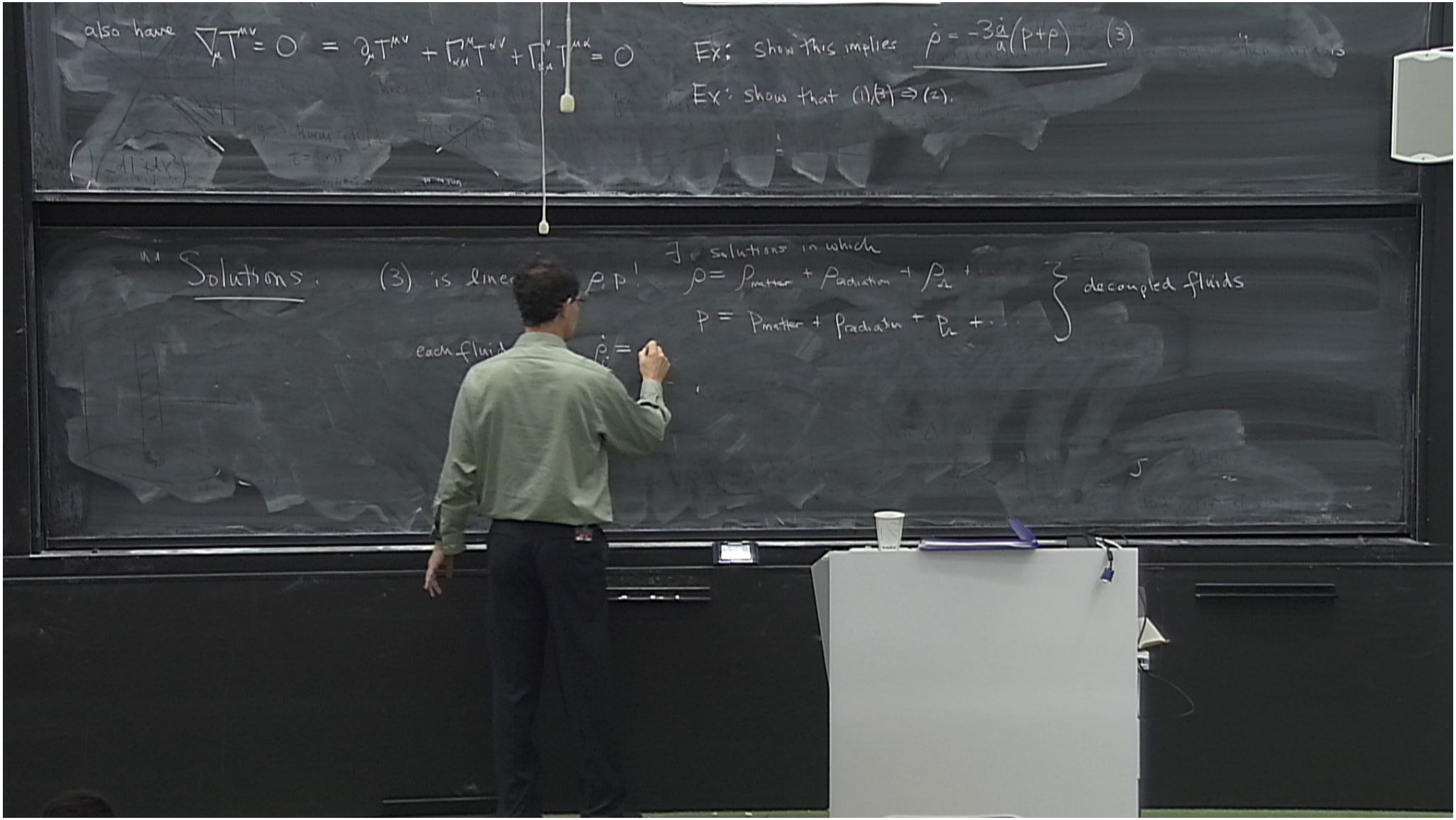
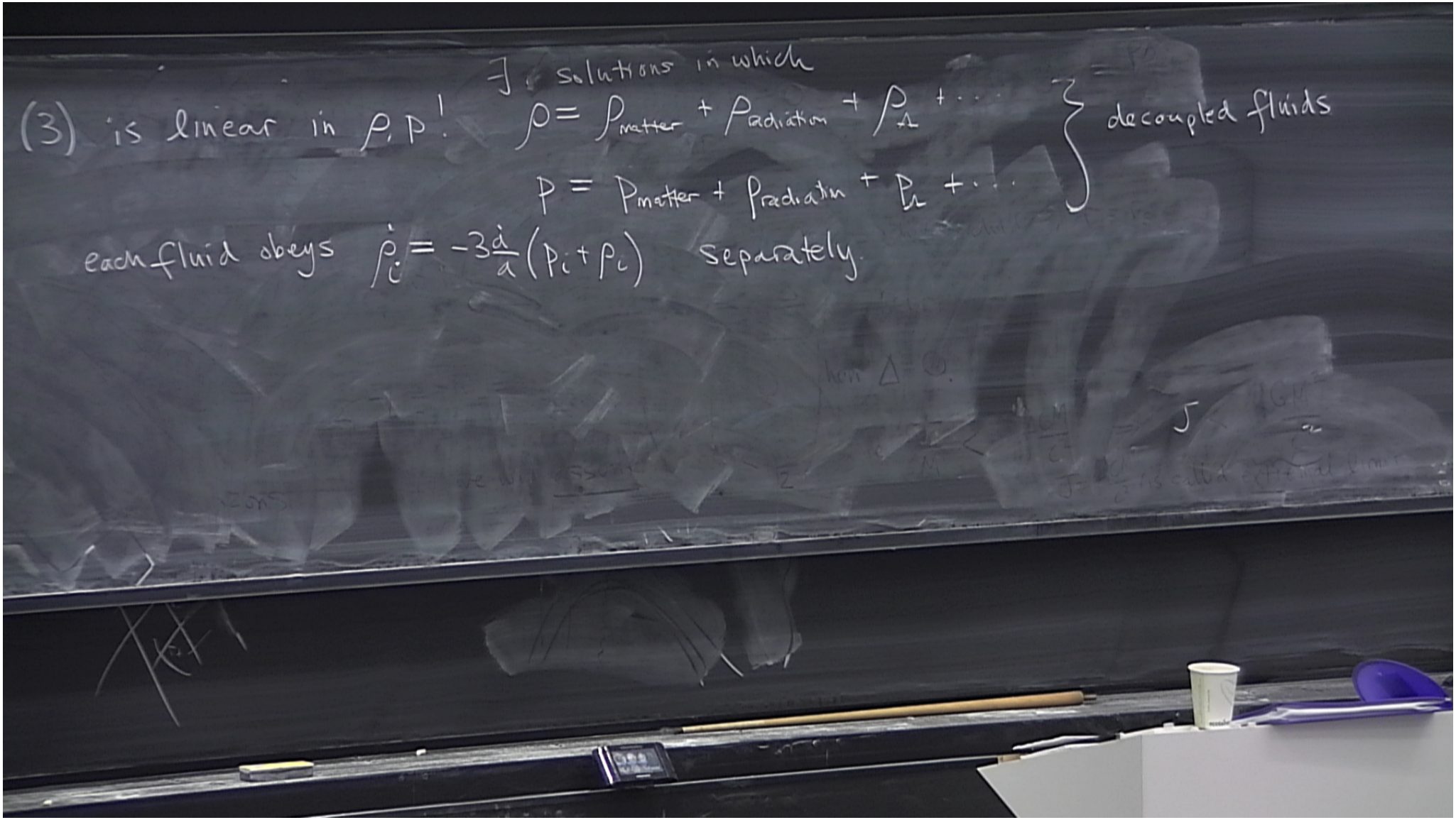
Title: Relativity - Lecture 15

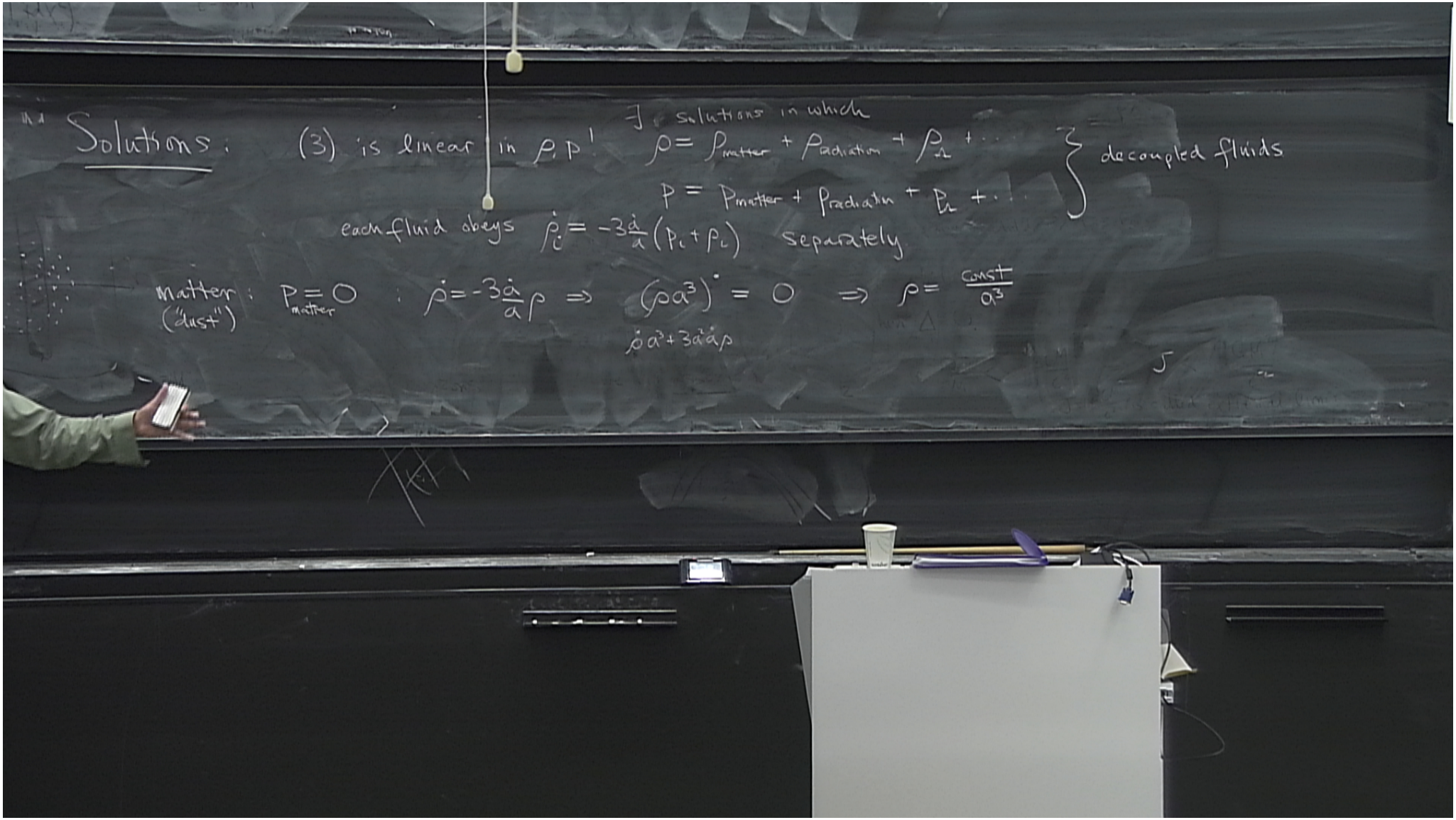
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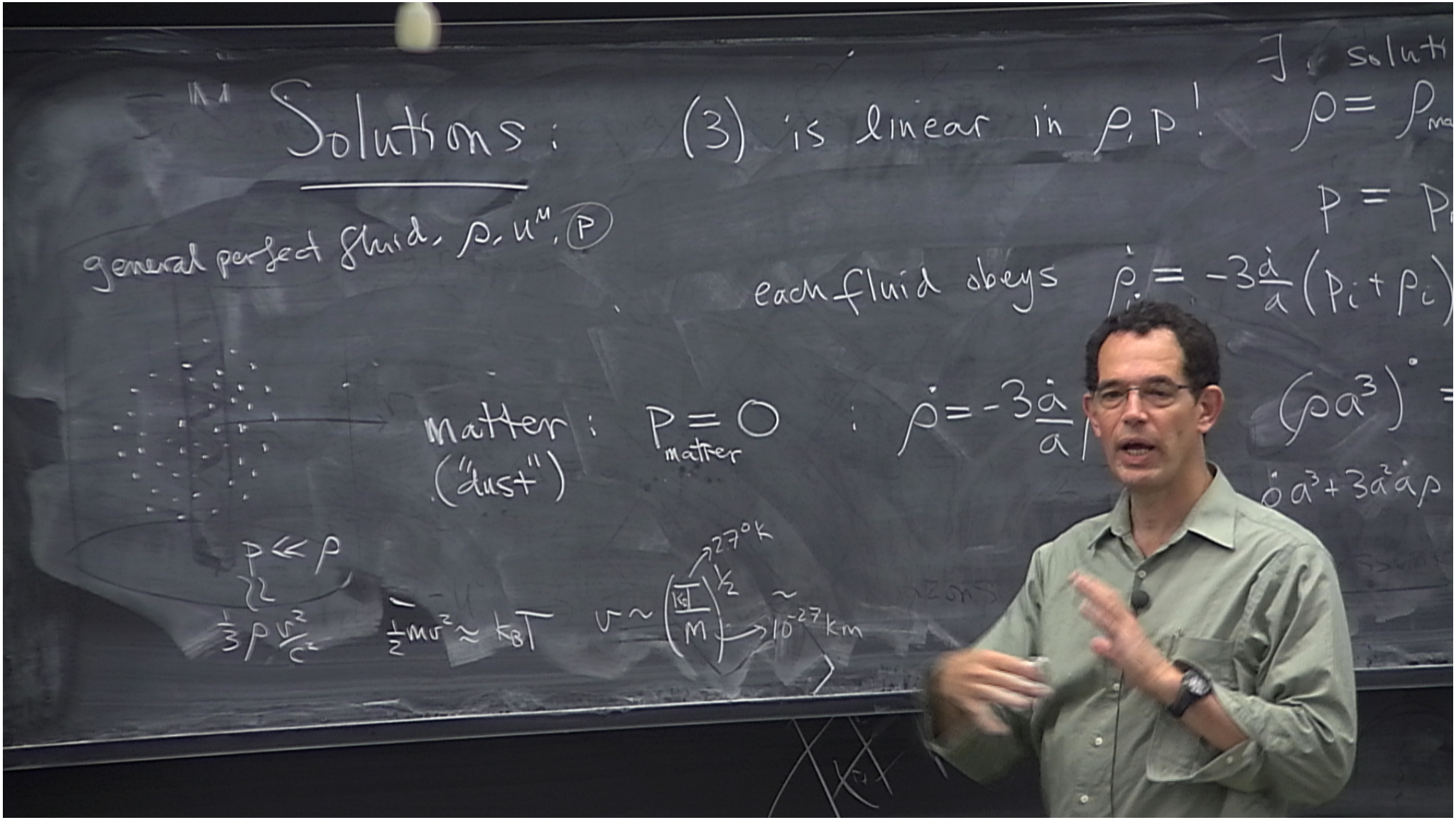
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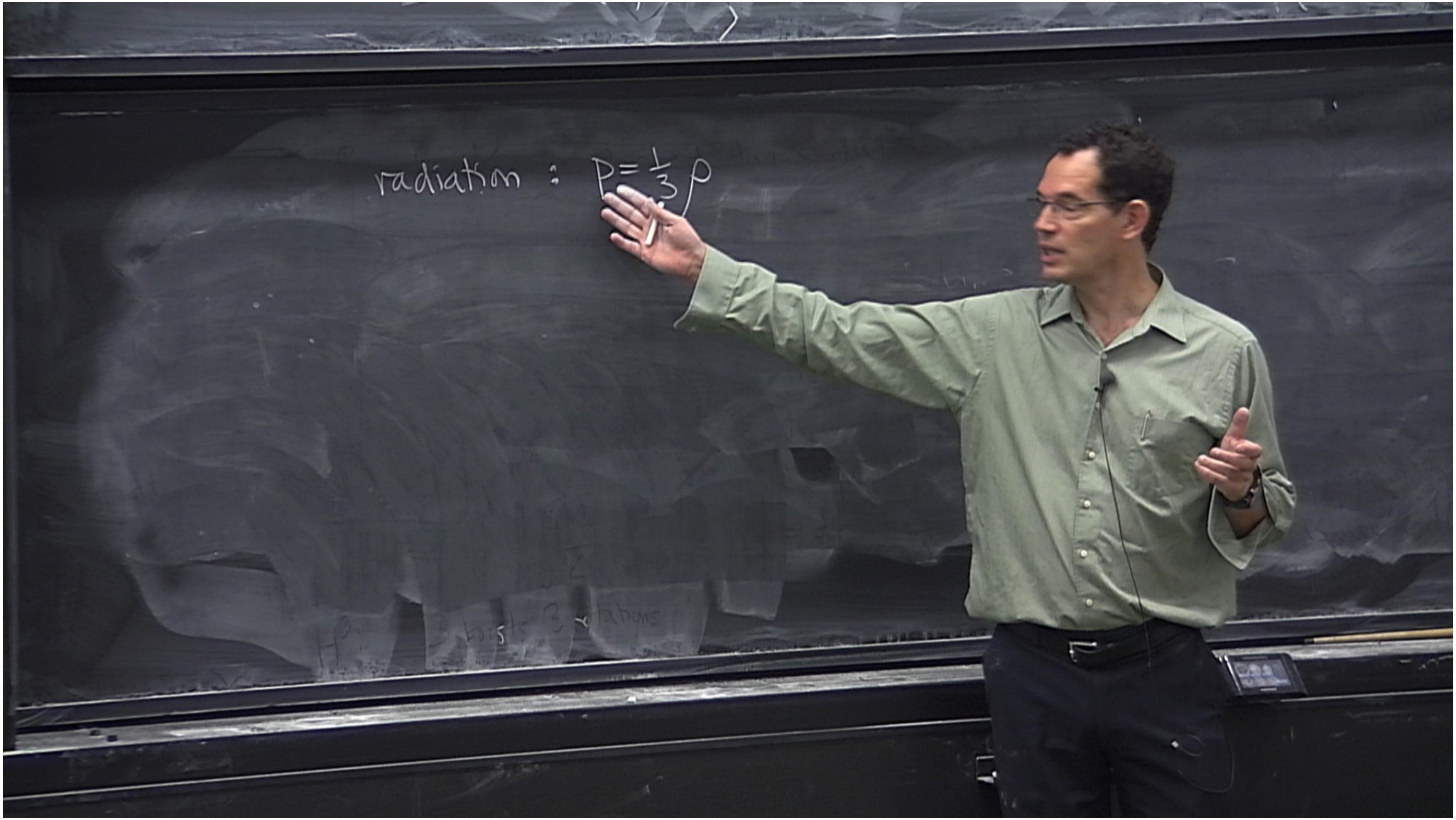
Abstract:

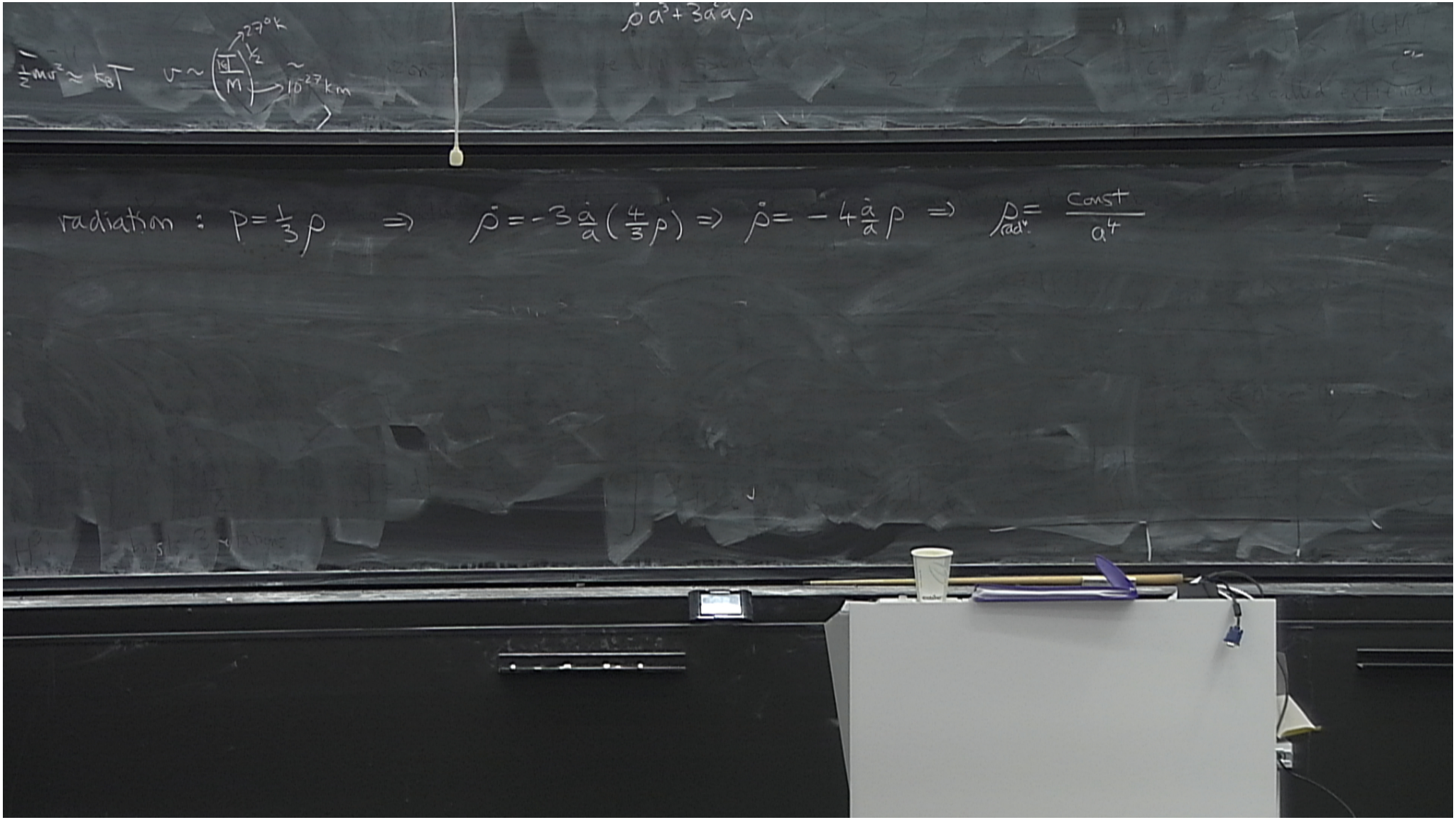

also have $\nabla_\mu T^{\mu\nu} = 0 = \partial_\mu T^{\mu\nu} + \Gamma^\mu_{\nu\mu} T^{\alpha\nu} + \Gamma^\nu_{\alpha\mu} T^{\mu\alpha} = 0$
Ex: Show this implies $\dot{\rho} = -3\frac{\dot{a}}{a}(p+\rho)$ (3)
Ex: Show that (1),(2) $\Rightarrow$ (3).
Solutions: (3) is linear in ρ, p!
$\rho = \rho_{matter} + \rho_{radiation} + \rho_\Lambda + \ldots$
$p = p_{matter} + p_{radiation} + p_\Lambda + \ldots$
} decoupled fluids


(3) is linear in ρ, p! ∃ solutions in which
ρ = ρ_matter + ρ_radiation + ρ_Λ + ...
p = p_matter + p_radiation + p_Λ + ...
} decoupled fluids
each fluid obeys ρ̇_i = −3(ȧ/a)(p_i + ρ_i) separately.


