetric
tensor: ε_{0123}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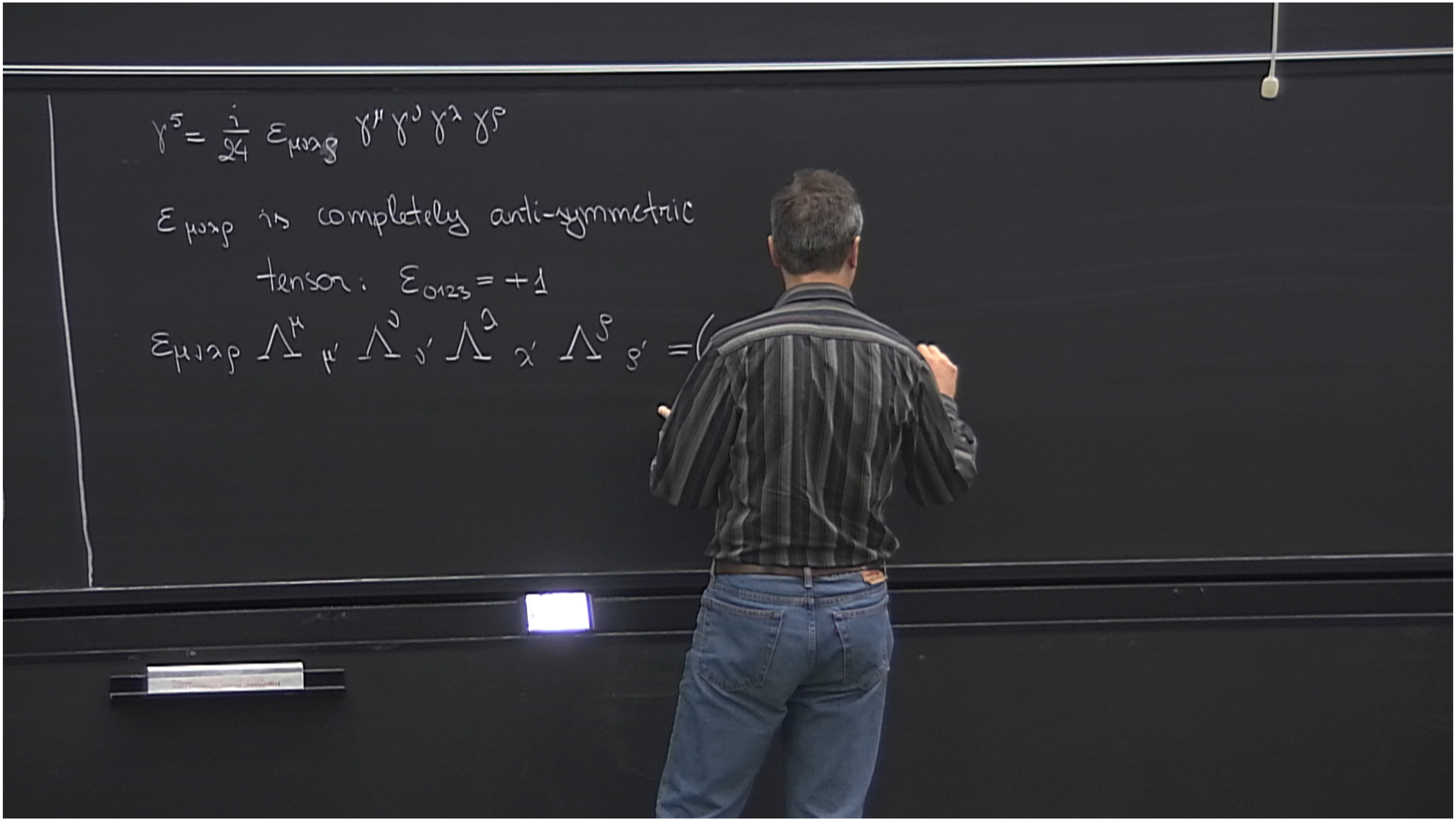
tensor: $\varepsilon_{0123} = +1$
$\varepsilon_{\mu\nu\lambda\rho} \Lambda^\mu{}_{\mu'} \Lambda^\nu{}_{\nu'} \Lambda^\lambda{}_{\lambda'} \Lambda^\rho{}_{\rho'}$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$$\gamma^5 = \frac{i}{24}\varepsilon_{\mu\nu\lambda\rho}\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho$$

$\varepsilon_{\mu\nu\lambda\rho}$ is completely anti-symmetric tensor: $\varepsilon_{0123} = +1$

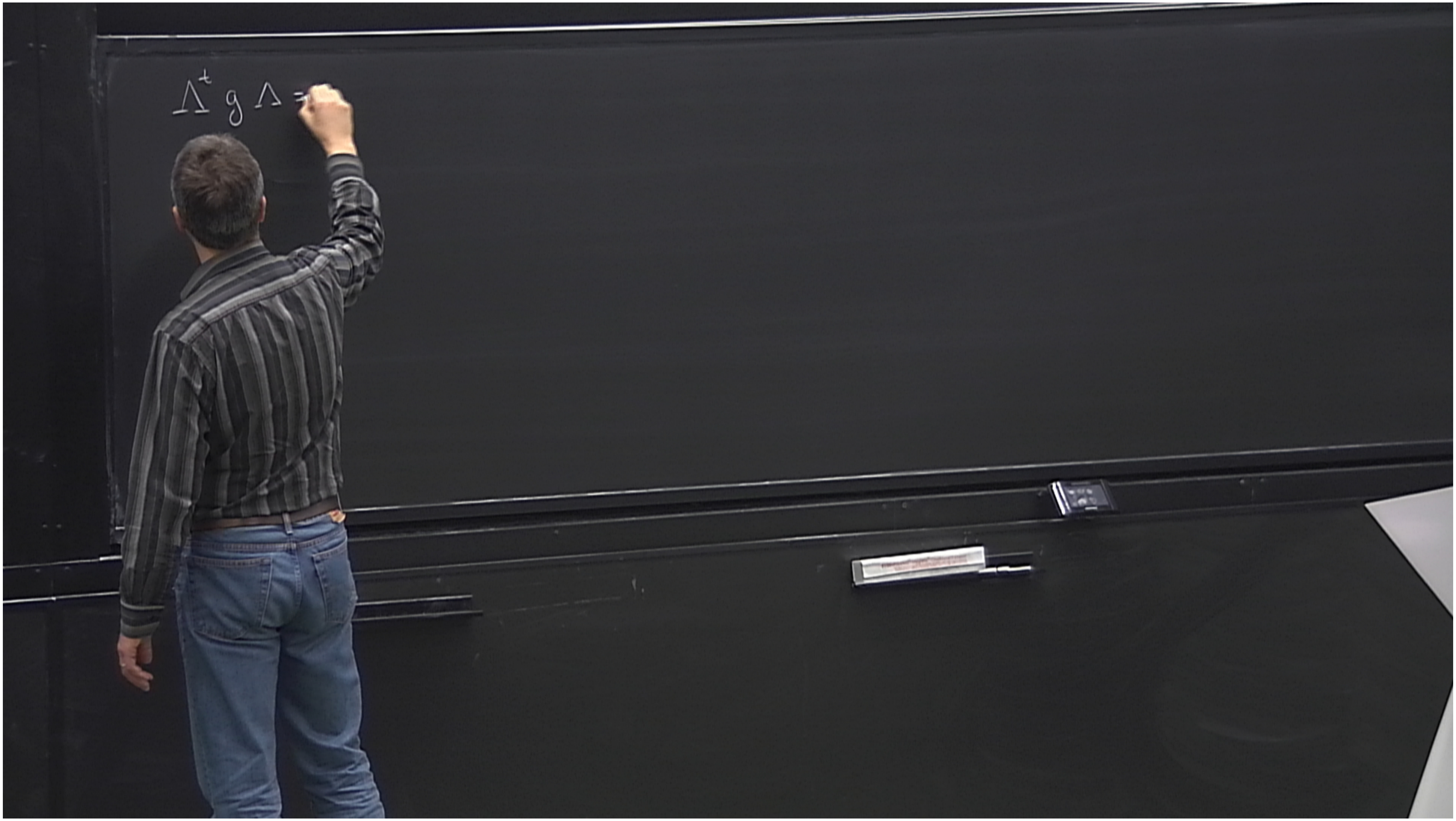
$$\varepsilon_{\mu\nu\lambda\rho}\,\Lambda^\mu{}_{\mu'}\,\Lambda^\nu{}_{\nu'}\,\Lambda^\lambda{}_{\lambda'}\,\Lambda^\rho{}_{\rho'} = (\det\Lambda)\,\varepsilon_{\mu'\nu'\lambda'\rho'}$$

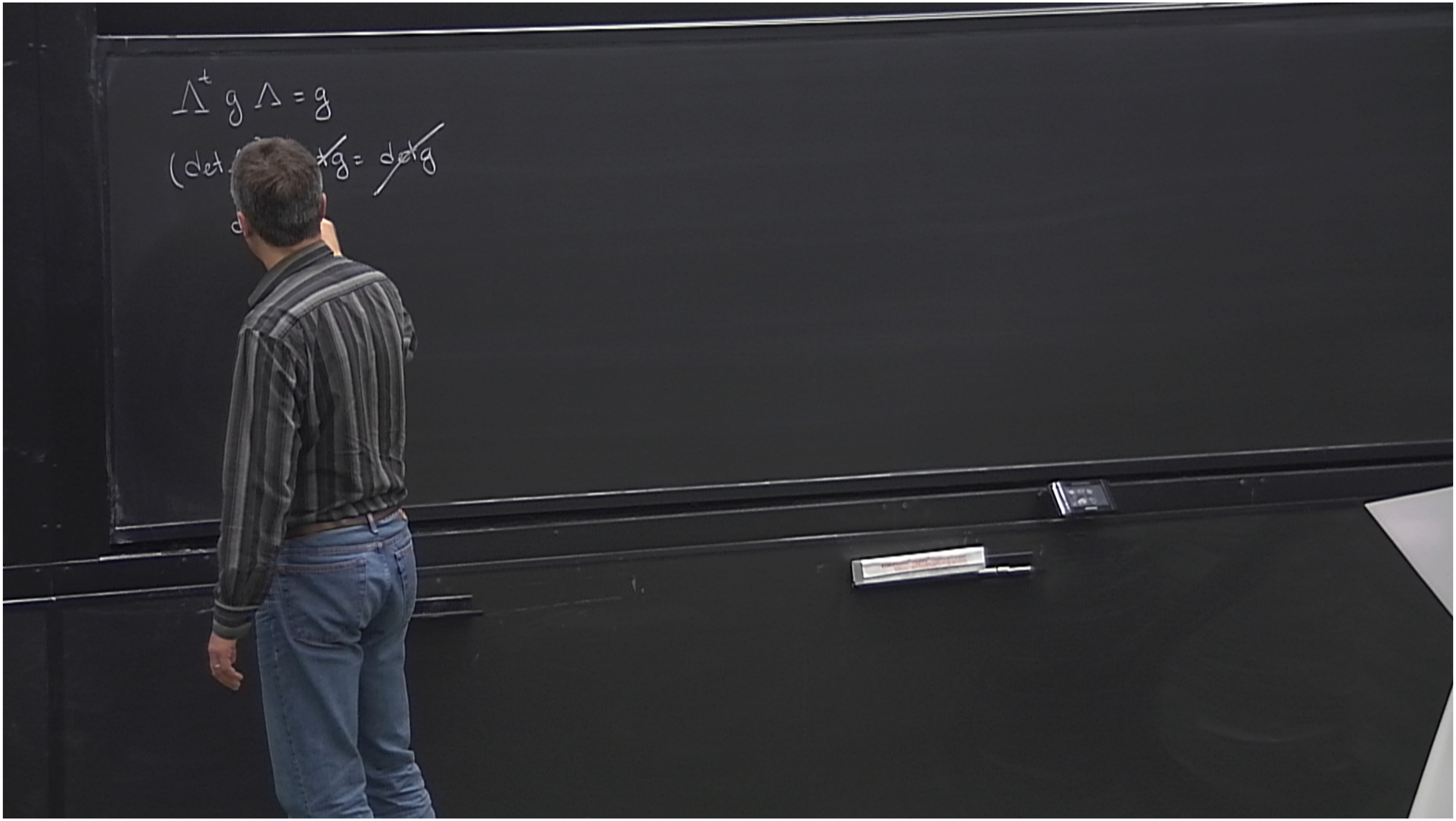
$\varepsilon_{\mu\nu\lambda\rho}$ is completely anti-symmetric

tensor: $\varepsilon_{0123} = +1$

$$\varepsilon_{\mu\nu\lambda\rho} \Lambda^{\mu}{}_{\mu'} \Lambda^{\nu}{}_{\nu'} \Lambda^{\lambda}{}_{\lambda'} \Lambda^{\rho}{}_{\rho'} = (\det \Lambda)\, \varepsilon_{\mu'\nu'\lambda'\rho'}$$

Only transforms as a tensor if $\det \Lambda = +1$


$\Lambda^t g \Lambda =$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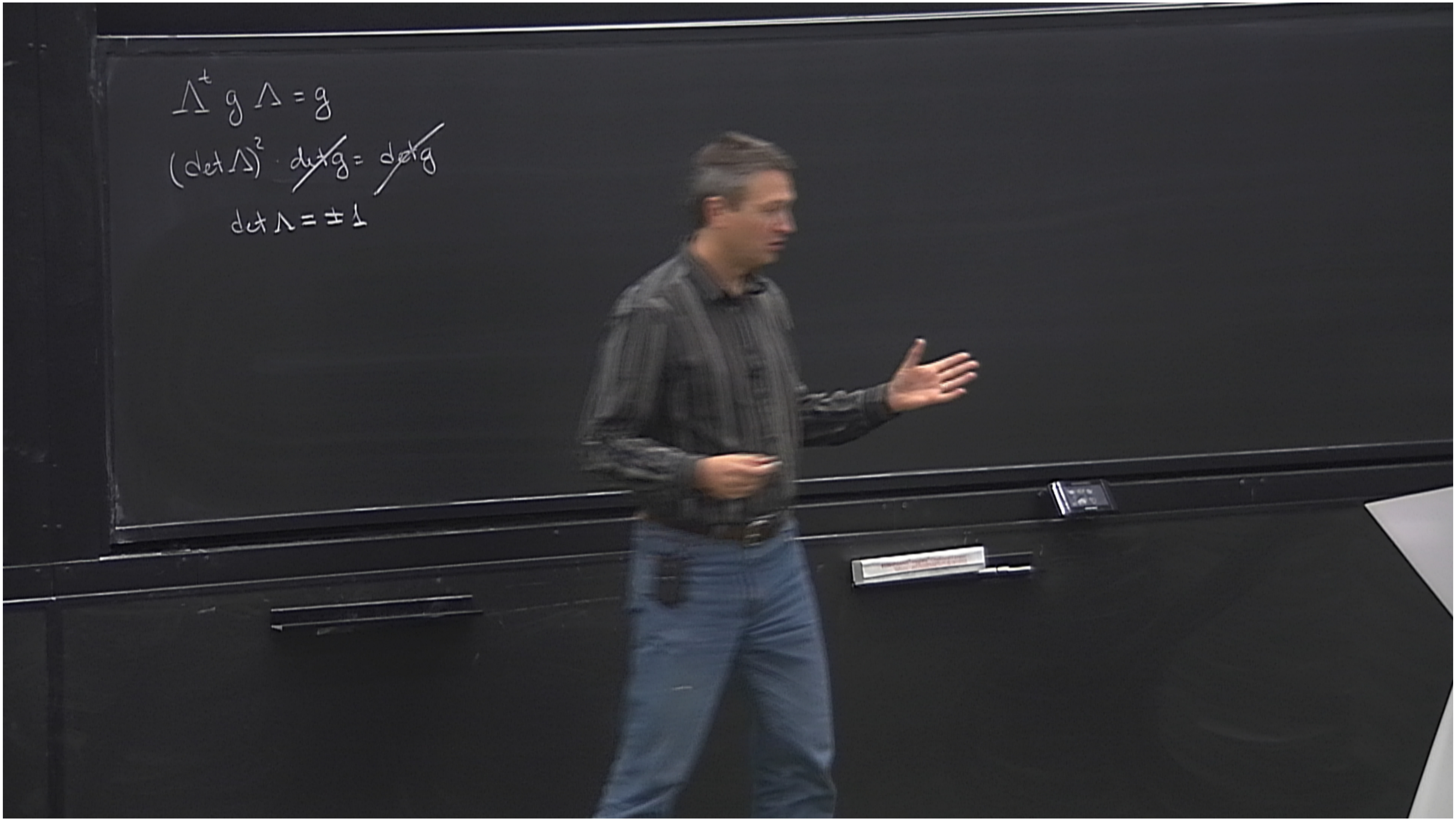
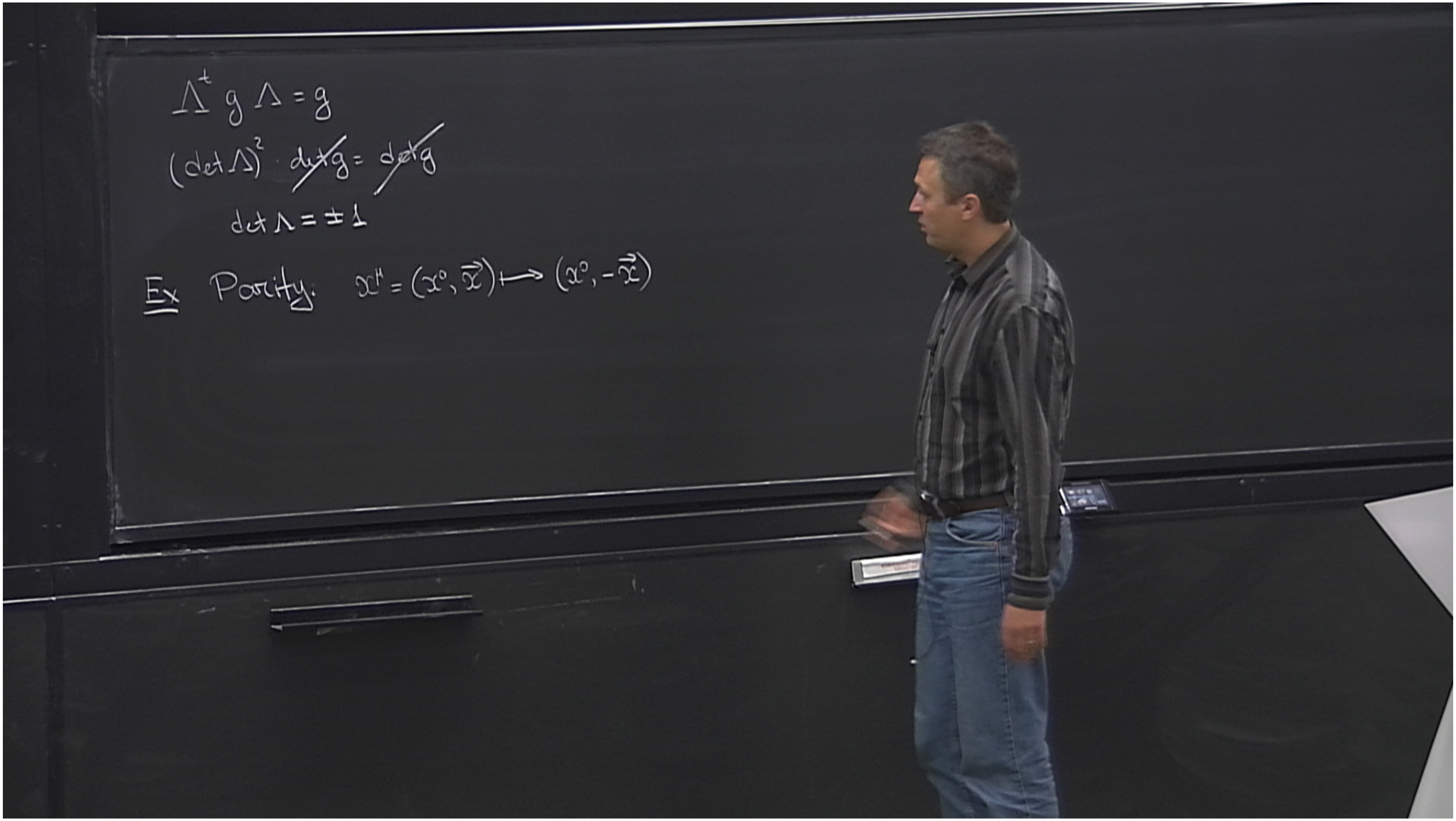

$\Lambda^t g \Lambda = g$
$(\det \Lambda)^2 \cdot \cancel{\det g} = \cancel{\det g}$
$\det \Lambda = \pm 1$


