Title: Building Fractional Topological Insulators

Date: Sep 14, 2011 02:30 PM

URL: http://pirsa.org/11090070

Abstract: Time-reversal invariant band insulators can be separated into two categories: `ordinary' insulators and `topological' insulators. Topological band insulators have low-energy edge modes that cannot be gapped without violating time-reversal symmetry, while ordinary insulators do not. A natural question is whether more exotic time-reversal invariant insulators (insulators not connected adiabatically to band insulators) can also exhibit time-reversal protected edge modes. In 2 dimensions, one example of this is the fractional spin Hall insulator (essentially a spin-up and spin-down copy of a fractional quantum Hall insulator, with opposite effective magnetic fields for each spin). I will discuss another family of strongly interacting insulators, which exist in both 2 and 3 dimensions, that can have time-reversal protected edge modes. This gives a new set of examples of `fractional' topological insulators.

Building Fractional Topological Insulators

Collaborators:

Michael Levin

Maciej Kosh-Janusz

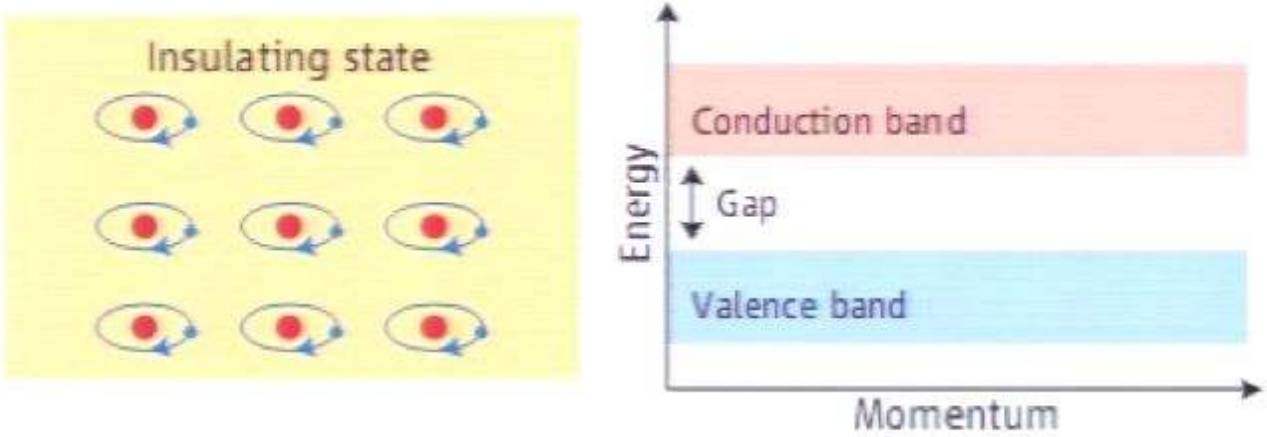
Ady Stern

The program

- Background:
  - Topological insulators
  - Fractionalization
- Exactly solvable Hamiltonians for "fractional topological insulators"
- When are they really fractional topological insulators?

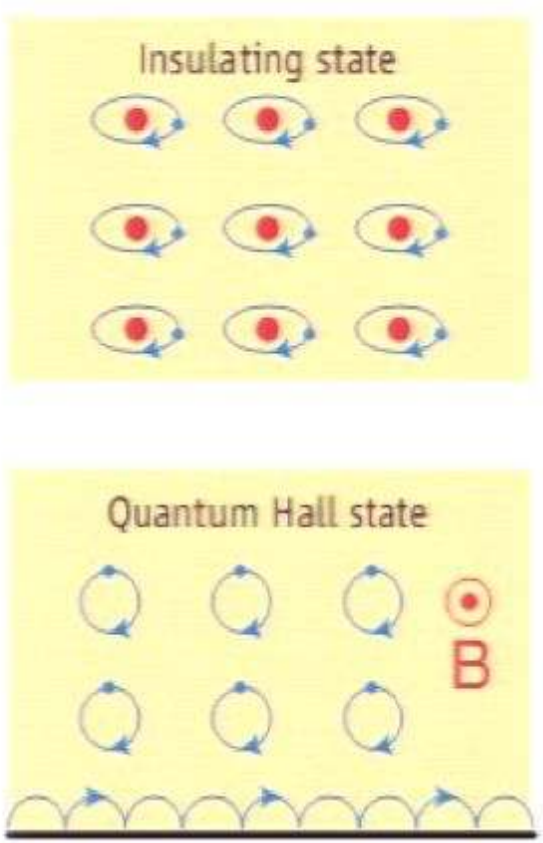
Ordinary Insulators

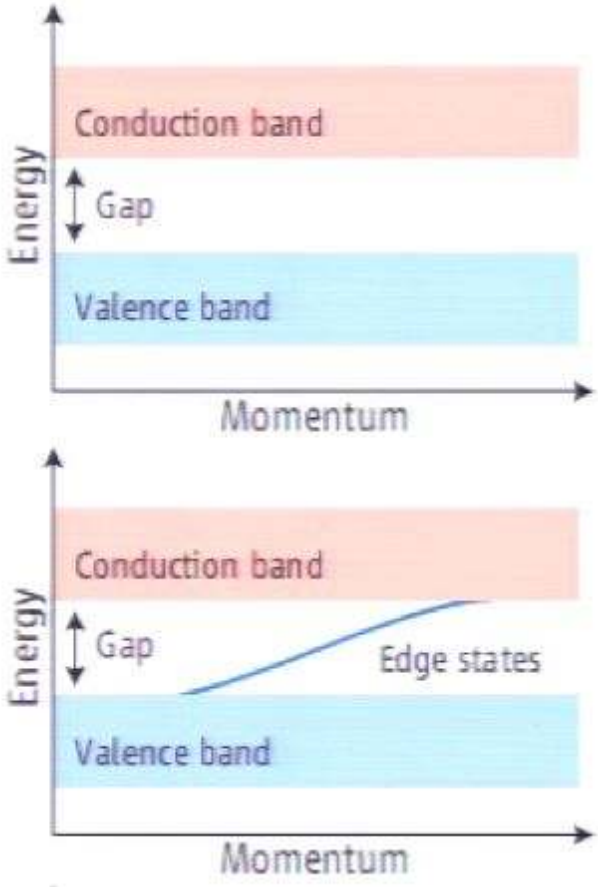
- all states are local OR
- Can be deformed into a product of local states without going through a phase transition



Chern insulators

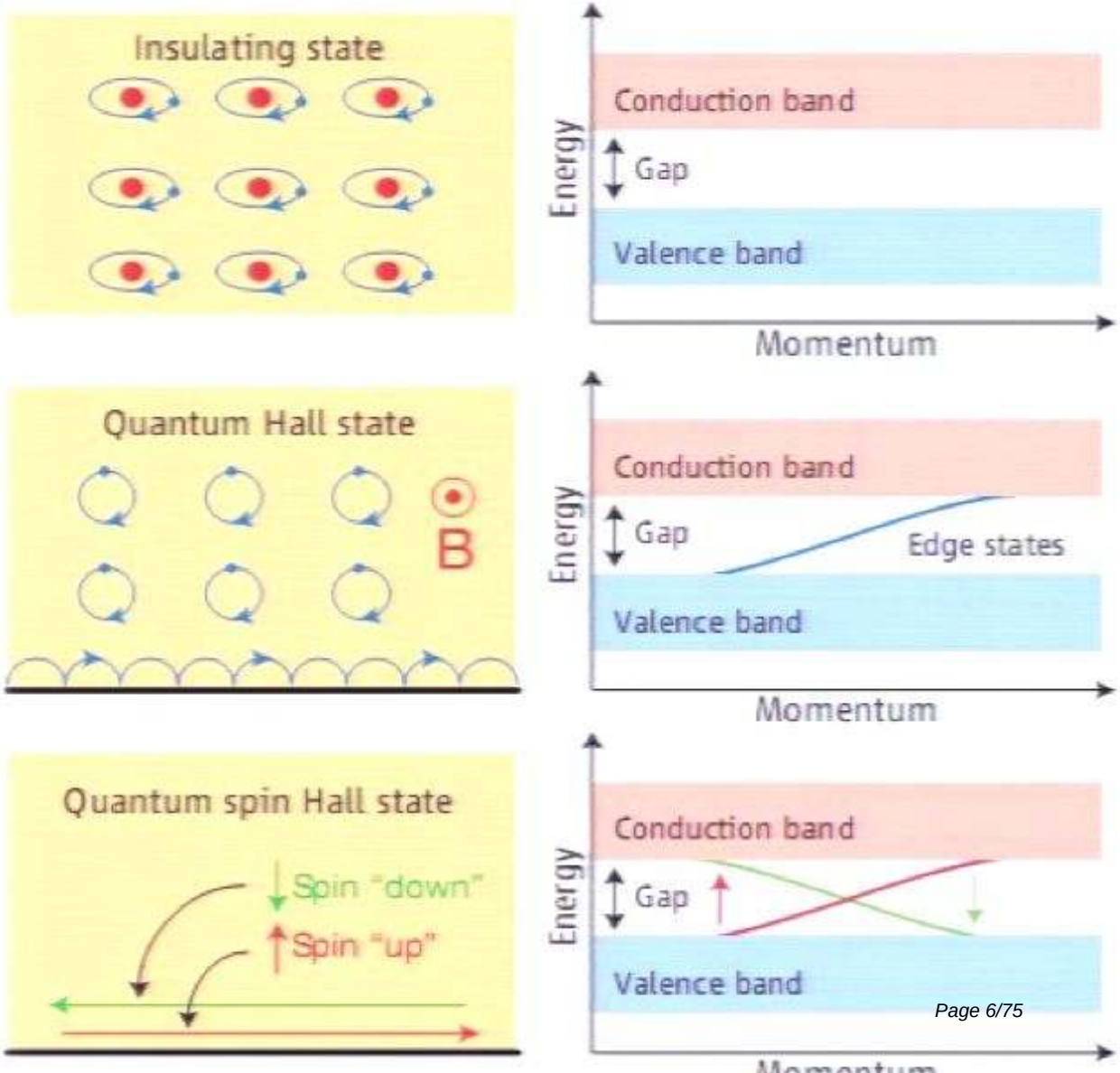
- Cannot localize all states without a phase transition
- Gapless edge modes





Topological insulators

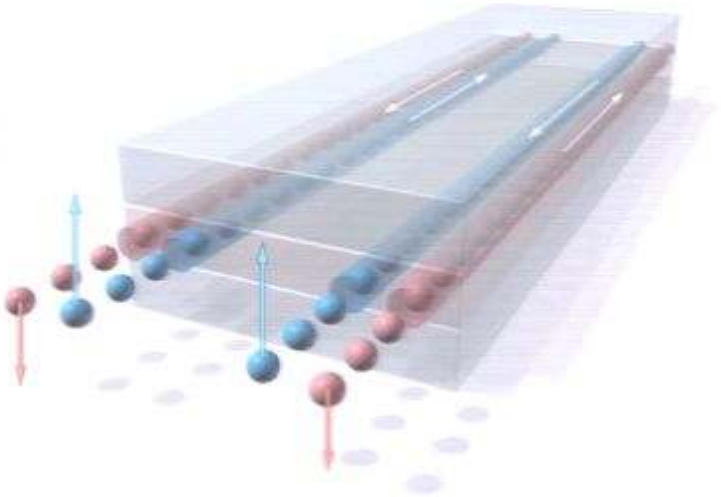
- Cannot localize all states without
  - a phase transition
    OR
  - breaking T
- Time-reversal protected gapless edge modes



Topological insulators

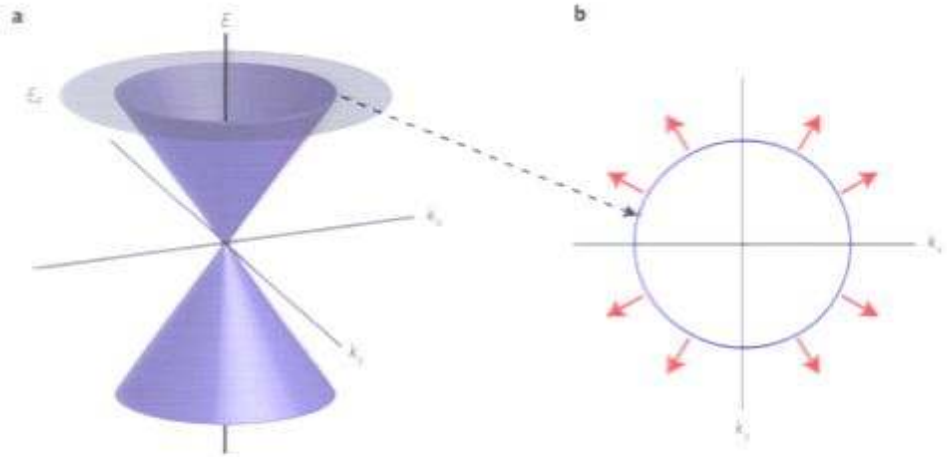
- 2D: quantum spin Hall effect

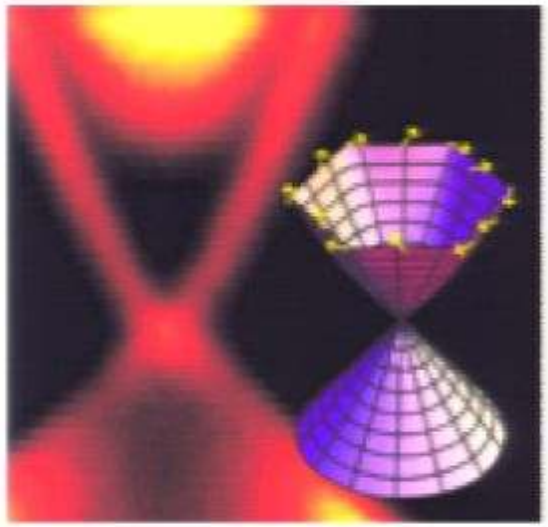
(HgTe; Bernevig Hughes Zhang; Konig et al)



- 3D: single surface Dirac cone

(Moore Balents; Fu Kane; Roy; Hsieh et al)



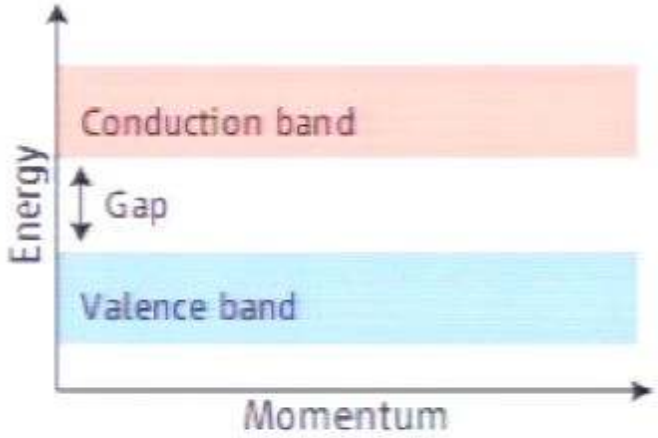


Moore, Nature physics 2009

What's the big deal?

- Even without interactions, there are different kinds of insulators

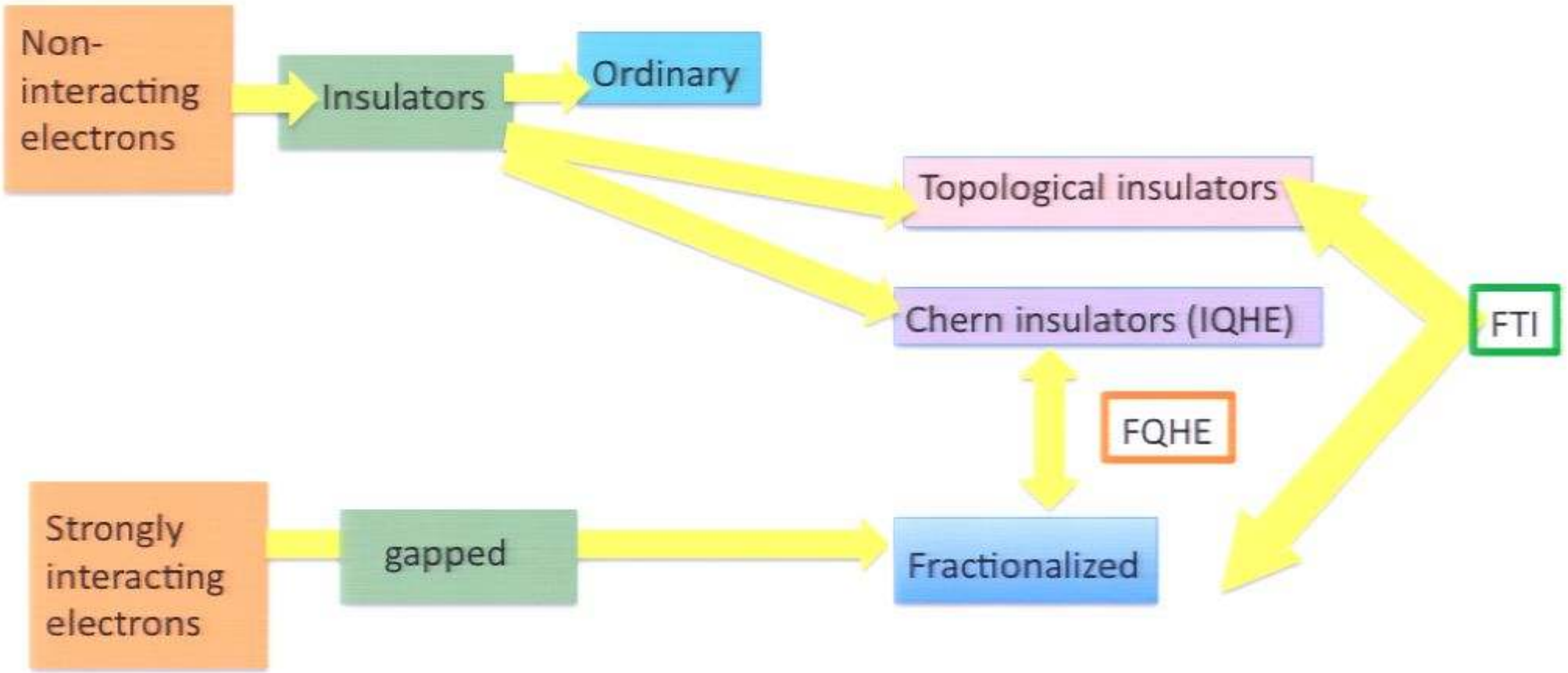
- Bulk properties are all ordinary
  - (gapped, incompressible states)



- Distinguished by properties of boundary
  - (Quantized transport of charge, spin)

Fractionalization

- What?
  - Excitations carry a fraction of quantum numbers of constituent particles.
- How?
  - Strong interactions
- Example
  - Fractional QHE


Non-interacting electrons
Insulators
Ordinary
Topological insulators
Chern insulators (IQHE)
FTI
FQHE
Strongly interacting electrons
gapped
Fractionalized

Fractional topological insulators

- Can you make a model with a single surface Dirac cone of fractionally charged fermions?
  - Exactly solvable lattice model
  - Fractionally charged fermionic excitations
  - Band structure.
- Is it a topological insulator?
  - Time reversal protected edge modes

