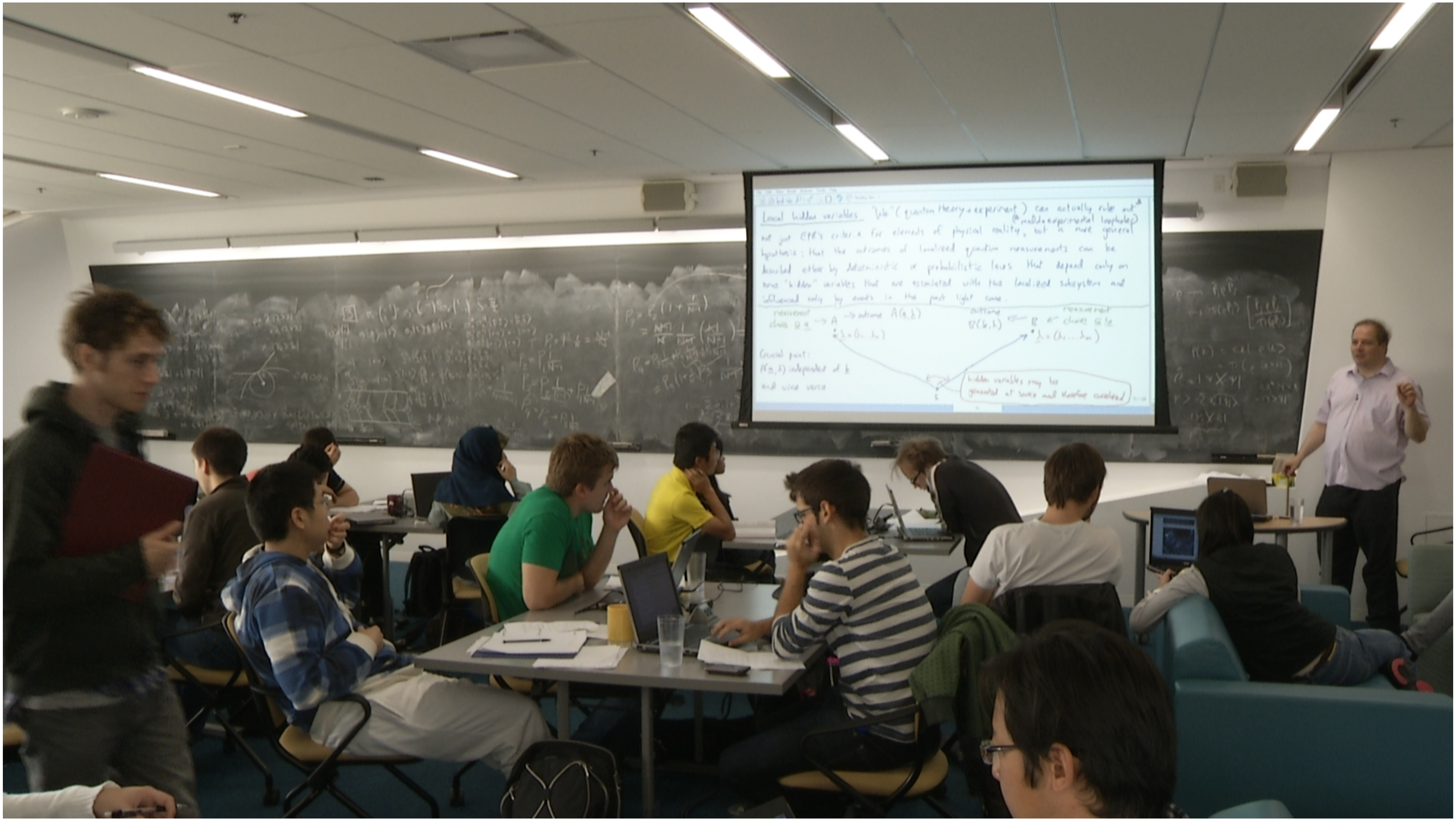
Title: Quantum Theory - Lecture 13

Date: Sep 27, 2011 01:30 PM

URL: http://pirsa.org/11090055

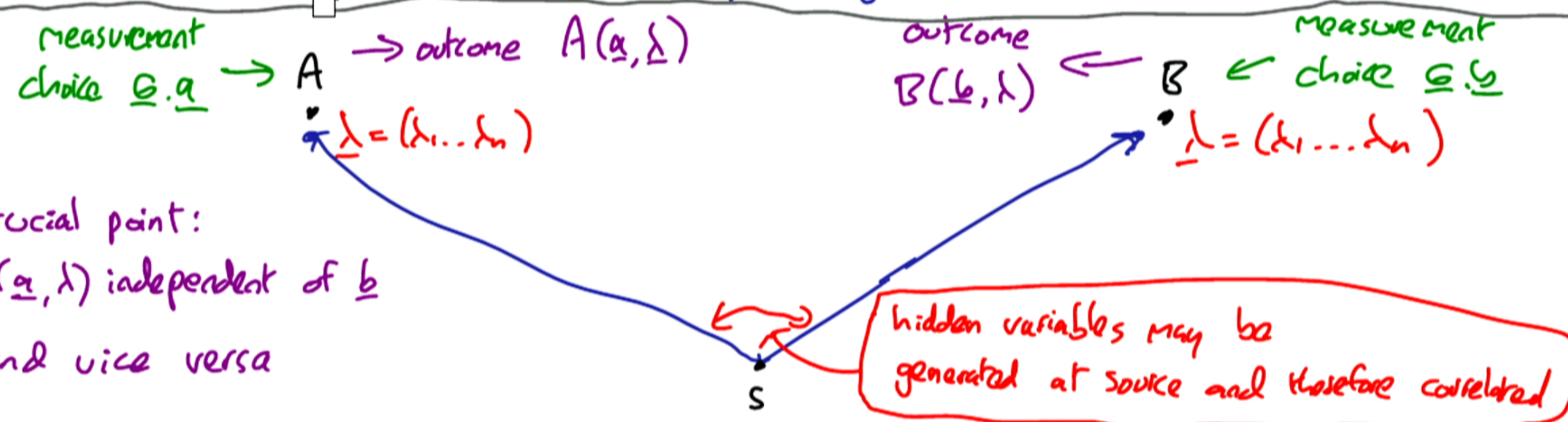
Abstract:


Local hidden variables
"We" (quantum theory + experiment) can actually rule out
not just EPR's criteria for elements of physical reality, but a more general
hypothesis: that the outcomes of localized quantum measurements can be
described either by deterministic or probabilistic laws that depend only on
some "hidden" variables that are associated with the localized subsystem and
influenced only by events in the past light cone.
Crucial point:
and vice versa
hidden variables may be
generated at source and therefore correlated

Local hidden variables

"We" (quantum theory + experiment) can actually rule out* not just EPR's criteria for elements of physical reality, but a more general hypothesis: that the outcomes of localized quantum measurements can be described either by deterministic or probabilistic laws that depend only on some "hidden" variables that are associated with the localized subsystem and influenced only by events in the past light cone.

(* modulo experimental loopholes)



Crucial point:

$A(\underline{a}, \lambda)$ independent of $\underline{b}$

and vice versa

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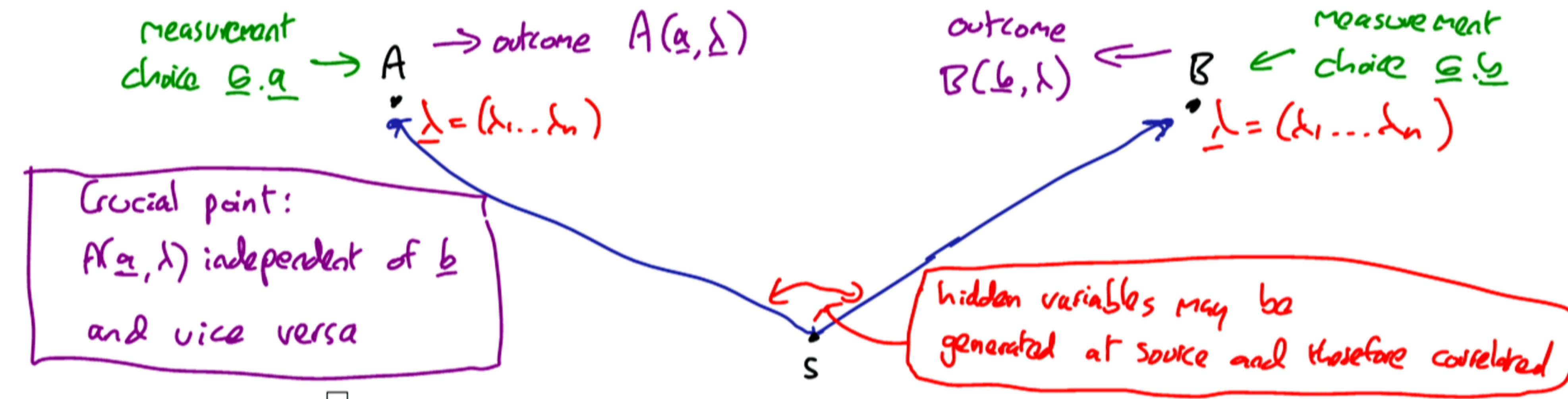
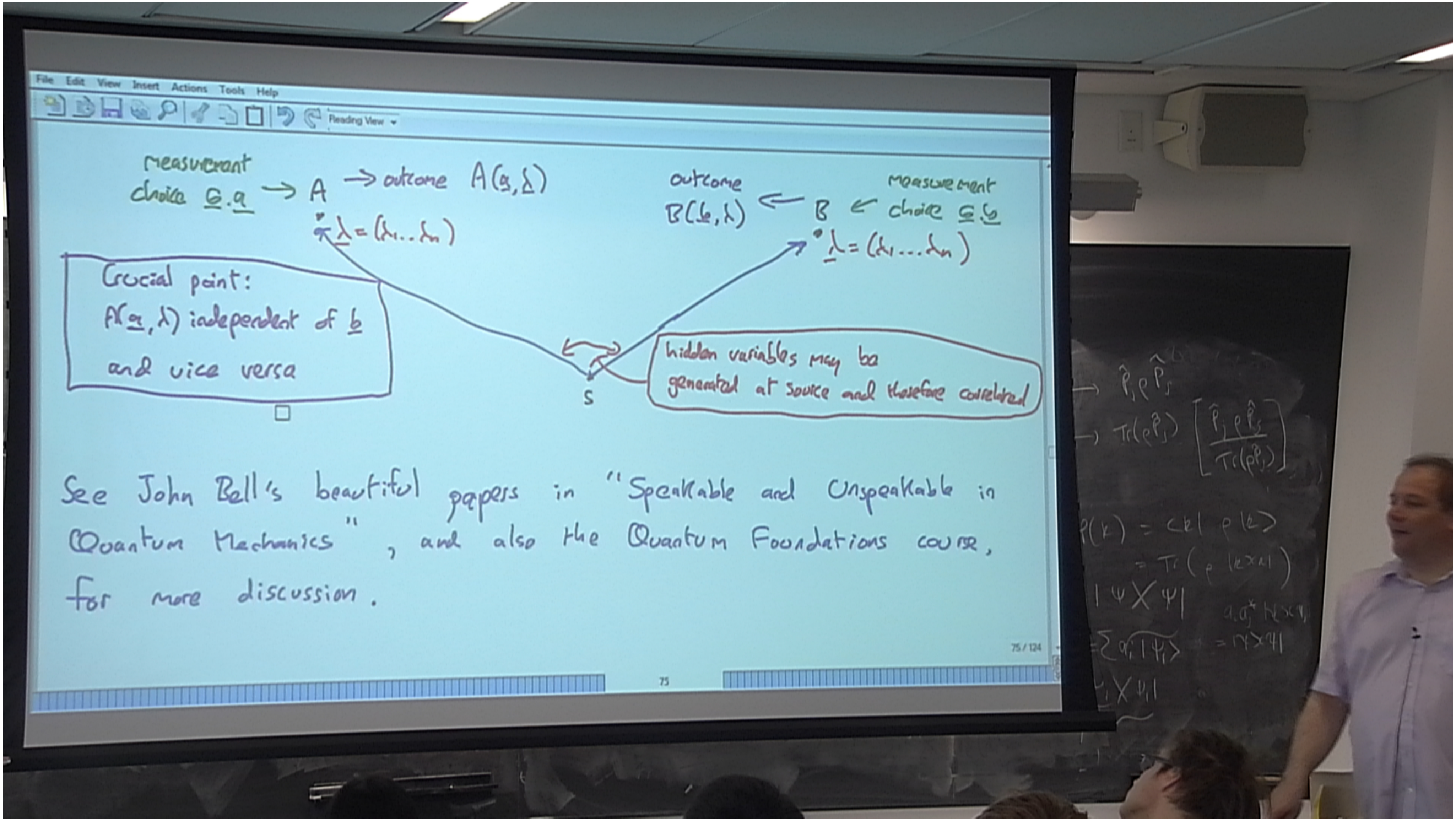


Note (1) Since the hidden variables could be correlated, the most general possibility is to allow both particles to carry the full list $\underline{\lambda}$ of all relevant hidden variables for both experiments.

(2) Doesn't matter if $A(\underline{a},\lambda)$, $B(\underline{b},\lambda)$ are probabilistic rather than deterministic. We can suppose all probabilistic choices for all possible measurements $\underline{\sigma}.\underline{a}$, $\underline{\sigma}.\underline{b}$ are already made at source S, and include these in our definition of $\underline{\lambda}$.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measurement choice e.g. a → A → outcome A(a, λ)
λ = (λ1 ... λn)
outcome B(b, λ) ⟵ B ⟵ measurement choice e.g. b
λ = (λ1 ... λn)
Crucial point:
A(a, λ) independent of b
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S
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See John Bell's beautiful papers in "Speakable and Unspeakable in Quantum Mechanics", and also the Quantum Foundations course, for more discussion.
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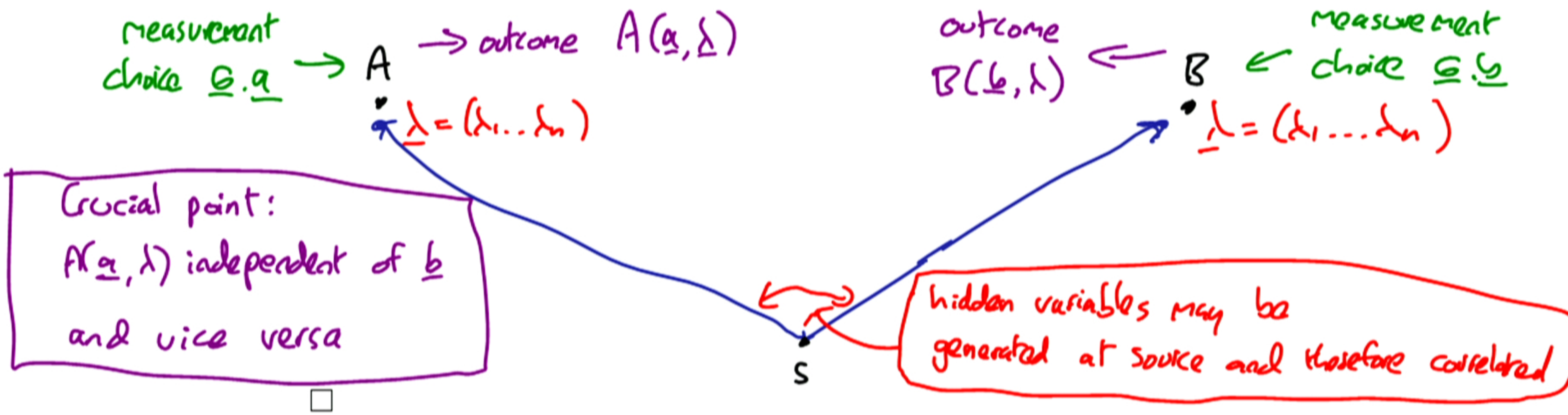

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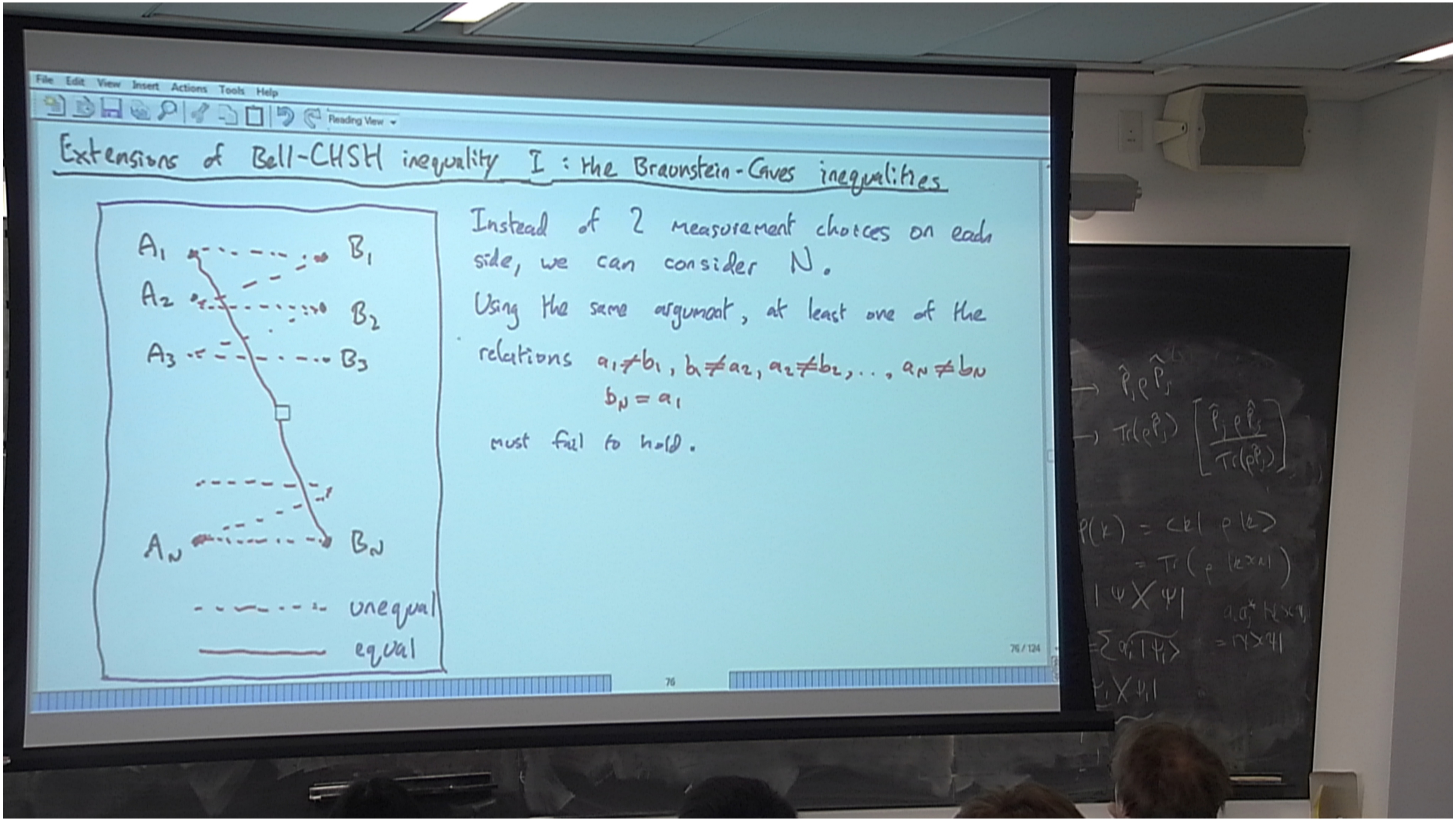
Extensions of Bell-CHSH inequality I : the Braunstein-Caves inequalities
Instead of 2 measurement choices on each side, we can consider N.
Using the same argument, at least one of the relations a1≠b1, b1≠a2, a2≠b2, . . . , aN≠bN
bN = a1
must fail to hold.
A1
B1
A2
B2
A3
B3
AN
BN
unequal
equal
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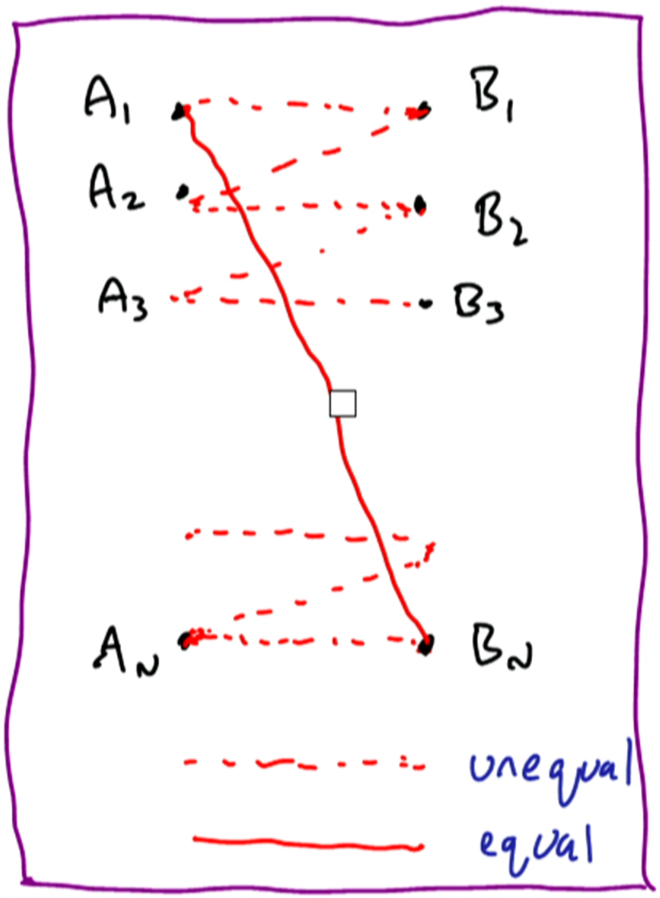
Extensions of Bell-CHSH inequality I: the Braunstein-Caves inequalities



Instead of 2 measurement choices on each side, we can consider N.

Using the same argument, at least one of the relations $a_1 \neq b_1$, $b_1 \neq a_2$, $a_2 \neq b_2$, ..., $a_N \neq b_N$, $b_N = a_1$ must fail to hold.

