Title: Line Operators on S^1 x R^3 and Quantization of the Hitchin Moduli Space

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Abstract: We perform an exact localization calculation for the expectation value of Wilson-'t Hooft line operators in N=2 gauge theories on S^1 x R^3.
The expectation values form a quantum mechanically deformed algebra of functions on the Hitchin moduli space by Moyal multiplication. We demonstrate that these expectation values are the Weyl transform of the Verlinde operators, which acts on conformal blocks as difference operators. Our results are also in exact match with the predictions from wall-crossing in the IR effective theory.

Line operators on $S^1 \times \mathbb{R}^3$ and quantization of the Hitchin moduli space

Work in progress with Y. Ito & M. Taki

- Intro
- Set-up
- Summary of results
- Localization, monopole moduli space
- Conclusions

· Localization, monopole moduli space

- Conclusions

Set-up

- Localization, monopole moduli space
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<u>Set-up</u>

$\mathcal{N}=2$ gauge theories

$S^1 \times \mathbb{R}^3$

Coulomb branch

$\langle L \rangle$

- Localization, monopole moduli space
- Conclusions

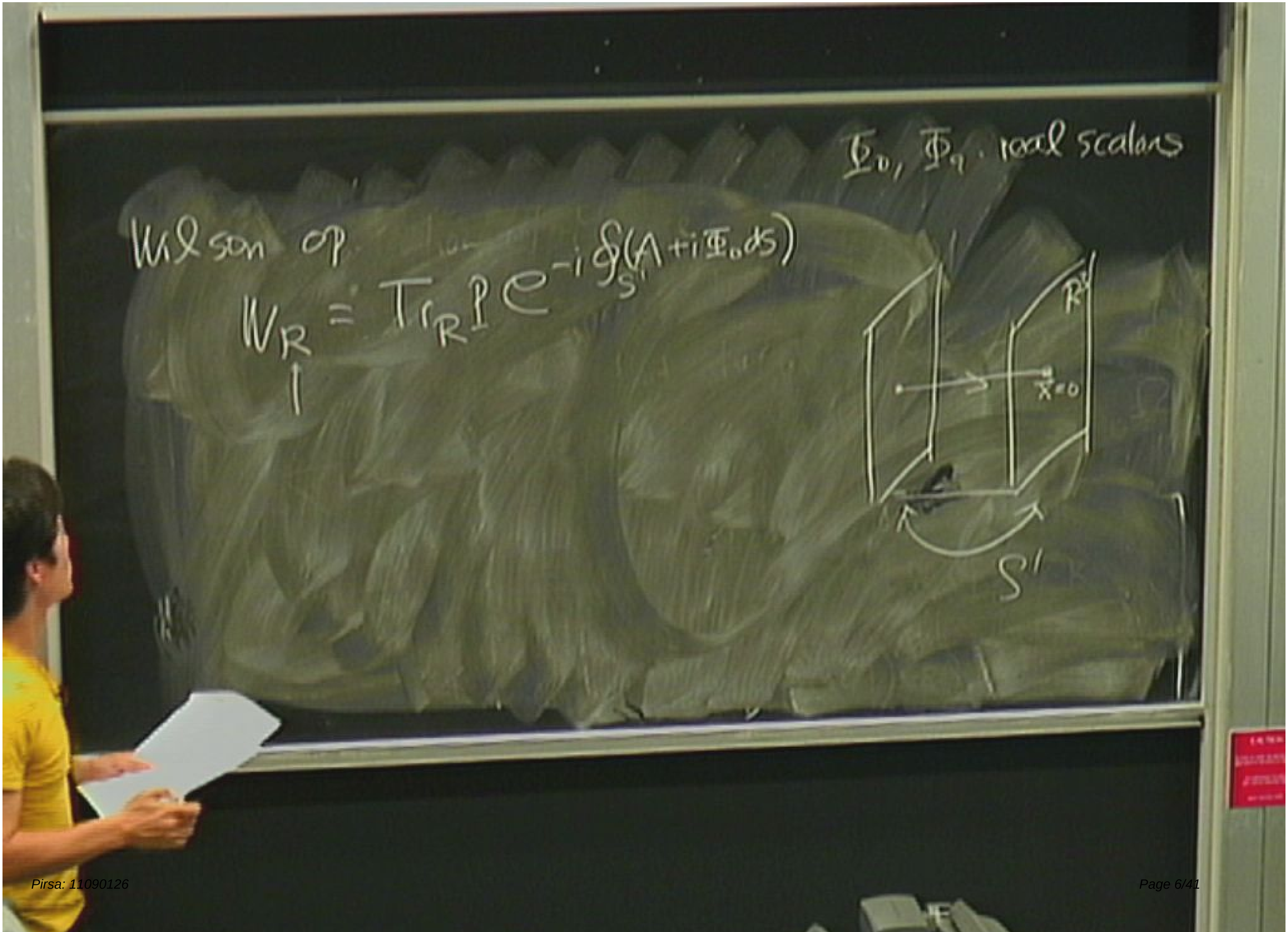
Set-up

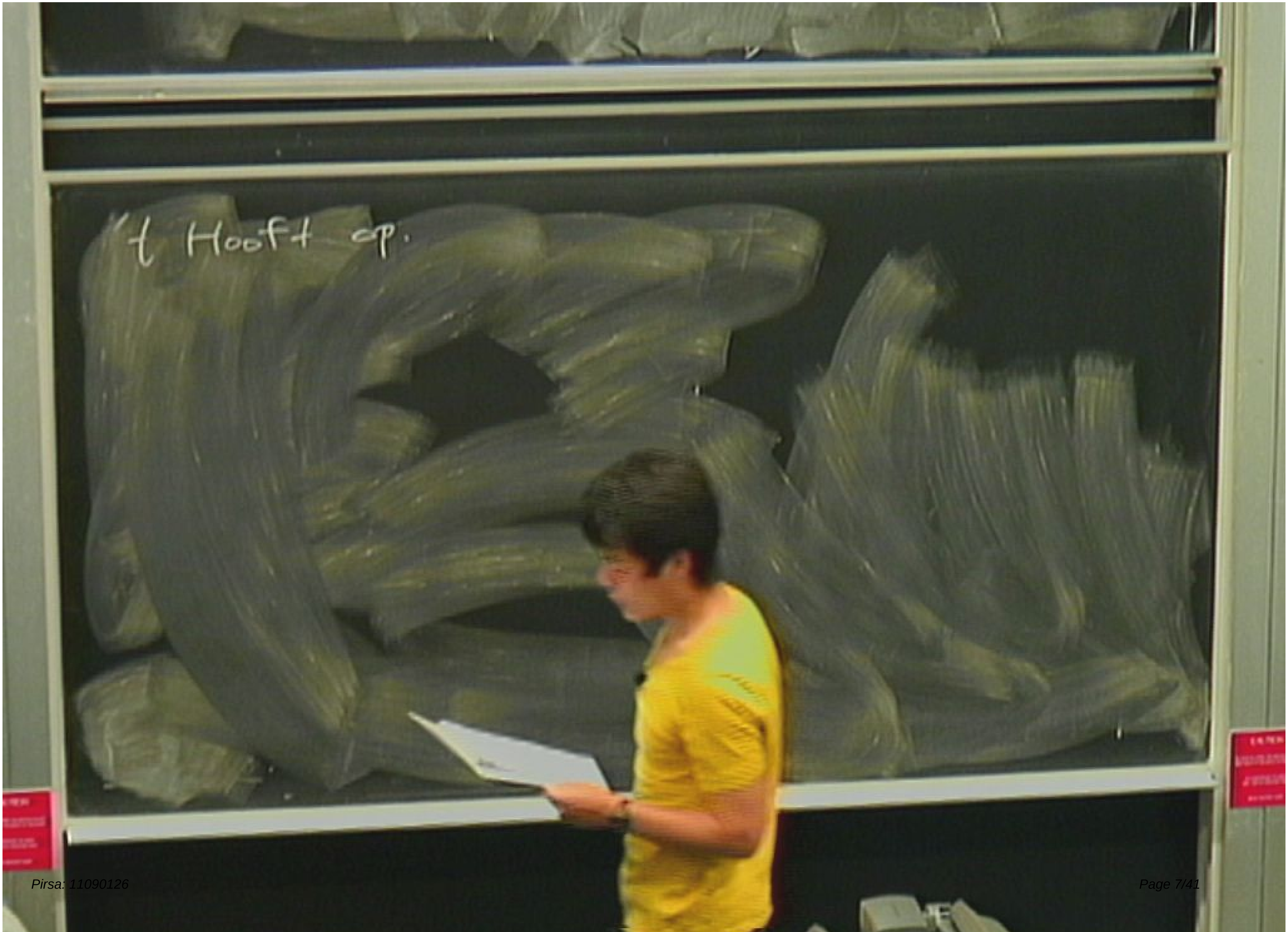
$\mathcal{N}=2$ gauge theories

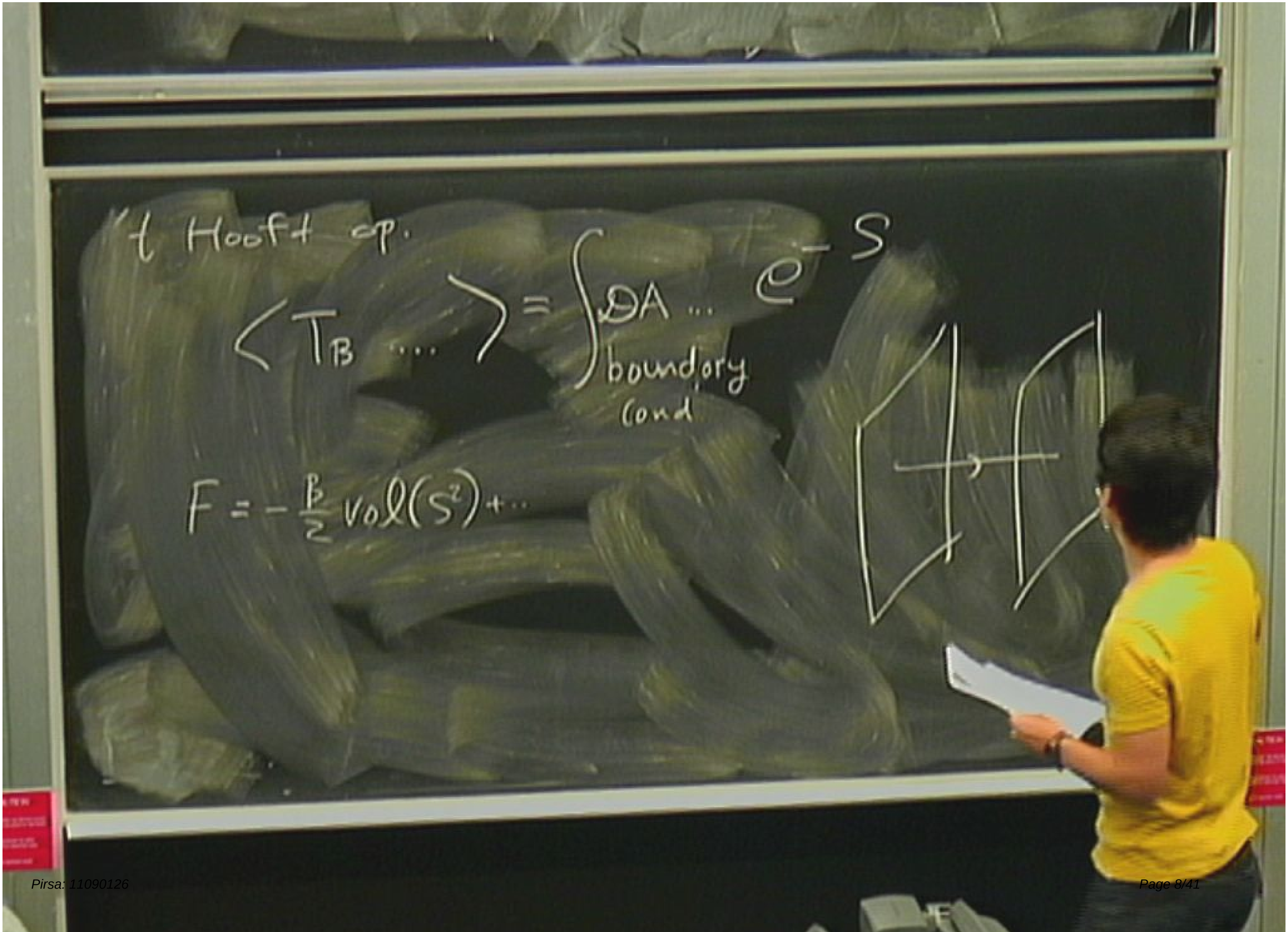
$S^1 \times \mathbb{R}^3$

Coulomb branch

$\langle L \rangle$

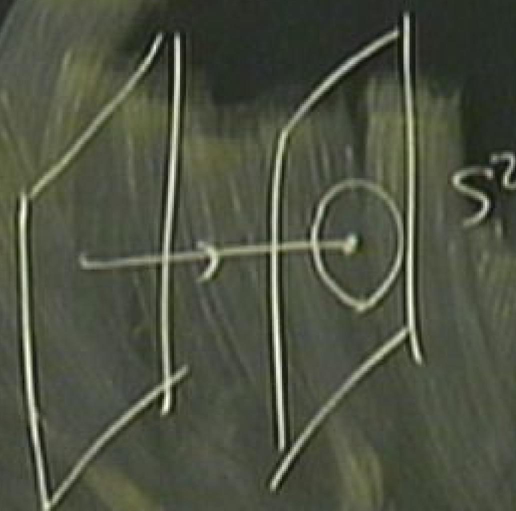

Φ_0, Φ_9 : real scalars
Wilson op.
$W_R = \mathrm{Tr}_R P e^{-i \oint_{S^1} (A + i \Phi_0 ds)}$
R^3
$\vec{x}=0$
S^1


't Hooft op.


't Hooft op.
$\langle T_B \ldots \rangle = \int_{\text{boundary cond}} \mathcal{D}A \ldots e^{-S}$
$F = -\frac{\beta}{2} \mathrm{vol}(S^2) + \ldots$

't Hooft op.

$$\langle T_B \ldots \rangle = \int_{\substack{\text{boundary} \\ \text{cond}}} \mathcal{D}A \ldots \, e^{-S}$$



$$F = -\frac{\beta}{2} \underline{Vol(S^2)} + \ldots$$

$B \in$

't Hooft op.

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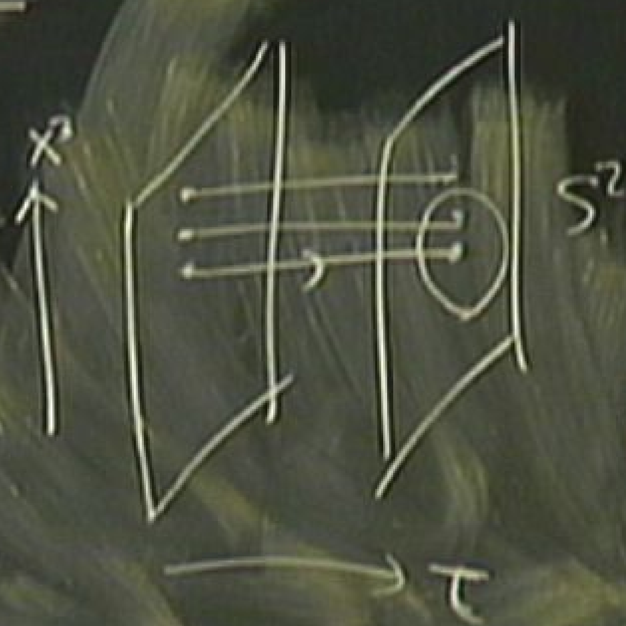

$$F = -\frac{B}{2}\,\underline{\mathrm{vol}(S^2)} + \ldots$$

$$\Phi_q = \frac{B}{2|\vec{x}|} + \ldots$$

$$B \in \underbrace{\Lambda_{\text{coweight}}}_{\cap\ \mathfrak{t}} = \Lambda^*_{\text{root}}$$

't Hooft op.

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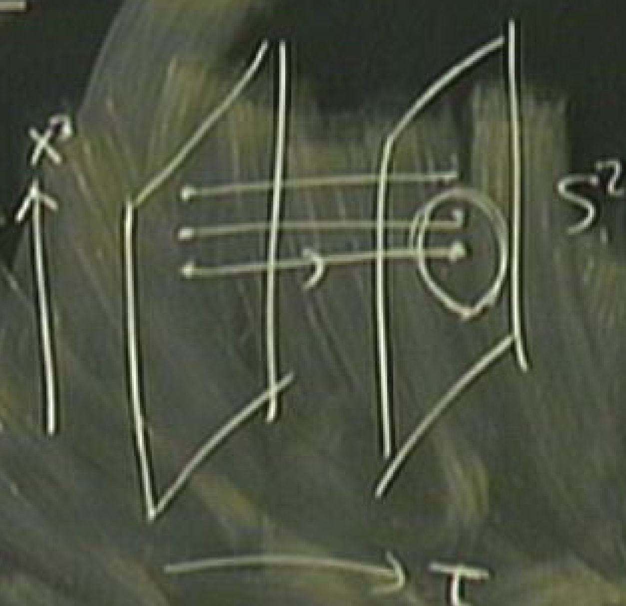

