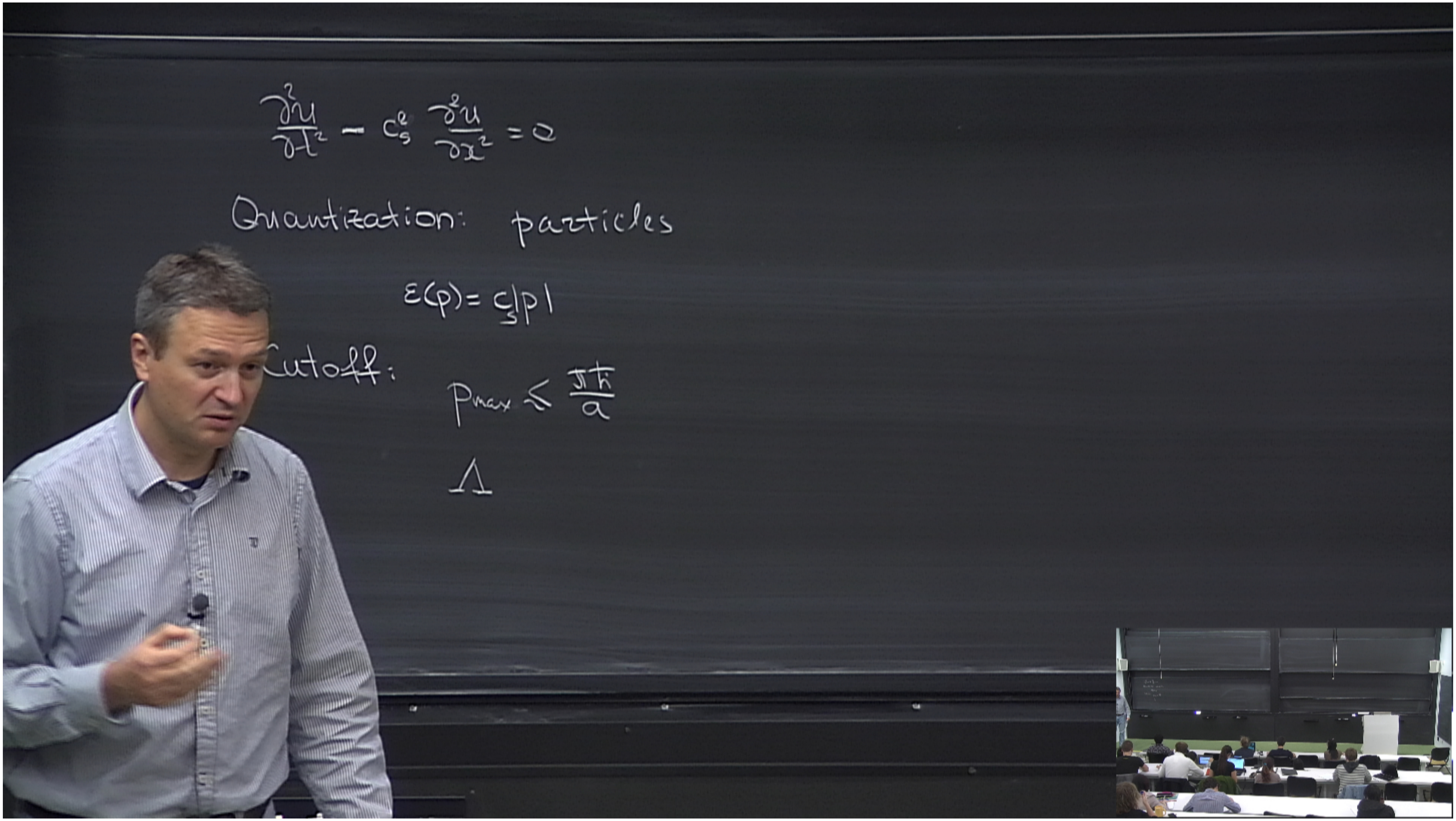
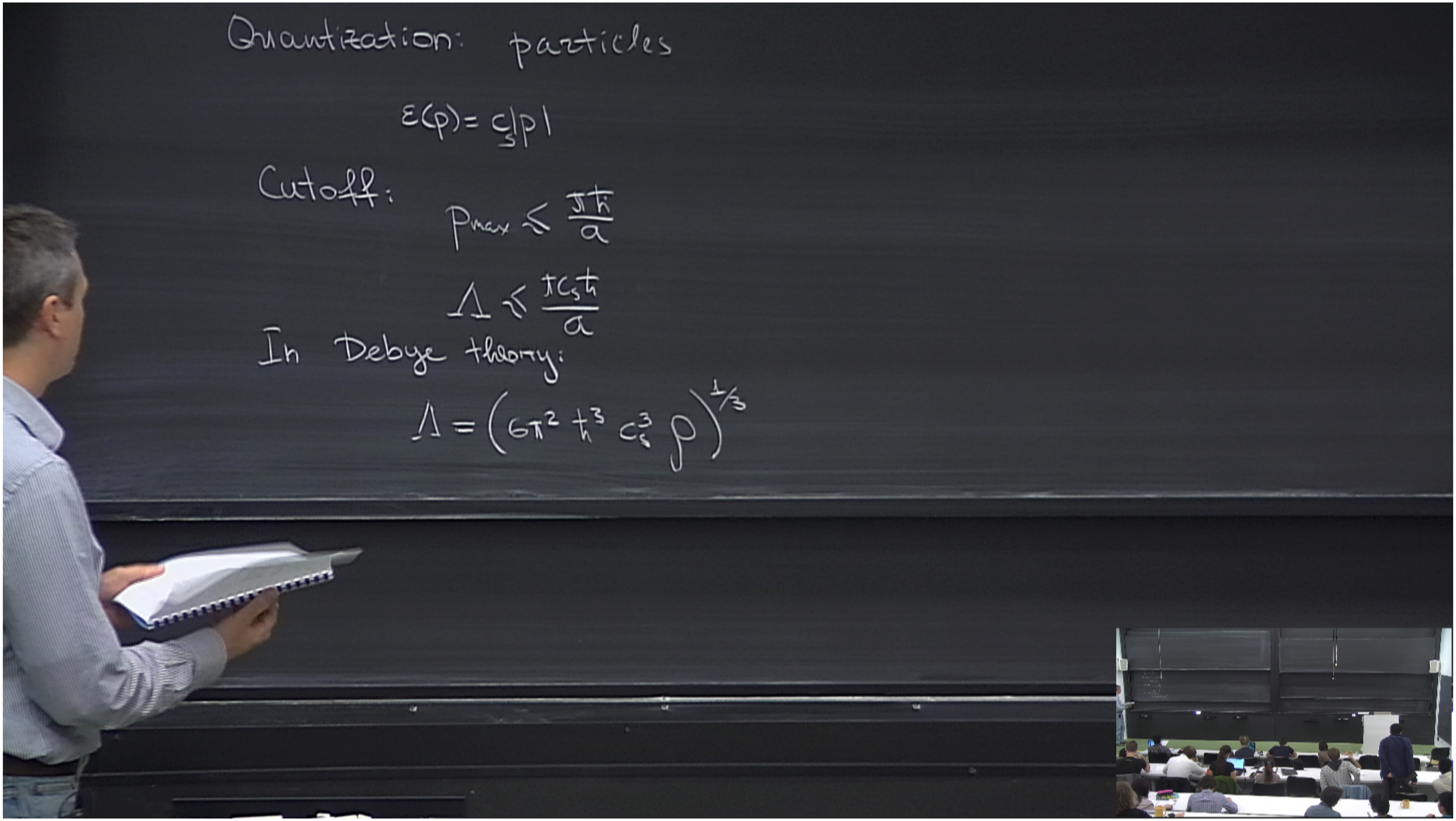
Title: Quantum Field Theory I - Lecture 3

