Title: Towards Matter Inflation in Heterotic String Theory

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Abstract: I will discuss the phenomenologically interesting scenario of matter inflation in supersymmetric hybrid inflation models. The inflaton resides in a gauge non-singlet matter multiplet and the eta-problem is solved by a "Heisenberg" symmetry. This symmetry relates the inflaton with a modulus field and we stabilize this modulus via corrections to the KÃ¤hler potential. The Heisenberg symmetry arises naturally for the untwisted matter fields in heterotic orbifolds. A way to embed matter inflation into heterotic orbifold compactifications is suggested and moduli stabilization in the extended setup is discussed. I argue that the corrections from moduli stabilization may not spoil the flatness of the inflaton potential at large radius.

Towards Matter Inflation in Heterotic String Theory

Sebastian Halter

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Perimeter Institute
September 6, 2011

Based on work with
S. Antusch, K. Dutta, J. Erdmenger
arXiv 1102.0093 & work in progress

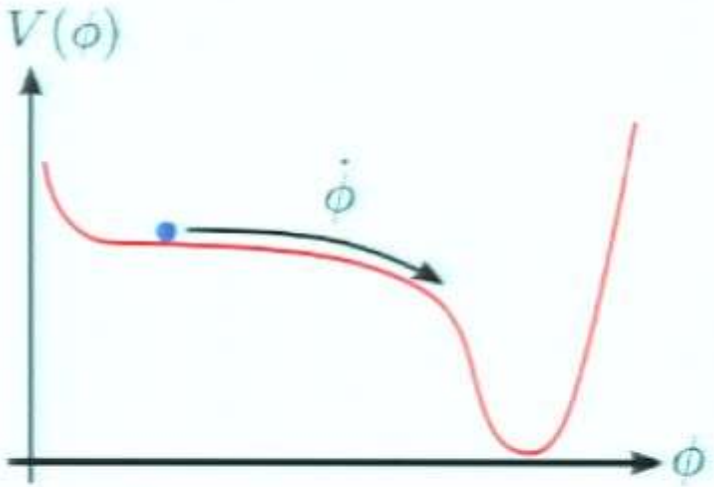
Slow-roll inflation

"Standard" realization: slowly rolling scalar field ϕ

$$ds^2 \approx -dt^2 + a(t)^2 d\vec{x}^2 \ , \ a(t) \approx e^{\mathcal{H}t} \ , \ \mathcal{H} \approx \sqrt{\frac{V(\phi)}{3M_P^2}}$$

$V(\phi)$ must satisfy

$$\epsilon \sim M_P^2 \frac{V'^2}{V^2} \ll 1 \ \& \ \eta \sim M_P^2 \frac{V''}{V} \sim \frac{m_\phi^2}{\mathcal{H}^2} \ll 1$$



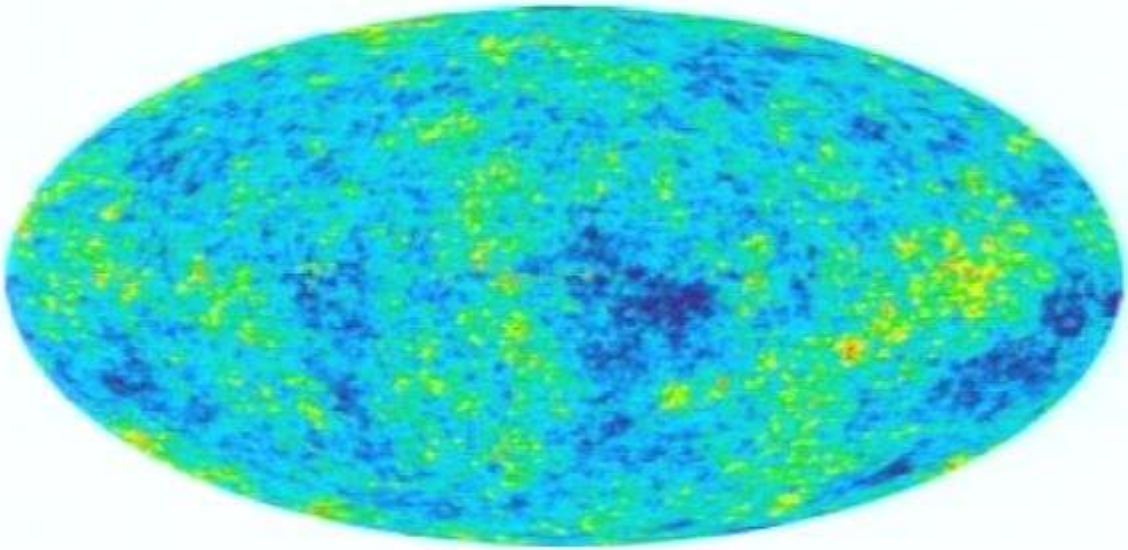
Inflationary paradigm

Inflation = Period of exponential expansion in very early universe.

