Title: Entanglement Spectra and Trace Index of Topological Insulators

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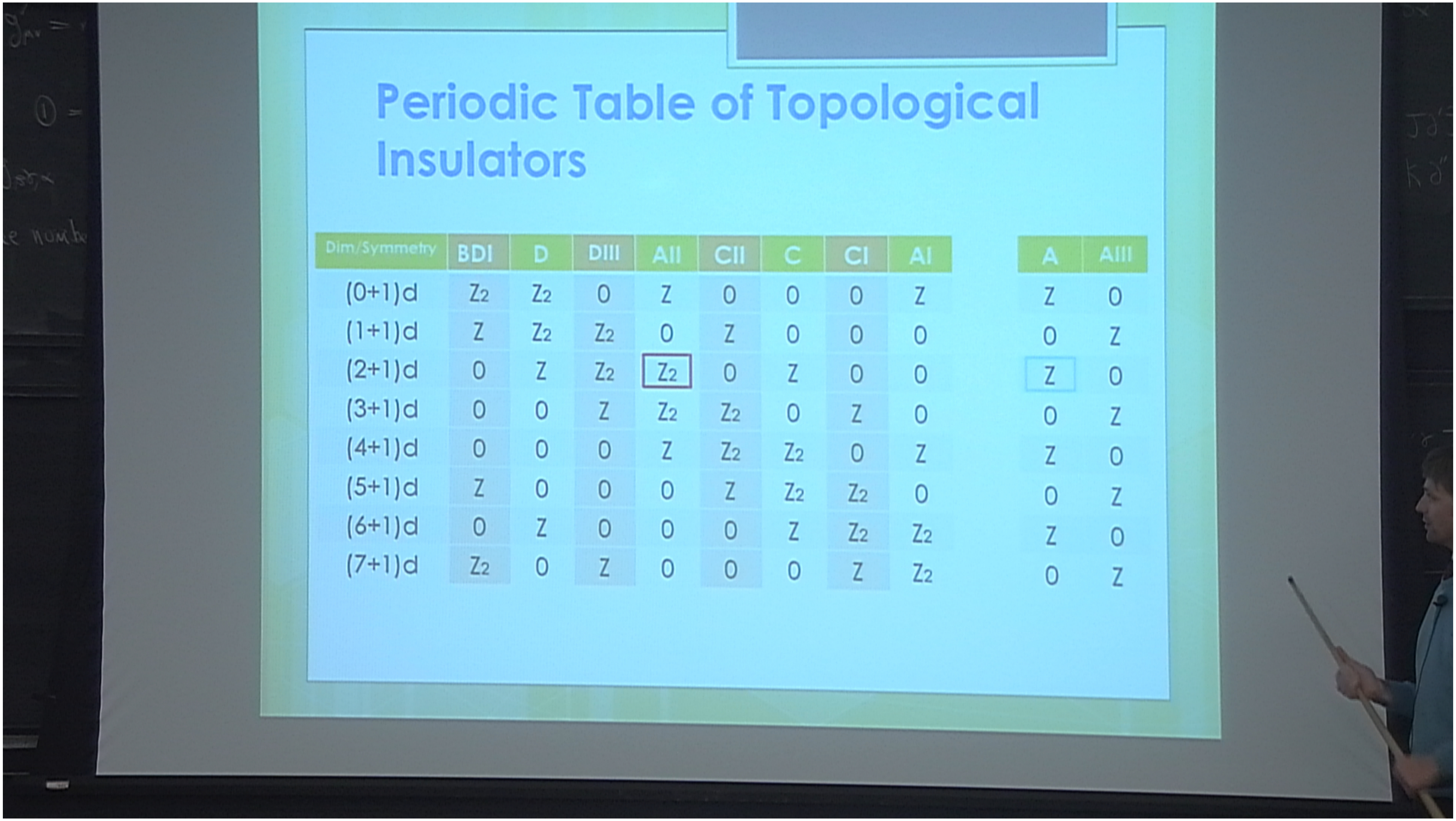
Abstract: We investigate the entanglement spectra of topological insulators which have gapless edge states on their spatial boundaries. In the physical energy spectrum, a subset of the edge states that intersect the Fermi level translates to discontinuities in the trace of the single-particle entanglement spectrum, which we call a `trace index'. We find that any free-fermion topological insulator that exhibits spectral flow has a non-vanishing trace index, which provides us with a new description of topological invariants. In addition, we identify the signatures of spectral flow in the single-particle and many-body entanglement spectrum; in the process we present new methods to extract topological invariants and establish a connection between entanglement and quantum Hall physics in translationally invariant and disordered systems.

Periodic Table of Topological Insulators

Dim/Symmetry	BDI	D	DIII	AII	CII	C	CI	AI	A	AIII
(0+1)d	Z_2	Z_2	0	Z	0	0	0	Z	Z	0
(1+1)d	Z	Z_2	Z_2	0	Z	0	0	0	0	Z
(2+1)d	0	Z	Z_2	Z_2	0	Z	0	0	Z	0
(3+1)d	0	0	Z	Z_2	Z_2	0	Z	0	0	Z
(4+1)d	0	0	0	Z	Z_2	Z_2	0	Z	Z	0
(5+1)d	Z	0	0	0	Z	Z_2	Z_2	0	0	Z
(6+1)d	0	Z	0	0	0	Z	Z_2	Z_2	Z	0
(7+1)d	Z_2	0	Z	0	0	0	Z	Z_2	0	Z



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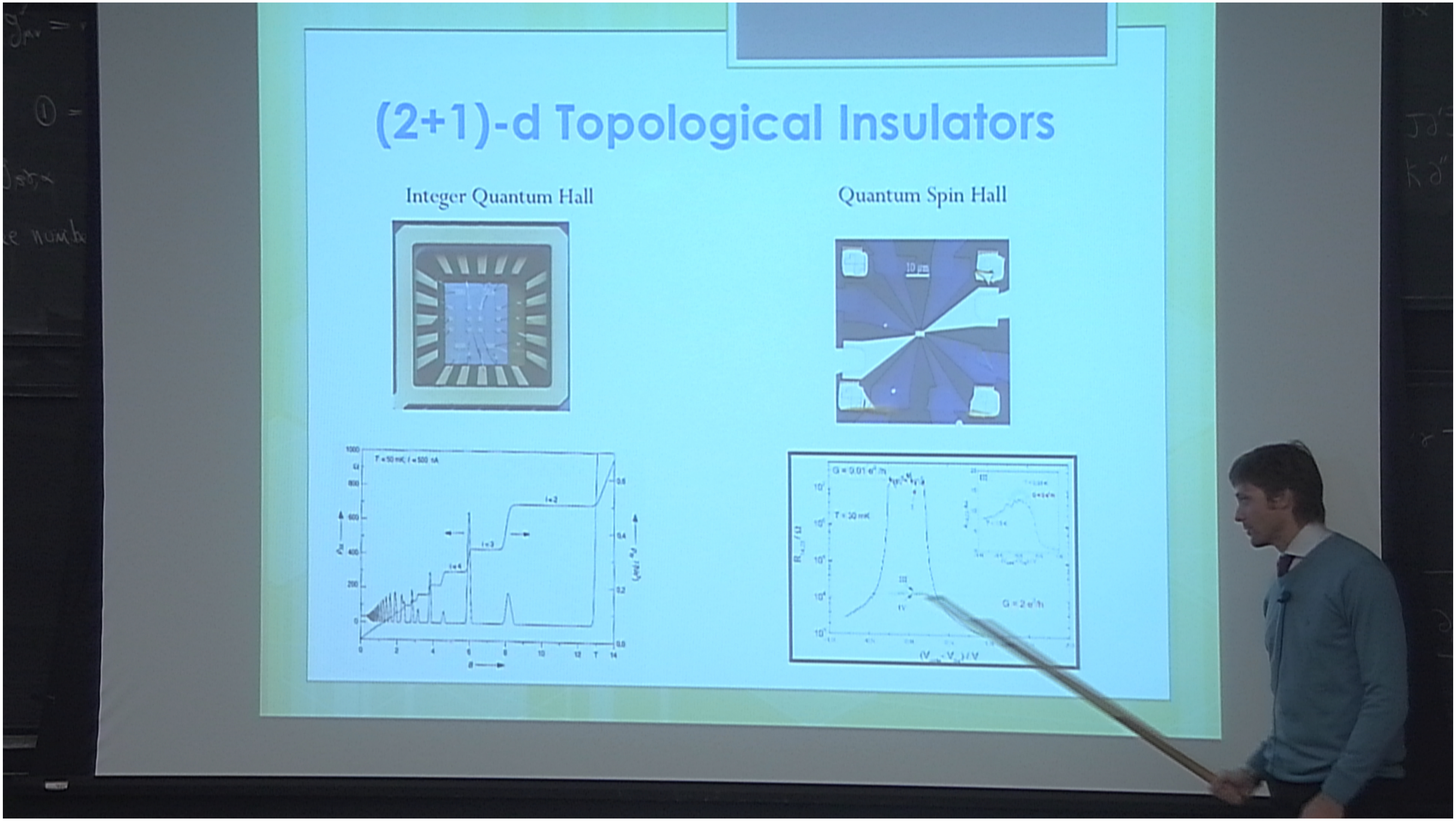
(2+1)-d Topological Insulators
Integer Quantum Hall
Quantum Spin Hall
10 μm

(2+1)-d Topological Insulators

Integer Quantum Hall

Bulk is gapped and described by an integer topological invariant.

Chiral Edge States on the Edge

Unitary Class (requires no special symmetries)

Quantum Spin Hall

Bulk is gapped and described by a Z_2 topological invariant.

Chiral and Anti-chiral Edge States on the Edge (a time-reversed pair)

Symplectic Class (Requires T symmetry)

Response to Background Fields

How does a 2D topological insulator respond to external applied electromagnetic fields?

Example Analogy: Ising model in an external B-field

$$H[B, \sigma_i] = \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + \sum_i B(i) \sigma_i^z$$

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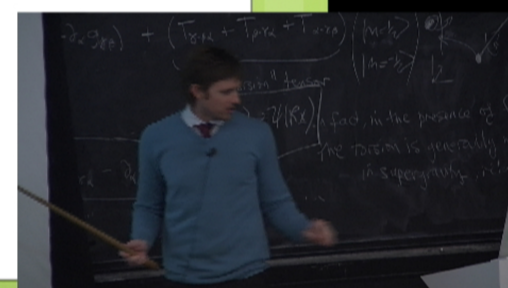

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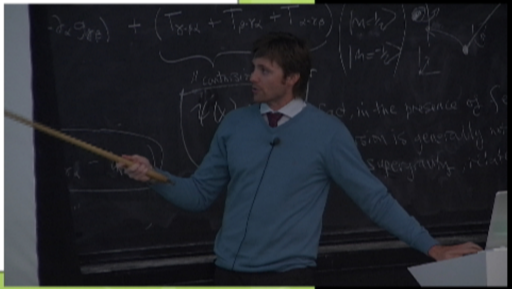

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$$M = \frac{\partial F}{\partial B}$$

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$$F[B] = -\frac{1}{\beta}\log Z[B]$$

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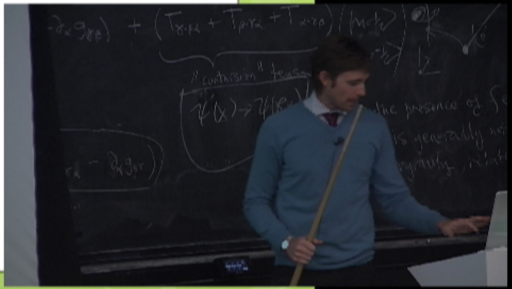
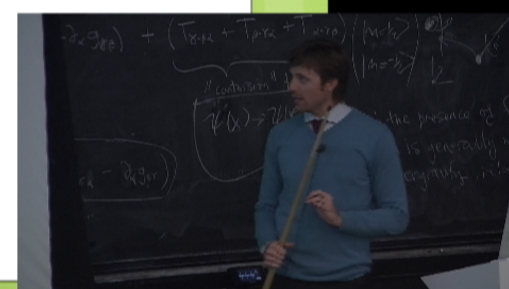


Response to Background Fields

How does a topological insulator respond to electromagnetic fields?

$$S[A, \psi, \bar{\psi}] = \int dt \, d^2x \, \bar{\psi} \left[\gamma^\mu \left(-i\partial_\mu + eA_\mu \right) - m \right] \psi$$