Solutions: (3) is linear in ρ, p
solutions in which
ρ = ρ_matter + ρ_radiation + ρ_Λ + ...
p = p_matter + p_radiation + p_Λ + ...
} decoupled fluids
each fluid obeys ρ̇_i = -3(ȧ/a)(p_i + ρ_i) separately
Matter ("dust"): p_matter = 0 : ρ̇ = -3(ȧ/a)ρ ⇒ (ρa³)˙ = 0 ⇒ ρ = const/a³
ρ̇a³ + 3ȧ²aρ


Solutions: (3) is linear in ρ, P!
general perfect fluid, ρ, u^μ, P
each fluid obeys
matter ("dust"): P_matter = 0
P ≪ ρ
v/c
km


radiation : $p = \frac{1}{3}\rho$


$\dot{\rho}a^3 + 3a^2\dot{a}\rho$
$\frac{1}{2}mv^2 \approx k_B T$
$v \sim \left(\frac{T}{M}\right)^{1/2}$
$\sim 10^{-27}$ km
$\sim 27°$K
radiation : $p = \frac{1}{3}\rho \Rightarrow \dot{\rho} = -3\frac{\dot{a}}{a}\left(\frac{4}{3}\rho\right) \Rightarrow \dot{\rho} = -4\frac{\dot{a}}{a}\rho \Rightarrow \rho_{rad} = \frac{\text{const}}{a^4}$

radiation : $p = \frac{1}{3}\rho \Rightarrow \dot{\rho} = -3\frac{\dot{a}}{a}\left(\frac{4}{3}\rho\right) \Rightarrow \dot{\rho} = -4\frac{\dot{a}}{a}\rho$



$\lambda \propto a(t)$

$\Rightarrow E_\gamma = h\nu = \frac{hc}{\lambda}$

$\propto \frac{1}{a}.$

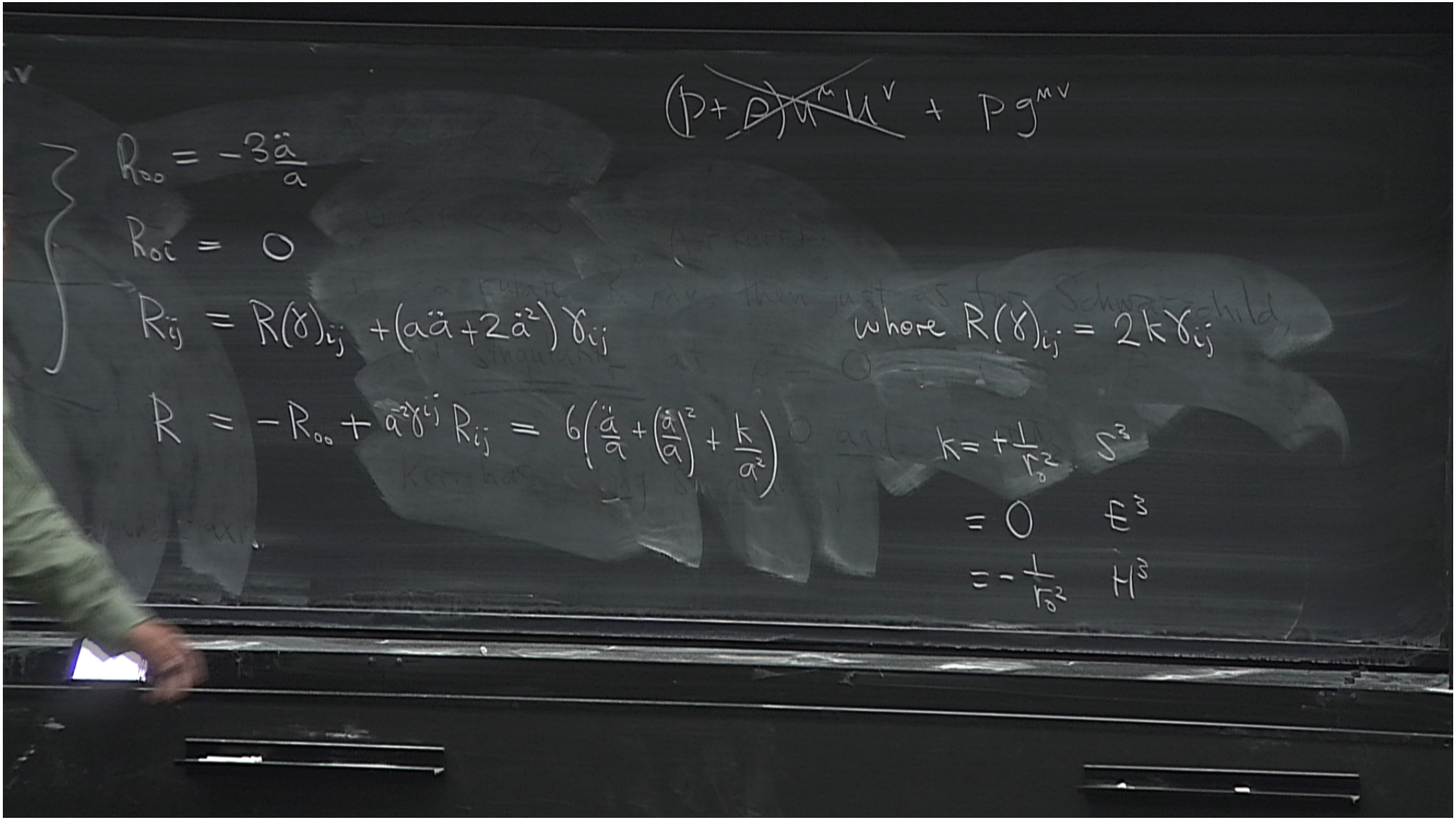
radiation : $p = \frac{1}{3}\rho \Rightarrow \dot{\rho} = -3\frac{\dot{a}}{a}\left(\frac{4}{3}\rho\right) \Rightarrow \dot{\rho} = -4\frac{\dot{a}}{a}\rho$

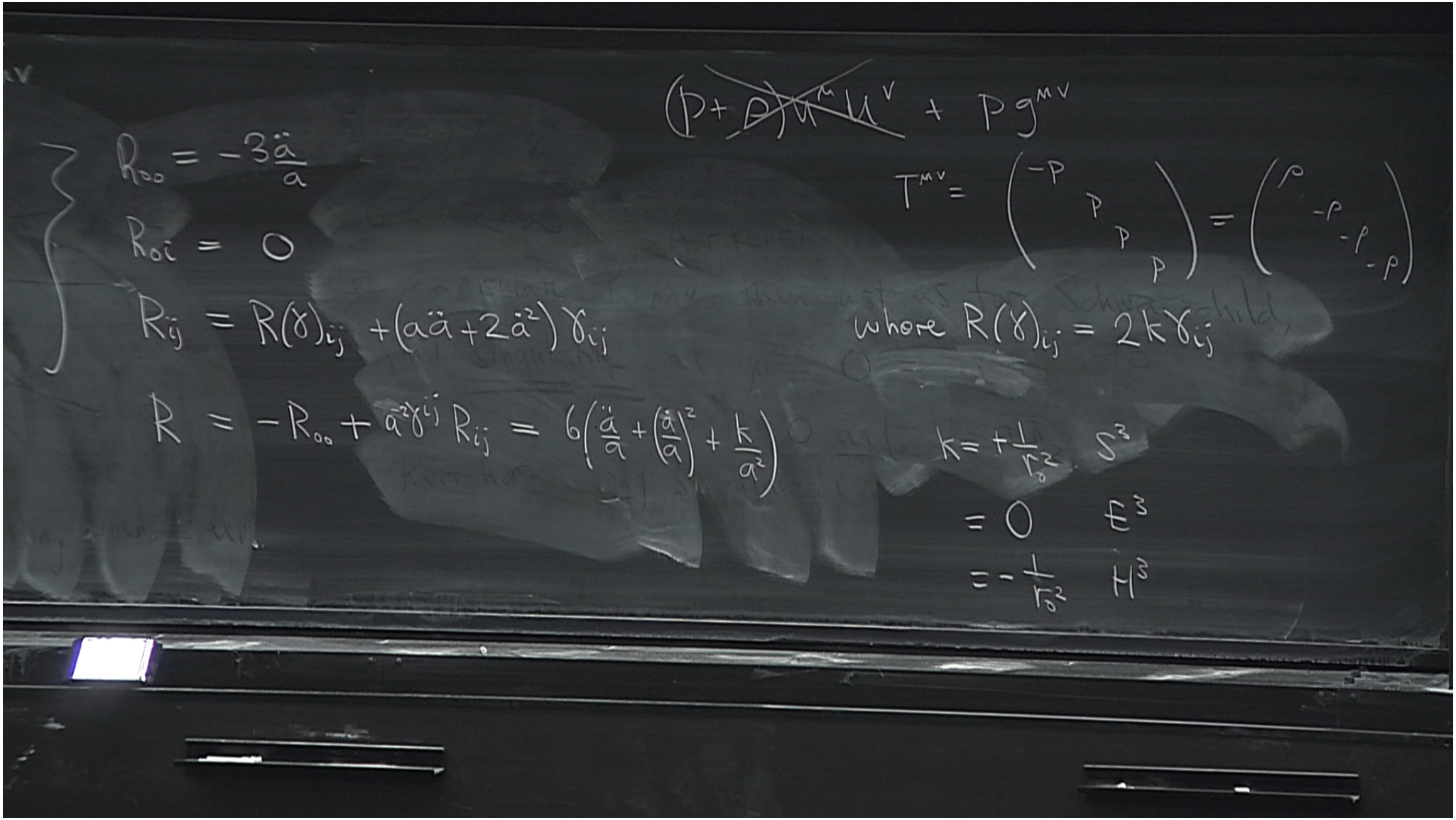


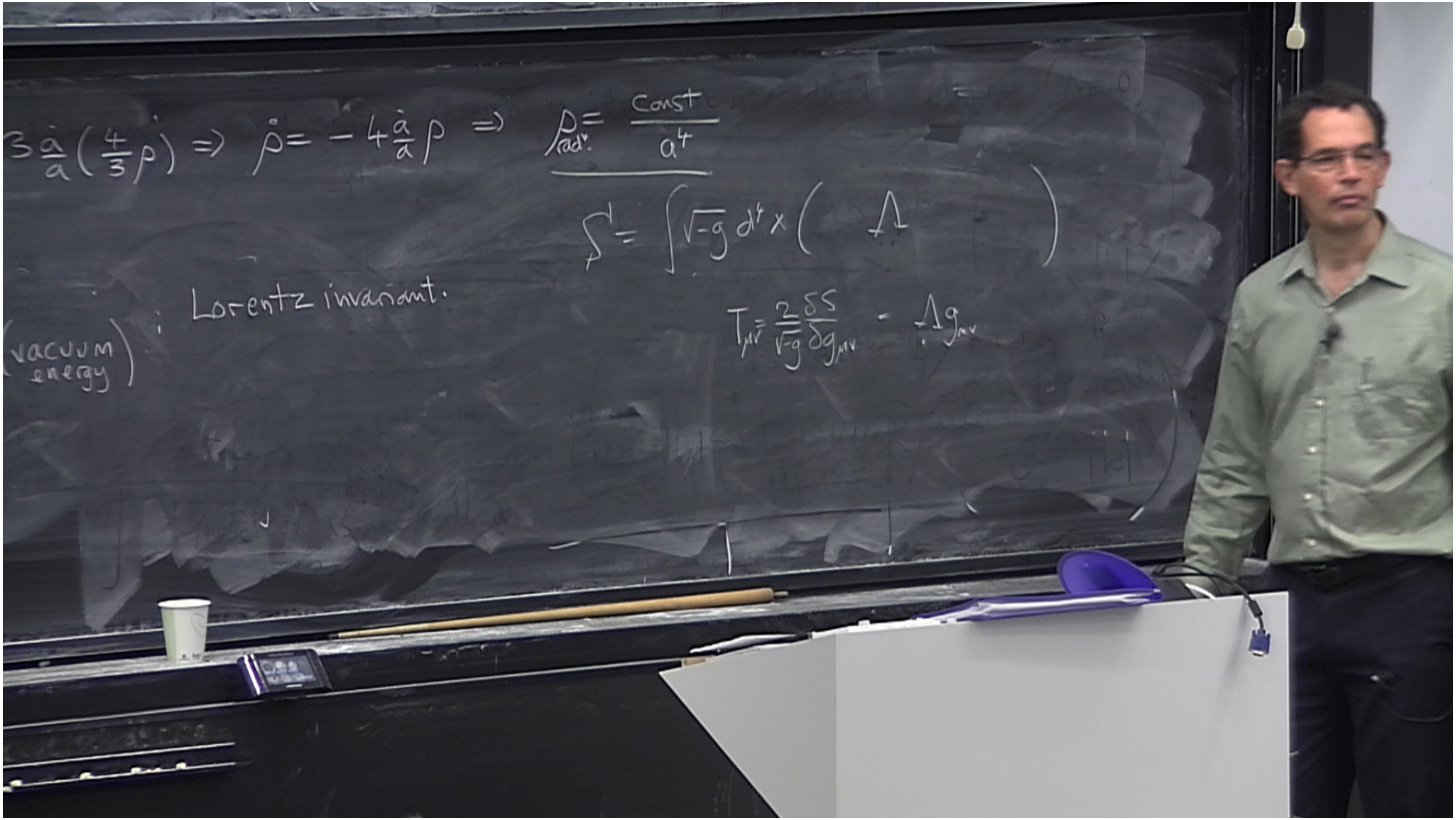
$\lambda \propto a(t)$

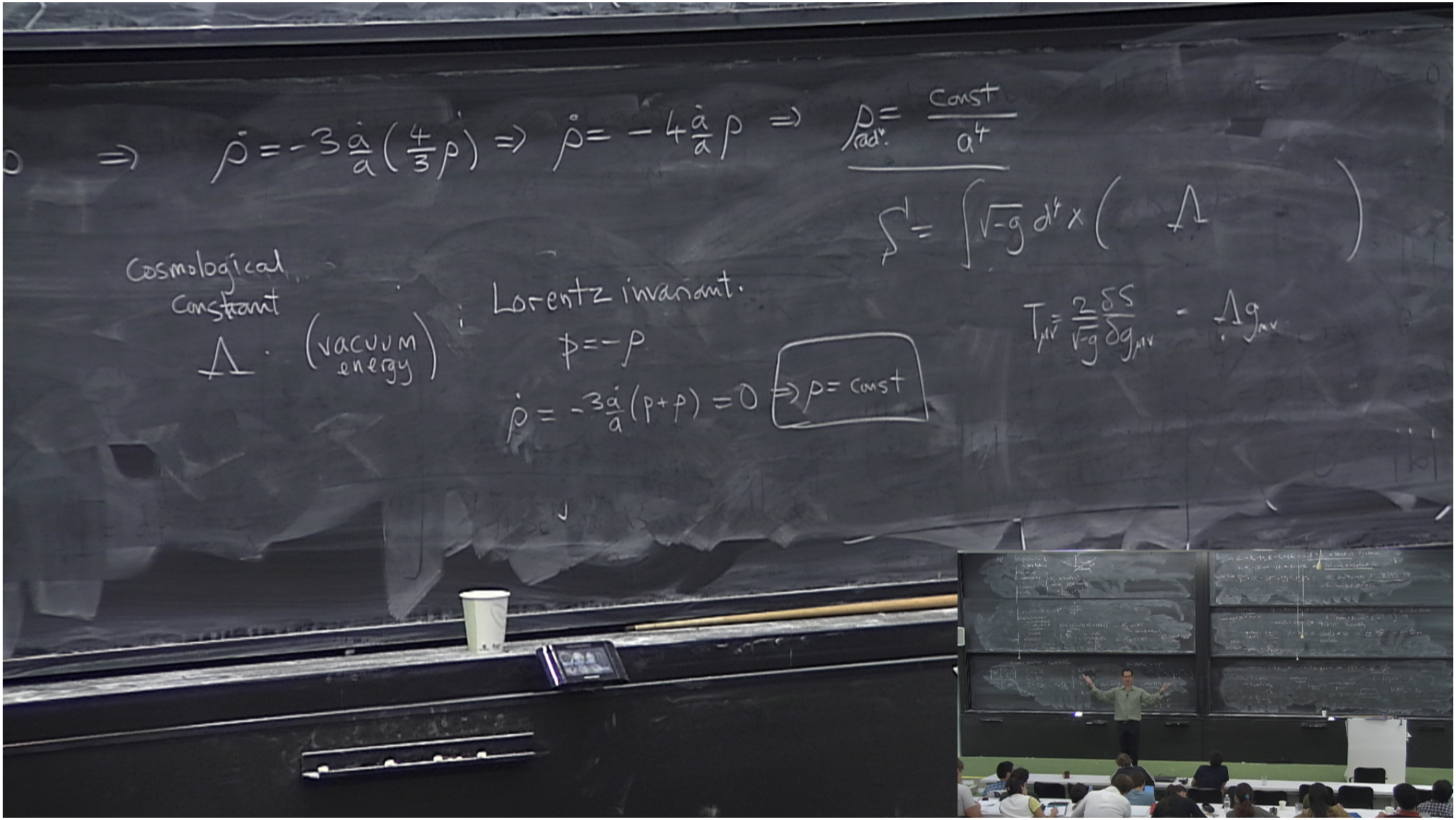
$\Rightarrow E_\gamma = h\nu = \frac{hc}{\lambda} \propto \frac{1}{a}.$

$\rho \propto \frac{E_\gamma}{a^3} \propto \frac{1}{a^4}.$


$(\rho + \cancel{p})u^\mu u^\nu + p g^{\mu\nu}$
$R_{00} = -3\frac{\ddot{a}}{a}$
$R_{0i} = 0$
$R_{ij} = R(\gamma)_{ij} + (a\ddot{a} + 2\dot{a}^2)\gamma_{ij}$
where $R(\gamma)_{ij} = 2k\gamma_{ij}$
$R = -R_{00} + a^{-2}\gamma^{ij} R_{ij} = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right)$
$k = +\frac{1}{r_0^2}$ S^3
$= 0$ E^3
$= -\frac{1}{r_0^2}$ H^3


$R_{00} = -3\frac{\ddot{a}}{a}$
$R_{0i} = 0$
$R_{ij} = R(\gamma)_{ij} + (a\ddot{a} + 2\dot{a}^2)\gamma_{ij}$
$R = -R_{00} + a^{-2}\gamma^{ij}R_{ij} = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right)$
$(\rho + p)u^\mu u^\nu + p g^{\mu\nu}$
$T^{\mu\nu} = \begin{pmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix} = \begin{pmatrix} \rho & & & \\ & -\rho & & \\ & & -\rho & \\ & & & -\rho \end{pmatrix}$
where $R(\gamma)_{ij} = 2k\gamma_{ij}$
$k = +\frac{1}{r_0^2}$ S^3
$= 0$ E^3
$= -\frac{1}{r_0^2}$ H^3


$3\frac{\dot{a}}{a}\left(\frac{4}{3}\rho\right) \Rightarrow \dot{\rho} = -4\frac{\dot{a}}{a}\rho \Rightarrow$
$\rho_{rad.} = \frac{Const}{a^4}$
$S' = \int \sqrt{-g}\, d^4x \left(\Lambda \right)$
Lorentz invariant.
(vacuum energy)
$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} = \Lambda g_{\mu\nu}$


$\Rightarrow \quad \dot{\rho} = -3\frac{\dot{a}}{a}\left(\frac{4}{3}\rho\right) \Rightarrow \dot{\rho} = -4\frac{\dot{a}}{a}\rho \Rightarrow \rho_{rad.} = \frac{const}{a^4}$
$S = \int \sqrt{-g}\, d^4x \left(\Lambda \right)$
Cosmological Constant
Λ (vacuum energy) : Lorentz invariant.
$p = -\rho$
$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) = 0 \Rightarrow \rho = const$
$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} = \Lambda g_{\mu\nu}$

radiation : $p = \frac{1}{3}\rho \Rightarrow \dot{\rho} = -3\frac{\dot{a}}{a}(\ldots)$



$\lambda \propto a(t)$

$\Rightarrow E_\gamma = h\nu = \frac{hc}{\lambda} \propto \frac{1}{a}$.

Cosmological Constant

Λ · (vacuu… energ…

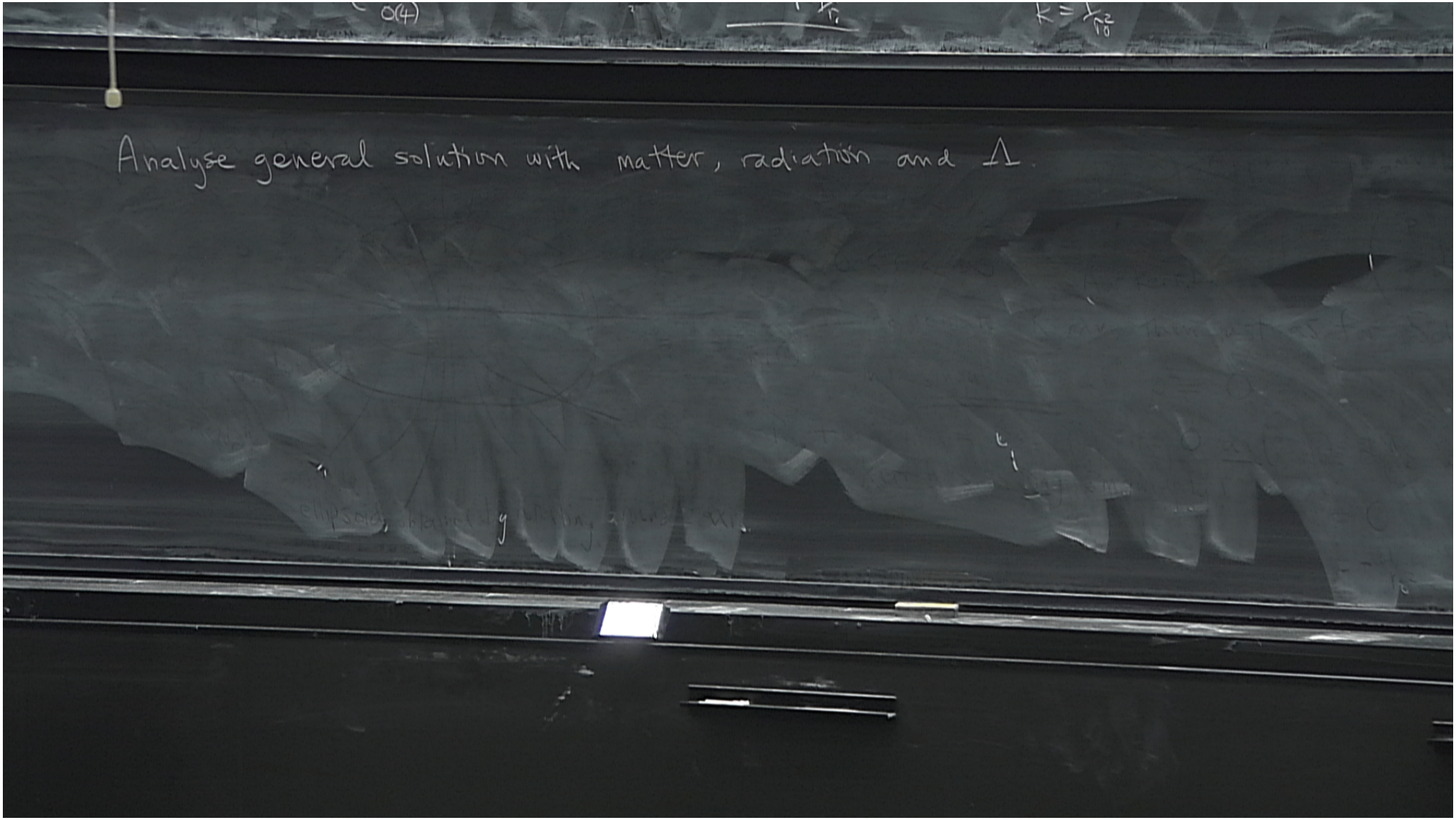
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$.

