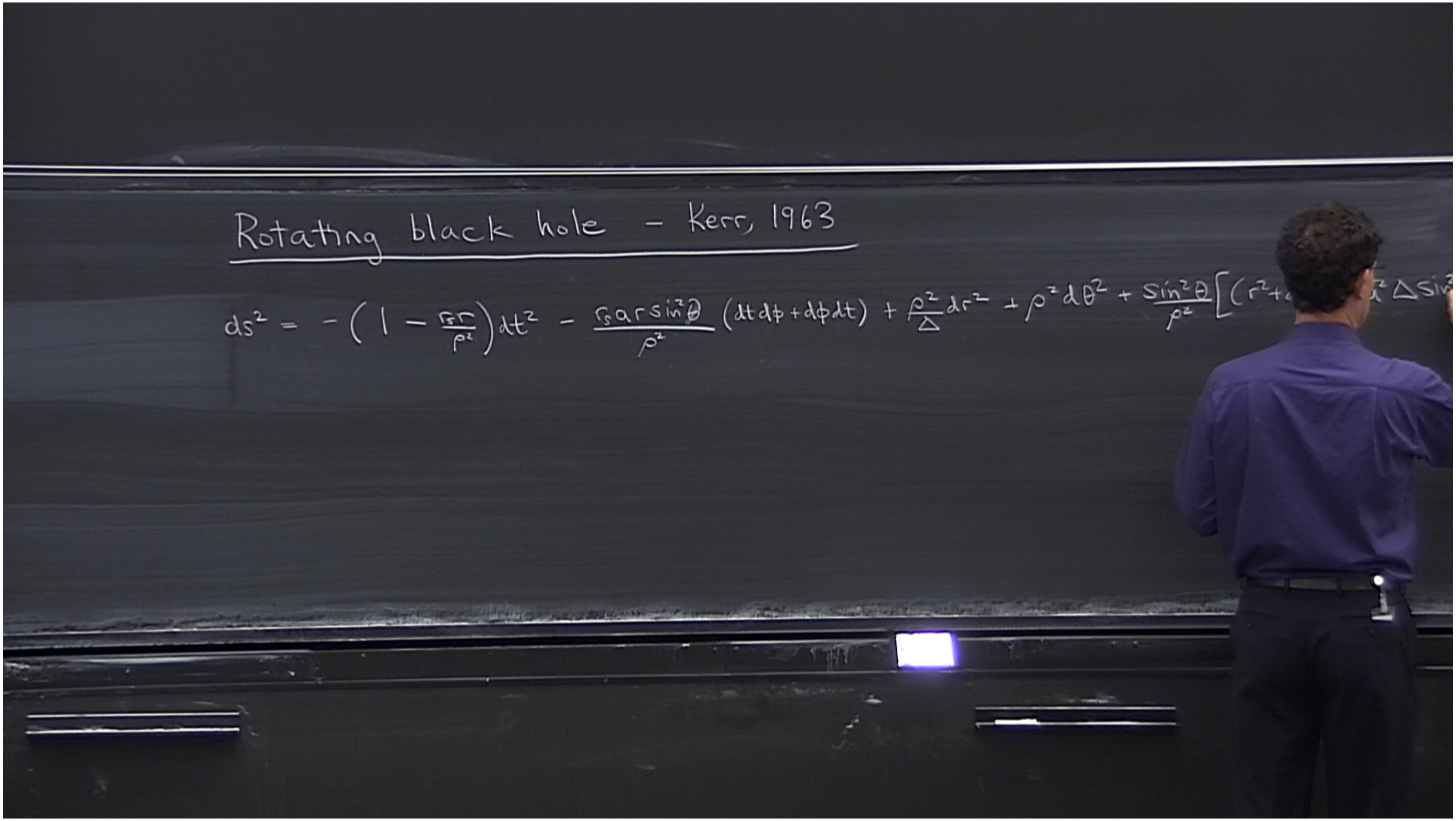
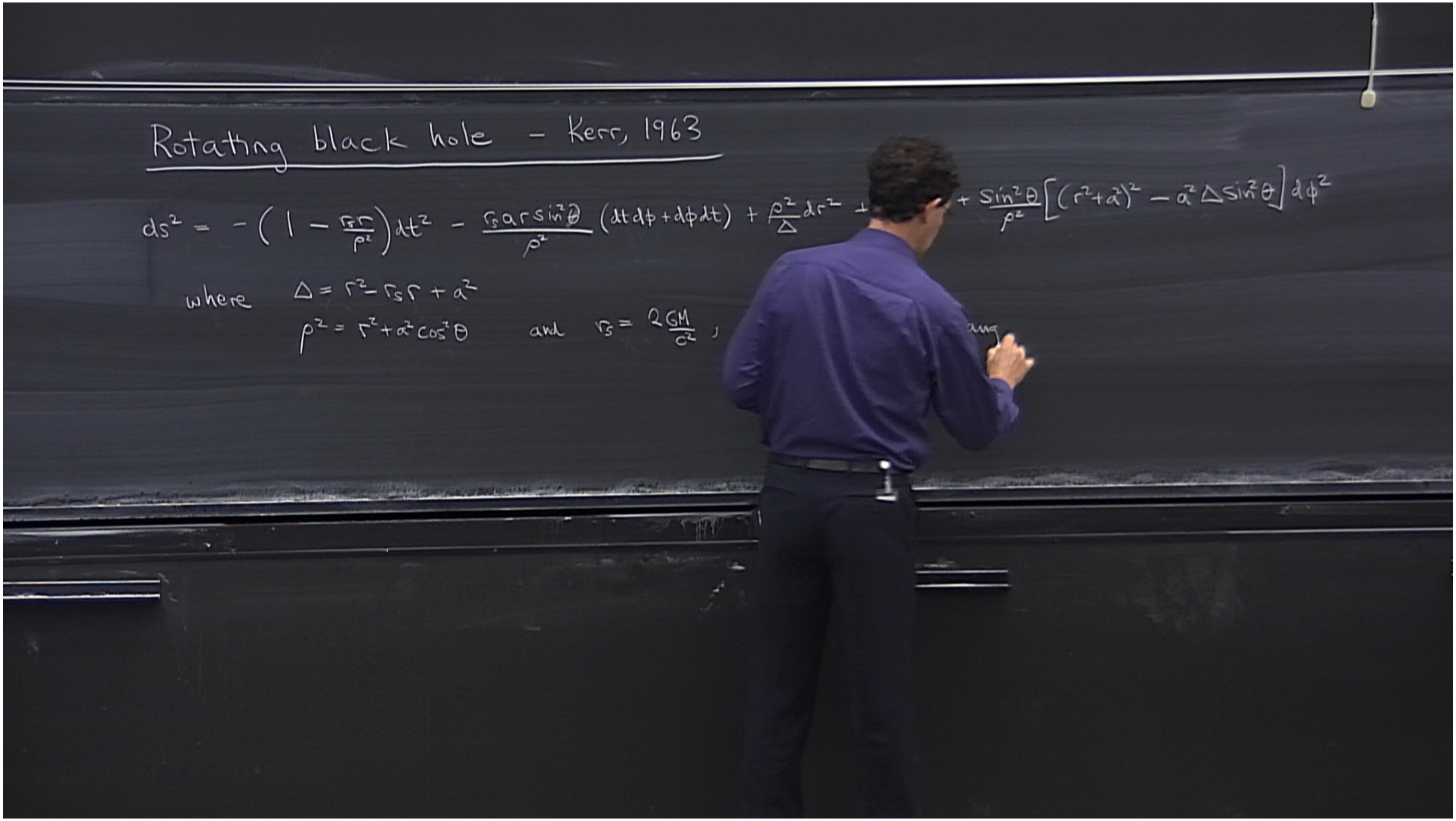
Title: Relativity - Lecture 13

Date: Sep 29, 2011 10:30 AM

URL: http://pirsa.org/11090041

Abstract:


Rotating black hole - Kerr, 1963
$ds^2 = -\left(1 - \frac{r_s r}{\rho^2}\right)dt^2 - \frac{r_s a r \sin^2\theta}{\rho^2}(dt\,d\phi + d\phi\,dt) + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\Big[(r^2 +$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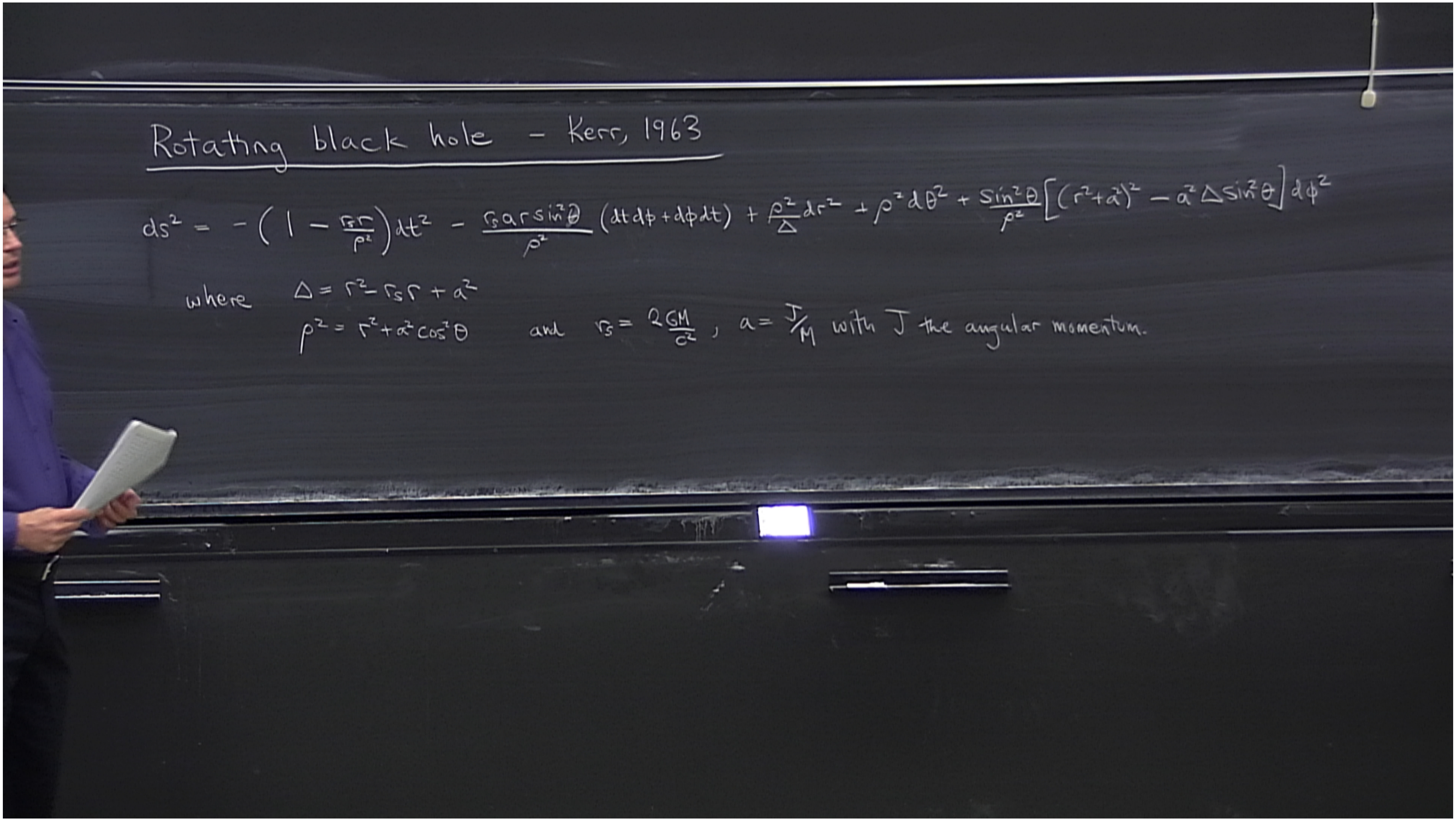
where $\Delta = r^2 - r_s r + a^2$
$\rho^2 = r^2 + a^2\cos^2\theta$ and $r_s = \frac{2GM}{c^2}$,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where $\Delta = r^2 - r_s r + a^2$
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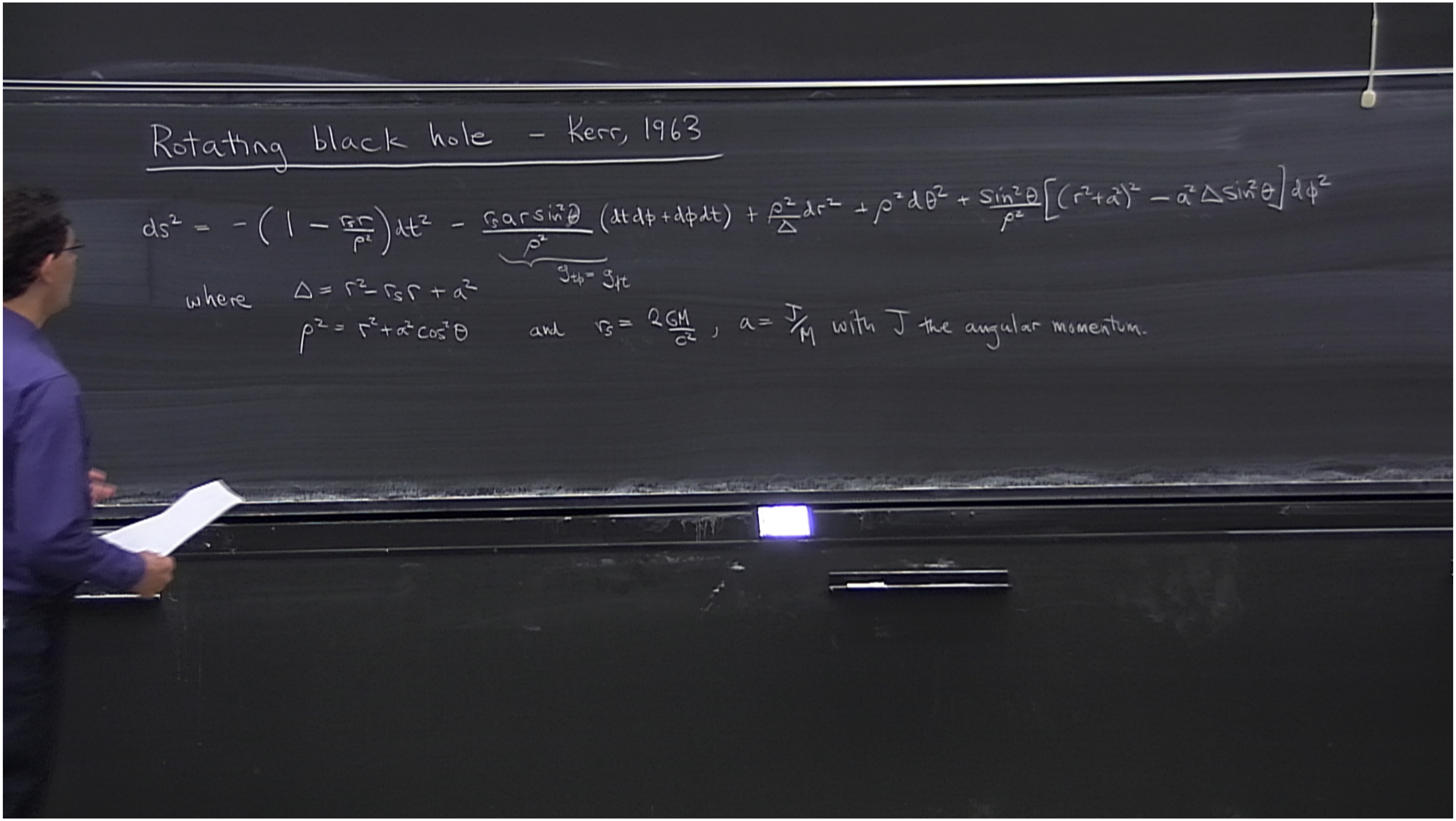
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$g_{t\phi} = g_{\phi t}$
where $\Delta = r^2 - r_s r + a^2$
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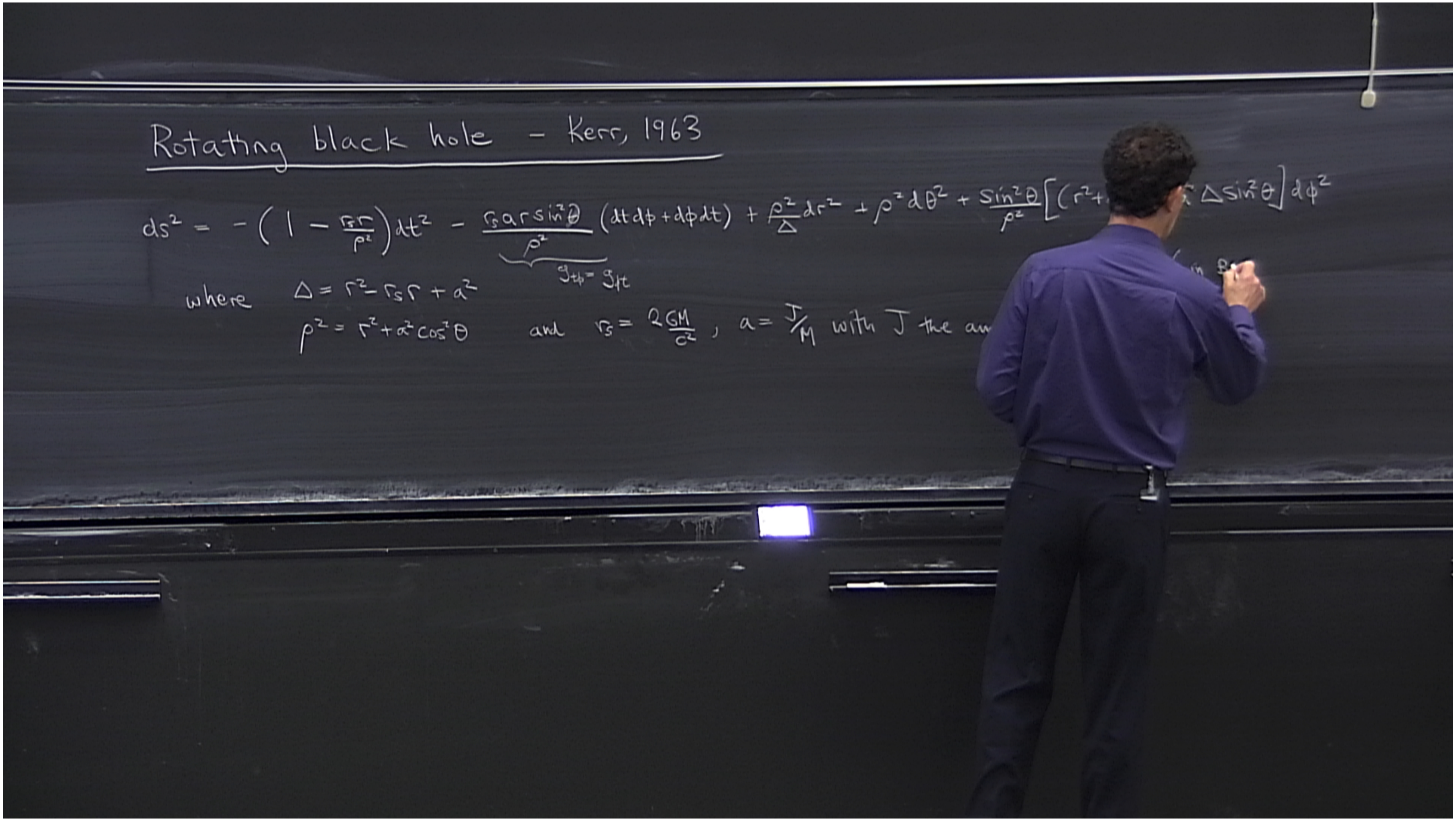
Rotating black hole – Kerr, 1963
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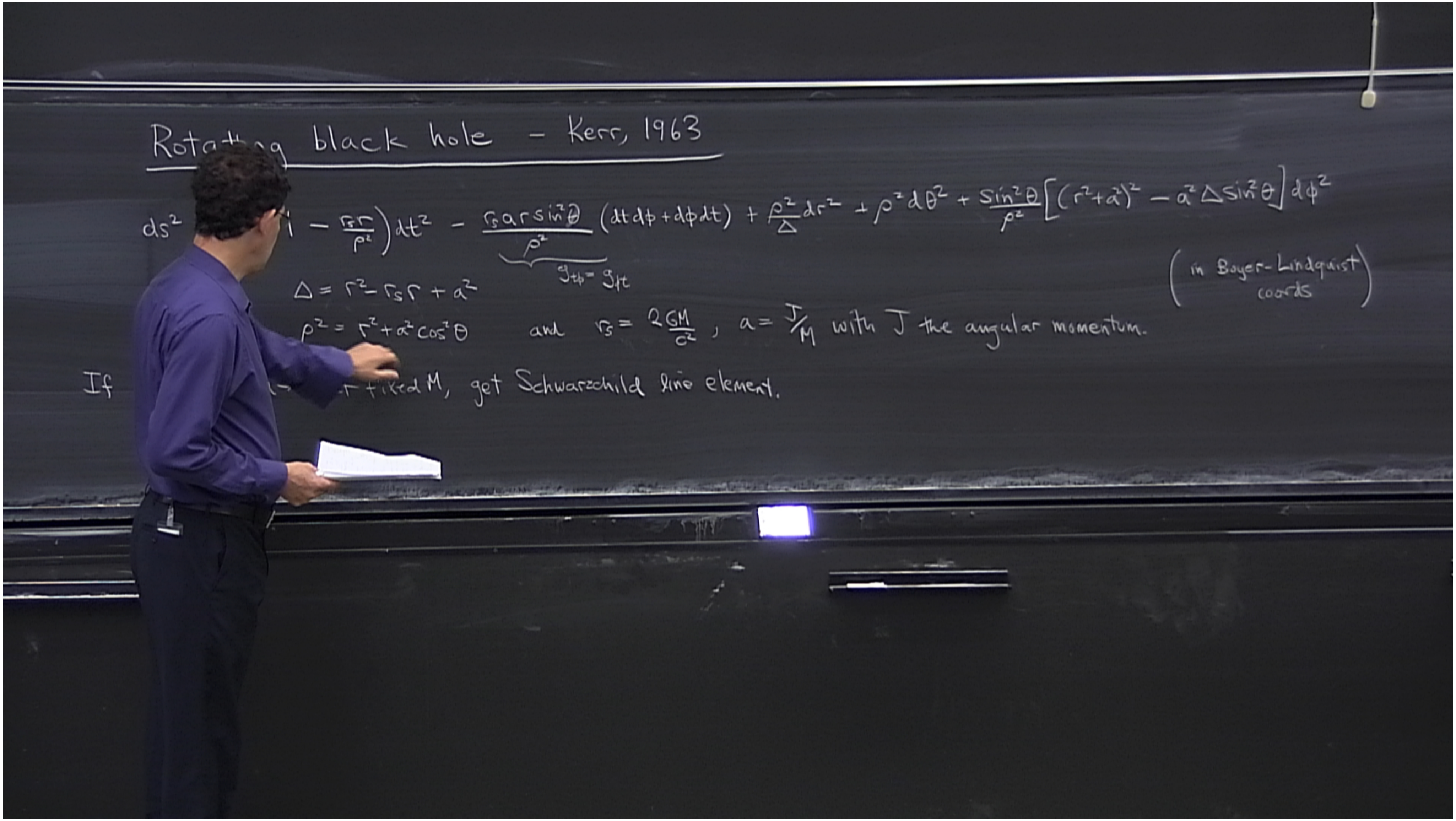
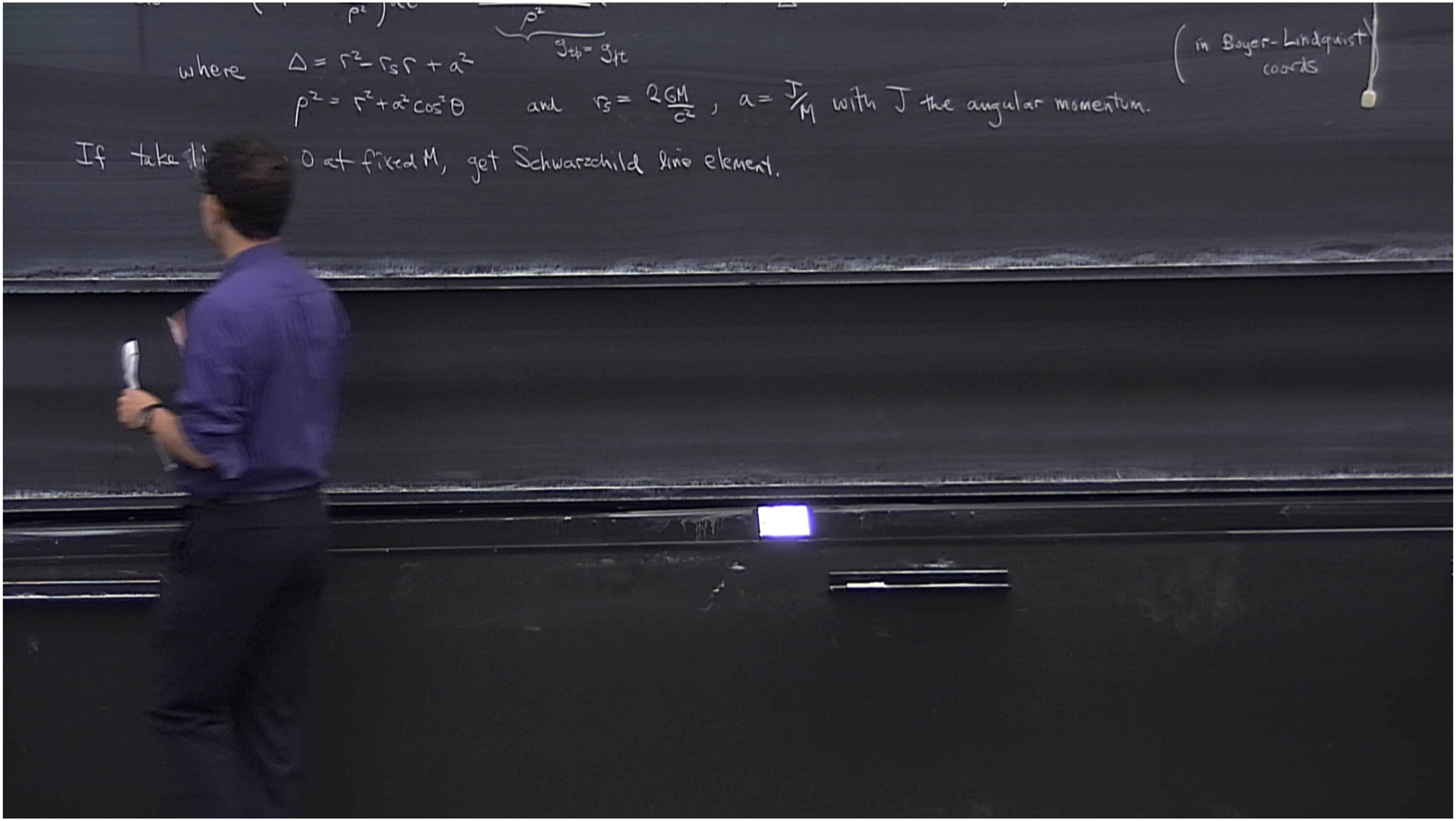
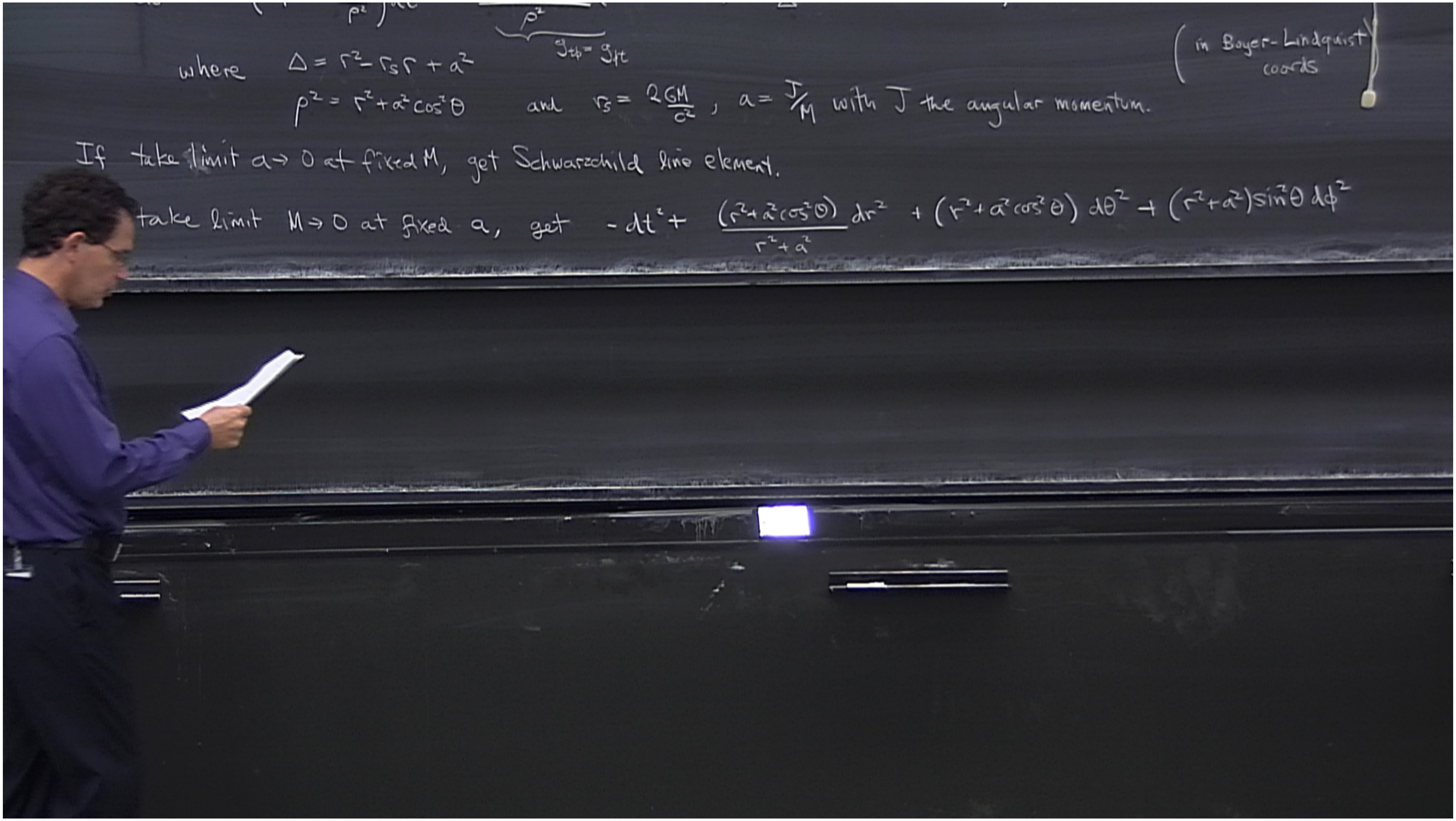
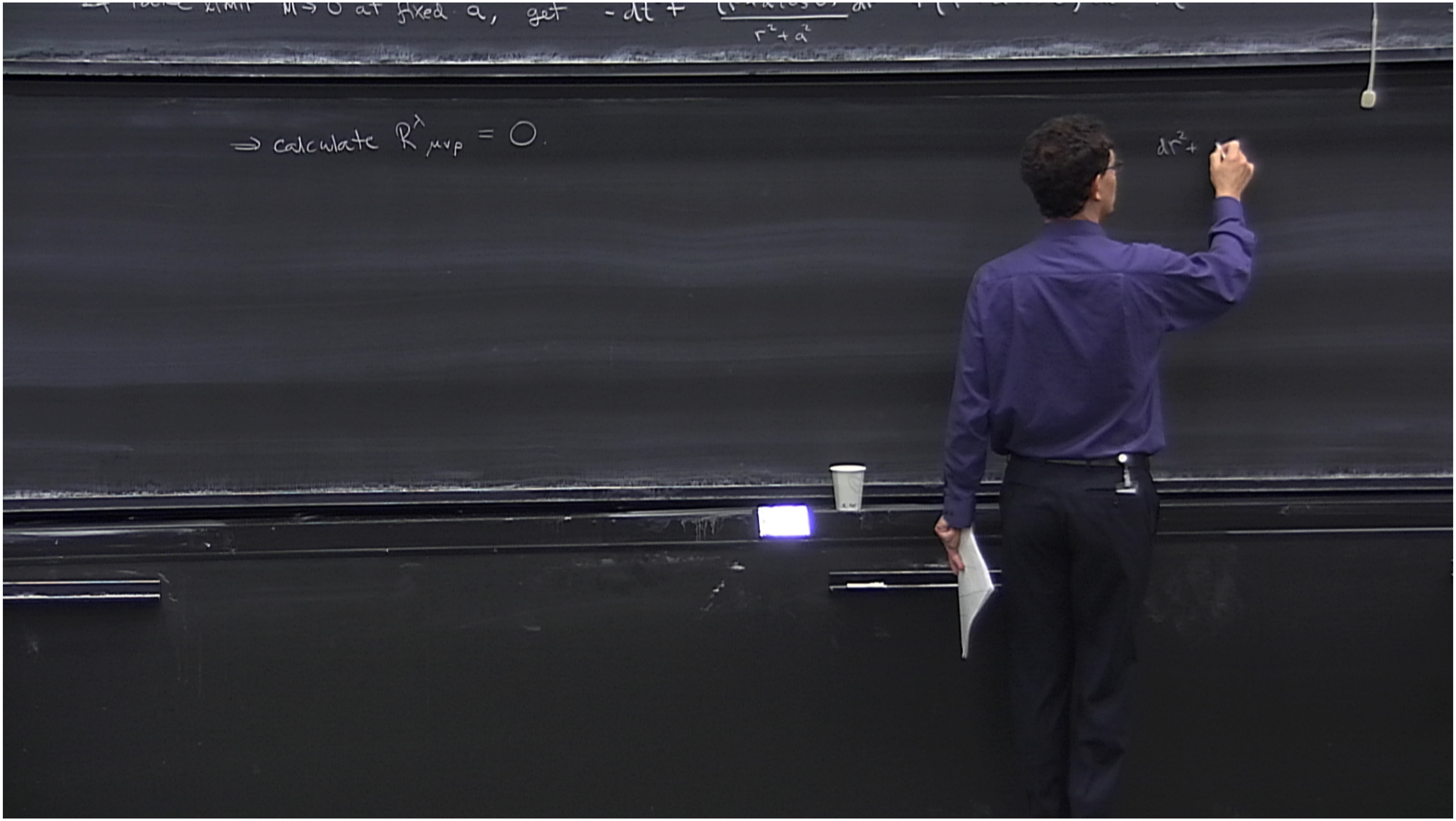
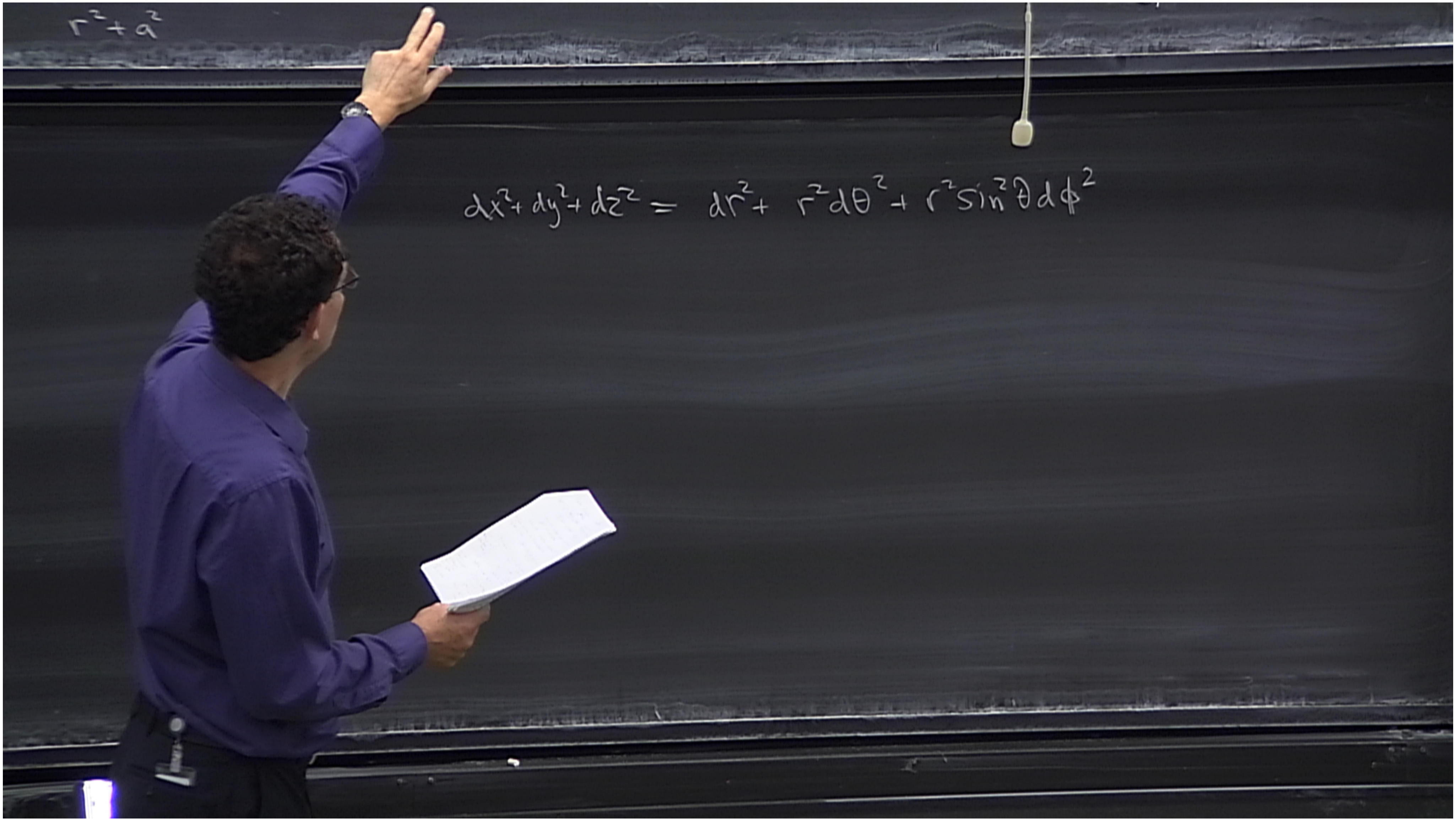

black hole – Kerr, 1963
$\Delta = r^2 - r_s r + a^2$
$\rho^2 = r^2 + a^2\cos^2\theta$ and $r_s = \frac{2GM}{c^2}$, $a = J/M$ with J the angular momentum.
(in Boyer-Lindquist coords)
$g_{t\phi} = g_{\phi t}$
get Schwarzchild line element.



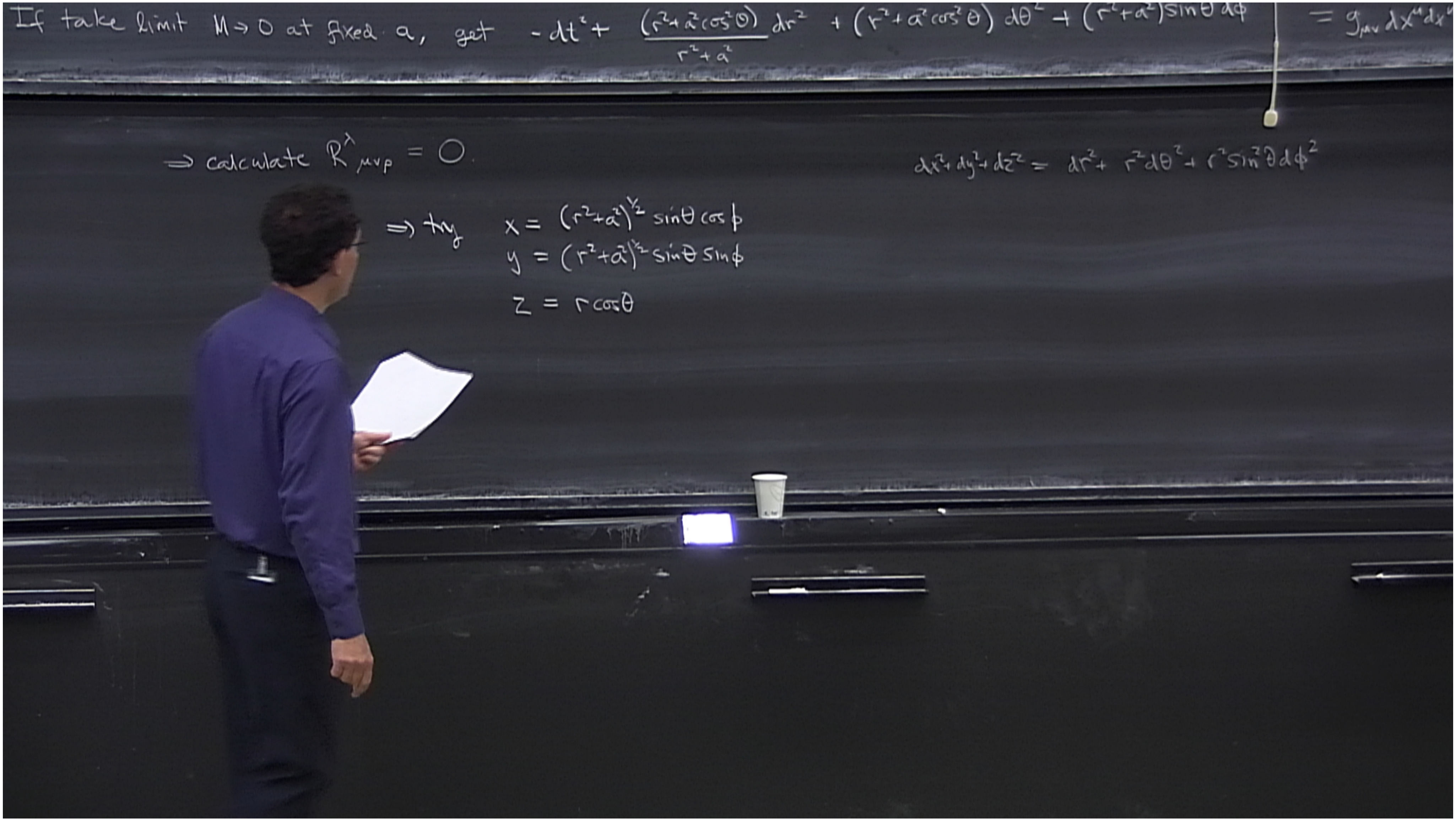
$g_{t\phi} = g_{\phi t}$
where $\Delta = r^2 - r_s r + a^2$
$\rho^2 = r^2 + a^2\cos^2\theta$ and $r_s = 2\frac{GM}{c^2}$, $a = J/M$ with J the angular momentum.
(in Boyer-Lindquist coords)
If take
0 at fixed M, get Schwarzchild line element.

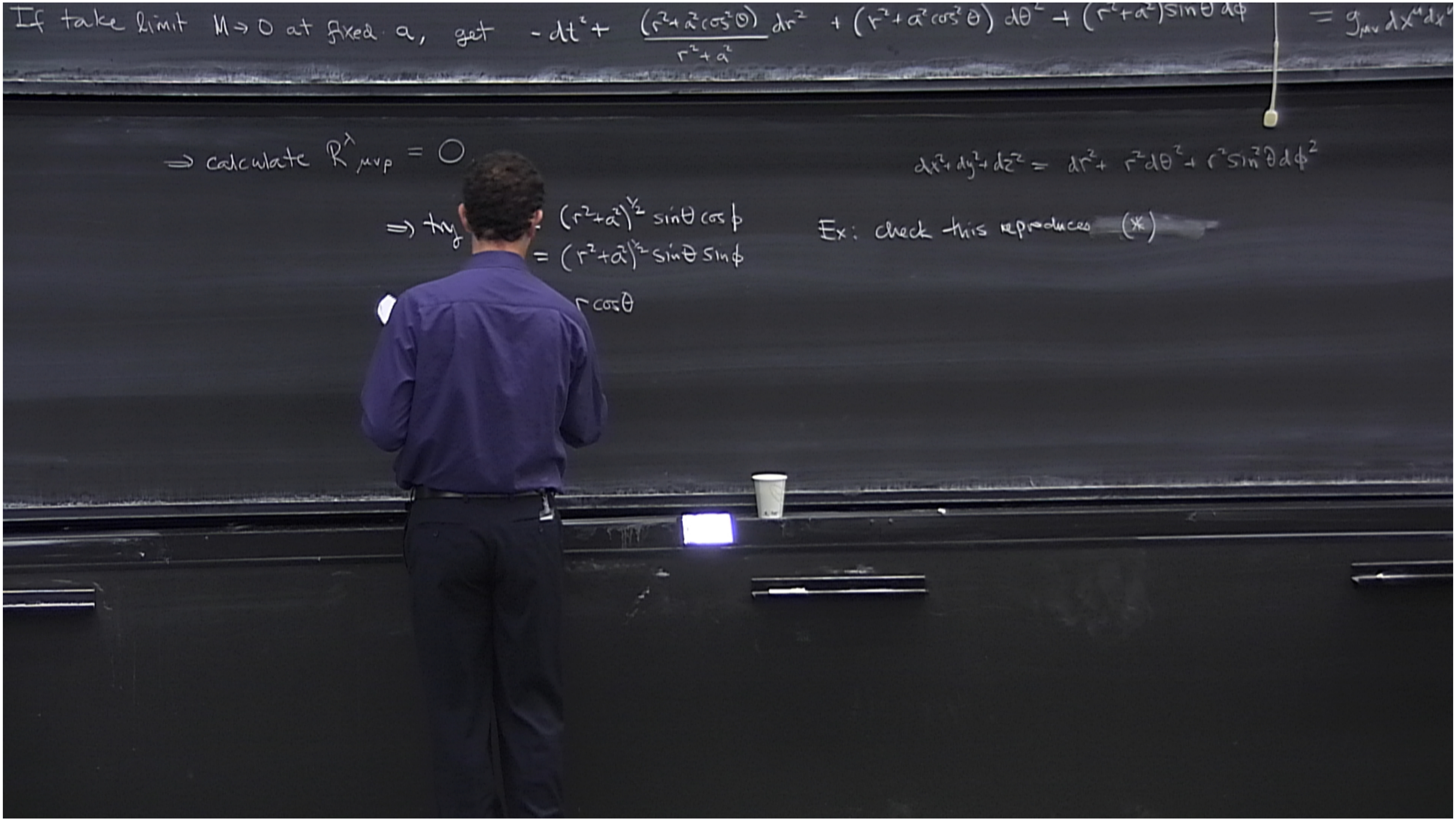

$g_{t\phi} = g_{\phi t}$
(in Boyer-Lindquist coords)
where $\Delta = r^2 - r_s r + a^2$
$\rho^2 = r^2 + a^2\cos^2\theta$ and $r_s = \frac{2GM}{c^2}$, $a = J/M$ with J the angular momentum.
If take limit $a \to 0$ at fixed M, get Schwarzchild line element.
take limit $M \to 0$ at fixed a, get $-dt^2 + \frac{(r^2+a^2\cos^2\theta)}{r^2+a^2}dr^2 + (r^2+a^2\cos^2\theta)\,d\theta^2 + (r^2+a^2)\sin^2\theta\,d\phi^2$

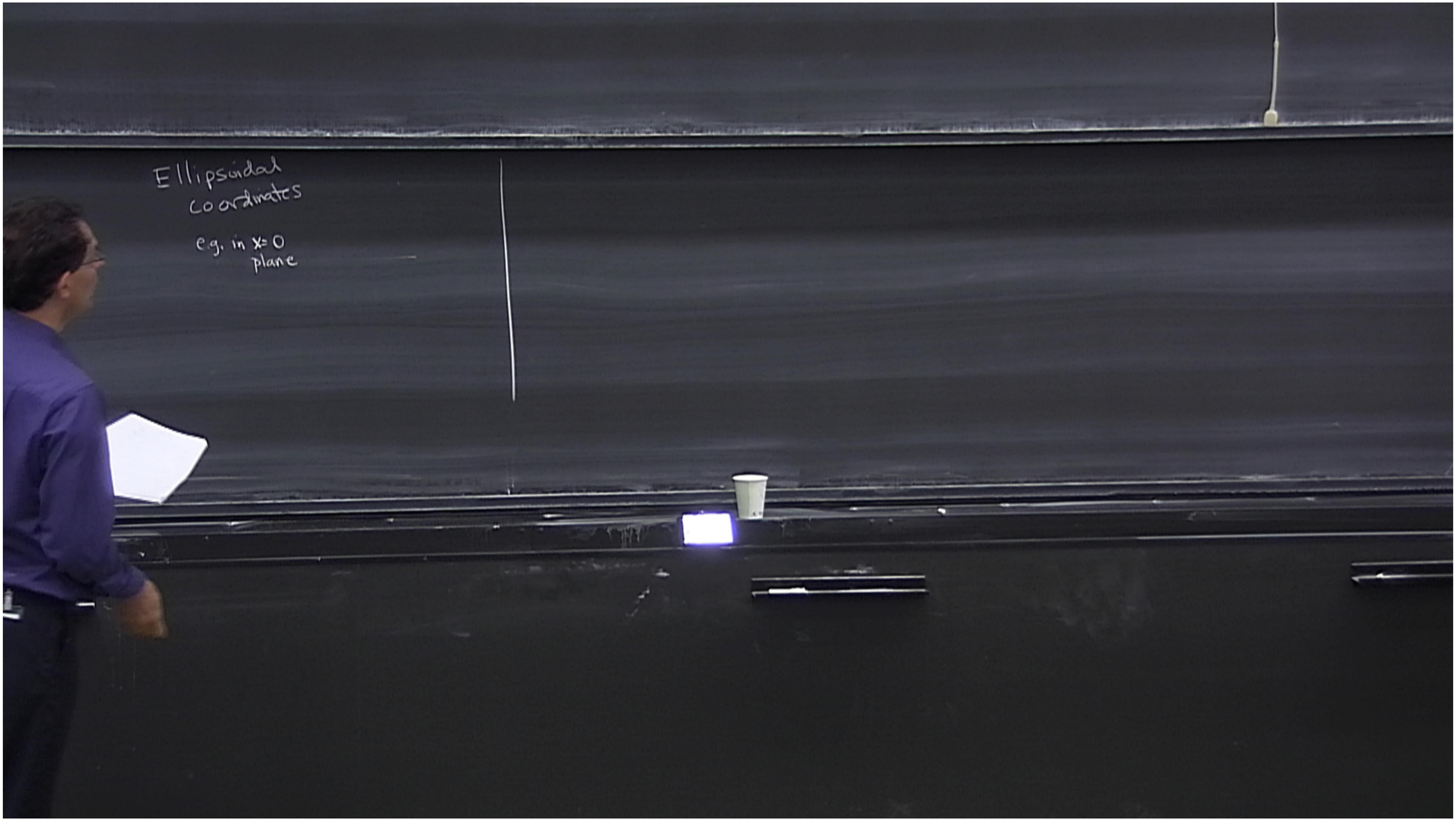

fixed a, get -dt +
r²+a²
⇒ calculate R^λ_μνρ = 0.
dr²+

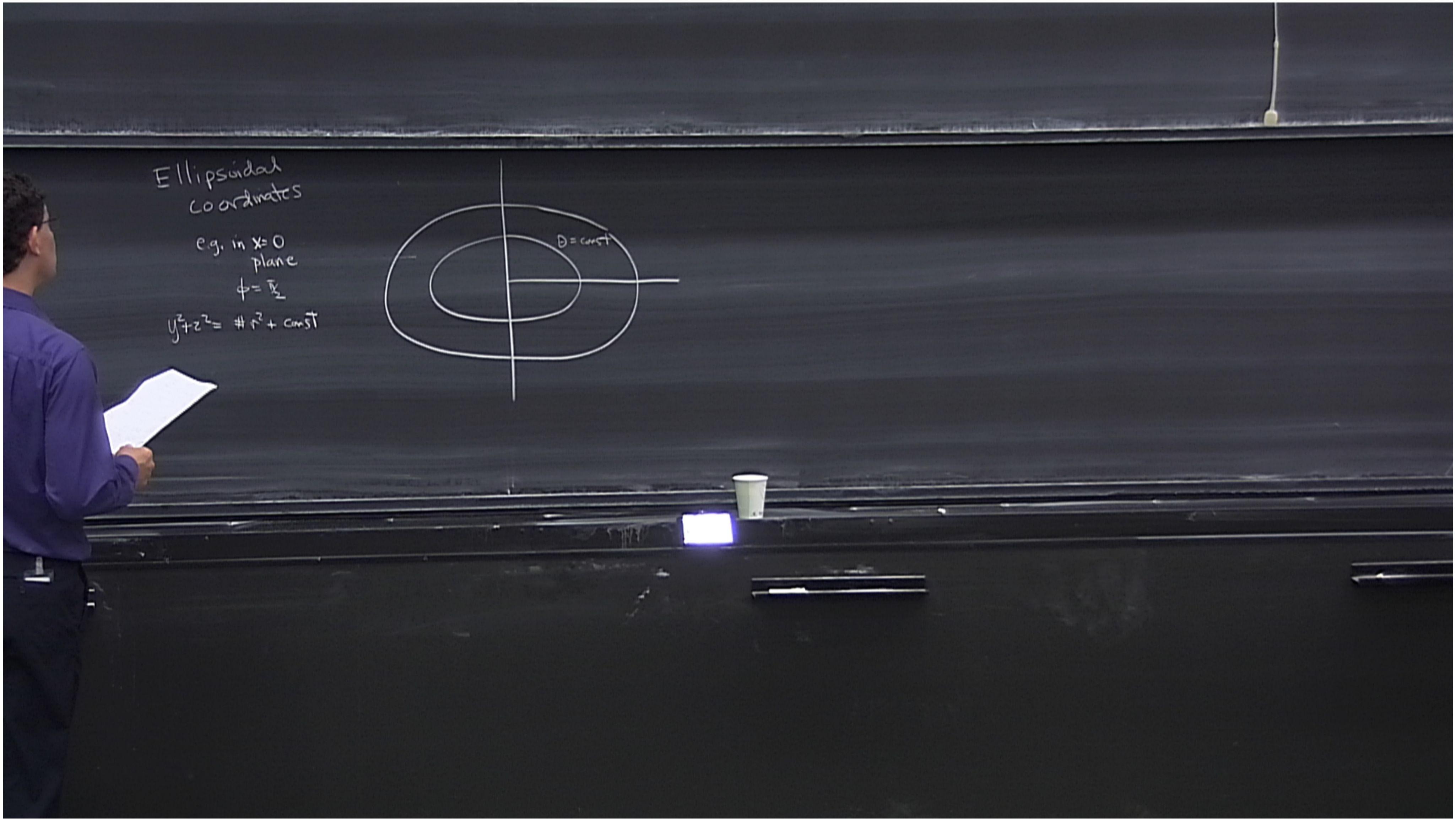


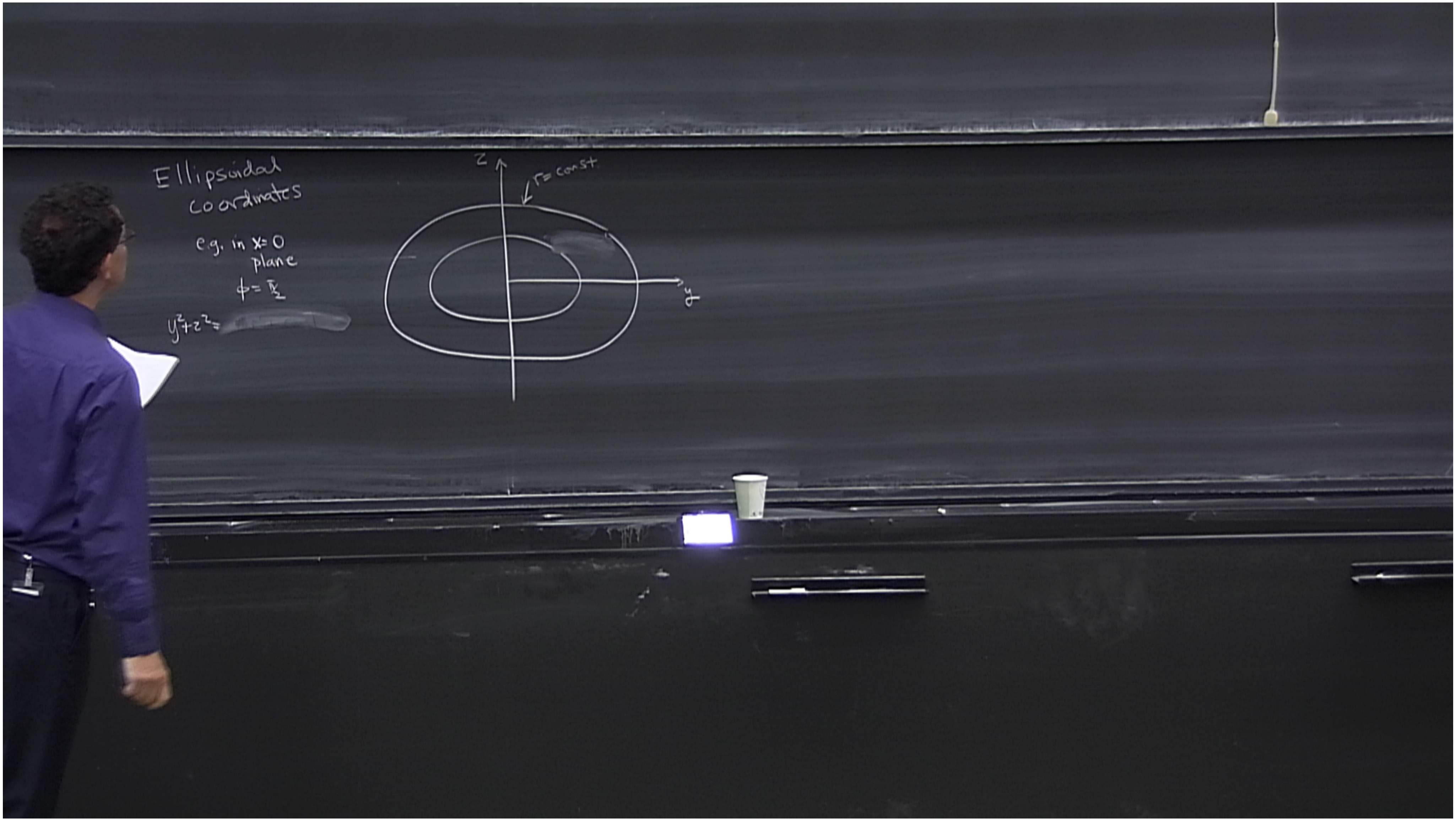
r^2+a^2
$dx^2+dy^2+dz^2 = dr^2+ r^2d\theta^2+ r^2\sin^2\theta\, d\phi^2$


