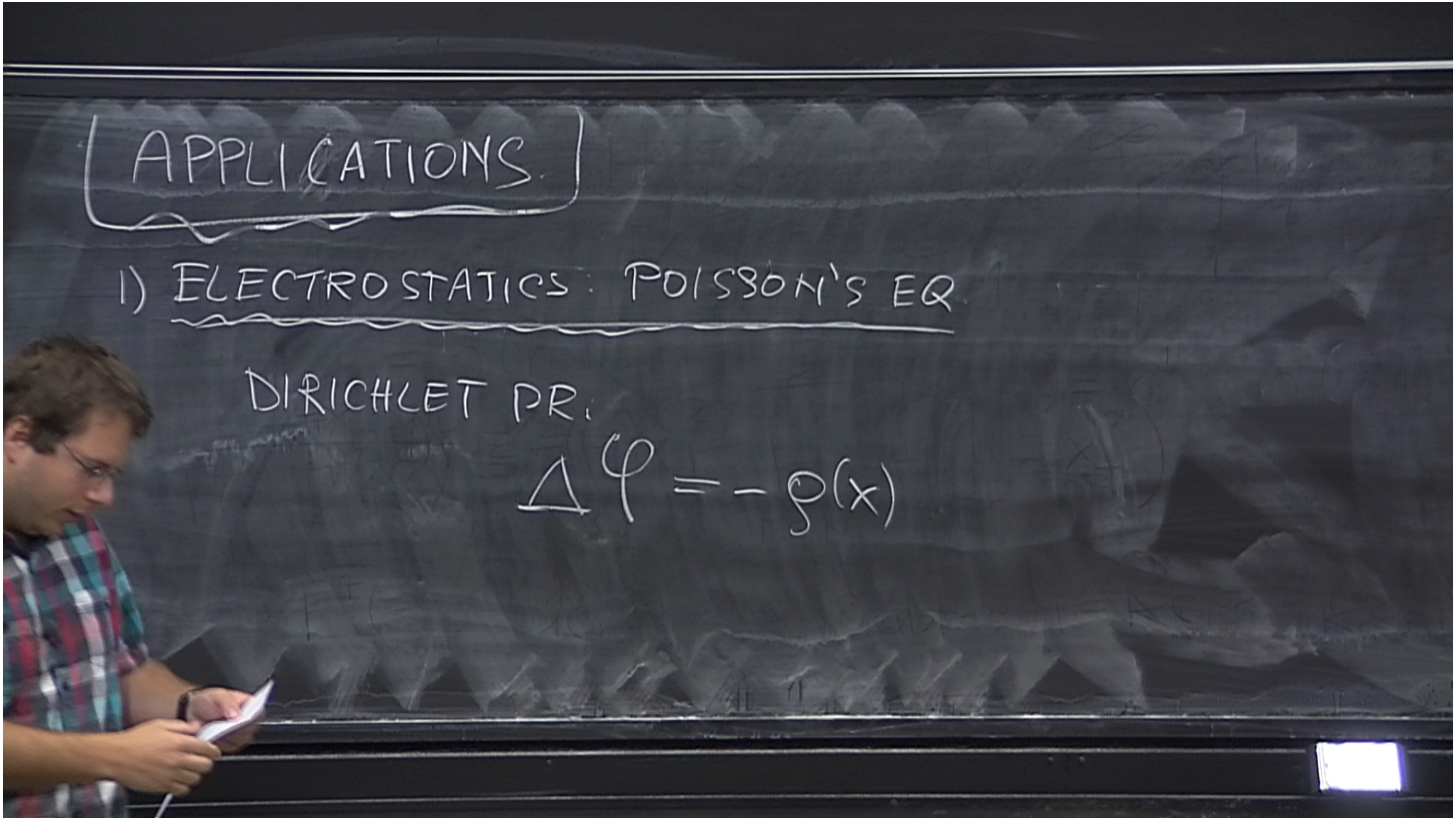
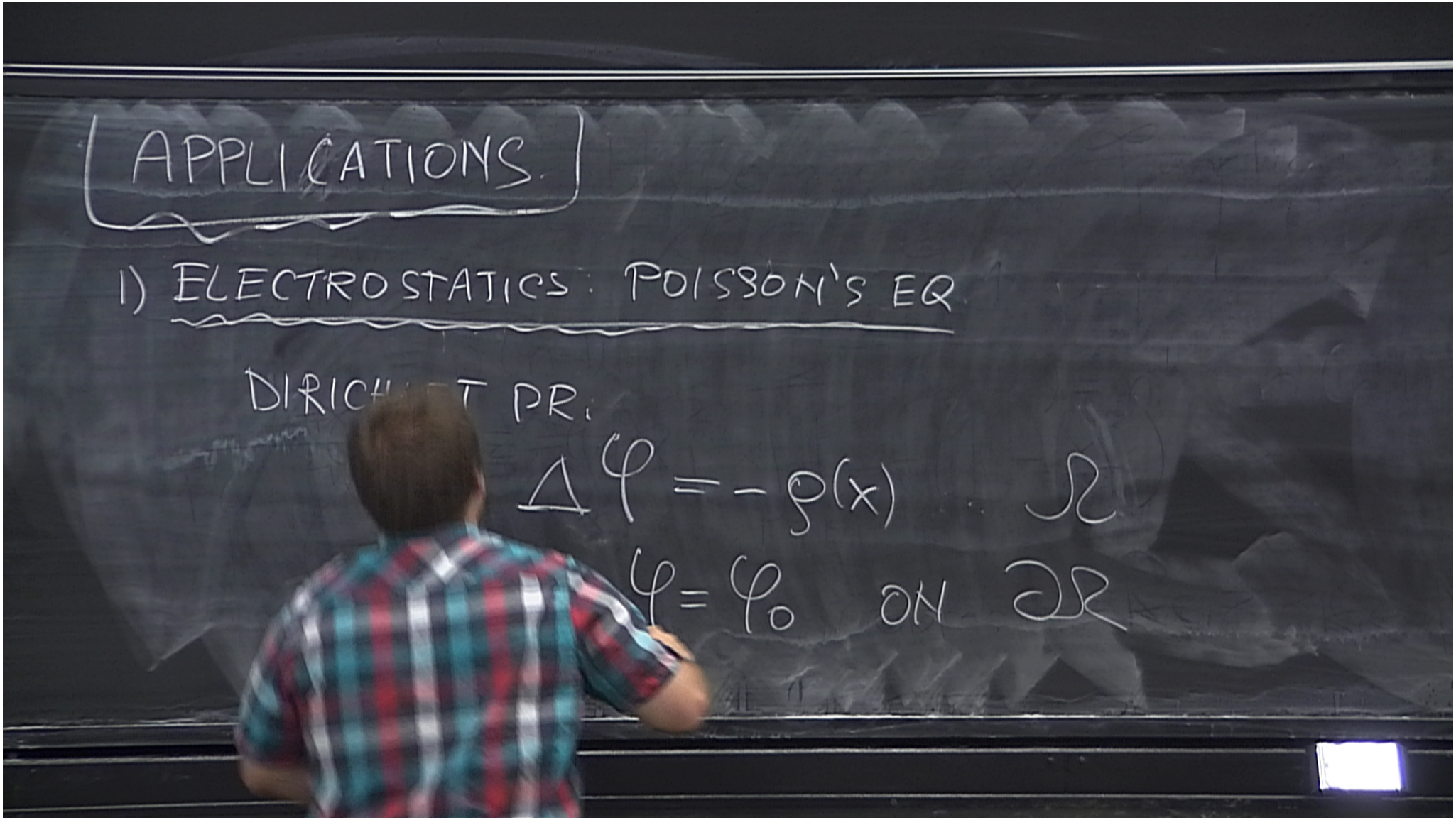
Title: Integral Transforms and Green's Functions - Lecture 4

Date: Sep 09, 2011 10:30 AM

URL: http://pirsa.org/11090051

Abstract:


APPLICATIONS.
1) ELECTROSTATICS: POISSON'S EQ.
DIRICHLET PR.
$\Delta\varphi = -\rho(x)$


APPLICATIONS.
1) ELECTROSTATICS: POISSON'S EQ
DIRICHLET PR.
$\Delta\varphi = -\rho(x)$ Ω
$\varphi = \varphi_0$ ON $\partial\Omega$

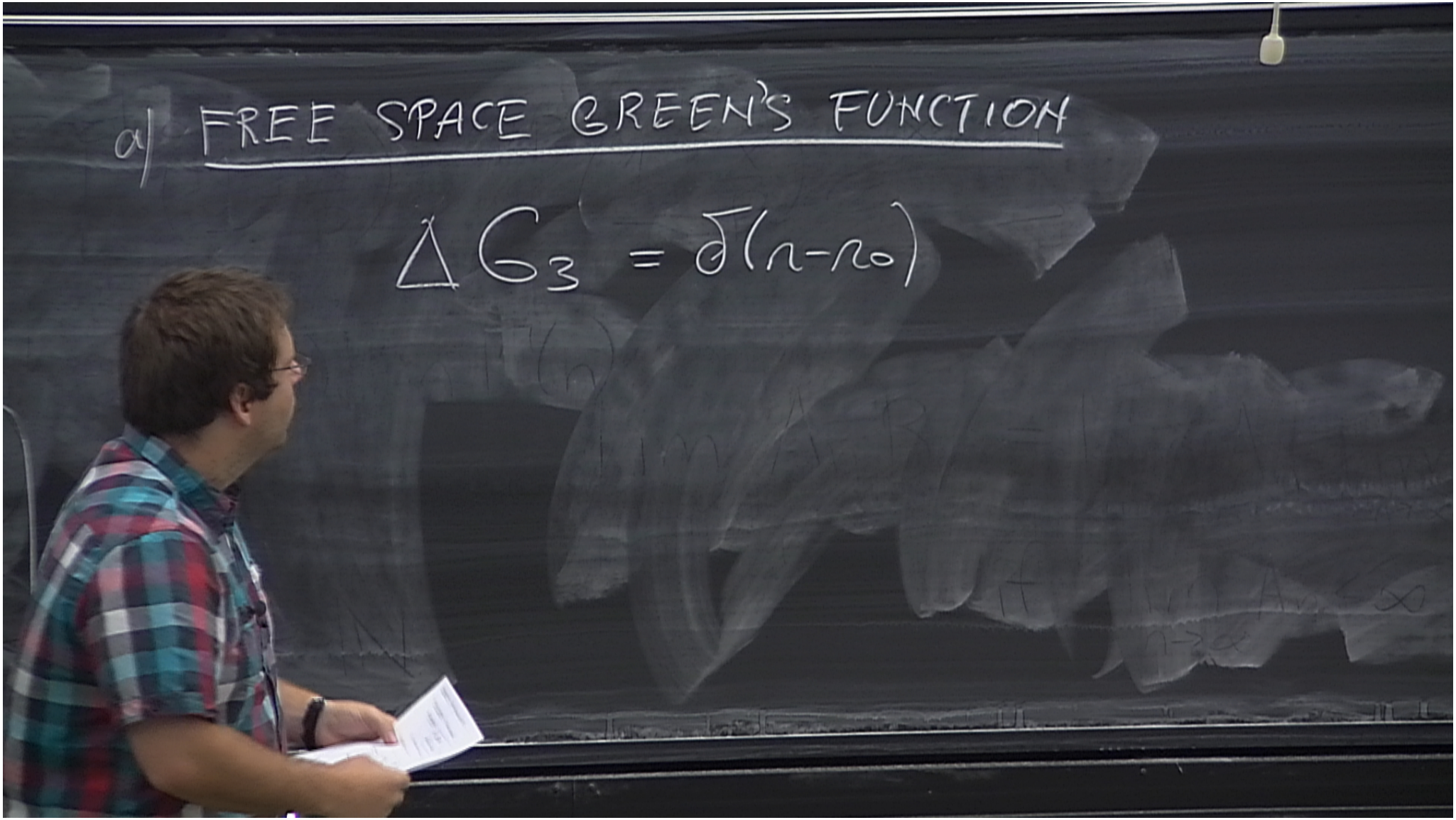
APPLICATIONS.

1) ELECTROSTATICS: POISSON'S EQ.

(DP) DIRICHLET PR.

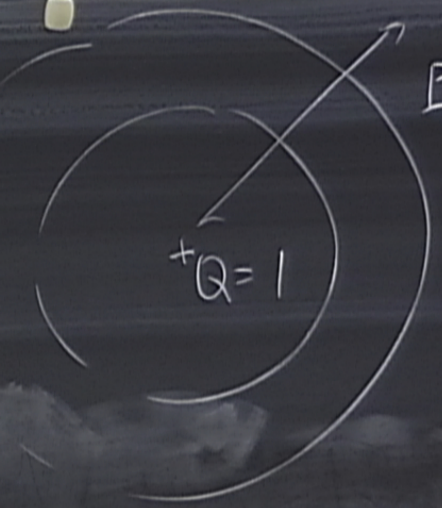
$$\Delta \varphi = -\rho(x) \quad \Omega$$

$$\varphi = \varphi_0 \quad \text{ON} \quad \partial\Omega$$


a) FREE SPACE GREEN'S FUNCTION
$\Delta G_3 = \delta(r - r_0)$

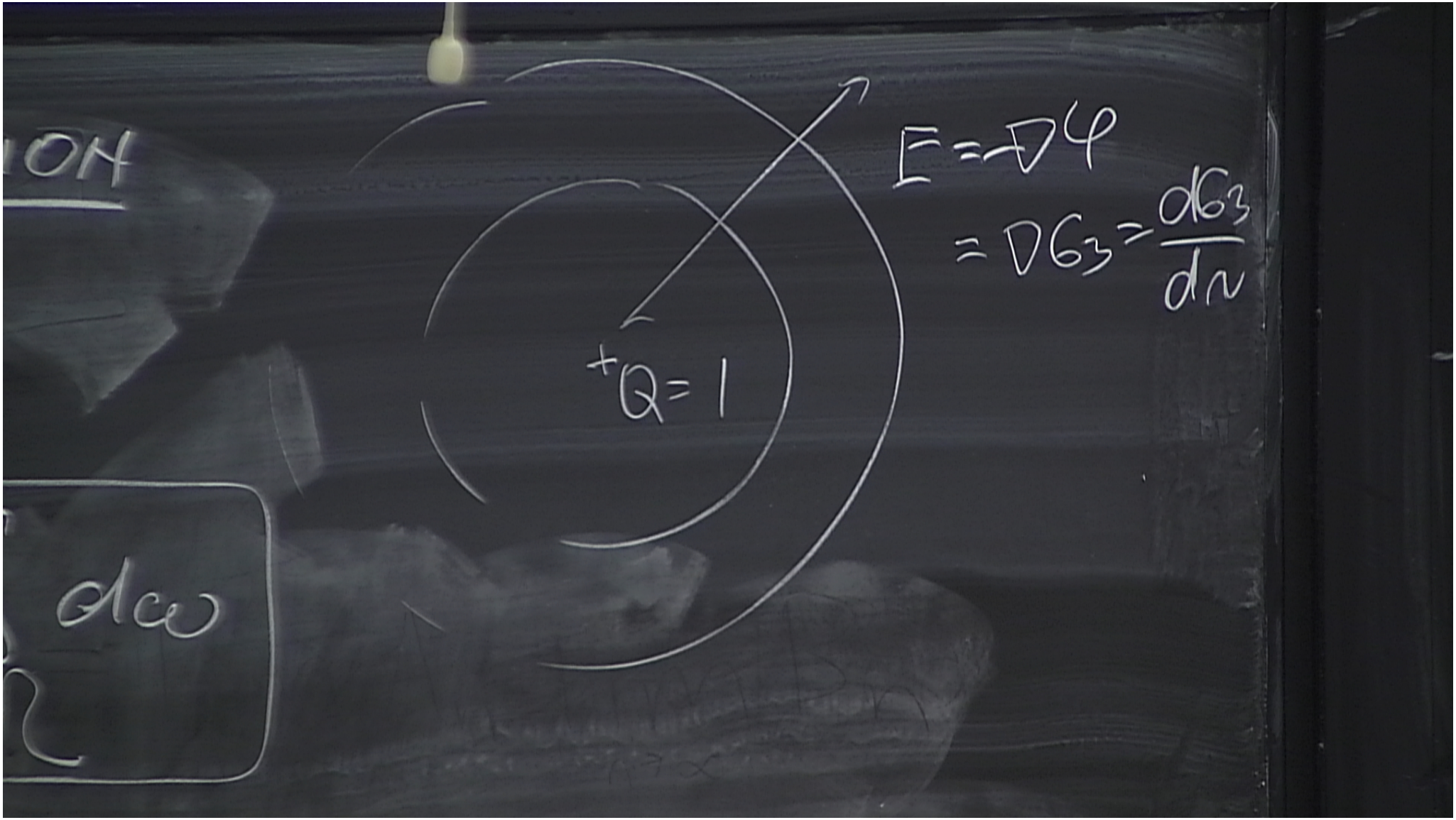
a) FREE SPACE GREEN'S FUNCTION

$$\Delta G_3 = \delta(r - r_0)$$



$E = -\nabla\varphi$

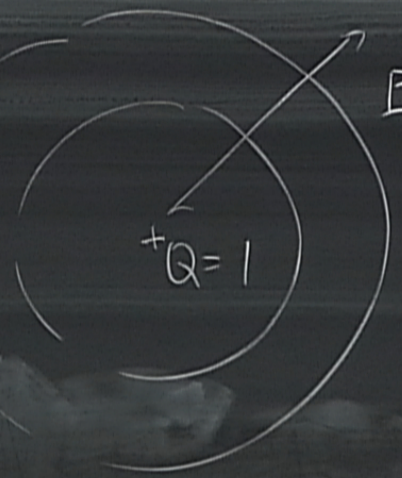
$= \nabla G_3 =$


$E = -\nabla\varphi$
$= \nabla G_3 = \frac{dG_3}{dn}$
$+Q = 1$
$d\omega$

a) FREE SPACE GREEN'S FUNCTION

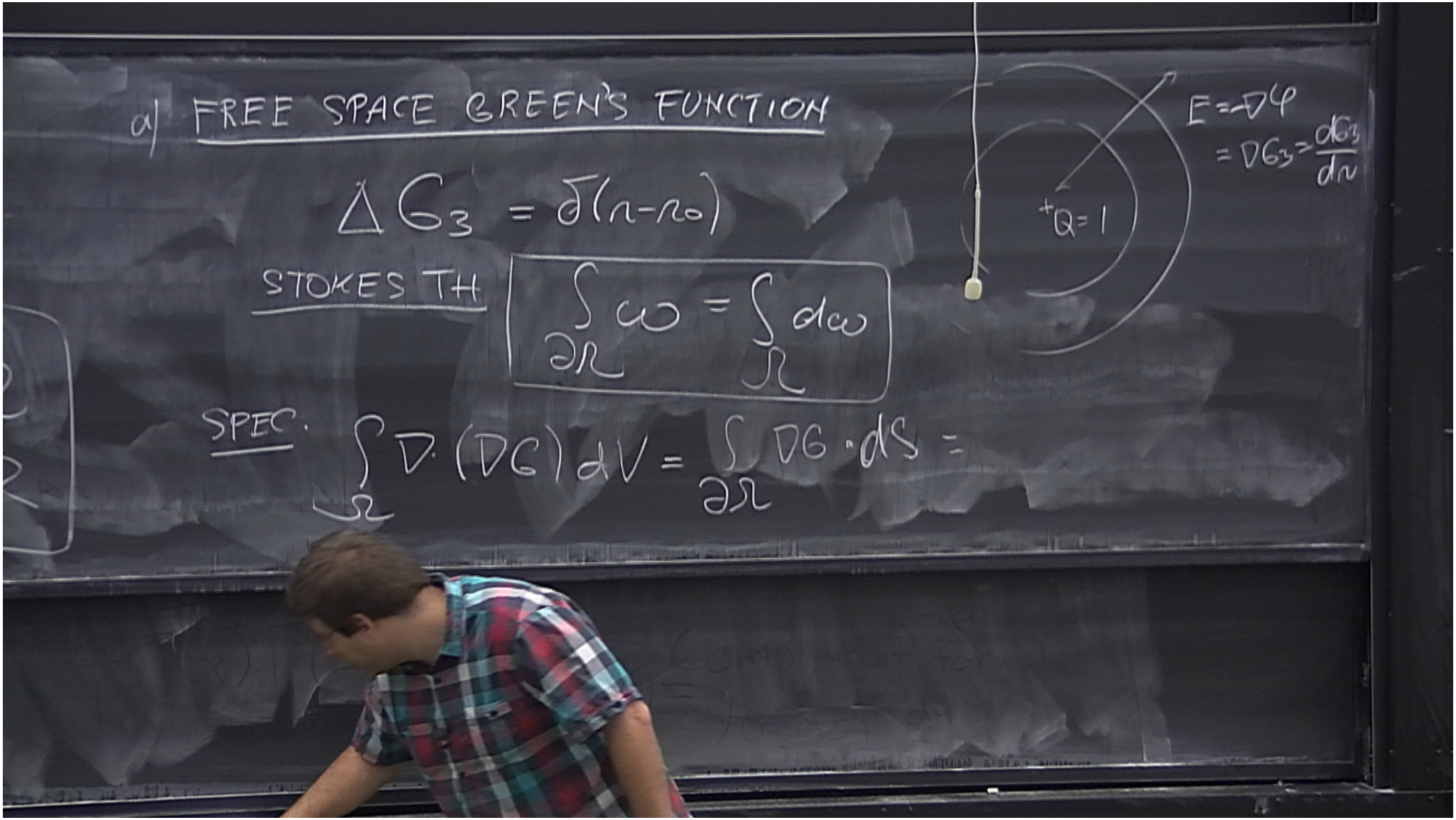
$$\Delta G_3 = \delta(r - r_0)$$

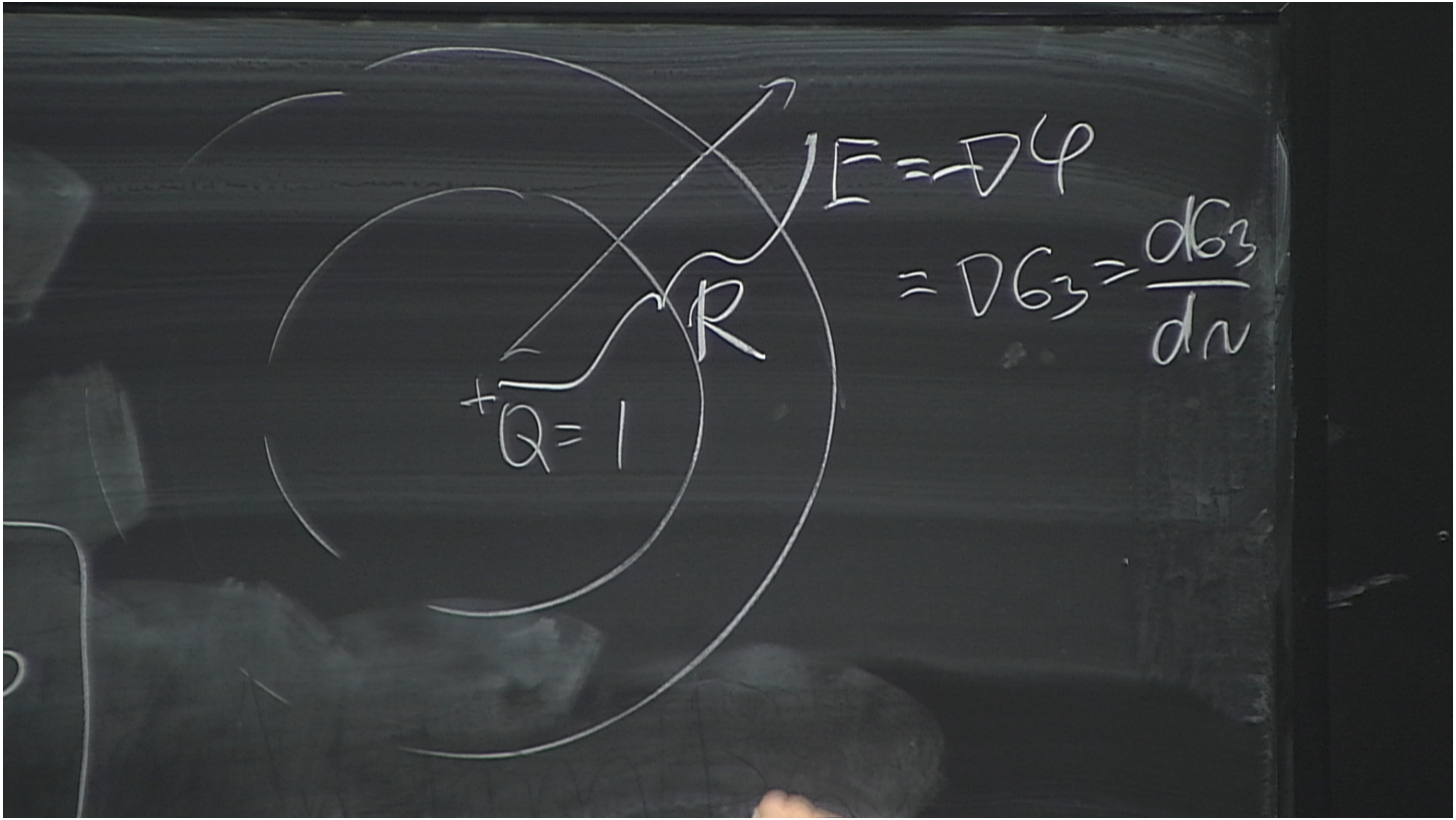
STOKES TH $\int_{\partial R} \omega = \int_R d\omega$



$E = -D\varphi$

$= DG_3 = \frac{dG_3}{dn}$


a) FREE SPACE GREEN'S FUNCTION
$\Delta G_3 = \delta(r - r_0)$
STOKES TH
$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega$
SPEC.
$\int_{\Omega} \nabla \cdot (\nabla G)\, dV = \int_{\partial\Omega} \nabla G \cdot dS =$
$+Q = 1$
$E = -\nabla\varphi$
$= \nabla G_3 = \frac{dG_3}{dn}$

