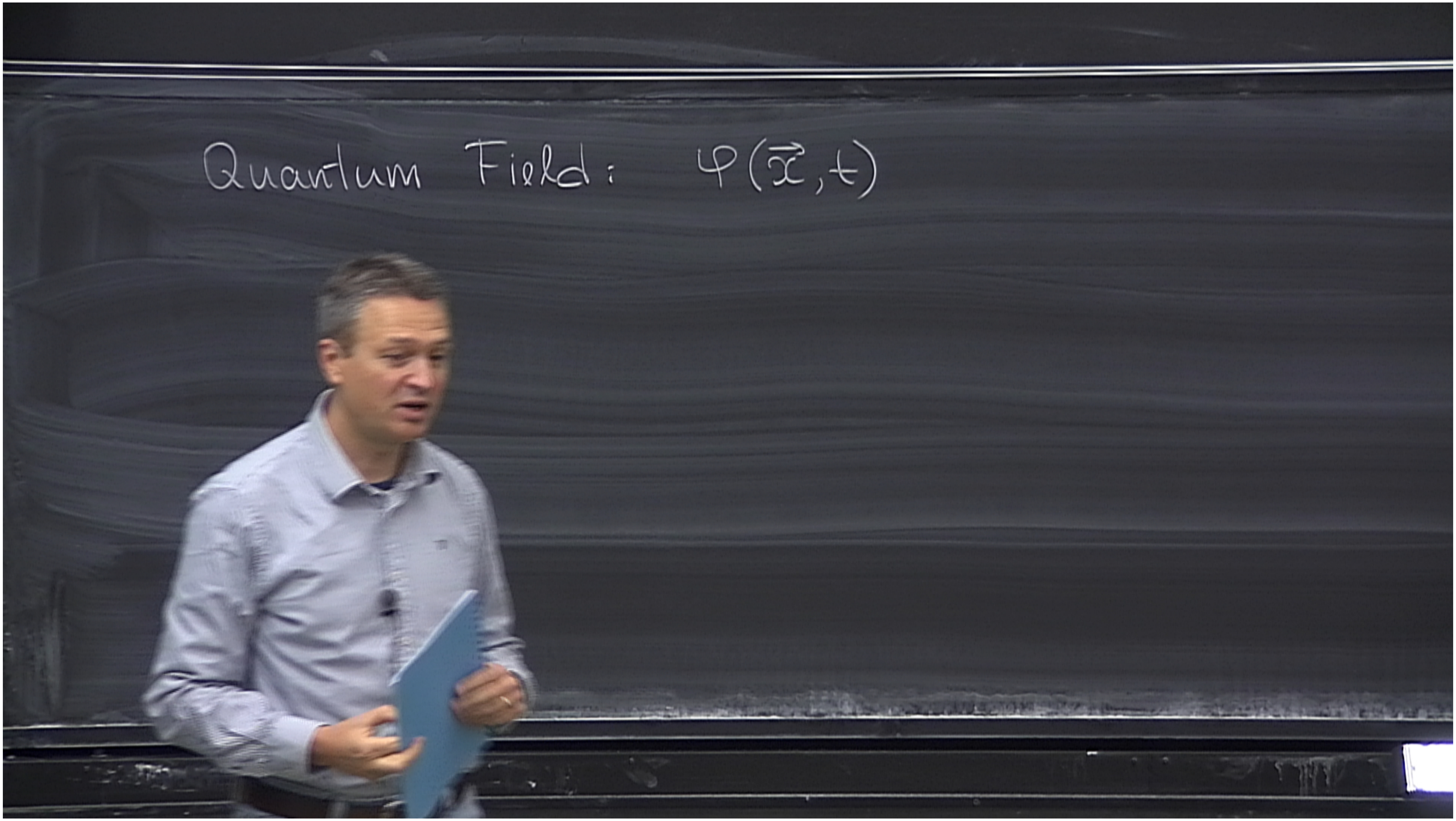
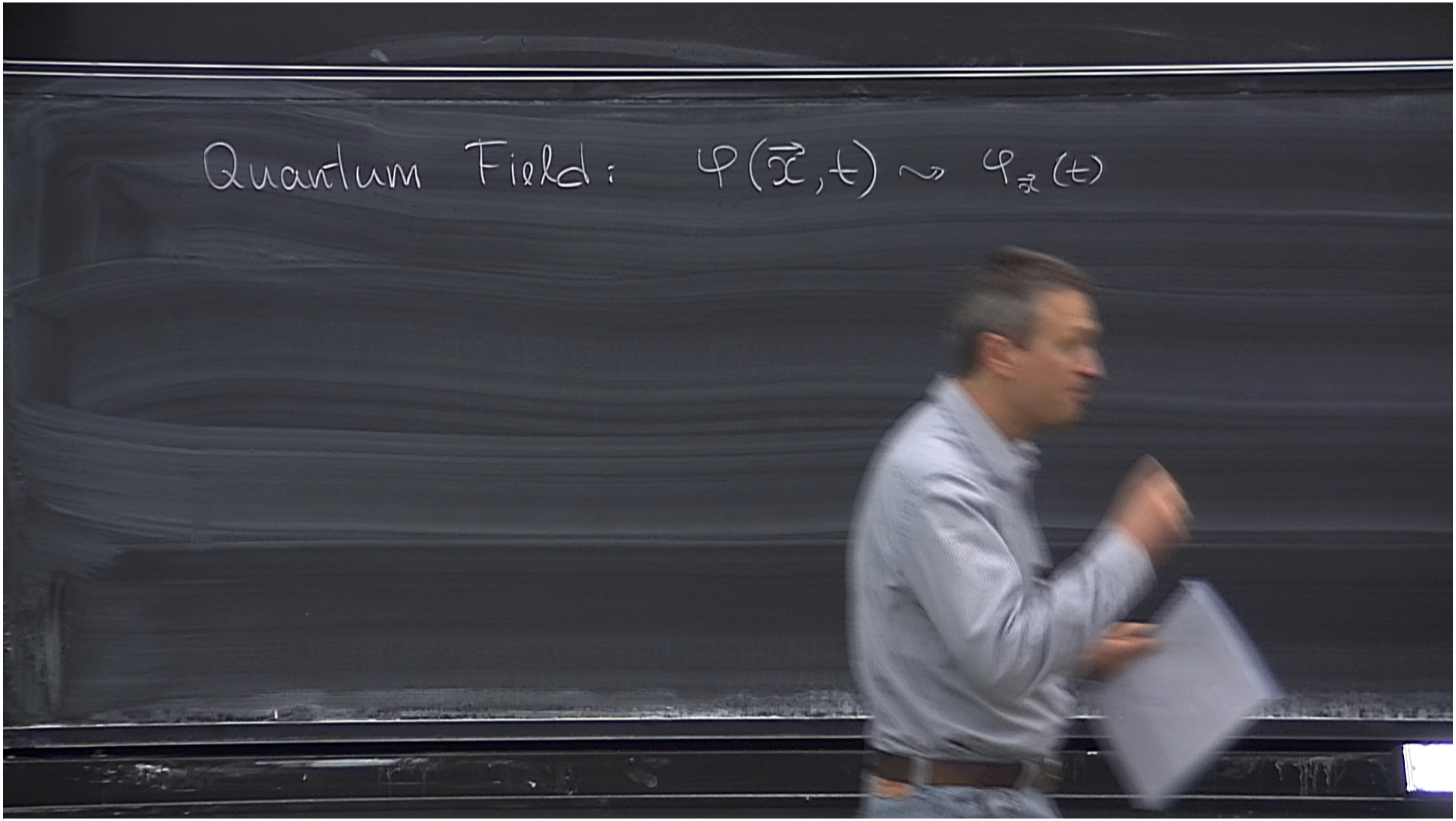
Title: Quantum Field Theory I - Lecture 1

Date: Oct 03, 2011 09:00 AM

URL: http://pirsa.org/11100006

Abstract:


Quantum Field: $\varphi(\vec{x}, t)$


Quantum Field: $\varphi(\vec{x},t) \rightsquigarrow \varphi_{\vec{x}}(t)$

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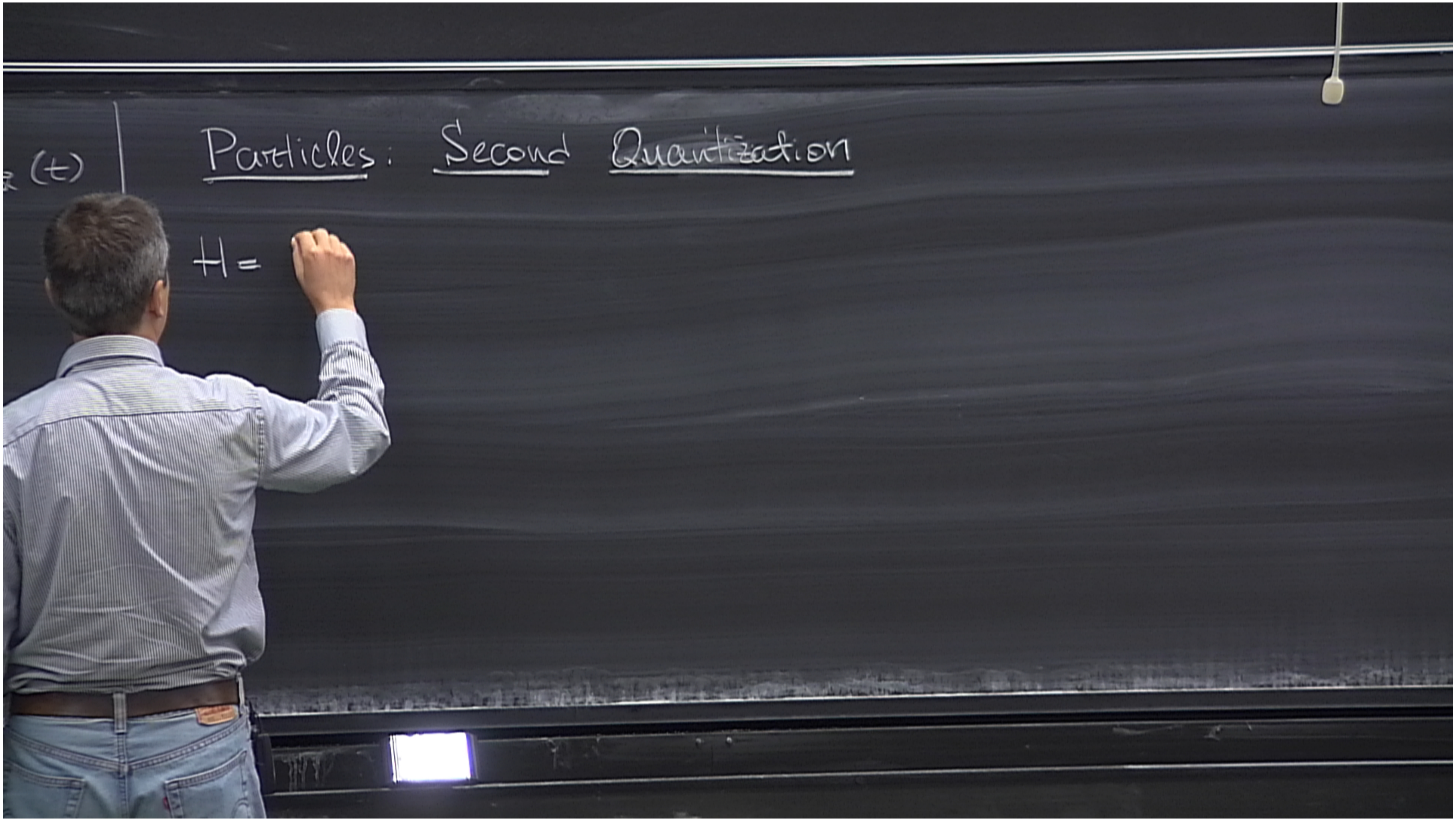
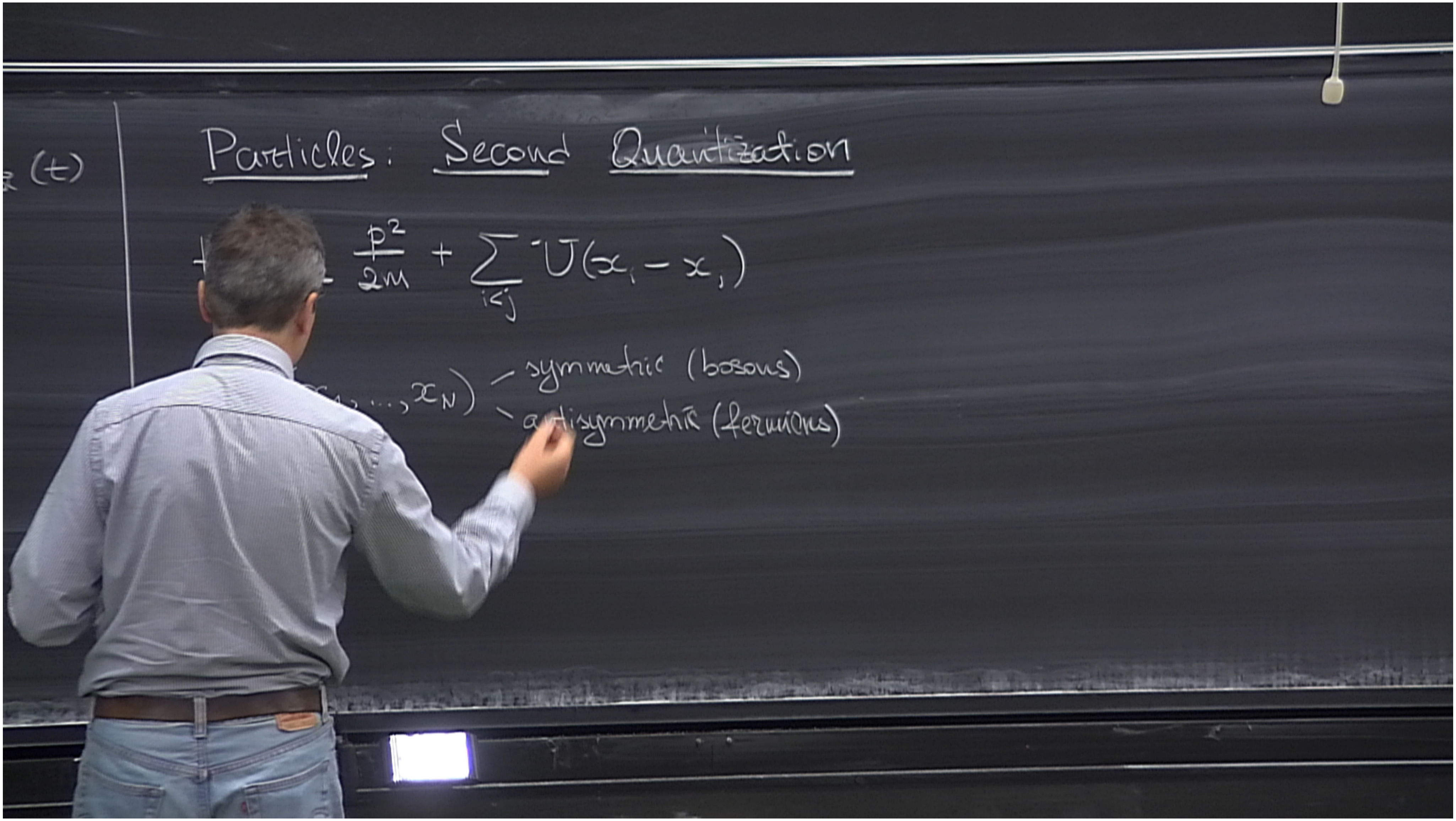
waves $\longleftrightarrow$ particles $\longleftrightarrow$ forces


(t)
Particles: Second Quantization
H =


Particles: Second Quantization
symmetric (bosons)
antisymmetric (fermions)

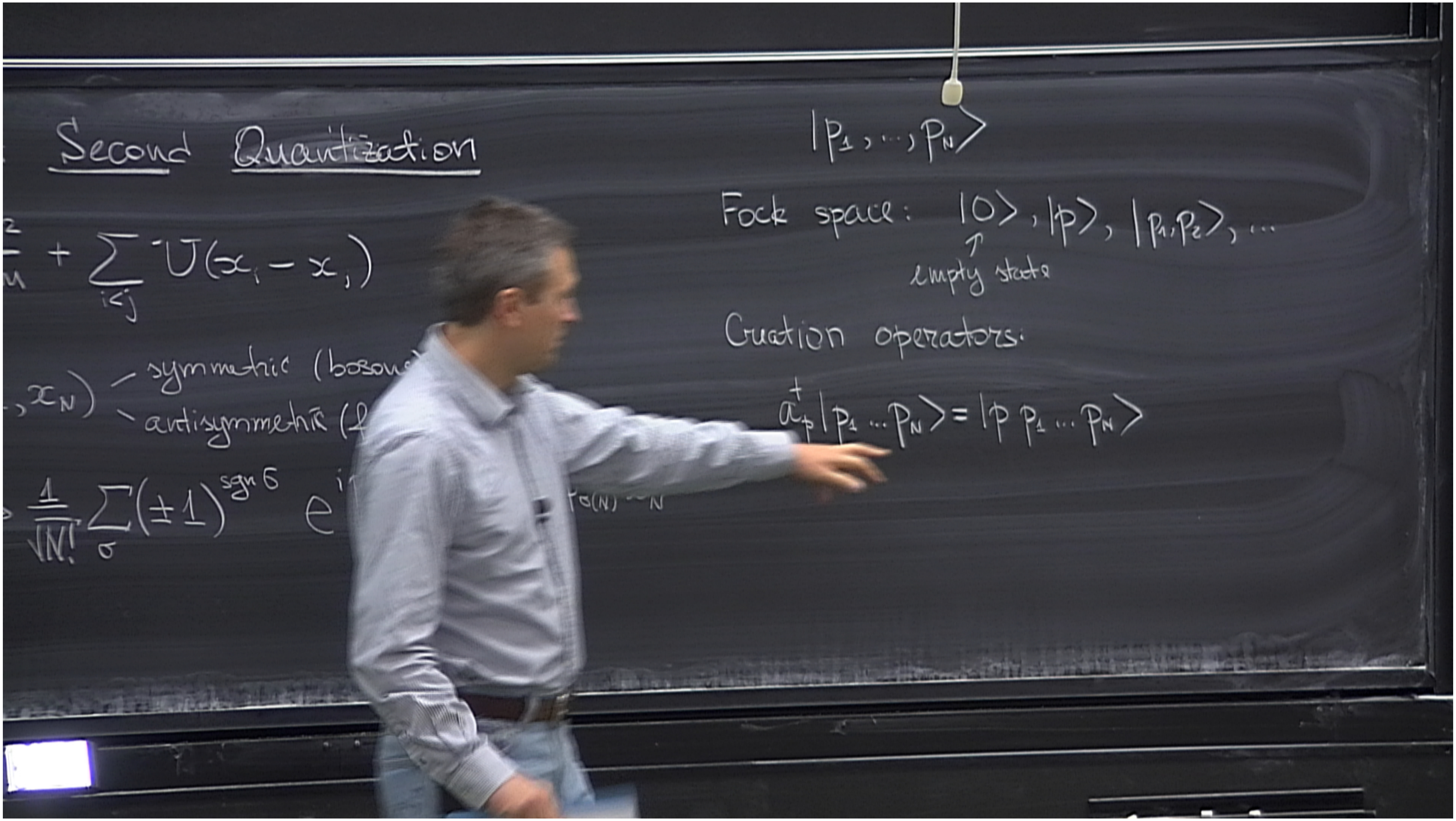
(t)