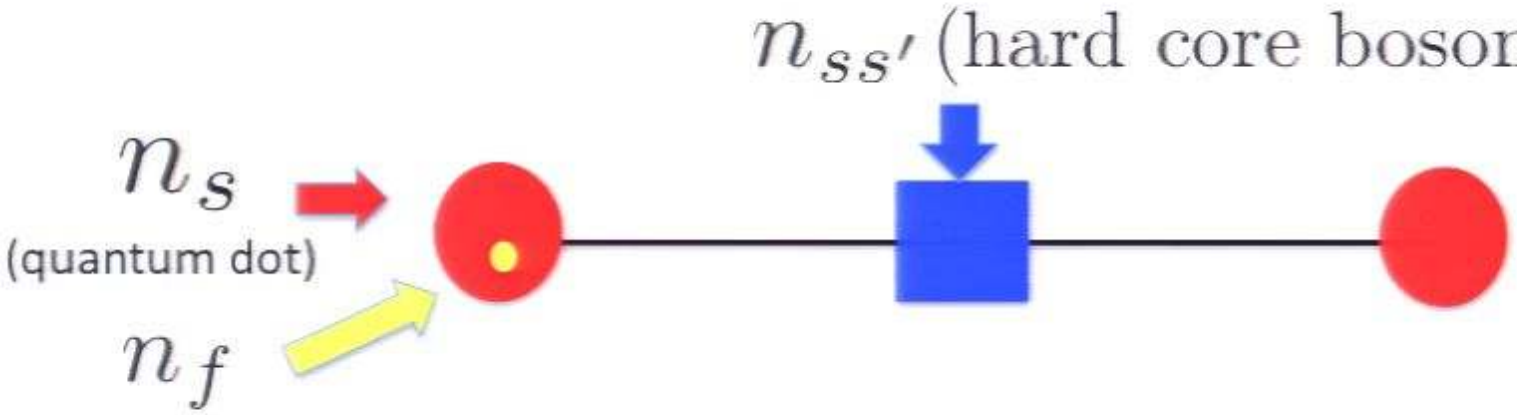
Solvable models with fractionally charged fermions

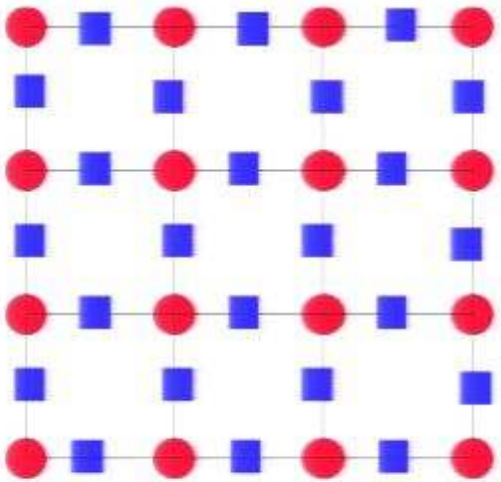
- Strongly interacting charged bosons ($\frac{q_B}{p}$)
  (RVB; Toric code; Senthil & Motrunich)
  - Charge conserving, exactly solvable

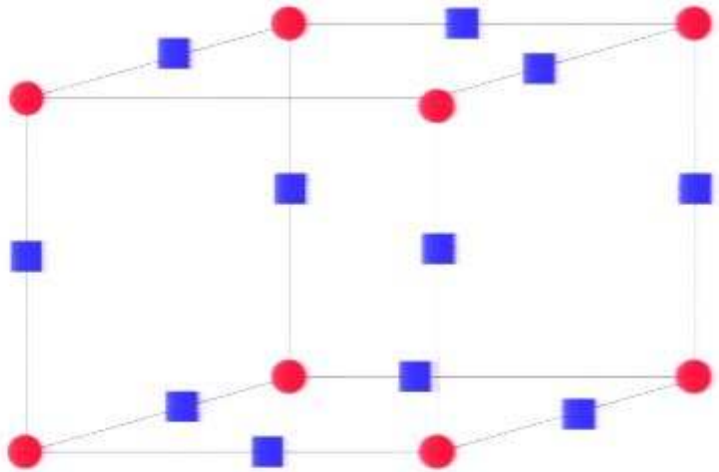
- Couple to fermions $q_f = e + k\frac{q_B}{p}$

- Band structure

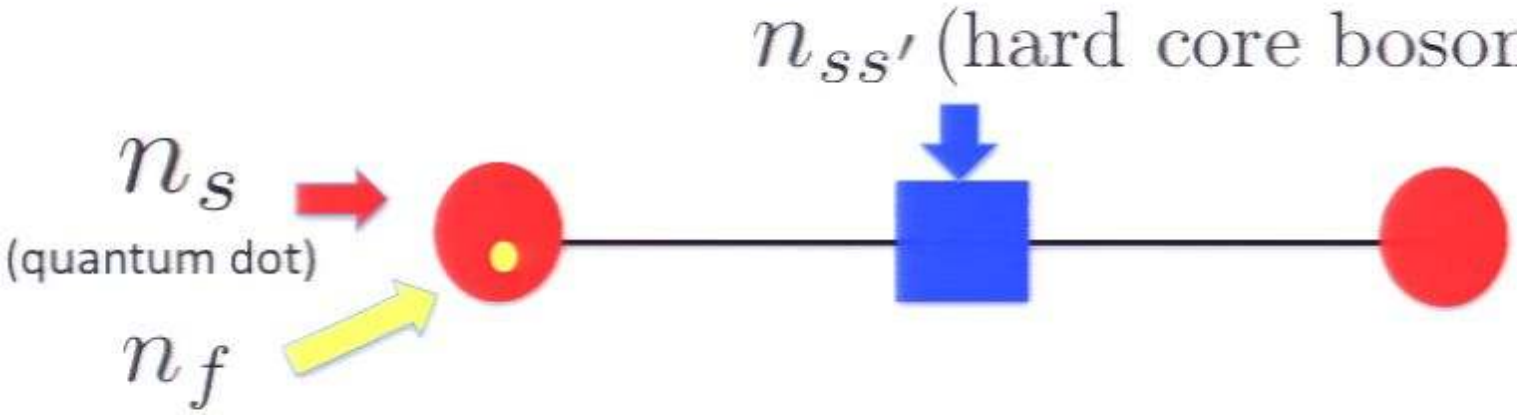
Solvable models with fractionally charged fermions

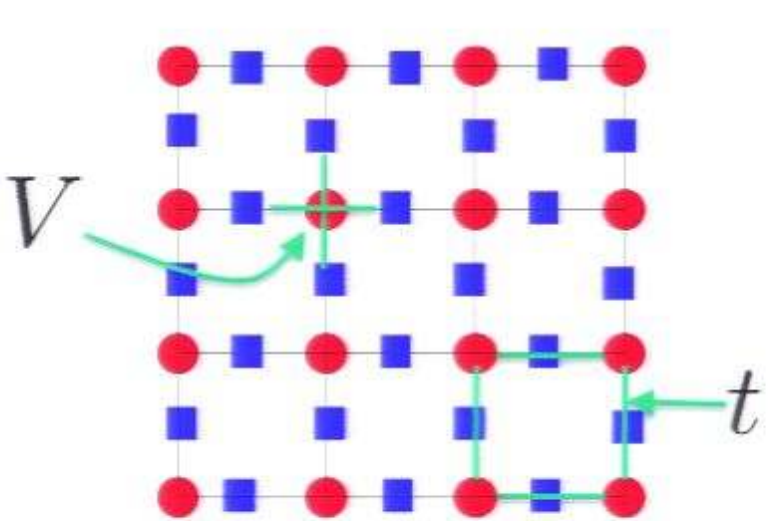


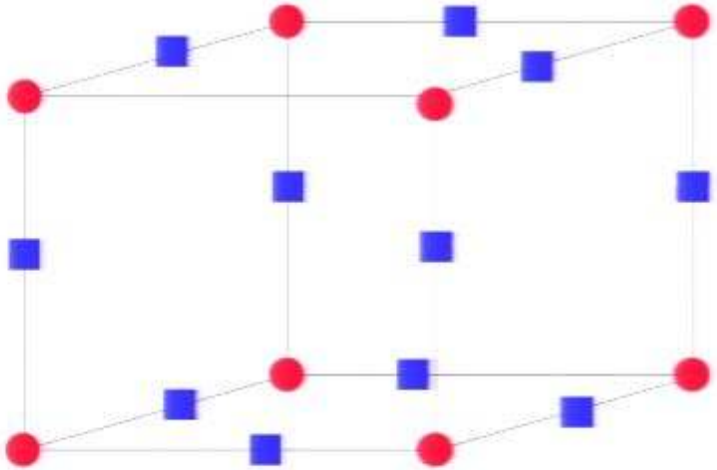




Solvable models with fractionally charged fermions

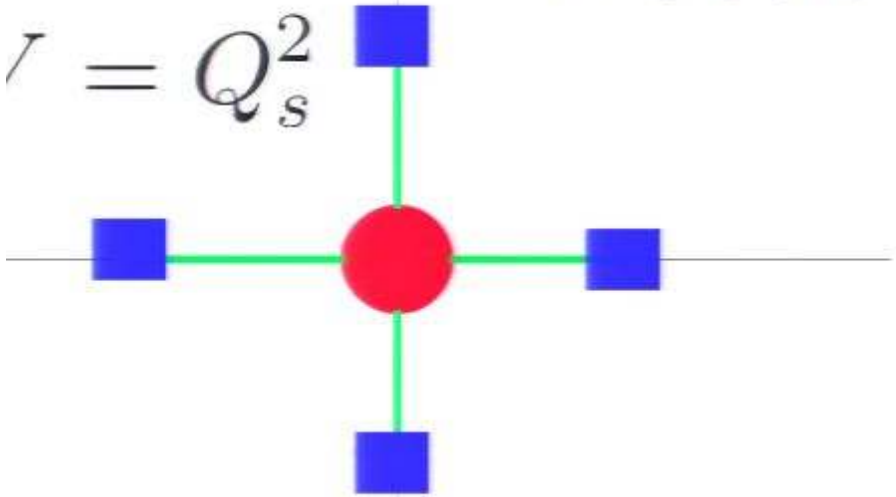






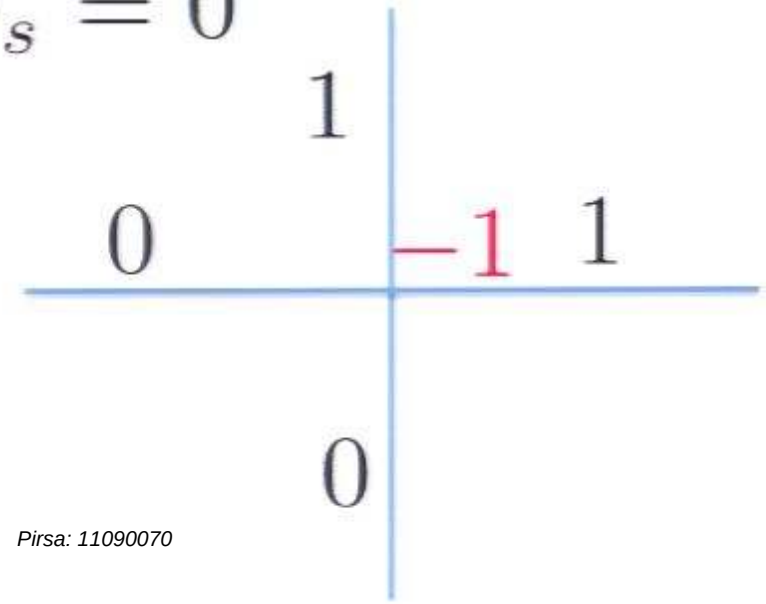
Fractionally charged bosons

$\Gamma = Q_s^2$

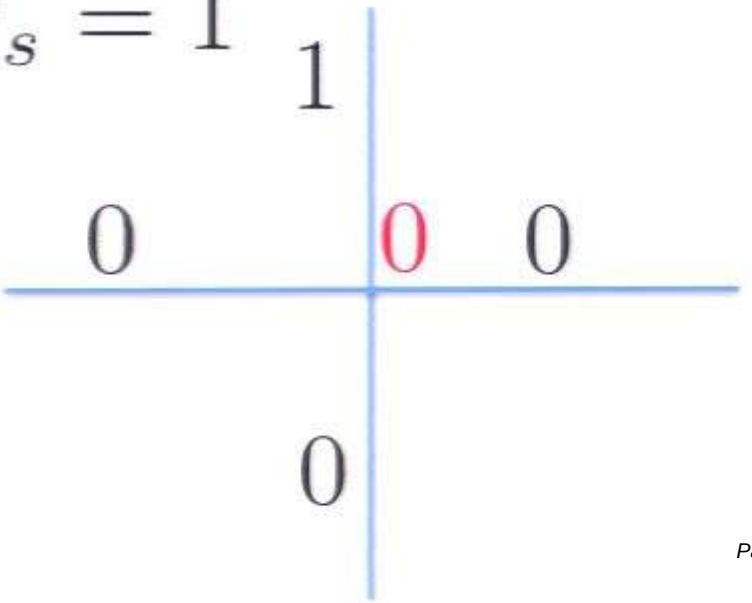


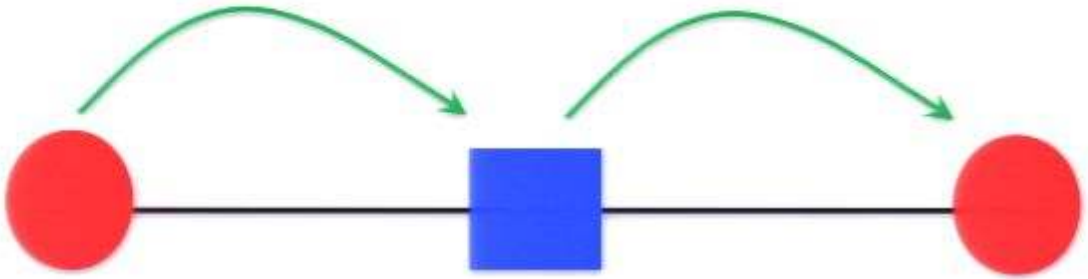
$$Q_s = \sum_{s'} n_{ss'} + 2n_s$$

$Q_s = 0$

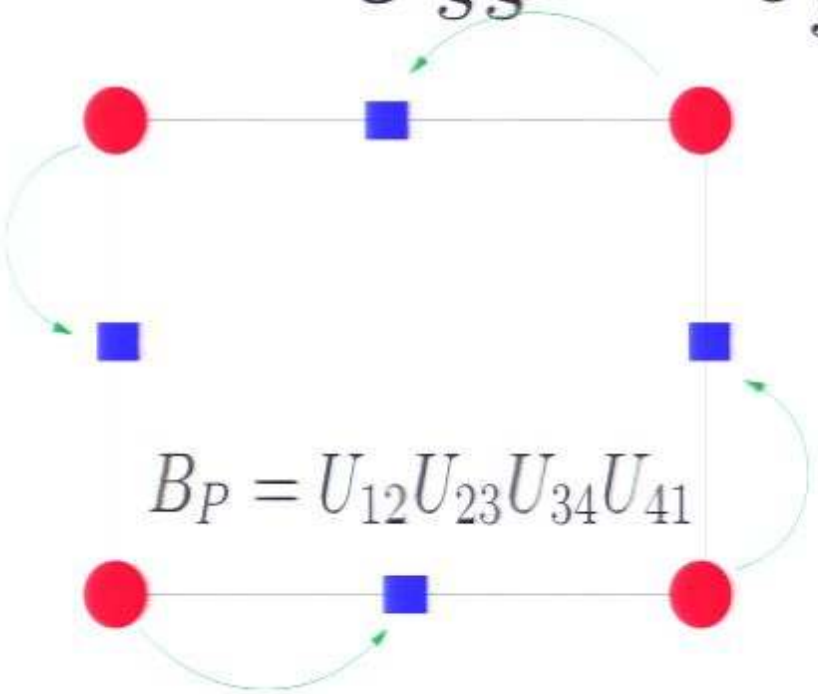


$Q_s = 1$





$$U_{ss'} = b^\dagger_{s'} b_{ss'} + b^\dagger_{ss'} b_s$$



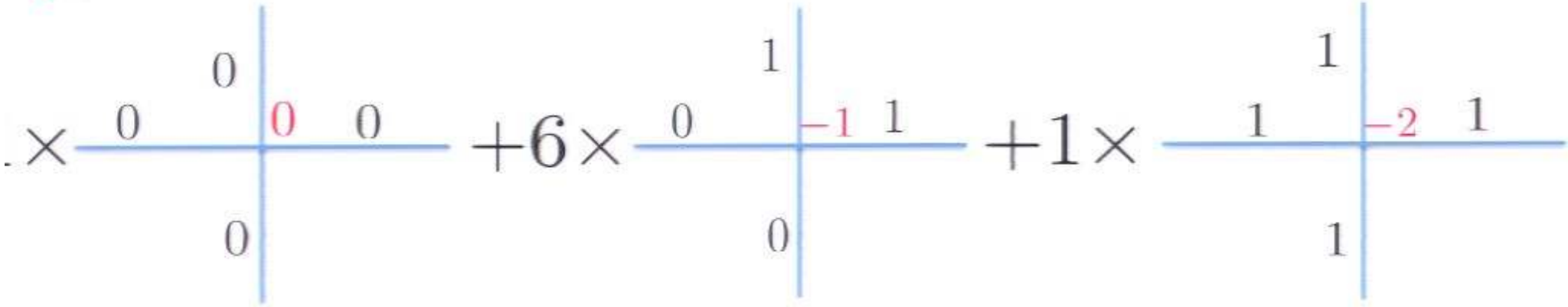
$$B_P = U_{12} U_{23} U_{34} U_{41}$$

$$[Q_s, B_P] = 0$$

- Equal amplitude superposition of all link occupation numbers compatible with Q_s

$$H = V \sum_s Q_s^2 - \frac{u}{2} \sum_P \left(B_P + B_P^\dagger \right)$$

$Q_s = 0$



links : 2 ; sites :-1

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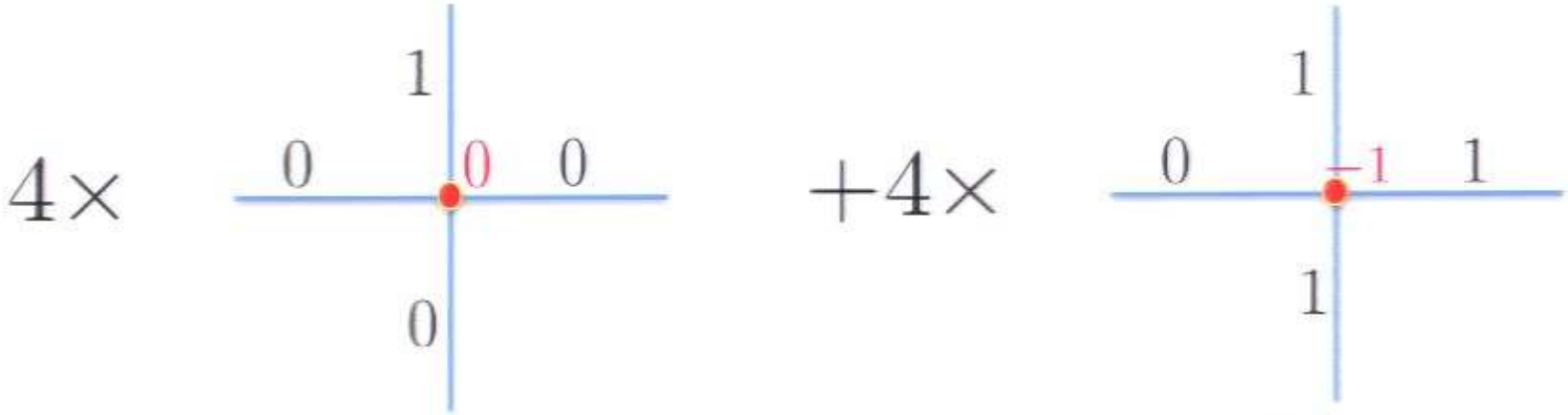
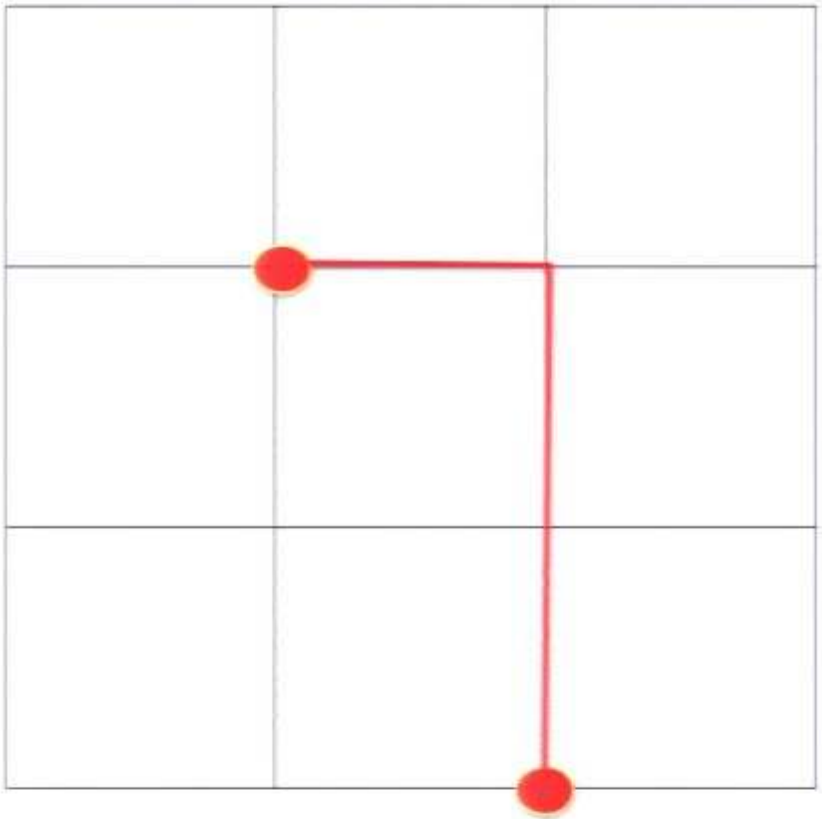


