Title: Loop Gravity as the Dynamics of Topological Defects

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Abstract: A charged particle can detect the presence of a magnetic field confined into a solenoid. The strength of the effect depends only on the phase shift experienced by the particle's wave function, as dictated by the Wilson loop of the Maxwell connection around the solenoid. In this seminar I'll show that Loop Gravity has a structure analogous to the one relevant in the Aharonov-Bohm effect described above: it is a quantum theory of connections with curvature vanishing everywhere, except on a 1d network of topological defects. Loop states measure the flux of the gravitational magnetic field through a defect line. A feature of this reformulation is that the space of states of Loop Gravity can be derived from an ordinary QFT quantization of a classical diffeomorphism-invariant theory defined on a manifold. I'll discuss the role quantum geometry operators play in this picture, and the perspective of formulating the Spin Foam dynamics as the local interaction of topological defects.

Loop Gravity as the dynamics of topological defects

Eugenio Bianchi

Perimeter Institute

Quantum Gravity
seminar series
Time Room: 4pm
Sept 21, 2011

Loop Quantum Gravity

⇓ GR + QM + " a technical assumption about Loops "

* mathematically robust quantum theory
* rich Planck-scale quantum geometry
* evidence: GR in the classical limit

Present formulations:

a) Loop representation [Rovelli-Smolin '88]
➡ Ashtekar-Lewandowski measure for generalized connections

b) Discrete classical theory, "twisted geometries" [Freidel-Speziale '10]
Quantization ➡ Spin Foams

In this talk:

new formulation of Loop Gravity that uses only Field Theoretical Methods

c) Classical field theory of locally-flat connections with topological defects
Quantization ➡ Loop Gravity and Spin Foams [EB, arXiv:0907.4388]

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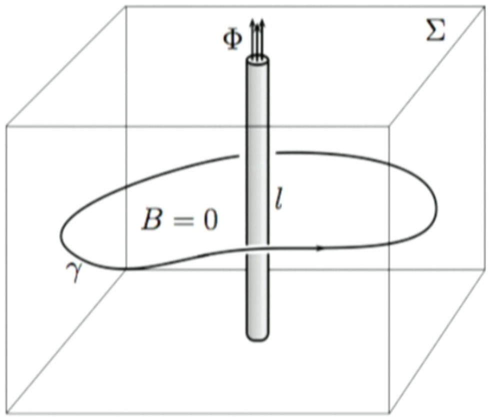
Loop Gravity as the dynamics of topological defects

Space = 3-manifold Σ of trivial topology



- Erase from the manifold a line l
  = defect

- Locally-flat connections $A_a^i(x)$ on $\Sigma - l$

  ➡ finite number of d.o.f. , just one modulus,

  captured by a Wilson loop around the defect

- Similarly for a network of line-defects

- Quantization of this system ➡ Hilbert space of LQG, sector associated to a graph

  [cf. 2+1 QG with point particles]

- Dynamics = local interaction of defects, locally Lorentz invariant and Diff invariant

  ➡ Topological theory with many d.o.f.
  GR as Effective description ?

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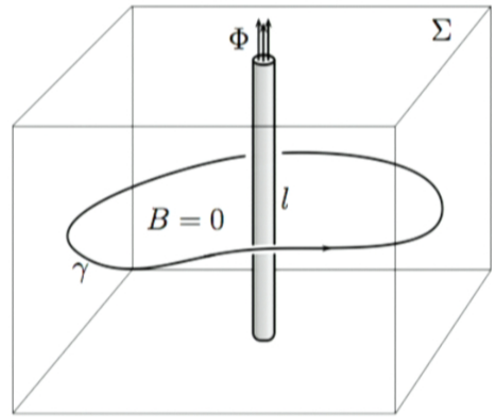

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The "Loop" assumption of Loop Quantum Gravity

Microscopic d.o.f. of Gravity are gravitational connections *A*
with *distributional magnetic field supported on defect lines.*

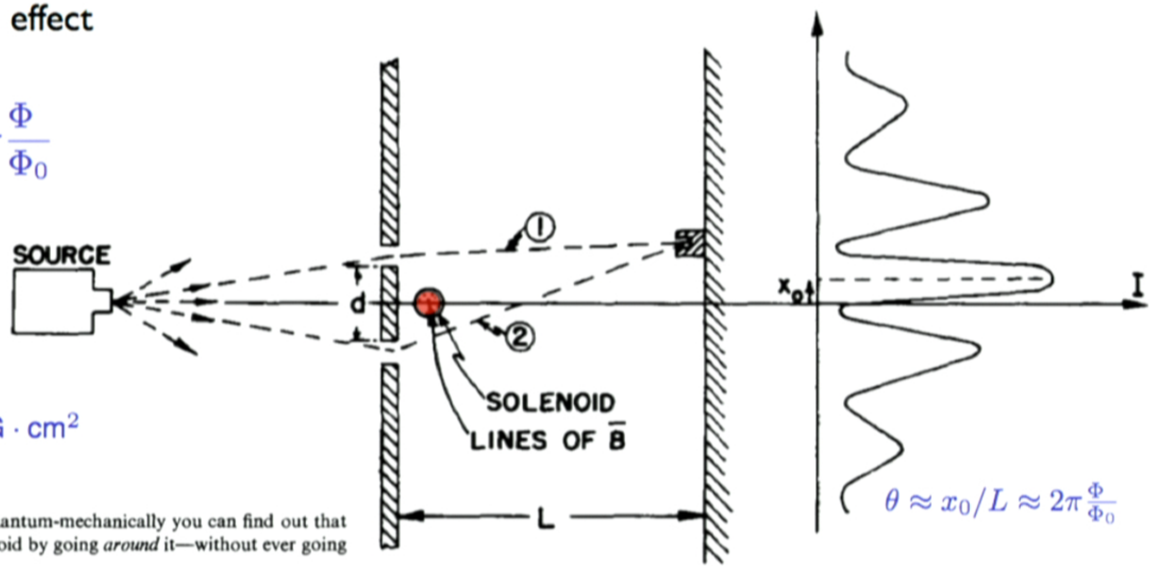
Finite number of gravitational d.o.f., completely captured by *Wilson loops*

Diff-invariant truncation of GR. It is not a UV cutoff, nor an IR one.