Particles: Second Quantization

$$H = \sum_i \frac{p^2}{2m} + \sum_{i<j} U(x_i - x_j)$$

$\Psi(x_1, \ldots, x_N)$ — symmetric (bosons) / antisymmetric (fermions)

$$U = 0 \;\longrightarrow\; \frac{1}{\sqrt{N!}} \sum_\sigma (\pm 1)^{\operatorname{sgn}\sigma}\, e^{i p_{\sigma(1)} x_1/\hbar + \ldots + i p_{\sigma(N)} x_N}$$


Second Quantization
$|p_1, \dots, p_N\rangle$
Fock space: $|0\rangle, |p\rangle, |p_1, p_2\rangle, \dots$
empty state
Creation operators:
$a_p^\dagger |p_1 \dots p_N\rangle = |p\, p_1 \dots p_N\rangle$
$+ \sum_{i<j} U(x_i - x_j)$
symmetric (bosons
antisymmetric (f
$\frac{1}{\sqrt{N!}} \sum_\sigma (\pm 1)^{\mathrm{sgn}\,\sigma}$

Second Quantization

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$$|p_1, \ldots, p_N\rangle$$

Fock space: $|0\rangle, |p\rangle, |p_1, p_2\rangle, \ldots$

empty state

Creation operators:

$$a_p^\dagger |p_1 \ldots p_N\rangle = |p\, p_1 \ldots p_N\rangle$$

$$|p_1 \ldots p_N\rangle = a_{p_1}^\dagger \ldots a_{p_N}^\dagger |0\rangle$$

$$a_p \, |p_1 \dots p_N\rangle = (2\pi\hbar)^3 \sum_{k=1}^{N} \delta(p - p_k)\,(\pm 1)^k \, |p_1 \dots \hat{p}_k \dots p_N\rangle$$

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$$[a_p, a_q^\dagger] = (2\pi\hbar)^3 \, \delta(p-q) \quad \text{(bosons)}$$

$$\{a_p, a_q^\dagger\} = (2\pi\hbar)^3 \, \delta(p-q) \quad \text{(fermions)}$$

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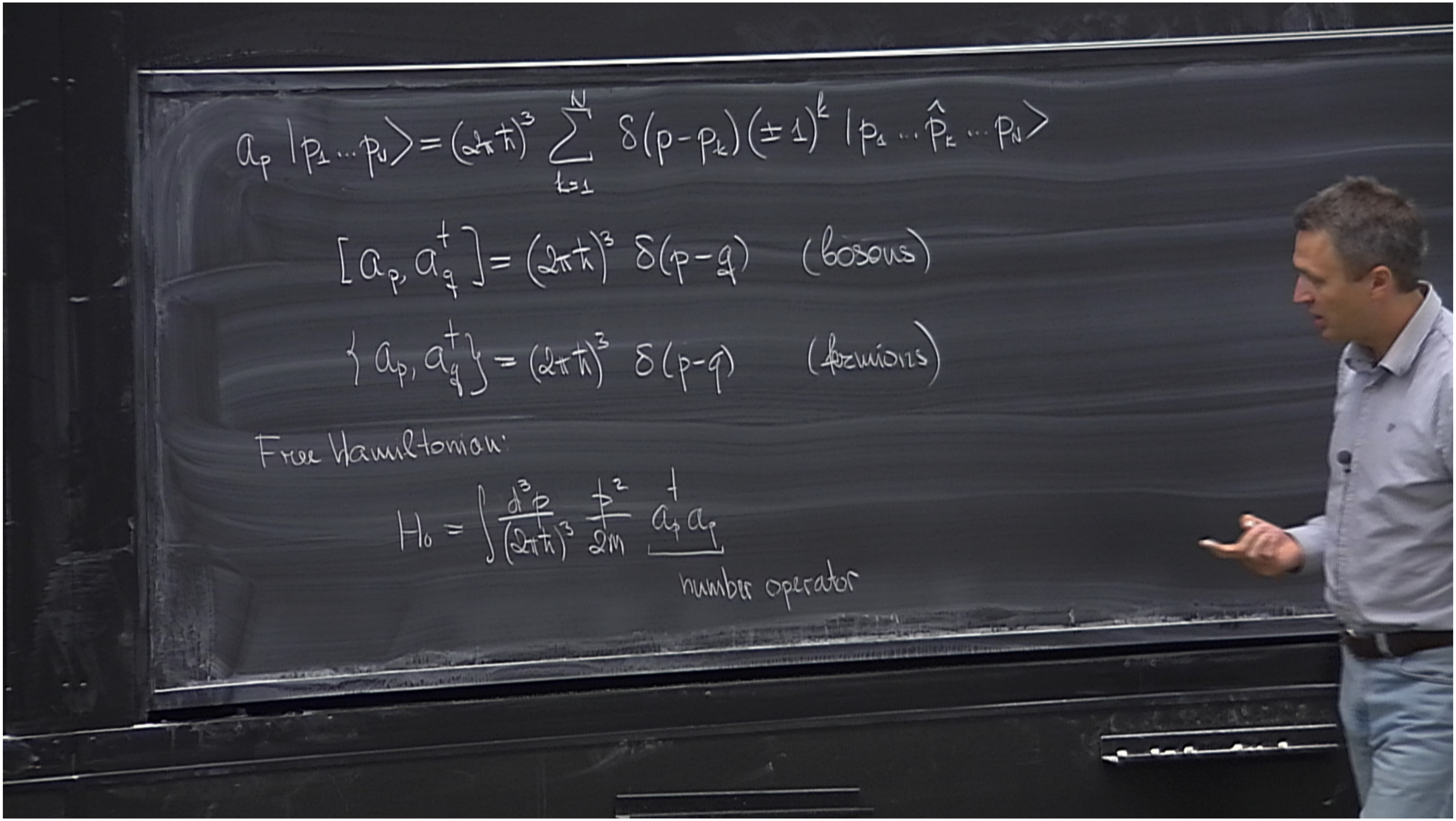
Free Hamiltonian:

$$H_0 = \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p^2}{2m} a_p^\dagger a_p$$