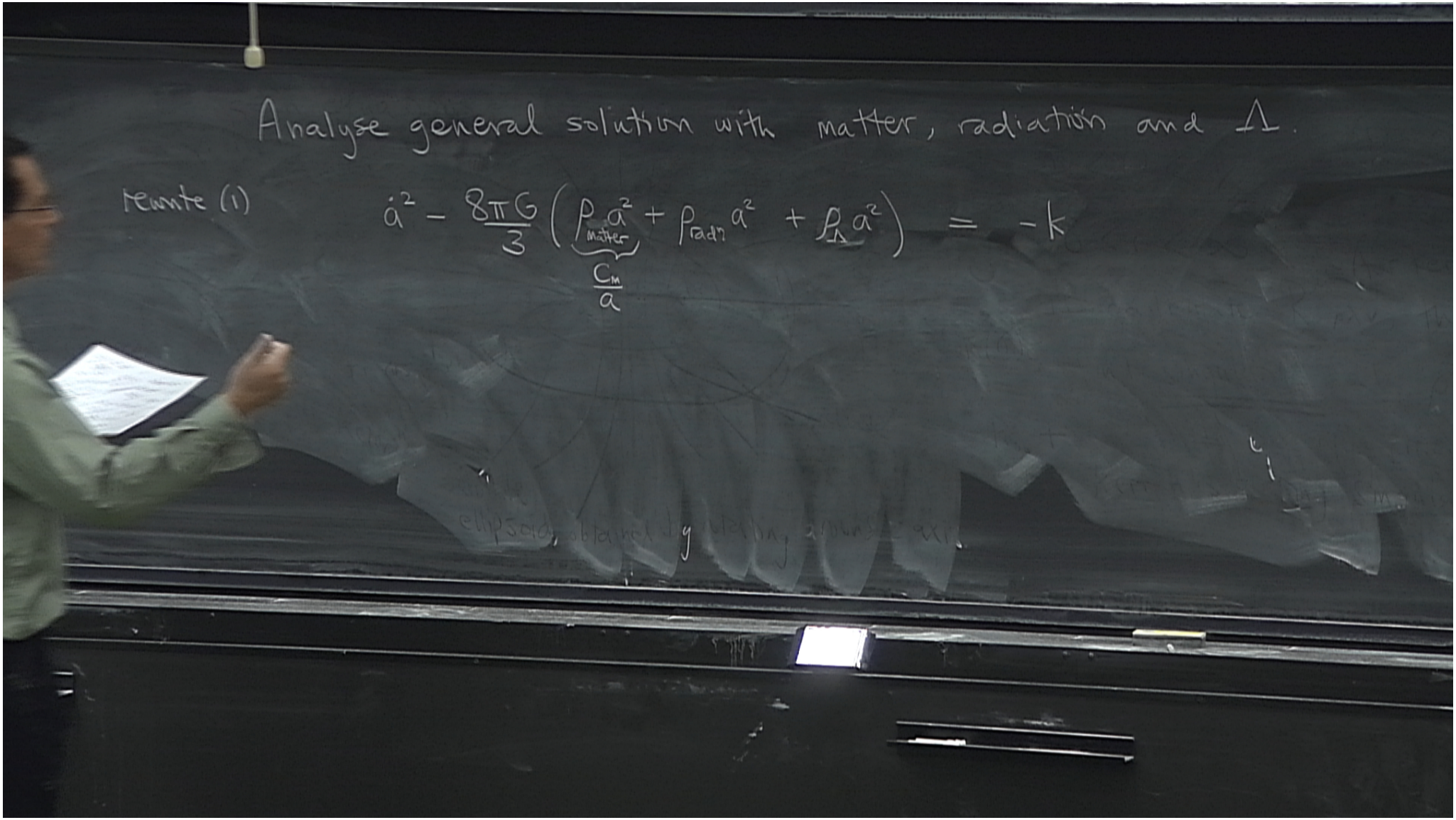
$= \text{const.} \equiv H_0^2$

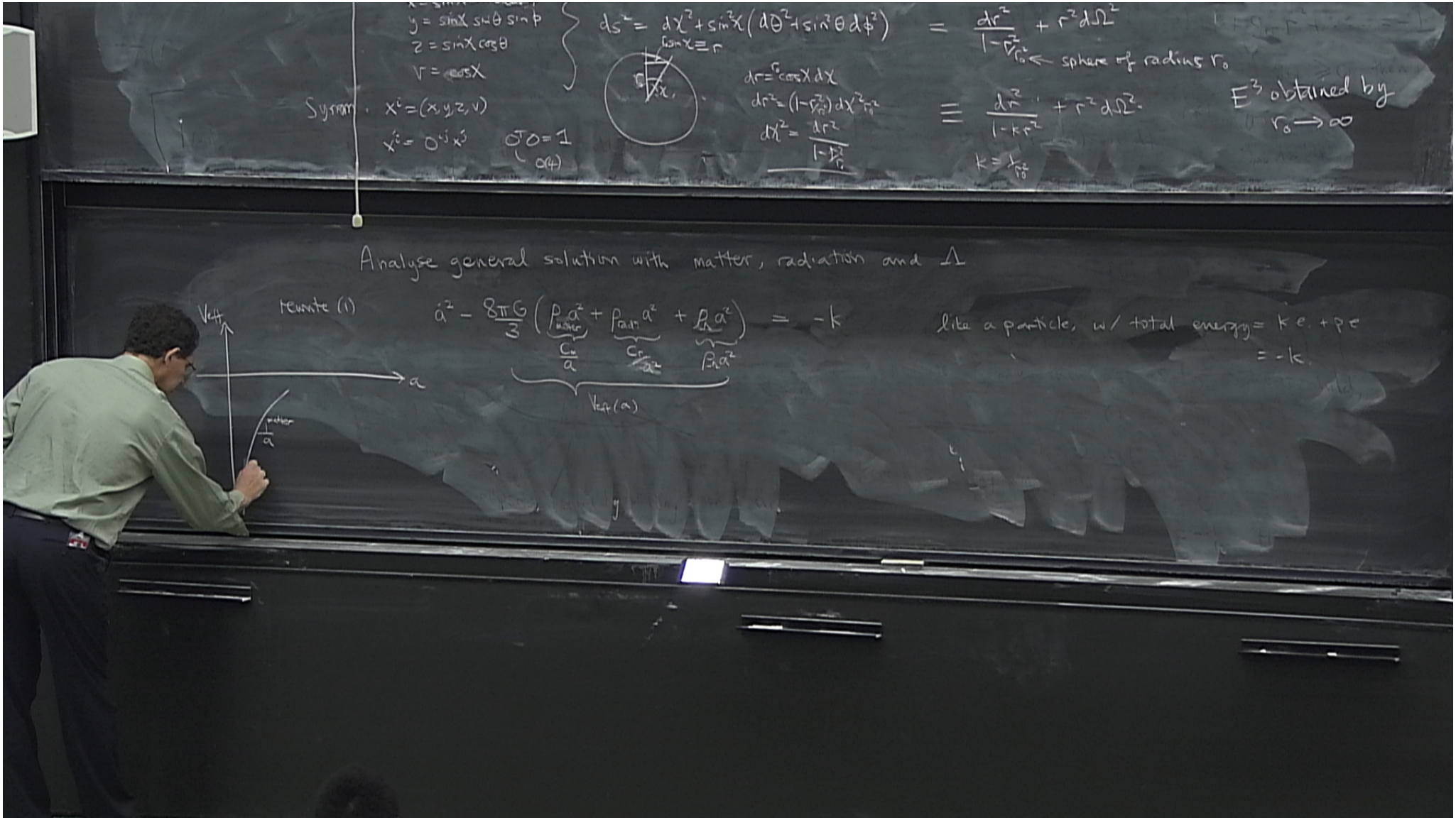
$\rho \propto \frac{E_\gamma}{a^3} \propto \frac{1}{a^4}$.

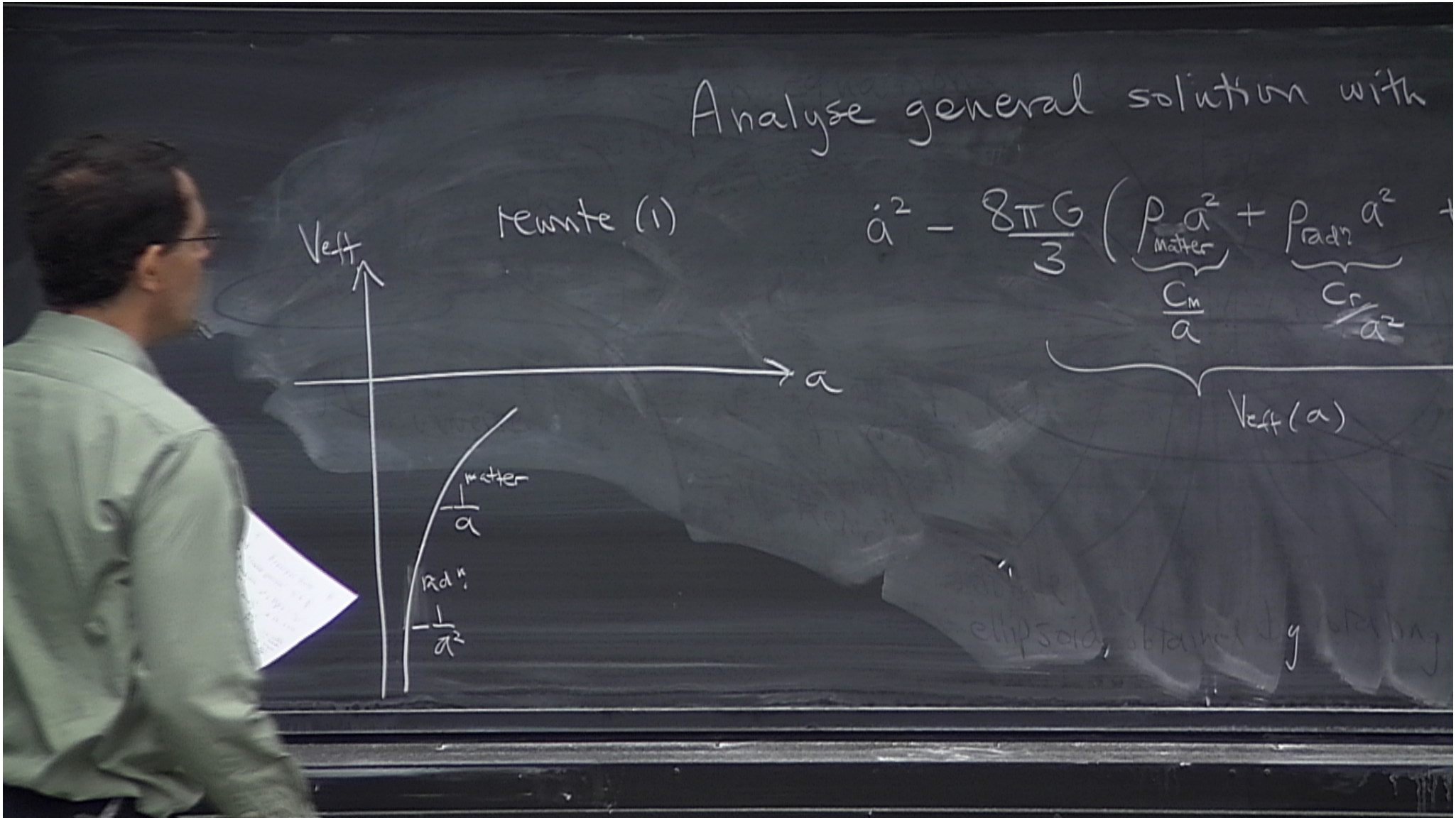
$\Rightarrow \boxed{a \propto e^{H_0 t}}$

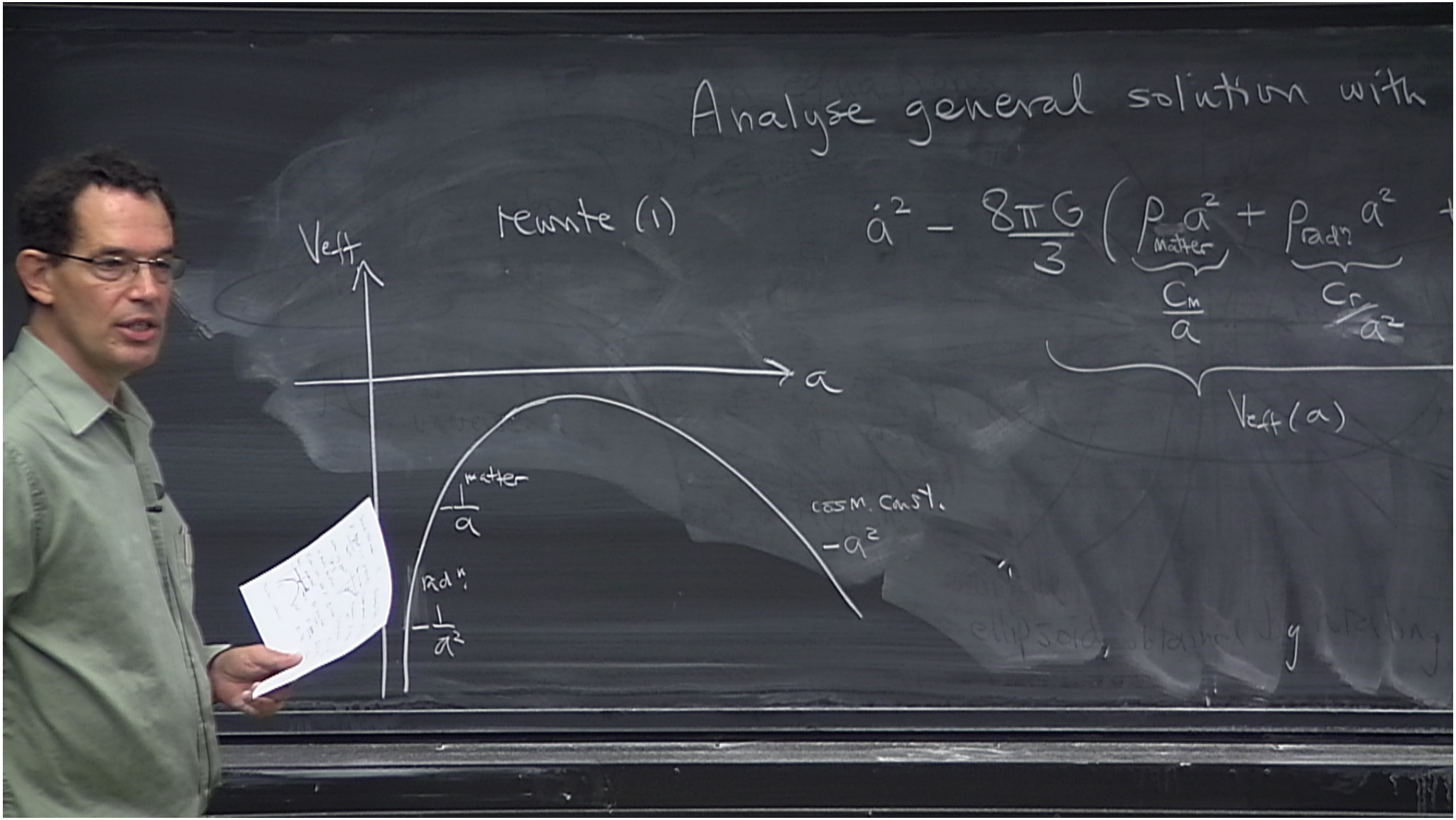
H… 3 boosts 3 rotations


O(4)
Analyse general solution with matter, radiation and Λ.


Analyse general solution with matter, radiation and Λ.
rewrite (1)
$\dot{a}^2 - \frac{8\pi G}{3}\left(\underbrace{\rho_{matter}\, a^2}_{\frac{C_m}{a}} + \rho_{rad}\, a^2 + \rho_\Lambda a^2\right) = -k$


$y = \sin\chi \sin\theta \sin\phi$
$z = \sin\chi \cos\theta$
$w = \cos\chi$
System: $x^i = (x, y, z, w)$
$x'^i = O^i{}_j x^j$
$O^T O = 1$ (O(4))
$ds^2 = d\chi^2 + \sin^2\chi (d\theta^2 + \sin^2\theta\, d\phi^2)$
$= \frac{dr^2}{1 - r^2/r_0^2} + r^2 d\Omega^2$
$\sin\chi \equiv r$
$dr = r_0 \cos\chi\, d\chi$
$dr^2 = (1 - r^2/r_0^2)\, d\chi^2 r_0^2$
$d\chi^2 = \frac{dr^2}{1 - r^2/r_0^2}$
$\equiv \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2$
$k = 1/r_0^2$
sphere of radius r_0
E^3 obtained by $r_0 \to \infty$
Analyse general solution with matter, radiation and Λ
rewrite (1)
$\dot{a}^2 - \frac{8\pi G}{3}\left(\underbrace{\rho_{matter} a^2}_{\frac{C_M}{a}} + \underbrace{\rho_{rad} a^2}_{\frac{C_r}{a^2}} + \underbrace{\rho_\Lambda a^2}_{\rho_\Lambda a^2}\right) = -k$
$V_{eff}(a)$
like a particle, w/ total energy = k.e. + p.e. $= -k$
V_{eff}
a
matter
$\frac{1}{a}$


Analyse general solution with
rewrite (1)
$\dot{a}^2 - \frac{8\pi G}{3}\left(\rho_{matter} a^2 + \rho_{radn} a^2 + \right.$
$\frac{C_M}{a}$
C_r/a^2
$V_{eff}(a)$
V_{eff}
a
matter
$-\frac{1}{a}$
Radn
$-\frac{1}{a^2}$

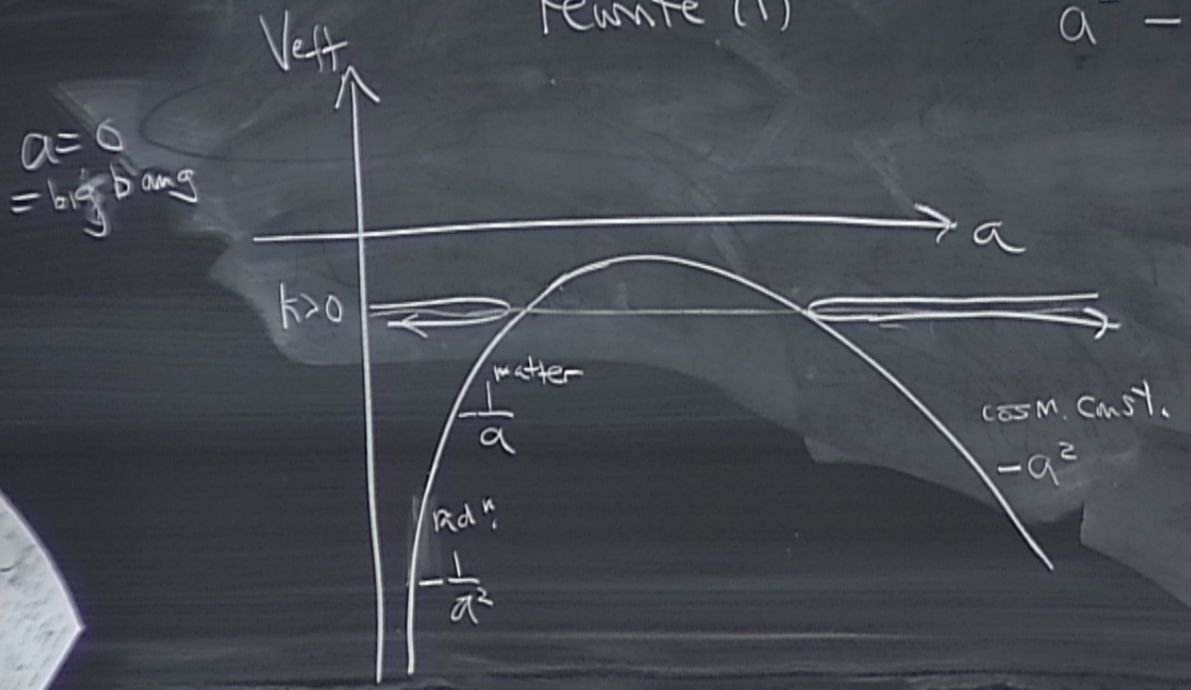

Analyse general solution with
rewrite (1)
$\dot{a}^2 - \frac{8\pi G}{3}\left(\rho_{matter}\, a^2 + \rho_{rad?}\, a^2 + \right.$
$\frac{C_M}{a}$
C_r/a^2
$V_{eff}(a)$
V_{eff}
a
matter
$-\frac{1}{a}$
cosm. const.
$-a^2$
Rad n.
$-\frac{1}{a^2}$

Analyse general solution with

remite (1)

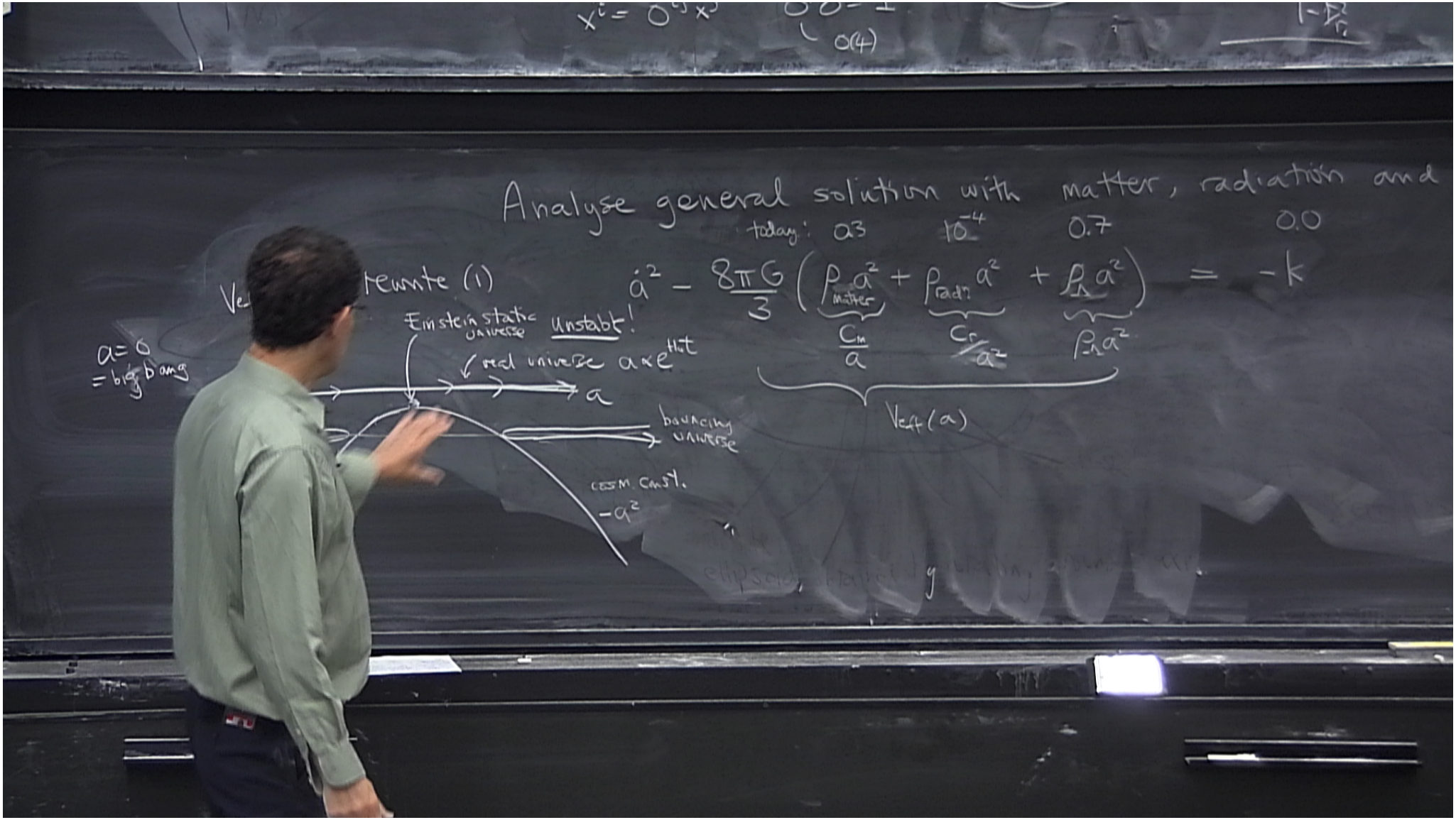
$$\dot{a}^2 - \frac{8\pi G}{3}\left(\underbrace{\rho_{\text{matter}}\, a^2}_{\frac{C_m}{a}} + \underbrace{\rho_{\text{radn}}\, a^2}_{\frac{C_r}{a^2}} + \right)$$