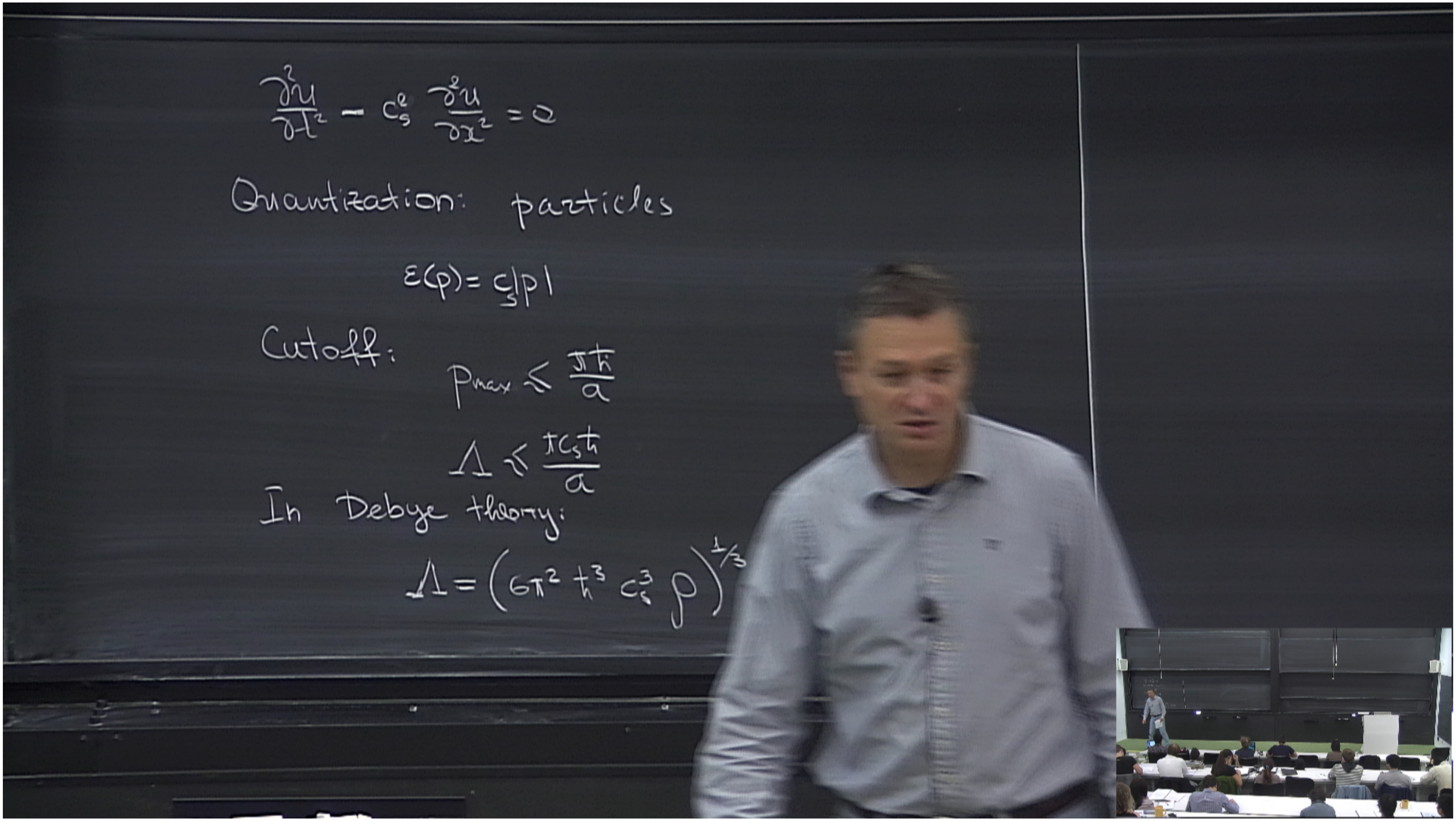
Date: Oct 05, 2011 09:00 AM

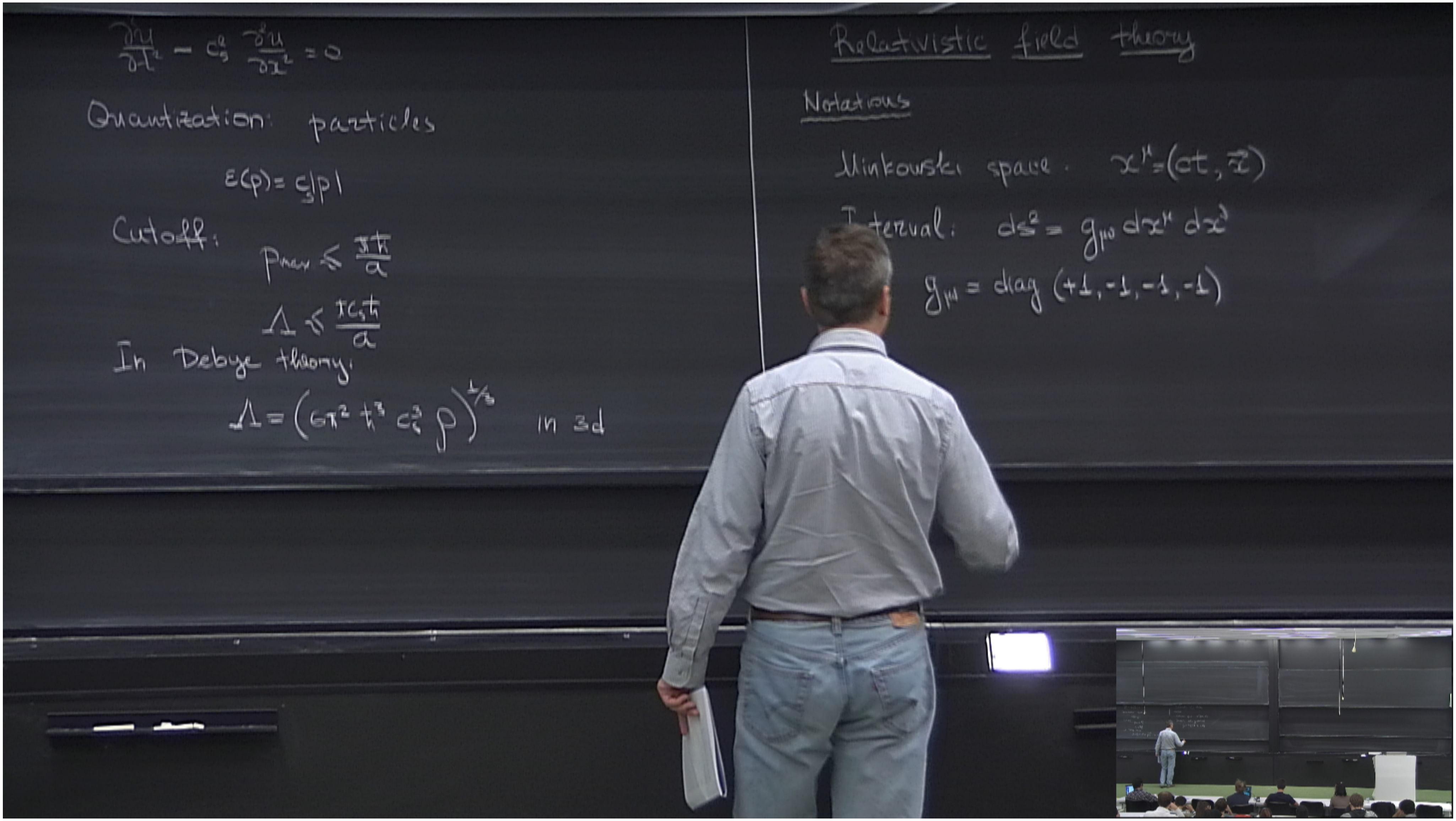
URL: http://pirsa.org/11100008

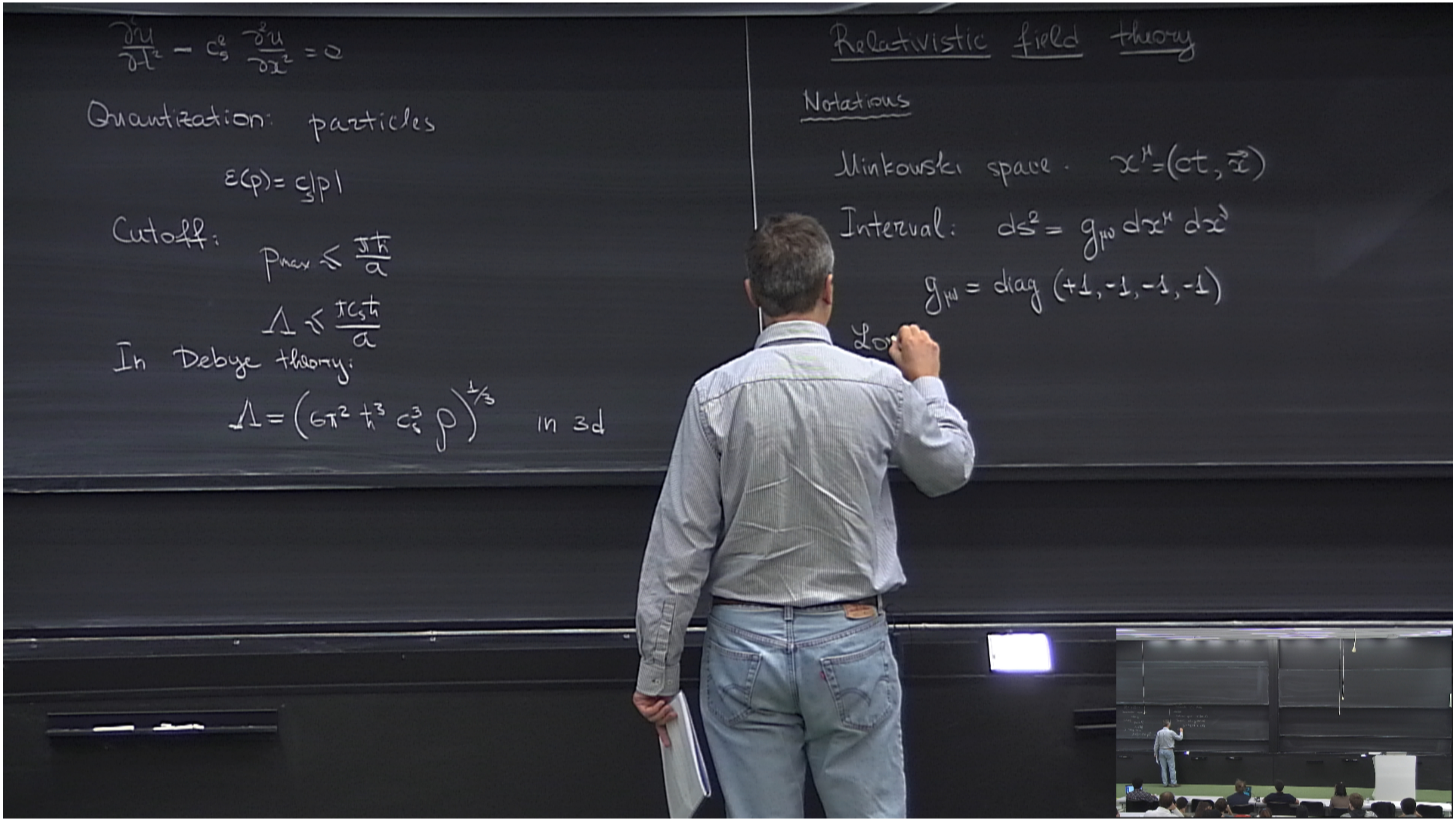
Abstract:

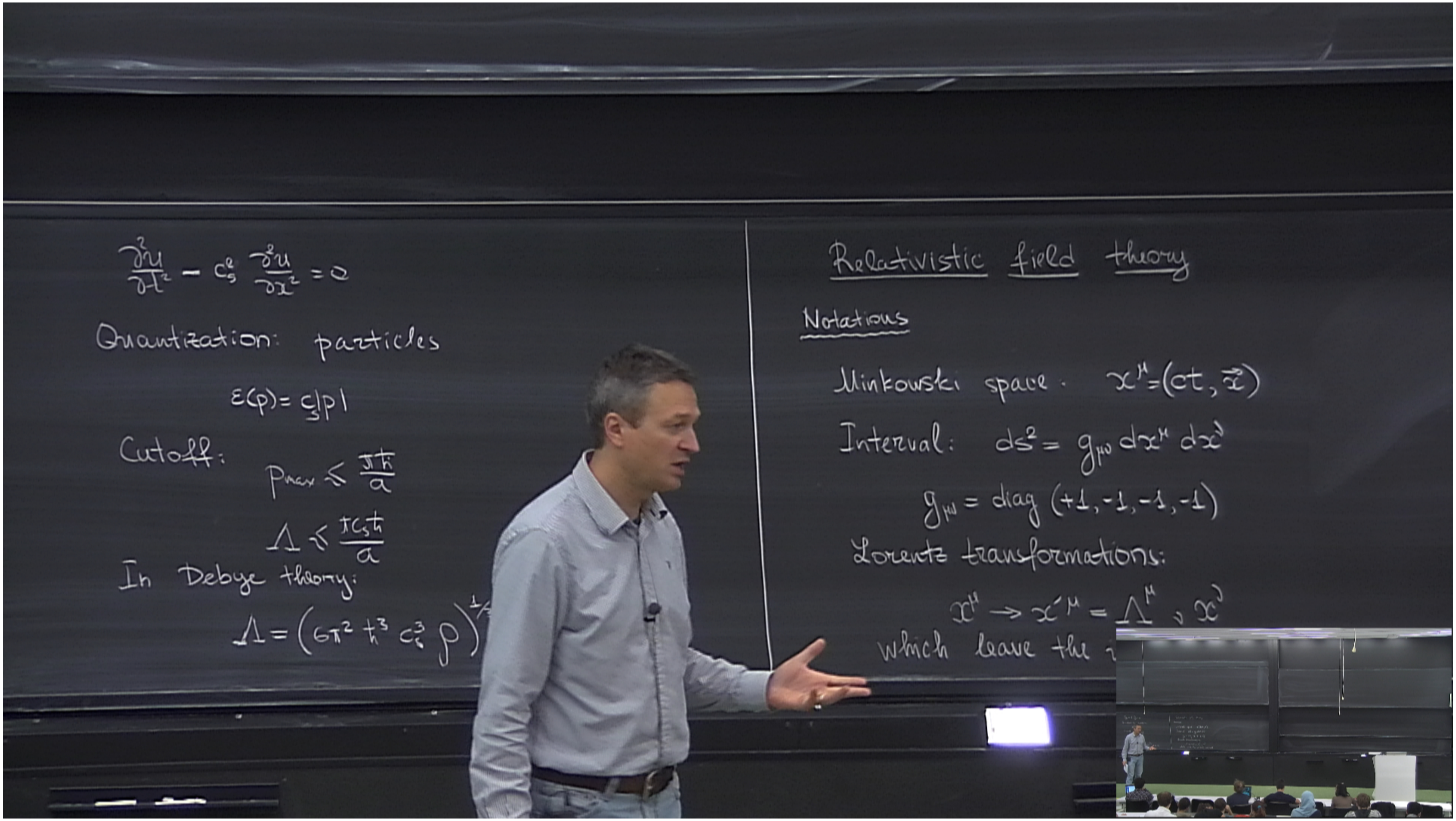

$\frac{\partial^2 u}{\partial t^2} - c_s^2 \frac{\partial^2 u}{\partial x^2} = 0$
Quantization: particles
$\varepsilon(p) = c_s|p|$
Cutoff:
$p_{max} \lesssim \frac{\pi\hbar}{a}$
Λ


Quantization: particles
$\varepsilon(p) = c_s|p|$
Cutoff:
$p_{max} \lesssim \frac{\pi\hbar}{a}$
$\Lambda \lesssim \frac{\pi c_s \hbar}{a}$
In Debye theory:
$\Lambda = \left(6\pi^2 \hbar^3 c_s^3 \rho\right)^{1/3}$


$\frac{\partial^2 u}{\partial t^2} - c_s^2 \frac{\partial^2 u}{\partial x^2} = 0$
Quantization: particles
$\varepsilon(p) = c_s|p|$
Cutoff:
$p_{max} \lesssim \frac{\pi\hbar}{a}$
$\Lambda \lesssim \frac{\pi c_s \hbar}{a}$
In Debye theory:
$\Lambda = (6\pi^2 \hbar^3 c_s^3 \rho)^{1/3}$


$\frac{\partial^2 u}{\partial t^2} - c_s^2 \frac{\partial^2 u}{\partial x^2} = 0$
Quantization: particles
$\varepsilon(p) = c_s|p|$
Cutoff: $p_{max} \leq \frac{\pi\hbar}{a}$
$\Lambda \leq \frac{\pi c_s \hbar}{a}$
In Debye theory:
$\Lambda = (6\pi^2 \hbar^3 c_s^3 \rho)^{1/3}$ in 3d
Relativistic field theory
Notations
Minkowski space. $x^\mu = (ct, \vec{x})$
Interval: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
$g_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$


$\frac{\partial^2 u}{\partial t^2} - c_s^2 \frac{\partial^2 u}{\partial x^2} = 0$
Quantization: particles
$\varepsilon(p) = c_s|p|$
Cutoff: $p_{max} \lesssim \frac{\pi\hbar}{a}$
$\Lambda \lesssim \frac{\pi c_s \hbar}{a}$
In Debye theory:
$\Lambda = (6\pi^2 \hbar^3 c_s^3 \rho)^{1/3}$ in 3d
Relativistic field theory
Notations
Minkowski space. $x^\mu = (ct, \vec{x})$
Interval: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
$g_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$


$\frac{\partial^2 u}{\partial t^2} - c_s^2 \frac{\partial^2 u}{\partial x^2} = 0$
Quantization: particles
$\varepsilon(p) = c_s|p|$
Cutoff: $p_{max} \lesssim \frac{\pi\hbar}{a}$
$\Lambda \lesssim \frac{\pi c_s \hbar}{a}$
In Debye theory:
$\Lambda = (6\pi^2 \hbar^3 c_s^3 \rho)^{1/3}$
Relativistic field theory
Notations
Minkowski space. $x^\mu = (ct, \vec{x})$
Interval: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$
$g_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$
Lorentz transformations:
$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$
which leave the

Relativistic field theory

Notations

Minkowski space: $x^\mu = (ct, \vec{x})$

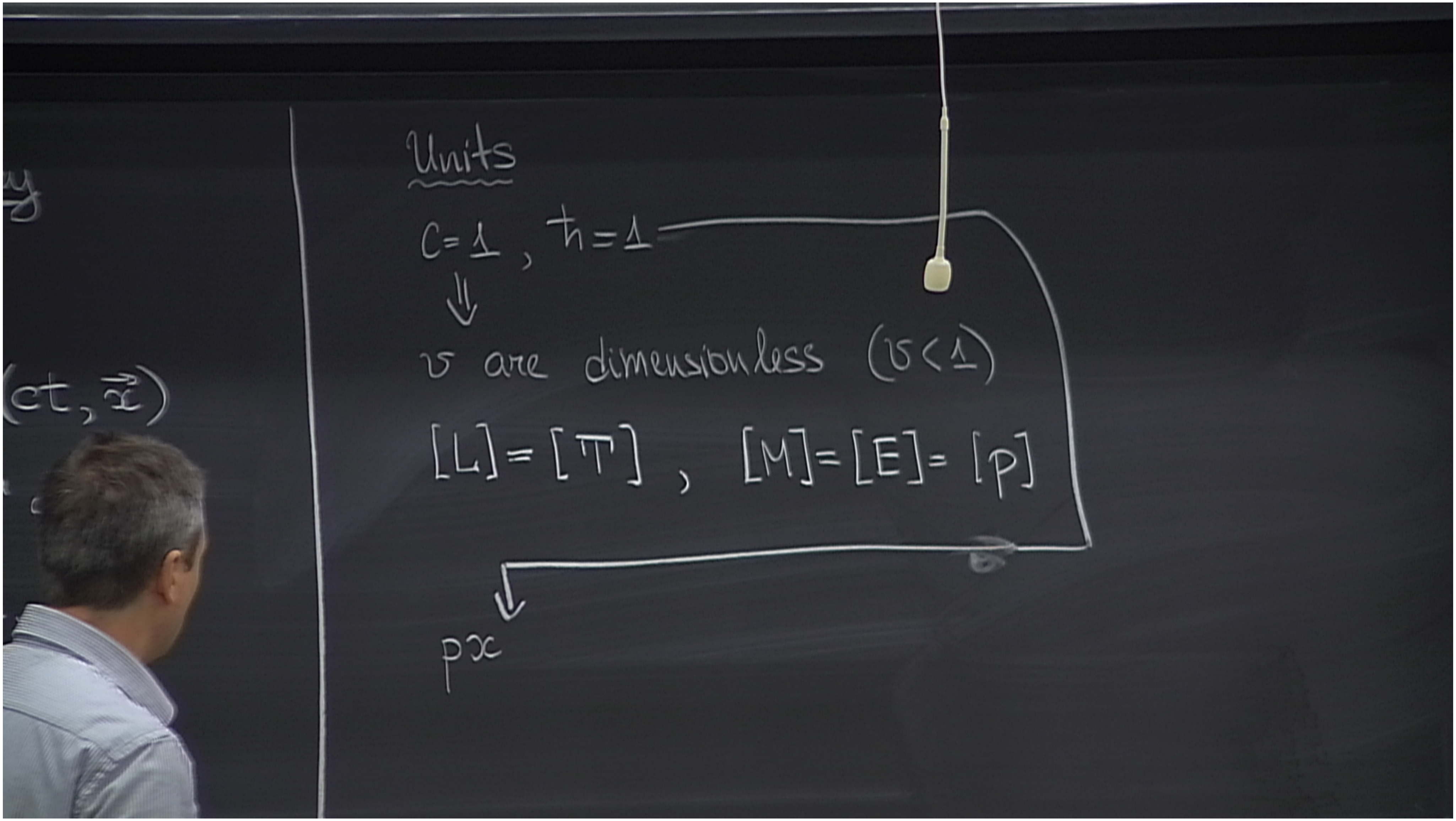
Interval: $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$

$$g_{\mu\nu} = \mathrm{diag}\,(+1, -1, -1, -1)$$

Lorentz transformations:

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu\, x^\nu$$

which leave the interval invariant


Units
c = 1 , ħ = 1
⇓
v are dimensionless (v < 1)
[L] = [T] , [M] = [E] = [p]
px
(ct, x⃗)

$(ct, \vec{x})$

dx^ν

$, -1)$

x^ν

Units

$c = 1\ ,\ \hbar = 1$

$\Downarrow$

v are dimensionless $(v < 1)$

$[L] = [T]\ ,\ \ [M] = [E] = [p]$

$\Downarrow$

$p \cdot x$ - dimensionless

$E \cdot t$ - dimensionless

theory

$x^\mu = (ct, \vec{x})$

$g_{\mu\nu}\, dx^\mu\, dx^\nu$

$1, -1, -1, -1)$

ations:

$= \Lambda^\mu{}_\nu\, x^\nu$

interval invariant

Units

$c = 1\ ,\ \hbar = 1$

$\Downarrow$

v are dimensionless $(v < 1)$

$[L] = [T]\ ,\ [M] = [E] = [p]$

$\Downarrow$

$p \cdot x$ - dimensionless

$E \cdot t$ - dimensionless

$[L^{-1}] = [T^{-1}] = [E] = [p] = [M]$

$x^\mu = (ct, \vec{x})$

$g_{\mu\nu}\, dx^\mu dx^\nu$

$1, -1, -1, -1)$

ations:

$= \Lambda^\mu{}_\nu x^\nu$

e interval invariant

$c = 1$, $\hbar = 1$

$\Downarrow$

v are dimensionless ($v < 1$)

$[L] = [T]$, $[M] = [E] = [p]$

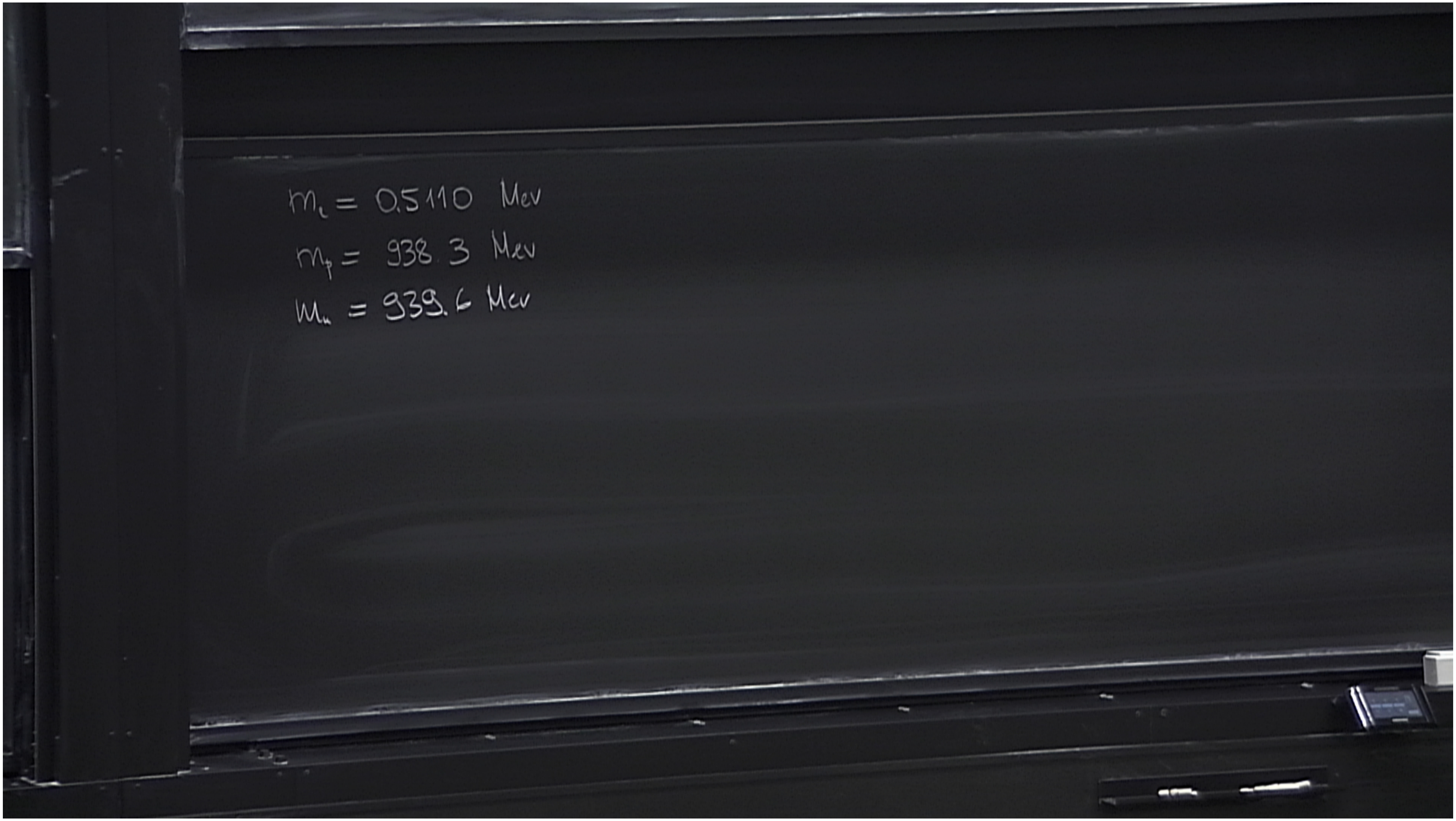
$\Downarrow$

$p \cdot x$ - dimensionless

$E \cdot t$ - dimensionless

$[L^{-1}] = [T^{-1}] = [E] = [p] = [M]$

ev - electronvolts (energy units)


$m_e = 0.5110$ MeV
$m_p = 938.3$ MeV
$m_n = 939.6$ MeV

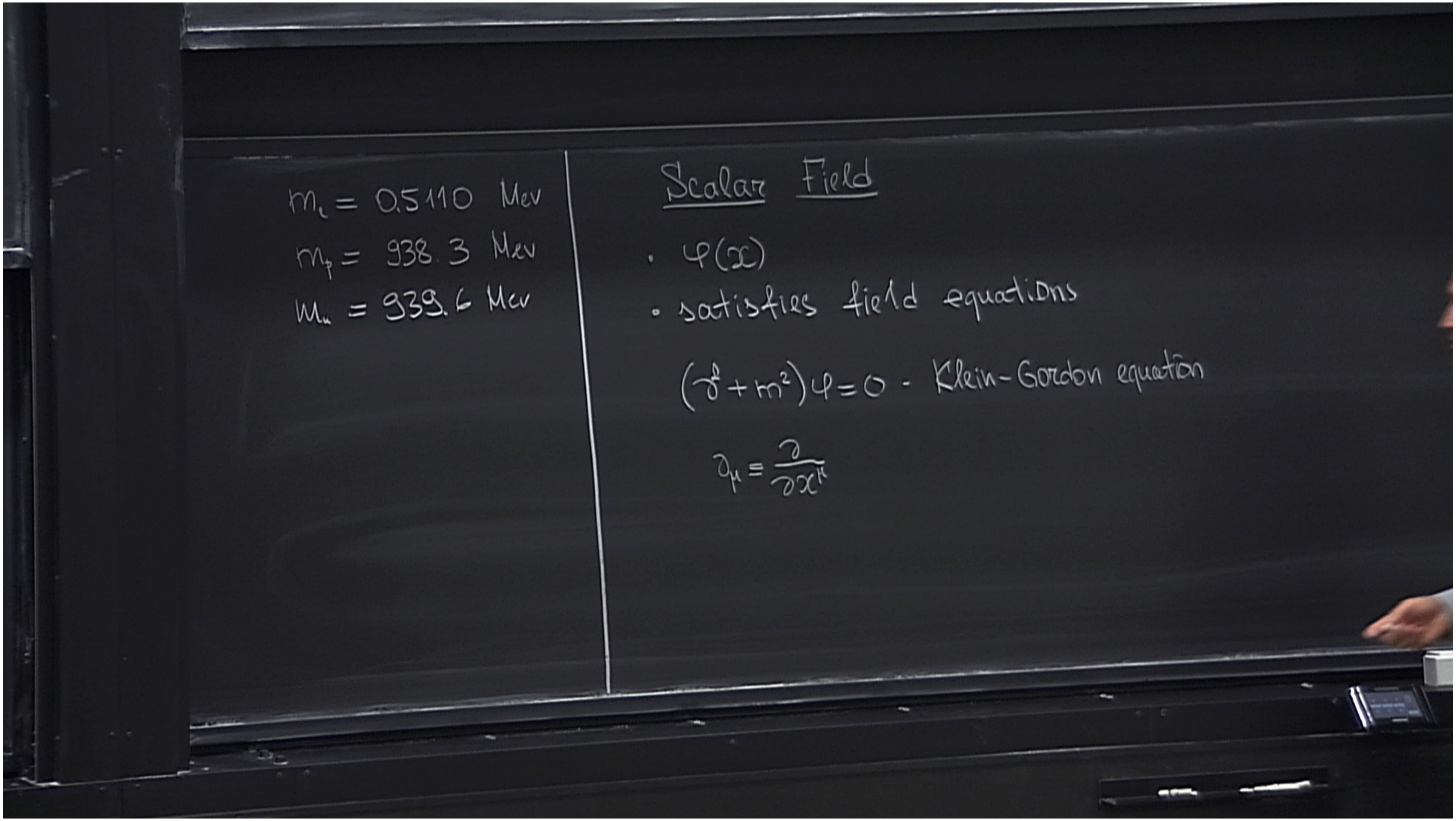
$m_e = 0.5110$ MeV

$m_p = 938.3$ MeV

$m_n = 939.6$ MeV

Scalar Field

- $\varphi(x)$
- satisfies field equations


$m_e = 0.5110$ Mev
$m_p = 938.3$ Mev
$m_n = 939.6$ Mev
Scalar Field
• $\varphi(x)$
• satisfies field equations
$(\partial^2 + m^2)\varphi = 0$ - Klein-Gordon equation
$\partial_\mu = \frac{\partial}{\partial x^\mu}$

$m_e = 0.5110$ MeV

$m_p = 938.3$ MeV

$m_n = 939.6$ MeV

Scalar Field

- $\varphi(x)$
- satisfies field equations

$(\partial^2 + m^2)\varphi = 0$ - Klein-Gordon equation

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$$

Plane-wave solutions:

$$(\partial^2 + m^2) e^{ipx} = 0 \Rightarrow \boxed{p^2 - m^2 = 0}$$

$m_e = 0.5110$ MeV

$m_p = 938.3$ MeV

$m_n = 939.6$ MeV

Scalar Field

- $\varphi(x)$
- satisfies field equations

$$(\partial^2 + m^2)\varphi = 0$$ - Klein-Gordon equation

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$$

Plane-wave solutions:

$$(\partial^2 + m^2)e^{ipx} = 0 \Rightarrow \boxed{p^2 - m^2 = 0}$$

Scalar Field

- $\varphi(x)$
- satisfies field equations

$$(\partial^2 + m^2)\varphi = 0$$ - Klein-Gordon equation

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$$

Plane-wave solutions:

$$(\partial^2 + m^2) e^{ipx} = 0 \Rightarrow \boxed{p^2 - m^2 = 0}$$

$$\varphi(t, \vec{x}) \sim \varphi_{\vec{x}}(t)$$

Action:

$$S = \frac{1}{2} \int d^4x \left(\partial_\mu \varphi \, \partial^\mu \varphi - m^2 \varphi^2 \right)$$