Guth '81, Linde '82, Albrecht, Steinhardt '82

Inflation is a successful paradigm which

- solves the flatness & horizon problem ($T \approx 2.7K$)
- provides a seed for structure formation ($\frac{\delta T}{T} \sim 10^{-5}$)



WMAP 7 year full sky temperature map

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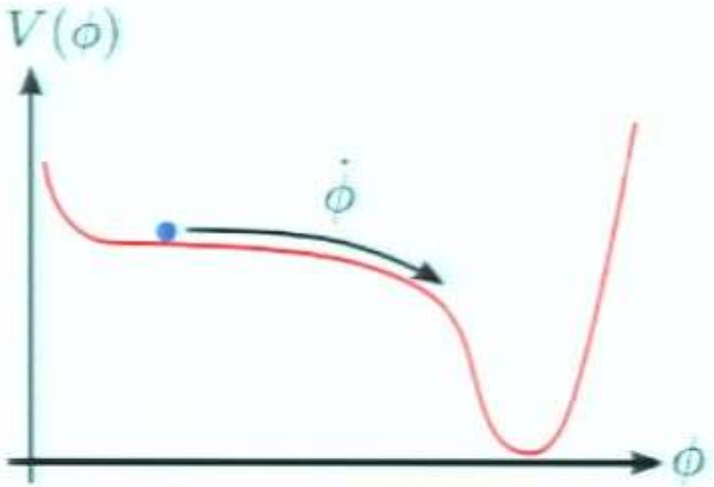

η-problem

Slow-roll inflation sensitive to Planck-scale physics:

$$\delta V = c\,\mathcal{O}_4 \frac{\phi^2}{M_P^2} \;\&\; \langle \mathcal{O}_4 \rangle \sim \langle V \rangle \Rightarrow \eta \sim c$$

e.g. F-term inflation in supergravity:

$$V_F = e^{K/M_P^2}\left(K^{i\bar{\jmath}} D_i W\, D_{\bar{\jmath}} \overline{W} - 3\frac{|W|^2}{M_P^2} \right) \;,\; D_i W = W_i + \frac{K_i W}{M_P^2}$$

with $K = |\Phi|^2 + |X|^2 + \ldots$ & only $\langle W_X \rangle \neq 0$

Copeland, Liddle, Lyth, Stewart, Wands '94; Dine, Randall, Thomas '95

$$V_F = |\langle W_X \rangle|^2 \left(1 + \frac{|\phi|^2}{M_P^2} + \ldots \right) \Rightarrow \eta \sim 1$$

Solution: fine-tune against dots or impose symmetry

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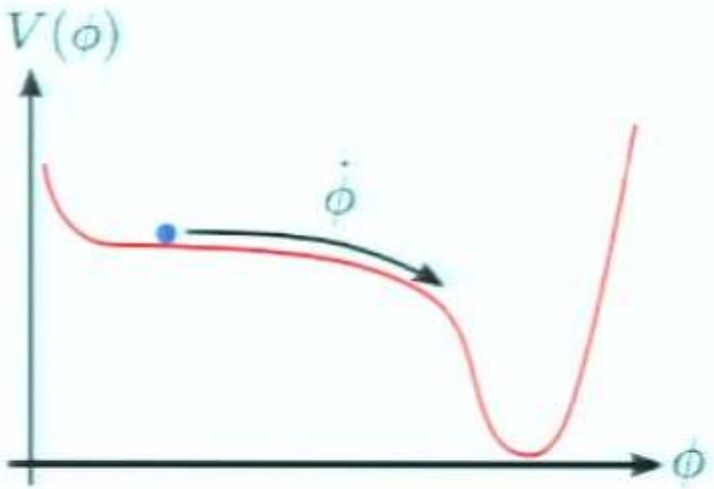

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Example: F-term Hybrid Inflation

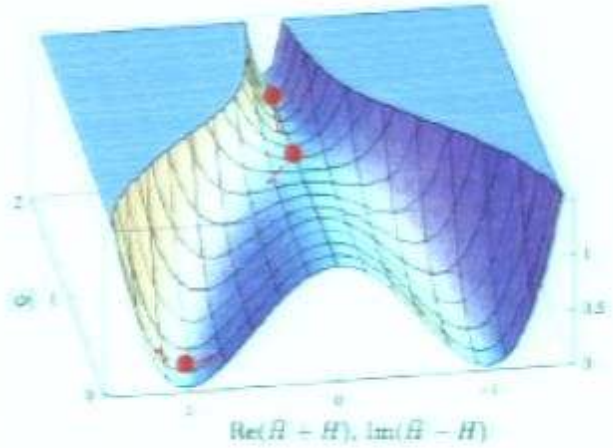
- Minimal W & K:

$$W = \kappa \Phi \left(H^2 - M^2 \right) \ , \ K = |\Phi|^2 + |H|^2$$

- Tree-level: ϕ^2-term in V_F cancels accidentally

  Copeland, Liddle, Lyth, Stewart, Wands '94

- 1-loop: slope from Coleman-Weinberg potential
- Pro: works with field values $\ll M_P$
- Con: implicit fine tuning of e.g. $\delta K = \frac{c}{M_P^2} |\Phi|^4$



Superpotential

Requirements:

1. Solve η-problem by special form of K
   — during inflation W should fulfill Stewart '95

$$\langle W \rangle \simeq \langle W_\Phi \rangle \simeq 0 \ , \ \langle W_X \rangle \neq 0$$

2. Inflation ends via hybrid mechanism Linde '93, Dvali, Shafi, Schaefer '94
   — at $\langle \phi \rangle = \phi_{cr}$ a tachyonic direction appears

Minimal form of W: Arkani-Hamed, Cheng, Creminelli, Randall '03, Antusch, Dutta, Kostka '09

$$W = \kappa X (H^2 - M^2) + \lambda f(\Phi) H^2$$

During inflation: $\langle X \rangle \simeq \langle H \rangle \simeq 0$

Kähler Potential

Usual choice: shift symmetry

e.g. Kawasaki, Yamaguchi, Yanagida '00; Arkani-Hamed, Cheng, Creminelli, Randall '03

$$\Phi \to \Phi + i\alpha$$

Alternative: "Heisenberg symmetry"

Binetruy, Gaillard '87; Ellwanger, Schmidt '87; Gaillard, Murayama, Olive '95; Gaillard, Lyth, Murayama '98

$$T \to T + i\alpha$$
$$\Phi \to \Phi + \beta$$
$$T \to T + \bar{\beta}\,\Phi + \frac{1}{2}|\beta|^2$$

Invariant combination: $\rho \equiv T + \bar{T} - |\Phi|^2$

$$\text{e.g. } K = -3\ln\rho + k(\rho)|X|^2 + \dots$$

Why Matter Inflation?

