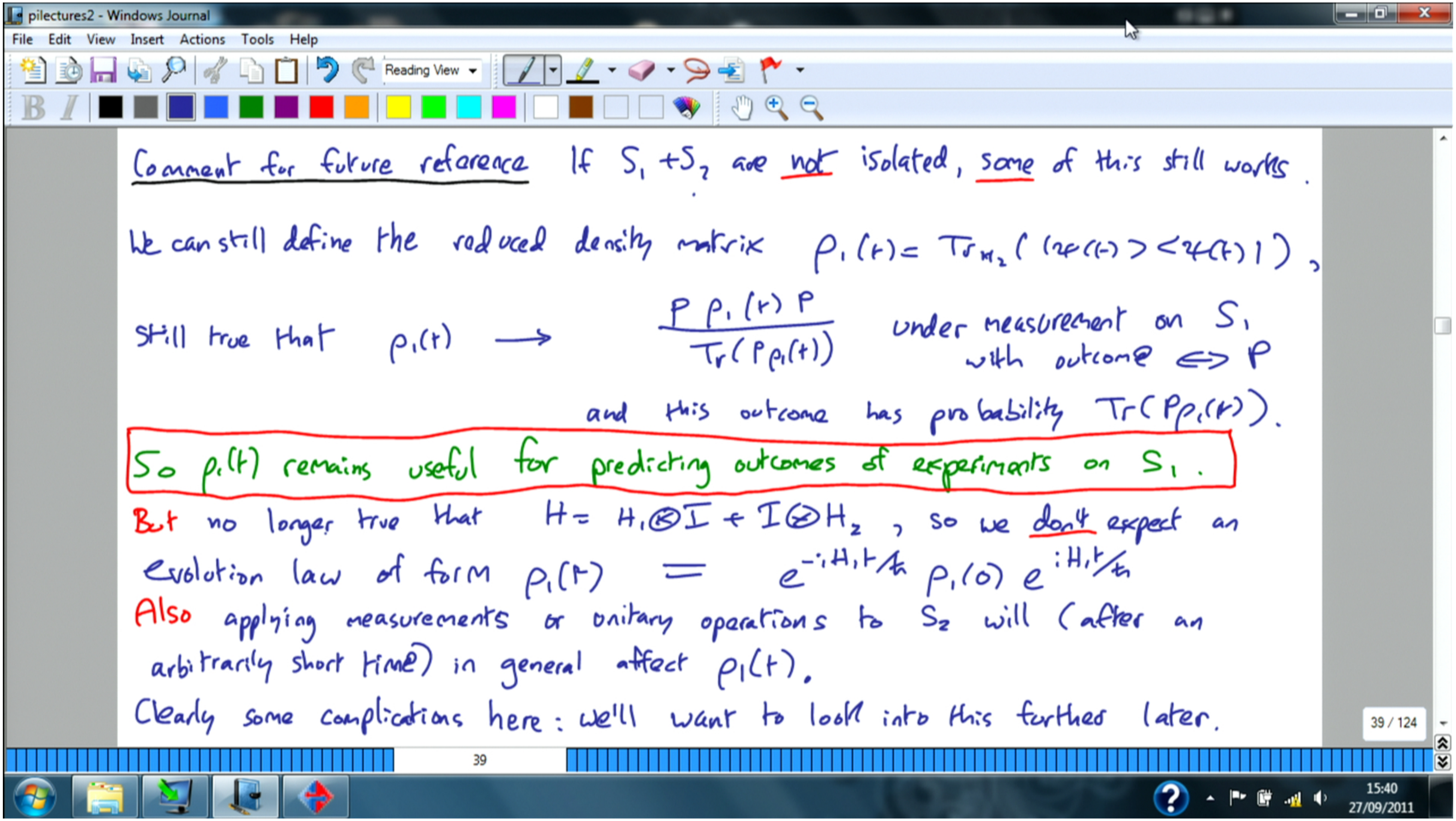
Title: Quantum Theory - Lecture 12

Date: Sep 27, 2011 09:00 AM

URL: http://pirsa.org/11090080

Abstract:



Comment for future reference If $S_1 + S_2$ are not isolated, some of this still works.

We can still define the reduced density matrix $\rho_1(t) = \mathrm{Tr}_{H_2}(\,|\psi(t)\rangle\langle\psi(t)|\,)$,

Still true that $\rho_1(t) \longrightarrow \dfrac{P\,\rho_1(t)\,P}{\mathrm{Tr}(P\rho_1(t))}$ under measurement on S_1 with outcome $\Leftrightarrow P$

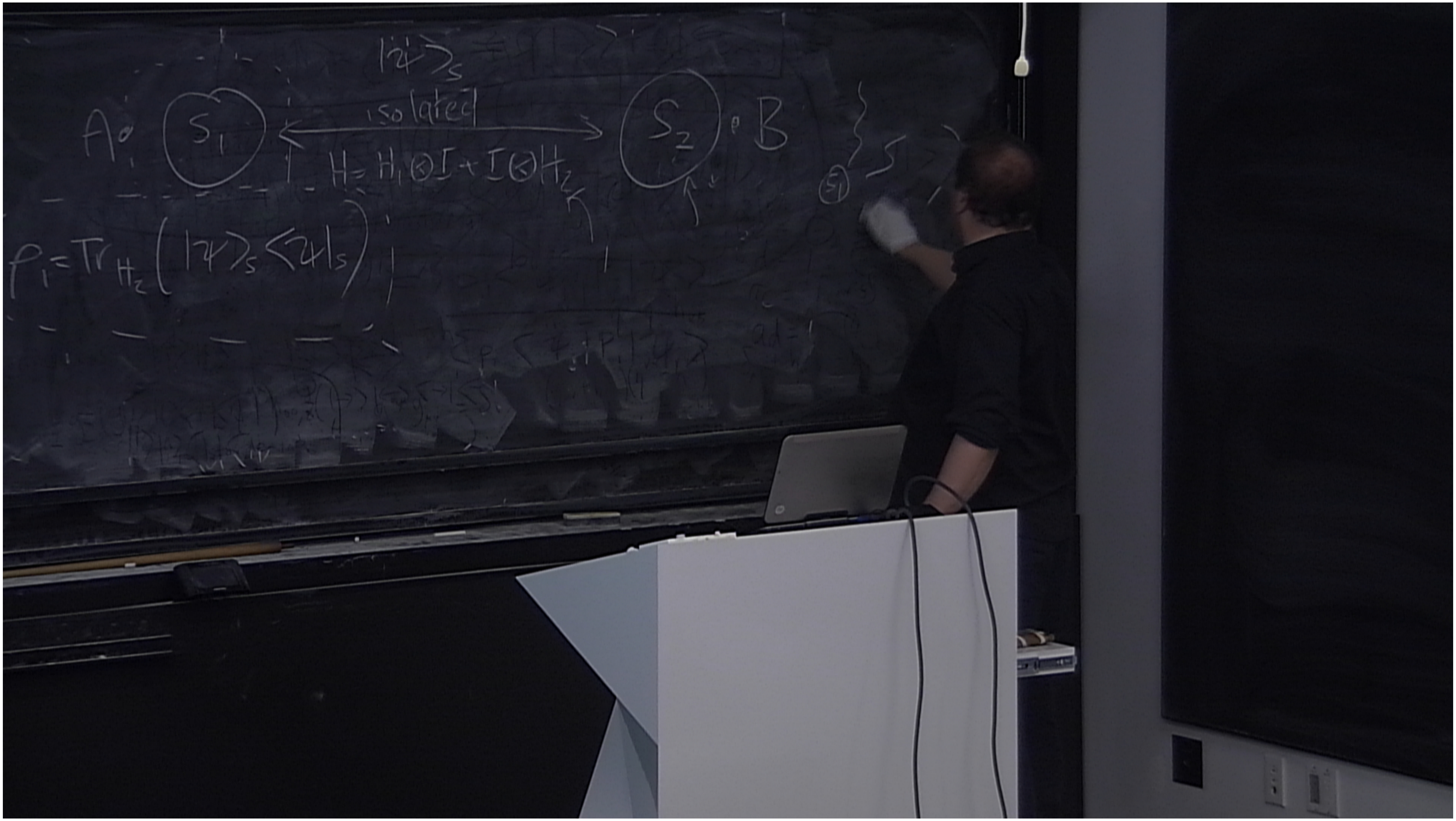
and this outcome has probability $\mathrm{Tr}(P\rho_1(t))$.

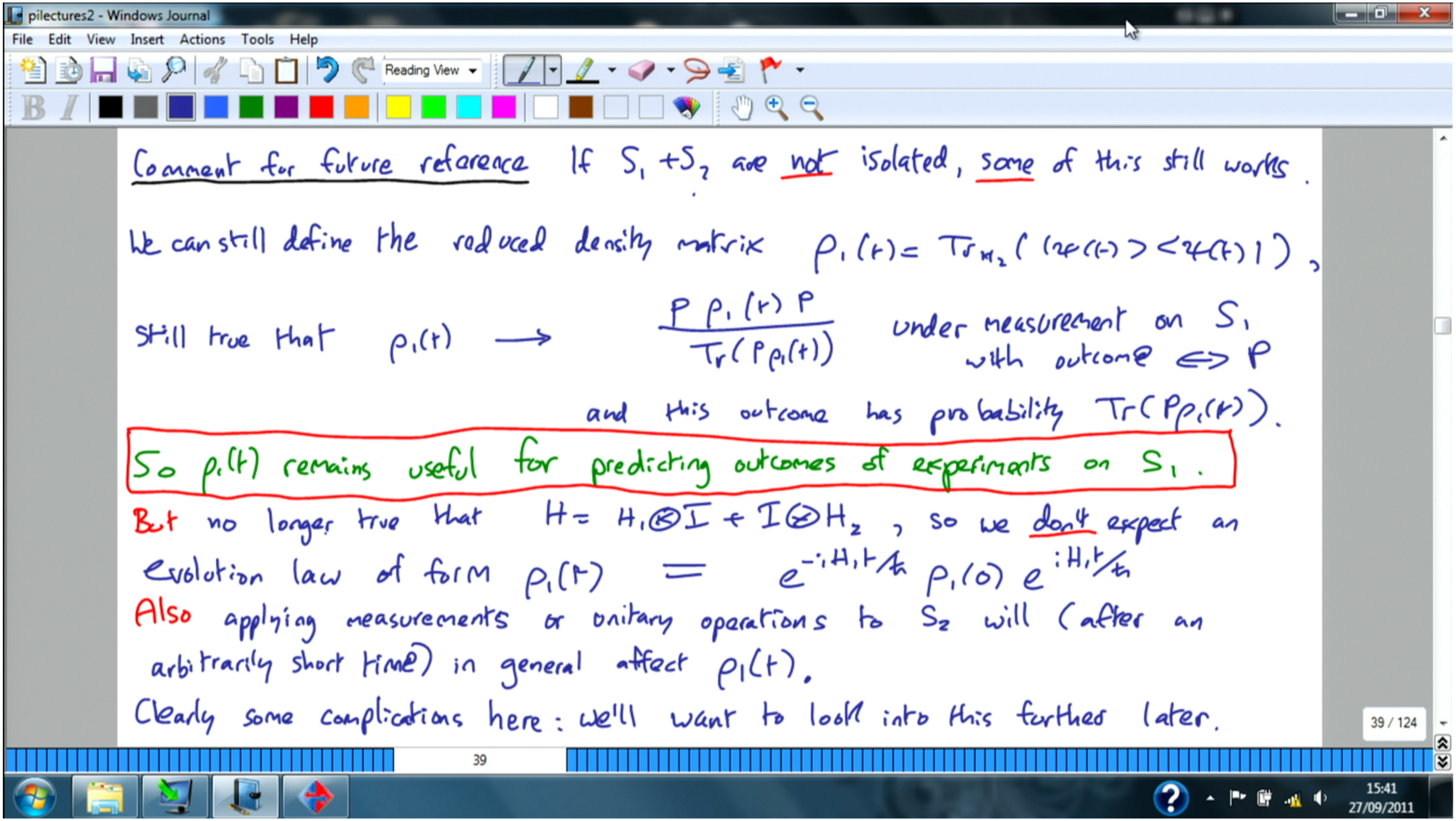
$$\boxed{\text{So } \rho_1(t) \text{ remains useful for predicting outcomes of experiments on } S_1.}$$

But no longer true that $H = H_1 \otimes I + I \otimes H_2$, so we don't expect an evolution law of form $\rho_1(t) = e^{-iH_1t/\hbar}\,\rho_1(0)\,e^{iH_1t/\hbar}$

Also applying measurements or unitary operations to S_2 will (after an arbitrarily short time) in general affect $\rho_1(t)$.

Clearly some complications here: we'll want to look into this further later.


|ψ⟩_S
A
S_1
isolated
S_2
B
H = H_1 ⊗ I + I ⊗ H_2
ρ_1 = Tr_{H_2}(|ψ⟩_S ⟨ψ|_S)



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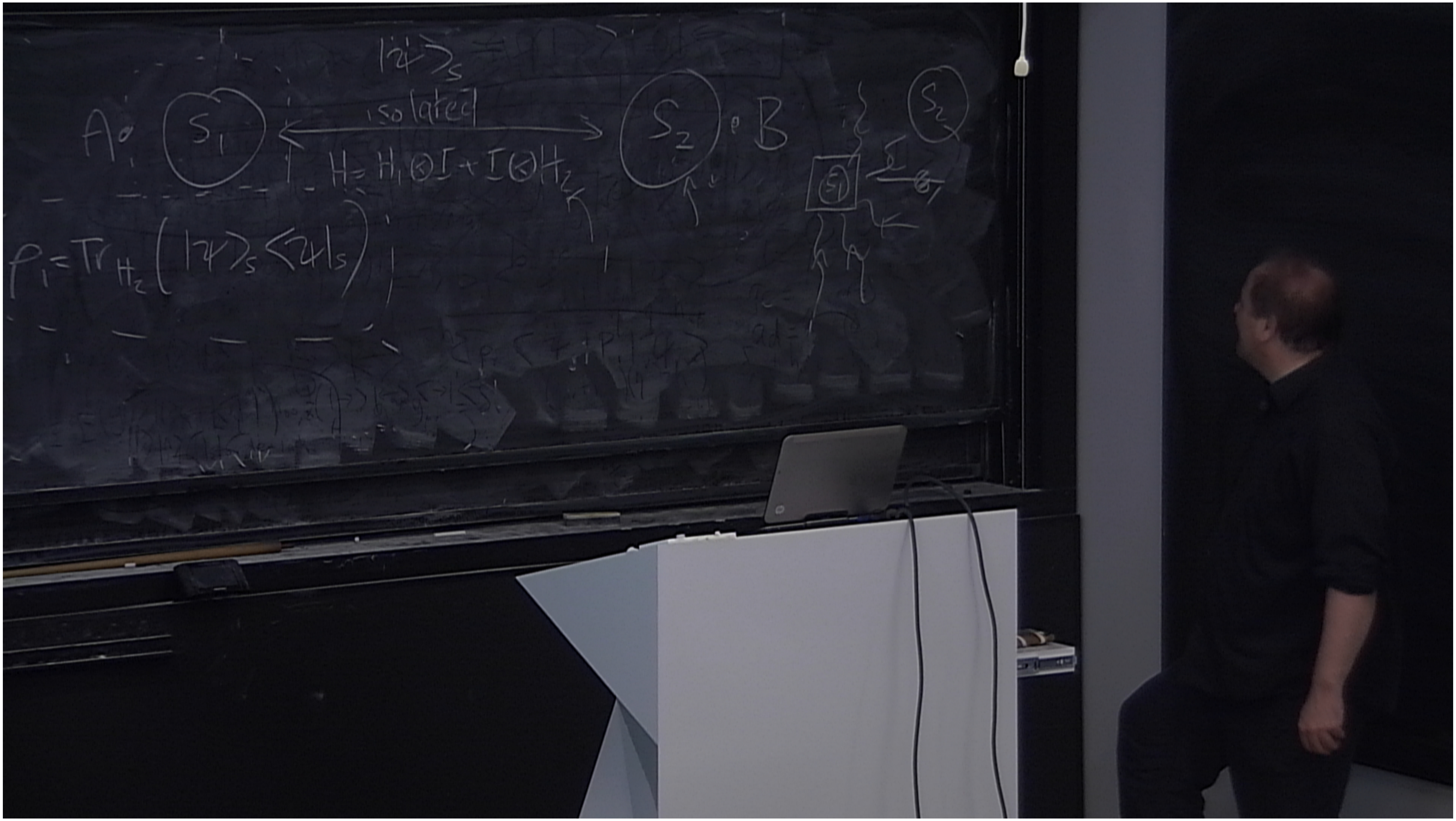
$|\psi\rangle_S$
A
S_1
isolated
S_2
B
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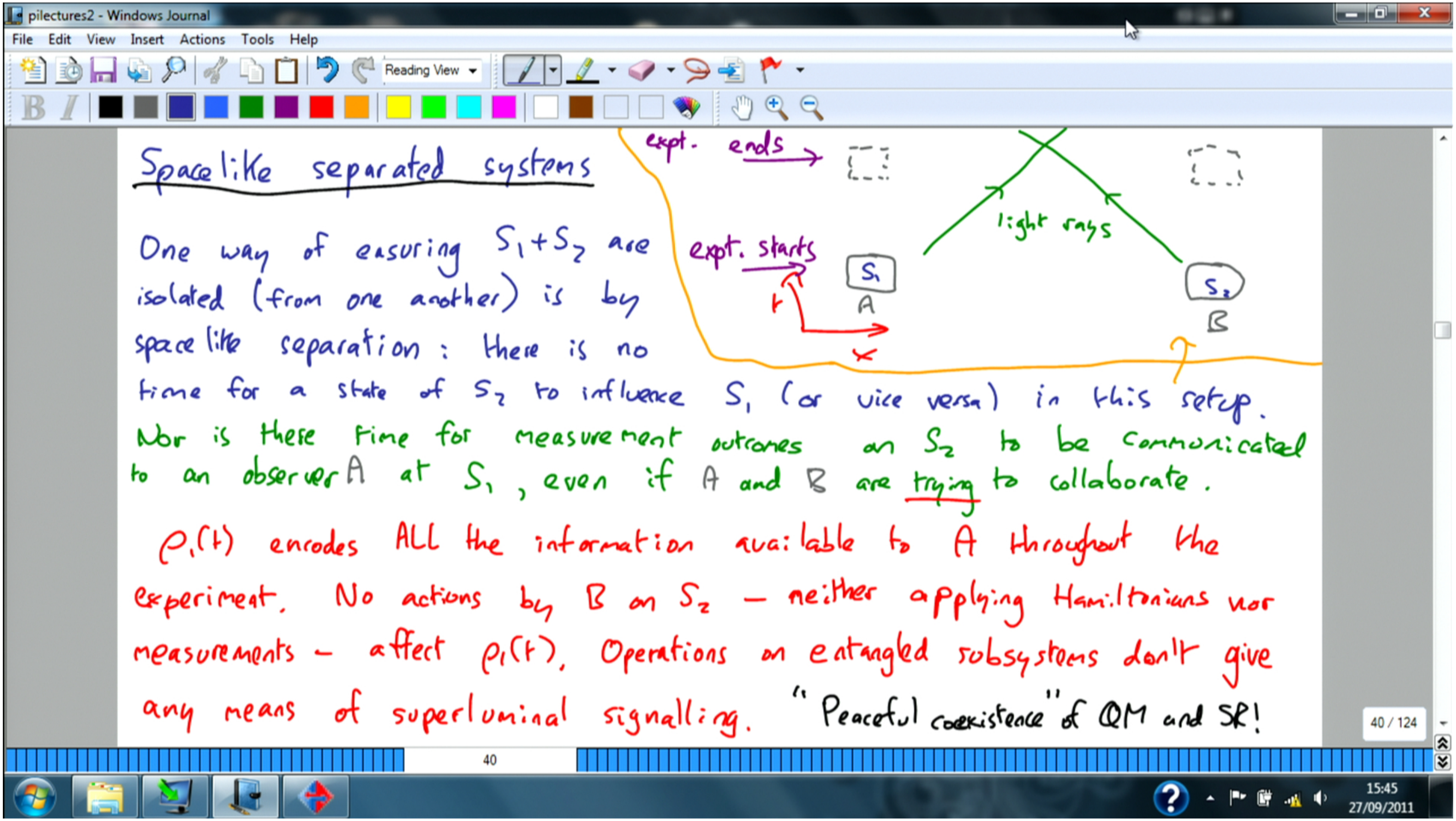
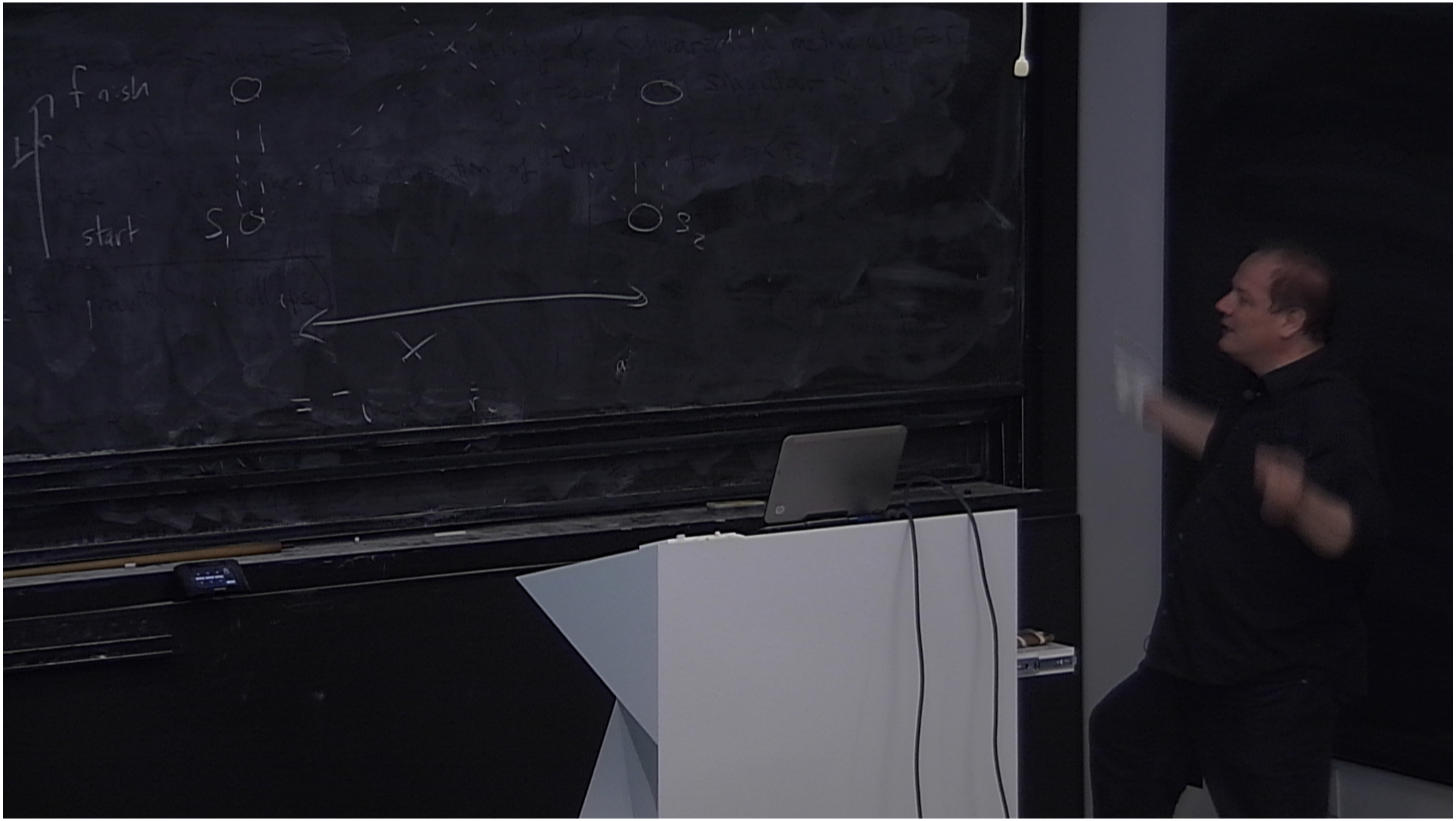
$\rho_1 = \mathrm{Tr}_{H_2}\left(|\psi\rangle_S \langle\psi|_S\right)$
S_1
S_2



Spacelike separated systems

One way of ensuring $S_1 + S_2$ are isolated (from one another) is by space like separation: there is no time for a state of S_2 to influence S_1 (or vice versa) in this setup. Nor is there time for measurement outcomes on S_2 to be communicated to an observer A at S_1, even if A and B are trying to collaborate.