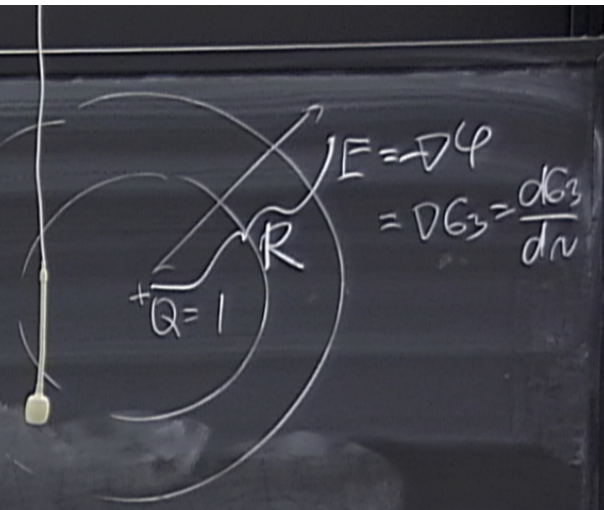

E = -∇φ
= DG3 = dG3/dn
R
+
Q = 1

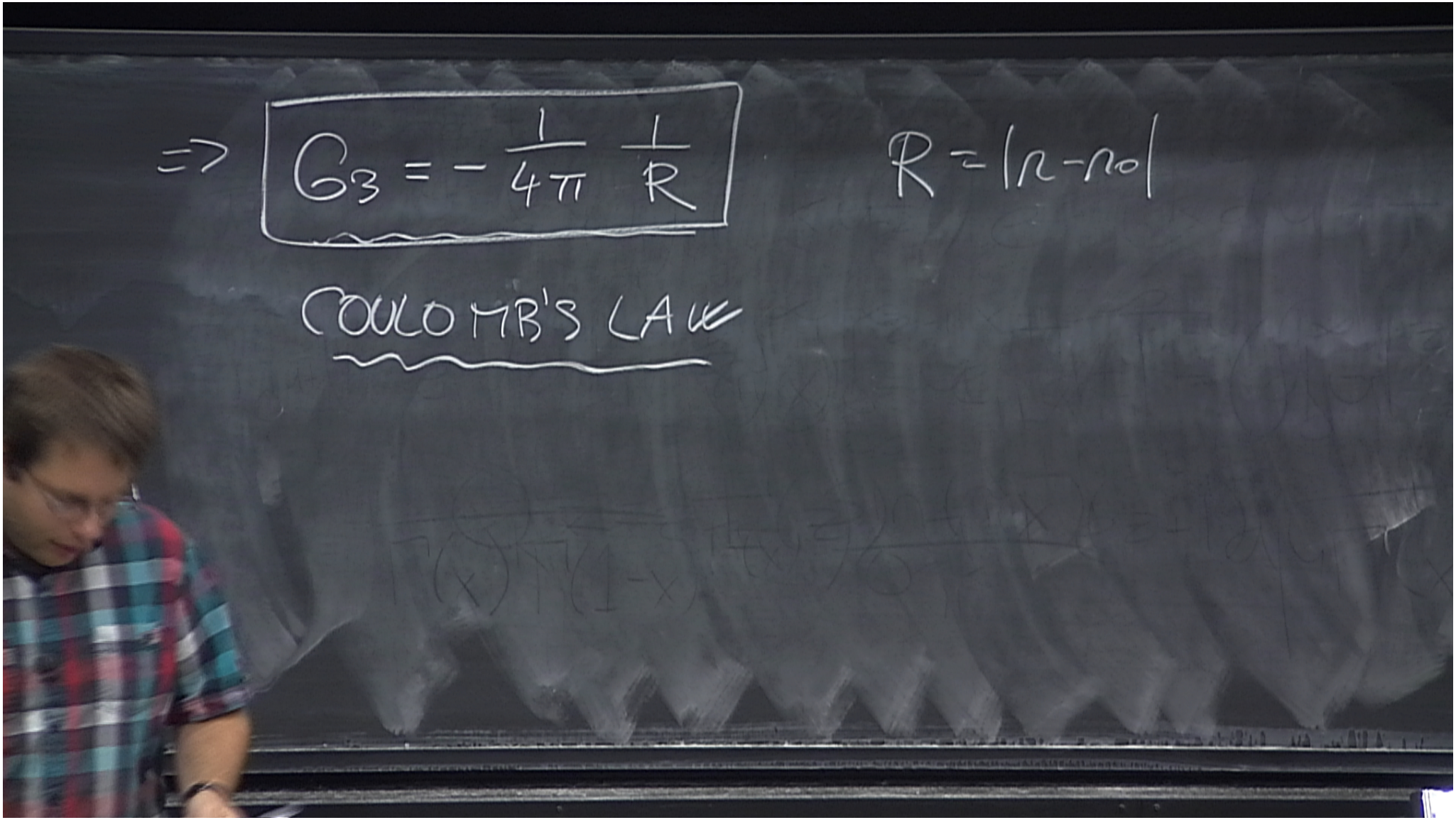
a/ FREE SPACE GREEN'S FUNCTION

$$\Delta G_3 = \delta(r - r_0)$$

STOKES TH $\quad \int_{\partial\Omega} \omega = \int_{\Omega} d\omega$

SPEC. $\quad \int_{\Omega} \nabla \cdot (\nabla G)\, dV = \int_{\partial\Omega} \nabla G \cdot dS = 4\pi R^2 \frac{dG_3}{dn} = 1$




=> $G_3 = -\frac{1}{4\pi}\frac{1}{R}$
$R = |r - r_0|$
COULOMB'S LAW

$$\Rightarrow \boxed{G_3 = -\frac{1}{4\pi}\frac{1}{R}} \qquad R = |r - r_0|$$

COULOMB'S LAW

GREEN'S IDENTITIES

$$\int_{\Omega} \nabla \cdot (\phi \nabla \chi)$$

$\varphi = \varphi_0$ ON $\partial\Omega$

SPEC. $\int_\Omega \nabla\cdot(D G)\, dV$

$\Rightarrow \boxed{G_3 = -\frac{1}{4\pi}\frac{1}{R}}$ $\quad R = |x - x_0|$

COULOMB'S LAW

b) GREEN'S IDENTITIES

$$\int_\Omega \nabla\cdot(\phi \nabla\chi)\, dV = \int_{\partial\Omega} (\phi \nabla\chi)\cdot d\vec{S} = \int_\Omega (\phi\, \Delta\chi + \nabla\phi\cdot\nabla\chi)\, dV$$

$\varphi = \varphi_0$ ON $\partial\Omega$

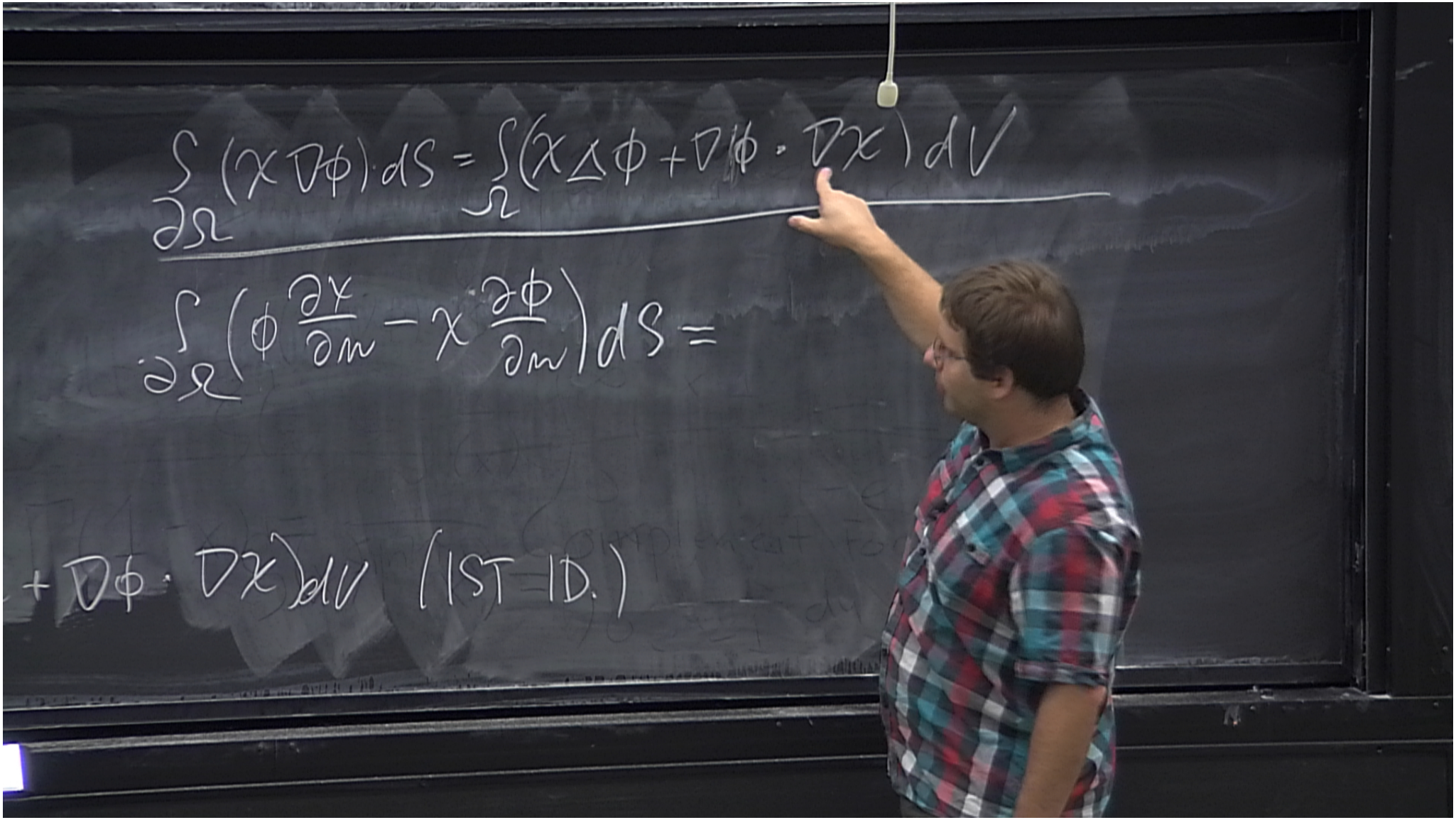
SPEC. $\int_\Omega \nabla\cdot(D G)\,dV = \int_{\partial\Omega} D$

$\Rightarrow \boxed{G_3 = -\frac{1}{4\pi}\frac{1}{R}}$ $\quad R = |r - r_0|$

COULOMB'S LAW

GREEN'S IDENTITIES

$$\int_\Omega \nabla\cdot(\phi\,\nabla\chi)\,dV = \int_{\partial\Omega} \underbrace{(\phi\nabla\chi)}_{\phi\frac{\partial\chi}{\partial n}}\cdot dS = \int_\Omega (\phi\,\Delta\chi + \nabla\phi\cdot\nabla\chi)\,dV \quad (1\text{ST ID.})$$

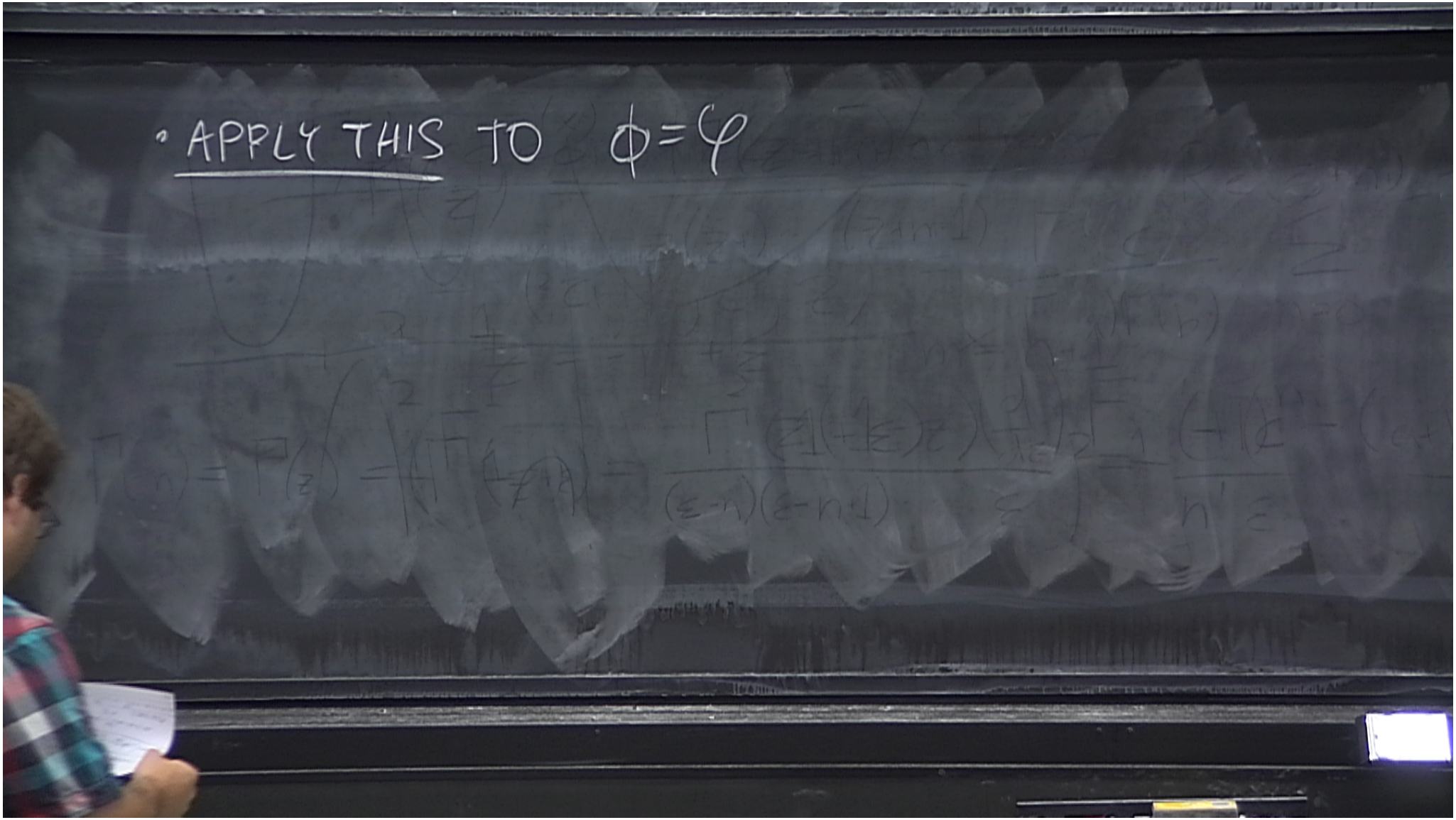

$\int_{\partial\Omega} (\chi \nabla\phi)\cdot dS = \int_{\Omega} (\chi \Delta\phi + \nabla\phi \cdot \nabla\chi)\, dV$
$\int_{\partial\Omega} \left(\phi \frac{\partial\chi}{\partial n} - \chi \frac{\partial\phi}{\partial n}\right) dS =$
$+ \nabla\phi \cdot \nabla\chi) dV$ (1ST ID.)

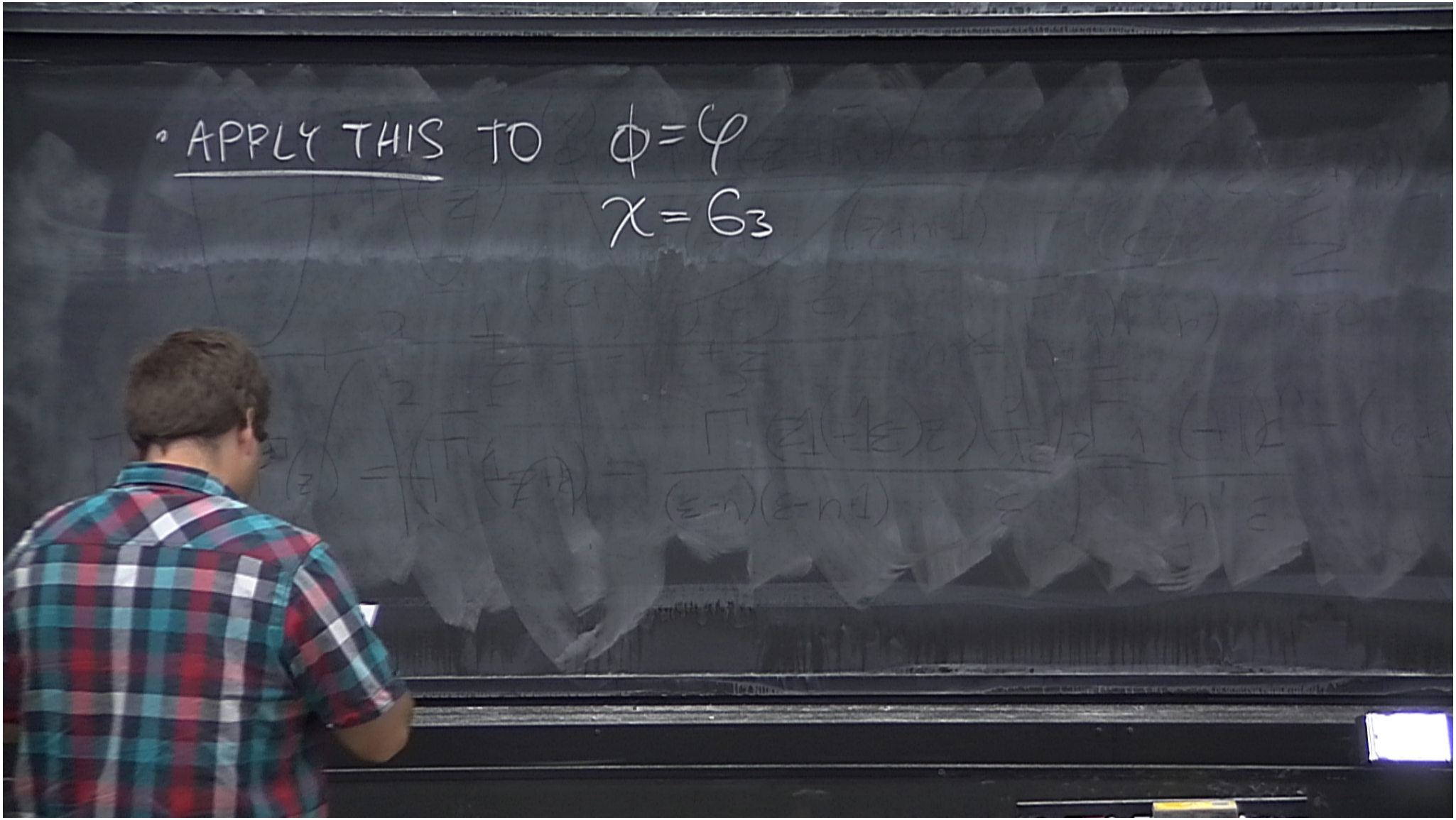
$$\int_{\partial\Omega} (\chi \nabla\phi)\cdot dS = \int_{\Omega} (\chi \Delta\phi + \nabla\phi \cdot \nabla\chi)\, dV$$

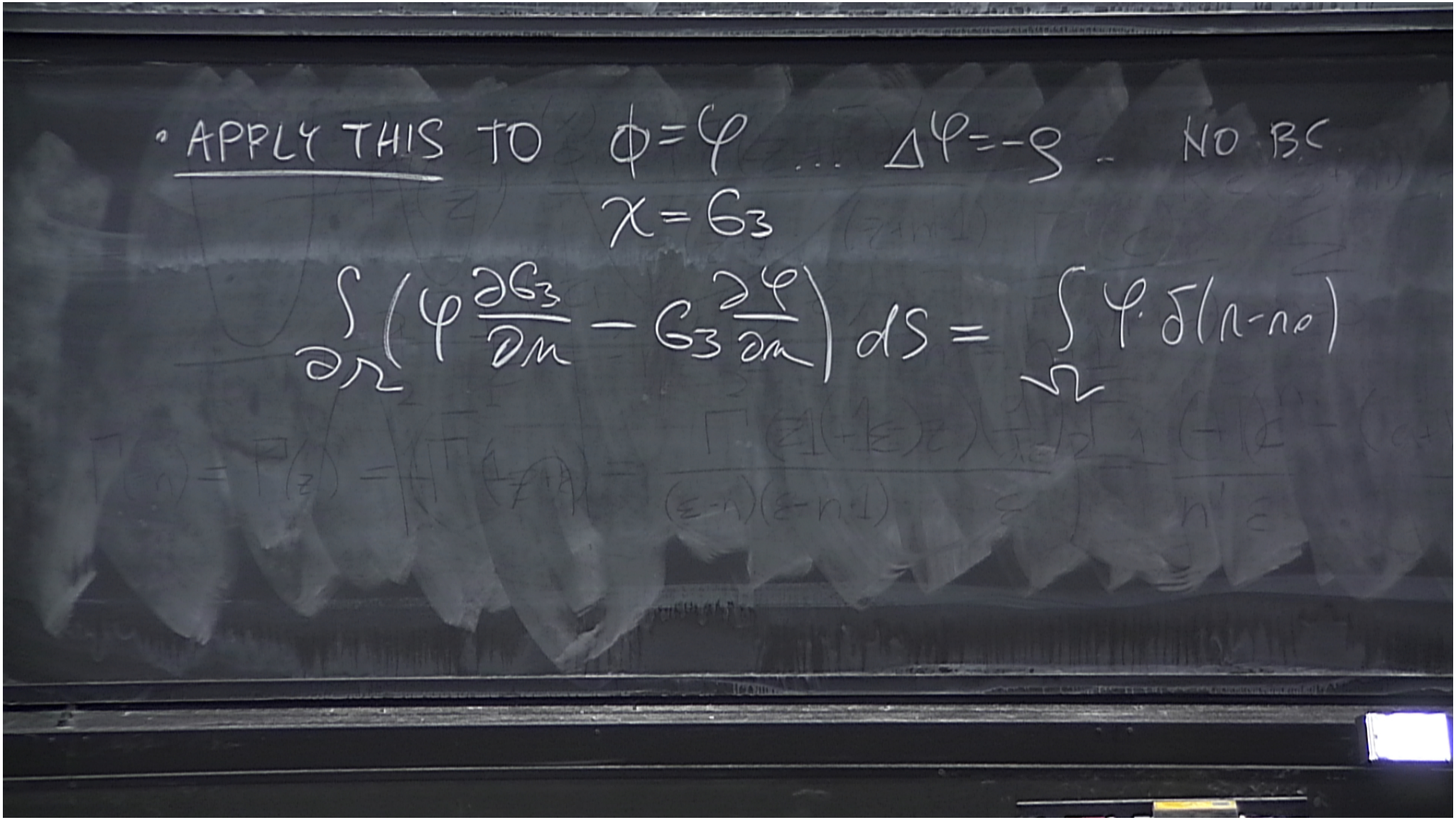
$$\int_{\partial\Omega} \left(\phi \frac{\partial\chi}{\partial n} - \chi \frac{\partial\phi}{\partial n}\right) dS = \int_{\Omega} (\phi \Delta\chi - \chi \Delta\phi)\, dV$$

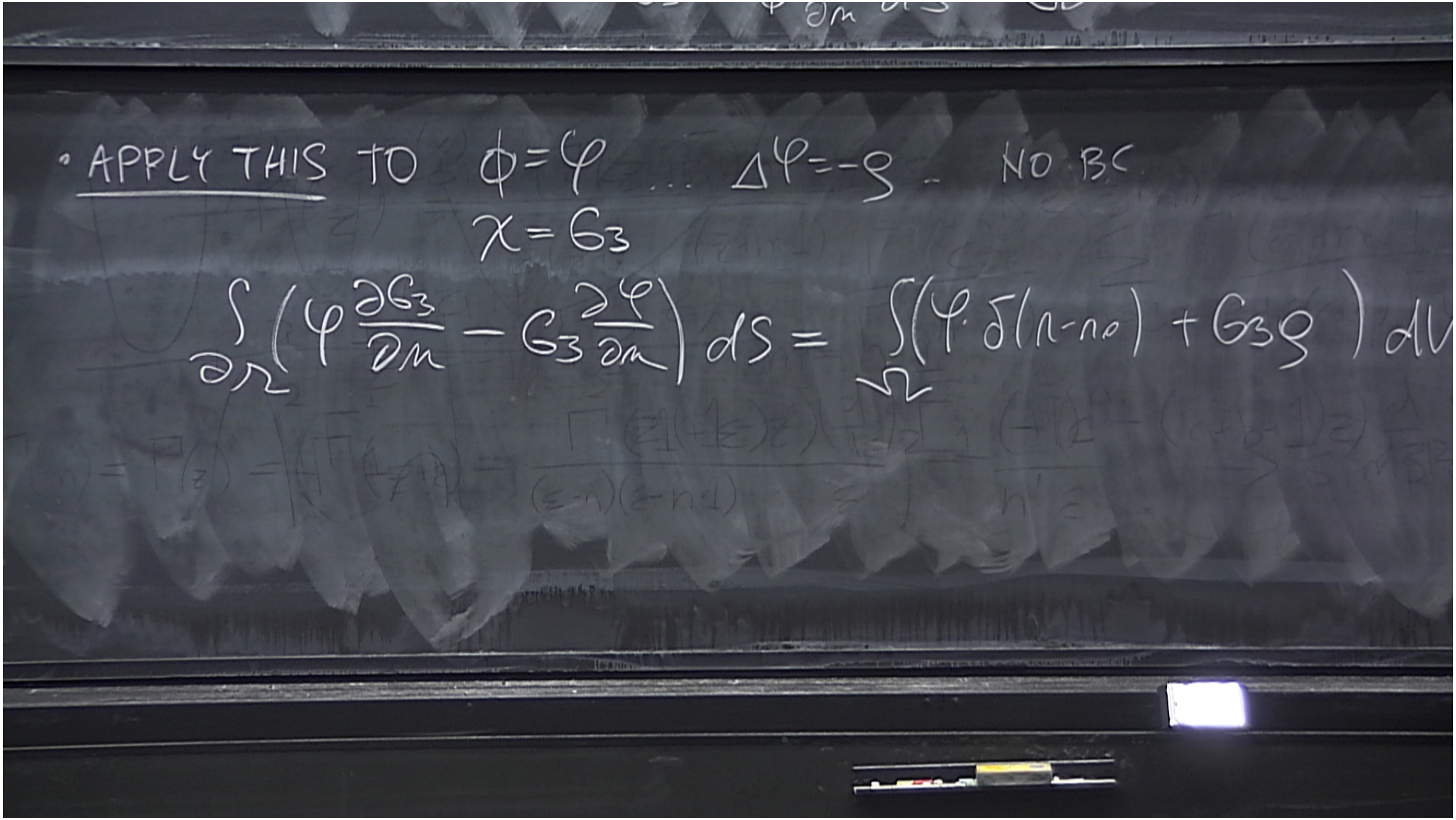
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• APPLY THIS TO $\phi = \varphi$


• APPLY THIS TO $\phi = \varphi$
$\chi = G_3$


• APPLY THIS TO $\phi = \varphi$ ... $\Delta\varphi = -\rho$. NO BC
$\chi = G_3$
$\int_{\partial\Omega} \left(\varphi \frac{\partial G_3}{\partial n} - G_3 \frac{\partial \varphi}{\partial n} \right) dS = \int_{\Omega} \varphi \cdot \delta(n - n_0)$


• APPLY THIS TO $\phi = \varphi$ ... $\Delta\varphi = -\varrho$ .. NO BC
$\chi = G_3$
$\int_{\partial\Omega} \left(\varphi \frac{\partial G_3}{\partial n} - G_3 \frac{\partial \varphi}{\partial n} \right) dS = \int_{\Omega} \left(\varphi \cdot \delta(\vec{r} - \vec{r}_0) + G_3 \varrho \right) dV$

$\varphi = \varphi_0$ ON $\partial\Omega$

$\int_\Omega \nabla\cdot(\nabla G)\,dV = \int_{\partial\Omega} \nabla G\cdot dS = -4\pi K$

$\Rightarrow \boxed{G_3 = -\frac{1}{4\pi}\frac{1}{R}}$ $\quad R = |r - r_0|$

COULOMB'S LAW

$\int_{\partial\Omega}(\chi\nabla\phi)\,dS = \int_\Omega(\chi\Delta\phi + \nabla\phi\cdot\nabla\chi)\,dV$

$\boxed{\int_{\partial\Omega}\left(\phi\frac{\partial\chi}{\partial n} - \chi\frac{\partial\phi}{\partial n}\right)dS = \int_\Omega(\phi\Delta\chi - \chi\Delta\phi)\,dV}$

(2ND Id)

b) GREEN'S IDENTITIES

$\int_\Omega \nabla\cdot(\phi\nabla\chi)\,dV = \int_{\partial\Omega}\underbrace{(\phi\nabla\chi)}_{\phi\frac{\partial\chi}{\partial n}}\cdot dS = \int_\Omega(\phi\Delta\chi + \nabla\phi\cdot\nabla\chi)\,dV$ (1ST ID.)

