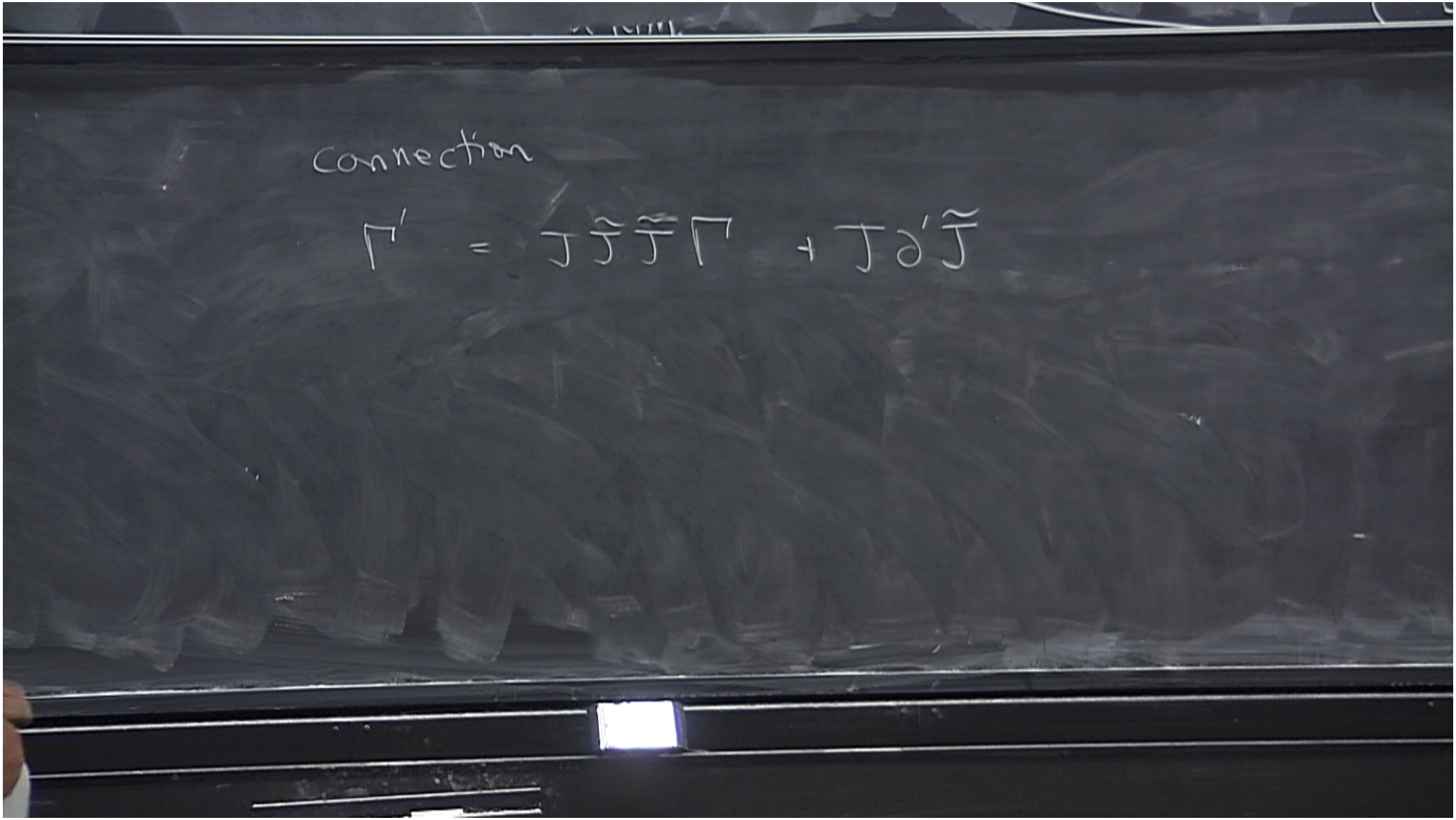
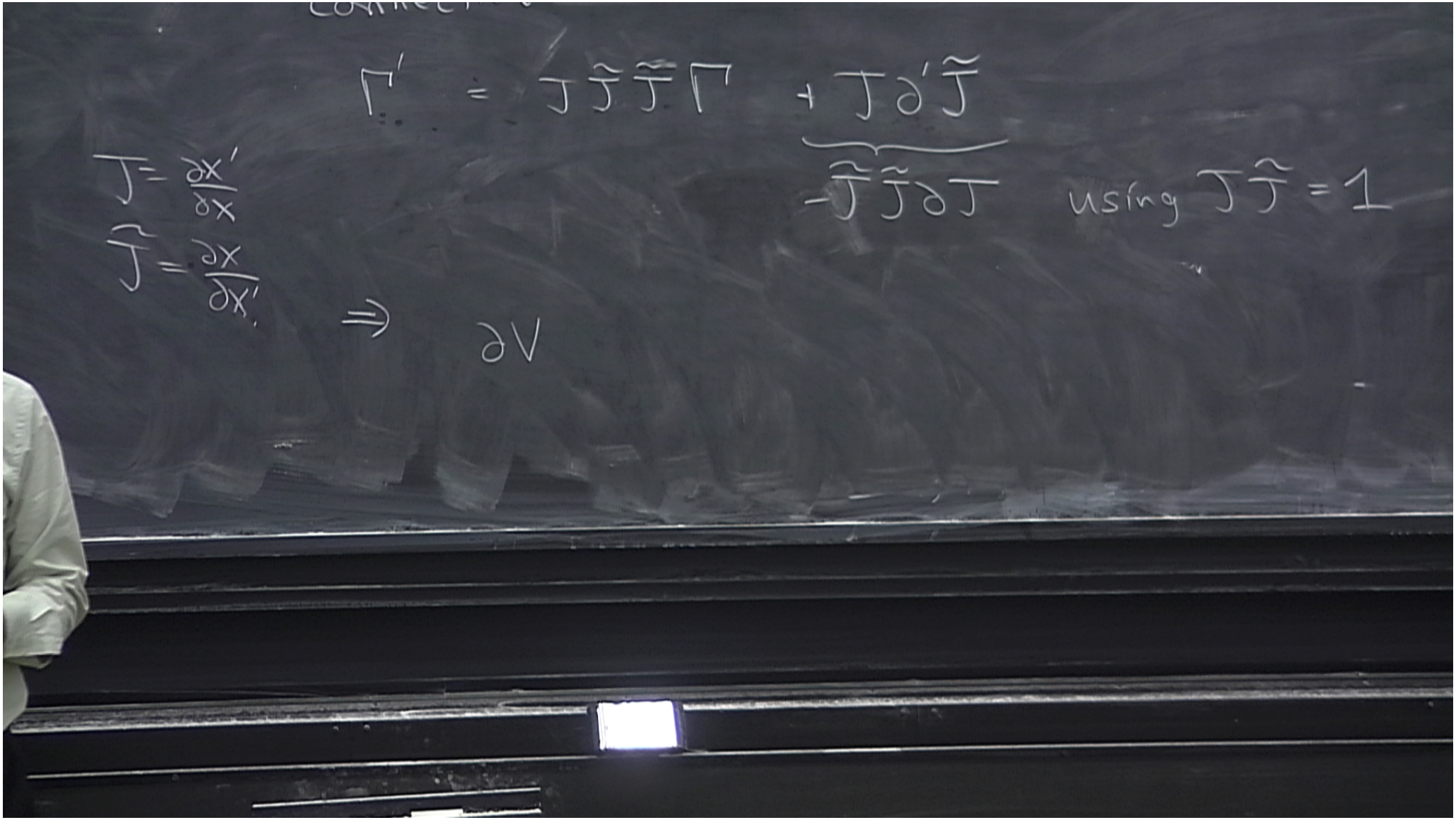
Title: Relativity - Lecture 5

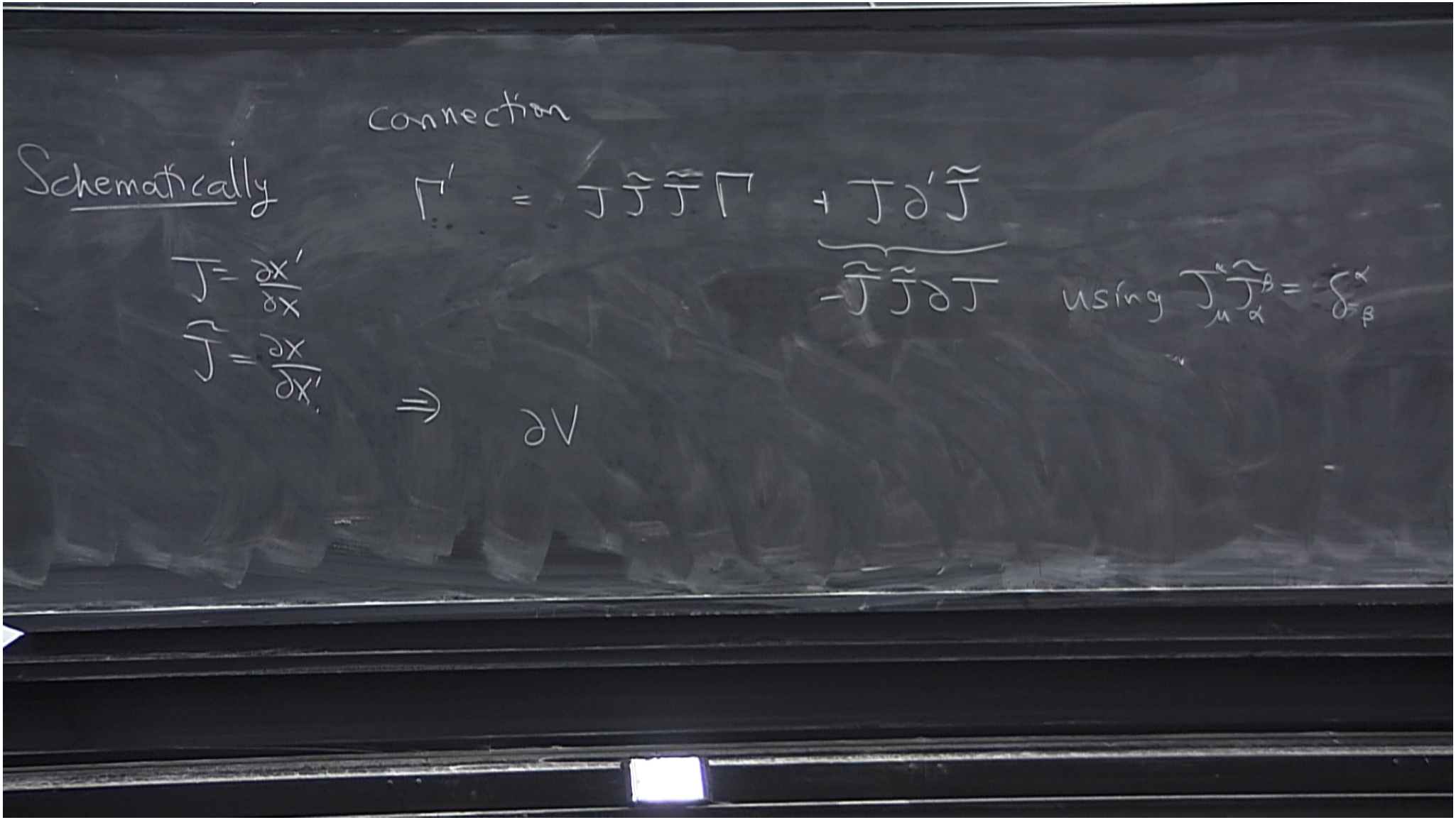
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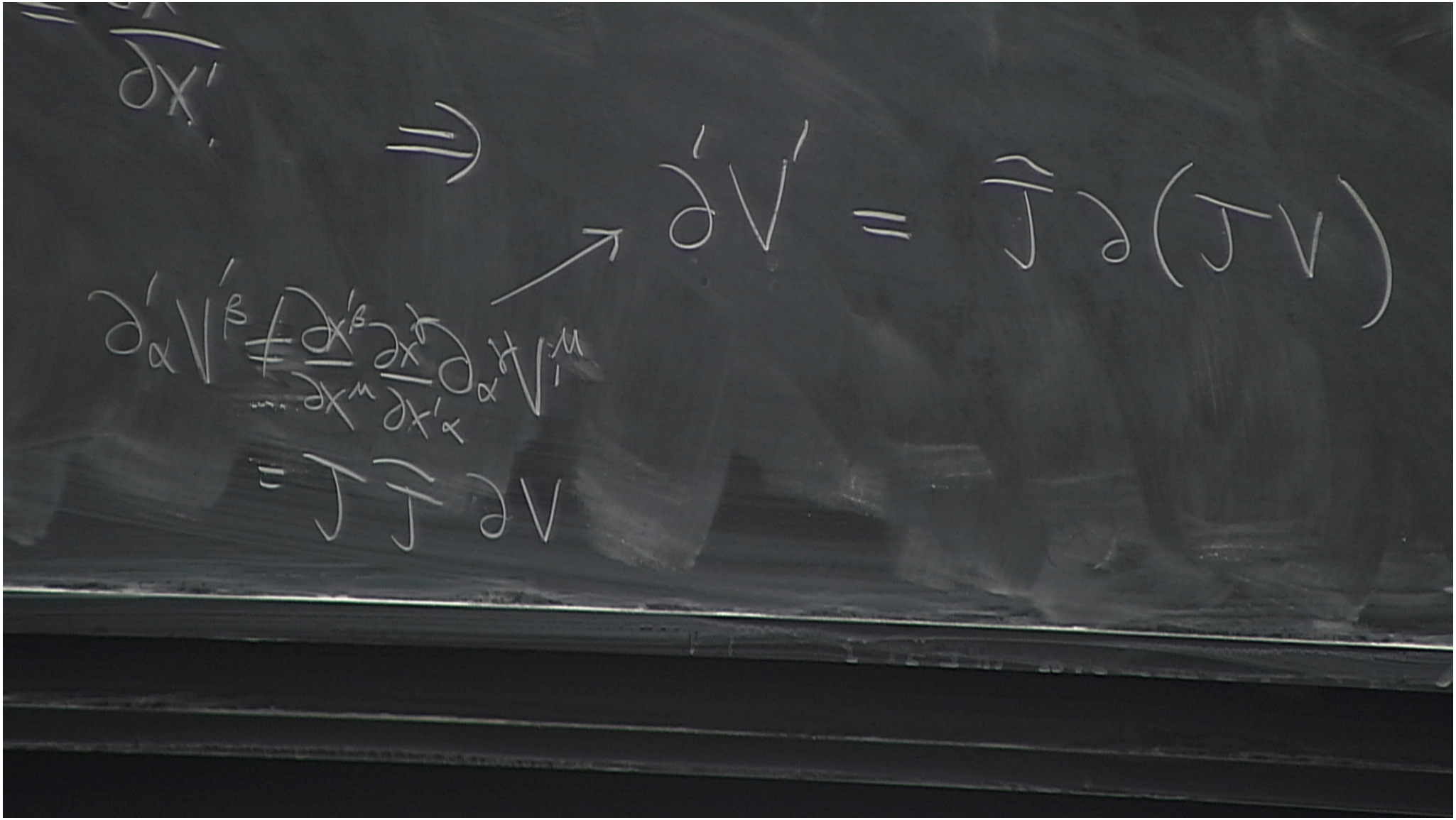
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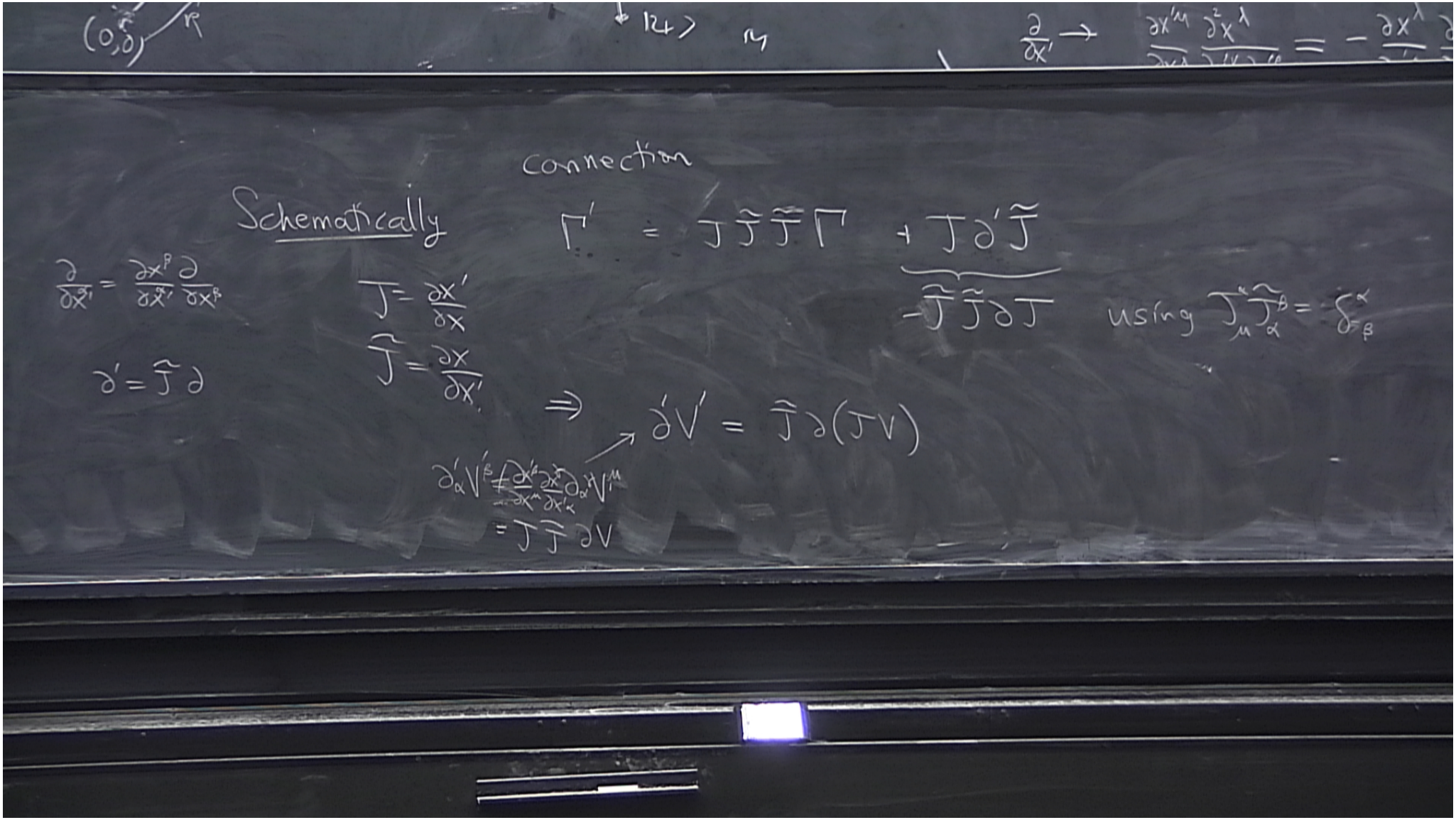
Abstract:

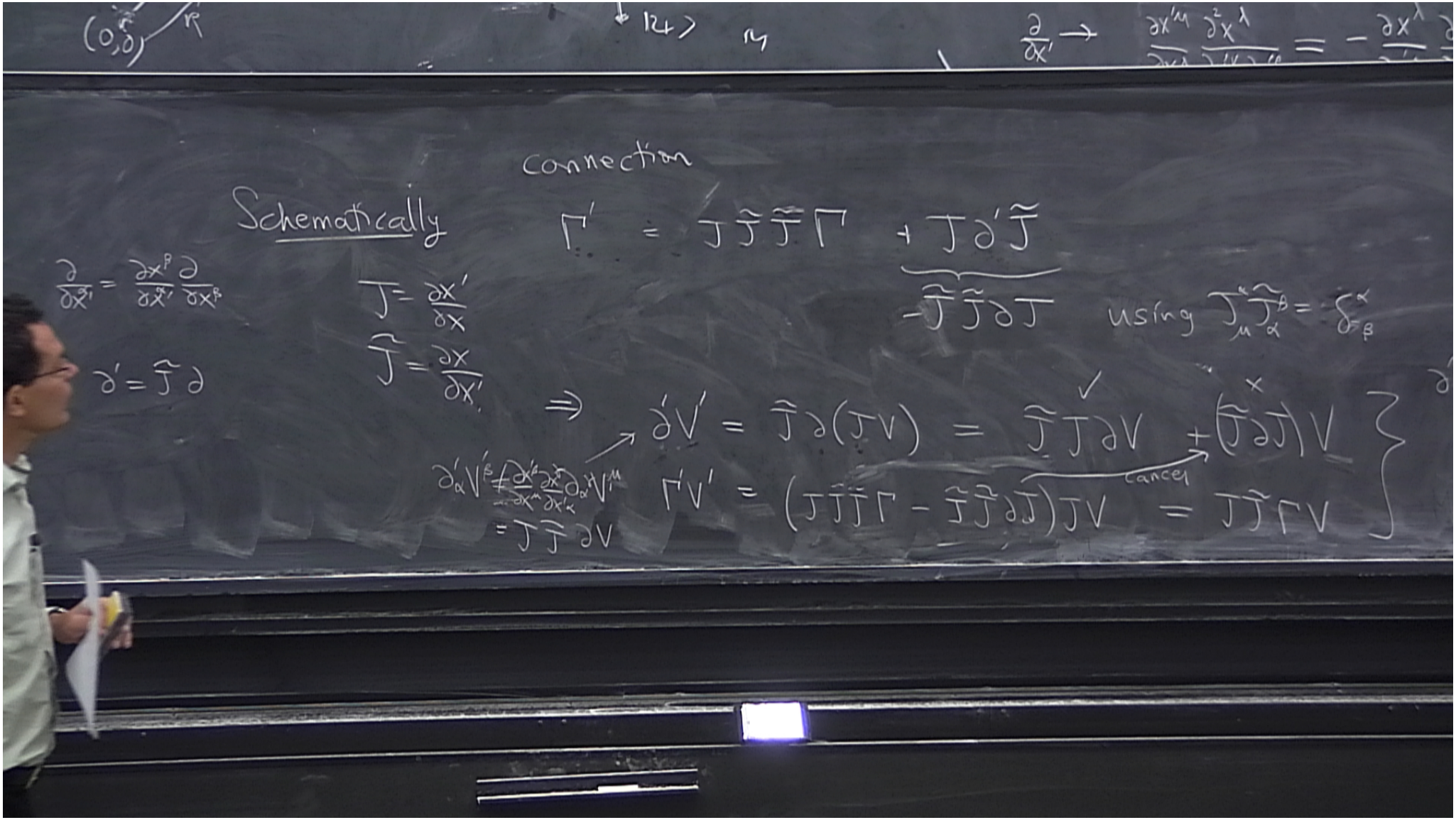

Connection

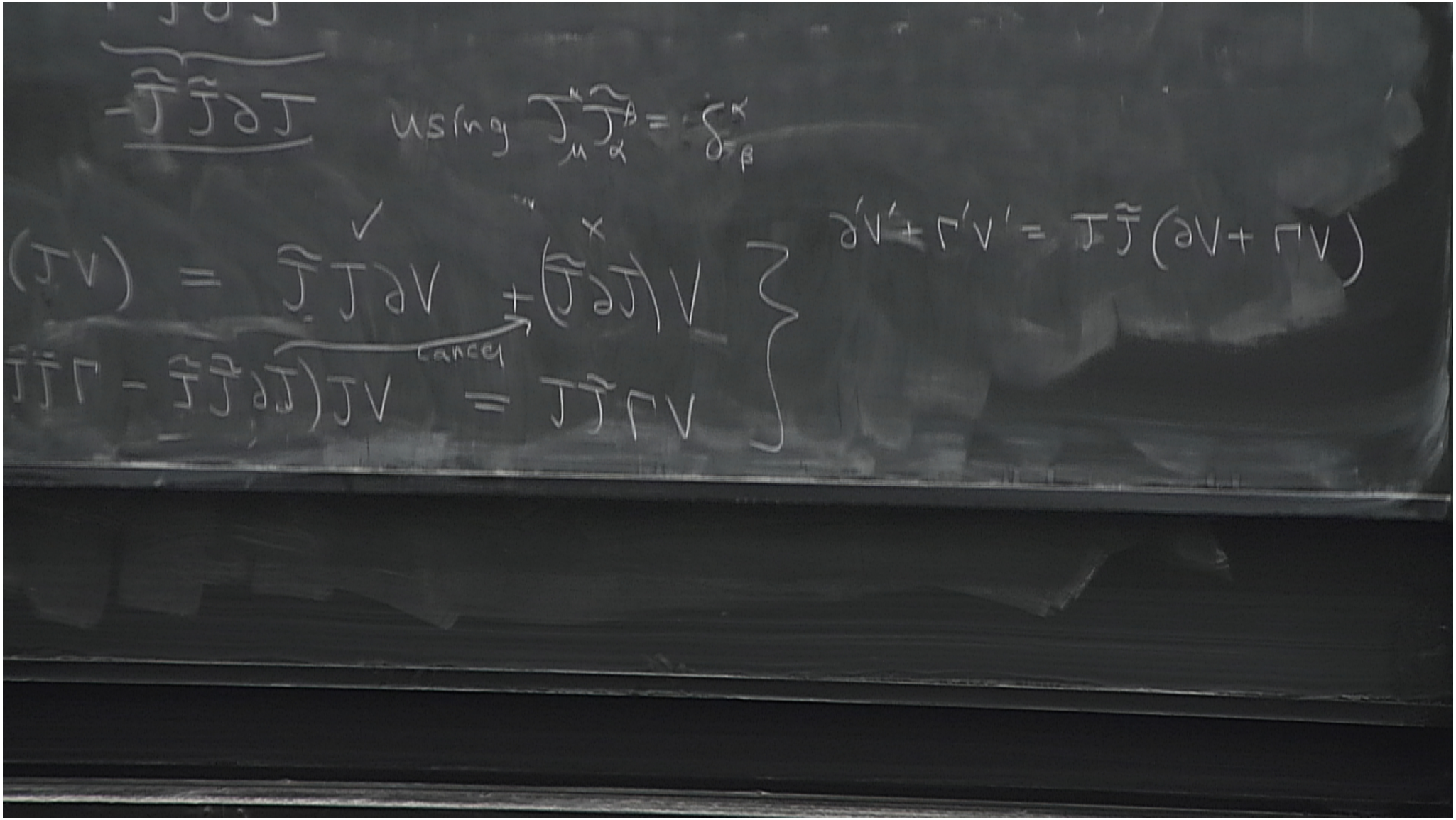

$\Gamma' = J\tilde{J}\tilde{\tilde{J}}\Gamma + J\partial'\tilde{J}$
$-\tilde{J}\tilde{J}\partial J$ using $J\tilde{J} = 1$
$J = \frac{\partial x'}{\partial x}$
$\tilde{J} = \frac{\partial x}{\partial x'}$
$\Rightarrow$ ∂V

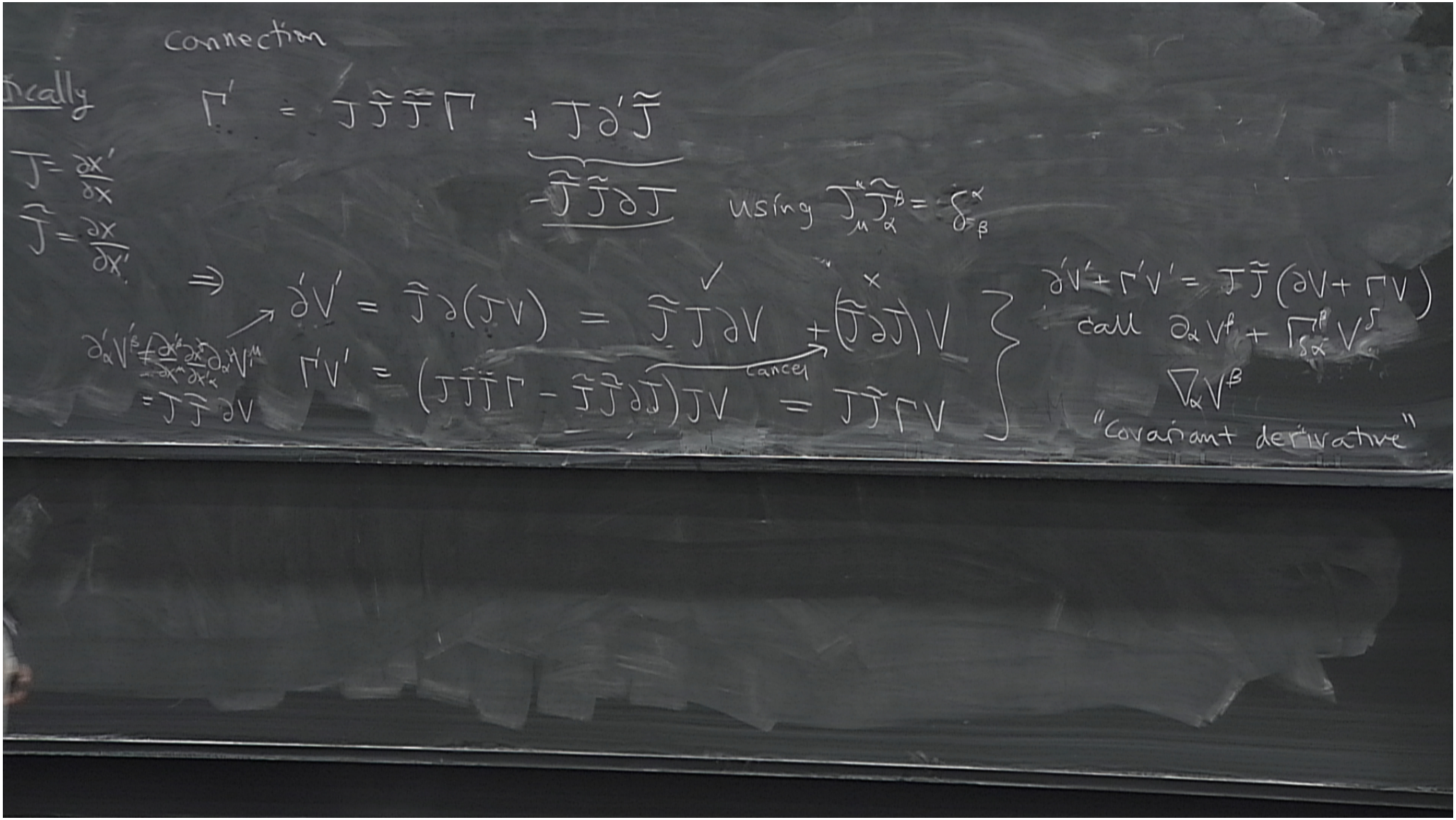

Schematically
connection
$\Gamma' = J\tilde{J}\tilde{\Gamma}\Gamma + J\partial'\tilde{J}$
$-\tilde{\Gamma}\tilde{J}\partial J$
using $J^{\alpha}_{\mu}\tilde{J}^{\beta}_{\alpha} = \delta^{\alpha}_{\beta}$
$J = \frac{\partial x'}{\partial x}$
$\tilde{J} = \frac{\partial x}{\partial x'}$
$\Rightarrow$
∂V

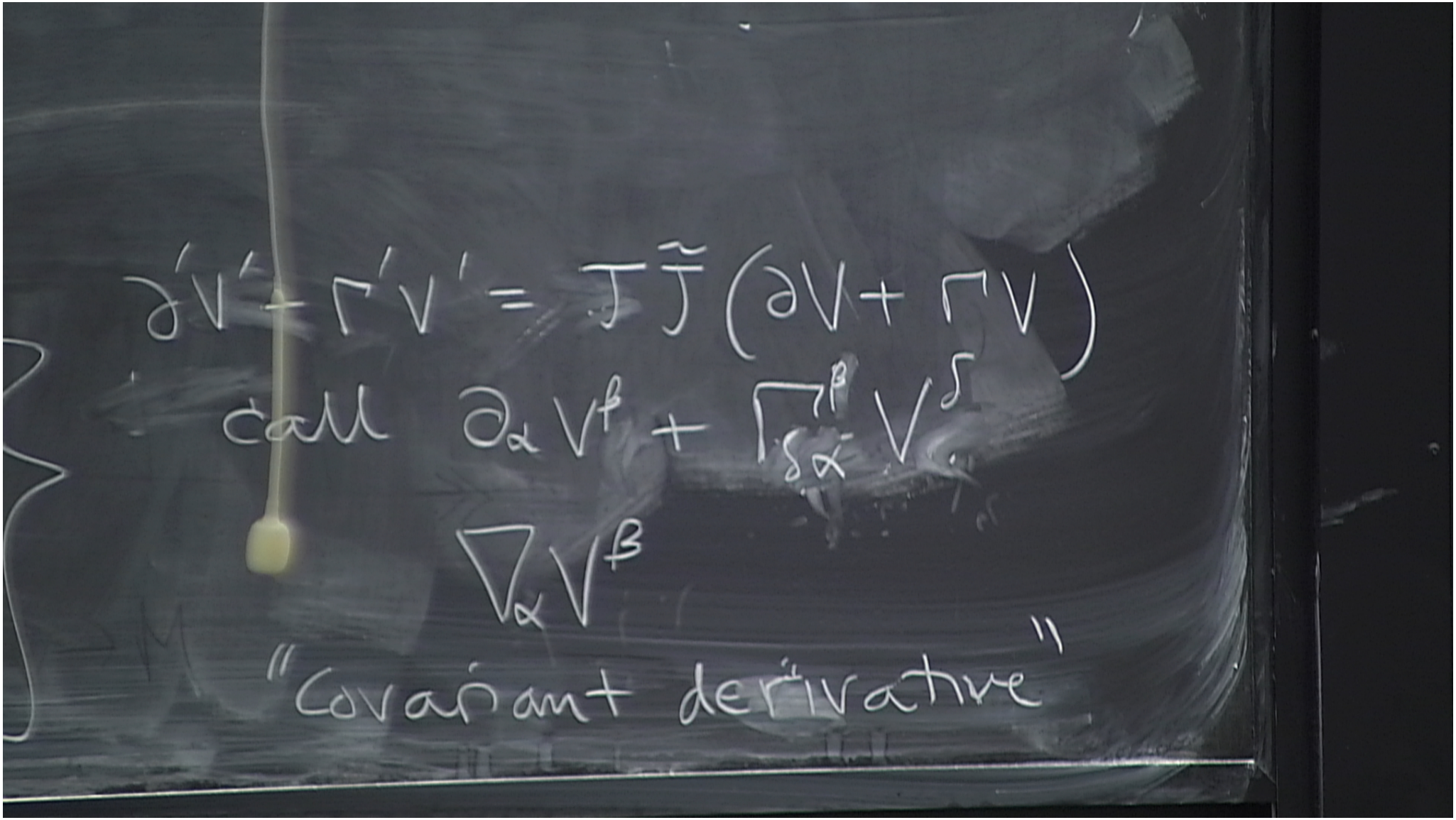

$\frac{\partial}{\partial x'_\alpha}$
$\Rightarrow$
$\partial' V' = \tilde{J}\,\partial(JV)$
$\partial'_\alpha V'^\beta = \frac{\partial x'^\beta}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\alpha}\partial_\nu V^\mu$
$= J\tilde{J}\,\partial V$