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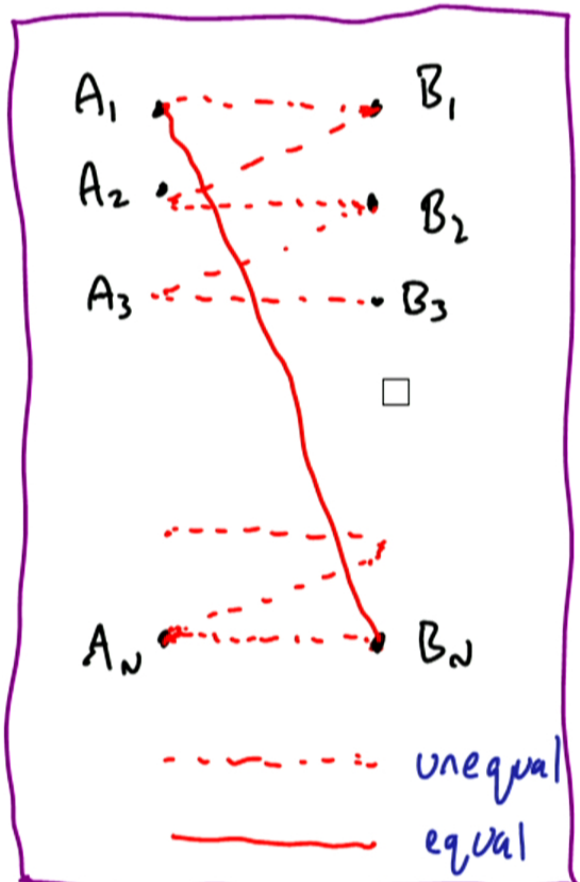
Braunstein-Caves inequality (cont.)



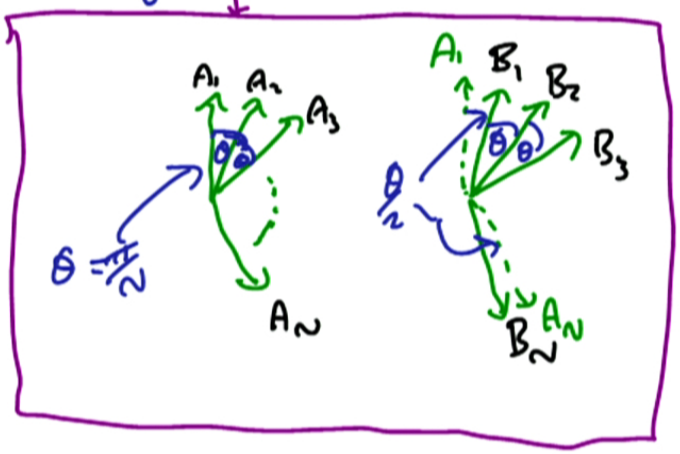
Not all these relations can simultaneously hold.

If we do 2N random tests we expect at least 1 violation, according to EPR / LHV.

I.e. $P(\text{violation}) \geq \frac{1}{2N}$

But QM predicts prob. of violation $\sim 1/N^2$

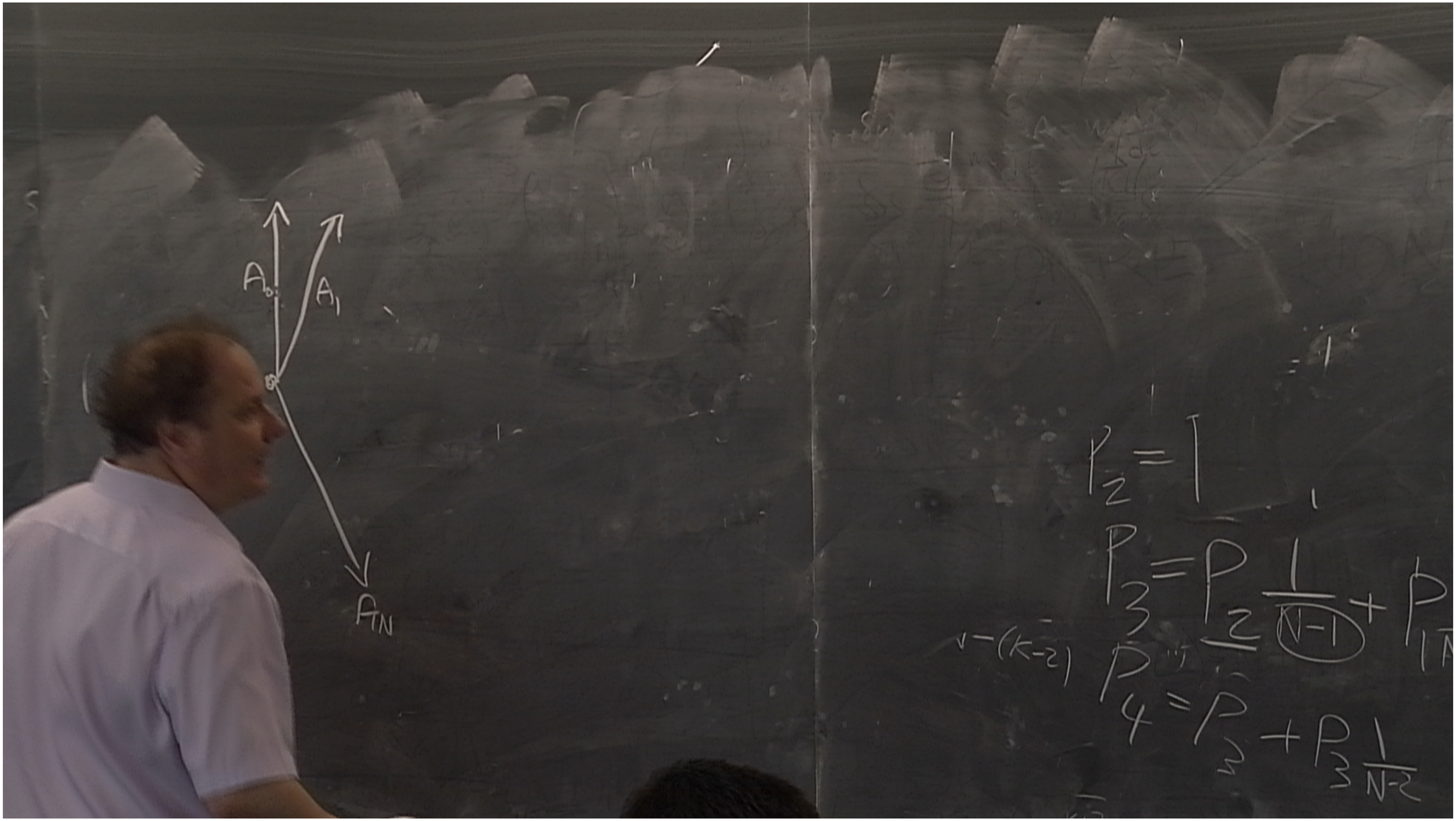
for eg. this choice of measurements on the singlet

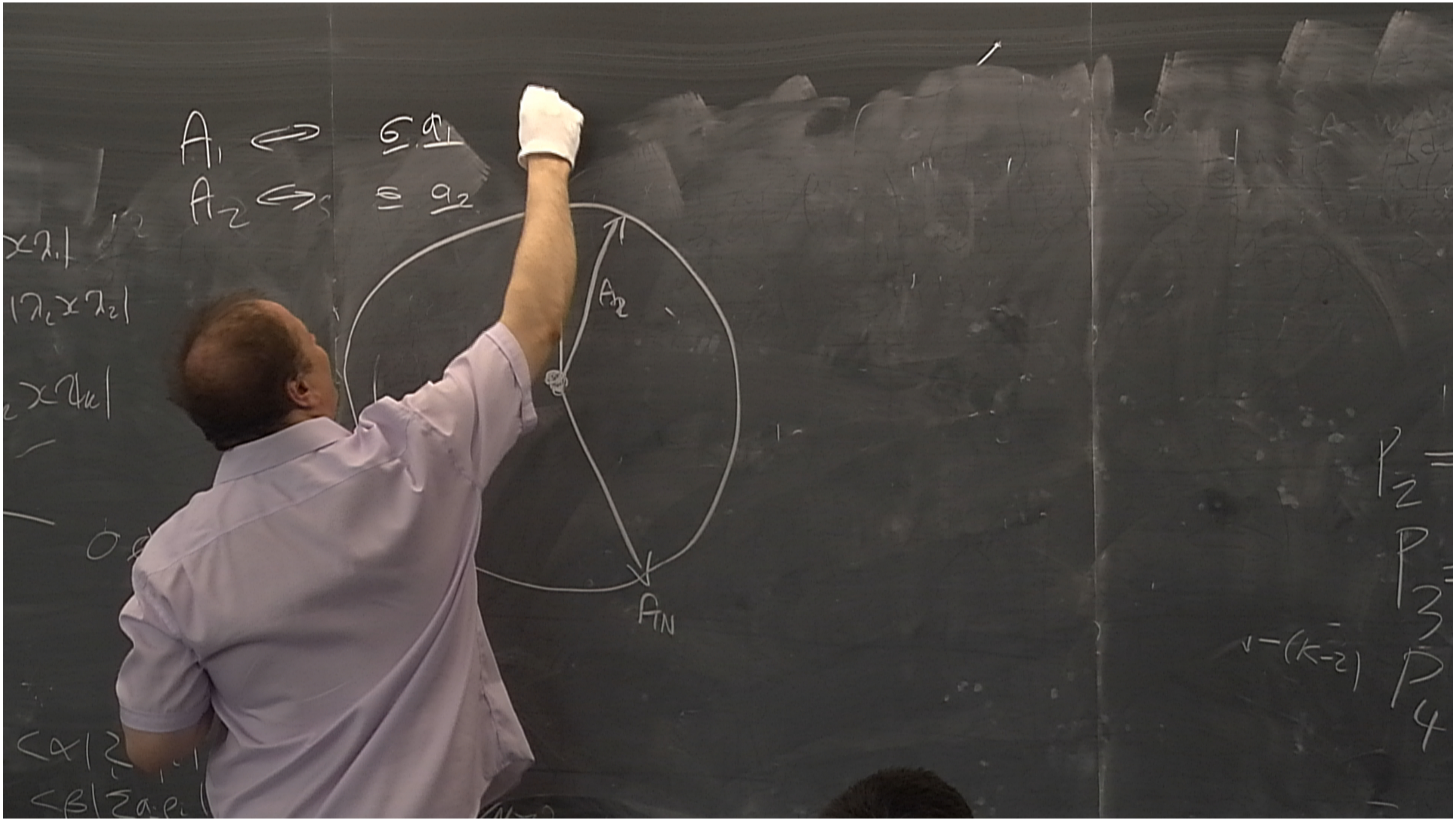


$P(A_1 \neq B_1) = \cos^2\left(\frac{\pi}{4N}\right)$

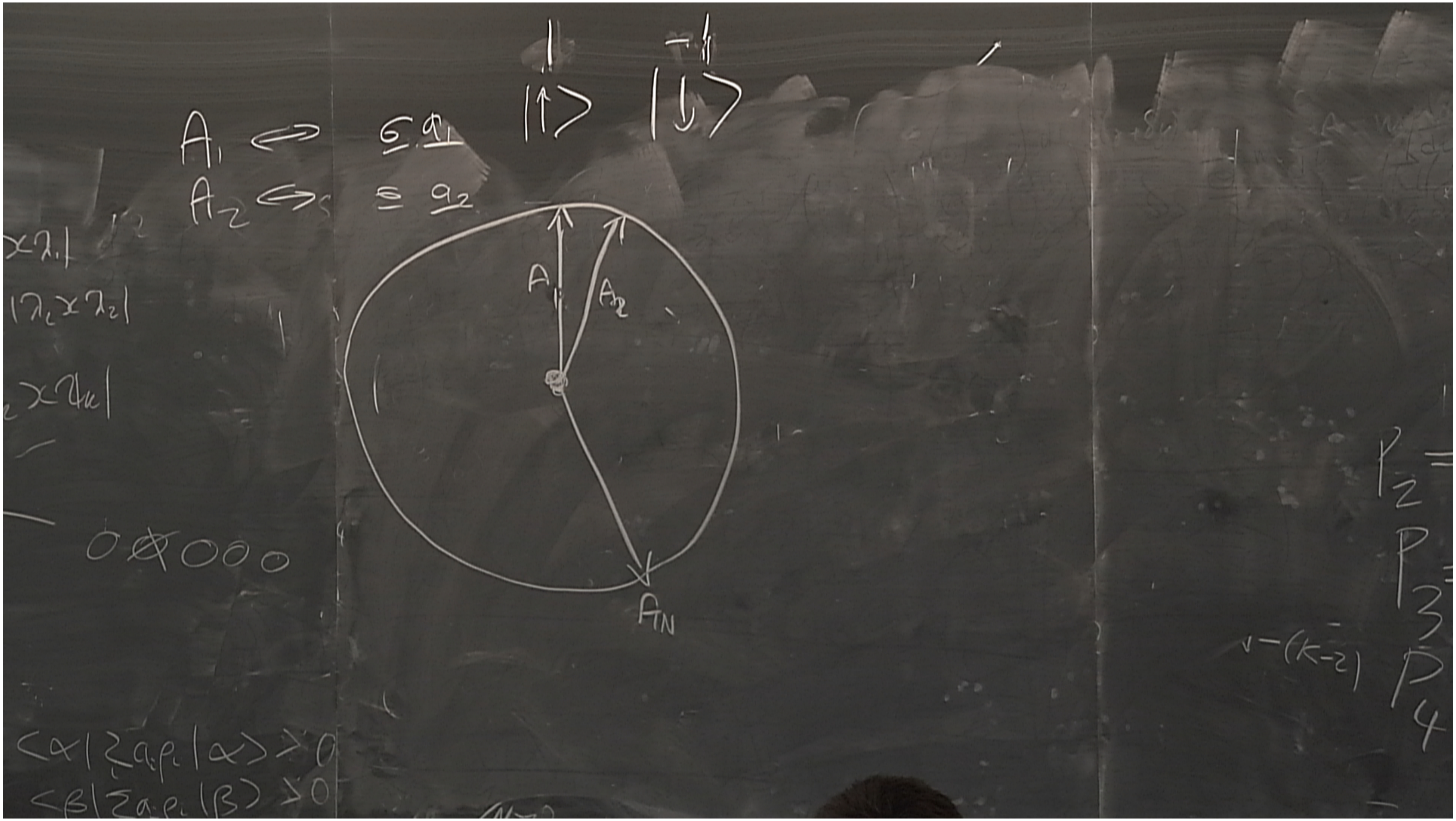
$P(\text{violation}) \doteq \frac{\pi^2}{32N^2}$

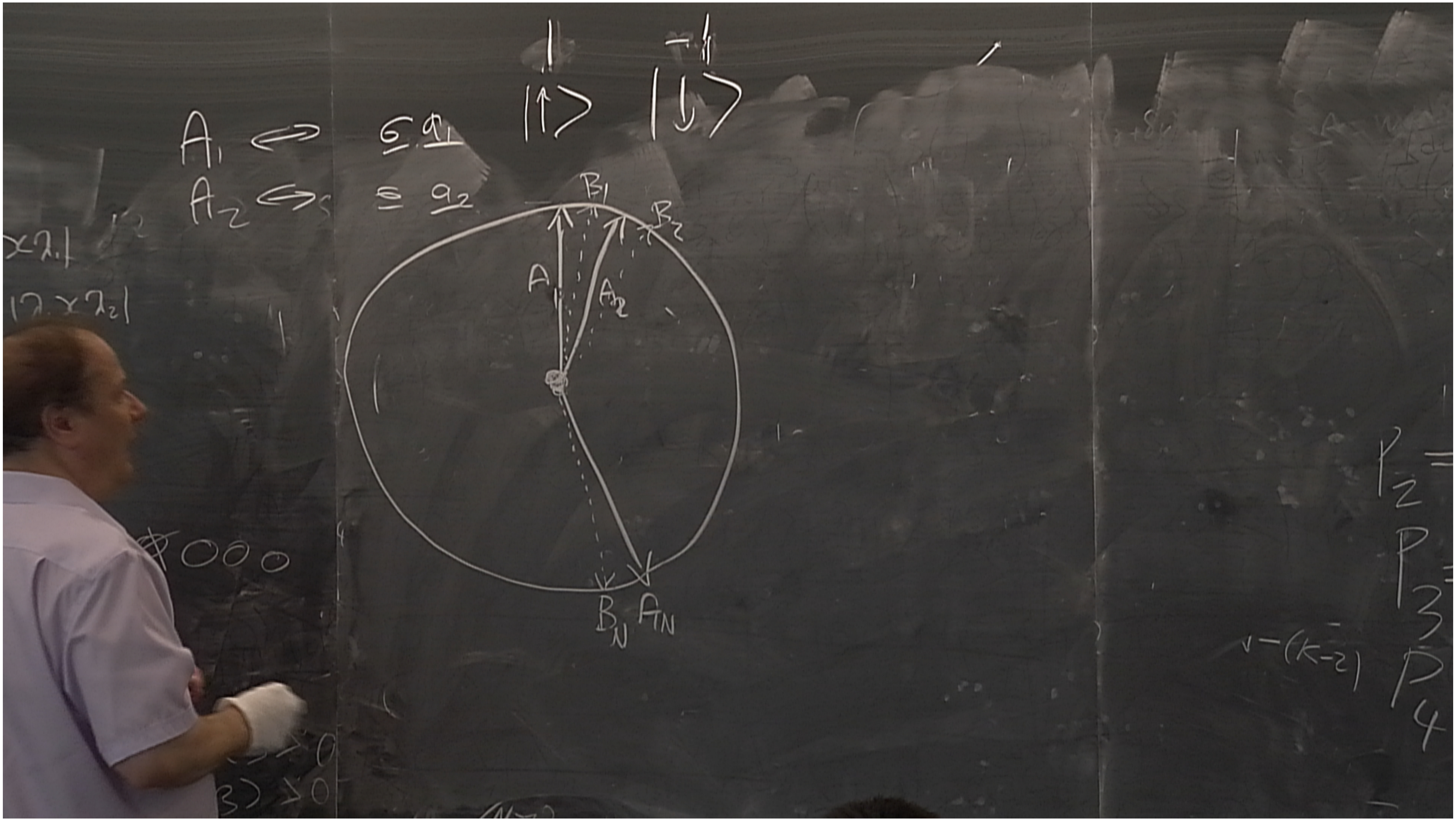
and same holds true for each (in)equality.

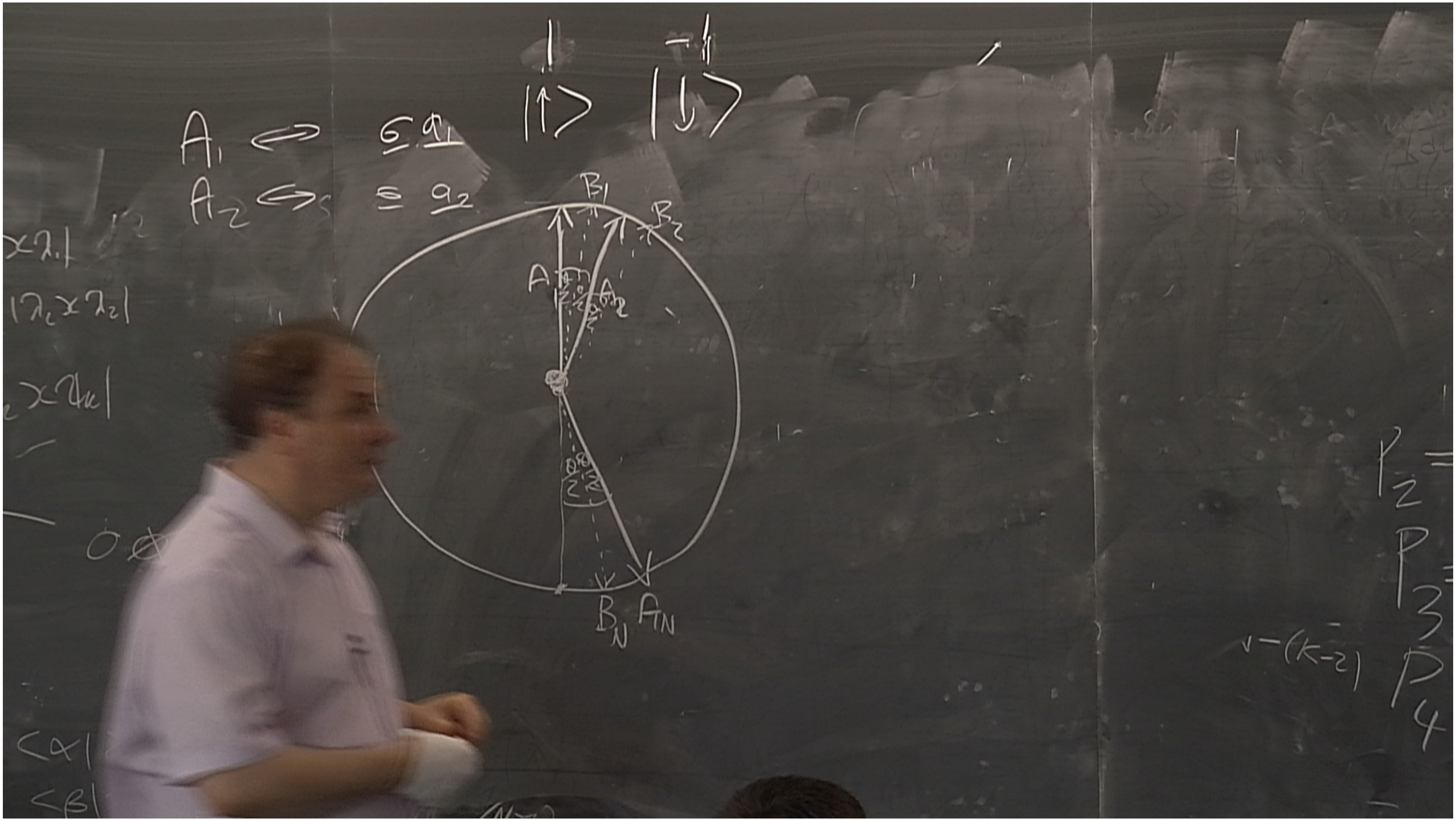

A_0
A_1
A_N
$P_2 = 1$
$P_3 = P_2 \frac{1}{N-1} + P_1$
$P_4 = P_3 + P_3 \frac{1}{N-2}$