If take limit $M \to 0$ at fixed a, get $-dt^2 + \frac{(r^2+a^2\cos^2\theta)}{r^2+a^2} dr^2 + (r^2+a^2\cos^2\theta)\, d\theta^2 + (r^2+a^2)\sin\theta\, d\phi$
$= g_{\mu\nu} dx^\mu dx^\nu$
$\Rightarrow$ calculate $R^\lambda{}_{\mu\nu\rho} = 0$.
$dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$
$\Rightarrow$ try $x = (r^2+a^2)^{1/2} \sin\theta \cos\phi$
$y = (r^2+a^2)^{1/2} \sin\theta \sin\phi$
$z = r\cos\theta$

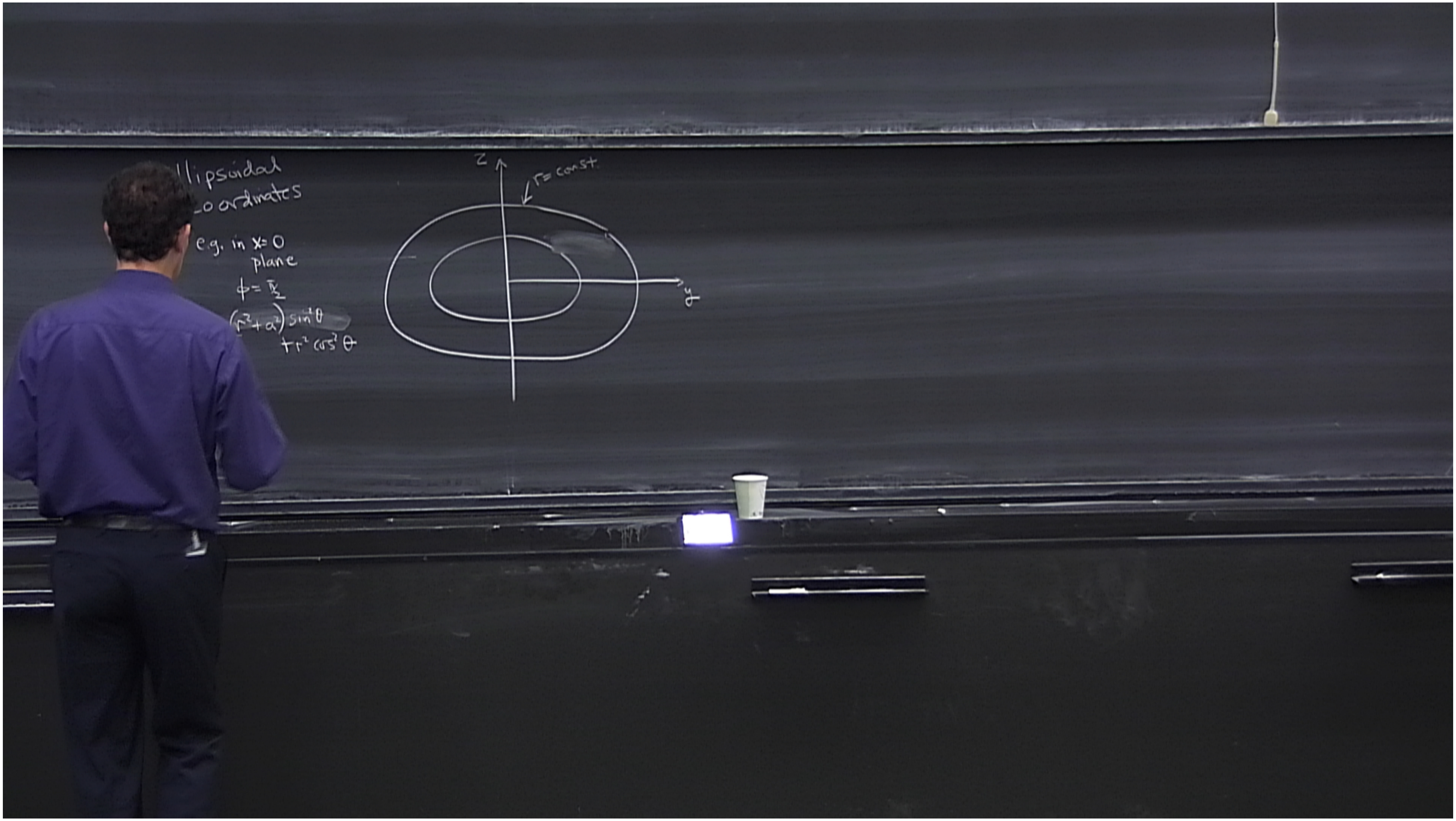

If take limit M → 0 at fixed a, get -dt² + (r²+a²cos²θ)/(r²+a²) dr² + (r²+a²cos²θ) dθ² + (r²+a²)sin²θ dφ²
= g_μν dx^μ dx^ν
⇒ calculate R^λ_μνρ = 0
⇒ try
(r²+a²)^½ sinθ cos φ
= (r²+a²)^½ sinθ sin φ
r cosθ
dx²+dy²+dz² = dr² + r²dθ² + r²sin²θ dφ²
Ex: check this reproduces (*)

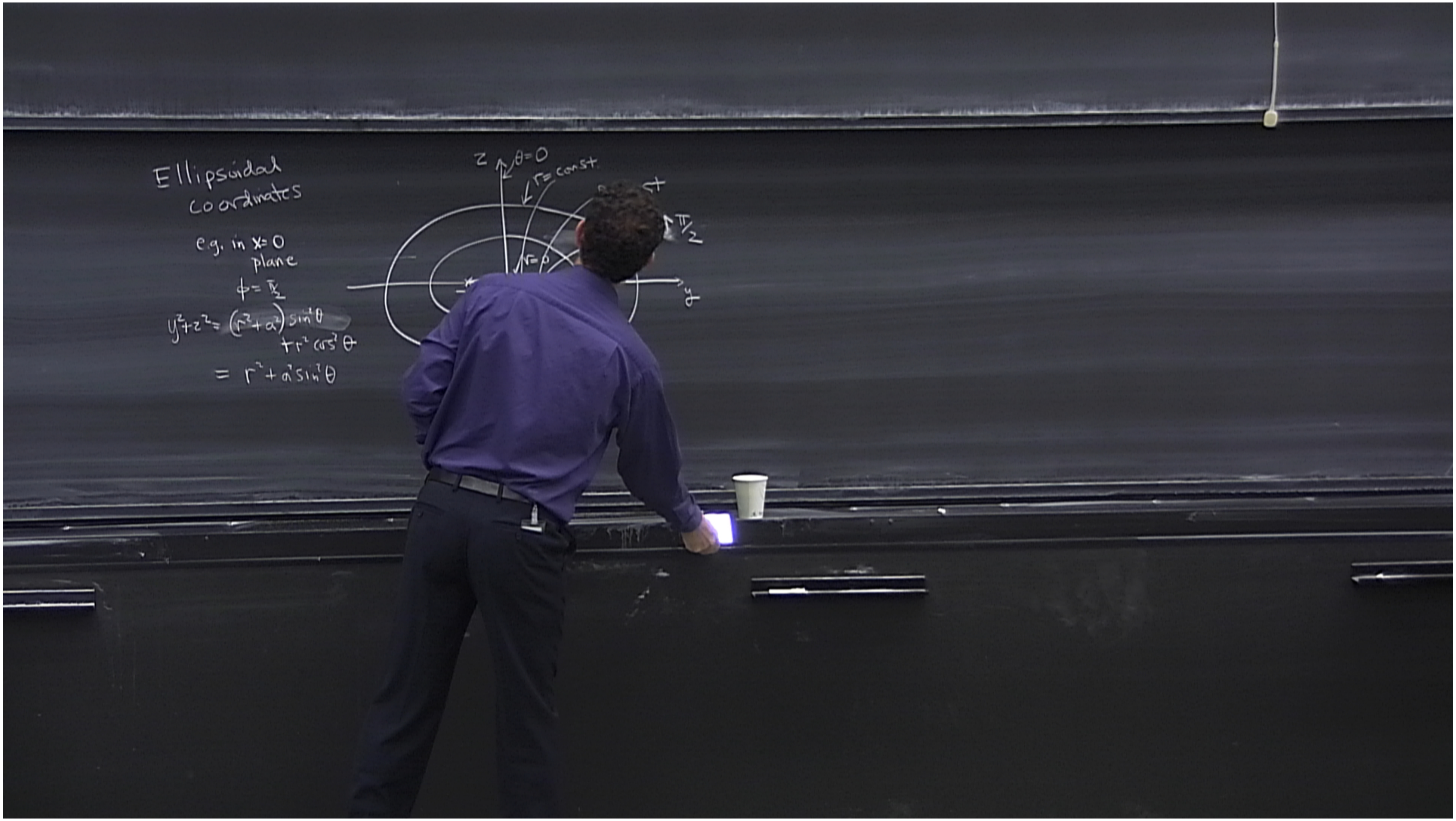

Ellipsoidal
coordinates
e.g. in x=0
plane

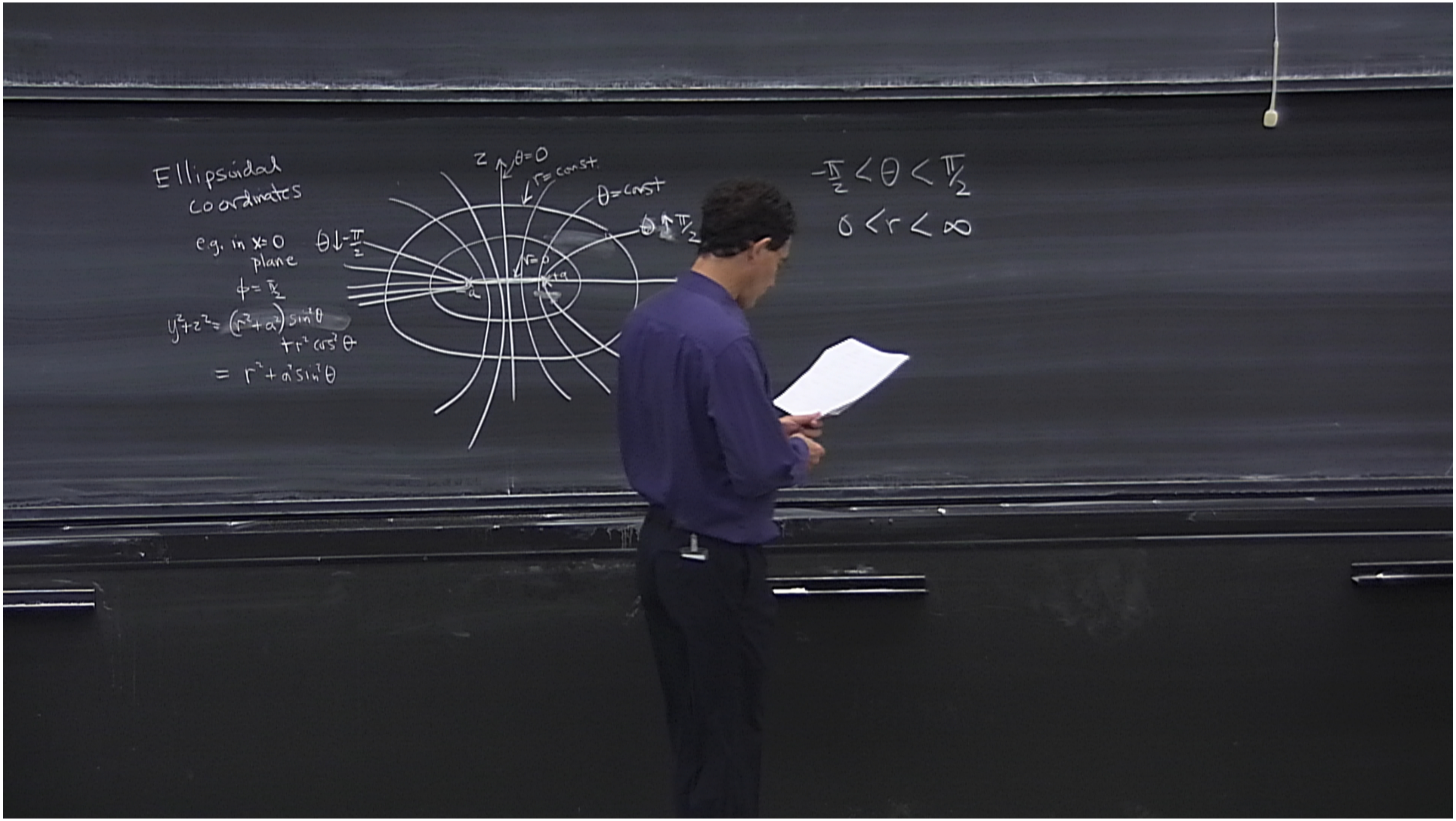

Ellipsoidal coordinates
e.g. in x=0 plane
φ = π/2
y²+z² = # r² + const
θ = const



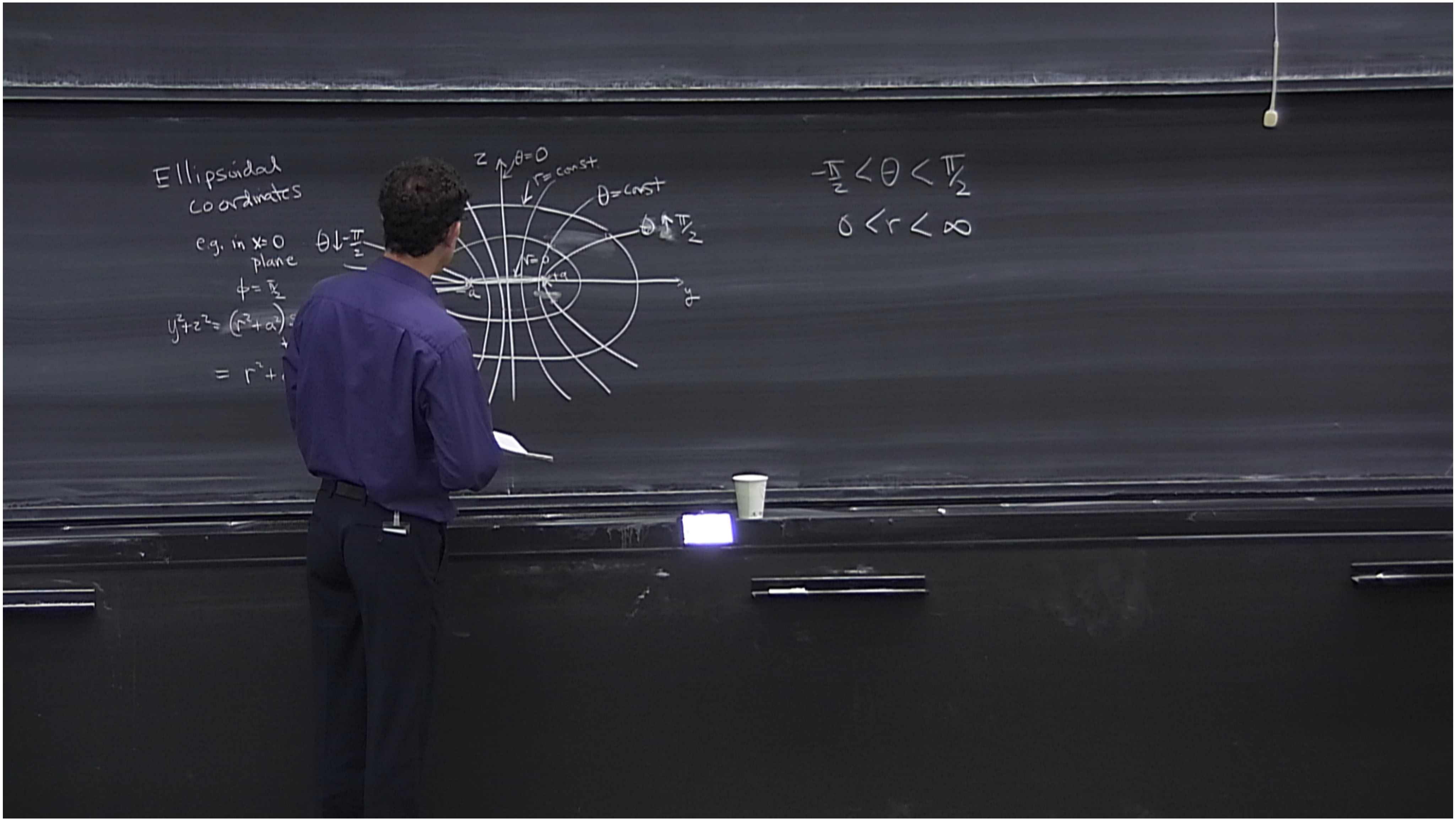
Ellipsoidal coordinates
e.g. in x=0 plane
$\phi = \frac{\pi}{2}$
$y^2+z^2=$
z
y
r= const

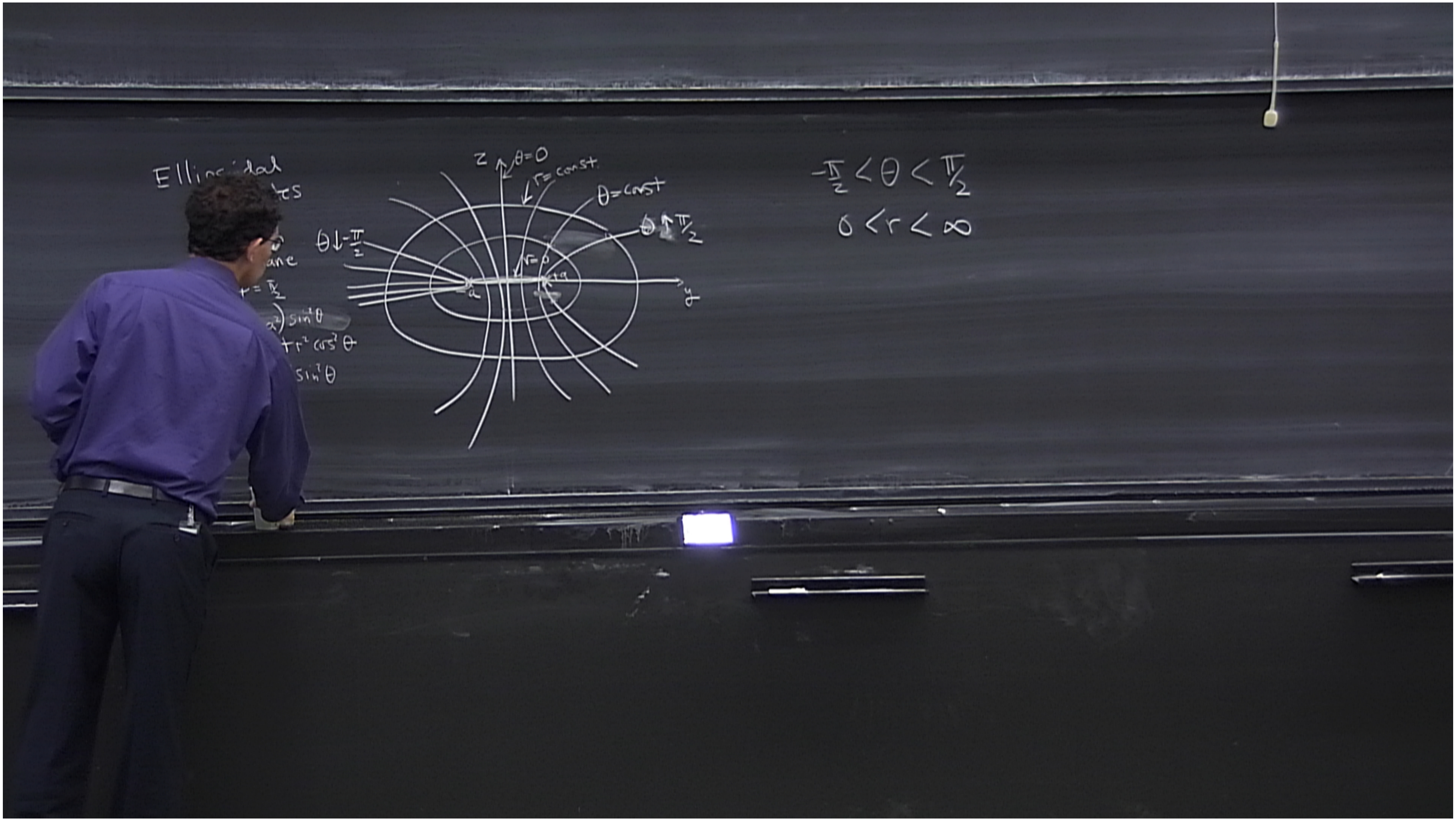

llipsoidal
coordinates
e.g. in x=0
plane
z
r= const
y

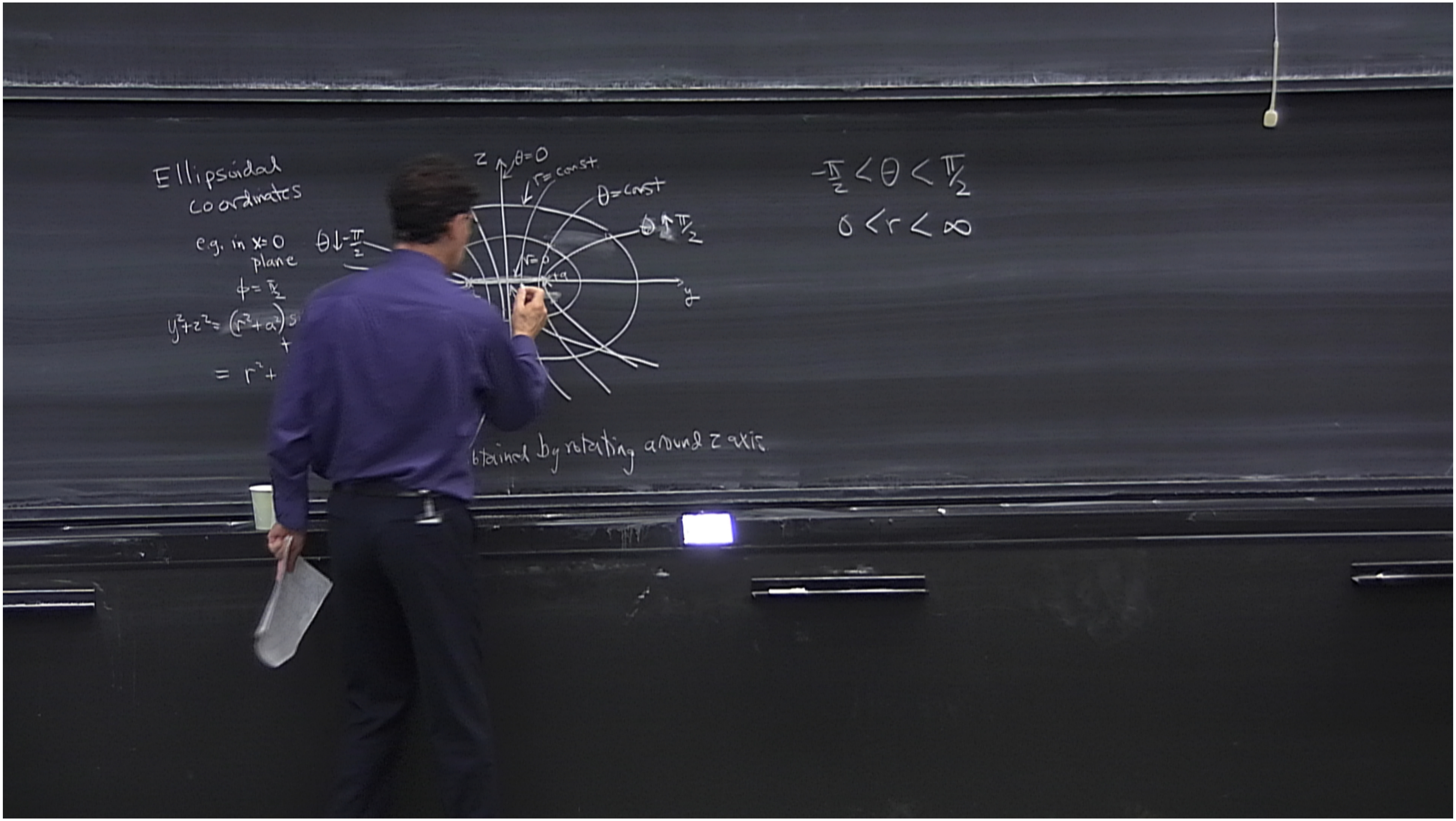

Ellipsoidal
coordinates
e.g. in x=0
plane
φ = π/2
y²+z² = (r²+a²) sin²θ
+ r² cos² θ
= r² + a² sin²θ
z
θ=0
r=const
π/2
y

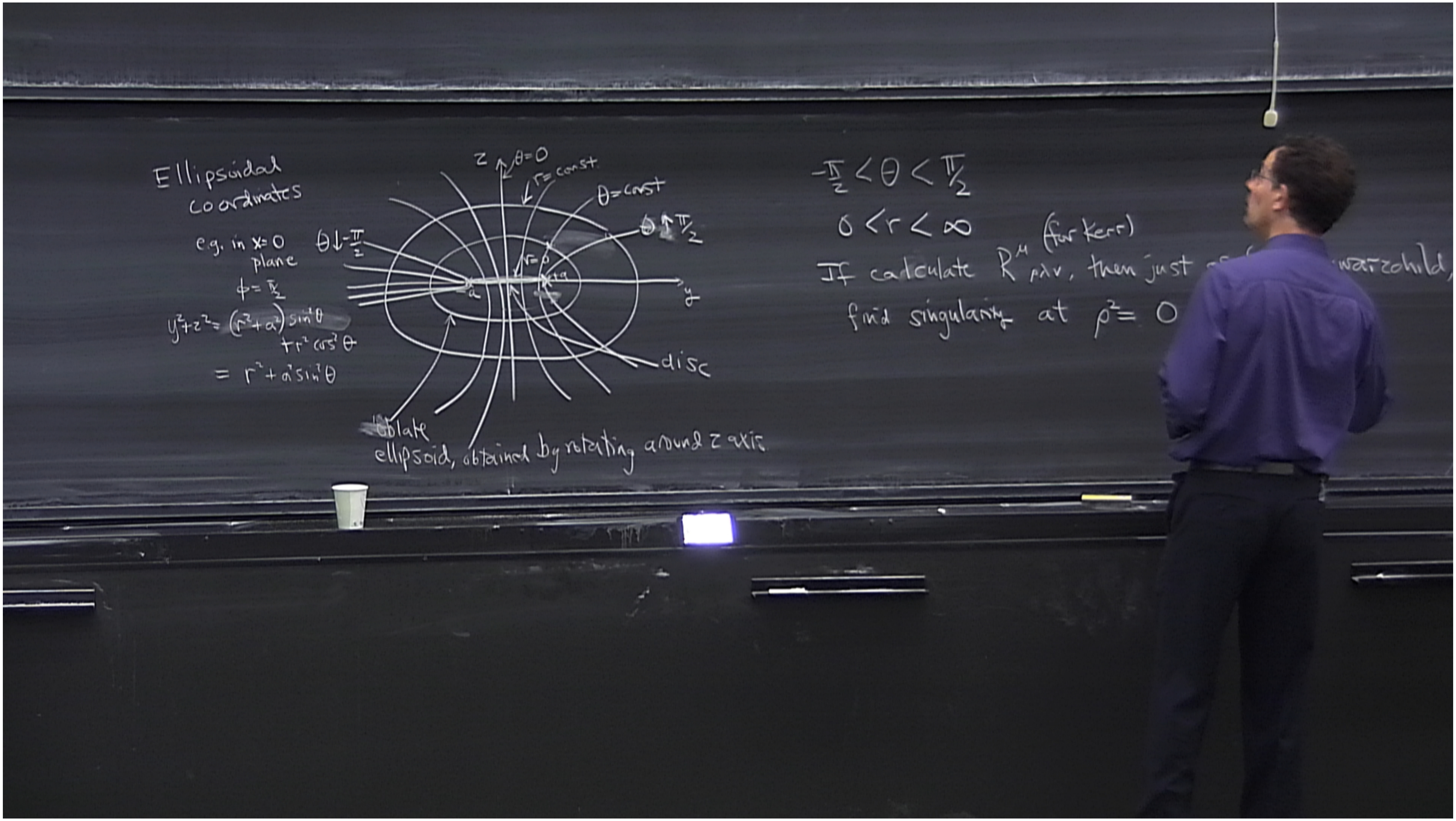


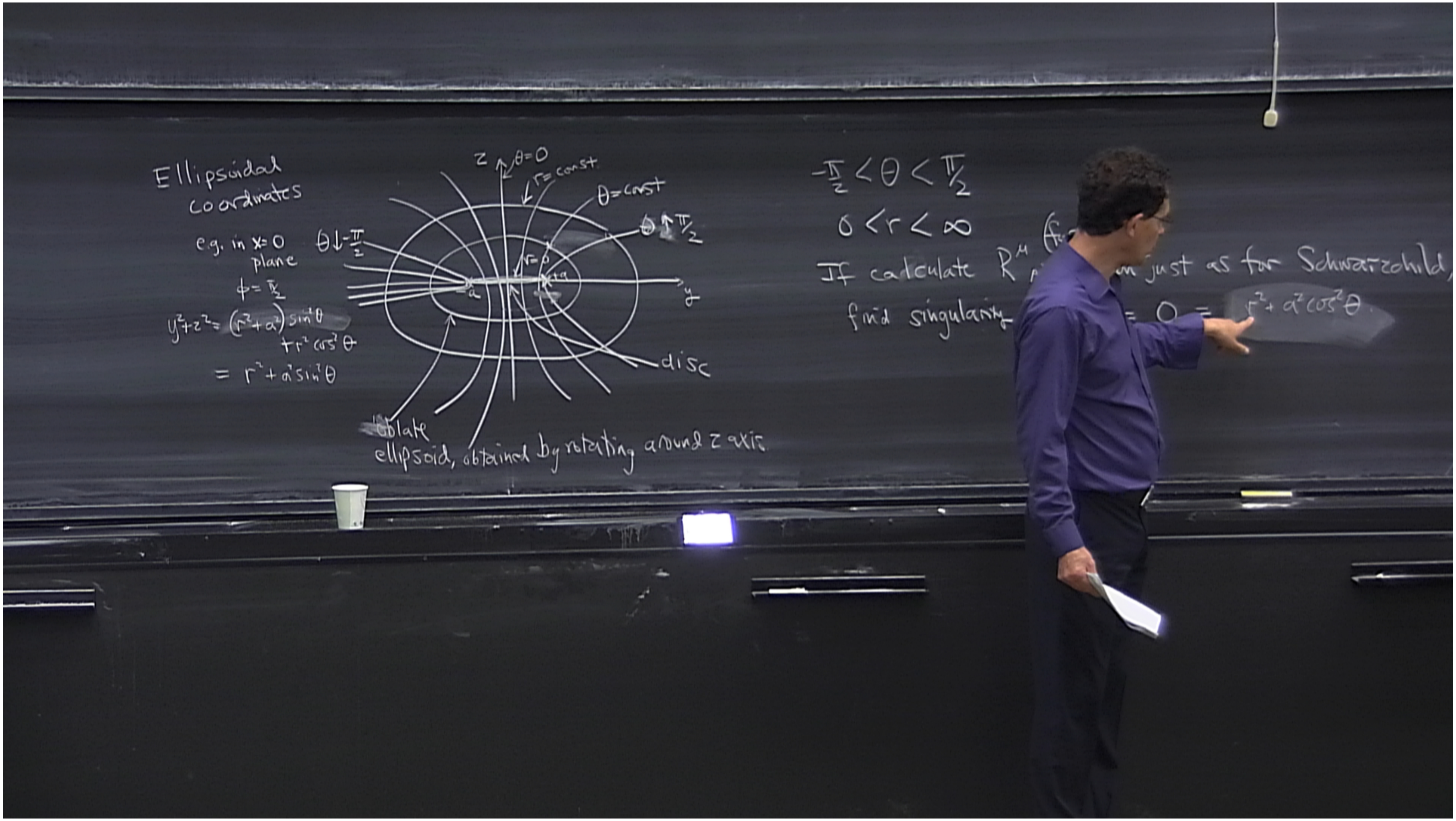
Ellipsoidal coordinates
e.g. in x=0 plane
$\phi = \frac{\pi}{2}$
$y^2+z^2 = (r^2+a^2)\sin^2\theta + r^2\cos^2\theta$
$= r^2 + a^2\sin^2\theta$
z
θ=0
r=const
θ=const
$\theta \downarrow -\frac{\pi}{2}$
$\theta \uparrow \frac{\pi}{2}$
r=0
−a
+a
$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$0 < r < \infty$


Ellipsoidal
coordinates
e.g. in x=0
plane
$\phi = \frac{\pi}{2}$
$y^2+z^2 = (r^2+a^2)$
$= r^2 +$
z
$\theta = 0$
r = const
θ = const
$\theta \downarrow -\frac{\pi}{2}$
$\theta \uparrow \frac{\pi}{2}$
r = 0
+a
-a
y
$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$0 < r < \infty$


θ=0
r=const
θ=const
θ↓-π/2
θ↑π/2
r=0
-a
+a
z
y
-π/2 < θ < π/2
0 < r < ∞


Ellipsoidal coordinates
e.g. in x=0 plane
φ = π/2
θ=0
r=const
θ=const
θ↓-π/2
θ↑π/2
r=0
y
z
-π/2 < θ < π/2
0 < r < ∞
btained by rotating around z axis


Ellipsoidal coordinates
e.g. in x=0 plane
φ = π/2
y²+z² = (r²+a²) sin²θ + r² cos²θ
= r² + a² sin²θ
z
θ=0
r=const
θ=const
θ↓-π/2
θ↑π/2
y
disc
Oblate ellipsoid, obtained by rotating around z axis
-π/2 < θ < π/2
0 < r < ∞
(for Kerr)
If calculate R^μ_ρλν, then just
find singularity at ρ² = 0



Ellipsoidal coordinates
e.g. in x=0 plane
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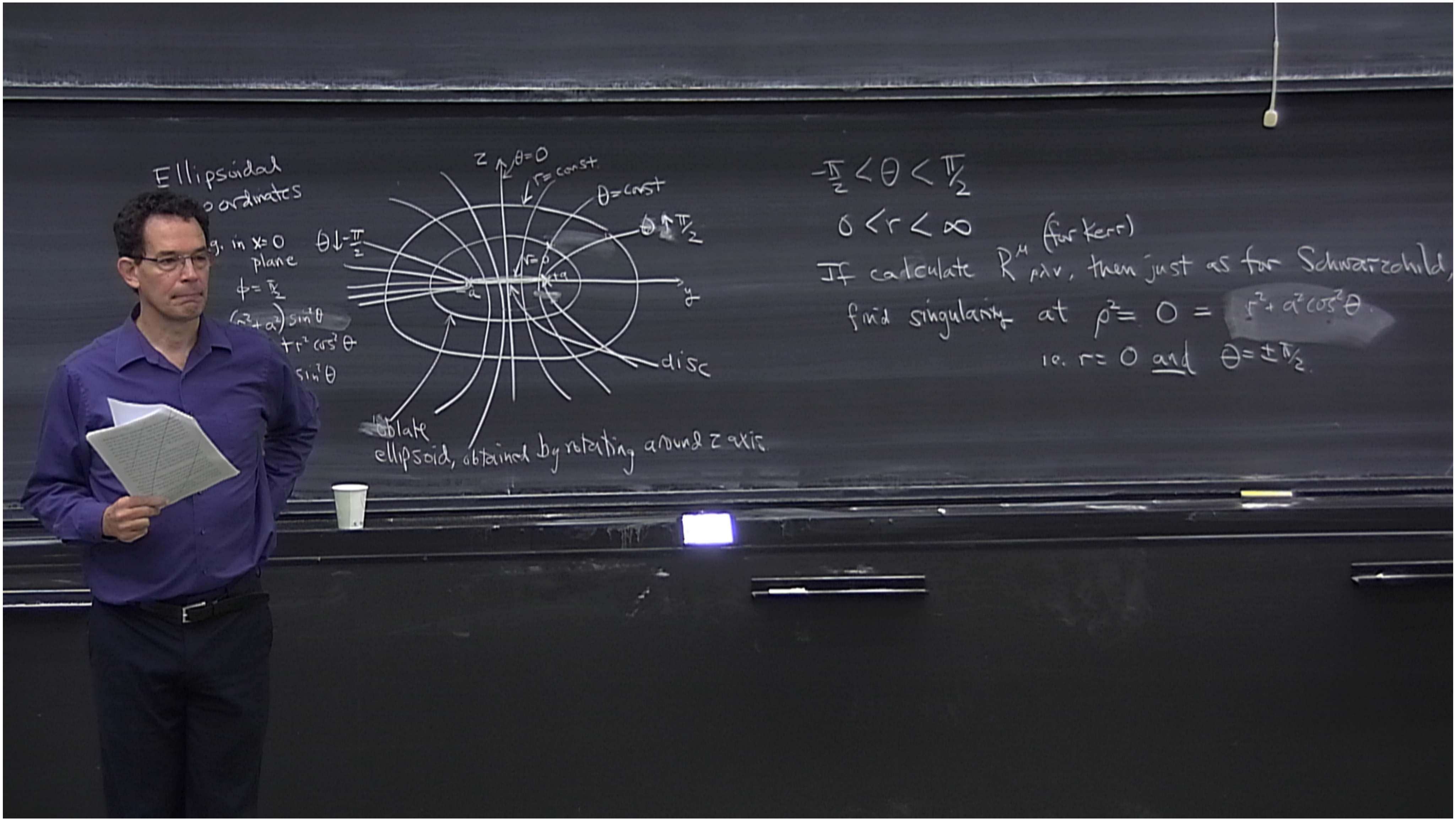
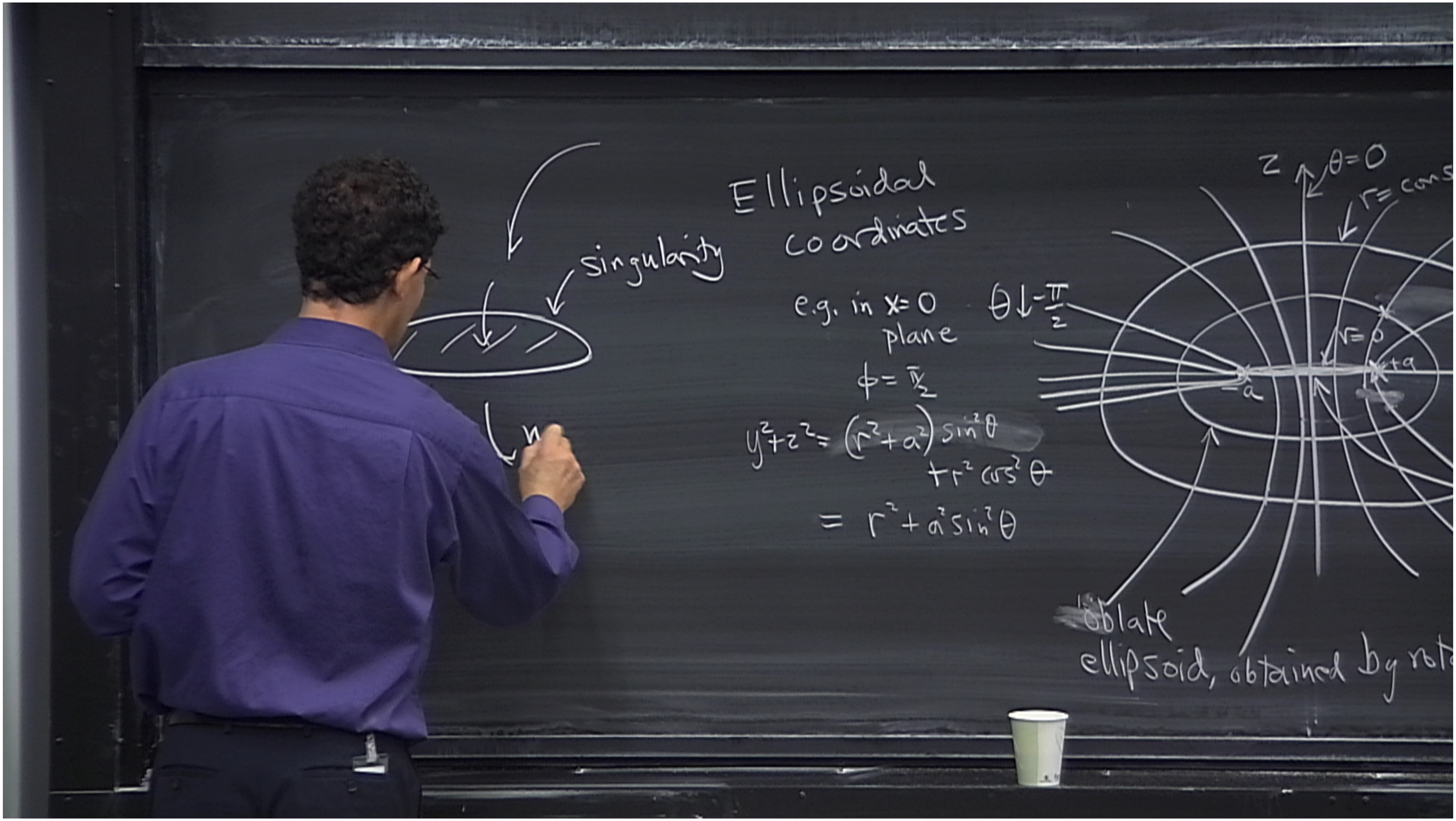
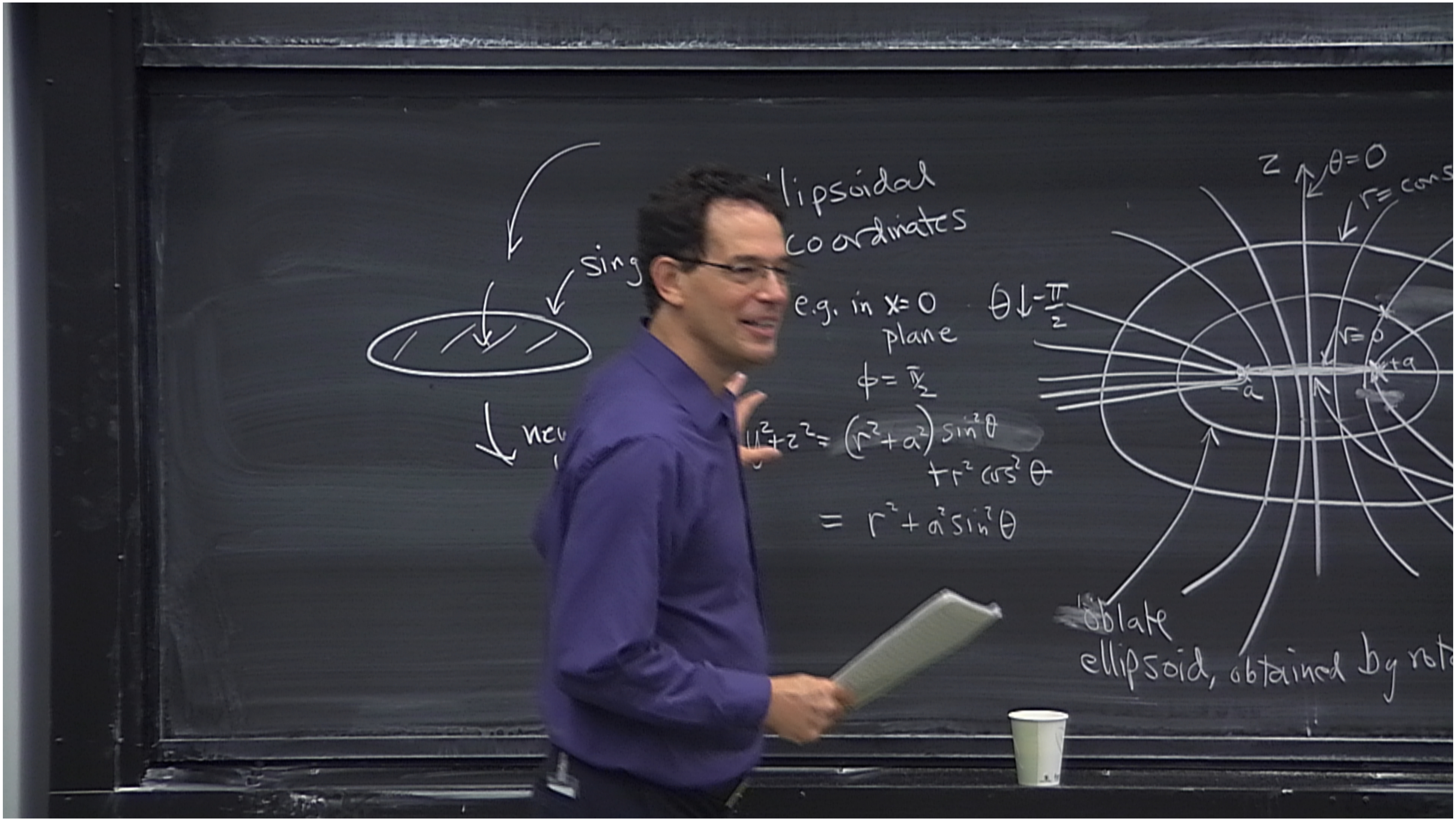
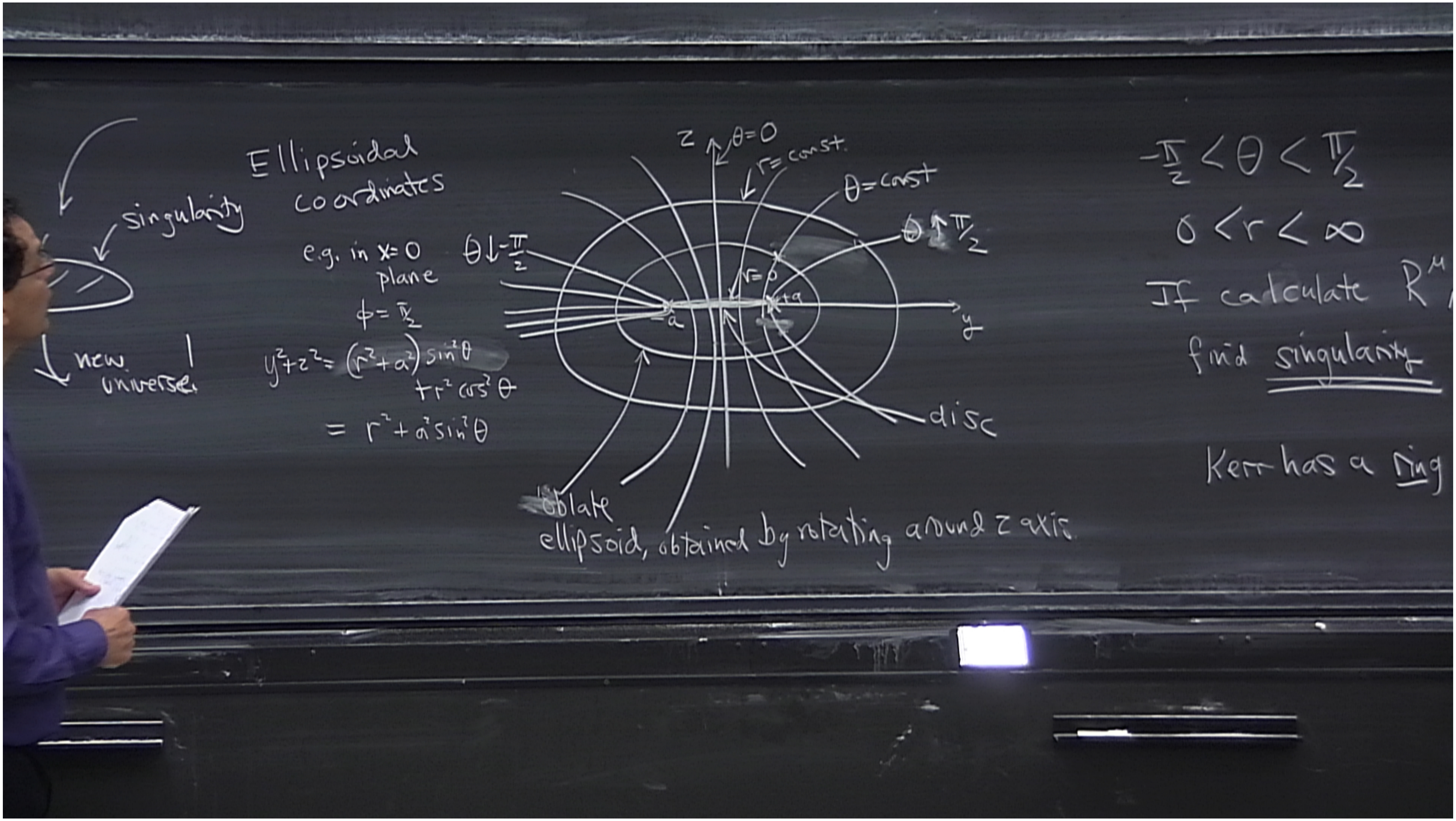
y²+z² = (r²+a²) sin²θ + r² cos²θ
= r² + a² sin²θ
θ=0
r=const
θ=const
disc
oblate ellipsoid, obtained by rotating around z axis
-π/2 < θ < π/2
0 < r < ∞
If calculate R just as for Schwarzschild,
find singularity
= 0 = r² + a² cos²θ

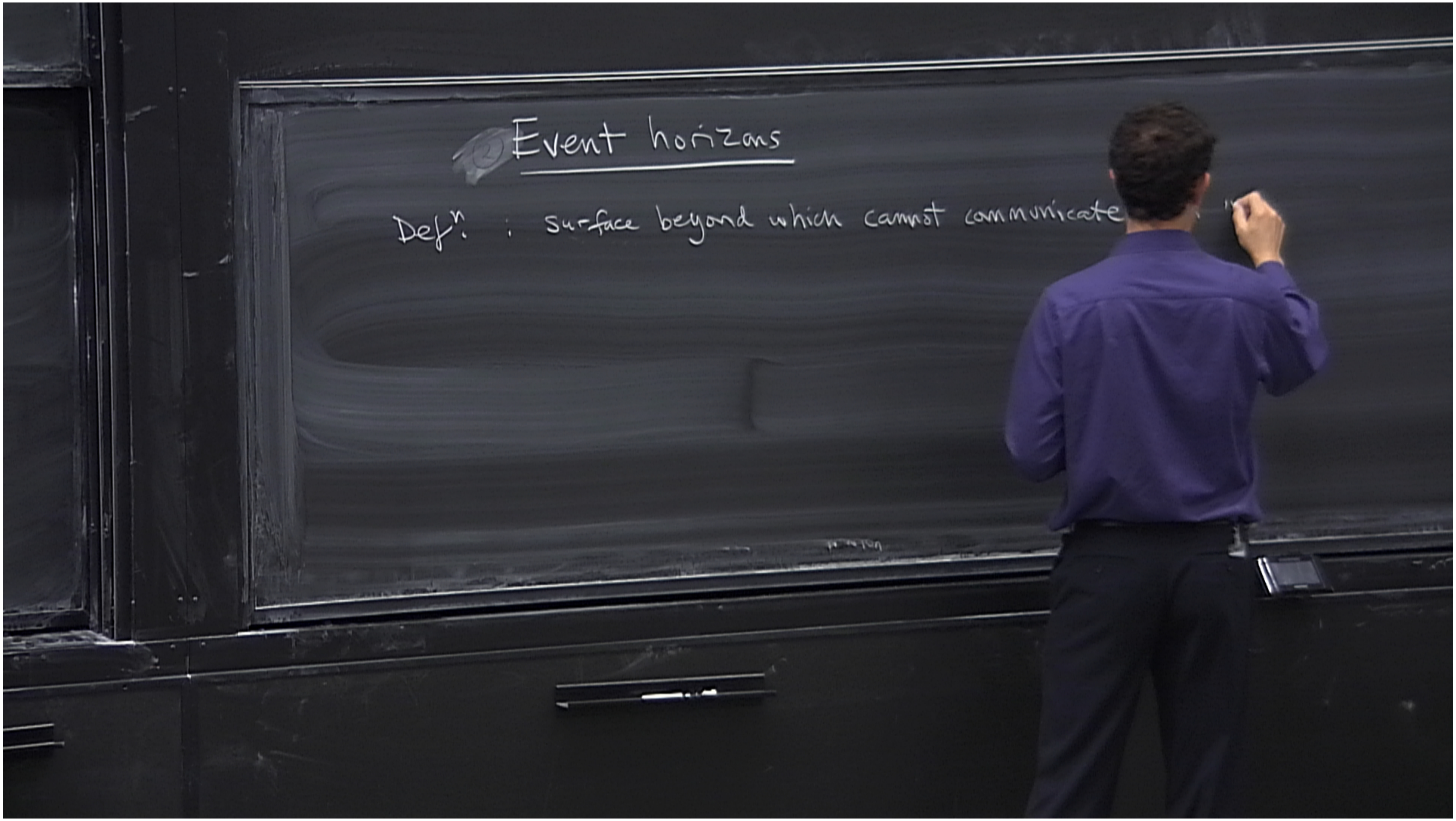

Ellipsoidal
coordinates
$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$0 < r < \infty$ (for Kerr)
If calculate $R^{\mu}{}_{\rho\lambda\nu}$, then just as for Schwarzschild,
find singularity at $\rho^2 = 0 = r^2 + a^2\cos^2\theta$.
i.e. r = 0 and $\theta = \pm\pi/2$
disc
oblate
ellipsoid, obtained by rotating around z axis

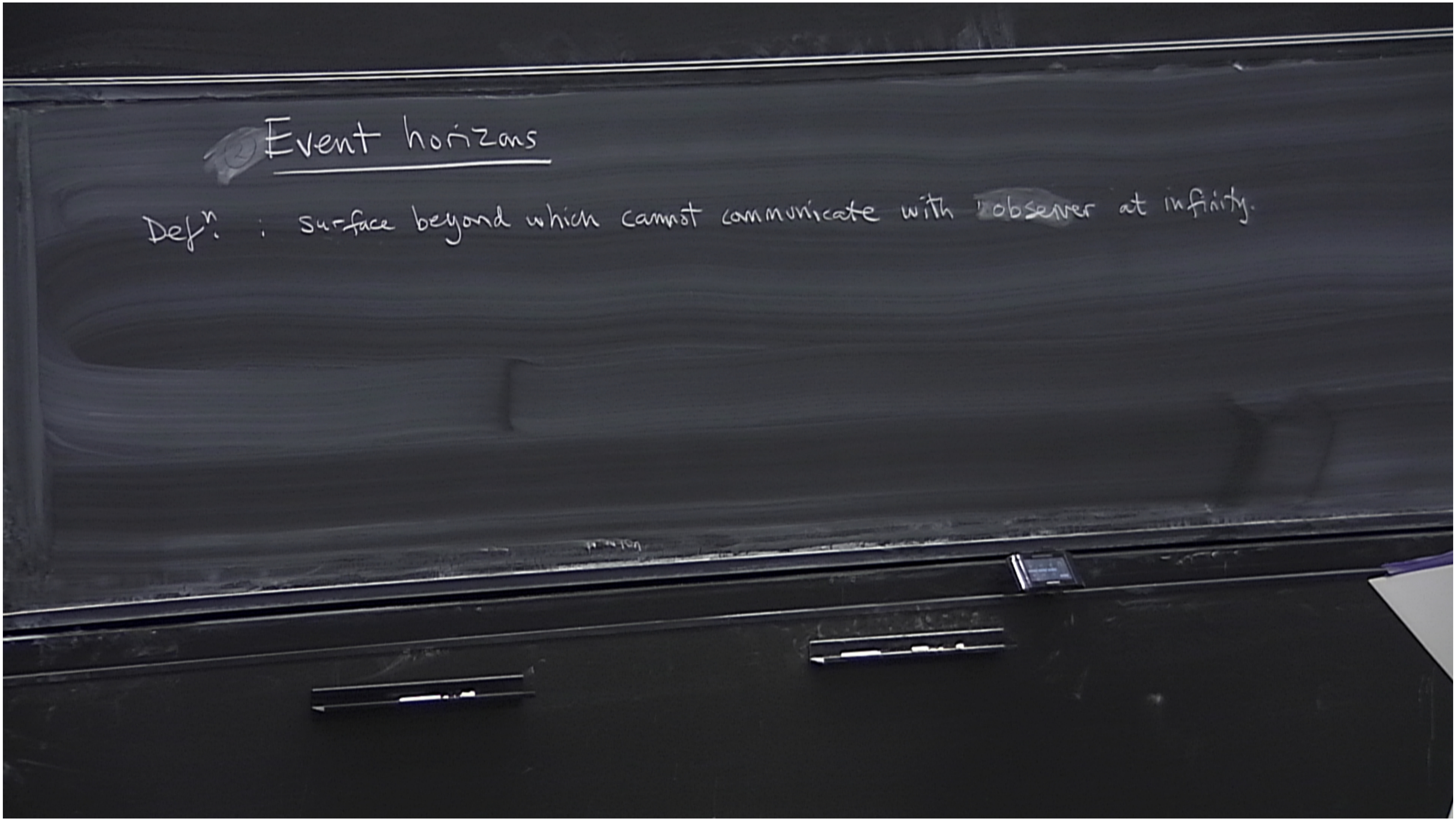


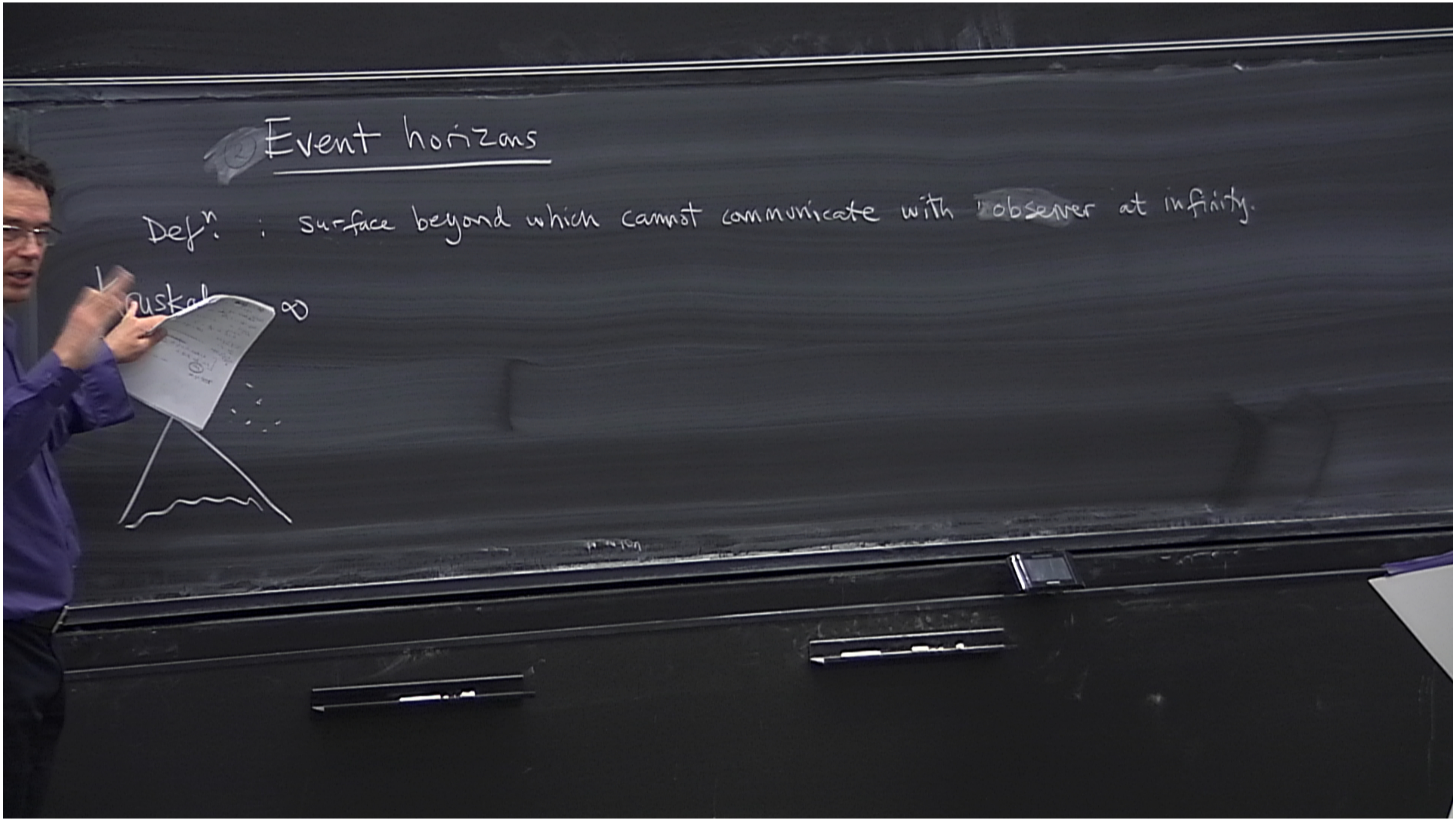
singularity
Ellipsoidal coordinates
e.g. in x=0 plane
φ = π/2
y²+z² = (r²+a²) sin²θ + r² cos²θ
= r² + a² sin²θ
θ ↓ -π/2
z
θ=0
r= cons
r=0
-a
+a
oblate ellipsoid, obtained by rot


llipsoidal
coordinates
e.g. in x=0 plane
ϕ = π/2
= r² + a² sin² θ
oblate
ellipsoid, obtained by rot


singularity
new universe!
Ellipsoidal coordinates
e.g. in x=0 plane
φ = π/2
y²+z² = (r²+a²) sin²θ + r² cos²θ
= r² + a² sin²θ
z
θ=0
r=const.
θ=const
θ↓-π/2
θ↑π/2
r=0
-a
+a
y
disc
oblate ellipsoid, obtained by rotating around z axis
-π/2 < θ < π/2
0 < r < ∞
If calculate R
find singularity
Kerr has a ring


Event horizons
Defn: : surface beyond which cannot communicate


Event horizons
Def^n : surface beyond which cannot communicate with observer at infinity.