Scalar Field

- $\varphi(x)$
- satisfies field equations

$$(\partial^2 + m^2)\varphi = 0$$ - Klein-Gordon equation

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$$

Plane-wave solutions:

$$(\partial^2 + m^2) e^{ipx} = 0 \Rightarrow \boxed{p^2 - m^2 = 0}$$

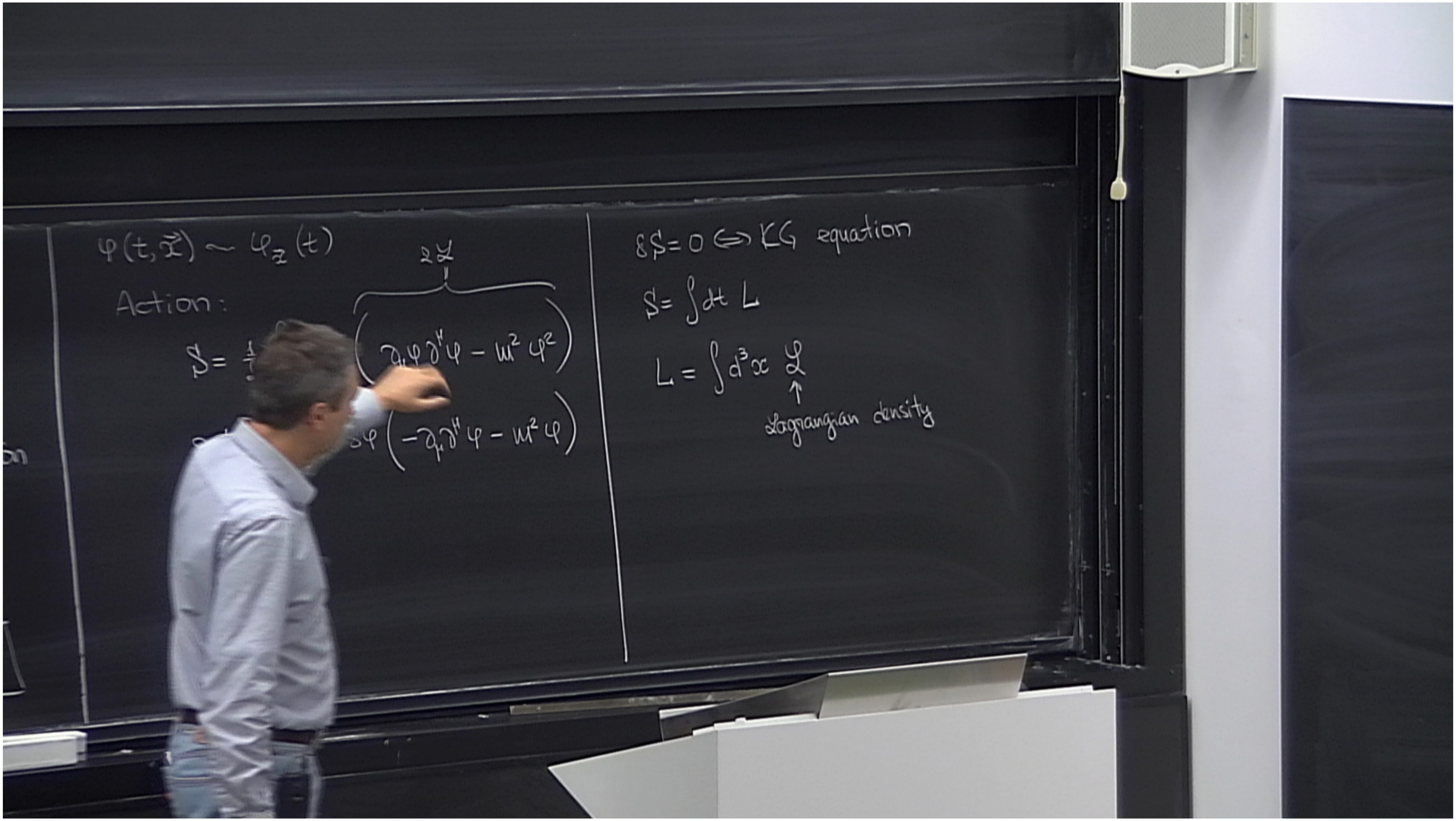
$$\varphi(t, \vec{x}) \sim \varphi_{\vec{x}}(t)$$

Action:

$$S = \frac{1}{2}\int d^4x \left(\partial_\mu\varphi\, \partial^\mu\varphi - m^2\varphi^2\right)$$

$$\delta S = \int d^4x\, \delta\varphi \left(-\partial_\mu\partial^\mu\varphi - m^2\varphi\right)$$

$\delta S = 0 \Leftrightarrow$ KG equ


$\varphi(t,\vec{x}) \sim \varphi_{x}(t)$
Action:
$2\mathcal{L}$
$S = \frac{1}{\cdot} \left(\partial_\mu\varphi\,\partial^\mu\varphi - m^2\varphi^2\right)$
$\delta\varphi\left(-\partial_\mu\partial^\mu\varphi - m^2\varphi\right)$
$\delta S = 0 \Leftrightarrow$ KG equation
$S = \int dt\, L$
$L = \int d^3x\, \mathcal{L}$
Lagrangian density

$\varphi(t,\vec{x}) \sim \varphi_{\vec{x}}(t)$

Action:

$$S = \frac{1}{2}\int d^4x \overbrace{\left(\partial_\mu\varphi\,\partial^\mu\varphi - m^2\varphi^2\right)}^{2\mathcal{L}}$$

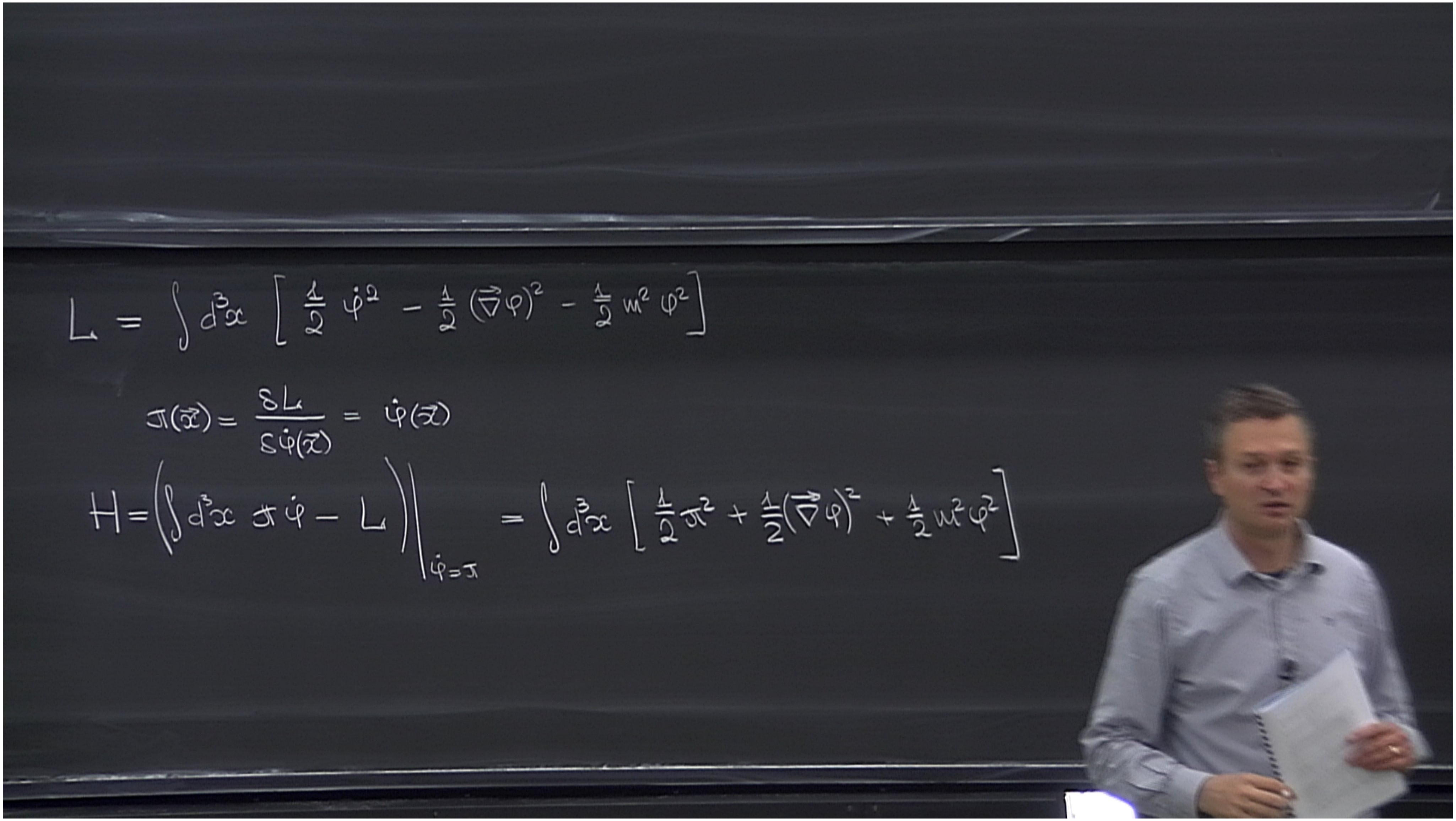
$$\delta S = \int d^4x\, \delta\varphi\left(-\partial_\mu\partial^\mu\varphi - m^2\varphi\right)$$

$\delta S = 0 \Leftrightarrow$ KG equation

$$S = \int dt\, L$$

$$L = \int d^3x\, \mathcal{L}$$

Lagrangian density



$L = \int d^3x \left[\frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2 - \frac{1}{2}m^2\varphi^2 \right]$
$\pi(\vec{x}) = \frac{\delta L}{\delta \dot{\varphi}(\vec{x})} = \dot{\varphi}(\vec{x})$
$H = \left(\int d^3x\, \pi\dot{\varphi} - L \right) \Big|_{\dot{\varphi}=\pi} = \int d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2 \right]$

$$L = \int d^3x \left[\frac{1}{2} \dot{\varphi}^2 - \frac{1}{2} (\vec{\nabla}\varphi)^2 - \frac{1}{2} m^2 \varphi^2 \right]$$

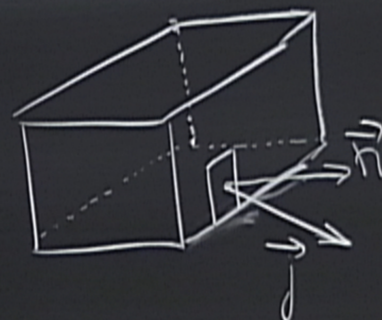
$$\pi(\vec{x}) = \frac{\delta L}{\delta \dot{\varphi}(\vec{x})} = \dot{\varphi}(\vec{x})$$

$$H = \left(\int d^3x \, \pi \dot{\varphi} - L \right) \Big|_{\dot{\varphi} = \pi} = \int d^3x \left[\frac{1}{2} \pi^2 + \frac{1}{2} (\vec{\nabla}\varphi)^2 + \frac{1}{2} m^2 \varphi^2 \right]$$

$\ldots \vec{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2\Big]$

ρ - density of conserved charge

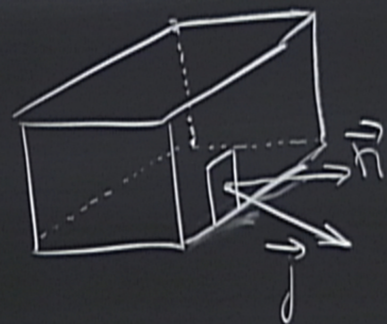
$\vec{j}$ - density of flux



$$\ldots(\vec{\nabla}\varphi)^2 + \tfrac{1}{2} m^2 \varphi^2 \Big]$$

ρ - density of conserved charge

$\vec{j}$ - density of flux

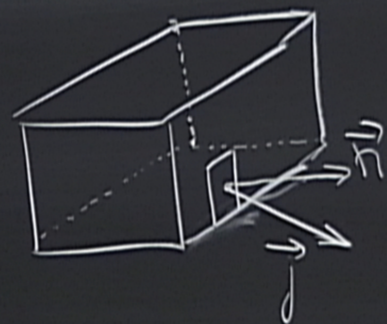


$$\frac{\partial}{\partial t} \int_V d^3x \, \rho = - \oint$$

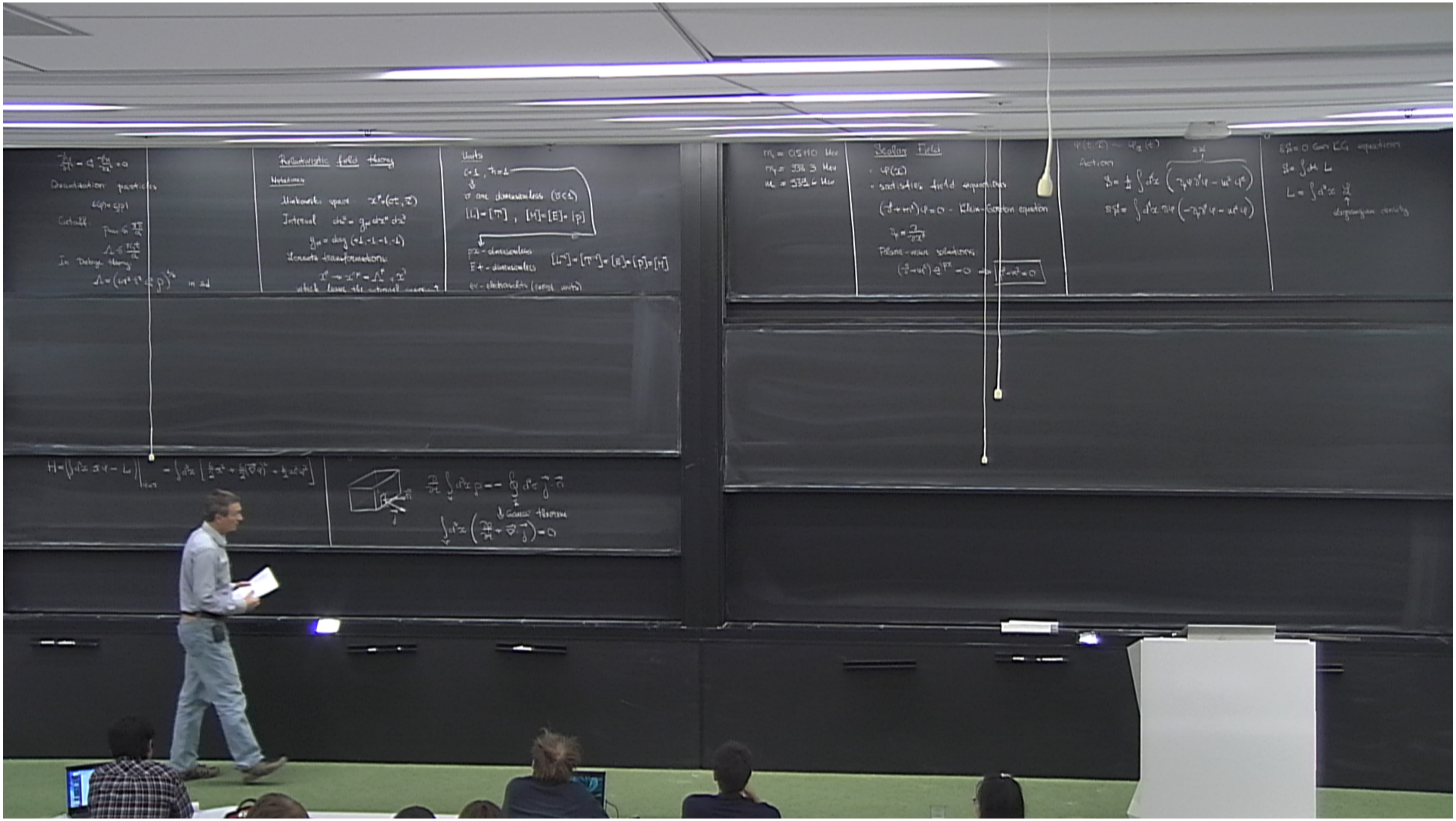
$$(\vec{\nabla}\varphi)^2 + \frac{1}{2} m^2 \varphi^2 \Big]$$

