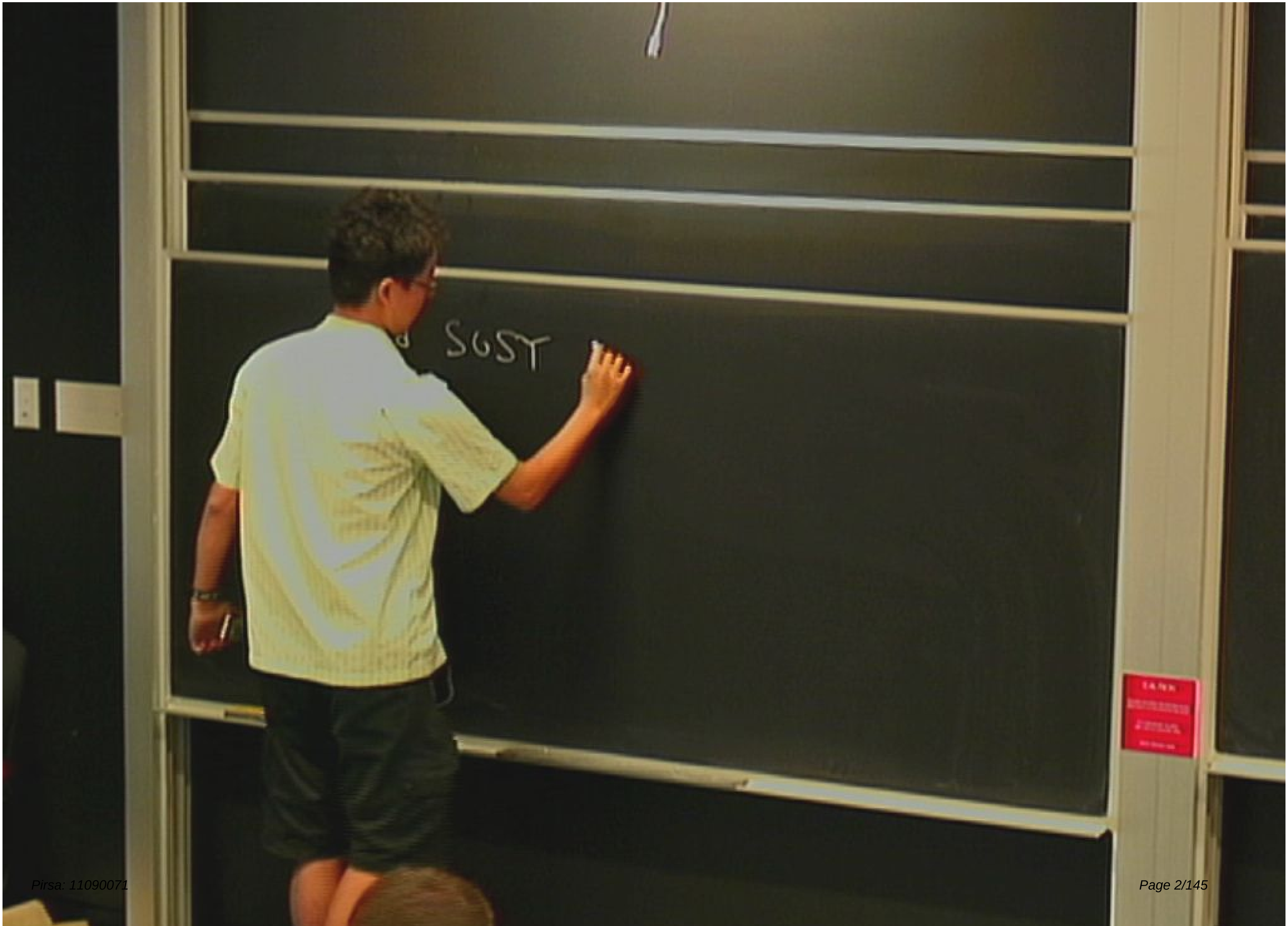
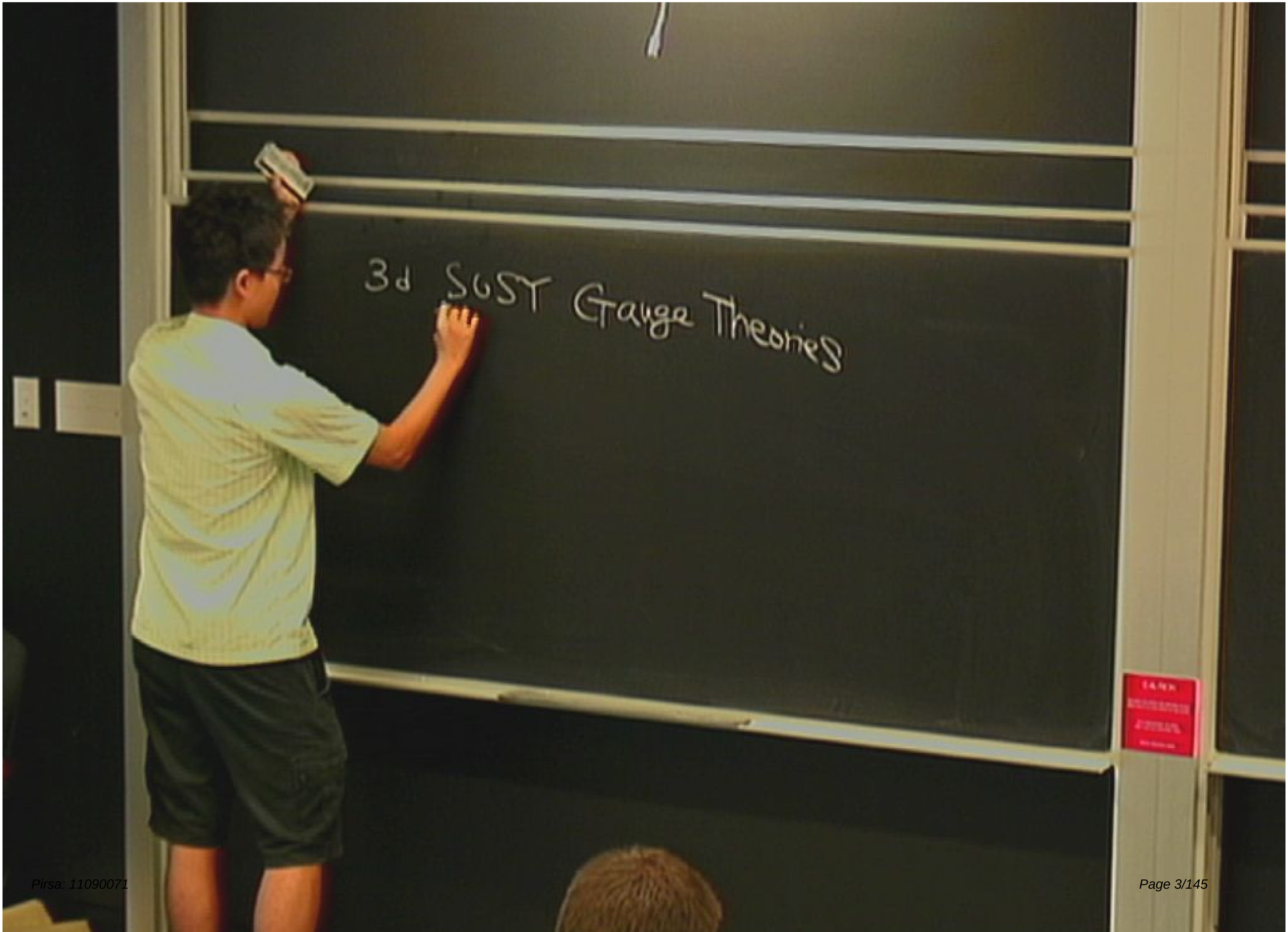
Title: 3-Manifolds and 3D Gauge Theories

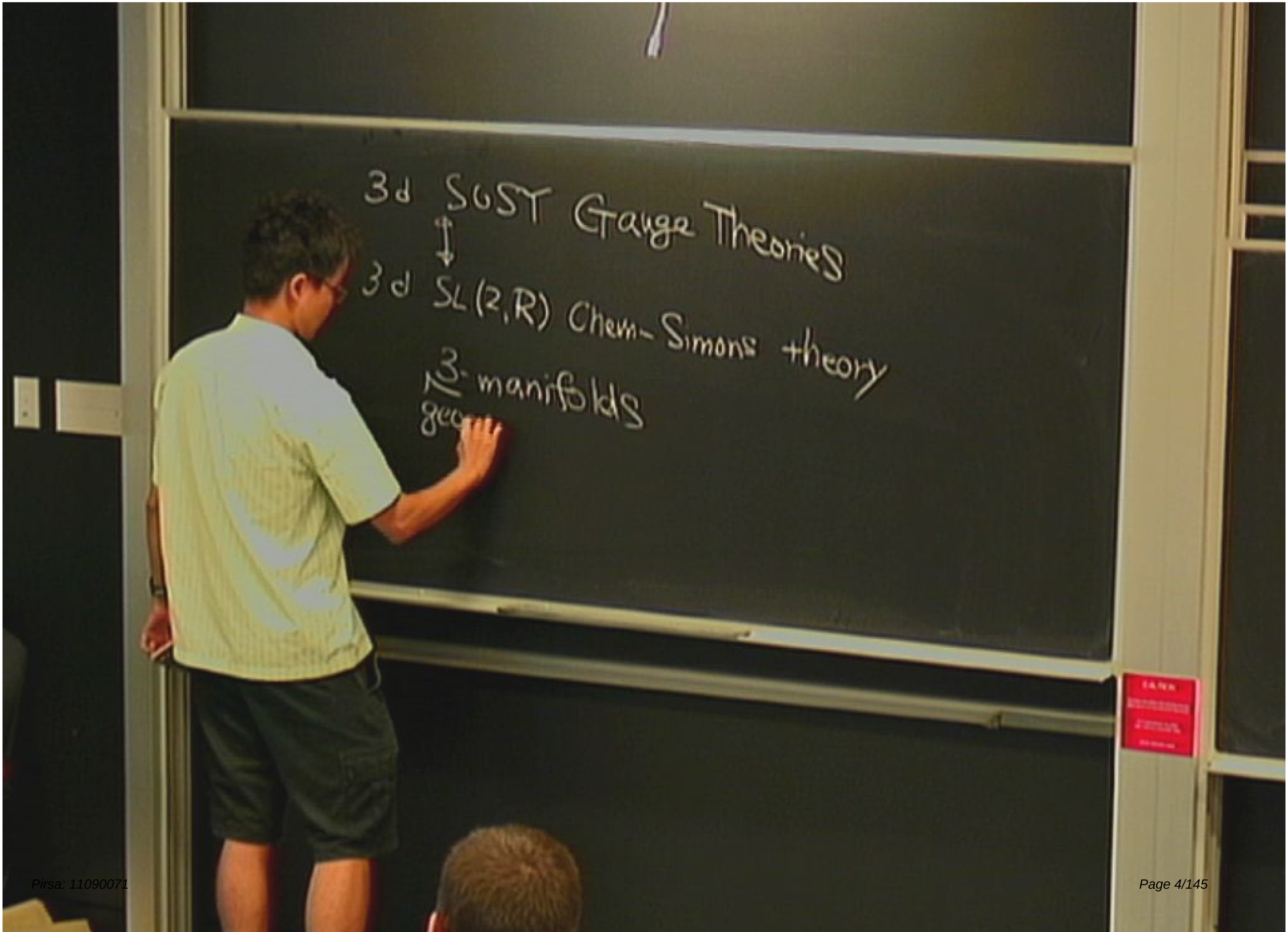
Date: Sep 02, 2011 11:00 AM

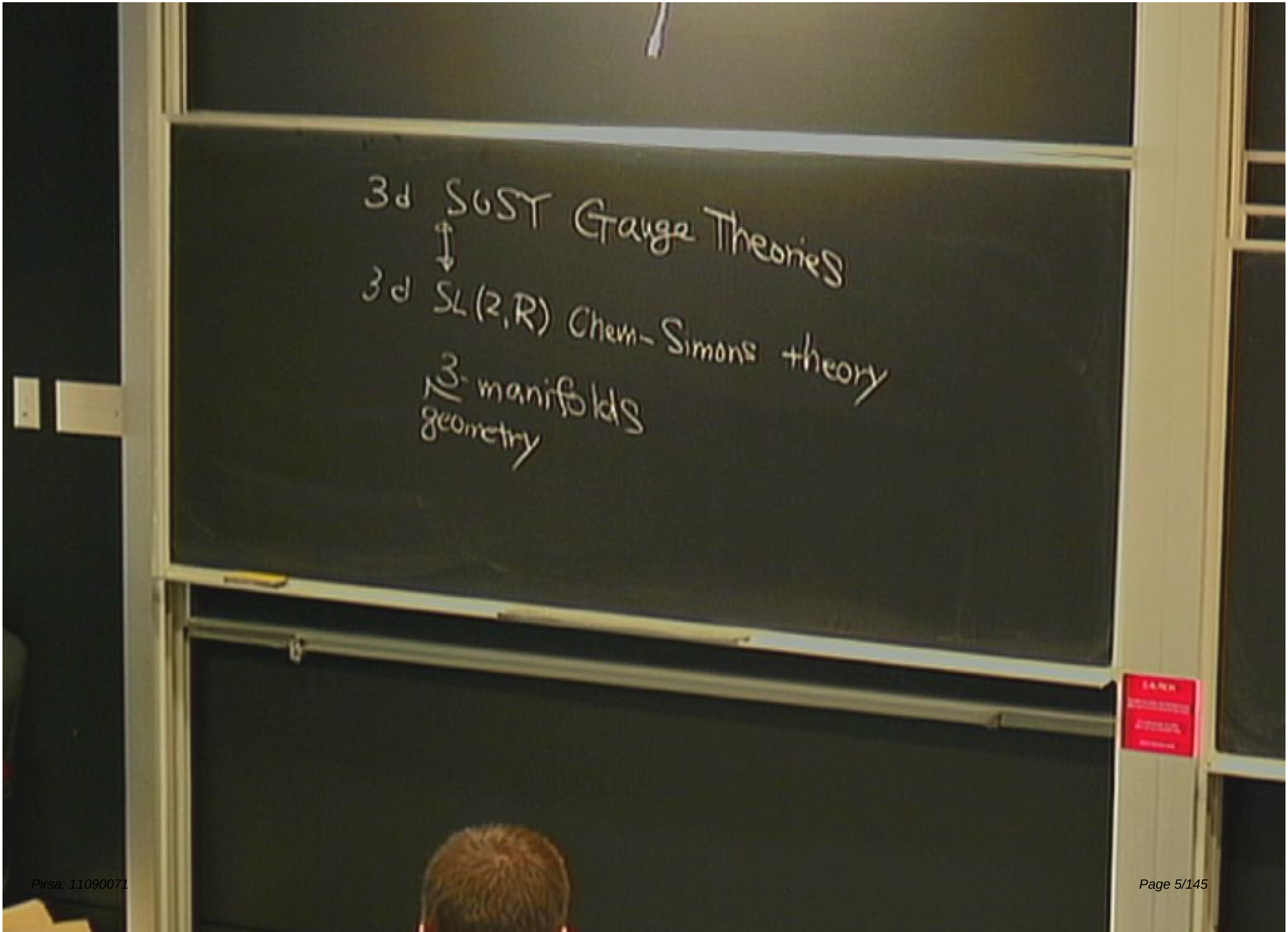
URL: http://pirsa.org/11090071

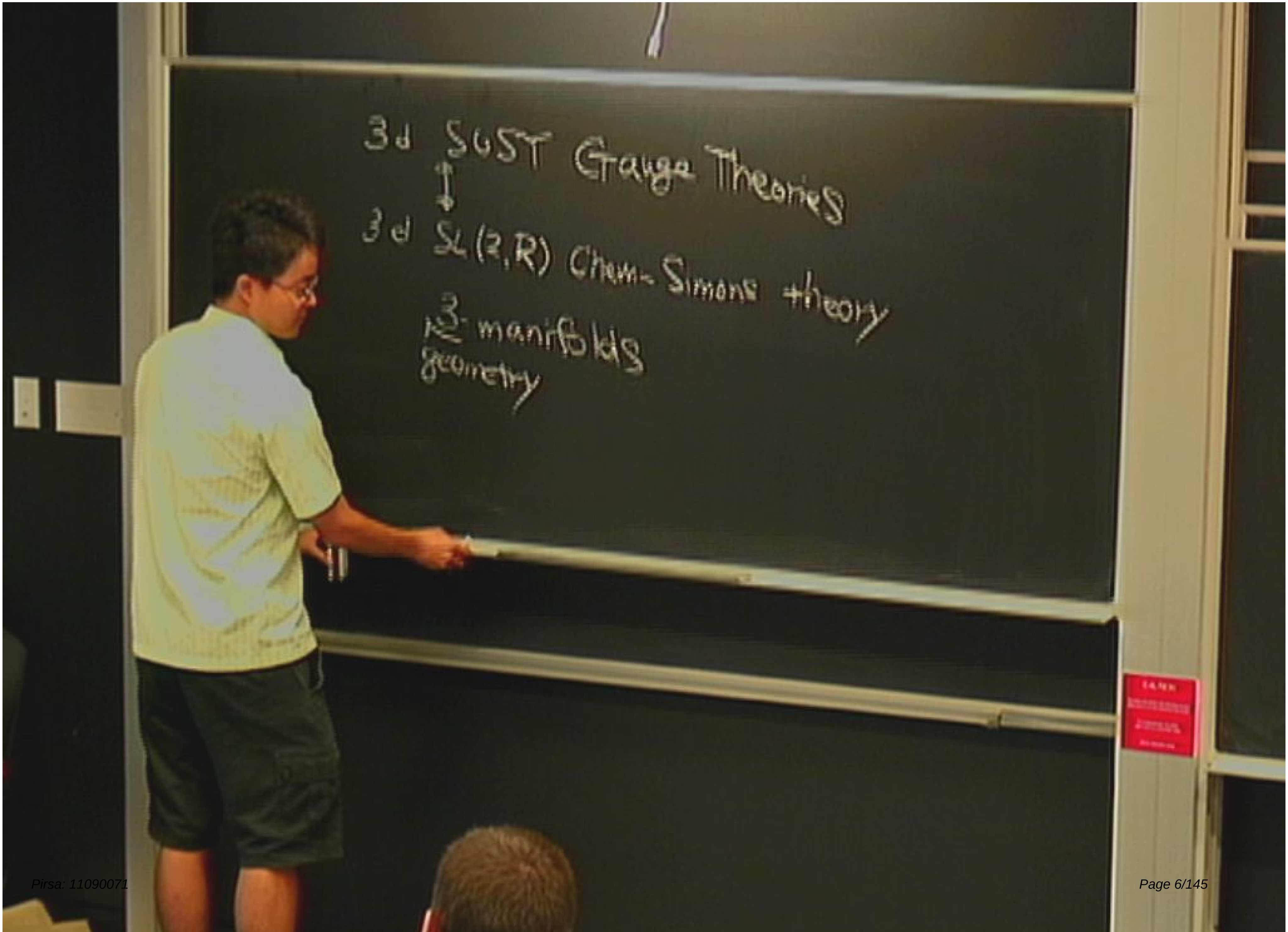
Abstract: I will first summarize recent exact localization computations of supersymmetric gauge theories, and then discuss curious connections between SUSY/non-SUSY theories coming from 6d (2,0) theories. I particular, I will focus on our recent proposal relating 3d N=2 theories and 3d SL(2,R) Chern-Simons theories (or more mathematically, geometry of 3-manifolds).

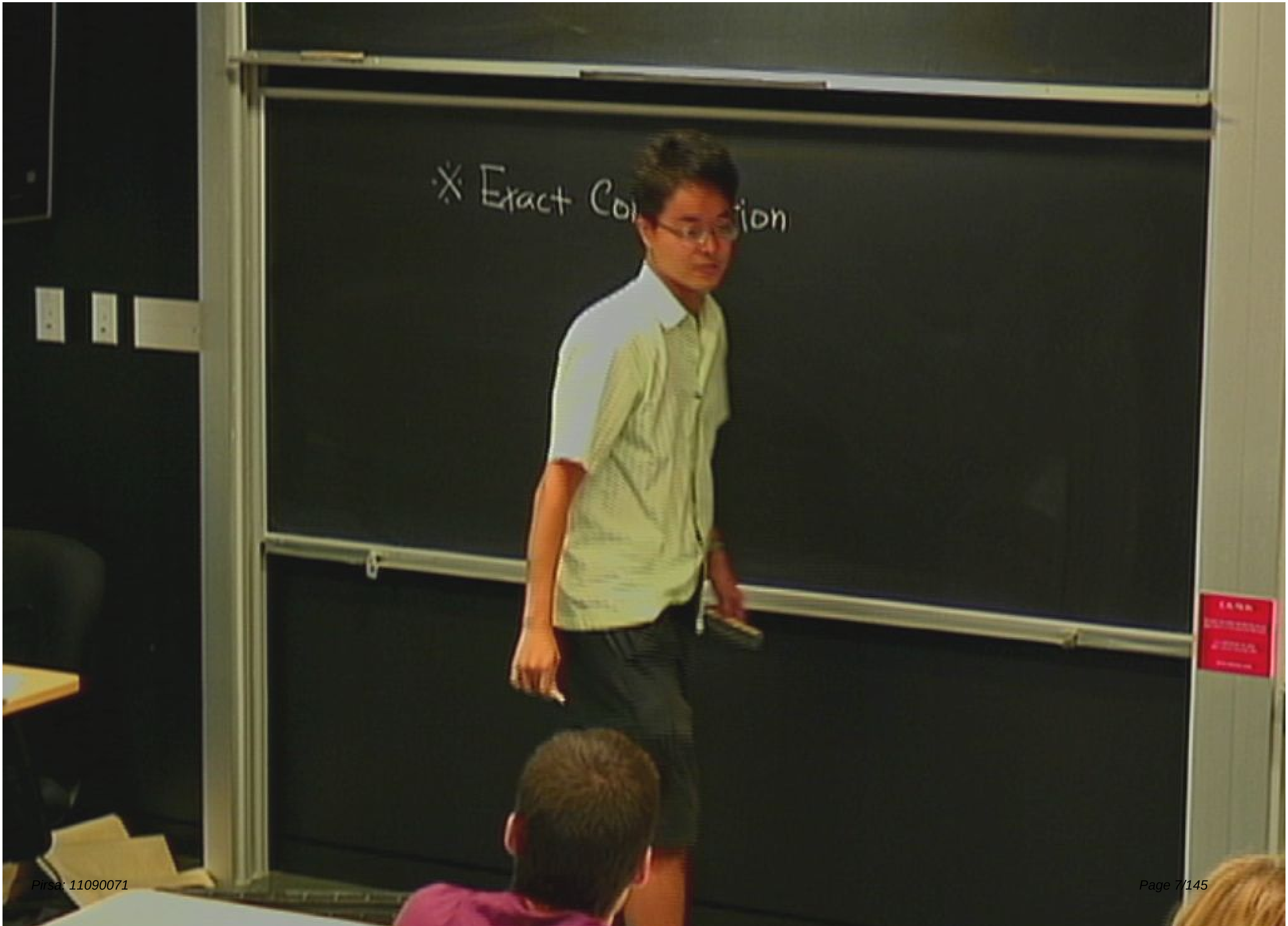

SUSY


3d SUSY Gauge Theories

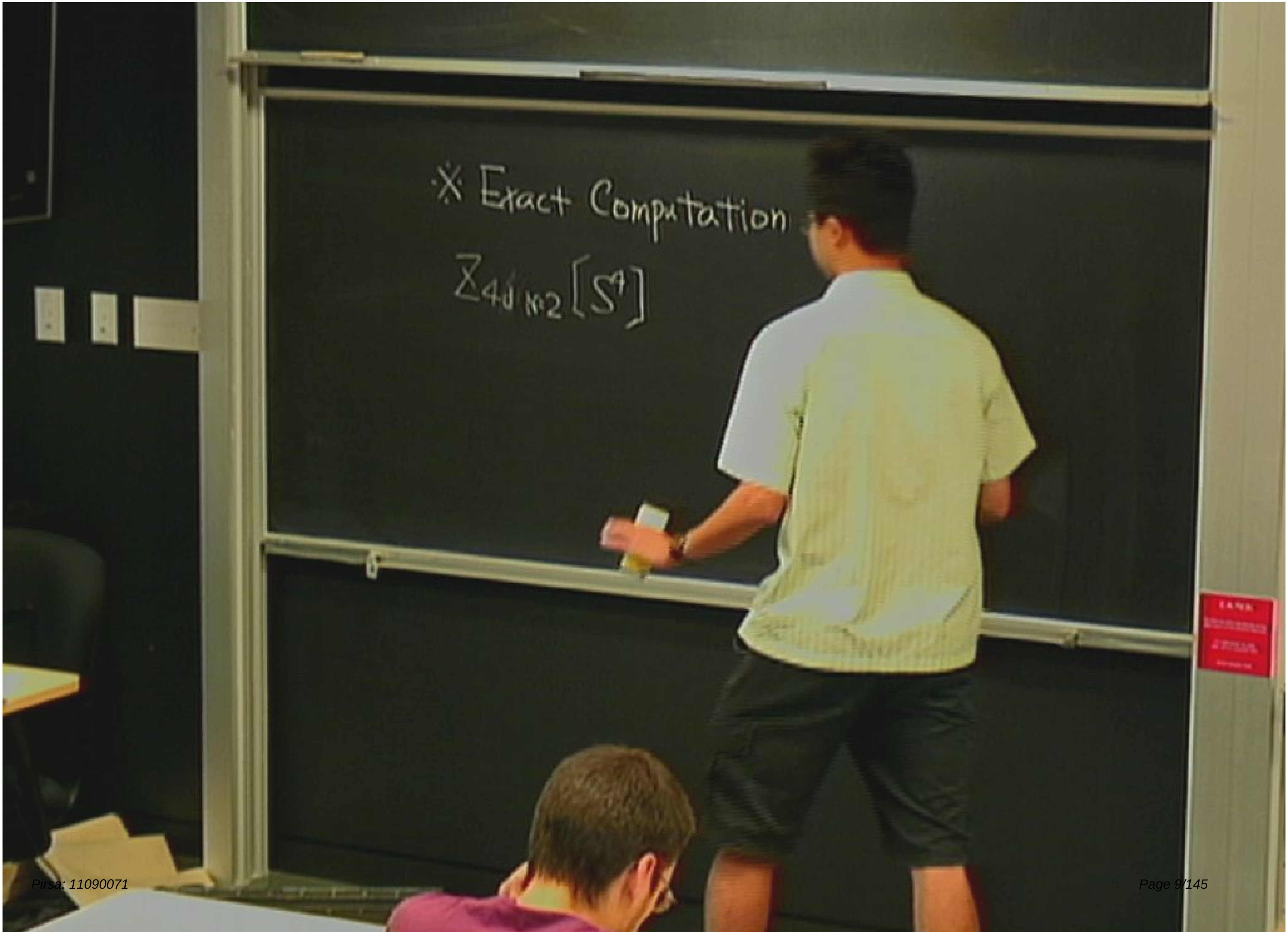

3d SUSY Gauge Theories
3d SL(2,R) Chern-Simons theory
3-manifolds

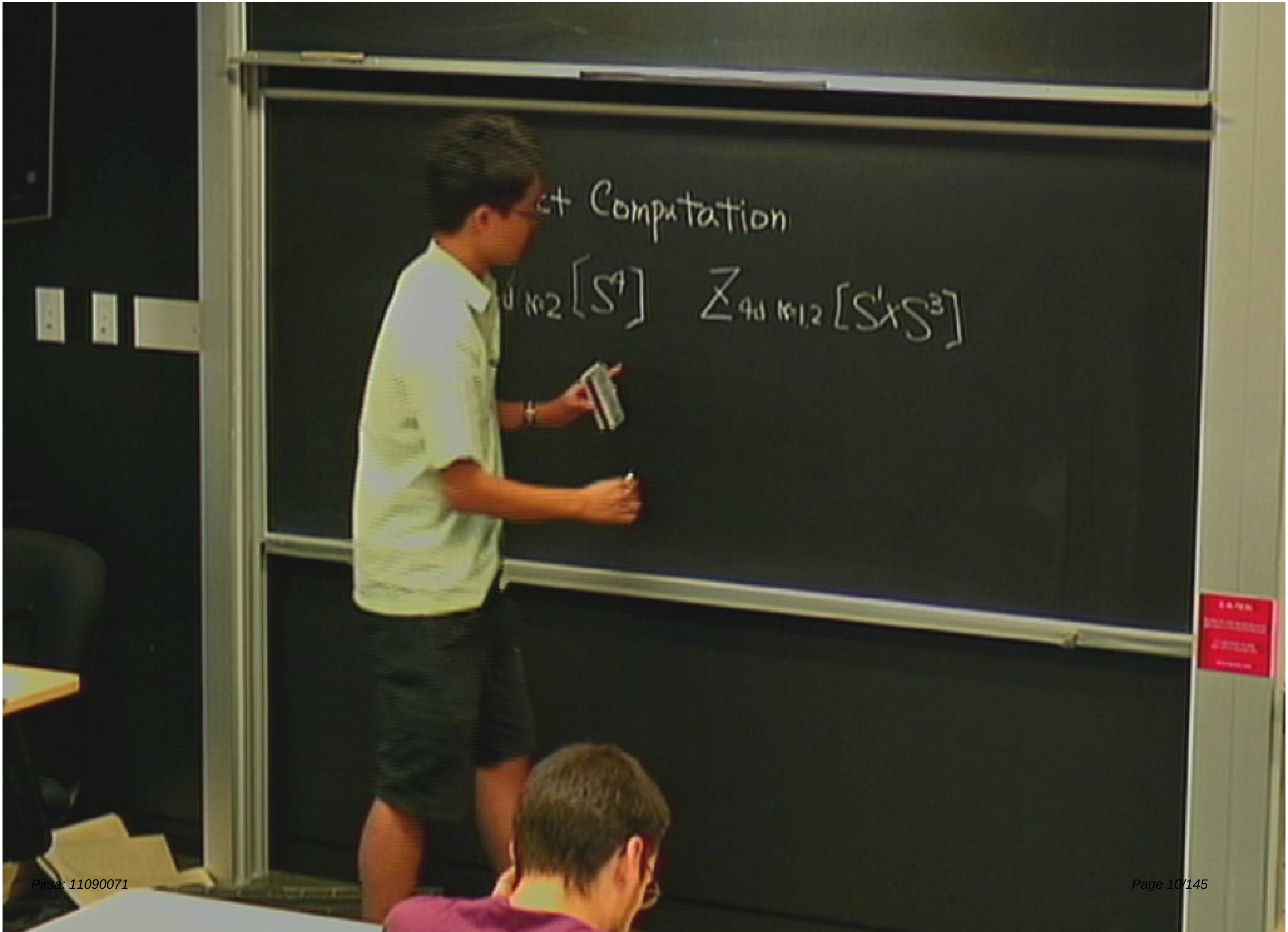

3d SUSY Gauge Theories
3d SL(2,R) Chern-Simons theory
3-manifolds
geometry

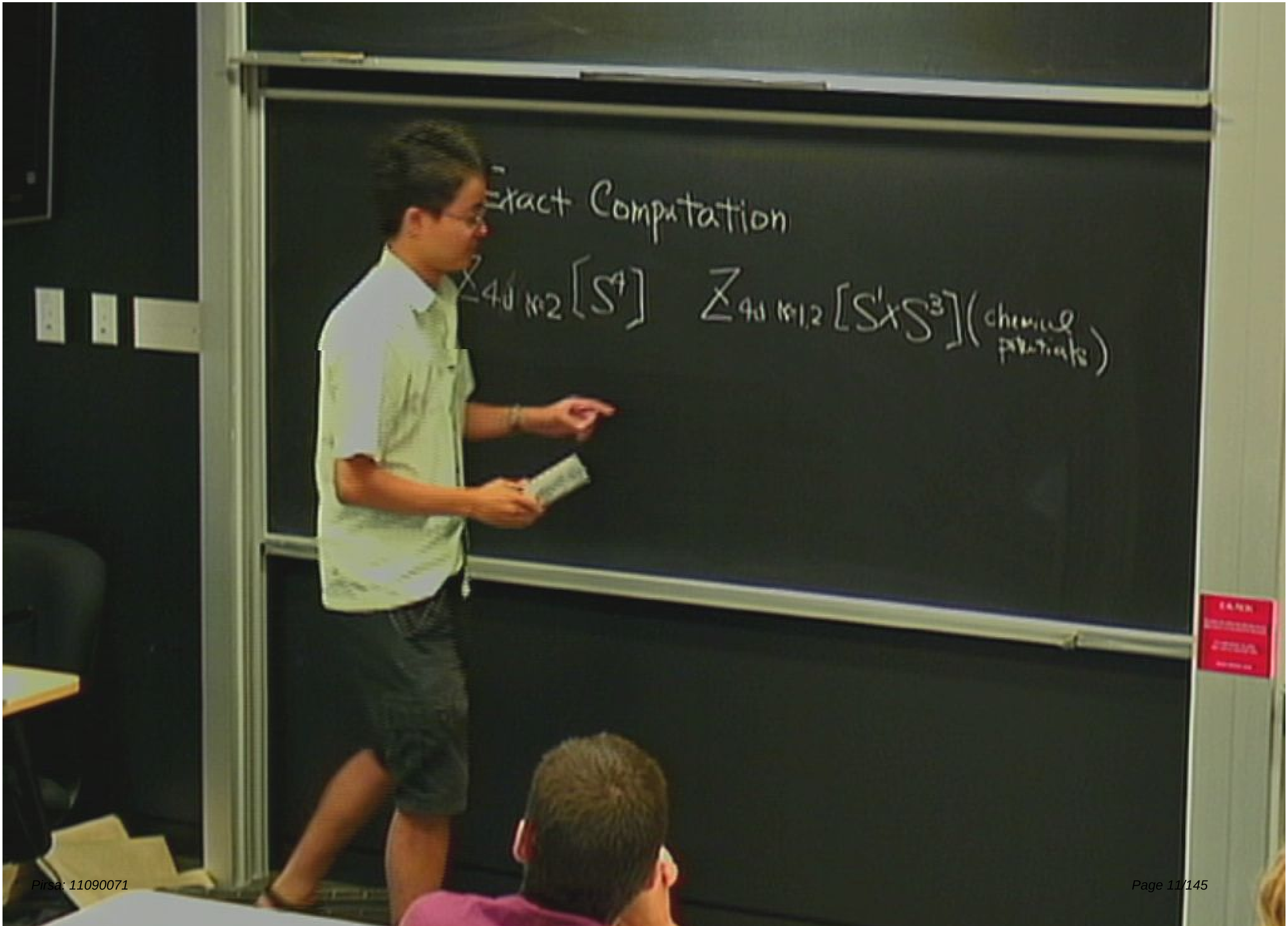

3d SUSY Gauge Theories
3d SL(2,R) Chern-Simons theory
3-manifolds
geometry

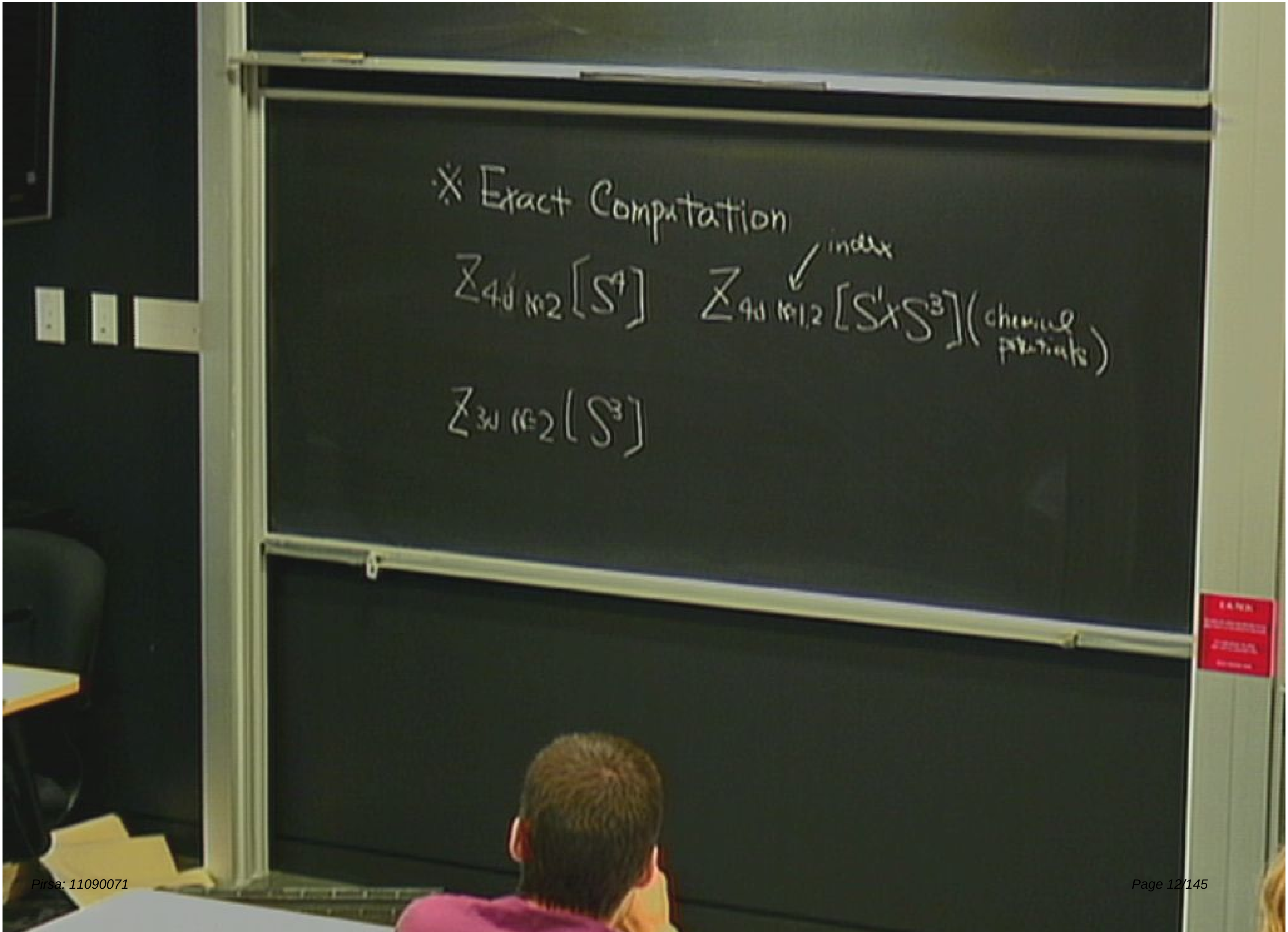

Exact

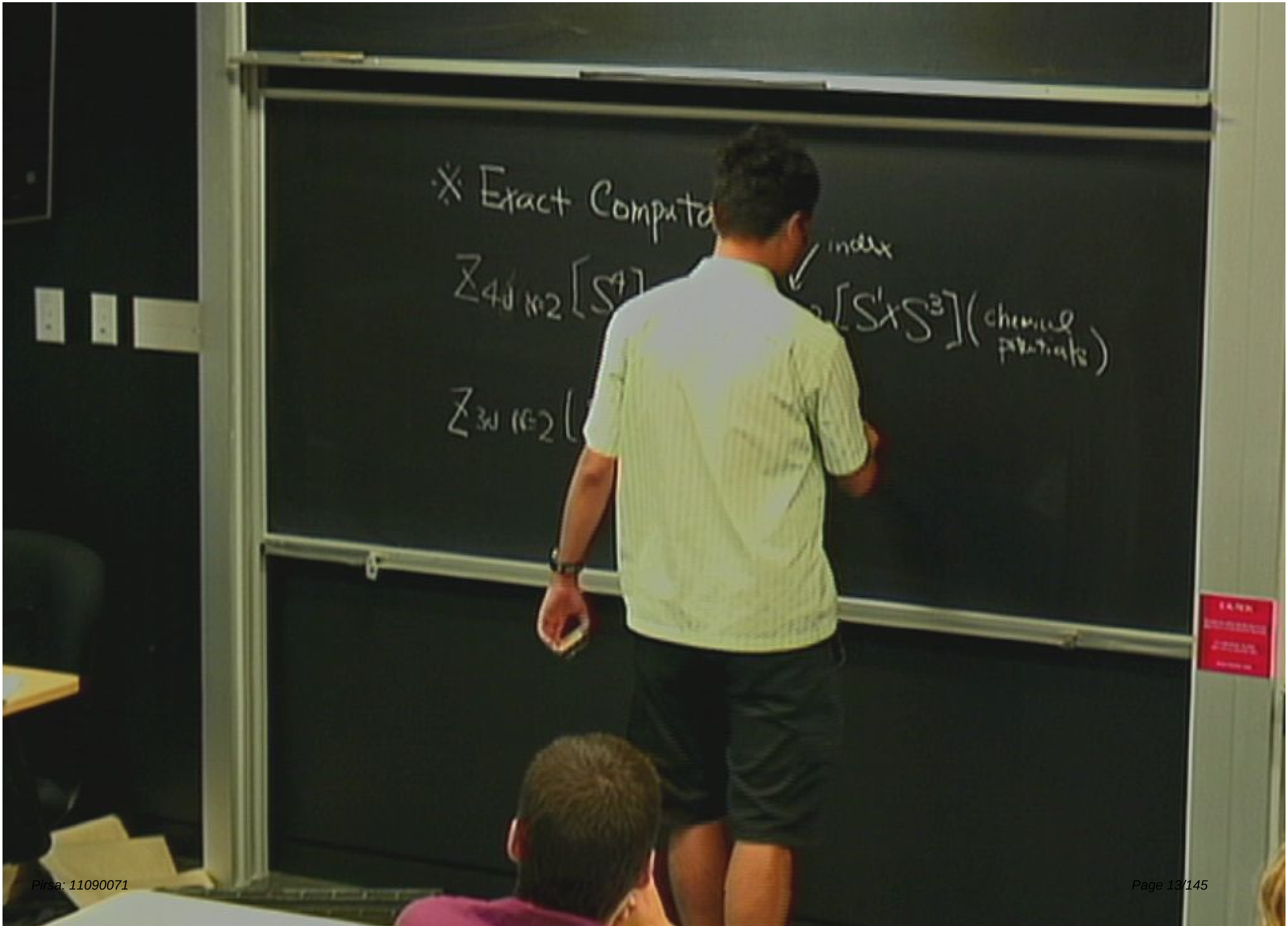

Exact Computation


※ Exact Computation
$Z_{4d\ N=2}[S^4]$


Computation
$[S^4]$
$Z_{4d\ N=1,2}[S^1 \times S^3]$


Exact Computation
Z 4d N=2 [S4]
Z 4d N=1,2 [S1×S3] (chemical potentials)

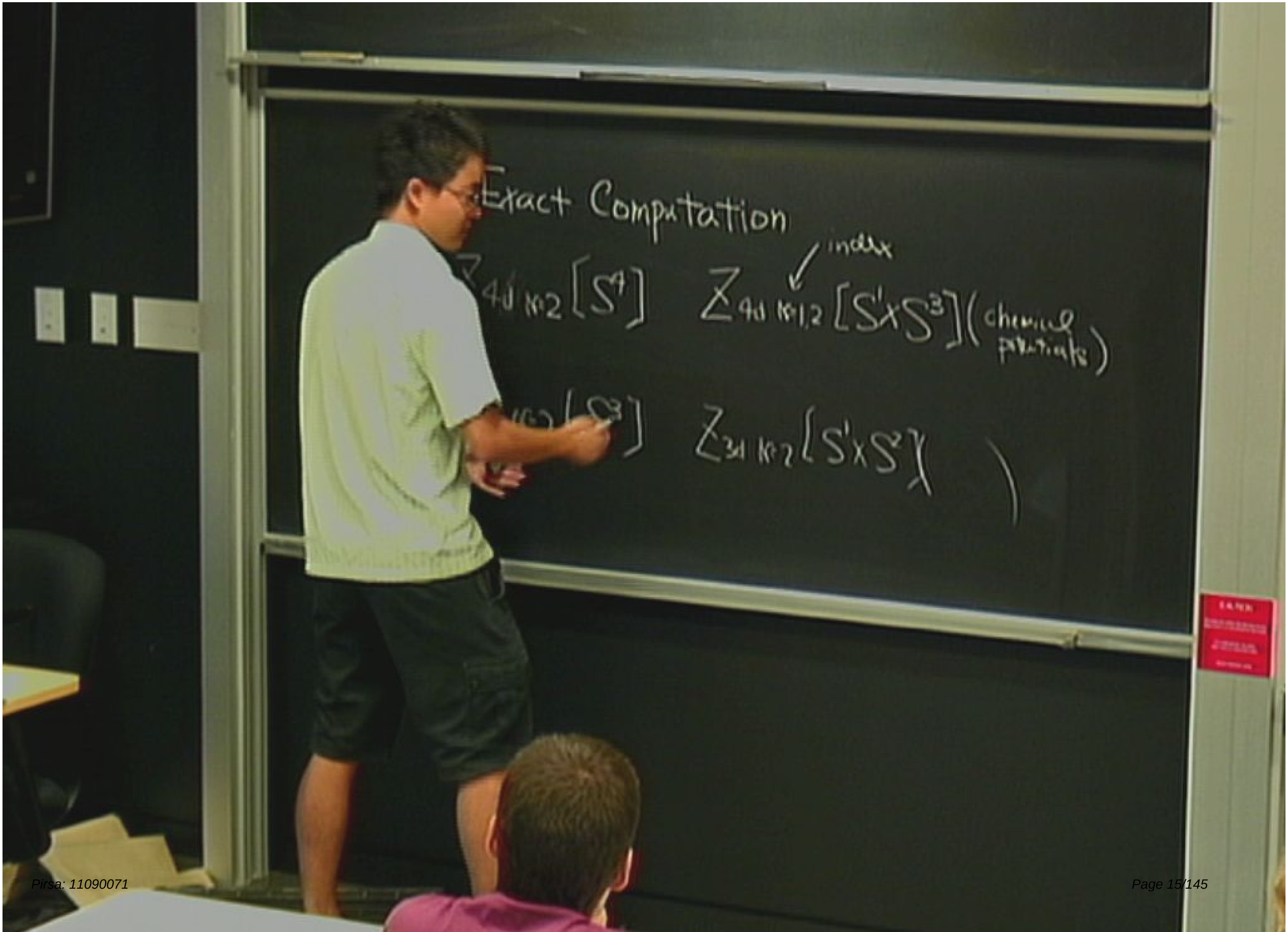

※ Exact Computation
$Z_{4d\ N=2}[S^4]$
index
$Z_{4d\ N=1,2}[S^1 \times S^3]$ (chemical potentials)
$Z_{3d\ N=2}[S^3]$

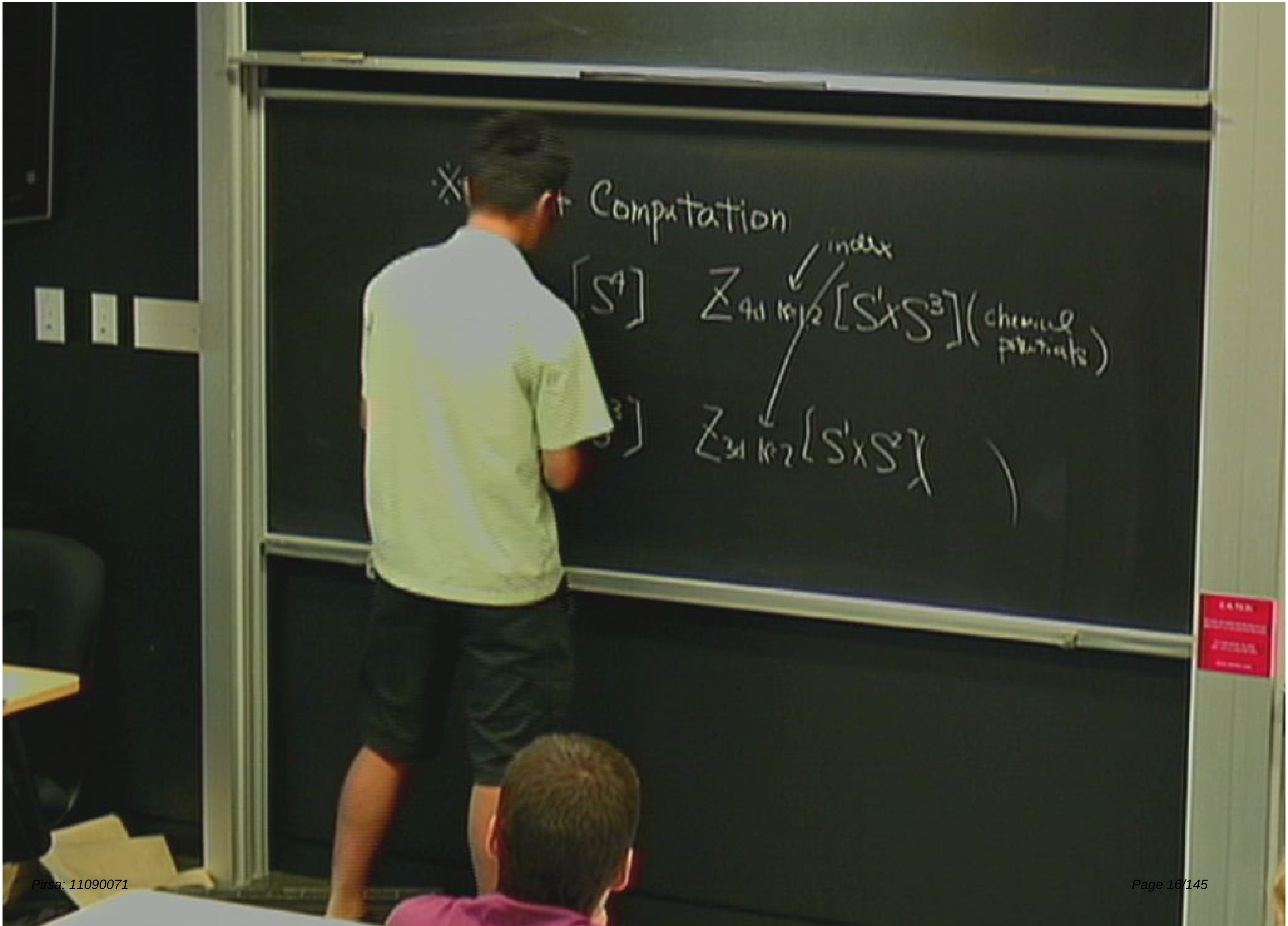

Exact
index
chemical
potentials

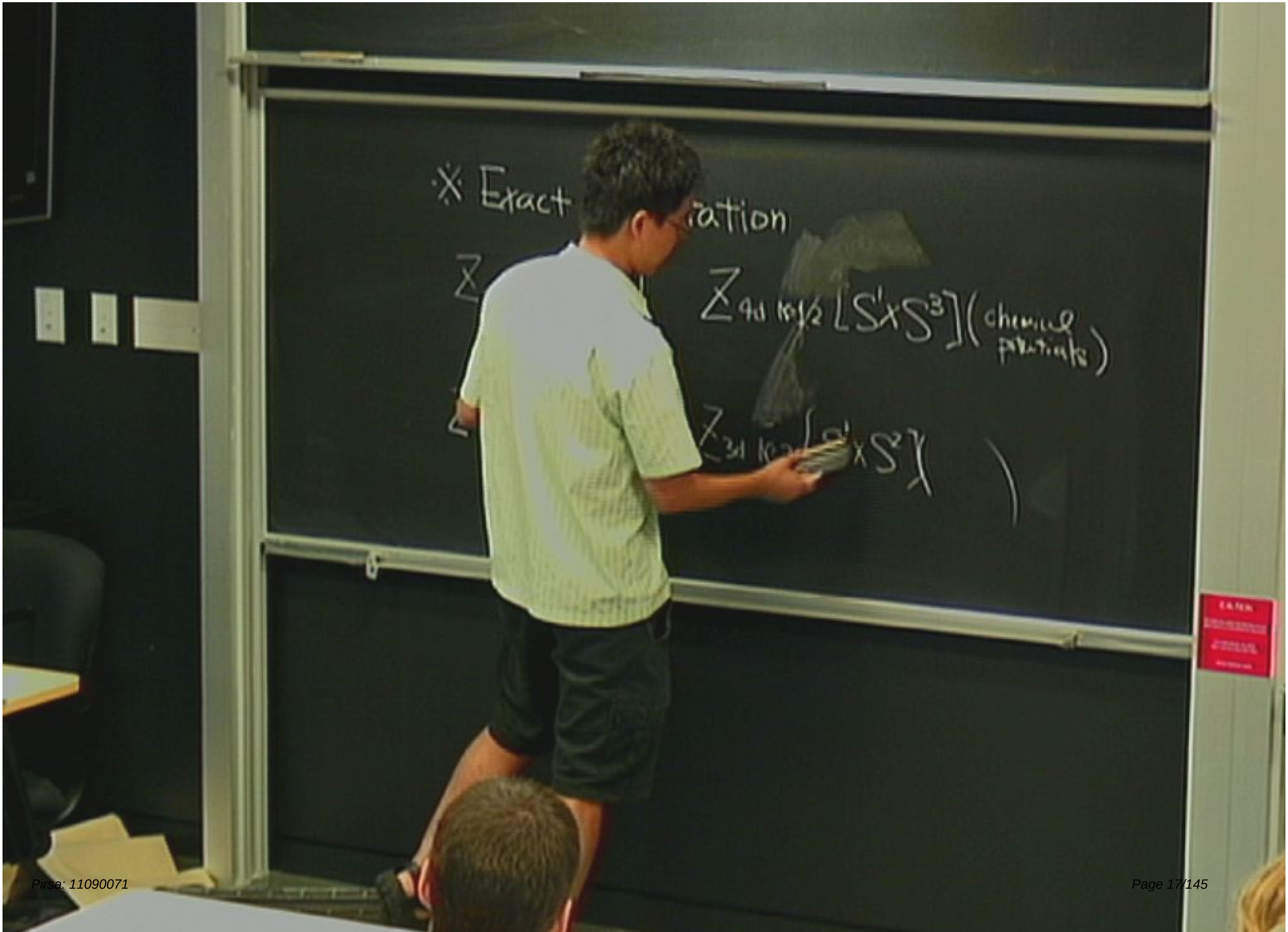
※ Exact Computation

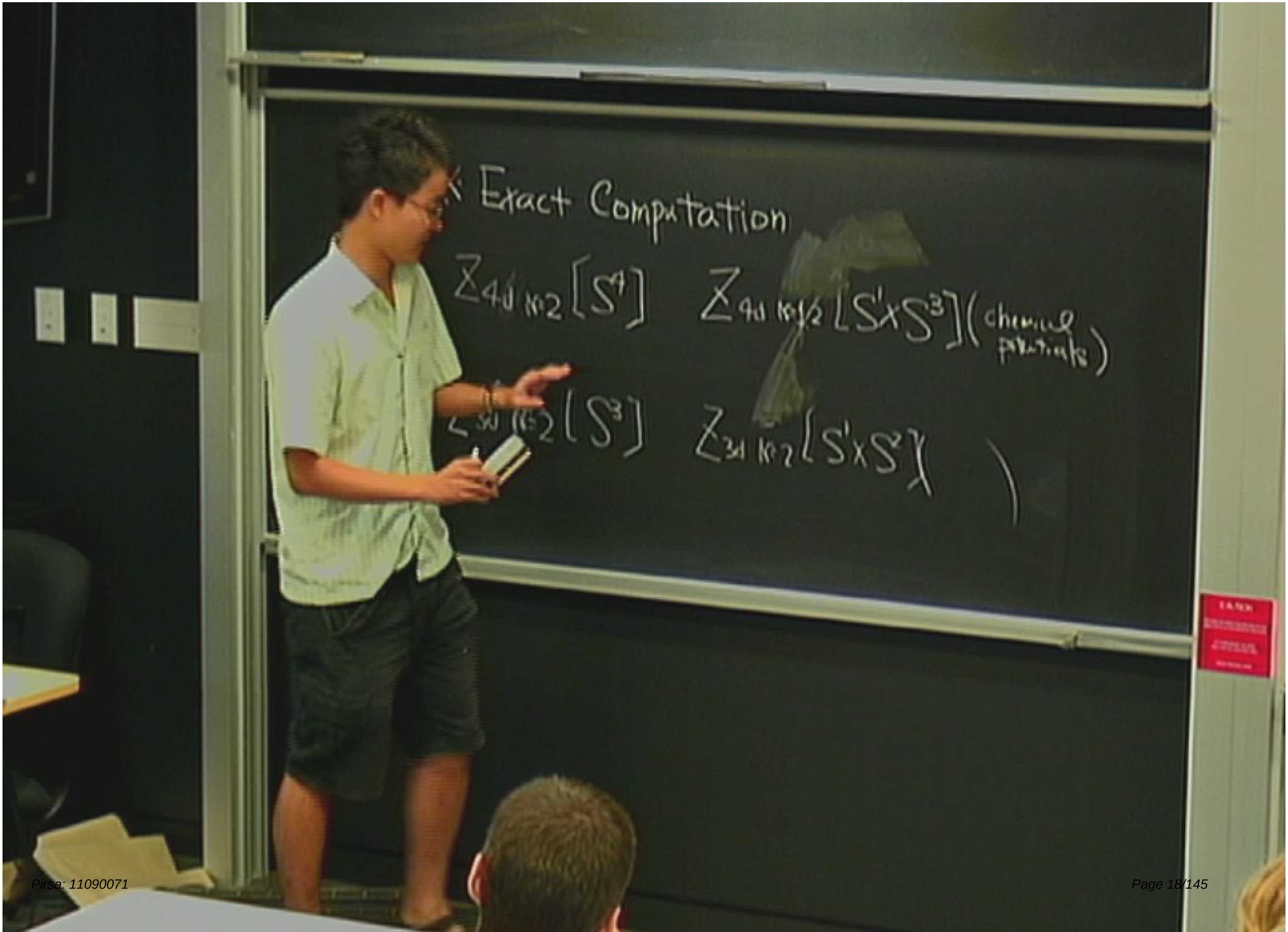
$$Z_{4d\ N=2}[S^4] \qquad \overset{\text{index}}{Z_{4d\ N=1,2}[S^1\times S^3]} \ (\text{chemical potentials})$$

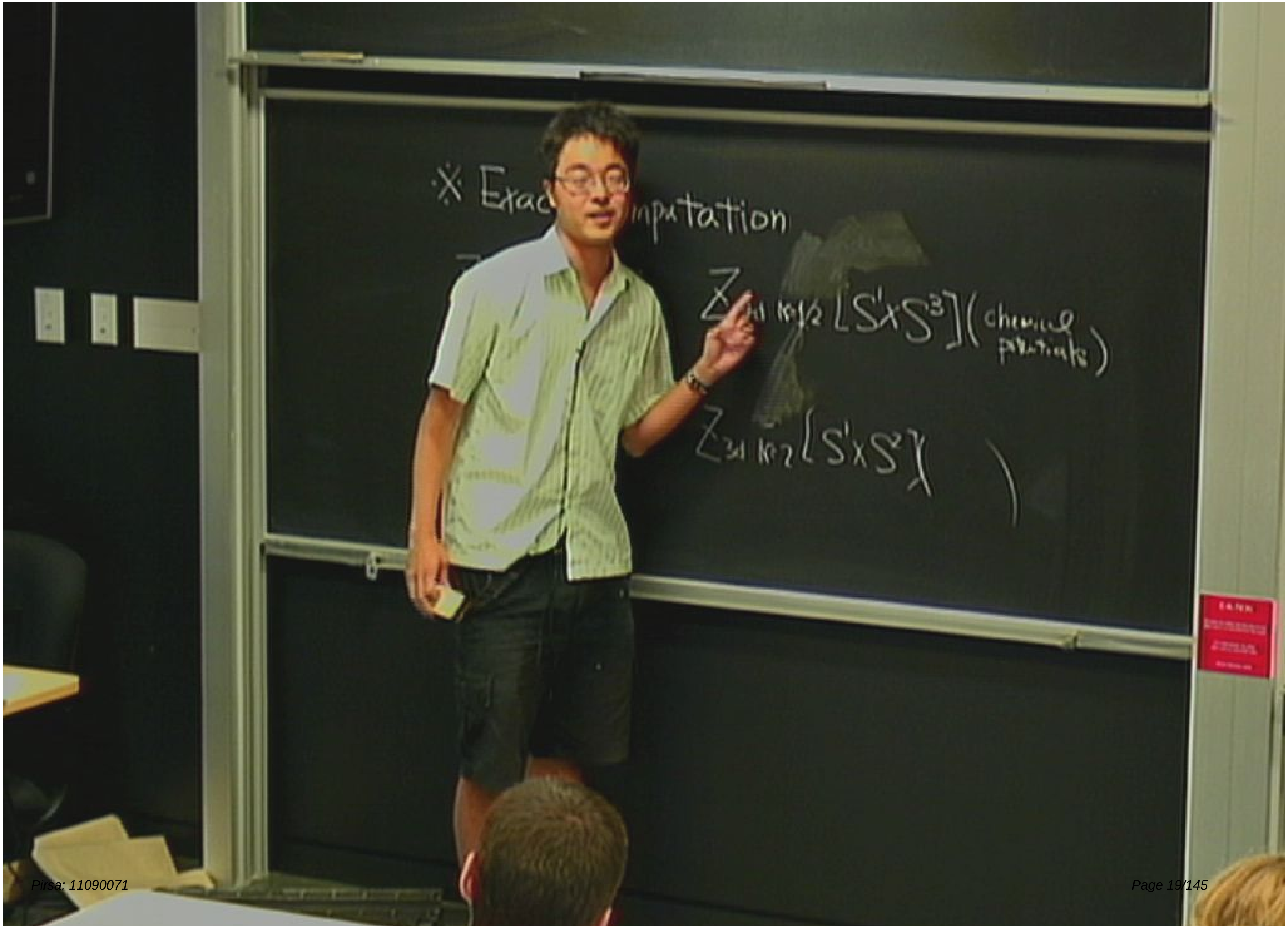
$$Z_{3d\ N=2}[S^3] \qquad Z_{3d\ N=2}[S^1\times S^2]$$

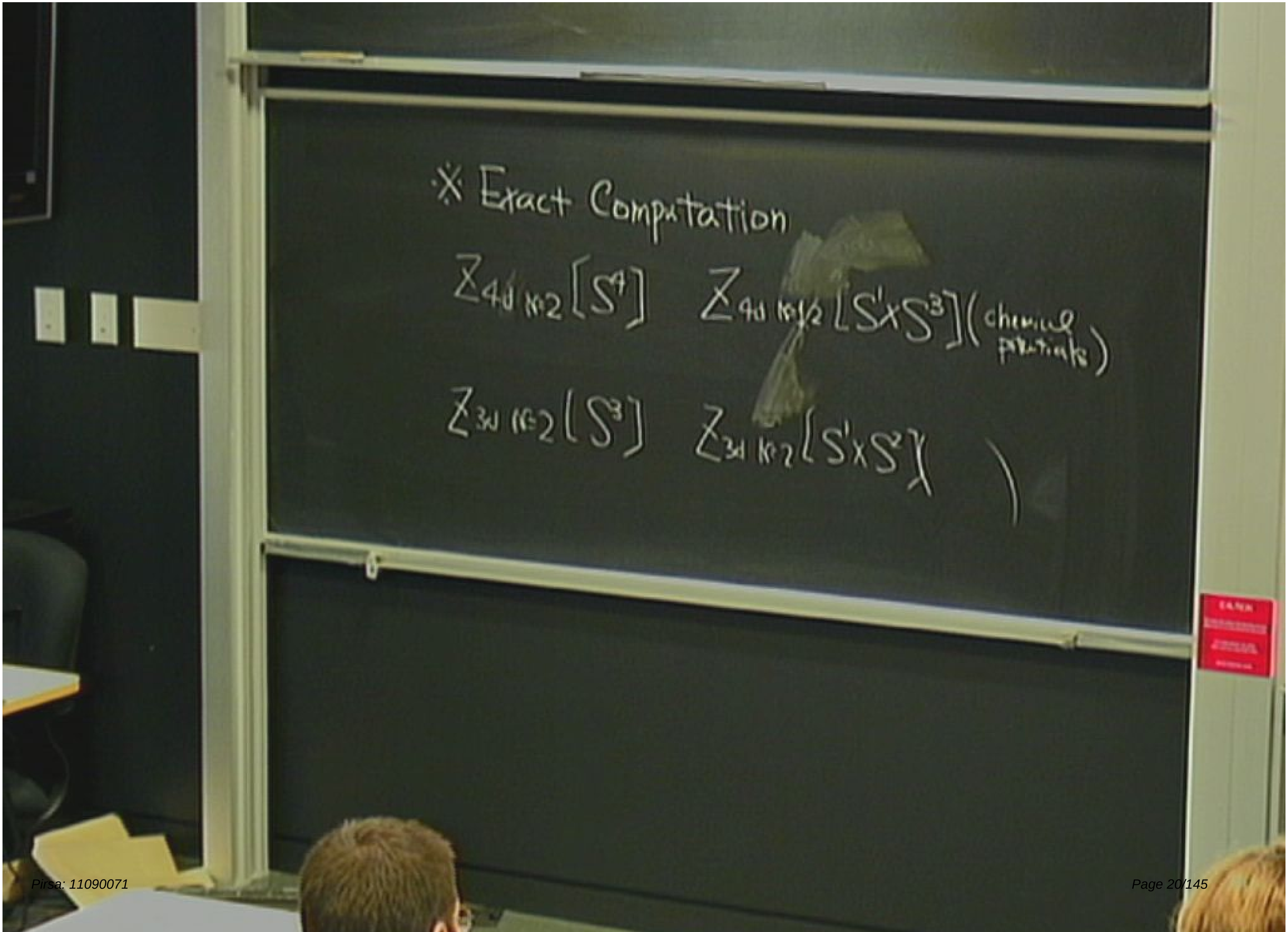

Exact Computation
index
4d N=2 [S^4]
Z 4d N=1,2 [S^1xS^3]
(chemical potentials)
Z 3d N=2 [S^1xS^2]