cf. Aharonov-Bohm effect

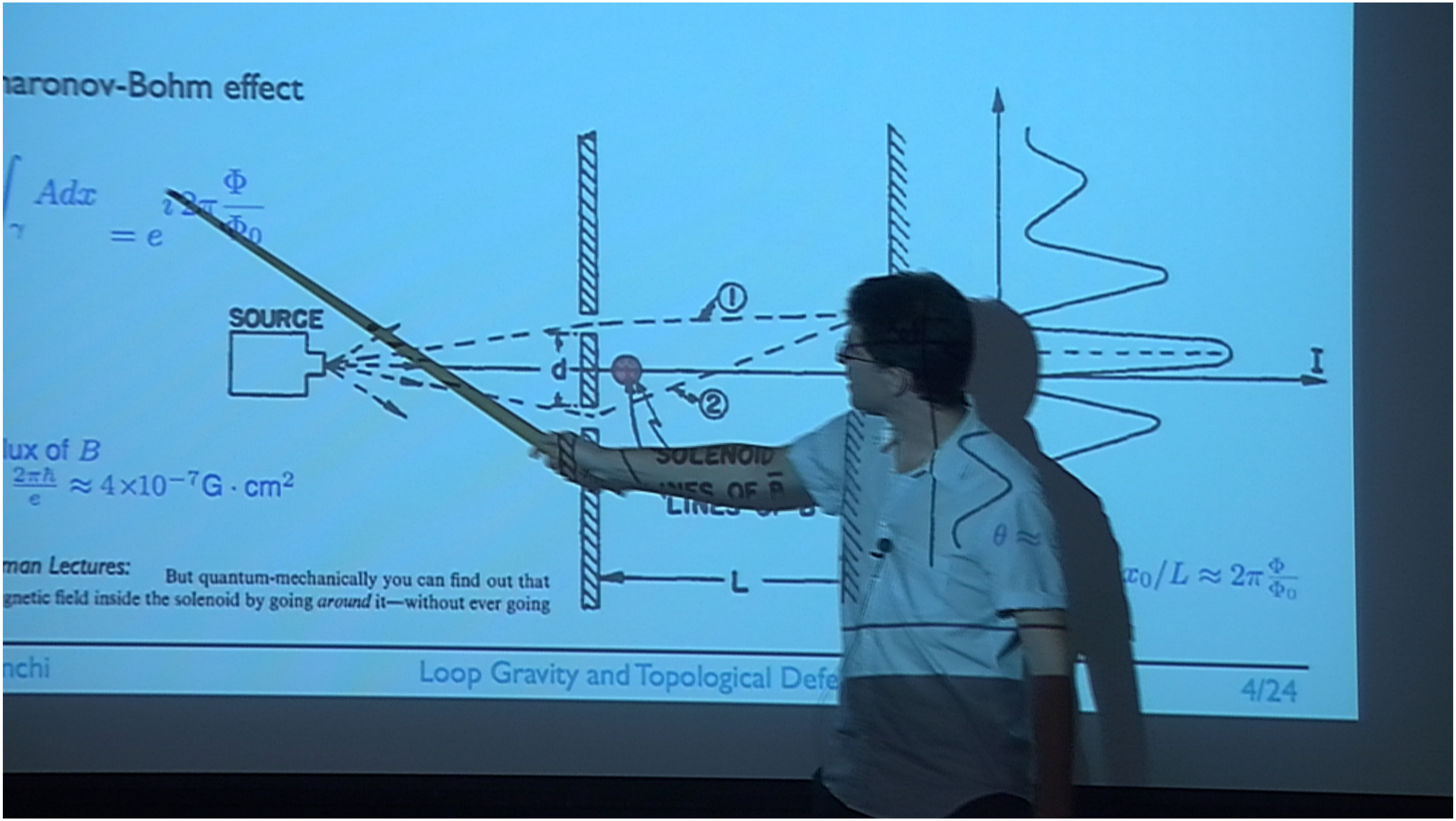
$$e^{\frac{i}{\hbar} e \int_\gamma A dx} = e^{i\, 2\pi \frac{\Phi}{\Phi_0}}$$

$\Phi =$ flux of B
$\Phi_0 = \frac{2\pi\hbar}{e} \approx 4\times 10^{-7}\,\mathrm{G}\cdot\mathrm{cm}^2$



$\theta \approx x_0/L \approx 2\pi \frac{\Phi}{\Phi_0}$

From Feynman Lectures: But quantum-mechanically you can find out that there is a magnetic field inside the solenoid by going *around* it—without ever going close to it!



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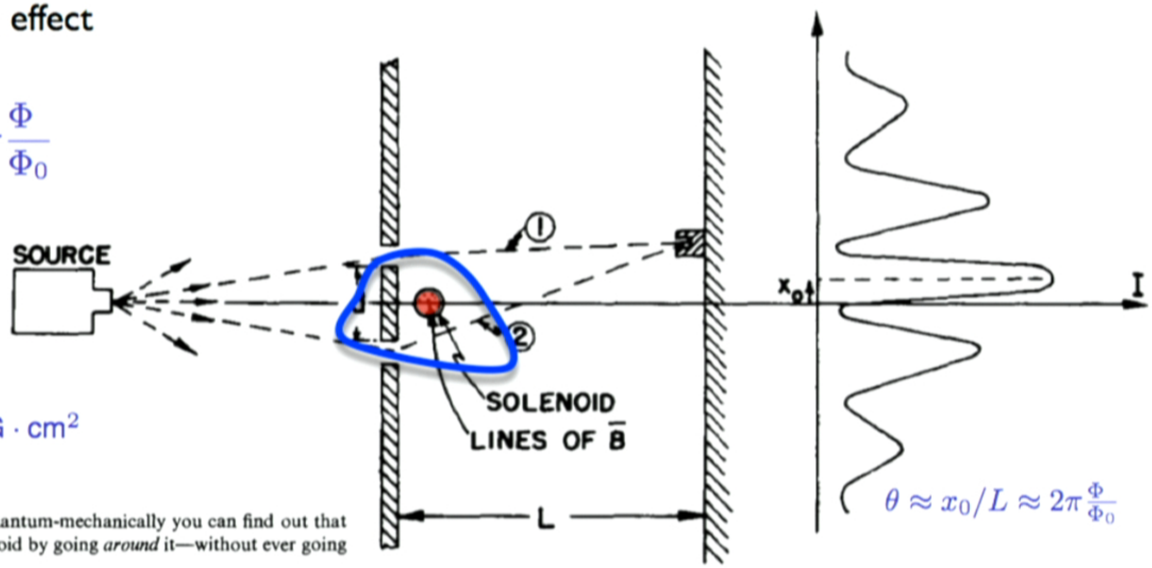

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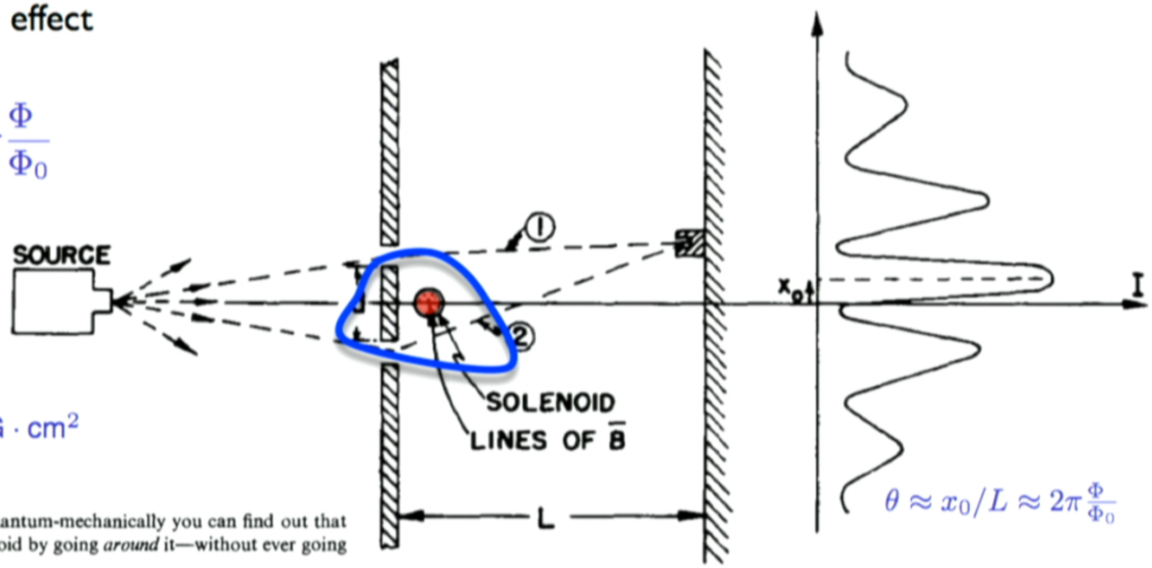

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Loop Gravity as the dynamics of topological defects