Why is it interesting to have the inflaton in the matter sector?

- Direct link between particle physics & inflation
- Hybrid phase transition and GUT breaking?
  — Typically $\langle H \rangle \simeq M \sim M_{GUT}$
- Inflaton in visible sector, e.g. right-handed sneutrino?
  — Relate inflation to leptogenesis
- Extra constraints on inflaton potential from particle physics
  — Minimally coupled SM Higgs excluded by EWSB vs. CMB

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Matter Fields as Inflatons

Heisenberg symmetry & structure of W
— Gauge non-singlet matter field as inflaton

Antusch, Bastero-Gil, Baumann, Dutta, King, Kostka '10

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\begin{aligned}
W &= \kappa X(HH^c - M^2) + \frac{\lambda}{\Lambda}\Phi\Phi^c HH^c \\
K &= -3\ln\rho + |X|^2(1 - \beta\rho - \gamma|X|^2) + |H|^2 + |H^c|^2 \\
\rho &\equiv T + \bar{T} - |\Phi|^2 - |\Phi^c|^2
\end{aligned}
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- D-flat trajectory: $\langle\Phi\rangle, \langle\Phi^c\rangle \neq 0$, $\langle H\rangle \simeq \langle H^c\rangle \simeq \langle X\rangle \simeq 0$
- Example(s): sneutrino inflation $\langle\Phi\rangle = \nu_R$, $\langle\Phi^c\rangle = \nu_R^c$ in
  - $SU(4)_c \times SU(2)_L \times SU(2)_R$ Pati-Salam model
  - $SO(10)$ GUT model

1-Loop Corrections

So far: inflaton potential flat at tree-level

— Slope provided at 1-loop by Coleman-Weinberg potential

$$V_{\text{1-loop}}(\phi) = \frac{1}{64\pi^2}\,\text{STr}\left[\mathcal{M}(\phi)^4\left(\ln\left(\frac{\mathcal{M}(\phi)^2}{Q^2}\right)-\frac{3}{2}\right)\right]$$

Contributions from 2 sectors:

- "Waterfall" sector $m_s^2 \sim \frac{\lambda^2}{\Lambda^2}\phi^4 \pm \kappa^2 M^2$, $m_f^2 \sim \frac{\lambda^2}{\Lambda^2}\phi^4$
- Gauge sector
  - Inflaton singlet under unbroken gauge group $\rightarrow m_A \sim g\phi$
  - Direct SUGRA gaugino masses $(G = K + \ln|W|^2)$

$$m^2_{\lambda,ab} \sim e^G G_i \left(G^{-1}\right)^{i\bar{j}} \frac{\partial \bar{f}_{ab}}{\partial \bar{\Phi}^{\bar{j}}} \sim e^K W_X \frac{\partial \bar{f}_{ab}}{\partial \bar{X}} + \mathcal{O}(W, X, W_{i\neq X})$$

— gauge sector contribution negligible if $\left\langle \frac{\partial \bar{f}_{ab}}{\partial \bar{X}} \right\rangle \simeq 0$

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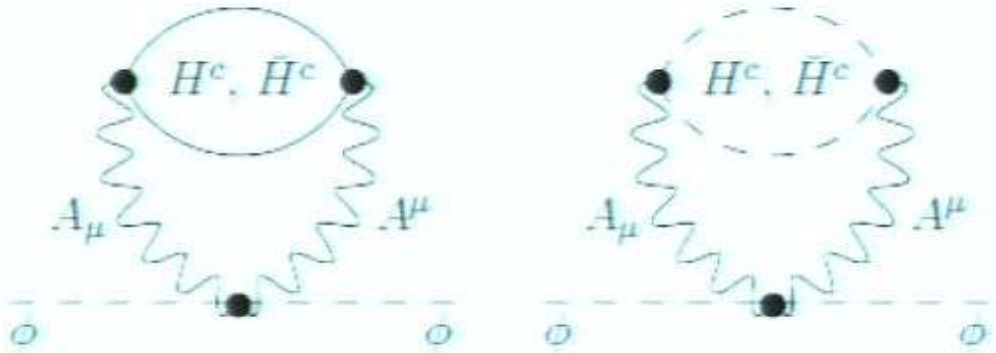
2-Loop Corrections

If $W \supset \kappa X(HH^c - M^2) + m_H^2 HH^c$ & ϕ gauge non-singlet

$\rightarrow \delta m^2 \sim \frac{g^4}{(4\pi)^4} \frac{|W_X|^2}{m_H^2} \gtrsim \mathcal{H}^2$ since $\mathcal{H}^2 \sim \frac{|W_X|^2}{M_P^2}$ Dvali '95

However: here no problem

- ϕ charged only under massive gauge bosons
  $\rightarrow$ extra factors of m_A
- universal suppression of $\frac{\delta m^2}{\mathcal{H}^2}$ by $\frac{\kappa^2}{(4\pi)^4}$