links : 2 ; sites $:-\frac{1}{2}$

Fractional Charge

$$\langle N_S \rangle_{Q_s=1} - \langle N_s \rangle_{Q_s=0} = \frac{1}{p}$$



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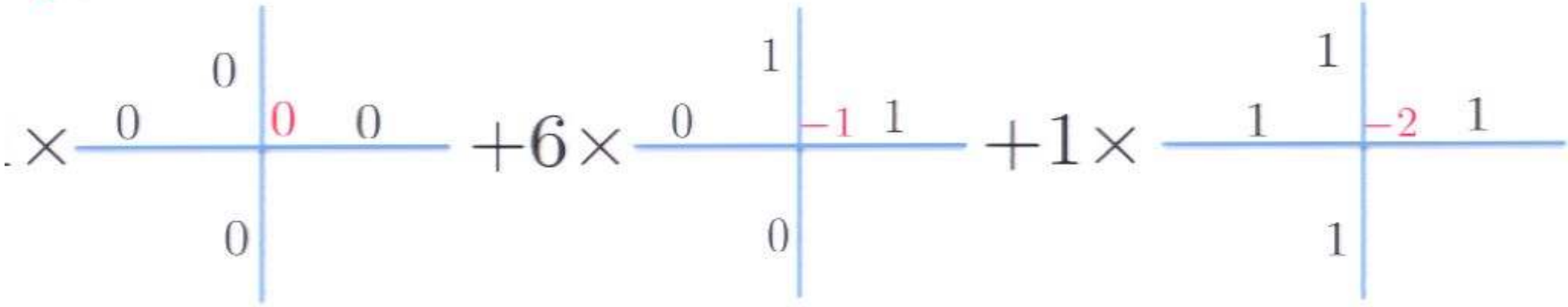

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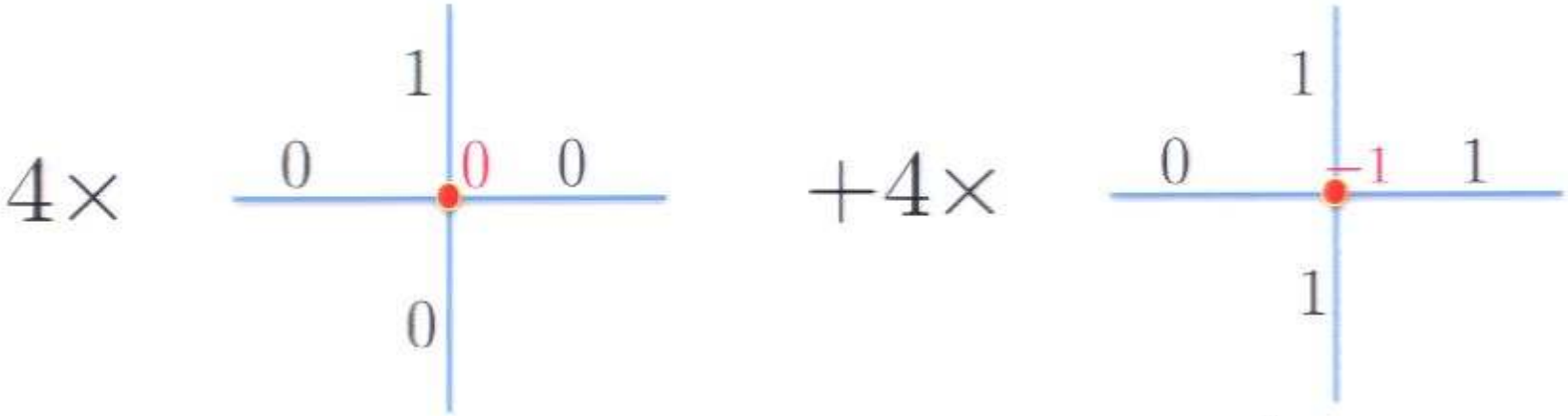

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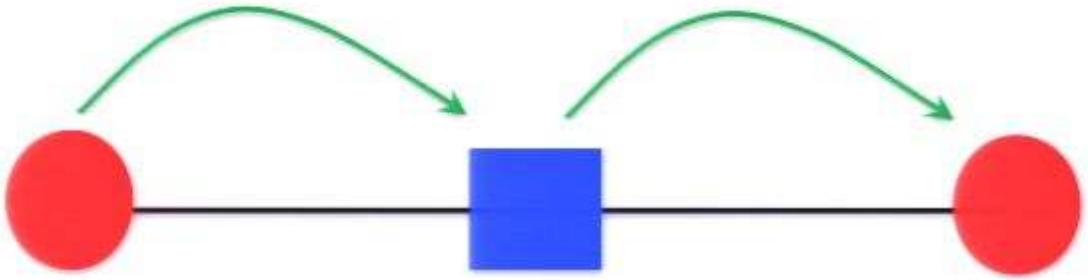

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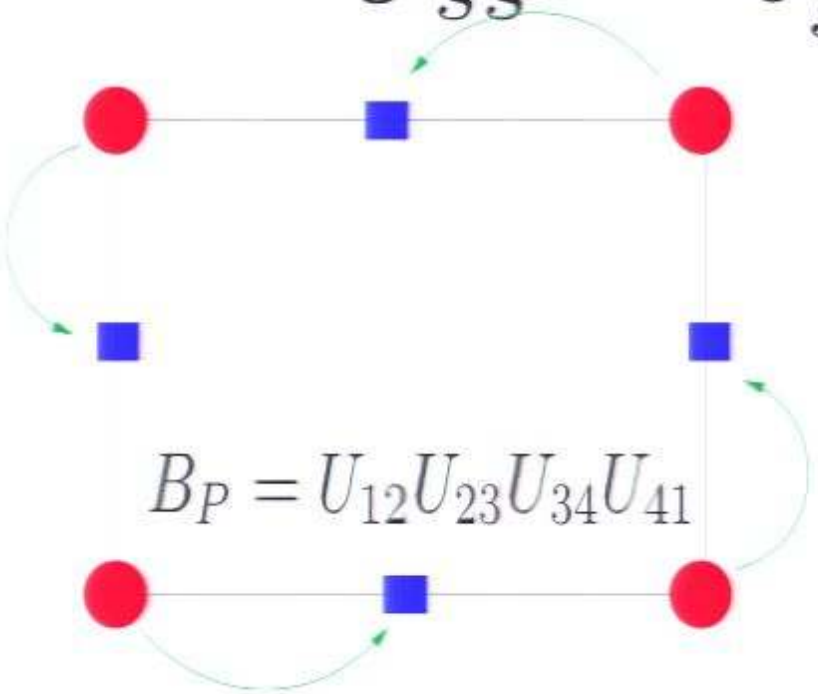

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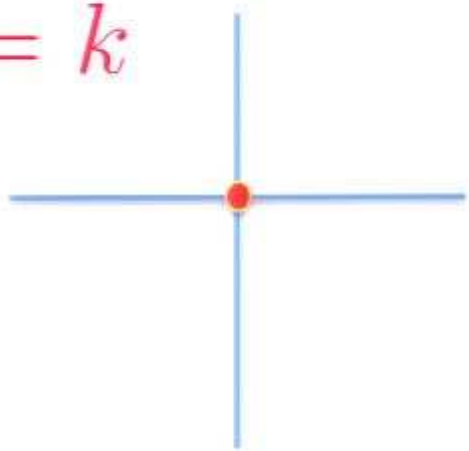
$$H = V \sum_s Q_s^2 - \frac{u}{2} \sum_P \left(B_P + B_P^\dagger \right)$$

Fractionally charged fermions

$$\tilde{Q}_s = Q_s - k n_{f,s}$$

$$Q_s = k$$



$$q_f = \frac{(p+2k)e}{p}$$

Fractional Charge

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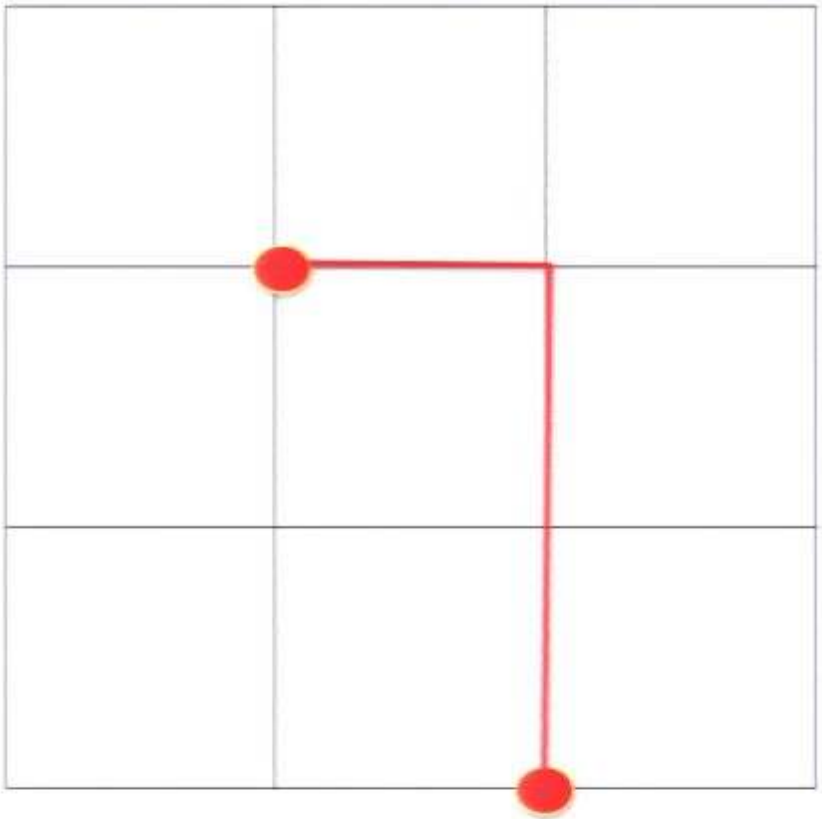

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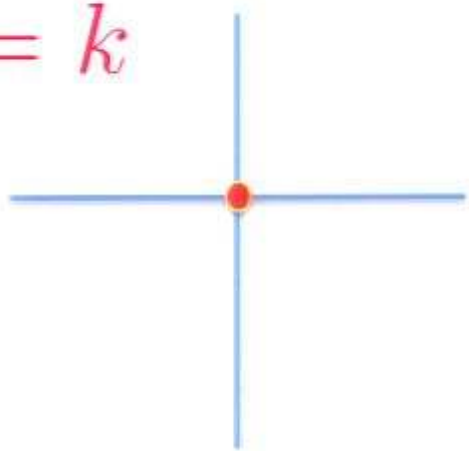

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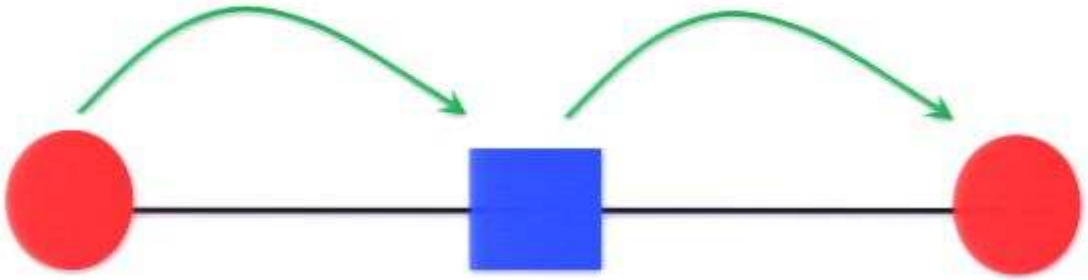

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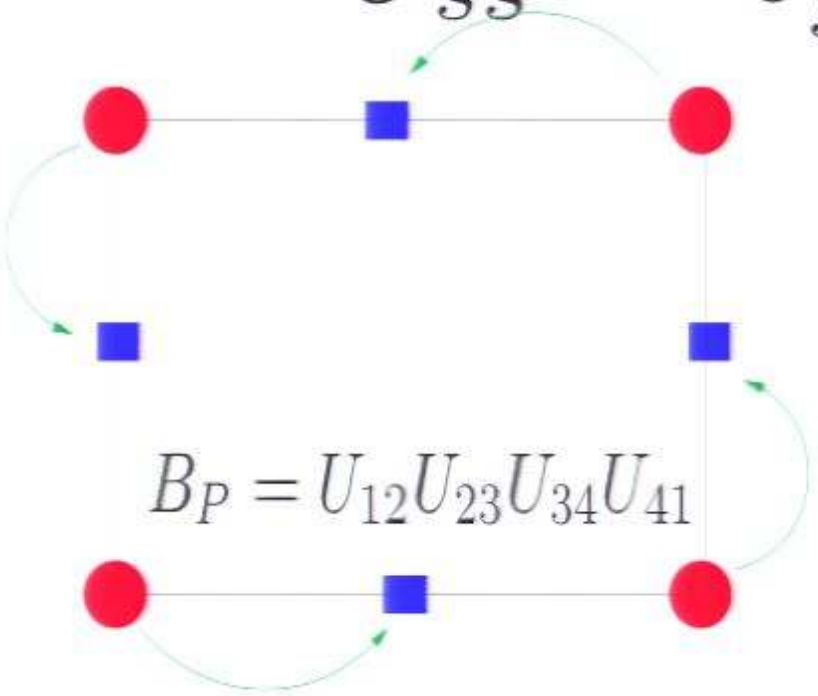

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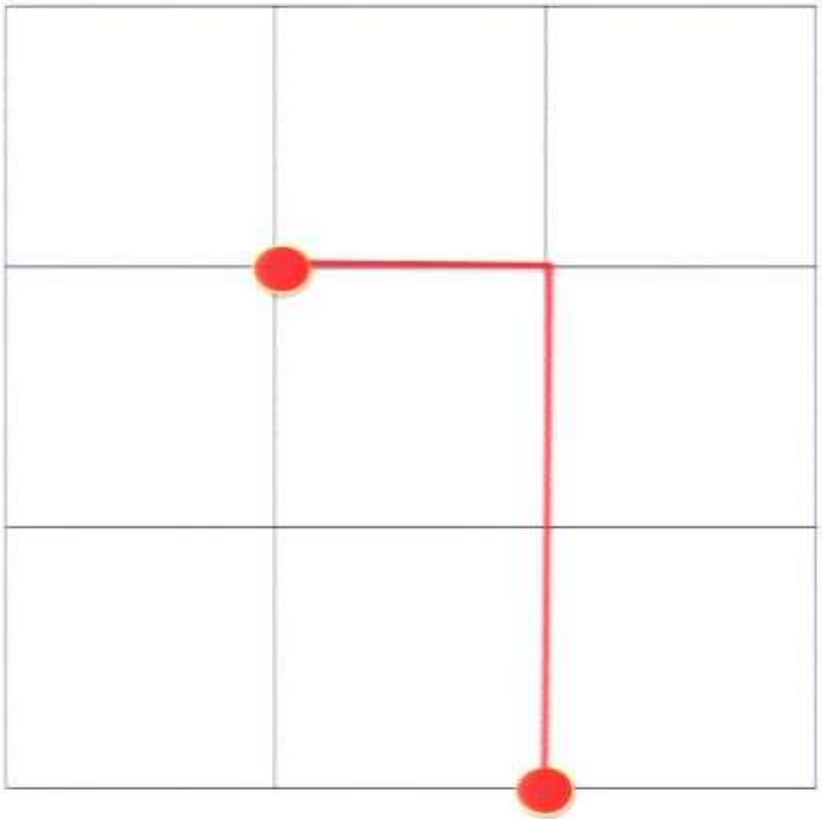

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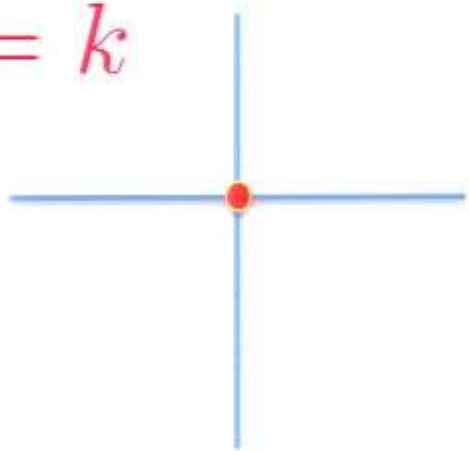

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Fractionally charged fermions

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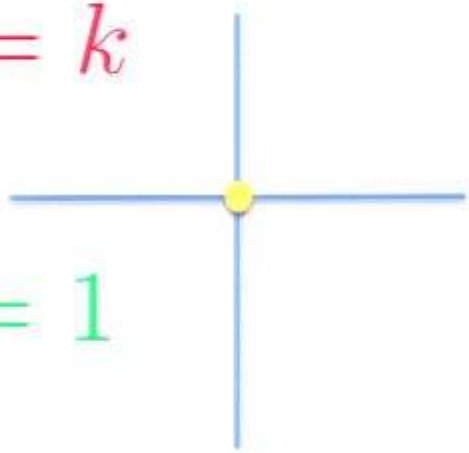
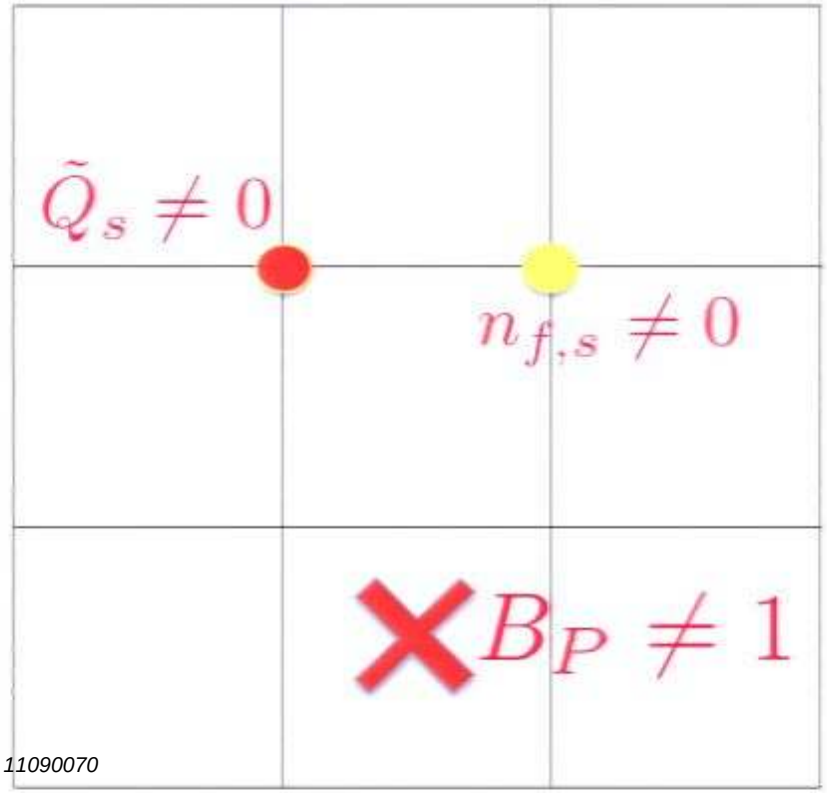


$$n_f = 1$$

$$q_f = \frac{(p+2k)e}{p}$$

$$H = V \sum_s \tilde{Q}_s^2 - \frac{u}{2} \sum_P \left(B_P + B_p^\dagger \right) + \mu \sum_s n_{f,s}$$



$$u, V \gg \mu$$

- Low-energy excitations: composite fermions (fractionally charged)