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$$S_{eff}[A] = -i \log Z[A]$$

$$j^\mu = \frac{\delta S_{eff}[A]}{\delta A_\mu}$$

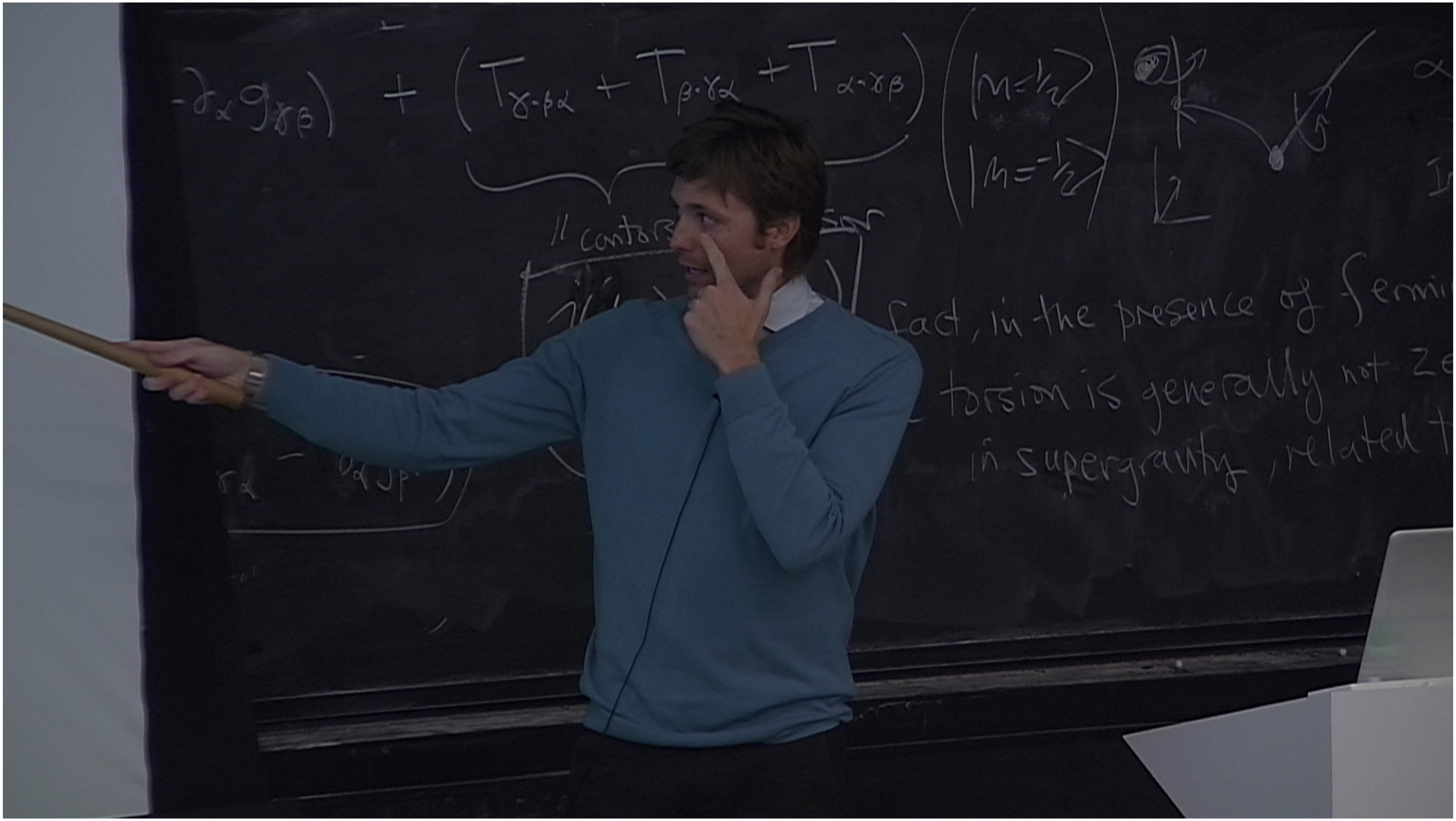
Electromagnetic Response (QAHE)

Electromagnetic linear response:

$$S_{eff}[A_\mu] = \frac{n}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

$$j^i = \frac{ne^2}{h} \epsilon^{ij} E_j$$

$$j^0 = \frac{ne^2}{h} B$$


fact, in the presence of
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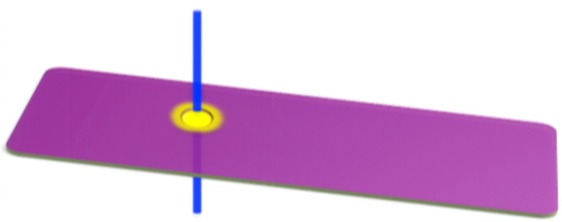

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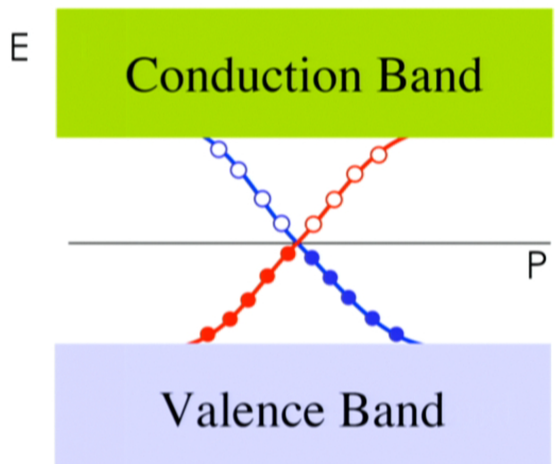
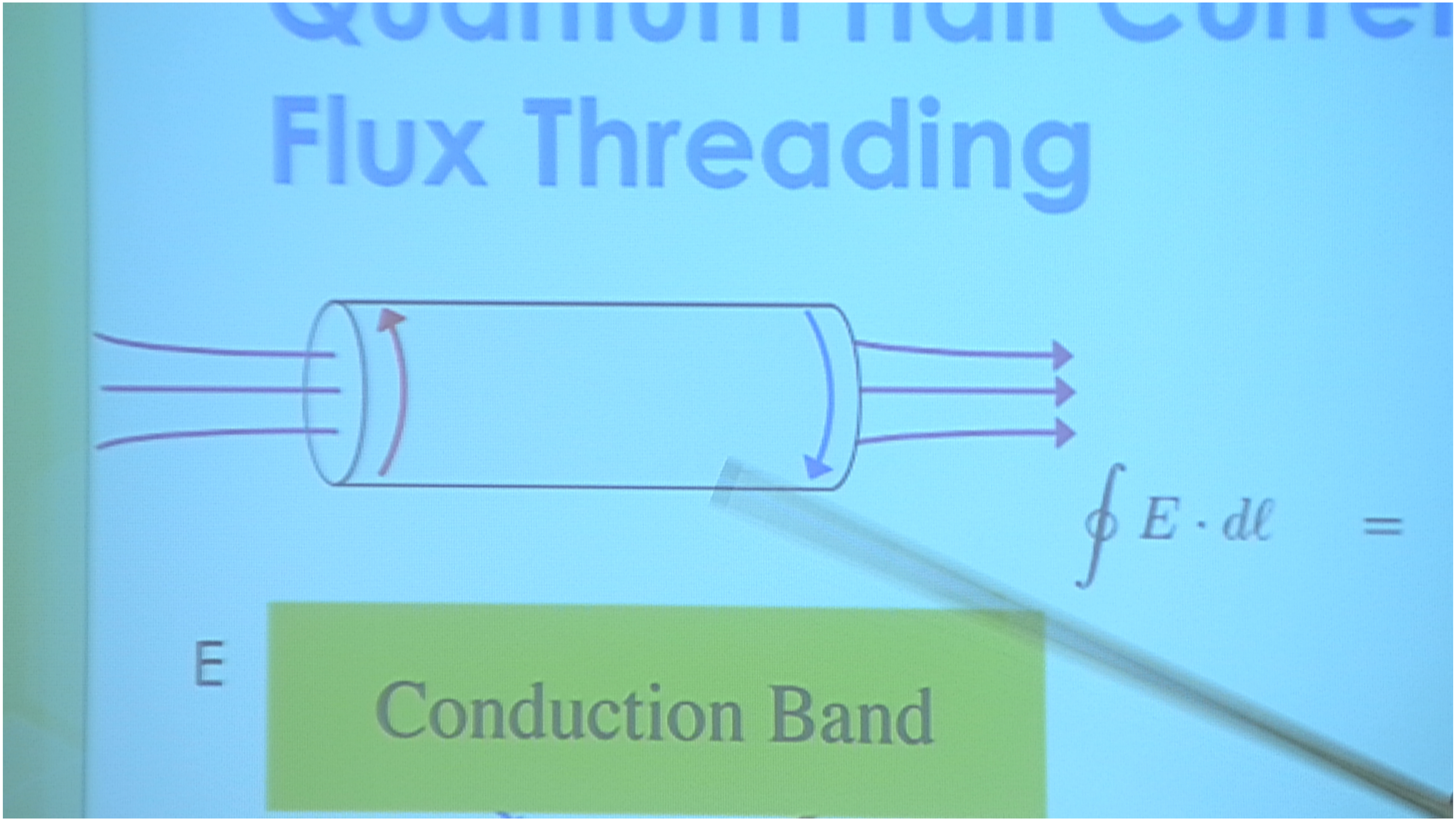
Trivial insulator has n=0 and j=0

Quantum Hall Current From Flux Threading

$$\oint E \cdot d\ell = -\frac{\partial \Phi}{\partial t}$$




Flux Threading
∮ E · dℓ =
E
Conduction Band

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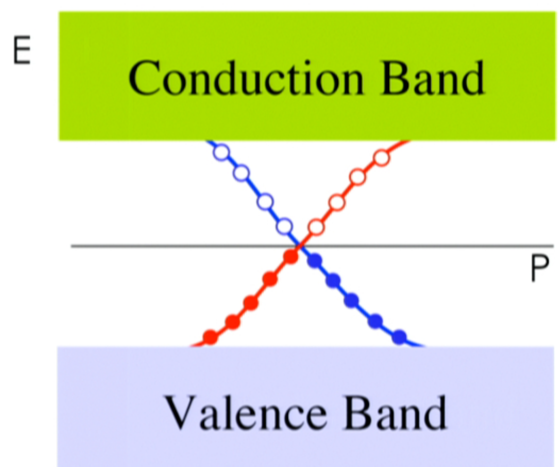

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$$\Longrightarrow -\int_0^T \oint \frac{dA}{dt}\cdot d\ell = -\int_0^T dt\frac{d\Phi}{dt}$$

$$\Longrightarrow \oint \Delta A\cdot d\ell = \Delta\Phi = \frac{h}{e}$$

$$\Longrightarrow \Delta A_y = \frac{h}{eL}$$

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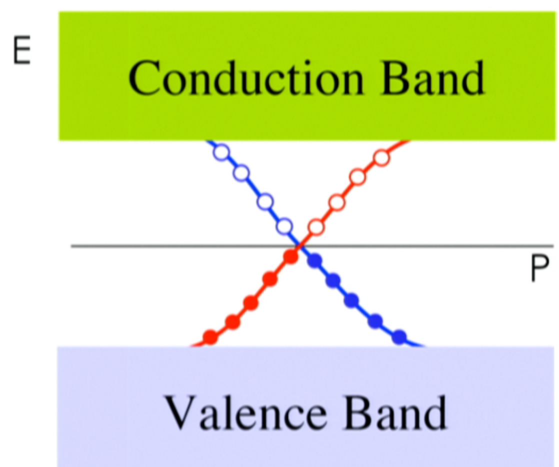

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$P \rightarrow P + eA$

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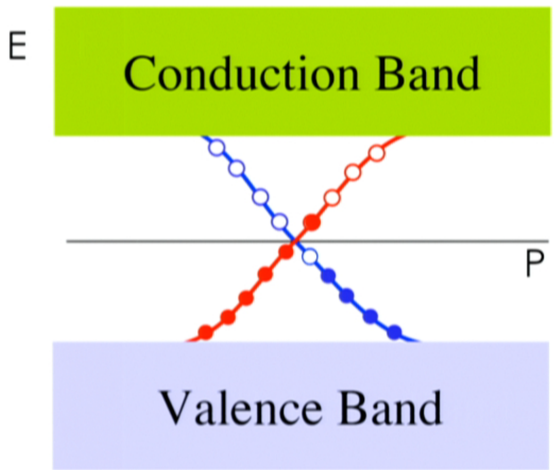

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Outline

- Introduction to topological insulators
  - I focus on (2+1)-d models of the quantum Hall effect and quantum spin Hall effect
- Discuss single-particle and many-particle entanglement spectra of these systems
- Discuss a topological invariant derived from the entanglement spectrum: trace index

The Rise of Entanglement in Condensed Matter

- The goal: Give me a many-body quantum ground state and I can tell you the state of matter.

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- The goal: Give me a many-body quantum ground state and I can tell you the state of matter.
- Easy to do if state has a broken symmetry e.g. a magnet $\langle\psi|M|\psi\rangle \neq 0$
- If the system is featureless what can we do? Are all "featureless" systems identical?

The Rise of Entanglement in Condensed Matter

- The answer is **no.** Quantum systems can exhibit *topological* order which globally distinguish them from truly featureless systems.

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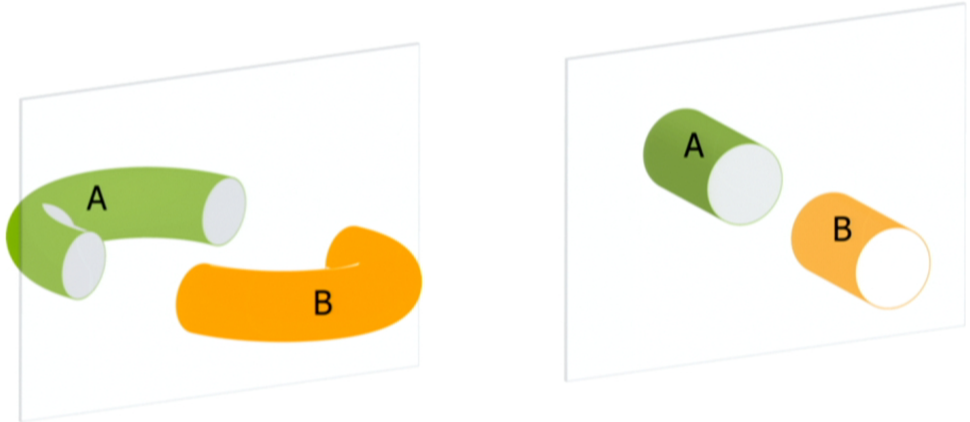
- The answer is **no.** Quantum systems can exhibit *topological* order which globally distinguish them from truly featureless systems.
- Entanglement is great for diagnosing the subtle features due to non-local or long-range correlations.
- Practical advantage: The potential to learn everything from the ground state without having to calculate the entire many-body excitation spectrum (costly in numerics)

Entanglement in Real-Space



$$|\psi\rangle = \sum_i e^{-\frac{1}{2}\xi_i} |\psi_A^i\rangle \otimes |\psi_B^i\rangle$$

Procedure:

(i) Take many-body ground state ψ and construct the density matrix ρ

(ii) Partition space into regions A and B and "trace-out" B to get the reduced density matrix ρ_A. The trace procedure is not trivial.