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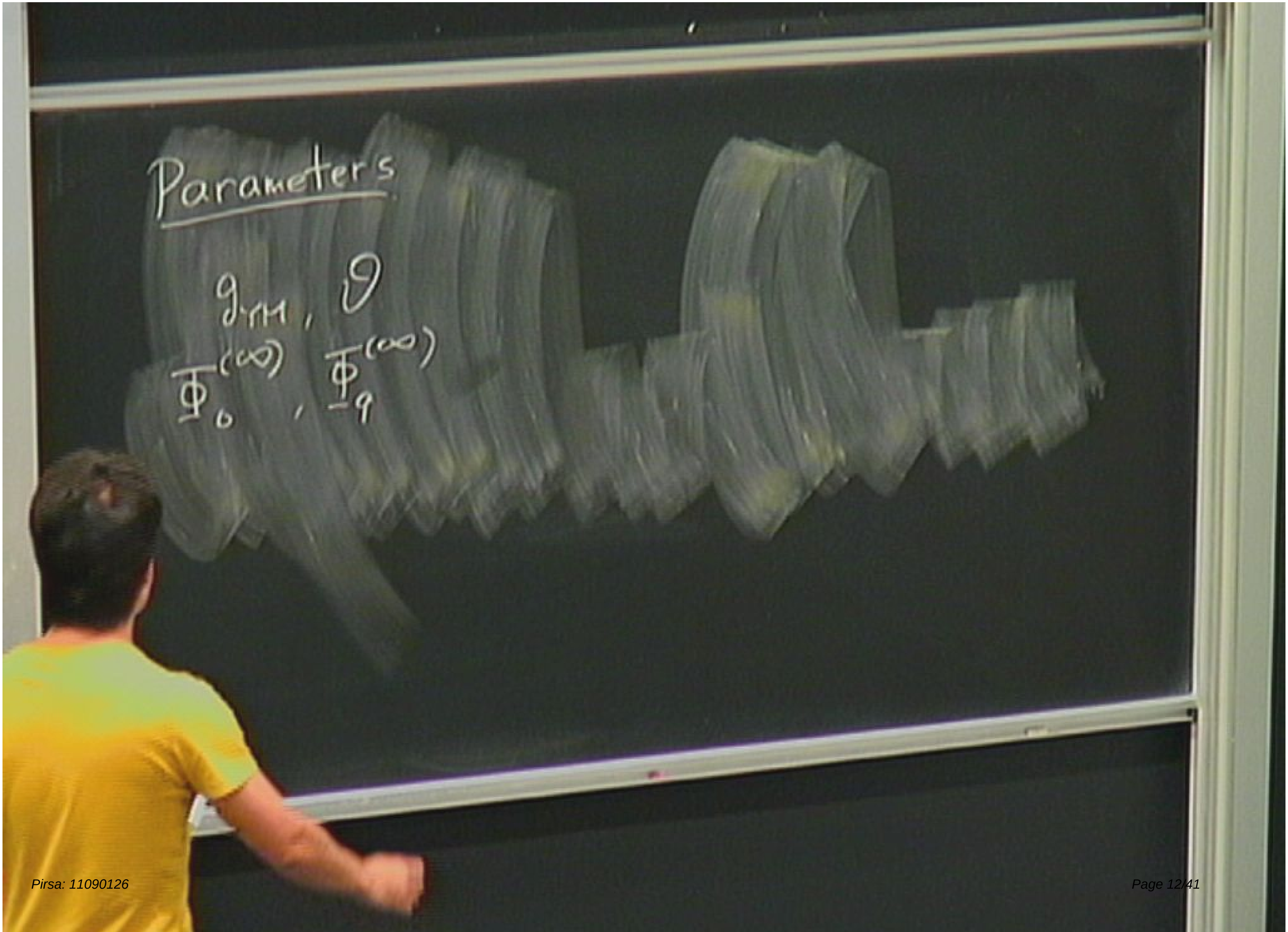