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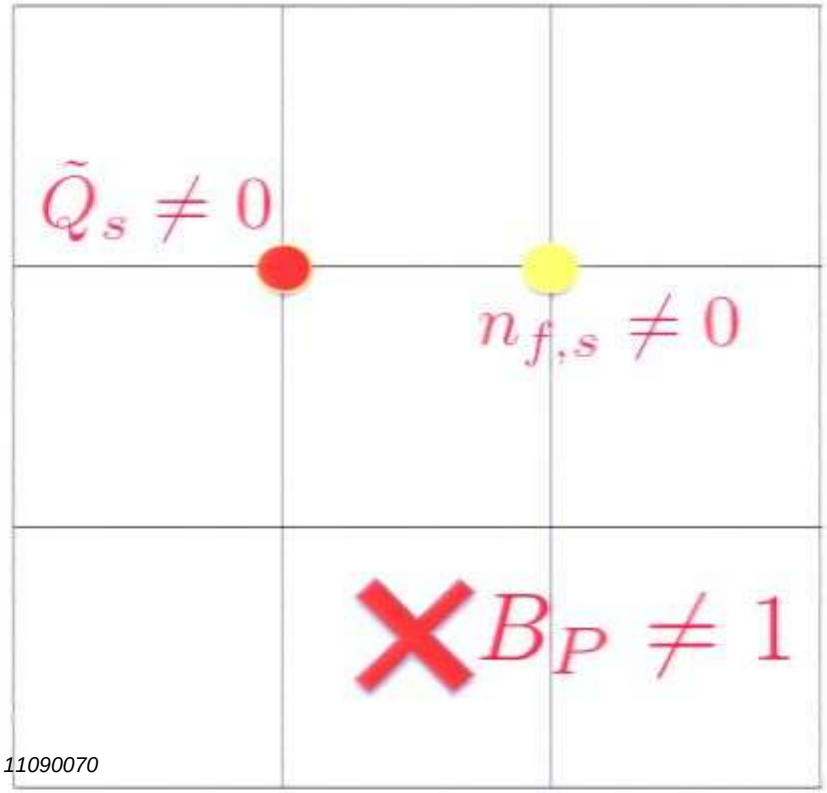

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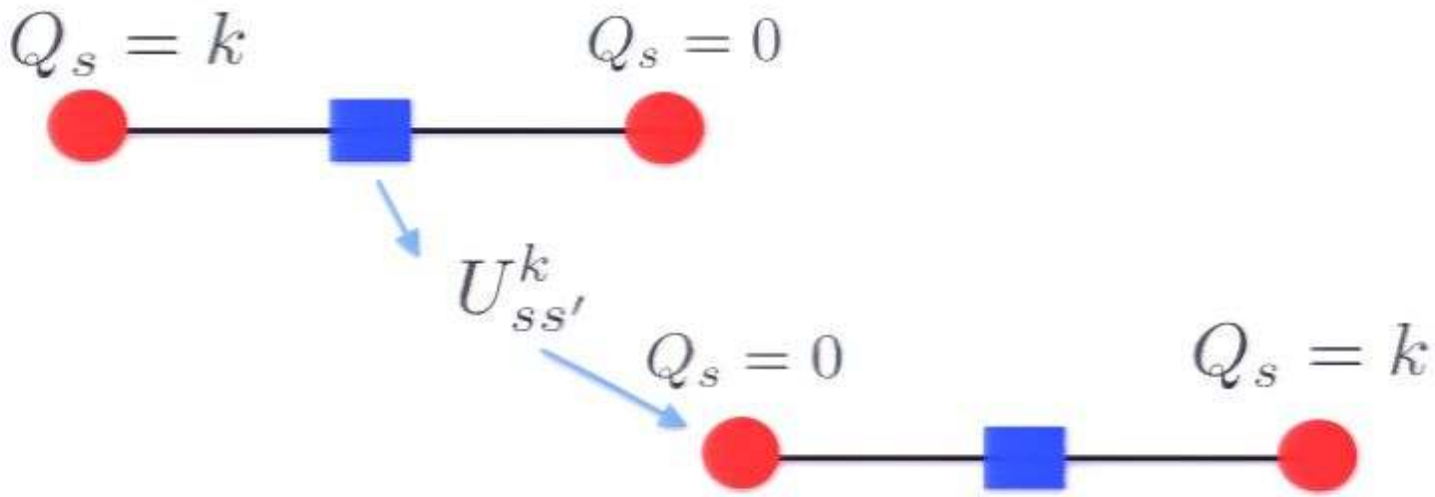
- Low-energy excitations: composite fermions (fractionally charged)

Band structure

$$H_{kin} = -\sum_{ss'} t_{ss'\sigma\sigma'} c^{\dagger}_{s'\sigma'} c_{s\sigma} U^{k}_{ss'}$$

- Hops one fermion and k fractionally charged bosonic excitations



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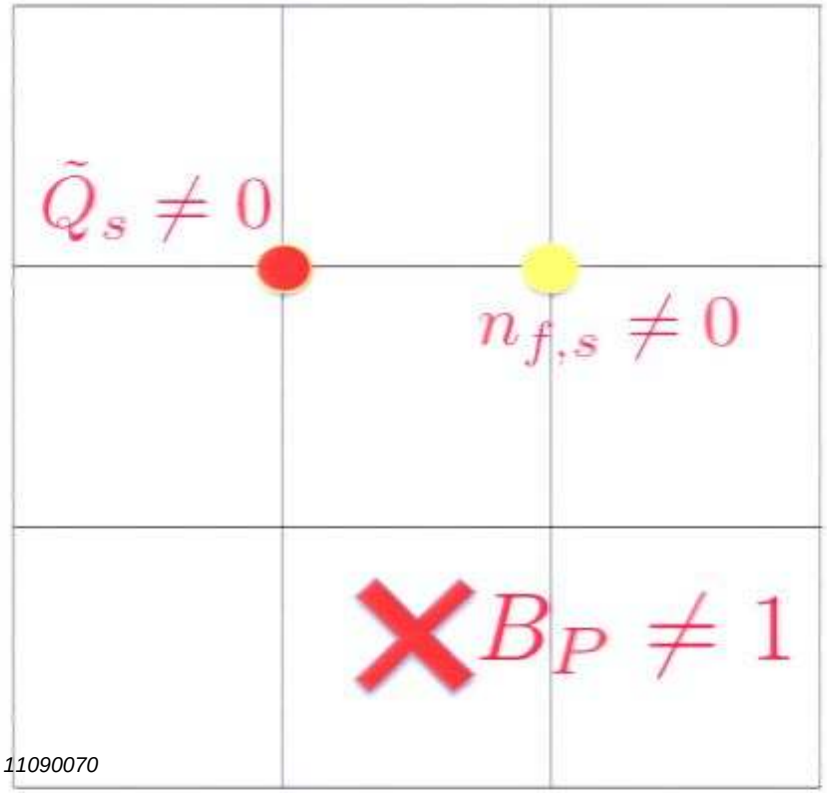

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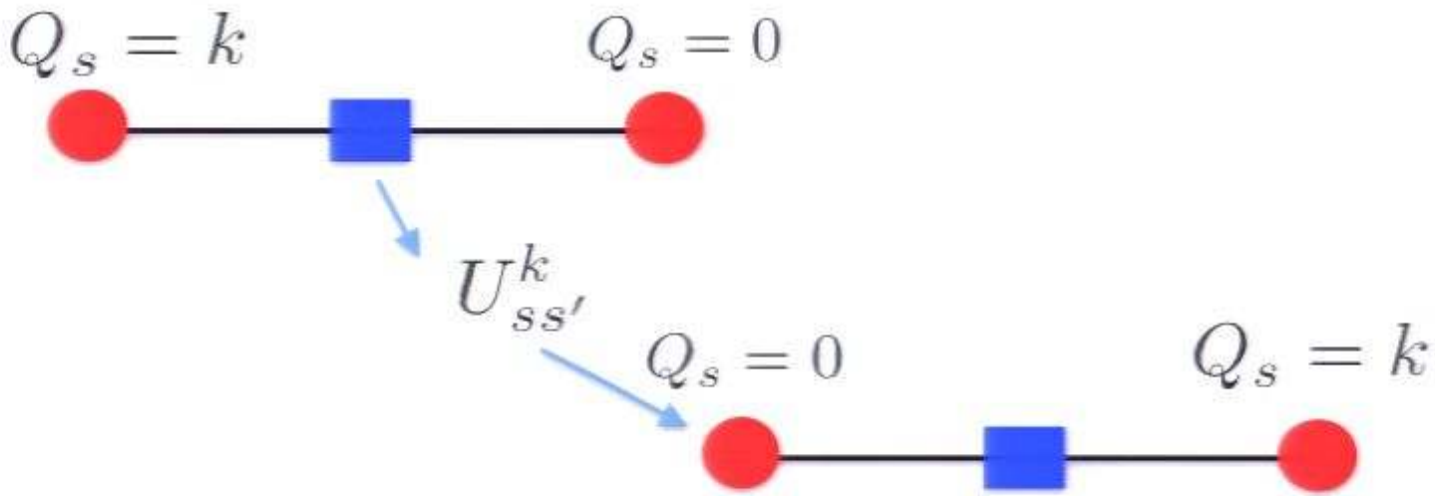
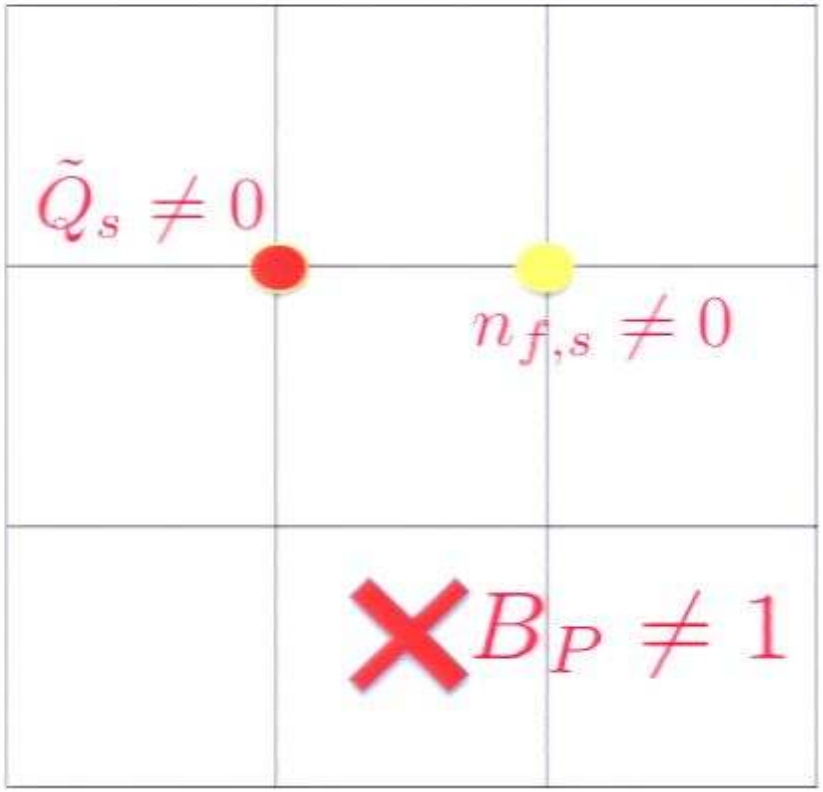


$$H = V \sum_s \tilde{Q}_s^2 - \frac{u}{2} \sum_P \left(B_p + B_P^\dagger \right) + H_{kin}$$



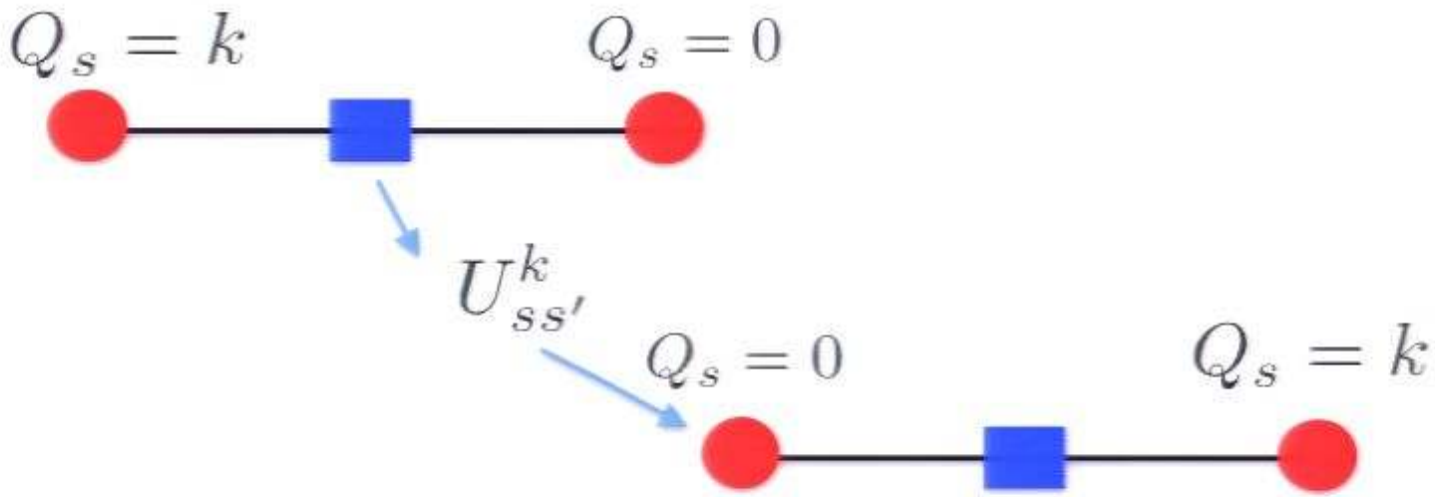
$$[H_{kin}, \tilde{Q}_s] = 0$$

$$[H_{kin}, B_P] = 0$$

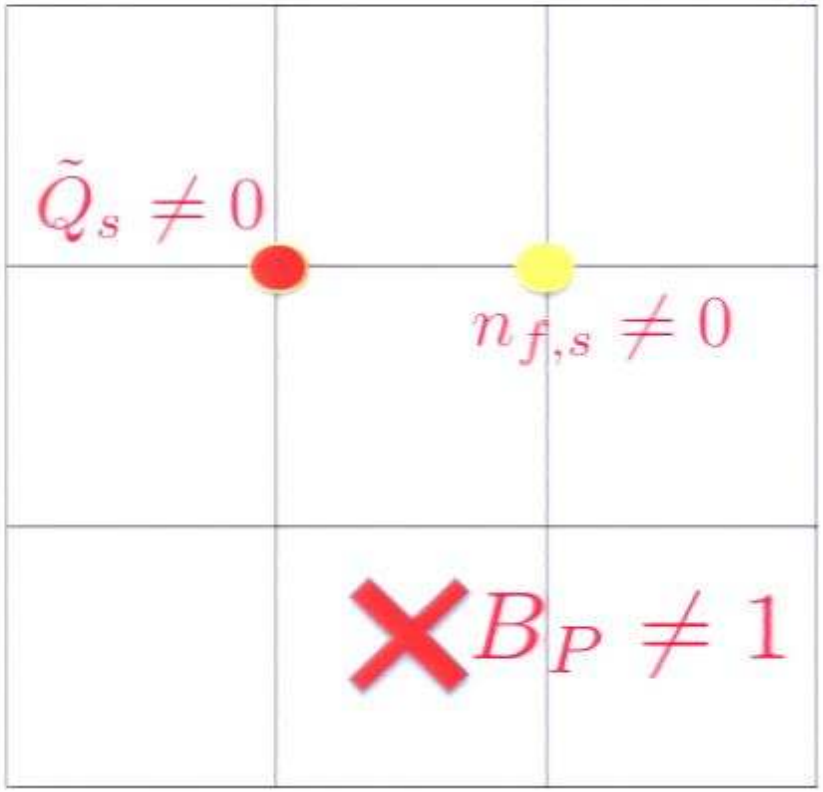
Band structure

$$H_{kin} = -\sum_{ss'} t_{ss'\sigma\sigma'} c^{\dagger}_{s'\sigma'} c_{s\sigma} U^{k}_{ss'}$$

- Hops one fermion and k fractionally charged bosonic excitations



$$H = V \sum_s \tilde{Q}_s^2 - \frac{u}{2} \sum_P \left(B_p + B_P^\dagger \right) + H_{kin}$$



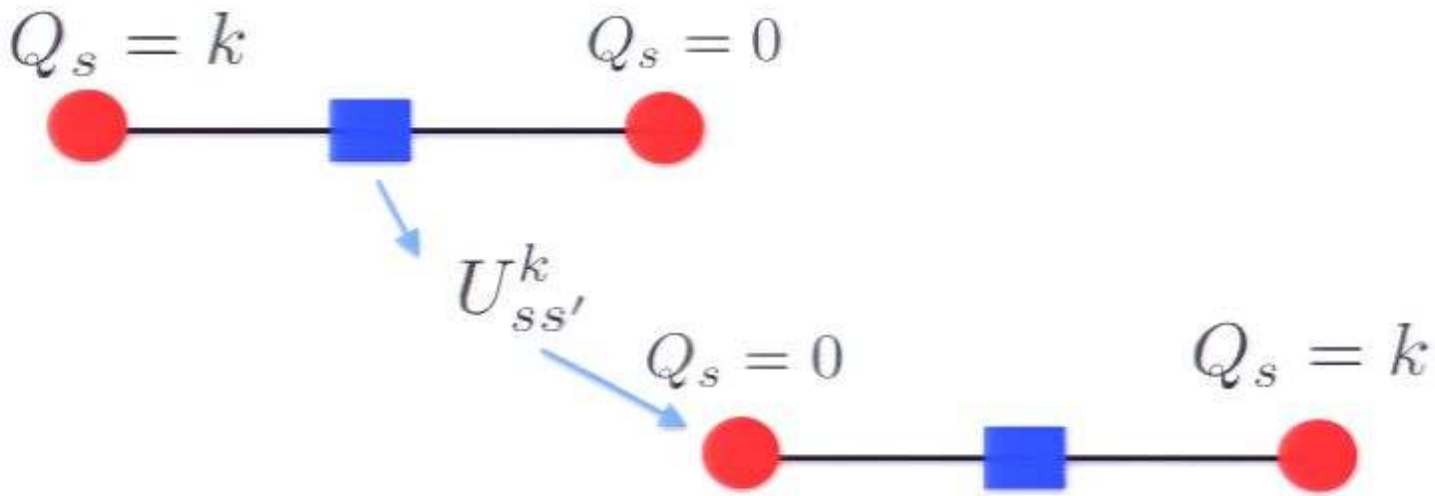
$$[H_{kin}, \tilde{Q}_s] = 0$$

$$[H_{kin}, B_P] = 0$$

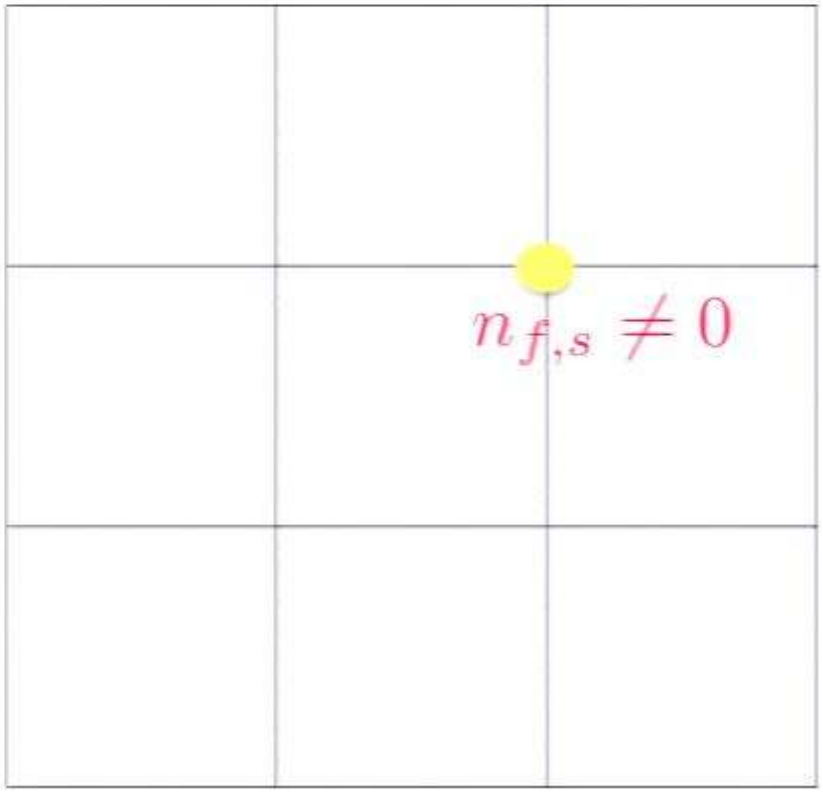
Band structure

$$H_{kin} = -\sum_{ss'} t_{ss'\sigma\sigma'} c^{\dagger}_{s'\sigma'} c_{s\sigma} U^{k}_{ss'}$$