$\rho_1(t)$ encodes ALL the information available to A throughout the experiment. No actions by B on S_2 — neither applying Hamiltonians nor measurements — affect $\rho_1(t)$. Operations on entangled subsystems don't give any means of superluminal signalling. "Peaceful coexistence" of QM and SR!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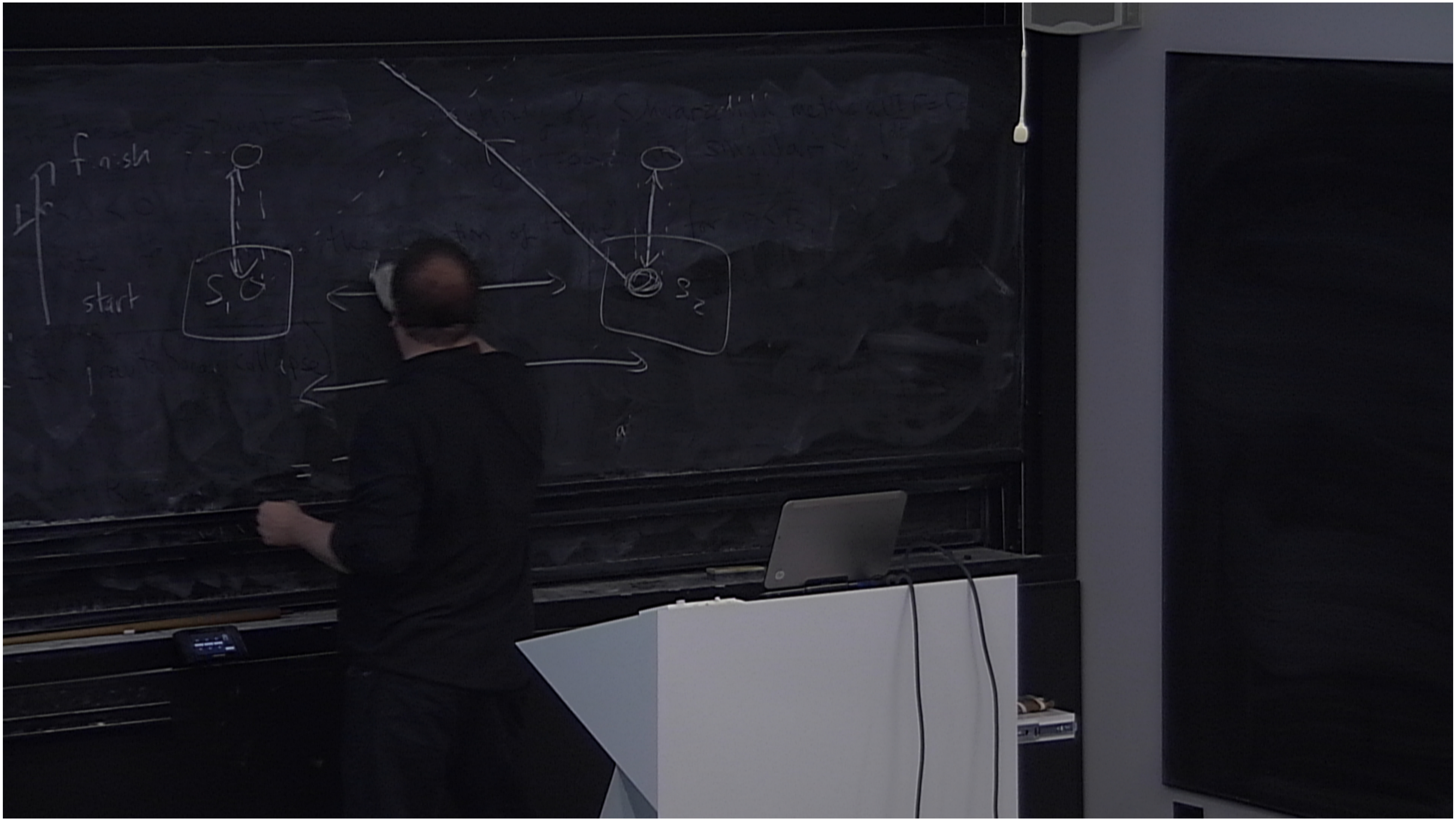
start
S_1
S_2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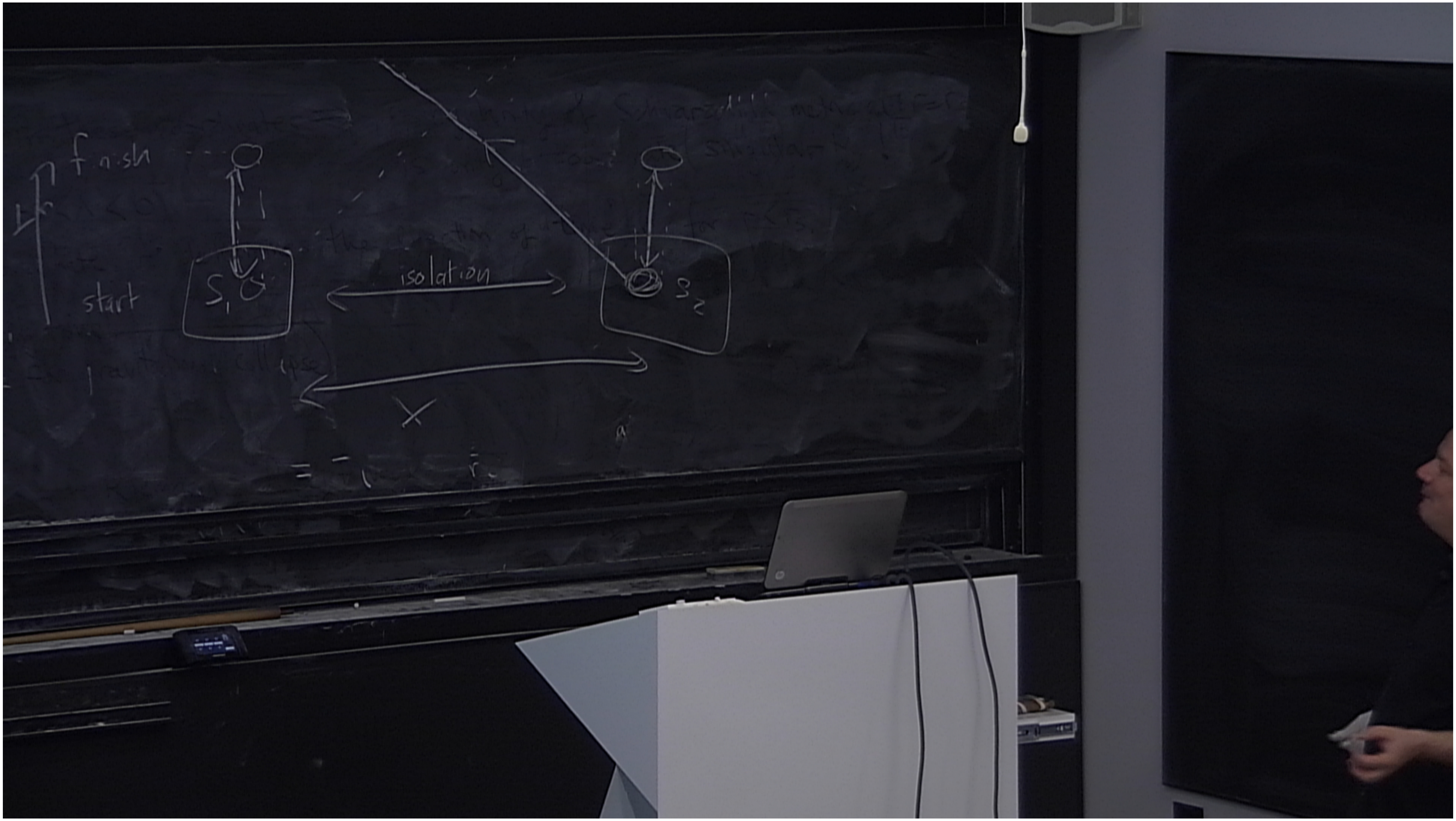
start
S_1
S_2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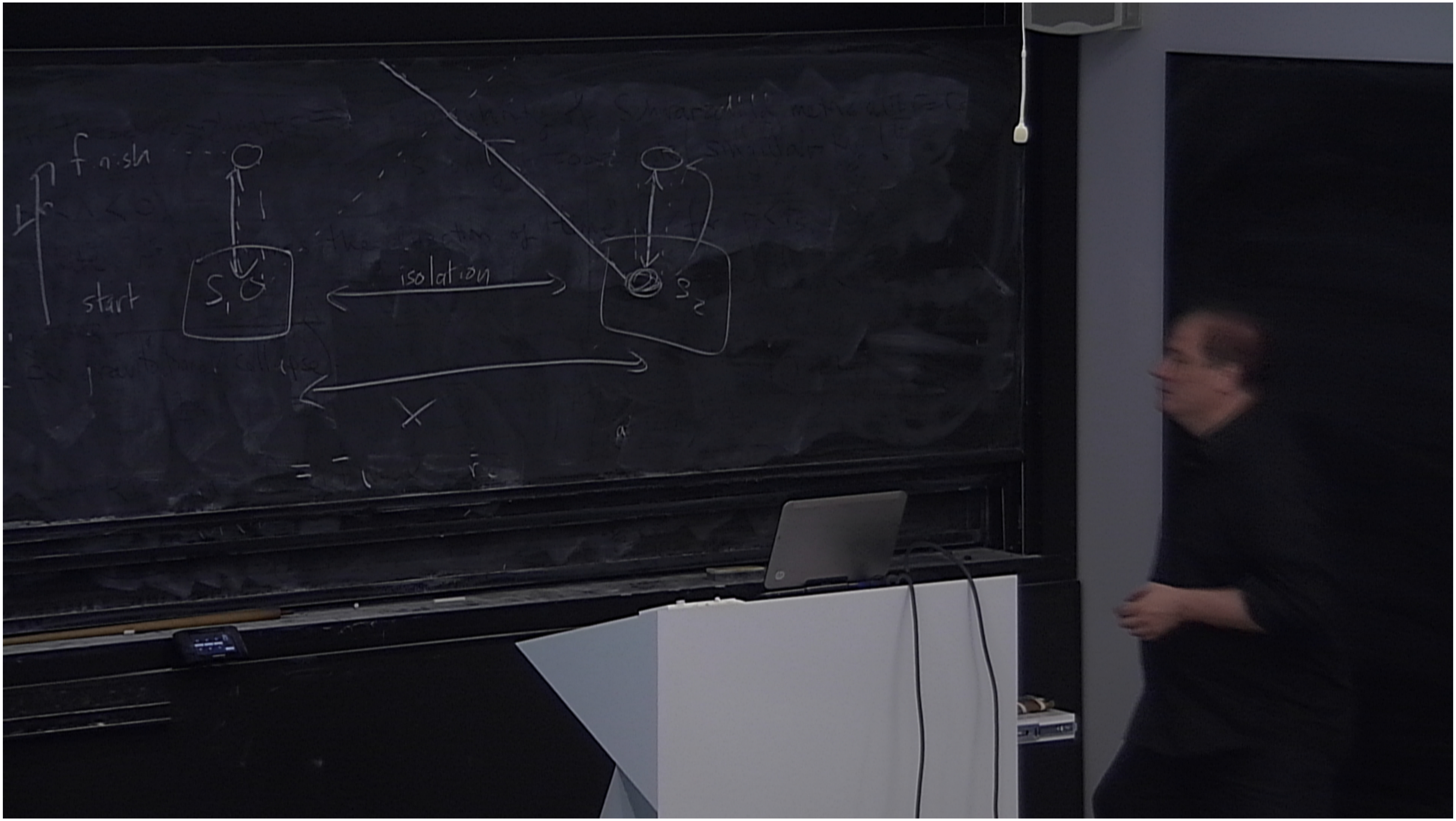
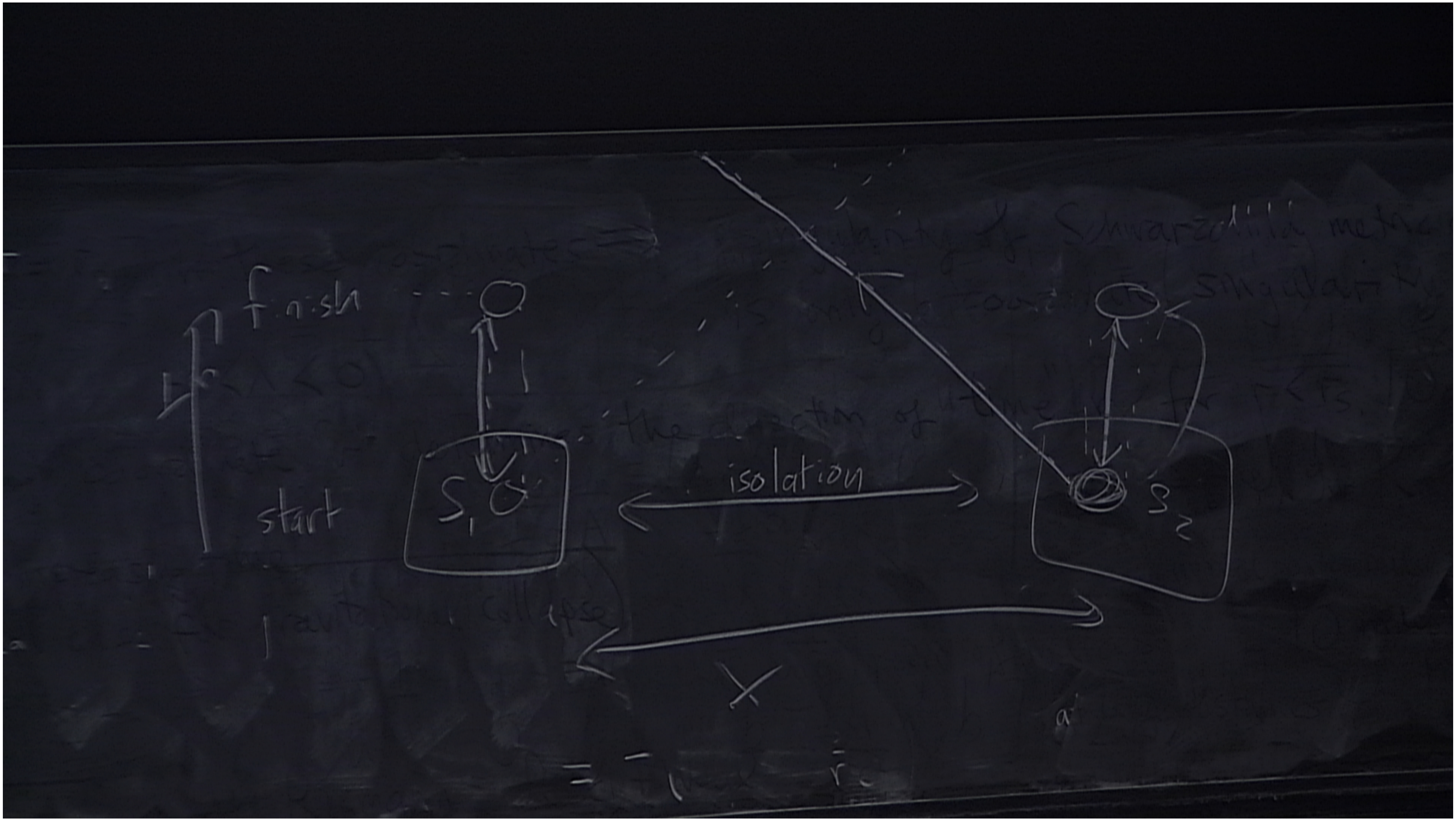
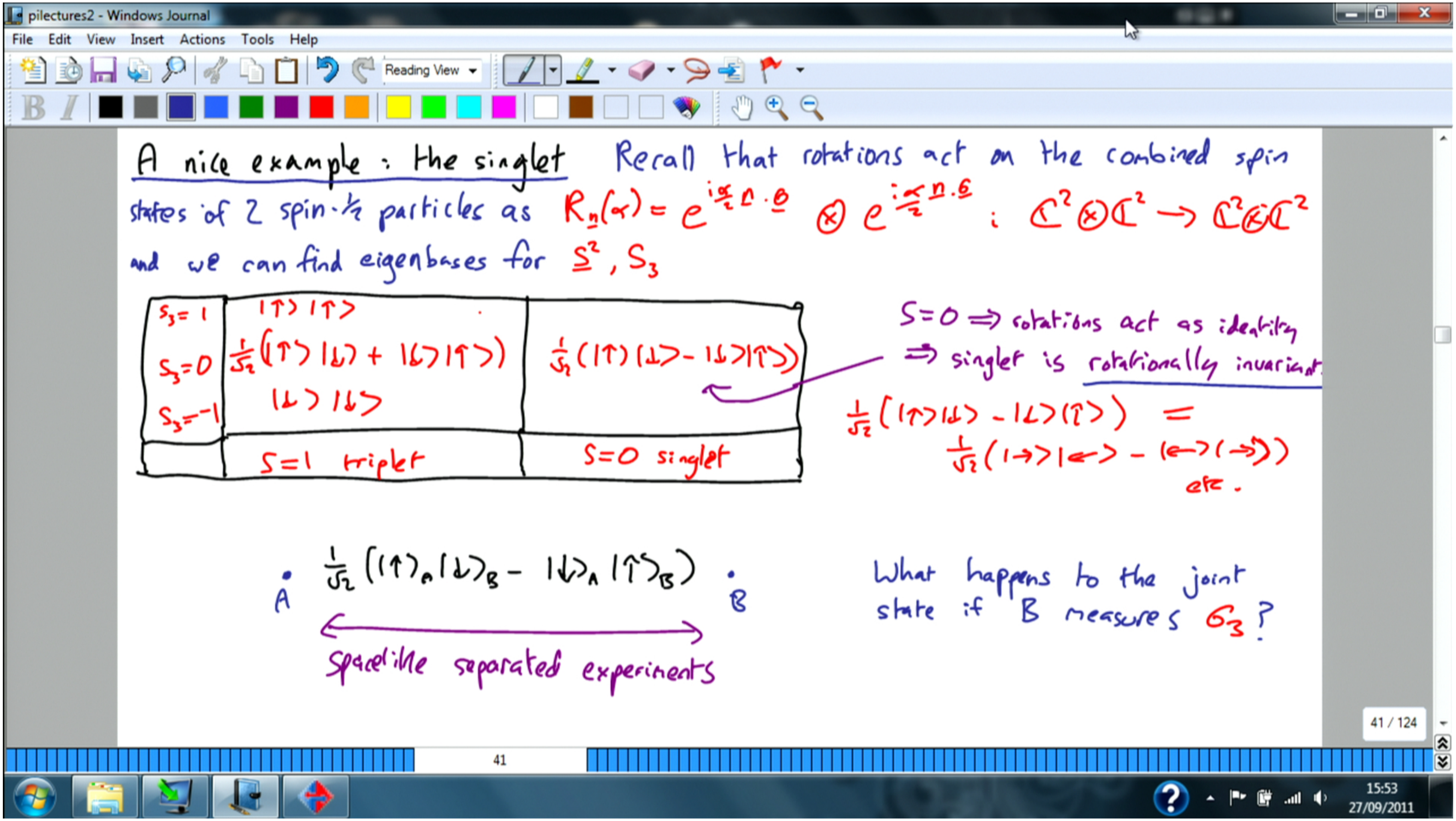
start
isolation
S_1
S_2


finish
start
S1
isolation
S2
gravitational collapse


finish
start
S_1
isolation
S_2



A nice example : the singlet

Recall that rotations act on the combined spin states of 2 spin-½ particles as $R_{\underline{n}}(\alpha) = e^{\frac{i\alpha}{2}\underline{n}\cdot\underline{\sigma}} \otimes e^{\frac{i\alpha}{2}\underline{n}\cdot\underline{\sigma}} : \mathbb{C}^2 \otimes \mathbb{C}^2 \to \mathbb{C}^2 \otimes \mathbb{C}^2$ and we can find eigenbases for $\underline{S}^2, S_3$

$S_3 = 1$	$\vert\uparrow\rangle\vert\uparrow\rangle$	
$S_3 = 0$	$\frac{1}{\sqrt{2}}(\vert\uparrow\rangle\vert\downarrow\rangle + \vert\downarrow\rangle\vert\uparrow\rangle)$	$\frac{1}{\sqrt{2}}(\vert\uparrow\rangle\vert\downarrow\rangle - \vert\downarrow\rangle\vert\uparrow\rangle)$
$S_3 = -1$	$\vert\downarrow\rangle\vert\downarrow\rangle$	
	$S=1$ triplet	$S=0$ singlet

$S=0 \Rightarrow$ rotations act as identity

$\Rightarrow$ singlet is rotationally invariant

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle) = \frac{1}{\sqrt{2}}(|\rightarrow\rangle|\leftarrow\rangle - |\leftarrow\rangle|\rightarrow\rangle)$$

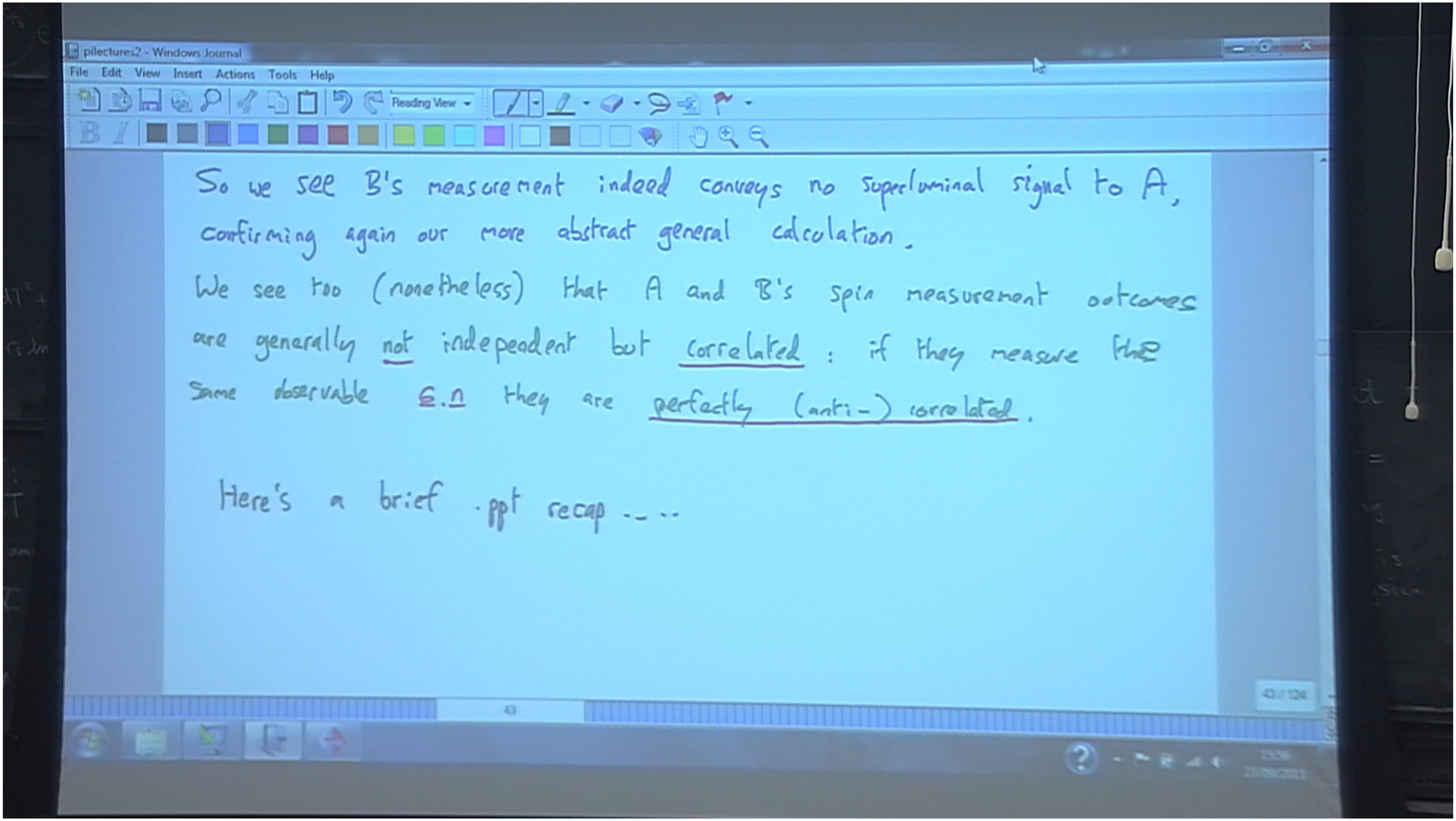
etc.

$$\overset{\bullet}{A} \quad \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B) \quad \overset{\bullet}{B}$$

$\longleftrightarrow$

spacelike separated experiments

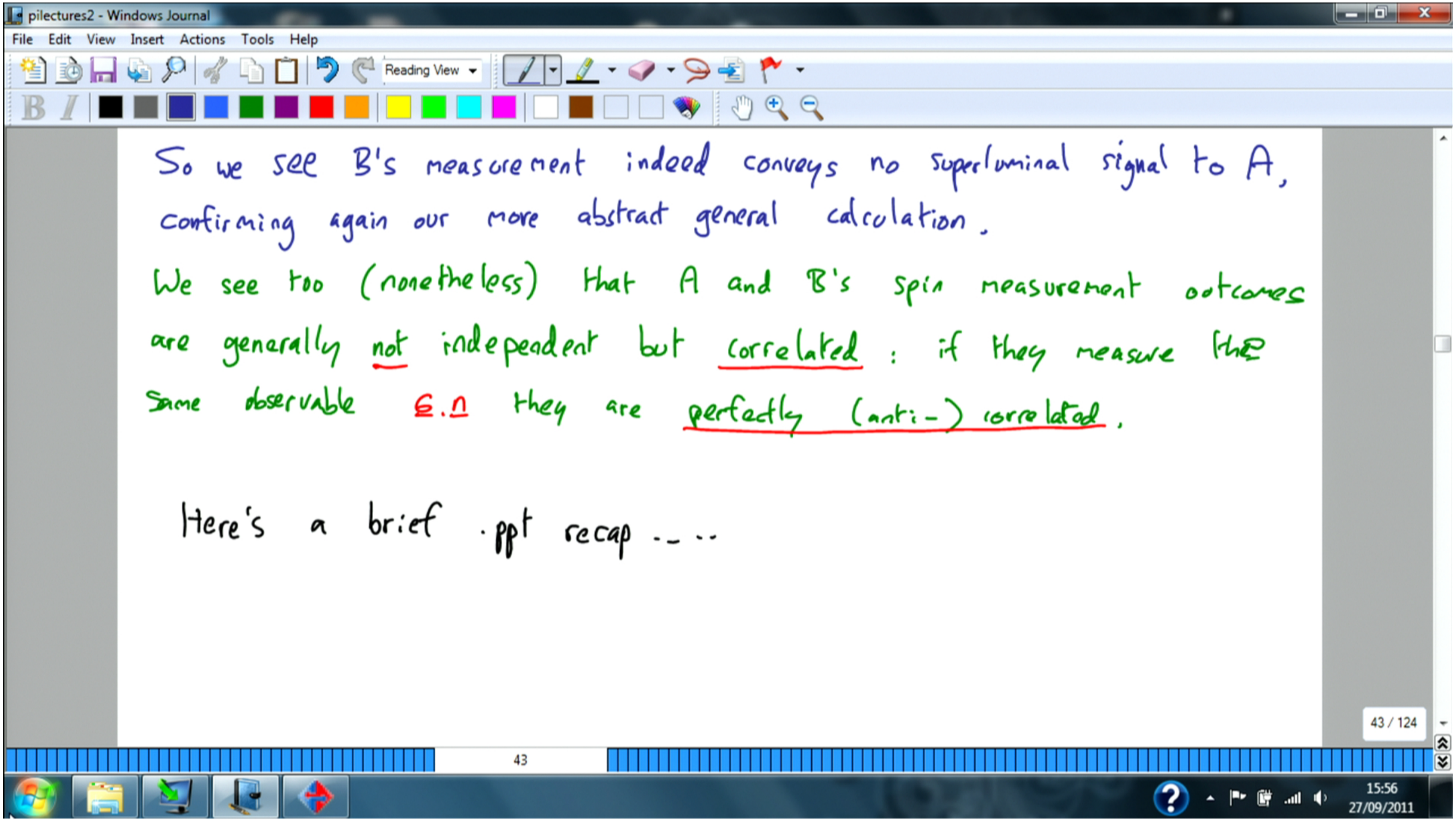
What happens to the joint state if B measures σ_3?



So we see B's measurement indeed conveys no superluminal signal to A, confirming again our more abstract general calculation.

We see too (nonetheless) that A and B's spin measurement outcomes are generally not independent but correlated: if they measure the same observable $\underline{\sigma}.\underline{n}$ they are perfectly (anti-) correlated.

Here's a brief .ppt recap .....



