Title: Cosmology Meets Quantum Gravity: a Chiral Signature in the CMB?

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Abstract: Cosmology and quantum gravity have not always had the smoothest of interactions. As a case in point I'll summarize the calculation behind the prediction of tensor modes in inflationary universes and discuss the difficulties found in recasting this calculation in terms of Ashtekar-Barbero-Immirzi variables. Contrary to the belief that ``inflation is shielded from quantum gravity", novelties are found, leading to the interesting prediction of a chiral signature in the gravitational wave background, proportional to the imaginary part of the Immirzi parameter. This would leave a distinctive imprint in the polarization of the cosmic microwave background. On a more theoretical level, our remarks shed light on matters permeating quantum gravity, such as the inner product, the ground state (which we prove is NOT the Kodama state) and ordering issues.

Could Quantum Gravity leave a chiral imprint in the Universe?

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Based on:

- **arXiv: 1108.0816, (Bethke, JM)**
- **PRD 84: 024014, 2011 (Bethke, JM)**
- PRL **106: 121302, 2011** (JM, Benincasa)
- **PRL 101: 141101,2008** (Contaldi, JM, Smolin)

Quantum gravity and cosmology: a happy family

The bad side of both families

- Cosmologists can be embarrassingly naïve (e.g. their obsession with scalar fields)
- Within the quantum gravity community (but not only), attempting to make contact with reality is sometimes considered to be "in bad taste".

And yet, a marriage (of convenience?) is highly advisable

- It's unlikely (but possible) that quantum gravity can be tested in any other way
- Even theoretically:
  - Cosmology brings focus to QG calculations
  - QG (or equivalent) is a requirement to put cosmological calculations on safe grounds.

Worries of modern cosmology

- The trans-Planckian menace…
- Do we really know the vacuum state?
- The perception that "inflation is insulated from quantum gravity" is merely a dogma, or at best "wishful thinking"

A moment of historical reflexion

- Inflation was never meant to stand alone.
- Historically, quantum cosmology was expected to fill in the gaps.
- Somehow the ADM canonical quantization program failed to deliver the goods.
- Inflation was left to fend for itself…

A summary of Ashtekar quantum gravity for cosmologists, I

- Take the Palatini-Kibble formulation of GR:

$$e^J \qquad \Gamma^I_J$$

1. Trade in the metric for four 1form tetrad fields (the two are equivalent up to degenerate metrics)

2. Allow the connection (represented by six 1forms) to start off an independent variables (becoming equivalent to the Christoffel connection as a result of a field equation in the absence of torsion/spinors)

A summary of Ashtekar quantum gravity for cosmologists, II

- Add a topological, or surface term to the action:

$$S = -\frac{1}{2l_P^2}\int \Sigma^{IJ} \wedge \left(F_{IJ} + \frac{1}{\gamma}{}^*F_{IJ}\right)$$

where $\Sigma^{IJ} = e^I \wedge e^J$

- Here γ is the Immirzi parameter, which can be complex → the need for reality conditions.

(For $\gamma = \pm i$ the connection becomes self-dual/ anti-self-dual: the Ashtekar connection)

So what?

- Classically: so nothing! Indeed.
- Quantum mechanically the story is very different! Examples:

Instanton effects (Mercury **arXiv:1007.3732**, Mercury, Randono **arXiv:1005.1291**)

Topological interpretation (Alexander, Calcagni, **arXiv: 0806.4382, 0807.0225**, Date, Kaul, Sengupta **Phys.Rev. D79 (2009) 044008)**

Or equivalently, in the Hamiltonian formulation

- Take bog standard Palatini theory in Hamiltonian framework:

$$\{K_a^i(\mathbf{x}), E_j^b(\mathbf{y})\} = l_p^2 \delta_a^b \delta_j^i \delta(\mathbf{x} - \mathbf{y})$$

- Apply a canonical transformation

$$A^i = \Gamma^i + \gamma K^i = \Gamma^i + \gamma \Gamma^{0i}$$

$$\Gamma^i = -\frac{1}{2}\epsilon^{ijk}\Gamma^{jk}$$

- And indeed:

$$\{A_a^i(\mathbf{x}), E_j^b(\mathbf{y})\} = \gamma l_P^2 \delta_a^b \delta_j^i \delta(\mathbf{x} - \mathbf{y})$$

So what?

- Classically: nothing! QM: everything!
- Let the connection drive the quantization
- A particular solution has been found in deSitter: the Kodama state.

$$\Phi = \mathcal{N} \exp\left(\frac{i\gamma}{2l_P^2 H^2} S_{CS}\right)$$

Is this the wave function of the Universe during inflation?

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This is somewhat old-fashioned but…

- In fact it's the holonomies of the connection (Wilson loops) that drive quantization, but…
- "Some recent developments in quantum gravity is just sweeping the problems under the carpet" (private communication…)

A 5 minute crash course in fluctuations

Fluctuations can be:

- Scalar (density fluctuations; gradient flows)
- Vector (flows with vorticity)
- Tensors (gravitational waves)

Use tensor fluctuations in de Sitter as a test case for QG:

TT

Metric: $$ds^2 = a^2[-d\eta^2 + (\delta_{ab} + h_{ab})dx^a dx^b]$$

With expansion factor $$a = -\frac{1}{H\eta}$$ $$H^2 = \Lambda/3$$ $$\eta < 0$$

Perturb Einstein equations.
Translate into cosmology speak be means of variable:

$$v(\mathbf{k}, \eta) = \sqrt{\frac{\epsilon^{ij}\bar{\epsilon}_{ij}}{32\pi G}}\, a h(\mathbf{k}, \eta)$$

Formally, we get the same equation for scalar and tensor fluctuations:

$$v'' + \left(k^2 - \frac{2}{\eta^2}\right)v = 0$$

I II

A harmonic oscillator with a variable(negative) mass. 2 regimes:

- Term I dominates: A regular free harmonic oscillator; Modes inside the horizon, dominated by microphysics.
- Term II dominates: An inverted harmonic oscillator; Jeans/gravitational instability, modes outside the horizon, dominated by expansion

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Follow up vacuum quantum fluctuations

- Consider first the regime $k|\eta| \gg 1$

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}$$

- With this normalization when we second quantize the amplitudes become creation /annihilation operators

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}} a$$

- A miracle happens near deSitter

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)a$$

- Compute the vacuum expectation value

$$< 0|\hat{v}^2|0 >= v^2 < 0|a^\dagger a + \frac{1}{2}|0 >$$

- In the limit $k|\eta| \ll 1$ we get:

$$< 0|\hat{v}^2|0 >\propto k^{-3}$$

This is very dodgy at the very least:

- What is the vacuum?
- Can we really second quantize metric fluctuations without full knowledge of quantum gravity?
- Is the calculation indifferent to the details of quantum gravity?

The punch line:

- The particle spectrum of gravitons is the same for right and left handed gravitons, but:
- their vacuum energy and vacuum fluctuations are not.

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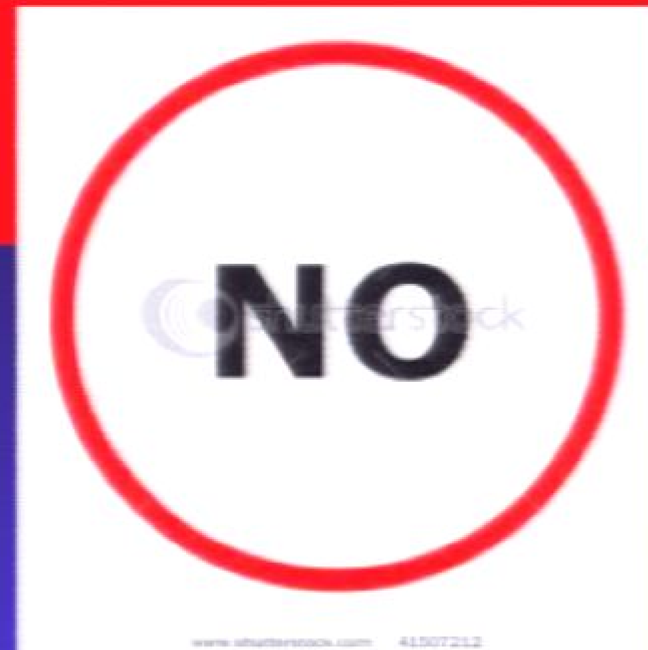

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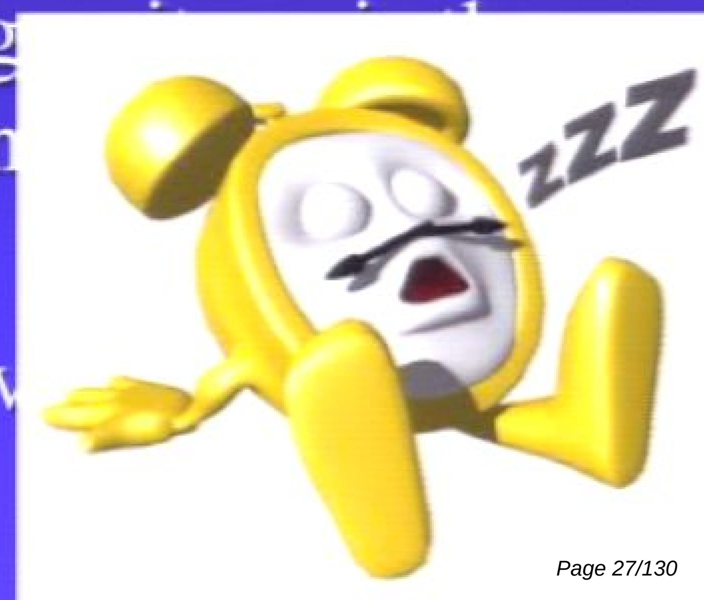

Not easy to recover Cosm. Pert. Th. within Ashtekar's formalism

- Even in usual Einstein-Hilbert formalism transitioning to Hamiltonians is non-trivial

Eg. D. Langlois, Class. Quantum Grav. 11 (1994) 389407.

- The Ashtekar formalism is ideal for non-perturbative work, cumbersome for perturbative work.
- But it's an important check!

It exposes past "misunderstandings":

- Reality conditions constrain graviton states ✗
- Spurious pump terms ✗
- Helicity aligns with duality ✗
- The Kodama state can be used as the base (ground?) state of quantum gravity ✗

A. Ashtekar, C. Rovelli and L. Smolin, Phys. Rev. D44, 1740, 1991.
L. Freidel and L. Smolin, Class. Quant. Grav. 21: 3831-3844, 2004. L. Smolin, hep-th/0209079.
E. Witten, "A Note on the Chern-Simons and Kodama wave functions", gr-qc/0306083.