Parameters
g_{YM} , ϑ
$\Phi_0^{(\infty)}$, $\Phi_q^{(\infty)}$

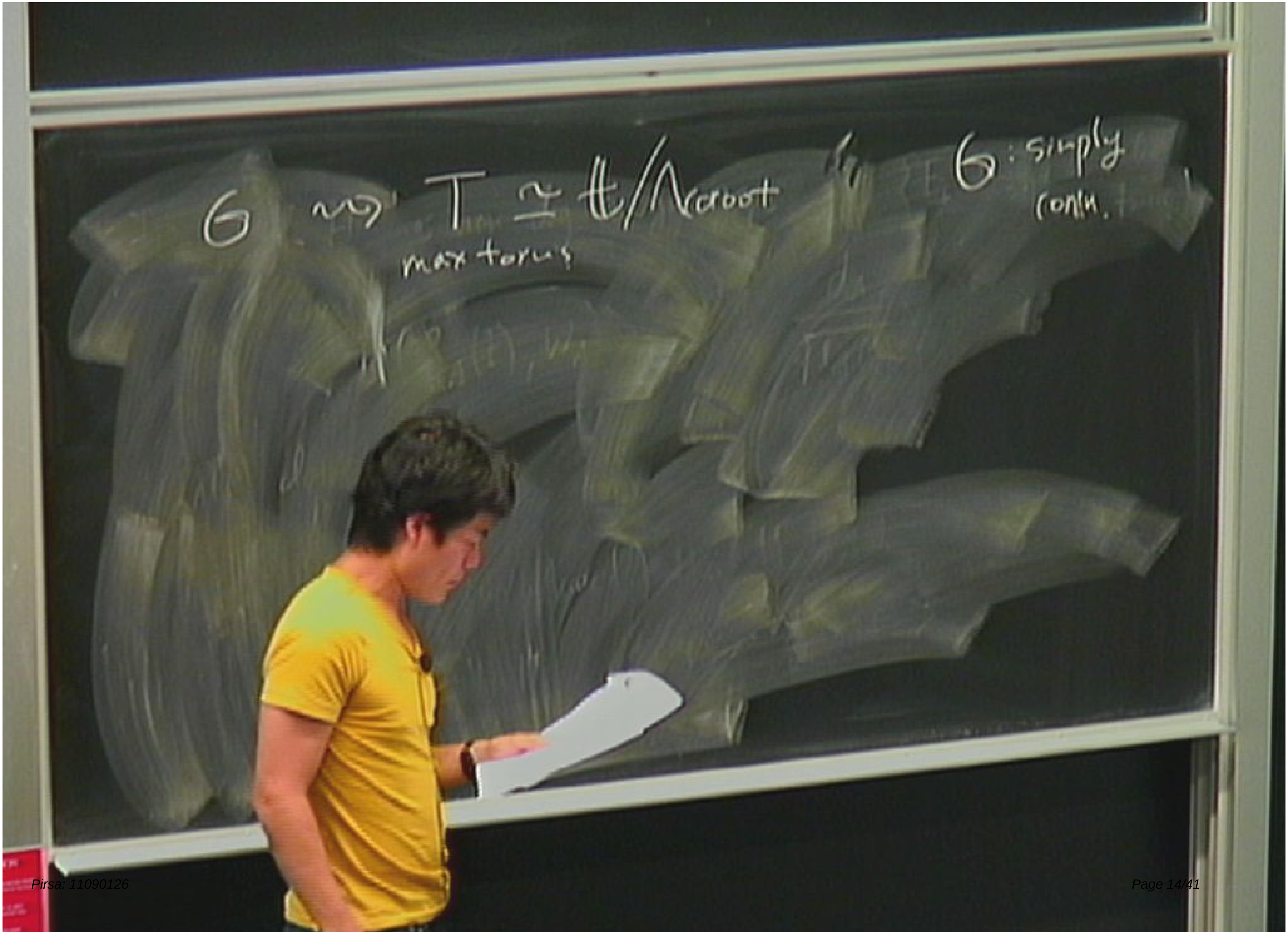
Parameters

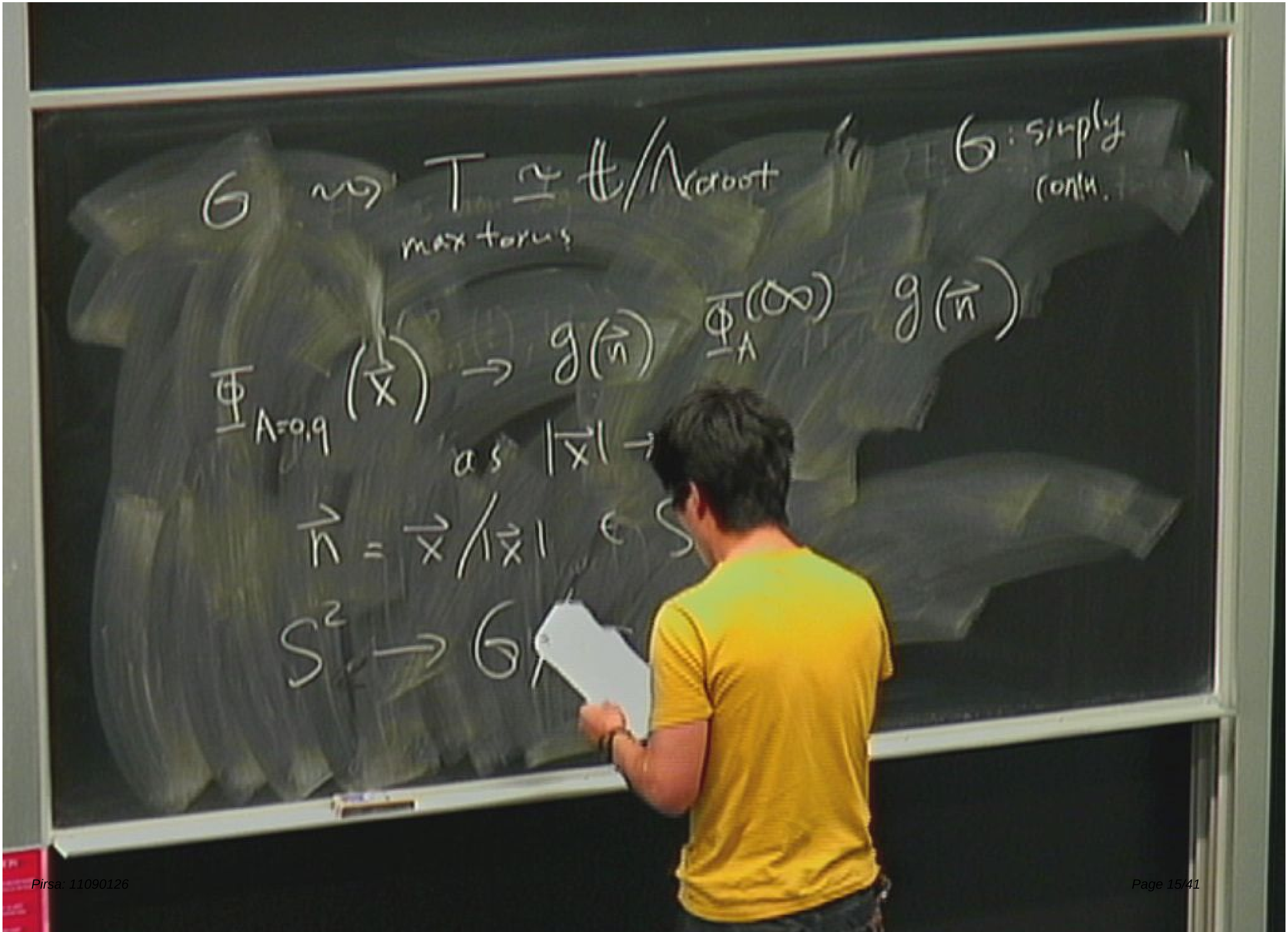
g_{YM}, ϑ

$\Phi_0^{(\infty)}$, $\Phi_9^{(\infty)}$

$A_\tau^{(\infty)}$

- Magnetic Wilson line.


G ⇝ T ≅ t/Λcoroot
max torus
G : simply conn.


G ⇝ T ≅ t/Λroot
max torus
G: simply conn.
$\Phi_{A=a,9}(\vec{x}) \to g(\vec{n})$
as $|\vec{x}| \to$
$\Phi_A(\infty)$ $g(\vec{n})$
$\vec{n} = \vec{x}/|\vec{x}| \in S$
$S^2 \to G/$

$G \rightsquigarrow T \cong \mathfrak{t}/\Lambda_{coroot}$; G : simply conn.

max torus

$$\Phi_{A=a,a}(\vec{x}) \to g(\vec{n})\, \Phi_A(\infty)\, g(\vec{n})$$

as $|\vec{x}| \to \infty$

$$\vec{n} = \vec{x}/|\vec{x}| \in S^2$$

$$S^2 \to G/T \Rightarrow \text{Top. sectors } \nu \in \pi_2(G/T)$$

$$= \pi_1(T) = \Lambda_{coroot}$$

$G \rightsquigarrow T \simeq \mathfrak{t}/\Lambda_{coroot}$; G : simply conn.

max torus

$\Phi_A(\infty)$ $g(\vec{n})$

$\Phi_{A=a,q}(\vec{x}) \to g(\vec{n})$

as $|\vec{x}| \to \infty$

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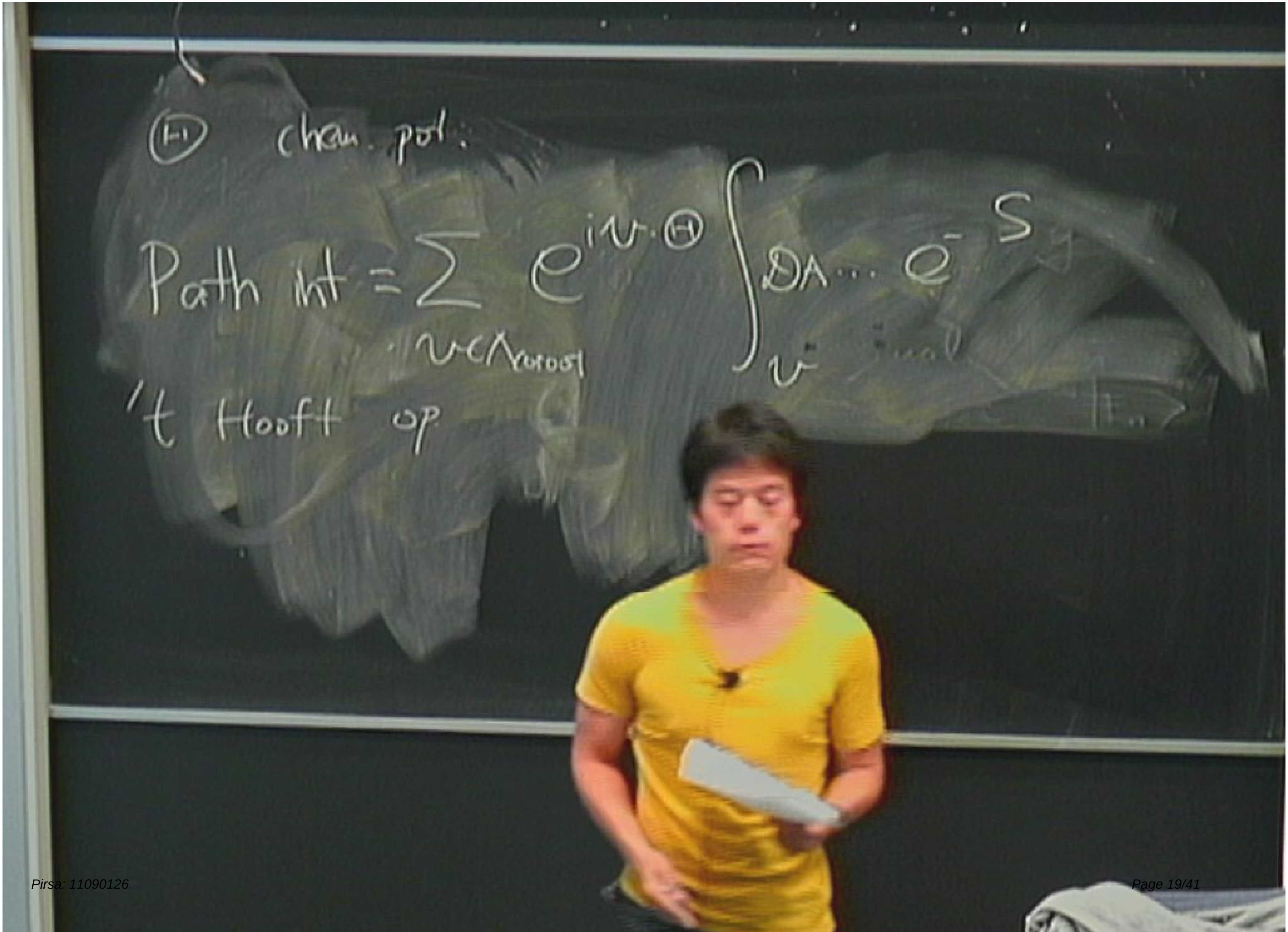
$S^2 \to G/T \Rightarrow$ Top. sectors $\nu \in \pi_2(G/T)$

$= \pi_1(T) = \Lambda_{coroot}$

Θ chem. pot.

$$\text{Path int} = \sum_{v \in \Lambda_{root}} e^{i v \cdot \Theta} \int_{v} \mathcal{D}A \cdots e^{-S_g}$$


Θ chem. pot.
Path int = $\sum_{v \in \Lambda_{coroot}} e^{iv \cdot \Theta} \int_{v} \mathcal{D}A \ldots e^{-S}$
't Hooft op

Twist param λ

spatial & R-sym rotations

Twist param λ

spatial & R-sym rotations

radius S^1

$$\langle L \rangle_{S^1 \times \mathbb{R}^3} = \mathrm{Tr}_{\mathcal{H}(L)} (-)^{\mathbb{F}} e^{-2\pi R \cdot H} e^{2\pi i \lambda (J_3 + I_3}$$

spatial

R-

Twist param λ

spatial & R-sym rotations

radius S^1

$$\langle L \rangle_{S^1\times\mathbb{R}^3} = \mathrm{Tr}_{\mathcal{H}(L)} (-)^{F} e^{-2\pi R\cdot H} e^{2\pi i \lambda (J_3 + I_3)} e^{-2\pi i \mu F}$$