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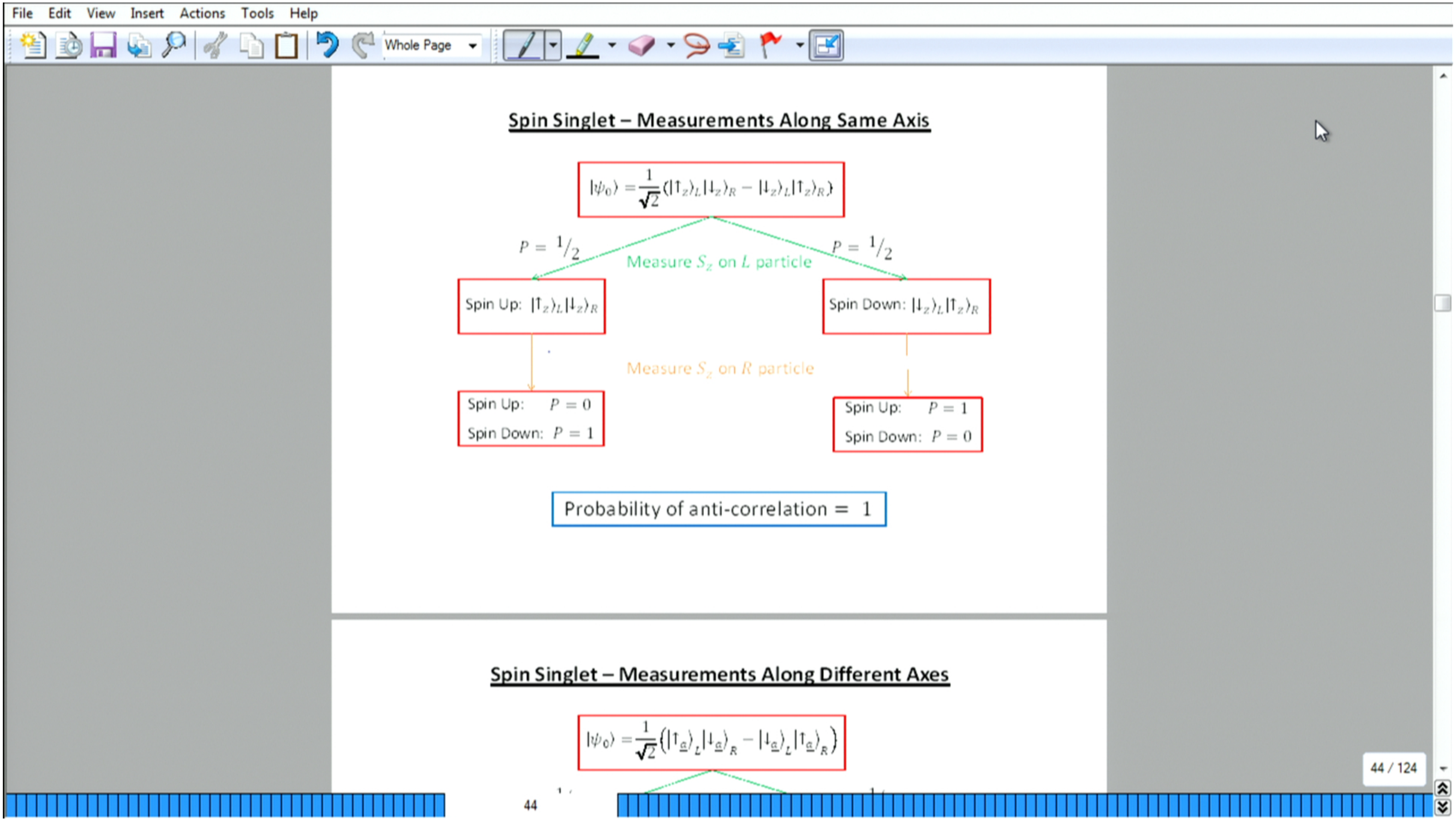


Spin Singlet – Measurements Along Same Axis

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_L|\downarrow_z\rangle_R - |\downarrow_z\rangle_L|\uparrow_z\rangle_R)$$

Measure S_z on L particle

$P = {}^1/_2$ Spin Up: $|\uparrow_z\rangle_L|\downarrow_z\rangle_R$

$P = {}^1/_2$ Spin Down: $|\downarrow_z\rangle_L|\uparrow_z\rangle_R$

Measure S_z on R particle

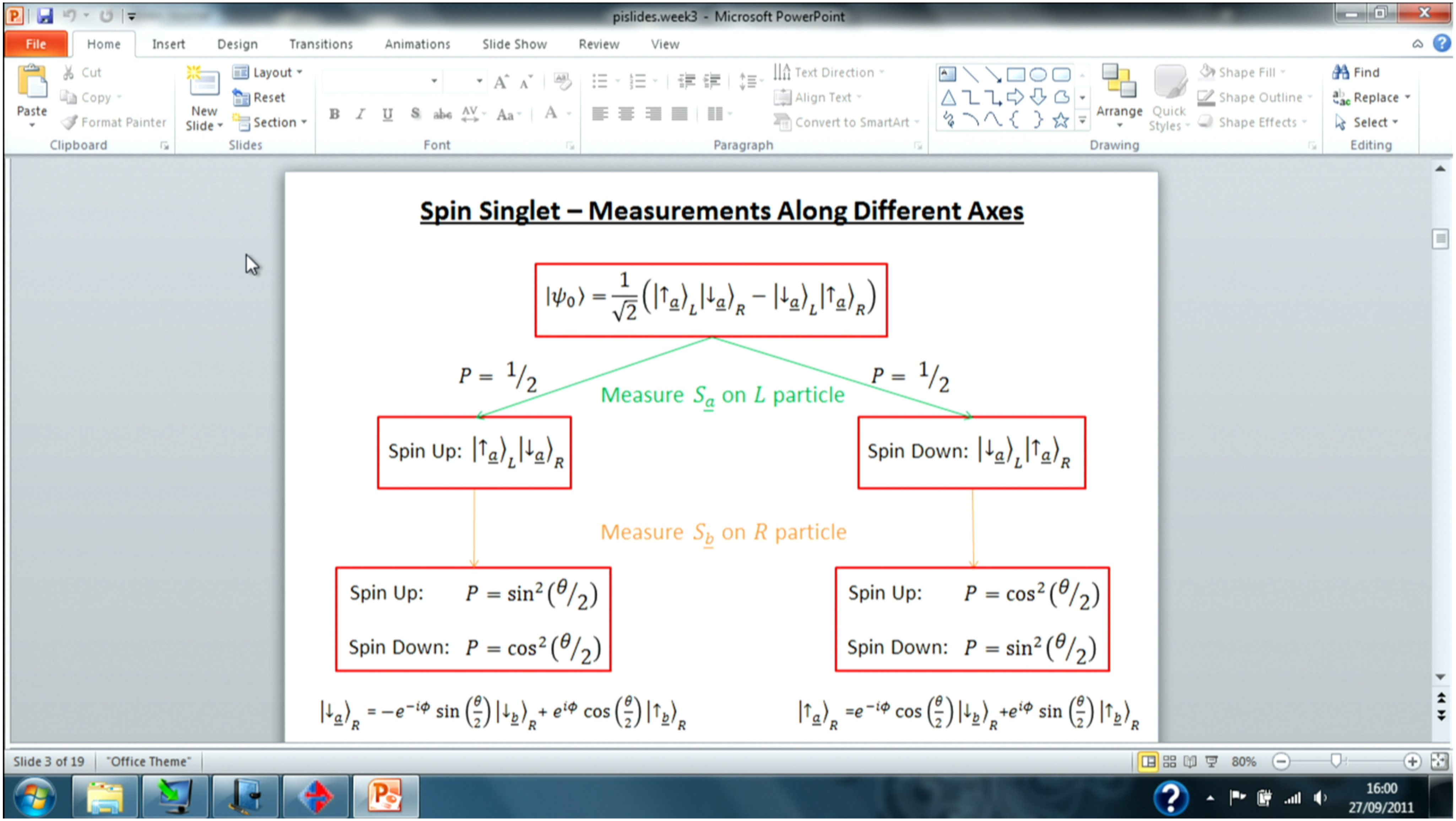
Spin Up: $P = 0$
Spin Down: $P = 1$

Spin Up: $P = 1$
Spin Down: $P = 0$

Probability of anti-correlation = 1

Spin Singlet – Measurements Along Different Axes

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_{\underline{a}}\rangle_L|\downarrow_{\underline{a}}\rangle_R - |\downarrow_{\underline{a}}\rangle_L|\uparrow_{\underline{a}}\rangle_R\right)$$



Spin Singlet – Measurements Along Different Axes

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_{\underline{a}}\rangle_L|\downarrow_{\underline{a}}\rangle_R - |\downarrow_{\underline{a}}\rangle_L|\uparrow_{\underline{a}}\rangle_R\right)$$

$P = {}^1\!/_2$

Measure $S_{\underline{a}}$ on L particle

$P = {}^1\!/_2$

Spin Up: $|\uparrow_{\underline{a}}\rangle_L|\downarrow_{\underline{a}}\rangle_R$

Spin Down: $|\downarrow_{\underline{a}}\rangle_L|\uparrow_{\underline{a}}\rangle_R$

Measure $S_{\underline{b}}$ on R particle

Spin Up: $P = \sin^2({}^{\theta}\!/_2)$

Spin Down: $P = \cos^2({}^{\theta}\!/_2)$

Spin Up: $P = \cos^2({}^{\theta}\!/_2)$

Spin Down: $P = \sin^2({}^{\theta}\!/_2)$

$|\downarrow_{\underline{a}}\rangle_R = -e^{-i\phi}\sin\left(\frac{\theta}{2}\right)|\downarrow_{\underline{b}}\rangle_R + e^{i\phi}\cos\left(\frac{\theta}{2}\right)|\uparrow_{\underline{b}}\rangle_R$

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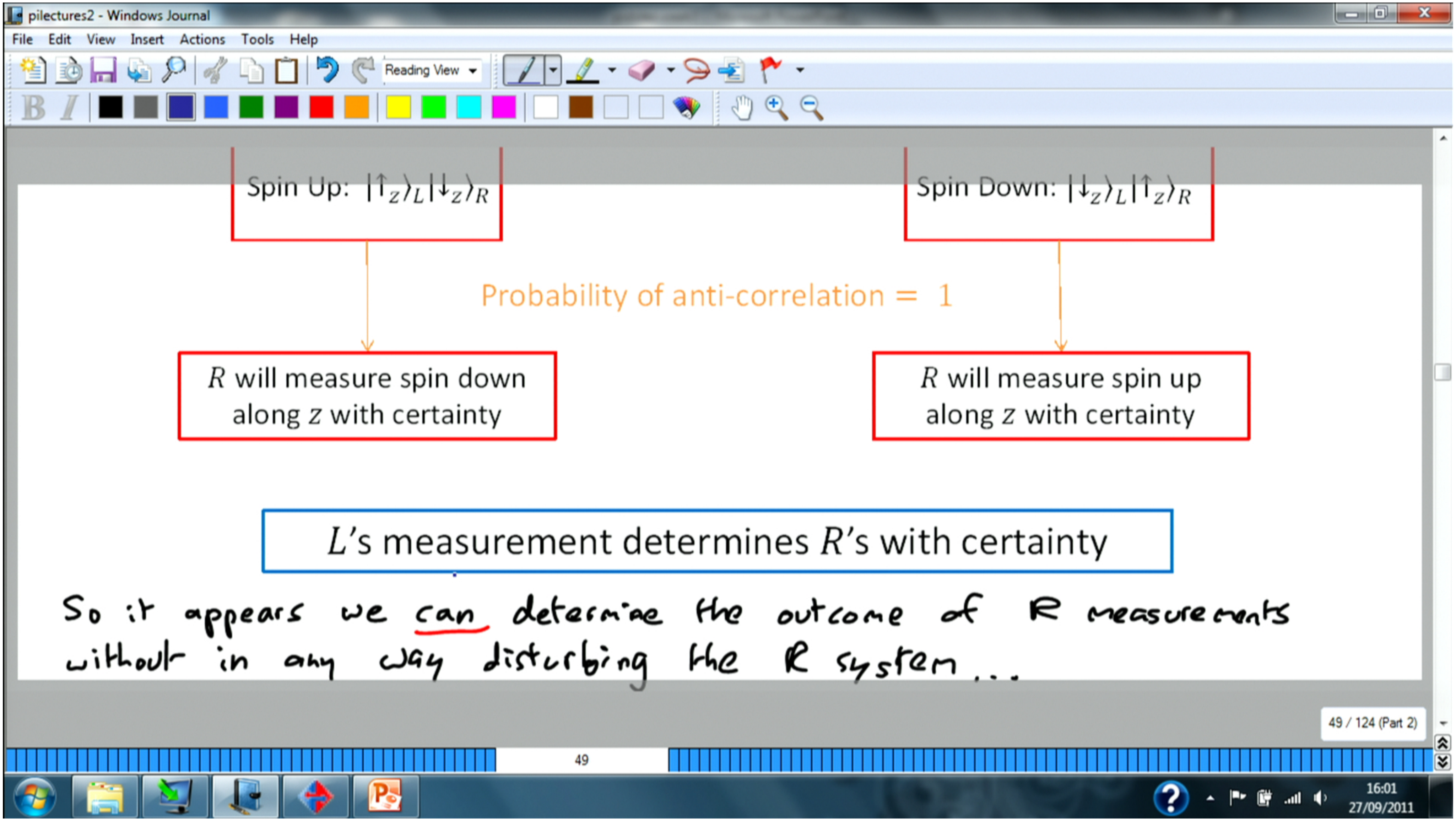

Spin Up: $|\uparrow_z\rangle_L|\downarrow_z\rangle_R$

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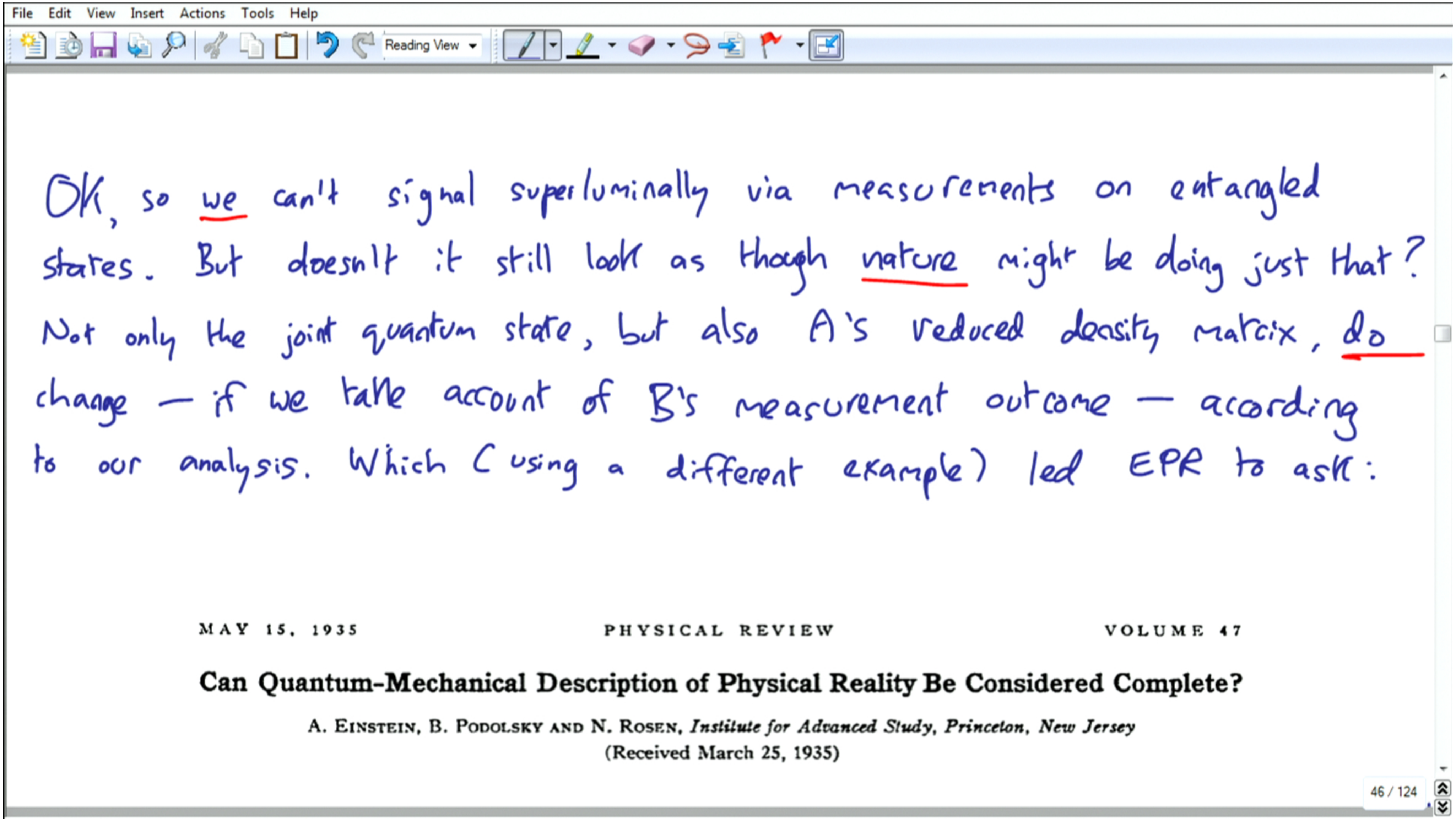
R will measure spin down along z with certainty

R will measure spin up along z with certainty

L's measurement determines R's with certainty

So it appears we can determine the outcome of R measurements without in any way disturbing the R system...



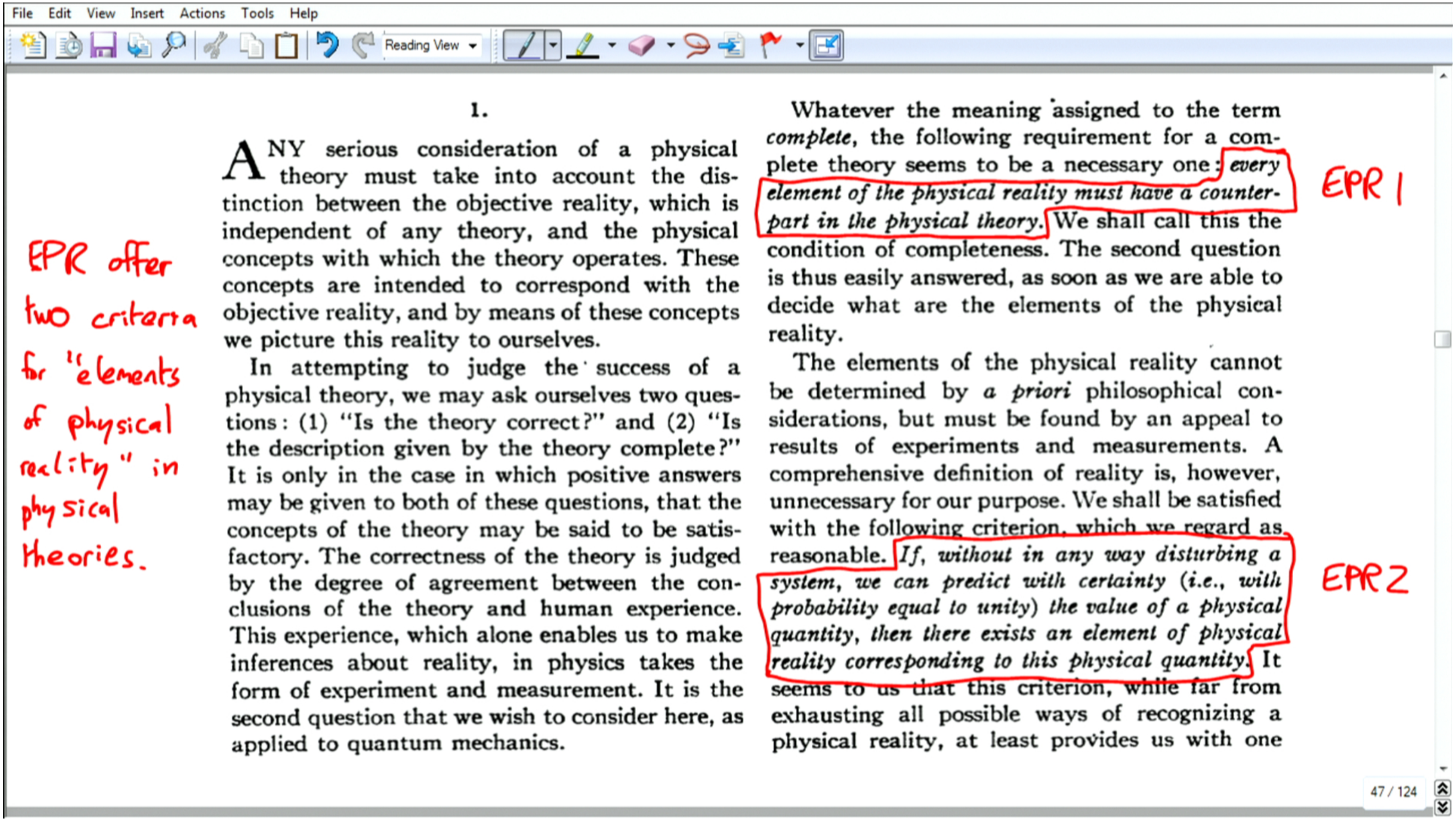
OK, so we can't signal superluminally via measurements on entangled states. But doesn't it still look as though nature might be doing just that? Not only the joint quantum state, but also A's reduced density matrix, do change — if we take account of B's measurement outcome — according to our analysis. Which (using a different example) led EPR to ask:

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)



EPR offer two criteria for "elements of physical reality" in physical theories.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

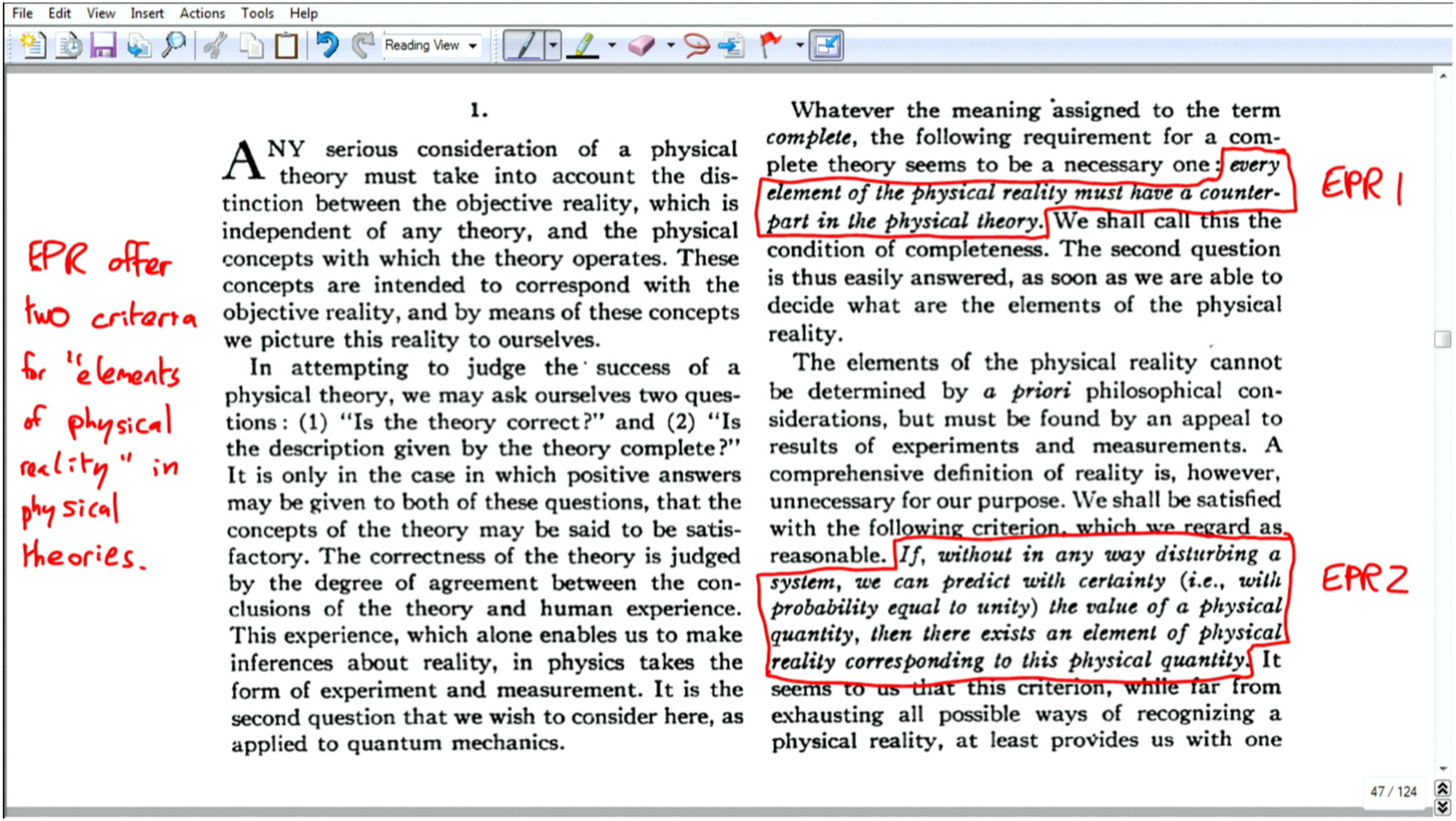
In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.* We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

EPR 1

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty* (*i.e.*, *with probability equal to unity*) *the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

EPR 2



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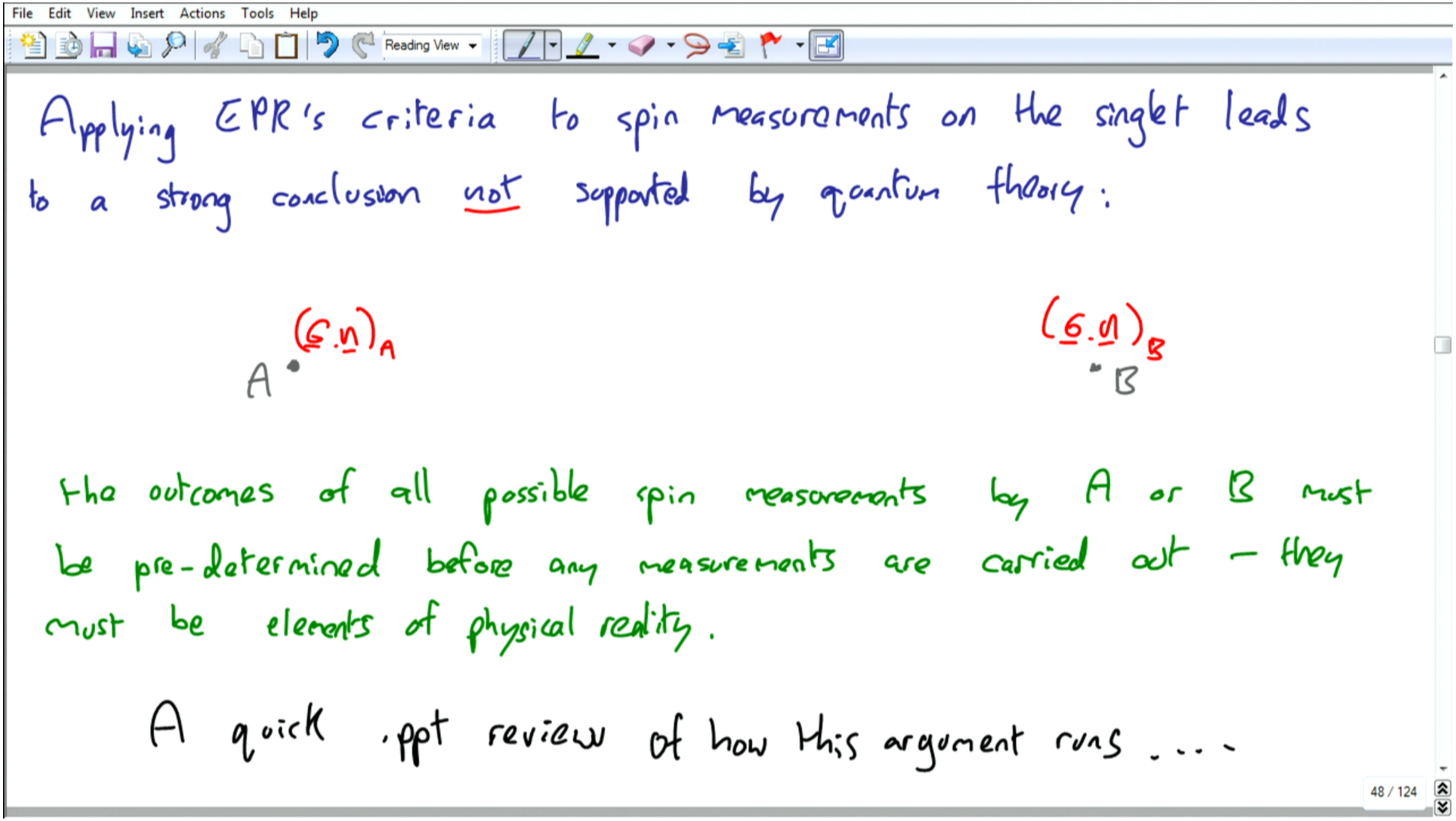


Applying EPR's criteria to spin measurements on the singlet leads to a strong conclusion not supported by quantum theory:

$(\underline{\sigma}.\underline{n})_A$

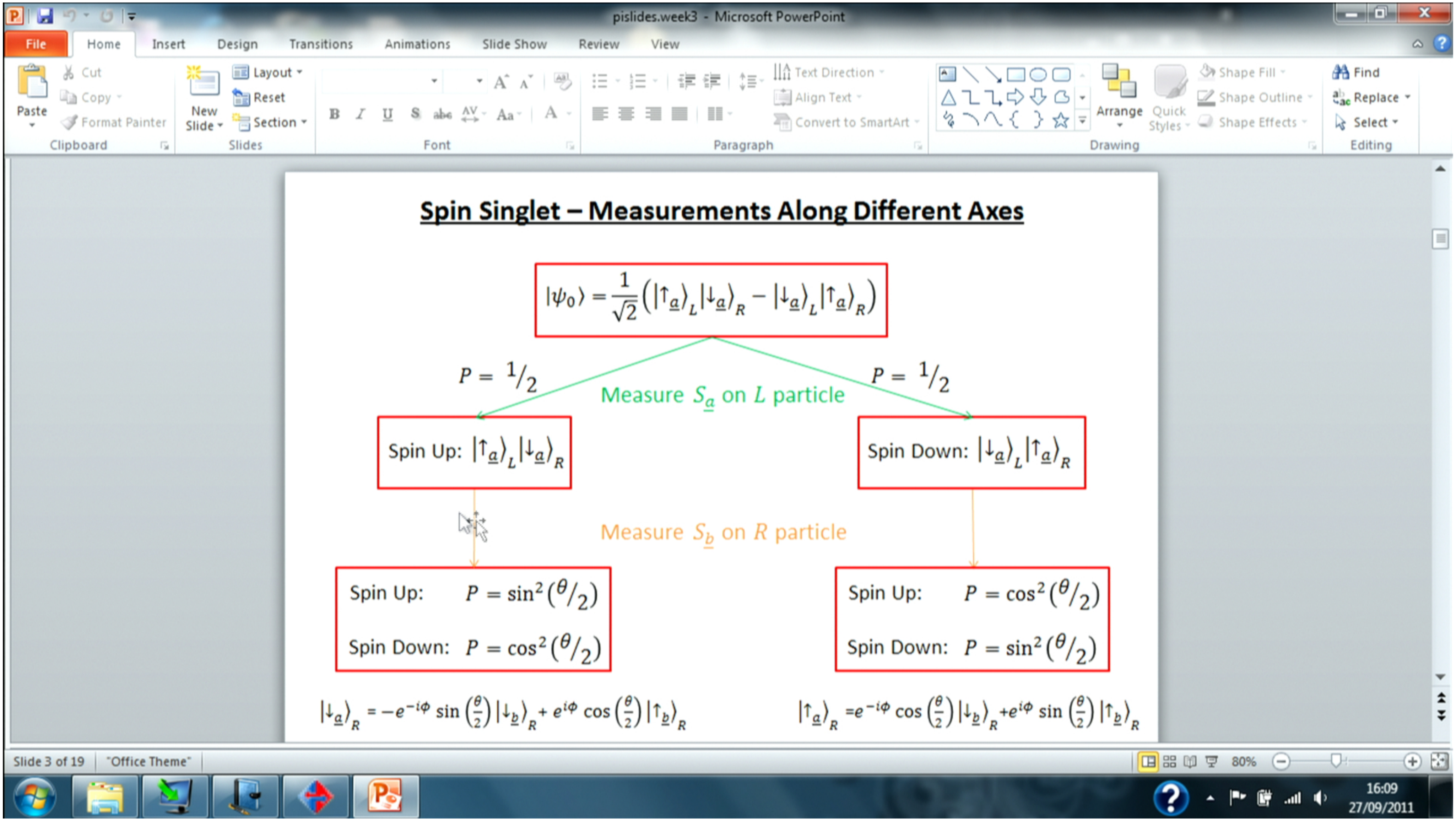
A

$(\underline{\sigma}.\underline{n})_B$

B

the outcomes of all possible spin measurements by A or B must be pre-determined before any measurements are carried out – they must be elements of physical reality.