Schematically
connection
$\Gamma' = J\tilde{J}\tilde{J}\Gamma + J\partial'\tilde{J}$
$-\tilde{J}\tilde{J}\partial J$
using $J^{\alpha}_{\mu}\tilde{J}^{\beta}_{\alpha} = \delta^{\alpha}_{\beta}$
$J = \frac{\partial x'}{\partial x}$
$\tilde{J} = \frac{\partial x}{\partial x'}$
$\partial' = \tilde{J}\partial$
$\Rightarrow \partial' V' = \tilde{J}\partial(JV)$
$= J\tilde{J}\partial V$


Schematically
connection
using
cancel

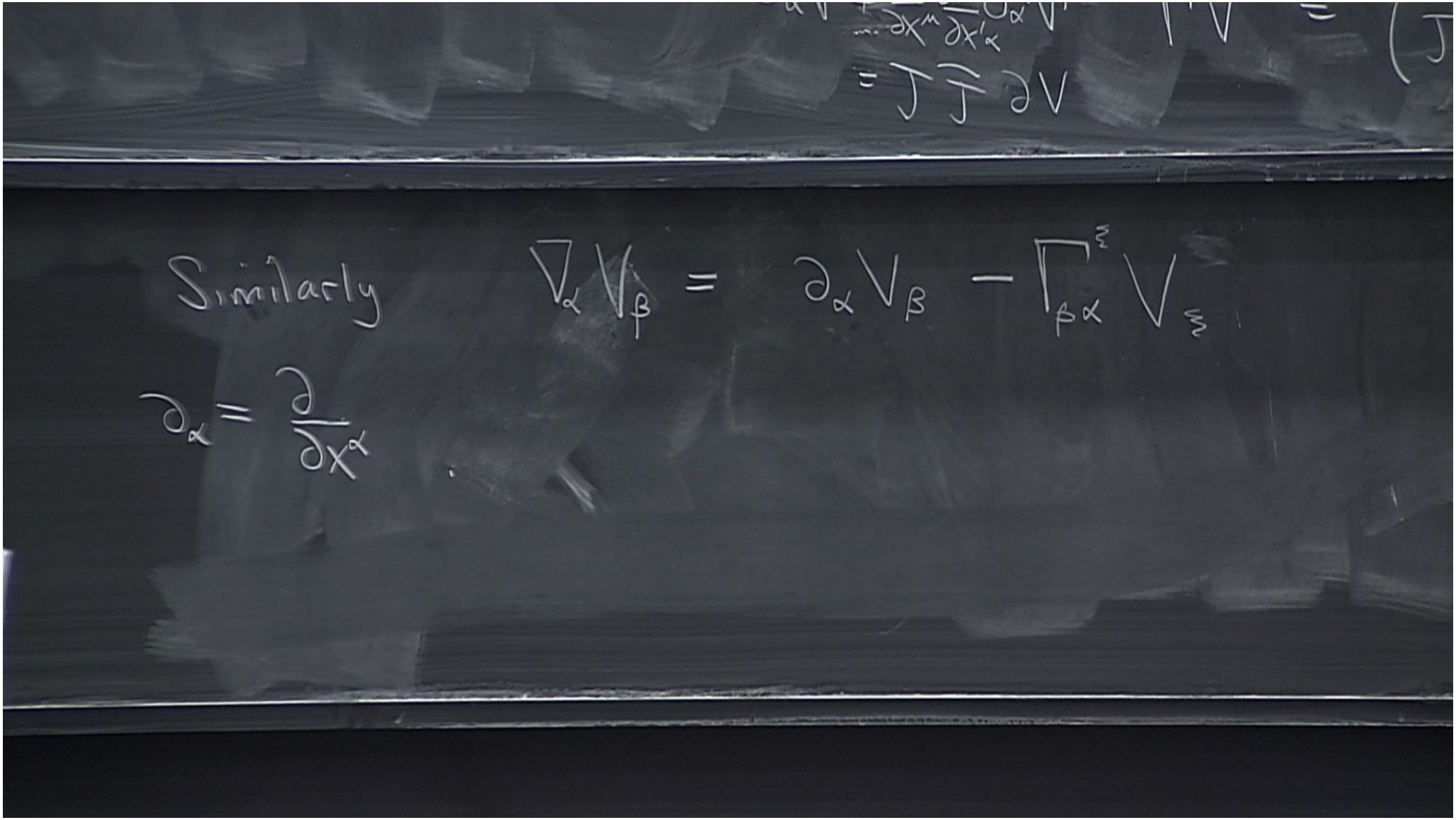

using
cancel

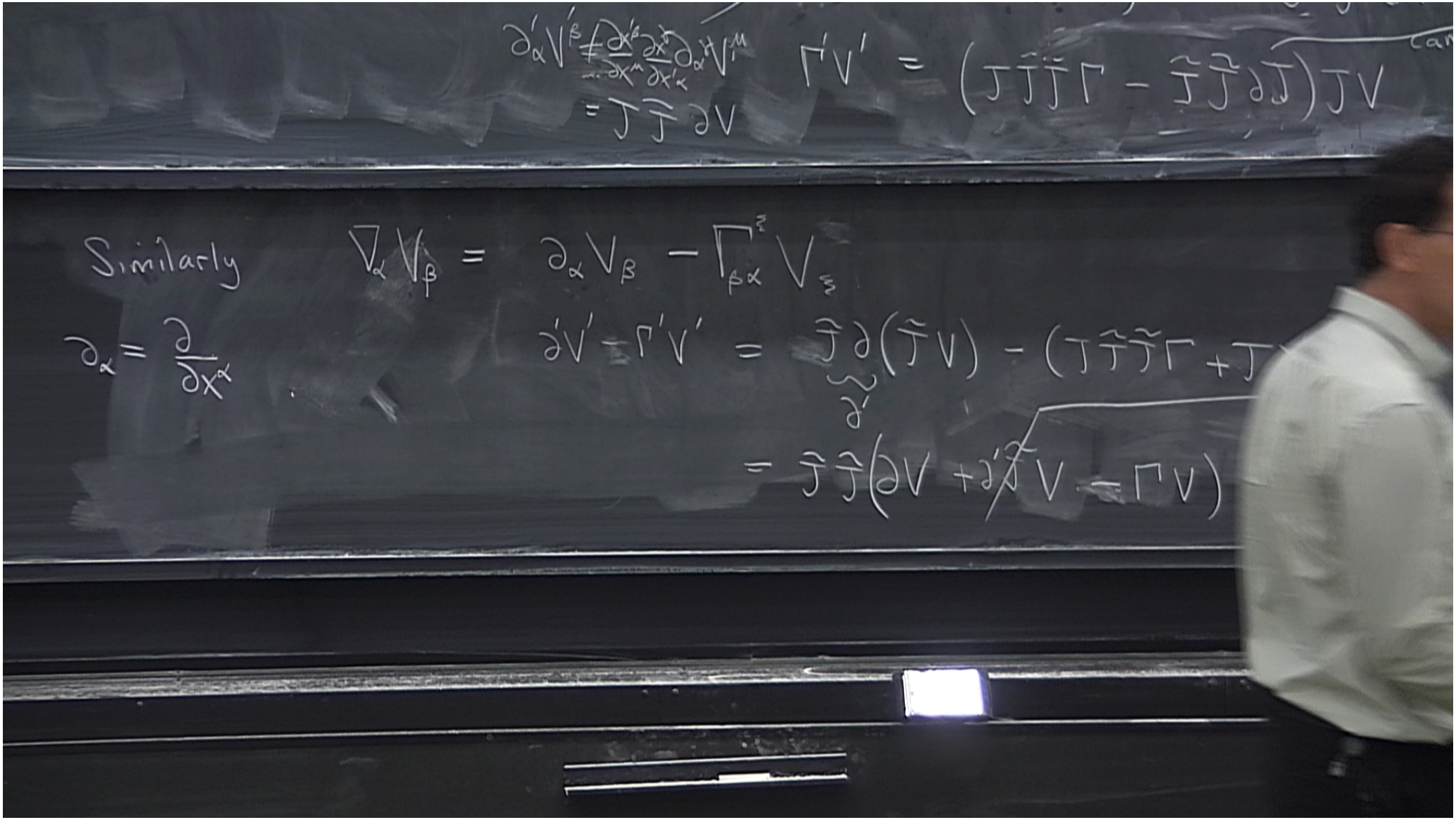

Connection
Locally
$\Gamma' = J\tilde{J}\tilde{\partial}\Gamma + J\partial'\tilde{J} - \tilde{J}\tilde{J}\partial J$
using $J^{\mu}_{\alpha}\tilde{J}^{\beta}_{\mu} = \delta^{\alpha}_{\beta}$
$J = \frac{\partial x'}{\partial x}$
$\tilde{J} = \frac{\partial x}{\partial x'}$
$\Rightarrow$
$\partial' V' = \tilde{J}\partial(JV) = \tilde{J}J\partial V + (\tilde{J}\partial J)V$
$\partial'_\alpha V'^\beta \neq \frac{\partial x'^\beta}{\partial x^\mu}\frac{\partial x^\nu}{\partial x'^\alpha}\partial_\nu V^\mu$
$= J\tilde{J}\partial V$
$\Gamma' V' = (J\tilde{J}\tilde{J}\Gamma - \tilde{J}\tilde{J}\partial J)JV = J\tilde{J}\Gamma V$
cancel
$\partial' V' + \Gamma' V' = J\tilde{J}(\partial V + \Gamma V)$
call $\partial_\alpha V^\beta + \Gamma^\beta_{\delta\alpha} V^\delta$
$\nabla_\alpha V^\beta$
"covariant derivative"

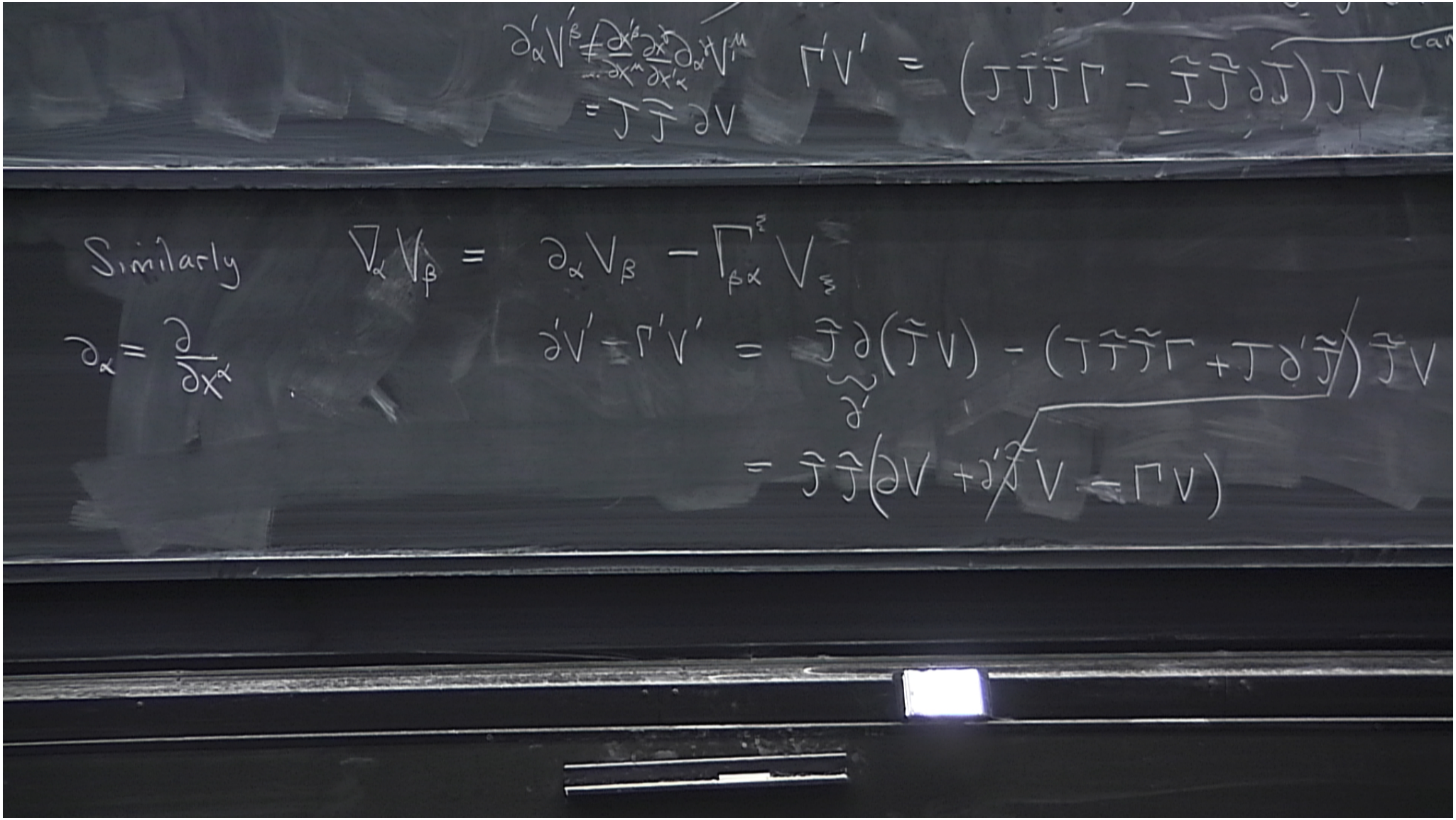

$\partial' V' + \Gamma' V' = J \tilde{J} (\partial V + \Gamma V)$
call $\partial_\alpha V^\beta + \Gamma^\beta_{\delta\alpha} V^\delta$
$\nabla_\alpha V^\beta$
"Covariant derivative"

Similarly $\nabla_\alpha V_\beta = \partial_\alpha V_\beta$

$$\partial_\alpha = \frac{\partial}{\partial x^\alpha}$$


$= J \tilde{J} \partial V$
Similarly $\nabla_\alpha V_\beta = \partial_\alpha V_\beta - \Gamma^m_{\beta\alpha} V_m$
$\partial_\alpha = \frac{\partial}{\partial x^\alpha}$

