

Title: A Natural Language for AdS/CFT Correlators

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Abstract: TBA

A Natural Language for ADS/CFT correlators  
ArXiv: 1107.1499 with Fitepletok, Kaplan, Raju, Van Rees

A Natural Language for ADS/CFT correlators  
Arxiv: 1107.1499 with Filipiak, Kaplan, Raju, Kim Rees

Motivation

A Natural Language for AdS/CFT correlators  
Arxiv: 1107.1499 with Filipiak, Kaplan, Raju, and Rees

Motivation



A Natural Language for AdS/CFT correlators  
Arxiv: 1107.1499 with Filipiak, Kaplan, Raju, and Rees

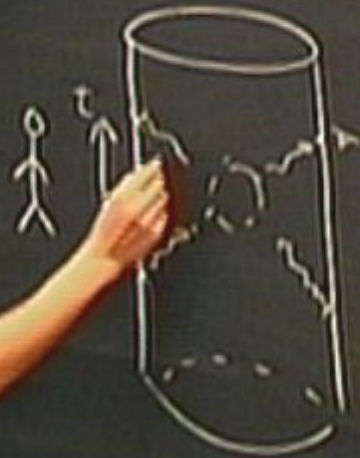
AdS/CFT



# A Natural Language for AdS/CFT correlators

Arxiv: 1107.1499 with Filipiak, Kaplan, Raju, van Rees

introduction



A Natural Language for AdS/CFT correlators  
Arxiv: 1107.1499 with Filipiak, Kaplan, Raju, van Rees

Motivation



AdS scattering amplitudes  
||  
CFT correlators

# Mellin Amplitudes



Mellin Amplituden [Mack 01]



Mellin Amplituden [Mack 07]

$$\langle \mathcal{O}_1(k_1) \dots \mathcal{O}_n(k_n) \rangle$$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int [ds]$$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int [dS]$$

$$(x_{ij}^2)^{-\epsilon}$$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int [ds] \prod_{i,j} \Gamma(s_{ij}) (x_{ij}^2)^{-s_{ij}}$$



Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int [ds] \underbrace{M(s_{ij})}_{\text{Mellin amplitude}} \prod_{i < j}^n P(s_{ij}) (x_{ij}^2)^{-\delta_{ij}}$$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int [ds] \underbrace{M(s_{ij})}_{\text{Mellin amplitude}} \prod_{i < j}^n P(s_{ij}) (x_{ij}^2)^{-s_{ij}}$$



Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int_{-\infty}^{\infty} [ds] \underbrace{M(s, s)} \prod_{i=1}^n P(s_i) (x_{ij}^2)^{-\delta_{ij}}$$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int_{-i\infty}^{i\infty} [ds] \underbrace{M(s_{ij})}_{\sum_{ij} \delta_{ij} = \Delta_i} \prod_{ij} P(s_{ij}) (x_{ij}^2)^{-s_{ij}}$$

$$\sum_{ij} \delta_{ij} = \Delta_i$$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int_{\gamma-i\infty}^{\gamma+i\infty} [ds] \underbrace{M(s_{ij})}_{\text{Mellin Amplitude}} \prod_{i,j} \Gamma(s_{ij}) (x_{ij}^2)^{-s_{ij}}$$

$$\sum_{j \neq i} \delta_{ij} = \Delta_i \quad \text{constraint}$$

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int_{\mathcal{T}=\text{const}} [ds] \underbrace{M(s_{ij})}_{\text{const.}} \prod_{i,j} P(s_{ij}) (x_{ij}^2)^{-s_{ij}}$$

$$\sum_{j \neq i} \delta_{ij} = \text{const.}$$

$$\frac{n(n-1)}{2}$$

$$\frac{n(n-3)}{2}$$

int. variables

Mellin Amplituden [Mack 07]

$x_i \in \mathbb{R}^d$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \int_{\mathcal{T}=\text{IBO}} [ds] \underbrace{M(s_{ij})}_{\text{Mellin Amplituden}} \prod_{i,j} P(s_{ij}) (x_{ij}^2)^{-s_{ij}}$$

$$\sum_{j \neq i} \delta_{ij} = \Delta_i \quad \text{constraint}$$

$$\frac{h-1}{2} - h = \frac{h(h-3)}{2} \quad \text{int. Variablen}$$

Introduce  $P_i$   
-  $P_i^2 = \Delta_i$  on-shell

$$\sum P_i = 0$$

Write  $\delta_{ij} = P_i \cdot P_j$

Introduce  $P_i$   
-  $P_i^2 = \Delta_i$  shell

$$\sum P_i = 0$$

Write  $\delta_{ij} = P_i$

"Mandelstam invariants"

Introduce  $P_i$

$$-P_i^2 = \Delta_i \quad \text{on-shell}$$

$$\sum P_i = 0$$

Write  $\delta_{ij} = P_i \cdot P_j$

↑ "Mandelstam invariants"



Introduce  $P_i$

$$-P_i^2 = \Delta_i \quad \text{on-shell}$$

$$\sum P_i = 0$$

Write  $\delta_{ij} = P_i \cdot P_j$

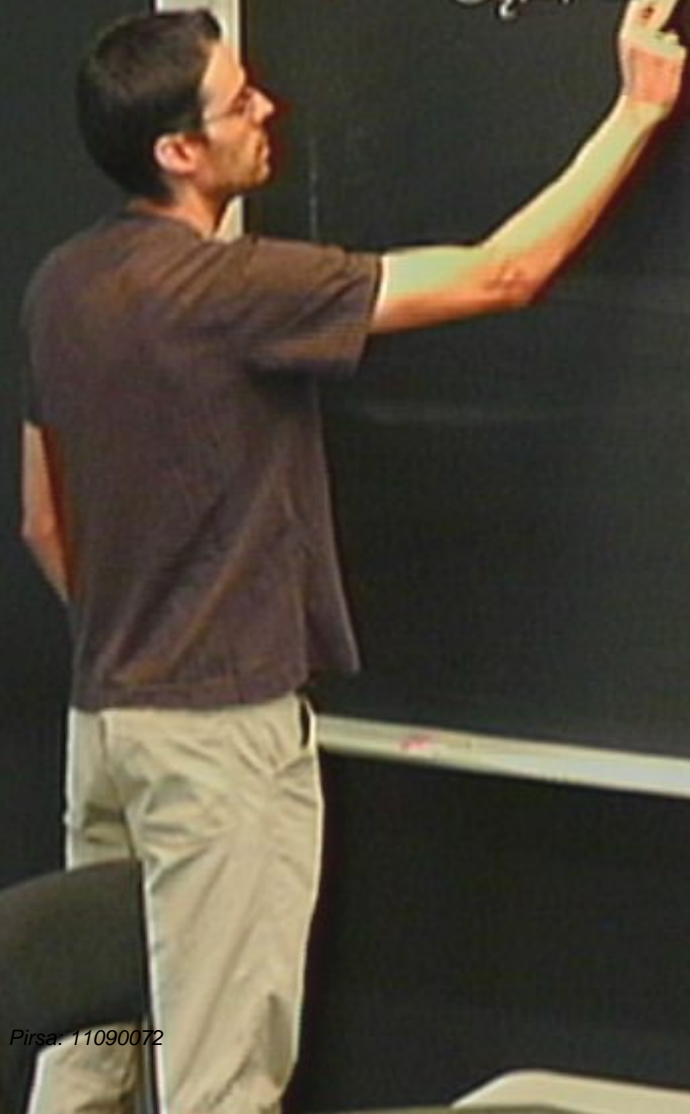
"Mandelstam invariants" :  $S_{ij} = -(P_i + P_j)^2 = \Delta_i + \Delta_j - 2\delta_{ij}$

OPE



OPE

$O_2(x) O_1$



OPE  $K_L \rightarrow x_1 = 0$

$\mathcal{O}_2(x) \mathcal{O}_1(0)$

OPE

$$k_L \rightarrow x_1 = 0$$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{-\Delta_P - \Delta_1 - \Delta_2} \mathcal{O}_P(0)$$

OPE  $k_2 \rightarrow k_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{\Delta_P - \Delta_1 - \Delta_2} \{ \mathcal{O}_P(0) + \dots \}$$

OPE

$$K_L \rightarrow x_1 = 0$$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{-\Delta_P - \Delta_1 - \Delta_2} \left[ \mathcal{O}_P(0) + \text{disc.} \right]$$

OPE  $K_L \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{-\Delta_p - \Delta_1 - \Delta_2} \left[ \mathcal{O}_p(0) + \dots \right]$$





OPE  $\kappa_2 \rightarrow \kappa_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{\Delta_p - \Delta_1 - \Delta_2} [\mathcal{O}_p(0) + \text{disc.}]$$

$h=4$

$$\langle \mathcal{O}_p(0) \mathcal{O}_2(x) \mathcal{O}_1(x) \rangle =$$

OPE  $\kappa_2 \rightarrow \kappa_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{\Delta_p - \Delta_1 - \Delta_2} [\mathcal{O}_p(0) + \text{disc.}]$$

$h=4$

$$\langle \mathcal{O}_p \rangle \mathcal{O}_2(x) \mathcal{O}_1(0) \int d\delta_{11} d\delta_{12} M(\delta_{11}, \delta_{12})$$

OPE  $k_2 \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{\Delta_p - \Delta_1 - \Delta_2} \{ \mathcal{O}_p(0) + \dots \}$$

$h=4$

$$\langle \mathcal{O}_p(0) \mathcal{O}_1(x) \mathcal{O}_2(y) \rangle = \int d\delta \delta(\delta_{11}, \delta_{12}) |x|^{-2\delta_{12}} \langle \mathcal{O}_p(\delta_{12}) \rangle$$

OPE  $x_2 \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{\Delta_p - \Delta_1 - \Delta_2} [\mathcal{O}_p(0) + \dots]$$

$h=4$

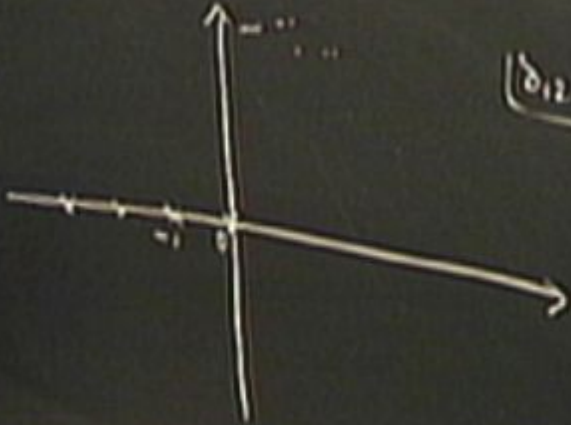
$$\langle \mathcal{O}_p(0) \mathcal{O}_1(x) \mathcal{O}_2(x) \rangle = \int d\delta_{11} d\delta_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\Delta_1} \prod_{i=1,2} \Gamma(\delta_{i1}, \delta_{i2})$$

