Title: Relativity - Lecture 12

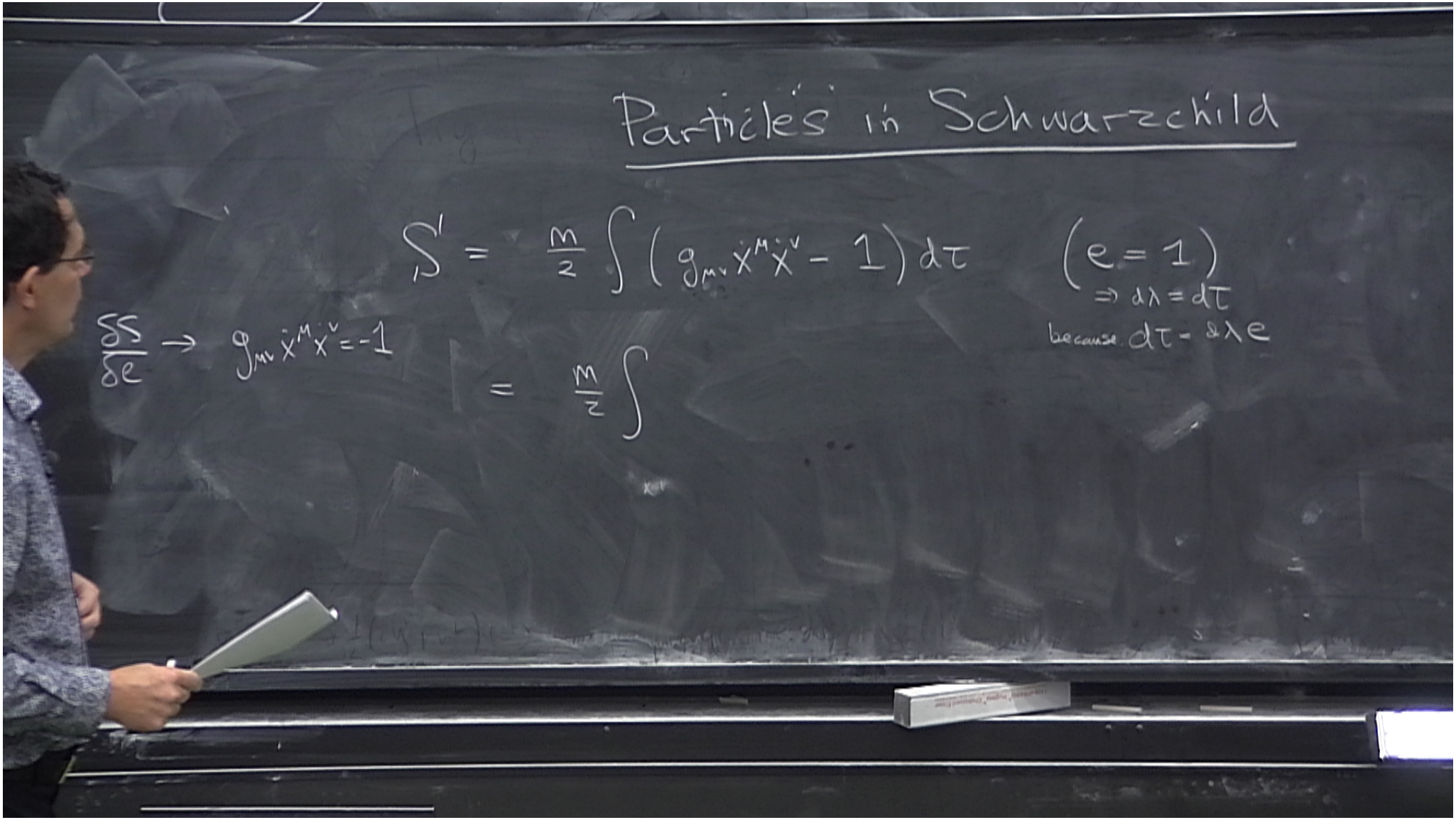
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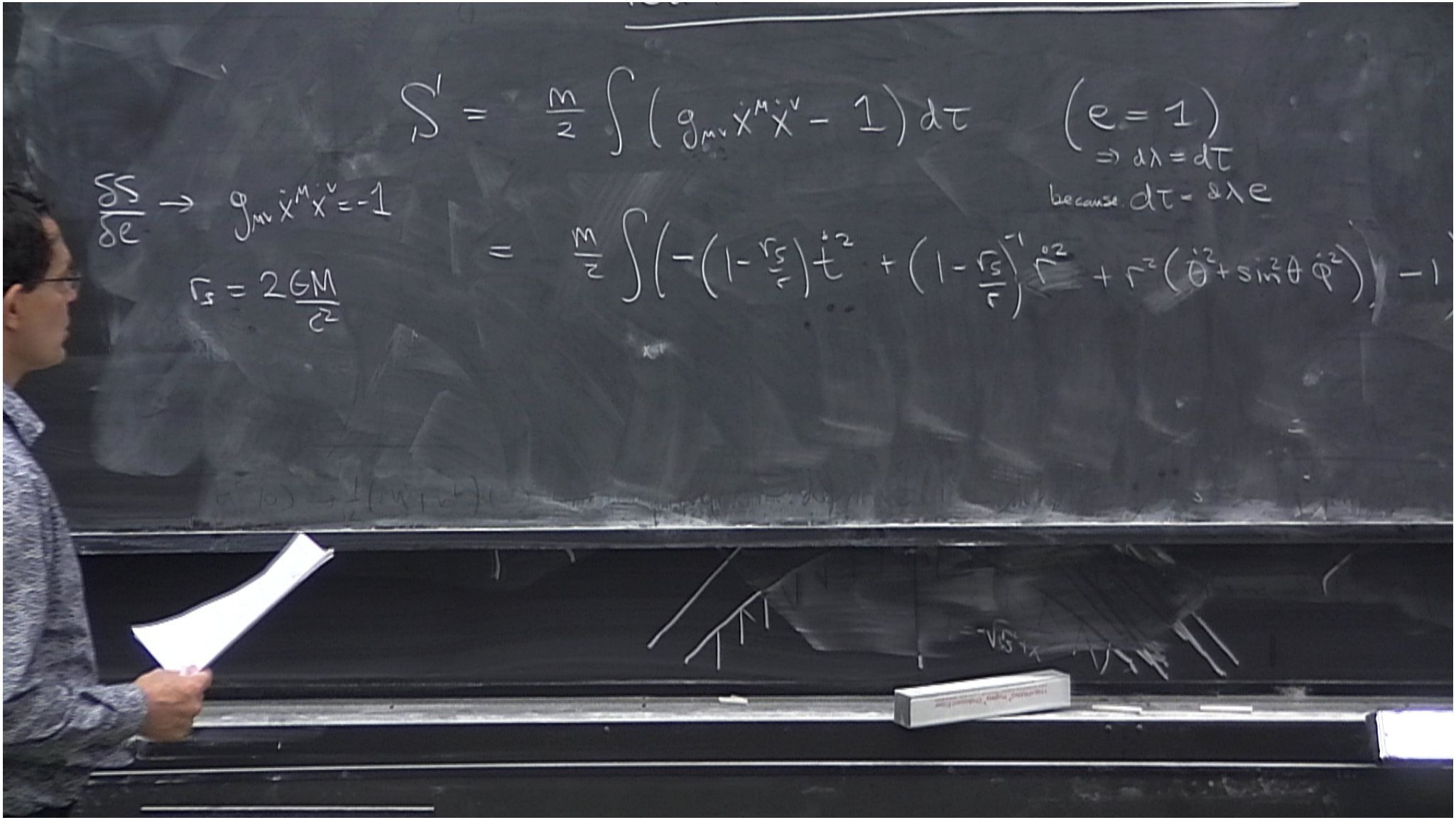
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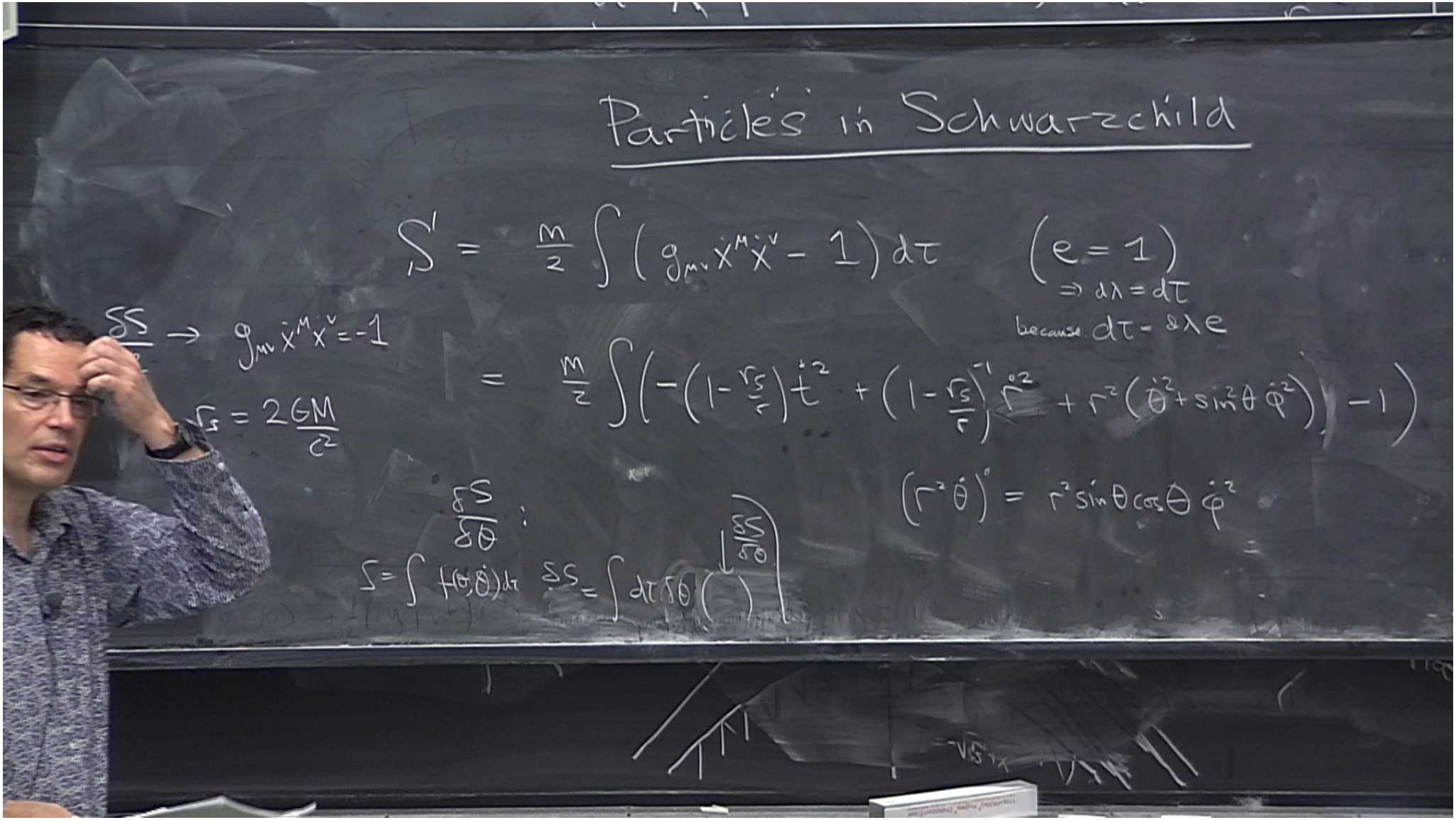
Abstract:

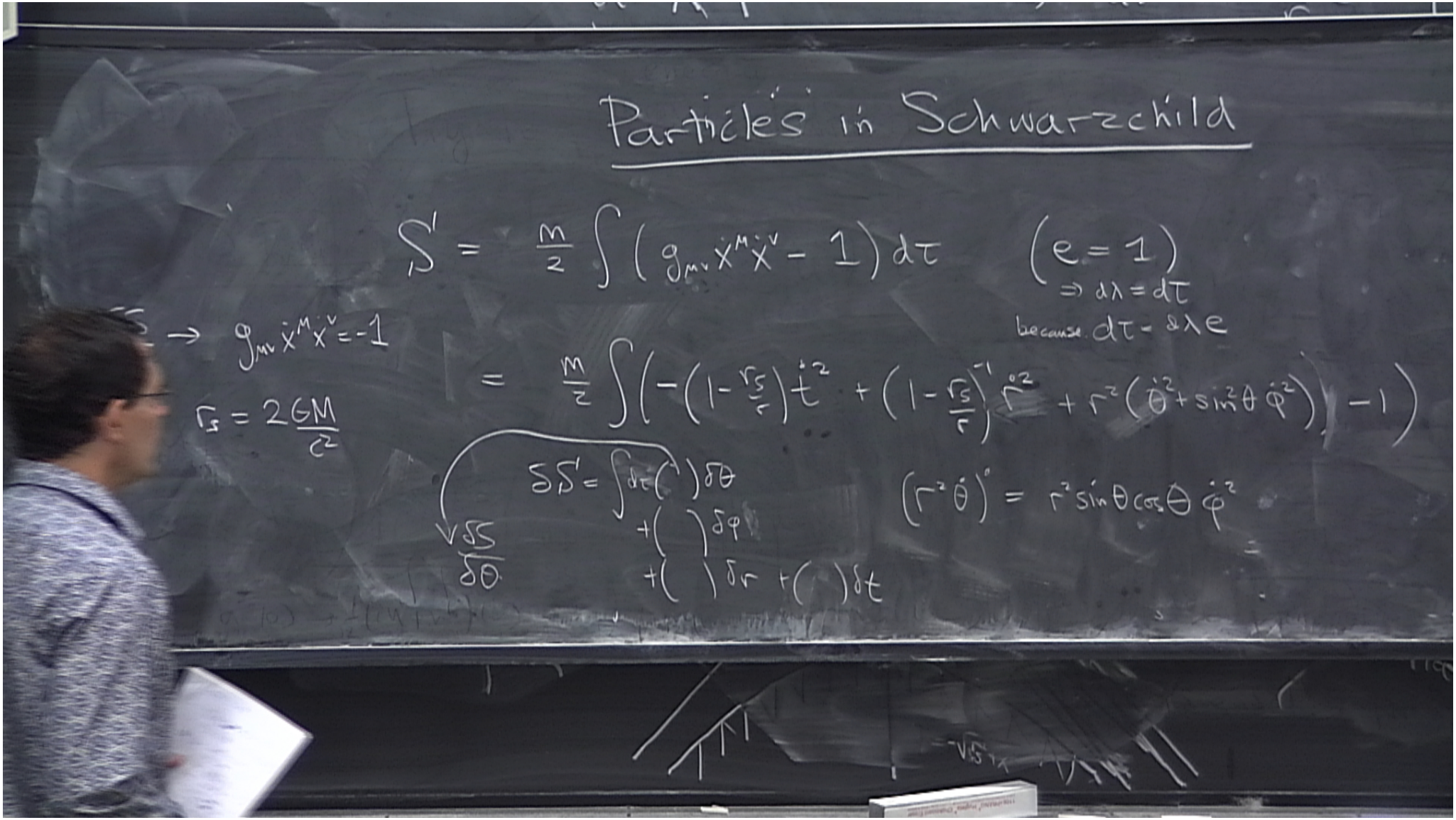
Particles in Schwarzchild

$$S = \frac{m}{2}\int \left(g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} - 1 \right) d\tau \qquad (e=1)$$


Particles in Schwarzchild
$S' = \frac{m}{2}\int (g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu - 1)\,d\tau$
$(e=1)$
$\Rightarrow d\lambda = d\tau$
because $d\tau = d\lambda e$
$\frac{\delta S}{\delta e} \rightarrow g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$
$= \frac{m}{2}\int$


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because $d\tau = e\,d\lambda$
$\frac{\delta S}{\delta e} \to g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$
$r_s = \frac{2GM}{c^2}$
$= \frac{m}{2}\int \left(-\left(1-\frac{r_s}{r}\right)\dot{t}^2 + \left(1-\frac{r_s}{r}\right)^{-1}\dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2)\right) - 1$


Particles in Schwarzschild
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$\frac{\delta S}{\delta\theta}$:
$S = \int H(\theta,\dot{\theta})\,d\tau$
$\delta S = \int d\tau\, \delta\theta \left(\frac{\delta S}{\delta\theta}\right)$
$(r^2\dot{\theta})^{\cdot} = r^2 \sin\theta\cos\theta\,\dot{\varphi}^2$


Particles in Schwarzschild
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$\delta S' = \int dt\,(\quad)\delta\theta + (\quad)\delta\varphi + (\quad)\delta r + (\quad)\delta t$
$\frac{\delta S}{\delta\theta}$
$(r^2\dot{\theta})^{\cdot} = r^2\sin\theta\cos\theta\,\dot{\varphi}^2$

$$\left(\dot{X}^\mu \dot{X}^\nu - 1\right) d\tau \qquad (e = 1)$$

$$\Rightarrow d\lambda = d\tau$$

because $d\tau = d\lambda e$

$$\left(1 - \frac{r_s}{r}\right)\dot{t}^2 + \left(1 - \frac{r_s}{r}\right)^{-1}\dot{r}^2 + r^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2\right) = -1$$

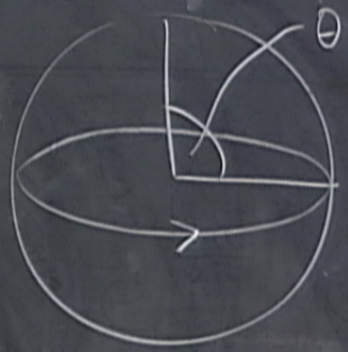
$$\frac{\delta S}{\delta \theta} = m\left[-\left(r^2\dot{\theta}\right)^{\cdot} + r^2 \sin\theta\cos\theta\,\dot{\varphi}^2\right] = 0$$

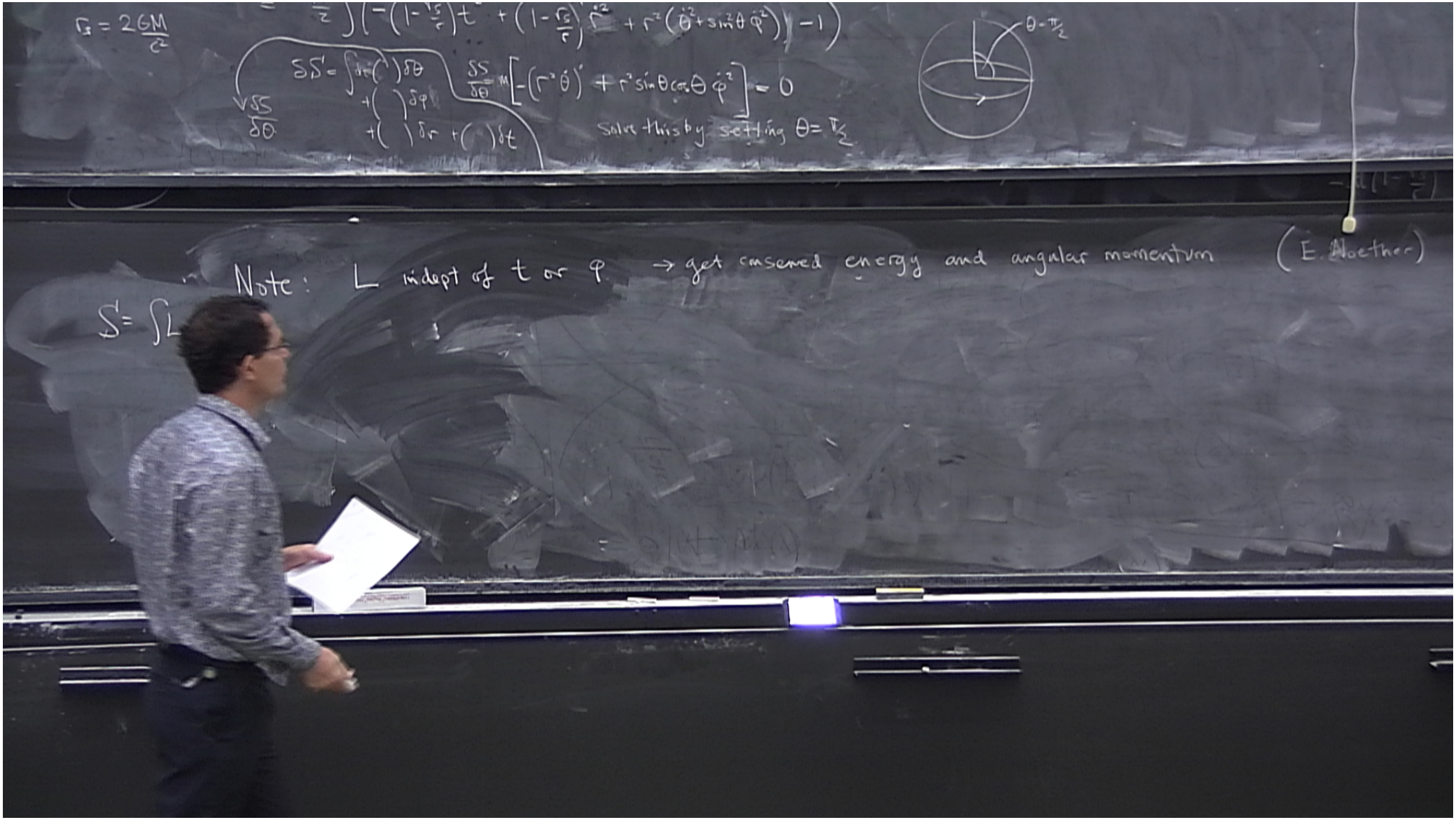
Solve this by setting $\theta = \pi/2$.

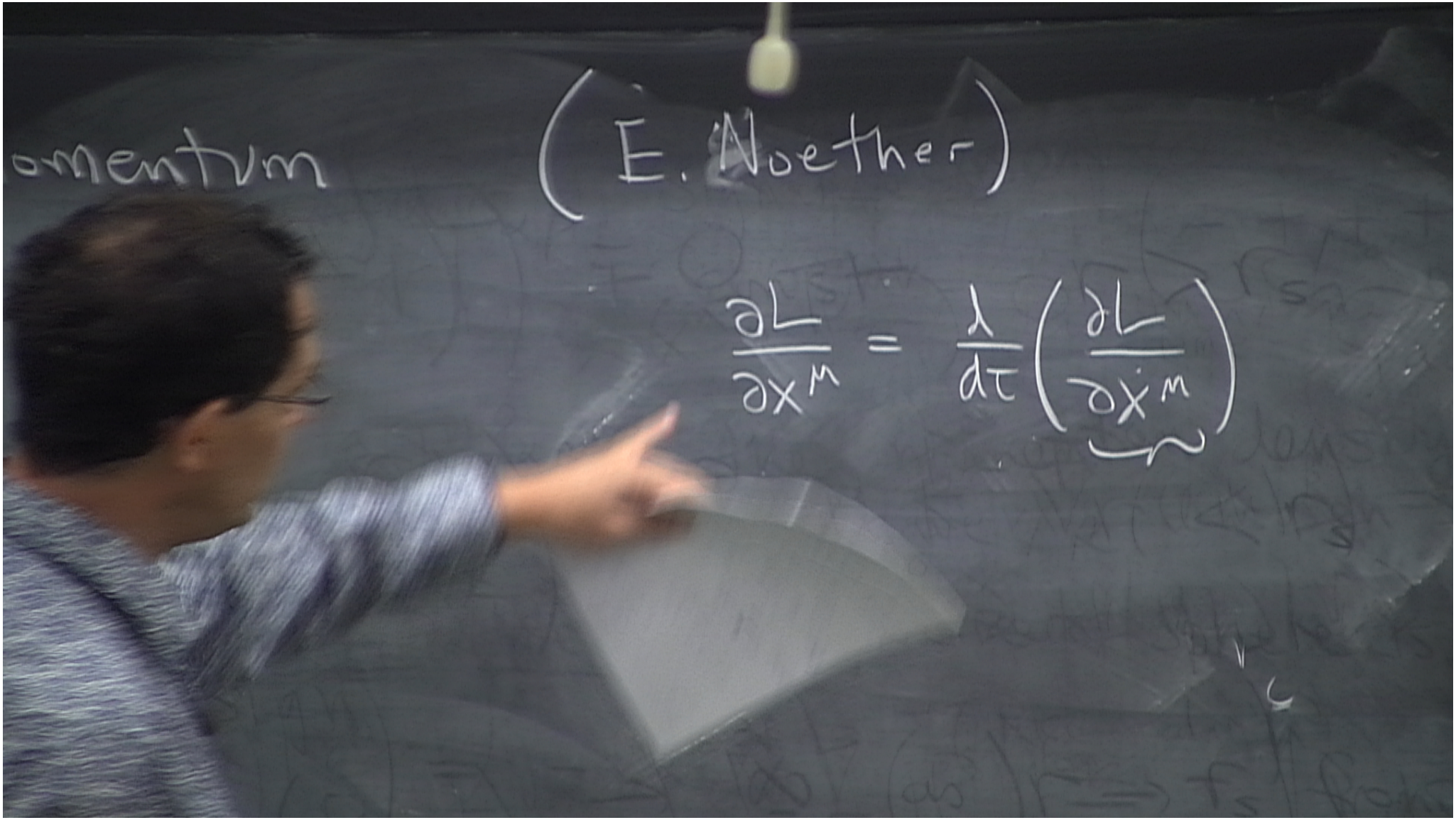
$\delta\theta$

$\delta\varphi$

$\delta r + (\quad)\delta t$




$r_s = \frac{2GM}{c^2}$
$\theta = \pi/2$
$\frac{\partial S}{\partial \theta} = m\left[-(r^2\dot{\theta})^{\cdot} + r^2 \sin\theta\cos\theta\,\dot{\varphi}^2\right] = 0$
Solve this by setting $\theta = \pi/2$
Note: L indept of t or φ → get conserved energy and angular momentum (E. Noether)
$S = \int L$


omentum
(E. Noether)
$\frac{\partial L}{\partial x^\mu} = \frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{x}^\mu}\right)$

ntum (E. Noether)

$$\frac{\partial L}{\partial x^\mu} = \frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{x}^\mu}\right)$$

$$= 0$$

$$P_\mu = \frac{\partial L}{\partial \dot{x}^\mu}$$

is conserved.

