Title: Corrections to the Apparent Value of the Cosmological Constant Due to Local Inhomogeneities

Date: Sep 29, 2011 01:00 PM

URL: http://www.pirsa.org/11090133

Abstract: Supernovae observations strongly support the presence of a cosmological constant, but its value, which we will call apparent, is normally determined assuming that the Universe can be accurately described by a homogeneous model. Even in the presence of a cosmological constant we cannot exclude nevertheless the presence of a small local inhomogeneity which could affect the apparent value of the cosmological constant. Neglecting the presence of the inhomogeneity can in fact introduce a systematic misinterpretation of cosmological data, leading to the distinction between an apparent and true value of the cosmological constant. We establish the theoretical framework to calculate the corrections to the apparent value of the cosmological constant by modeling the local inhomogeneity with a ΛLTB solution. Our assumption to be at the center of a spherically symmetric inhomogeneous matter distribution correspond to effectively calculate the monopole contribution of the non linear inhomogeneities surrounding us, which we expect to be the dominant one, because of other observations supporting a high level of isotropy of the Universe around us. By performing a local Taylor expansion we analyze the number of independent degrees of freedom which determine the local shape of the inhomogeneity, and consider the issue of central smoothness, showing how the same correction can correspond to different inhomogeneity profiles. Contrary to previous attempts to fit data using large void models our approach is quite general. The correction to the apparent value of the cosmological constant is in fact present for local inhomogeneity of any size, and should always be taken appropriately into account both theoretically and observationally.

Outline

1. Dark Energy and inhomogeneities
2. Solving the inversion problem for $D_L(z)$ in ΛLTB spaces
3. Relation between Ω_Λ^{app} and Ω_Λ^{true}

- Expanding the r.h.s. of the geodesics equation we can easily integrate the corresponding polynomial in $q(z), p(z)$, to get $r(z)$ and $\eta(z)$.
- It can be easily shown that in order to obtain $D_L(z)$ **to the fourth order** and $mn(z)$ **the fifth** we need to expand $r(z)$ **to the fourth order and** $\eta(z)$ **to the third**.
- In order to have an solution which is **analytical everywhere** we will should use the following expressions for $k(r)$ and $t_b(r)$:

$$
\begin{aligned}
k(r) &= k_0 + k_2 r^2 + k_4 r^4 \\
t_b(r) &= t_0^b + t_2^b r^2 + t_4^b r^4
\end{aligned}
$$

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What can be the consequences of ignoring large scale inhomogeneities

- If we try to fit cosmological data with a homogeneous and isotropic model we can miss important effects from large scale inhomogeneities
- Local gravitational red-shift due to large scale inhomogeneities can in fact be mistaken for evolving dark energy
- The effects can be important even for relatively small inhomogeneities compatible with the inflation theory

FLRW case

- $$\left(\frac{\dot{a}_F}{a_F}\right)^2 = -\frac{k}{a_F^2} + \frac{\rho_0}{3a_F^3} + \frac{\Lambda}{3}\,.$$

  It is convenient to introduce the conformal time η such that $d\eta = dt/a_F$,

$$a_F(\eta) = \frac{\rho_0 L^2}{3\phi(\frac{\eta}{2L}; g_2, g_3) + kL^2}\,; \quad g_2 = \frac{4}{3}k^2 L^4\,,$$

$$g_3 = \frac{4}{27}\left(2k^3 - \Lambda\rho_0^2\right) L^6\,,$$

- Where $\phi(x; g_2, g_3)$ is the Weierstrass elliptic function satisfying the differential equation,

$$\left(\frac{d\phi}{dx}\right)^2 = 4\phi^3 - g_2\phi - g_3 \,.$$

and we have explicitly introduced the length scale L to make the equations dimensionally consistent.