$\Lambda^t g \Lambda = g$
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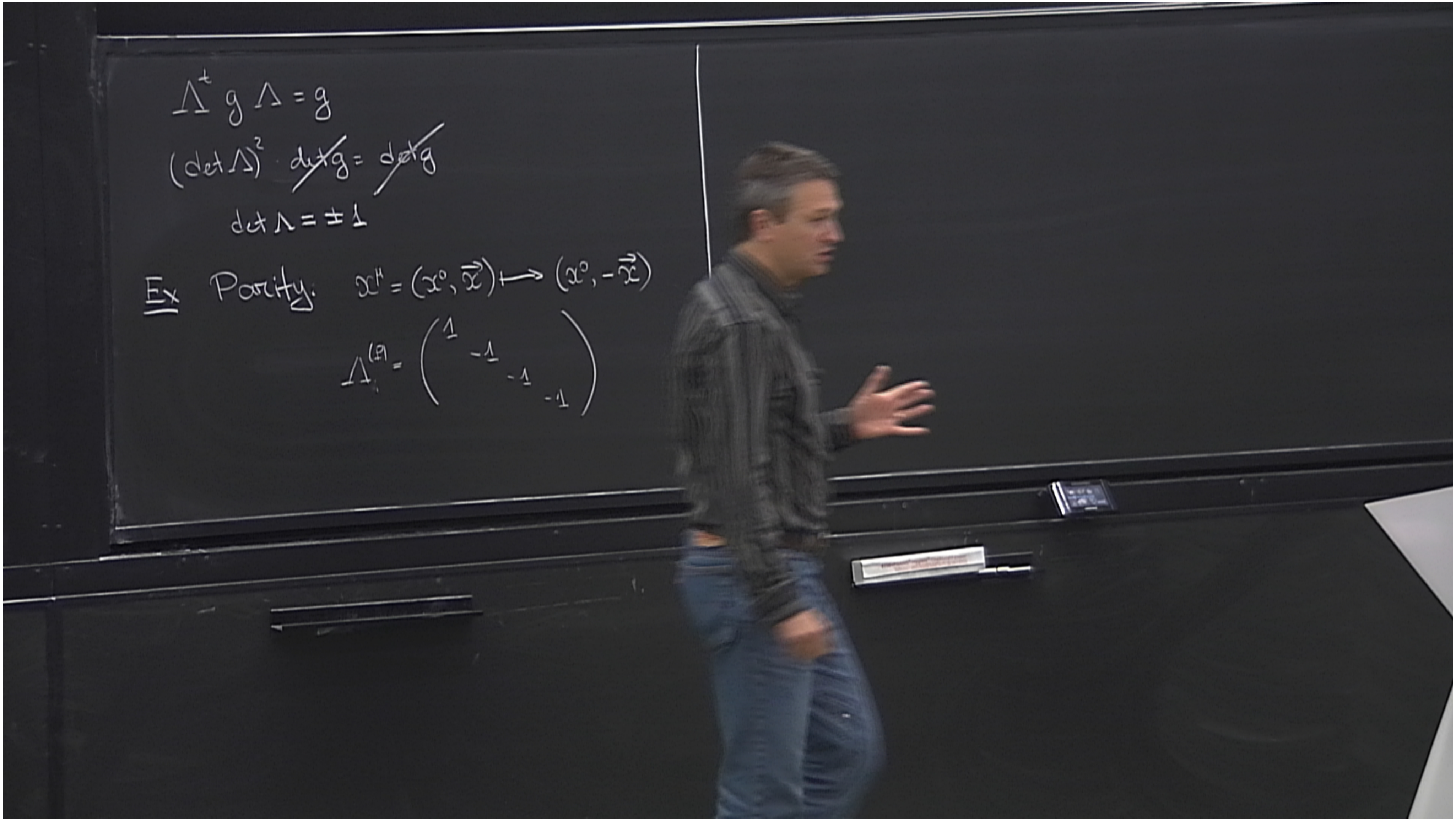
Ex Parity: $x^\mu = (x^0, \vec{x}) \longmapsto (x^0, -\vec{x})$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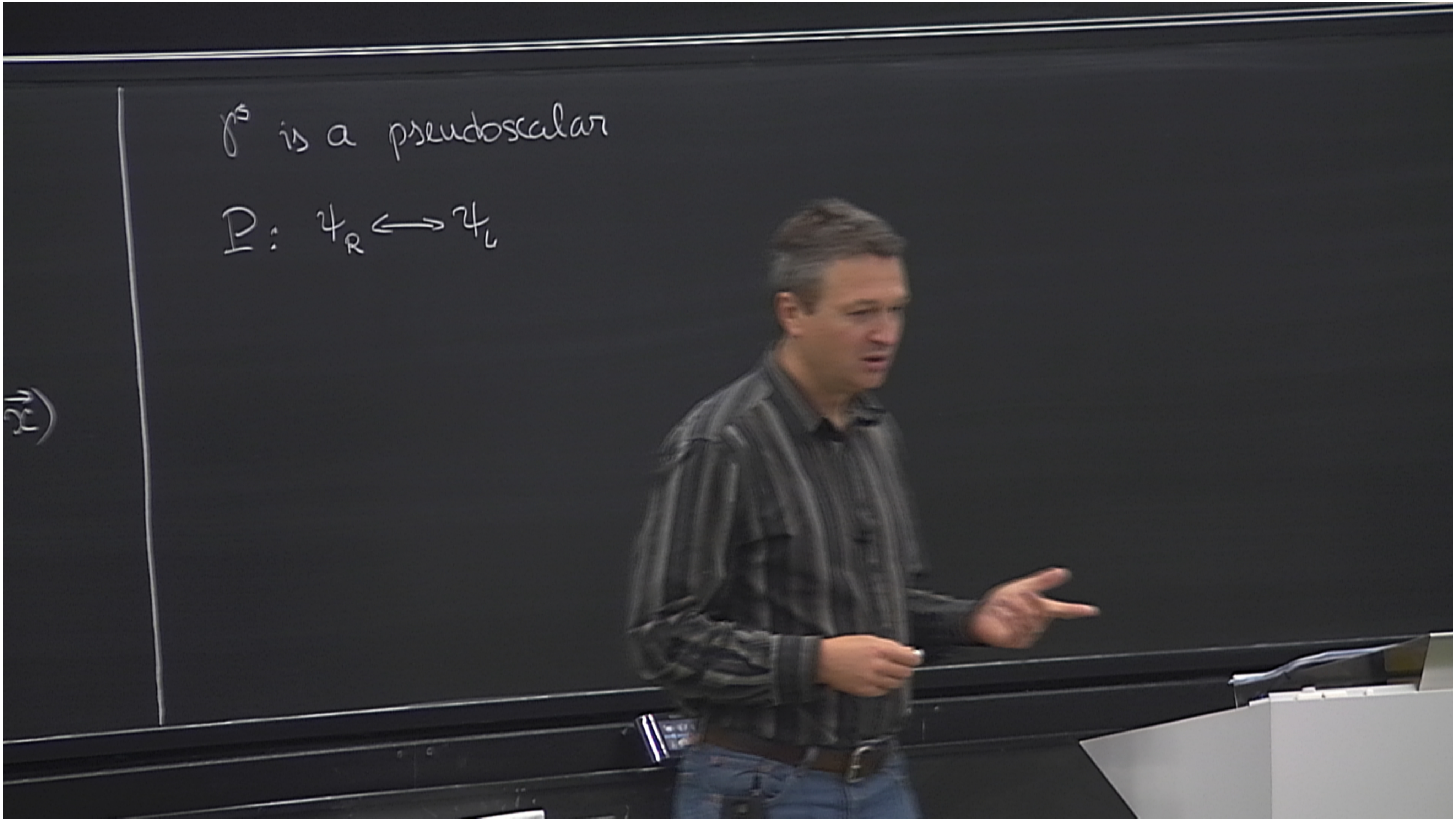
γ^5 is a pseudoscalar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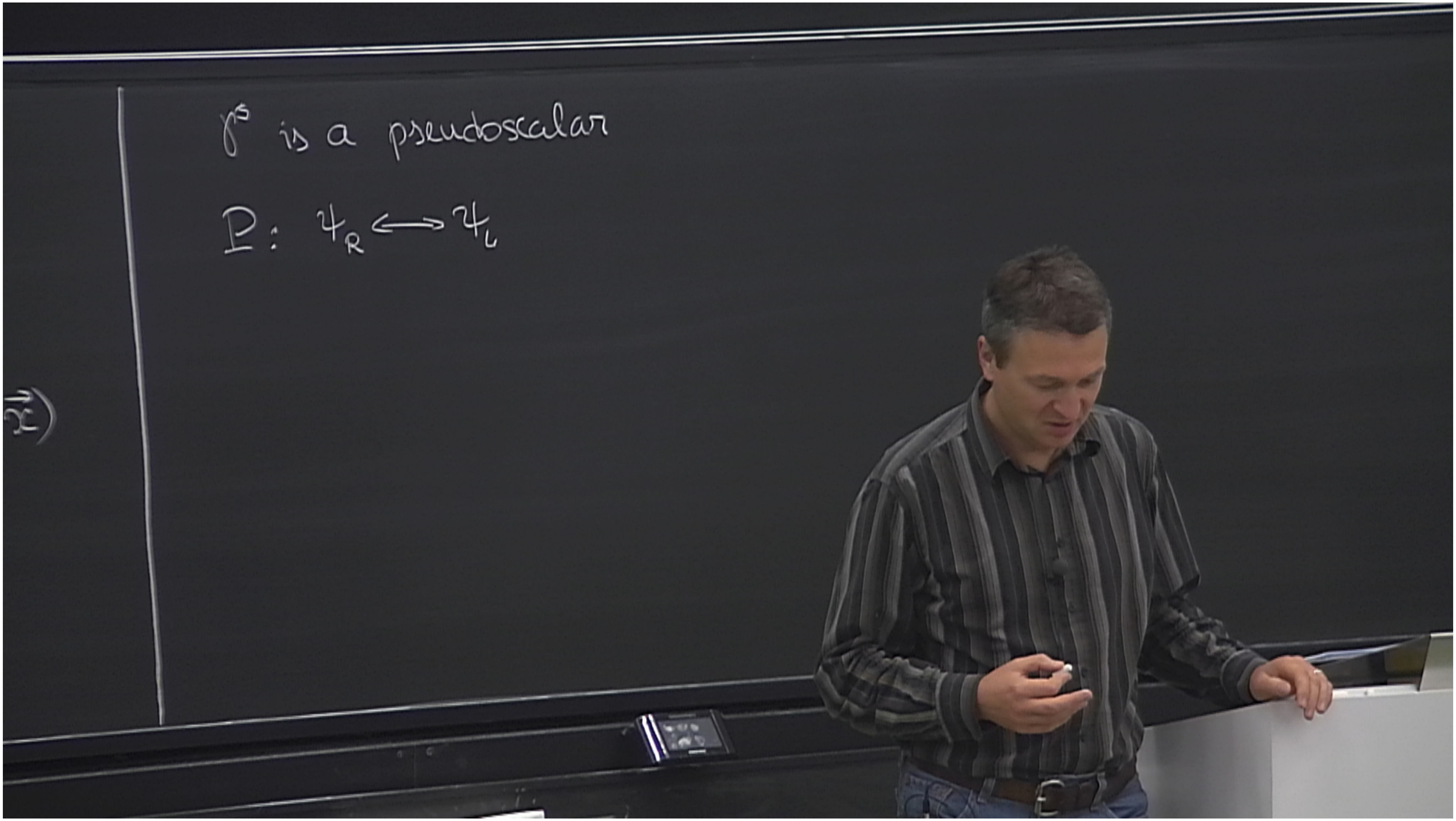
$P: \psi_R \longleftrightarrow \psi_L$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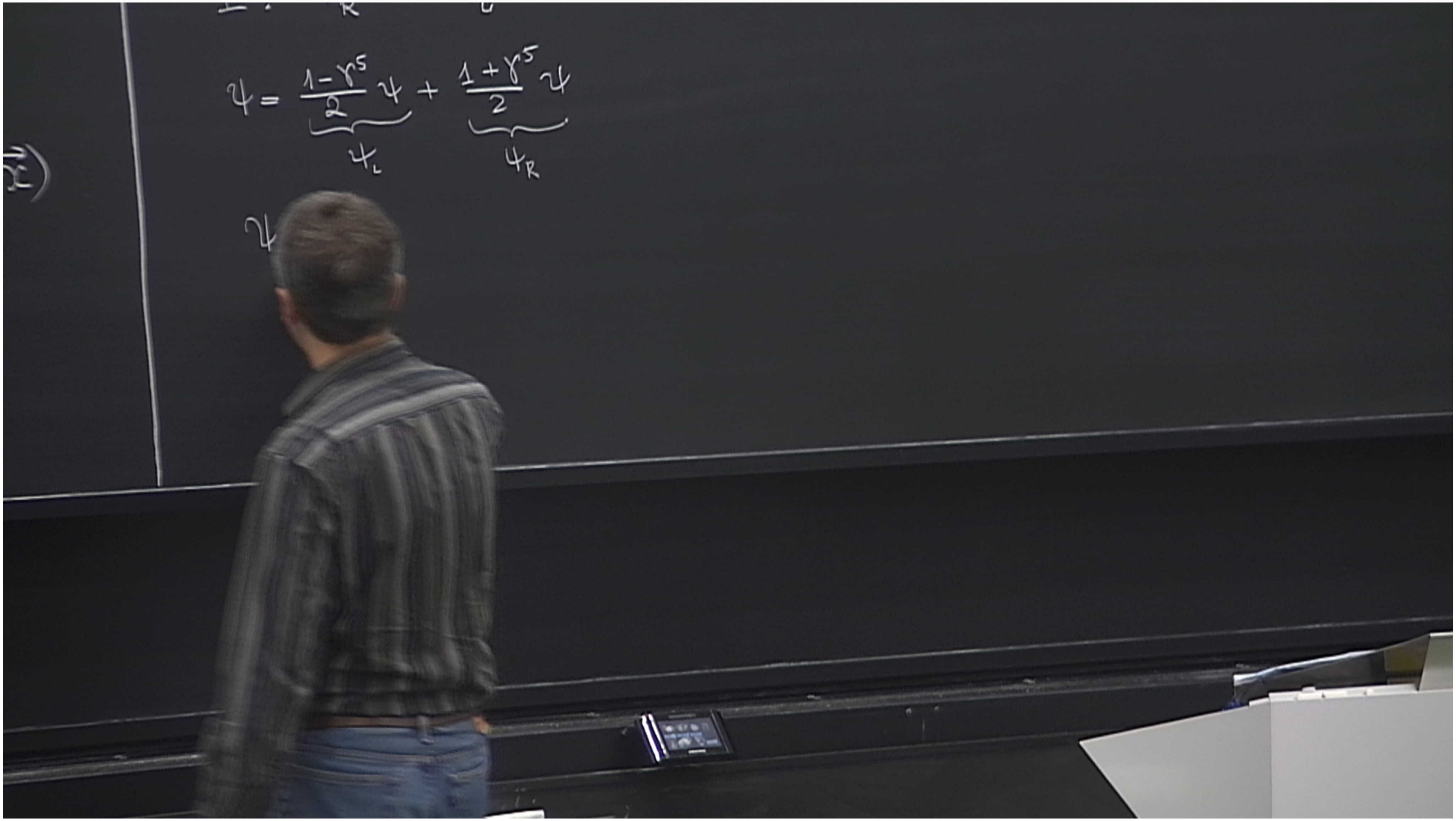
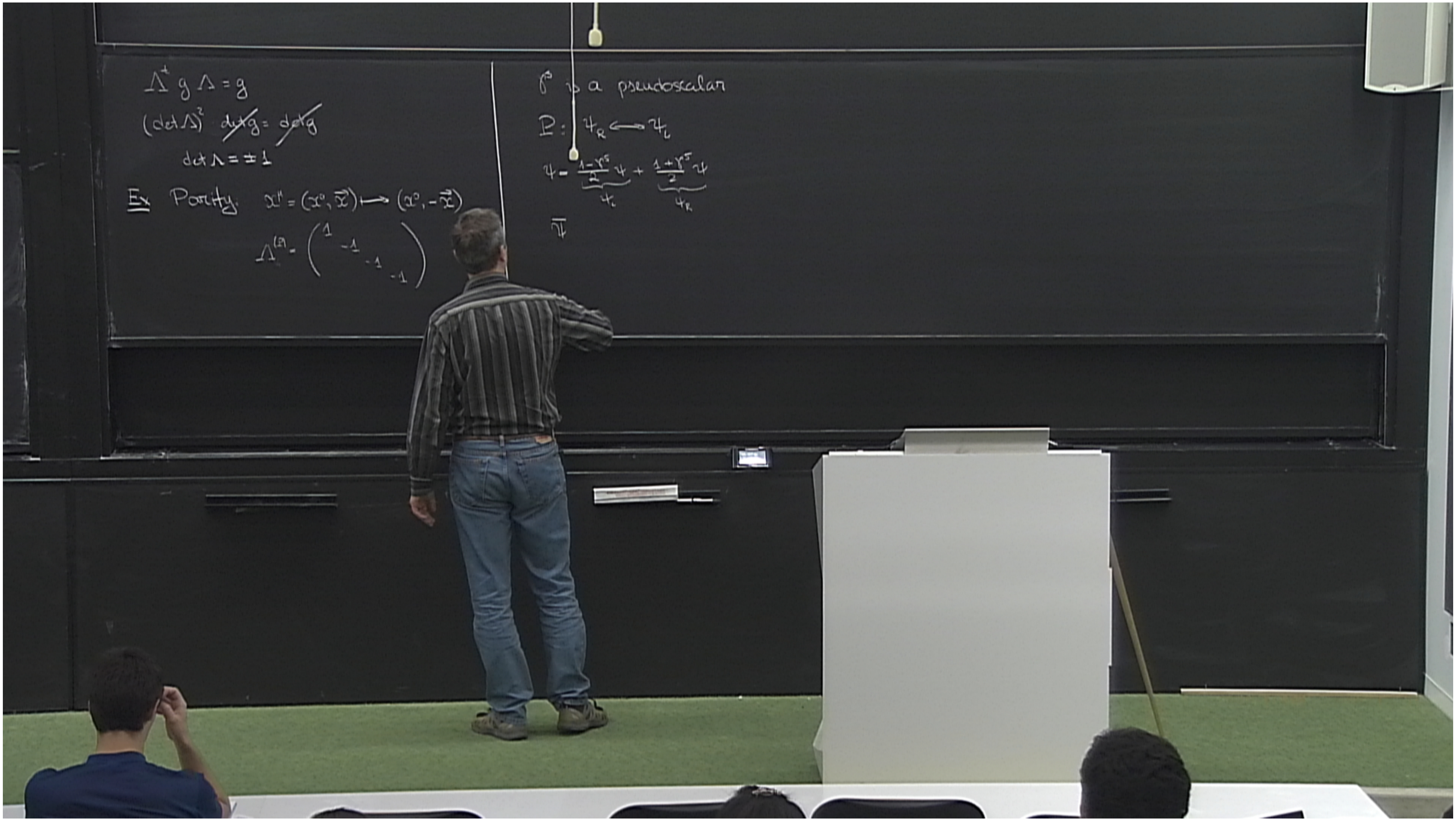

$\psi = \underbrace{\frac{1-\gamma^5}{2}\psi}_{\psi_L} + \underbrace{\frac{1+\gamma^5}{2}\psi}_{\psi_R}$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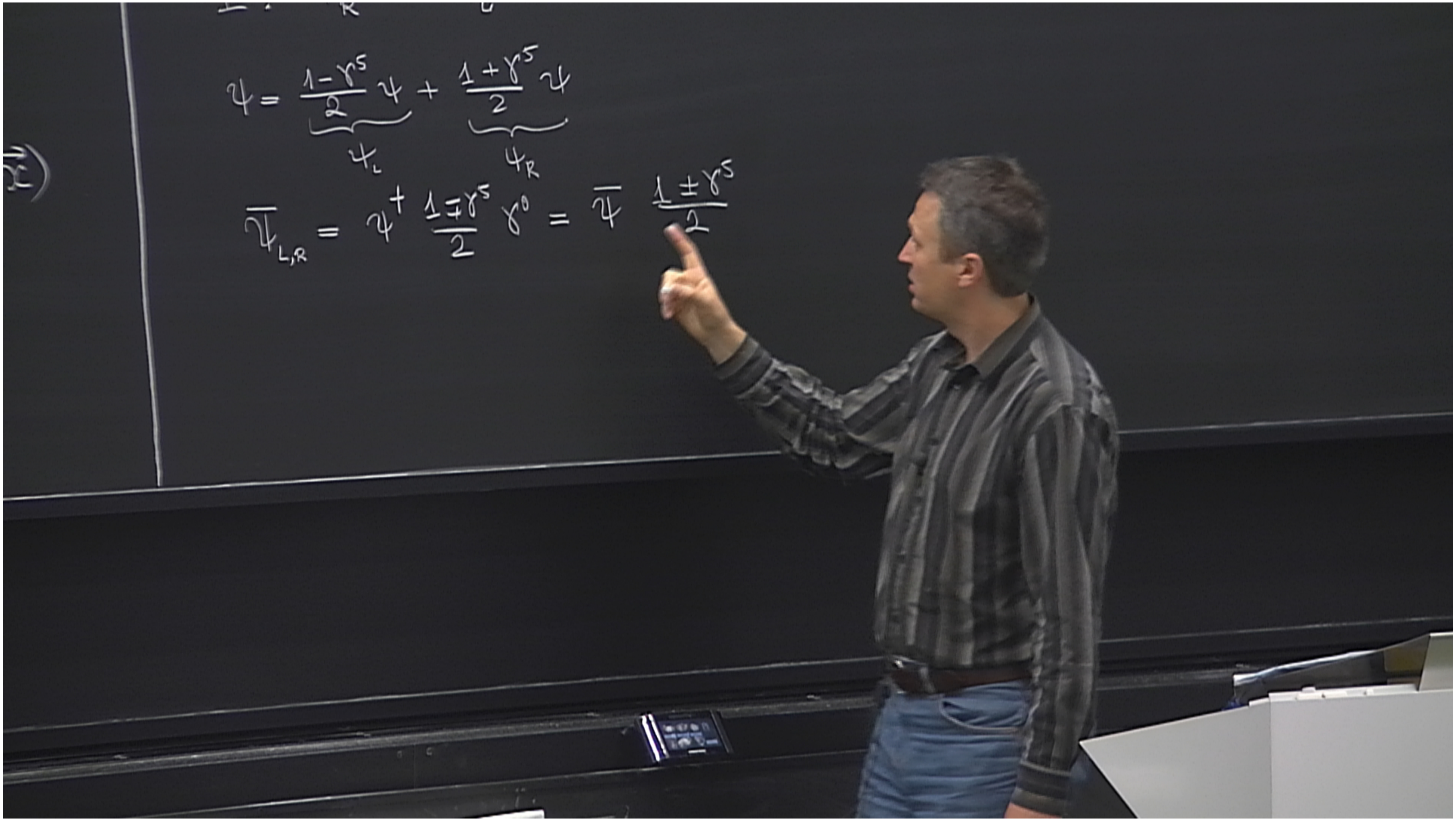
$\bar{\psi}$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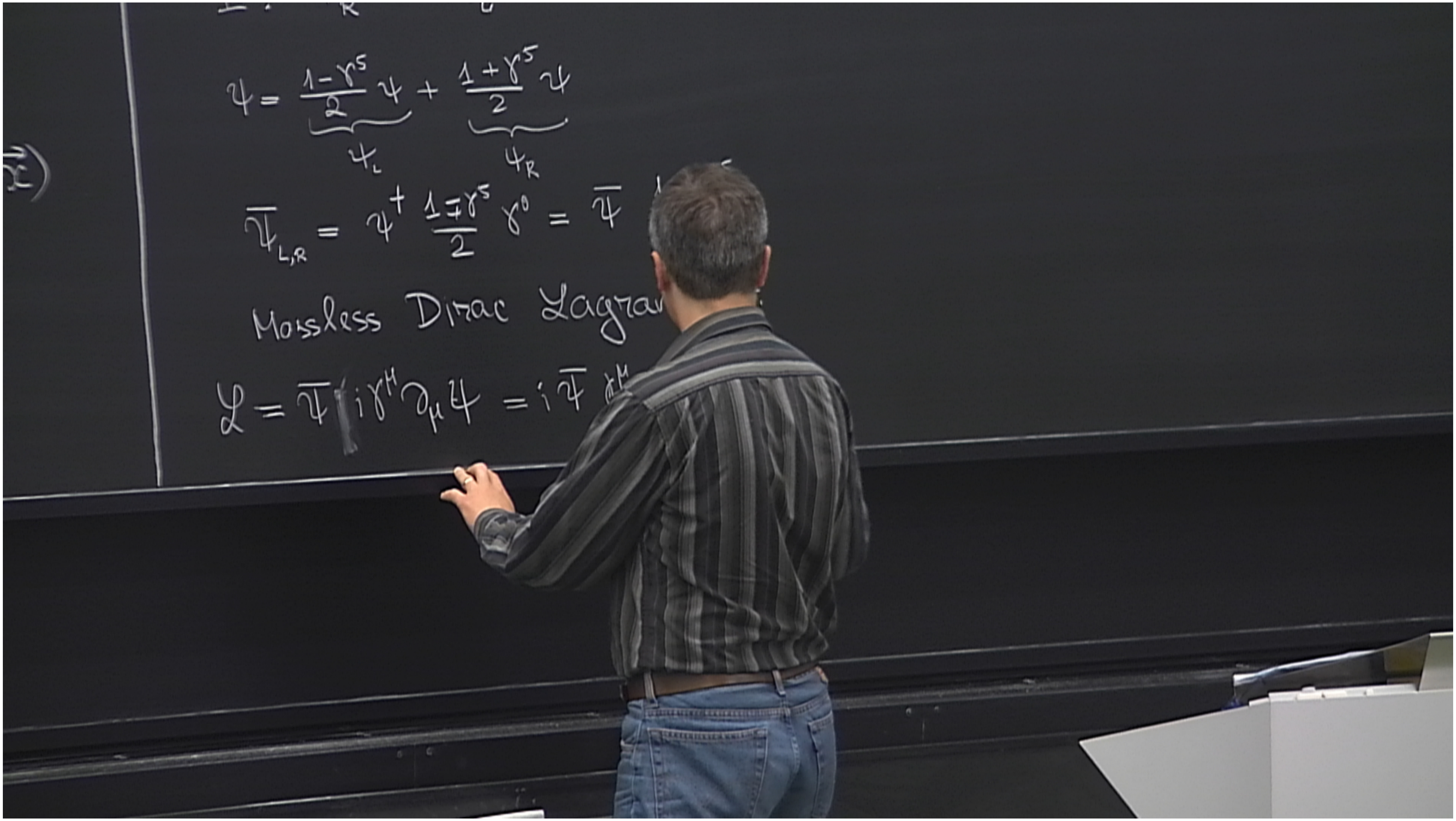
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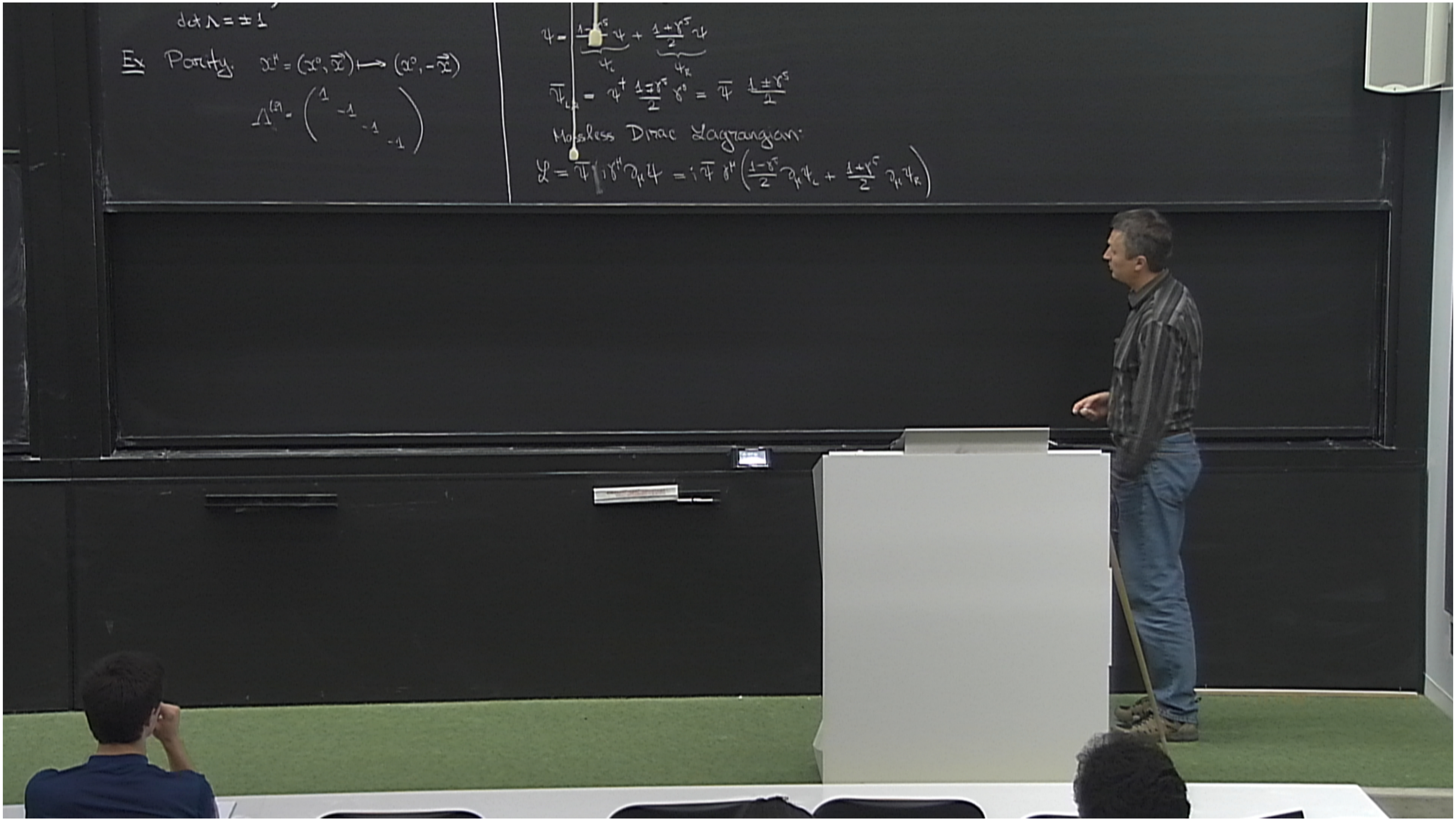
Massless Dirac Lagran
$\mathcal{L} = \bar\psi i\gamma^\mu\partial_\mu\psi = i\bar\psi\gamma^\mu$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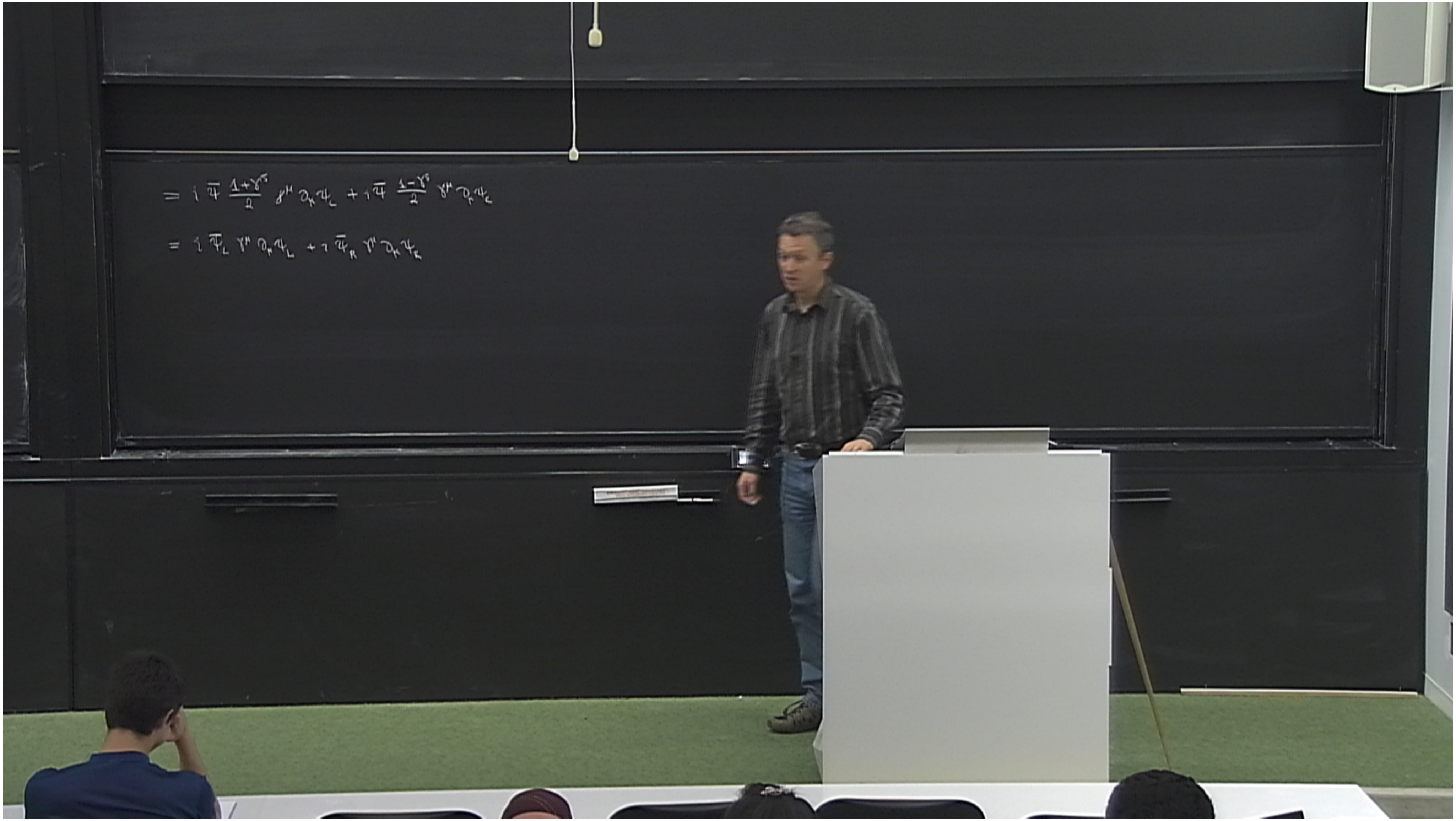
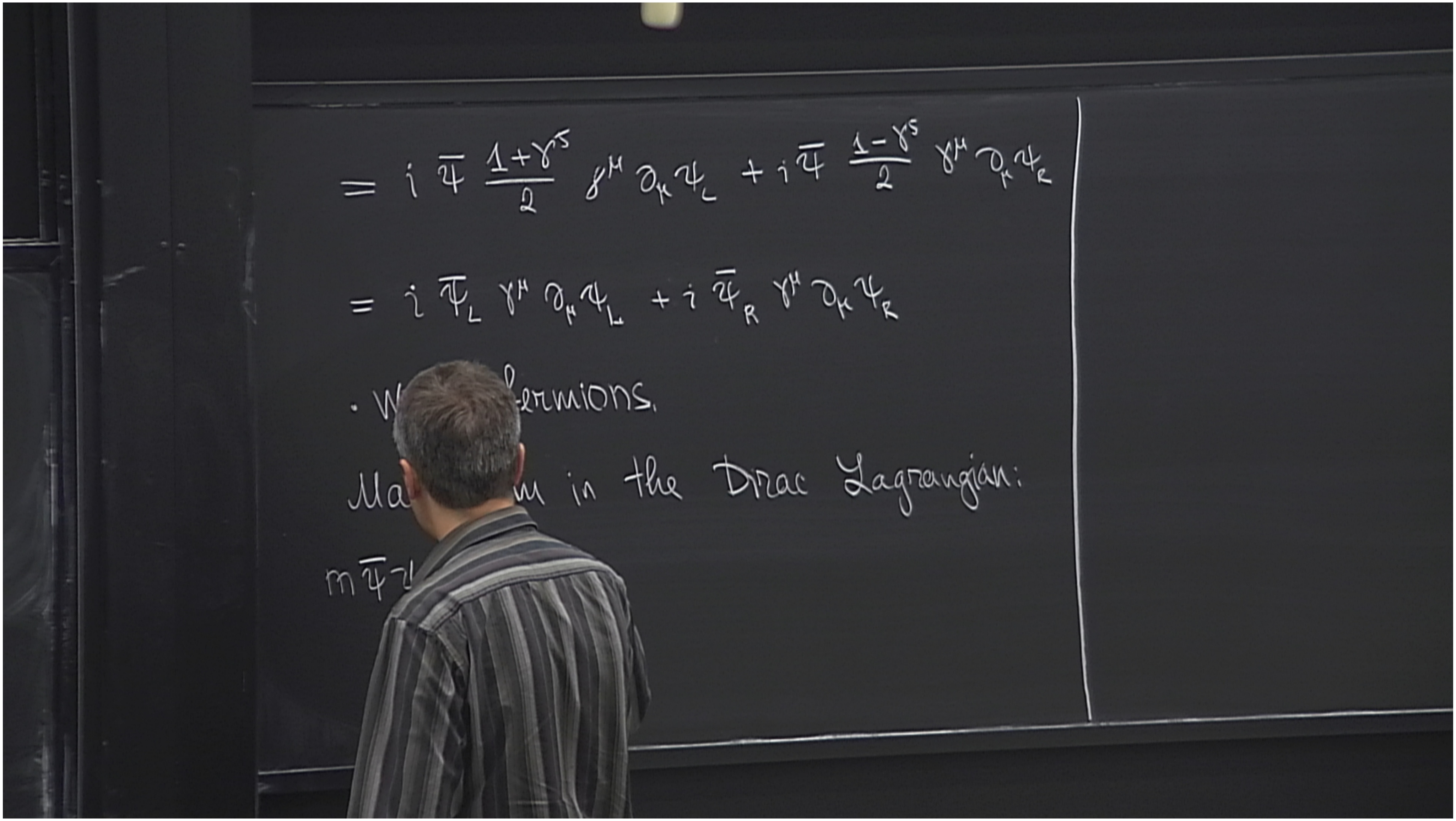
Massless Dirac Lagrangian:
$\mathcal{L} = \bar{\psi} i\gamma^\mu \partial_\mu \psi = i\bar{\psi}\gamma^\mu\left(\frac{1-\gamma^5}{2}\partial_\mu\psi_L + \frac{1+\gamma^5}{2}\partial_\mu\psi_R\right)$