→ get conserved energy and angular m

(Symmetries → conservation laws).

- indept of t or φ. → get conserved energy and angular

(Symmetries → conservation laws).

① $\frac{\partial L}{\partial \varphi} = 0 \Rightarrow \left(r^2\dot{\varphi}\right)^{\cdot} = 0 \Rightarrow L \equiv r^2\dot{\varphi} = \text{const.}$

② $\frac{\partial L}{\partial t} = 0 \Rightarrow \left(\dot{t}\left(1-\frac{r_s}{r}\right)\right)^{\cdot} = 0 \Rightarrow E = \dot{t}\left(1-\frac{r_s}{r}\right) = \text{const}$

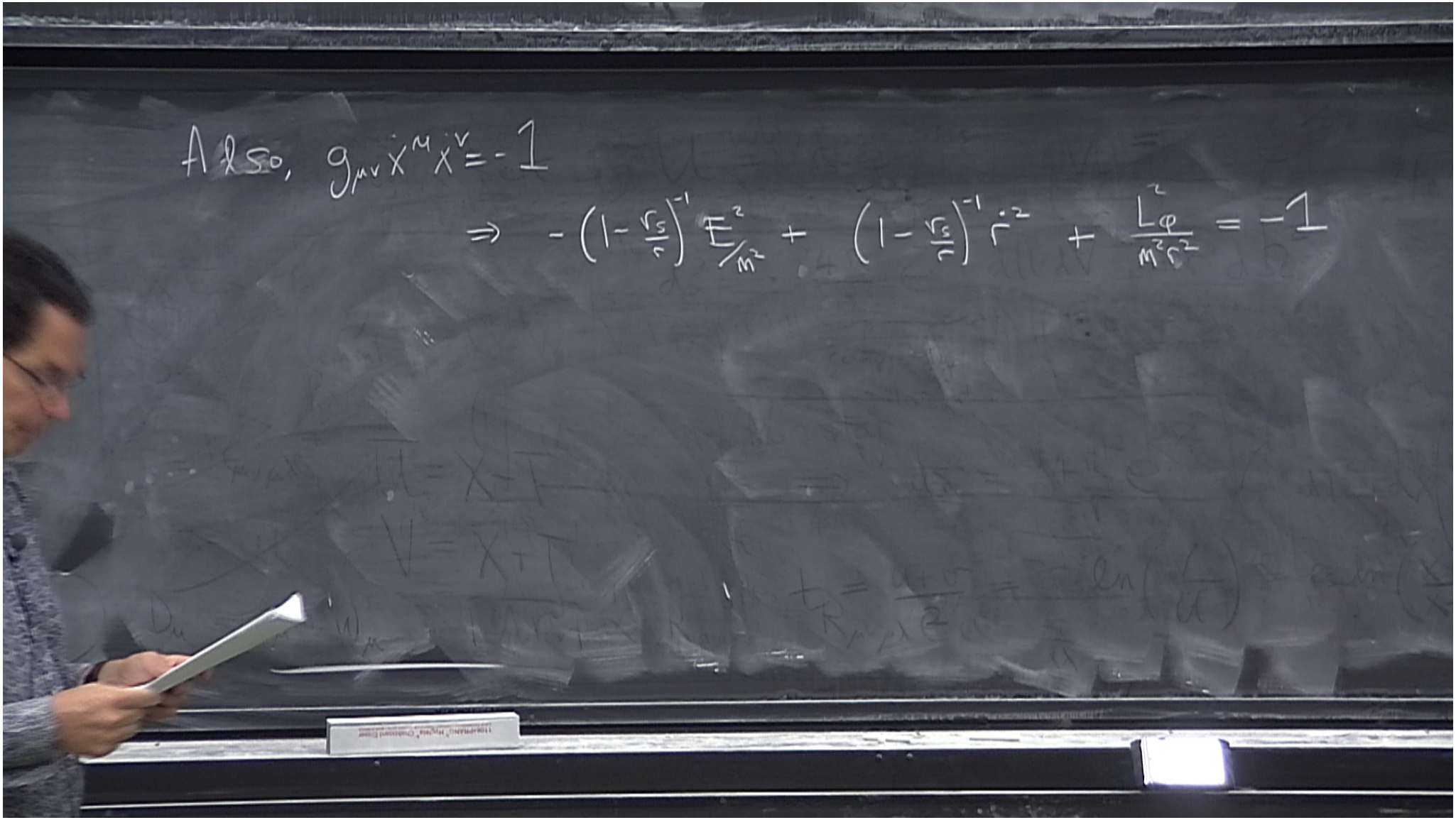
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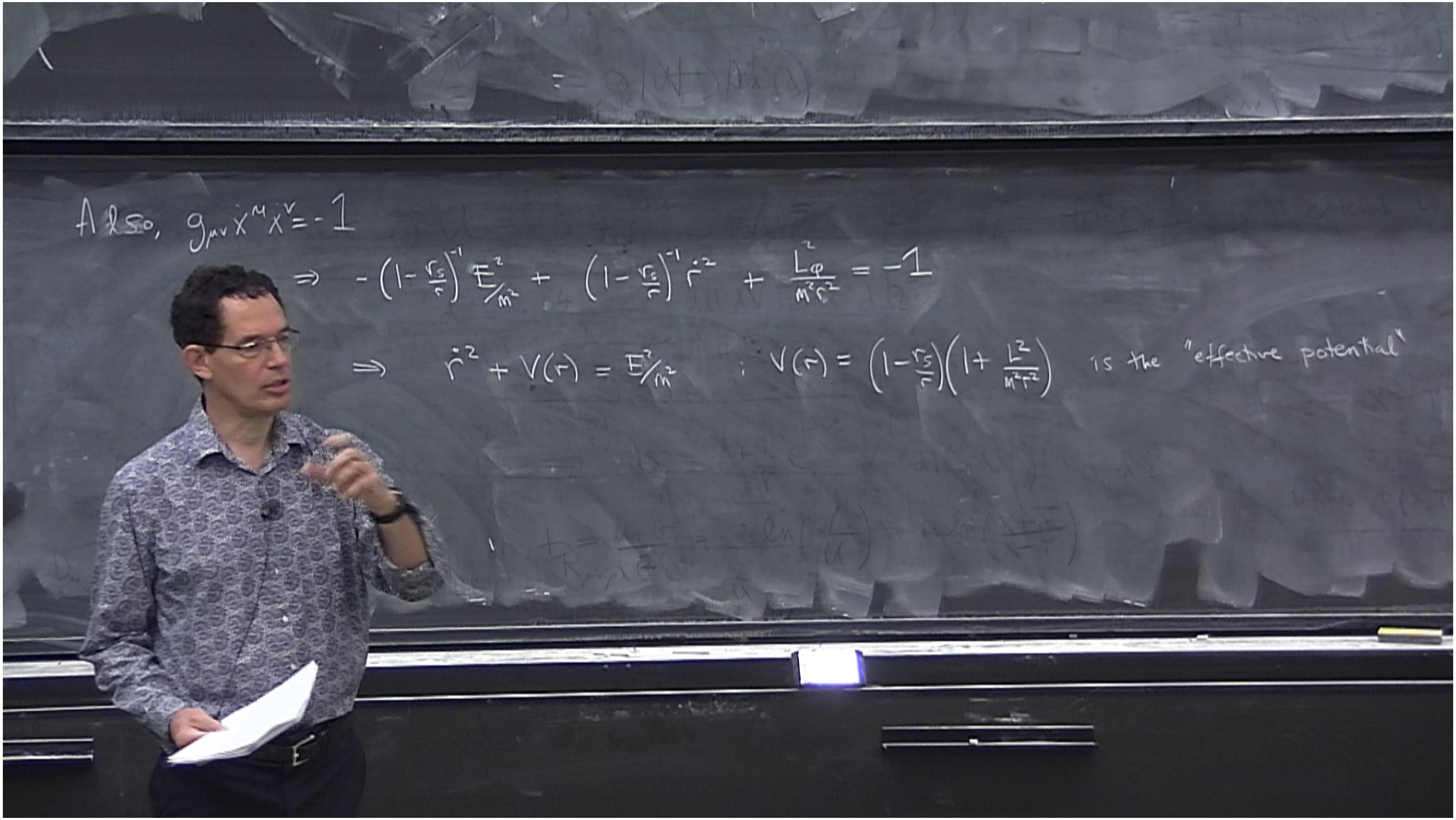
(Symmetries → conservation laws).

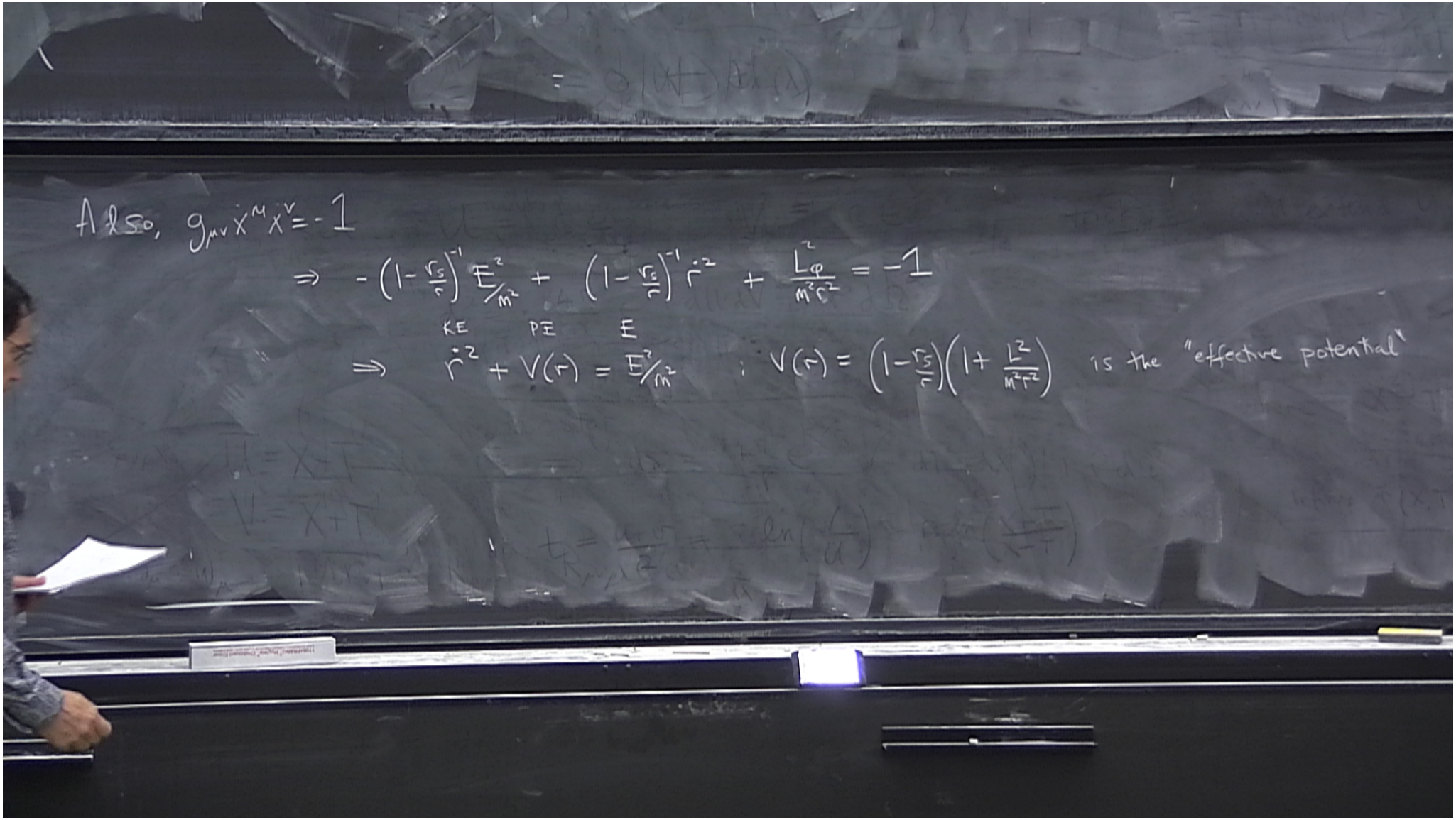
① $\frac{\partial L}{\partial \varphi} = 0 \Rightarrow \left(r^2\dot{\varphi}\right)^{\cdot} = 0 \Rightarrow L_\varphi \equiv mr^2\dot{\varphi} = \text{const.}$

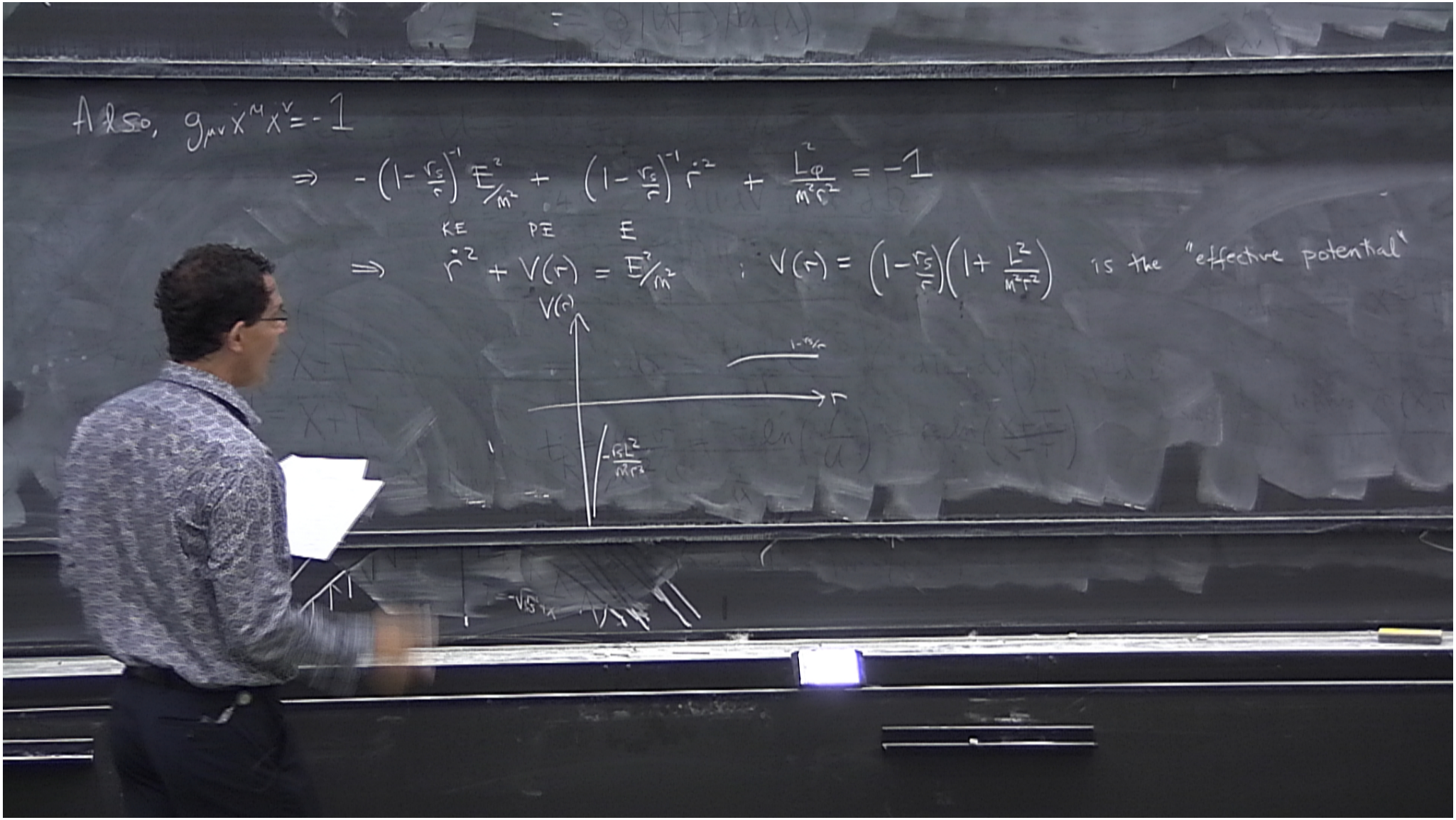
② $\frac{\partial L}{\partial t} = 0 \Rightarrow \left(\dot{t}\left(1-\frac{r_s}{r}\right)\right)^{\cdot} = 0 \Rightarrow E = m\dot{t}\left(1-\frac{r_s}{r}\right) = \text{const}$

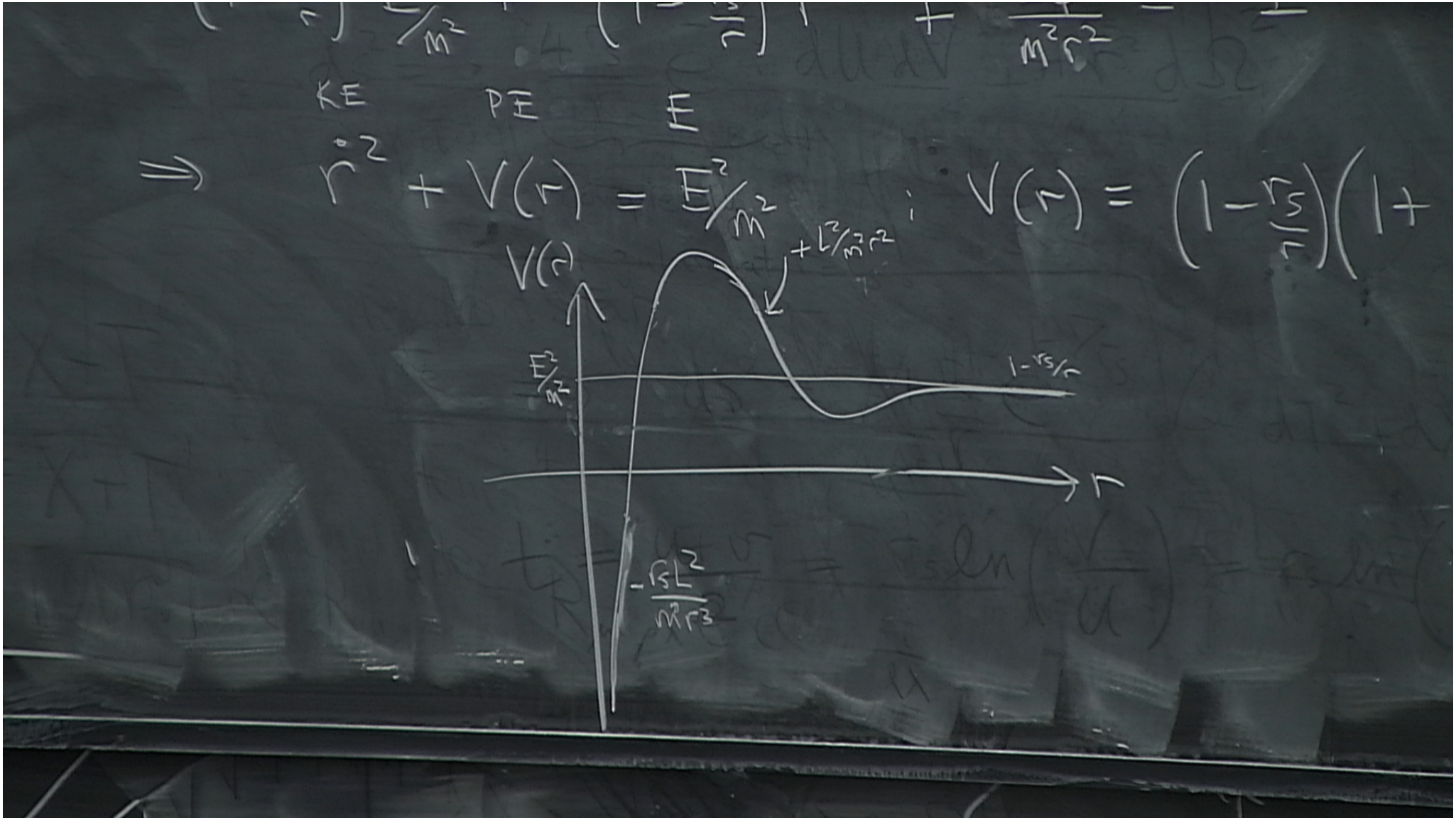


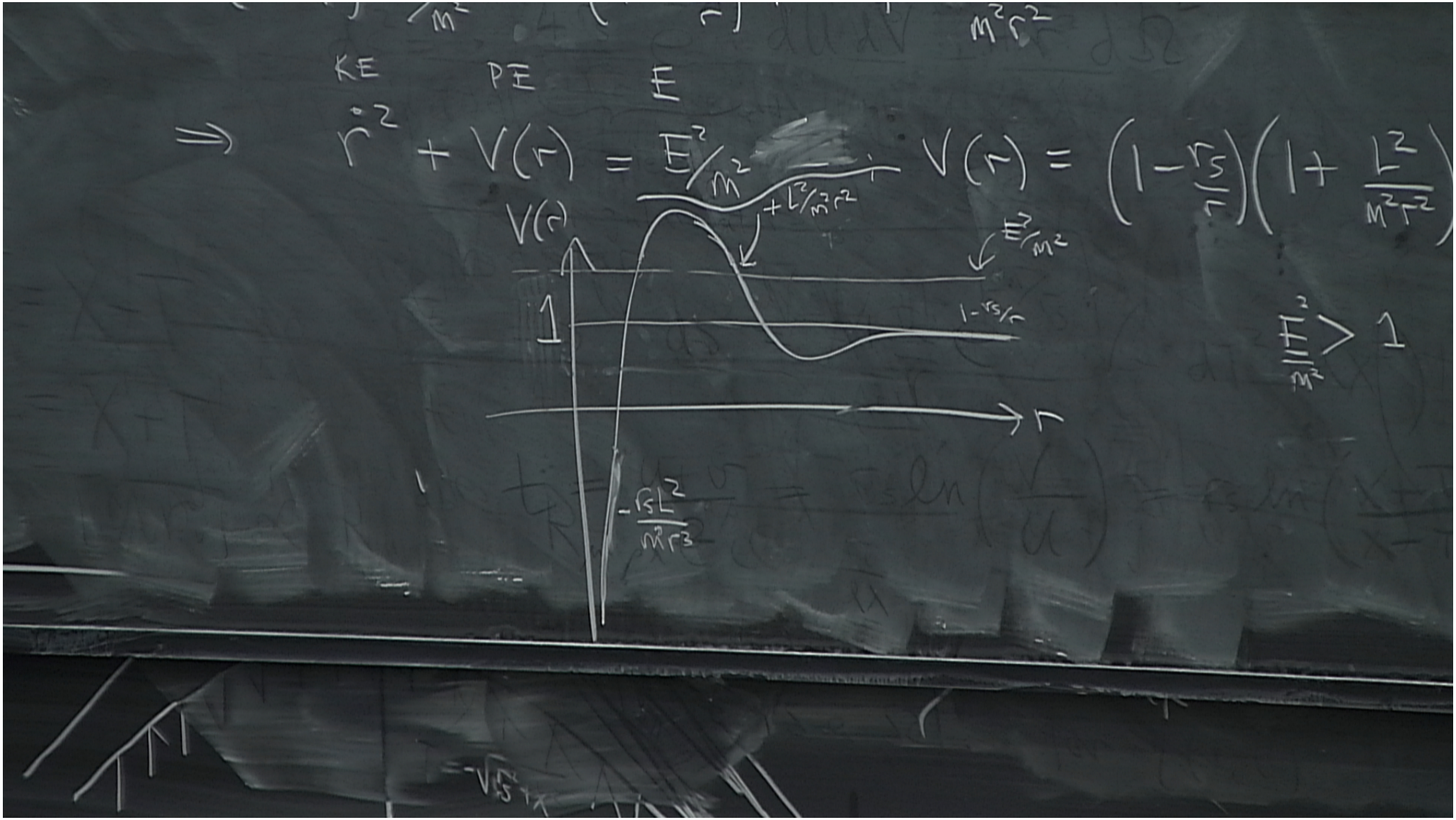

Also, $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$
$\Rightarrow -\left(1-\frac{r_s}{r}\right)^{-1}\frac{E^2}{m^2} + \left(1-\frac{r_s}{r}\right)^{-1}\dot{r}^2 + \frac{L_\varphi^2}{m^2 r^2} = -1$