A quick .ppt review of how this argument runs . . . .



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$P = {}^1/_2$

Measure $S_{\underline{a}}$ on L particle

$P = {}^1/_2$

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Measure $S_{\underline{b}}$ on R particle

Spin Up: $P = \sin^2({}^\theta/_2)$

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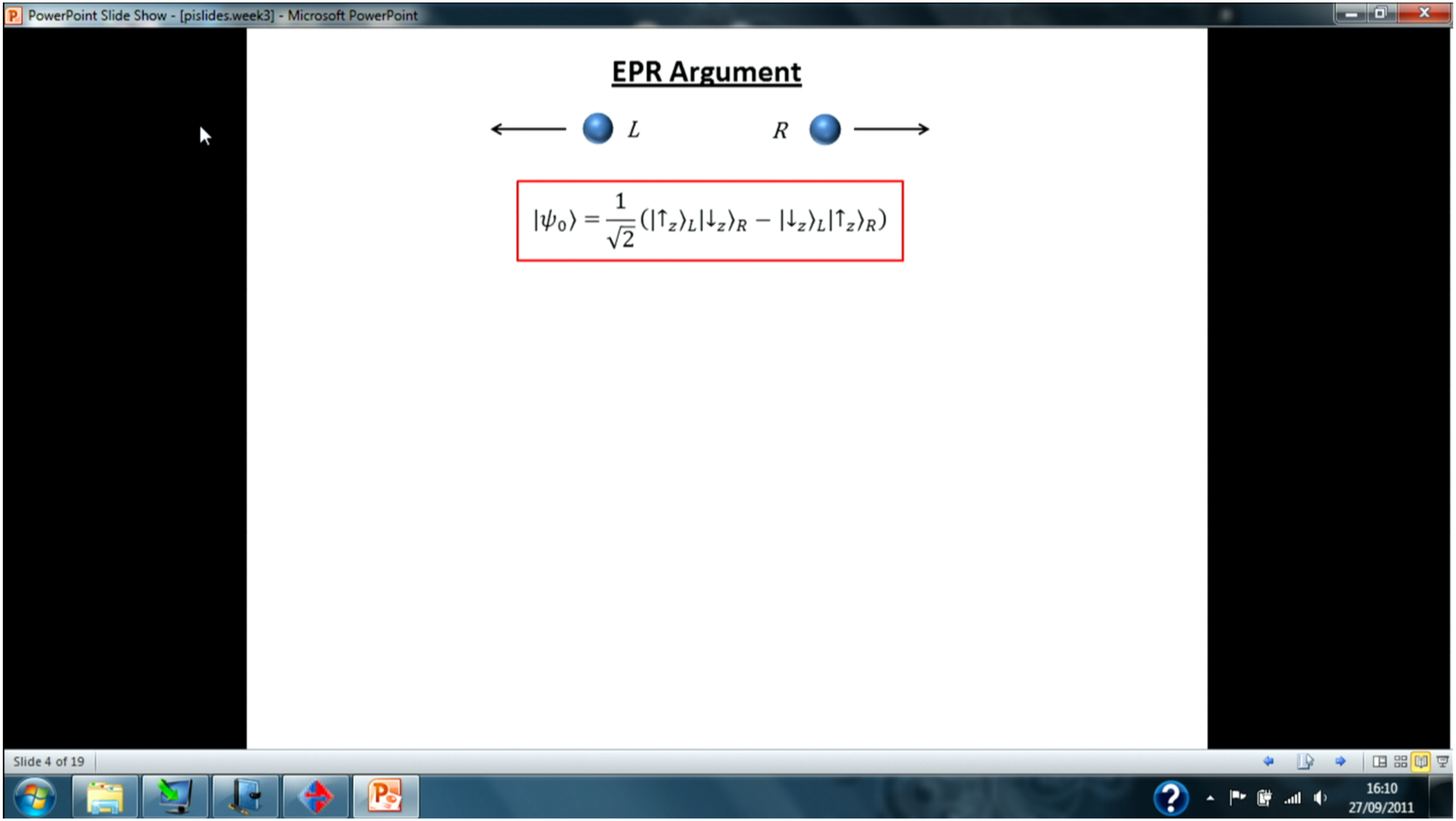
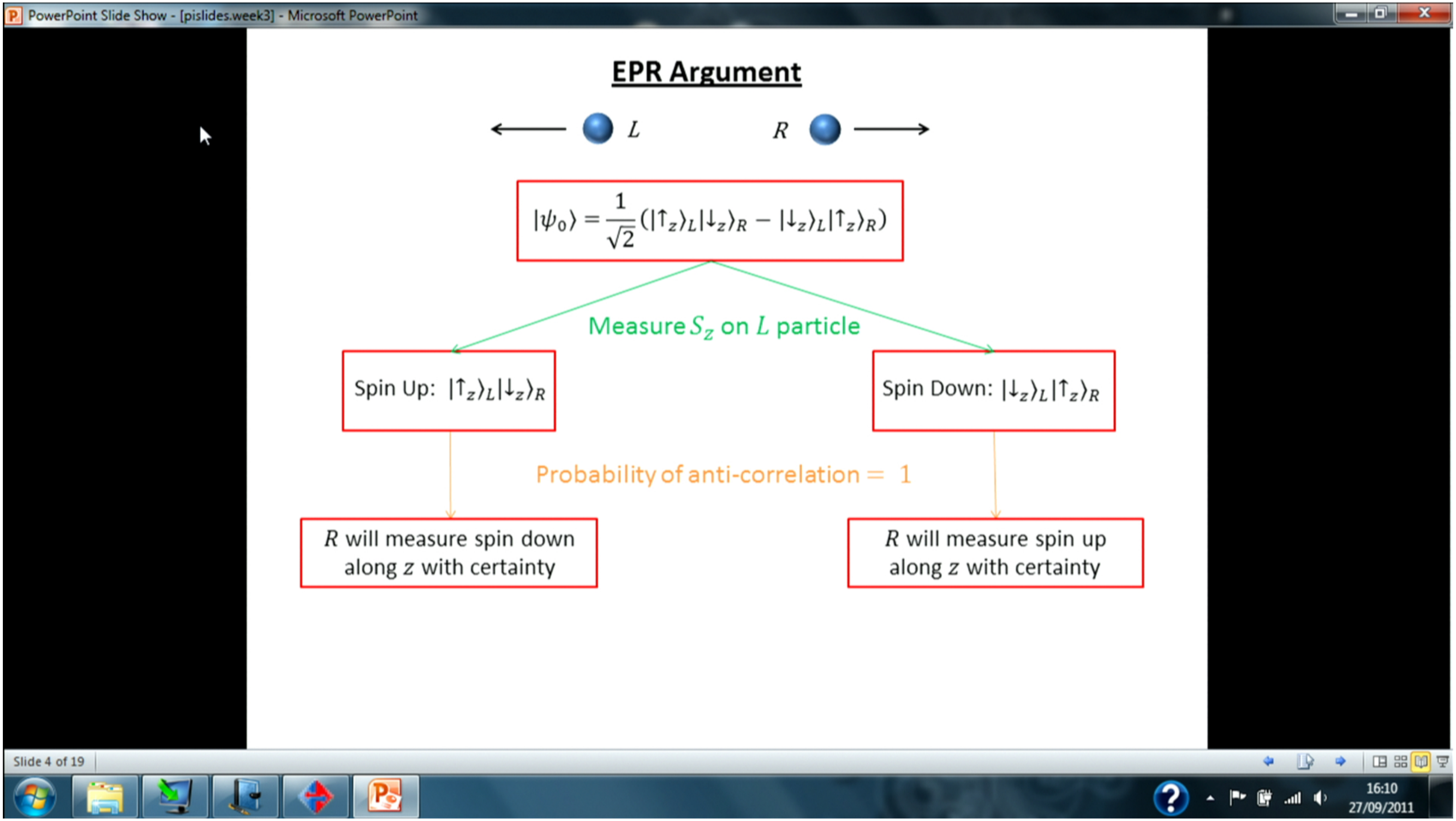


EPR Argument

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_z\rangle_L|\downarrow_z\rangle_R - |\downarrow_z\rangle_L|\uparrow_z\rangle_R\right)$$



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Measure S_z on L particle

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Probability of anti-correlation = 1

R will measure spin down along z with certainty

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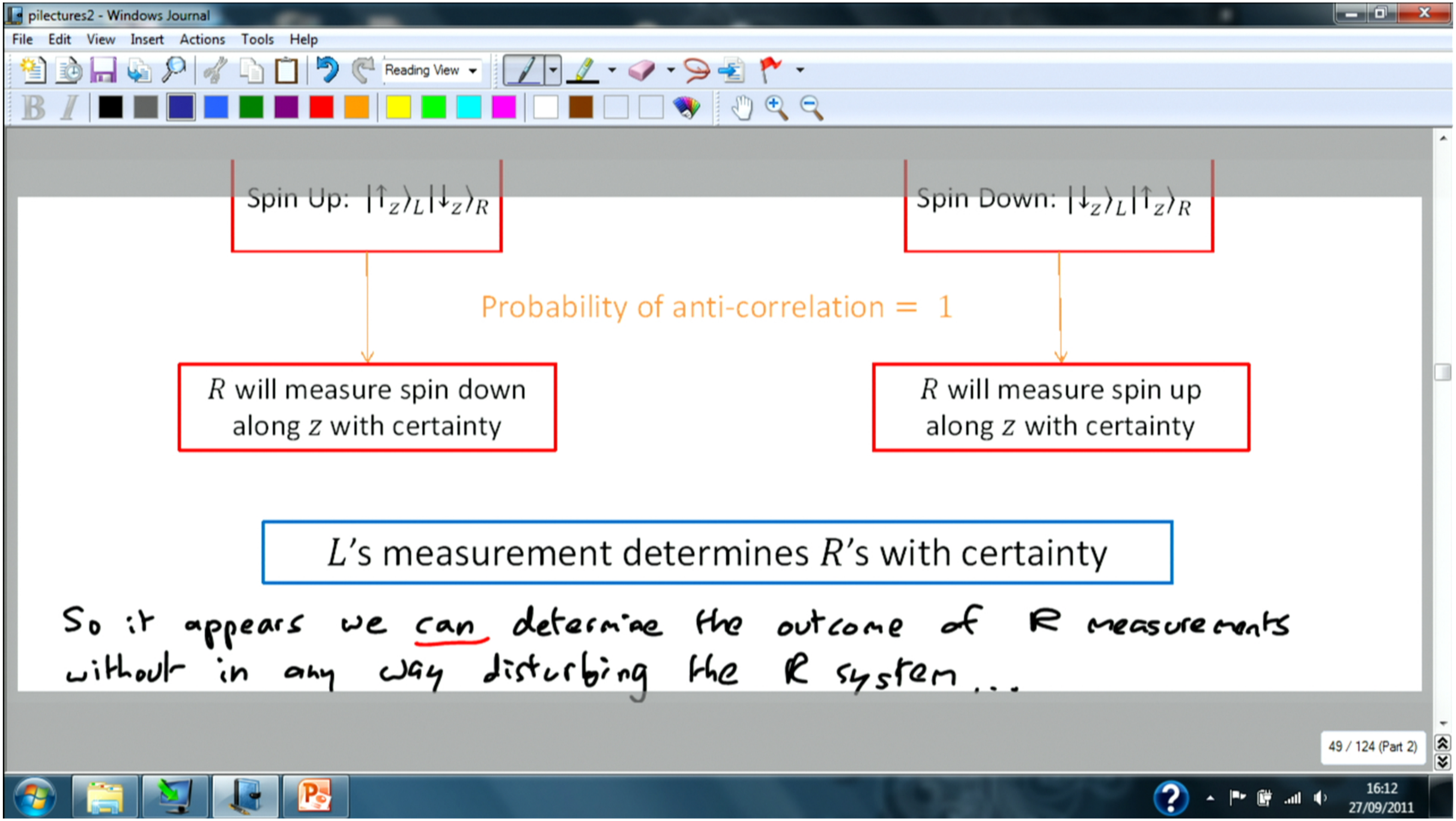

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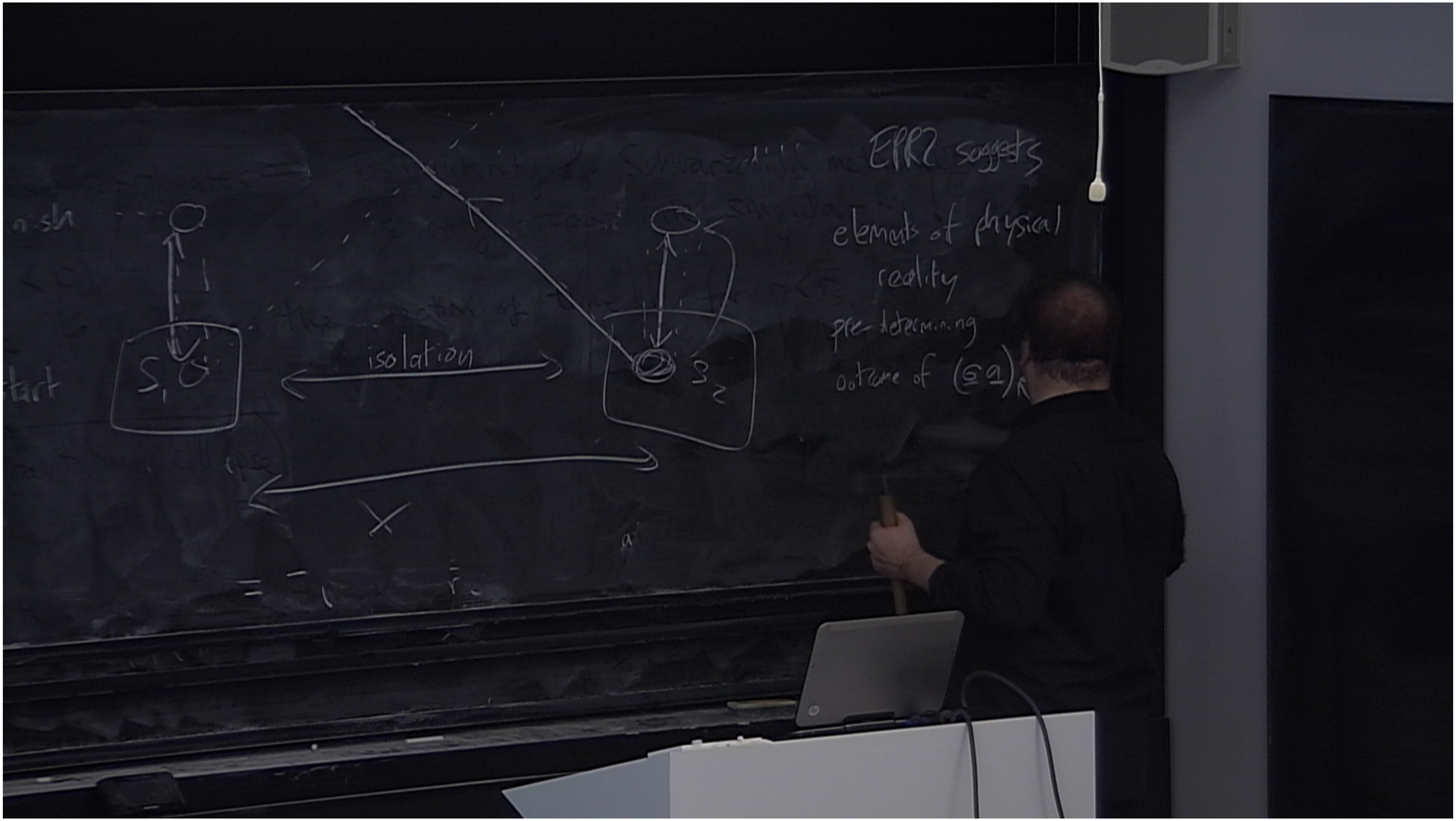
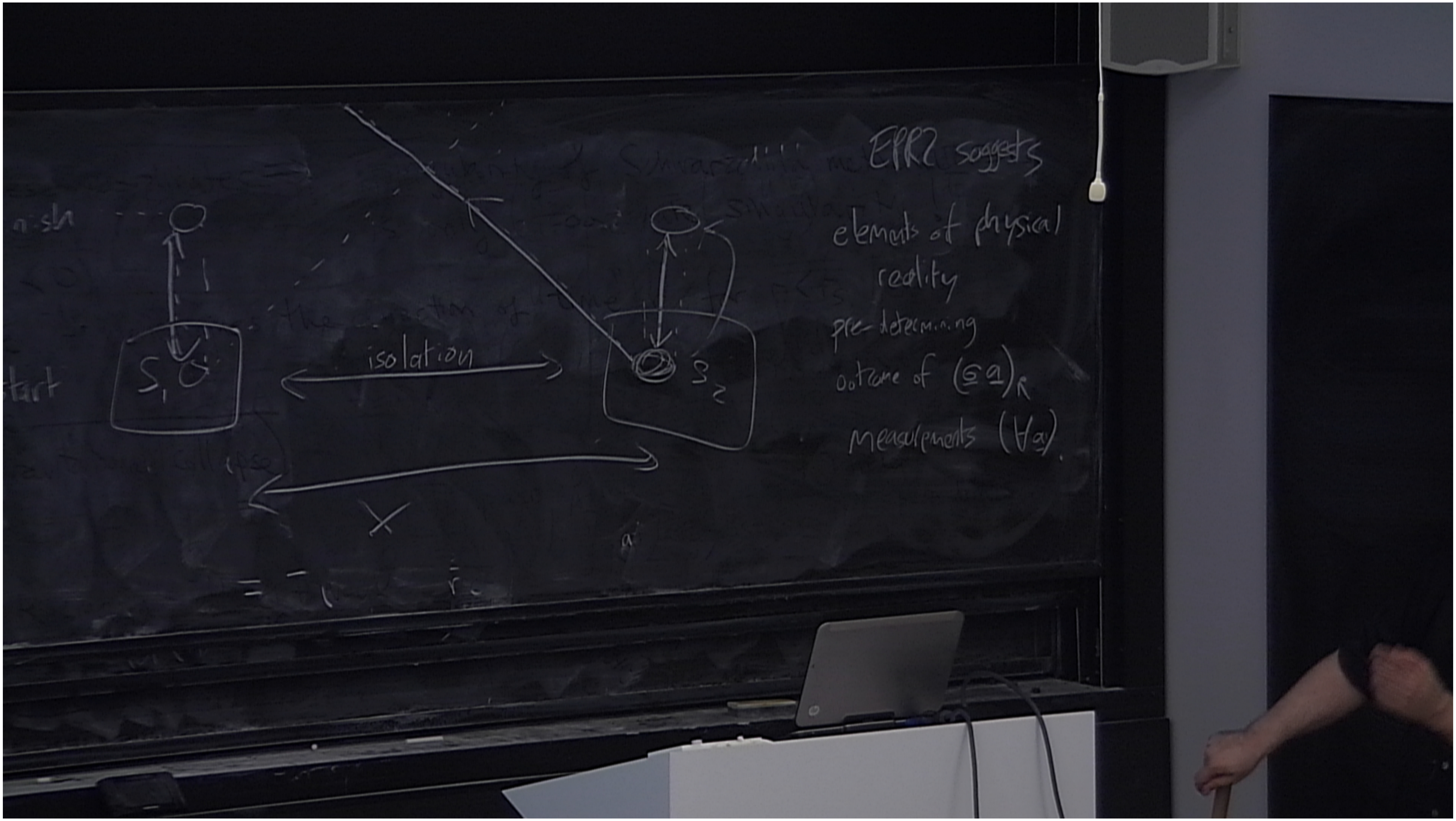

EPR2 suggests
elements of physical reality
pre-determining
outcome of (σ·a)
isolation
S1
S2
start


EPR2 suggests
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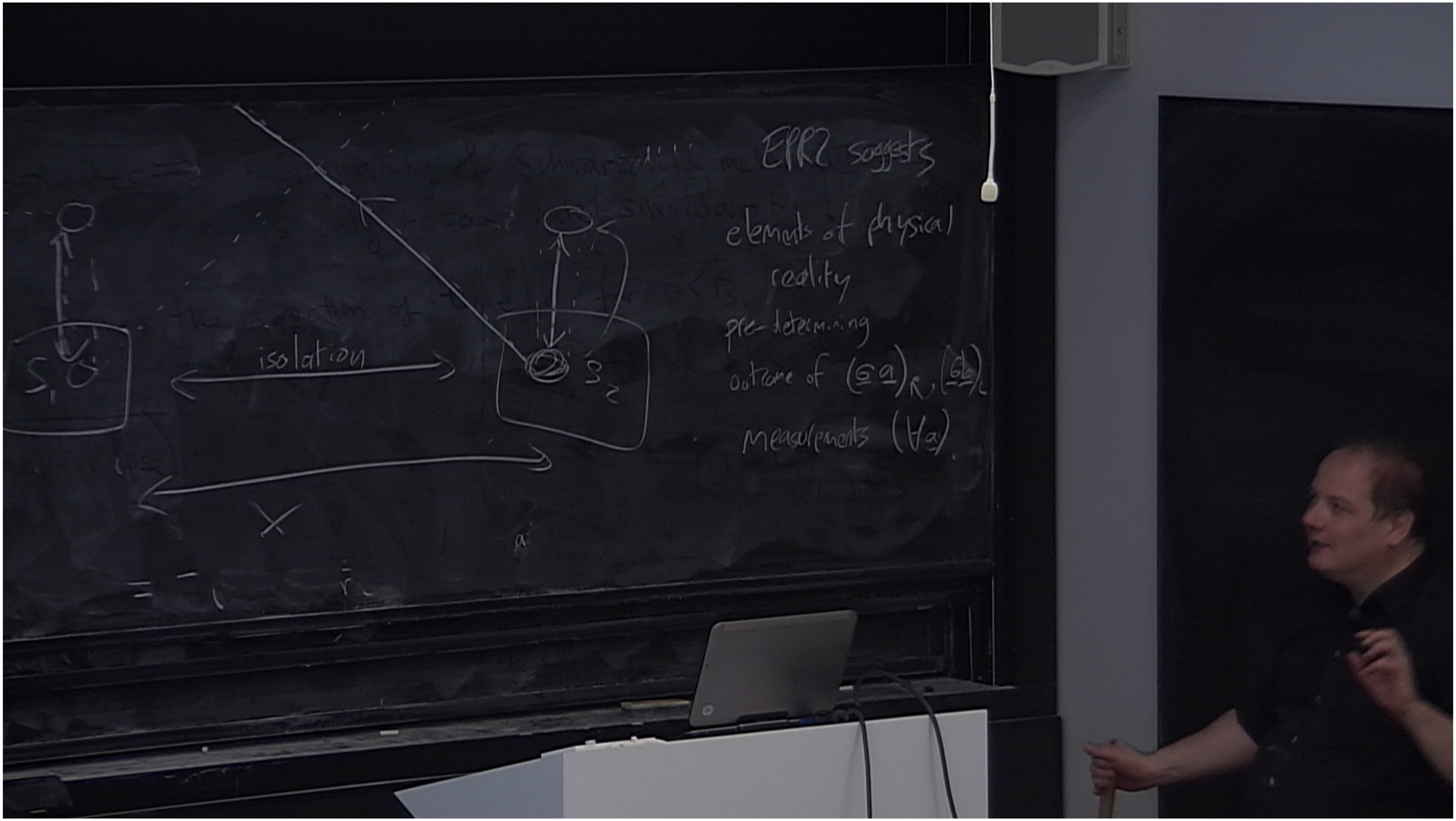
measurements $(\forall \underline{a})$.
isolation
S_1
S_2
start



EPR2 suggests

elements of physical
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pre-determining
outcome of $(\underline{\sigma}\cdot\underline{a})_R, (\underline{\sigma}\cdot\underline{b})_L$
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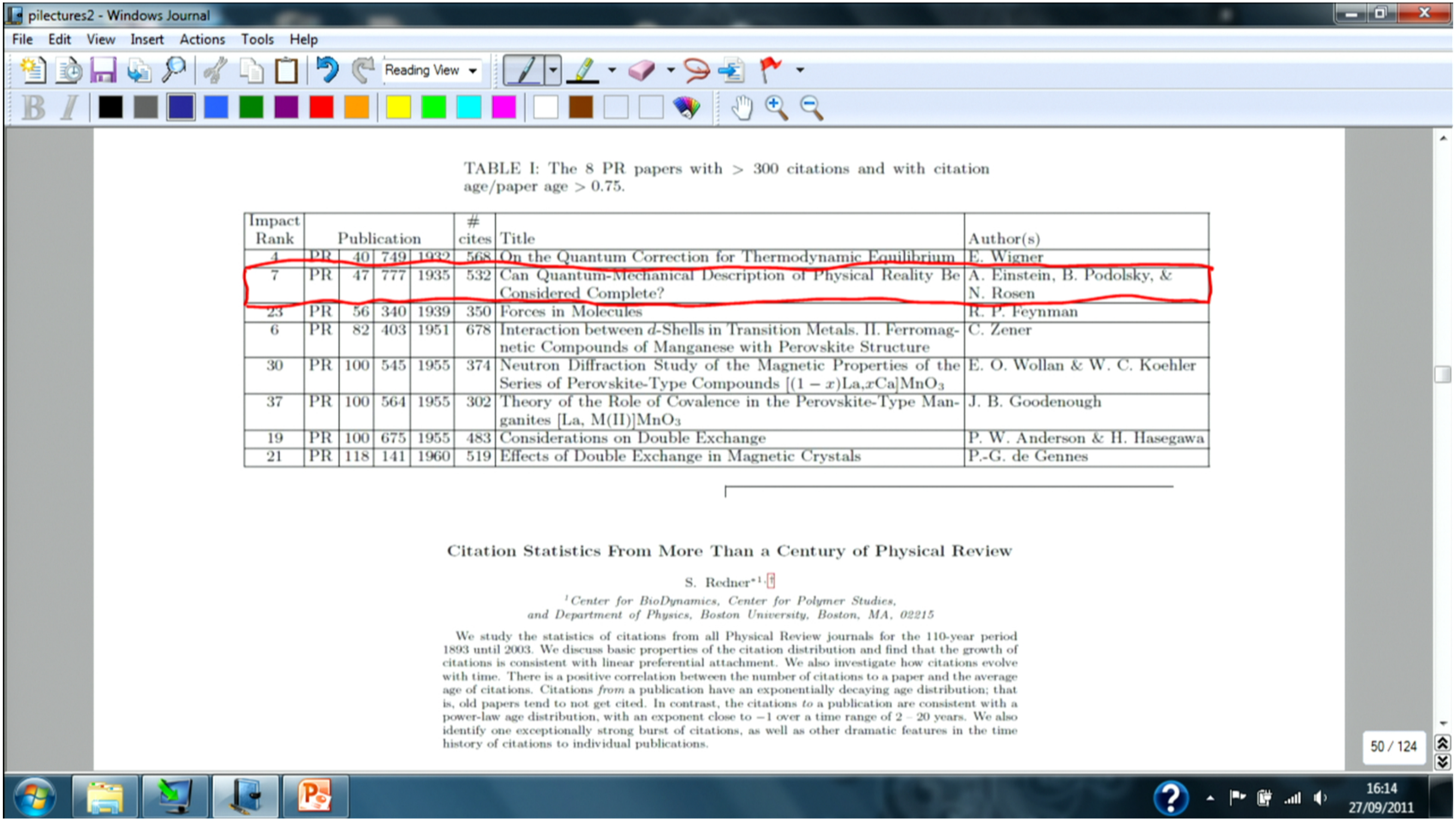


TABLE I: The 8 PR papers with $>$ 300 citations and with citation age/paper age $>$ 0.75.