$a_p |p_1 \dots p_N\rangle = (2\pi\hbar)^3 \sum_{k=1}^{N} \delta(p-p_k) (\pm 1)^k |p_1 \dots \hat{p}_k \dots p_N\rangle$
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Free Hamiltonian:
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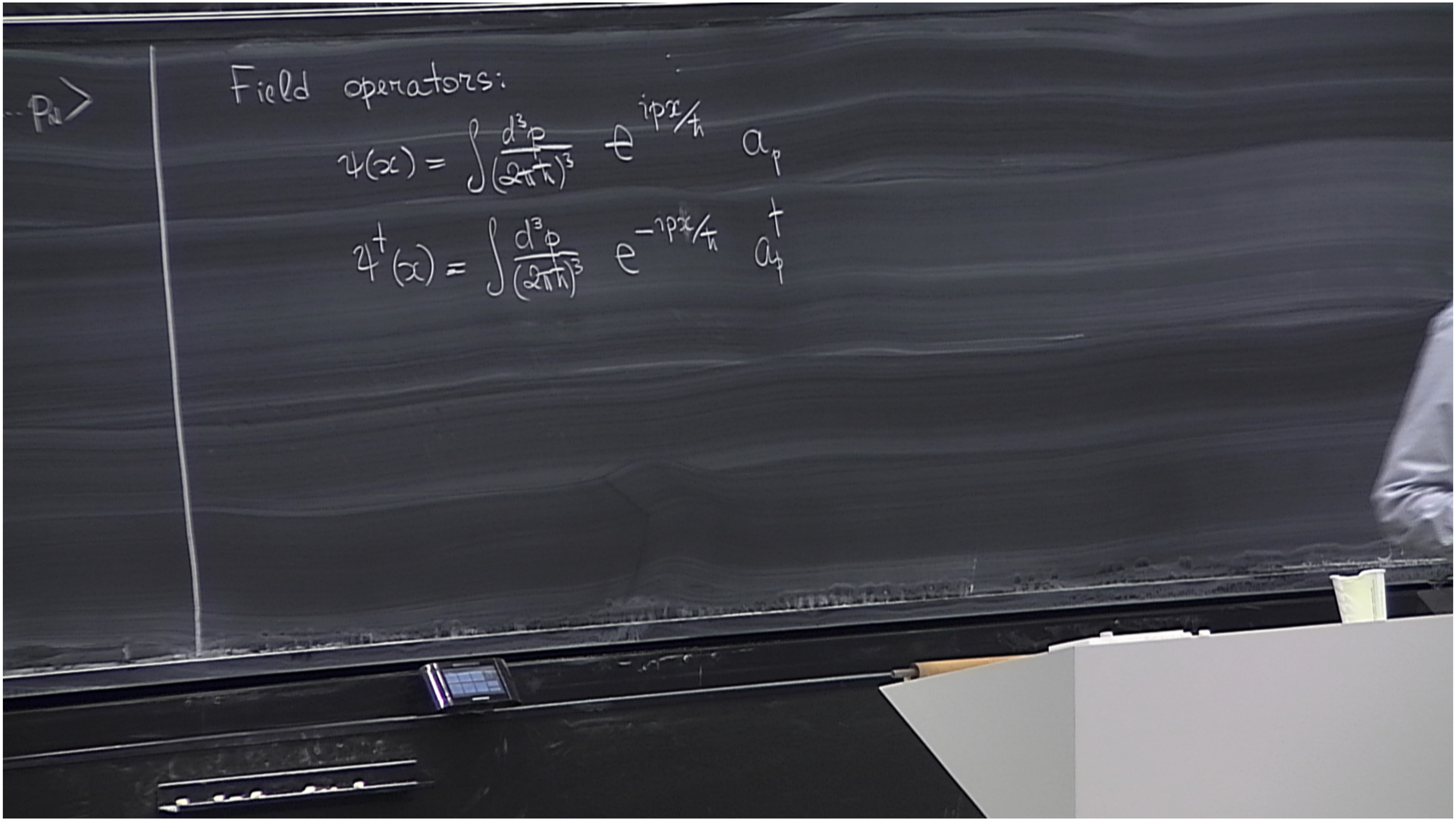
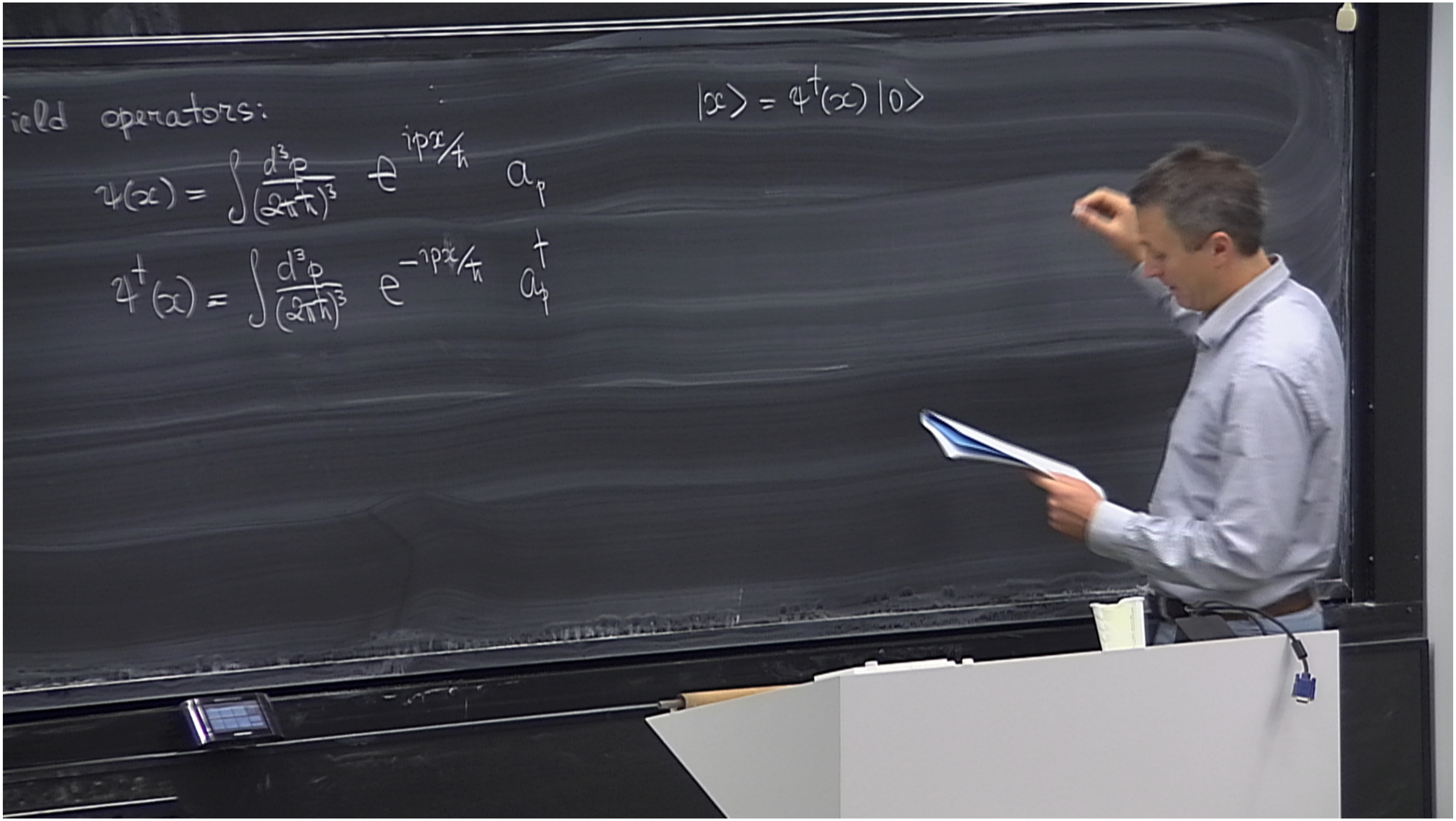