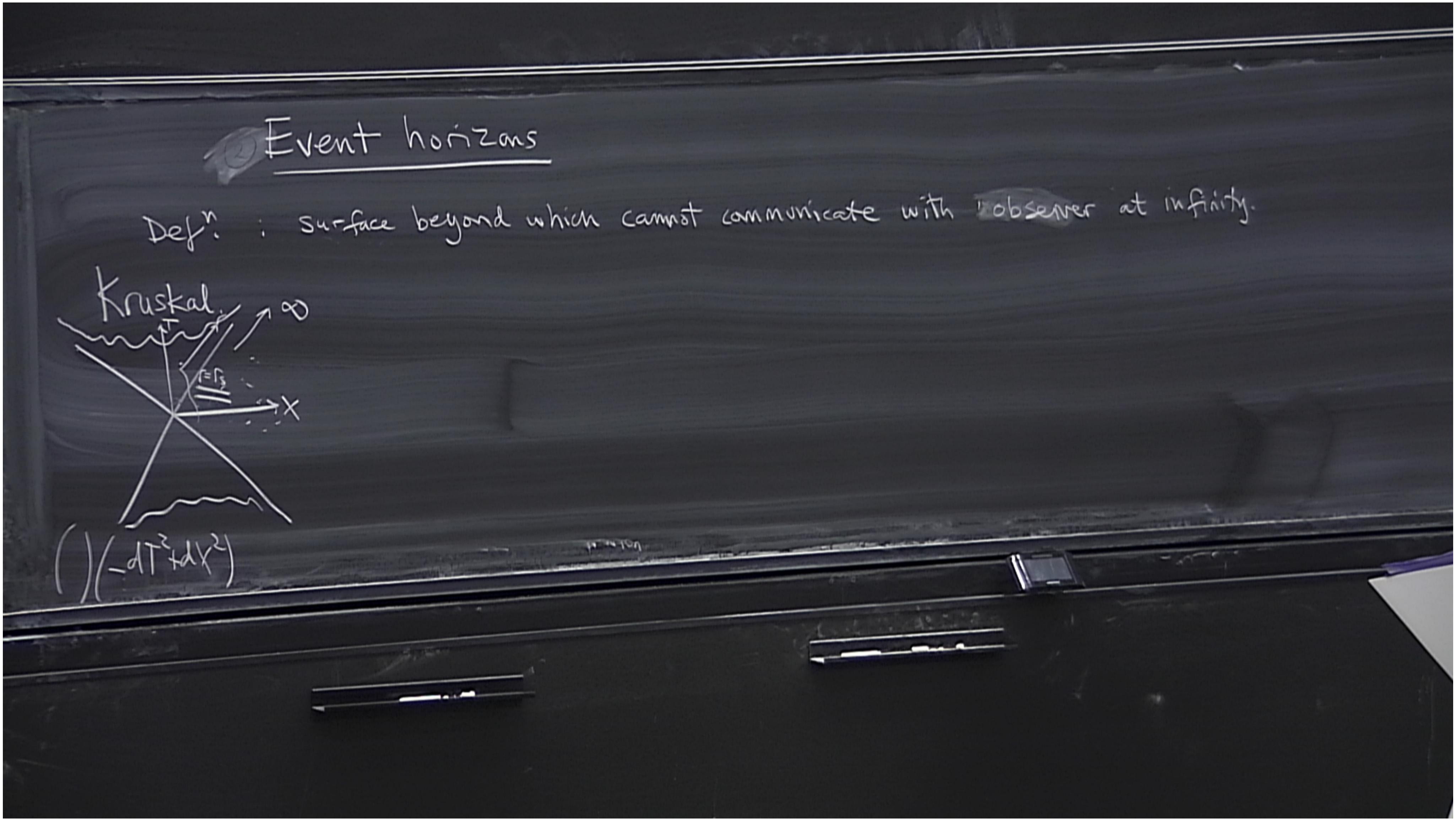

Event horizons
Defn : surface beyond which cannot communicate with observer at infinity.
∞

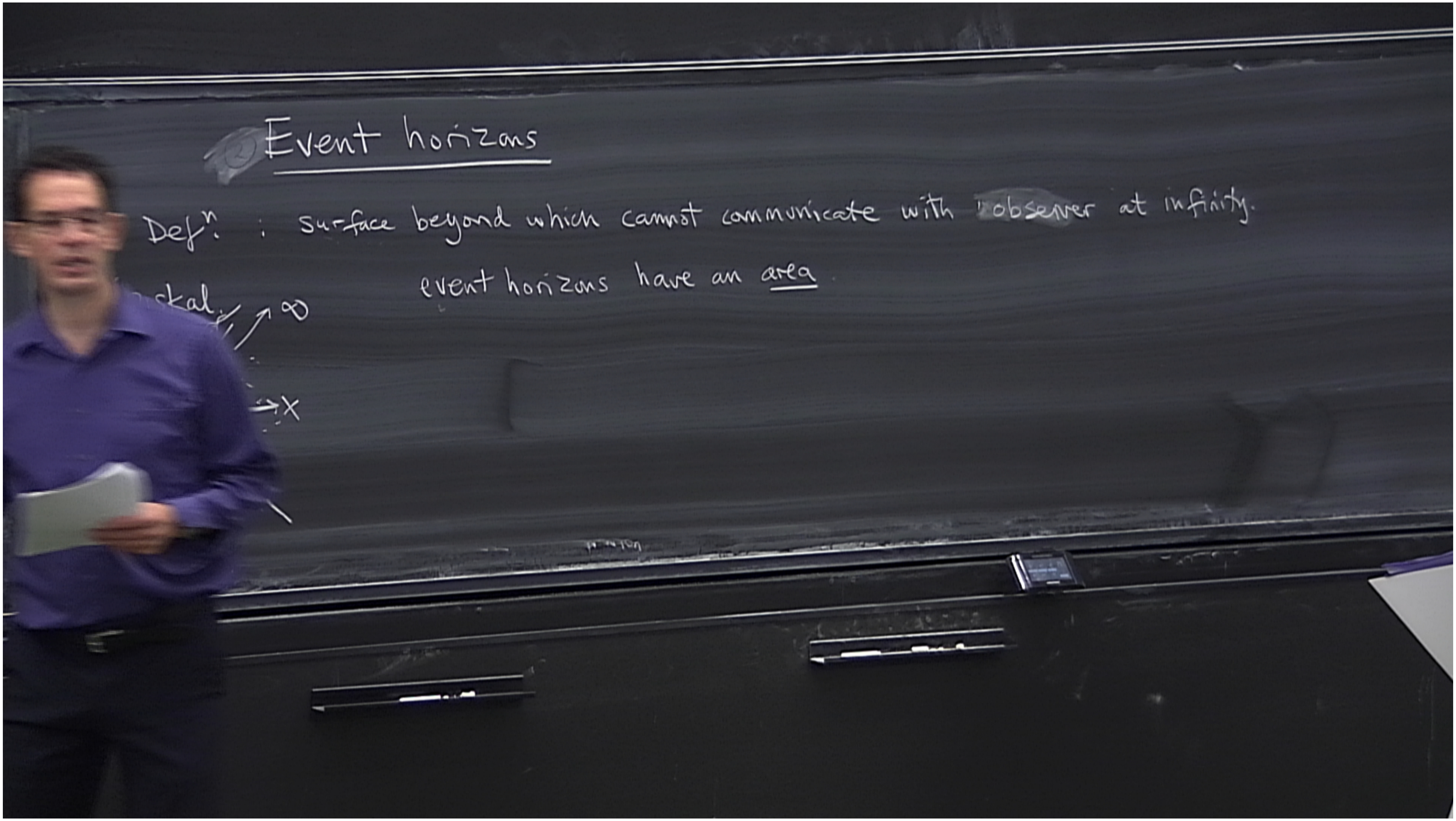
Event horizons

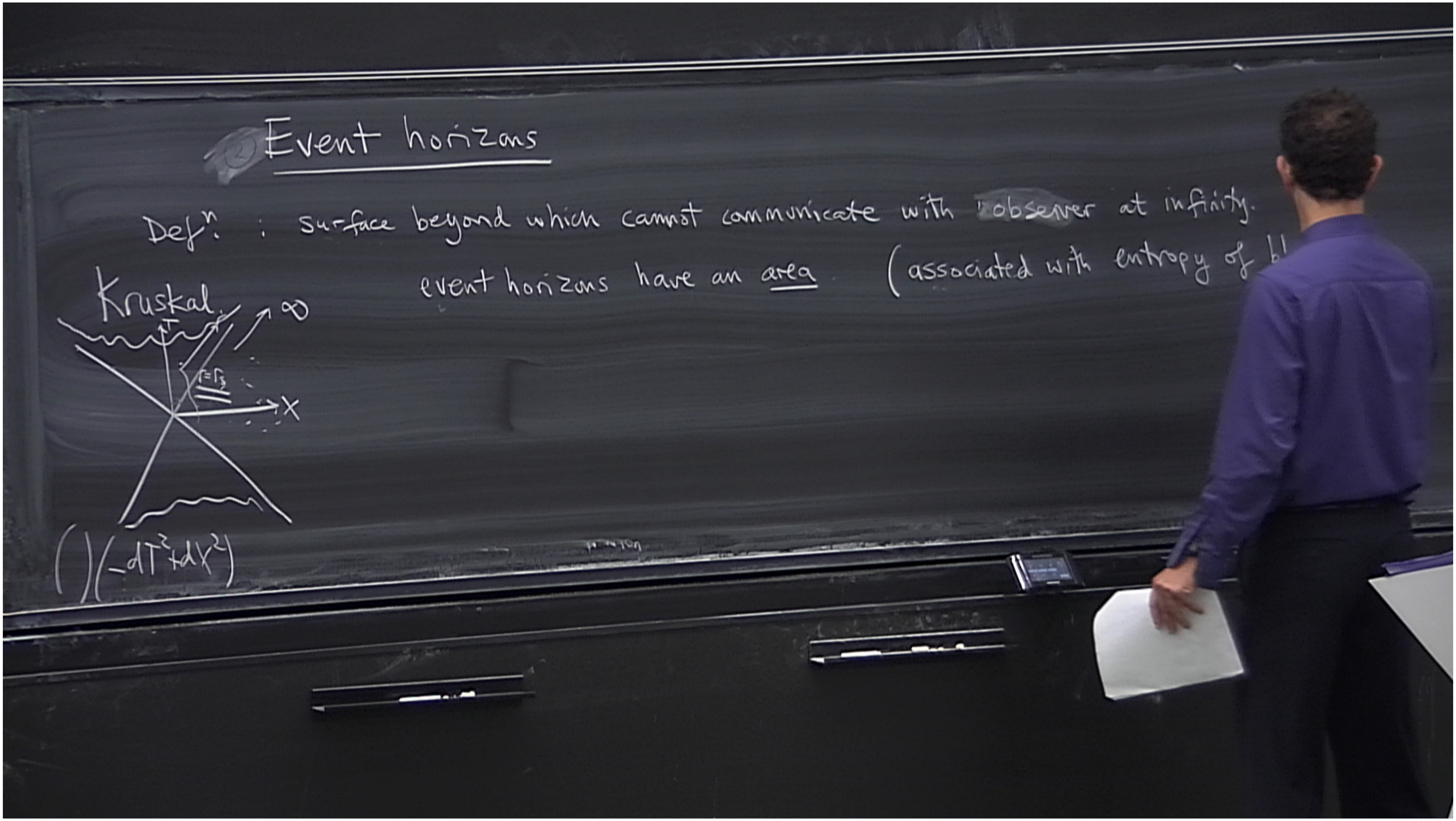
Def^n: surface beyond which cannot communicate with observer at infinity.

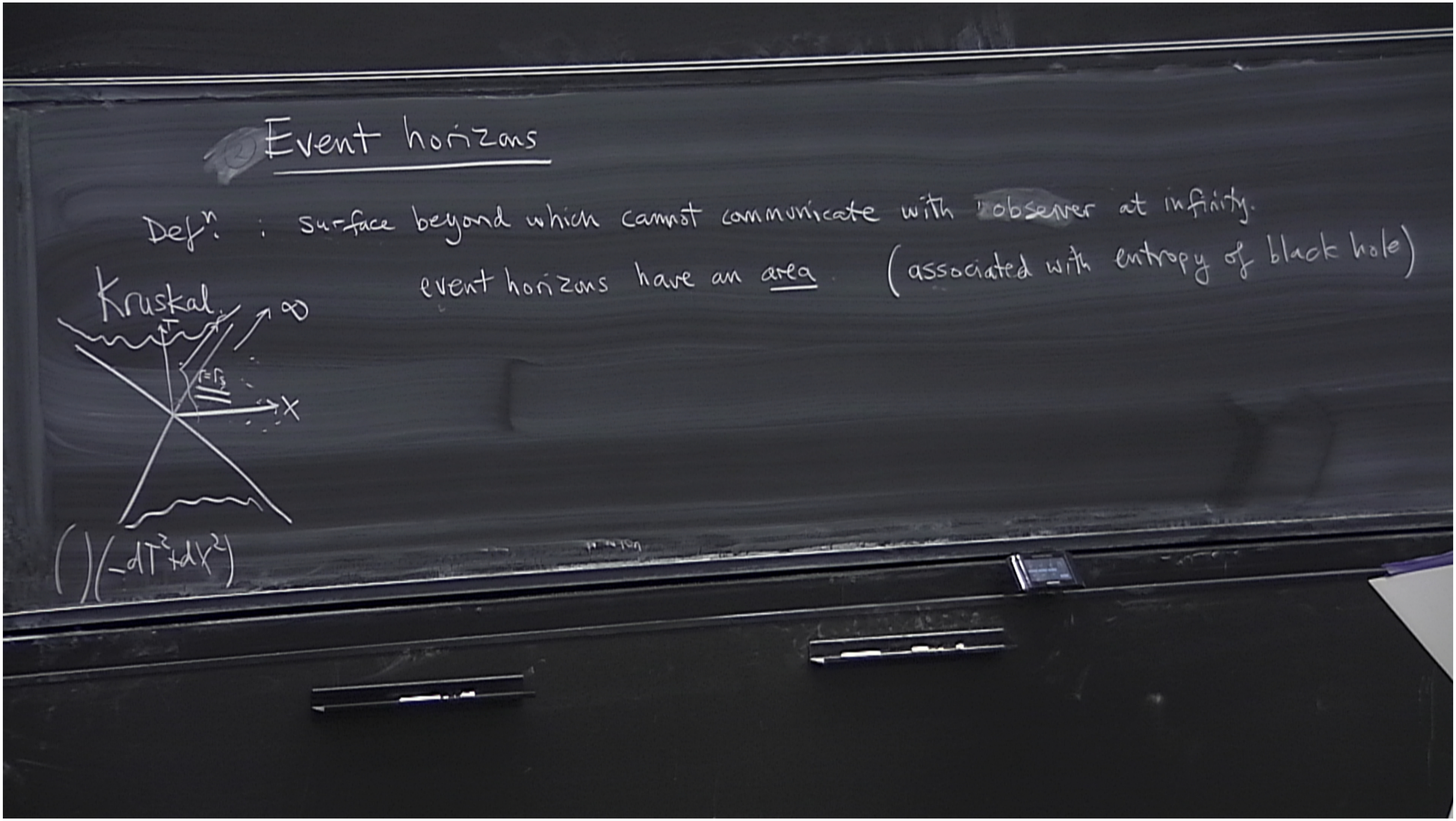
Kruskal

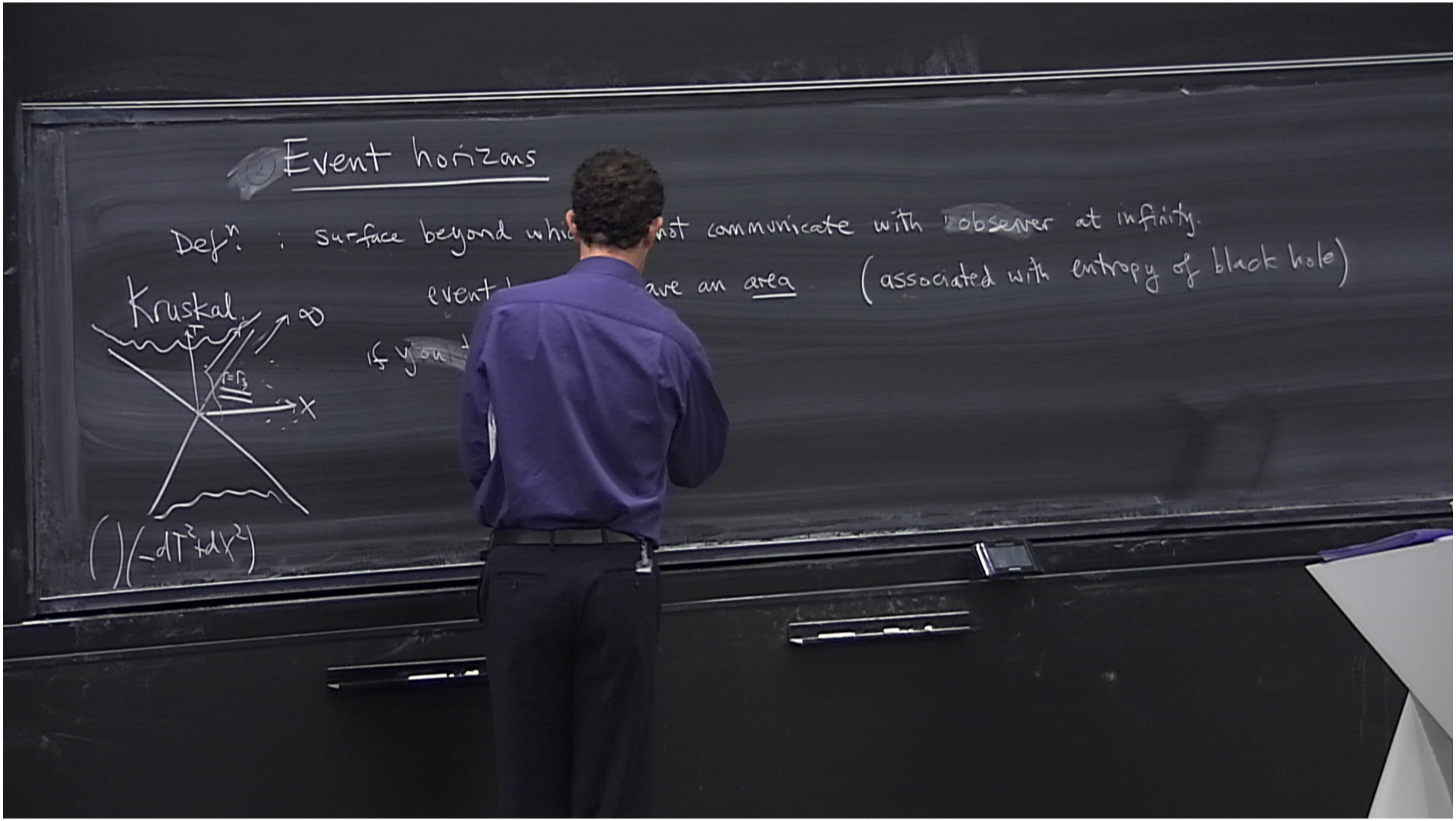
$(\ )(-dT^2+dX^2)$

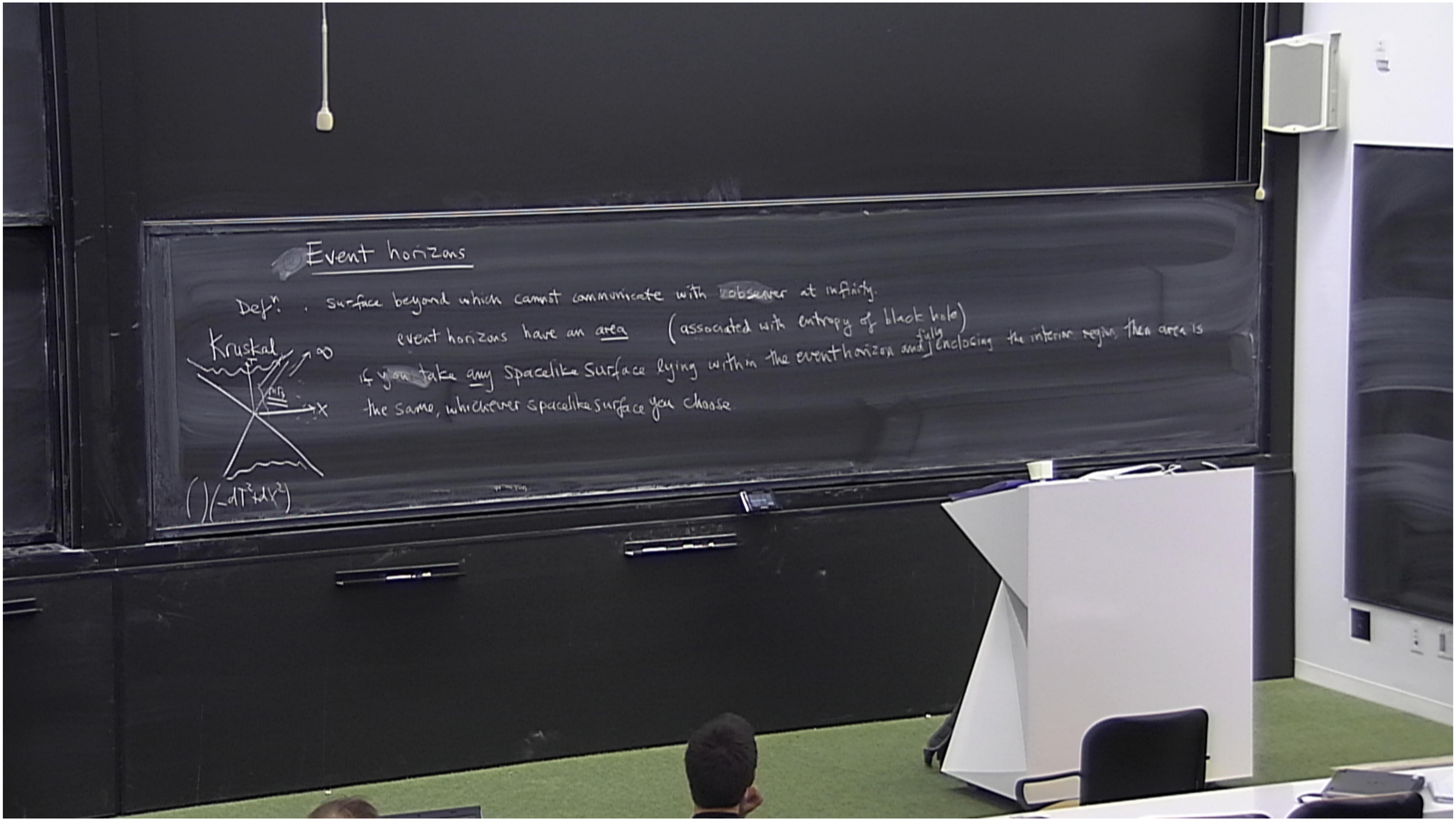

Event horizons
Defn : surface beyond which cannot communicate with observer at infinity.
Kruskal
T
∞
r=rs
X
( )(-dT²+dX²)

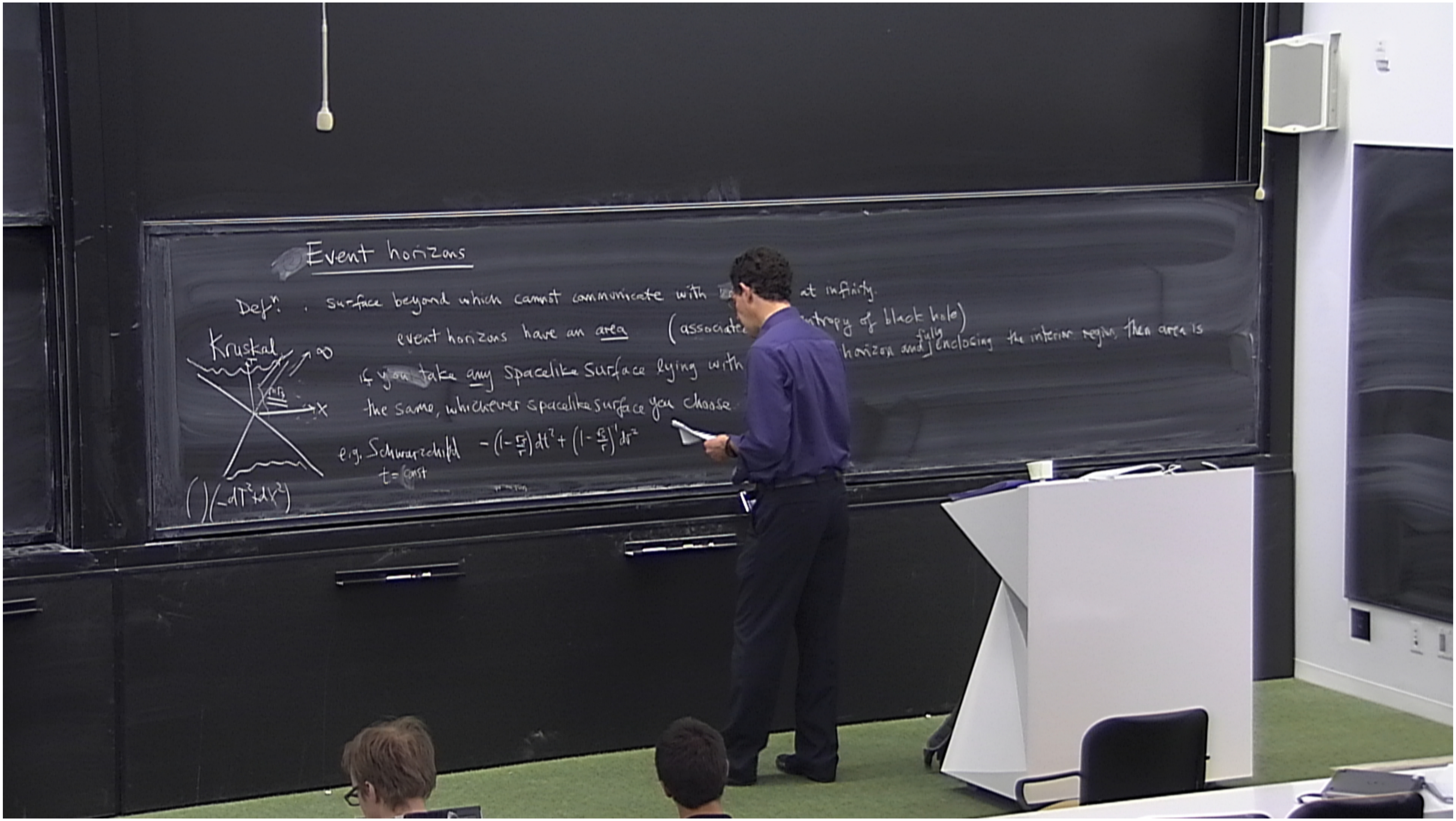

Event horizons
Defn: surface beyond which cannot communicate with observer at infinity.
event horizons have an area

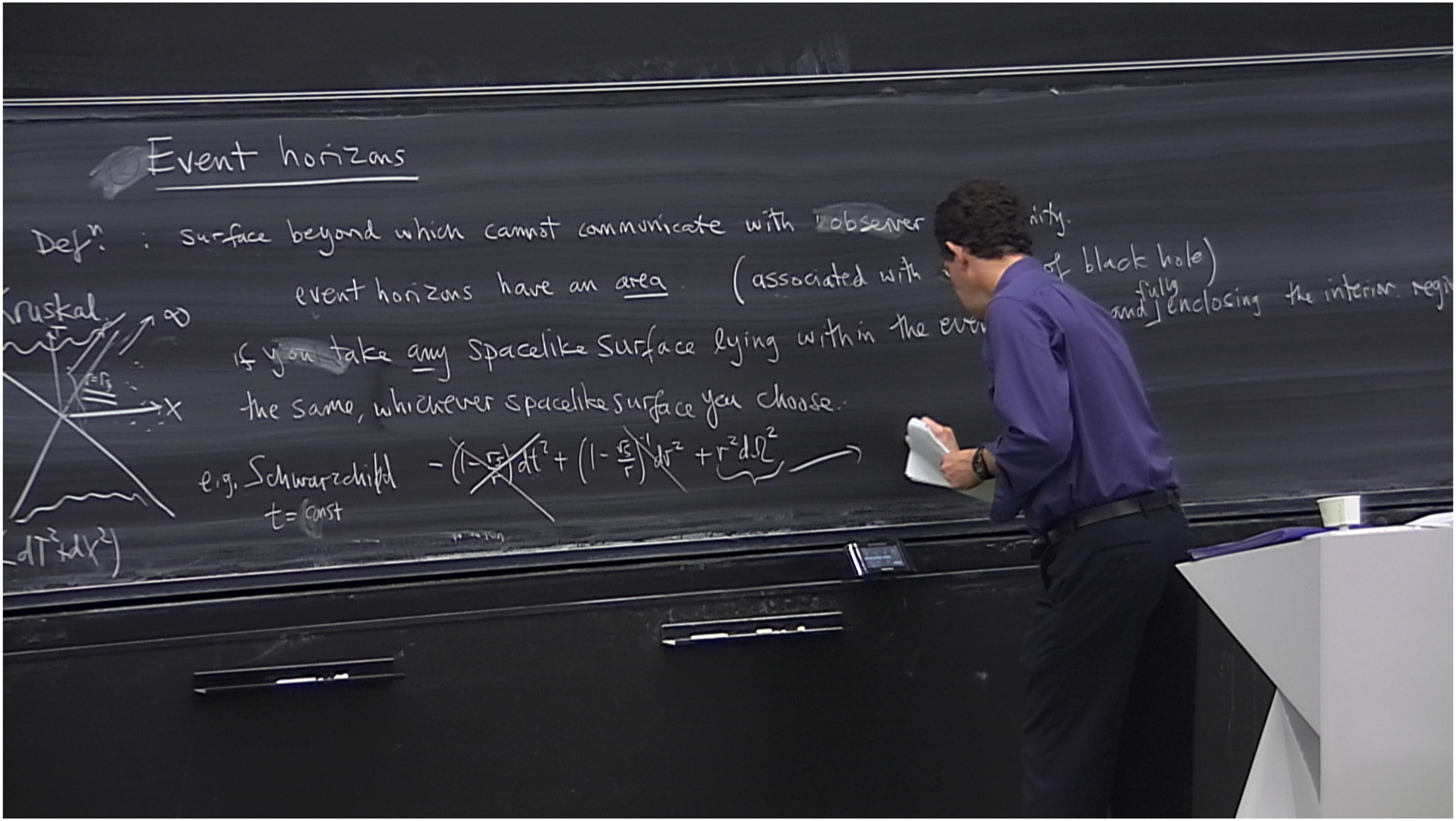

Event horizons
Def^n : surface beyond which cannot communicate with observer at infinity.
event horizons have an area (associated with entropy of b
Kruskal
T
∞
X
r=r_s
()(-dT²+dX²)

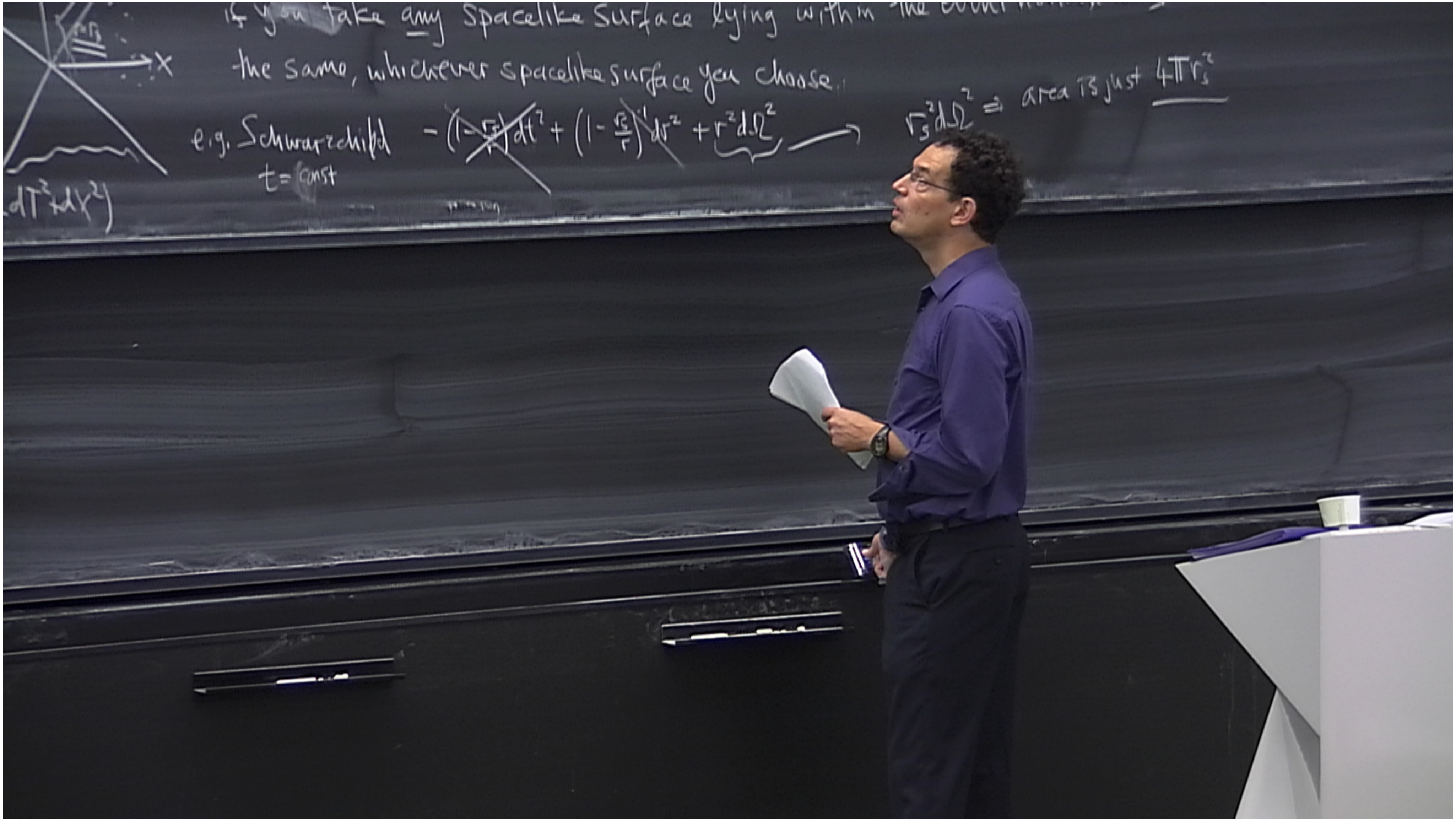

Event horizons
Defn: surface beyond which cannot communicate with observer at infinity.
event horizons have an area (associated with entropy of black hole)
Kruskal
∞
T
X
r = r_s
( )(-dT² + dX²)

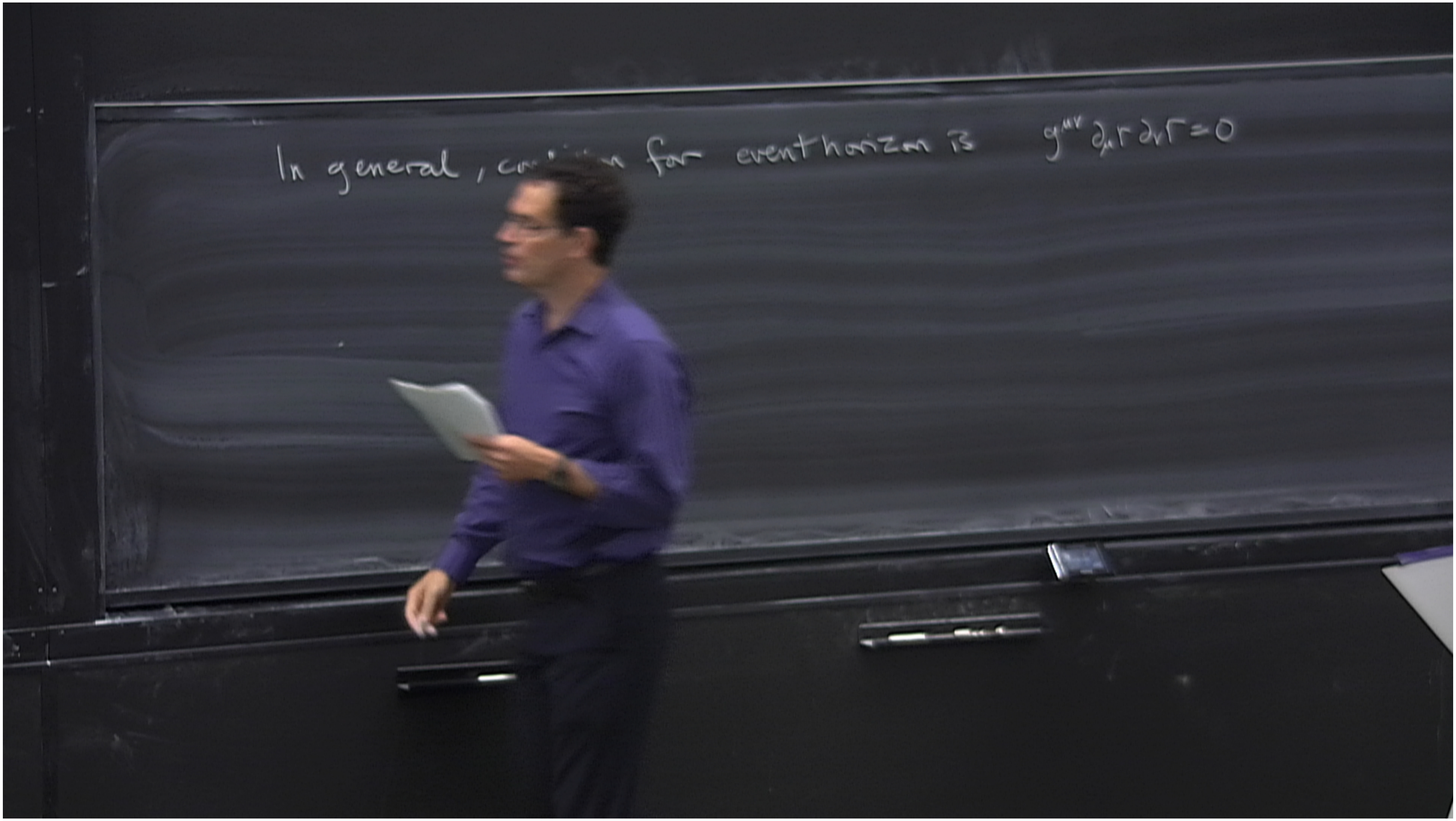

Event horizons
Defn: surface beyond whic... not communicate with observer at infinity.
event ... ave an area (associated with entropy of black hole)
Kruskal
∞
T
X
r = rs
If you ...
( )(-dT² + dX²)

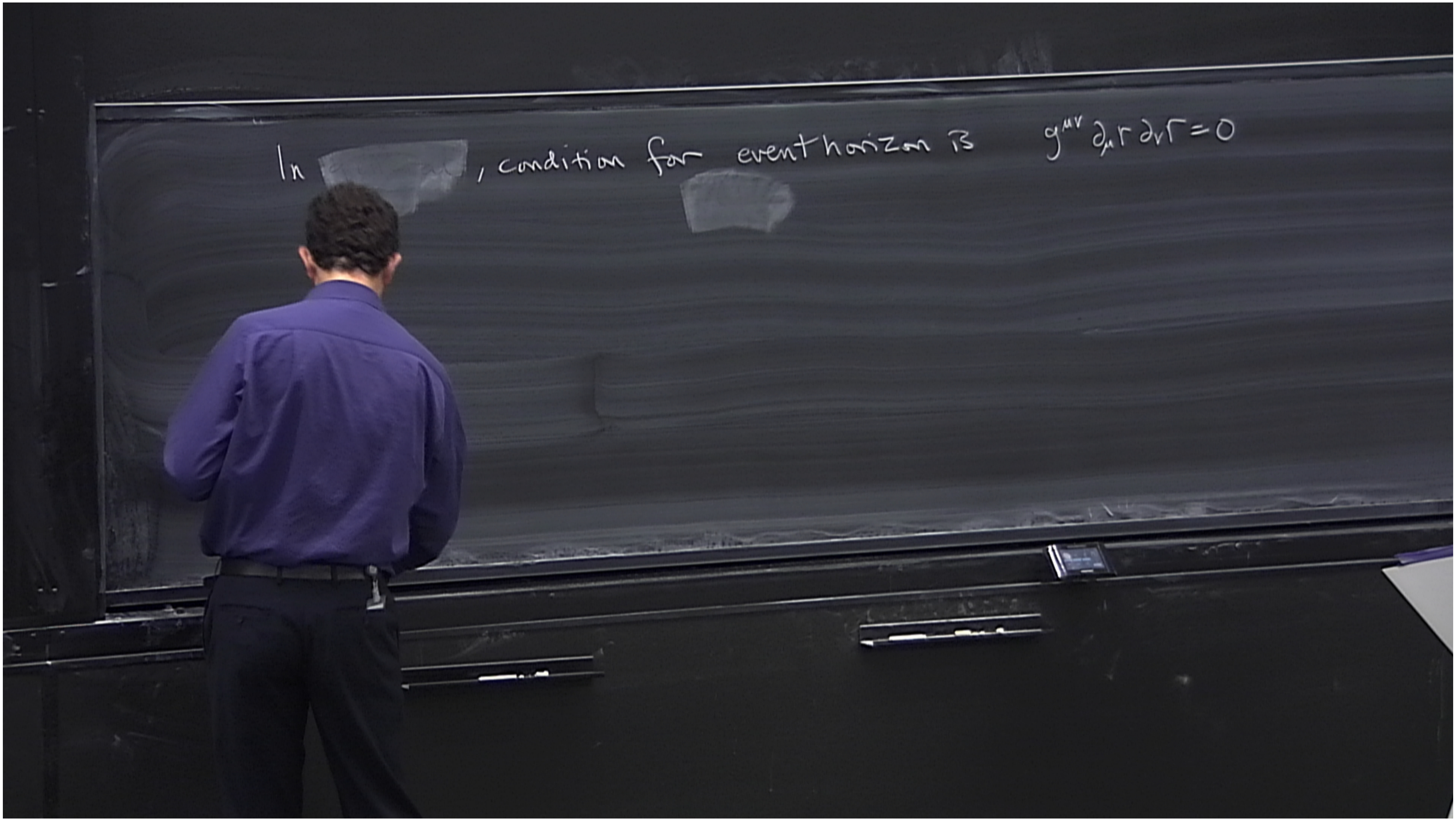

Event horizons
Def^n: . surface beyond which cannot communicate with observer at infinity.
event horizons have an area (associated with entropy of black hole)
if you take any spacelike surface lying within the event horizon and fully enclosing the interior region then area is
the same, whichever spacelike surface you choose
Kruskal
∞
X
( )(-dT²+dX²)

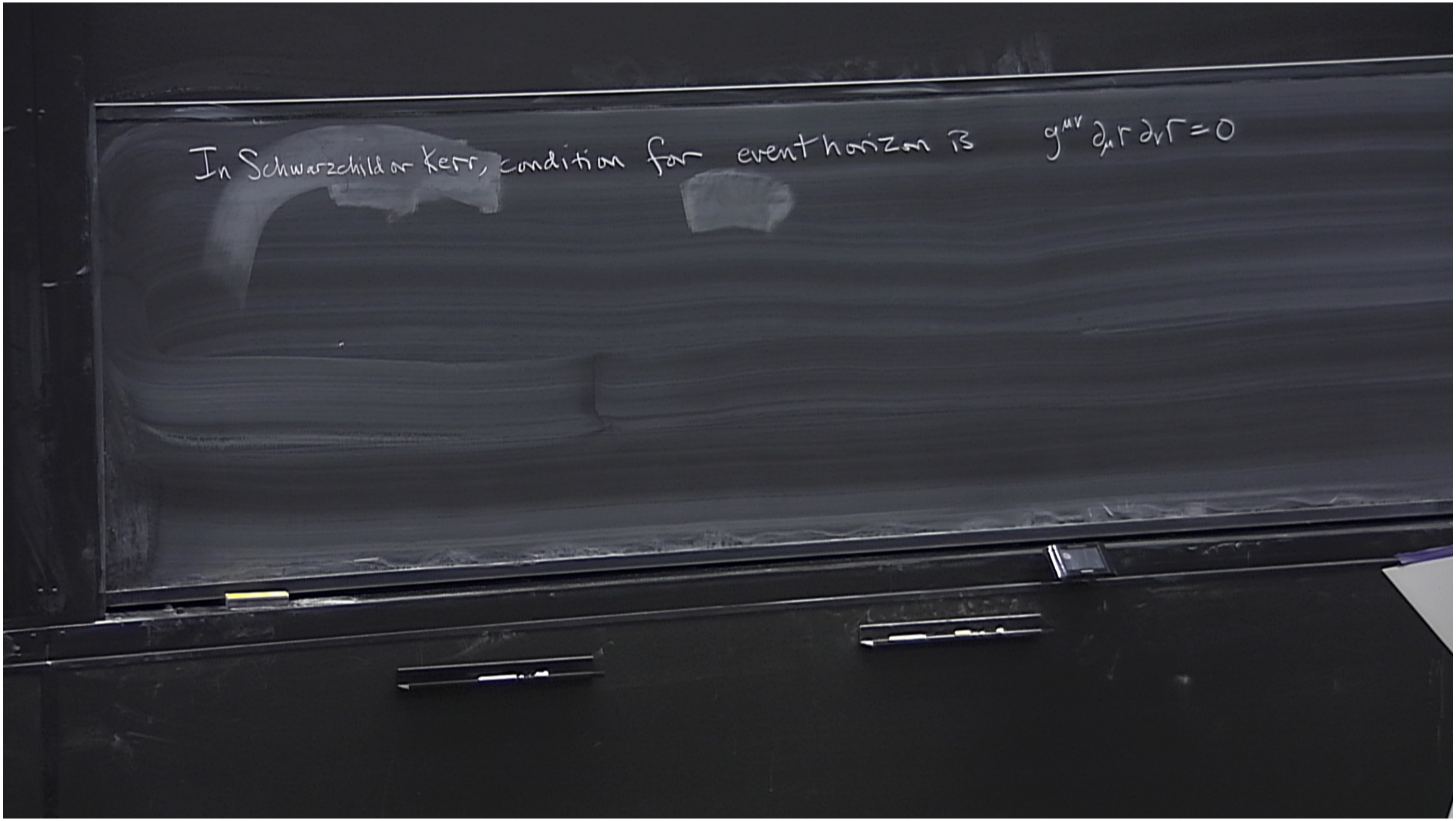

Event horizons
Def^n: surface beyond which cannot communicate with at infinity.
event horizons have an area (associate ntropy of black hole)
Kruskal
if you take any spacelike surface lying with horizon and fully enclosing the interior region, then area is
the same, whichever spacelike surface you choose
e.g. Schwarzschild $-(1-\frac{r_s}{r})dt^2+(1-\frac{r_s}{r})^{-1}dr^2$
t = const
$(\ )(-dT^2+dX^2)$

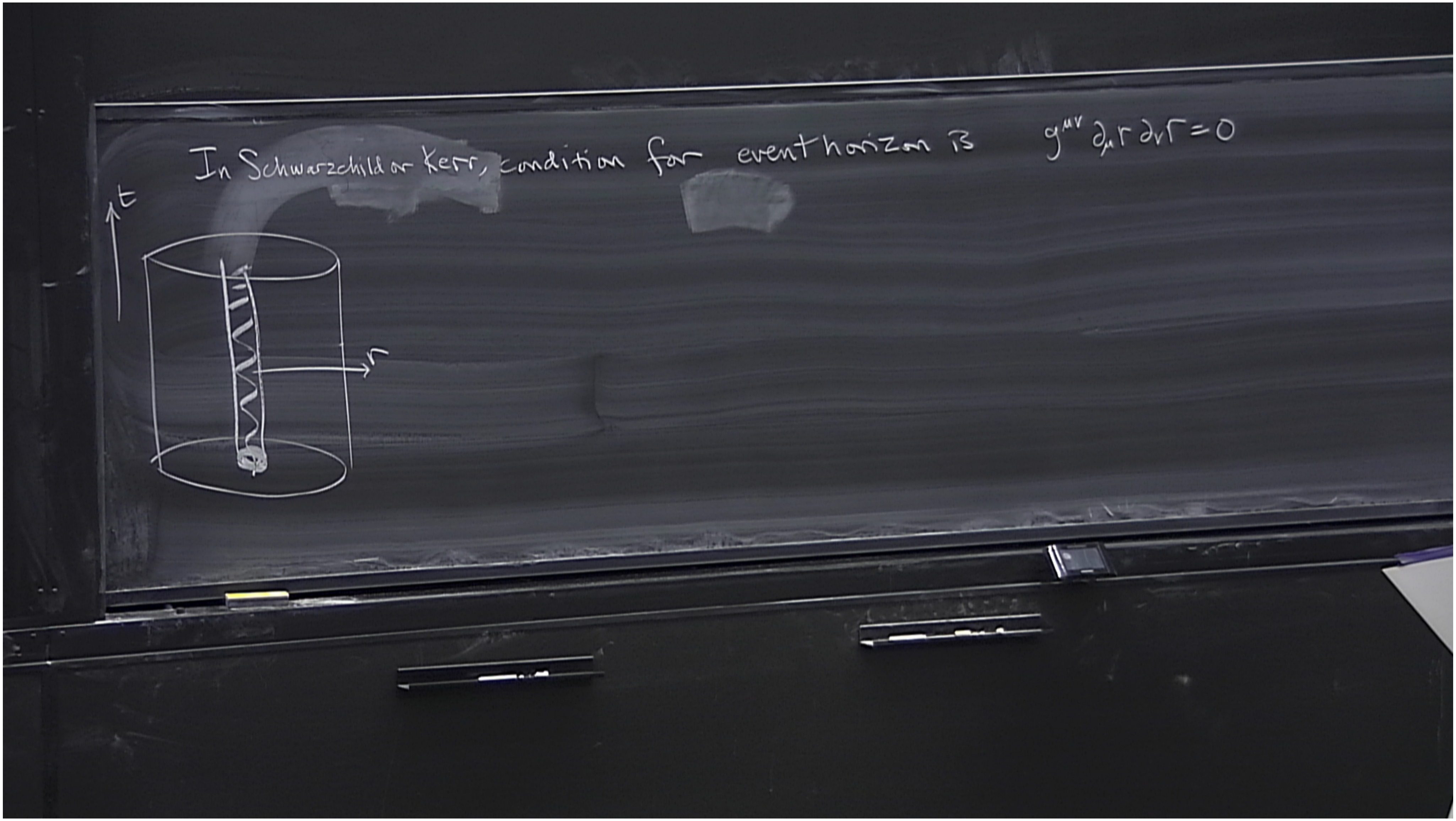

Event horizons
Defn: surface beyond which cannot communicate with observer
event horizons have an area (associated with
of black hole)
fully enclosing the interior
If you take any spacelike surface lying within the
the same, whichever spacelike surface you choose.
e.g. Schwarzchild
t = const
Kruskal
X

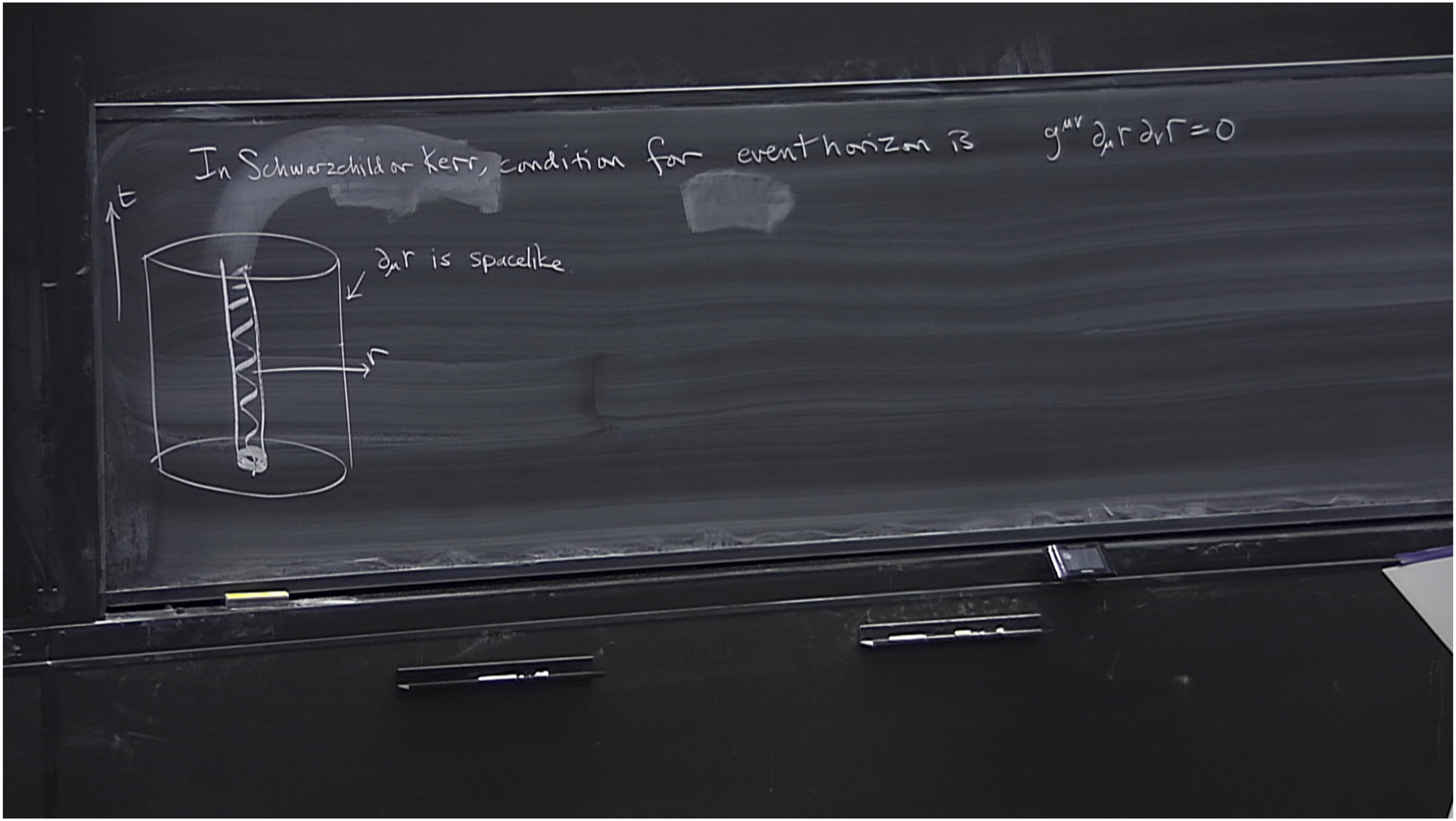

If you take any spacelike surface lying within the
the same, whichever spacelike surface you choose.
e.g. Schwarzschild
t = const
area is just $4\pi r_s^2$