First of all expand correctly:

$$A_a^i = \gamma H a \delta_a^i + \frac{a_a^i}{a}$$

$$E_i^a \;=\; a^2 \delta_i^a - a \delta e_i^a$$

- Include positive and negative frequencies
- Make sure they are independent before reality conditions are imposed: a graviton and an anti-graviton off-shell
- Identify physical direction of motion and polarization correctly

Not nice, but right…

$$\delta e_{ij} = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \sum_r \epsilon^r_{ij}(\mathbf{k})\tilde{\Psi}_e(\mathbf{k},\eta)e_{r+}(\mathbf{k})$$
$$+\epsilon^{r\star}_{ij}(\mathbf{k})\tilde{\Psi}^\star_e(\mathbf{k},\eta)e^\dagger_{r-}(\mathbf{k})$$
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$$\tilde{\Psi}(\mathbf{k},\eta) = \Psi(k,\eta)e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\Psi(k,\eta) \sim e^{-ik\eta}$$

The reality conditions applied to these expansions:

- In contrast to previous work, they never relate different graviton states.
- They merely equate graviton and anti-graviton metric modes:

$$e_{r+}(\mathbf{k}) = e_{r-}(\mathbf{k})$$

- For the connection:

$$\begin{aligned} \tilde{a}_{r+}(\mathbf{k},\eta) + \tilde{a}_{r-}(\mathbf{k},\eta) &= 2rk\tilde{e}_{r+}(\mathbf{k},\eta) \\ \tilde{a}^{\dagger}_{r+}(\mathbf{k},\eta) + \tilde{a}^{\dagger}_{r-}(\mathbf{k},\eta) &= 2rk\tilde{e}^{\dagger}_{r-}(\mathbf{k},\eta) \end{aligned}$$

Solving Lagrange equations we find that duality and helicity don't align:

$k|\eta| \gg 1$

$$\Psi_a^{rp} = (r - ip\gamma)k\Psi_e \; .$$

	$r = +$ [R]	$r = -$ [L]
$p = +$ $[G]$	SD	ASD
$p = -$ $[\overline{G}]$	ASD	SD

A. Ashtekar, J. Math.Phys. 27, 824, 1986.

- SD has the right graviton positive frequency and the left anti-graviton negative frequency (contrast with $E \pm iB$ in Yang-Mills)

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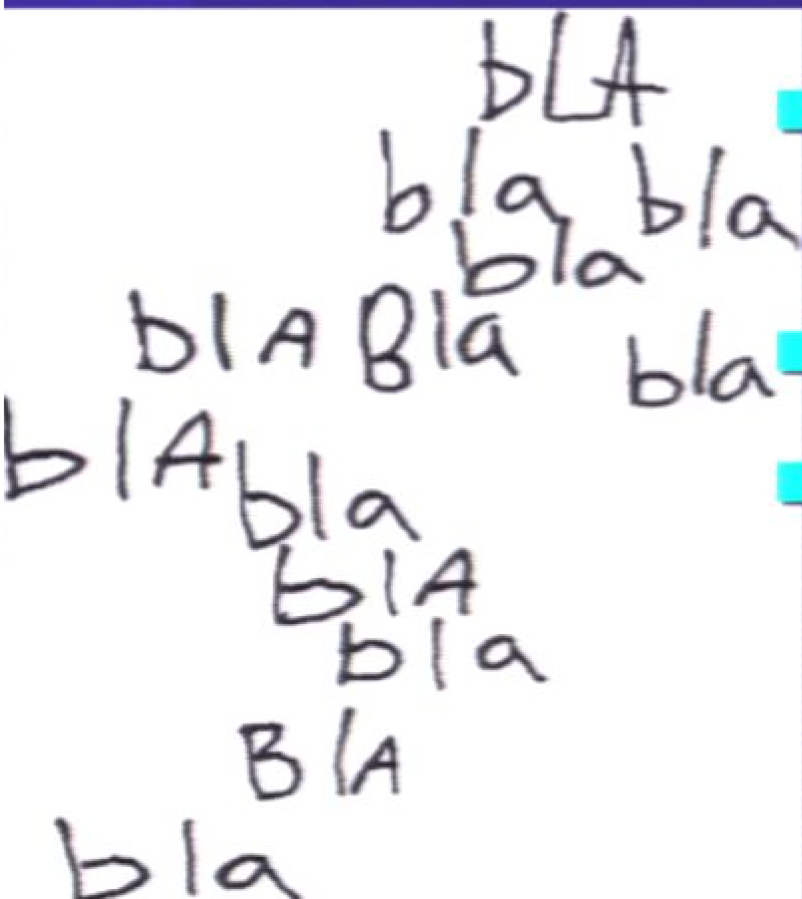
Classically we should recover Cosm. Pert. Theory within QG formalism:



- "It has all been done before…"
- "An exercise for the student"
- If not done carefully the canonical formalism actually fails the test!!!!!!!!!!!!

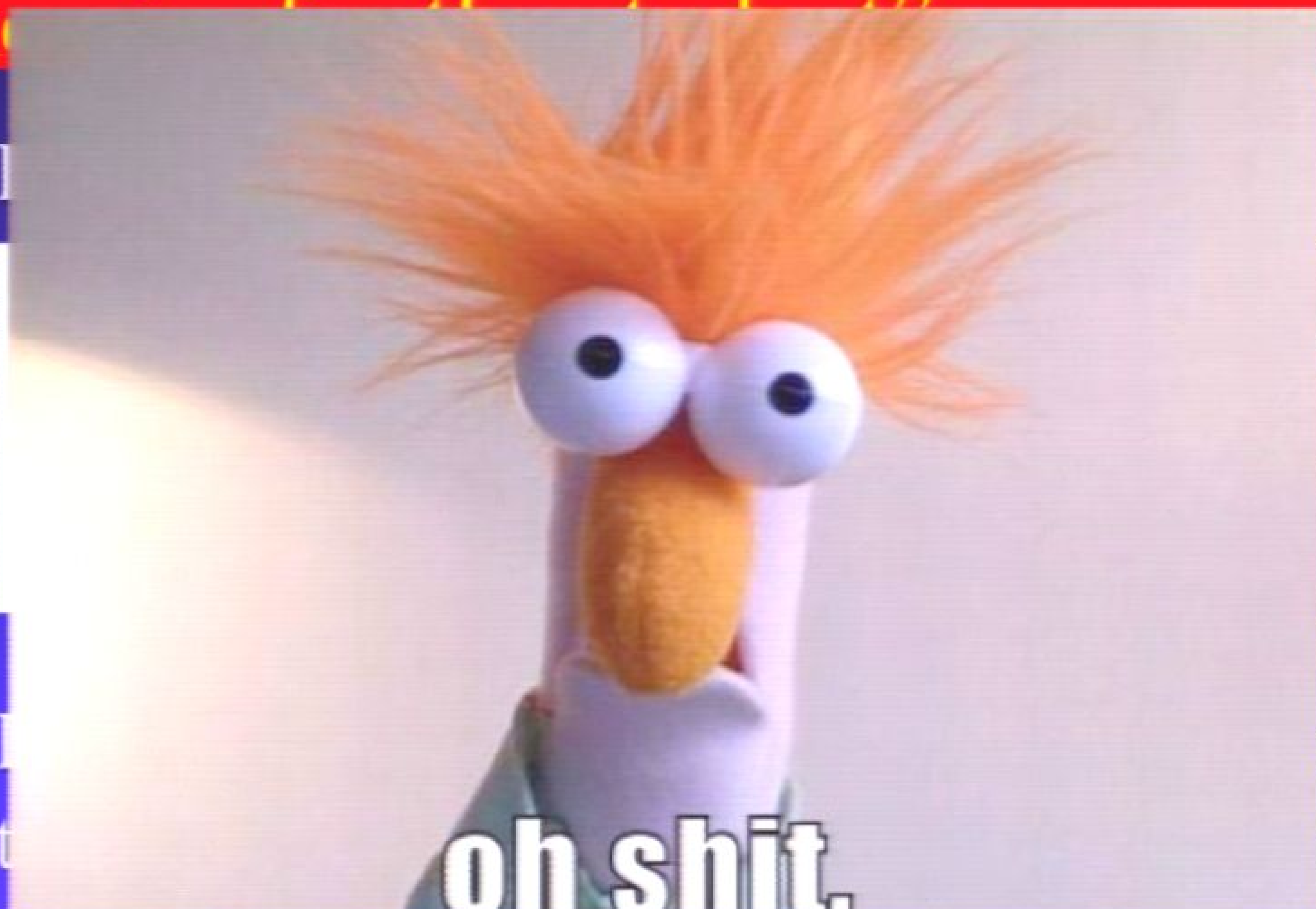
Please perform the following "exercise for the student":

- Insert expansions into the Hamiltonian

$$\mathcal{H} = \frac{1}{2l_P^2}\int d^3x N E_i^a E_j^b \Big[\epsilon_{ijk}(F_{ab}^k + H^2 \epsilon_{abc} E_k^c) -2(1+\gamma^2) K_{[a}^i K_{b]}^j\Big]$$

- Find to your horror that it doesn't reduce to the Hamiltonian of our beloved "v" variable

Please perform the following “ex



ce to

iable

This is due to a few simple subtleties:

- GR's boundary term: ignore it at your peril
- Perturbing is really a time dependent canonical transformation:

$$A_a^i = \gamma H a \delta_a^i + \frac{a_a^i}{a}$$

$$E_i^a = a^2 \delta_i^a - a \delta e_i^a$$

$$\mathcal{K} = \mathcal{H} + \frac{\partial F}{\partial \eta}$$

* Etc… (it's good to have PhD students.)

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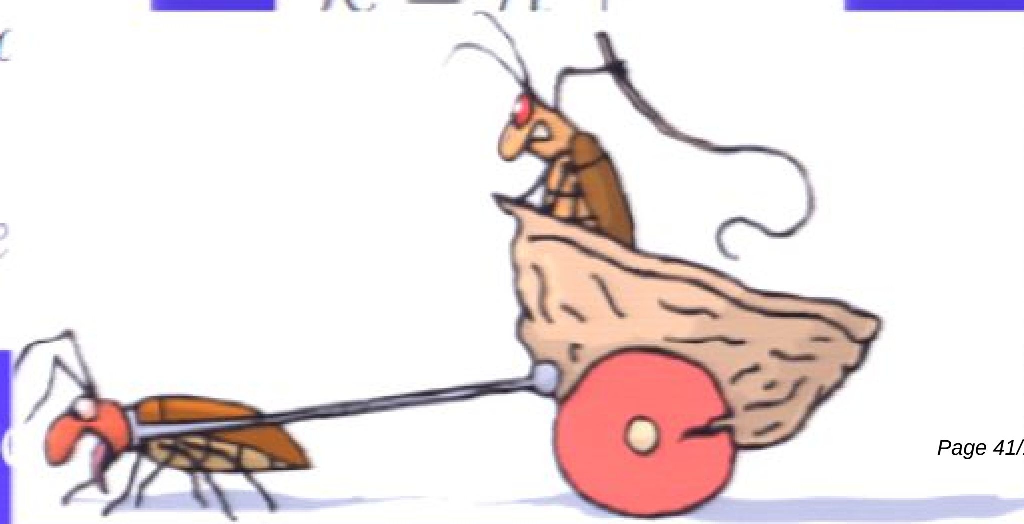

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After much labour we do recover cosmology classically. Eg, (A)SD:

$$\mathcal{H}_{eff} = \frac{1}{2l_P^2}\int d^3x[-a_{ij}a_{ij} - 2\epsilon_{ijk}(\partial_j\delta e_{li})a_{kl} - 2H^2a^2\delta e_{ij}\delta e_{ij}]$$

$$a'_{ij} = 2\gamma H^2 a^2 \delta e_{ij} - \gamma\epsilon_{inm}\partial_n a_{mj}$$

$$\delta e'_{ij} = -\gamma(a_{ij} - \epsilon_{inm}\partial_n \delta e_{mj}) \, .$$

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$$\delta e''_{ij} - \left(\partial^2 + \frac{2}{\eta^2}\right)\delta e_{ij} = 0$$

It all goes through, except that…

- The Hamiltonian constraint doesn't apply to the perturbations. It is enforced by the second order back reaction. A Russian doll kind of trick:

$$\mathcal{H} \approx 0$$

$$^2\mathcal{H} = {}^2_1\mathcal{H} + {}^2_2\mathcal{H}$$

My CAN50c contribution to the problem of time in quantum gravity:

- Time is an illusion of perturbation theory, but that's the set up where we live, so don't knock it.