• APPLY THIS TO $\phi = \varphi$ $\quad \Delta\varphi = -\rho$ NO BC

$\chi = G_3$

$\int_{\partial\Omega}\left(\varphi\frac{\partial G_3}{\partial n} - G_3\frac{\partial\varphi}{\partial n}\right)dS = \int_\Omega(\varphi\,\delta(r-r_0) + G_3\rho)\,dV = \varphi(r_0) + \int_\Omega G_3\rho$

$\boxed{\varphi(r_0) = -\int_\Omega G_3\rho + \int_{\partial\Omega}\left(\varphi\frac{\partial G_3}{\partial n} - G_3\frac{\partial\varphi}{\partial n}\right)dS}$

$$\int_\Omega \nabla\cdot(\phi\nabla\chi)\,dV = \int_{\partial\Omega} (\phi\nabla\chi)\cdot dS = \int_\Omega (\phi\,\Delta\chi + \nabla\phi\cdot\nabla\chi)\,dV \quad \text{(1ST ID.)}$$

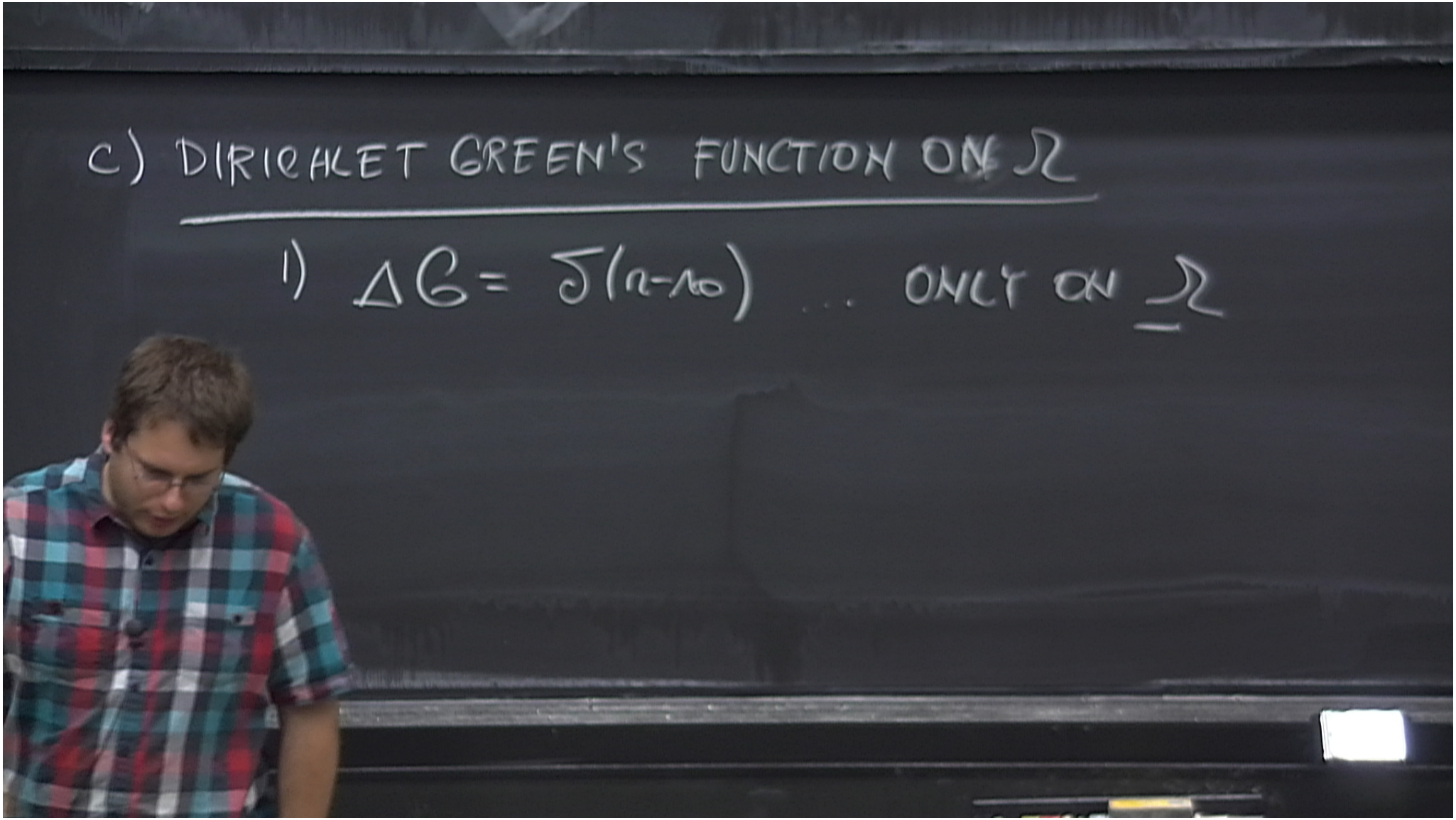
$\phi\frac{\partial\chi}{\partial n}\,dS$

- APPLY THIS TO $\phi=\varphi$ ... $\Delta\varphi=-\rho$ - NO BC

$\chi = G_3$

$$\int_{\partial\Omega}\left(\varphi\frac{\partial G_3}{\partial n} - G_3\frac{\partial\varphi}{\partial n}\right)dS = \int_\Omega (\varphi\cdot\delta(r-r_0) + G_3\rho)\,dV = \varphi(r_0) + \int_\Omega G_3\rho$$

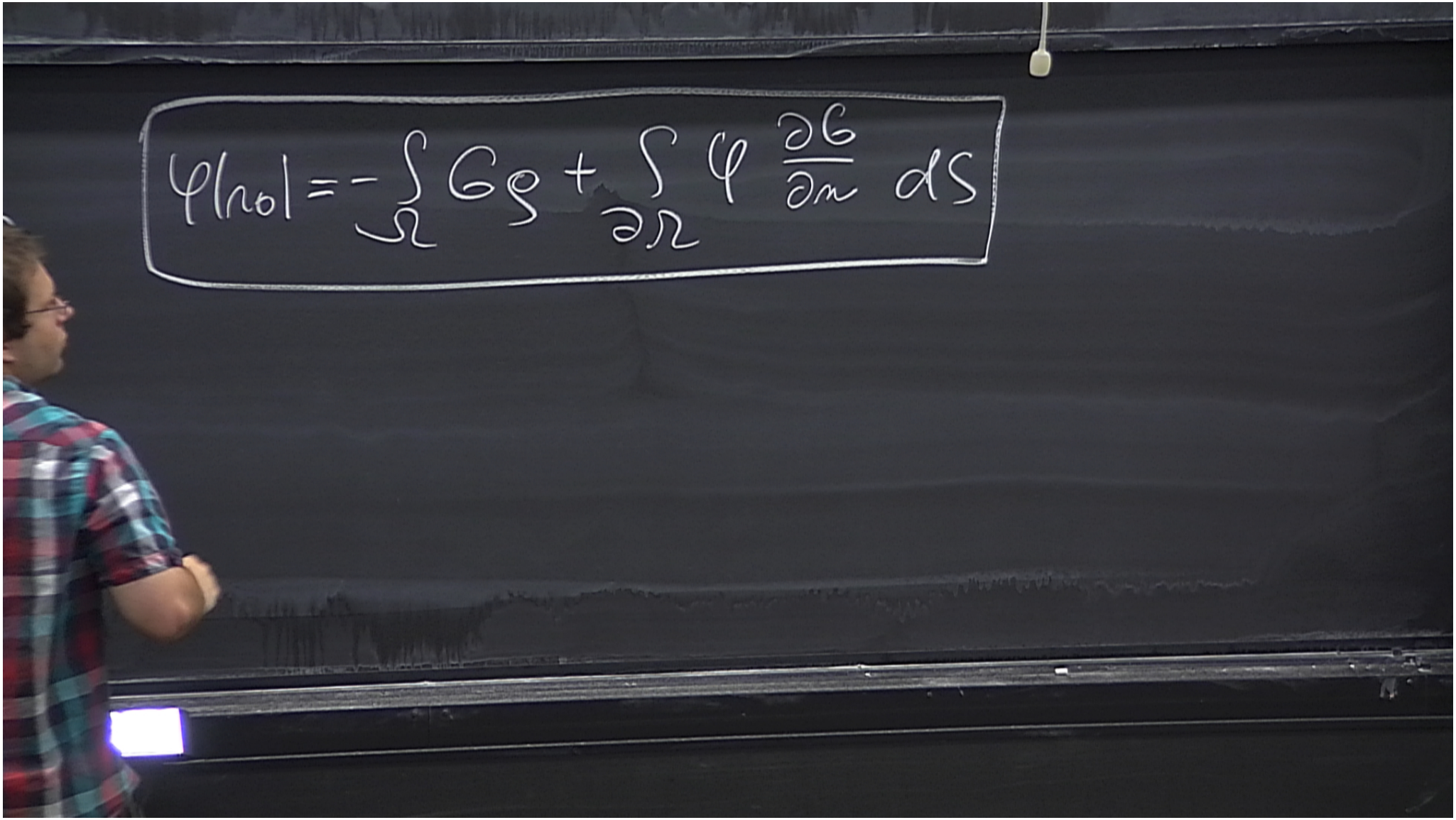
$$\boxed{\varphi(r_0) = -\int_\Omega G_3\rho + \int_{\partial\Omega}\left(\varphi\frac{\partial G_3}{\partial n} - G_3\frac{\partial\varphi}{\partial n}\right)dS}$$


c) DIRICHLET GREEN'S FUNCTION ON Ω
1) ΔG = δ(x-x0) ... ONLY ON Ω

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1) $\Delta G = \delta(r - r_0)$ ... ONLY ON Ω

2) $G(r, r_0)\big|_{\partial\Omega} = 0$


$\varphi(x_0) = -\int_\Omega G\rho + \int_{\partial\Omega} \varphi \frac{\partial G}{\partial n} dS$

• APPLY THIS TO $\phi = \varphi$ $\quad \Delta\varphi = -\rho$ NO BC

$\chi = G_3$

$$\int_{\partial\Omega} \left(\varphi \frac{\partial G_3}{\partial n} - G_3 \frac{\partial \varphi}{\partial n} \right) dS = \int_{\Omega} \left(\varphi \cdot \delta(r - r_0) + G_3 \rho \right) dV = \varphi(r_0) + \int_{\Omega} G_3 \rho$$

$$\boxed{\varphi(r_0) = -\int_{\Omega} G_3 \rho + \int_{\partial\Omega} \left(\varphi \frac{\partial G_3}{\partial n} - G_3 \frac{\partial \varphi}{\partial n} \right) dS}$$ (3RD Id) DEFINONS!

c) DIRICHLET GREEN'S FUNCTION ON Ω

1) $\Delta G = \delta(r - r_0)$ ONLY ON Ω

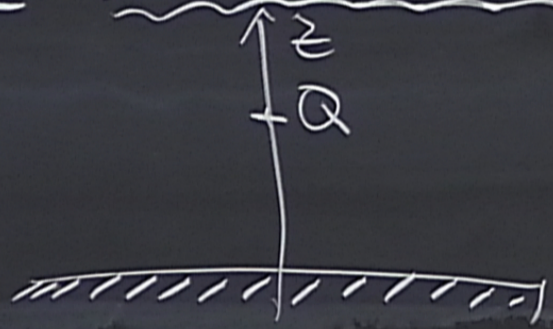
2) $G(r, r_0)\big|_{\partial\Omega} = 0$

$$\boxed{\varphi(r_0) = -\int_{\Omega} G \rho + \int_{\partial\Omega} \varphi \frac{\partial G}{\partial n} \, dS}$$ ... SOLUTION OF (DP)

$$\varphi(x_0) = -\int_\Omega G g + \int_{\partial\Omega} \varphi \frac{\partial G}{\partial n} \, dS$$

... SOLUTION OF (DP)

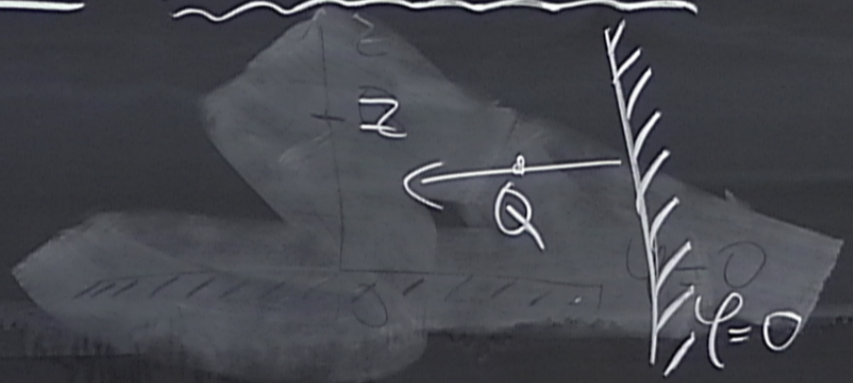
EXAMPLE: METHOD OF IMAGES



$$\varphi(x_0) = -\int_\Omega G g + \int_{\partial\Omega} \varphi \frac{\partial G}{\partial n} \, dS$$

... SOLUTION OF (DP)

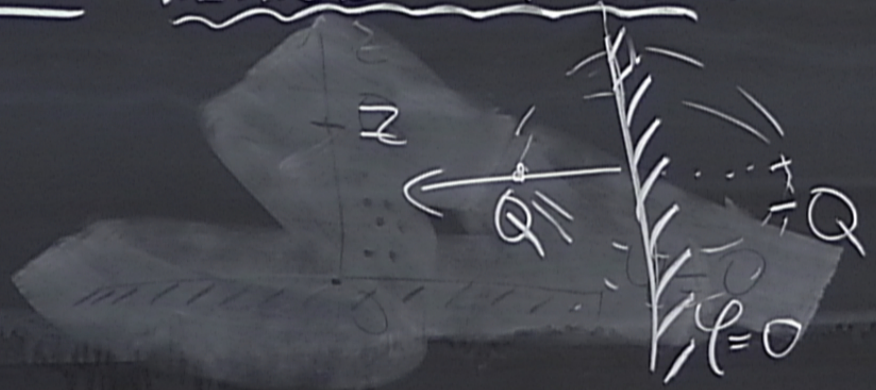
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... SOLUTION OF (DP)

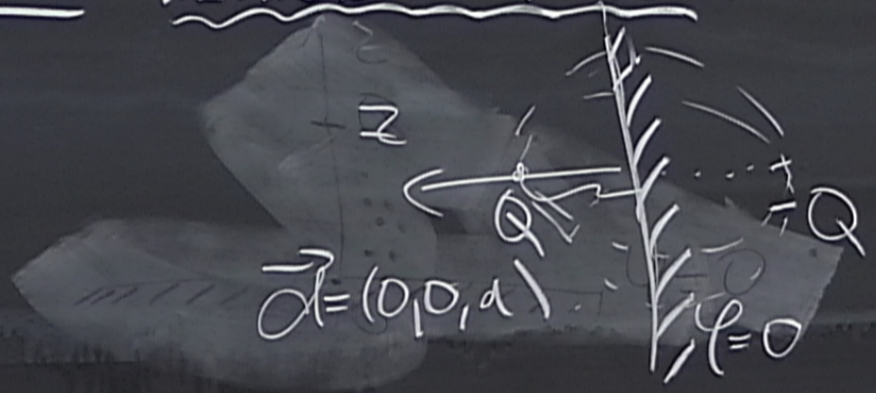
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$$\varphi(x_0) = -\int_\Omega G g + \int_{\partial\Omega} \varphi \frac{\partial G}{\partial n} \, dS$$

... SOLUTION OF (DP)

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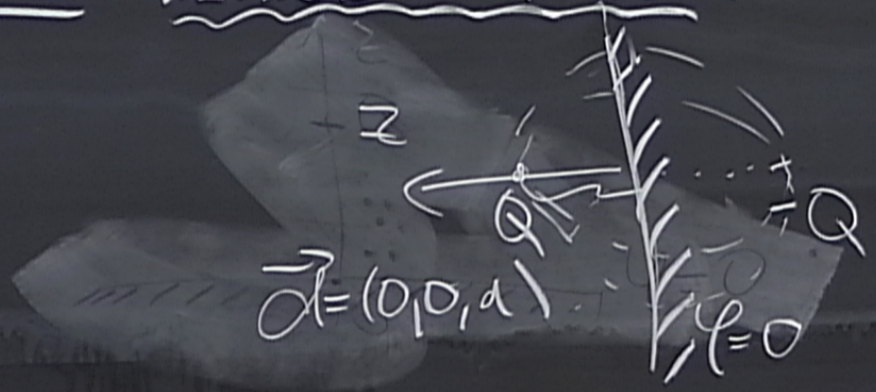


$$\varphi(x_0) = -\int_\Omega G g + \int_{\partial\Omega} \varphi \frac{\partial G}{\partial n}\, dS$$

... SOLUTION OF (DP)

EXAMPLE: METHOD OF IMAGES

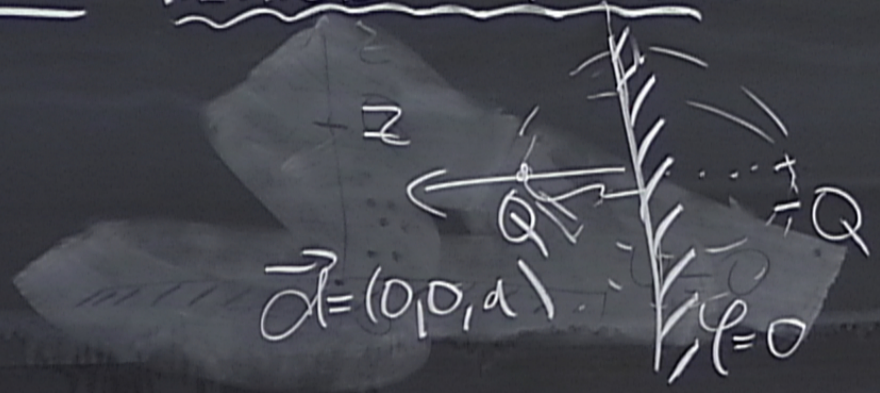
$$G = -\frac{1}{4\pi} \frac{1}{|\vec{x} - \vec{\xi}|}$$



$$\varphi(x_0) = -\int_\Omega G\varrho + \int_{\partial\Omega} \varphi \frac{\partial G}{\partial n}\, dS$$

... SOLUTION OF (DP)

EXAMPLE: METHOD OF IMAGES

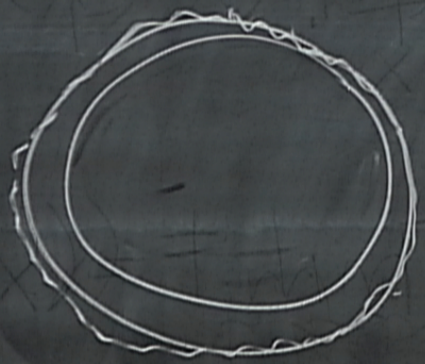


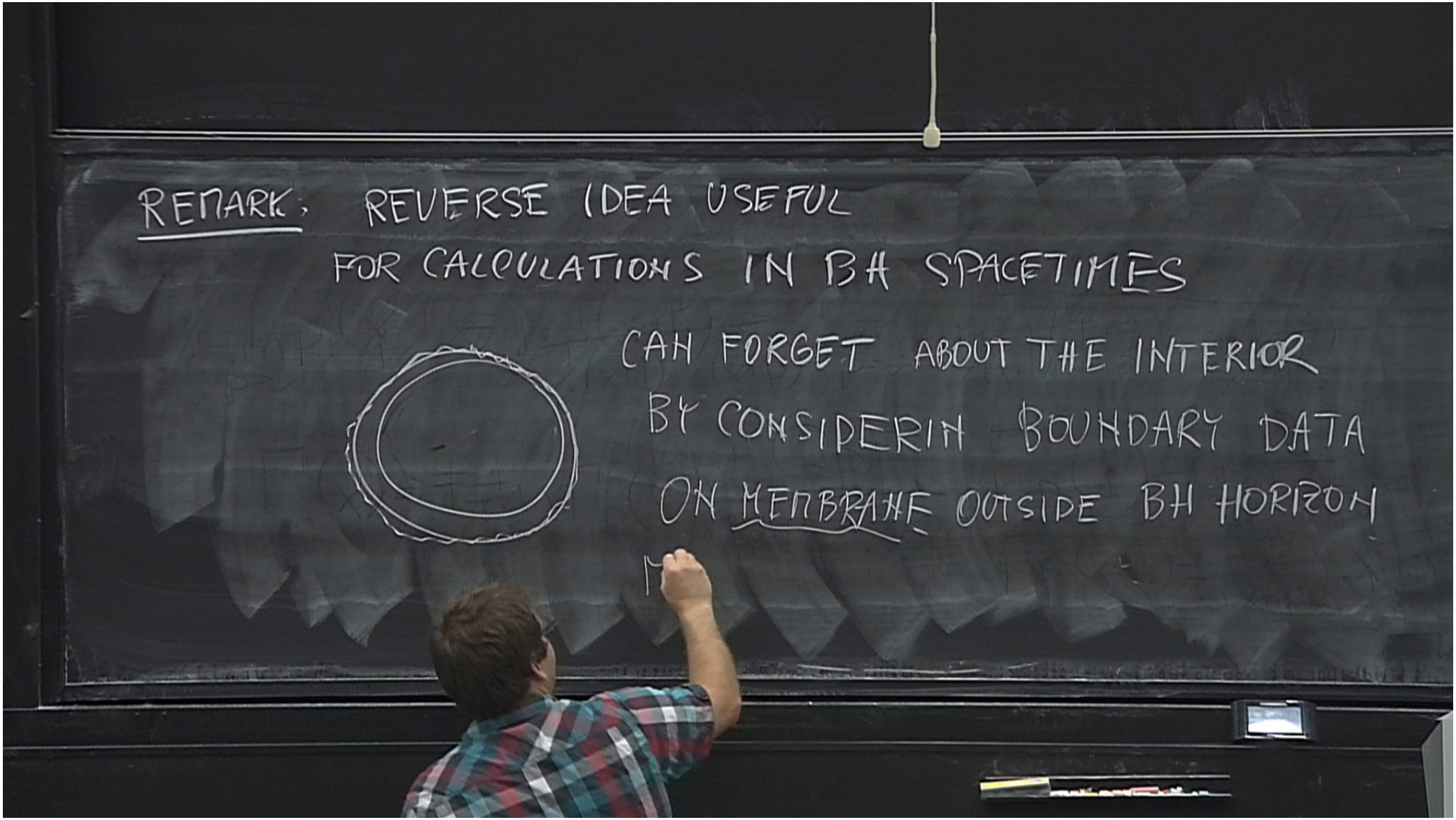
$$G = -\frac{1}{4\pi}\frac{1}{|\vec{x}-\vec{a}|} + \frac{1}{4\pi}\frac{1}{|\vec{x}+\vec{a}|}$$

$z>0$ $\quad \Delta G = \delta(\vec{x}-\vec{a})$

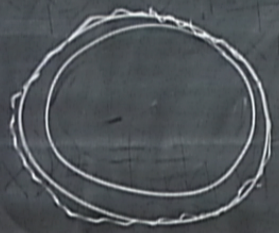
$G|_{z=0} = 0$

REMARK: REVERSE IDEA USEFUL FOR CALCULATIONS IN BH SPACETIMES

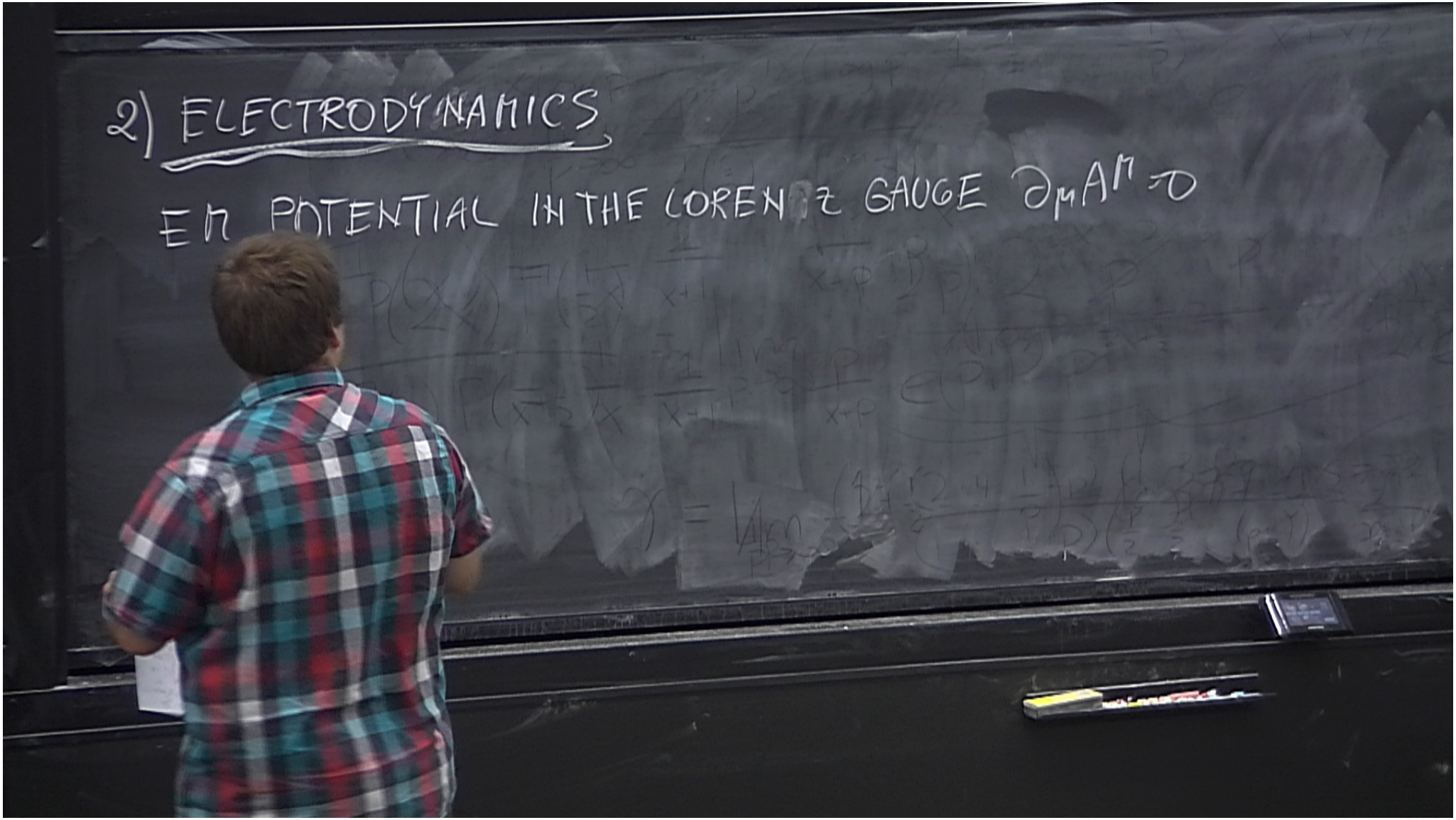


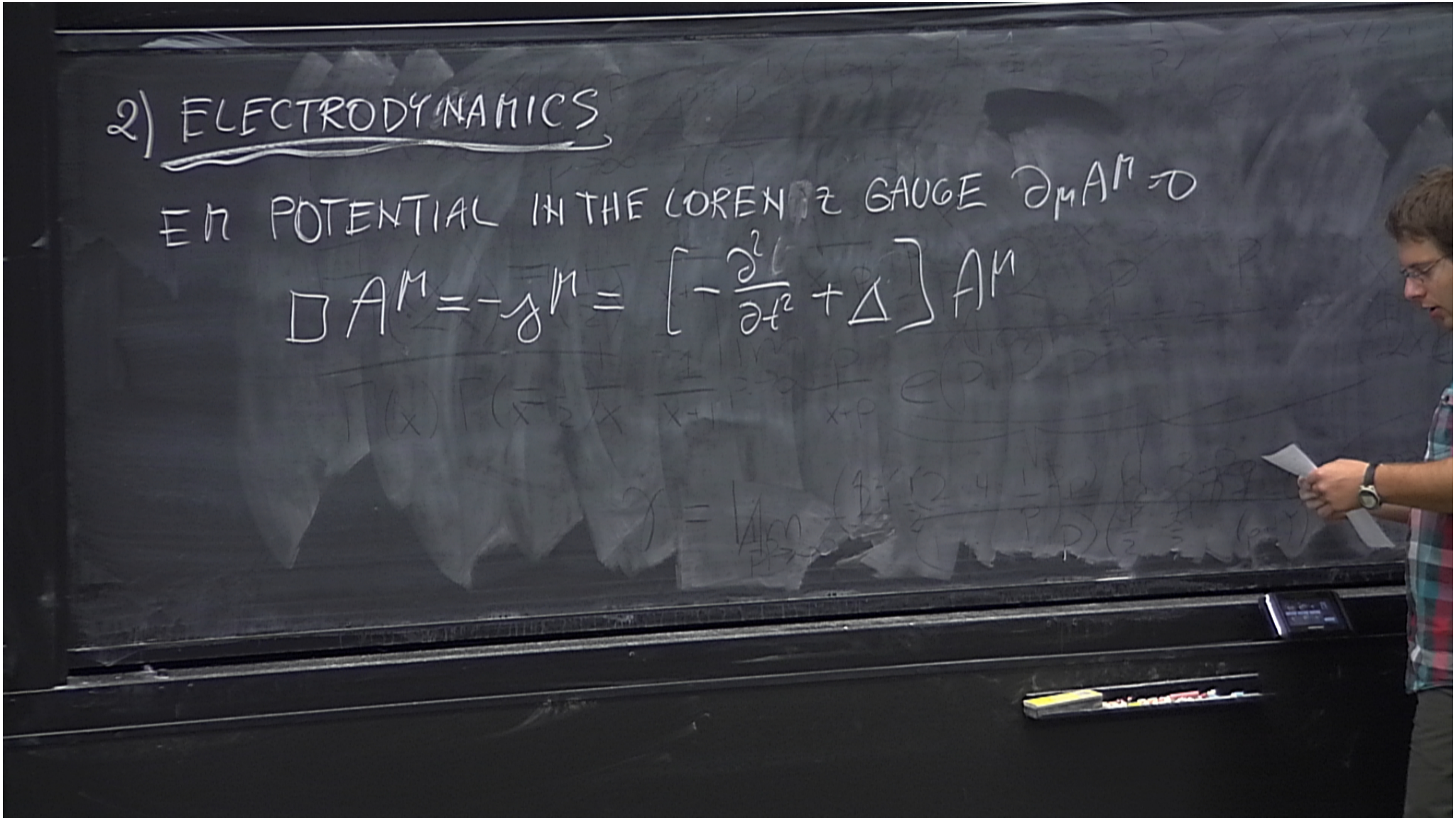

REMARK: REVERSE IDEA USEFUL
FOR CALCULATIONS IN BH SPACETIMES
CAN FORGET ABOUT THE INTERIOR
BY CONSIDERIN BOUNDARY DATA
ON MEMBRANE OUTSIDE BH HORIZON

REVERSE IDEA USEFUL
FOR CALCULATIONS IN BH SPACETIMES



CAN FORGET ABOUT THE INTERIOR
BY CONSIDERING BOUNDARY DATA
ON MEMBRANE OUTSIDE BH HORIZON
MEMBRANE PARADIGM → PHYS. PROP. (VISCOSITY, CONDUCTIVE $\frac{1}{4\pi}$, CHARGE DENSITY.