(iii) Using ρ_A calculate your favorite entanglement measure

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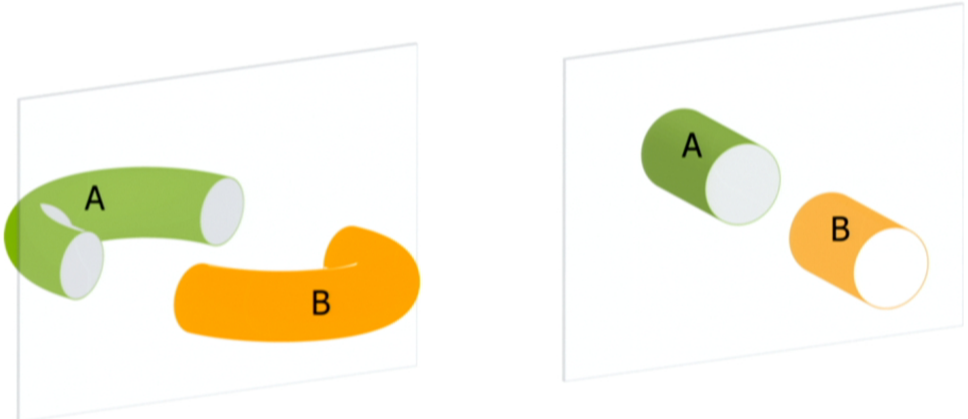

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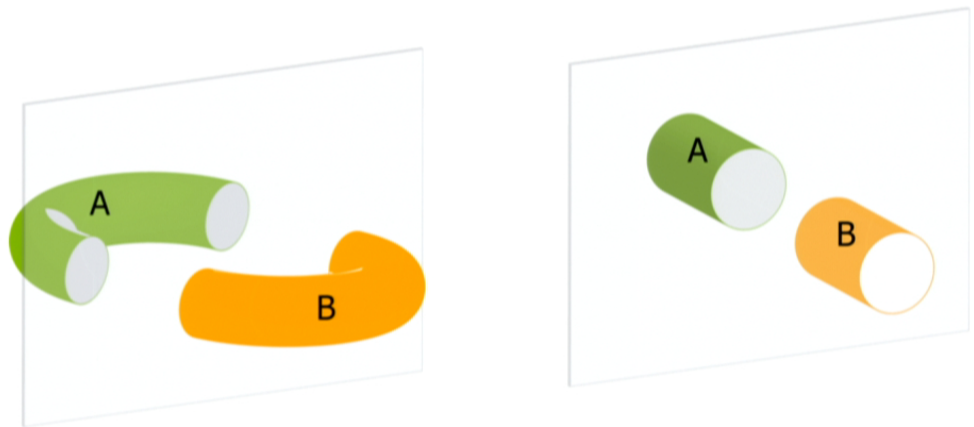

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Measures of Entanglement

- Entanglement Spectrum: Look at the entire set of eigenvalues of ρ_A in each quantum number sector
- Von-Neumann Entanglement entropy: Condense the eigenvalues of ρ_A into a single number

$$S = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$

- Renyi entropies: "Moments" of the entanglement spectrum containing the Von-Neumann entropy as a special case.

$$S(\alpha) = \frac{1}{1-\alpha} \log\left[\mathrm{Tr}\rho_A^{\alpha}\right]$$

Entanglement of Free Fermions

- We can get the *single-particle* entanglement spectrum of any free-fermion Hamiltonian by using Peschel's method:

I. Peschel *J. Phys. A:* **36** L205

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Calculate: $$\hat{C}_{mn} = \langle\Omega|c_m^\dagger c_n|\Omega\rangle$$

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Entanglement "Hamiltonian": $$H = \ln[(1-C)/C]$$

Where C is the two-point correlation function restricted to sites in region A

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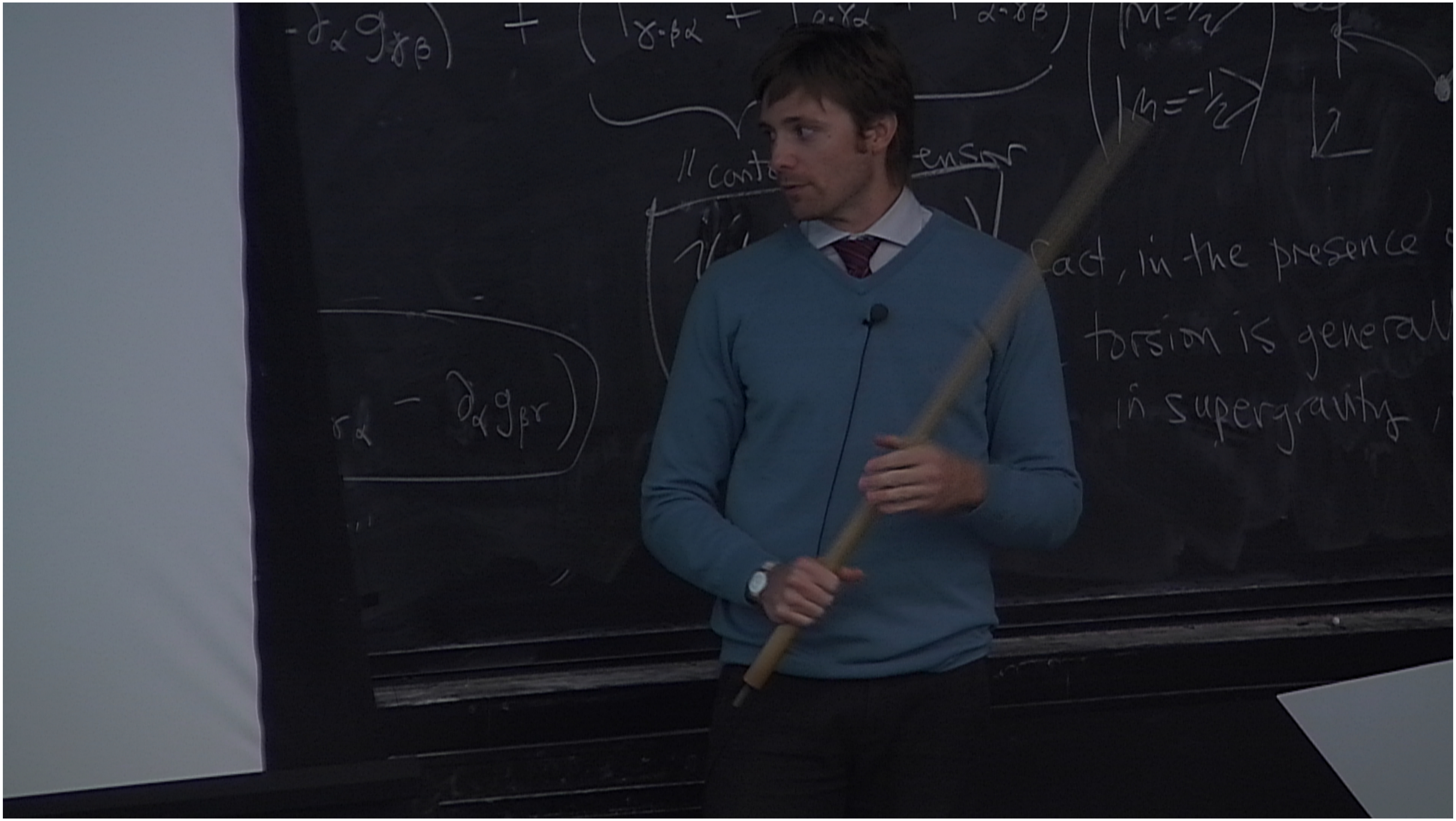
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$$-\begin{pmatrix} 0 & t & 0 & 0 & 0 & t \\ t & 0 & t & 0 & 0 & 0 \\ 0 & t & 0 & t & 0 & 0 \\ 0 & 0 & t & 0 & t & 0 \\ 0 & 0 & 0 & t & 0 & t \\ t & 0 & 0 & 0 & t & 0 \end{pmatrix}$$

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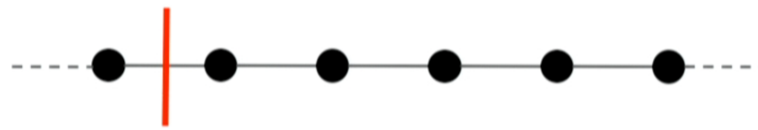

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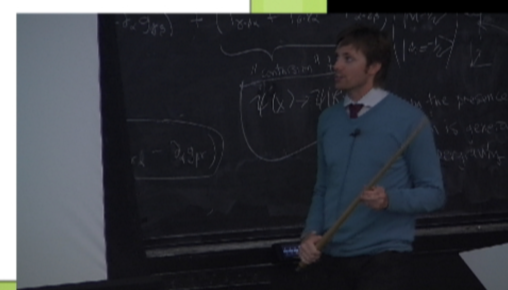

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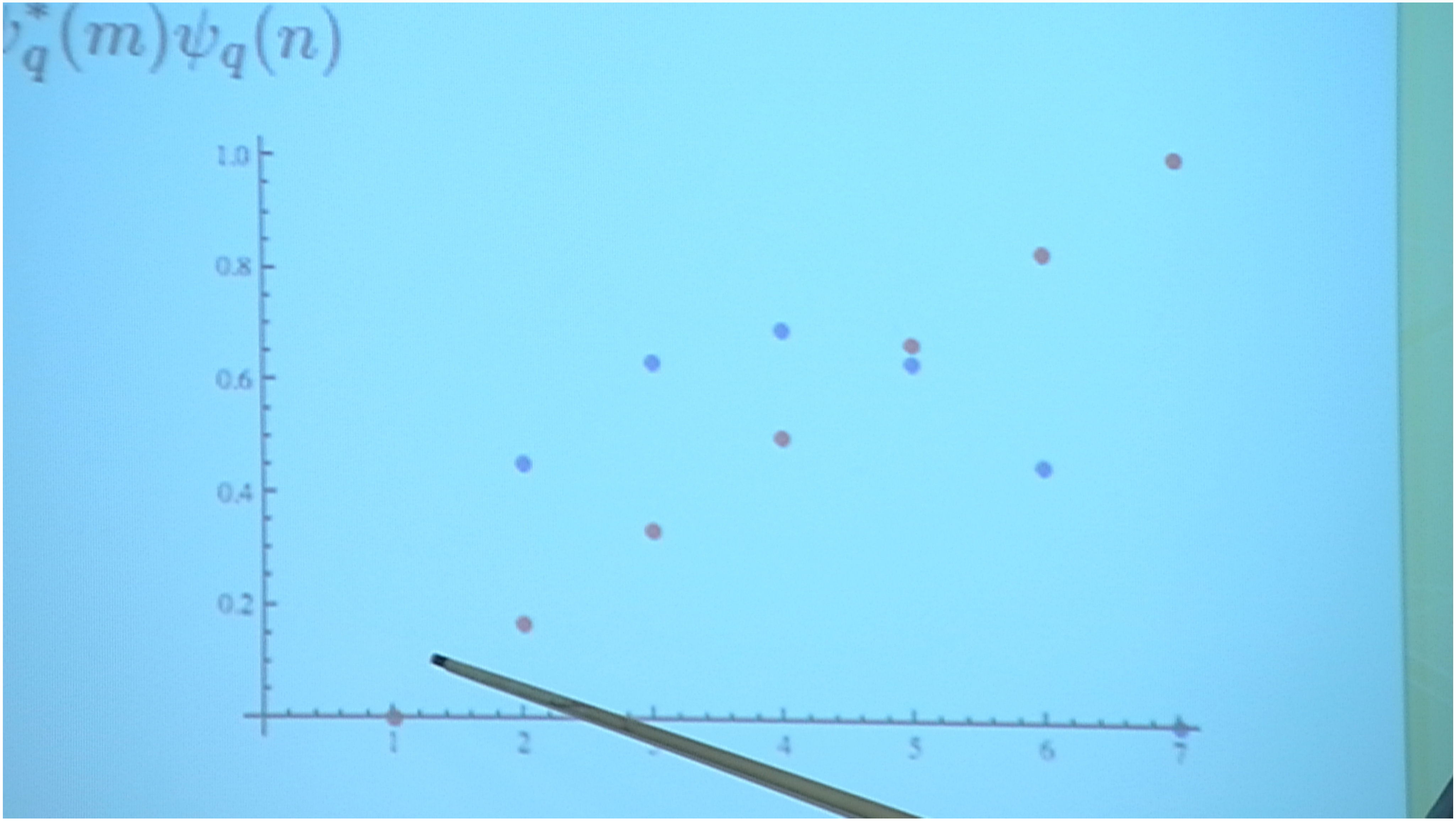
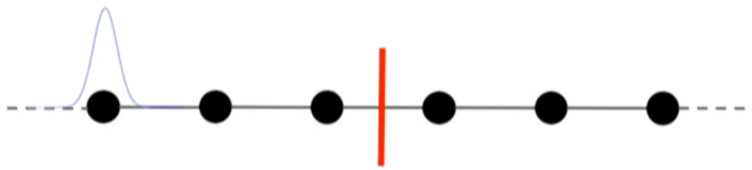

$^*_q(m)\psi_q(n)$
1.0
0.8
0.6
0.4
0.2
1
2
4
5
6
7

Entanglement of Free Fermions



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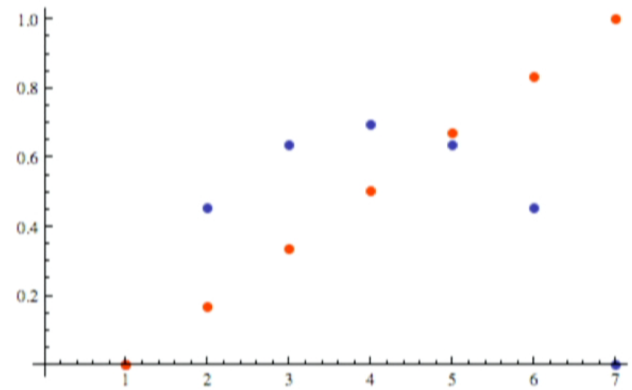

Entanglement eigenvalues of 0, and 1 give no entanglement entropy

Topological Insulators and Entanglement

- Are TI's topologically ordered?
  - No. They have non-degenerate ground states, always. Thus there is no topological entanglement entropy.