- Hops one fermion and k fractionally charged bosonic excitations



$$H = \sum t_{ss'\sigma\sigma'} d^{\dagger}_{s'\sigma'} d_{s\sigma}$$

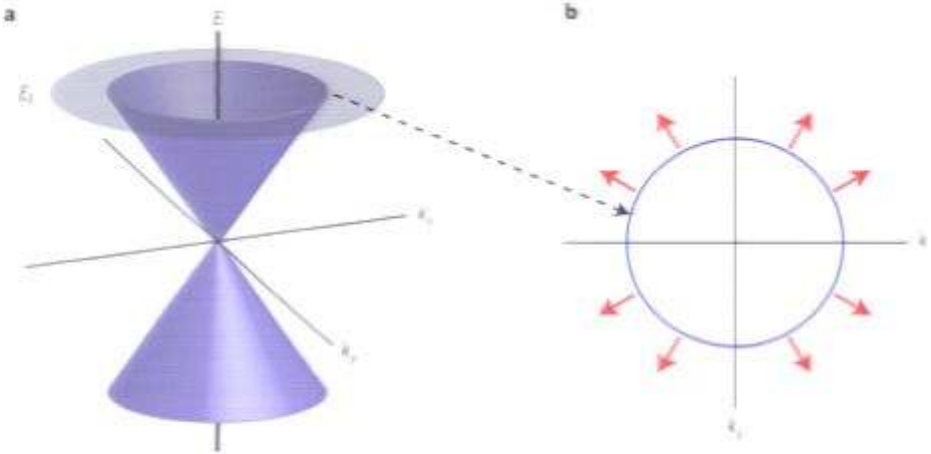


$$[H_{kin}, \tilde{Q}_s] = 0$$

$$[H_{kin}, B_P] = 0$$

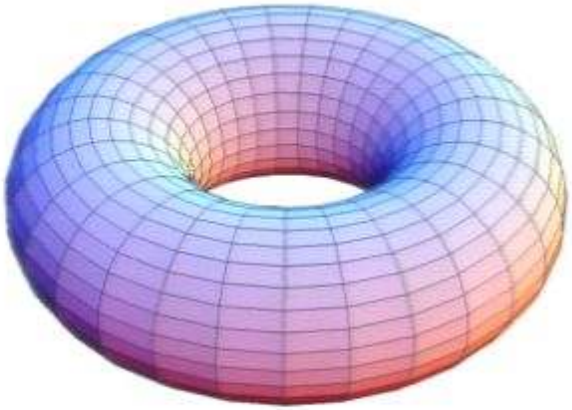
Fractional topological insulators

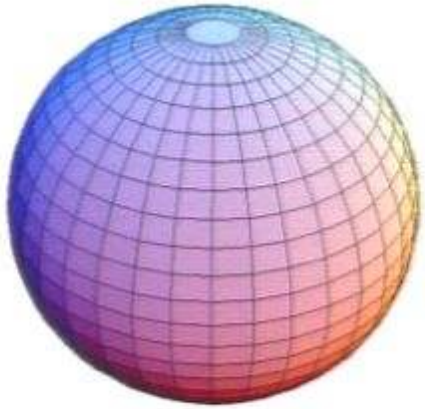
- We can choose any band structure
- Choose TI band structure
- Fractionally charged surface states



A few interesting properties

- Topological order



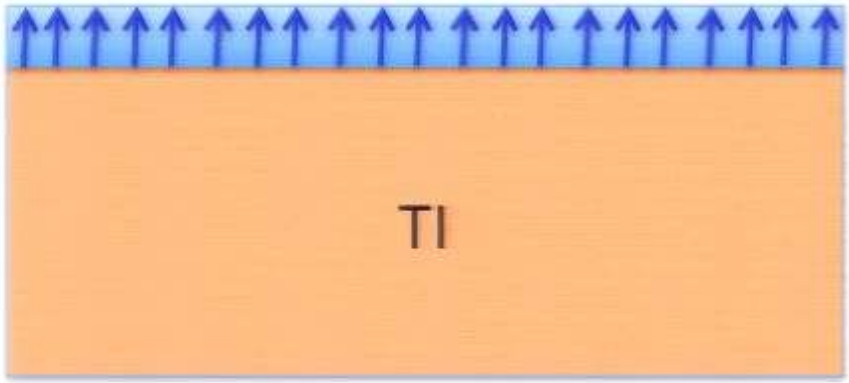


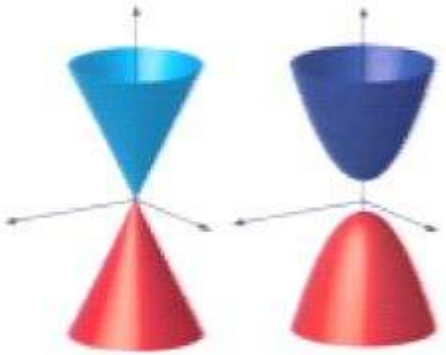
$p^2 \quad (2D)$ $\qquad$ 1

$p^3 \quad (3D)$

A few interesting properties

- Magnetoelectric effect
  - Break T on the surface
  - ½ integer Hall conductivity

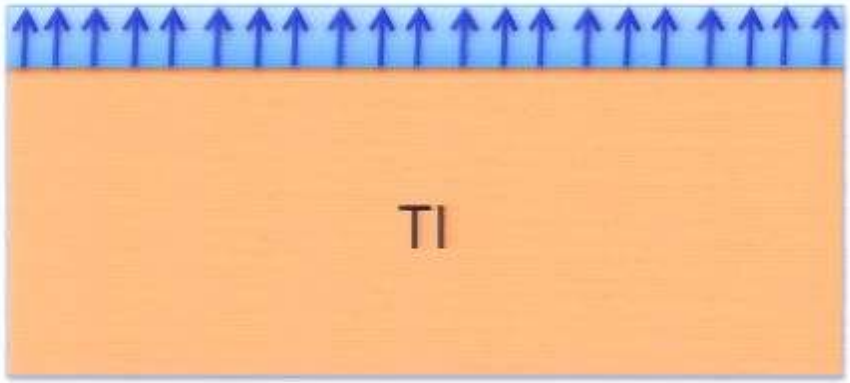


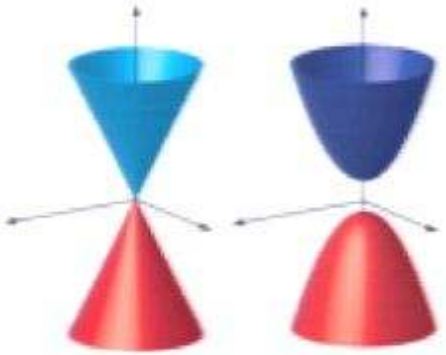


$$\sigma_{xy} = \frac{1}{2}\frac{e^2}{h}$$

A few interesting properties

- Fractional magnetoelectric effect
  - Break T on the surface
  - ½ integer Hall conductivity





$$\sigma_{xy} = \frac{1}{2}\frac{q_f^2}{h}$$

Fractional topological insulators

- Can you make a model with a single surface Dirac cone of fractionally charged fermions?
  - Exactly solvable lattice model
  - Fractionally charged fermionic excitations
  - Band structure.
- Is it a topological insulator?
  - Time reversal protected edge modes

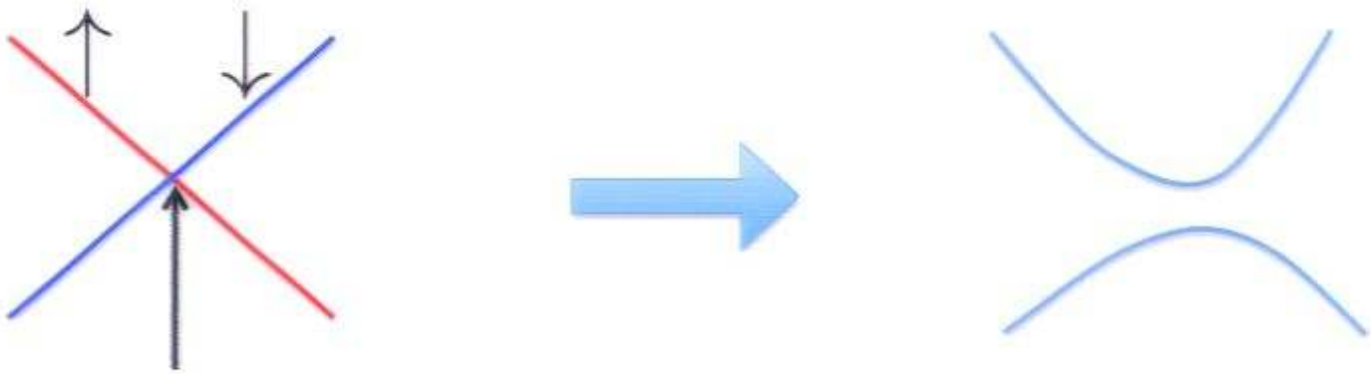
Time-reversal protected gapless boundary modes

- Kramers theorem: if

$$T : k \to -k \equiv k$$

$$T^2 = -1$$

- then states must be degenerate.



$k = 0, \pi$

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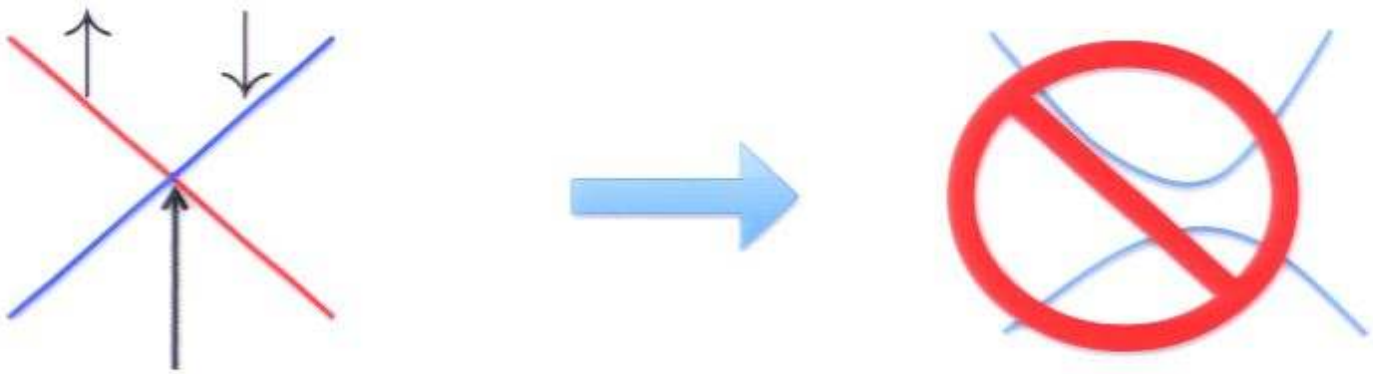
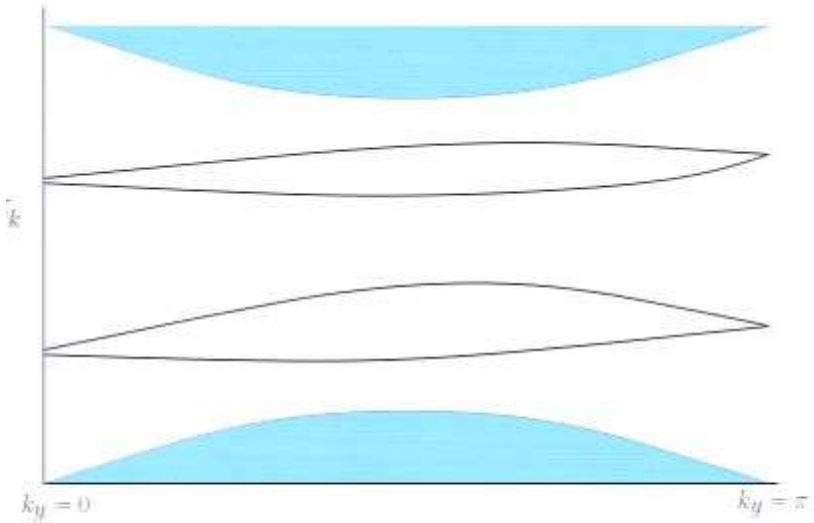


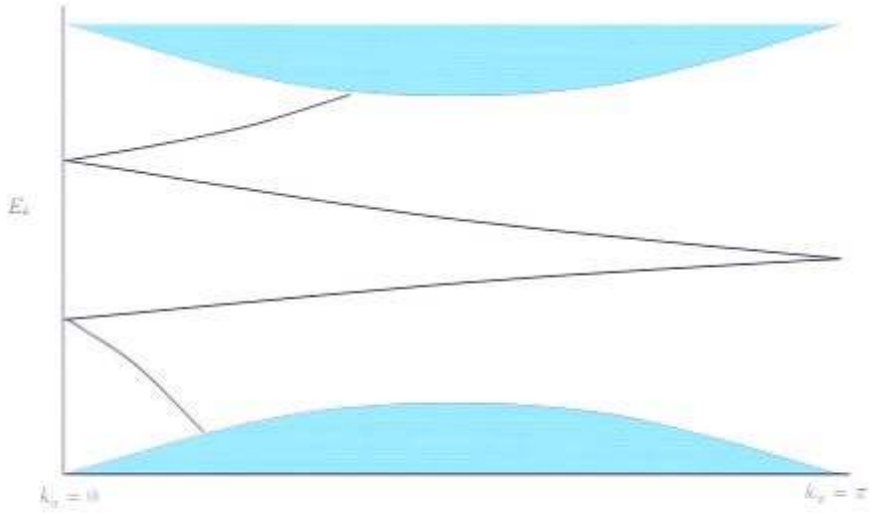
$k = 0, \pi$

Topological insulators:

- odd number of Kramers pairs in the edge spectrum



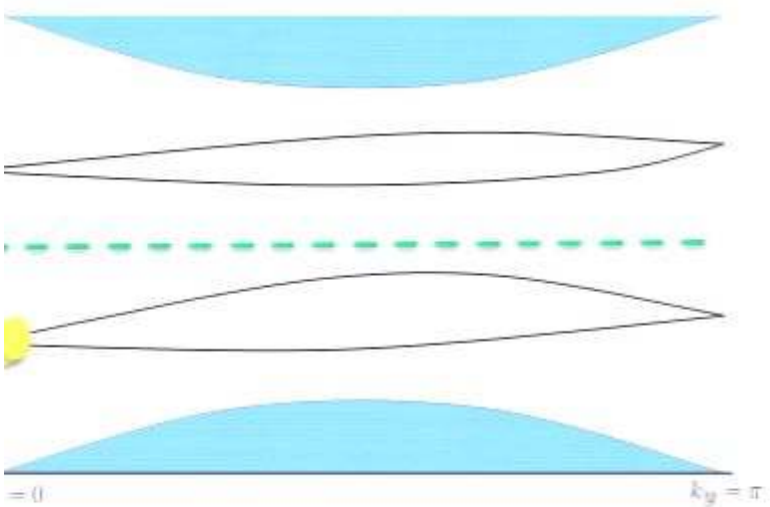
Ordinary insulator

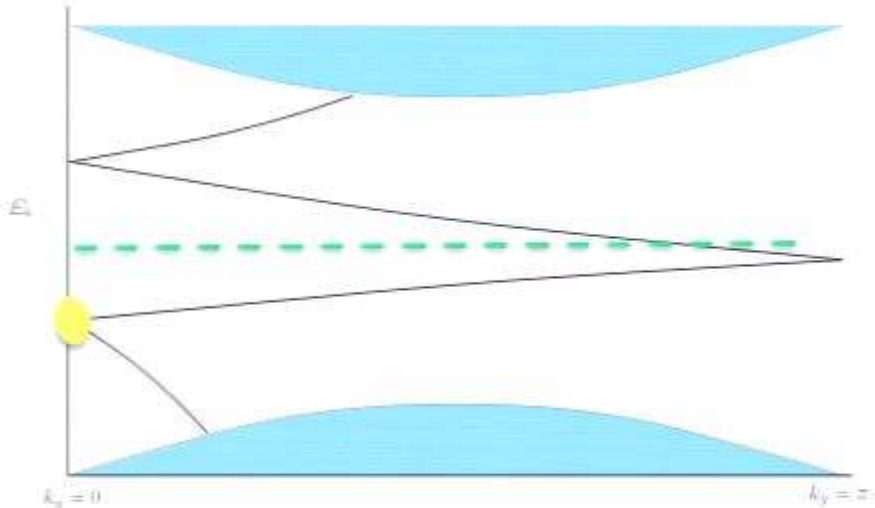


Topological insulator

Interacting equivalent:

$$\Phi = 0$$





(Fu and Kane '06)

Interacting equivalent:

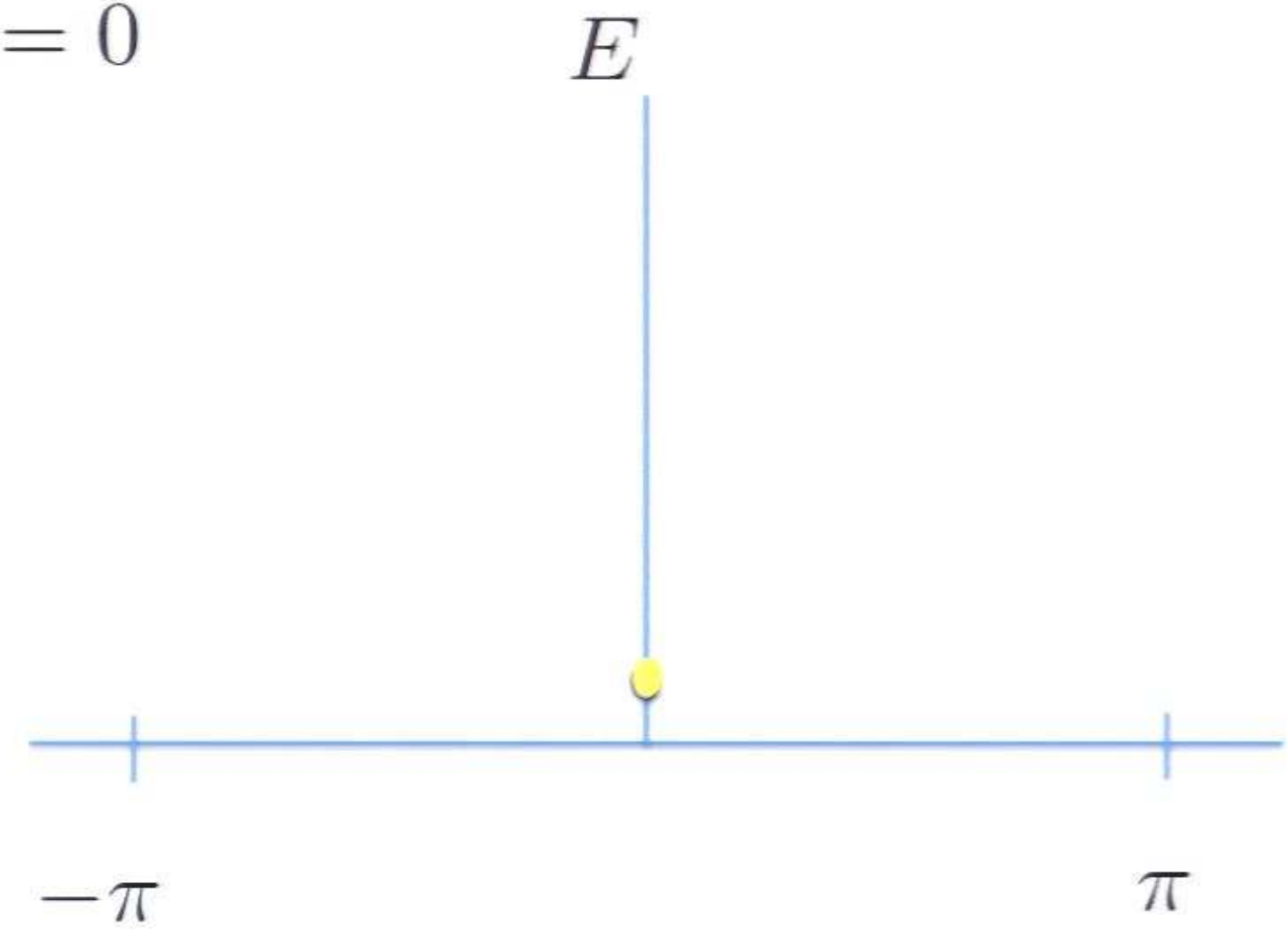
- $\Phi = 0, \pi$: Time-reversal invariant
- Kramers degeneracies:

	TI	Non-TI
$\Phi = 0$	Yes	No
$\Phi = \pi$	No	No
$\Phi = 0$	No	Yes
$\Phi = \pi$	Yes	Yes
	Odd	Even

(Fu and Kane '06)

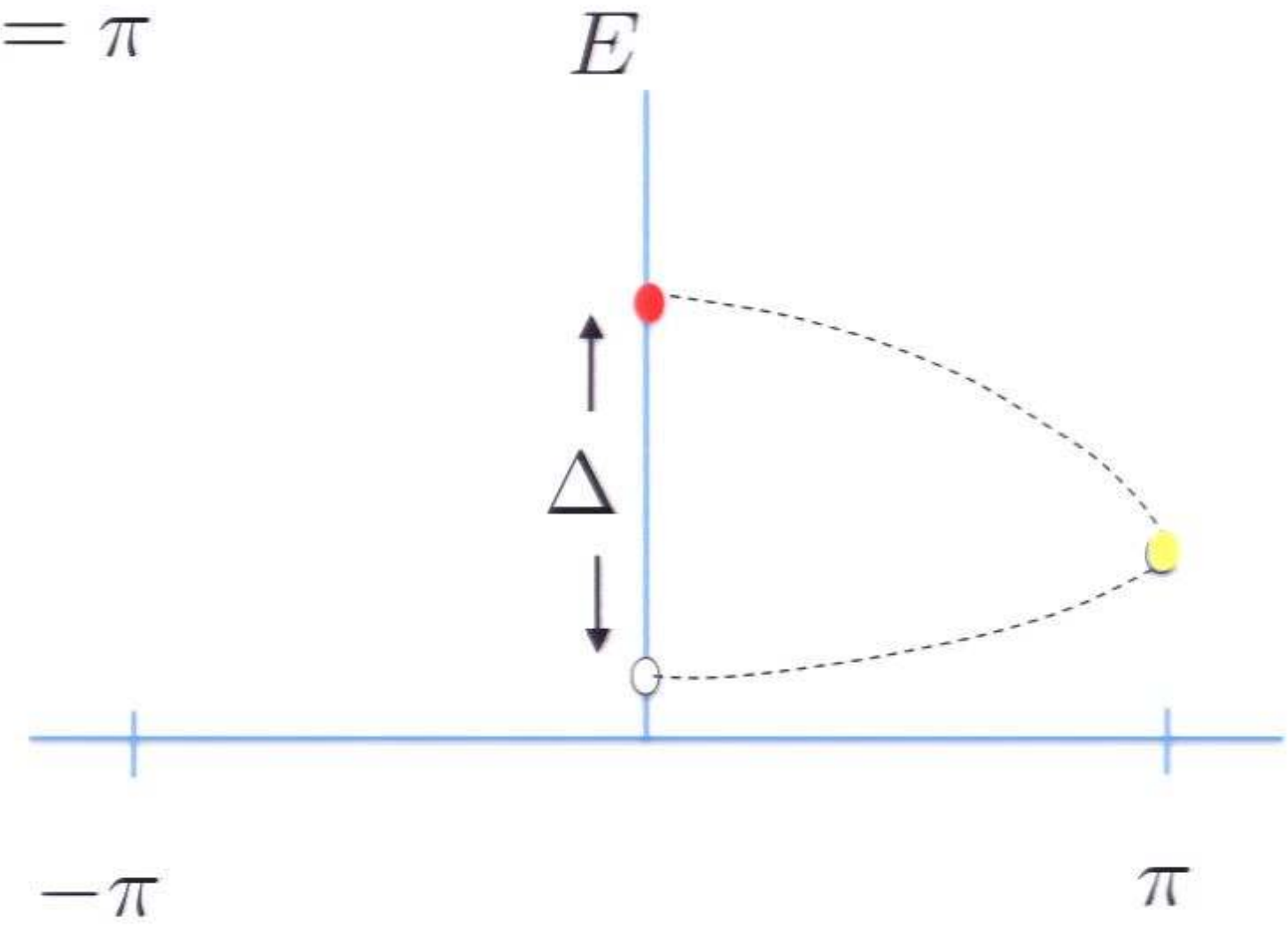
Constructing low-lying excited states

$\Phi = 0$



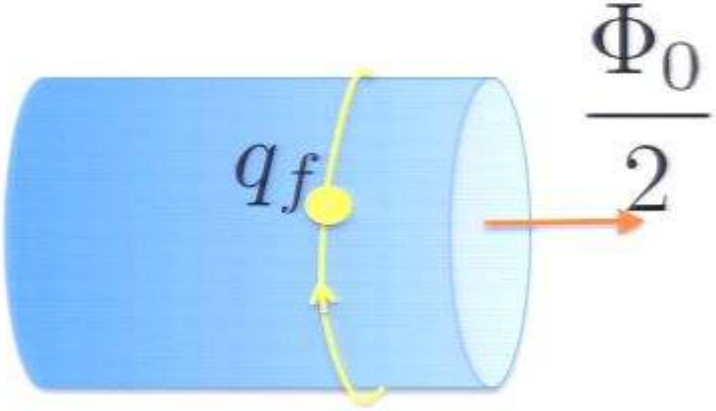
Constructing low-lying excited states

$\Phi = \pi$



Flux insertion in fractional insulators

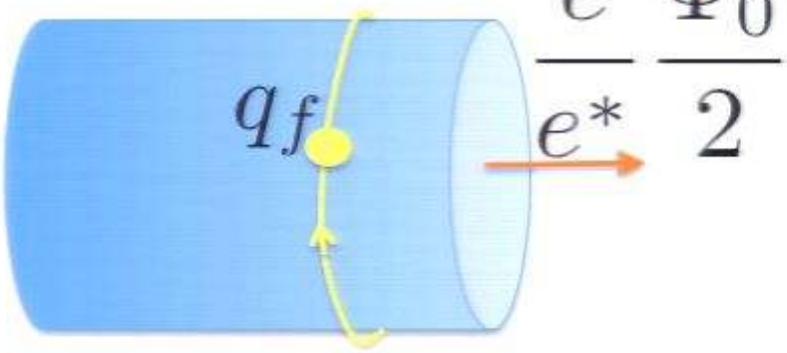
Levin and Stern '09



- Inserting $\frac{\Phi_0}{2}$: a different ground state

Flux insertion in fractional insulators

Levin and Stern '09



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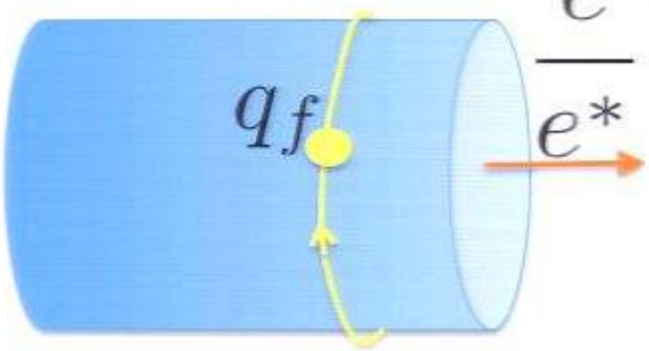


$\frac{e}{\quad}\ \frac{\Phi_0}{\quad}$

2

$\frac{q_f}{e^*}$ odd: Kramers $\rightarrow$ No Kramers

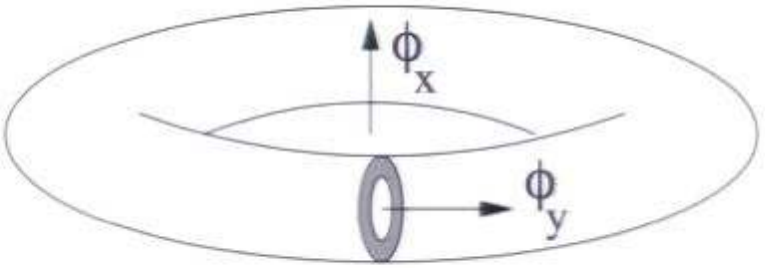
$\frac{q_f}{e^*}$ even: No change

$$\frac{q_f}{e^*} = \begin{cases} p + 2k & (p \text{ odd}) \\ \frac{1}{2}(p + 2k) & (p \text{ even}) \end{cases}$$

2D Fractional topological insulators

- $\frac{q_f}{e^*}$ odd : protected edge modes
- $\frac{q_f}{e^*}$ even : Not protected!
  - Can add interactions that preserve T and gap the edge

Fractional 3D insulators



- $\frac{e}{e^*}\frac{\Phi_0}{2}$: same ground state sector
  : changes Kramers degeneracy if $\frac{q_f}{e^*}$ odd
- $\frac{q_f}{e^*}$ odd : protected surface modes
- $\frac{q_f}{e^*}$ even : ?

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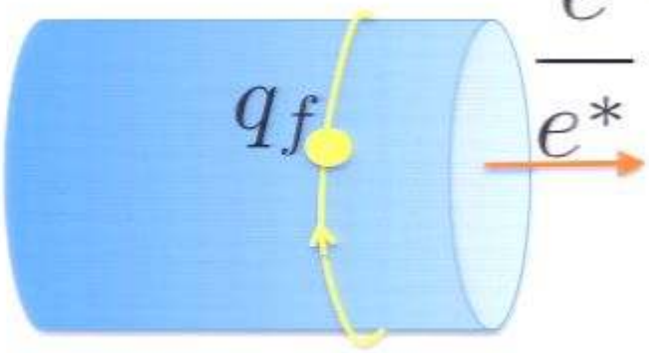

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$\frac{2}{}$

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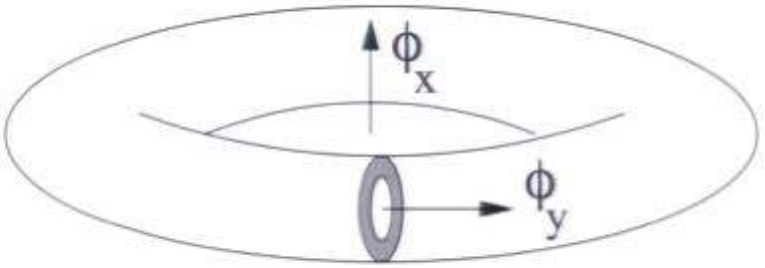

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Conclusions

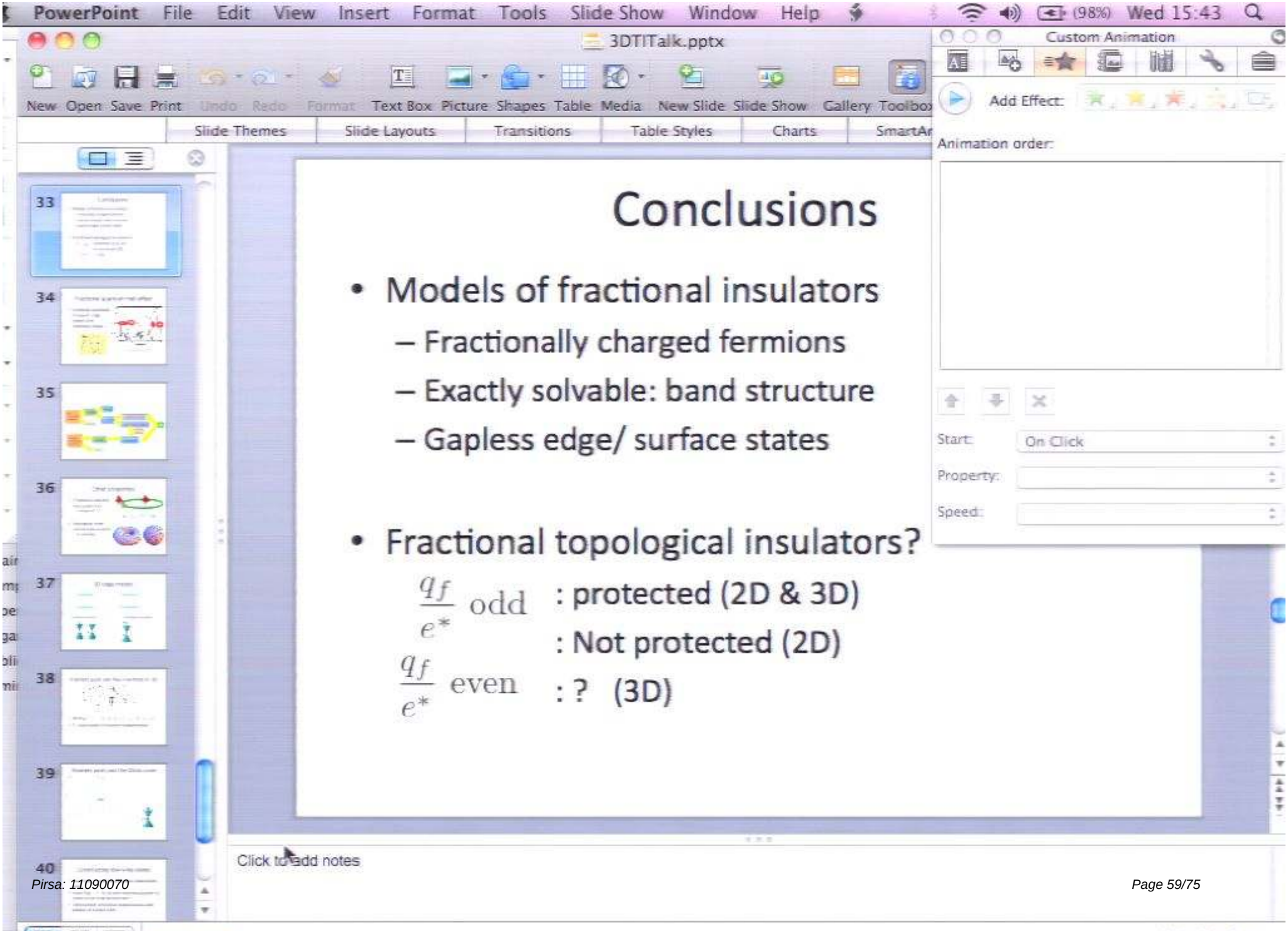
- Models of fractional insulators
  - Fractionally charged fermions
  - Exactly solvable: band structure
  - Gapless edge/ surface states

- Fractional topological insulators?

$\frac{q_f}{e^*}$ odd : protected (2D & 3D)

: Not protected (2D)