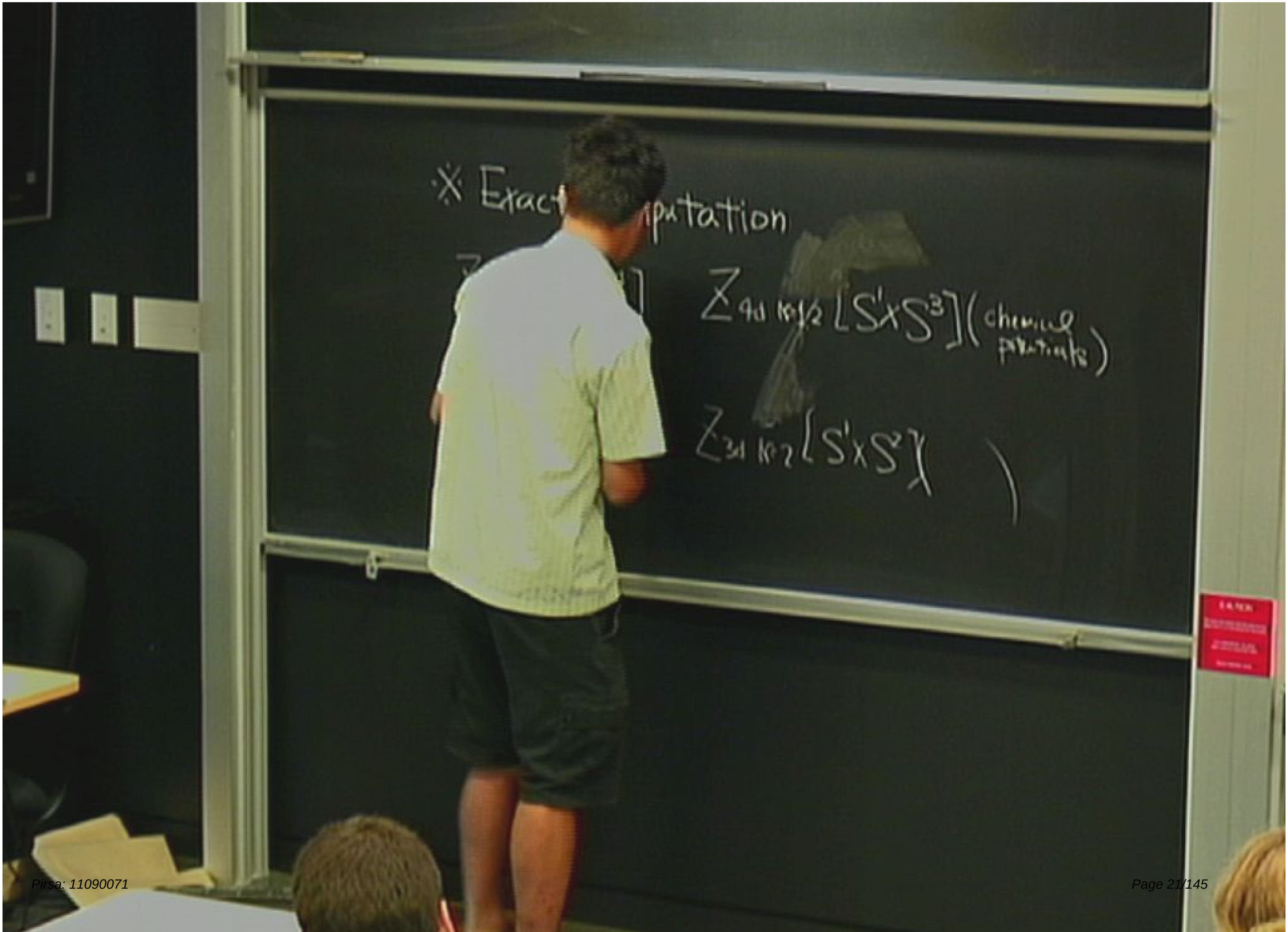

Computation
index
chemical potentials


Exact
ation
(chemical potentials)

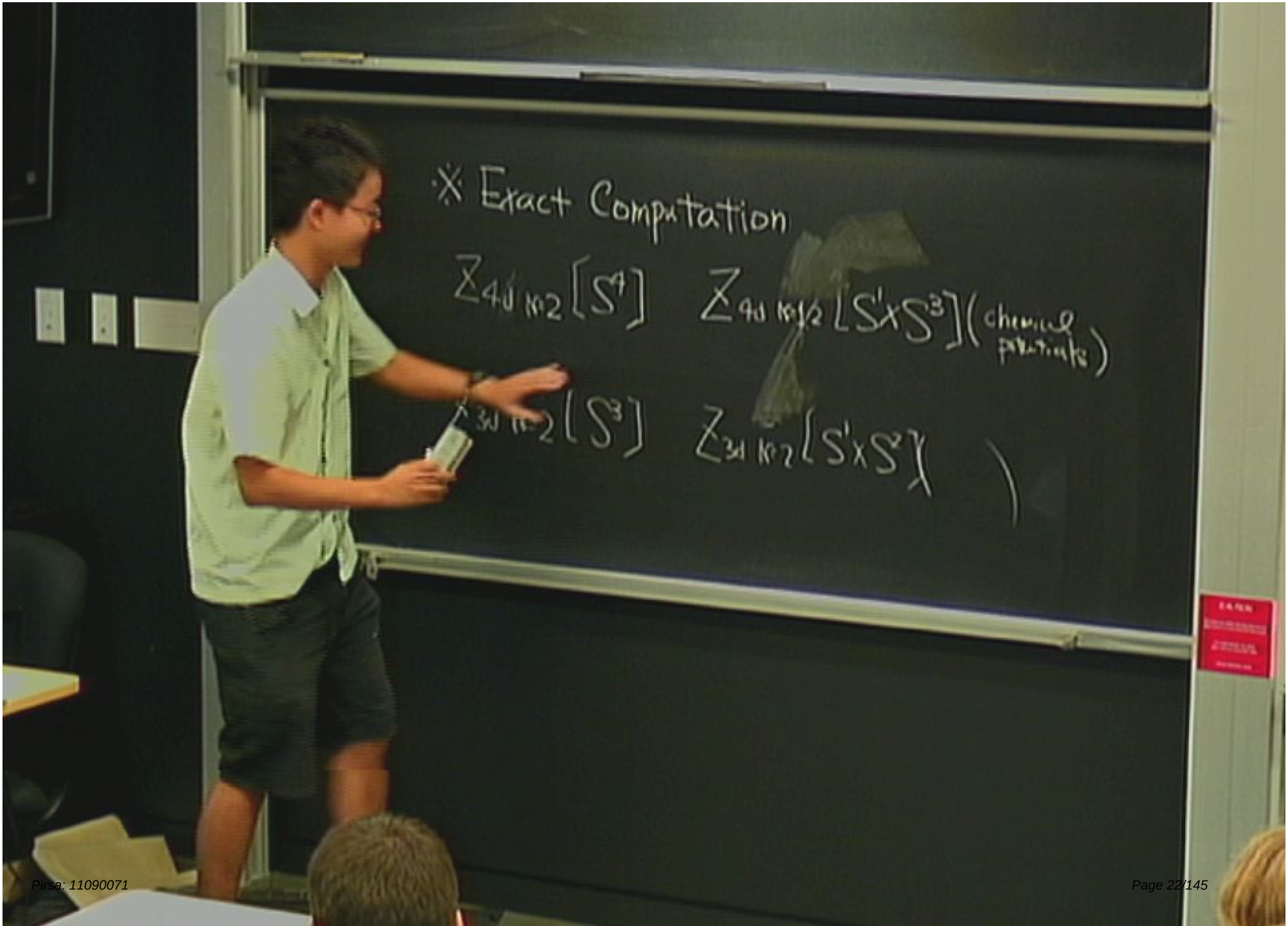

Exact Computation
$Z_{4d\ N=2}[S^4]$
$Z_{4d\ N=2}[S^1 \times S^3]$ (chemical potentials)
$Z_{3d\ N=2}[S^3]$
$Z_{3d\ N=2}[S^1 \times S^2]$

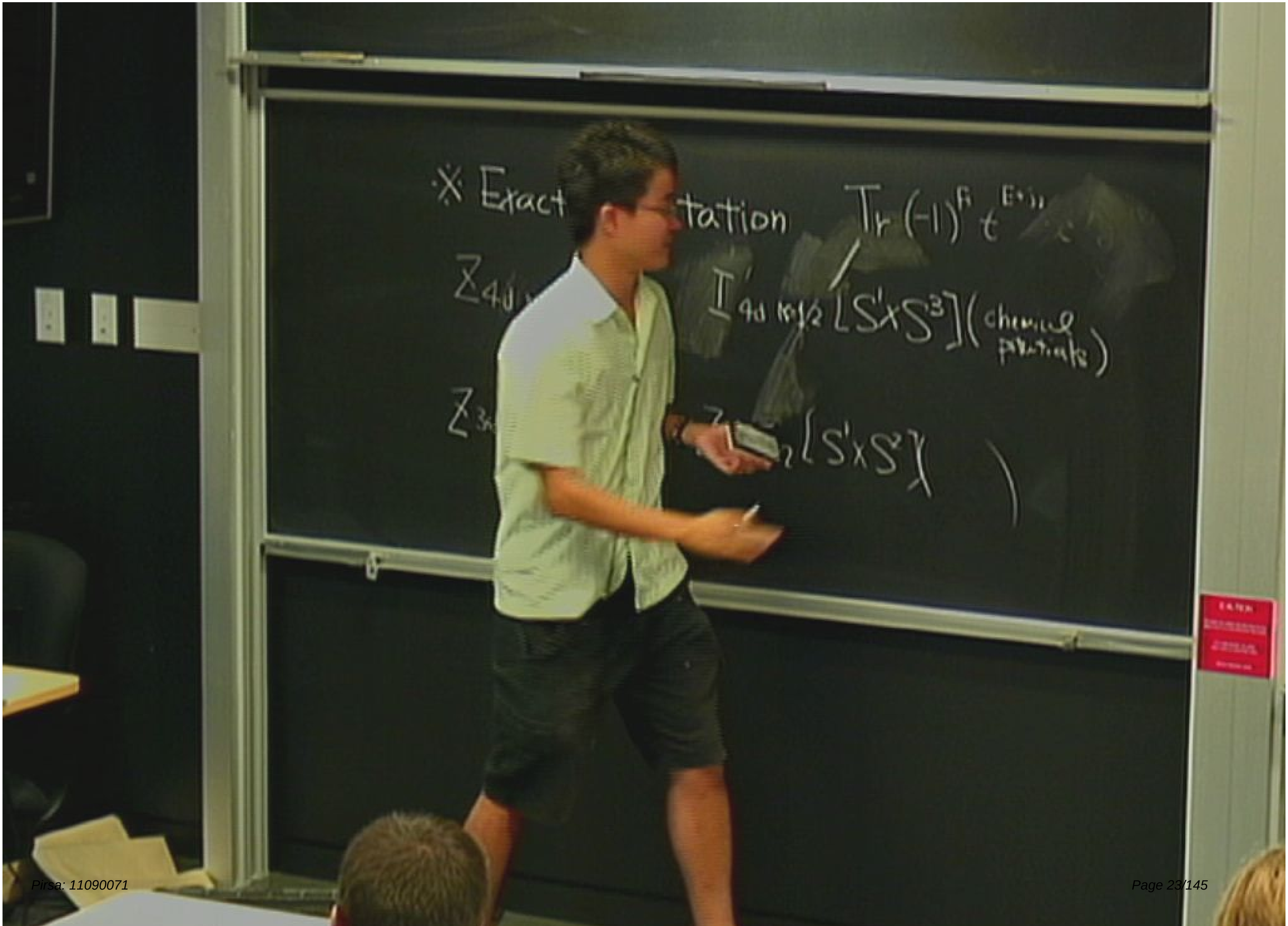

※ Exac mputation
(chemical potentials)

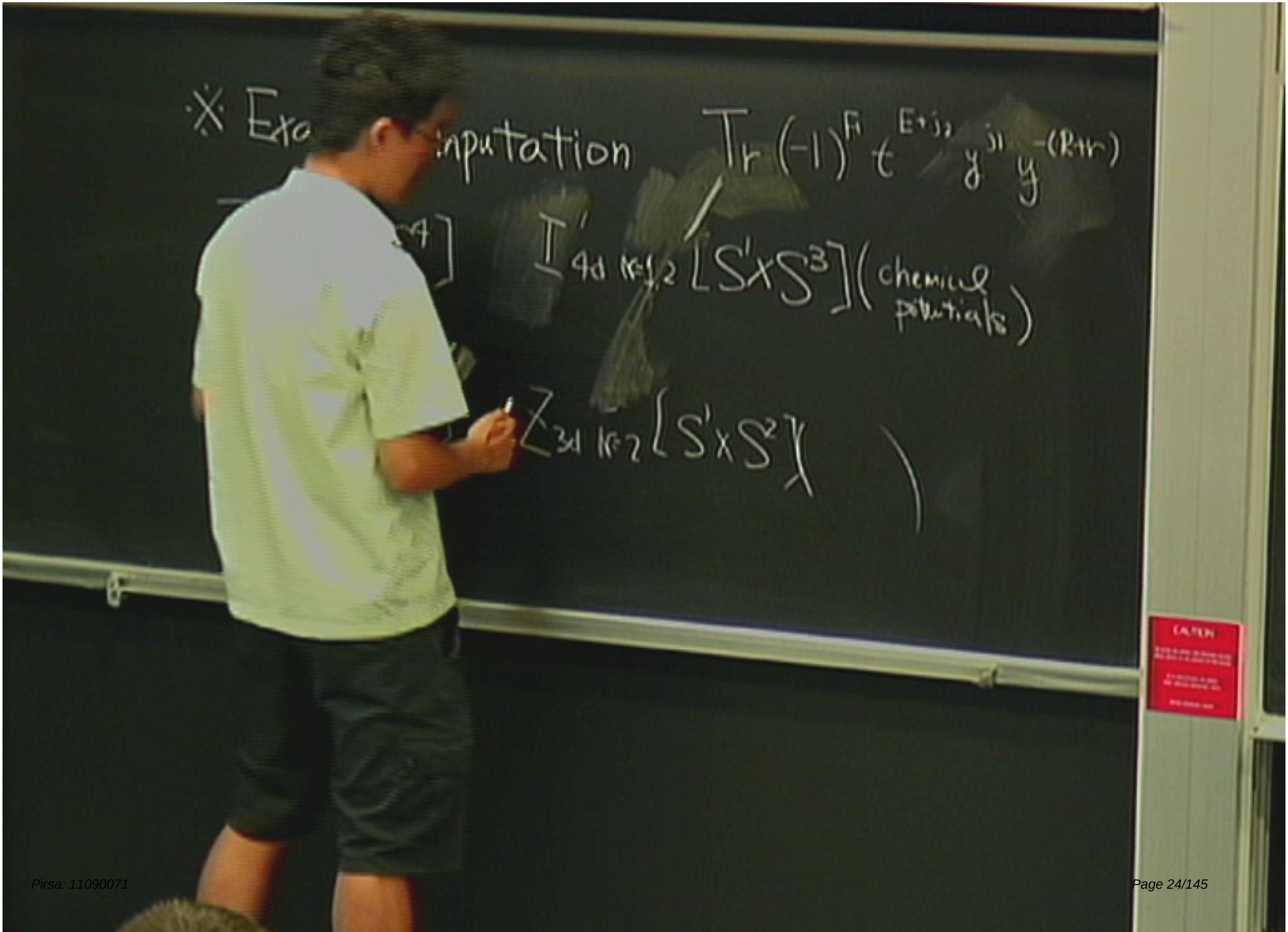

※ Exact Computation
$Z_{4d\ N=2}[S^4]$
$Z_{4d\ N=2}[S^1 \times S^3]$ (chemical potentials)
$Z_{3d\ N=2}[S^3]$
$Z_{3d\ N=2}[S^1 \times S^2]$



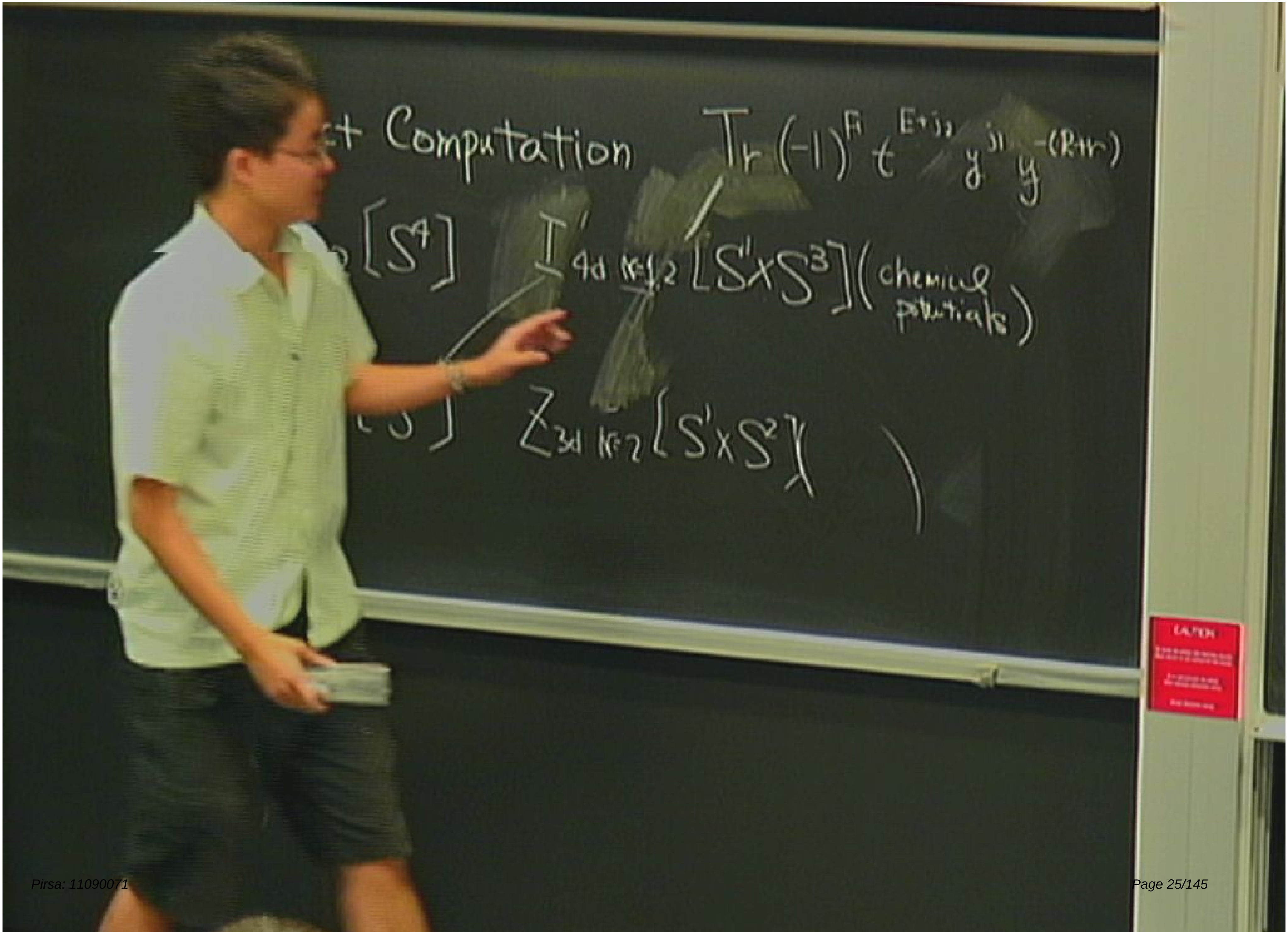
※ Exact ... putation
Z 4d N=2 [S'xS^3] (chemical potentials)
Z 3d N=2 [S'xS^2]


※ Exact Computation
Z 4d N=2 [S4]
Z 4d N=2 [S1×S3] (chemical potentials)
Z 3d N=2 [S3]
Z 3d N=2 [S1×S2]

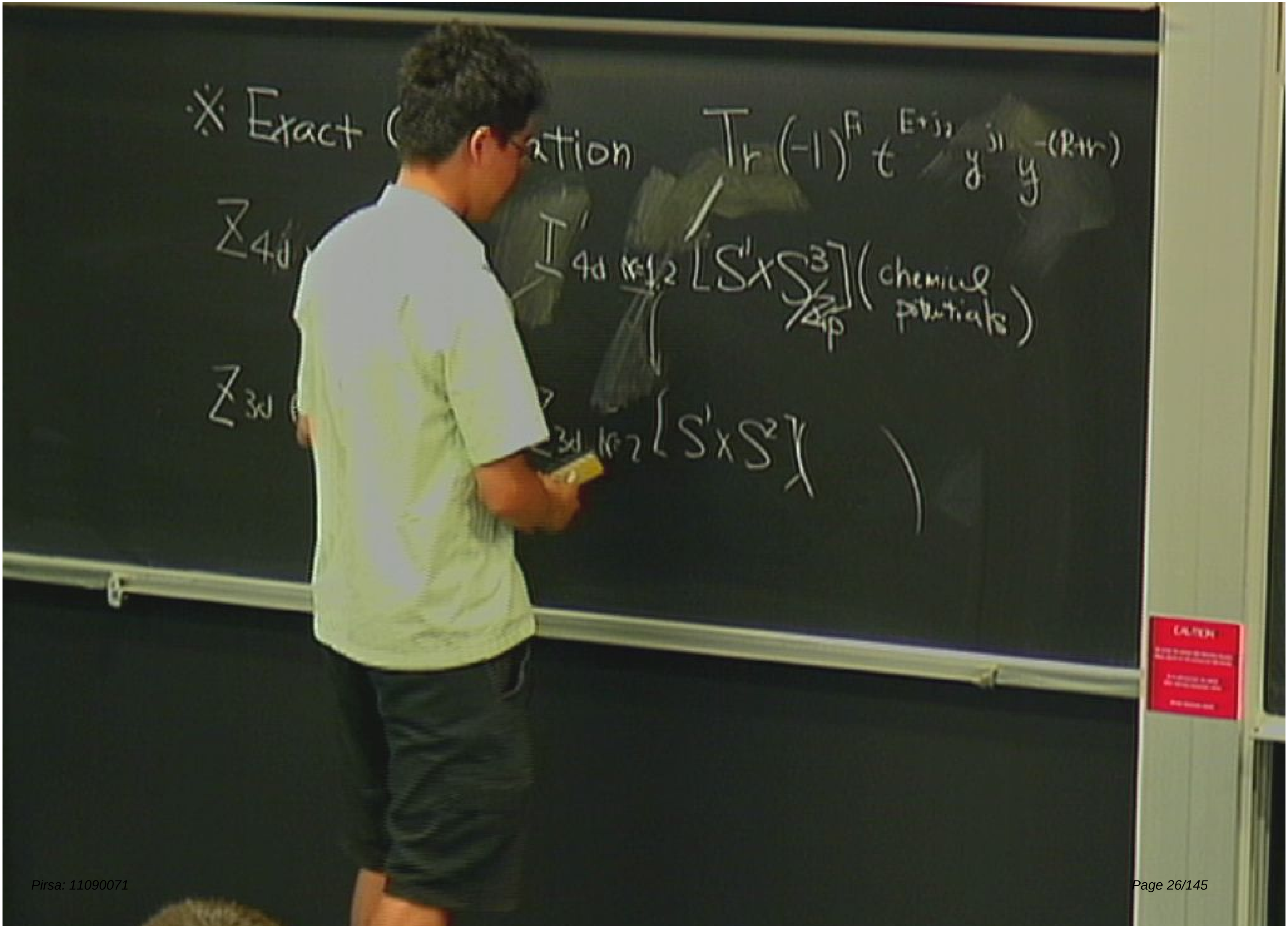

※ Exact
tation
Tr
[S¹×S³]
(chemical potentials)
[S¹×S²]



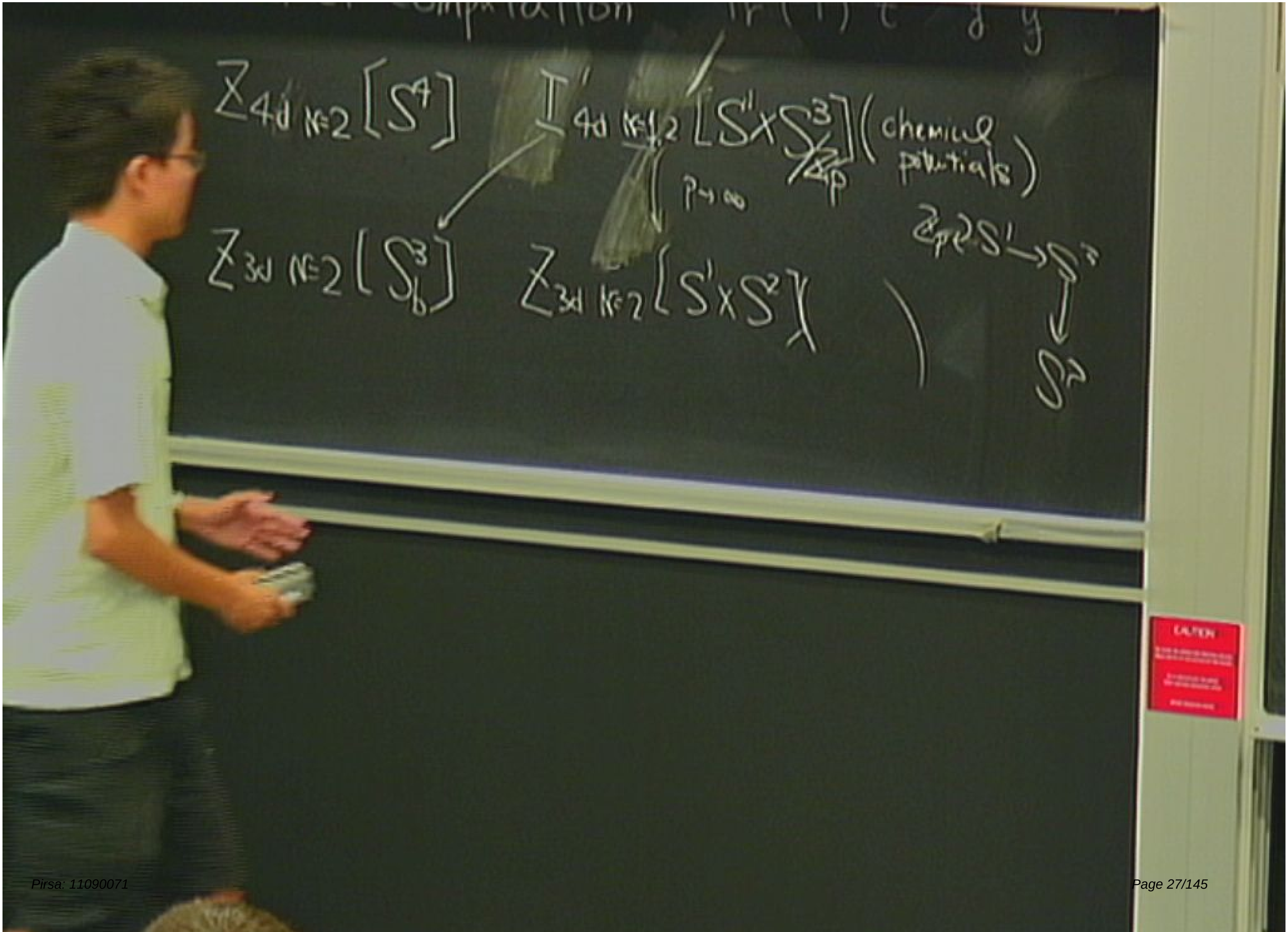
Exa mputation
Tr (-1)^F t^{E+j_2} y^{j_1} v^{-(R+r)}
I_{4d N=1,2} [S^1×S^3] (chemical potentials)
Z_{3d N=2} [S^1×S^2]



Computation
Tr (-1)^F t^{E+j2} y^{j1} v^{-(R+r)}
[S^4]
I 4d N=1,2 [S^1×S^3] (chemical potentials)
Z 3d N=2 [S^1×S^2]



Exact
ation
Tr
chemical
potentials



$Z_{4d\ N=2}[S^4]$
$I_{4d\ N=1,2}[S^1 \times S^3/\mathbb{Z}_p]$ (chemical potentials)
$p \to \infty$
$Z_{3d\ N=2}(S^3_b)$
$Z_{3d\ N=2}[S^1 \times S^2]$
$\mathbb{Z}_p \subset S^1 \to S^3$
S^2

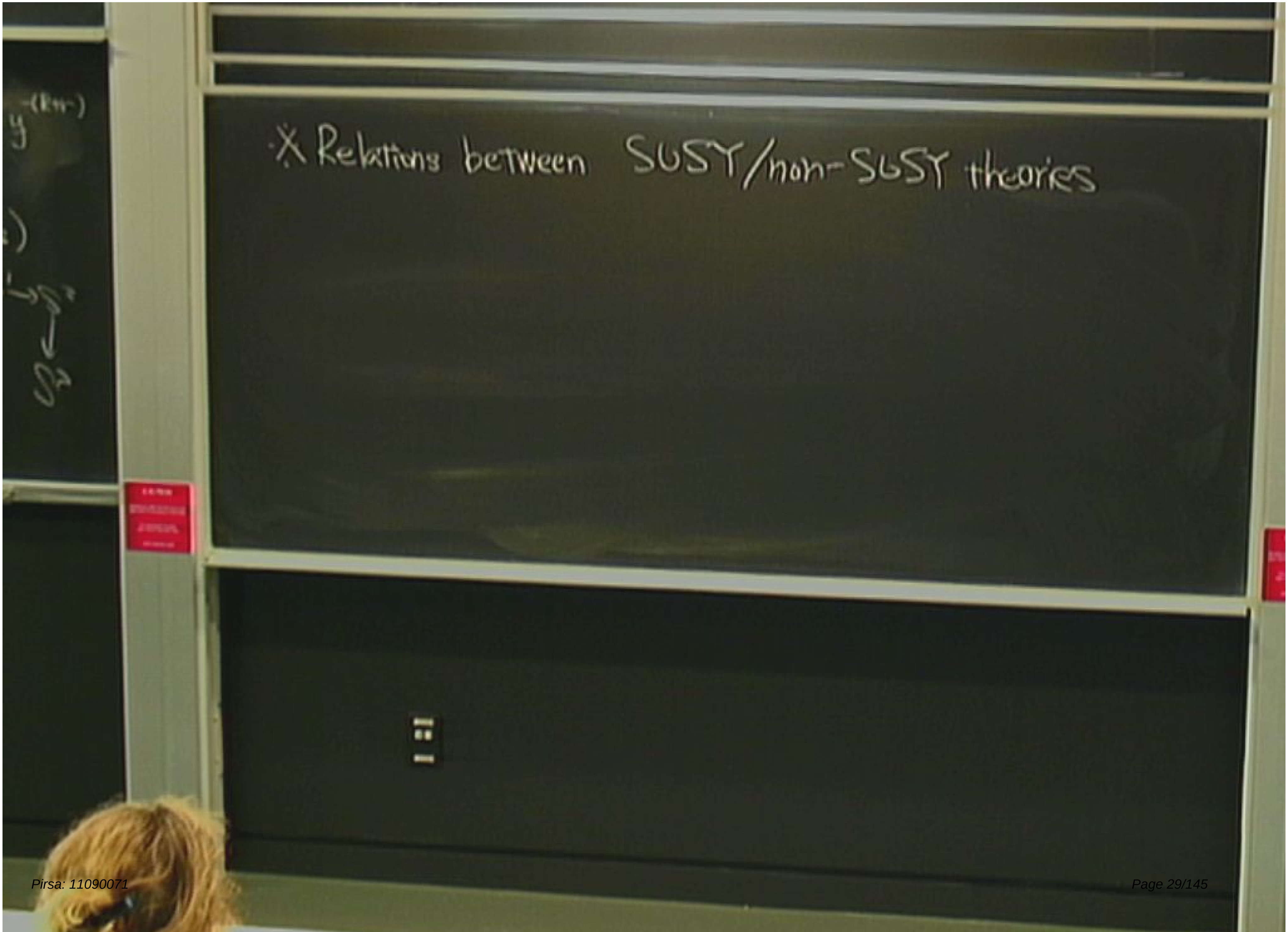
※ Exact Computation $\mathrm{Tr}(-1)^{F} t^{E+j_2} y^{j_1} y^{-(R+r)}$

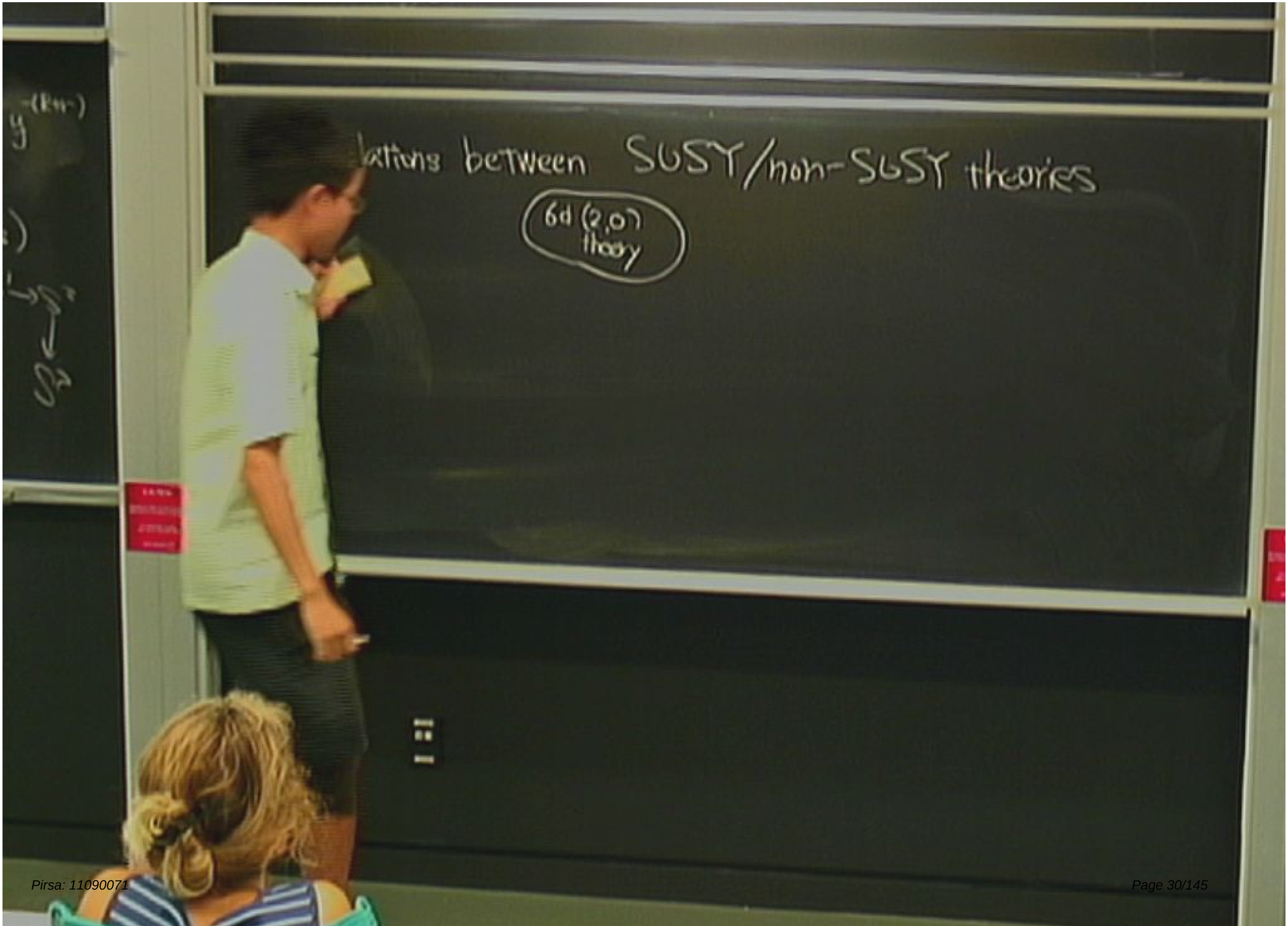
$Z_{4d\ N=2}[S^4]$ $\quad I_{4d\ N=2}[S^1 \times S^3/\mathbb{Z}_p]$ (chemical potentials)

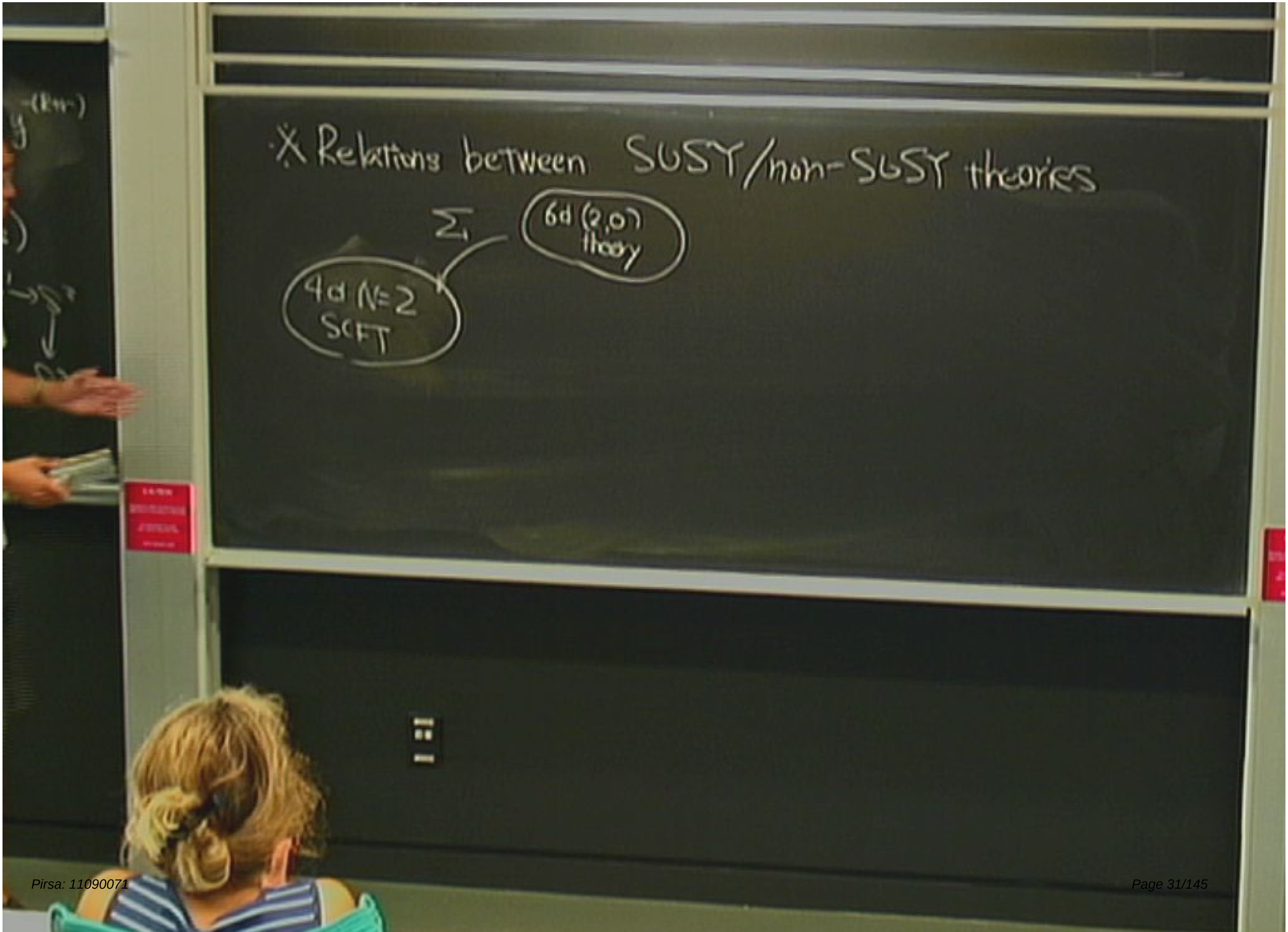
$p \to \infty$

$Z_{3d\ N=2}(S^3_b)$ $\quad Z_{3d\ N=2}[S^1 \times S^2]$ $($ $)$

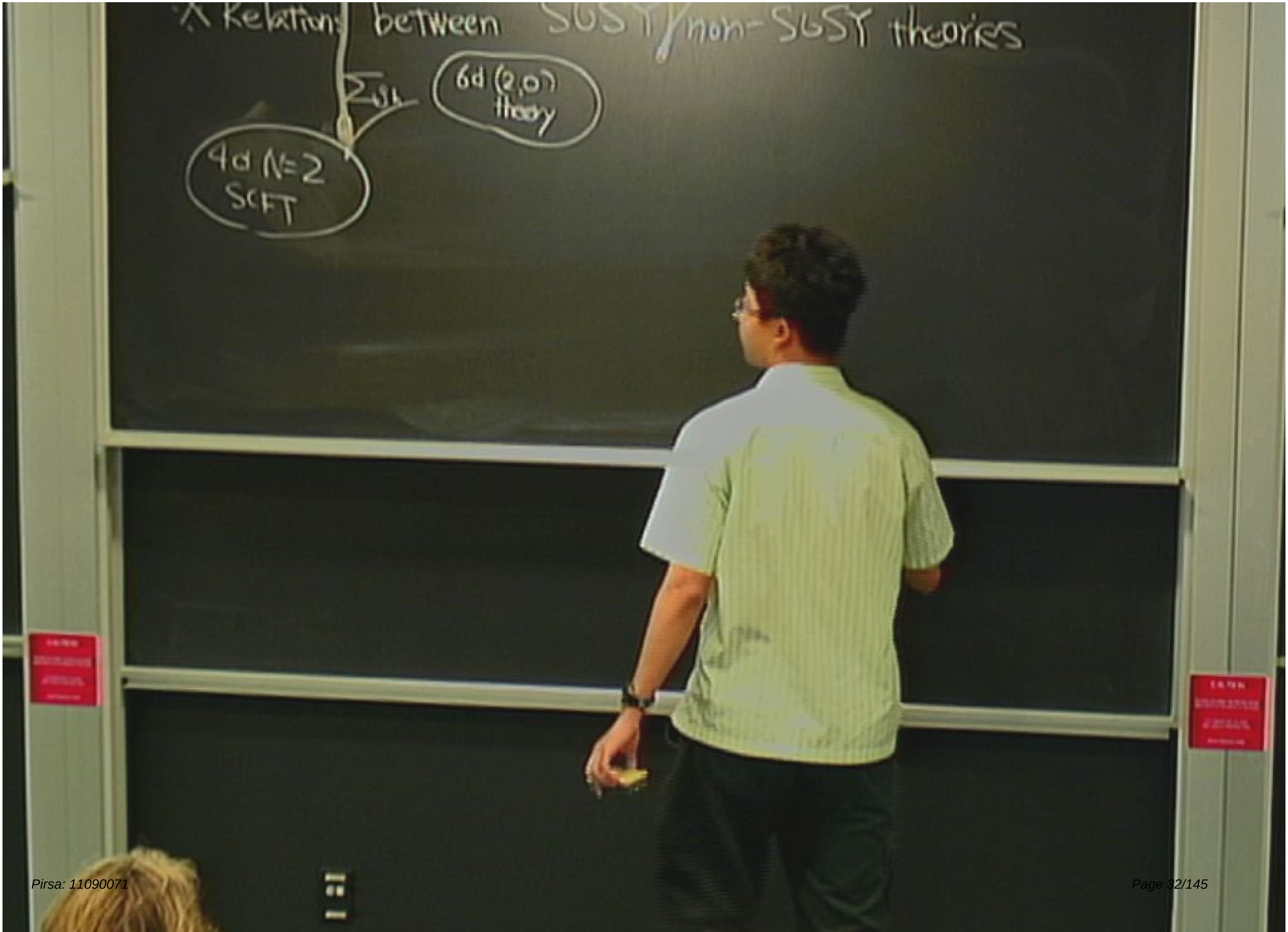
$\mathbb{Z}_p \curvearrowright S^1 \to S^3 \to S^2$

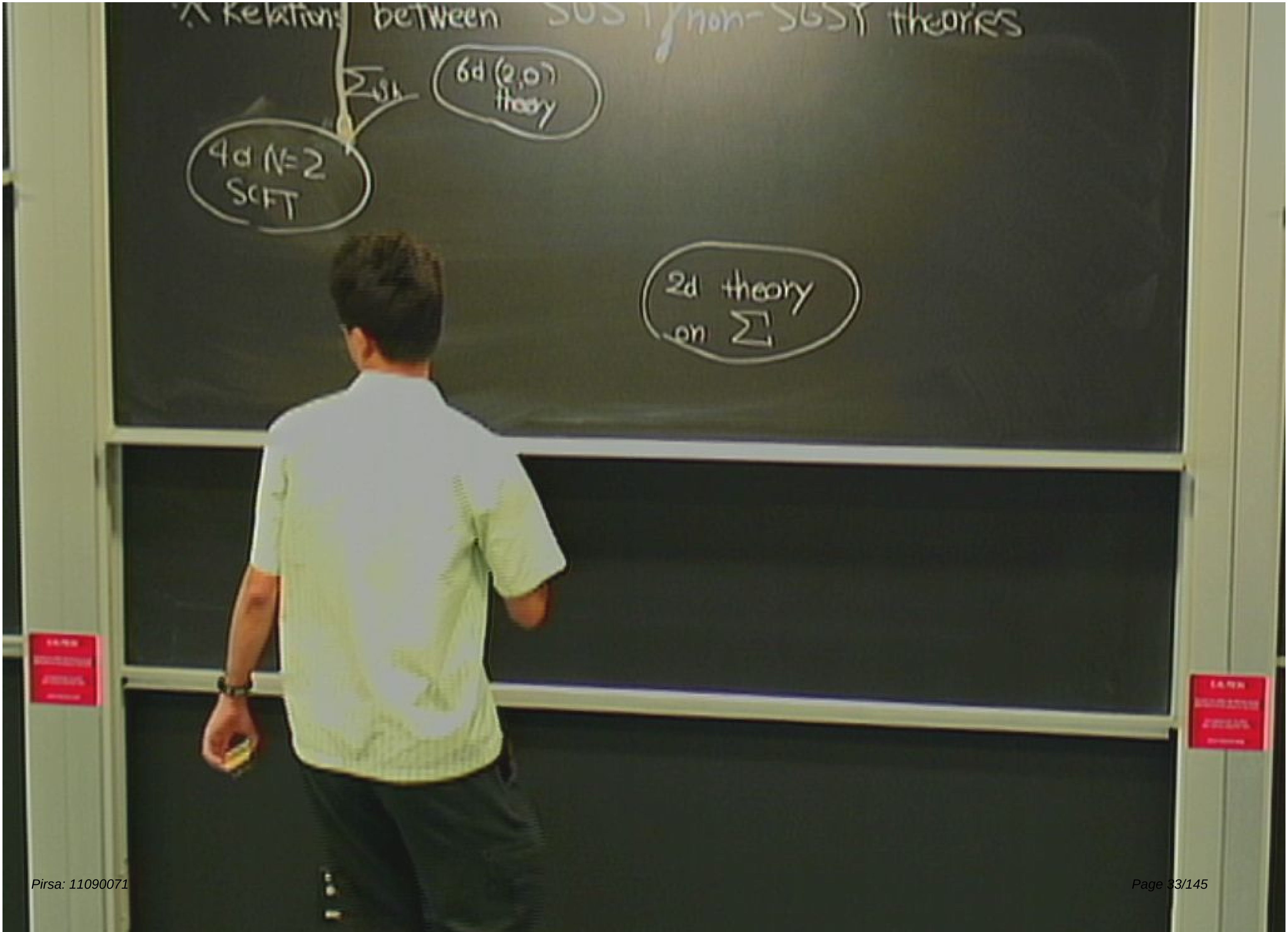

* Relations between SUSY/non-SUSY theories


lations between SUSY/non-SUSY theories
6d (2,0) theory



Relations between SUSY/non-SUSY theories
Σ
6d (2,0) theory
4d N=2 SCFT


* Relations between SUSY / non-SUSY theories
6d (2,0) theory
Σg,h
4d N=2 SCFT


Relations between SUSY/non-SUSY theories
6d (2,0) theory
4d N=2 SCFT
2d theory on Σ

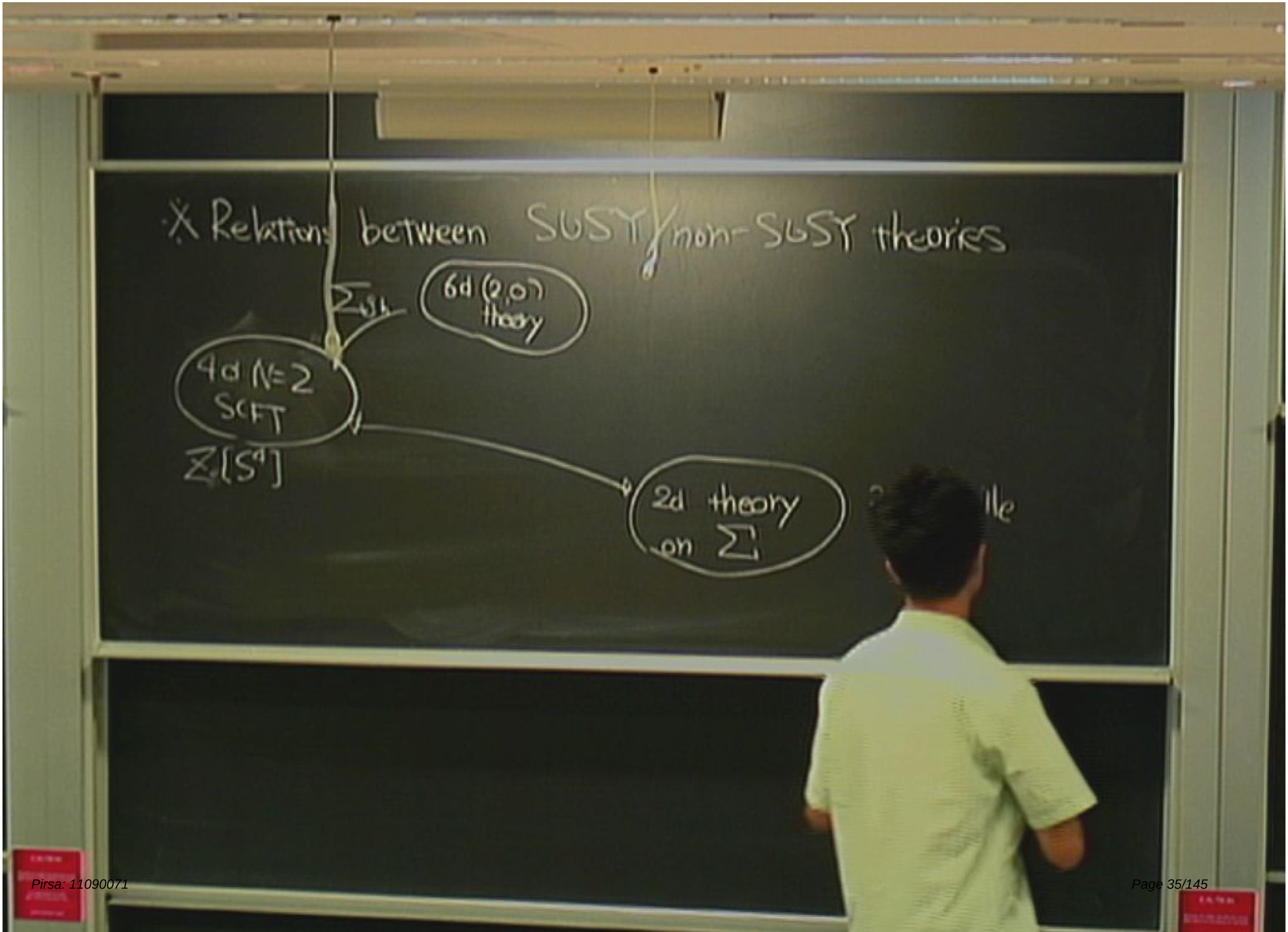
6d (2,0) theory

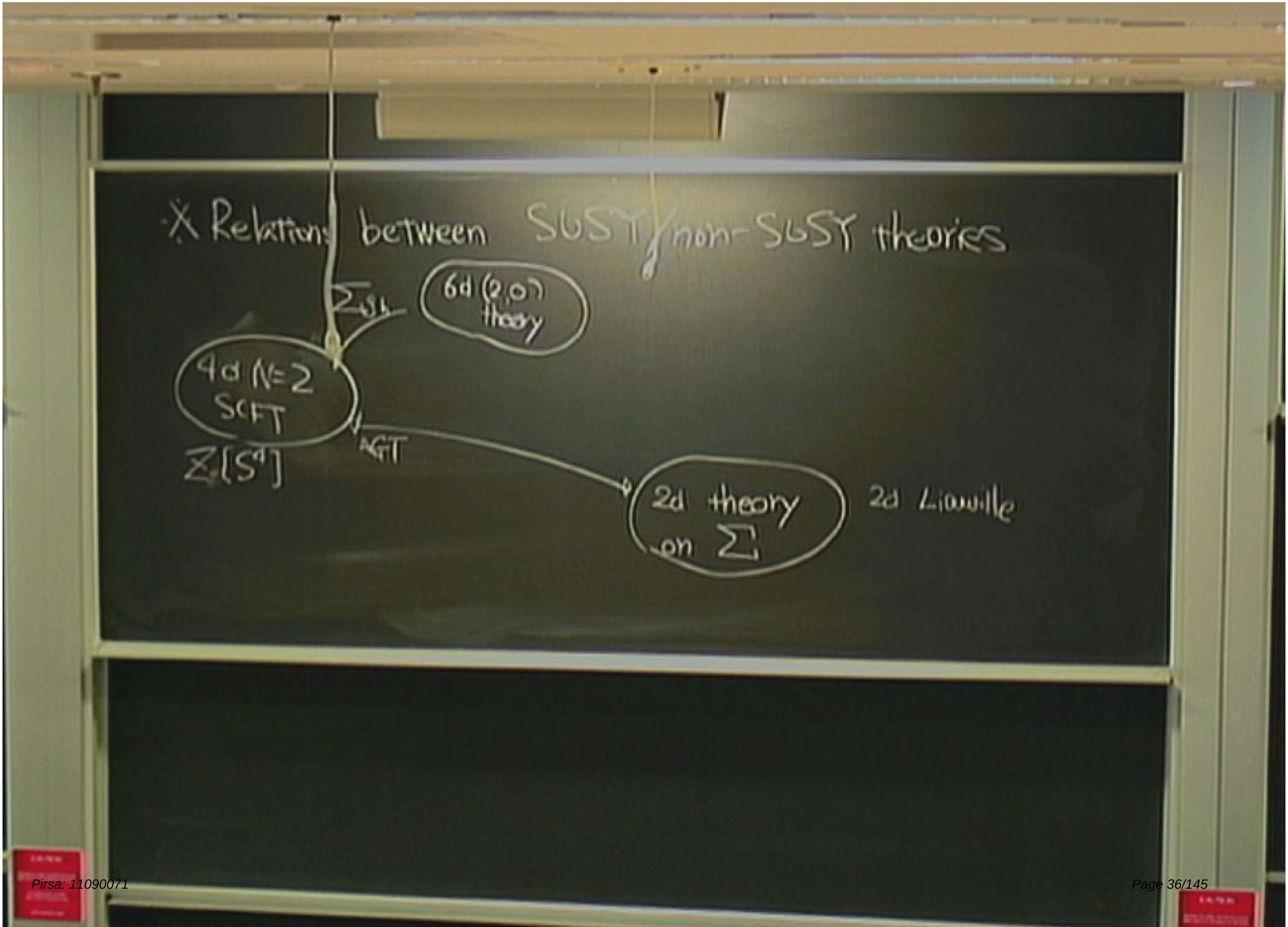
$\Sigma_{g,h}$

4d $\mathcal{N}=2$ SCFT

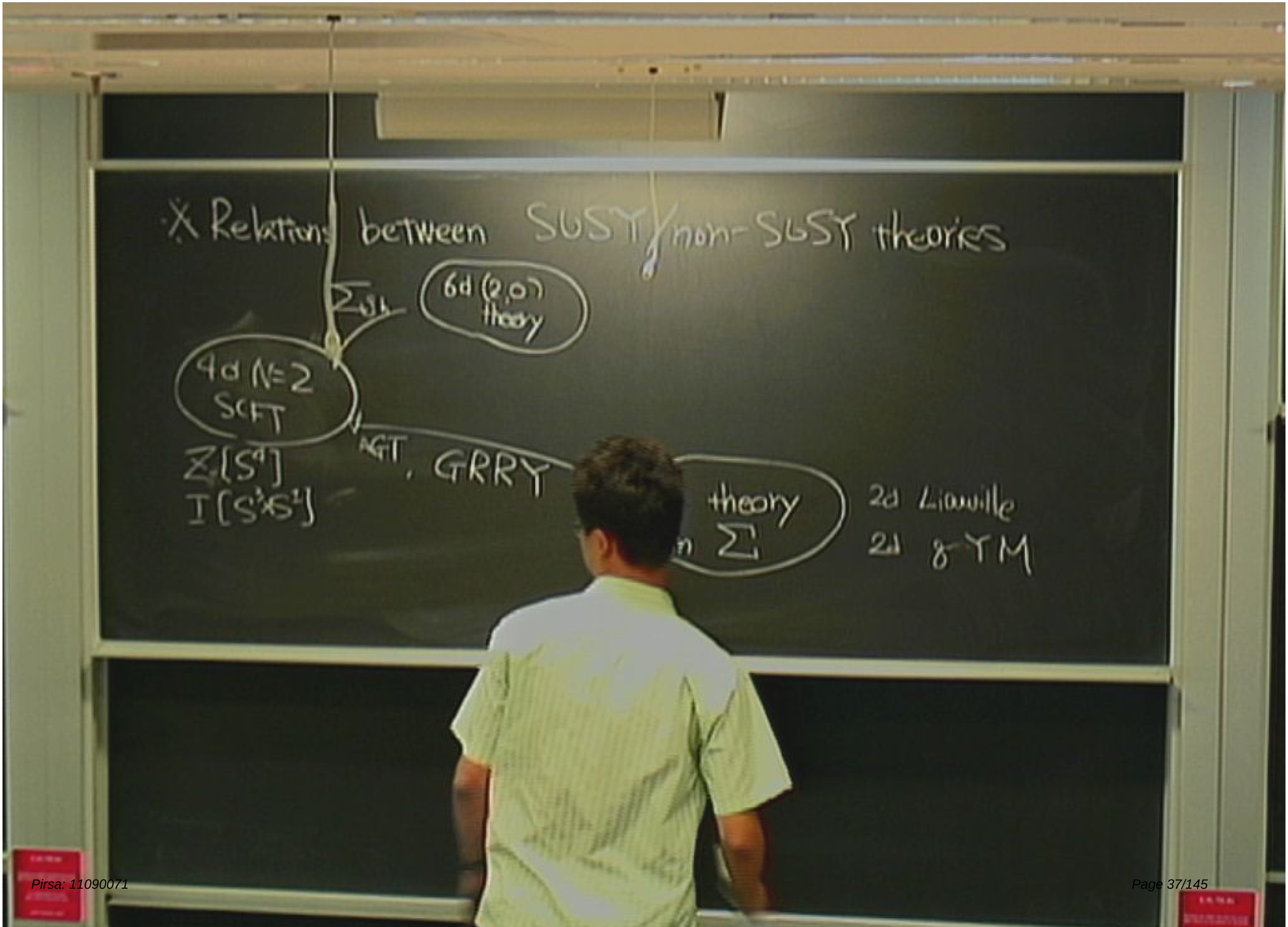
$Z[S^4]$

$y^{-(R+r)}$

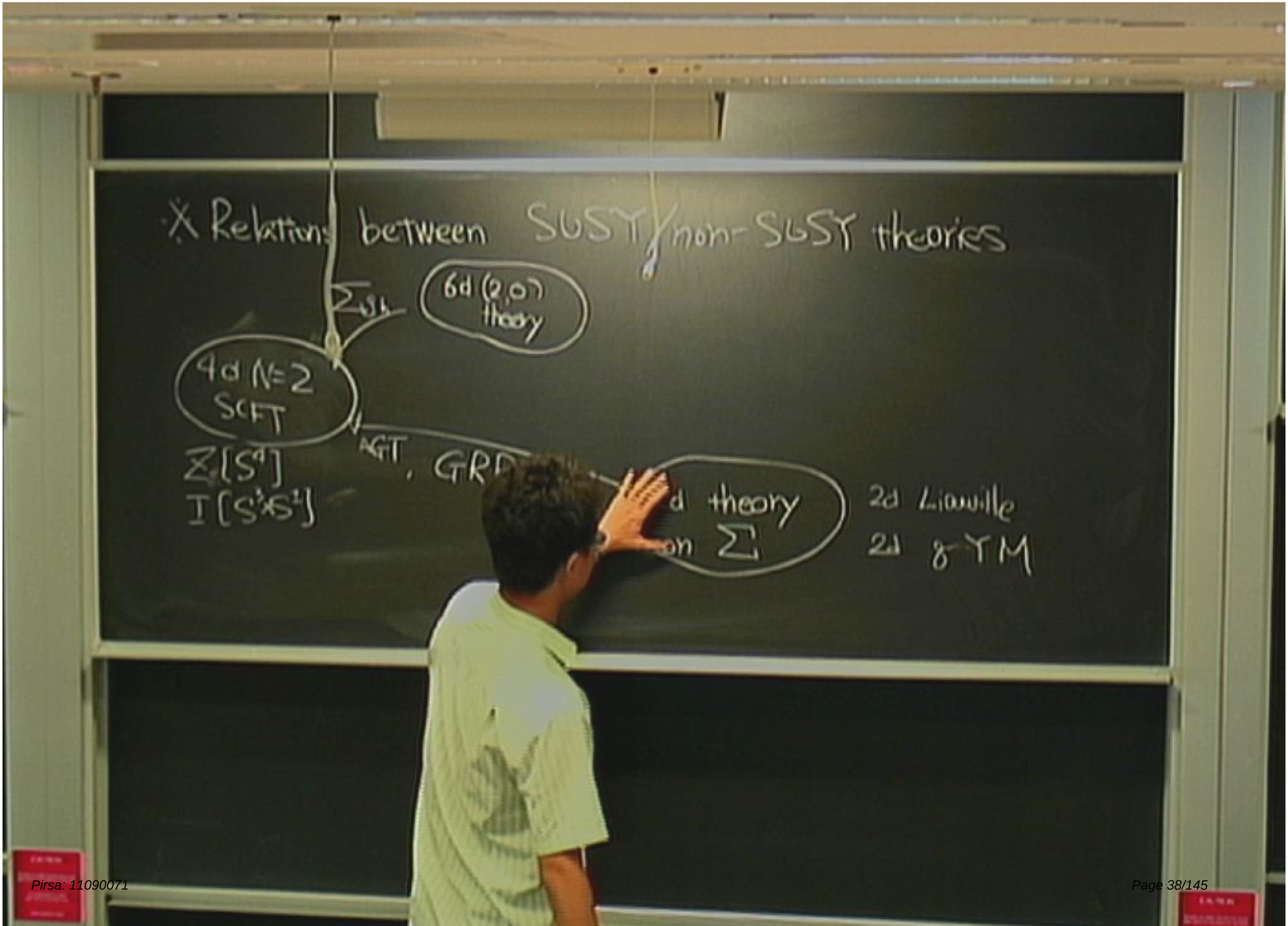

Relations between SUSY/non-SUSY theories
6d (2,0) theory
4d N=2 SCFT
Z[S^4]
2d theory on Σ