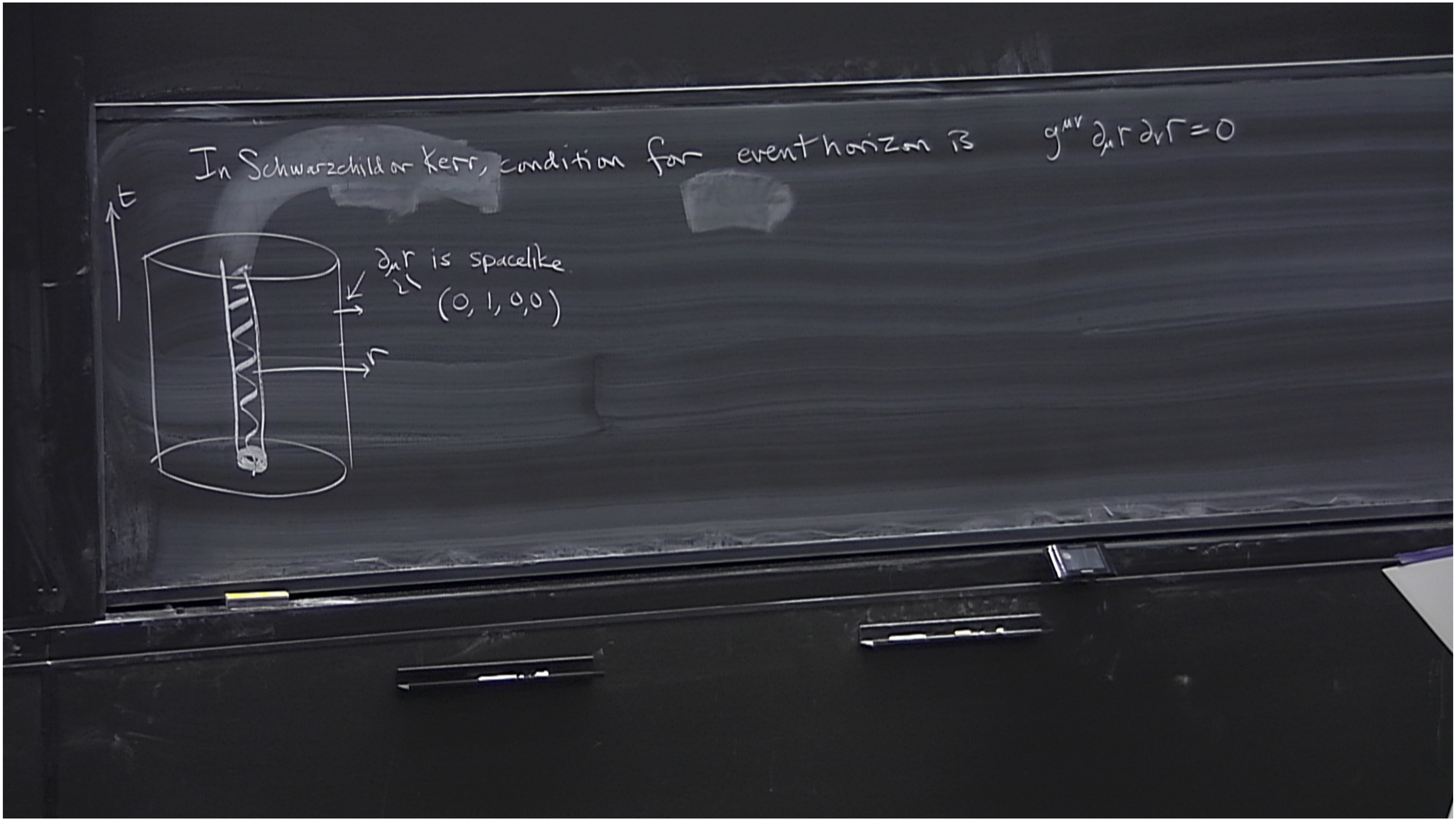

In general,
for event horizon is
$g^{\mu\nu}\partial_\mu r\,\partial_\nu r = 0$

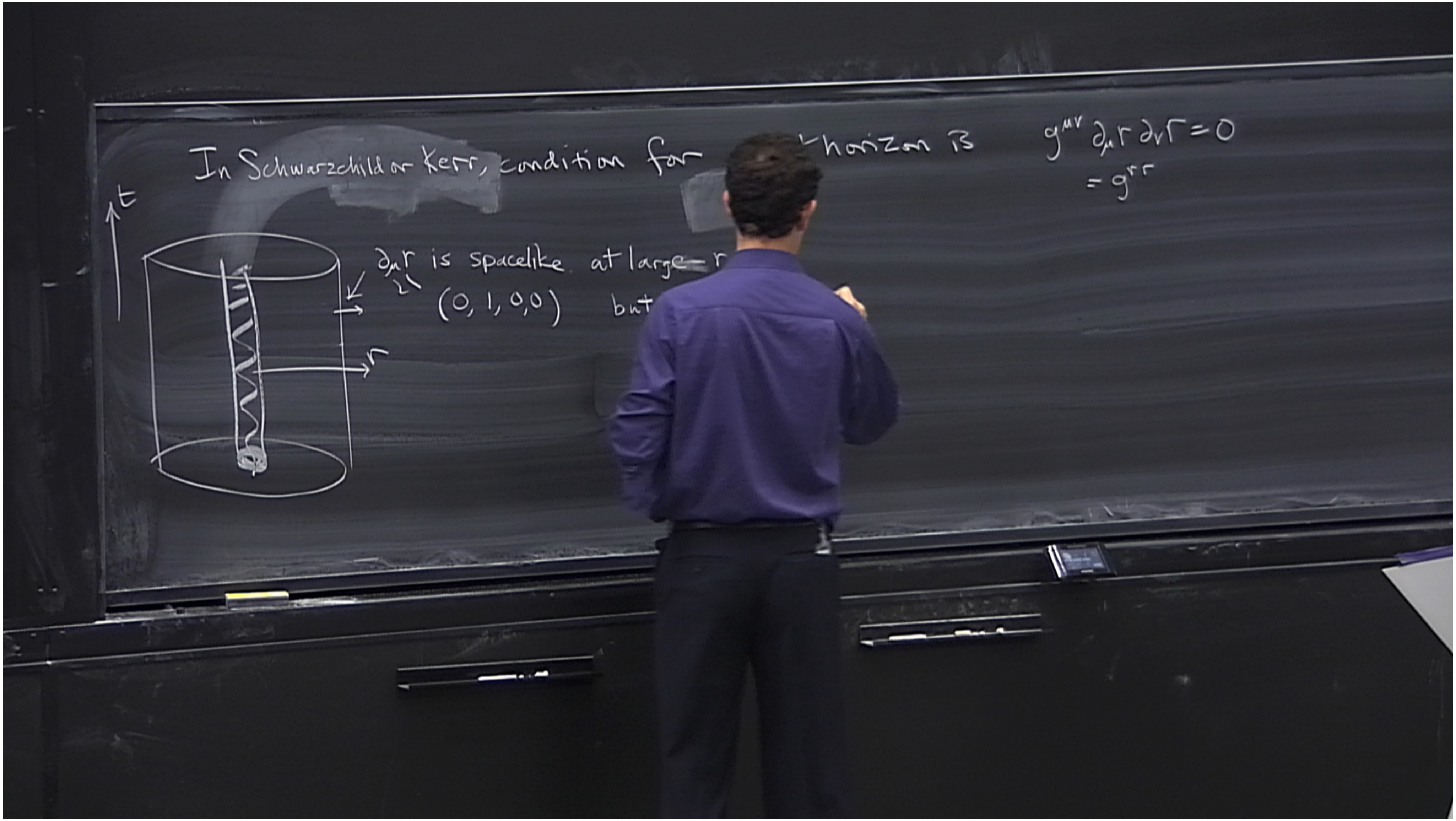

In , condition for event horizon is $g^{\mu\nu}\partial_\mu r \partial_\nu r = 0$

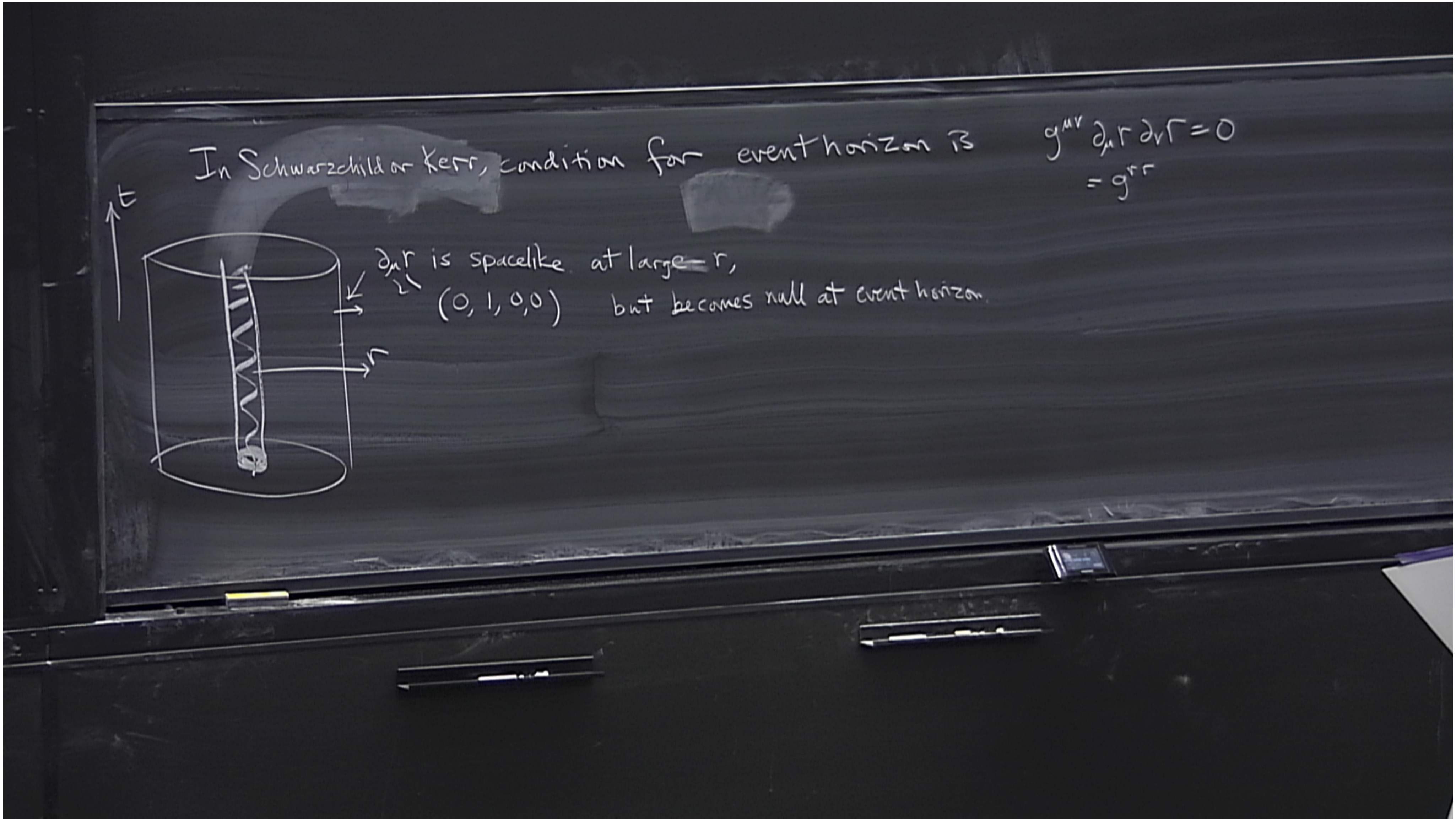

In Schwarzschild or Kerr, condition for event horizon is $g^{\mu\nu}\partial_\mu r\,\partial_\nu r = 0$

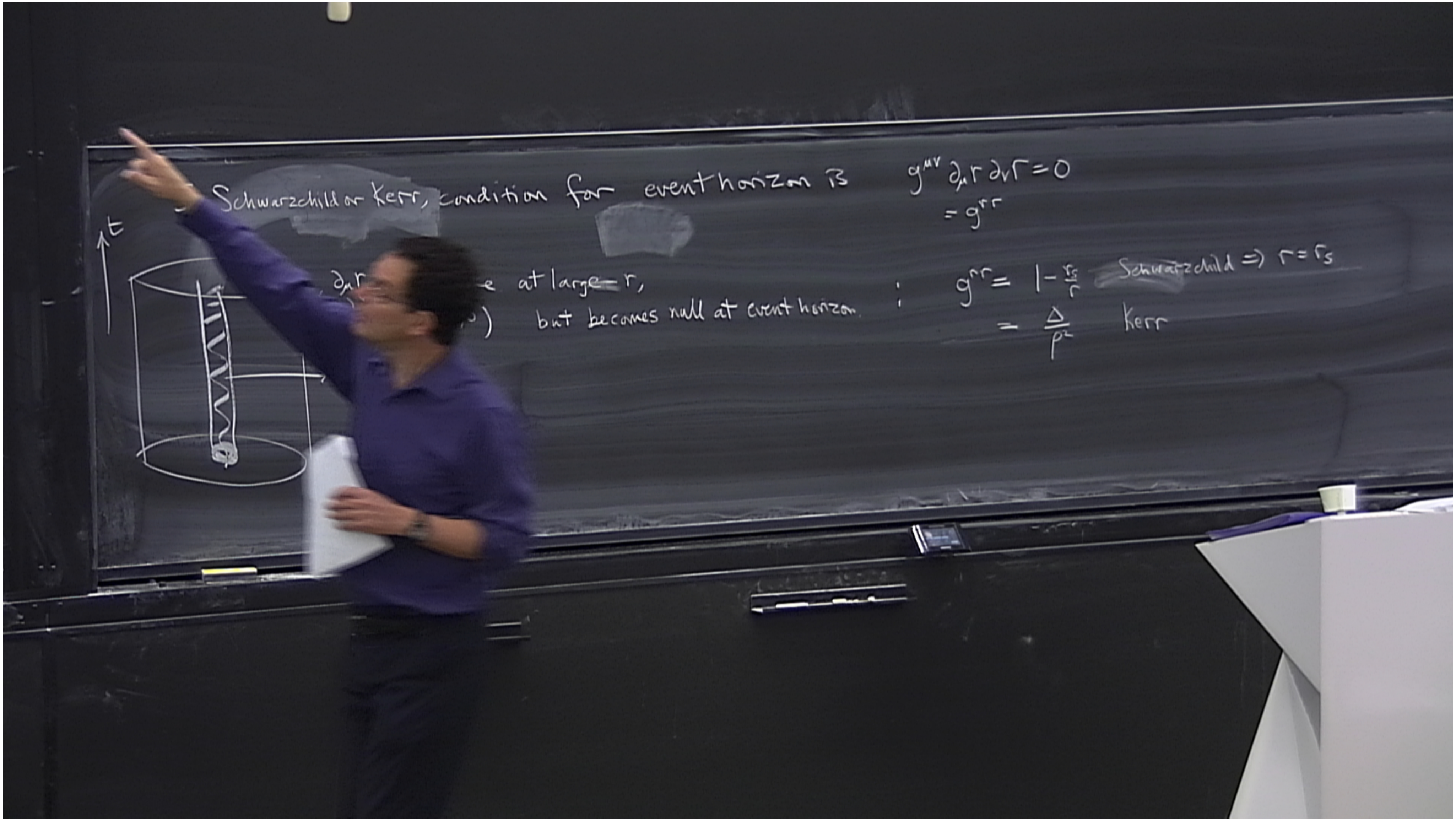

In Schwarzschild or Kerr, condition for event horizon is $g^{\mu\nu}\partial_\mu r\,\partial_\nu r = 0$
t
r

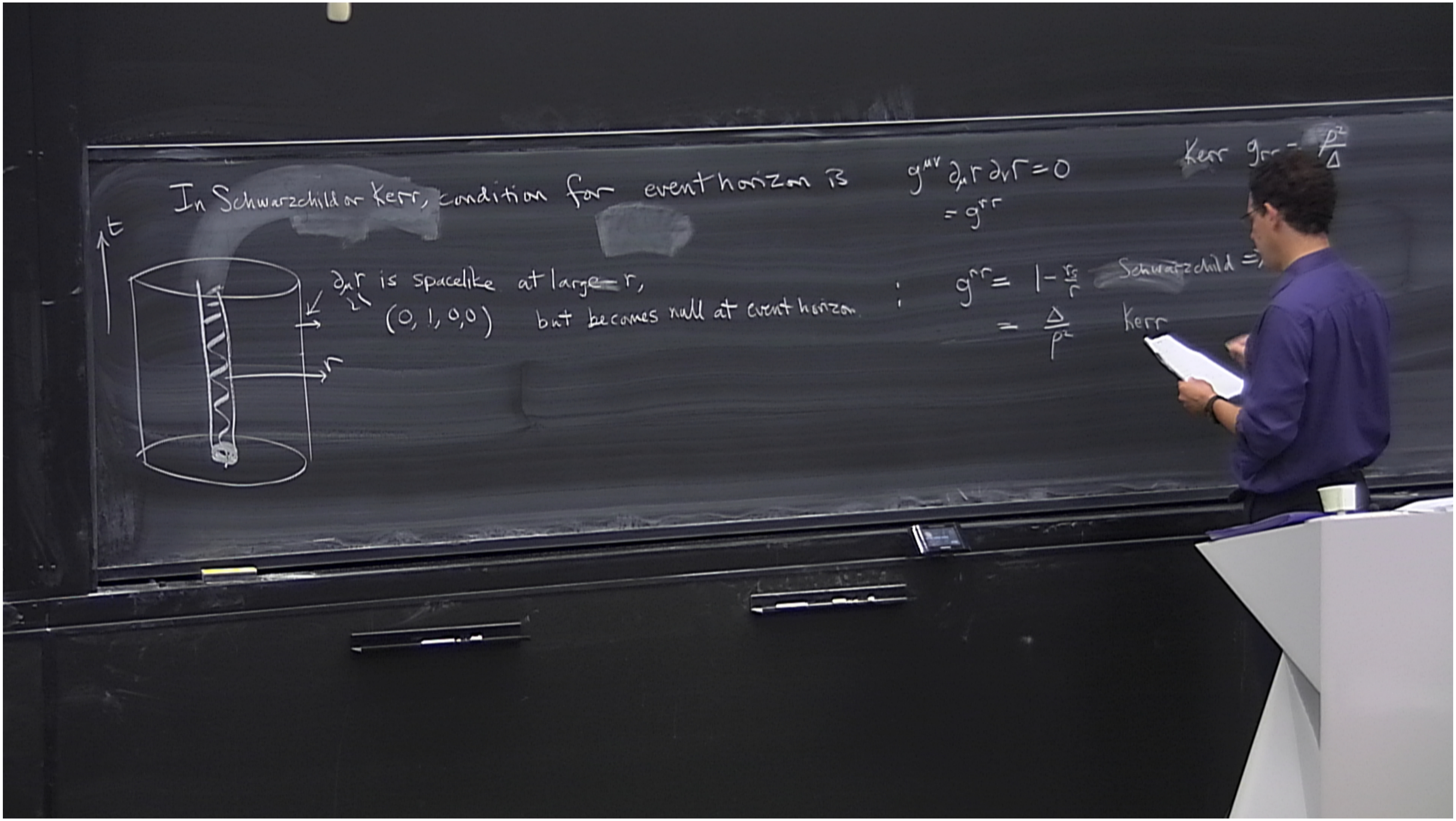

In Schwarzchild or Kerr, condition for event horizon is $g^{\mu\nu}\partial_\mu r \partial_\nu r = 0$
t
$\partial_\mu r$ is spacelike
r


In Schwarzchild or Kerr, condition for event horizon is $g^{\mu\nu}\partial_\mu r \partial_\nu r = 0$
$\partial_\mu r$ is spacelike
(0, 1, 0, 0)
t
r


In Schwarzschild or Kerr, condition for
horizon is
$g^{\mu\nu} \partial_\mu r \partial_\nu r = 0$
$= g^{rr}$
t
$\partial_\mu r$ is spacelike at large r
(0, 1, 0, 0) but
r


In Schwarzchild or Kerr, condition for event horizon is $g^{\mu\nu}\partial_\mu r\,\partial_\nu r = 0$
$= g^{rr}$
$\partial_\mu r$ is spacelike at large r,
(0, 1, 0, 0) but becomes null at event horizon
t
r


Schwarzchild or Kerr, condition for event horizon is
g^μν ∂μr ∂νr = 0
= g^rr
at large r,
but becomes null at event horizon
g^rr = 1 - rs/r
Schwarzchild ⇒ r = rs
= Δ/ρ²
Kerr
t


In Schwarzschild or Kerr, condition for event horizon is
g^μν ∂_μ r ∂_ν r = 0
= g^rr
∂_μ r is spacelike at large r,
(0, 1, 0, 0)
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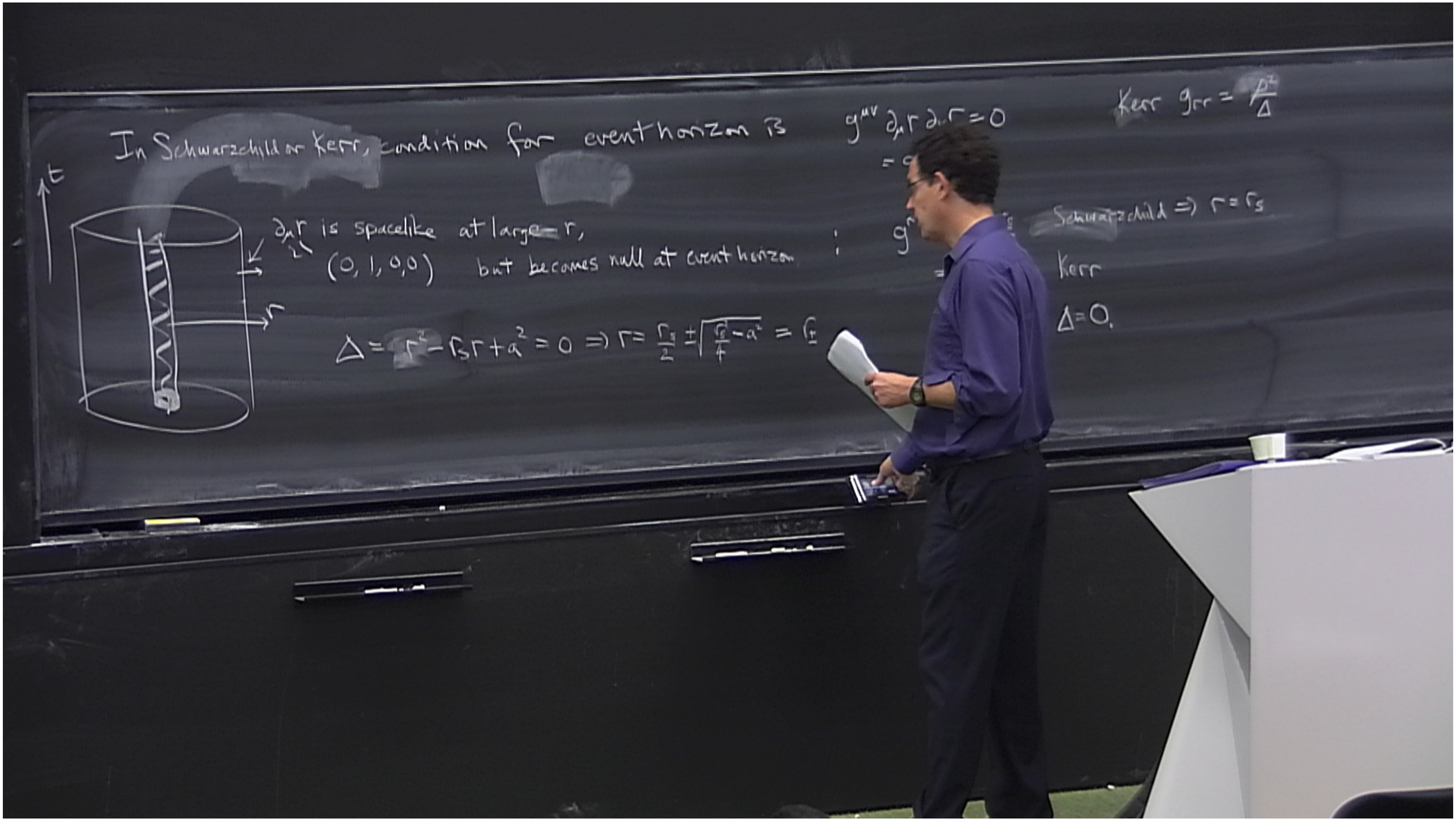
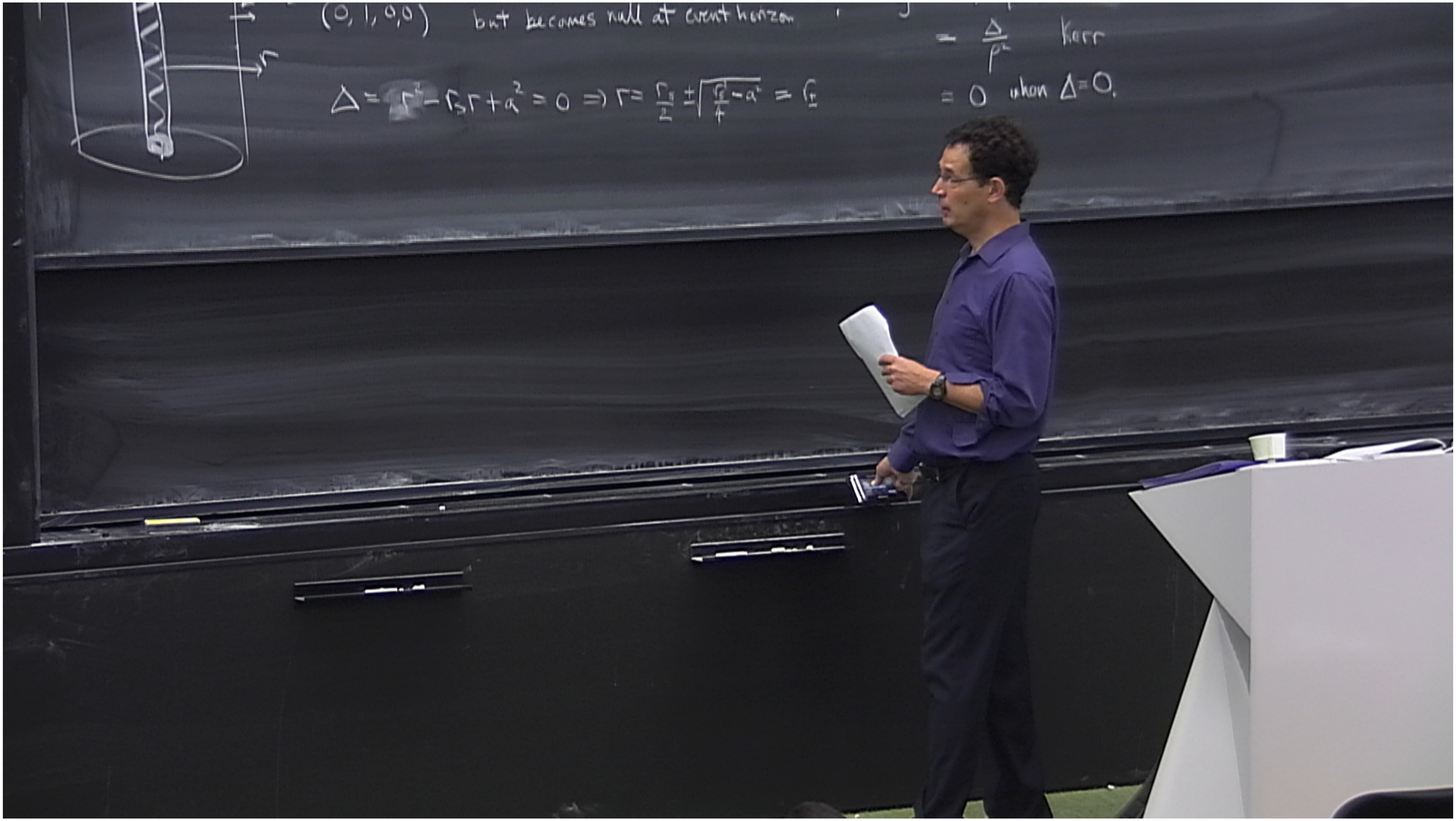
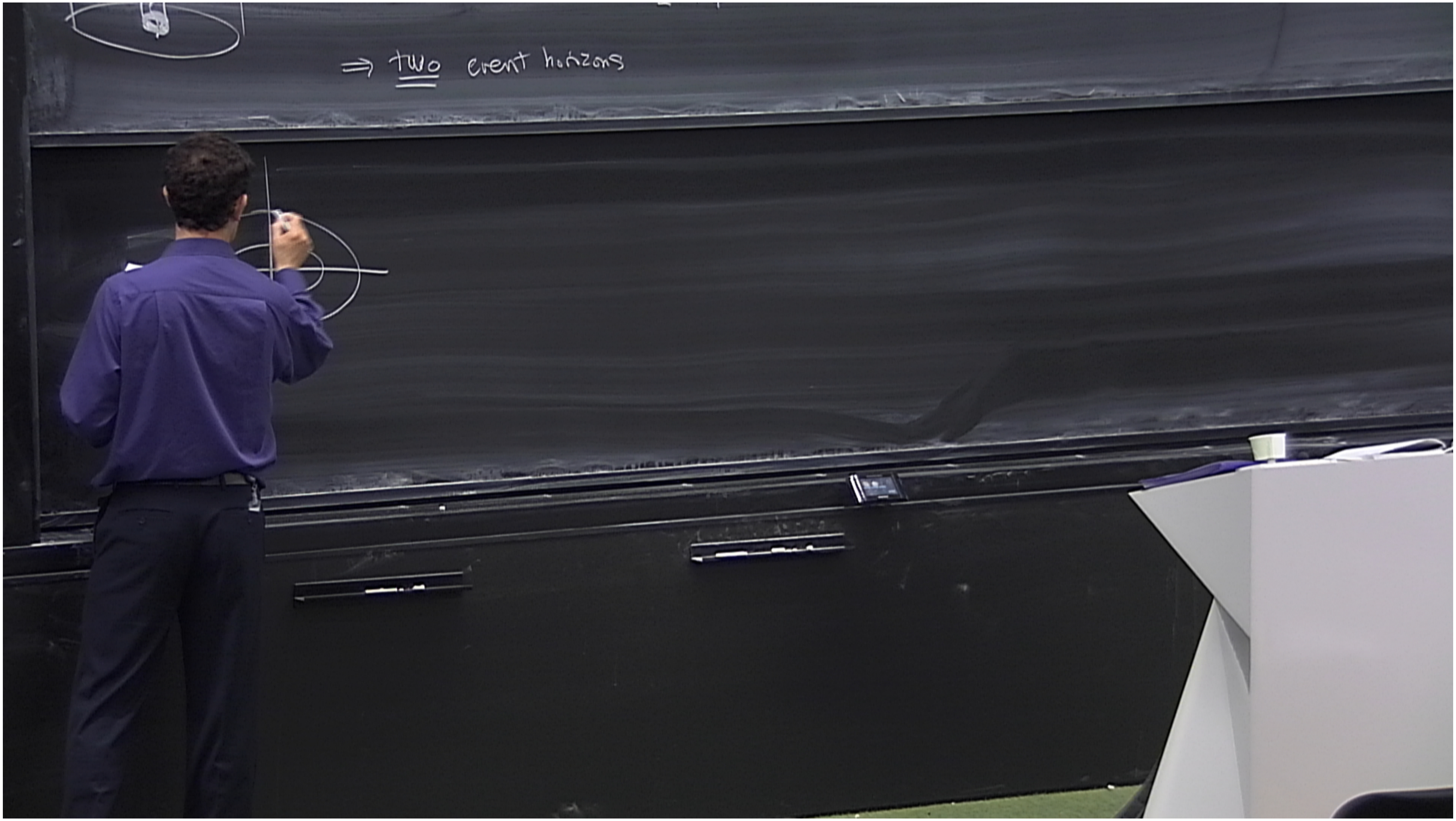
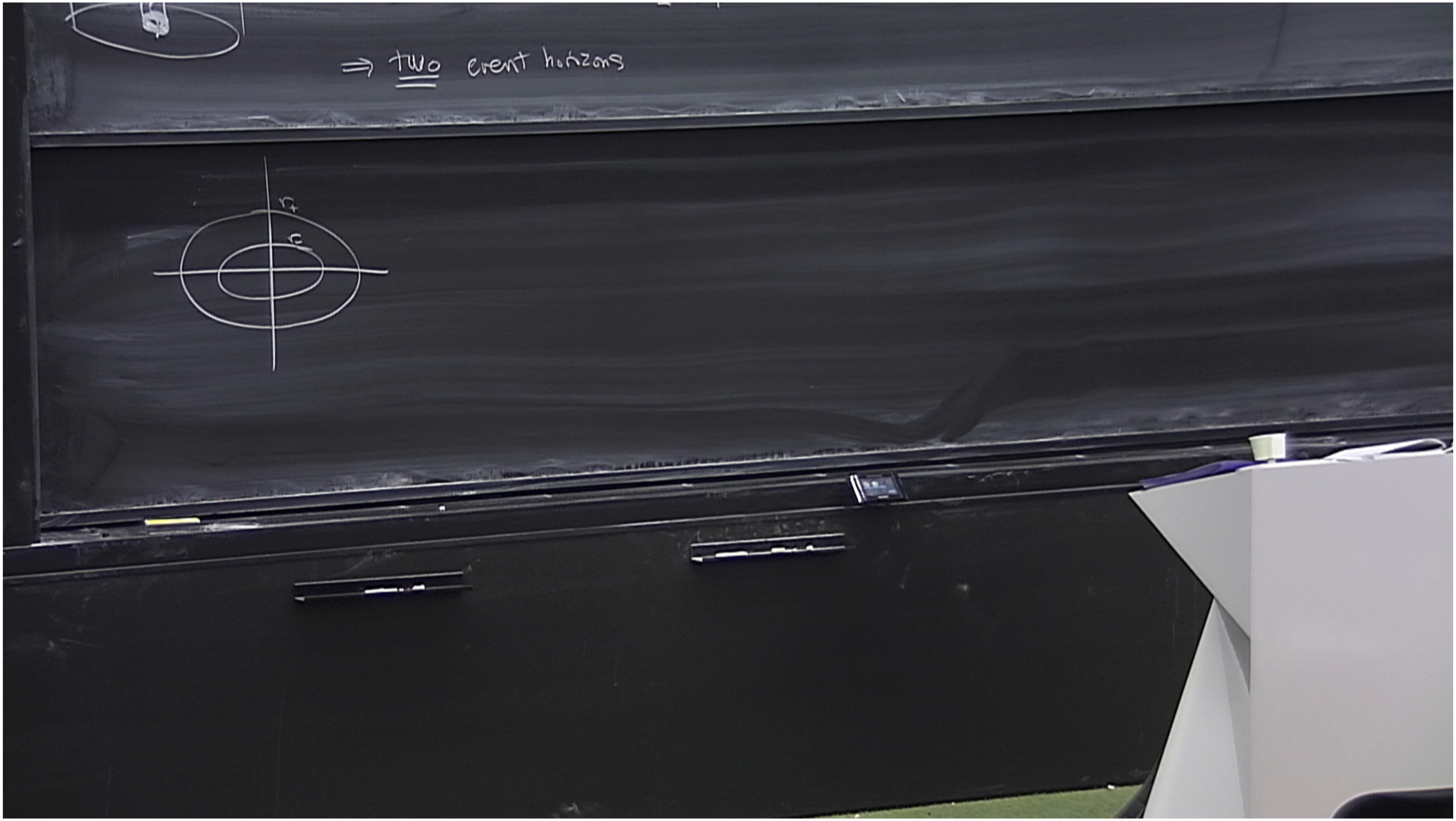
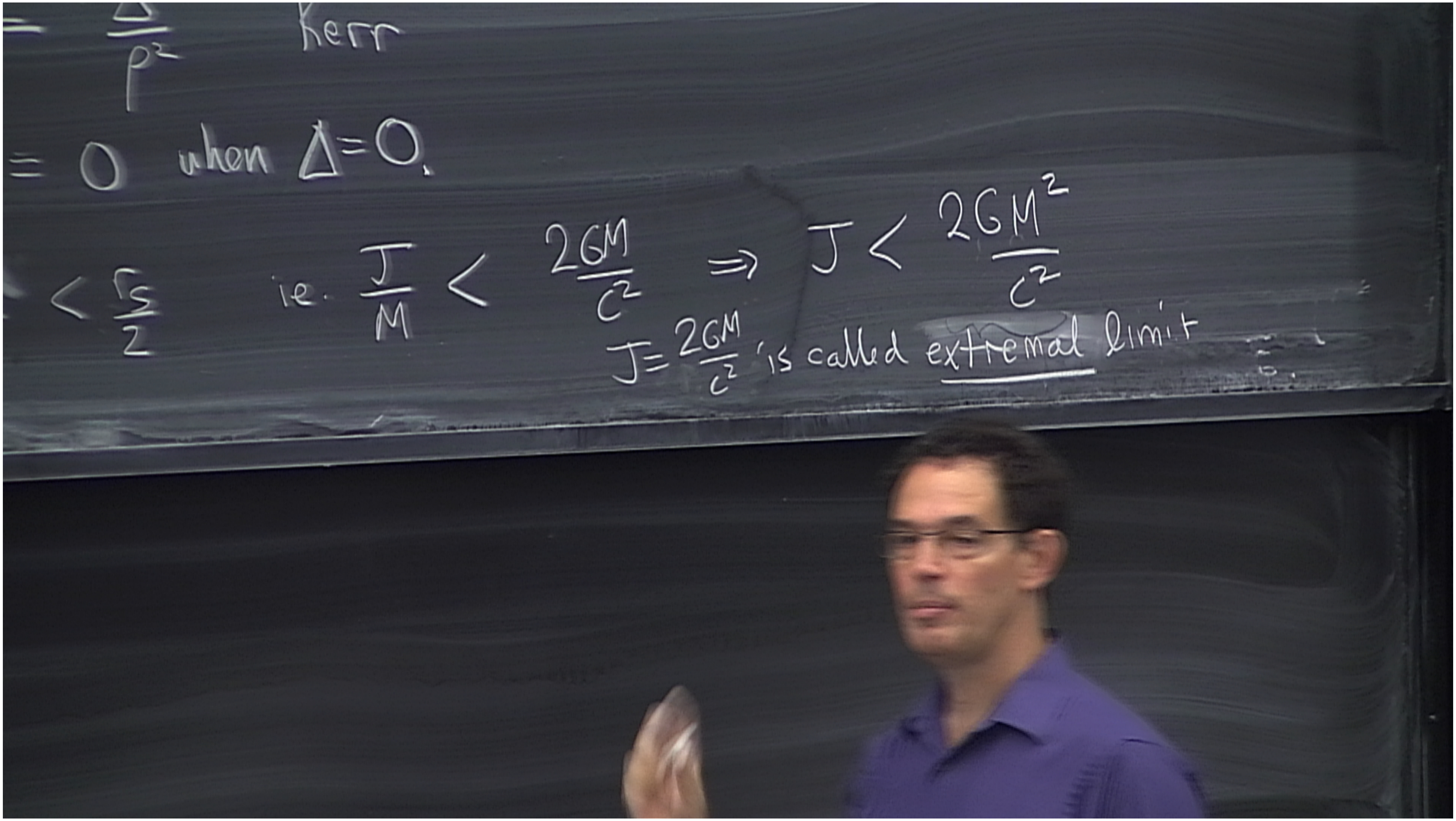
g^rr = 1 − r_s/r
Schwarzschild
= Δ/ρ²
Kerr
Kerr g_rr = ρ²/Δ
t
r

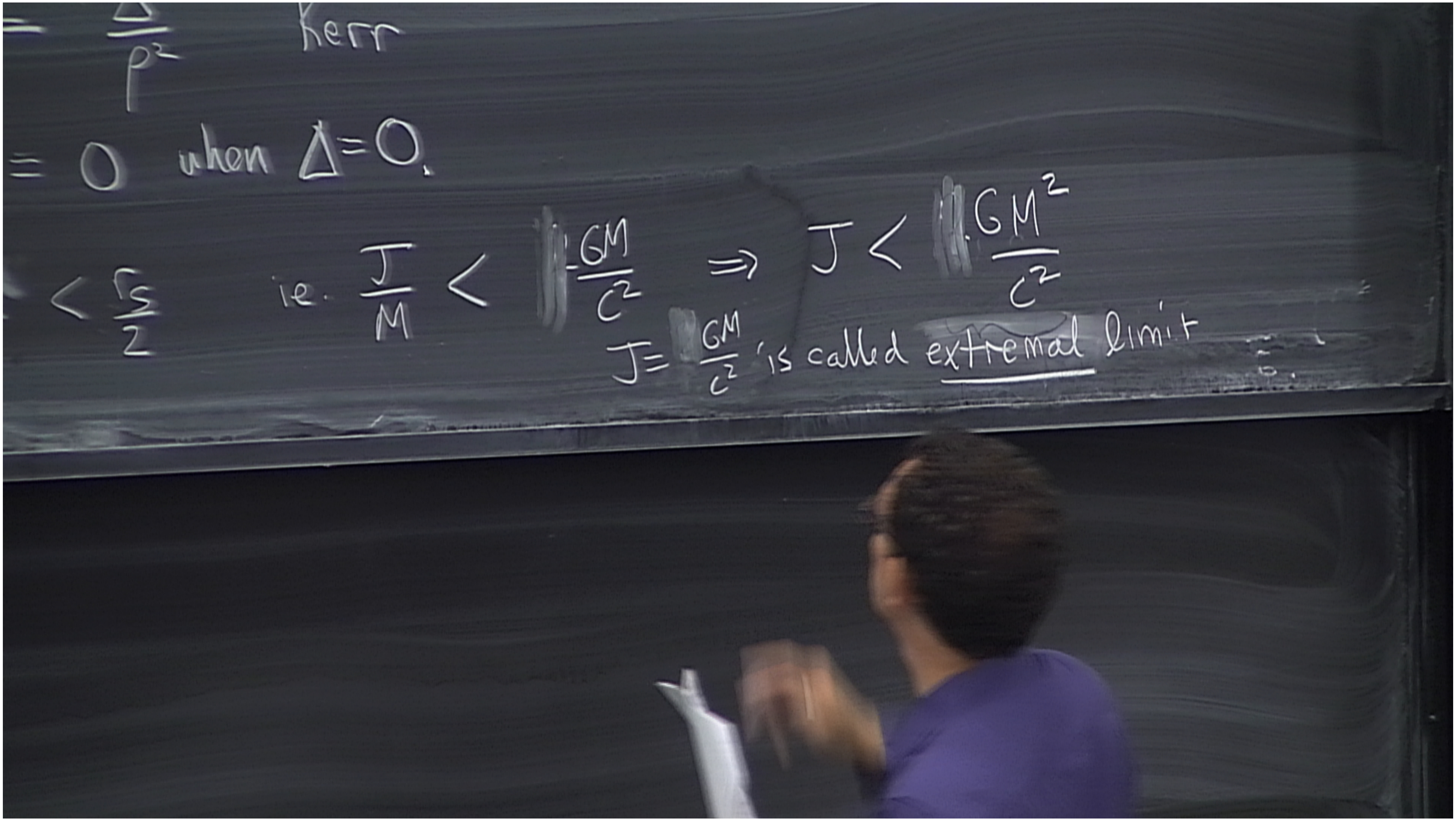

In Schwarzschild or Kerr, condition for event horizon is
g^{μν} ∂_μ r ∂_ν r = 0
Kerr g_rr = ρ²/Δ
∂_μ r is spacelike at large r,
(0, 1, 0, 0) but becomes null at event horizon
Schwarzschild ⇒ r = r_s
Kerr
Δ = 0.
Δ = r² − r_s r + a² = 0 ⇒ r = r_s/2 ± √(r_s²/4 − a²) = r_+
t
r


(0,1,0,0) but becomes null at event horizon
= Δ/ρ² Kerr
Δ = r² − r_s r + a² = 0 ⇒ r = r_s/2 ± √(r_s²/4 − a²) = r_±
= 0 when Δ = 0.


⇒ two event horizons


⇒ two event horizons
r_+
r_-


Kerr
= 0 when Δ=0.
< r_s/2 ie. J/M < 2GM/c^2 ⇒ J < 2GM^2/c^2
J = 2GM/c^2 is called extremal limit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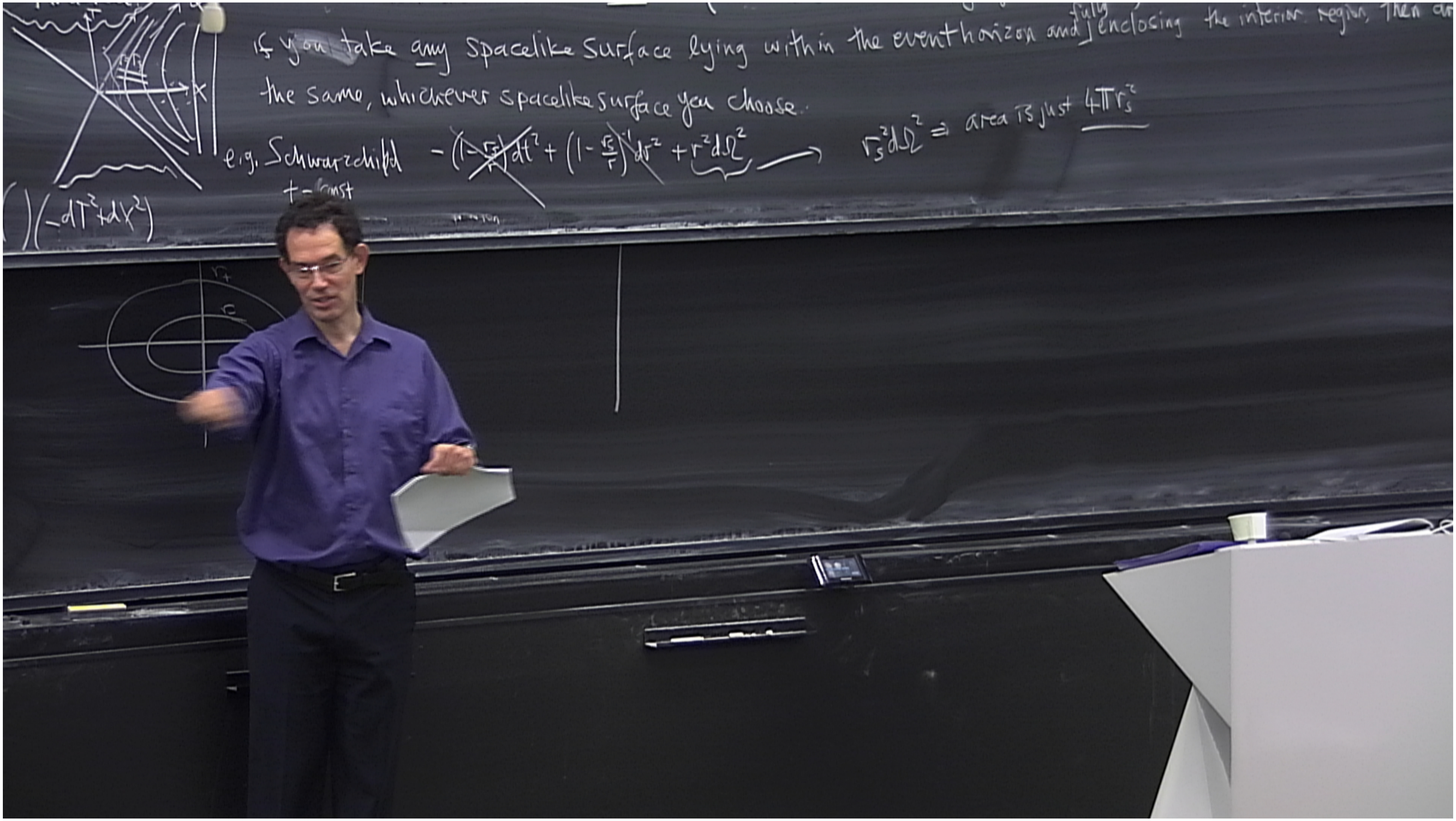
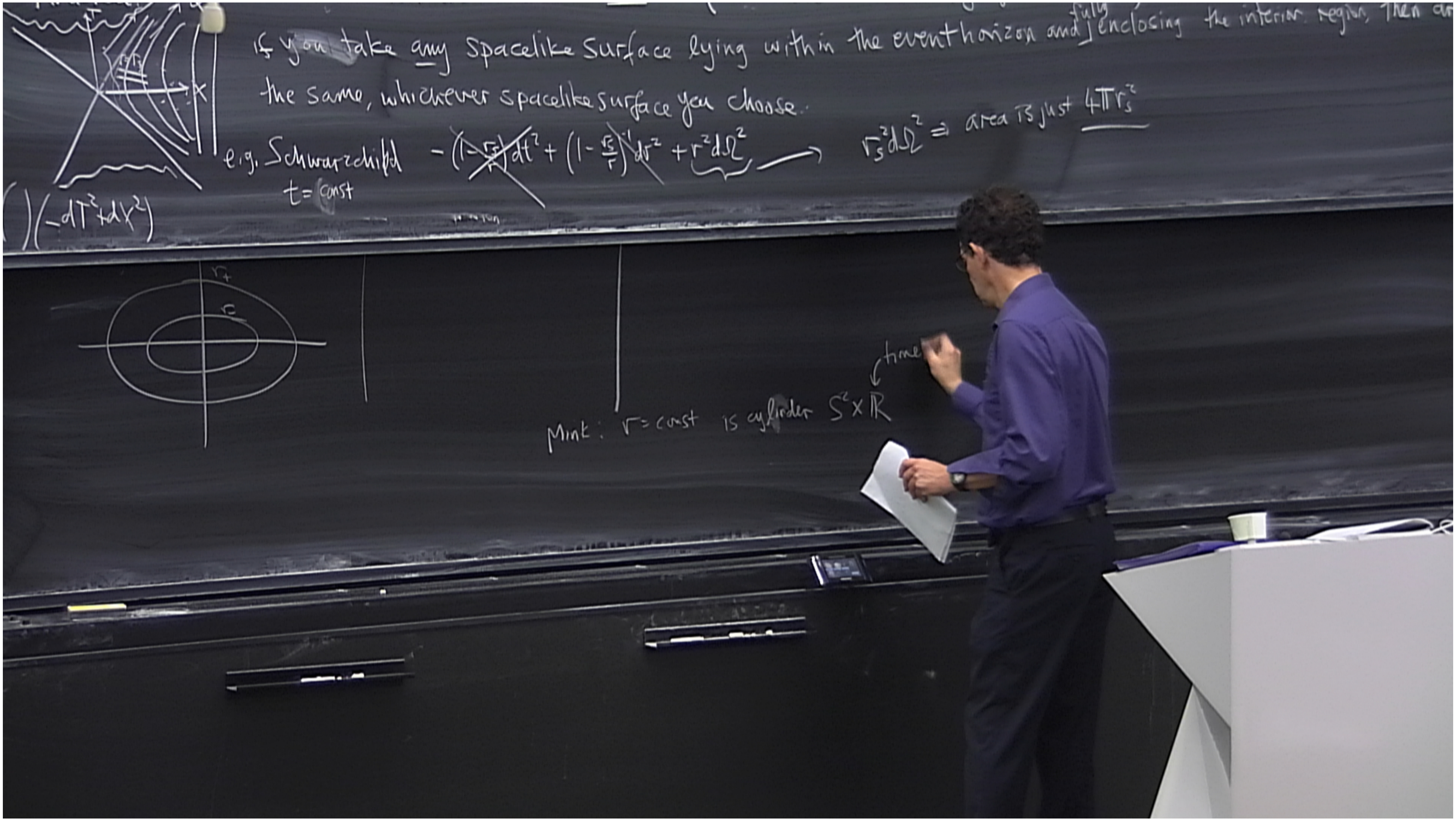
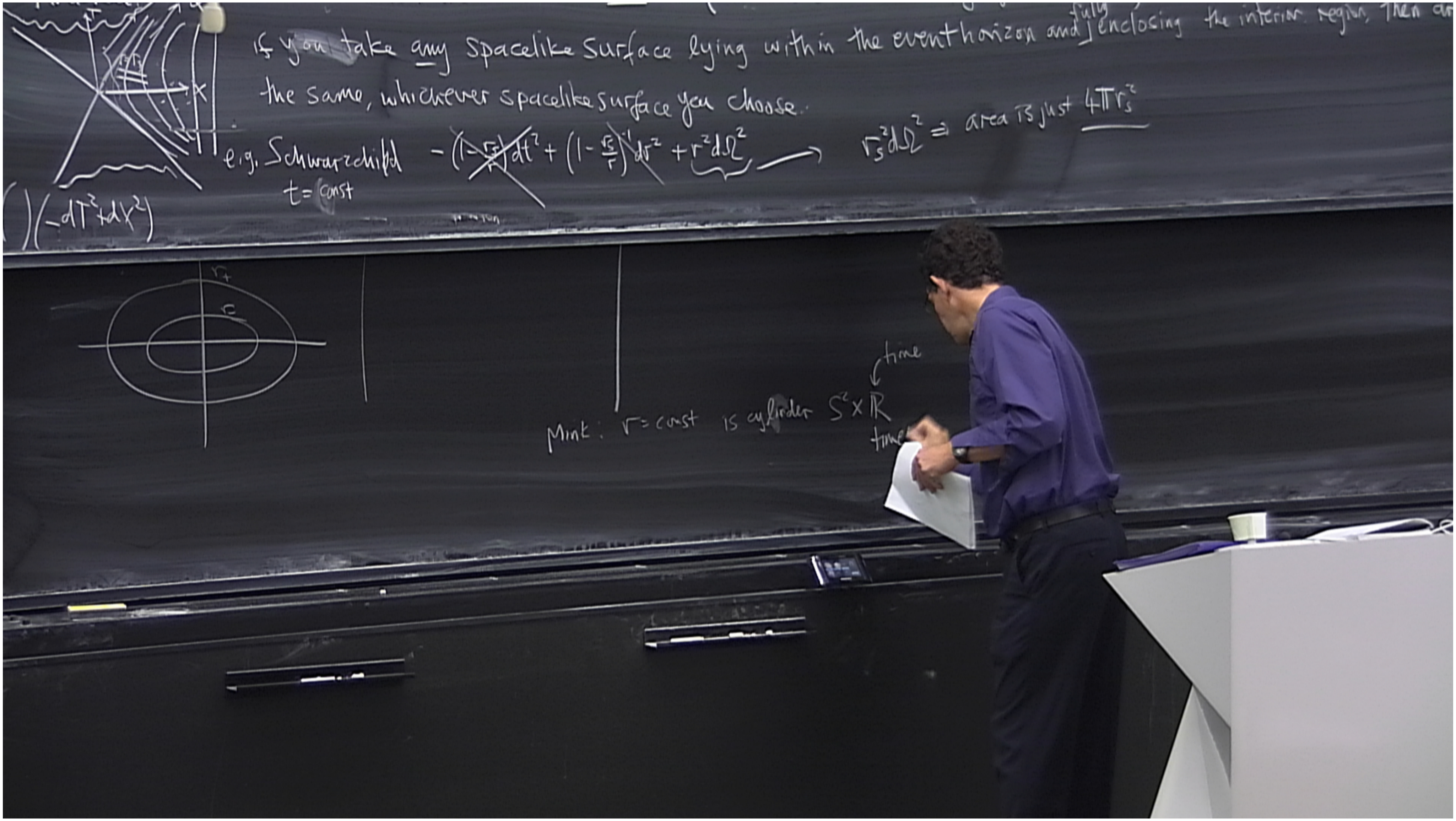

if you take any spacelike surface lying within the event horizon and fully enclosing the interior region, then
the same, whichever spacelike surface you choose.
e.g. Schwarzchild
area is just 4π