- [Could also say that the problem of time is an illusion of the non-perturbative theory]

Sure, but WHAT is new?

Quantization is different: a sort of uncertainty relations between metric and connection

$$\{A_a^i(\mathbf{x}), E_j^b(\mathbf{y})\} = \gamma l_P^2 \delta_a^b \delta_j^i \delta(\mathbf{x}-\mathbf{y})$$

It begs the question: then, what's the graviton made of?

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Quantization is different: a sort of uncertainty relations between metric and connection

$$[\tilde{a}_{rp}(\mathbf{k}), \tilde{e}^{\dagger}_{sq}(\mathbf{k}')] = -i\gamma p \frac{l_P^2}{2} \delta_{rs}\delta_{p\bar{q}}\delta(\mathbf{k}-\mathbf{k}') \, .$$

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The Hamiltonian reveals special graviton operators (for SD/ASD)

$$\mathcal{H}_{eff} = \frac{1}{l_P^2}\int d^3k \sum_r g_{r-}(\mathbf{k})g_{r+}(-\mathbf{k}) + g_{r-}(\mathbf{k})g^\dagger_{r-}(\mathbf{k})$$
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They inherit a funny algebra:

- Something is wrong with half the modes:

$$[g_{rp}(\mathbf{k}), g^{\dagger}_{sq}(\mathbf{k}')] = -i\gamma l_P^2 (pr) k \delta_{rs}\delta_{pq}\delta(\mathbf{k}-\mathbf{k}')$$

- These are the modes that don't exist classically (e.g. for SD connection, the R anti-graviton and L graviton)
- Upon identifying the inner product, they turn out to be non-normalizable: non-physical

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Can be seen using the Bargman (or holomorphic) representation:

$$[g_{rp}(\mathbf{k}), g^{\dagger}_{sq}(\mathbf{k}')] = -i\gamma l_P^2 (pr) k \delta_{rs}\delta_{pq}\delta(\mathbf{k}-\mathbf{k}')$$

Diagonalize the creation operator

$$g^{\dagger}_{rp}\Phi(z) = z_{rp}\Phi(z)$$

$$g_{rp}\Phi = -i\gamma l_P^2 (pr) k \frac{\partial \Phi}{\partial z_{rp}}$$

The reality conditions are second class constraints:

$$\tilde{a}_{r+}(\mathbf{k},\eta) + \tilde{a}_{r-}(\mathbf{k},\eta) \;=\; 2rk\tilde{e}_{r+}(\mathbf{k},\eta)$$
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$$e_{r+}(\mathbf{k}) = e_{r-}(\mathbf{k})$$

don't close under the Poisson bracket algebra

Solution 1: work out the Dirac bracket

Solution 2: Note that they amount to a definition of a formal definition of

$$g^{\dagger}_{rp} = (g_{rp})^{\dagger}$$

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$$e_{r+}(\mathbf{k}) = e_{r-}(\mathbf{k})$$

don't close under the Poisson bracket algebra

Solution 1: work out the Dirac bracket

Solution 2: Note that they amount to a definition of a formal definition of

$$g^{\dagger}_{rp} = (g_{rp})^{\dagger}$$

The inner product then implements the reality conditions:

With ansatz:

$$\langle \Phi_1 | \Phi_2 \rangle = \int dz d\bar{z} e^{\mu(z,\bar{z})} \bar{\Phi}_1(\bar{z}) \Phi_2(z)$$

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A familiar story:

- Worked with a larger space of states
- Pathological modes (negative energies) don't belong to the physical subspace: they are not normalizable
- Physical Hamiltonian becomes the familiar:

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- Eg: for the SD connection only the L graviton has a vacuum energy:

$$\mathcal{H}^{ph}_{eff} \approx \frac{1}{l_P^2} \int d\mathbf{k} \, (g_L^{ph} g_L^{ph\dagger} + g_R^{ph\dagger} g_R^{ph})$$

- This traces to the ordering prescription used in the Hamiltonian, here EEF (effect is erased with EFE or EEF+FEE)

Furthermore:

- It propagates into the vacuum two-point function, with similar chiral behaviour:

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\begin{aligned}
A_R^{ph}(\mathbf{k}) &= a_{R+}(\mathbf{k})e^{-ik\cdot x} = g_{R+}(\mathbf{k})e^{-ik\cdot x} \\
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For other values of the Barbero-Immirzi parameter:

- It's a mess… (see our paper)
- Again half the modes are unphysical. Others have the same spectrum as usual, but the vacuum energy and fluctuations acquire chirality. For EA ordering:

$$\frac{V_R - V_L}{V_R + V_L} = i\gamma$$

$$\frac{P_R - P_L}{P_R + P_L} = \frac{2i\gamma}{1-\gamma^2}$$

Quantum gravity does correct the inflationary calculation

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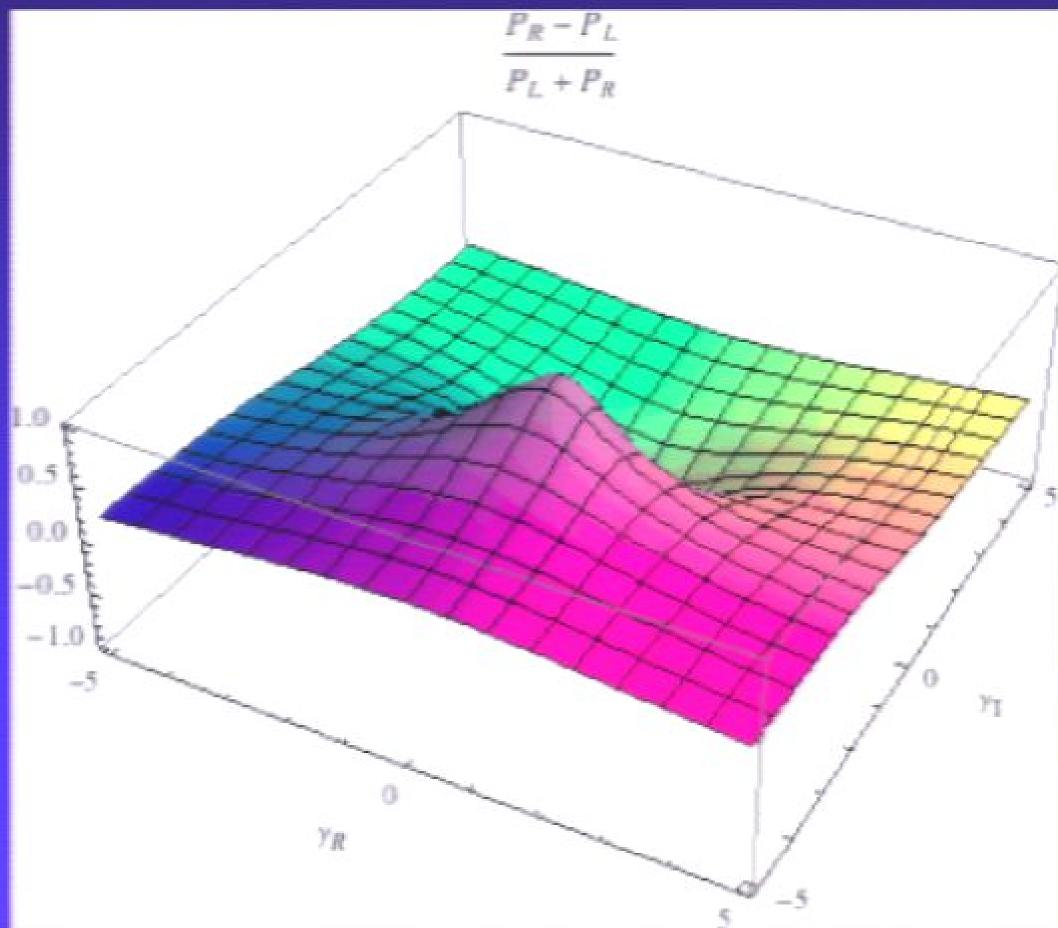
What if the Immirzi parameter is real? No chirality!



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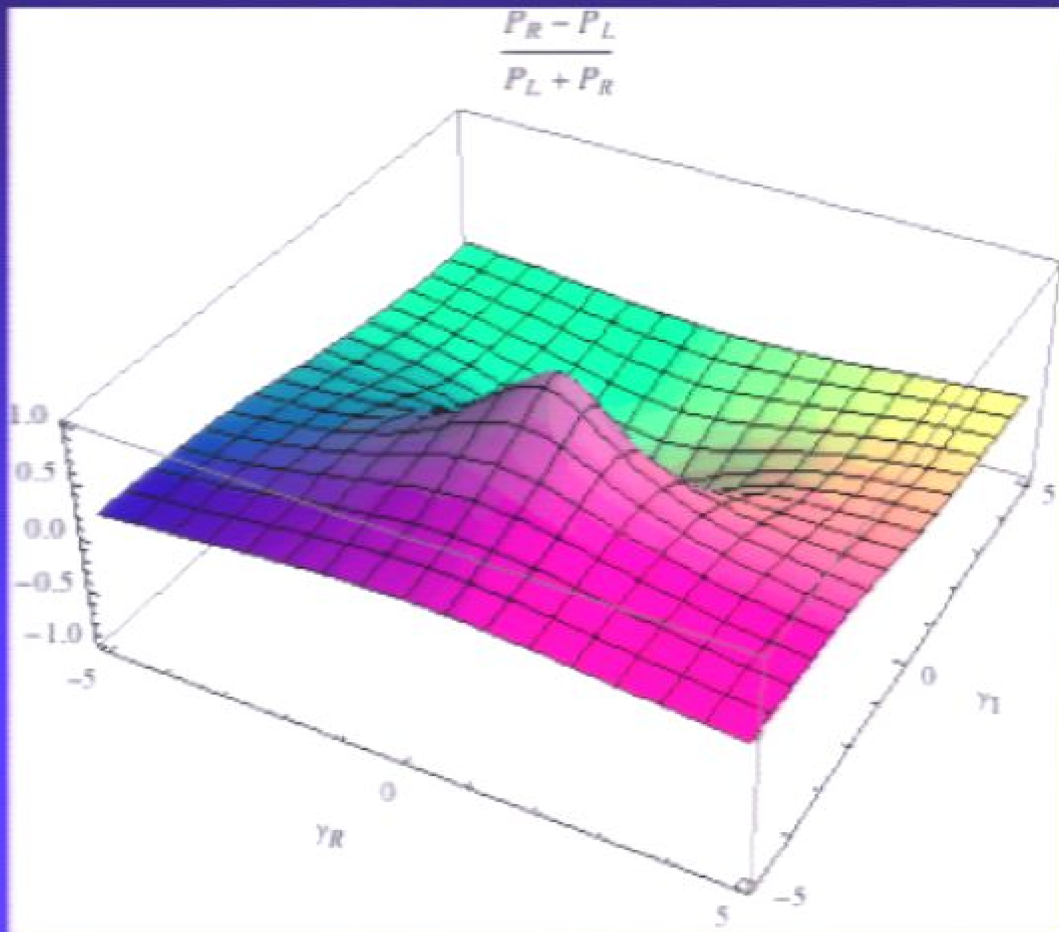
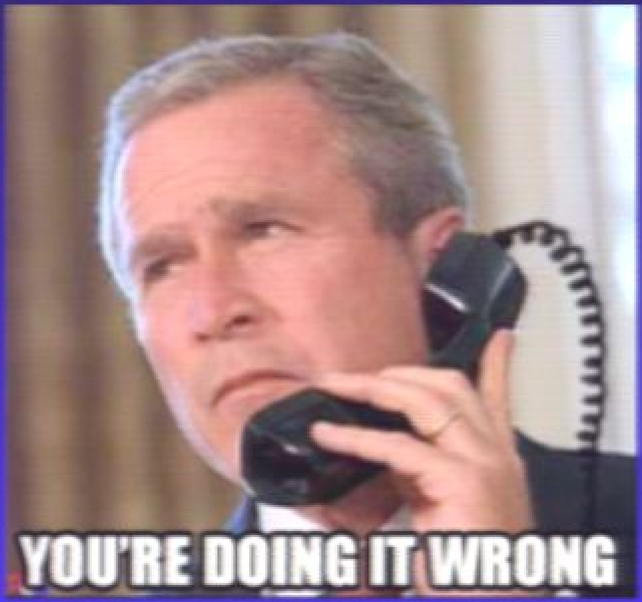