Impact Rank	Publication				# cites	Title	Author(s)
4	PR	40	749	1932	568	On the Quantum Correction for Thermodynamic Equilibrium	E. Wigner
7	PR	47	777	1935	532	Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?	A. Einstein, B. Podolsky, & N. Rosen
23	PR	56	340	1939	350	Forces in Molecules	R. P. Feynman
6	PR	82	403	1951	678	Interaction between *d*-Shells in Transition Metals. II. Ferromagnetic Compounds of Manganese with Perovskite Structure	C. Zener
30	PR	100	545	1955	374	Neutron Diffraction Study of the Magnetic Properties of the Series of Perovskite-Type Compounds $[(1-x)\text{La}, x\text{Ca}]\text{MnO}_3$	E. O. Wollan & W. C. Koehler
37	PR	100	564	1955	302	Theory of the Role of Covalence in the Perovskite-Type Manganites $[\text{La}, \text{M(II)}]\text{MnO}_3$	J. B. Goodenough
19	PR	100	675	1955	483	Considerations on Double Exchange	P. W. Anderson & H. Hasegawa
21	PR	118	141	1960	519	Effects of Double Exchange in Magnetic Crystals	P.-G. de Gennes

Citation Statistics From More Than a Century of Physical Review

S. Redner*[1,†]

[1] *Center for BioDynamics, Center for Polymer Studies, and Department of Physics, Boston University, Boston, MA, 02215*

We study the statistics of citations from all Physical Review journals for the 110-year period 1893 until 2003. We discuss basic properties of the citation distribution and find that the growth of citations is consistent with linear preferential attachment. We also investigate how citations evolve with time. There is a positive correlation between the number of citations to a paper and the average age of citations. Citations *from* a publication have an exponentially decaying age distribution; that is, old papers tend to not get cited. In contrast, the citations *to* a publication are consistent with a power-law age distribution, with an exponent close to -1 over a time range of 2 – 20 years. We also identify one exceptionally strong burst of citations, as well as other dramatic features in the time history of citations to individual publications.

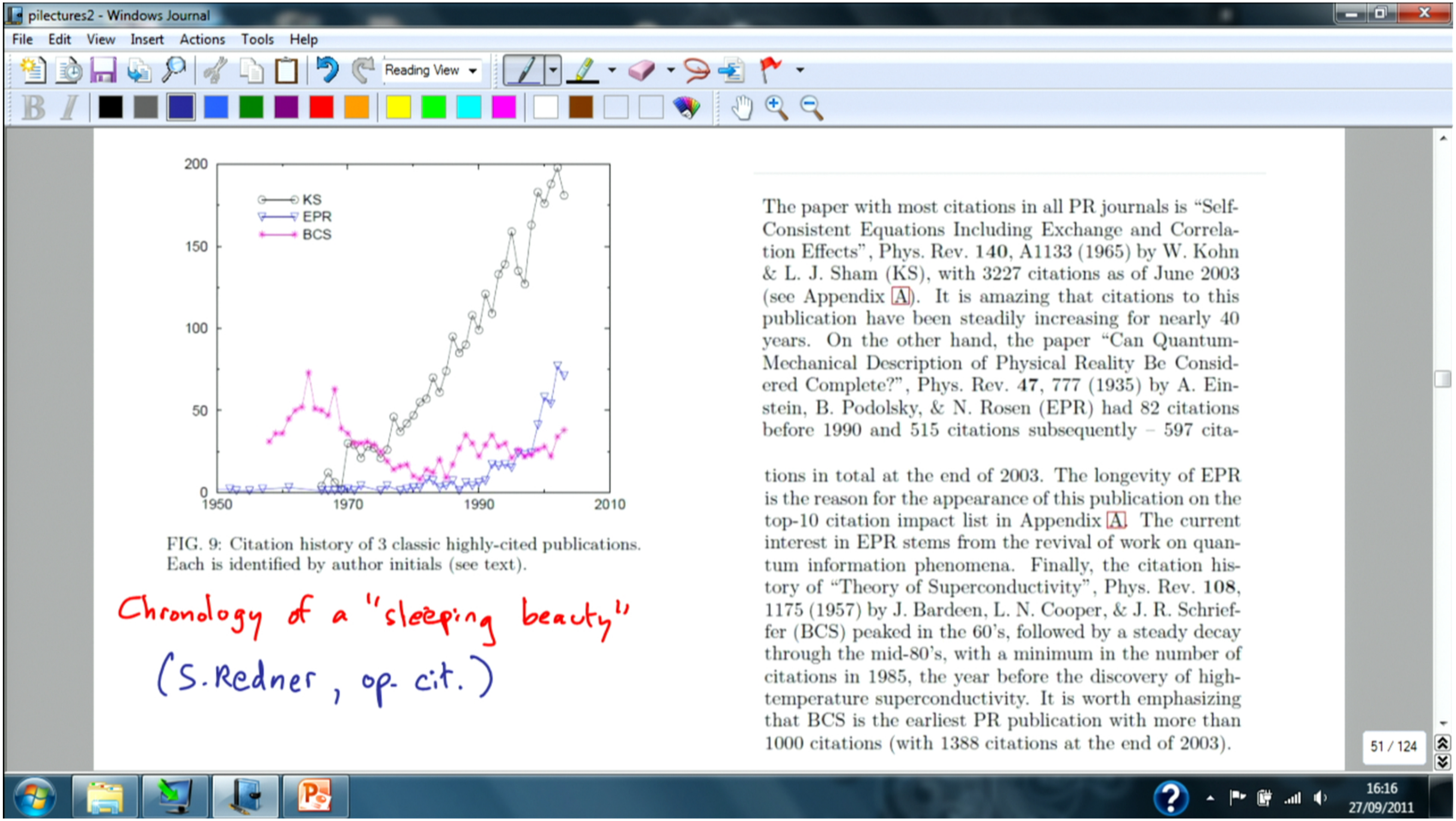
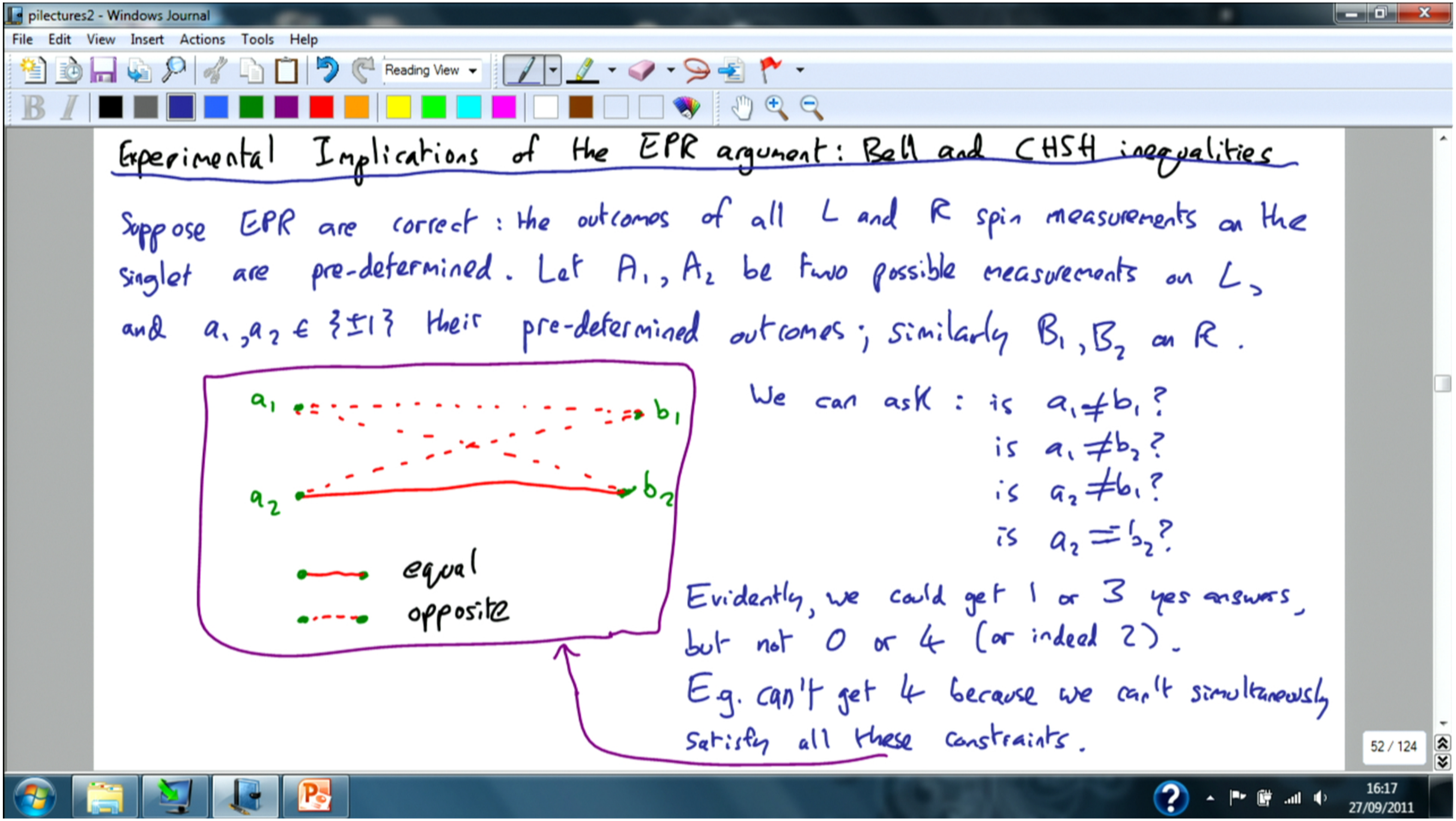


FIG. 9: Citation history of 3 classic highly-cited publications. Each is identified by author initials (see text).

Chronology of a "sleeping beauty"

(S. Redner, op. cit.)

The paper with most citations in all PR journals is "Self-Consistent Equations Including Exchange and Correlation Effects", Phys. Rev. **140**, A1133 (1965) by W. Kohn & L. J. Sham (KS), with 3227 citations as of June 2003 (see Appendix A). It is amazing that citations to this publication have been steadily increasing for nearly 40 years. On the other hand, the paper "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", Phys. Rev. **47**, 777 (1935) by A. Einstein, B. Podolsky, & N. Rosen (EPR) had 82 citations before 1990 and 515 citations subsequently – 597 citations in total at the end of 2003. The longevity of EPR is the reason for the appearance of this publication on the top-10 citation impact list in Appendix A. The current interest in EPR stems from the revival of work on quantum information phenomena. Finally, the citation history of "Theory of Superconductivity", Phys. Rev. **108**, 1175 (1957) by J. Bardeen, L. N. Cooper, & J. R. Schrieffer (BCS) peaked in the 60's, followed by a steady decay through the mid-80's, with a minimum in the number of citations in 1985, the year before the discovery of high-temperature superconductivity. It is worth emphasizing that BCS is the earliest PR publication with more than 1000 citations (with 1388 citations at the end of 2003).



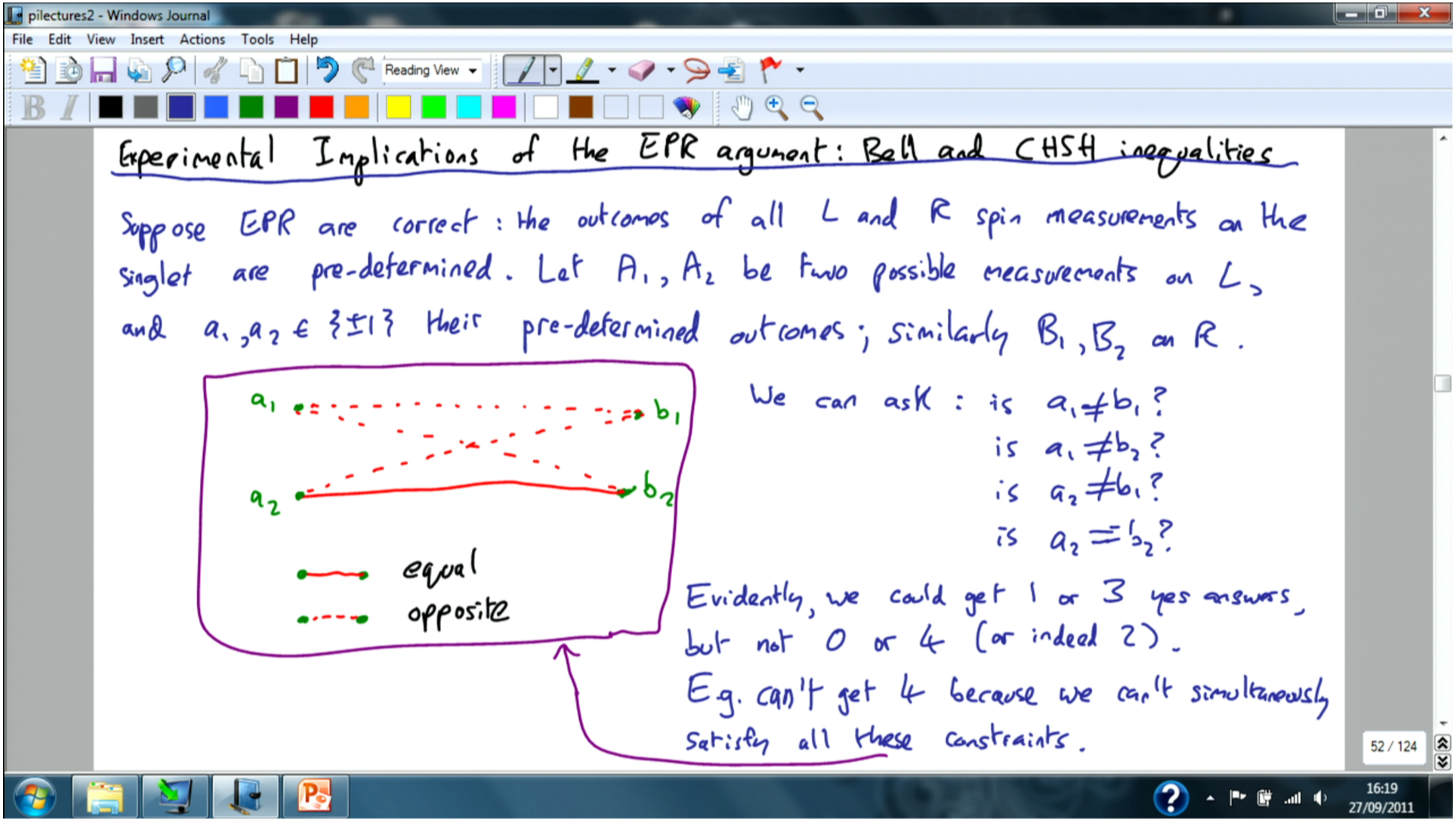
Experimental Implications of the EPR argument: Bell and CHSH inequalities

Suppose EPR are correct: the outcomes of all L and R spin measurements on the singlet are pre-determined. Let A_1, A_2 be two possible measurements on L, and $a_1, a_2 \in \{\pm 1\}$ their pre-determined outcomes; similarly B_1, B_2 on R.

We can ask: is $a_1 \neq b_1$?
is $a_1 \neq b_2$?
is $a_2 \neq b_1$?
is $a_2 = b_2$?

Evidently, we could get 1 or 3 yes answers, but not 0 or 4 (or indeed 2).

E.g. can't get 4 because we can't simultaneously satisfy all these constraints.



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equal

opposite

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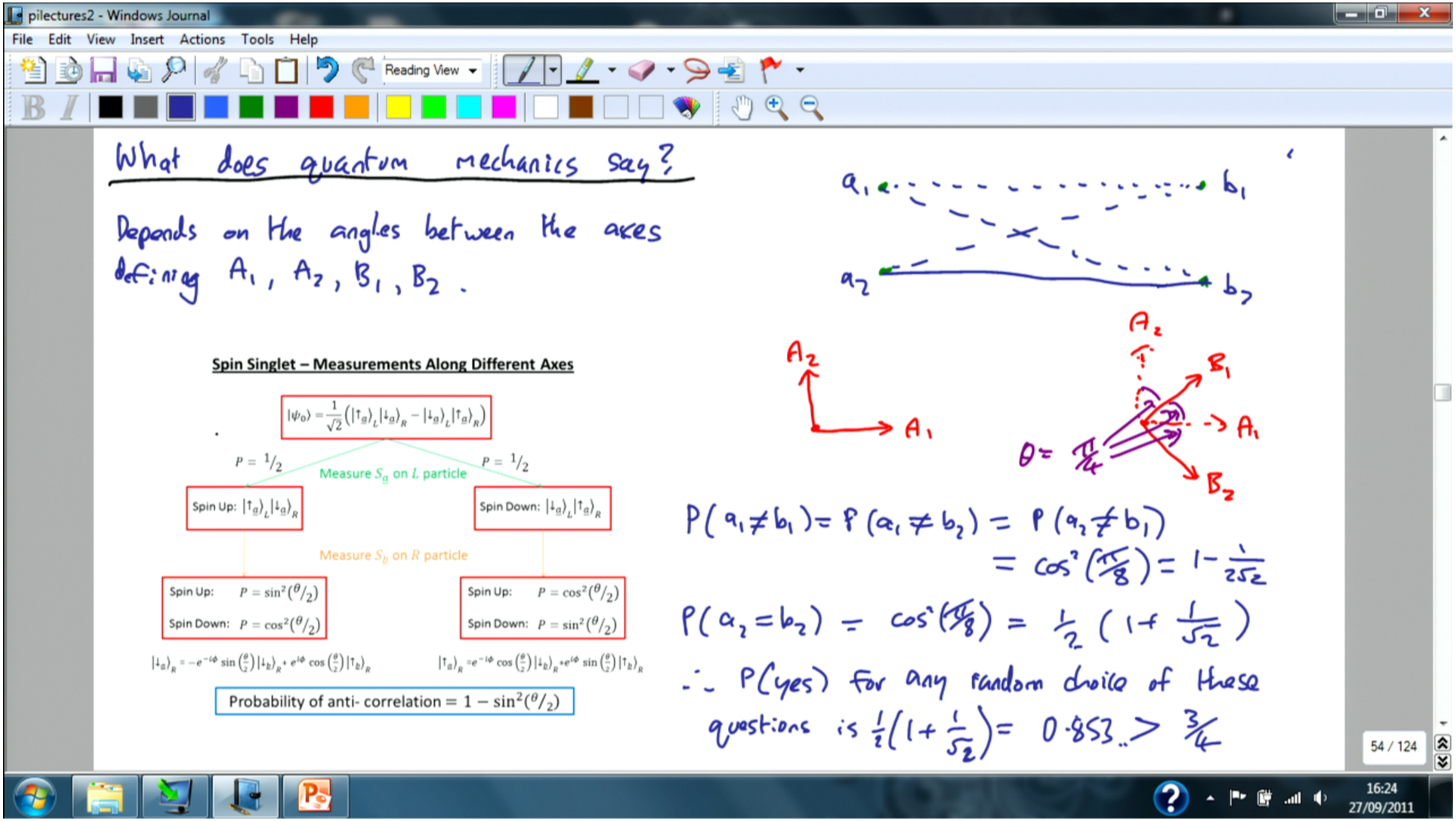

What does quantum mechanics say?

Depends on the angles between the axes defining A_1, A_2, B_1, B_2.

Spin Singlet – Measurements Along Different Axes

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_{\underline{a}}\rangle_L|\downarrow_{\underline{a}}\rangle_R - |\downarrow_{\underline{a}}\rangle_L|\uparrow_{\underline{a}}\rangle_R\right)$$

$P = {}^1/_2$

Measure $S_{\underline{a}}$ on L particle

$P = {}^1/_2$

Spin Up: $|\uparrow_{\underline{a}}\rangle_L|\downarrow_{\underline{a}}\rangle_R$

Spin Down: $|\downarrow_{\underline{a}}\rangle_L|\uparrow_{\underline{a}}\rangle_R$

Measure $S_{\underline{b}}$ on R particle

Spin Up: $P = \sin^2({}^\theta/_2)$

Spin Down: $P = \cos^2({}^\theta/_2)$

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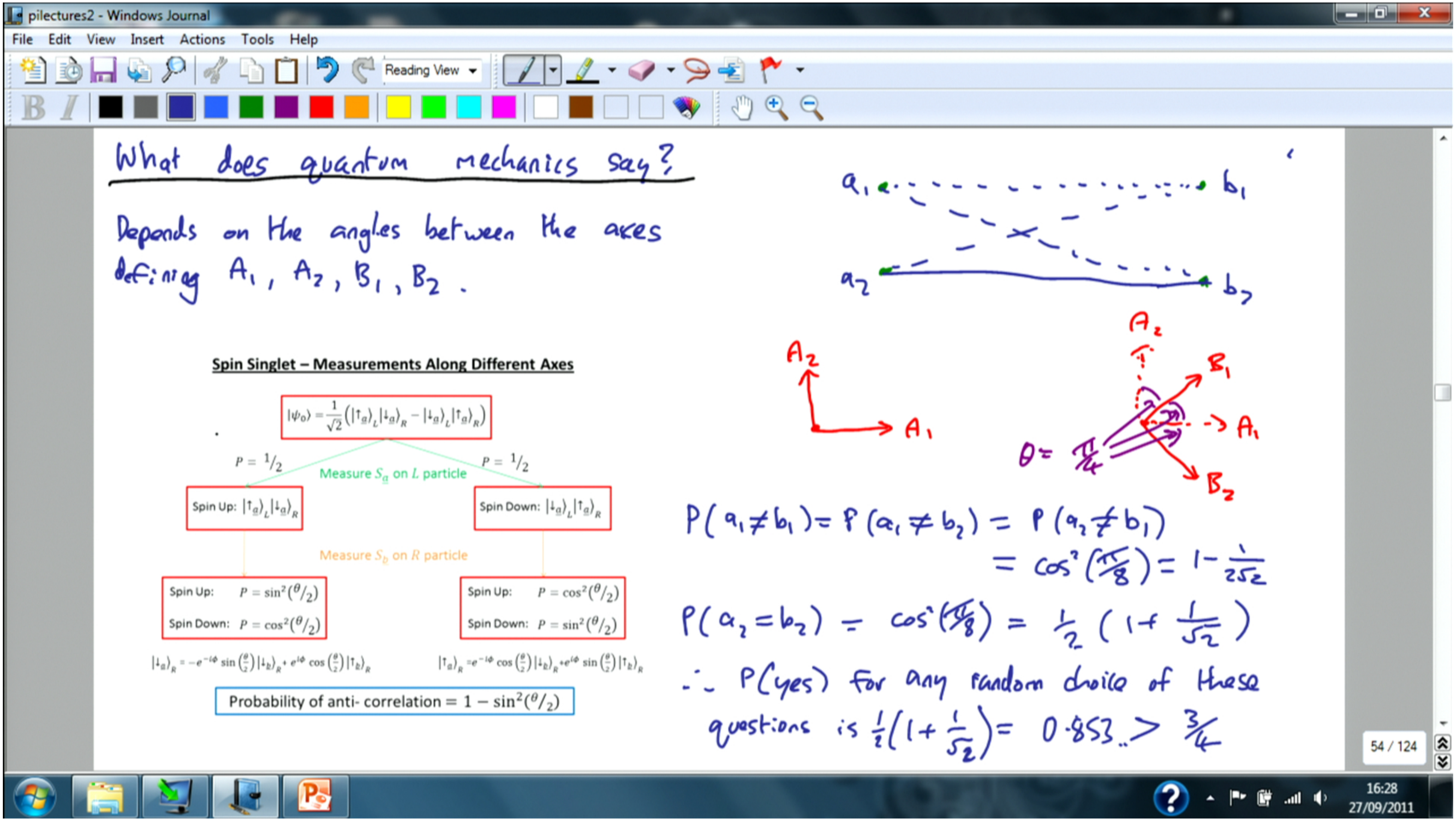
$\therefore$ P(yes) for any random choice of these questions is $\frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right) = 0.853.. > \frac{3}{4}$



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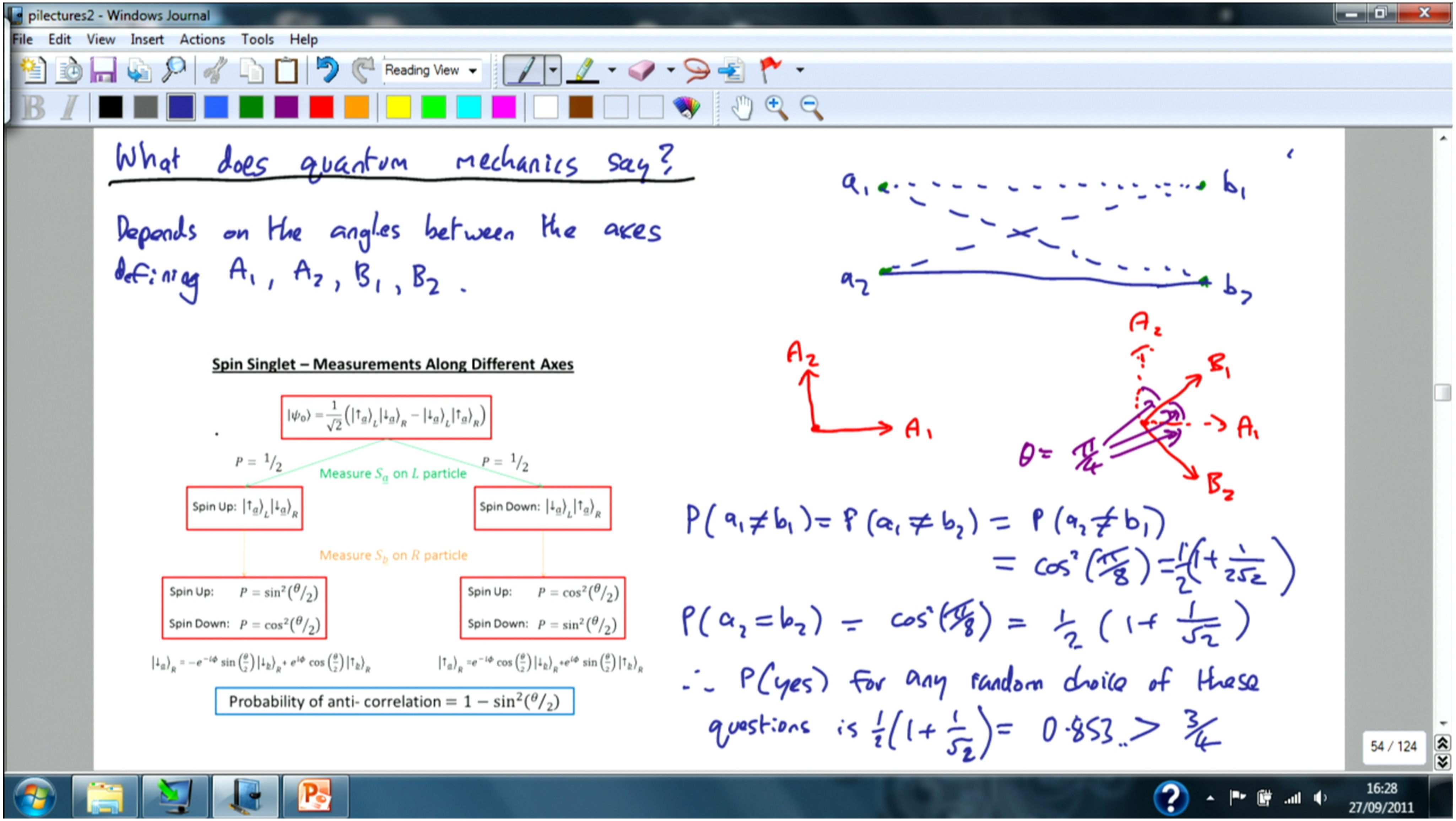