$\ldots p_N \rangle$
Field operators:
$\psi(x) = \int \frac{d^3p}{(2\pi\hbar)^3} e^{ipx/\hbar} a_p$
$\psi^\dagger(x) = \int \frac{d^3p}{(2\pi\hbar)^3} e^{-ipx/\hbar} a_p^\dagger$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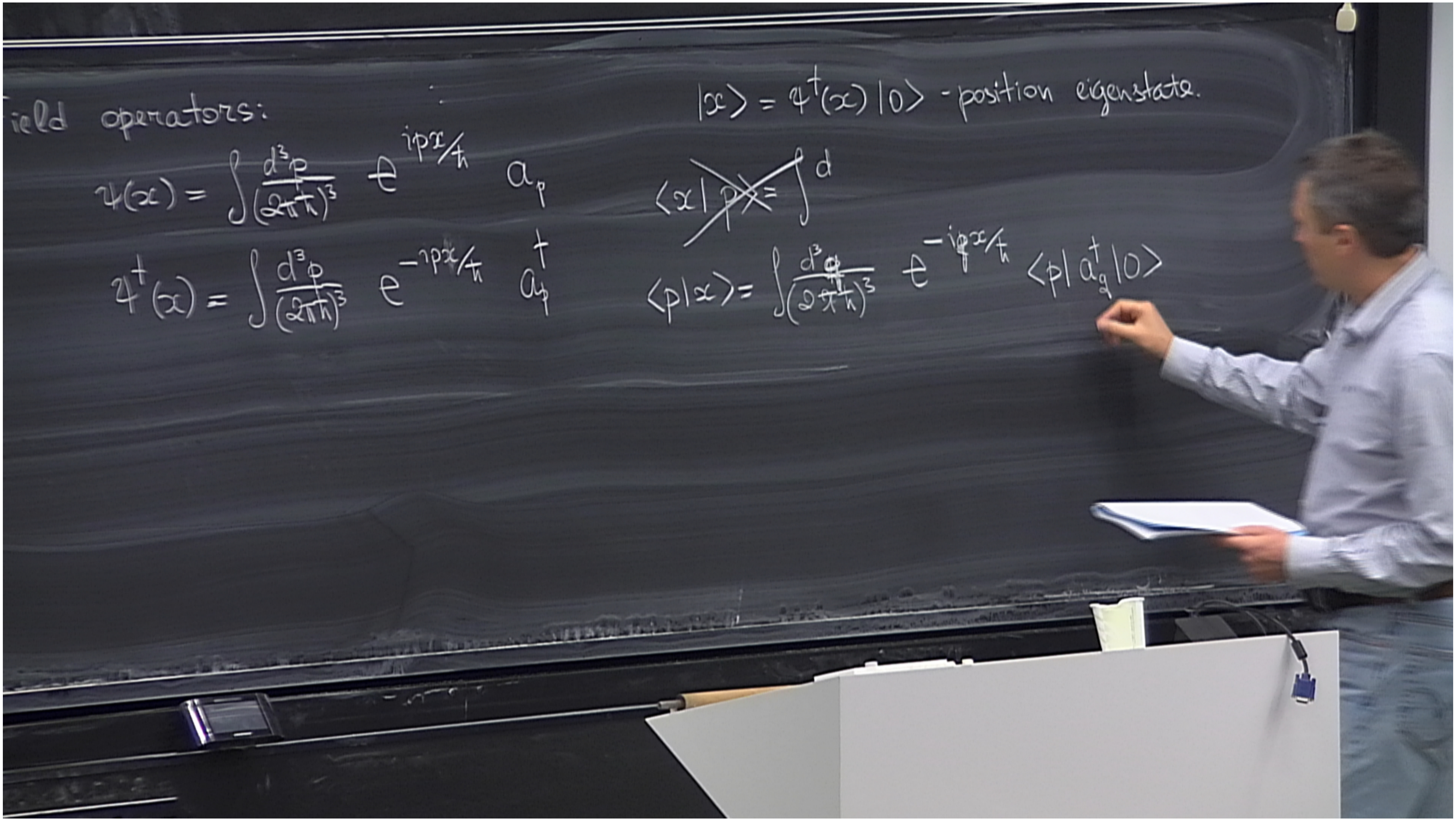
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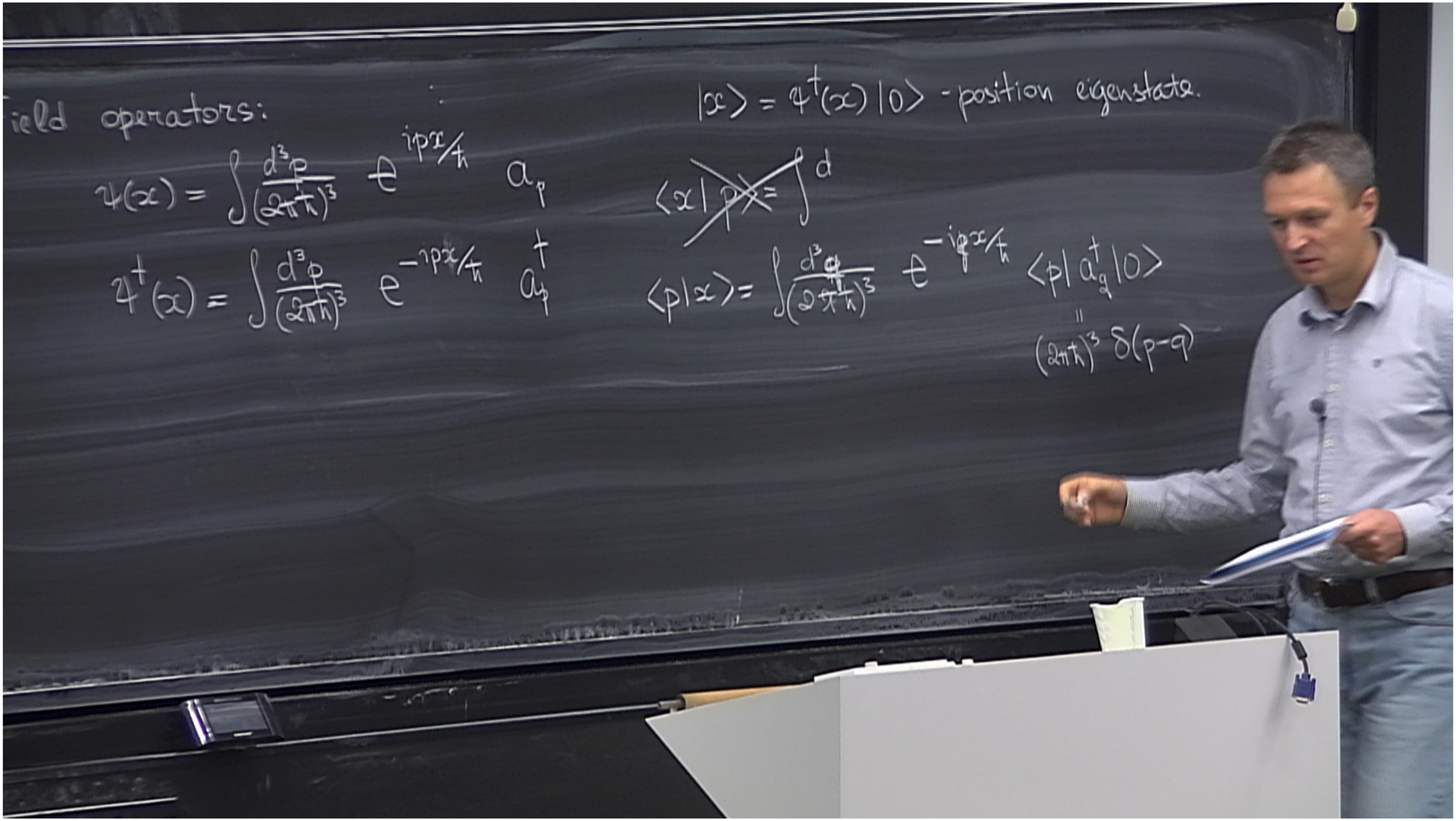
$|x\rangle = \psi^\dagger(x)|0\rangle$ - position eigenstate.
$\langle x|\cancel{p\rangle =} \int d$
$\langle p|x\rangle = \int \frac{d^3q}{(2\pi\hbar)^3} e^{-iqx/\hbar} \langle p|a_q^\dagger|0\rangle$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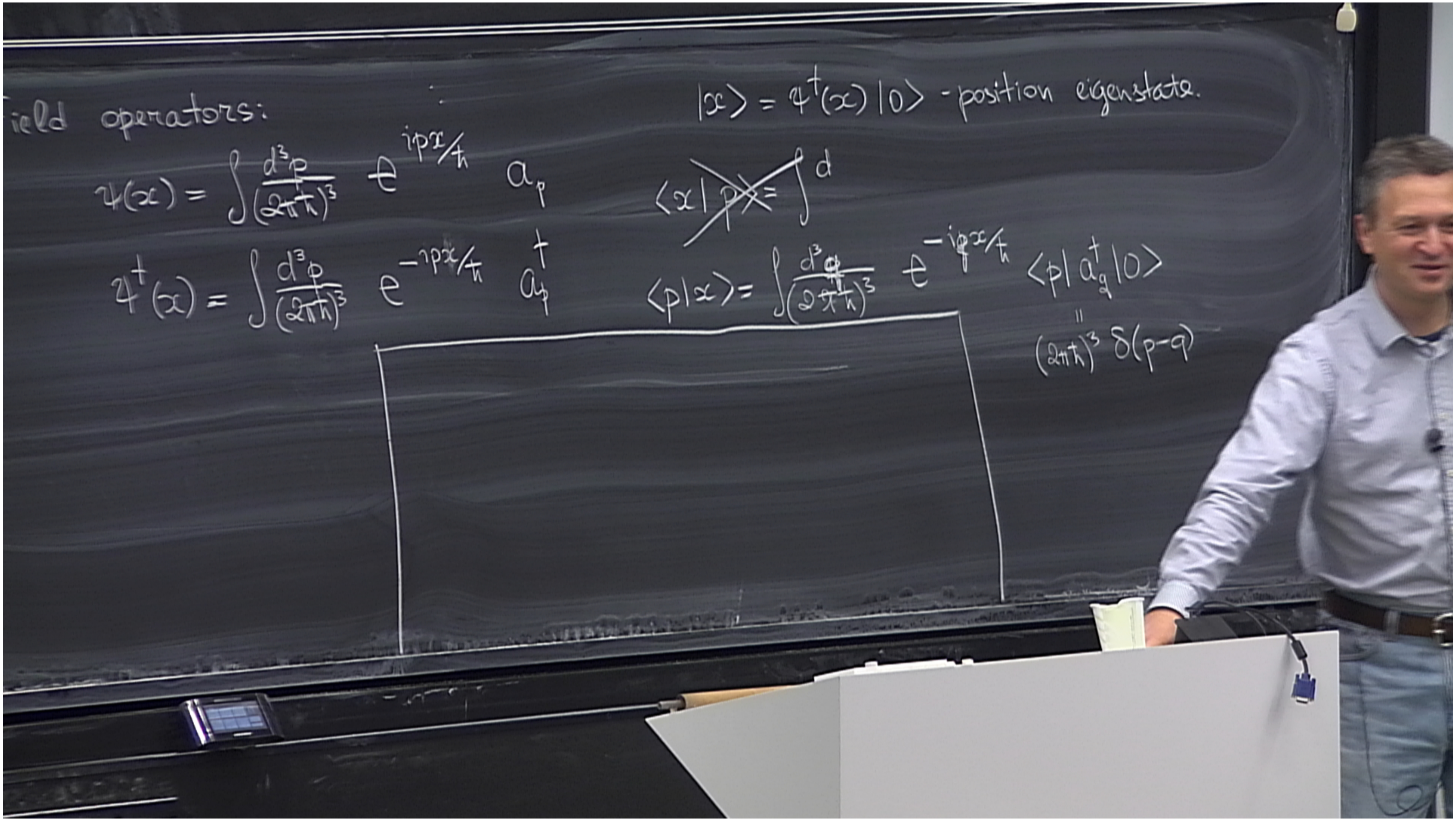
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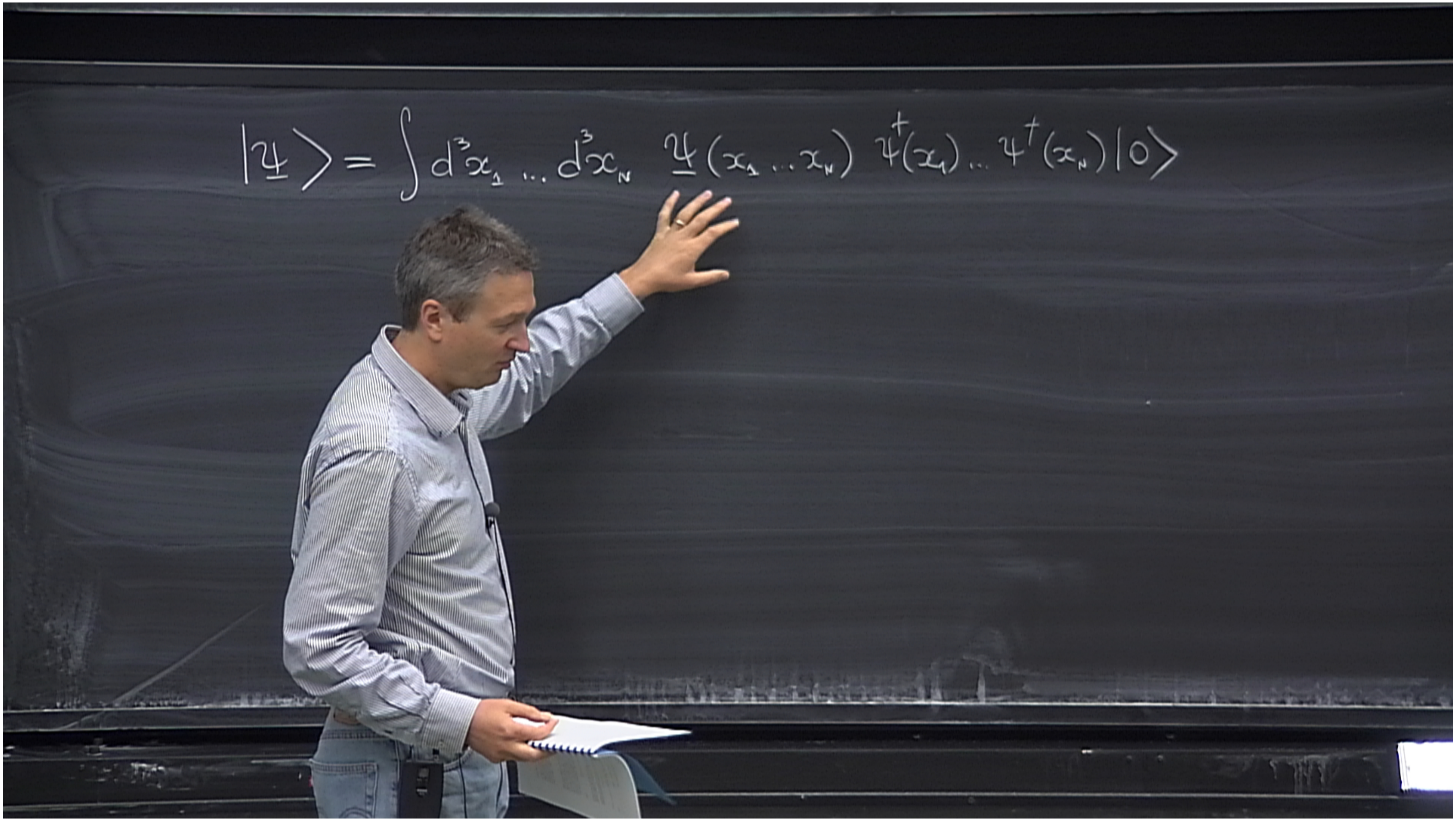
$(2\pi\hbar)^3\delta(p-q)$