Therefore..........

- SO MUCH FOR INFLATION BEING BLIND TO QUANTUM GRAVITY
- SO MUCH FOR THE BUNCH DAVIES VACUUM BEING THE SELF-EVIDENT GOD'S CHOICE

Discrediting the Kodama state:



- We have identified a vacuum state for a theory that contains gravitons. Could it be the perturbed Kodama state?
- NO!

Discrediting the Kodama state:

$$\Phi = \mathcal{N} \exp\left(\frac{i\gamma}{2l_P^2 H^2} S_{CS}\right)$$

- A solution to the self-dual equation:

$$\mathcal{S}\Phi = 0 \qquad (\widehat{B}^{ka} + H^2 \widehat{E}^{ka})\Phi = 0$$

- In the connection representation

$$\begin{aligned}\hat{A}_a^i(\mathbf{x})\Phi(A_a^i) &= A_a^i(\mathbf{x})\Phi(A_a^i)\\ \hat{E}_j^b(\mathbf{x})\Phi(A_a^i) &= -i\gamma l_P^2 \frac{\delta}{\delta A_b^j(\mathbf{x})}\Phi(A_a^i)\end{aligned}$$

- Assuming an ordering with Es to left of As

$$\mathcal{H} = EE\mathcal{S}$$

As with the Ham. Const. perturbation theory plays a Russian doll trick

$$\mathcal{H} = E E \mathcal{S}$$

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So the perturbed Kodama state:

- Isn't even an eigenstate of the perturbative Hamiltonian
- Represents self-dual perturbations: these cannot be gravitons!
- More specifically its fluctuations are a combination of the positive frequency of the right-handed graviton and negative frequency of the left-handed graviton
- Unsurprisingly NOT a physical state

An important lesson:

- The Kodama state is a semi-classical solution to quantum gravity
- We now learn that this is never the path to phenomenology, e.g. gravitons. These may be perturbative, but are never semi-classical: they are fully quantum!

The million dollar question:

- With the help of cosmology, we have identified an alternative perturbative ground state for quantum gravity
- This is not the perturbed Kodama state
- What is the full, non-perturbative state from which it derives?

On a more pedestrian level…

- Here's a testable prediction of quantum gravity: TB polarization!

PRL101141101,2008 (Contaldi, JM, Smolin)

We are waiting for the polarization!

- Besides measuring the CMB temperature T we have access to its polarization. This has 2 modes E B
- The famous C_ℓ apply to all possible pairs.
- Even-parity ones: TT TE EE BB
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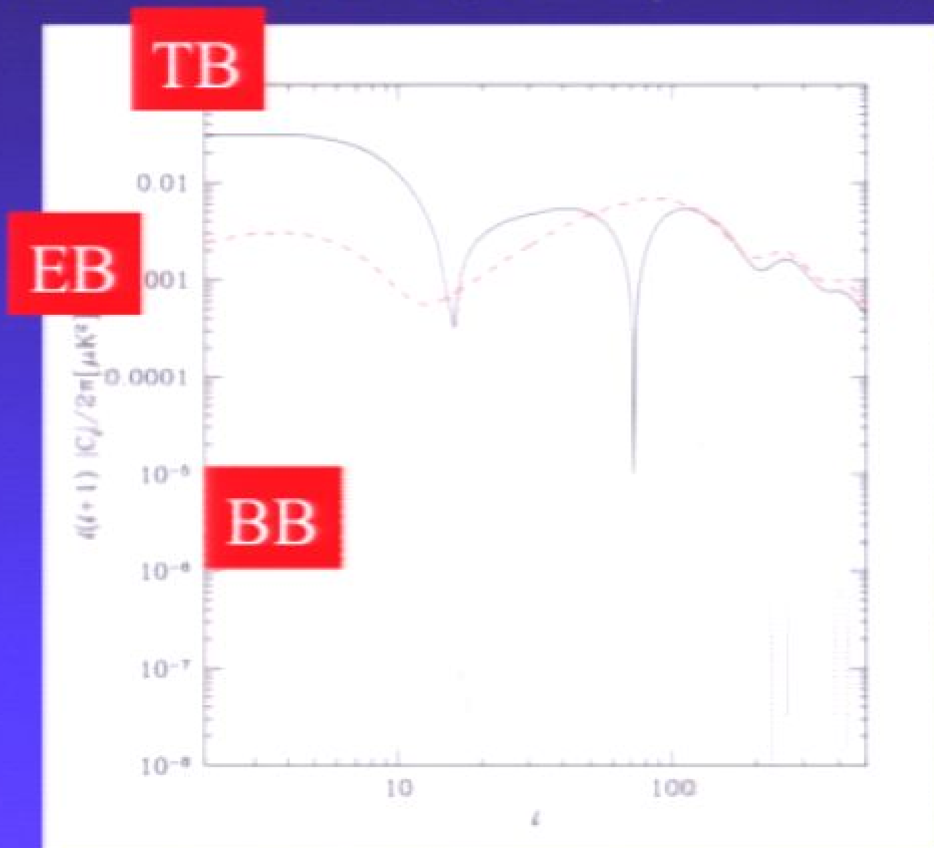

The signature in TB (and EB) is typically much larger than in BB

Killing two pigeons with one stone

- Obviously it may be that there are no tensor modes.
- But if they do exist they will be easier to detect via chirality (TB) for a wide range of Immirzi parameters:

$$\frac{1}{800} < |Im\gamma| < 800$$

Conclusions:

- It is possible to recover cosmological perturbation theory within Ashtekar's formalism but you have to sweat
- The exercise suggests a new base (ground?) state for QG: it is chiral.
- QG quantization corrects the usual inflationary calculation.
- The implications are observationally striking!

"Gravity's bias for left may be writ in the sky"

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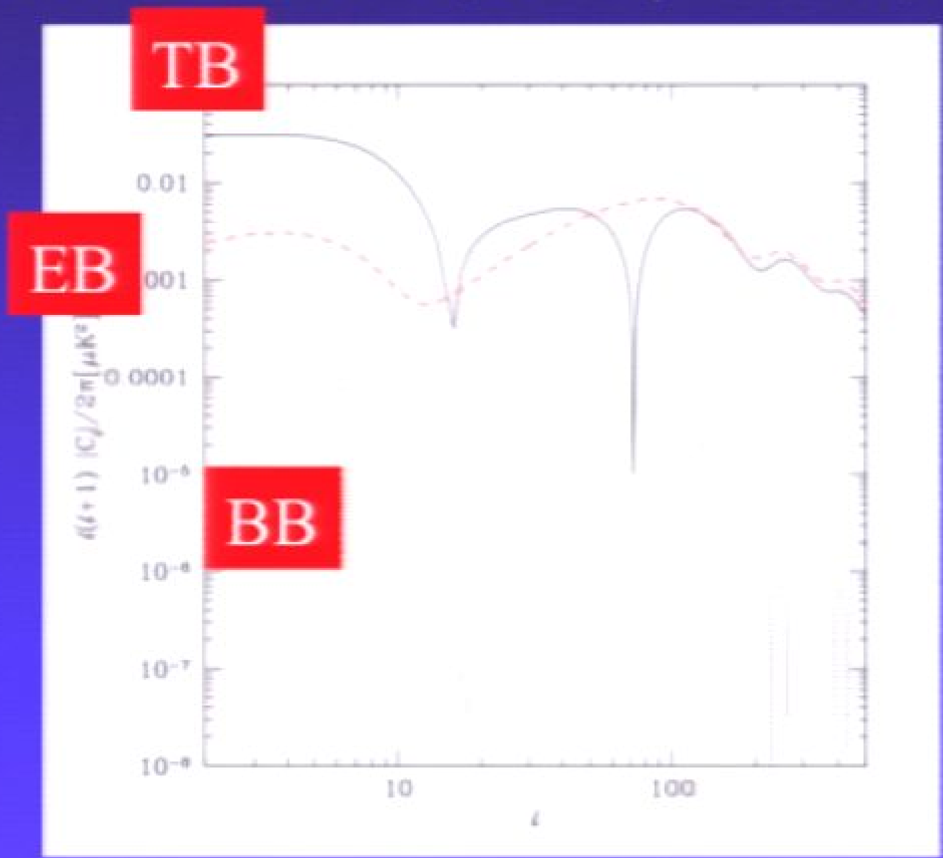

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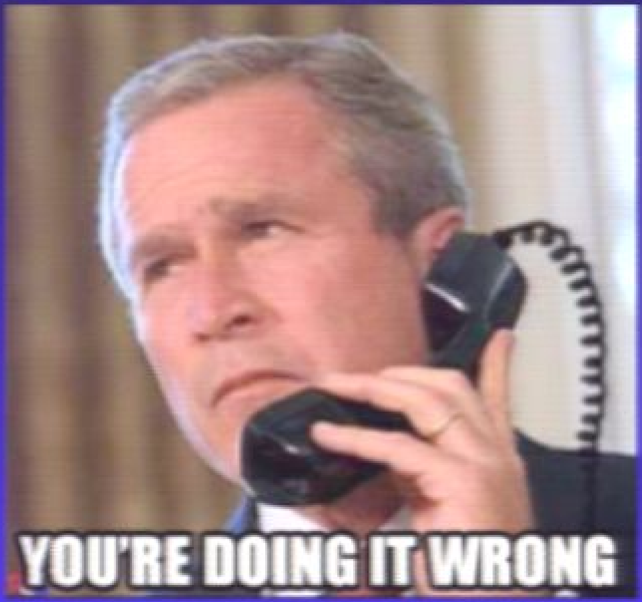

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Can be seen using the Bargman (or holomorphic) representation:

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Sure, but WHAT is new?