Some Generalization

Generalization of previous model: Antusch, Dutta, Erdmenger, Halter '11

$$W = a(T_i)\, X \left(b(T_i)\, HH^c - \langle \Sigma^2 \rangle\right) + c(T_i)\, f(\Phi_{3,\alpha})\, HH^c + \dots$$

$$K = -\sum_{i=1}^{3} \ln \rho_i + \left(\prod_{i=1}^{2} \rho_i^{-q_{i,X}}\right) |X|^2 \left(1 + d(\rho_3) - \gamma |X|^2\right)$$

$$+ \left(\prod_{i=1}^{3} \rho_i^{-q_{i,H}}\right) |H|^2 + \left(\prod_{i=1}^{3} \rho_i^{-q_{i,H^c}}\right) |H^c|^2 + \dots$$

with $\rho_i \equiv T_i + \bar{T}_i - \sum_\alpha |\Phi_{i,\alpha}|^2$ and $0 \leq q_{i,\alpha} < 1$

During inflation: $\langle X \rangle \simeq \langle H \rangle \simeq \langle H^c \rangle \simeq \langle \Phi_{1,\alpha} \rangle \simeq \langle \Phi_{2,\alpha} \rangle \simeq 0$

Some Comments

- $f(\Phi_{3,\alpha})$ = gauge invariant (D-flat) product of Φ's
  e.g. $N \times \bar{N}$ of $SU(N)$ or $27 \times 27 \times 27$ of E_6 etc.
- Need $K \supset -\gamma|X|^4$ to ensure $m_X \gtrsim \mathcal{H}$ Kawasaki, Yamaguchi, Yanagida '00
- Geometric interpretation: Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca '06
  If $\langle X \rangle \simeq 0$, $\langle W_X \rangle \neq 0$
  — $\langle R_{X\bar{X}X\bar{X}} \rangle < 0$ necessary for de Sitter vacua
- Non-canonical kinetic terms $\propto (T + \bar{T} - |\Phi|^2)^{-q}$
- Moduli-dependent superpotential couplings $\sim e^{-aT}$
- $\langle W_X \rangle \propto \langle \Sigma \rangle^2$ with $\langle \Sigma \rangle$ generated dynamically
  — $\langle \Sigma \rangle$ carries moduli dependence

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Moduli-dependent D-term VEVs

A D-term example:

$$D_a = \xi + \sum_\alpha Q_{a,\alpha} \frac{\partial K}{\partial \bar{\psi}_\alpha} \psi_\alpha = \xi + \sum_\alpha Q_{a,\alpha} \left(\prod_{i=1}^{3} \rho_i^{-q_{i,\alpha}} \right) |\psi_\alpha|^2$$

Simplest solution to $D_a = 0$:

$$\langle |\psi|^2 \rangle \sim -\frac{\xi}{Q_a} \left(\prod_{i=1}^{3} \rho_i^{q_i} \right) \text{ with } Q_a \xi < 0$$

More general solution to $D_a = 0$:

$$\left\langle \prod_\beta \psi_\beta^{n_\beta} \right\rangle \neq 0 \text{ with } \sum_\beta n_\beta Q_{a,\beta} \xi < 0$$

Relative size fixed e.g. by further D-term equations

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Moduli-dependent F-term VEVs

An F-term example:

$$W \sim e^{-aT} X \chi \phi \psi + Y \phi' \psi' (1 + e^{-bT} \chi \phi' \psi')$$

consider $\langle \phi \rangle, \langle \phi' \rangle, \langle \psi \rangle, \langle \psi' \rangle \neq 0, \langle X \rangle, \langle Y \rangle \simeq 0$