OPE  $k_2 \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{-\Delta_P - \Delta_1 - \Delta_2} \{ \mathcal{O}_P(0) + \dots \}$$

$\hbar = 4$

$$\langle \mathcal{O}_p \rangle \mathcal{O}_1(x) \mathcal{O}_2(0) = \int ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{11}} \prod_{i=1,2} \Gamma(\delta_{i1}, \delta_{i2})$$



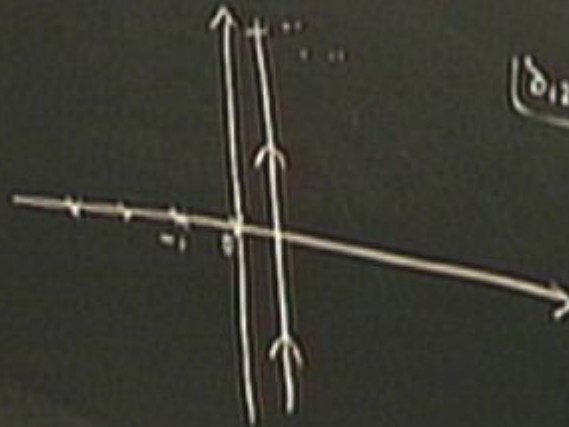
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OPE  $k_2 \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{\Delta_p - \Delta_1 - \Delta_2} \{ \mathcal{O}_p(0) + \dots \}$$

$h=4$

$$\langle \mathcal{O}_p \rangle \mathcal{O}_1(x) \mathcal{O}_2(y) \mathcal{O}_3(z) = \int ds_{11} ds_{12} M(s_{11}, s_{12}) |x|^{-2s_{11}} \Gamma(s_{12}) \prod_{i=1}^3 \Gamma(s_{1i}) \omega_{i,1}^2$$



OPE  $k_2 \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{-\Delta_P - \Delta_1 - \Delta_2} \{ \mathcal{O}_P(0) + \dots \}$$

$h=4$

$$\langle \mathcal{O}_p \mathcal{O}_1(x) \mathcal{O}_2(y) \mathcal{O}_3(z) \rangle = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{11}} \prod_{i=1}^3 \Gamma(\delta_{i1}) \Gamma(\delta_{i2})$$

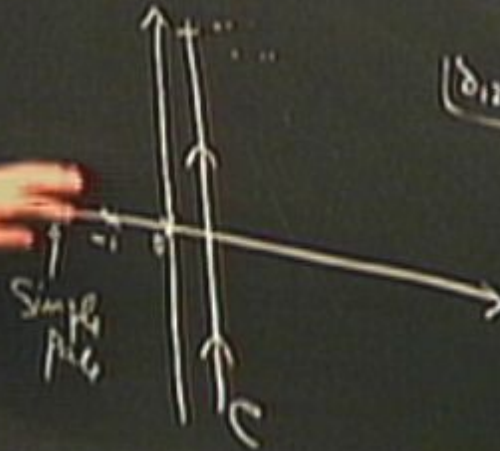


OPE  $k_2 \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{-\Delta_P - \Delta_1 - \Delta_2} \left[ \mathcal{O}_P(0) + \dots \right]$$

$h=4$

$$\langle \mathcal{O}_p \mathcal{O}_1(x) \mathcal{O}_2(y) \mathcal{O}_k(z) \rangle = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{11}} \prod_{i=1,2} \Gamma(\delta_{i1}) \prod_{i=1,2} \Gamma(\delta_{i2}) \omega_{i1}^{\delta_{i1}}$$



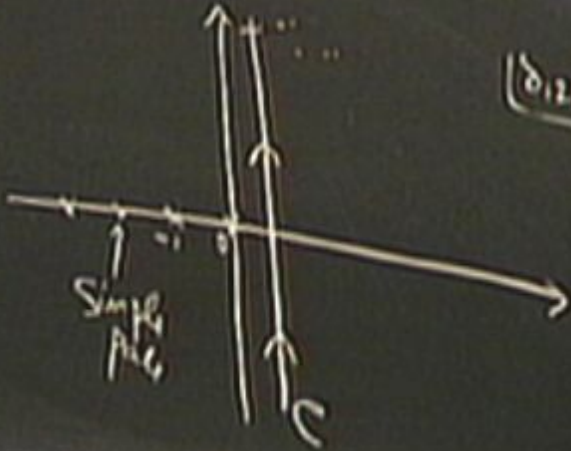


OPE  $\kappa_2 \rightarrow x_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{-\Delta_P - \Delta_1 - \Delta_2} \{ \mathcal{O}_P(0) + \dots \}$$

$h=4$

$$\langle \mathcal{O}_p \mathcal{O}_1(x) \mathcal{O}_2(y) \mathcal{O}_3(z) \rangle = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{11}} \prod_{i=1}^3 \Gamma(\delta_{1i}) \Gamma(\delta_{2i}) \Gamma(\delta_{3i})^2$$

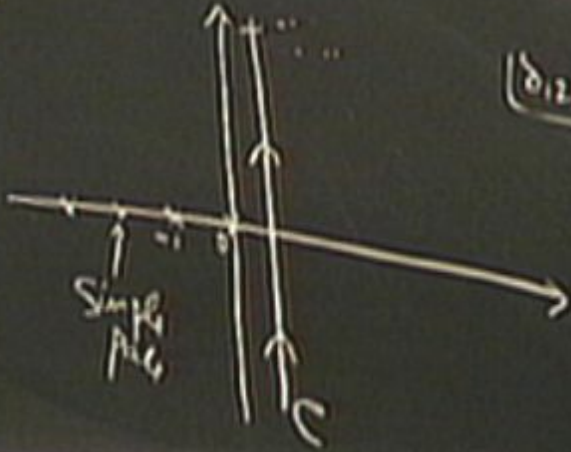


OPE  $\kappa_2 \rightarrow \kappa_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{\Delta_P - \Delta_1 - \Delta_2} \{ \mathcal{O}_P(0) + \dots \}$$

$h=4$

$$\langle \mathcal{O}_p \mathcal{O}_1(x) \mathcal{O}_2(y) \rangle = \int_C ds_{11}, ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{12}} \prod_{i=1,2} \Gamma(\delta_{i1}) \Gamma(\delta_{i2})$$



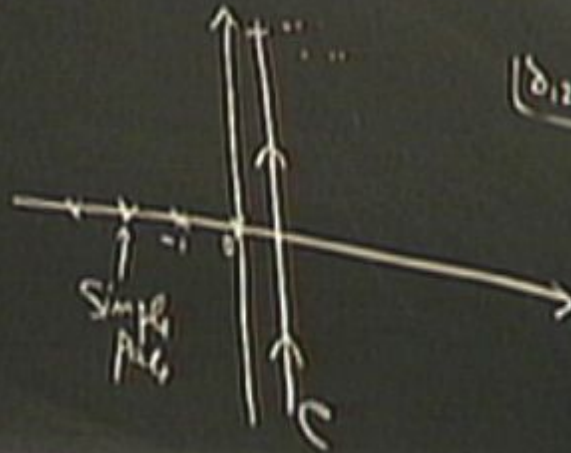
OPE  $\kappa_2 \rightarrow \kappa_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{\Delta_p - \Delta_1 - \Delta_2} \{ \mathcal{O}_p(0) + \dots \}$$

$h=4$

$$\langle \mathcal{O}_p(0) \mathcal{O}_1(x) \mathcal{O}_2(0) \rangle = \int_C ds_{11}, ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{12}} \prod_{i=1,2} \Gamma(\delta_{i1}) \Gamma(\delta_{i2})^2$$

only has simple poles



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OPE  $\kappa_L \rightarrow \kappa_1 = 0$

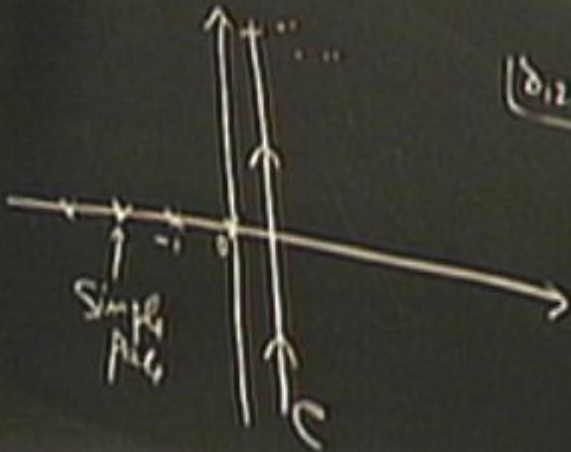
$$\langle \mathcal{O}_2(x) \mathcal{O}_1(0) \rangle \sim \sum_P |x|^{-\Delta_P - \Delta_1 - \Delta_2} \left[ \langle \mathcal{O}_P(0) \rangle + \dots \right]$$

$h=4$

$$\langle \mathcal{O}_P(0) \mathcal{O}_1(x) \mathcal{O}_2(0) \rangle \sim \int_C d\delta_{11} d\delta_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{12}} \prod_{i=1,2} \Gamma(\delta_{i1} + \nu_{i1}^2)$$

• M

simple poles



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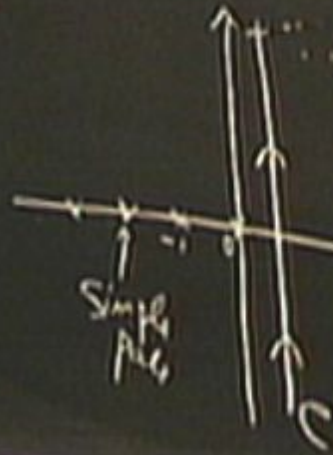
OPE  $k_2 \rightarrow k_1 = 0$

$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{-\Delta_p - \Delta_1 - \Delta_2} \{ \mathcal{O}_p(0) + \dots \}$$

$h=4$

$$\langle \mathcal{O}_p(0) \mathcal{O}_1(x) \mathcal{O}_2(x) \rangle = \int_C ds_{11}, ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{12}} \prod_{i=1,2} \Gamma(\delta_{i1}) \prod_{i=1,2} \Gamma(\delta_{i2}) \Gamma(\delta_{12})^2$$

has simple poles



$|\delta_{12}$

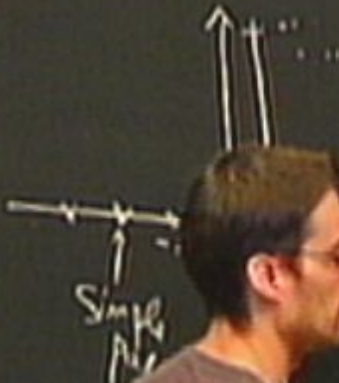
Pole at  $\delta_{12} = -m$

$\Delta_p = \Delta_1 + \Delta_2 + 2m$

$n=4$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle = \int_C d\delta_{11} d\delta_{12} M(\delta_{11}, \delta_{12}) |\lambda|^{-2\delta_{12}} \Gamma(\delta_{11}) \prod_{i=1}^4 \Gamma(\delta_{1i}) \omega_{i1}^2$$