Quantization is different: a sort of uncertainty relations between metric and connection

$$\{A_a^i(\mathbf{x}), E_j^b(\mathbf{y})\} = \gamma l_P^2 \delta_a^b \delta_j^i \delta(\mathbf{x} - \mathbf{y})$$

It begs the question: then, what's the graviton made of?

They inherit a funny algebra:

- Something is wrong with half the modes:

$$[g_{rp}(\mathbf{k}), g^{\dagger}_{sq}(\mathbf{k}')] = -i\gamma l_P^2 (pr) k \delta_{rs}\delta_{pq}\delta(\mathbf{k}-\mathbf{k}')$$

- These are the modes that don't exist classically (e.g. for SD connection, the R anti-graviton and L graviton)
- Upon identifying the inner product, they turn out to be non-normalizable: non-physical

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The reality conditions are second class constraints:

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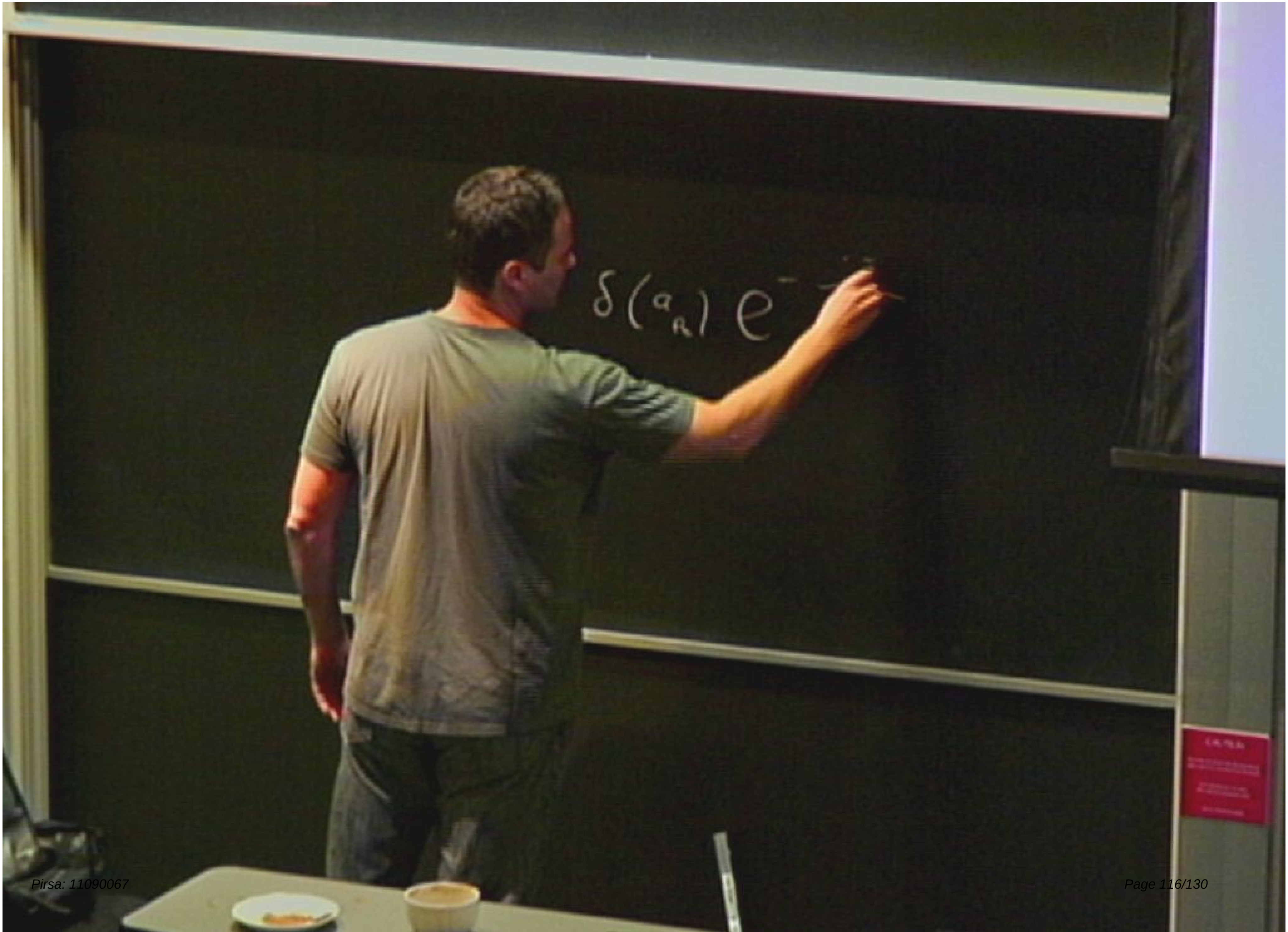
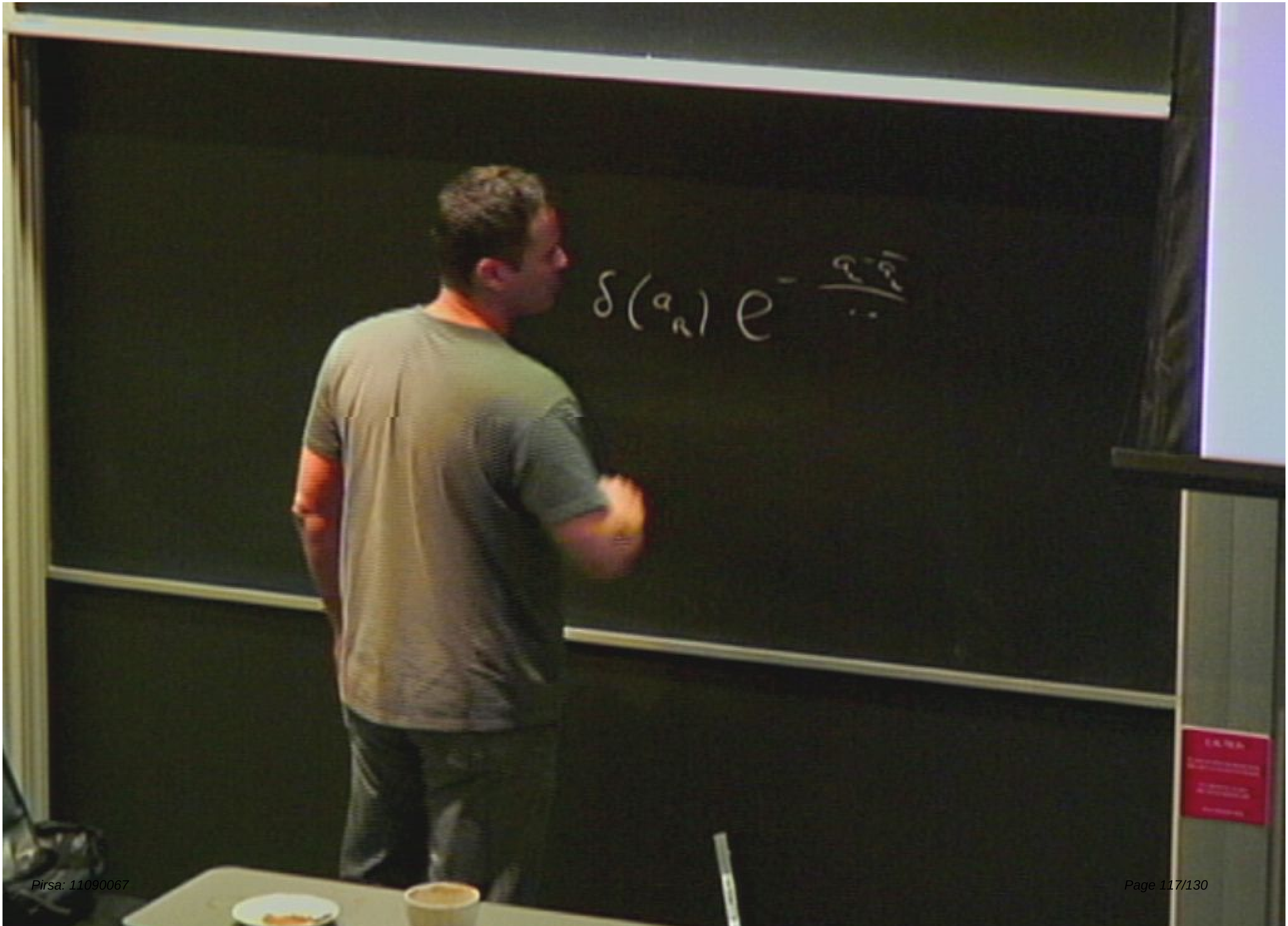
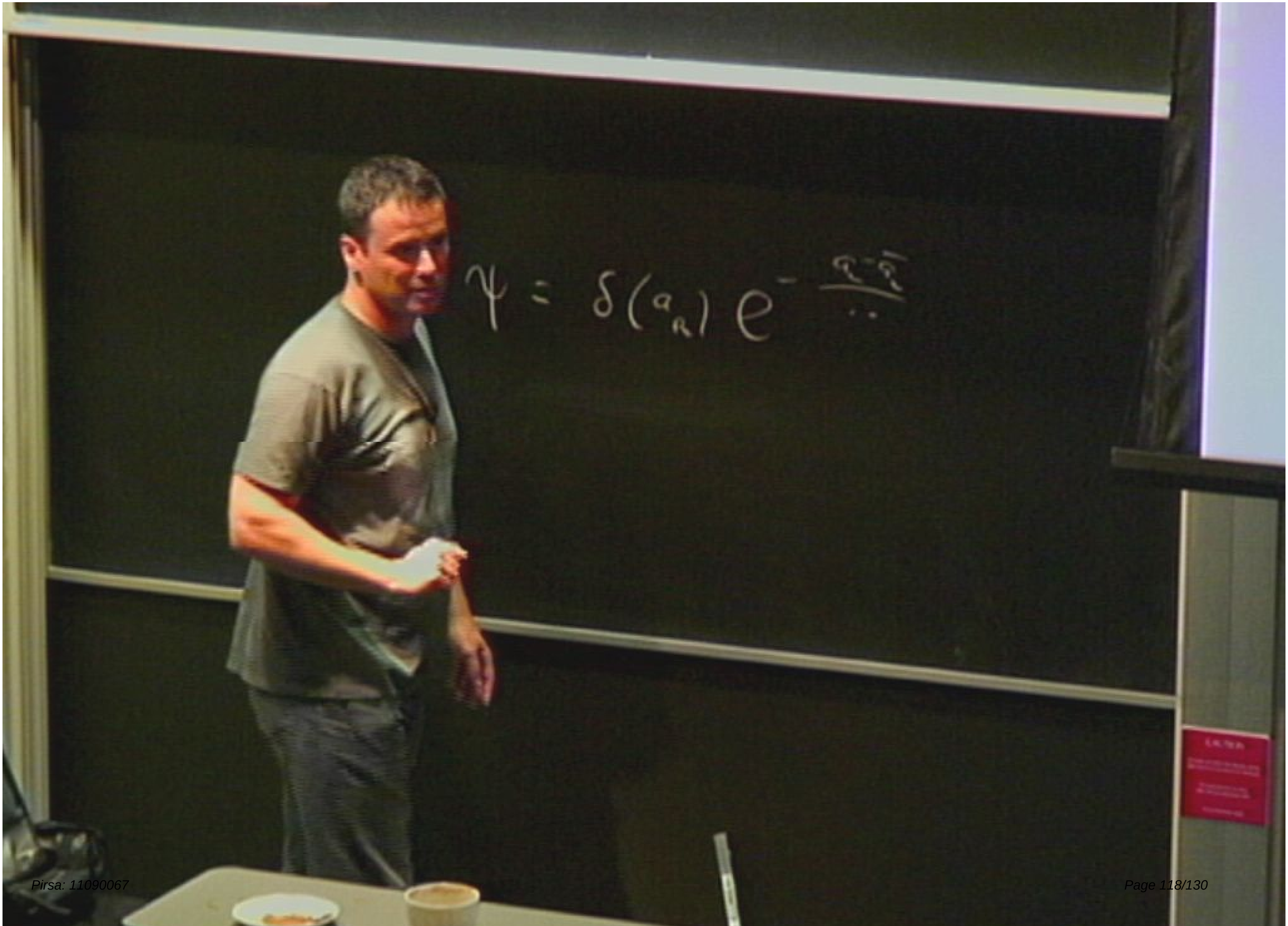
\langle 0|A_L^{ph\dagger}(\mathbf{k})A_L^{ph}(\mathbf{k}')|0\rangle &= \langle 0|g_{L-}(\mathbf{k})g_{L-}^{\dagger}(\mathbf{k})|0\rangle \neq 0
\end{aligned}
$$