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$\Rightarrow \dot{r}^2 + V(r) = E^2/m^2 \quad ; \quad V(r) = \left(1-\frac{r_s}{r}\right)\left(1+\frac{L^2}{m^2r^2}\right)$ is the "effective potential"


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KE PE E
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V(r)
$1-r_s/r$
$-\frac{r_s L^2}{m^2 r^3}$
r


KE
PE
E
$\Rightarrow \dot{r}^2 + V(r) = E^2/m^2$
$V(r) = \left(1 - \frac{r_s}{r}\right)\left(1 + \right.$
$V(r)$
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E^2/m^2
$1 - r_s/r$
r
$-\frac{r_s L^2}{m^2 r^3}$


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$\dot{r}^2 + V(r) = E^2/m^2$
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$+L^2/m^2r^2$
E^2/m^2
$1 - r_s/r$
$\frac{E^2}{m^2} > 1$
$-\frac{r_s L^2}{m^2 r^3}$

$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$

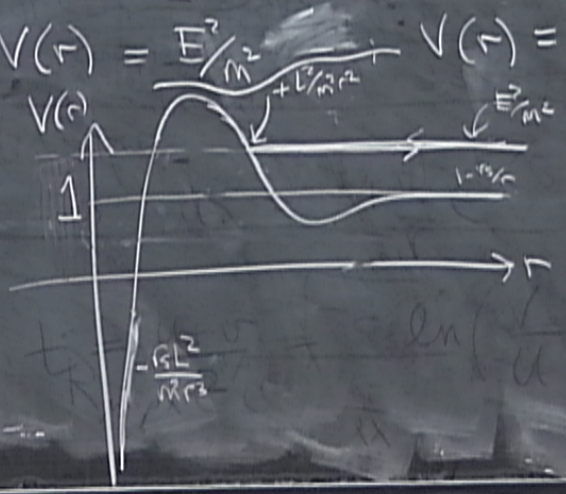
$$\Rightarrow \quad -\left(1-\frac{r_s}{r}\right)^{-1}\frac{E^2}{m^2} + \left(1-\frac{r_s}{r}\right)^{-1}\dot{r}^2 + \frac{L_\varphi^2}{m^2r^2} = -1$$

c.f Newton $1 - \frac{r_s}{r} + \frac{L^2}{m^2r^2}$

KE PE E

$$\Rightarrow \quad \dot{r}^2 + V(r) = \frac{E^2}{m^2}$$

$$V(r) = \left(1-\frac{r_s}{r}\right)\left(1+\frac{L^2}{m^2r^2}\right)$$ is the "effective potenti



$\frac{E^2}{m^2} > 1 \Rightarrow$

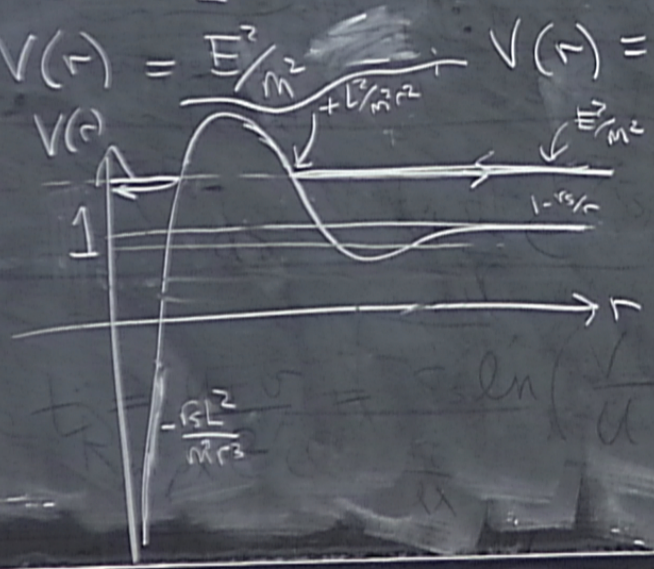
$$\left(1-\frac{r_s}{r}\right)^{-1}\frac{E^2}{m^2}+\left(1-\frac{r_s}{r}\right)^{-1}\dot{r}^2+\frac{L^2}{m^2r^2}=-1$$

cf Newton $1-\frac{r_s}{r}+\frac{L^2}{m^2r^2}$

KE PE E

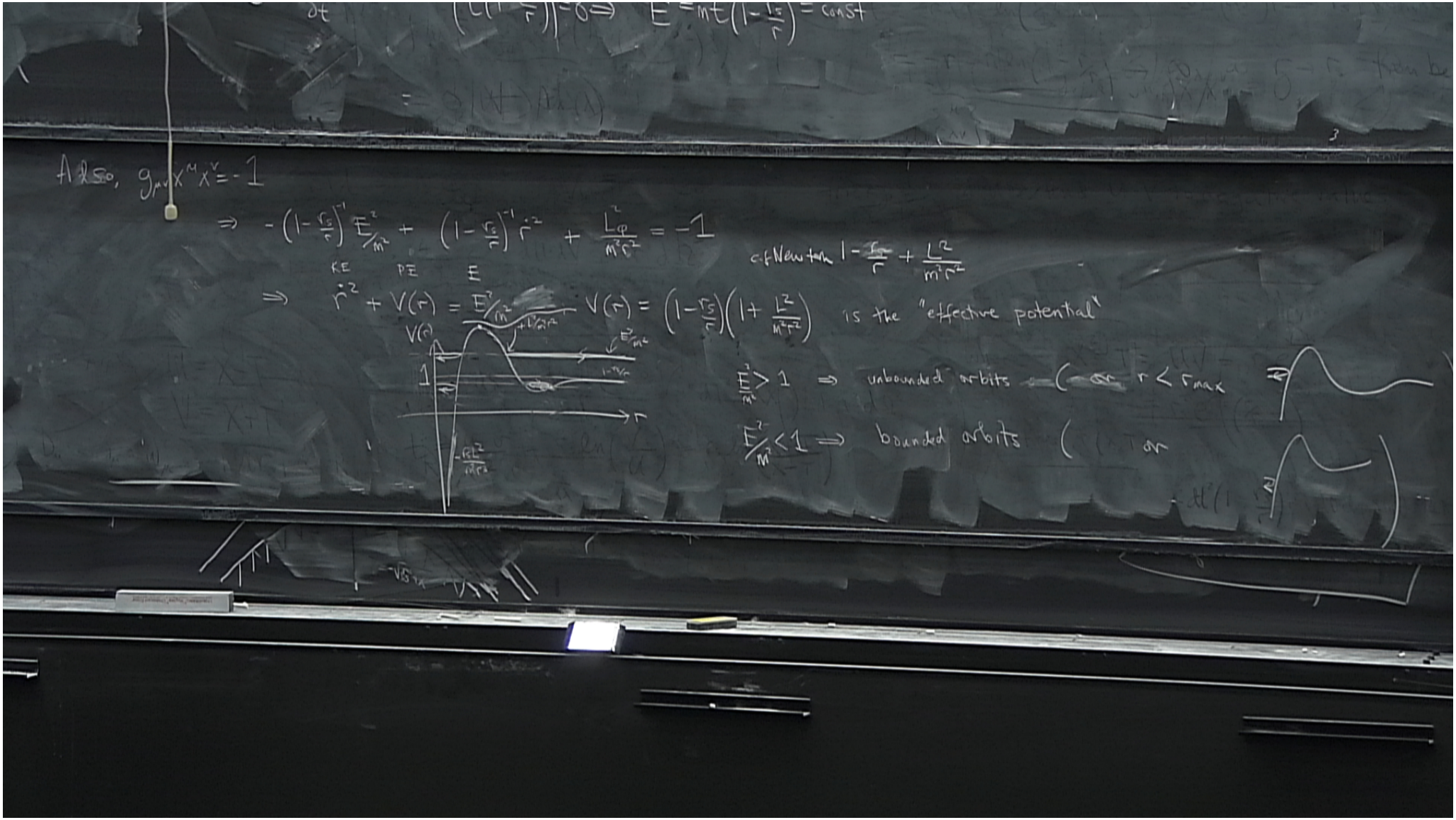
$$\dot{r}^2+V(r)=\frac{E^2}{m^2}$$

$$V(r)=\left(1-\frac{r_s}{r}\right)\left(1+\frac{L^2}{m^2r^2}\right)$$ is the "effective potential"



$\frac{E^2}{m^2}>1$ ⇒ unbounded orbits

$\frac{E^2}{m^2}<1$


Also, $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$
$\Rightarrow -\left(1-\frac{r_s}{r}\right)^{-1}\frac{E^2}{m^2} + \left(1-\frac{r_s}{r}\right)^{-1}\dot{r}^2 + \frac{L_\phi^2}{m^2r^2} = -1$
KE PE E
$\Rightarrow \dot{r}^2 + V(r) = \frac{E^2}{m^2}$
$V(r) = \left(1-\frac{r_s}{r}\right)\left(1+\frac{L^2}{m^2r^2}\right)$ is the "effective potential"
c.f. Newton $1-\frac{r_s}{r}+\frac{L^2}{m^2r^2}$
$\frac{E^2}{m^2} > 1 \Rightarrow$ unbounded orbits (or $r < r_{max}$
$\frac{E^2}{m^2} < 1 \Rightarrow$ bounded orbits (or

$$\left(1-\frac{r_s}{r}\right)^{-1}\frac{E^2}{m^2}+\left(1-\frac{r_s}{r}\right)^{-1}\dot{r}^2+\frac{L^2}{m^2r^2}=-1$$

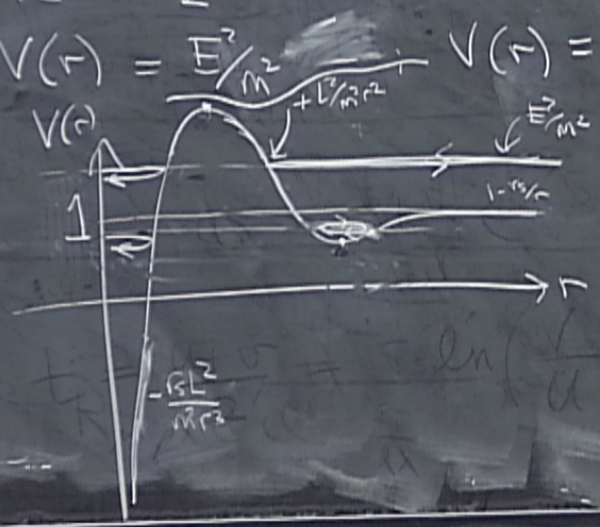
KE PE E

$$\dot{r}^2+V(r)=\frac{E^2}{m^2}$$

$$V(r)=\left(1-\frac{r_s}{r}\right)\left(1+\frac{L^2}{m^2r^2}\right)$$

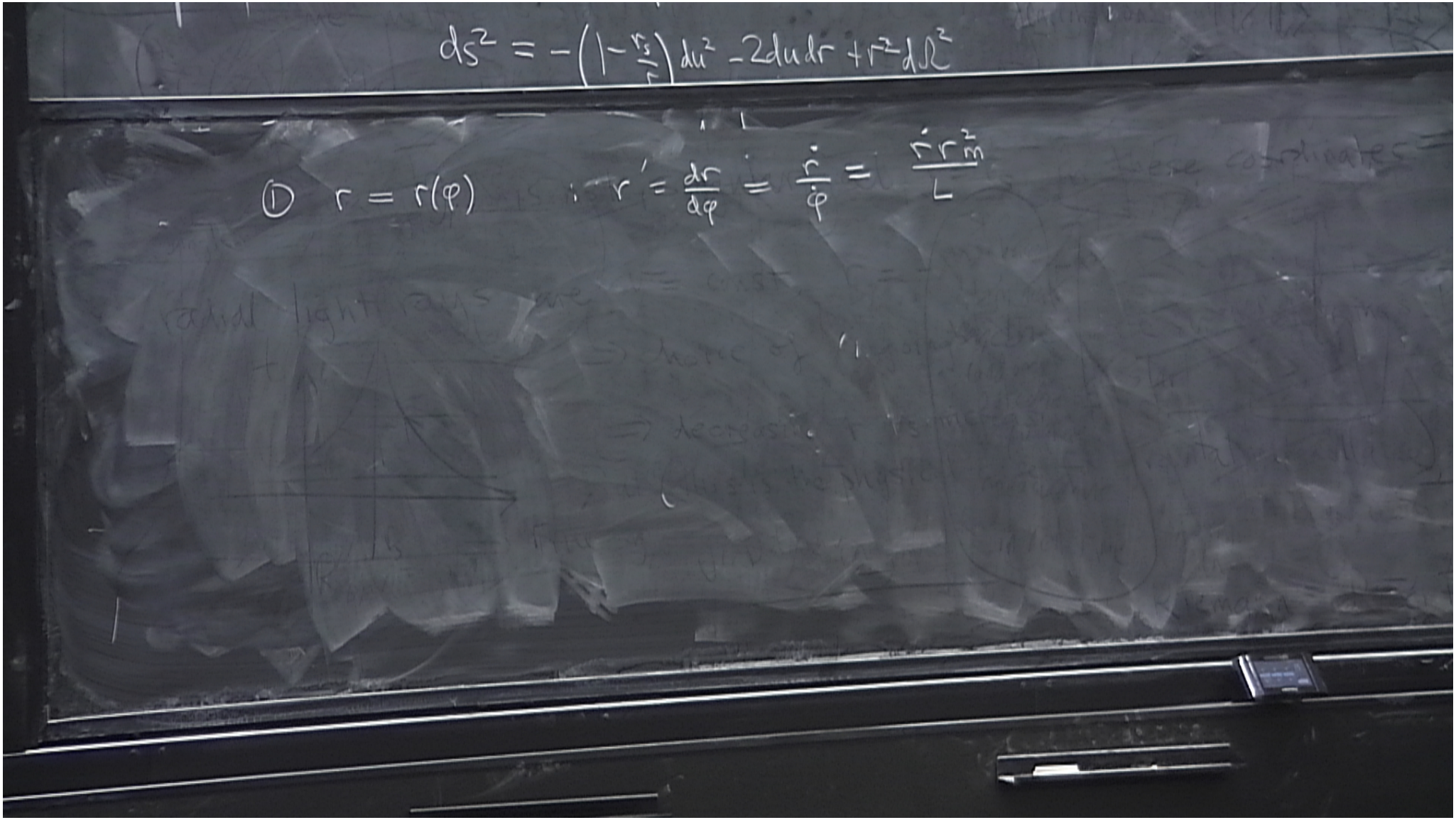
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is the "effective potential"



$\frac{E^2}{m^2}>1 \Rightarrow$ unbounded orbits (or $r<r_{max}$

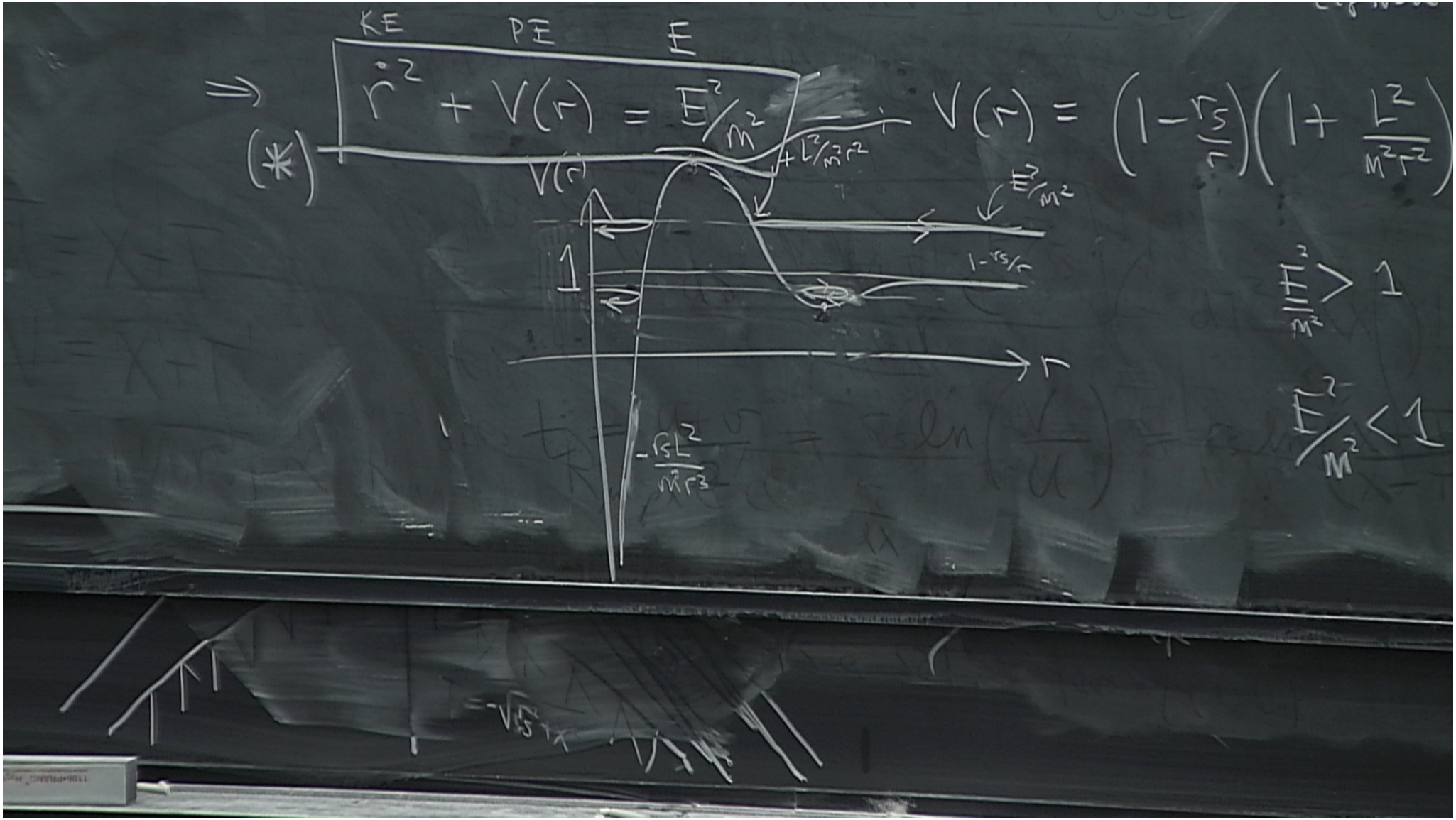
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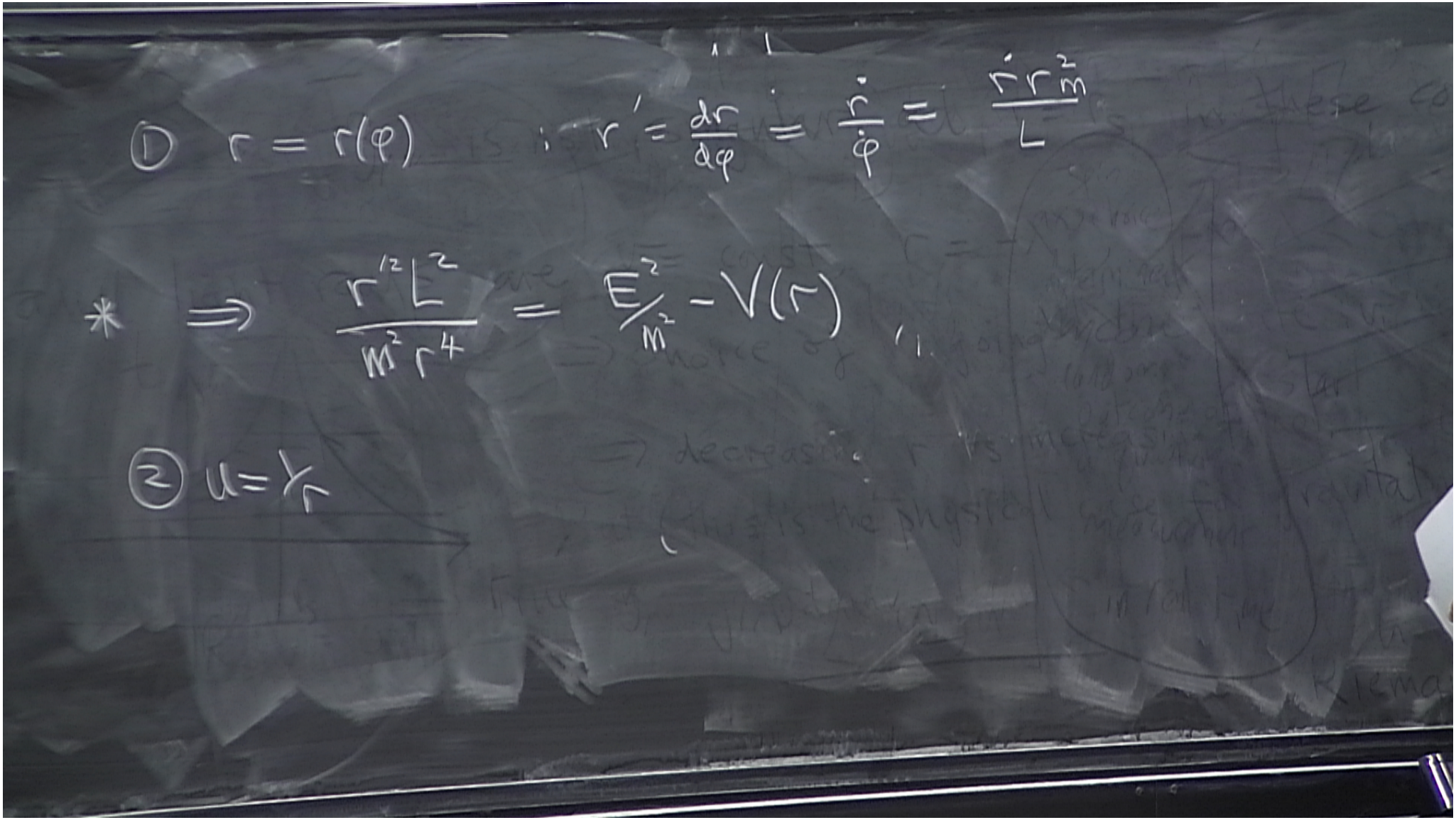

$ds^2 = -\left(1 - \frac{r_s}{r}\right) du^2 - 2du\,dr + r^2 d\Omega^2$
① $r = r(\varphi)$; $r' = \frac{dr}{d\varphi} = \frac{\dot{r}}{\dot{\varphi}} = \frac{\dot{r} r^2}{L}$