spatial

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flavor

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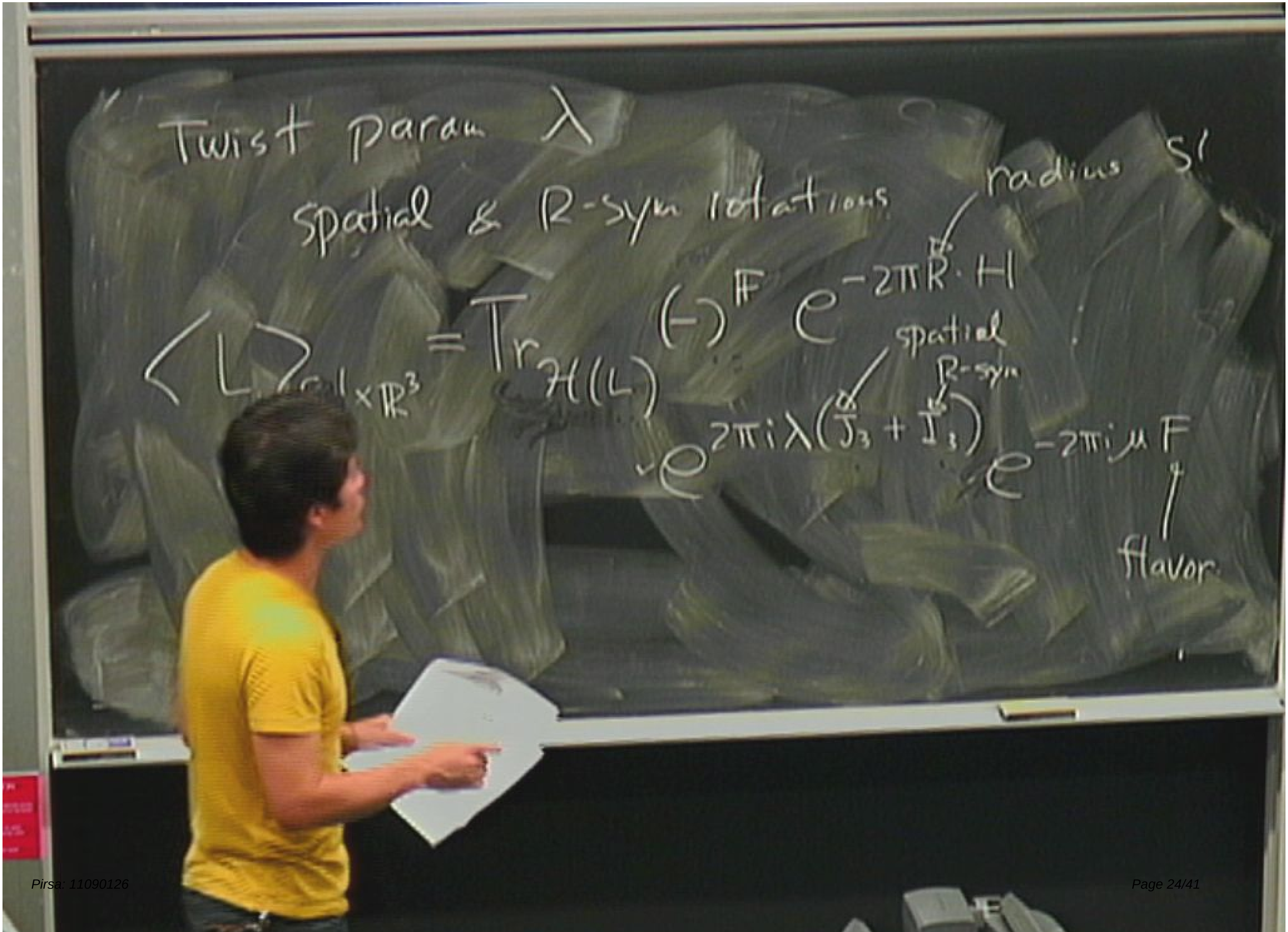
Twist param λ
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$\langle L \rangle_{S^1\times\mathbb{R}^3} = \mathrm{Tr}_{\mathcal{H}(L)} (-)^{\mathbb{F}} e^{-2\pi R\cdot H} e^{2\pi i\lambda(J_3 + I_3)} e^{-2\pi i \mu F}$
spatial
R-sym
flavor

$$a \equiv R\left(A_\tau^{(\infty)} + i\,\Phi_0^{(\infty)}\right) \in \mathfrak{t}_{\mathbb{C}}$$

$$b \equiv \frac{\Theta}{2\pi} - \frac{4\pi i}{g^2}\,\Phi_9^{(\infty)} + \frac{\theta}{2\pi}\,a \in \mathfrak{t}_{\mathbb{C}}$$

$$m = \mu + i R M \quad \in \mathbb{C}$$

$$a \equiv R\left(A_\tau^{(\infty)} + i\,\Phi_0^{(\infty)}\right) \in \mathfrak{t}_{\mathbb{C}}$$

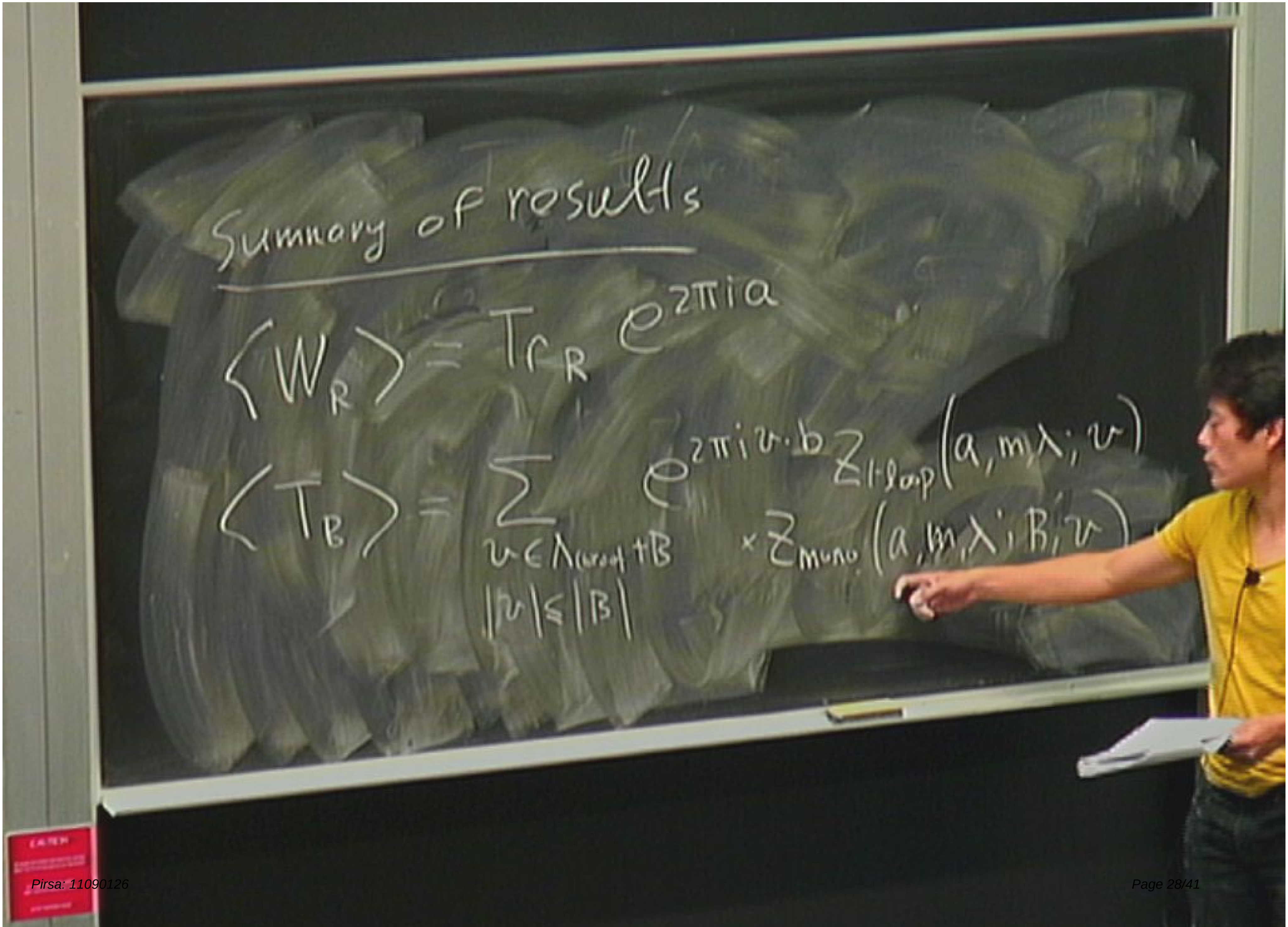
$$b \equiv \frac{\Theta}{2\pi} - \frac{4\pi i}{g^2}\,\Phi_q^{(\infty)} + \frac{\vartheta}{2\pi}\,a \in \mathfrak{t}_{\mathbb{C}}$$

$$m = \mu + i R \underbrace{M_0}_{\text{mass}} \in \mathbb{C}$$

Summary of results

$$\langle W_R \rangle = \mathrm{Tr}_R \, e^{2\pi i a}$$

$$\langle T_B \rangle = \sum_{\substack{v \in \Lambda_{(\mathrm{cr},g)} + B \\ |v| \leq |B|}} e^{2\pi i v \cdot b}$$


Summary of results
$\langle W_R \rangle = \mathrm{Tr}_R\, e^{2\pi i a}$
$\langle T_B \rangle = \sum_{\substack{v \in \Lambda_{\mathrm{cr}(\mathrm{root})} + B \\ |v| \le |B|}} e^{2\pi i v\cdot b}\, Z_{\text{1-loop}}(a, m, \lambda; v) \times Z_{\text{mono}}(a, m, \lambda; B, v)$

Summary of results

$$\langle W_R \rangle = \mathrm{Tr}_R\, e^{2\pi i a}$$

$$\langle T_B \rangle = \sum_{\substack{v \in \Lambda_{cr(wt)} + B \\ |v| \leq |B|}} e^{2\pi i v \cdot b}\, Z_{1\text{-loop}}(a, m, \lambda; v) \times Z_{\text{mono}}(a, m, \lambda; B, v)$$