$$\langle |\chi| \rangle \sim \frac{e^{bT}}{\langle |\phi' \psi'| \rangle}$$

Parametrize moduli dependence of $\langle \Sigma \rangle$

$$\langle \Sigma \rangle \propto \prod_{i=1}^{3} \rho_i^{p_i} e^{b_i T_i}$$

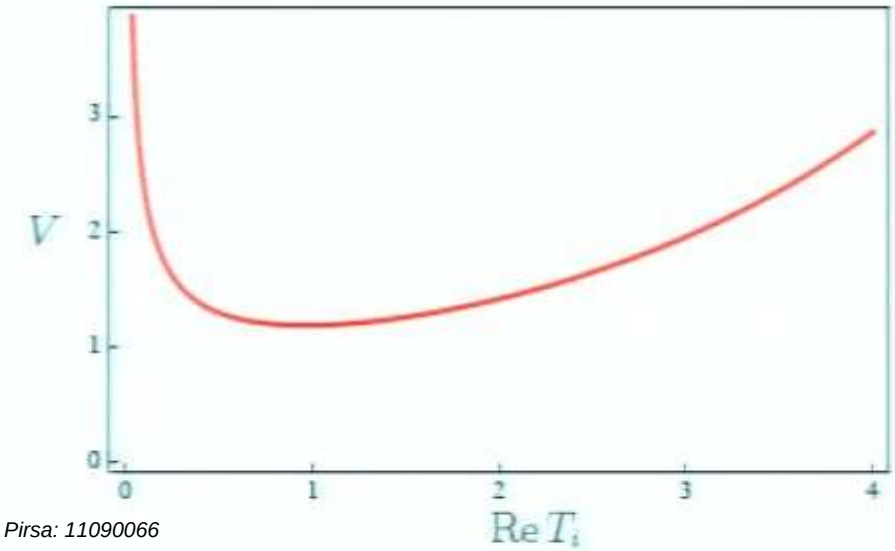
Moduli Stabilization

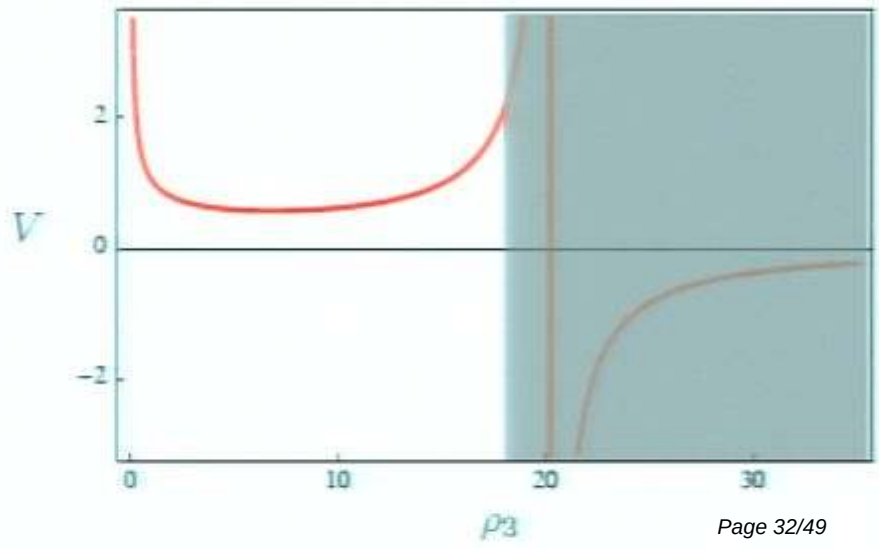
Moduli stabilized for suitable choice of $a(T_i)$, $\langle\Sigma\rangle$ and $d(\rho_3)$

$$\text{e.g. } a(T_i)\langle\Sigma^2\rangle \sim M^2 e^{a_1 T_1 + a_2 T_2} \;,\; d(\rho_3) \sim -\beta\rho_3$$

$$\rightarrow V \sim \frac{M^4 |e^{a_1 T_1 + a_2 T_2}|^2}{(T_1+\bar{T}_1)^{n_1}(T_2+\bar{T}_2)^{n_2}\rho_3^{n_3}(1-\beta\rho_3)}$$

$\rightarrow \langle \text{Re}\, T_{1,2}\rangle \sim \mathcal{O}(1)$ and $\langle\rho_3\rangle \sim \beta^{-1}$ with masses $\sim \mathcal{H}$





Some Generalization

Generalization of previous model: Antusch, Dutta, Erdmenger, Halter '11

$$W = a(T_i)\, X \left(b(T_i)\, HH^c - \langle \Sigma^2 \rangle\right) + c(T_i)\, f(\Phi_{3,\alpha})\, HH^c + \dots$$

$$K = -\sum_{i=1}^{3} \ln \rho_i + \left(\prod_{i=1}^{2} \rho_i^{-q_{i,X}}\right) |X|^2 \left(1 + d(\rho_3) - \gamma |X|^2\right)$$

$$+ \left(\prod_{i=1}^{3} \rho_i^{-q_{i,H}}\right) |H|^2 + \left(\prod_{i=1}^{3} \rho_i^{-q_{i,H^c}}\right) |H^c|^2 + \dots$$

with $\rho_i \equiv T_i + \bar{T}_i - \sum_\alpha |\Phi_{i,\alpha}|^2$ and $0 \leq q_{i,\alpha} < 1$

During inflation: $\langle X \rangle \simeq \langle H \rangle \simeq \langle H^c \rangle \simeq \langle \Phi_{1,\alpha} \rangle \simeq \langle \Phi_{2,\alpha} \rangle \simeq 0$

Some Comments

- $f(\Phi_{3,\alpha})$ = gauge invariant (D-flat) product of Φ's
  e.g. $N \times \bar{N}$ of $SU(N)$ or $27 \times 27 \times 27$ of E_6 etc.
- Need $K \supset -\gamma|X|^4$ to ensure $m_X \gtrsim \mathcal{H}$ Kawasaki, Yamaguchi, Yanagida '00
- Geometric interpretation: Covi, Gomez-Reino, Gross, Louis, Palma, Scrucca '06
  If $\langle X \rangle \simeq 0$, $\langle W_X \rangle \neq 0$
  — $\langle R_{X\bar{X}X\bar{X}} \rangle < 0$ necessary for de Sitter vacua
- Non-canonical kinetic terms $\propto (T + \bar{T} - |\Phi|^2)^{-q}$
- Moduli-dependent superpotential couplings $\sim e^{-aT}$
- $\langle W_X \rangle \propto \langle \Sigma \rangle^2$ with $\langle \Sigma \rangle$ generated dynamically
  — $\langle \Sigma \rangle$ carries moduli dependence

Moduli-dependent F-term VEVs

An F-term example:

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consider $\langle \phi \rangle, \langle \phi' \rangle, \langle \psi \rangle, \langle \psi' \rangle \neq 0, \langle X \rangle, \langle Y \rangle \simeq 0$