ρ - density of conserved charge

$\vec{j}$ - density of flux



$$\frac{\partial}{\partial t}\int_V d^3x\, \rho = -\oint_\Sigma d^2\sigma\, \vec{j}\cdot\vec{n}$$


Relativistic field theory
Units
Scalar Field
Klein-Gordon equation
Plane-wave solutions
Action
Lagrangian density
Gauss' theorem

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

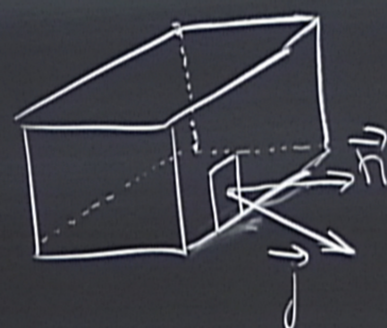
Et - dimensionless

$$\ldots\varphi)^2 + \frac{1}{2} m^2 \varphi^2 \Big]$$

Conservation laws

ρ - density of conserved charge

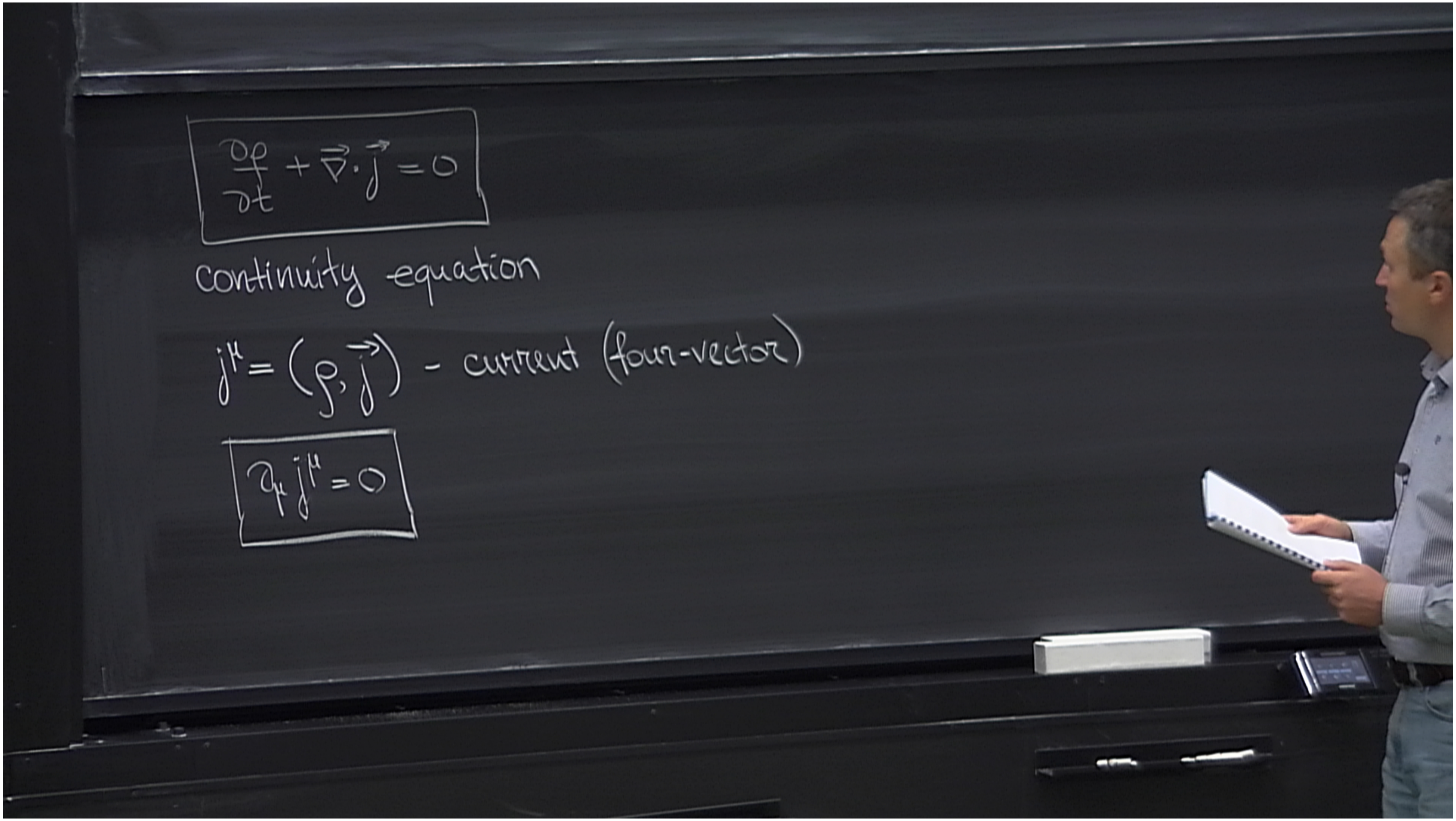
$\vec{j}$ - density of flux



$$\frac{\partial}{\partial t} \int_V d^3x \, \rho = - \oint_\Sigma d^2\sigma \, \vec{j} \cdot \vec{n}$$

$\Downarrow$ Gauss' theorem

$$\int_V d^3x \left(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} \right) = 0$$


$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$
continuity equation
$j^\mu = (\rho, \vec{j})$ - current (four-vector)
$\partial_\mu j^\mu = 0$

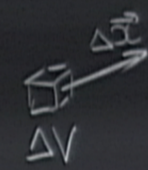
$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0}$$

continuity equation

$j^\mu = (\rho, \vec{j})$ - current (four-vector)

$$\boxed{\partial_\mu j^\mu = 0}$$

Δt :



$$\rho = \frac{\Delta Q}{\Delta t}$$

$$\vec{j} = \frac{\Delta Q}{\Delta t} \cdot \frac{\Delta \vec{x}}{\Delta t}$$

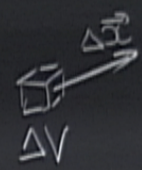
$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0}$$

continuity equation

$j^\mu = (\rho, \vec{j})$ - current (four-vector)

$$\boxed{\partial_\mu j^\mu = 0}$$

Δt :



$$\rho = \frac{\Delta Q}{\Delta V} = \frac{\Delta Q}{\Delta V \cdot \Delta t} \cdot \Delta t$$

$$\vec{j} = \frac{\Delta Q}{\Delta V} \cdot \frac{\Delta \vec{x}}{\Delta t} = \frac{\Delta Q}{\Delta V \, \Delta t}\, \Delta\vec{x}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0}$$

continuity equation

$j^\mu = (\rho, \vec{j})$ - current (four-vector)

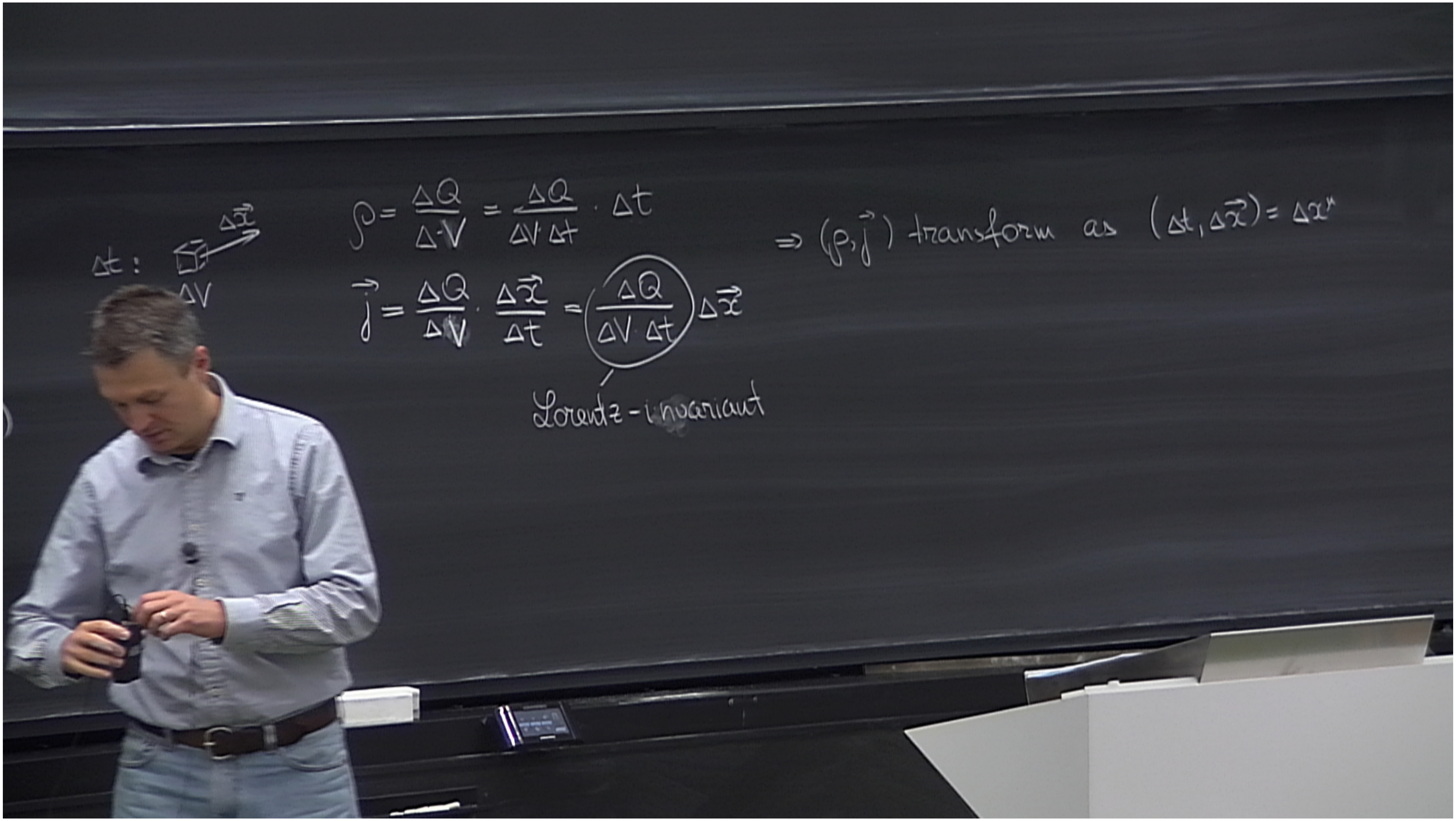
$$\boxed{\partial_\mu j^\mu = 0}$$

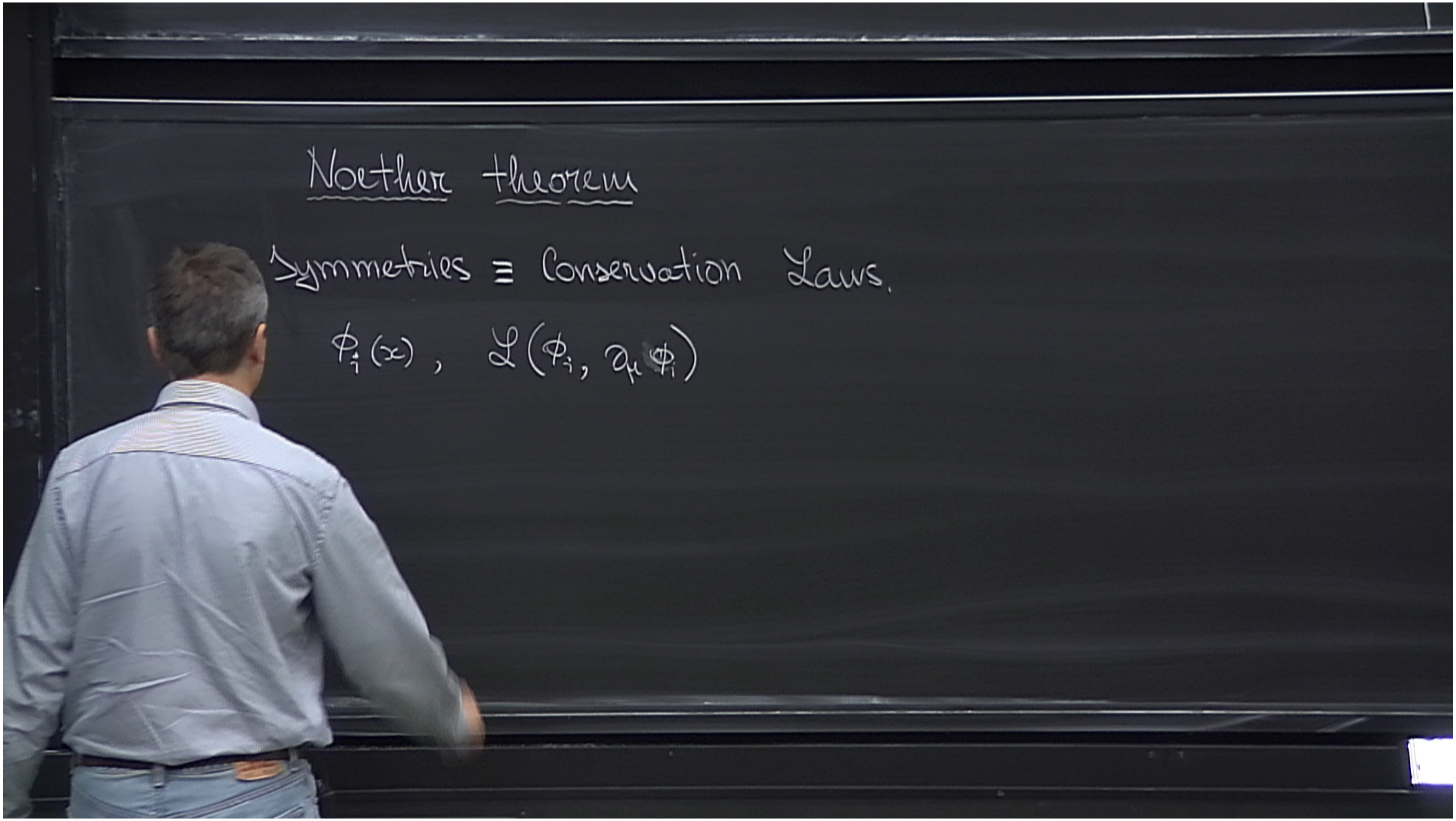
Δt : ΔV, $\Delta\vec{x}$

$$\rho = \frac{\Delta Q}{\Delta V} = \frac{\Delta Q}{\Delta V \cdot \Delta t}\cdot \Delta t$$

$$\vec{j} = \frac{\Delta Q}{\Delta V}\cdot\frac{\Delta\vec{x}}{\Delta t} = \left(\frac{\Delta Q}{\Delta V\cdot\Delta t}\right)\Delta\vec{x}$$