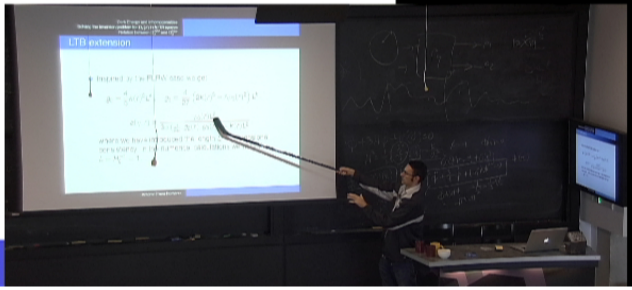
LTB extension

- Inspired by the FLRW case we get

$$g_2 = \frac{4}{3}k(r)^2L^4 , \quad g_3 = \frac{4}{27}\left(2k(r)^3 - \Lambda\rho_0(r)^2\right)L^6 .$$

$$a(\eta, r) = \frac{\rho_0(r)L^2}{3\phi\left(\frac{\eta}{2L}; g_2(r), g_3(r)\right) + k(r)L^2} .$$

where we have introduced the length L for dimensional consistency. In the numerical calculations we will set $L = H_0^{-1} = 1$.



Geodesic equations

- Taking advantage of the anaytical solution we can write the geodesics equations

$$\frac{d\eta}{dz} = -\frac{\partial_r t(\eta,r) + F(\eta,r)}{(1+z)\partial_\eta F(\eta,r)} \equiv p(\eta,r)\,,$$

$$\frac{dr}{dz} = \frac{a(\eta,r)}{(1+z)\partial_\eta F(\eta,r)} \equiv q(\eta,r)\,,$$

where

$$F(\eta,r) \equiv \frac{R_{,r}}{\sqrt{1+2E(r)}} =$$

$$= \frac{\partial_r(a(\eta,r)r) - a^{-1}\partial_\eta(a(\eta,r)r)\,\partial_r t(\eta,r)}{\sqrt{1-k(r)r^2}}\,.$$

It is important that the functions p, q, F have explicit analytical forms.

For a matter of computational convenience we will choose K_0, K_1 as free parameters and express all the other in terms of them. For example from :

$$D_2^{\Lambda CDM} = D_2^{\Lambda LTB},$$

we can get

$$\Omega_\Lambda^{app}(\Omega_\Lambda, K_0, K_1),$$

from

$$D_3^{\Lambda CDM} = D_3^{\Lambda LTB},$$

we can get

$$K_2(\Omega_\Lambda^{app}, K_0, K_1),$$

and in general from

$$D_i^{\Lambda CDM} = D_i^{\Lambda LTB},$$

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$$K_{i-1}(\Omega_\Lambda^{app}, K_0, K_1).$$

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Local expansion

- We need to expand locally the relevant functions:

$$
\begin{aligned}
k(r) &= k_0 + k_1 r + k_2 r^2 + .. \\
t(\eta, r) &= b_0(\eta) + b_1(\eta) r + b_2(\eta) r^2 + ..
\end{aligned}
$$

- After expanding the geodesics equation we can then get the final formula for the luminosity distance:

$$
\begin{aligned}
D_L^{\Lambda LTB}(z) &= (1+z)^2 r(z) a^{\Lambda LTB}(\eta(z), r(z)) \\
&= D_1^{\Lambda LTB} z + D_2^{\Lambda LTB} z^2 + D_3^{\Lambda LTB} z^3 + .. \\
D_1^{\Lambda LTB} &= \frac{1}{H_0}, \\
D_2^{\Lambda LTB} &= \frac{1}{36 H_0 (\Omega_\Lambda^{true} - 1)} \Big[54 B_1 (\Omega_\Lambda^{true} - 1)^2 + 18 B_1' (\Omega_\Lambda^{true} - 1) - 18 h_{0,r} (\Omega_\Lambda^{true})^2 \\
&\quad + 30 h_{0,r} \Omega_\Lambda^{true} - 12 h_{0,r} + 6 K_1 \Omega_\Lambda^{true} - 10 K_1 + 27 (\Omega_\Lambda^{true})^2 - 18 \Omega_\Lambda^{true} - 9 \Big],
\end{aligned}
$$