2) ELECTRODYNAMICS
EM POTENTIAL IN THE LORENTZ GAUGE $\partial_\mu A^\mu = 0$


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EM POTENTIAL IN THE LORENTZ GAUGE $\partial_\mu A^\mu = 0$
$\Box A^\mu = -j^\mu = \left[-\frac{\partial^2}{\partial t^2} + \Delta\right] A^\mu$

2) ELECTRODYNAMICS

EM POTENTIAL IN THE LORENTZ GAUGE $\partial_\mu A^\mu = 0$

$$\Box A^\mu = -j^\mu = \left[-\frac{\partial^2}{\partial t^2} + \Delta\right] A^\mu$$

- $\Box G(R,T) = -\delta(R)\delta(T)$

$$\frac{1}{(2\pi)^4}$$

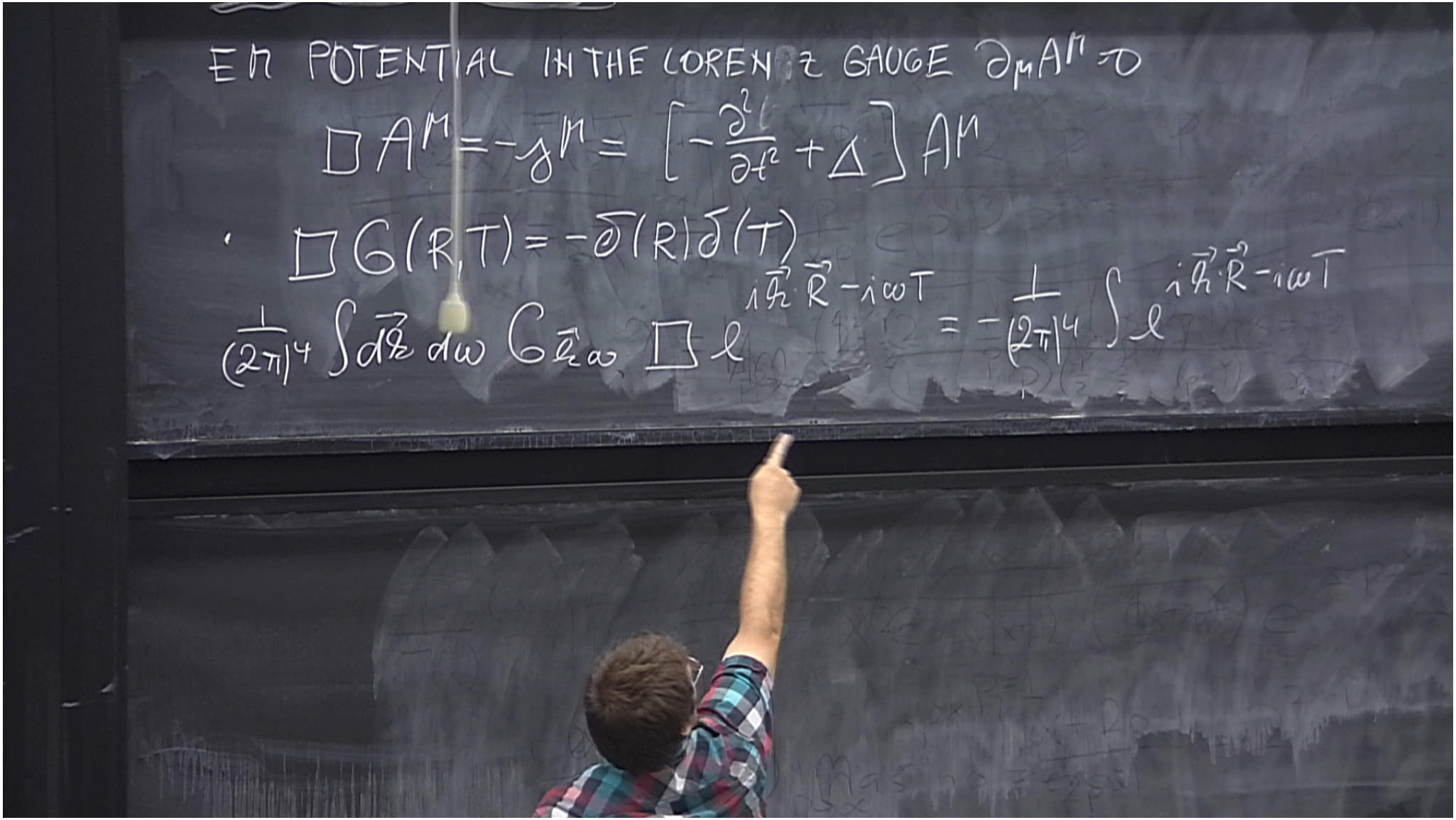
2) ELECTRODYNAMICS

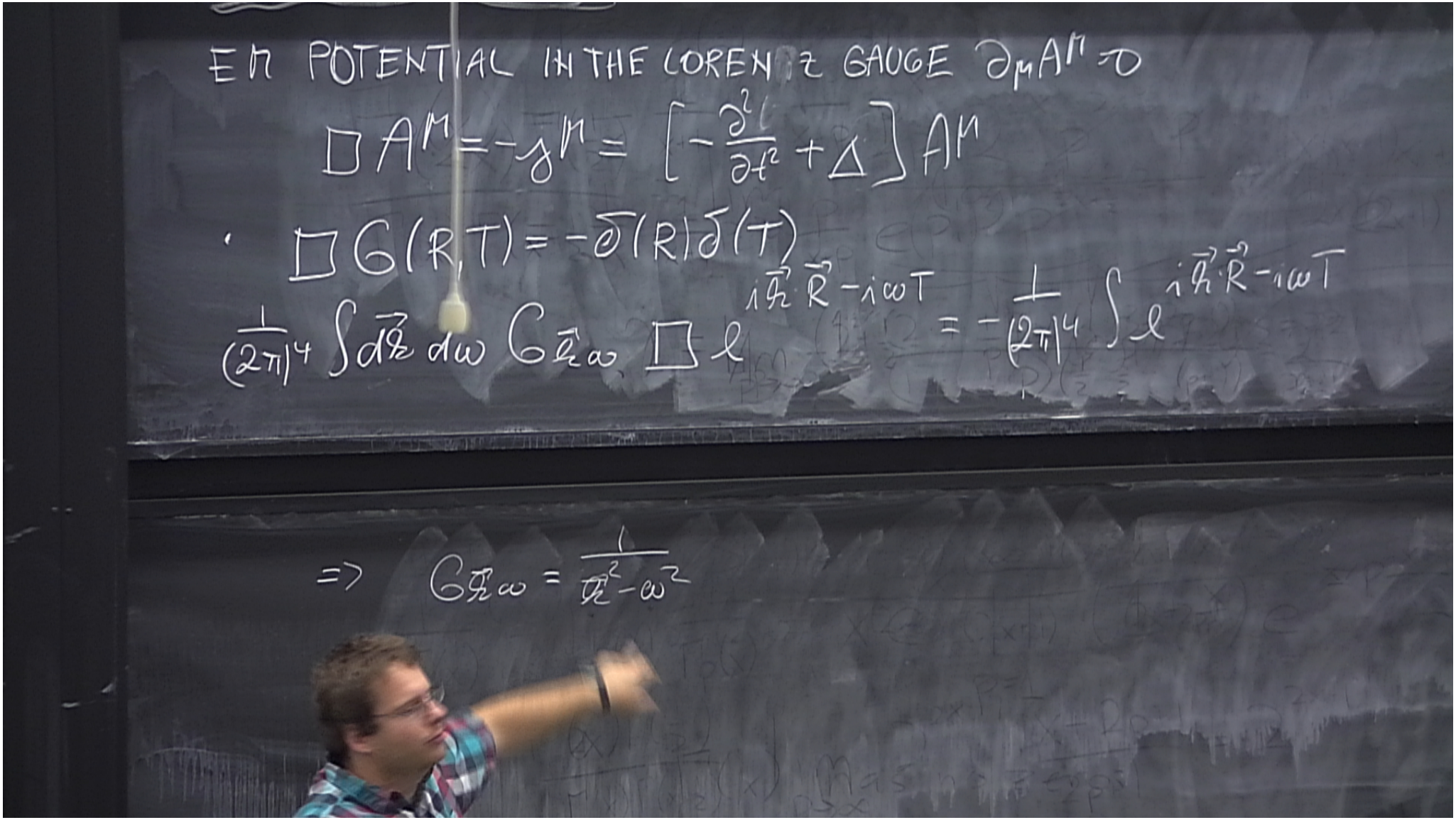
EM POTENTIAL IN THE LORENTZ GAUGE $\partial_\mu A^\mu = 0$

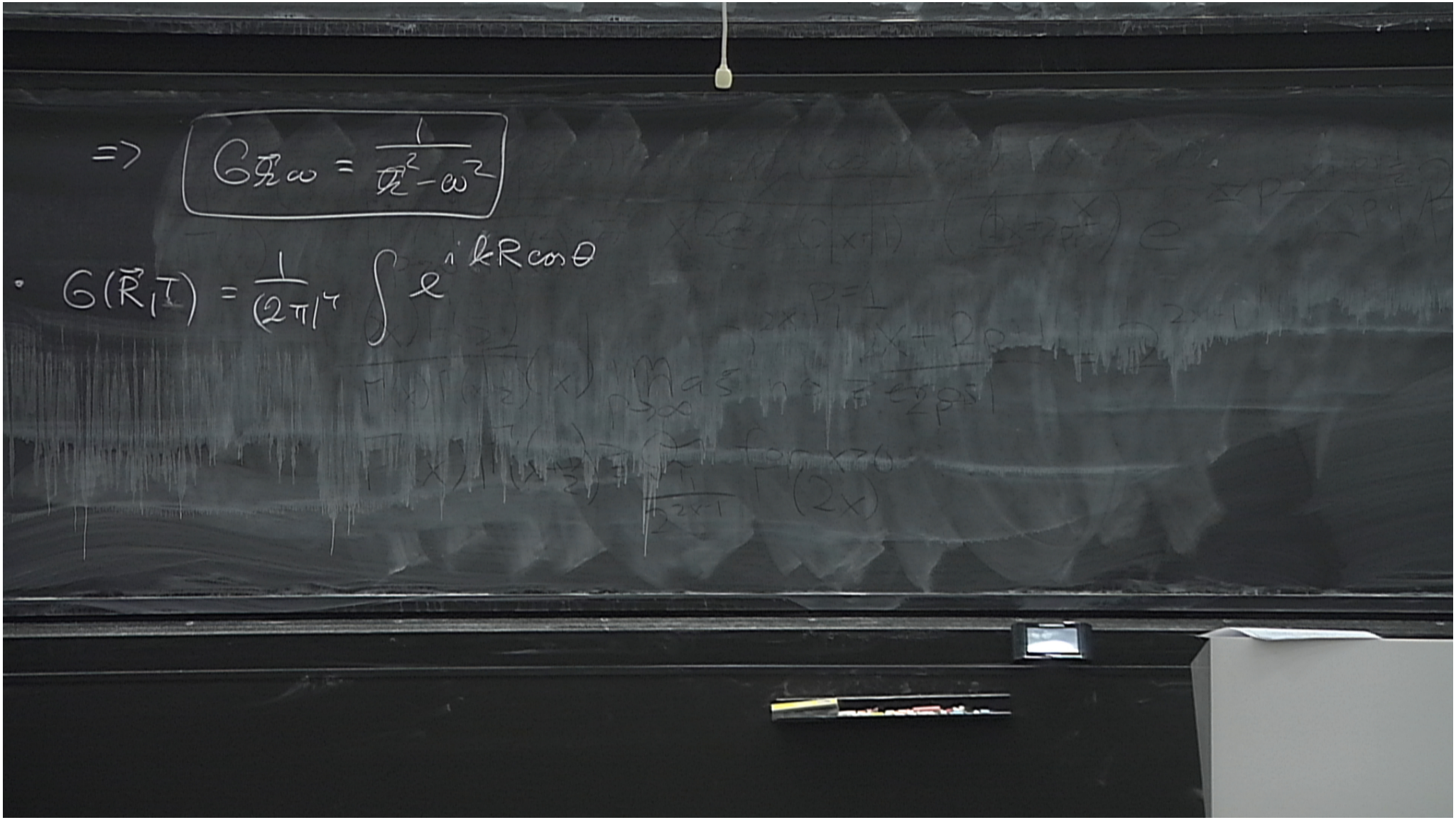
$$\Box A^\mu = -j^\mu = \left[-\frac{\partial^2}{\partial t^2} + \Delta\right] A^\mu$$

- $\Box G(R,T) = -\delta(R)\delta(T)$

$$\frac{1}{(2\pi)^4}\int d\vec{k}\, d\omega\, G_{\vec{k}\omega}\, \Box\, e^{i\vec{k}\cdot\vec{R} - i\omega T} =$$


EM POTENTIAL IN THE LORENTZ GAUGE $\partial_\mu A^\mu = 0$
$\Box A^\mu = -j^\mu = \left[-\frac{\partial^2}{\partial t^2} + \Delta\right] A^\mu$
$\Box G(R,T) = -\delta(R)\delta(T)$
$\frac{1}{(2\pi)^4}\int d\vec{k}\, d\omega\, G_{\vec{k}\omega}\, \Box\, e^{i\vec{k}\cdot\vec{R} - i\omega T} = -\frac{1}{(2\pi)^4}\int e^{i\vec{k}\cdot\vec{R} - i\omega T}$


EM POTENTIAL IN THE LORENTZ GAUGE $\partial_\mu A^\mu = 0$
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$\Rightarrow \quad G_{\vec{k}\omega} = \frac{1}{\vec{k}^2 - \omega^2}$


$\Rightarrow \boxed{G_{\vec{k}}\omega = \frac{1}{k^2 - \omega^2}}$
$G(\vec{R},T) = \frac{1}{(2\pi)^4} \int e^{ikR\cos\theta}$

EM POTENTIAL IN THE LORENTZ GAUGE $\partial_\mu A^\mu = 0$

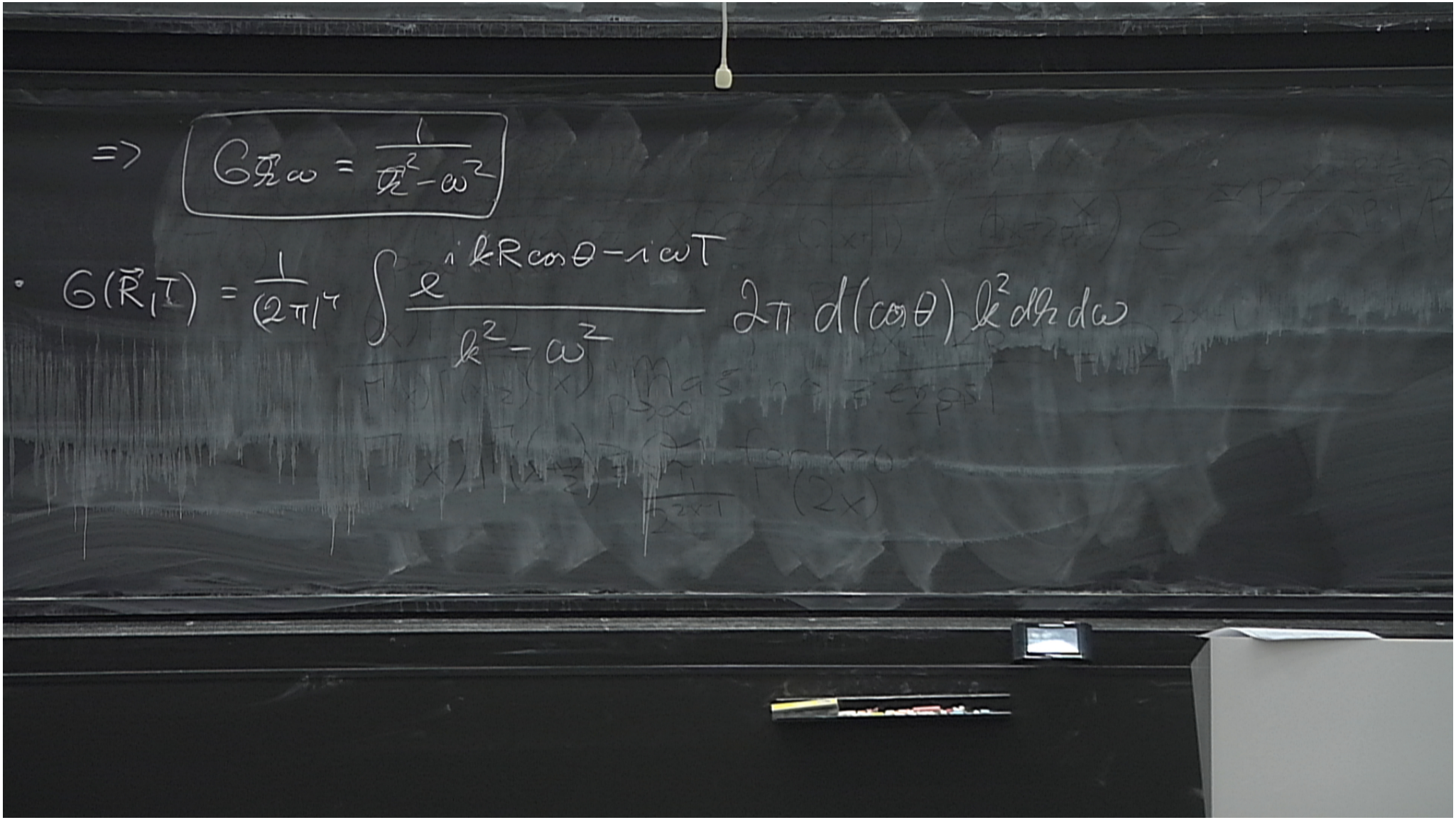
$$\Box A^\mu = -j^\mu = \left[-\frac{\partial^2}{\partial t^2} + \Delta\right] A^\mu$$

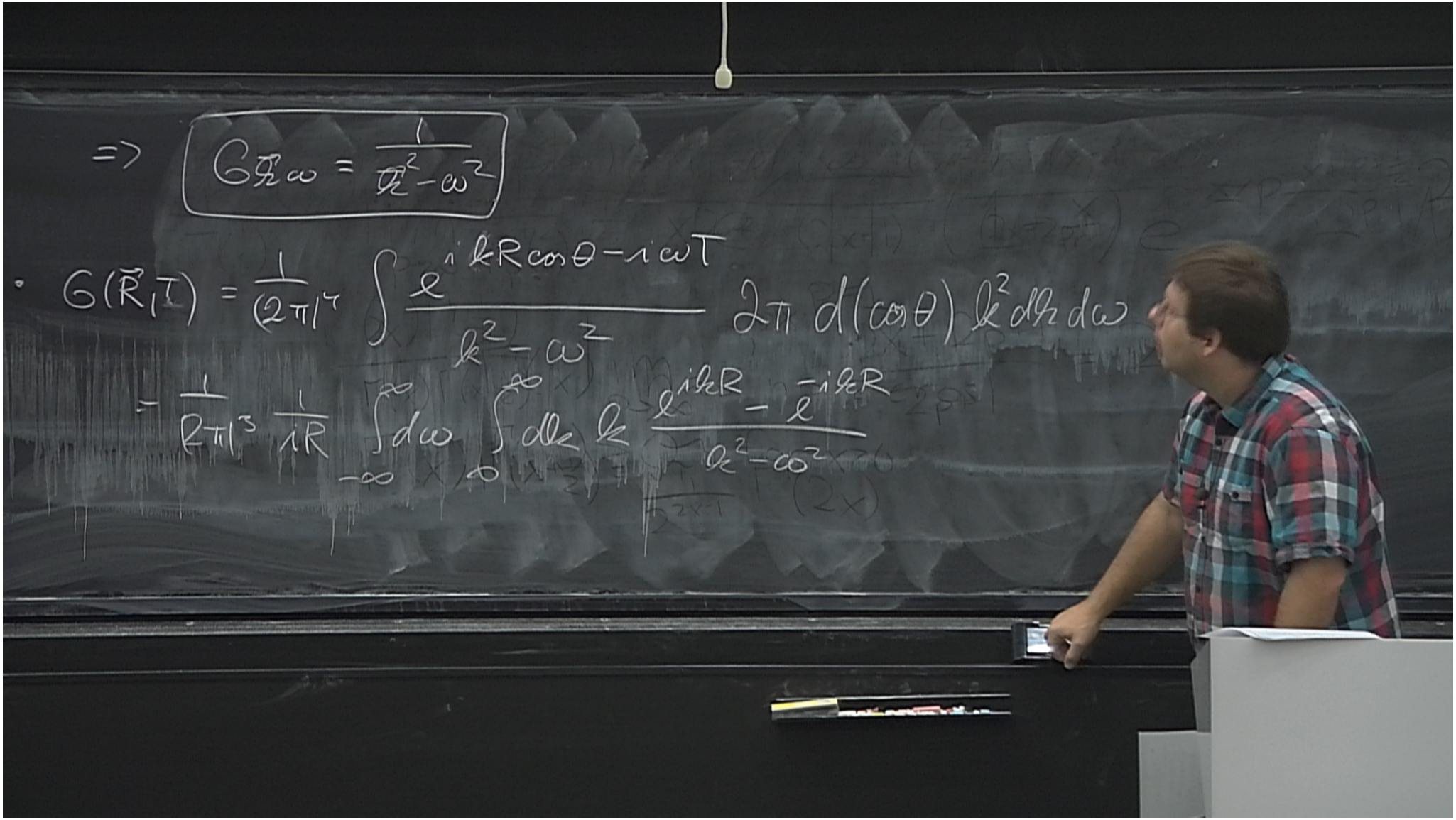
- $\Box G(R,T) = -\delta(R)\delta(T)$

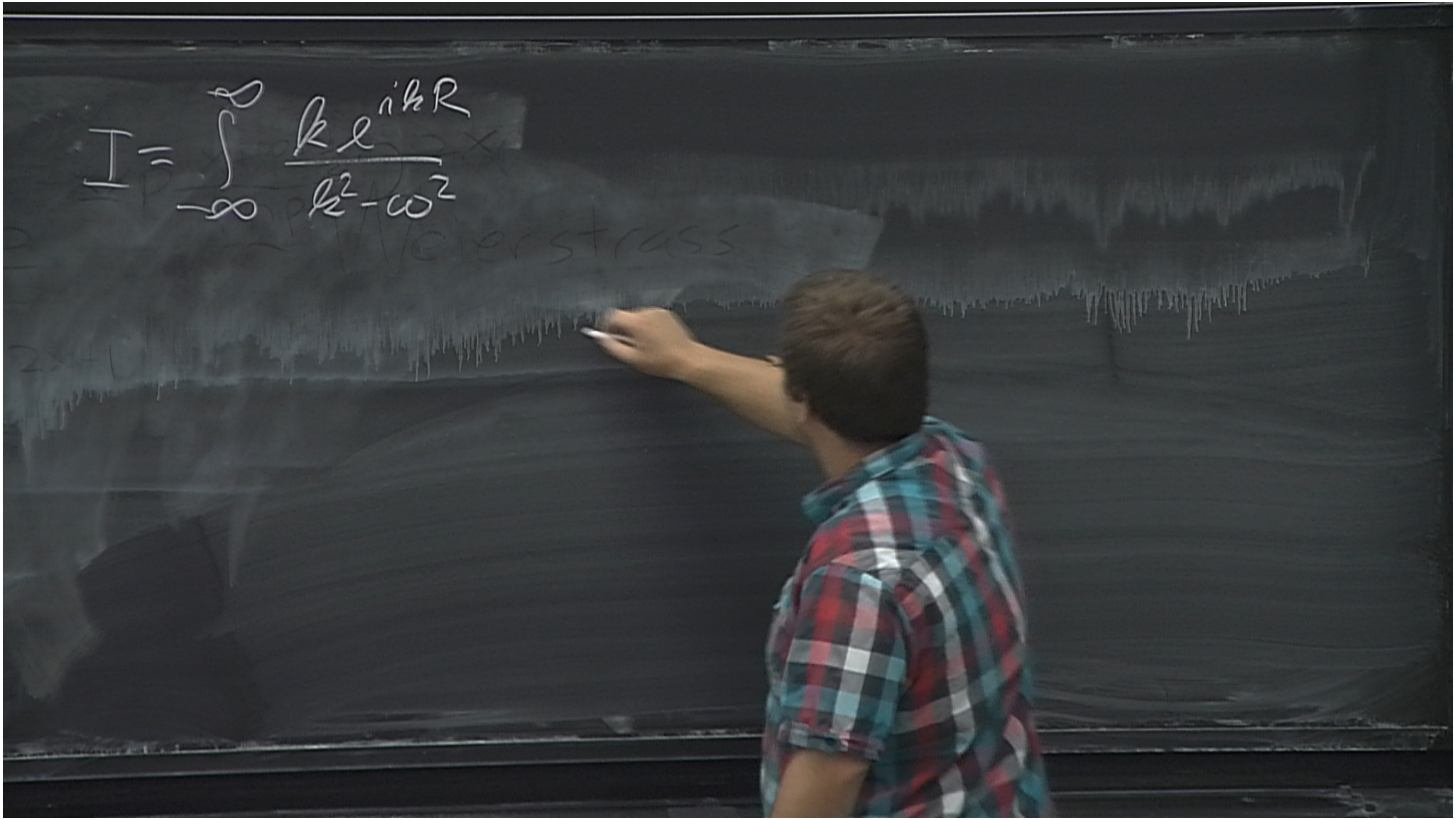
$$\frac{1}{(2\pi)^4}\int d\vec{k}\, d\omega\, G_{\vec{k}\omega}\, \Box\, e^{i\vec{k}\cdot\vec{R} - i\omega T} = -\frac{1}{(2\pi)^4}\int e^{i\vec{k}\cdot\vec{R} - i\omega T}$$