- Eg: for the SD connection only the L graviton has vacuum fluctuations


δ(a_R) e





Furthermore:

- It propagates into the vacuum two-point function, with similar chiral behaviour:

$$
\begin{aligned}
A_R^{ph}(\mathbf{k}) &= a_{R+}(\mathbf{k})e^{-ik\cdot x} = g_{R+}(\mathbf{k})e^{-ik\cdot x} \\
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\end{aligned}
$$

$$
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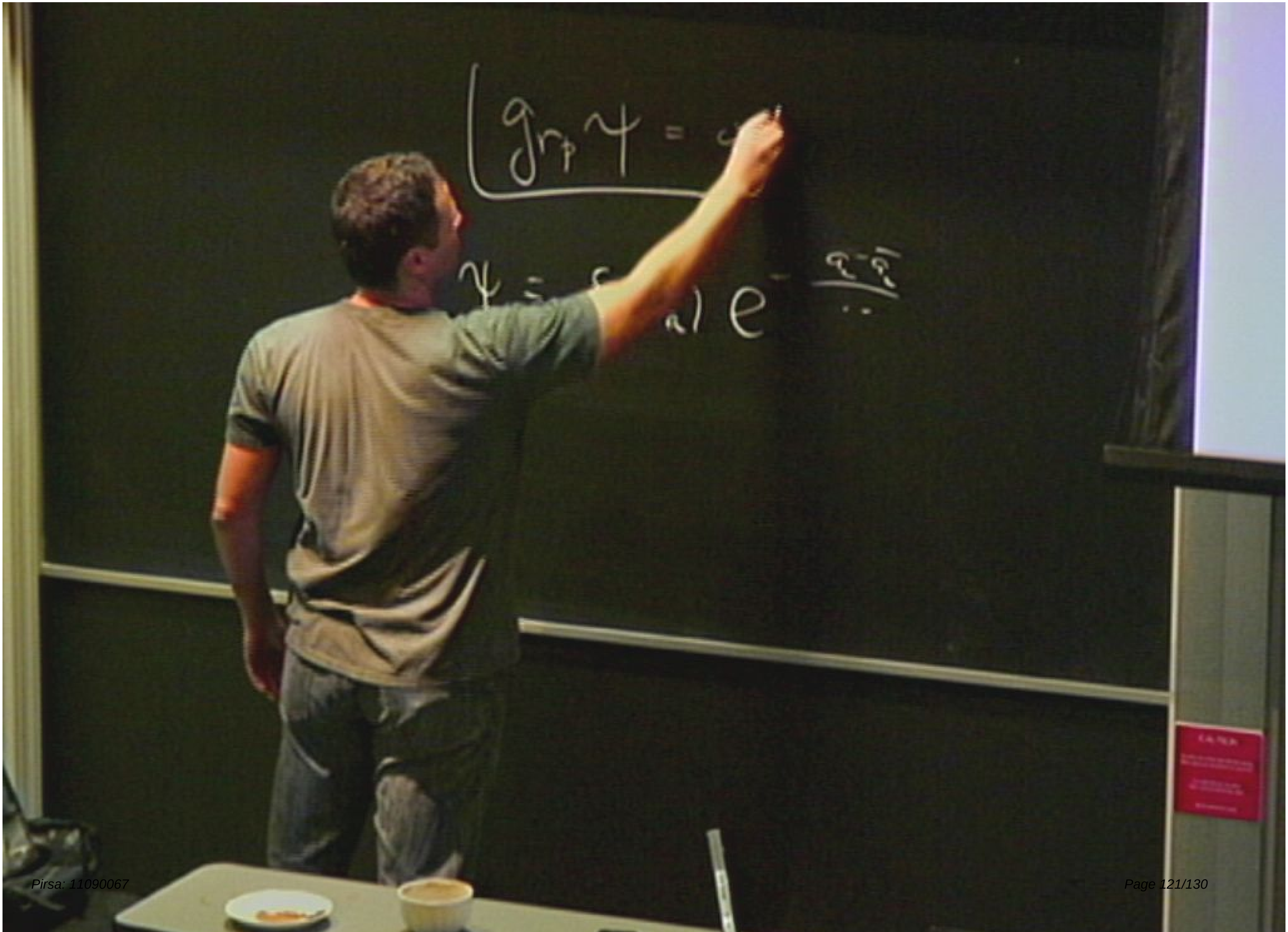

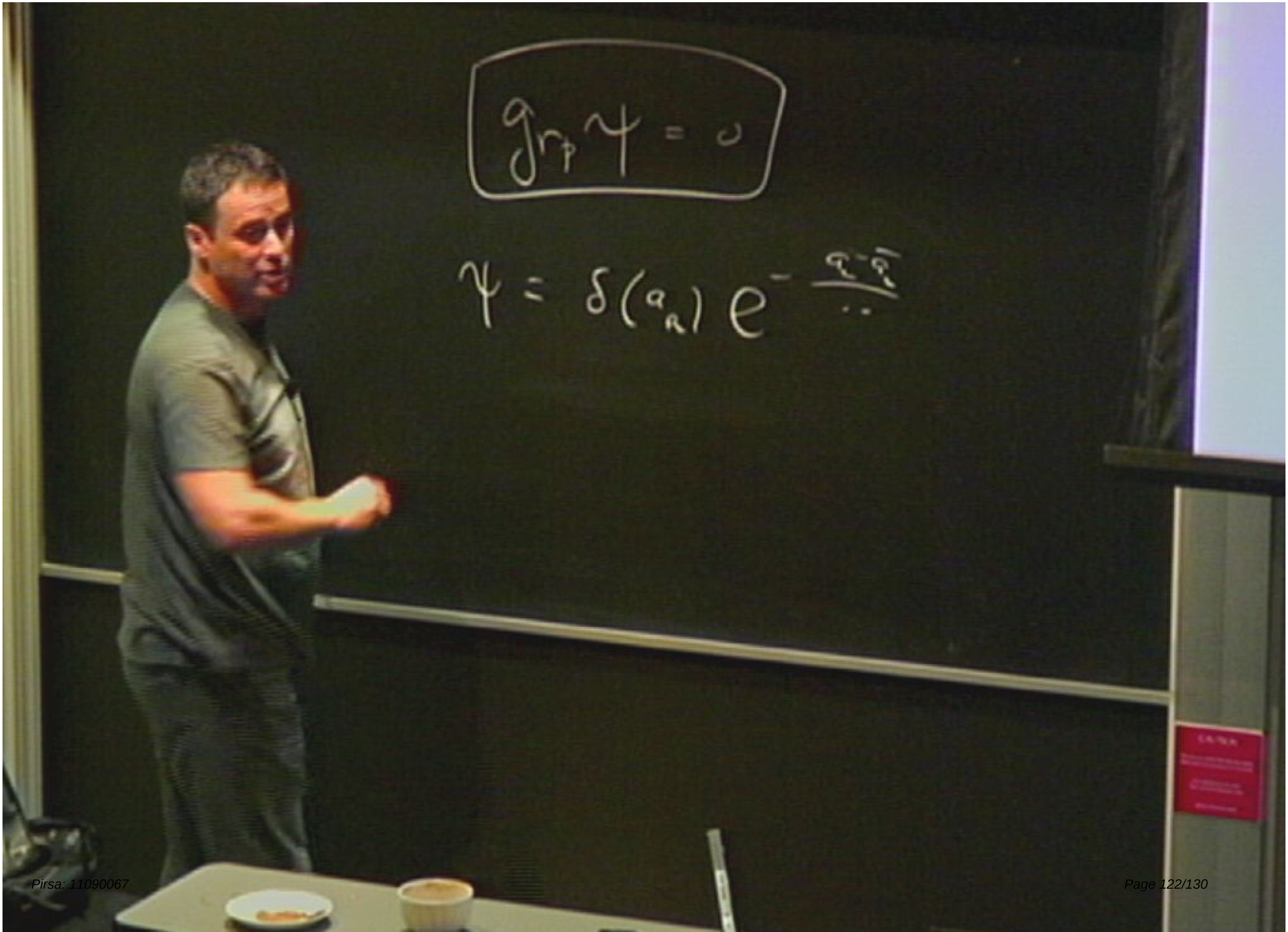
$g r_p \psi = 0$
$\psi = \delta(a_R)\, e^{-}$

