Title: Quantum Theory - Lecture 15

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Abstract:

Measurement and Decoherence

We have started exploring the physics of quantum subsystems in terms of their reduced density matrices.

This makes sense for interacting subsystems as well as for isolated ones.

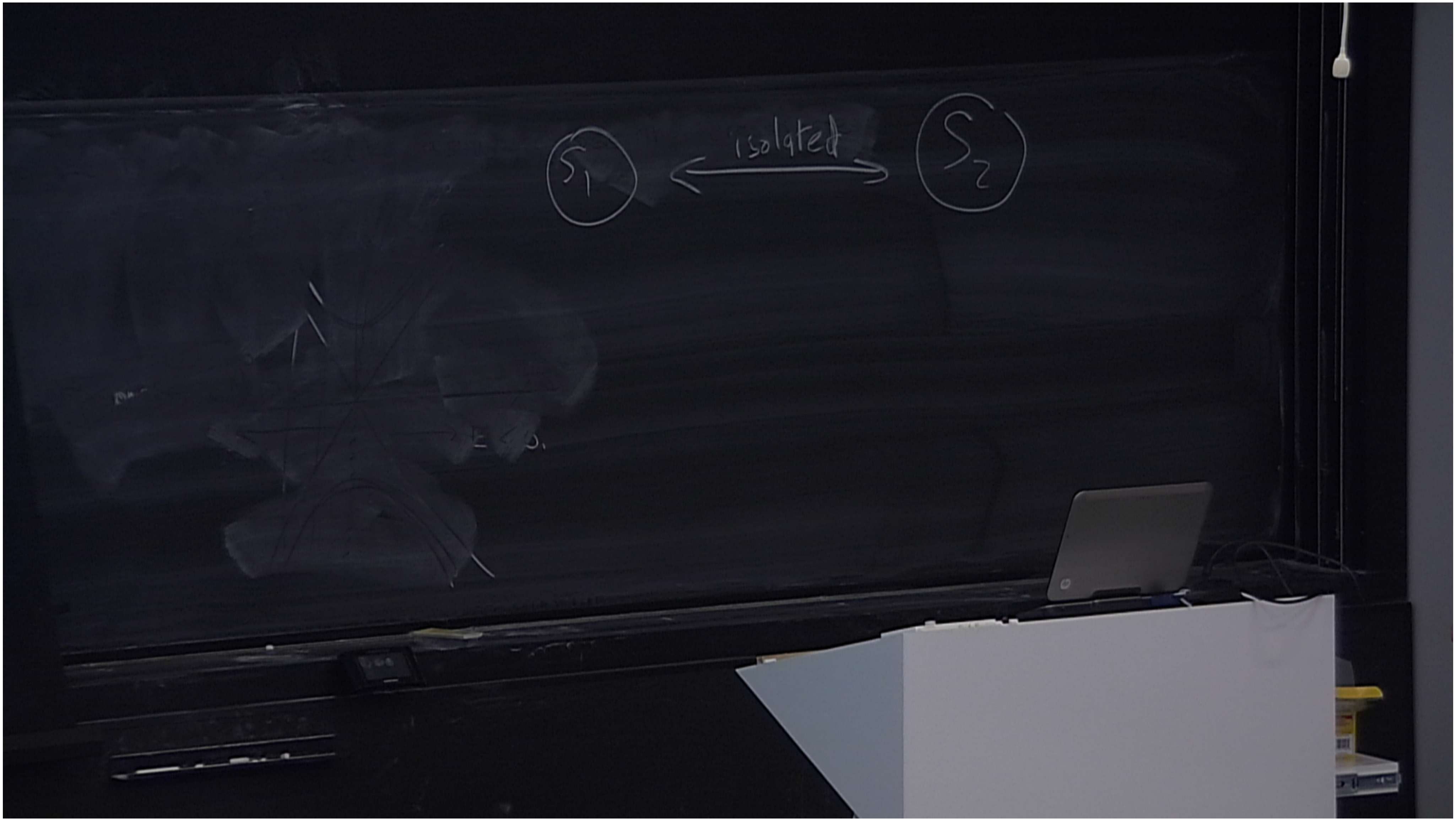
We need to think about what this says about subsystems interacting with an apparatus,

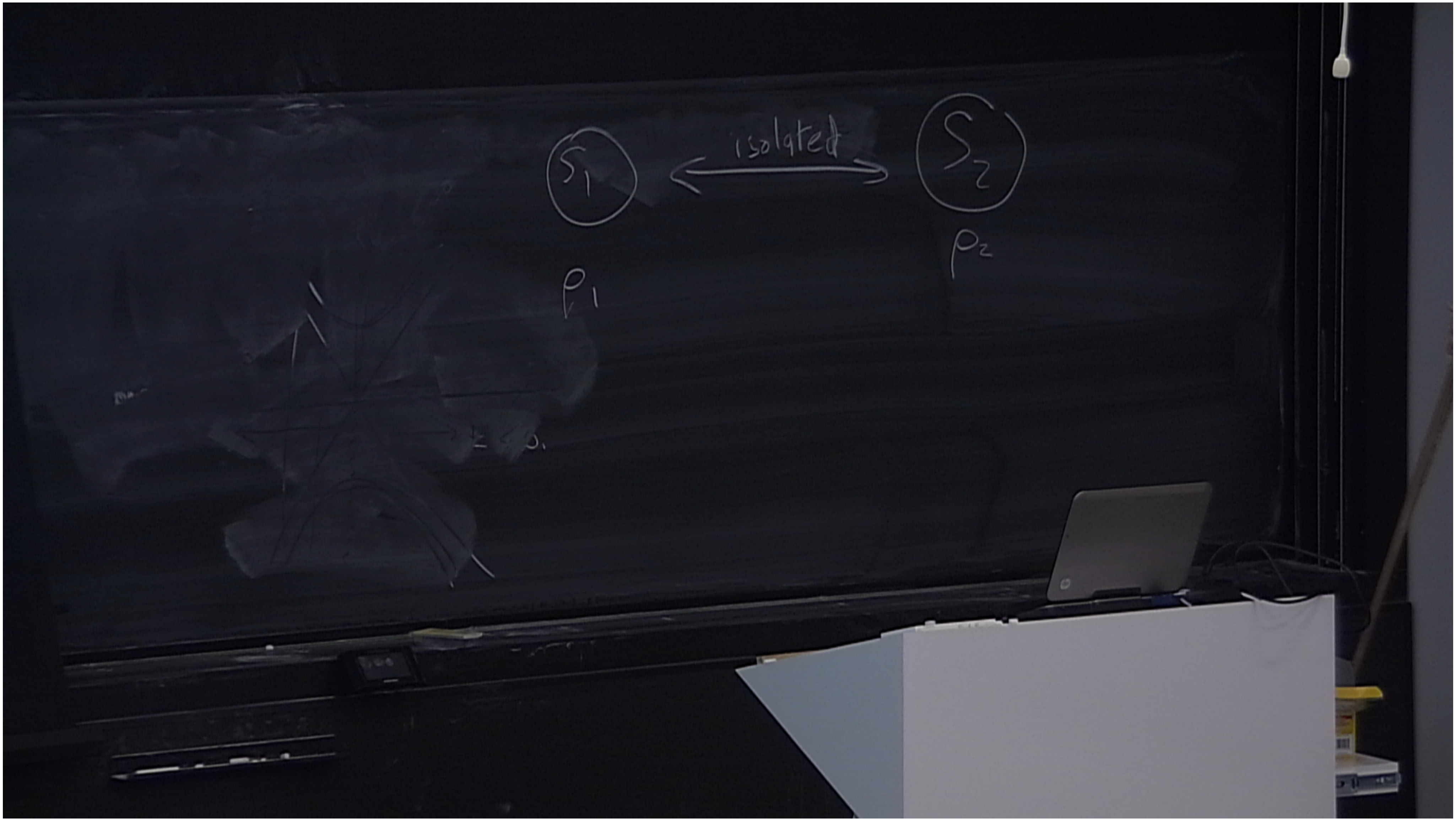
or with the environment.

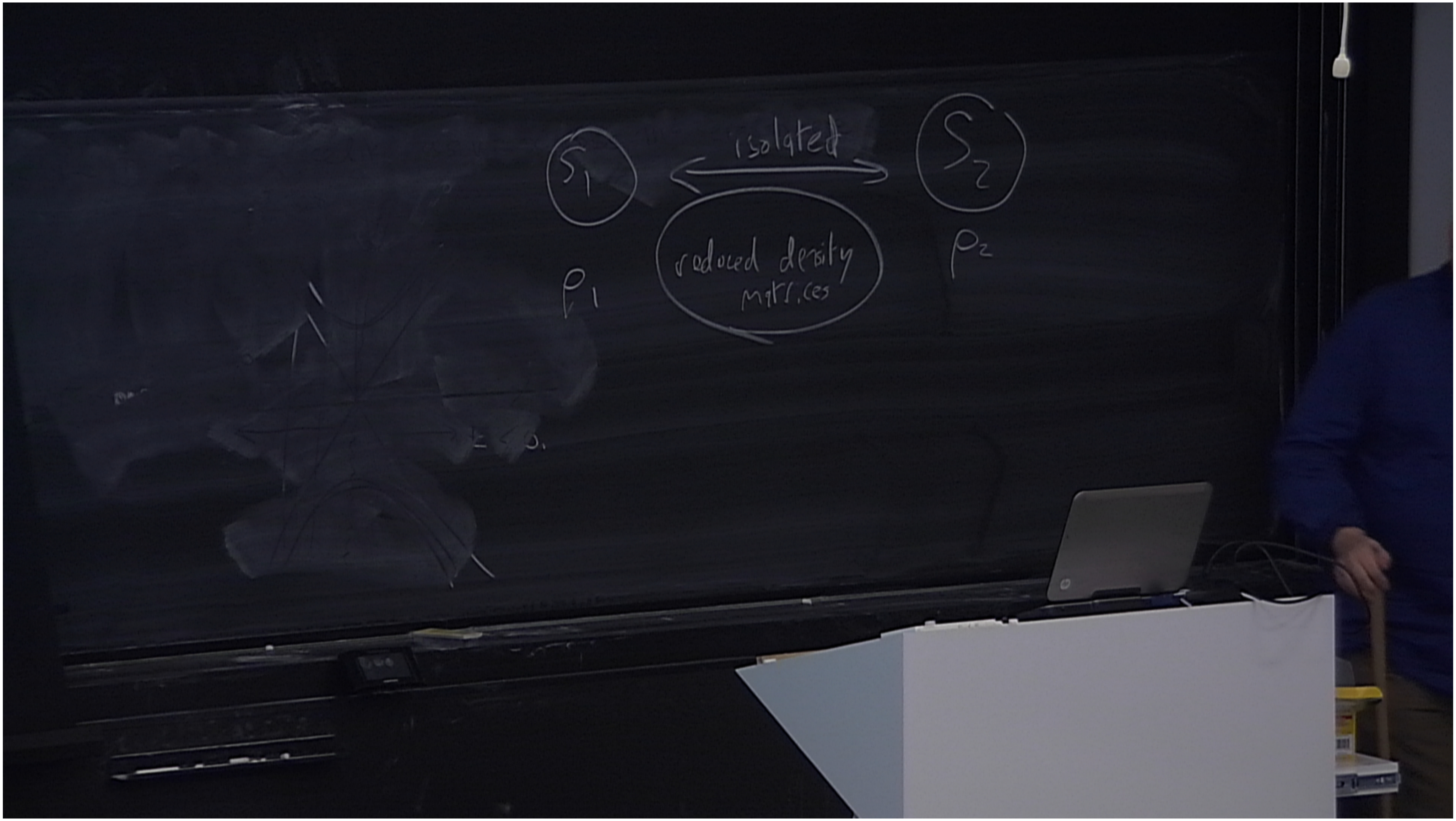
Why? —

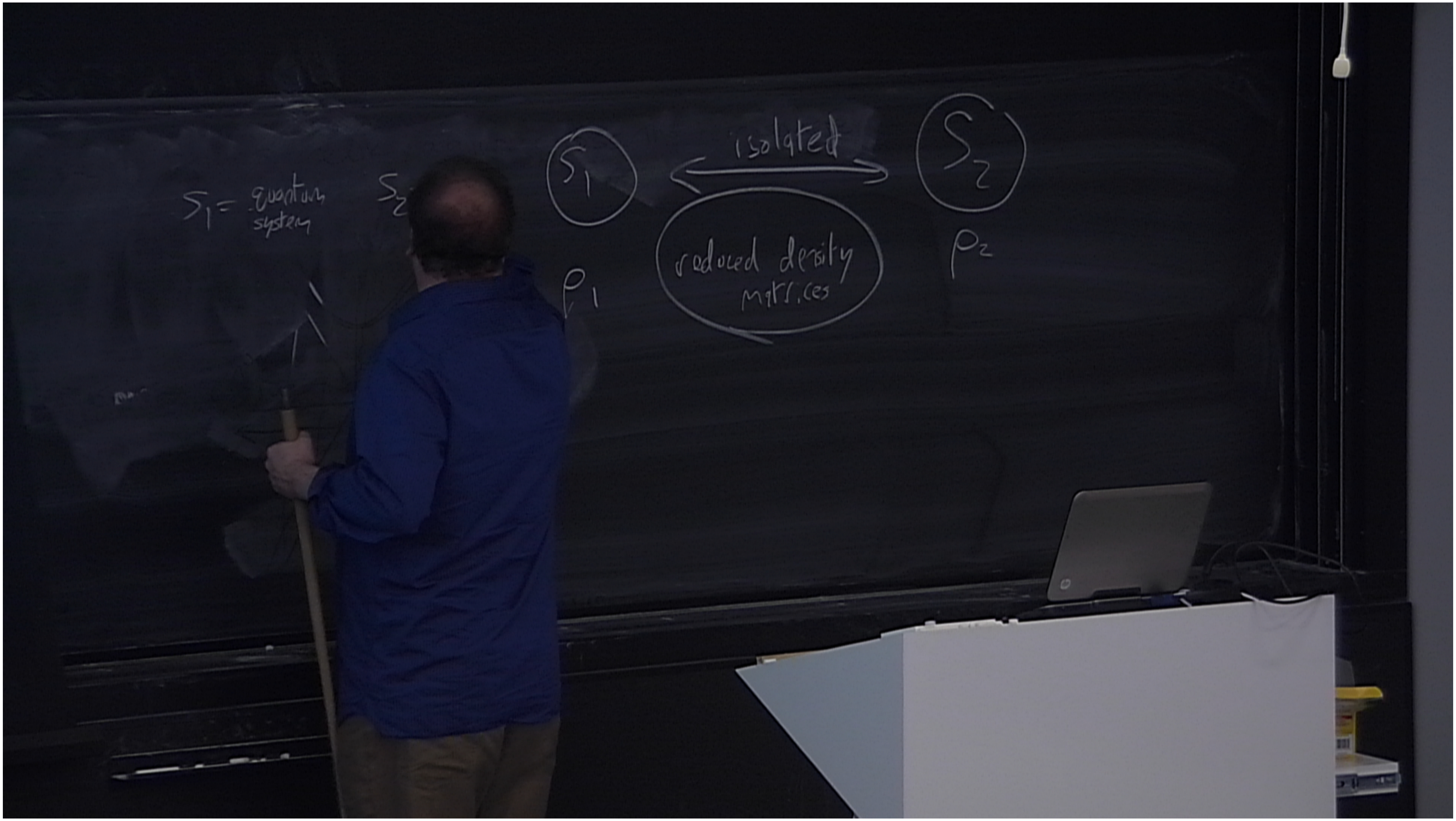
It's fruitful! Gives us new formalism for measurements, dynamical laws for open systems, understanding of the physics of decoherence, ...