- Space of states: line-defects in space and Spin Networks

- Dynamics: surface-defects in space-time and Spin Foams

Kinematics of General Relativity in Ashtekar-Barbero variables

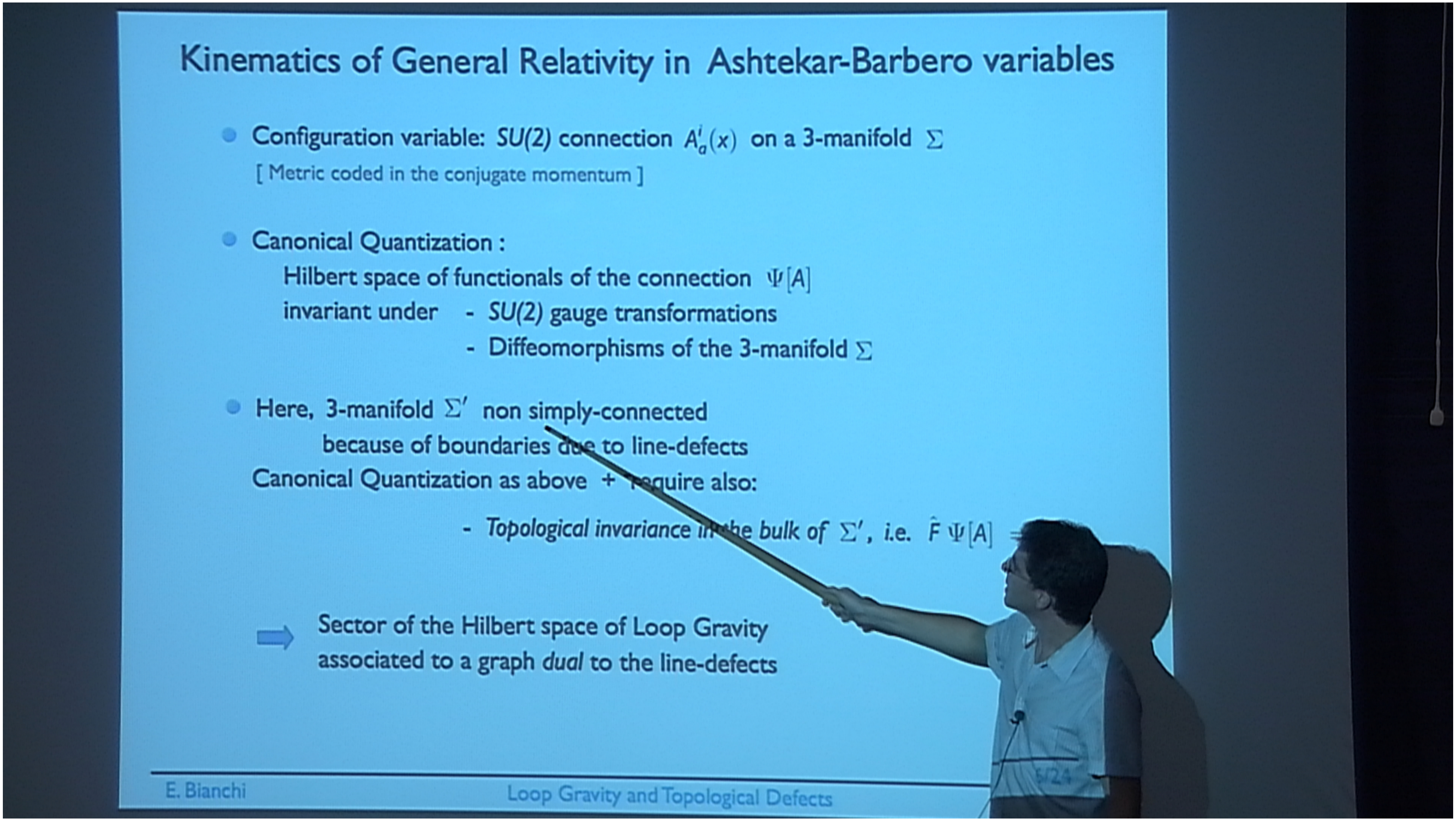
- Configuration variable: *SU(2)* connection $A_a^i(x)$ on a 3-manifold Σ
  [Metric coded in the conjugate momentum]

- Canonical Quantization :
  Hilbert space of functionals of the connection $\Psi[A]$
  invariant under - *SU(2)* gauge transformations
  - Diffeomorphisms of the 3-manifold Σ

- Here, 3-manifold Σ' non simply-connected
  because of boundaries due to line-defects
  Canonical Quantization as above + require also:

  - *Topological invariance in the bulk of* Σ', *i.e.* $\hat{F}\,\Psi[A] = 0$

⇒ Sector of the Hilbert space of Loop Gravity
associated to a graph *dual* to the line-defects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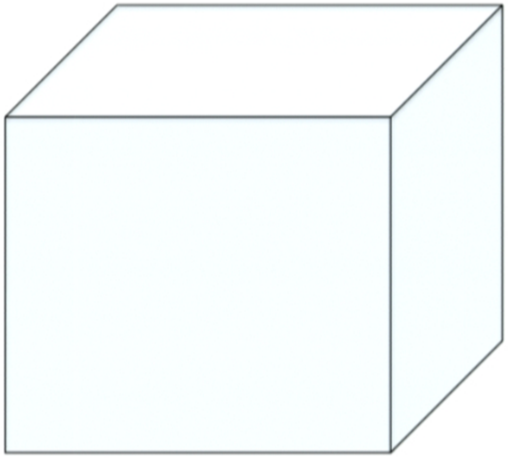
Loop Gravity and Topological Defects

Configuration space of connections with distributional magnetic field

Space = 3-manifold Σ of given topology

Consider a topological decomposition of Σ in cells T

e.g. a triangulation



- Erase from the manifold the 1-skeleton of T

  ⇒ new manifold $\Sigma' = \Sigma - T_1$
  with a network of line-defects

  notice: non-trivial $\pi_1(\Sigma')$

- Locally-flat connections $A_a^i(x)$ on Σ'

  $$\mathcal{A}_{LF} = \{ A : \Sigma' \to SU(2) \,|\, F(A) = 0 \}$$

  ⇒ distributional magnetic field on T_1

Modulo gauge transf., finite dimensional and compact config. space (moduli space)

$$\mathcal{N} \;=\; \mathcal{A}_{LF}/\mathcal{G} = \mathrm{Hom}(\pi_1(\Sigma'), SU(2)) \;/\; SU(2)$$

Coordinates (*moduli*): $(m_1, \ldots, m_R) \in \mathcal{N}$

- Phase space: $(m_r, p^r) \in T^*\mathcal{N}$

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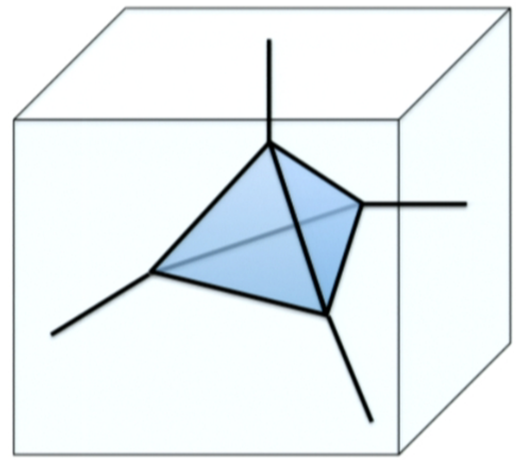

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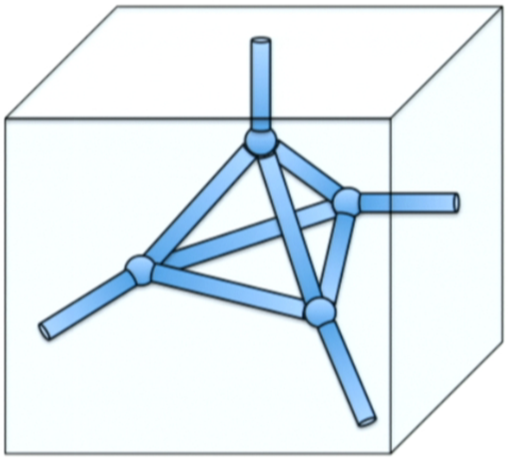

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Wavefunctions and the Scalar Product

The functional integral $\int D[A]$ in the scalar product reduces to an ordinary integral:

use $A = g^{-1}\bar{A}(m_r)g + g^{-1}dg$

$$\langle 1|2\rangle = \int_{\mathcal{A}_{LF}/\mathcal{G}} D[A]\ \overline{\Psi_1[A]}\ \Psi_2[A]$$

gauge-fixing + Faddeev-Popov det.