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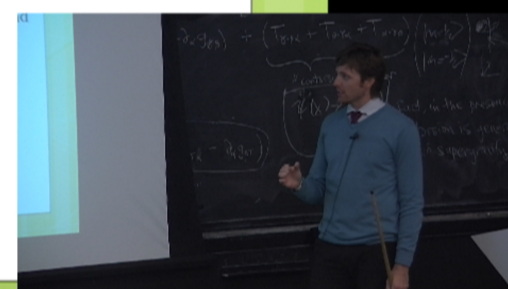

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Interestingly, the Kitaev honeycomb model has both, and they are completely decoupled from each other.

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Heuristic Picture:

Topological Insulators and Entanglement

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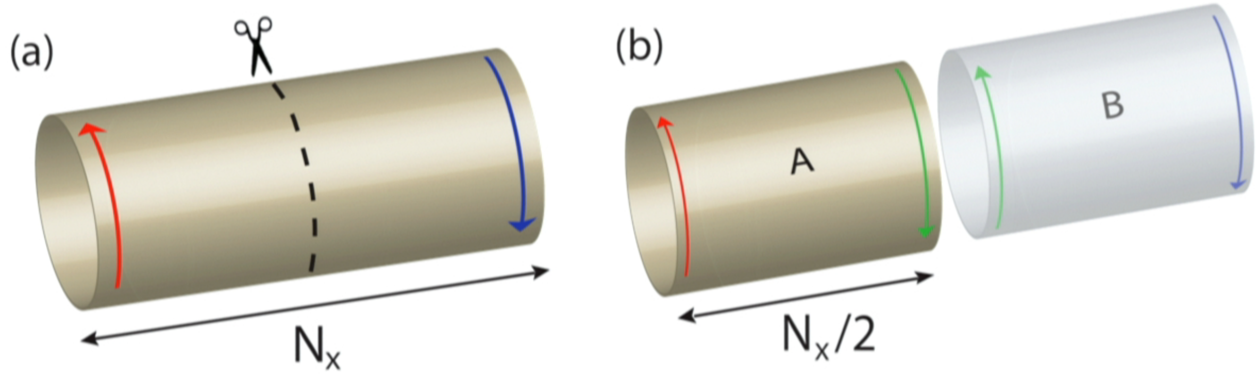

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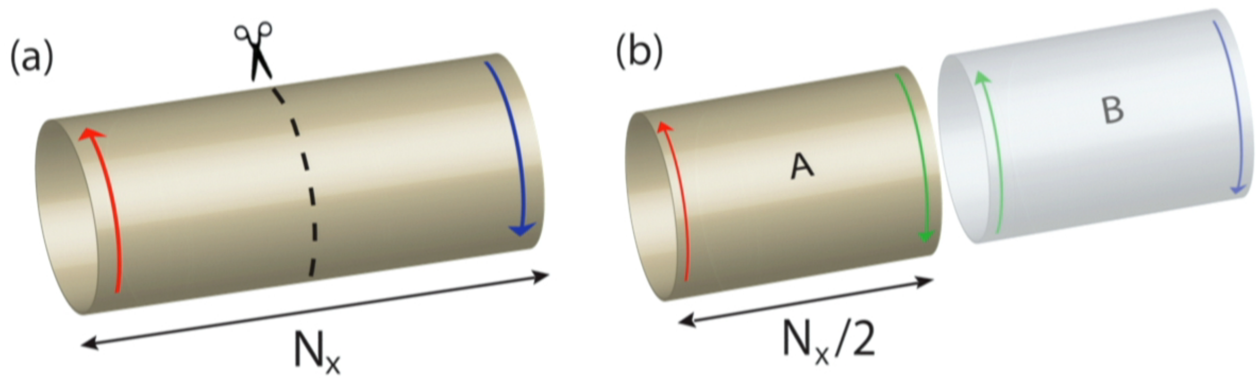

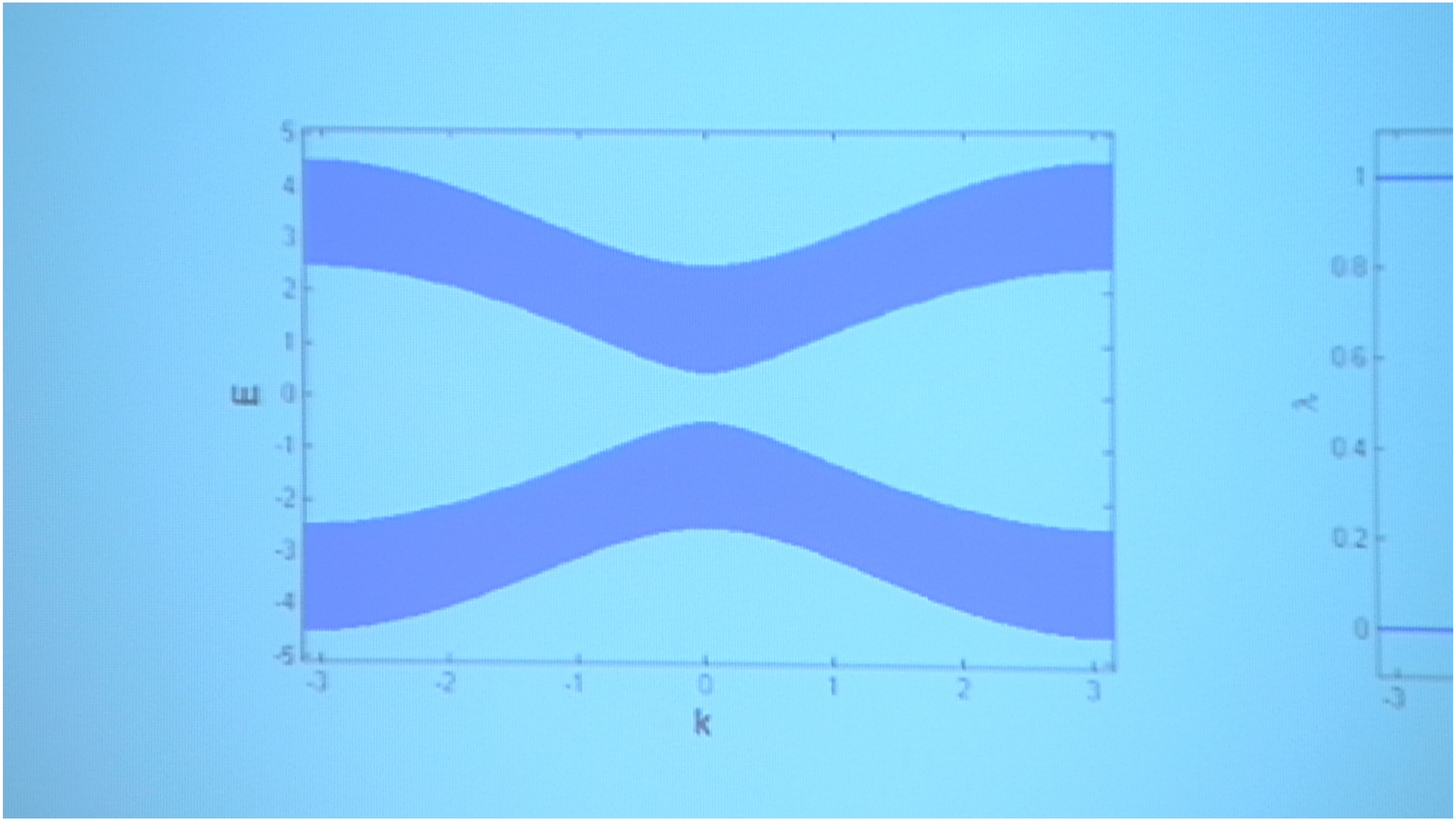
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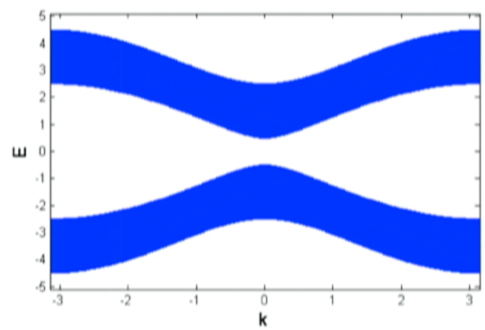
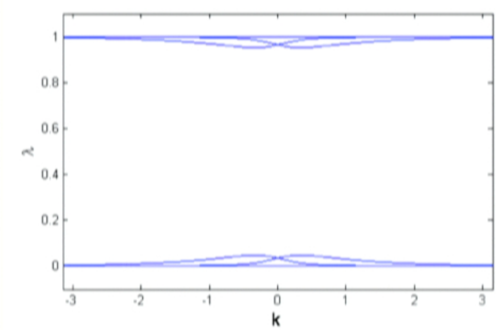
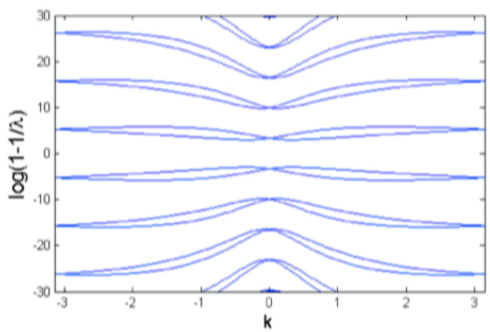
Entanglement Spectrum of a Trivial Insulator

Physical Energy Spectrum



Eigenvalues of C





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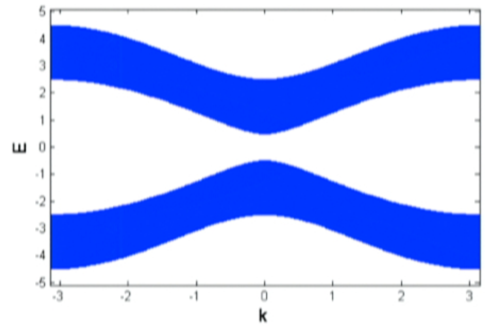

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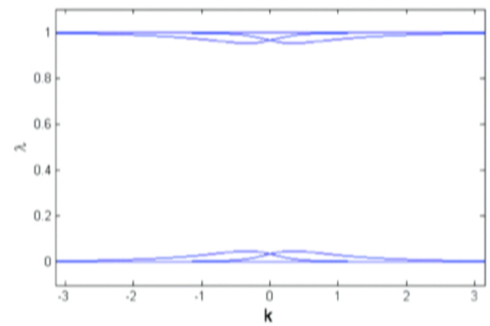
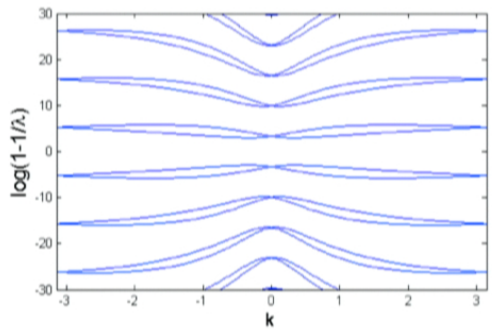
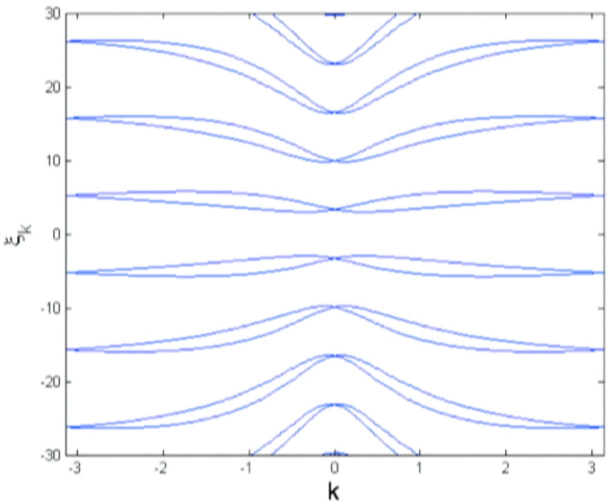


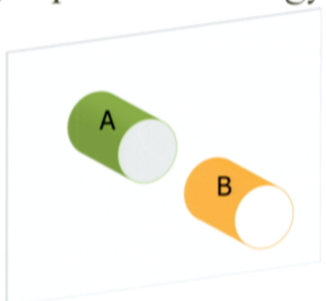


Entanglement Energies

Many-body Entanglement Spectrum

- There is an easy way to get the many-body entanglement spectrum analogous to filling the single-particle energy states