if you take any spacelike surface lying within the event horizon and fully enclosing the interior region, then
the same, whichever spacelike surface you choose.
e.g. Schwarzschild
t = const
area is just 4πr_s^2
Mink: r = const is cylinder S^2 × R
time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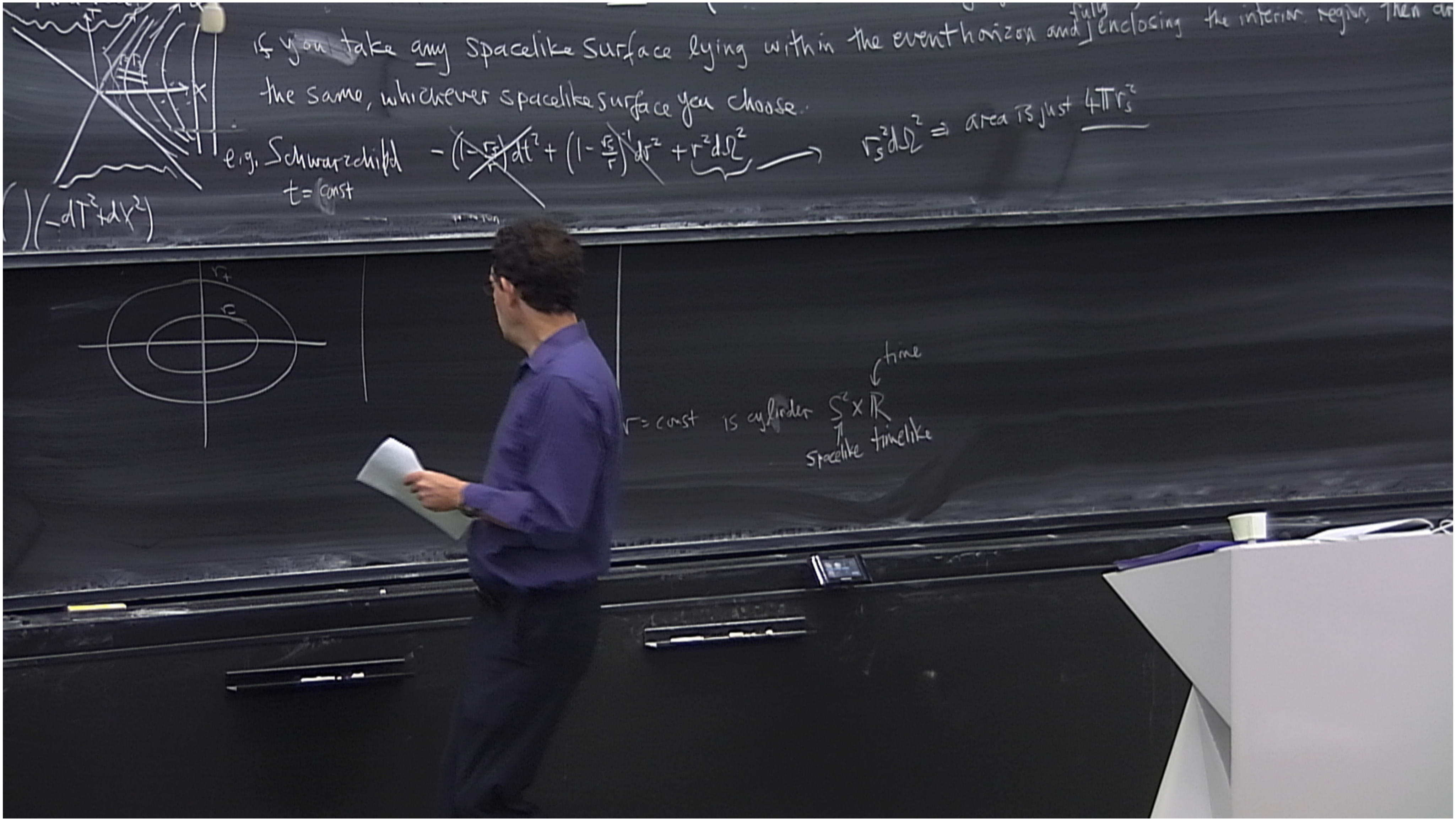
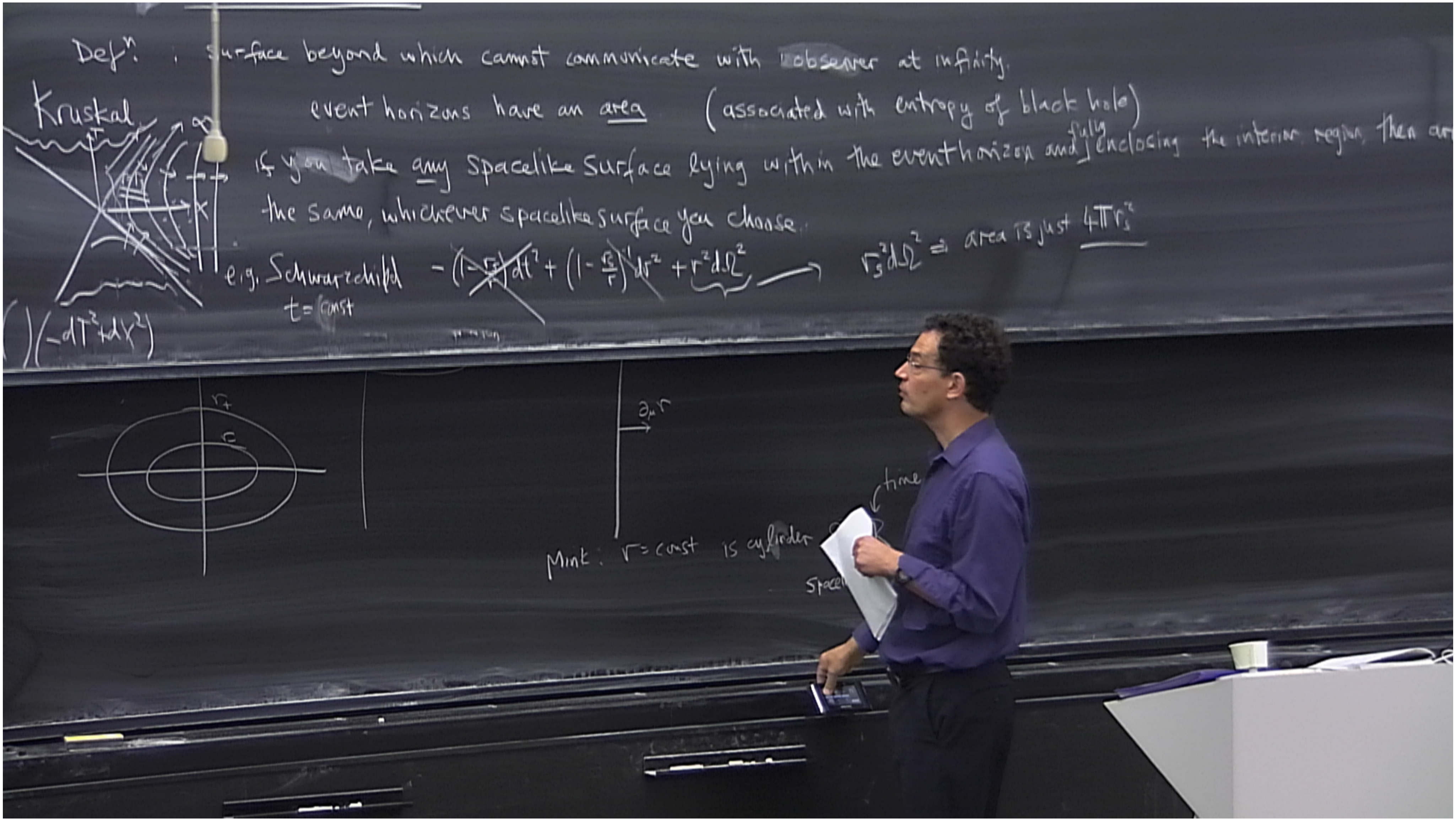
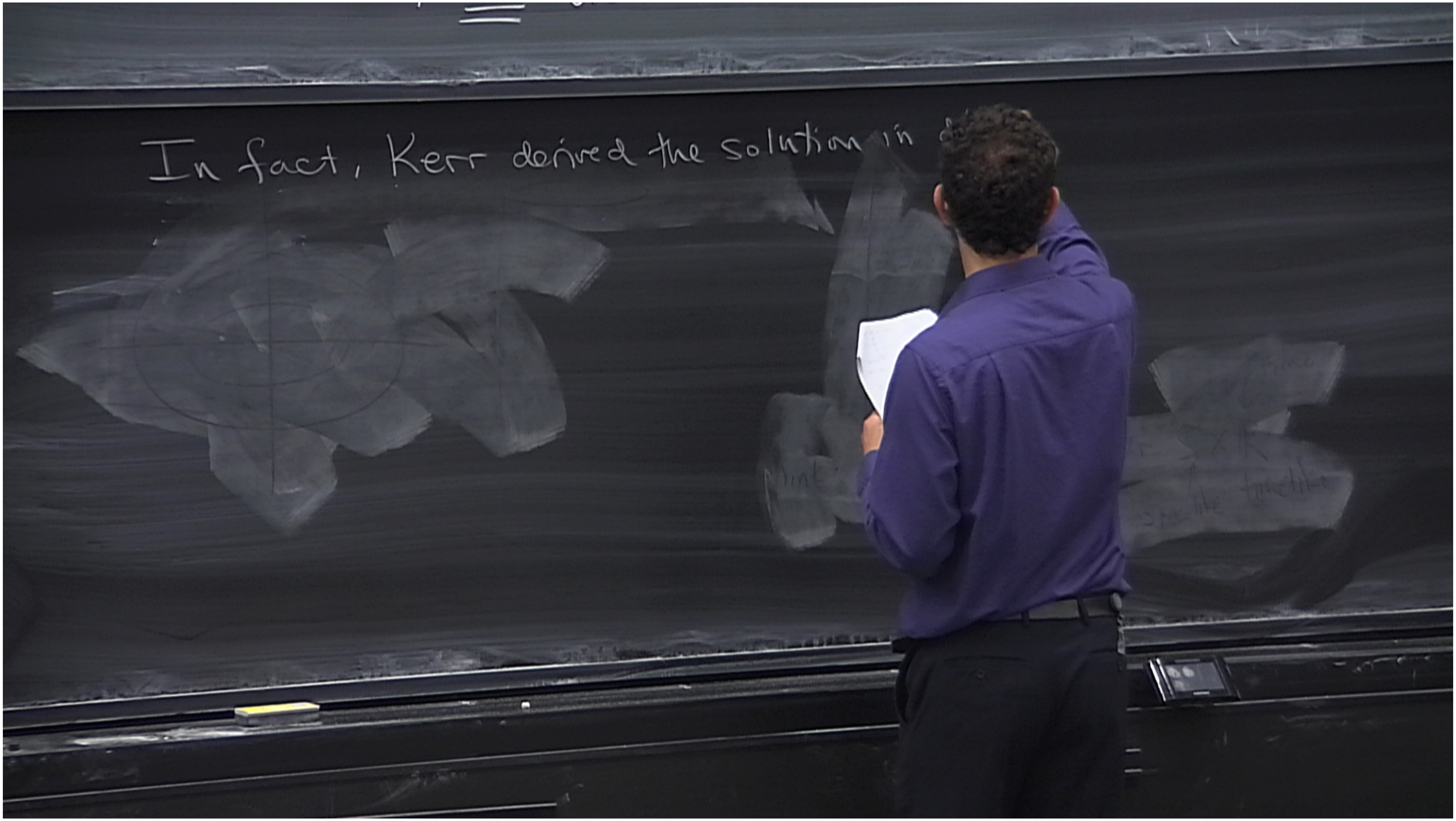
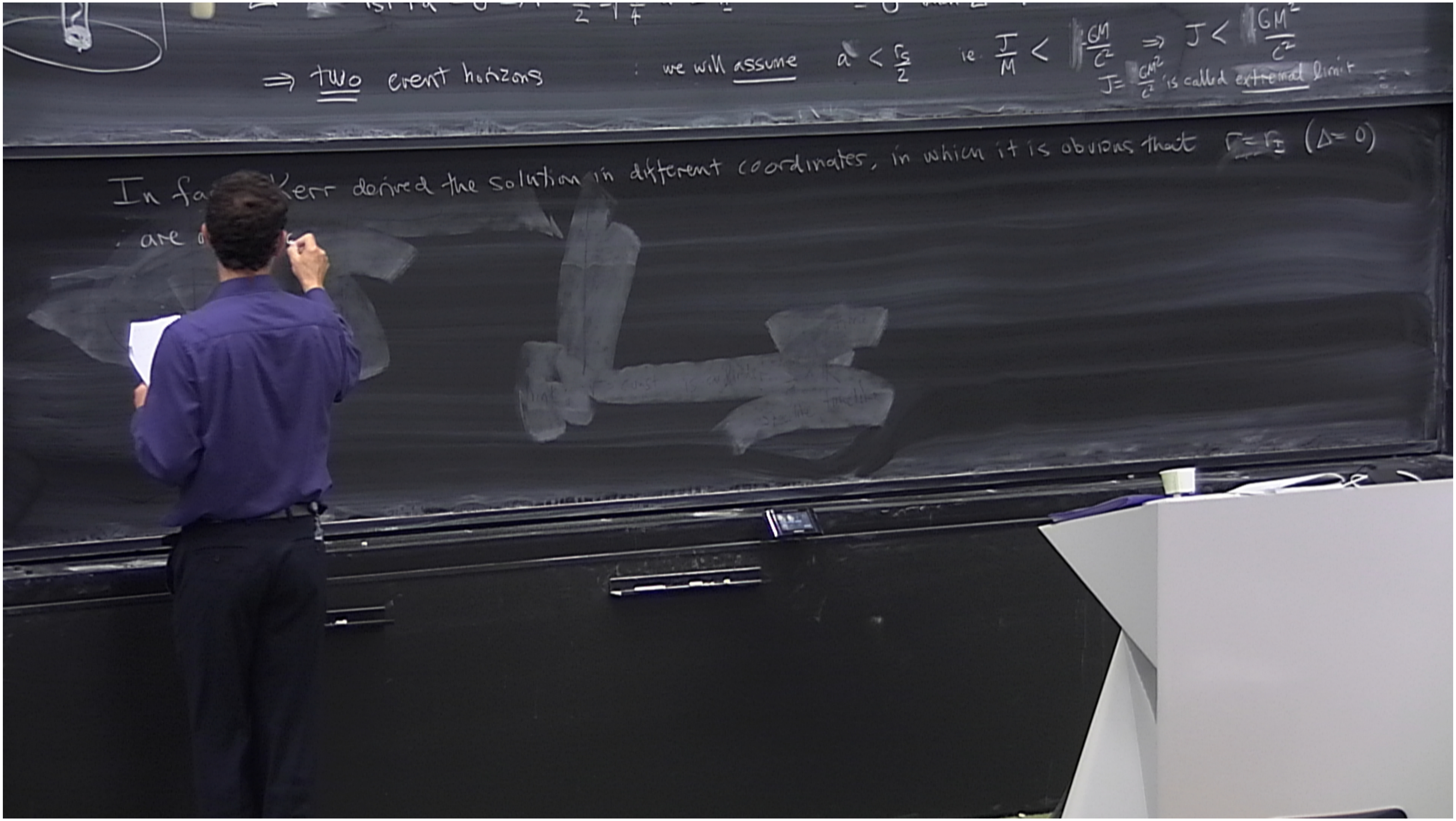
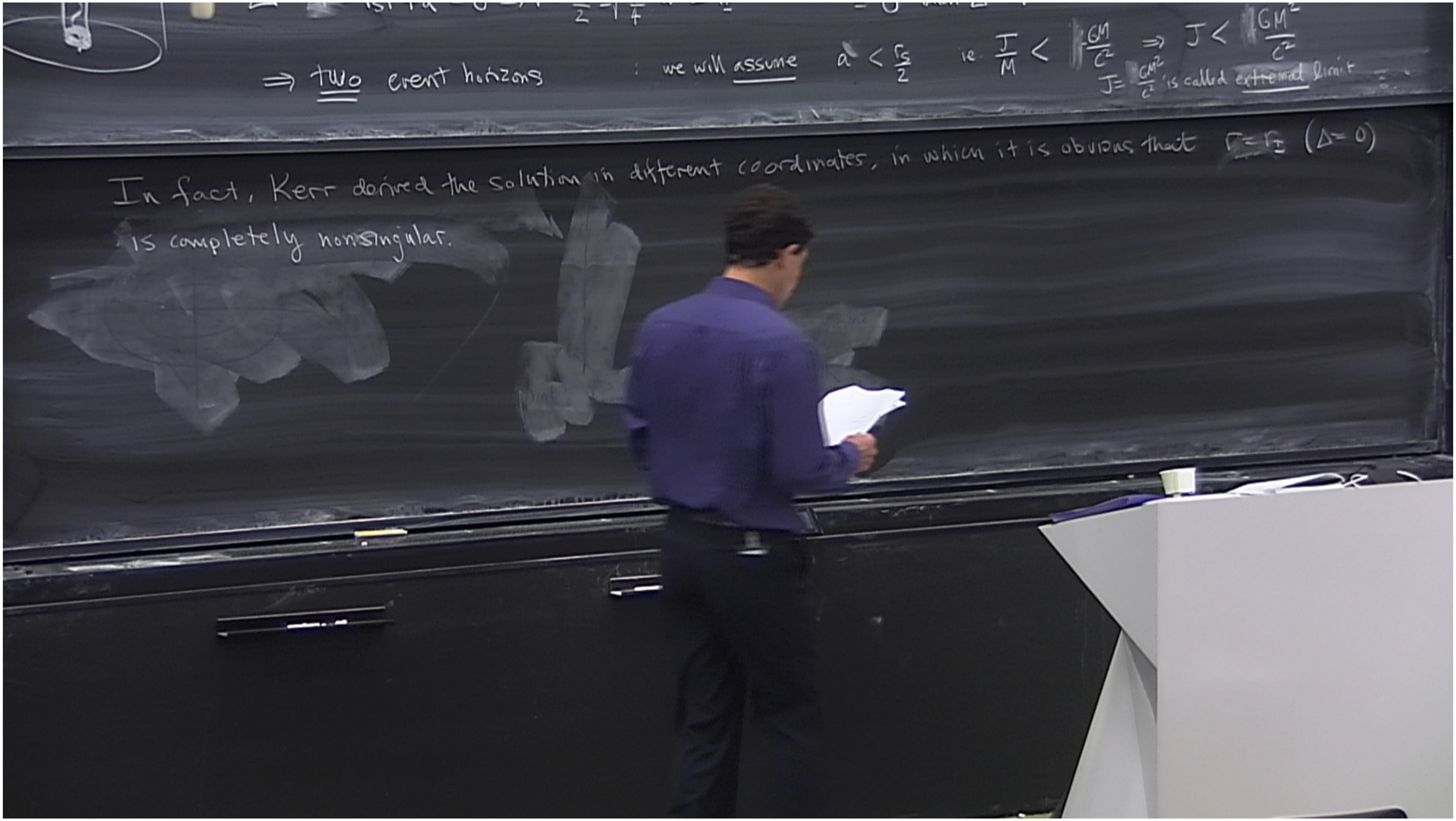
time

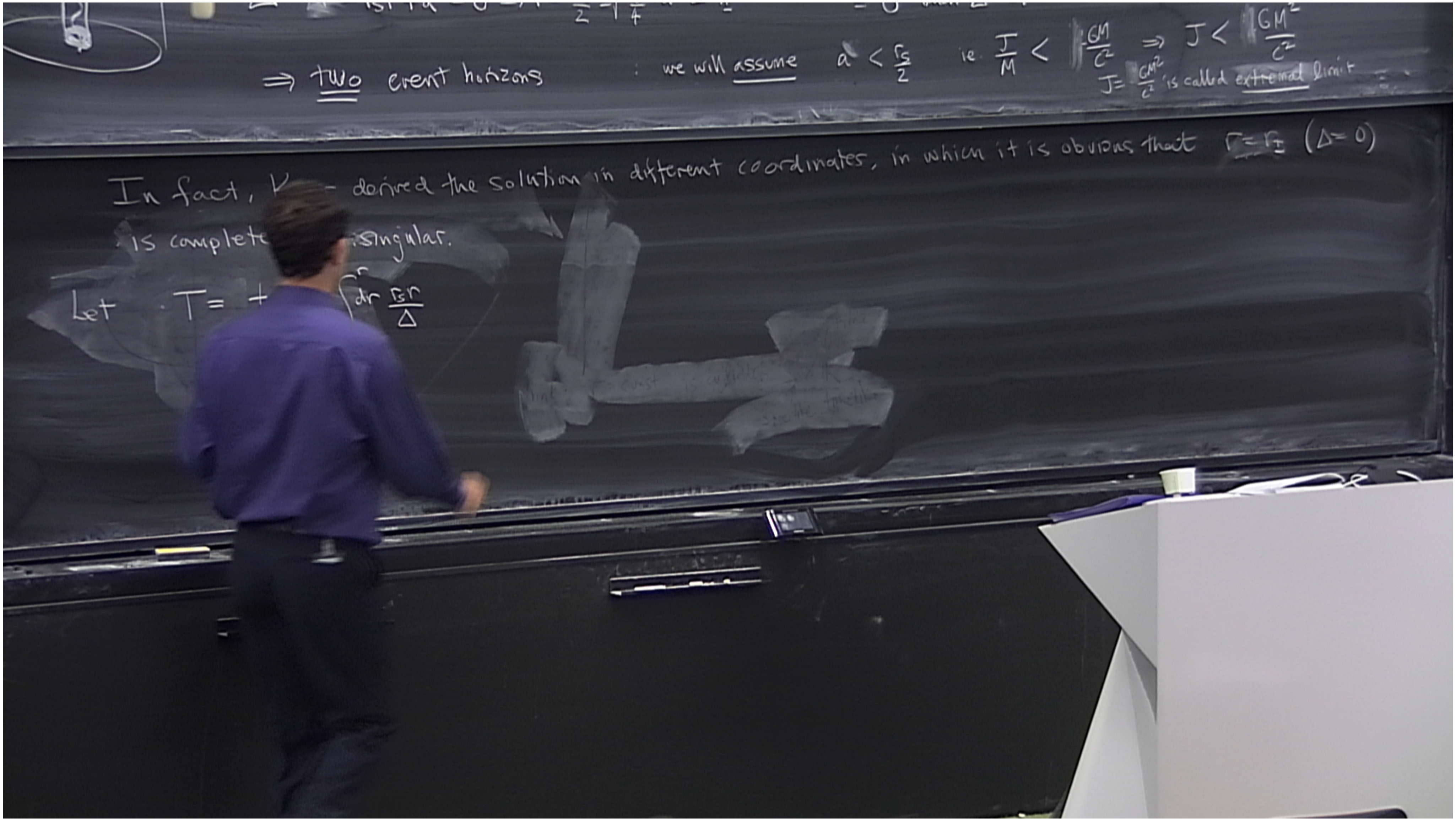

if you take any spacelike surface lying within the event horizon and [fully] enclosing the interior region, then ar
the same, whichever spacelike surface you choose.
e.g. Schwarzschild
t = const
area is just $4\pi r_s^2$
r = const is cylinder $S^2 \times \mathbb{R}$
time
spacelike
timelike


Defn: surface beyond which cannot communicate with observer at infinity.
Kruskal
event horizons have an area (associated with entropy of black hole)
If you take any spacelike surface lying within the event horizon and fully enclosing the interior region, then
the same, whichever spacelike surface you choose.
area is just 4πrs²
e.g. Schwarzschild
t = const
Mink: r = const is cylinder
time


In fact, Kerr derived the solution in


⇒ two event horizons
we will assume a < r_s/2 ie. J/M < GM/c^2 ⇒ J < GM^2/c^2
J = GM^2/c^2 is called extremal limit
Kerr derived the solution in different coordinates, in which it is obvious that r = r_± (Δ = 0)
are


⇒ two event horizons
we will assume $a < \frac{r_S}{2}$ i.e. $\frac{J}{M} < \frac{GM}{c^2}$ ⇒ $J < \frac{GM^2}{c^2}$
$J = \frac{GM^2}{c^2}$ is called extremal limit
In fact, Kerr derived the solution in different coordinates, in which it is obvious that $r = r_\pm$ ($\Delta = 0$)
is completely nonsingular.


⇒ two event horizons
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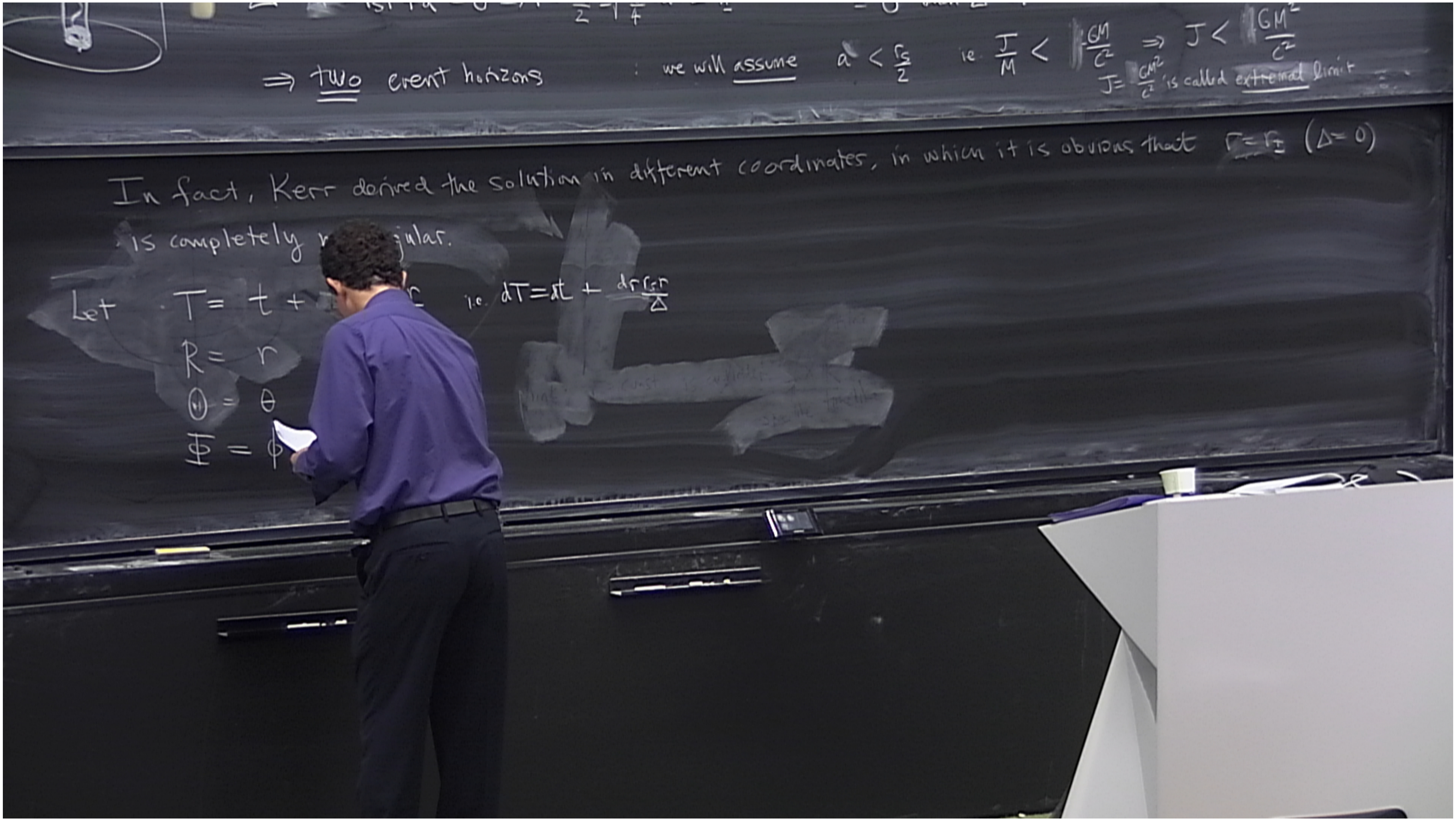
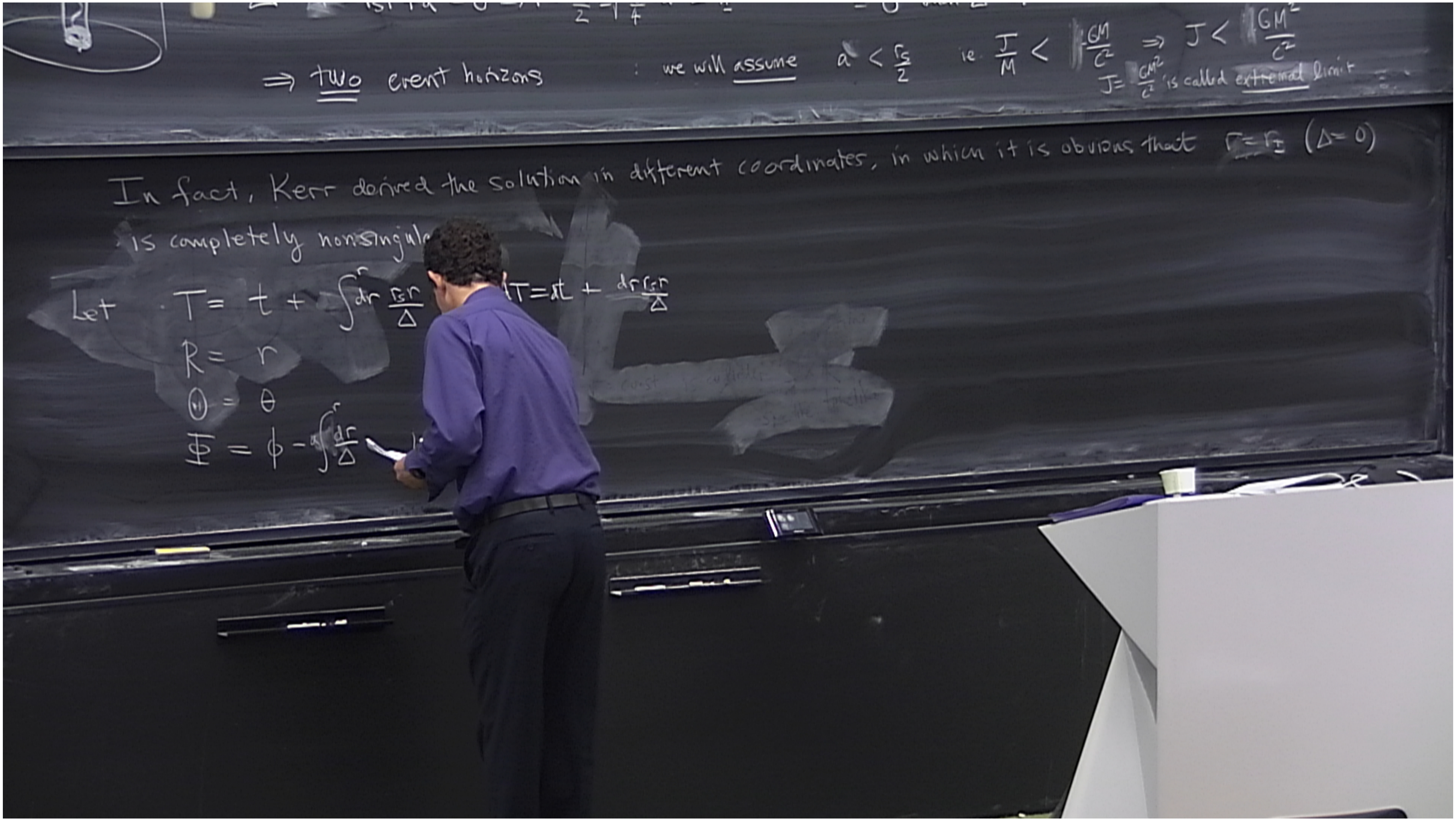
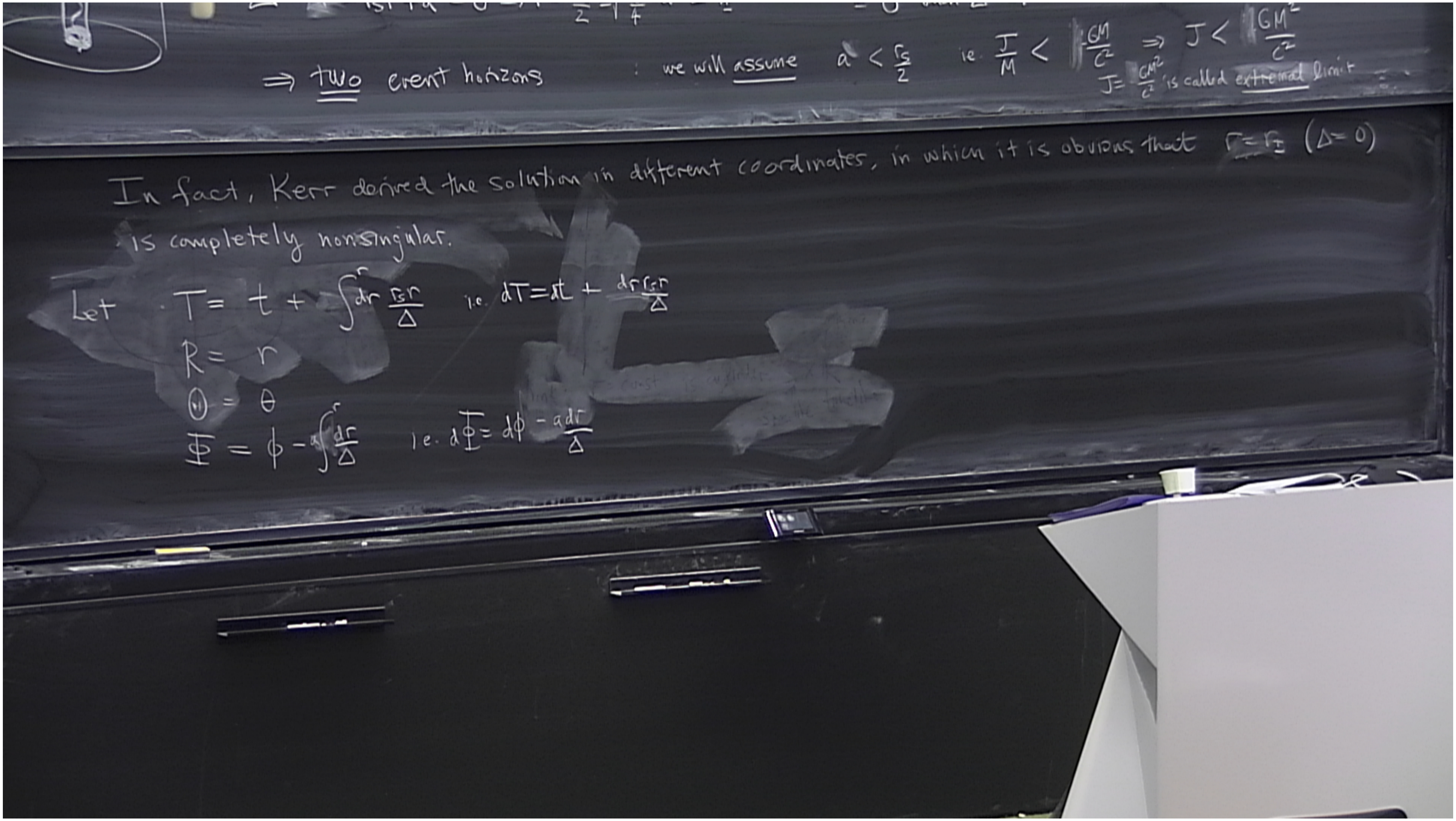
J = GM²/c² is called extremal limit
In fact, K derived the solution in different coordinates, in which it is obvious that r = r± (Δ = 0)
is complete singular.
Let T = t ∫dr rsr/Δ


⇒ two event horizons
we will assume
ie.
is called extremal limit
In fact, Kerr derived the solution in different coordinates, in which it is obvious that
is completely
Let
i.e.


⇒ two event horizons
we will assume $a < \frac{r_s}{2}$ ie. $\frac{J}{M} < \frac{GM}{c^2} \Rightarrow J < \frac{GM^2}{c^2}$
$J = \frac{GM^2}{c^2}$ is called extremal limit
In fact, Kerr derived the solution in different coordinates, in which it is obvious that $r = r_\pm$ $(\Delta = 0)$
is completely
Let $T = t + \int^r dr \frac{r_s r}{\Delta}$
$dT = dt + \frac{dr\, r_s r}{\Delta}$
$R = r$
$\Theta = \theta$
$\Phi = \phi - a\int^r \frac{dr}{\Delta}$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In fact, Kerr derived the solution in different coordinates, in which it is obvious that r = r_± (Δ = 0)
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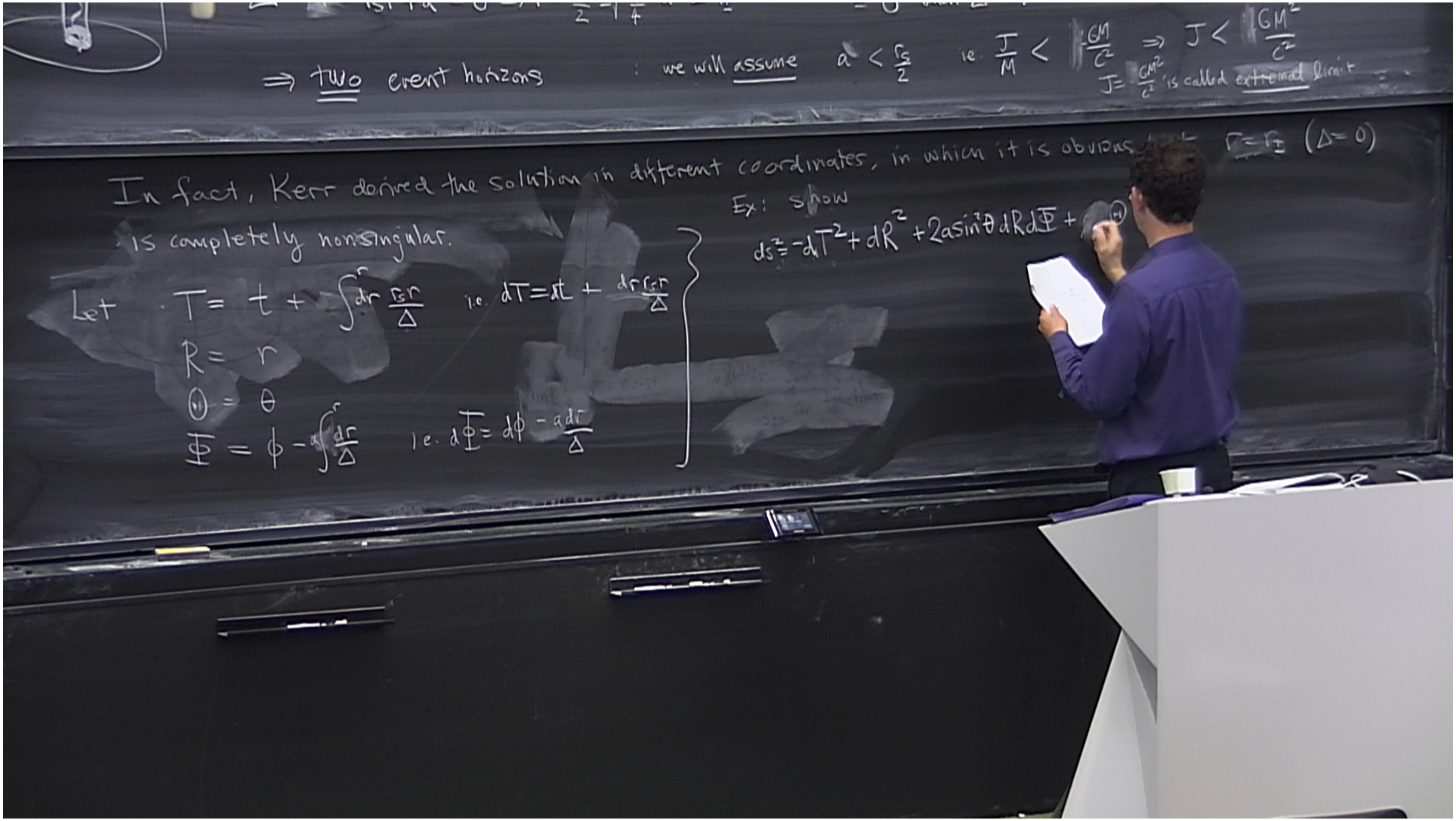
Let T = t + ∫^r dr r_s r/Δ
i.e. dT = dt + dr r_s r/Δ
R = r
Θ = θ
Φ = φ − a∫^r dr/Δ
i.e. dΦ = dφ − a dr/Δ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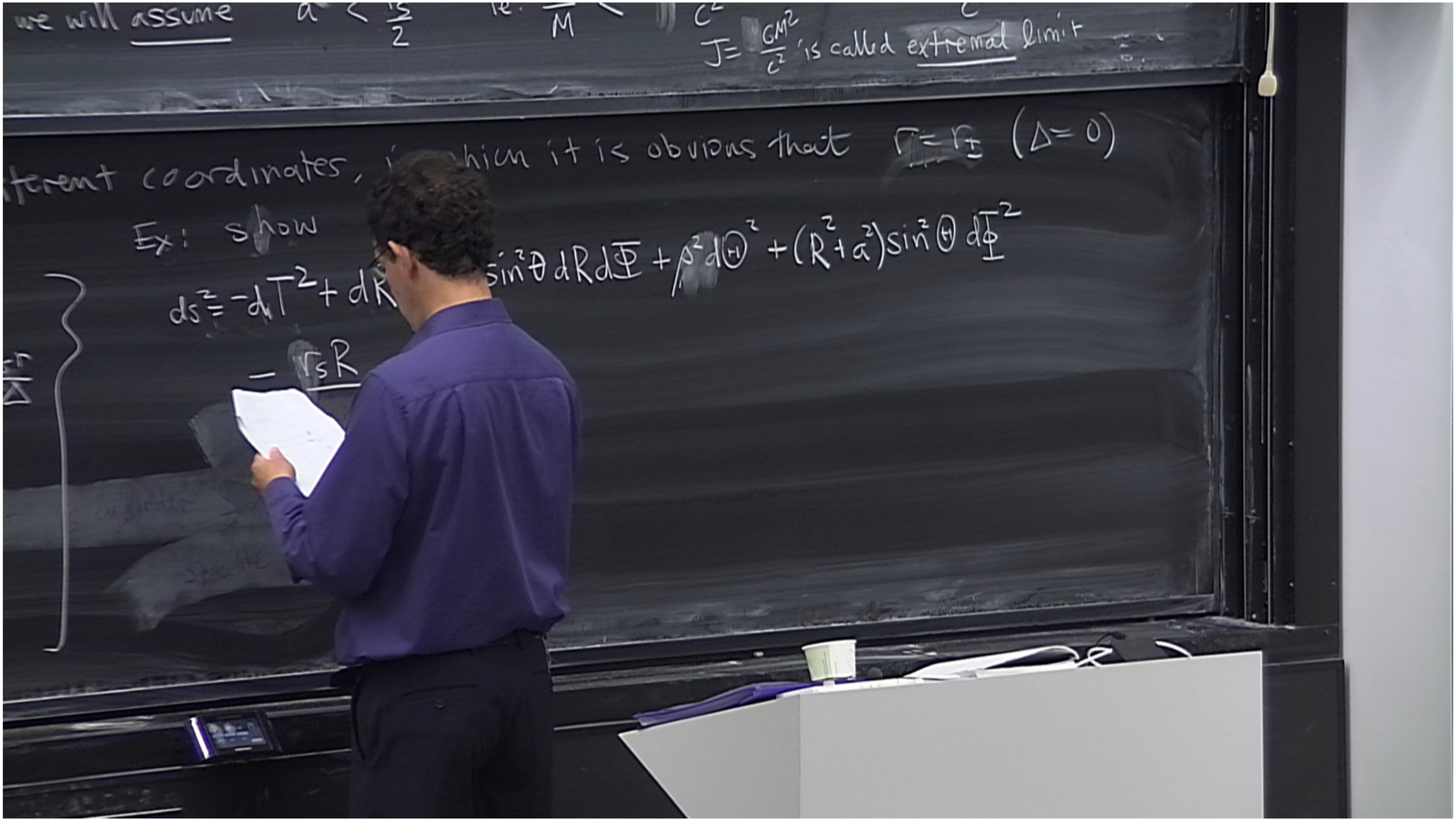
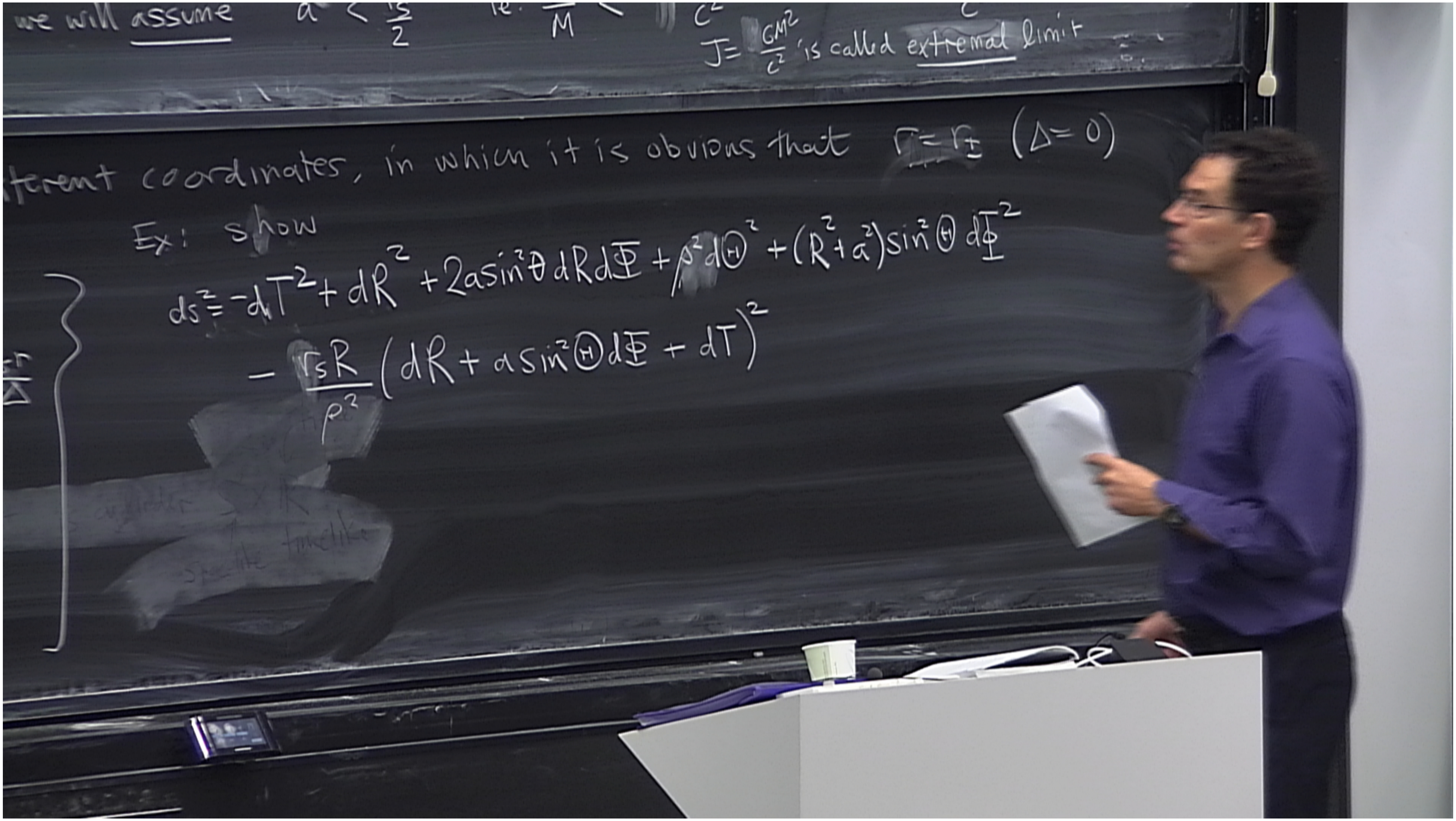
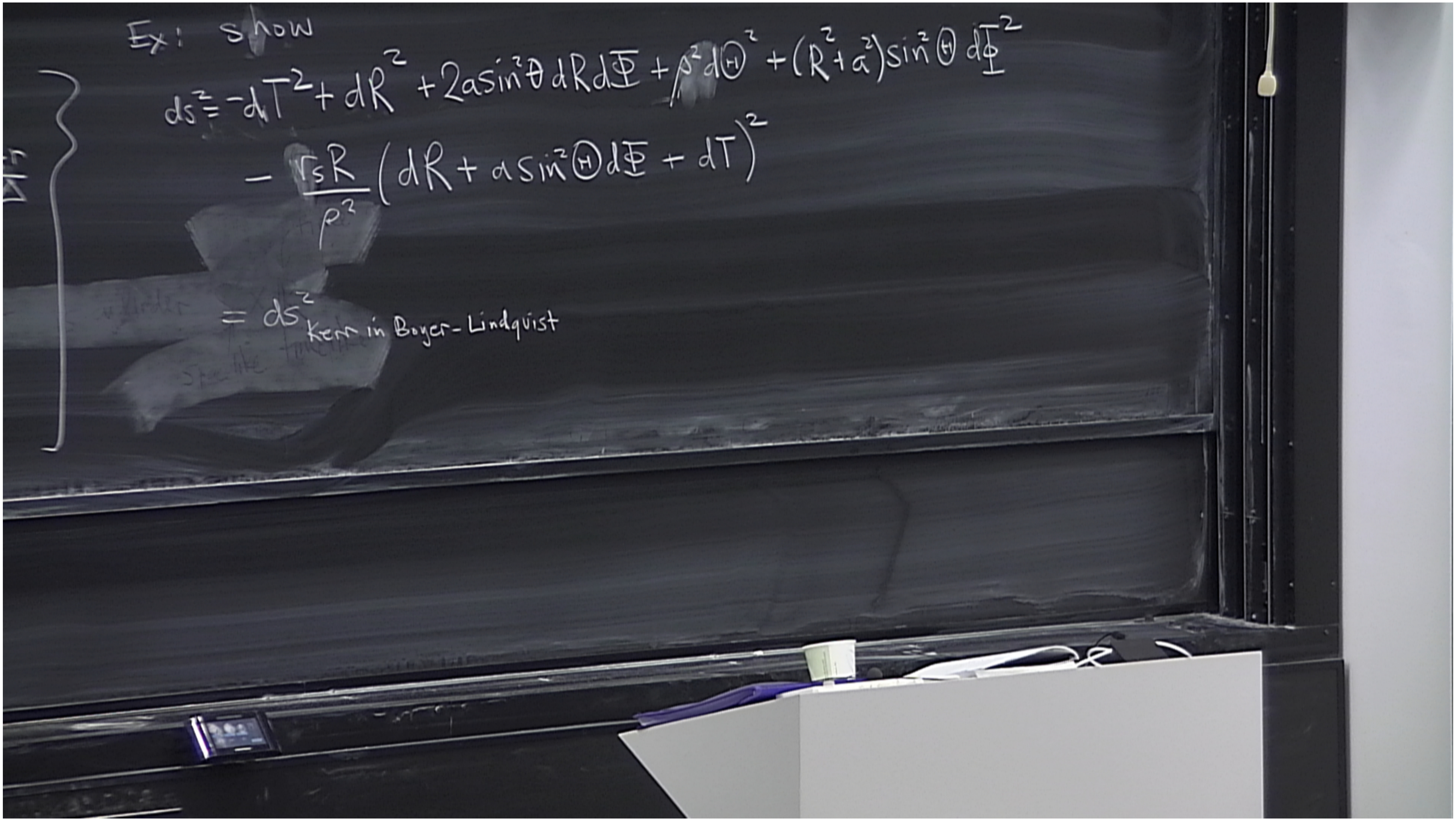
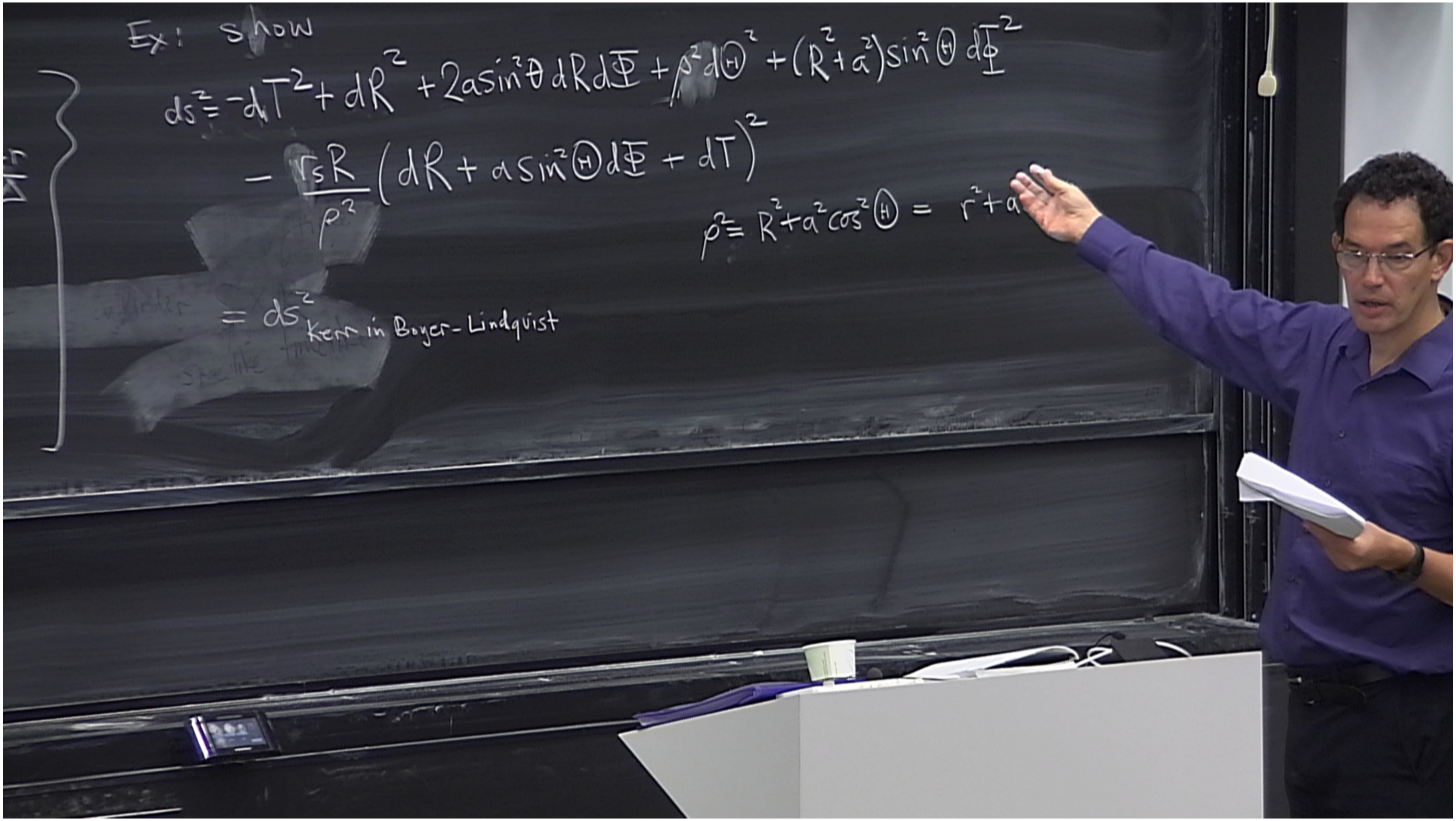
Let T = t + ∫dr rs r/Δ i.e. dT = dt + dr rs r/Δ
R = r
Θ = θ
Φ = φ − a∫dr/Δ i.e. dΦ = dφ − a dr/Δ
Ex: show
ds² = −dT² + dR² + 2a sin²θ dR dΦ +

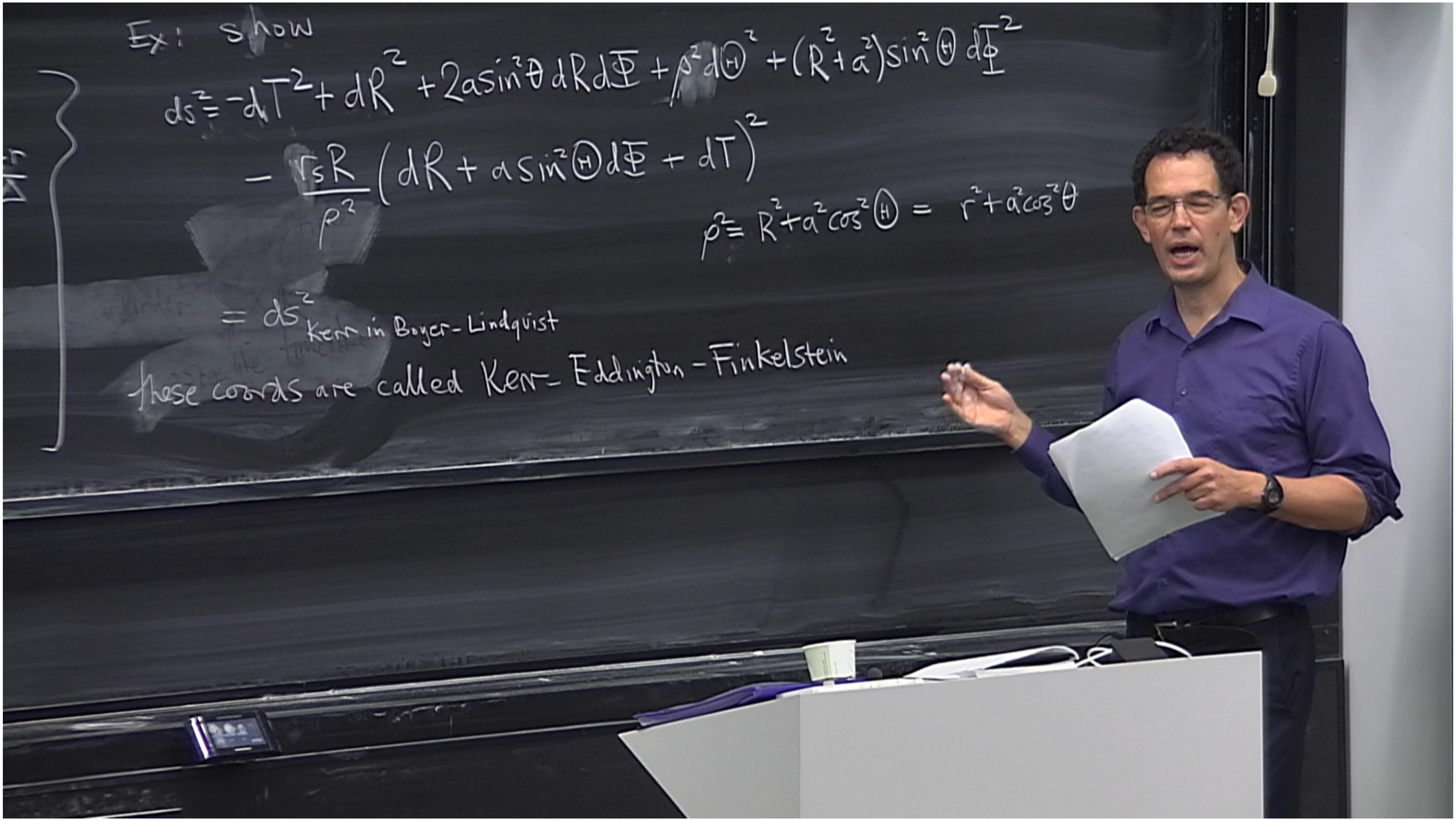

we will assume
is called extremal limit
coordinates,
it is obvious that
Ex: show


we will assume
J= GM²/c² is called extremal limit
fferent coordinates, in which it is obvious that r = r± (Δ=0)
Ex: show
ds² = -dT² + dR² + 2a sin²θ dR dΦ + ρ² dΘ² + (R²+a²) sin²Θ dΦ²
- rsR/ρ² (dR + a sin²Θ dΦ + dT)²



Ex: show
$ds^2 = -dT^2 + dR^2 + 2a\sin^2\Theta\, dR\, d\Phi + \rho^2 d\Theta^2 + (R^2+a^2)\sin^2\Theta\, d\Phi^2$
$- \frac{r_s R}{\rho^2}\left(dR + a\sin^2\Theta\, d\Phi + dT\right)^2$
$= ds^2_{\text{Kerr in Boyer-Lindquist}}$


Ex: show
ds² = -dT² + dR² + 2a sin²θ dR dΦ + ρ² dΘ² + (R² + a²) sin²Θ dΦ²
- (r_s R / ρ²) (dR + a sin²Θ dΦ + dT)²
ρ² = R² + a² cos²Θ = r² + a
= ds² Kerr in Boyer-Lindquist


Ex: show
ds² = -dT² + dR² + 2a sin²θ dR dΦ + ρ² dΘ² + (R² + a²) sin²Θ dΦ²
− r_s R / ρ² (dR + a sin²Θ dΦ + dT)²
ρ² ≡ R² + a² cos²Θ = r² + a² cos²θ
= ds² Kerr in Boyer-Lindquist
these coords are called Kerr-Eddington-Finkelstein