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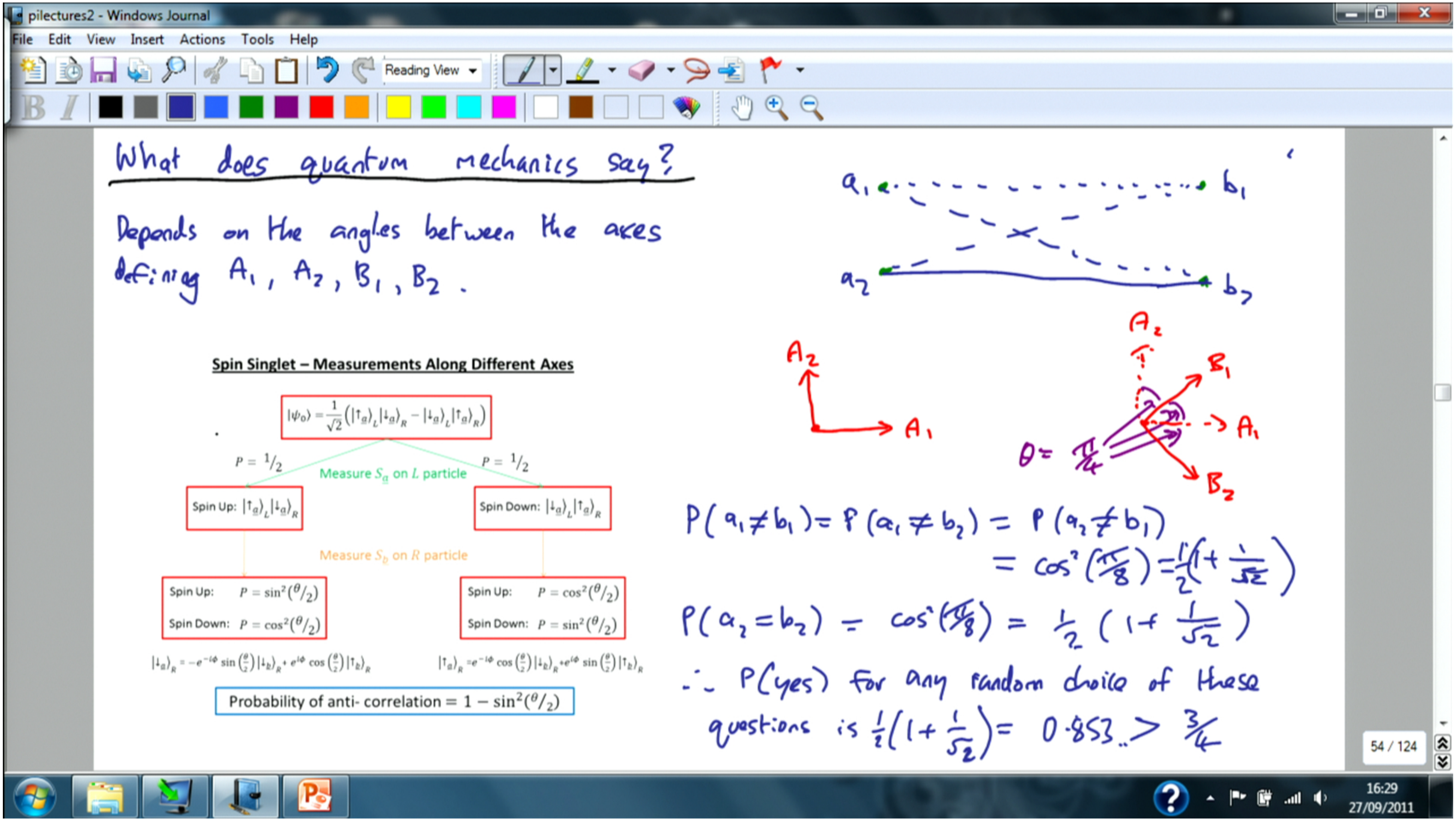

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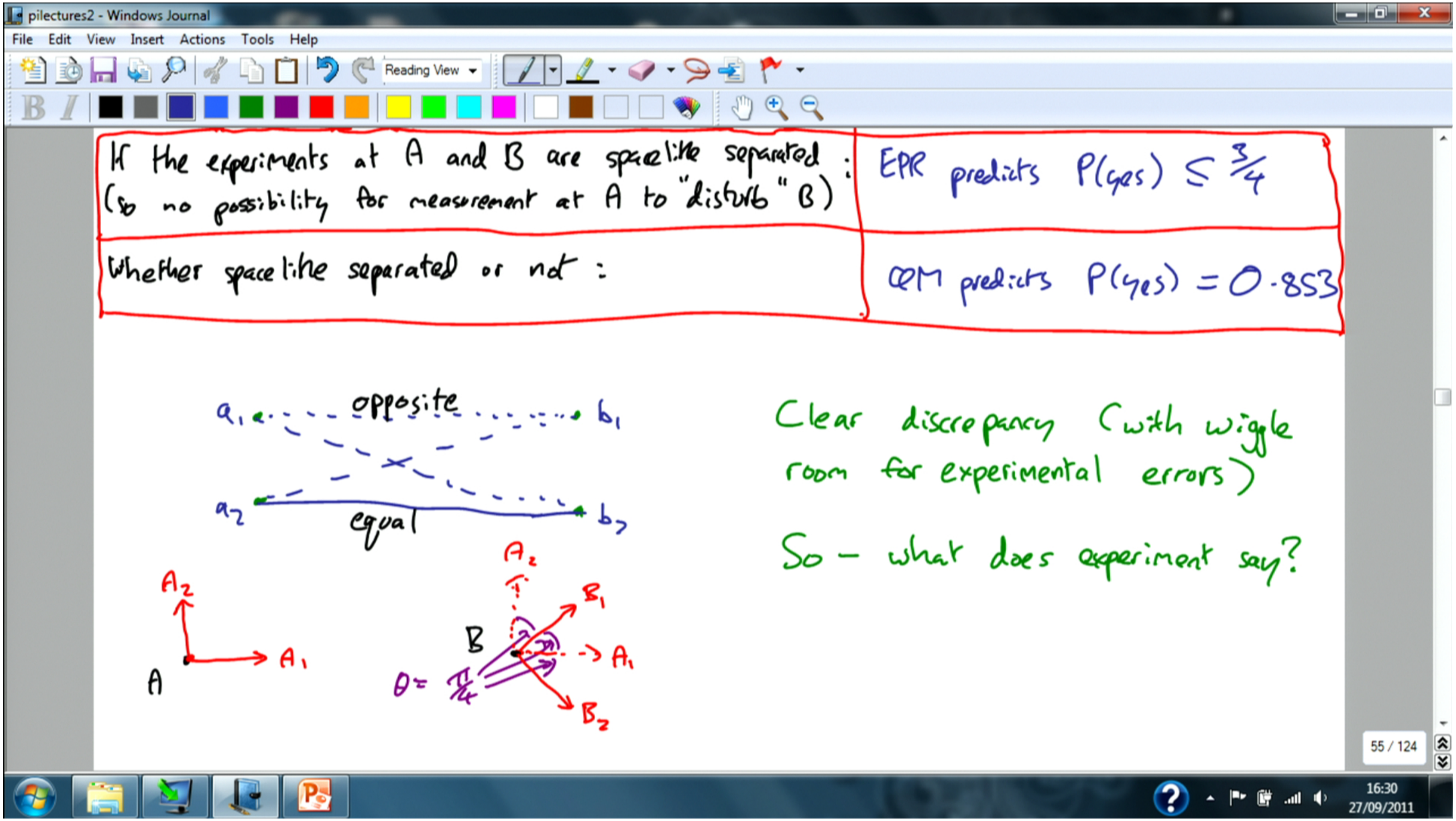

If the experiments at A and B are spacelike separated: (so no possibility for measurement at A to "disturb" B)	EPR predicts $P(\text{yes}) \leq 3/4$
Whether spacelike separated or not:	QM predicts $P(\text{yes}) = 0.853$

Clear discrepancy (with wiggle room for experimental errors)

So – what does experiment say?

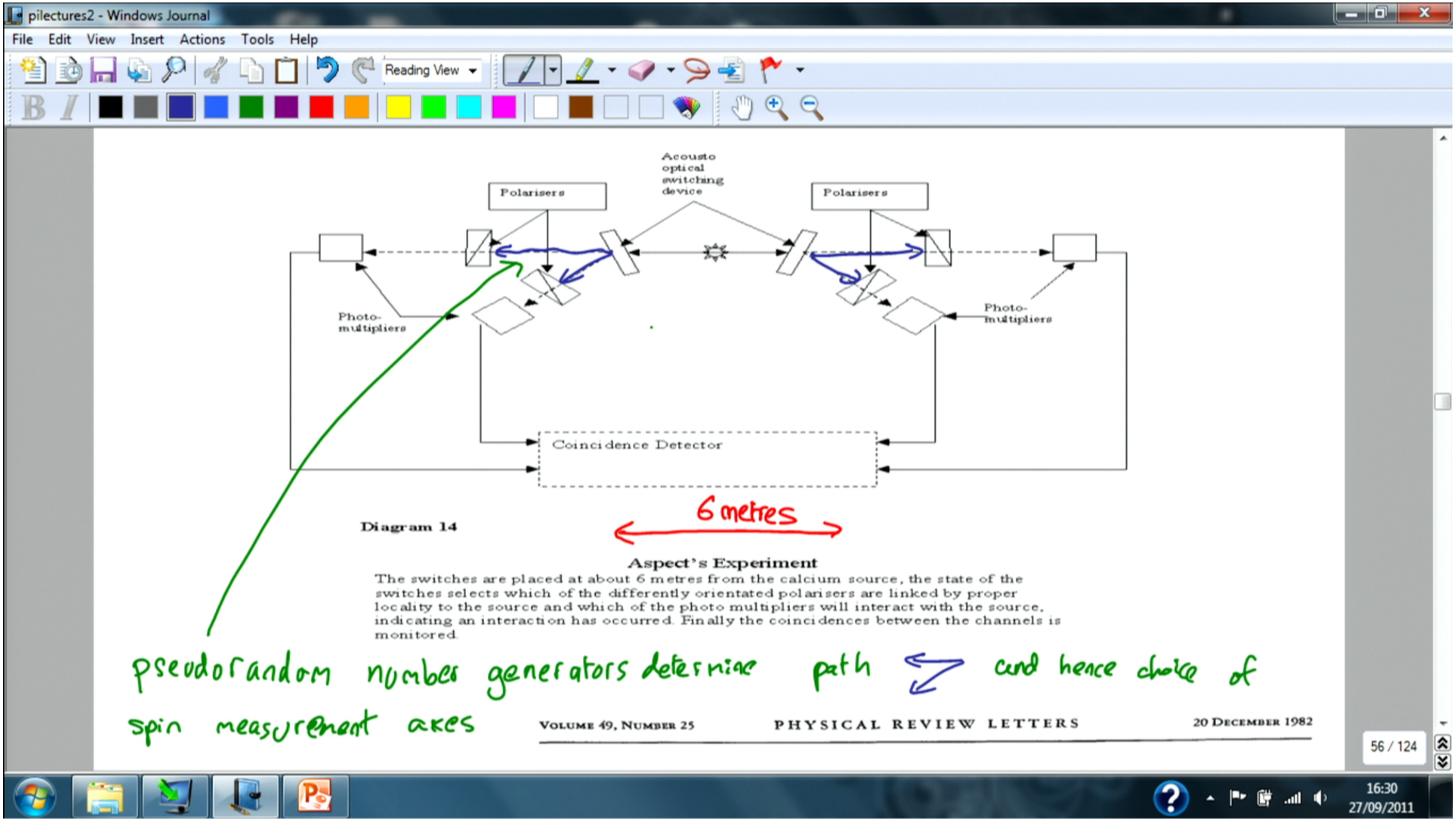

Diagram 14

6 metres

Aspect's Experiment

The switches are placed at about 6 metres from the calcium source, the state of the switches selects which of the differently orientated polarisers are linked by proper locality to the source and which of the photo multipliers will interact with the source, indicating an interaction has occurred. Finally the coincidences between the channels is monitored.

pseudorandom number generators determine path ⇄ and hence choice of spin measurement axes

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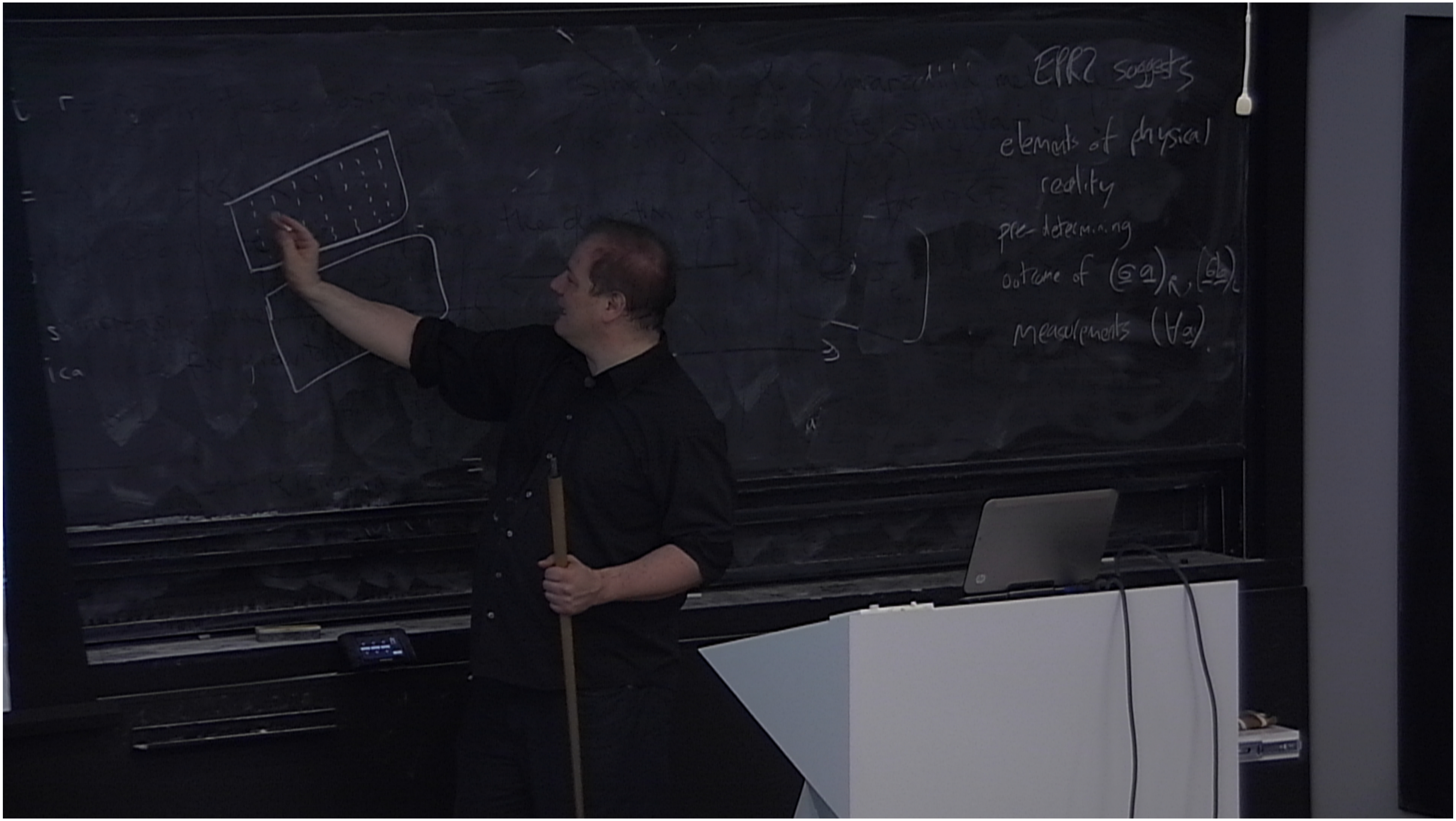
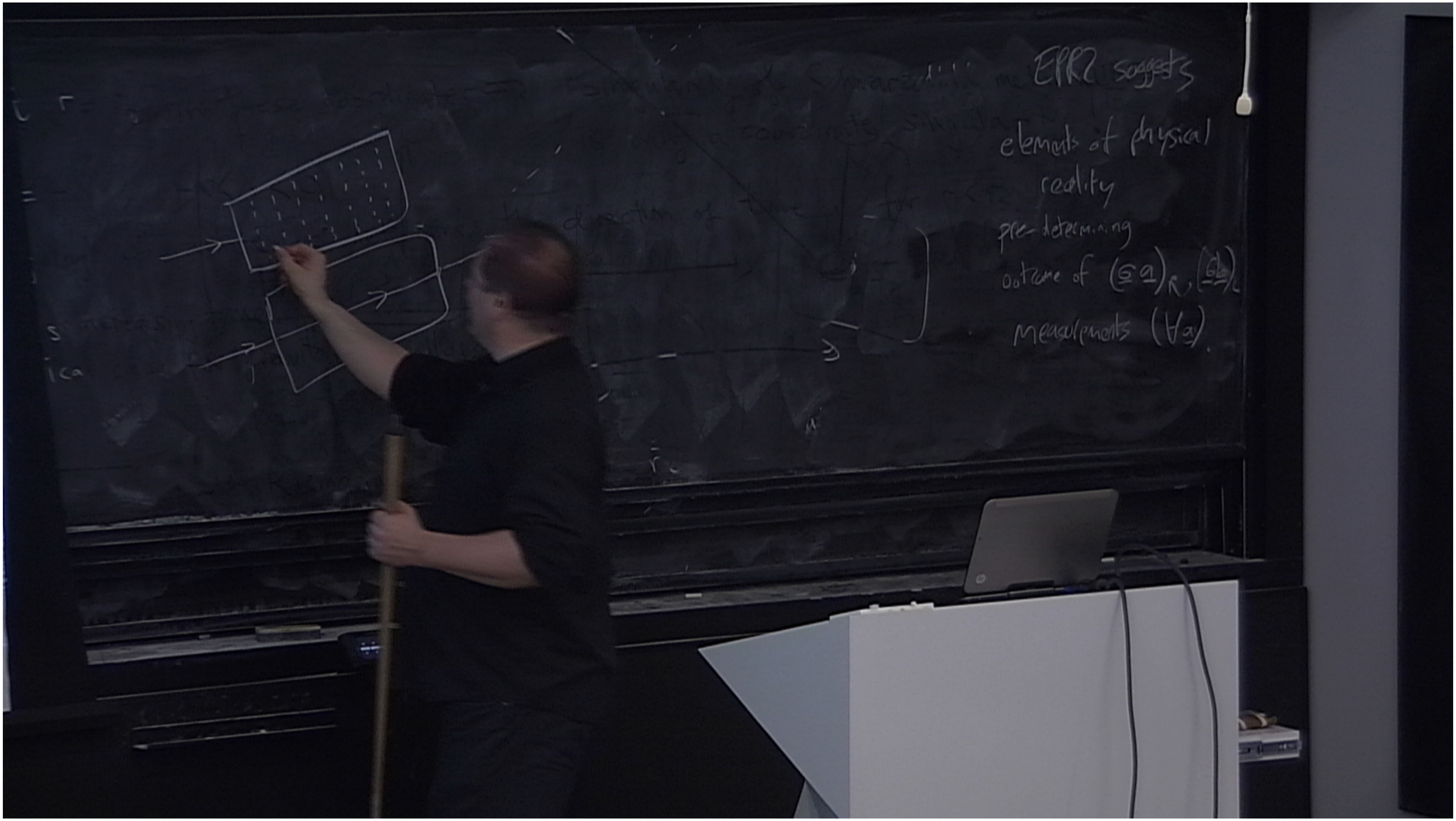


EPR2 suggests

elements of physical
reality
pre-determining
outcome of $(\underline{\sigma}\cdot\underline{a})_R, (\underline{\sigma}\cdot\underline{b})_L$
measurements $(\forall \underline{a})$.



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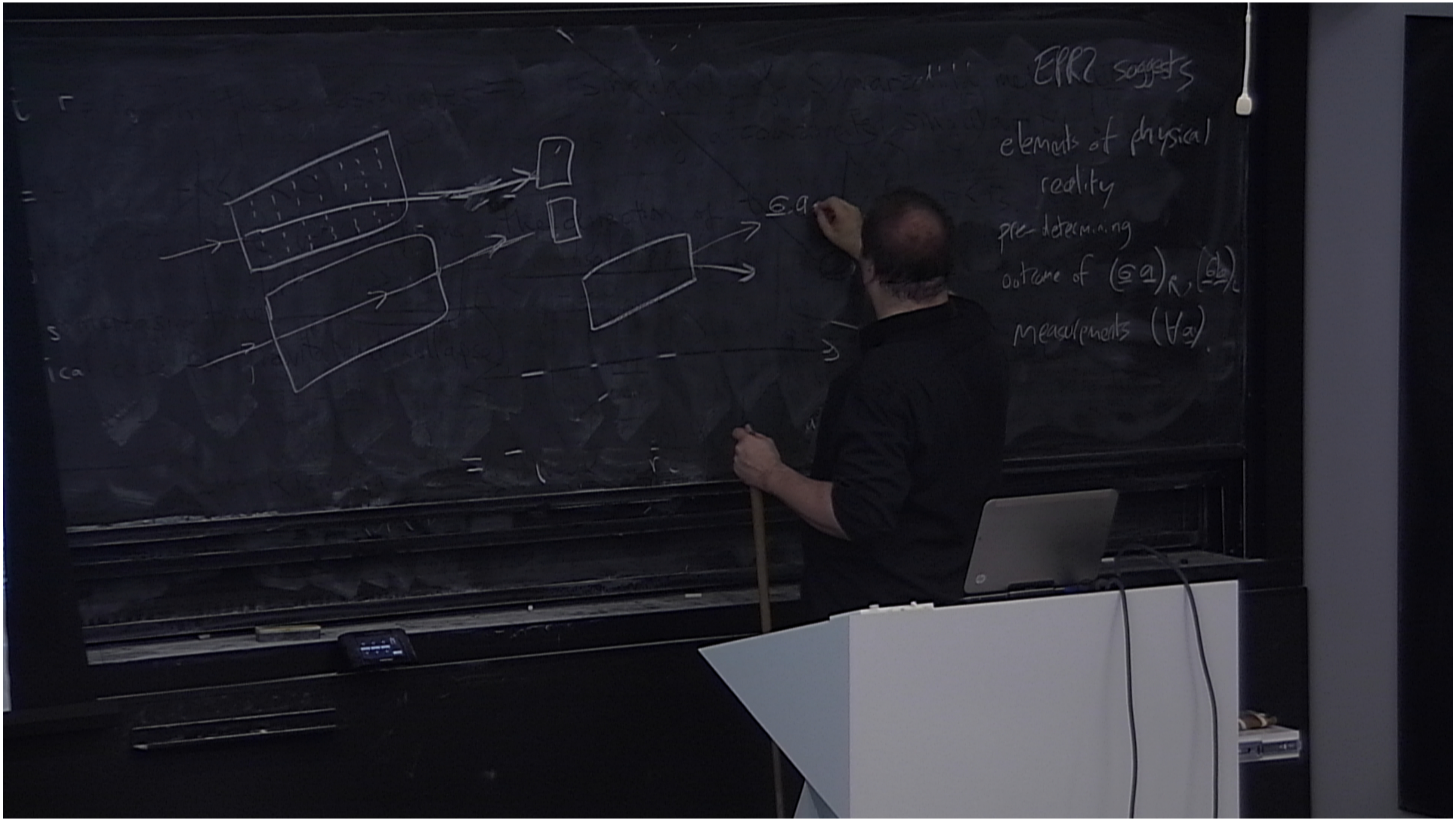
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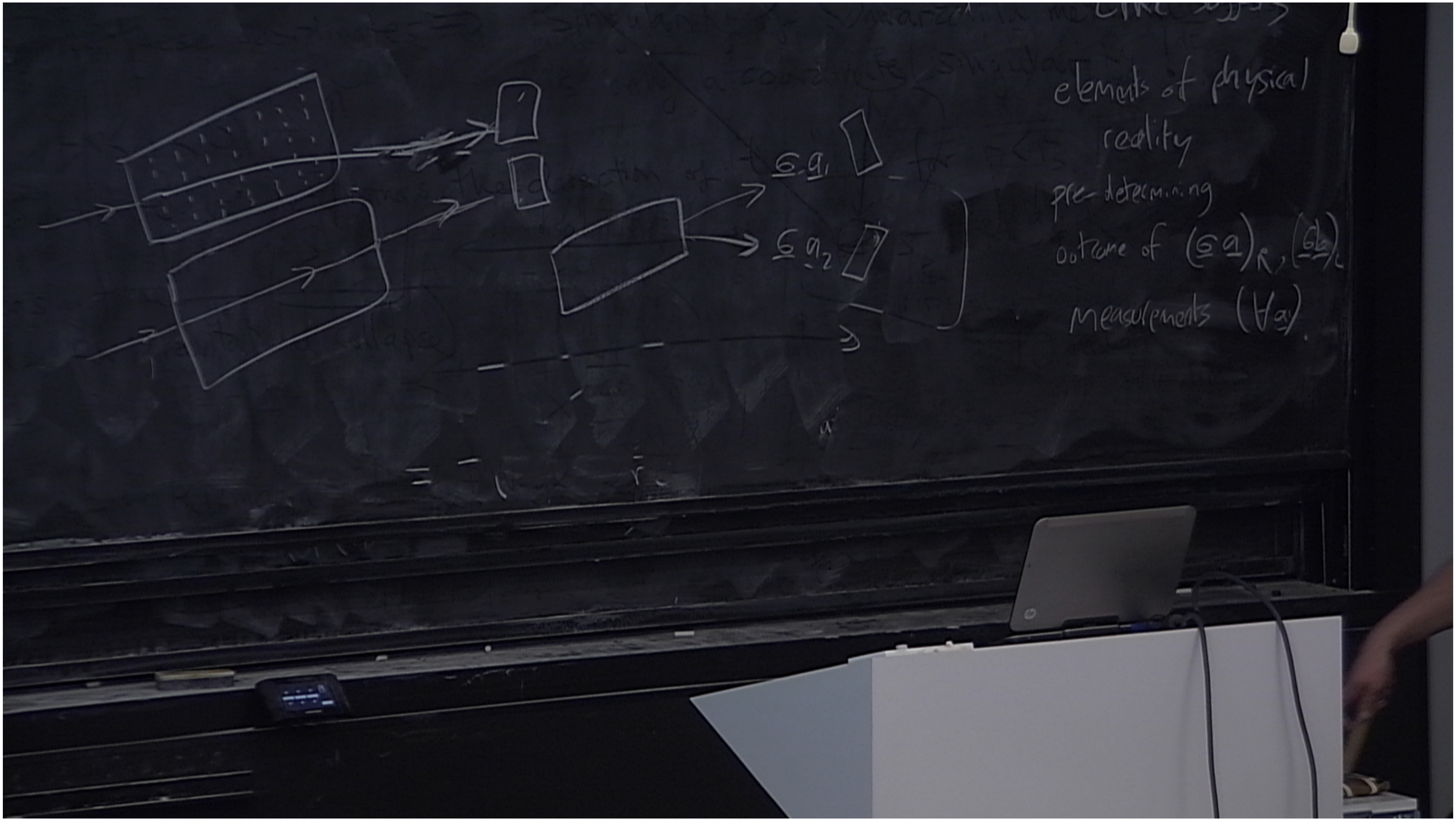

