Title: Building Fractional Topological Insulators

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Abstract: Time-reversal invariant band insulators can be separated into two categories: `ordinary' insulators and `topological' insulators. Topological band insulators have low-energy edge modes that cannot be gapped without violating time-reversal symmetry, while ordinary insulators do not. A natural question is whether more exotic time-reversal invariant insulators (insulators not connected adiabatically to band insulators) can also exhibit time-reversal protected edge modes. In 2 dimensions, one example of this is the fractional spin Hall insulator (essentially a spin-up and spin-down copy of a fractional quantum Hall insulator, with opposite effective magnetic fields for each spin). I will discuss another family of strongly interacting insulators, which exist in both 2 and 3 dimensions, that can have time-reversal protected edge modes. This gives a new set of examples of `fractional' topological insulators.

Building Fractional Topological Insulators

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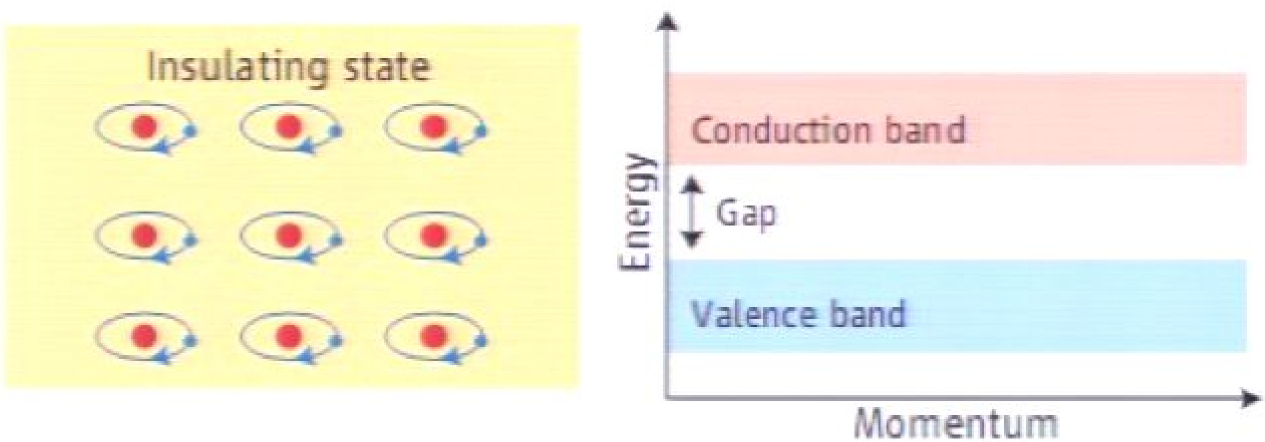
The program

- Background:
  - Topological insulators
  - Fractionalization
- Exactly solvable Hamiltonians for "fractional topological insulators"
- When are they really fractional topological insulators?

Ordinary Insulators

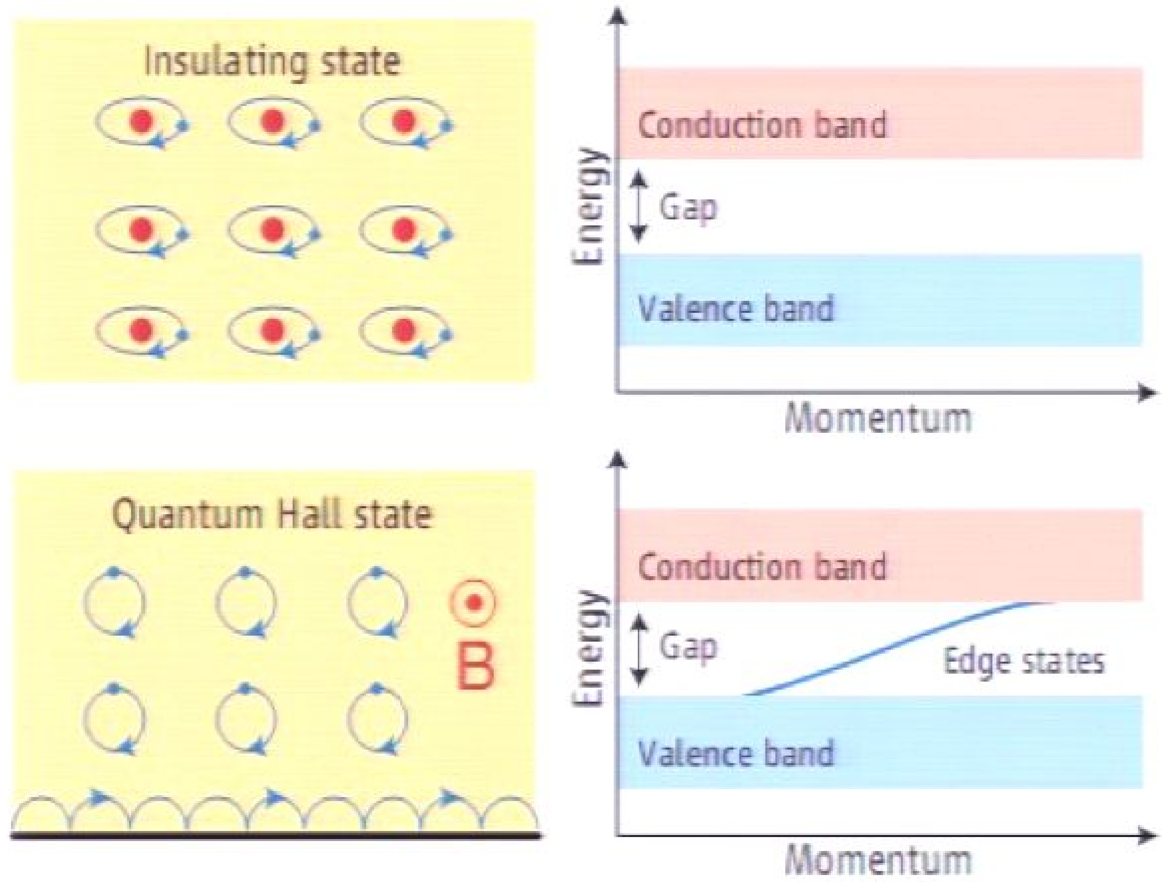
- all states are local OR
- Can be deformed into a product of local states without going through a phase transition



Chern insulators

- Cannot localize all states without a phase transition

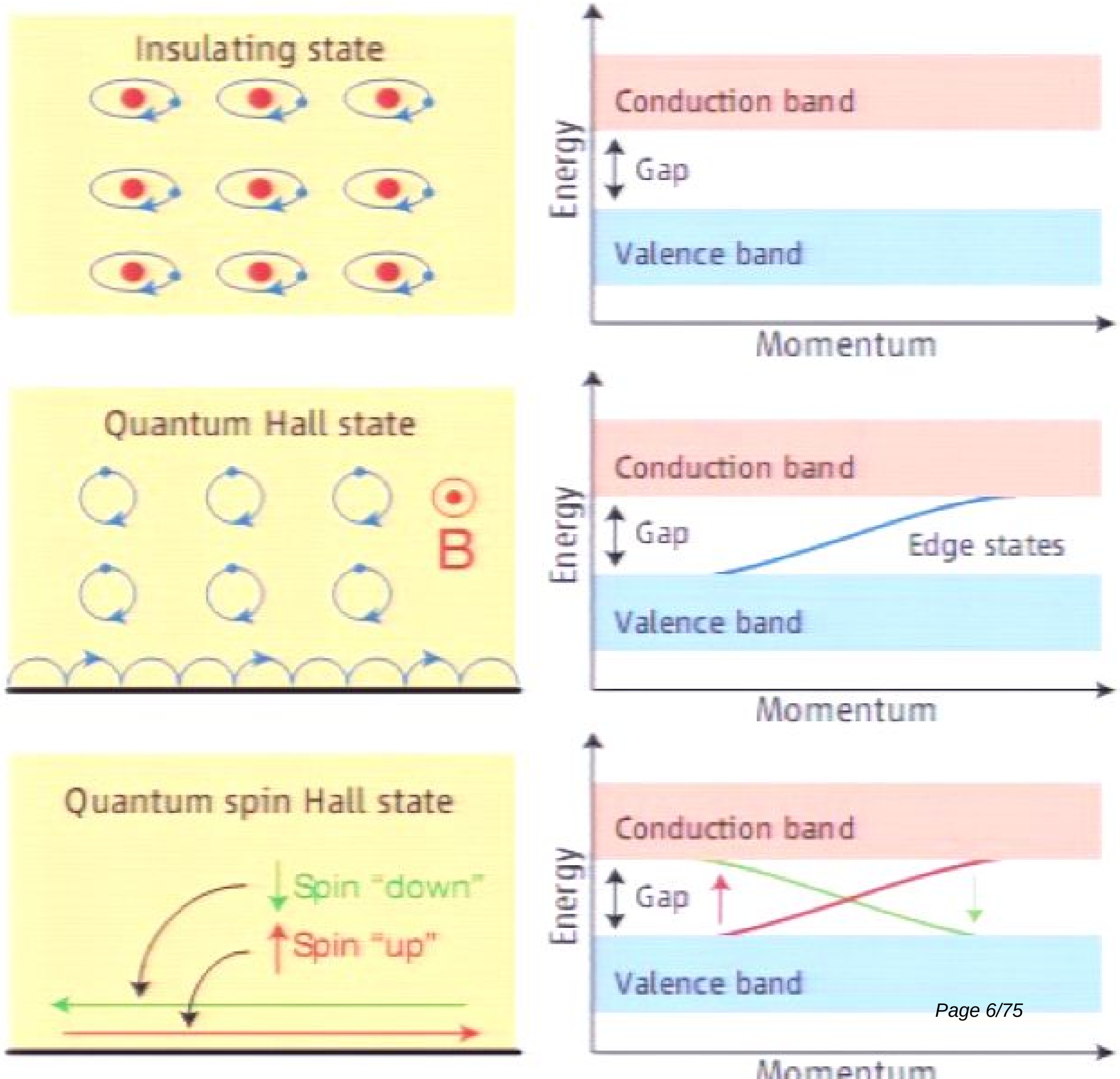
- Gapless edge modes



Topological insulators

- Cannot localize all states without
  a phase transition
  OR
  breaking T

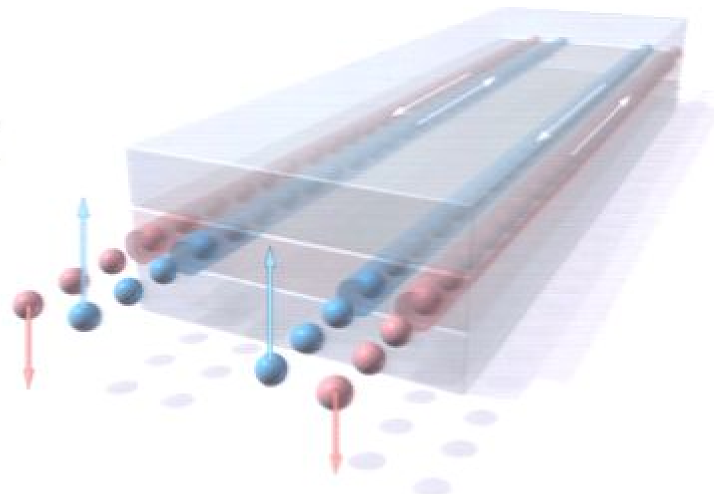
- Time-reversal protected gapless edge modes



Topological insulators

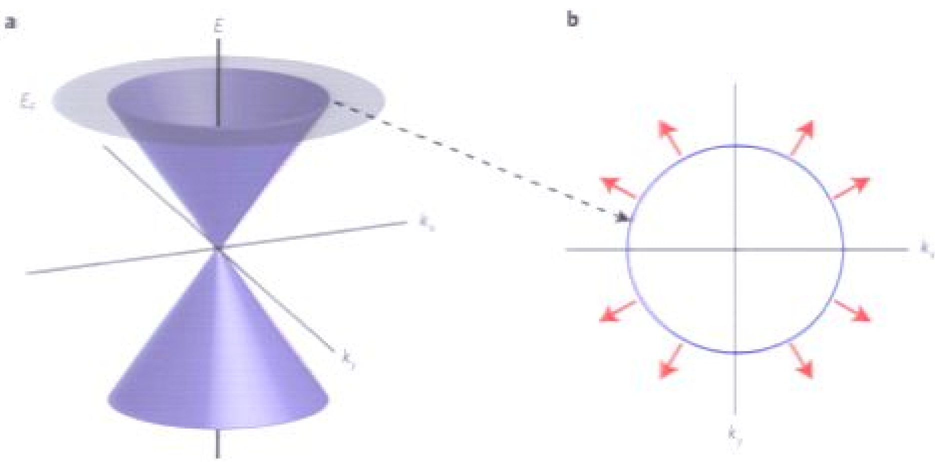
- 2D: quantum spin Hall effect

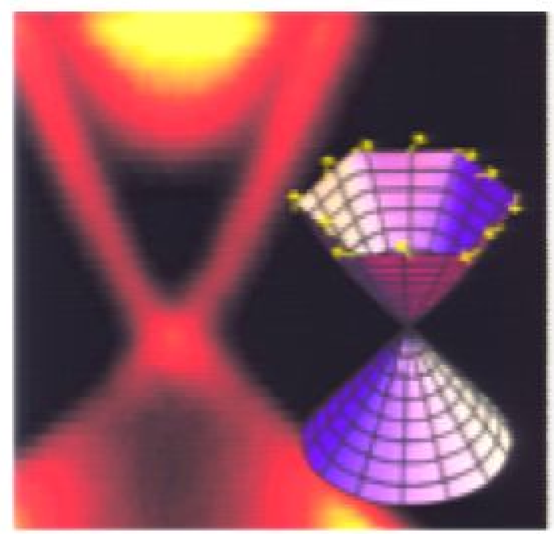
  (HgTe; Bernevig Hughes Zhang; Konig et al)



- 3D: single surface Dirac cone

  (Moore Balents; Fu Kane; Roy; Hsieh et al)



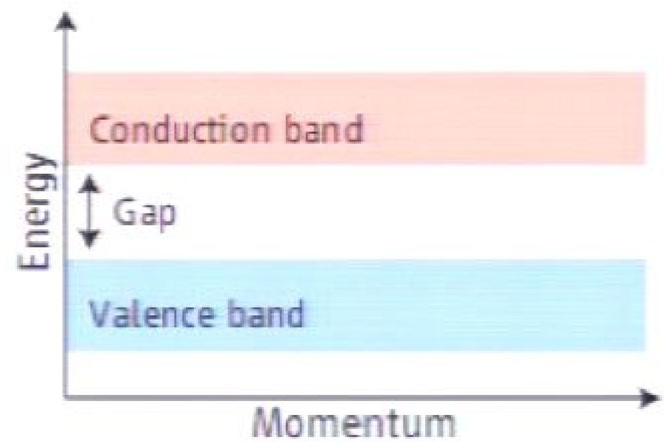


Moore, Nature physics 2009

What's the big deal?

- Even without interactions, there are different kinds of insulators

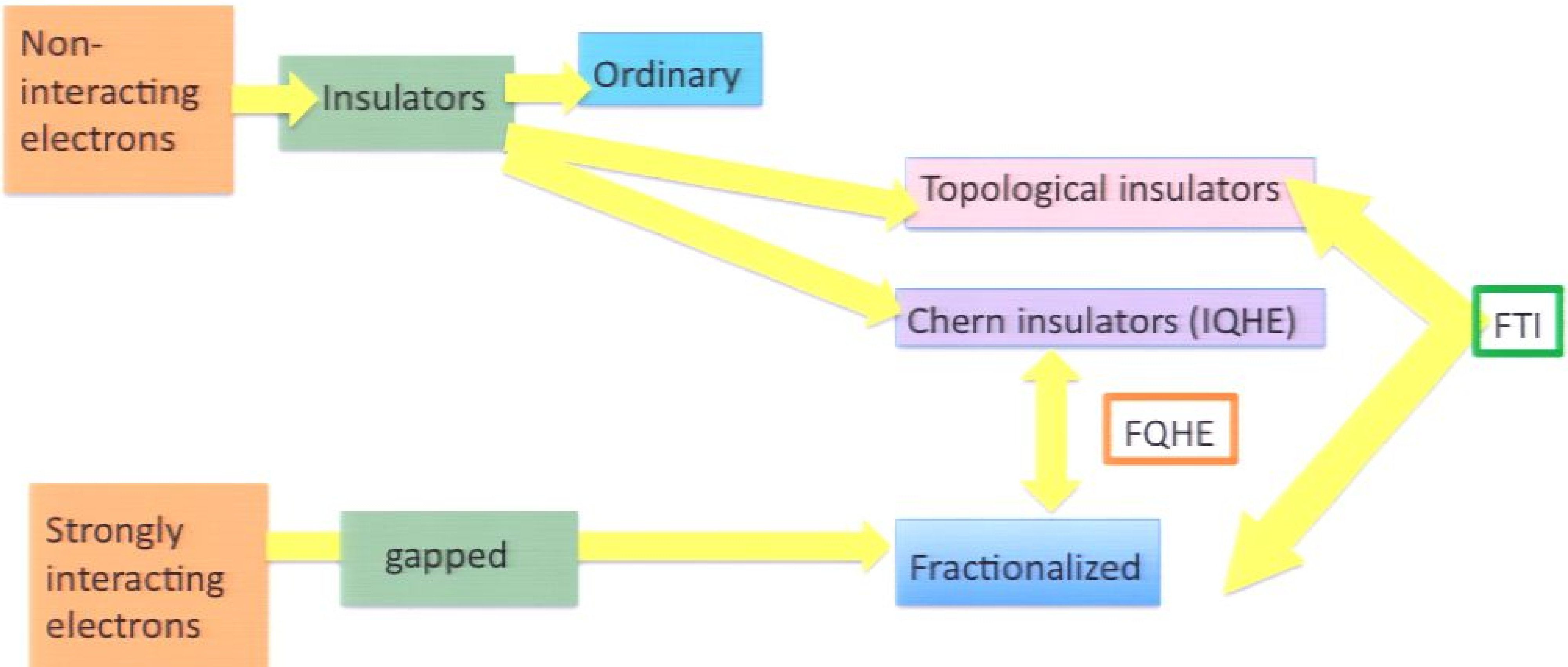
- Bulk properties are all ordinary
  - (gapped, incompressible states)



- Distinguished by properties of boundary
  - (Quantized transport of charge, spin)

Fractionalization

- What?
  - Excitations carry a fraction of quantum numbers of constituent particles.
- How?
  - Strong interactions
- Example
  - Fractional QHE


Non-interacting electrons
Insulators
Ordinary
Topological insulators
Chern insulators (IQHE)
FTI
FQHE
Strongly interacting electrons
gapped
Fractionalized

Fractional topological insulators

- Can you make a model with a single surface Dirac cone of fractionally charged fermions?
  - Exactly solvable lattice model
  - Fractionally charged fermionic excitations
  - Band structure.
- Is it a topological insulator?
  - Time reversal protected edge modes

Solvable models with fractionally charged fermions

- Strongly interacting charged bosons ($\frac{q_B}{p}$)

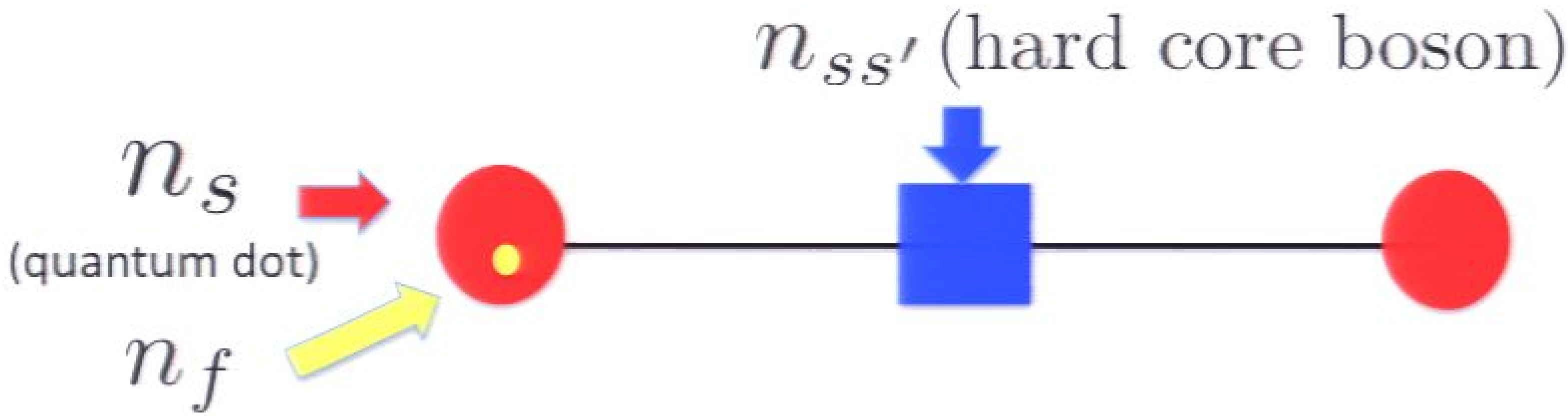
  (RVB; Toric code; Senthil & Motrunich)