•  $M$  only has simple poles

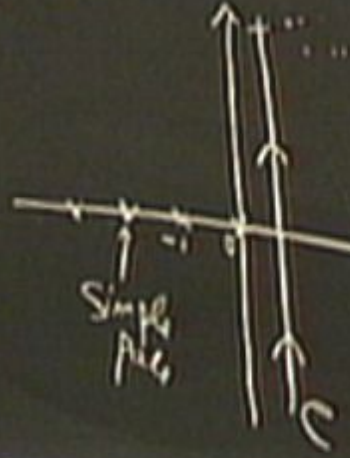


$\delta_{12}$   
pole at  $\delta_{12} = -m$   
 $\Delta_1 = \Delta_1 + \Delta_2 + 2m$   
 $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$

$n=4$

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |X|^{-2\delta_{12}} \prod_{i=1}^2 \Gamma(\delta_{1i}) \prod_{i=1}^2 \Gamma(\delta_{1i}) \omega_{i,1}^2$$

•  $M$  only has simple poles



$\delta_{12}$   
 Pole at  $\delta_{12} = -m$   
 $\Delta_1 = \Delta_2 = 2m$   
 $\mathcal{O}_1, \mathcal{O}_2$

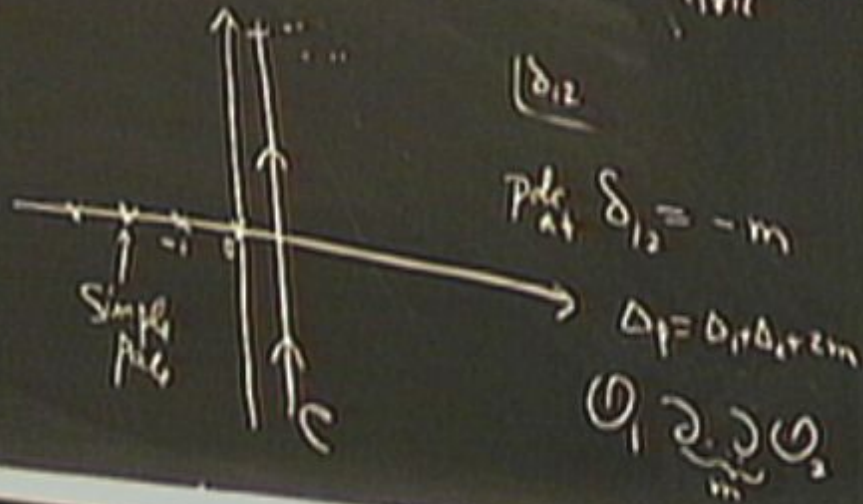


$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_p |x|^{\Delta_p - \Delta_1 - \Delta_2} [\mathcal{O}_p(0) + \dots]$$

$h=4$

$$\langle \mathcal{O}_p \rangle \mathcal{O}_1(x) \mathcal{O}_2(0) = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{12}} \prod_{i=1}^2 \Gamma(\delta_{i1}) \Gamma(\delta_{i2}) \omega_{i1}^{\delta_{i1}}$$

- $M$  only has simple poles



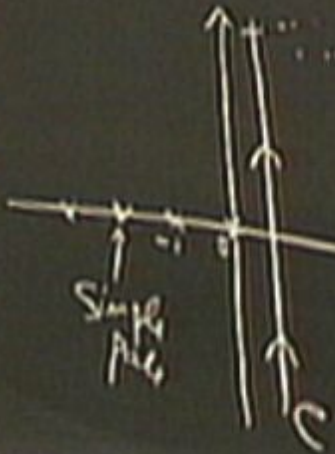


$$\mathcal{O}_2(x) \mathcal{O}_1(0) \sim \sum_P |x|^{\Delta_P - \Delta_1 - \Delta_2} \{ \mathcal{O}_P(0) + \dots \}$$

$n=4$

$$\langle \mathcal{O}_P(0) \mathcal{O}_1(x) \mathcal{O}_2(0) \rangle = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\Delta_1} \prod_{i=1}^2 \Gamma(\delta_{1i}) \prod_{i=1}^2 \Gamma(\delta_{1i} - \Delta_i)$$

- $M$  only has simple poles associated with singularities of



$|\delta_{12}$

Pole at  $\delta_{12} = -m$

$$\Delta_P = \Delta_1 + \Delta_2 + 2m$$

$$\mathcal{O}_1 \mathcal{O}_2 \rightarrow \mathcal{O}_P$$

$$M \sim \overline{S_{12} - (\Delta_1 - \frac{\ell}{2} + 2m)}$$

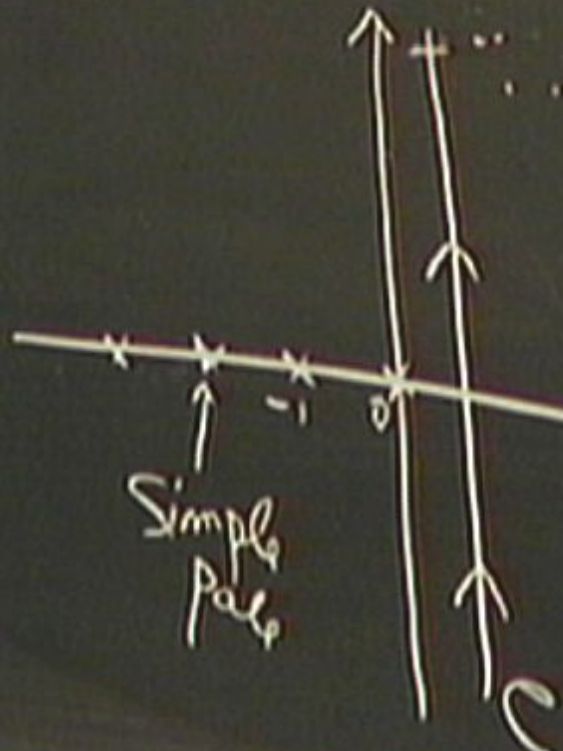
$$M \sim \frac{S_{12} - (\Delta_1 - \theta + 2im)}{im}$$

$$x_2 \rightarrow x_1 = 0 \quad c_{12p}$$

$$(x) \mathcal{O}_1(0) \sim \sum_p \frac{1}{|x|} \Delta_p - \Delta_1 - \Delta_2 \left[ \mathcal{O}_p(0) + \text{desc.} \right]$$

$$\langle \mathcal{O}_2(x) \mathcal{O}_1(0) \rangle = \int d\delta_{12} d\delta_{13} M(\delta_{12}, \delta_{13}) |x|^{-2\delta_{12}} \Gamma(\delta_{12}) \prod_{i \neq 12} \Gamma(\delta_{i,j})$$

ly has simple poles  
inted with single-trace ops



poles at  $\delta_{12} = -m$

$$\Delta_p = \Delta_1 + \Delta_2 + 2$$

$h=4$

$$M \sim \frac{C_{12P} C_{31P} P}{S_{12} - (\Delta_1 - \underset{\substack{\uparrow \\ \text{spin}}}{\ell} + 2m)}$$

$m = 0, 1, 2, \dots$

$h=4$

$$M \sim \frac{C_{12P} C_{31P} P_{\ell, m}(S_{12})}{S_{12} - (\Delta_1 - \underset{\substack{\uparrow \\ \text{spin}}}{\ell} + 2m)}$$

$$m = 0, 1, 2, \dots$$

$$\boxed{n=4}$$

$$M \sim \frac{C_{12P} C_{31P} P_{\ell,m}(s_{12})}{s_{12} - (\Delta_1 - \ell + 2m)}$$

$\uparrow$   
spin

$$m = 0, 1, 2, \dots$$



$h=4$

$$M \sim \frac{C_{12P} C_{31P} P_{\ell, m}(S_{12})}{S_{12} - (\Delta_1 - \underset{\substack{\uparrow \\ \text{spin}}}{\ell} + 2m)}$$