A1
A2
A2
AN


A1
A2
|↑>
|↓>
AN


|↑>
|↓>
B1
B2
A1
A2
BN
AN


|↑>
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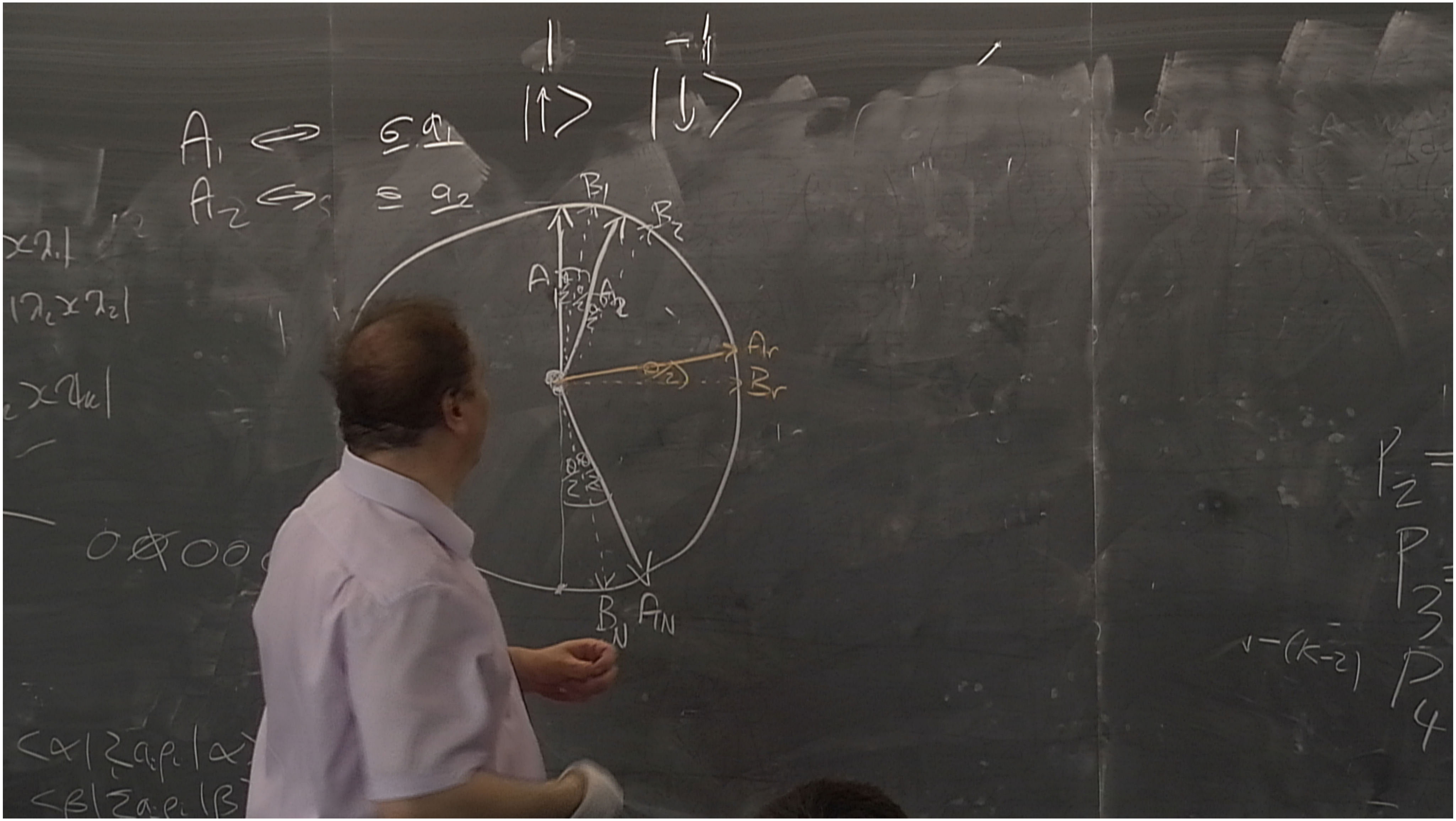
B1
B2
BN
AN


|↑>
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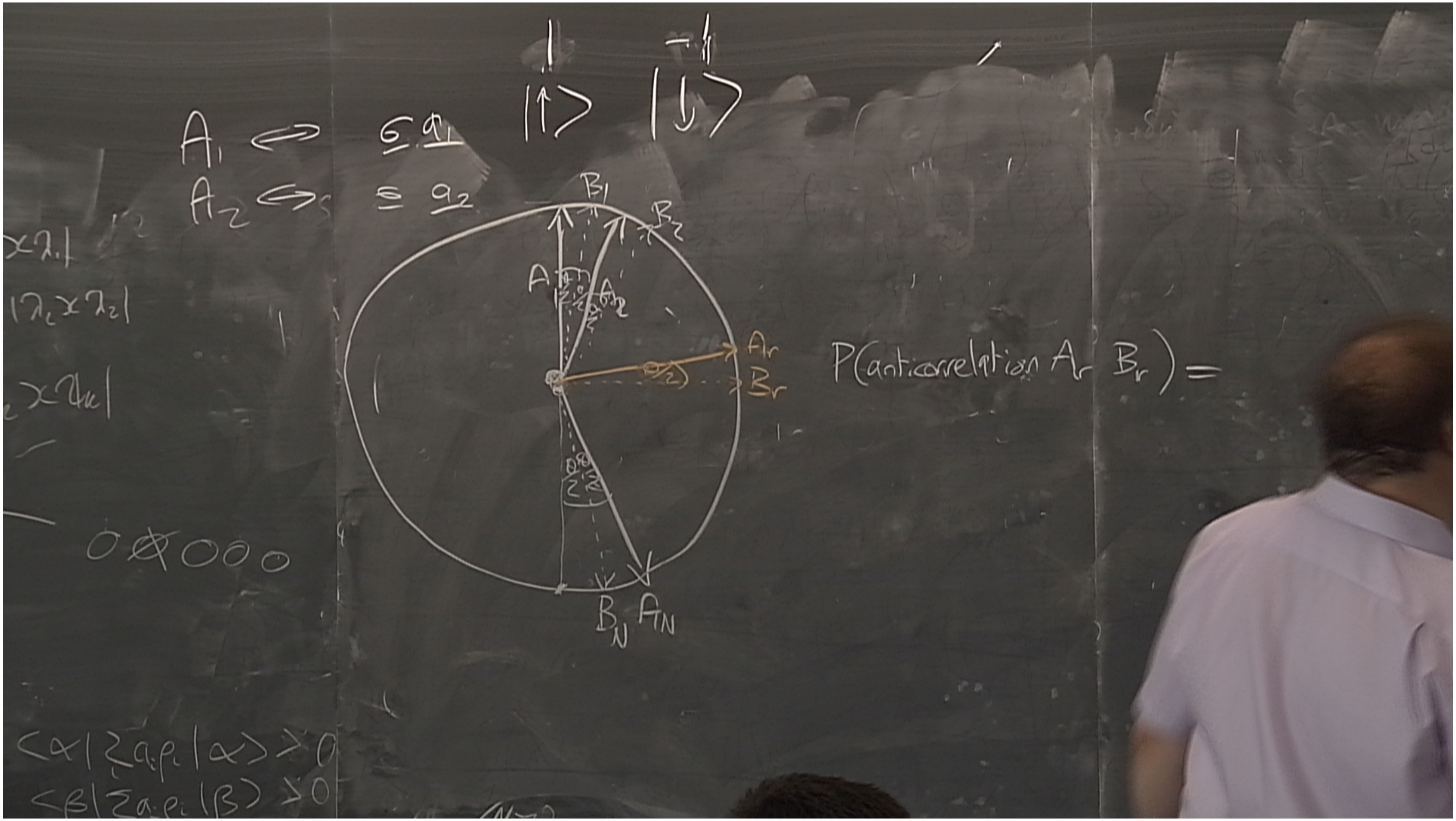
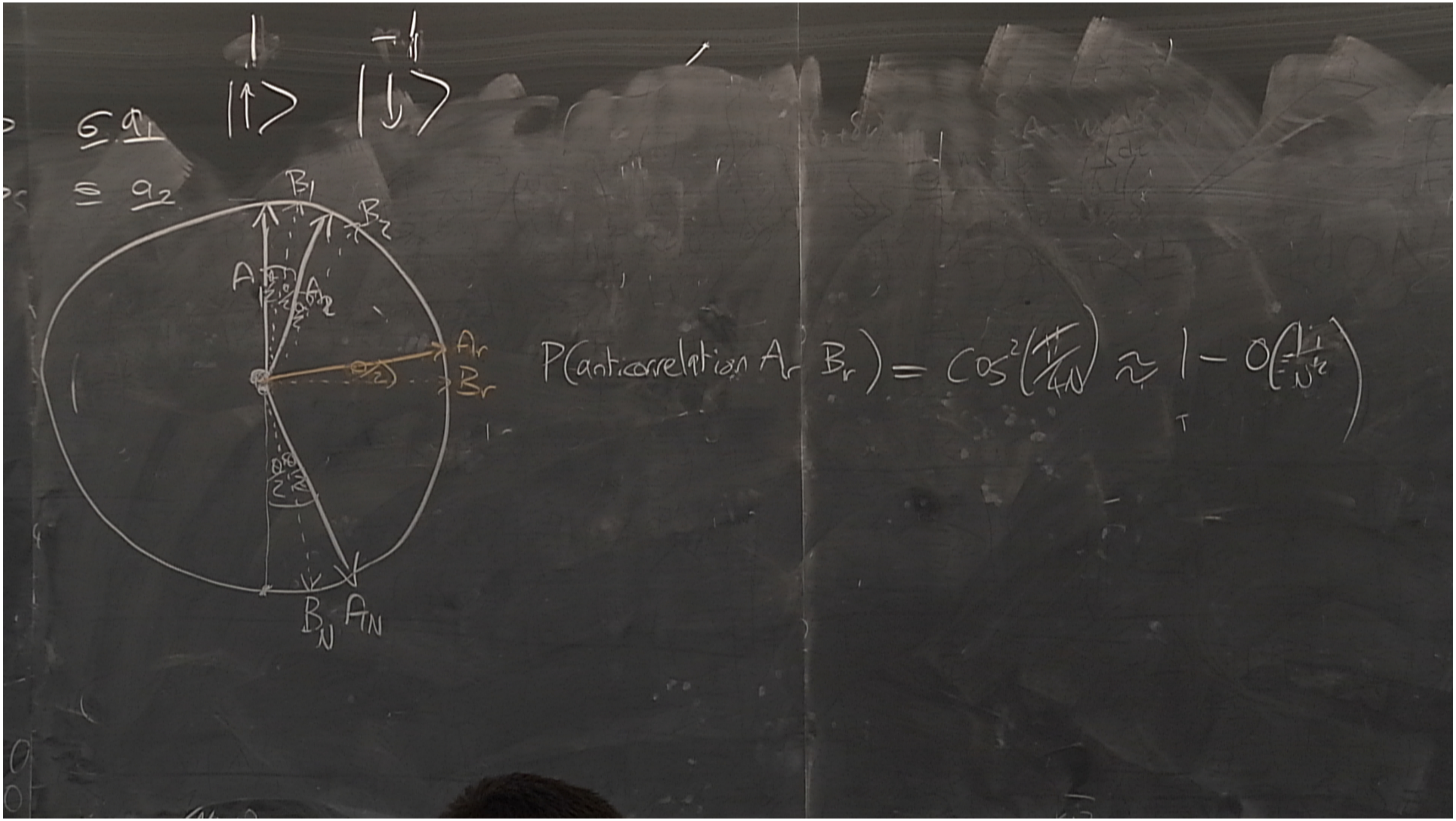
B1
B2
Ar
Br
BN
AN



A1
a1
A2
a2
|↑>
|↓>
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) =


|↑>
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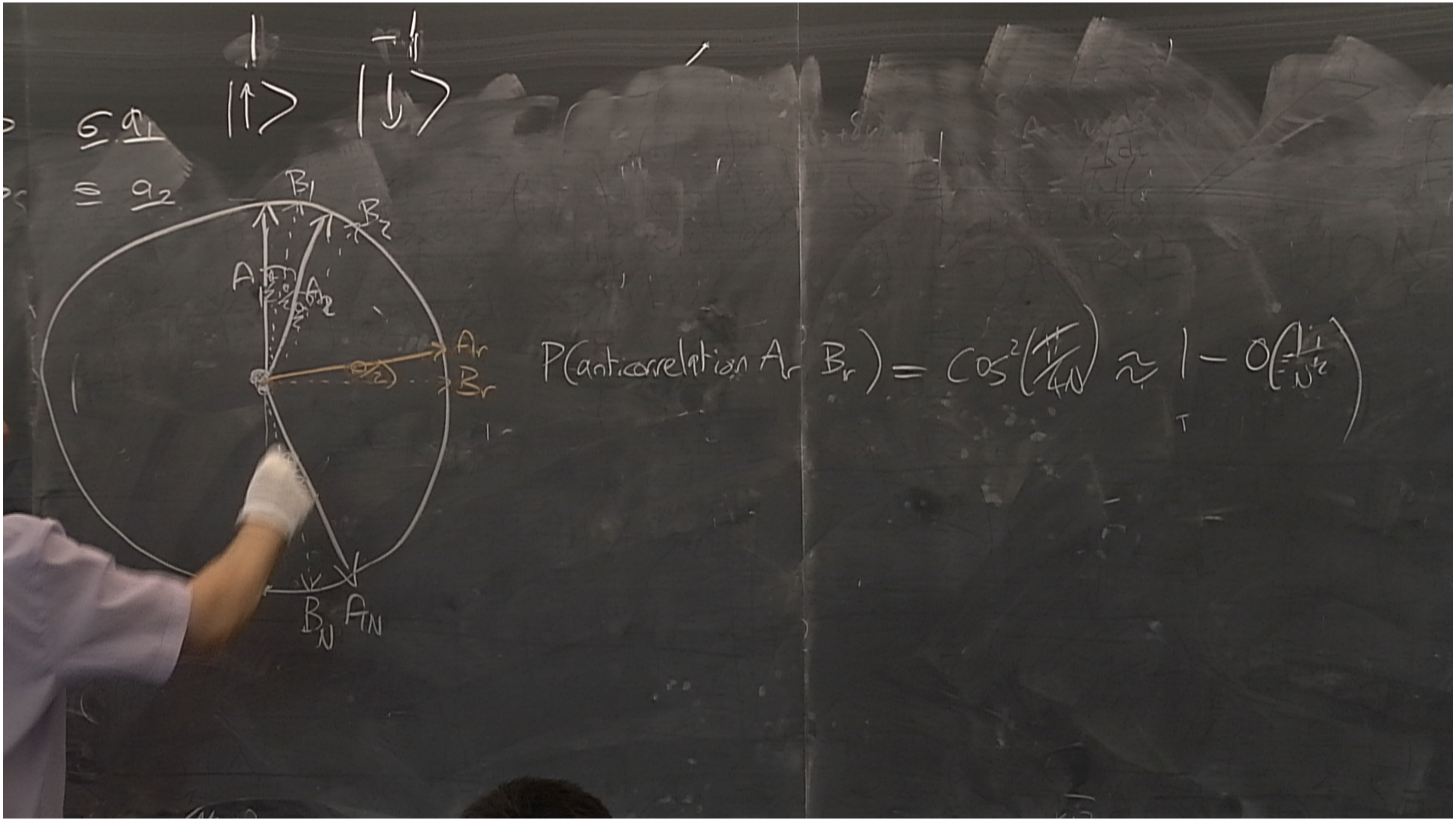
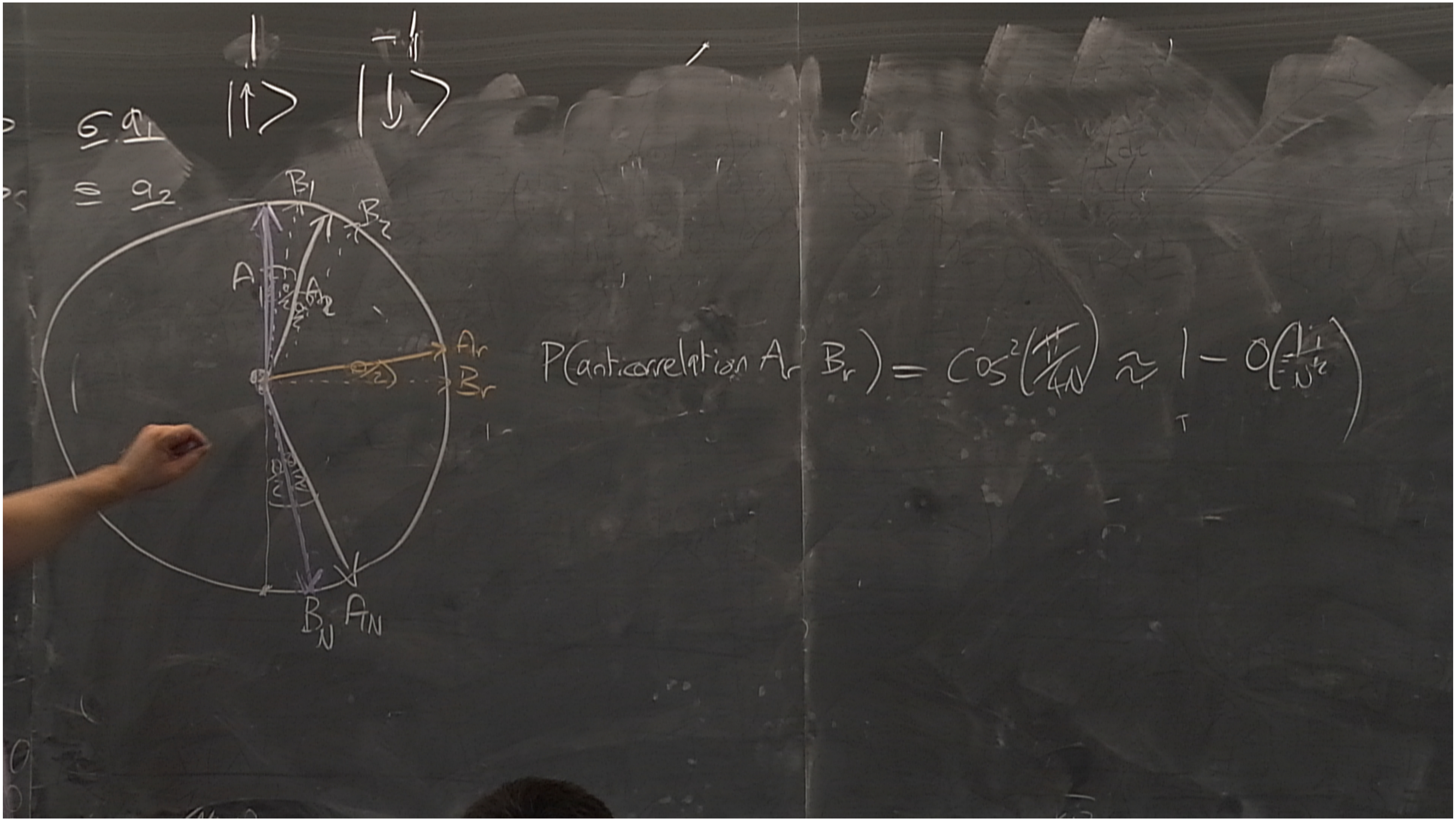
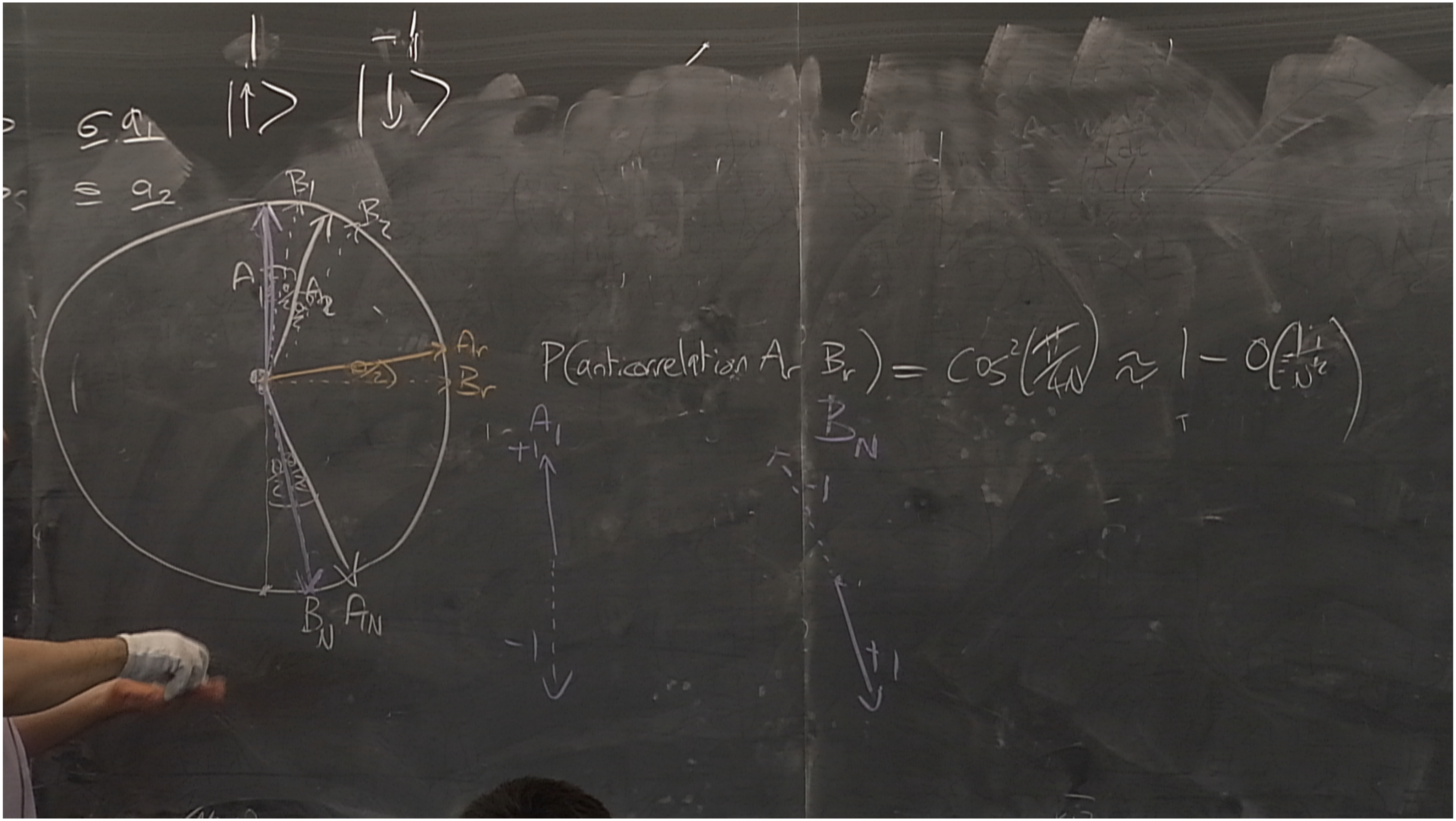
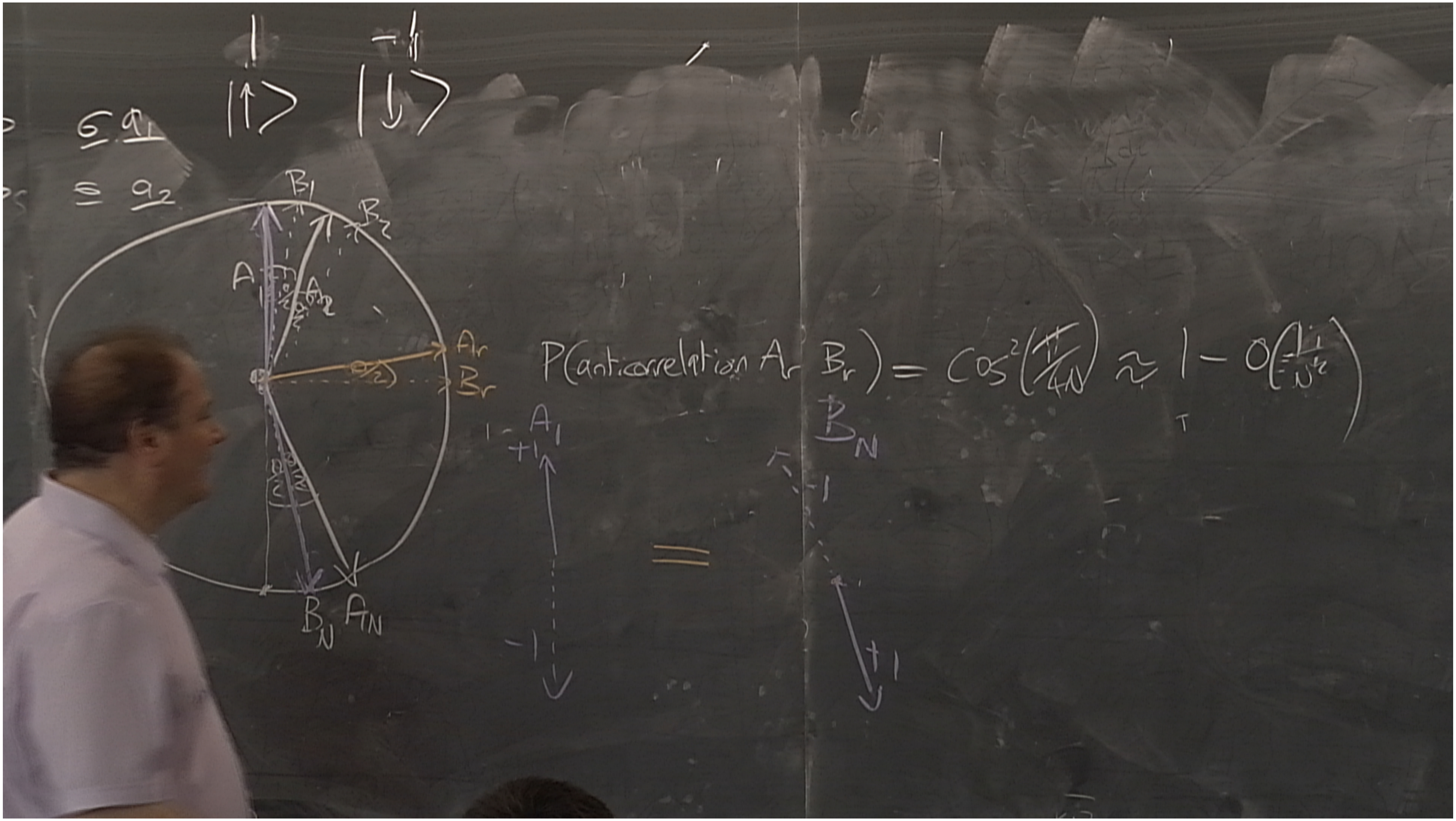
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) = cos²(π/4N)