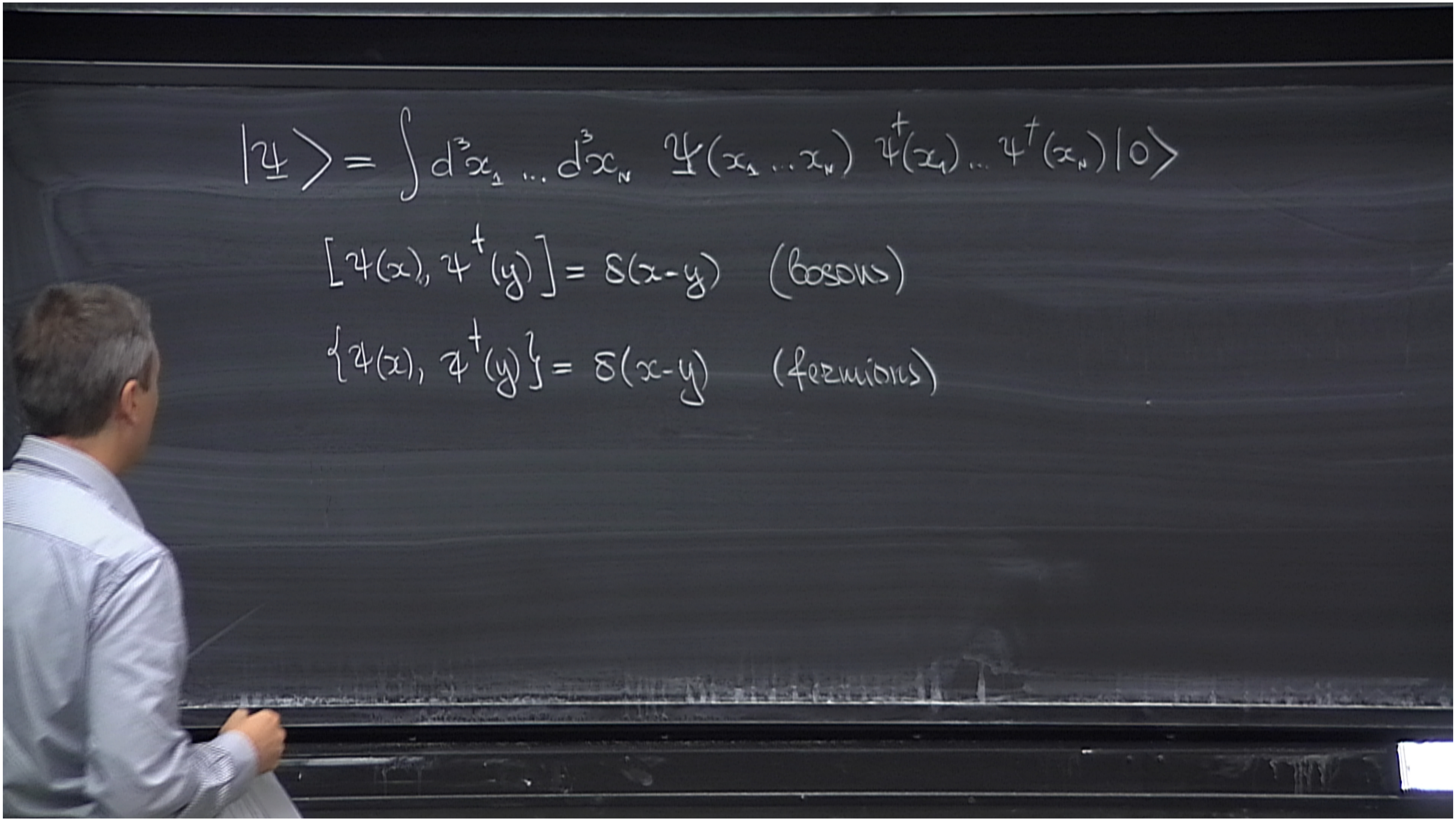
$$= \int \frac{d^3p}{(2\pi\hbar)^3}\, e^{-ipx/\hbar}\, a_p^\dagger$$

$$\langle p|x\rangle = \int \frac{d^3q}{(2\pi\hbar)^3}\, e^{-iqx/\hbar}\, \langle p|a_q^\dagger|0\rangle$$

$$\langle p|a_q^\dagger|0\rangle = (2\pi\hbar)^3\,\delta(p-q)$$

$$e^{-ipx/\hbar}$$


$|\underline{\Psi}\rangle = \int d^3x_1 \dots d^3x_N \; \underline{\Psi}(x_1 \dots x_N) \; \Psi^\dagger(x_1) \dots \Psi^\dagger(x_N) |0\rangle$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$[\psi(x), \psi^\dagger(y)] = \delta(x-y)$ (bosons)
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$\ldots_N)|0\rangle$