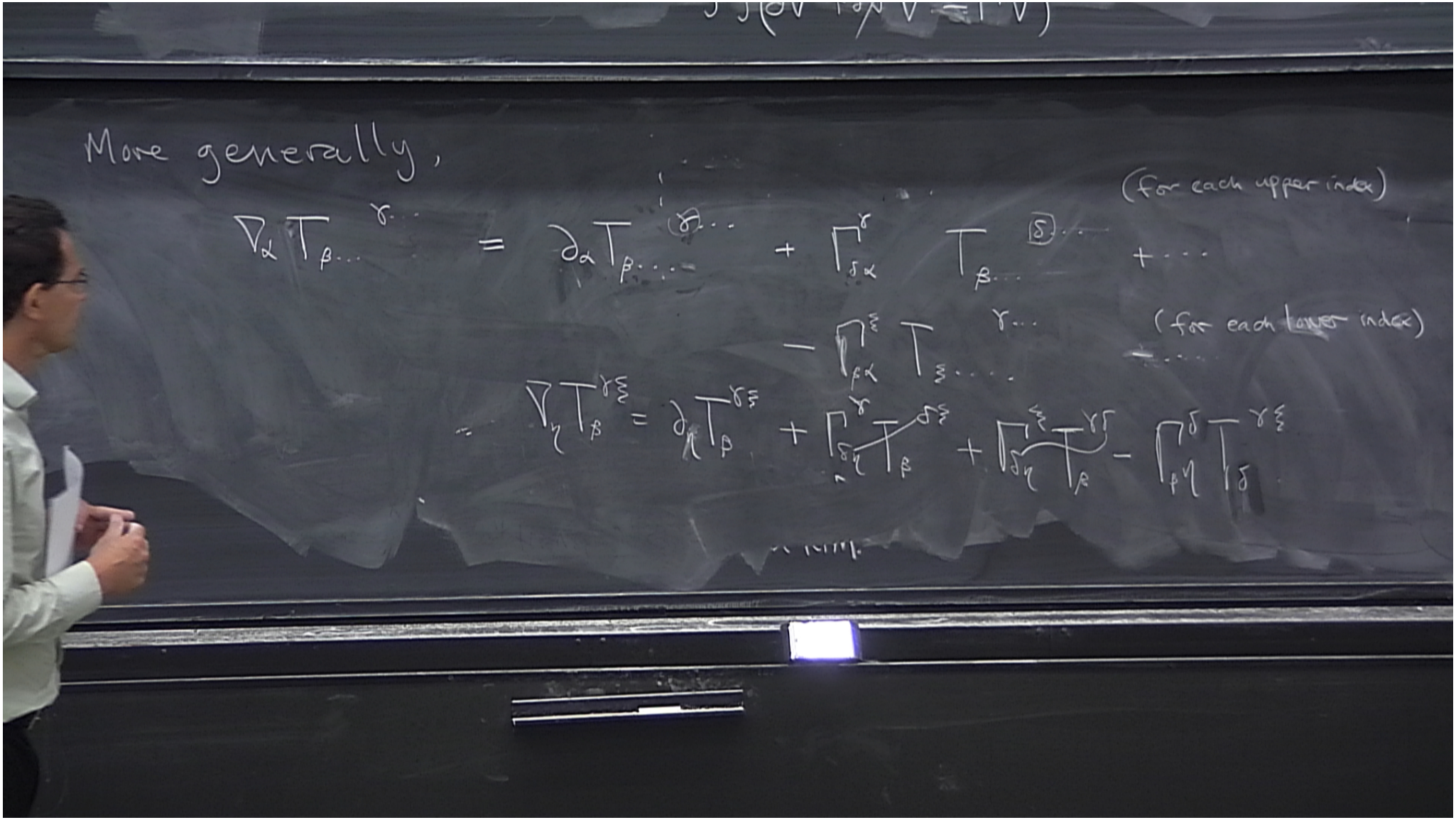

Similarly
$\nabla_\alpha V_\beta = \partial_\alpha V_\beta - \Gamma^{\gamma}_{\beta\alpha} V_\gamma$
$\partial_\alpha = \frac{\partial}{\partial x^\alpha}$

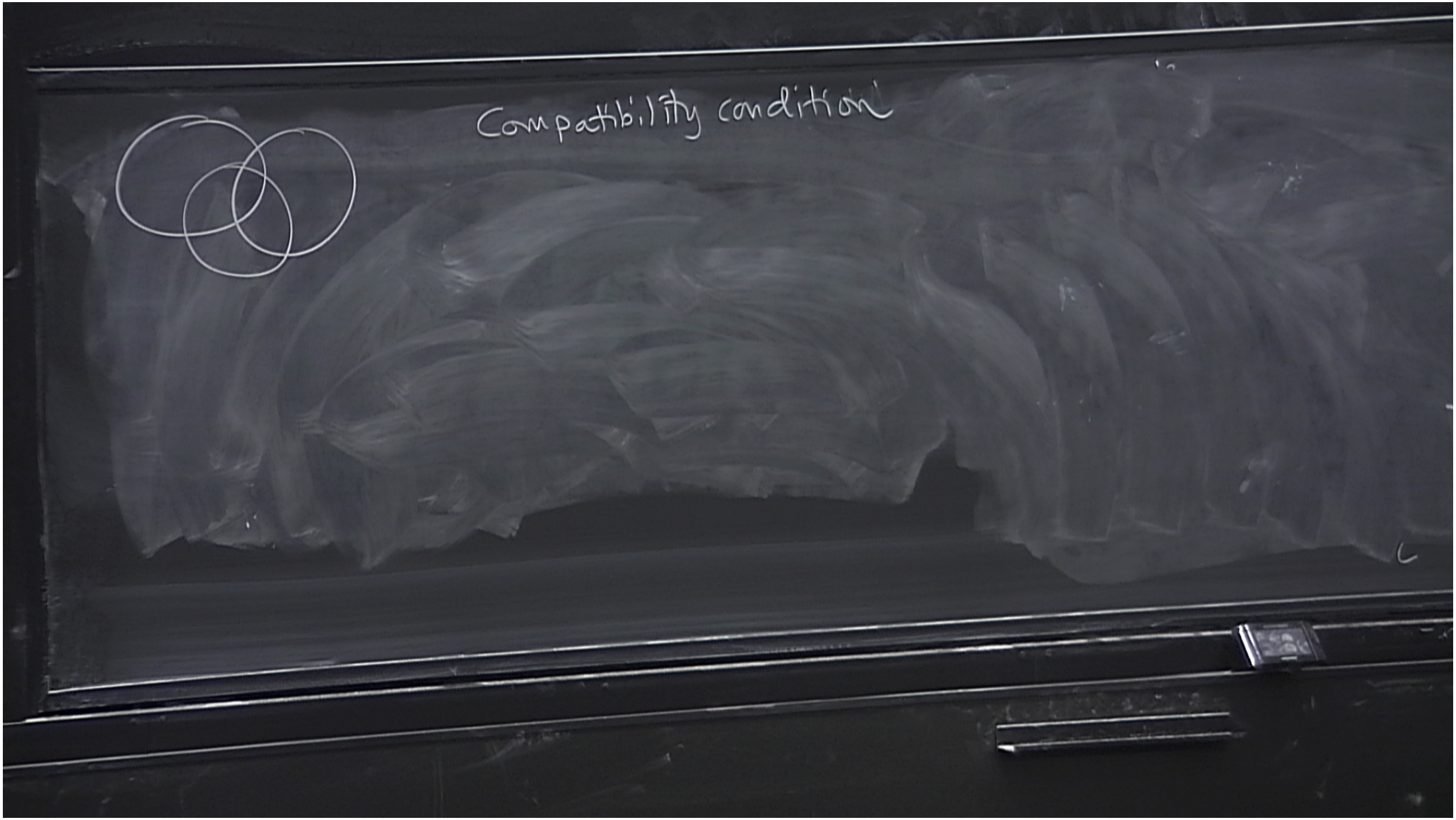

Similarly
$\nabla_\alpha V_\beta = \partial_\alpha V_\beta - \Gamma^{\gamma}_{\beta\alpha} V_\gamma$
$\partial_\alpha = \frac{\partial}{\partial x^\alpha}$

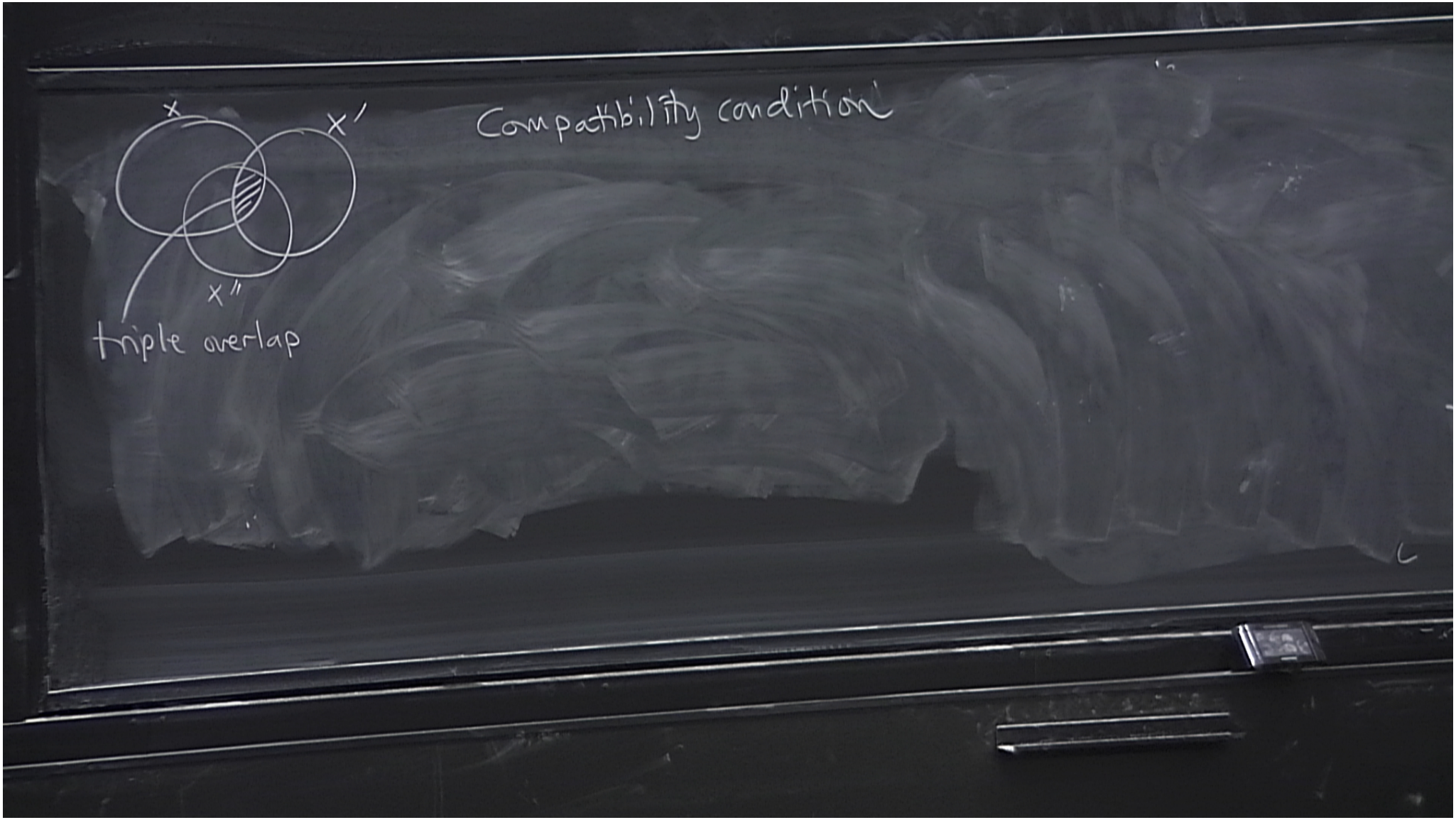
More generally,

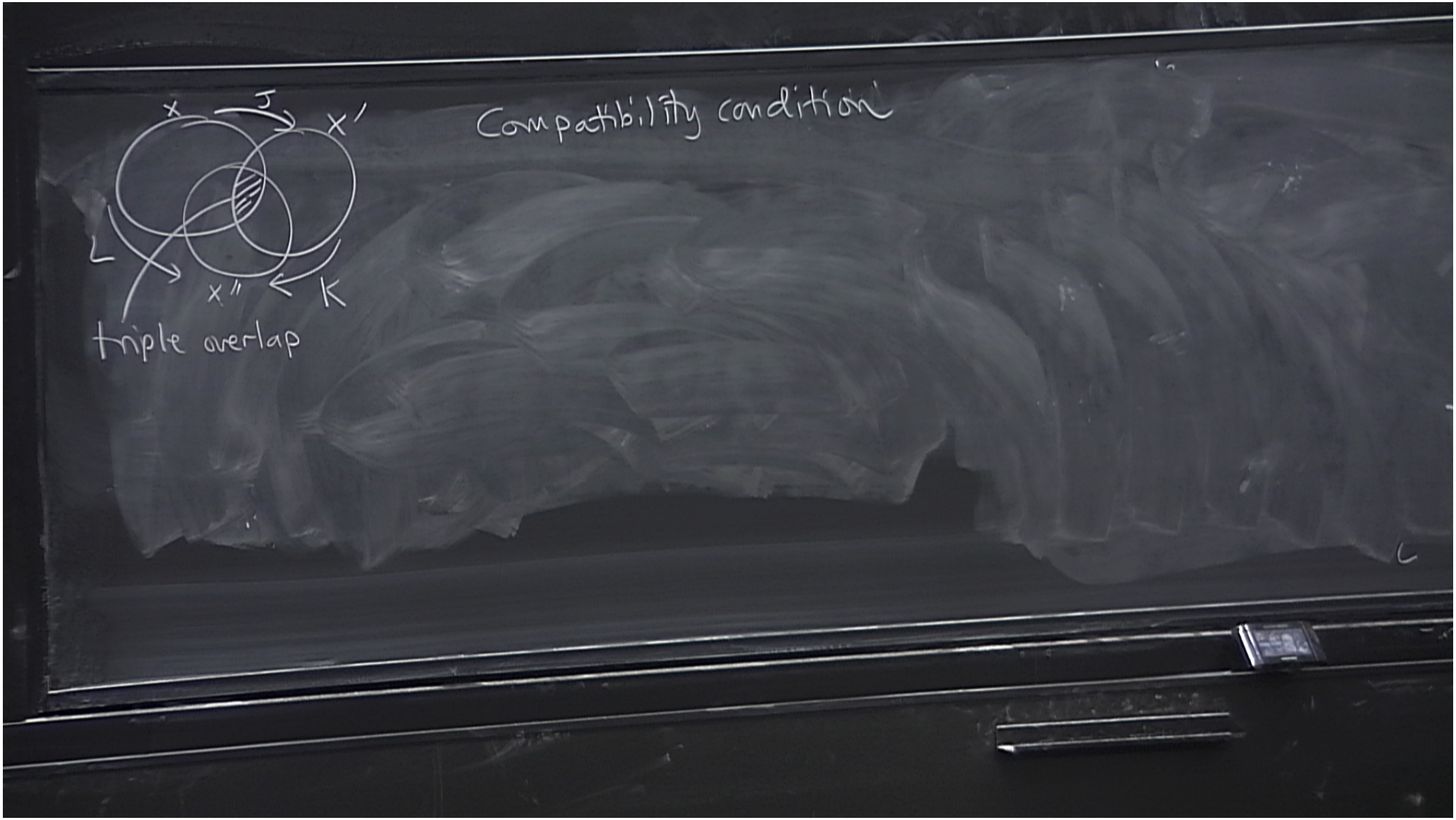
$$\nabla_\alpha T_{\beta\ldots}{}^{\gamma\ldots} = \partial_\alpha T_{\beta\ldots}{}^{\gamma\ldots} + \Gamma^{\gamma}_{\delta\alpha} T_{\beta\ldots}{}^{\delta\ldots} + \ldots \quad \text{(for each upper index)}$$

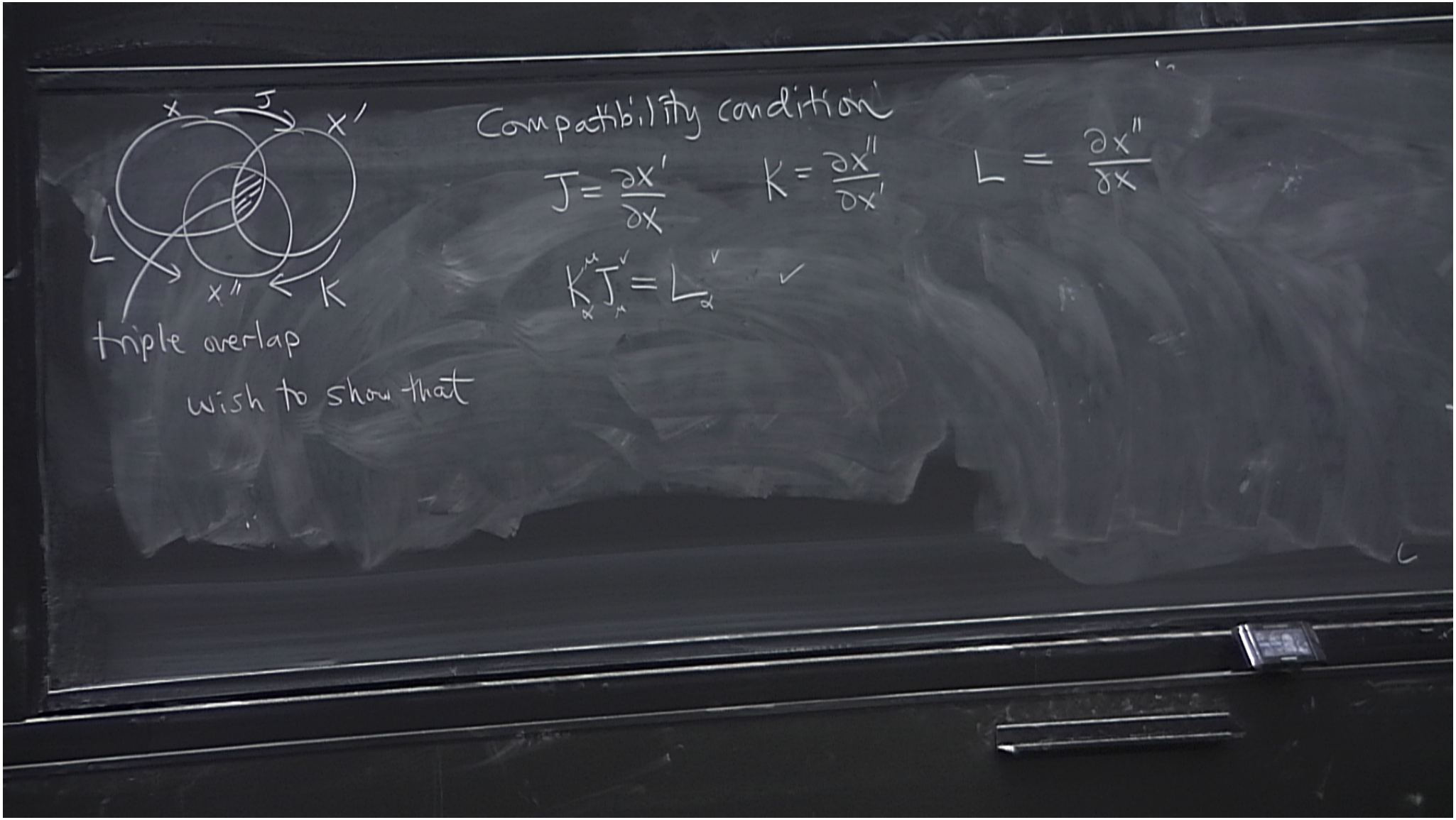
$$- \Gamma^{\xi}_{\beta\alpha} T_{\xi\ldots}{}^{\gamma\ldots} - \ldots \quad \text{(for each lower index)}$$