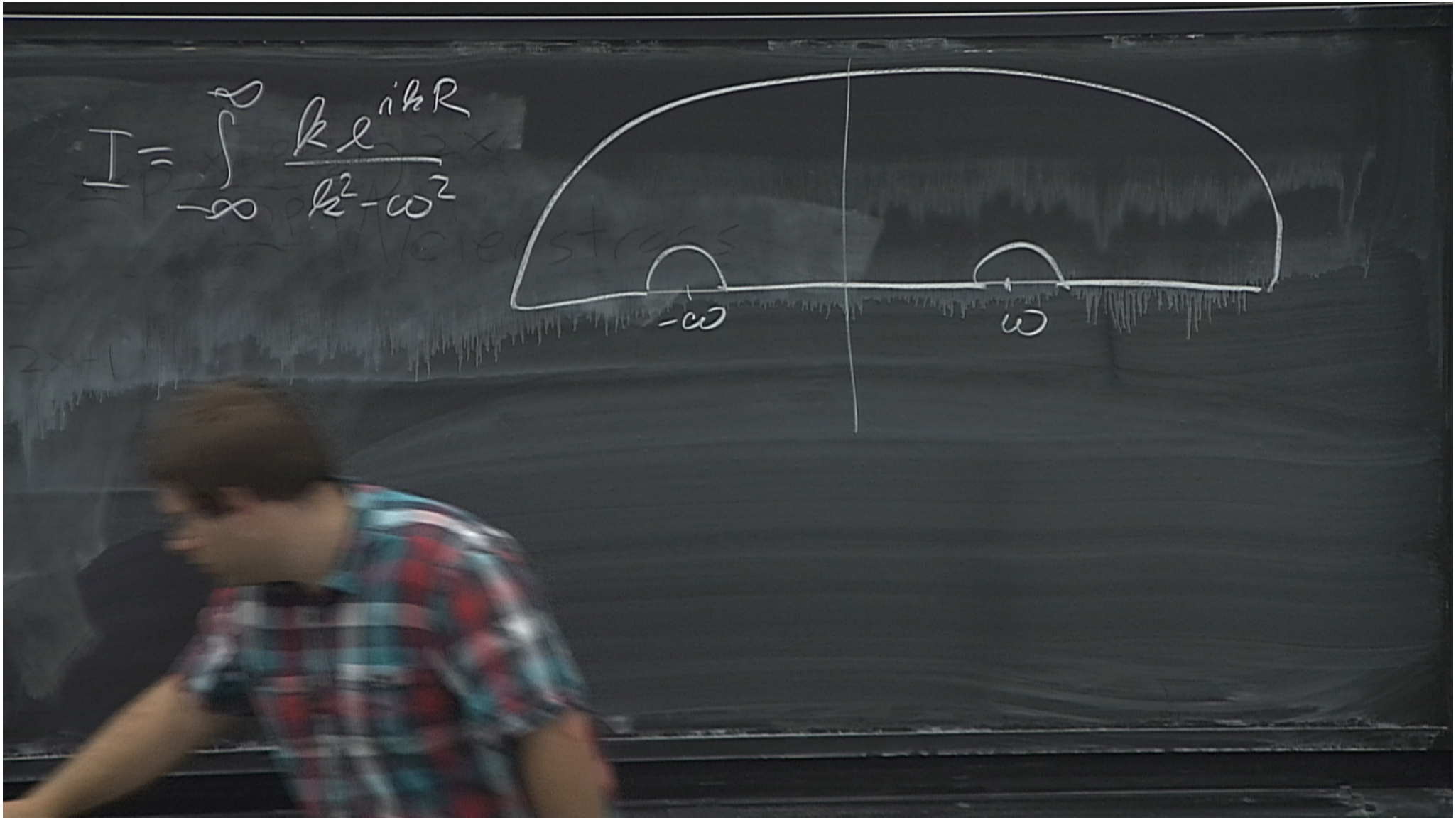
$$\Rightarrow \boxed{G_{\vec{k}\omega} = \frac{1}{\vec{k}^2 - \omega^2}}$$

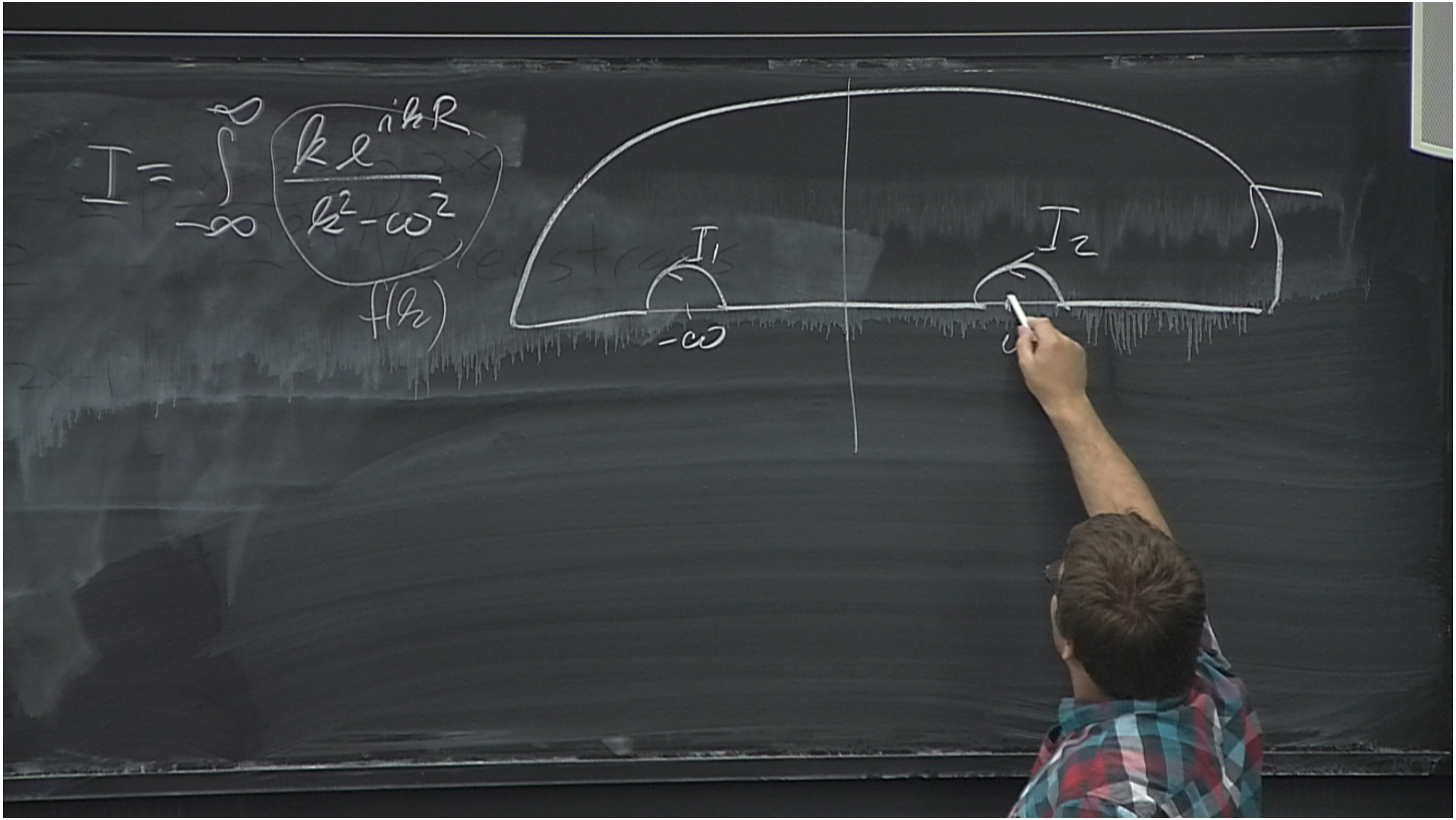
- $$G(\vec{R},T) = \frac{1}{(2\pi)^4}\int \frac{e^{ikR\cos\theta - i\omega T}}{k^2 - \omega^2}\, 2\pi\, d(\cos\theta)\, k^2\, dk$$

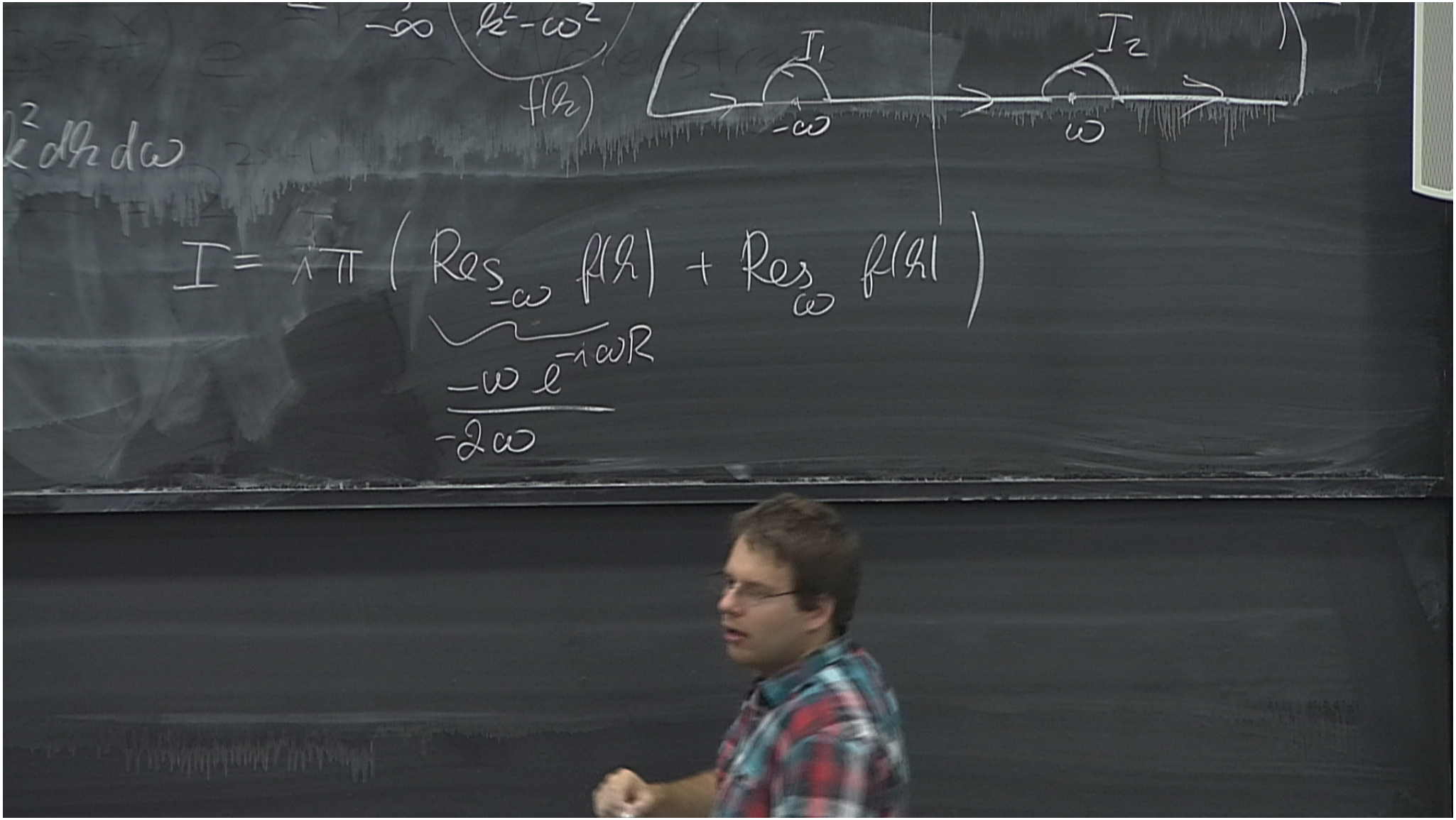

$\Rightarrow \boxed{G(\vec{k},\omega) = \frac{1}{k^2-\omega^2}}$
$G(\vec{R},T) = \frac{1}{(2\pi)^4} \int \frac{e^{ikR\cos\theta - i\omega T}}{k^2-\omega^2} 2\pi\, d(\cos\theta)\, k^2\, dk\, d\omega$


$\Rightarrow \boxed{G_{k\omega} = \frac{1}{k^2 - \omega^2}}$
$G(\vec{R}, T) = \frac{1}{(2\pi)^4} \int \frac{e^{ikR\cos\theta - i\omega T}}{k^2 - \omega^2} 2\pi\, d(\cos\theta)\, k^2\, dk\, d\omega$
$= \frac{1}{R\,(2\pi)^3} \frac{1}{iR} \int_{-\infty}^{\infty} d\omega \int_{0}^{\infty} dk\, k \frac{e^{ikR} - e^{-ikR}}{k^2 - \omega^2}$

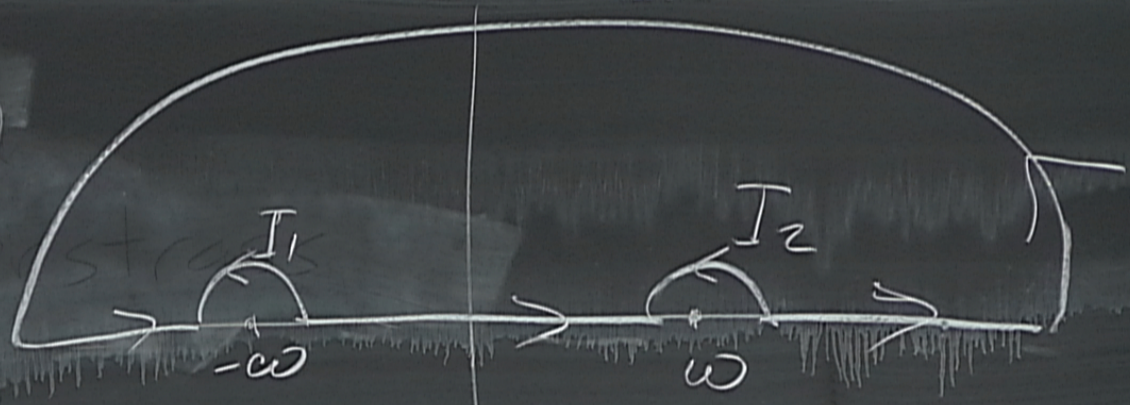

$I = \int_{-\infty}^{\infty} \frac{k e^{ikR}}{k^2 - \omega^2}$


$I = \int_{-\infty}^{\infty} \frac{k e^{ikR}}{k^2 - \omega^2}$
$-\omega$
ω

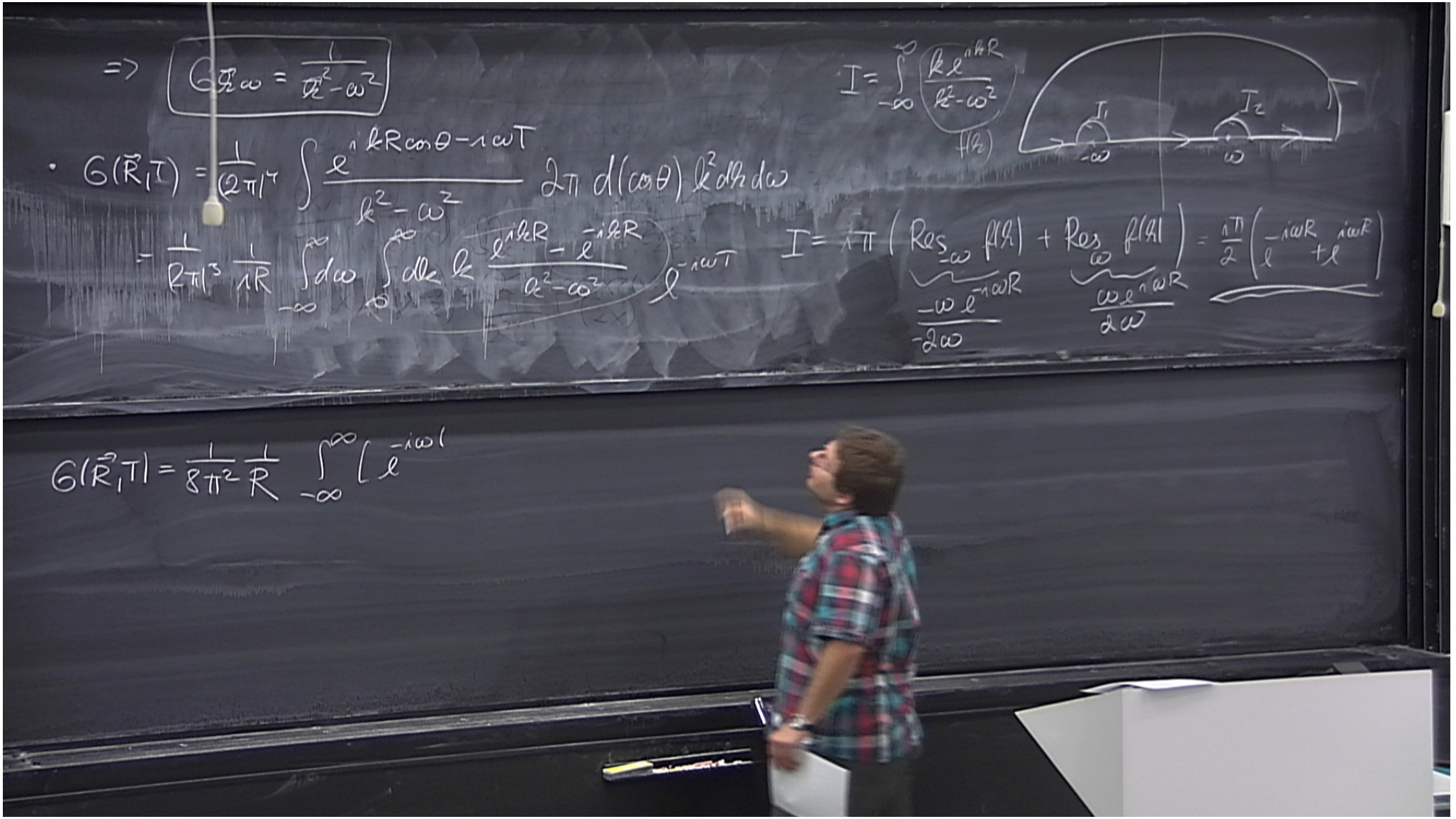

$I = \int_{-\infty}^{\infty} \frac{k e^{ikR}}{k^2 - \omega^2} dk$
$f(k)$
I_1
I_2
$-\omega$


$k^2 - \omega^2$
$f(k)$
I_1
I_2
$-\omega$
ω
$k^2 dk\, d\omega$
$I = \lambda\pi \left(\mathrm{Res}_{-\omega} f(k) + \mathrm{Res}_{\omega} f(k) \right)$
$\frac{-\omega e^{-i\omega R}}{-2\omega}$

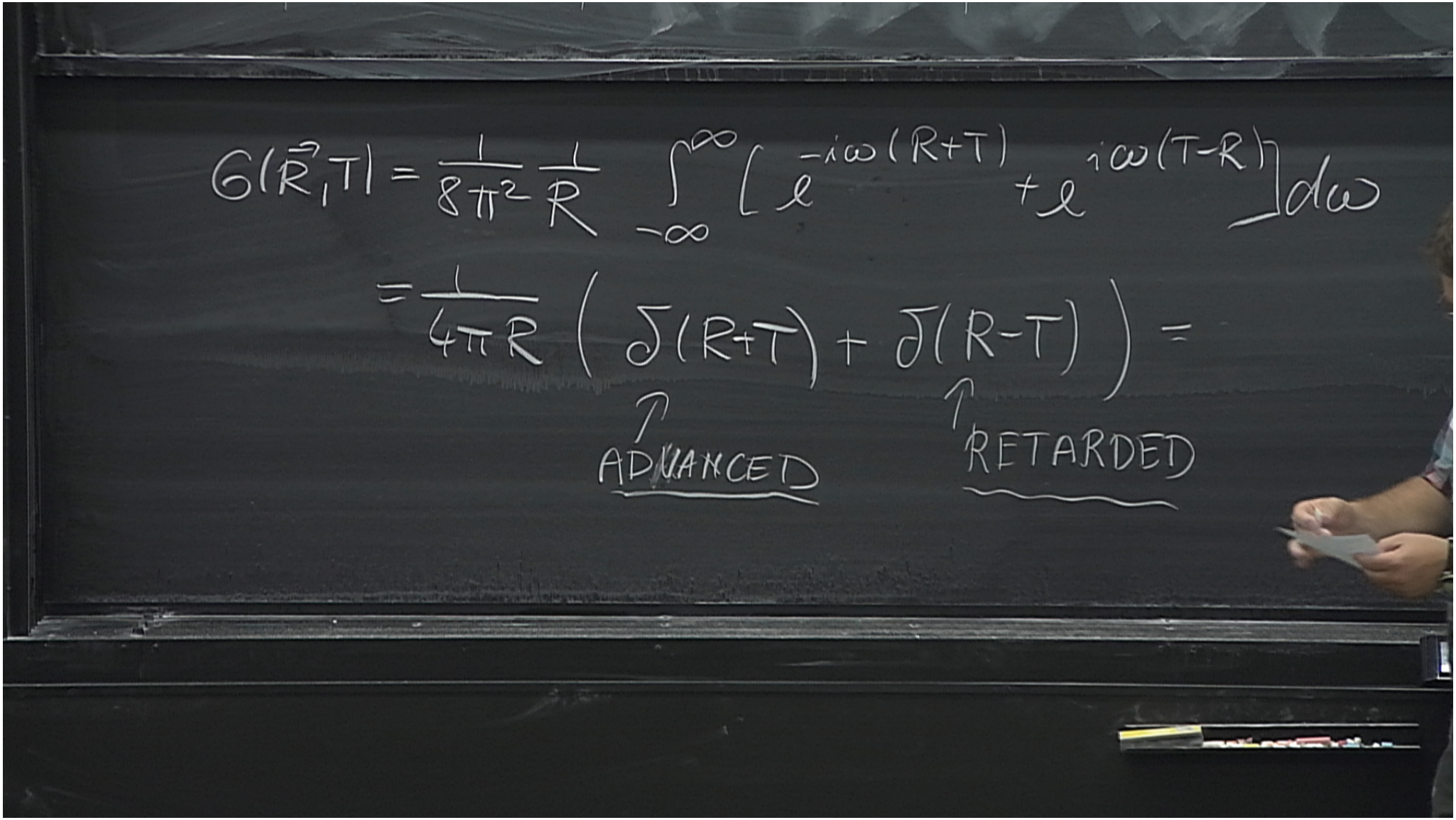
$$I = \int_{-\infty}^{\infty} \underbrace{\frac{k e^{ikR}}{k^2 - \omega^2}}_{f(k)}$$



$$I = \lambda\pi \left(\underbrace{\operatorname{Res}_{-\omega} f(k)}_{\frac{-\omega e^{-i\omega R}}{-2\omega}} + \underbrace{\operatorname{Res}_{\omega} f(k)}_{\frac{\omega e^{i\omega R}}{2\omega}} \right) = \frac{i\pi}{2} \left(\right.$$


$\Rightarrow \boxed{G(\vec{k},\omega) = \frac{1}{k^2-\omega^2}}$
$G(\vec{R},T) = \frac{1}{(2\pi)^4}\int \frac{e^{ikR\cos\theta - i\omega T}}{k^2-\omega^2} 2\pi\, d(\cos\theta)\, k^2 dk\, d\omega$
$= \frac{1}{8\pi^3}\frac{1}{iR}\int_{-\infty}^{\infty} d\omega \int_0^{\infty} dk\, k \frac{e^{ikR} - e^{-ikR}}{\alpha^2-\omega^2} e^{-i\omega T}$
$I = \int_{-\infty}^{\infty} \frac{k e^{ikR}}{k^2-\omega^2}$
$I = i\pi\left(\underbrace{Res_{-\omega} f(k)}_{\frac{-\omega e^{-i\omega R}}{-2\omega}} + \underbrace{Res_{\omega} f(k)}_{\frac{\omega e^{i\omega R}}{2\omega}}\right) = \frac{i\pi}{2}\left(e^{-i\omega R} + e^{i\omega R}\right)$
I_1
I_2
$-\omega$
ω
$G(\vec{R},T) = \frac{1}{8\pi^2}\frac{1}{R}\int_{-\infty}^{\infty}\left[e^{-i\omega}\right.$

$$G(\vec{R},T) = \frac{1}{8\pi^2}\frac{1}{R}\int_{-\infty}^{\infty}\left[e^{-i\omega(R+T)} + e^{i\omega(T-R)}\right]d\omega$$


$G(\vec{R},T) = \frac{1}{8\pi^2}\frac{1}{R}\int_{-\infty}^{\infty}\left[e^{-i\omega(R+T)} + e^{i\omega(T-R)}\right]d\omega$
$= \frac{1}{4\pi R}\left(\delta(R+T) + \delta(R-T)\right) =$
ADVANCED
RETARDED

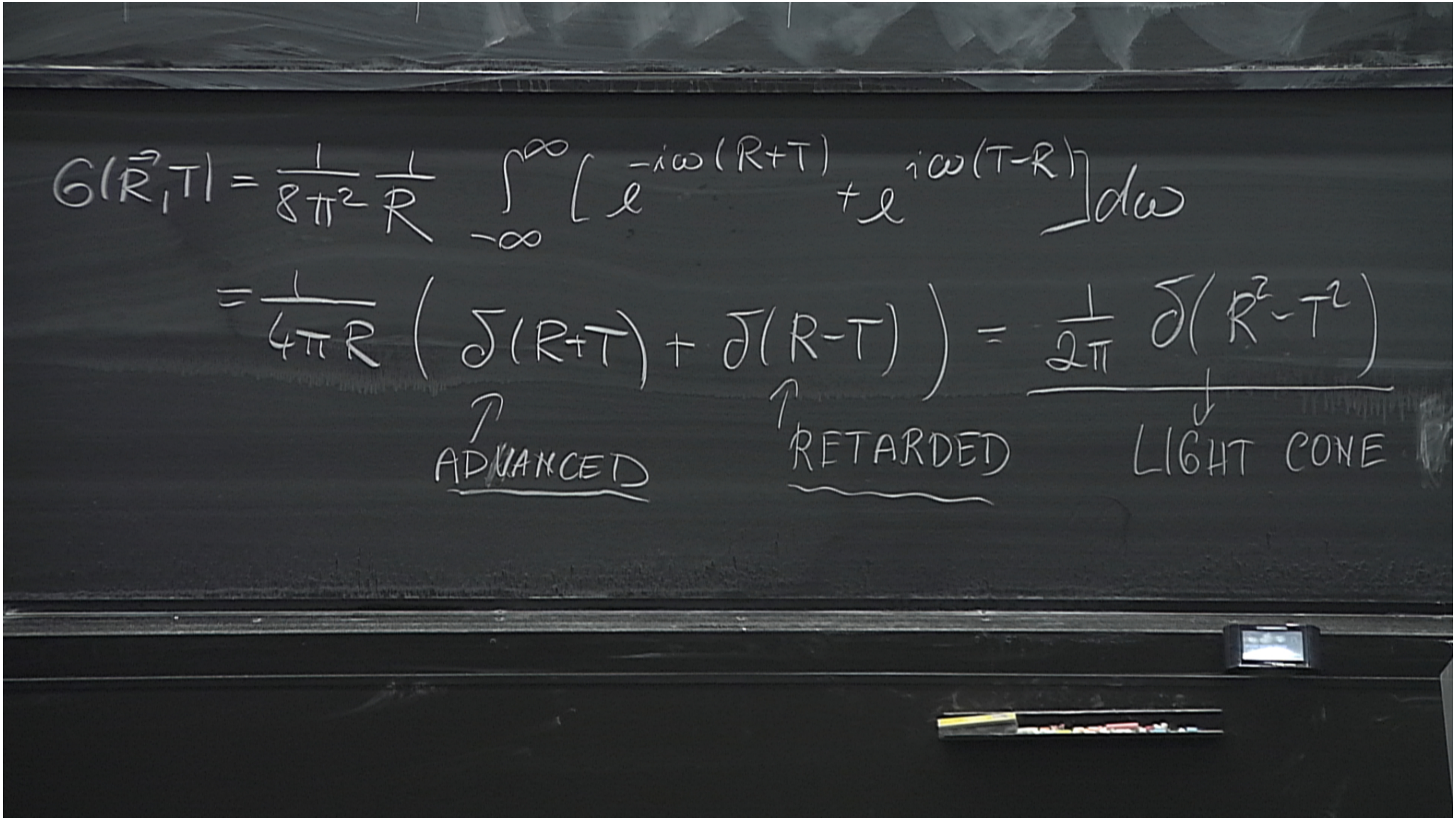
$$G(\vec{R},T) = \frac{1}{8\pi^2}\frac{1}{R}\int_{-\infty}^{\infty}\left[e^{-i\omega(R+T)} + e^{i\omega(T-R)}\right]d\omega$$

$$= \frac{1}{4\pi R}\left(\delta(R+T) + \delta(R-T)\right) = \frac{1}{2\pi}\delta(R^2-T^2)$$