$= i\bar{\psi}\frac{1+\gamma^5}{2}\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}\frac{1-\gamma^5}{2}\gamma^\mu\partial_\mu\psi_R$
$= i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R$

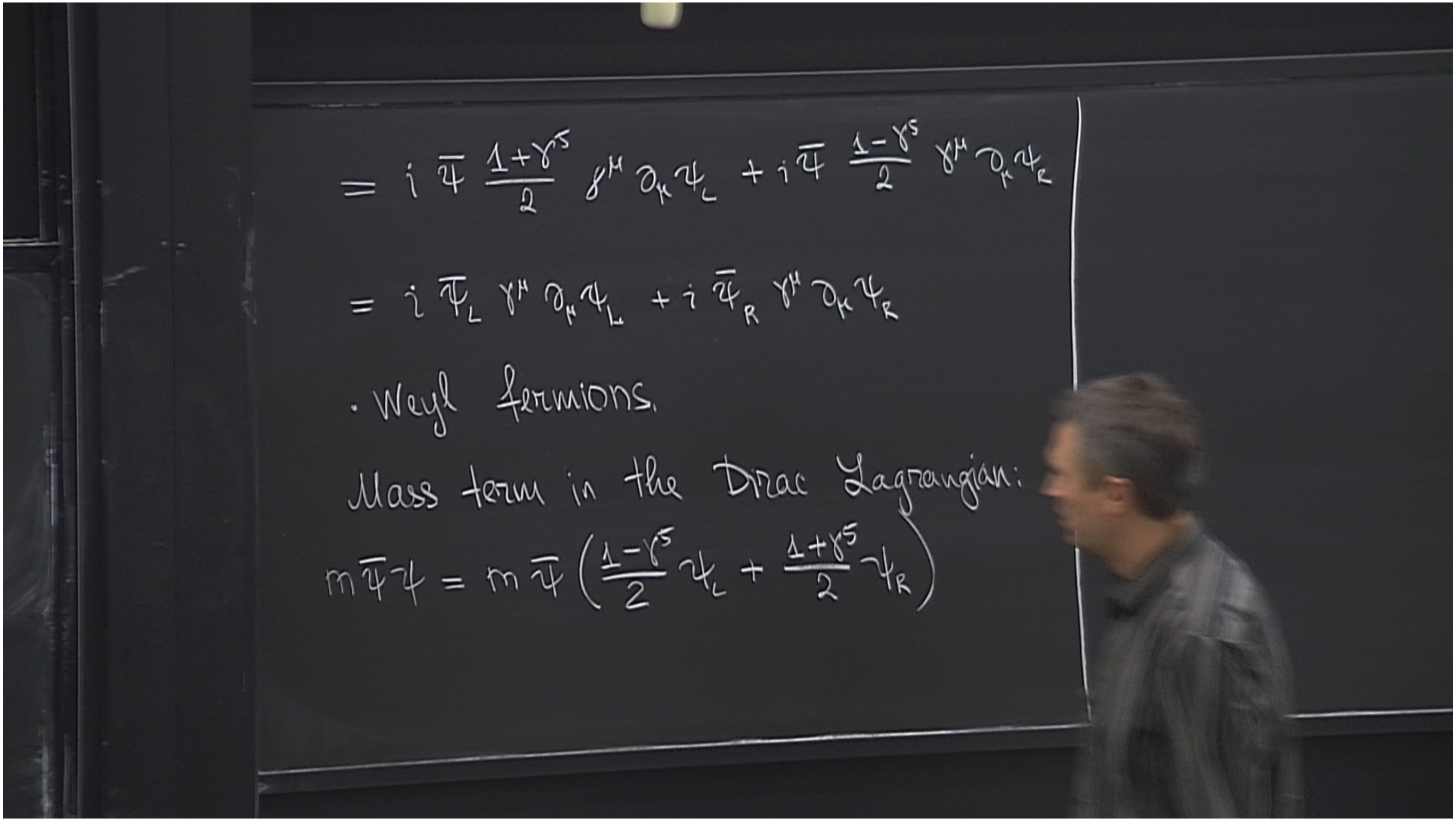
$$= i\,\bar{\Psi}\,\frac{1+\gamma^5}{2}\,\gamma^\mu \partial_\mu \psi_L + i\,\bar{\Psi}\,\frac{1-\gamma^5}{2}\,\gamma^\mu \partial_\mu \psi_R$$

$$= i\,\bar{\Psi}_L\,\gamma^\mu \partial_\mu \psi_L + i\,\bar{\Psi}_R\,\gamma^\mu \partial_\mu \psi_R$$

- Weyl fermions.



$= i\,\bar{\Psi}\,\frac{1+\gamma^5}{2}\,\gamma^\mu \partial_\mu \Psi_L + i\,\bar{\Psi}\,\frac{1-\gamma^5}{2}\,\gamma^\mu \partial_\mu \Psi_R$
$= i\,\bar{\Psi}_L \gamma^\mu \partial_\mu \Psi_L + i\,\bar{\Psi}_R \gamma^\mu \partial_\mu \Psi_R$
in the Dirac Lagrangian:



$= i\bar{\psi}\frac{1+\gamma^5}{2}\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}\frac{1-\gamma^5}{2}\gamma^\mu\partial_\mu\psi_R$
$= i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R$
· Weyl fermions.
Mass term in the Dirac Lagrangian:
$m\bar{\psi}\psi = m\bar{\psi}\left(\frac{1-\gamma^5}{2}\psi_L + \frac{1+\gamma^5}{2}\psi_R\right)$

$$= i\bar{\psi}\,\frac{1+\gamma^5}{2}\,\gamma^\mu \partial_\mu \psi_L + i\bar{\psi}\,\frac{1-\gamma^5}{2}\,\gamma^\mu \partial_\mu \psi_R$$

$$= i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + i\bar{\psi}_R \gamma^\mu \partial_\mu \psi_R$$

- Weyl fermions.

Mass term in the Dirac Lagrangian:

$$m\bar{\psi}\psi = m\bar{\psi}\left(\frac{1-\gamma^5}{2}\psi_L + \frac{1+\gamma^5}{2}\psi_R\right)$$

$$= m\bar{\psi}_R\psi_L + m\bar{\psi}$$

$$= i\,\bar{\Psi}\,\frac{1+\gamma^5}{2}\,\gamma^\mu \partial_\mu \psi_L + i\,\bar{\Psi}\,\frac{1-\gamma^5}{2}\,\gamma^\mu \partial_\mu \psi_R$$

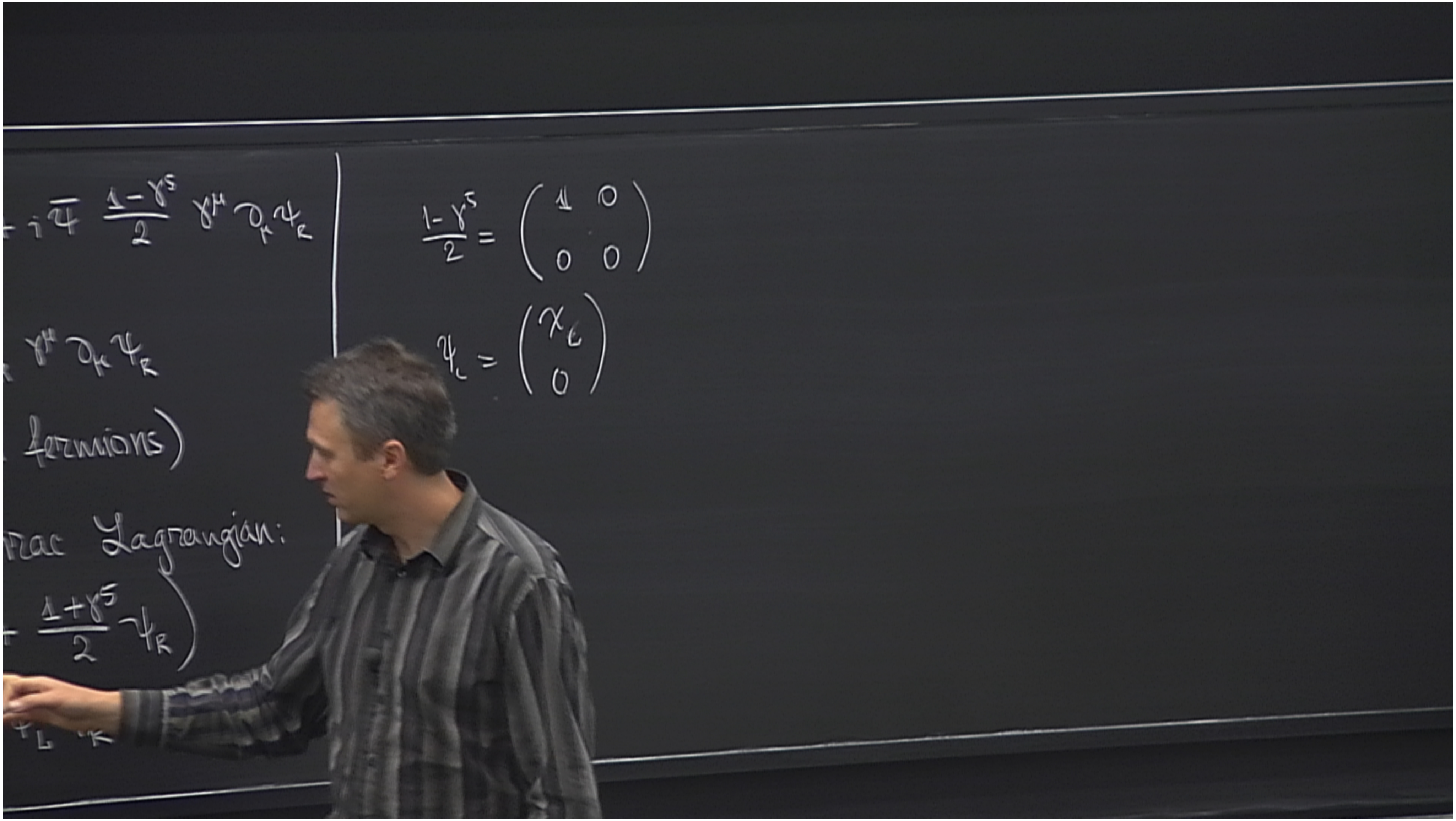
$$= i\,\bar{\Psi}_L\, \gamma^\mu \partial_\mu \psi_L + i\,\bar{\Psi}_R\, \gamma^\mu \partial_\mu \psi_R$$

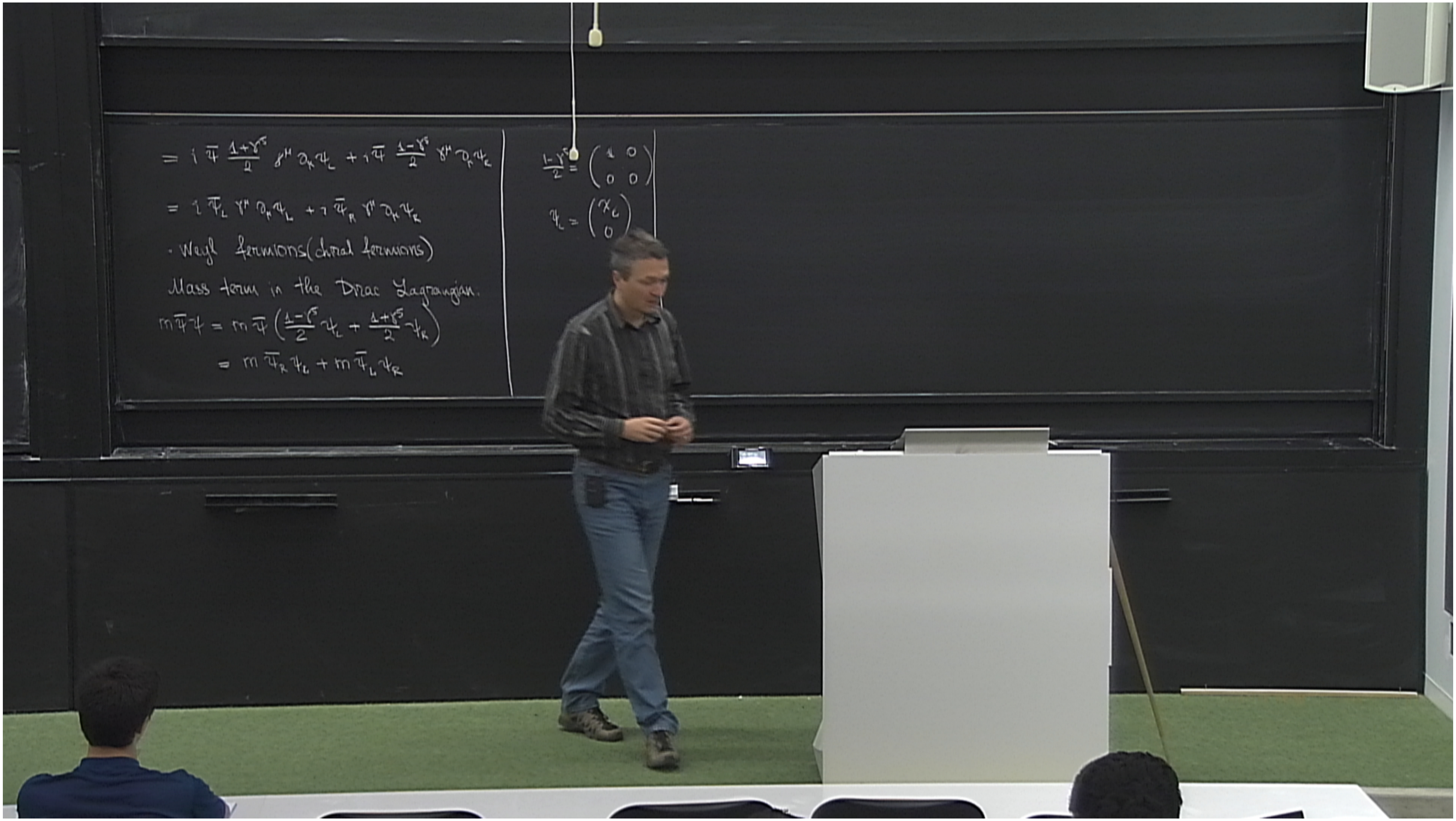
- Weyl fermions (chiral fermions)

Mass term in the Dirac Lagrangian:

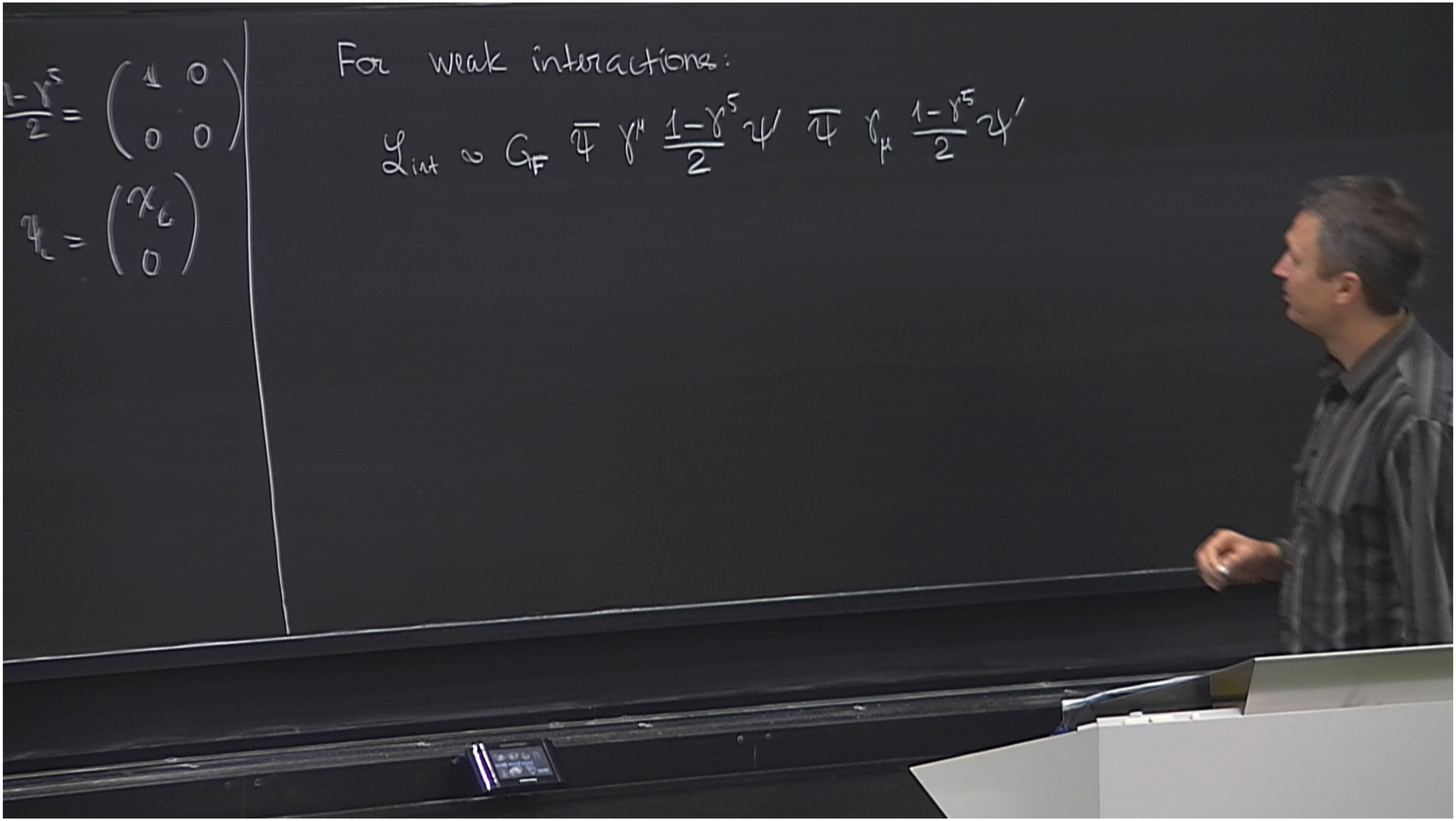
$$m\bar{\Psi}\psi = m\bar{\Psi}\left(\frac{1-\gamma^5}{2}\psi_L + \frac{1+\gamma^5}{2}\psi_R\right)$$

$$= m\bar{\Psi}_R\psi_L + m\bar{\Psi}_L\psi_R$$


$i\bar{\psi}\,\frac{1-\gamma^5}{2}\,\gamma^\mu \partial_\mu \psi_R$
$\gamma^\mu \partial_\mu \psi_R$
fermions)
rac Lagrangian:
$\frac{1+\gamma^5}{2}\psi_R$)
$\frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$\psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$