$$= \int_{\mathcal{A}} D[A]\ \delta[F(A)]\ \delta[\chi(A)]\ \Delta_{FP}[A]\ \overline{\Psi_1[A]}\ \Psi_2[A]$$

$$= \int d\bar{A}(m_r)\ \Delta_{FP}[\bar{A}(m_r)]\ \overline{\Psi_1[\bar{A}(m_r)]}\ \Psi_2[\bar{A}(m_r)]$$

change of variables to *moduli*

$$= \int_{\mathcal{N}} dm_1 \cdots dm_R\ J(m_r)\ \Delta_{FP}(m_r)\ \overline{\psi_1(m_r)}\ \psi_2(m_r)$$

- QFT measure $d\mu_{QFT}(m_r) = J(m_r)\,\Delta_{FP}(m_r)\,dm_r$

  compare to
  string measure: Polyakow 1981, Alvarez 1983
  2+1 gravity with and w/o particles: Carlip 1995, Cantini Menotti 2002
  simplicial measure: Jevicki Ninomiya 1986, Menotti Peirano 1995

- Faddeev-Popov determinant: in general, very difficult to determine
  Here, advantage of Gauge connections: ⟹ *LQG*
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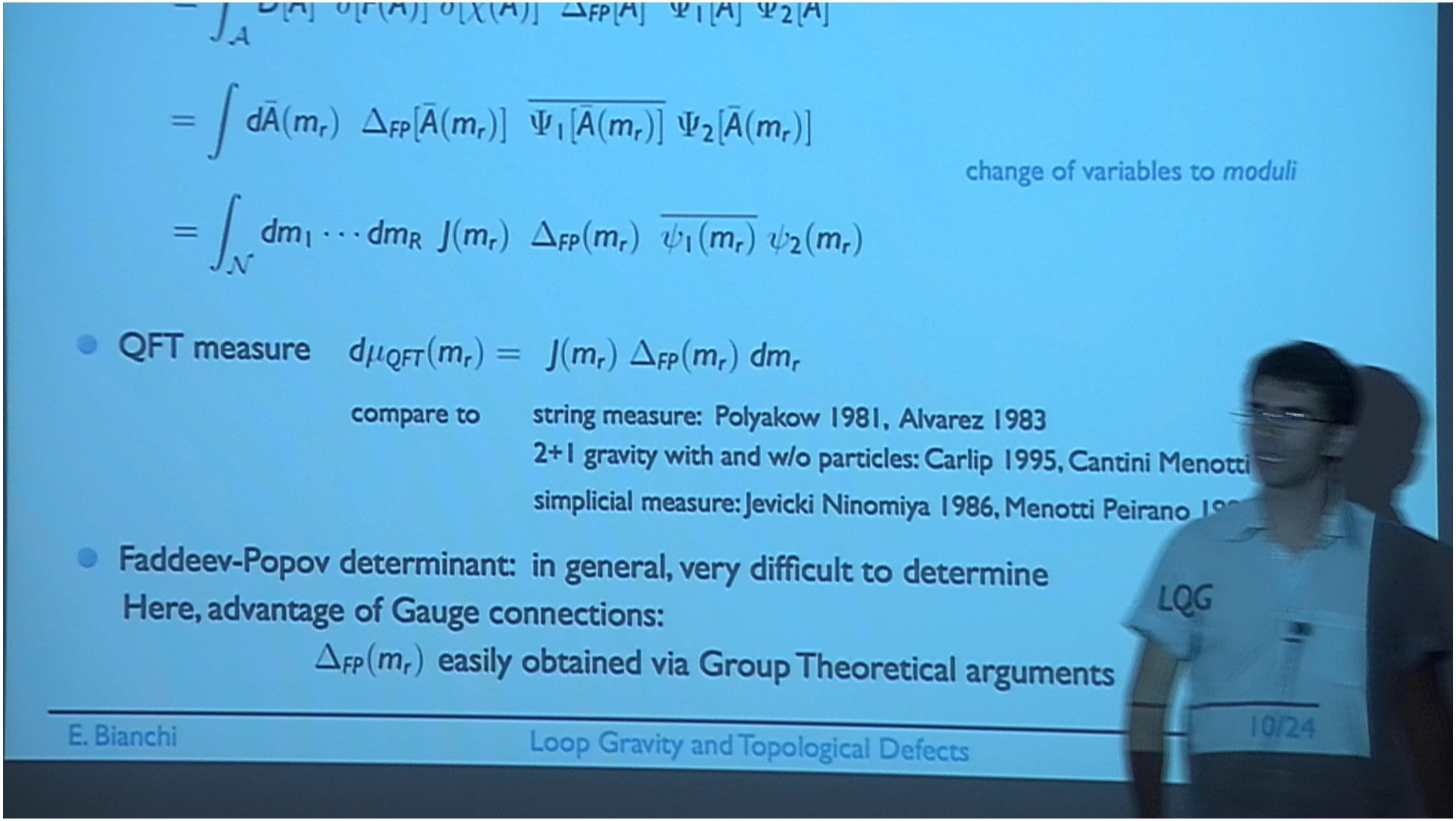
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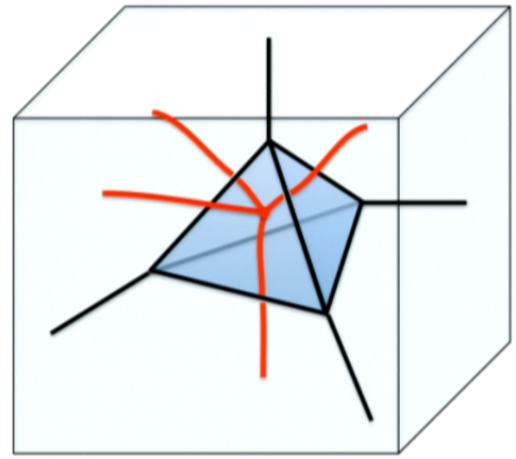
Wavefunctions and *Spin-Network states*

Consider a graph Γ *dual* to the cellular decomposition T



- The edge-path group of Γ is isomorphic to the first homotopy group of $\Sigma' = \Sigma - T_1$

$$\pi(\Gamma) \sim \pi_1(\Sigma')$$

Moduli space $\mathcal{N}$ equivalently described by

$$\mathcal{N} = \mathrm{Hom}(\,\pi(\Gamma)\,, SU(2))\,/\,SU(2)$$

- Spin-Network state: functional of A labeled by Γ and a function $f : SU(2)^L/SU(2)^N \to \mathbb{C}$

$$\Psi_{\Gamma,f}[A] = f(h_{l_1}[A], \ldots, h_{l_L}[A])$$

where l is a link of the graph and $h_l[A] = P\exp i\int_l A$

- Diff-invariance: largely independent from the embedding of Γ, only sensitive to the knotting with the defects

After gauge-fixing, reduces to a function of moduli $f(h_{l_1}[A], \ldots, h_{l_L}[A]) = \psi(m_r)$

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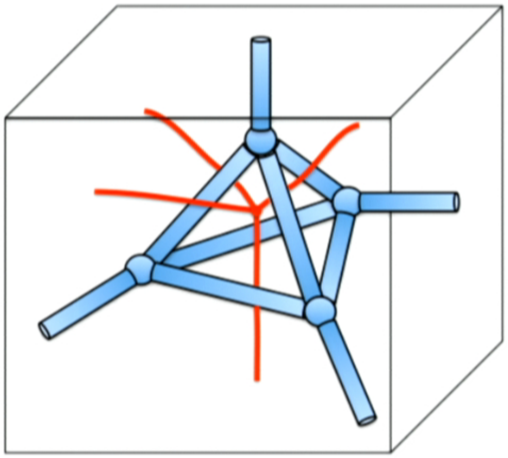

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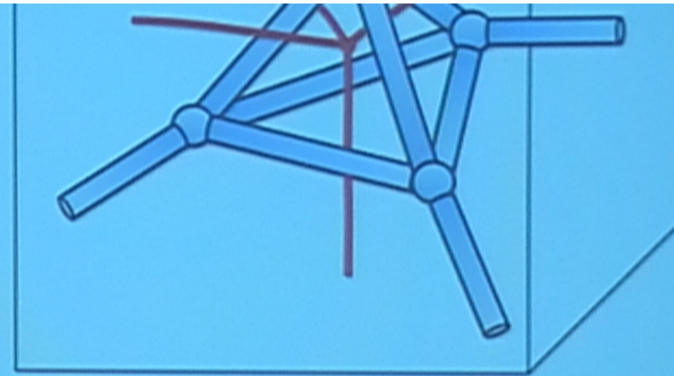