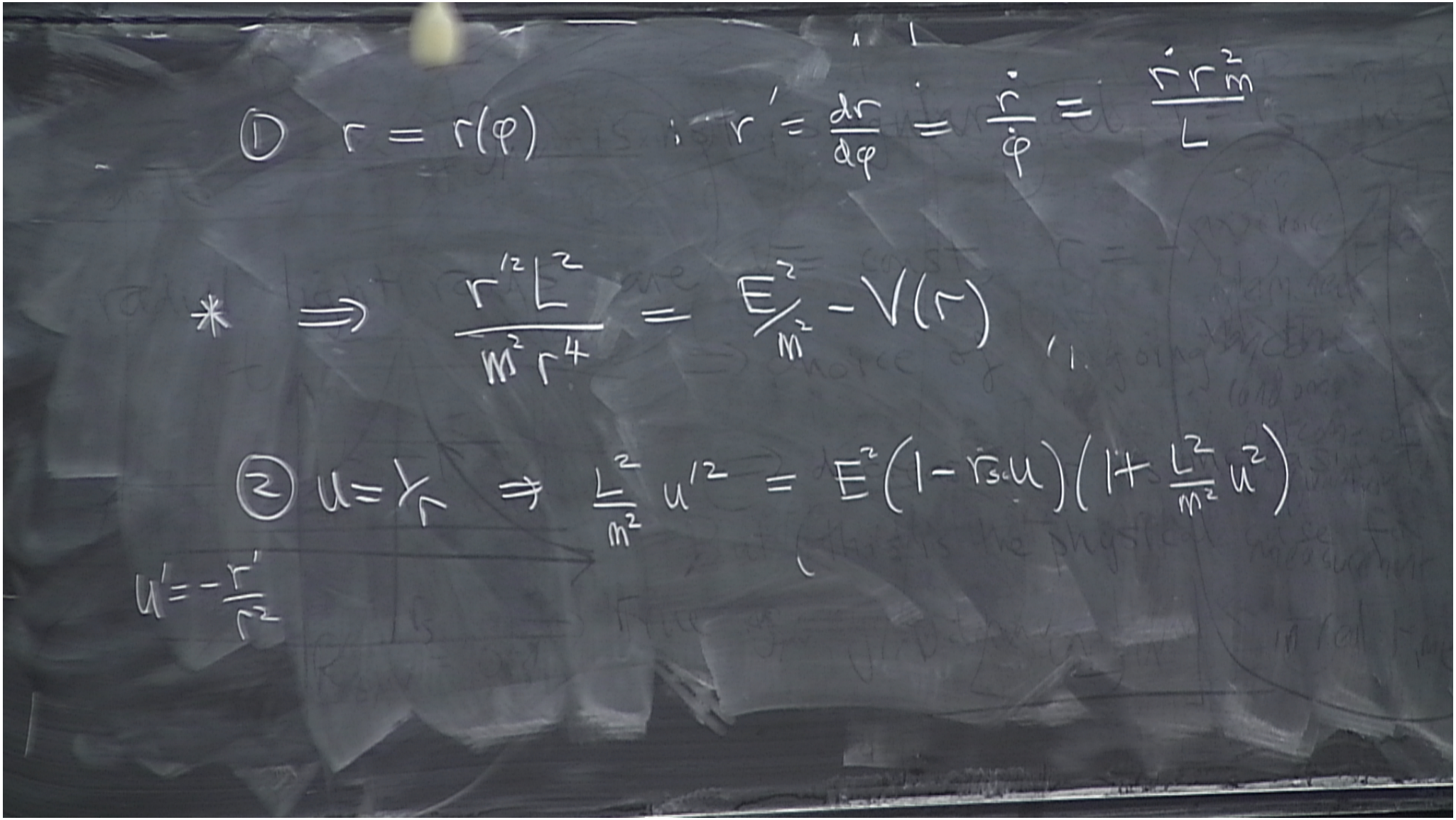
$$ds^2 = -\left(1 - \frac{r_s}{r}\right) du^2 - 2du\,dr + r^2 d\Omega^2$$

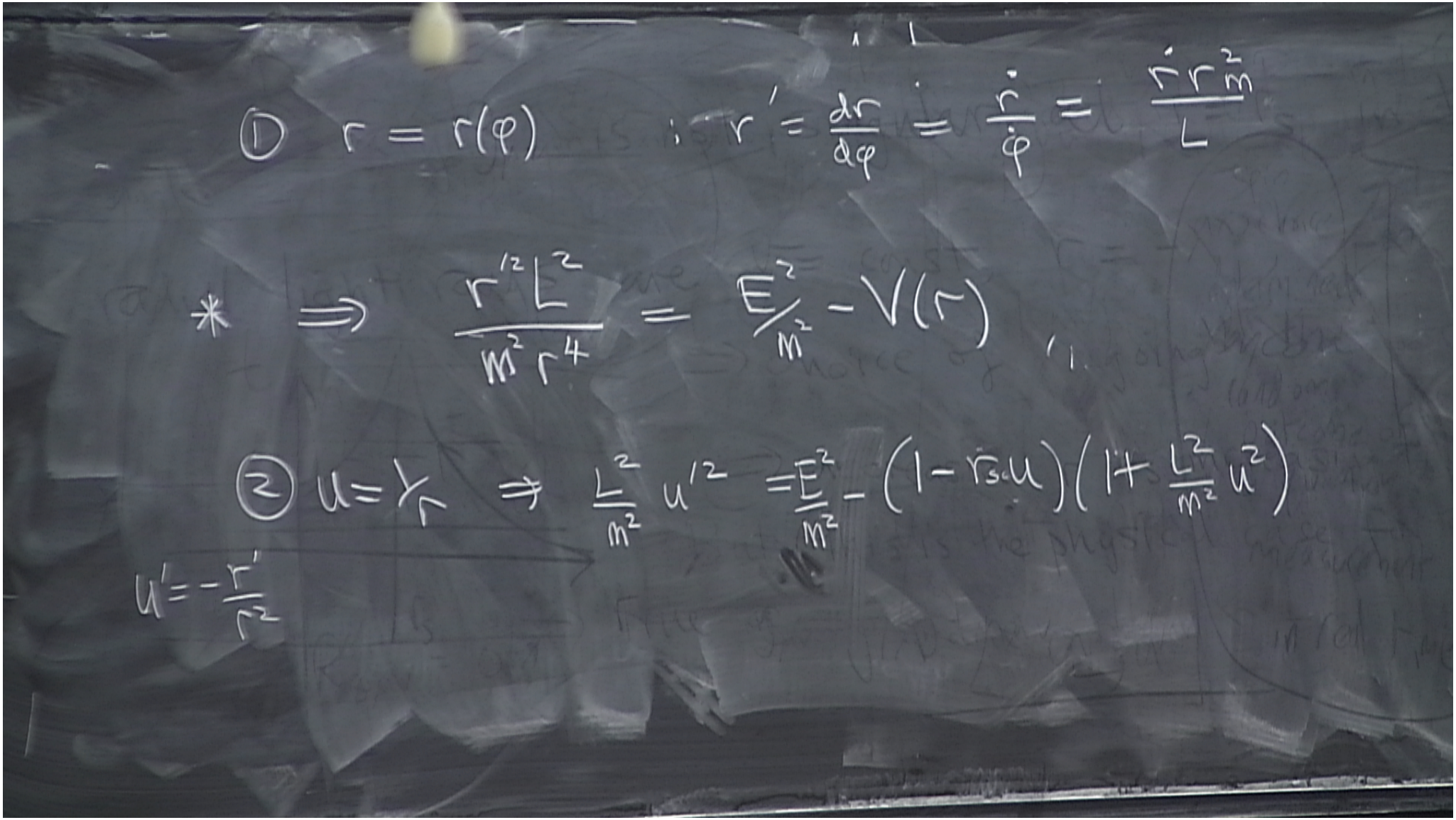
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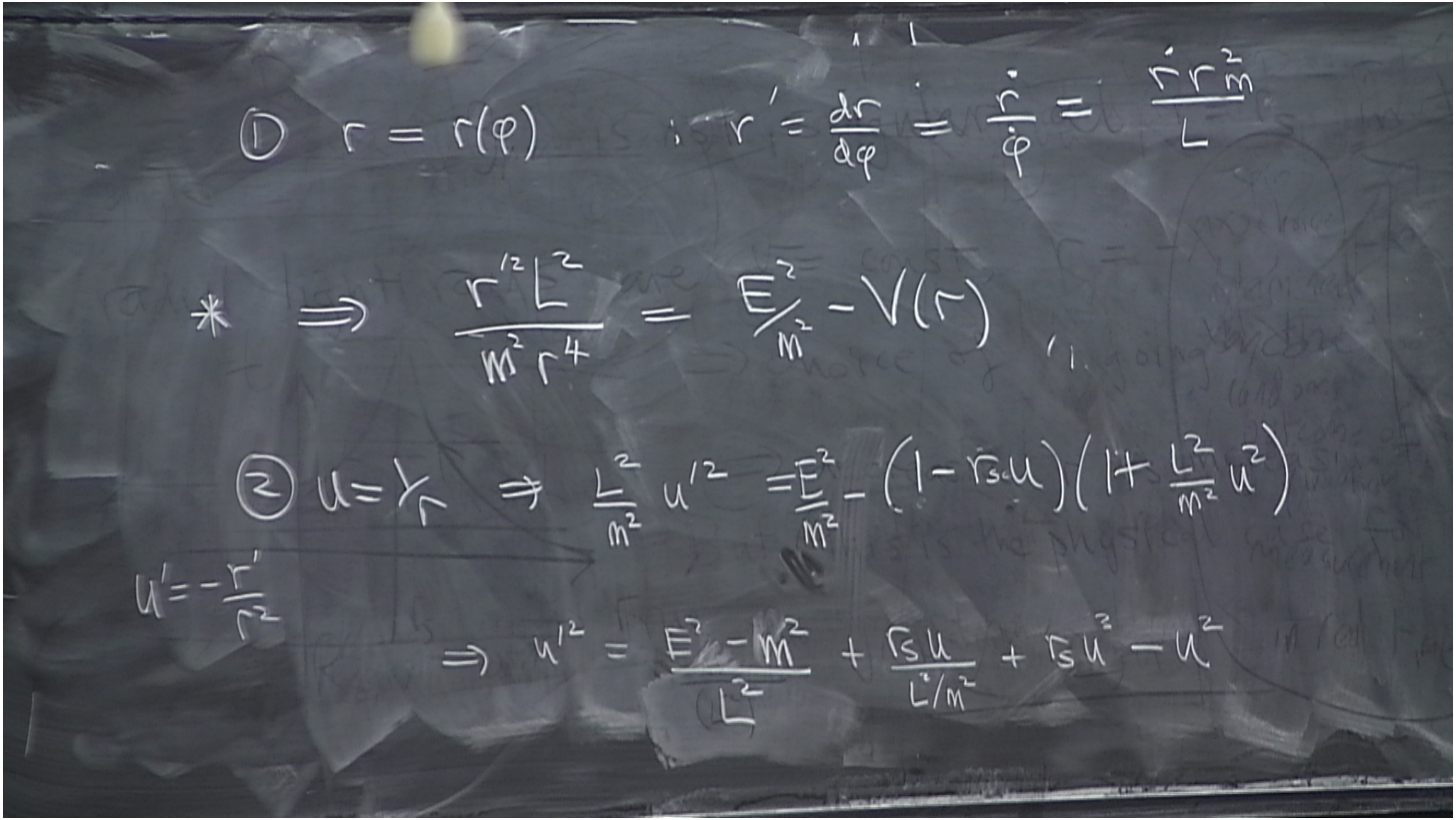
$$* \Rightarrow \frac{r'^2 L^2}{m^2 r^4} = \frac{E^2}{m^2} - V(r)$$

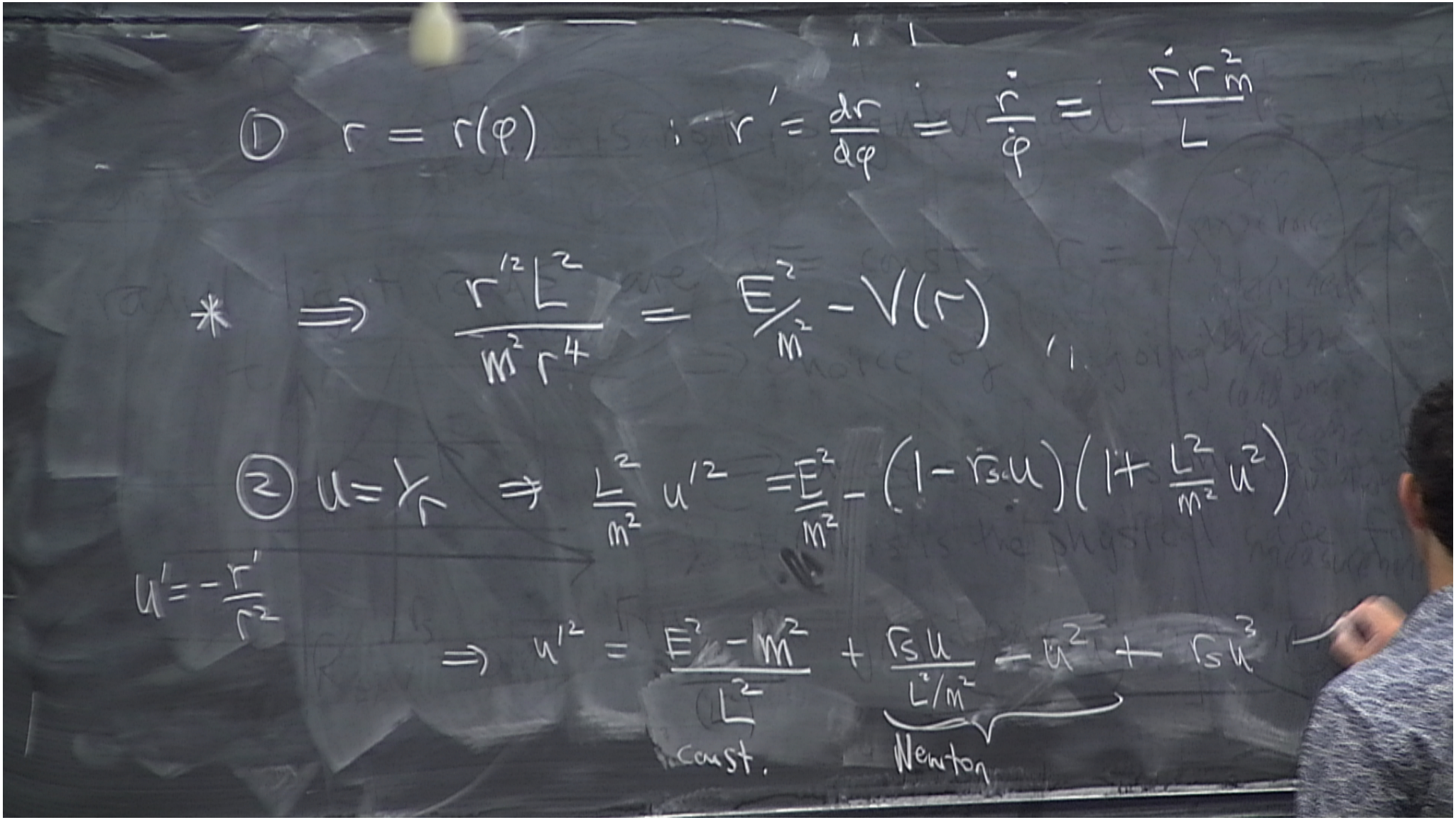

KE
PE
E
$\dot{r}^2 + V(r) = E^2/m^2$
(*)
$V(r) = \left(1 - \frac{r_s}{r}\right)\left(1 + \frac{L^2}{m^2 r^2}\right)$
$V(r)$
$+L^2/m^2r^2$
E^2/m^2
1
$1 - r_s/r$
r
$-\frac{r_s L^2}{m^2 r^3}$
$\frac{E^2}{m^2} > 1$
$\frac{E^2}{m^2} < 1$

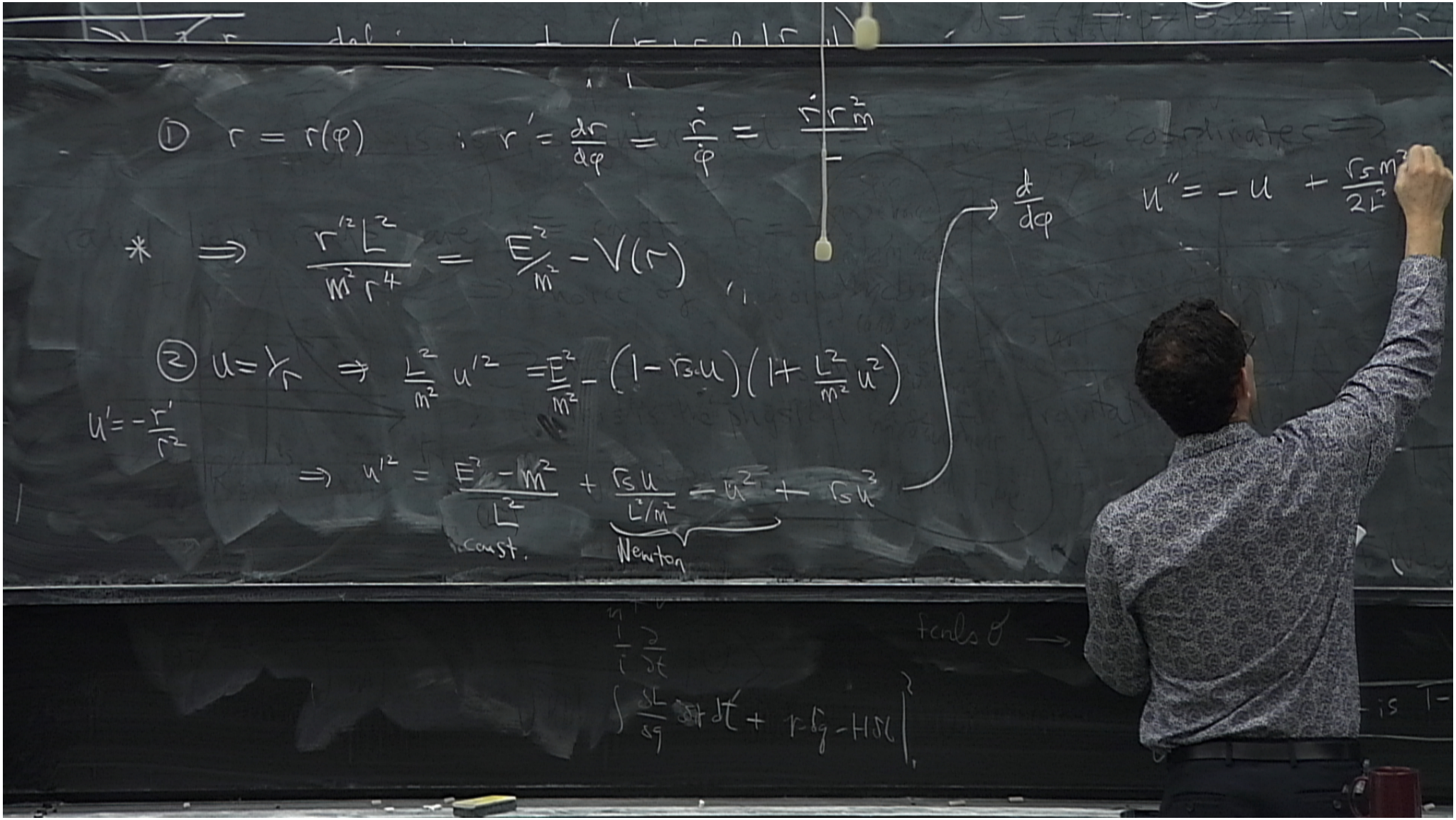

① $r = r(\varphi)$; $r' = \frac{dr}{d\varphi} = \frac{\dot{r}}{\dot{\varphi}} = \frac{\dot{r} r m^2}{L}$
$* \Rightarrow \frac{r^2 L^2}{m^2 r^4} = \frac{E^2}{m^2} - V(r)$
② $u = 1/r$

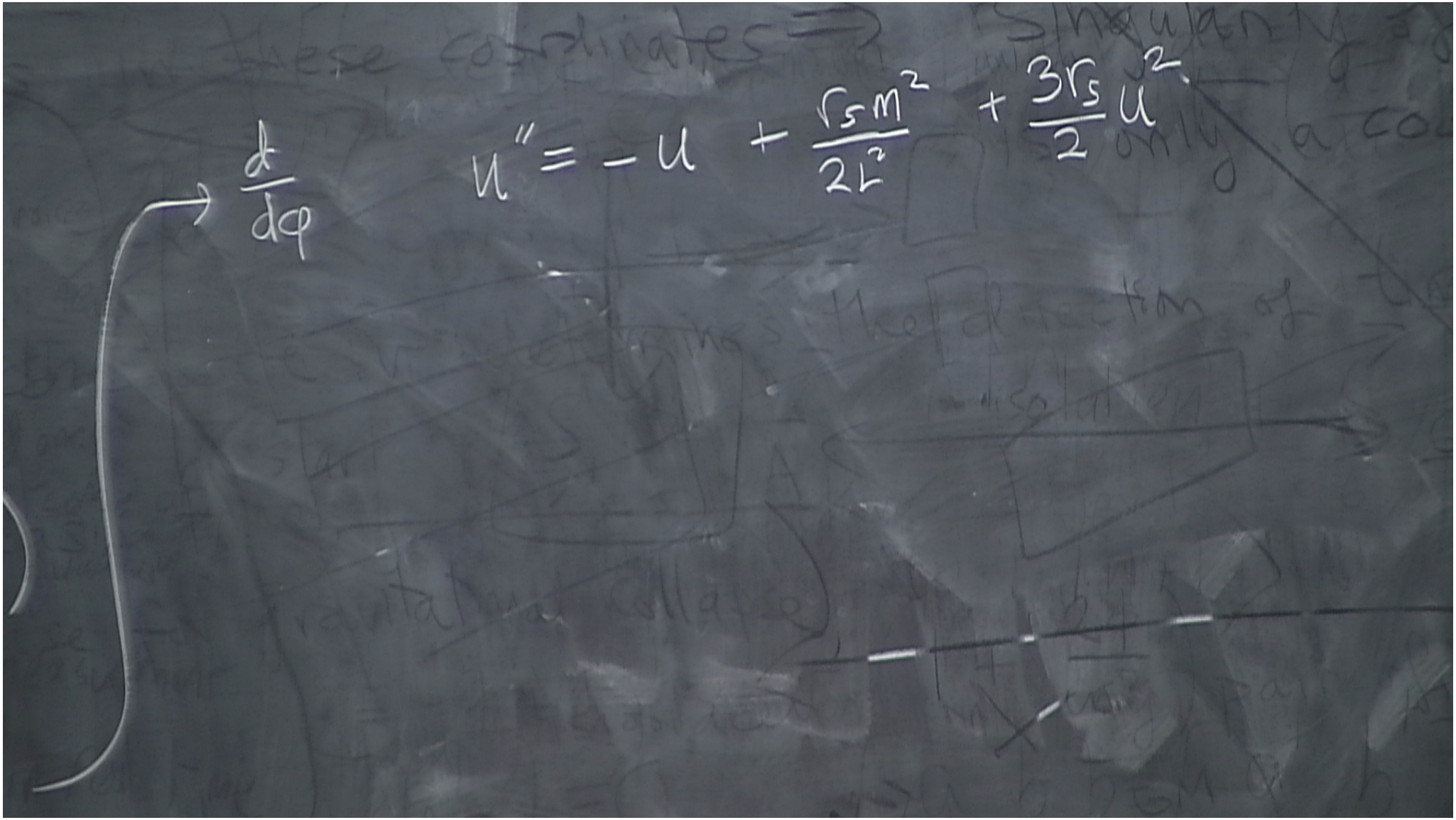

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$* \Rightarrow \frac{r'^2 L^2}{m^2 r^4} = \frac{E^2}{m^2} - V(r)$
② $u = \frac{1}{r} \Rightarrow \frac{L^2}{m^2} u'^2 = E^2 \left(1 - r_s u\right)\left(1 + \frac{L^2}{m^2} u^2\right)$
$u' = -\frac{r'}{r^2}$