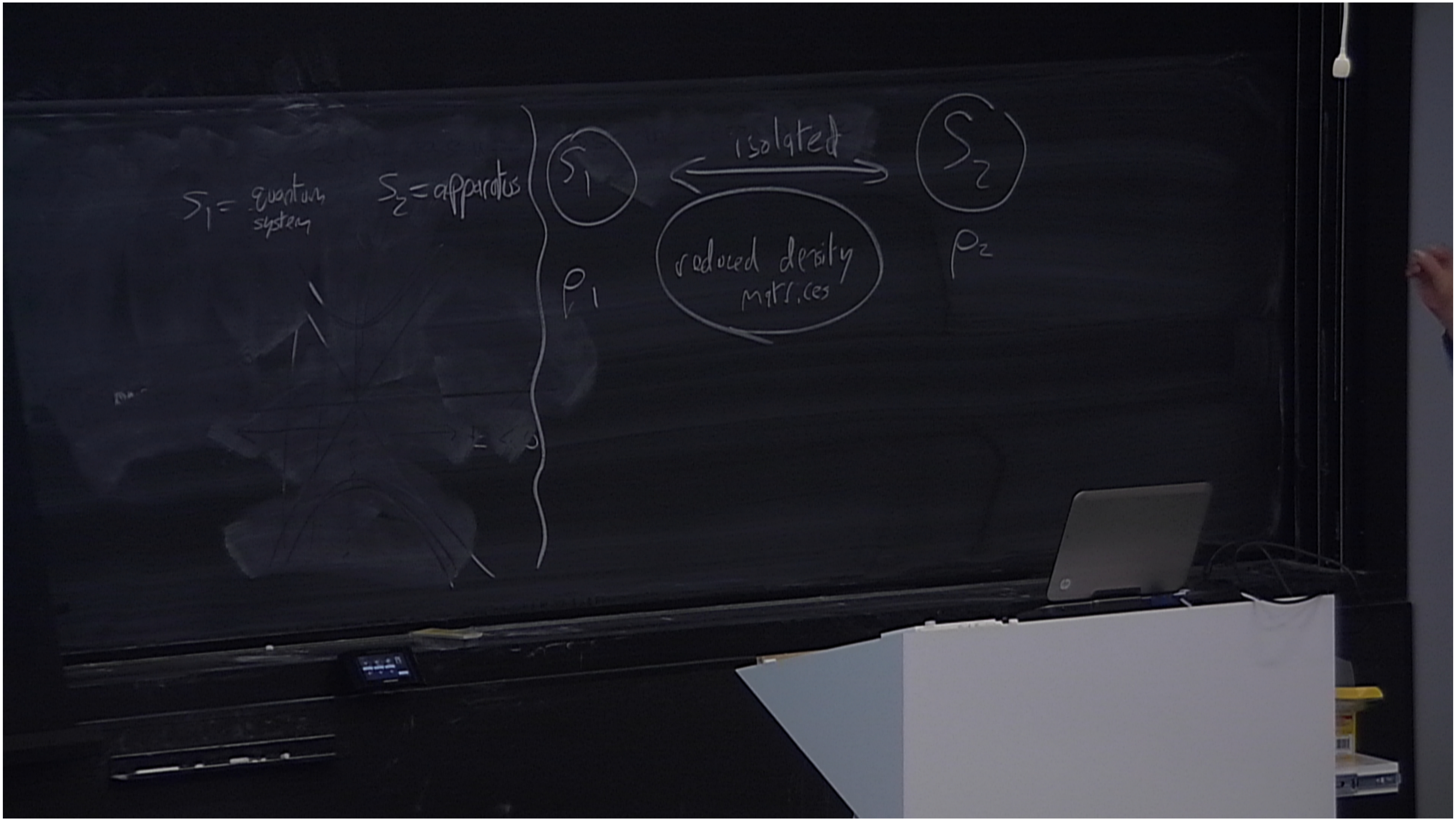
But NB our analysis will not resolve all problems in quantum theory: the so-called measurement problem (a.k.a. reality problem, a.k.a. problem of the appearance of a classical world in quantum theory) will remain a live issue.

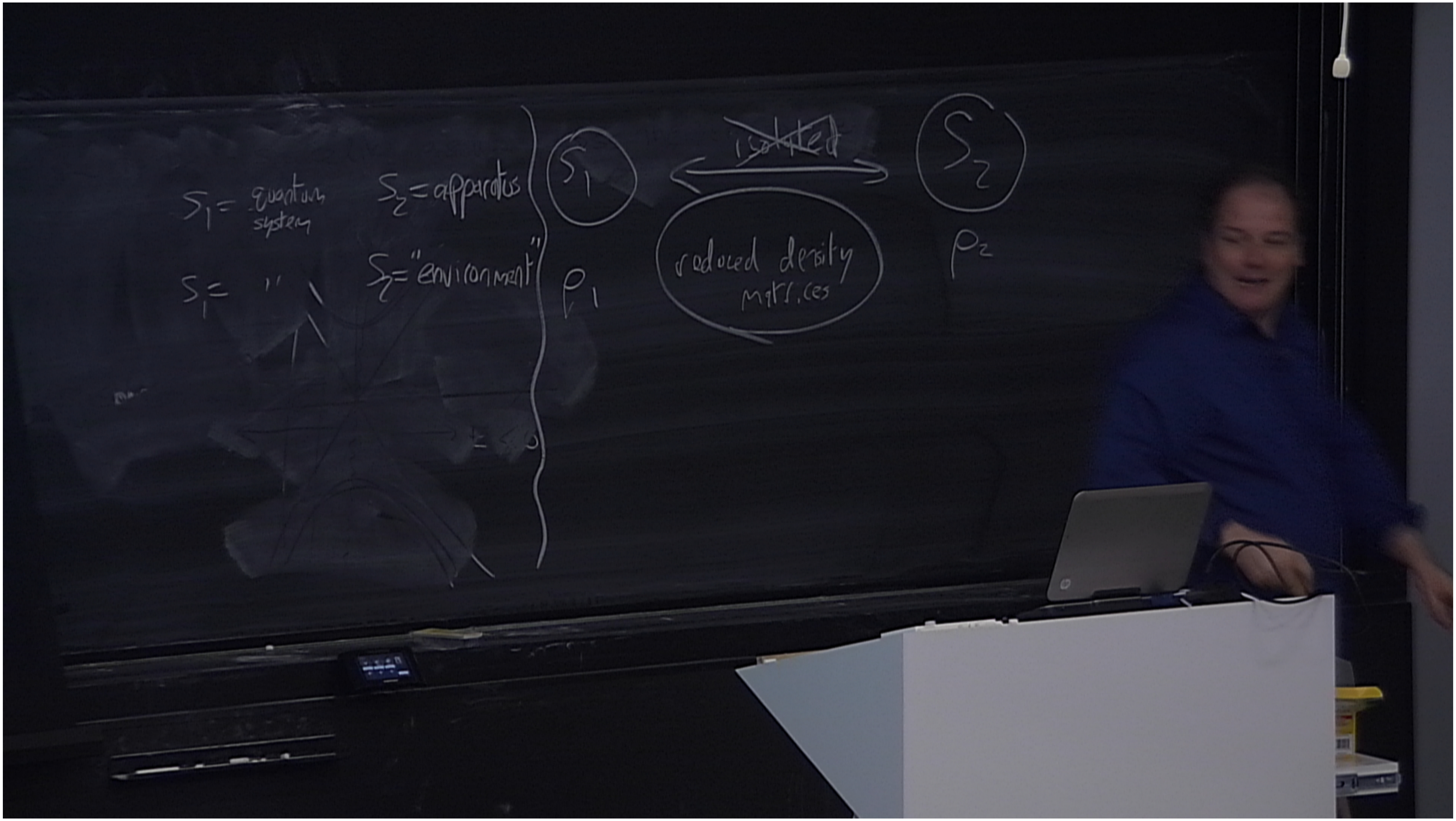

S_1
isolated
S_2


S1
isolated
S2
ρ1
ρ2


S_1
isolated
S_2
reduced density matrices
ρ_1
ρ_2


S_1 = quantum system
S_1
isolated
S_2
ρ_1
reduced density matrices
ρ_2


S_1 = quantum system
S_2 = apparatus
S_1
isolated
S_2
reduced density matrices
ρ_1
ρ_2


S1 = quantum system
S2 = apparatus
S1 = "
S2 = "environment"
S1
~~isolated~~
S2
reduced density matrices
ρ1
ρ2

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Reading View

<u>von Neumann's model of measurement</u> We suppose an apparatus A is designed to measure some observable $M = M^\dagger$ on a quantum system S.

M has eigenstates $\{|m\rangle\}$ in H_S.

Suppose total Hamiltonian is

$$\underbrace{0 \otimes I}_{\text{no system self-interaction}} + \underbrace{H_{int}}_{\text{system-apparatus interaction}} + \underbrace{I \otimes 0}_{\text{no apparatus self-interaction}}.$$

$$H_{int} = \sum_m |m\rangle\langle m| \otimes A_m$$

operators on H_A

projectors onto eigenstates of M

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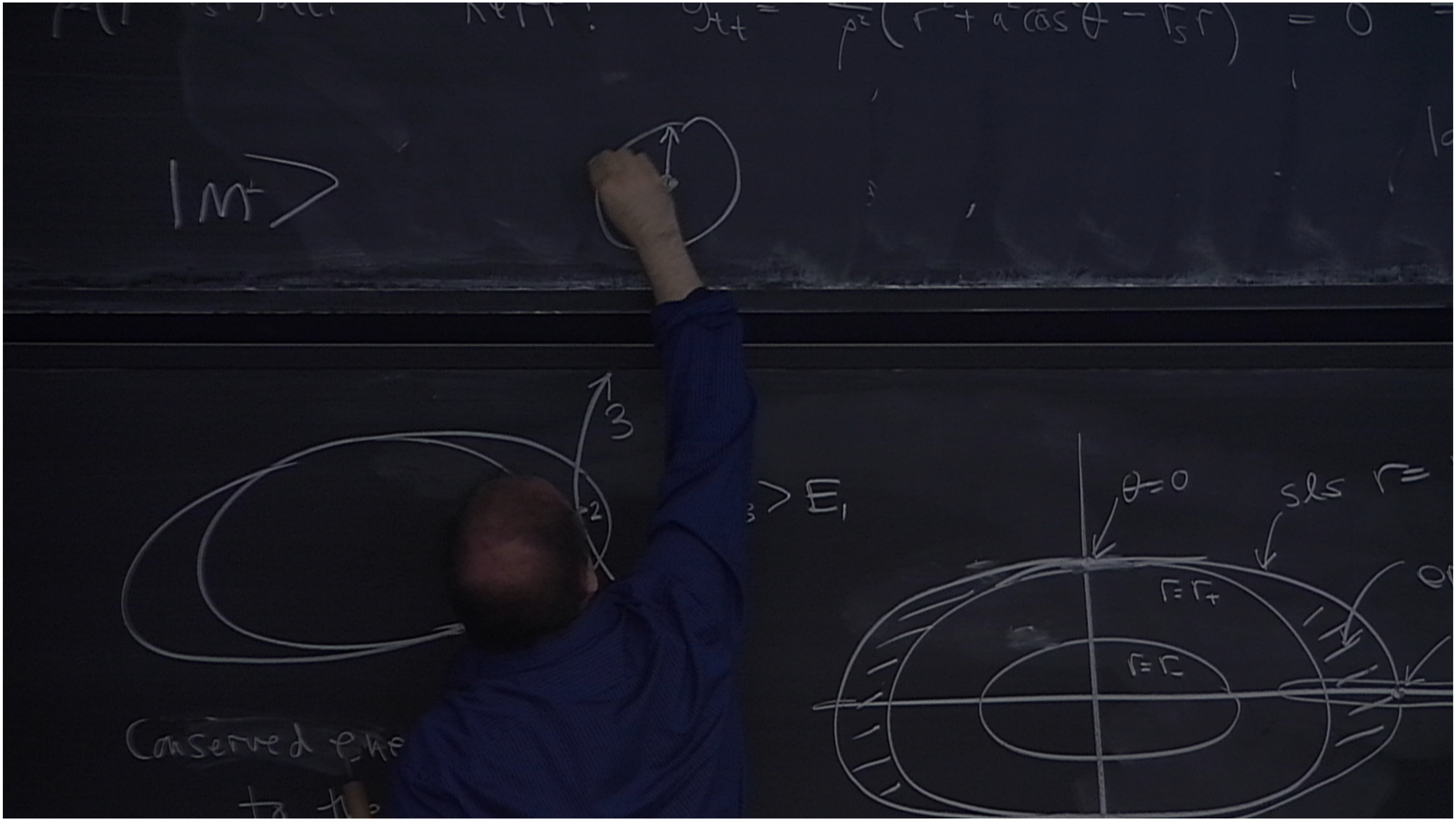
$$H = H_{int} = \sum_m |m\rangle\langle m| \otimes A_m$$

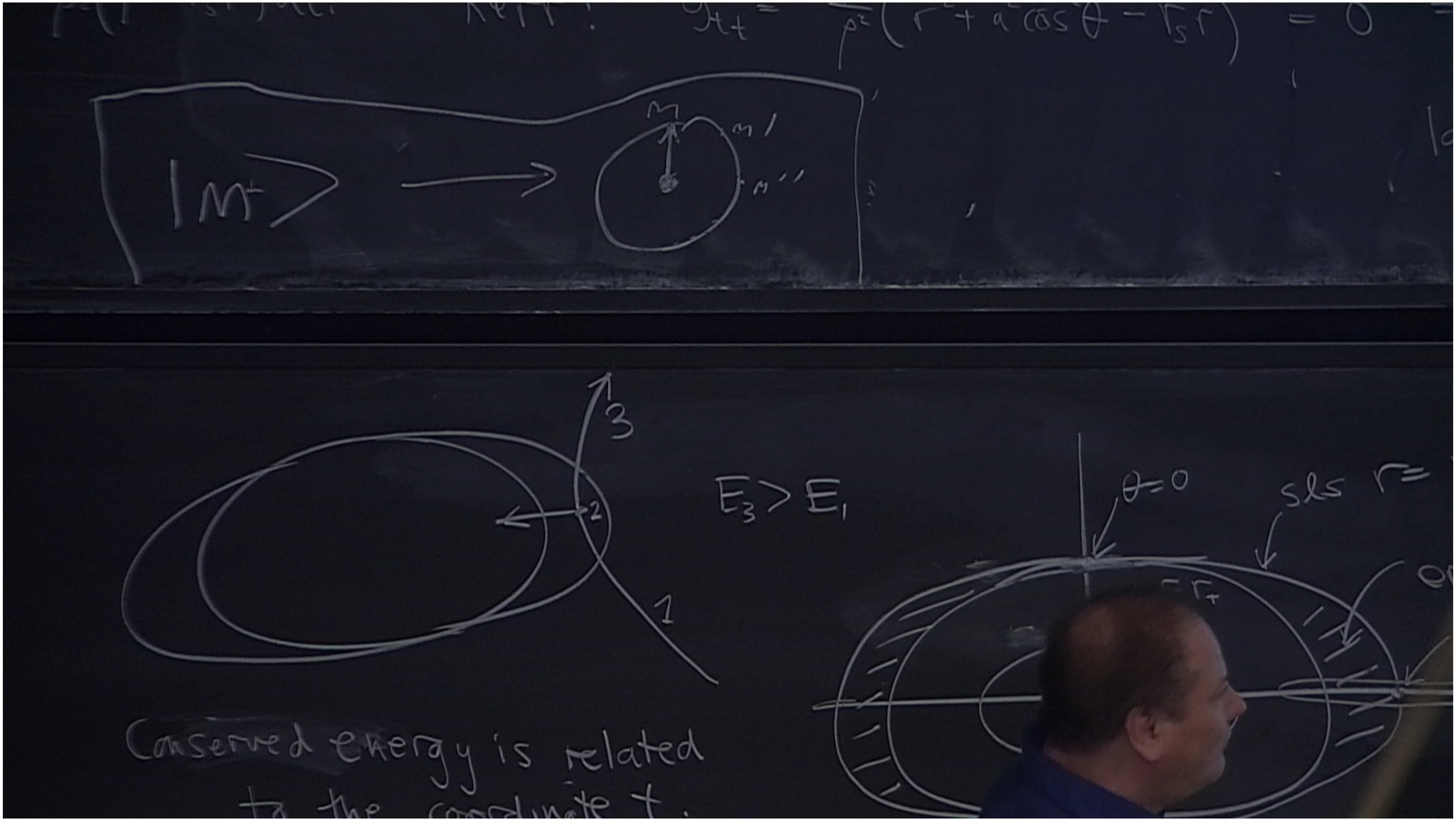
Initial state of system at $t=0$ is $\sum_m c_m |m\rangle$, of apparatus is $|\Phi_0\rangle$.

$$\sum_m c_m |m\rangle \otimes |\Phi_0\rangle \xrightarrow{e^{-iHt/\hbar}} \sum_m c_m |m\rangle \otimes |\Phi_m(t)\rangle$$

where $|\Phi_m(t)\rangle = \exp(-iA_m t/\hbar)\, |\Phi_0\rangle$

In a perfect measurement (using 1950s technology), the state $|\Phi_m(t)\rangle$ could include a pointer on a dial pointing to the number m on a readout. Even if this is not (even approximately) the case, the states $|\Phi_m(t)\rangle$ are often called pointer states.