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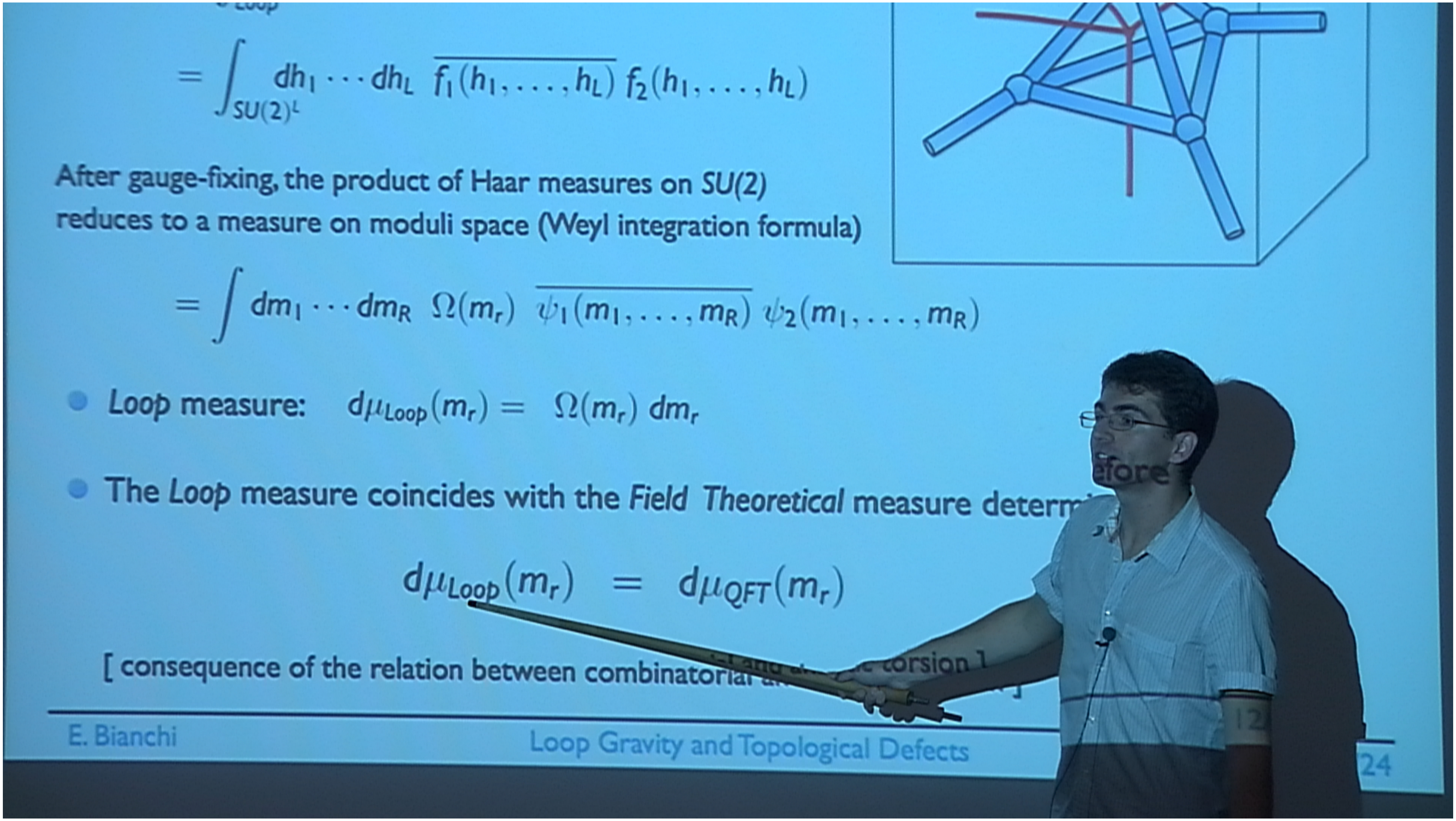

$= \int_{SU(2)^L} dh_1 \cdots dh_L \; \overline{f_1(h_1, \ldots, h_L)} \, f_2(h_1, \ldots, h_L)$
After gauge-fixing, the product of Haar measures on SU(2)
reduces to a measure on moduli space (Weyl integration formula)
$= \int dm_1 \cdots dm_R \; \Omega(m_r) \; \overline{\psi_1(m_1, \ldots, m_R)} \, \psi_2(m_1, \ldots, m_R)$
Loop measure: $d\mu_{Loop}(m_r) = \Omega(m_r) \, dm_r$
The Loop measure coincides with the Field Theoretical measure determ
$d\mu_{Loop}(m_r) = d\mu_{QFT}(m_r)$
[consequence of the relation between combinatorial
torsion]
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Loop Gravity as the dynamics of topological defects

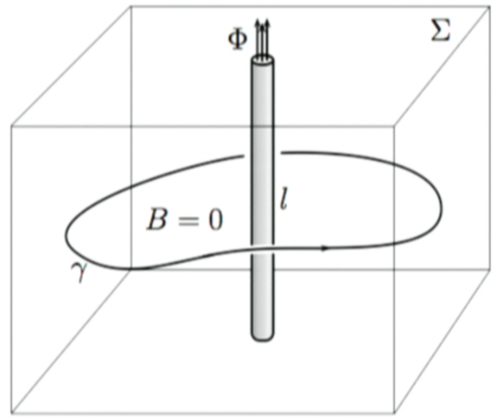
- An example: single line-defect and the Wilson loop

A single defect-line

Gauge-fixed locally-flat connection Σ'

$$A^i_a(x) = \frac{\Phi^i}{2\pi}\alpha_a(x) \qquad \alpha_a(x) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right)$$

[Coulomb gauge $\chi^i = \delta^{ab}\partial_a A^i_a = 0$]