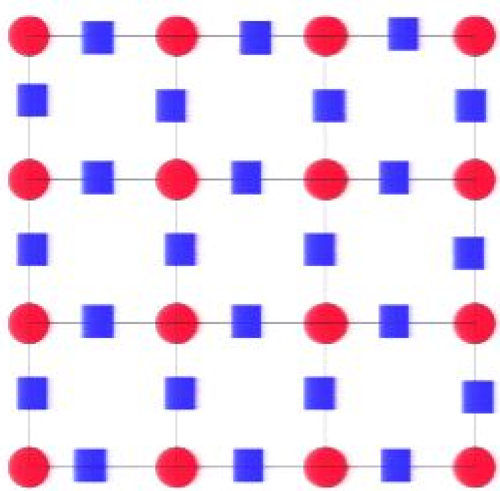
  - Charge conserving, exactly solvable

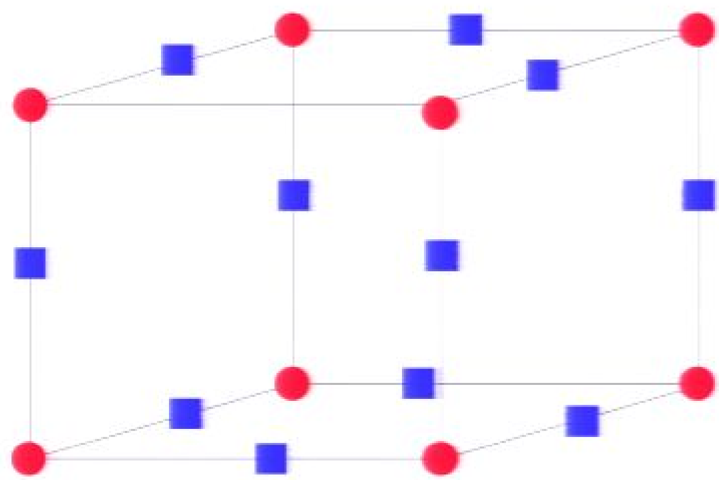
- Couple to fermions $q_f = e + k\frac{q_B}{p}$

- Band structure

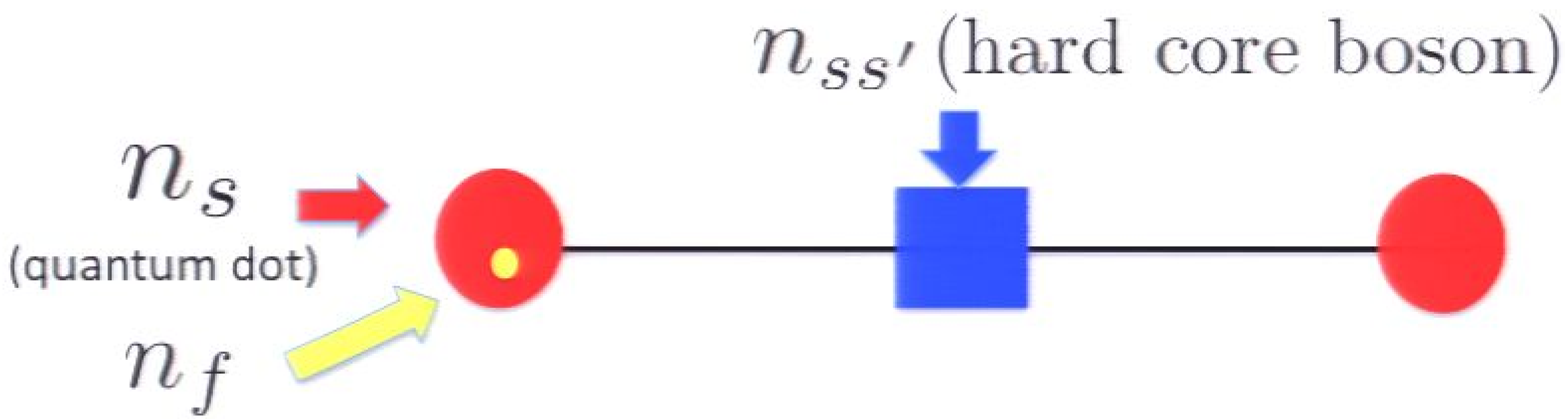
Solvable models with fractionally charged fermions

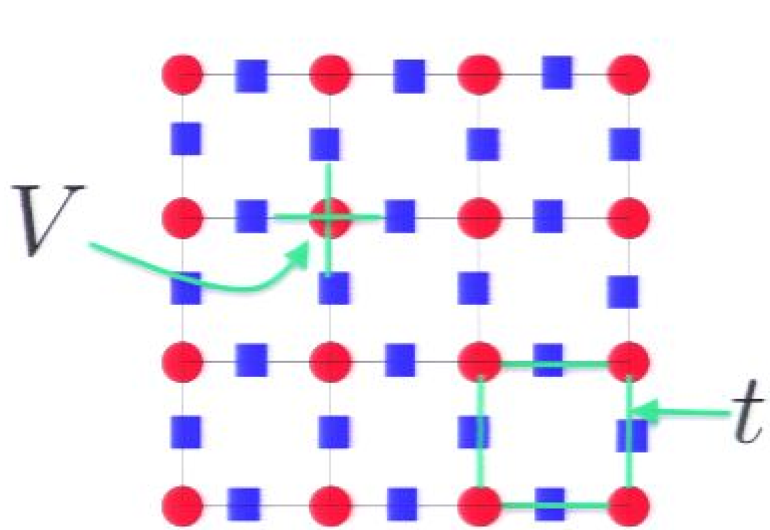


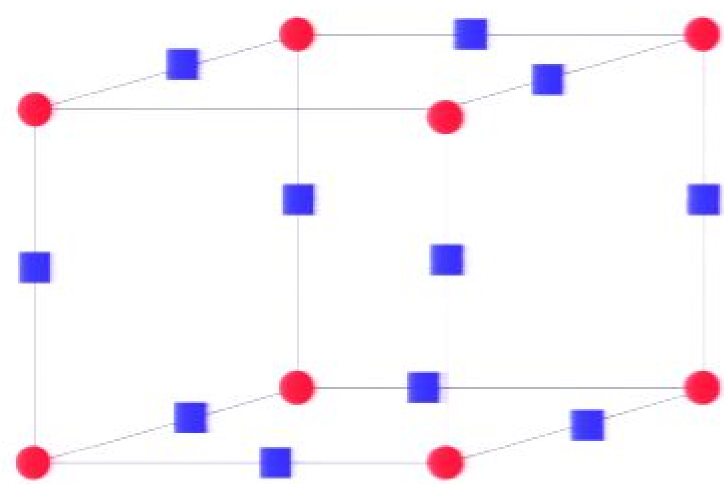




Solvable models with fractionally charged fermions

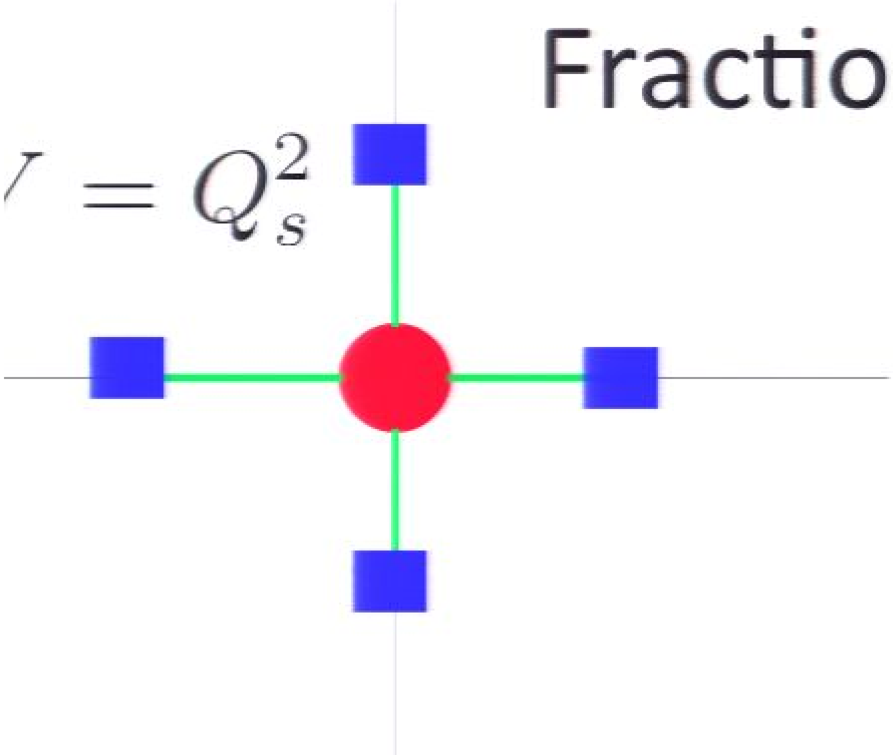






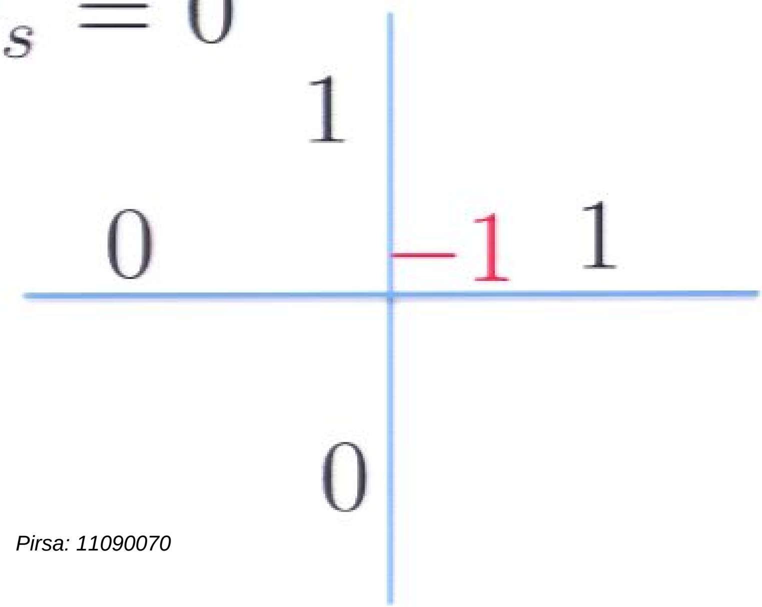
Fractionally charged bosons

$V = Q_s^2$

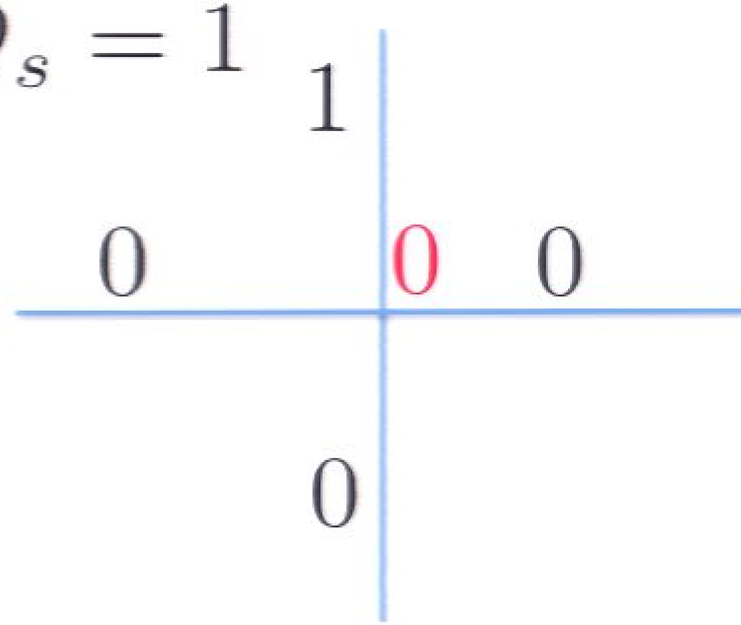


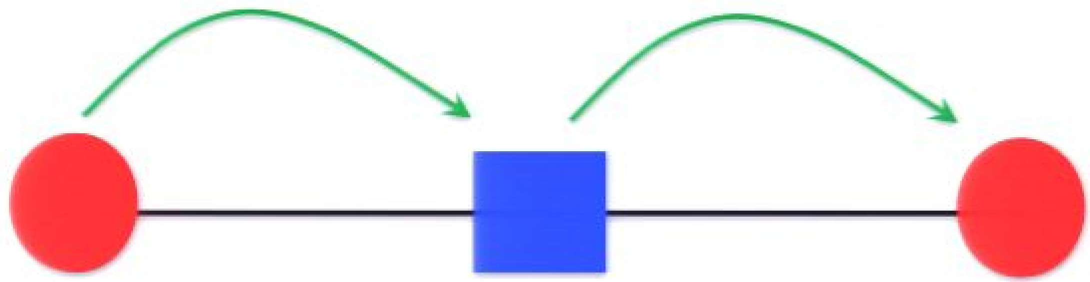
$$Q_s = \sum_{s'} n_{ss'} + 2n_s$$

$Q_s = 0$

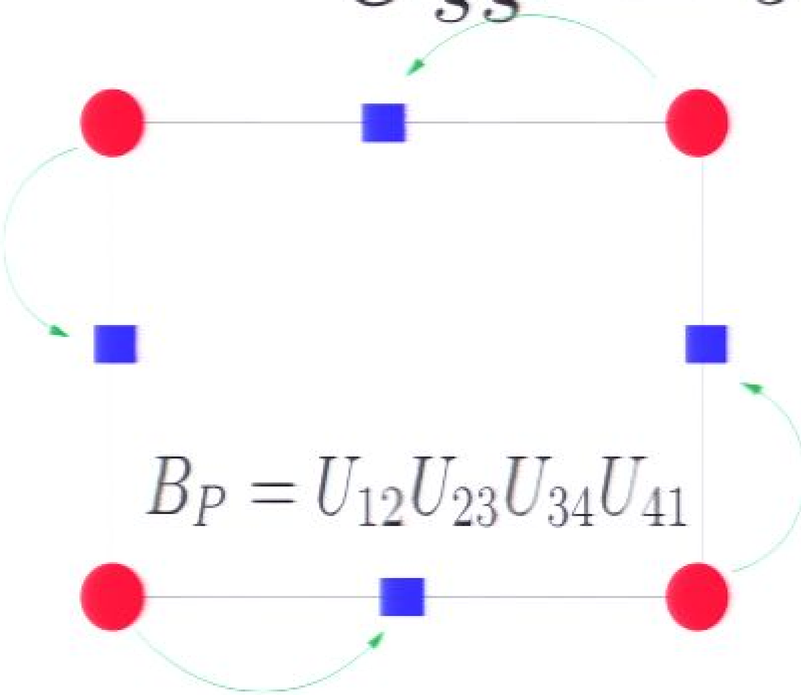


$Q_s = 1$





$$U_{ss'} = b^\dagger_{s'} b_{ss'} + b^\dagger_{ss'} b_s$$



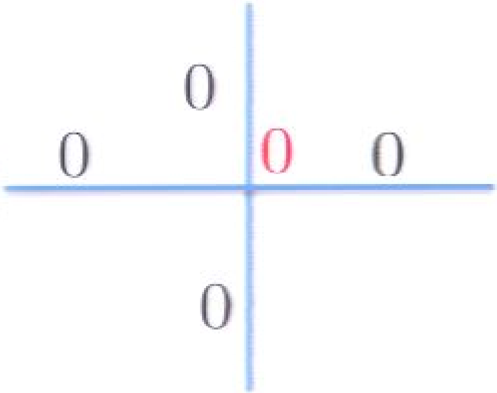
$$B_P = U_{12} U_{23} U_{34} U_{41}$$

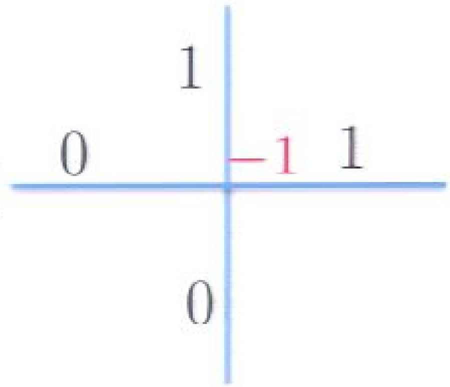
$$[Q_s, B_P] = 0$$

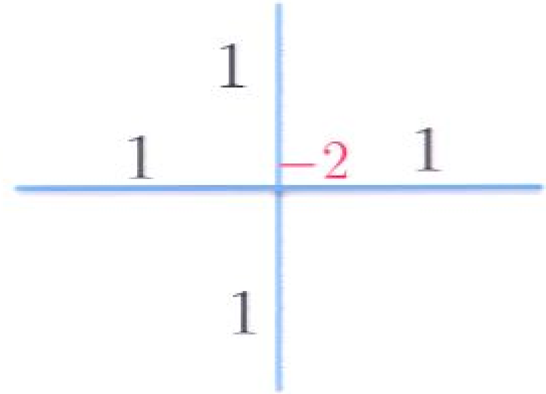
- Equal amplitude superposition of all link occupation numbers compatible with Q_s