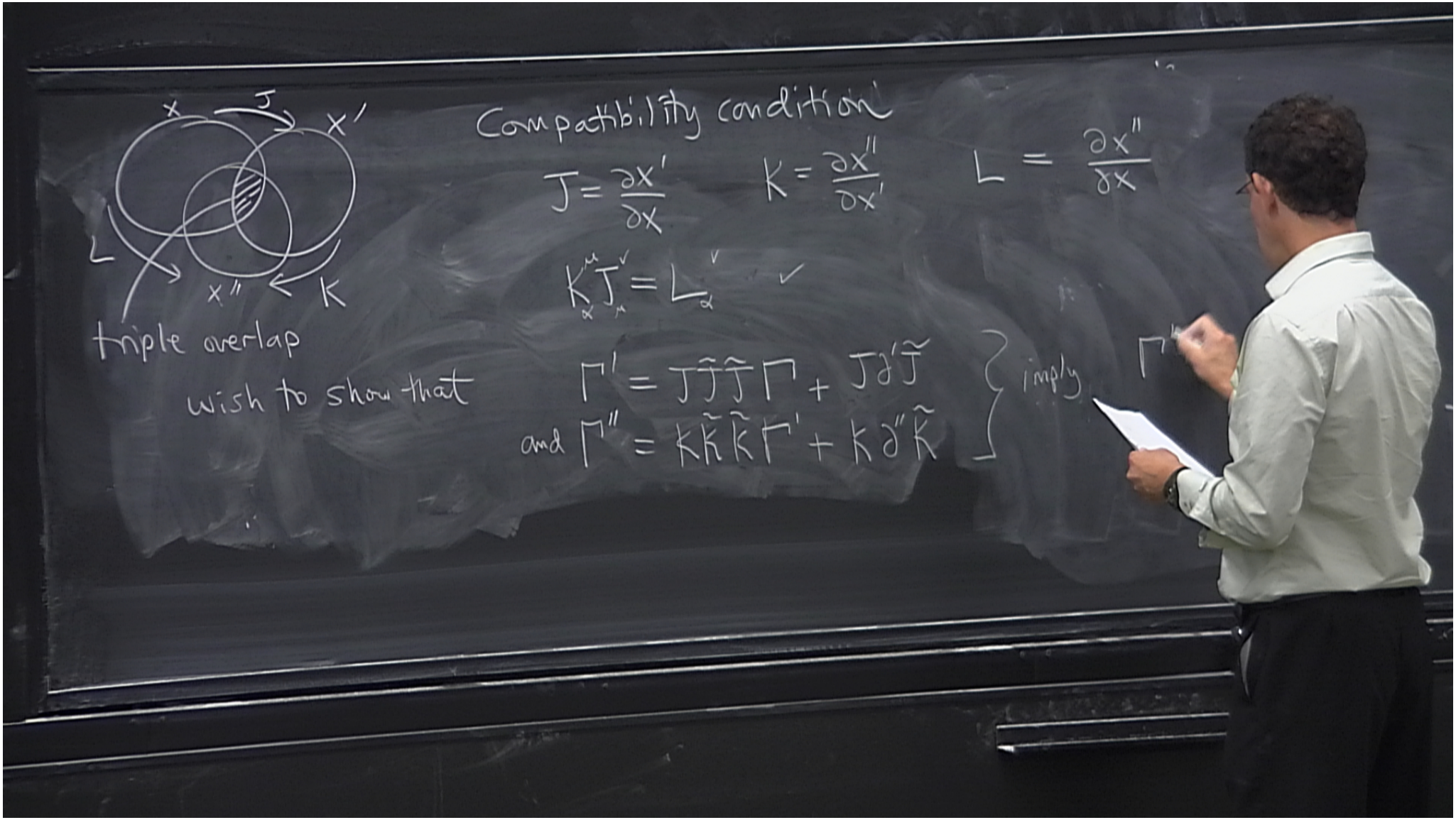

More generally,
(for each upper index)
(for each lower index)


Compatibility condition

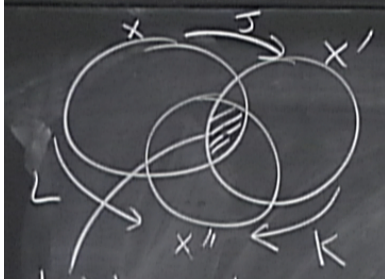

x
x'
x''
triple overlap
Compatibility condition


x
J
x'
L
x''
K
triple overlap
Compatibility condition


x
J
x'
L
x''
K
triple overlap
wish to show that
Compatibility condition
$J = \frac{\partial x'}{\partial x}$
$K = \frac{\partial x''}{\partial x'}$
$L = \frac{\partial x''}{\partial x}$
$K^{\mu}_{\alpha} J^{\nu}_{\mu} = L^{\nu}_{\alpha}$


x
J
x'
L
x''
K
triple overlap
wish to show that
Compatibility condition
$J = \frac{\partial x'}{\partial x}$
$K = \frac{\partial x''}{\partial x'}$
$L = \frac{\partial x''}{\partial x}$
$K^{\mu}_{\alpha} J^{\nu}_{\mu} = L^{\nu}_{\alpha}$
$\Gamma' = J\tilde{J}\tilde{J}\Gamma + J\partial'\tilde{J}$
and $\Gamma'' = K\tilde{K}\tilde{K}\Gamma' + K\partial''\tilde{K}$
imply


Discovery



triple overlap

Compatibility condition

$$J = \frac{\partial x'}{\partial x} \qquad K = \frac{\partial x''}{\partial x'} \qquad L = \frac{\partial x''}{\partial x}$$

$$K^{\mu}_{\alpha'} J^{\nu}_{\mu} = L^{\nu}_{\alpha} \quad \checkmark$$

wish to show that

$$\Gamma' = J\tilde{J}\tilde{J}\Gamma + J\partial'\tilde{J} \quad (1)$$

and

$$\Gamma'' = K\tilde{K}\tilde{K}\Gamma' + K\partial''\tilde{K} \quad (2)$$

} imply

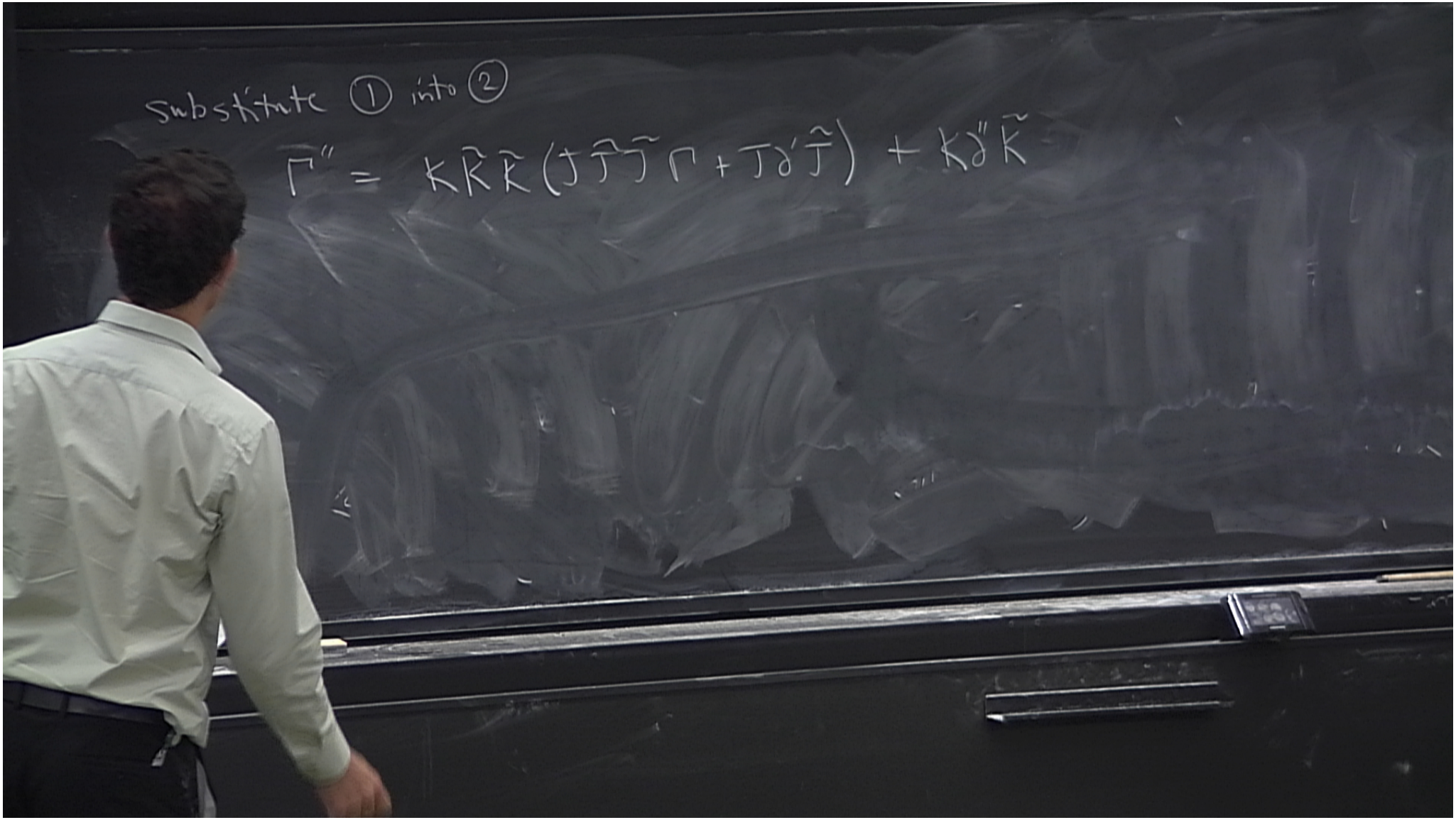
$$\Gamma'' = L\tilde{L}\tilde{L}\Gamma + L\partial''\tilde{L} \quad (3)$$