N=total # of particles

ρ_A breaks up into particle sectors from N_A=0…N_{MAX}

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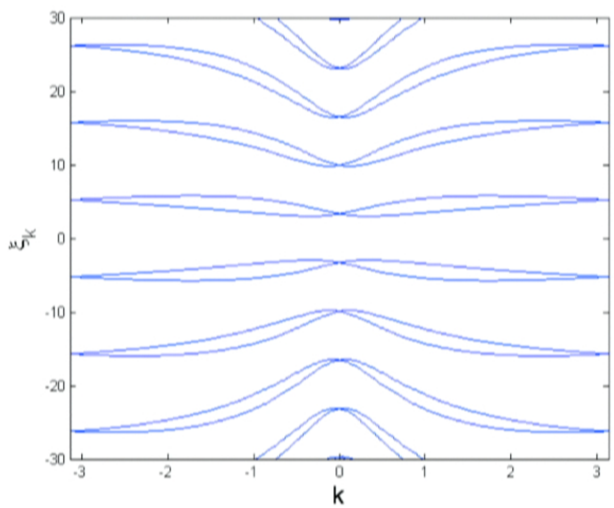

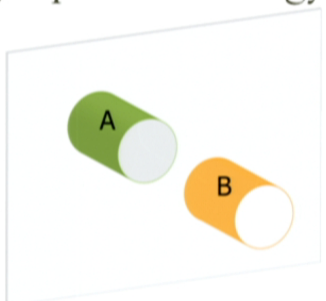

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The particle sector we want defines what an entanglement "Fermi-level" means

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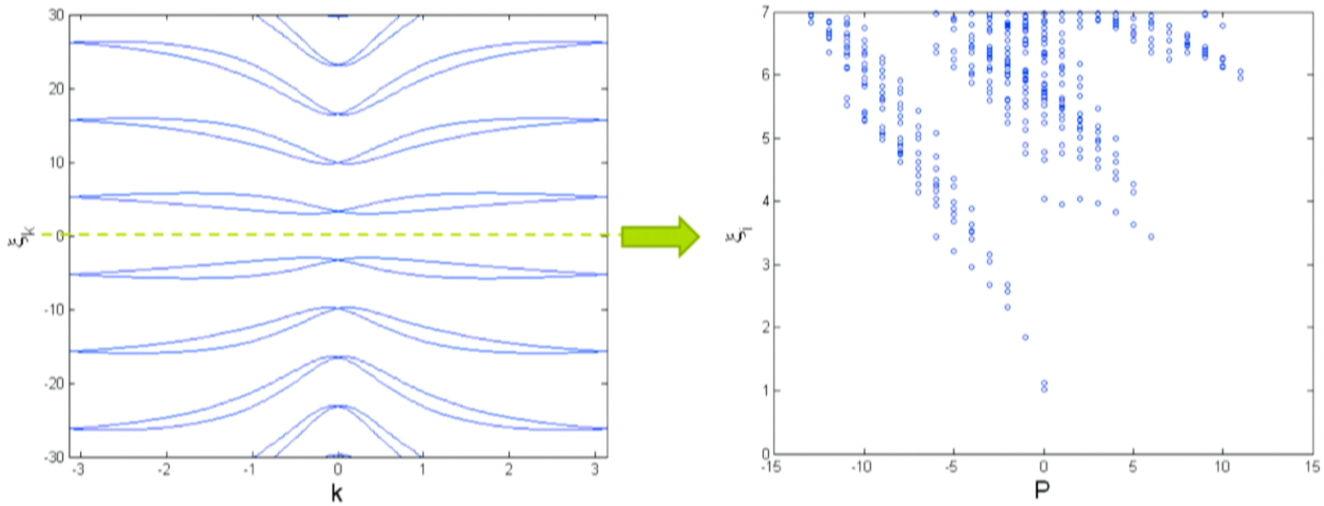
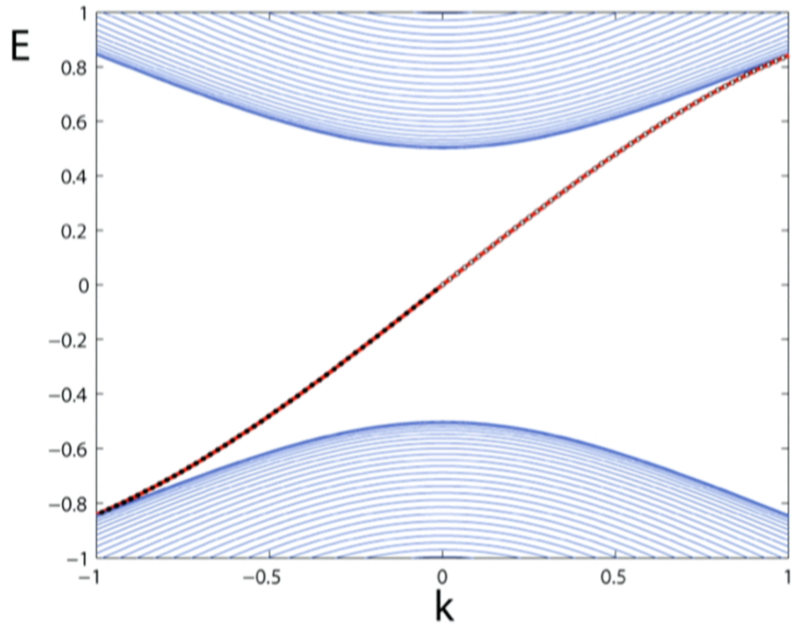


Energy Spectrum of the QAHE

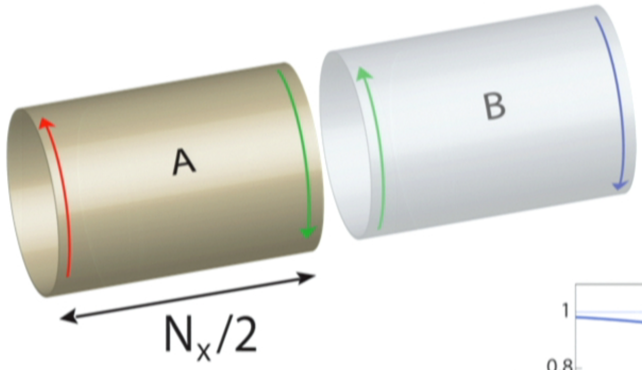
Solve for energy spectrum on a cylinder

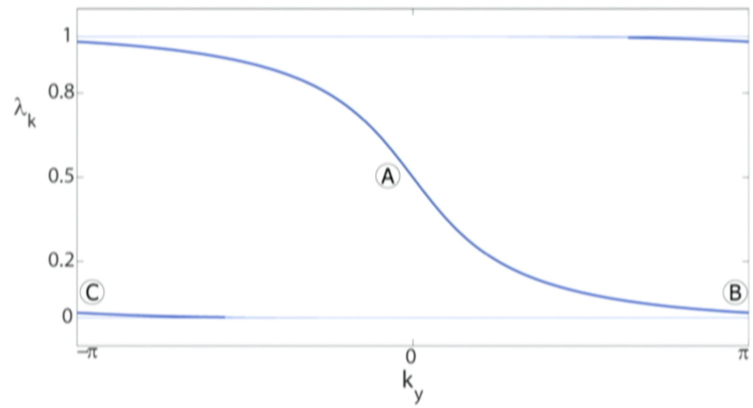


Open BC's

Entanglement Spectrum of QAHE

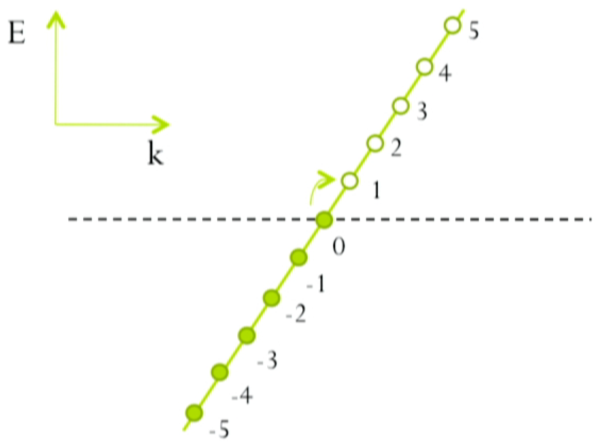
- We will cut in a cylinder geometry so there is only one "cut" boundary





Edge/Cut State Correspondence

- Li and Haldane showed that low-energy many-body entanglement spectra of FQHE states have the same structure as the real edge states. Let's see what we get for the state counting here:



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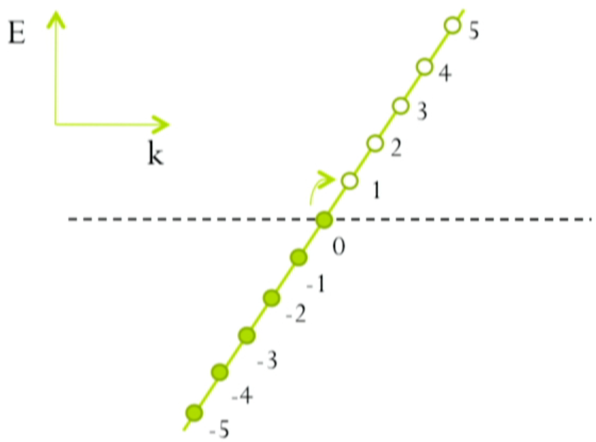
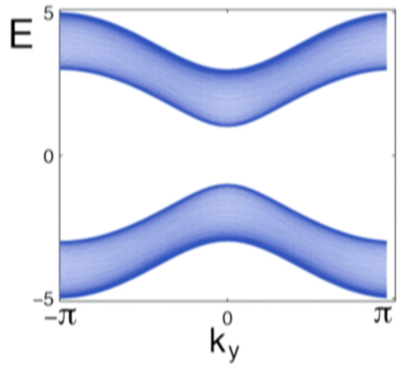


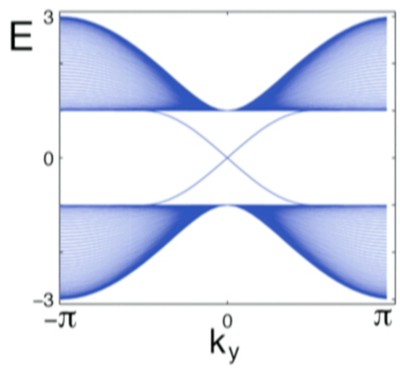
Outline

- Introduction to topological insulators
  - I focus on (2+1)-d models of the quantum Hall effect and quantum spin Hall effect
- Discuss single-particle and many-particle entanglement spectra of these systems
- Discuss a topological invariant derived from the entanglement spectrum: trace index

Discontinuity in Entanglement Trace

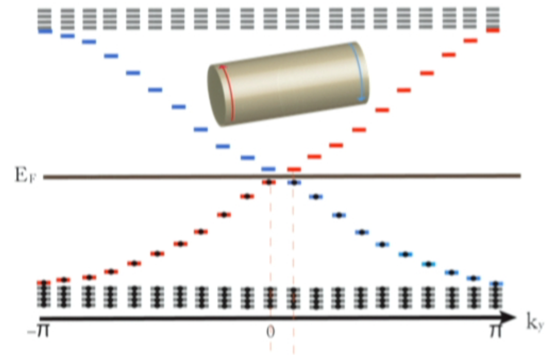
- We want to look at: $\mathrm{Tr}\left[\hat{C}_{mn}(k_y)\right]$





Microscopic Picture of Discontinuity

$$\mathrm{Tr}\left[\hat{C}_{mn}(k_y)\right] = \sum_{m\in A} \langle\Omega|c^\dagger_{mk_y} c_{mk_y}|\Omega\rangle$$
$$= \langle N_A(k_y)\rangle$$



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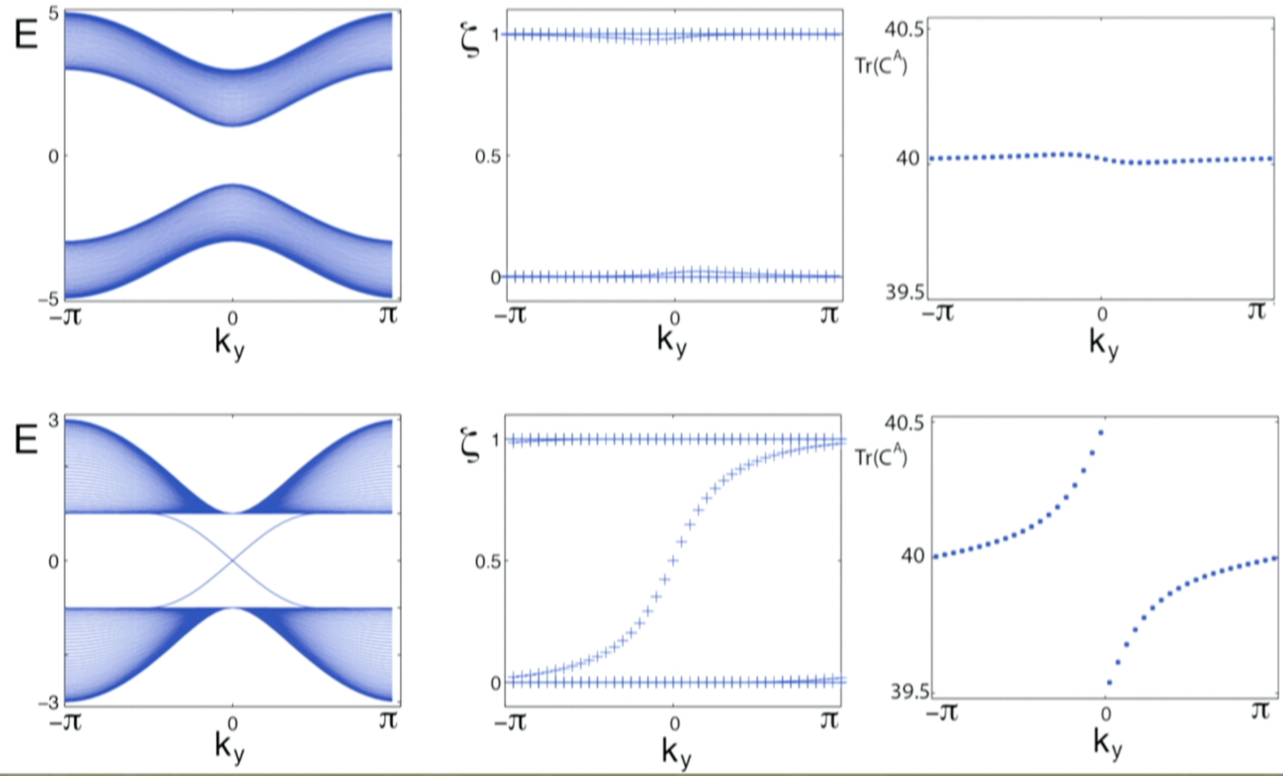