$$H = V \sum_s Q_s^2 - \frac{u}{2} \sum_P \left(B_P + B_P^\dagger \right)$$

$Q_s = 0$

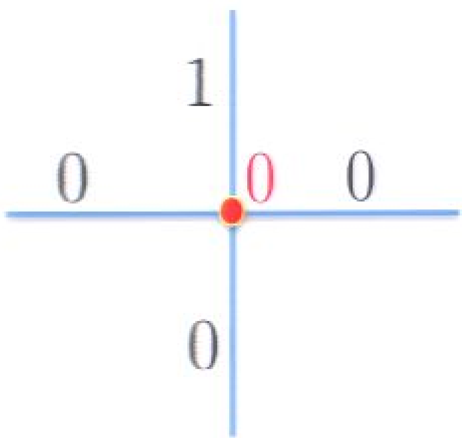
. $\times$ 

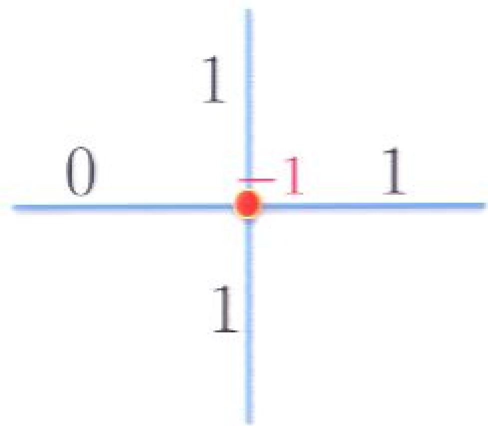
$+6\times$ 

$+1\times$ 

links : 2 ; sites : -1

$Q_s = 1$

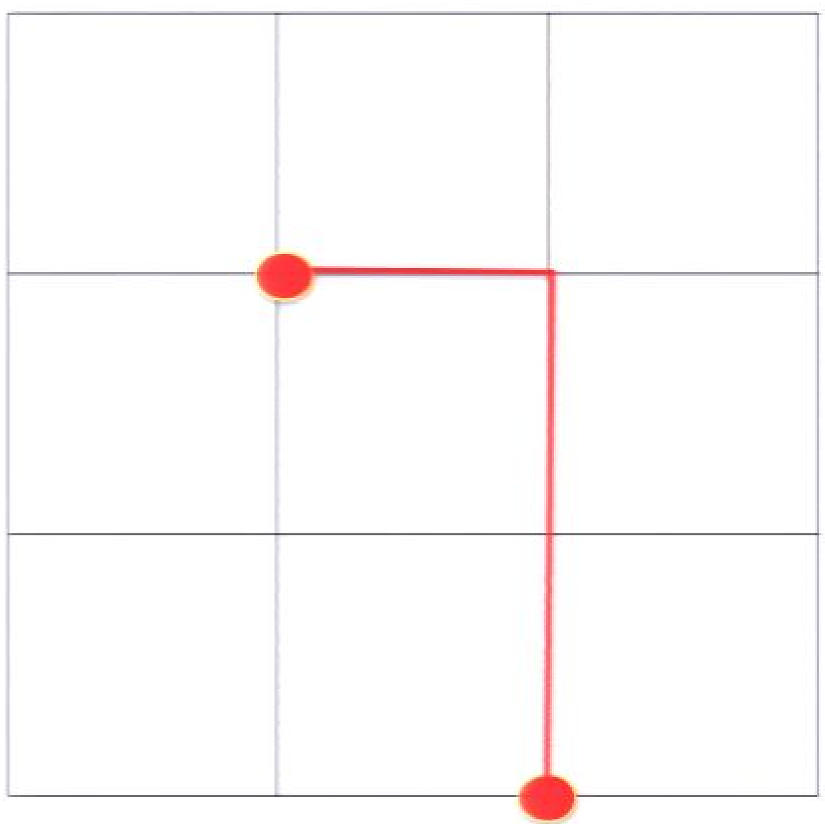
$4\times$ 

$+4\times$ 

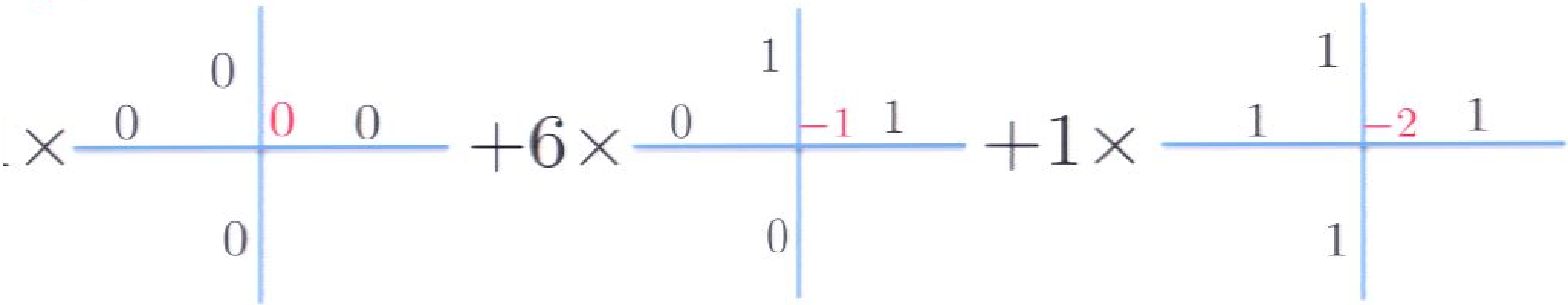
links : 2 ; sites : $-\frac{1}{2}$

Fractional Charge

$$\langle N_S \rangle_{Q_s=1} - \langle N_s \rangle_{Q_s=0} = \frac{1}{p}$$



$Q_s = 0$



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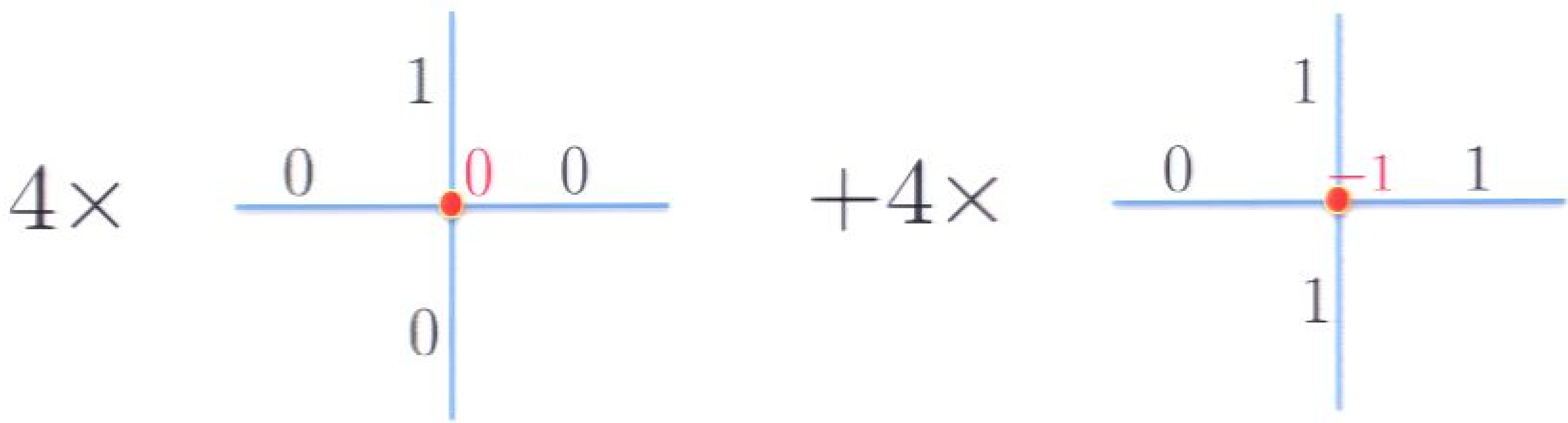

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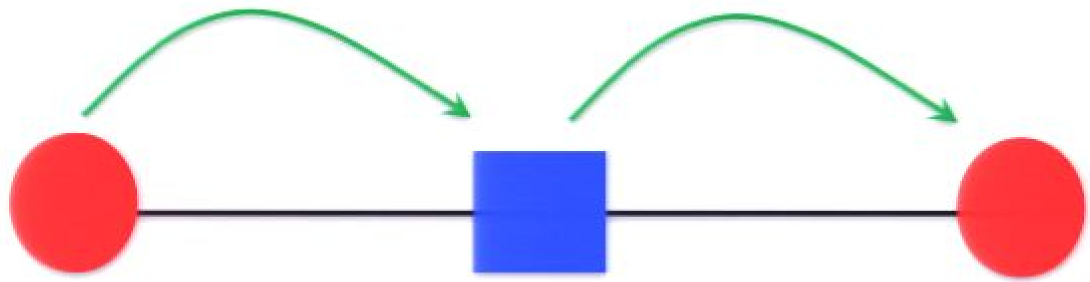

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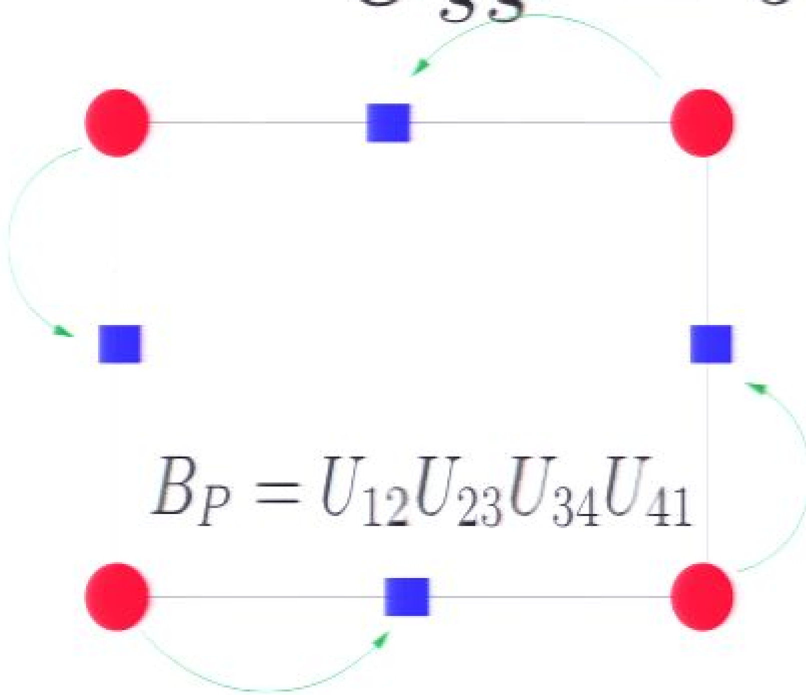

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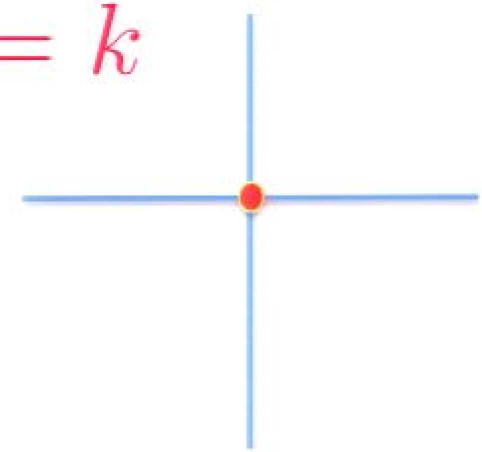
Fractionally charged fermions

$$\tilde{Q}_s = Q_s - kn_{f,s}$$

$$Q_s = k$$



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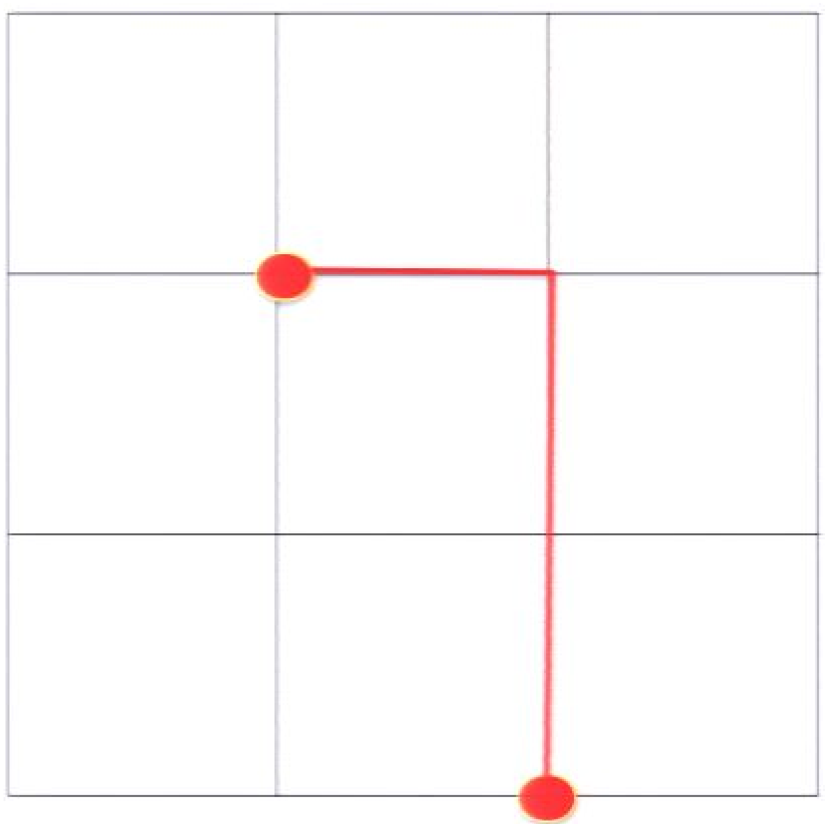

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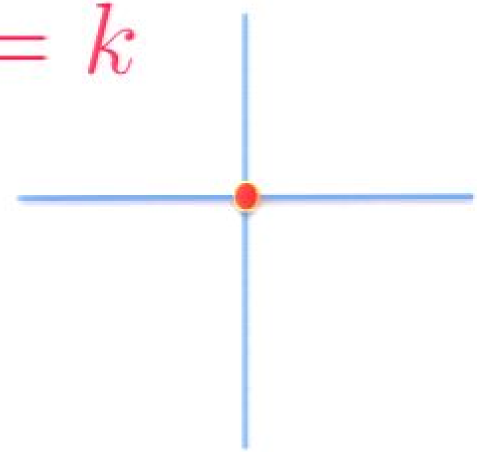

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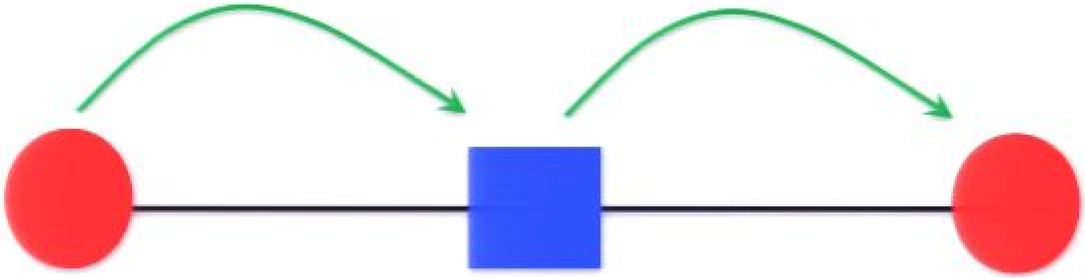

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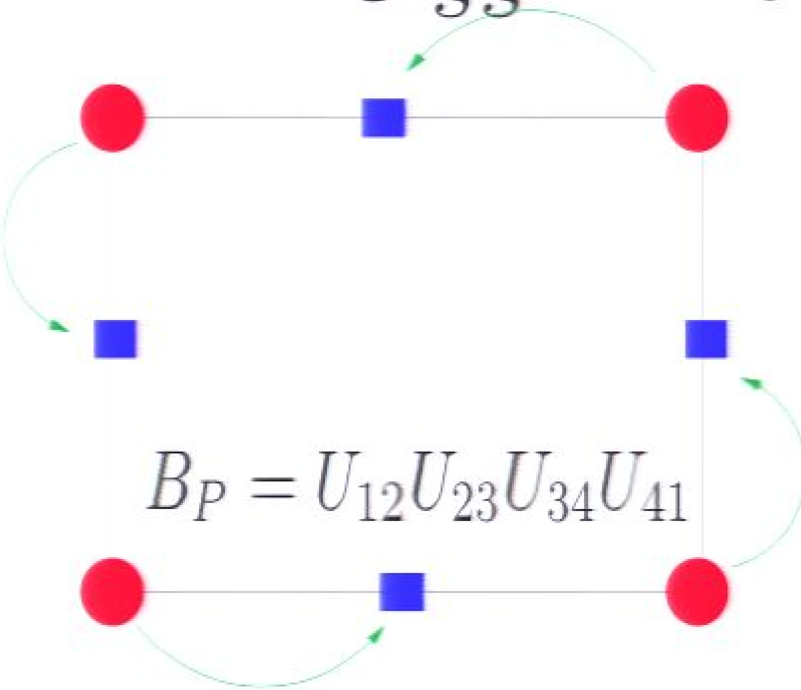

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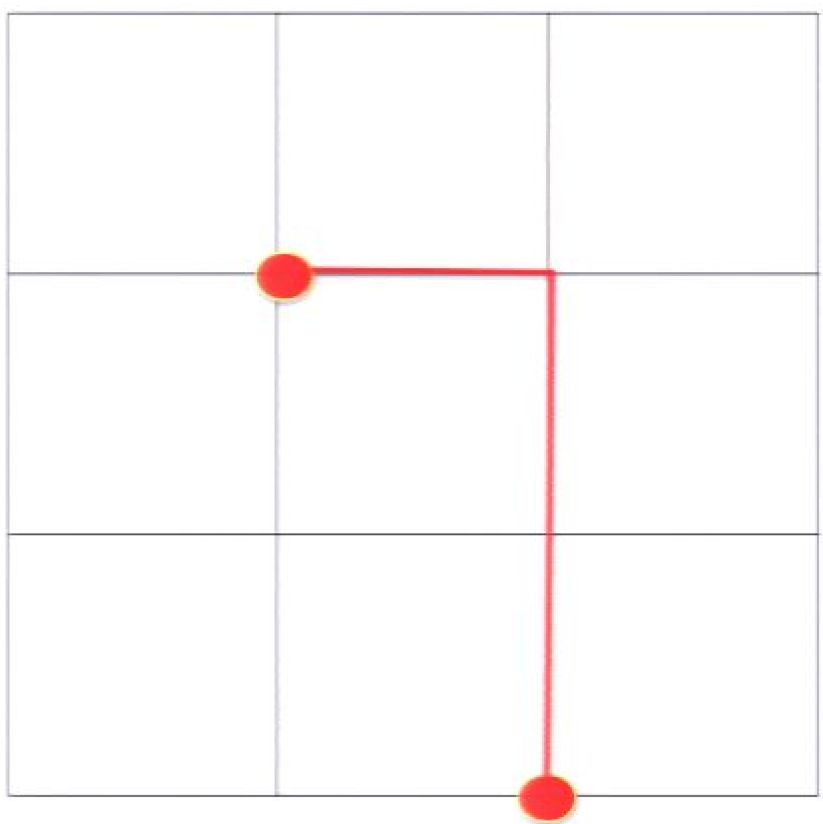

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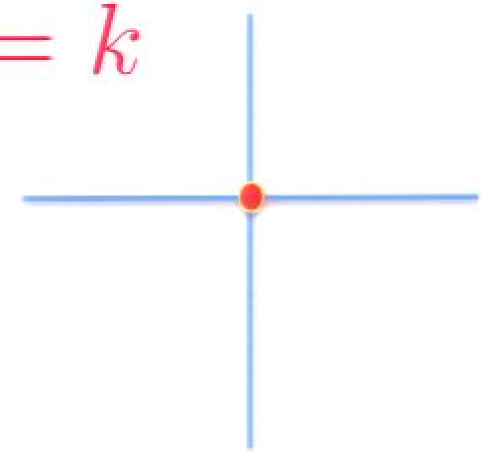

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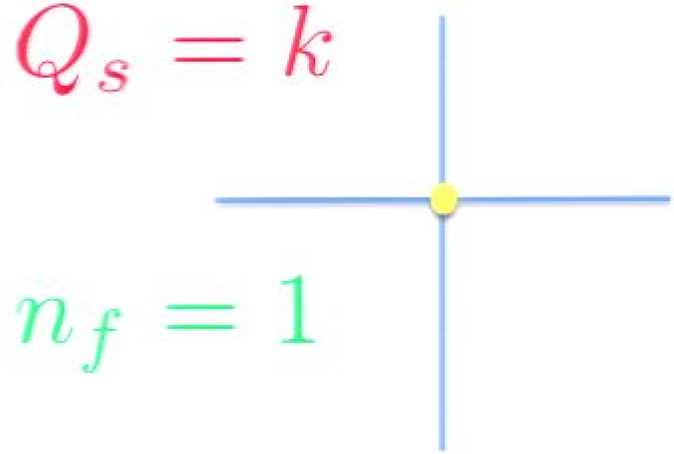
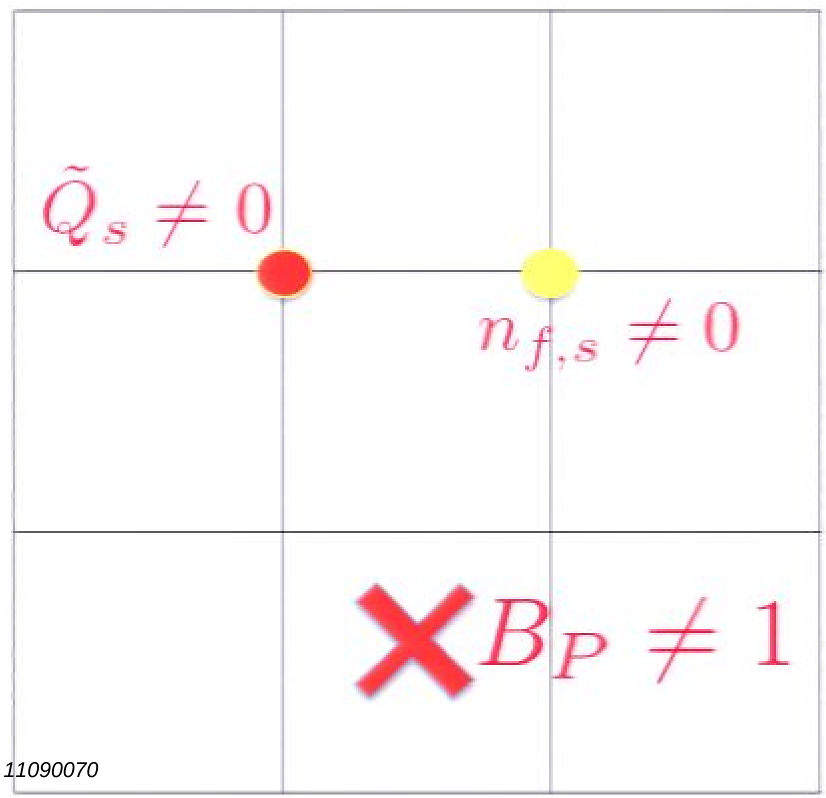


$$H = V\sum_s \tilde{Q}_s^2 - \frac{u}{2}\sum_P \left(B_P + B_p^\dagger\right) + \mu \sum_s n_{f,s}$$