- Holonomy along a loop around the defect

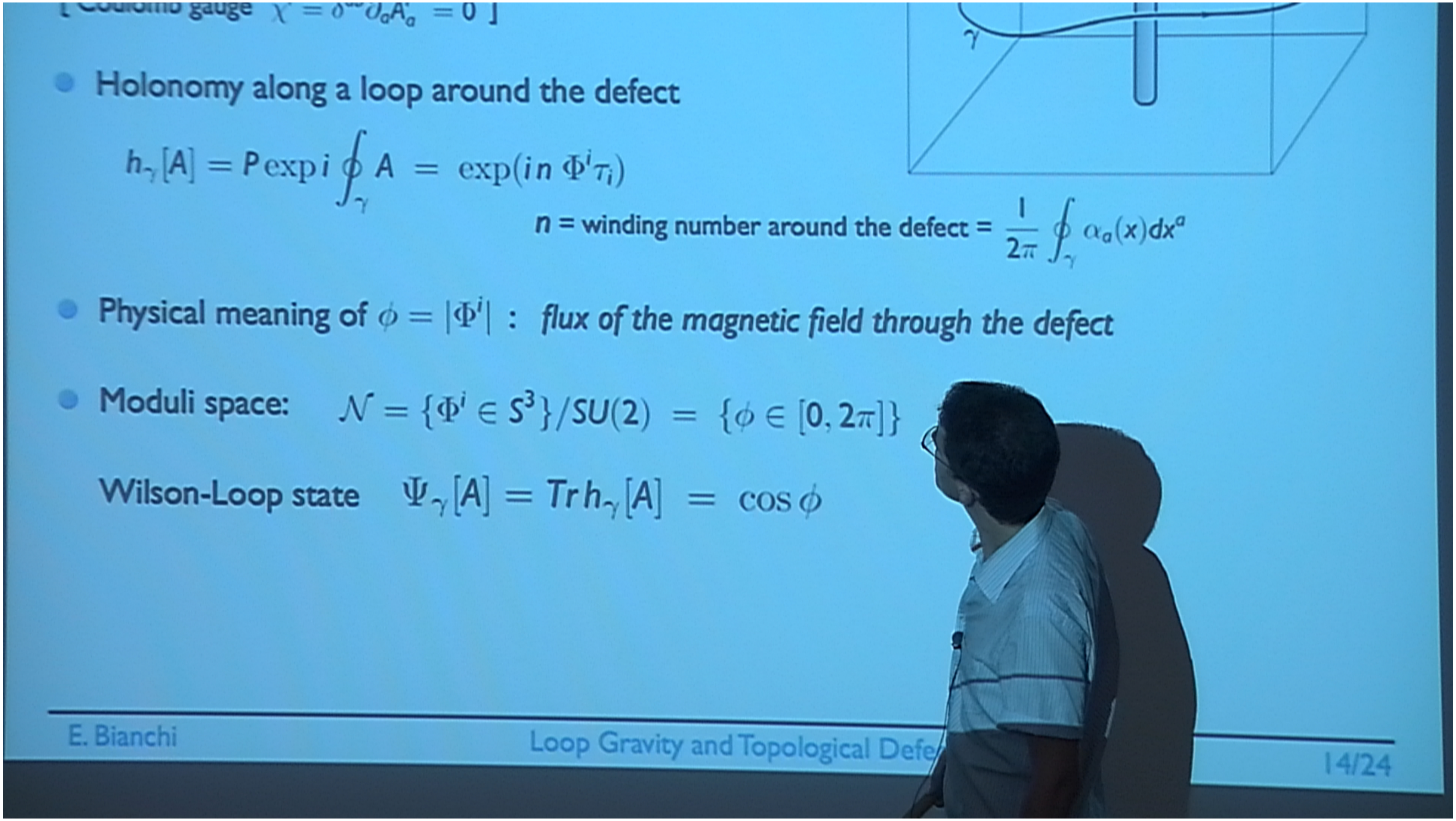
$$h_\gamma[A] = P\exp i\oint_\gamma A = \exp(i n\, \Phi^i \tau_i)$$

n = winding number around the defect = $\frac{1}{2\pi}\oint_\gamma \alpha_a(x)dx^a$

- Physical meaning of $\phi = |\Phi^i|$: *flux of the magnetic field through the defect*

- Moduli space: $\mathcal{N} = \{\Phi^i \in S^3\}/SU(2) = \{\phi \in [0, 2\pi]\}$

  Wilson-Loop state $\Psi_\gamma[A] = Tr\, h_\gamma[A] = \cos\phi$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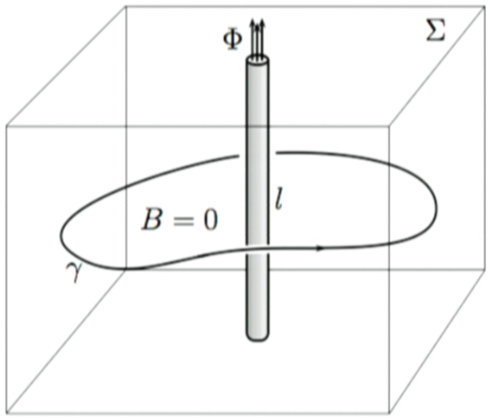

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Momentum conjugate to ϕ, gauge-invariant part of the electric field, *role of the metric*

$\hat{p} = -i\frac{\partial}{\partial\phi} - \frac{i}{2}\cot\frac{\phi}{2}$ ⇒ discrete spectrum $p_j = j(j+1) + \frac{1}{4}$

A single defect-line: scalar product

- Loop scalar product: $d\mu_{Loop}(\phi) = \frac{1}{\pi}\sin^2\frac{\phi}{2}\, d\phi$

  Haar measure on class functions reduces to

  $$\int_{SU(2)} dh\, \overline{f_1(h)}\, f_2(h) \;= \frac{1}{\pi}\int_0^{2\pi} d\phi\, \sin^2\frac{\phi}{2}\, \overline{\psi_1(\phi)}\, \psi_2(\phi)$$

- QFT measure on moduli space $d\mu_{QFT}(\phi) = J(\phi)\,\Delta_{FP}(\phi)\, d\phi$

  Jacobian easily determined: $d^3\Phi^i = \phi^2 d\phi d^2 v^i$, so that $J(\phi) = \phi^2$

  Faddeev-Popov term: determinant of the operator $K(\Phi^i)$

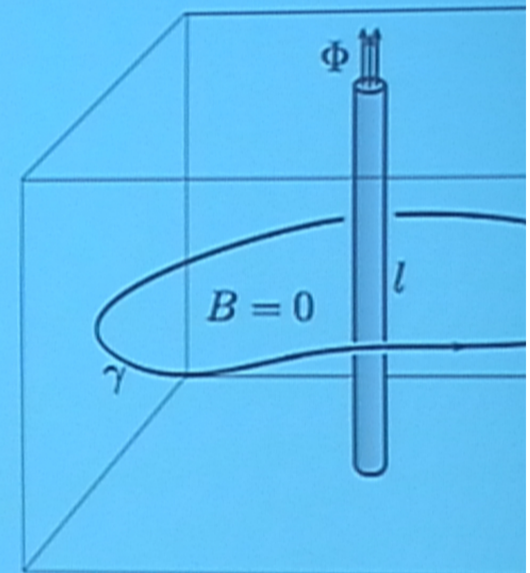
  $$K_{ij}(\Phi^i) = \frac{\delta\chi_i}{\delta\xi^j} = -\delta_{ij}\Delta - \varepsilon_{ijk}\frac{\Phi^k}{2\pi}$$

  Eigenvalues $\lambda_n = n^2 + n\frac{\phi}{2\pi}$ with $n = \pm1, \pm2, \ldots$ and twice degenerate

  Φ^i dependence extracted considering the appropriately regularized ratio

  $$\Delta_{FP}(\phi) = c\,\frac{\text{Det}K(\Phi^i)}{\text{Det}K(0)} = c\,\frac{\prod_{n=1}^{\infty}\left(\lambda_n(\phi)\right)^2\left(\lambda_{-n}(\phi)\right)^2}{\prod_{n=1}^{\infty}\left(\lambda_n(0)\right)^2\left(\lambda_{-n}(0)\right)^2} =$$

  $$= c\left(\prod^{\infty}\left(1-\left(\frac{\phi}{\;}\right)^2\right)\right)^2 = c\left(\sin \ldots\right.$$



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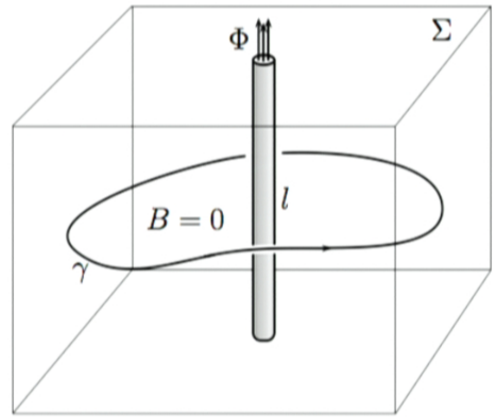

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  $$= c\left(\prod_{n=1}^{\infty}\left(1-\left(\frac{\phi}{2\pi n}\right)^2\right)\right)^2 = c\left(\frac{\sin\phi/2}{\phi/2}\right)^2$$

  The constant c is undetermined and is fixed so that the measure of $\mathcal{N}$ is one

  Therefore: $d\mu_{QFT}(\phi) = \frac{1}{\pi}\sin^2\frac{\phi}{2}\, d\phi$

Loop Gravity as the dynamics of topological defects

- Space of states: line-defects in space and Spin Networks

- Dynamics: surface-defects in space-time and Spin Foams

Spin Foams: the defect-network description

General Relativity = a topological theory + constraints

[Plebanski '77]

$$S[B,\omega] = \int B_{IJ} \wedge F^{IJ}(\omega) \qquad B_{IJ} = \frac{1}{16\pi G}\left(\frac{1}{2}\epsilon_{IJKL}\, e^K \wedge e^L + \frac{1}{\gamma} e_I \wedge e_J \right)$$