- We have introduced the dimensionless quantities :

$$
\begin{aligned}
H_0 &= \left(\frac{\partial_t, a(t,r)}{a(t,r)}\right)^2\Bigg|_{t=t_0,r=0} = \left(\frac{\partial_\eta a(\eta,r)}{a(\eta,r)^2}\right)^2\Bigg|_{\eta=\eta_0,r=0}, \\
K_0 &= k_0(a_0 H_0)^{-2}, \\
K_1 &= k_1(a_0 H_0)^{-3}, \\
B_1(\eta) &= b_1(\eta) a_0^{-1}, \\
B_1 &= b_1(\eta_0) a_0^{-1}, \\
B_1' &= \frac{\partial B_1(\eta)}{\partial \eta}\Bigg|_{\eta=\eta_0} (a_0 H_0)^{-2}, \\
h_{0,r} &= \frac{1}{a_0 H_0} \frac{\partial_r a(\eta,r)}{a(\eta,r)}\Bigg|_{\eta=\eta_0,r=0}, \\
t_0 &= t(\eta_0, 0),
\end{aligned}
$$

- Note that apart from the central curvature term K_0, the inhomogeneity of the *LTB* space is expressed in h_0, r, which encodes the radial dependence of the scale factor.
- As expected a_0 is not present in the formula for $D_L(z)$ since its value is arbitrary
- We are denoting as Λ^{true} the cosmological constant appearing in the LTB solution because we are assuming that the Universe is inhomogeneous, so that it is correctly described by the ΛLTB metric rather than FLRW.

- The metric of a ΛCDM model is the FLRW metric, a special case of LTB solution, where :

$$\begin{aligned} \rho_0(r) &\propto const, \\ k(r) &= 0, \\ t_b(r) &= 0, \\ a(t,r) &= a(t). \end{aligned}$$

- We will calculate independently the expansion of the luminosity distance for the case of a flat ΛCDM, to clearly show the meaning of our notation, and in particular the distinction between Ω_Λ^{app} and Ω_Λ^{true}.
- One of the Einstein equation can be expressed as:

$$\begin{aligned} H^{\Lambda CDM}(z) &= H_0\sqrt{(1-\Omega_\Lambda^{app})\left(\frac{a_0}{a}\right)^3+\Omega_\Lambda^{app}} \\ &= H_0\sqrt{(1-\Omega_\Lambda^{app})(1+z)^3+\Omega_\Lambda^{app}}. \end{aligned}$$

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- We can then calculate the luminosity distance using the following relation, which is only valid assuming flatness:

$$D_L^{\Lambda CDM}(z) = (1+z)\int_0^z \frac{dz'}{H^{\Lambda CDM}(z')} = D_1^{\Lambda CDM} z + D_2^{\Lambda CDM} z^2 + D_3^{\Lambda CDM} z^3 + \ldots$$

$$D_1^{\Lambda CDM} = \frac{1}{H_0},$$
$$D_2^{\Lambda CDM} = \frac{3\Omega_\Lambda^{app} + 1}{4H_0}.$$

- We can check the consistency between these formulae and the ones derived in the case of LTB by setting:

$$K_1 = B_1 = B_1' = K_0 = h_{0,r} = 0,$$

which corresponds to the case in which $\Omega_\Lambda^{app} = \Omega_\Lambda^{true}$.

- In order to find the relation between the apparent and true value of the cosmological constant we need in fact to match the terms in the red-shift expansion :

$$D_i^{\Lambda CDM} = D_i^{\Lambda LTB} \quad , \quad 1 \leq i \leq 2 ,$$

- From the above relations we can finally derive :