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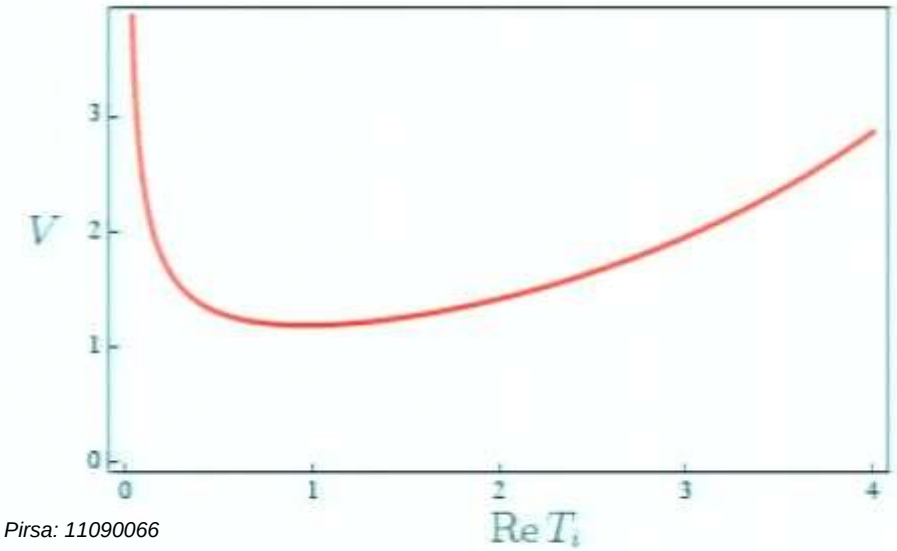
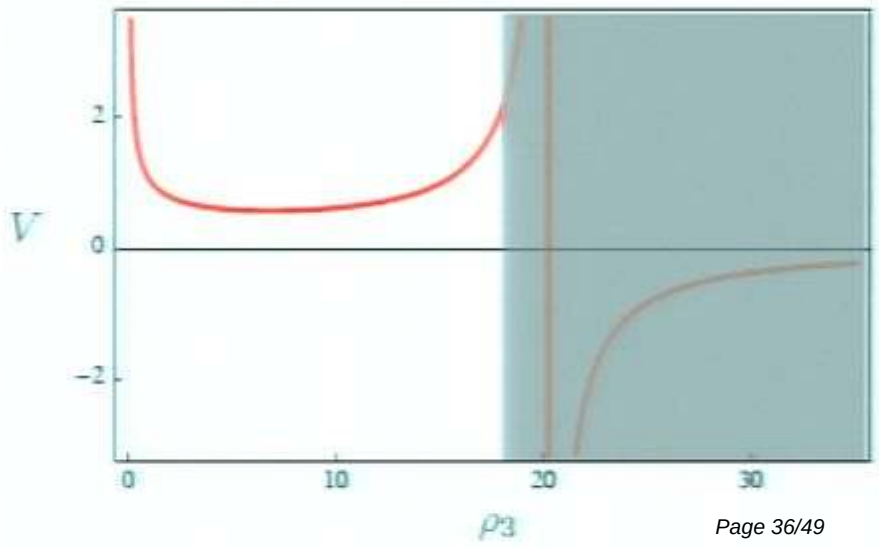




Heterotic Orbifolds

- Heterotic string theory = theory of closed strings
- Contains gauge group $SO(32)$ or $E_8 \times E_8$
- Orbifolds are "toy models" of Calabi-Yau compactifications: obtained as $T^6/\mathbb{Z}_N$
- Strings on oribfolds can be
  - "untwisted" — closed in T^6 and $T^6/\mathbb{Z}_N$
  - "twisted" — closed only in $T^6/\mathbb{Z}_N$
- Corresponds to fields living in

  $10D$ bulk — full orbifold — untwisted
  $4D$ brane — fixed point — twisted
  $6D$ brane — fixed torus — twisted

Heterotic Orbifolds

- MSSM-like models exist, e.g. heterotic "mini-landscape" based on $T^6/\mathbb{Z}_6$ or $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

  Buchmuller, Hamaguchi, Lebedev, Ratz '05 - '06; Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter '06 - '08; Blaszczyk, Groot-Nibbelink, Ratz, Ruhle, Trapletti, Vaudrevange '09

  — viable models contain anomalous $U(1)_a$
- Identification of field content could be

  T_i — 3 "universal" untwisted Kähler moduli
  Φ^i_α — associated untwisted matter fields
  X — twisted sector field
- Technical challenge: twisted matter field VEVs
  — (partial) "blow-up" of orbifold singularities
  — How to compute reliably?

Heisenberg Symmetry

Heisenberg symmetry at tree-level for untwisted matter fields

$$T_i \sim R_i^2 + iB_i \xrightarrow{\Phi_{i,\alpha}\neq 0} T_i \sim R_i^2 + iB_i + |\Phi_{i,\alpha}|^2$$

$10D$ picture

$g \to 0$ limit: for λ_M^a harmonic Ellwanger, Schmidt '87

$$\begin{aligned} A_M^a &\to A_M^a + \lambda_M^a \\ B_{MN} &\to B_{MN} - A_{[M}^a \lambda_{N]}^a \end{aligned}$$

Limit $g \to 0 \leftrightarrow$ limit $W \to 0$

Worldsheet picture

Accidental symmetry for $A_M^a \ll 1$ Cvetič, Molera, Ovrut '89

Target Space Modular Invariance

Low-energy effective action constrained by "modular invariance"

$$T \to \frac{aT + ib}{icT + d} \,, \quad ab - cd = 1 \,, \quad a, b, c, d \in \mathbb{Z}$$

Kähler potential transforms as

$$K = -\ln(T + \bar{T}) \to -\ln\left(\frac{T + \bar{T}}{(icT + d)(-ic\bar{T} + d)}\right)$$

Invariance of supergravity action $\Rightarrow$ superpotential also transforms

$$W \to (icT + d)^{-1} W$$

$\exists$ One such symmetry for each T_i

Target Space Modular Invariance

Matter fields also transform

$$\Phi_\alpha - \prod_{i=1}^{3} (ic_i T_i + d_i)^{-q_{i,\alpha}} \Phi_\alpha$$

with "modular weights" $q_{i,\alpha}$ determined by action of $\mathbb{Z}_N$ on T^6