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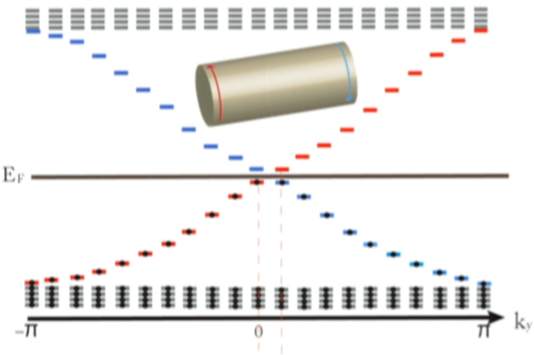

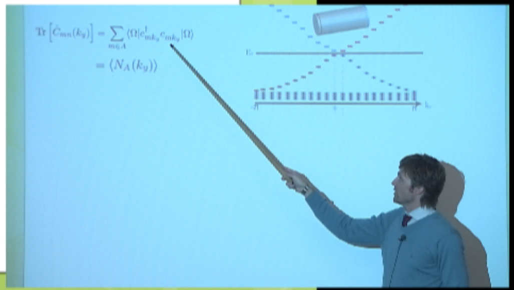

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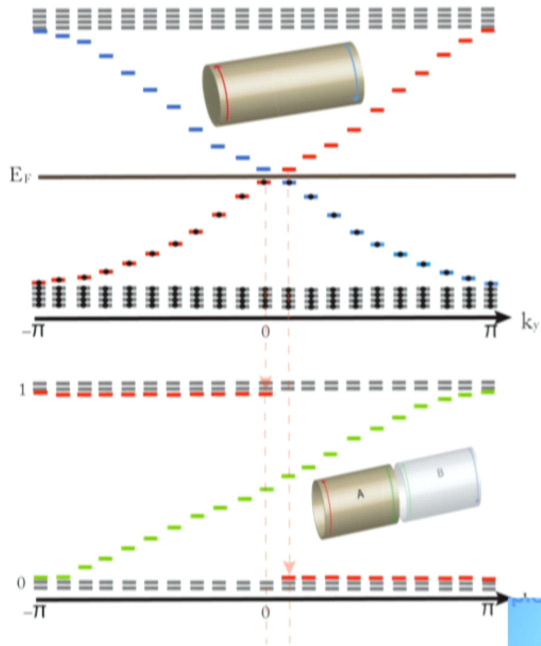

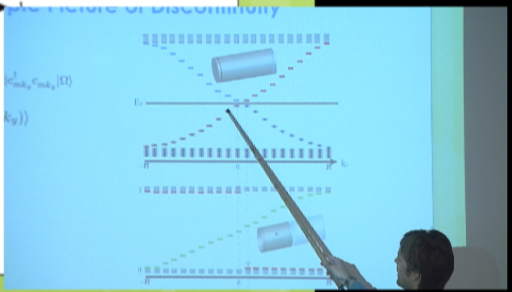

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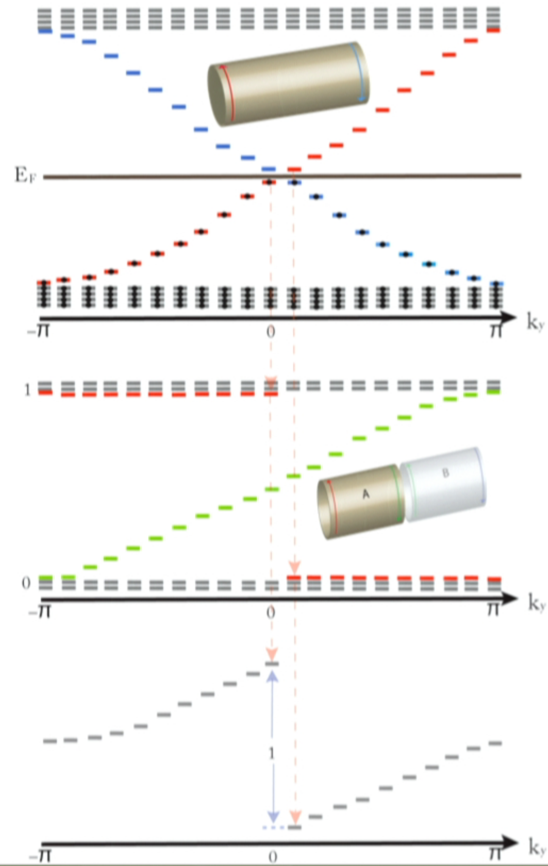

It's the physical edge state that leads to the discontinuity

Macroscopic Picture of Discontinuity

- Let us take a look at the quantity:

$$A^{\Phi} = \sum_{k_y} \mathrm{Tr}\left[\hat{C}_{\Phi_0}(k_y)\right] - \sum_{k_y} \mathrm{Tr}\left[\hat{C}_0(k_y)\right] = \langle N_A \rangle_{\Phi_0} - \langle N_A \rangle_0$$

We are imagining two ground states related by adiabatically threading flux over a long time T

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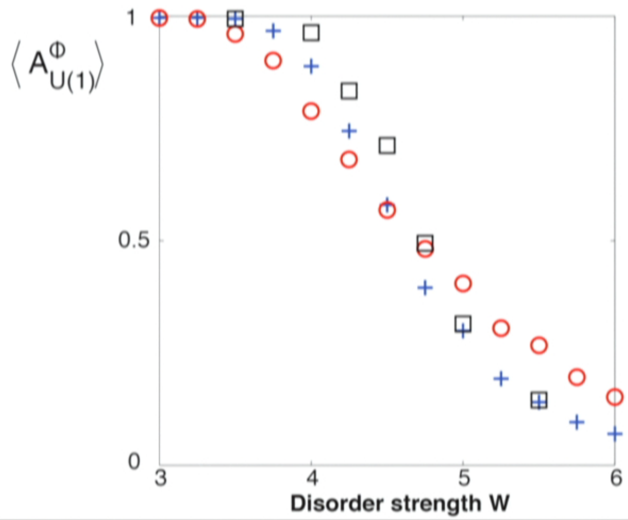
$$\langle N_A \rangle_{\Phi_0} - \langle N_A \rangle_0 = \frac{N_y}{e}\int_0^T j_x dt = C_1$$

Trace index equivalent to Chern number topological invariant

See related work by Fidkowski, Jackson, Klich arxiv: 1101.0320

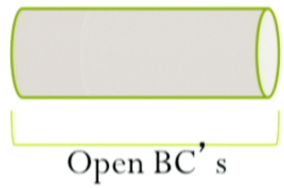
Calculating the Chern Number in Disordered Systems

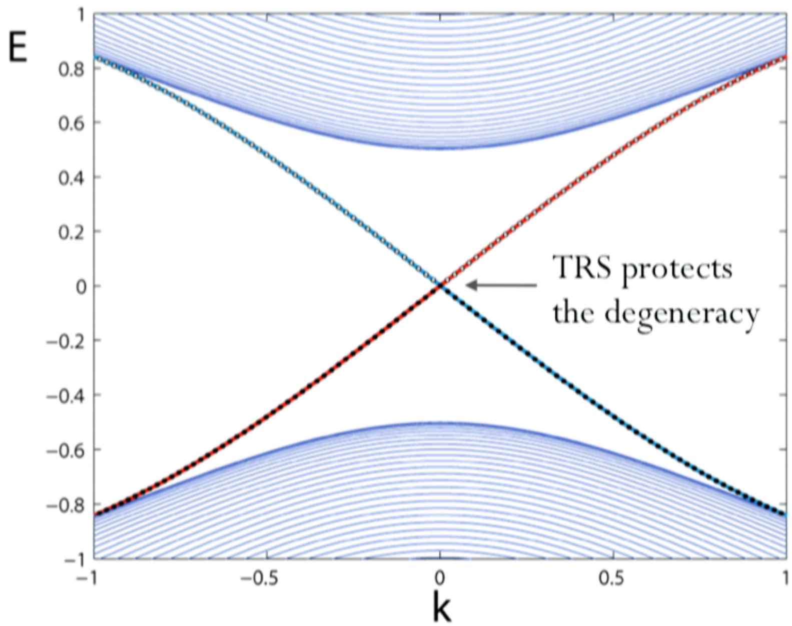
- The trace index gives a new method to calculate the Chern number
- Only have to twist boundary conditions in one direction, and no numerical integrations
- Example: Anderson transition in site-disordered QAHE



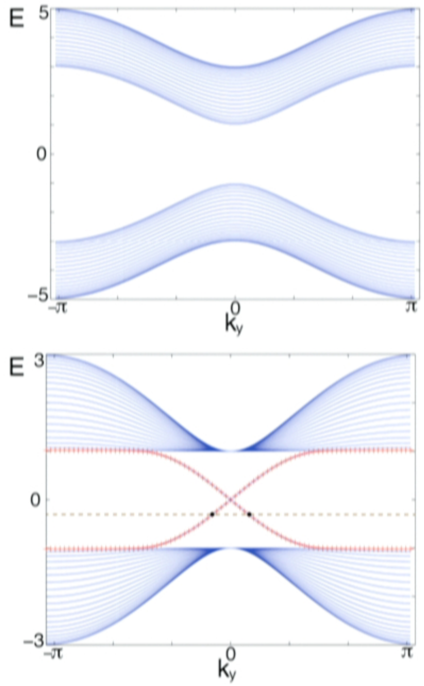
Energy Spectrum of QSH

Solve for energy spectrum on a cylinder





Z_2 Trace Index



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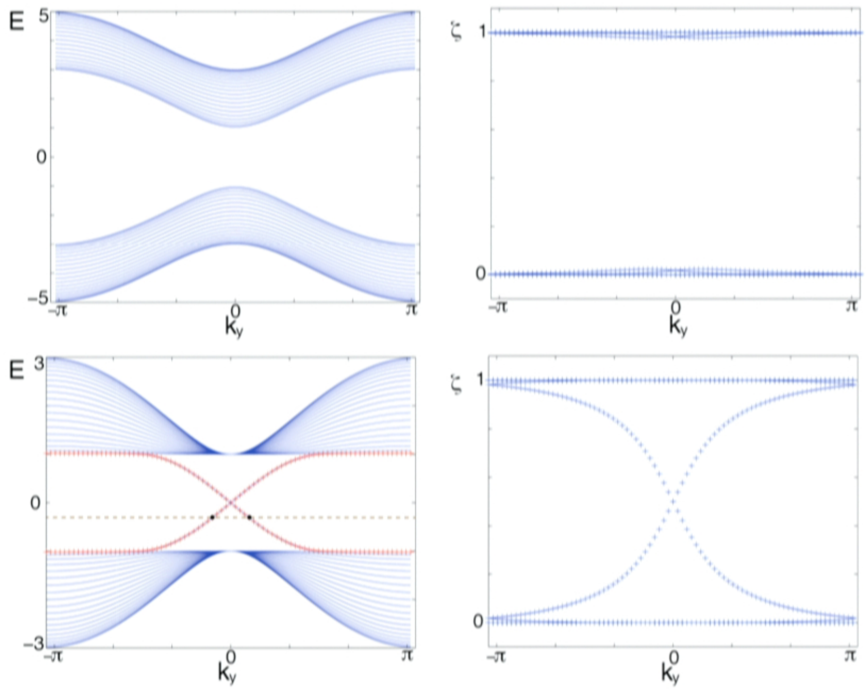

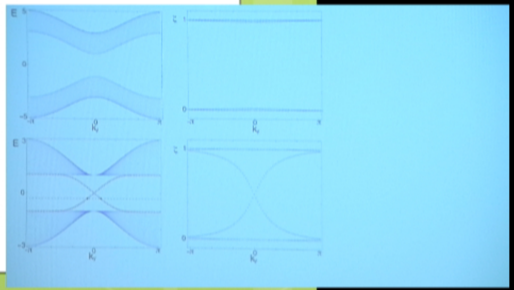

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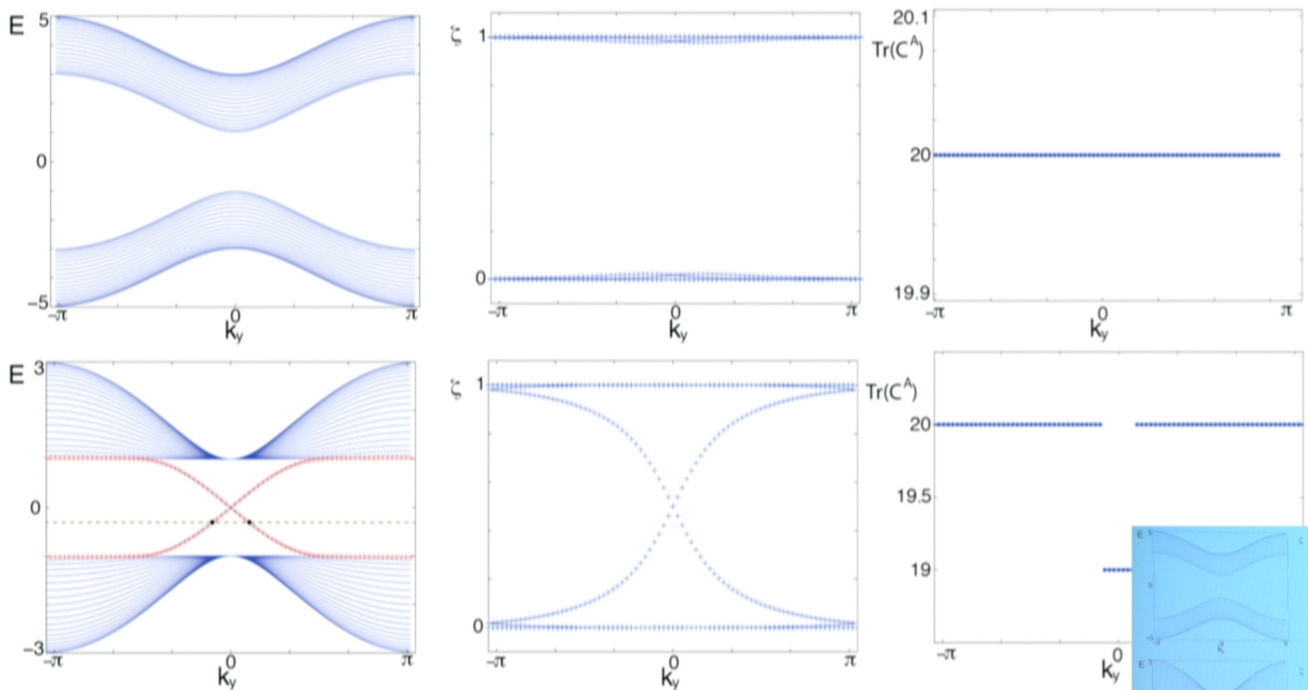