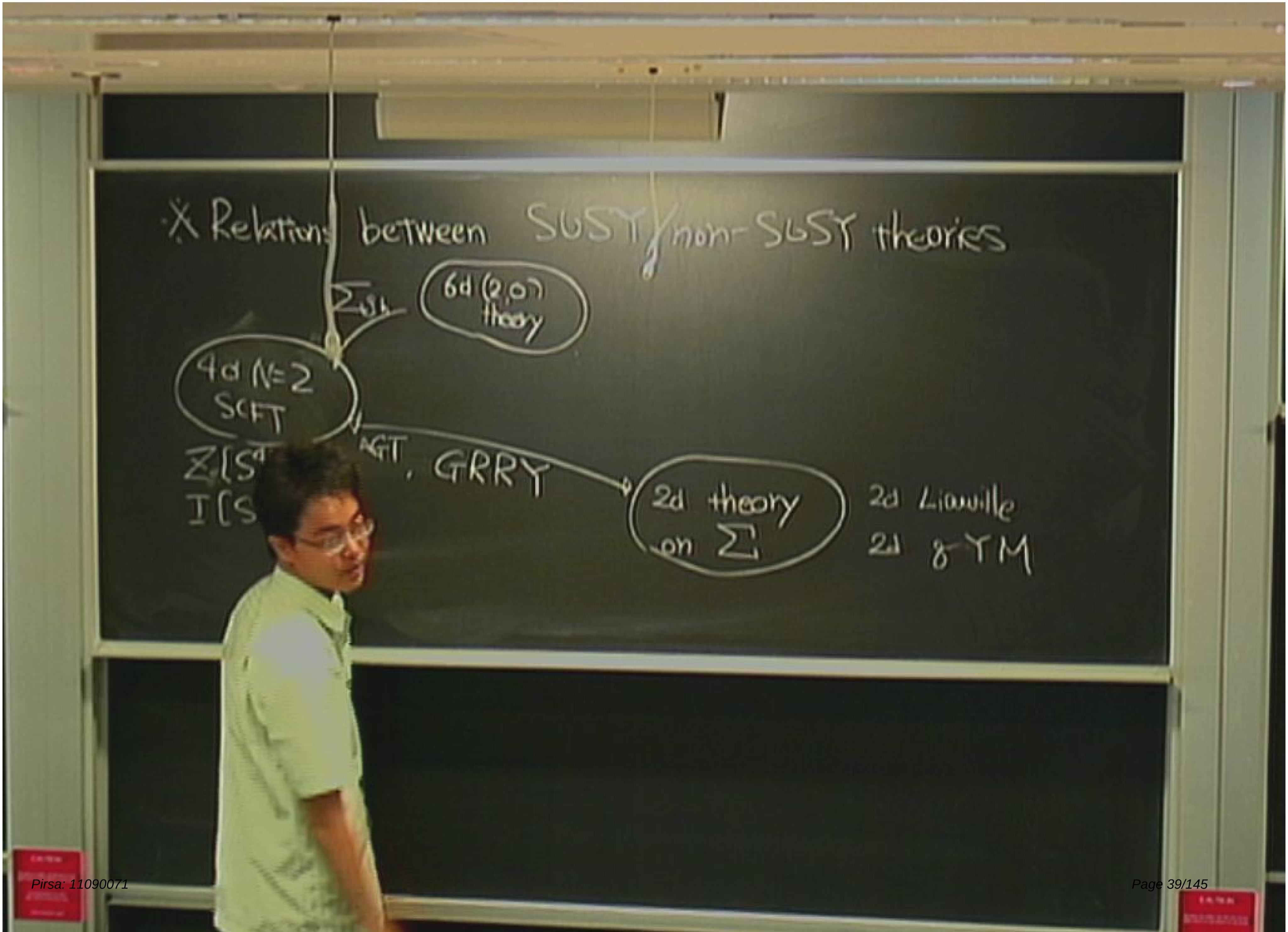


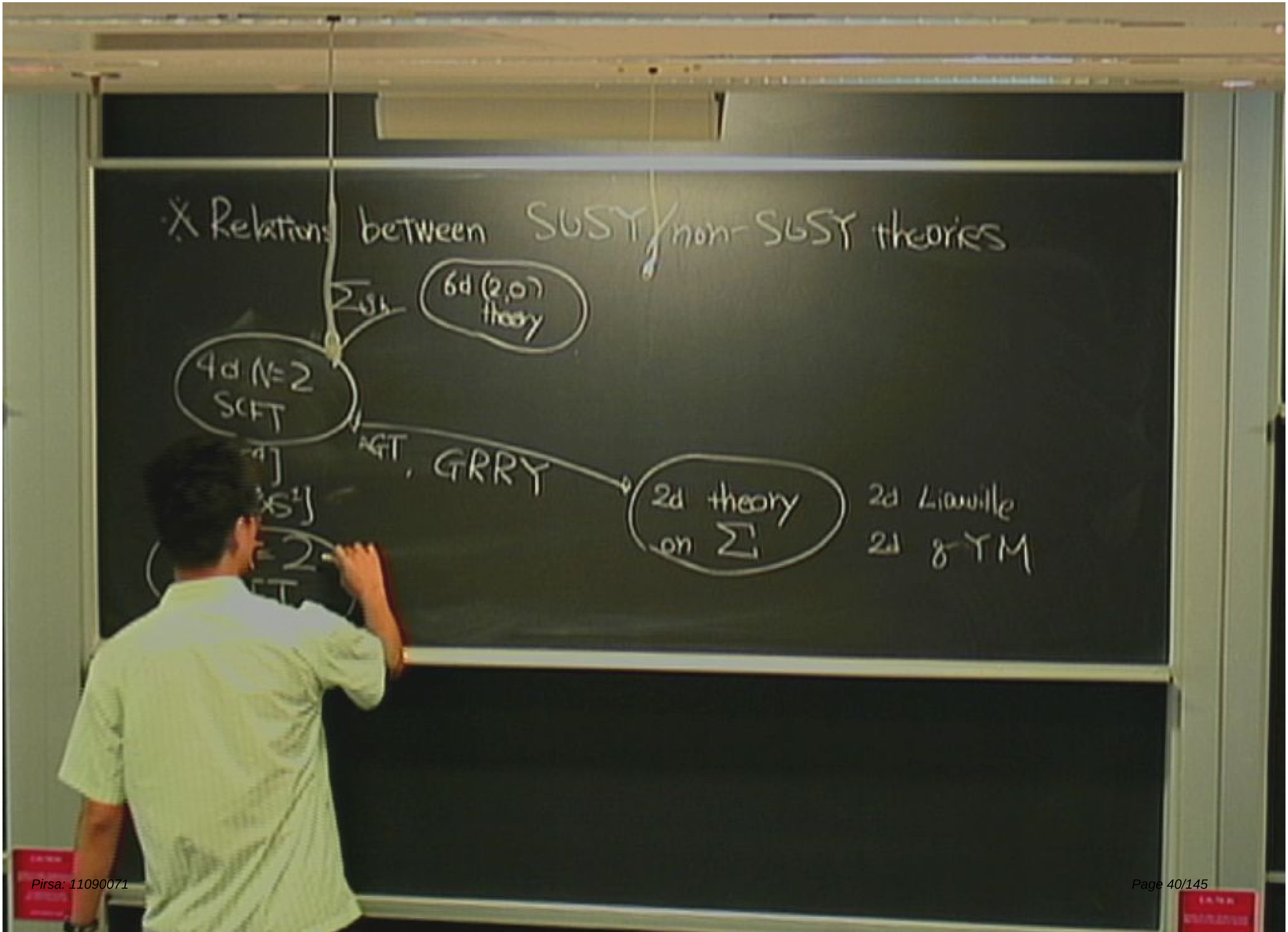
* Relations between SUSY/non-SUSY theories
6d (2,0) theory
Σ
4d N=2 SCFT
Z[S⁴]
AGT
2d theory on Σ
2d Liouville



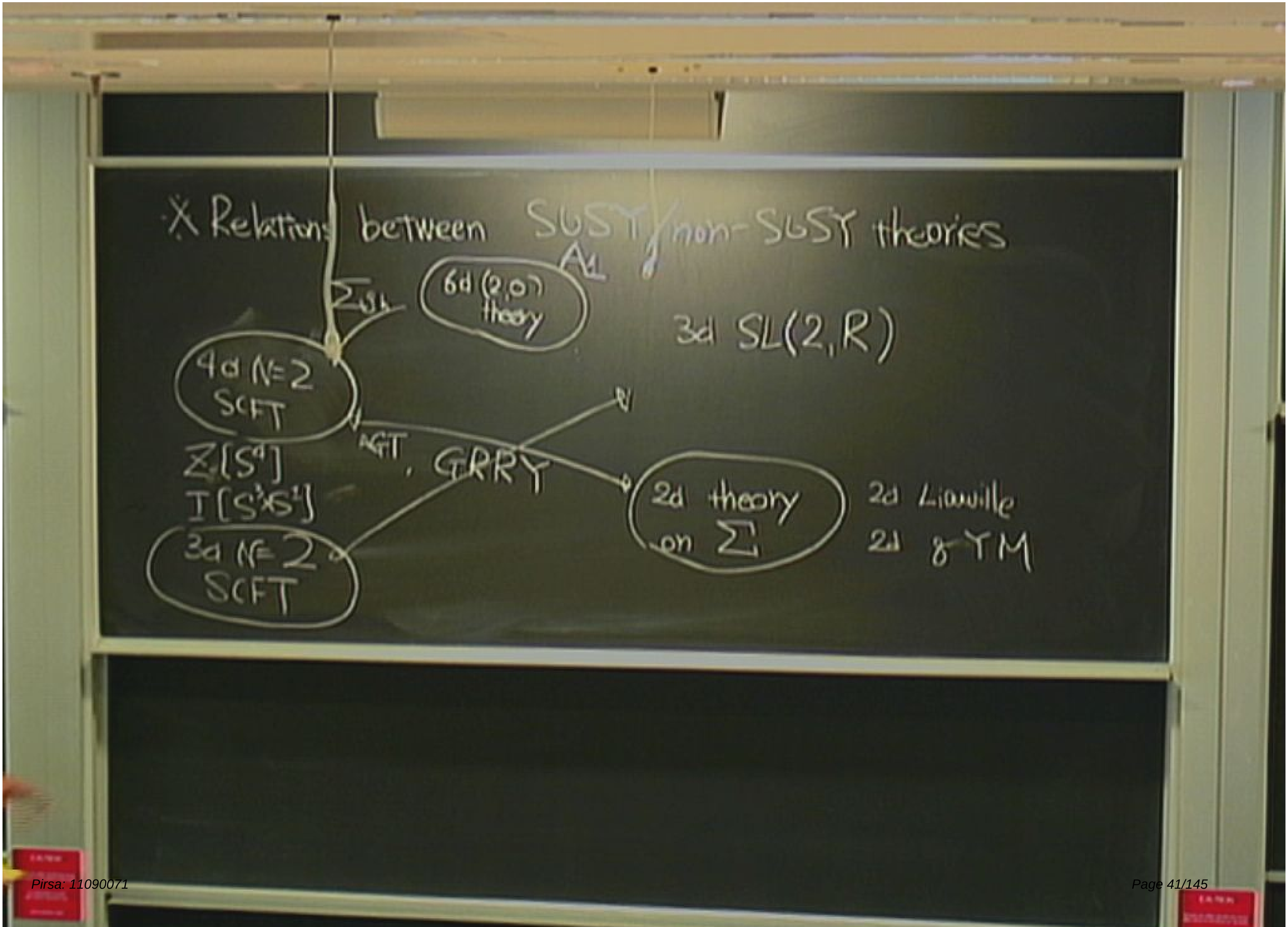
Relations between SUSY/non-SUSY theories
6d (2,0) theory
Σ
4d N=2 SCFT
Z[S^4]
I[S^3×S^1]
AGT, GRRY
theory
Σ
2d Liouville
2d q-YM


Relations between SUSY / non-SUSY theories
6d (2,0) theory
4d N=2 SCFT
Z[S^4]
I[S^1×S^3]
AGT, GR
theory
on Σ
2d Liouville
2d q-YM

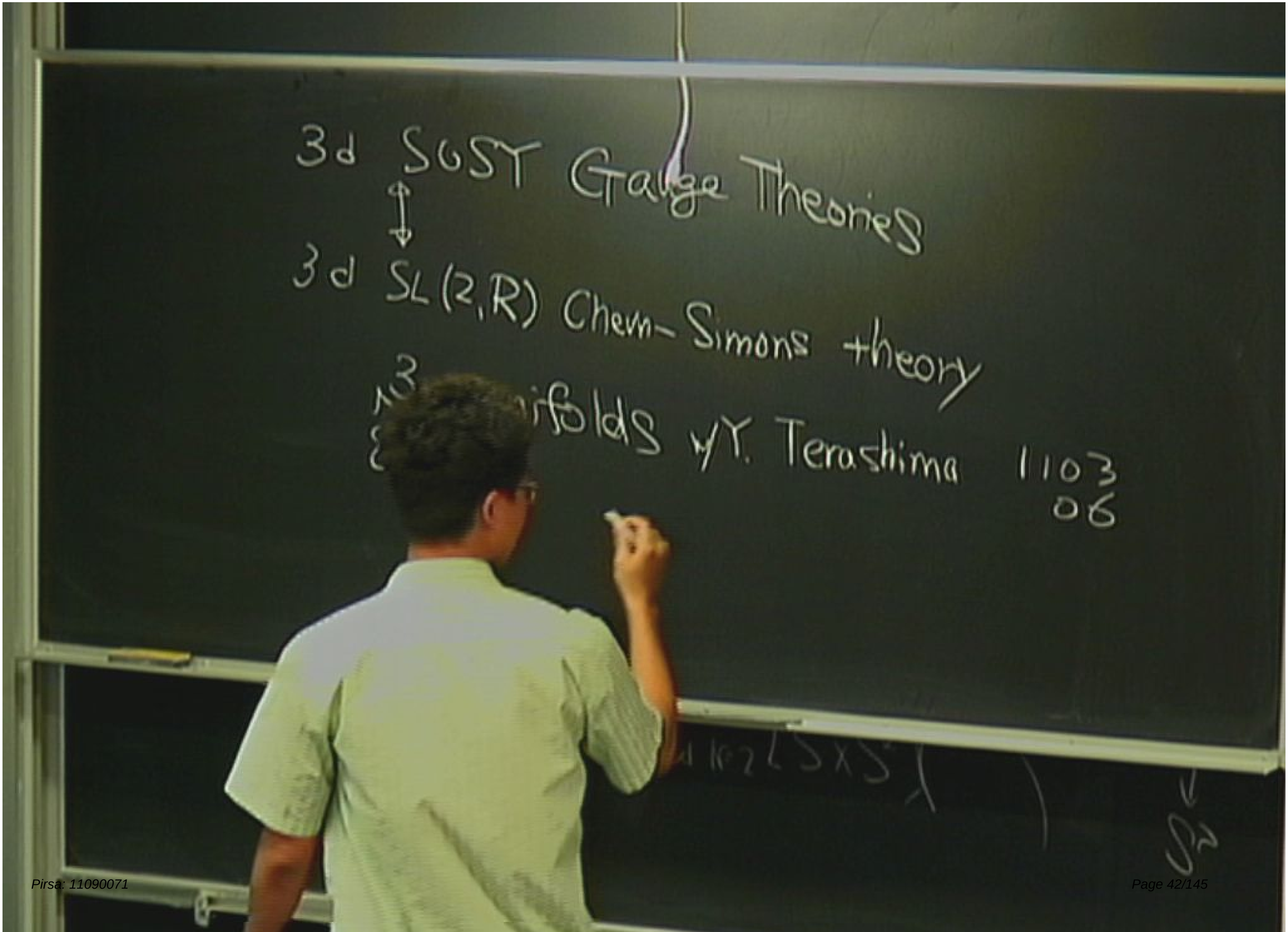

* Relations between SUSY / non-SUSY theories
6d (2,0) theory
4d N=2 SCFT
AGT, GRRY
2d theory on Σ
2d Liouville
2d q-YM

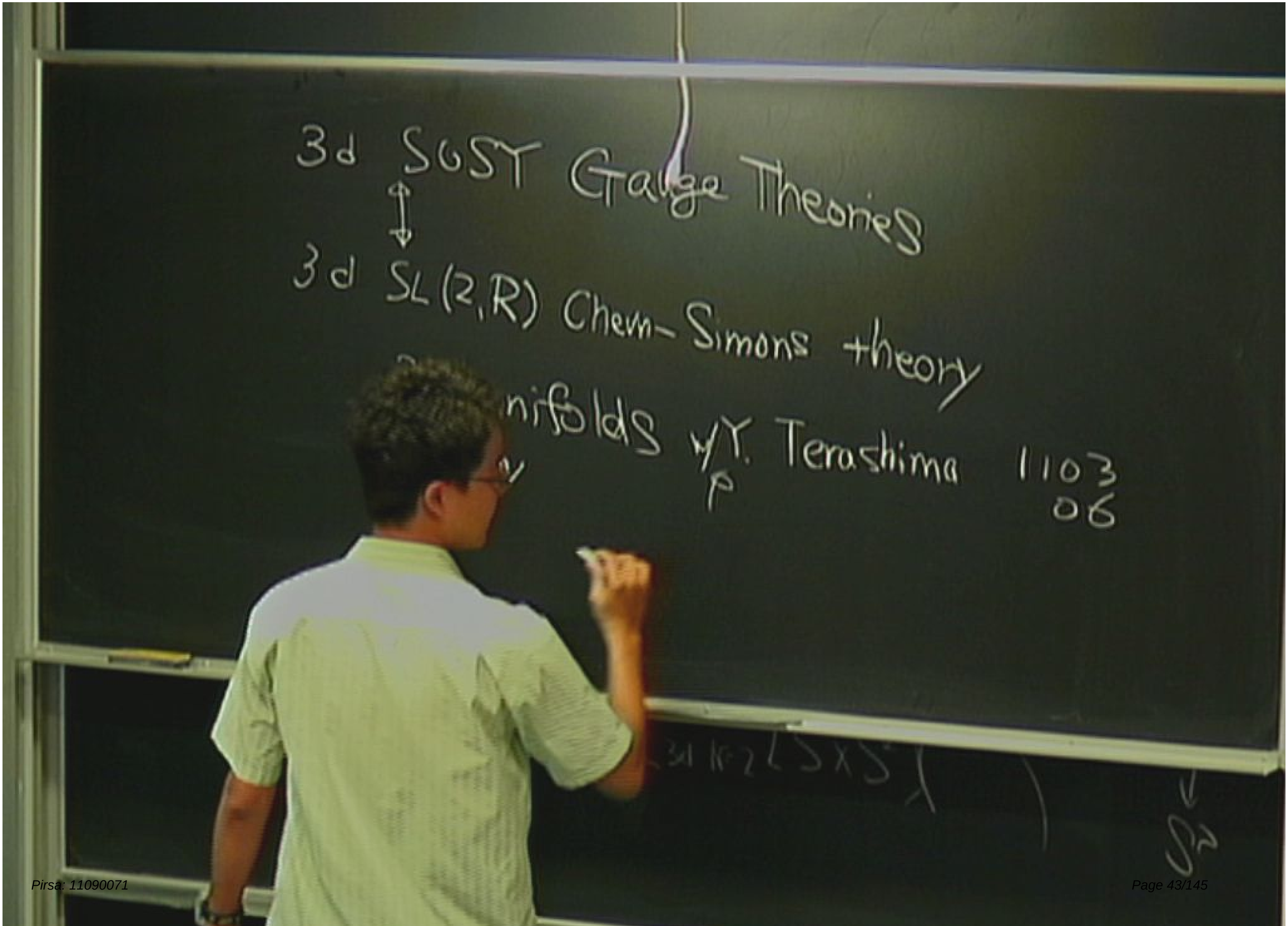


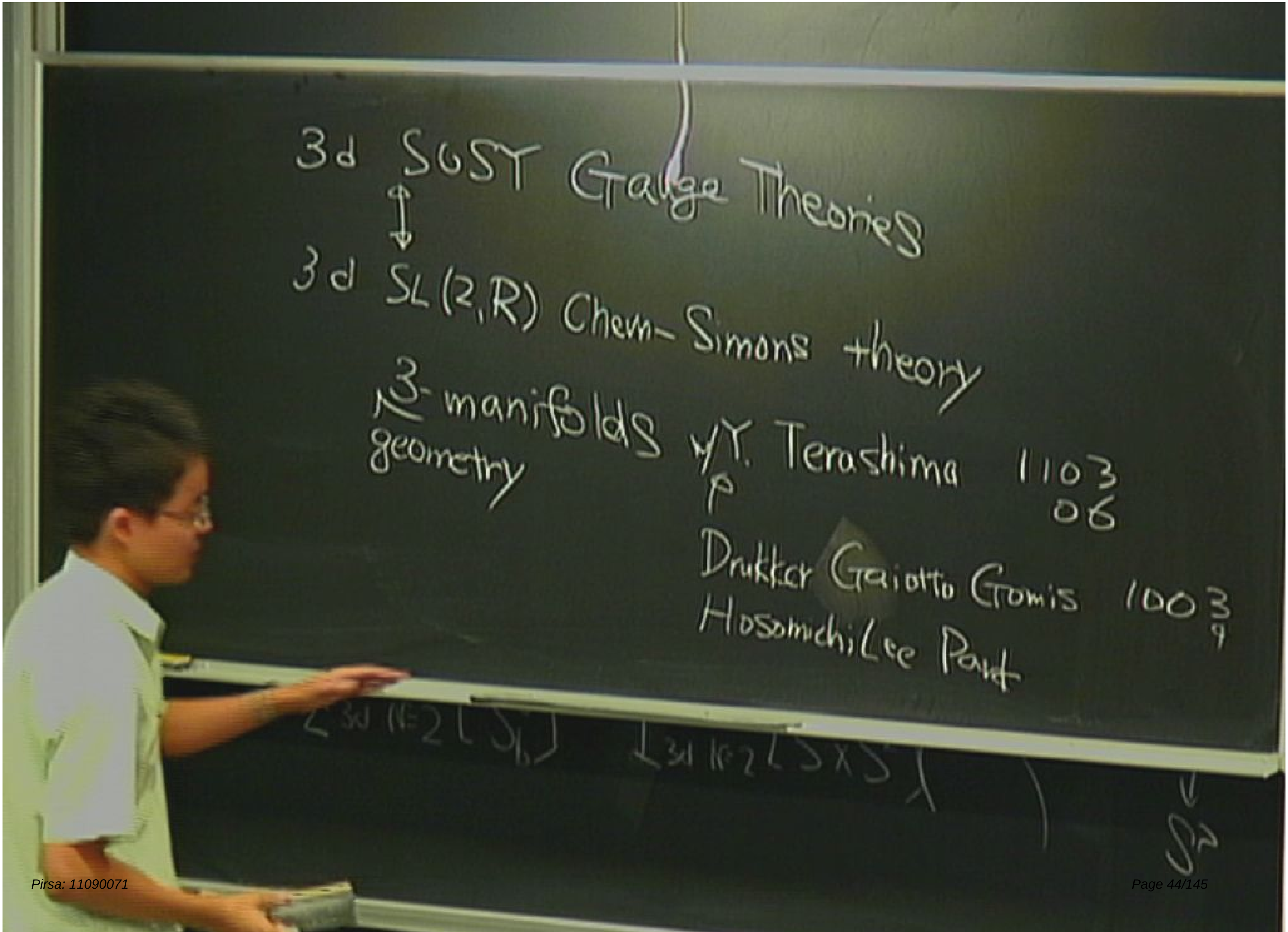
Relations between SUSY/non-SUSY theories
6d (2,0) theory
4d N=2 SCFT
AGT, GRRY
2d theory on Σ
2d Liouville
2d q-YM



Relations between SUSY/non-SUSY theories
A1
6d (2,0) theory
$\Sigma_{g,h}$
4d N=2 SCFT
$Z[S^4]$
$I[S^3\times S^1]$
AGT, GRRY
3d N=2 SCFT
3d SL(2,R)
2d theory on Σ
2d Liouville
2d q-YM


3d SUSY Gauge Theories
3d SL(2,R) Chern-Simons theory
w/Y. Terashima
1103
06


3d SUSY Gauge Theories
3d SL(2,R) Chern-Simons theory
w/Y. Terashima 1103
06


3d SUSY Gauge Theories
3d SL(2,R) Chern-Simons theory
3-manifolds w/ Y. Terashima 1103
geometry
06
Drukker Gaiotto Gomis 1003
Hosomichi Lee Park

3d SUSY Gauge Theories

$\Updownarrow$

3d $SL(2,R)$ Chern-Simons theory

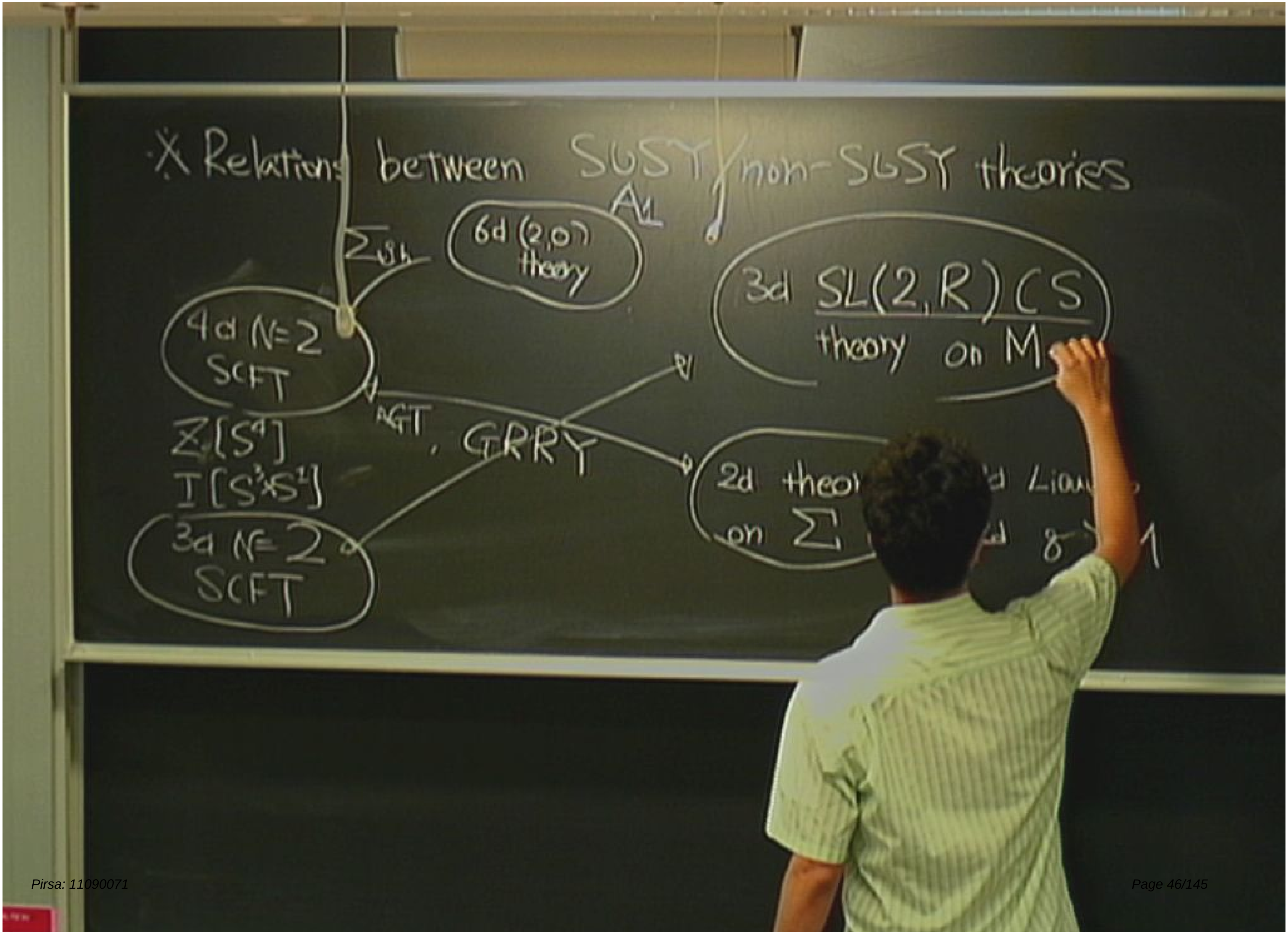
3-manifolds geometry w/ Y. Terashima 1103 06

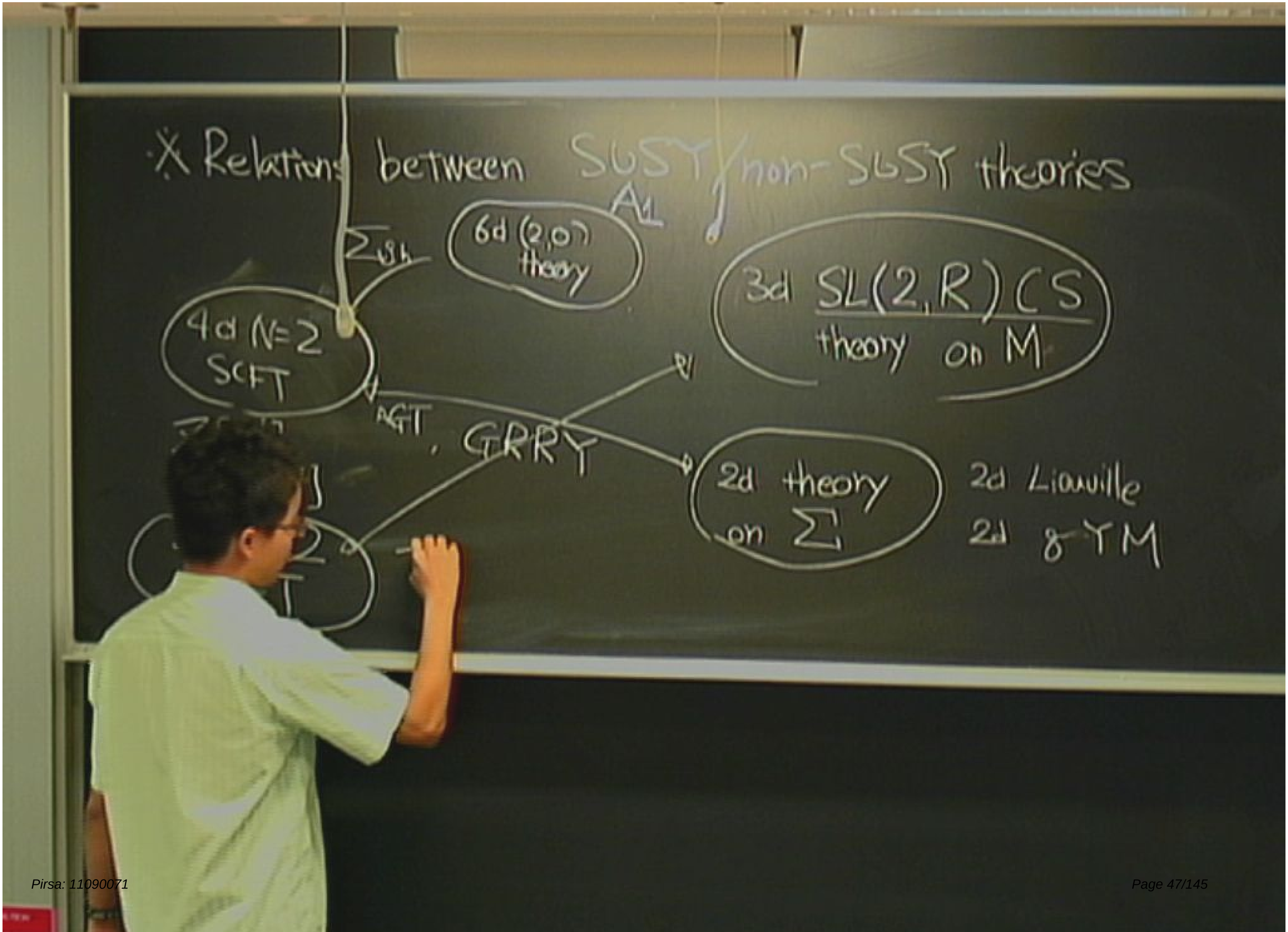
Drukker Gaiotto Gomis 1003 9

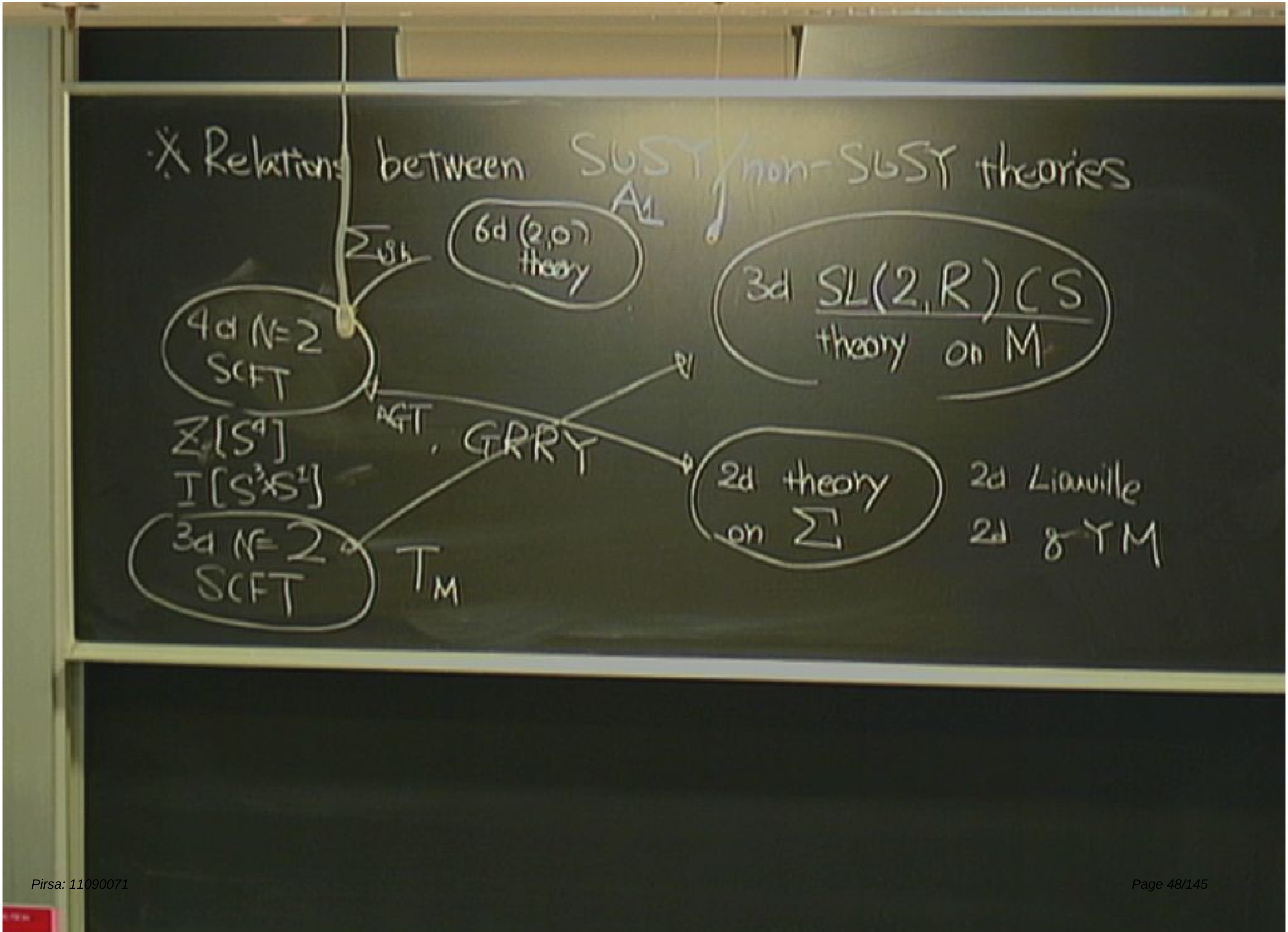
Hosomichi Lee Park 1009

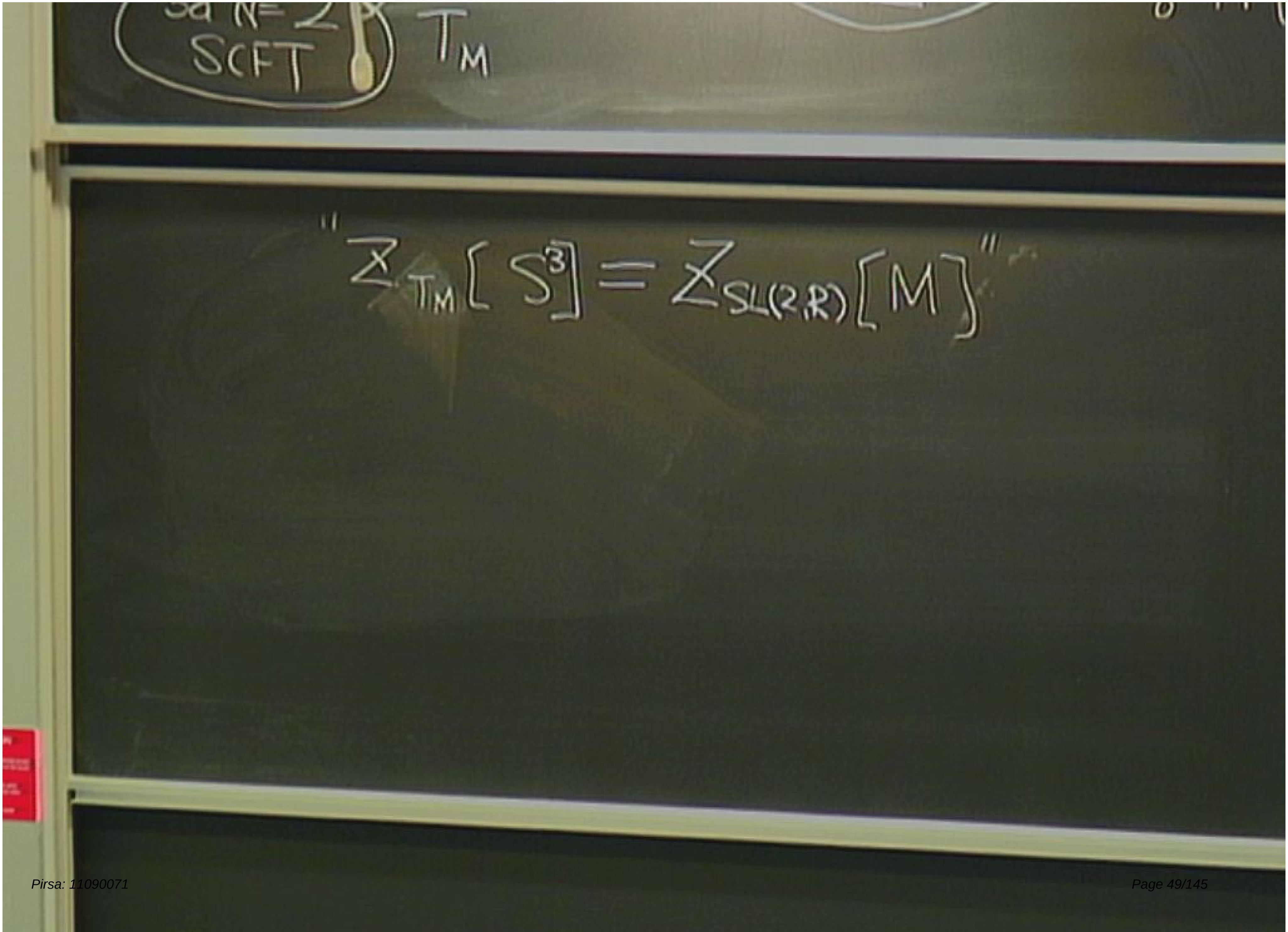
$Z_{3d\,N=2}[S^3_b]$ $\quad$ $I_{3d\,N=2}[S^1\times S^2]$

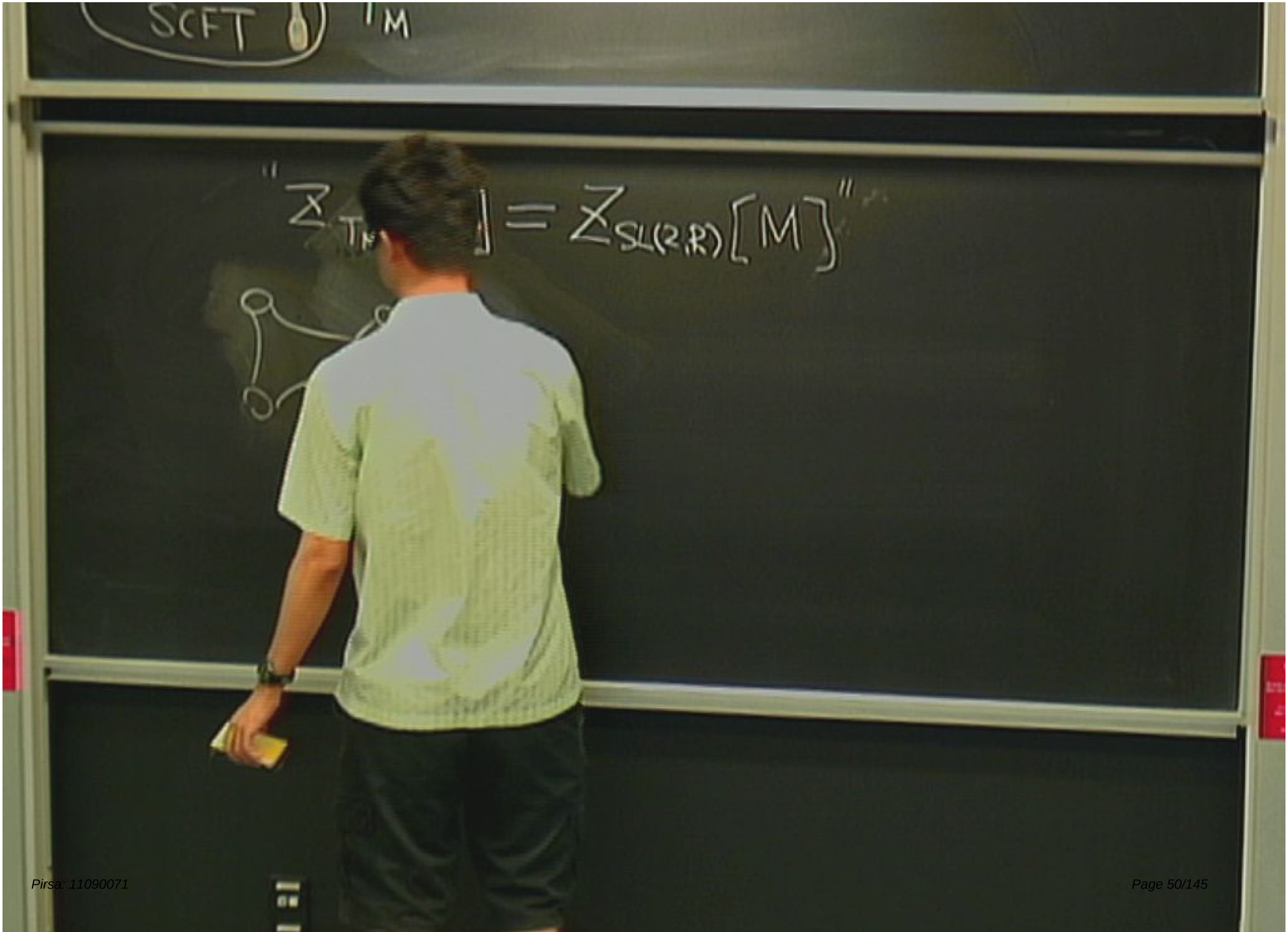
S^2

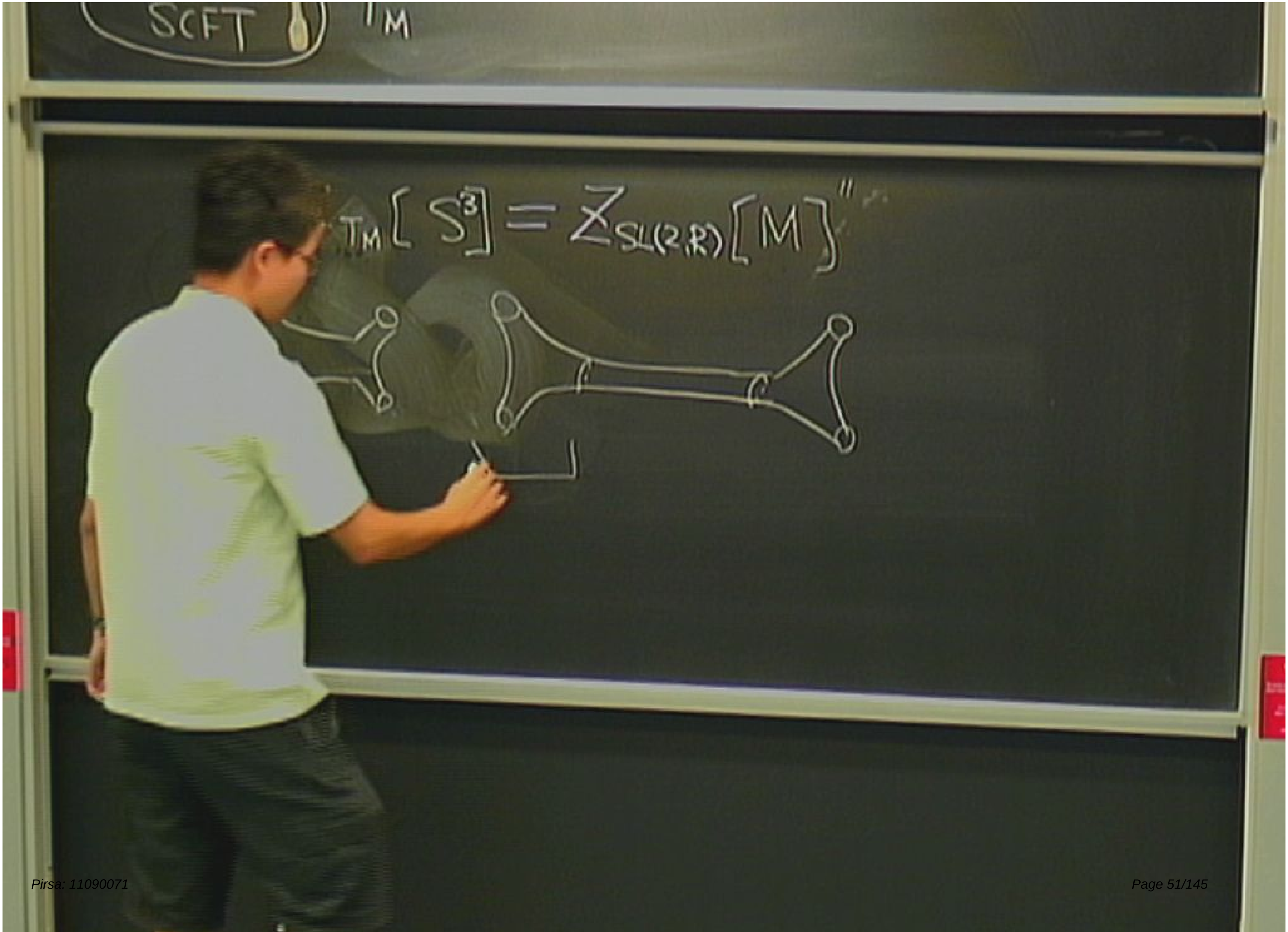

Relations between SUSY/non-SUSY theories
6d (2,0) theory
A1
Σg,h
4d N=2 SCFT
3d SL(2,R) CS theory on M
AGT, GRRY
Z[S4]
I[S3×S1]
3d N=2 SCFT
2d theo
on Σ

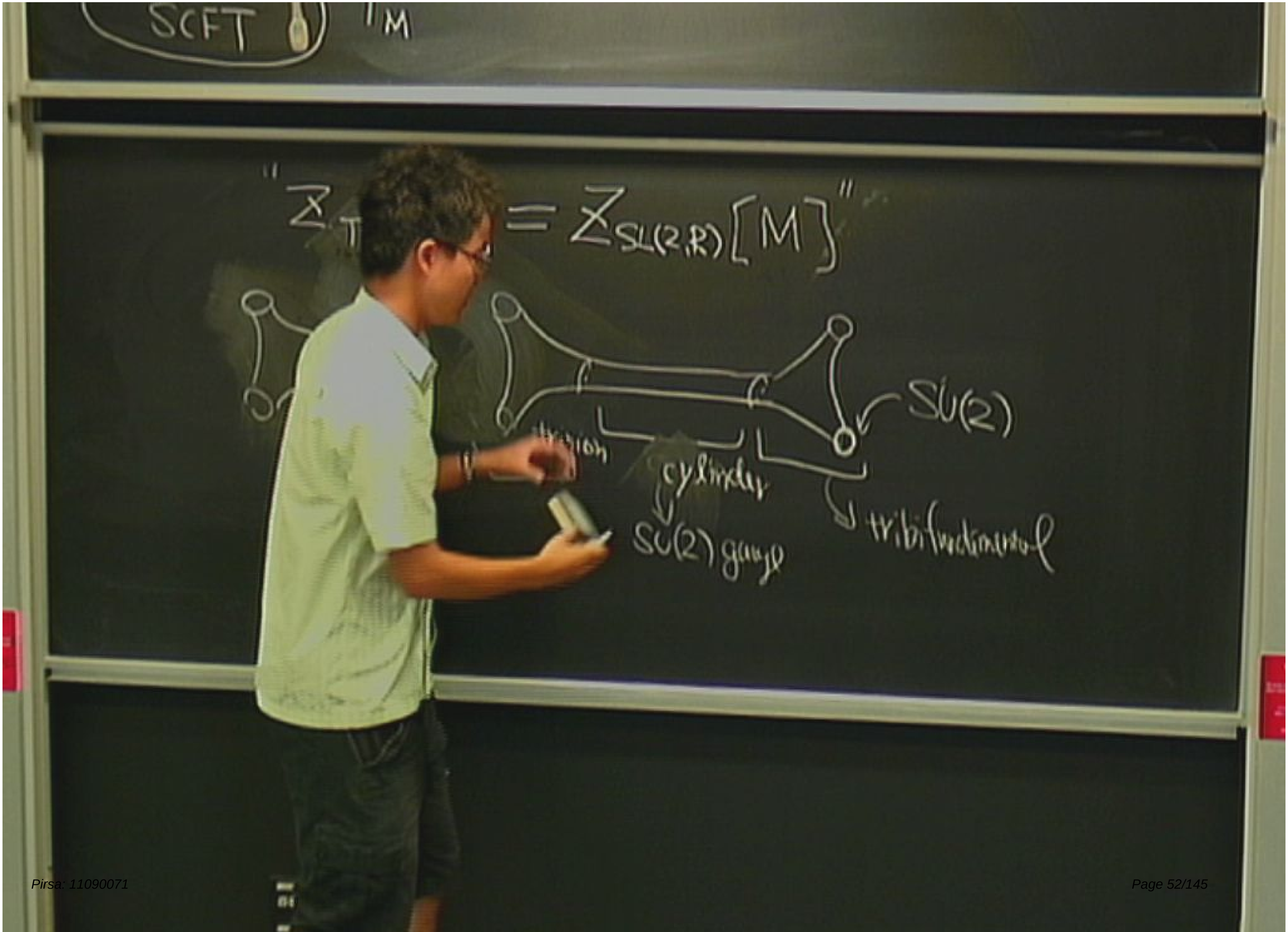

Relations between SUSY / non-SUSY theories
6d (2,0) theory
A1
4d N=2 SCFT
AGT, GRRY
3d SL(2,R) CS theory on M
2d theory on Σ
2d Liouville
2d q-YM


※ Relations between SUSY / non-SUSY theories
A_1
6d (2,0) theory
$\Sigma_{g,h}$
4d N=2 SCFT
$Z[S^4]$
$I[S^3 \times S^1]$
AGT, GRRY
3d N=2 SCFT
T_M
3d SL(2,R) CS theory on M
2d theory on Σ
2d Liouville
2d q-YM


SCFT
T_M
"Z_{T_M}[S^3] = Z_{SL(2,R)}[M]"

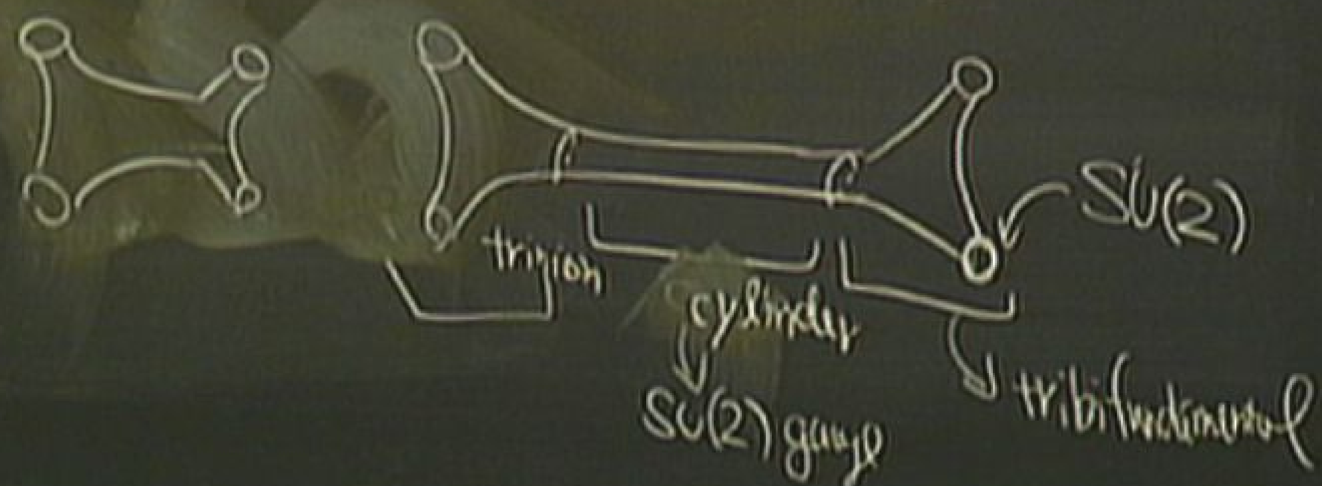

SCFT
T_M
"$Z_{T_M}[\ldots] = Z_{SL(2,\mathbb{R})}[M]$"

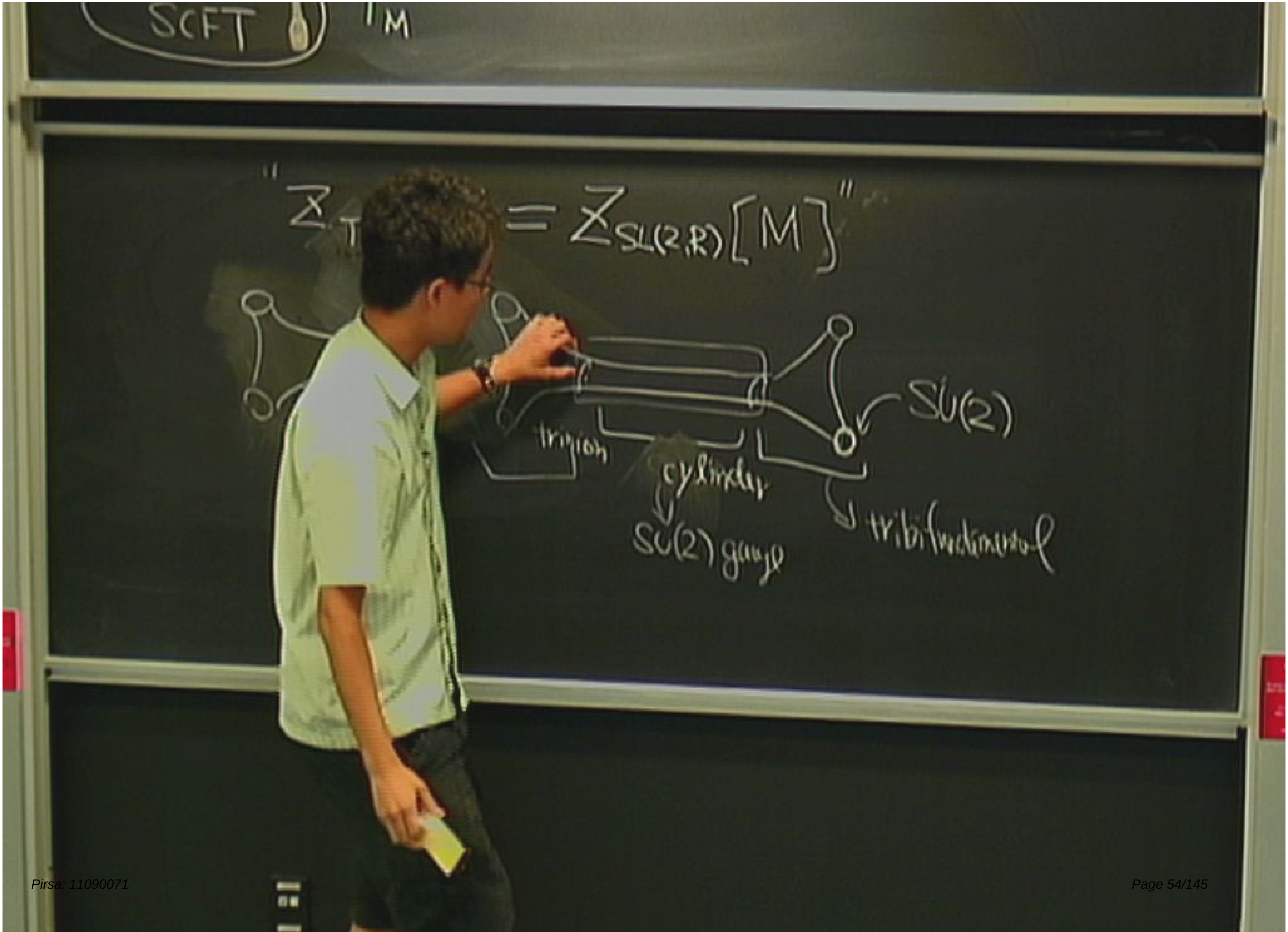

SCFT
T_M
$T_M[S^3] = Z_{SL(2,\mathbb{R})}[M]$"

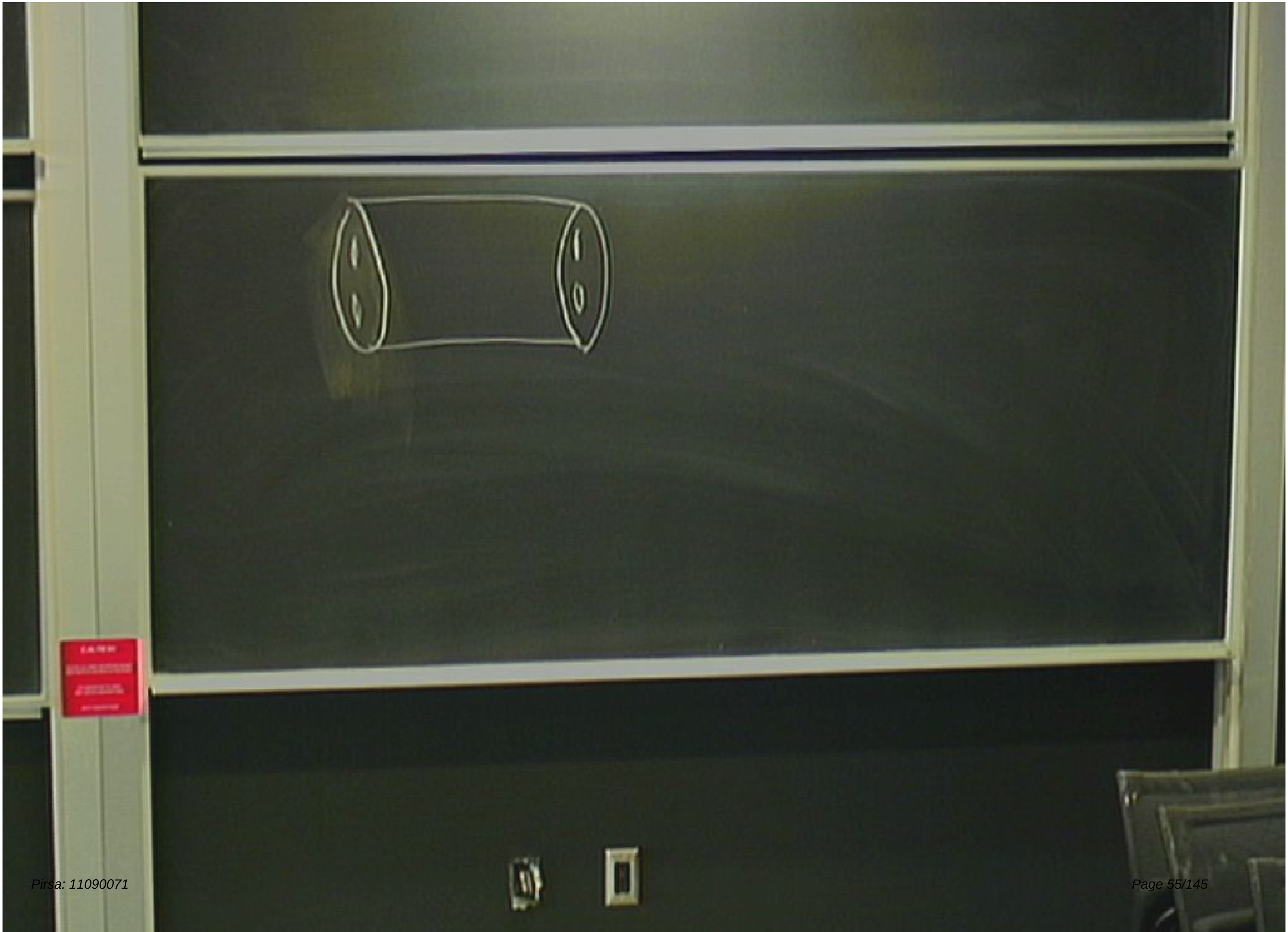

SCFT
M
" Z_T = Z_SL(2,R)[M] "
SU(2)
cylinder
SU(2) gauge
tribifundamental

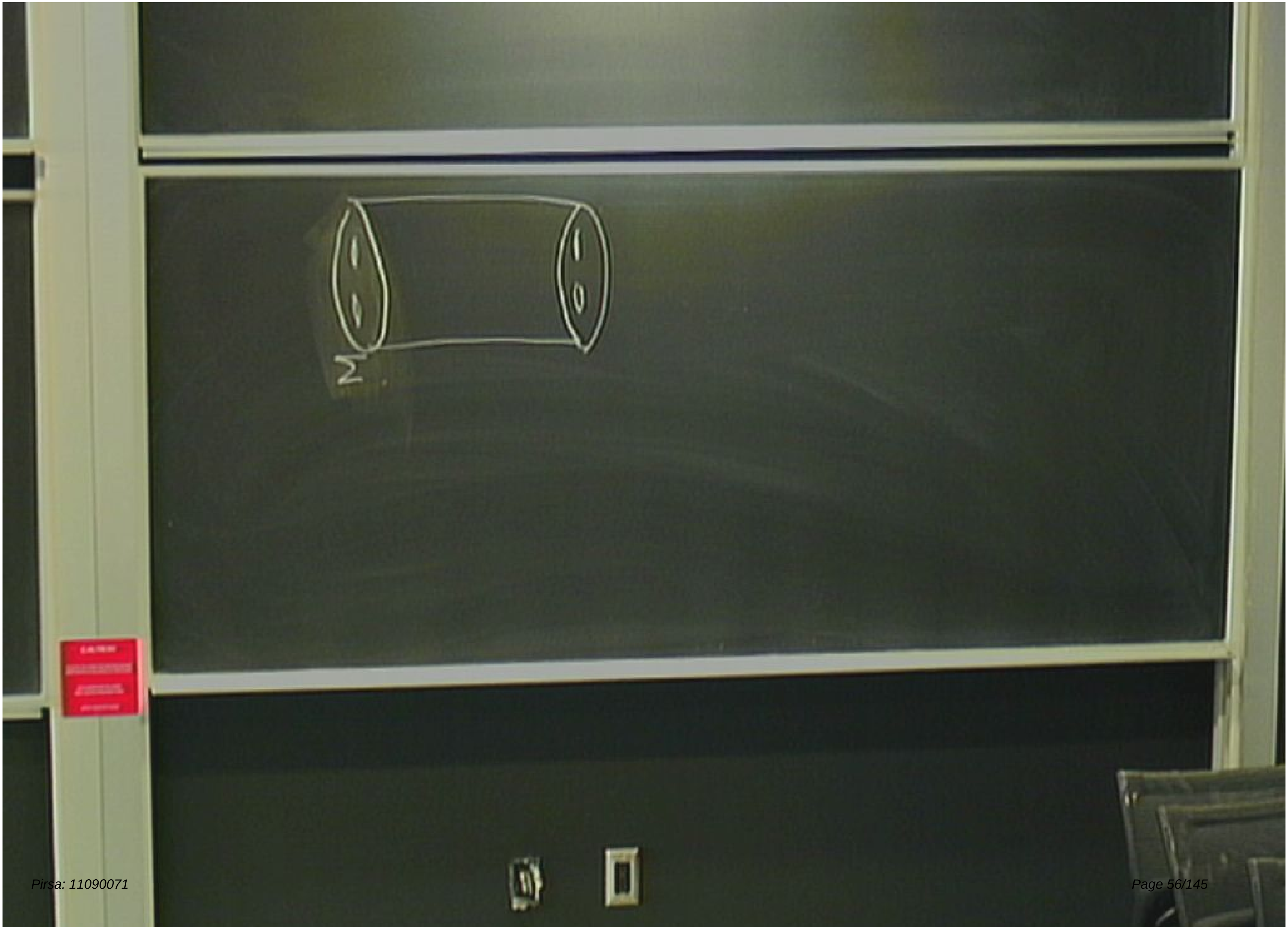
SCFT T_M

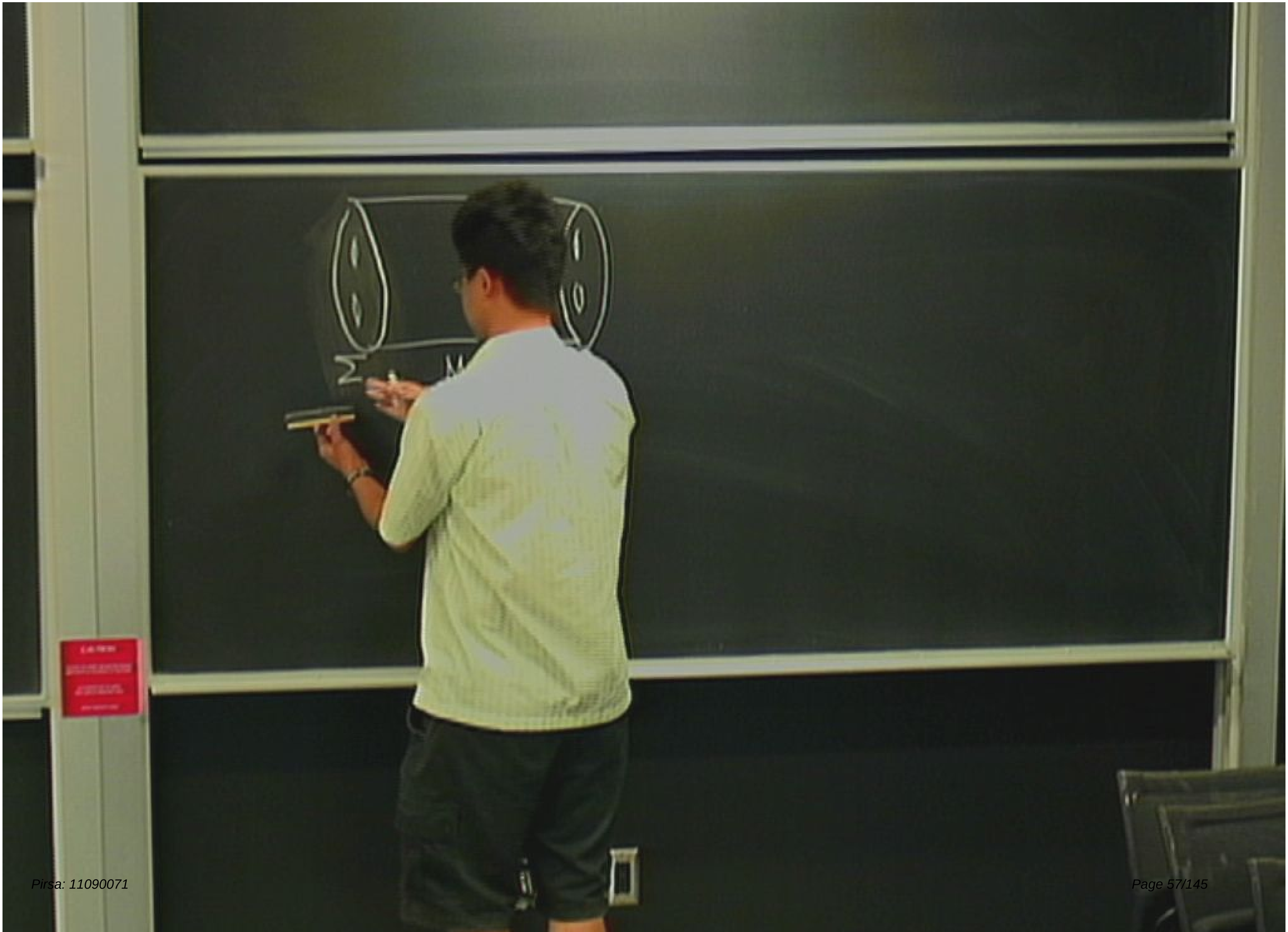
$$\text{"} Z_{T_M}[S^3] = Z_{SL(2,\mathbb{R})}[M] \text{"}$$