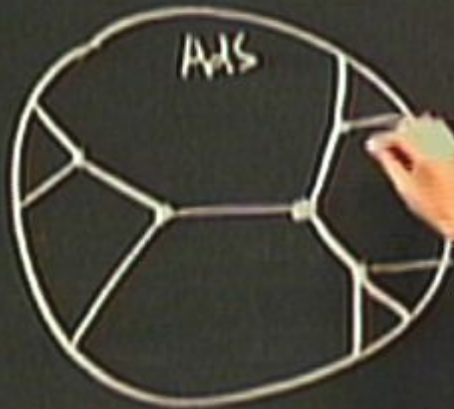
$$m = 0, 1, 2, \dots$$



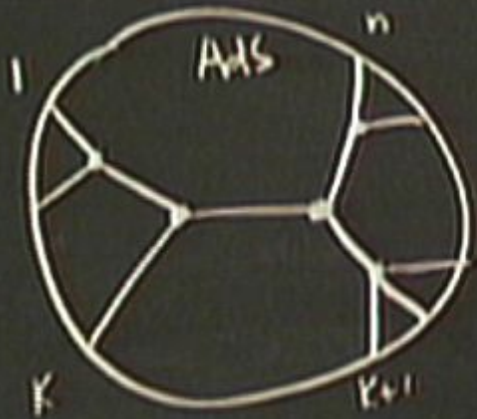
Tree-level Witten diagrams



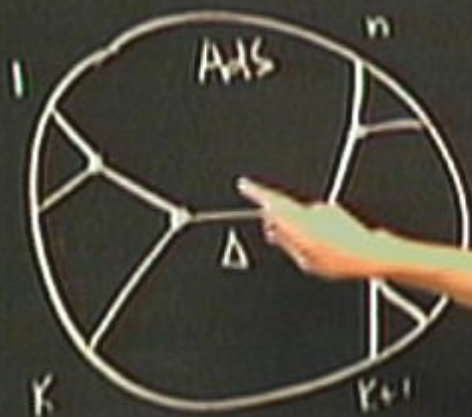
# Tree-level Witten diagrams



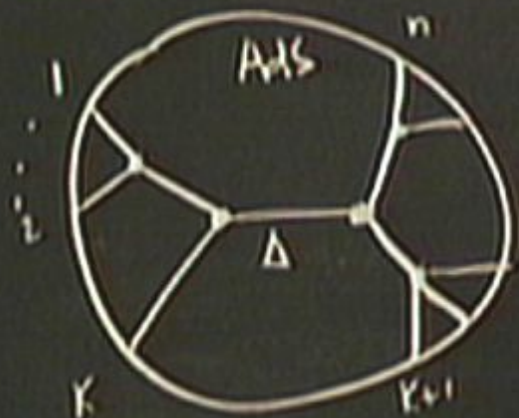
# Tree-level Witten diagrams



# Tree-level Witten diagrams



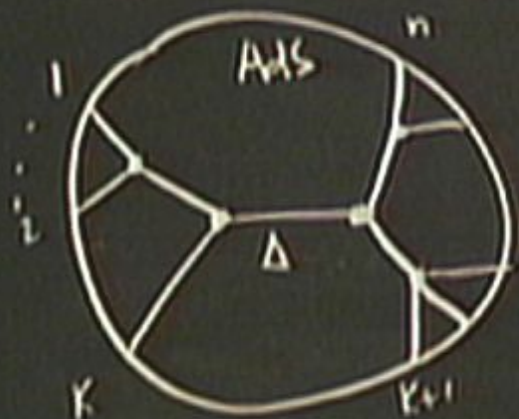
# Tree-level Witten diagrams



→  $M$  has poles at

-

# Tree-level Witten diagrams

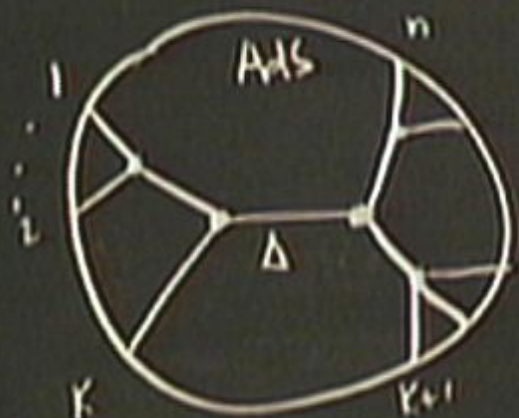


→  $M$  has poles at

$$-(p_i + p_n)' = \sum_{i=1}^K \Delta_i - 2 \sum_{i=1}^K \delta_{ij} =$$



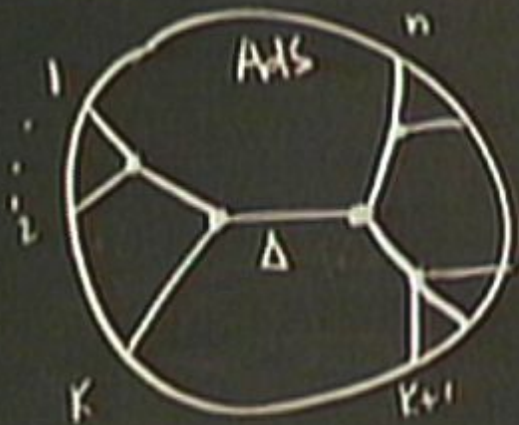
# Tree-level Witten diagrams



→  $M$  has poles at  $m=0, 1, \dots$

$$-(p_1 + \dots + p_n)^2 = \sum_{i=1}^K \Delta_i - 2 \sum_{i < j}^K \delta_{ij} = \Delta + 2m$$

# Tree-level Witten diagrams

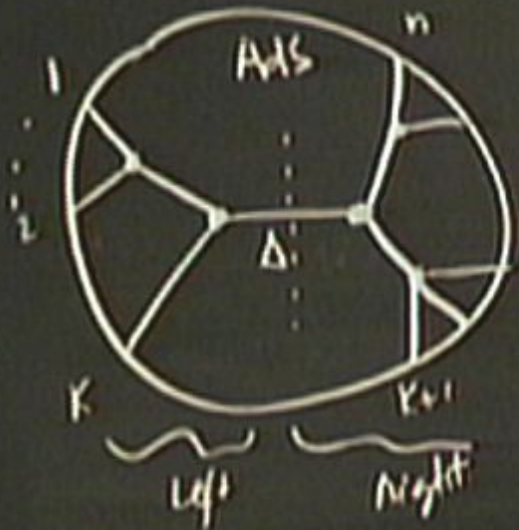


$\rightarrow M$  has poles at  $m=0,1,\dots$   
 $-(p_1 + \dots + p_n)^2 = \sum_{i=1}^K \Delta_i - 2 \sum_{i < j}^K \delta_{ij} = \Delta + 2m$   
 Residue





# Tree-level Witten diagrams



→  $M$  has poles at  $m=0, 1, \dots$

$$-(p_i + p_k)^2 = \sum_{i=1}^k \Delta_i - 2 \sum_{i < j}^k \delta_{ij} = \Delta + 2m$$

Residue

$$M^L(S_{ij})$$

# Tree-level Witten diagrams



→  $M$  has poles at

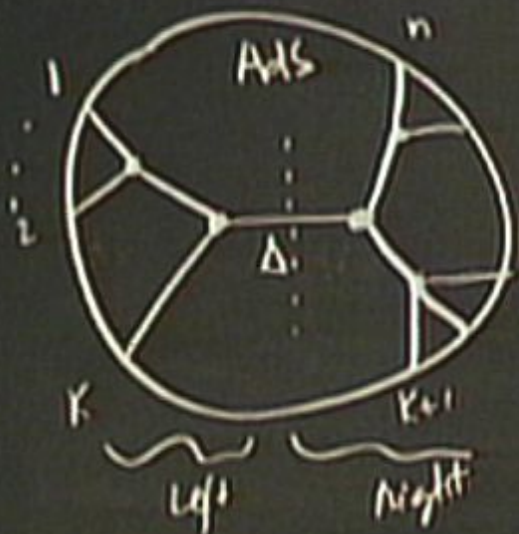
$$-(p_1 + \dots + p_n)^2 = \sum_{i=1}^k \Delta_i - 2 \sum_{\substack{j=1 \\ (i,j)}}^k \Delta_{ij} = \Delta + 2m$$

$m=0,1,\dots$

Residue

$$M^L(s_{ij}) \times M^R(s_{ij}), \quad m=0$$

tree-level Wilson diagrams

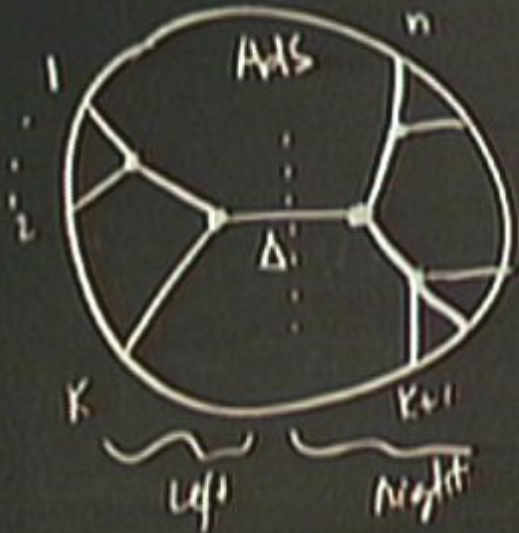


→  $M$  has poles at  $m=0, 1, \dots$

$$-(p_1 + \dots + p_n)^2 = \sum_{i=1}^K \Delta_i - 2 \sum_{i,j=1}^K \delta_{ij} = \Delta + 2m$$

Residue  $M^L(\delta_{ij}) \times M^R(\delta_{ij}), m=0$



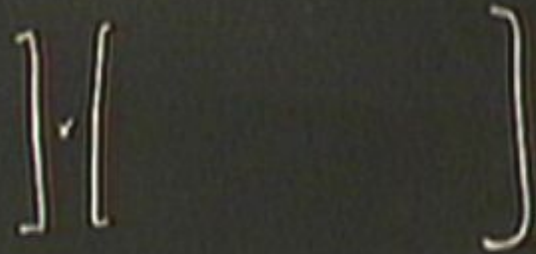


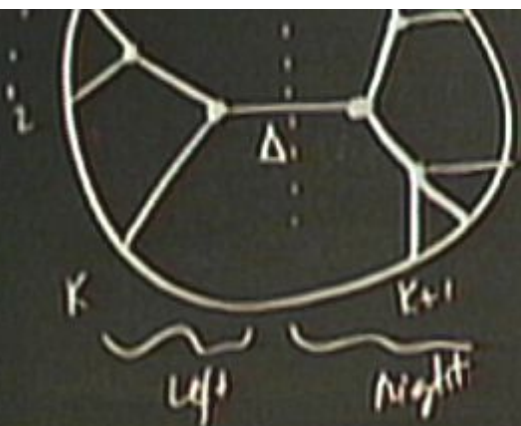
→  $M$  has poles at

$$-(p_1 + \dots + p_n)^2 = \sum_{i=1}^K \Delta_i - 2 \sum_{i,j=1}^K \delta_{ij} = \left( \begin{matrix} \uparrow \\ m=0,1,\dots \end{matrix} \right) (s+2m)$$

Residue

$$M^L(s_{ij}) \times M^R(s_{ij}), \quad m=0$$





→  $M$  has poles at  $m=0, 1, \dots$

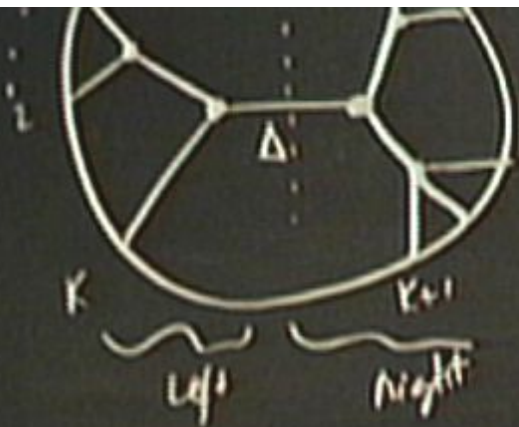
$$-(P_1 + P_2)^2 = \sum_{i=1}^K \Delta_i - 2 \sum_{i,j=1}^K \delta_{ij} = \Delta + 2m$$

Residue  $M^L(s_{ij}) \times M^R(s_{ij}), m=0$

$$\left[ \sum_{\sum h_{ij} = m} M^L(s_{ij}, h_{ij}) \frac{(s_{ij})_{w_{ij}}}{h_{ij}!} \right] \cdot \left[ \dots \right]$$