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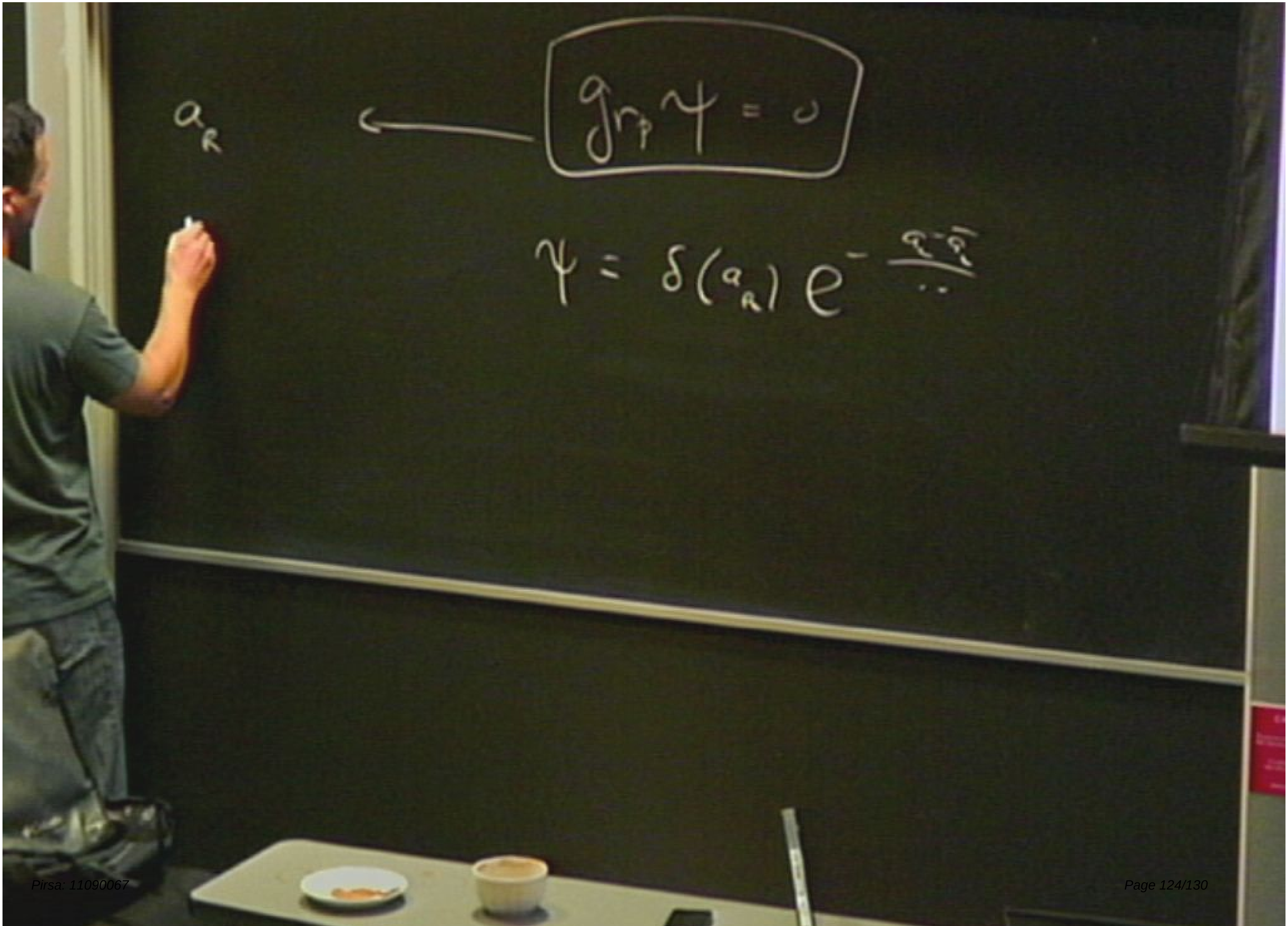
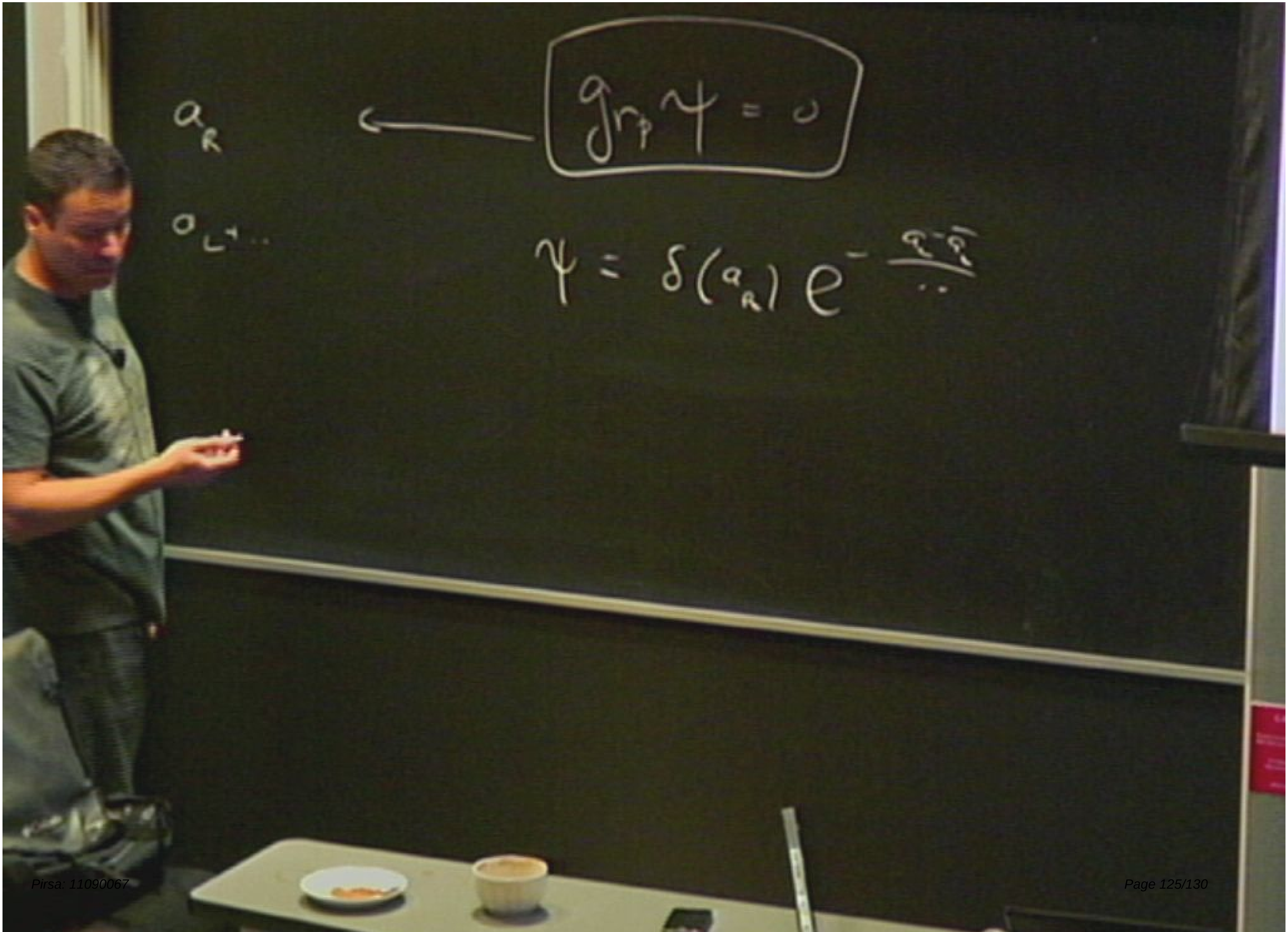
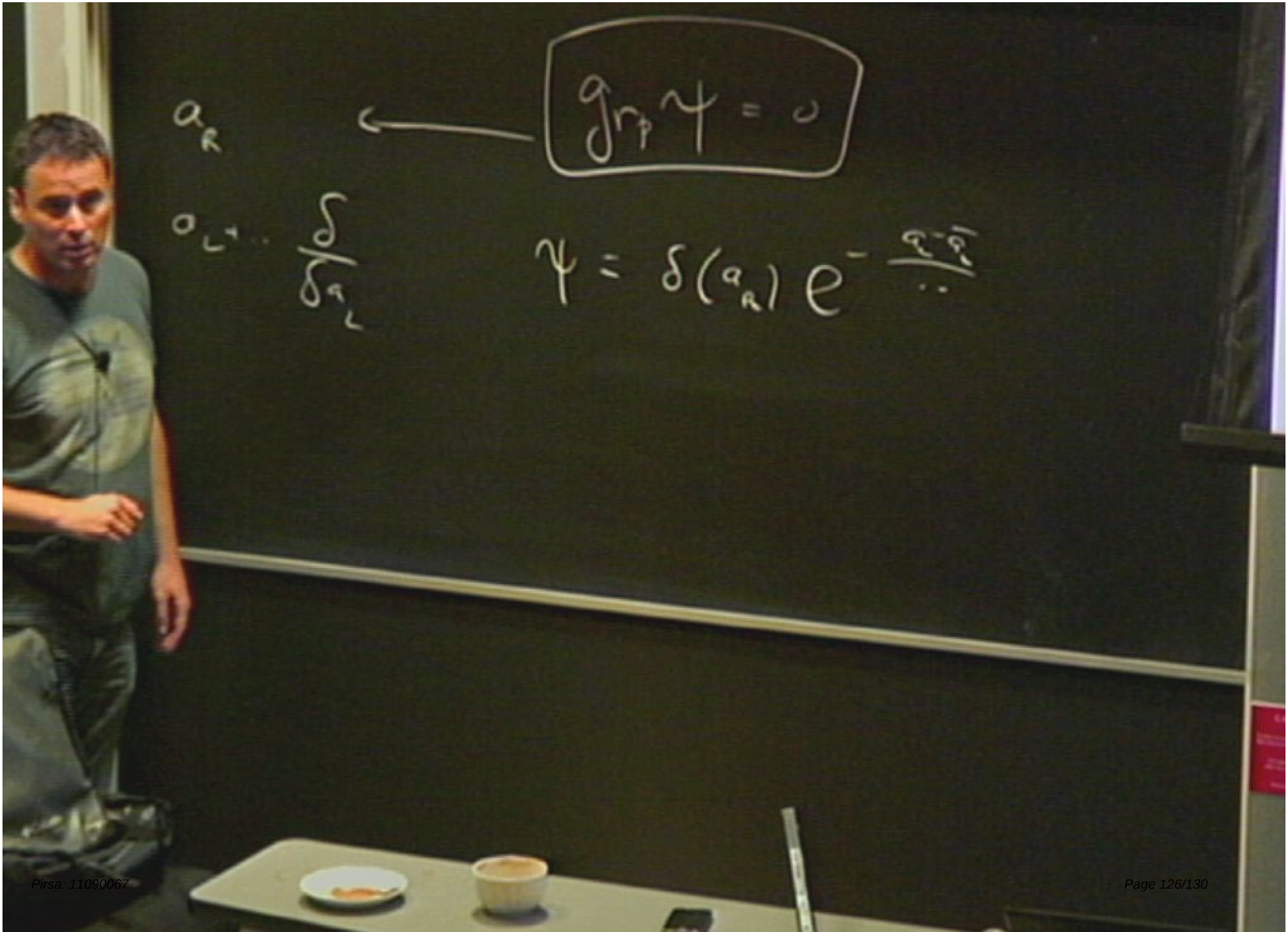
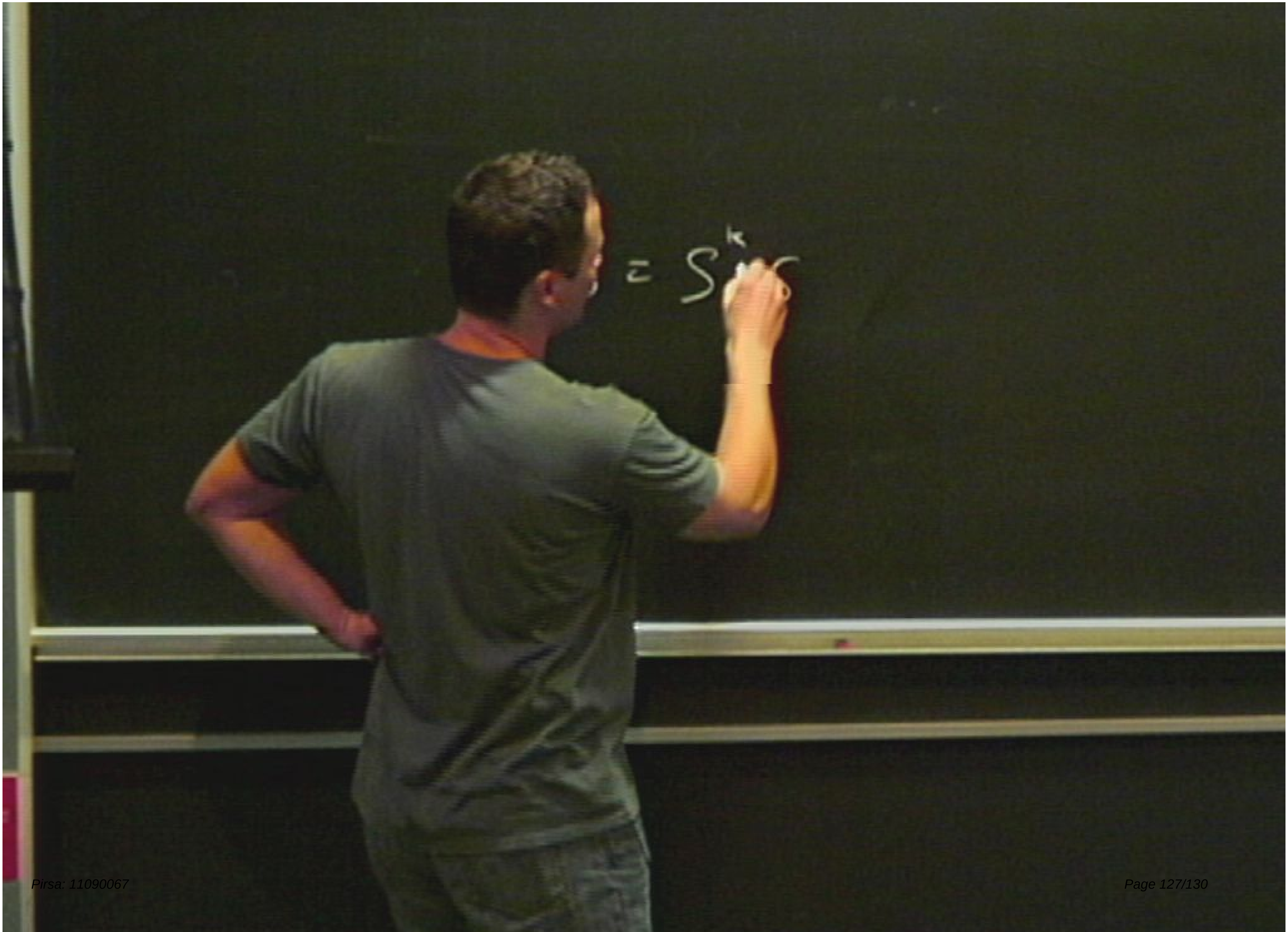
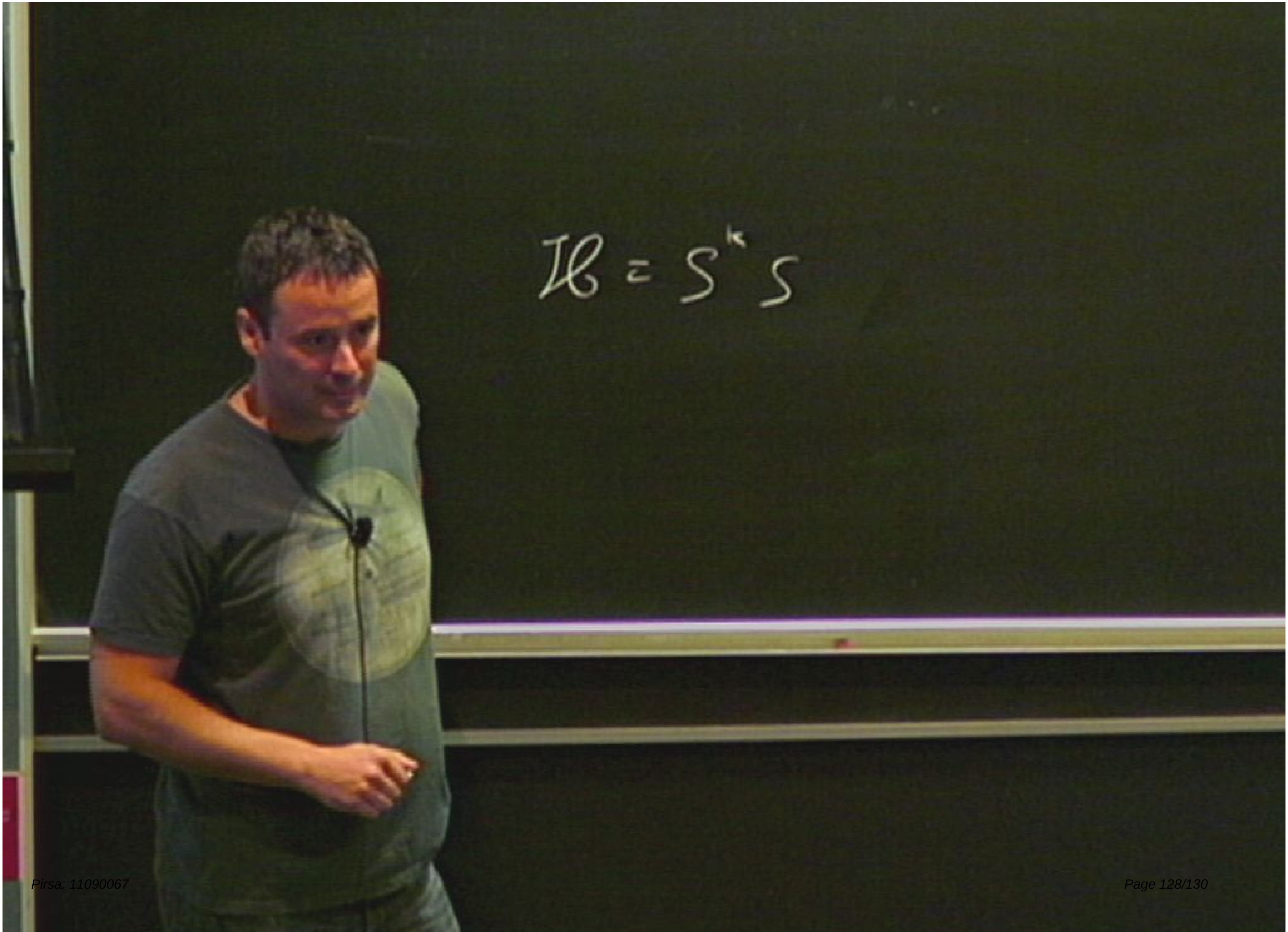