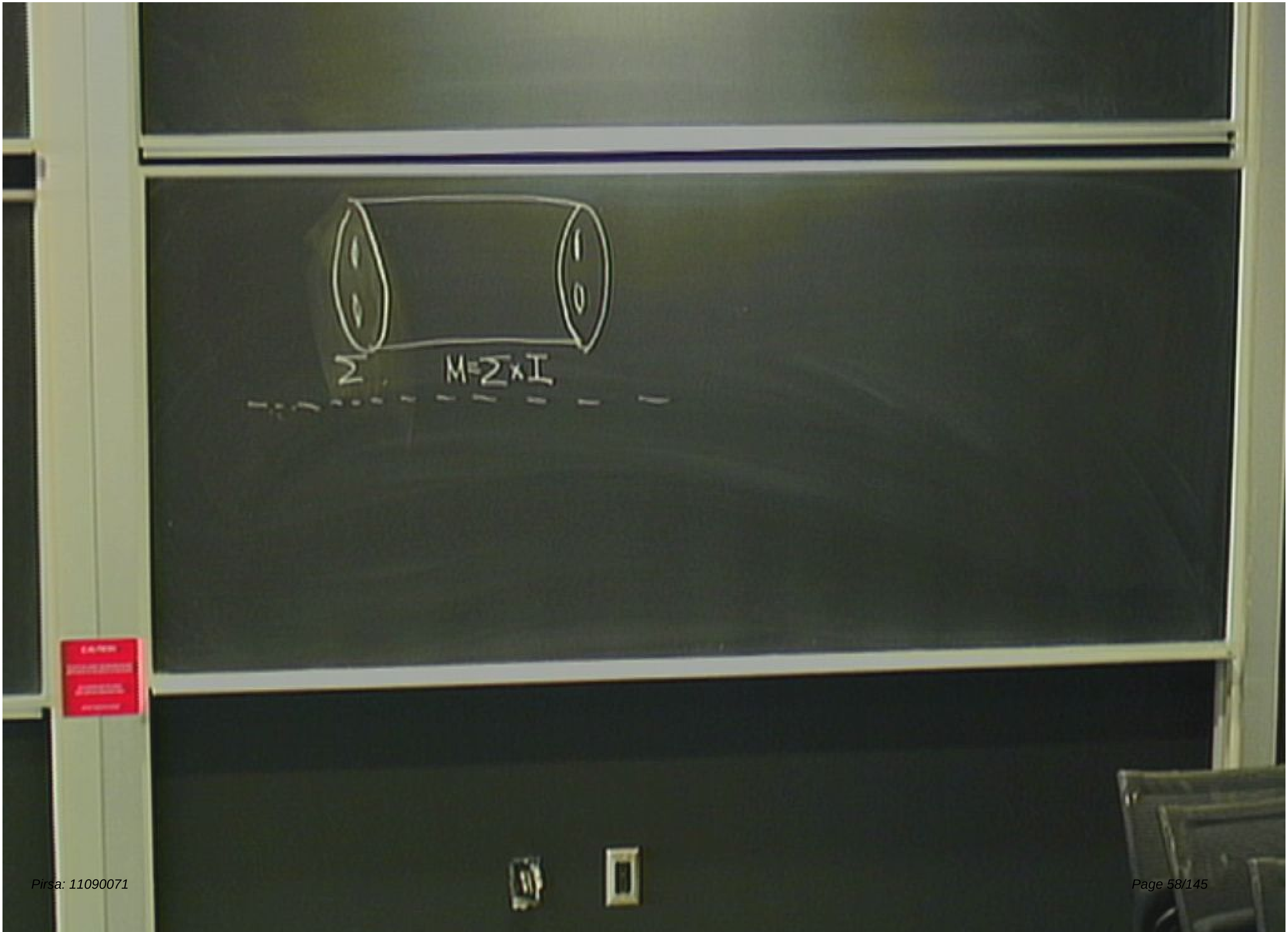


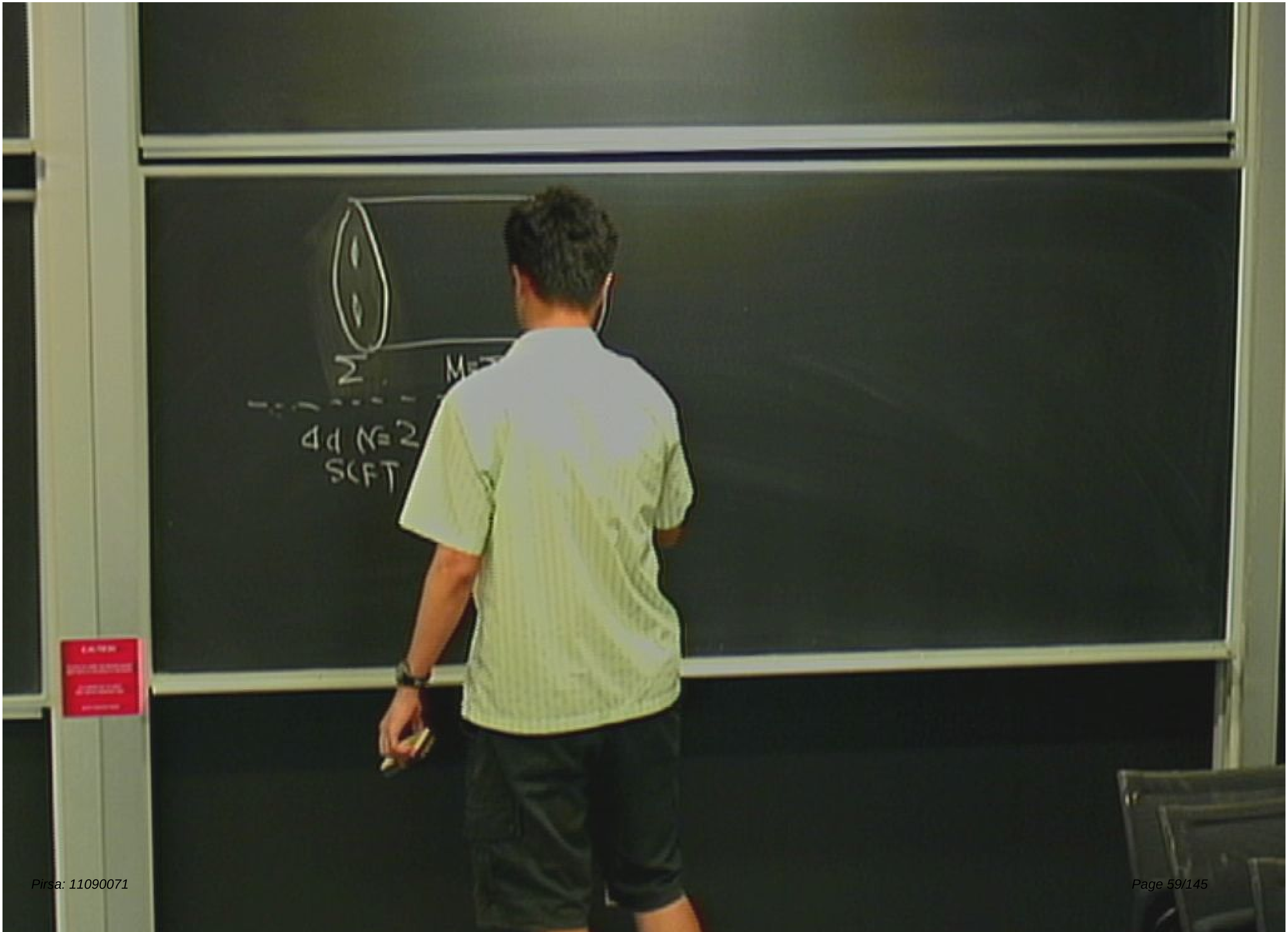

SCFT
M
"Z_T = Z_{SL(2,R)}[M]"
trinion
cylinder
SU(2) gauge
SU(2)
tribifundamental

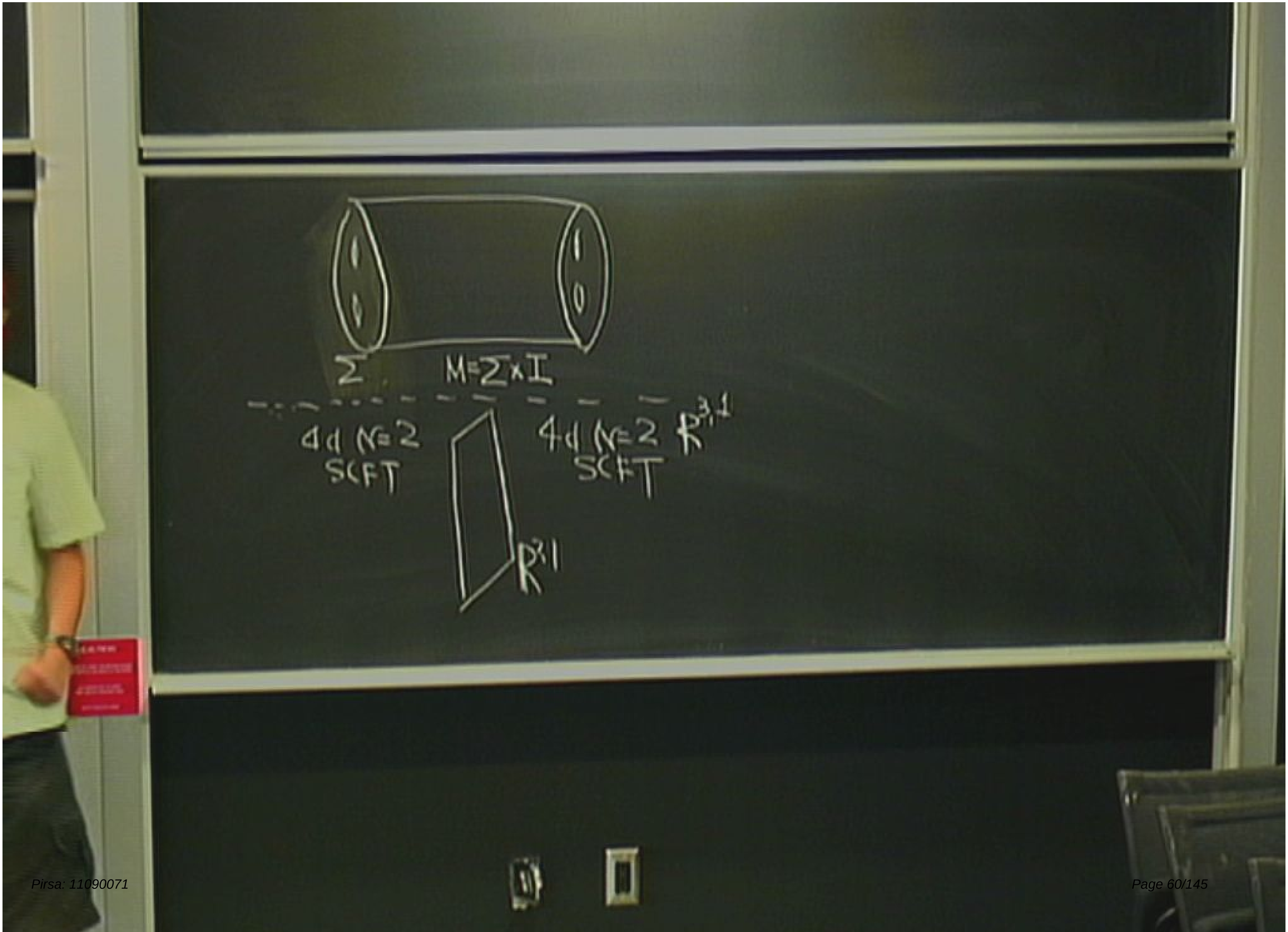


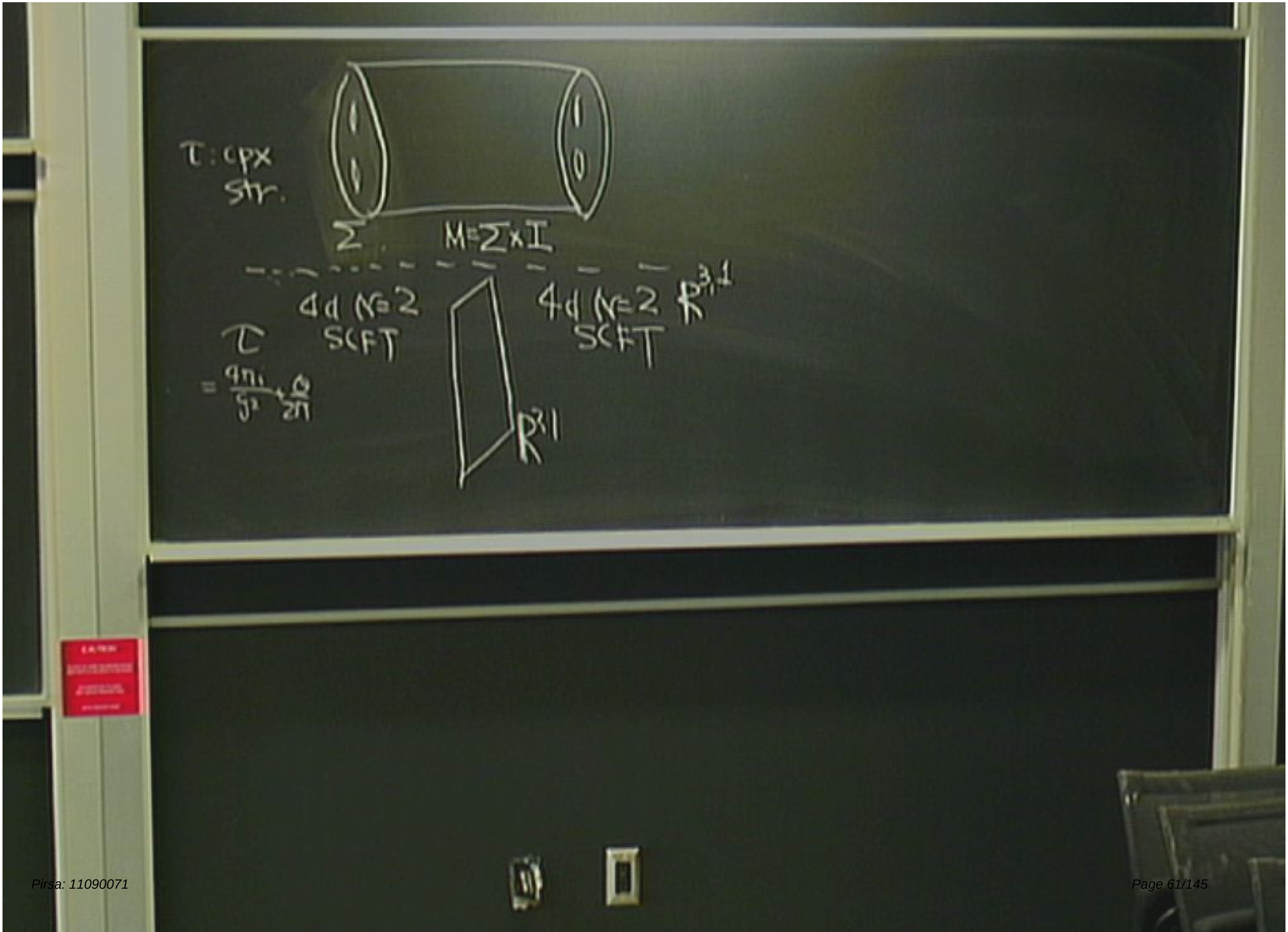


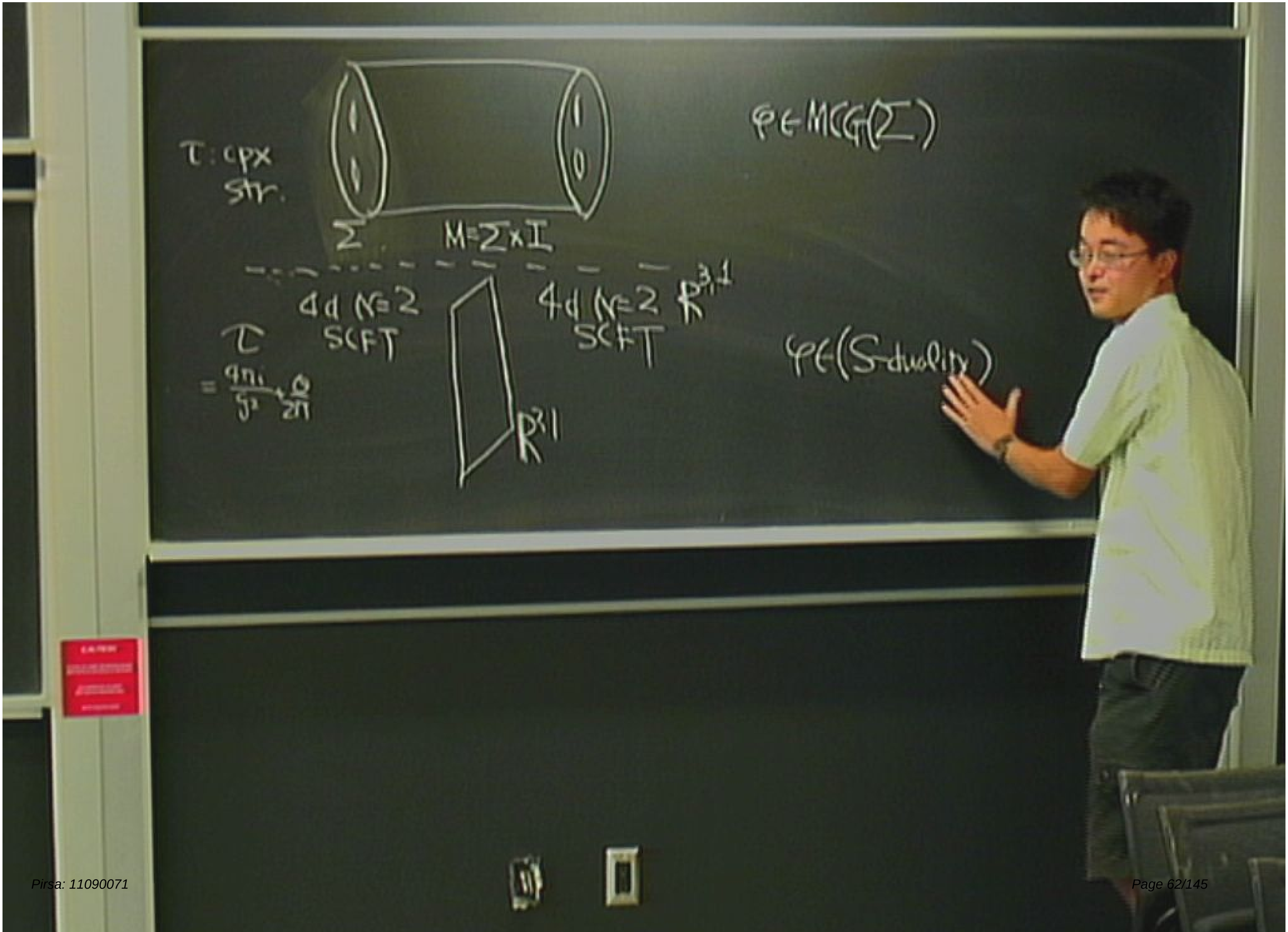

Σ
M

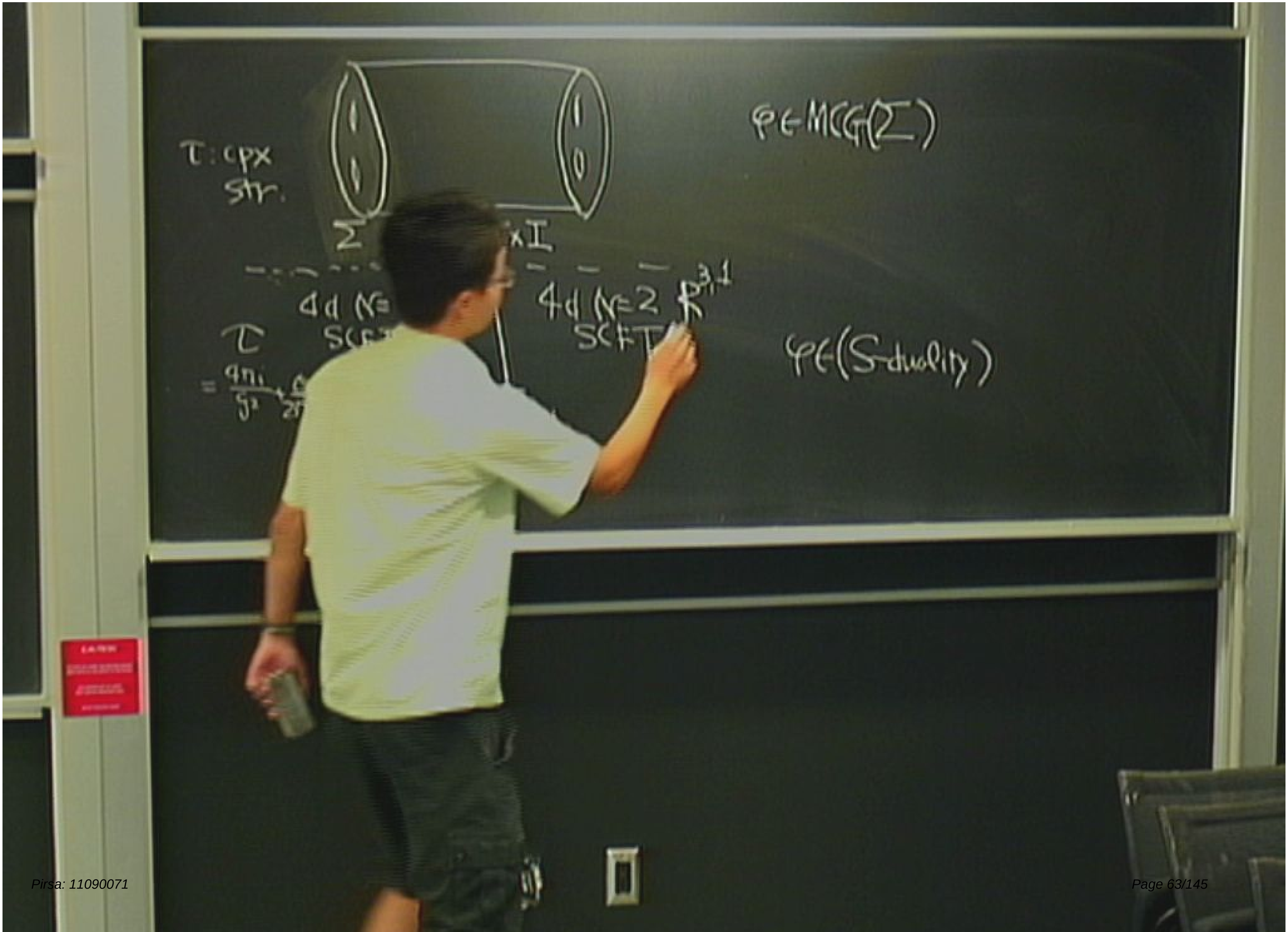

Σ
$M = \Sigma \times I$

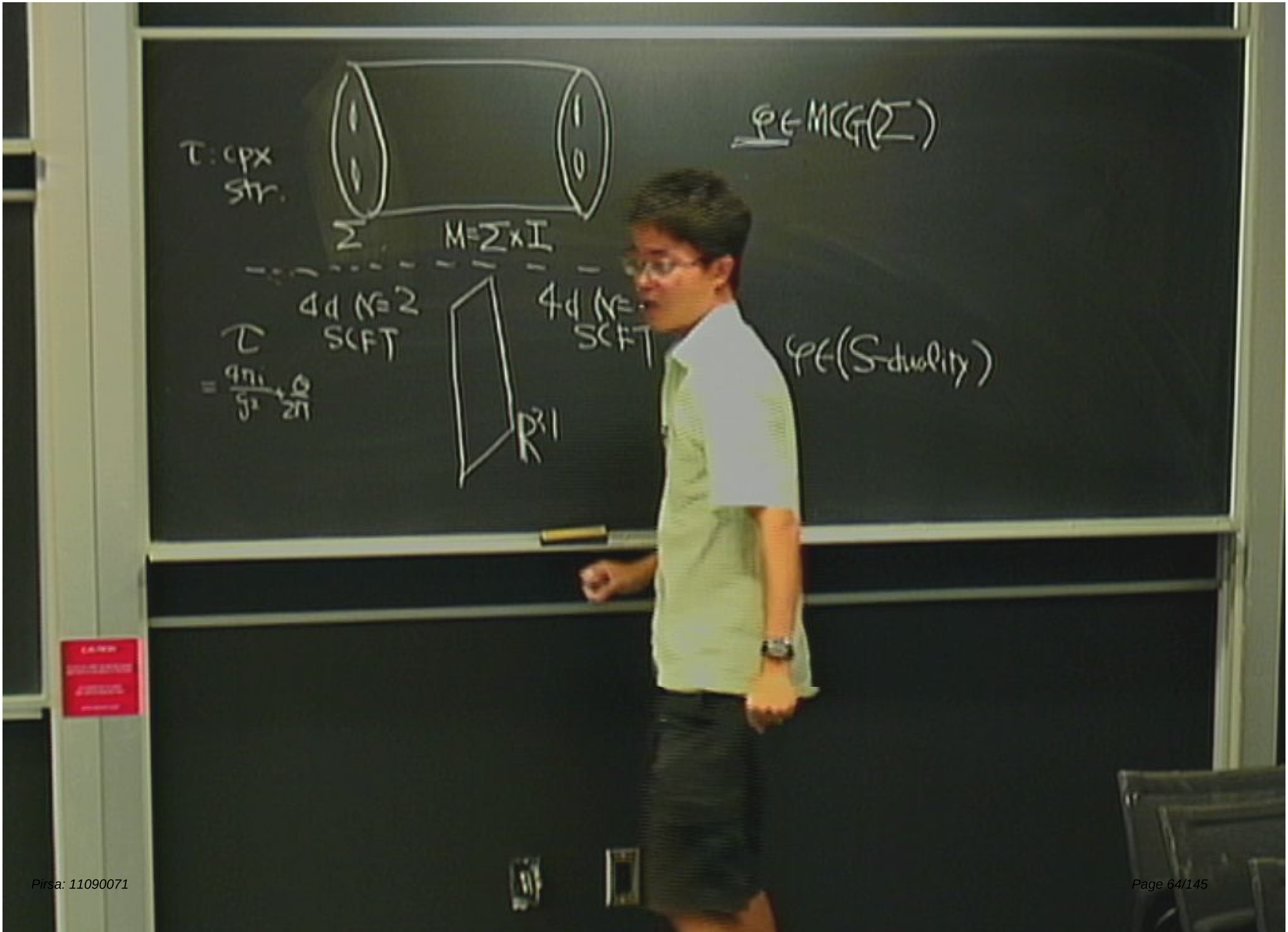

Σ
4d N=2
SCFT

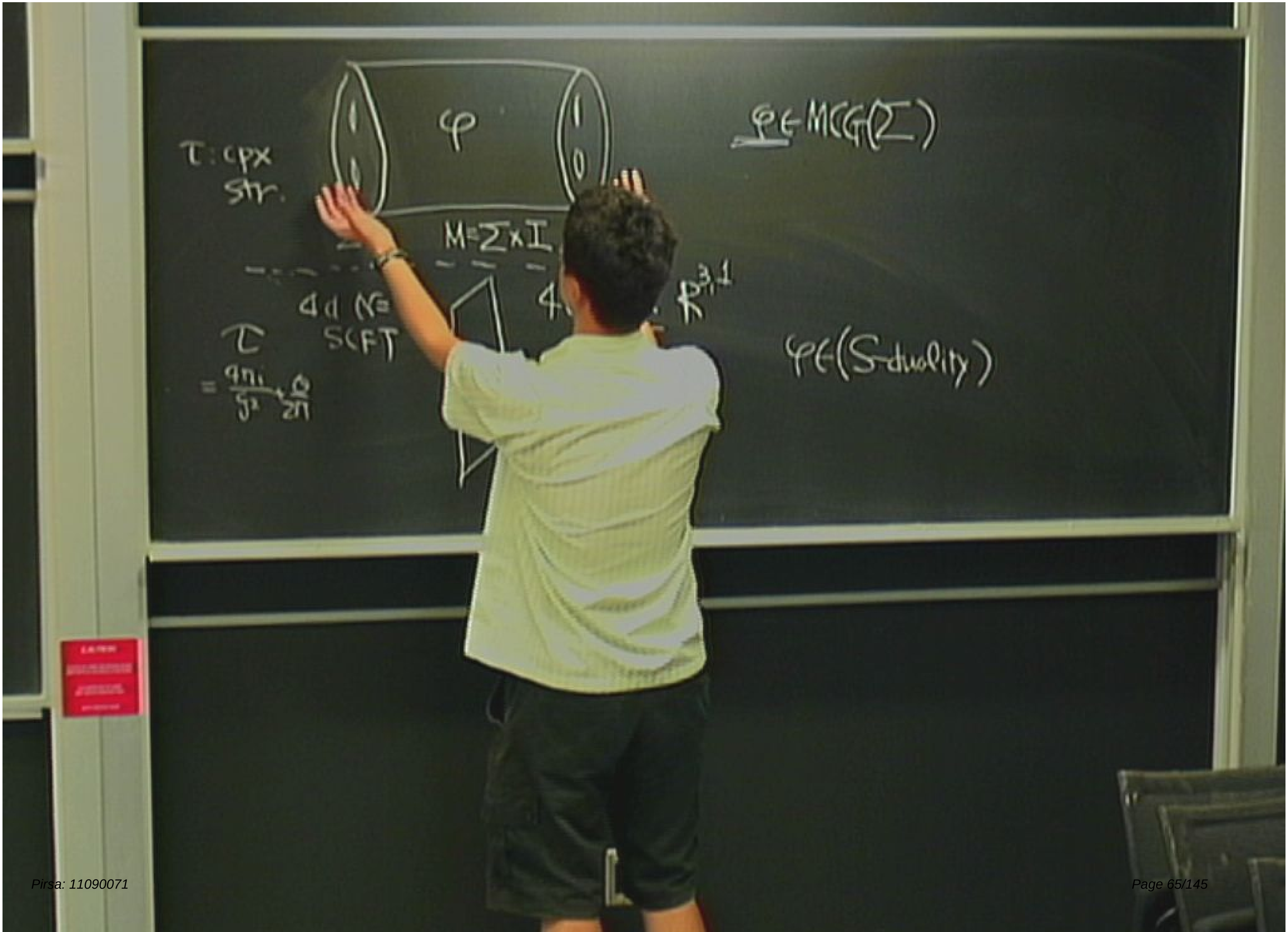

Σ
M=Σ×I
4d N=2 SCFT
4d N=2 SCFT R^3,1
R^3,1

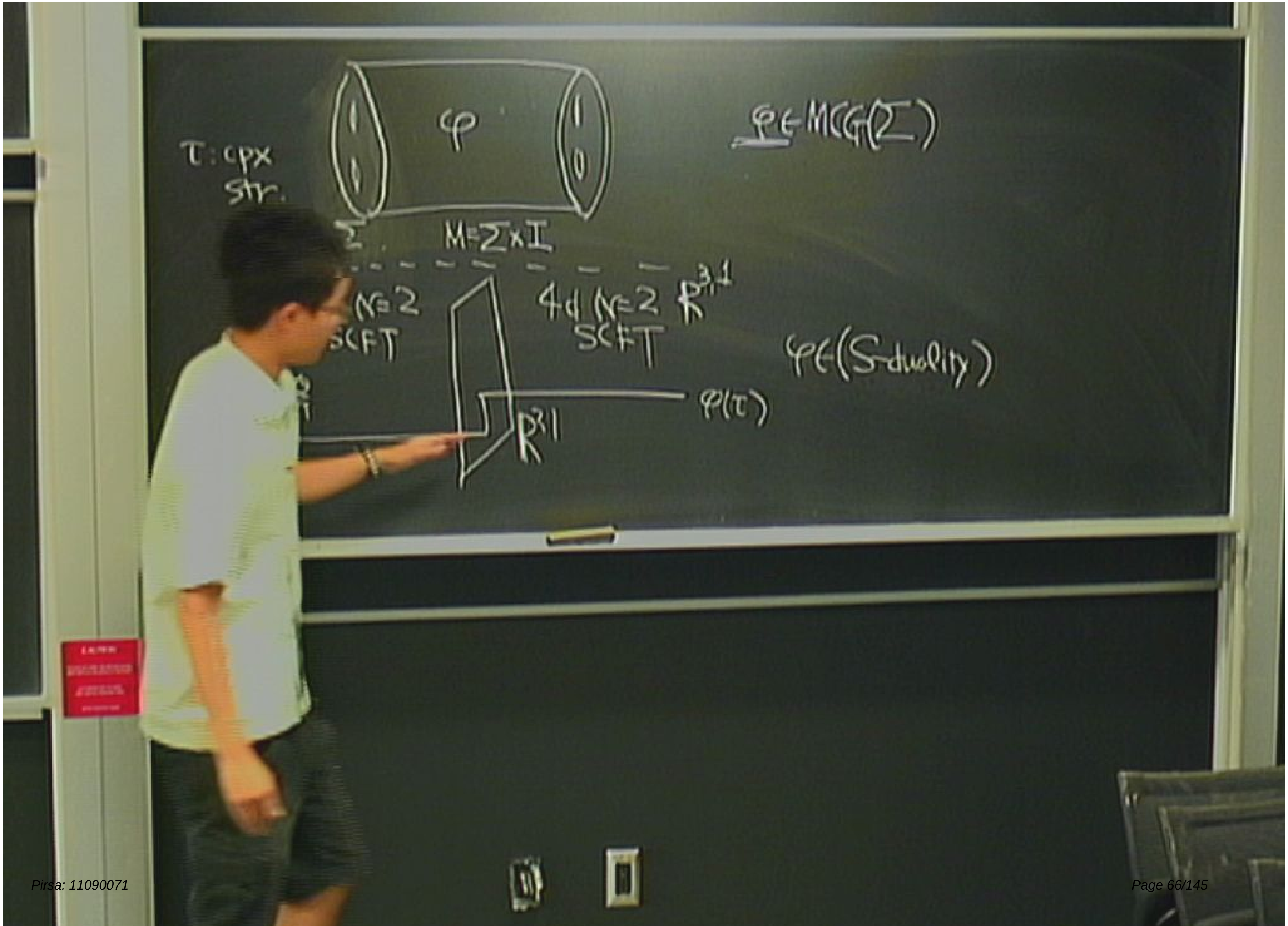

τ: cpx str.
Σ
M=Σ×I
4d N=2 SCFT
4d N=2 SCFT
R3,1
R3,1
τ
= 4πi/g² + θ/2π

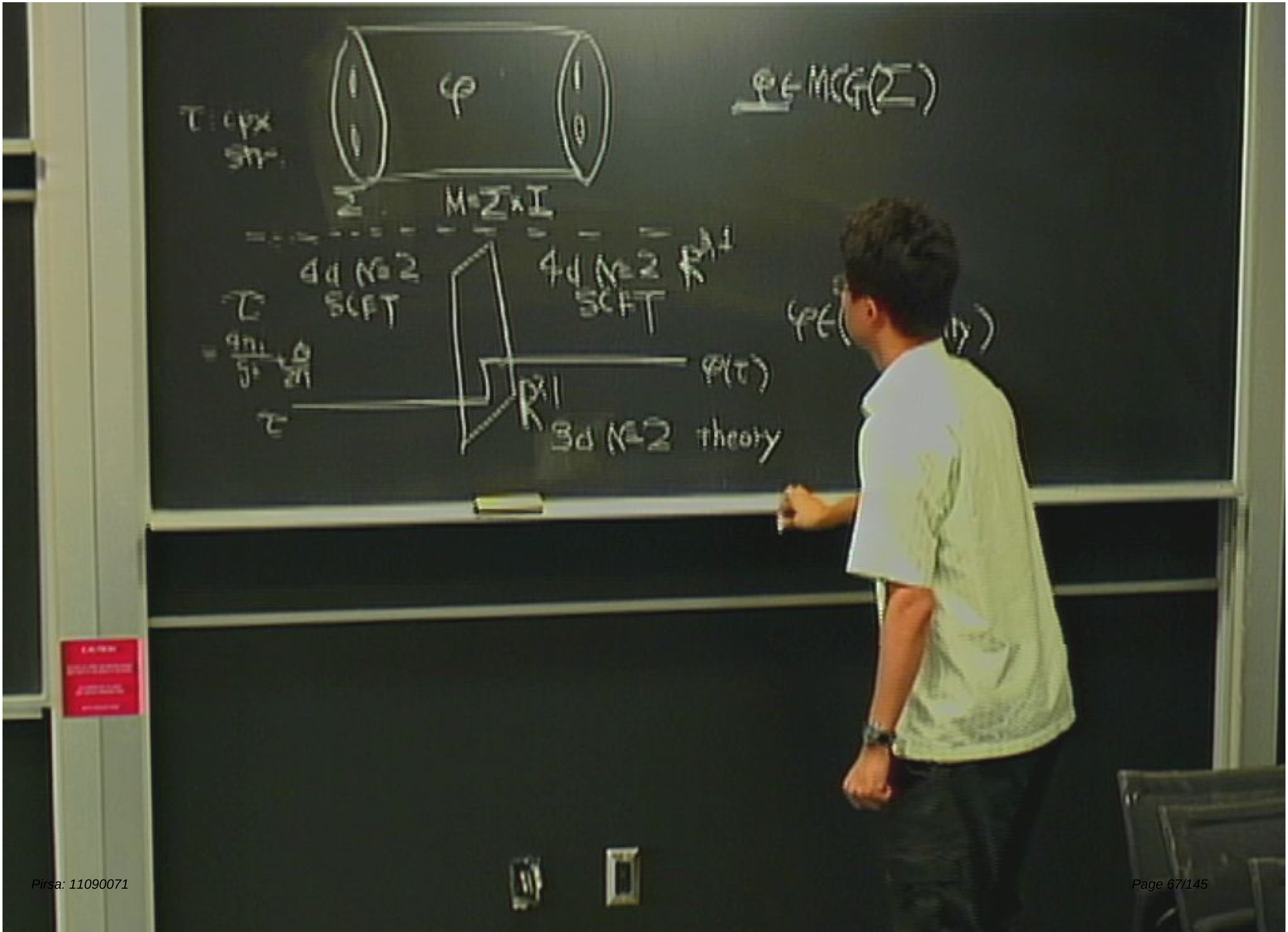

τ: cpx str.
Σ
M=Σ×I
φ∈MCG(Σ)
4d N=2 SCFT
4d N=2 SCFT
R^{3,1}
τ
= 4πi/g² + θ/2π
R^{3,1}
φ∈(S-duality)


τ : cpx
str.
Σ
×I
4d N=2 $\mathbb{R}^{3,1}$
SCFT
φ∈MCG(Σ)
φ∈(S-duality)

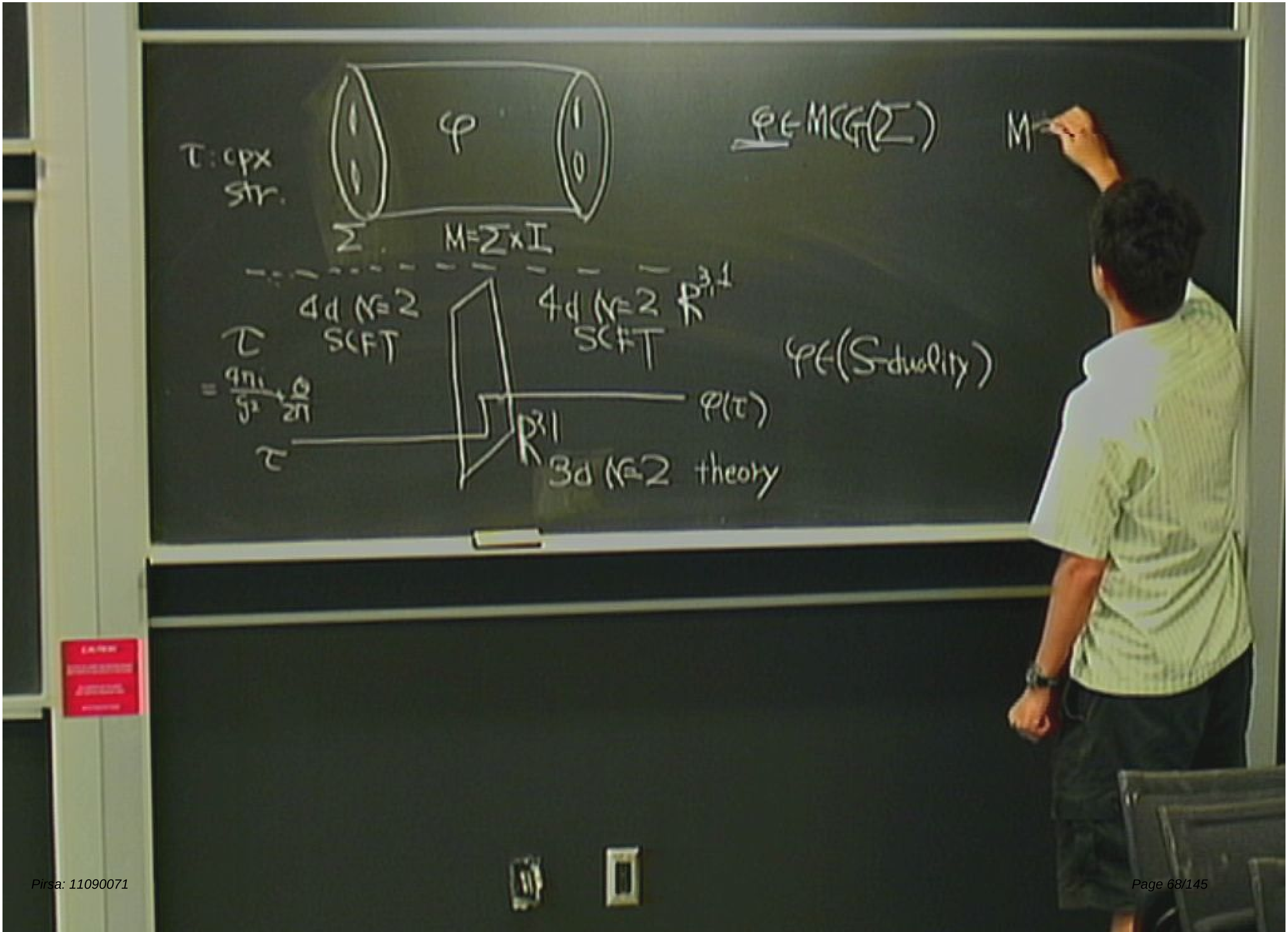

τ : cpx
str.
Σ
M=Σ×I
φ∈MCG(Σ)
4d N=2
SCFT
4d N=
SCFT
τ
= 4πi/g² + θ/2π
R³'¹
φ∈(S-duality)

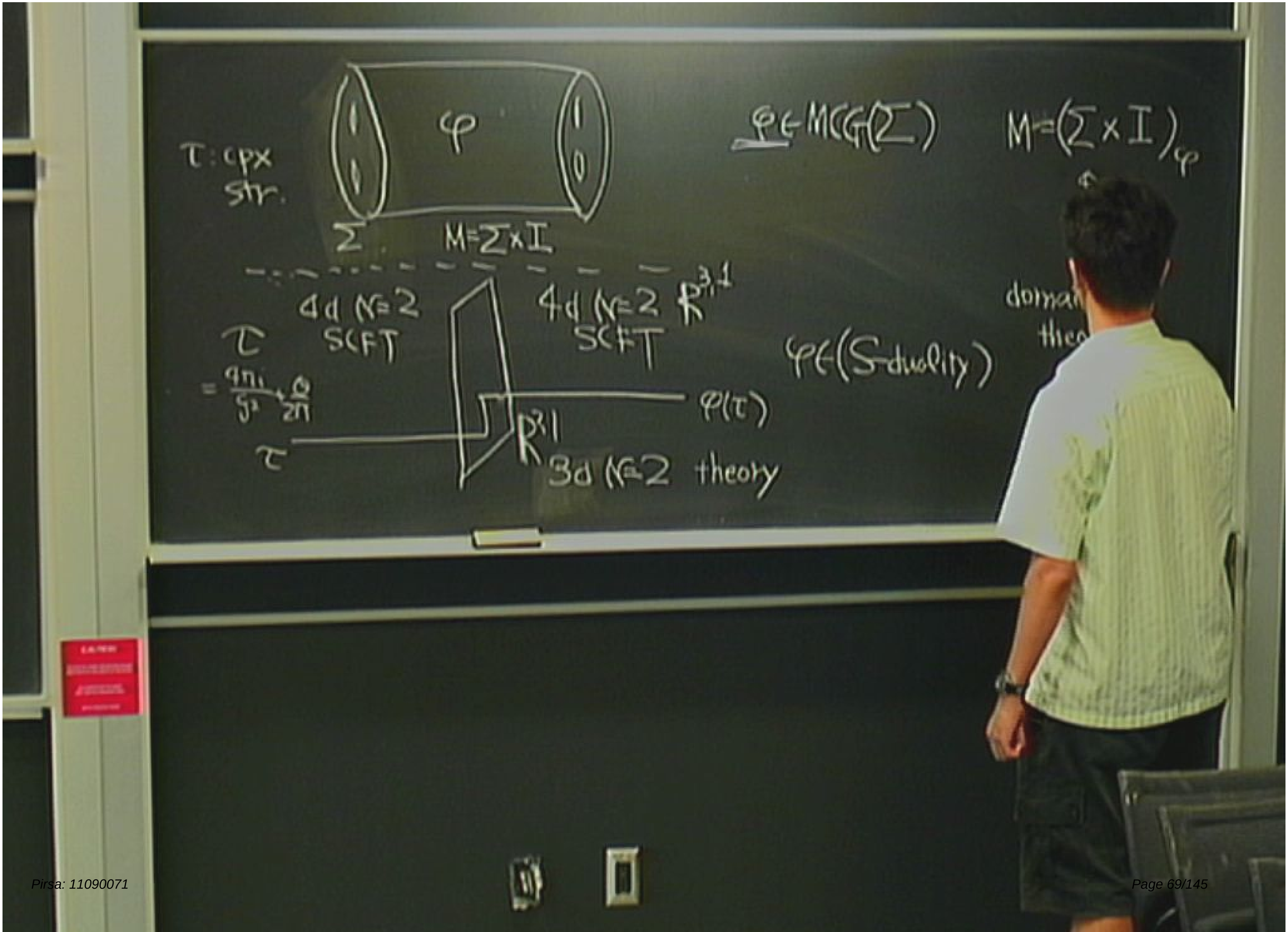

τ: cpx str.
φ
φ ∈ MCG(Σ)
M=Σ×I
4d N=
SCFT
τ
= 4πi/g² + θ/2π
R^{3,1}
φ ∈ (S-duality)


τ: cpx
str.
φ
φ ∈ MCG(Σ)
Σ
M=Σ×I
4d N=2 R^{3,1}
SCFT
N=2
SCFT
φ ∈ (S-duality)
φ(τ)
R^{3,1}

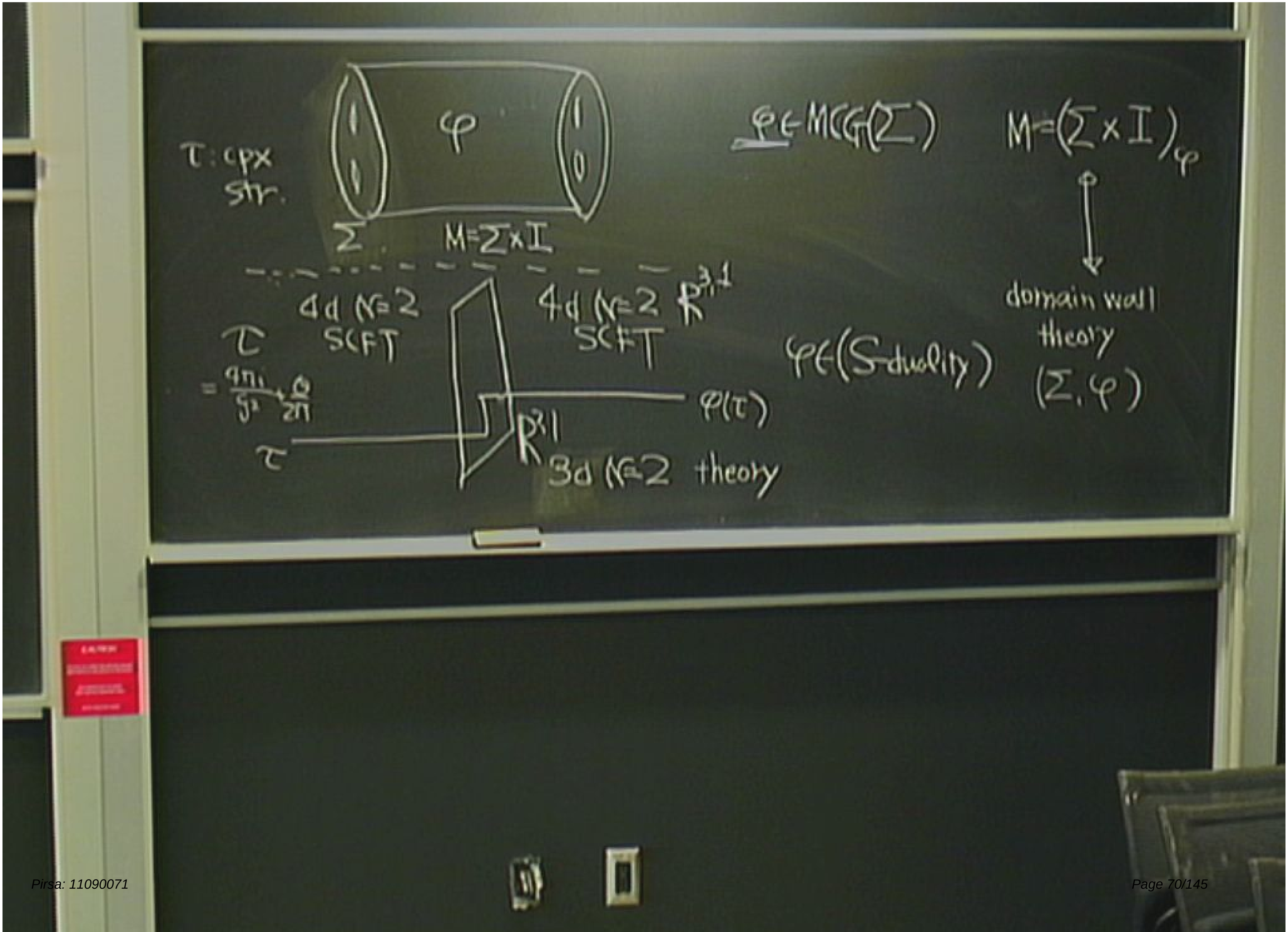


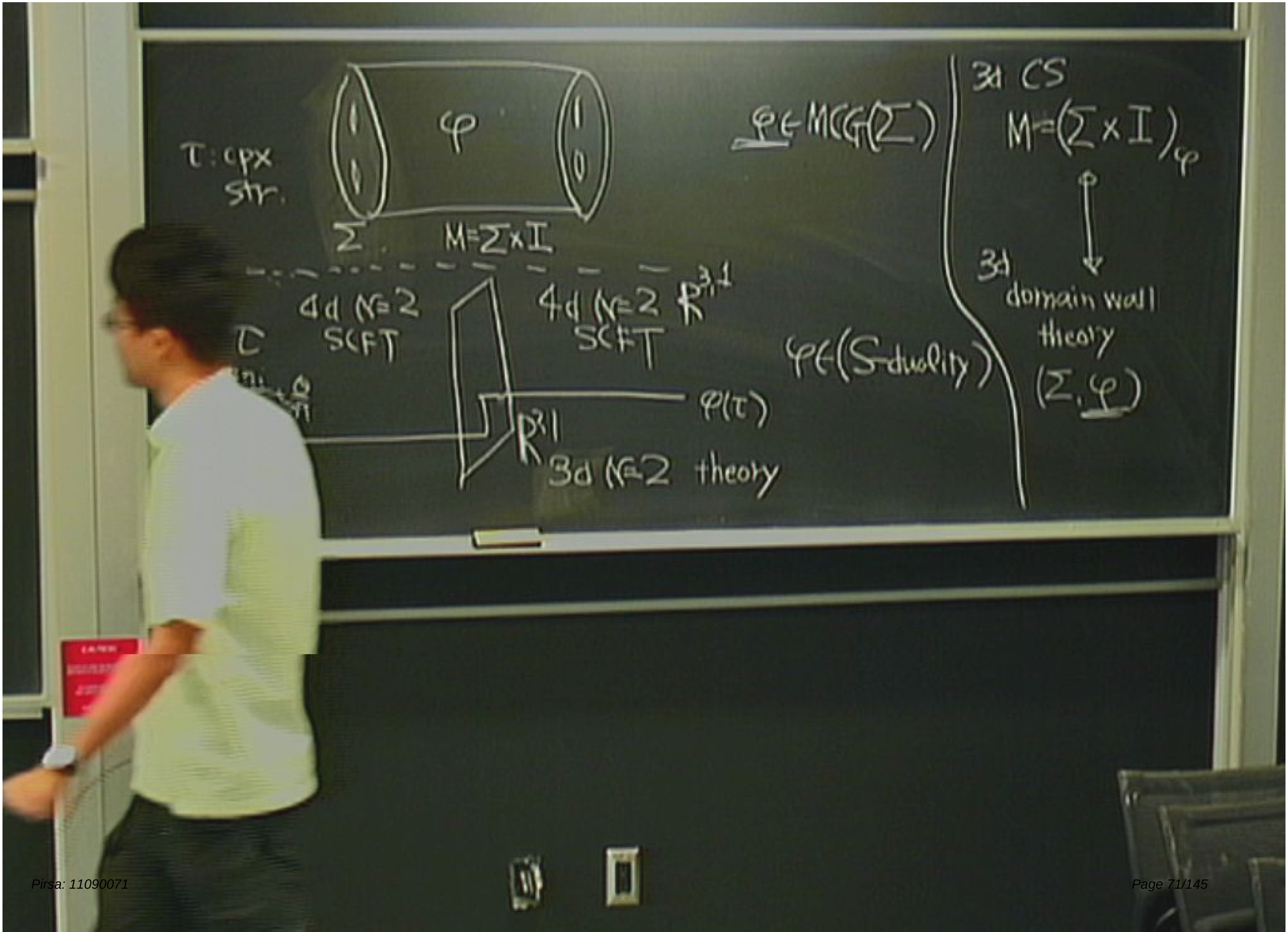
T : cpx str.
φ
φ ∈ MCG(Σ)
Σ
M = Σ × I
4d N=2 SCFT
4d N=2 SCFT
τ
τ
φ(τ)
3d N=2 theory

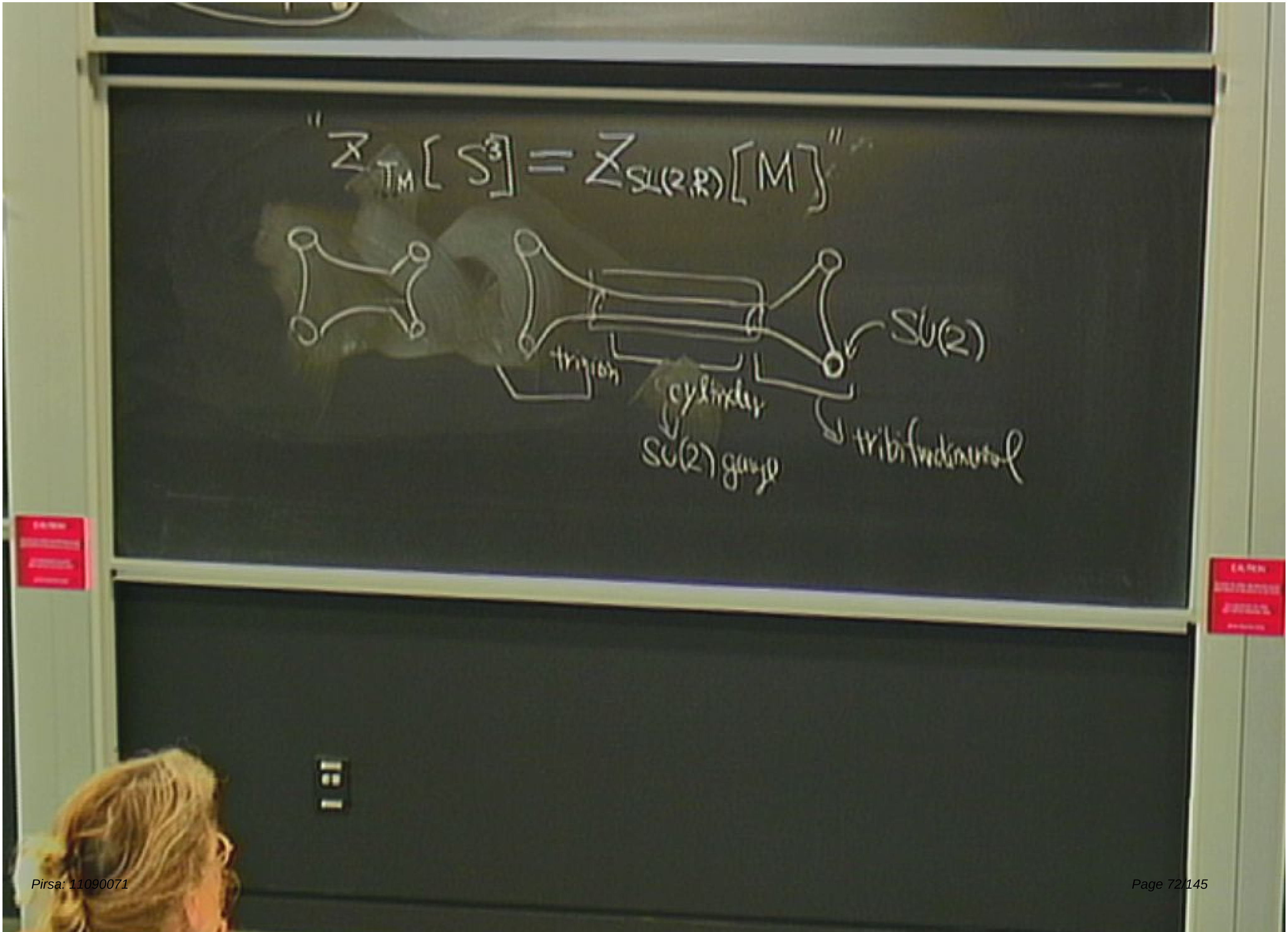

τ : cpx str.
φ
Σ
M = Σ x I
φ ∈ MCG(Σ)
4d N=2 SCFT
4d N=2 SCFT
R^{3,1}
τ
= 4πi/g² + θ/2π
φ ∈ (S-duality)
φ(τ)
R^{3,1}
τ
3d N=2 theory

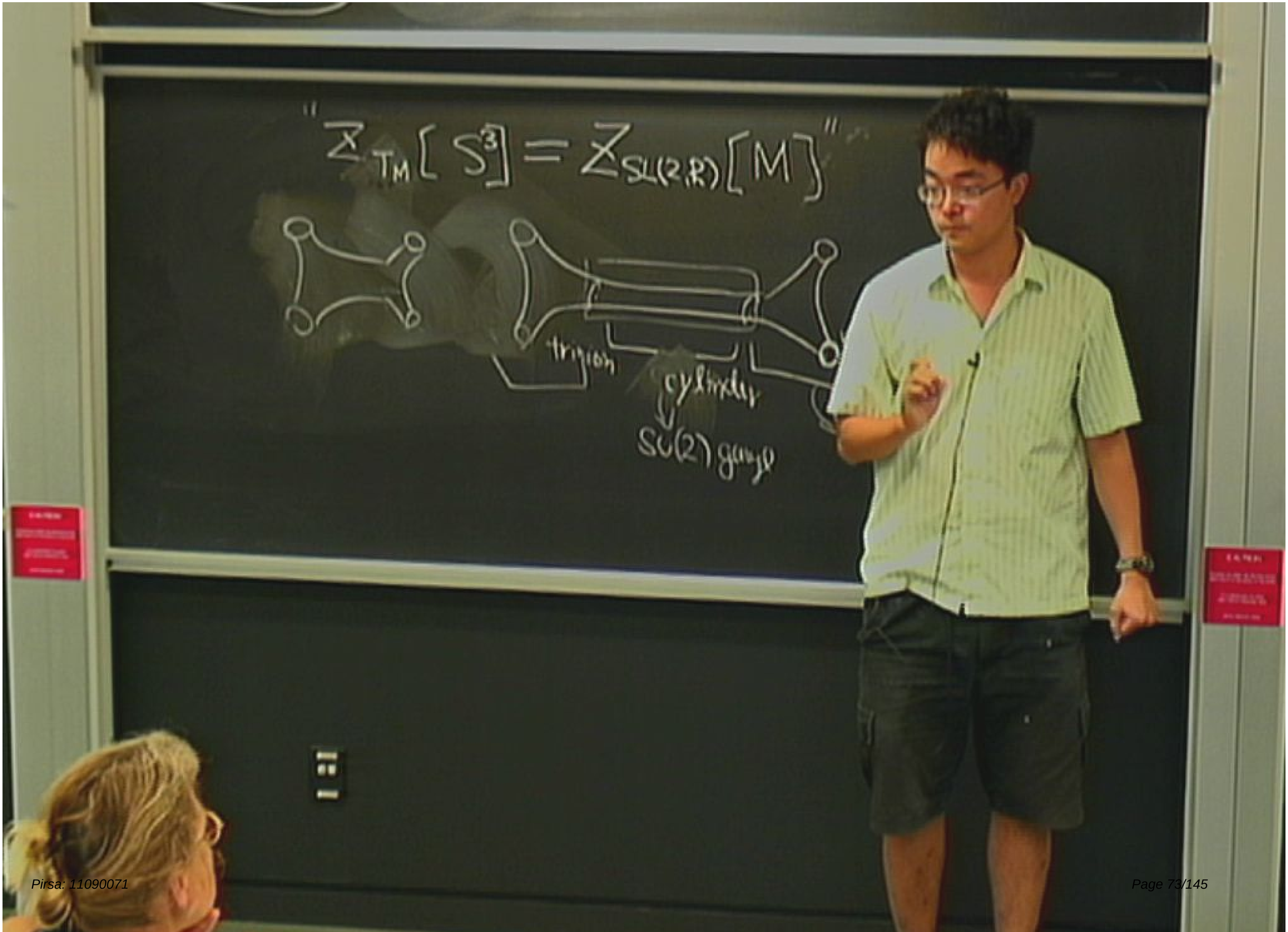


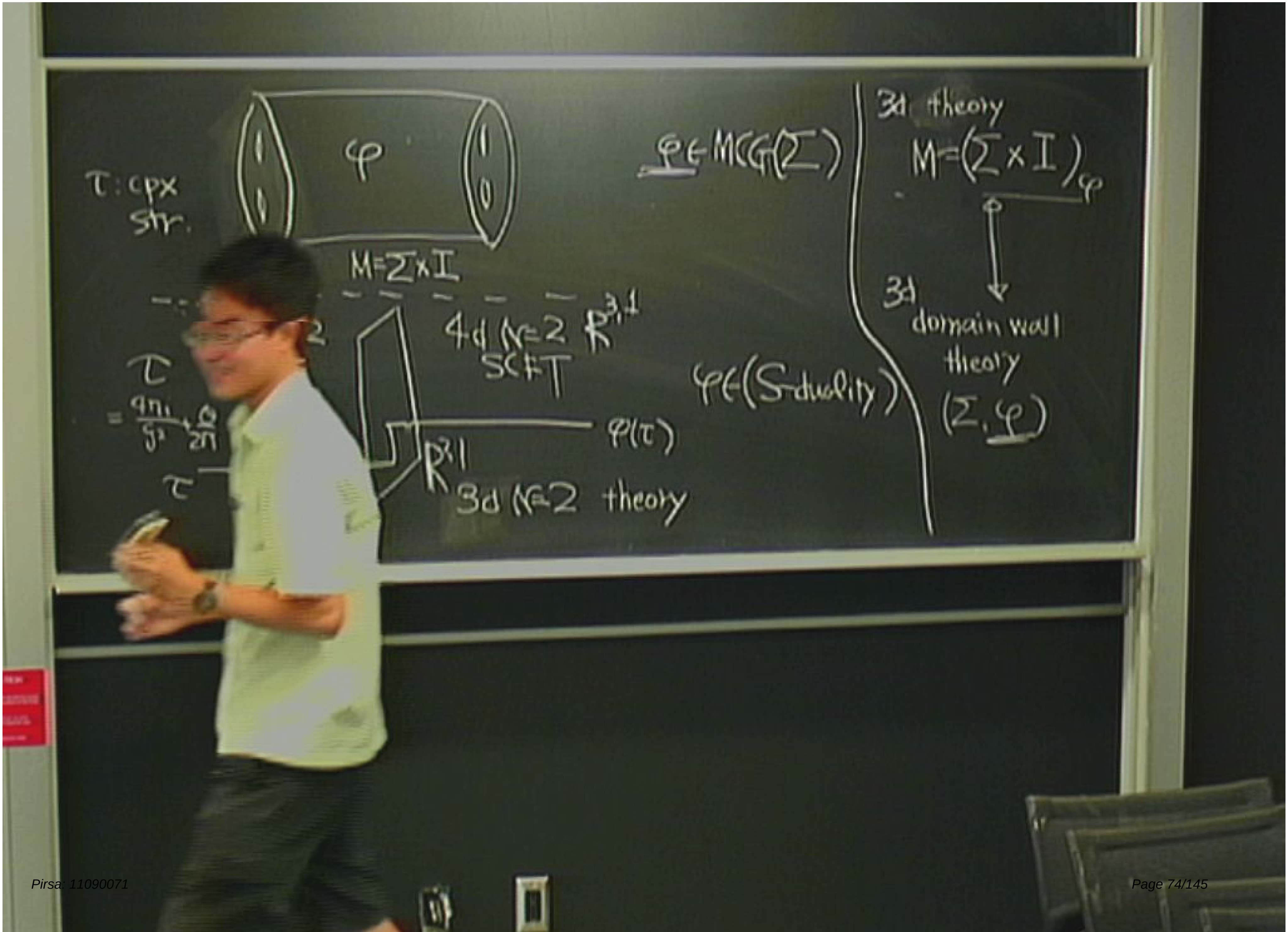
τ : cpx str.
φ
Σ
M=Σ×I
φ ∈ MCG(Σ)
M=(Σ×I)_φ
4d N=2 SCFT
4d N=2 SCFT
R^{3,1}
τ = 4πi/g² + θ/2π
φ ∈ (S-duality)
φ(τ)
R^{2,1}
τ
3d N=2 theory