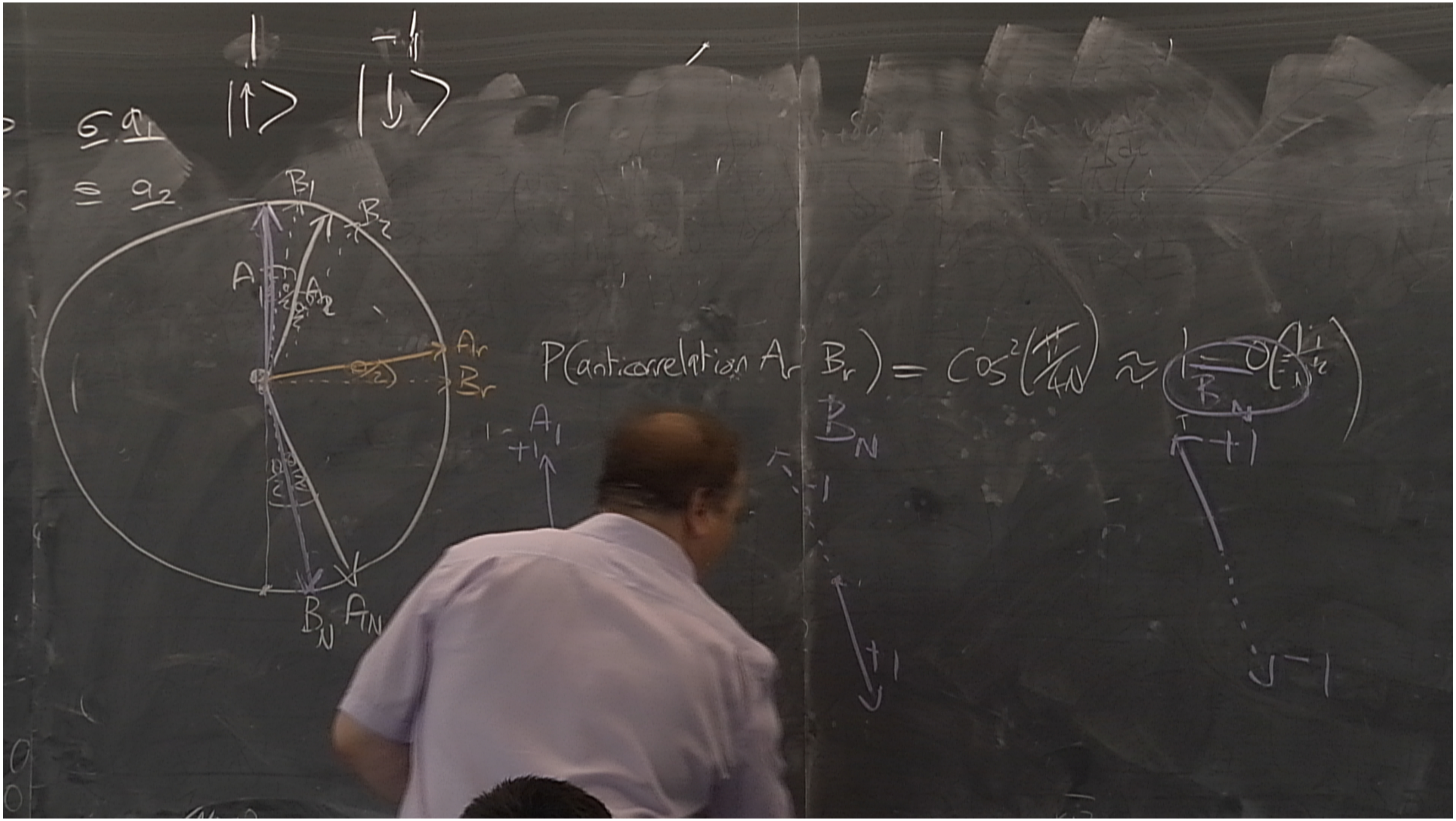

|↑⟩ |↓⟩
P(anticorrelation A_r B_r) = cos²(π/4N) ≈ 1 − O(1/N²)
A_r
B_r
B_N A_N

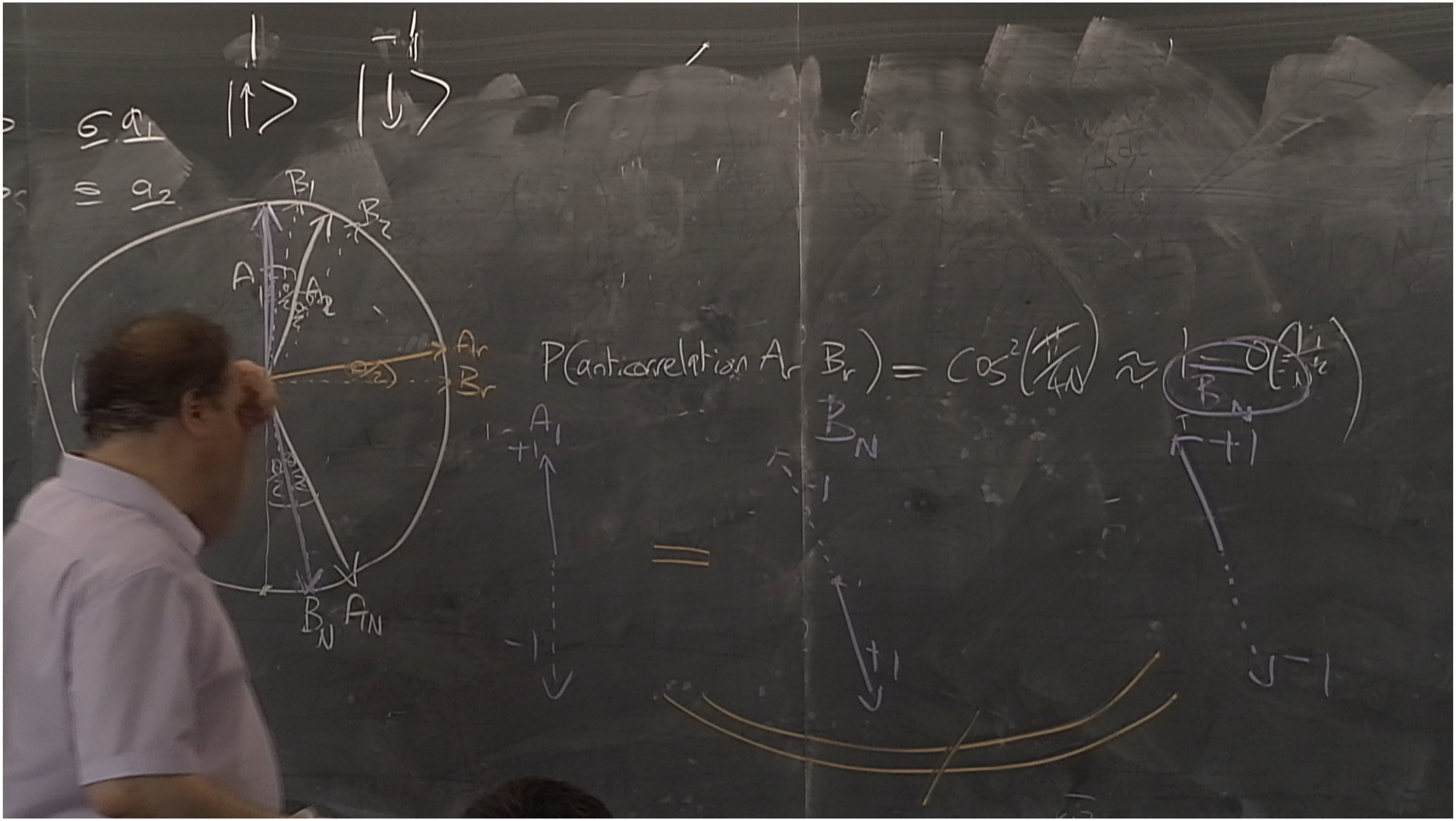

|↑>
|↓>
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) = cos²(π/4N) ≈ 1 − O(1/N²)

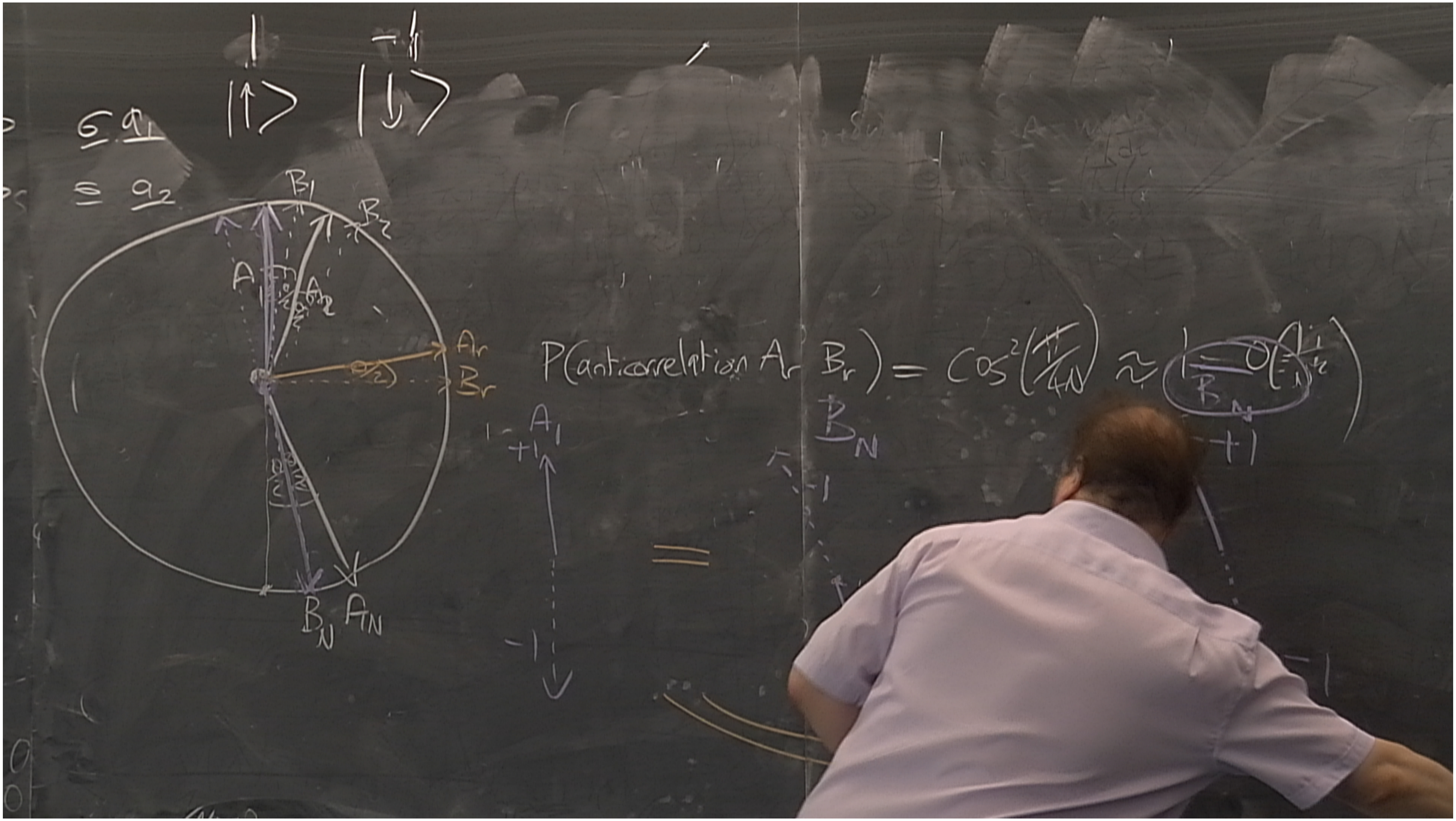

|↑>
|↓>
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) = cos²(π/4N) ≈ 1 − O(1/N²)
A1
BN
+1
−1
−1
+1

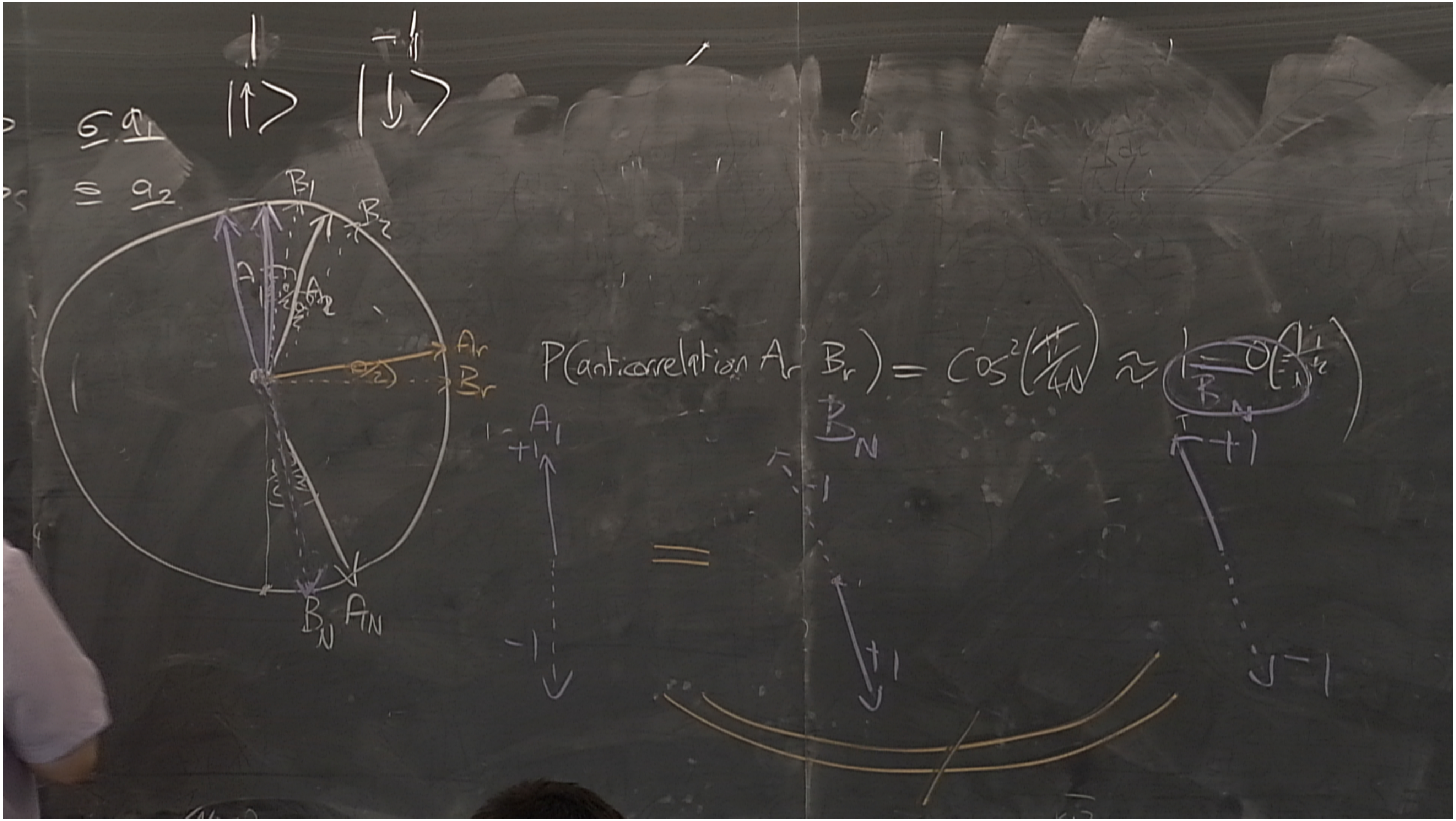


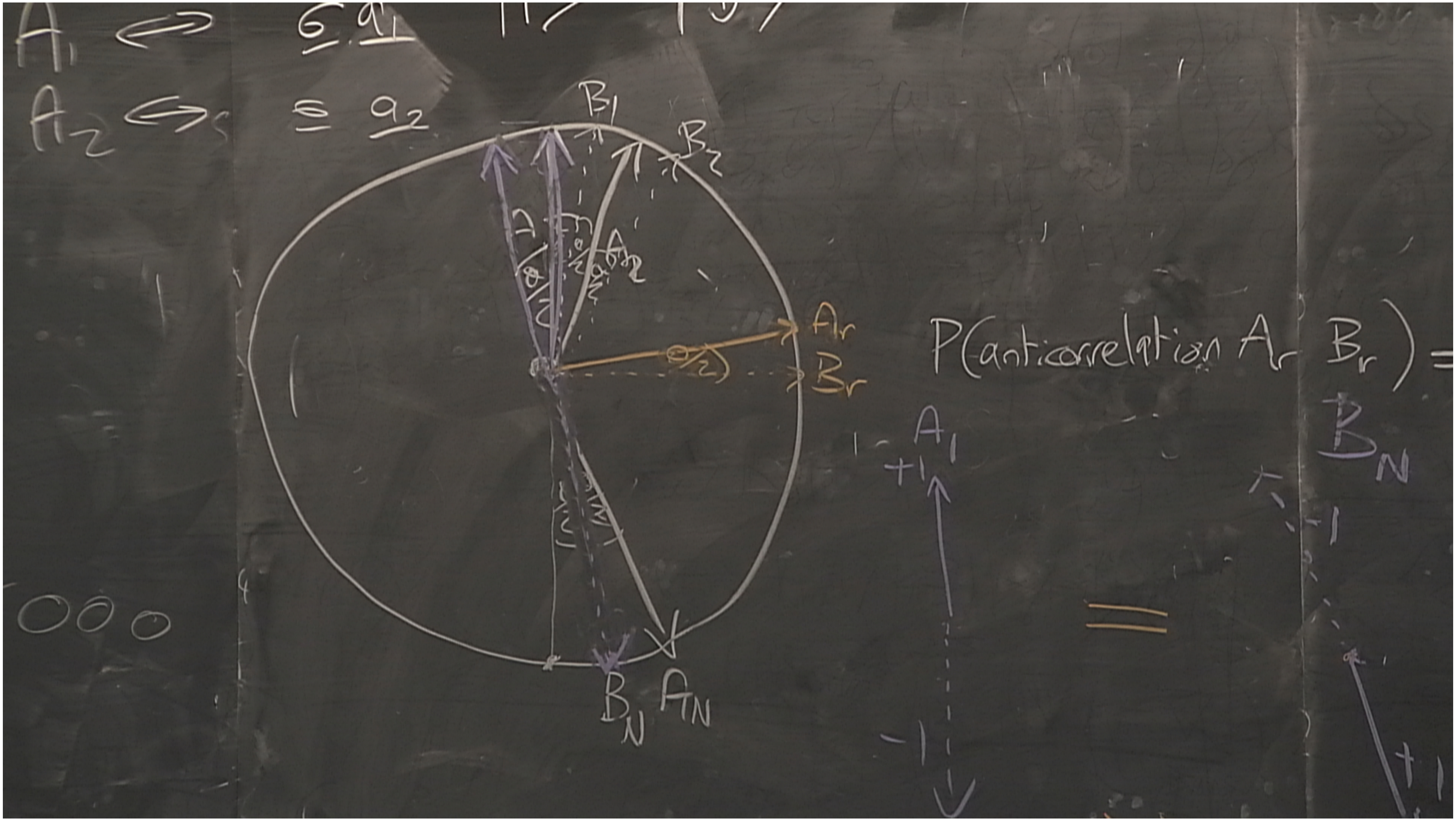
|↑> |↓>
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) = cos²(π/4N) ≈ 1 − O(1/N²)


|↑>
|↓>
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) = cos²(π/4N)
+1
-1


|↑>
|↓>
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) = cos²(π/4N)
+1
-1


|↑>
|↓>
B1
B2
A1
A2
Ar
Br
BN
AN
P(anticorrelation Ar Br) = cos²(π/4N)