ADVANCED

RETARDED



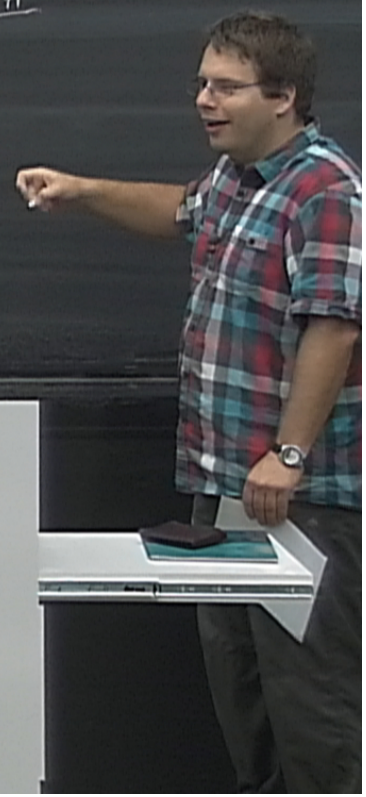

$G(\vec{R},T) = \frac{1}{8\pi^2}\frac{1}{R}\int_{-\infty}^{\infty}\left[e^{-i\omega(R+T)} + e^{i\omega(T-R)}\right]d\omega$
$= \frac{1}{4\pi R}\left(\delta(R+T) + \delta(R-T)\right) = \frac{1}{2\pi}\delta(R^2-T^2)$
ADVANCED
RETARDED
LIGHT CONE

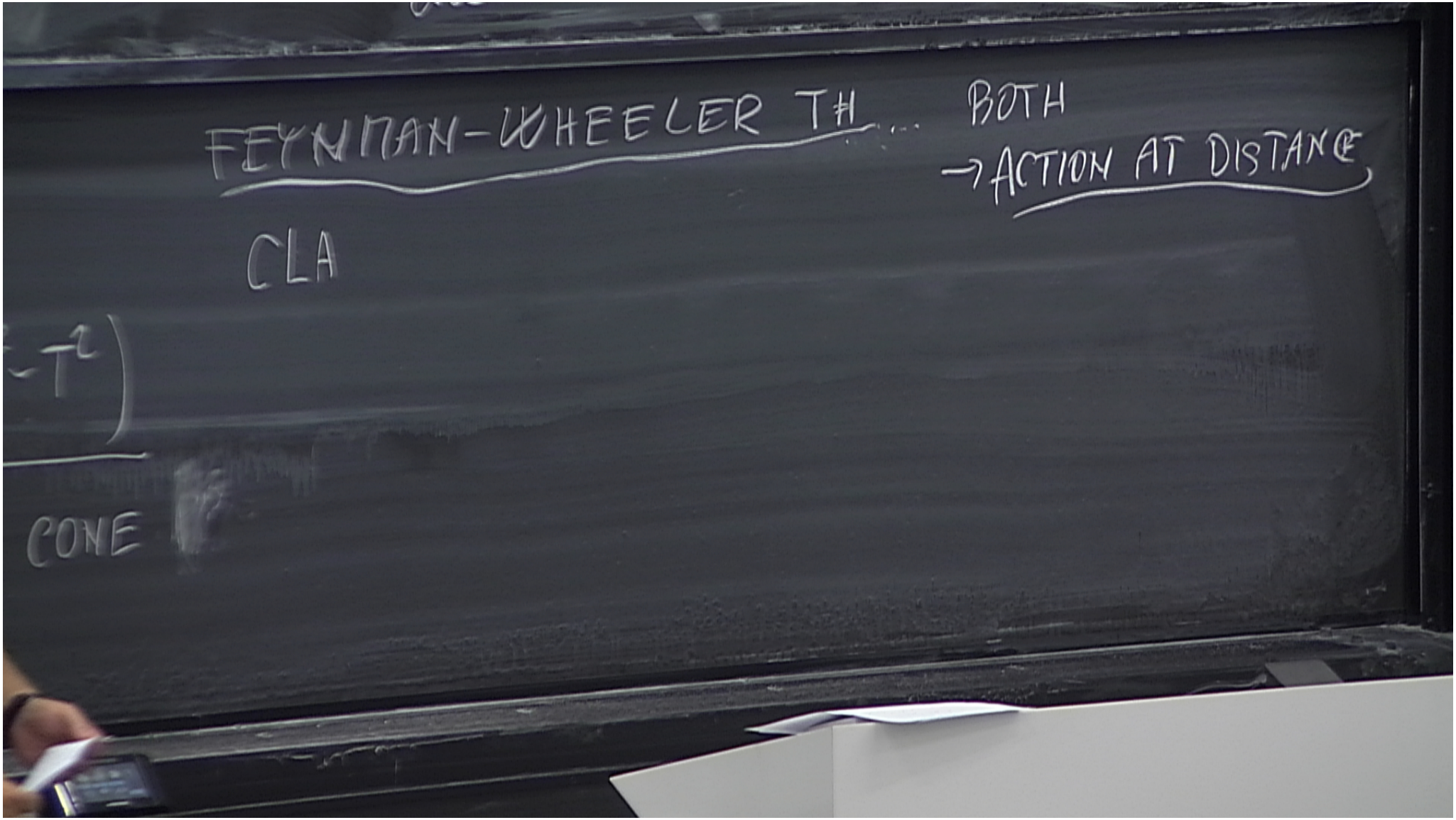
$$G(\vec{R},T) = \frac{1}{8\pi^2}\frac{1}{R}\int_{-\infty}^{\infty}\left[e^{-i\omega(R+T)} + e^{i\omega(T-R)}\right]d\omega$$

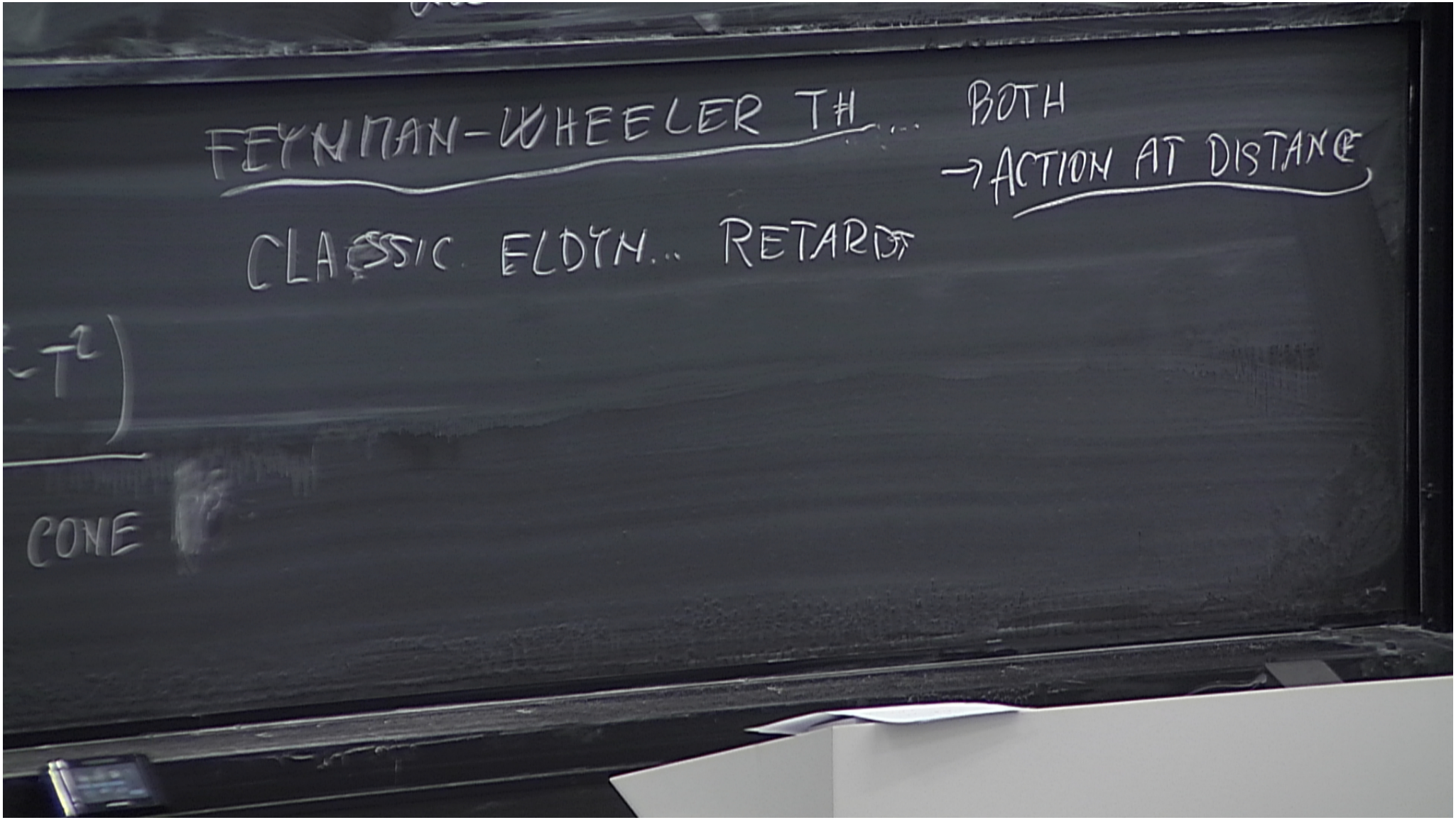
$$= \frac{1}{4\pi R}\left(\delta(R+T) + \delta(R-T)\right) = \frac{1}{2\pi}\delta(R^2-T^2)$$

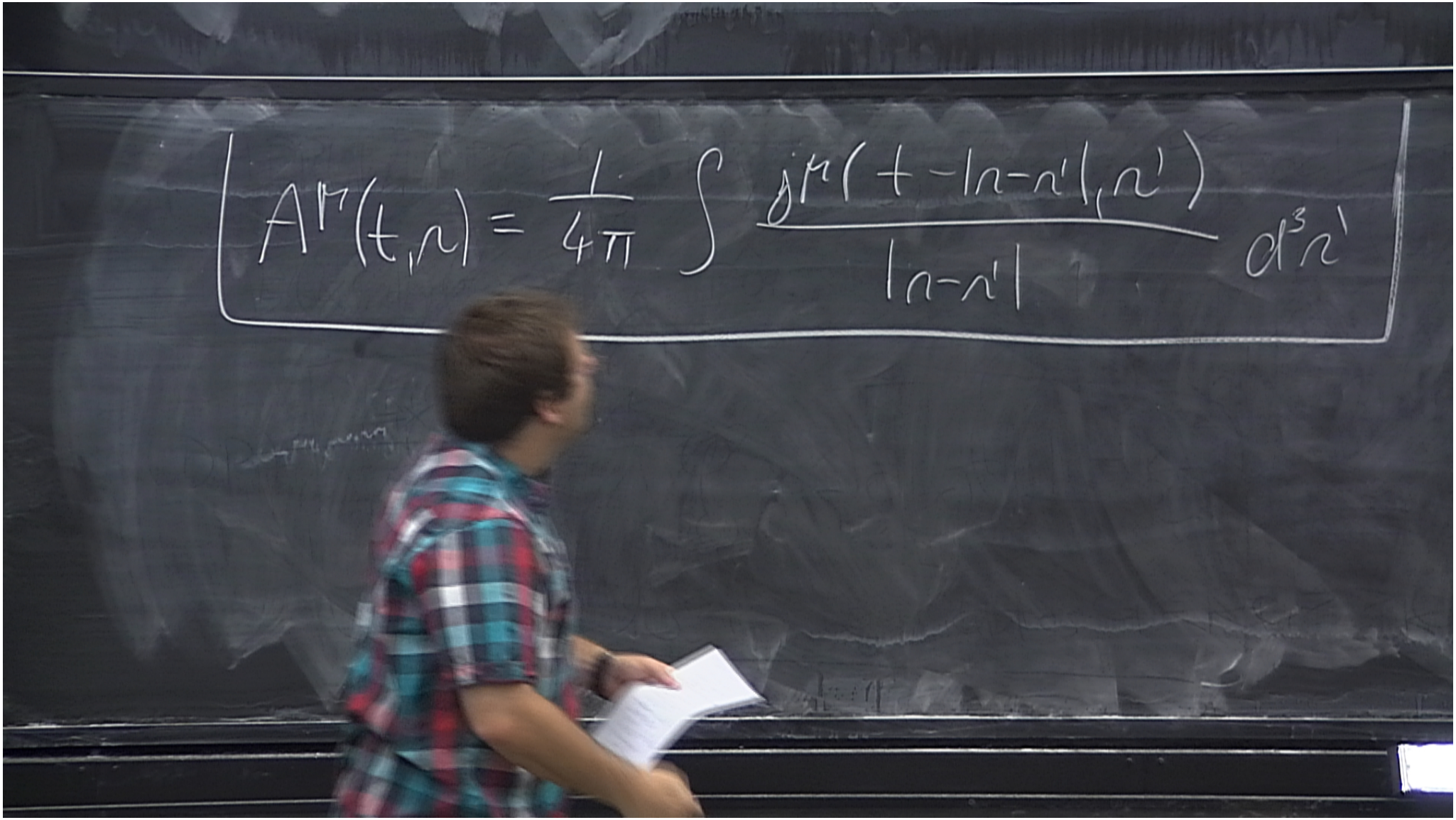
ADVANCED — RETARDED — LIGHT CONE

FEYNMAN-WHEELER TH




FEYNMAN-WHEELER TH...
BOTH
→ ACTION AT DISTANCE
CLA
CONE


FEYNMAN-WHEELER TH... BOTH
→ ACTION AT DISTANCE
CLASSIC ELDTH... RETARD
CONE


$A^\mu(t,\vec{r}) = \frac{1}{4\pi} \int \frac{j^\mu(t-|\vec{r}-\vec{r}'|,\vec{r}')}{|\vec{r}-\vec{r}'|} d^3\vec{r}'$

$$\int \left(\varphi \frac{\partial G_3}{\partial n} - G_3 \frac{\partial \varphi}{\partial n}\right) dS = \int \left(\varphi \cdot \delta(n-n_0) + \right.$$

$$A^\mu(t,n) = \frac{1}{4\pi} \int \frac{j^\mu(t-|n-n'|, n')}{|n-n'|} d^3n'$$

3) LIGHT CONE & GAUGE FIELDS

RULE:

$$\frac{\partial}{\partial r} = 0$$

$\chi = G_3$

$$\int \left(\varphi \frac{\partial G_3}{\partial n} - G_3 \frac{\partial \varphi}{\partial n}\right) dS = \int \left(\varphi \cdot \delta(n-n_0) + G_3 \varrho\right) dV = \varphi$$

$$A^\mu(t,n) = \frac{1}{4\pi} \int \frac{j^\mu(t-|n-n'|, n')}{|n-n'|} d^3n'$$

3) LIGHT CONE & GAUGE FIELDS

RULE: IN GAUGE THIS: # REAL DOF = # OF APPARENT

$\partial_n \chi = 0$

• APPLY THIS TO $\phi = \varphi$ ... $\Delta\varphi = -\rho$ - NO BC
$\chi = G_3$

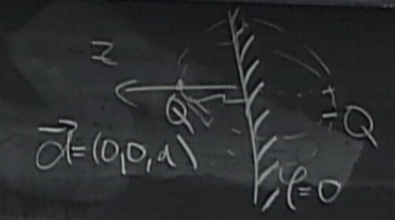
$$\int \left(\varphi \frac{\partial G_3}{\partial n} - G_3 \frac{\partial \varphi}{\partial n}\right) dS = \int \left(\varphi\, \delta(n-n_0) + G_3 \rho\right) dV = \varphi(n_0) + \int G_3 \rho$$