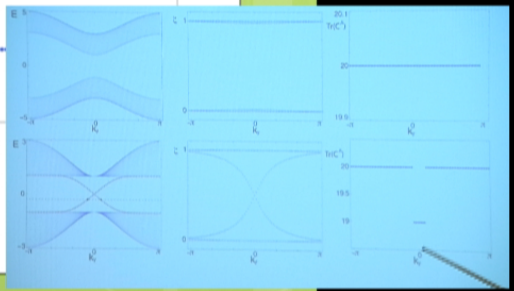

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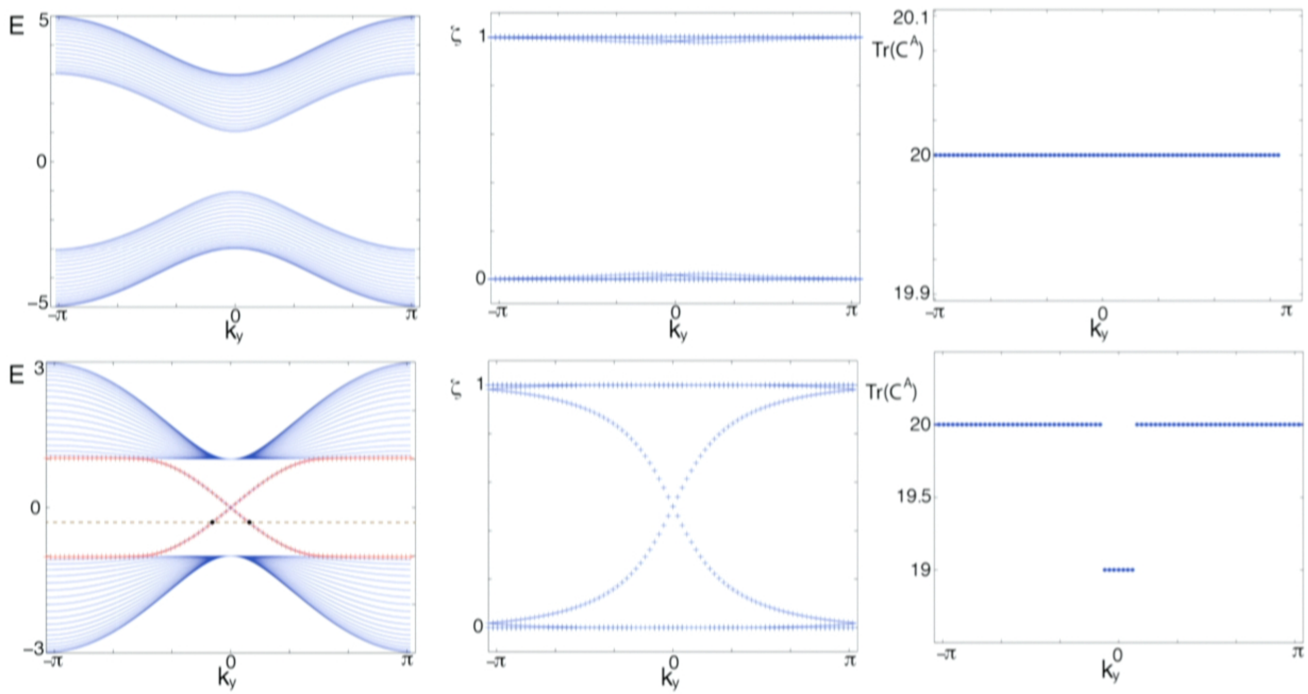
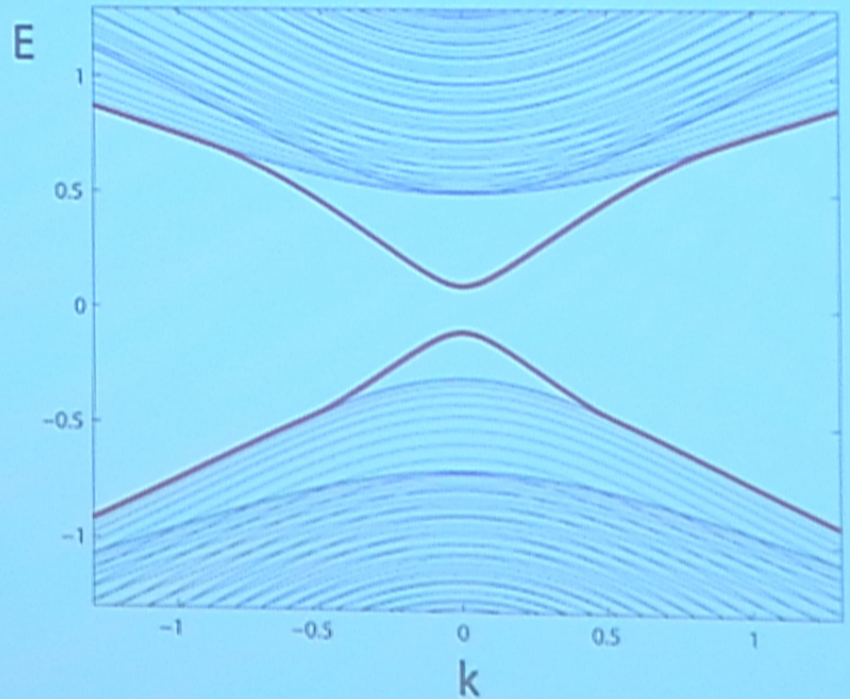


Physics connected with Z_2 pumping of Fu and Kane:
Threading π-flux gives a charge transfer ($C_1 = 0$)

Question: Protected physical edge states imply protected entanglement cut states, but if there are no protected edge states can there be protected cut states?

Example: Take QSH and apply a magnetic (Zeeman only) field. Z_2 Invariant no longer protected and a gap opens in the edge states



Breaking Time-Reversal Symmetry

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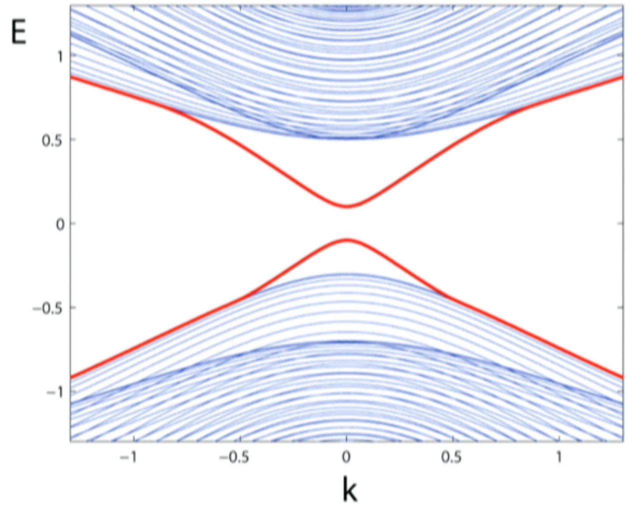

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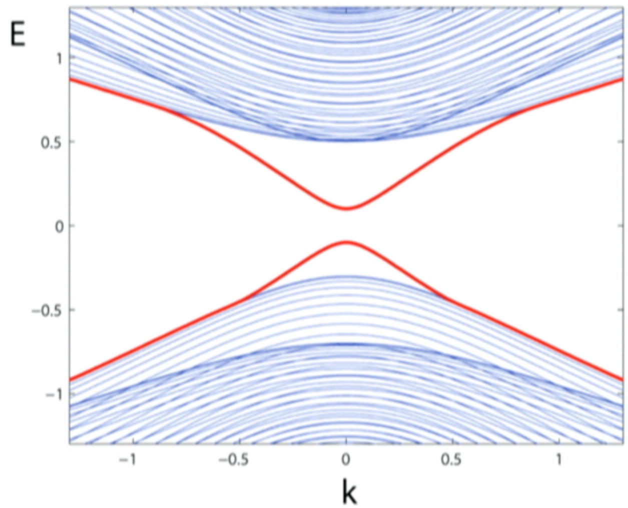
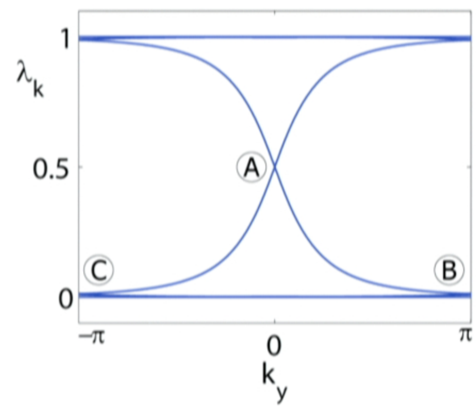
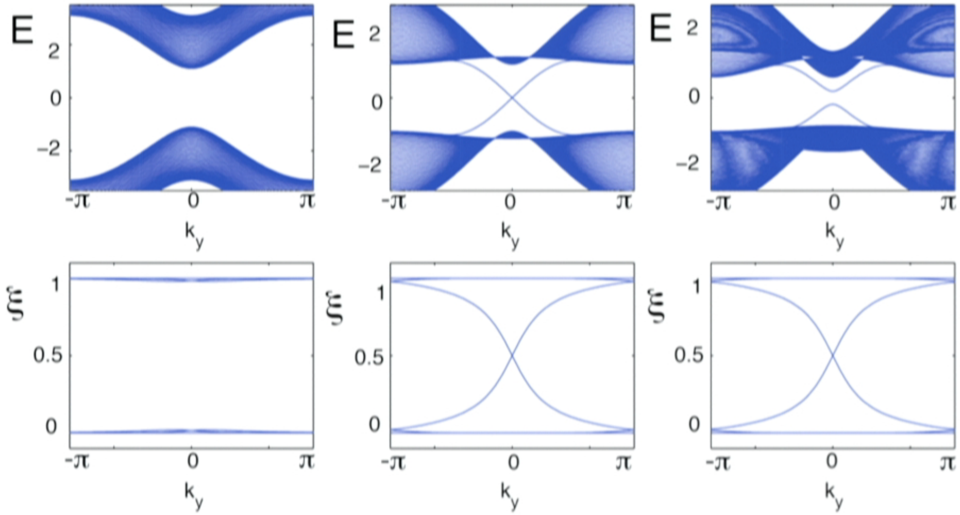




Still gapless?

What is going on in the entanglement spectrum?