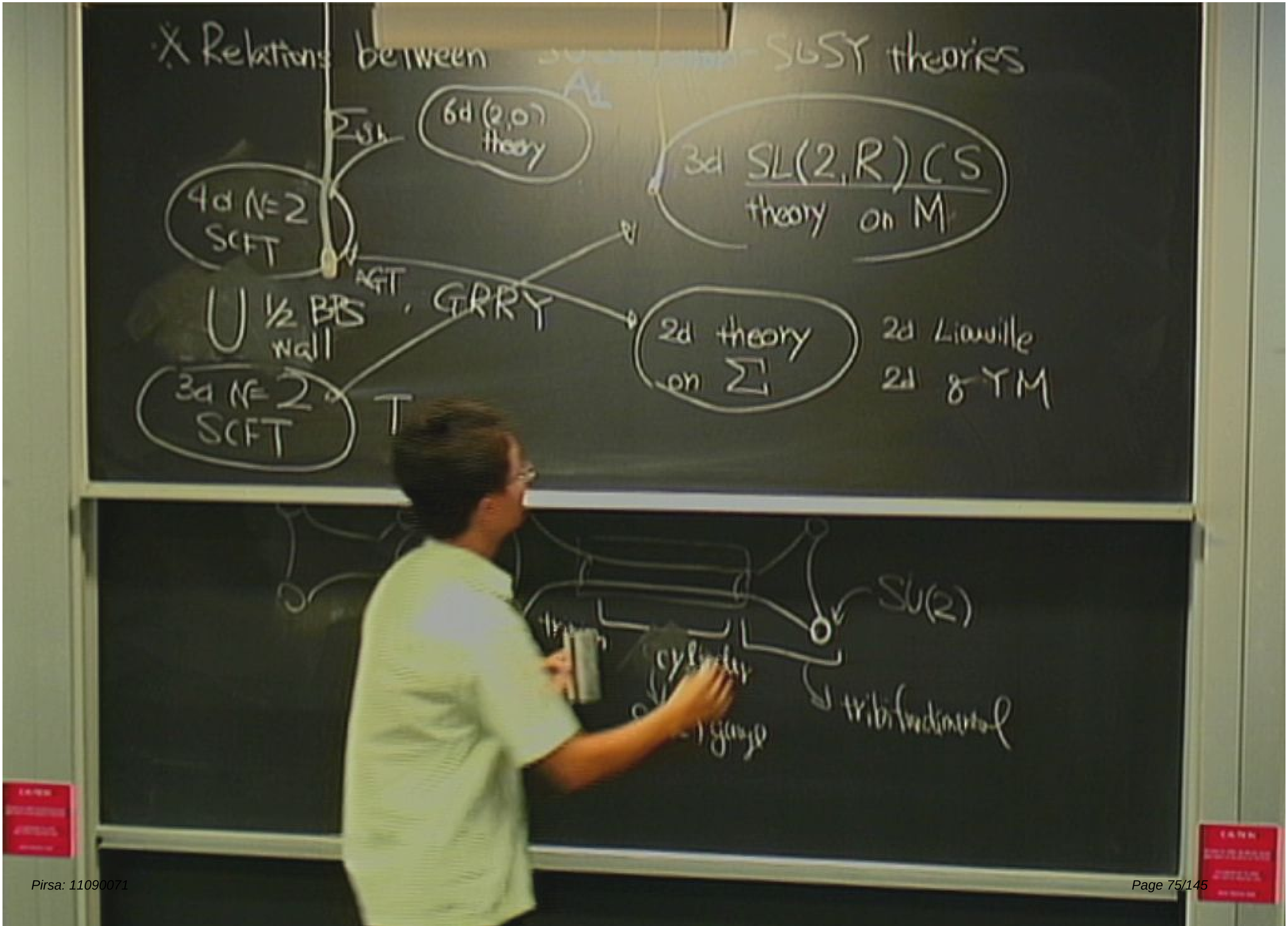

τ : cpx str.
φ
Σ
M = Σ × I
4d N=2 SCFT
4d N=2 SCFT
$\mathbb{R}^{3,1}$
τ = 4πi/g² + θ/2π
τ
φ(τ)
$\mathbb{R}^{3,1}$
3d N=2 theory
φ ∈ MCG(Σ)
M = (Σ × I)_φ
domain wall theory (Σ, φ)
φ ∈ (S-duality)


τ: cpx str.
φ
Σ
M=Σ×I
φ∈MCG(Σ)
3d CS
M=(Σ×I)φ
3d domain wall theory
(Σ, φ)
4d N=2 SCFT
4d N=2 SCFT R3,1
φ∈(S-duality)
φ(τ)
R3,1
3d N=2 theory

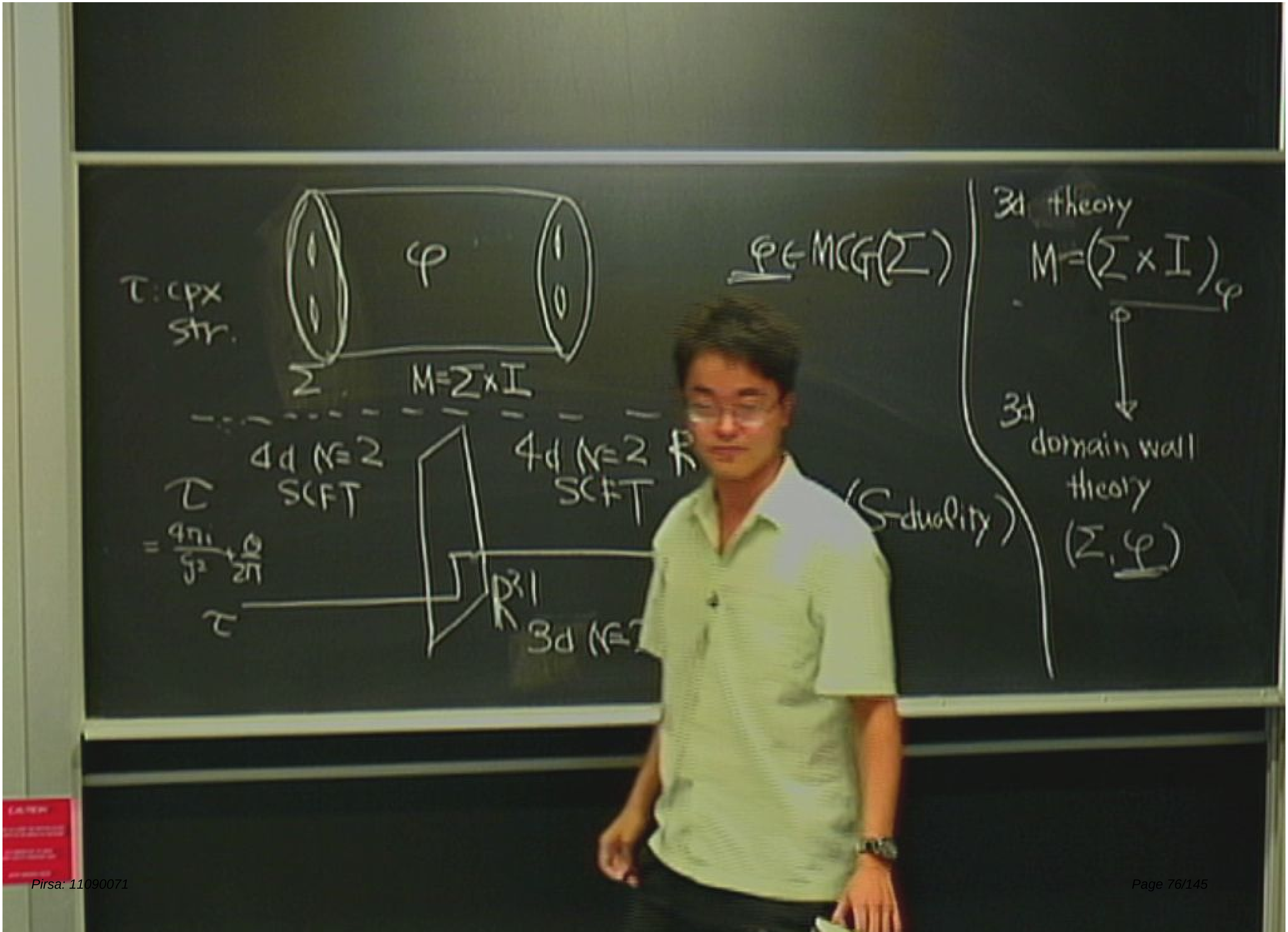

"$Z_{T_M}[S^3] = Z_{SL(2,\mathbb{R})}[M]$"
SU(2)
cylinder
SU(2) gauge
trifundamental

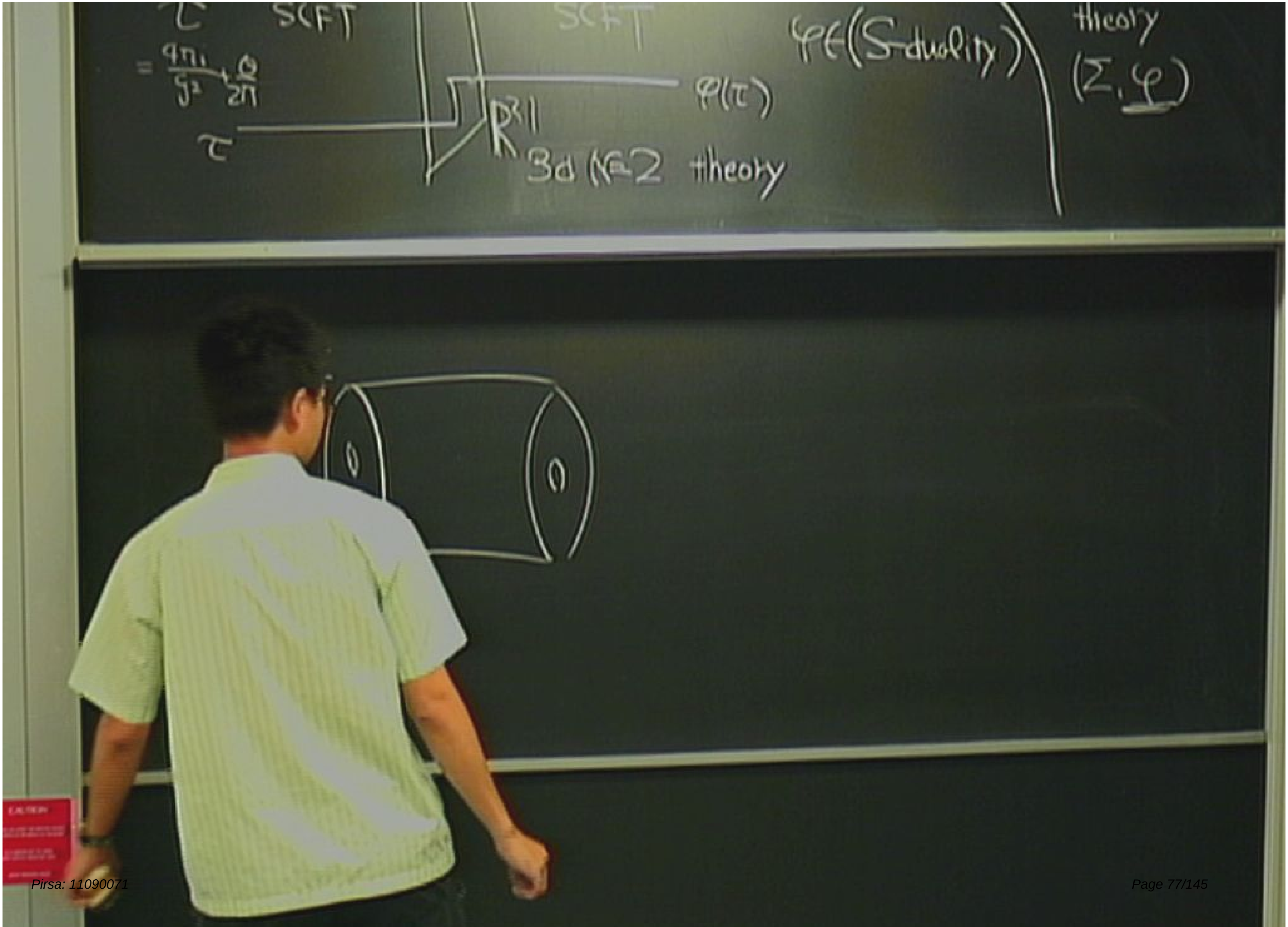

"Z_TM[S^3] = Z_SL(2,R)[M]"
trinion
cylinder
SU(2) gauge


τ: cpx str.
φ
M=Σ×I
4d N=2 R^{3,1}
SCFT
φ(τ)
R^{3,1}
3d N=2 theory
φ∈MCG(Σ)
φ∈(S-duality)
3d theory
M=(Σ×I)_φ
3d domain wall theory
(Σ, φ)

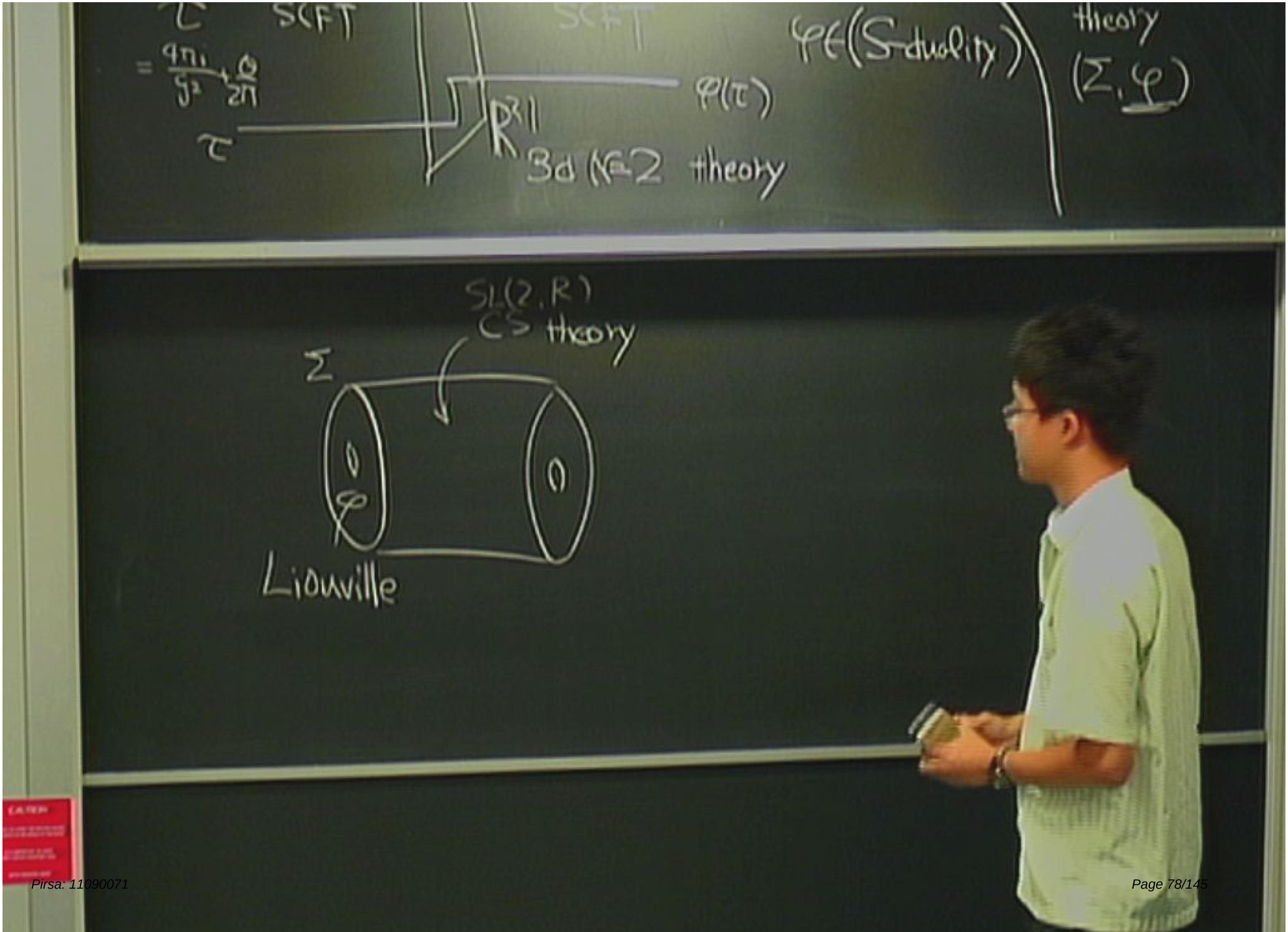


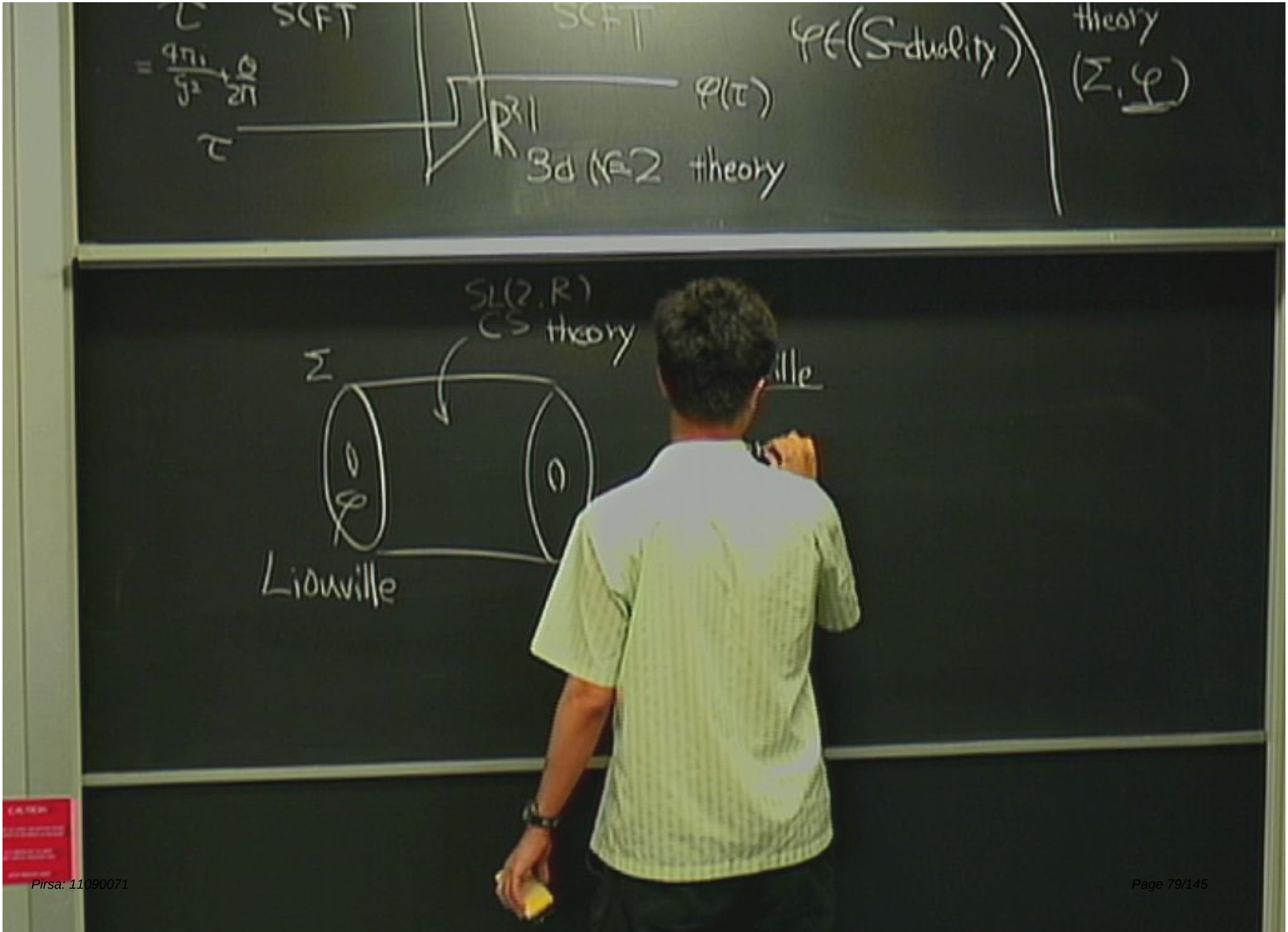
Relations between SUSY theories
6d (2,0) theory
$\Sigma_{3,h}$
3d SL(2,R) CS theory on M
4d N=2 SCFT
AGT
∪ ½ BPS wall
GRRY
2d theory on Σ
2d Liouville
2d q-YM
3d N=2 SCFT
T
SU(2)
cylinder
gauge
trifundamental

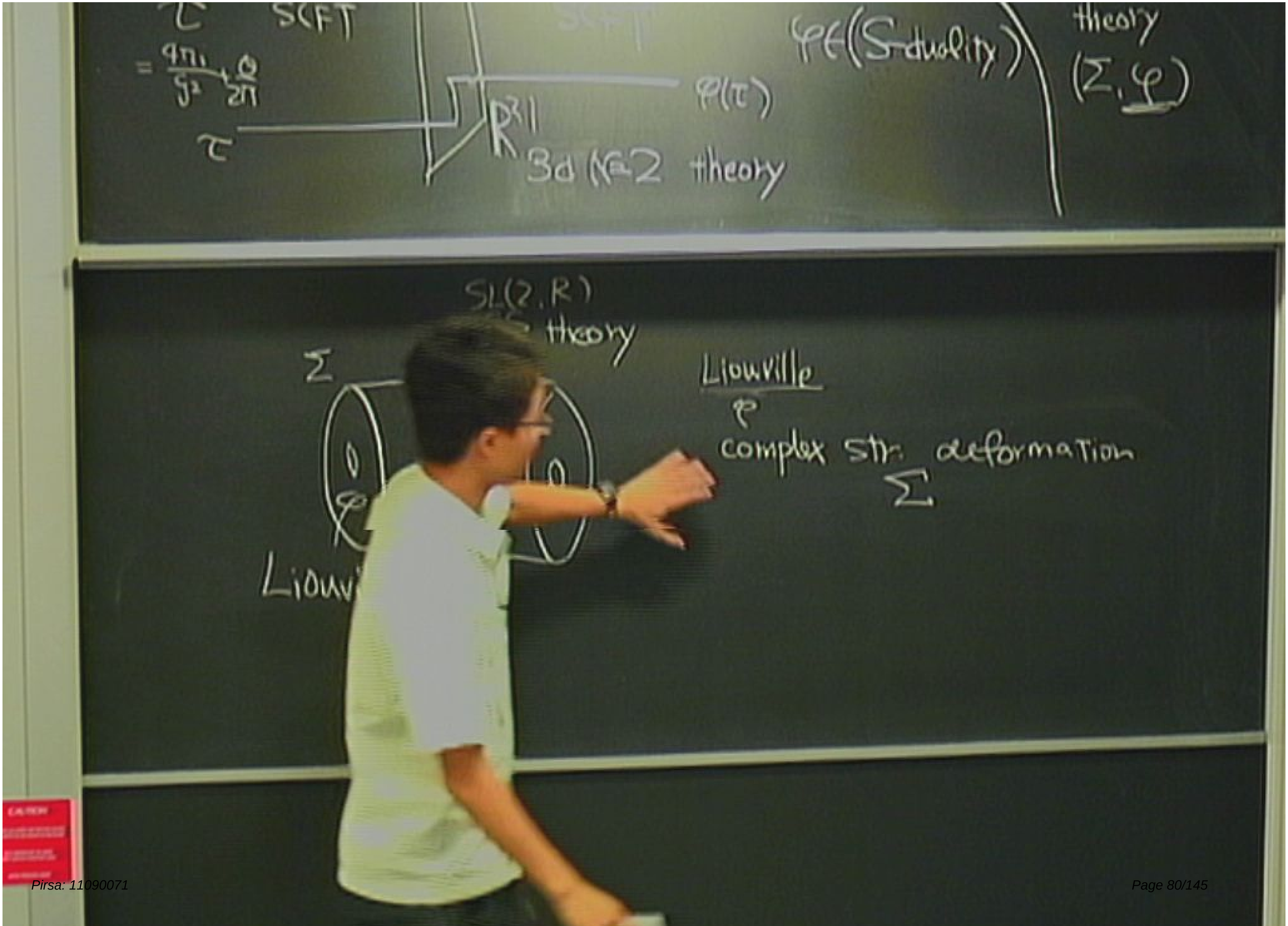

τ: cpx str.
Σ
φ
M=Σ×I
φ ∈ MCG(Σ)
3d theory
M=(Σ×I)_φ
3d domain wall theory
(Σ, φ)
τ
= 4πi/g² + θ/2π
4d N=2 SCFT
4d N=2 SCFT
(S-duality)
R³×I
3d N=2
τ

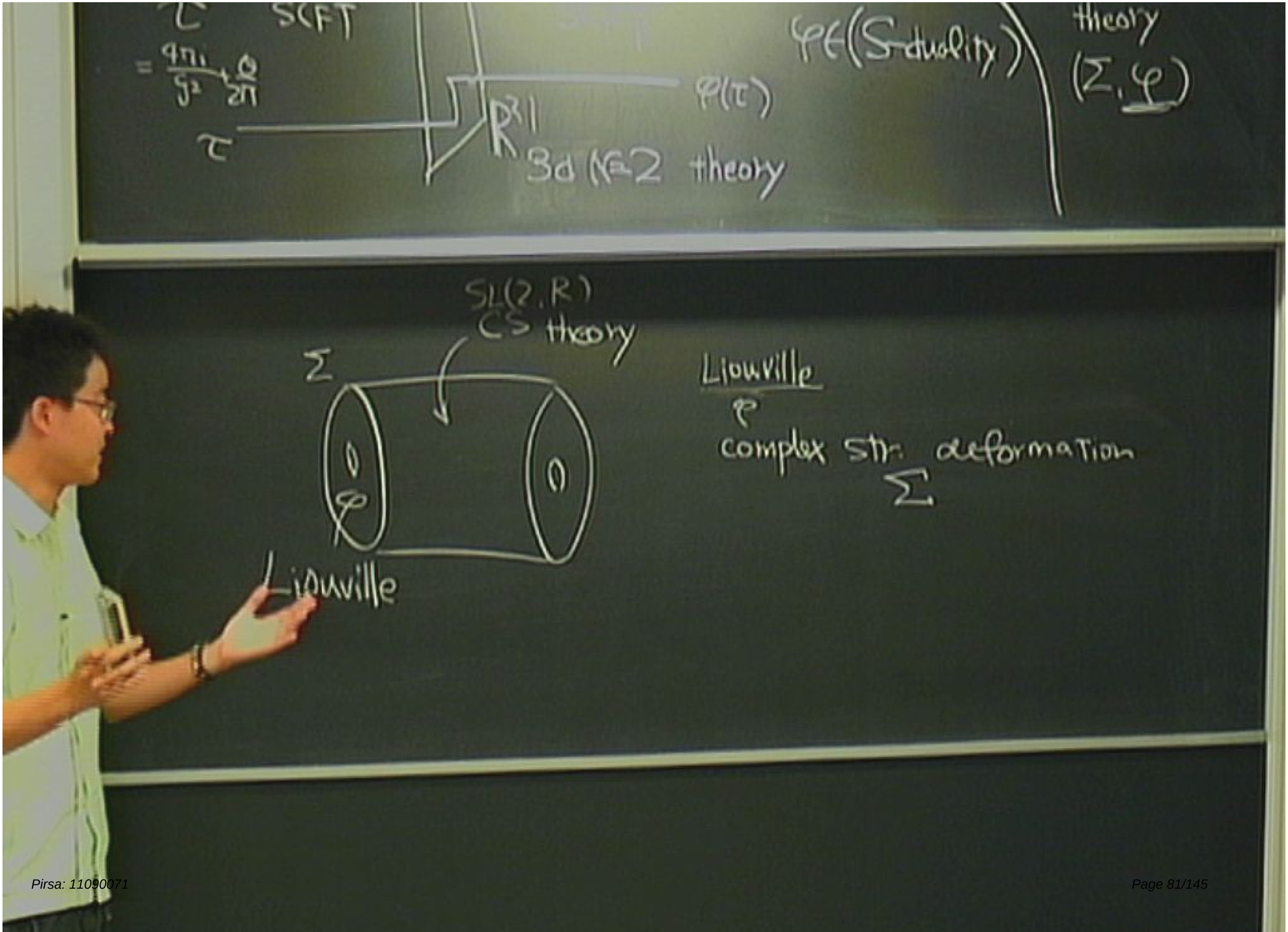


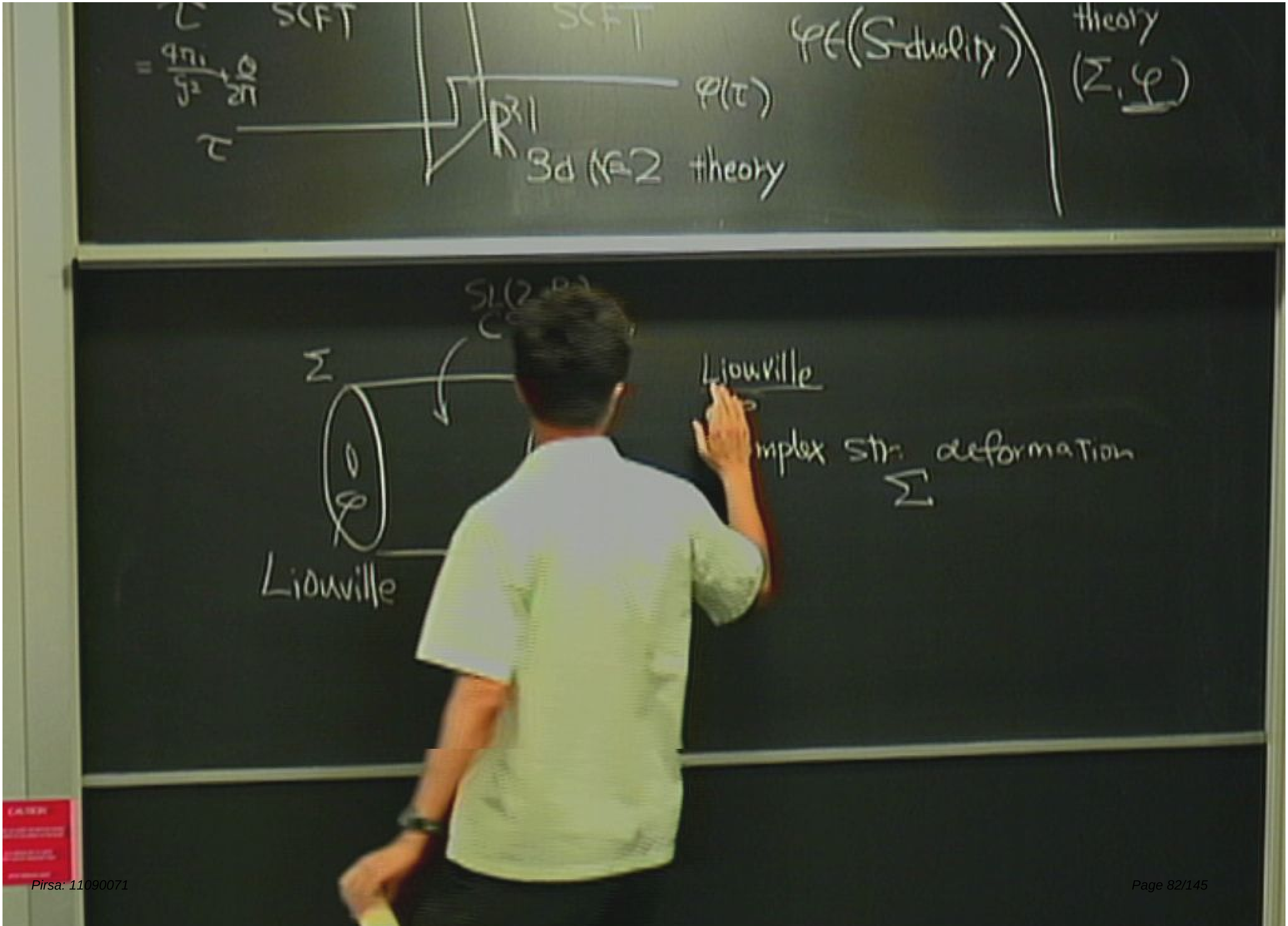
SCFT
SCFT
$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$
τ
$\varphi \in (S\text{-duality})$
theory
(Σ, φ)
$\varphi(\tau)$
$\mathbb{R}^{3,1}$
3d $\mathcal{N}=2$ theory

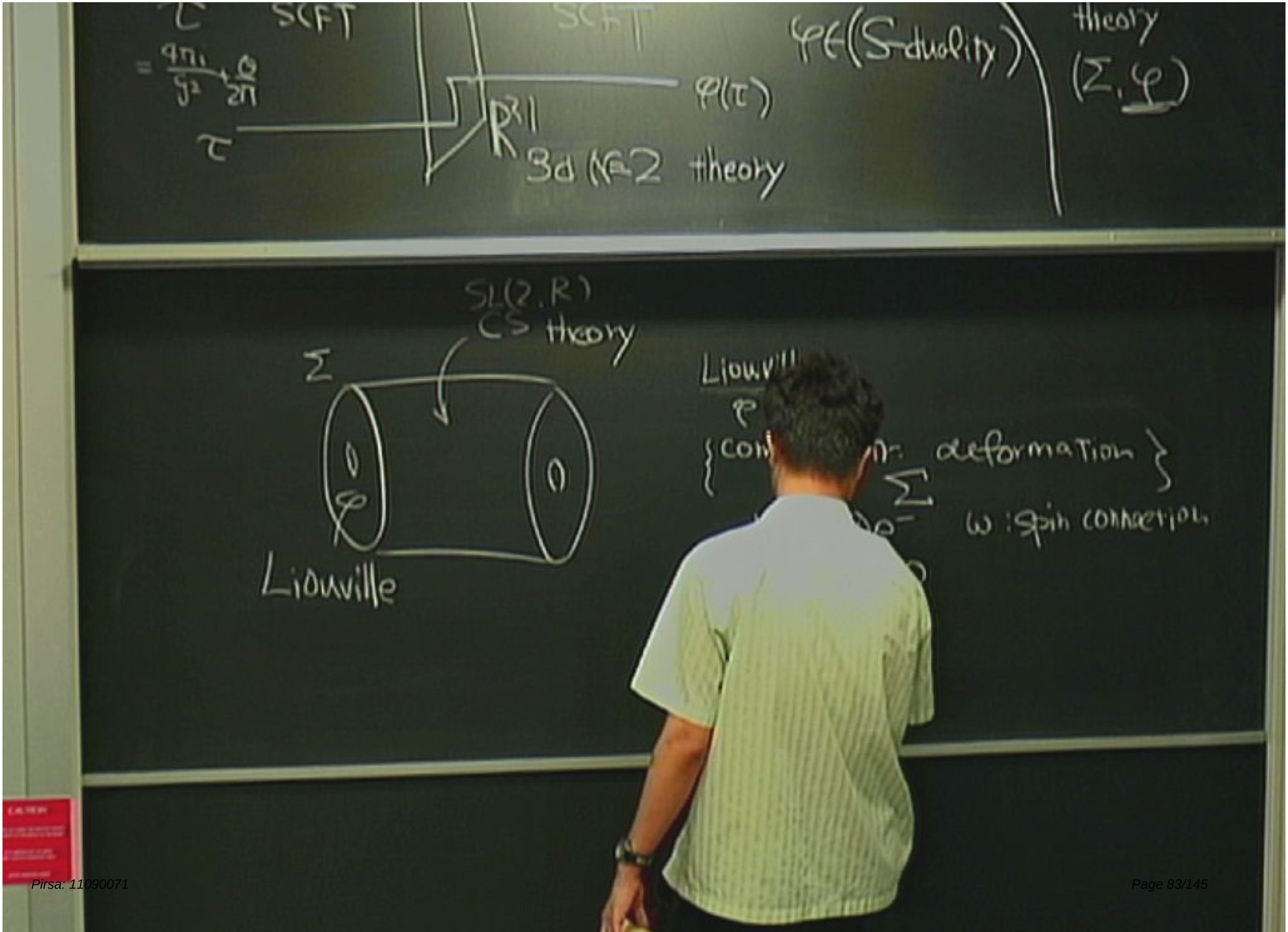

τ
SCFT
SCFT
φ ∈ (S-duality)
theory
(Σ, φ)
= 4πi/g² + θ/2π
τ
φ(τ)
3d N=2 theory
SL(2,R)
CS theory
Σ
φ
Liouville

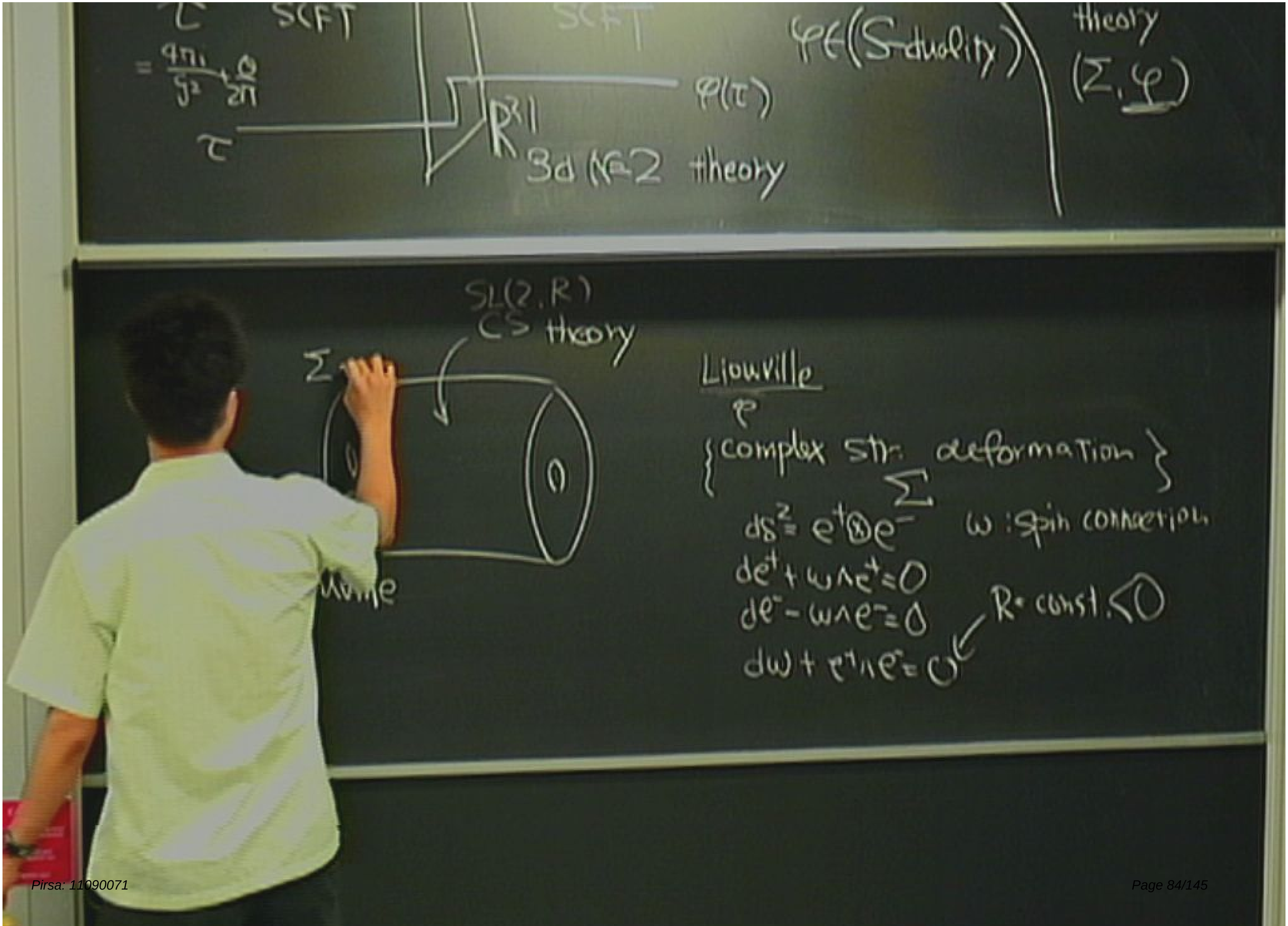

SCFT
SCFT
theory
φ ∈ (S-duality)
(Σ, φ)
φ(τ)
τ
3d N=2 theory
SL(2,R)
CS theory
Σ
Liouville

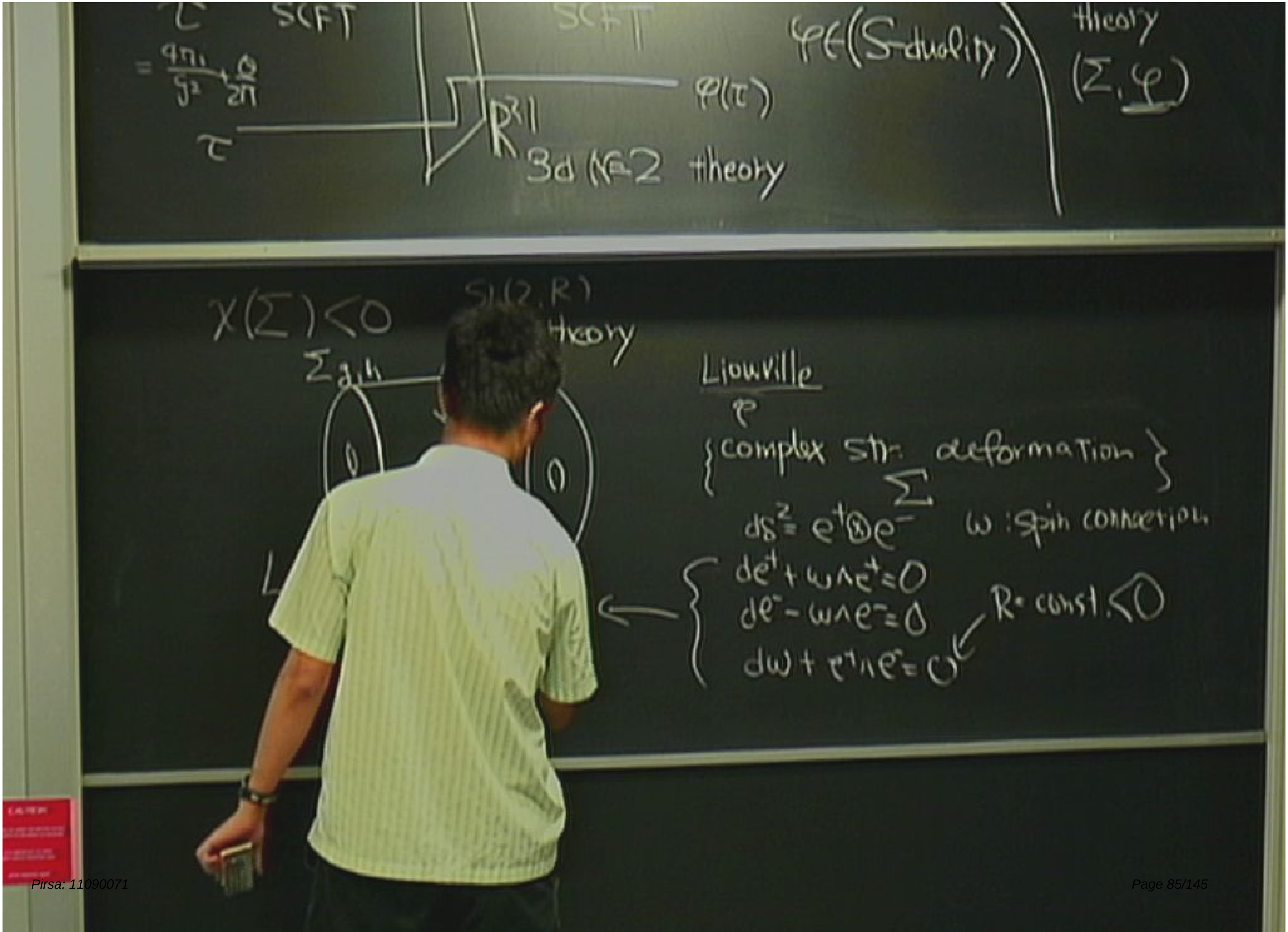

SCFT
theory
(Σ, φ)
φ(τ)
3d N=2 theory
SL(2,R)
theory
Σ
Liouville
φ
complex str. deformation
Σ

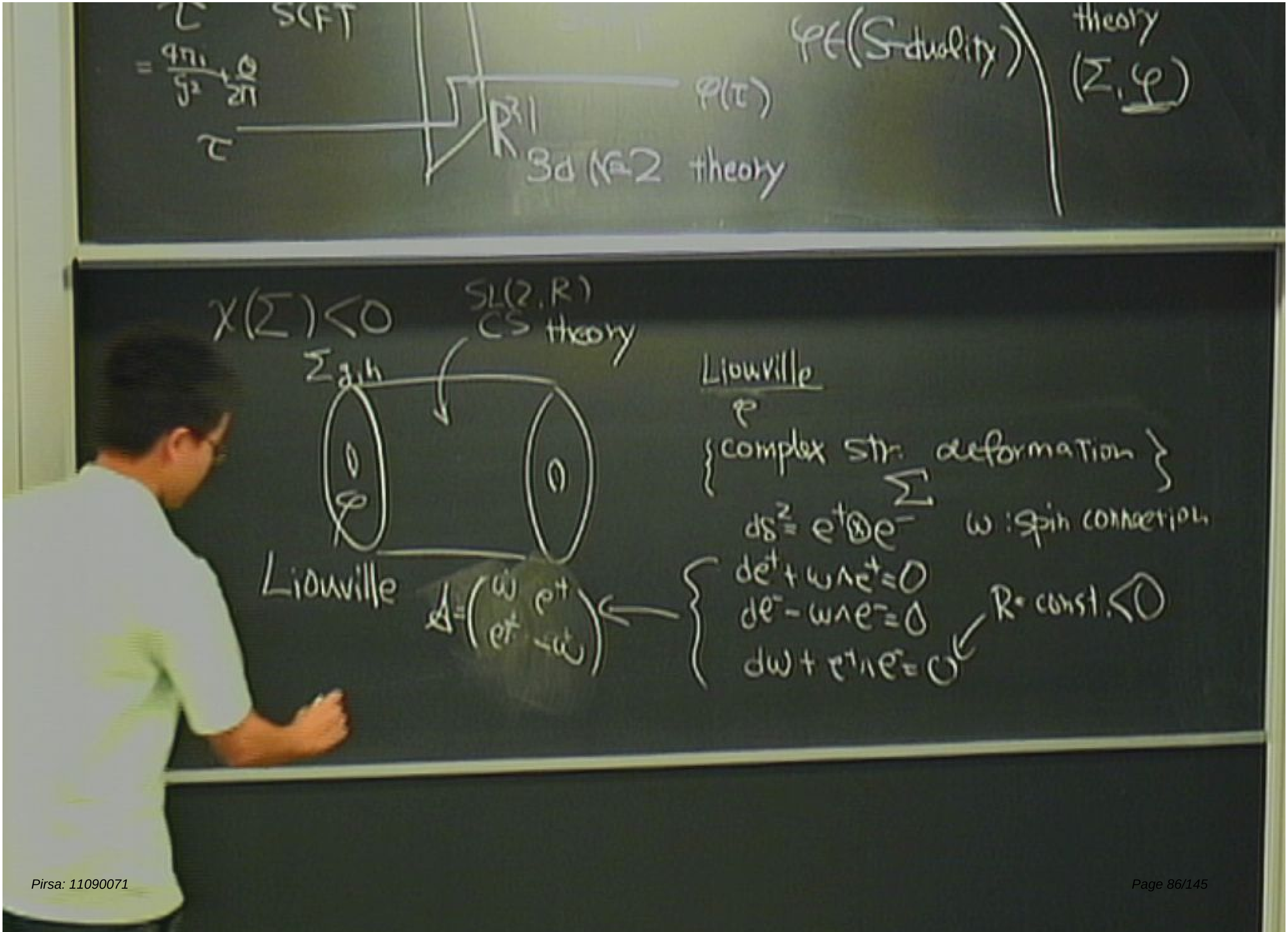

SCFT
φ ∈ (S-duality)
theory
(Σ, φ)
φ(τ)
3d N=2 theory
SL(2, R)
CS theory
Σ
Liouville
φ
complex str. deformation
Σ
φ
Liouville

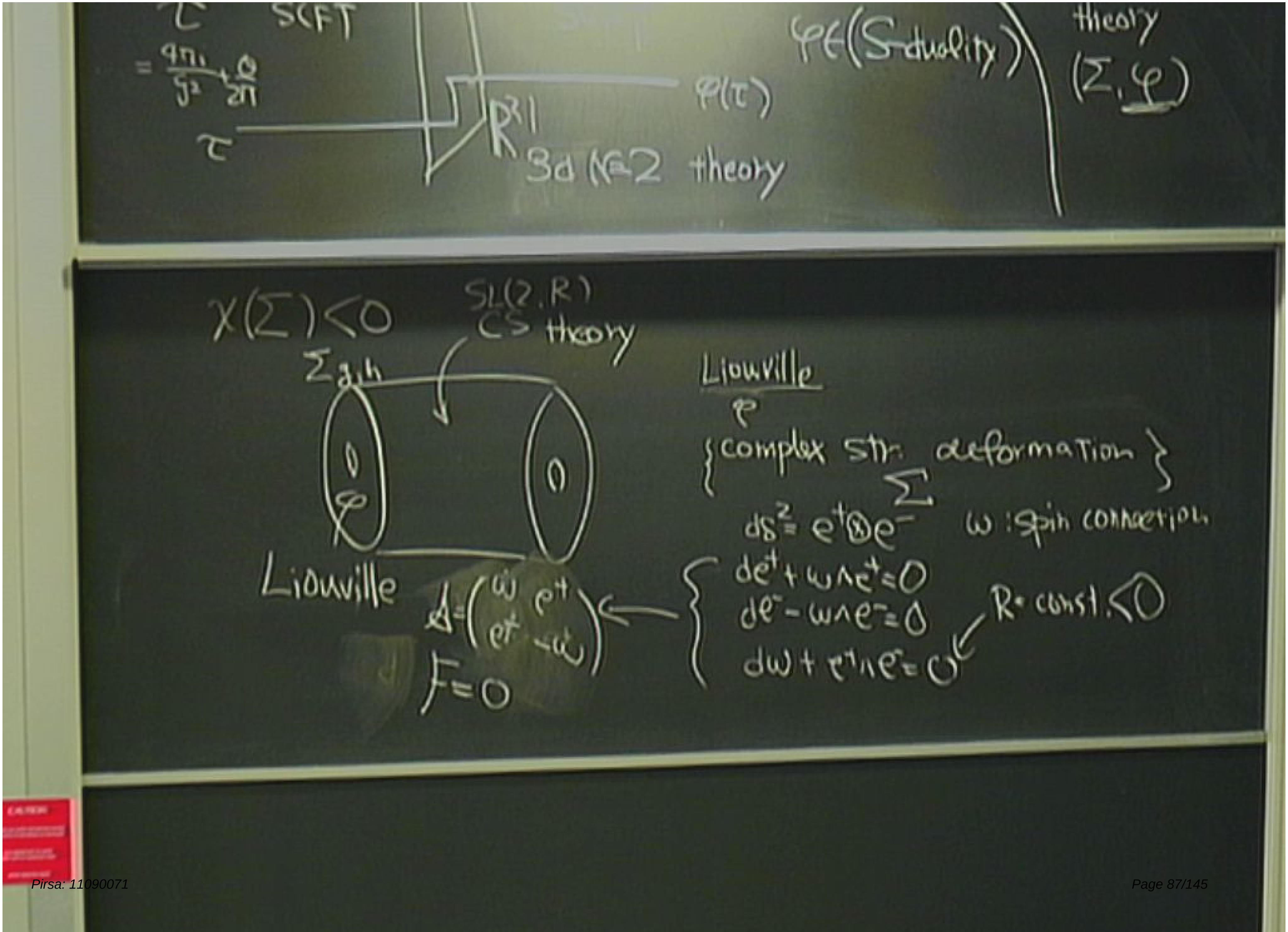

SCFT
SCFT
theory
(Σ, φ)
$\varphi \in$ (S-duality)
$\varphi(\tau)$
3d N=2 theory
Σ
Liouville
Liouville
mplex str. deformation
Σ

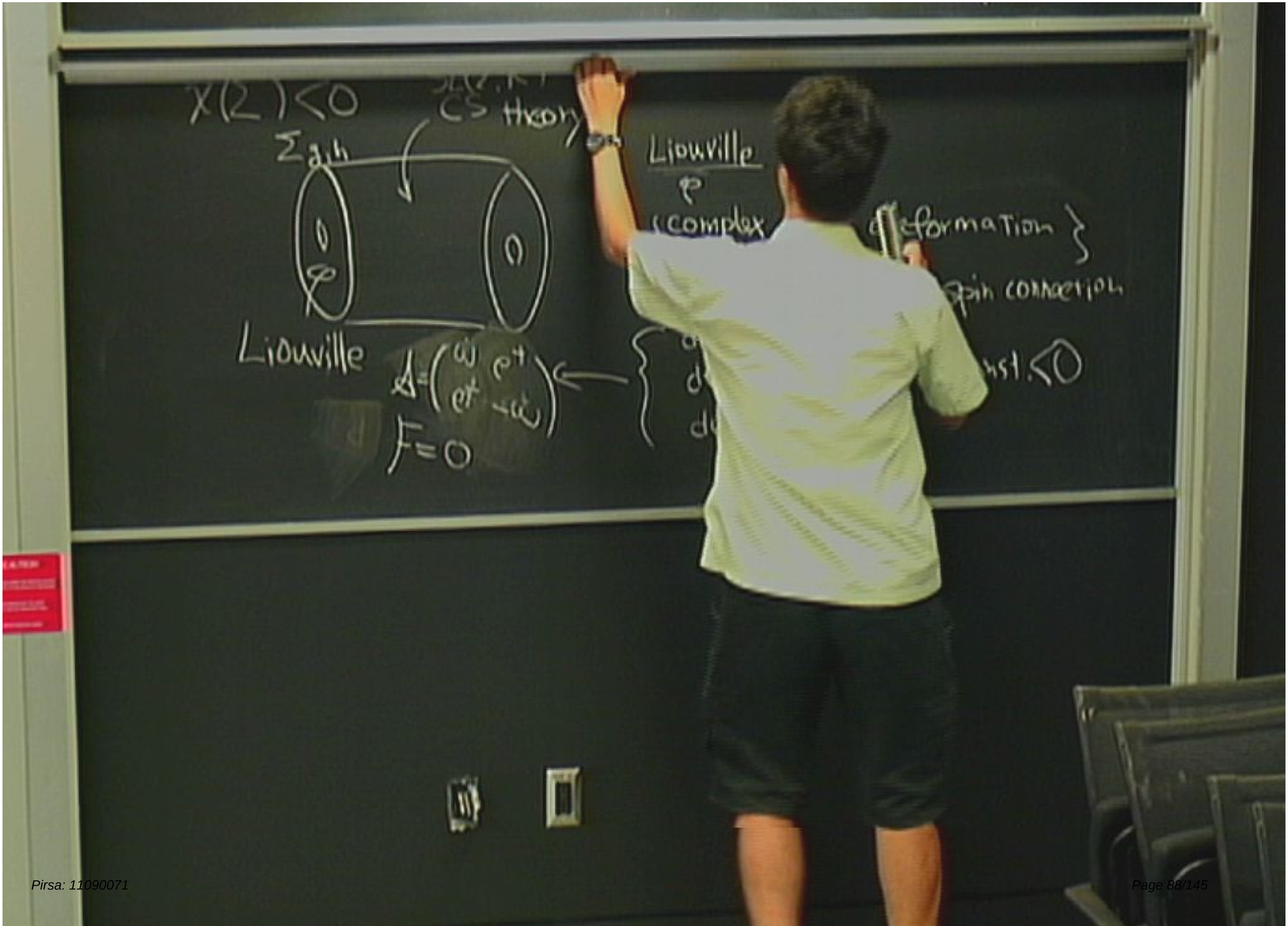

SCFT
theory
(S-duality)
3d N=2 theory
SL(2,R)
CS theory
Liouville
deformation
ω : spin connection
Liouville

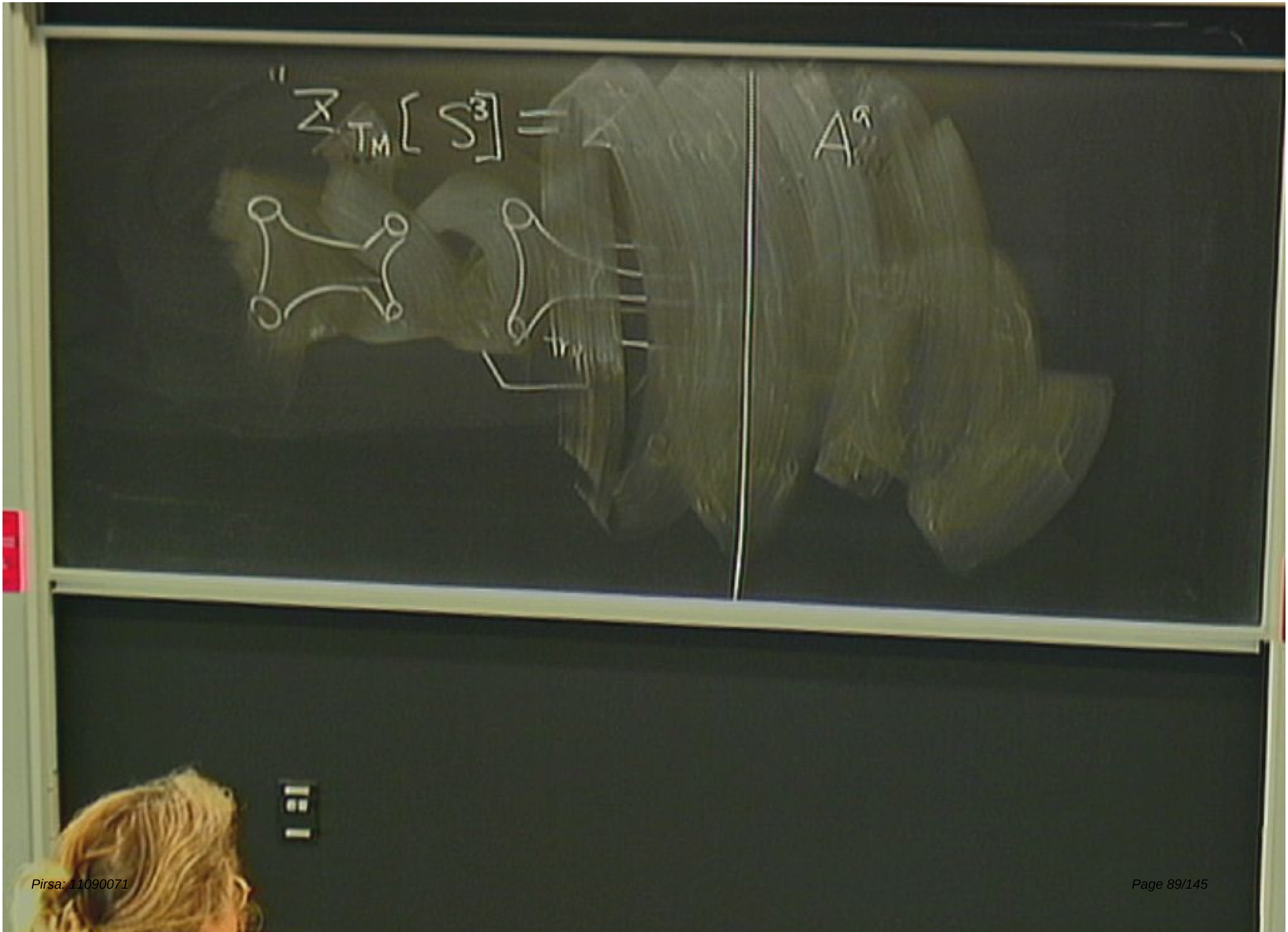

τ
SCFT
SCFT
= 4πi/g² + θ/2π
τ
φ(τ)
$R^3 \times I$
3d N=2 theory
φ∈(S-duality)
theory
(Σ, φ)
SL(2,R)
CS theory
Σ
Liouville
φ
{complex str. deformation}
Σ
$ds^2 = e^+ \otimes e^-$
ω : spin connection
$de^+ + \omega \wedge e^+ = 0$
$de^- - \omega \wedge e^- = 0$
$d\omega + e^+ \wedge e^- = 0$
R = const. < 0

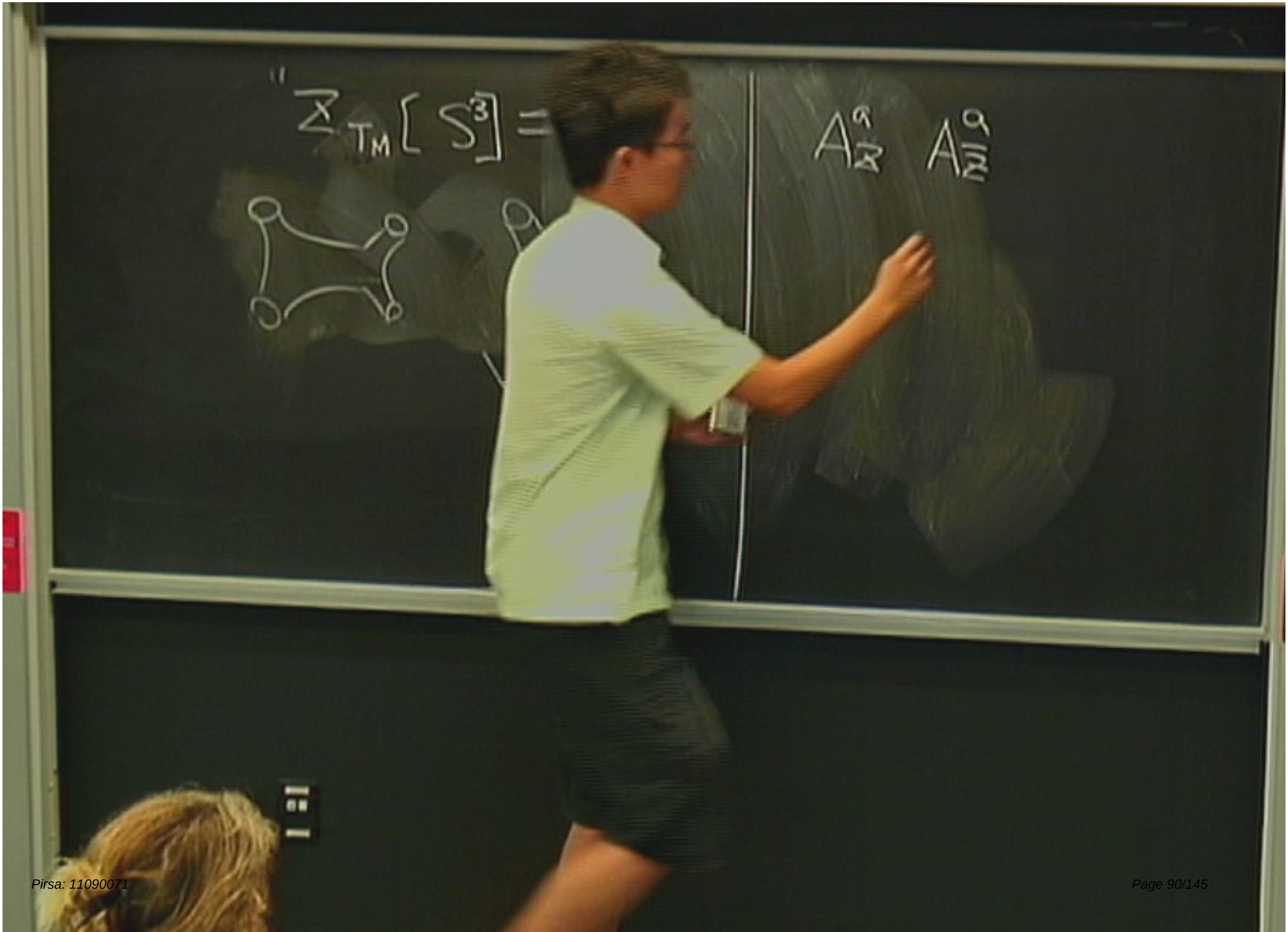

SCFT
SCFT
φ ∈ (S-duality)
theory
(Σ, φ)
φ(τ)
3d N=2 theory
χ(Σ) < 0
SL(2,R) theory
Liouville
{complex str. deformation}
ds² = e⁺ ⊗ e⁻
ω : spin connection
de⁺ + ω∧e⁺ = 0
de⁻ − ω∧e⁻ = 0
dω + e⁺∧e⁻ = 0
R = const. < 0