$\frac{q_f}{e^*}$ even : ? (3D)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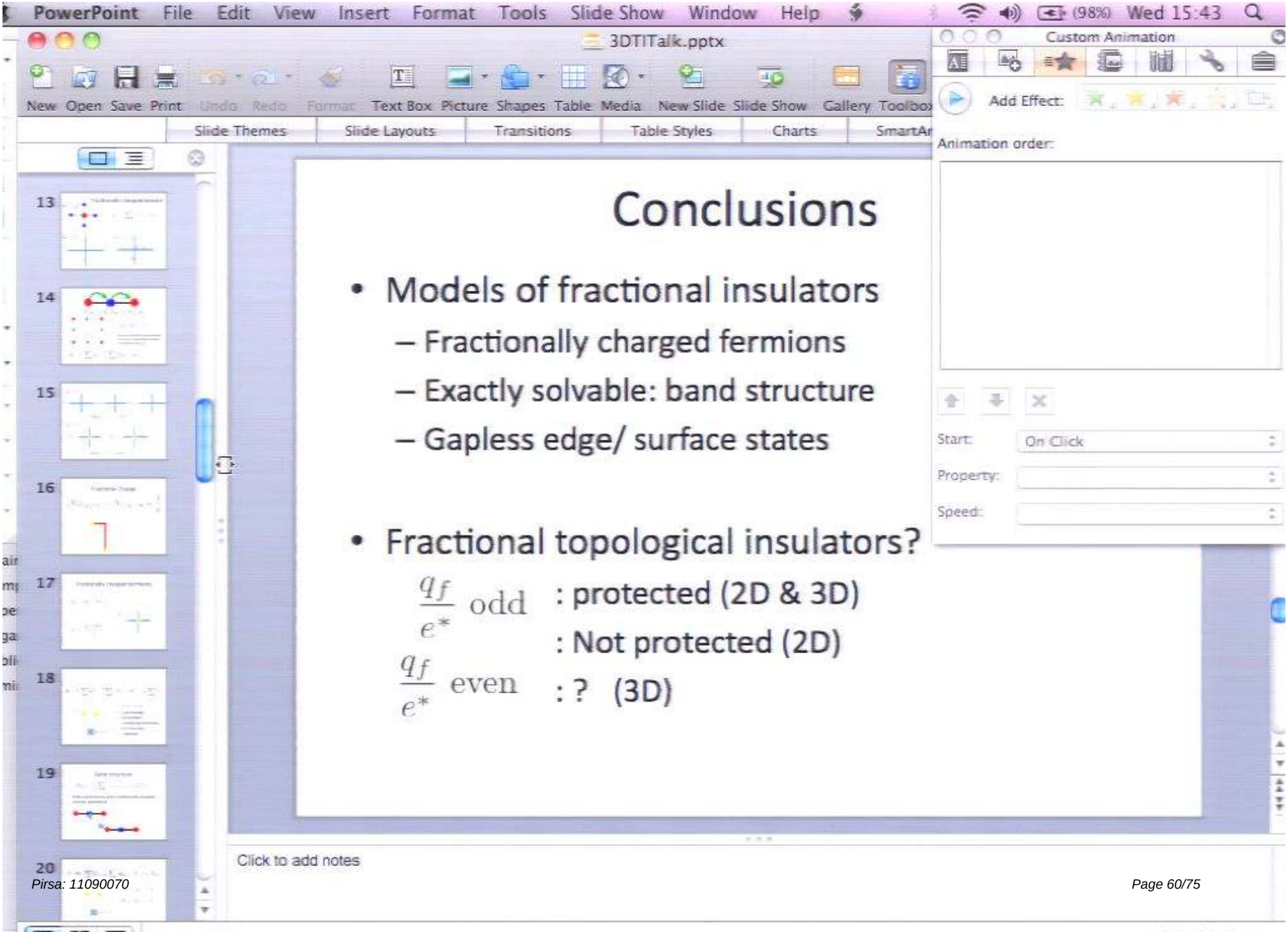
Click to add notes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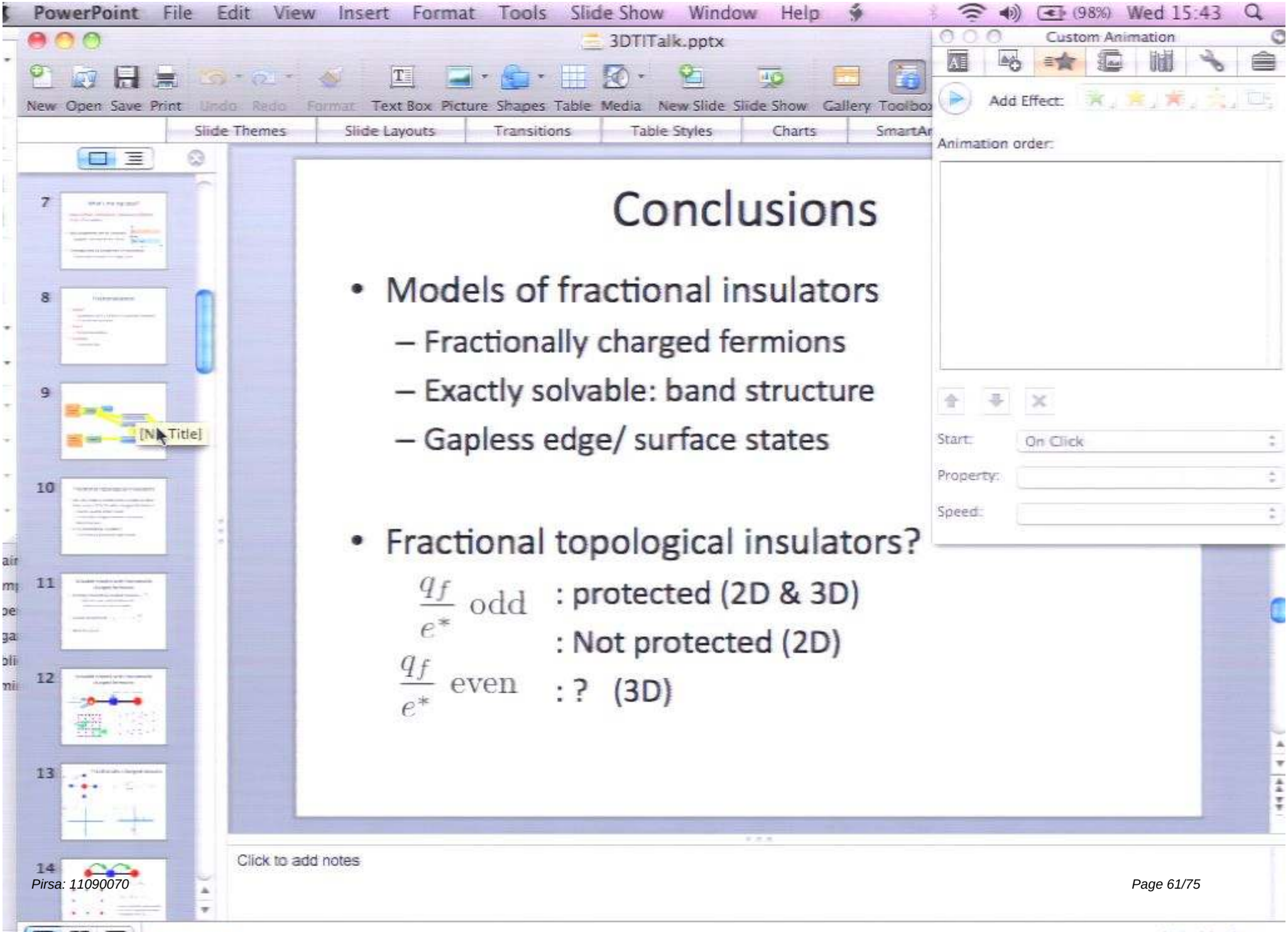
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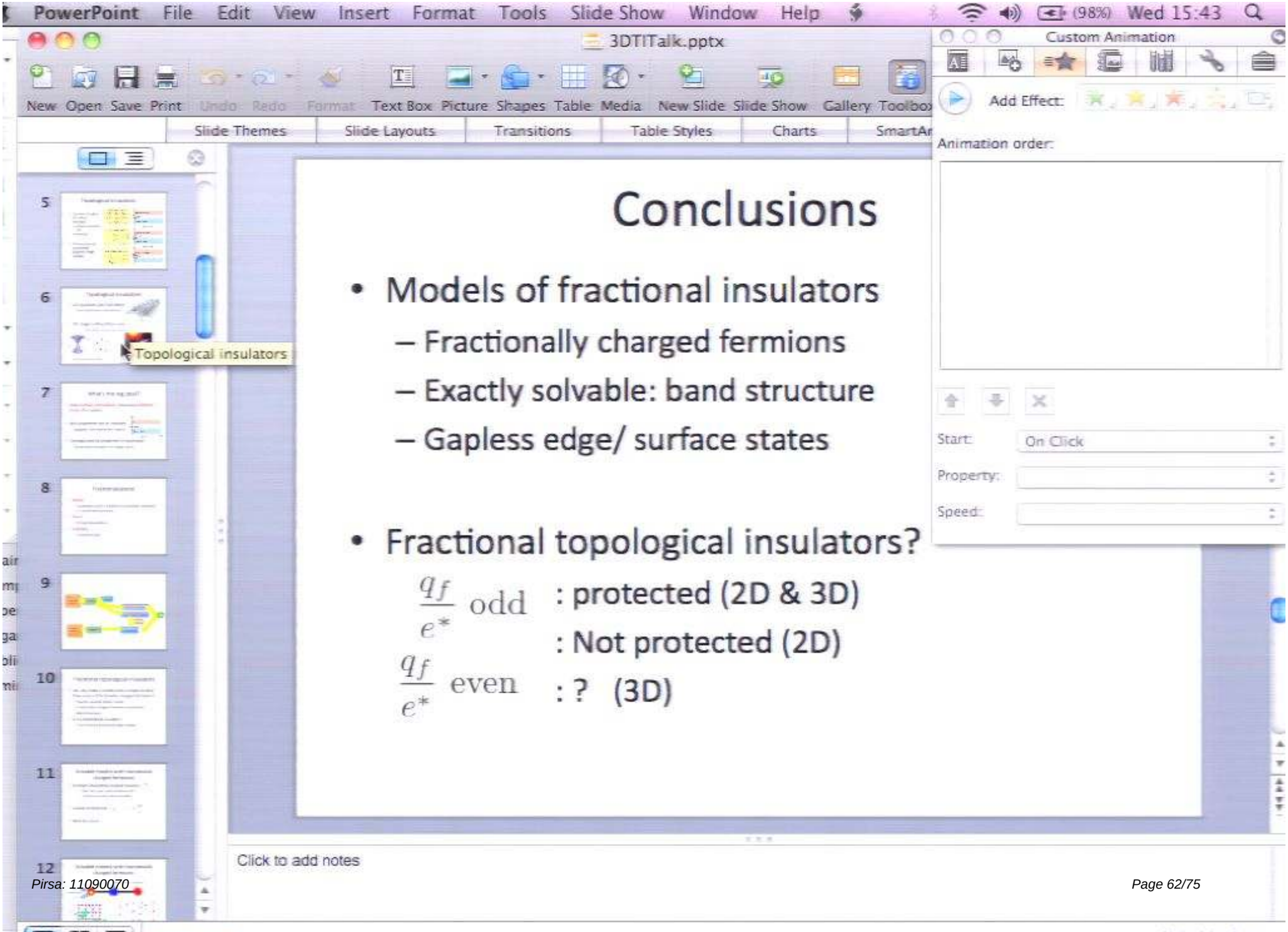
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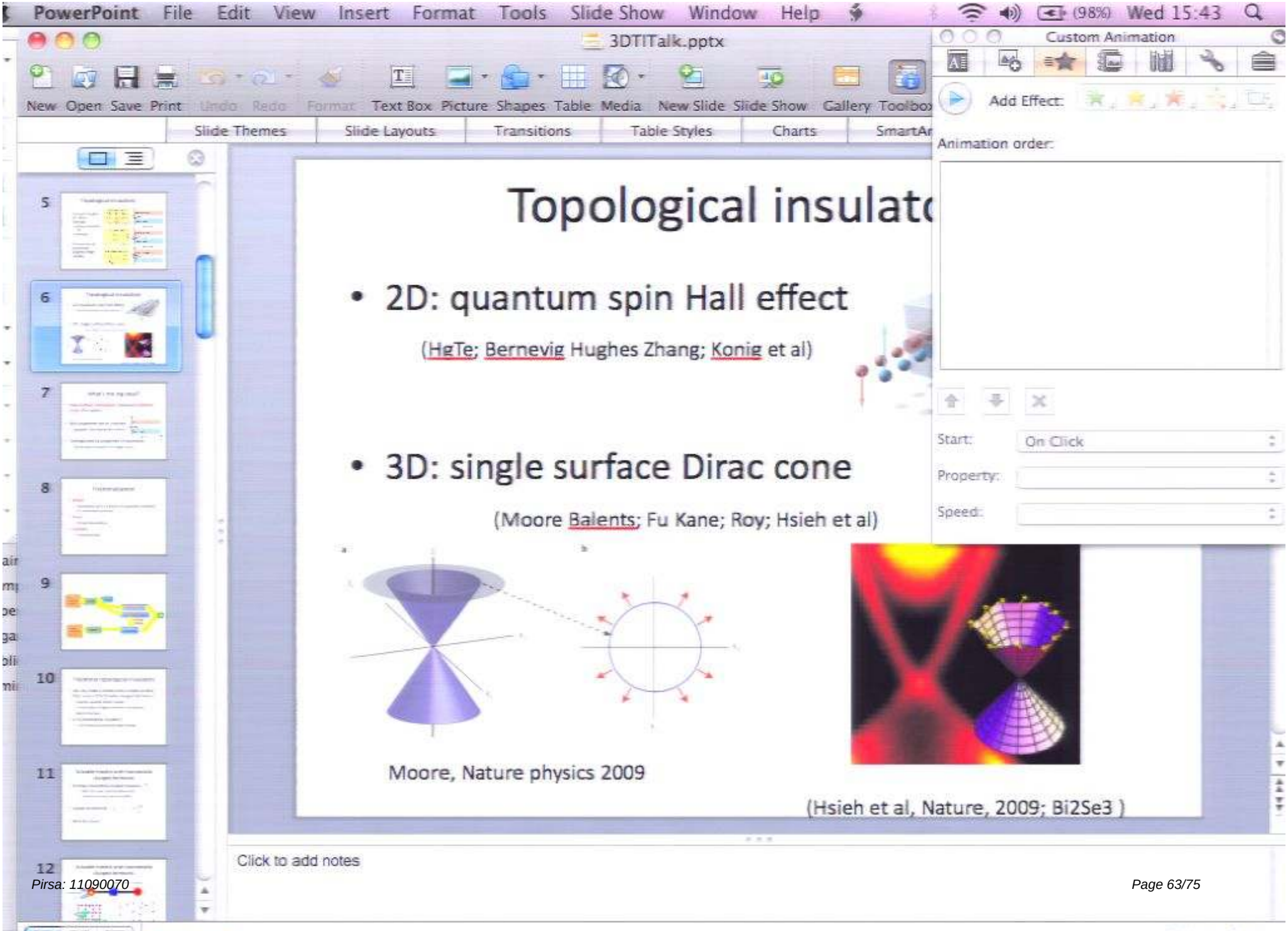
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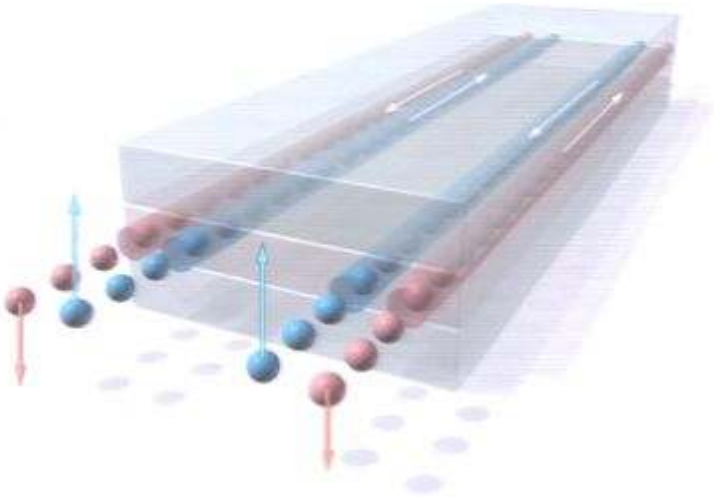
Topological insulato
• 2D: quantum spin Hall effect
(HgTe; Bernevig Hughes Zhang; Konig et al)
• 3D: single surface Dirac cone
(Moore Balents; Fu Kane; Roy; Hsieh et al)
Moore, Nature physics 2009
(Hsieh et al, Nature, 2009; Bi2Se3)
Click to add notes

Topological insulators

- 2D: quantum spin Hall effect

  (HgTe; Bernevig Hughes Zhang; Konig et al)



- 3D: single surface Dirac cone

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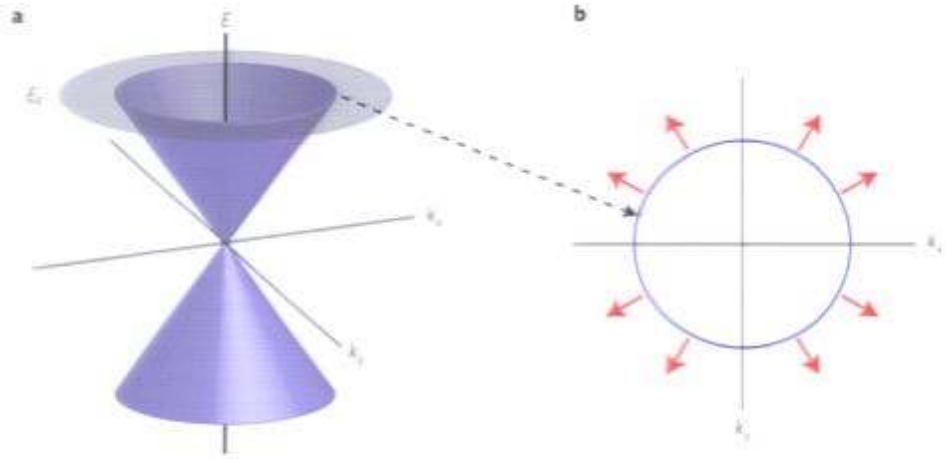
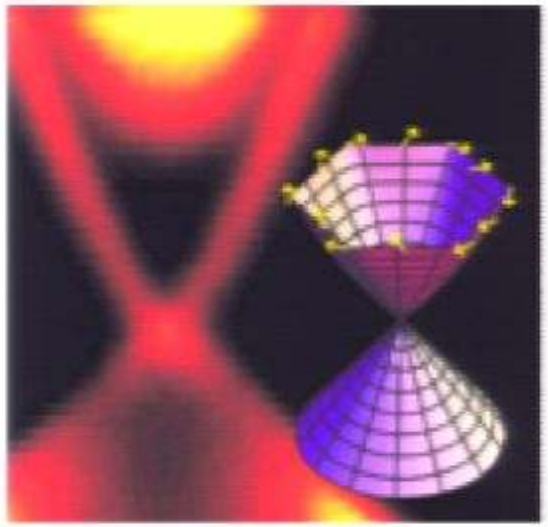
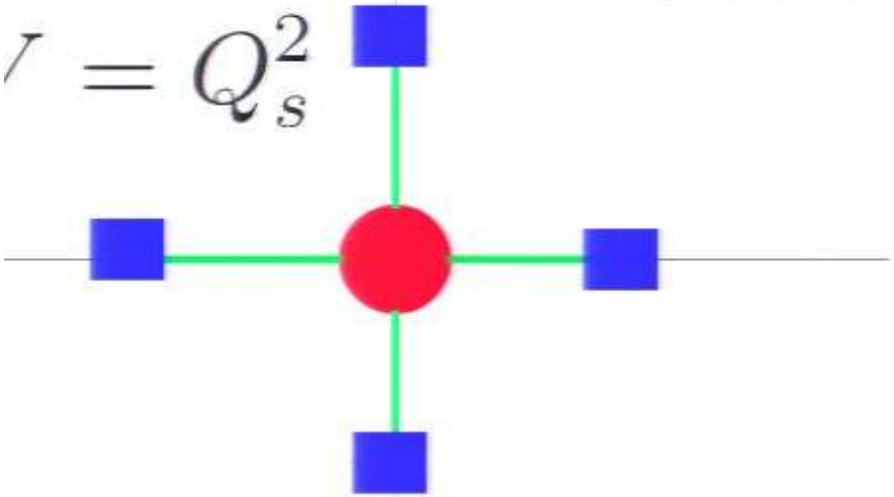