Im ⟩
3
$>E_1$
$\theta=0$
sls r=
$r=r_+$
$r=r_-$
Conserved ene


$|m\rangle$
3
2
1
$E_3 > E_1$
$\theta = 0$
sls $r =$
r_+
Conserved energy is related
to the coordinate t.

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$$\sum_m c_m |m\rangle \otimes |\Phi_0\rangle \xrightarrow{e^{-iHt/\hbar}} \sum_m c_m |m\rangle \otimes |\Phi_m(t)\rangle$$

System reduced density matrix at time 0 is

$$\rho_s(0) = \mathrm{Tr}_A\left(\sum_{mm'} c_m |m\rangle |\Phi_0\rangle c^*_{m'} \langle m'| \langle \Phi_0| \right)$$

$$= \sum_{mm'} c_m c^*_{m'} |m\rangle\langle m'| .$$

At time t $\rho_s(0) \to \rho_s(t) = \mathrm{Tr}_A\left(\sum_{mm'} c_m |m\rangle |\Phi_m(t)\rangle c^*_{m'} \langle m'| \langle \Phi_{m'}(t)| \right)$

$$= \sum_{mm'} c_m c^*_{m'} \langle \Phi_{m'}(t) | \Phi_m(t)\rangle |m\rangle\langle m'|$$

IF $\langle \Phi_{m'}(t) | \Phi_m(t)\rangle = \delta_{m'm}$ this $= \sum_{mm'} |c_m|^2 |m\rangle\langle m|$

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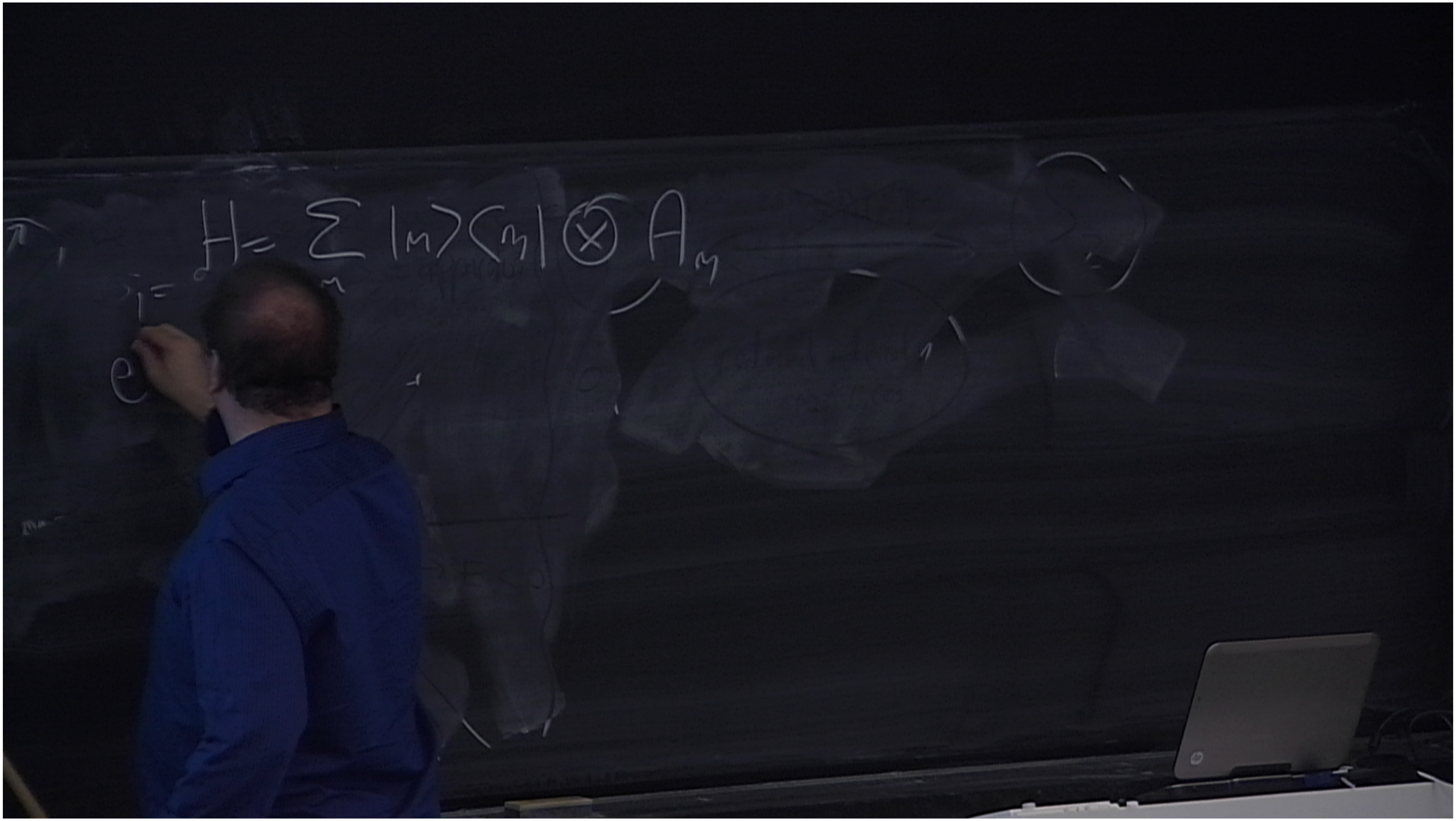
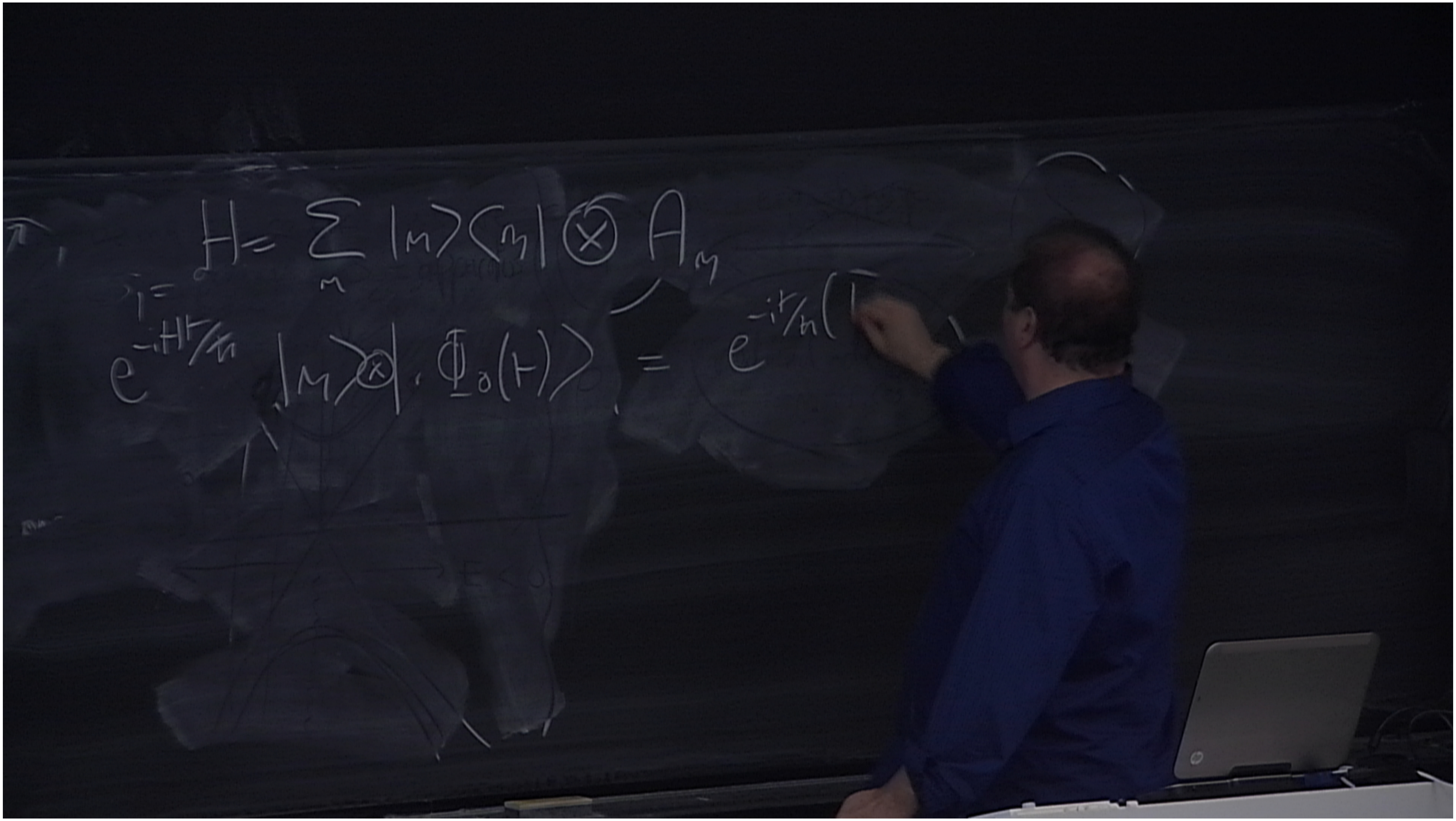

$H = \sum_m |m\rangle\langle m| \otimes A_m$



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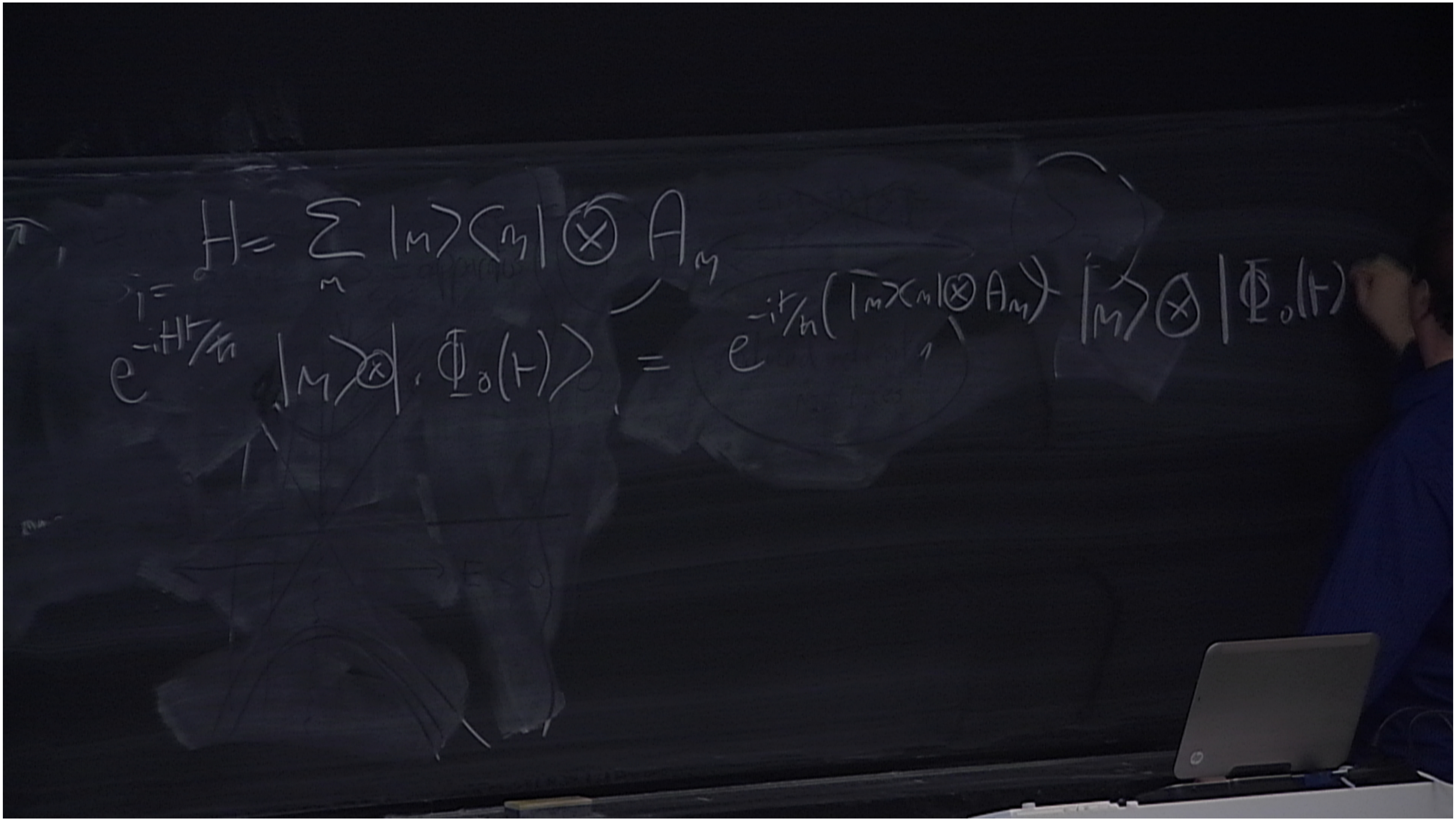
$H = \sum_m |m\rangle\langle m| \otimes A_m$
$e^{-iHt/\hbar} |m\rangle \otimes |\Phi_0(t)\rangle = e^{-it/\hbar(|m\rangle\langle m| \otimes A_m)} |m\rangle \otimes |\Phi_0(t)\rangle$

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Reading View

$$\sum_m c_m |m\rangle \otimes |\Phi_0\rangle \xrightarrow{e^{-iHt/\hbar}} \sum_m c_m |m\rangle \otimes |\Phi_m(t)\rangle$$

System reduced density matrix at time 0 is

$$\rho_s(0) = \mathrm{Tr}_A\Big(\sum_{mm'} c_m |m\rangle |\Phi_0\rangle c^*_{m'} \langle m'| \langle \Phi_0|\Big)$$