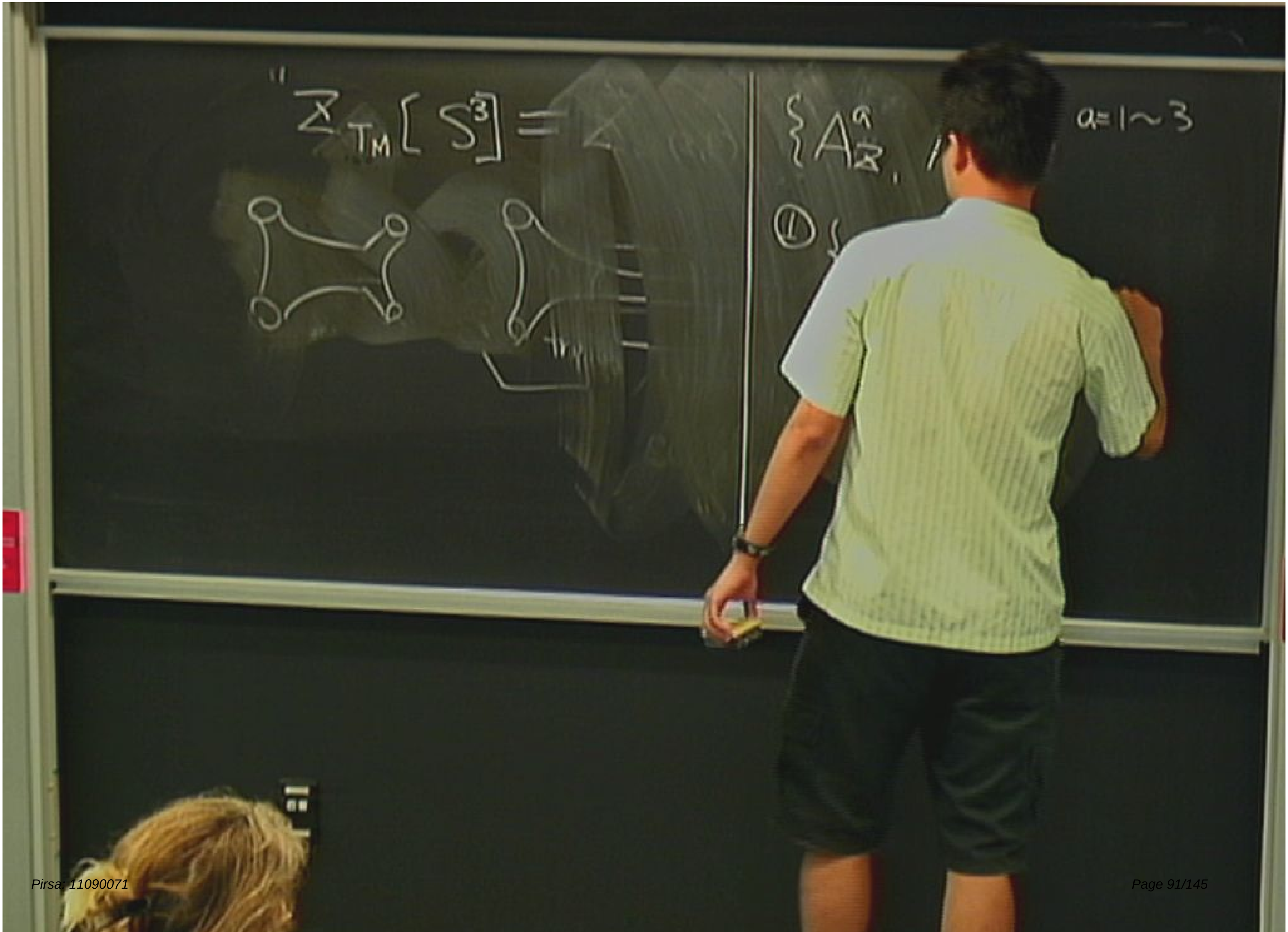

SCFT
τ
$= \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$
$\mathbb{R}^3 \times I$
3d N=2 theory
$\varphi(\tau)$
$\varphi \in$ (S-duality)
theory
(Σ, φ)
$\chi(\Sigma) < 0$
SL(2,R)
CS theory
$\Sigma_{g,h}$
φ
Liouville
Liouville
φ
{complex str. deformation}
Σ
$ds^2 = e^+ \otimes e^-$
ω : spin connection
$de^+ + \omega \wedge e^+ = 0$
$de^- - \omega \wedge e^- = 0$
$d\omega + e^+ \wedge e^- = 0$
R = const. < 0
$A = \begin{pmatrix} \omega & e^+ \\ e^- & -\omega \end{pmatrix}$

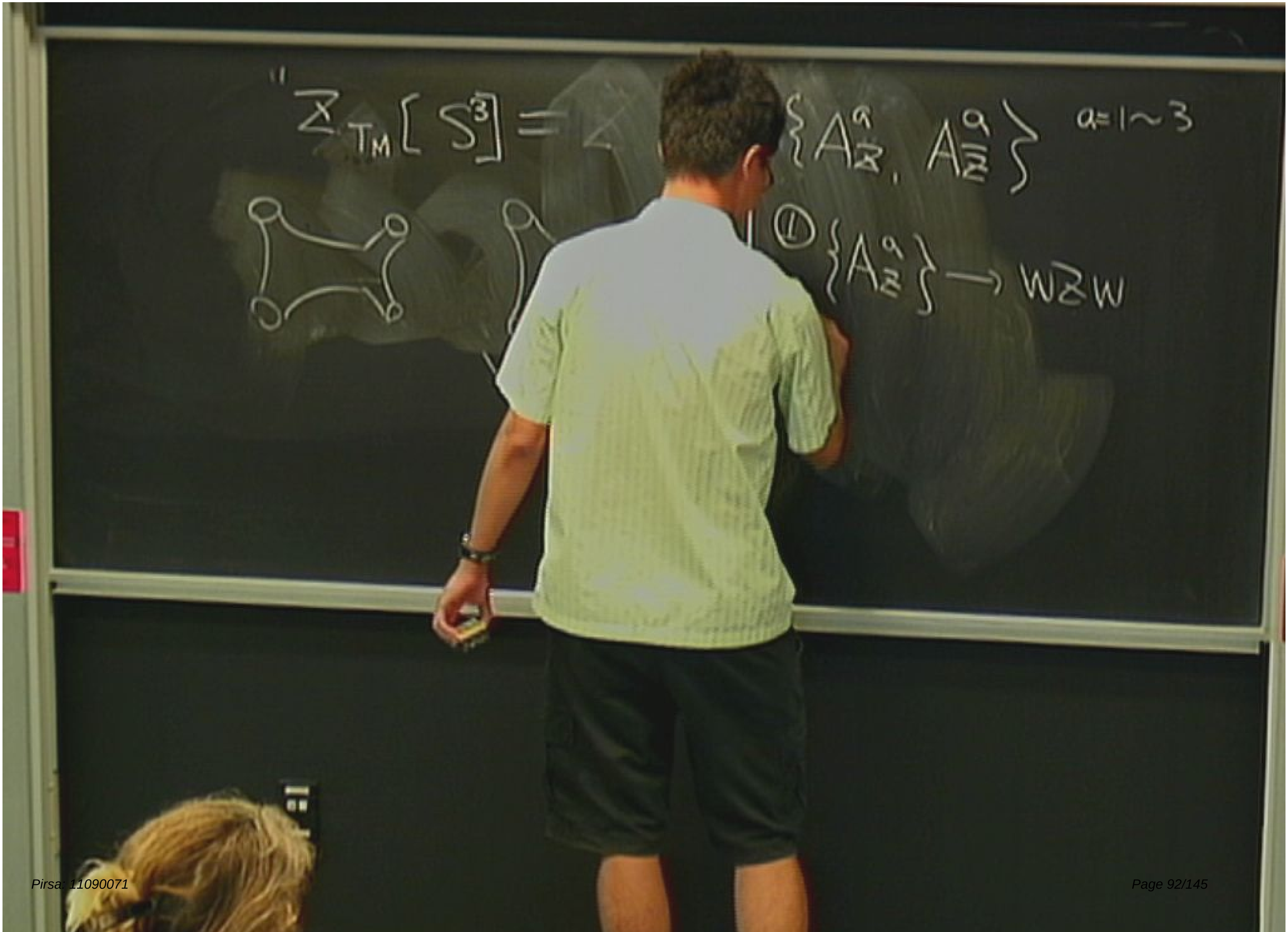

SCFT
= 4πi/g² + θ/2π
τ
φ(τ)
φ ∈ (S-duality)
R³1
3d N=2 theory
theory
(Σ, φ)
χ(Σ) < 0
SL(2,R)
CS theory
Σ_{g,h}
Liouville
Liouville
φ
{complex str. deformation}
Σ
ds² = e⁺⊗e⁻
ω : spin connection
de⁺ + ω∧e⁺ = 0
de⁻ − ω∧e⁻ = 0
dω + e⁺∧e⁻ = 0
R = const. < 0
A = (ω e⁺ ; e⁺ −ω)
F = 0


CS theory
Liouville
complex
deformation
spin connection
Liouville
F=0

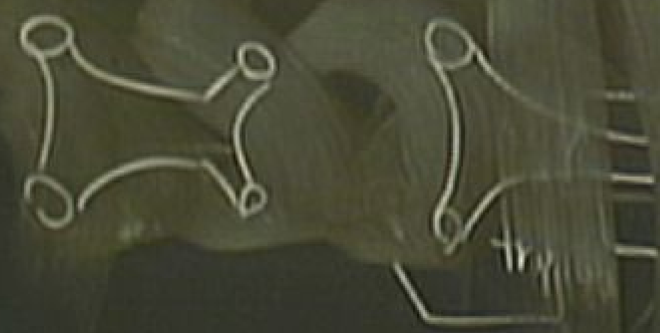

"Z_TM[S^3] =
A^q
tr_V


"Z_TM [S³] =
A^a_z A^a_z̄


"Z_TM[S³] =
{A^a_z,
a=1~3


"$Z_{T_M}[S^3] = Z$
$\{A^a_z, A^a_{\bar{z}}\}$
$a = 1 \sim 3$
① $\{A^a_z\} \longrightarrow$ WZW

" $Z_{TM}[S^3] =$



$\{A^a_z, A^a_{\bar{z}}\}$ $a = 1 \sim 3$

① $\{A^a_z\} \longrightarrow$ WZW

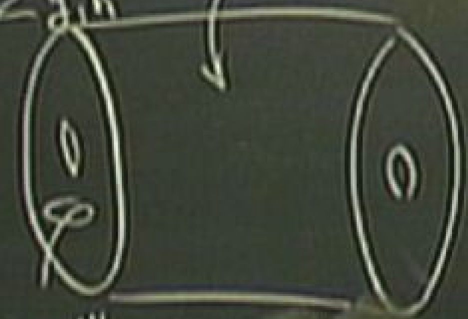
② $\{e^+_z, e^+_{\bar{z}}, \omega_z\}$

3d $\mathcal{N}=2$ theory

$\chi(\Sigma) < 0$

$SL(2,\mathbb{R})$ CS theory

$\Sigma_{g,h}$



Liouville

$$A = \begin{pmatrix} \omega & e^{+} \\ e^{-} & -\omega \end{pmatrix} \longleftarrow \begin{cases} de^{+} + \omega \wedge e^{+} = 0 \\ de^{-} - \omega \wedge e^{-} = 0 \\ d\omega + e^{+} \wedge e^{-} = 0 \end{cases}$$

$F = 0$

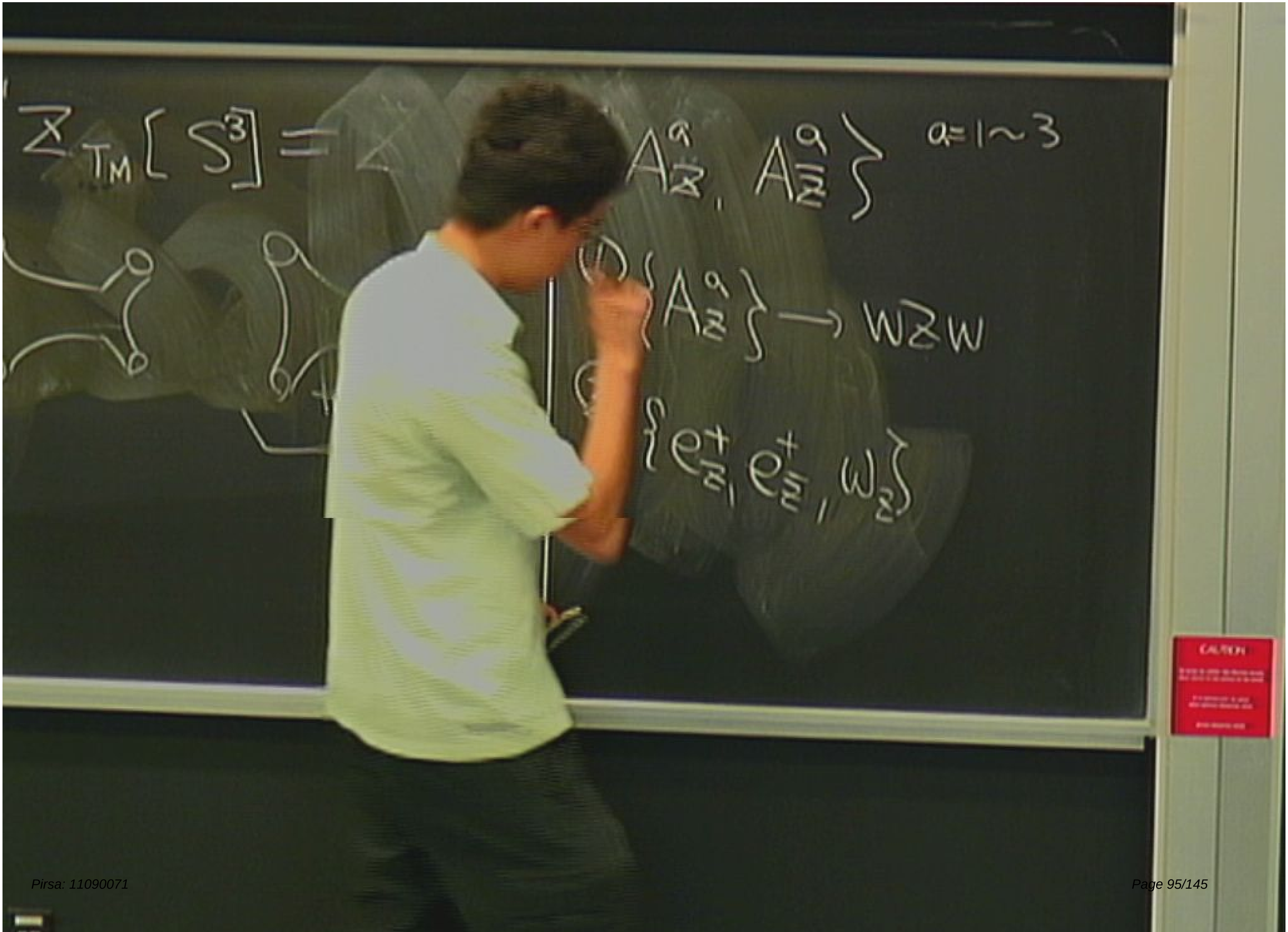
Liouville

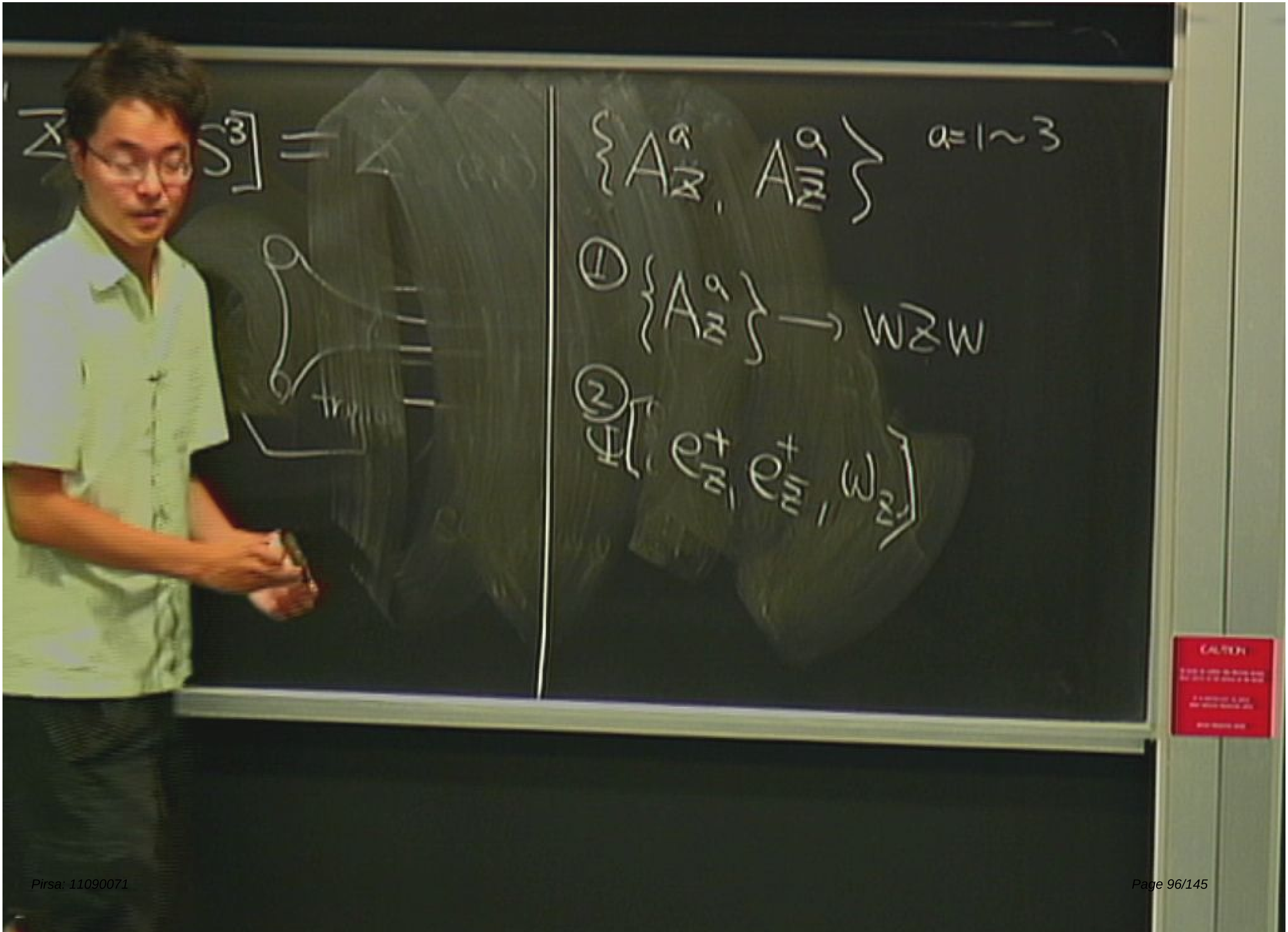
φ

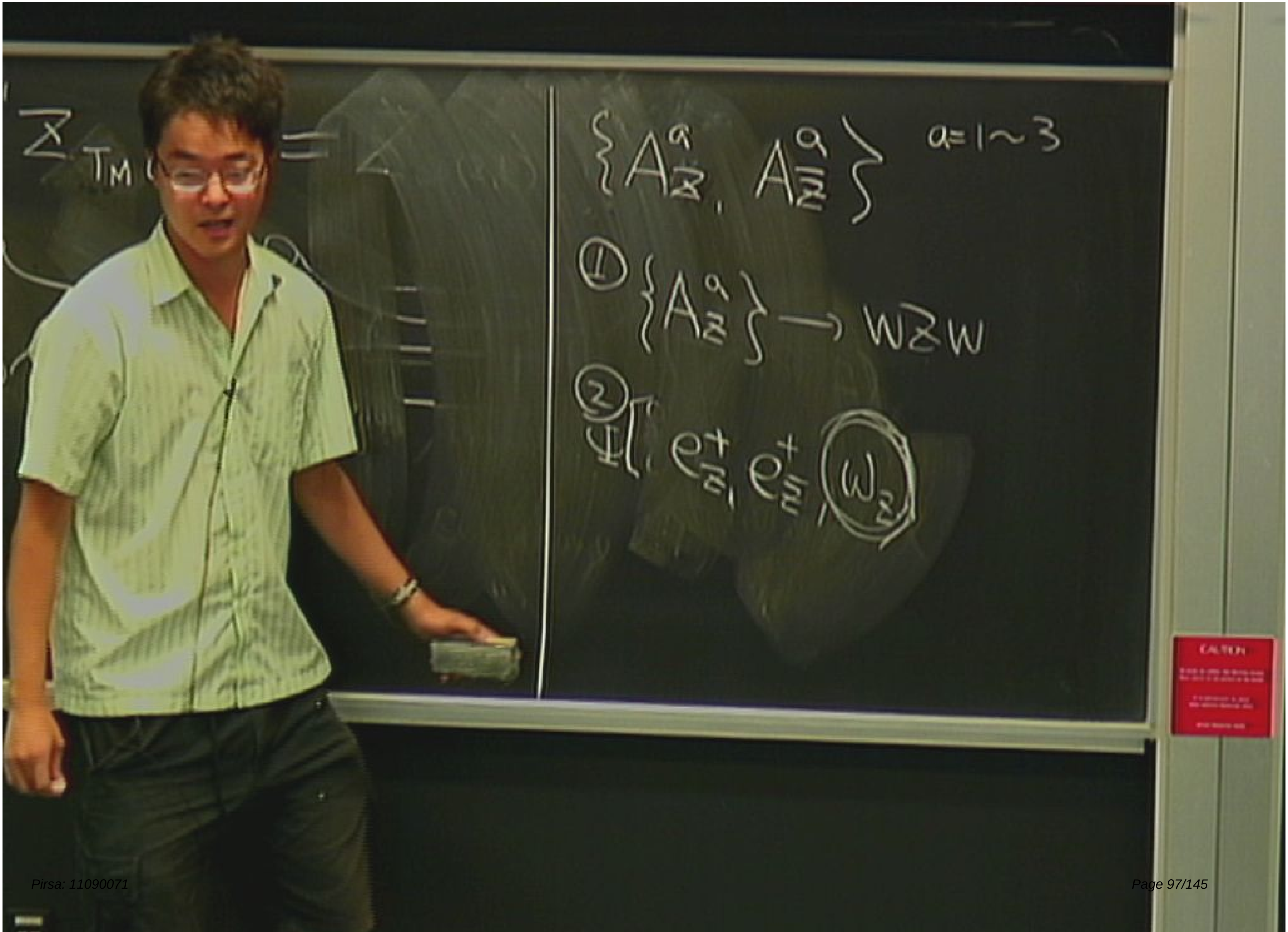
$\{$ complex str. deformation Σ $\}$

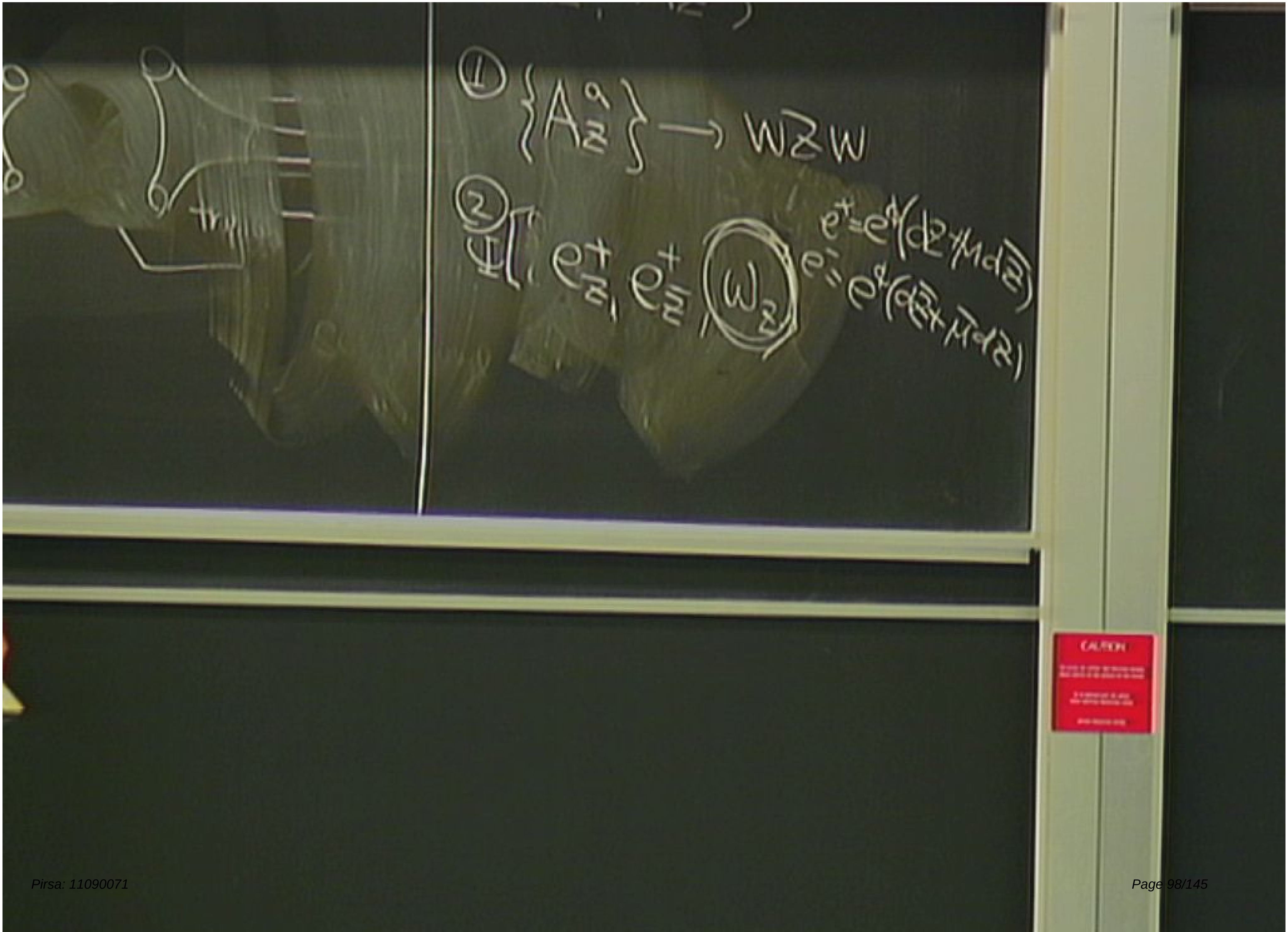
$ds^2 = e^{+} \otimes e^{-}$ ω : spin connection

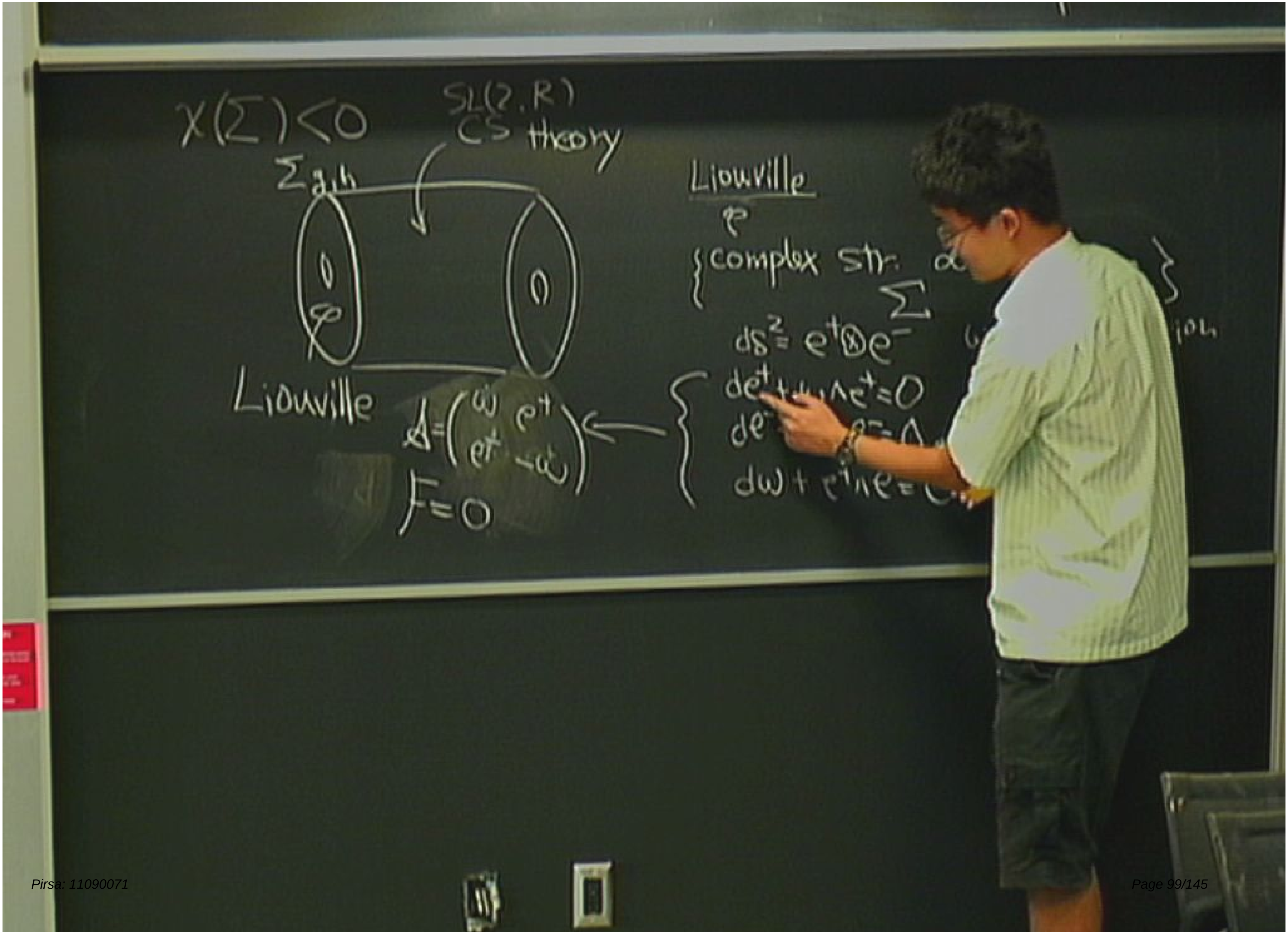
$R = \text{const.} < 0$


$Z_{TM}[S^3] = \int \{A^a_z, A^a_{\bar{z}}\}$ $a = 1 \sim 3$
$\mathcal{D}\{A^a_z\} \longrightarrow$ WZW
$\{e^+_z, e^+_{\bar{z}}, \omega_z\}$
CAUTION


$Z[S^3] =$
$\{A^a_z, A^a_{\bar z}\}$
$a = 1 \sim 3$
① $\{A^a_z\} \longrightarrow$ WZW
② $\Phi[e^+_z, e^+_{\bar z}, \omega_z]$
CAUTION


{A^a_z, A^a_z̄} a=1~3
① {A^a_z} → WZW
② Φ[e^+_z, e^+_z̄, ω_z


① $\{A^a_z\} \longrightarrow$ WZW
② $\Phi[\,, e^+_z, e^+_{\bar z}, \omega_z]$
$e^+ = e^\phi(dz + \mu\, d\bar z)$
$e^- = e^\phi(d\bar z + \bar\mu\, dz)$

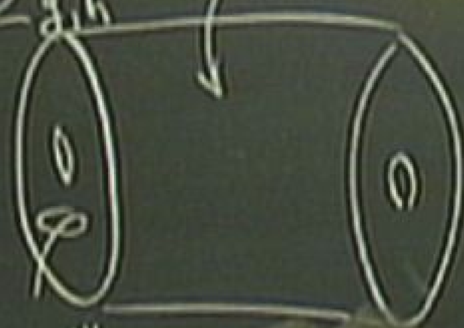


χ(Σ)<0
SL(2,R)
CS theory
Liouville
complex str.
Liouville
F=0

$\chi(\Sigma)<0$

SL(2,R) CS theory

$\Sigma_{g,h}$



Liouville

$$A=\begin{pmatrix}\omega & e^{+}\\ e^{-} & -\omega\end{pmatrix}$$

$F=0$

Liouville

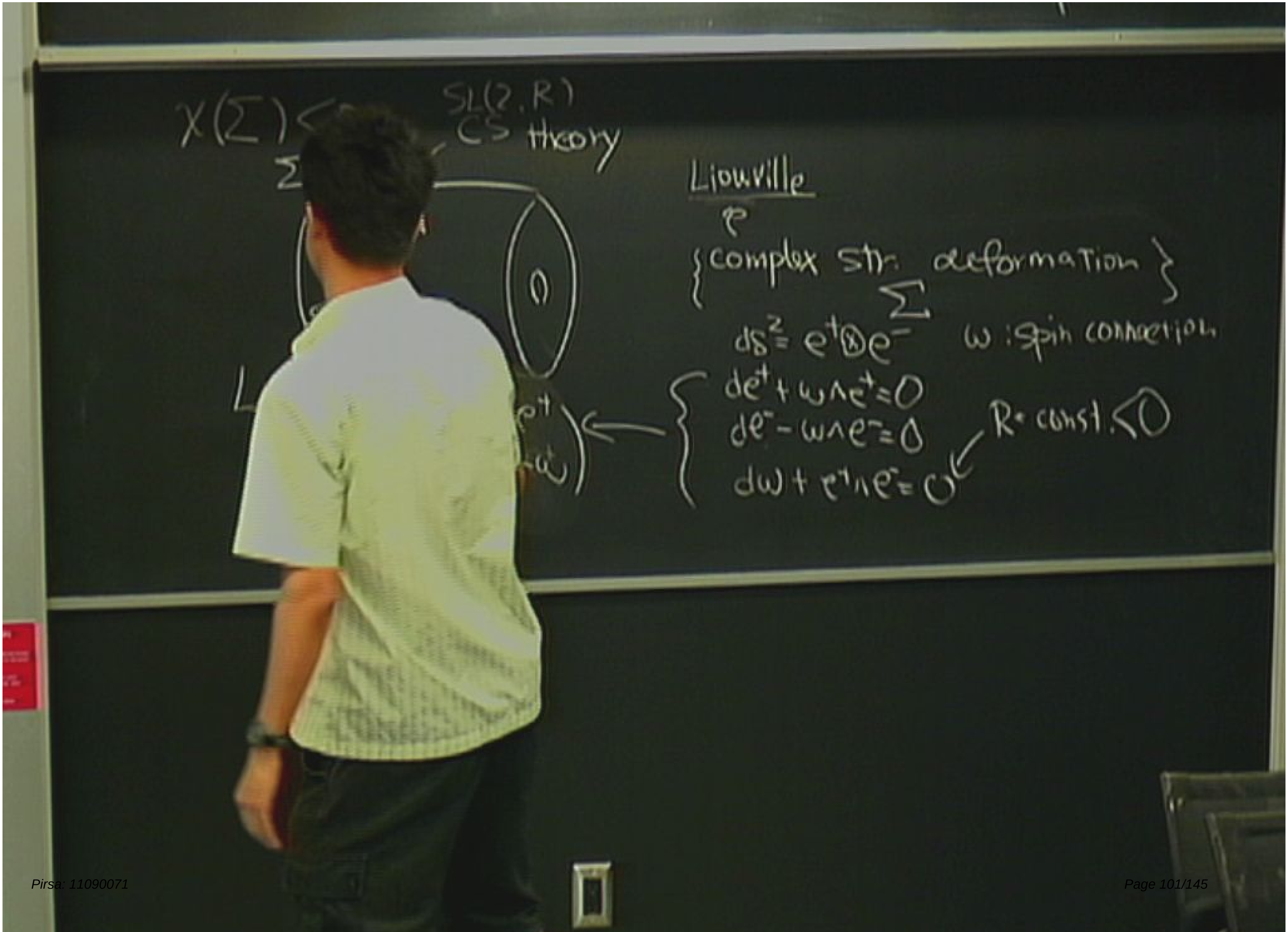
$\Updownarrow$

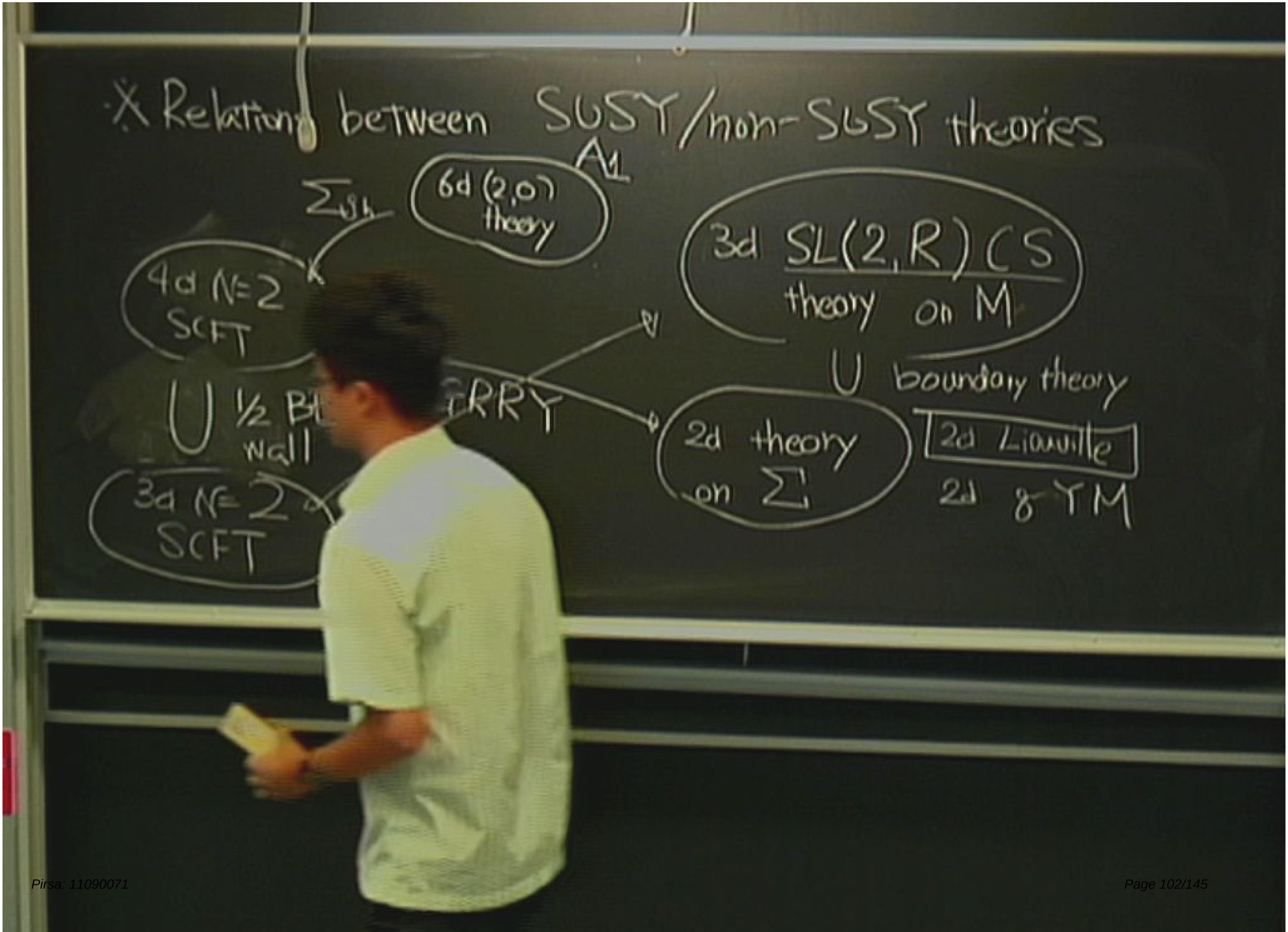
{complex str. deformation} Σ

$ds^2 = e^{+}\otimes e^{-}$ $\quad\omega$: spin connection

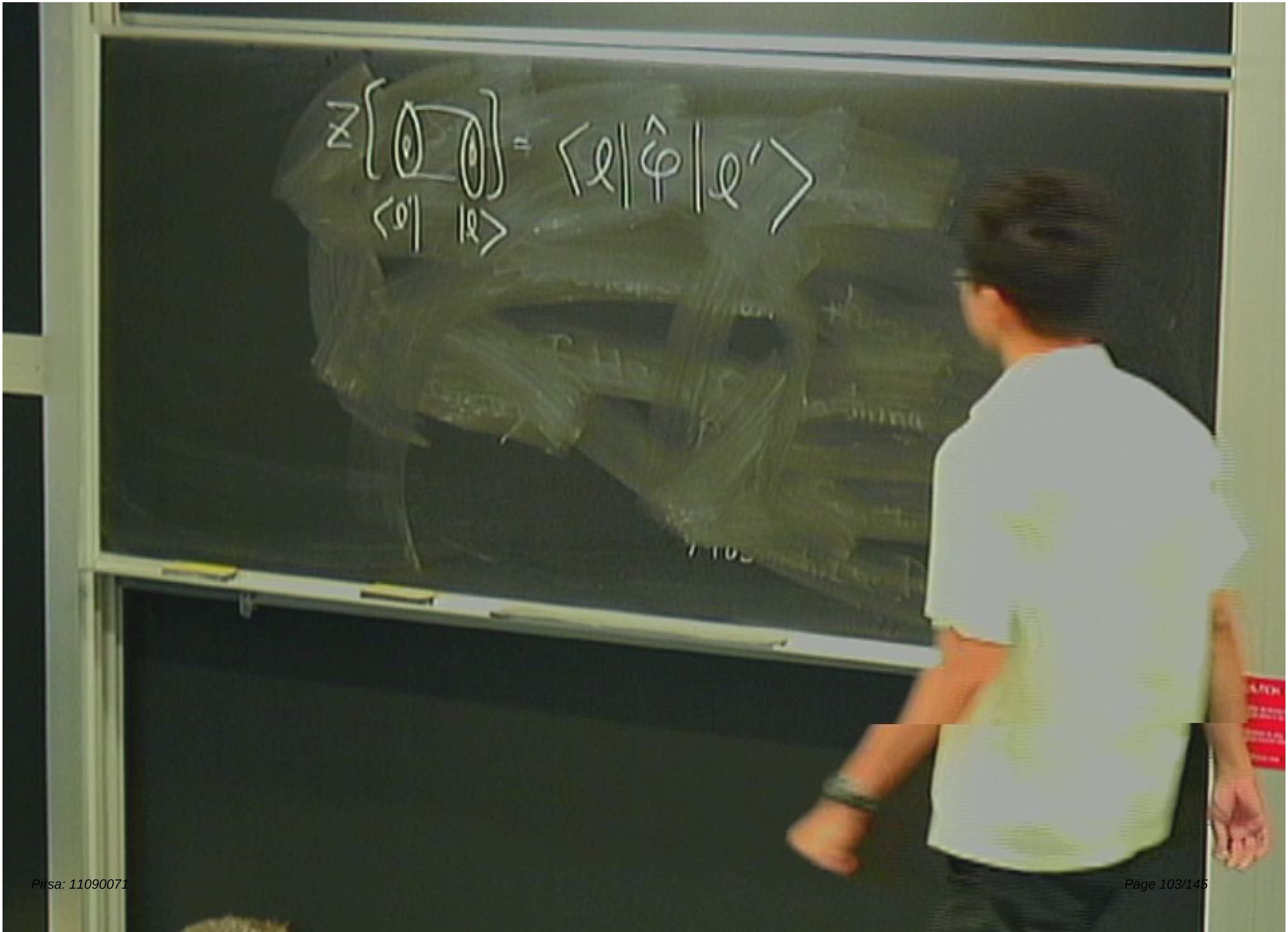
$$\begin{cases} de^{+}+\omega\wedge e^{+}=0\\ de^{-}-\omega\wedge e^{-}=0\\ d\omega+e^{+}\wedge e^{-}=0\end{cases}$$

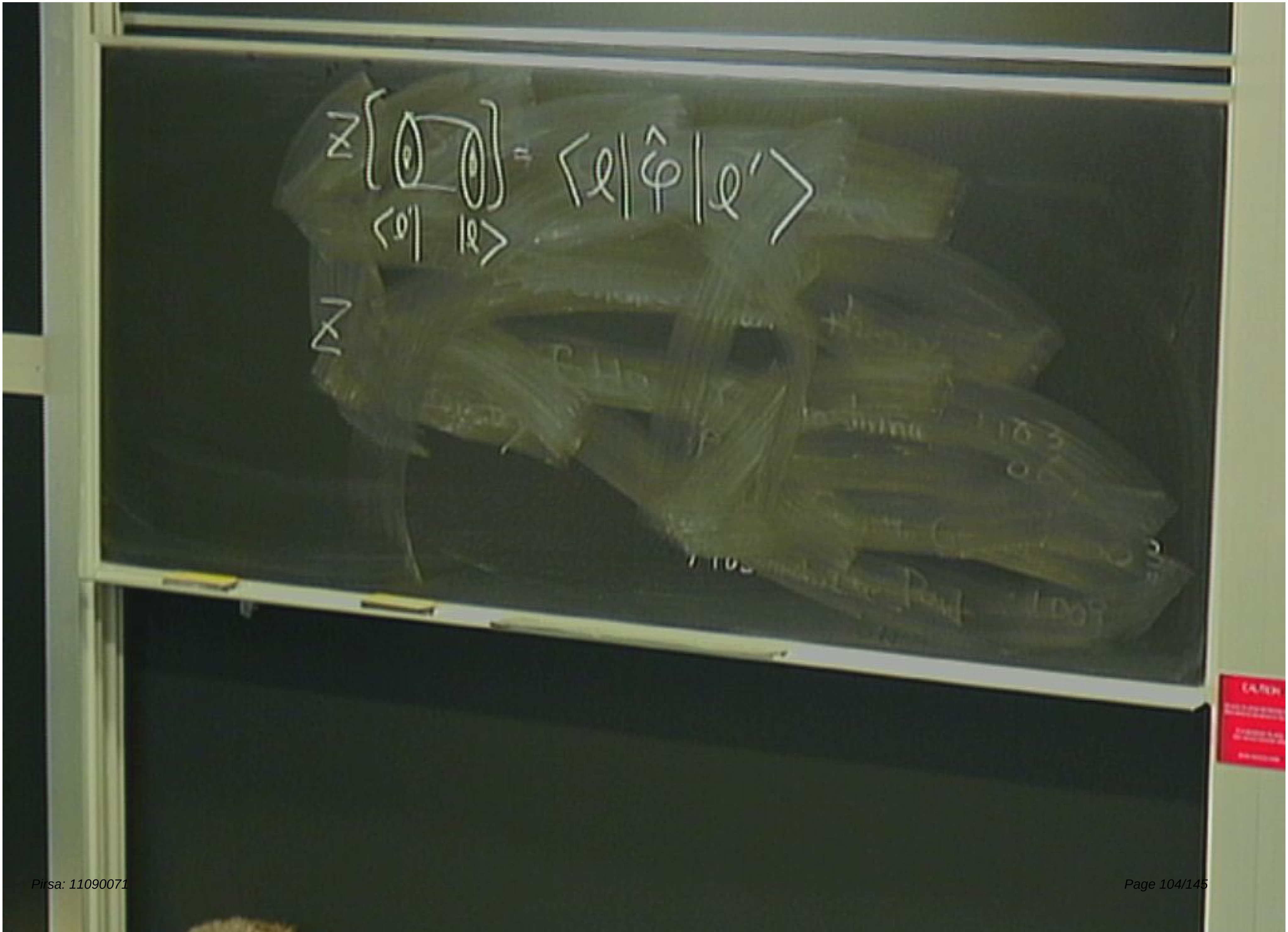
$R=\text{const.}<0$


$\chi(\Sigma)$
$SL(2,R)$
CS theory
Liouville
φ
{complex str. deformation}
Σ
$ds^2 = e^+ \otimes e^-$
ω : spin connection
$de^+ + \omega \wedge e^+ = 0$
$de^- - \omega \wedge e^- = 0$
$R = \text{const.} < 0$
$d\omega + e^+ \wedge e^- = 0$

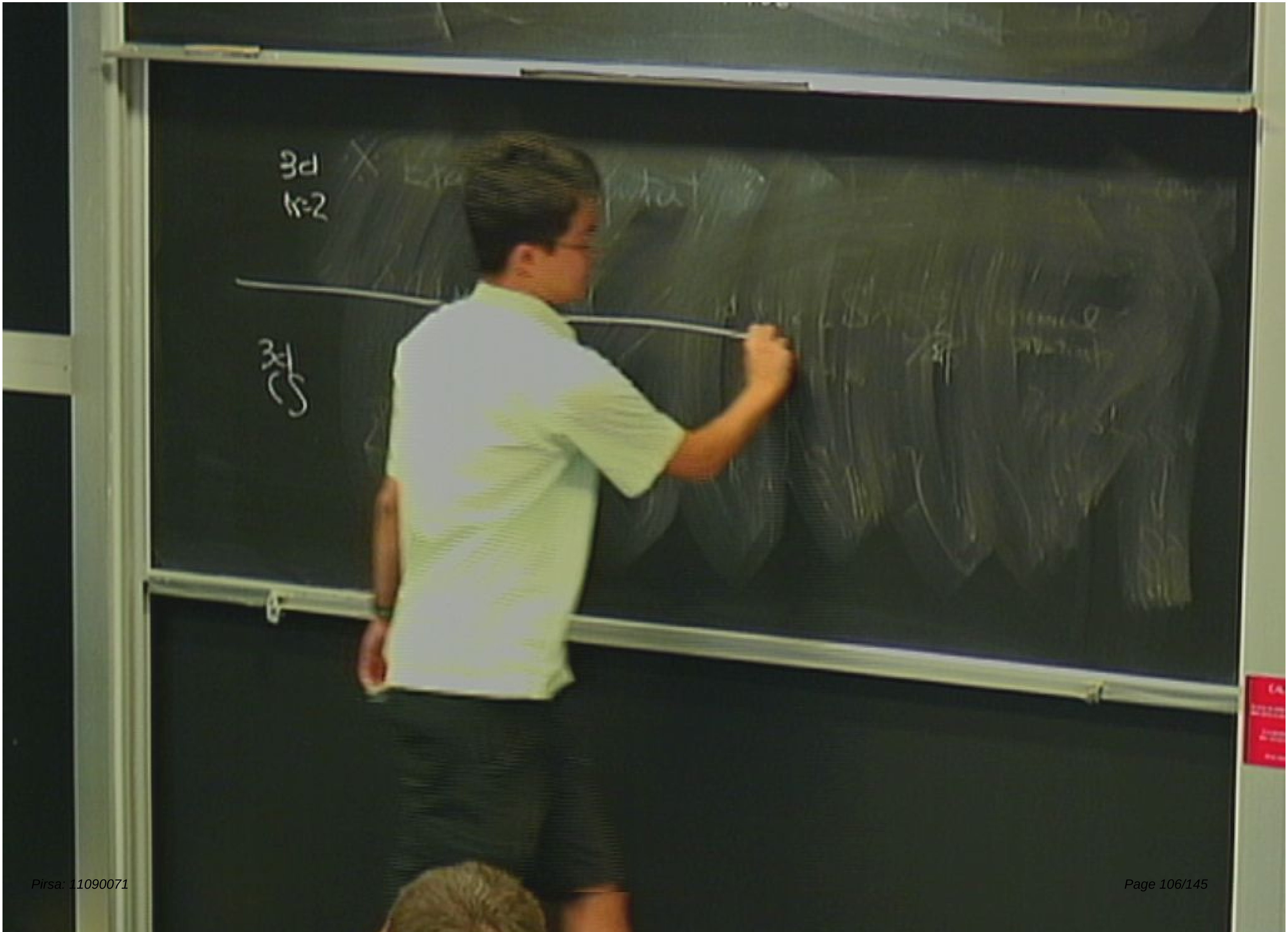


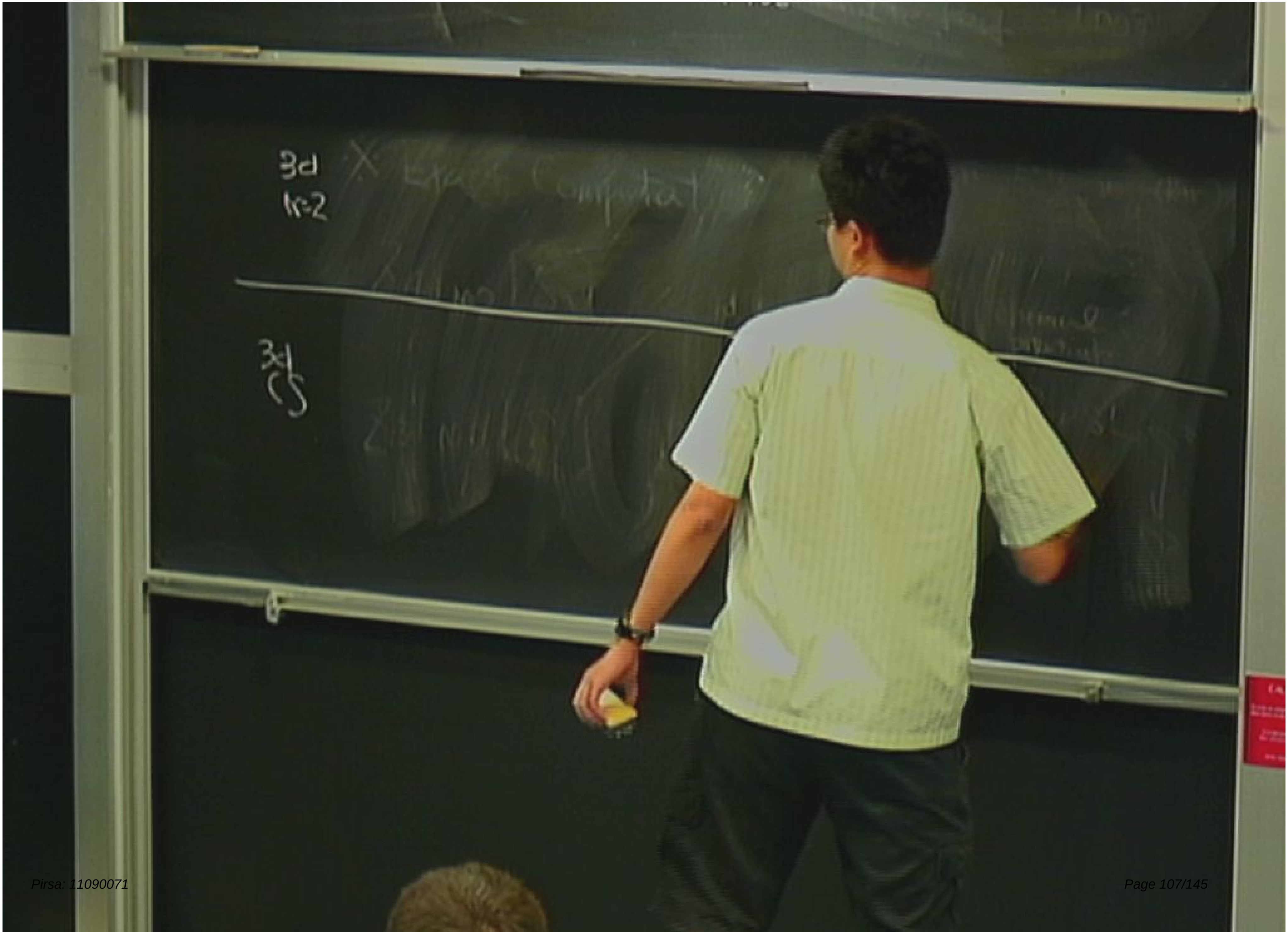
※ Relations between SUSY/non-SUSY theories
A_1
6d (2,0) theory
$\Sigma_{g,h}$
4d N=2 SCFT
U ½ BPS wall
3d N=2 SCFT
3d SL(2,R) CS theory on M
U boundary theory
2d theory on Σ
2d Liouville
2d q-YM

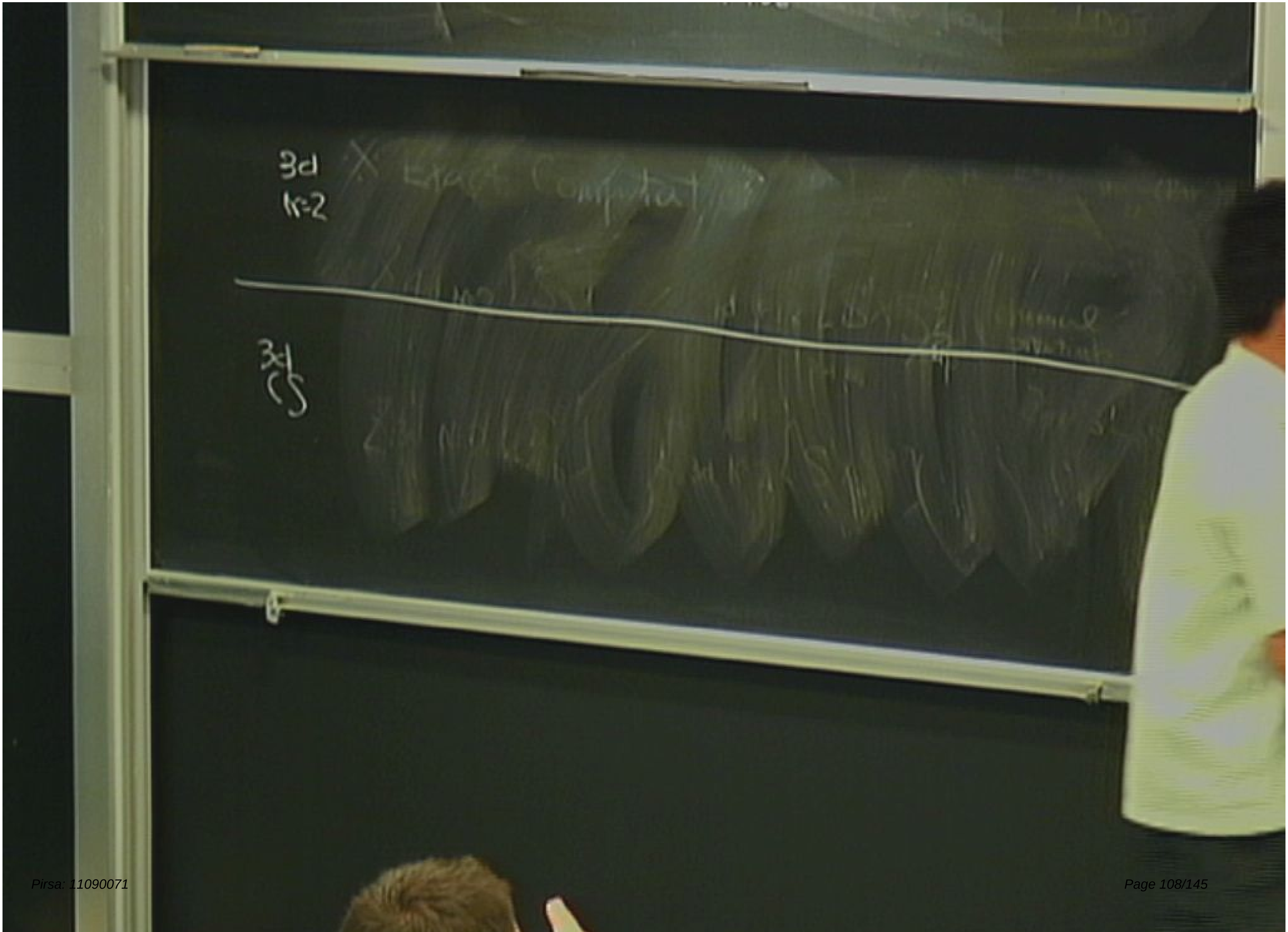

$Z\left[\ \right] = \langle \ell | \hat{\varphi} | \ell' \rangle$
$\langle \ell' |$ $| \ell \rangle$

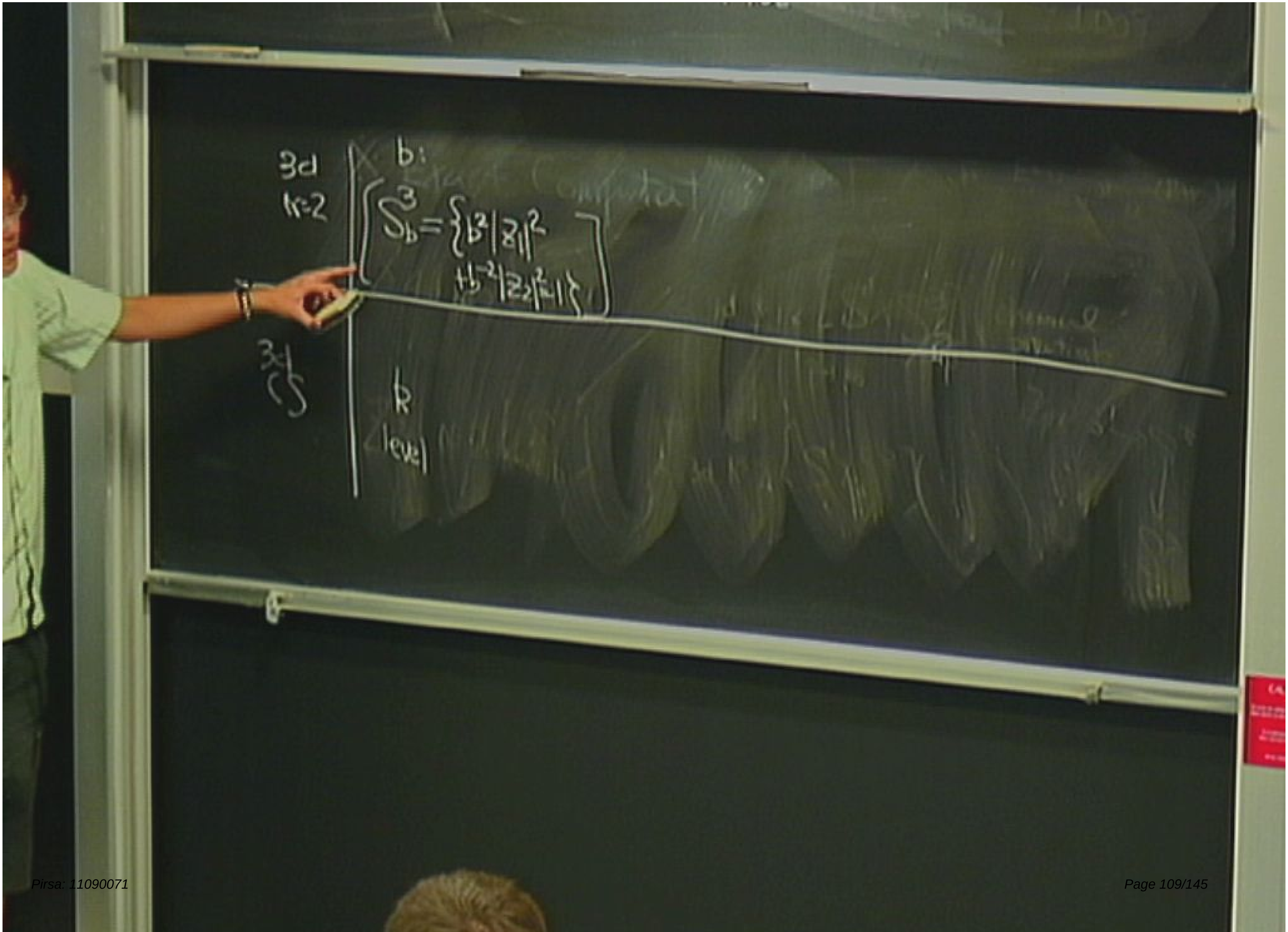

Z
= ⟨ℓ|φ̂|ℓ'⟩
⟨ℓ'| |ℓ⟩
Z

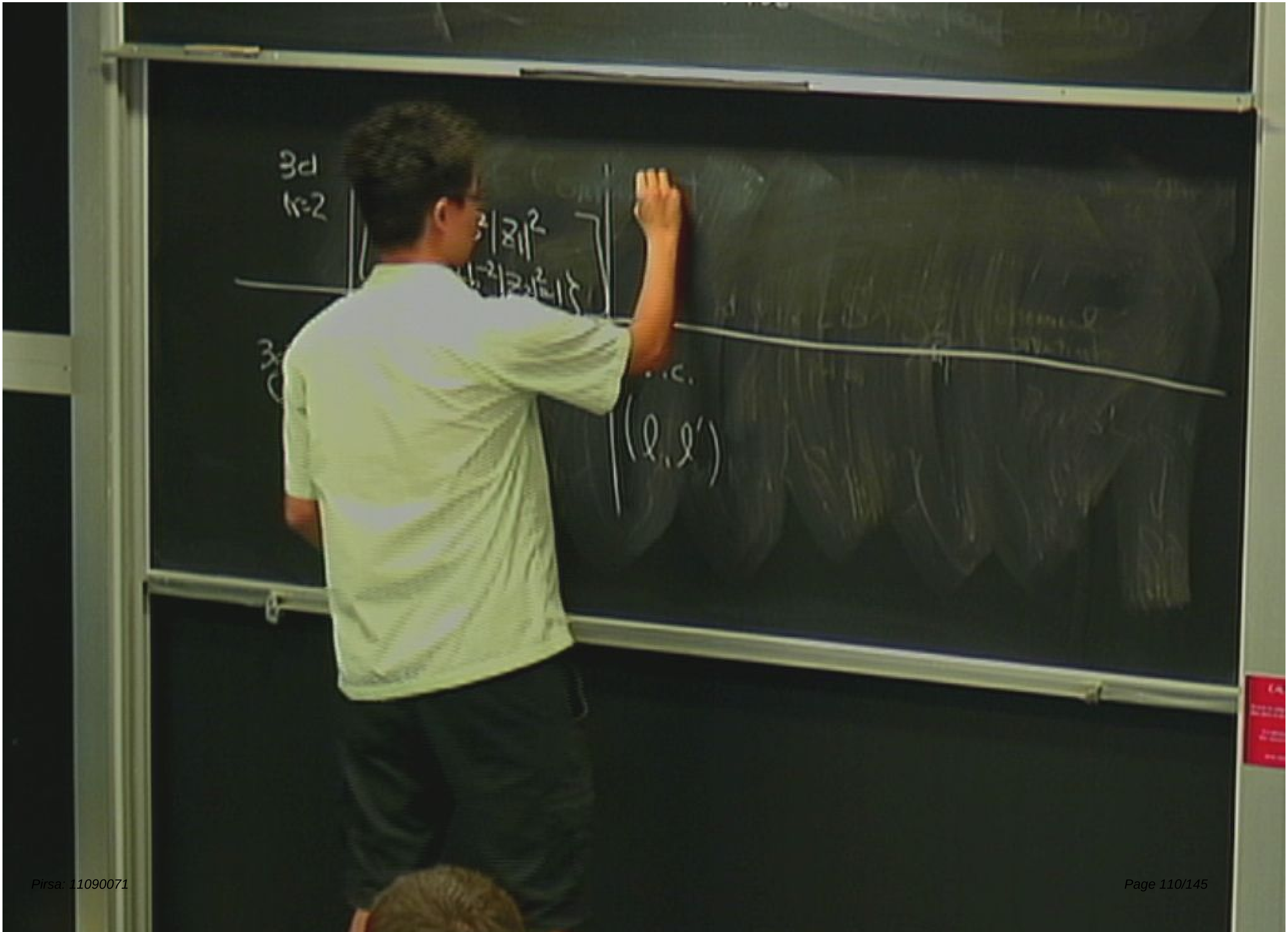

$Z = \langle \ell | \hat{\varphi} | \ell' \rangle$
$\langle \ell' |$ $| \ell \rangle$