$u, V \gg \mu$

- Low-energy excitations: composite fermions (fractionally charged)

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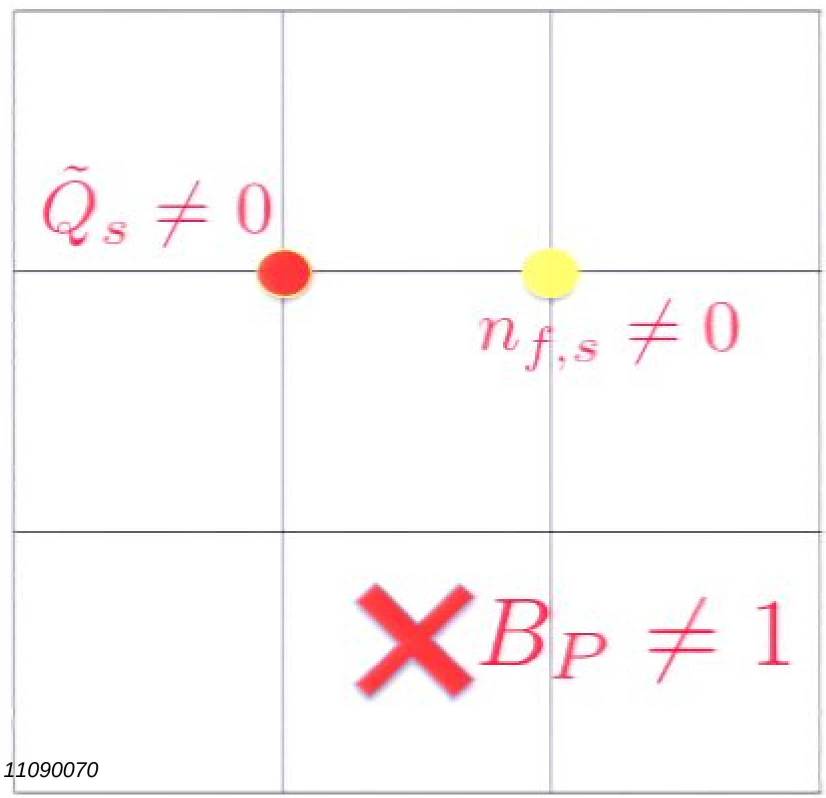

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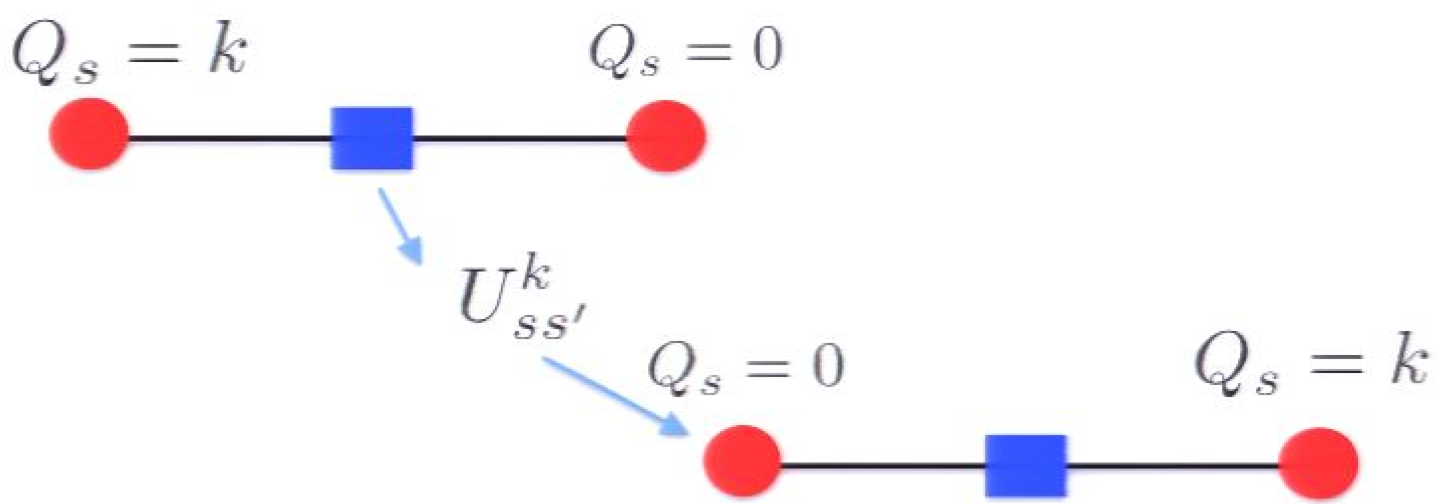
- Low-energy excitations: composite fermions (fractionally charged)

Band structure

$$H_{kin} = -\sum_{ss'} t_{ss'\sigma\sigma'} c^{\dagger}_{s'\sigma'} c_{s\sigma} U^k_{ss'}$$

- Hops one fermion and k fractionally charged bosonic excitations



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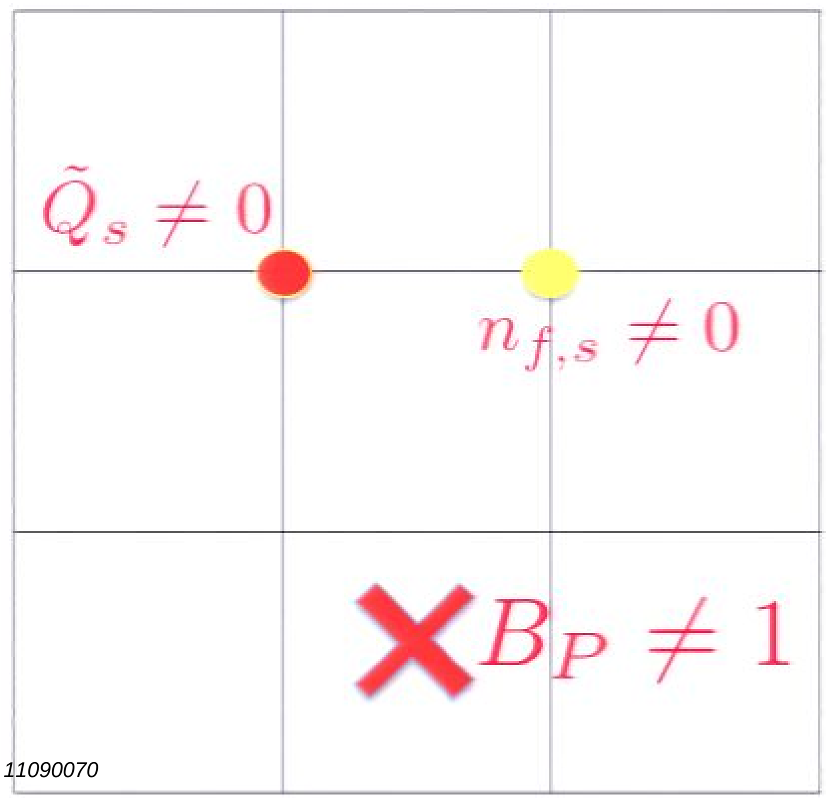

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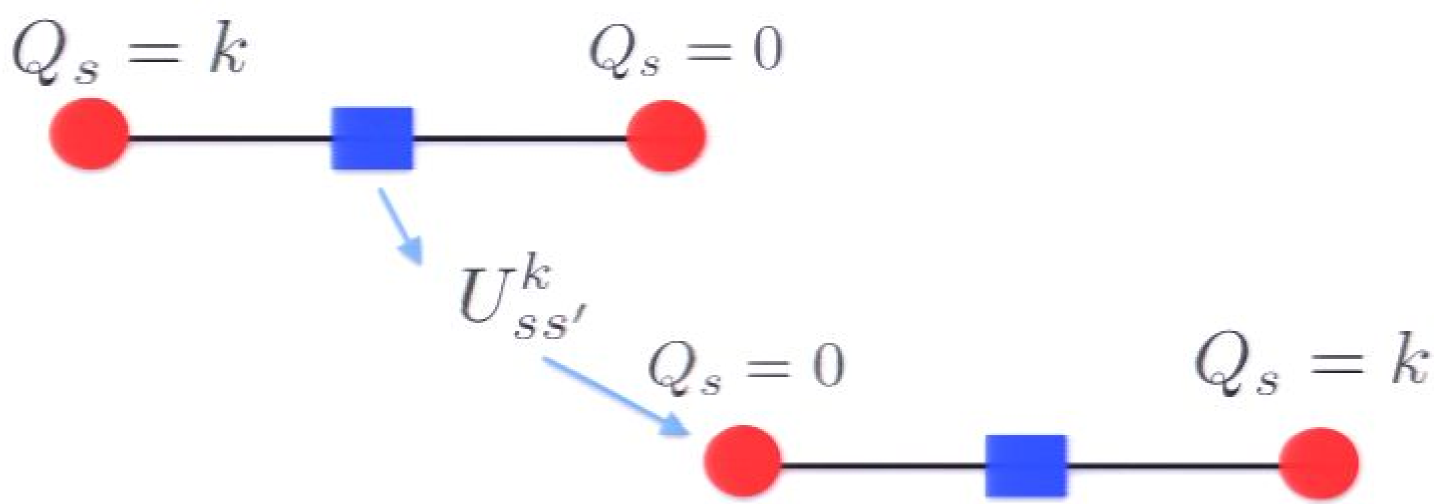
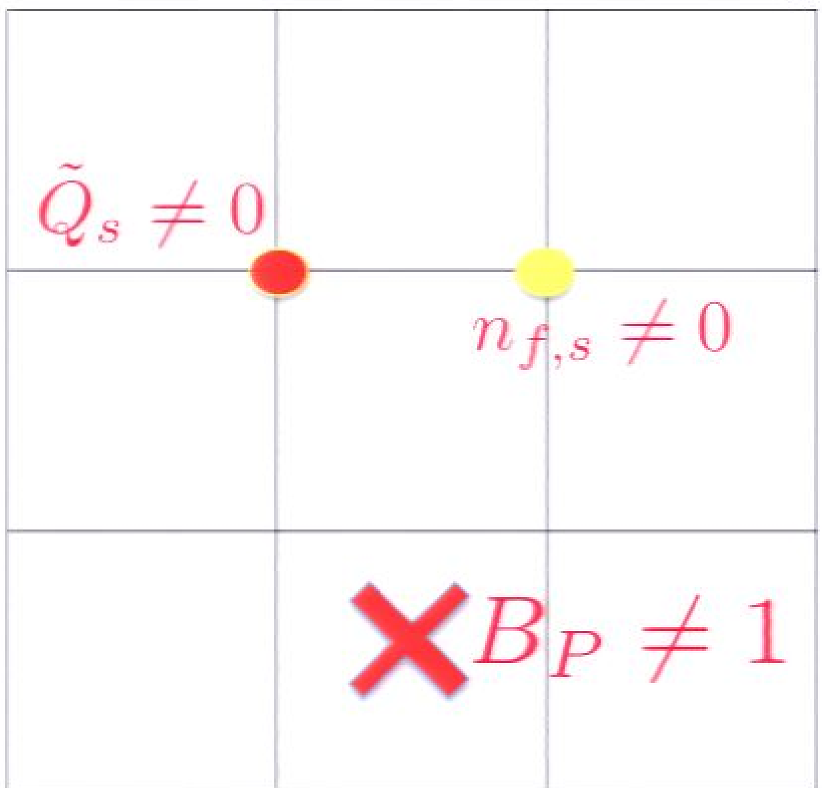


$$H = V\sum_s \tilde{Q}_s^2 - \frac{u}{2}\sum_P \left(B_p + B_P^\dagger\right) + H_{kin}$$



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$$[H_{kin}, B_P] = 0$$

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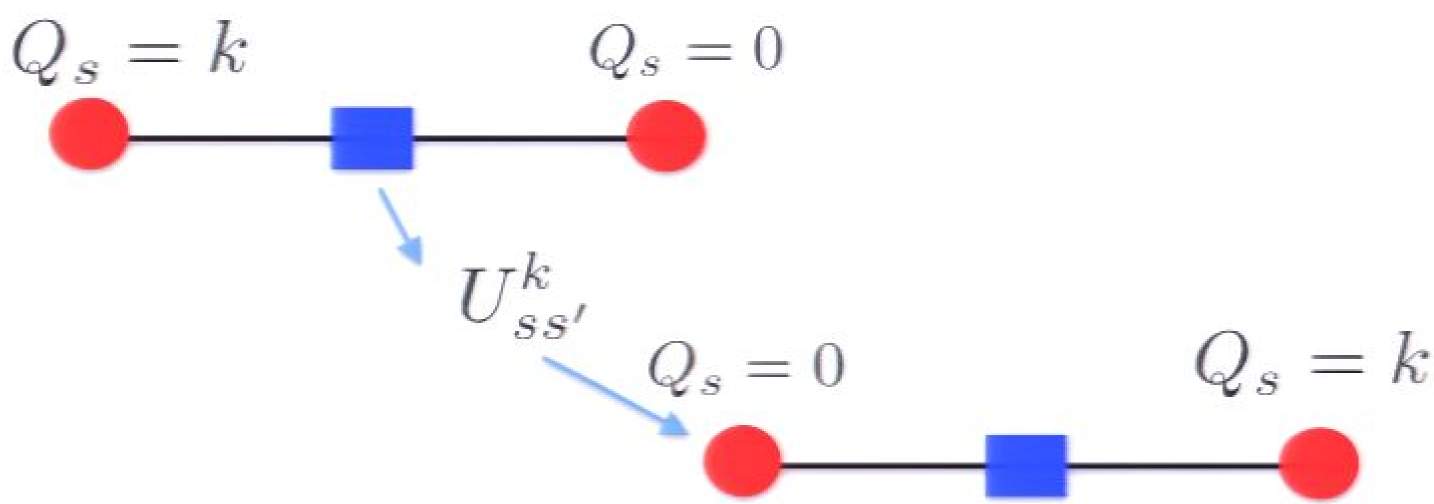

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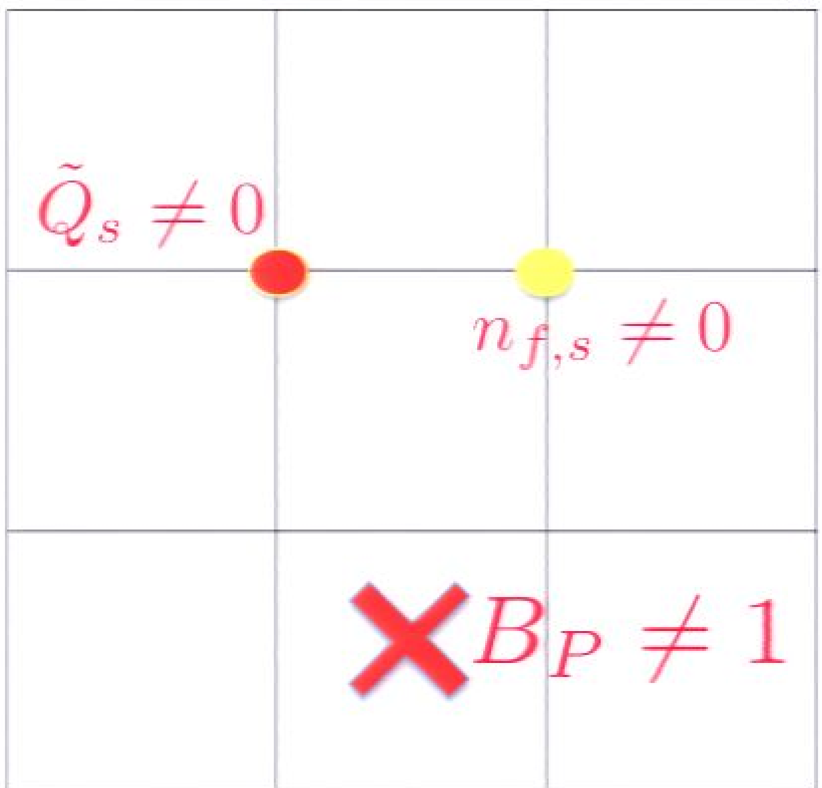