Ex: show

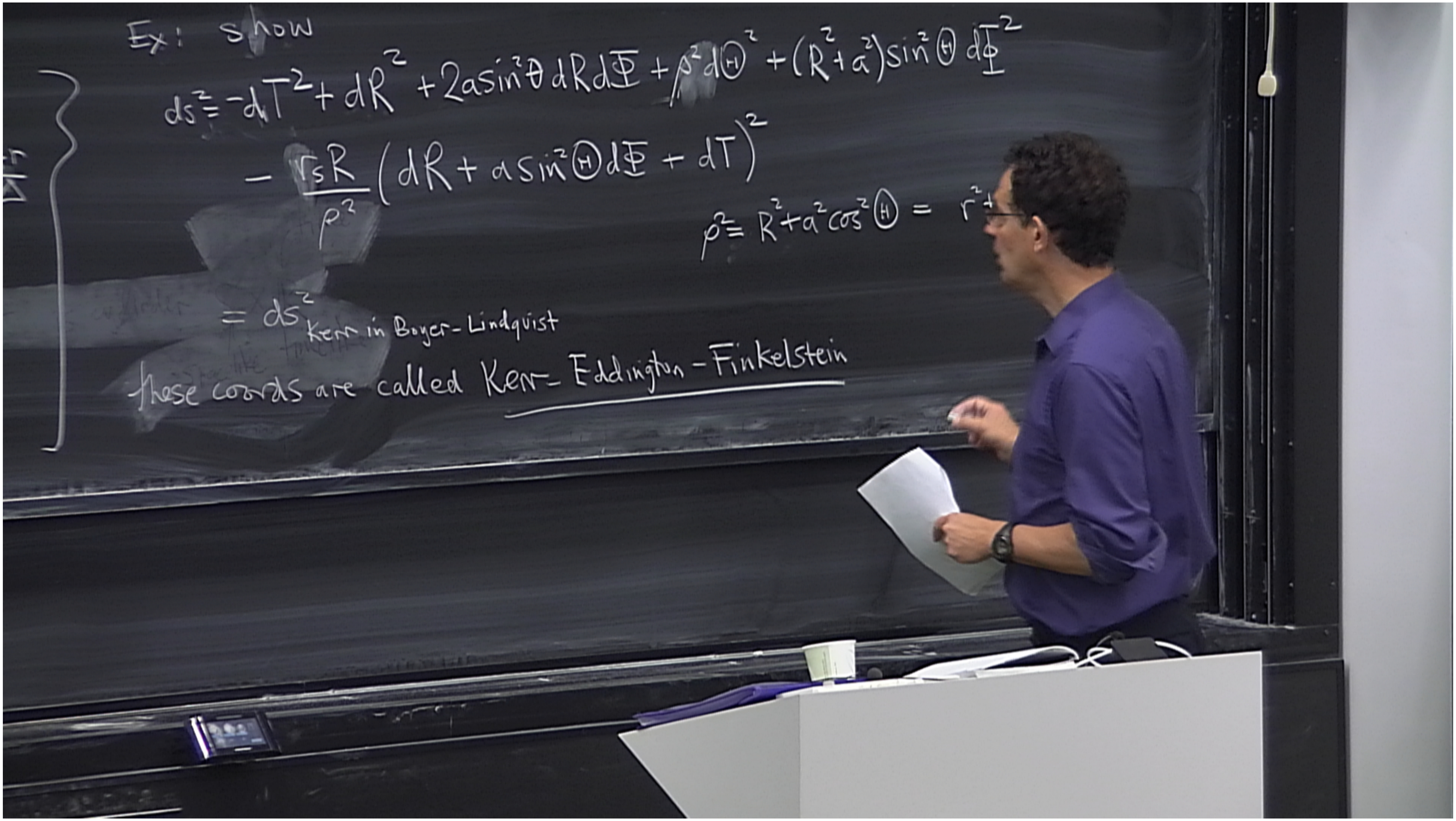
$$ds^2 = -dT^2 + dR^2 + 2a\sin^2\theta\, dR\, d\Phi + \rho^2 d\Theta^2 + (R^2+a^2)\sin^2\Theta\, d\Phi^2$$

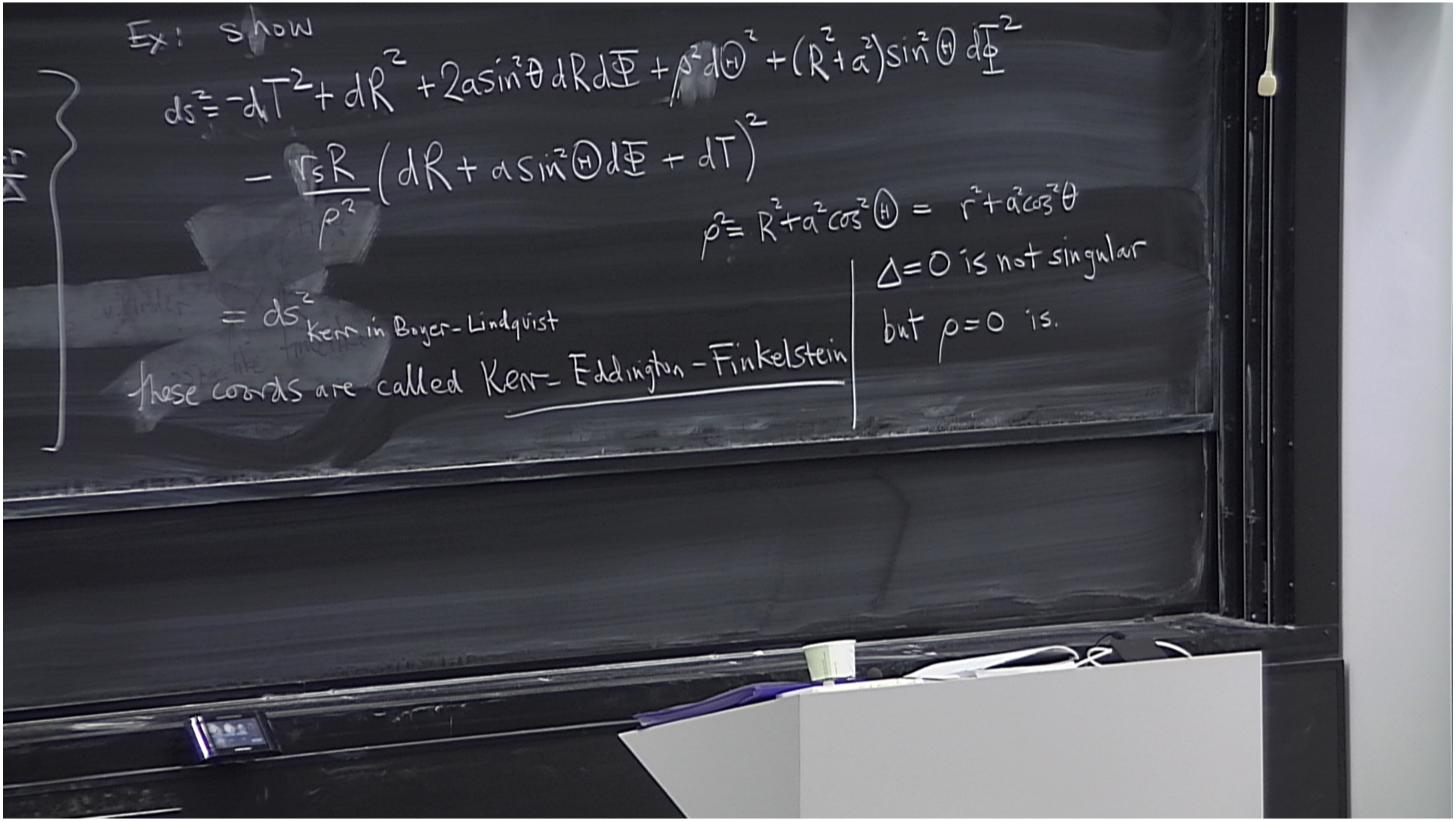
$$- \frac{r_s R}{\rho^2}\left(dR + a\sin^2\Theta\, d\Phi + dT\right)^2$$

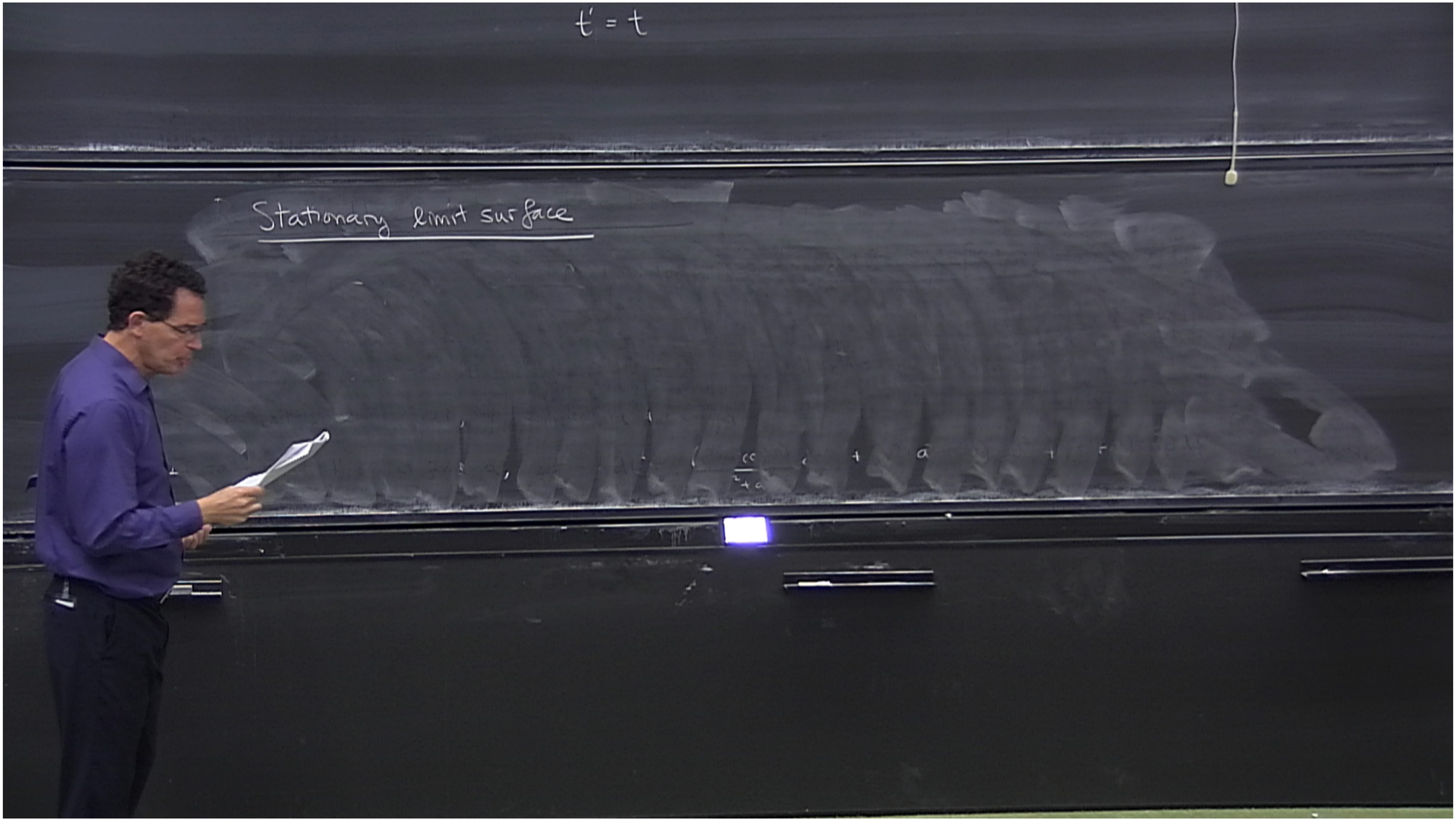
$$\rho^2 = R^2 + a^2\cos^2\Theta = r^2 + a^2\cos^2\theta$$

$$= ds^2_{\text{Kerr in Boyer-Lindquist}}$$

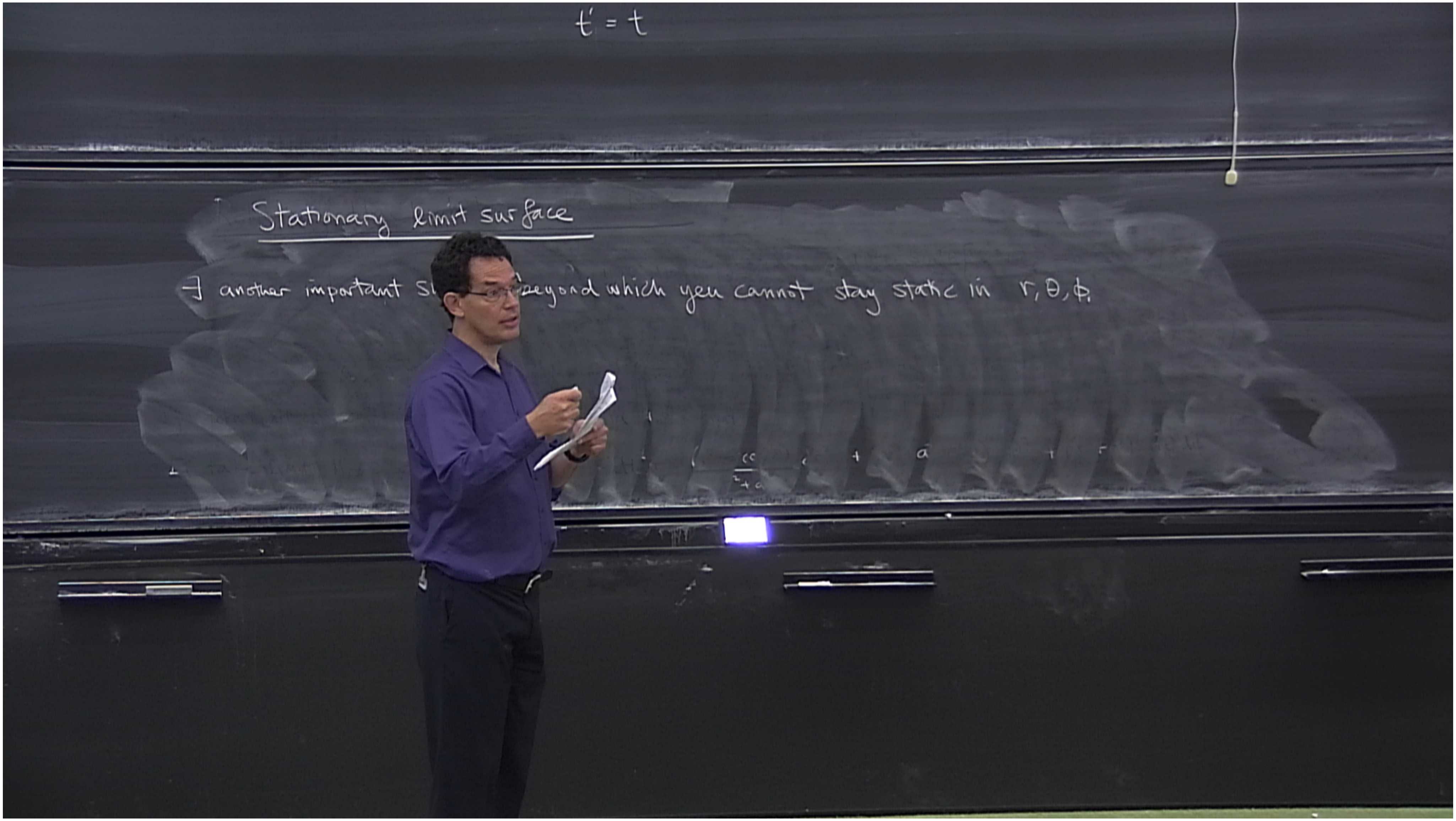
these coords are called Kerr-Eddington-Finkelstein

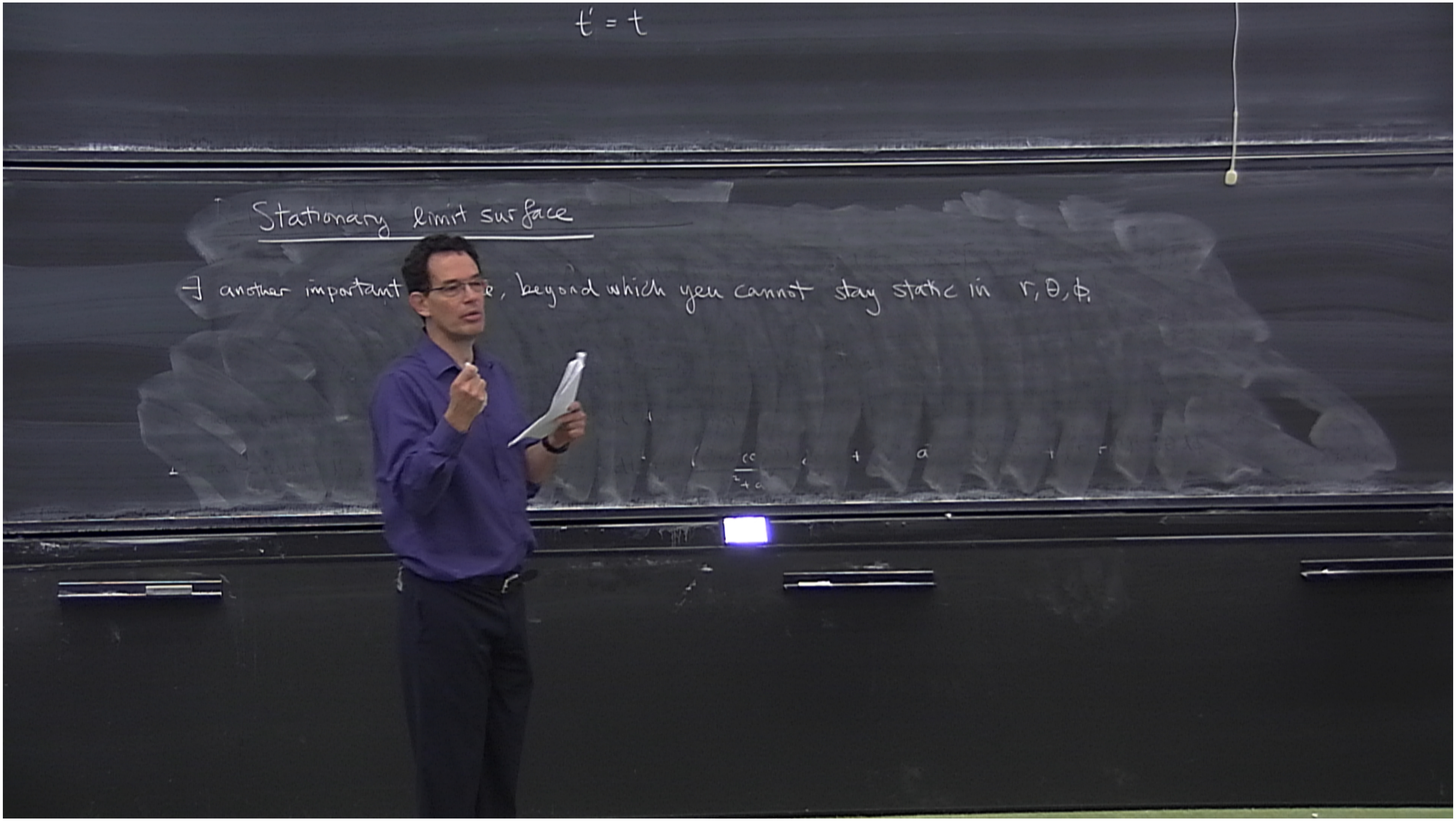

Ex: show
$ds^2 = -dT^2 + dR^2 + 2a\sin^2\Theta\, dR\, d\Phi + \rho^2 d\Theta^2 + (R^2+a^2)\sin^2\Theta\, d\Phi^2$
$- \frac{r_s R}{\rho^2}\left(dR + a\sin^2\Theta\, d\Phi + dT\right)^2$
$\rho^2 \equiv R^2 + a^2\cos^2\Theta = r^2 +$
$= ds^2_{\text{Kerr in Boyer-Lindquist}}$
these coords are called Kerr-Eddington-Finkelstein


Ex: show
$ds^2 = -dT^2 + dR^2 + 2a\sin^2\theta\, dR\, d\Phi + \rho^2 d\Theta^2 + (R^2+a^2)\sin^2\Theta\, d\Phi^2$
$- \frac{r_s R}{\rho^2}\left(dR + a\sin^2\Theta\, d\Phi + dT\right)^2$
$\rho^2 = R^2 + a^2\cos^2\Theta = r^2 + a^2\cos^2\theta$
$= ds^2_{\text{Kerr in Boyer-Lindquist}}$
these coords are called Kerr-Eddington-Finkelstein
$\Delta = 0$ is not singular
but $\rho = 0$ is.



t' = t
Stationary limit surface


t' = t
Stationary limit surface
∃ another important
beyond which you cannot stay static in r, θ, φ.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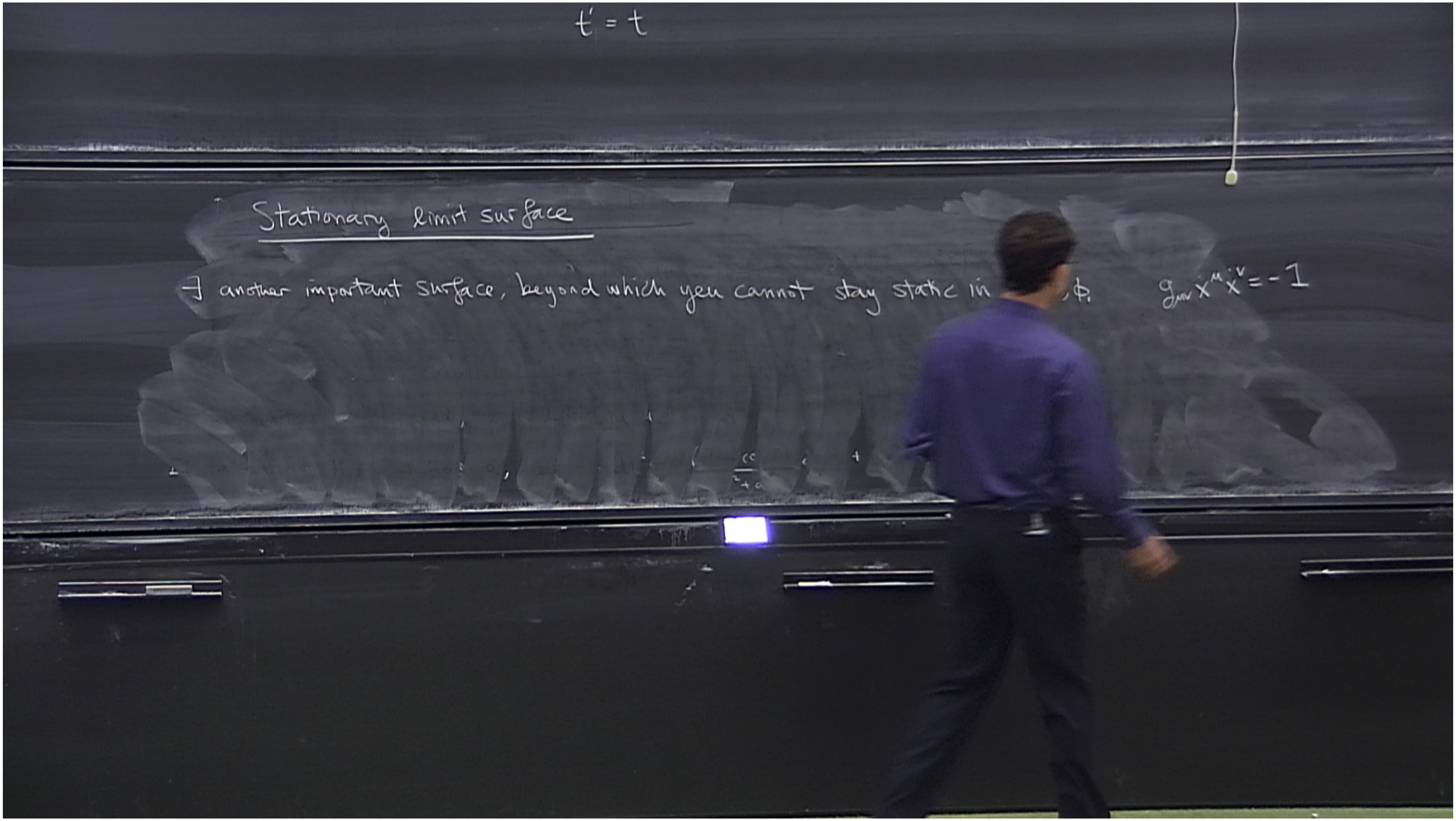
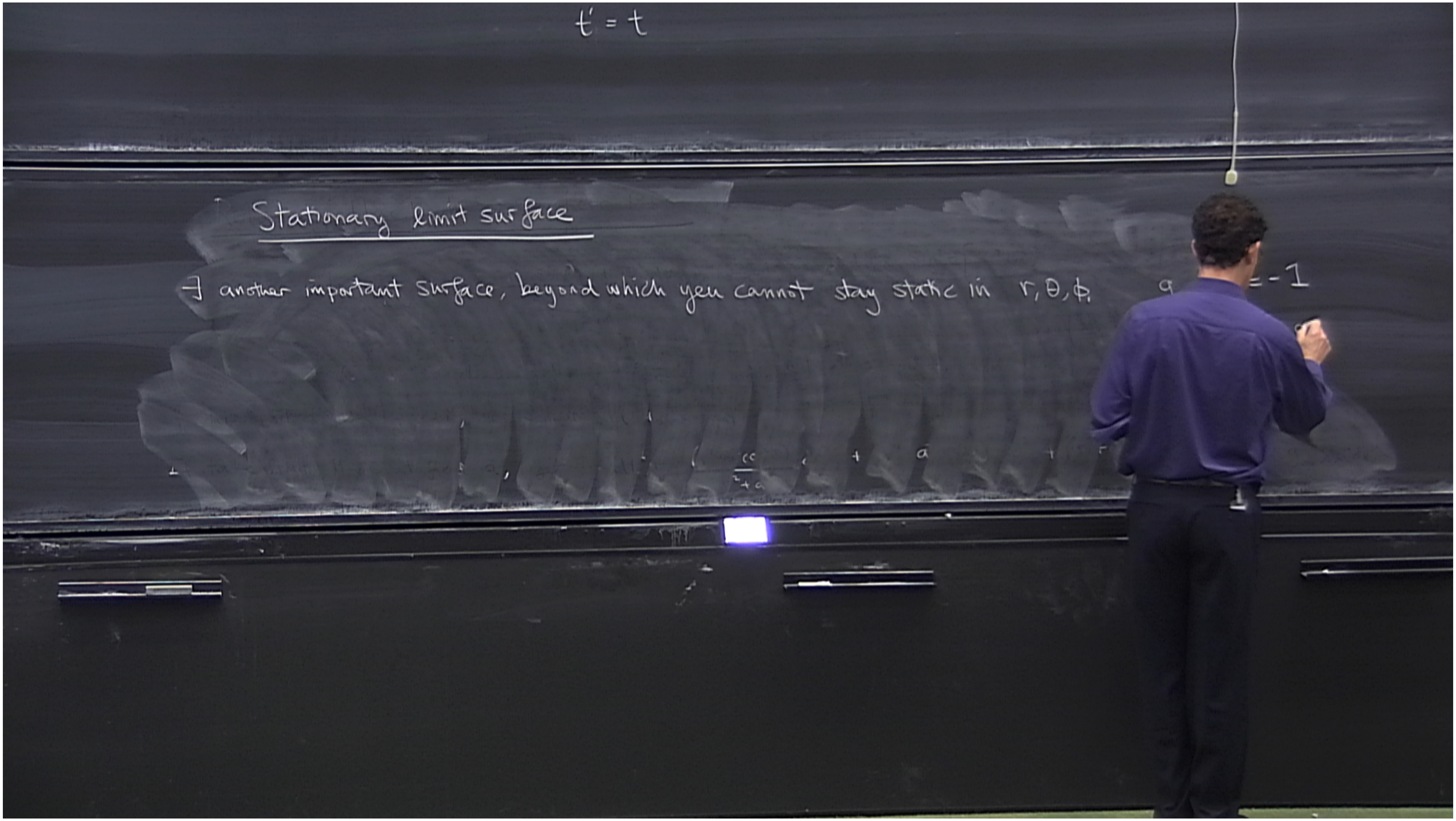
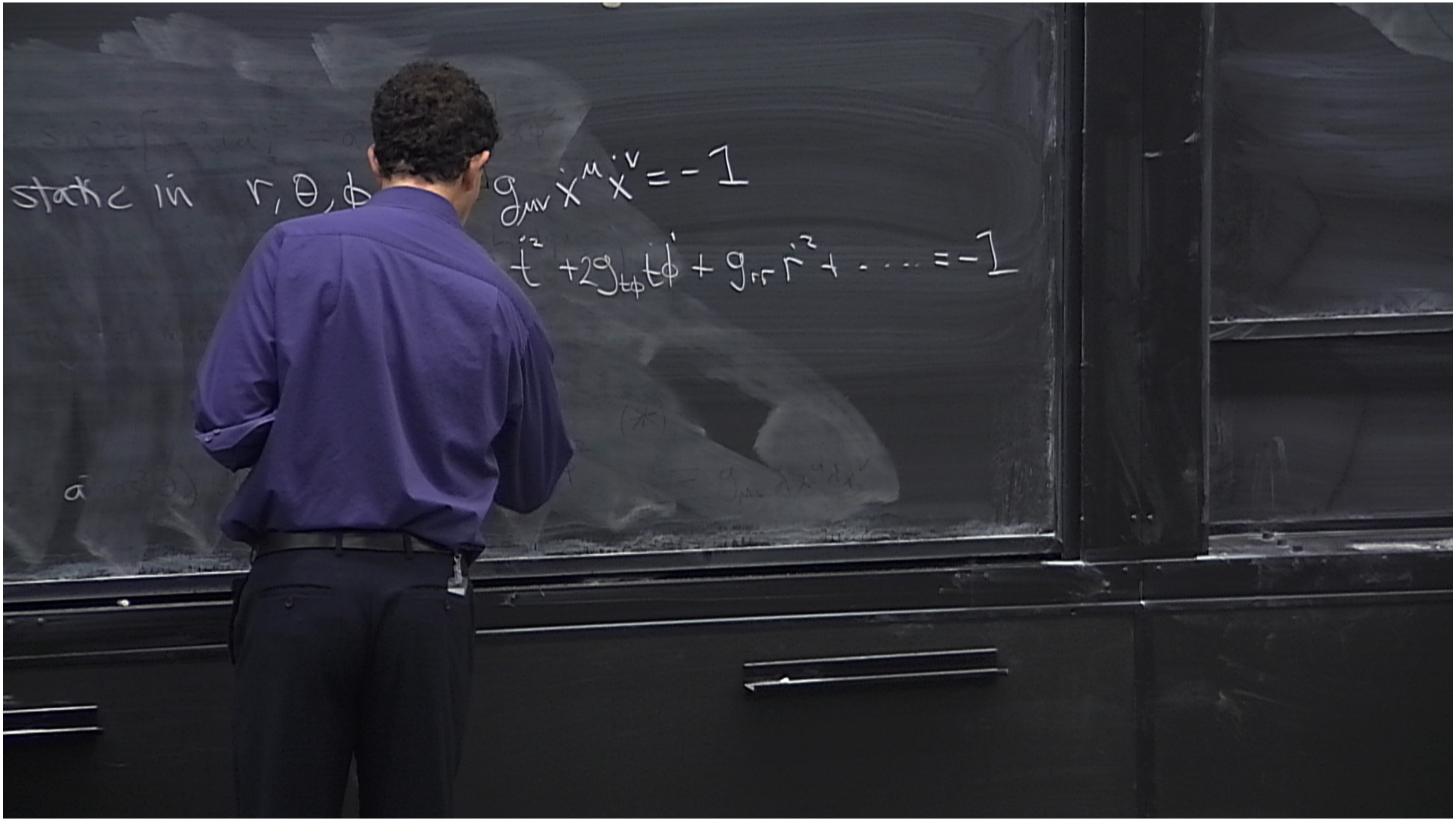

$t' = t$
Stationary limit surface
∃ another important surface, beyond which you cannot stay static in ... ϕ.
$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$


t' = t
Stationary limit surface
∃ another important surface, beyond which you cannot stay static in r, θ, φ.

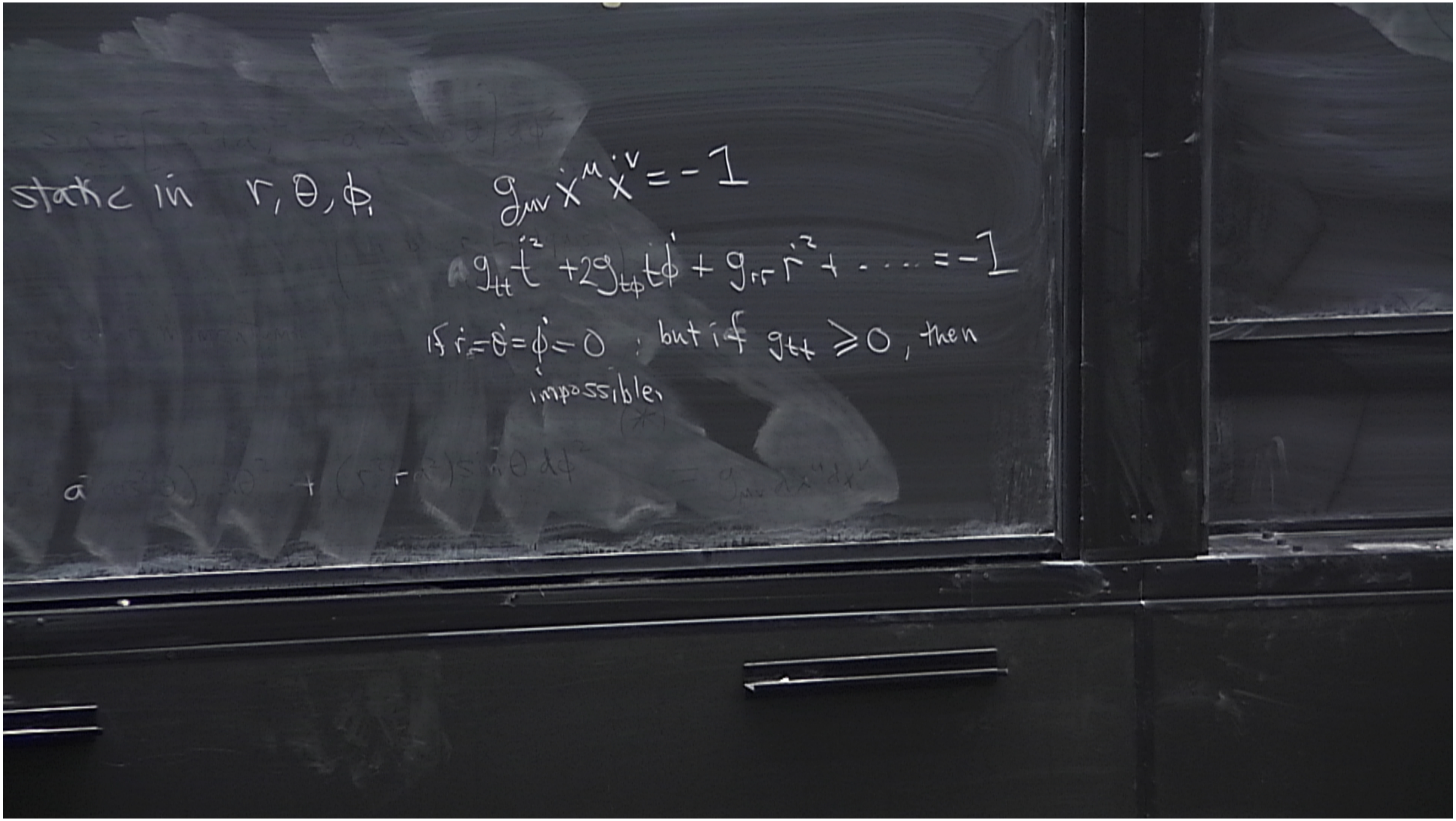

static in r, θ, φ
$g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = -1$
$\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^2 + \ldots = -1$

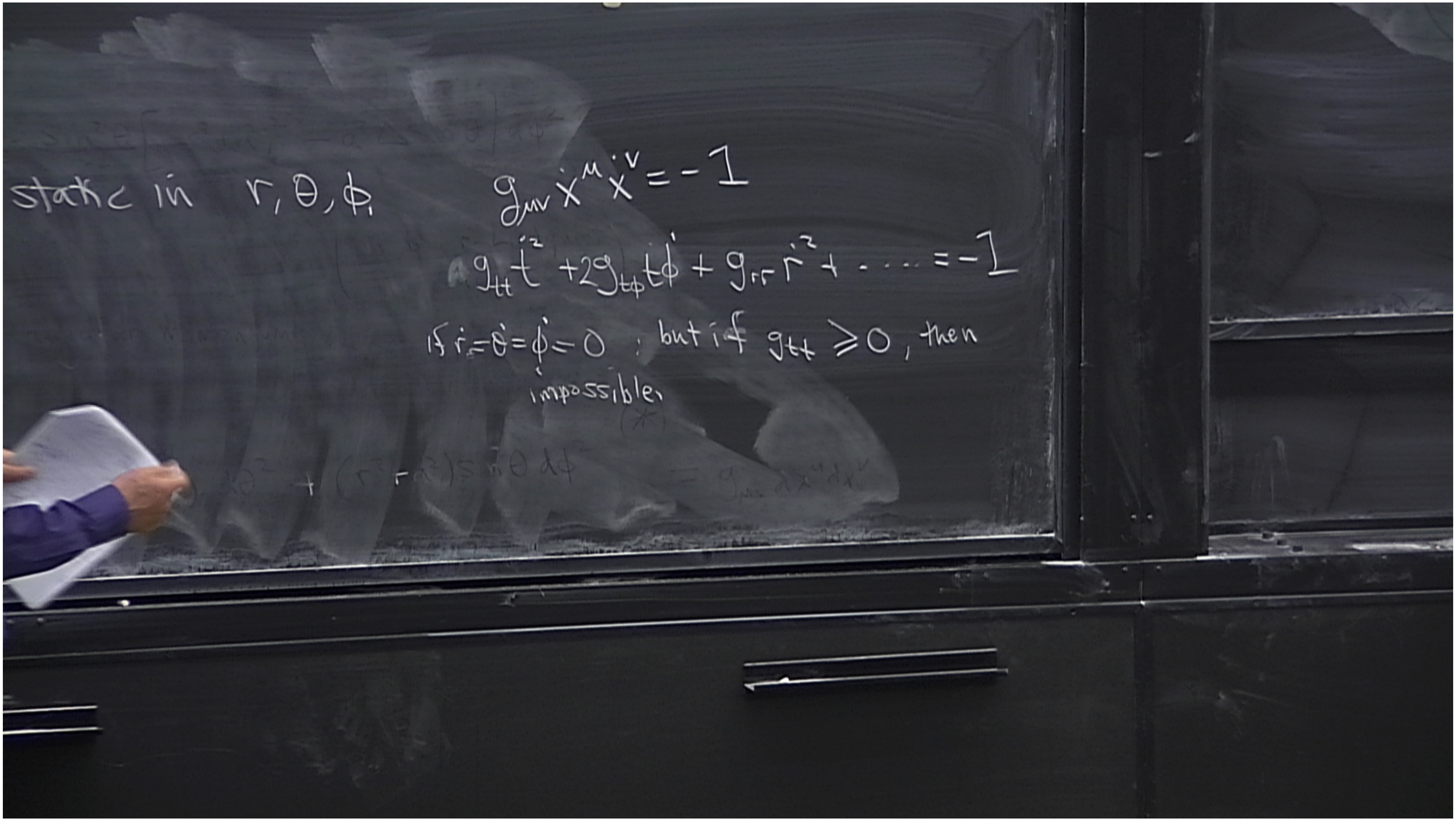
static in r, θ, ϕ.

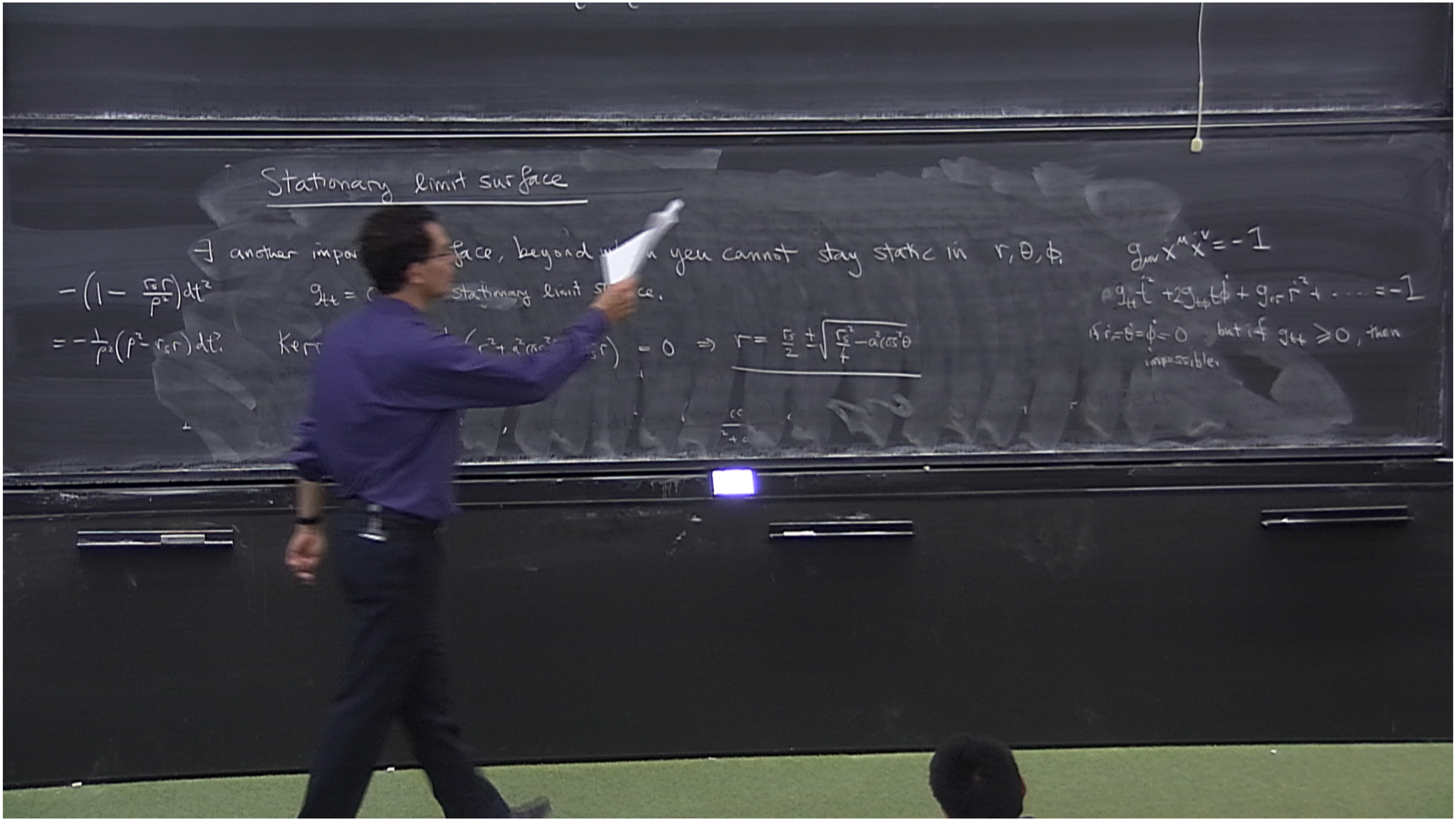
$$g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = -1$$

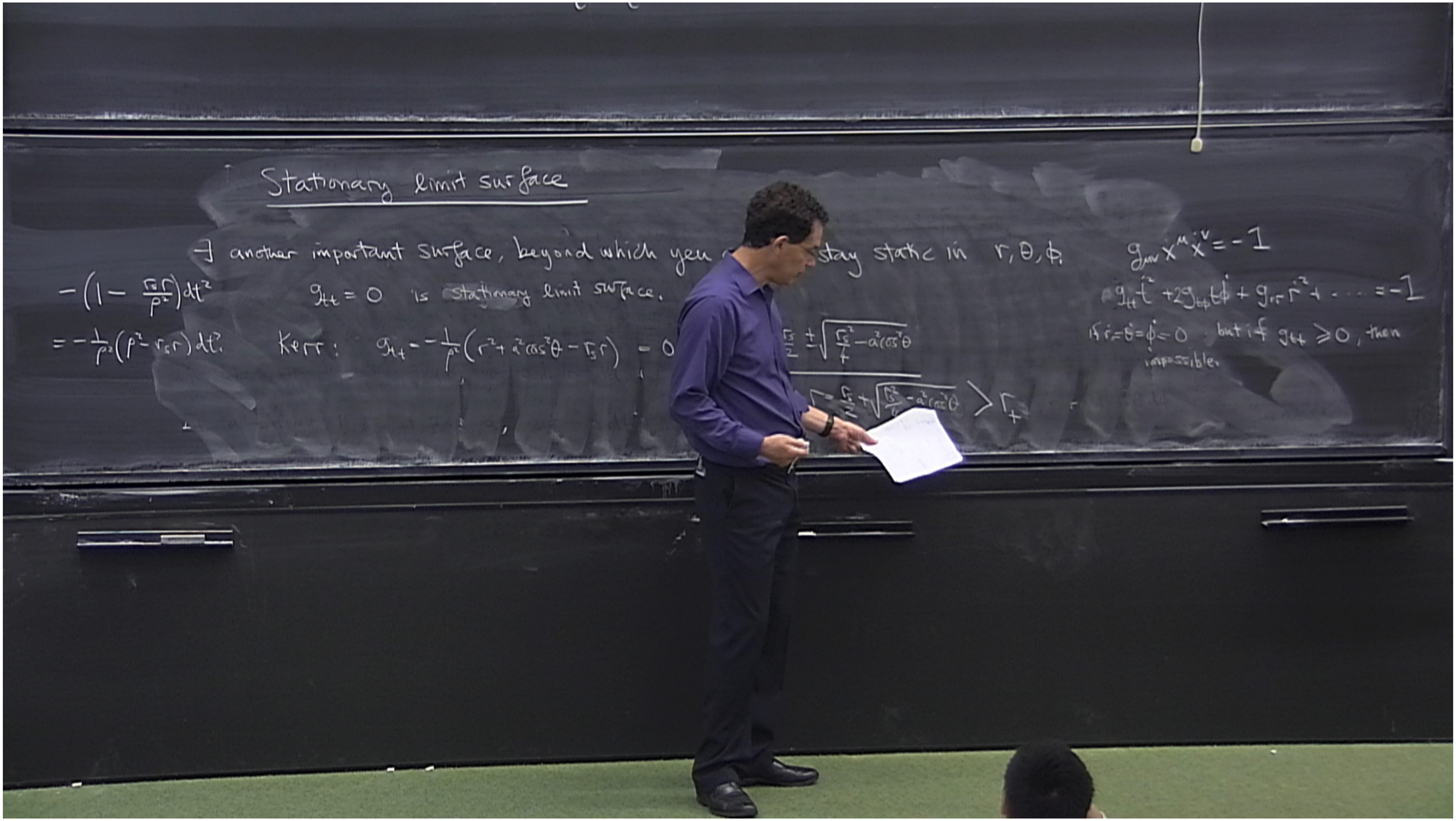
$$g_{tt}\dot{t}^{2} + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^{2} + \dots = -1$$

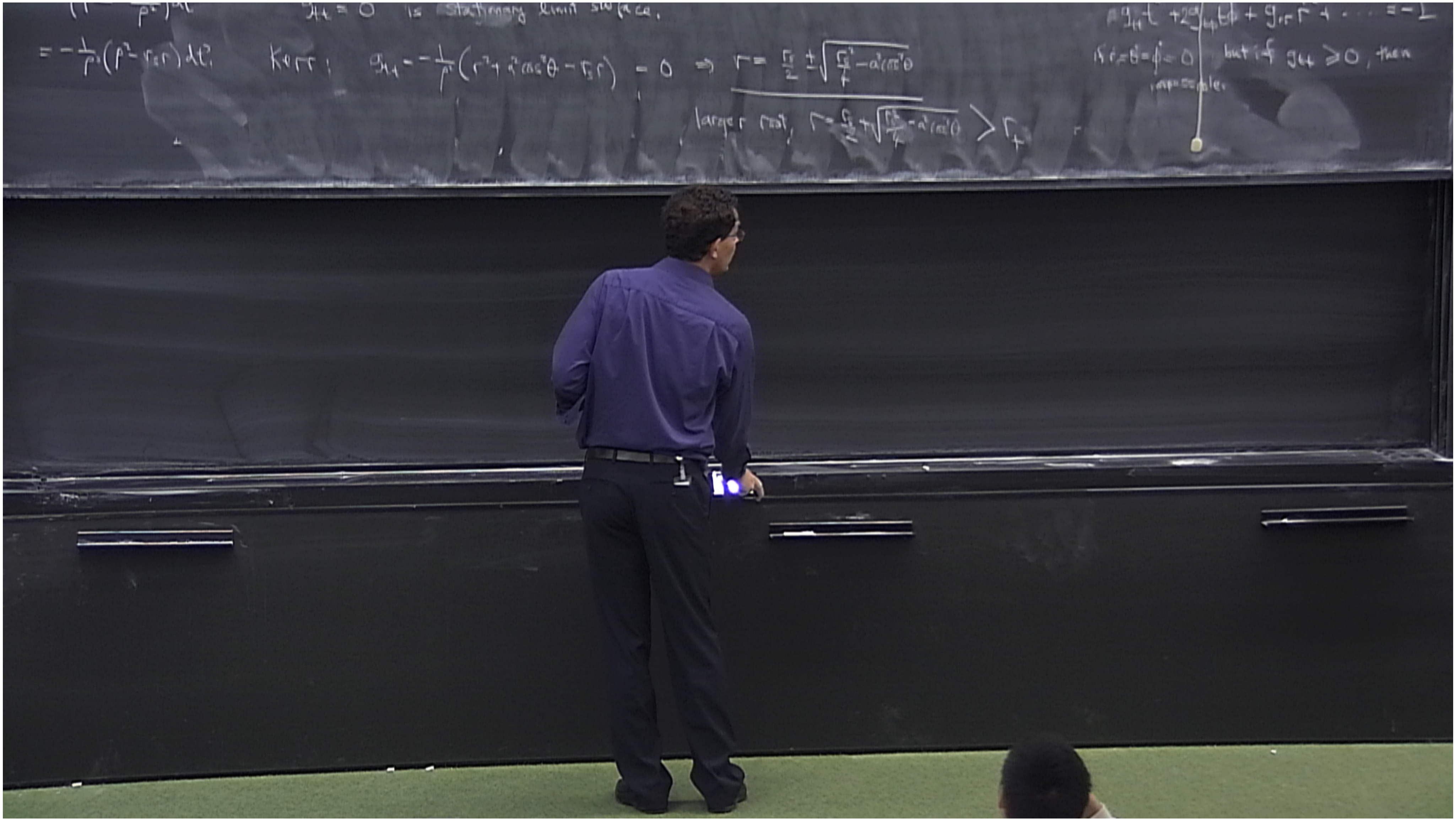
if $\dot{r} = \dot{\theta} = \dot{\phi} = 0$

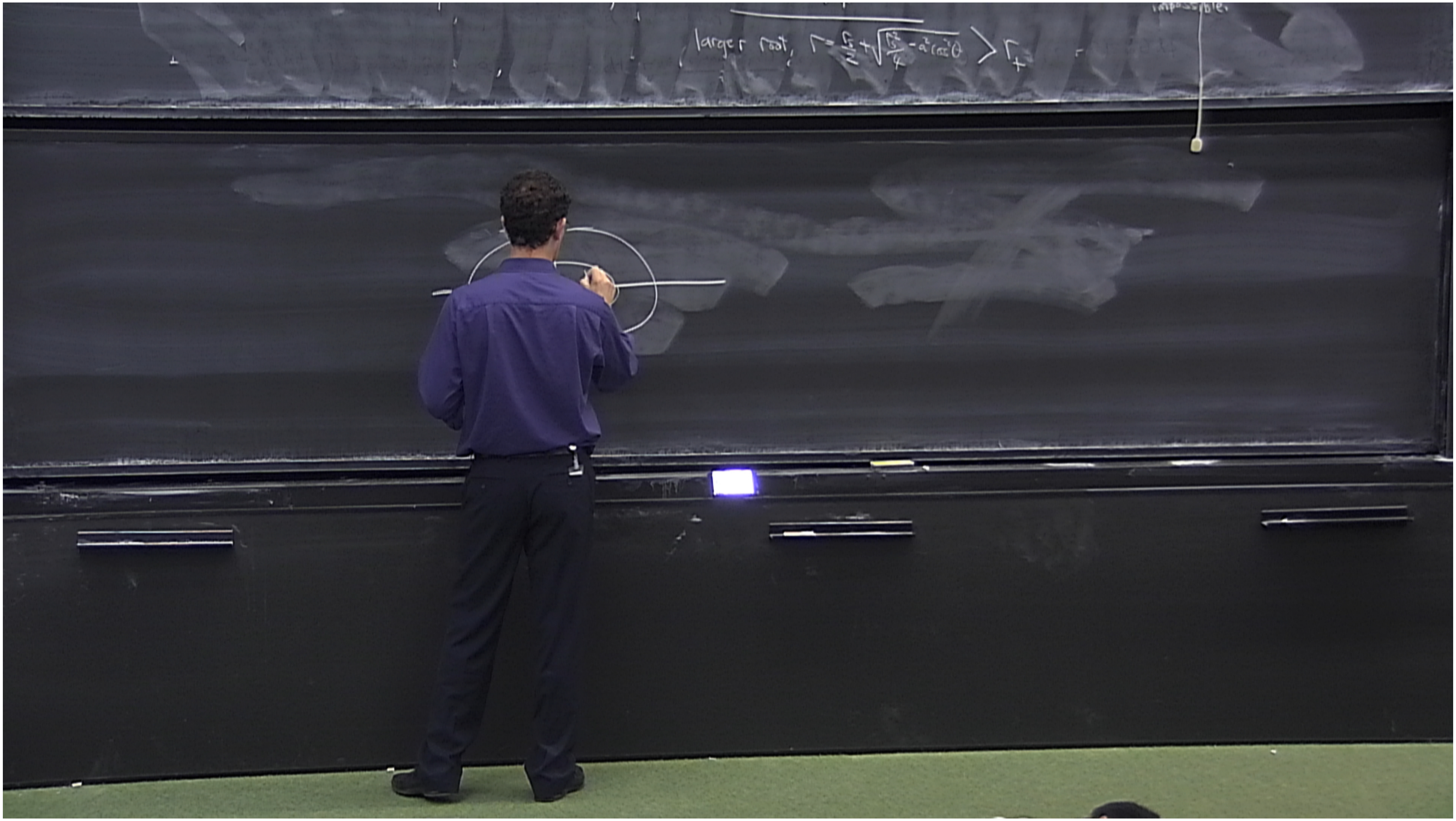

static in r, θ, φ.
$g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = -1$
$g_{tt}\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^2 + \ldots = -1$
if $\dot{r} = \dot{\theta} = \dot{\phi} = 0$; but if $g_{tt} \geq 0$, then
impossible.

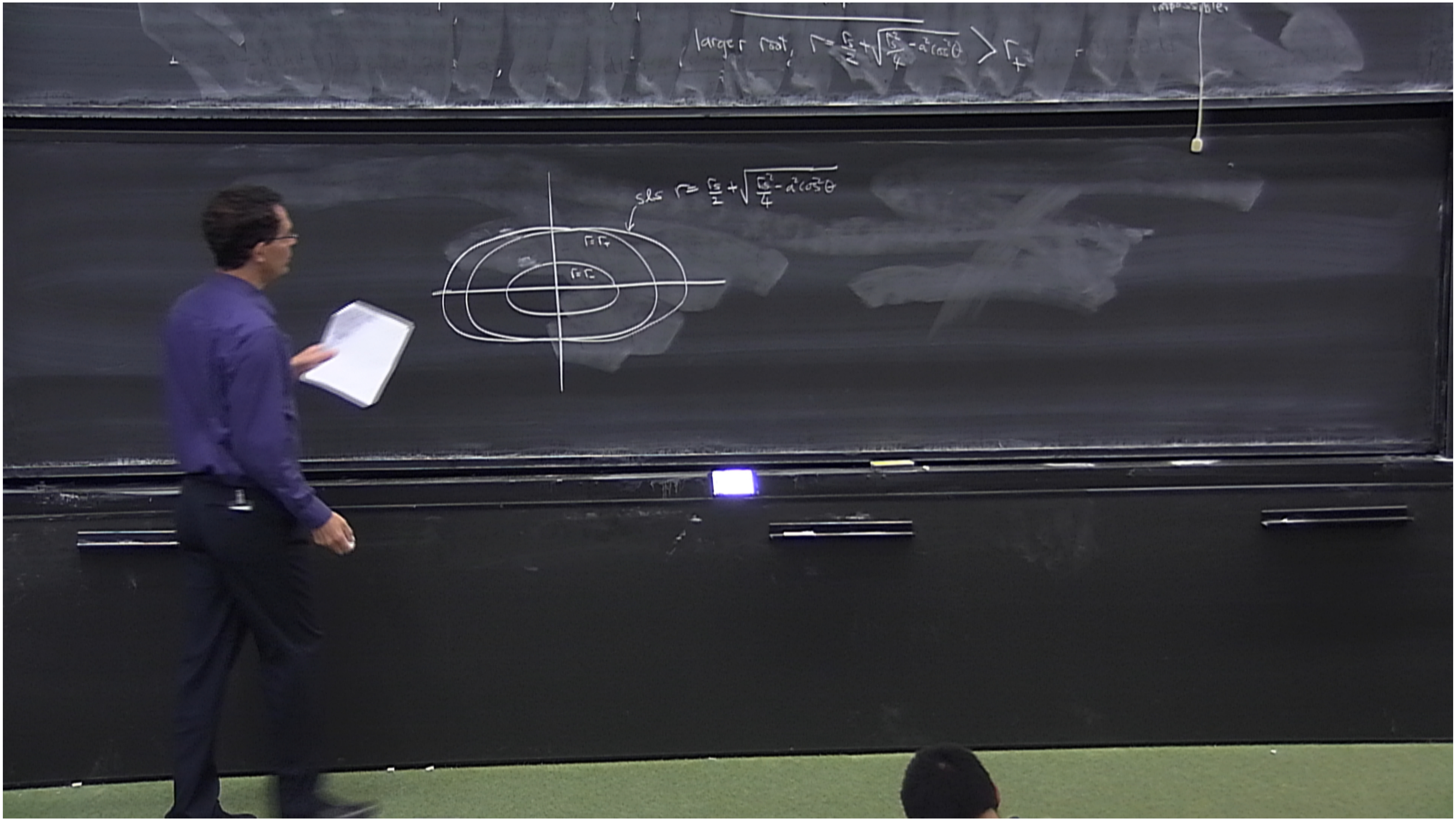

static in r, θ, φ.
$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$
$g_{tt}\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^2 + \dots = -1$
if $\dot{r} = \dot{\theta} = \dot{\phi} = 0$; but if $g_{tt} \geq 0$, then
impossible.