Moore, Nature physics 2009

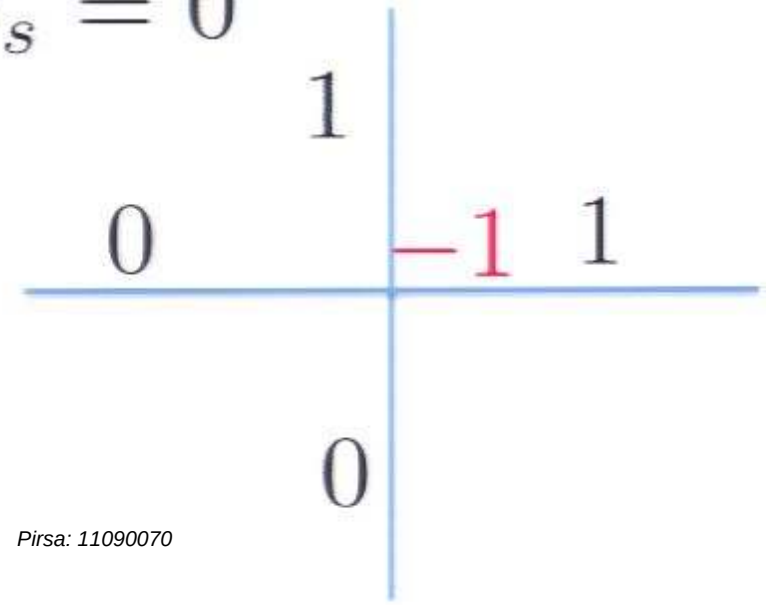
Fractionally charged bosons

$\gamma = Q_s^2$

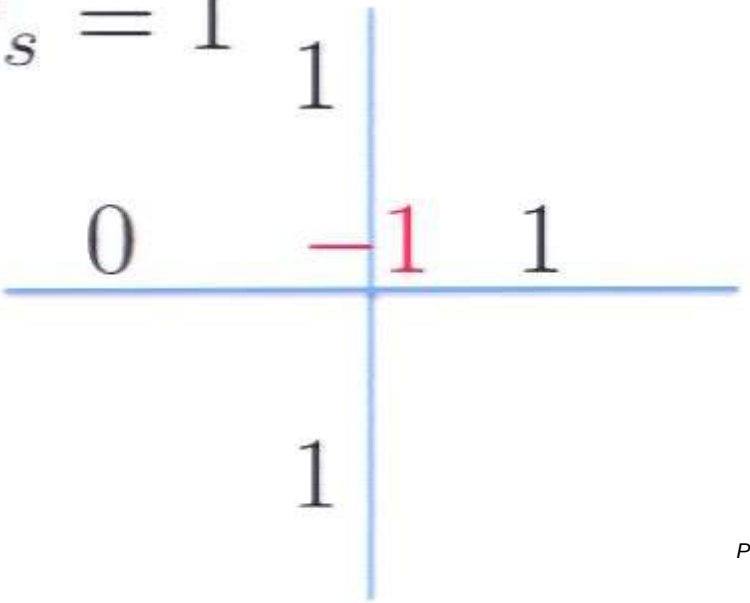


$$Q_s = \sum_{s'} n_{ss'} + 2n_s$$

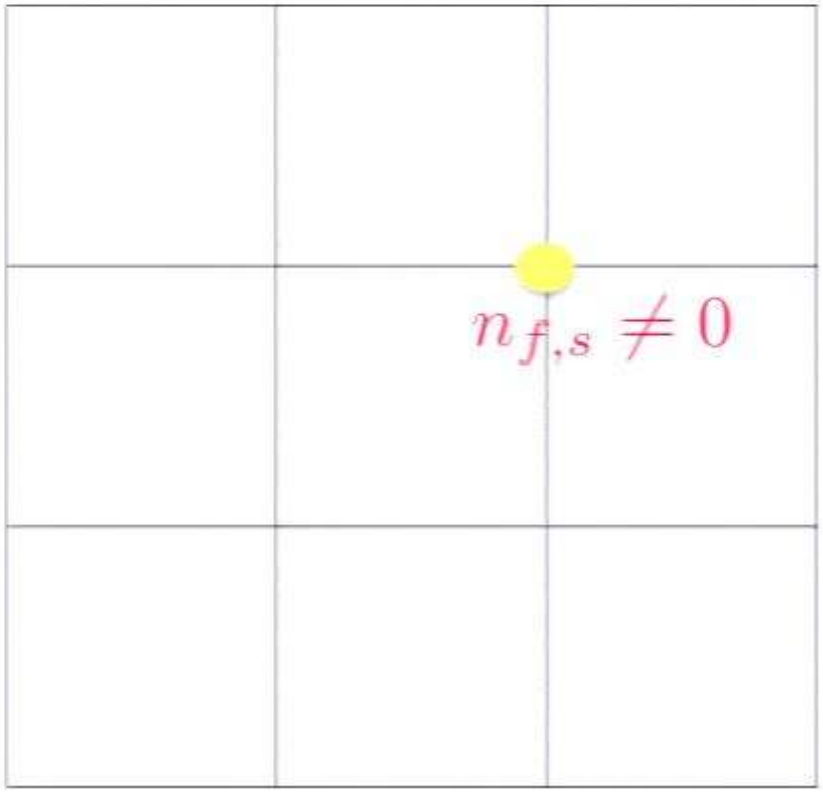
$Q_s = 0$



$Q_s = 1$



$$H = \sum t_{ss'\sigma\sigma'} d^{\dagger}_{s'\sigma'} d_{s\sigma}$$



$$[H_{kin}, \tilde{Q}_s] = 0$$

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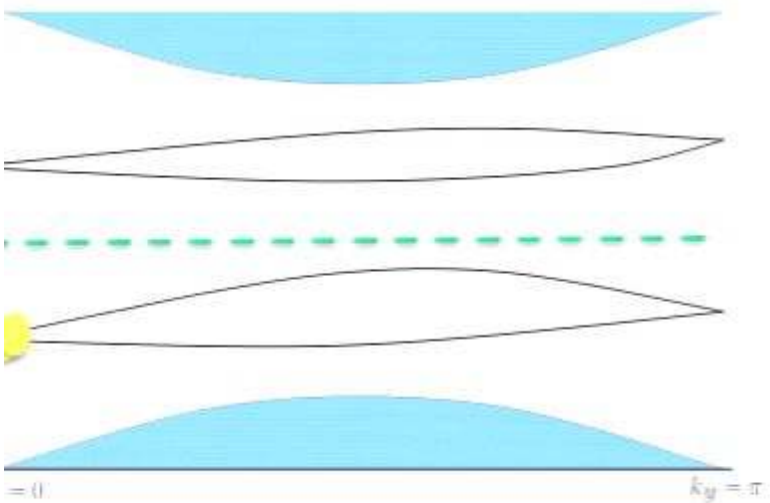
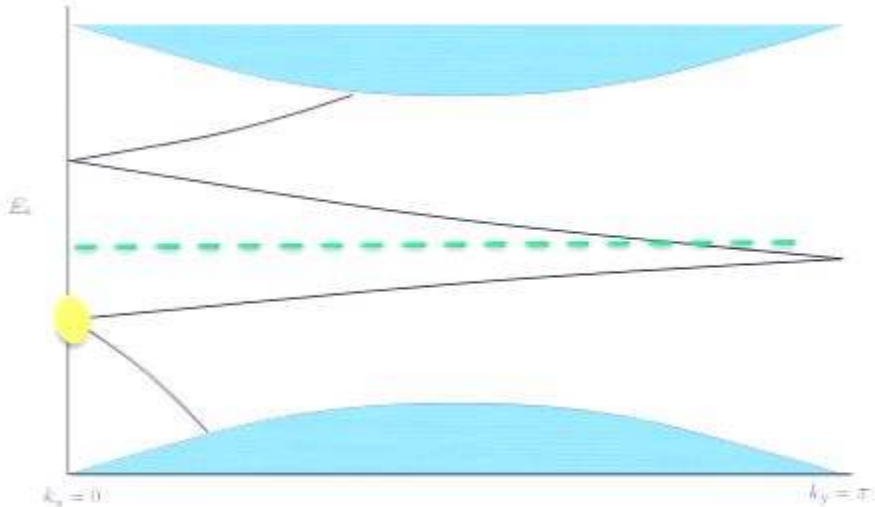
$$\Phi = \pi$$

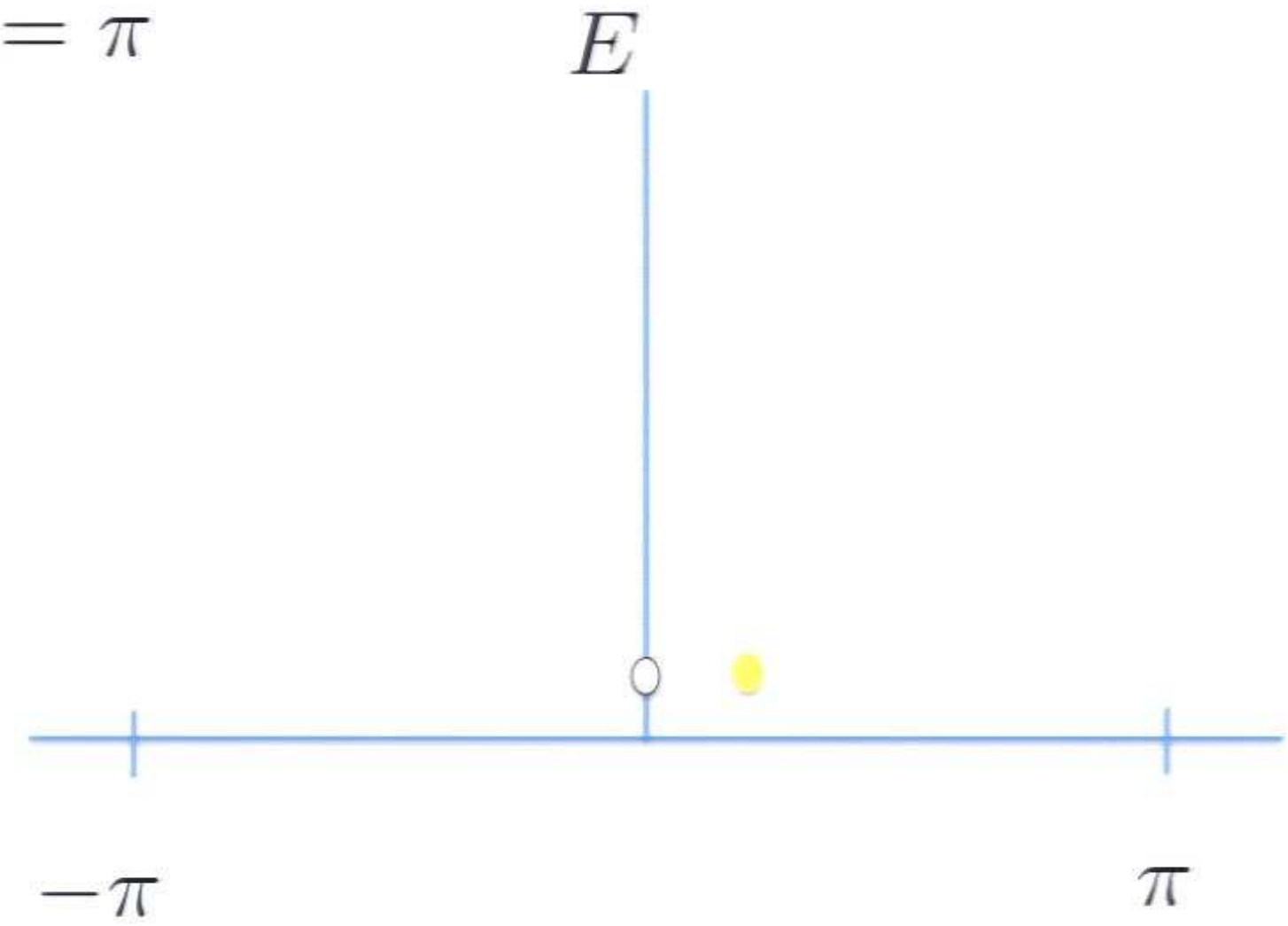


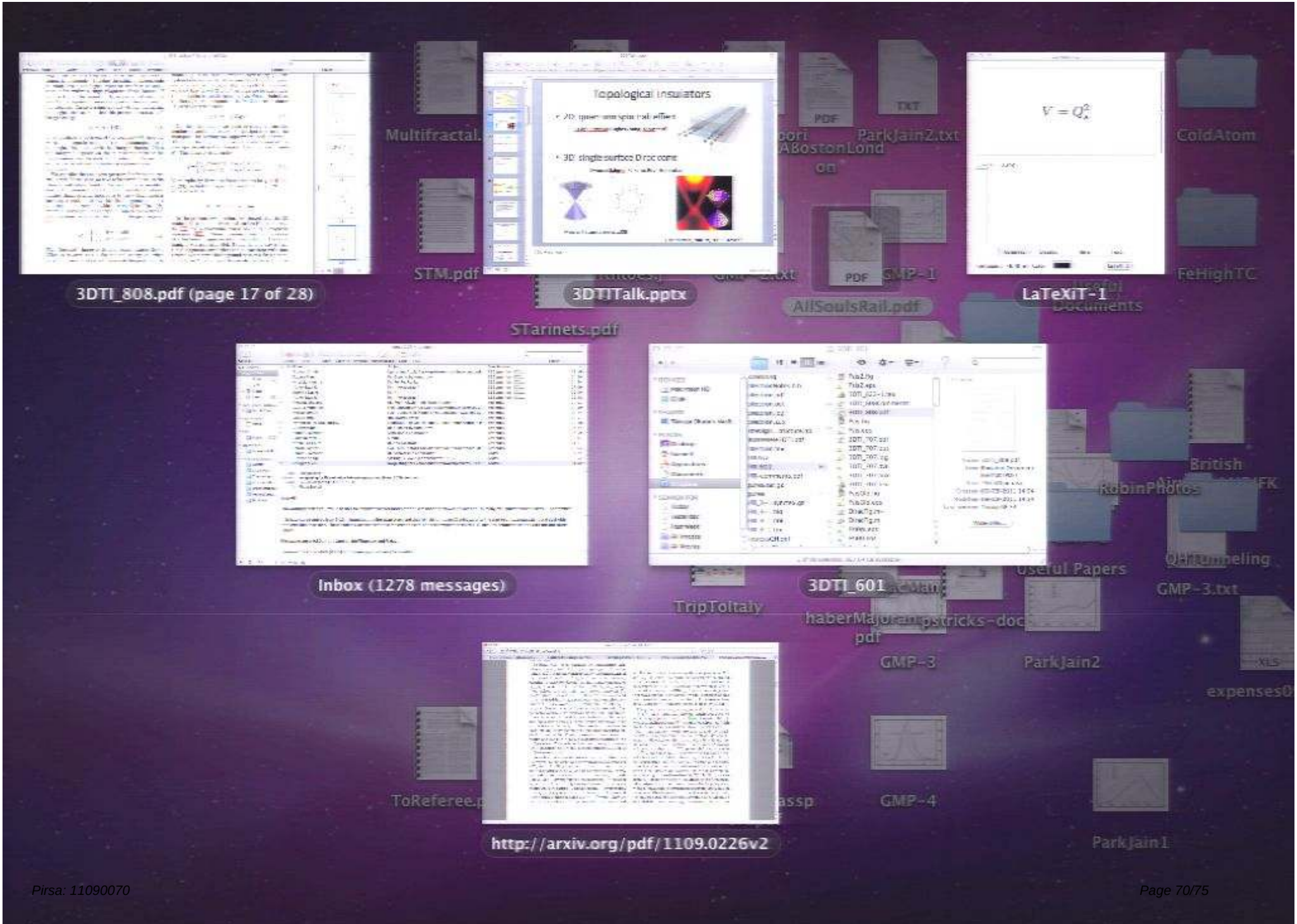


(Fu and Kane '06)

Constructing low-lying excited states

$\Phi = \pi$




3DTI_808.pdf (page 17 of 28)
Topological Insulators
3DTITalk.pptx
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http://arxiv.org/pdf/1109.0226v2
Multifractal
STM.pdf
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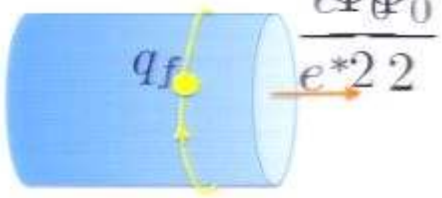
Flux insertion in fractional insulators

Levin and Stern '09

- Inserting $\frac{\Phi_0}{2}$: a different ground state



$\frac{e\Phi\Phi_0}{2}$

$\frac{q_f}{e^*}$ odd: Kramers $\rightarrow$ No Kramers

$\frac{q_f}{e^*}$ even: No change

$$\frac{q_f}{e^*} = \begin{cases} p + 2k & (p \text{ odd}) \\ \frac{1}{2}(p + 2k) & (p \text{ even}) \end{cases}$$

30
31
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34 Conclusions
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36
37

Flux insertion in fractional insulators

Levin and Stern '09

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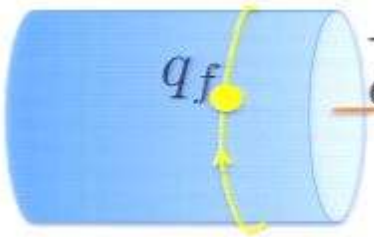

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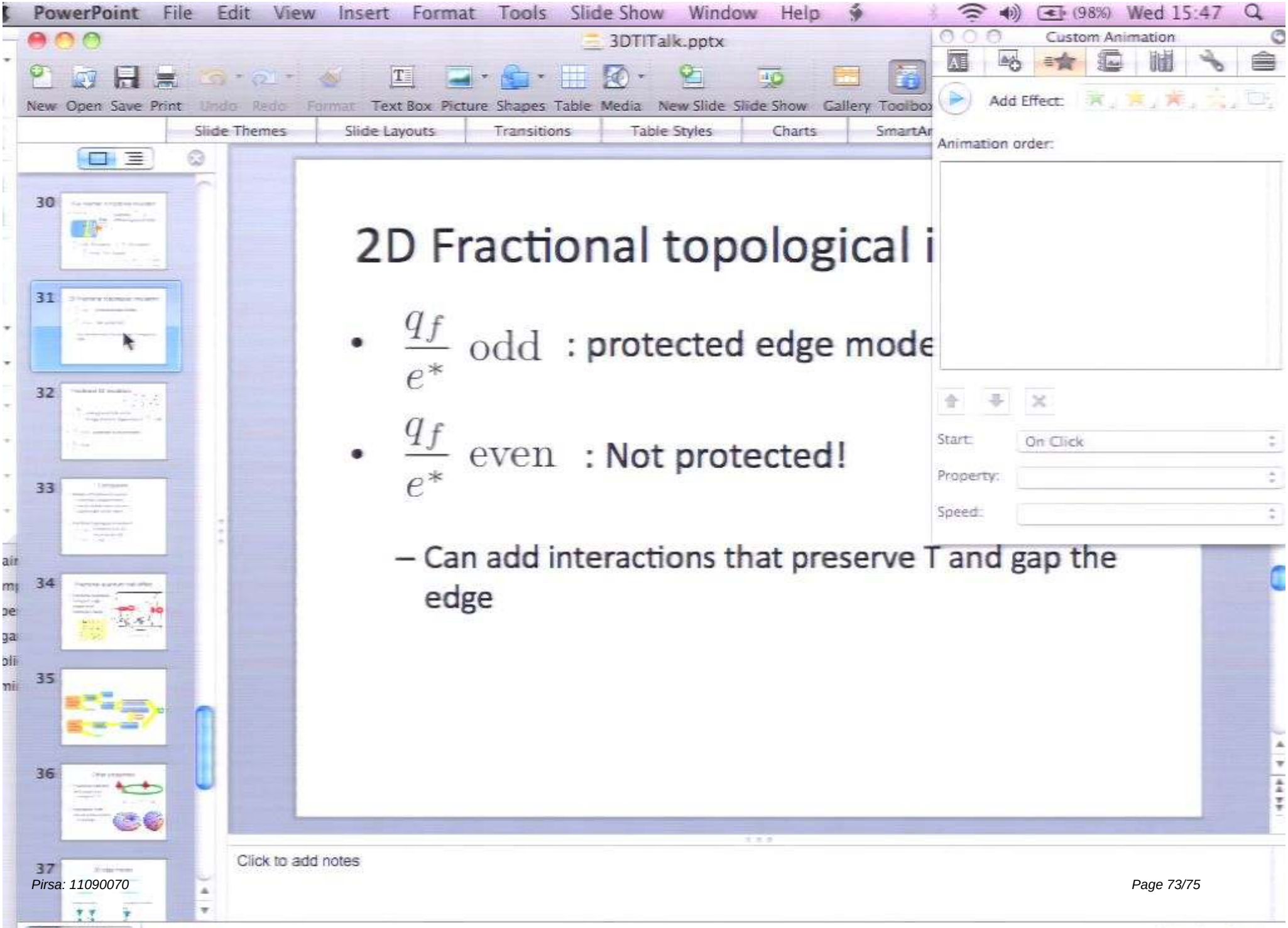
Click to add notes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New Open Save Print Undo Redo Format Text Box Picture Shapes Table Media New Slide Slide Show Gallery Toolbox
Slide Themes Slide Layouts Transitions Table Styles Charts
Custom Animation
Add Effect:
Animation order:
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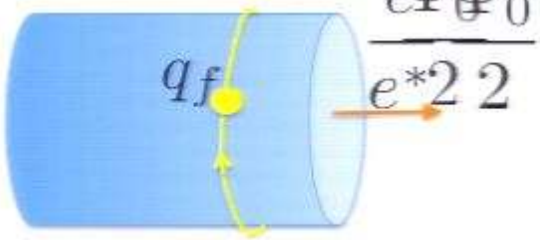
Speed:
2D Fractional topological i
$\frac{q_f}{e^*}$ odd : protected edge mode
$\frac{q_f}{e^*}$ even : Not protected!
– Can add interactions that preserve T and gap the edge
Click to add notes

Flux insertion in fractional i

Levin and Stern '09

- Inserting
different gr



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Click to add notes

Flux insertion in fractional insulators

Levin and Stern '09

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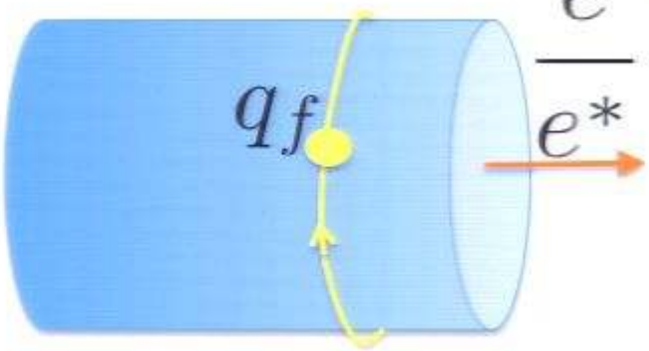

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