$= i\bar{\Psi}\frac{1+\gamma^5}{2}\gamma^\mu\partial_\mu\Psi_L + i\bar{\Psi}\frac{1-\gamma^5}{2}\gamma^\mu\partial_\mu\Psi_R$
$= i\bar{\Psi}_L\gamma^\mu\partial_\mu\Psi_L + i\bar{\Psi}_R\gamma^\mu\partial_\mu\Psi_R$
· Weyl fermions (chiral fermions)
Mass term in the Dirac Lagrangian.
$m\bar{\Psi}\Psi = m\bar{\Psi}\left(\frac{1-\gamma^5}{2}\Psi_L + \frac{1+\gamma^5}{2}\Psi_R\right)$
$= m\bar{\Psi}_R\Psi_L + m\bar{\Psi}_L\Psi_R$
$\frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$\Psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$

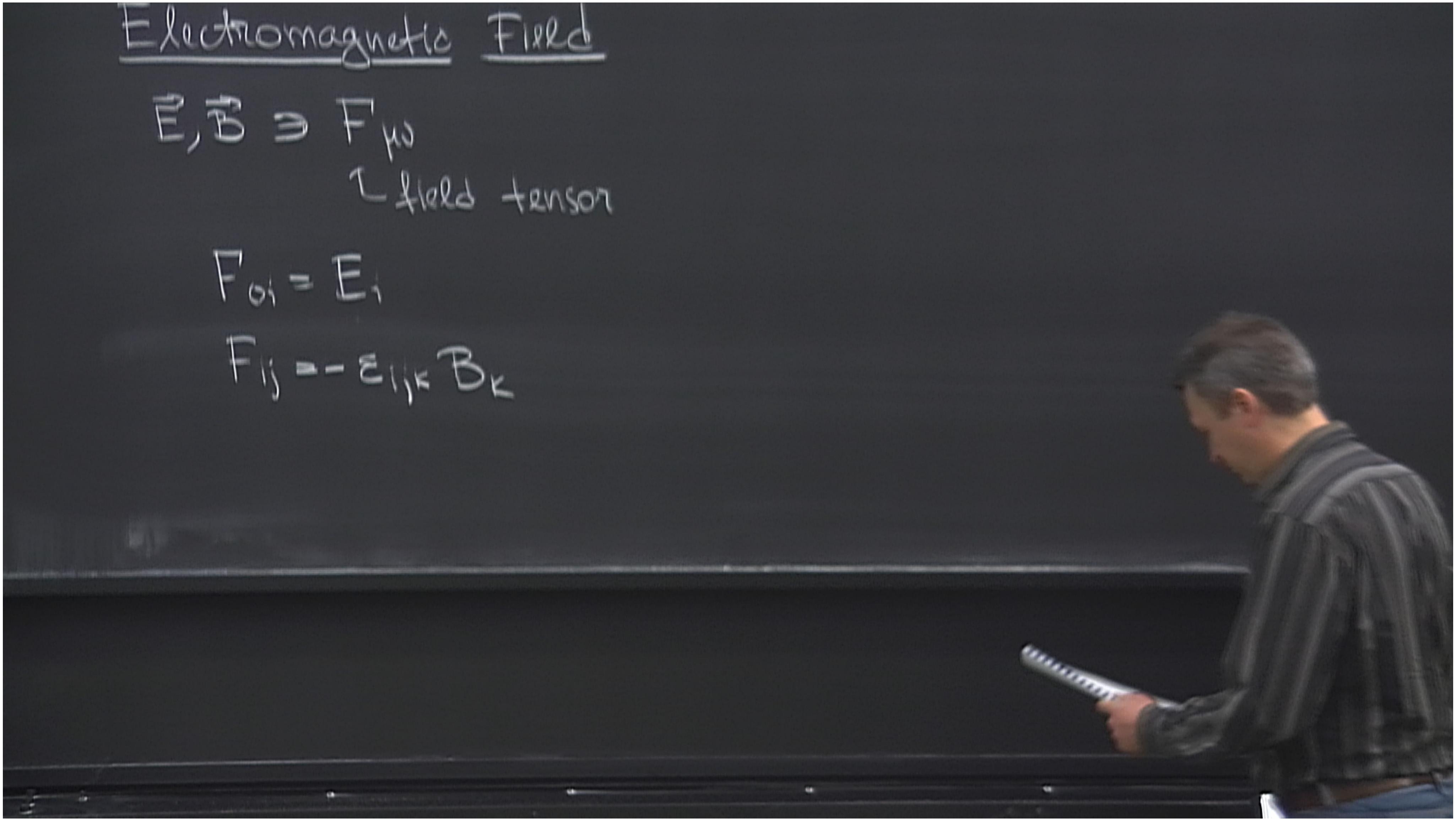

$\frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
$\psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$
For weak interactions:
$\mathcal{L}_{int} \propto G_F \, \bar{\Psi} \gamma^\mu \frac{1-\gamma^5}{2} \psi' \, \bar{\Psi} \gamma_\mu \frac{1-\gamma^5}{2} \psi'$

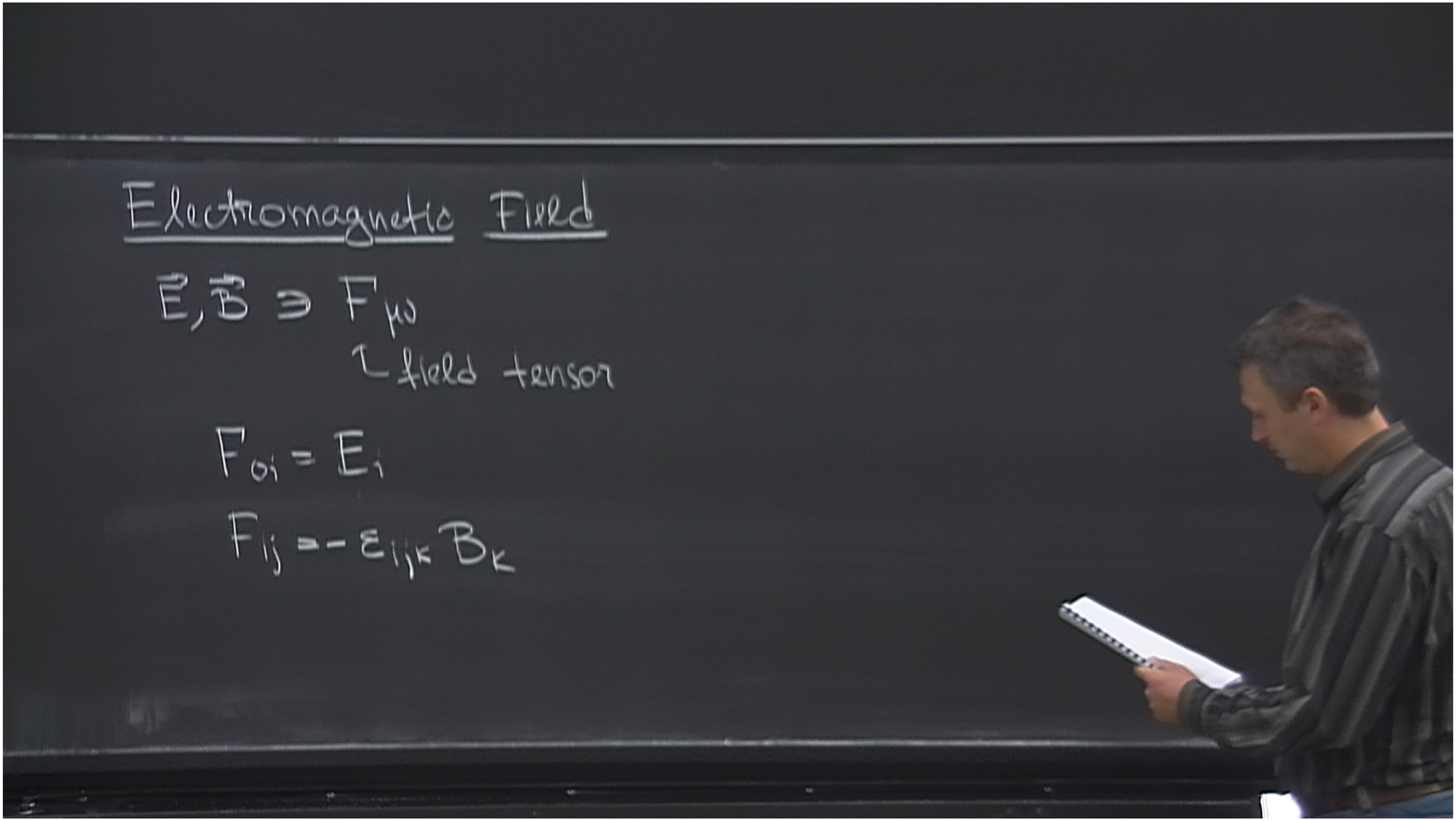
$$\frac{1-\gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\psi_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}$$

For weak interactions:

$$\mathcal{L}_{int} \propto G_F \, \bar{\Psi} \underbrace{\gamma^\mu \frac{1-\gamma^5}{2}}_{V^\mu - A^\mu} \psi' \; \bar{\Psi} \gamma_\mu \frac{1-\gamma^5}{2} \psi'$$


Electromagnetic Field
$\vec{E}, \vec{B} \ni F_{\mu\nu}$
field tensor
$F_{0i} = E_i$
$F_{ij} = -\varepsilon_{ijk} B_k$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Maxwell's equations:

$$\partial_\mu F^{\mu\nu} = 0$$

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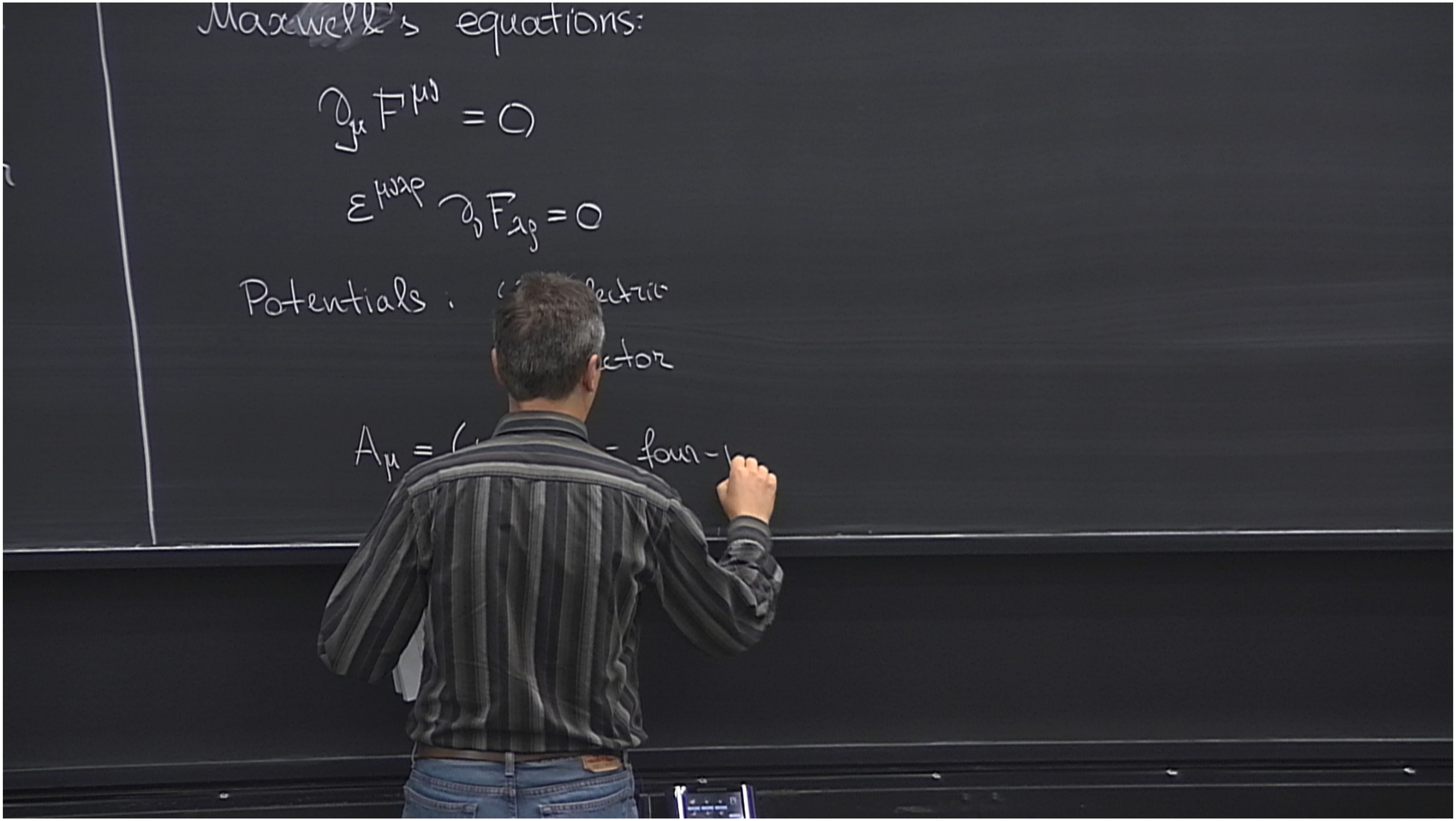
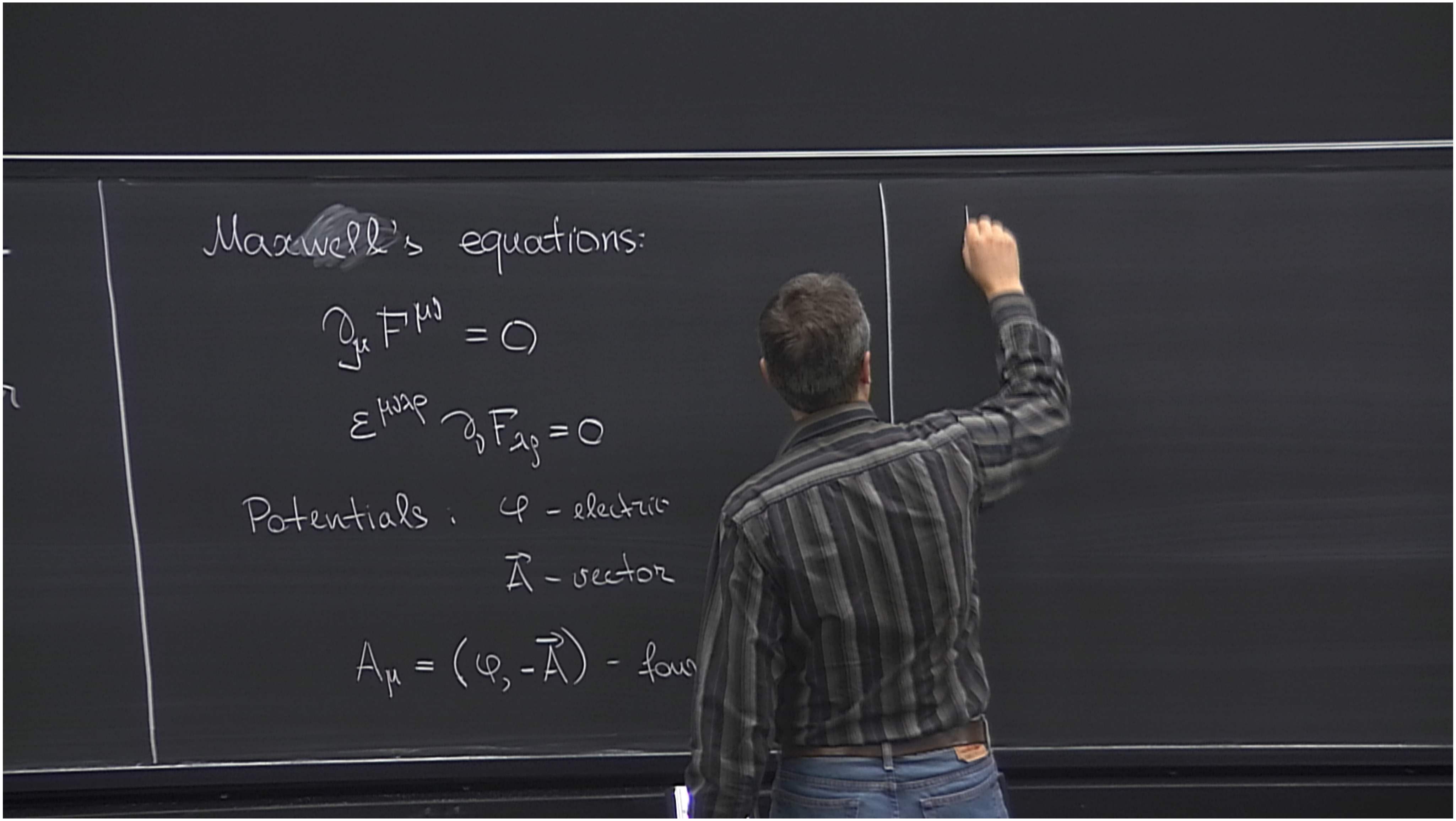
$$\partial_\mu F^{\mu\nu} = 0$$

$$\varepsilon^{\mu\nu\lambda\rho}\,\partial_\nu F_{\lambda\rho} = 0$$


Maxwell's equations:
$\partial_\mu F^{\mu\nu} = 0$
$\varepsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho} = 0$
Potentials:
lectric
ctor
$A_\mu =$
four-


Maxwell's equations:
$\partial_\mu F^{\mu\nu} = 0$
$\varepsilon^{\mu\nu\lambda\rho} \partial_\nu F_{\lambda\rho} = 0$
Potentials : φ - electric
$\vec{A}$ - vector
$A_\mu = (\varphi, -\vec{A})$ - four

Maxwell's equations:

$$\partial_\mu F^{\mu\nu} = 0 \leftarrow \text{equation of motion}$$

$$\varepsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = 0 \leftarrow \text{identity}$$

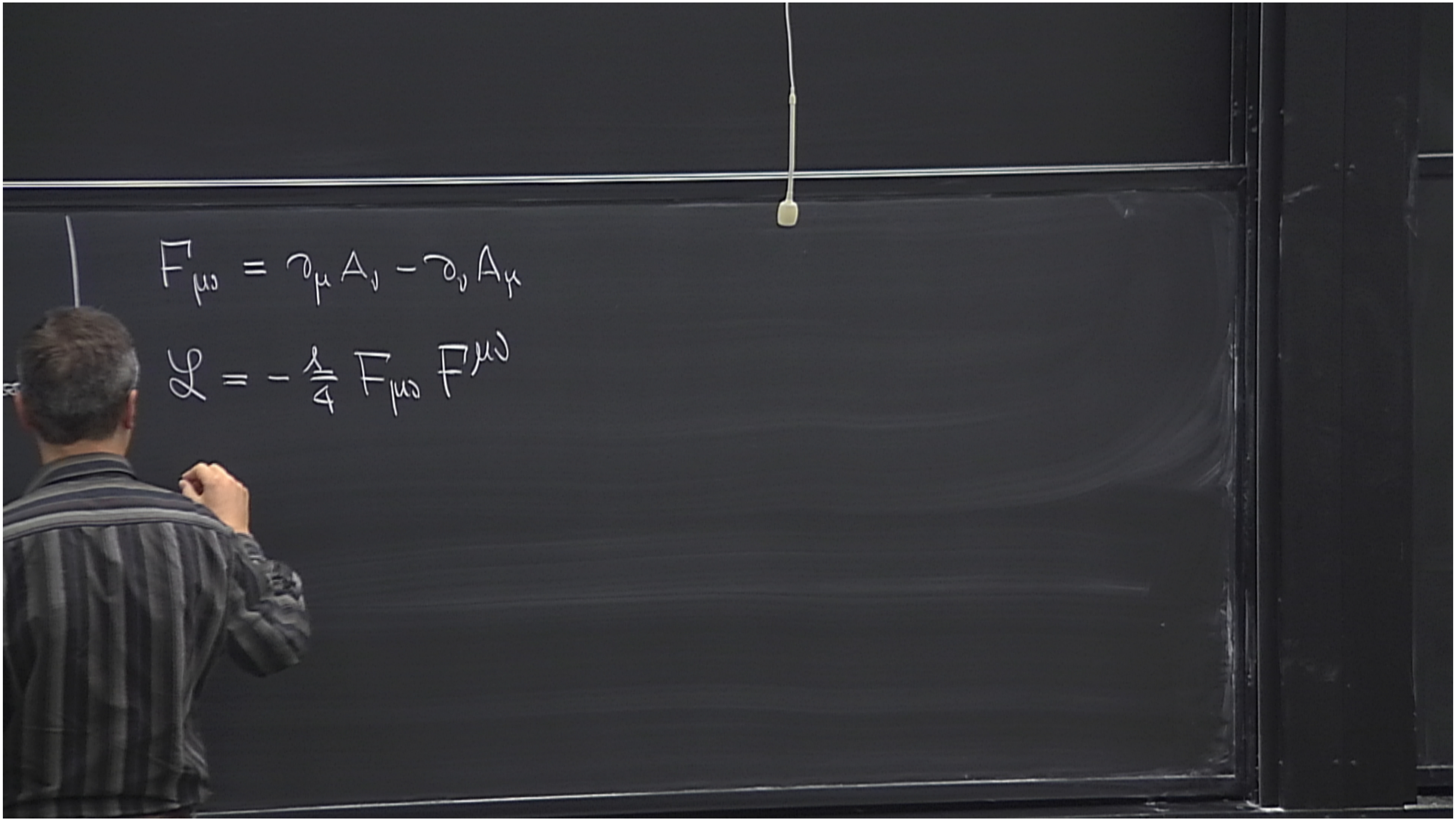
Potentials: φ - electric

$\vec{A}$ - vector

$$A_\mu = (\varphi, -\vec{A}) \text{ - four-potential}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$