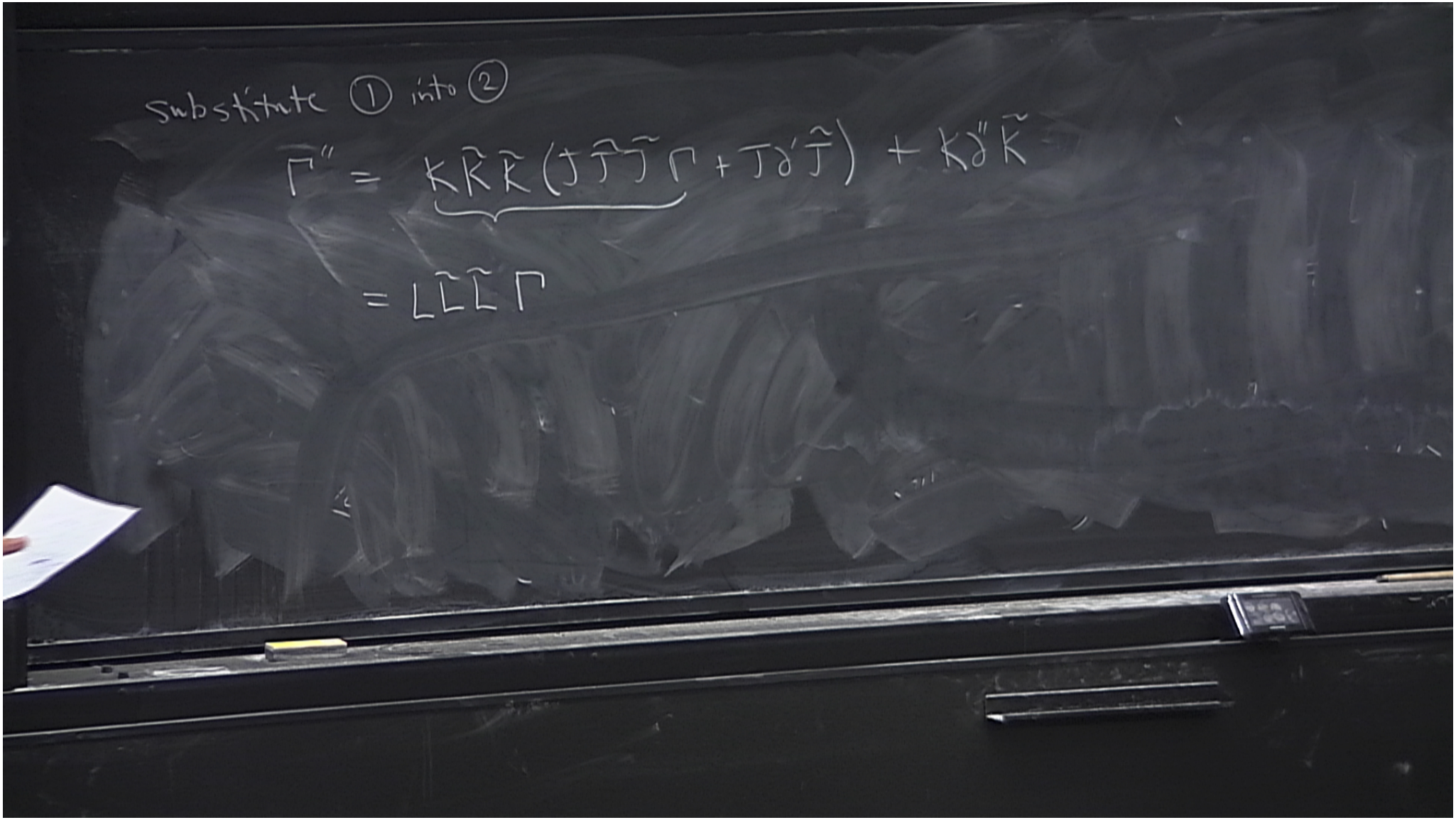
Ex: deSitter

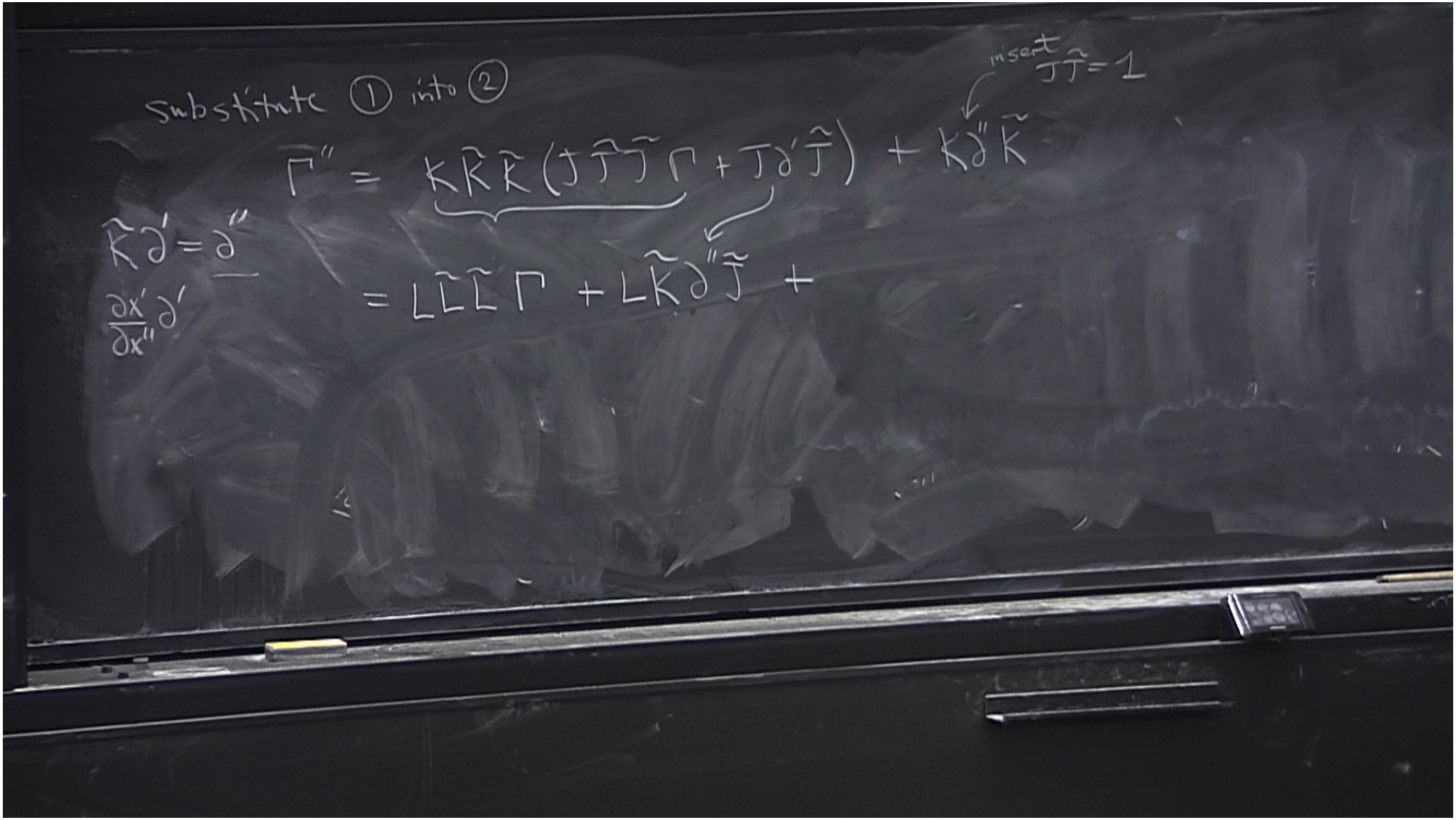
$$-x^0 + x^1 + x^2 + x^3 + x^4 =$$

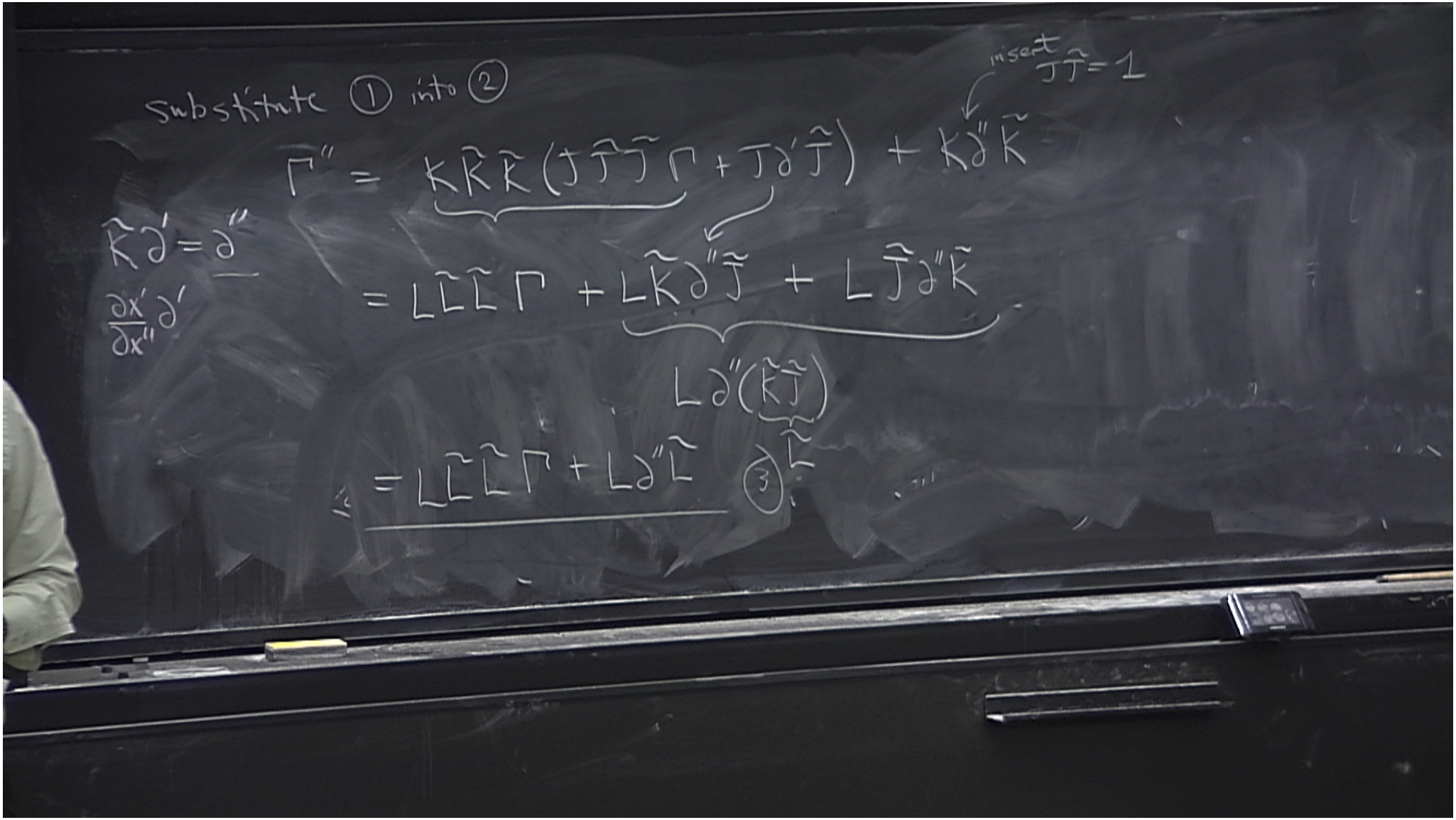
work out the metric

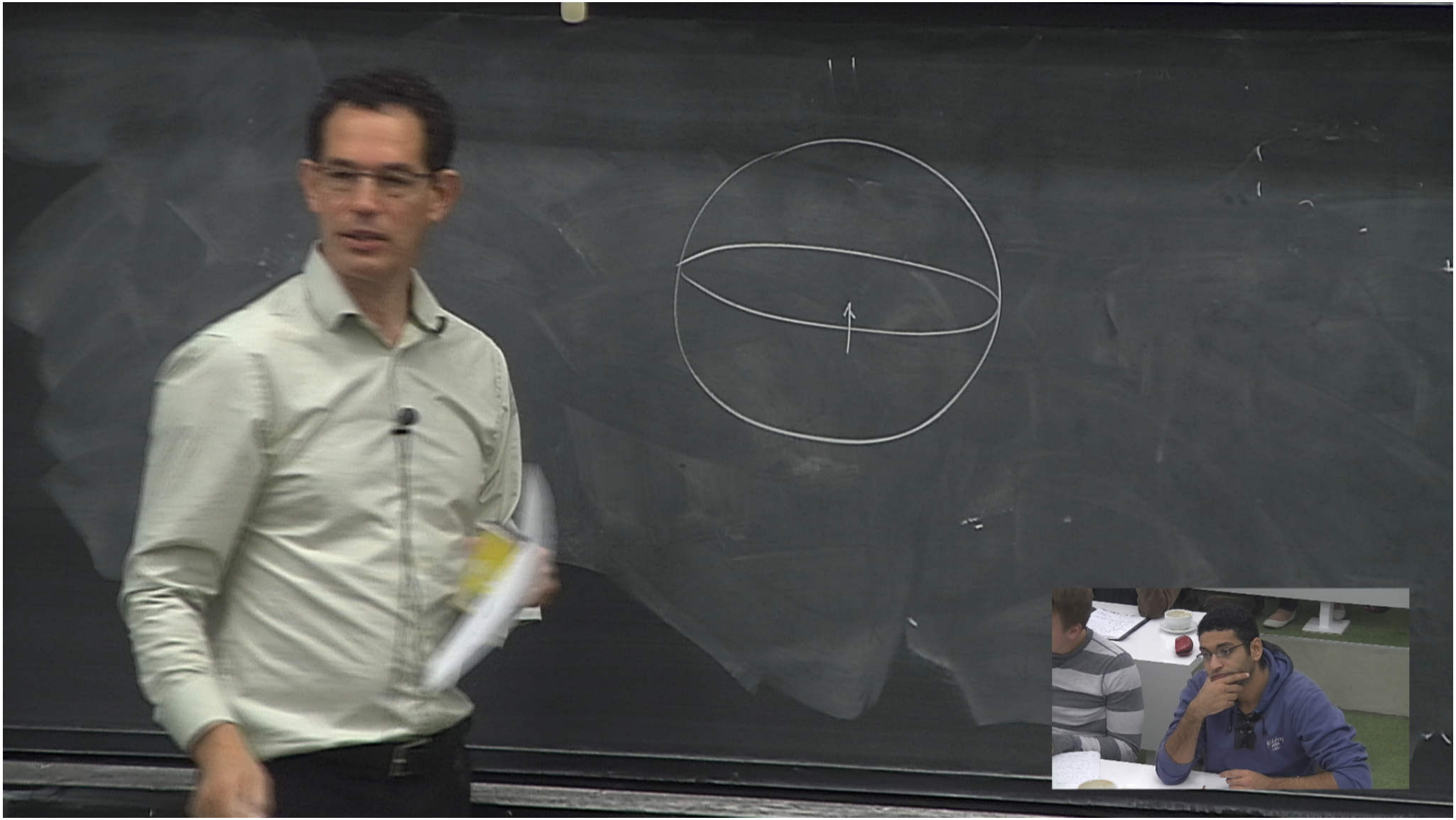
$$\left(-l(l+1) - M(M-1)\right)\frac{1}{n^2}\langle \ldots | \ldots \rangle$$

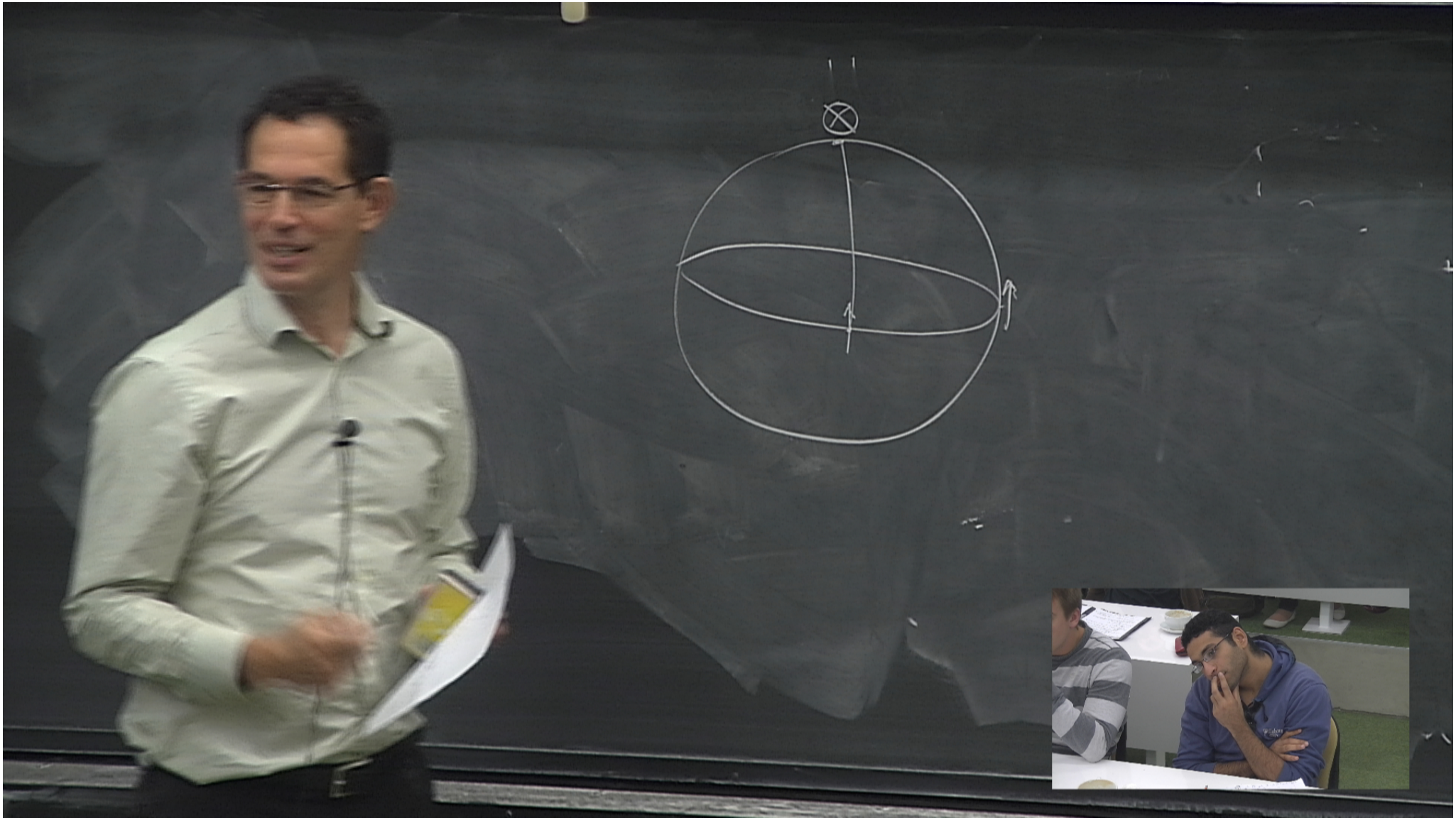

Substitute ① into ②


Substitute ① into ②

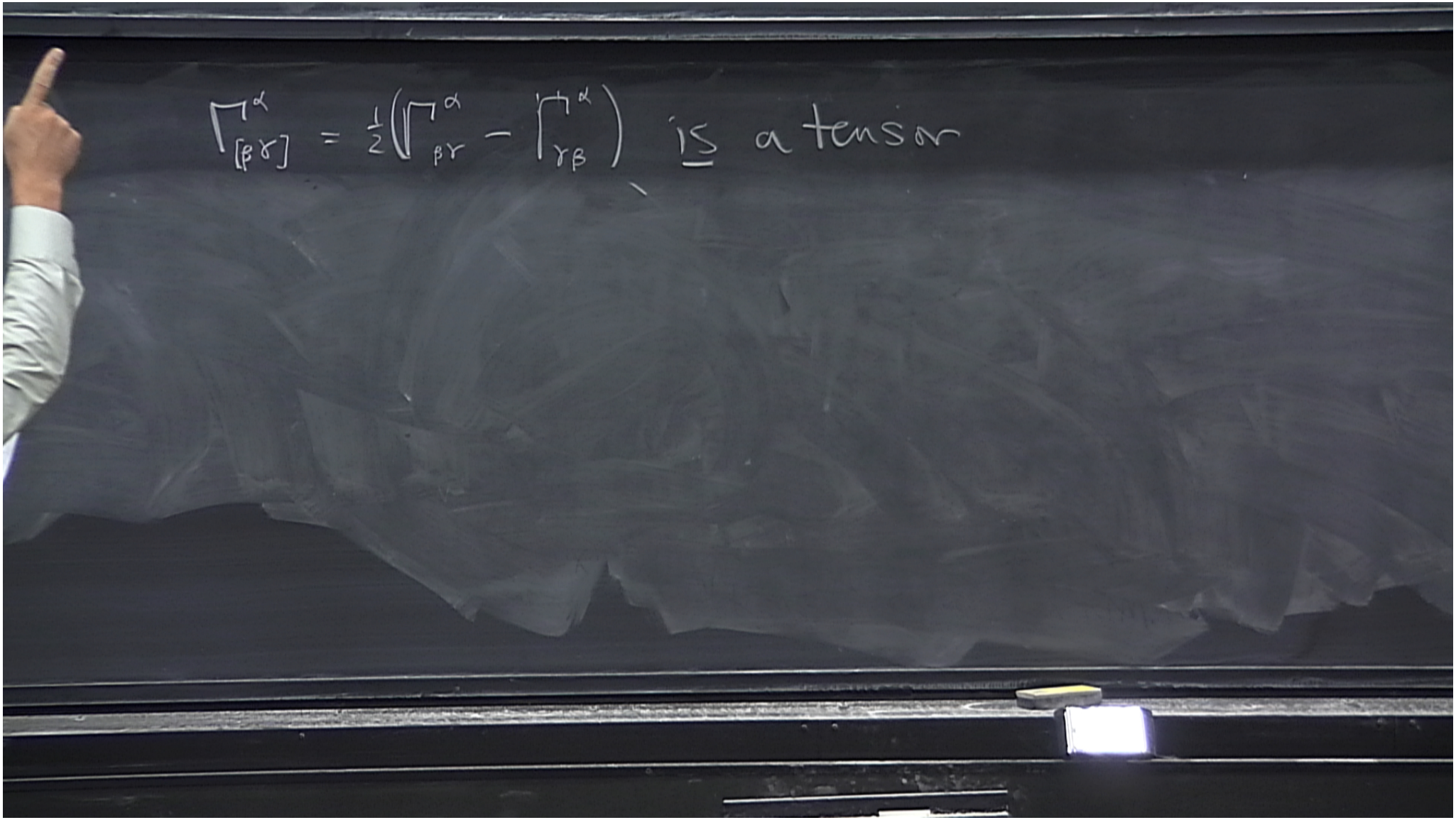

Substitute ① into ②
insert $J\tilde{J}=1$
$\Gamma'' = K\tilde{K}\tilde{K}(J\tilde{J}\tilde{J}\Gamma + J\partial'\tilde{J}) + K\partial''\tilde{K}$
$\tilde{K}\partial' = \partial''$
$\frac{\partial x'}{\partial x''}\partial'$
$= L\tilde{L}\tilde{L}\Gamma + L\tilde{K}\partial''\tilde{J} +$