P(anticorrelation A_r B_r) = $\cos^2(\frac{\pi}{4N})$ ≈ 1 − O($\frac{1}{N^2}$)
A_r
B_r
B_1
B_2
A_1
A_2
B_N
A_N
|↑⟩
|↓⟩
+1
−1



$A_1 \Leftrightarrow$
$A_2 \Leftrightarrow$
B_1
B_2
A_2
A_r
B_r
P(anticorrelation A_r B_r) =
A_1
B_N
B_N
A_N
+1
-1
=

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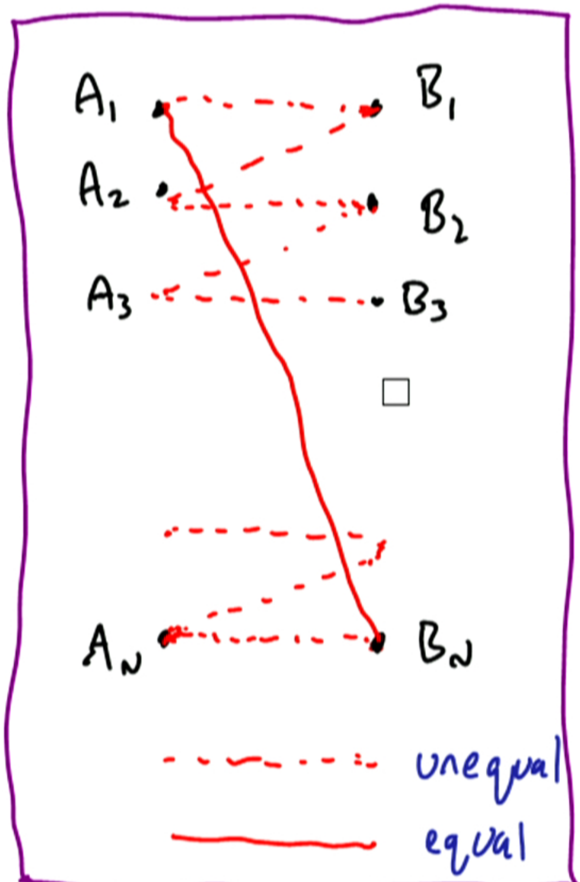
Braunstein-Caves inequality (cont.)

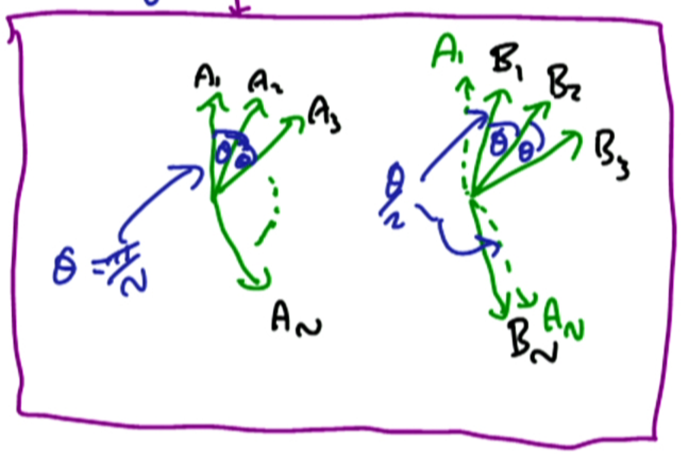


Not all these relations can simultaneously hold.

If we do 2N random tests we expect at least 1 violation, according to EPR/LHV.

I.e. $P(\text{violation}) \geq \frac{1}{2N}$

But QM predicts prob. of violation $\sim 1/N^2$ for e.g. this choice of measurements on the singlet



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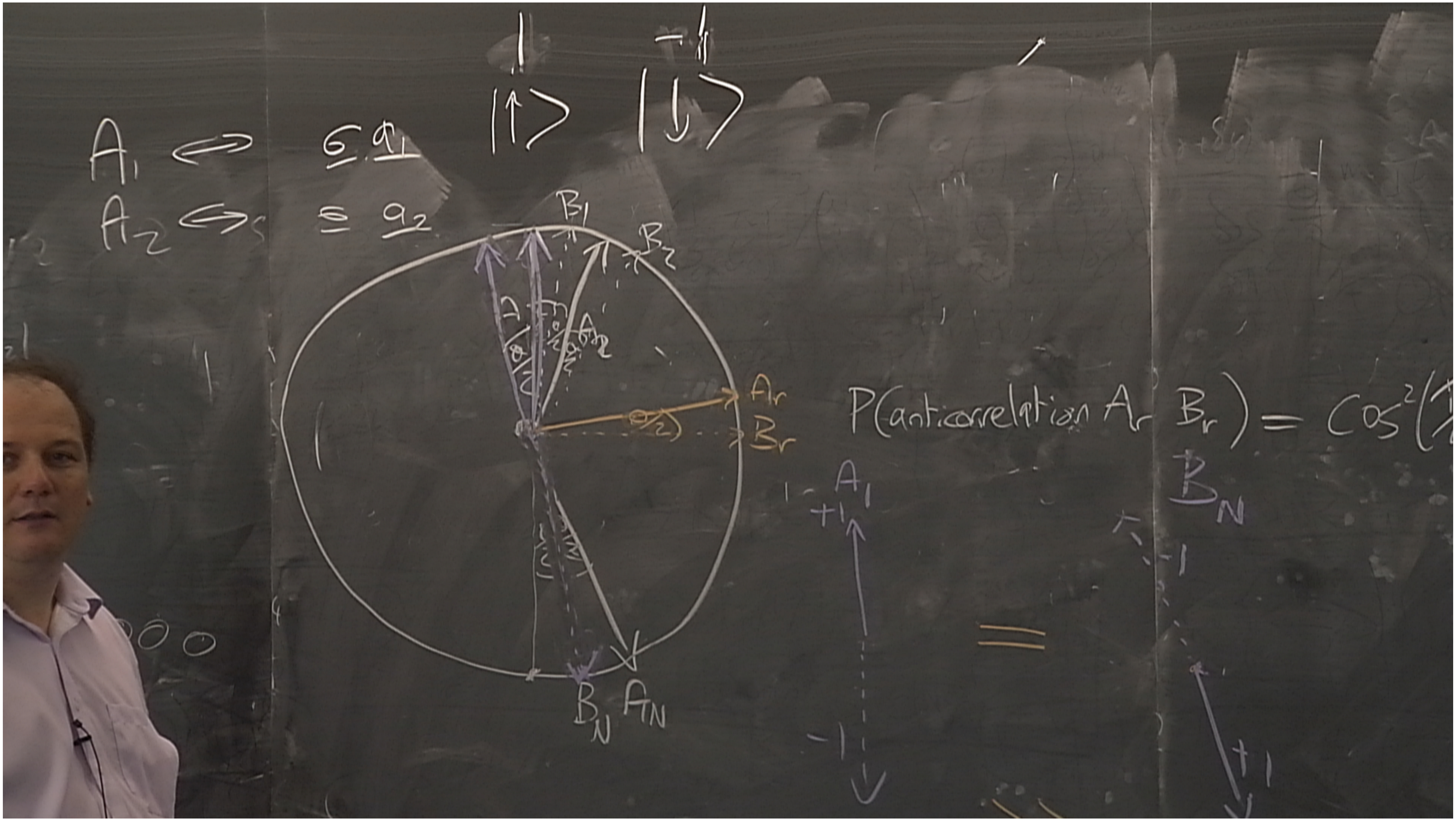

$A_1 \Leftrightarrow \underline{\sigma}.\underline{a}_1$
$A_2 \Leftrightarrow \underline{\sigma}\ \underline{a}_2$
$|\uparrow\rangle$
$|\downarrow\rangle$
B_1
B_2
A_r
B_r
B_N
A_N
P(anticorrelation A_r B_r) = $\cos^2($
A_1
+1
-1
B_N
-1
+1

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Extensions of Bell-CHSH II: the GHZ state

We can produce an even more striking contradiction between the EPR argument and quantum theory if we consider 3 subsystems of spin ½ particles in the state $|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \otimes |\uparrow\rangle_3 - |\downarrow\rangle_1 \otimes |\downarrow\rangle_2 \otimes |\downarrow\rangle_3 \right)$

Key facts: ① the operators $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$ all commute.

For example $(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)$

$$= (\sigma_x^1 \sigma_y^1 \otimes \sigma_y^2 \sigma_x^2 \otimes \sigma_y^3 \sigma_y^3)$$

$$= (-1)^2 (\sigma_y^1 \sigma_x^1 \otimes \sigma_x^2 \sigma_y^2 \otimes \sigma_y^3 \sigma_y^3)$$

$$= (\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$$

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Braunstein-Caves inequality (cont.)

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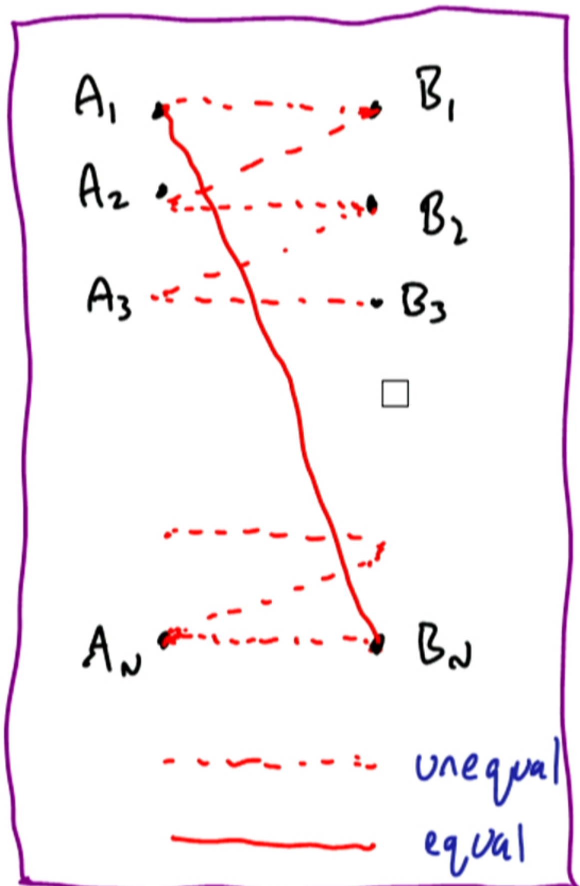

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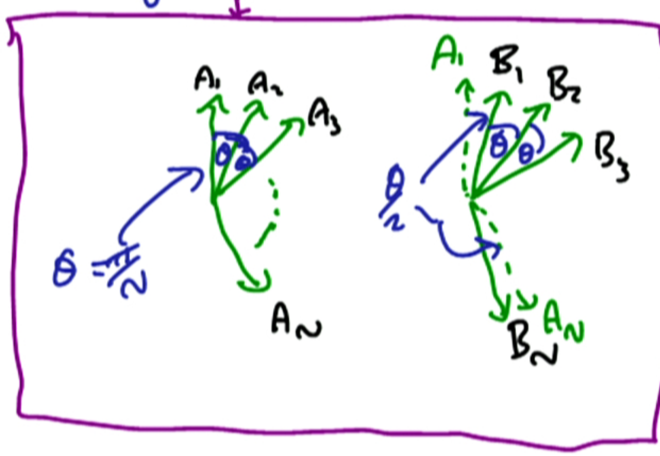

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$$= (\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$$

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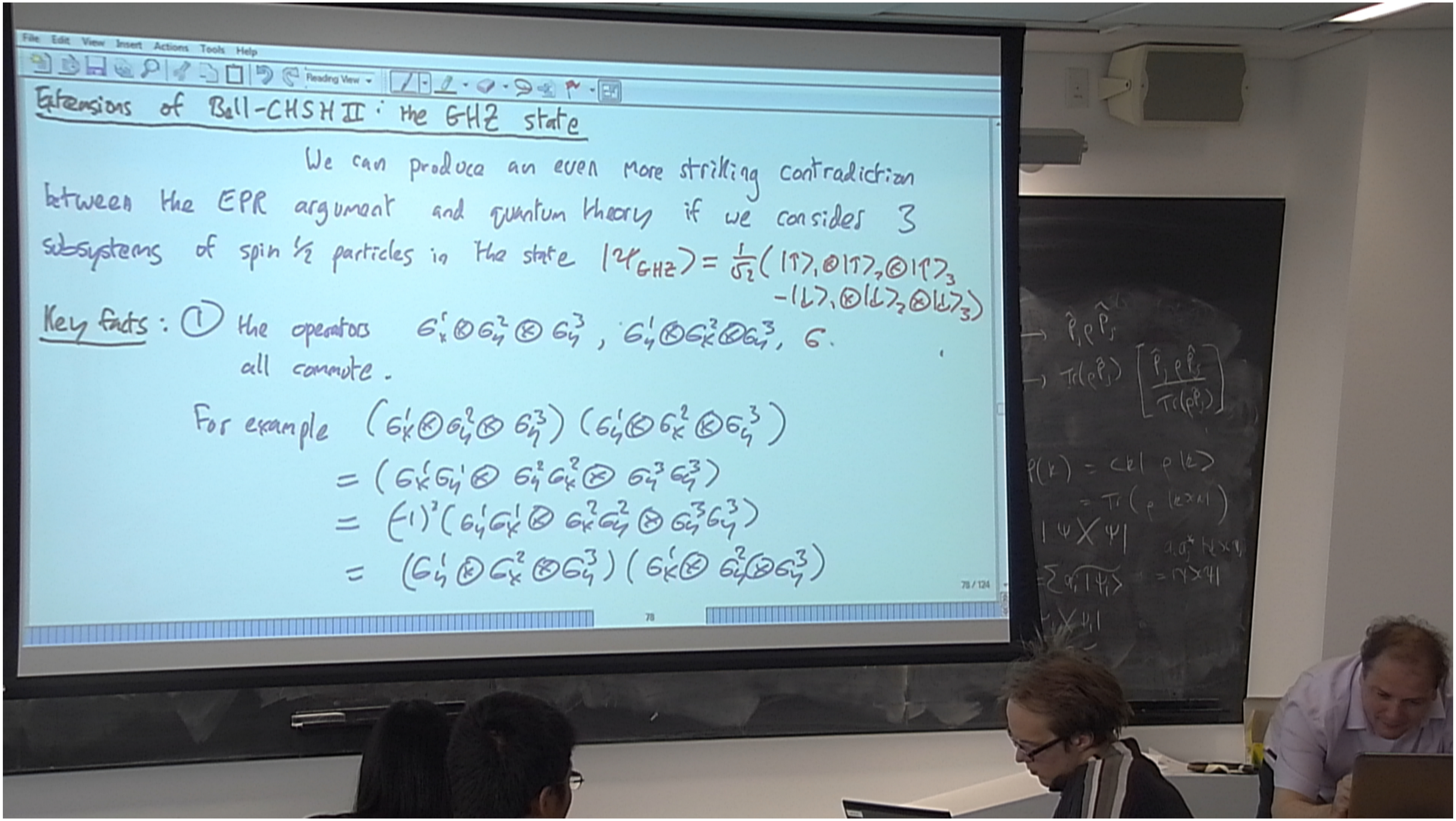
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$= (-1)^2 (\sigma_y^1 \sigma_x^1 \otimes \sigma_x^2 \sigma_y^2 \otimes \sigma_y^3 \sigma_y^3)$
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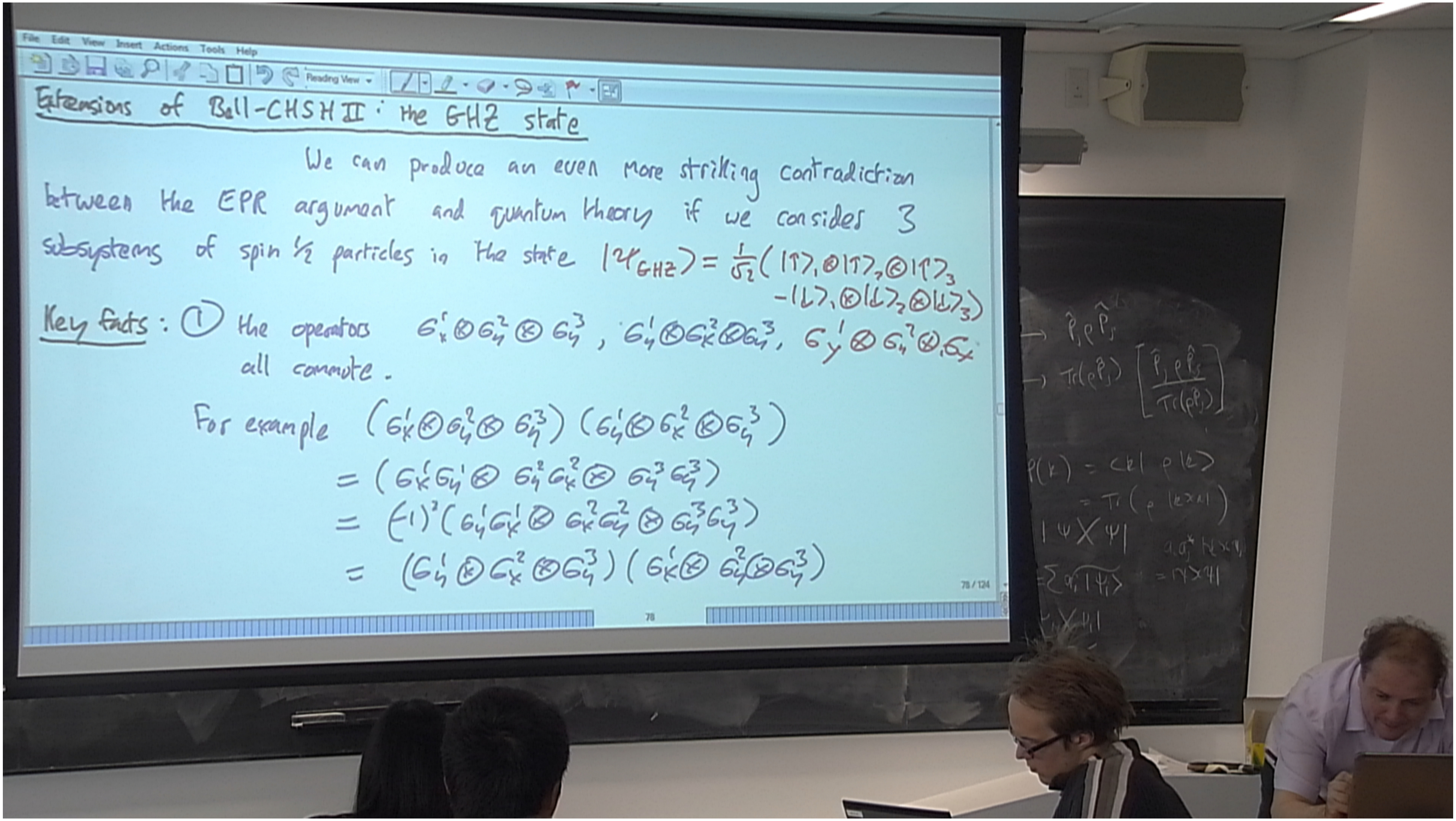
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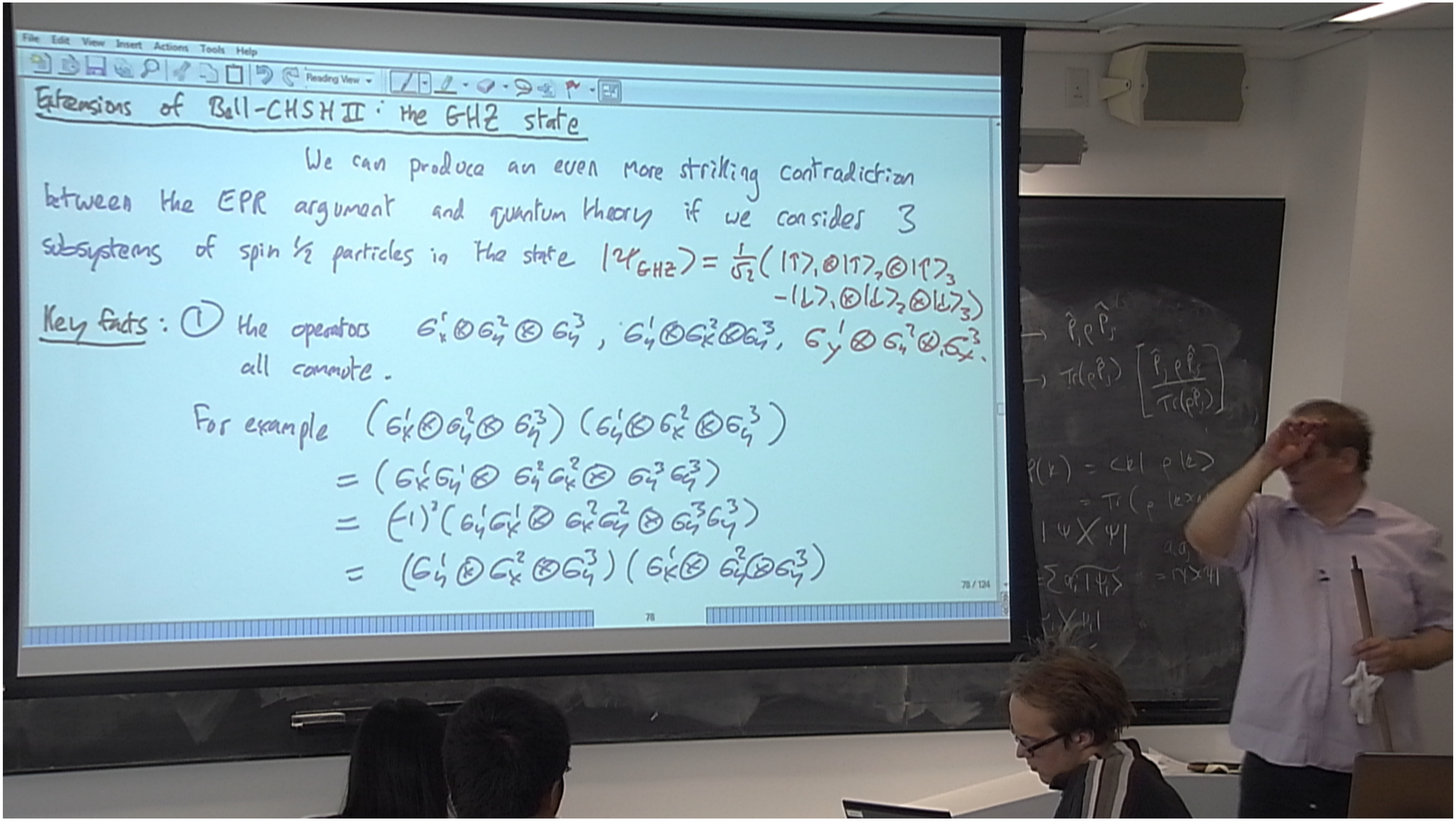
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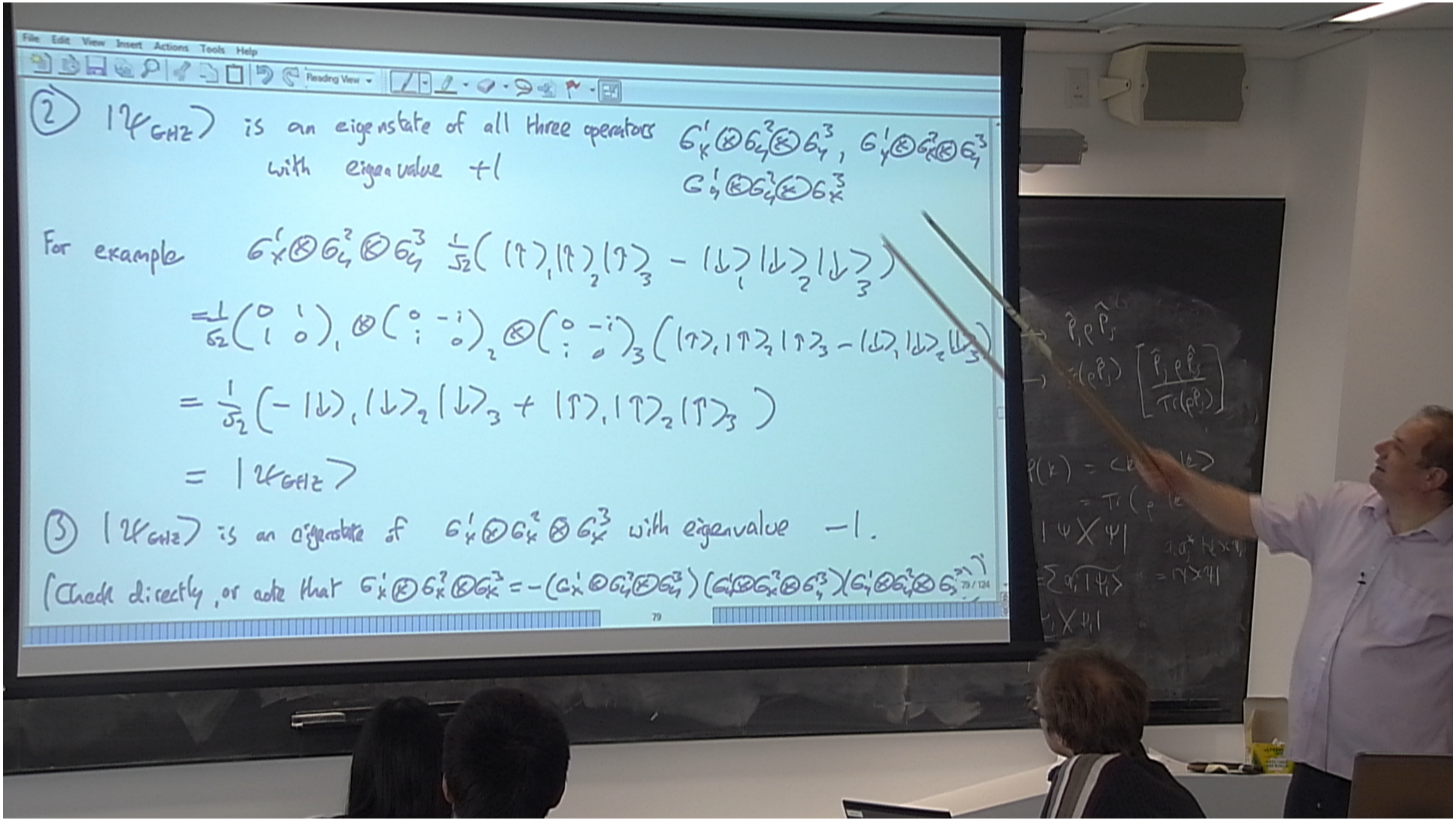
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② $|\psi_{GHZ}\rangle$ is an eigenstate of all three operators $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$
with eigenvalue +1
For example $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3 \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3)$
$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_1 \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_2 \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_3 (|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3)$
$= \frac{1}{\sqrt{2}}(-|\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3 + |\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3)$
$= |\psi_{GHZ}\rangle$
③ $|\psi_{GHZ}\rangle$ is an eigenstate of $\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$ with eigenvalue −1.
(Check directly, or note that $\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3 = -(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3)$
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(2) $|\psi_{GHZ}\rangle$ is an eigenstate of all three operators $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$ with eigenvalue $+1$