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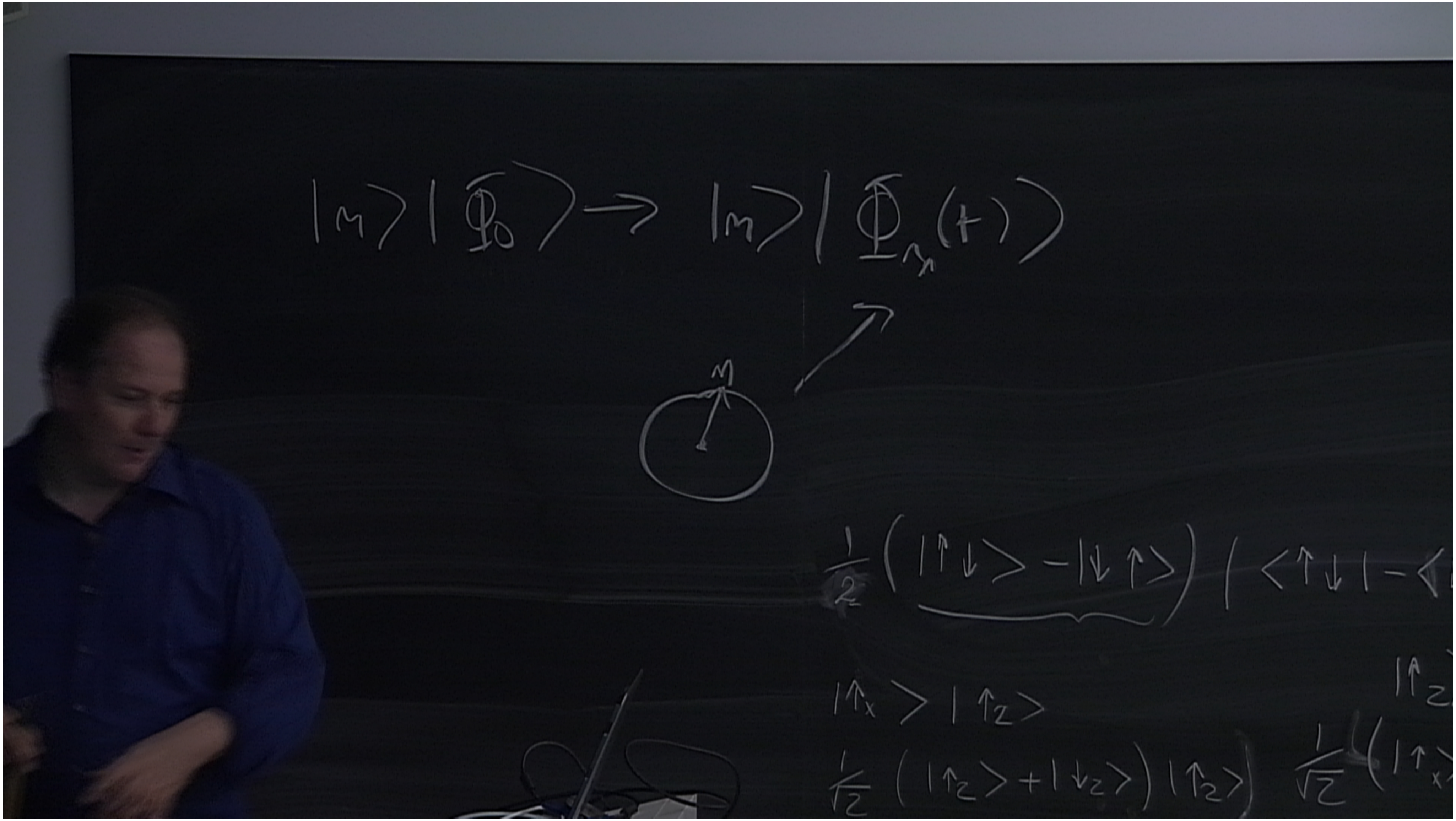
|m⟩|Φ₀⟩ → |m⟩|Φ_m(t)⟩
m
1/2 (|↑↓⟩ − |↓↑⟩)(⟨↑↓| −
|↑_x⟩|↑_z⟩
1/√2 (|↑_z⟩ + |↓_z⟩)|↑_z⟩
1/√2

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$$\rho_s(0) = \mathrm{Tr}_A\Big(\sum_{mm'} c_m |m\rangle |\Phi_0\rangle c^*_{m'} \langle m'| \langle \Phi_0| \Big)$$

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IF $\langle \Phi_{m'}(t) | \Phi_m(t)\rangle = \delta_{m'm}$ $\Rightarrow$ $= \sum_{mm'} |c_m|^2 |m\rangle\langle m|$

which is true if these literally are pointer states

Then $\rho_s(t)$ is diagonal in the observable eigenbasis : interaction has killed the off-diagonal terms

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Reading View

$$\rho_s(0) = \mathrm{Tr}_A \Big(\sum_{MM'} c_M |M\rangle |\Phi_0\rangle c^*_{M'} \langle M'| \langle \Phi_0| \Big)$$

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Then $\rho_s(t)$ is diagonal in the observable eigenbasis : interaction has killed the off-diagonal terms

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Reading View

Suppose this indeed holds: $\rho_S(t) = \sum_m |c_m|^2 \, |m\rangle\langle m|$.

The reduced density matrix $\rho_S(t)$ — an improper mixed state —

is physically indistinguishable (by experiments on S alone)

from the proper mixed state defined by $\{\{|c_m|^2\}, \{|m\rangle\}\}$

probabilities states

I.e. the density matrix has the same mathematical form as the one we would derive if we assumed the projection postulate for the projective measurement $\{|m\rangle\langle m|\}$ on S and calculated from the perspective of an observer aware the measurement has taken place but ignorant of the outcome.

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What does this explain? $\rho_s(t) = \sum |c_m|^2 \, |m\rangle\langle m|$.

Explains why experiments on S at time t (and later times, if the interaction is not reversed and S is then isolated) give outcomes consistent with describing the state of S as a proper mixture. For instance, we will find no interference between components $|m\rangle$ and $|m'\rangle$ if we measure $P = \frac{1}{2}(|m\rangle + |m'\rangle)(\langle m| + \langle m'|)$

$$\mathrm{Tr}(P\rho_s(t)) = \frac{1}{2}\mathrm{Tr}\Big((|m\rangle + |m'\rangle)(\langle m| + \langle m'|)\sum_{m''}|c_{m''}|^2 \, |m''\rangle\langle m''|\Big)$$

$$= \frac{1}{2}\left(|c_m|^2 + |c_{m'}|^2\right)$$

no interference

whereas
$$\mathrm{Tr}(P\rho_s(0)) = \frac{1}{2}(\langle m| + \langle m'|)\sum_{m''m'''} c_{m''}c^*_{m'''} \, |m''\rangle\langle m'''| \, (|m\rangle + |m'\rangle)$$

$$= \frac{1}{2}\left(|c_m|^2 + |c_{m'}|^2 + 2\,\mathrm{Re}(c_m c^*_{m'})\right)$$

interference term

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What does this not explain? $\rho_s(t) = \sum |c_m|^2 |m\rangle\langle m|$.

Does not explain the fact that we actually see one definite outcome m. We still need to apply the projection postulate to conclude that: we haven't derived it from our analysis.

Recall: $\rho_s(t)$ is an improper mixed state, obtained as the partial trace of a pure entangled state of S+A.

We haven't justified treating it as a proper mixture.

Still, intriguing! We obtained some physical predictions (loss of interference, form of density matrix) for the post-interaction state of S by looking at unitary evolution of S+A alone. Next project: explore further! generalize!

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von Neumann's model of measurement assumed a very specific Hamiltonian evolution

$$H = O \otimes I + H_{int} + I \otimes O, \quad H_{int} = \sum_m |m\rangle\langle m| \otimes A_m$$

projectors onto eigenstates of M

operators on H_A

and that the pointer states $|\Phi_m(t)\rangle = \exp(-iA_m t/\hbar)\,|\Phi_0\rangle$

are orthogonal

Suppose now we consider a general interaction $U \in U(t)$ between S and A, but still assume A has a pointer which defines orthogonal pointer states $|m\rangle_A$.

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Reading View

Suppose now we consider a general interaction $U_{SA} \equiv U_{SA}(t)$ between S and A, but still assume A has a pointer which defines orthogonal pointer states $|k\rangle_A$.

$$|\psi_0\rangle_S |\Phi_0\rangle_A \rightarrow U_{SA}(|\psi_0\rangle|\Phi_0\rangle).$$

After this evolution, the system reduced density matrix is

$$\rho_S^{final} = Tr_A\left((U_{SA}|\psi_0\rangle|\Phi_0\rangle)(\langle\psi_0|\langle\Phi_0|U_{SA}^\dagger)\right)$$

$$= \sum_k \underline{\langle k|U_{SA}|\psi_0\rangle|\Phi_0\rangle} \langle\psi_0|\langle\Phi_0|U_{SA}^\dagger|k\rangle$$

Look at this expression. It's a state in S, depending linearly on $|\psi_0\rangle$

i.e. can write as $A_k|\psi_0\rangle$

where A_k is some linear operator depending on k, U_{SA}, $|\Phi_0\rangle$ — a Kraus operator

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Suppose now we consider a general interaction $U_{SA} \equiv U_{SA}(t)$ between S and A, but still assume A has a pointer which defines orthogonal pointer states $|k\rangle_A$, and that these form a complete basis for H_A.

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Reading View

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$$|\psi_0\rangle_S |\Phi_0\rangle_A \rightarrow U_{SA}(|\psi_0\rangle|\Phi_0\rangle).$$

After this evolution, the system reduced density matrix is

$$I_A = \sum_k |k\rangle\langle k|$$

$$\rho_S^{final} = Tr_A((U_{SA}|\psi_0\rangle|\Phi_0\rangle)(\langle\psi_0|\langle\Phi_0|U_{SA}^{\dagger}))$$

$$= \sum_k \underline{\langle k|U_{SA}|\psi_0\rangle|\Phi_0\rangle} \langle\psi_0|\langle\Phi_0|U_{SA}^{\dagger}|k\rangle$$

Look at this expression. It's a state in S, depending linearly on $|\psi_0\rangle$

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$$\rho_S^{final} = Tr_A\left((U_{SA}|\Psi_0\rangle|\Phi_0\rangle)(\langle\Psi_0|\langle\Phi_0|U_{SA}^\dagger) \right)$$

$$= \sum_K \langle K|U_{SA}|\Psi_0\rangle|\Phi_0\rangle\langle\Psi_0|\langle\Phi_0|U_{SA}^\dagger|K\rangle$$

$$= \sum_K A_K|\Psi_0\rangle\langle\Psi_0|A_K^\dagger = \sum_K A_K\,\rho_S^{initial}A_K^\dagger$$

The Kraus operators A_K define an evolution law $\boxed{\rho_S \to \sum_K A_K \rho_S A_K^\dagger}$ for the system reduced density matrix. We derived this for $\rho_S = |\Psi_0\rangle\langle\Psi_0|$ pure.

But it's linear in density matrix input — works for any ρ_S, pure or mixed.

$$1 = Tr(\rho_S^{final}) = Tr\left(\sum_K A_K \rho_S^{initial} A_K^\dagger\right) = Tr\left(\left(\sum_K A_K^\dagger A_K\right)\rho_S^{initial}\right)$$

True for any $\rho_S^{initial}$, so $\boxed{\sum_K A_K^\dagger A_K = I}$ (Completeness equation for Kraus operators)

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Kraus operators and generalized measurements

$$\rho_S \to \sum_k A_k \rho_S A_k^\dagger$$

$$\sum A_k^\dagger A_k = I$$

Recall that $\quad A_k |\psi_0\rangle_S = \langle k | U_{SA} |\psi_0\rangle_S |\Phi_0\rangle_A$