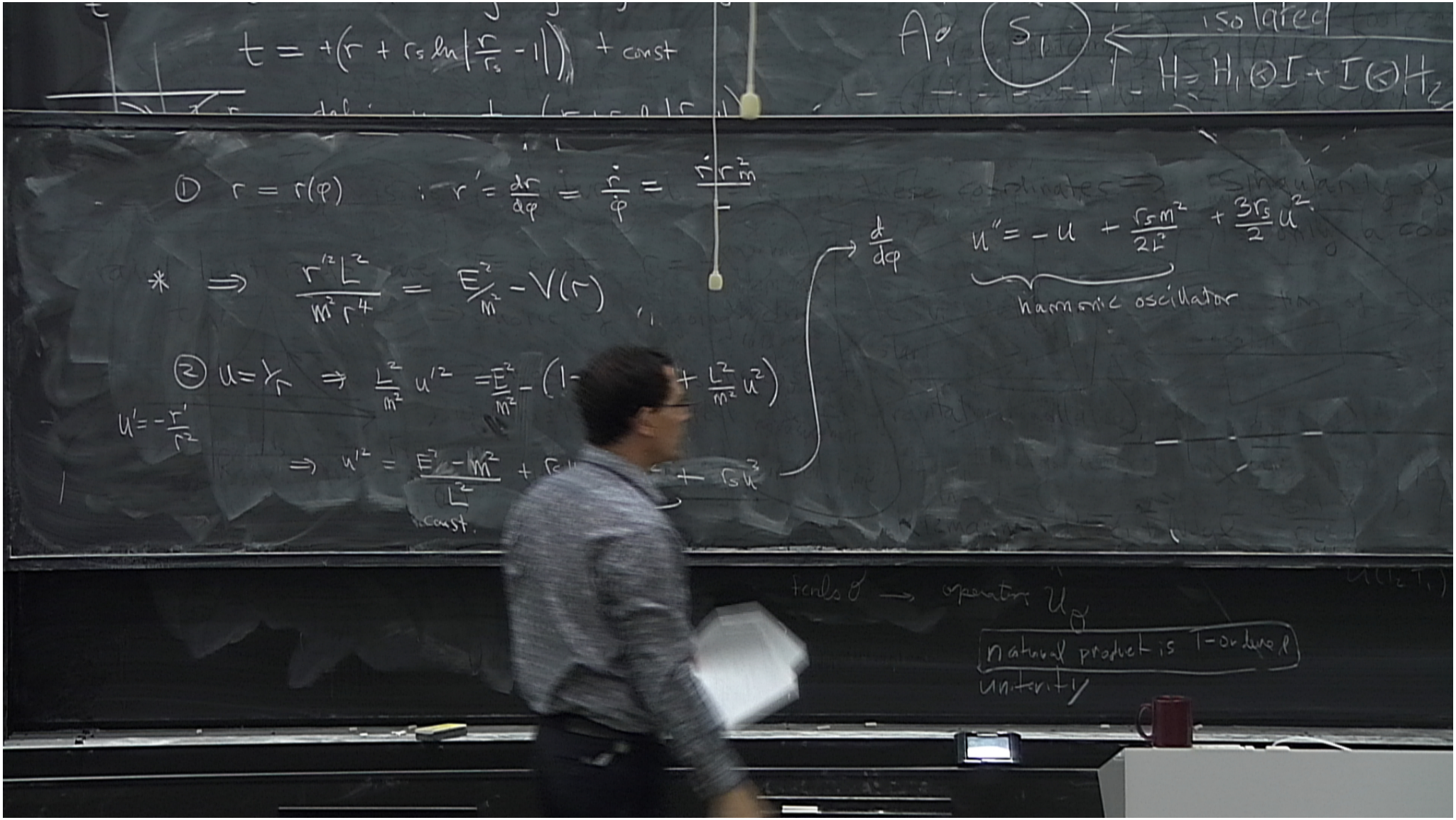

① $r = r(\varphi)$; $r' = \frac{dr}{d\varphi} = \frac{\dot{r}}{\dot{\varphi}} = \frac{\dot{r} r^2 m}{L}$
$* \Rightarrow \frac{r'^2 L^2}{m^2 r^4} = \frac{E^2}{m^2} - V(r)$
② $u = 1/r \Rightarrow \frac{L^2}{m^2} u'^2 = \frac{E^2}{m^2} - (1 - r_s u)(1 + \frac{L^2}{m^2} u^2)$
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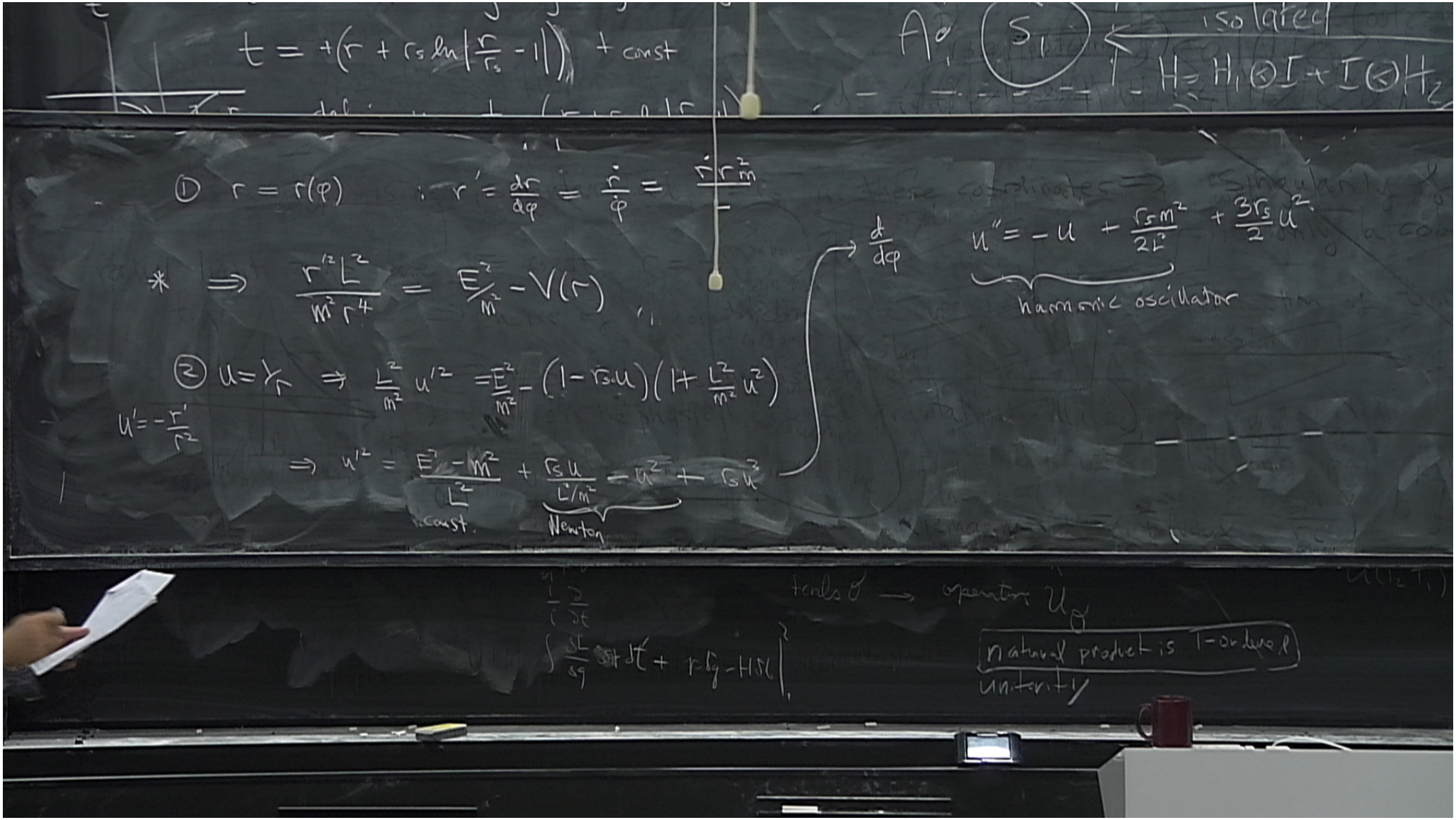

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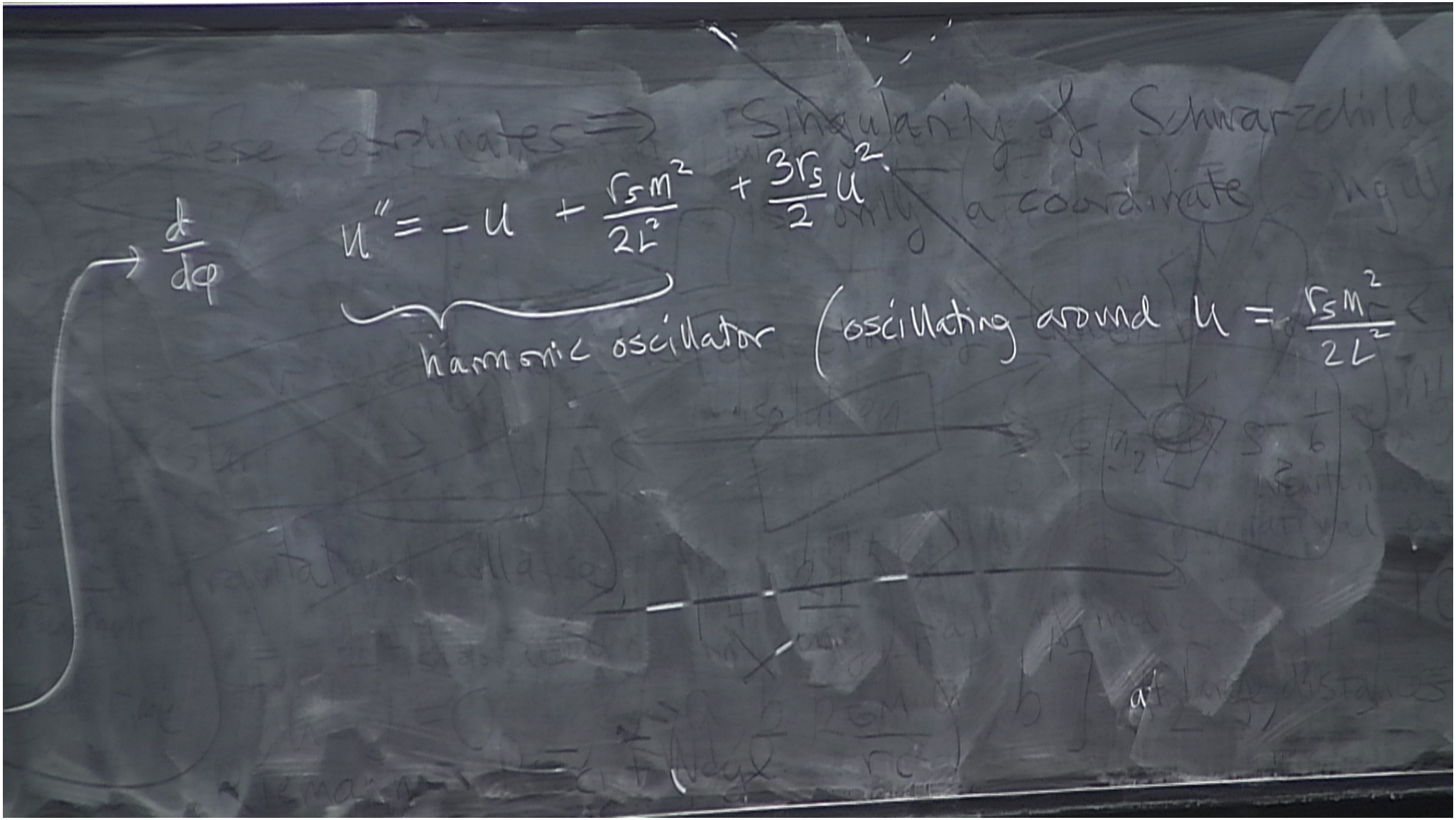

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Const.
Newton

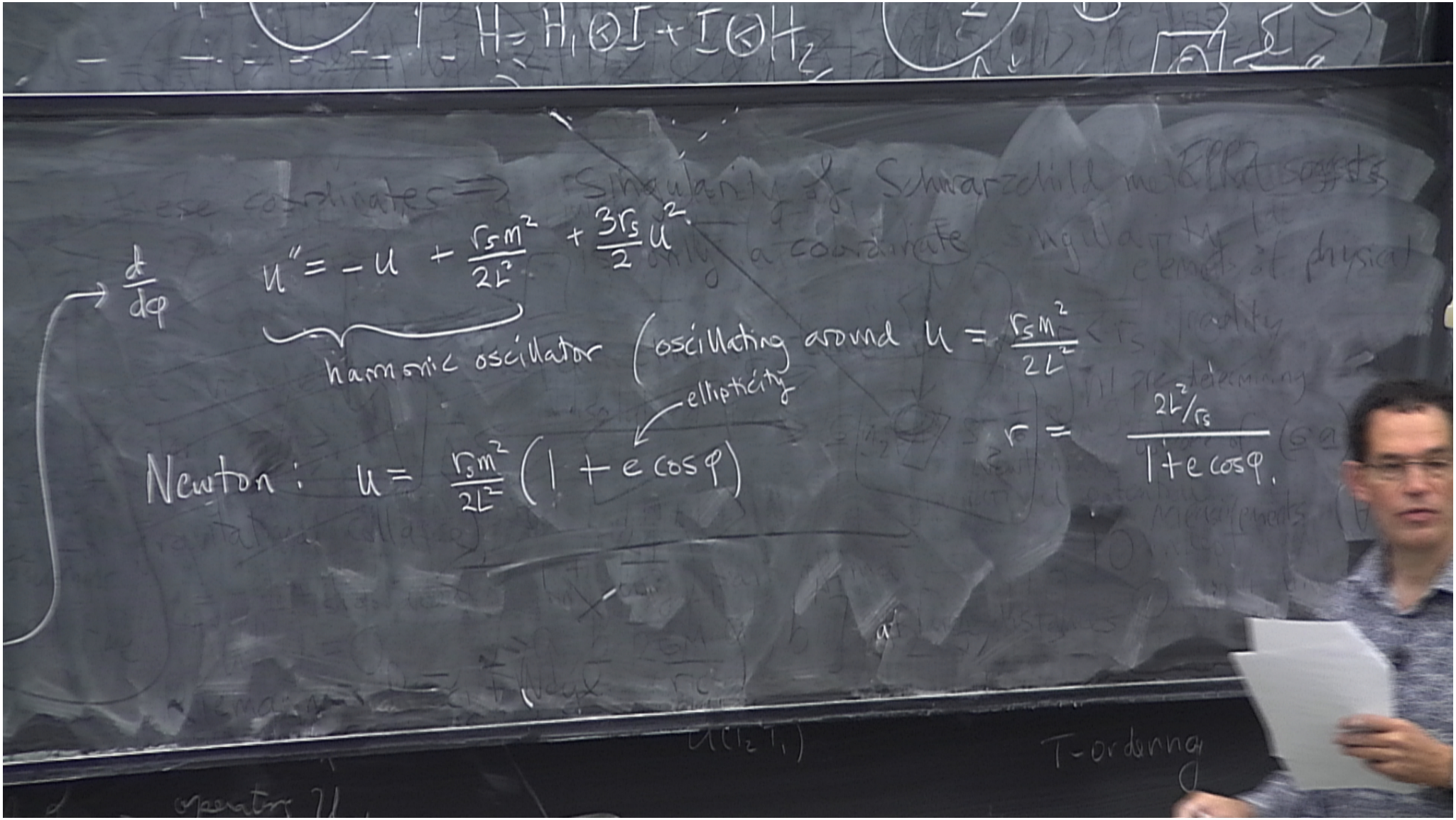

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const.
Newton
$\frac{1}{i}\frac{\partial}{\partial t}$
$\int \frac{\partial L}{\partial \dot q} \dot q\, dt + r\dot\theta - H\delta t$

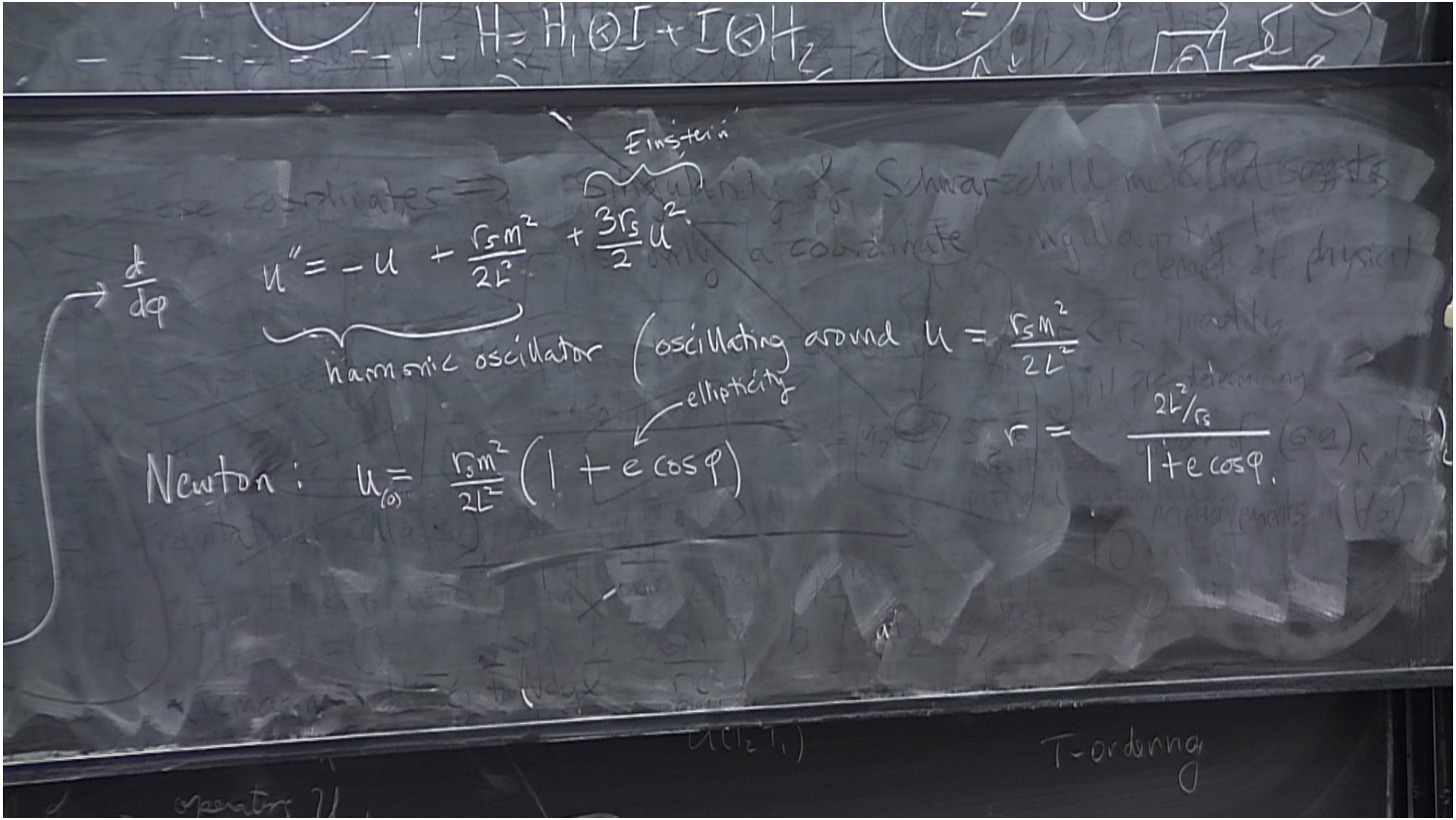

$\frac{d}{d\varphi}$
$u'' = -u + \frac{r_s m^2}{2L^2} + \frac{3r_s}{2}u^2$

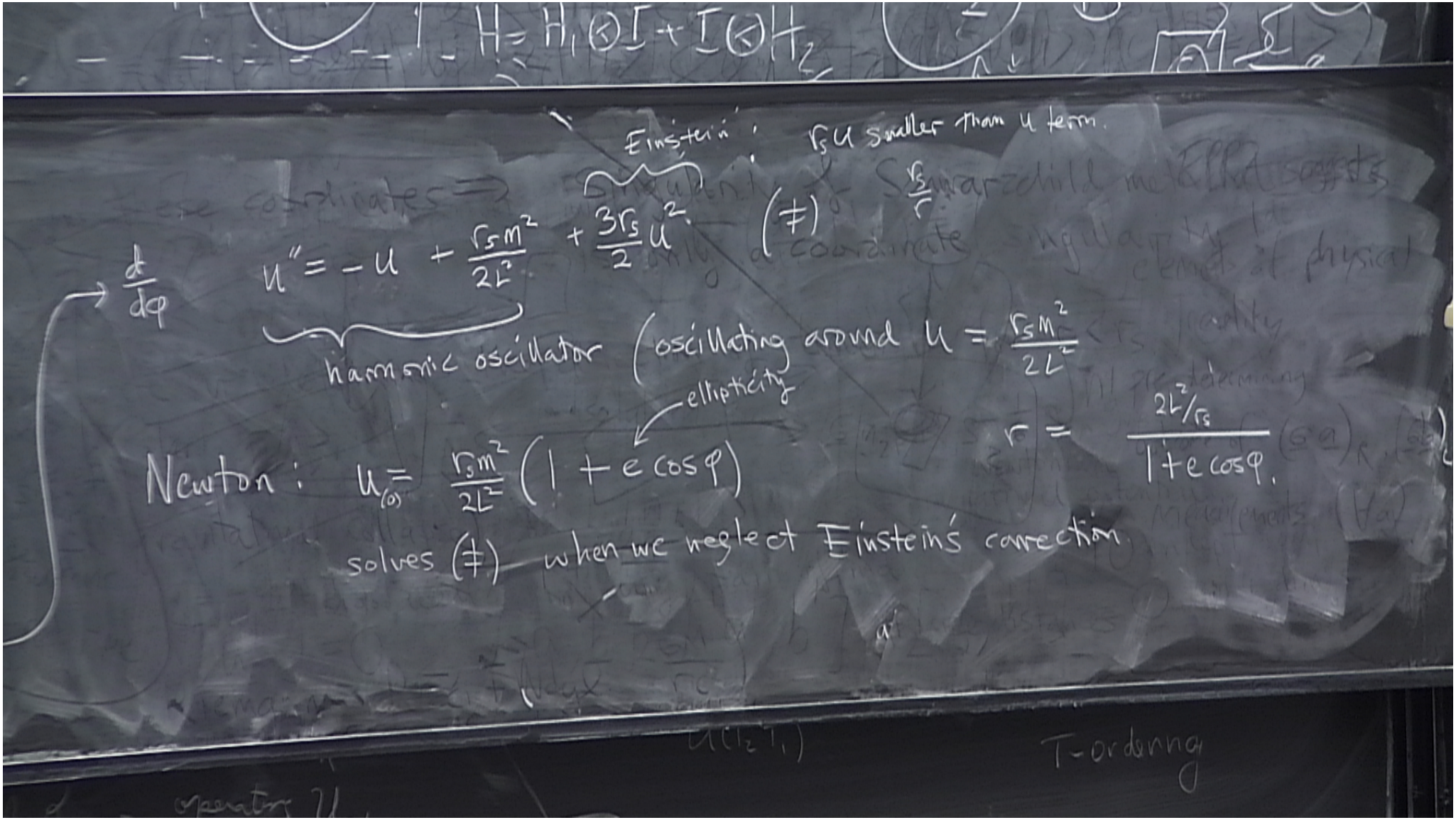

$t = +\left(r + r_s \ln\left|\frac{r}{r_s} - 1\right|\right) + \text{const}$
isolated
$H = H_1 \otimes I + I \otimes H_2$
① $r = r(\varphi)$: $r' = \frac{dr}{d\varphi} = \frac{\dot{r}}{\dot{\varphi}} = \frac{\dot{r} r^2 m}{L}$
$\frac{d}{d\varphi}$
$u'' = -u + \frac{r_s m^2}{2L^2} + \frac{3 r_s}{2} u^2$
harmonic oscillator
$* \Rightarrow \frac{r'^2 L^2}{m^2 r^4} = \frac{E^2}{m^2} - V(r)$
② $u = \frac{1}{r} \Rightarrow \frac{L^2}{m^2} u'^2 = \frac{E^2}{m^2} - \left(1 - \right) + \frac{L^2}{m^2} u^2$
$u' = -\frac{r'}{r^2}$
$\Rightarrow u'^2 = \frac{E^2 - m^2}{L^2} + r_s u^3$
const.
states ψ → operator U_g
natural product is 1-or-zero
unitarity