For example $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3 \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 - |\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3 \right)$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_1 \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_2 \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_3 \left(|\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 - |\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3 \right)$$

$$= \frac{1}{\sqrt{2}} \left(-|\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3 + |\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 \right)$$

$$= |\psi_{GHZ}\rangle$$

(3) $|\psi_{GHZ}\rangle$ is an eigenstate of $\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$ with eigenvalue -1.

(Check directly, or note that $\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3 = -(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3))$

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④ The EPR criteria suggest there are elements of physical reality $M_x^i, M_y^i = \pm 1$ determining the outcomes of measurements of σ_x^i, σ_y^i

⑤ If so,

$\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$	has determined value	$M_x^1 M_y^2 M_y^3 = +1$
$\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$	" "	$M_y^1 M_x^2 M_y^3 = +1$
$\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$	" "	$M_y^1 M_y^2 M_x^3 = +1$
$\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$	" "	$M_x^1 M_x^2 M_x^3 = -1$

⑥ A logical contradiction! $-1 = M_x^1 M_x^2 M_x^3 = (M_x^1 M_y^2 M_y^3)(M_y^1 M_x^2 M_y^3)(M_y^1 M_y^2 M_x^3)$

$$= +1.$$

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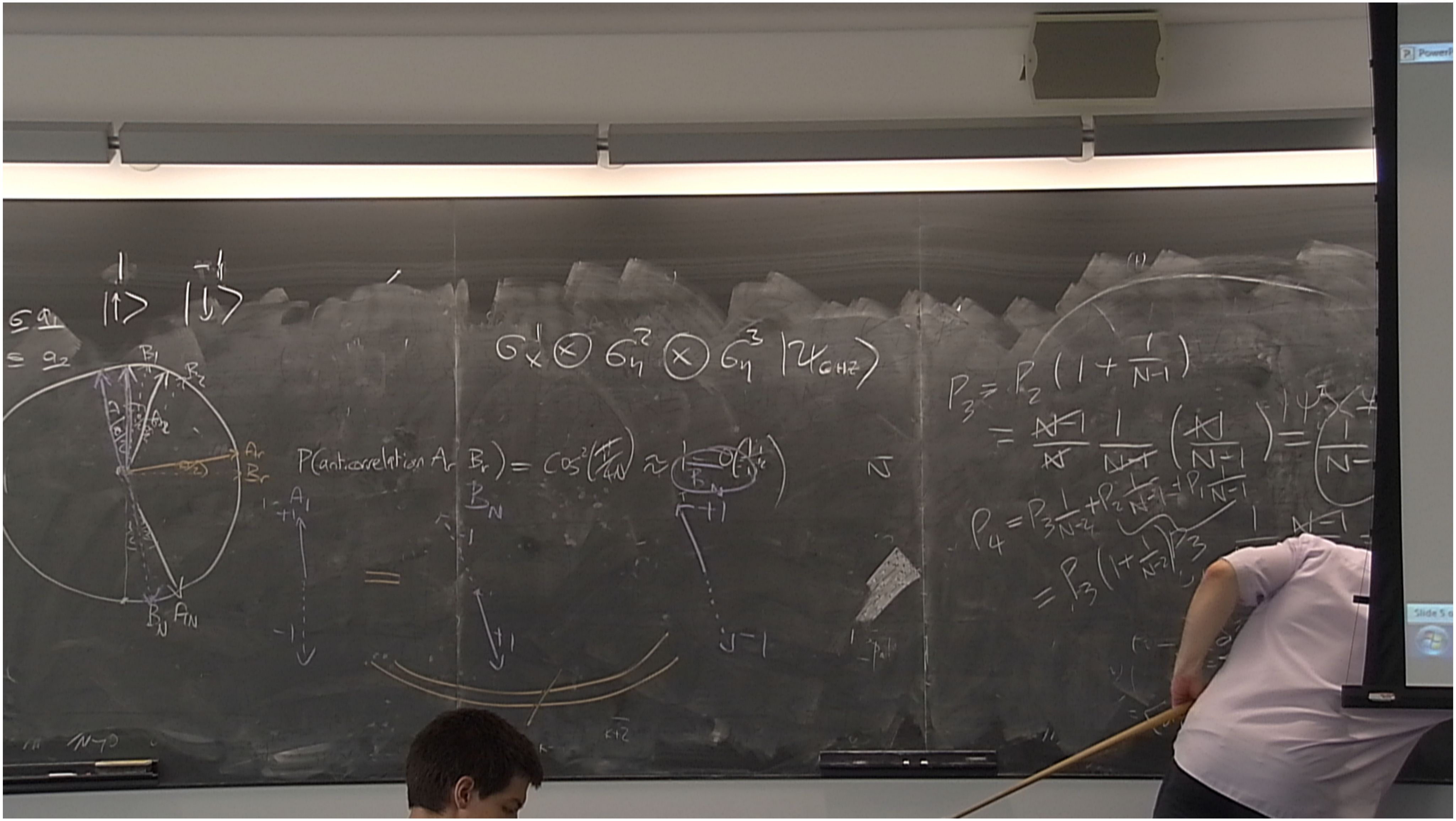
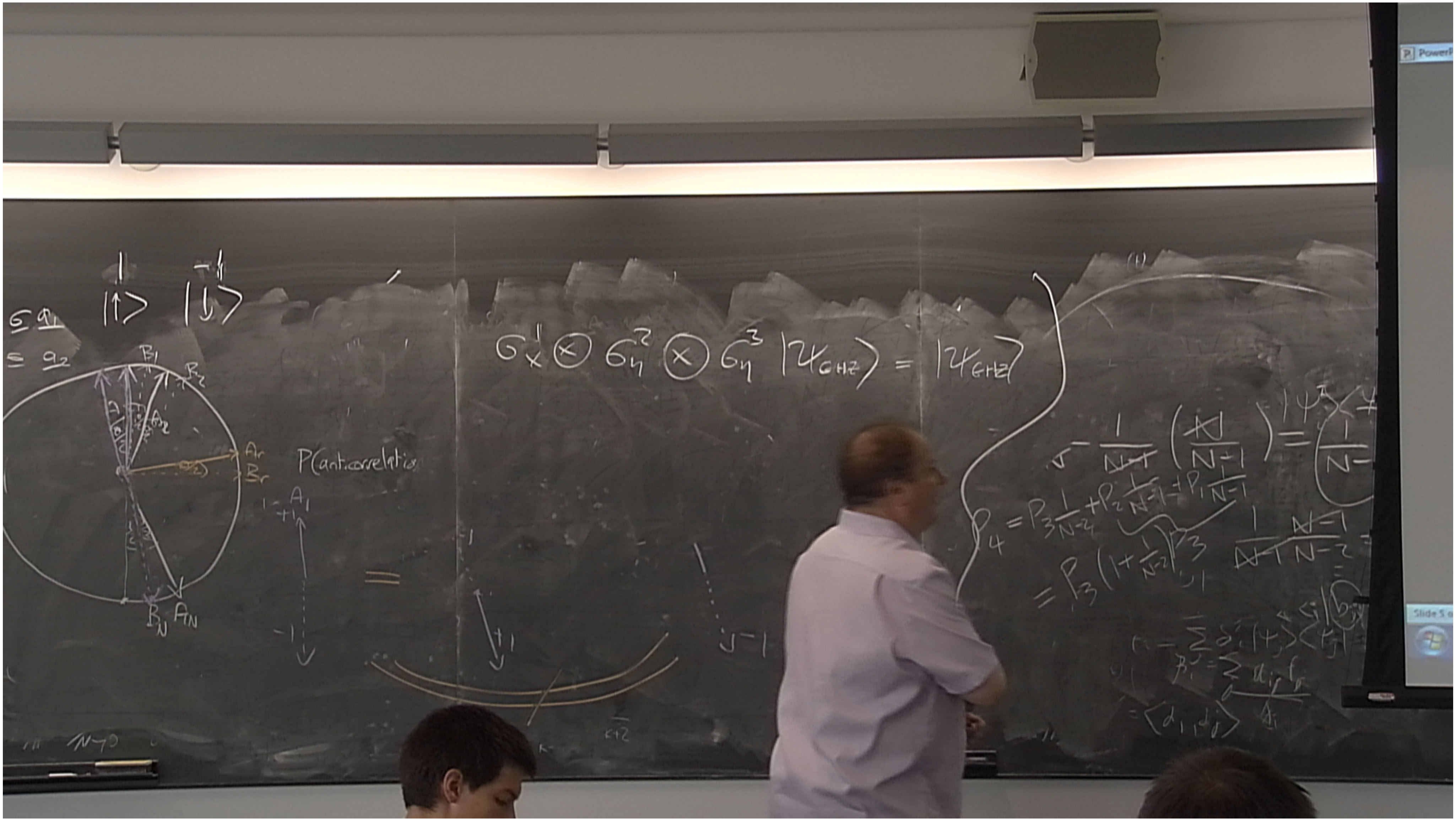

P(anticorrelation A_r B_r) = cos²(π/4N)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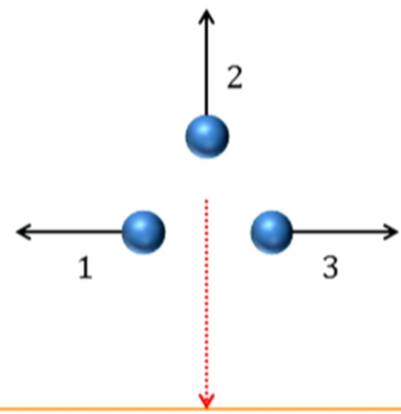
GHZ Argument



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\uparrow\rangle_2 |\uparrow\rangle_3 - |\downarrow\rangle_1 |\downarrow\rangle_2 |\downarrow\rangle_3\right)$$

$$\sigma_x^1 \sigma_y^2 \, \sigma_y^3 |\psi\rangle = |\psi\rangle$$

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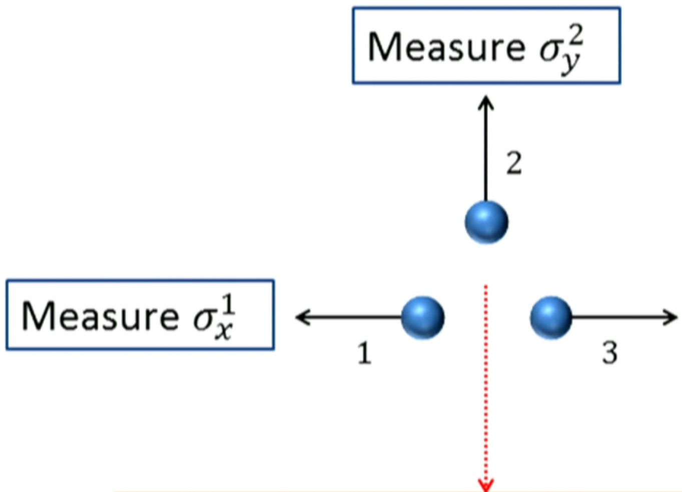
GHZ Argument



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3)$$

$$\sigma_x^1\sigma_y^2\;\sigma_y^3|\psi\rangle = |\psi\rangle$$

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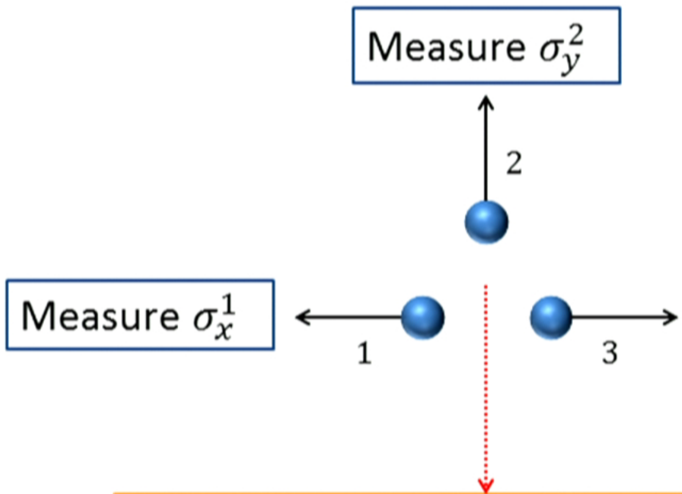
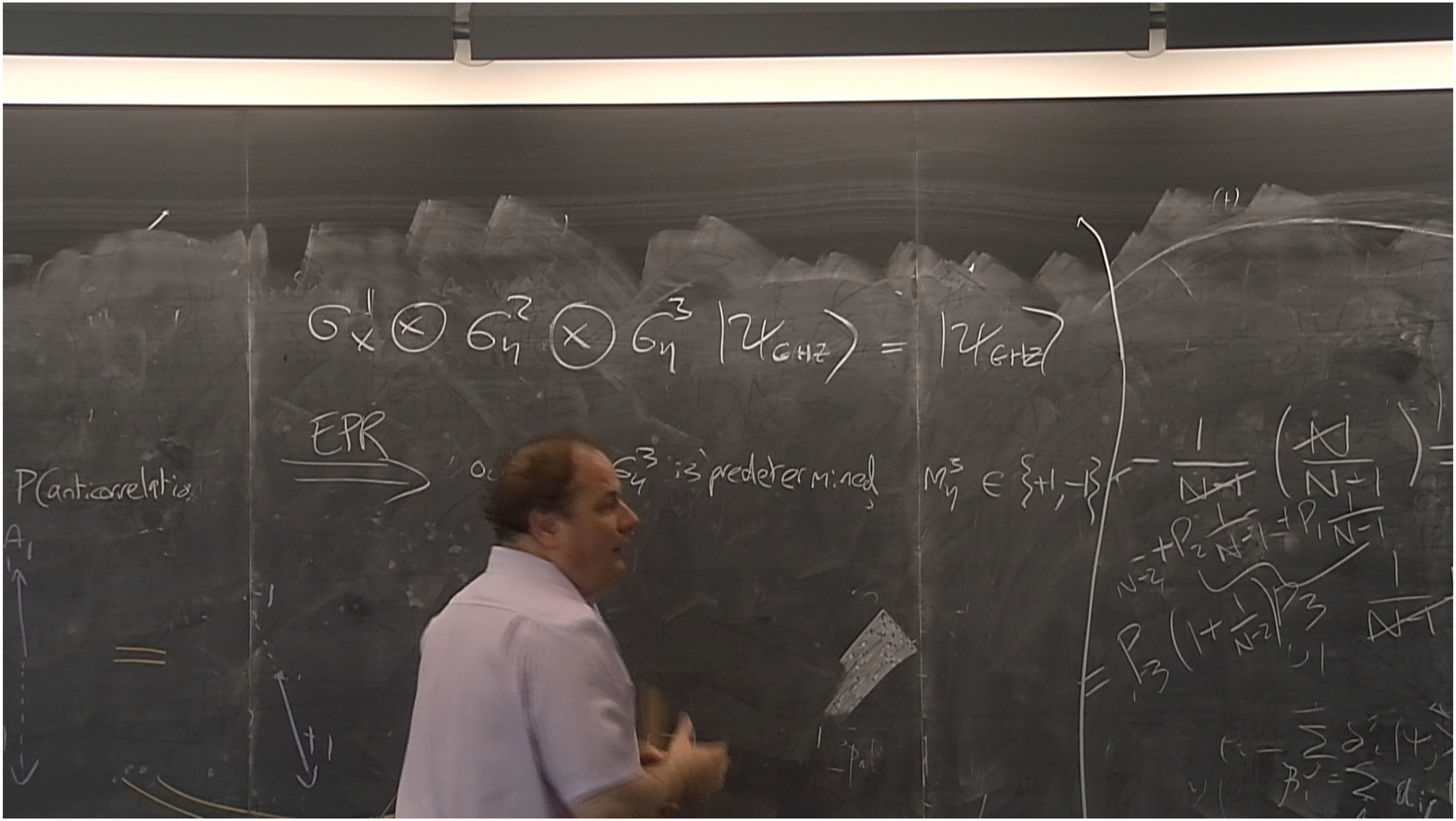
GHZ Argument