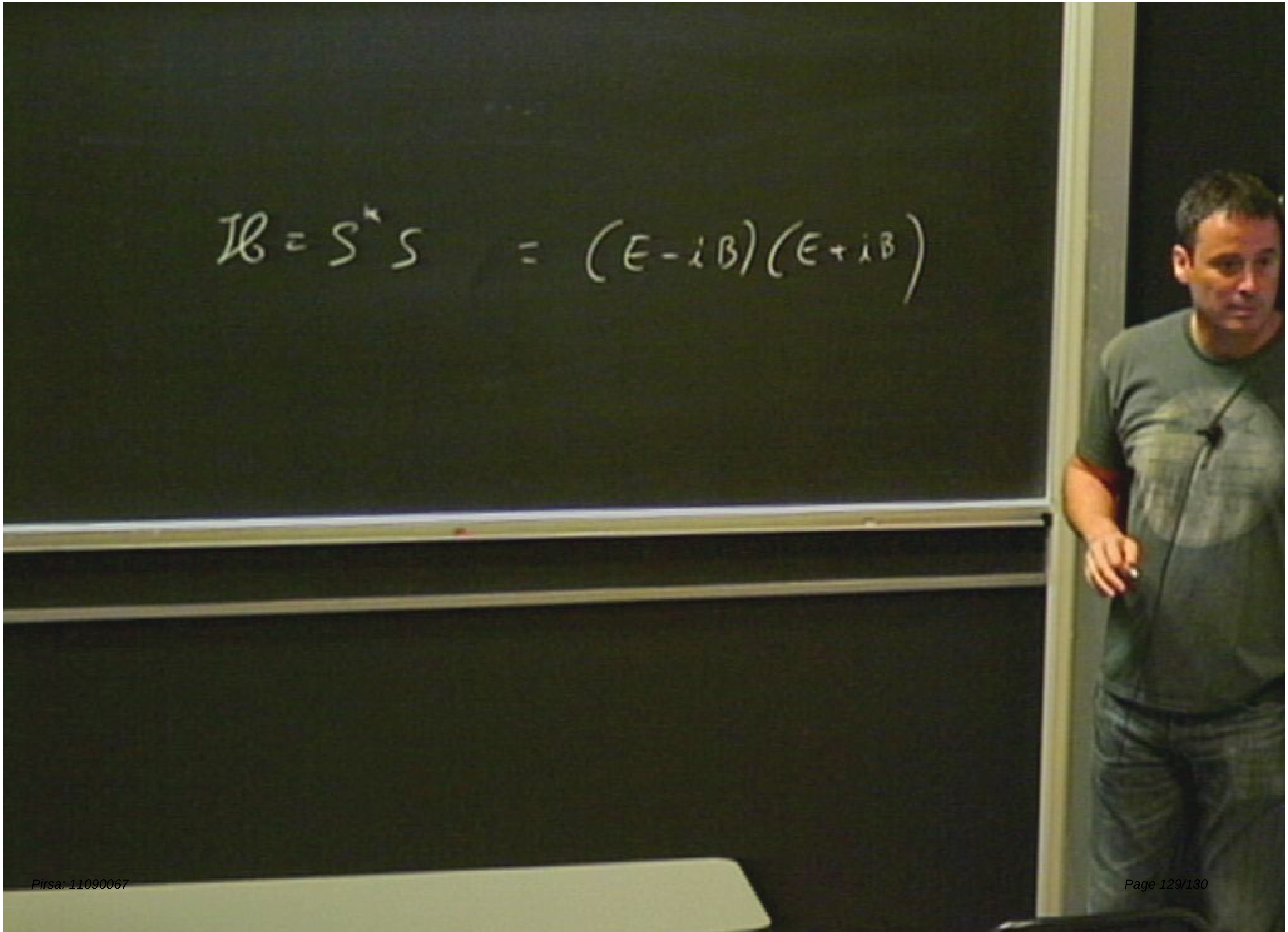

a_R
$gr_p \psi = 0$
$\psi = \delta(a_R)\, e^{-}$


a_R
a_L + ...
gr_p ψ = 0
ψ = δ(a_R) e^-


a_R
g_{r_p} ψ = 0
δ/δa_L
ψ = δ(a_R) e^-




$\mathcal{H} = S^* S$


𝔹 = S* S = (E - iB)(E + iB)


= (E - iB)(E + iB)