$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

motion

ntial

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

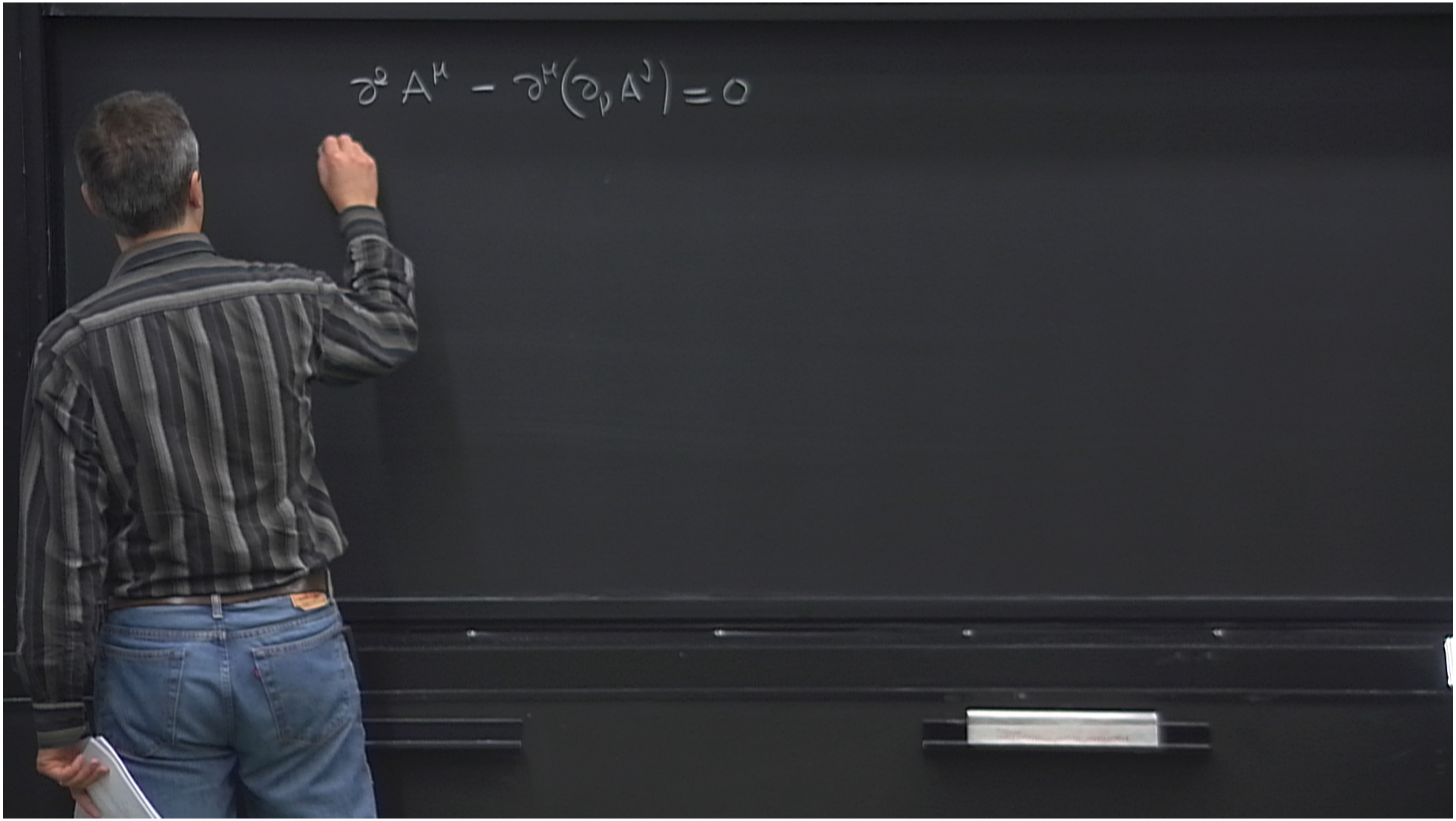
Gradient (gauge

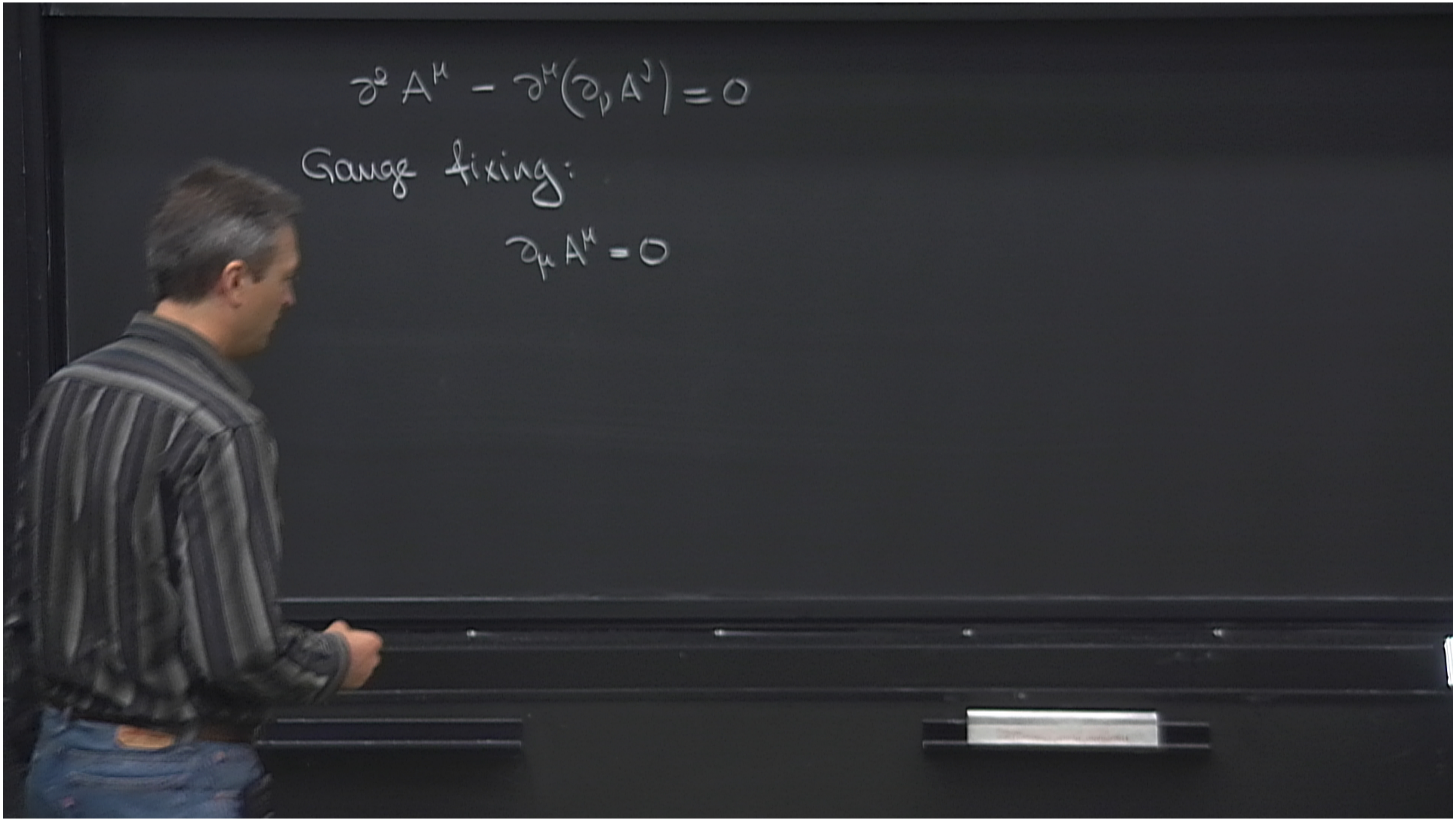
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

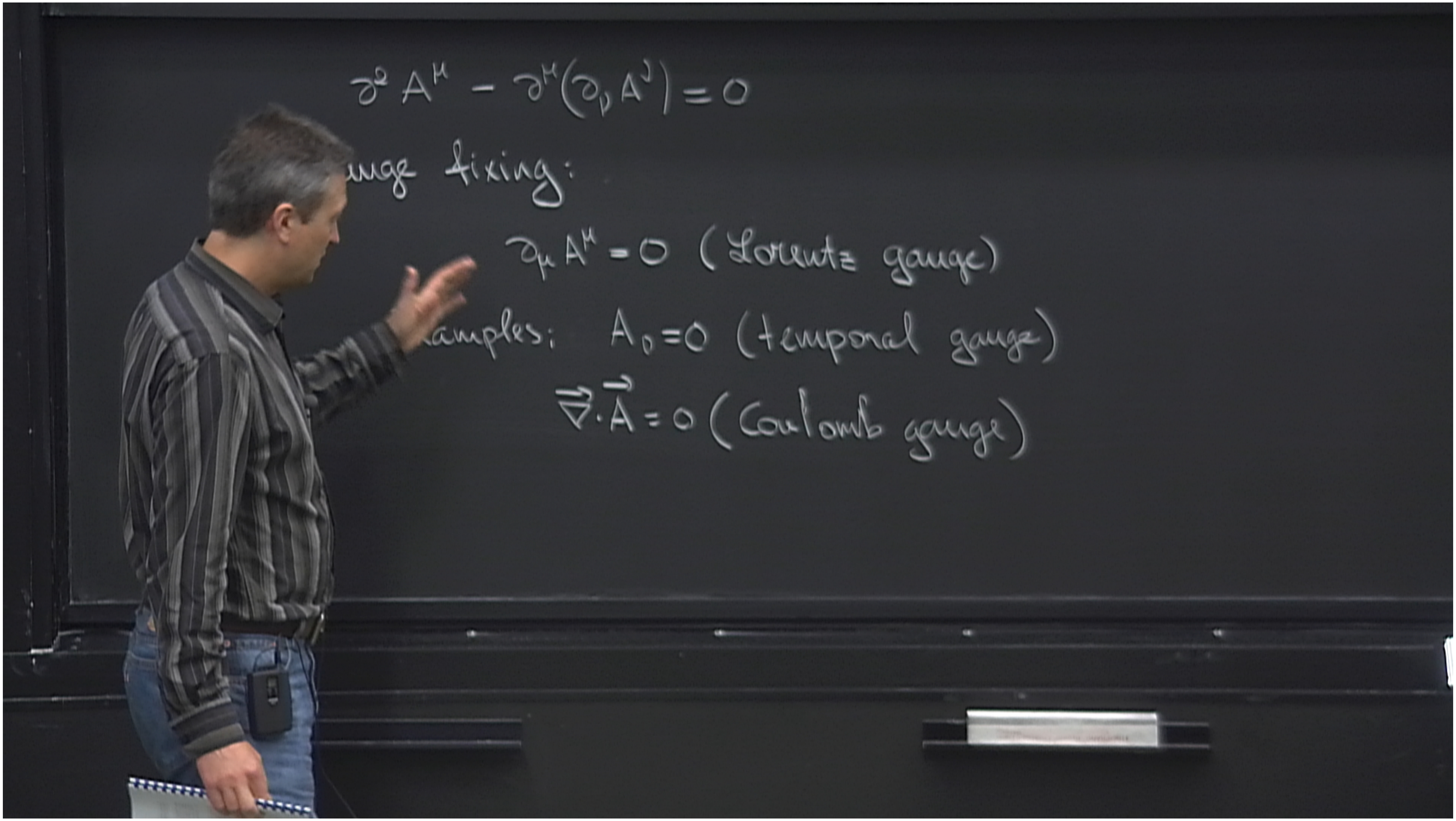
Gradient (gauge) invariance:

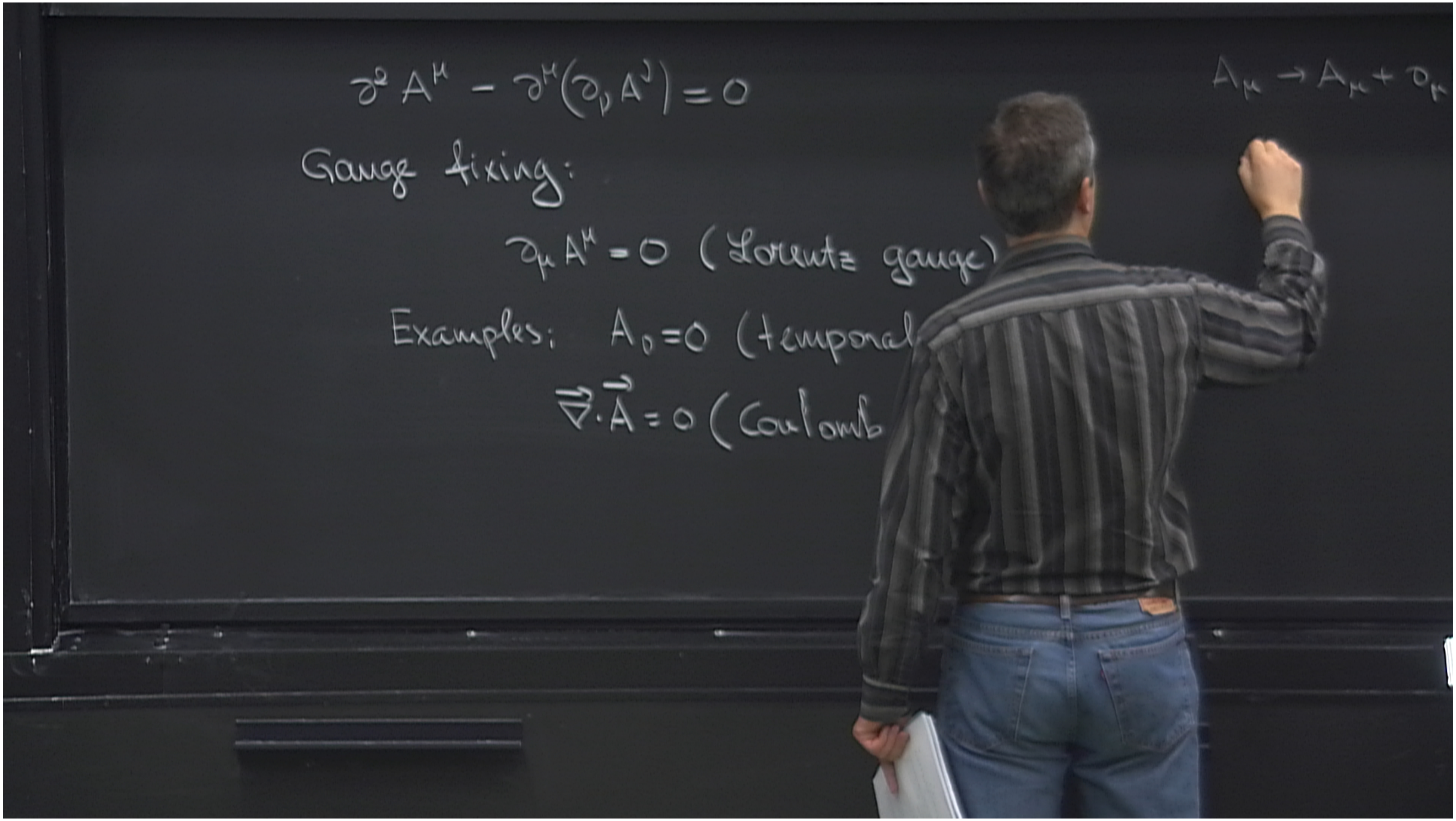
$$A_\mu \rightarrow A_\mu + \partial_\mu \omega$$

ω - any function


$\partial^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0$


$\partial^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0$
Gauge fixing:
$\partial_\mu A^\mu = 0$


$\partial^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0$
uge fixing:
$\partial_\mu A^\mu = 0$ (Lorentz gauge)
amples: $A_0 = 0$ (temporal gauge)
$\vec{\nabla} \cdot \vec{A} = 0$ (Coulomb gauge)


$\partial^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = 0$
Gauge fixing:
$\partial_\mu A^\mu = 0$ (Lorentz gauge)
Examples: $A_0 = 0$ (temporal
$\vec{\nabla}\cdot\vec{A} = 0$ (Coulomb
$A_\mu \to A_\mu + \partial_\mu$

$A_\mu \to A_\mu + \partial_\mu \omega$

$\partial^2 \omega = 0$ for the new potential to satisfy the Lorentz gauge condition

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$A_\mu = (\varphi, -\vec{A})$ - four-potential

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...ntz gauge)

...mporal gauge)

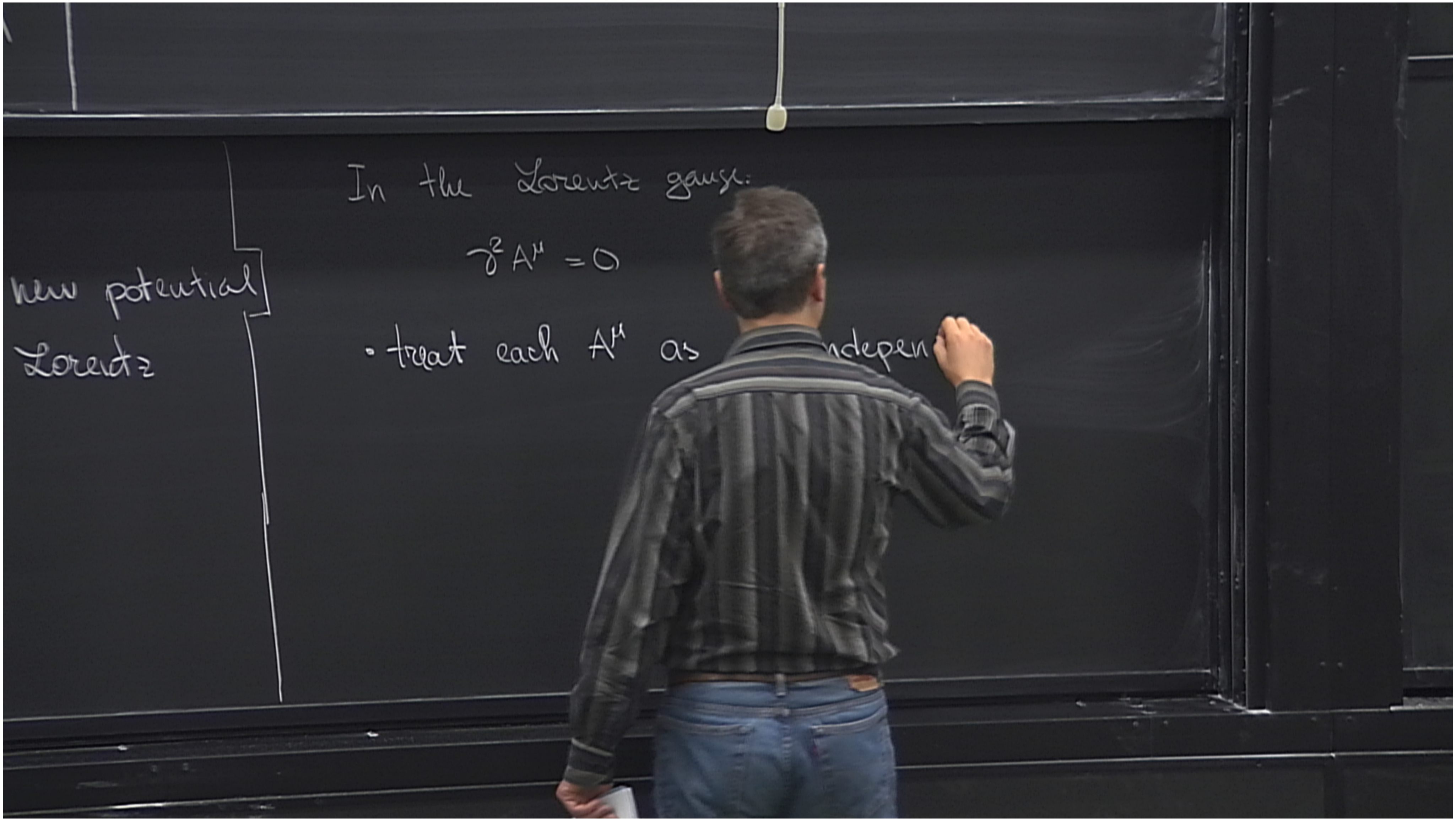
...lomb gauge)

new potential

Lorentz

In the Lorentz gauge:

$$\partial^2 A^\mu = 0$$


In the Lorentz gauge:
$\partial^2 A^\mu = 0$
new potential
Lorentz
· treat each A^μ as

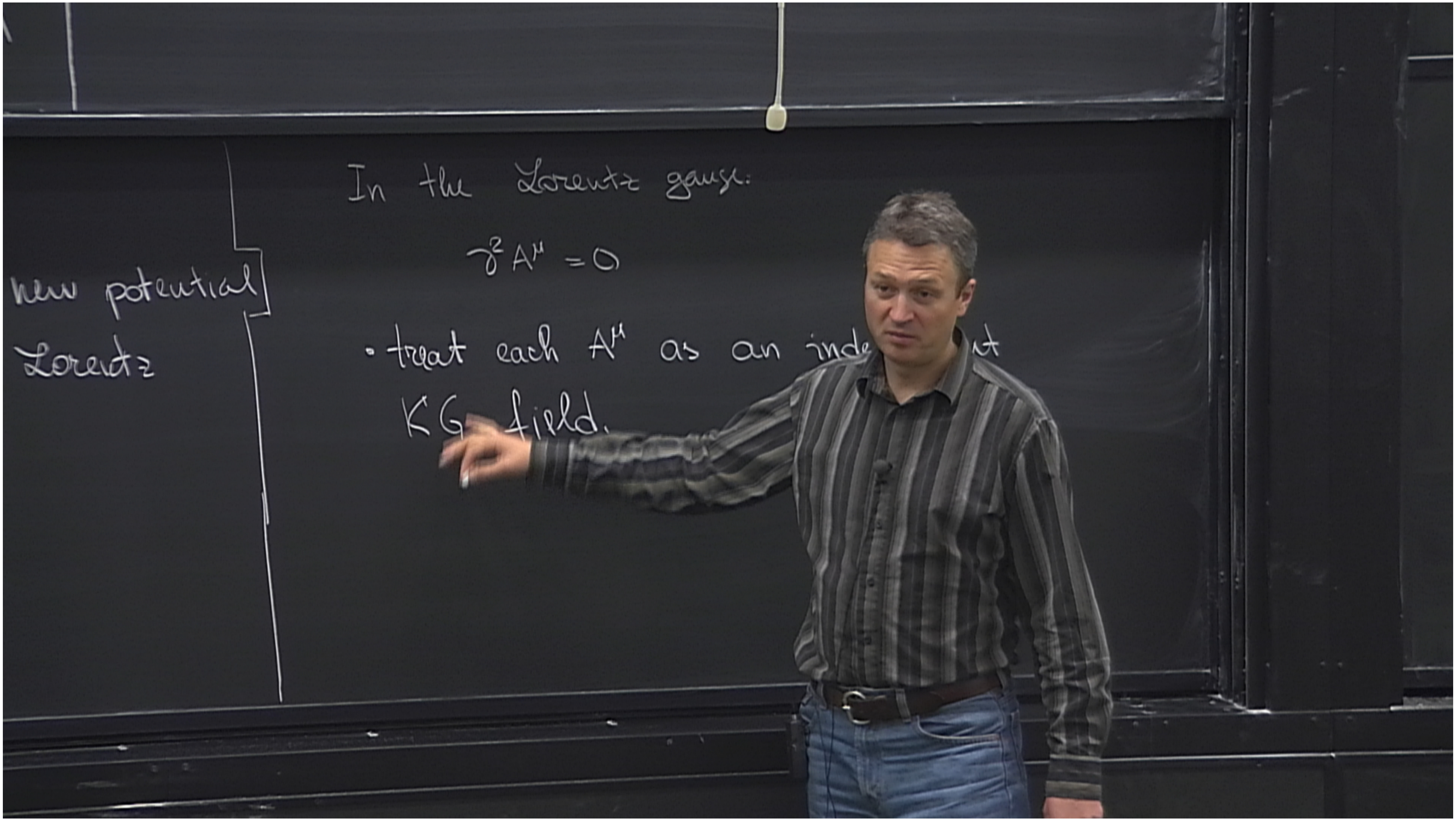
new potential

Lorentz

In the Lorentz gauge:

$$\partial^2 A^\mu = 0$$

- treat each A^μ as an independent KG field.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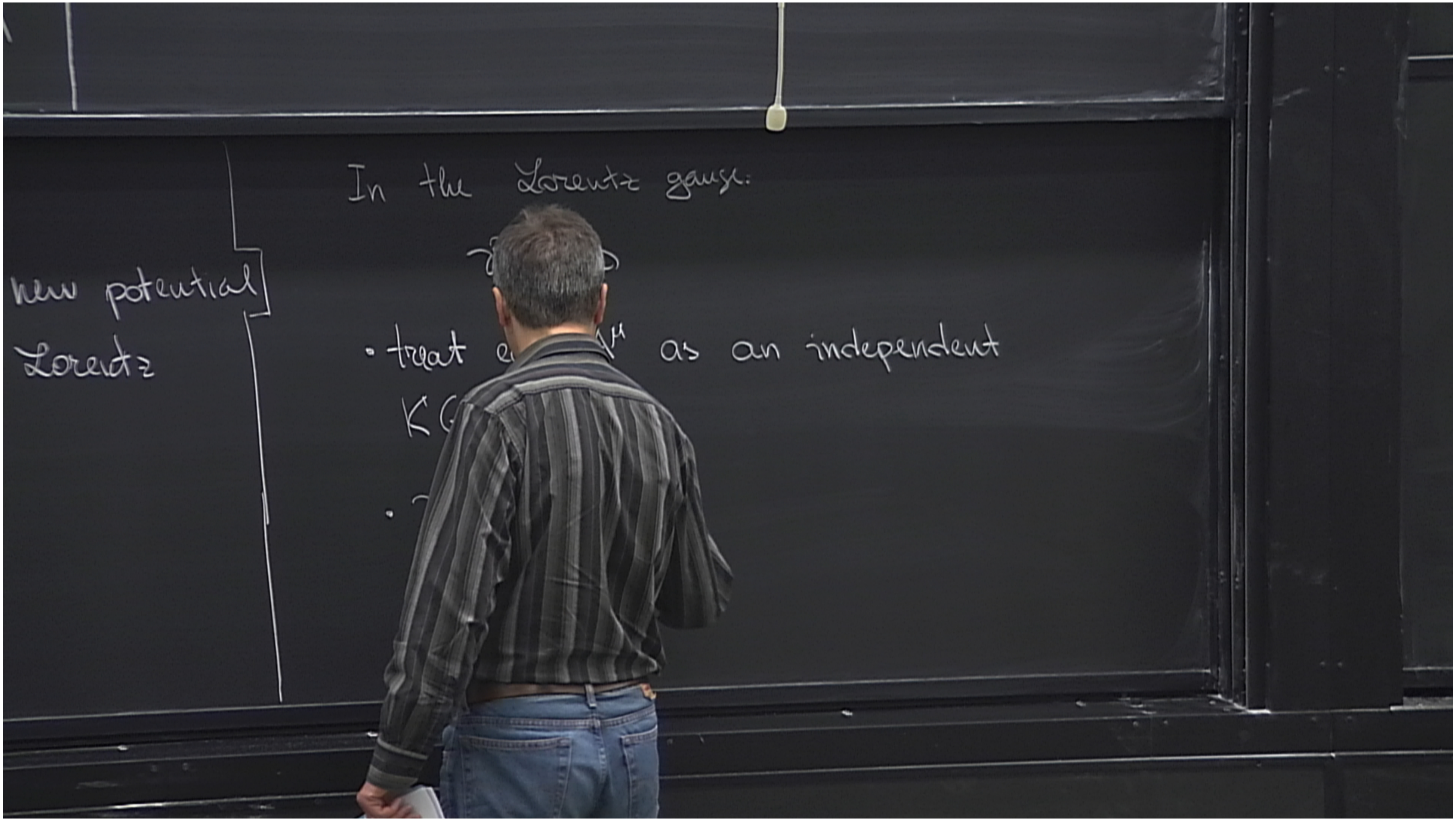
KG field.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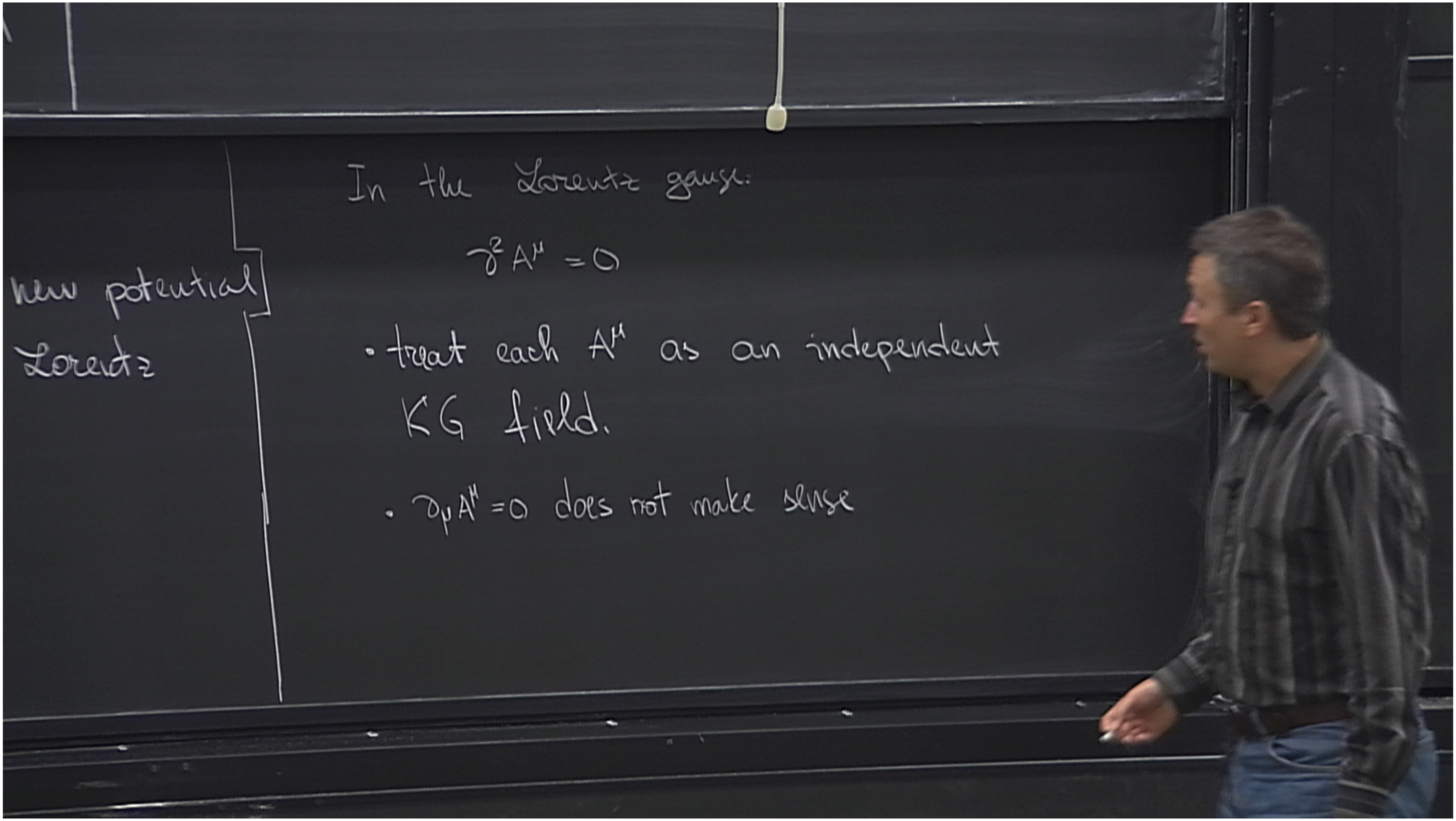
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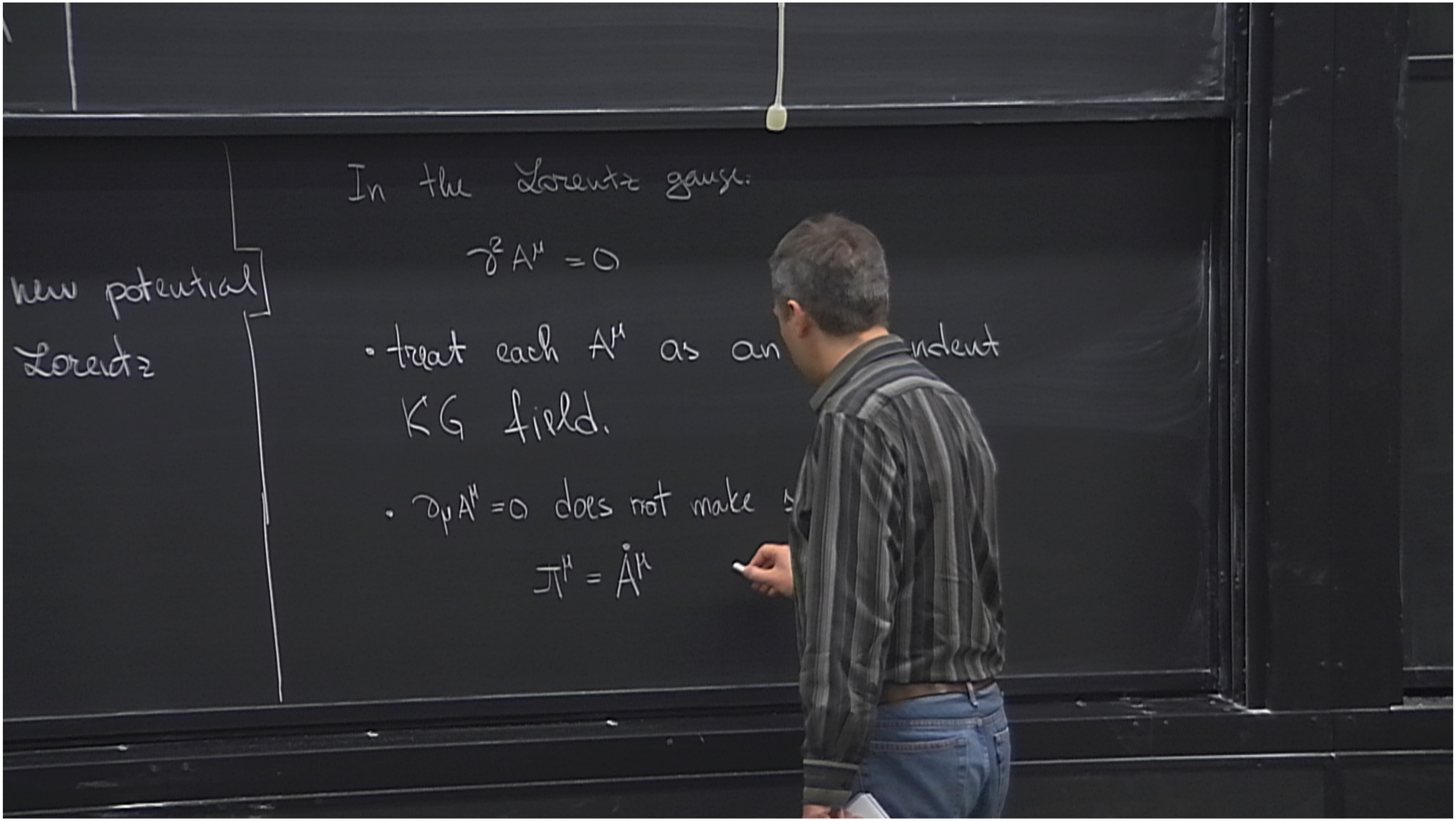
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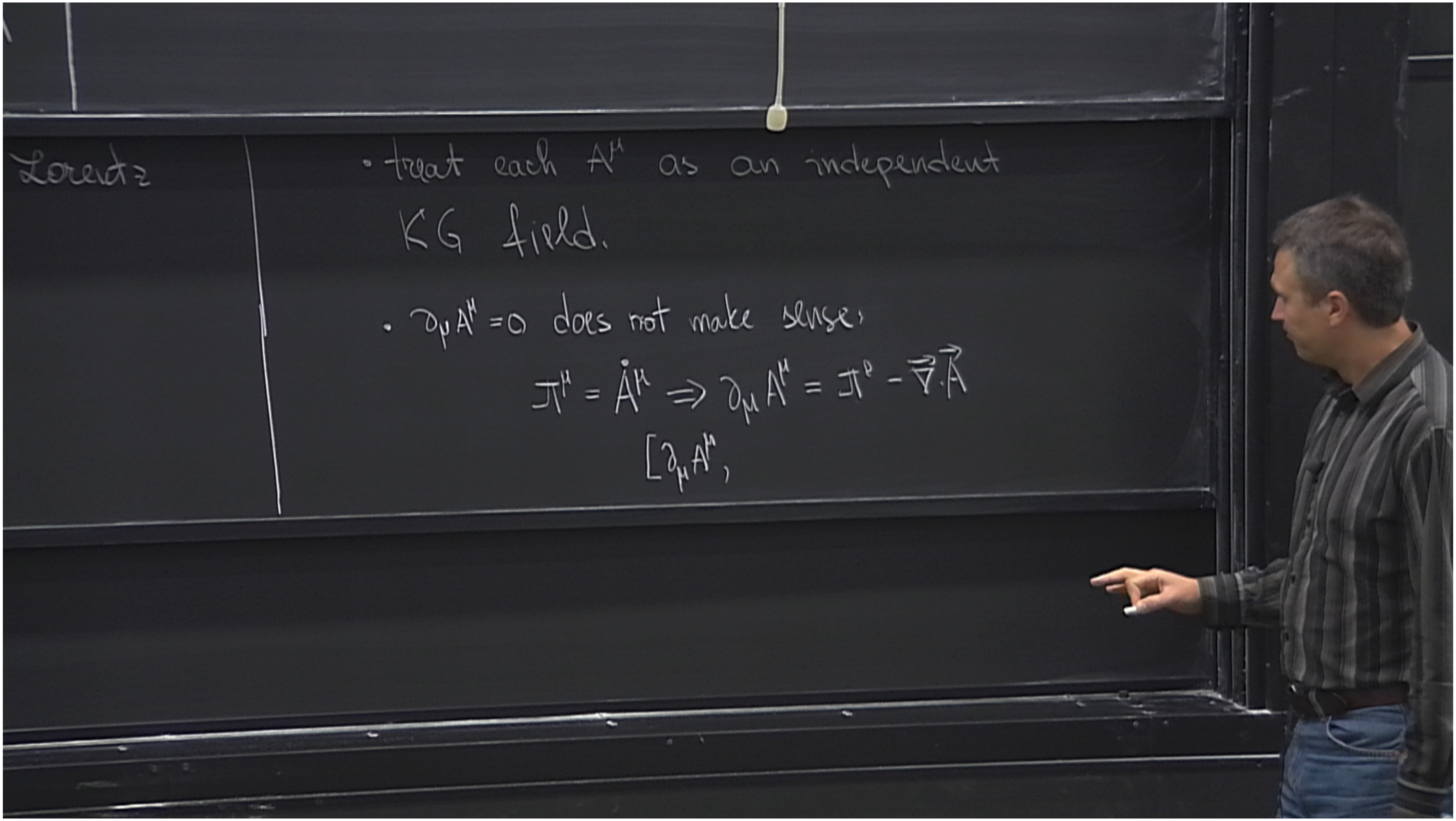
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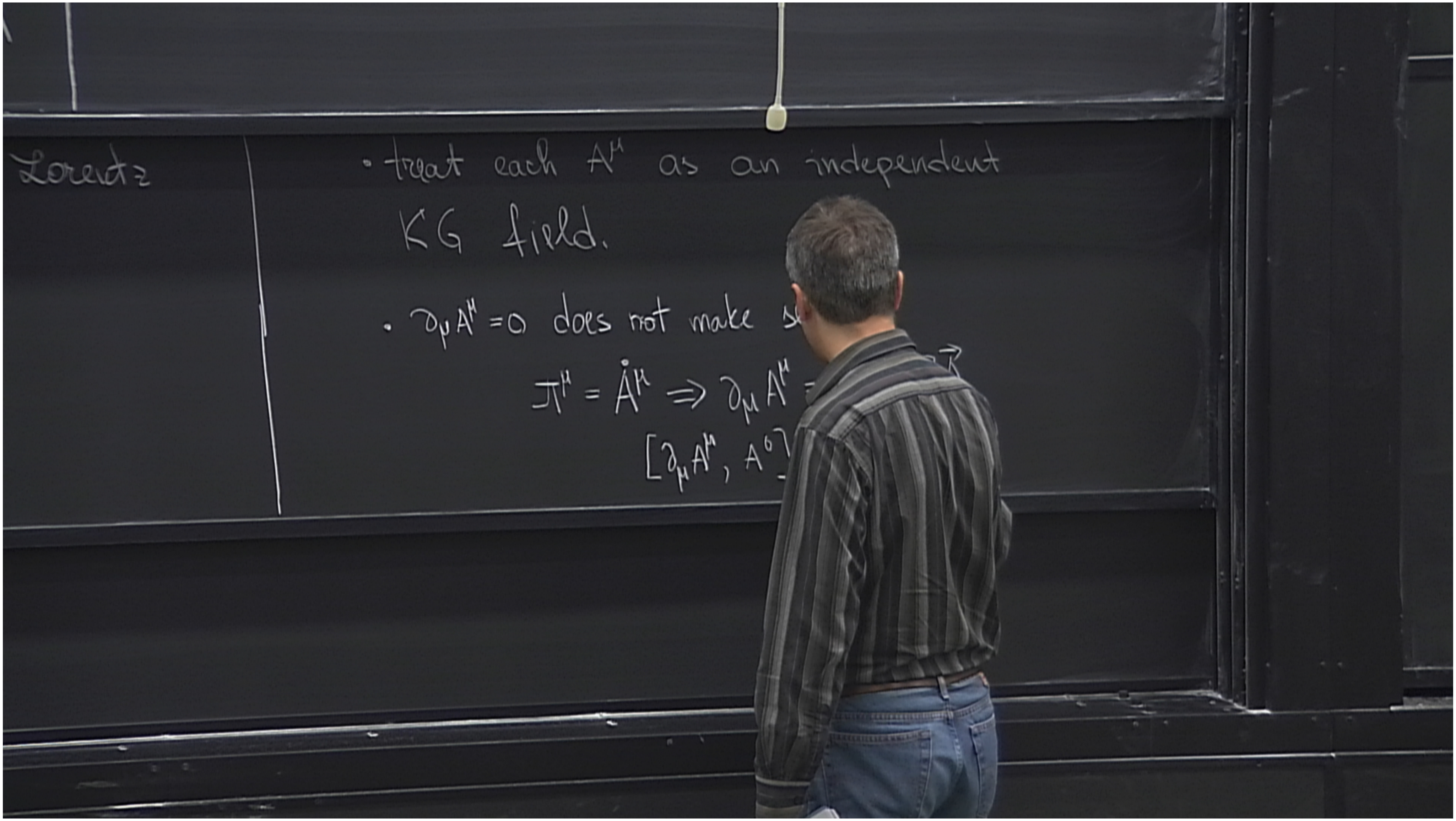
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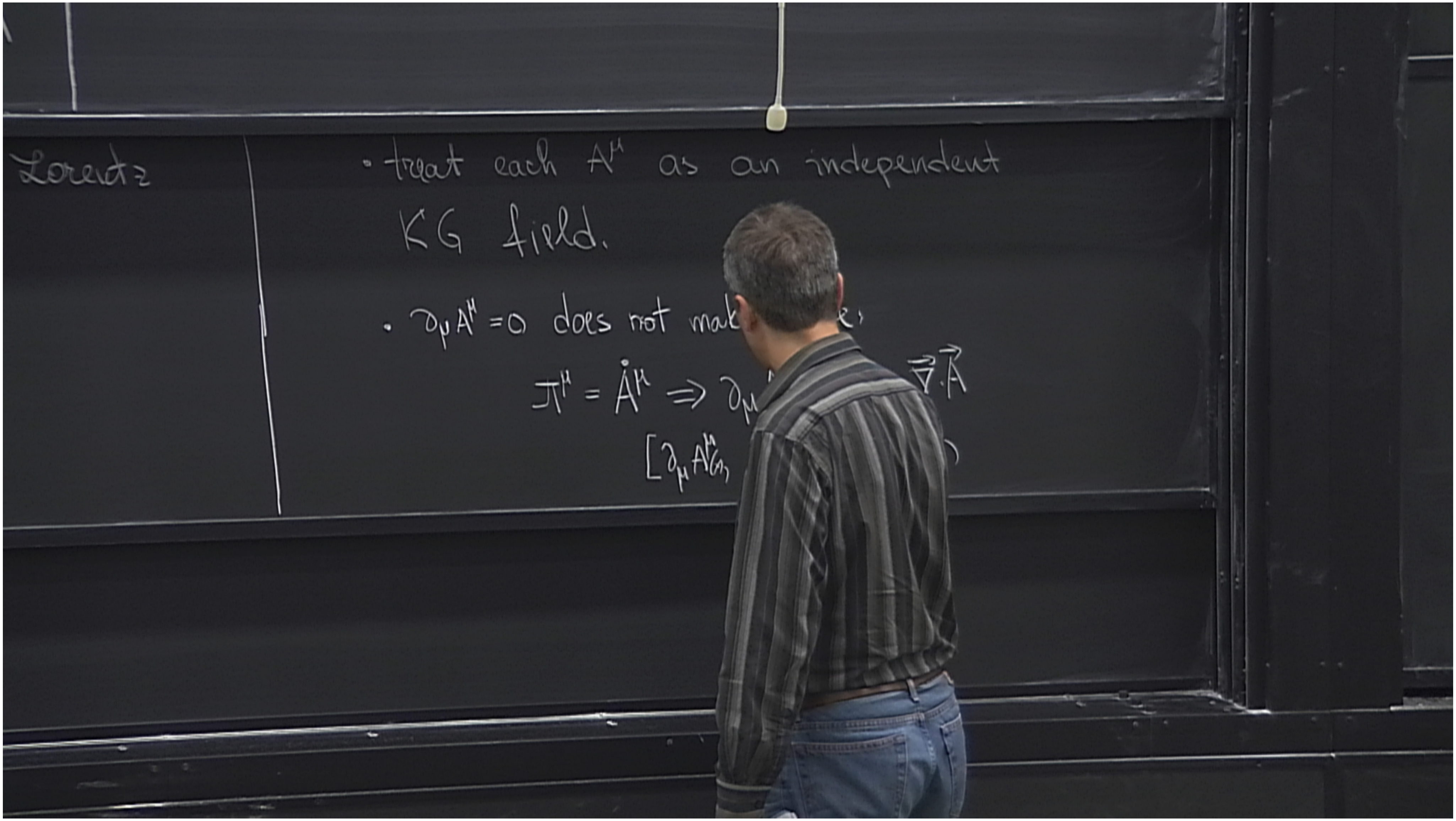
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$\vec{\nabla}\cdot\vec{A}$
$[\partial_\mu A^\mu(x),$

$[\partial_\mu A^\mu(x), A^0(y)] \sim i\delta(x-y)$

tion

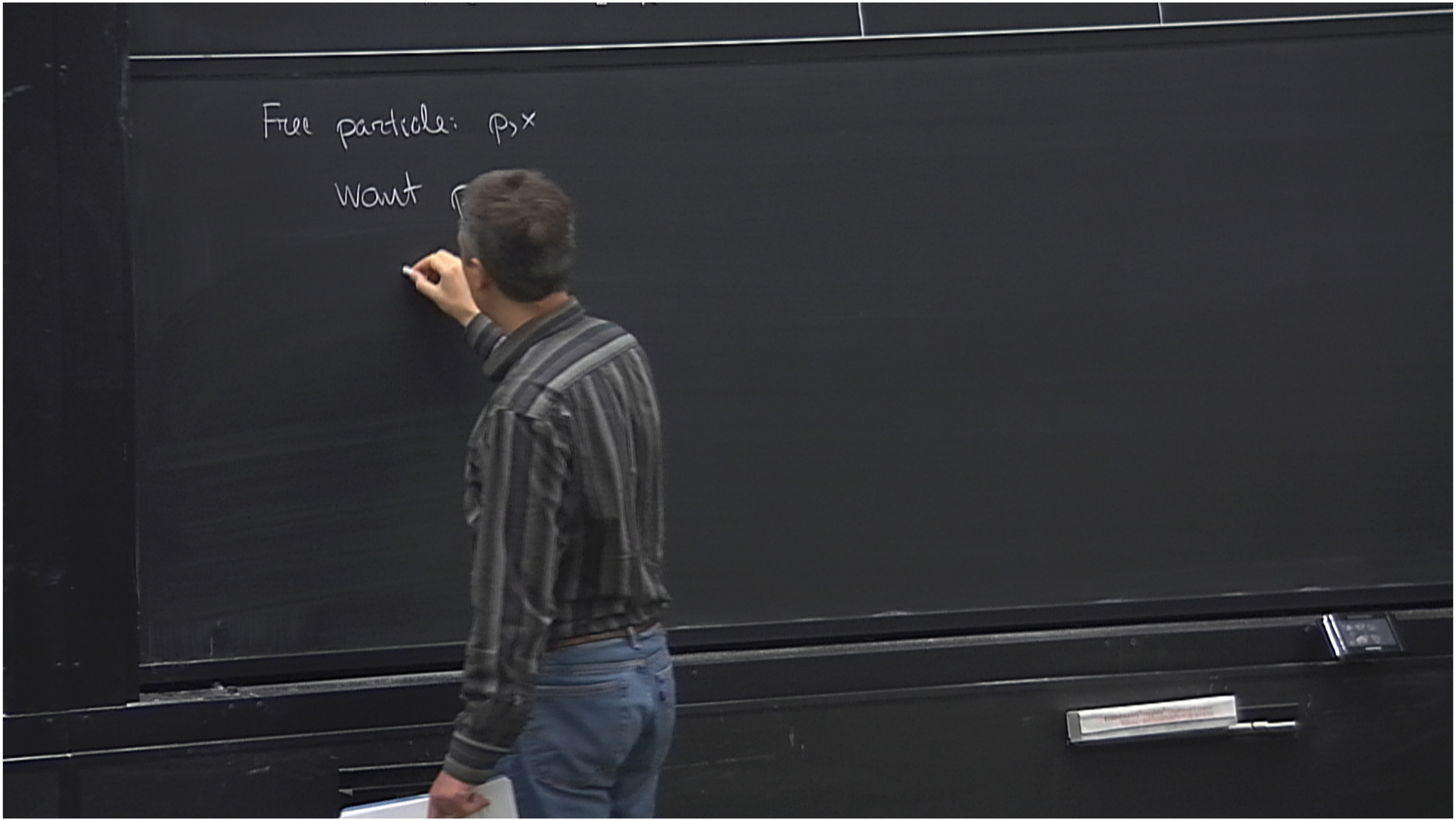
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

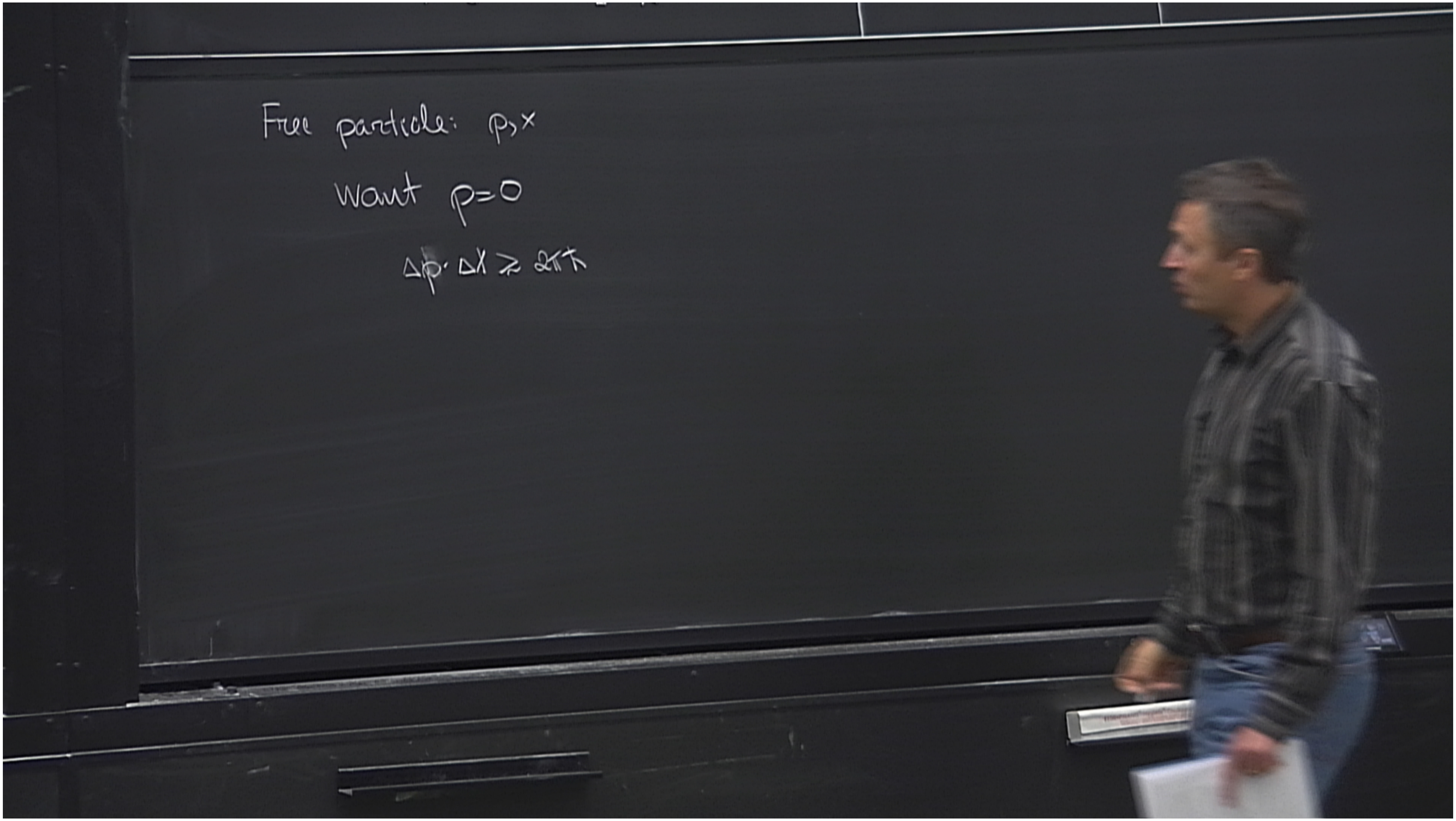
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