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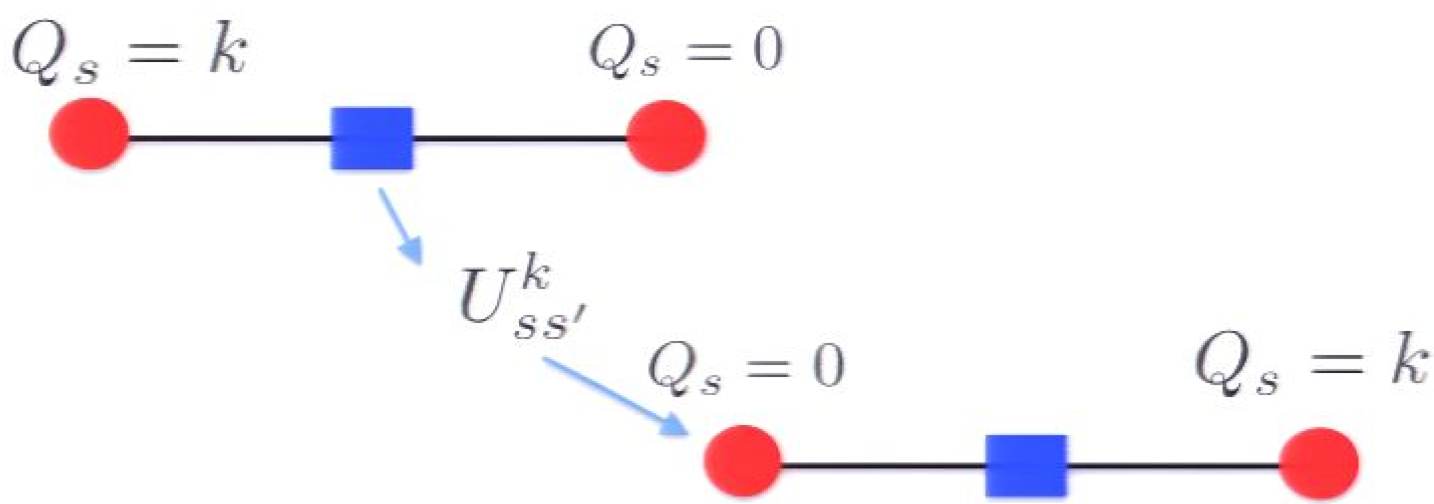
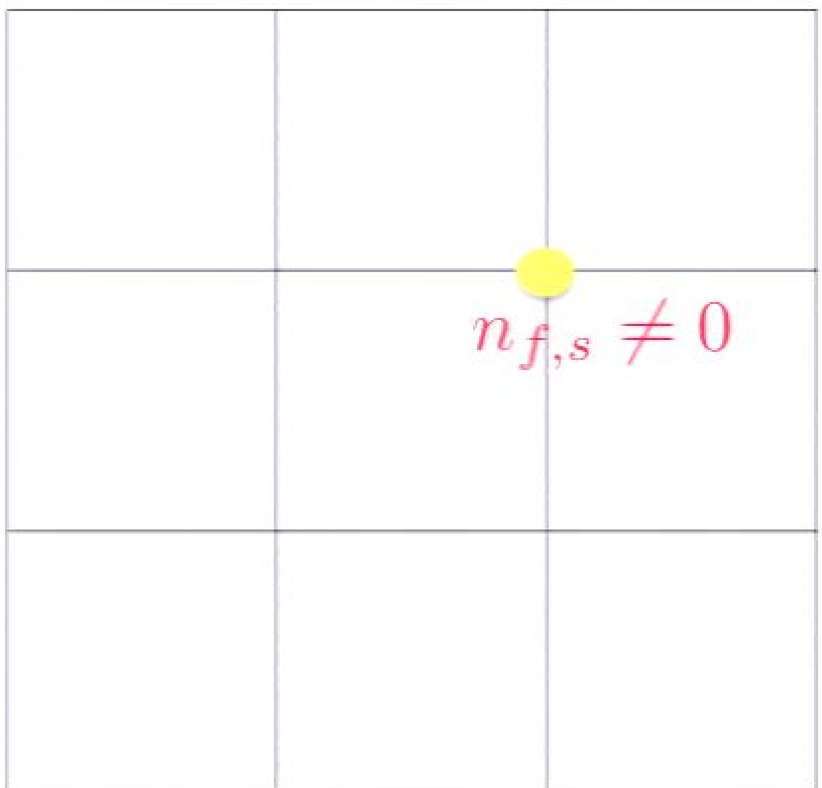


$$H = \sum t_{ss'\sigma\sigma'} d^{\dagger}_{s'\sigma'} d_{s\sigma}$$



$$[H_{kin}, \tilde{Q}_s] = 0$$

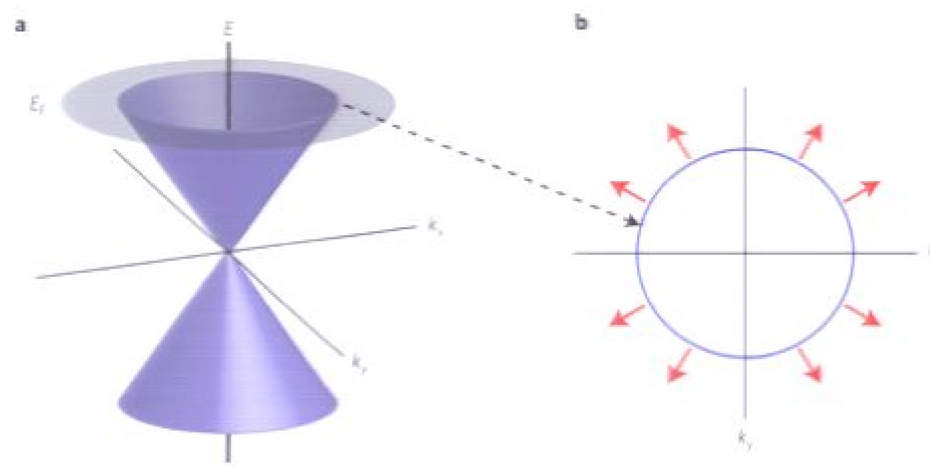
$$[H_{kin}, B_P] = 0$$

Fractional topological insulators

- We can choose any band structure

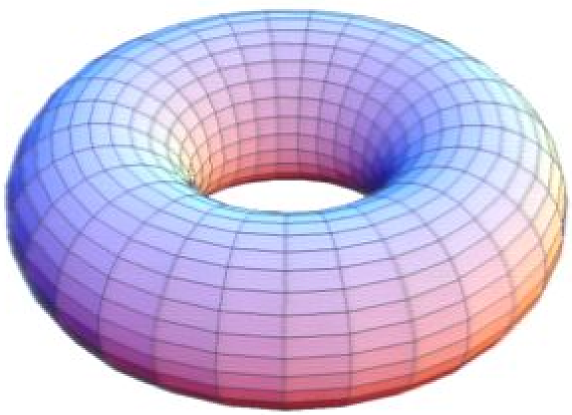
- Choose TI band structure

- Fractionally charged surface states



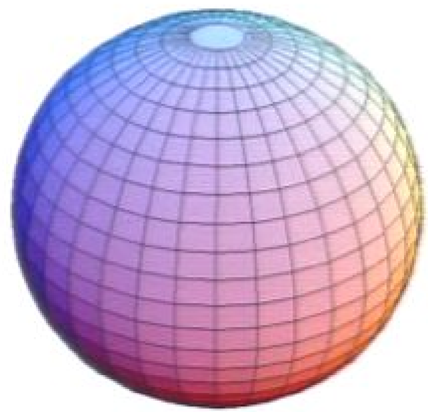
A few interesting properties

- Topological order



$p^2 \quad (2D)$

$p^3 \quad (3D)$



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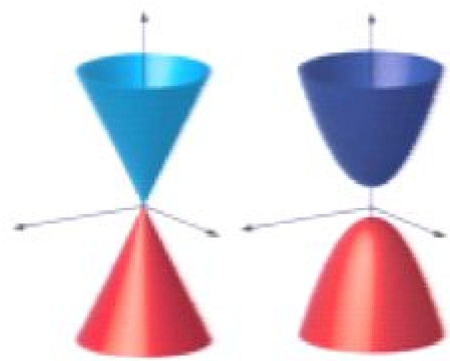
A few interesting properties

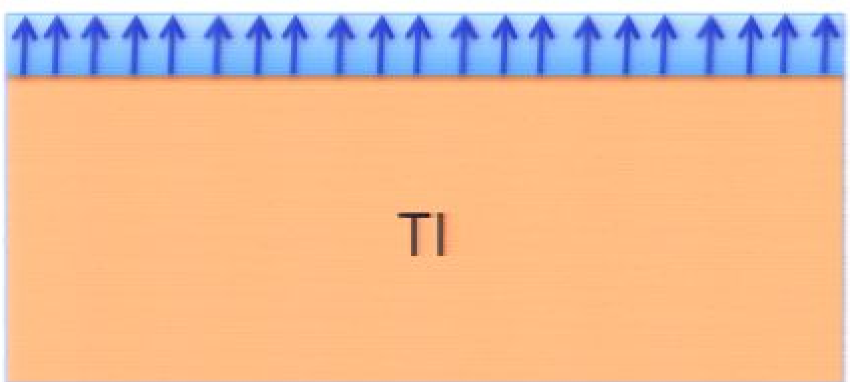
- Magnetoelectric effect
  - Break T on the surface
  - ½ integer Hall conductivity

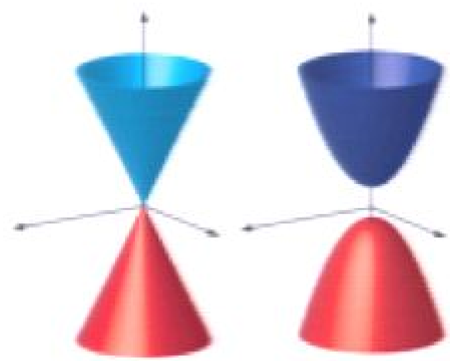


$$\sigma_{xy} = \frac{1}{2}\frac{e^2}{h}$$

A few interesting properties

- Fractional magnetoelectric effect
  - Break T on the surface
  - ½ integer Hall conductivity





$$\sigma_{xy} = \frac{1}{2}\frac{q_f^2}{h}$$

Fractional topological insulators

- Can you make a model with a single surface Dirac cone of fractionally charged fermions?
  - Exactly solvable lattice model
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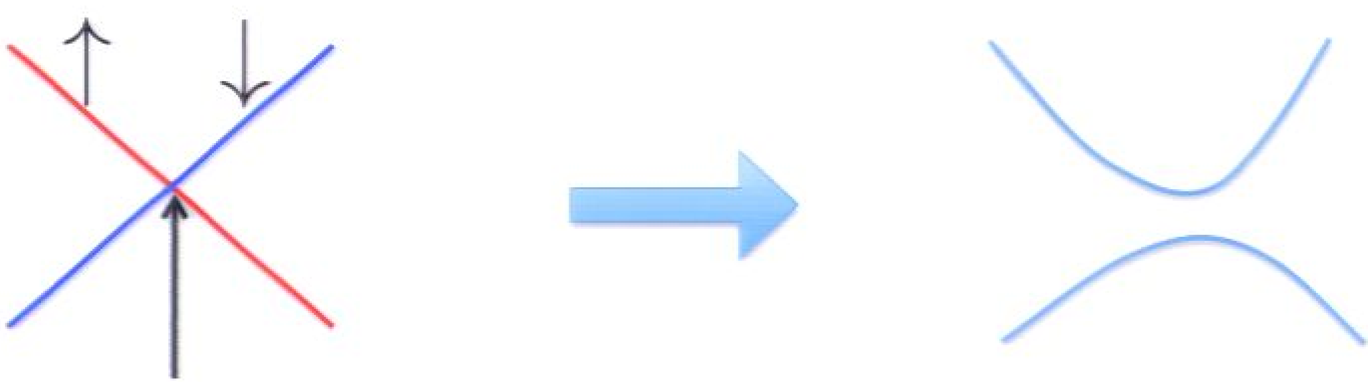
  - Time reversal protected edge modes

Time-reversal protected gapless boundary modes

- Kramers theorem: if

$$T : k \to -k \equiv k$$
$$T^2 = -1$$

- then states must be degenerate.



$$k = 0, \pi$$

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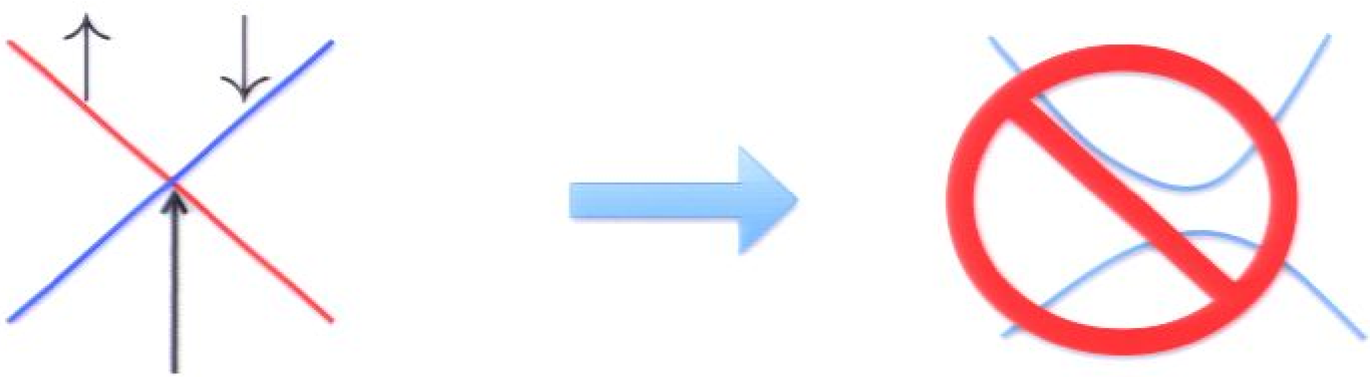
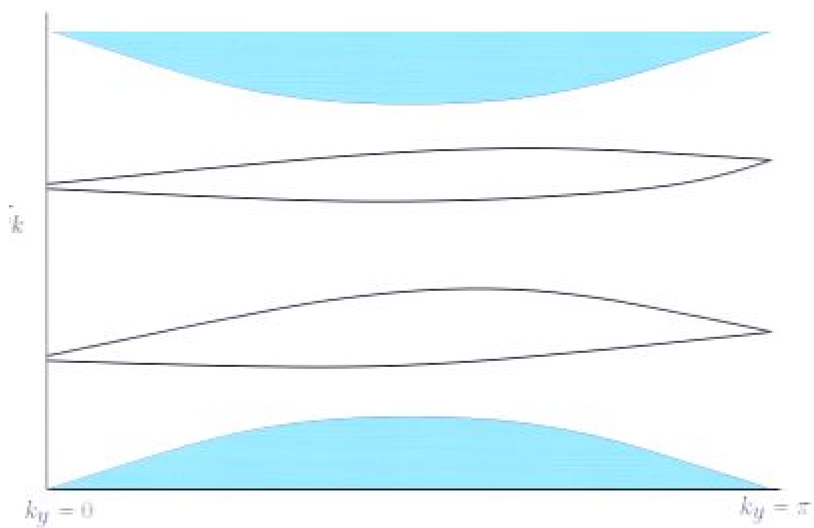


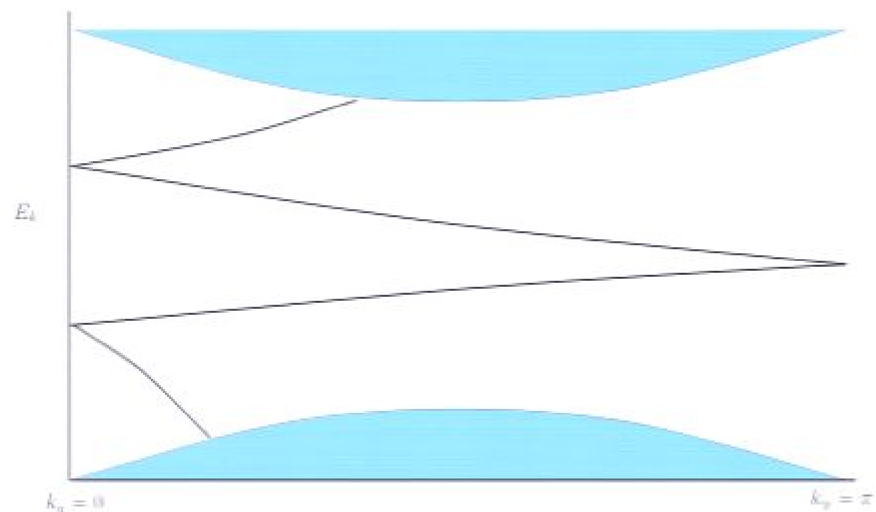
$k = 0, \pi$

Topological insulators:

- odd number of Kramers pairs in the edge spectrum



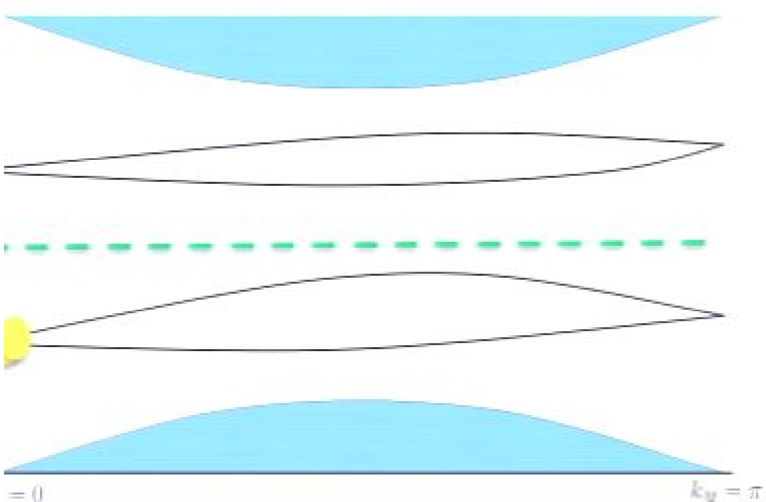
Ordinary insulator

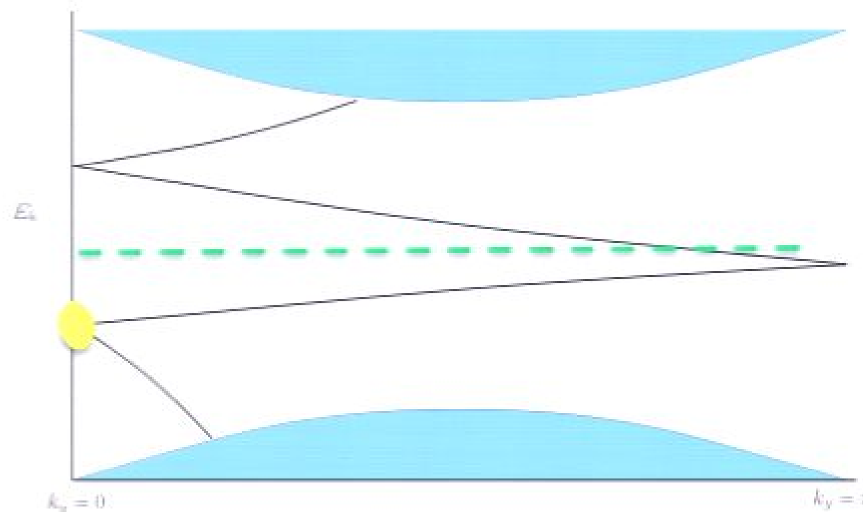


Topological insulator

Interacting equivalent:

$$\Phi = 0$$





(Fu and Kane '06)

Interacting equivalent:

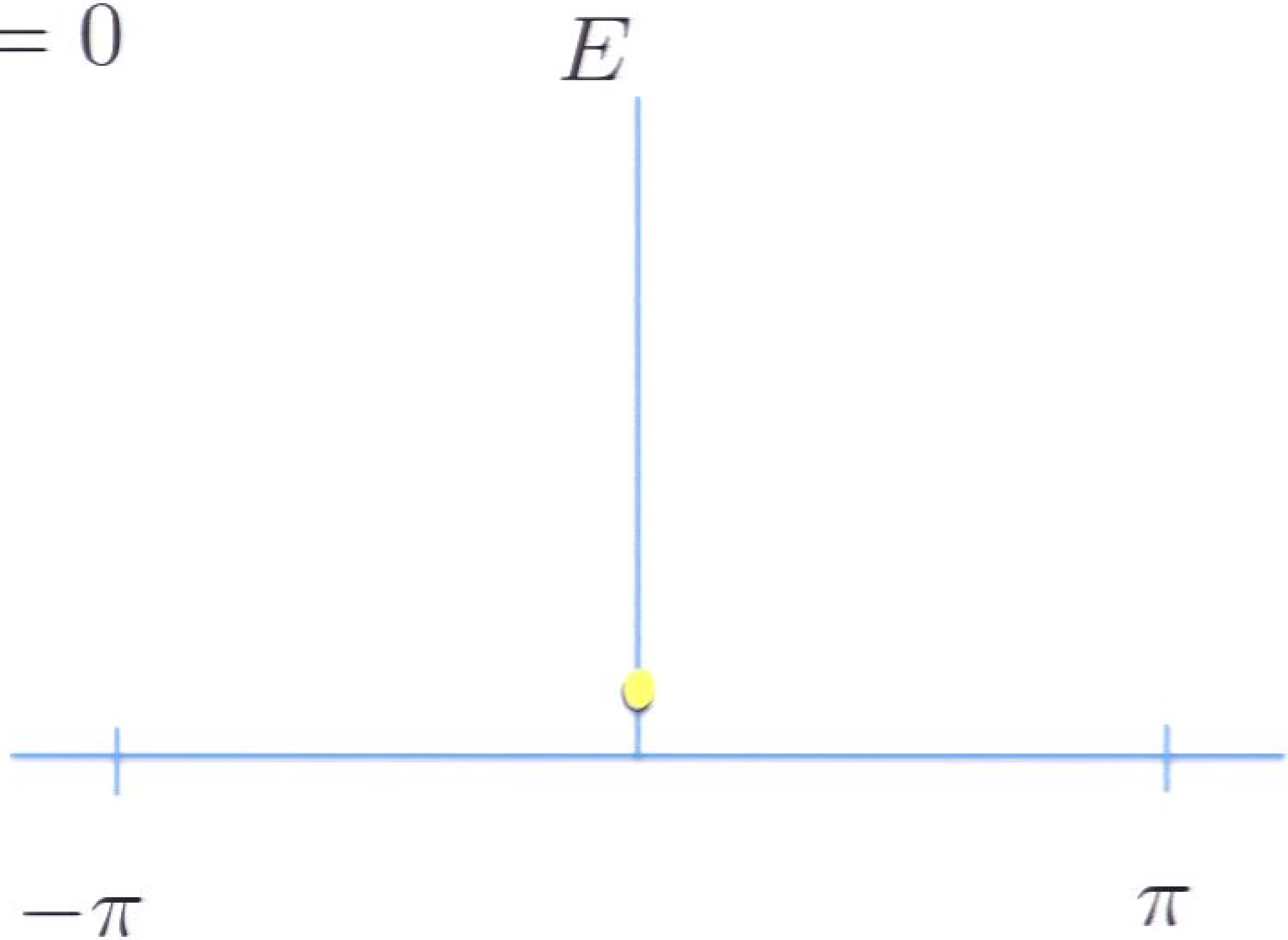
- $\Phi = 0, \pi$: Time-reversal invariant
- Kramers degeneracies:

	TI	Non-TI
$\Phi = 0$	Yes	No
$\Phi = \pi$	No	No
$\Phi = 0$	No	Yes
$\Phi = \pi$	Yes	Yes
	Odd	Even

(Fu and Kane '06)

Constructing low-lying excited states

$\Phi = 0$



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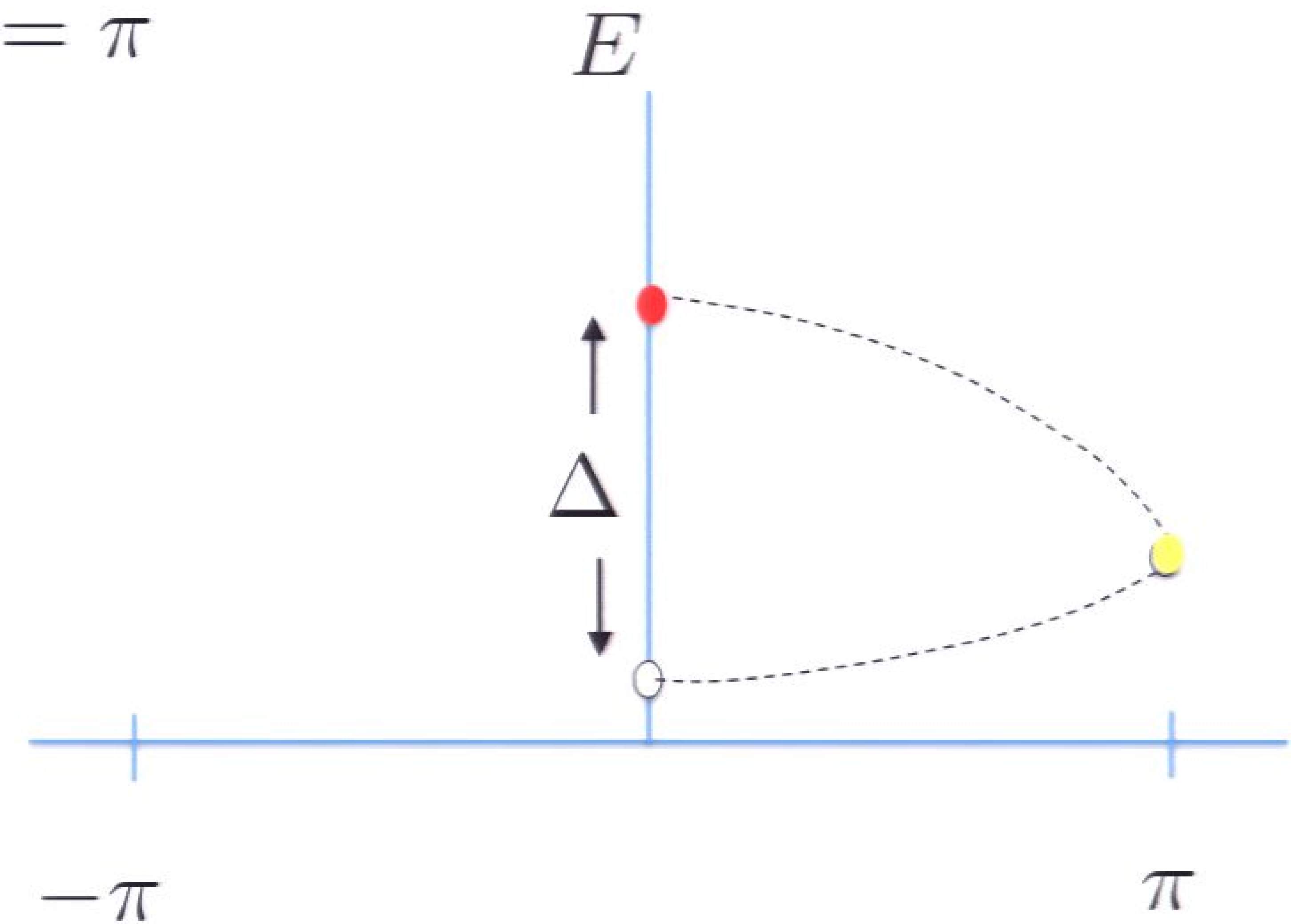
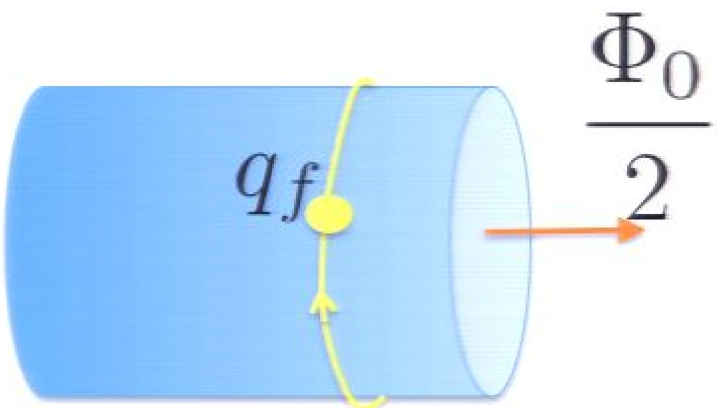


Flux insertion in fractional insulators

Levin and Stern '09



- Inserting $\frac{\Phi_0}{2}$: a different ground state

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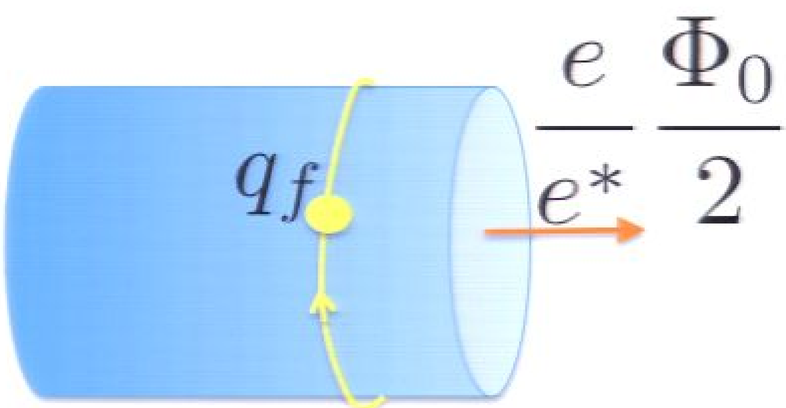

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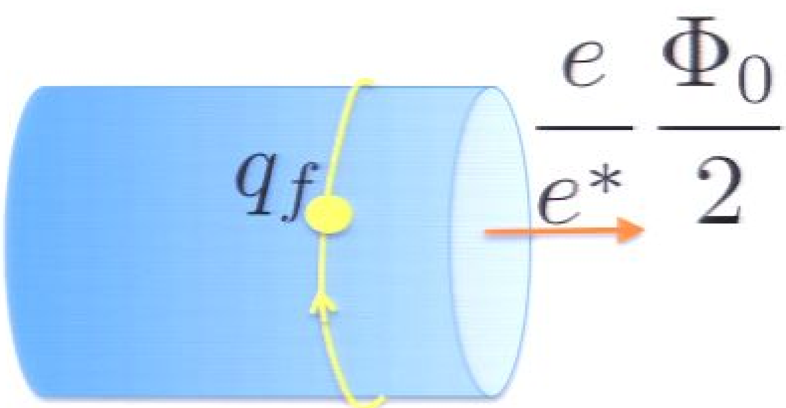


- Inserting $\frac{\Phi_0}{2}$: a different ground state

$$\frac{q_f}{e^*} \text{ odd: Kramers } \rightarrow \text{ No Kramers}$$

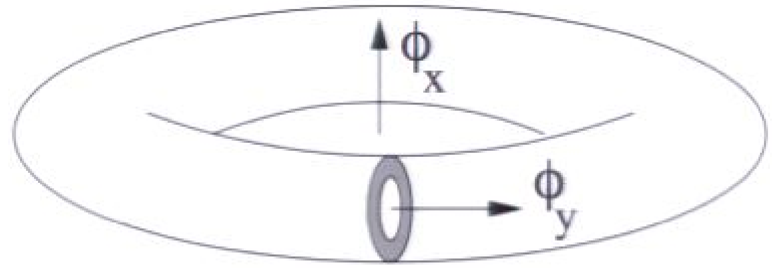
$$\frac{q_f}{e^*} \text{ even: No change}$$

$$\frac{q_f}{e^*} = \begin{cases} p + 2k & (p \text{ odd}) \\ \frac{1}{2}(p+2k) & (p \text{ even}) \end{cases}$$

2D Fractional topological insulators

- $\frac{q_f}{e^*}$ odd : protected edge modes
- $\frac{q_f}{e^*}$ even : Not protected!
  - Can add interactions that preserve T and gap the edge

Fractional 3D insulators



- $\frac{e}{e^*}\frac{\Phi_0}{2}$: same ground state sector
  : changes Kramers degeneracy if $\frac{q_f}{e^*}$ odd

- $\frac{q_f}{e^*}$ odd : protected surface modes

- $\frac{q_f}{e^*}$ even : ?

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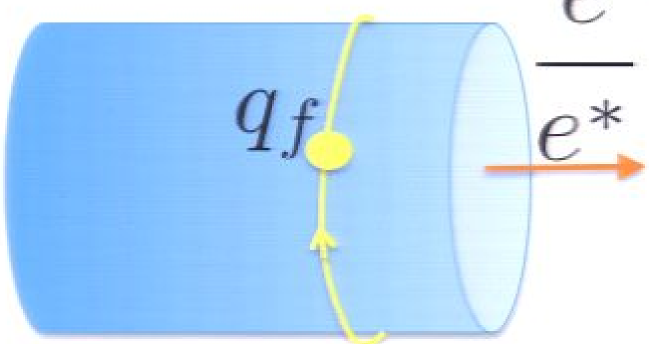

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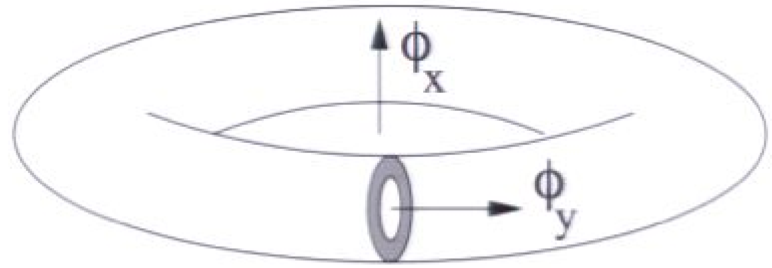

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Conclusions

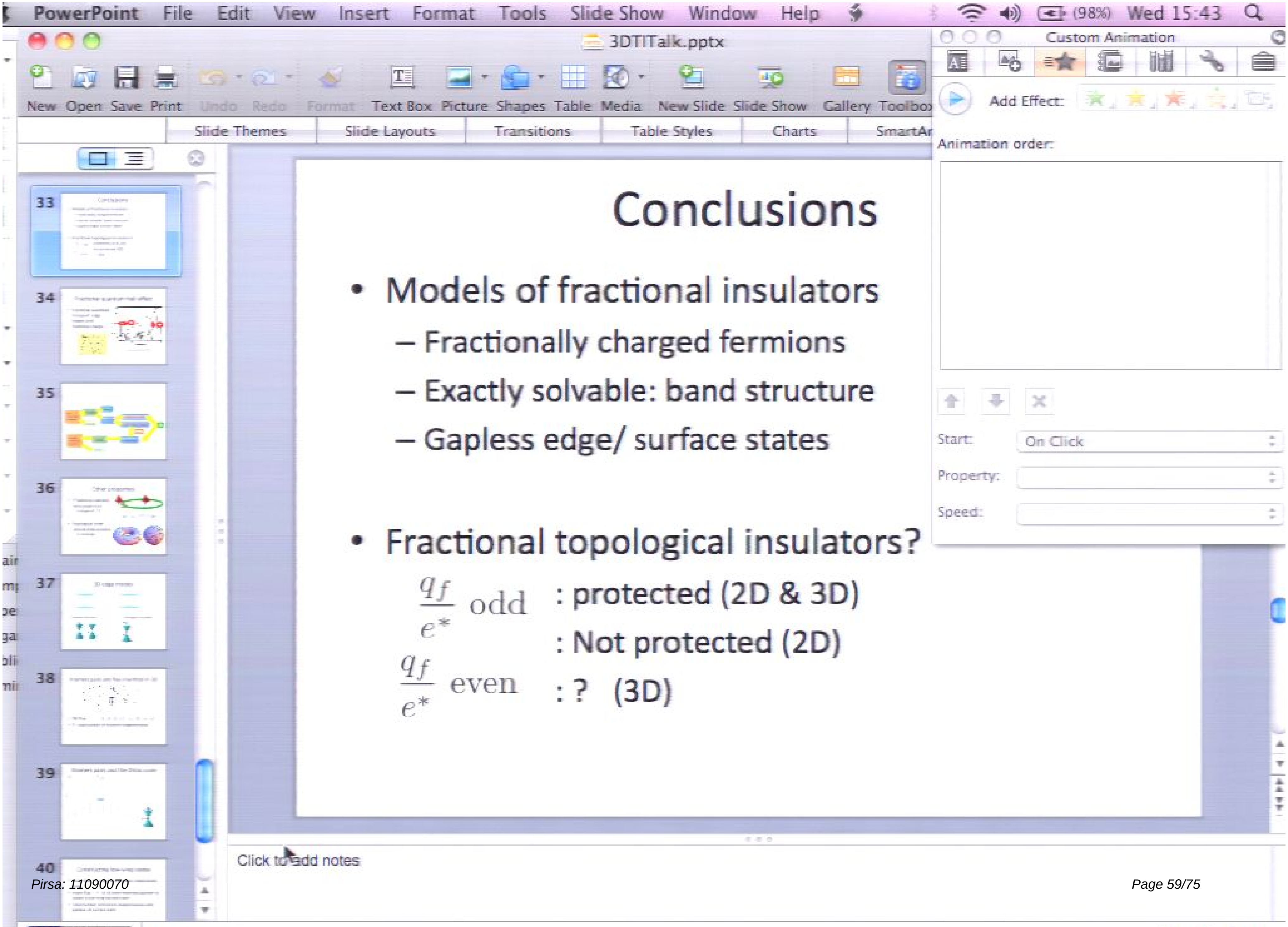
- Models of fractional insulators
  - Fractionally charged fermions
  - Exactly solvable: band structure
  - Gapless edge/ surface states

- Fractional topological insulators?

$\frac{q_f}{e^*}$ odd : protected (2D & 3D)

$\frac{q_f}{e^*}$ even : Not protected (2D)