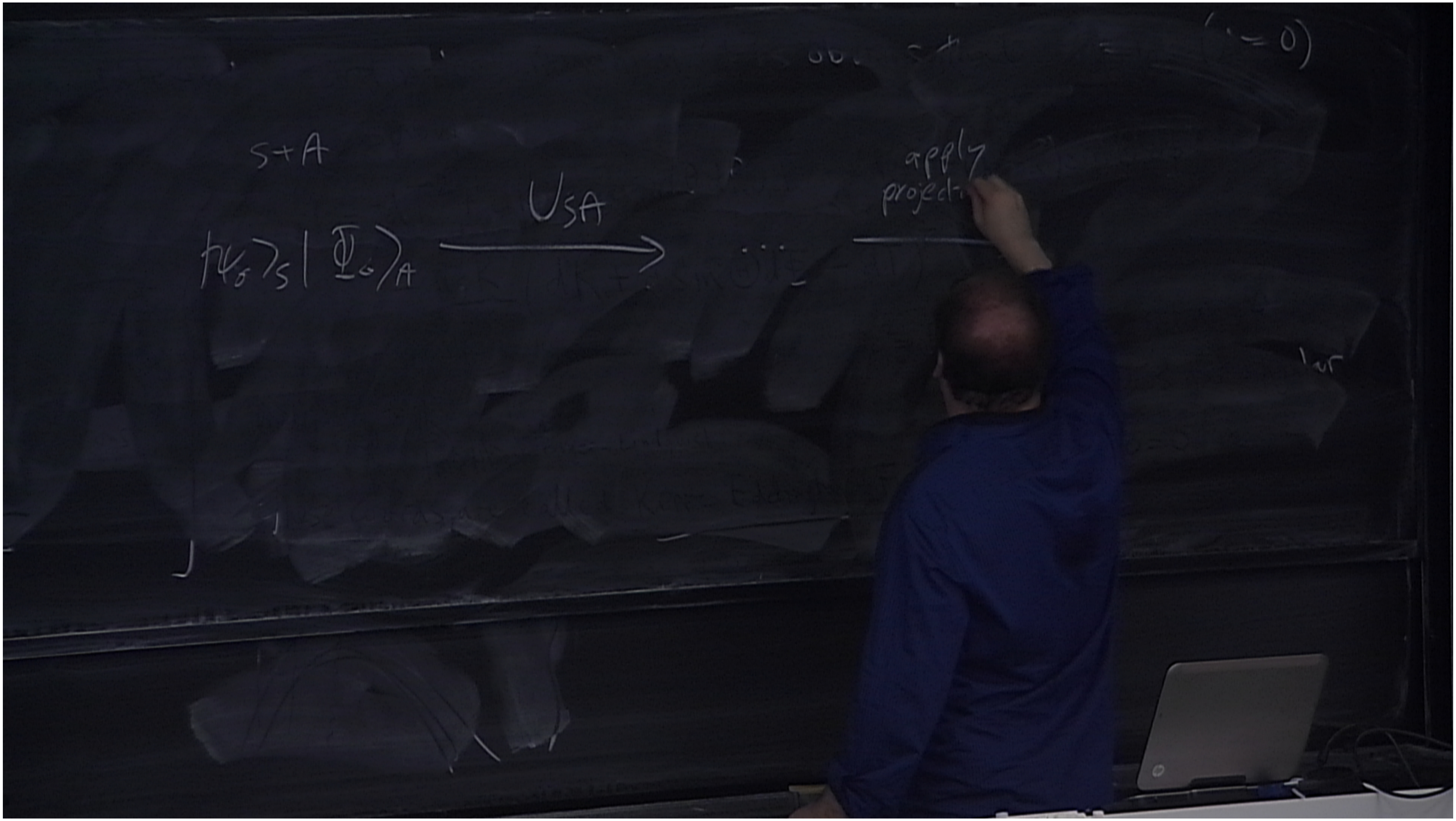
pointer state k

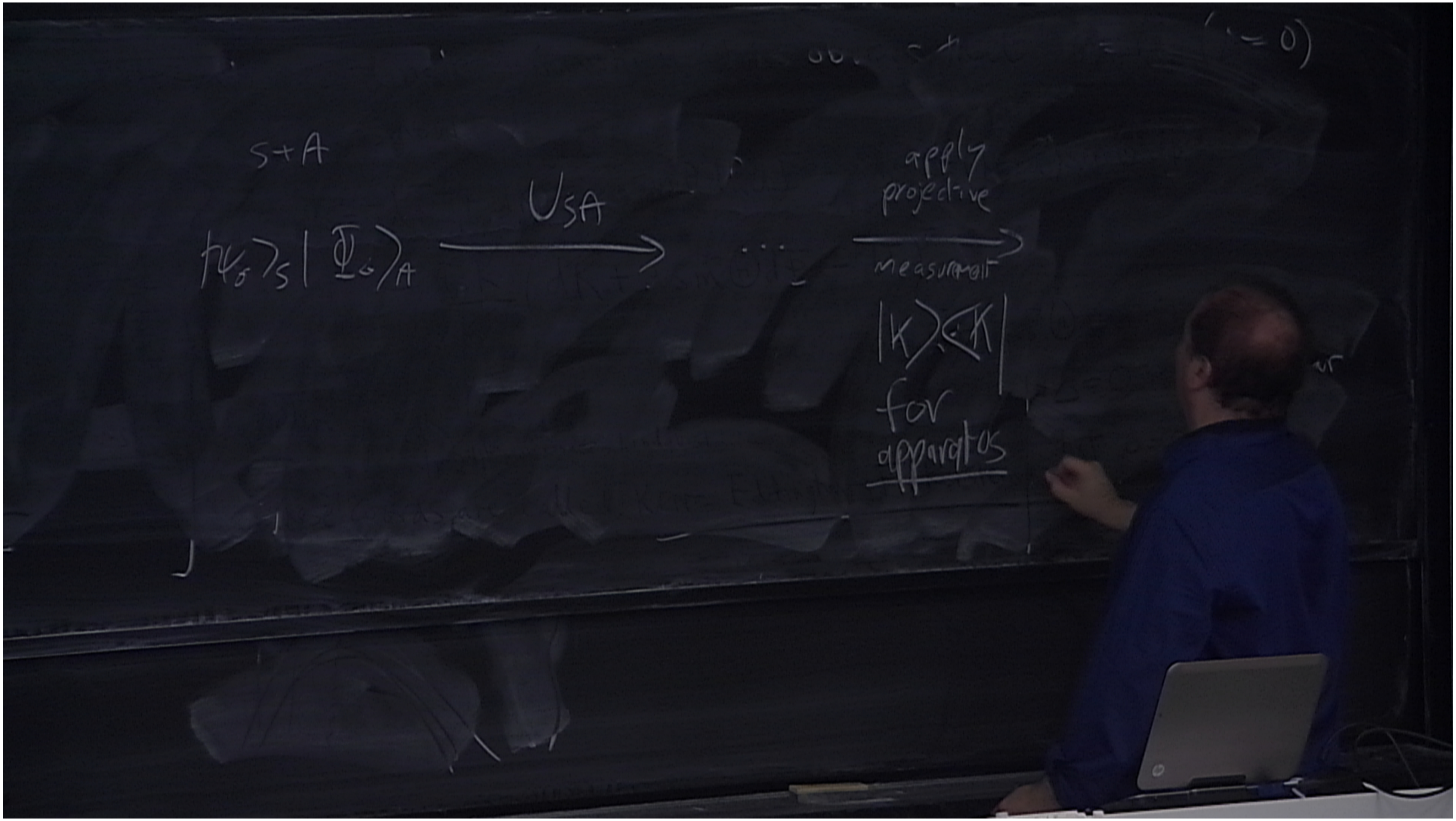
If we now observe the pointer's position, we select one value of K.

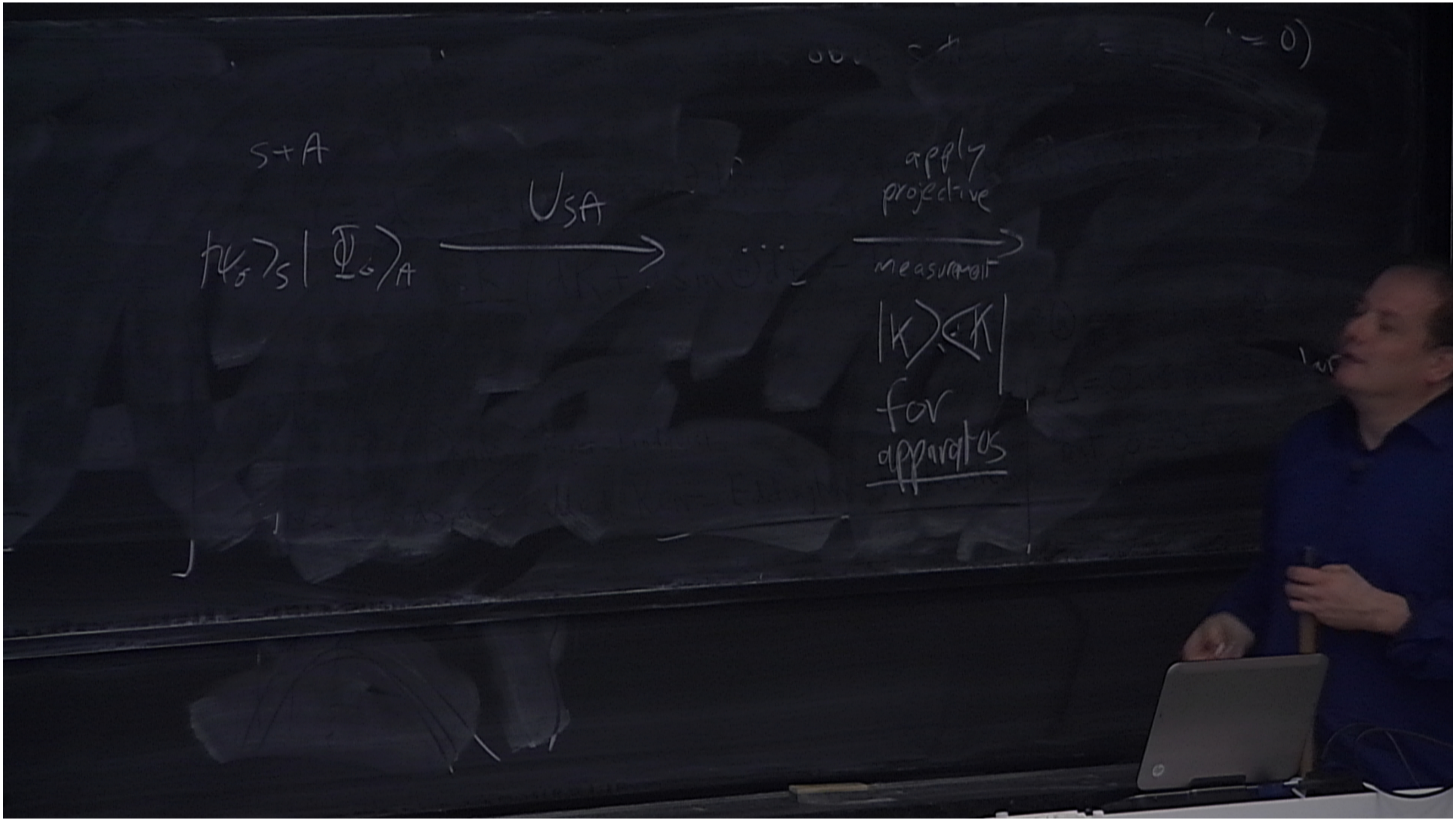
We have defined a generalised type of measurement on the system: apply a completely general U_{SA}, then look at the apparatus pointer reading.

$$\text{Prob(outcome } k) = \mathrm{Tr}\left(A_k |\psi_0\rangle_S \langle\psi_0|_S A_k^\dagger\right) = \mathrm{Tr}\left(A_k \rho_S A_k^\dagger\right)$$

$$\text{Normalised post-measurement state} = \frac{A_k |\psi_0\rangle_S \langle\psi_0|_S A_k^\dagger}{\mathrm{Tr}\left(A_k |\psi_0\rangle_S \langle\psi_0|_S A_k^\dagger\right)} = \frac{A_k \rho_S A_k^\dagger}{\mathrm{Tr}\left(A_k \rho_S A_k^\dagger\right)}$$


S+A
apply


S+A
U_{SA}
$|\psi_0\rangle_S |\Phi_0\rangle_A$
apply projective
measurement
$|k\rangle\langle k|$
for apparatus


S+A
U_{SA}
$|\psi_0\rangle_S |\Phi_0\rangle_A$
...
apply projective
measurement
$|k\rangle\langle k|$
for
apparatus

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Reading View

Kraus operators and generalized measurements

$$\rho_S \to \sum_k A_k \rho_S A_k^\dagger$$

$$\sum A_k^\dagger A_k = I$$

Recall that $\quad A_k |\psi_0\rangle_S = \langle k| U_{SA} |\psi_0\rangle_S |\Phi_0\rangle_A$

pointer state k

If we now observe the pointer's position, we select one value of k.

We have defined a generalised type of measurement on the system: apply a completely general U_{SA}, then look at the apparatus pointer reading.

$$\text{Prob(outcome } k) = \mathrm{Tr}\left(A_k |\psi_0\rangle_S \langle\psi_0|_S A_k^\dagger\right) = \mathrm{Tr}\left(A_k \rho_S A_k^\dagger\right)$$

$$\text{Normalised post-measurement state} = \frac{A_k |\psi_0\rangle_S \langle\psi_0|_S A_k^\dagger}{\mathrm{Tr}\left(A_k |\psi_0\rangle_S \langle\psi_0|_S A_k^\dagger\right)} = \frac{A_k \rho_S A_k^\dagger}{\mathrm{Tr}\left(A_k \rho_S A_k^\dagger\right)}$$

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Reading View

Kraus operators and generalized measurements We obtained the Kraus operators A_k from a specific construction: $A_k |\psi_0\rangle_S = \langle k| U_{SA} |\psi_0\rangle_S |\Phi_0\rangle_A$ and showed normalisation requires $\sum_k A_k^\dagger A_k = I$.

But – important though simple result – the converse holds: given any set of linear operators on H_S obeying $\sum_k A_k^\dagger A_k = 1$ we can find an "apparatus space" H_A, a unitary evolution U_{SA}, an initial "apparatus state" $|\Phi_0\rangle_A$ and an orthonormal "pointer basis" $|k\rangle$ such that

$$A_k |\psi_0\rangle_S = \langle k| U_{SA} |\psi_0\rangle_S |\Phi_0\rangle_A$$

I.e. for any set of Kraus operators $\sum A_k^\dagger A_k = I$ we can find a way of implementing the corresponding generalized measurement.

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Reading View

Claim for any set of Kraus operators $\sum_k A_k^\dagger A_k = I$ we can find a way of implementing the corresponding generalized measurement.

Sketch proof Take H_A to have orthonormal basis $\{\underbrace{|0\rangle}_{\text{extra state}}\} \cup \{\underbrace{|k\rangle}_{\text{one for each } A_k}\}$

Define $U_{SA}|\psi_0\rangle_S|0\rangle_A = \sum (A_k|\psi_0\rangle_S)|k\rangle$ (any $|\psi_0\rangle_S$)

(Notice this is consistent since $|RHS|^2 = \langle\psi_0|_S \sum A_k^\dagger A_k |\psi_0\rangle_S = 1$.)

Complete the definition of the unitary operator U_{SA} by mapping

{orthogonal states to those considered on LHS} → {orthogonal states to those on RHS}

(Doesn't matter precisely how: not hard to show this can always be done.)

So now: $|\psi_0\rangle_S|0\rangle_A \xrightarrow{U_{SA}} \sum_k (A_k|\psi_0\rangle_S)|k\rangle_A \xrightarrow[\text{basis}]{\text{measure in } |k\rangle_A} A_k|\psi_0\rangle_S$ (for some outcome k) QED

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Reading View

Kraus operators and the evolution of quantum subsystems

We saw previously that the Kraus operators A_k define an evolution law $\boxed{\rho_s \to \sum_k A_k \rho_s A_k^\dagger}$ for the single-time evolution of $\rho_s(0)$ to $\rho_s(t)$, when S is interacting with some other subsystem A.

Let's think abstractly about the most general possible sequence of operations that can act on a quantum system, with the effect $\rho_s \to \mathcal{E}(\rho_s)$

$\mathcal{E}$ must satisfy:

1. linear $p\rho_s^1 + (1-p)\rho_s^2 \to p\,\mathcal{E}(\rho_s^1) + (1-p)\,\mathcal{E}(\rho_s^2)$

   so $\mathcal{E}(p\rho_s^1 + (1-p)\rho_s^2) = p\,\mathcal{E}(\rho_s^1) + (1-p)\,\mathcal{E}(\rho_s^2)$

2. trace-preserving $\mathrm{Tr}(\mathcal{E}(\rho_s)) = 1$ if $\mathrm{Tr}(\rho_s) = 1$

3. positive $\mathcal{E}(\rho_s)$ is a positive (semidefinite) operator whenever ρ_s is