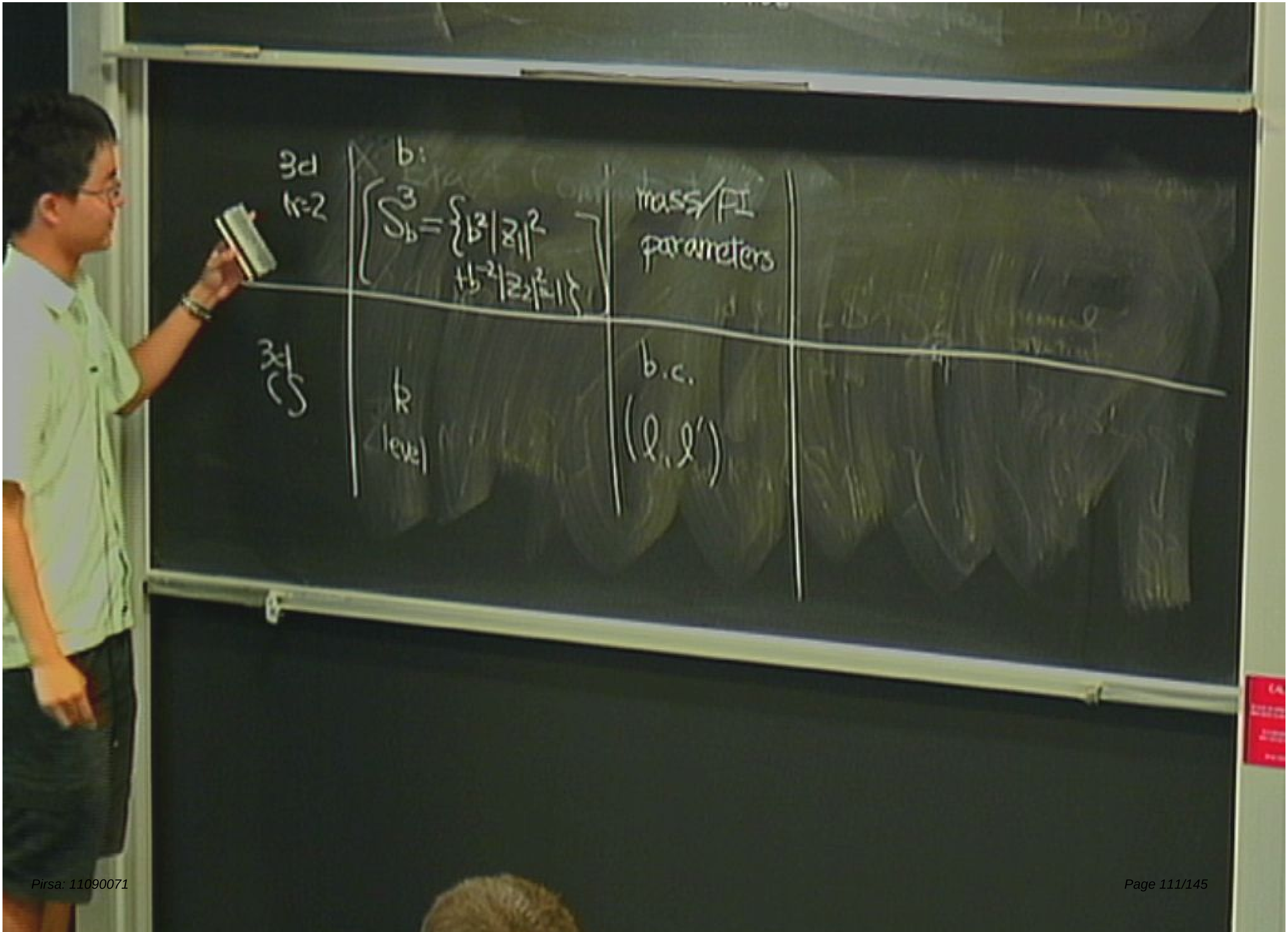

3d
k=2
3d
CS

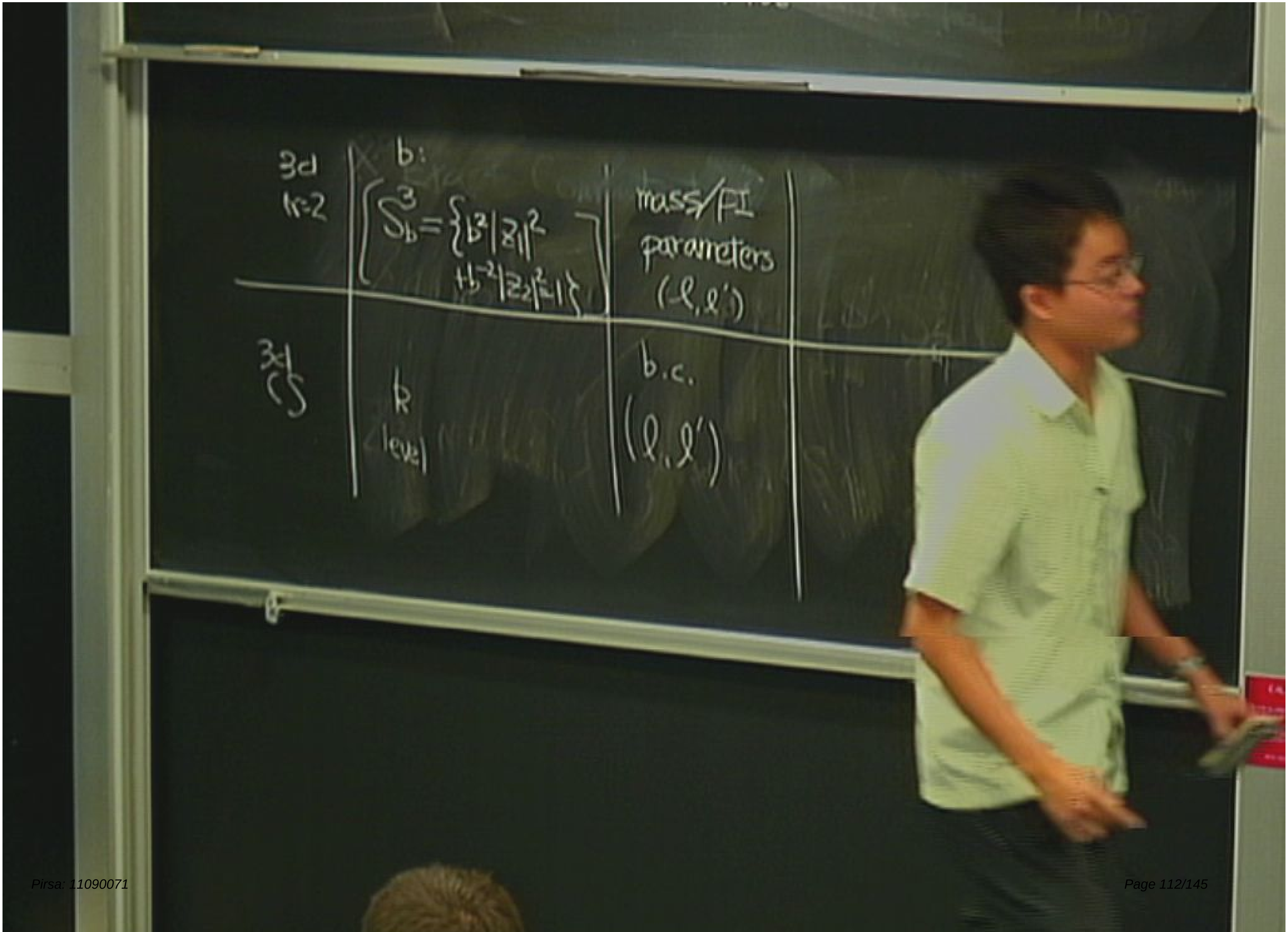

3d
N=2
3d
CS


3d
k=2
3d
CS

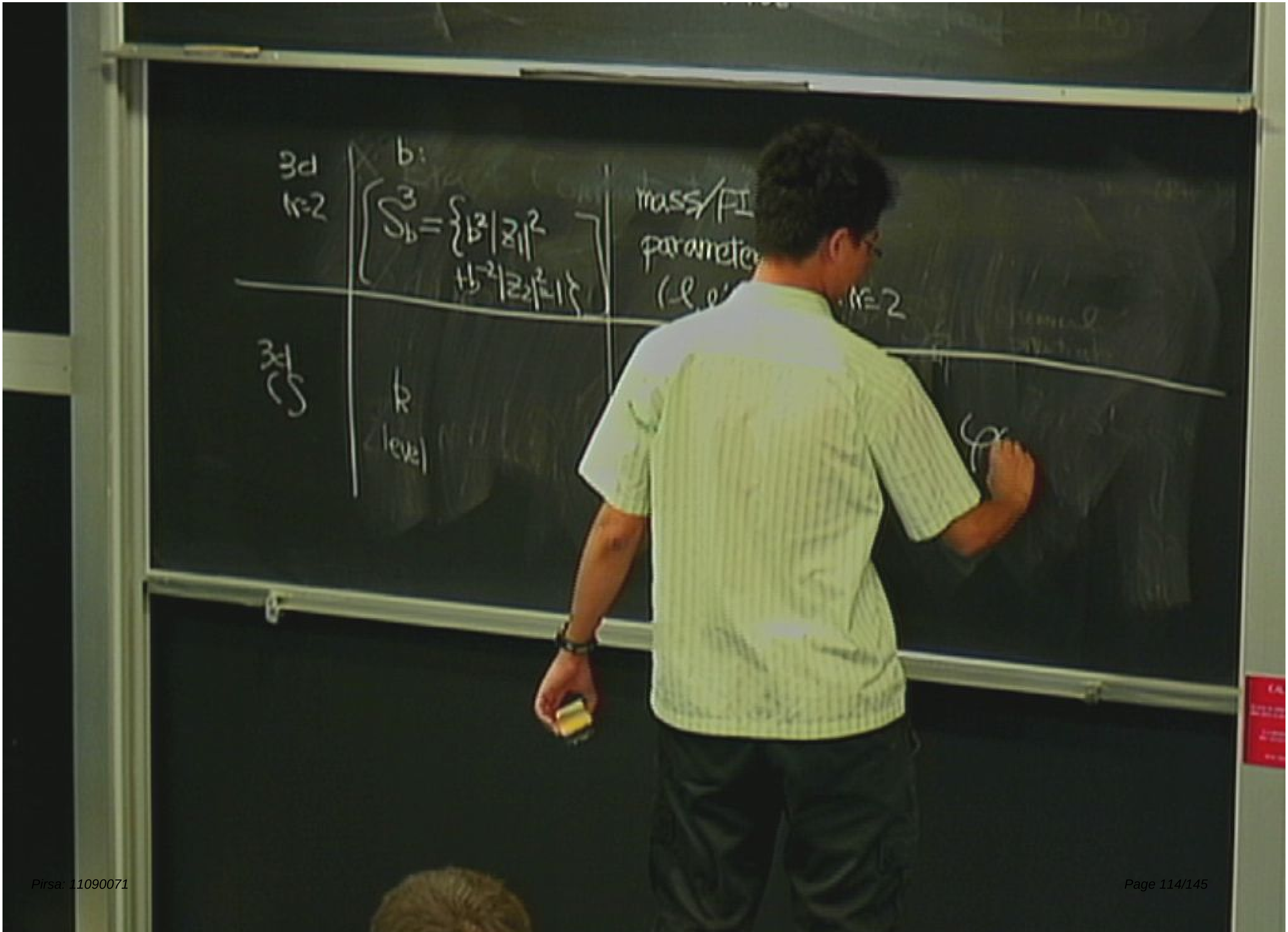

3d
k=2
b:
$S^3_b = \{b^2|z_1|^2 + b^{-2}|z_2|^2 = 1\}$
3d CS
k
level

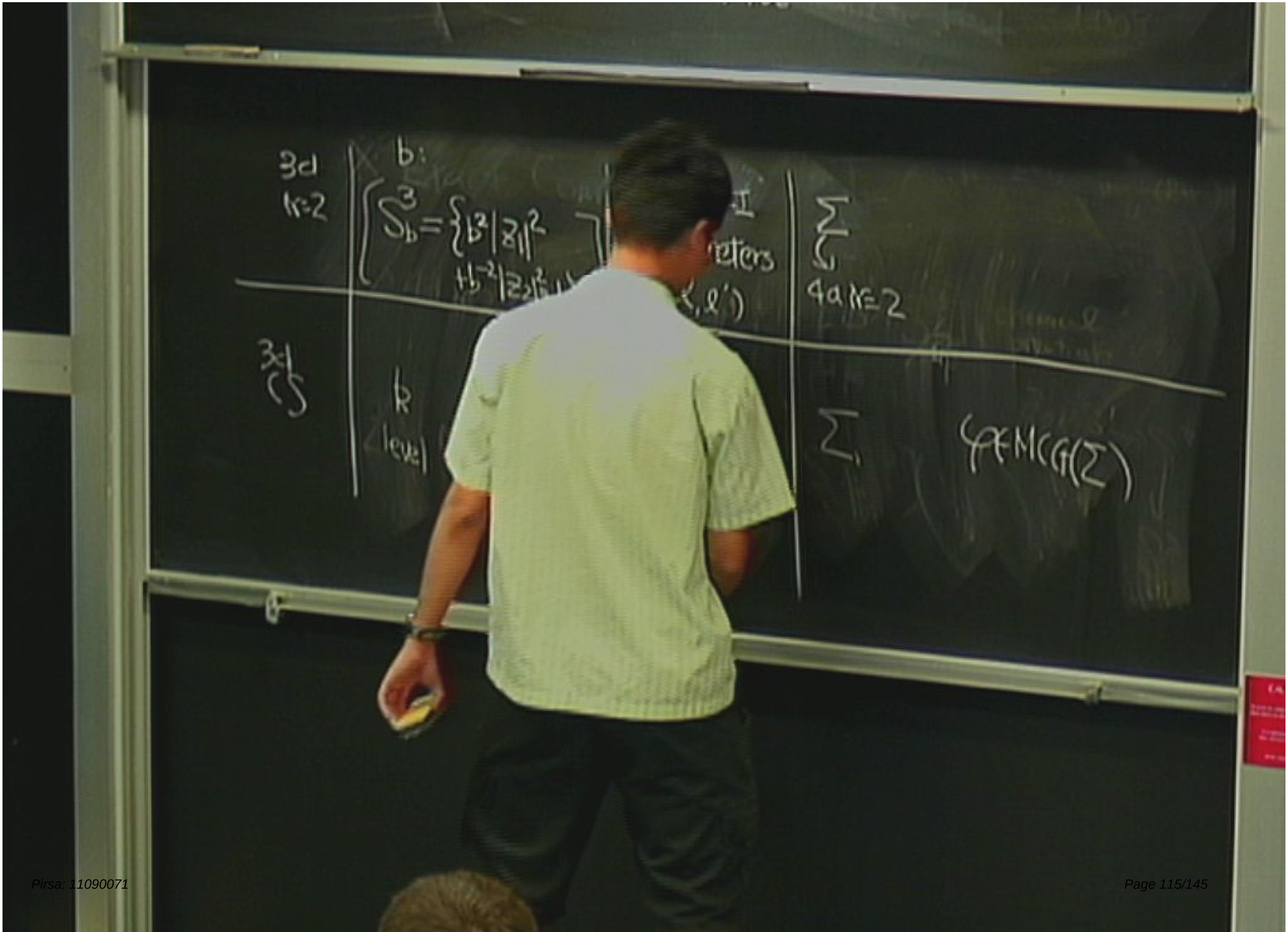

3d
k=2
(ℓ, ℓ')

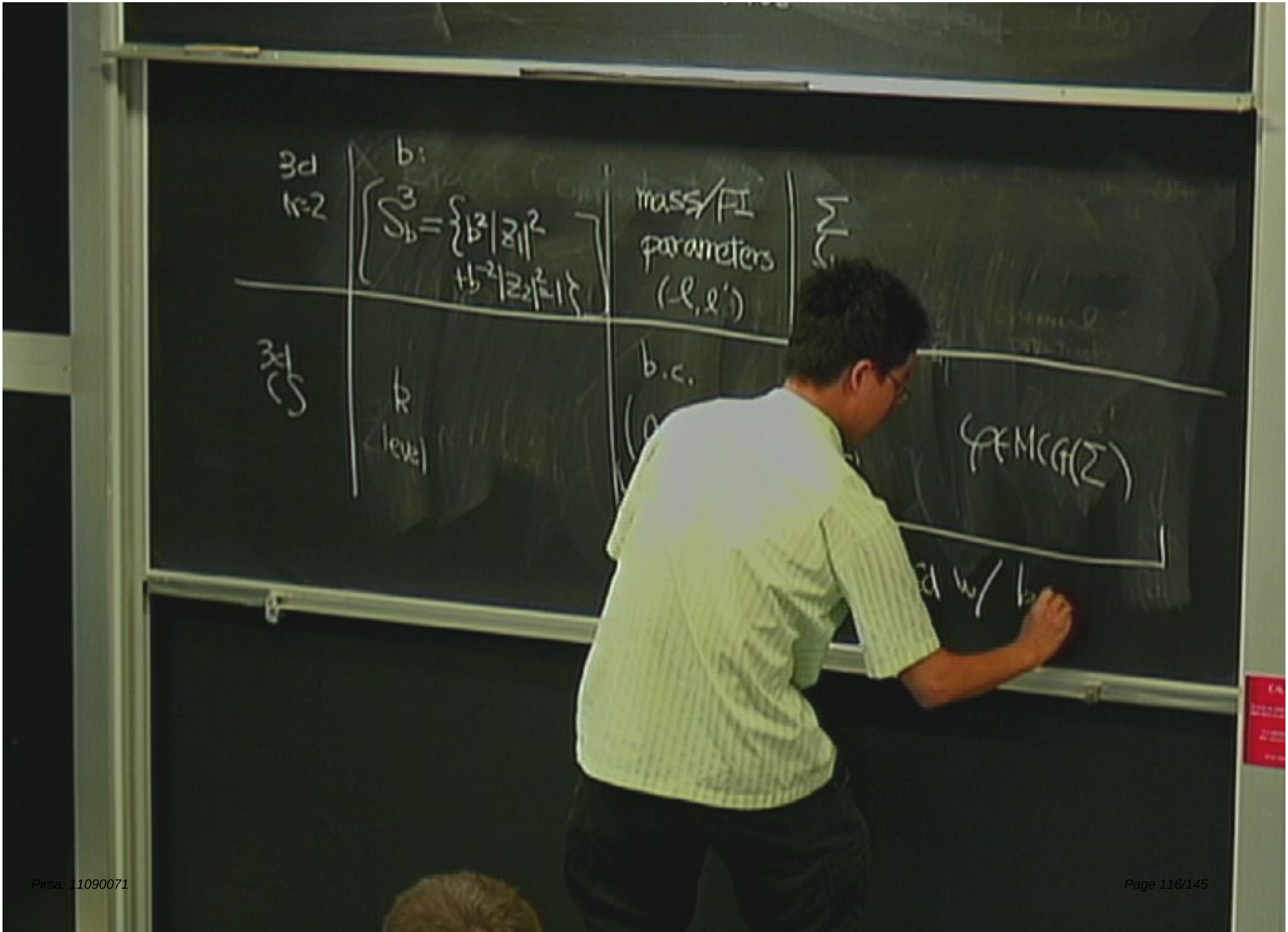

3d
N=2
b:
$S^3_b = \{b^2|z_1|^2 + b^{-2}|z_2|^2 = 1\}$
mass/FI
parameters
3d
CS
k
level
b.c.
(ℓ, ℓ')

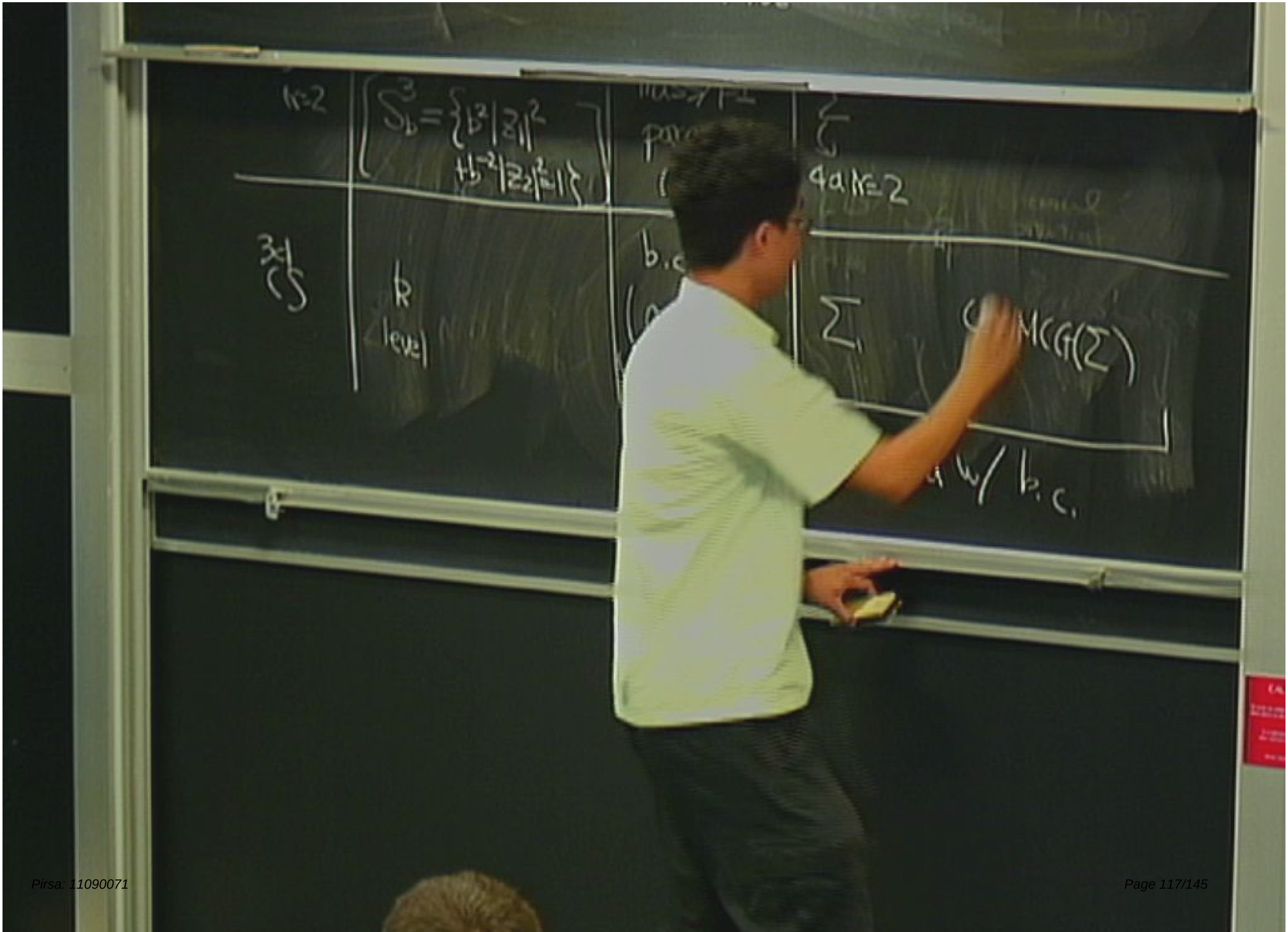

3d
N=2
b:
$S_b^3 = \{b^2|z_1|^2 + b^{-2}|z_2|^2 = 1\}$
mass/FI
parameters
(ℓ, ℓ')
3d
CS
k
level
b.c.
(ℓ, ℓ')

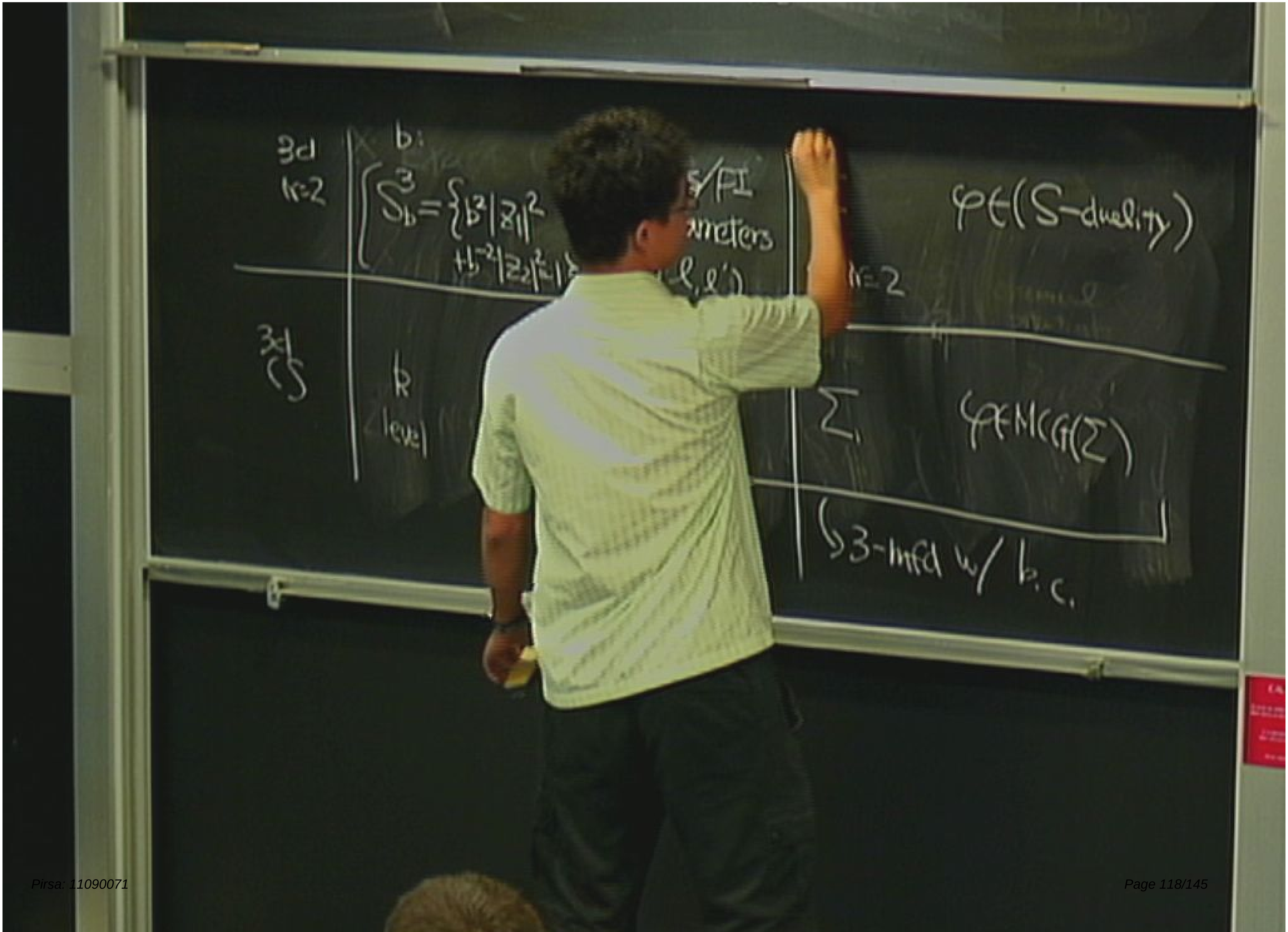
3d $\mathcal{N}=2$	b: $S^3_b=\{b^2\|z_1\|^2+b^{-2}\|z_2\|^2=1\}$	mass/FI parameters (ℓ,ℓ')	Σ_1 4d $\mathcal{N}=2$
3d CS	k level	b.c. (ℓ,ℓ')	Σ_1

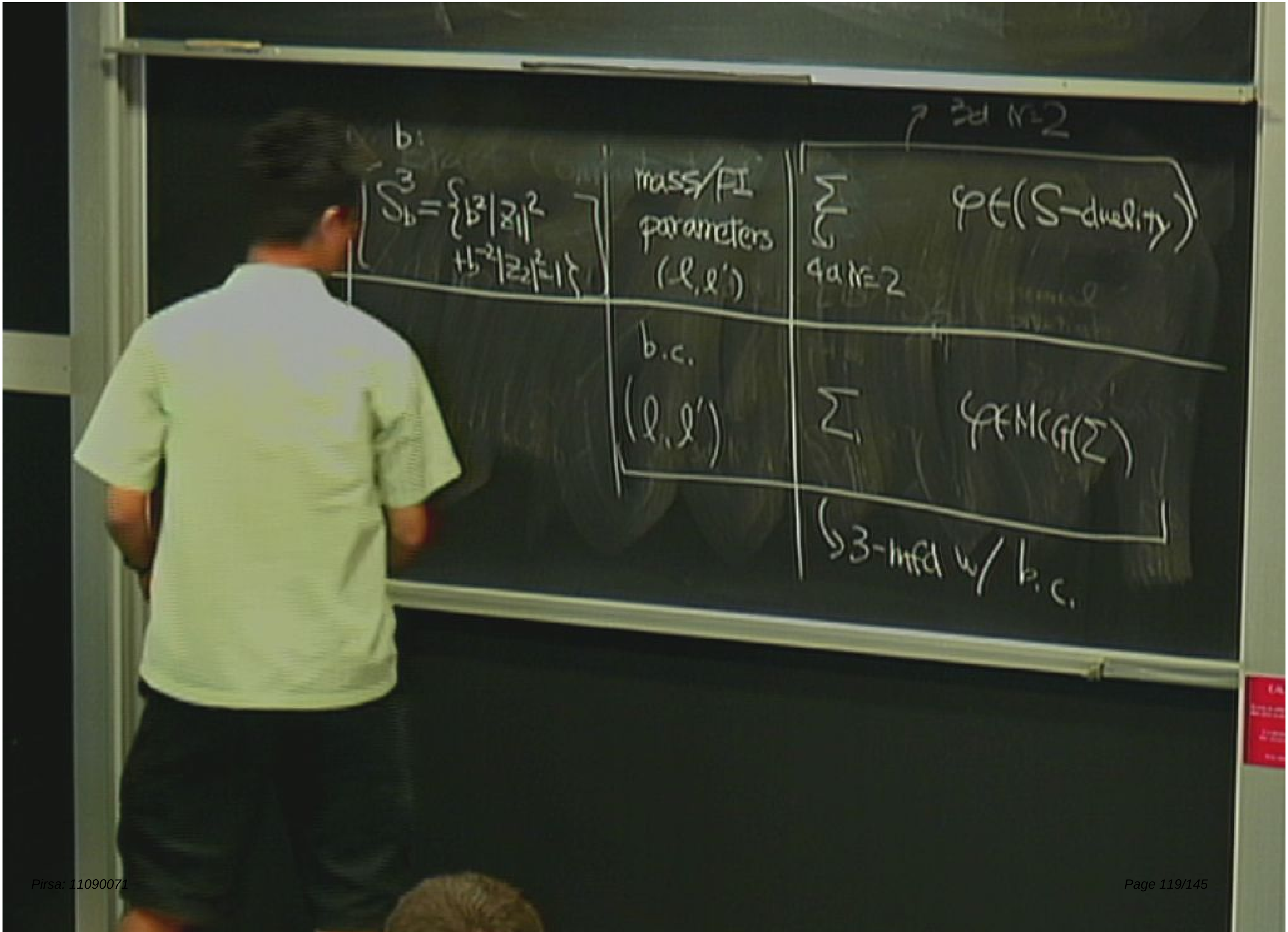

3d
N=2
b:
$S^3_b = \{b^2|z_1|^2 + b^{-2}|z_2|^2 = 1\}$
mass/FI
paramete
N=2
3d
CS
k
level

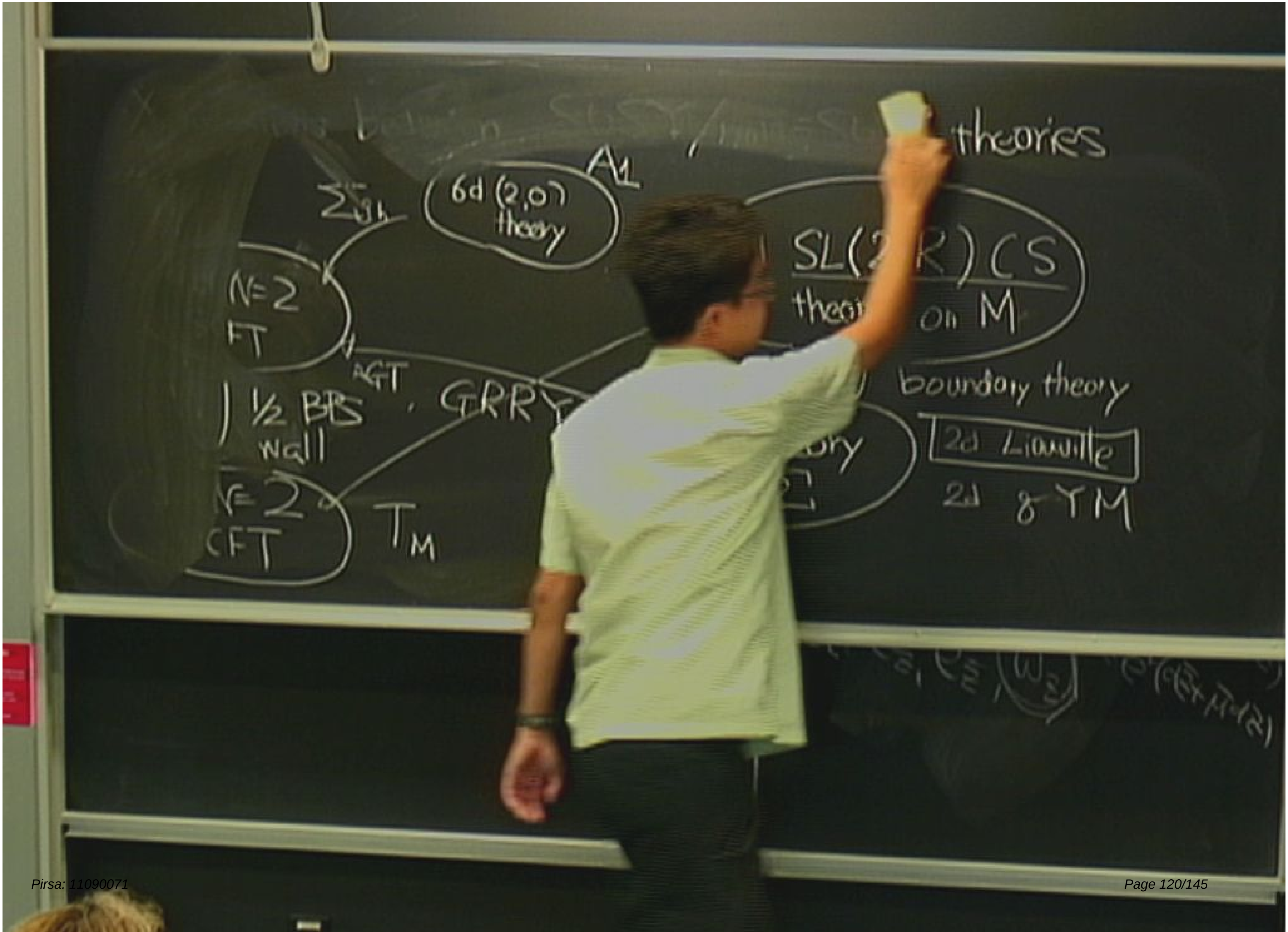

3d
k=2
b:
4a k=2
3d
CS
k
level

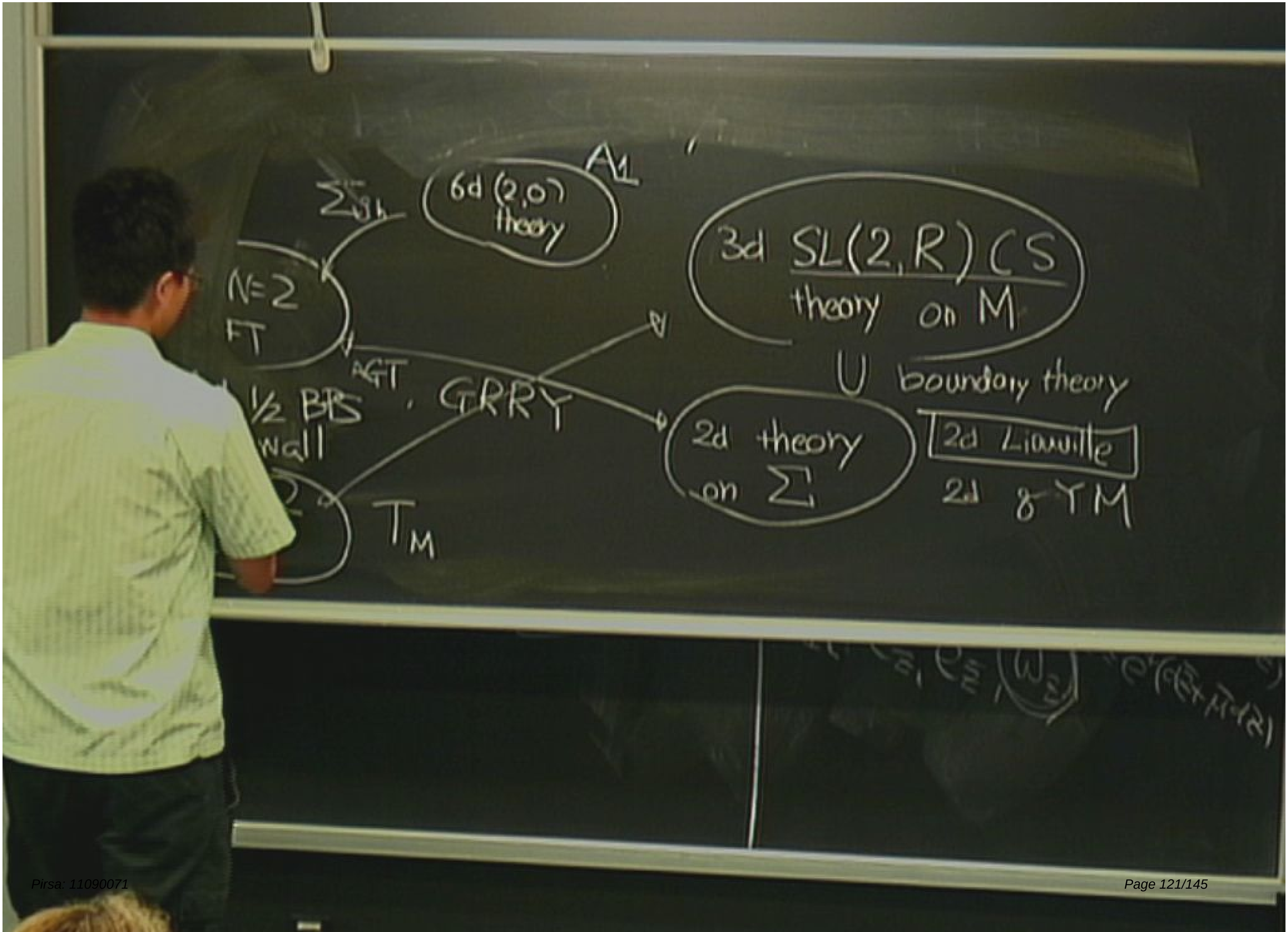

3d
k=2
b:
mass/FI
parameters
(ℓ, ℓ')
3d
CS
k
level
b.c.
φ∈MCG(Σ)

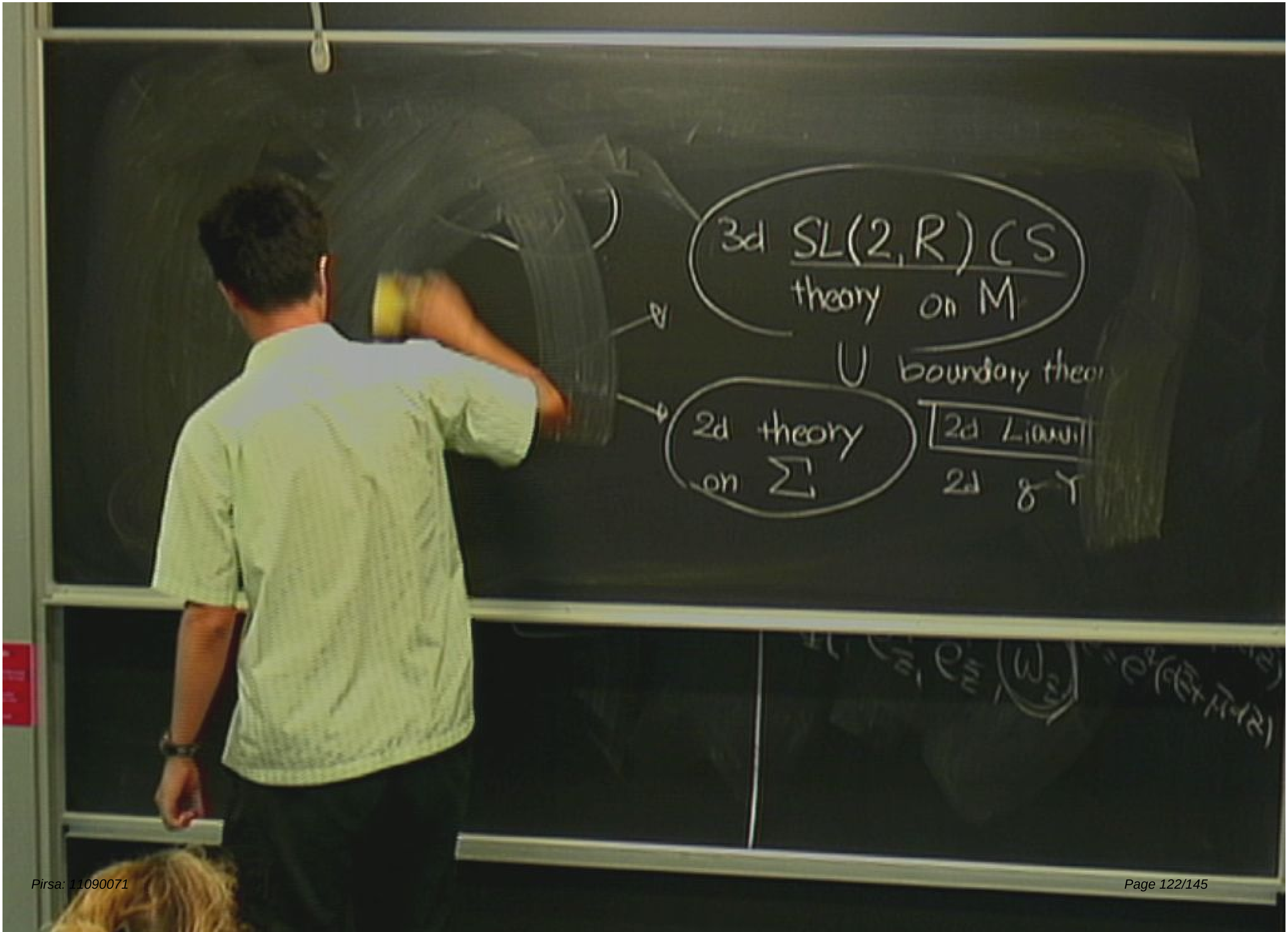

k=2
4d k=2
b.c.
k
level
w/ b.c.

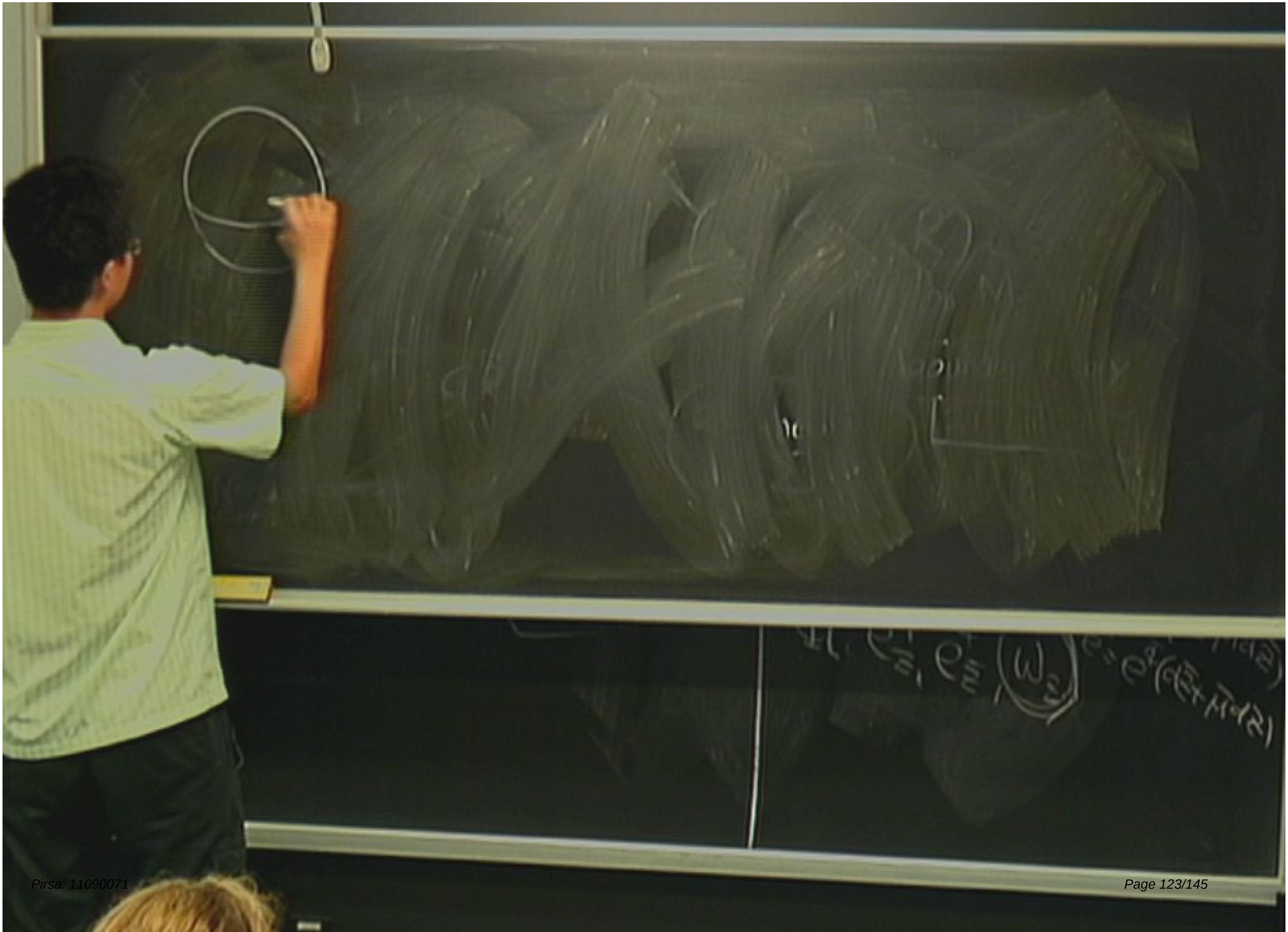

3d
k=2
b:
3d
CS
k
level
φ ∈ (S-duality)
Σ
φ ∈ MCG(Σ)
3-mfd w/ b.c.

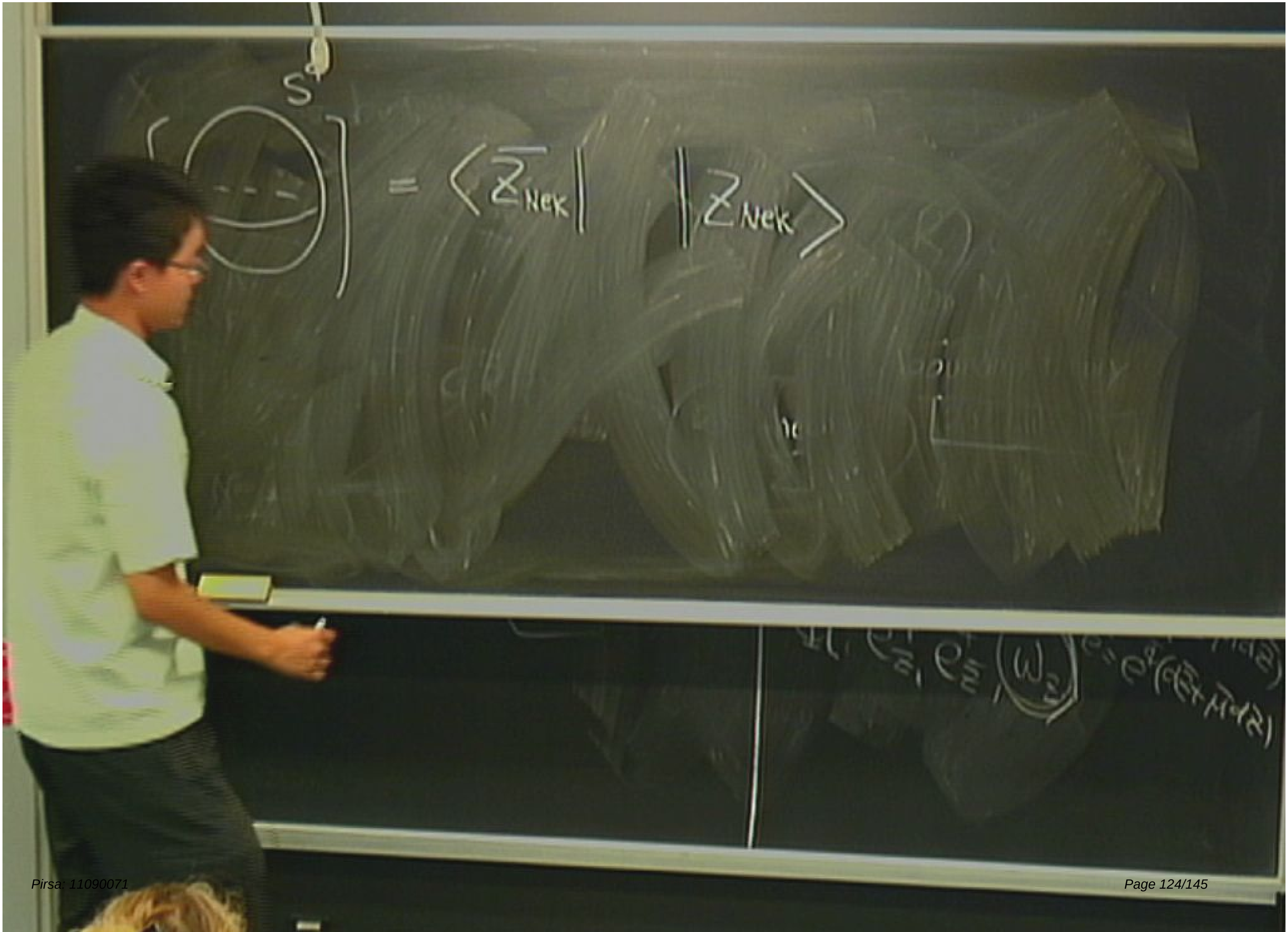

b:
$S_b^3 = \{b^2|z_1|^2 + b^{-2}|z_2|^2 = 1\}$
mass/FI parameters
(ℓ, ℓ')
3d N=2
Σ
φ ∈ (S-duality)
4d N=2
b.c.
(ℓ, ℓ')
Σ,
φ ∈ MCG(Σ)
3-mfd w/ b.c.

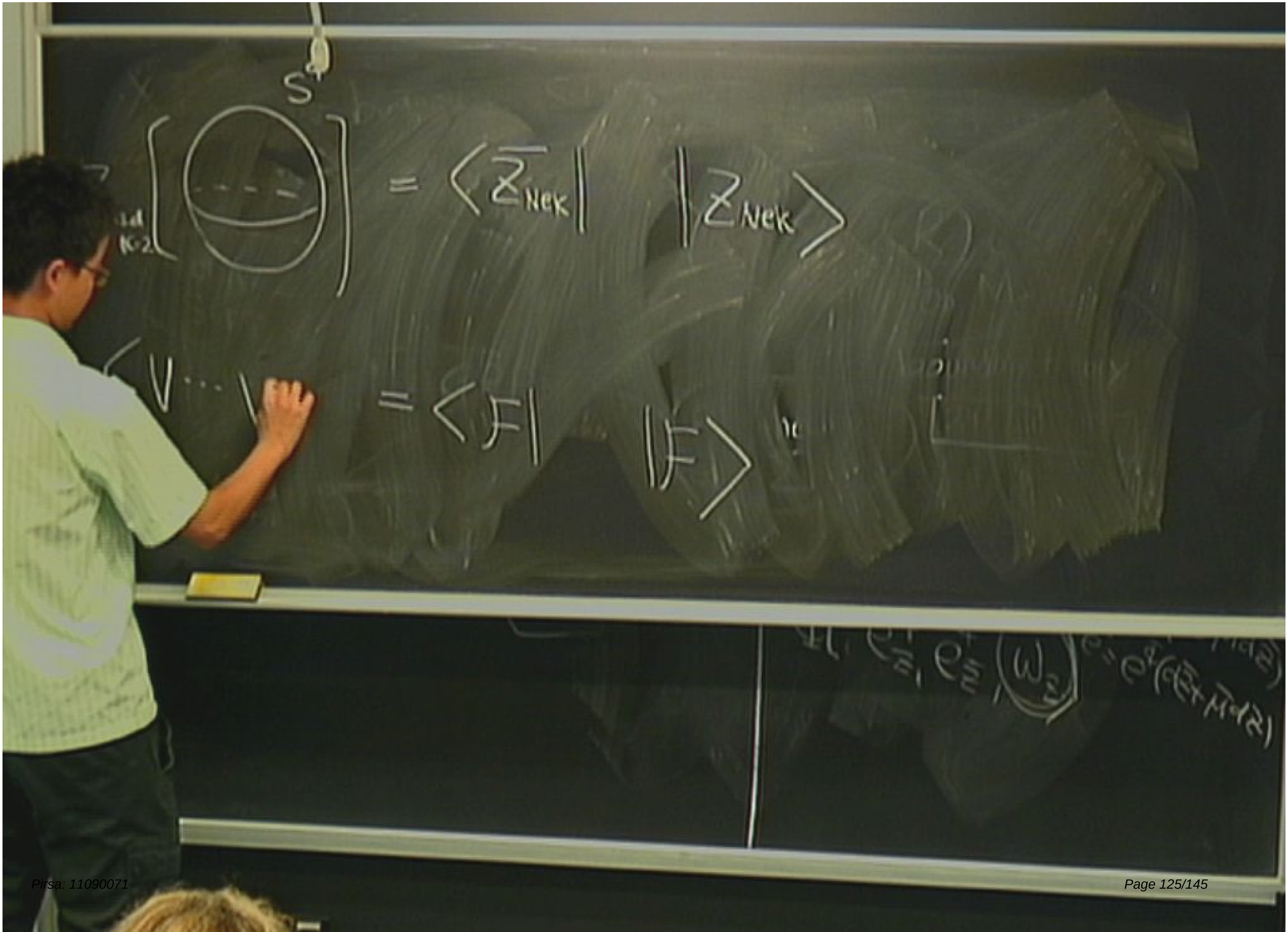

theories
6d (2,0) theory
N=2
FT
AGT
GRRY
1/2 BPS
wall
N=2
CFT
T_M
SL(2,R) CS
theory on M
boundary theory
2d Liouville
2d q YM

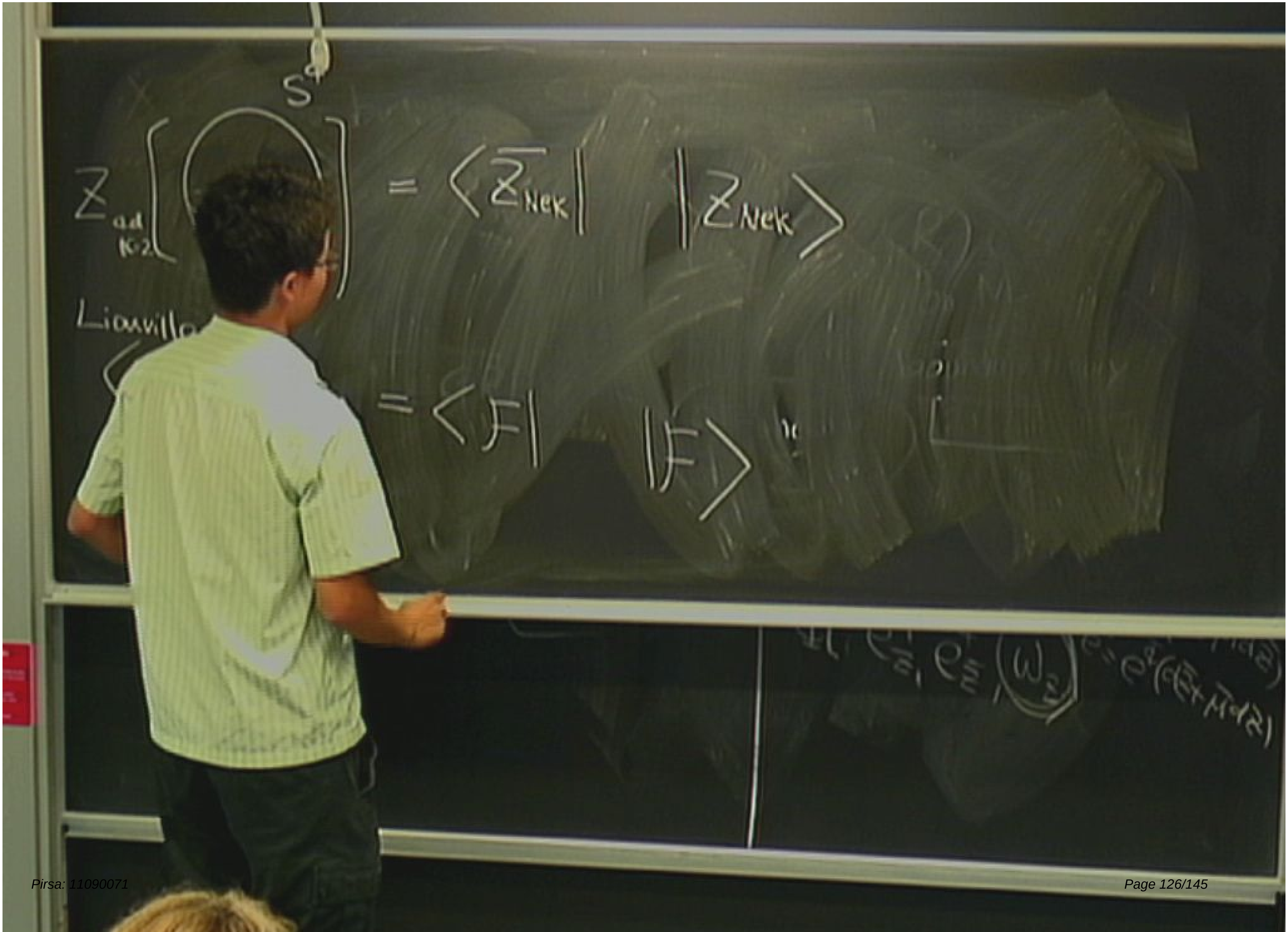

6d (2,0) theory
A_1
$\Sigma_{g,h}$
N=2 FT
AGT
1/2 BPS wall
GRRY
T_M
3d SL(2,R) CS theory on M
∪ boundary theory
2d theory on Σ
2d Liouville
2d q-YM

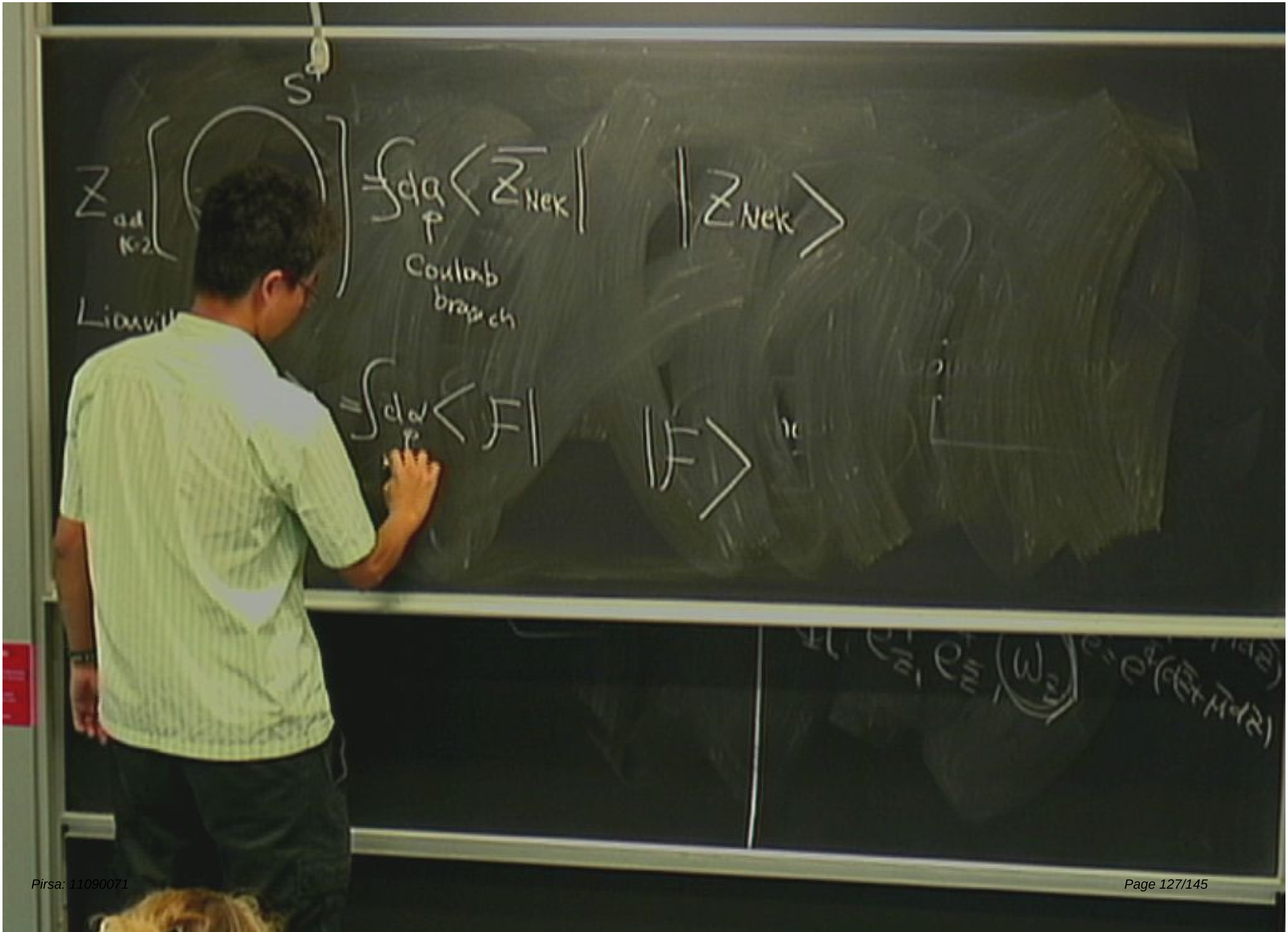

3d SL(2,R) CS
theory on M
U boundary theo
2d theory
on Σ
2d Liouvil




S^4
$= \langle \bar{Z}_{Nek} | \quad | Z_{Nek} \rangle$


S
= ⟨Z̄Nek|
|ZNek⟩
= ⟨F|
|F⟩


S^4
Z_{Nek}
Z_{Nek}


Coulomb
branch

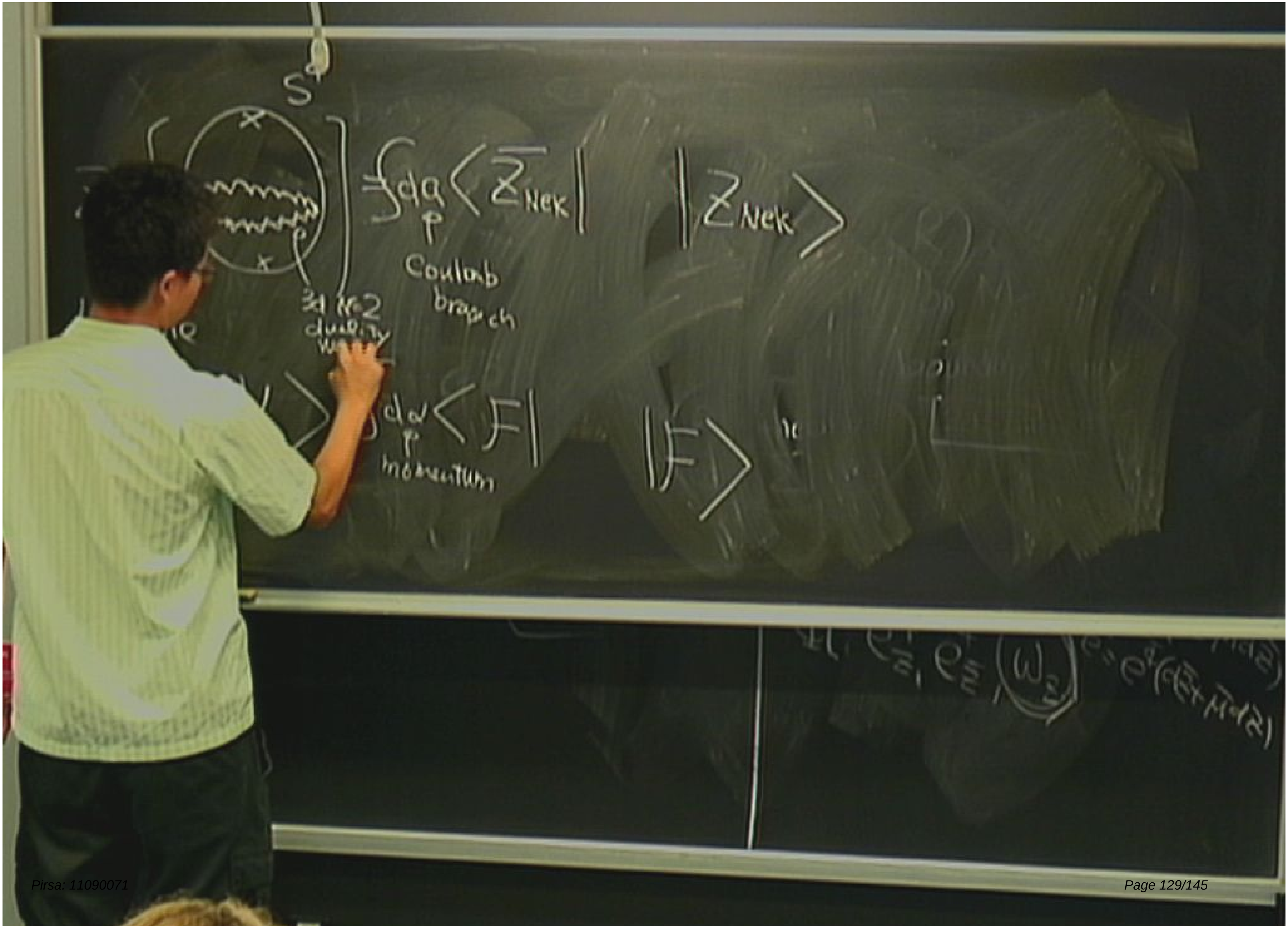
$$Z_{\substack{ad\\ K\text{-}2}}\left[S^4 \right] = \int da_p \, \langle \bar{Z}_{Nek} | \qquad | Z_{Nek} \rangle$$