$$A^\mu(t,n) = \frac{1}{4\pi} \int \frac{j^\mu(t - |n-n'|, n')}{|n-n'|} d^3n'$$

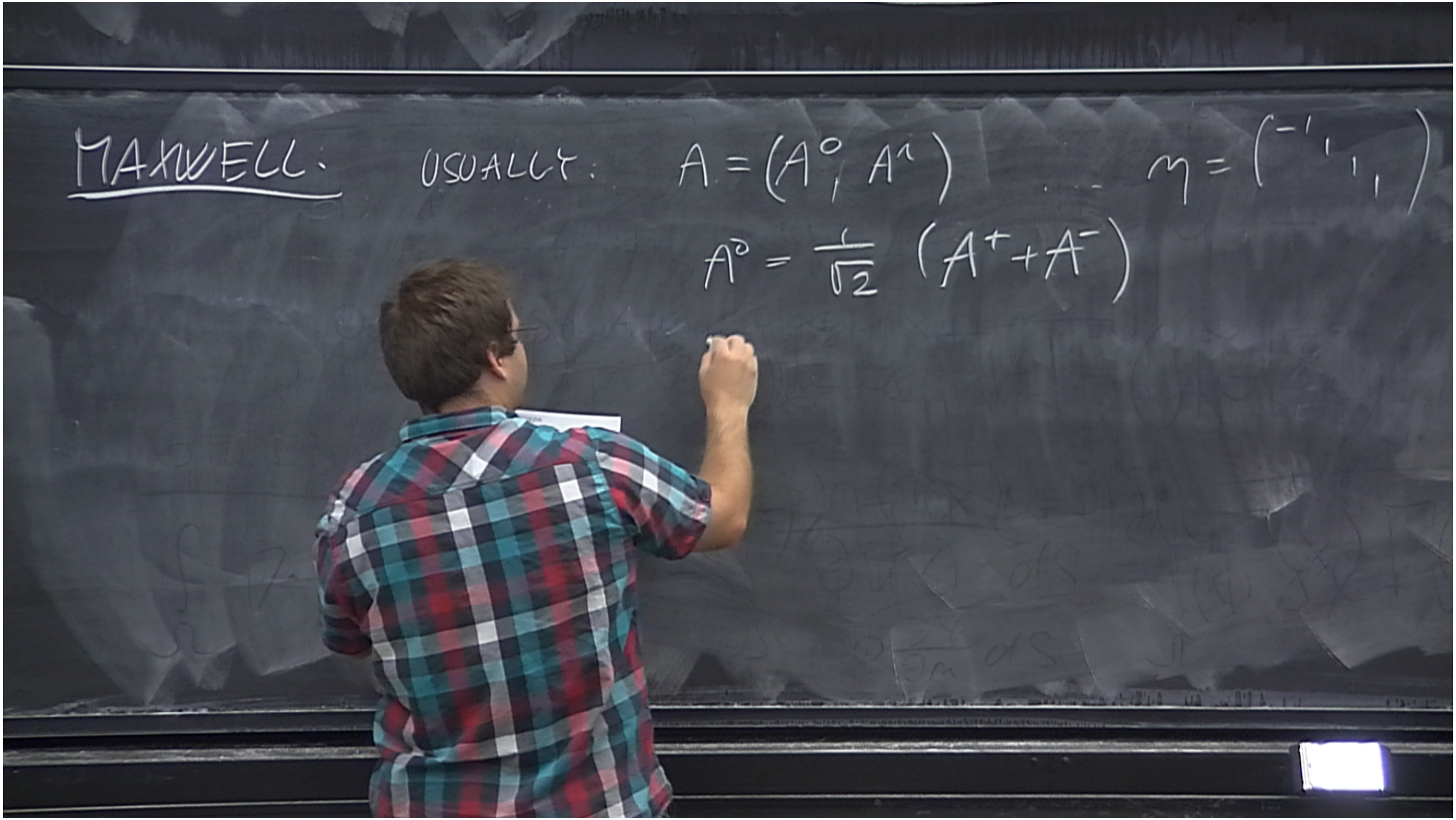
3) LIGHT CONE & GAUGE FIELDS

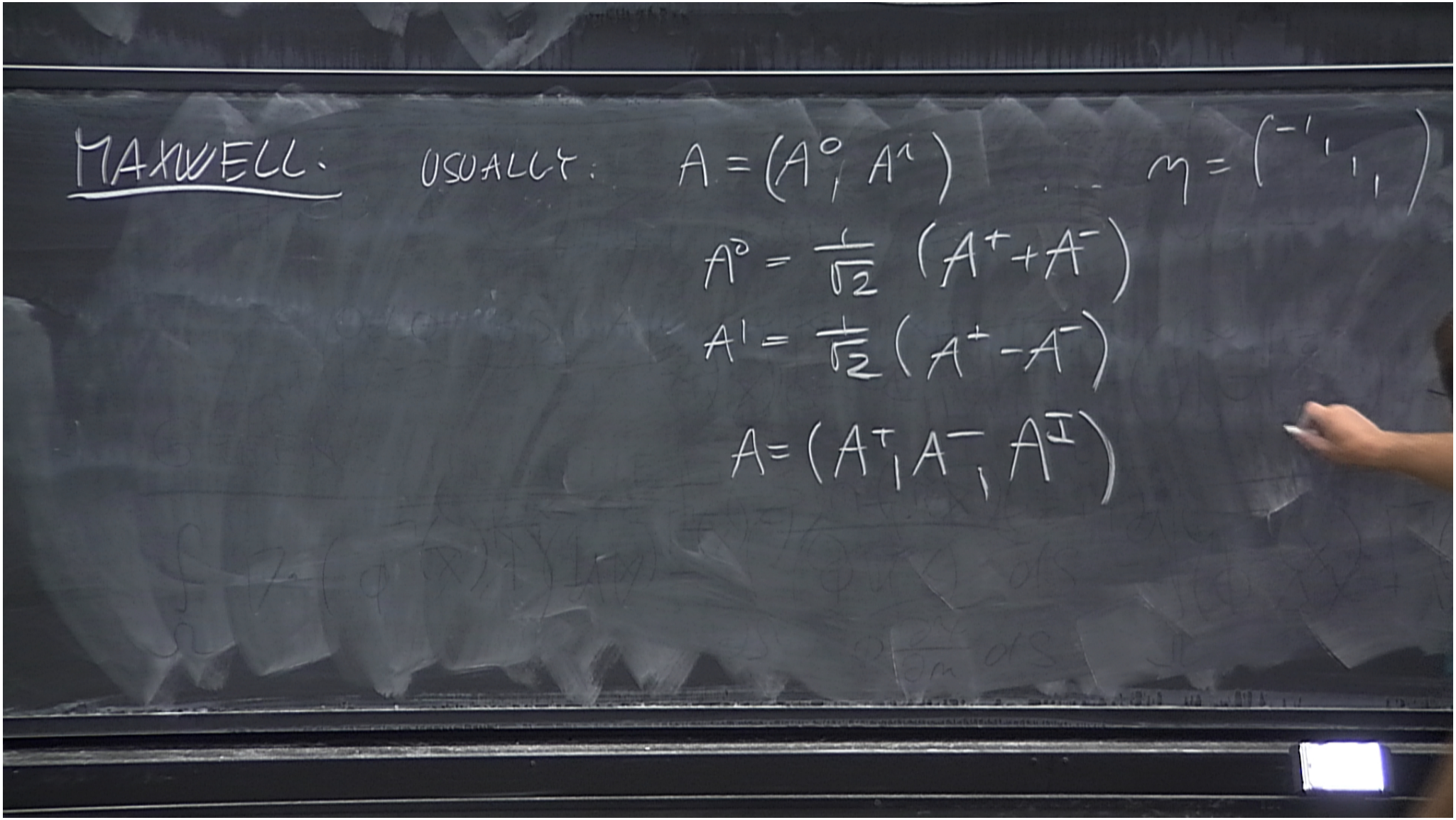
LE. IN GAUGE THIS: # REAL DOF = # OF APPARENT DOF - 2x DOF OF GAUGE FUNCTION

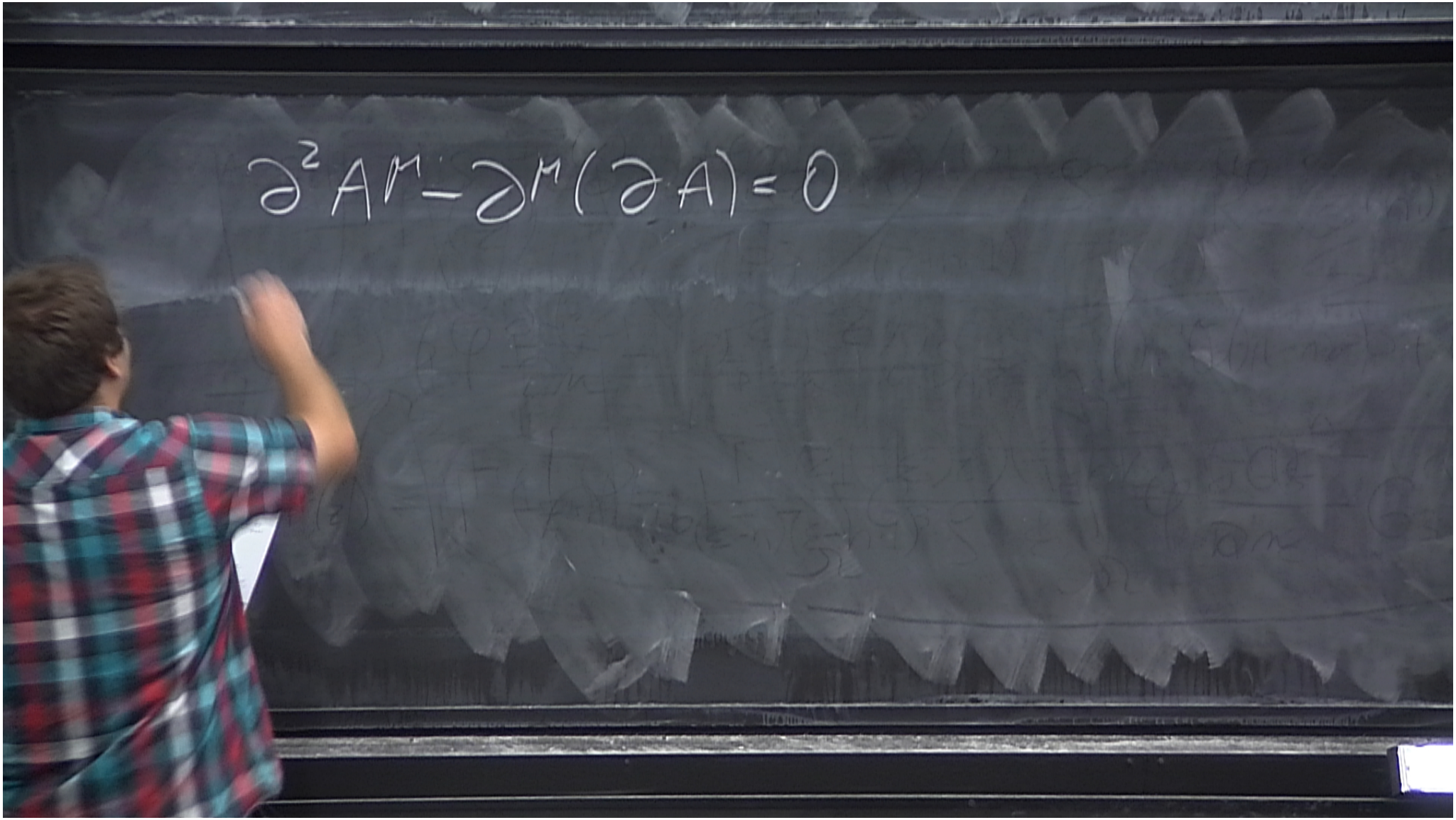
$\partial_z = 0$

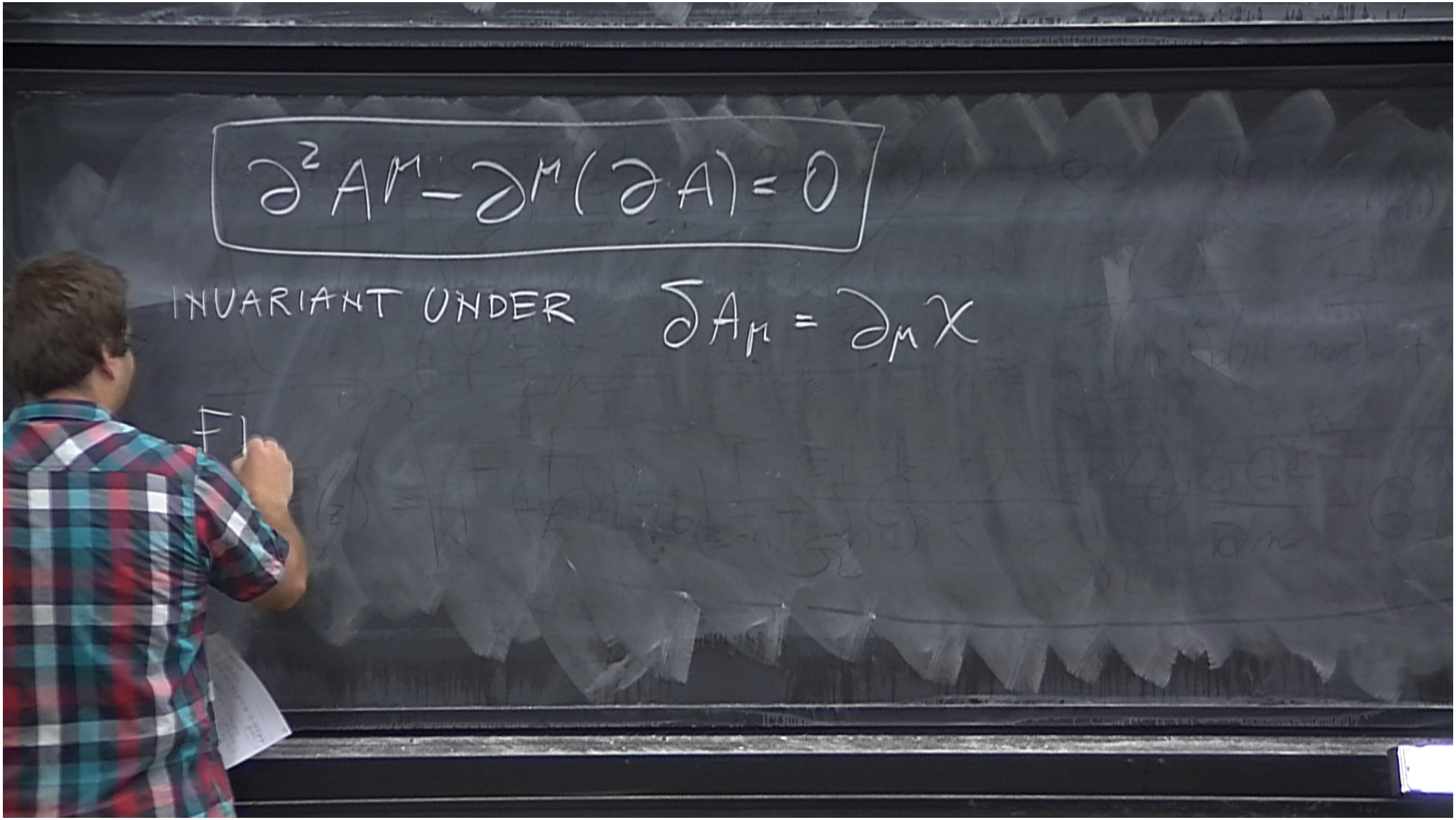


$z > 0$ $\quad \Delta G = \delta(\vec{r} - \vec{d})$
$G|_{z=0} = 0$


MAXWELL.
USUALLY: $A = (A^0, A^1)$ ... $\eta = (-1, 1, 1, 1)$
$A^0 = \frac{1}{\sqrt{2}}(A^+ + A^-)$


MAXWELL.
USUALLY: $A = (A^0, A^1)$ ... $\eta = (-1, 1, 1, 1)$
$A^0 = \frac{1}{\sqrt{2}}(A^+ + A^-)$
$A^1 = \frac{1}{\sqrt{2}}(A^+ - A^-)$
$A = (A^+, A^-, A^I)$

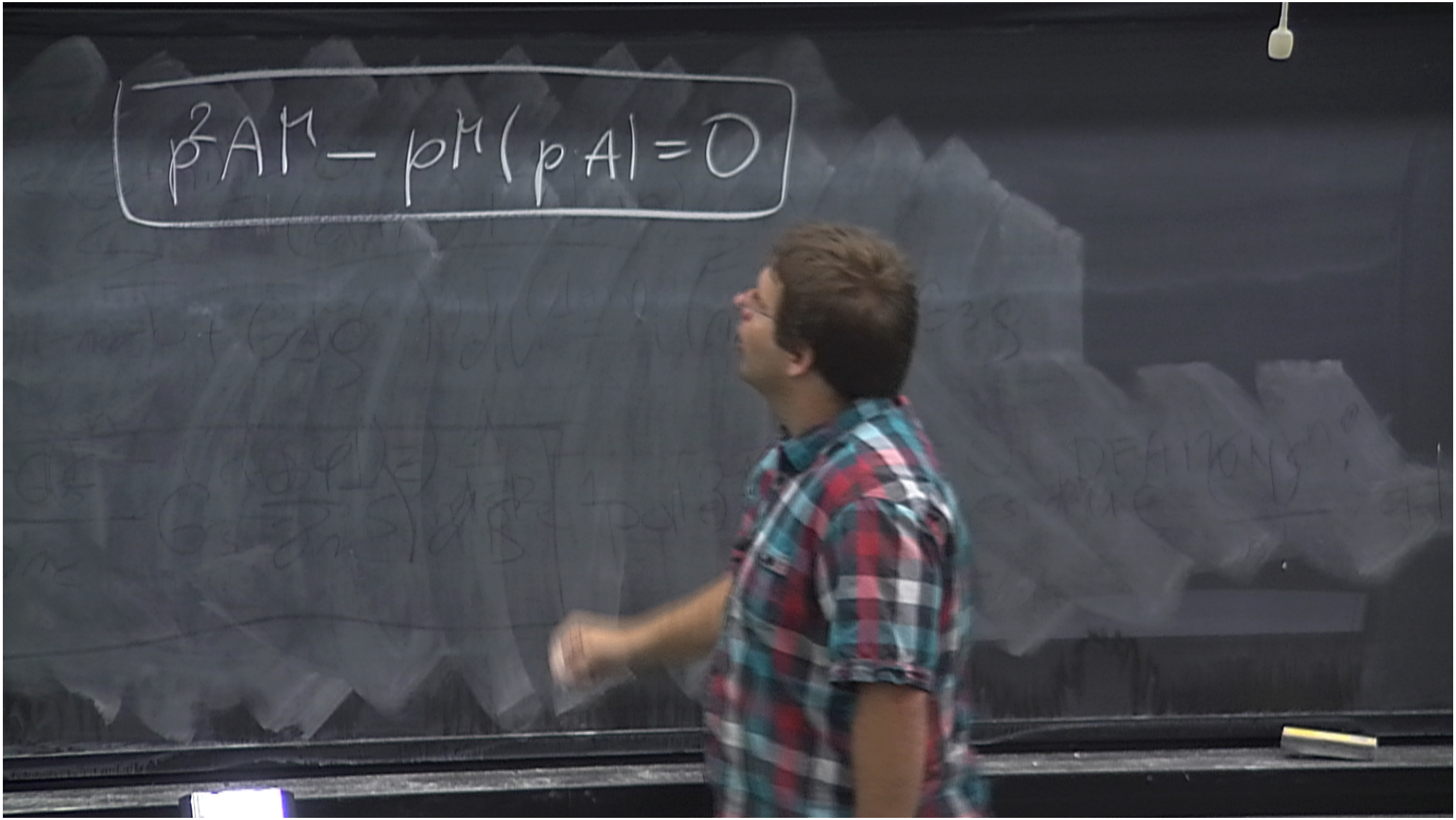

$\partial^2 A^\mu - \partial^\mu(\partial A) = 0$

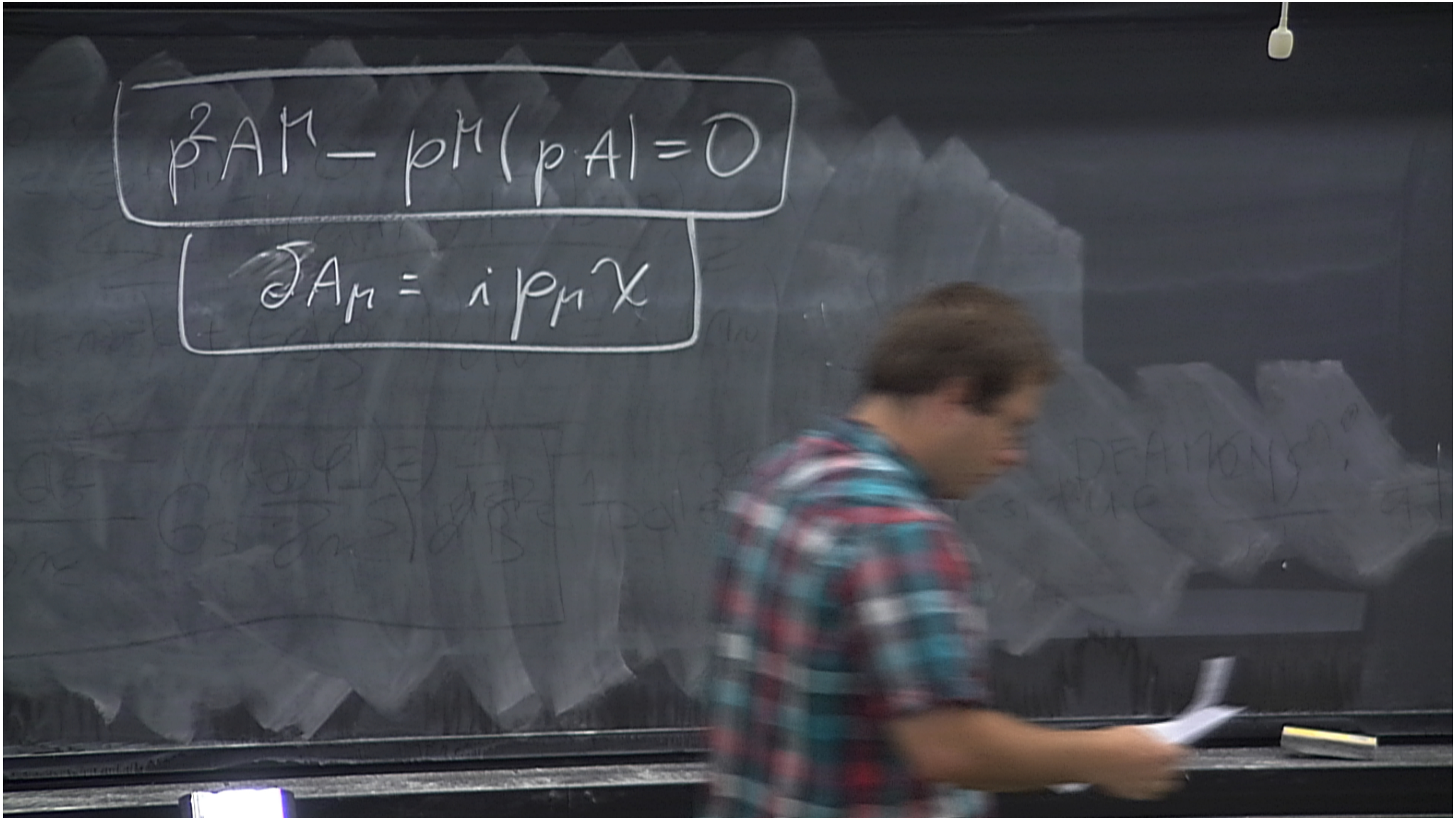

$\partial^2 A^\mu - \partial^\mu(\partial A) = 0$
INVARIANT UNDER $\delta A_\mu = \partial_\mu \chi$

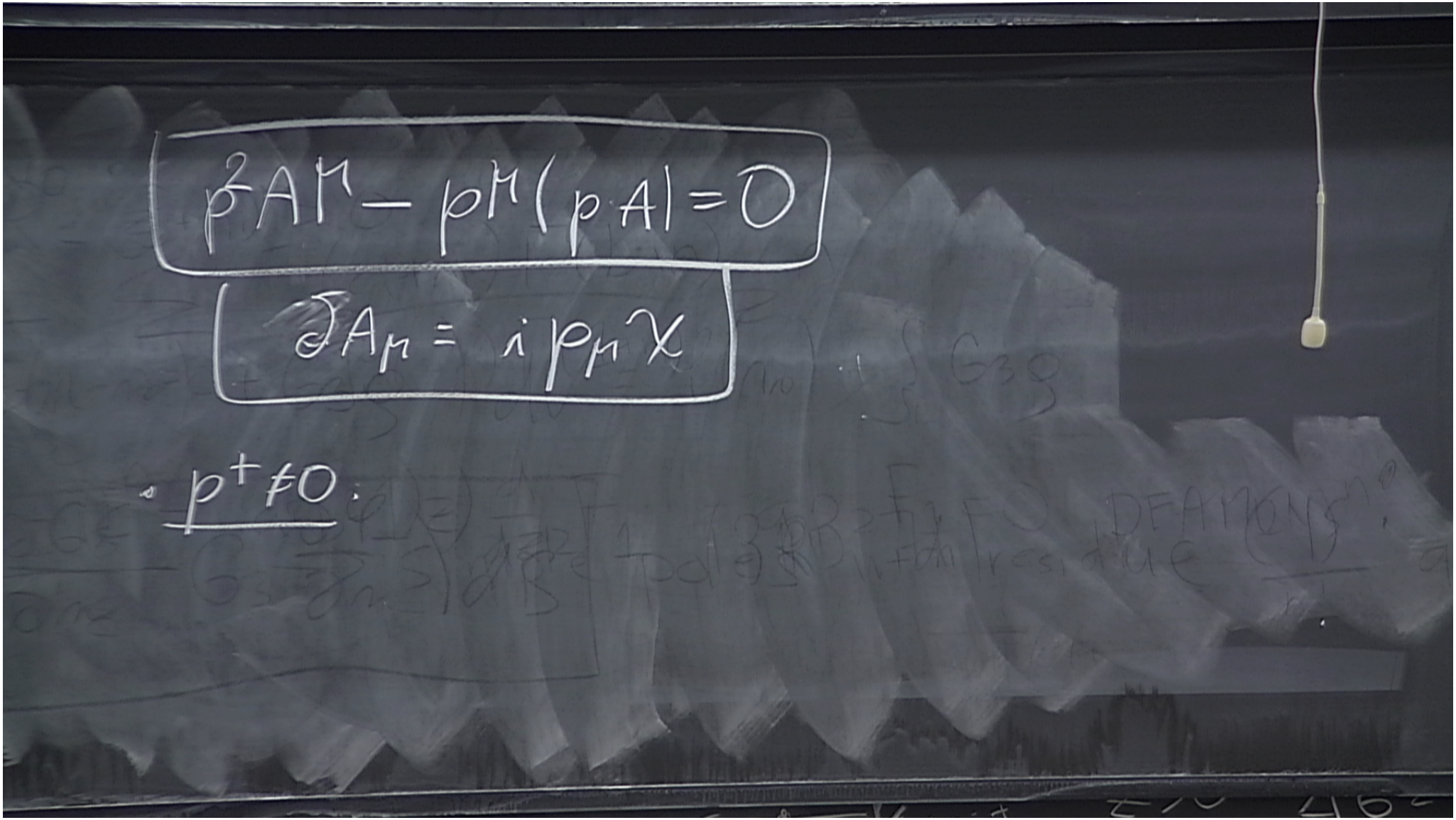
$$\partial^2 A^\mu - \partial^\mu (\partial \cdot A) = 0$$

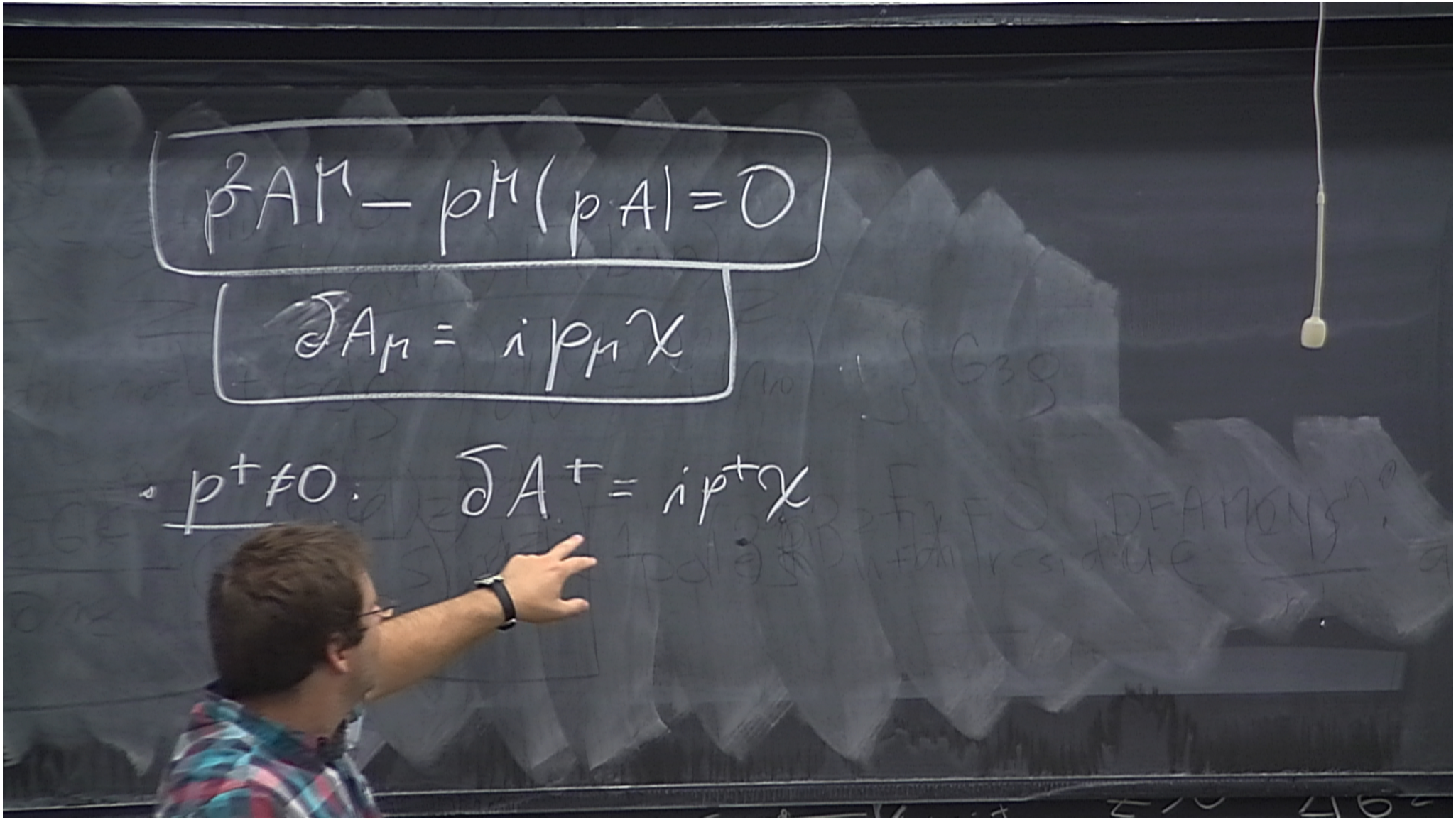
INVARIANT UNDER $\delta A_\mu = \partial_\mu \chi$

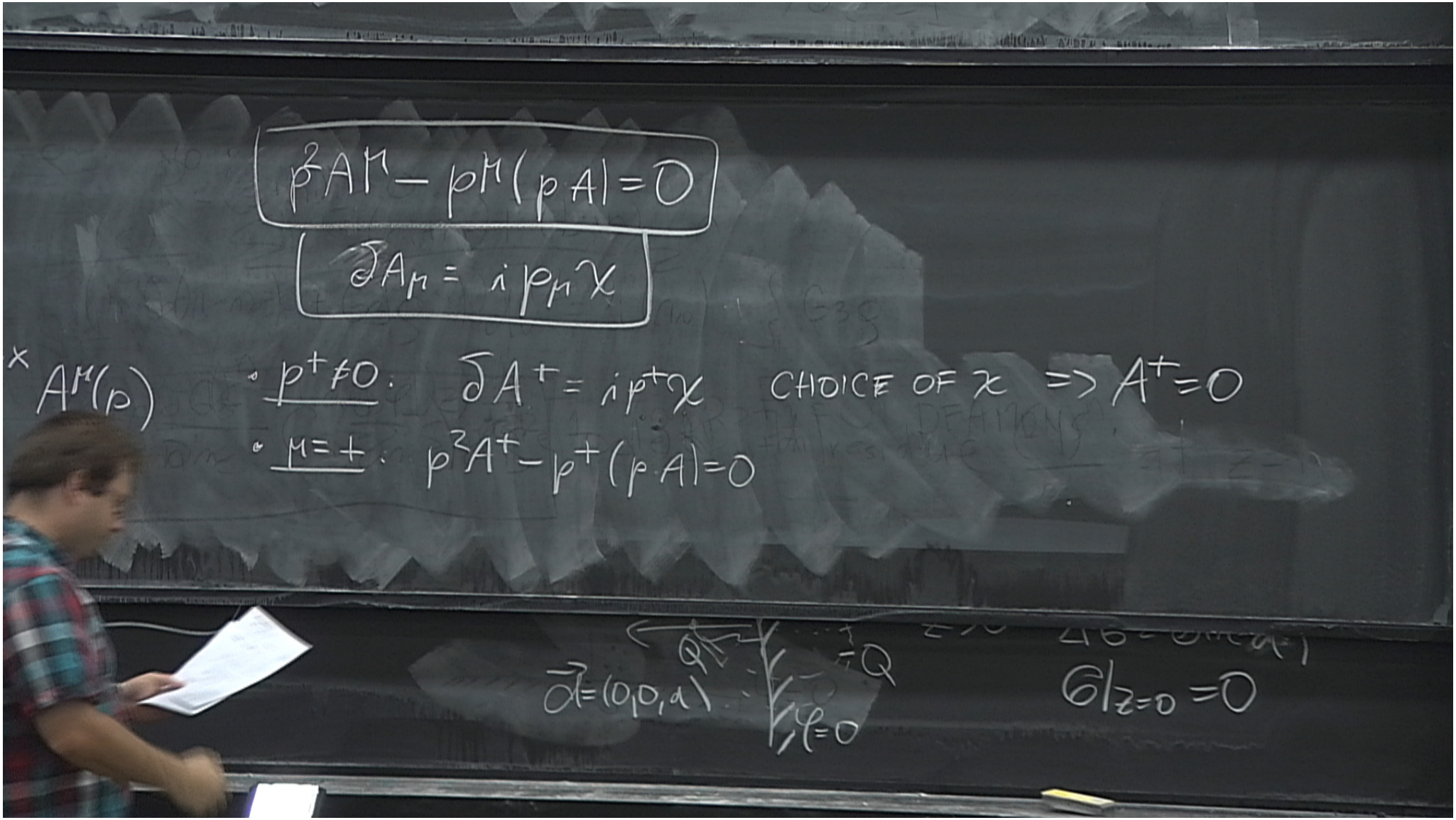
FT: $A^\mu(x) = \frac{1}{(2\pi)^D} \int d^D p \, e^{ip\cdot x} A^\mu(p)$

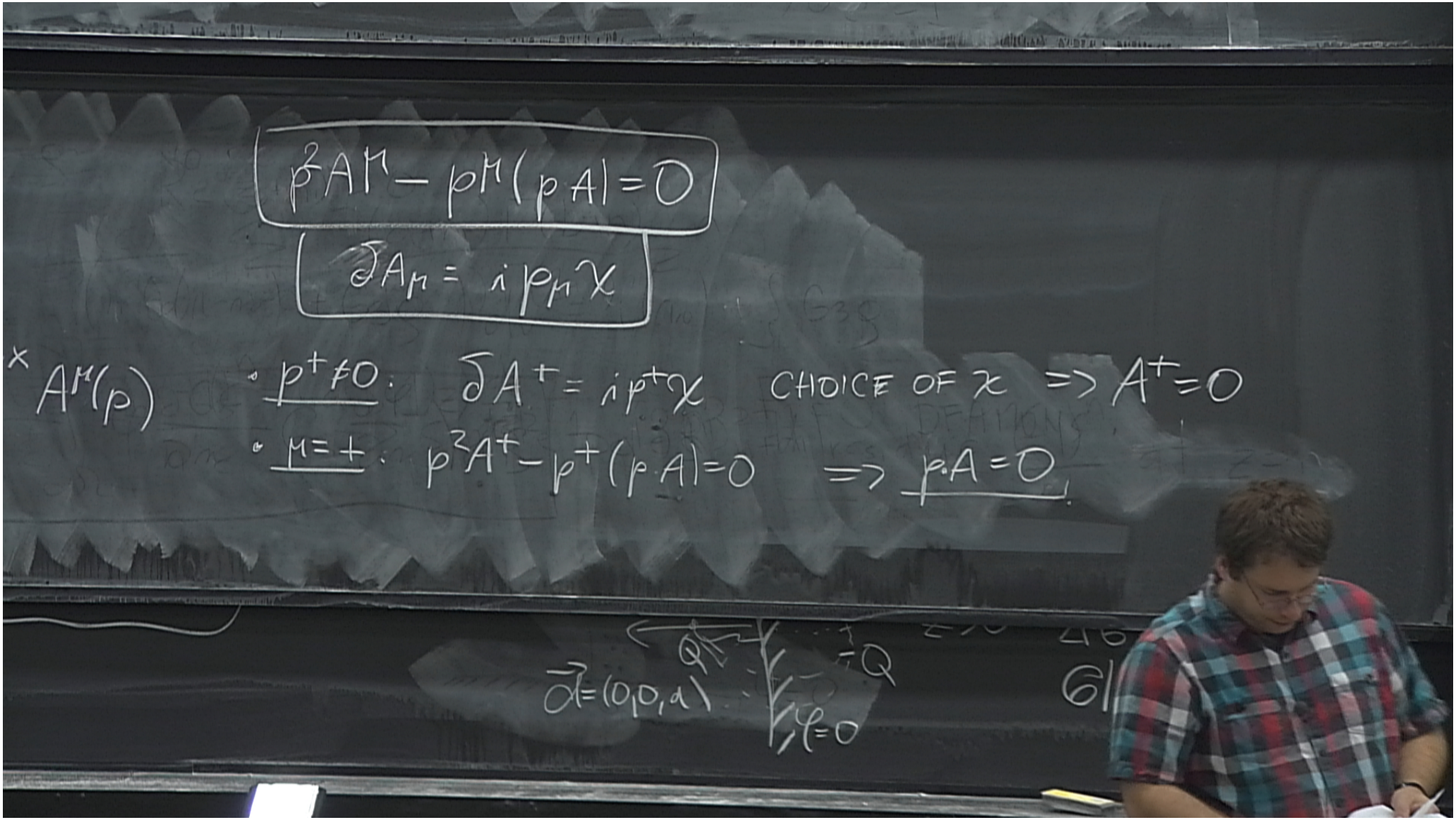

$p^2 A^\mu - p^\mu (p \cdot A) = 0$


$p^2 A^\mu - p^\mu (p \cdot A) = 0$
$\delta A_\mu = i p_\mu \chi$


$p^2 A^\mu - p^\mu (p \cdot A) = 0$
$\partial A_\mu = i p_\mu \chi$
$p^+ \neq 0$:


$p^2 A^\mu - p^\mu (p \cdot A) = 0$
$\delta A_\mu = i p_\mu \chi$
$p^+ \neq 0$: $\delta A^+ = i p^+ \chi$


$p^2 A^\mu - p^\mu (p\cdot A) = 0$
$\delta A_\mu = i p_\mu \chi$
$A^\mu(p)$
$p^+ \neq 0$: $\delta A^+ = i p^+ \chi$ CHOICE OF χ $\Rightarrow A^+ = 0$
$\mu = +$: $p^2 A^+ - p^+ (p\cdot A) = 0$
$\vec{d} = (0,0,a)$
Q
$\rho = 0$
$G|_{z=0} = 0$

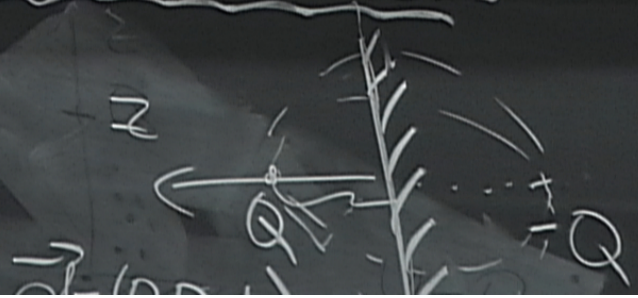

$p^2 A^\mu - p^\mu (p\cdot A) = 0$
$\delta A_\mu = i p_\mu \chi$
$A^\mu(p)$
$p^+ \neq 0$: $\delta A^+ = i p^+ \chi$ CHOICE OF χ $\Rightarrow A^+ = 0$
$\mu = +$: $p^2 A^+ - p^+ (p\cdot A) = 0$ $\Rightarrow p\cdot A = 0$
$\vec{a} = (0,0,a)$
Q
Q
$\ell = 0$

$\partial A_\mu = i p_\mu \chi$

- $p^+ \neq 0$: $\partial A^+ = i p^+ \chi$ CHOICE OF χ $\Rightarrow A^+ = 0$
- $\mu = +$: $p^2 A^+ - p^+ (p \cdot A) = 0 \Rightarrow p \cdot A = 0$

$-p^+ A^- - p^- A^+ + p^I A^I = 0$

METHOD OF IMAGES



$G = -\frac{1}{4\pi|\vec{r}-\vec{d}|} + \frac{1}{4\pi|\vec{r}|}$

$z > 0$ $\Delta G = \delta(\vec{r}-\vec{d})$

$p \cdot A = 0$

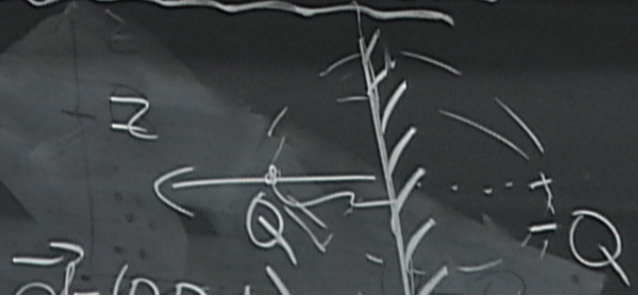
$\partial A_\mu = i p_\mu \chi$

- $\underline{p^+ \neq 0}$: $\partial A^+ = i p^+ \chi$ CHOICE OF χ $\Rightarrow A^+ = 0$
- $\underline{\mu = +}$: $p^2 A^+ - p^+ (p \cdot A) = 0 \Rightarrow \underline{p \cdot A = 0}$

$-p^+ A^- - \underline{p^- A^+} + p^I A^I = 0$

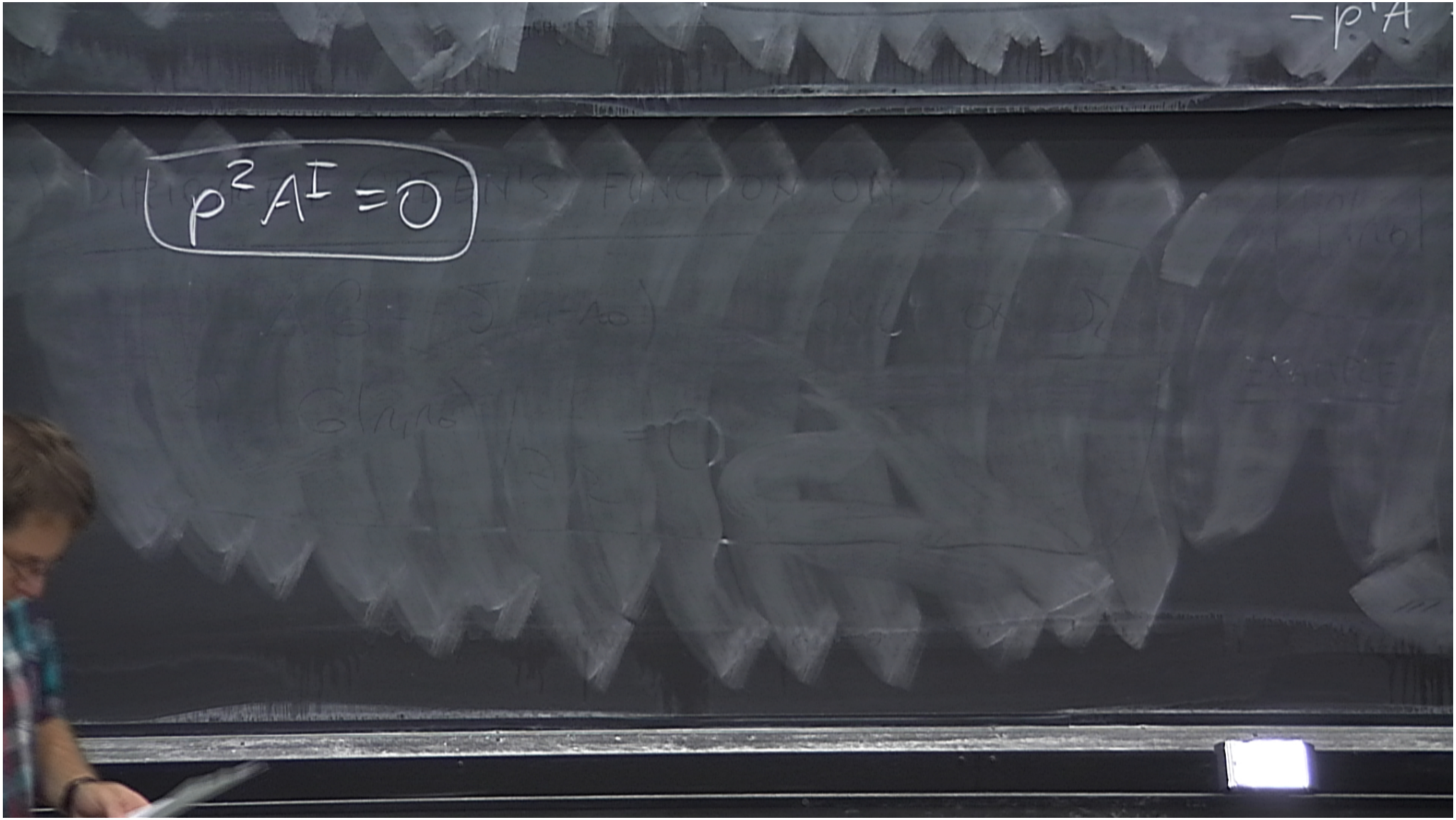
$\boxed{A^- = \frac{1}{p^+} p^I A^I}$

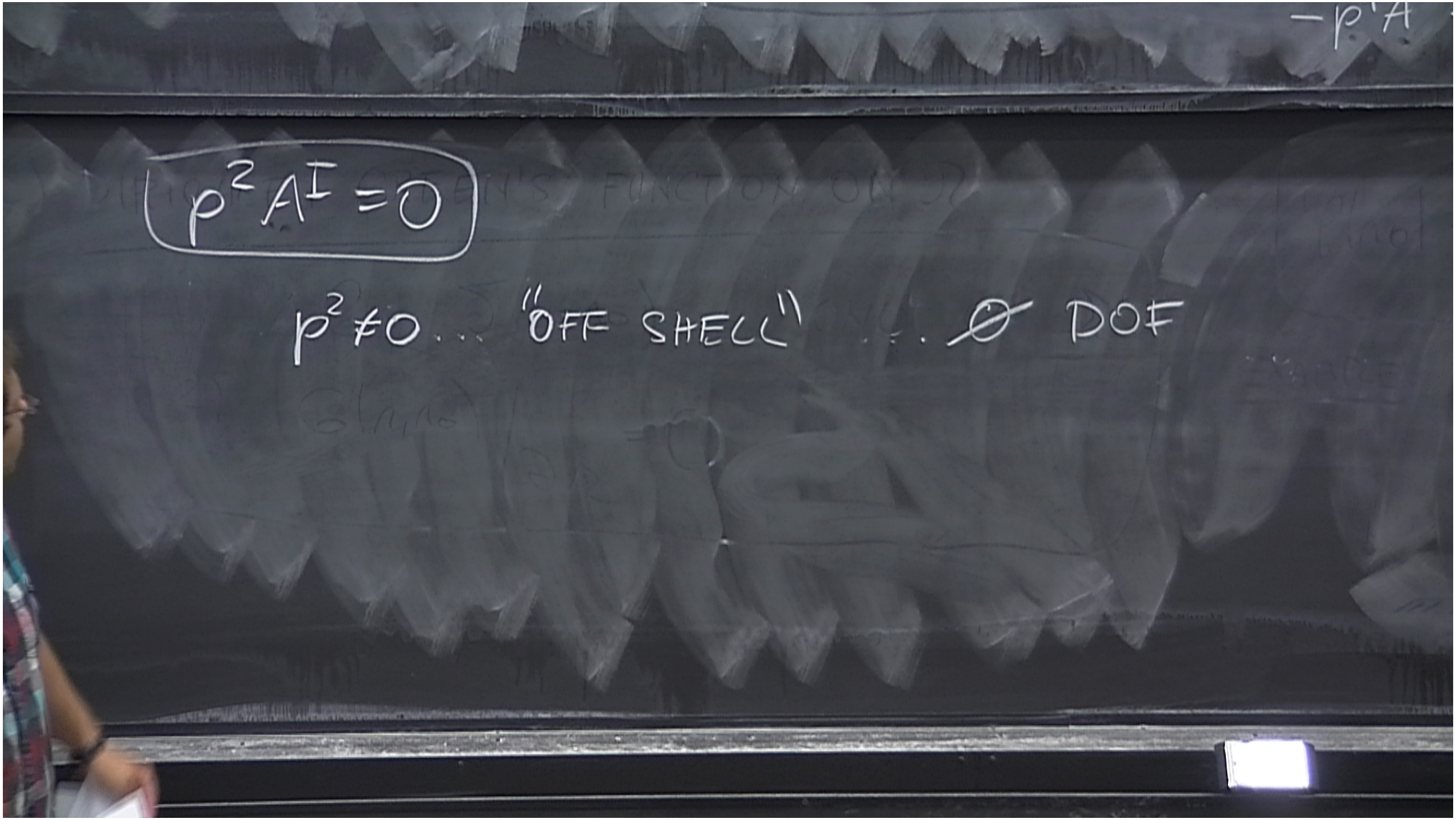
EXAMPLE: METHOD OF IMAGES

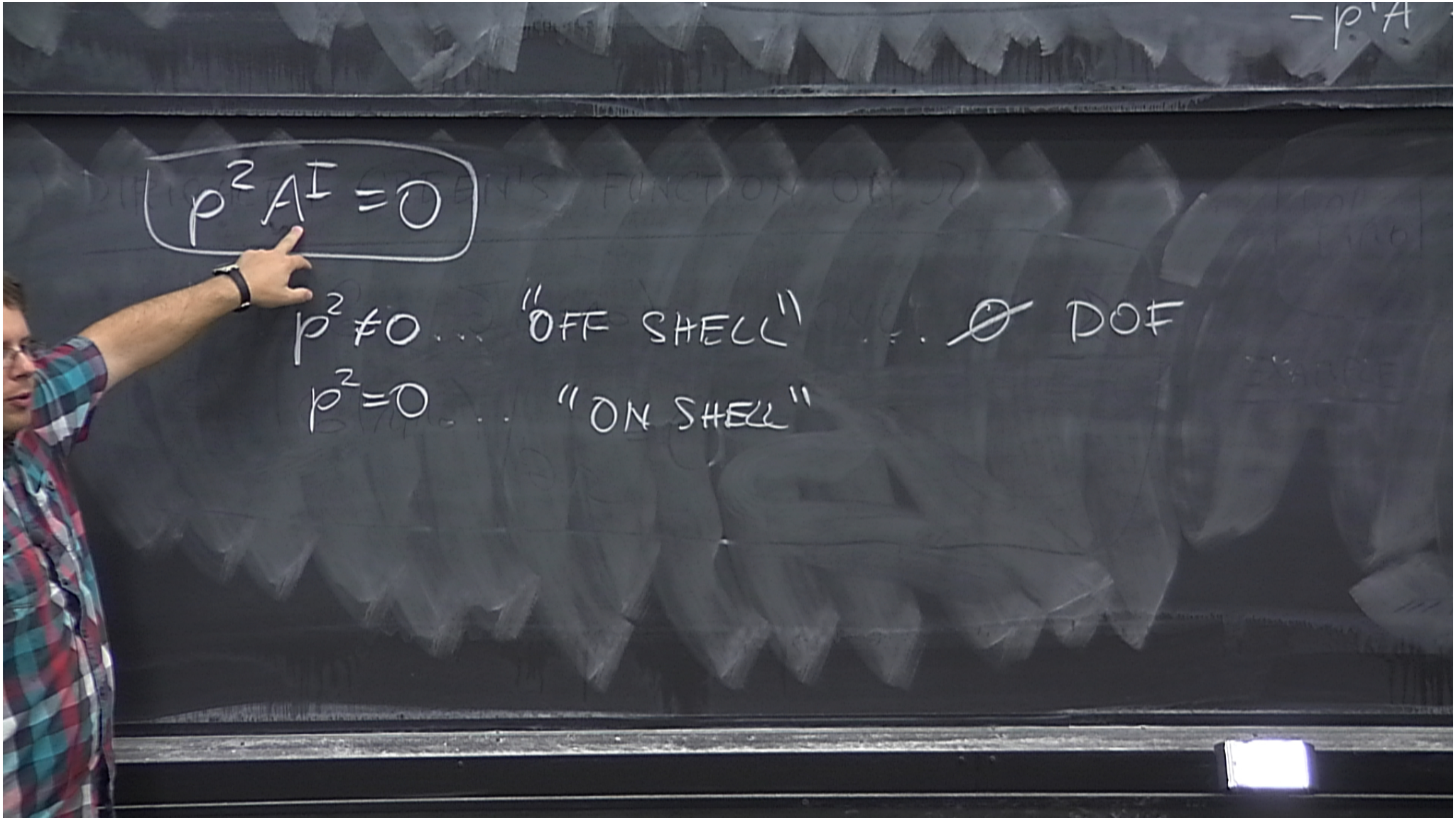


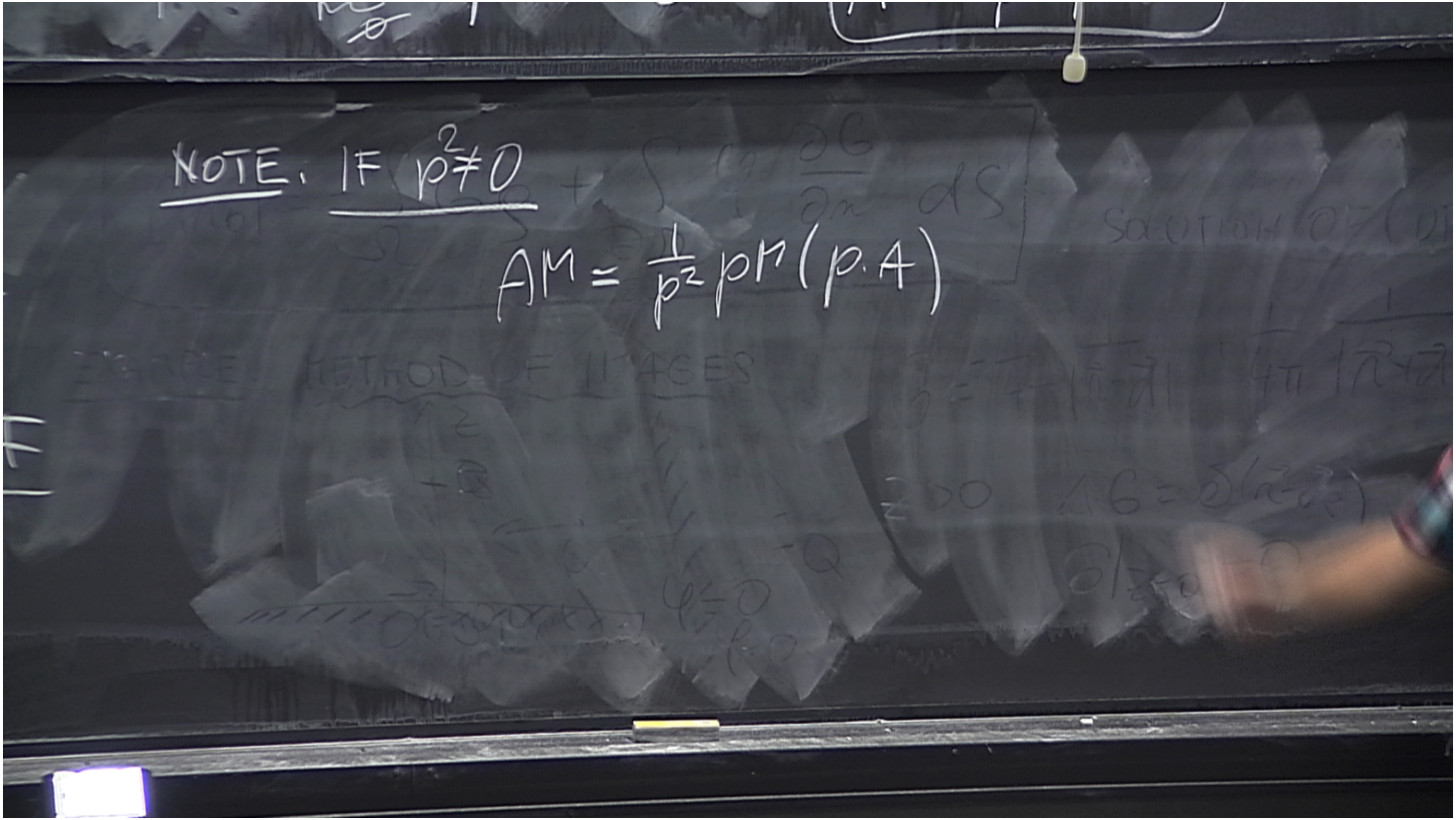
$G = -\frac{1}{4\pi |\vec{r} - \vec{r}'|} + \frac{1}{4\pi |r$

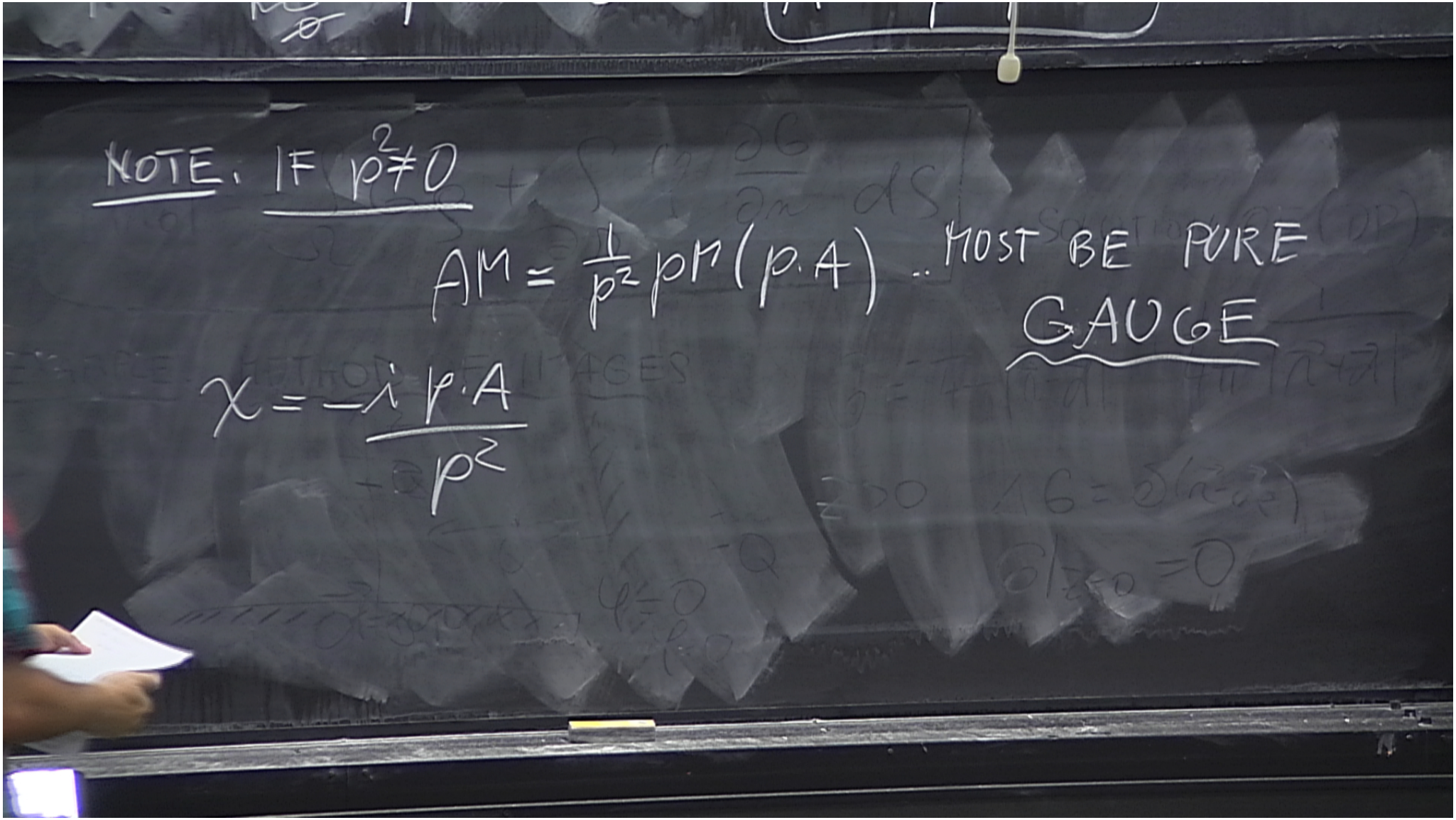
$z > 0 \quad \Delta G = \delta(\vec{r} - \vec{r}')$

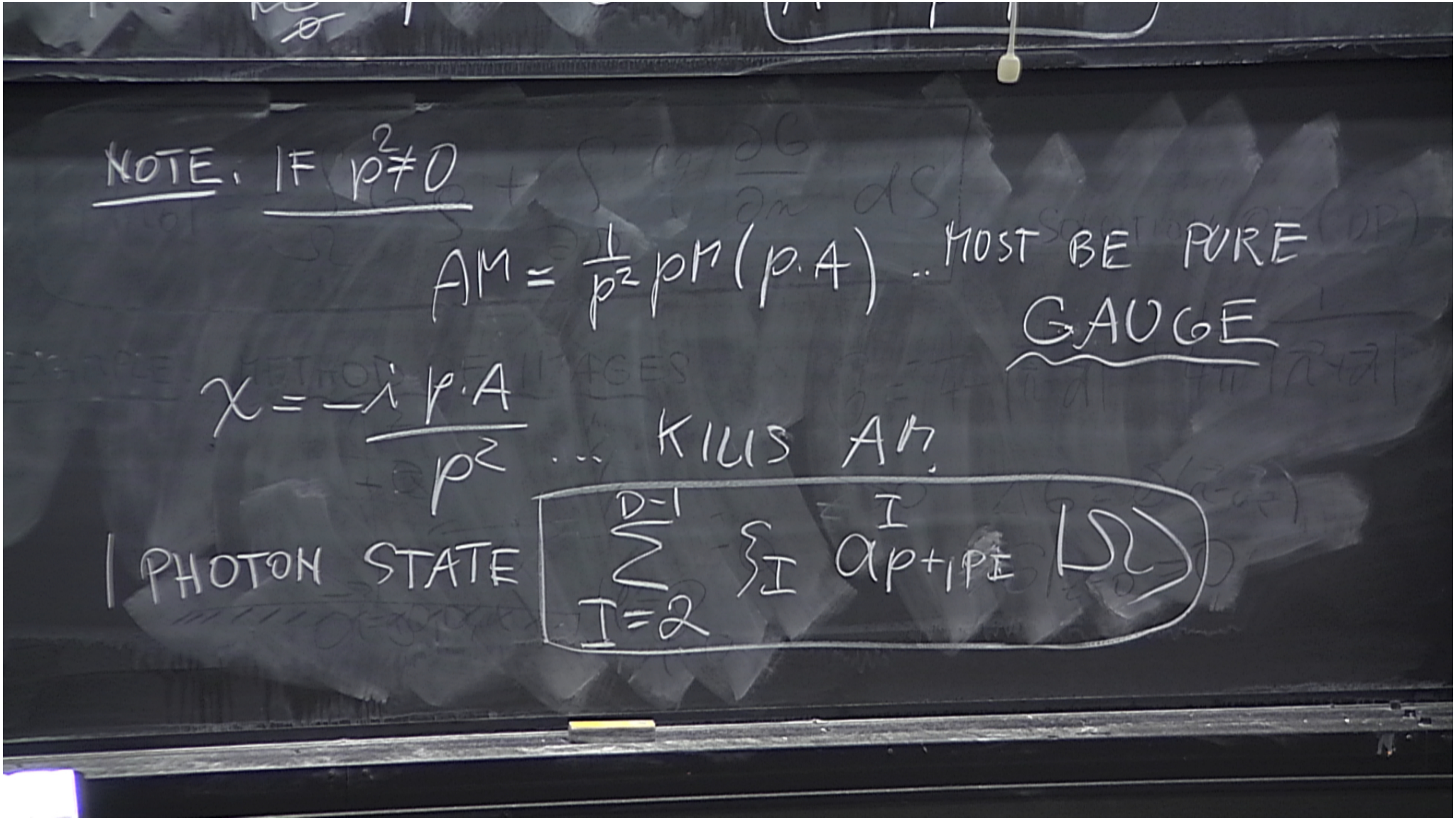

$-p^2A$
$p^2A^I = 0$


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$p^2 \neq 0$ ... "OFF SHELL" ... ~~0~~ DOF


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$p^2 \neq 0$ ... "OFF SHELL" ... ~~0~~ DOF
$p^2 = 0$ ... "ON SHELL"


NOTE. IF $p^2 \neq 0$
$A^\mu = \frac{1}{p^2} p^\mu (p \cdot A)$


NOTE. IF $p^2 \neq 0$
$A^\mu = \frac{1}{p^2} p^\mu (p \cdot A)$ .. MUST BE PURE GAUGE
$\chi = -\lambda \frac{p \cdot A}{p^2}$


NOTE. IF $p^2 \neq 0$
$A^\mu = \frac{1}{p^2} p^\mu (p \cdot A)$ .. MUST BE PURE GAUGE
$\chi = -i \frac{p \cdot A}{p^2}$ ... KILLS A^μ
1 PHOTON STATE $\sum_{I=2}^{D-1} \xi_I^I a^\dagger_{p+} |p\rangle$