Summary of results

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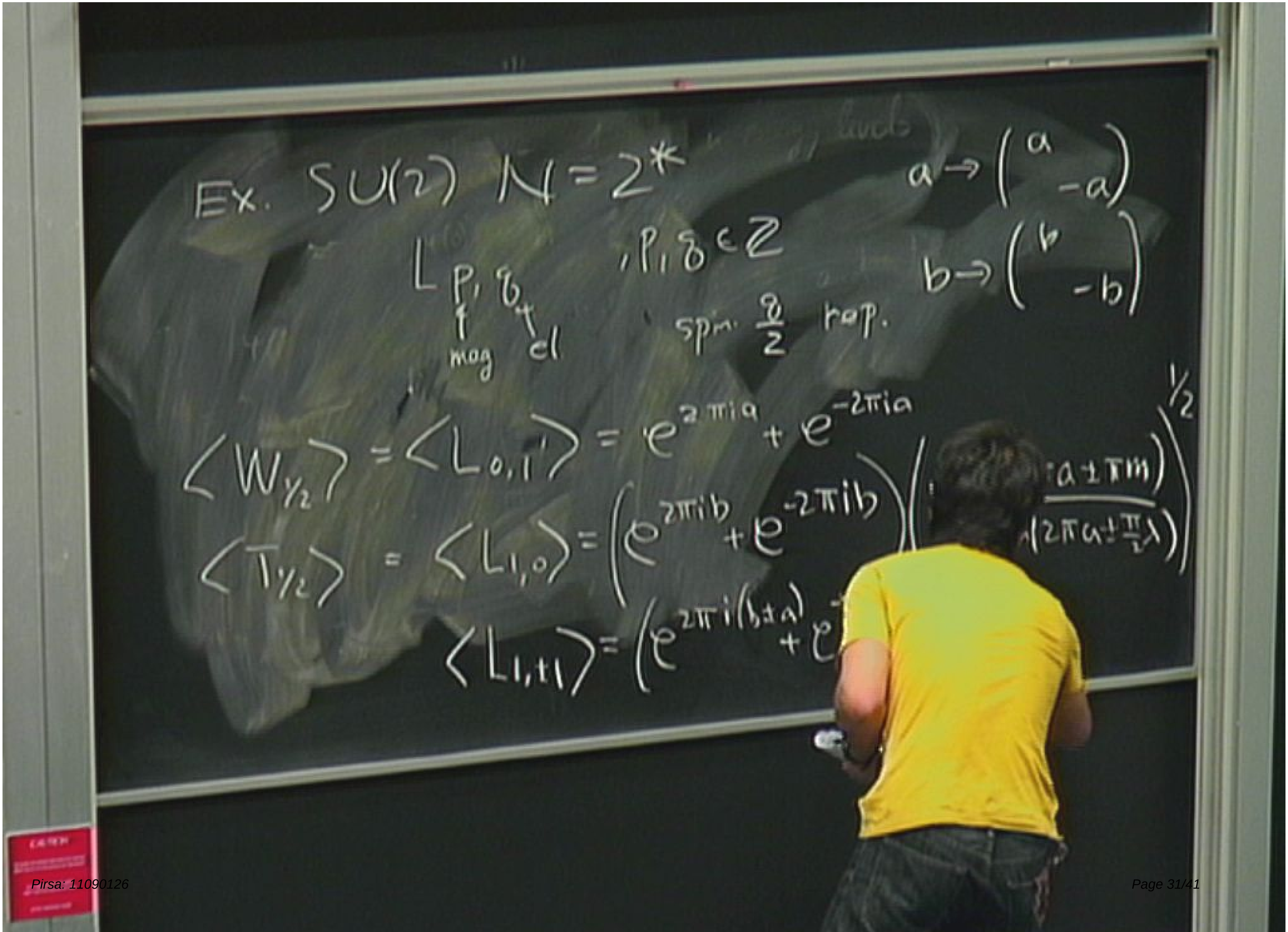

Ex. SU(2) N=2*
a → (a, -a)
b → (b, -b)
L p,q , p,q ∈ Z
mag el
spin q/2 rep.
⟨W 1/2⟩ = ⟨L 0,1⟩ = e^{2πia} + e^{-2πia}
⟨T 1/2⟩ = ⟨L 1,0⟩ = (e^{2πib} + e^{-2πib})
⟨L 1,±1⟩ = (e^{2πi(b±a)} + e

Ex. $SU(2)$ $\mathcal{N}=2^*$

$$a \to \begin{pmatrix} a & \\ & -a \end{pmatrix} \qquad b \to \begin{pmatrix} b & \\ & -b \end{pmatrix}$$

$$L_{\underset{\text{mag}}{p},\,\underset{\text{el}}{q}}\,, \quad p,q \in \mathbb{Z}$$

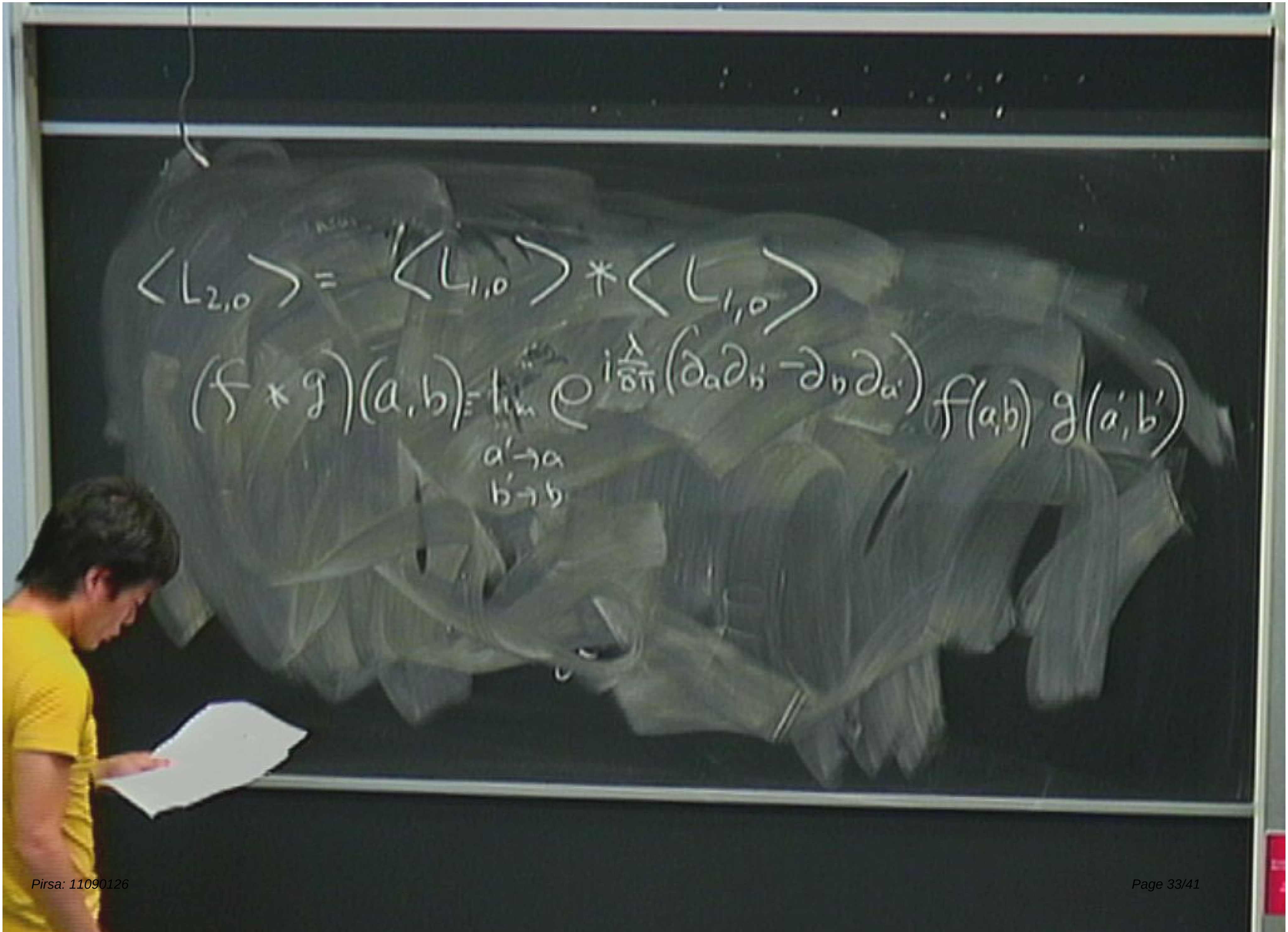
spin $\frac{q}{2}$ rep.