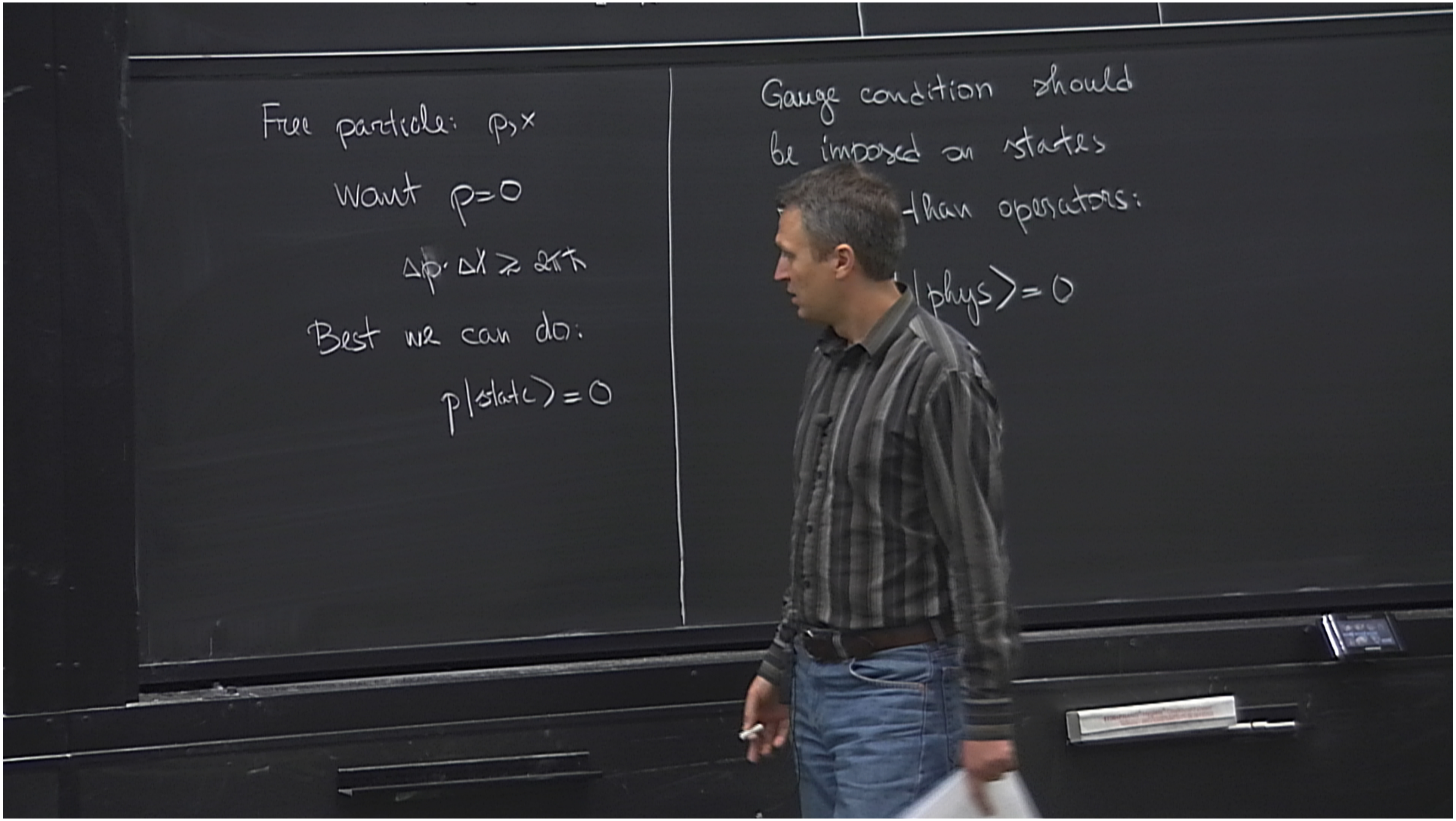
Gradient (gauge) invariance:

$$A_\mu \longrightarrow A_\mu + \partial_\mu \omega$$

ω - any function


Free particle: p, x
want


Free particle: p, x
Want p=0
Δp · Δx ≳ 2πħ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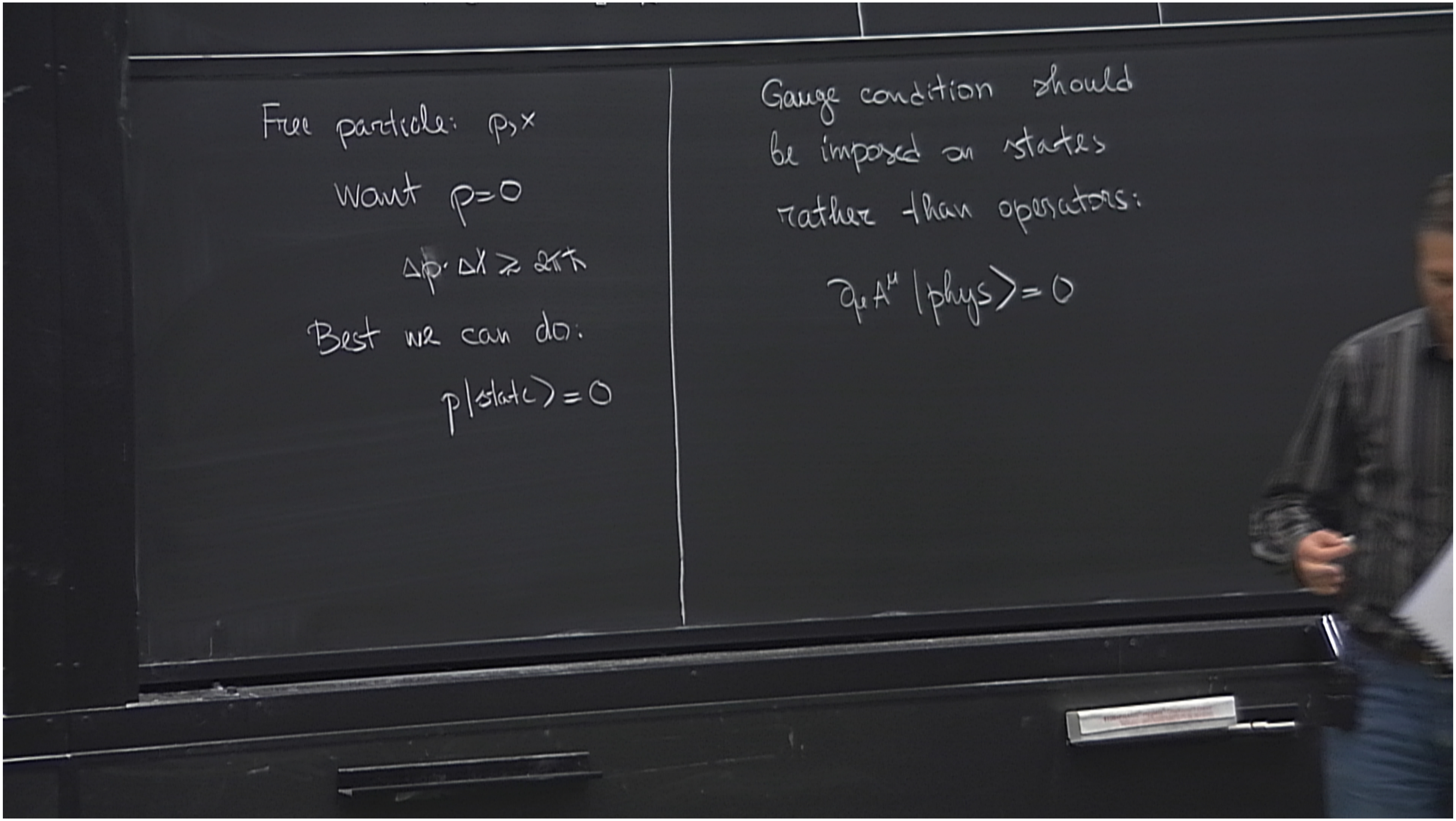
Best we can do:
p|state> = 0
Gauge condition should
be imposed on states
than operators:
|phys> = 0


Free particle: p, x
Want $p=0$
$\Delta p \cdot \Delta x \gtrsim 2\pi\hbar$
Best we can do:
$p|\text{state}\rangle = 0$
Gauge condition should be imposed on states rather than operators:
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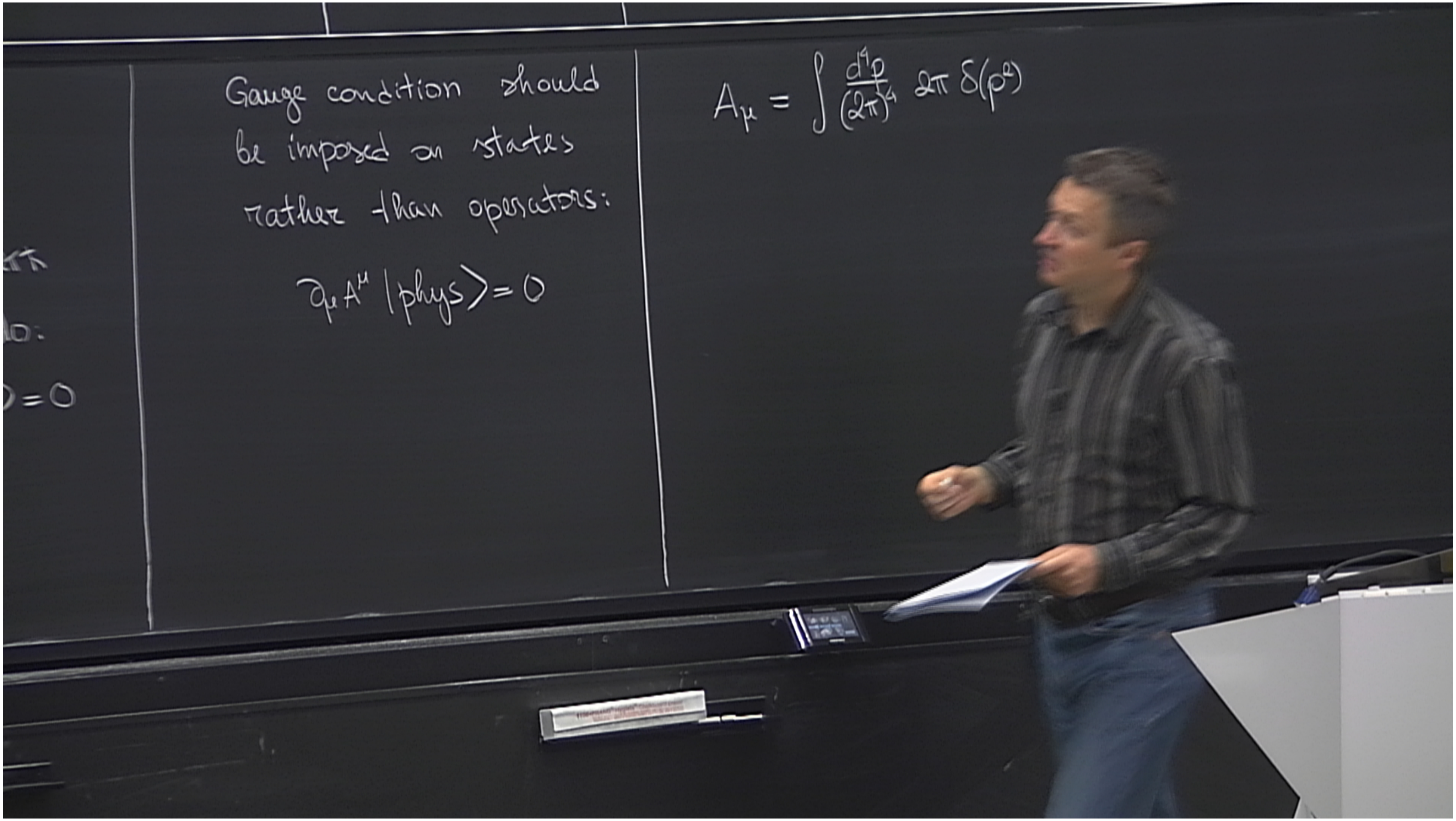
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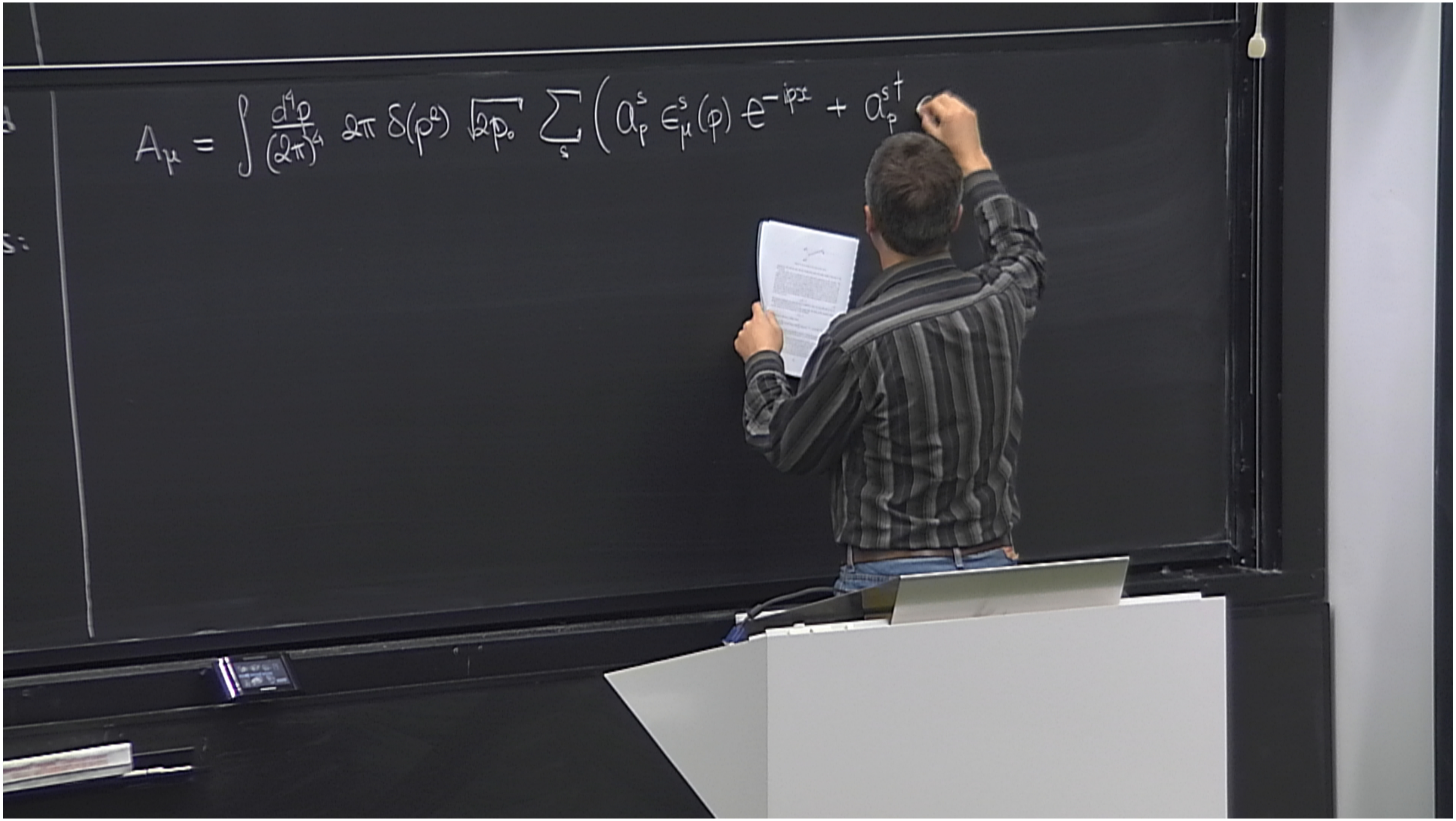
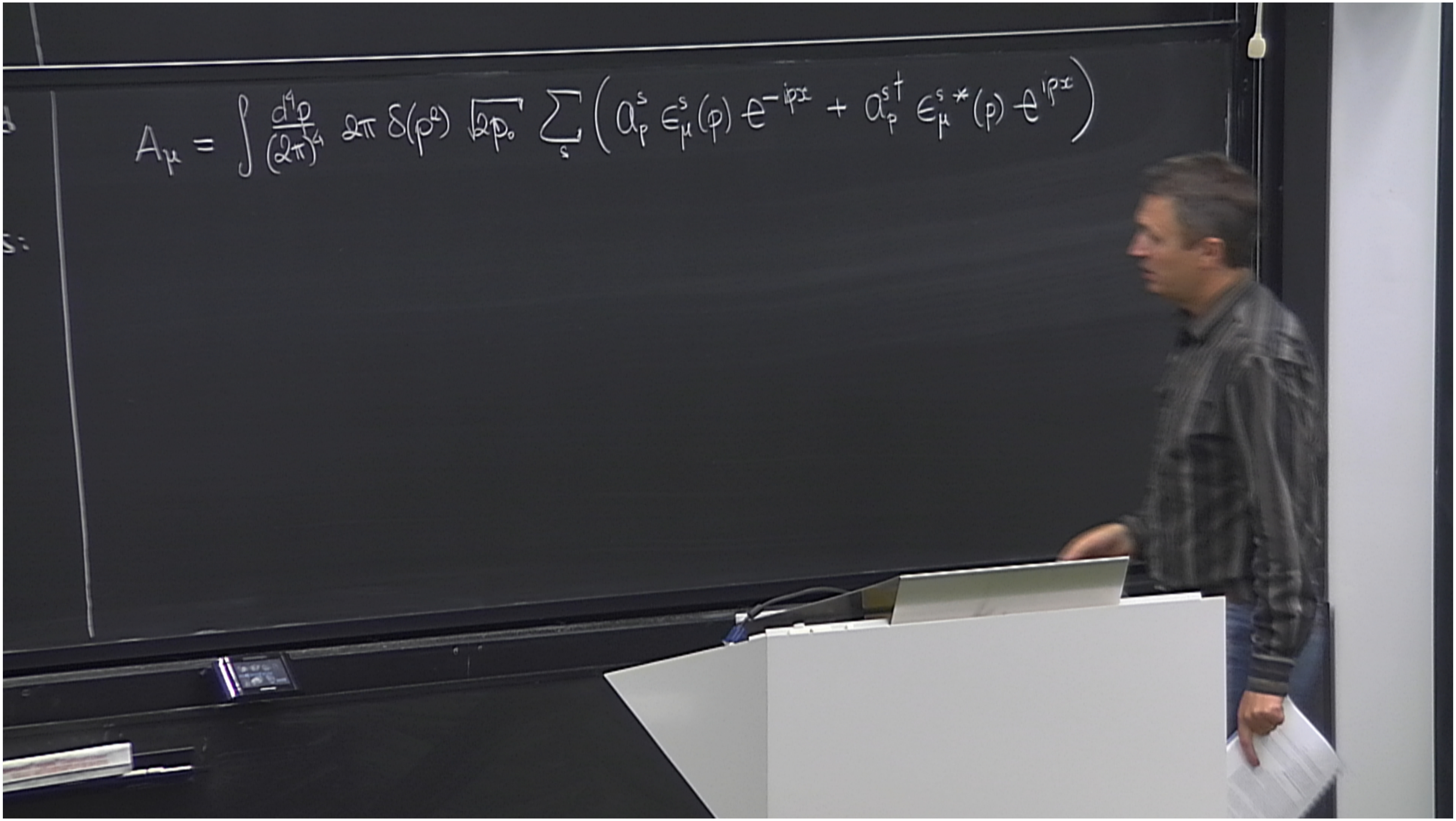