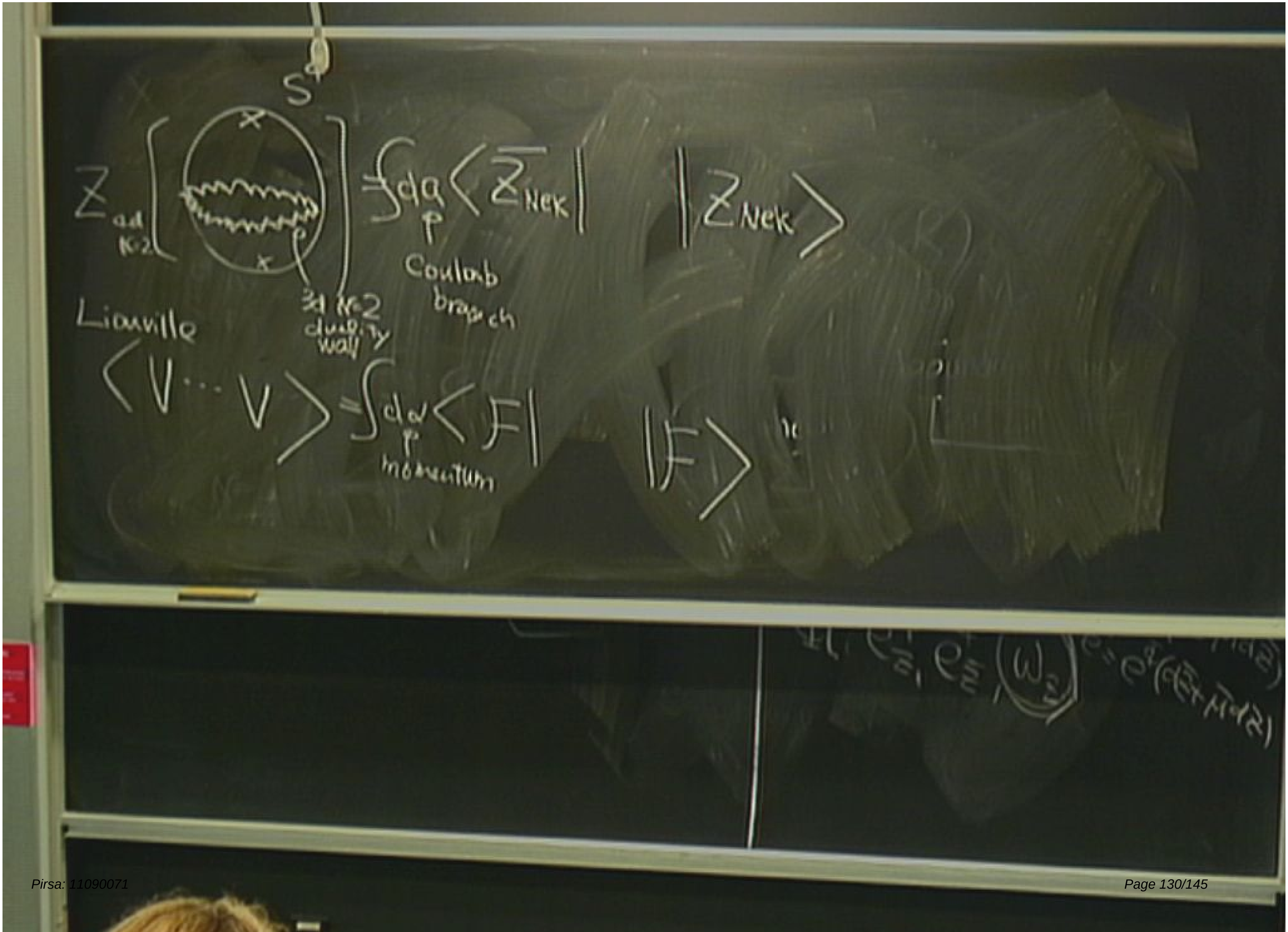
Coulomb branch

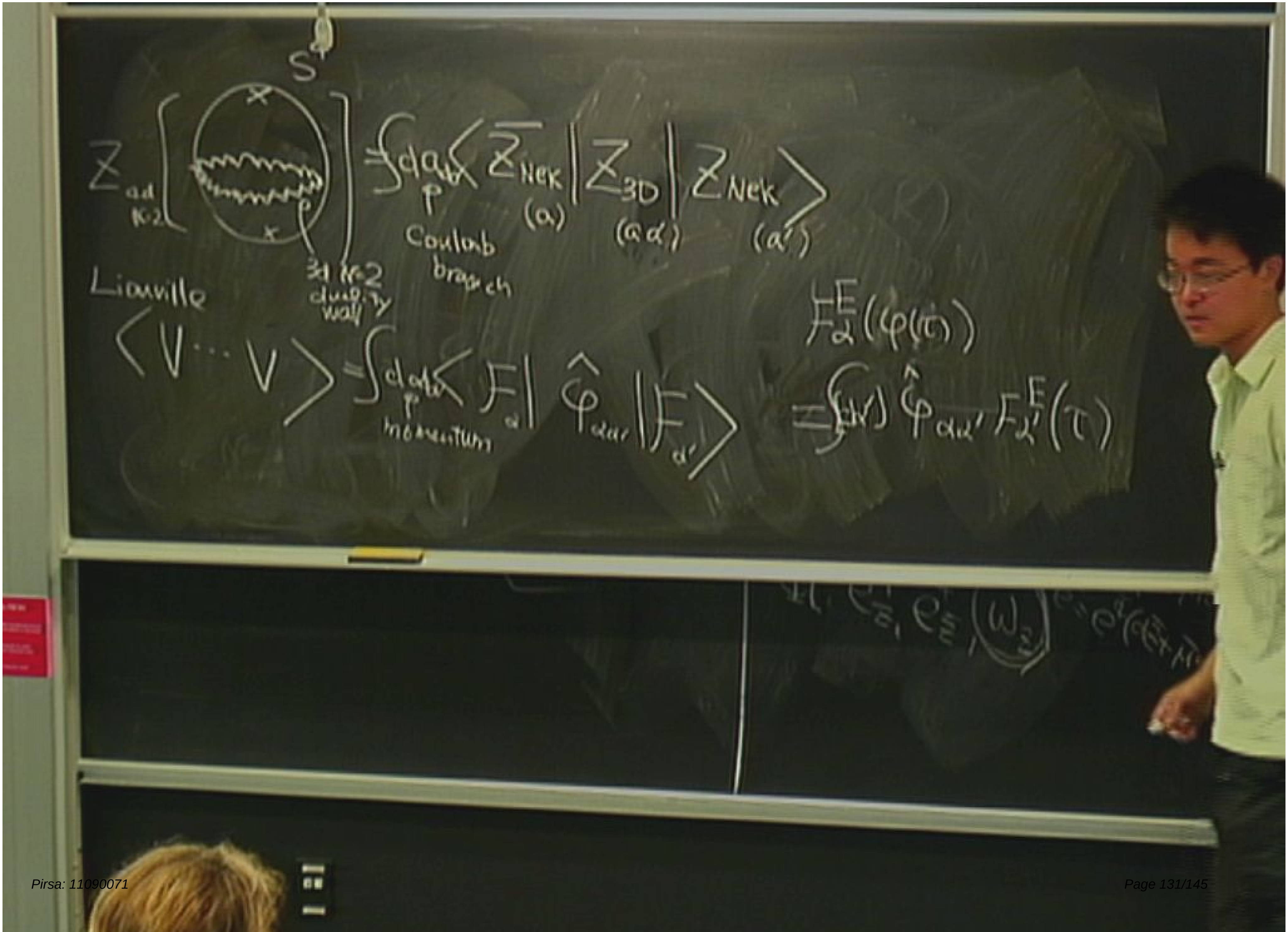
Liouville

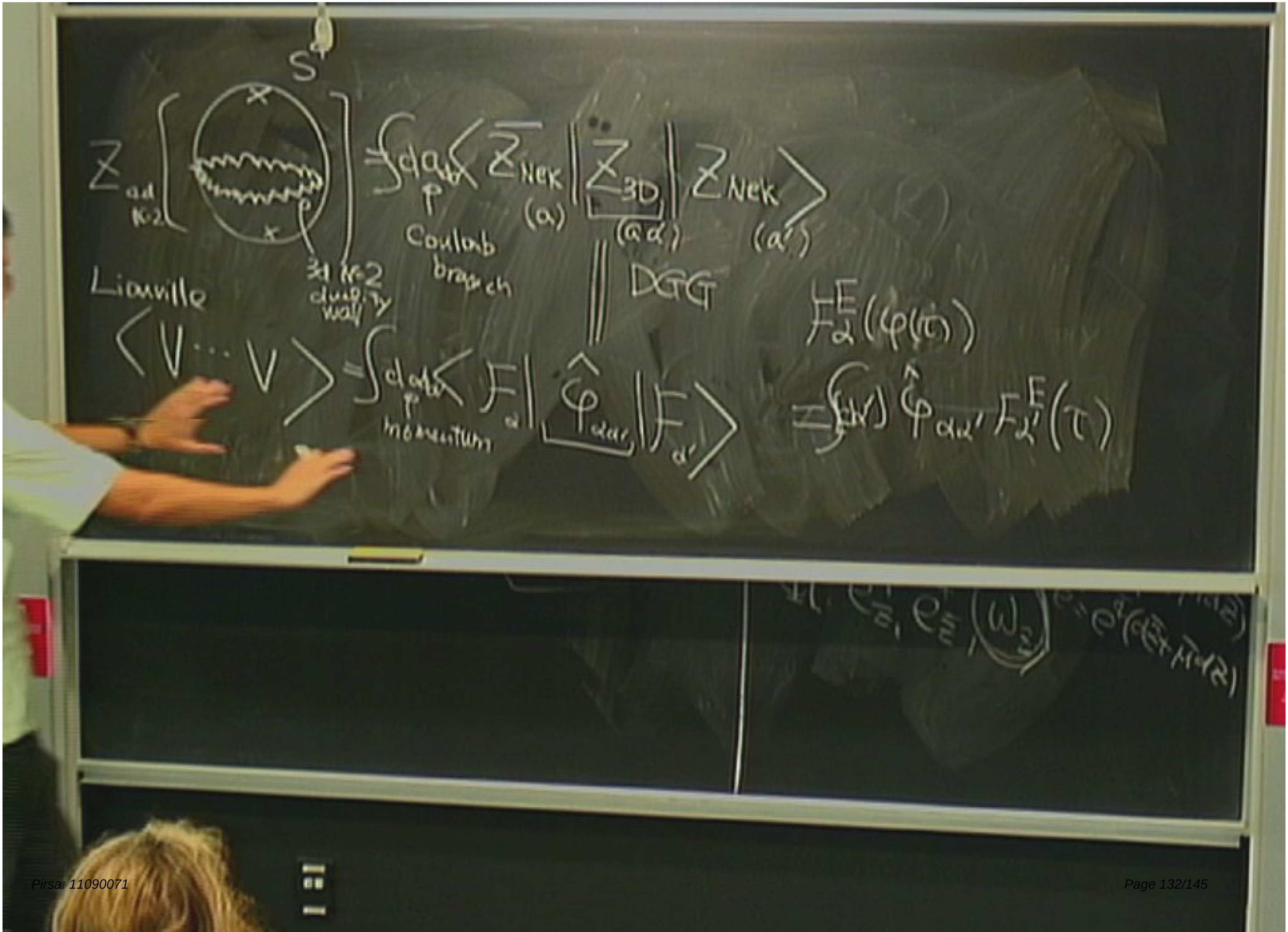
$$\langle V \cdots V \rangle = \int d\alpha_p \, \langle F | \qquad | F \rangle$$

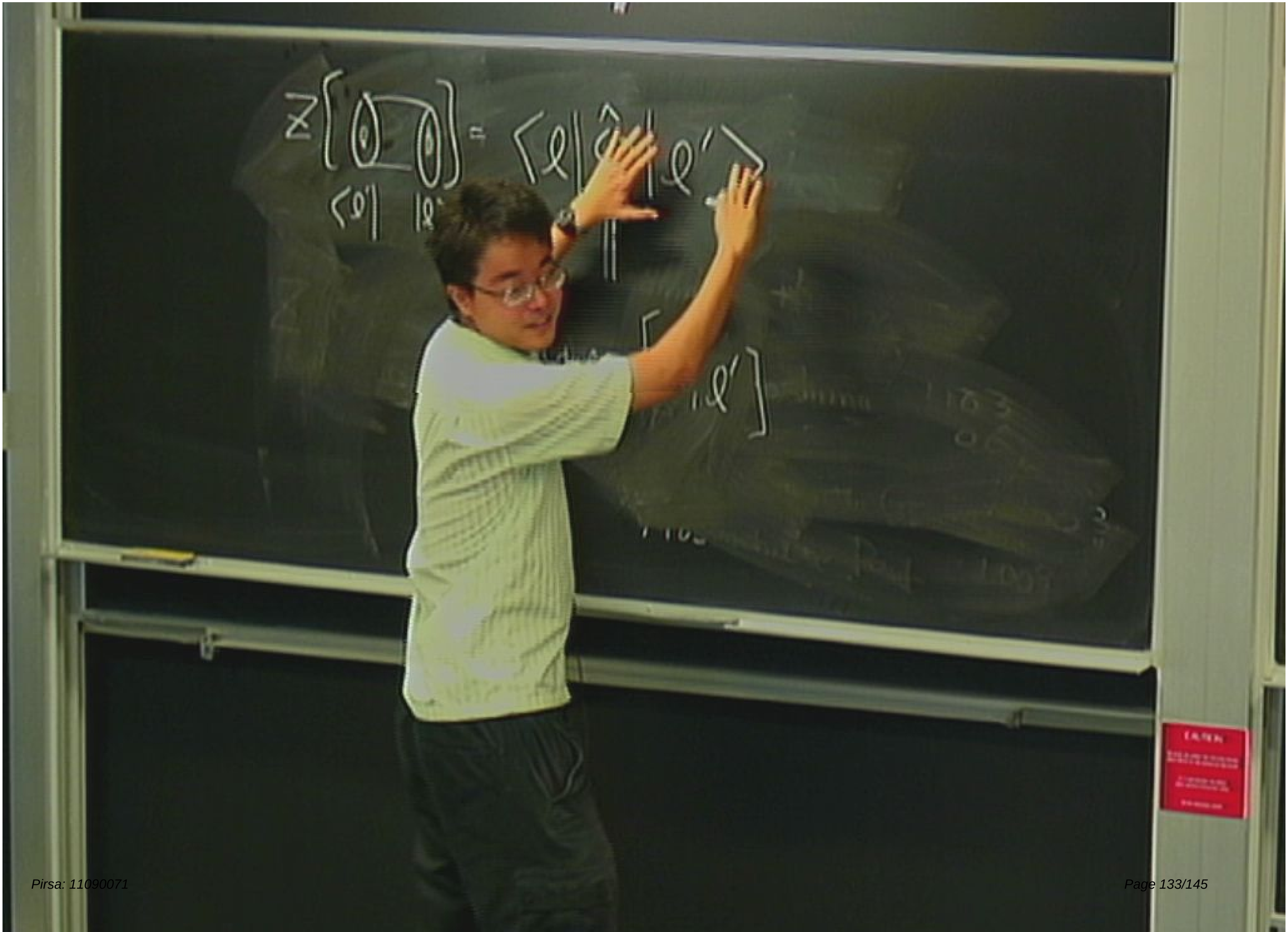
momentum

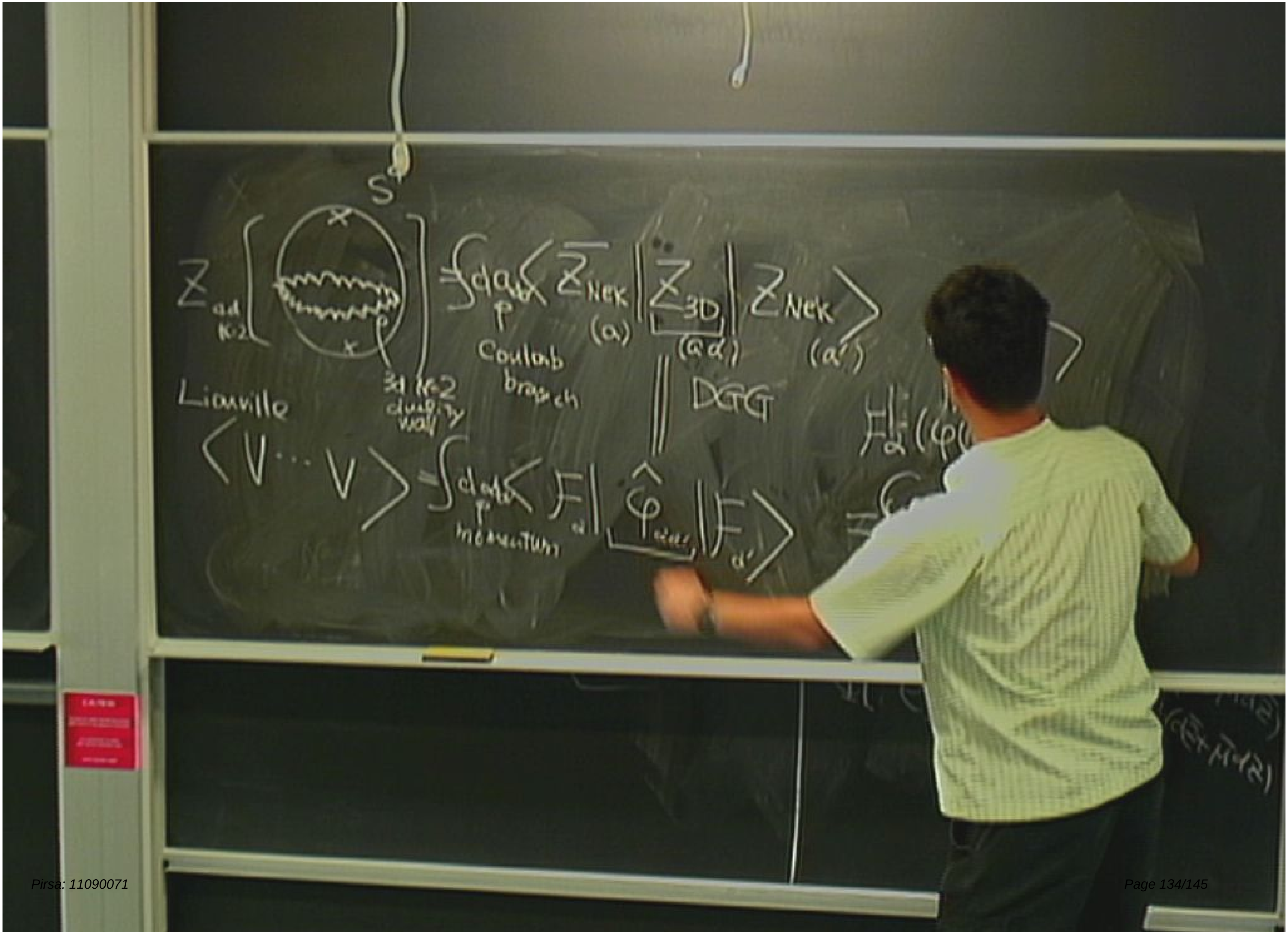

S4
Coulmb
branch
momentum

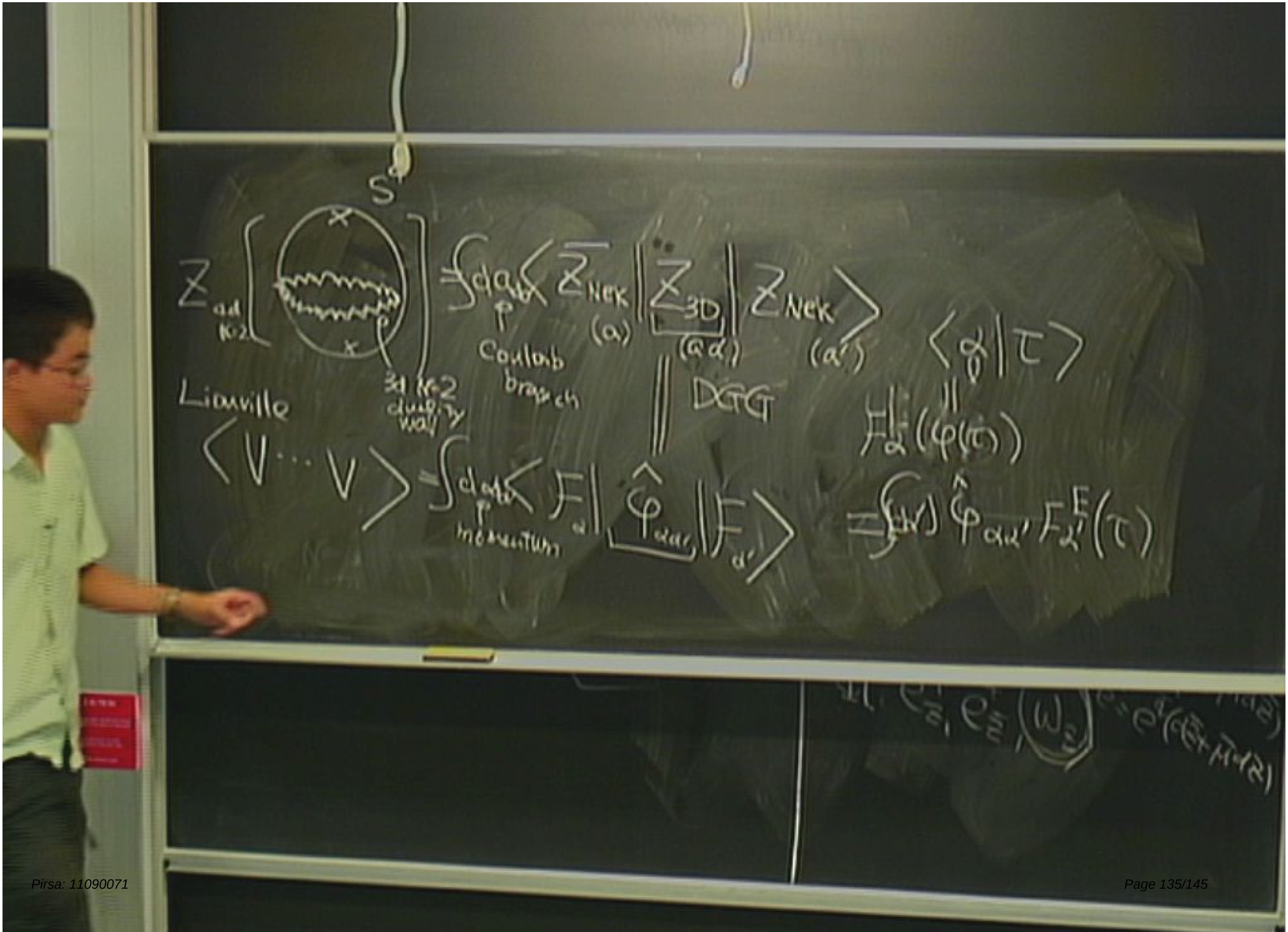

S^4
$Z_{ad,\,N=2}[\;\;] = \int da_p \langle \bar{Z}_{Nek} | \;\; | Z_{Nek} \rangle$
Coulomb branch
Liouville
3d N=2 duality wall
$\langle V \cdots V \rangle = \int d\alpha_p \langle F | \;\; | F \rangle$
momentum

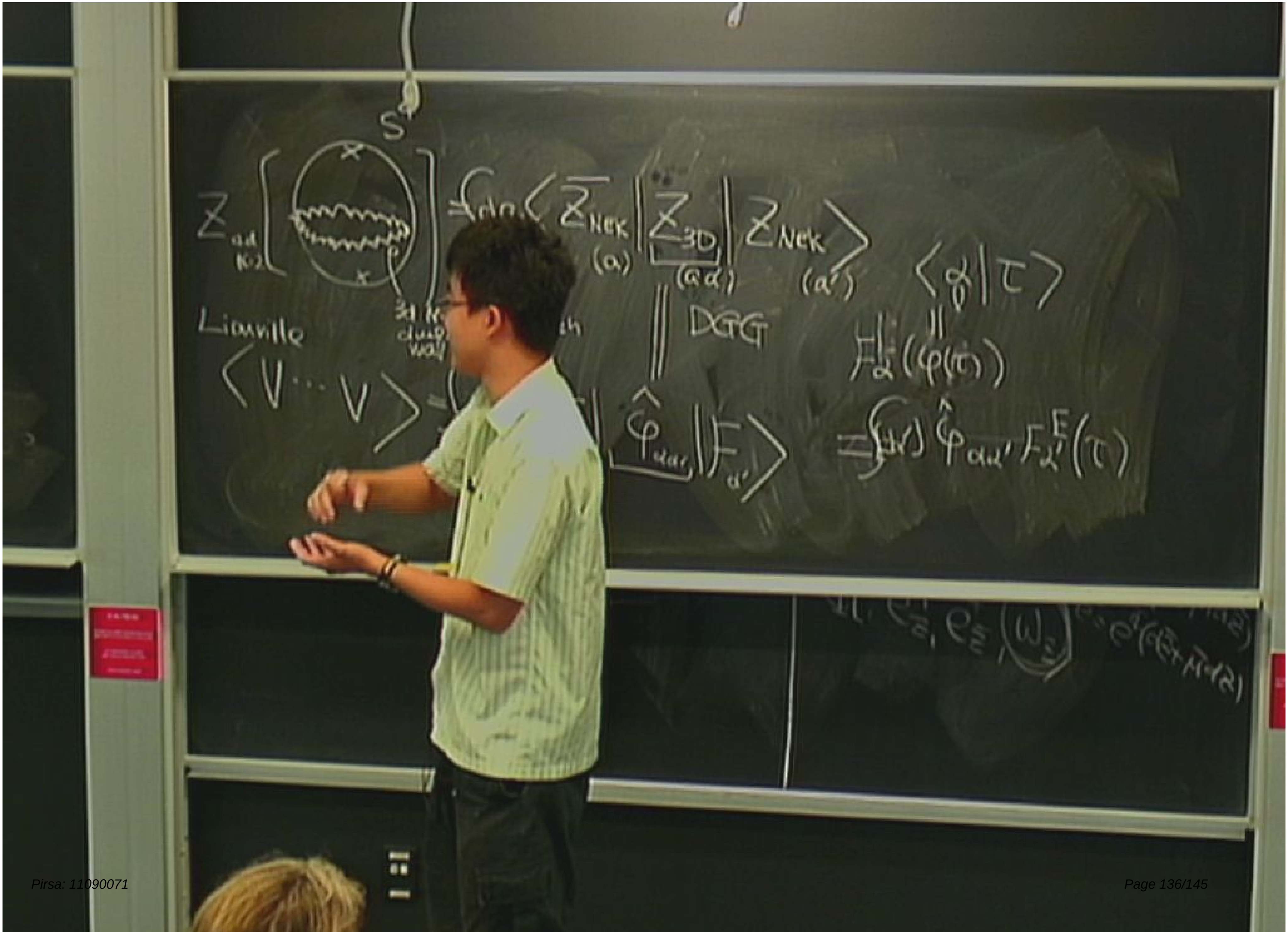

S1
Liouville
Coulomb branch
3d N=2 duality wall
momentum

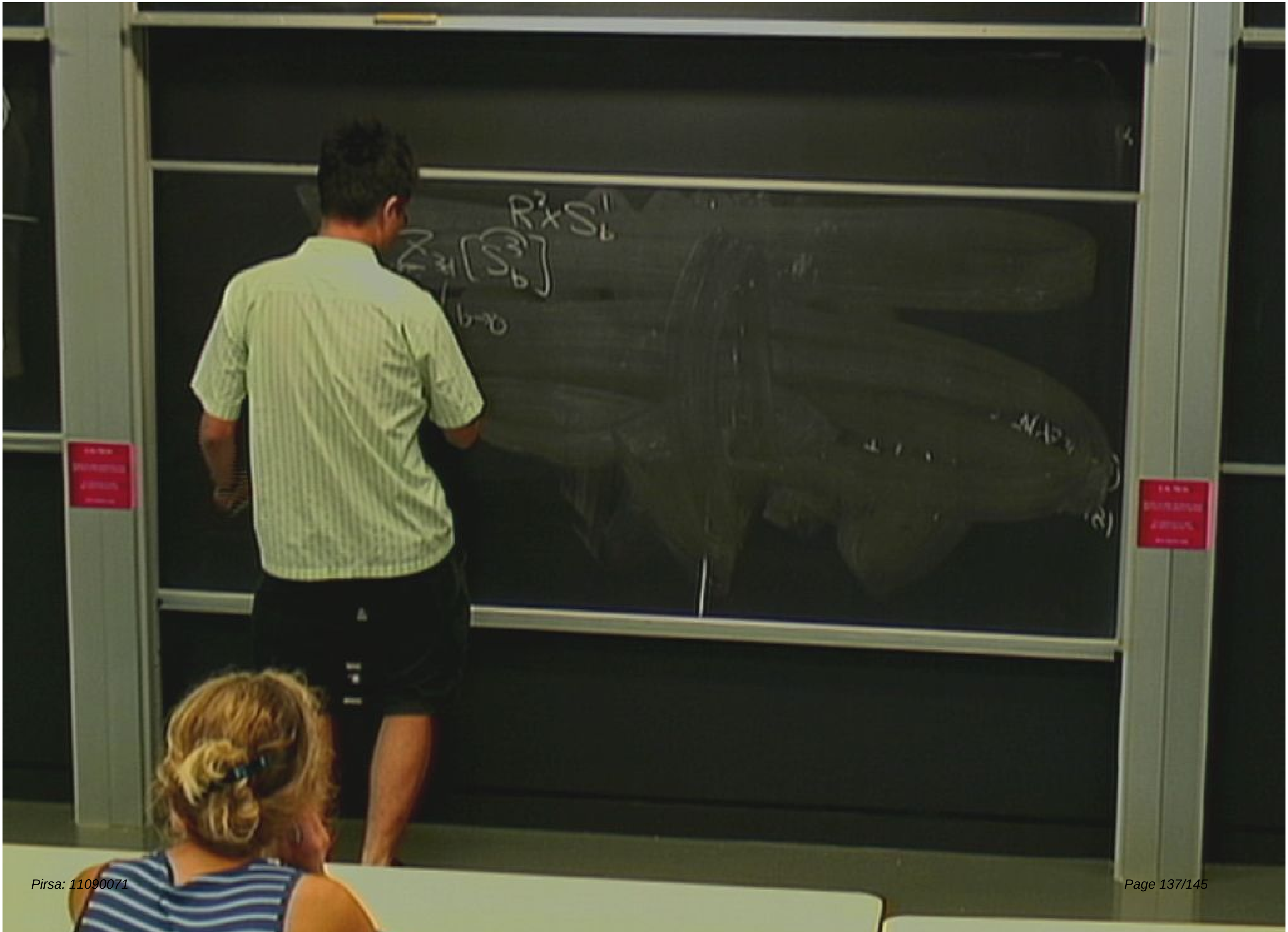

S4
Coulomb branch
DGG
Liouville
3d N=2 duality wall
momentum

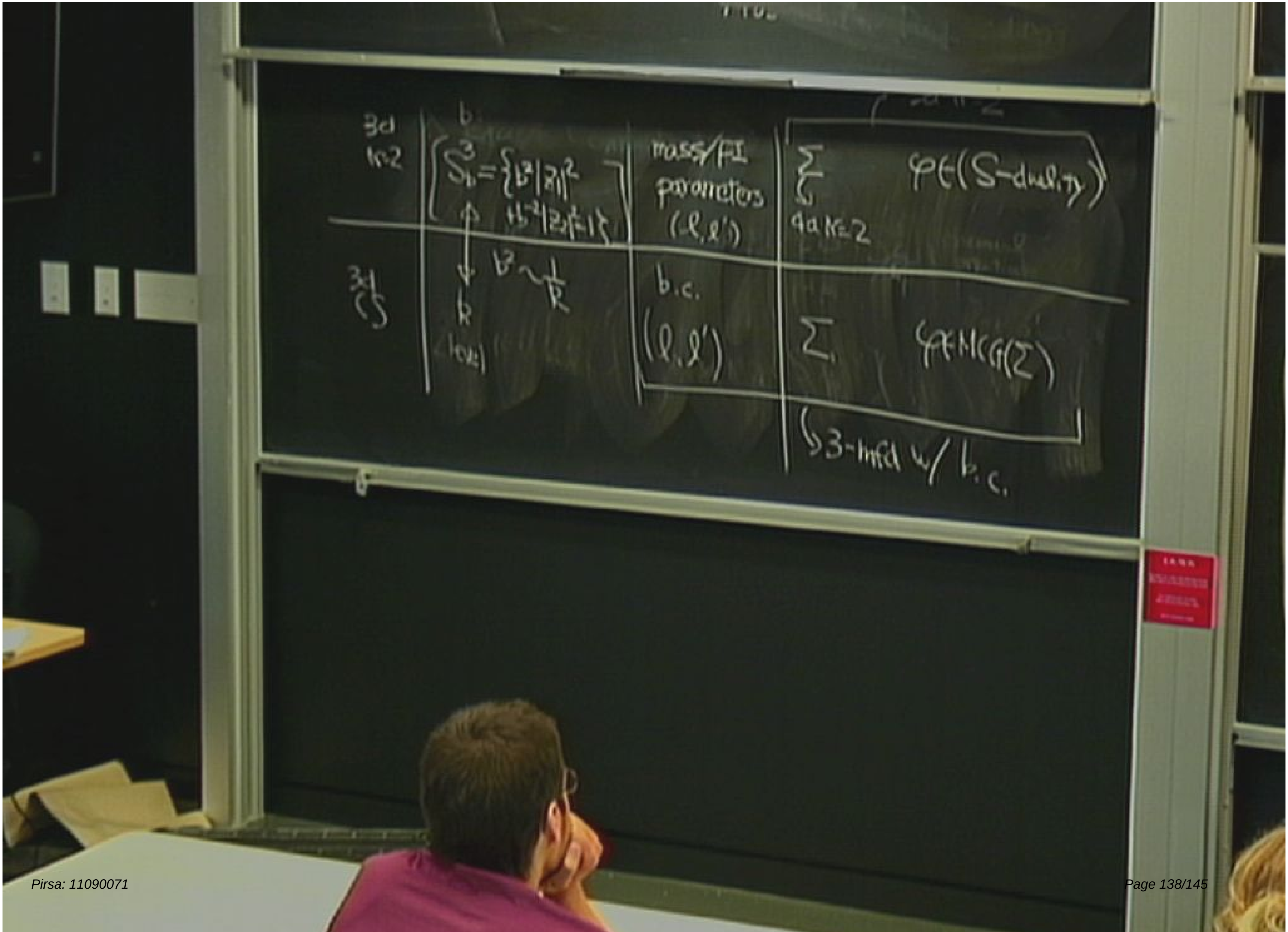


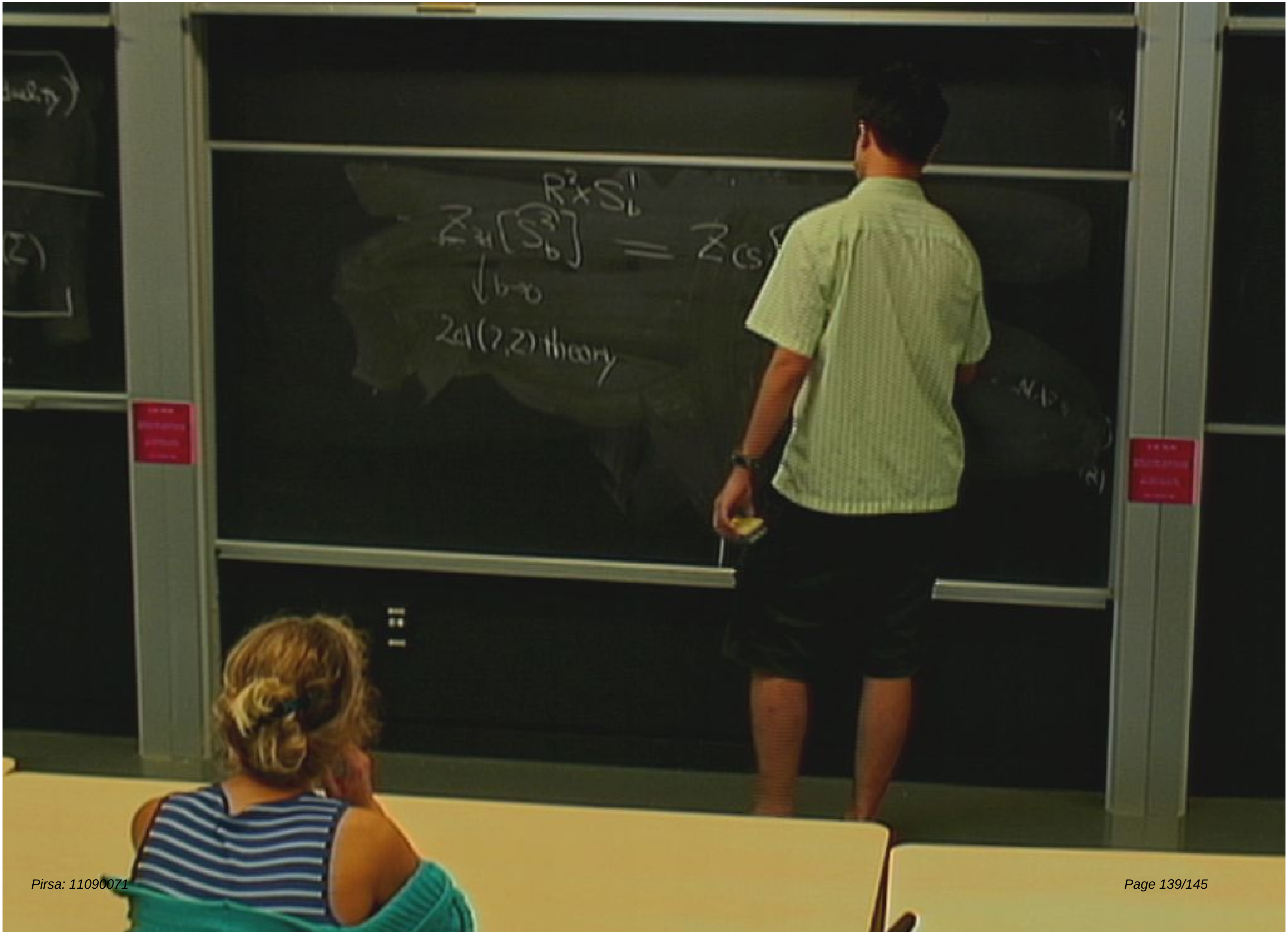

Liouville
Coulomb branch
momentum
DGG
duality wall


S^1
Z_{ad, N=2}
= ∫ da ⟨ Z_Nek (a) | Z_3D (a a') | Z_Nek (a') ⟩
Coulomb branch
3d N=2 duality wall
Liouville
DGG
⟨ V ··· V ⟩ = ∫ dα ⟨ F_α | φ̂_αα' | F_α' ⟩
momentum
⟨ φ | τ ⟩
= ∫ dα' φ̂_αα' F_α'^E (τ)

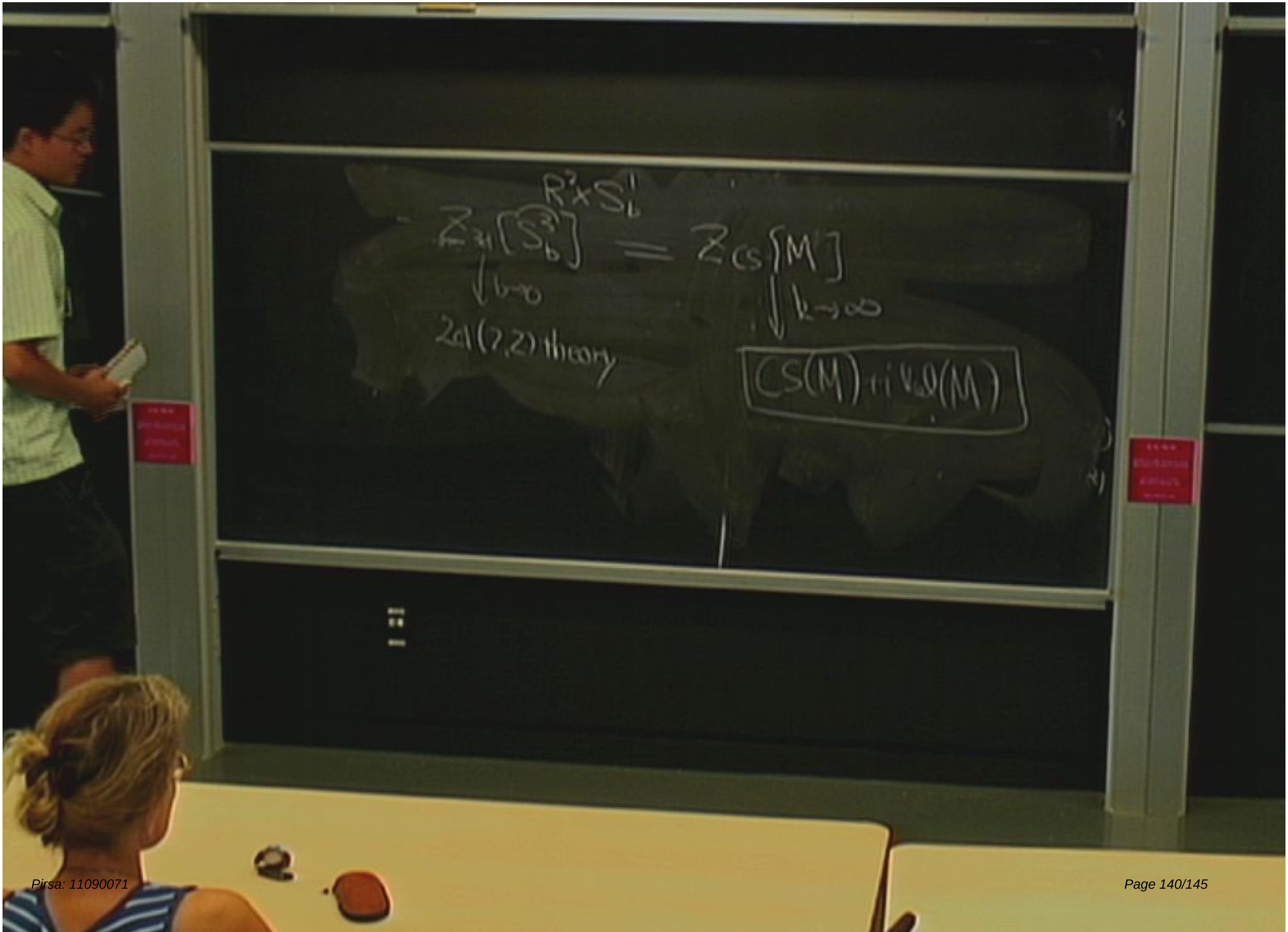

Liouville
DGG

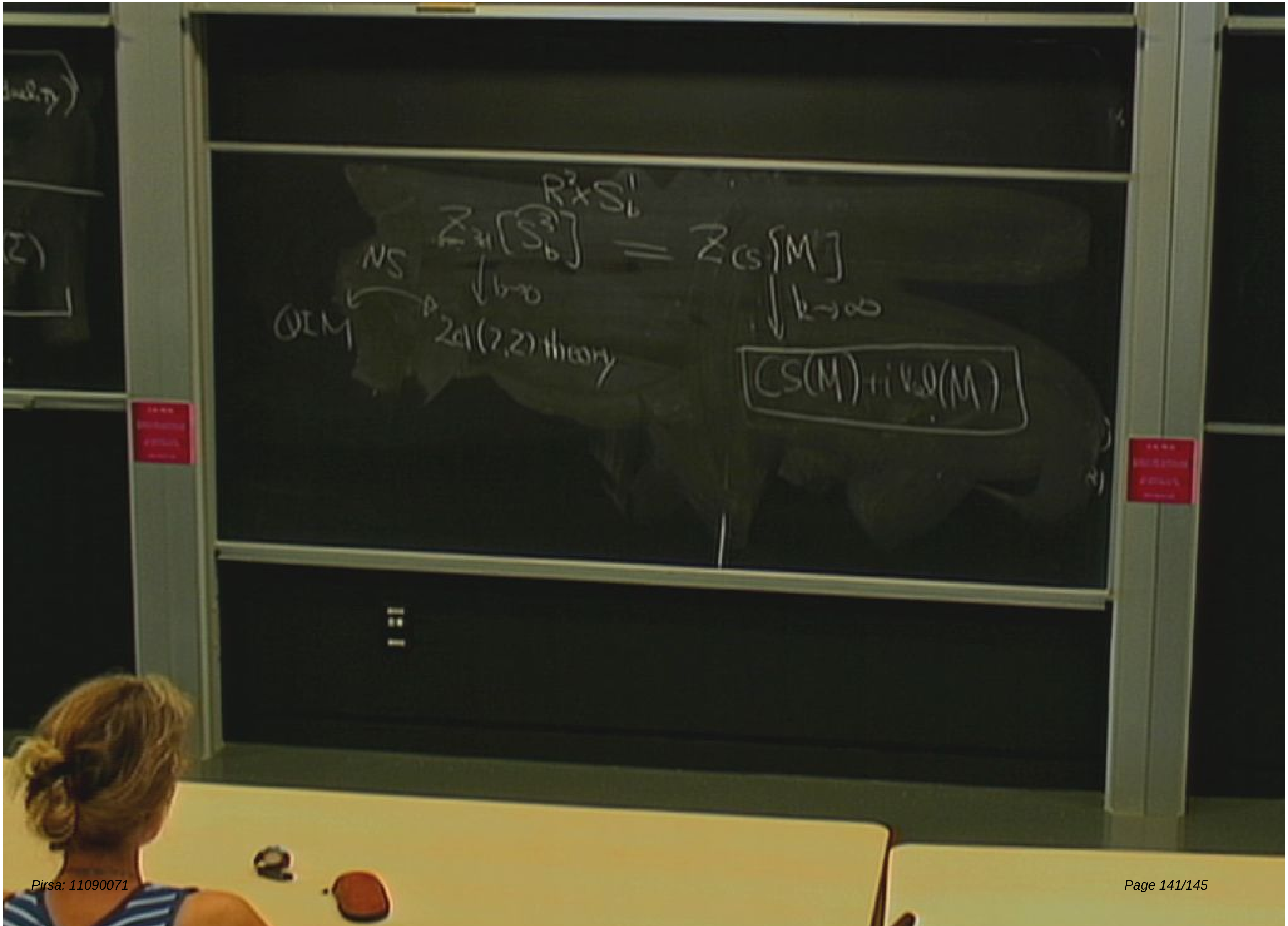


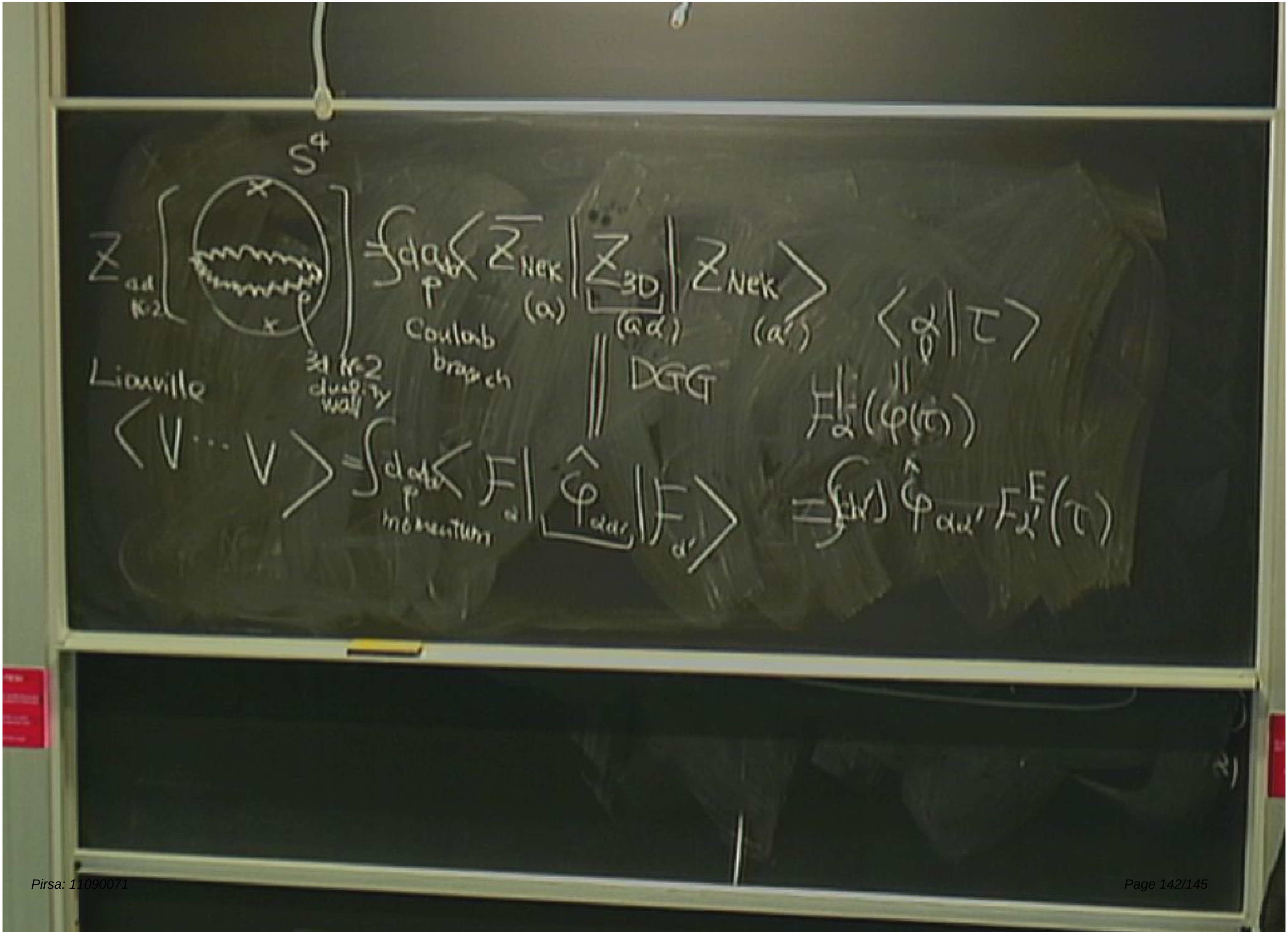

mass/FI
parameters
b.c.
(S-duality)
3-mfd w/ b.c.

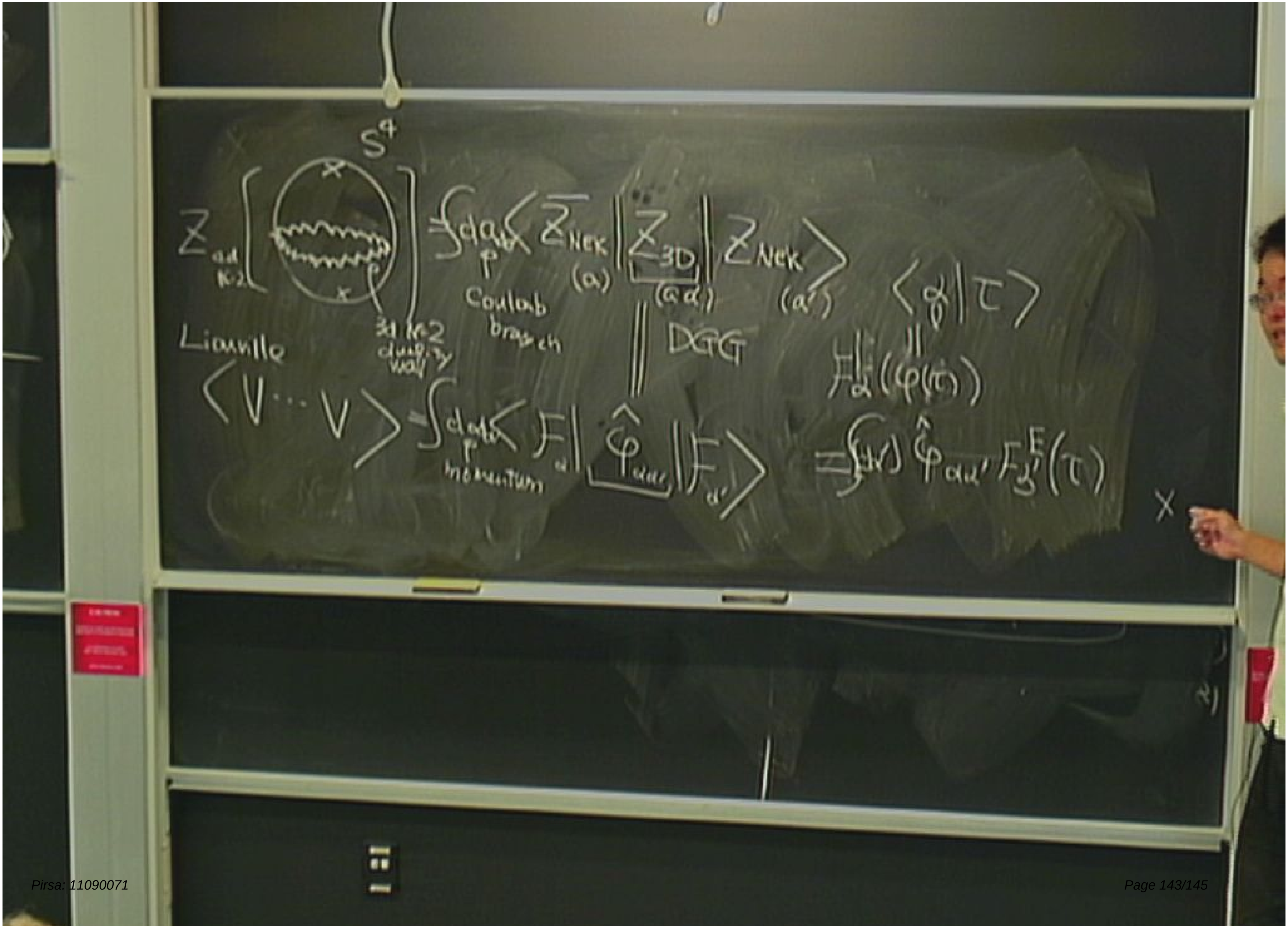


R²×S¹b
Z₄₁[S³b] = Z(S
↓b→0
2d (2,2) theory

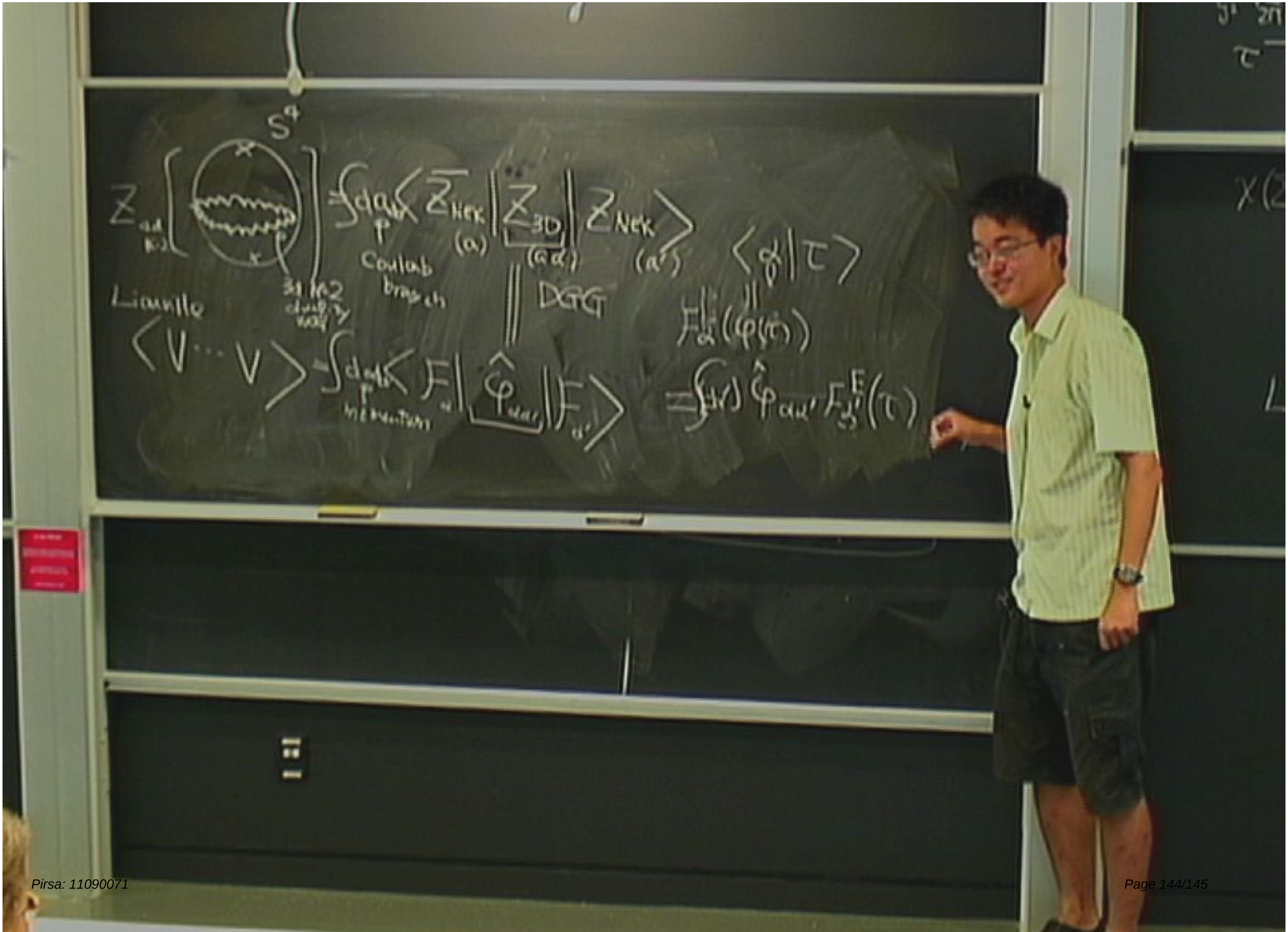

R²×S¹_b
Z_3d[S³_b] = Z_CS[M]
b→0
k→∞
2d (2,2) theory
CS(M) + i Vol(M)


R²×S¹_b
Z_3d[S³_b] = Z_CS[M]
NS
b→0
k→∞
QLM
2d (2,2) theory
CS(M) + i Vol(M)

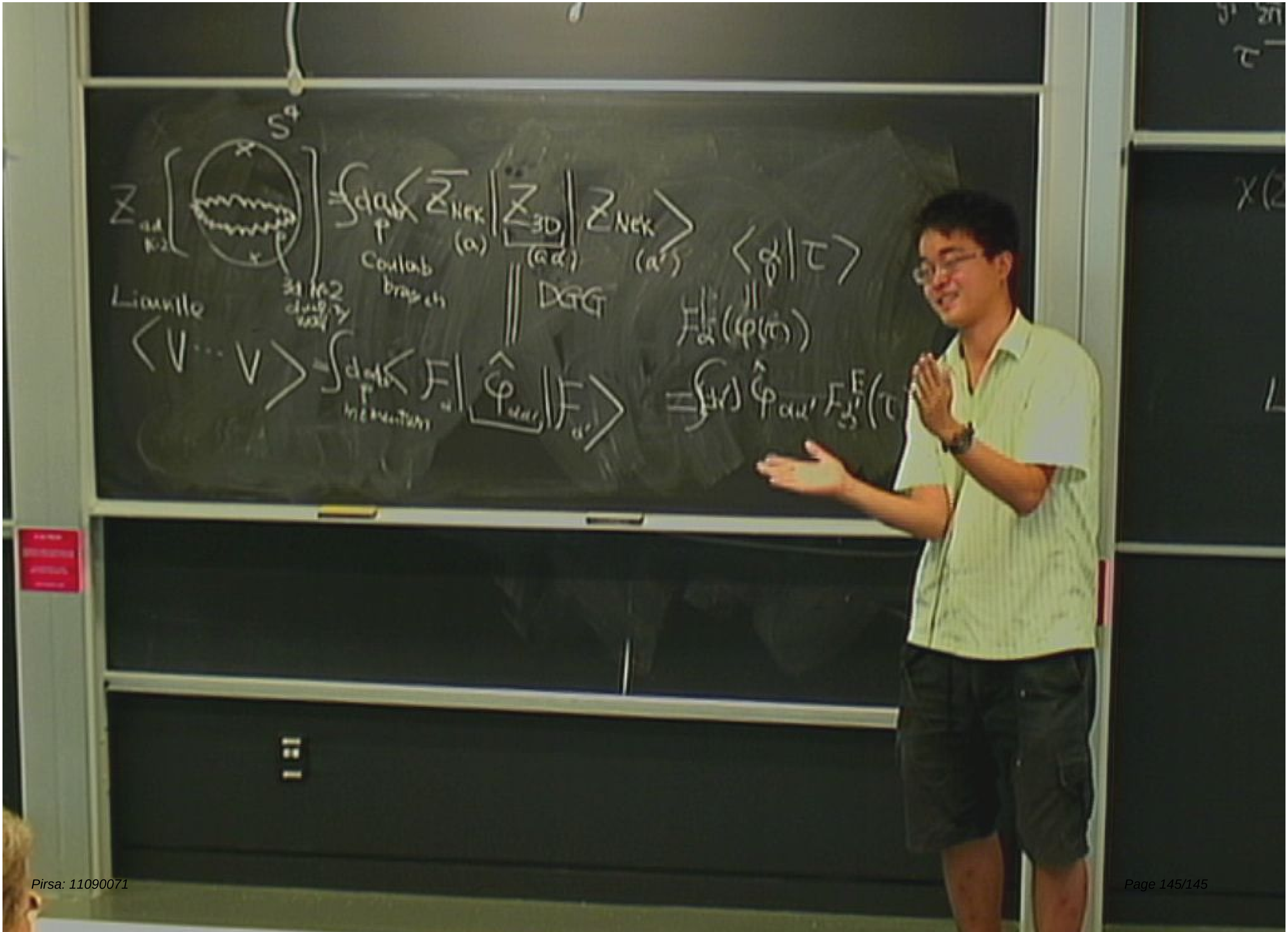

S4
Coulomb branch
Liouville
3d N=2 duality wall
DGG
momentum



S4
Z
N=2
Coulomb branch
3d N=2 duality wall
Liouville
DGG
NEK
3D
momentum



Liouville
Coulomb branch
DGG


S4
Coulomb branch
3d N=2 duality wall
Liouville
DGG
momentum