4. Completely positive If S, S' are two subsystems then $\mathcal{E} \otimes I_{s'}(\rho_{ss'})$ must be positive

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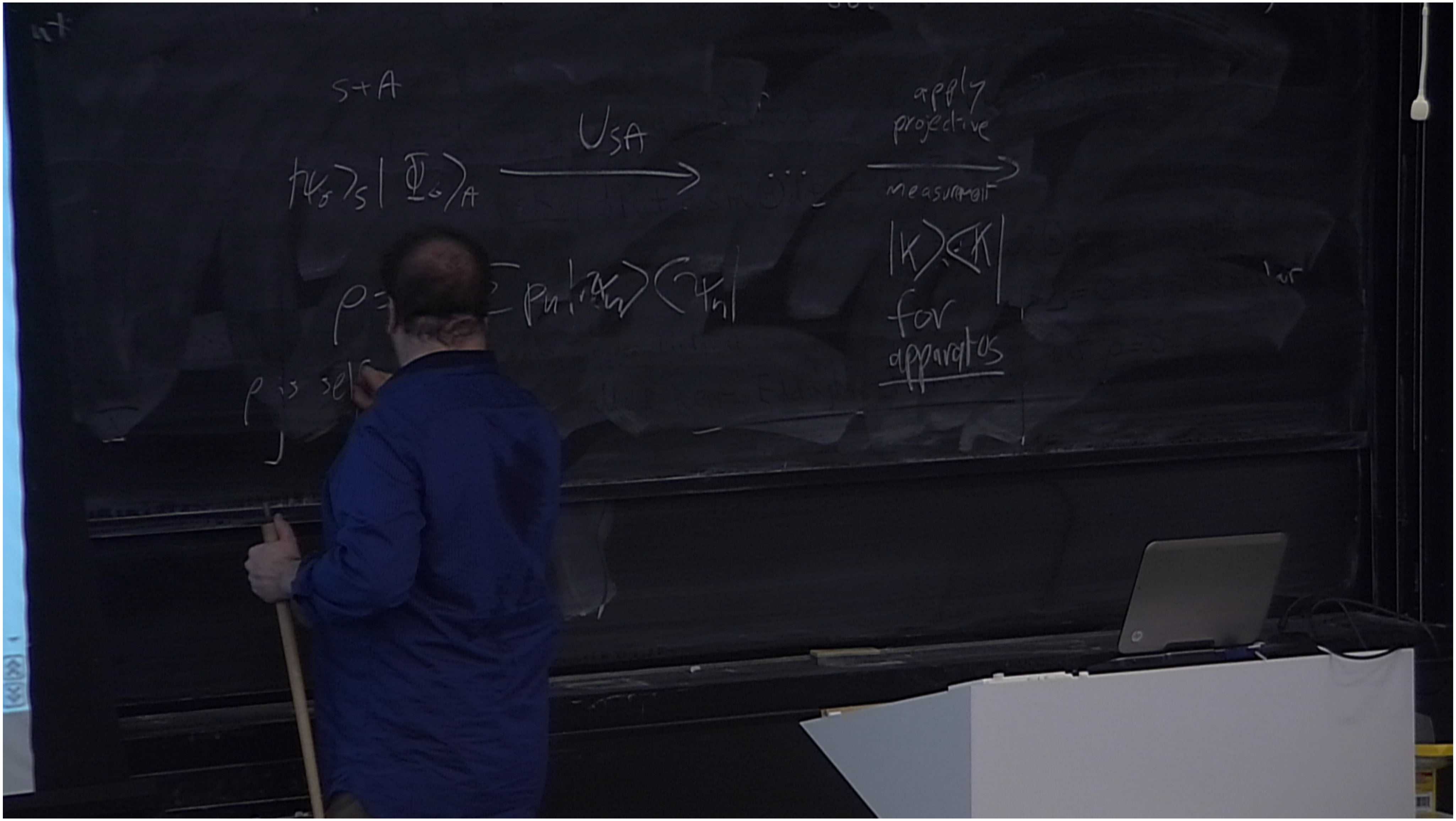
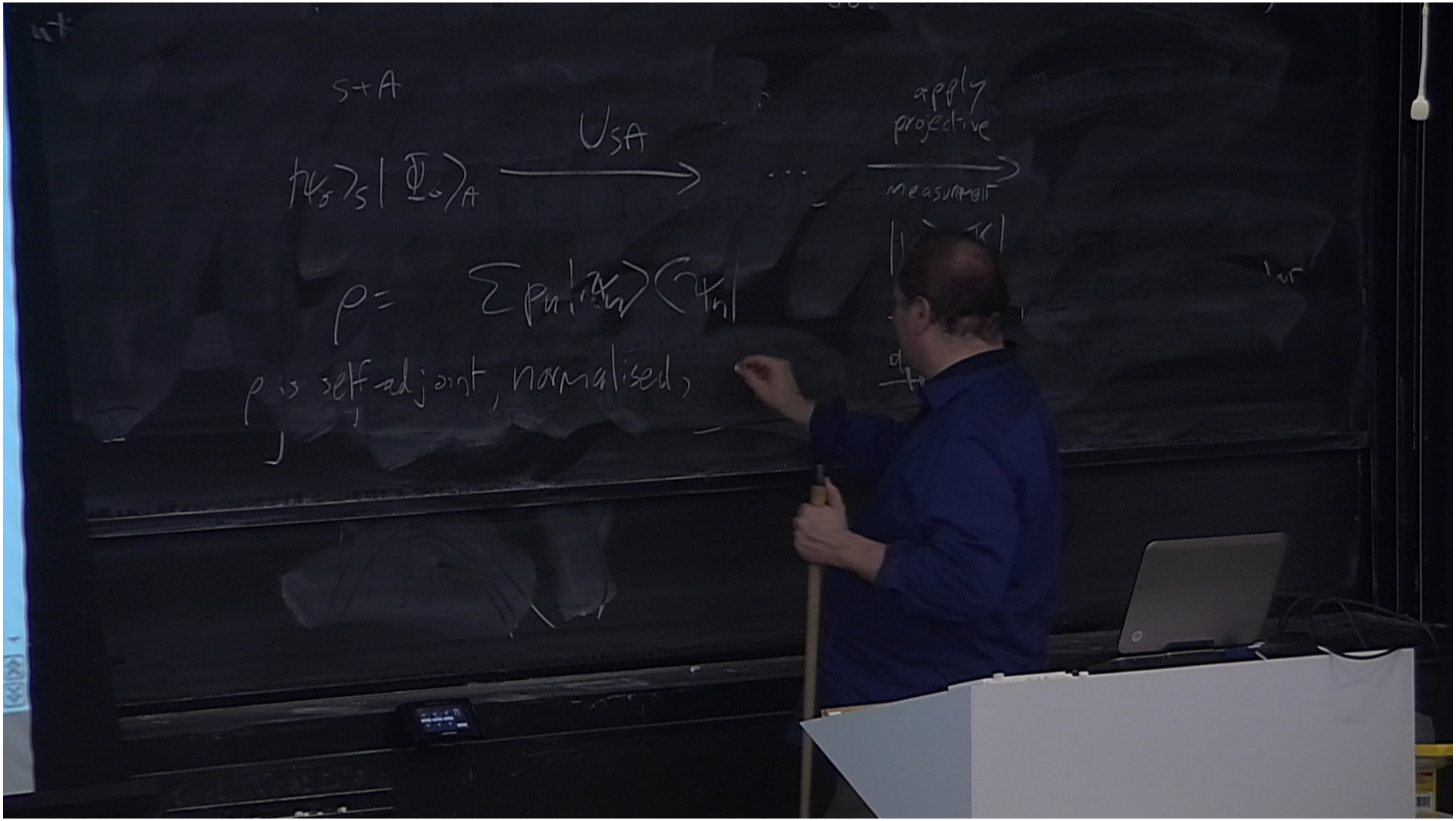

S+A
U_SA
apply
projective
measurement
|K⟩⟨K|
for
apparatus
ρ is self


S+A
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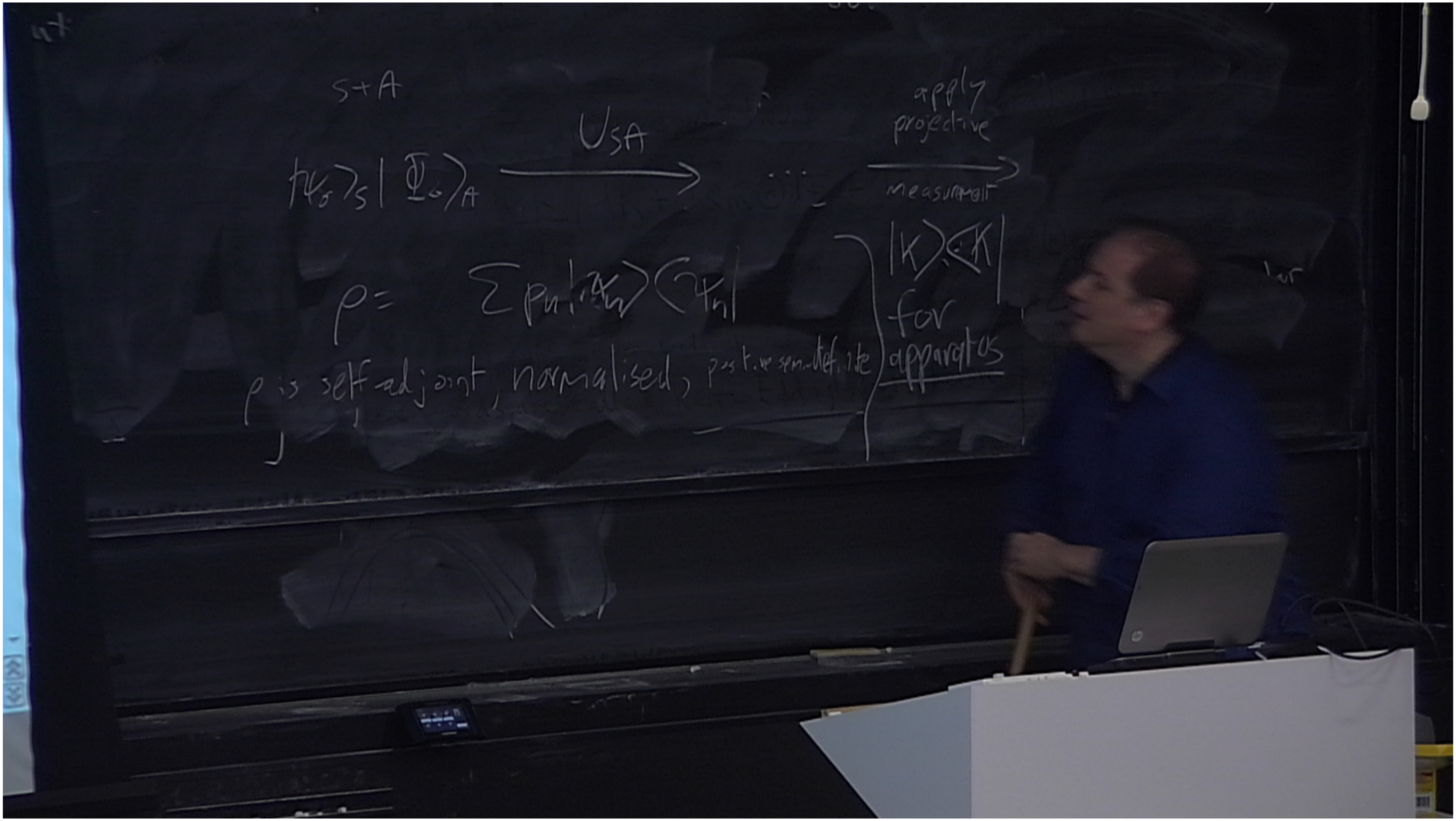
ρ is self-adjoint, normalised,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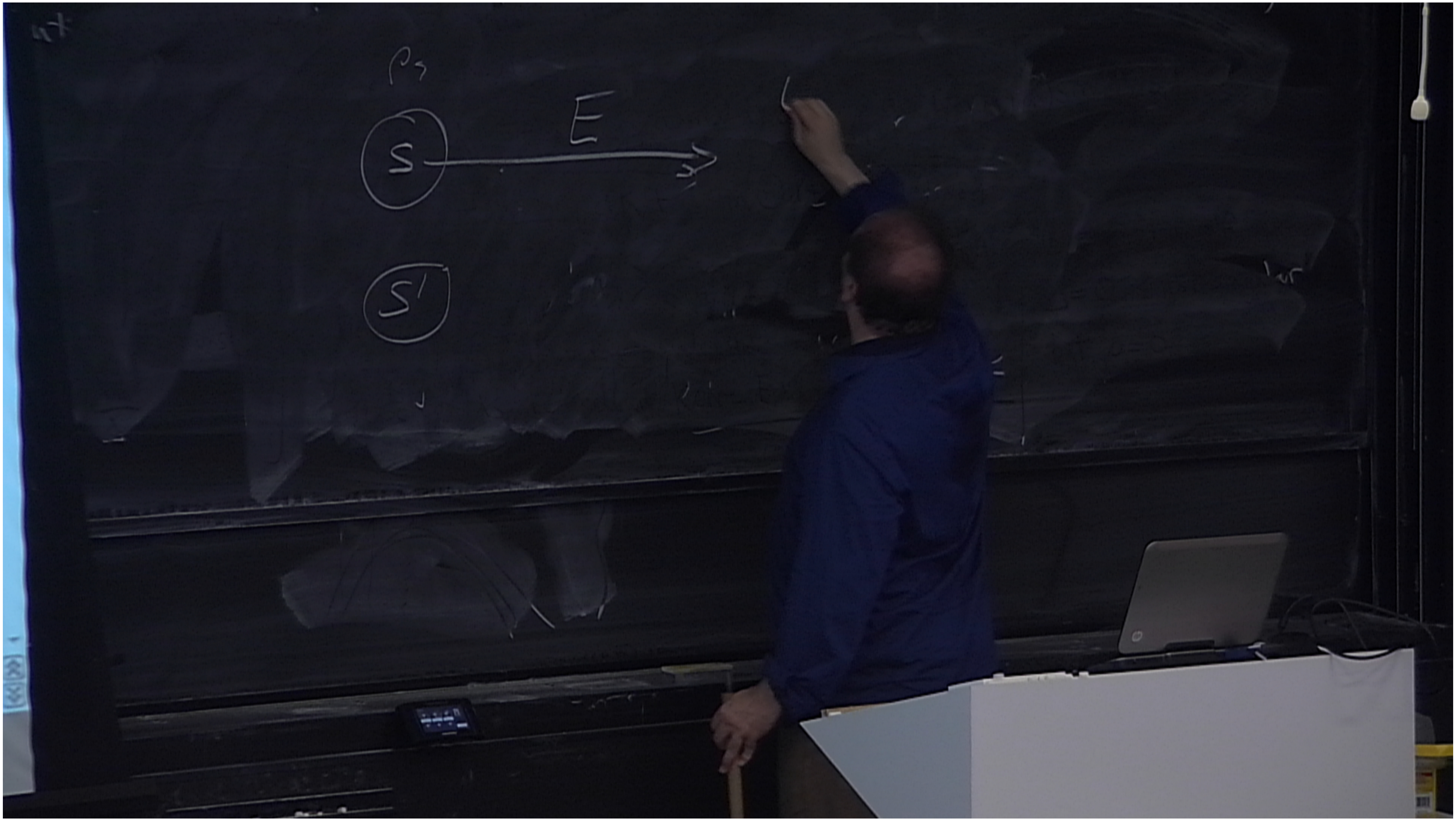
ρ=
ρ is self adjoint, normalised, positive semi-definite



S
E
S'


ρs
S
E
E(ρs)
S'
I

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Let's think abstractly about the most general possible sequence of operations that can act on a quantum system, with the effect $\rho_S \to \mathcal{E}(\rho_S)$

$\mathcal{E}$ must satisfy:

1. linear $p\rho_S^1 + (1-p)\rho_S^2 \to p\,\mathcal{E}(\rho_S^1) + (1-p)\,\mathcal{E}(\rho_S^2)$

   so $\mathcal{E}(p\rho_S^1 + (1-p)\rho_S^2) = p\,\mathcal{E}(\rho_S^1) + (1-p)\,\mathcal{E}(\rho_S^2)$

2. trace-preserving $\mathrm{Tr}(\mathcal{E}(\rho_S)) = 1$ if $\mathrm{Tr}(\rho_S) = 1$

3. positive $\mathcal{E}(\rho_S)$ is a positive (semidefinite) operator whenever ρ_S is

4. Completely positive If S, S' are two subsystems then $\mathcal{E} \otimes I_{S'}(\rho_{SS'})$ must be positive

Theorem (Stinespring dilation theorem) Given any such $\mathcal{E}$, there exist Kraus operators A_k obeying $\sum A_k^\dagger A_k = I$ such that $\mathcal{E}(\rho) = \sum_k A_k \rho A_k^\dagger$.

And so, as previously noted, we can find some system A, state $|0\rangle \in H_A$ and unitary U_{SA} such that $\mathcal{E}(\rho) = \mathrm{tr}_A\, U_{SA}(\rho \otimes |0\rangle\langle 0|)U_{SA}^\dagger$.

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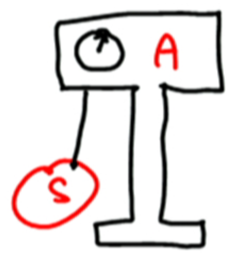
Reading View

Models of decoherence Although we began our discussion with a model of a system S interacting with an apparatus A, we ended up considering a completely general interaction U_{SA}, assuming nothing about its form.

Our discussion applies equally well to any quantum subsystem S interacting with its environment E



Clearly practically important – no system is completely isolated, and often (for instance in quantum computing) the environmental effects are very significant.

Also of fundamental interest: what effects do environmental interactions generically have? could they resolve/shed light on the problems of quantum theory?

We can get some good insights from simple models.