: ? (3D)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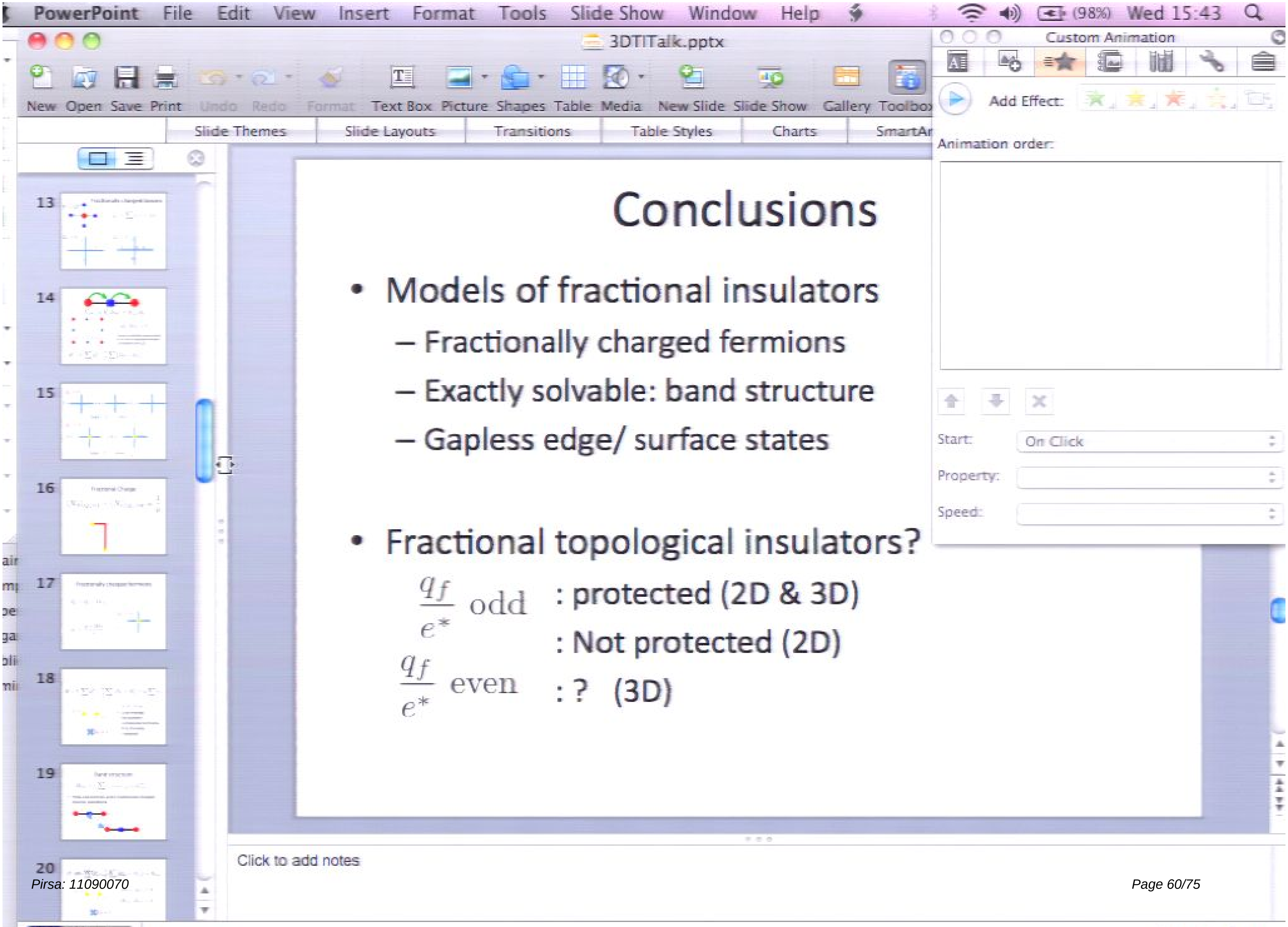
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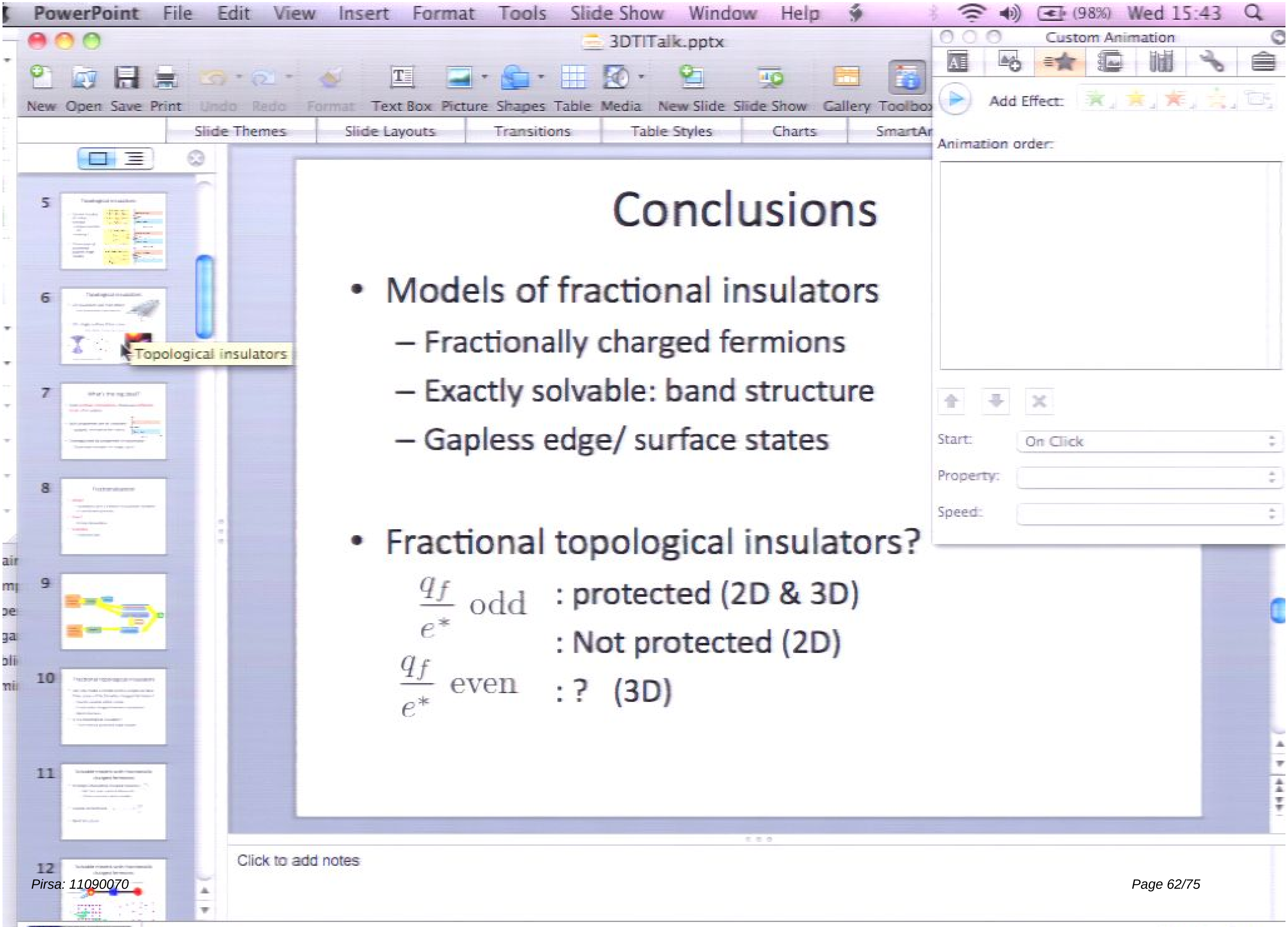
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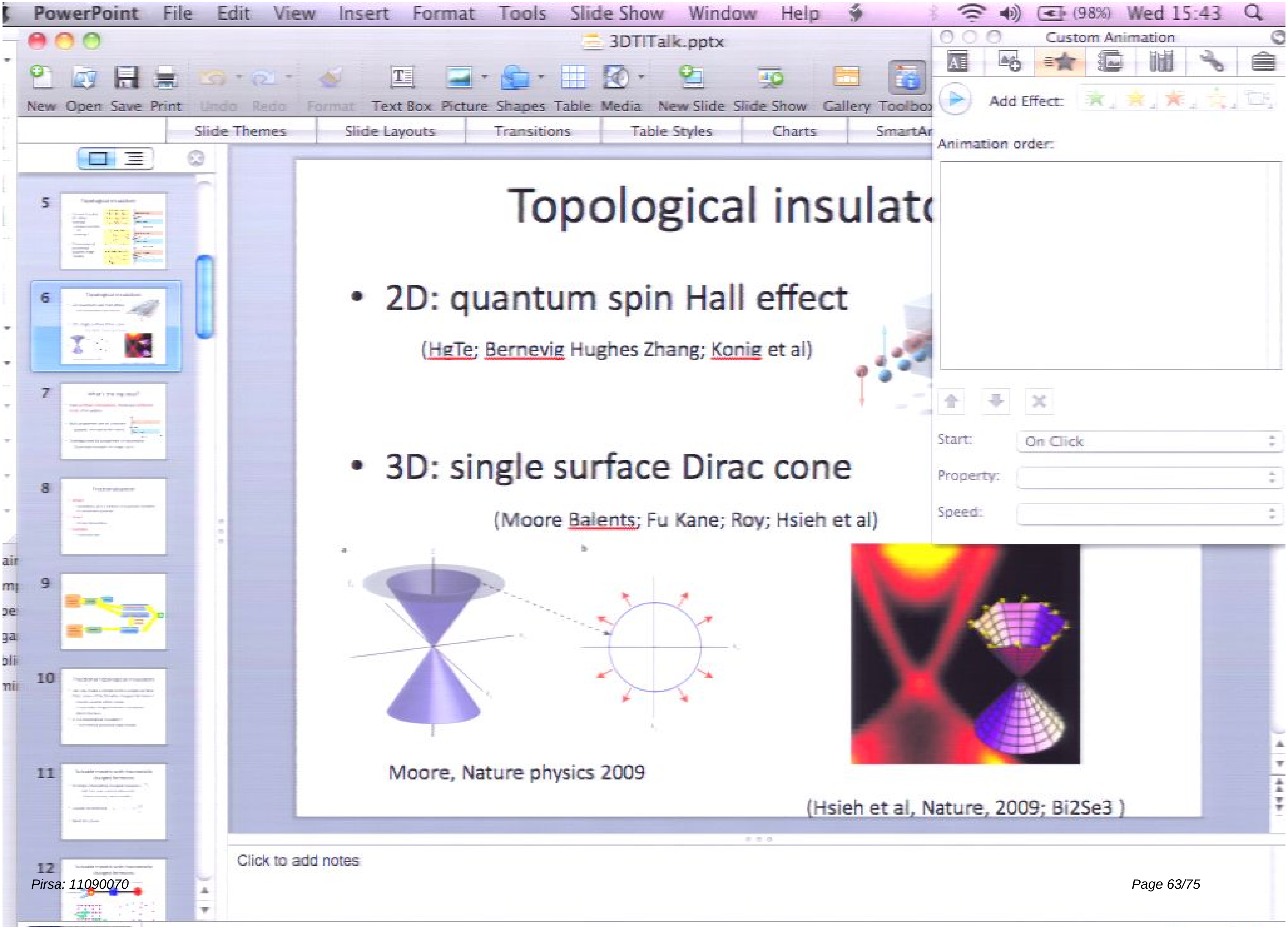
Topological insulators
Click to add notes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Topological insulato
2D: quantum spin Hall effect
(HgTe; Bernevig Hughes Zhang; Konig et al)
3D: single surface Dirac cone
(Moore Balents; Fu Kane; Roy; Hsieh et al)
Moore, Nature physics 2009
(Hsieh et al, Nature, 2009; Bi2Se3)
Click to add notes

Topological insulators

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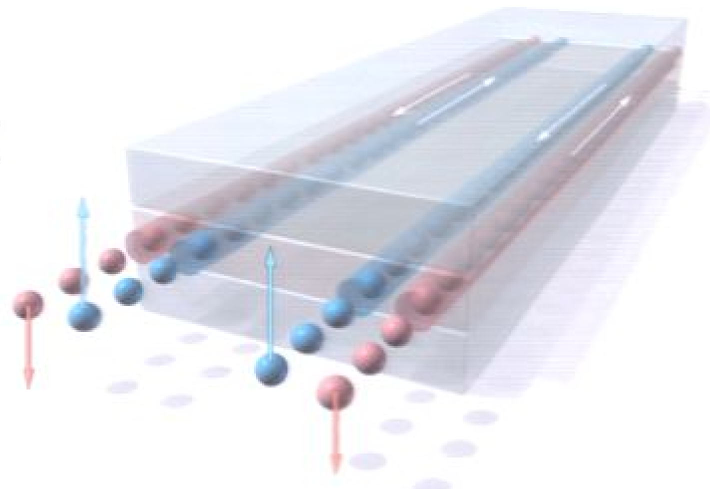

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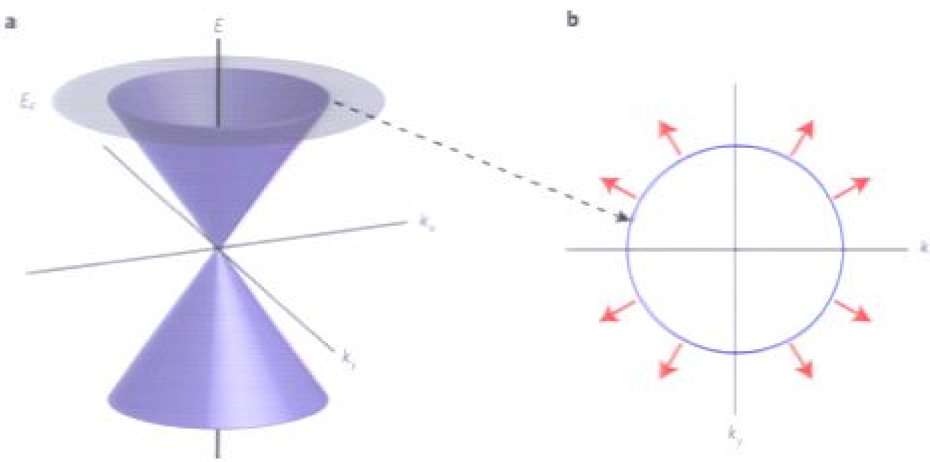
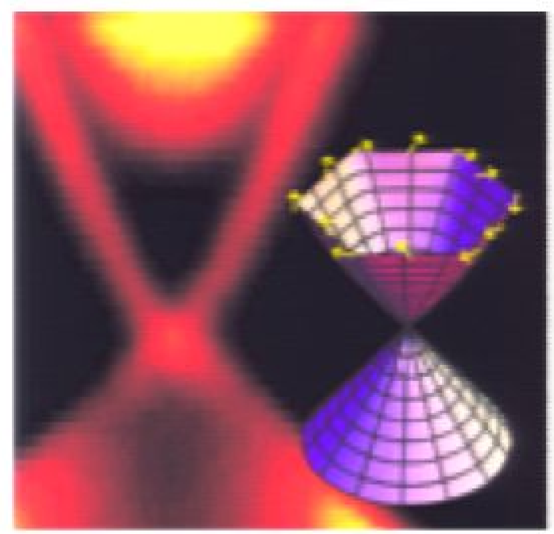
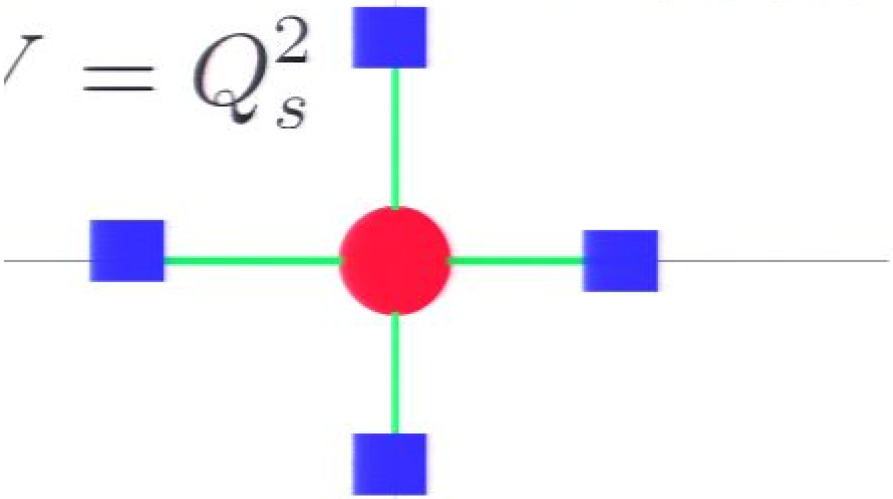




Moore, Nature physics 2009

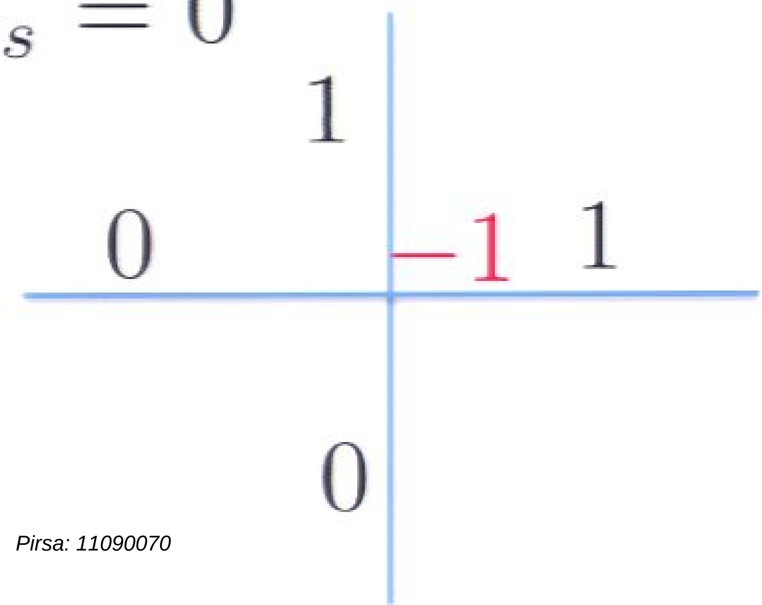
Fractionally charged bosons

$V = Q_s^2$

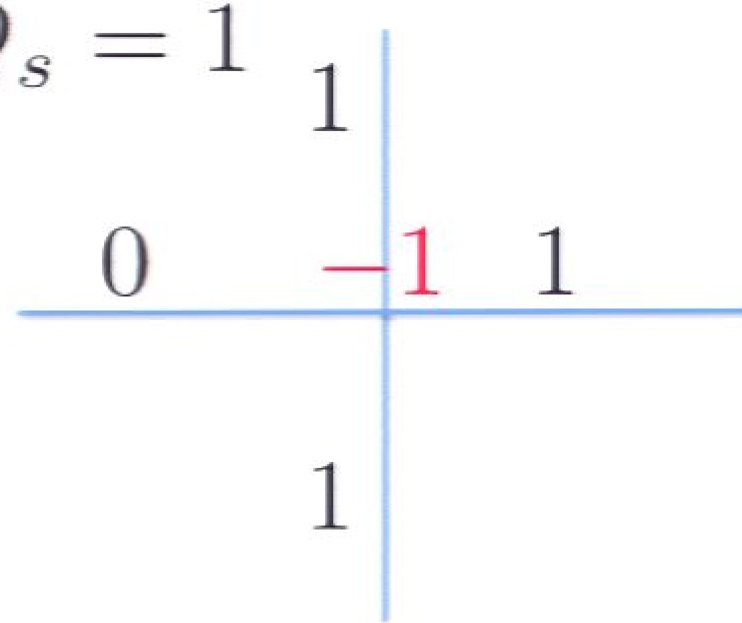


$$Q_s = \sum_{s'} n_{ss'} + 2n_s$$

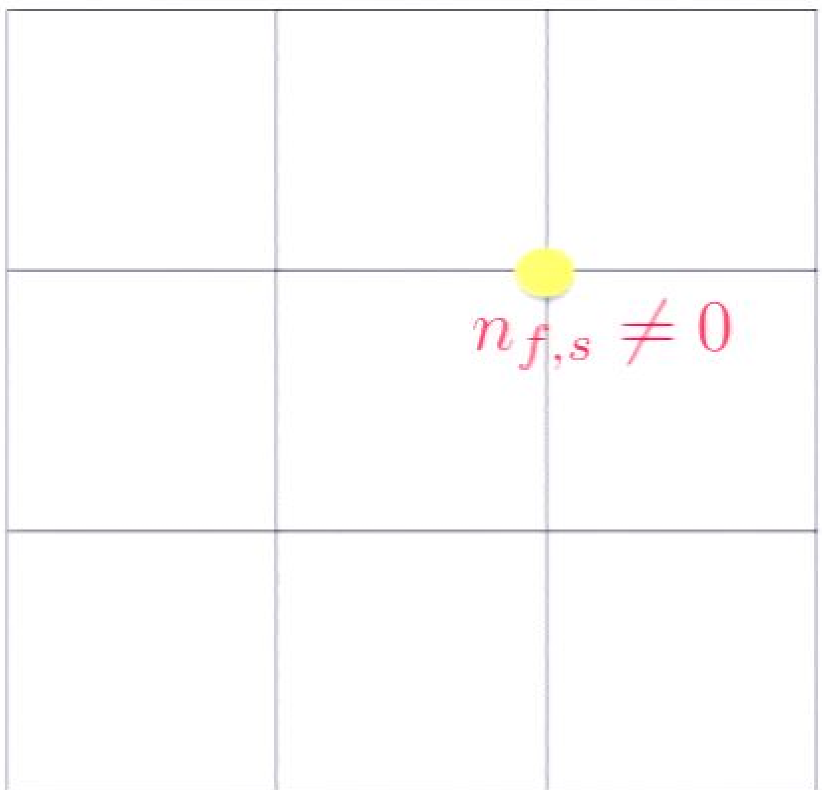
$Q_s = 0$



$Q_s = 1$



$$H = \sum t_{ss'\sigma\sigma'} d^\dagger_{s'\sigma'} d_{s\sigma}$$



$$[H_{kin}, \tilde{Q}_s] = 0$$

$$[H_{kin}, B_P] = 0$$

Fractional topological insulators

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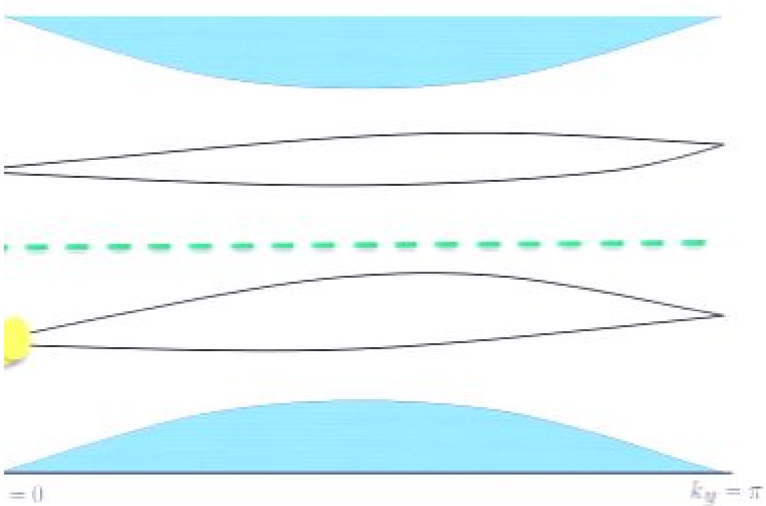

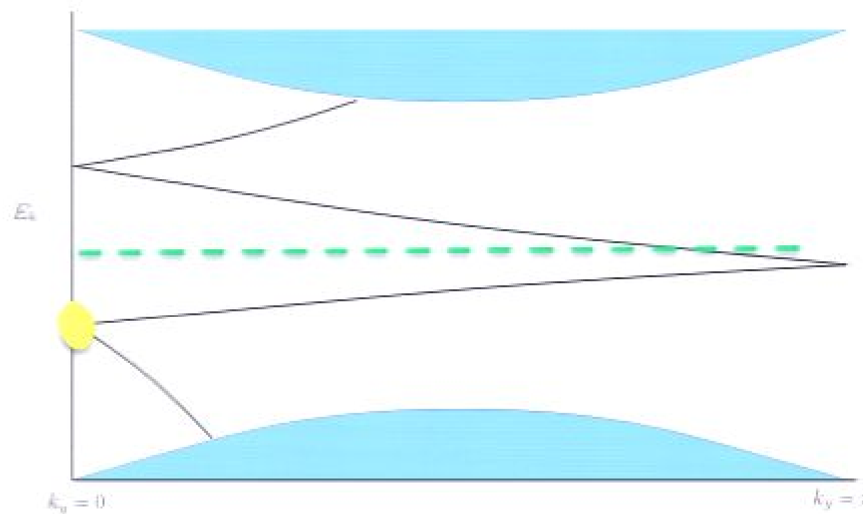

(Fu and Kane '06)