- Imposing the constraint everywhere unfreezes infinitely many d.o.f. of the connection ⇒ GR

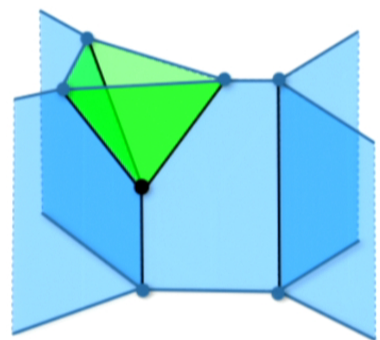
- Diff-Invariant theory with finitely many d.o.f.:
  - Topological decomposition of the space-time 4-manifold in cells
  - Impose the constraints only on the 2-skeleton of $\triangle$

  ⇒ Diff-Invariant truncation of GR

$F(\omega)$ supported on defect-surfaces

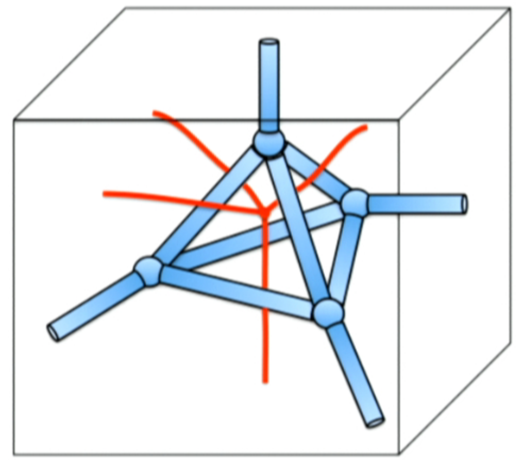
- Quantum Theory ⇒ Spin Foams

$$Z = \int_{\substack{C(B)=0 \ \ on \\ \partial(\mathcal{M}-\Delta_2)}} D[B] \int D[\omega] \ \ e^{\,i \int_{\mathcal{M}-\Delta_2} B^{IJ} \wedge F_{IJ}(\omega)}$$



Spin-Network states and the "Loop" Scalar Product

There is a natural group theoretical scalar product on the space of Spin-Networks



$$\langle 1|2\rangle = \int_{Loop} D[A]\ \overline{\Psi_{\Gamma,f_1}[A]}\ \Psi_{\Gamma,f_2}[A]$$

$$= \int_{SU(2)^L} dh_1 \cdots dh_L\ \overline{f_1(h_1,\ldots,h_L)}\ f_2(h_1,\ldots,h_L)$$

After gauge-fixing, the product of Haar measures on *SU(2)* reduces to a measure on moduli space (Weyl integration formula)

$$= \int dm_1 \cdots dm_R\ \ \Omega(m_r)\ \ \overline{\psi_1(m_1,\ldots,m_R)}\ \psi_2(m_1,\ldots,m_R)$$

- *Loop* measure: $d\mu_{Loop}(m_r) = \ \Omega(m_r)\ dm_r$
- The *Loop* measure coincides with the *Field Theoretical* measure determined before

$$d\mu_{Loop}(m_r) \quad = \quad d\mu_{QFT}(m_r)$$

[consequence of the relation between combinatorial and analytic torsion]

General Relativity = a topological theory + constraints [Plebanski '77]

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General Relativity: a topological theory + constraints

[Plebanski '77]

- Topological theory

$$S[B,\omega] = \int B_{IJ} \wedge F^{IJ}(\omega)$$

[Horowitz '89]

- Reduces to General Relativity if the *B-field* is constrained to be

$$B_{IJ} = \frac{1}{16\pi G}\left(\frac{1}{2}\epsilon_{IJKL}\, e^K \wedge e^L + \frac{1}{\gamma} e_I \wedge e_J \right)$$

- Decomposing B_{IJ} in rotations L^i and boosts K^i with respect to a timelike vector N^I the constraint implies that

(linear simplicity constraint)

$$\vec{L} + \gamma \vec{K} = 0$$

Engle-Pereira-Rovelli '07
Freidel-Krasnov '07
Livine-Speziale '07
Engle-Livine-Pereira-Rovelli '08

Topological QFT using a cellular decomposition

[Ooguri '92]

$$Z = \int \mathcal{D}B\, \mathcal{D}\omega\ \exp i\, S[B,\omega] = \int \mathcal{D}\omega\ \delta[F(\omega)]$$

- Topological decomposition of the space-time 4-manifold in 4-cells Δ
  Associate holonomies G_e to edges of the 2-complex $\mathcal{C} = \Delta^*$

- Partition function as a product over single cells

$$Z = \int \prod_{e\in\mathcal{C}_1} dG_e \prod_{f\in\mathcal{C}_2} \delta\Big(\prod_{e\in\partial f} G_e\Big) \quad = \int \prod_{ef} dG_{ef} \prod_{v\in\mathcal{C}_0} Z_\Delta(G_{ef})$$

- In Fourier transform (Peter-Weyl decomposition), Ooguri partition function:

$$Z = \int_{SL(2,\mathbb{C})\text{irrep},\mathcal{I}} \prod_f (\rho_f^2 + k_f^2) \prod_v \{ \bigotimes_{e\in v} \mathcal{I} \}$$

Some remarks on Loop Gravity and Topological Defects

Relation to other approaches:

* Quantum Gravity in 2+1 dimensions with particles = *point defects*
  classical and quantum theory both well understood
  Are the methods developed useful in 3+1 dimensions? ➡ Loop Gravity and line-defects

* 't Hooft, *A locally finite model of gravity,* `arXiv:0804.0328`
  Gravity from scattering of straight pieces of string, surrounded by locally flat sections of space-time.
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  group theoretical methods because of the central role of the connection

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Completion to infinitely many d.o.f.:

* The Spin Foam dynamics changes the network of defects in space

➡ larger Hilbert space defined à la Fock $\mathcal{F} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \cdots$

* Notion of *Locality*:

In TQFT, only global degrees of freedom.

In presence of defects, a finite number of d.o.f. is associated to each region of the manifold.

Dual Spin-Network graph has *no* non-local links: dual to a cellular decomposition

[cf Markopoulou Smolin, arXiv:gr-qc/0702044]

The discrete spectrum of quantum-geometry operators sets the scale of the region.

* Appealing scenario for Quantum Gravity:

No trans-Planckian d.o.f. because topological (and therefore finite) at small scales

At larger scales, finitely many d.o.f. which can be described effectively in terms of a local quantum field theory.