Stationary limit surface
you cannot stay static in r, θ, φ.
stationary limit
impossible.

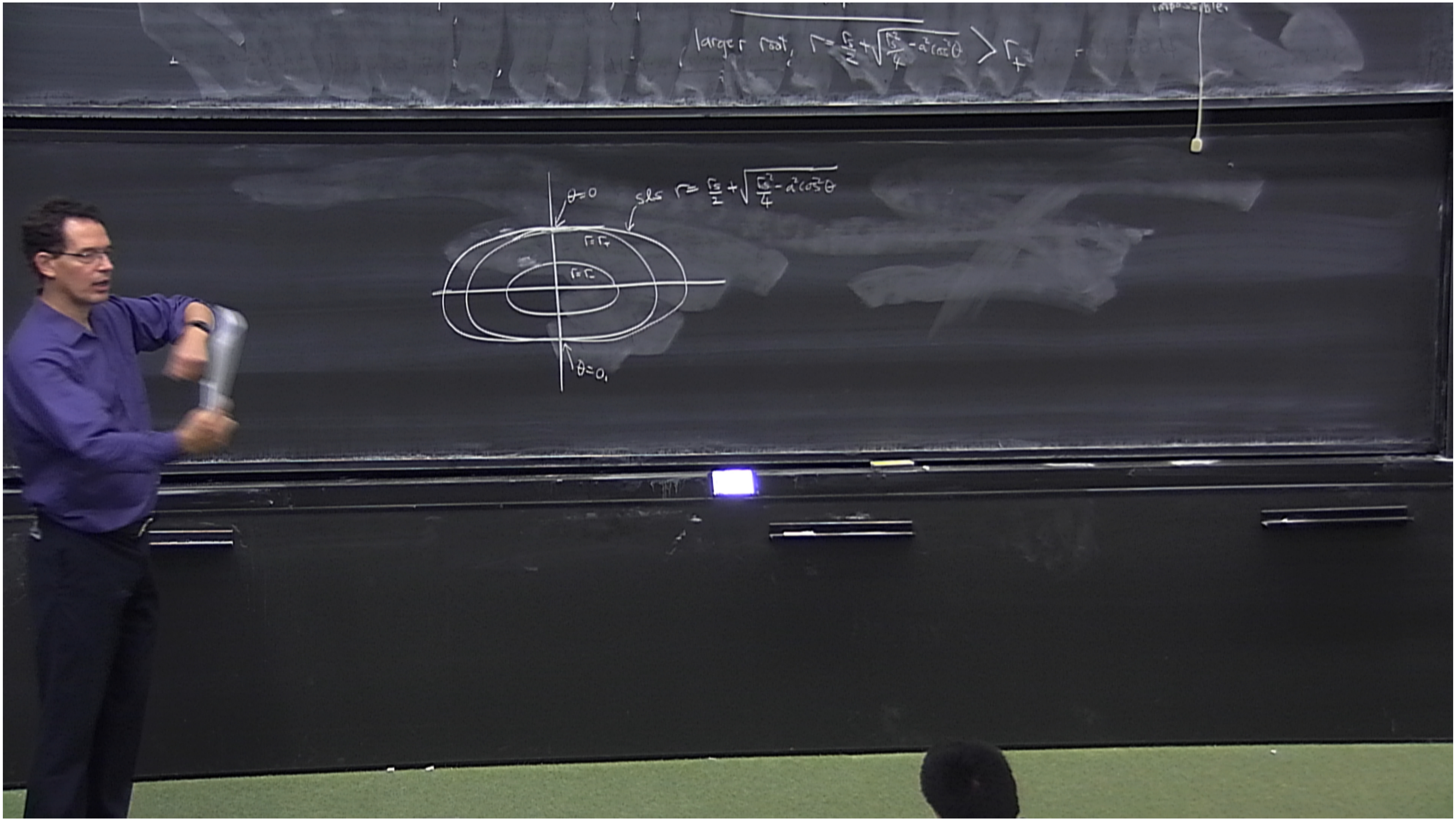

Stationary limit surface
∃ another important surface, beyond which you
stay static in r, θ, φ.
g_{tt} = 0 is stationary limit surface.
Kerr:
impossible.

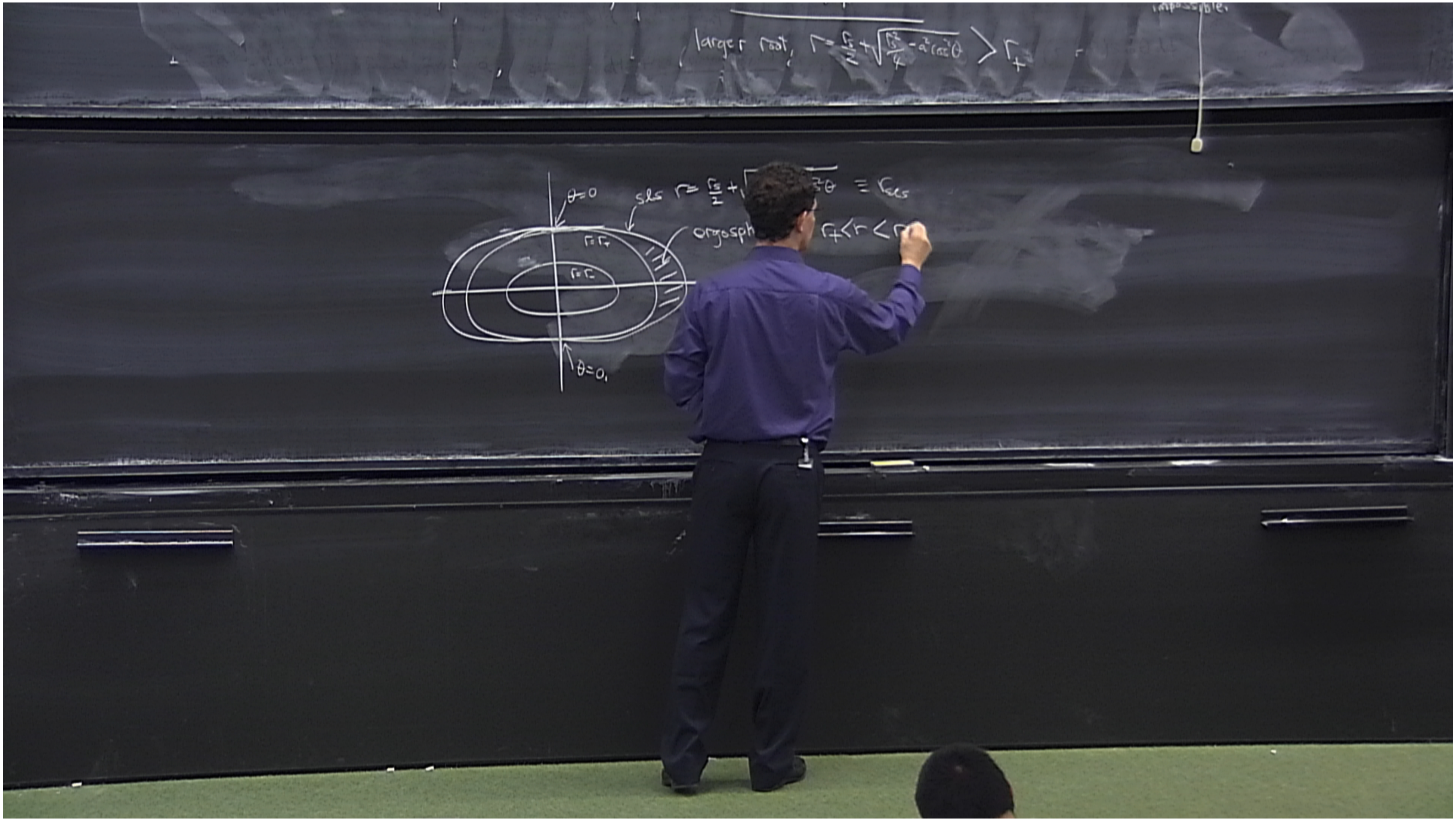

is stationary limit surface.
Kerr:
larger r root,
but if $g_{tt} \geq 0$, then
impossible.

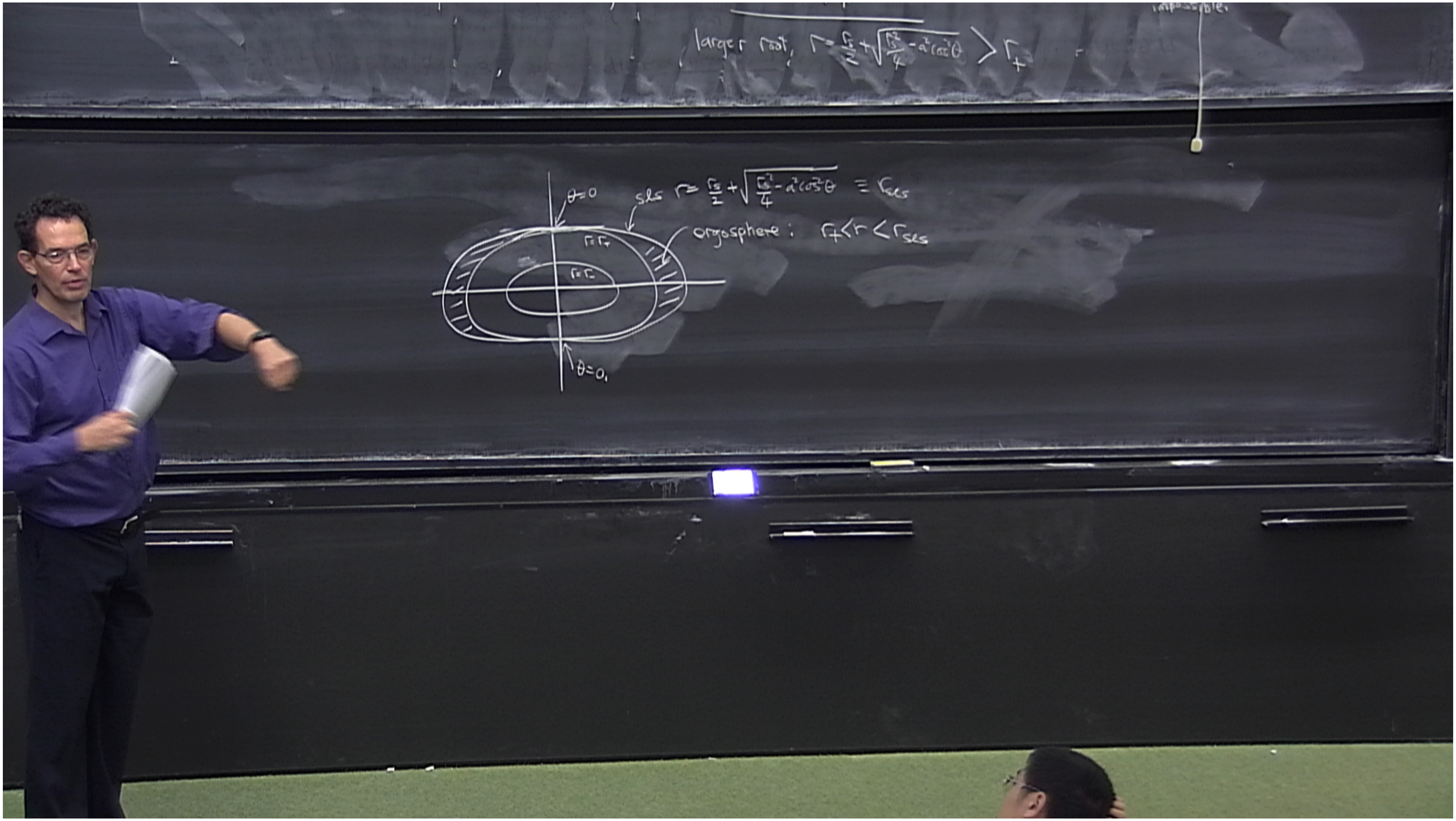

larger root, $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} > r_+$

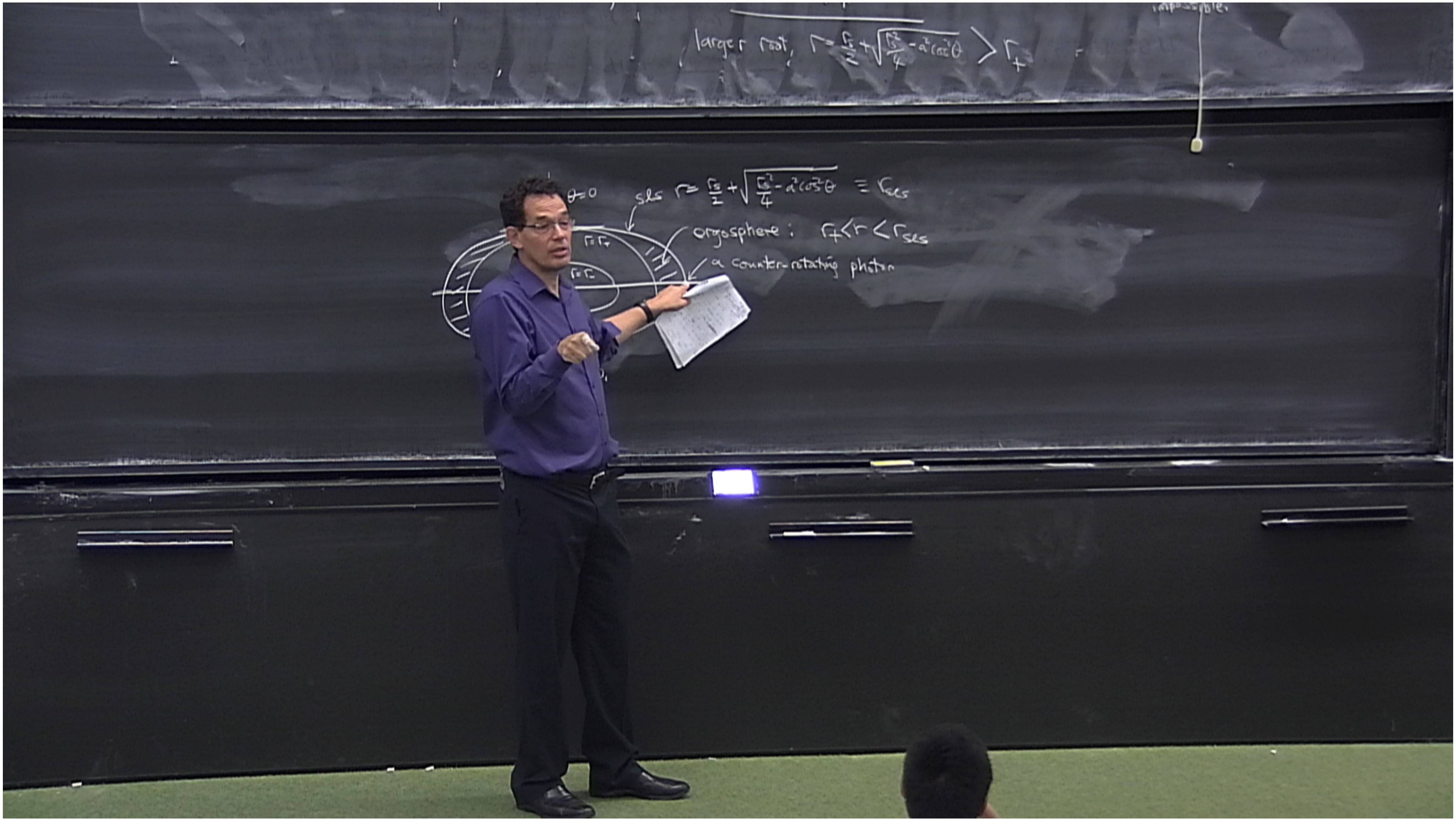


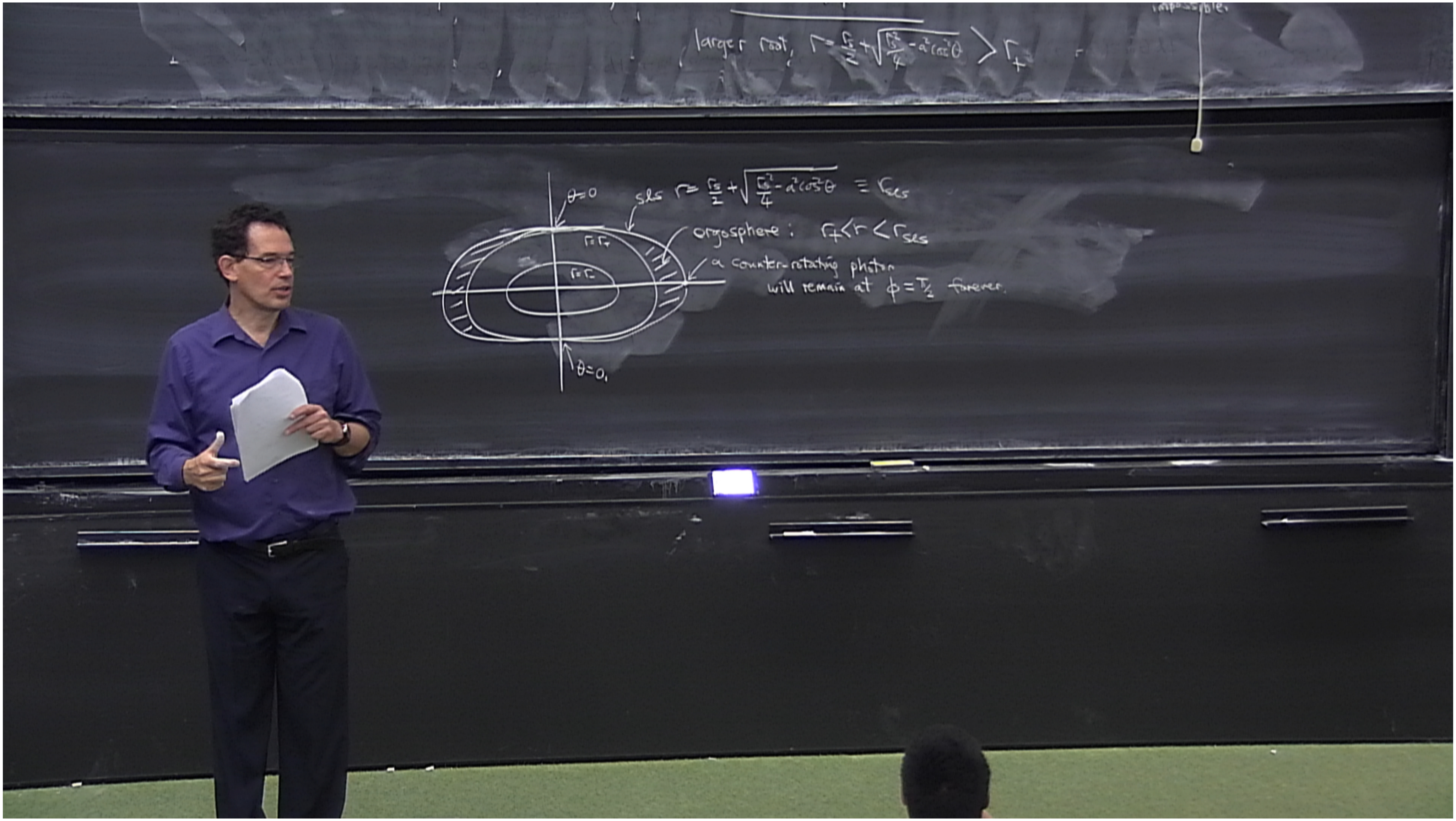
larger root, $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} > r_+$
SLS $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta}$
$r = r_+$
$r = r_-$

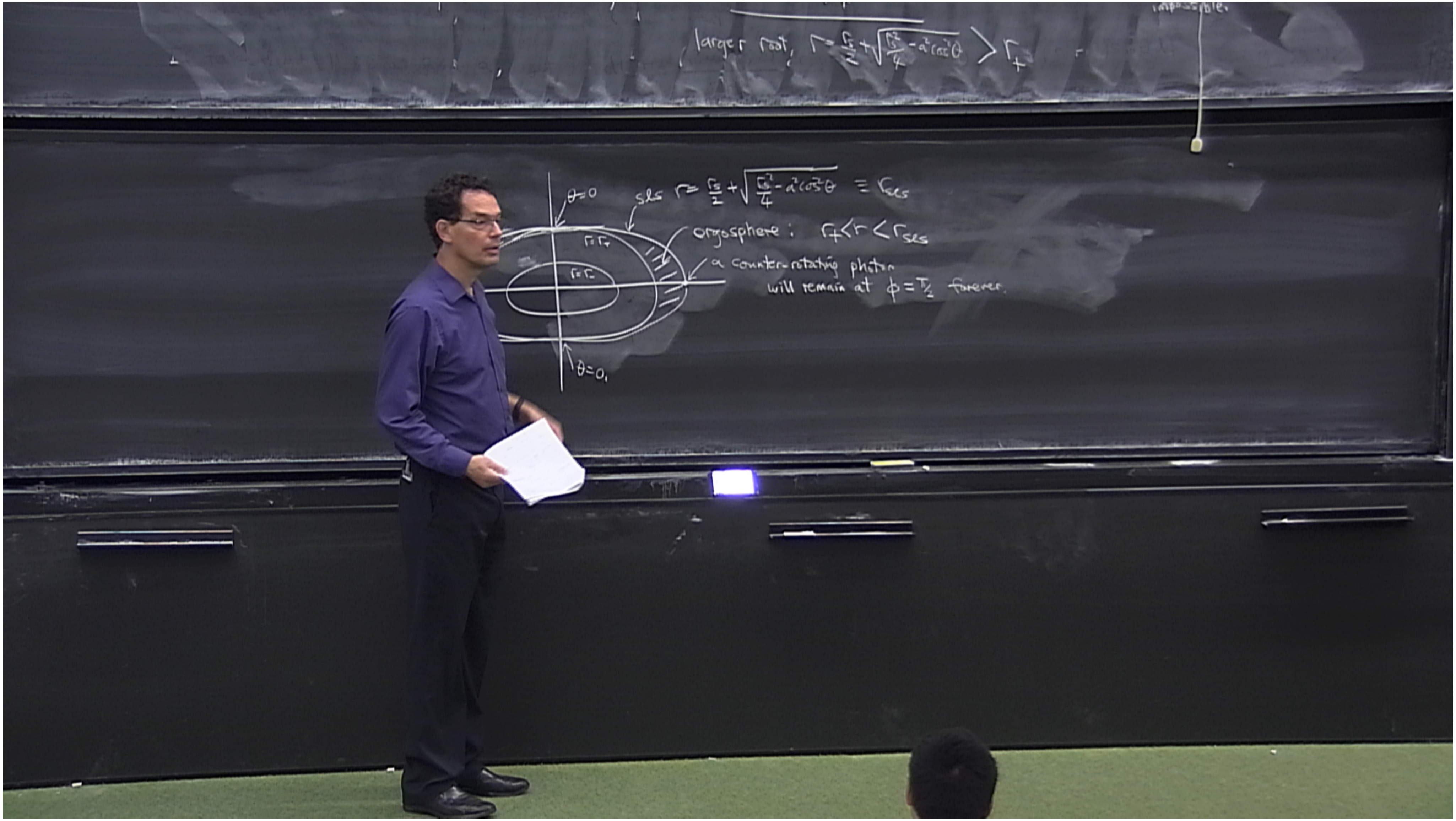

larger root, $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} > r_+$
$\theta = 0$
sls $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta}$
$r = r_+$
$r = r_-$
$\theta = 0.$

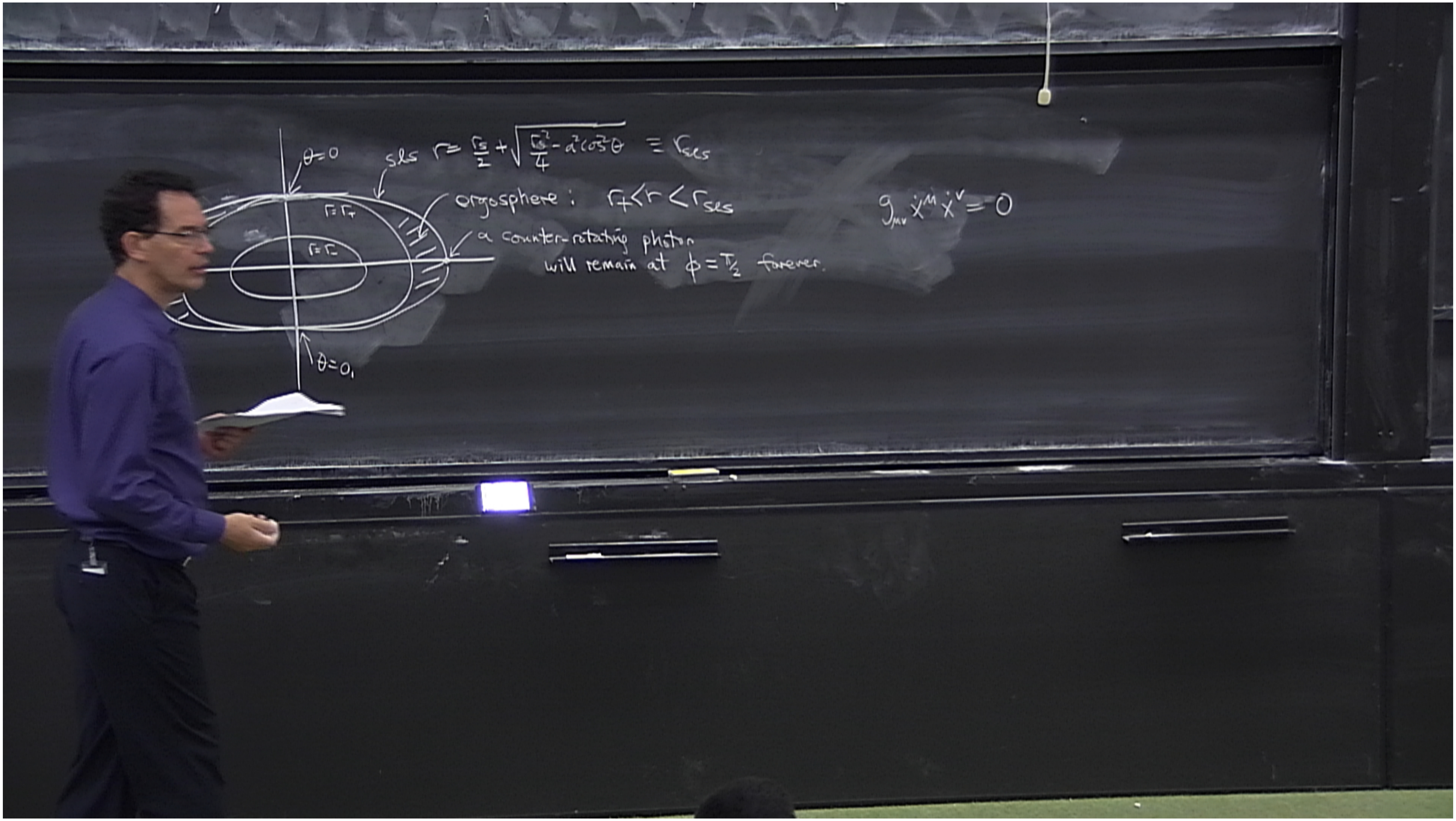

larger root, $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} > r_+$
$\theta = 0$
sls
$\equiv r_{sls}$
ergosph
$r_+ < r <$
$r = r_+$
$r = r_-$
$\theta = 0$.

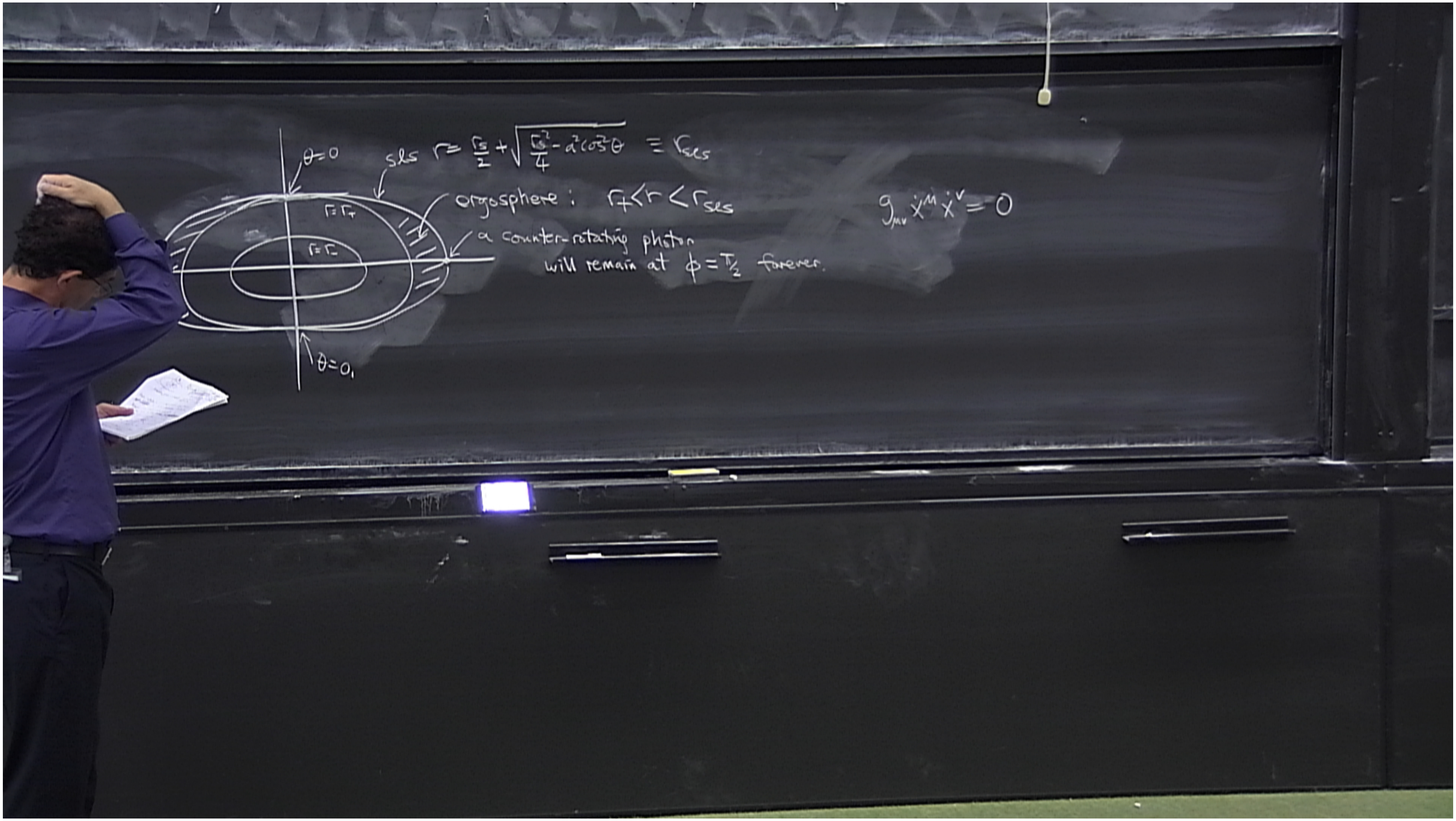

larger root, $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} > r_+$
sls $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} \equiv r_{sls}$
ergosphere : $r_+ < r < r_{sls}$
$\theta = 0$
$r = r_+$
$r = r_-$
$\theta = 0$.

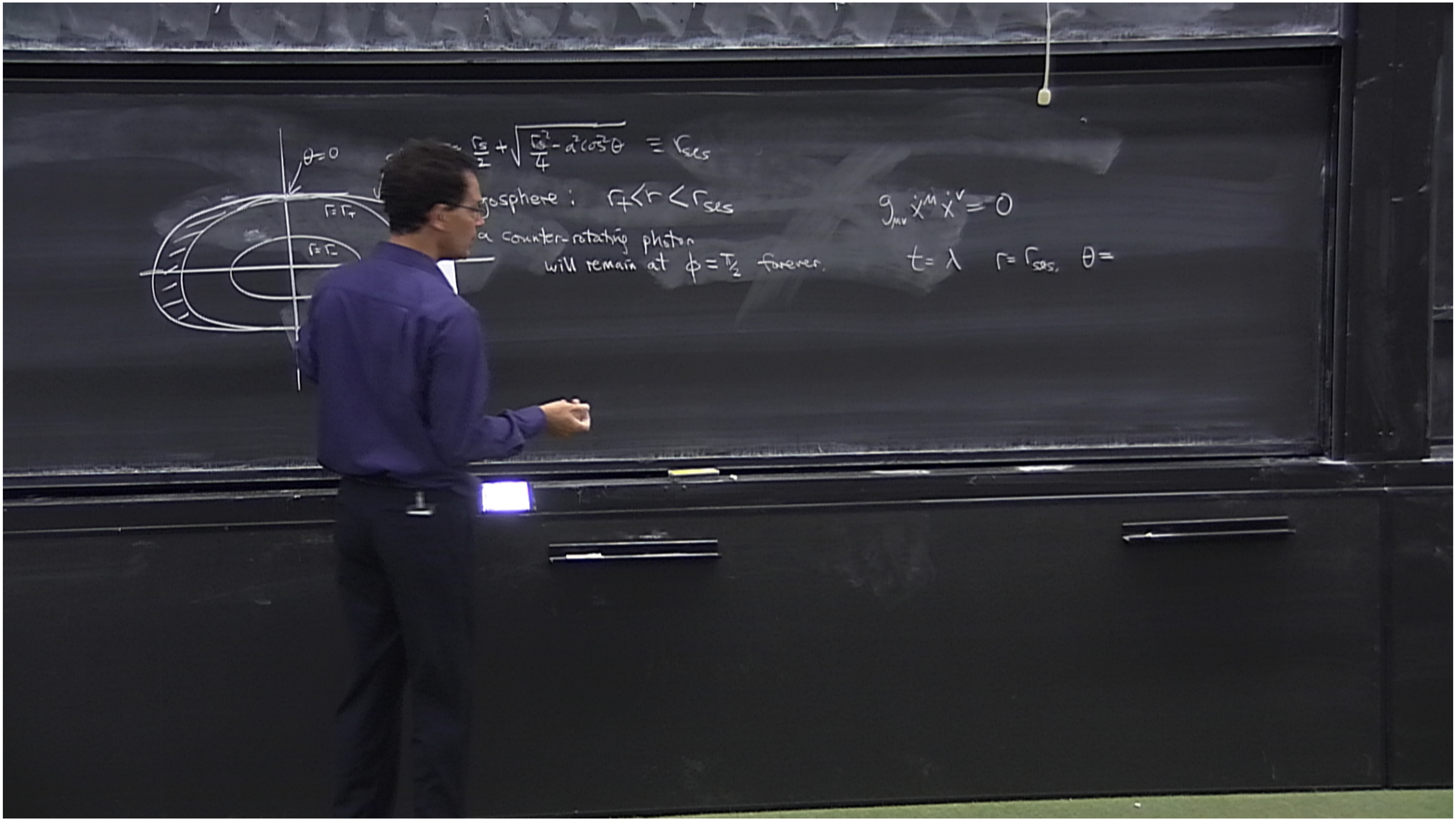

larger r root, $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} > r_+$
$\theta = 0$
sls $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} \equiv r_{sls}$
ergosphere : $r_+ < r < r_{sls}$
a counter-rotating photon
$r = r_+$
$r = r_-$


larger root, $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} > r_+$
$\theta=0$
sls $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} \equiv r_{sls}$
ergosphere : $r_+ < r < r_{sls}$
$r=r_+$
$r=r_-$
a counter-rotating photon
will remain at $\phi = \pi/2$ forever.
$\theta=0$.

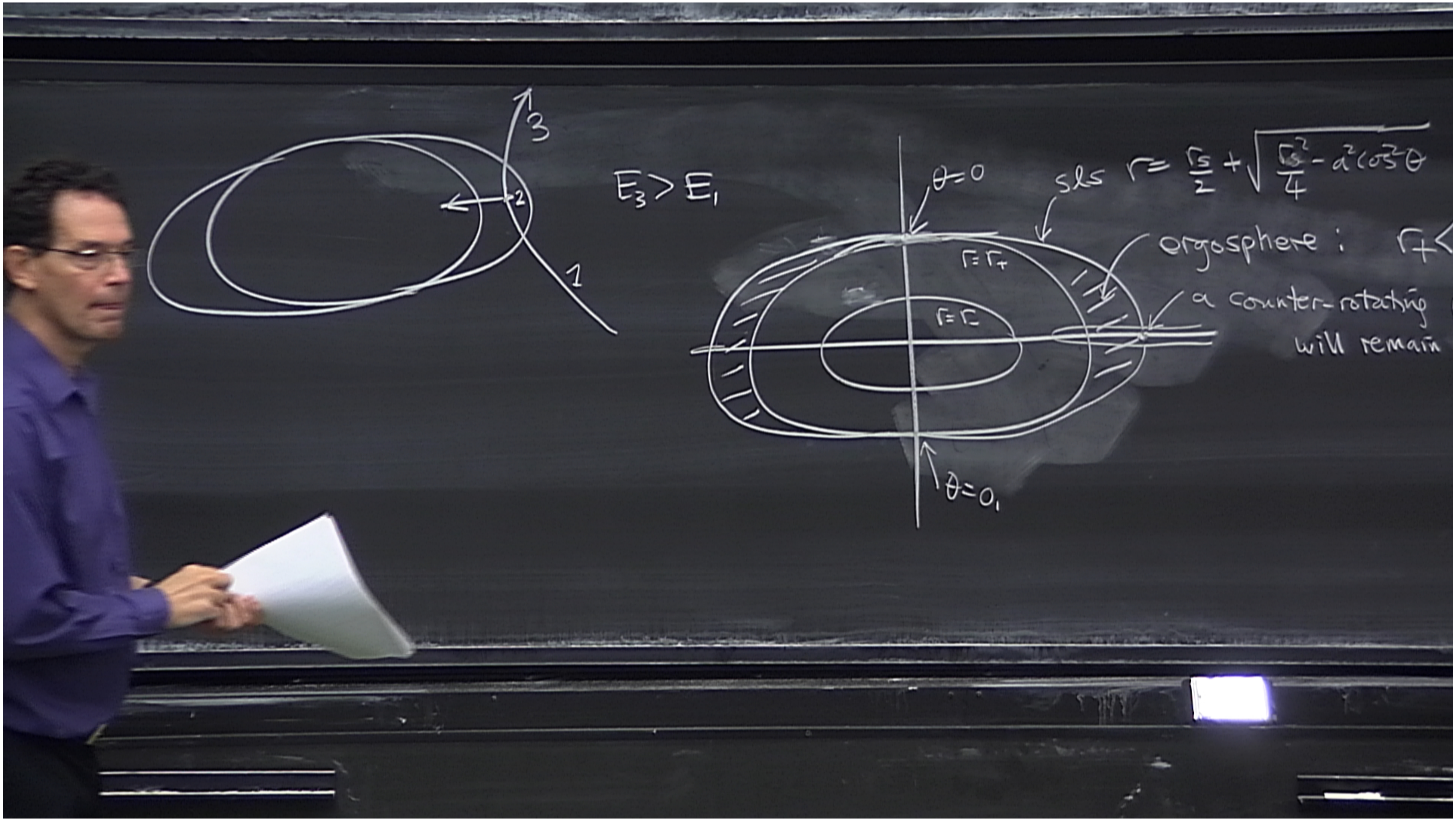

larger root, r = r_s/2 + √(r_s²/4 − a²cos²θ) > r_+
sls r = r_s/2 + √(r_s²/4 − a²cos²θ) ≡ r_sls
θ=0
r=r_+
r=r_-
ergosphere : r_+ < r < r_sls
a counter-rotating photon
will remain at φ = π/2 forever.
θ=0.


θ=0
sls $r = \frac{r_s}{2} + \sqrt{\frac{r_s^2}{4} - a^2\cos^2\theta} \equiv r_{sls}$
ergosphere : $r_+ < r < r_{sls}$
$g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = 0$
$r = r_+$
a counter-rotating photon
$r = r_-$
will remain at $\phi = \pi/2$ forever.
θ=0

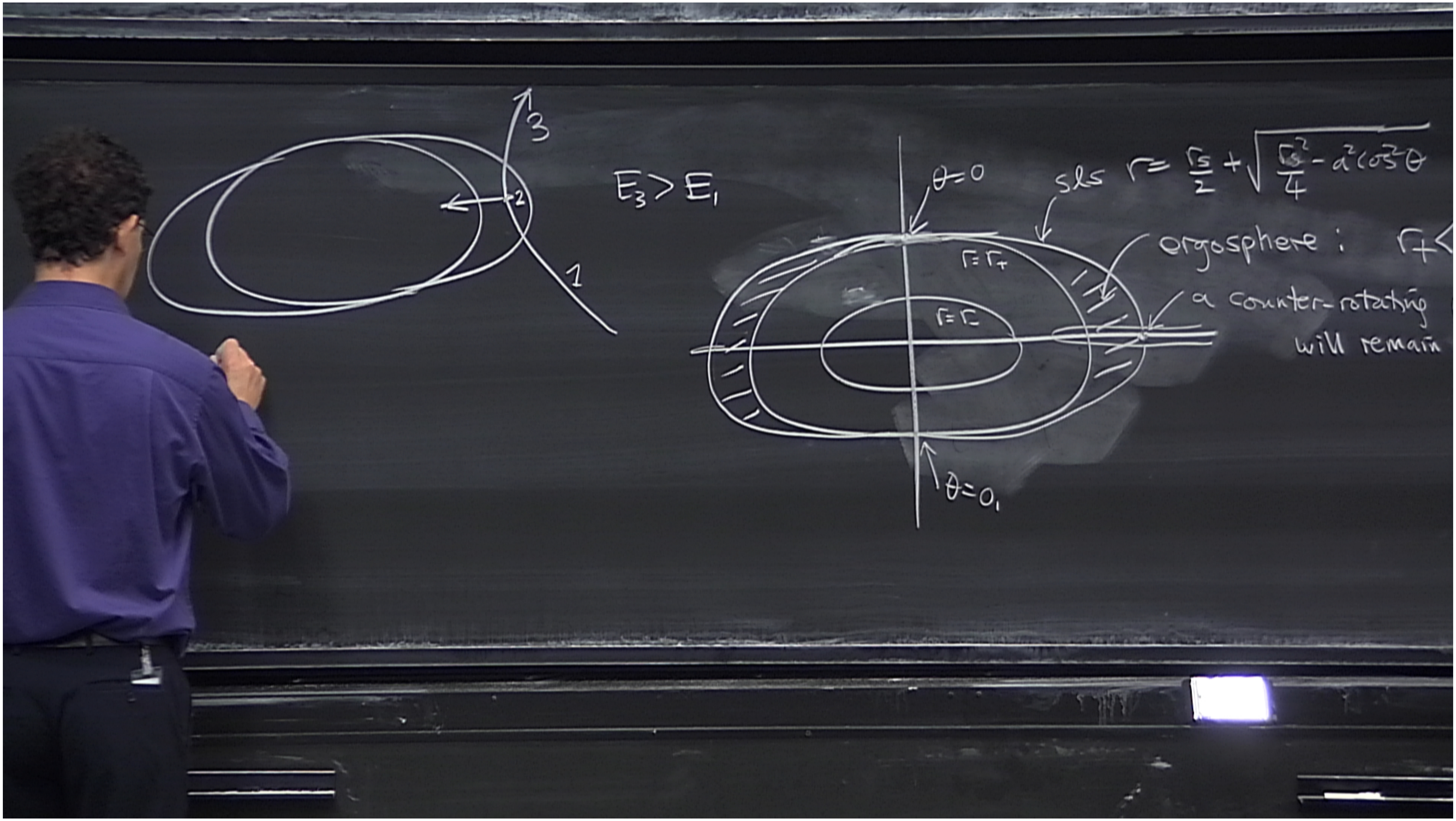

θ=0
sls r = rs/2 + √(rs²/4 − a²cos²θ) ≡ rses
orgosphere : r+ < r < rses
r=r+
r=r−
a counter-rotating photon
will remain at φ = π/2 forever.
gμν ẋ^μ ẋ^ν = 0
θ=0.



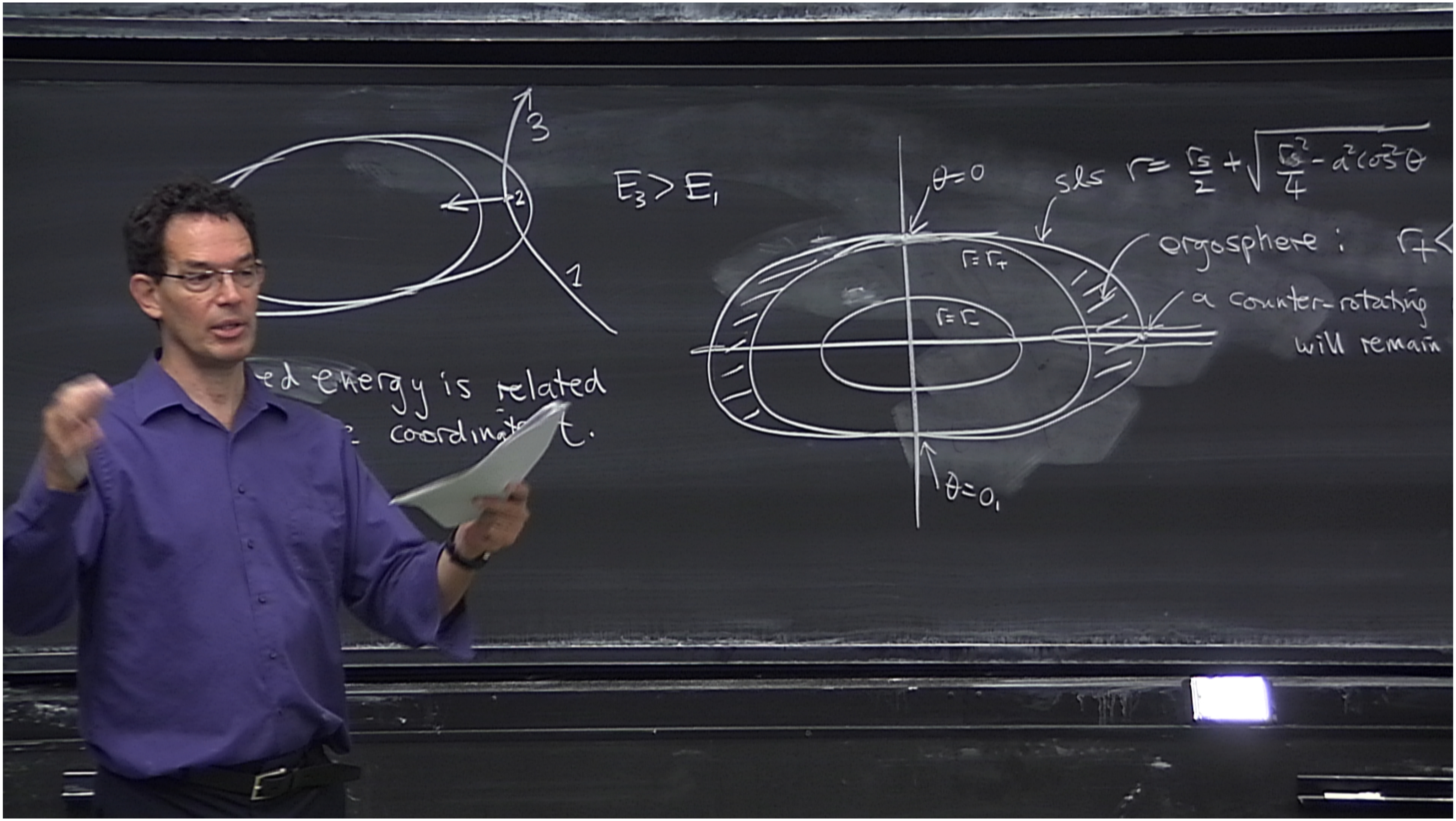
θ=0
r=r+
r=r−
= rses
gosphere : r+<r<rses
a counter-rotating photon
will remain at φ=π/2 forever.
gμν ẋμ ẋν = 0
t=λ r=rses. θ=

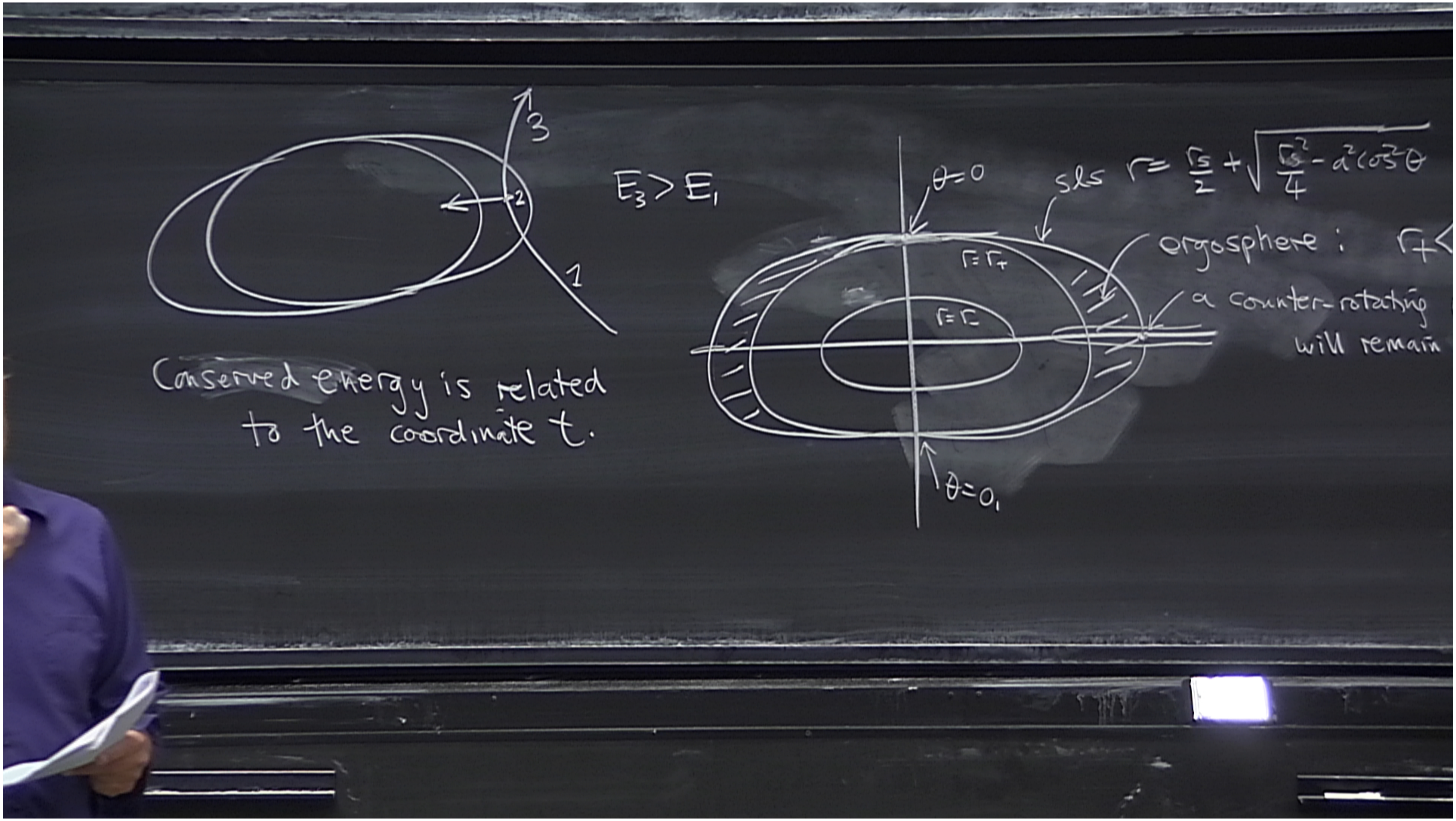


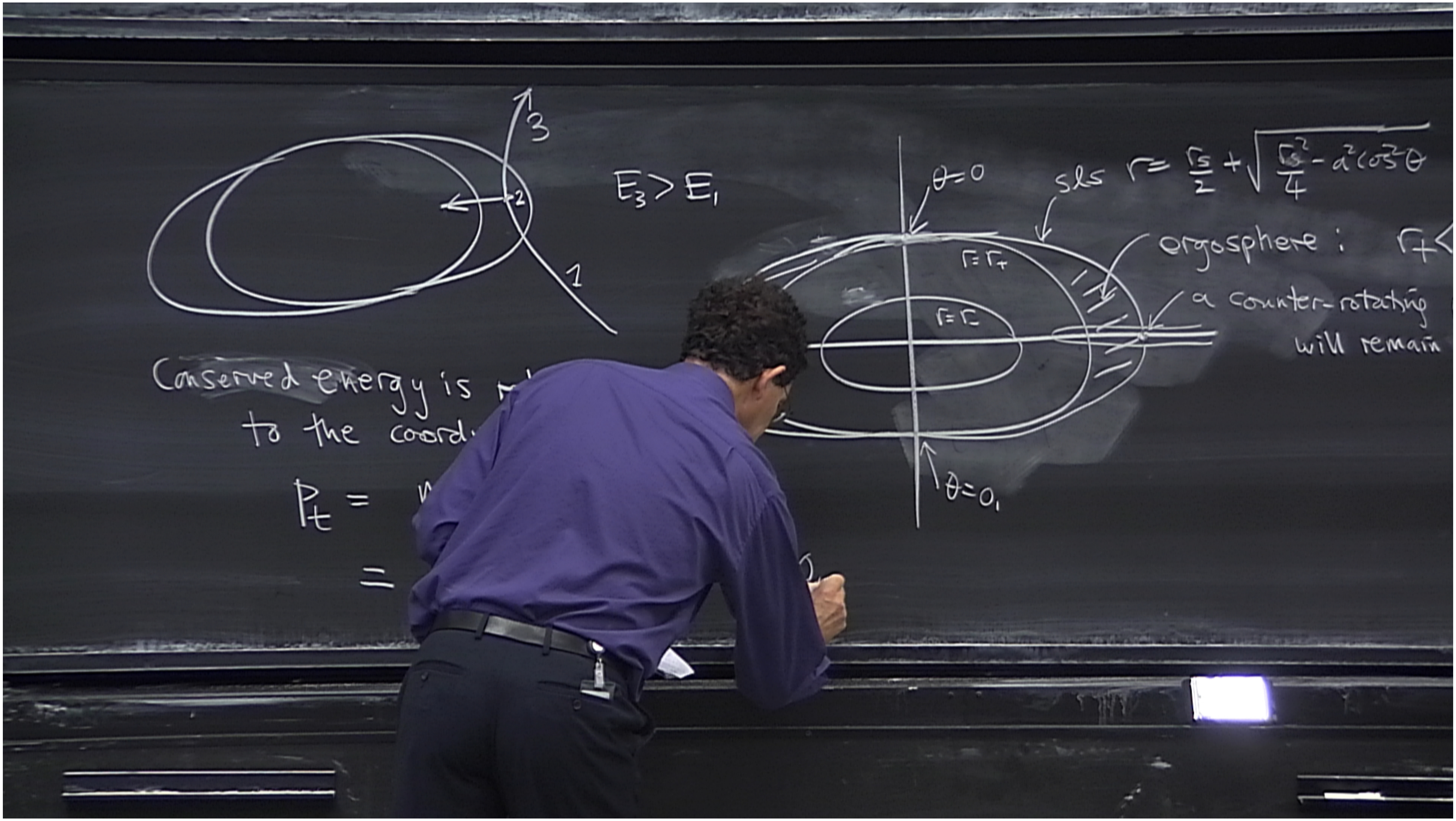
3
2
1
E3 > E1
θ=0
sls
r = rs/2 + √(rs²/4 − a²cos²θ)
ergosphere : r+ <
r = r+
r = r−
a counter-rotating
will remain
θ=0



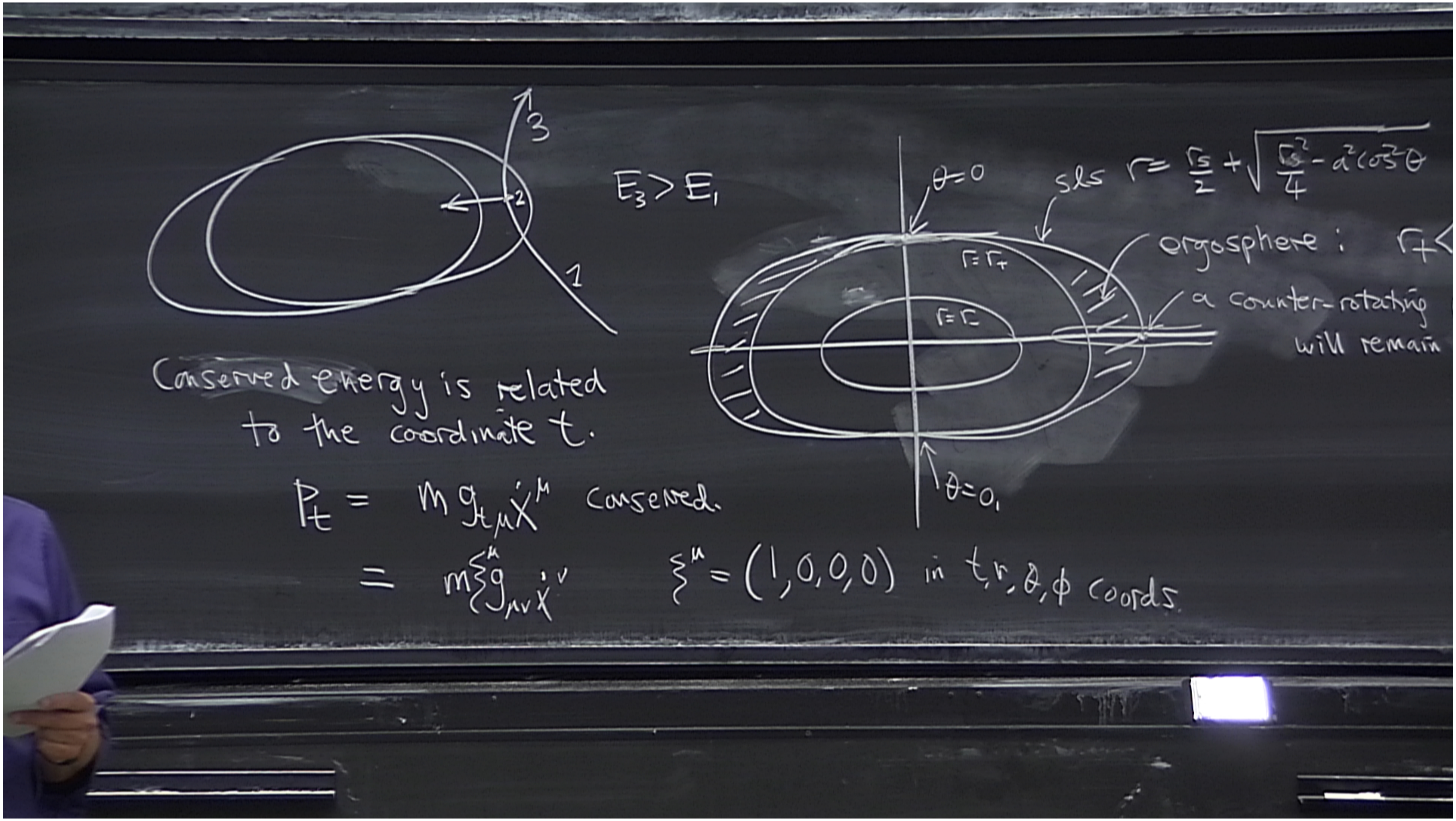
3
2
1
E₃ > E₁
θ=0
sls r = rs/2 + √(rs²/4 − a²cos²θ)
ergosphere : r₊ <
a counter-rotating
will remain
r = r₊
r = r₋
θ = 0₁