$A_\mu = \int \frac{d^4p}{(2\pi)^4} 2\pi \delta(p^2) \sqrt{2p_0} \sum_s \left(a_p^s \epsilon_\mu^s(p) e^{-ipx} + a_p^{s\dagger}\right.$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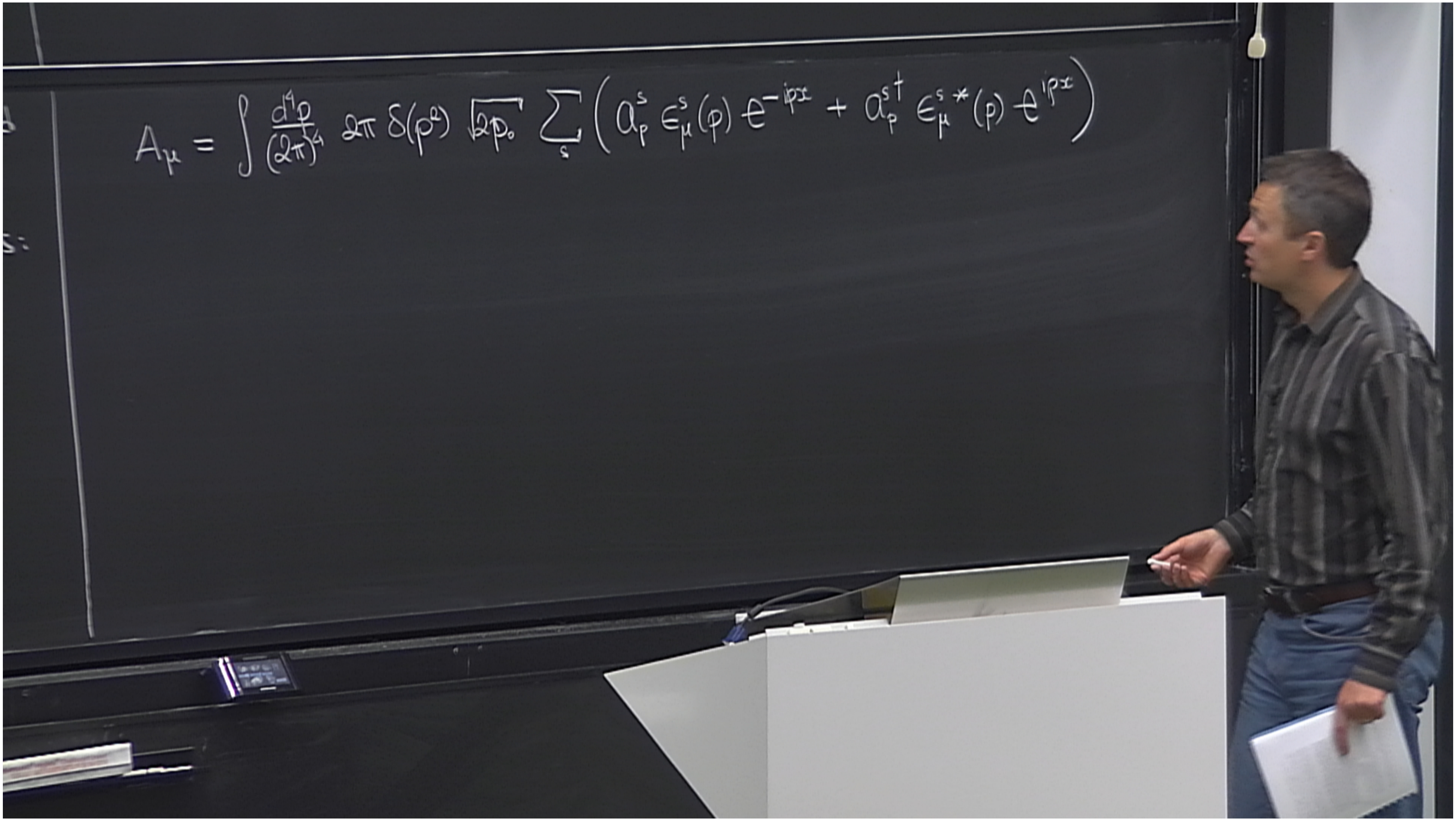
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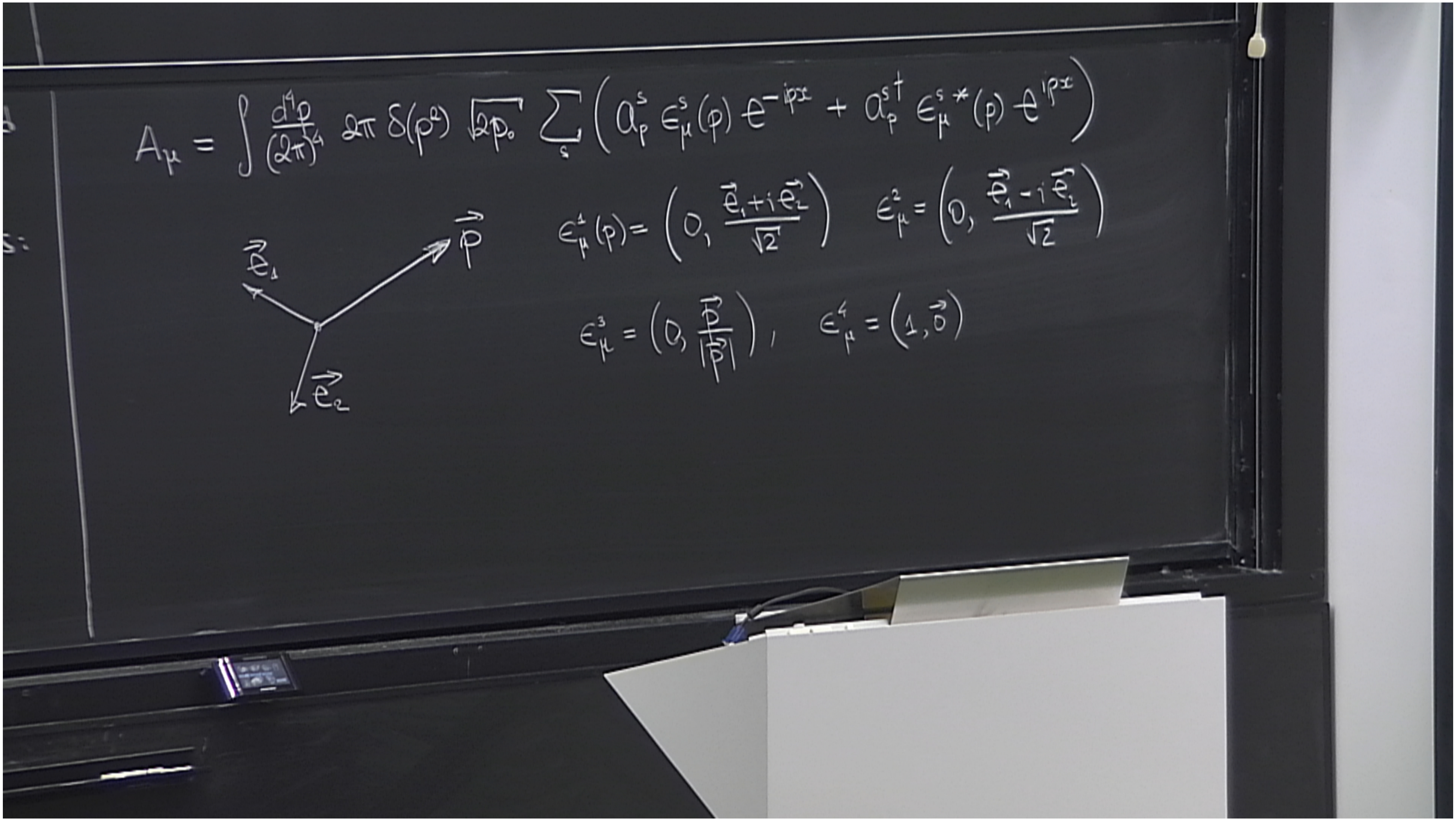
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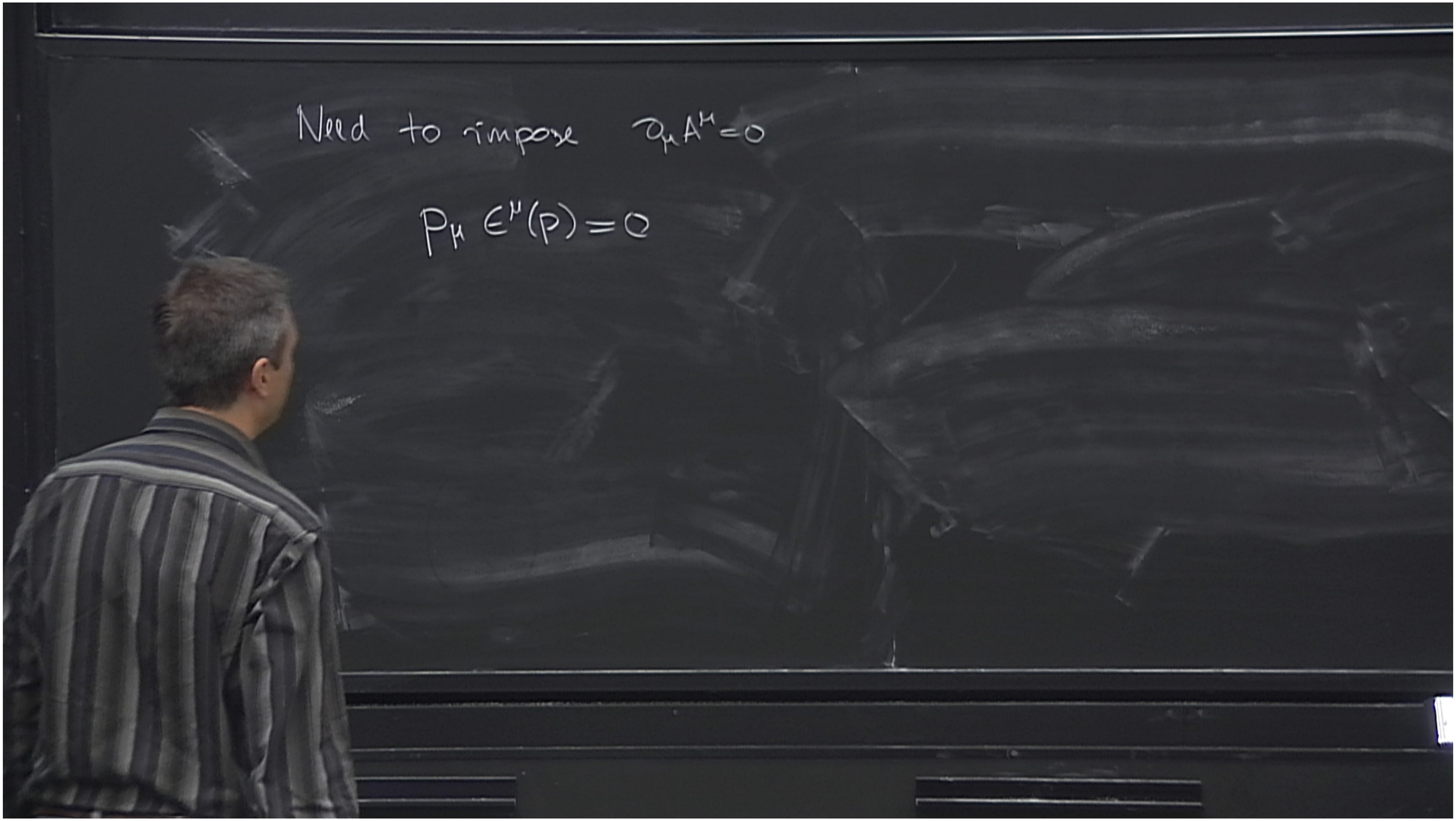
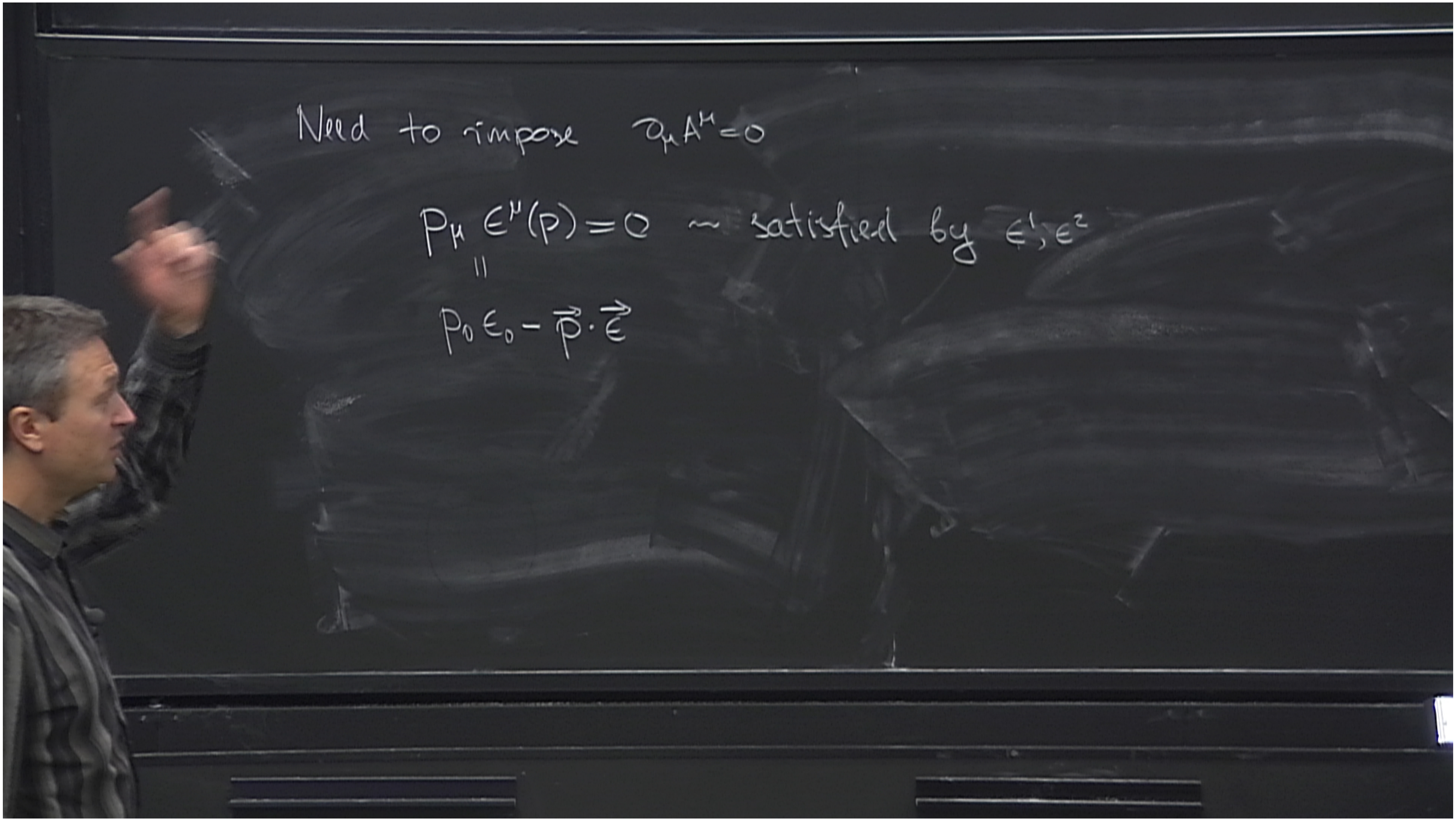
$\epsilon_\mu^1(p) = \left(0, \frac{\vec{e}_1 + i\vec{e}_2}{\sqrt{2}}\right)$
$\epsilon_\mu^2 = \left(0, \frac{\vec{e}_1 - i\vec{e}_2}{\sqrt{2}}\right)$
$\epsilon_\mu^3 = \left(0, \frac{\vec{p}}{|\vec{p}|}\right)$, $\epsilon_\mu^4 = (1, \vec{0})$
$\vec{e}_1$
$\vec{p}$
$\vec{e}_2$


Need to impose $\partial_\mu A^\mu = 0$
$p_\mu \epsilon^\mu(p) = 0$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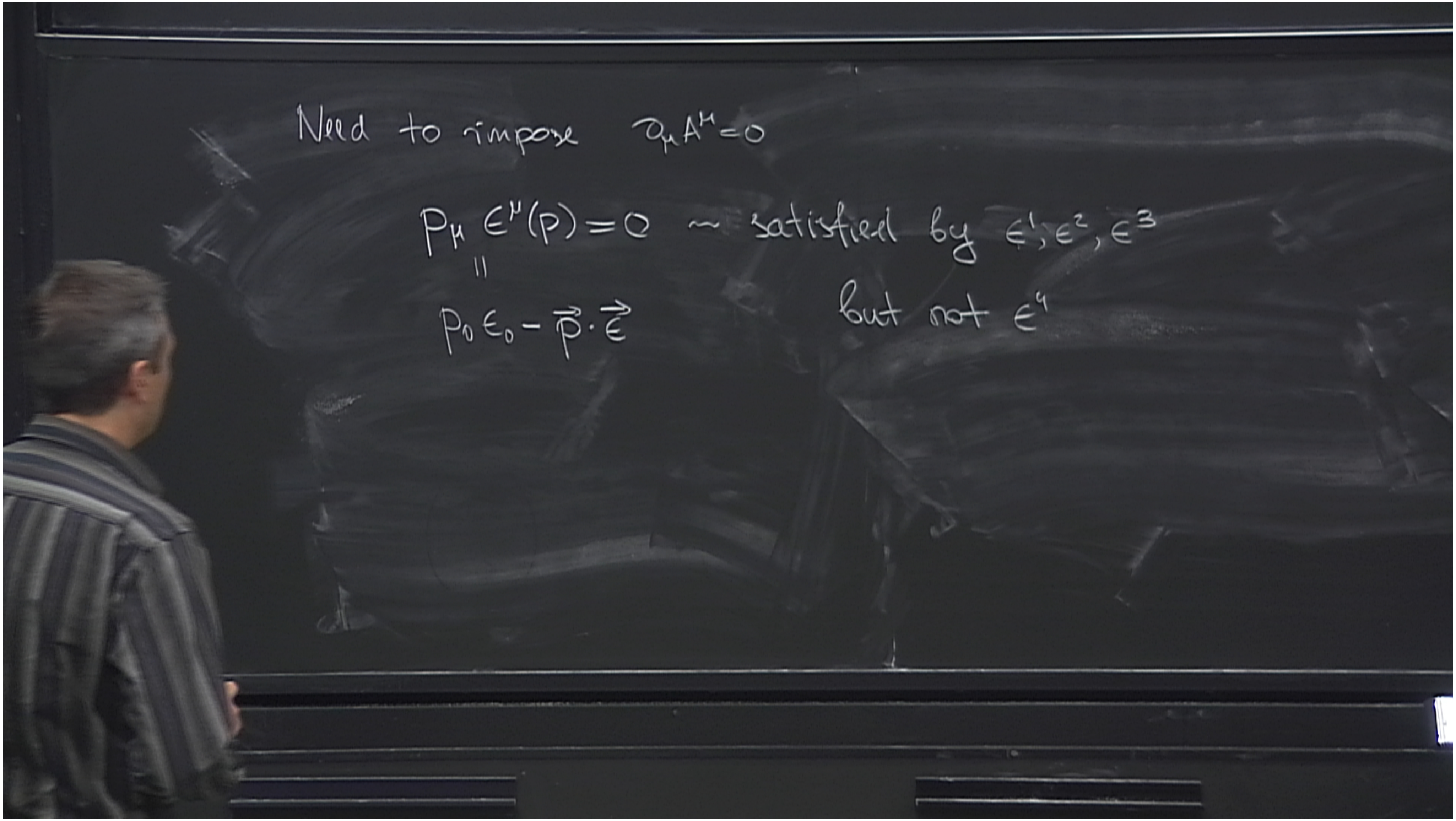
$\parallel$
$p_0 \epsilon_0 - \vec{p} \cdot \vec{\epsilon}$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$$\parallel$$

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$$p_\mu \epsilon^{3\,\mu} = p^2 = 0$$

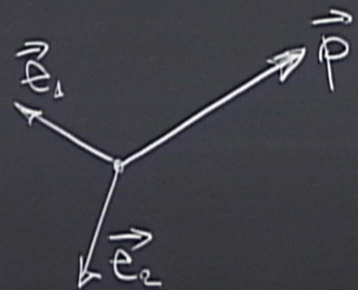
For weak interactions

ould

es

tors:

$$A_\mu = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\,\delta(p^2)\,\sqrt{2p_0}\,\sum_s \left(a_p^s\, \epsilon_\mu^s(p)\, e^{-ipx} + a_p^{s\dagger}\, \epsilon_\mu^{s*}(p)\, e^{ipx} \right)$$



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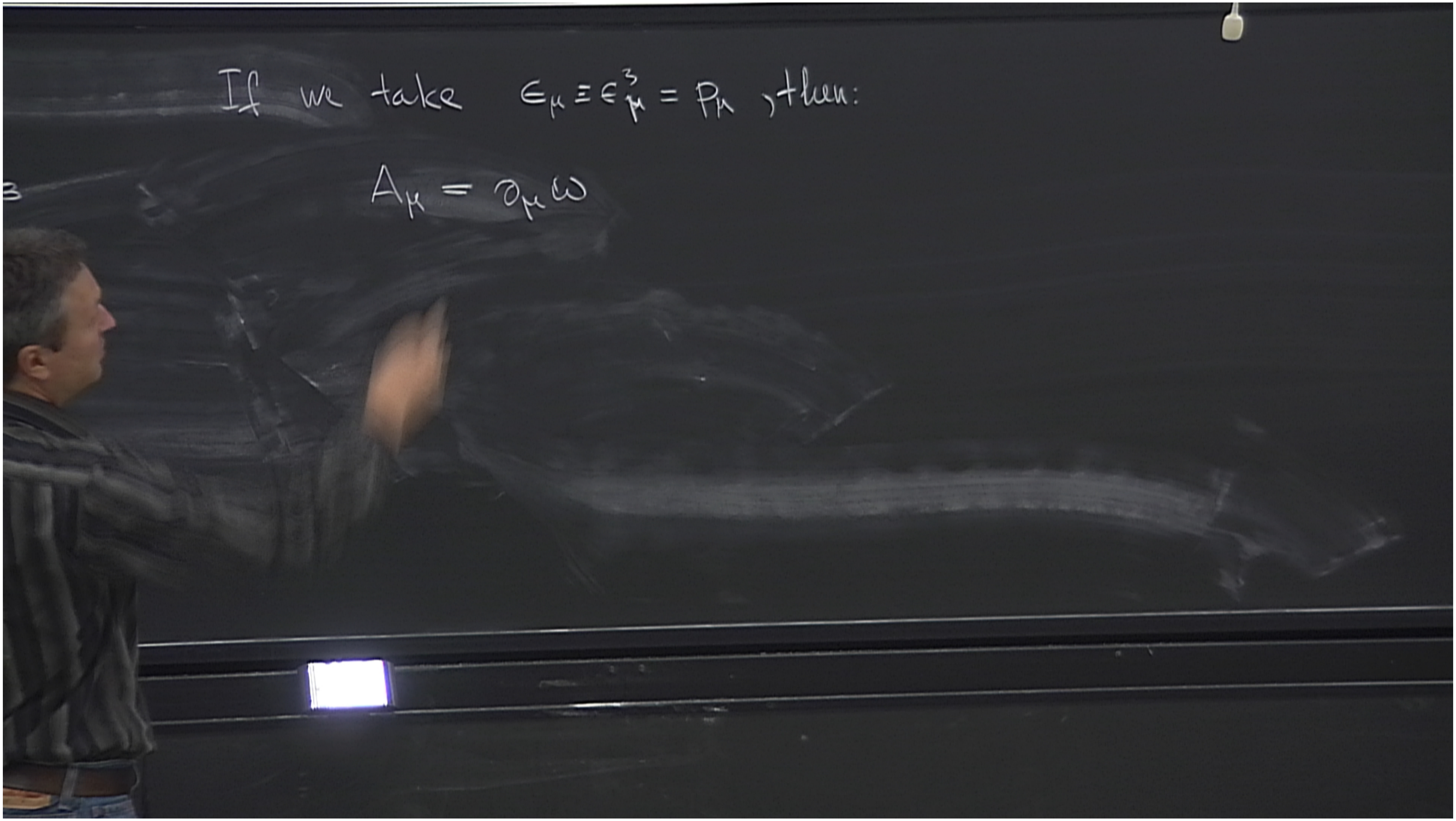
$$\cancel{\epsilon_\mu^3 = \left(0, \frac{\vec{p}}{|\vec{p}|}\right)}, \qquad \epsilon_\mu^4 = (1, \vec{0})$$

$$\epsilon_\mu^3 = p_\mu$$

pseudoscalar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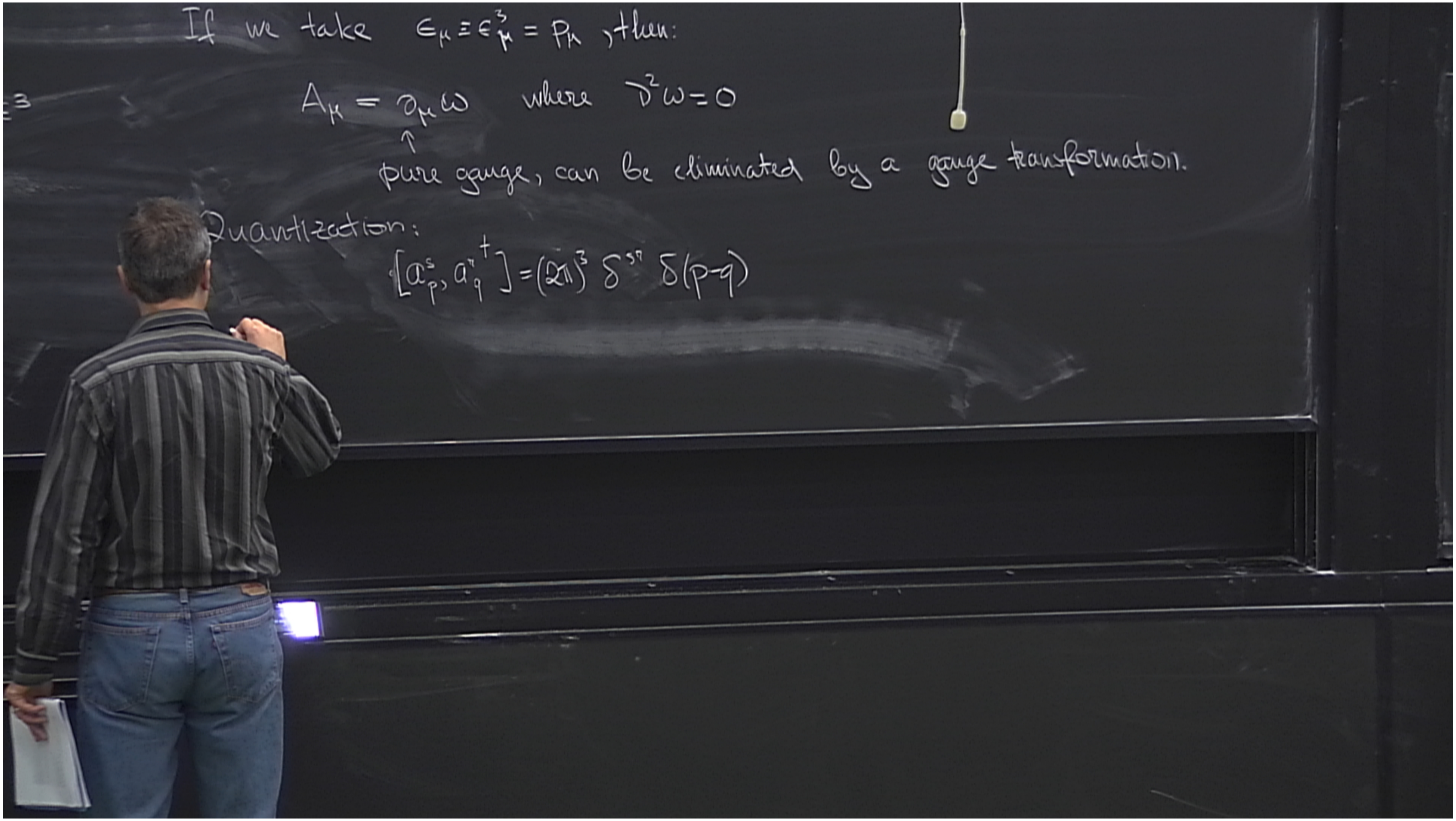
pure gauge, can be eliminated by a gauge transformation.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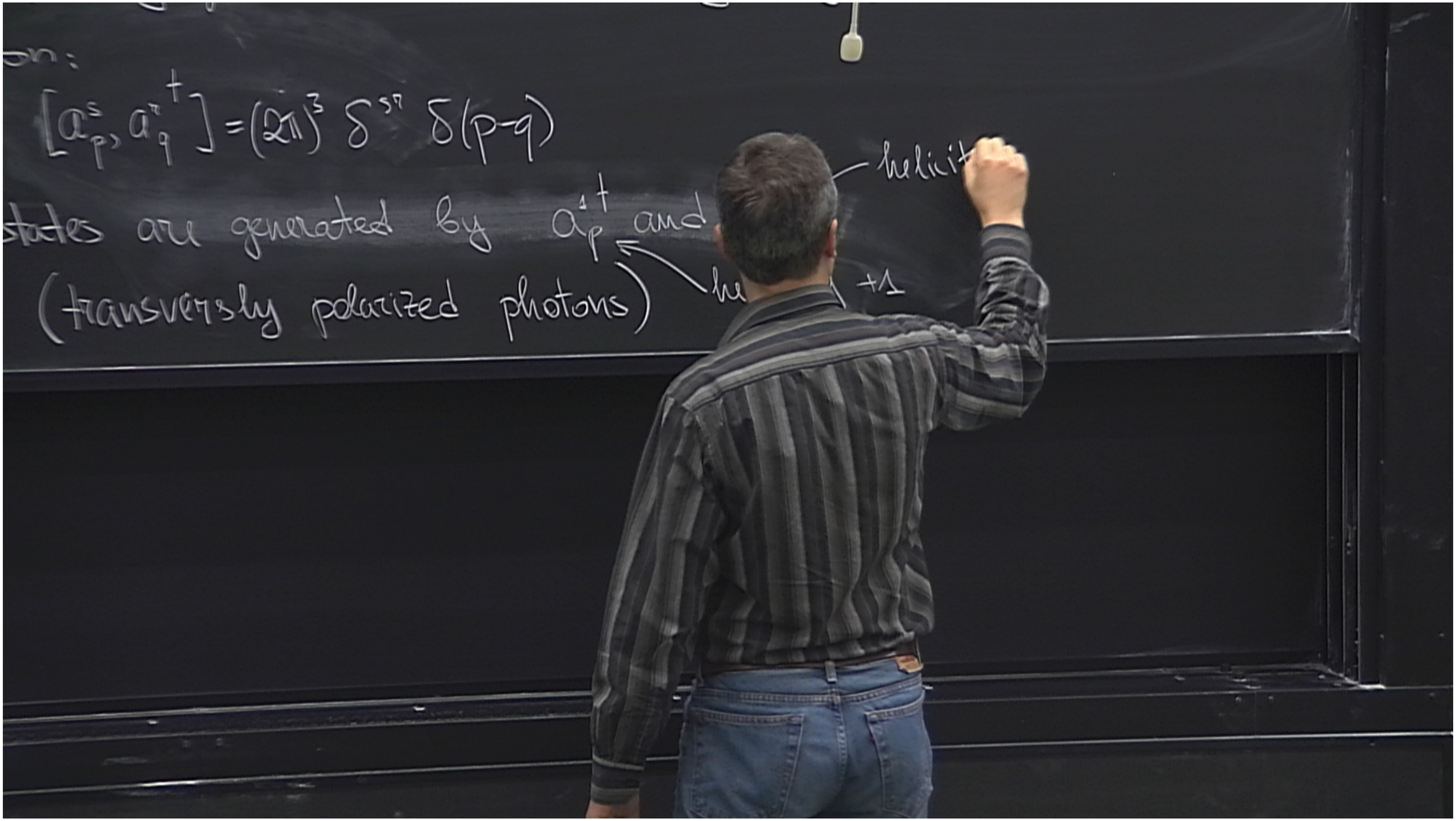
Quantization:
$[a^s_p, a^{r\dagger}_q] = (2\pi)^3 \delta^{sr} \delta(p-q)$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states are generated by $a^{1\dagger}_p$ and
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helicit
+1

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(transversly polarized photons) helicity $+1$