Density operator:

$$\rho(x) = \psi^\dagger(x)\,\psi(x)$$

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Interaction energy: $\rightsquigarrow \quad |\psi(x)|^2\, U(x-y)\, |\psi(y)|^2$

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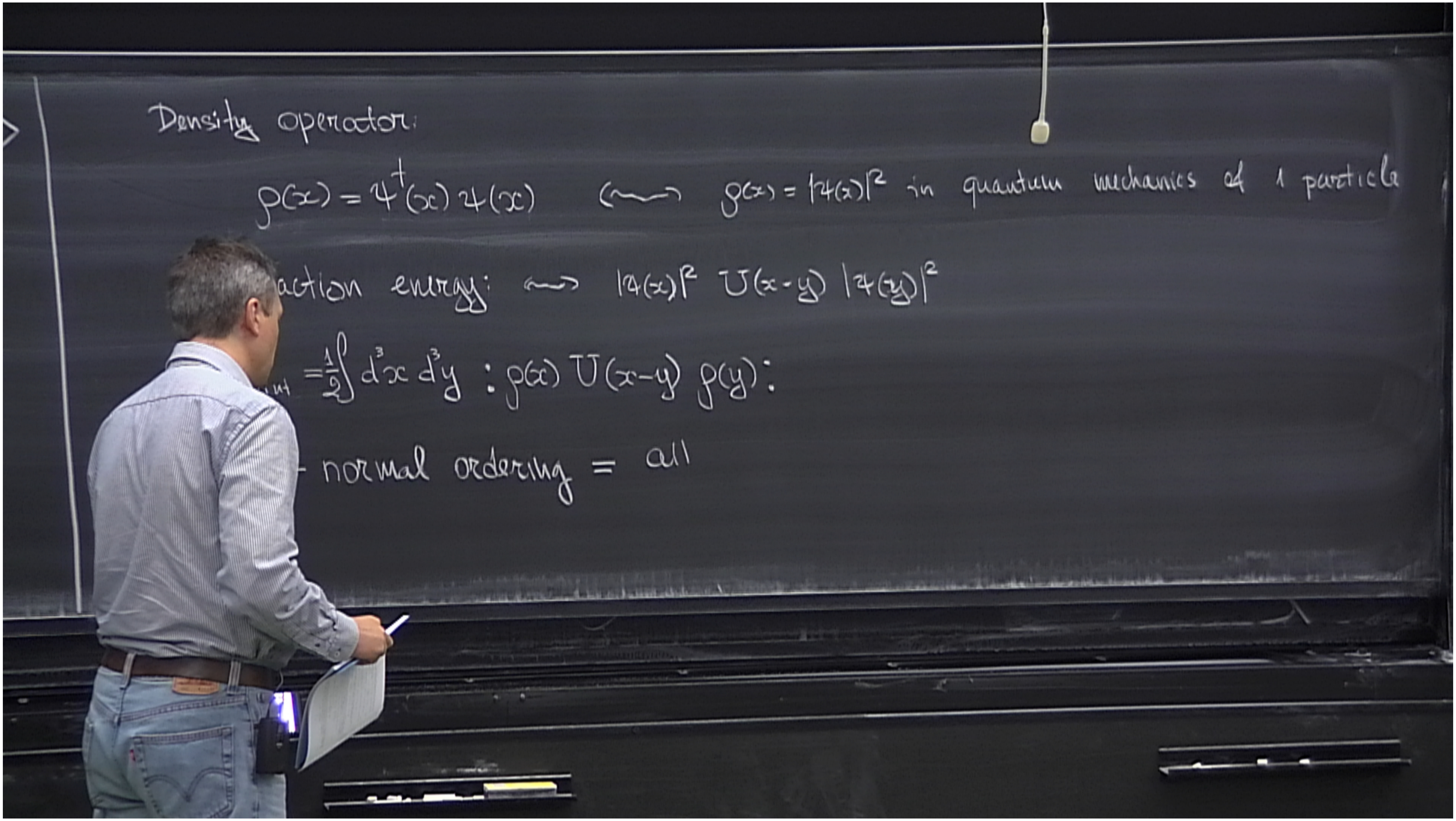
Interaction energy: $\rightsquigarrow \; |\psi(x)|^2 \, U(x-y) \, |\psi(y)|^2$

$$H_{int} = \frac{1}{2}\int d^3x \, d^3y \;\; \rho(x)\, U(x-y)\, \rho(y)$$


Density operator:
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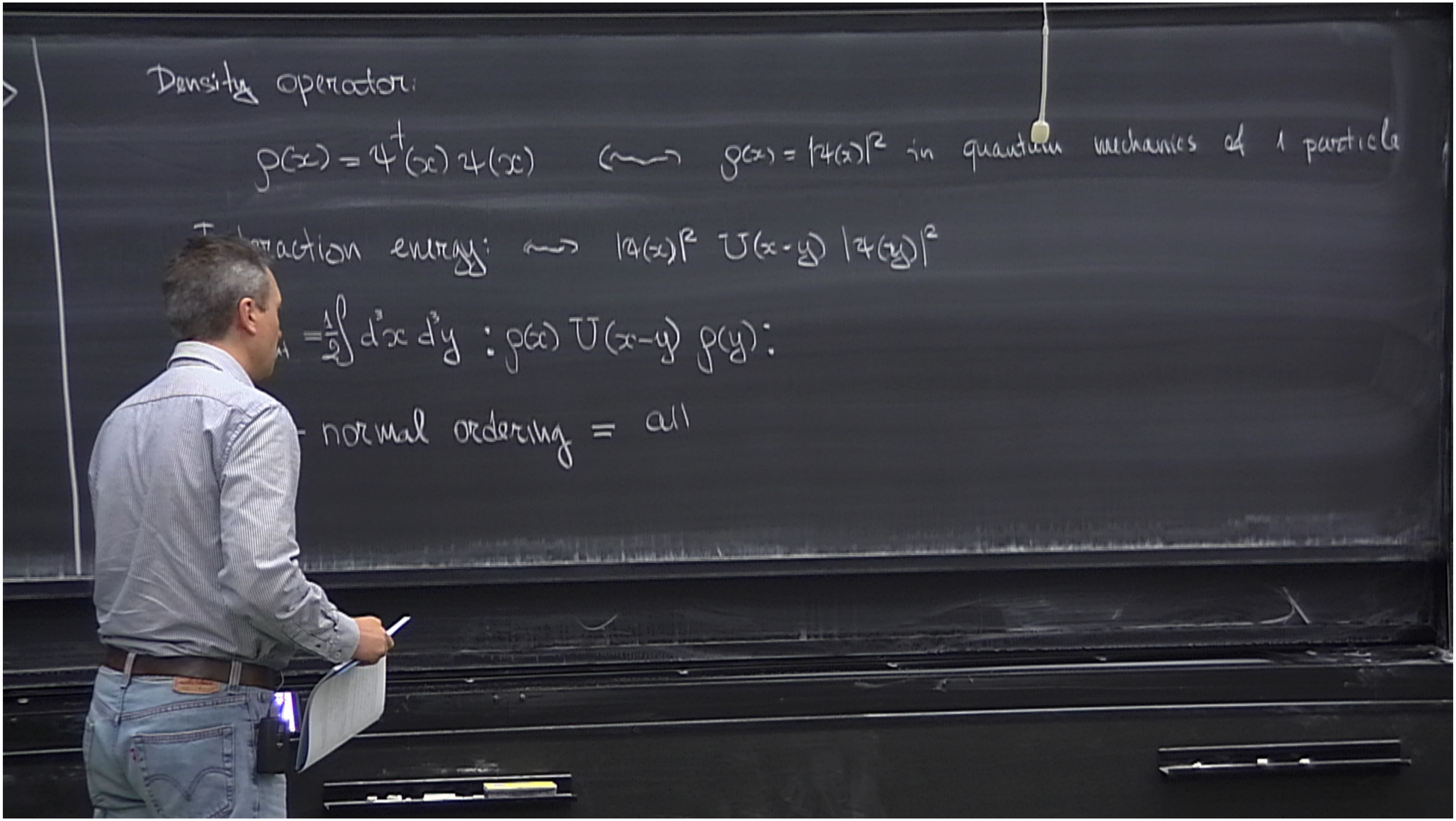
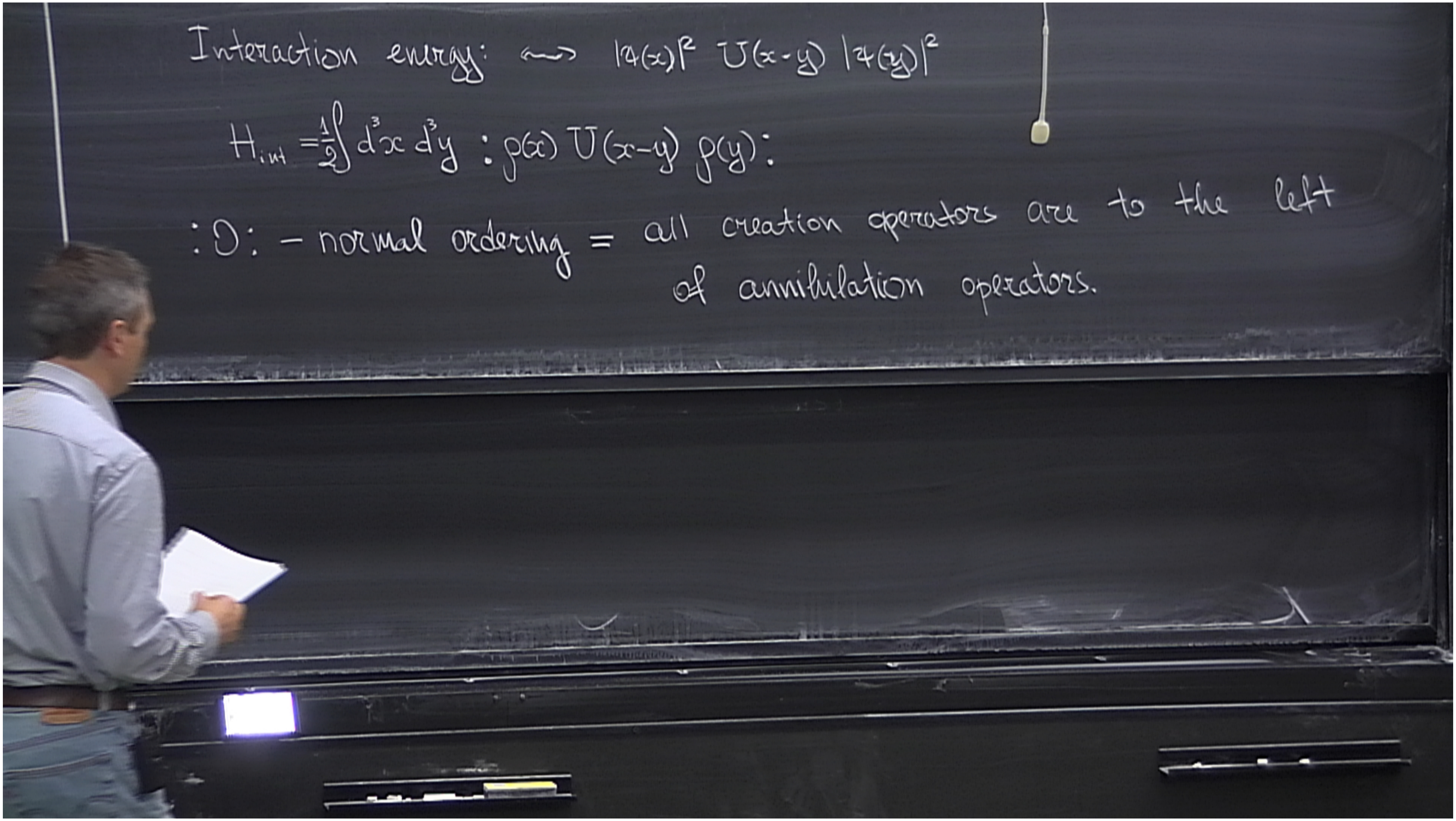
$_{int} = \frac{1}{2}\int d^3x\, d^3y \, :\rho(x)\, U(x-y)\, \rho(y):$
normal ordering = all