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3)$$

$$\sigma_x^1\sigma_y^2\,\sigma_y^3|\psi\rangle = |\psi\rangle$$


EPR
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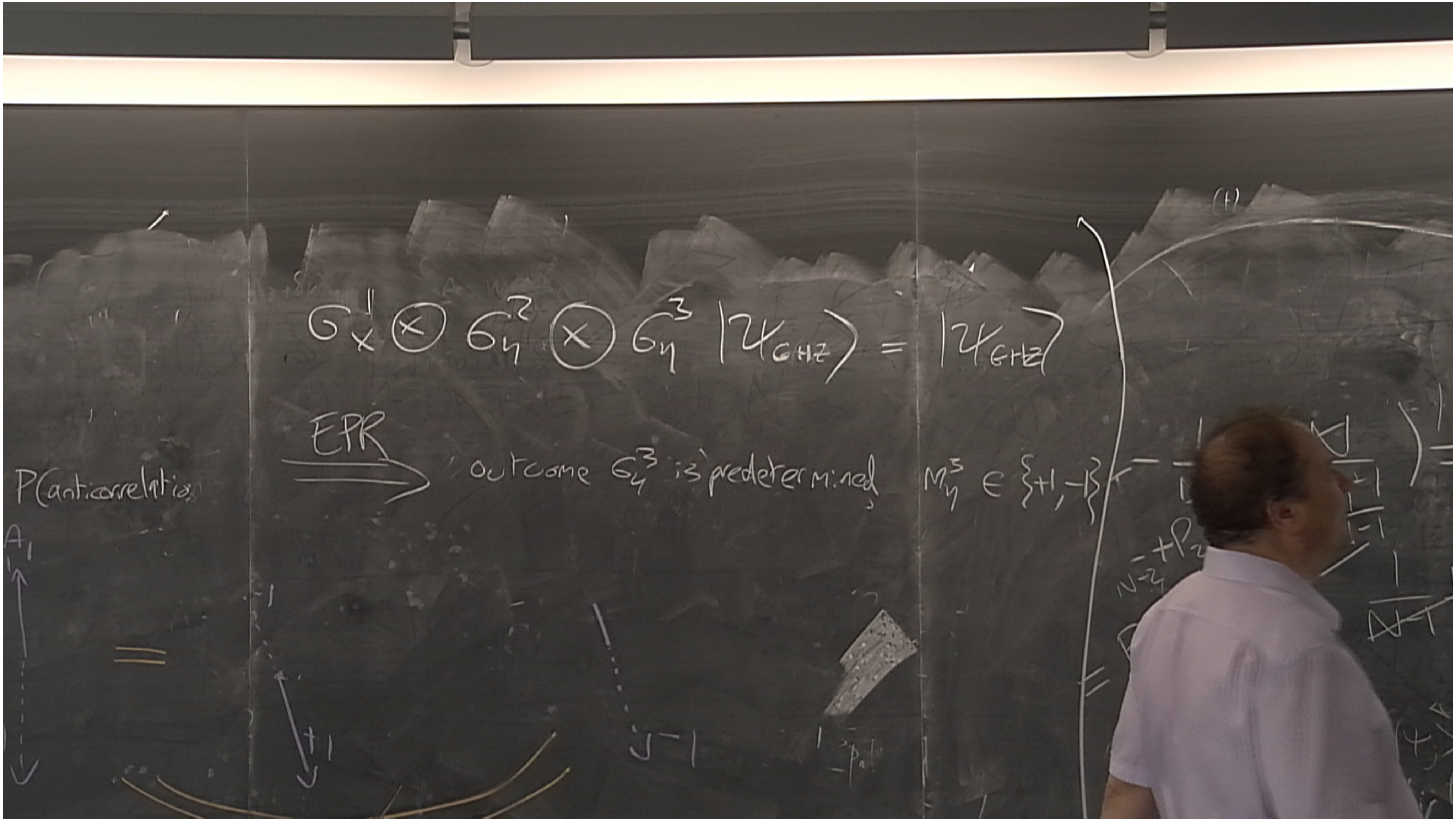
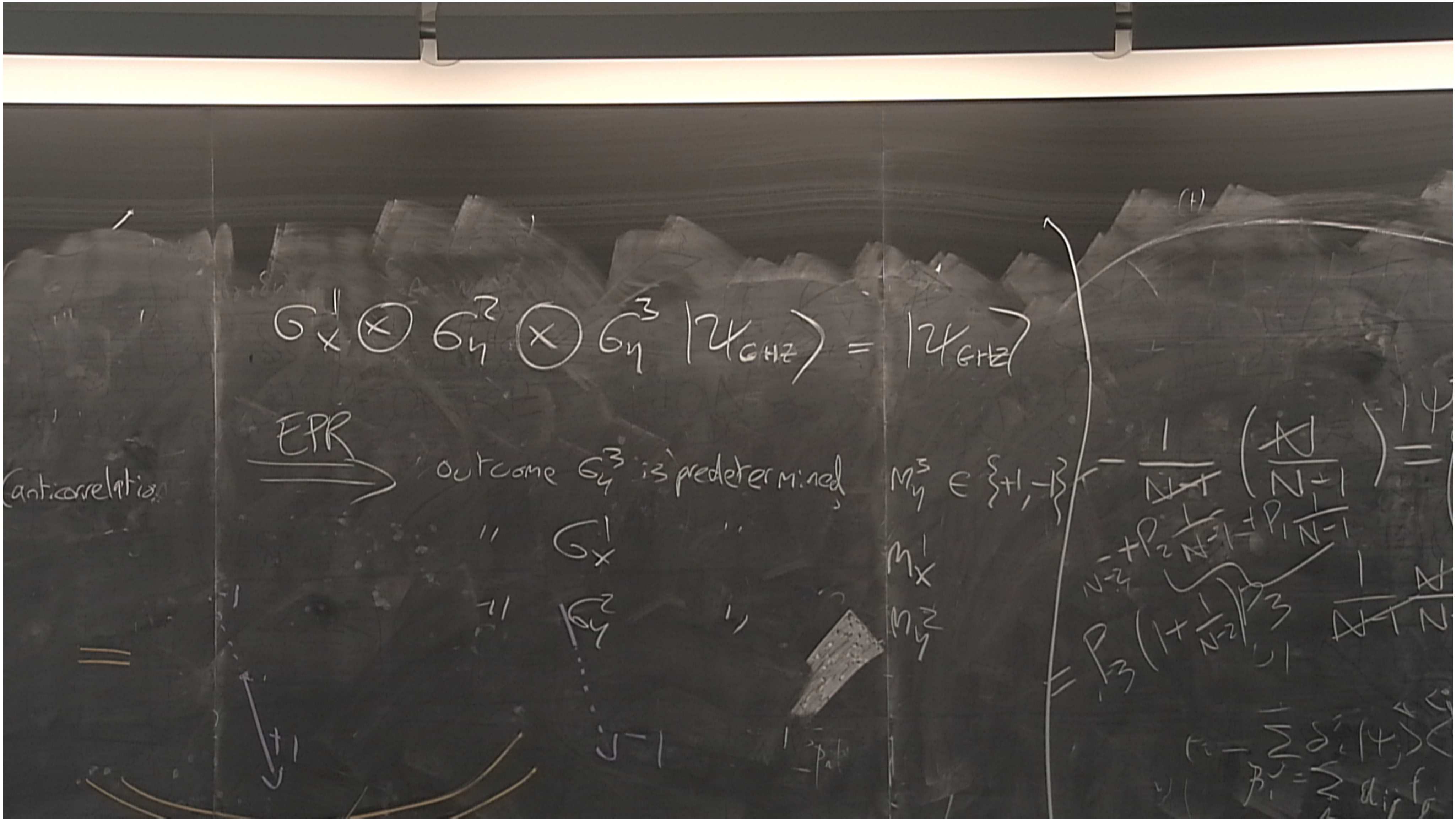
is predetermined


$\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3 |\psi_{GHZ}\rangle = |\psi_{GHZ}\rangle$
EPR
outcome σ_y^3 is predetermined $M_y^3 \in \{+1, -1\}$
P(anticorrelatio


GHZ Argument
Measure σ_y^2
Measure σ_x^1
$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3)$
$\sigma_x^1\sigma_y^2\sigma_y^3|\psi\rangle = |\psi\rangle$


σ_x^1 ⊗ σ_y^2 ⊗ σ_y^3 |ψ_GHZ⟩ = |ψ_GHZ⟩
EPR
anticorrelation
outcome σ_y^3 is predetermined
m_y^3 ∈ {+1, −1}
σ_x^1
m_x^1
σ_y^2
m_y^2

(4) The EPR criteria suggest there are elements of physical reality $m_x^i, m_y^i = \pm 1$ determining the outcomes of measurements of σ_x^i, σ_y^i

(5) If so, $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$ has determined value

$\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$ " "

$\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$ " "

$\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$ " "

$$m_x^1 m_y^2 m_y^3 = +1$$
$$m_y^1 m_x^2 m_y^3 = +1$$
$$m_y^1 m_y^2 m_x^3 = +1$$
$$m_x^1 m_x^2 m_x^3 = -1$$

(6) A logical contradiction! $-1 = m_x^1 m_x^2 m_x^3 = (m_x^1 m_y^2 m_y^3)(m_y^1 m_x^2 m_y^3)(m_y^1 m_y^2 m_x^3)$

$= +1.$

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Extensions of Bell-CHSH II: the GHZ state

We can produce an even more striking contradiction between the EPR argument and quantum theory if we consider 3 subsystems of spin ½ particles in the state $|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \otimes |\uparrow\rangle_3 - |\downarrow\rangle_1 \otimes |\downarrow\rangle_2 \otimes |\downarrow\rangle_3\right)$

Key facts: ① the operators $\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$, $\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$, all commute.

For example $(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)(\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)$

$$= (\sigma_x^1 \sigma_y^1 \otimes \sigma_y^2 \sigma_x^2 \otimes \sigma_y^3 \sigma_y^3)$$

$$= (-1)^2 (\sigma_y^1 \sigma_x^1 \otimes \sigma_x^2 \sigma_y^2 \otimes \sigma_y^3 \sigma_y^3)$$

$$= (\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3)(\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3)$$

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(4) The EPR criteria suggest there are elements of physical reality $m_x^i, m_y^i = \pm 1$ determining the outcomes of measurements of σ_x^i, σ_y^i

(5) If so,

$\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$	has determined value		$m_x^1 m_y^2 m_y^3 = +1$
$\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$	"	"	$m_y^1 m_x^2 m_y^3 = +1$
$\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$	"	"	$m_y^1 m_y^2 m_x^3 = +1$
$\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$	"	"	$m_x^1 m_x^2 m_x^3 = -1$

(6) A logical contradiction! $-1 = m_x^1 m_x^2 m_x^3 = (m_x^1 m_y^2 m_y^3)(m_y^1 m_x^2 m_y^3)(m_y^1 m_y^2 m_x^3)$

$= +1.$

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EPR offer two criteria for "elements of physical reality" in physical theories.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.* We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

EPR 1

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

EPR 2

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(4) The EPR criteria suggest there are elements of physical reality $m_x^i, m_y^i = \pm 1$ determining the outcomes of measurements of σ_x^i, σ_y^i

(5) If so,

$\sigma_x^1 \otimes \sigma_y^2 \otimes \sigma_y^3$	has determined	value	$m_x^1 m_y^2 m_y^3 = +1$
$\sigma_y^1 \otimes \sigma_x^2 \otimes \sigma_y^3$	"	"	$m_y^1 m_x^2 m_y^3 = +1$
$\sigma_y^1 \otimes \sigma_y^2 \otimes \sigma_x^3$	"	"	$m_y^1 m_y^2 m_x^3 = +1$
$\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3$	"	"	$m_x^1 m_x^2 m_x^3 = -1$

(6) A logical contradiction! $-1 = m_x^1 m_x^2 m_x^3 = (m_x^1 m_y^2 m_y^3)(m_y^1 m_x^2 m_y^3)(m_y^1 m_y^2 m_x^3)$

$= +1.$

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Nick Herbert (physicist)

From Wikipedia, the free encyclopedia

Nick Herbert is an American physicist and author, best known for his book *Quantum Reality*.

Herbert studied Engineering Physics at the Ohio State University, graduating in 1959. He received a Ph.D. in physics from Stanford University in 1967 for work on nuclear scattering experiments. After a one year teaching job at Monmouth College in Illinois, Herbert held a number of posts in industry. The most illustrious of these was senior physicist at Memorex Corporation in Santa Clara, California, where he developed new magnetic materials, as well as magnetic, electrostatic and optical measuring devices, and carried out theoretical work on Lorentz microscopy. He was also senior physicist at Smith-Corona Marchant Corporation in Palo Alto, California where he developed a new theory of xerographic process and worked on early developments in ink jet printing.[1][2]

While employed in industry, Herbert was part of the Fundamental Fysiks Group at Lawrence Berkeley National Laboratory, founded in May 1975 by Elizabeth Rauscher and George Weissmann.[3] The group's initial interest was in the interpretation of quantum mechanics, the EPR paradox, and Bell's inequality, but members pursued diverse interests that lay outside of mainstream physics, exploring psychedelic drugs, psi phenomena, of consciousness, and speculative connections of these areas with quantum physics. During the 1970s and 1980s, Herbert and Saul-Paul Sirag organized a yearly Esalen Seminar on the Nature of Reality, bringing together participants to discuss the interpretation of quantum mechanics.[4] With Richard Shoup of Xerox PARC, Herbert constructed a "Metaphase Typewriter", a "quantum operated" device whose purpose was "to communicate with disembodied spirits".[1] Despite many tests, including an attempt to contact the spirit of Harry Houdini on the hundredth anniversary of his birth, the group reported no success with the device.[5]

In 1981, Herbert proposed *FLASH*, a scheme for sending signals faster than the speed of light using quantum entanglement.[6] Of this proposal, quantum computing pioneer Asher Peres wrote, "I was the referee who approved the publication of Nick Herbert's *FLASH paper*, knowing perfectly well that it was wrong. I explain why my decision was the correct one, and I briefly review the progress to which it led." Chief among the results that Peres claimed stemmed from a refutation of Herbert's proposal was the no-cloning

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Nick Herbert's FLASH (First Light Amplification Superluminal Hookup) proposal

A and B share $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B \right)$

A measures in $\{|\uparrow\rangle_A, |\downarrow\rangle_A\}$ to signal 0, $\{|\rightarrow\rangle_A, |\leftarrow\rangle_A\}$ to signal 1.

B now makes a second copy of B's state. So from B's perspective:

if 0, B has $\frac{1}{2}\left(|\uparrow\rangle_B |\uparrow\rangle_B \langle\uparrow|_B \langle\uparrow|_B + |\downarrow\rangle_B |\downarrow\rangle_B \langle\downarrow|_B \langle\downarrow|_B \right)$

if 1, B has $\frac{1}{2}\left(|\rightarrow\rangle_B |\rightarrow\rangle_B \langle\rightarrow|_B \langle\rightarrow|_B + |\leftarrow\rangle_B |\leftarrow\rangle_B \langle\leftarrow|_B \langle\leftarrow|_B \right)$

These are distinguishable! So B can read A's signal, and we have an implementable superluminal signalling scheme

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Nick Herbert's FLASH (First Light Amplification Superluminal Hookup)

proposal

A and B share $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B\right)$

A measures in $\{|\uparrow\rangle_A, |\downarrow\rangle_A\}$ to signal 0, $\{|\rightarrow\rangle_A, |\leftarrow\rangle_A\}$ to signal 1.

B now makes a second copy of B's state. So from B's perspective:

if 0, B has $\frac{1}{2}\left(|\uparrow\rangle_B|\uparrow\rangle_B\langle\uparrow|_B\langle\uparrow|_B + |\downarrow\rangle_B|\downarrow\rangle_B\langle\downarrow|_B\langle\downarrow|_B\right)$

if 1, B has $\frac{1}{2}\left(|\rightarrow\rangle_B|\rightarrow\rangle_B\langle\rightarrow|_B\langle\rightarrow|_B + |\leftarrow\rangle_B|\leftarrow\rangle_B\langle\leftarrow|_B\langle\leftarrow|_B\right)$

These are distinguishable! So B can read A's signal, and we have an implementable superluminal signalling scheme

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The quantum no-cloning theorem (Wootters-Zurek, Dieks (1982))

It is impossible to build a quantum cloning machine which has the effect

$$|\psi\rangle\,|\psi_0\rangle\,|M_0\rangle \longrightarrow |\psi\rangle\,|\psi\rangle\,|M_\psi\rangle$$

- $|\psi\rangle$: unknown input state in $\mathbb{C}^d$
- $|\psi_0\rangle$: "blank" reference state in $\mathbb{C}^d$
- $|M_0\rangle$: initial machine state
- $|\psi\rangle$ (first on right): original input state
- $|\psi\rangle$ (second on right): faithful copy
- $|M_\psi\rangle$: final machine state (may depend on ψ)

Proof: Let $|\psi_1\rangle, |\psi_2\rangle$ be independent possible inputs, $|\psi_3\rangle = a|\psi_1\rangle + b|\psi_2\rangle$.

$$|\psi_1\rangle|\psi_0\rangle|M_0\rangle \rightarrow |\psi_1\rangle|\psi_1\rangle|M_{\psi_1}\rangle$$

$$|\psi_2\rangle|\psi_0\rangle|M_0\rangle \rightarrow |\psi_2\rangle|\psi_2\rangle|M_{\psi_2}\rangle$$

$$\therefore\ (a|\psi_1\rangle + b|\psi_2\rangle)|\psi_0\rangle|M_0\rangle \rightarrow a|\psi_1\rangle|\psi_1\rangle|M_{\psi_1}\rangle + b|\psi_2\rangle|\psi_2\rangle|M_{\psi_2}\rangle$$

$$\neq (a|\psi_1\rangle + b|\psi_2\rangle)(a|\psi_1\rangle + b|\psi_2\rangle)|M_{\psi_3}\rangle$$

The quantum no-cloning theorem (Wootters-Zurek, Dieks (1982))

It is impossible to build a quantum cloning machine which has the effect

$$|\psi\rangle |\psi_0\rangle |M_0\rangle \longrightarrow |\psi\rangle |\psi\rangle |M_\psi\rangle$$

Notes ① Our proof used only linearity of quantum theory (not even unitarity of evolution).

② But we did implicitly assume we can describe the entire cloning process by a linear quantum evolution (no explicit projective measurements).

That's (a) consistent with the Copenhagen view: we can postpone the projective "Heisenberg cut" till after the process is complete

(b) justifiable by the Stinespring dilation theorem: any quantum process on a system can be represented by a unitary evolution on a suitable larger system. (See later for more on this.)

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The Church of the larger Hilbert space

The Church of the larger Hilbert space

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Introduction

John Smolin coined the phrase "Going to the Church of the Larger Hilbert Space" for the dilation constructions of channels and states, which not only provide a neat characterization of the set of permissible quantum operations but are also a most useful tool in quantum information science.

According to **Stinespring's dilation theorem**, every completely positive and trace-preserving map, or channel, can be built from the basic operations of (1) tensoring with a second system in a specified state, (2) unitary transformation, and (3) reduction to a subsystem. Thus, any quantum operation can be thought of as arising from a unitary evolution on a larger (*dilated*) system. The auxiliary system to which one has to couple the given one is usually called the ancilla of the channel. Stinespring's representation comes with a bound on the dimension of the ancilla system, and is unique up to unitary equivalence.

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Extending the no-cloning theorem I: no perfect discrimination of non-orthogonal states.

New (much simpler) task: you're given a randomly chosen one of two known non-orthogonal states $|\psi_1\rangle, |\psi_2\rangle$: $|\langle\psi_1|\psi_2\rangle| < 1$

You need to build a machine that will always correctly decide which. Note ① if we can do this, we can certainly clone $|\psi_1\rangle, |\psi_2\rangle$.

② But.... the linearity argument of the no-cloning proof doesn't work here.

If $|\psi_1\rangle |R_0\rangle |M_0\rangle \rightarrow |\psi_1\rangle |\psi_1\rangle |M_1\rangle$

$|\psi_2\rangle |R_0\rangle |M_0\rangle \rightarrow |\psi_2\rangle |\psi_2\rangle |M_2\rangle$

then $(a|\psi_1\rangle + b|\psi_2\rangle) |R_0\rangle |M_0\rangle \rightarrow$ ..... but it wouldn't matter!

Machine only needs to work for the given states $|\psi_1\rangle, |\psi_2\rangle$

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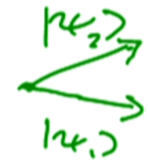
Task: you're given a randomly chosen one of two known non-orthogonal states $|\psi_1\rangle, |\psi_2\rangle$: $|\langle\psi_1|\psi_2\rangle| < 1$. You need to build a machine that will always correctly decide which.



<u>Impossibility proof</u>: If this works, once we know which state we have, we can make any number N copies. So effectively

$$(1)\quad |\psi_1\rangle|R_0\rangle\ldots|R_0\rangle|M_0\rangle \rightarrow |\psi_1\rangle|\psi_1\rangle\ldots|\psi_1\rangle|M_1\rangle$$

$$(2)\quad \underbrace{|\psi_2\rangle}_{\text{input}}\underbrace{|R_0\rangle\ldots|R_0\rangle}_{N\text{ reserve states}}\underbrace{|M_0\rangle}_{\text{initial machine state}} \rightarrow \underbrace{|\psi_2\rangle|\psi_2\rangle\ldots|\psi_2\rangle}_{N+1\text{ copies}}\underbrace{|M_2\rangle}_{\text{final machine state}}$$

Now $|\langle LHS_1|LHS_2\rangle| = |\langle\psi_1|\psi_2\rangle|$

$|\langle RHS_1|RHS_2\rangle| = |\langle\psi_1|\psi_2\rangle^{N+1}||\langle M_1|M_2\rangle|$

Contradiction! Violates unitarity!