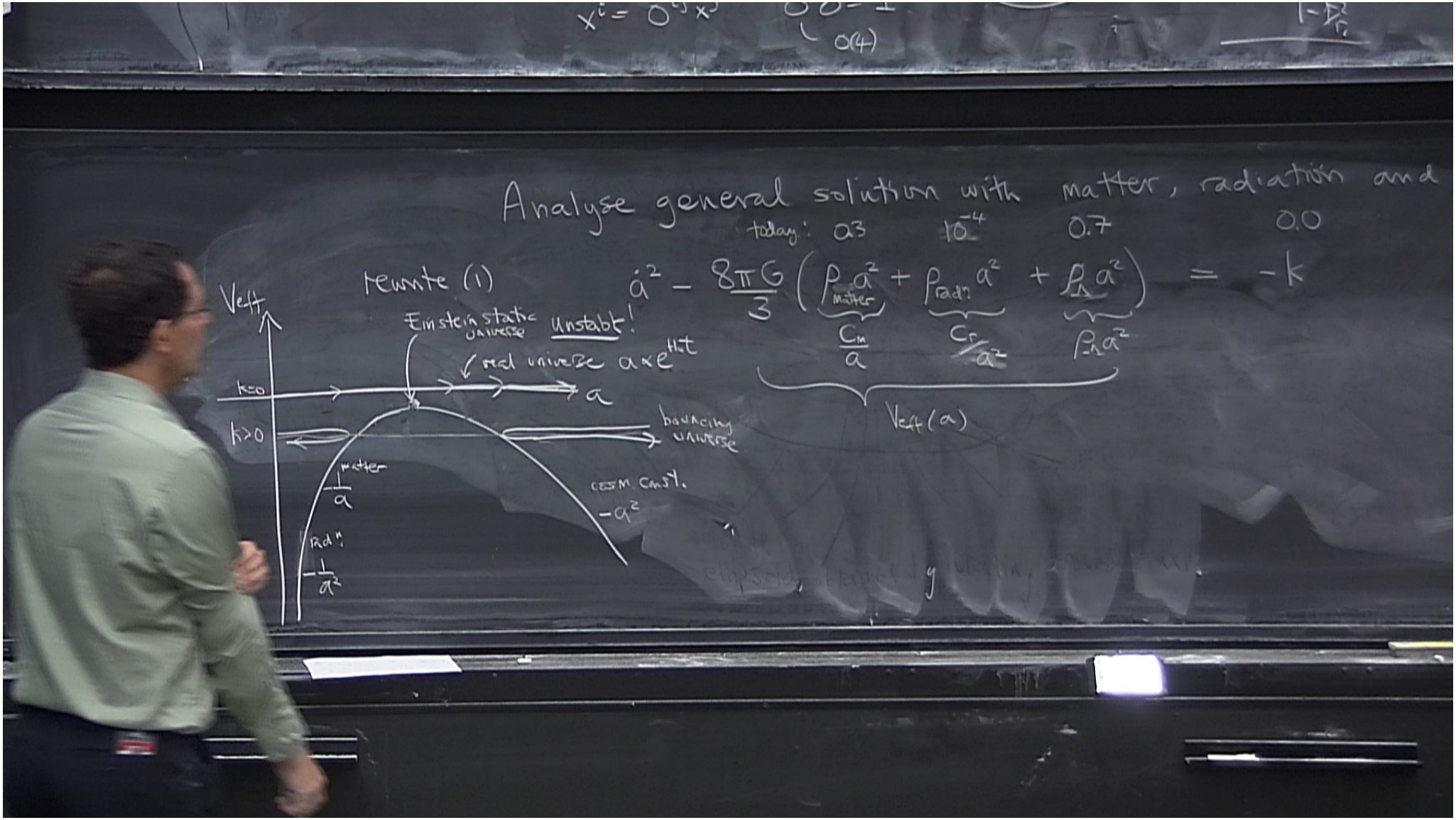
$V_{\text{eff}}(a)$

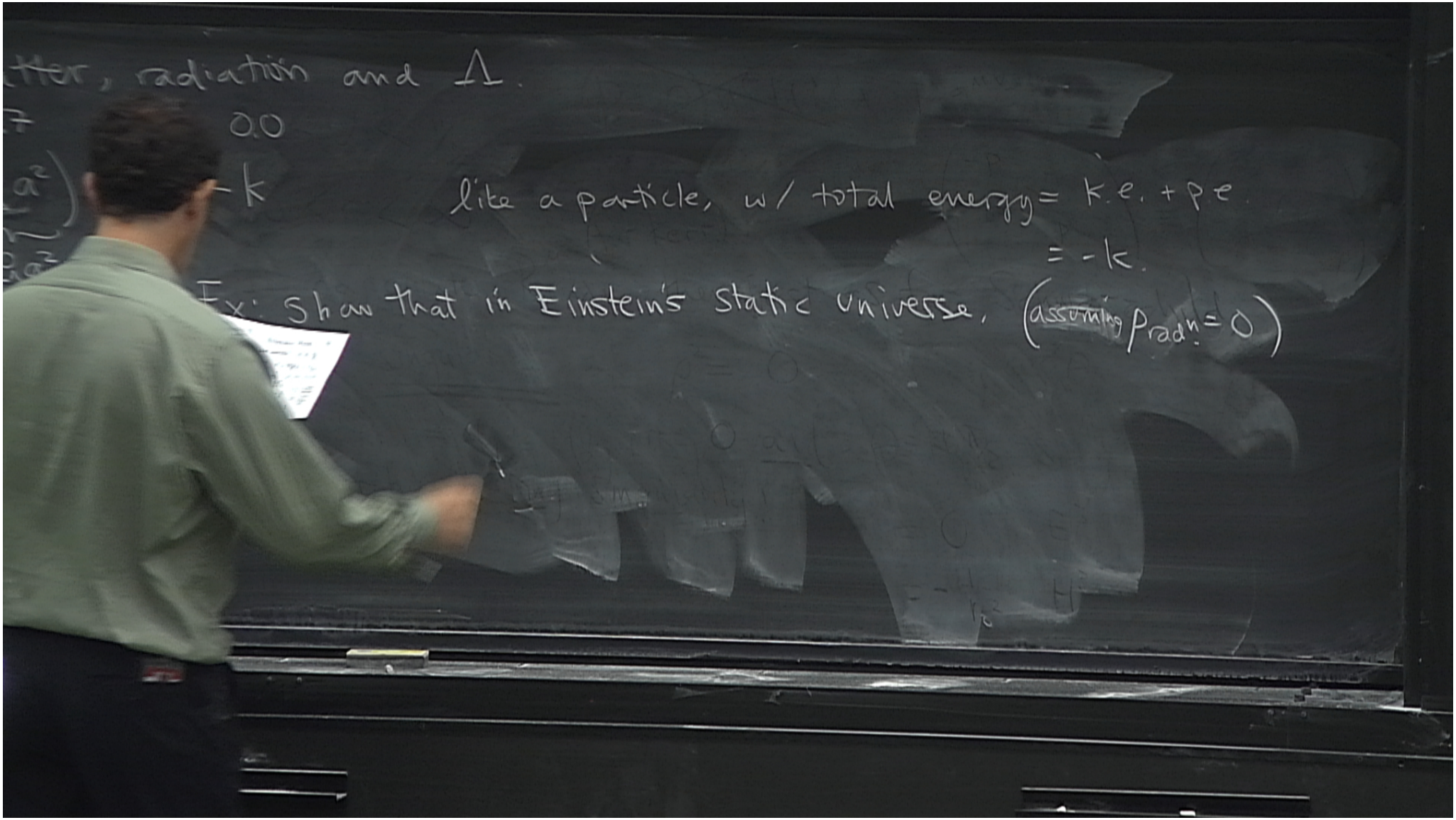


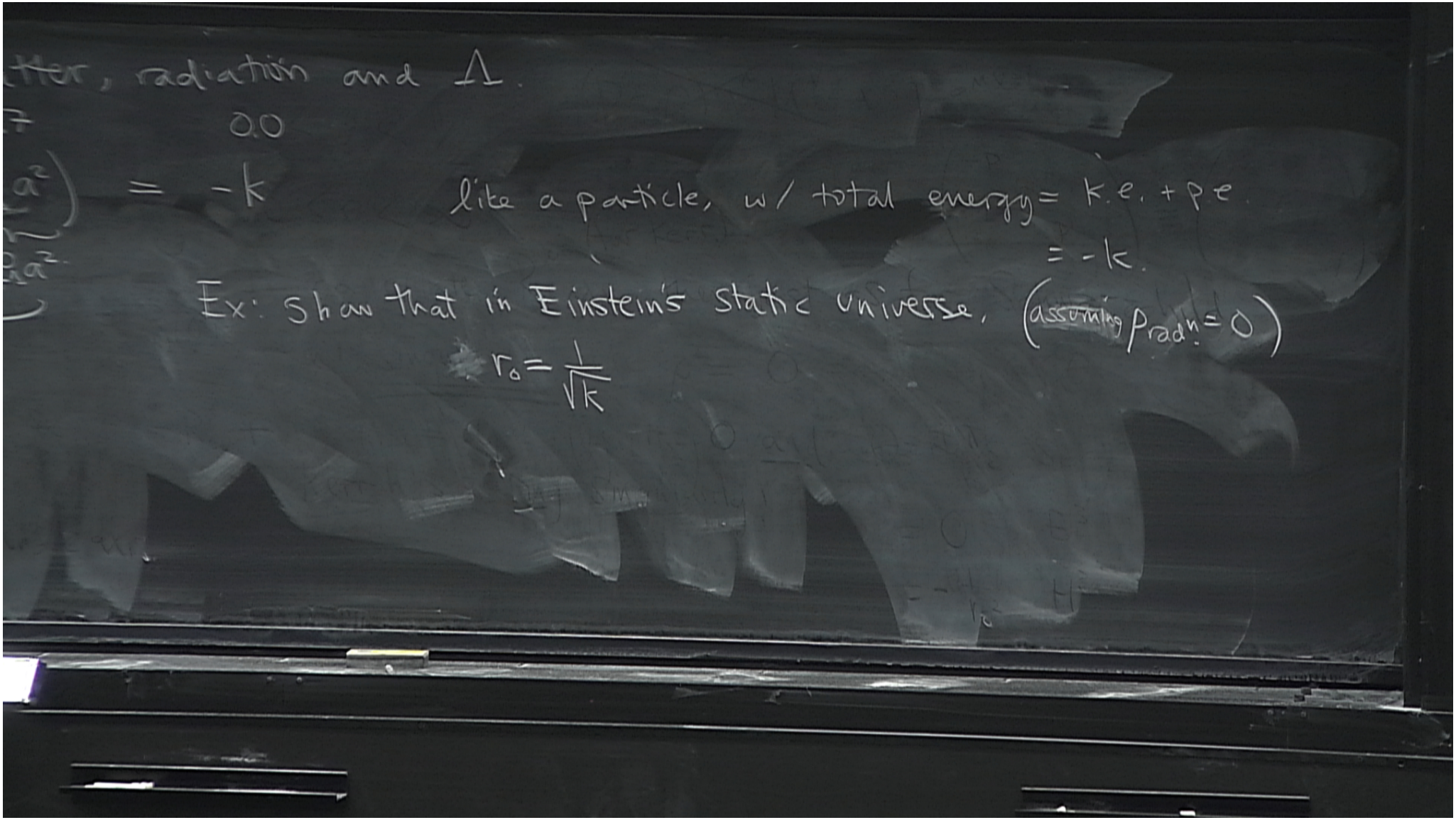
$a=0$
= big bang

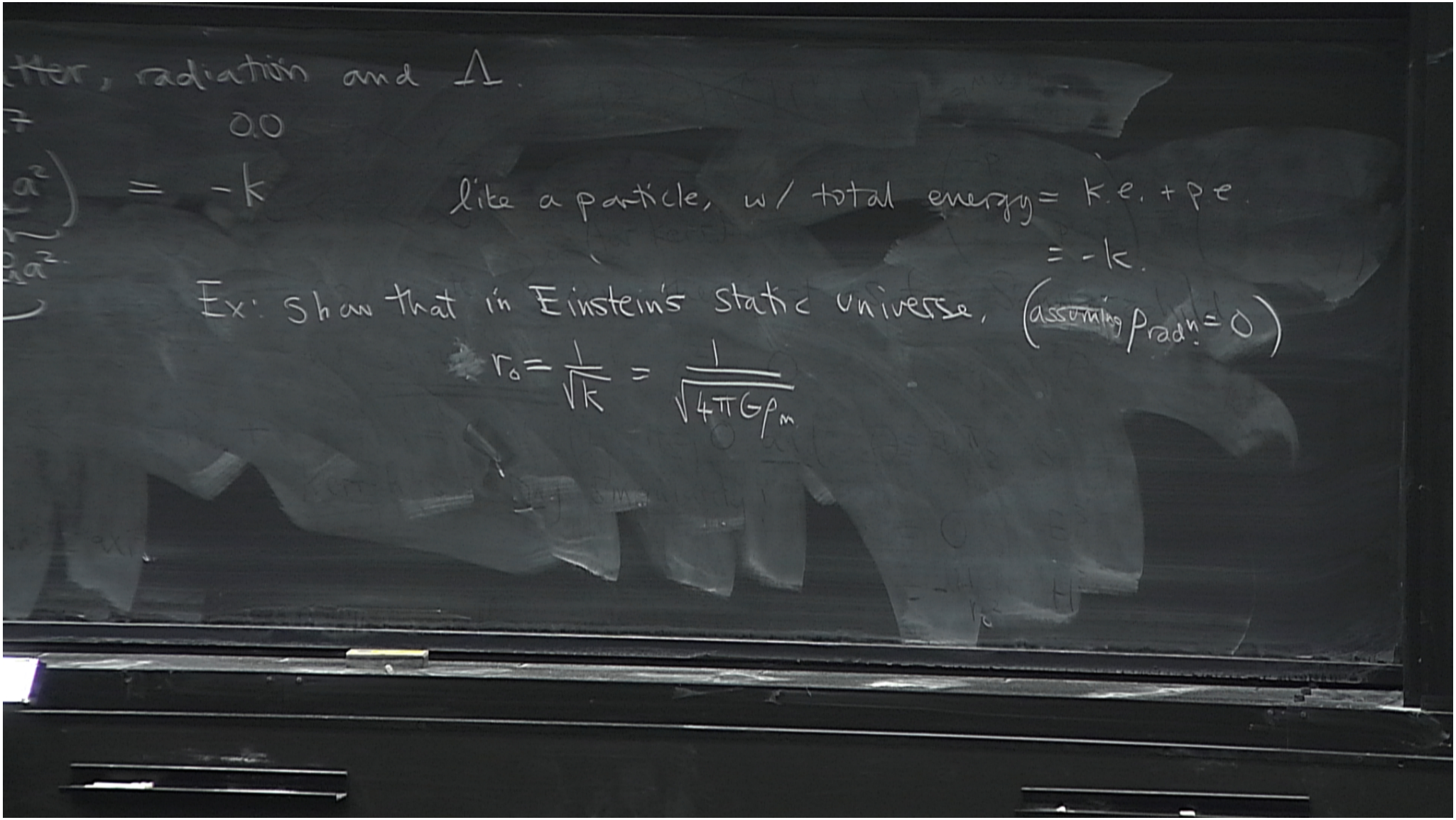
ellipsoid, obtained by stretching

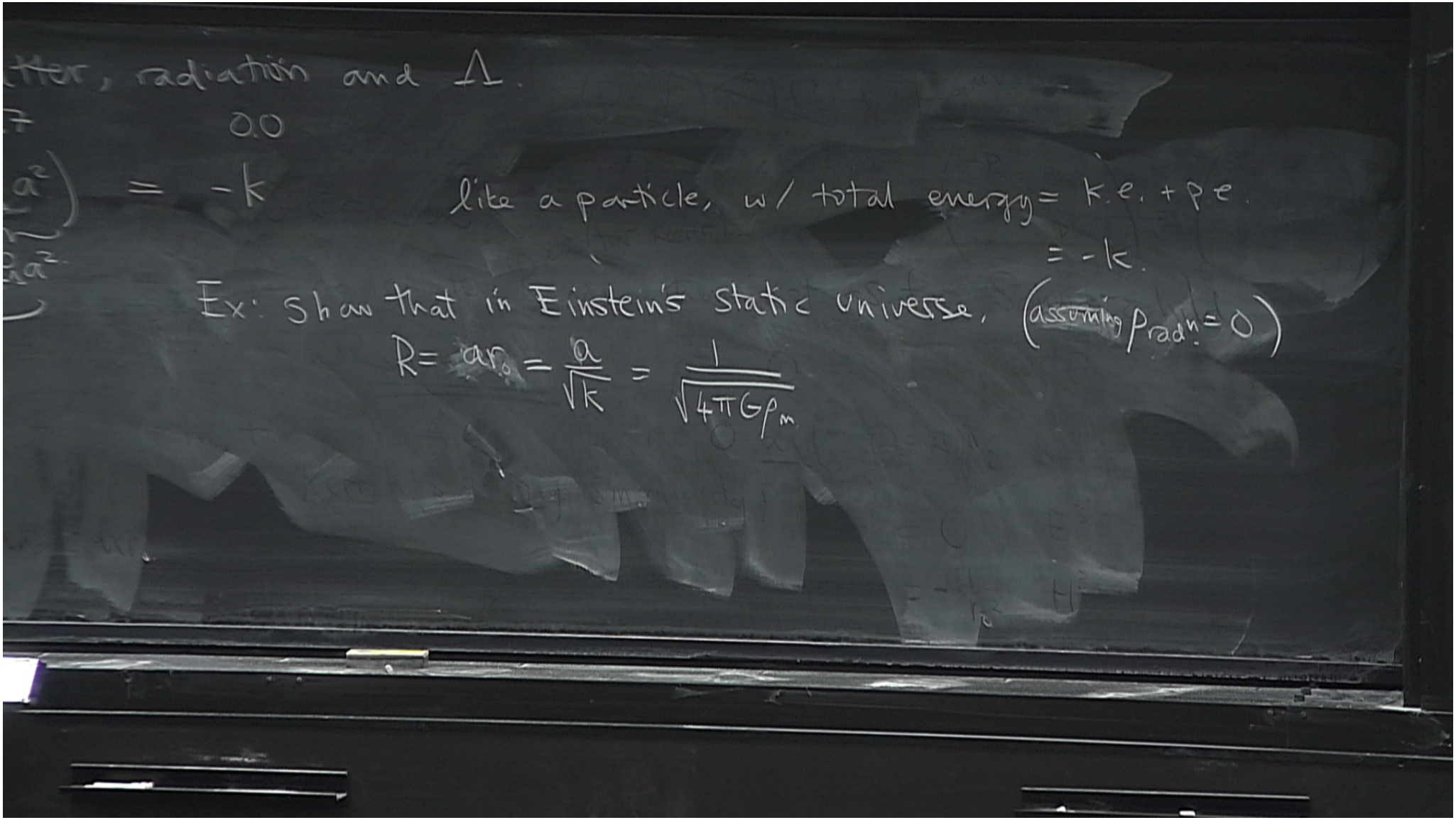

Analyse general solution with matter, radiation and
today: 0.3 10^{-4} 0.7 0.0
$\dot{a}^2 - \frac{8\pi G}{3}\left(\rho_{matter}a^2 + \rho_{rad^n}a^2 + \rho_\Lambda a^2\right) = -k$
$\frac{C_m}{a}$ $\frac{C_r}{a^2}$ $\rho_\Lambda a^2$
$V_{eff}(a)$
rewrite (1)
Einstein static universe Unstable!
real universe ave
$a=0$ = big bang
a
bouncing universe
cosm const.
$-a^2$

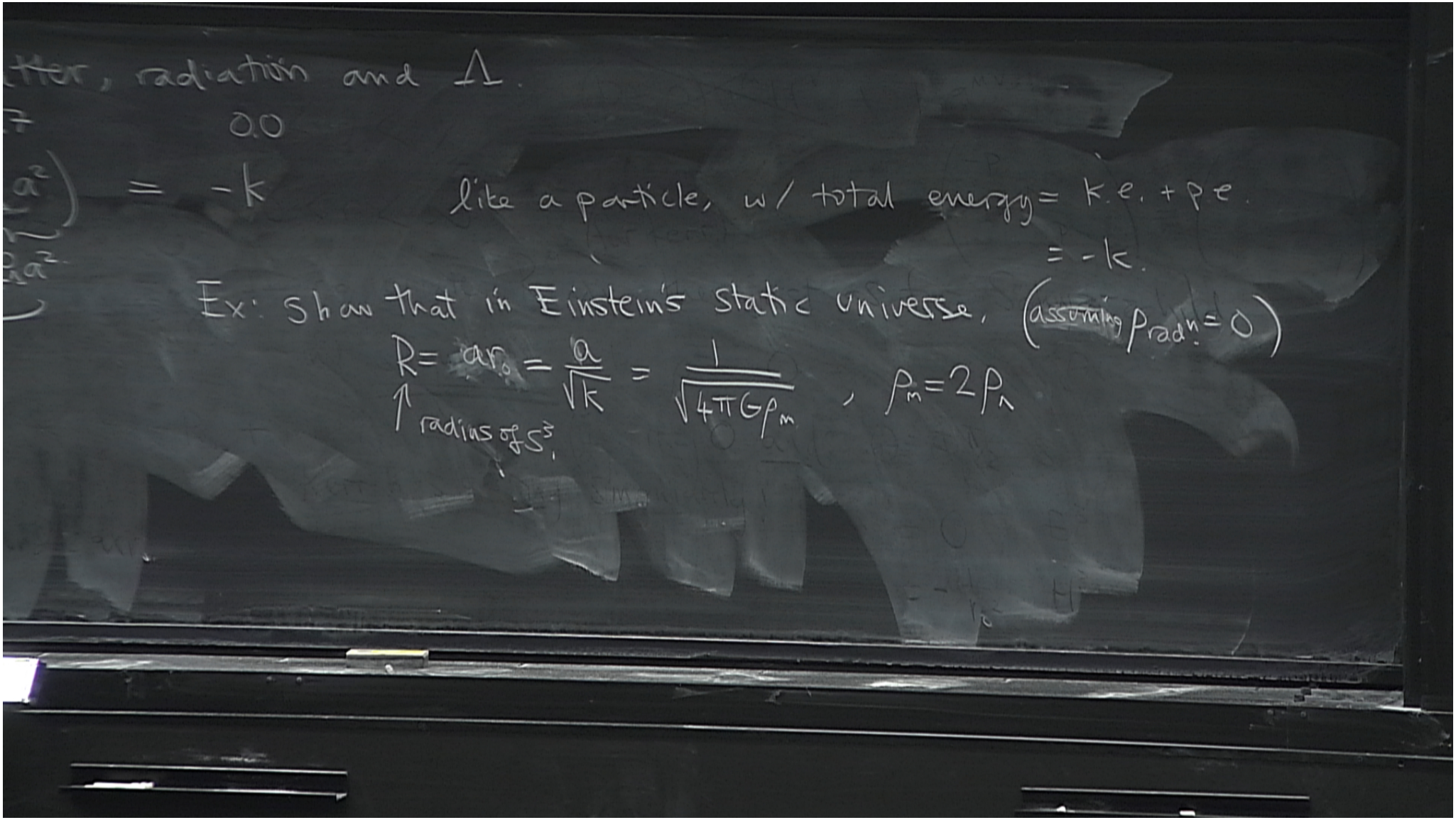

Analyse general solution with matter, radiation and
today: Ω3 10^-4 0.7 0.0
rewrite (1)
$\dot{a}^2 - \frac{8\pi G}{3}\left(\rho_{matter}a^2 + \rho_{radn}a^2 + \rho_\Lambda a^2\right) = -k$
$\frac{C_m}{a}$ $\frac{C_r}{a^2}$ $\rho_\Lambda a^2$
$V_{eff}(a)$
V_{eff}
Einstein static universe
unstable!
real universe a ∝ e^Ht
k=0
k>0
a
bouncing universe
matter
$-\frac{1}{a}$
Radn.
$-\frac{1}{a^2}$
cosm cnst.
$-a^2$

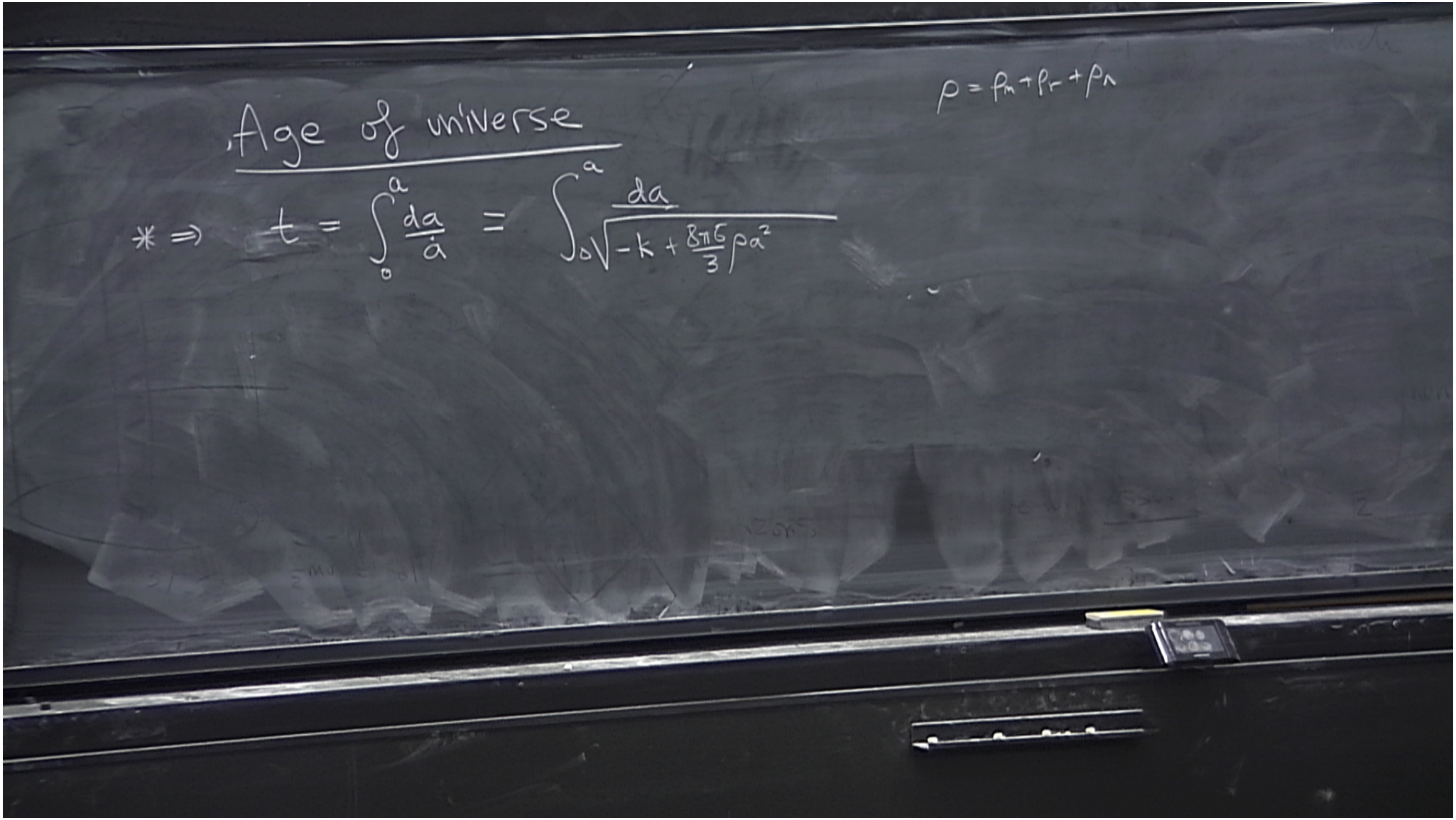

tter, radiation and Λ.
0.0
-k
like a particle, w/ total energy = k.e. + p.e.
= -k.
Ex: Show that in Einstein's static universe, (assuming ρrad = 0)


tter, radiation and Λ.
0.0
$= -k$
like a particle, w/ total energy = k.e. + p.e.
$= -k$.
Ex: Show that in Einstein's static universe, (assuming $\rho_{rad} = 0$)
$r_0 = \frac{1}{\sqrt{k}}$


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Ex: Show that in Einstein's static universe, (assuming $\rho_{rad} = 0$)
$r_0 = \frac{1}{\sqrt{k}} = \frac{1}{\sqrt{4\pi G \rho_m}}$


tter, radiation and Λ.
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Ex: Show that in Einstein's static universe, (assuming $\rho_{rad} = 0$)
$R = ar_0 = \frac{a}{\sqrt{k}} = \frac{1}{\sqrt{4\pi G \rho_m}}$


tter, radiation and Λ.
0.0
$= -k$
like a particle, w/ total energy = k.e. + p.e.
$= -k$.
Ex: Show that in Einstein's static universe, (assuming ρ_{rad} = 0)
$R = ar_0 = \frac{a}{\sqrt{k}} = \frac{1}{\sqrt{4\pi G \rho_m}}$, $\rho_m = 2\rho_\Lambda$
↑ radius of S^3.