$$\langle W_{1/2}\rangle = \langle L_{0,1}\rangle = e^{2\pi i a} + e^{-2\pi i a}$$

$$\langle T_{1/2}\rangle = \langle L_{1,0}\rangle = \left(e^{2\pi i b} + e^{-2\pi i b}\right)\left(\prod_{\pm} \frac{\sin(2\pi a \pm \pi m)}{\sin(2\pi a \pm \frac{\pi}{2}\lambda)}\right)^{1/2}$$

$$\langle L_{1,\pm 1}\rangle = \left(e^{2\pi i (b\pm a)} + e^{-2\pi i (b\pm a)}\right)\left(\; ; \; \right)$$

$$\langle L_{2,0}\rangle = \dots$$

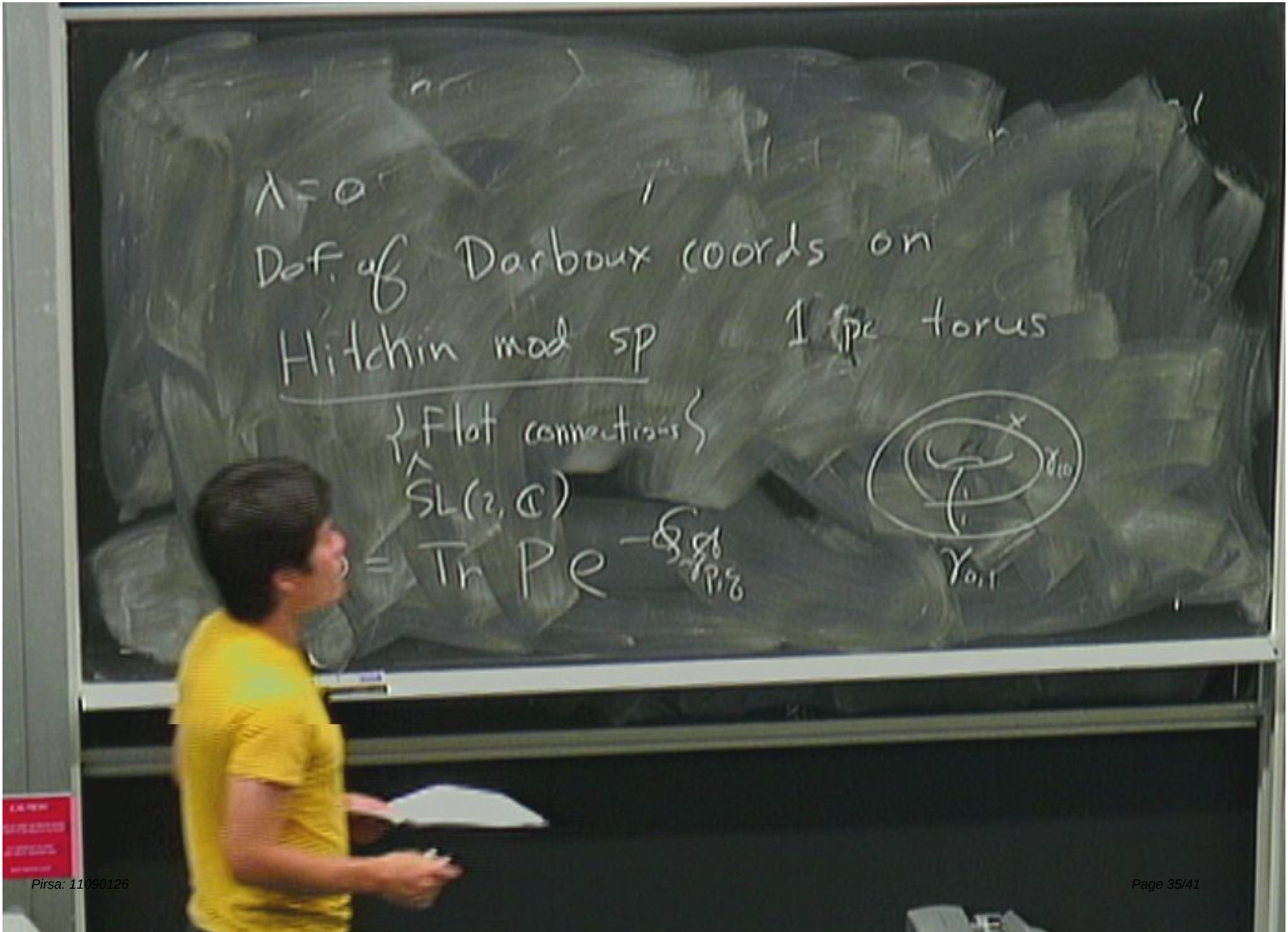

$\langle L_{2,0} \rangle = \langle L_{1,0} \rangle * \langle L_{1,0} \rangle$
$(f * g)(a,b) = \lim_{\substack{a' \to a \\ b' \to b}} e^{i \frac{\hbar}{8\pi} (\partial_a \partial_{b'} - \partial_b \partial_{a'})} f(a,b)\, g(a',b')$

$$\langle L_{2,0} \rangle = \langle L_{1,0} \rangle * \langle L_{1,0} \rangle$$

$$(f * g)(a,b) = \lim_{\substack{a' \to a \\ b' \to b}} e^{i\frac{\lambda}{8\pi}(\partial_a \partial_{b'} - \partial_b \partial_{a'})} f(a,b)\, g(a',b')$$

$$\hbar = \frac{\lambda}{2\pi}$$

Symplectic form $2\, da \wedge db$


λ=0
Def. of Darboux coords on
Hitchin mod sp
1 pc torus
{Flat connections}
SL(2, ℂ)
= Tr P e
γ1,0
γ0,1

$\lambda = 0$

Def. of Darboux coords on

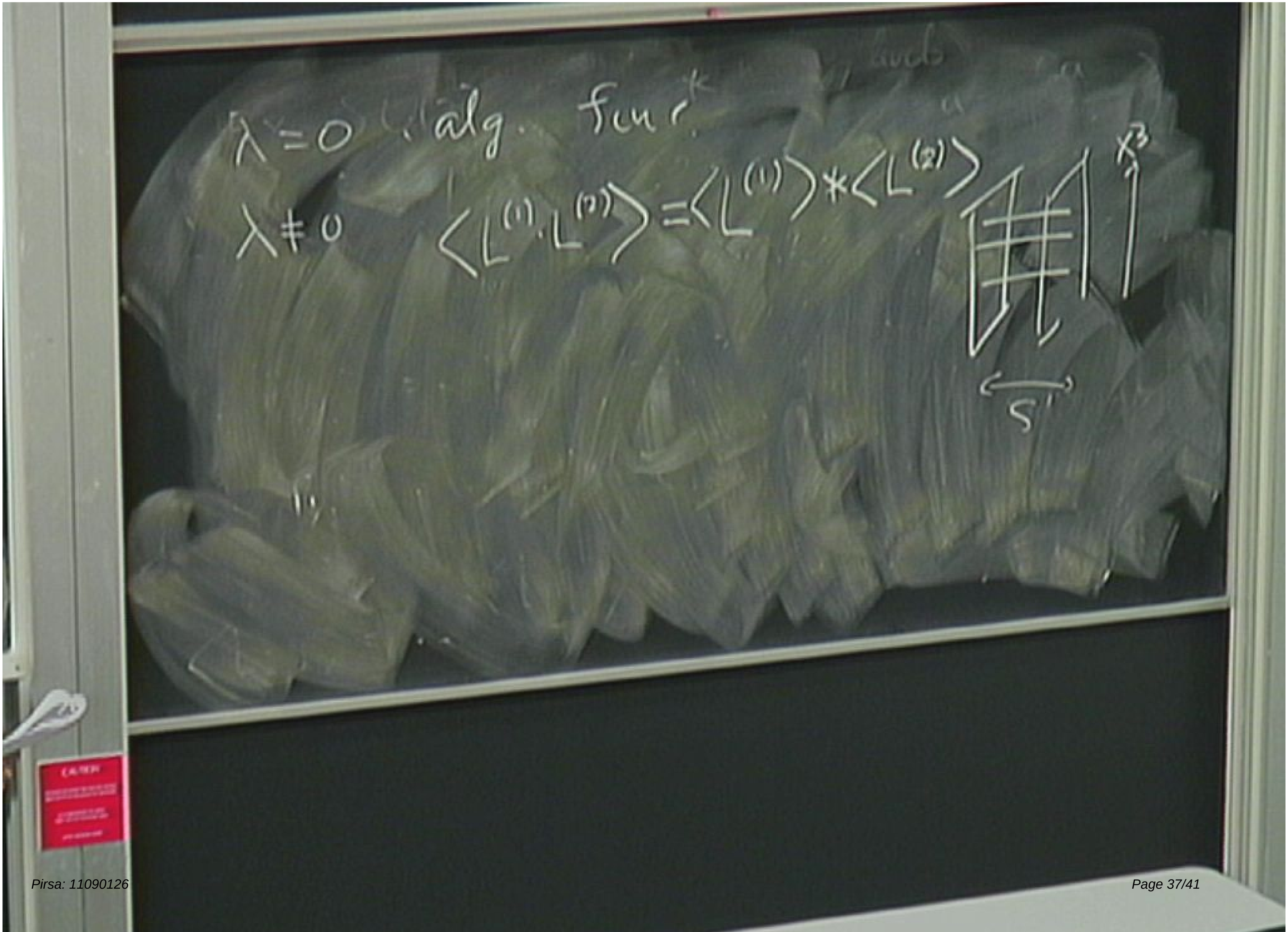
Hitchin mod sp 1 pc torus

$\{\text{Flat connections}\}$

$\widehat{SL}(2,\mathbb{C})$

$$\langle L_{p,q} \rangle = \mathrm{Tr}\, P\, e^{-\oint_{\gamma_{p,q}} \mathcal{A}}$$

$$\otimes = \begin{pmatrix} e^{2\pi i \alpha} & \\ & e^{-2\pi i \alpha} \end{pmatrix}$$