$(a)_n$





→  $M$  has poles at  $m=0, 1, \dots$

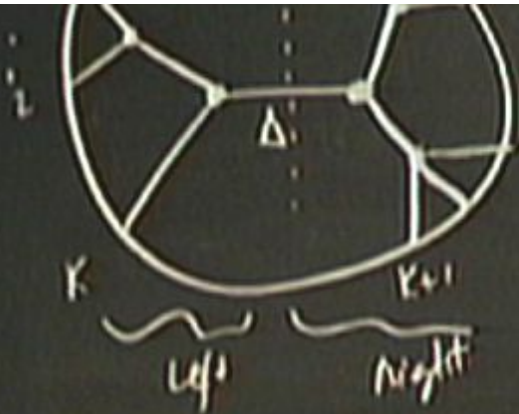
$$-(T_1 + \dots + T_n)^2 = \sum_{i=1}^K \Delta_i - 2 \sum_{i,j=1}^K \delta_{ij} = \Delta + 2m$$

Residue

$$M^L(\delta_{ij}) \times M^R(\delta_{ij}), \quad m=0$$

$$\left[ \sum_{\sum h_{ij} = m} M^L(\delta_{ij} + h_{ij}) \frac{(\delta_{ij})_{w_{ij}}}{h_{ij}!} \right] \cdot \left[ \dots \right]$$

(a)<sub>n</sub>



$$\rightarrow \prod_{i=1}^K (p_i + 1) \dots$$

$$-(p_i + 1 + p_k)^2 = \sum_{i=1}^K \Delta_i - 2 \sum_{i,j} \delta_{ij} = \Delta + 2m$$

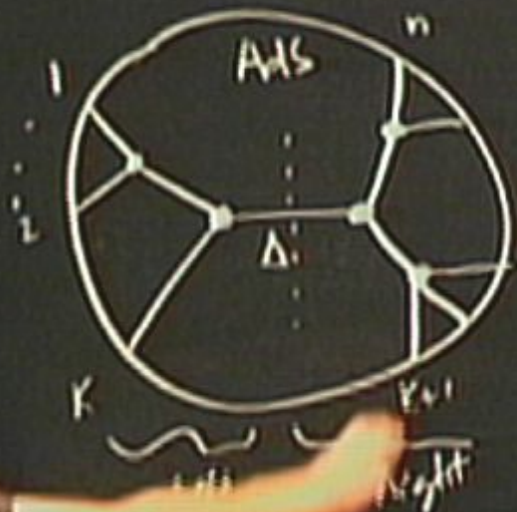
Residue

$$M^L(\delta_{ij}) \times M^R(\delta_j), m=0$$

$$\left[ \sum_{\substack{2n_{ij} = m \\ n_{ij} \geq 0}} M^L(\delta_{ij} + n_{ij}) \frac{(\delta_{ij})_{n_{ij}}}{n_{ij}!} \right] \cdot \left[ \dots \right]$$

$$(a)_n = \Gamma(a+n)/\Gamma(a)$$

# Tree-level Witten diagrams



→  $M$  has poles at  $m=0, 1, \dots$

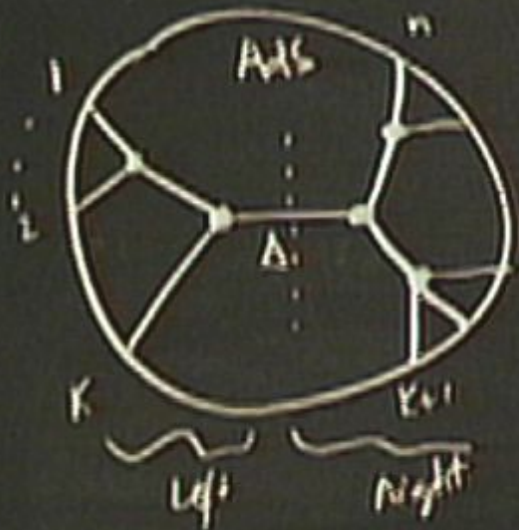
$$-(\tau_i + \tau_k)' = \sum_{i=1}^k \Delta_i - 2 \sum_{i=1}^k \delta_{ij} = \Delta + 2m$$

Residue  $M^L(s_{ij}) \times M^R(s_{ij})$ ,  $m=0$

$$\left[ \sum_{\substack{2n_{ij} = m \\ n_{ij} \geq 0}} M^L(s_{ij}, n_{ij}) \frac{(s_{ij})_{n_{ij}}}{n_{ij}!} \right] \cdot \left[ \frac{(\alpha)_n = \Gamma(\alpha+n)/\Gamma(\alpha)}{\dots} \right]$$



# Tree-level Witten diagrams



→  $M_n$  has poles at

$$-(\tau_1 + \tau_2)^2 = \sum \Delta_i - 2 \sum_{i < j}^k \delta_{ij} = \Delta + 2m$$

msq... ↑

Residue

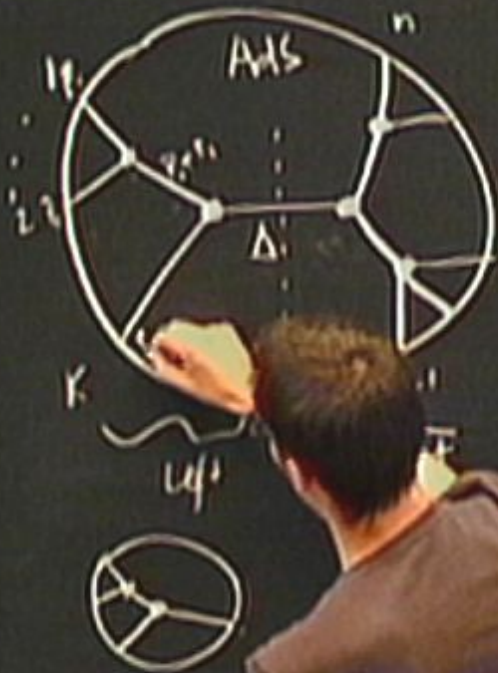
$$M_{\Delta_i}^L(\delta_{ij}) \times M_{\Delta_i}^R(\delta_{ij})$$

$$\left[ \sum_{\substack{2n_{ij} = m \\ n_{ij} \geq 0}} M_{\Delta_i}^L(\delta_{ij}, n_{ij}) \frac{(\delta_{ij})_{n_{ij}}}{n_{ij}!} \right] \cdot \left[ \dots \right]$$

$(\alpha)_n = \prod_{a=1}^n \alpha$



# Tree-level Witten diagrams



→  $M_n$  has poles at  $m=0, 1, \dots$

$$-(p_i + p_n)^2 = \sum_{i=1}^k \Delta_i - 2 \sum_{i=1}^k \delta_{ij} = \Delta + 2m$$

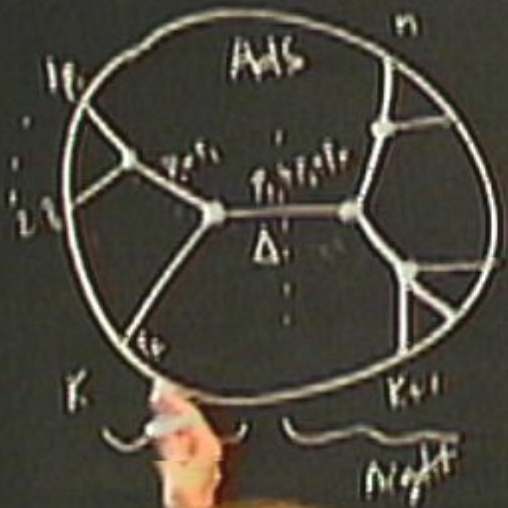
Residue

$$M_{k+1}^L(s_{ij}) \times M_{k+1}^R(s_j), \quad m=0$$

$$\sum_{\substack{i,j=1 \\ i \neq j \\ n_{ij} \geq 0}} M^L(s_{ij}, h_{ij}) \frac{(s_{ij})_{n_{ij}}}{n_{ij}!} \cdot \left[ \dots \right]$$

$$(\alpha)_n = \Gamma(\alpha+n)/\Gamma(\alpha)$$

# Tree-level Witten diagrams



→  $M_n$  has poles at

$$-(\tau_i + \tau_{i+1}) = \sum_{i=1}^k \Delta_i - 2 \sum_{i=1}^k s_{ij} = \Delta + 2m$$

$m=0, 1, \dots$

Residue

$$M_{n, \Delta_i}^L(s_{ij}) \times M_{n, \Delta_{i+1}}^R(s_{ij}), \quad m=0$$

$$\left[ \sum_{\substack{2 \leq i_1 < i_2 < \dots < i_m \\ n_{ij} \geq 0}} M_{n, \Delta_i}^L(s_{ij}, n_{ij}) \frac{(s_{ij})_{n_{ij}}}{n_{ij}!} \right] \cdot \left[ \dots \right]$$

$$(a)_n = \Gamma(a+n)/\Gamma(a)$$

## Feynman Rules

- momentum conservation at each vertex
- propagator (only internal)

# Feynman Rules

- Momentum conservation at each vertex
- propagator (only internal)

$$\frac{1}{\Delta, m}$$

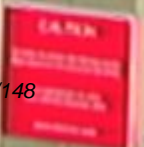
# Feynman Rules

- Momentum conservation at each vertex

Propagator (only internal)

$$\frac{1}{\Delta, m}$$

$$\frac{\Gamma(1+\Delta-\frac{d}{2})}{m^{2\Delta}} \frac{1}{\Delta+2m}$$



# Feynman Rules

- momentum conservation at each vertex

propagator (only internal)

$$\frac{1}{\Delta, m}$$

$$\frac{\Gamma(1 + \Delta - \frac{d}{2} + i\epsilon)}{(\epsilon p_i)^2 + \Delta + 2m}$$

## Feynman Rules

- momentum conservation at each vertex
- propagator (only internal)

$$\frac{1}{\Delta, m}$$

$$\frac{\Gamma(1 + \Delta - \frac{d}{2}) / m!}{(\epsilon p_i)^2 + \Delta + 2m}$$

- Vertices



# Feynman Rules

- Momentum conservation at each vertex
- Propagator (only internal)

$$\frac{1}{\Delta, m}$$

$$\frac{\Gamma(1 + \Delta - \frac{d}{2}) / m!}{(\epsilon p_i)^2 + \Delta + 2m}$$

- Vertices  $\lambda_i \phi^i$



# Feynman Rules

- Momentum conservation at each vertex
- Propagator (only internal)

$$\frac{1}{\Delta, m}$$

$$\frac{\Gamma(1 + \Delta - \frac{d}{2}) / m!}{(\epsilon p_i)^2 + \Delta + 2m}$$

- Vertices  $\lambda_i \phi^i$



$$\rightarrow V(\{\Delta_i, m_i\})$$

# Feynman Rules

- momentum conservation at each vertex
- propagator (only internal)

$$\frac{\text{---}}{\Delta, m}$$

$$\frac{\Gamma(1 + \Delta - \frac{d}{2}) / m!}{(\epsilon p_i)^2 + \Delta + 2m}$$

Vertices  $\lambda_i \phi^i$  [M. Peskin 1107...]