Age of universe
$\rho = \rho_m + \rho_r + \rho_\Lambda$
$* \Rightarrow \quad t = \int_0^a \frac{da}{\dot{a}} = \int_0^a \frac{da}{\sqrt{-k + \frac{8\pi G}{3}\rho a^2}}$

Age of universe

$\rho = \rho_m + \rho_r + \rho_\Lambda$

$* \Rightarrow \quad t = \int_0^a \frac{da}{\dot{a}} = \int_0^a \frac{da}{\sqrt{-k + \frac{8\pi G}{3}\rho a^2}} \Rightarrow t(a)$

Expansion rate: metric $-dt^2 + a^2 \gamma_{ij} dx^i dx^j$: separation of points at fixed $\underline{x}$ grows like $a(t)$.

$\Rightarrow d \propto a(t)$: $\frac{\dot{d}}{d} = \frac{\dot{a}}{a}$ Hubble's law.

$\dot{d} = H d$

Age of universe

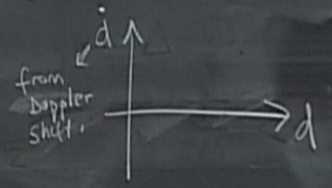
$\rho = \rho_m + \rho_r + \rho_\Lambda$

$* \Rightarrow \quad t = \int_0^a \frac{da}{\dot{a}} = \int_0^a \frac{da}{\sqrt{-k + \frac{8\pi G}{3}\rho a^2}} \Rightarrow t(a)$

...nsion rate: metric $-dt^2 + a^2 \gamma_{ij} dx^i dx^j$: separation of points at fixed $\underline{x}$ grows like $a(t)$.

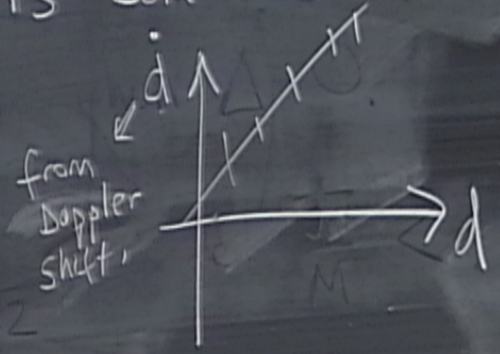
$\Rightarrow d \propto a(t)$: $\frac{\dot{d}}{d} = \frac{\dot{a}}{a}$ Hubble's law. H is sometimes called Hubble

$\dot{d} = H d$



fixed $\underline{x}$ grows like $a(t)$.

law. H is sometimes called Hubble constant.



Hubble 1928. $H = 550$ km s^{-1}/Mpc.

1pc $= 3.26$ lt years.

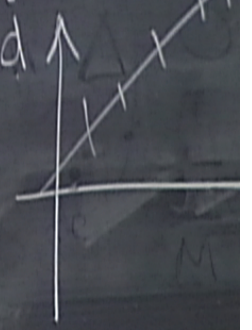
$\frac{GM}{c^2}$ is called extremal limit

$$\int_0^a \frac{da}{\sqrt{-k + \frac{8\pi G}{3}\rho a^2}} \Rightarrow t(a)$$

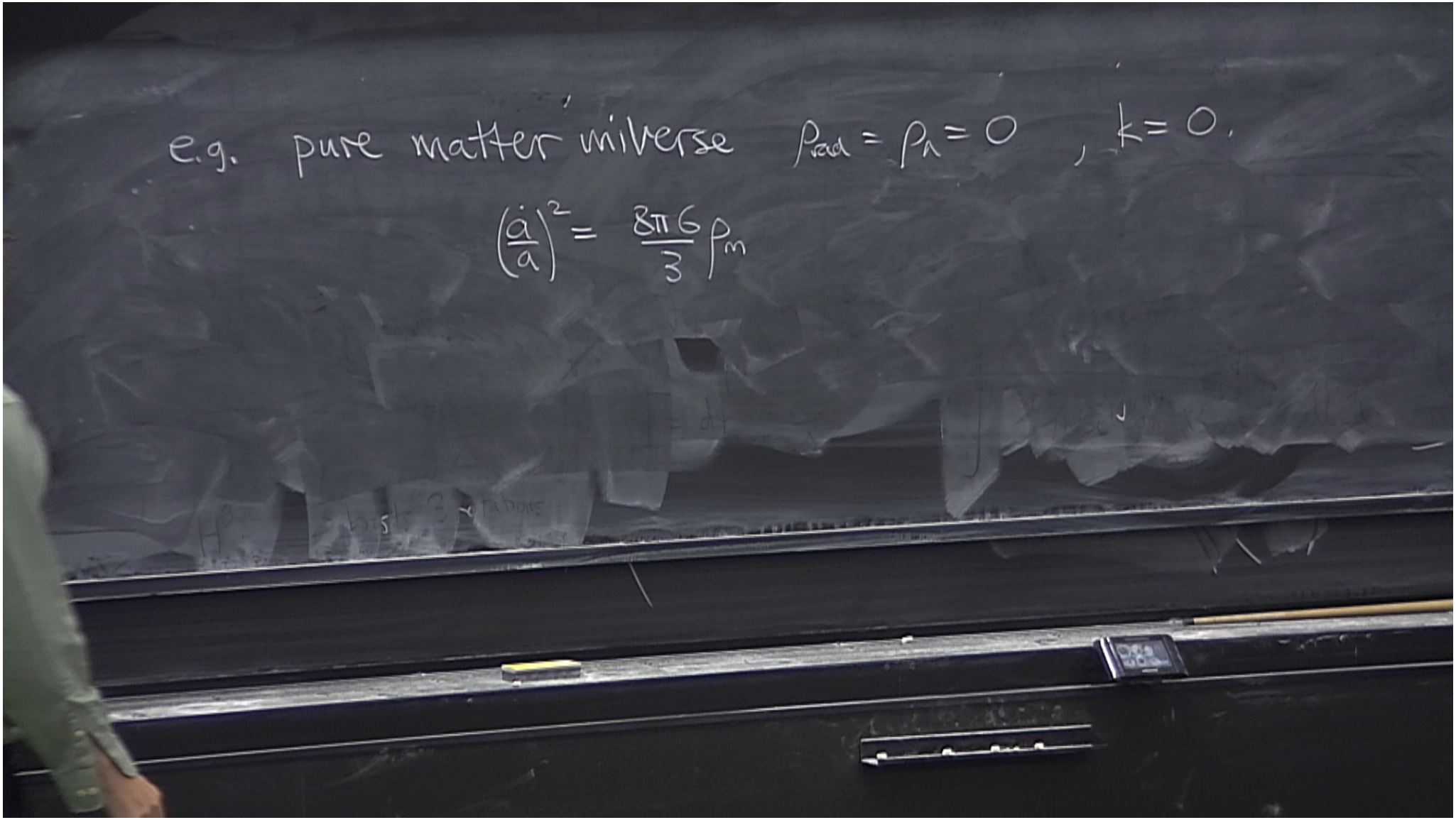
$dt^2 + a^2 \gamma_{ij} dx^i dx^j$: separation of points at fixed $\underline{x}$ grows like $a(t)$

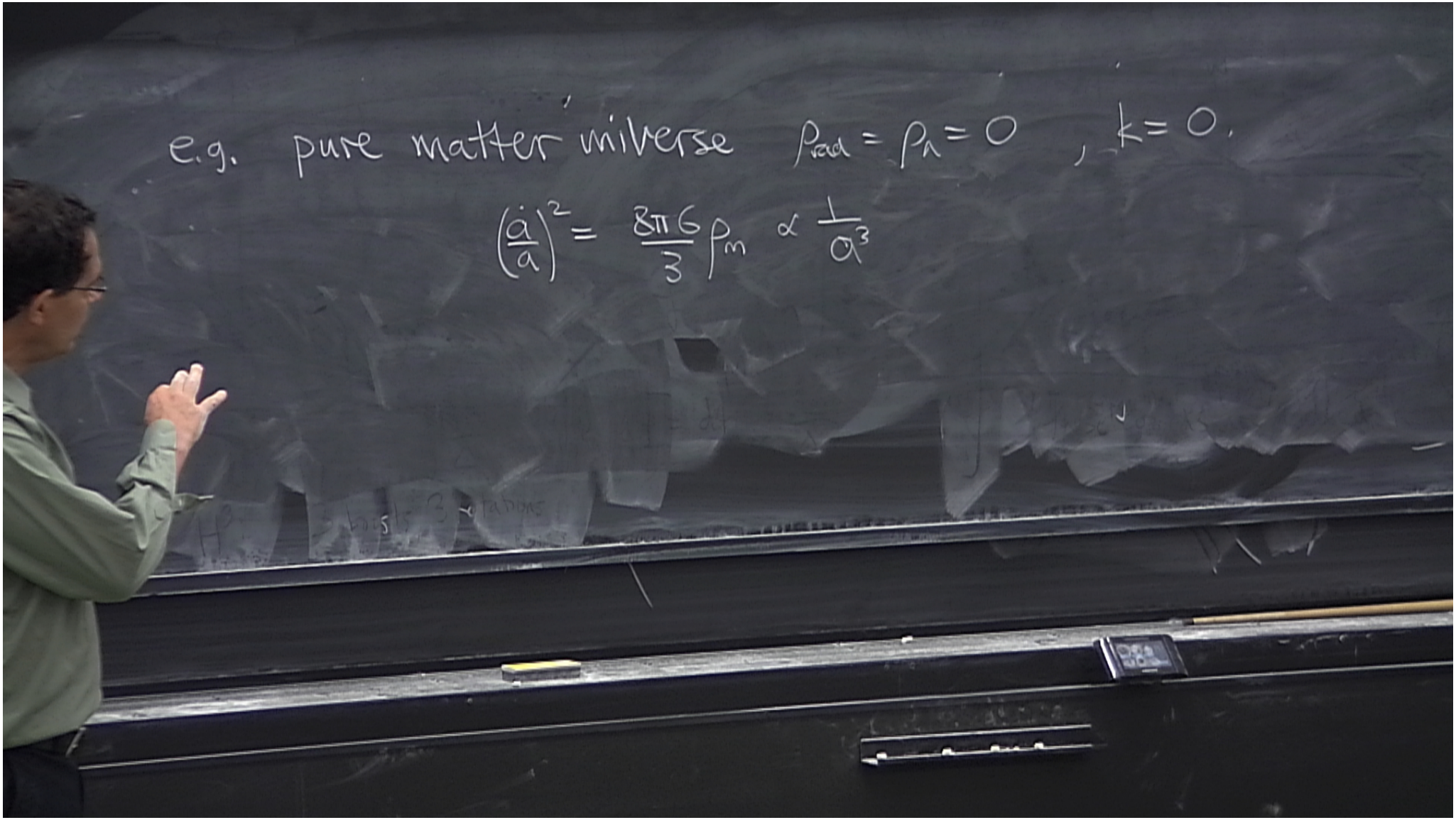
$d \propto a(t)$: $\frac{\dot{d}}{d} = \frac{\dot{a}}{a}$ Hubble's law. H is sometimes

$\dot{d} = H d$



Modern measurements give $H \approx 65 \pm 10 \text{ km s}^{-1}/\text{Mpc}$.


e.g. pure matter universe $\rho_{rad} = \rho_\Lambda = 0$, $k = 0$.
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m$


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$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{k}{a^2}$: using this, can get t.

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Doppler shift.

d

1pc

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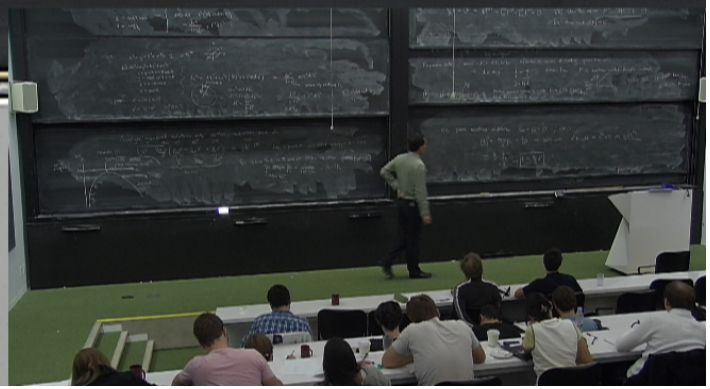
Doppler shift.

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$\Rightarrow \frac{\dot{a}}{a} = \frac{2}{3t} = H \Rightarrow \boxed{t = \frac{2}{3}H^{-1}}$



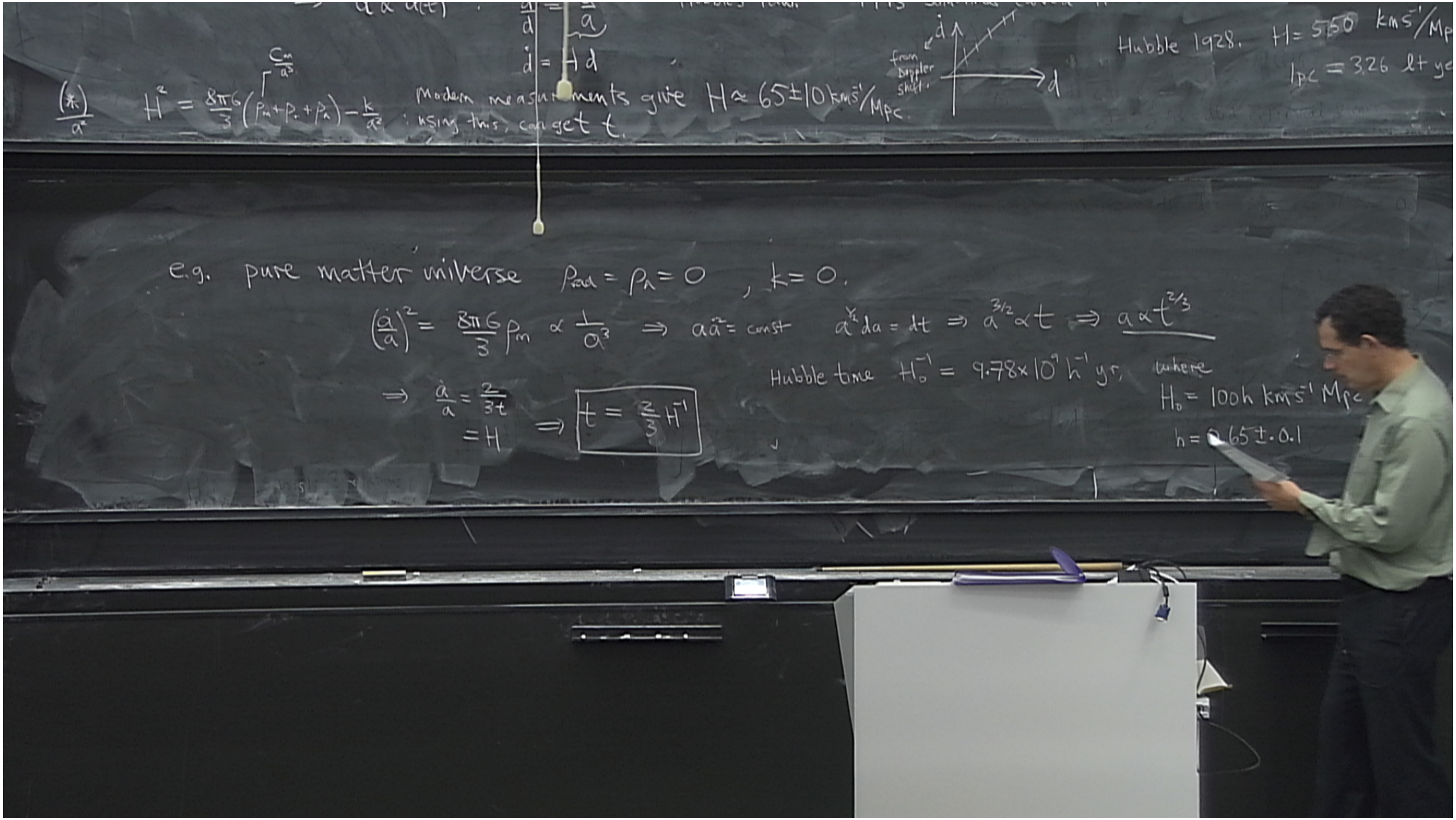
$k = 0$.

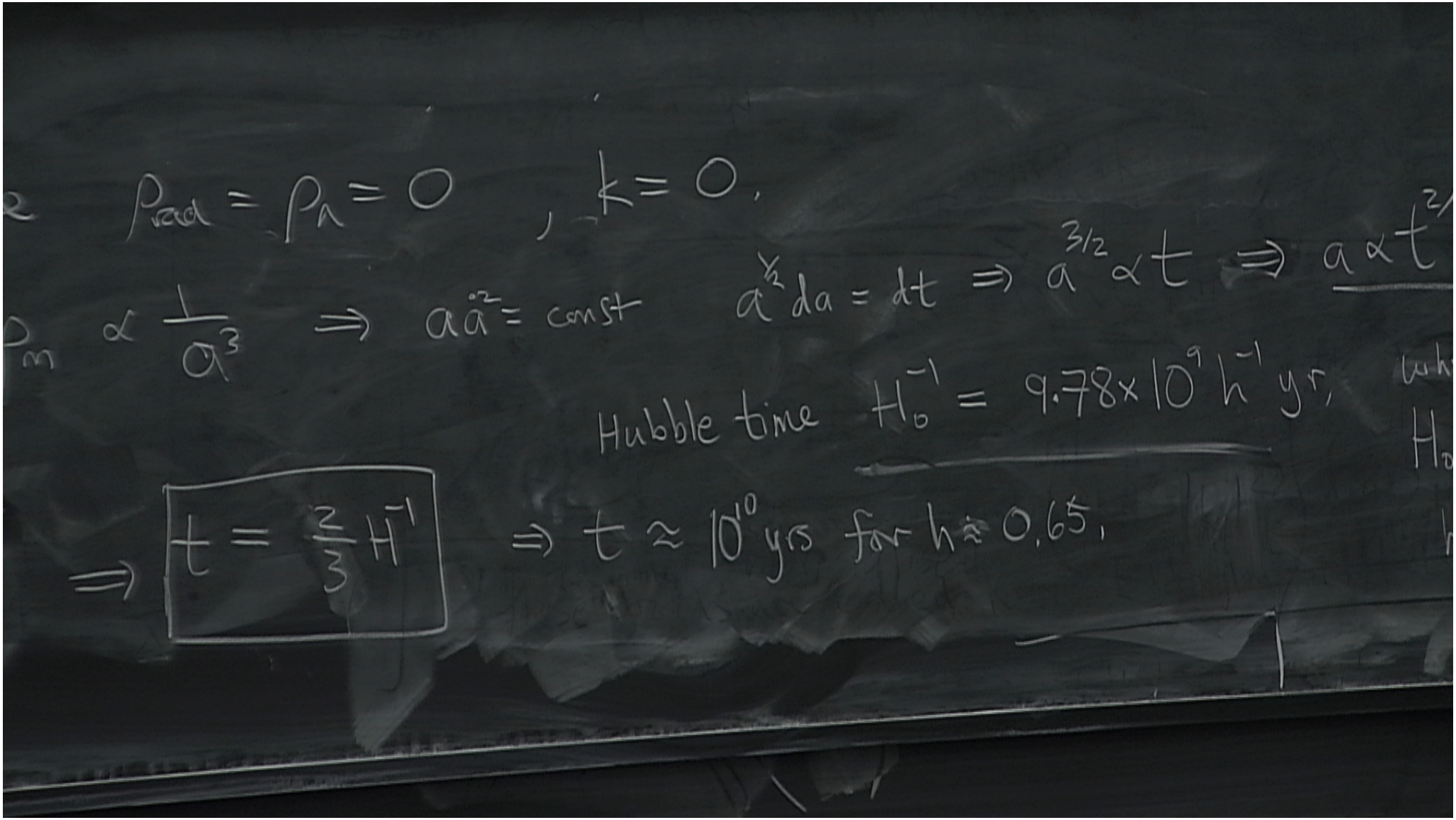
const $a^{1/2} da = dt \Rightarrow a^{3/2} \propto t \Rightarrow \underline{a \propto t^{2/3}}$

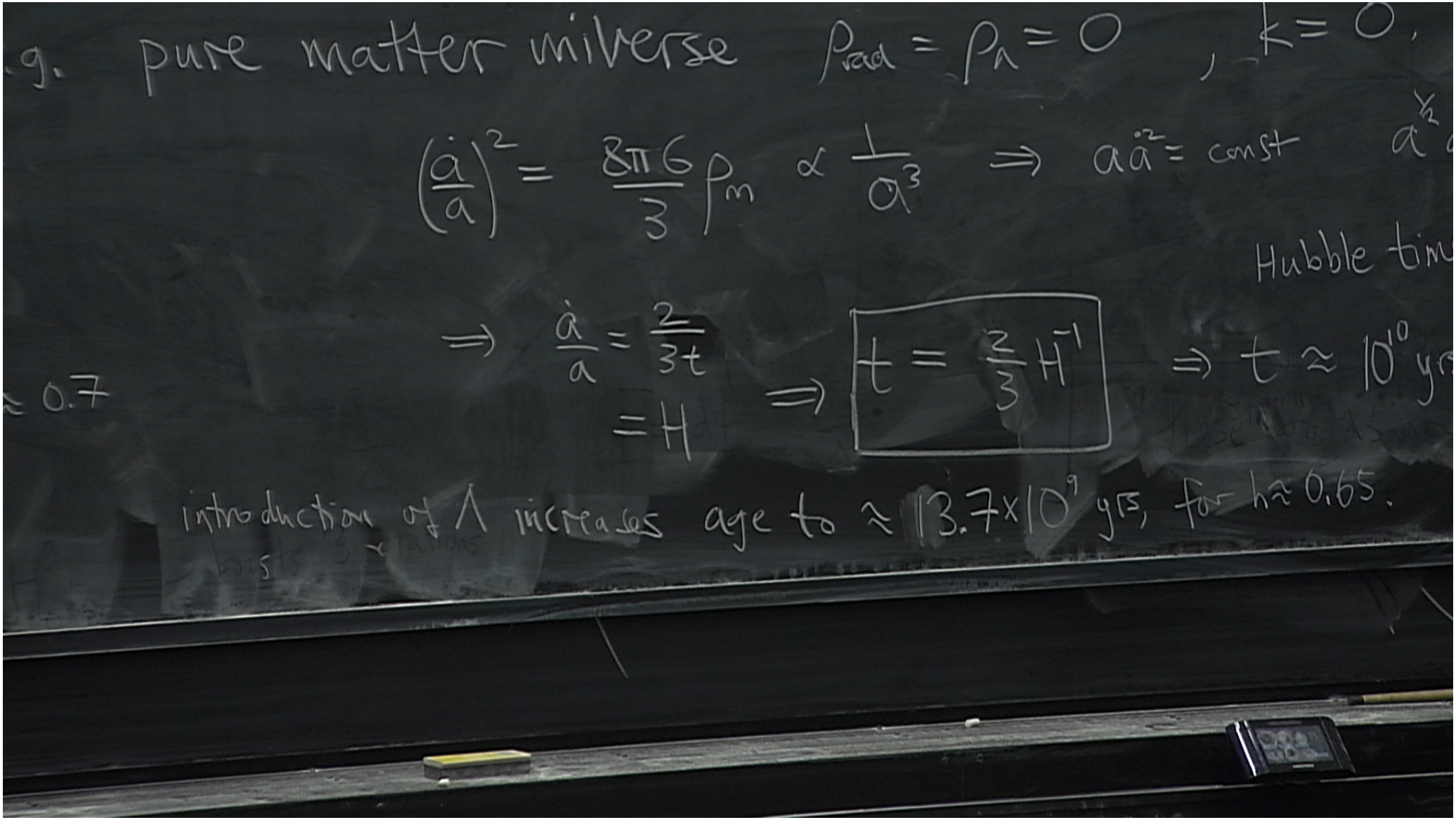
Hubble time $H_0^{-1} = 9.78 \times 10^9 h^{-1}$ yr; where

$H_0 = 100h$ km s^{-1} Mpc^{-1}

$h = 0.65 \pm 0.1$


$\frac{(*)}{a^2}$ $H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{k}{a^2}$
$\frac{C^m}{a^3}$
Modern measurements give $H \approx 65 \pm 10\ km s^{-1}/Mpc$.
using this, can get t.
$\dot{d} = H d$
from Doppler shift.
Hubble 1928. $H = 550\ km s^{-1}/Mp$
$1pc = 3.26\ lt\ ye$
e.g. pure matter universe $\rho_{rad} = \rho_\Lambda = 0$, $k = 0$.
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m \propto \frac{1}{a^3} \Rightarrow a\dot{a}^2 = const$ $a^{\frac{1}{2}} da = dt \Rightarrow a^{3/2} \propto t \Rightarrow a \propto t^{2/3}$
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Hubble time $H_0^{-1} = 9.78 \times 10^9 h^{-1}$ yrs,
$\Rightarrow \boxed{t = \frac{2}{3} H^{-1}}$
$\Rightarrow t \approx 10^{10}$ yrs for $h \approx 0.65$.


g. pure matter universe $\rho_{rad} = \rho_\Lambda = 0$, $k = 0$.
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m \propto \frac{1}{a^3} \Rightarrow a\dot{a}^2 = \text{const}$
$a^{\frac{1}{2}}$
Hubble tim
$\Rightarrow \frac{\dot{a}}{a} = \frac{2}{3t}$
$= H$
$\Rightarrow \boxed{t = \frac{2}{3}H^{-1}}$
$\Rightarrow t \approx 10^{0}$ yr
≈ 0.7
introduction of Λ increases age to $\approx 13.7 \times 10^{9}$ yrs, for $h \approx 0.65$.