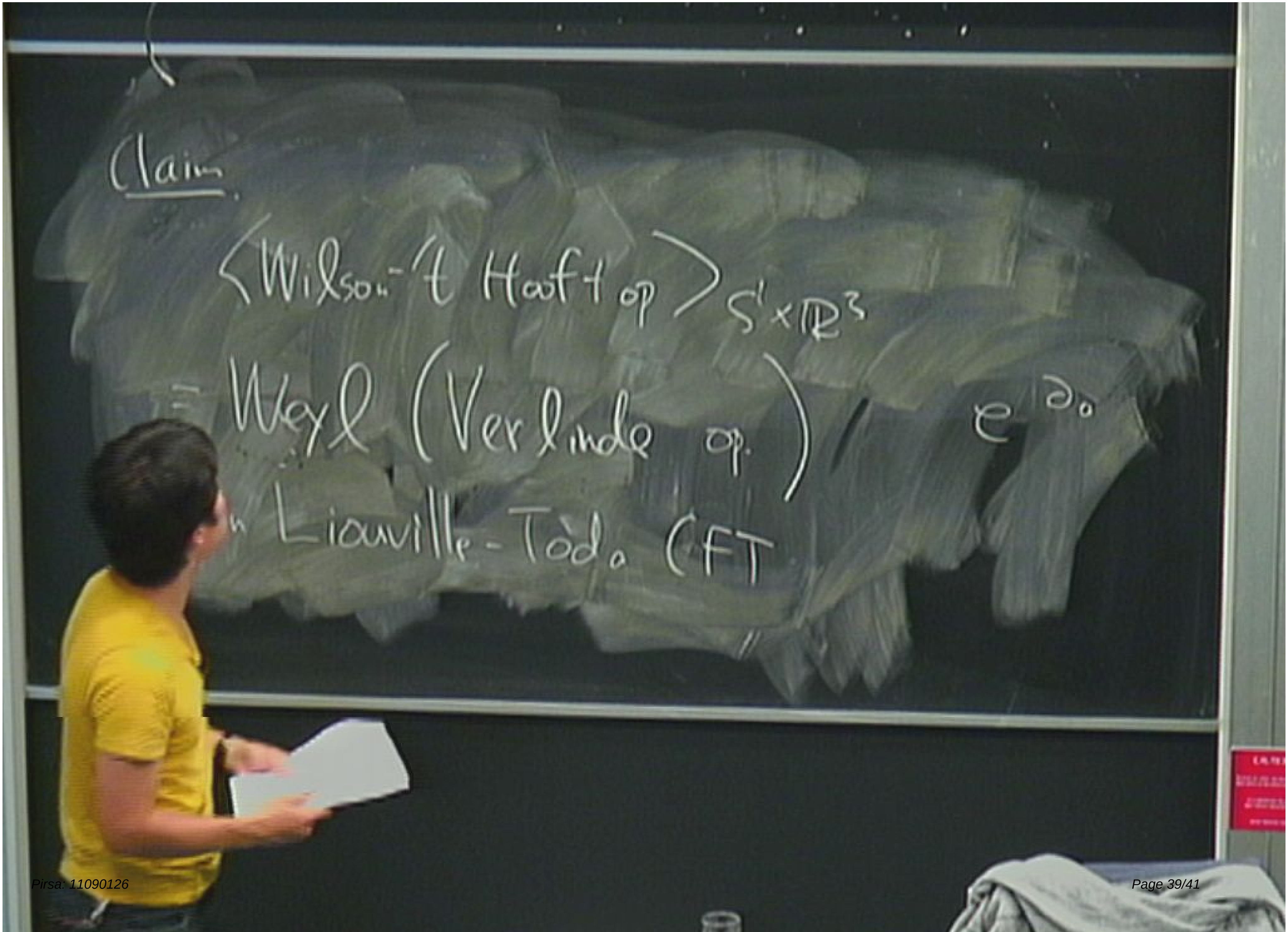

$\lambda = 0$ alg. func*
$\lambda \neq 0 \quad \langle L^{(1)} \cdot L^{(2)} \rangle = \langle L^{(1)} \rangle * \langle L^{(2)} \rangle$
x^3
S^1

$\lambda = 0$: alg. func

$\lambda \neq 0$ $\quad \langle L^{(1)} \cdot L^{(2)} \rangle = \langle L^{(1)} \rangle * \langle L^{(2)} \rangle$

Q: Hilb space.

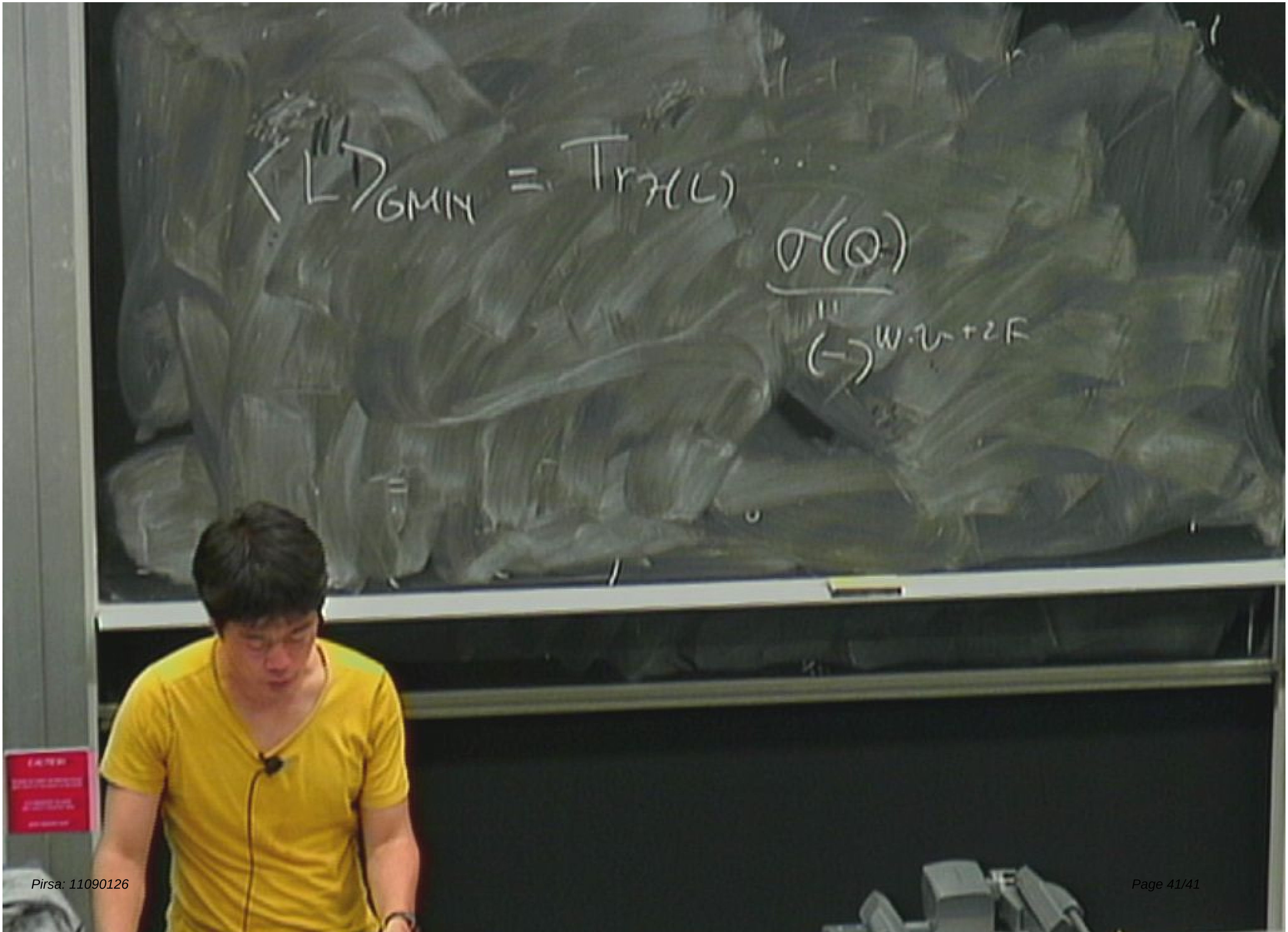
A: Space of conformal blocks


Claim
⟨Wilson-'t Hooft op⟩ S¹×ℝ³
= Weyl (Verlinde op.) e ∂₀
in Liouville-Toda CFT

Claim:

$$\langle \text{Wilson–'t Hooft op} \rangle_{S^1 \times \mathbb{R}^3}$$

$$= \text{Weyl}\left(\begin{array}{l}\text{Verlinde op.} \\ \text{in Liouville–Toda CFT}\end{array}\right) e^{\partial_0}$$


⟨L⟩GMN = Tr H(L) . . .
σ(Q)
(−)W·v+2F