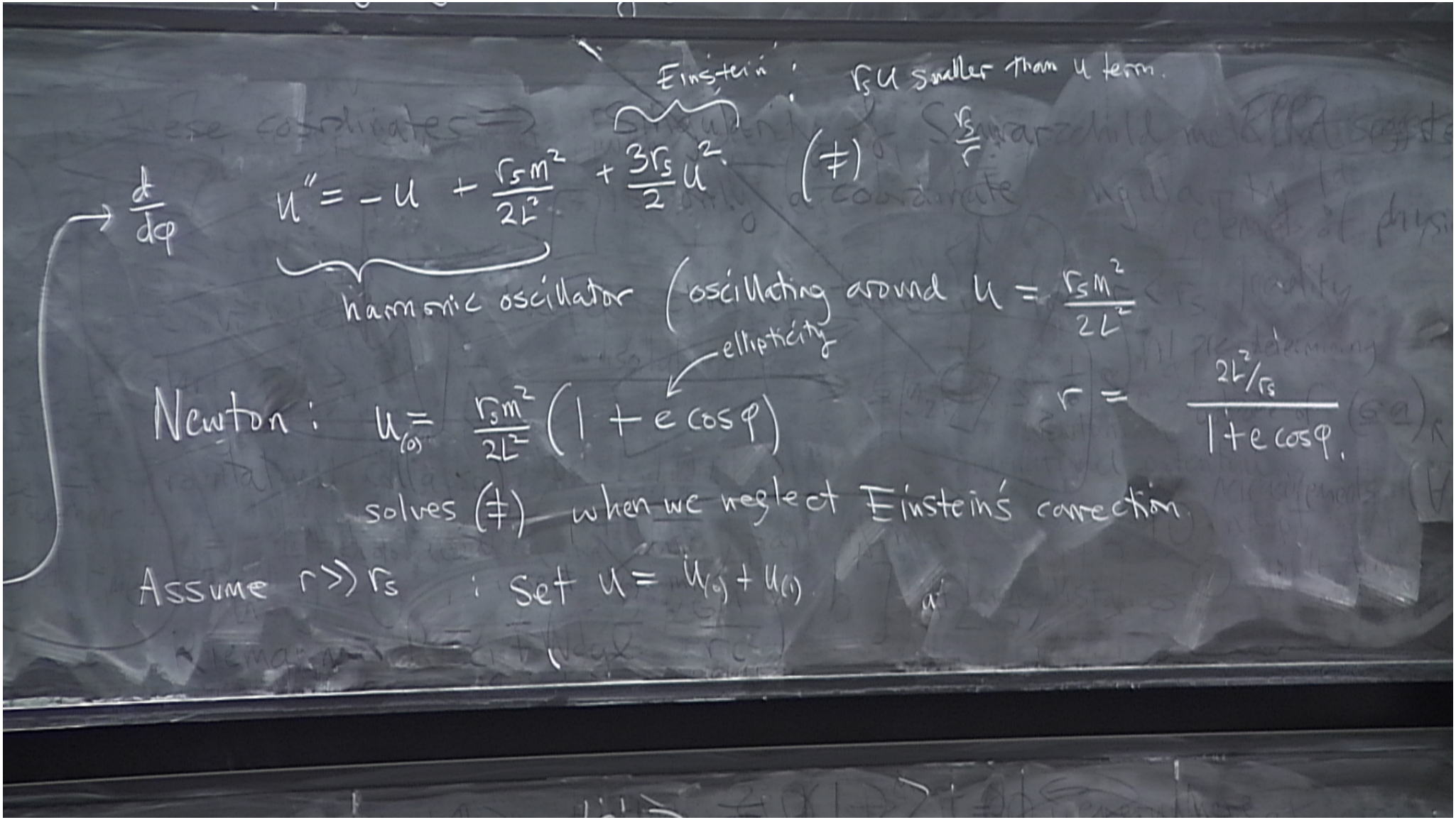

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const.
Newton
$\frac{1}{i}\frac{\partial}{\partial t}$
$\int \frac{\partial L}{\partial q} \delta t \, dt + r \dot{q} - H \delta t \Big|_1^2$
tends to $\rightarrow$ operator U_g
natural product is 1-cocycle
unitarity

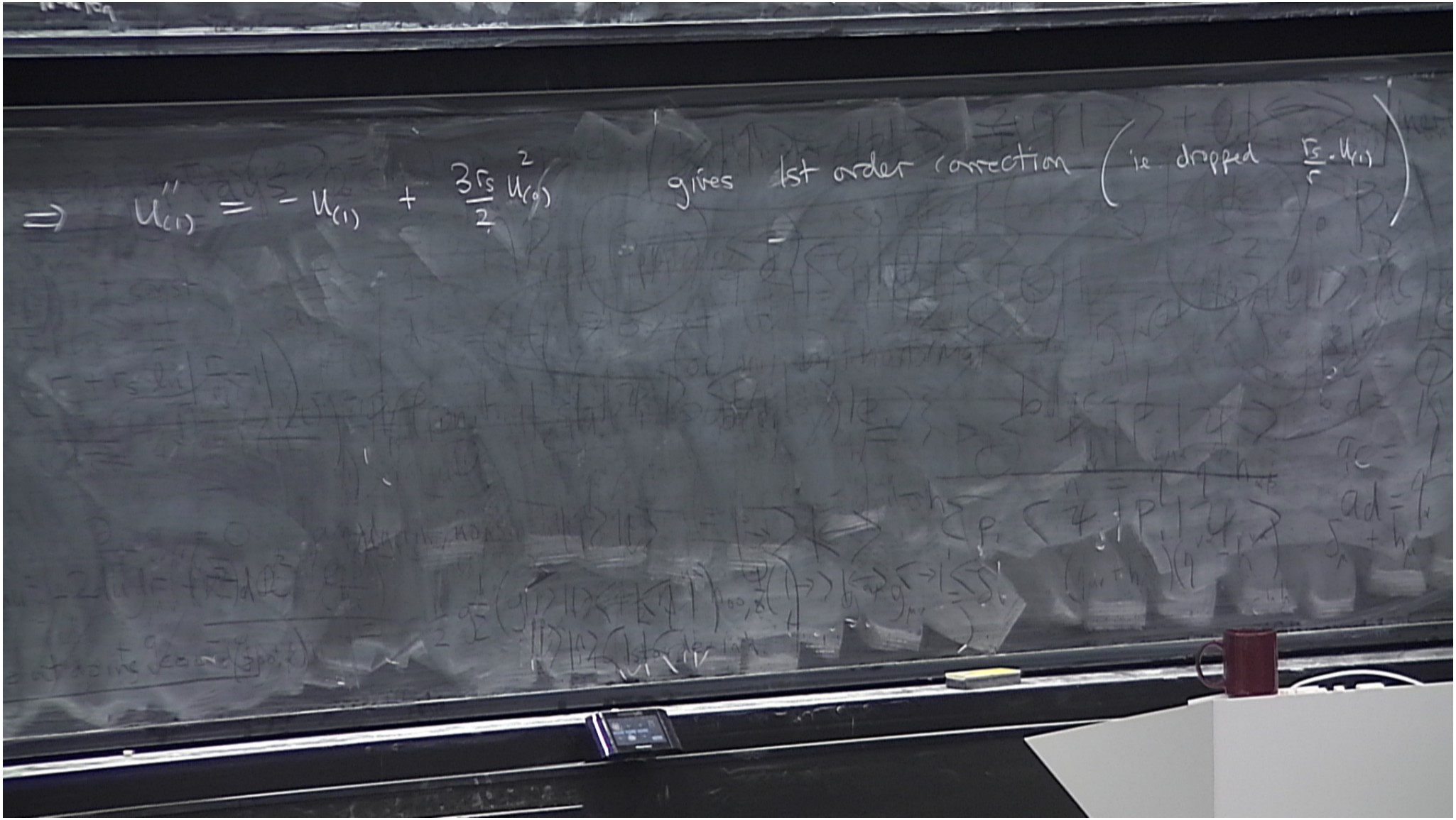

$\frac{d}{d\varphi}$
$u'' = -u + \frac{r_s M^2}{2L^2} + \frac{3r_s}{2}u^2$
harmonic oscillator (oscillating around $u = \frac{r_s M^2}{2L^2}$

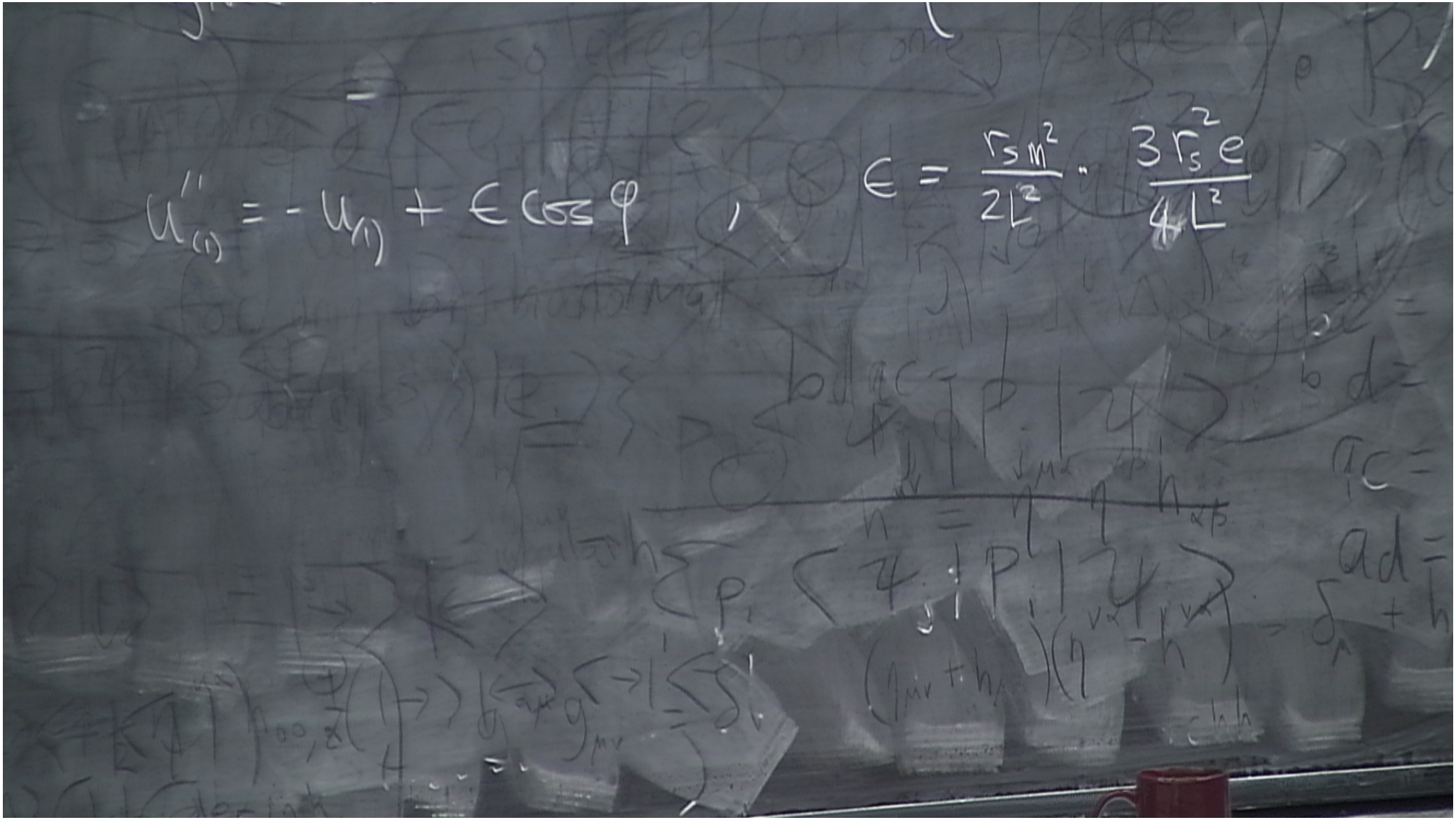

$H = H_1 \otimes I + I \otimes H_2$
$\frac{d}{d\varphi}$
$u'' = -u + \frac{r_s m^2}{2L^2} + \frac{3 r_s}{2} u^2$
harmonic oscillator
oscillating around $u = \frac{r_s m^2}{2L^2}$
ellipticity
Newton: $u = \frac{r_s m^2}{2L^2}(1 + e\cos\varphi)$
$r = \frac{2L^2/r_s}{1 + e\cos\varphi}$.
T-ordering

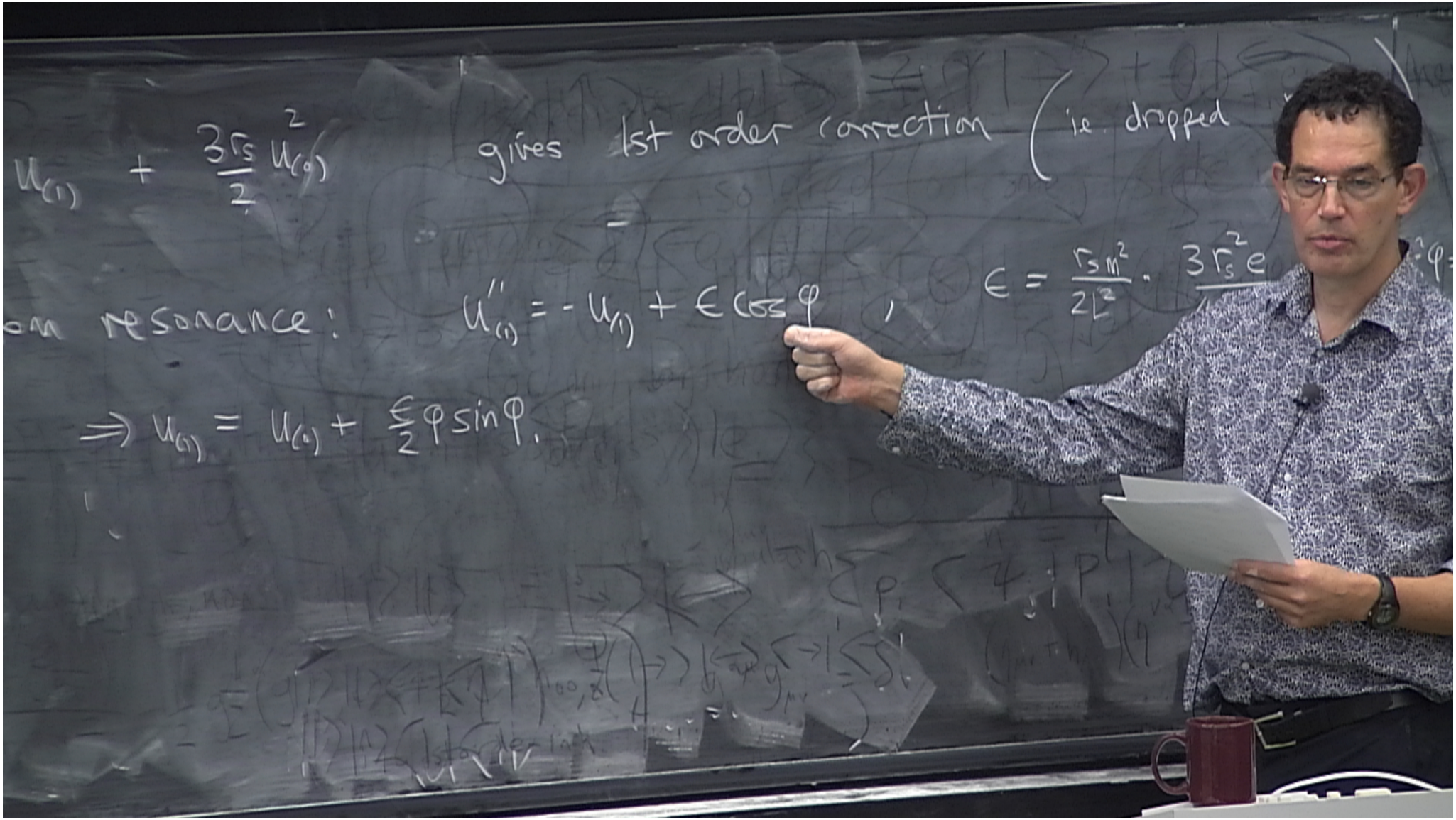

$H = H_1 \otimes I + I \otimes H_2$
Einstein
$\frac{d}{d\varphi}$
$u'' = -u + \frac{r_s m^2}{2L^2} + \frac{3r_s}{2}u^2$
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ellipticity
Newton: $u_{(0)} = \frac{r_s m^2}{2L^2}(1 + e\cos\varphi)$
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T-ordering

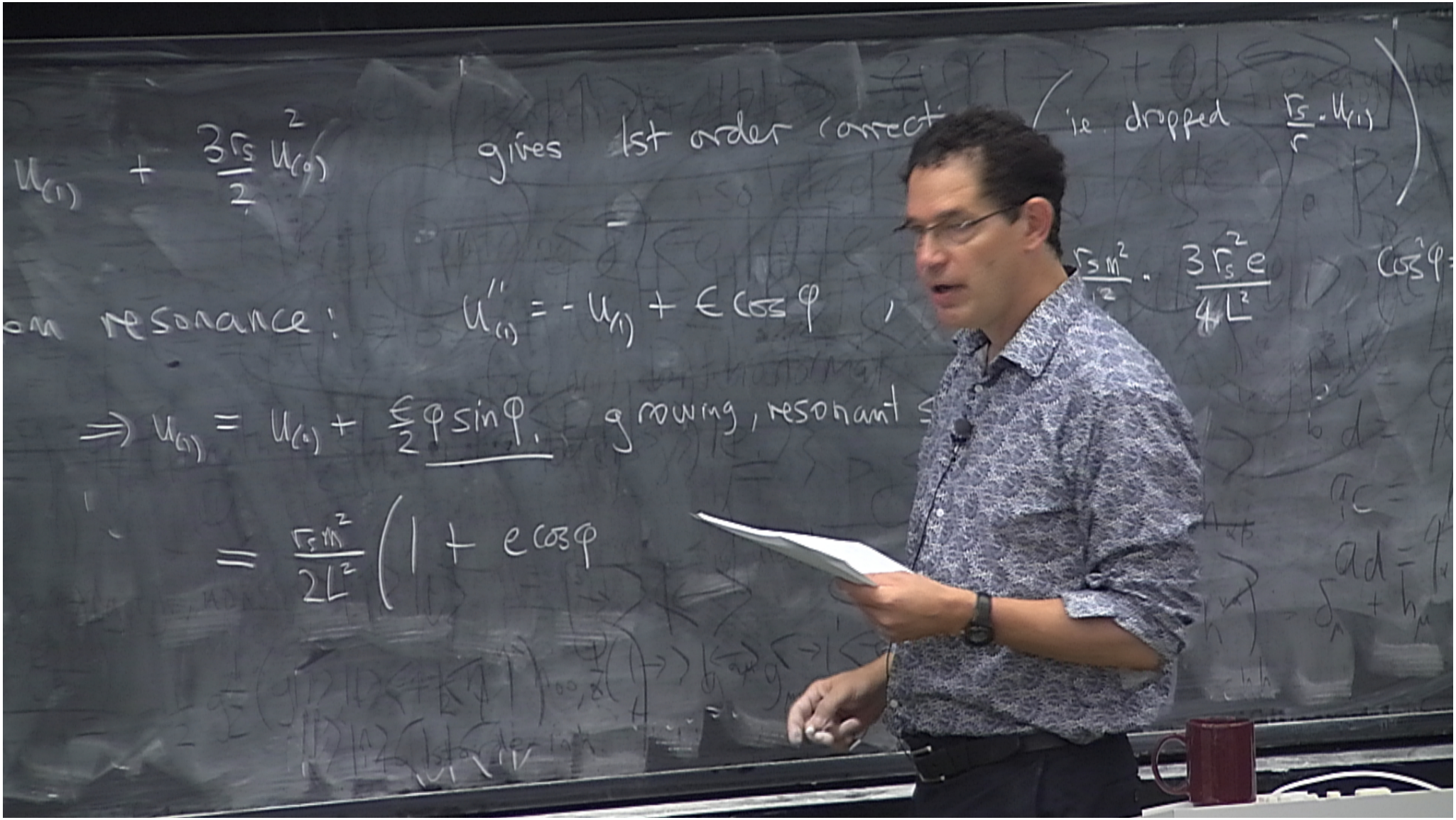

Einstein: $r_s u$ smaller than u term.
$\frac{d}{d\varphi}$
$u'' = -u + \frac{r_s m^2}{2L^2} + \frac{3 r_s}{2} u^2$ $(\neq)$ $\frac{r_s}{r}$
harmonic oscillator (oscillating around $u = \frac{r_s m^2}{2L^2}$
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Newton: $u_{(0)} = \frac{r_s m^2}{2L^2}(1 + e\cos\varphi)$
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solves $(\neq)$ when we neglect Einstein's correction
T-ordering


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solves $(\neq)$ when we neglect Einstein's correction.
Assume $r \gg r_s$: Set $u = u_{(0)} + u_{(1)}$


$\Rightarrow u_{(1)}'' = -u_{(1)} + \frac{3r_s}{2} u_{(0)}^2$
gives 1st order correction
(ie dropped $\frac{r_s}{r} \cdot u_{(1)}$)


$u''_{(1)} = -u_{(1)} + \epsilon \cos\varphi$, $\epsilon = \frac{r_s m^2}{2L^2} \cdot \frac{3 r_s^2 e}{4 L^2}$


$u_{(1)} + \frac{3r_s}{2} u_{(0)}^2$ gives 1st order correction (ie. dropped
on resonance: $u''_{(1)} = -u_{(1)} + \epsilon \cos\varphi$, $\epsilon = \frac{r_s m^2}{2L^2} \cdot \frac{3r_s^2 e}{}$
$\Rightarrow u_{(1)} = u_{(0)} + \frac{\epsilon}{2}\varphi \sin\varphi$.

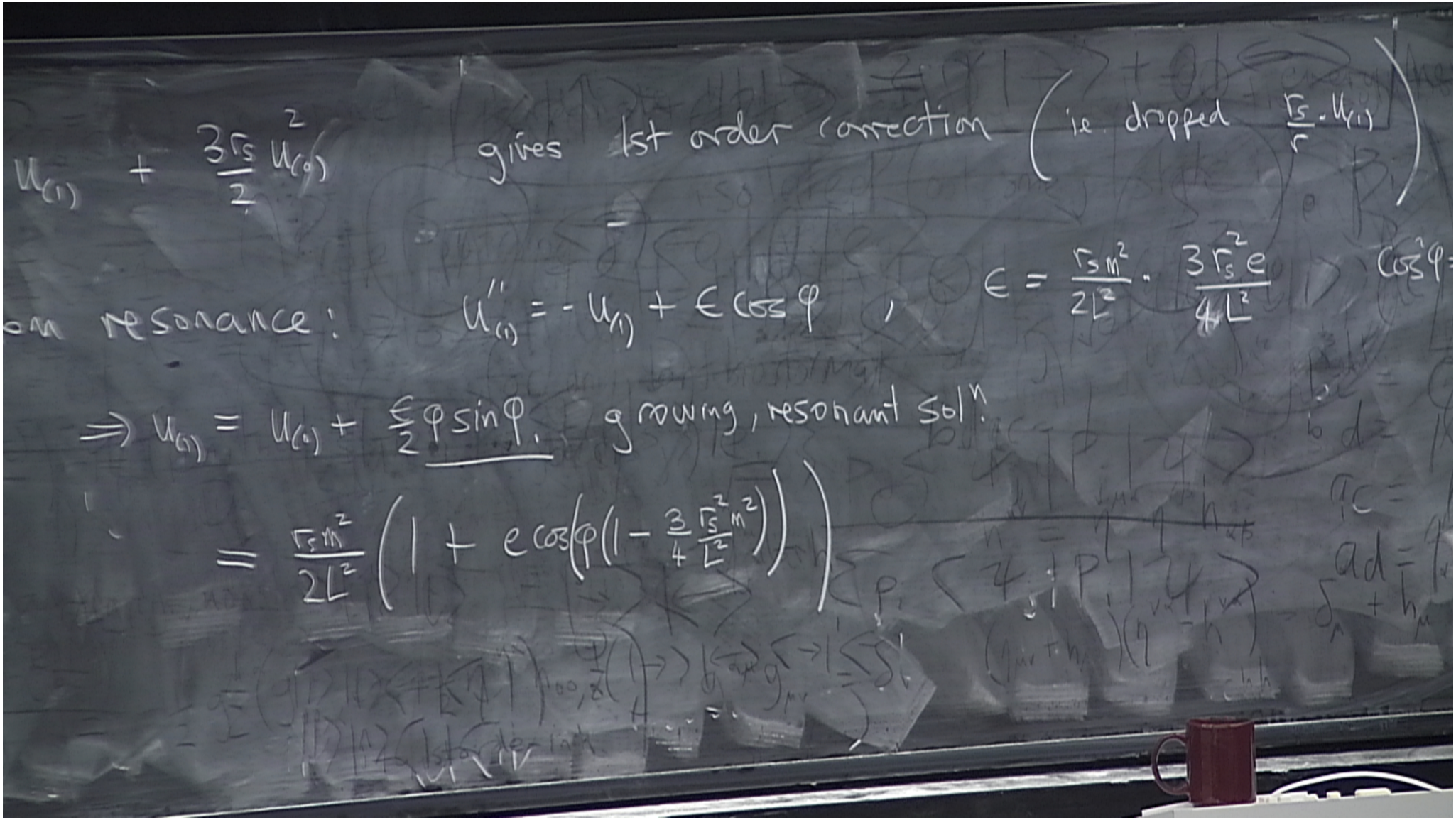

$u_{(1)} + \frac{3r_s}{2} u_{(0)}^2$ gives 1st order correct
(ie dropped $\frac{r_s}{r} \cdot u_{(1)}$)
on resonance: $u''_{(1)} = -u_{(1)} + \epsilon \cos\varphi$,
$\frac{r_s m^2}{2} \cdot \frac{3 r_s^2 e}{4 L^2}$ $\cos^2\varphi$
$\Rightarrow u_{(1)} = u_{(0)} + \frac{\epsilon}{2} \varphi \sin\varphi$. growing, resonant
$= \frac{r_s m^2}{2L^2} \left(1 + e \cos\varphi \right.$

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$\Rightarrow u_{(1)} = u_{(0)} + \frac{\epsilon}{2} \varphi \sin\varphi$. growing, resonant sol^n!