$$\begin{aligned}
H_0^{\Lambda LTB} &= H_0^{\Lambda CDM} , \\
\Omega_\Lambda^{app} &= \frac{1}{27(\Omega_\Lambda^{true} - 1)} \Big[54B_1 (\Omega_\Lambda^{true})^2 - 108B_1 \Omega_\Lambda^{true} + 54B_1 + 18B_1' \Omega_\Lambda^{true} - 18B_1' \\
&\quad - 18h_{0,r} (\Omega_\Lambda^{true})^2 + 30h_{0,r} \Omega_\Lambda^{true} - 12h_{0,r} + 6K_1 \Omega_\Lambda^{true} - 10K_1 \\
&\quad + 27\Omega_\Lambda^{true} (\Omega_\Lambda^{true} - 1) \Big] , \\
\Omega_\Lambda^{true} &= -\frac{1}{6(6B_1 - 2h_{0,r} + 3)} \Big[\Big((36B_1 - 6B_1' - 10h_{0,r} - 2K_1 + 9\Omega_\Lambda^{app} + 9)^2 + \\
&\quad - 4(6B_1 - 2h_{0,r} + 3)(54B_1 - 18B_1' - 12h_{0,r} - 10K_1 + 27\Omega_\Lambda^{app}) \Big)^{1/2} - 36B_1 \\
&\quad + 6B_1' + 10h_{0,r} + 2K_1 - 9(\Omega_\Lambda^{app} - 1) \Big] .
\end{aligned}$$

For a matter of computational convenience we will choose K_0, K_1 as free parameters and express all the other in terms of them. For example from :

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- We can also expand the above exact relations assuming that all the inhomogeneities, can be treated perturbatively respect to the ΛCDM , i.e. $\{K_1, B_1, B_1'\} \propto \epsilon$, where ϵ stands for a small deviation from *FLRW* solution :

$$\Omega_\Lambda^{true} = \Omega_\Lambda^{app} - \frac{2}{27(\Omega_\Lambda^{app}-1)}(27B_1(\Omega_\Lambda^{app}-1)^2 + 9B_1'(\Omega_\Lambda^{app}-1) - 9h_{0,r}(\Omega_\Lambda^{app})^2 + 15h_{0,r}\Omega_\Lambda^{app}$$
$$-6h_{0,r} + 3K_1\Omega_\Lambda^{app} - 5K_1) + O(\epsilon^2)\,.$$

- As expected all these relations reduce to

$$\Omega_\Lambda^{true} = \Omega_\Lambda^{app},$$

in the limit in which there is no inhomogeneity, i.e. when $K_1 = B_1 = B_1' = h_{0,r} = 0$.

Quantitative preliminary results

- Based on numerical and analytical calculations we set some quantitative bounds:
- These are only valid at low redshift < 0.2 since after that the Taylor expansion for ΛCDM is not sufficiently accurate anymore (more than 6% error at any order of expansion)
- A better expansion may be necessary or a full numerical inversion method
- A correction of order of 5% corresponds to a underdensity of order of 0.1%
- 20% only corresponds to only 0.4% underdensity respect to the center
- These results require a more careful numerical check, and are only preliminary now