$$V(\{\Delta_i, m_i\})$$

# Feynman Rules

- Momentum conservation at each vertex
- Propagator (only internal)

$$\frac{1}{\Delta, m}$$

$$\frac{\Gamma(1 + \Delta - \frac{d}{2} + i\epsilon)}{(\epsilon p_i)^2 + \Delta + 2m}$$

- Vertices  $\lambda_i \phi^{\Delta_i}$  [M. Peskin 1107...]



$$\rightarrow V(\{\Delta_i, m_i\})$$

- Sum over  $m_i = 0, 1, 2, \dots$

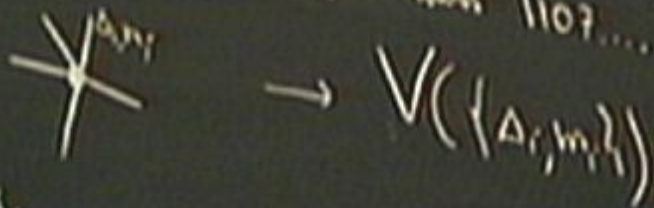
# Feynman Rules

- Momentum conservation at each vertex
- Propagator (only internal)

$$\frac{1}{\Delta, m}$$

$$\frac{\Gamma(1 + \Delta - \frac{d}{2} + i\epsilon)}{(\epsilon p_i)^2 + \Delta + 2m}$$

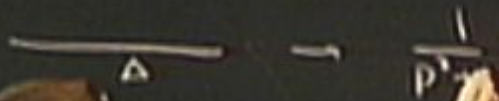
- Vertices  $\lambda_i \phi^{\Delta_i}$  [M. Peskin 1107...]



$$\rightarrow V(\{\Delta_i, m_i\})$$

- Sum over  $m_i = 0, 1, 2, \dots$

Example  $\Delta = 2$  ( $d < 6$ )



Example  $\Delta=2$  ( $d < 6$ )

$$\frac{\quad}{\Delta=2} \rightarrow \frac{1}{p^2+2}$$

$\times \propto \lambda_n$

Example  $\Delta=2$  ( $d < 6$ )  $\phi^3$

$$\frac{\text{---}}{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$$\propto \lambda_n$$



Example  $\Delta=2$  ( $d < 6$ ),  $\lambda \phi^3$

$$\frac{1}{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$\propto \lambda$

Example  $\Delta=2$  ( $d < 6$ ),  $\lambda\phi^3$

$$\frac{1}{p^2 + 2} \rightarrow \frac{1}{p^2 + 2}$$



Example  $\Delta=2$  ( $d < 6$ ),  $\lambda\phi^3$

$\rightarrow \frac{1}{p^2+2}$

$\propto \lambda$

Example  $\Delta=2$  ( $d < 6$ ),  $\lambda\phi^3$

$$\frac{1}{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$\propto \lambda$



$$\frac{1}{p^2+2}$$

$\infty$   $\lambda$



CAUTION  
DO NOT TOUCH THE SURFACE  
OF THE BOARD OR THE  
SURROUNDING AREA  
WHEN THE BOARD IS  
HOT

Flat Space Limit



CAUTION  
DO NOT TOUCH  
THE BOARD

# Flat Space Limit



CAUTION  
DO NOT TOUCH  
THE BOARD  
OR THE CHALK

# Flat Space Limit of AdS



$R \rightarrow \infty$  →



CAUTION  
PROHIBITED  
SMOKING  
NO DRINKING



# Flat Space Limit of AdS



$R \rightarrow \infty$   $\rightarrow$



CAUTION  
DO NOT TOUCH  
THE BOARD  
OR THE MARKERS

# Flat Space Limit of AdS



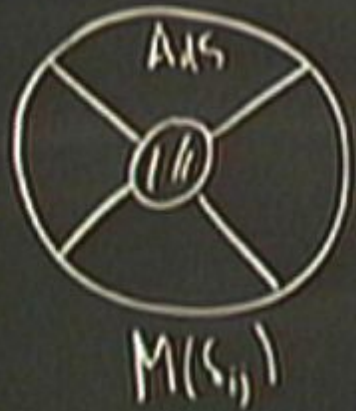
$R \rightarrow \infty$

A horizontal arrow pointing from the AdS diagram on the left to the flat space diagram on the right.



CAUTION  
DO NOT TOUCH  
EQUIPMENT

# Flat Space Limit of AdS

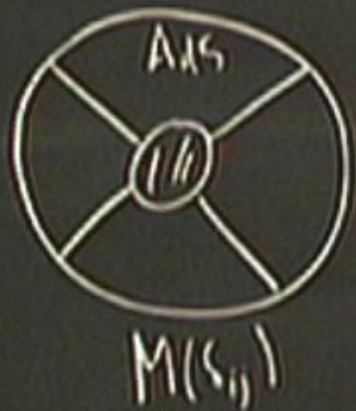


$R \rightarrow \infty$



$$S = 1$$

# Flat Space Limit of AdS



$R \rightarrow \infty$

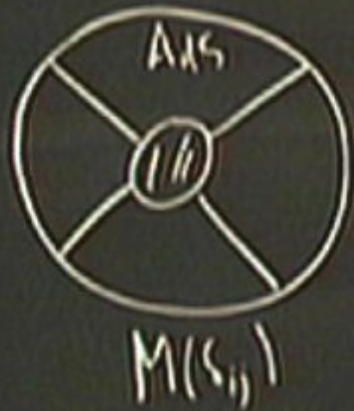


$$S = \mathbb{1} + i T$$



UNIVERSITY OF CALIFORNIA  
SANTA BARBARA

# Flat Space Limit of AdS

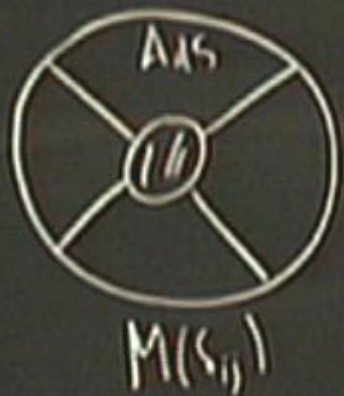


$R \rightarrow \infty$  →



$$S = \mathbb{1} + i T$$

# Flat Space Limit of AdS



$R \rightarrow \infty$

Emerges  $\gg \frac{1}{R}$



$$S = 1 + i T$$

# Flat Space Limit of AdS



$R \rightarrow \infty$   
 $\longrightarrow$



$E_{\text{mass}} \gg \frac{1}{R}$

$$S = 1 + i T$$

# Flat Space Limit of AdS



$M(S_{ij})$

$R \rightarrow \infty$   
→



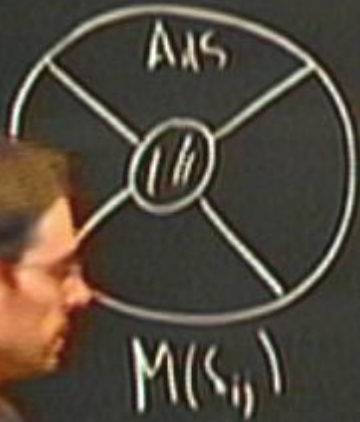
$T(P_i - P_i)$

Energy  $\gg \frac{1}{R}$   
 $\delta_{ij} \gg 1$

$$S = 1 + i T$$



# Flat Space Limit of AdS



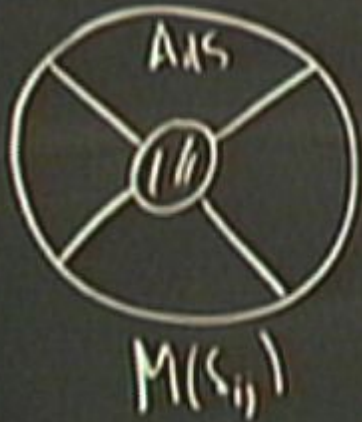
$R \rightarrow \infty$   
 $\rightarrow$



Energy  $\gg \frac{1}{R}$   
 $\delta_{ij} \gg 1$

$$S = \mathbb{1} + i T$$

# Flat Space Limit of AdS



$R \rightarrow \infty$

Energy  $\gg \frac{1}{R}$   
 $\delta_{ij} \gg 1$



$$S = \mathbb{1} + i T$$

# Flat Space Limit of Ads



$$M(s_{ij})$$

$$M(s_{ij}) \approx$$

$$R \rightarrow \infty \rightarrow$$

$$E_{\text{emission}} \gg |s_{ij}| \gg 1$$



$$T(p_{ij})$$

$$S = \ln \frac{1}{T}$$

# Flat Space Limit of AdS



$M(s_{ij})$

$M(s_{ij}) \approx$

$R \rightarrow \infty$

$E_{\text{mass}} \gg \frac{1}{R}$   
 $|s_{ij}| \gg 1$

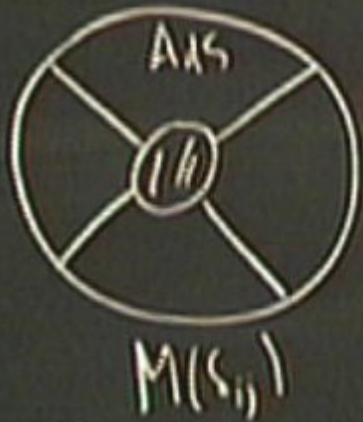
$$\int_0^{\infty} \frac{ds}{s^2}$$



$T(p_i \cdot p_j)$

$$S = \mathbb{1} + i T$$

# Flat Space Limit of AdS



$R \rightarrow \infty$



Energy  $\gg \frac{1}{R}$   
|δ<sub>ij</sub>|  $\gg 1$

$$S = 1 + i T$$

$$M(\delta_{ij}) \approx \int_0^\infty \frac{d\beta}{\beta} \beta^{\frac{\sum \Delta_i - d}{2}} e^{-\beta} T(p_i \cdot p_j = 2\beta \delta_{ij})$$

Example  $\Delta=2$  ( $d < 6$ ),  $\lambda \phi^3$

$$\frac{\text{---}}{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$\propto \lambda$



Example  $\Delta=2$  ( $d < 6$ ),  $\lambda \phi^3$

$$\overline{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



Example  $\Delta=2$  ( $d < 6$ ),  $\lambda\phi^3$

$$\overline{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$$N_{\text{us}}^2 = \frac{\Delta(\Delta-1)}{R^2}$$





Example  $\Delta=2$  ( $d < 6$ ),  $\lambda\phi^3$

$$\overline{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$$N_{\text{cut}}^2 = \frac{\Delta(\Delta-1)}{R^2}$$