Lorentz-invariant


Δt : ΔV $\Delta\vec{x}$
$\rho = \frac{\Delta Q}{\Delta V} = \frac{\Delta Q}{\Delta V \cdot \Delta t} \cdot \Delta t$
$\vec{j} = \frac{\Delta Q}{\Delta V} \cdot \frac{\Delta \vec{x}}{\Delta t} = \left(\frac{\Delta Q}{\Delta V \cdot \Delta t}\right) \Delta \vec{x}$
Lorentz-invariant
$\Rightarrow (\rho, \vec{j})$ transform as $(\Delta t, \Delta\vec{x}) = \Delta x^\mu$

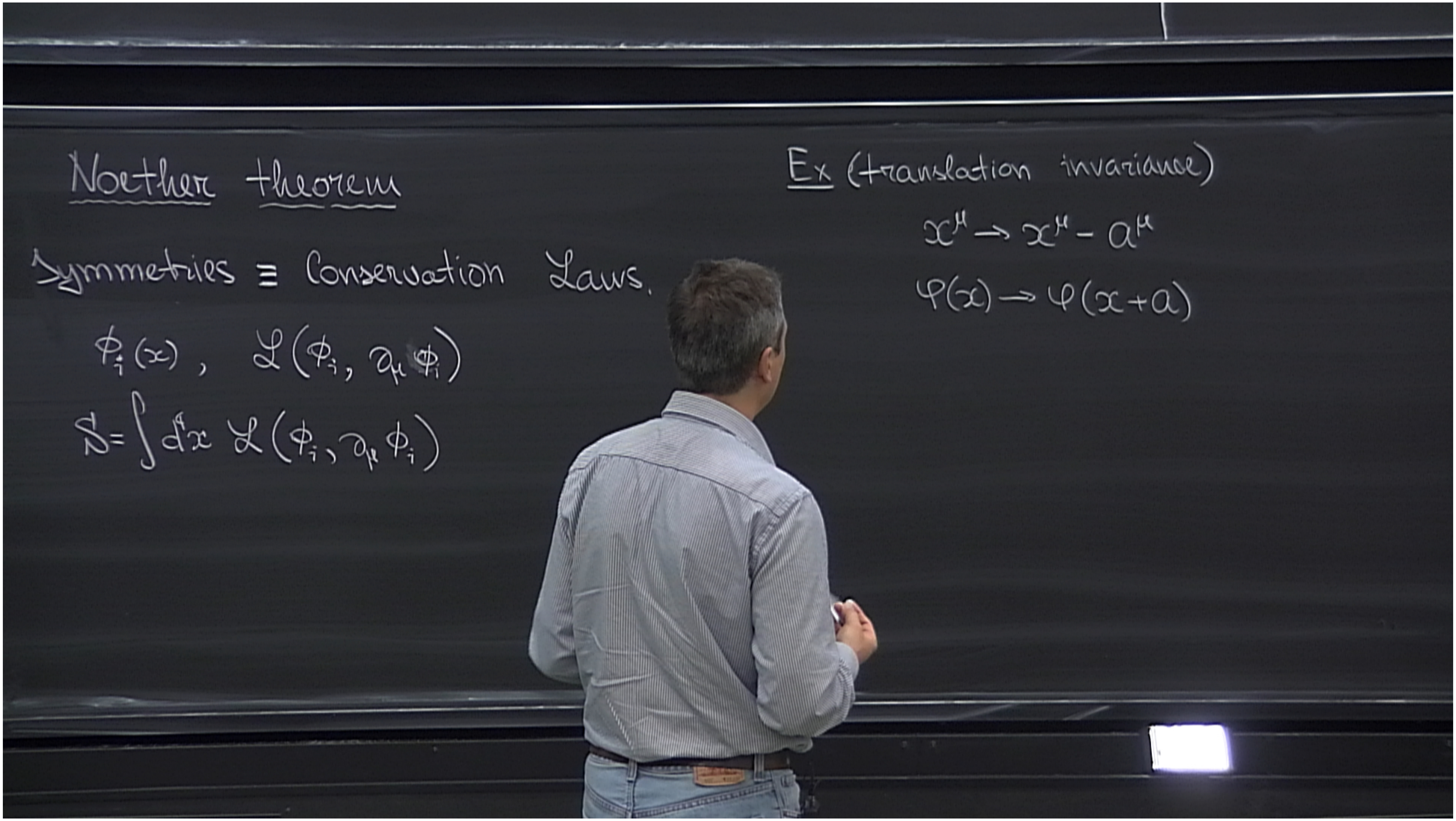

Noether theorem
Symmetries ≡ Conservation Laws.
$\phi_i(x)$, $\mathcal{L}(\phi_i, \partial_\mu \phi_i)$

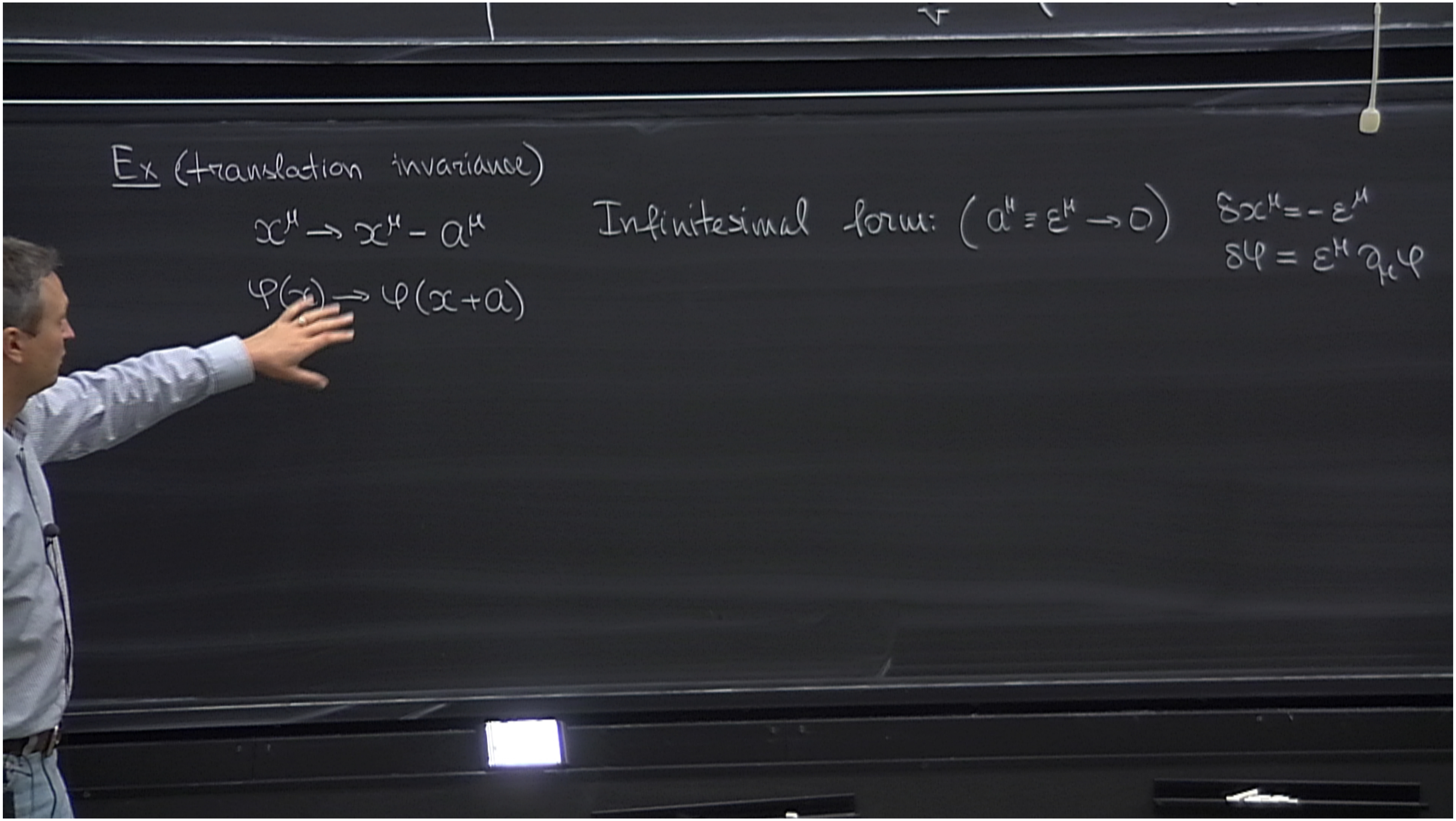
Noether theorem

Symmetries $\equiv$ Conservation Laws.

$\phi_i(x)$, $\mathcal{L}(\phi_i, \partial_\mu \phi_i)$

$S = \int d^4x \, \mathcal{L}(\phi_i, \partial_\mu \phi_i)$


Noether theorem
Symmetries ≡ Conservation Laws.
$\phi_i(x)$, $\mathcal{L}(\phi_i, \partial_\mu \phi_i)$
$S = \int d^4x \, \mathcal{L}(\phi_i, \partial_\mu \phi_i)$
Ex (translation invariance)
$x^\mu \to x^\mu - a^\mu$
$\varphi(x) \to \varphi(x+a)$


Ex (translation invariance)
$x^\mu \to x^\mu - a^\mu$
$\varphi(x) \to \varphi(x+a)$
Infinitesimal form: $(a^\mu \equiv \varepsilon^\mu \to 0)$
$\delta x^\mu = -\varepsilon^\mu$
$\delta\varphi = \varepsilon^\mu \partial_\mu \varphi$

Ex (translation invariance)

$$x^\mu \to x^\mu - a^\mu$$

$$\varphi(x) \to \varphi(x+a)$$

Infinitesimal form: $(a^\mu \equiv \varepsilon^\mu \to 0)$

$$\delta x^\mu = -\varepsilon^\mu$$

$$\delta\varphi = \varepsilon^\mu \partial_\mu \varphi$$

Ex (translation invariance)

$$x^\mu \to x^\mu - a^\mu$$

$$\varphi(x) \to \varphi(x+a)$$

Infinitesimal form: $(a^\mu \equiv \varepsilon^\mu \to 0)$

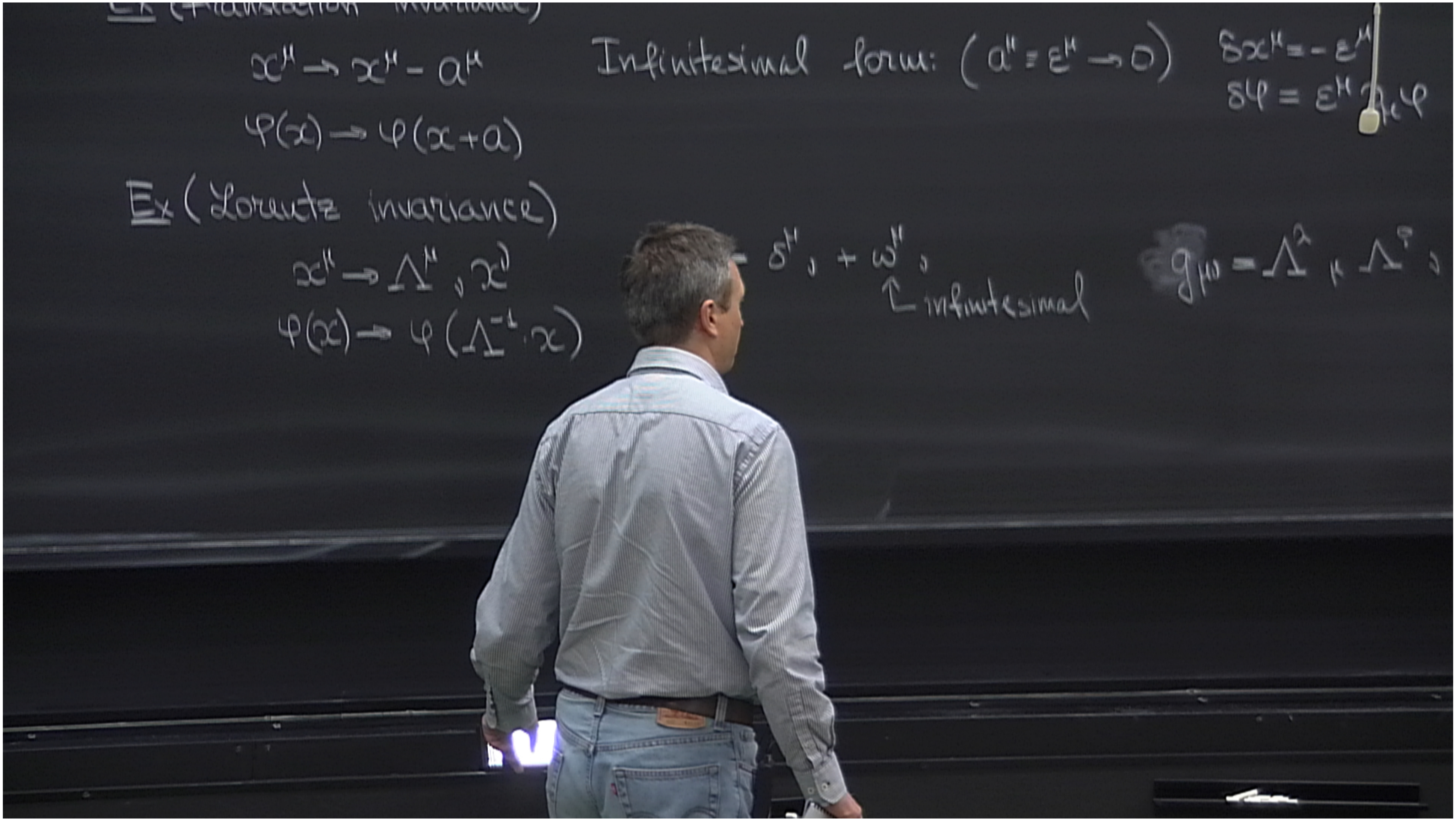
$$\delta x^\mu = -\varepsilon^\mu$$

$$\delta\varphi = \varepsilon^\mu \partial_\mu \varphi$$

Ex (Lorentz invariance)