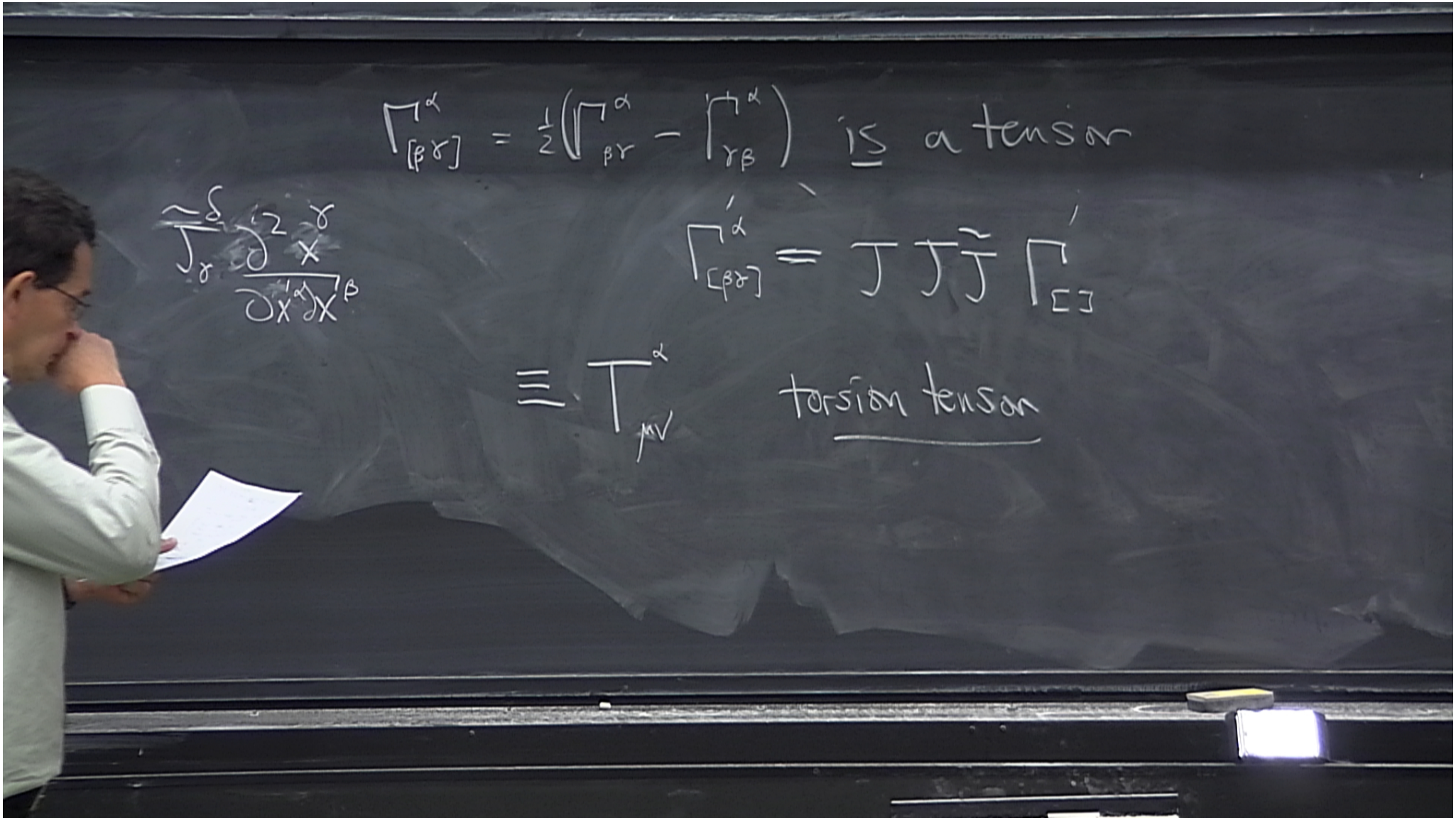

Substitute ① into ②
insert $J\tilde{J}=1$
$\tilde{K}\partial'=\partial''$
$\frac{\partial x'}{\partial x''}\partial'$
$\Gamma''=K\tilde{K}\tilde{K}(J\tilde{J}\tilde{J}\Gamma+J\partial'\tilde{J})+K\partial''\tilde{K}$
$=L\tilde{L}\tilde{L}\Gamma+L\tilde{K}\partial''\tilde{J}+L\tilde{J}\partial''\tilde{K}$
$L\partial''(\tilde{K}\tilde{J})$
$=L\tilde{L}\tilde{L}\Gamma+L\partial''\tilde{L}$
③

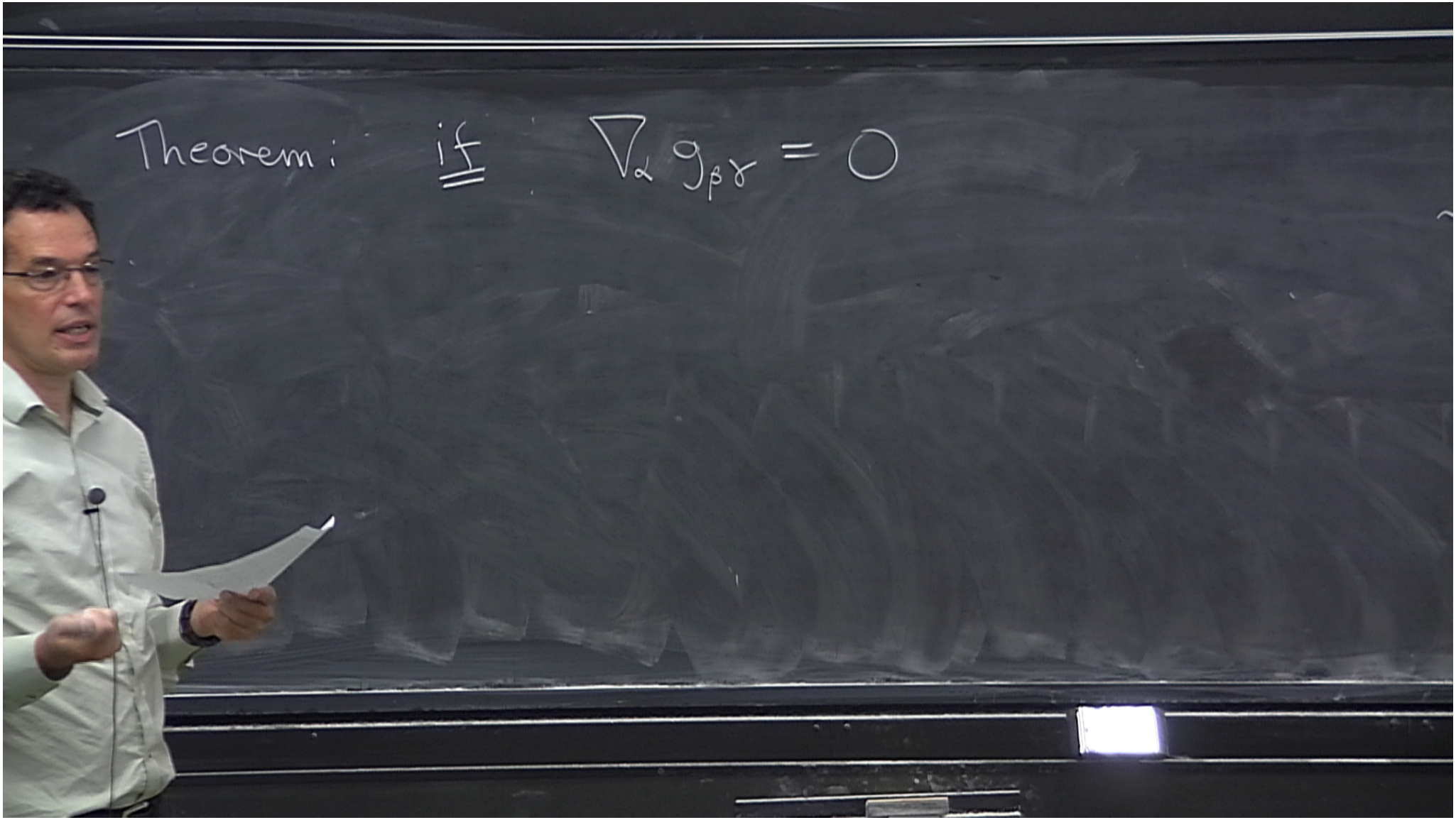





(for each upper
(for each lower


$\Gamma^{\alpha}_{[\beta\gamma]} = \frac{1}{2}\left(\Gamma^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\beta}\right)$ is a tensor

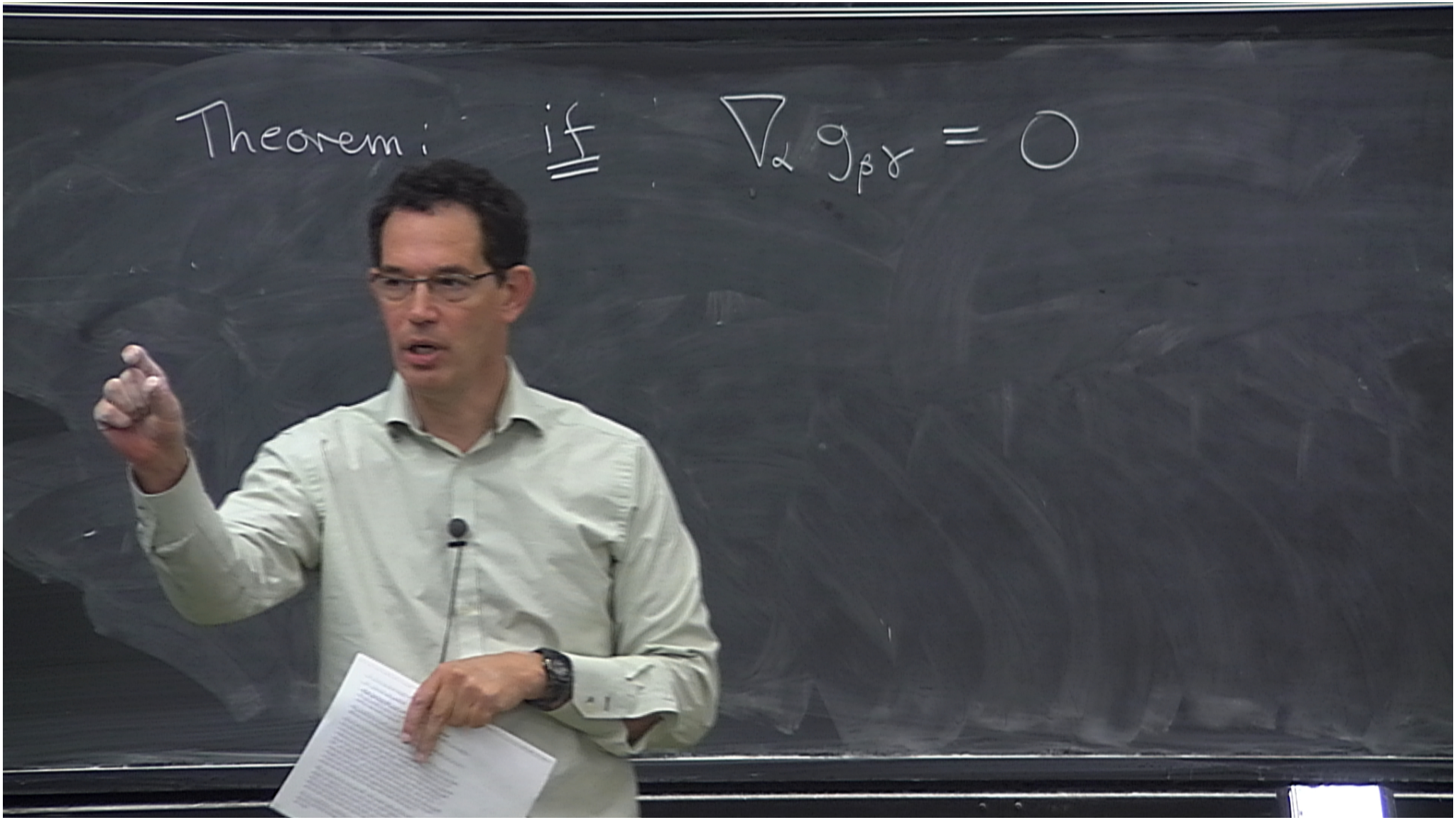

$\Gamma^{\alpha}_{[\beta\gamma]} = \frac{1}{2}\left(\Gamma^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\beta}\right)$ is a tensor
$\tilde{J}^{\delta}_{\gamma} = \frac{\partial^2 x^{\gamma}}{\partial x'^{\alpha} \partial x'^{\beta}}$
$\Gamma'^{\alpha}_{[\beta\gamma]} = J J \tilde{J} \Gamma'_{[\,]}$
$\equiv T^{\alpha}_{\mu\nu}$ torsion tensor