$= \frac{r_s m^2}{2L^2} \left(1 + e\cos\left(\varphi\left(1 - \frac{3}{4}\frac{r_s^2}{L^2}m^2\right)\right)\right)$


$u_{(1)} + \frac{3r_s}{2} u_{(0)}^2$ gives 1st order correction (ie dropped $\frac{r_s}{r} \cdot u_{(1)}$)
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$= \frac{r_s m^2}{2L^2}\left(1 + e\cos\left(\varphi\left(1 - \frac{3}{4}\frac{r_s^2 m^2}{L^2}\right)\right)\right)$

Const. Newton

$$\Rightarrow u''_{(1)} = -u_{(1)} + \frac{3r_s}{2}u_{(0)}^2$$ gives 1st order correction

Most interesting term comes from resonance: $u''_{(1)} = -u_{(1)} + \epsilon\cos\varphi$, $\epsilon =$

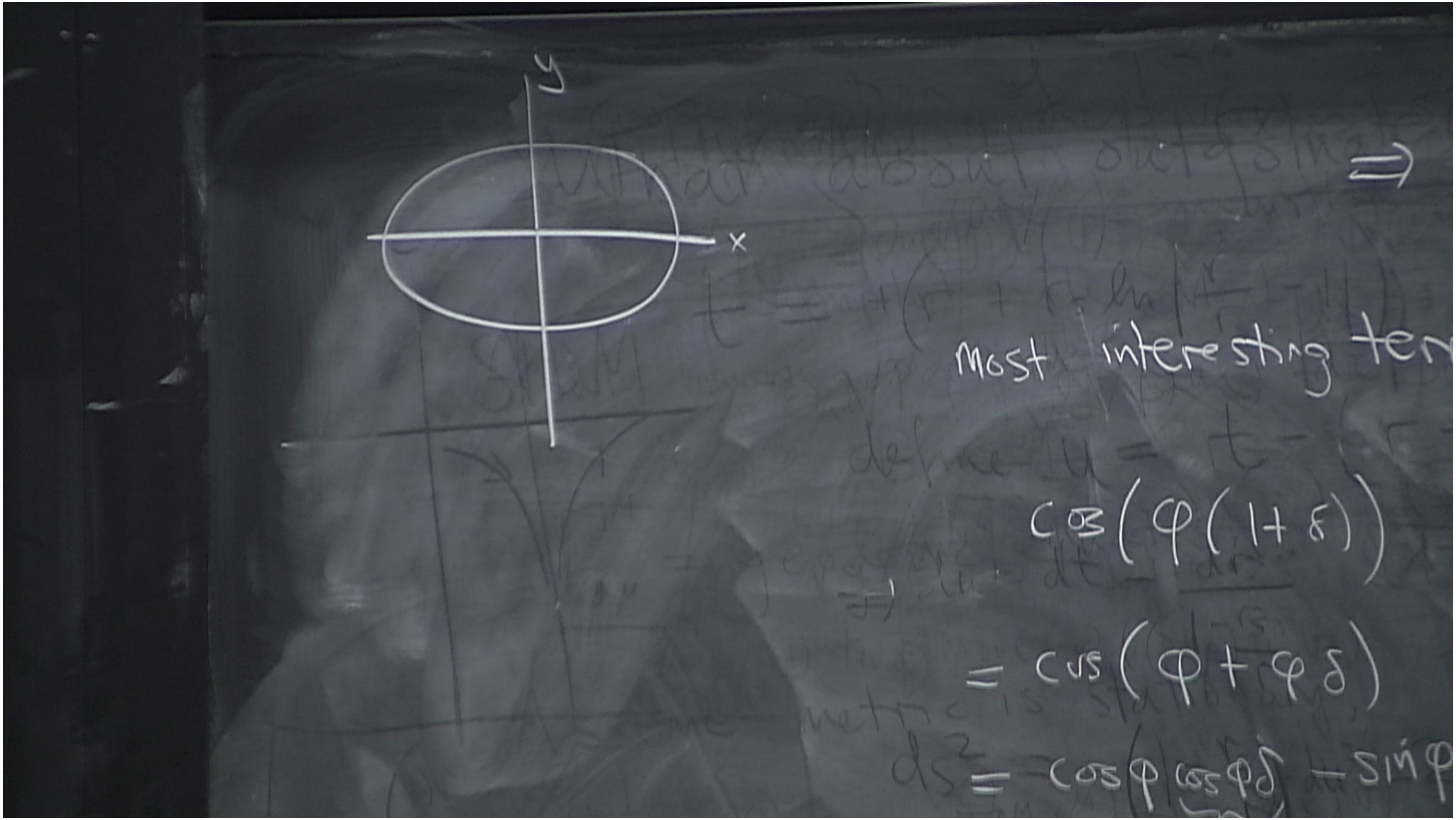
$$\cos\left(\varphi(1+\delta)\right)$$

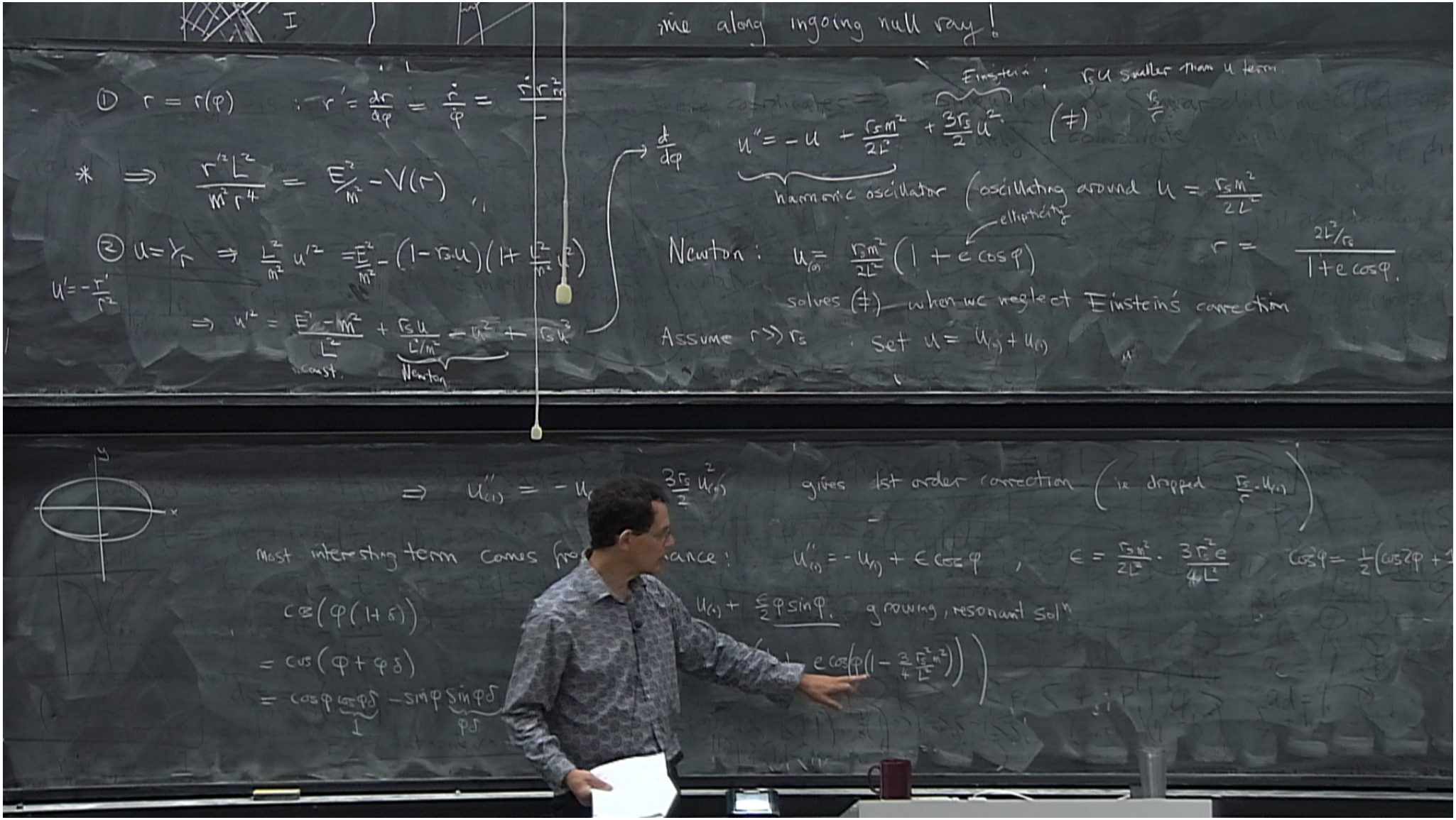
$$= \cos\left(\varphi + \varphi\delta\right)$$

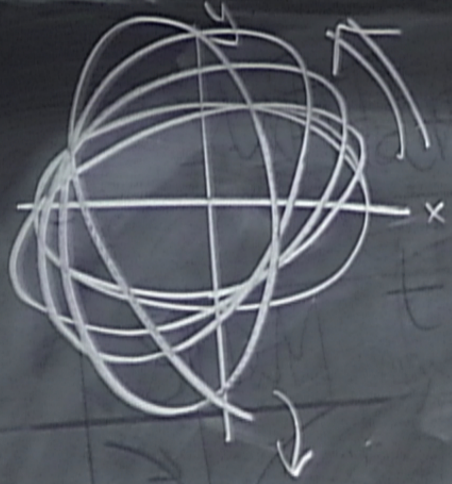
$$= \cos\varphi\underbrace{\cos\varphi\delta}_{1} - \sin\varphi\underbrace{\sin\varphi\delta}_{\varphi\delta}$$

$$\Rightarrow u = u_{(0)} + \frac{\epsilon}{2}\varphi\sin\varphi$$ growing, resonant sol'n

$$= \frac{r_s m^2}{2L^2}\left(1 + e\cos\left(\varphi\left(1 - \frac{3}{4}\frac{r_s^2 m^2}{L^2}\right)\right)\right)$$


y
x
Most interesting ten
cos(φ(1+δ))
= cos(φ + φδ)
ds² = cos φ cos φδ + sin φ


ine along ingoing null ray!
① $r = r(\varphi)$: $r' = \frac{dr}{d\varphi} = \frac{\dot r}{\dot\varphi} = \frac{\dot r}{L/r^2}$
$* \Rightarrow \frac{r'^2 L^2}{m^2 r^4} = \frac{E^2}{m^2} - V(r)$,
② $u = 1/r \Rightarrow \frac{L^2}{m^2} u'^2 = \frac{E^2}{m^2} - (1 - r_s u)(1 + \frac{L^2}{m^2} u^2)$
$u' = -\frac{r'}{r^2}$
$\Rightarrow u'^2 = \frac{E^2 - m^2}{L^2} + \frac{r_s u}{L^2/m^2} - u^2 + r_s u^3$
= const. Newton
$\frac{d}{d\varphi}$
$u'' = -u + \frac{r_s m^2}{2L^2} + \frac{3r_s}{2} u^2$ (≠)
Einstein : $r_s u$ smaller than u term. $\frac{r_s}{r}$
harmonic oscillator (oscillating around $u = \frac{r_s m^2}{2L^2}$
Newton : $u_{(0)} = \frac{r_s m^2}{2L^2}(1 + e\cos\varphi)$ ellipticity
$r = \frac{2L^2/r_s}{1 + e\cos\varphi}$.
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Assume $r \gg r_s$: set $u = u_{(0)} + u_{(1)}$
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Most interesting term comes from ... cance : $u''_{(1)} = -u_{(1)} + \epsilon\cos\varphi$, $\epsilon = \frac{r_s m^2}{2L^2} \cdot \frac{3r_s^2 e}{4L^2}$ $\cos^2\varphi = \frac{1}{2}(\cos 2\varphi + 1$
$u_{(1)} + \frac{\epsilon}{2}\varphi\sin\varphi$. growing, resonant sol'n
$e\cos(\varphi(1 - \frac{3r_s^2 m^2}{L^2}))$
$\cos(\varphi(1+\delta))$
$= \cos(\varphi + \varphi\delta)$
$= \cos\varphi \underbrace{\cos\varphi\delta}_{1} - \sin\varphi \underbrace{\sin\varphi\delta}_{\varphi\delta}$



precession of the perihelion

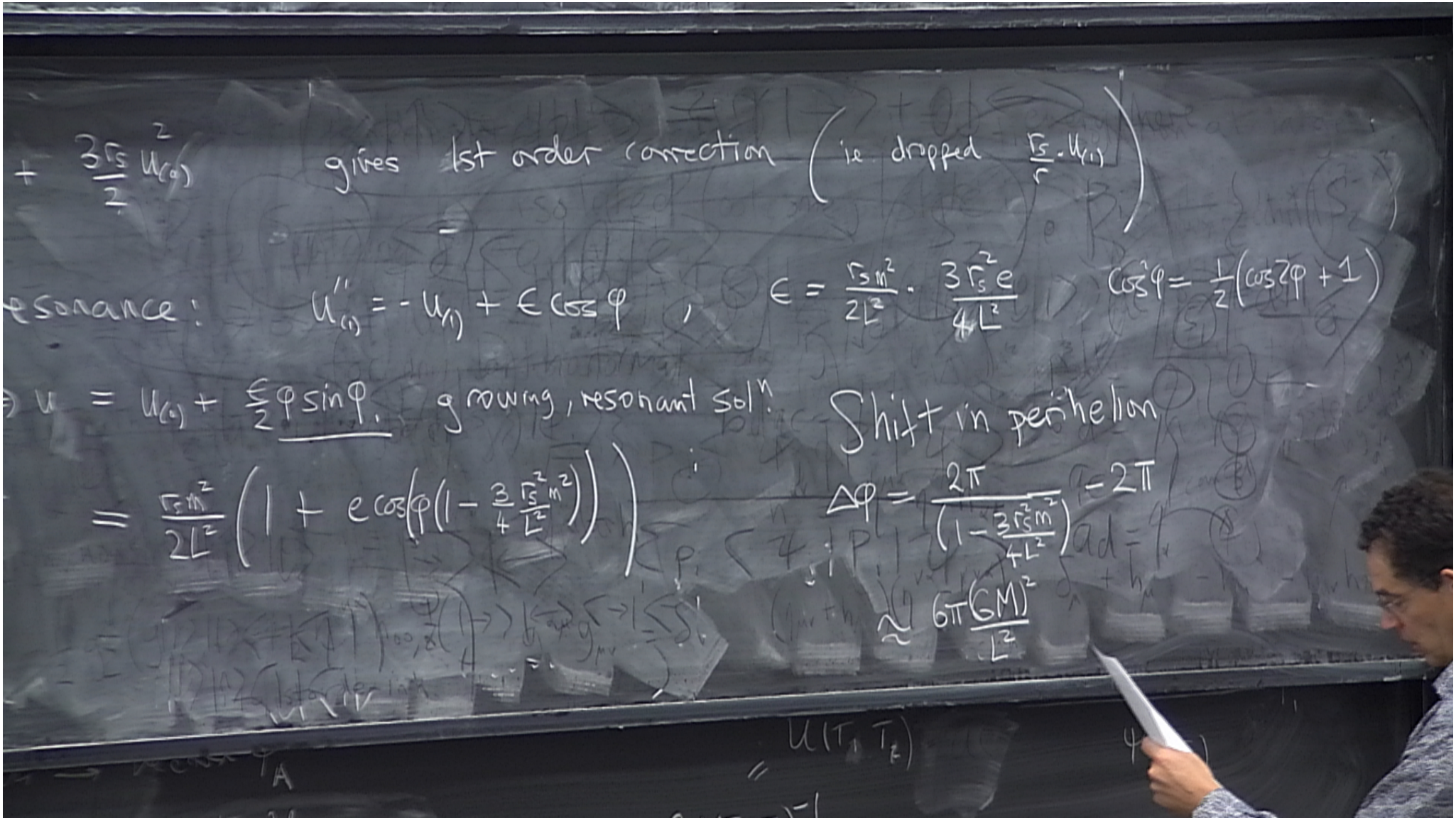
$$\Rightarrow \quad u''_{(1)} = 1 - u_{(1)}$$

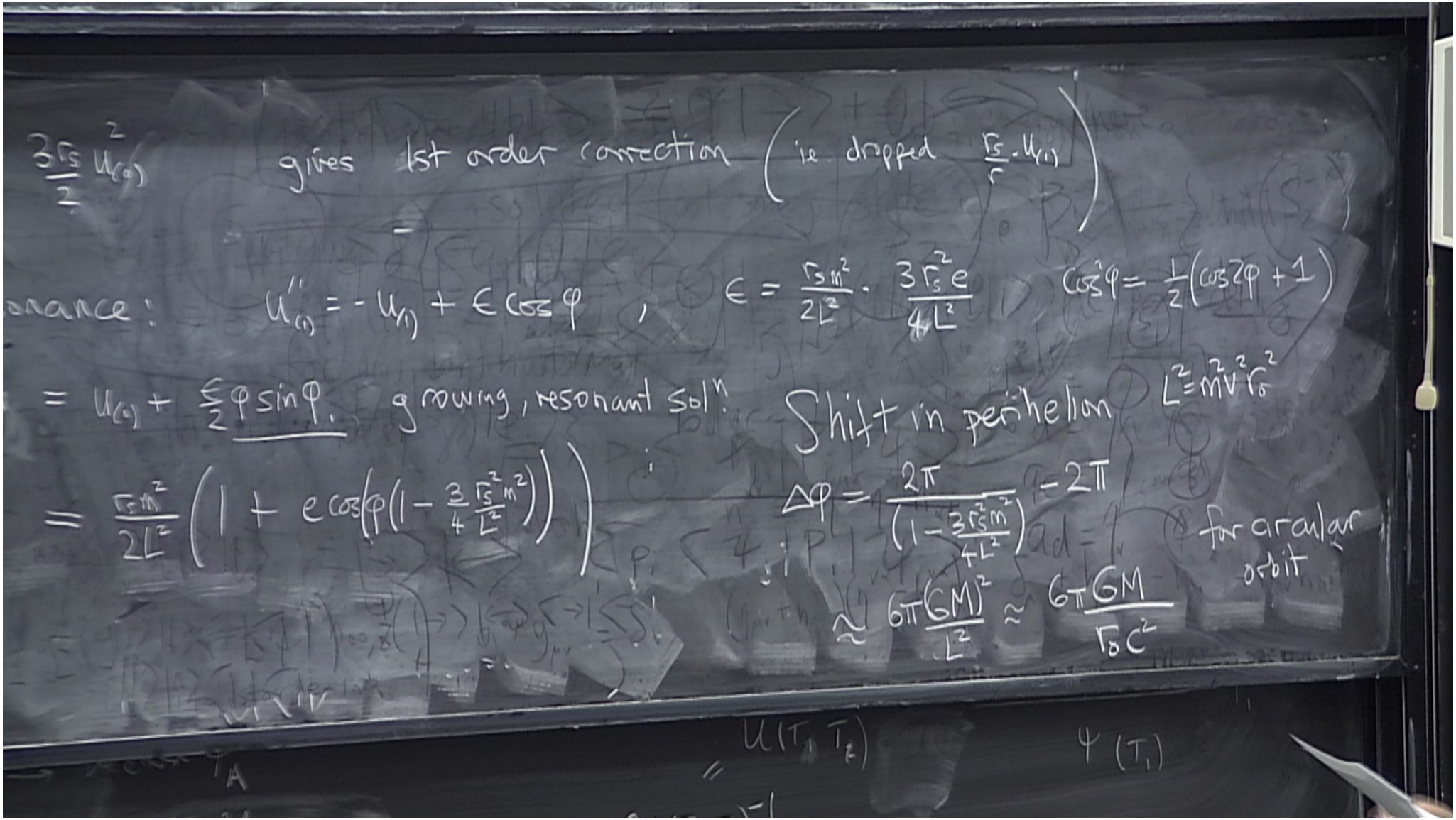
Most interesting term comes from

$$\cos\left(\varphi(1+\delta)\right)$$

$$= \cos\left(\varphi + \varphi\delta\right)$$

$$= \cos\varphi \underbrace{\cos\varphi\delta}_{1} - \sin\varphi \underbrace{\sin\varphi\delta}_{\varphi\delta}$$


+ $\frac{3r_s}{2}u_{(0)}^2$ gives 1st order correction (ie dropped $\frac{r_s}{r}\cdot u_{(1)}$)
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$u = u_{(0)} + \frac{\epsilon}{2}\varphi\sin\varphi$. growing, resonant soln!
$= \frac{r_s m^2}{2L^2}\left(1 + e\cos\left(\varphi\left(1 - \frac{3}{4}\frac{r_s^2 m^2}{L^2}\right)\right)\right)$
Shift in perihelion
$\Delta\varphi = \frac{2\pi}{\left(1 - \frac{3r_s^2 m^2}{4L^2}\right)} - 2\pi$
$\simeq 6\pi\frac{(GM)^2}{L^2}$


$\frac{3r_s}{2} u_{(0)}^2$ gives 1st order correction (ie dropped $\frac{r_s}{r} \cdot u_{(1)}$)
$u''_{(1)} = -u_{(1)} + \epsilon \cos\varphi$, $\epsilon = \frac{r_s m^2}{2L^2} \cdot \frac{3r_s^2 e}{4L^2}$ $\cos^2\varphi = \frac{1}{2}(\cos 2\varphi + 1)$
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Shift in perihelion $L^2 = m^2 v^2 r_0^2$
$\Delta\varphi = \frac{2\pi}{\left(1 - \frac{3r_s^2 m^2}{4L^2}\right)} - 2\pi$
$\simeq 6\pi\frac{(GM)^2}{L^2} \simeq 6\pi\frac{GM}{r_0 c^2}$ for circular orbit

$\frac{3r_s}{2}u_{(0)}^2$ gives 1st order correction $\left(\text{ie dropped } \frac{r_s}{r}\cdot u_{(1)}\right)$

$$u''_{(1)} = -u_{(1)} + \epsilon\cos\varphi \quad , \quad \epsilon = \frac{r_s m^2}{2L^2}\cdot\frac{3r_s^2 e}{4L^2} \qquad \cos^2\varphi = \frac{1}{2}(\cos 2\varphi + 1)$$

resonance:

$$= u_{(0)} + \frac{\epsilon}{2}\varphi\sin\varphi \quad \text{growing, resonant soln!}$$

$$= \frac{r_s m^2}{2L^2}\left(1 + e\cos\left(\varphi\left(1 - \frac{3}{4}\frac{r_s^2 m^2}{L^2}\right)\right)\right)$$