Age of universe

$\rho = \rho_m + \rho_r + \rho_\Lambda$

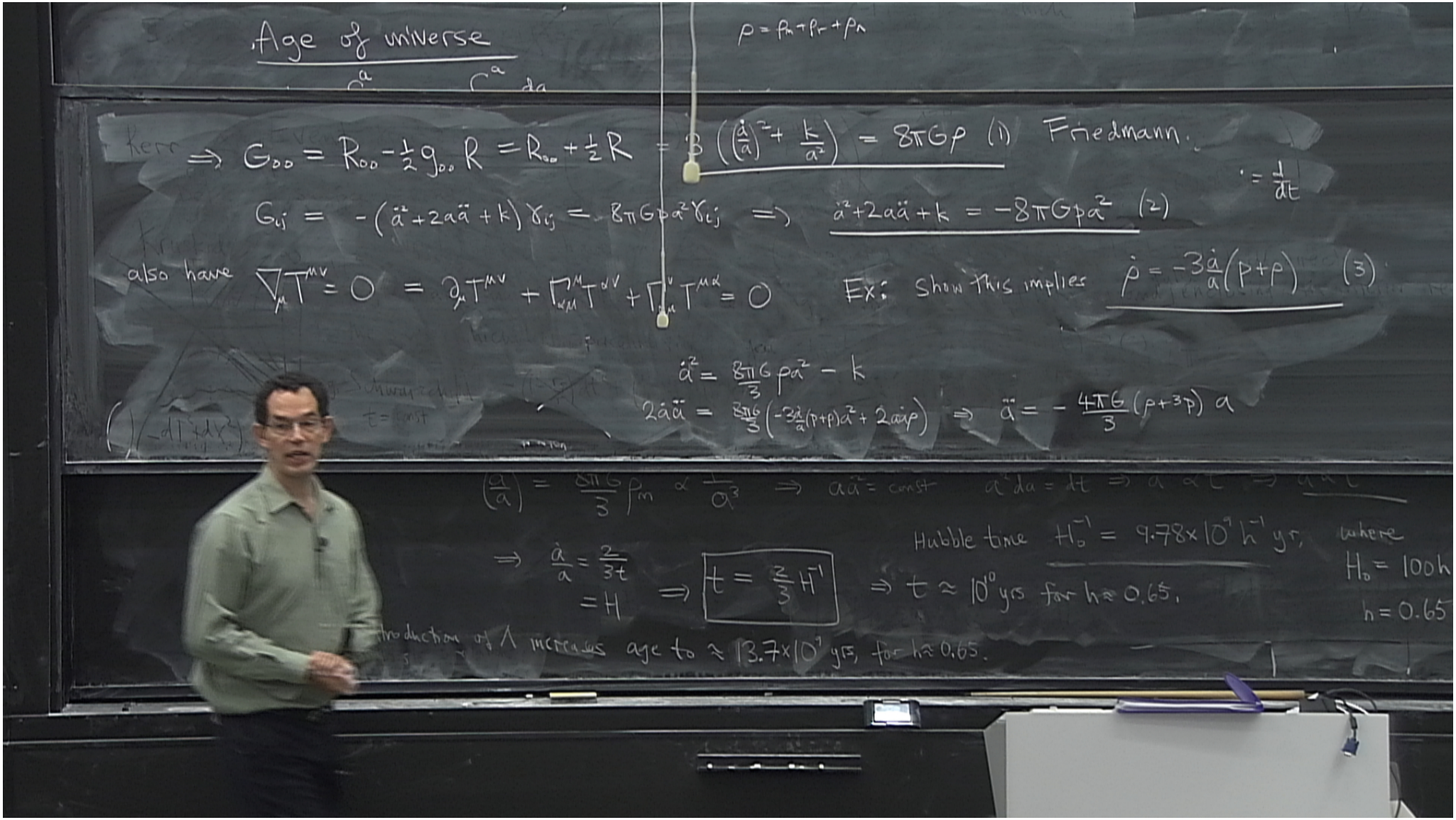
$\Rightarrow G_{00} = R_{00} - \frac{1}{2} g_{00} R = R_{00} + \frac{1}{2} R = 3\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) = 8\pi G\rho$ (1) Friedm

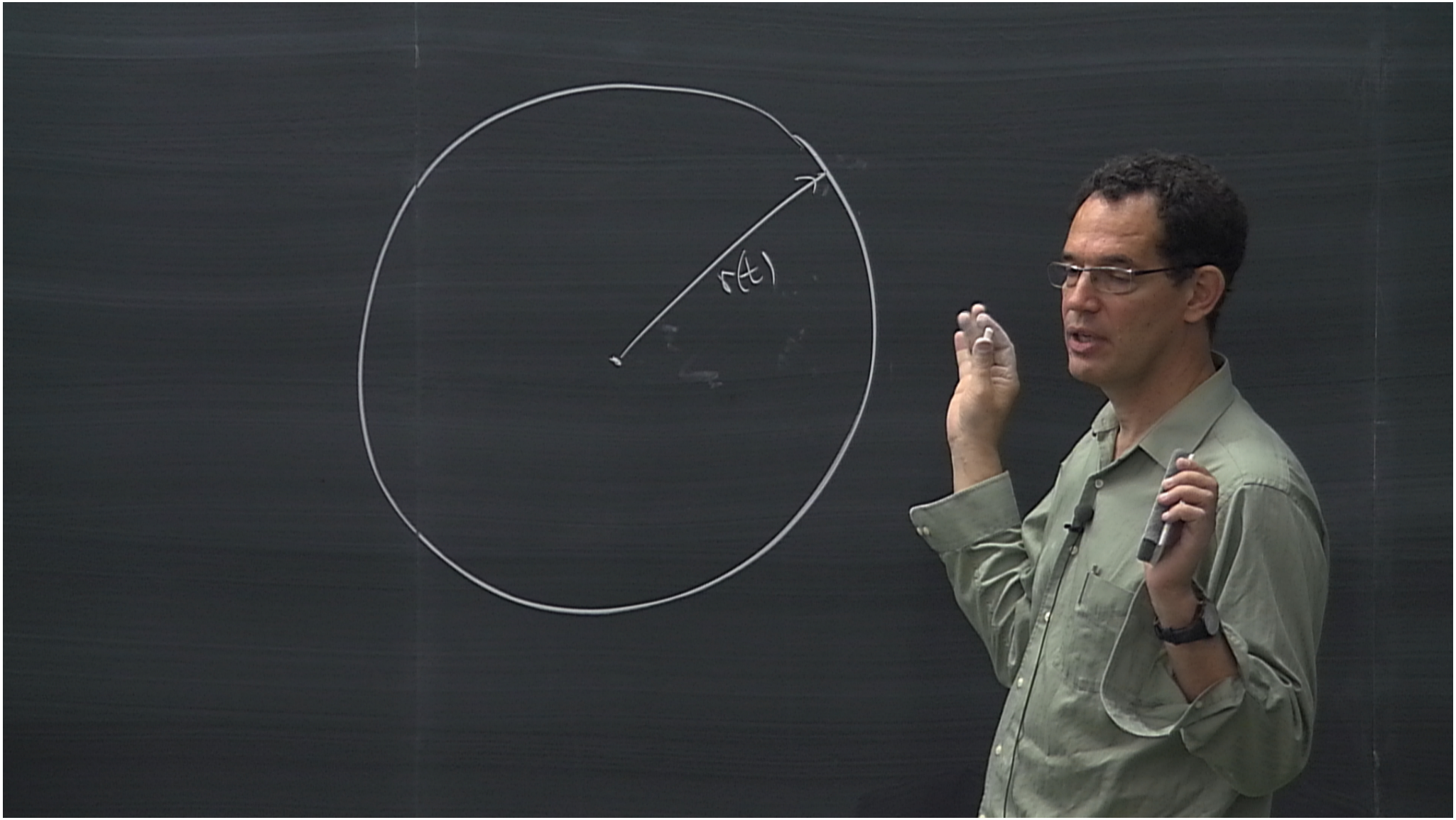
$G_{ij} = -\left(\dot{a}^2 + 2a\ddot{a} + k\right)\gamma_{ij} = 8\pi G\rho a^2 \gamma_{ij} \Rightarrow \dot{a}^2 + 2a\ddot{a} + k = -8\pi G\rho a^2$

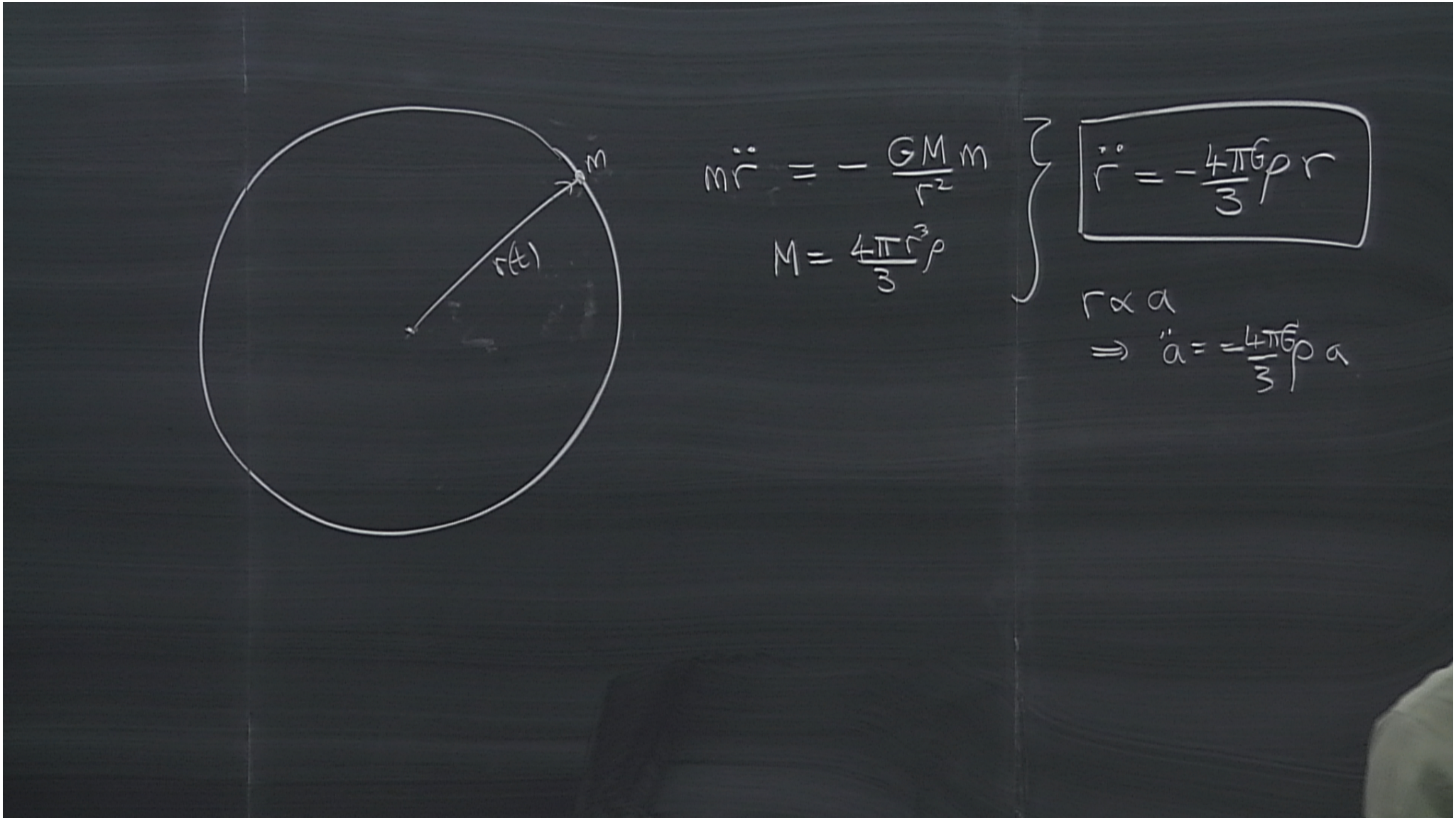
ave $\nabla_\mu T^{\mu\nu} = 0 = \partial_\mu T^{\mu\nu} + \Gamma^\mu_{\alpha\mu} T^{\alpha\nu} + \Gamma^\nu_{\alpha\mu} T^{\mu\alpha} = 0$ Ex: Show this implies $\dot{\rho}$

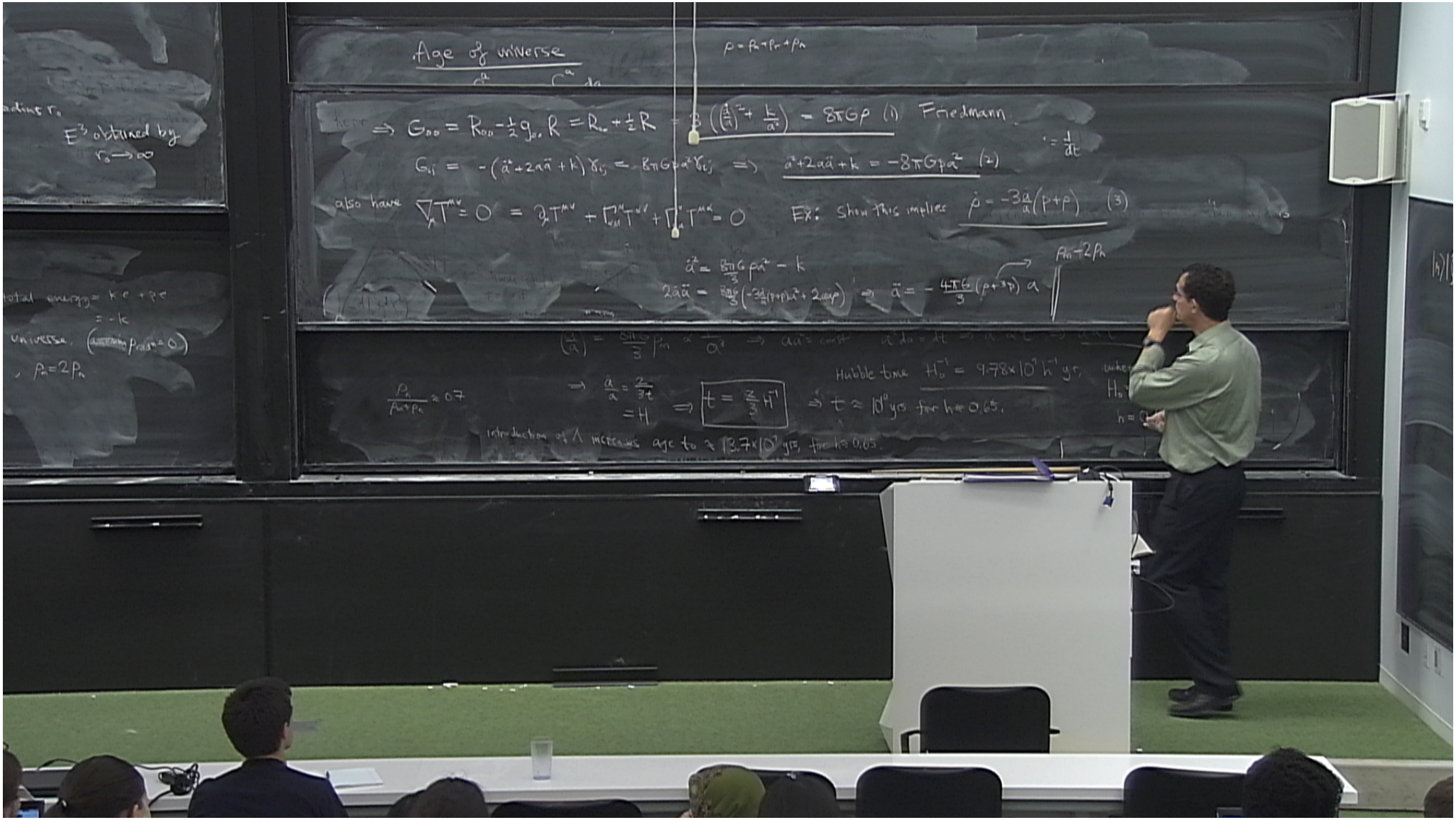
$\dot{a}^2 = \frac{8\pi}{3} G\rho a^2 - k$

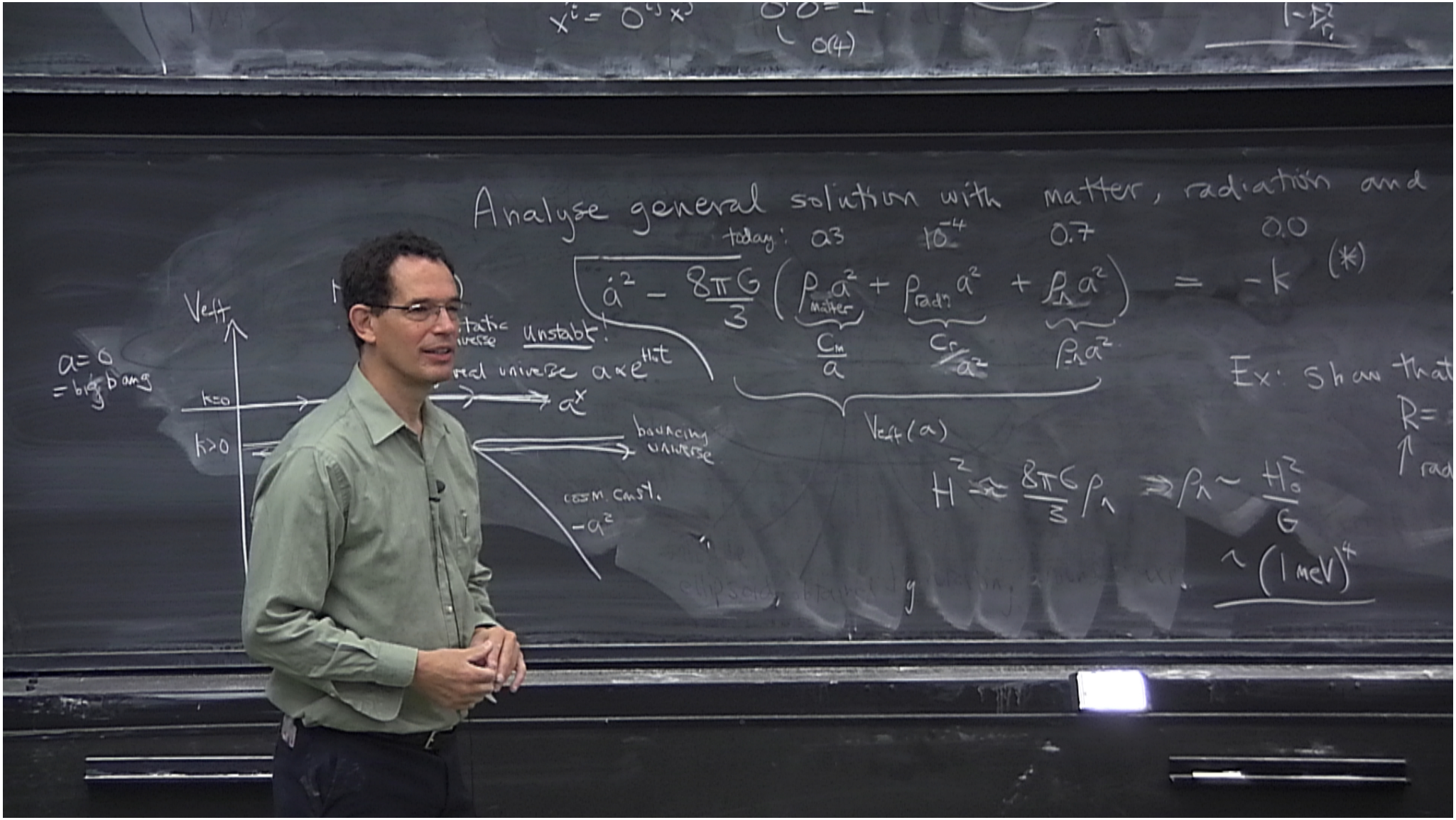
$2\dot{a}\ddot{a} = \frac{8\pi G}{3}\left(-3\frac{\dot{a}}{a}(p+\rho)a^2 + 2a\dot{a}\rho\right)$