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu$$

$$\varphi(x) \to \varphi(\Lambda^{-1}\cdot x)$$


$x^\mu \to x^\mu - a^\mu$
$\varphi(x) \to \varphi(x+a)$
Infinitesimal form: $(a^\mu = \varepsilon^\mu \to 0)$
$\delta x^\mu = -\varepsilon^\mu$
$\delta\varphi = \varepsilon^\mu \partial_\mu \varphi$
Ex (Lorentz invariance)
$x^\mu \to \Lambda^\mu{}_\nu x^\nu$
$\varphi(x) \to \varphi(\Lambda^{-1} x)$
$\delta^\mu{}_\nu + \omega^\mu{}_\nu$
infinitesimal
$g_{\mu\nu} = \Lambda^\lambda{}_\mu \Lambda^\rho{}_\nu$

$\Downarrow$ Gauss' theorem

$$\int_V d^3x \left(\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} \right) = 0$$

$\vec{j}$

Ex (translation invariance)

$$x^\mu \to x^\mu - a^\mu$$

$$\varphi(x) \to \varphi(x+a)$$

Infinitesimal form: $(a^\mu \equiv \varepsilon^\mu \to 0)$ $\quad \delta x^\mu = -\varepsilon^\mu$

$$\delta\varphi = \varepsilon^\mu \partial_\mu \varphi$$

Ex (Lorentz invariance)

$$x^\mu \to \Lambda^\mu{}_\nu x^\nu$$

$$\varphi(x) \to \varphi(\Lambda^{-1} \cdot x)$$

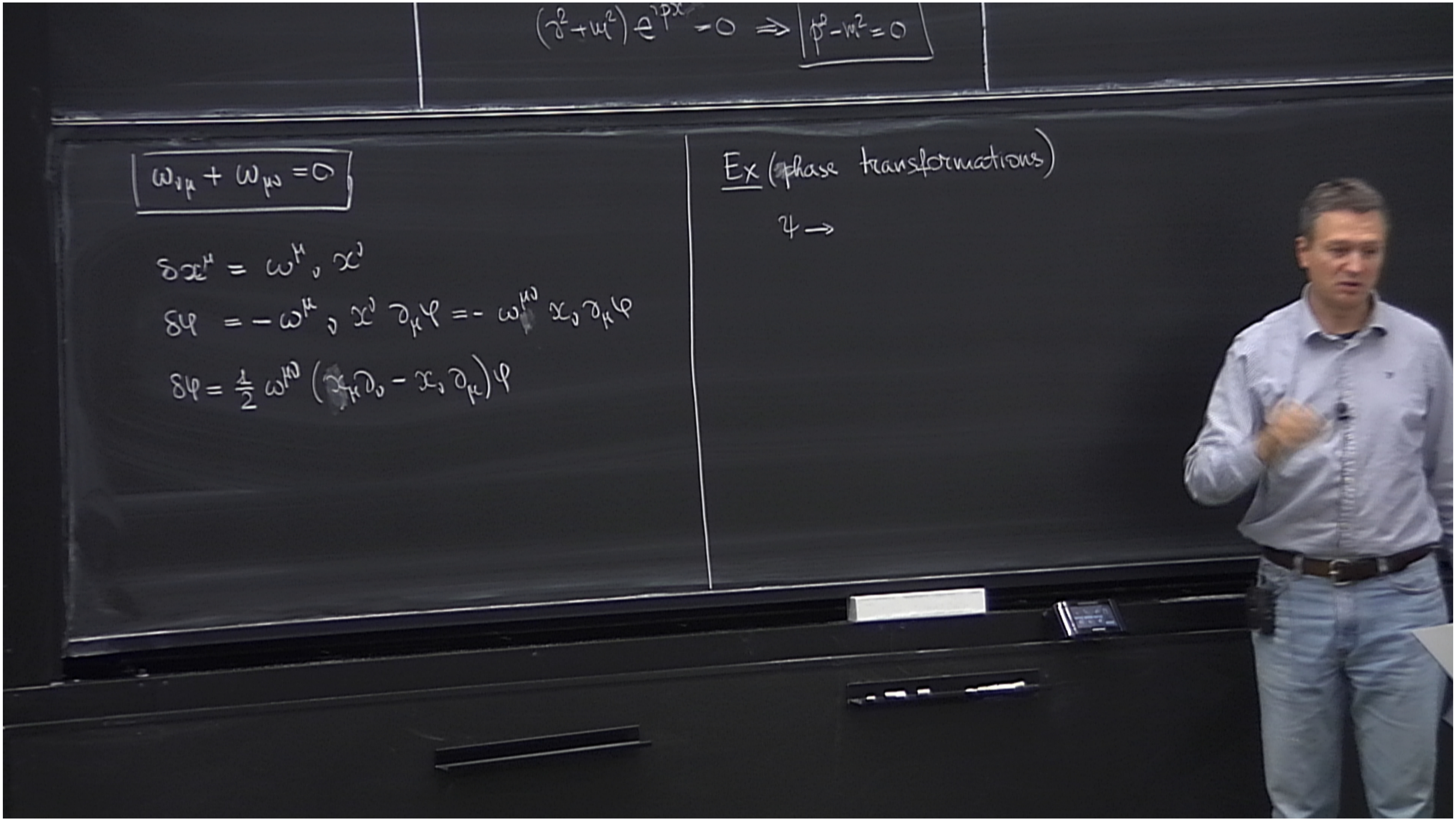
$$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu$$

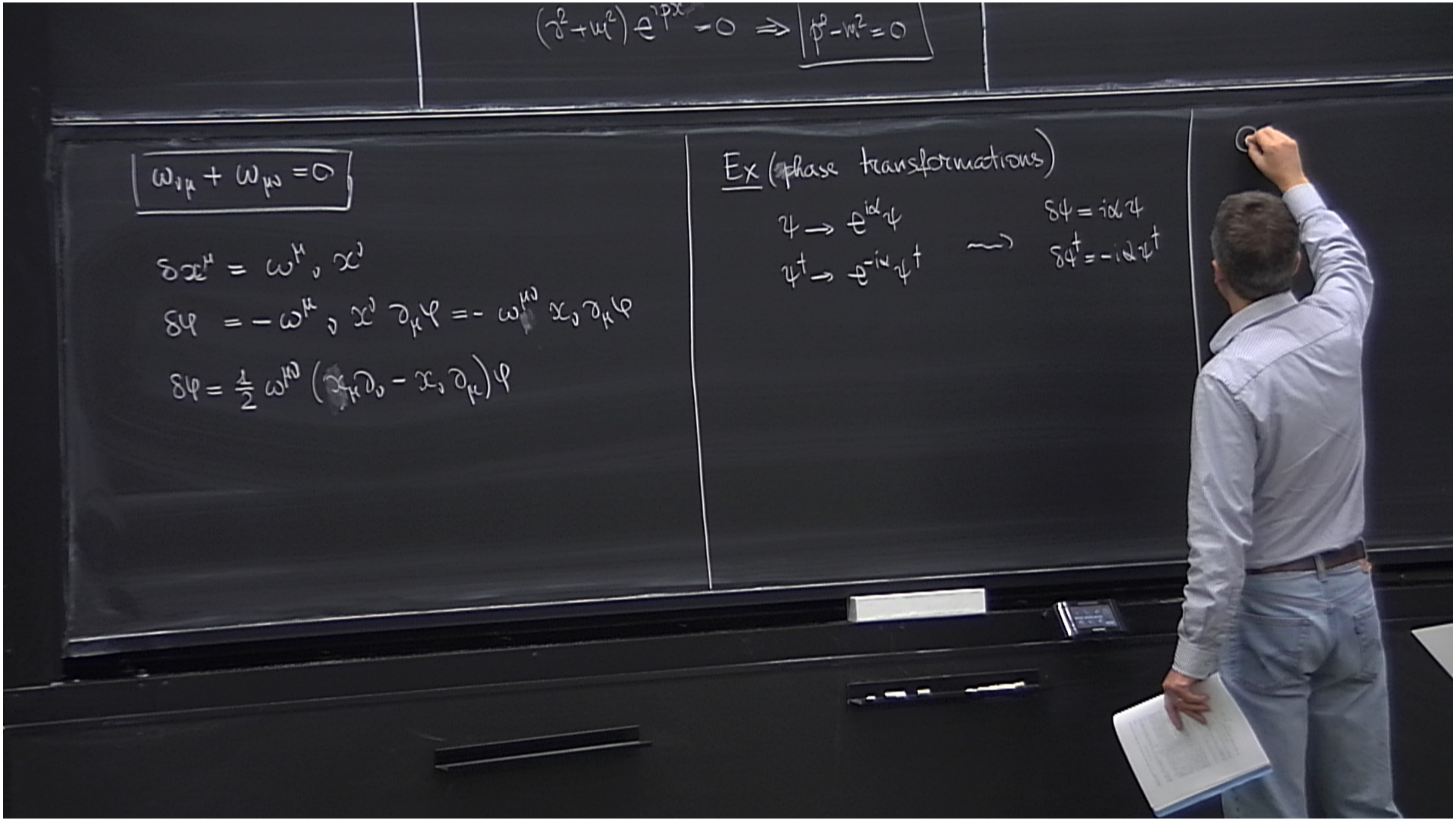
$\uparrow$ infinitesimal

$$g_{\mu\nu} = \Lambda^\lambda{}_\mu \Lambda^\rho{}_\nu g_{\lambda\rho}$$

$$g_{\mu\nu} = (\delta^\lambda{}_\mu + \omega^\lambda{}_\mu)(\delta^\rho{}_\nu + \omega^\rho{}_\nu) g_{\lambda\rho}$$

$$= g_{\mu\nu} + \underbrace{\omega^\lambda{}_\mu g_{\lambda\nu} + \omega^\rho{}_\nu g_{\mu\rho}}_{=0} + O(\omega^2)$$


$(\partial^2+m^2)e^{ipx}=0 \Rightarrow \boxed{p^2-m^2=0}$
$\boxed{\omega_{\nu\mu}+\omega_{\mu\nu}=0}$
$\delta x^\mu = \omega^\mu{}_\nu x^\nu$
$\delta\varphi = -\omega^\mu{}_\nu x^\nu \partial_\mu\varphi = -\omega^{\mu\nu} x_\nu \partial_\mu \varphi$
$\delta\varphi = \frac{1}{2}\omega^{\mu\nu}(x_\mu\partial_\nu - x_\nu\partial_\mu)\varphi$
Ex (phase transformations)
$\psi \rightarrow$


$(\partial^2 + m^2) e^{ipx} = 0 \Rightarrow \boxed{p^2 - m^2 = 0}$
$\boxed{\omega_{\nu\mu} + \omega_{\mu\nu} = 0}$
$\delta x^\mu = \omega^\mu{}_\nu x^\nu$
$\delta\varphi = -\omega^\mu{}_\nu x^\nu \partial_\mu \varphi = -\omega^{\mu\nu} x_\nu \partial_\mu \varphi$
$\delta\varphi = \frac{1}{2}\omega^{\mu\nu}(x_\mu\partial_\nu - x_\nu\partial_\mu)\varphi$
Ex (phase transformations)
$\psi \to e^{i\alpha}\psi$
$\psi^\dagger \to e^{-i\alpha}\psi^\dagger$
$\delta\psi = i\alpha\psi$
$\delta\psi^\dagger = -i\alpha\psi^\dagger$