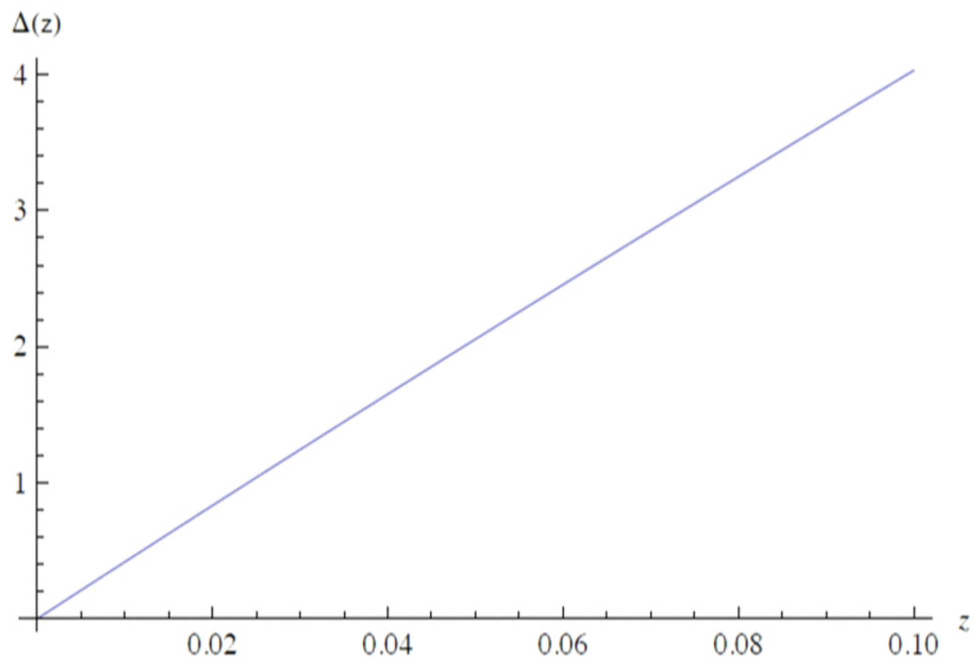

Figure: The percentual error $\Delta = 100\frac{D^{\Lambda CDM} - D^{\Lambda CDM}_{Taylor}}{D^{\Lambda CDM}}$ for a third order expansion is plotted as a function of the redshift. As it can be seen the error is already quite large at redshift 0.1. Higher order expansion does not improve the convergence.

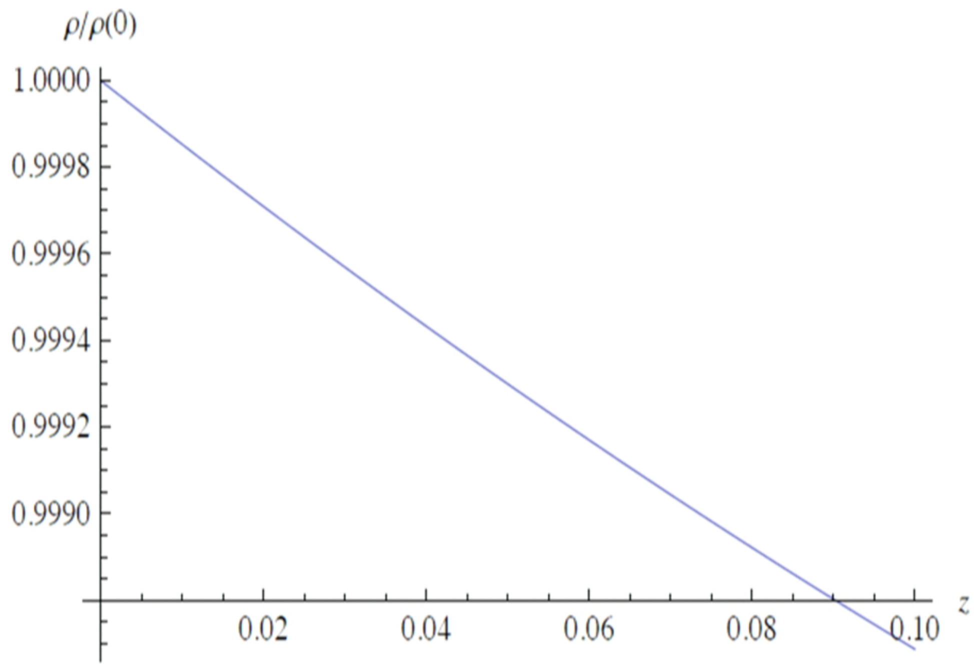

Figure: The ratio $\frac{\rho(t_0,r)}{\rho_0}$ of the density respect to the central density $\rho_0 = \rho(t_0, 0)$ at the present time t_0, is plotted as function of the radial coordinate r. The LTB model is the solution of the inversion problem corresponding to $K_0 = 0, \Omega_\Lambda^{true} = 0.95\% \, \Omega_\Lambda^{app}, \Omega_\Lambda^{app} = 0.72$.

Conclusions and future directions of investigation

- We have derived the relation between the **apparent** and **true** value of the **cosmological constant** due to the presence of a local large scale inhomogeneity
- The next step will be to obtain some quantitative bounds on the magnitude of the correction by using **experimental data**
- The same approach could be extended to other cosmological parameters
- In this case we assumed spherical symmetry for simplicity: this could be considered the **monopole contribution**, while a local inhomogeneity could have another less symmetric shape which would then induce some **additional anisotropy**, but CMB isotropy put some strong bounds on this.