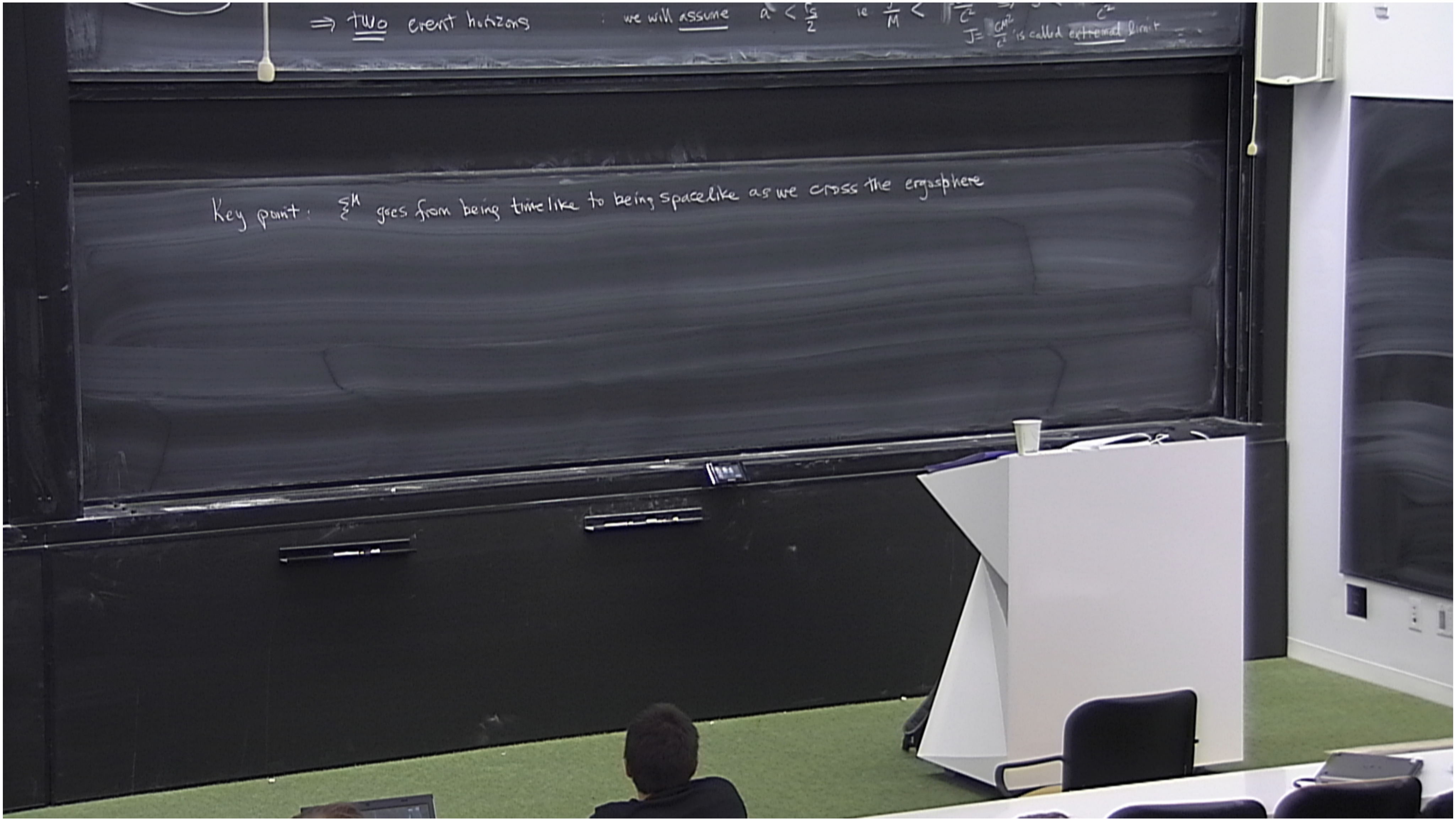

3
2
1
E3 > E1
ed energy is related
coordinate t.
θ=0
sls r = rs/2 + √(rs²/4 − a²cos²θ)
ergosphere : r+ <
r = r+
r = r−
a counter-rotating
will remain
θ=0

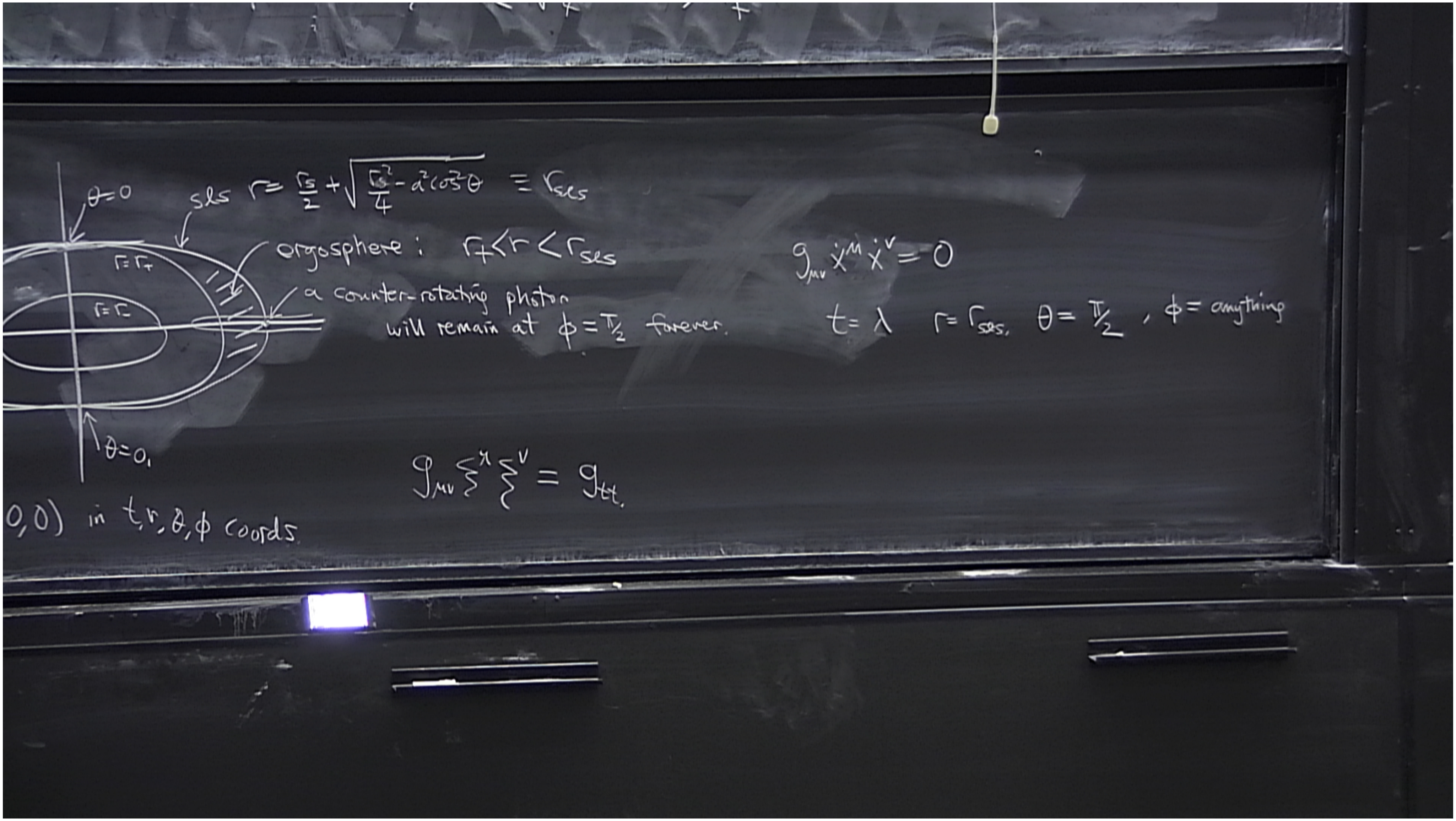

3
2
1
E3 > E1
Conserved energy is related to the coordinate t.
θ=0
s|s
r = rs/2 + √(rs²/4 − a²cos²θ)
r=r+
r=r−
ergosphere : r+<
a counter-rotating
will remain
θ=0,

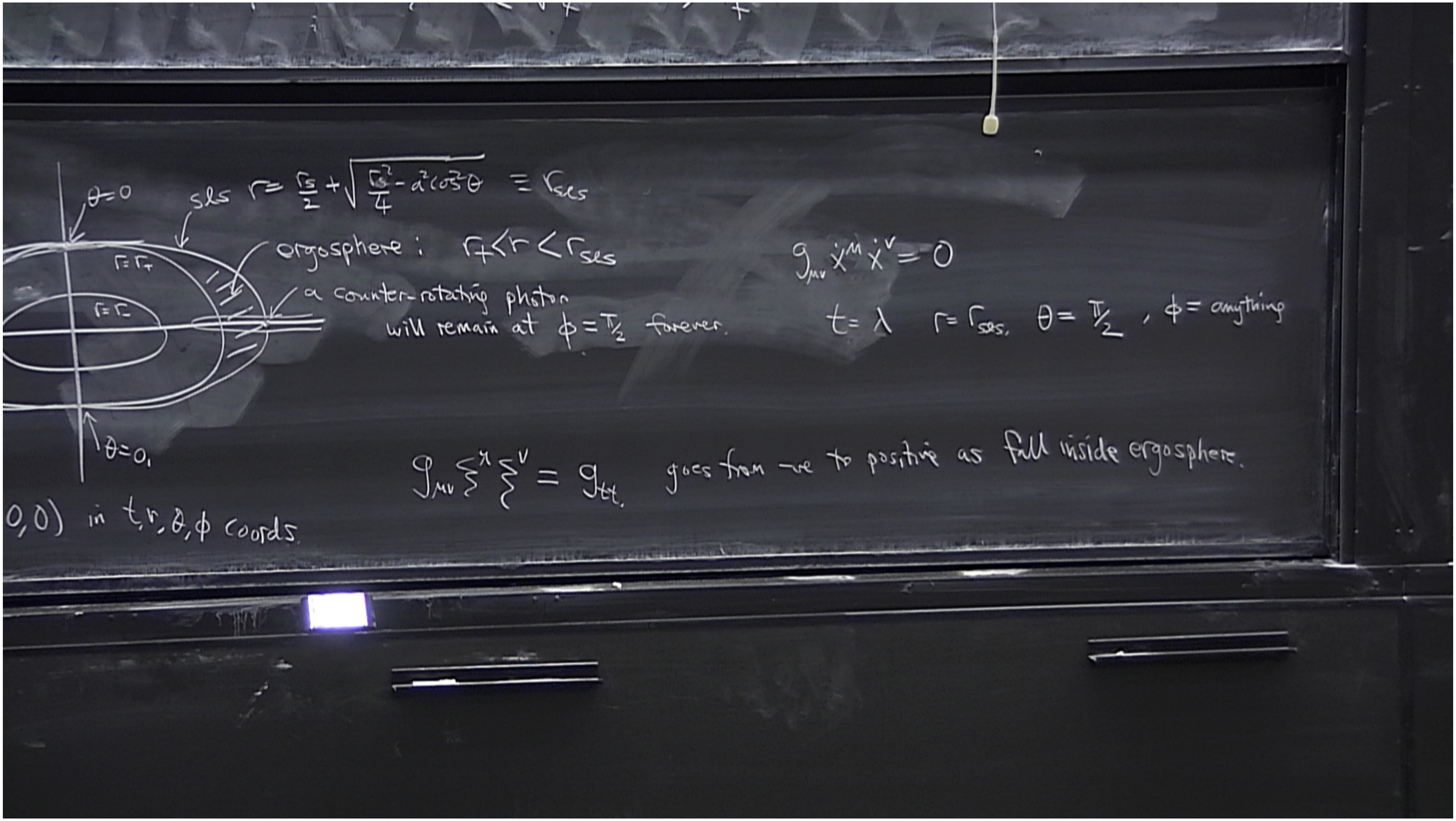


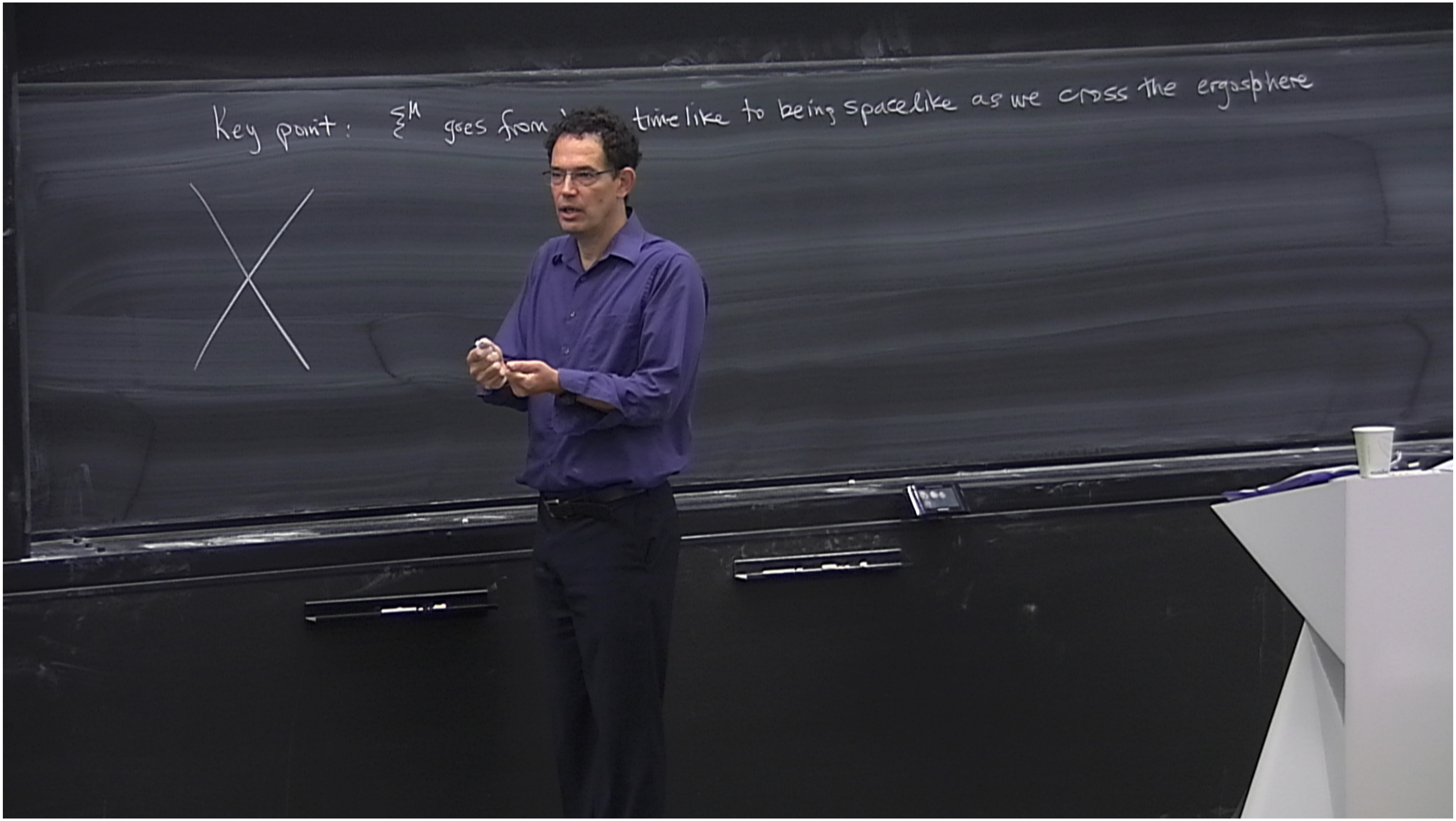
3
2
1
E3 > E1
θ=0
sls r = rs/2 + √(rs²/4 − a²cos²θ)
ergosphere : r+ <
a counter-rotating
will remain
r=r+
r=r−
θ=0,
Conserved energy is
to the coord
Pt =
=

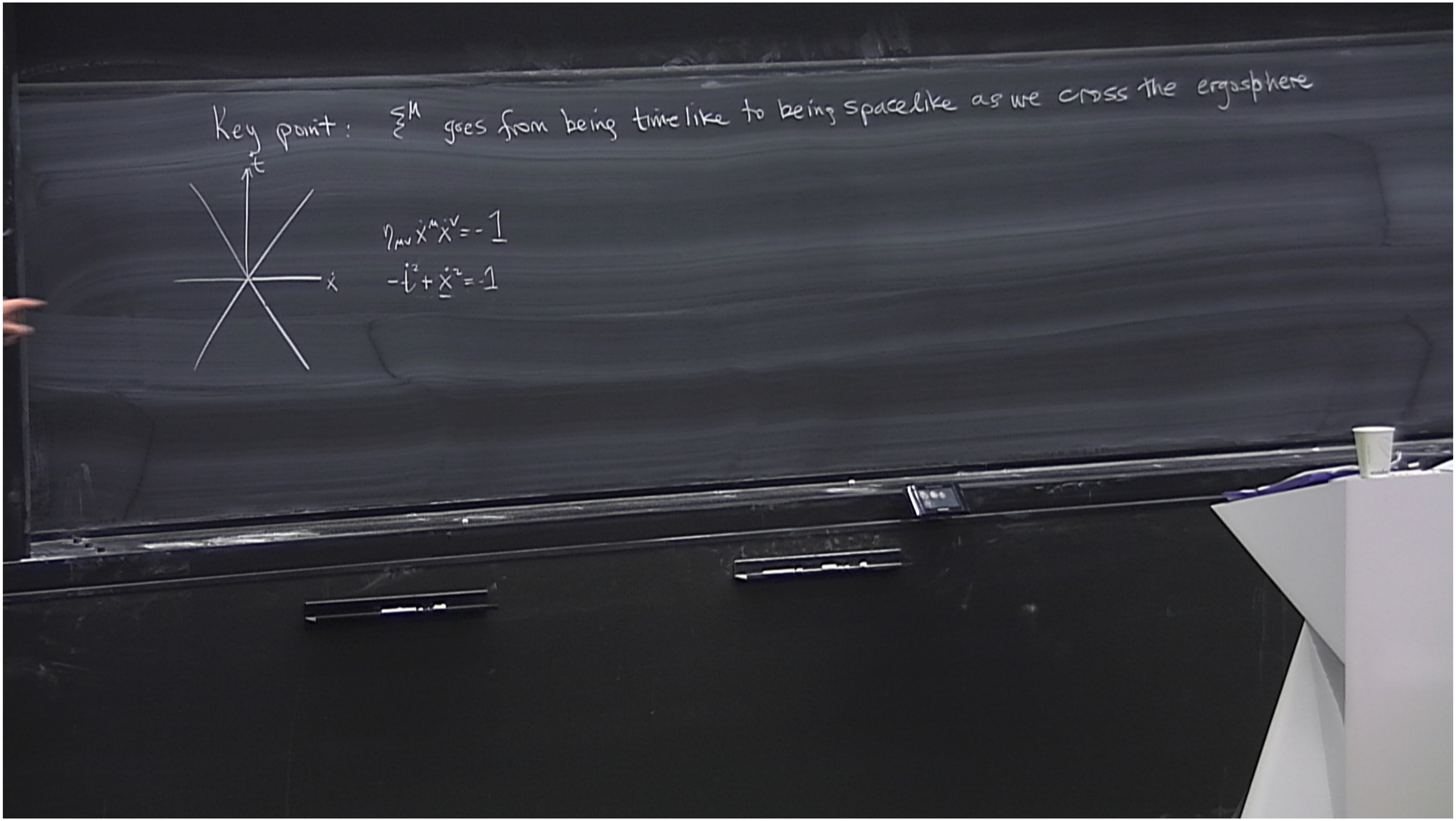

3
2
1
E3 > E1
Conserved energy is related to the coordinate t.
Pt = m g_tμ ẋ^μ conserved.
= m ξ^μ g_μν ẋ^ν
ξ^μ = (1,0,0,0) in t,r,θ,φ coords.
θ=0
s/s r = rs/2 + √(rs²/4 − a²cos²θ)
r=r+
r=r−
ergosphere : r+<
a counter-rotating
will remain
θ=0,

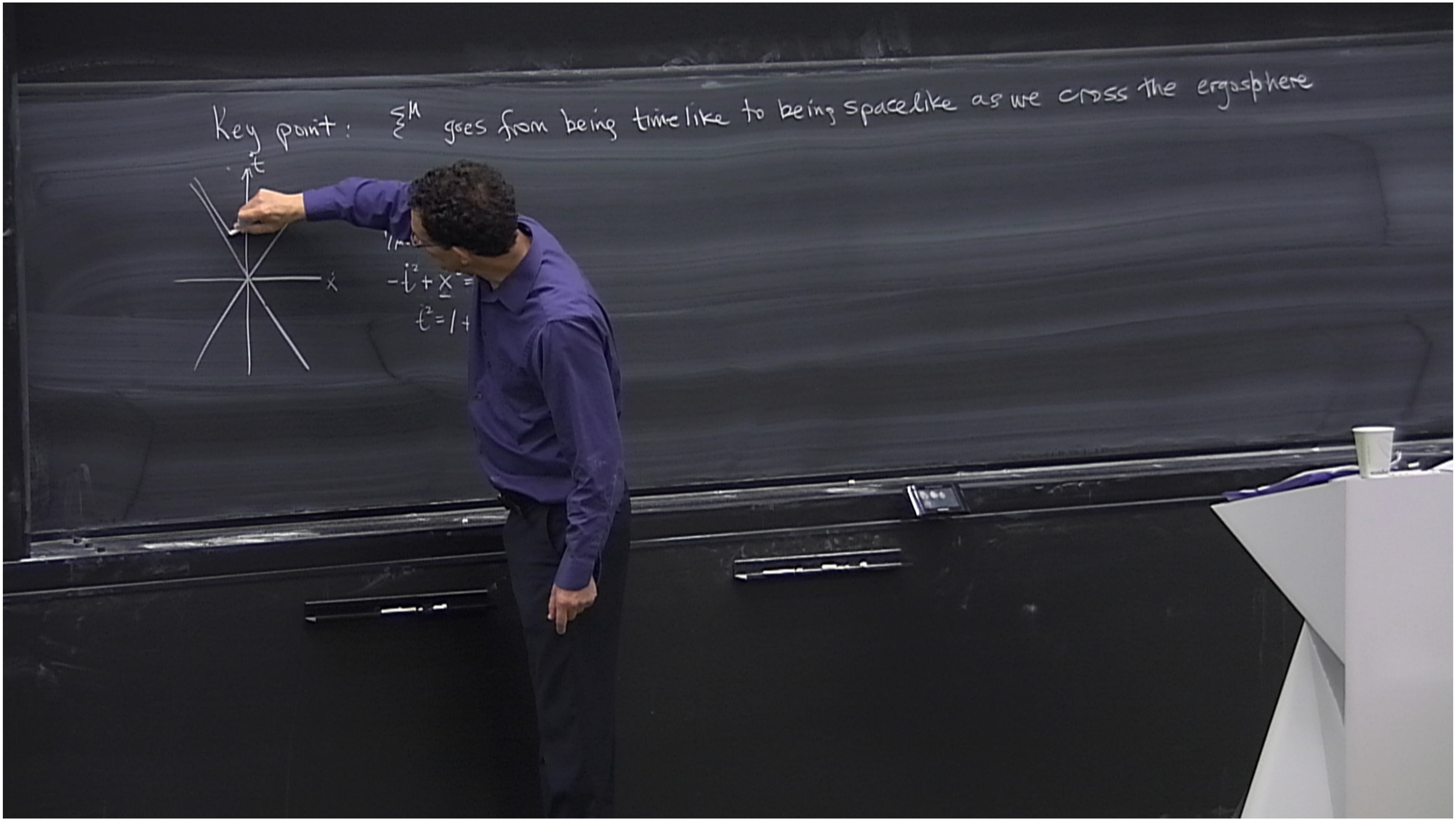

⇒ two event horizons
we will assume
J= GM²/c² is called extremal limit
Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere

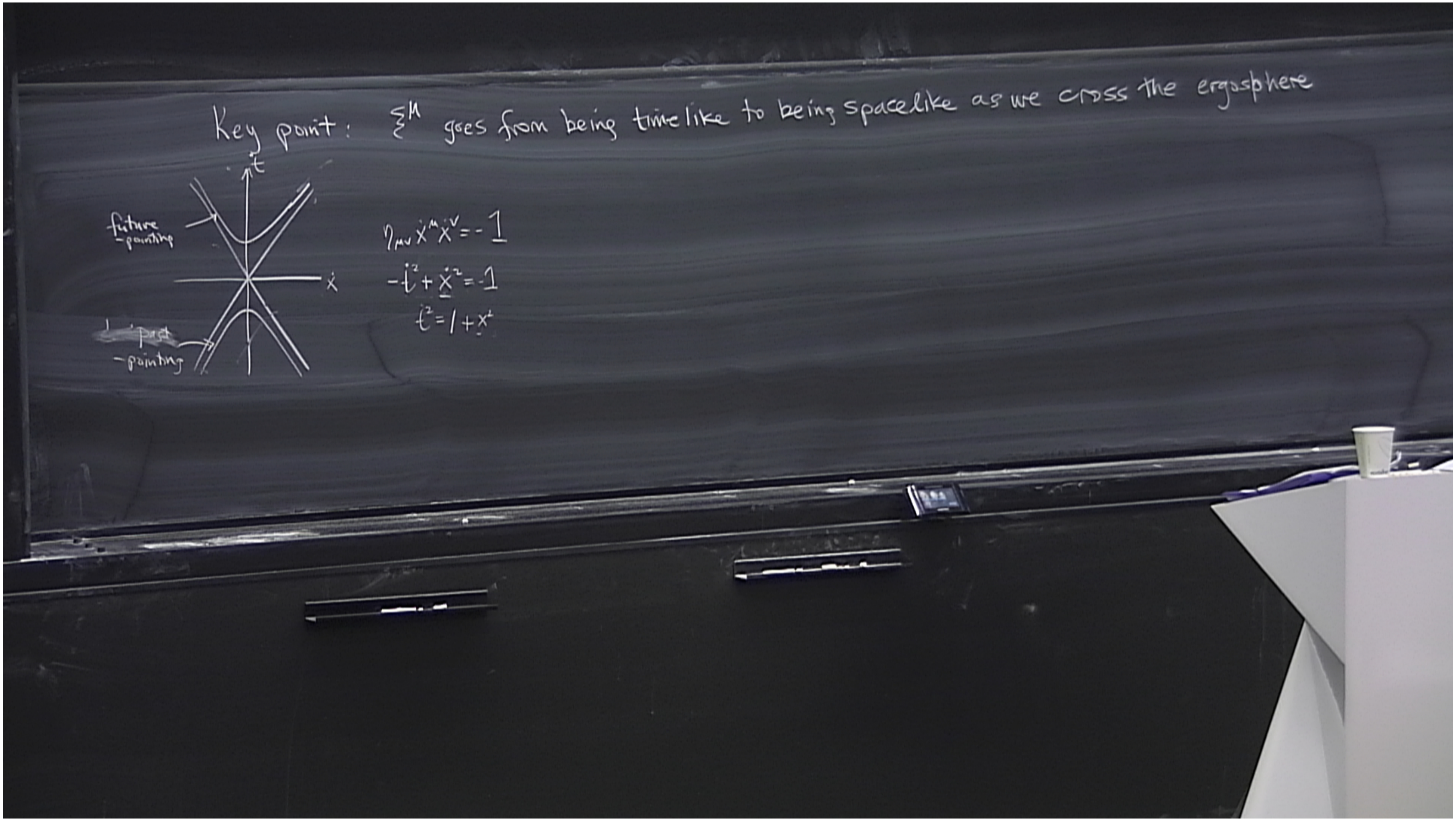

θ=0
sls r= rs/2 + √(rs²/4 - a²cos²θ) ≡ rsls
ergosphere: r+ < r < rsls
r=r+
a counter-rotating photon
will remain at φ=π/2 forever.
r=r-
θ=0,
gμν ẋμ ẋν = 0
t=λ r=rsls, θ=π/2, φ=anything
gμν ξμ ξν = gtt.
0,0) in t,r,θ,φ coords

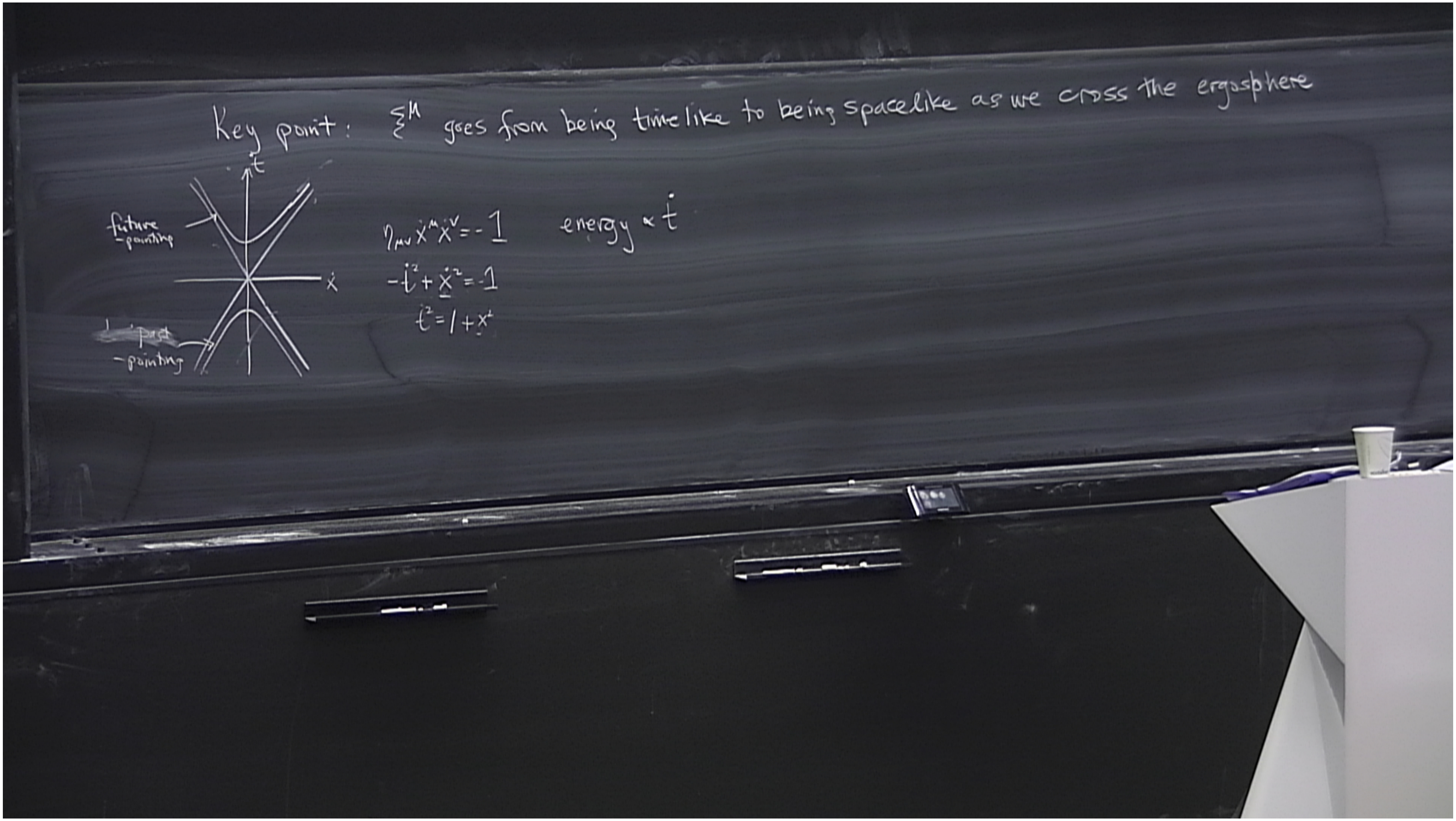

θ=0
sls r = rs/2 + √(rs²/4 − a²cos²θ) ≡ rsls
ergosphere : r+ < r < rsls
r=r+
a counter-rotating photon
will remain at φ = π/2 forever.
r=r−
gμν ẋ^μ ẋ^ν = 0
t = λ r = rsls, θ = π/2 , φ = anything
θ=0,
gμν ξ^μ ξ^ν = gtt. goes from −ve to positive as fall inside ergosphere.
0,0) in t, r, θ, φ coords

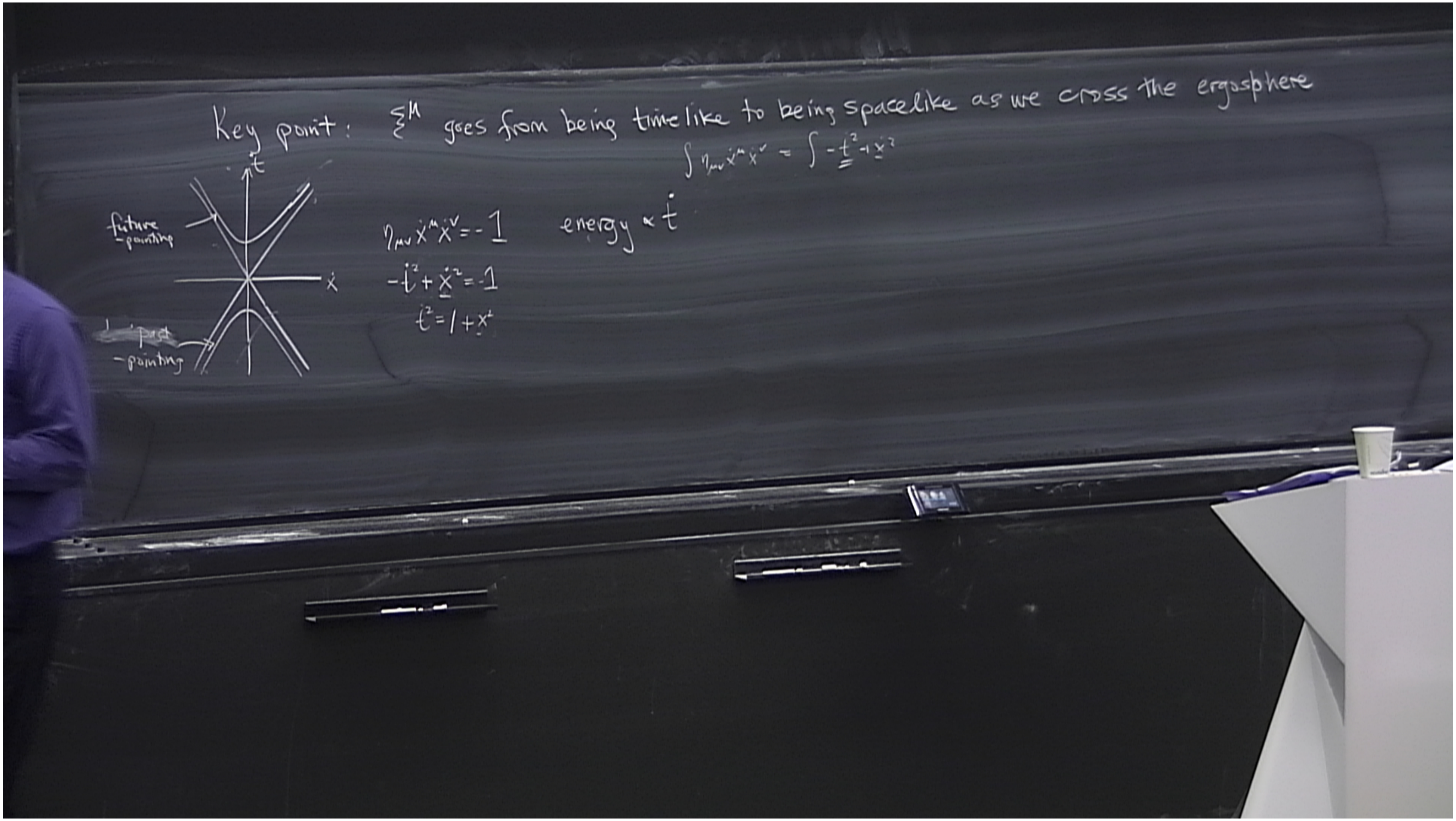

Key point: ξ^μ goes from
time like to being spacelike as we cross the ergosphere

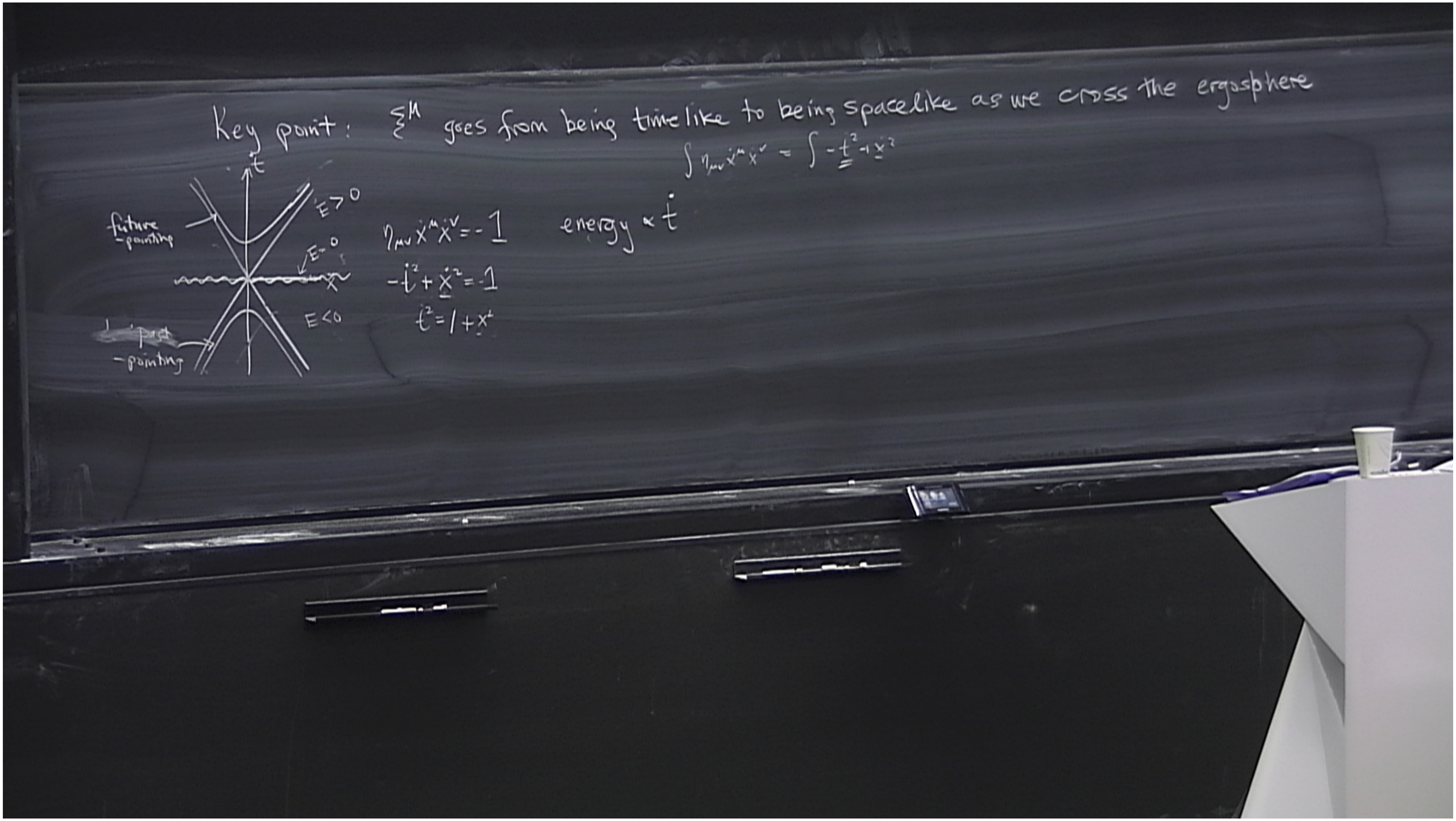

Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
t
x
$\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$
$-\dot{t}^2 + \dot{\underline{x}}^2 = -1$

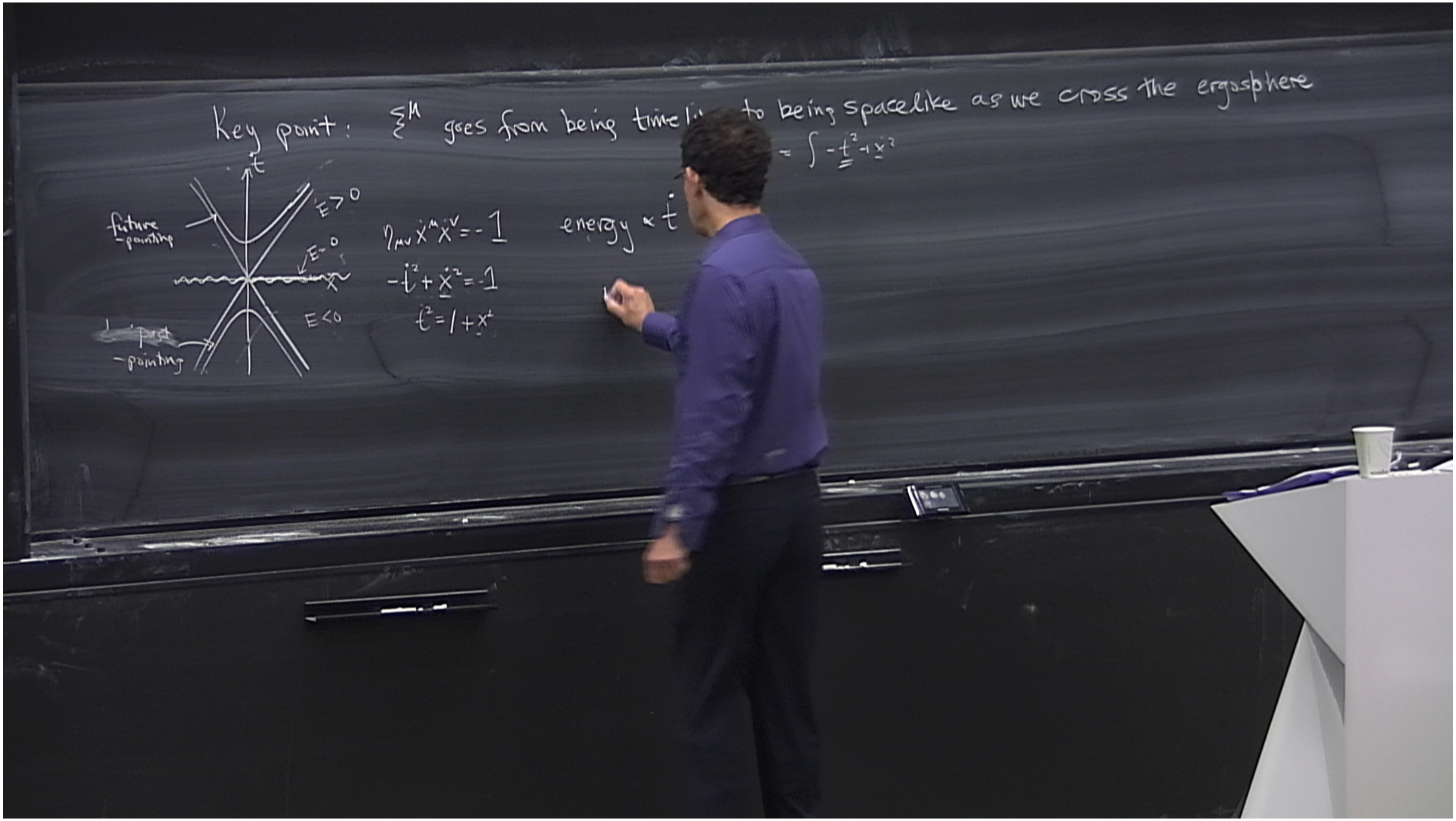

Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
t
x


Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
t
future -pointing
x
past -pointing
$\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$
$-\dot{t}^2+\dot{x}^2=-1$
$\dot{t}^2=1+\dot{x}^2$

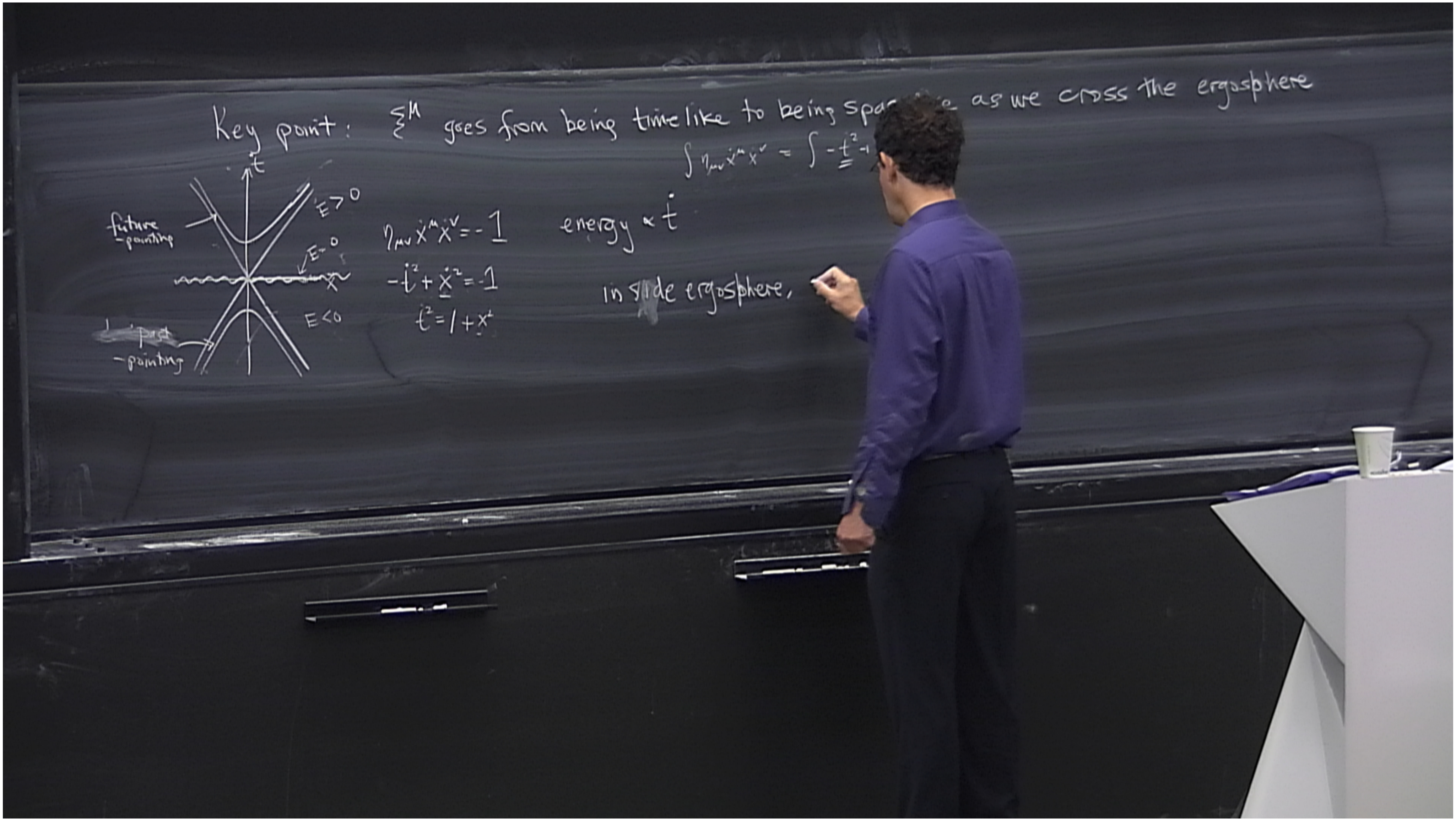

Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
t
x
future -pointing
past -pointing
η_μν ẋ^μ ẋ^ν = -1
energy ∝ ṫ
-ṫ² + ẋ² = -1
ṫ² = 1 + ẋ²

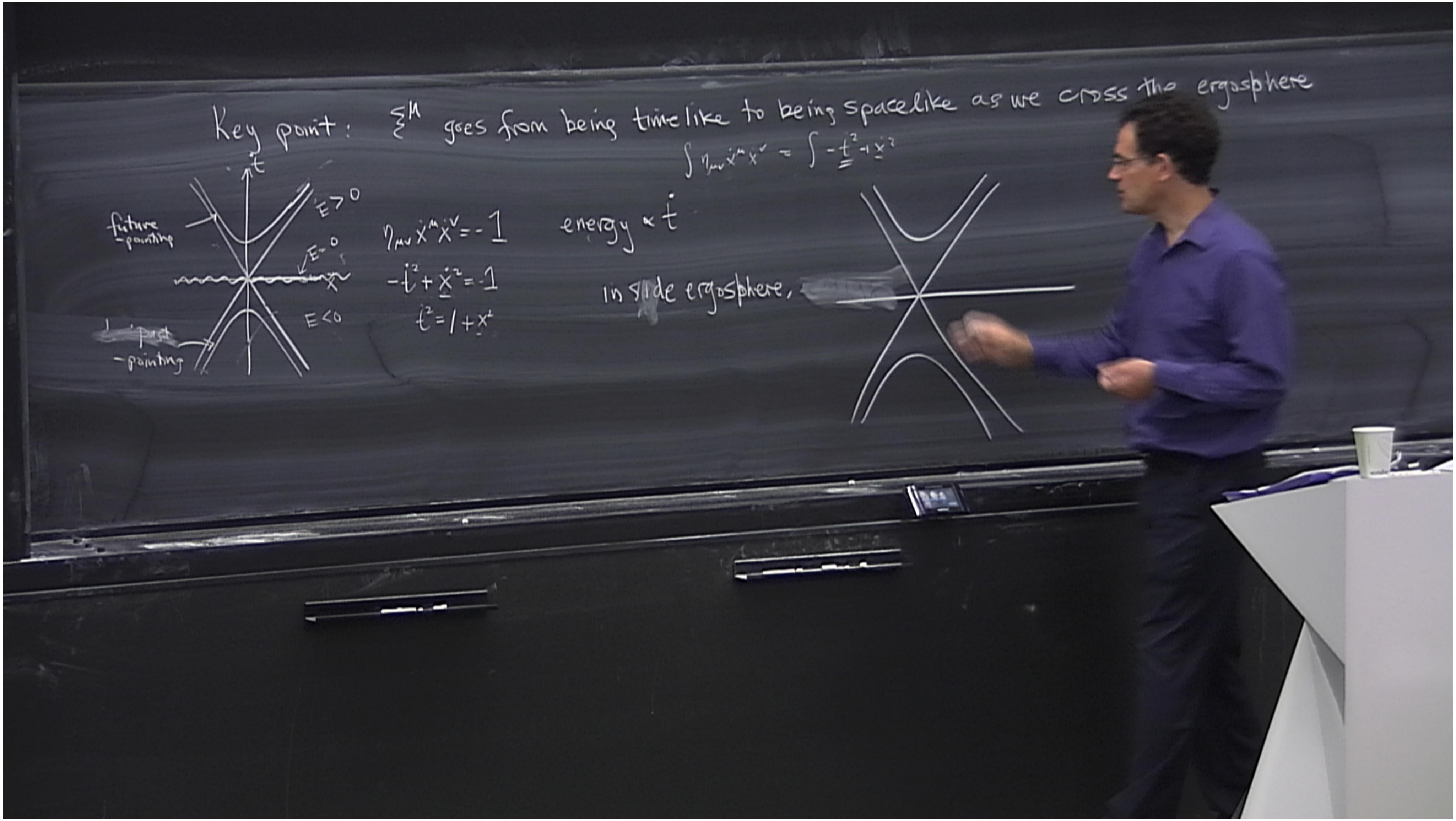

Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
∫ η_μν ẋ^μ ẋ^ν = ∫ -ṫ² + ẋ²
t
future -pointing
x
past -pointing
η_μν ẋ^μ ẋ^ν = -1
energy ∝ ṫ
-ṫ² + ẋ² = -1
ṫ² = 1 + ẋ²

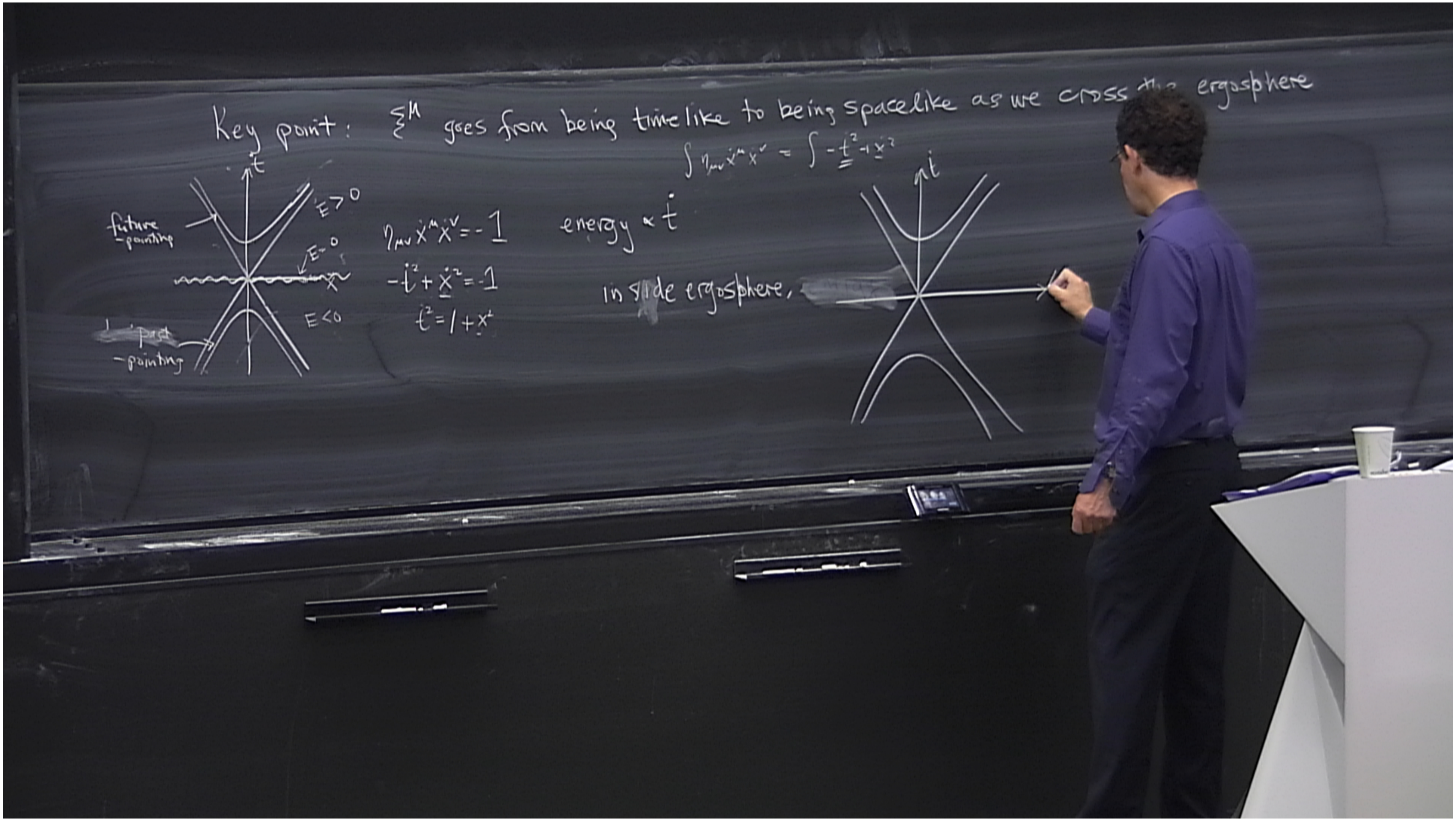

Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
∫ η_μν ẋ^μ ẋ^ν = ∫ -ṫ² + ẋ²
t
x
E>0
E=0
E<0
future-pointing
past-pointing
η_μν ẋ^μ ẋ^ν = -1
-ṫ² + ẋ² = -1
ṫ² = 1 + ẋ²
energy ∝ ṫ



Key point: ξ^μ goes from being time
to being spacelike as we cross the ergosphere
t
x
future -pointing
past -pointing
E>0
E=0
E<0
η_μν ẋ^μ ẋ^ν = -1
-ṫ² + ẋ² = -1
ṫ² = 1 + ẋ²
energy ∝ ṫ


Key point: ξ^μ goes from being time like to being spa... as we cross the ergosphere
future -pointing
past -pointing
E>0
E=0
E<0
t
x
η_μν ẋ^μ ẋ^ν = -1
-ṫ² + ẋ² = -1
ṫ² = 1 + ẋ²
energy ∝ ṫ
inside ergosphere,


Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
∫ η_μν ẋ^μ ẋ^ν = ∫ -ṫ² + ẋ²
t
E>0
future -pointing
E=0
x
E<0
past -pointing
η_μν ẋ^μ ẋ^ν = -1
-ṫ² + ẋ² = -1
ṫ² = 1 + ẋ²
energy ∝ ṫ
inside ergosphere,


Key point: ξ^μ goes from being timelike to being spacelike as we cross the ergosphere
future-pointing
past-pointing
E>0
E=0
E<0
η_μν ẋ^μ ẋ^ν = -1
-ṫ² + ẋ² = -1
ṫ² = 1 + ẋ²
energy ∝ ṫ
inside ergosphere,