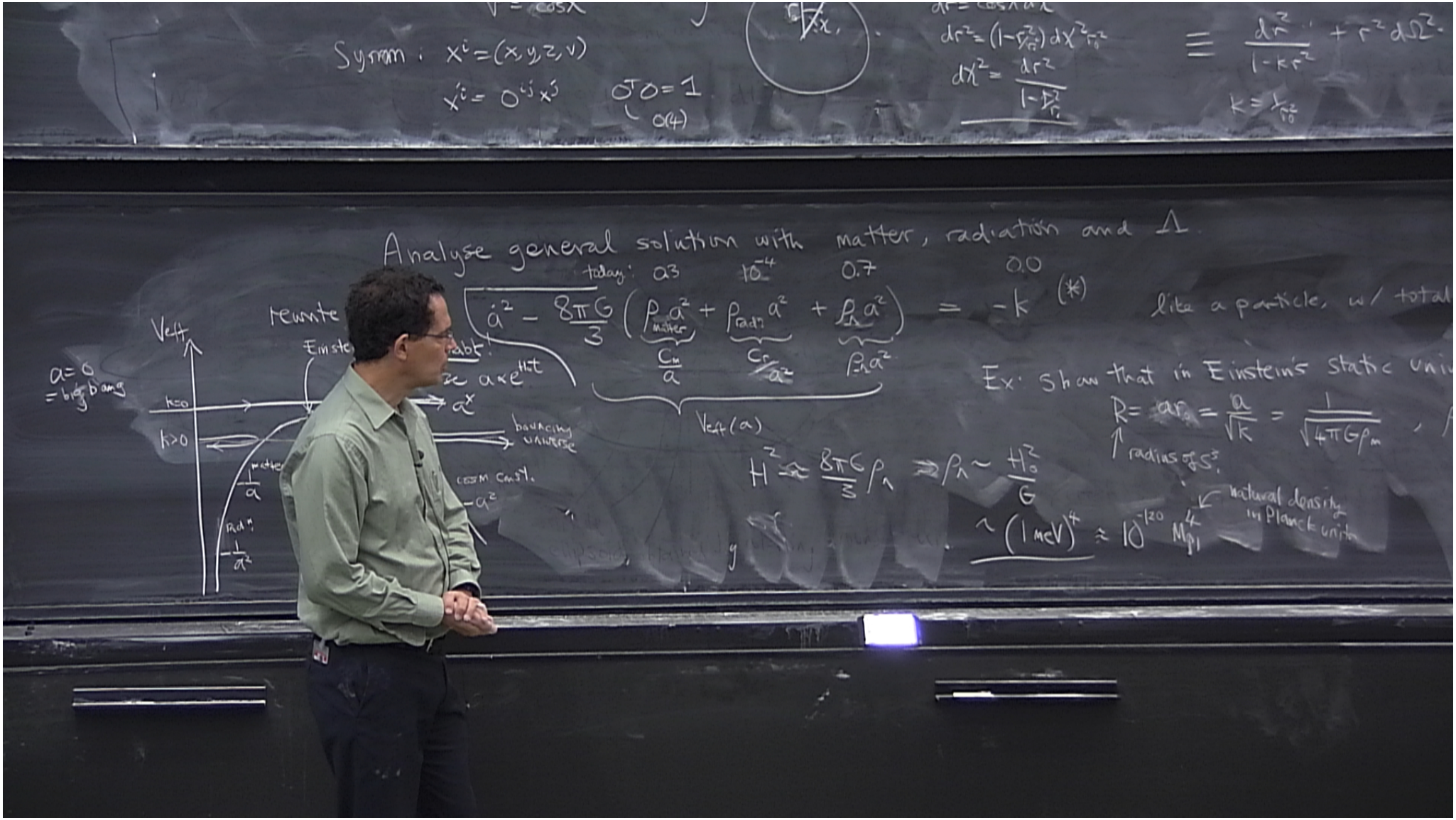

Age of universe
$\rho = \rho_m + \rho_r + \rho_\Lambda$
$\Rightarrow G_{00} = R_{00} - \frac{1}{2} g_{00} R = R_{00} + \frac{1}{2} R = 3\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) = 8\pi G\rho$ (1) Friedmann.
$\dot{} = \frac{1}{dt}$
$G_{ij} = -\left(\dot{a}^2 + 2a\ddot{a} + k\right)\gamma_{ij} = 8\pi G p a^2 \gamma_{ij} \Rightarrow \dot{a}^2 + 2a\ddot{a} + k = -8\pi G p a^2$ (2)
also have $\nabla_\mu T^{\mu\nu} = 0 = \partial_\mu T^{\mu\nu} + \Gamma^\mu_{\mu\lambda} T^{\lambda\nu} + \Gamma^\nu_{\mu\lambda} T^{\mu\lambda} = 0$ Ex: Show this implies $\dot{\rho} = -3\frac{\dot{a}}{a}(p+\rho)$ (3)
$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$
$2\dot{a}\ddot{a} = \frac{8\pi G}{3}\left(-3\frac{\dot{a}}{a}(p+\rho)a^2 + 2a\dot{a}\rho\right) \Rightarrow \ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)\, a$
$\left(\frac{\dot{a}}{a}\right) = \frac{8\pi G}{3}\rho_m \propto \frac{1}{a^3} \Rightarrow a\dot{a}^2 = \text{const}$
$\Rightarrow \frac{\dot{a}}{a} = \frac{2}{3t} = H \Rightarrow \boxed{t = \frac{2}{3}H^{-1}}$
Hubble time $H_0^{-1} = 9.78 \times 10^9\, h^{-1}$ yrs, where $H_0 = 100h$
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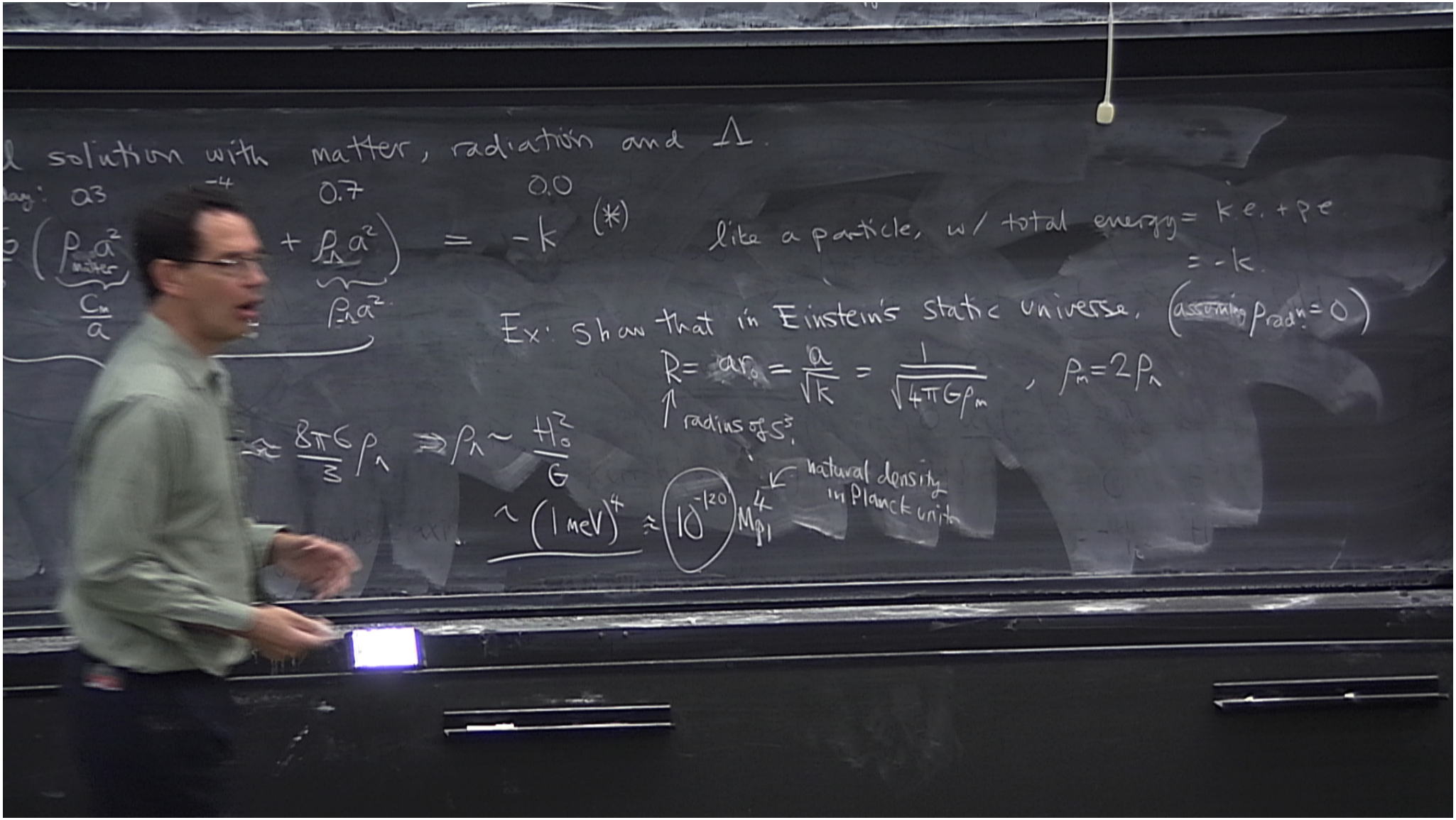

r(t)

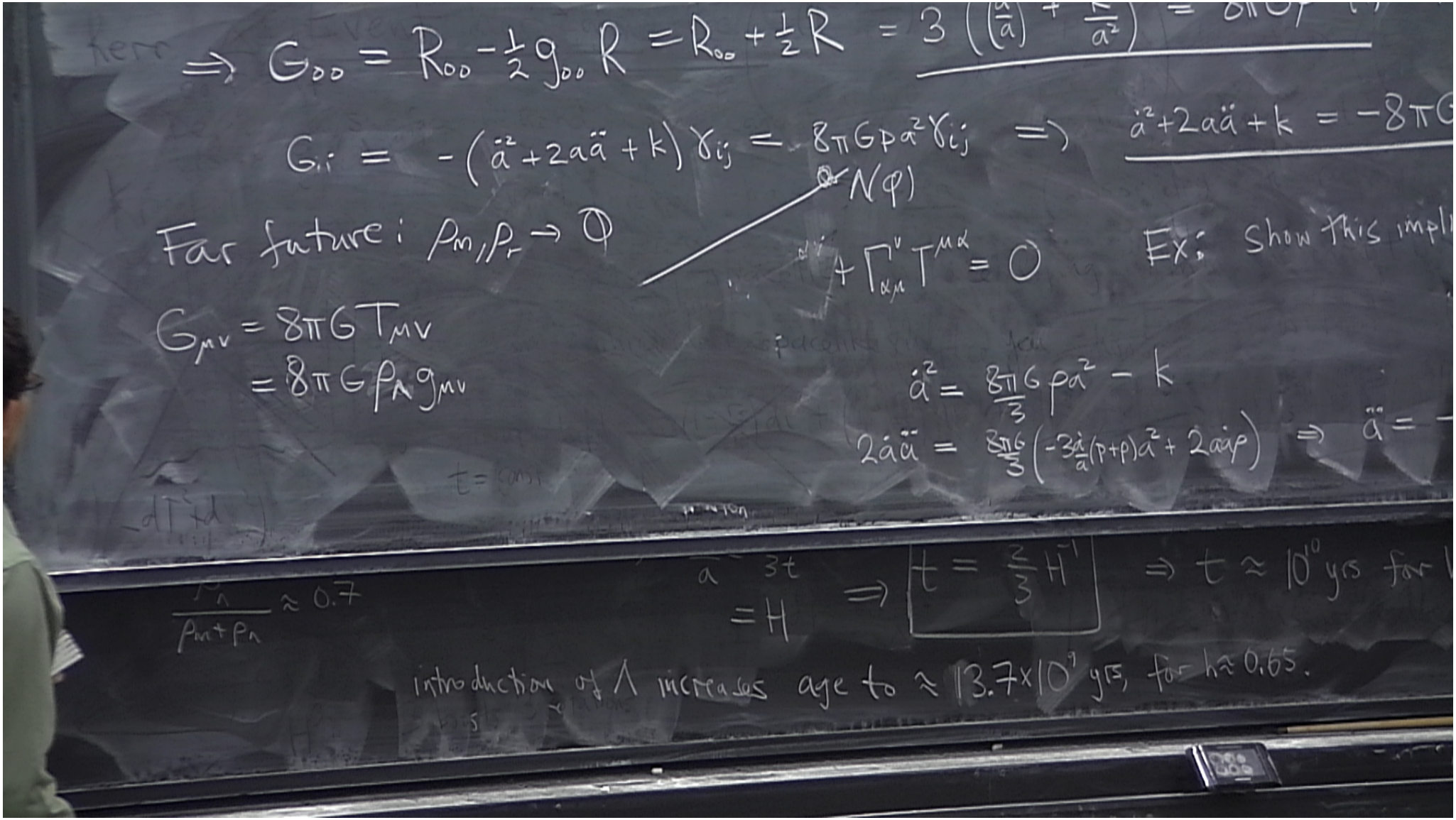

m
r(t)
$m\ddot{r} = -\frac{GMm}{r^2}$
$M = \frac{4\pi r^3}{3}\rho$
$\ddot{r} = -\frac{4\pi G}{3}\rho r$
$r \propto a$
$\Rightarrow \ddot{a} = -\frac{4\pi G}{3}\rho a$

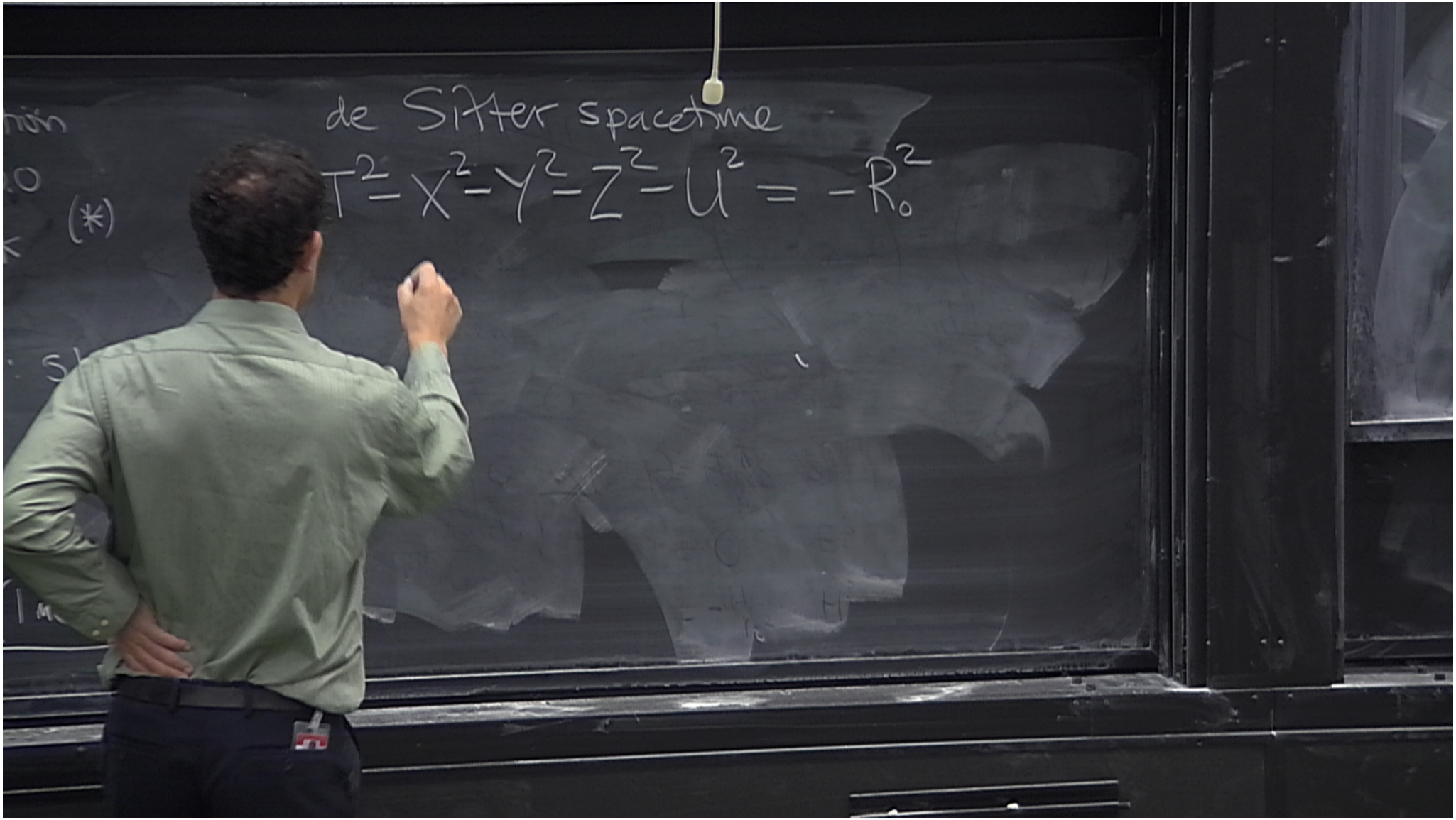

Age of universe
Friedmann
Ex: Show this implies
Hubble time
also have
total energy = ke + pe
universe

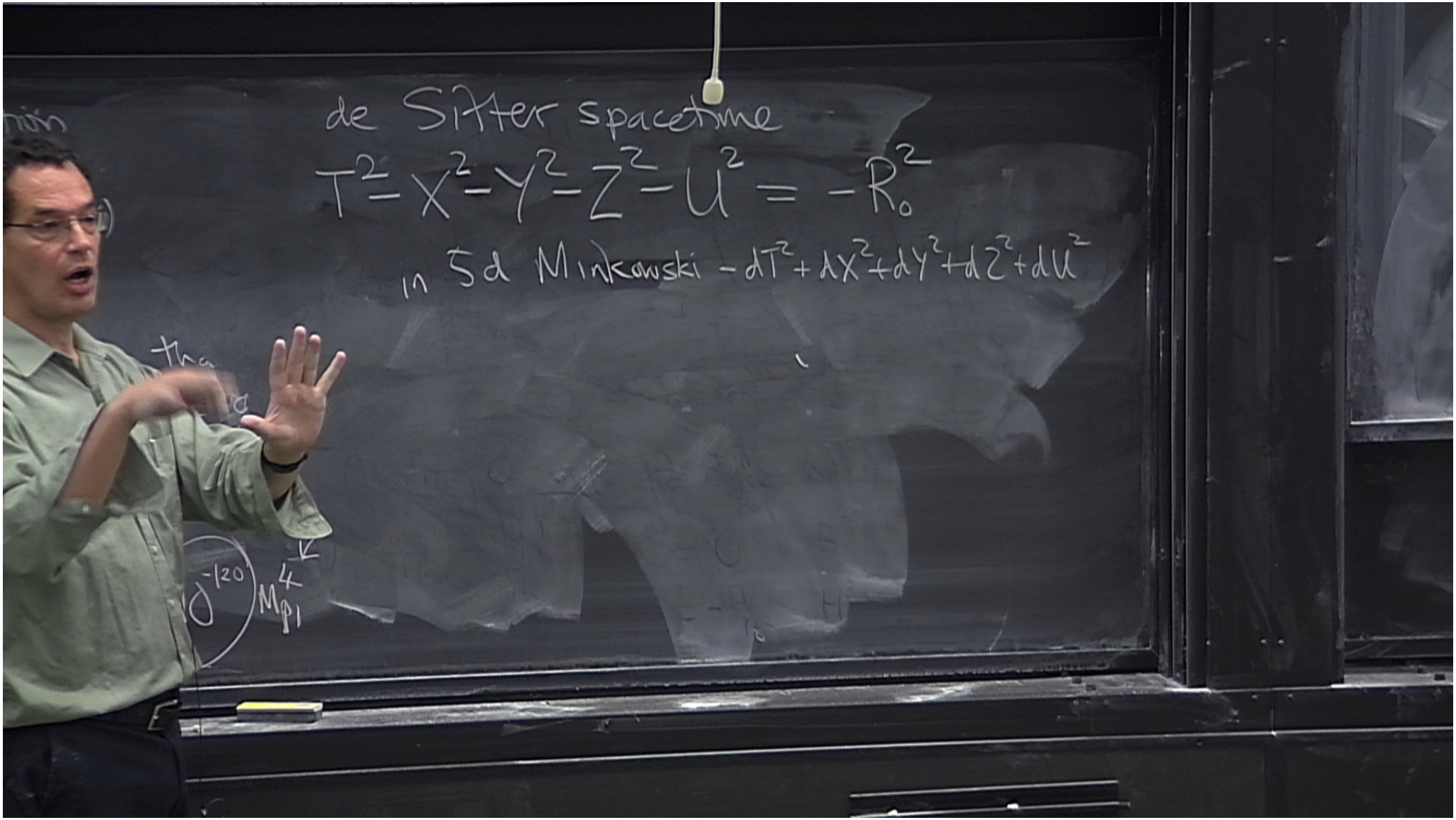

Analyse general solution with matter, radiation and
today: a3 10^{-4} 0.7 0.0
$\dot{a}^2 - \frac{8\pi G}{3}\left(\rho_{matter}a^2 + \rho_{rad}a^2 + \rho_\Lambda a^2\right) = -k$ (*)
$\frac{C_M}{a}$ $\frac{C_r}{a^2}$ $\rho_\Lambda a^2$
$V_{eff}(a)$
V_{eff}
$a=0$ = big bang
$k=0$
$k>0$
a^x
bouncing universe
cosm. const.
$-a^2$
Ex: Show that
$H^2 \approx \frac{8\pi G}{3}\rho_\Lambda \Rightarrow \rho_\Lambda \sim \frac{H_0^2}{G}$
$\sim (1\,meV)^4$

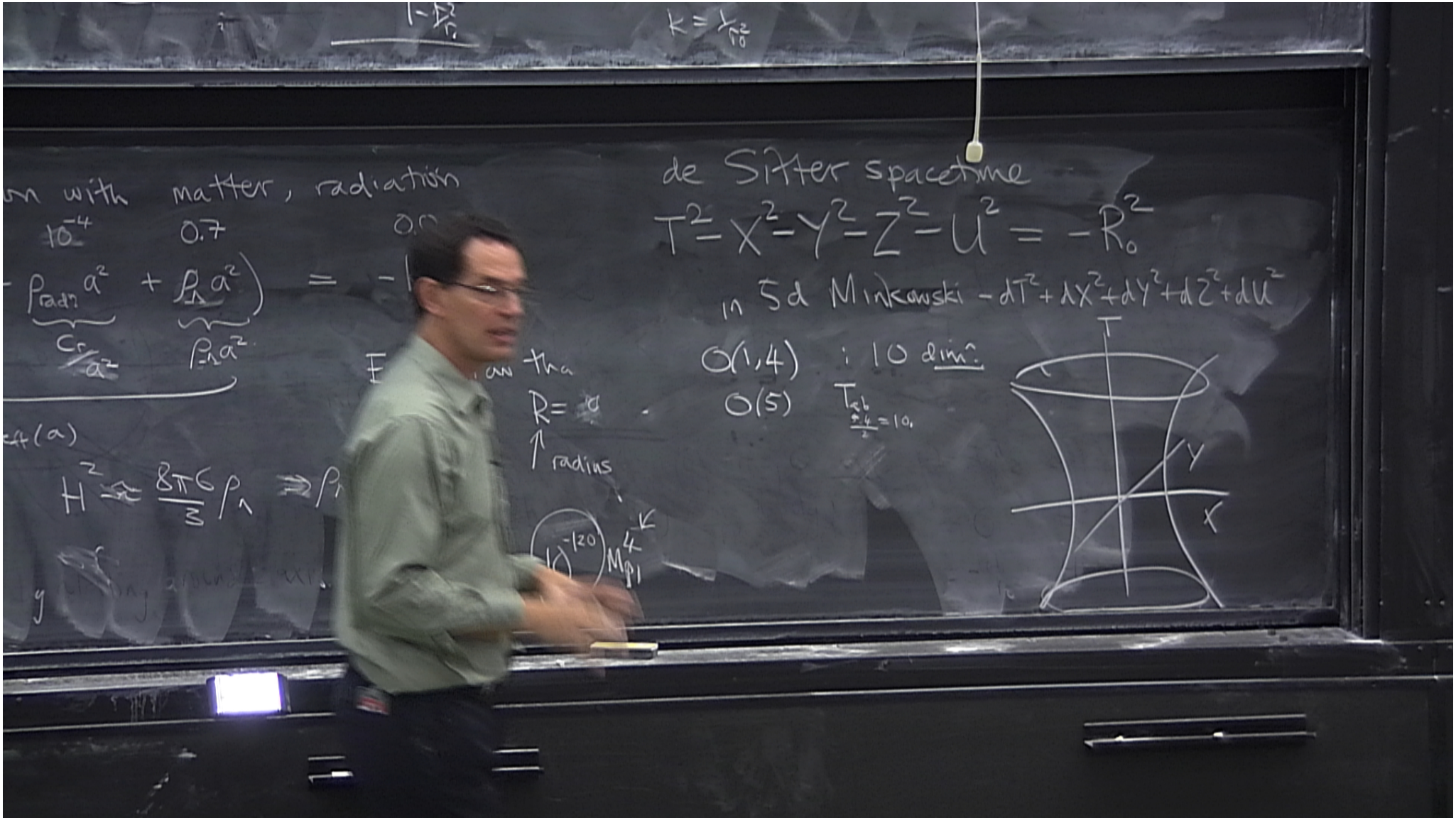

Symm: $x^i = (x, y, z, v)$
$x'^i = O^{ij} x^j$ $O^T O = 1$ $O(4)$
$dr^2 = (1 - r_{r_0}^2) dx^2 r_0^2$
$dx^2 = \frac{dr^2}{1 - r_{r_0}^2}$
$\equiv \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2$
$k = \frac{1}{r_0^2}$
Analyse general solution with matter, radiation and Λ.
today: a3 10^{-4} 0.7 0.0
$\dot{a}^2 - \frac{8\pi G}{3}\left(\rho_{m,0} a^2 + \rho_{rad,0} a^2 + \rho_\Lambda a^2\right) = -k$ (*)
$\frac{C_m}{a}$ $\frac{C_r}{a^2}$ $\rho_\Lambda a^2$
$V_{eff}(a)$
like a particle, w/ total
Ex: Show that in Einstein's static uni
$R = a r_0 = \frac{a}{\sqrt{k}} = \frac{1}{\sqrt{4\pi G \rho_m}}$
radius of S^3
$H^2 = \frac{8\pi G}{3}\rho_\Lambda \Rightarrow \rho_\Lambda \sim \frac{H_0^2}{G}$
$\sim (1\,\text{meV})^4 \approx 10^{-120} M_{Pl}^4$
natural density in Planck units
V_{eff}
$a = 0$ = big bang
$k = 0$
$k > 0$
a^x
bouncing universe
matter
$\frac{1}{a}$
rad n
$\frac{1}{a^2}$
rewrite
Einste
rabbit!
$a \propto e^{Ht}$
cosm const.
$-a^2$