# Flat Space Limit of AdS



$R \rightarrow \infty$



$$S = 1 + i T$$

$$\beta \frac{\sum \Delta_{ij}}{r} e^{-\beta} T(P_i \cdot P_j = 2\beta \delta_{ij})$$

$$\frac{1}{s_{11}}$$

# Feynman Rules

• momentum conservation at each vertex

• propagator (only internal)

$$\frac{\Gamma(1 + \Delta - \frac{d}{2} + m)}{m!}$$

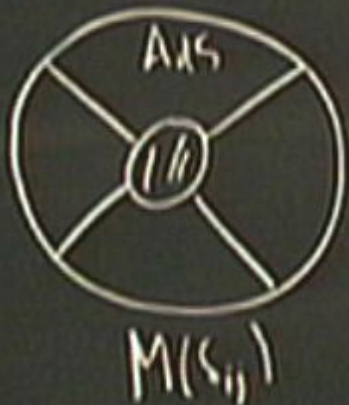
$$\frac{1}{k^2 + \Delta + 2m}$$

[L. Paulus 1107...]

$$\rightarrow V(\{\Delta_i, m_i\})$$

$$m_i = 0, 1, 2, \dots$$

# Flat Space Limit of AdS



$R \rightarrow \infty$   
→



Energy  $\gg \frac{1}{R}$   
 $|\delta_{ij}| \gg 1$

$$S = \mathbb{1} + i T$$

$$M(\delta_{ij}) \approx \int_0^\infty \frac{dp}{p} \beta^{\frac{S_{\text{AdS}}}{2}} e^{-\beta} T(p_i \cdot p_j = 2\beta \delta_{ij})$$

$\frac{1}{\beta} \approx \frac{1}{2M^2}$

Example  $\Delta=2$  ( $d < 6$ ),  $\lambda \phi^3$

$$\overline{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$$M_{\text{tree}}^2 = \frac{(\Delta-4)}{R^2}$$

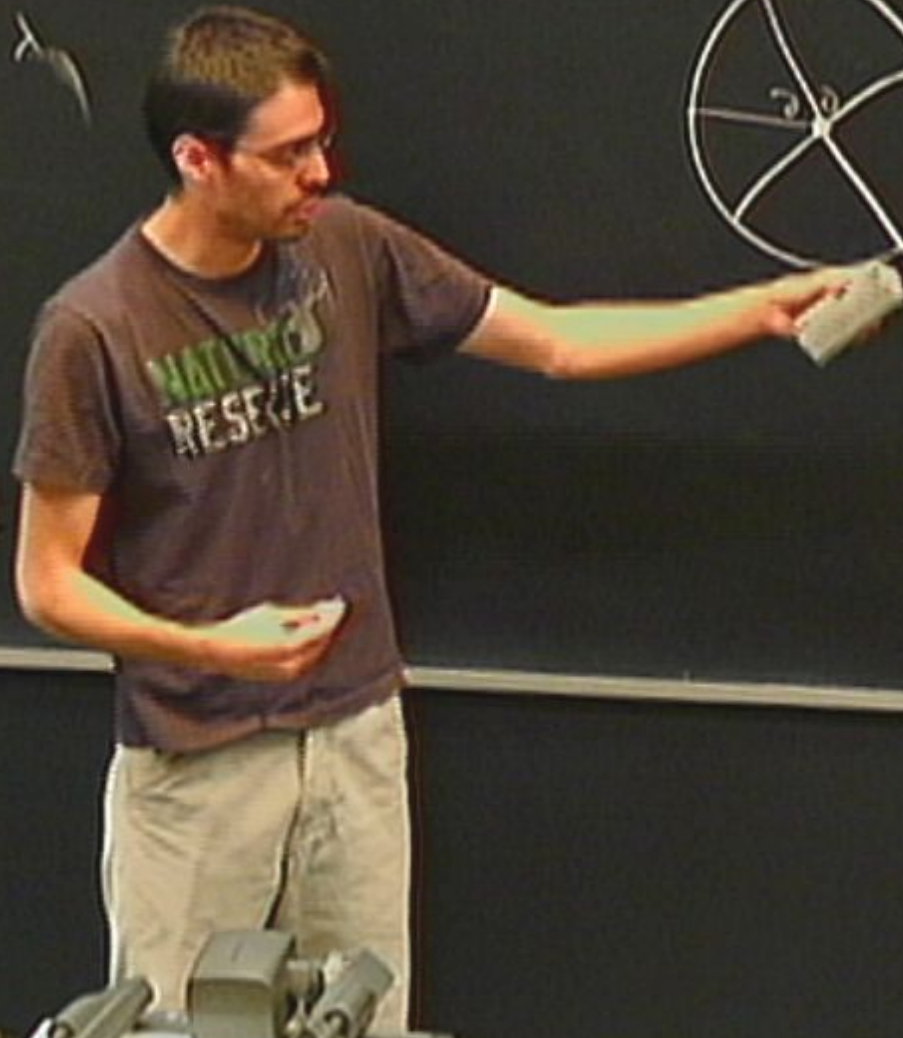


Example  $\Delta=2$  ( $d < 6$ ),  $\lambda \phi^3$

$$\frac{1}{p^2+2} \rightarrow \frac{1}{p^2+2}$$

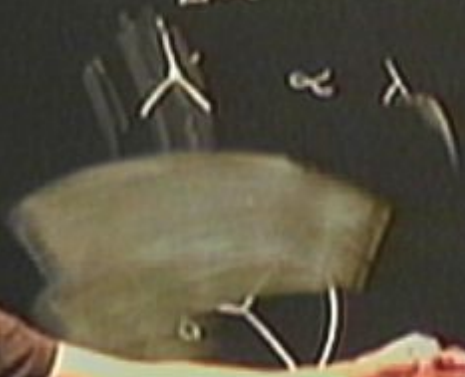


$$M_{11}^2 = \frac{(\Delta-4)}{R^2}$$



Example  $\Delta=2$  ( $d < 6$ ),  $\lambda \phi^3$

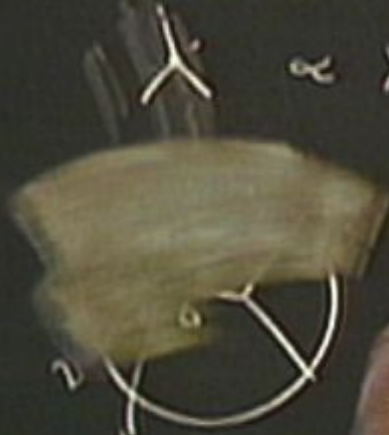
$$\overline{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$$N_{\text{loop}} = \frac{D(D-1)}{R^2}$$

Example  $\Delta=2$  ( $d < 6$ ),  $\lambda \phi^3$

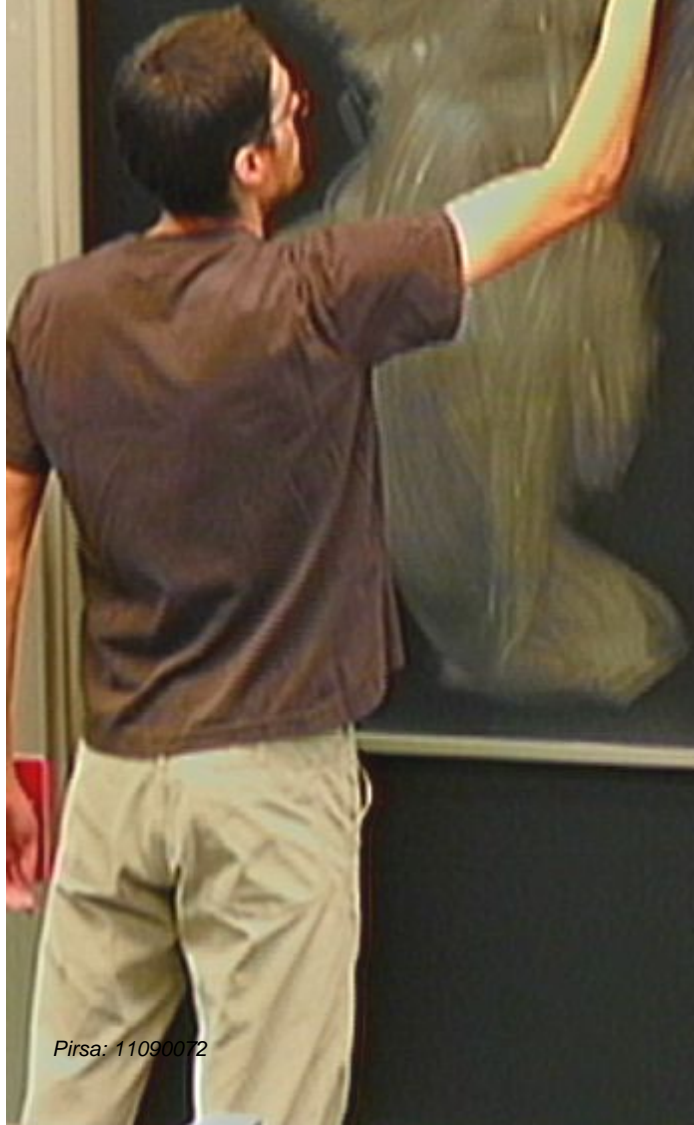
$$\overline{\Delta=2} \rightarrow \frac{1}{p^2+2}$$



$$N_{\text{us}} = \frac{\Delta \Delta}{p}$$



Open Question (100)



## Open Questions

- Loops in Ads - Feynman

Open Questions

- Loops in AdS - Feynman rules?

Open Questions

- Loops in AdS - Feynman rules?



Open Questions

- Loops in AdS - Feynman rules?



Open Questions

- Loops in AdS - Feynman rules?