-wave solutions:

$$(\partial^2+m^2)e^{ipx}=0 \Rightarrow \boxed{p^2-m^2=0}$$

$x_\nu \partial_\mu \varphi$

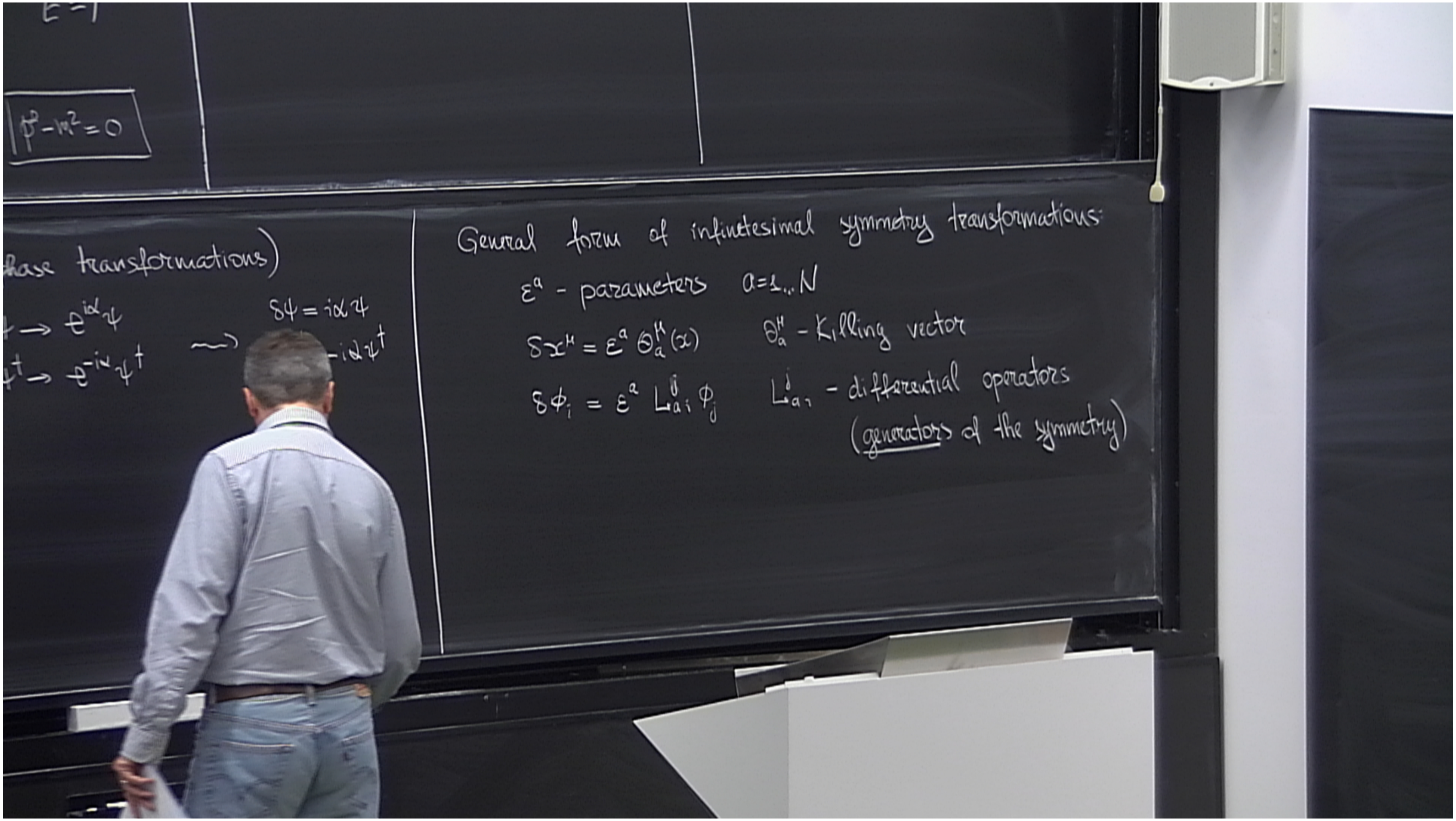
<u>Ex</u> (phase transformations)

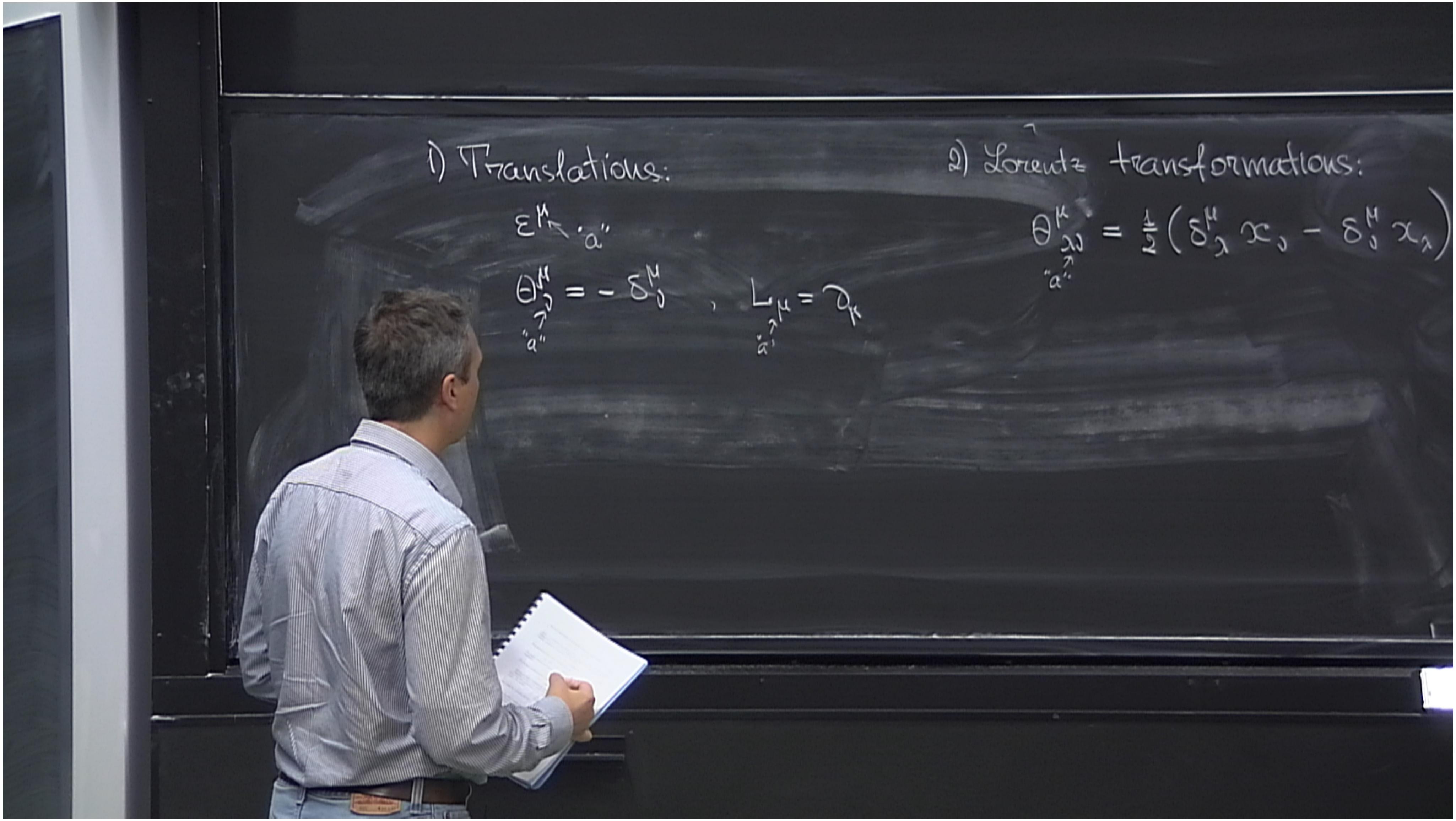
$$\psi \to e^{i\alpha}\psi \qquad \delta\psi = i\alpha\psi$$
$$\psi^\dagger \to e^{-i\alpha}\psi^\dagger \quad \rightsquigarrow \quad \delta\psi^\dagger = -i\alpha\psi^\dagger$$

General form of infinetesimal symmetry transformations:

ε^a - parameters $a=1,\dots,N$

$\delta x^\mu = \varepsilon^a \Theta_a^\mu(x)$ $\quad \Theta_a^\mu$ - Killing vector


$\boxed{p^2 - m^2 = 0}$
hase transformations)
$\psi \to e^{i\alpha}\psi$
$\delta\psi = i\alpha\psi$
$\psi^\dagger \to e^{-i\alpha}\psi^\dagger$
$-i\alpha\psi^\dagger$
General form of infinetesimal symmetry transformations:
ε^a - parameters $a = 1,..N$
$\delta x^\mu = \varepsilon^a \Theta^\mu_a(x)$
Θ^μ_a - Killing vector
$\delta\phi_i = \varepsilon^a L^j_{ai}\phi_j$
L^j_{ai} - differential operators
(generators of the symmetry)



1) Translations:
$\varepsilon^\mu \leftarrow$ "a"
$\Theta^\mu_\nu = - \delta^\mu_\nu$, $L_\mu = \partial_\mu$
"a"
"a"
2) Lorentz transformations:
$\Theta^\mu_{\lambda\nu} = \frac{1}{2}\left(\delta^\mu_\lambda x_\nu - \delta^\mu_\nu x_\lambda\right)$
"a"

1) Translations:

$$\varepsilon^\mu \leftarrow \text{"a"}$$

$$\underset{\text{"a"}}{\Theta^\mu_{\ \nu}} = -\delta^\mu_\nu \quad , \quad \underset{\text{"a"}}{L_\mu} = \partial_\mu$$

2) Lorentz transformations:

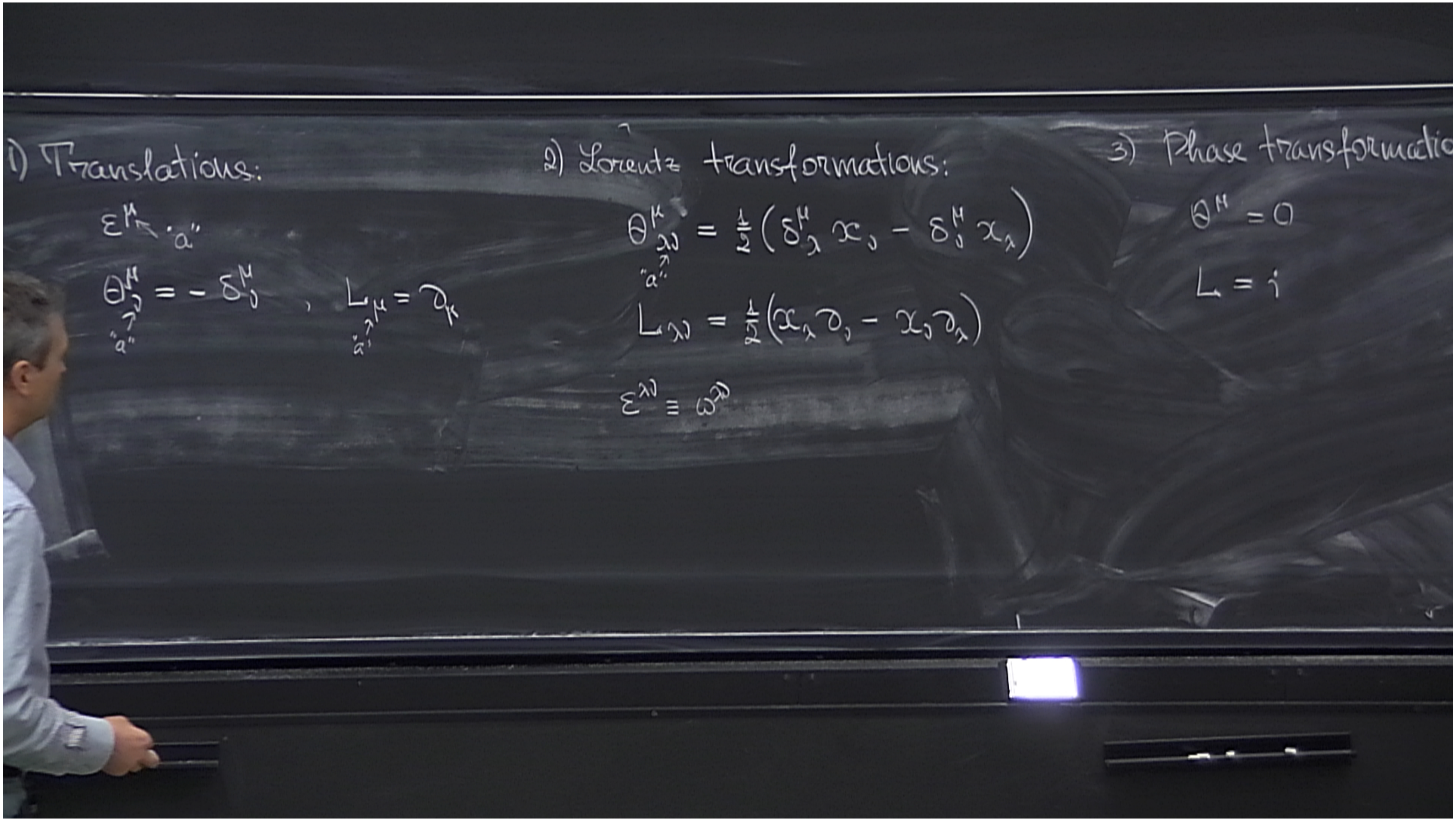
$$\underset{\text{"a"}}{\Theta^\mu_{\ \lambda\nu}} = \frac{1}{2}\left(\delta^\mu_\lambda x_\nu - \delta^\mu_\nu x_\lambda\right)$$

$$L_{\lambda\nu} = \frac{1}{2}\left(x_\lambda \partial_\nu - x_\nu \partial_\lambda\right)$$

3) Phase transformatio

$$\Theta^\mu = 0$$

$$L = i$$


1) Translations:
$\varepsilon^{\mu} \leftarrow$ "a"
$\Theta^{\mu}_{\nu} = -\delta^{\mu}_{\nu}$, $L_{\mu} = \partial_{\mu}$
"a" "a"
2) Lorentz transformations:
$\Theta^{\mu}_{\lambda\nu} = \frac{1}{2}\left(\delta^{\mu}_{\lambda} x_{\nu} - \delta^{\mu}_{\nu} x_{\lambda}\right)$
"a"
$L_{\lambda\nu} = \frac{1}{2}\left(x_{\lambda}\partial_{\nu} - x_{\nu}\partial_{\lambda}\right)$
$\varepsilon^{\lambda\nu} \equiv \omega^{\lambda\nu}$
3) Phase transformatio
$\Theta^{\mu} = 0$
$L = i$