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18

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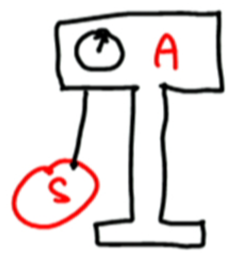

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Reading View

Simple model of heat bath decoherence

System initial state $\sum_n c_n |n\rangle$, Environment initial state $\rho = \frac{1}{N} \sum_{k=1}^{N} |\Phi_k\rangle\langle\Phi_k|$

Interaction $|n\rangle_S |\Phi_m\rangle_E \to |n\rangle_S |\Phi_{m+n}\rangle_E$ over time t.

Effect $\rho_{SE}(0) = \sum_{mn} c_m^* c_n |n\rangle\langle m| \otimes \frac{1}{N} \sum_{k=1}^{N} |\Phi_k\rangle\langle\Phi_k|$

$$\downarrow$$

$$\rho_{SE}(t) = \sum_{mn} c_m^* c_n |n\rangle\langle m| \otimes \frac{1}{N} \sum_{k=1}^{N} |\Phi_{k+n}\rangle\langle\Phi_{k+m}|$$

So $\rho_S(t) = \mathrm{Tr}_E(\rho_{SE}(t)) = \sum_n |c_n|^2 |n\rangle\langle n|$.

– just as in the von Neumann measurement model, but NB no pointer states here

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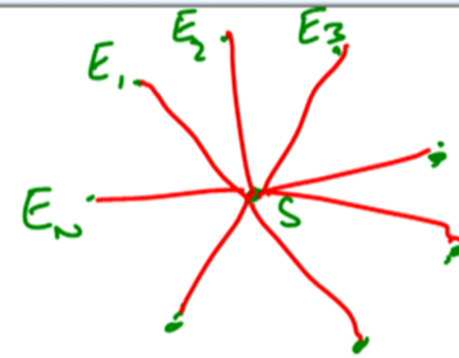
Reading View

Zurek's exactly solvable two state decoherence model



Spin ½ system S, states $|\Uparrow\rangle, |\Downarrow\rangle$

N spin ½ environment systems, states $|\uparrow_k\rangle, |\downarrow_k\rangle$

No self-interaction $H_S = H_{E_k} = 0$

No environment spin-spin interaction $H_{E_k E_{k'}} = 0$

Only SE coupling:

$$H = H_{SE} = \hbar (\sigma_z^S) \otimes \sum_k g_k \, I_1 \otimes \dots \otimes I_{k-1} \otimes \sigma_z^{E_k} \otimes I_{k+1} \otimes \dots \otimes I_N$$

Initial state $$|\psi(0)\rangle = (a|\Uparrow\rangle + b|\Downarrow\rangle) \bigotimes_{k=1}^{N} (\alpha_k |\uparrow_k\rangle + \beta_k |\downarrow_k\rangle)$$

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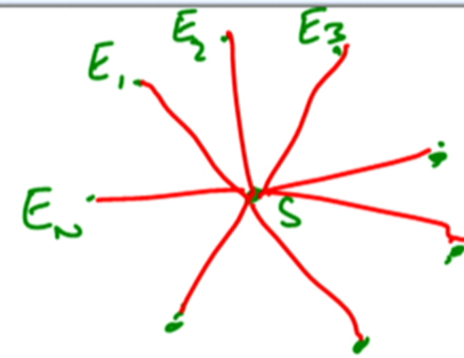
Zurek's exactly solvable two state decoherence model



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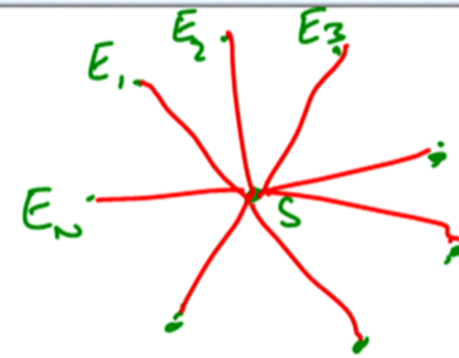

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So we get $|\psi(t)\rangle = a|\Uparrow\rangle|E_\Uparrow(t)\rangle + b|\Downarrow\rangle|E_\Downarrow(t)\rangle$

where $|E_\Uparrow(t)\rangle = |E_\Downarrow(-t)\rangle = \bigotimes_{k=1}^{N} \left(\alpha_k e^{-ig_k t} |\uparrow_k\rangle + \beta_k e^{ig_k t} |\downarrow_k\rangle \right).$

The reduced density matrix is $\rho_S(t) = \mathrm{Tr}_E \left(|\psi(t)\rangle\langle\psi(t)| \right)$

$$= |a|^2 |\Uparrow\rangle\langle\Uparrow| + |b|^2 |\Downarrow\rangle\langle\Downarrow|$$

$$+ z(t)\, a b^* |\Uparrow\rangle\langle\Downarrow| + z^*(t)\, a^* b \, |\Downarrow\rangle\langle\Uparrow|$$

where the interference coefficient $z(t) = \langle E_\Downarrow(t) | E_\Uparrow(t) \rangle$

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The reduced density matrix is $\rho_s(t) = \mathrm{Tr}_E\left(|\Psi(t)\rangle\langle\Psi(t)| \right)$

$$= |a|^2 |\Uparrow\rangle\langle\Uparrow| + |b|^2 |\Downarrow\rangle\langle\Downarrow| + z(t)\, a b^* |\Uparrow\rangle\langle\Downarrow| + z^*(t)\, a^* b\, |\Downarrow\rangle\langle\Uparrow|$$

where the interference coefficient $z(t) = \langle E_{\Downarrow}(t) | E_{\Uparrow}(t)\rangle$

Explicitly, $z(t) = \prod_{k=1}^{N} \left(|\alpha_k|^2 e^{-2ig_k t} + |\beta|^2 e^{2ig_k t} \right)$

So $$|z(t)|^2 = \prod_{k=1}^{N} \left(1 + \left((|\alpha_k|^2 - |\beta_k|^2)^2 - 1 \right) \sin^2 2g_k t \right)$$

We have $\langle \sin^2(2g_k t)\rangle \approx 1/2$ time-averaged value. If we assume α_k, β_k, g_k are independently randomly distributed, then for typical large times the factors are effectively independent: we can estimate $|z(t)|^2$ by replacing each term by its time average

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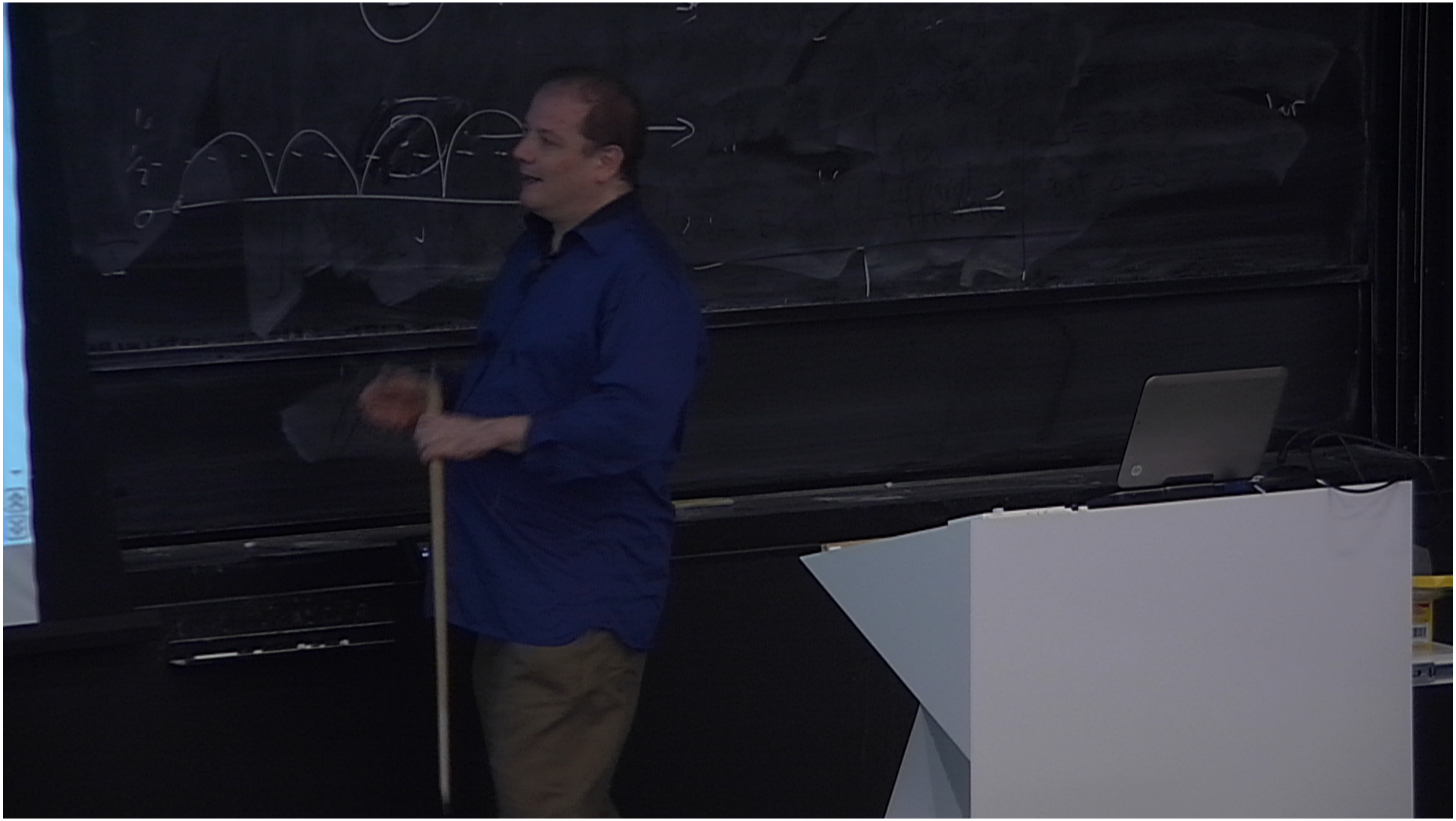

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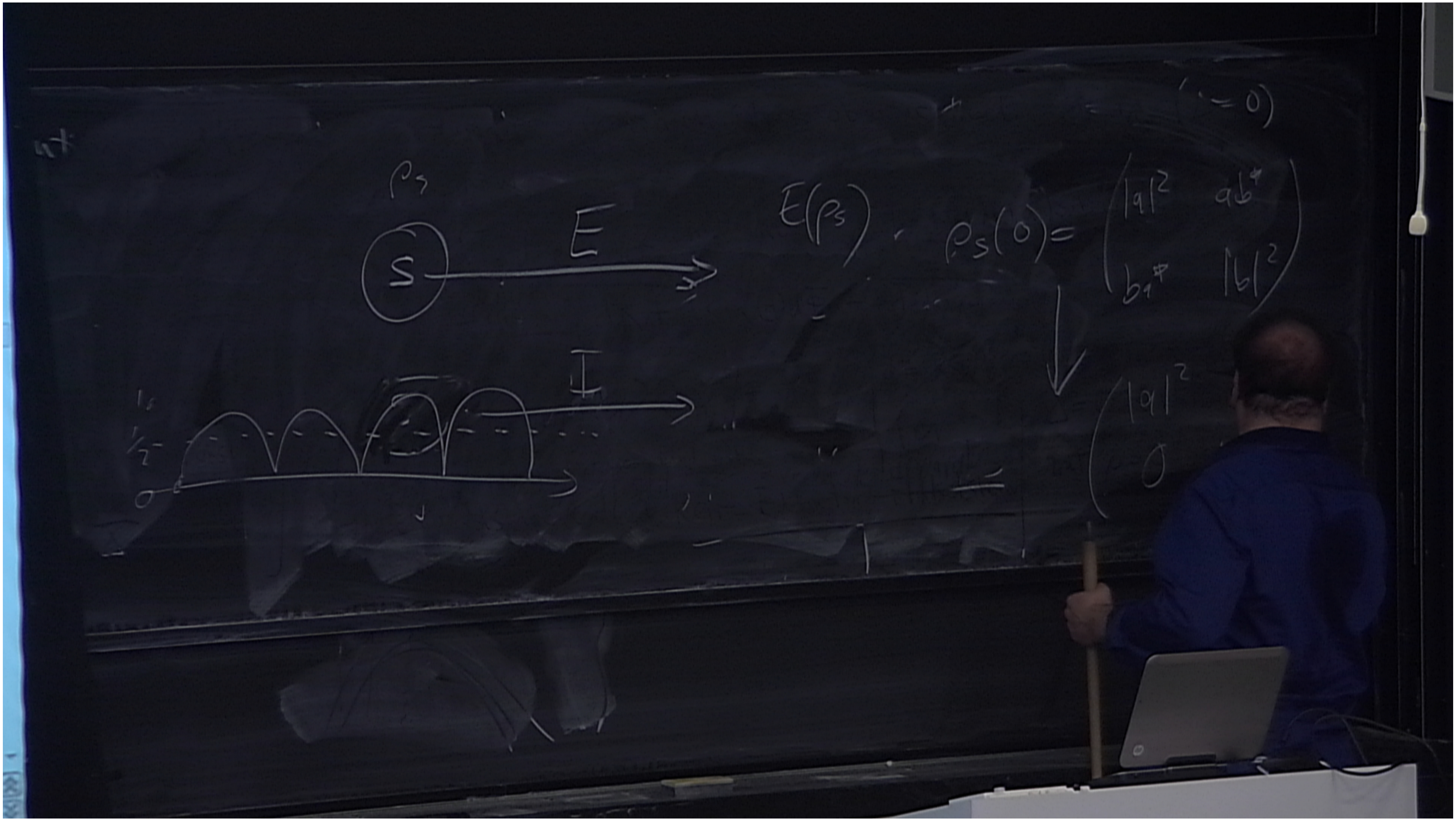
Which gives $\langle |z(t)|^2 \rangle_{t \to \infty} \sim \prod_{k=1}^{N} \left(\frac{1}{2} + \frac{1}{2} (|\alpha_k|^2 - |\beta_k|^2)^2 \right)$