$h d$



## Open Questions

- Loops in Ads - Feynman rules?



$$\delta_{ij} = \frac{h d}{(d+1)h} p_i \cdot p_j$$

## Open Questions

- Loops in Ads - Feynman rules?



$$\delta_{ij} = \int_{(H)} \frac{h^d}{P_i \cdot P_j}$$



## Open Questions

- Loops in AdS - Feynman rules?



$$\delta_{ij} = \frac{P_i \cdot P_j}{(d+1)h}$$

# Flat Space Limit of AdS



$R \rightarrow \infty$



$$S = 1 + i T$$

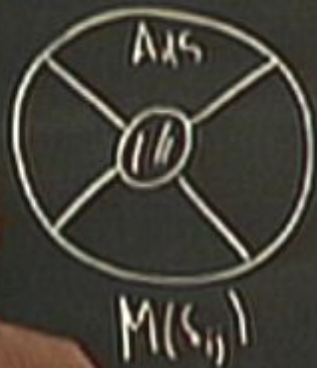
Energy  $\gg \frac{1}{R}$   
 $|\delta_{ij}| \gg 1$

$$M(\delta_{ij}) \approx \int_0^\infty \frac{dp}{p} \beta^{\frac{S_{AdS} - d}{2}} e^{-\beta} T(p_i, p_j = 2\beta \delta_{ij})$$

=

$$\frac{1}{\Lambda^{S_{AdS} + M^2}}$$

# Flat Space Limit of AdS



$R \rightarrow \infty$



$$S = 1 + i T$$

Energy  $\gg \frac{1}{R}$   
 $|\delta_{ij}| \gg 1$

$M(\delta_{ij}) \rightarrow$

$$\int_0^\infty \frac{dp}{p} \beta^{\frac{5d-4}{2}} e^{-\beta} T(p_i \cdot p_j = 2\beta \delta_{ij})$$

$$\frac{1}{\Lambda^{2d+1} M^2}$$

## Open Questions

- Loops in Ads - Feynman rules?

• Fundamental Operators w

Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
[M. Paulos]



$$\delta_{ij} = \int_{(d+1)_h} p_i \cdot p_j$$

Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
(M. Paulos)



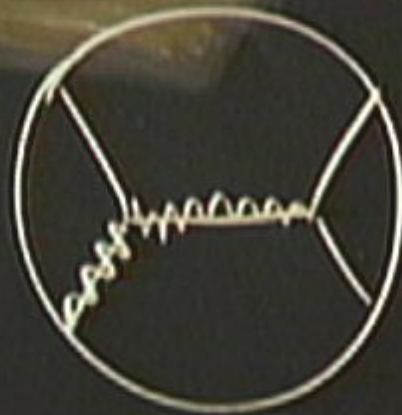
$$\int \frac{d^d p_i}{(2\pi)^d} p_i \cdot p_j$$



ators with spin  
]

$$\delta_{ij} = p_i \cdot p_j$$

$\downarrow$   
 $(d+1)n$



## Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
[M. Paulos]



$n, d$

$$\delta_{ij} = \frac{P_i \cdot P_j}{(d+1)n}$$





## Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
M. Paul



$$\delta_{ij} = \frac{P_i \cdot P_j}{(d+1)\pi}$$



## Open Questions

- Loops in AdS - Feynman rules?

- External Operators with spin  
[M. Paulos]

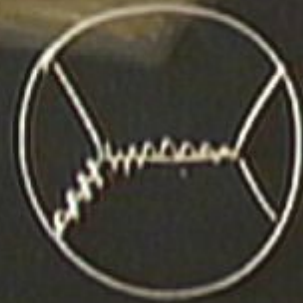
- BC



$h d$

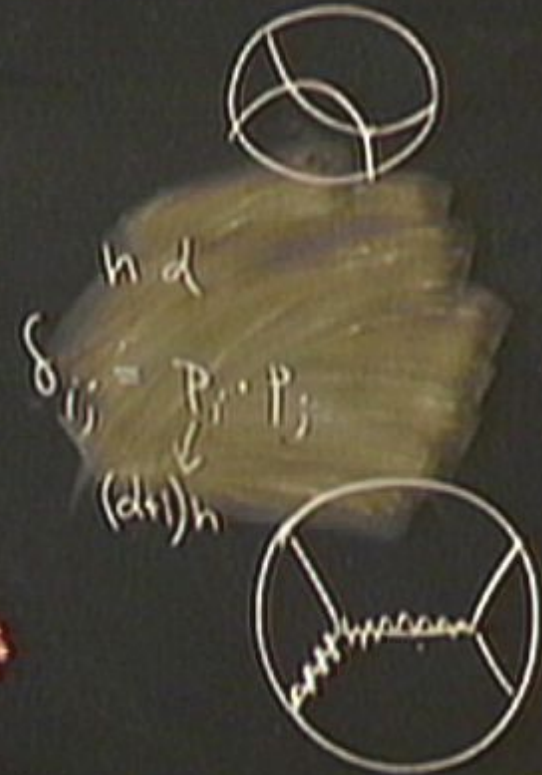
$$\delta_{ij} = P_i \cdot P_j$$

$(d+1)h$



## Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
[Paulos]



## Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
[M. Paulos]
- BCFW?
- Bootstrap in Mellin space?



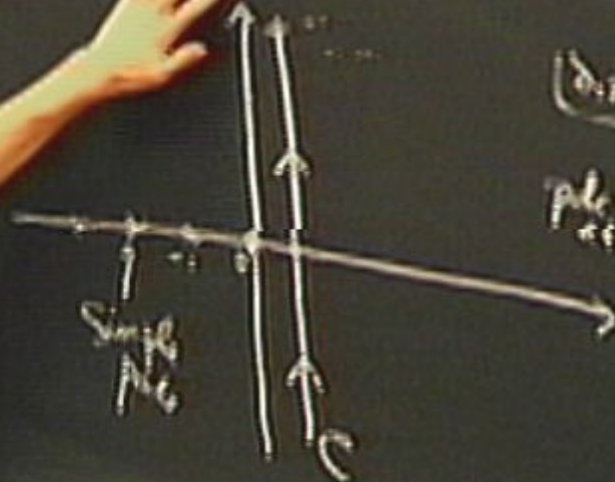
$$\delta_{ij} = \frac{P_i \cdot P_j}{(d-1)n}$$



$n=4$

$$\langle 0_p | 0_{p+1} | 0_{p+2} | \dots \rangle = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |X|^{-2\delta_{12}} \Gamma(\delta_{11}) \prod_{i=1}^n \Gamma(\delta_{12}) \omega_{i,1}^{\delta_{11}}$$

- $M$  only has simple poles associated with slingshot



$\delta_{12}$

Pole at  $\delta_{12} = -m$

$\Delta_p = 0, \Delta_{p+1} = m$

$\omega_1, \omega_2, \omega_3, \omega_4$

$\rightarrow V(\{\Delta_i, m_i\})$

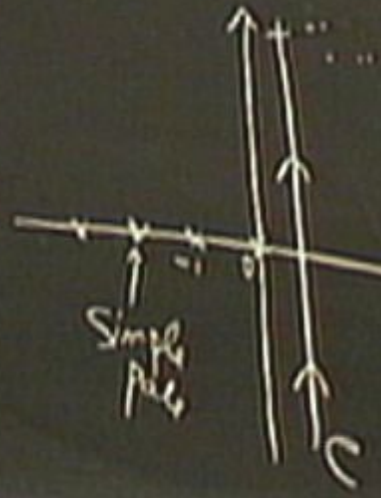
$i=0, 1, 2, \dots$

$$\langle \mathcal{O}_2(x) \mathcal{O}_1(0) \rangle \sim \sum_p \frac{1}{p} \quad (\langle \mathcal{O}_p(0) \rangle + \text{disc.})$$

$h=4$

$$\langle \mathcal{O}_1(0) \mathcal{O}_2(x) \mathcal{O}_3(x) \mathcal{O}_4(x) \rangle = \int_C ds_{11} ds_{12} M(\delta_{11}, \delta_{12}) |x|^{-2\delta_{12}} \Gamma(\delta_{12}) \prod_{i=1}^3 \Gamma(\delta_{1i}) \omega_{ii}^{2\delta_{1i}}$$

- $M$  only has simple poles associated with simple-line ops

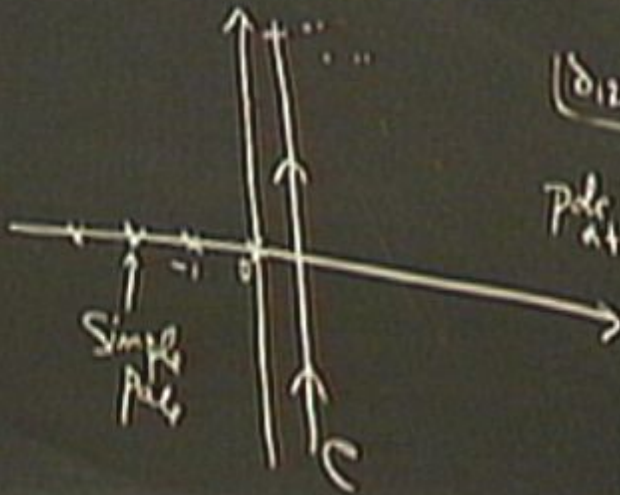


$\delta_{12}$   
 Pole at  $\delta_{12} = -m$   
 $\Delta_p = \Delta_1 + \Delta_2 + 2m$   
 $\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4$

$V(\{\Delta_i, m_i\})$

- Sum over  $m_i = 0, 1, 2, \dots$

- $M$  only has simple poles associated with singularity of



Poles at  $\delta_{12} = -m$

$\Delta_f = \Delta_1 + \Delta_2 + 2m$

$\mathcal{O}_1, \mathcal{O}_2, \dots, \mathcal{O}_m$

- Vertices  $\lambda_i^{\mu_i}$  [M. Taroni 1107...]

$\lambda_i^{\mu_i} \rightarrow V(\{\Delta_i, m_i^{\mu_i}\})$

- Sum over  $m_i = 0, 1, 2, \dots$

$$\prod_{i=1}^n \Gamma(\nu_i) \omega_i^{\nu_i}$$

$$\delta_{ij} = -m$$

$$\Delta_f = 0, \Delta_r = 1$$

$$\mathcal{O}_1 \dots \mathcal{O}_n$$

Open Questions

- Loops in AdS - Feynman rules?
- Ext. Operators with spin
- ...
- ... in Mellin space?
- Dalmauer, Osborn



$n, d$

$$\delta_{ij} = \frac{P_i \cdot P_j}{(d+1)^n}$$





$$\prod_{i=1}^n \Gamma(\nu_i) \omega_i^{\nu_i}$$

$$\delta_{ij} = -m$$

$$\Delta_f = 0, \delta, \epsilon, \dots$$

$$\mathcal{O}_1, \mathcal{O}_2, \dots$$

Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
[M Paulos]
- BCFW?
- Bootstrapping in Mellin space?  
(Dolan, Osborn)



$n, d$

$$\delta_{ij} = \frac{P_i \cdot P_j}{(d_i)_{j_i}}$$



## Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spin  
[M Paulos]
- BCFW?
- Bootstrapping in Mellin space?  
[Dolan, Osborn]
- Weak coupl.



$$\delta_{ij} = \int \frac{d^d p}{(2\pi)^d} p_i p_j$$



## Open Questions

- Loops in AdS - Feynman rules?
- External Operators with spins  
([M Paulos])
- BCFW?
- Bootstrapping in Mellin space?  
(Dolan, Osborn)
- Weak coupling



$$\delta_{ij} = \int_{(d^4)h} p_i \cdot p_j$$