Generic superpotential term with matter fields

$$\prod_\alpha \Phi_\alpha^{n_\alpha} \left(\prod_{i=1}^{3} \eta(T_i)^{2\sigma_i} \right)$$

$\sigma_i = -1 + \sum_\alpha n_\alpha q_{i,\alpha}$ & $\eta(T) = e^{-\frac{\pi T}{12}} \prod_n \left(1 - e^{-2\pi n T}\right) \simeq e^{-\frac{\pi T}{12}}$

— all matter couplings in W are $1 + e^{-aT} + \ldots$ or $e^{-aT} + \ldots$

Dilaton Stabilization

Dilaton S additional modulus

Non-perturbative corrections to K

"Kähler stabilization" $(g_s^{-2} \sim S + \bar{S})$

Shenker '90; Banks, Dine '94; Casas '96; Binetruy, Gaillard, Wu '96 & '97; Gaillard, Lyth, Murayama '98

$$K_{np} \simeq (A_0 + A_1 g_s^{-1} + \dots) e^{-\frac{B}{g_s}}$$

from worldsheet instanton corrections

Anomalous $U(1)_a$

FI-parameter $\xi \propto (S + \bar{S})^{-1}$ → moduli dependence of VEVs?

Gaillard, Lyth, Murayama '98

- D-term driven VEVs $\langle |\psi|^2 \rangle \propto (S + \bar{S})^{-1}$ → destabilizing
- F-term induced VEVs $\langle |\chi| \rangle \propto (S + \bar{S})^{p}$ → stabilizing

Kähler Moduli Stabilization

- T_1 & T_2 stabilized if Copeland, Liddle, Lyth, Stewart, Wands '94

$$\langle W_X \rangle \propto \eta(T_1)^{-p_1}\eta(T_2)^{-p_2} \sim e^{a_1 T_1 + a_2 T_2}$$

- Ideally: T_3 & $\Phi_{3,\alpha}$ enter V only through $\rho_3 \sim R_3^2$
- Avoid superpotential stabilization $\rightarrow \langle W_{T_3} \rangle \simeq \langle W_{\Phi_{3,\alpha}} \rangle \simeq 0$
- Alternatives:
  - α'-corrections? Candelas, De La Ossa, Green, Parkes '91

$$-\ln \mathcal{V} \rightarrow -\ln(\mathcal{V}+\xi)\ ,\ \xi \propto -\chi = 2\,(h^{1,1}-h^{2,1})$$

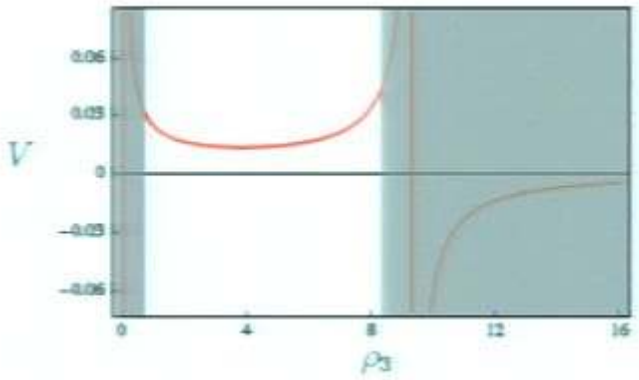
  - Moduli-dependent threshold corrections to $K_{X\bar{X}}$?
    Antoniadis, Gava, Narain, Taylor '92

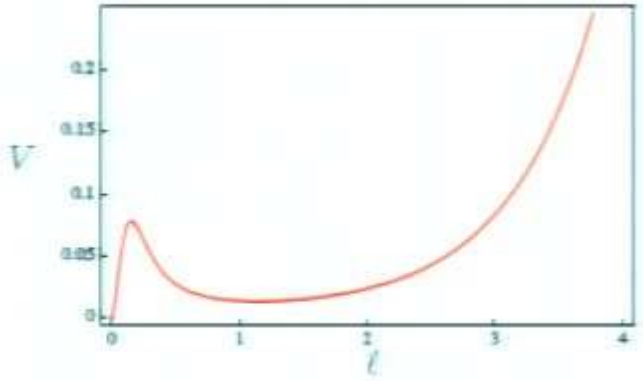
$$\langle \Phi_\alpha \rangle = 0 \rightarrow \ln|\eta(T)|^4(T+\bar{T}) \simeq \ln(T+\bar{T}) - \frac{\pi}{6}(T+\bar{T}) + \mathcal{O}(e^{-2\pi T})$$

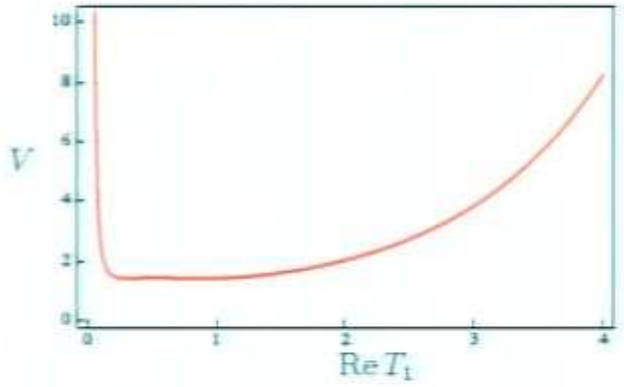
Moduli Stabilization with Threshold Corrections