Shift in perihelion

$$\Delta\varphi = \frac{2\pi}{\left(1 - \frac{3r_s^2 m^2}{4L^2}\right)} - 2\pi$$

$$\simeq 6\pi\frac{(GM)^2}{L^2} \simeq 6\pi\frac{GM}{r_0 c^2}$$

$L^2 = m^2 v^2 r_0^2 = \frac{GM}{r_0} m^2 r_0^2$

$\Rightarrow L^2 = GMr_0$ for circular orbit

$$\epsilon = \frac{r_s m^2}{2L^2} \cdot \frac{3 r_s^2 e}{4 L^2} \qquad \cos^2\varphi = \frac{1}{2}(\cos 2\varphi + 1)$$

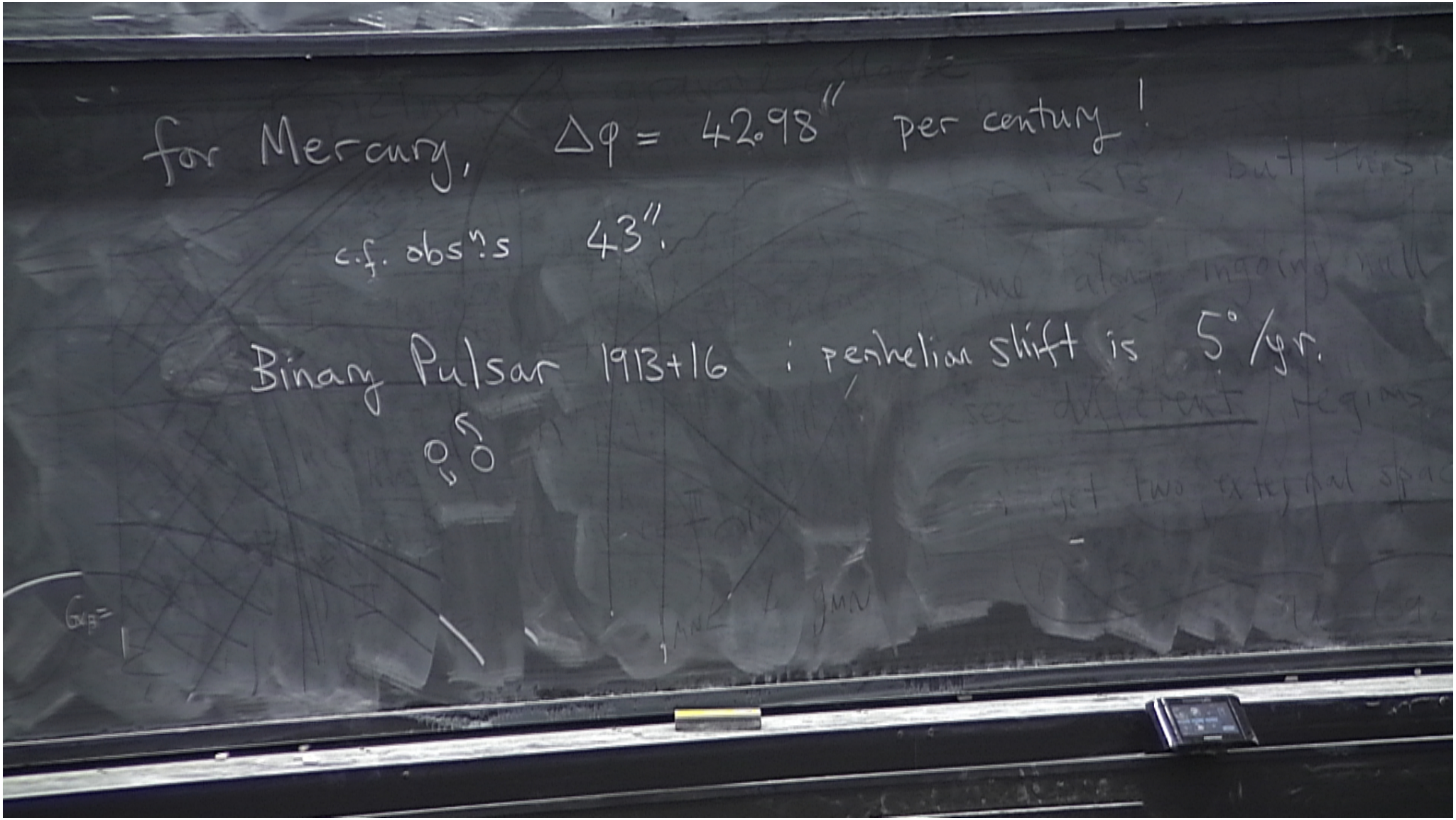
Shift in perihelion

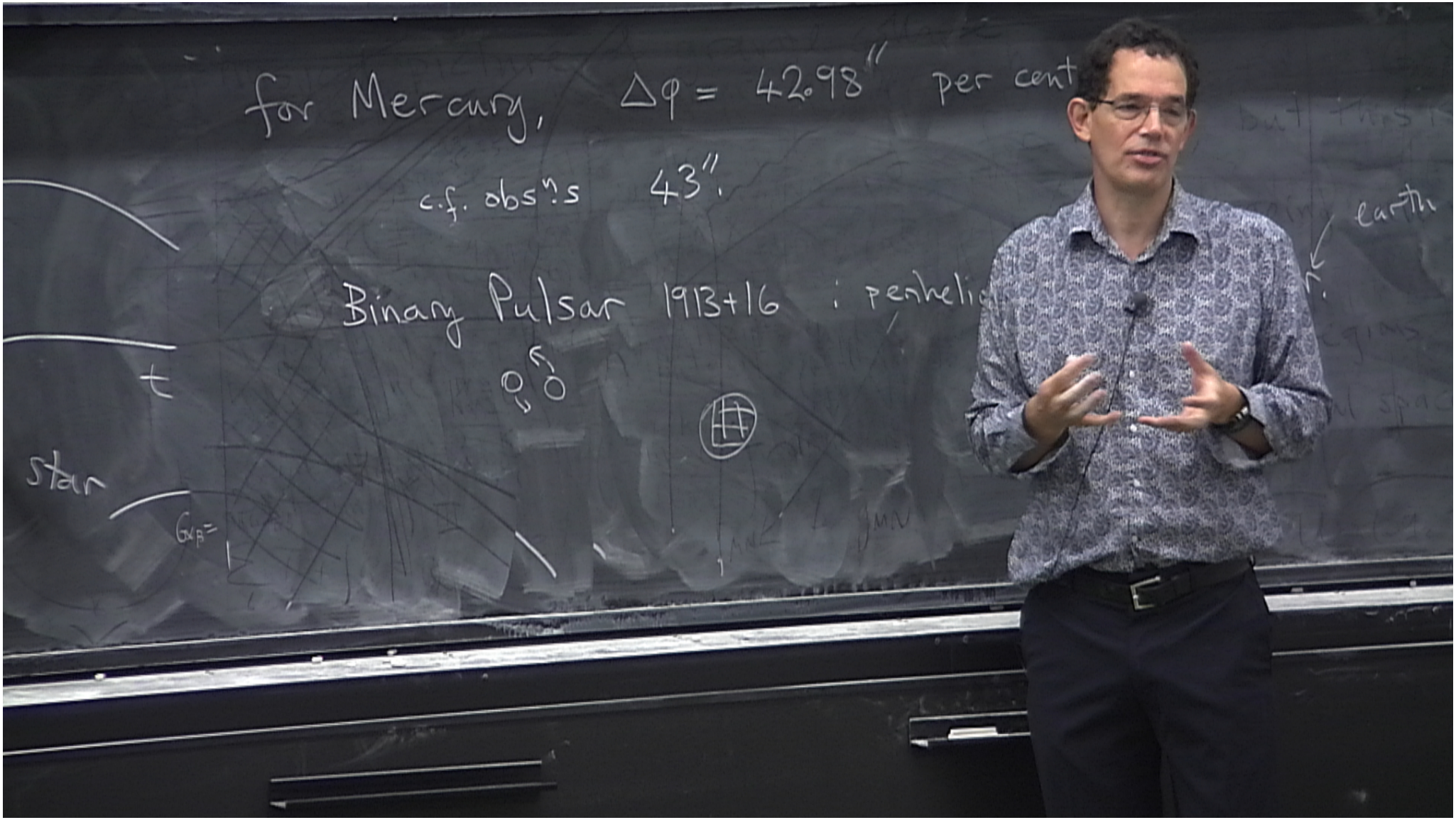
$$\Delta\varphi = \frac{2\pi}{\left(1 - \frac{3 r_s^2 m^2}{4L^2}\right)} \; 1 = 2\pi$$

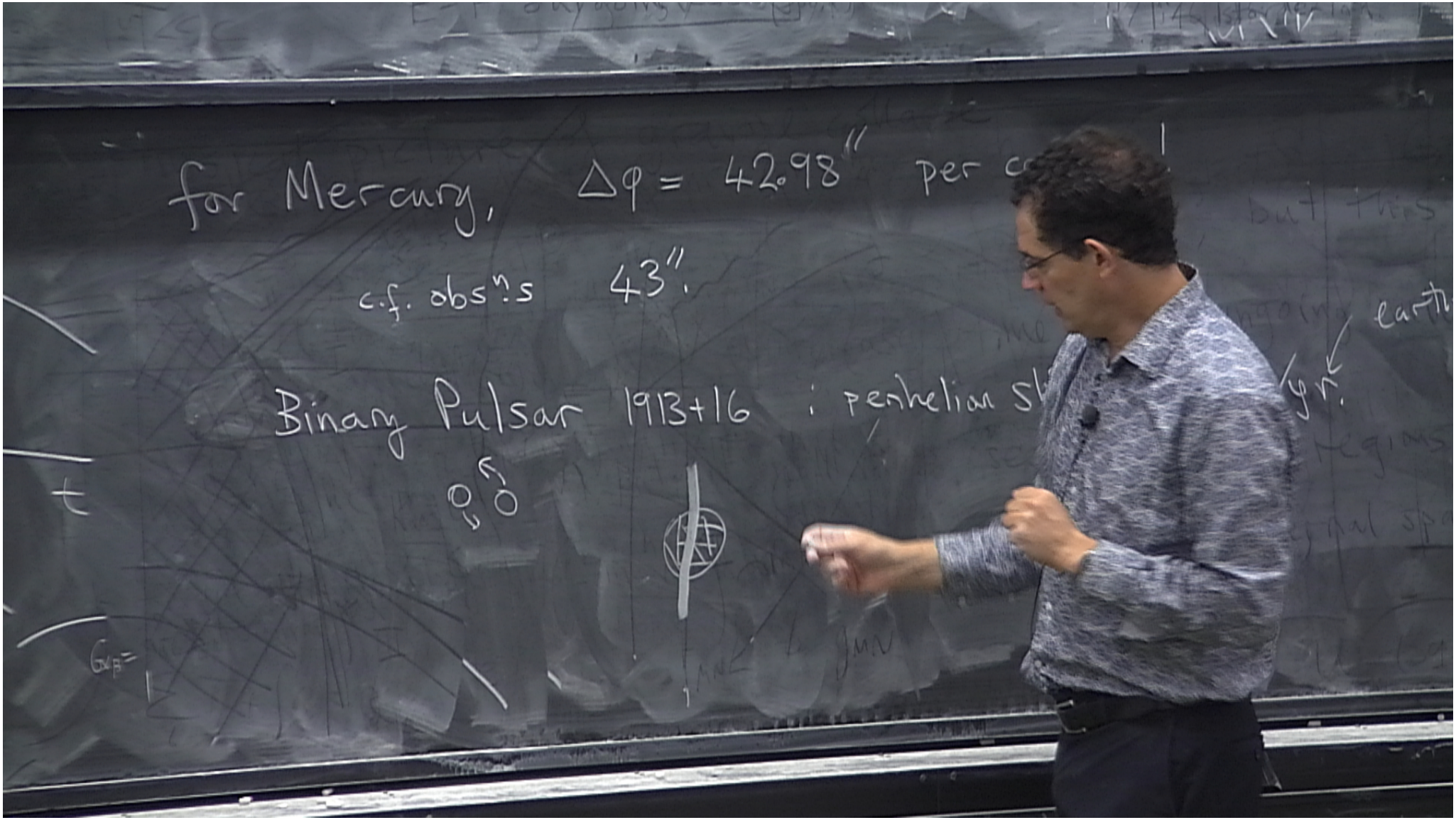
$$\simeq 6\pi \frac{(GM)^2}{L^2} \simeq 6\pi \frac{GM}{r_0 c^2} = 3\pi \frac{r_s}{r_0}.$$

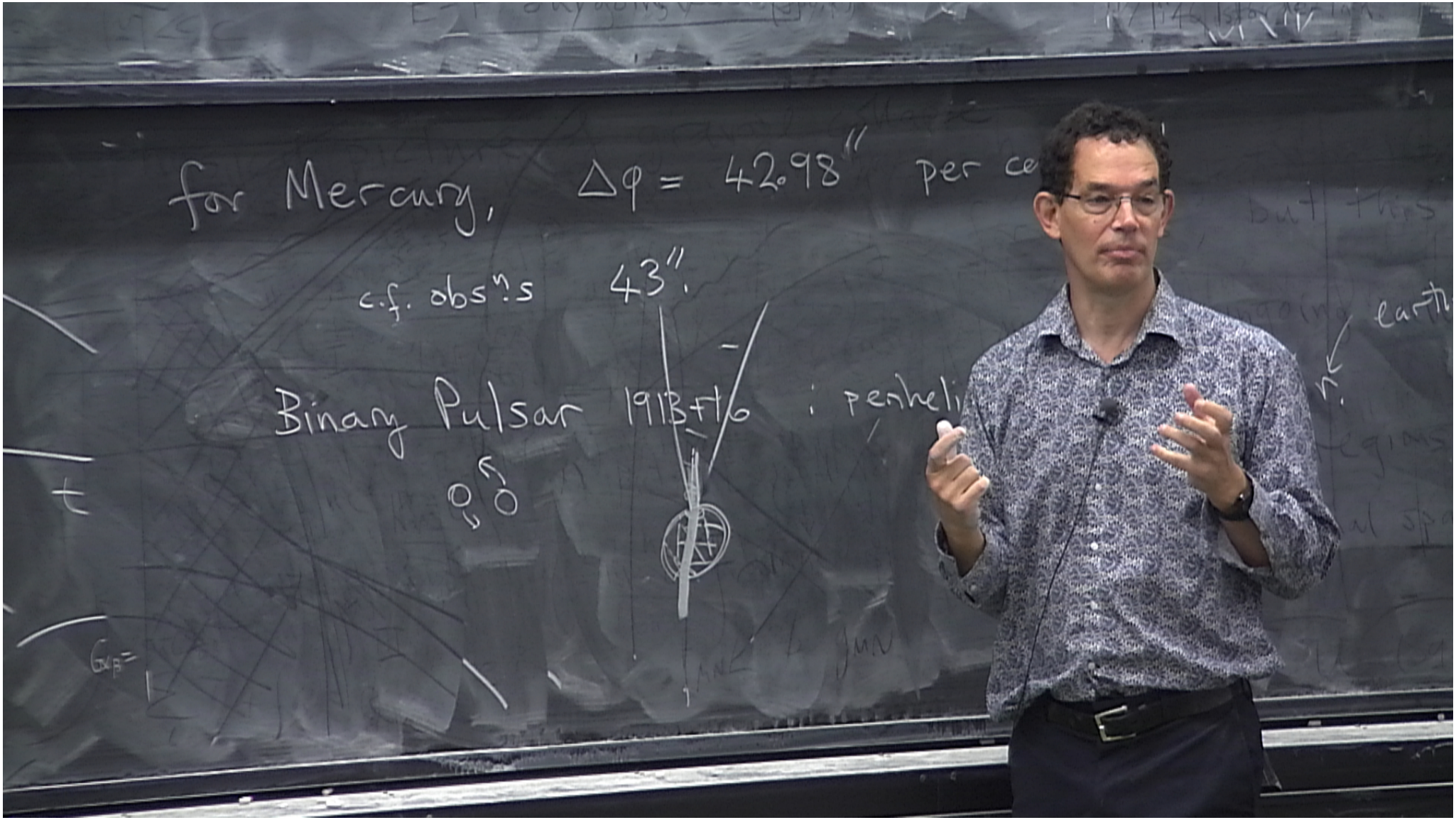
$$L^2 = m^2 v^2 r_0^2 = \frac{GM}{r_0} m^2 r_0^2 \Rightarrow L^2 = GM r_0$$

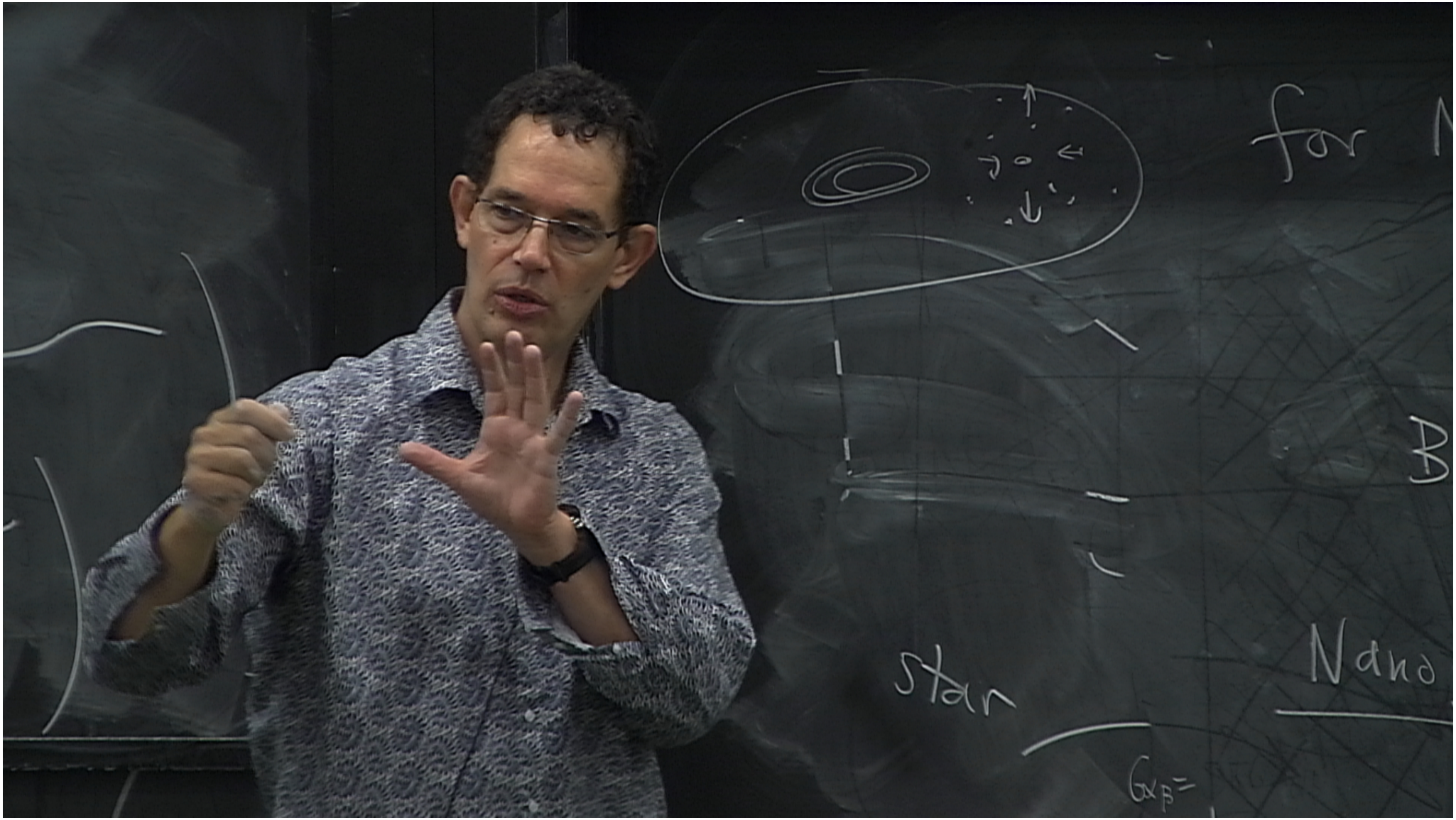
for circular orbit

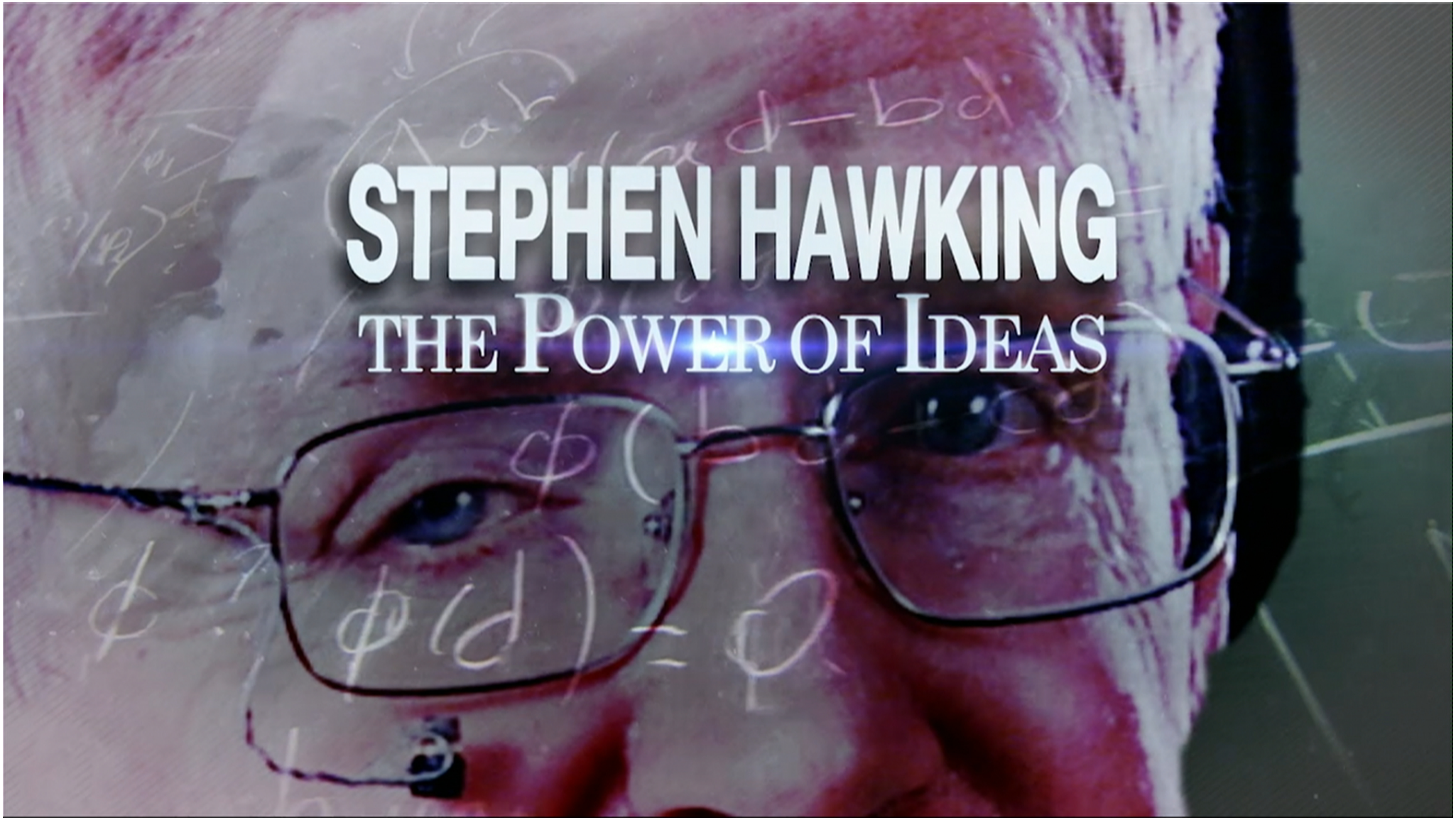

for Mercury, $\Delta\varphi = 42.98''$ per century!
c.f. obs^{n}s $43''$.
Binary Pulsar 1913+16 : perihelion shift is 5°/yr.


for Mercury, Δφ = 42.98″ per cent
c.f. obs^ns 43″.
Binary Pulsar 1913+16 : perhelic
earth
star
t


for Mercury, Δφ = 42.98″ per
c.f. obs?s 43″.
Binary Pulsar 1913+16 : perihelian
earth
yr.


for Mercury, Δφ = 42.98″ per ce
c.f. obs^n.s 43″.
Binary Pulsar 1913+16 : perihel
earth


for
Star
Nano


STEPHEN HAWKING
THE POWER OF IDEAS