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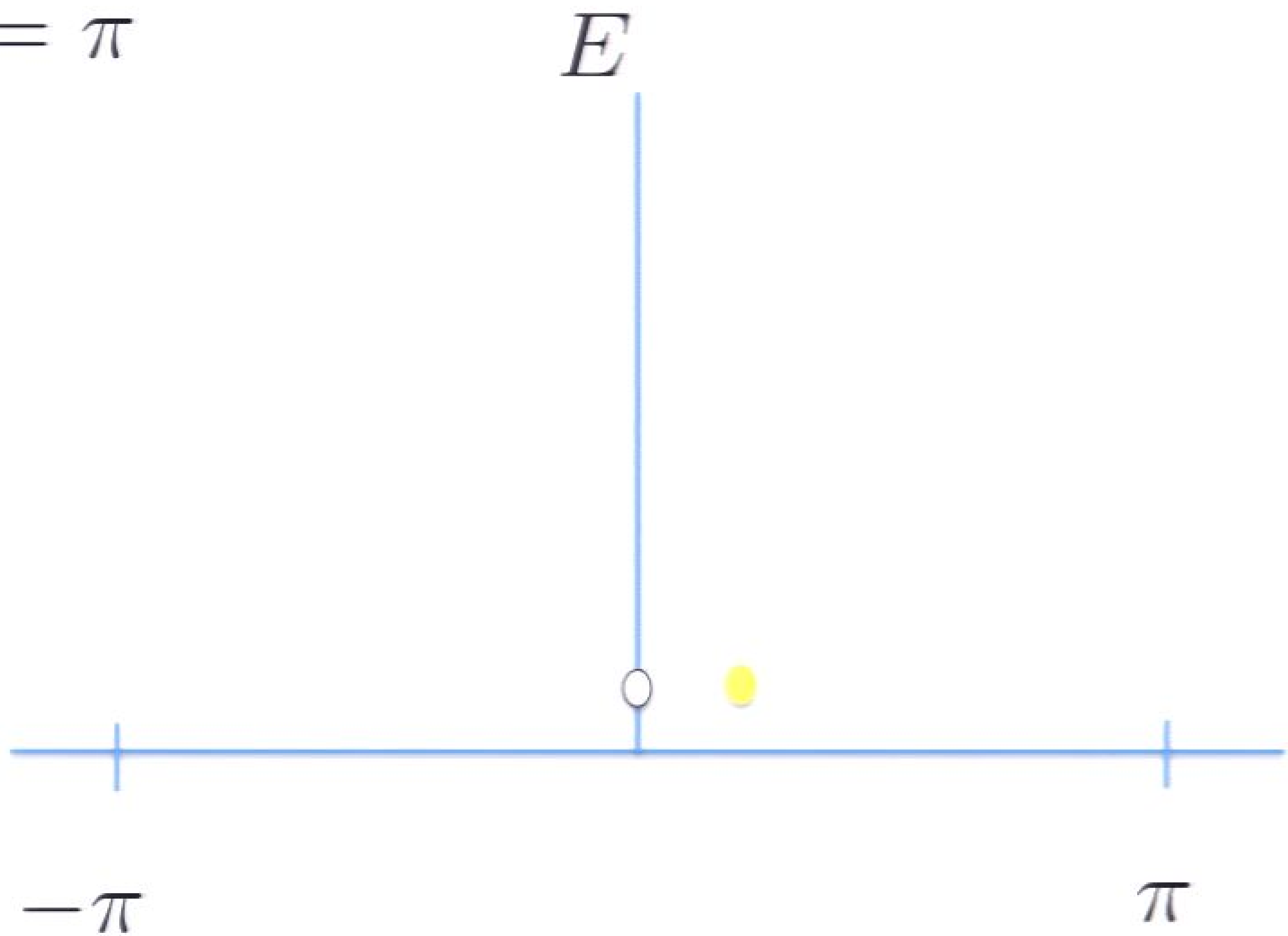

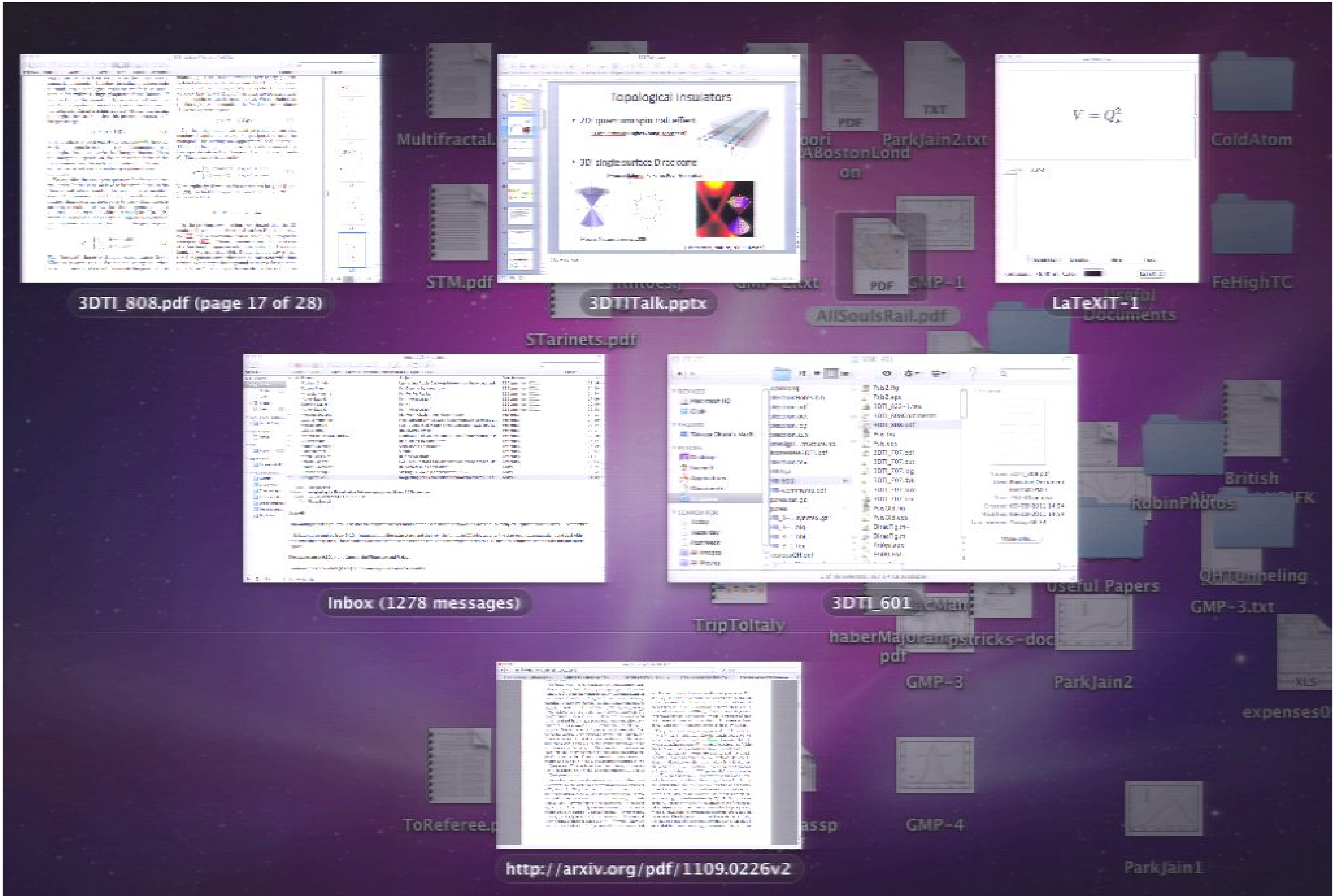
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Topological insulators
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$V = Q_*^2$
LaTeXiT-1
Multifractal.
STM.pdf
STarinets.pdf
ParkJain2.txt
ColdAtom
FeHighTC
AllSoulsRail.pdf
Inbox (1278 messages)
3DTI_601
TripToItaly
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ParkJain2
GMP-4
ParkJain1
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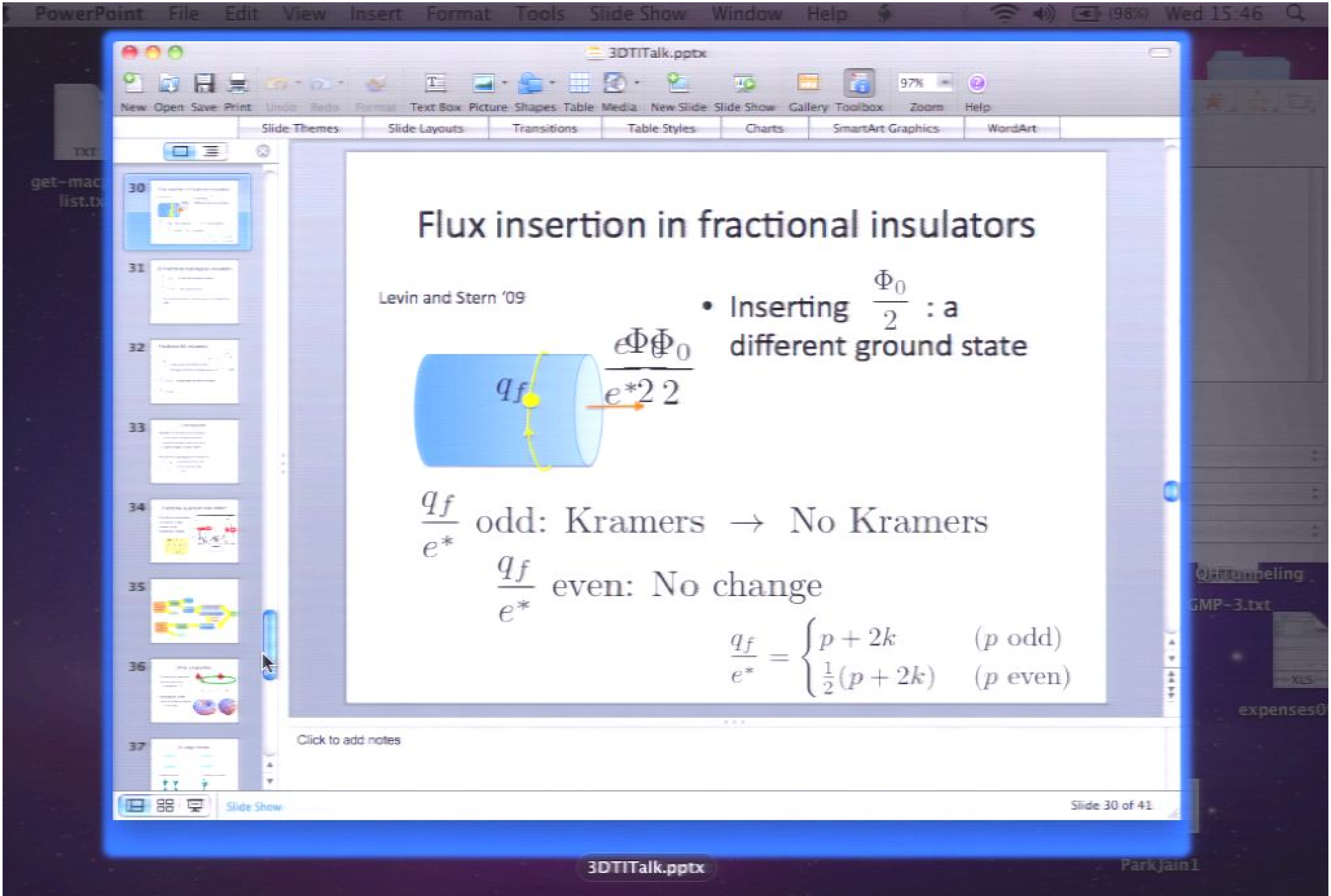
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Flux insertion in fractional insulators
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$\frac{e\Phi}{e^*2}$ $\frac{\Phi_0}{2}$
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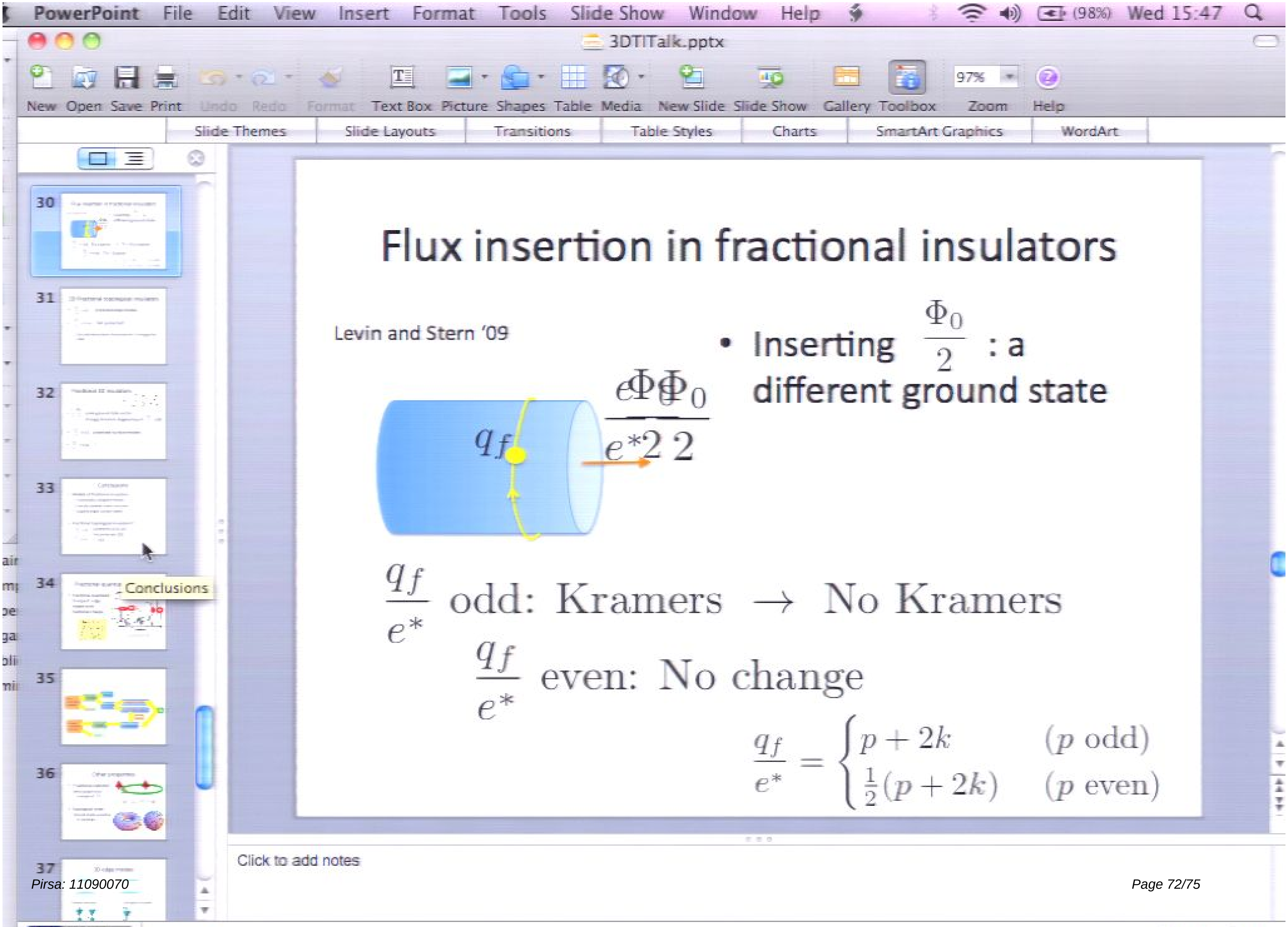
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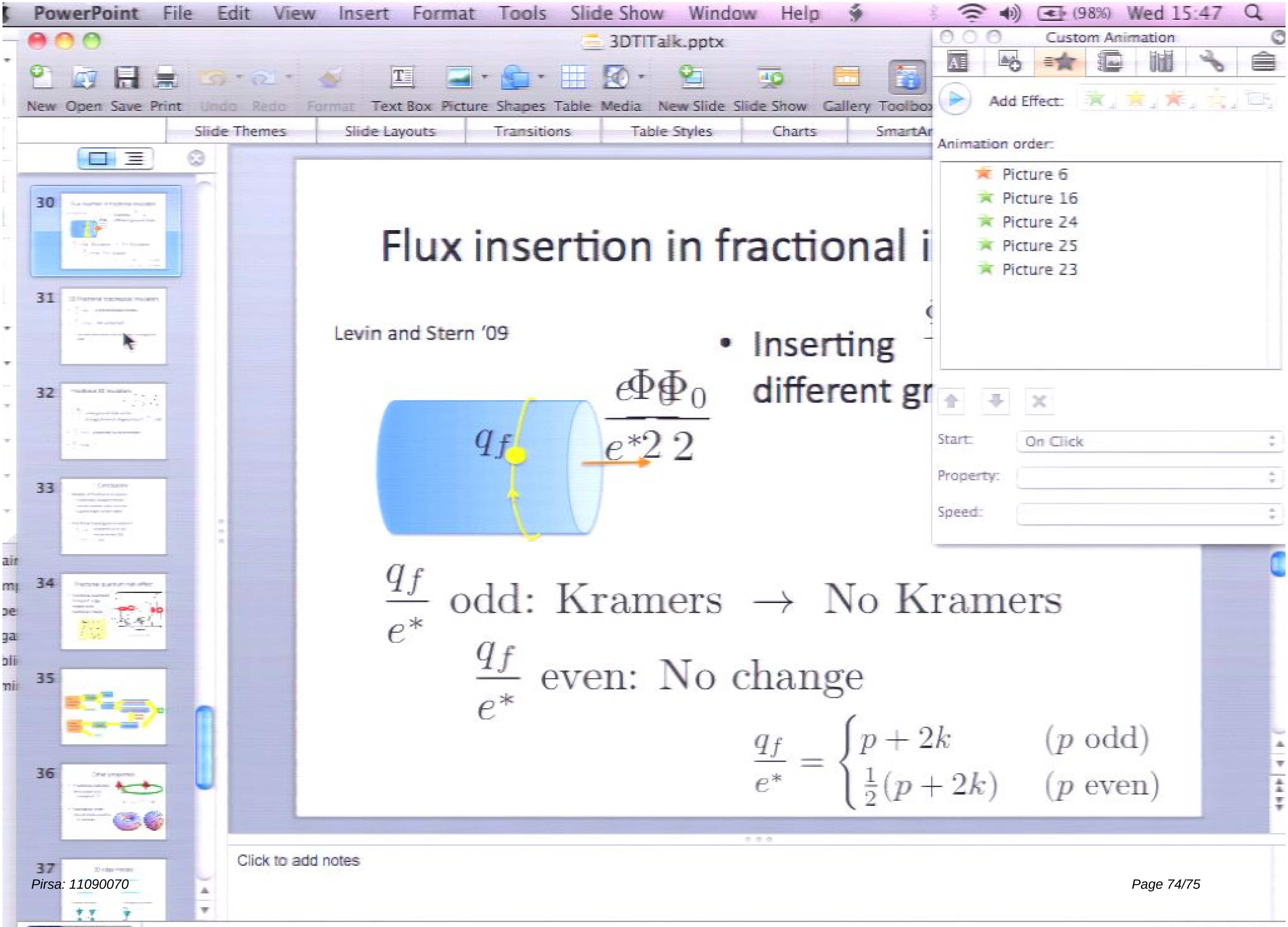
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Picture 6
Picture 16
Picture 24
Picture 25
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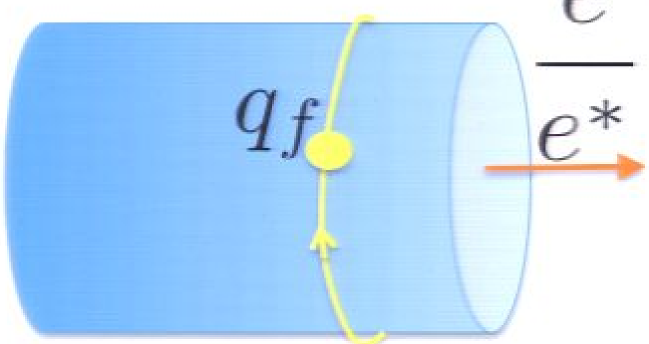

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