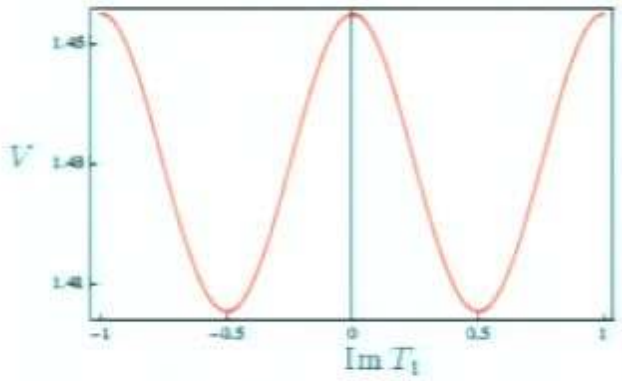
In principle: can stabilize T_1, T_2, ρ_3 and $\ell \sim 1/(S+\bar{S})$

$\rightarrow$ But: requires some tuning of parameters









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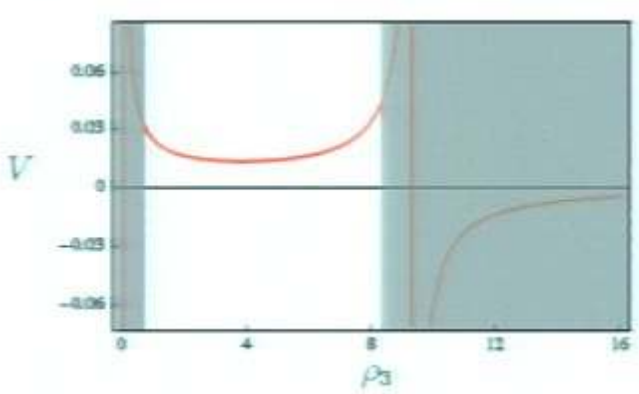

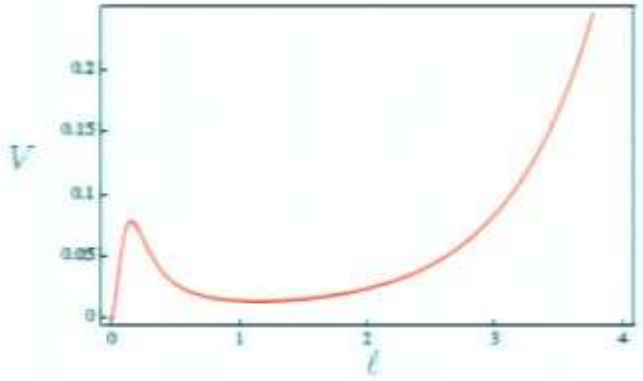

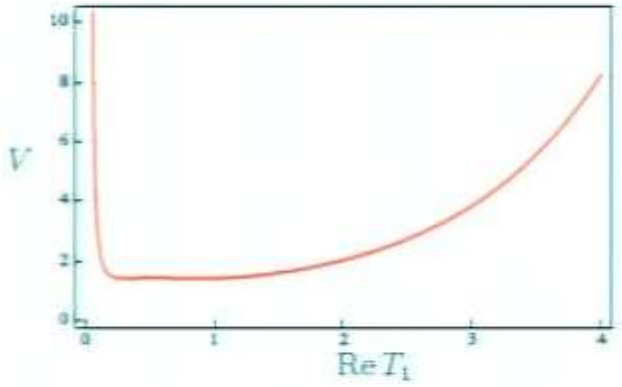

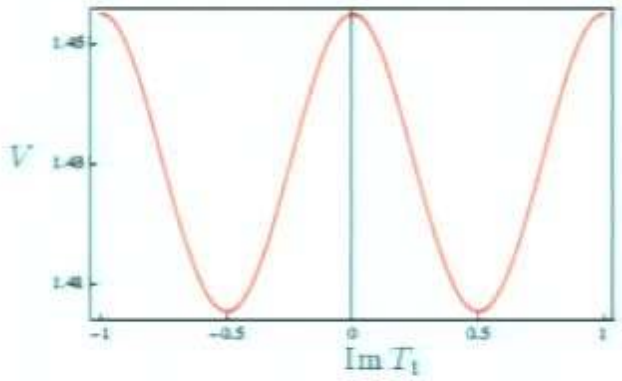

Slope for Inflaton

Various sources for inflaton slope

- Loop corrections from waterfall & gauge sector
- Gaugino condensate $W \supset A(T_i)e^{-cS}$
- Corrections from $\langle W \rangle \propto 0$, $\langle W_{i \neq X} \rangle \propto 0$ & $\langle X \rangle \propto 0$
- Corrections from $\langle D \rangle \propto 0$
- Threshold corrections not only depending on ρ_3
- α'-corrections

Requires systematic study to check if $|\eta| \ll 1$ (in progress)

Additionally:

- Corrections from complex structure stabilization?
- What if fluxes are turned on?

Moduli Stabilization after Inflation

Moduli stabilizing mechanism changes

- During inflation: moduli stabilization tied to $\langle W_X \rangle \neq 0$
- After waterfall phase transition: $\langle W_X \rangle \simeq 0$
  — Need extra terms e.g. gaugino condensate $W \supset A(T_i)e^{-cS}$

Possible issues

1. Cosmological moduli problem?
2. Overshooting problem?
3. Reheating through moduli decays?

Preliminary results

Simulate field evolution for toy model with only T, Φ, H, X

- To evade 1 & 2 seems to require fine tuning
- Typically $m_T \ll m_{\Phi,H,X}$ → Reheating through moduli decays

Conclusion

Matter inflation

- is phenomenologically interesting
- seems suitable to embed in heterotic compactifications
- stabilizes moduli differently during & after inflation
  — interesting possibility

Open issues

- Explicit realization in heterotic orbifolds?
- Better understanding of moduli stabilization
  — other ways to stabilize dilaton?
- Reheating & moduli stabilization after inflation?

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