<u>Note</u> we needed the stronger property (unitarity rather than linearity) to prove this stronger result.

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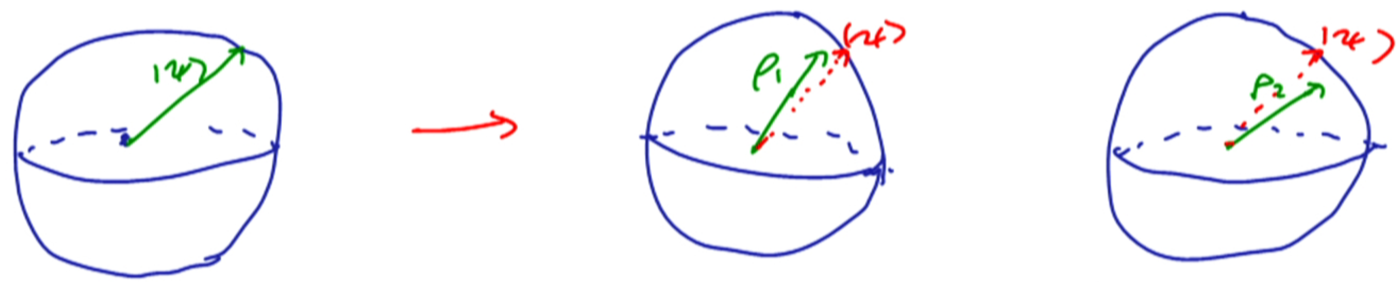
Extending the no-cloning theorem II

What about approximate quantum cloning?



$|\psi\rangle|\psi_0\rangle \longrightarrow \rho$ (possibly mixed) state of 2 qubits.

$Tr_2(\rho) = \rho_1 \quad , \quad Tr_1(\rho) = \rho_2$

ρ_1, ρ_2 close to $|\psi\rangle\langle\psi|$.

We need a measure of closeness: a natural one is the fidelity $\langle\psi|\rho_i|\psi\rangle$ which gives the probability of outcome "yes" if we measure $|\psi\rangle\langle\psi|$ on ρ_i.

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Task: you're given a randomly chosen one of two known non-orthogonal states $|\psi_1\rangle, |\psi_2\rangle$: $|\langle\psi_1|\psi_2\rangle| < 1$. You need to build a machine that will always correctly decide which.

Impossibility proof: If this works, once we know which state we have, we can make any number N copies. So effectively

(1) $|\psi_1\rangle|R_0\rangle\ldots|R_0\rangle|M_0\rangle \rightarrow |\psi_1\rangle|\psi_1\rangle\ldots|\psi_1\rangle|M_1\rangle$

(2) $|\psi_2\rangle|R_0\rangle\ldots|R_0\rangle|M_0\rangle \rightarrow |\psi_2\rangle|\psi_2\rangle\ldots|\psi_2\rangle|M_2\rangle$

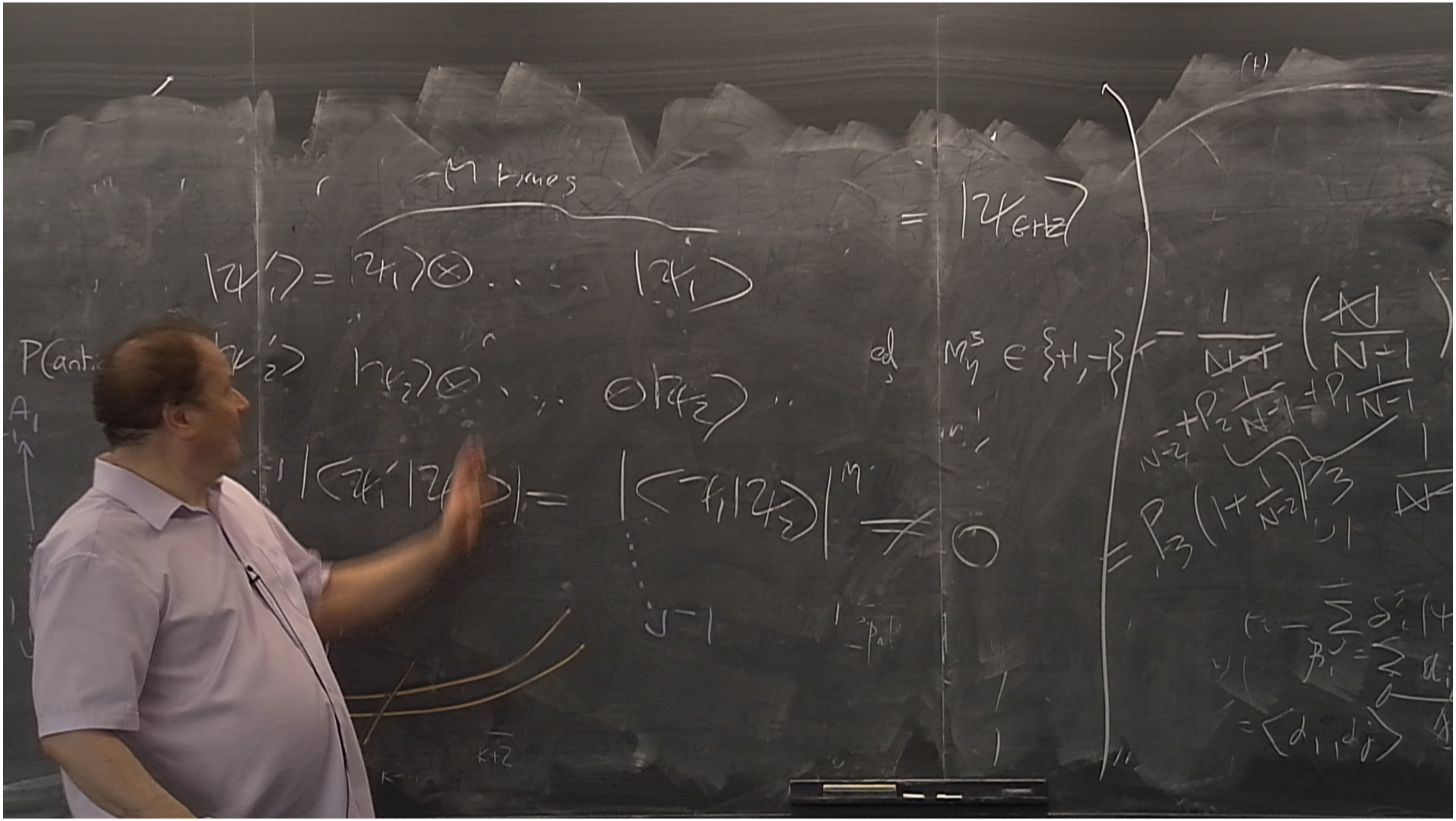
input — N reserve states — initial machine state — N+1 copies — final machine state

Now $|\langle LHS_1|LHS_2\rangle| = |\langle\psi_1|\psi_2\rangle|$

$|\langle RHS_1|RHS_2\rangle| = |\langle\psi_1|\psi_2\rangle|^{N+1}|\langle M_1|M_2\rangle|$

Contradiction! Violates unitarity!

Note we needed the stronger property (unitarity rather than linearity) to prove this stronger result.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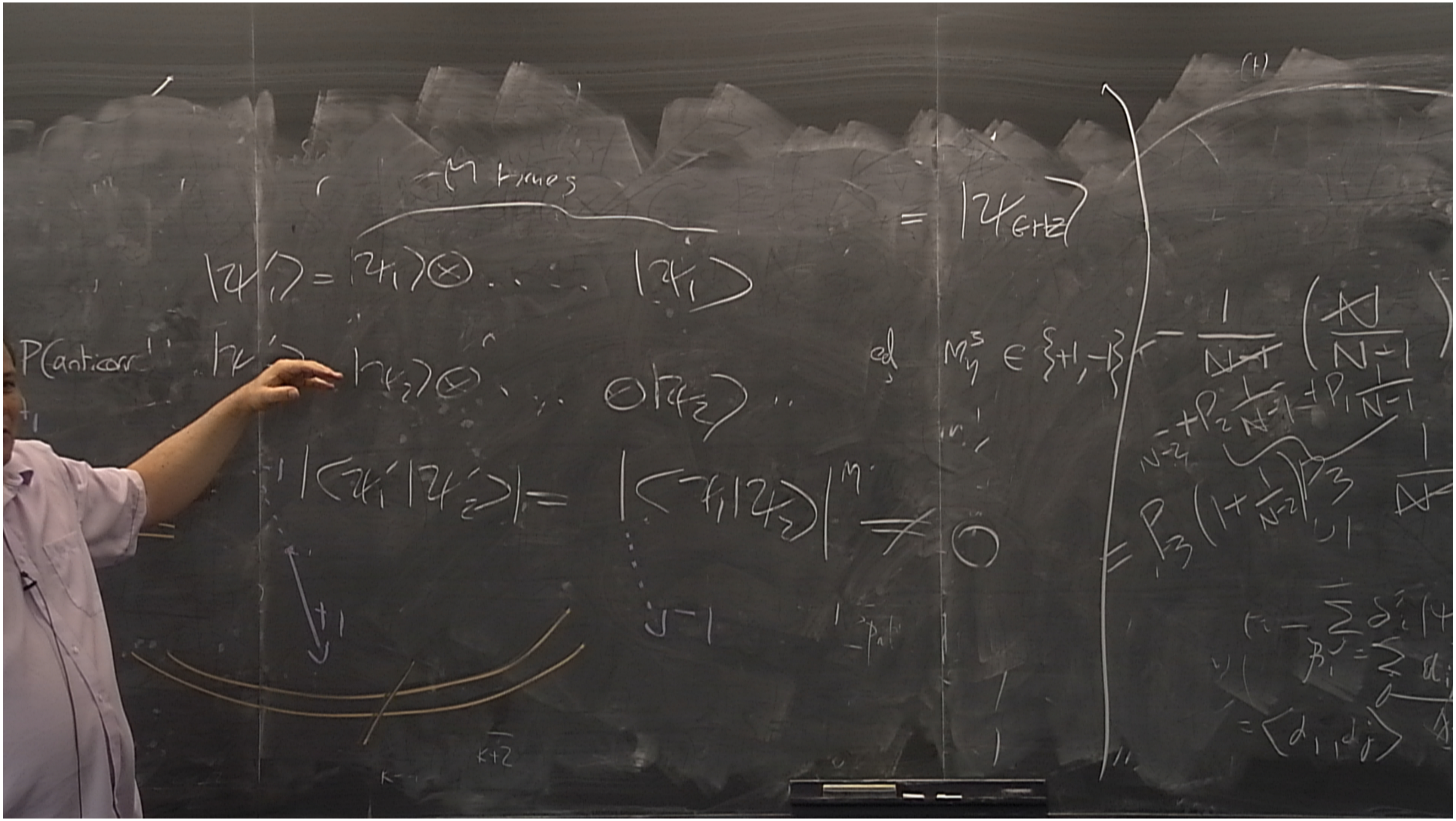
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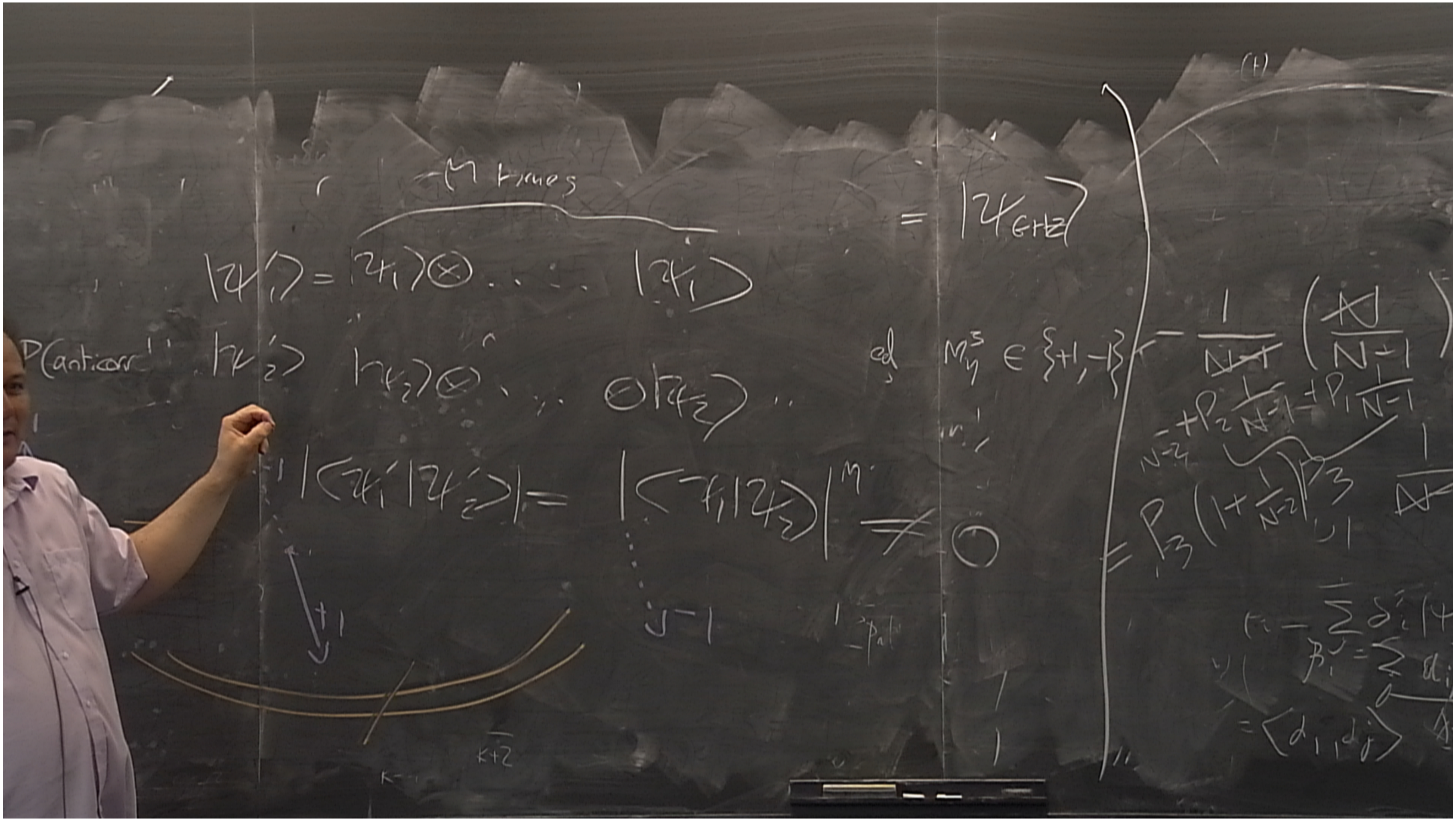
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The quantum no-cloning theorem (Wootters-Zurek, Dieks (1982))

It is impossible to build a quantum cloning machine which has the effect

$$|\psi\rangle |\psi_0\rangle |M_0\rangle \longrightarrow |\psi\rangle |\psi\rangle |M_\psi\rangle$$

Notes ① Our proof used only linearity of quantum theory (not even unitarity of evolution).

② But we did implicitly assume we can describe the entire cloning process by a linear quantum evolution (no explicit projective measurements).

That's (a) consistent with the Copenhagen view: we can postpone the projective "Heisenberg cut" till after the process is complete

(b) justifiable by the Stinespring dilation theorem: any quantum process on a system can be represented by a unitary evolution on a suitable larger system. (See later for more on this.)

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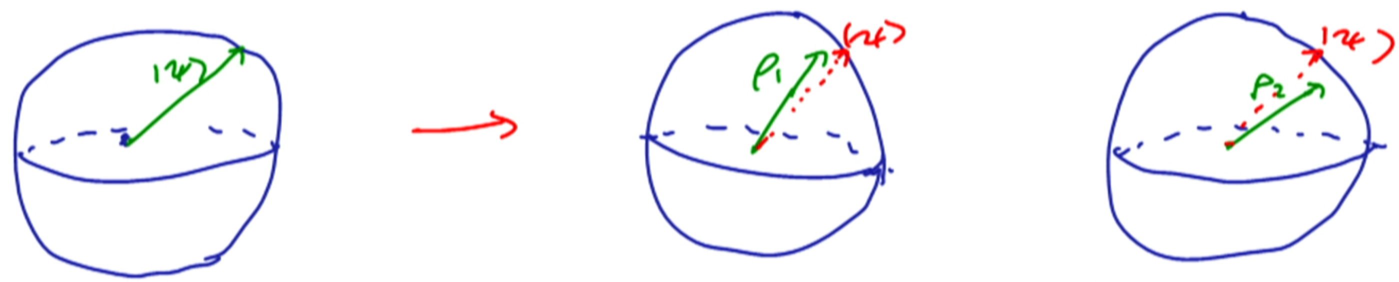
Extending the no-cloning theorem II What about approximate quantum cloning?



$|\psi\rangle|\psi_0\rangle \longrightarrow \rho$ (possibly mixed) state of 2 qubits.

$Tr_2(\rho) = \rho_1 \quad , \quad Tr_1(\rho) = \rho_2$

ρ_1, ρ_2 close to $|\psi\rangle\langle\psi|$.

We need a measure of closeness: a natural one is the fidelity $\langle\psi|\rho_i|\psi\rangle$ which gives the probability of outcome "yes" if we measure $|\psi\rangle\langle\psi|$ on ρ_i.

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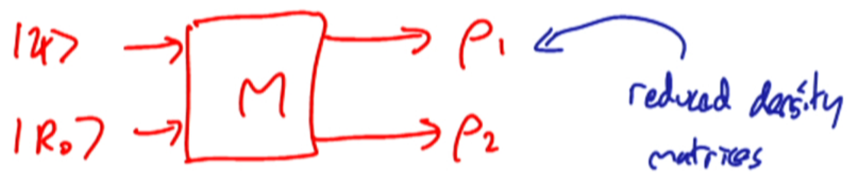

Could we arrange that $|\langle\psi|\rho_1|\psi\rangle| \geq 1-\varepsilon$, $|\langle\psi|\rho_2|\psi\rangle| \geq 1-\varepsilon$

for all possible inputs $|\psi\rangle$?

And can we arrange this for any $\varepsilon > 0$?

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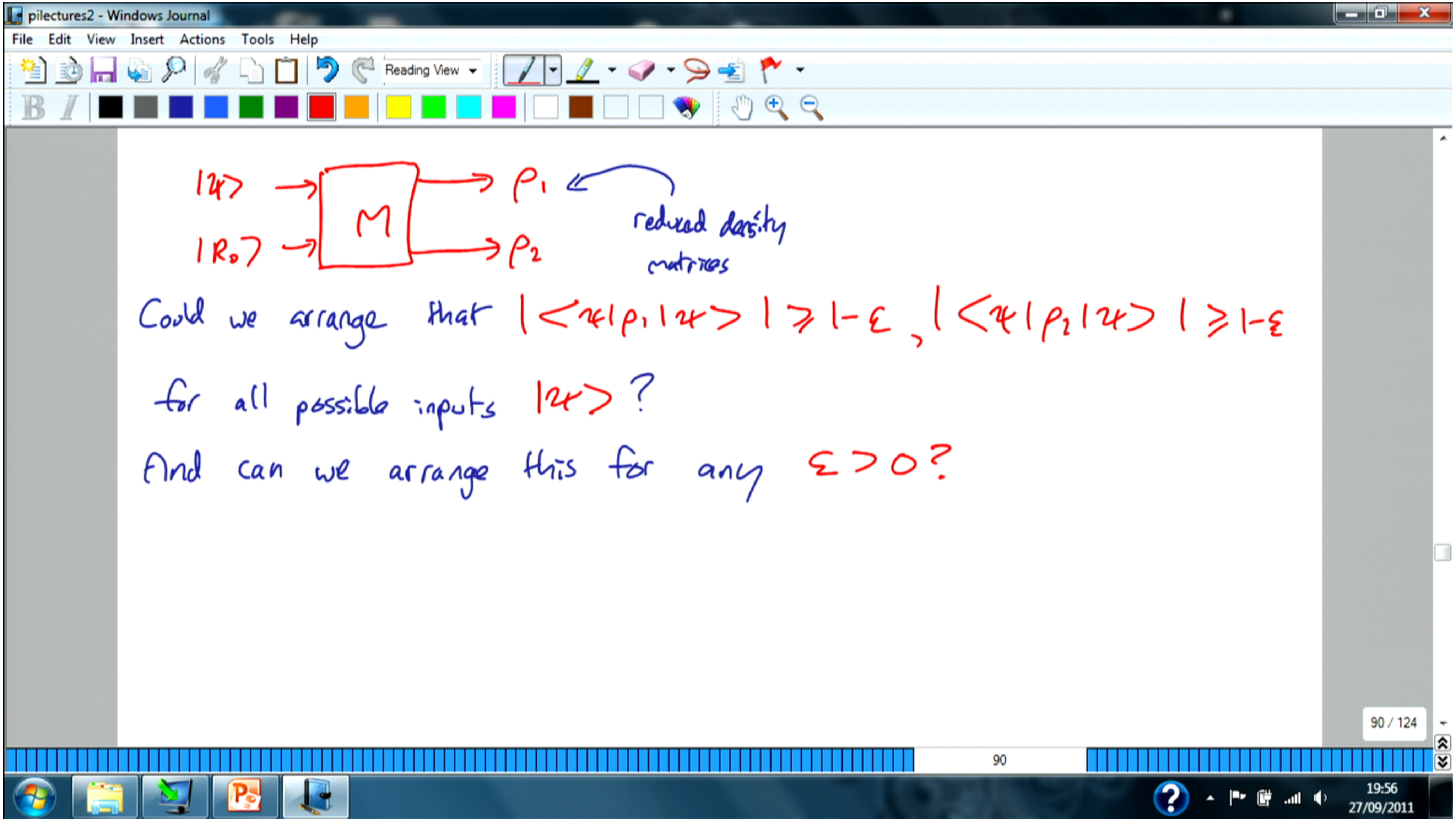
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$|\psi\rangle$
$|R_0\rangle$
M
ρ_1
ρ_2
reduced density matrices
Could we arrange that $|\langle\psi|\rho_1|\psi\rangle| \geq 1-\varepsilon$, $|\langle\psi|\rho_2|\psi\rangle| \geq 1-\varepsilon$
for all possible inputs $|\psi\rangle$?
And can we arrange this for any $\varepsilon > 0$?
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