$$\sim 2^{-N} \prod_{k=1}^{N} \left(1 + (|\alpha_k|^2 - |\beta_k|^2)^2 \right)$$

$$\to 0 .$$

on average

The off-diagonal interference terms in $\rho_S(t)$ are strongly damped for large N, t.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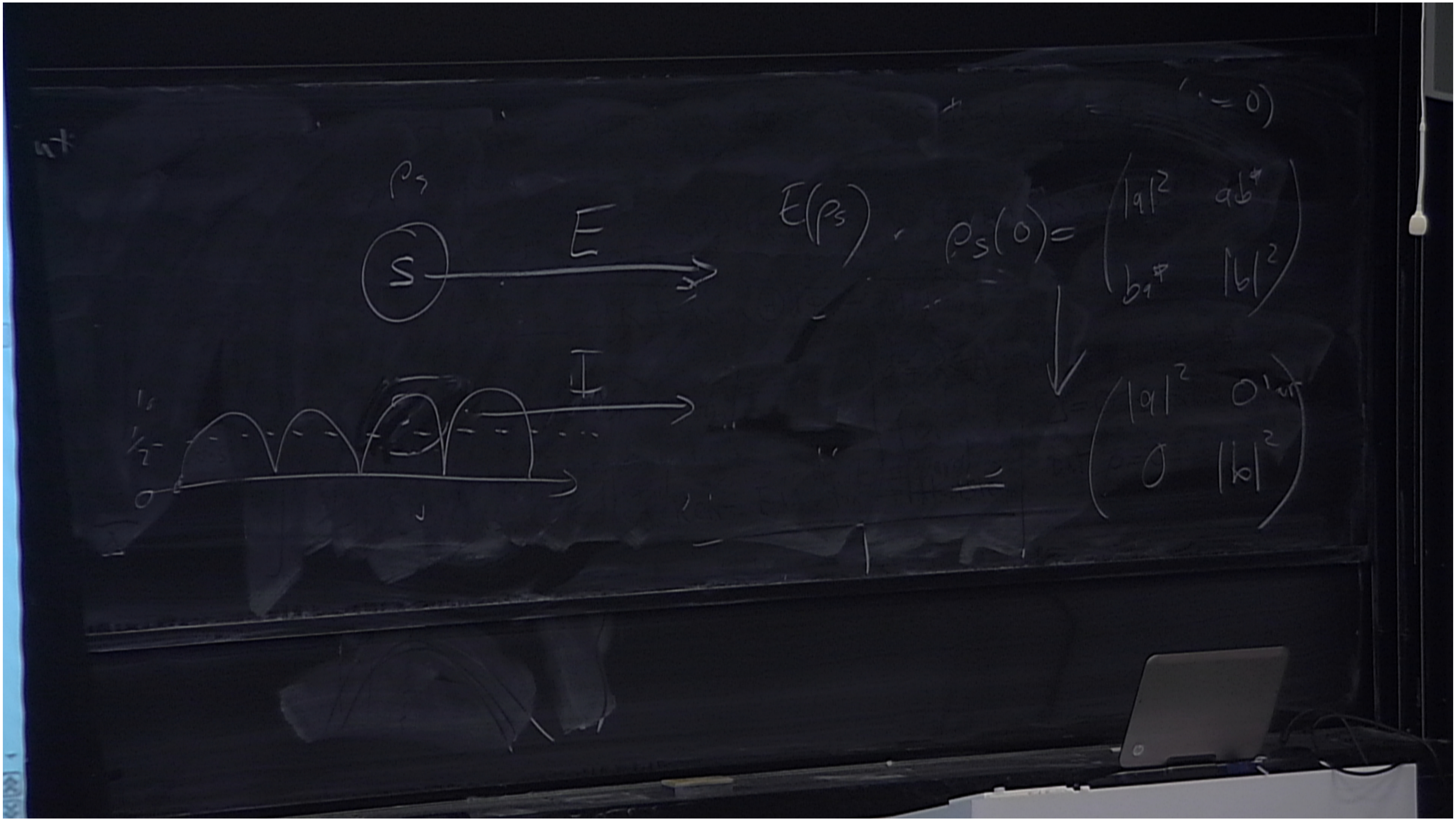
E
E(ρs)
ρs(0)=
|a|2 ab*
ba* |b|2
I
|a|2
0


ρ_s
S
E
I
$E(\rho_s)$
$\rho_s(0) = \begin{pmatrix} |a|^2 & ab^* \\ ba^* & |b|^2 \end{pmatrix}$
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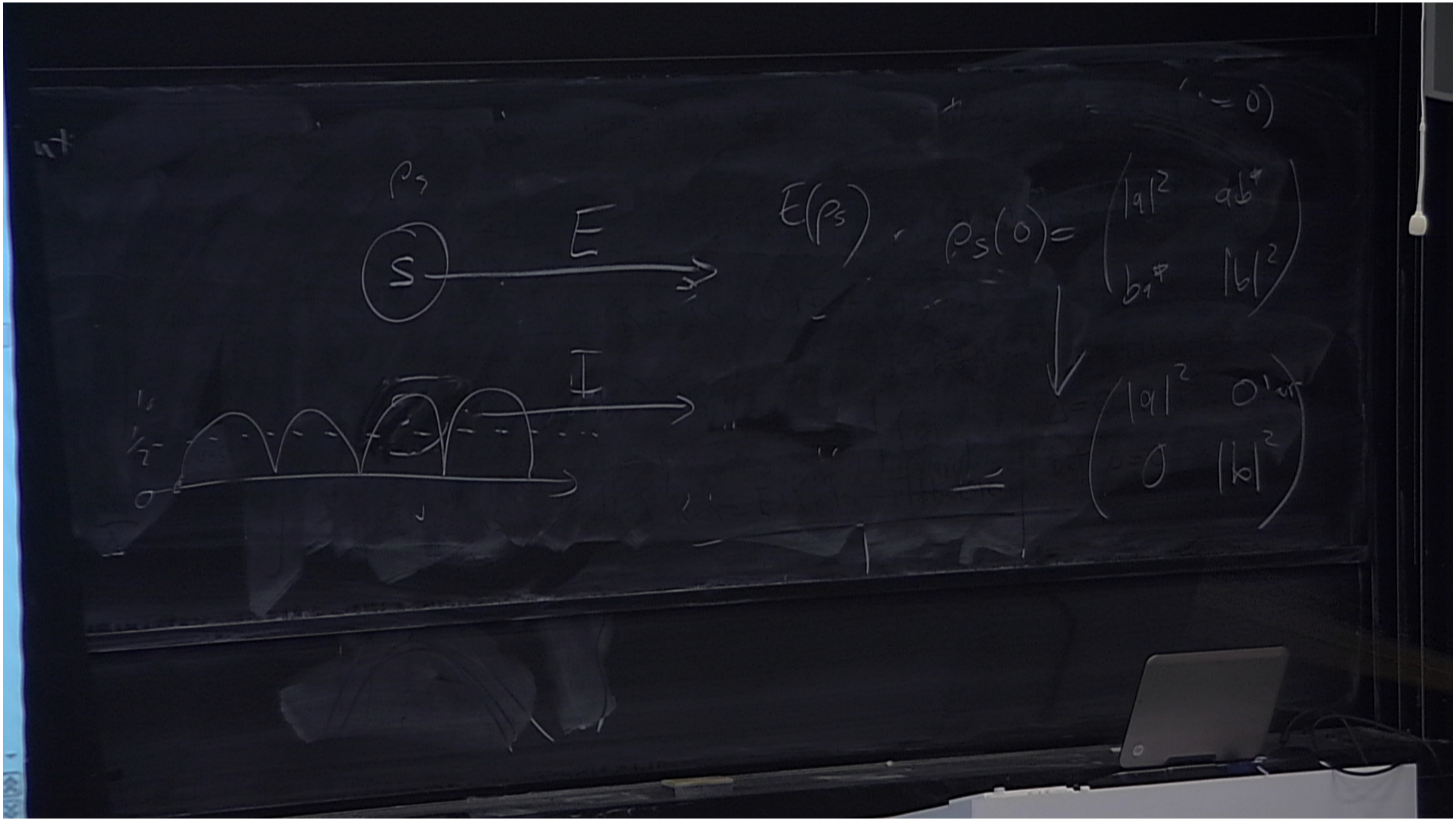
ρ_s
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Reading View

To quote some figures (from Joos, Chapter 3 in Joos et al. (2003))

$$\rho(x, x', t) \doteq \rho(x, x', 0)\exp(-\Lambda t (x - x')^2)$$

where the "localization rate" Λ in $cm^{-2}s^{-1}$ is estimated as:

E \ S	10^{-3} cm dust particle	10^{-5} cm dust particle	10^{-6} cm large molecule
cosmic microwave background	10^{6}	10^{-6}	10^{-12}
300K photons	10^{19}	10^{12}	10^{6}
Sunlight on earth	10^{21}	10^{17}	10^{13}
air molecules	10^{36}	10^{32}	10^{30}
Laboratory vacuum (10^6 particles/cm)	10^{23}	10^{19}	10^{17}

(Joos + Zeh, 1985; similar estimates by Tegmark, 1993)

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Reading View

"Does decoherence solve the measurement problem? Clearly not. What decoherence tells us, is that certain objects appear classical when they are observed. But what is an observation? At some stage, we still have to apply the usual probability rules of quantum theory."

"One often finds explicit or implicit statements to the effect that the above processes are equivalent to the collapse of the wave function (or even solve the measurement problem). Such statements are certainly unfounded."

"Decoherence by itself does not yet solve the measurement problem (. . .). This argument is nonetheless found wide-spread in the literature. (. . .) It does seem that the measurement problem can only be resolved if the Schrodinger dynamics (. . .) is supplemented by a nonunitary collapse (. . .)."

(Joos (2000, p. 14). Kiefer and Joos (1999, p. 5), Zeh in Joos et al., 2003, Ch. 2; all quoted in Schlosshauer (2003))

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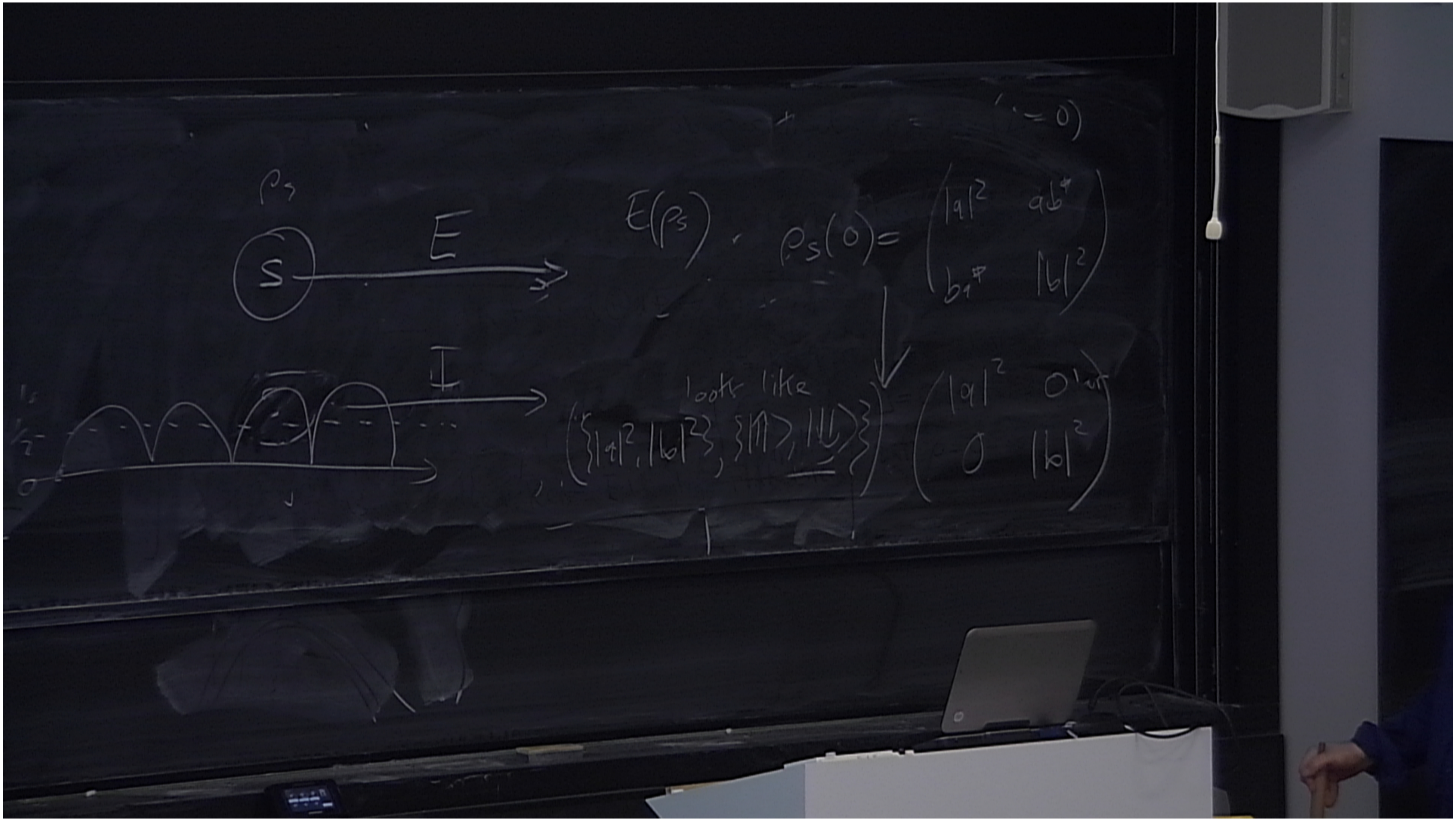
E
looks like

"Cosmologists, even more than laboratory physicists, must find the usual interpretive rules of quantum mechanics a bit frustrating [....]

It would seem that the theory is exclusively concerned with 'results of measurement' and has nothing to say about anything else. When the 'system' in question is the whole world where is the 'measurer' to be found? Inside, rather than outside, presumably. Was the world wave function waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer for some more highly qualified measurer -- with a Ph.D.? If the theory is to apply to anything but idealized laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time more or less everywhere?
Is there ever then a moment when there is no jumping and the Schrodinger equation applies?"

(J.S. Bell, *Quantum mechanics for cosmologists*.
In Quantum Gravity 2 (Isham, Penrose and Sciama, eds; OUP, 1981))

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Reading View

Everett on branching worlds and reality

“ In reply to a preprint of this article some correspondents have raised the question of the "transition from possible to actual," arguing that in "reality" there is — as our experience testifies — no such splitting of observers states, so that only one branch can ever actually exist. Since this point may occur to other readers the following is offered in explanation.

The whole issue of the transition from "possible" to "actual" is taken care of in the theory in a very simple way — there is no such transition, nor is such a transition necessary for the theory to be in accord with our experience. From the viewpoint of the theory all elements of a superposition (all "branches") are "actual", none any more "real" than the rest. ”

(Everett, Rev Mod Phys 1957., footnote added in proof)

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Reading View

Everett on branching worlds and reality

"Arguments that the world picture presented by this theory is contradicted by experience, because we are unaware of any branching process, are like the criticism of the Copernican theory that the mobility of the earth as a real physical fact is incompatible with the common sense interpretation of nature because we feel no such motion. In both cases the argument fails when it is shown that the theory itself predicts that our experience will be what it in fact is. "

(Everett, Rev Mod Phys 1957., footnote added in proof)

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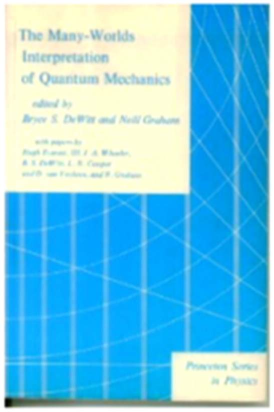
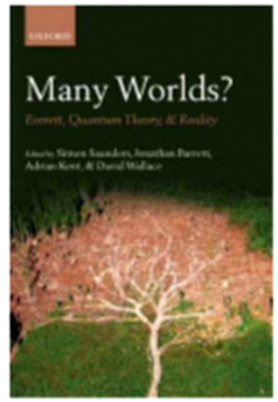
Further reading





B. DeWitt and N. Graham,
"The Many-Worlds Interpretation of Quantum Mechanics",
(Princeton University Press, 1973)

S. Saunders, J. Barrett, A. Kent, D. Wallace
"Many Worlds? Everett, Quantum Theory and Reality"
(Oxford University Press, 2010)

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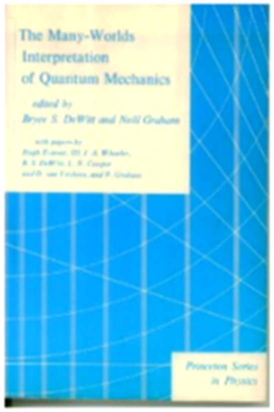
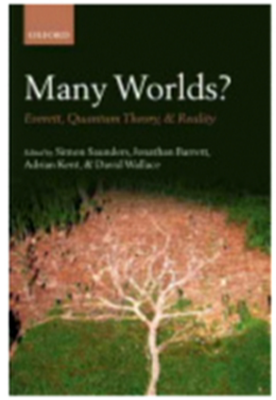
Further reading





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