Density operator:
ρ(x) = ψ†(x) ψ(x) ⟷ ρ(x) = |ψ(x)|² in quantum mechanics of 1 particle
raction energy: ⟶ |ψ(x)|² U(x-y) |ψ(y)|²
= 1/2 ∫ d³x d³y :ρ(x) U(x−y) ρ(y):
normal ordering = all


Interaction energy: $\rightsquigarrow$ $|\psi(x)|^2 \, U(x-y) \, |\psi(y)|^2$
$H_{int} = \frac{1}{2} \int d^3x \, d^3y \; :\rho(x) \, U(x-y) \, \rho(y):$
$:O:$ - normal ordering = all creation operators are to the left
of annihilation operators.

Ex

$$:\rho(x)\,\rho(y): \;=\; :\psi^\dagger(x)\,\psi(x)\,\psi^\dagger(y)\,\psi(y): \;=\; \pm\,\psi^\dagger(x)\,\psi^\dagger(y)\,\psi(x)\,\psi(y)$$

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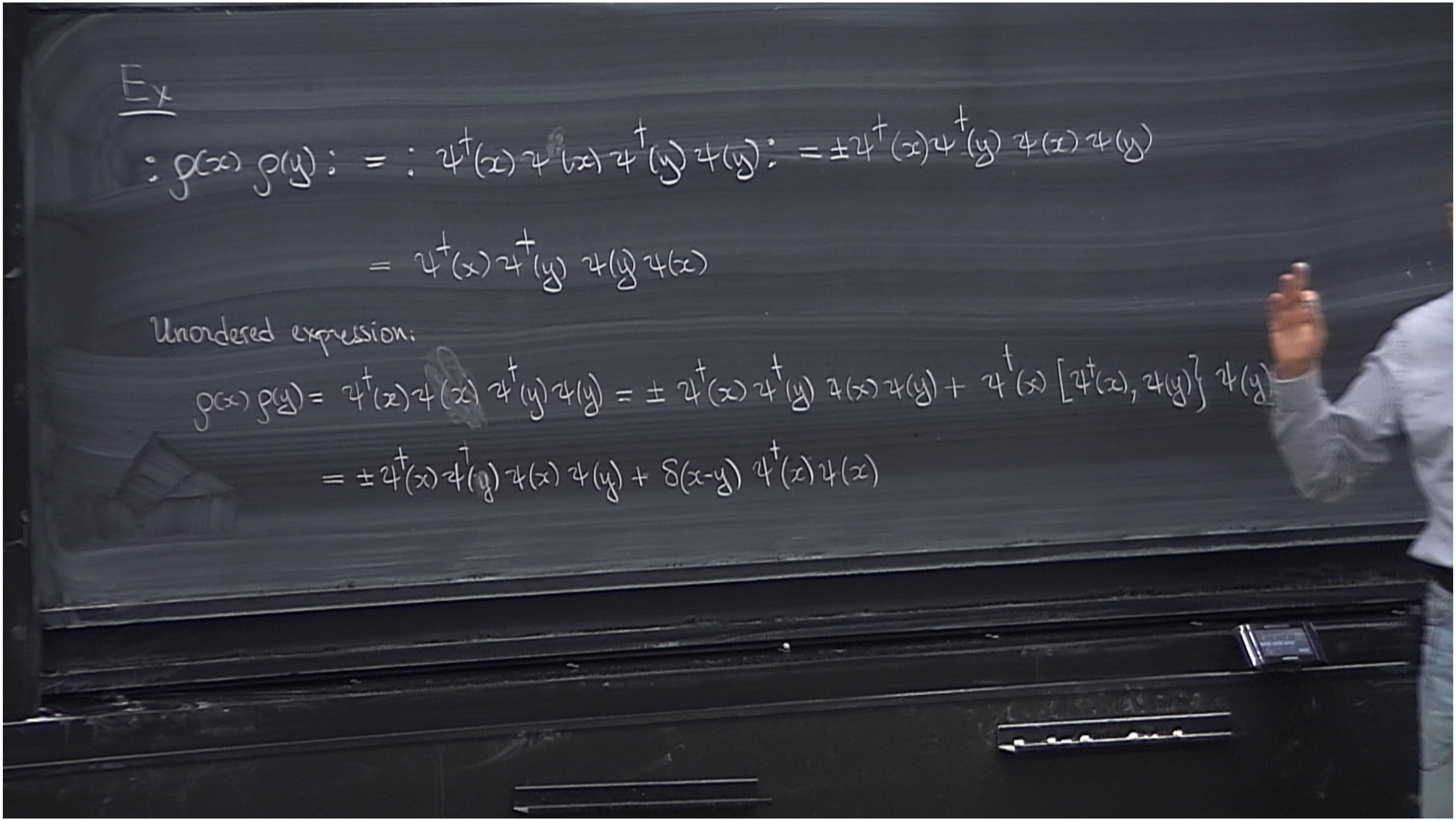
Unordered expression:

$$\rho(x)\,\rho(y) = \psi^\dagger(x)\,\psi(x)\,\psi^\dagger(y)\,\psi(y) = \pm\,\psi^\dagger(x)\,\psi^\dagger(y)\,\psi(x)\,\psi(y) + \psi^\dagger(x)\,[\psi^\dagger(x), \psi(y)\}\,\psi(y)$$


Ex
$:\rho(x)\rho(y): \; = \; :\psi^\dagger(x)\psi(x)\psi^\dagger(y)\psi(y): \; = \pm\psi^\dagger(x)\psi^\dagger(y)\psi(x)\psi(y)$
$= \psi^\dagger(x)\psi^\dagger(y)\psi(y)\psi(x)$
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$= \pm\psi^\dagger(x)\psi^\dagger(y)\psi(x)\psi(y) + \delta(x-y)\,\psi^\dagger(x)\psi(x)$

Ex

$$:\rho(x)\,\rho(y): \;=\; :\psi^\dagger(x)\,\psi(x)\,\psi^\dagger(y)\,\psi(y): \;=\; \pm\,\psi^\dagger(x)\,\psi^\dagger(y)\,\psi(x)\,\psi(y)$$

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