Breaking Time-Reversal Symmetry



Hughes, Prodan, Bernevig PRB, **83** 245132 (2011)

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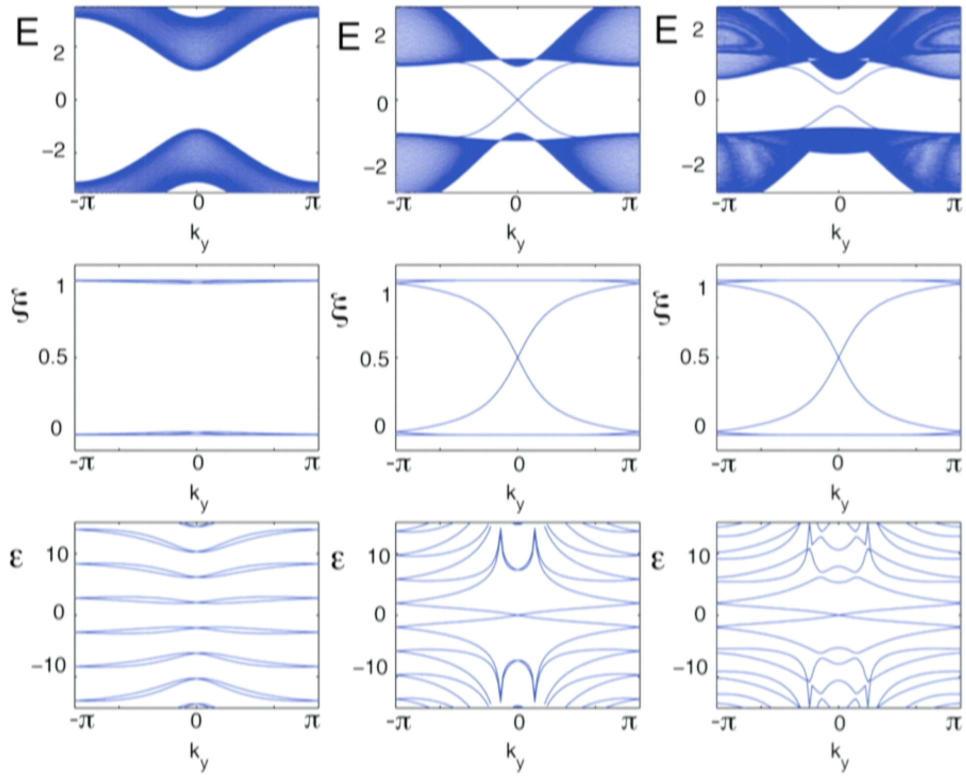

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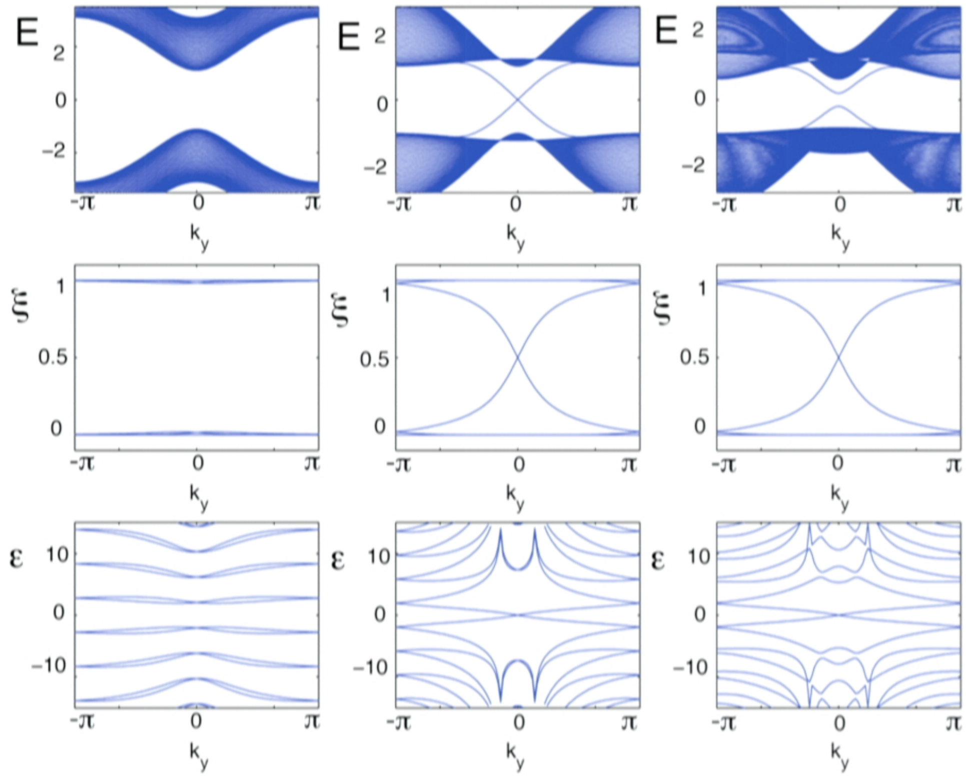

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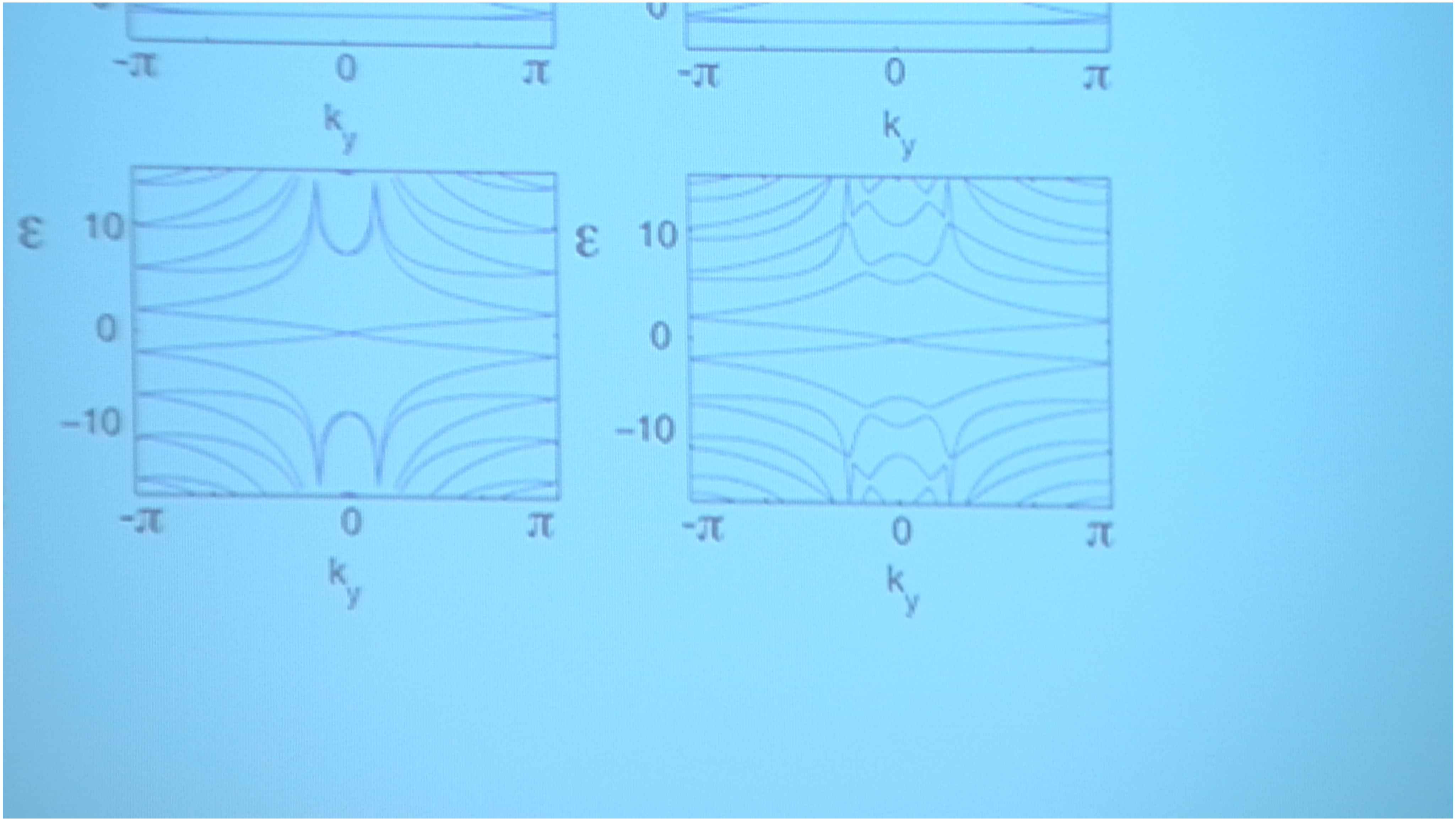

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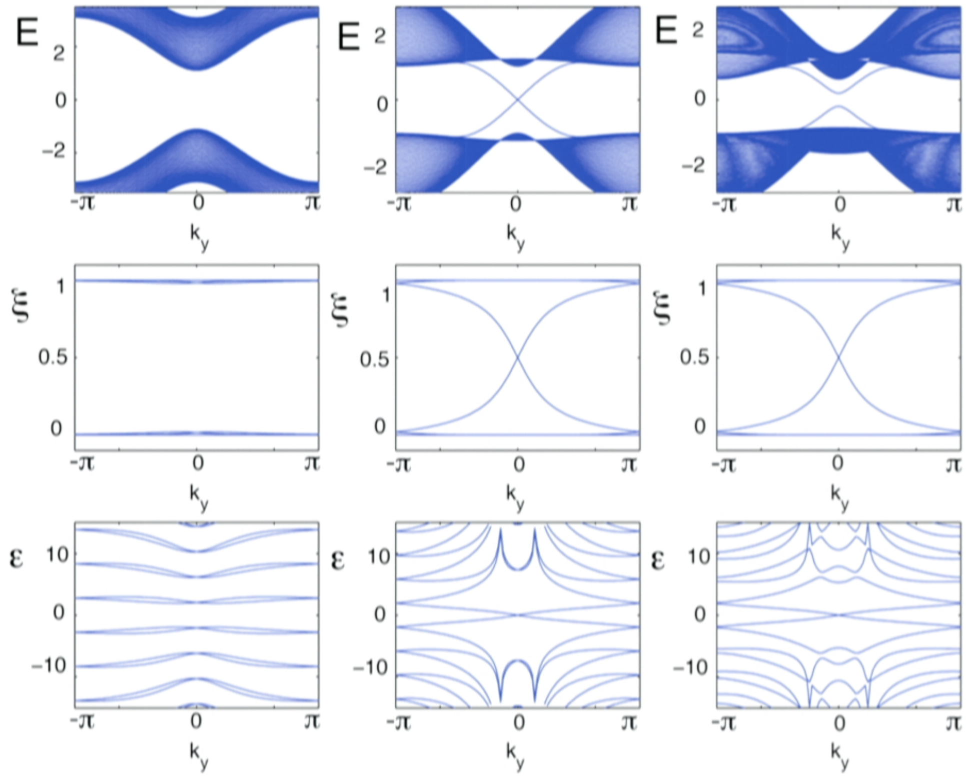
Summary

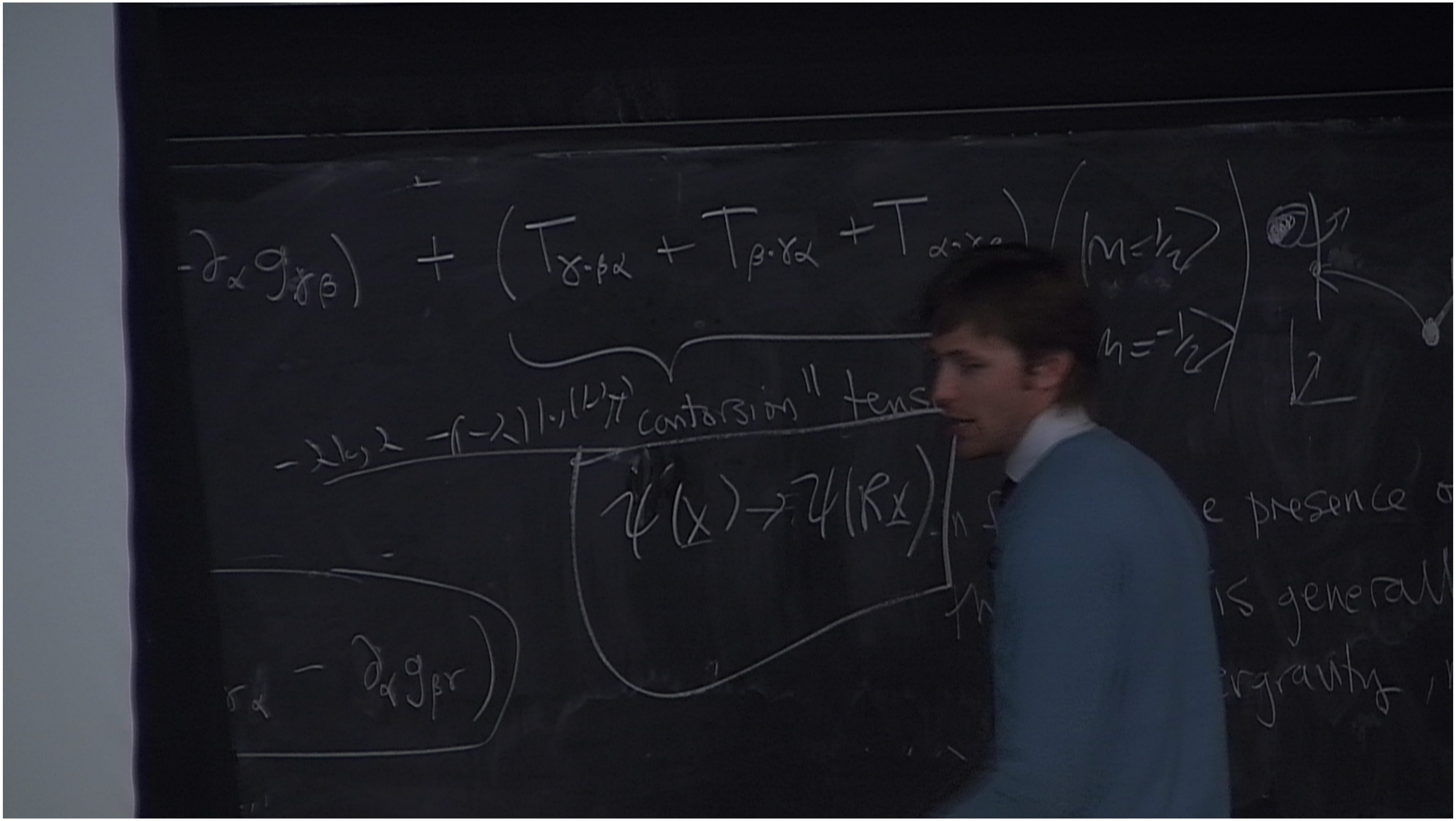
- Entanglement spectra of topological insulators has revealed interesting features
- Open Questions:
  - Entanglement in disordered fermionic systems
  - Topological invariants in 1D and 3D
  - Classification of protected entanglement modes for all point-group symmetries
  - Other ways to cut the system

Breaking Time-Reversal Symmetry



We clearly see here the signature of the TRS breaking, the spectral flow is cut-off. Every degeneracy protected by TRS is lifted *except* one. This last degeneracy is protected by *inversion* symmetry.

Hughes, Prodan, Bernevig PRB, **83** 245132 (2011)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