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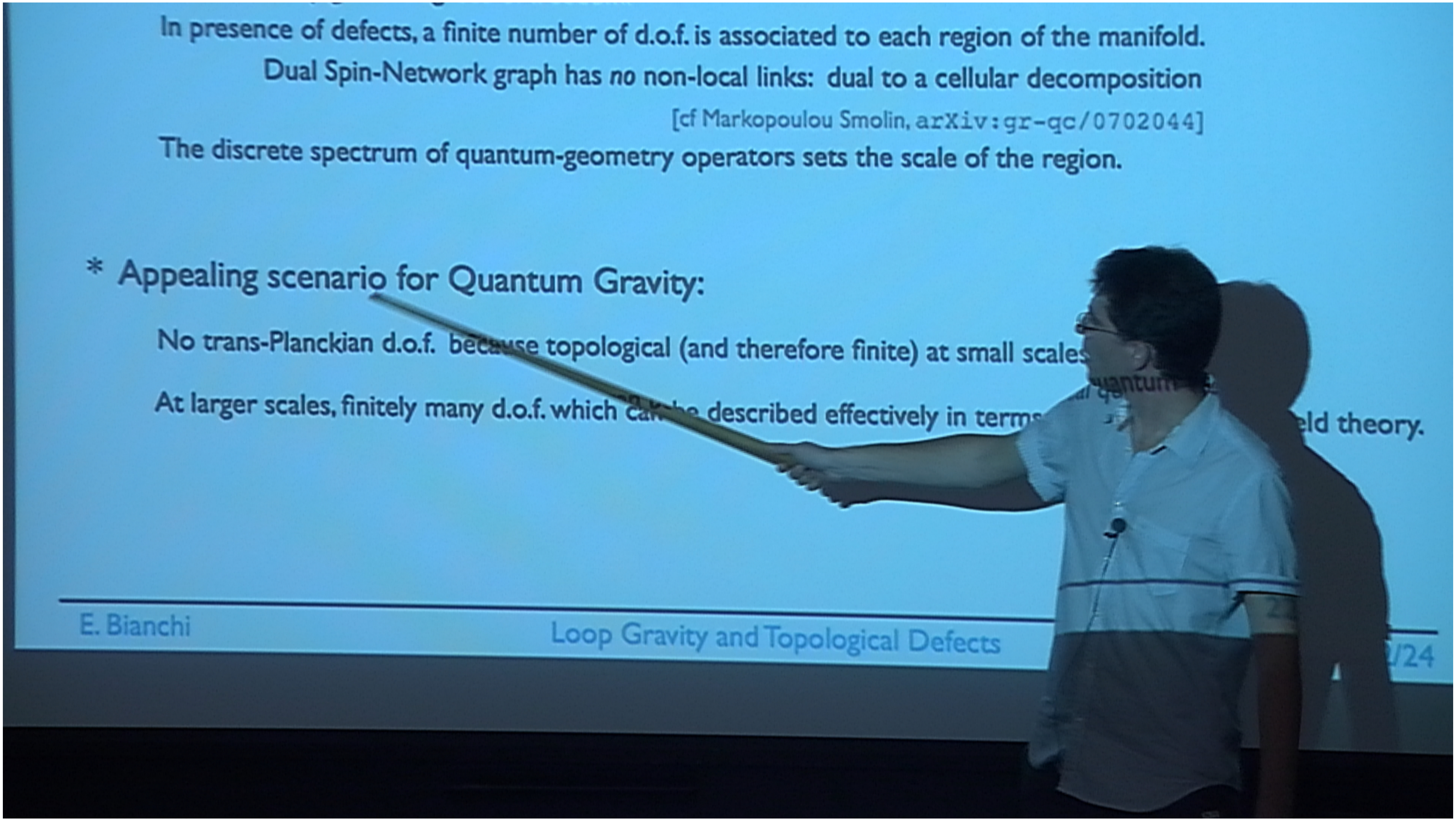
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E. Bianchi
Loop Gravity and Topological Defects
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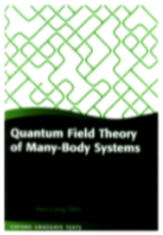
Some remarks on Loop Gravity and Topological Defects

From Wen's book:

Problem 7.1.1.

Deflecting without touching



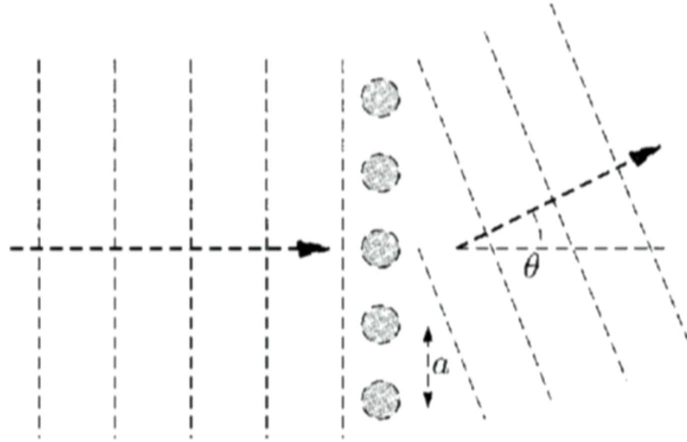


FIG. 7.3. A beam of particles passes through a grid of tubes, with a flux through each tube.

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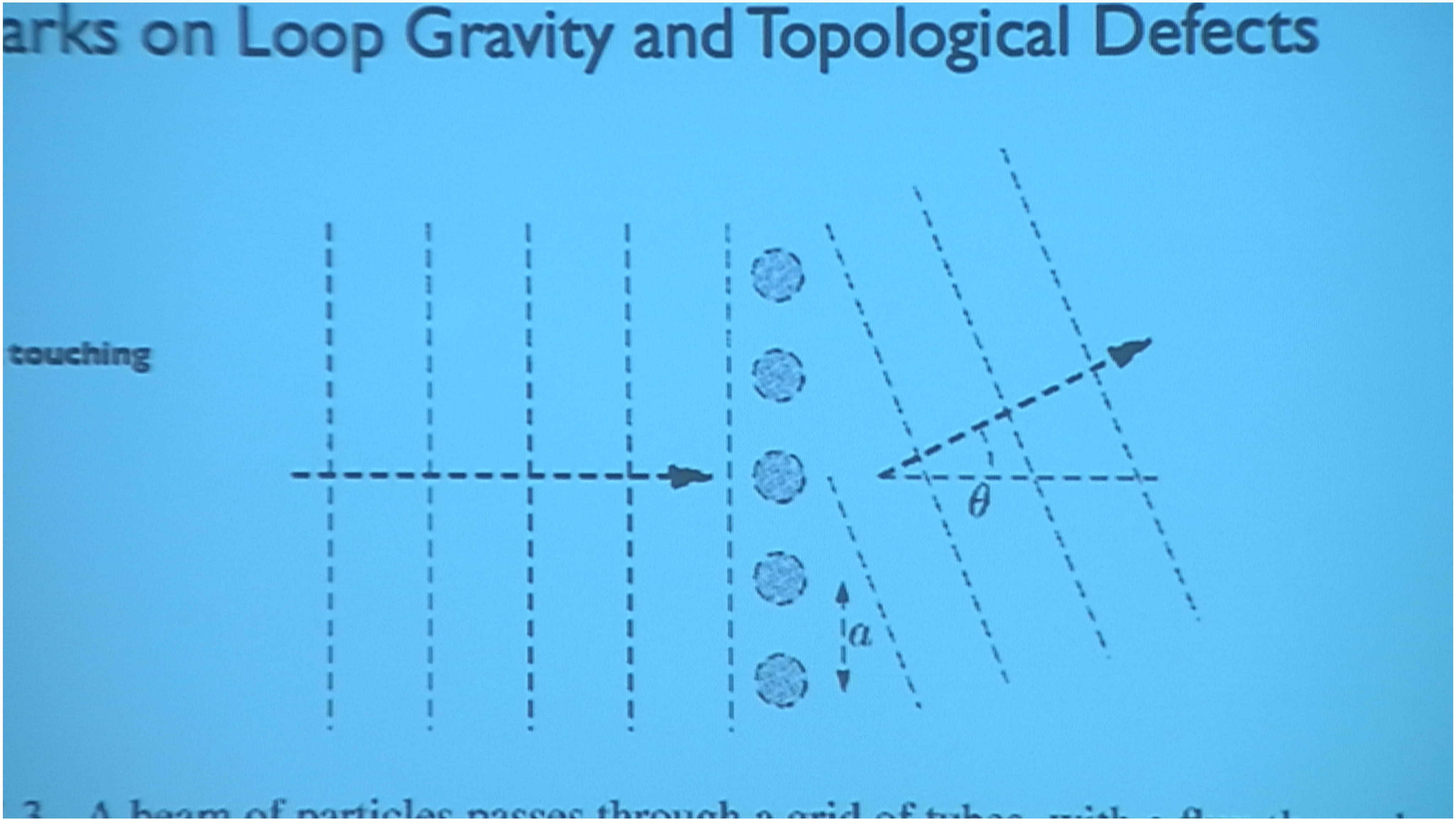
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Summary: Loop Gravity and Topological Defects

- Dual formulation of Loop Gravity:
  not in terms of Spin Networks and Spin Foams,
  ⇒ local Quantum Field Theory with Topological Defects

- Derivation of the Loop Gravity functional measure via QFT methods

- New light on the main technical assumption of Loop Gravity:
  the microscopic d.o.f. of classical and quantum Loop Gravity are
  gravitational connections A with *distributional magnetic field on defects*