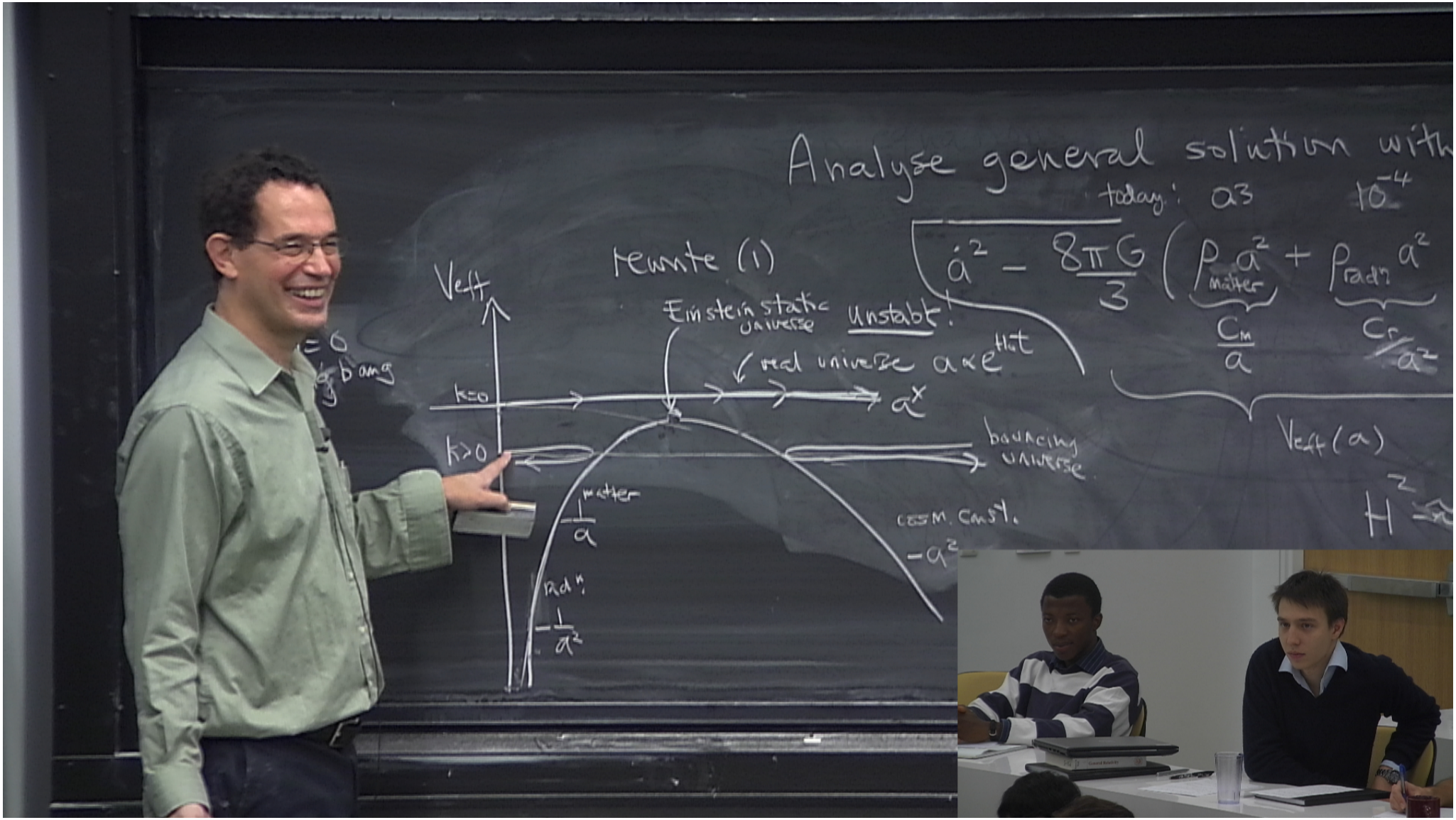

solution with matter, radiation and Λ.
like a particle, w/ total energy = k.e. + p.e
= -k.
Ex. Show that in Einstein's static universe, (assuming p_rad = 0)
R = a r_0 = a/√k = 1/√(4πGρ_m) , ρ_m = 2ρ_Λ
radius of S³
natural density in Planck units
~ (1 meV)⁴ ≈ 10^-120 M_Pl^4

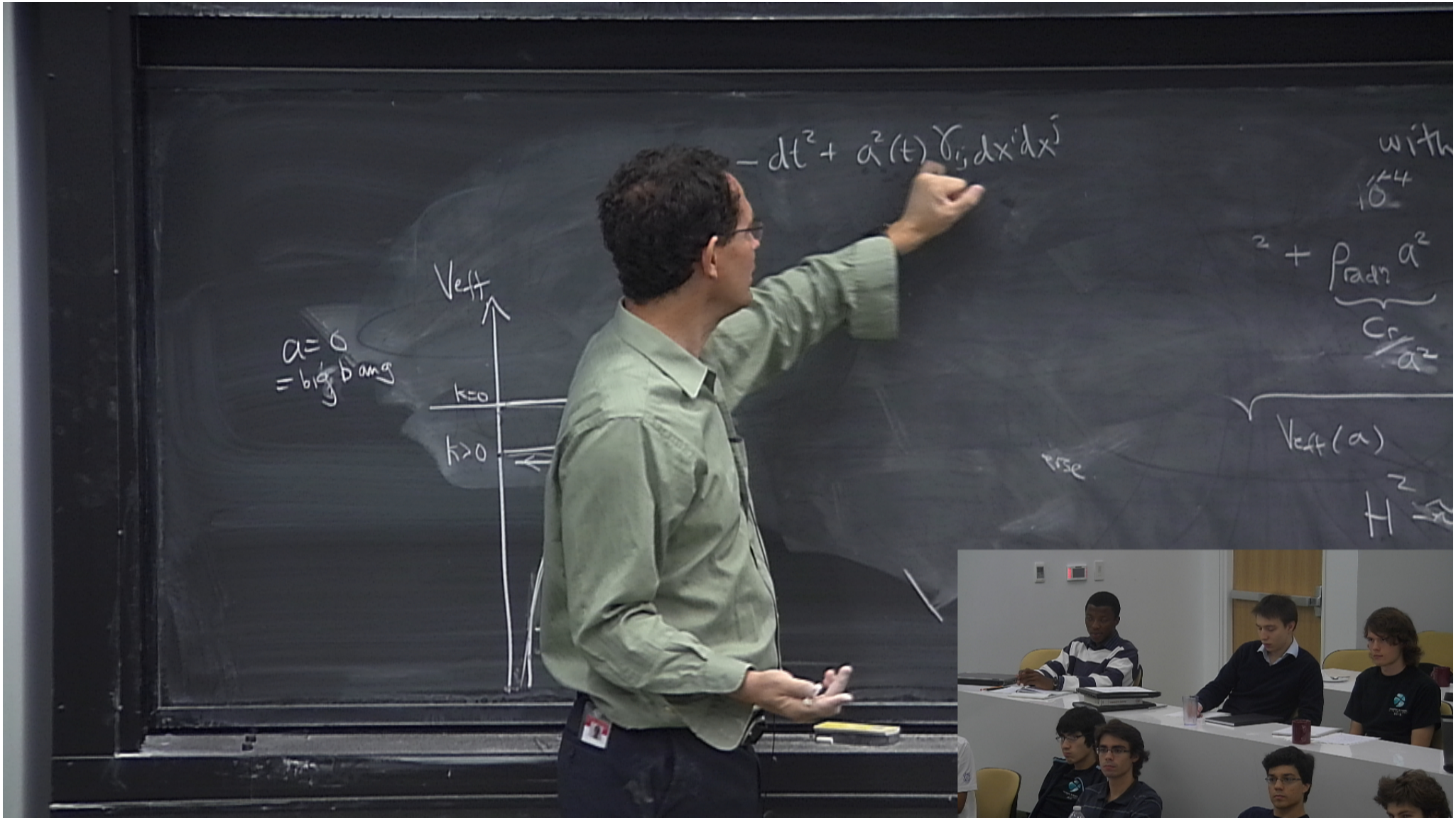

$\Rightarrow G_{00} = R_{00} - \frac{1}{2} g_{00} R = R_{00} + \frac{1}{2} R = 3\left(\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) = 8\pi G \rho$
$G_{ij} = -\left(\dot{a}^2 + 2a\ddot{a} + k\right) \gamma_{ij} = 8\pi G p a^2 \gamma_{ij} \Rightarrow \dot{a}^2 + 2a\ddot{a} + k = -8\pi G$
$\Lambda(\varphi)$
Far future: $\rho_m, \rho_r \to 0$
$+ \Gamma^{\nu}_{\alpha\mu} T^{\mu\alpha} = 0$
Ex: Show this impl
$G_{\mu\nu} = 8\pi G T_{\mu\nu}$
$= 8\pi G \rho_\Lambda g_{\mu\nu}$
$\dot{a}^2 = \frac{8\pi}{3} G \rho a^2 - k$
$2\dot{a}\ddot{a} = \frac{8\pi G}{3}\left(-3\frac{\dot{a}}{a}(\rho + p)a^2 + 2a\dot{a}\rho\right) \Rightarrow \ddot{a} =$
$\frac{\rho_\Lambda}{\rho_m + \rho_\Lambda} \approx 0.7$
$a \sim t^{\frac{2}{3}}$
$= H$
$\Rightarrow \boxed{t = \frac{2}{3} H^{-1}}$
$\Rightarrow t \approx 10^{10}$ yrs for
Introduction of Λ increases age to $\approx 13.7 \times 10^9$ yrs, for $h \approx 0.65$.

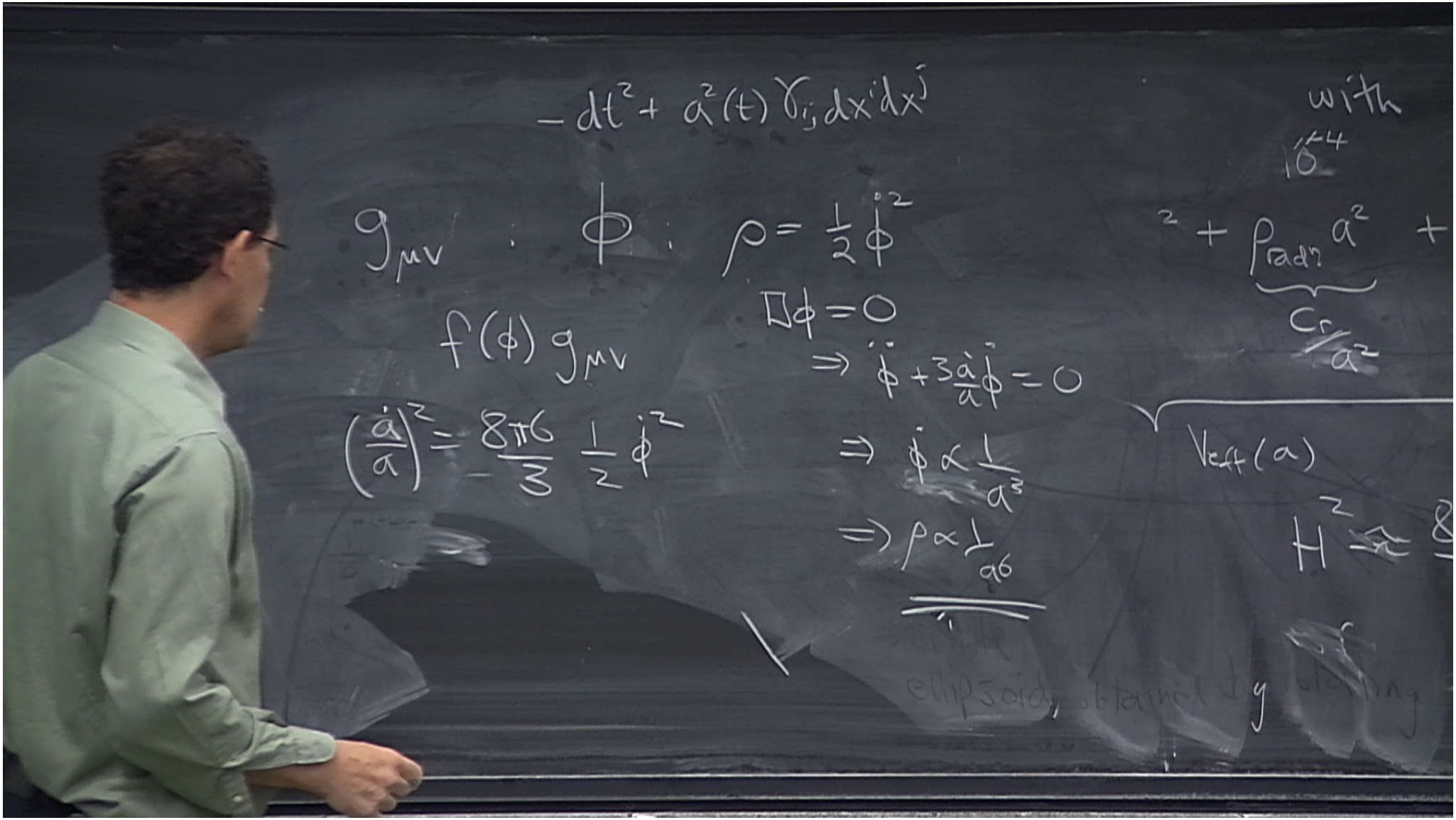

de Sitter spacetime
$T^2 - X^2 - Y^2 - Z^2 - U^2 = -R_0^2$

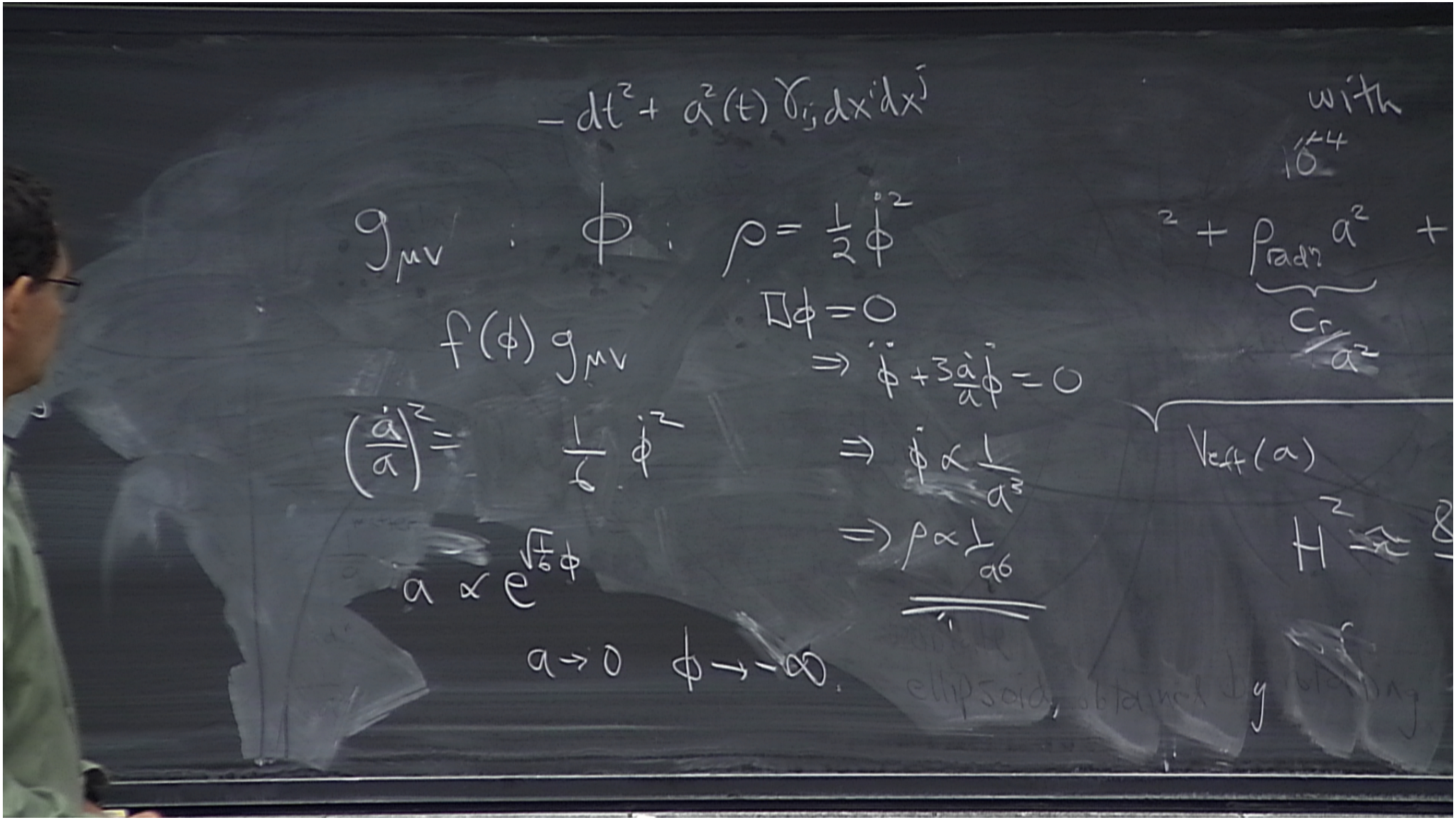

de Sitter spacetime
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in 5d Minkowski $-dT^2 + dX^2 + dY^2 + dZ^2 + dU^2$

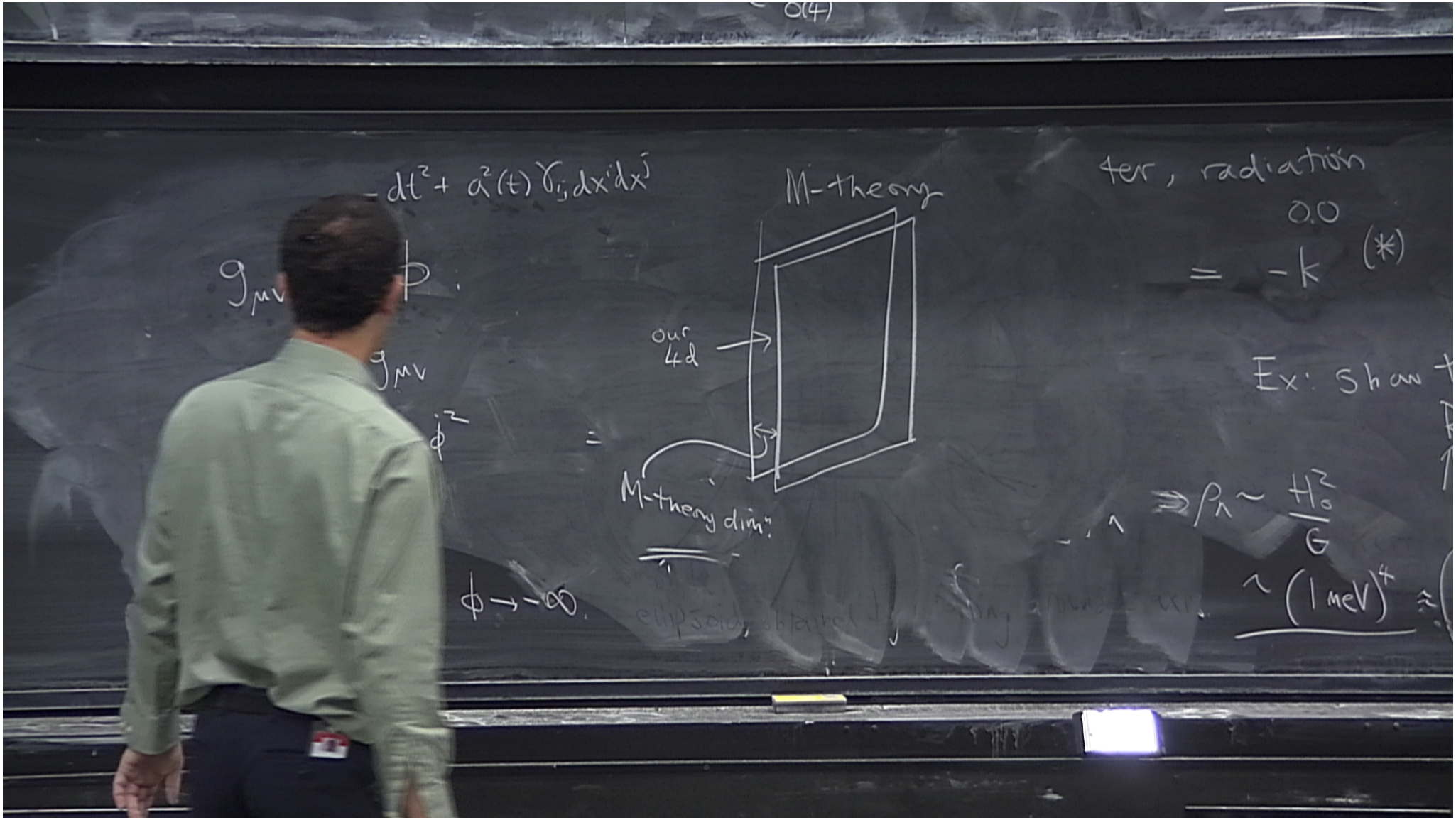

de Sitter spacetime
$T^2 - X^2 - Y^2 - Z^2 - U^2 = -R_0^2$
in 5d Minkowski $-dT^2 + dX^2 + dY^2 + dZ^2 + dU^2$
O(1,4) : 10 dim.
O(5)
matter, radiation
10^{-4}
0.7
$H^2 = \frac{8\pi G}{3}\rho$
R=
radius
T
Y
X


Analyse general solution with
today: a3 10^-4
remnte (1)
Veff
Einstein static universe
Unstabil!
real universe
bouncing universe
k=0
k>0
matter
rad.
Veff(a)


$-dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$
with
$\rho_{rad} a^2$
C_r/a^2
$V_{eff}(a)$
V_{eff}
$a=0$
= big bang
k=0
k>0


$-dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$
with
10^{-4}
$g_{\mu\nu}$: ϕ : $\rho = \frac{1}{2}\dot{\phi}^2$
$\Box\phi = 0$
$\Rightarrow \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = 0$
$f(\phi)\, g_{\mu\nu}$
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{1}{2}\dot{\phi}^2$
$\Rightarrow \dot{\phi} \propto \frac{1}{a^3}$
$\Rightarrow \rho \propto \frac{1}{a^6}$
$+ \rho_{rad}\, a^2 +$
C_r/a^2
$V_{eff}(a)$
$H^2 =$
ellipsoid, obtained by


$-dt^2 + a^2(t)\gamma_{ij}dx^i dx^j$
with
10^{-4}
$g_{\mu\nu}$: ϕ : $\rho = \frac{1}{2}\dot{\phi}^2$
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C_r/a^2
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$\Rightarrow \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = 0$
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{6}\dot{\phi}^2$
$V_{eff}(a)$
$\Rightarrow \dot{\phi} \propto \frac{1}{a^3}$
$H^2 =$
$\Rightarrow \rho \propto \frac{1}{a^6}$
$a \propto e^{\sqrt{\frac{1}{6}}\phi}$
$a \to 0 \quad \phi \to -\infty$


M-theory
our 4d
M-theory dim'n
ter, radiation
0.0
= -k (*)
Ex: Show
$\Rightarrow \rho_\Lambda \sim \frac{H_0^2}{G}$
$\sim (1\,\mathrm{meV})^4$
$\phi \to -\infty$.