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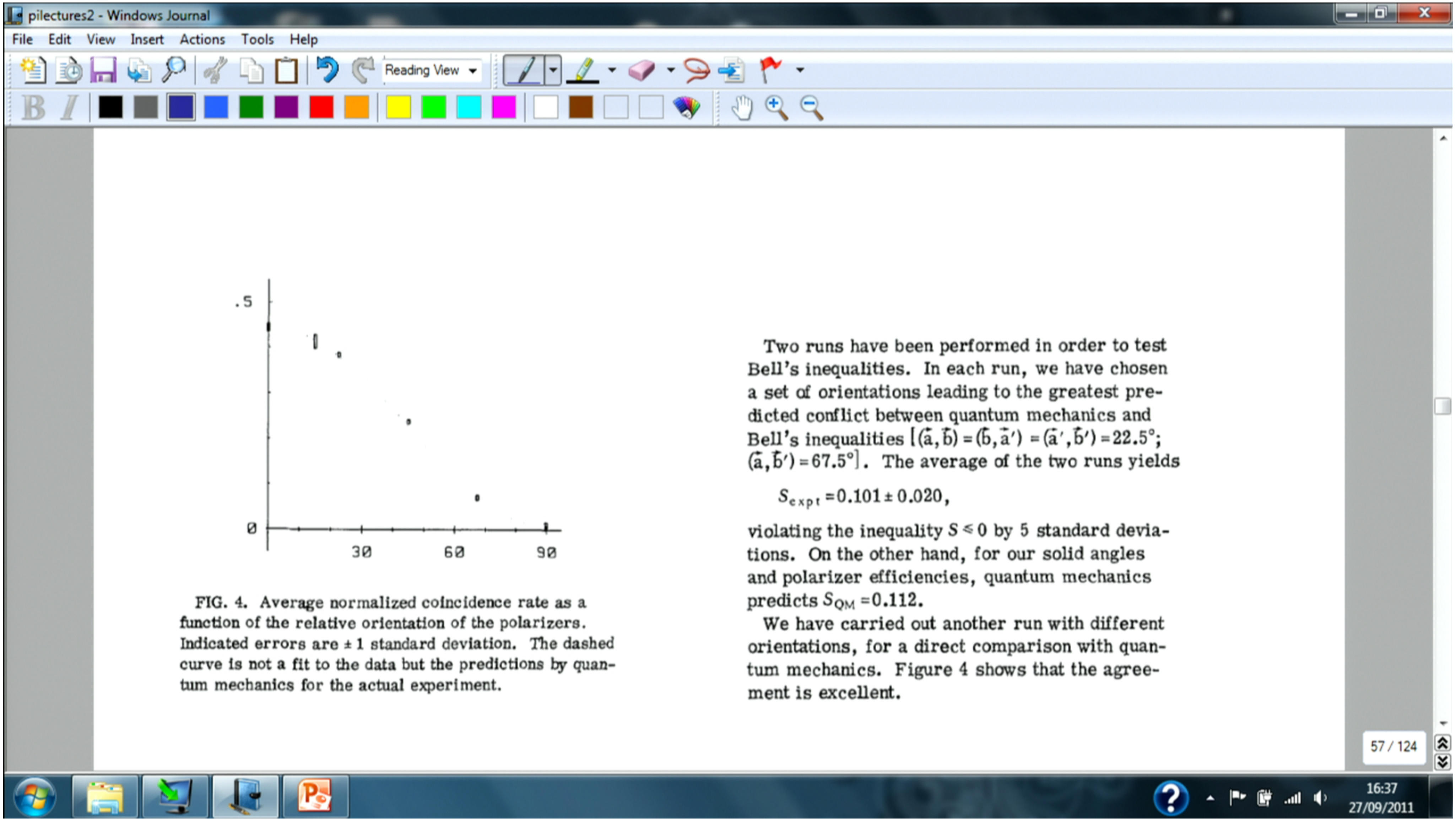


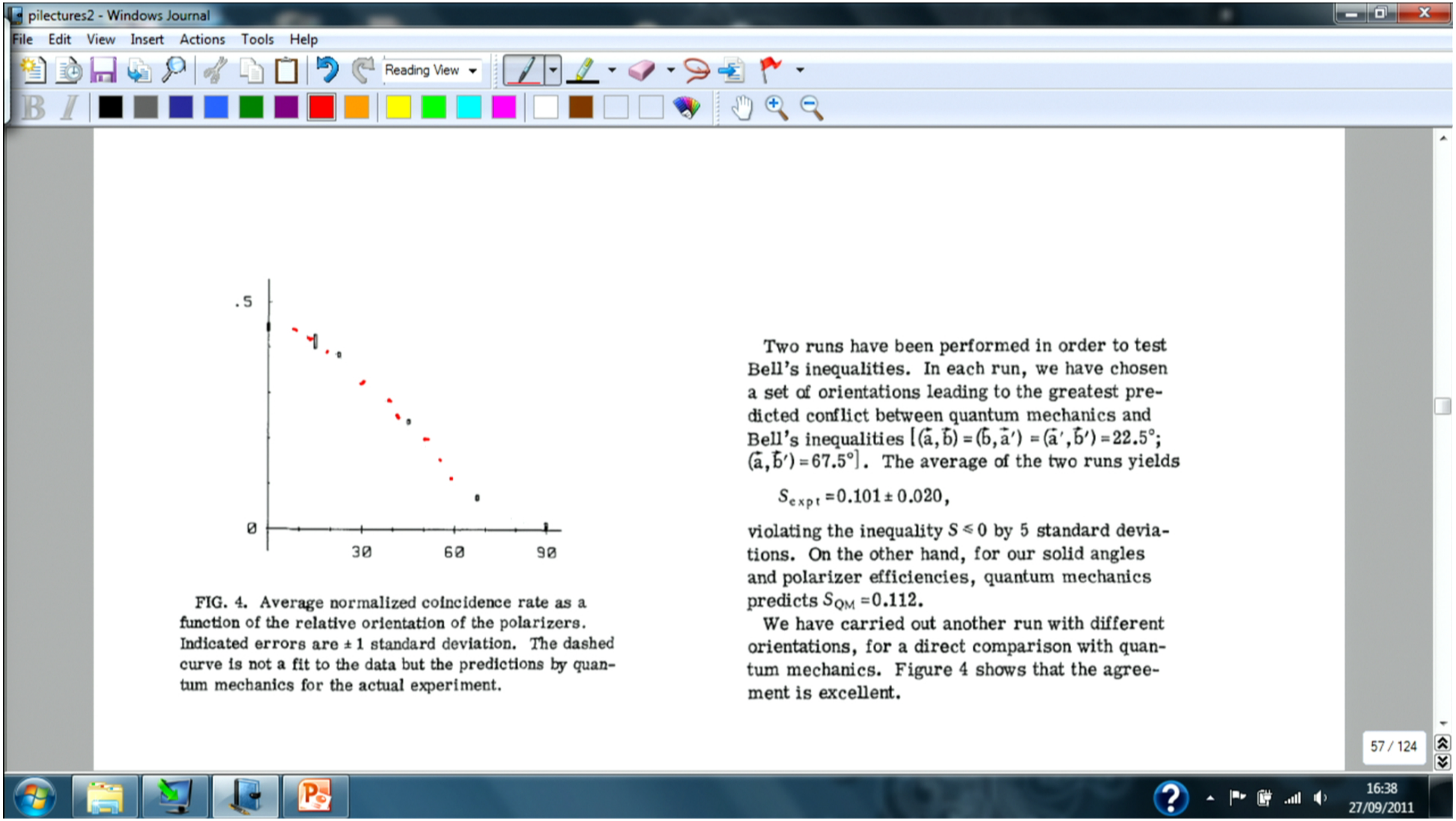
FIG. 4. Average normalized coincidence rate as a function of the relative orientation of the polarizers. Indicated errors are ± 1 standard deviation. The dashed curve is not a fit to the data but the predictions by quantum mechanics for the actual experiment.

Two runs have been performed in order to test Bell's inequalities. In each run, we have chosen a set of orientations leading to the greatest predicted conflict between quantum mechanics and Bell's inequalities $[(\vec{a},\vec{b}) = (\vec{b},\vec{a}') = (\vec{a}',\vec{b}') = 22.5°;$ $(\vec{a},\vec{b}') = 67.5°]$. The average of the two runs yields

$$S_{\mathrm{expt}} = 0.101 \pm 0.020,$$

violating the inequality $S \leq 0$ by 5 standard deviations. On the other hand, for our solid angles and polarizer efficiencies, quantum mechanics predicts $S_{\mathrm{QM}} = 0.112$.

We have carried out another run with different orientations, for a direct comparison with quantum mechanics. Figure 4 shows that the agreement is excellent.



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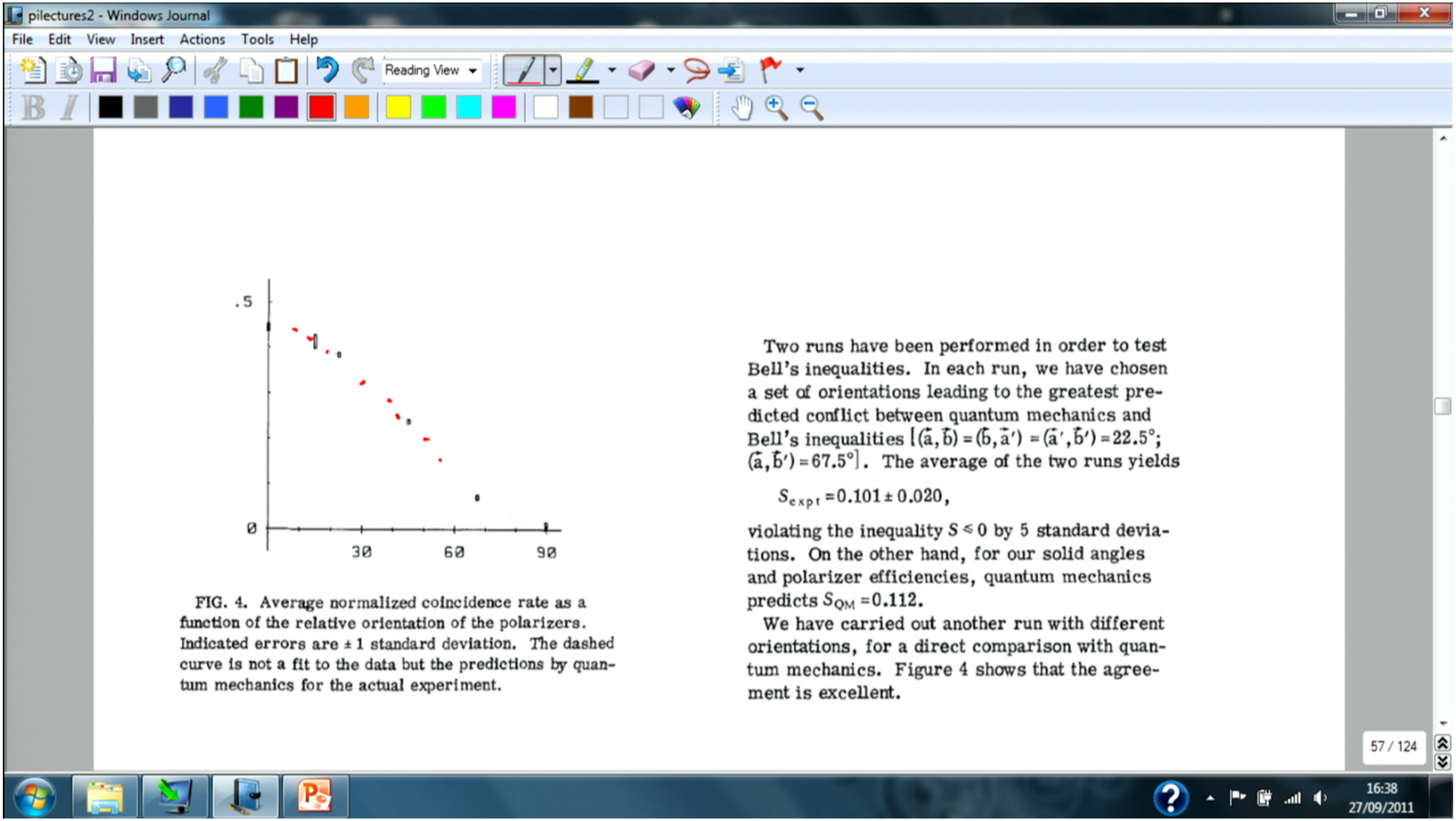

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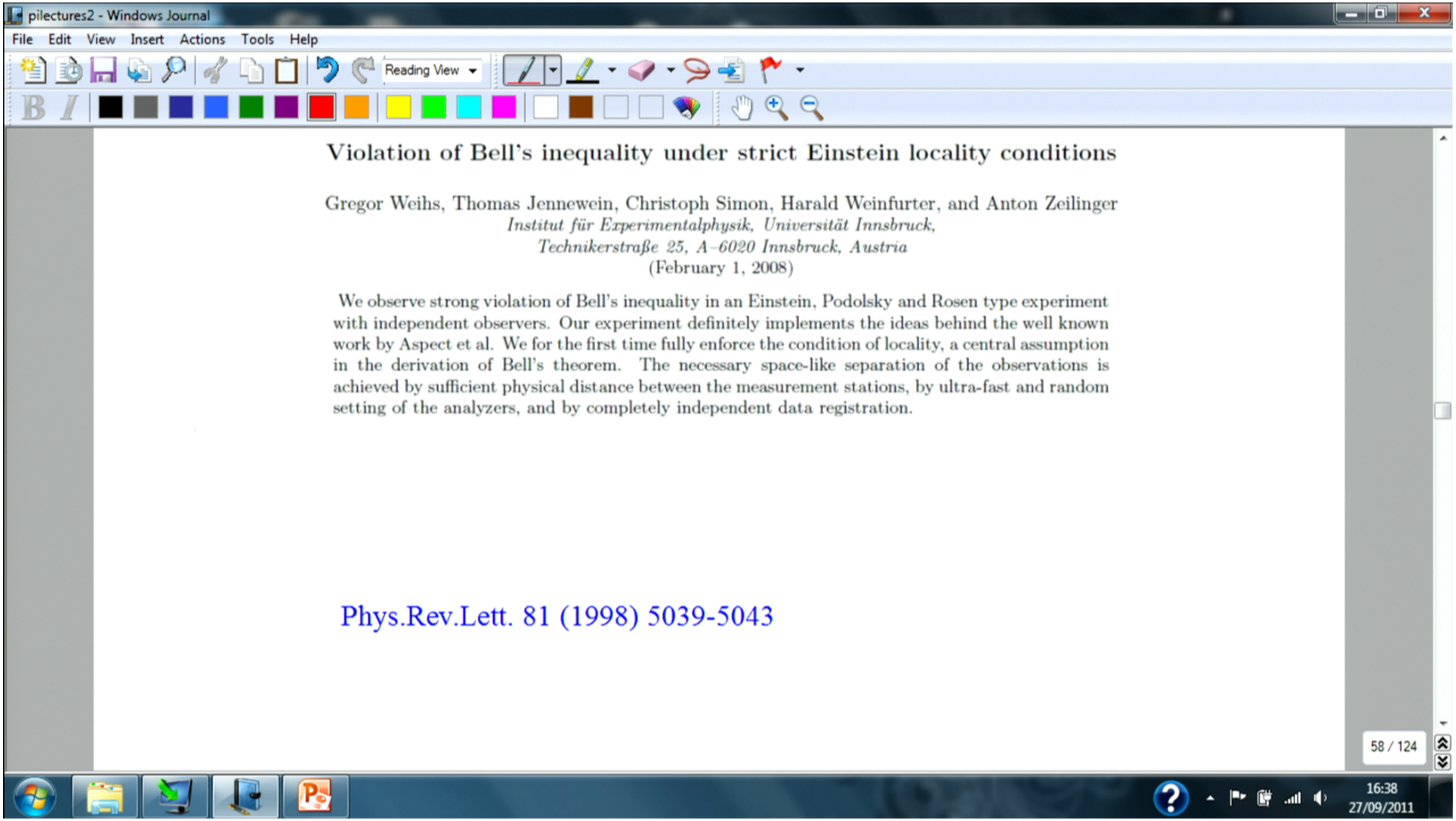
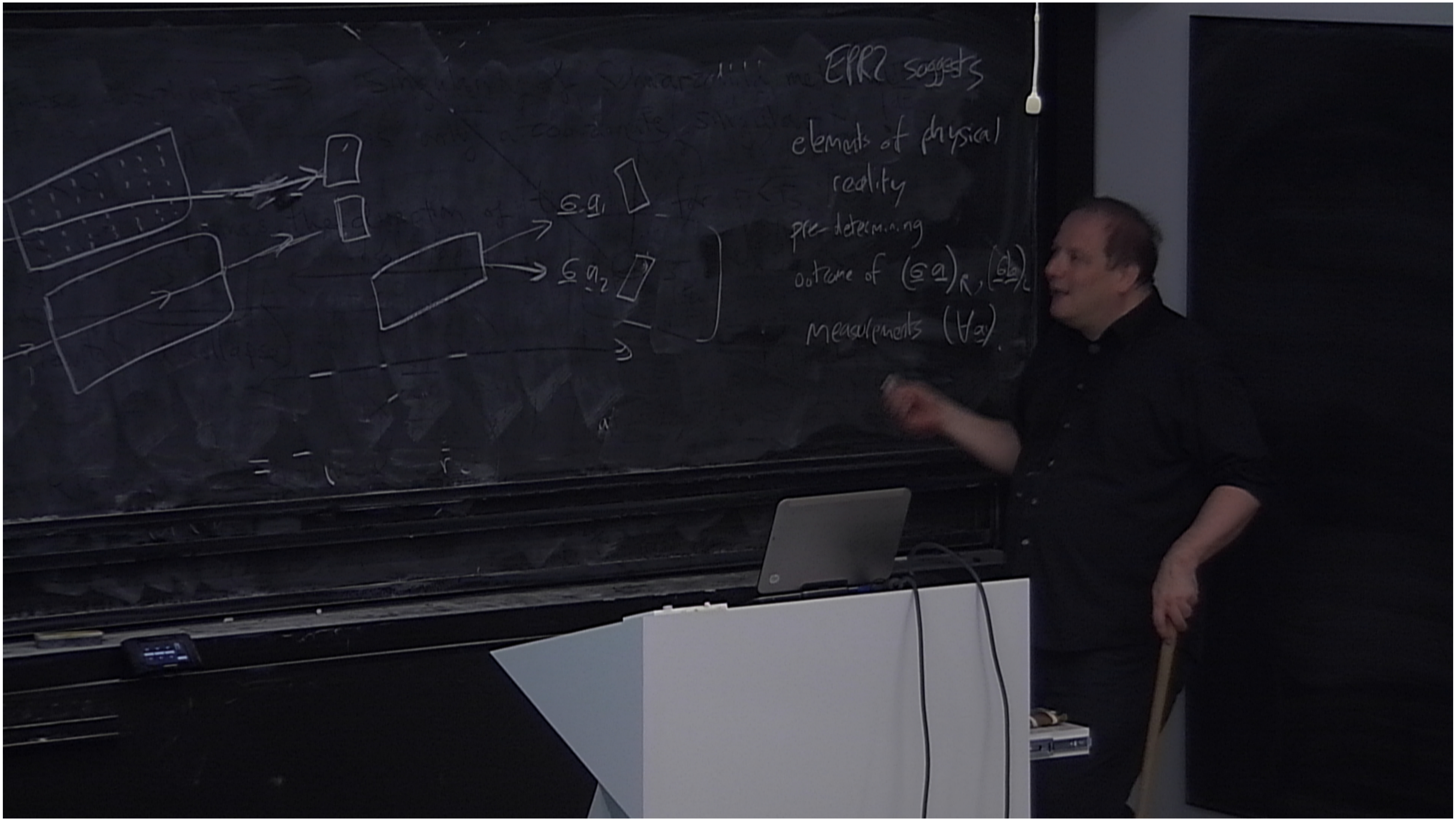


Violation of Bell's inequality under strict Einstein locality conditions

Gregor Weihs, Thomas Jennewein, Christoph Simon, Harald Weinfurter, and Anton Zeilinger
Institut für Experimentalphysik, Universität Innsbruck,
Technikerstraße 25, A–6020 Innsbruck, Austria
(February 1, 2008)

We observe strong violation of Bell's inequality in an Einstein, Podolsky and Rosen type experiment with independent observers. Our experiment definitely implements the ideas behind the well known work by Aspect et al. We for the first time fully enforce the condition of locality, a central assumption in the derivation of Bell's theorem. The necessary space-like separation of the observations is achieved by sufficient physical distance between the measurement stations, by ultra-fast and random setting of the analyzers, and by completely independent data registration.

Phys.Rev.Lett. 81 (1998) 5039-5043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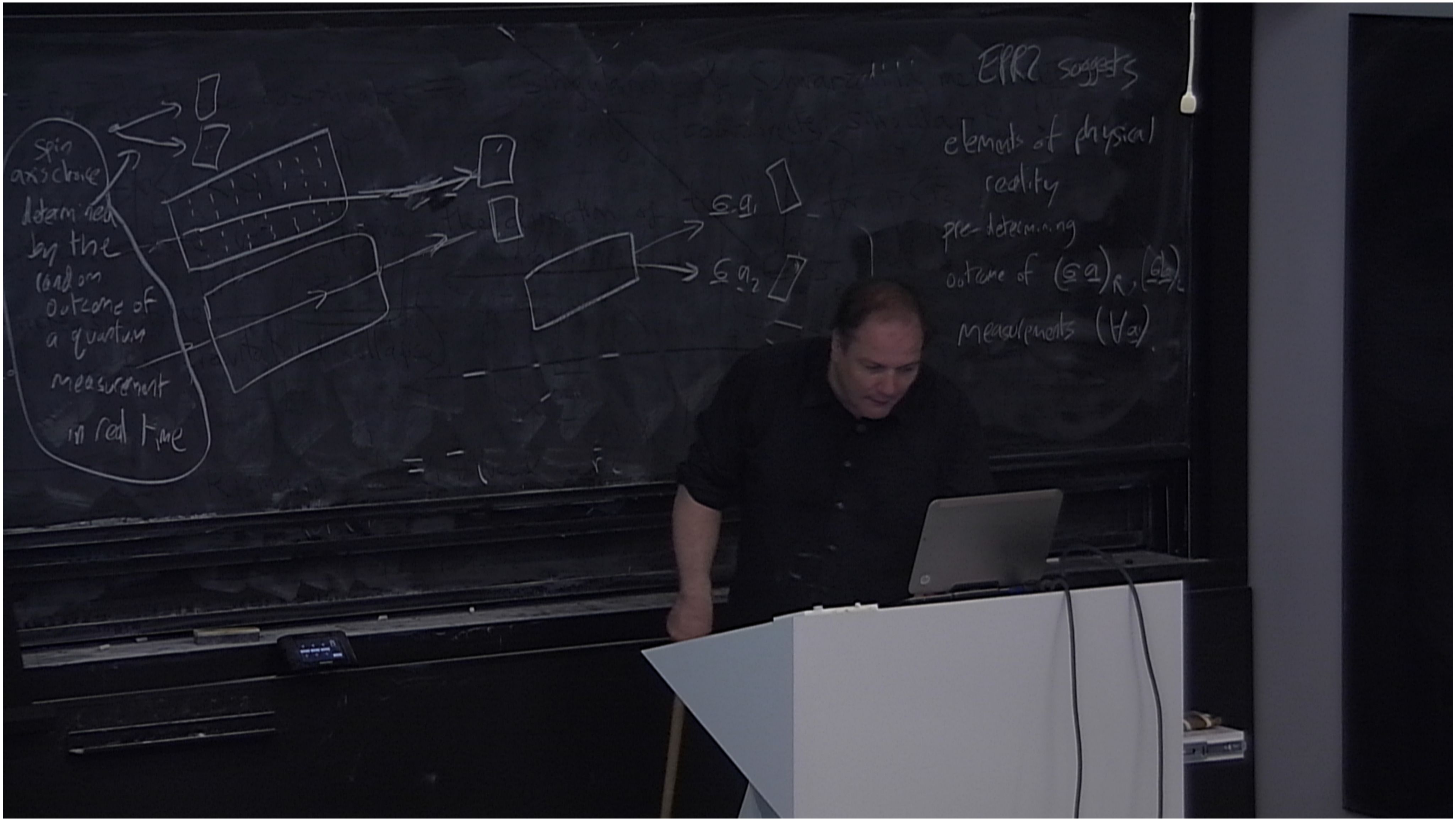
σ a_1
σ a_2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spin axis choice determined by the random outcome of a quantum measurement in real time
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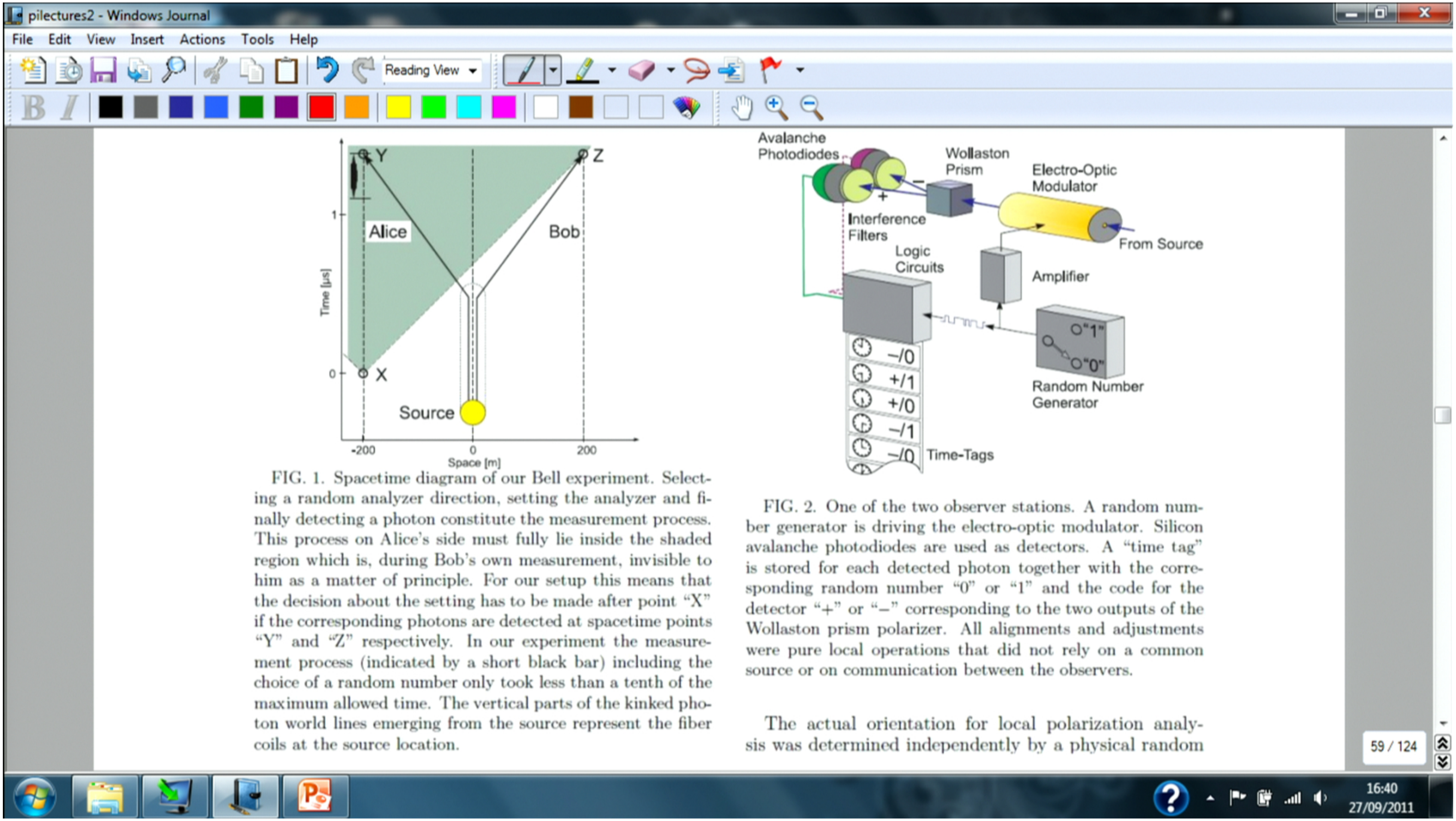
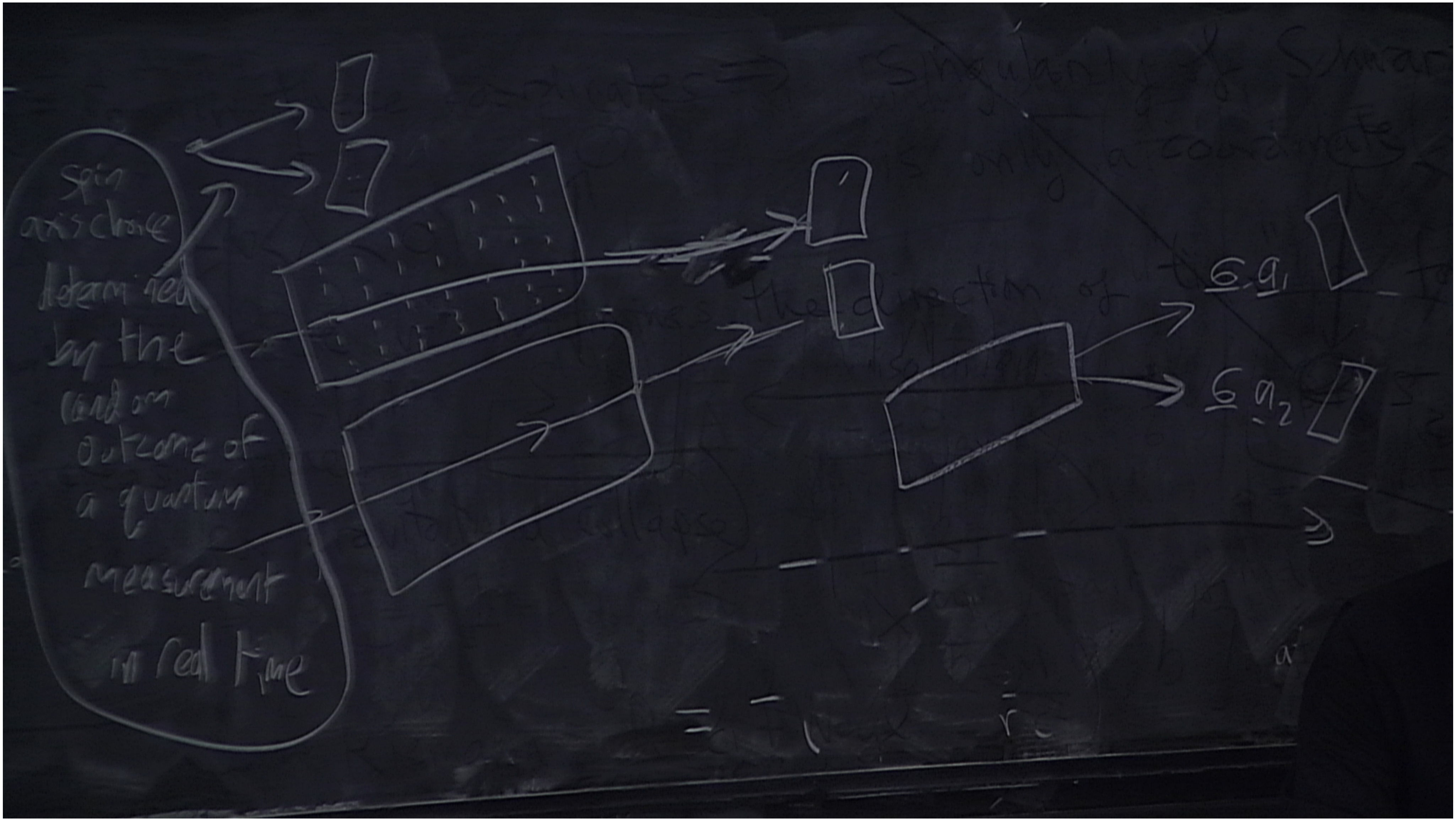


FIG. 1. Spacetime diagram of our Bell experiment. Selecting a random analyzer direction, setting the analyzer and finally detecting a photon constitute the measurement process. This process on Alice's side must fully lie inside the shaded region which is, during Bob's own measurement, invisible to him as a matter of principle. For our setup this means that the decision about the setting has to be made after point "X" if the corresponding photons are detected at spacetime points "Y" and "Z" respectively. In our experiment the measurement process (indicated by a short black bar) including the choice of a random number only took less than a tenth of the maximum allowed time. The vertical parts of the kinked photon world lines emerging from the source represent the fiber coils at the source location.

FIG. 2. One of the two observer stations. A random number generator is driving the electro-optic modulator. Silicon avalanche photodiodes are used as detectors. A "time tag" is stored for each detected photon together with the corresponding random number "0" or "1" and the code for the detector "+" or "−" corresponding to the two outputs of the Wollaston prism polarizer. All alignments and adjustments were pure local operations that did not rely on a common source or on communication between the observers.

The actual orientation for local polarization analysis was determined independently by a physical random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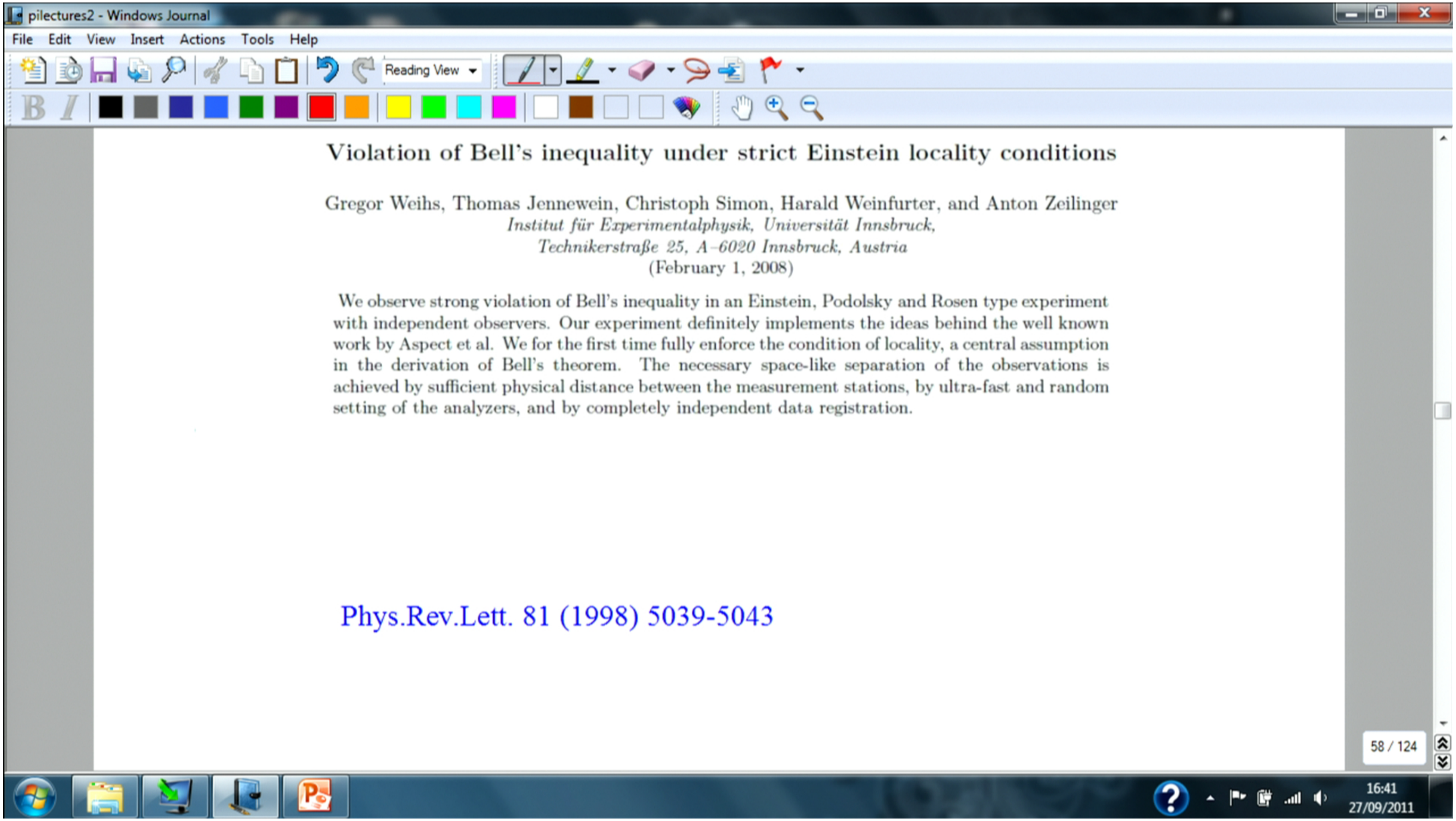

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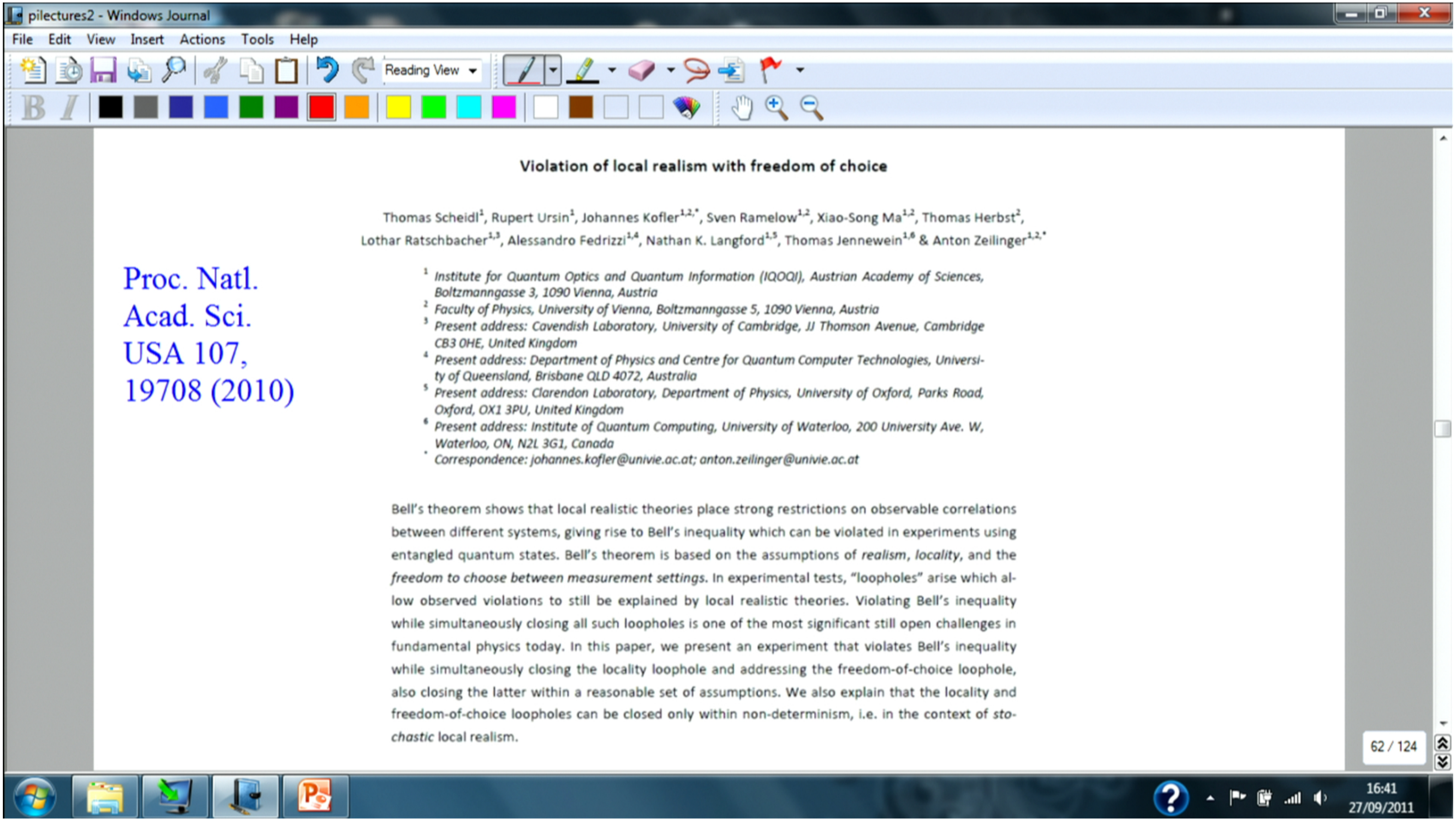

Violation of local realism with freedom of choice

Thomas Scheidl[1], Rupert Ursin[1], Johannes Kofler[1,2,*], Sven Ramelow[1,2], Xiao-Song Ma[1,2], Thomas Herbst[2],
Lothar Ratschbacher[1,3], Alessandro Fedrizzi[1,4], Nathan K. Langford[1,5], Thomas Jennewein[1,6] & Anton Zeilinger[1,2,*]

[1] *Institute for Quantum Optics and Quantum Information (IQOQI), Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria*
[2] *Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria*
[3] *Present address: Cavendish Laboratory, University of Cambridge, JJ Thomson Avenue, Cambridge CB3 0HE, United Kingdom*
[4] *Present address: Department of Physics and Centre for Quantum Computer Technologies, University of Queensland, Brisbane QLD 4072, Australia*
[5] *Present address: Clarendon Laboratory, Department of Physics, University of Oxford, Parks Road, Oxford, OX1 3PU, United Kingdom*
[6] *Present address: Institute of Quantum Computing, University of Waterloo, 200 University Ave. W, Waterloo, ON, N2L 3G1, Canada*
[*] *Correspondence: johannes.kofler@univie.ac.at; anton.zeilinger@univie.ac.at*

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Bell's theorem shows that local realistic theories place strong restrictions on observable correlations between different systems, giving rise to Bell's inequality which can be violated in experiments using entangled quantum states. Bell's theorem is based on the assumptions of *realism*, *locality*, and the *freedom to choose between measurement settings*. In experimental tests, "loopholes" arise which allow observed violations to still be explained by local realistic theories. Violating Bell's inequality while simultaneously closing all such loopholes is one of the most significant still open challenges in fundamental physics today. In this paper, we present an experiment that violates Bell's inequality while simultaneously closing the locality loophole and addressing the freedom-of-choice loophole, also closing the latter within a reasonable set of assumptions. We also explain that the locality and freedom-of-choice loopholes can be closed only within non-determinism, i.e. in the context of *stochastic* local realism.

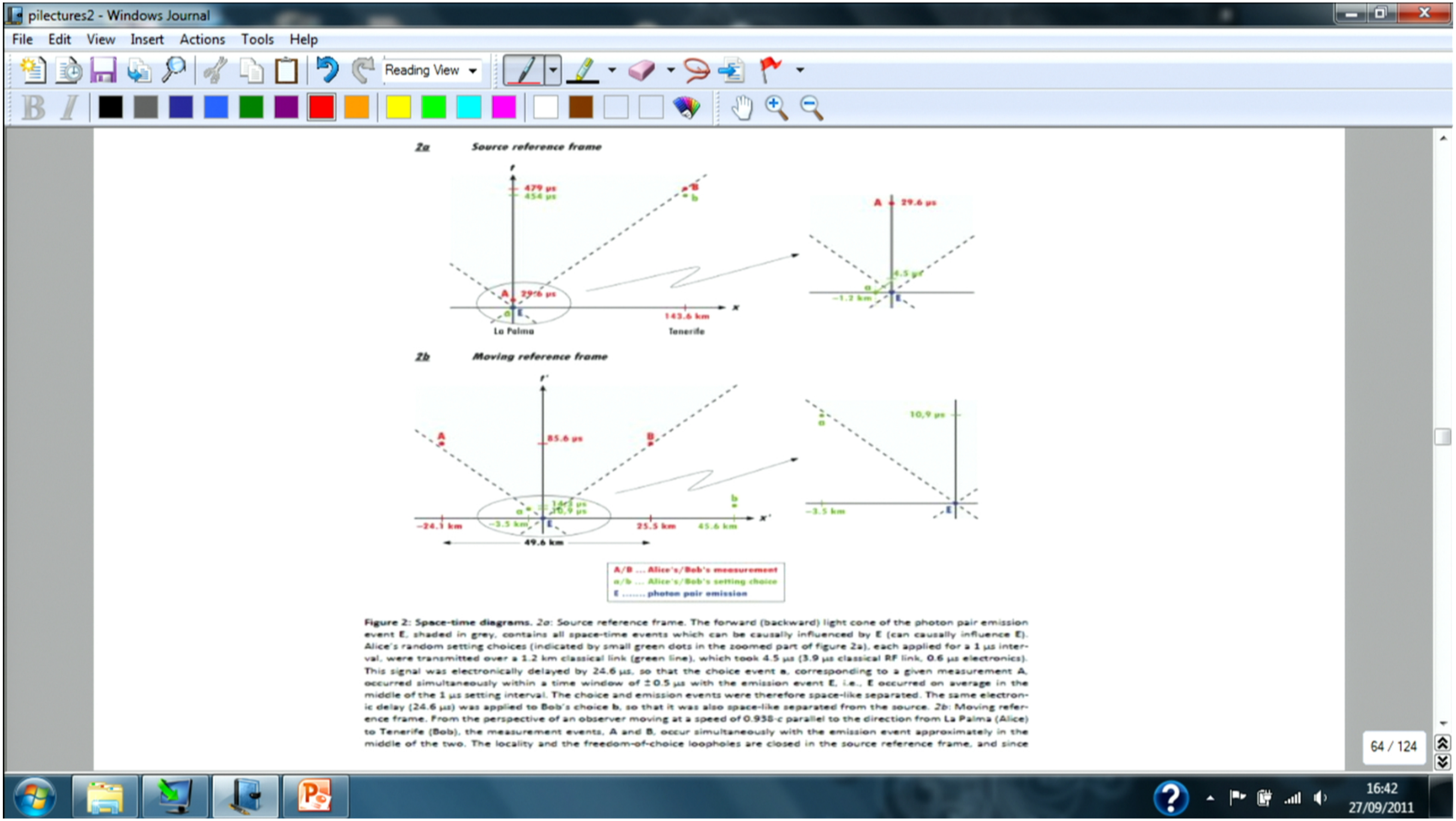
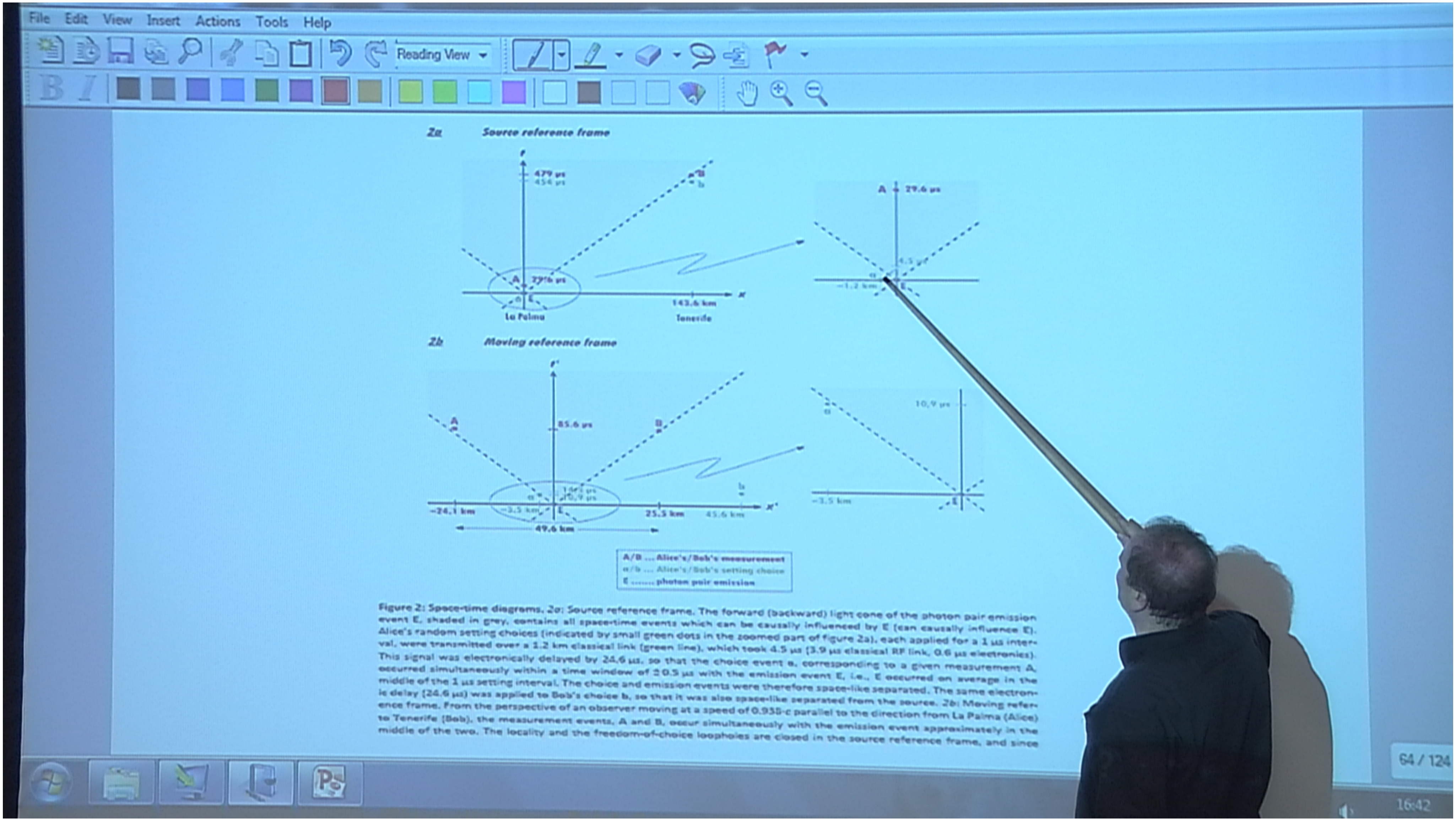


Figure 2: Space-time diagrams. *2a*: Source reference frame. The forward (backward) light cone of the photon pair emission event E, shaded in grey, contains all space-time events which can be causally influenced by E (can causally influence E). Alice's random setting choices (indicated by small green dots in the zoomed part of figure 2a), each applied for a 1 μs interval, were transmitted over a 1.2 km classical link (green line), which took 4.5 μs (3.9 μs classical RF link, 0.6 μs electronics). This signal was electronically delayed by 24.6 μs, so that the choice event a, corresponding to a given measurement A, occurred simultaneously within a time window of ±0.5 μs with the emission event E, i.e., E occurred on average in the middle of the 1 μs setting interval. The choice and emission events were therefore space-like separated. The same electronic delay (24.6 μs) was applied to Bob's choice b, so that it was also space-like separated from the source. *2b*: Moving reference frame. From the perspective of an observer moving at a speed of 0.938·*c* parallel to the direction from La Palma (Alice) to Tenerife (Bob), the measurement events, A and B, occur simultaneously with the emission event approximately in the middle of the two. The locality and the freedom-of-choice loopholes are closed in the source reference frame, and since



A/B ... Alice's/Bob's measurement
a/b ... Alice's/Bob's setting choice
E .... photon pair emission

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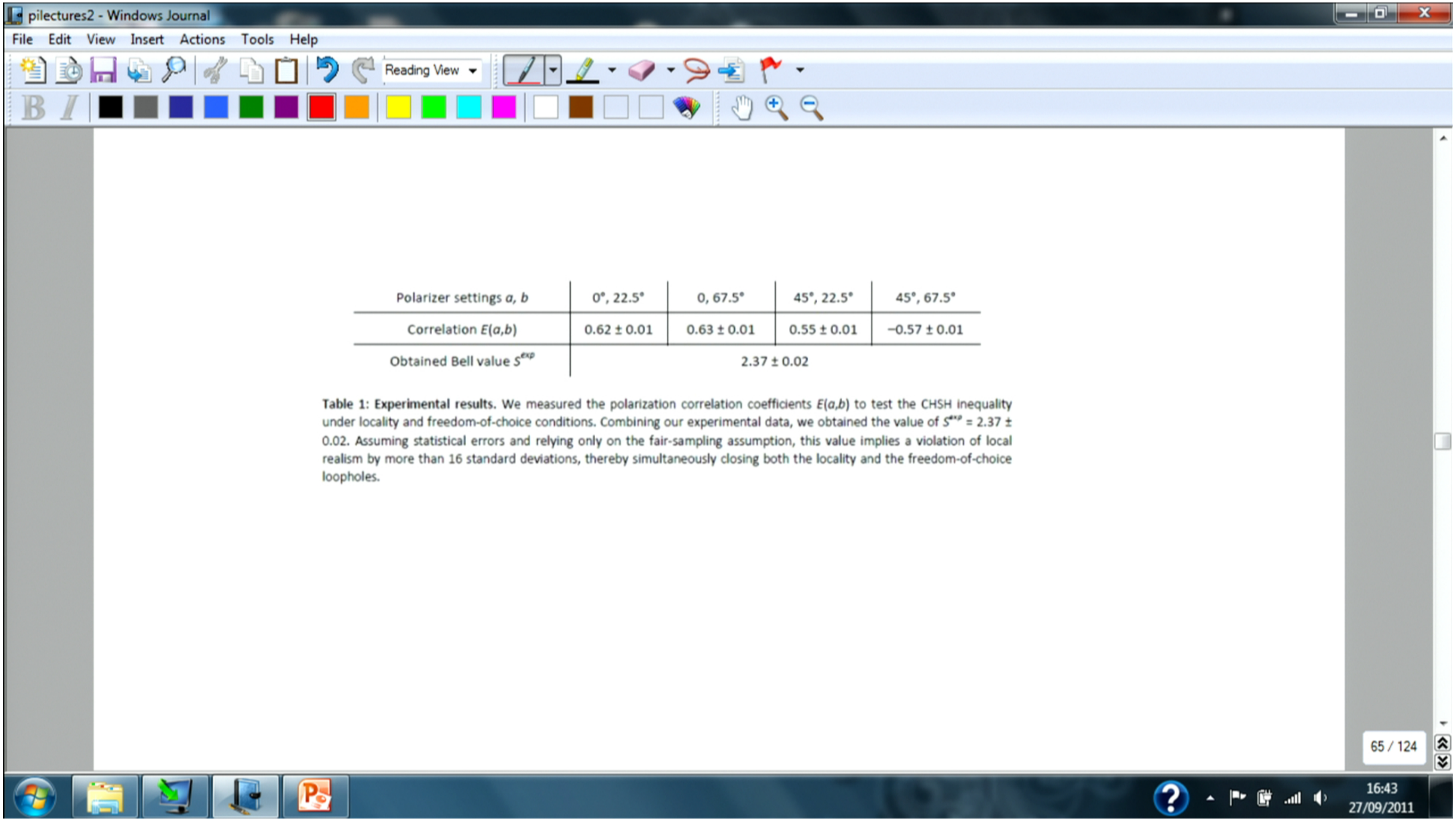

Polarizer settings a, b	0°, 22.5°	0, 67.5°	45°, 22.5°	45°, 67.5°
Correlation $E(a,b)$	0.62 ± 0.01	0.63 ± 0.01	0.55 ± 0.01	−0.57 ± 0.01
Obtained Bell value S^{exp}	2.37 ± 0.02			

Table 1: Experimental results. We measured the polarization correlation coefficients $E(a,b)$ to test the CHSH inequality under locality and freedom-of-choice conditions. Combining our experimental data, we obtained the value of $S^{exp} = 2.37 \pm 0.02$. Assuming statistical errors and relying only on the fair-sampling assumption, this value implies a violation of local realism by more than 16 standard deviations, thereby simultaneously closing both the locality and the freedom-of-choice loopholes.