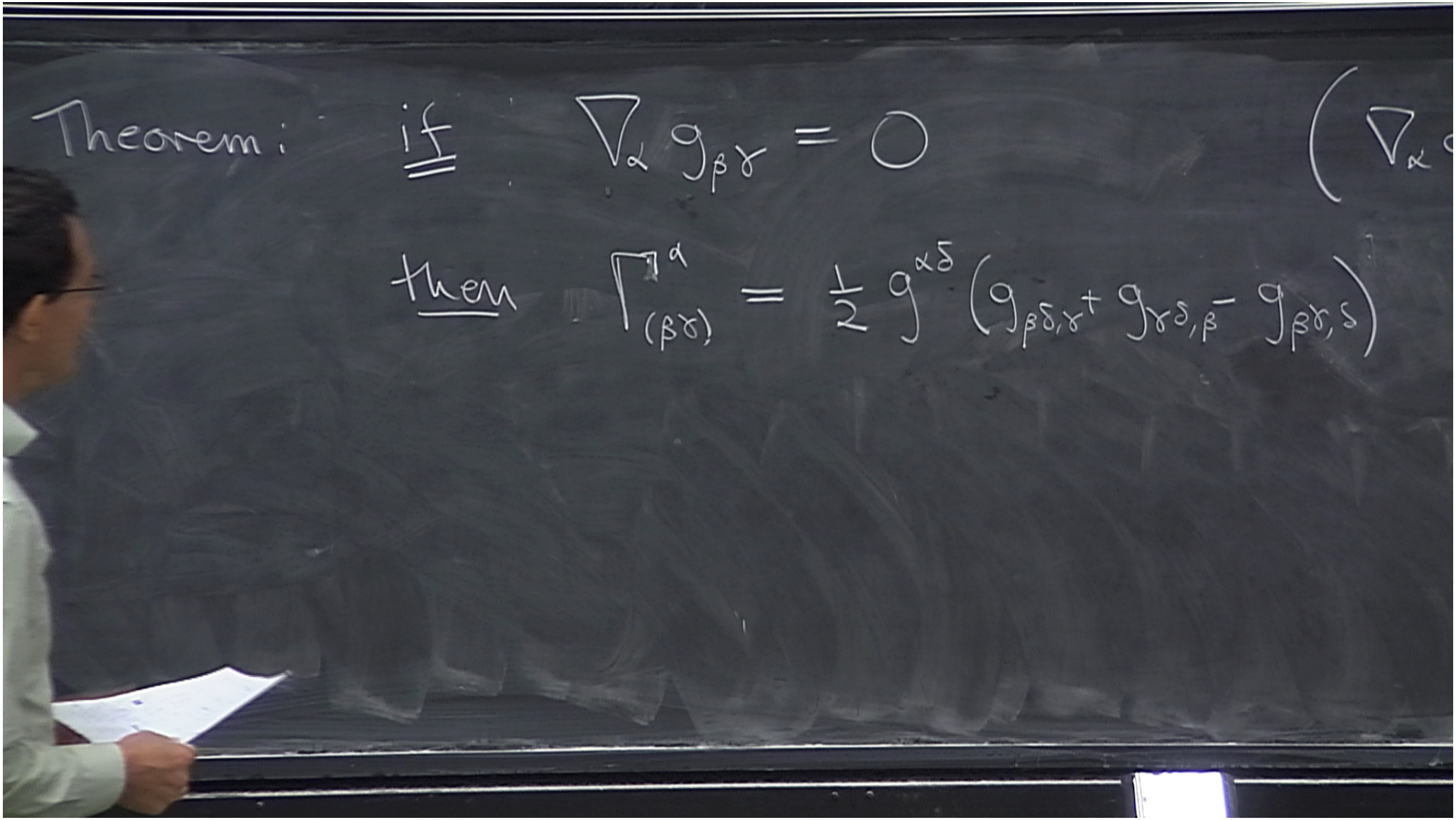

Theorem: if $\nabla_\alpha g_{\beta\gamma} = 0$

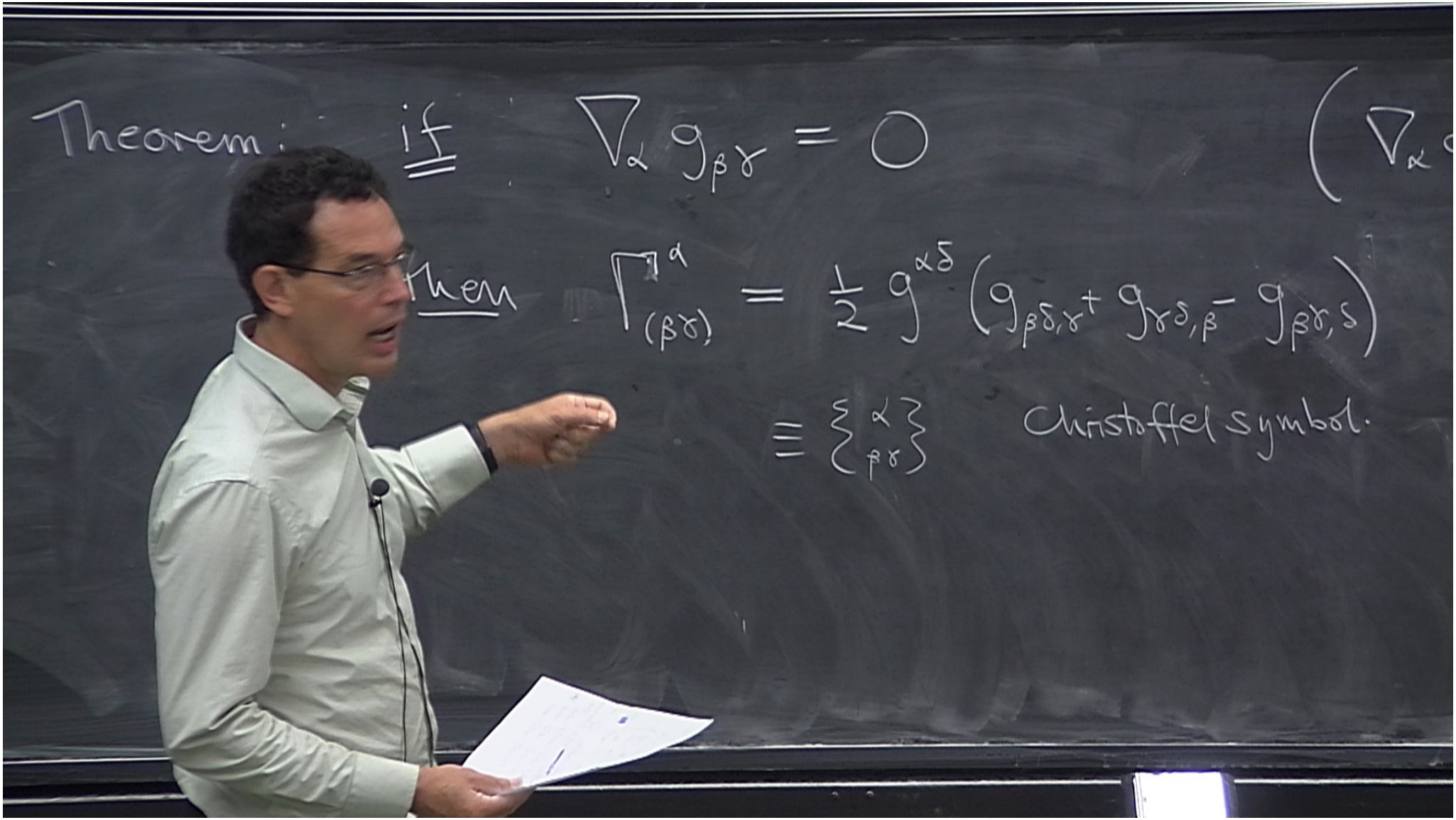
Theorem: if $\nabla_\alpha g_{\beta\gamma} = 0$ ($\nabla_\alpha g_{\beta\gamma}$ is a tensor, "metricity")

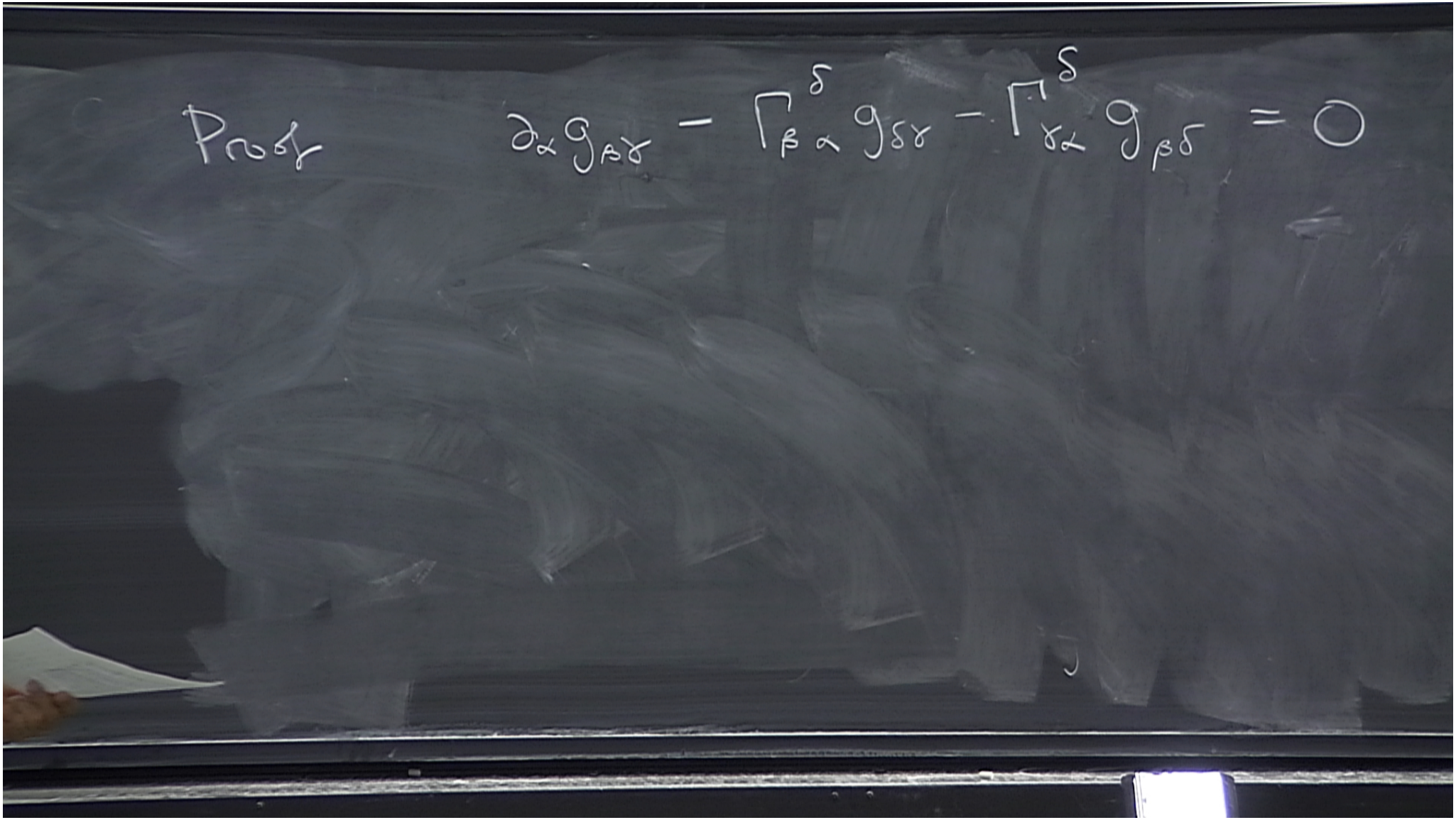
$(\nabla_\alpha g_{\beta\gamma}$ is a tensor, "metricity")

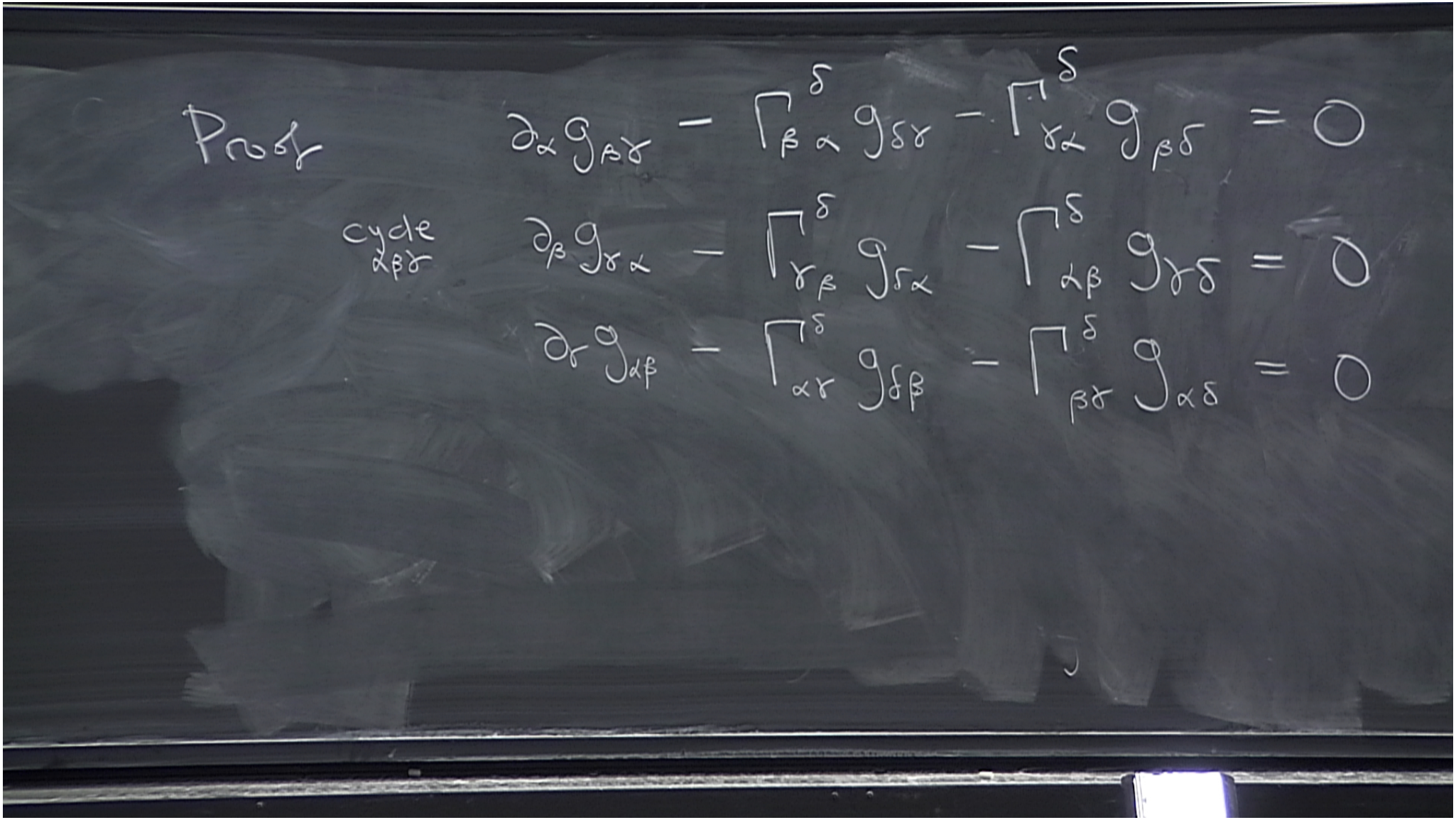
Ex: think about a theory in which metricity would b
dynamically determined.

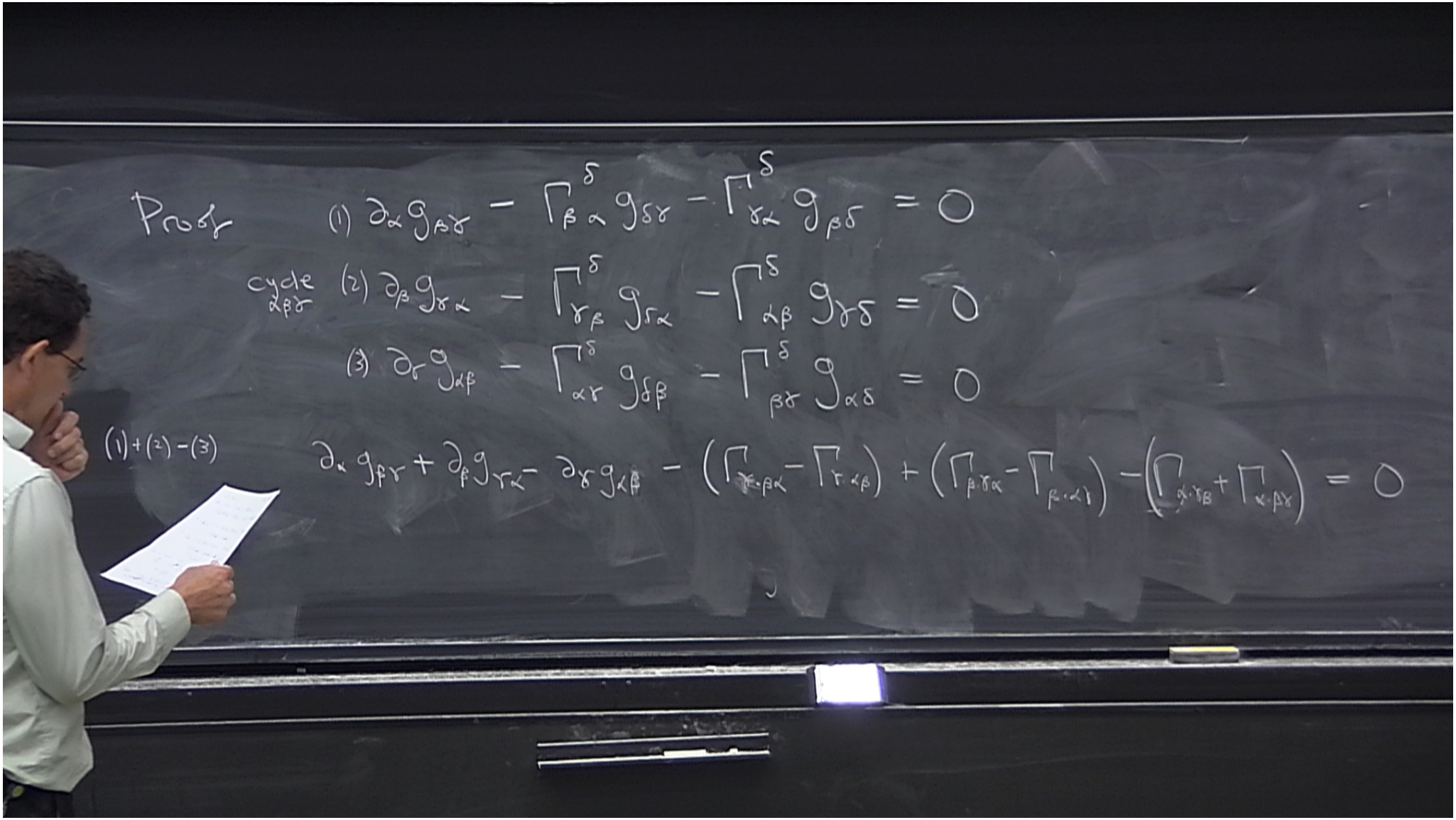

Theorem: if $\nabla_\alpha g_{\beta\gamma} = 0$

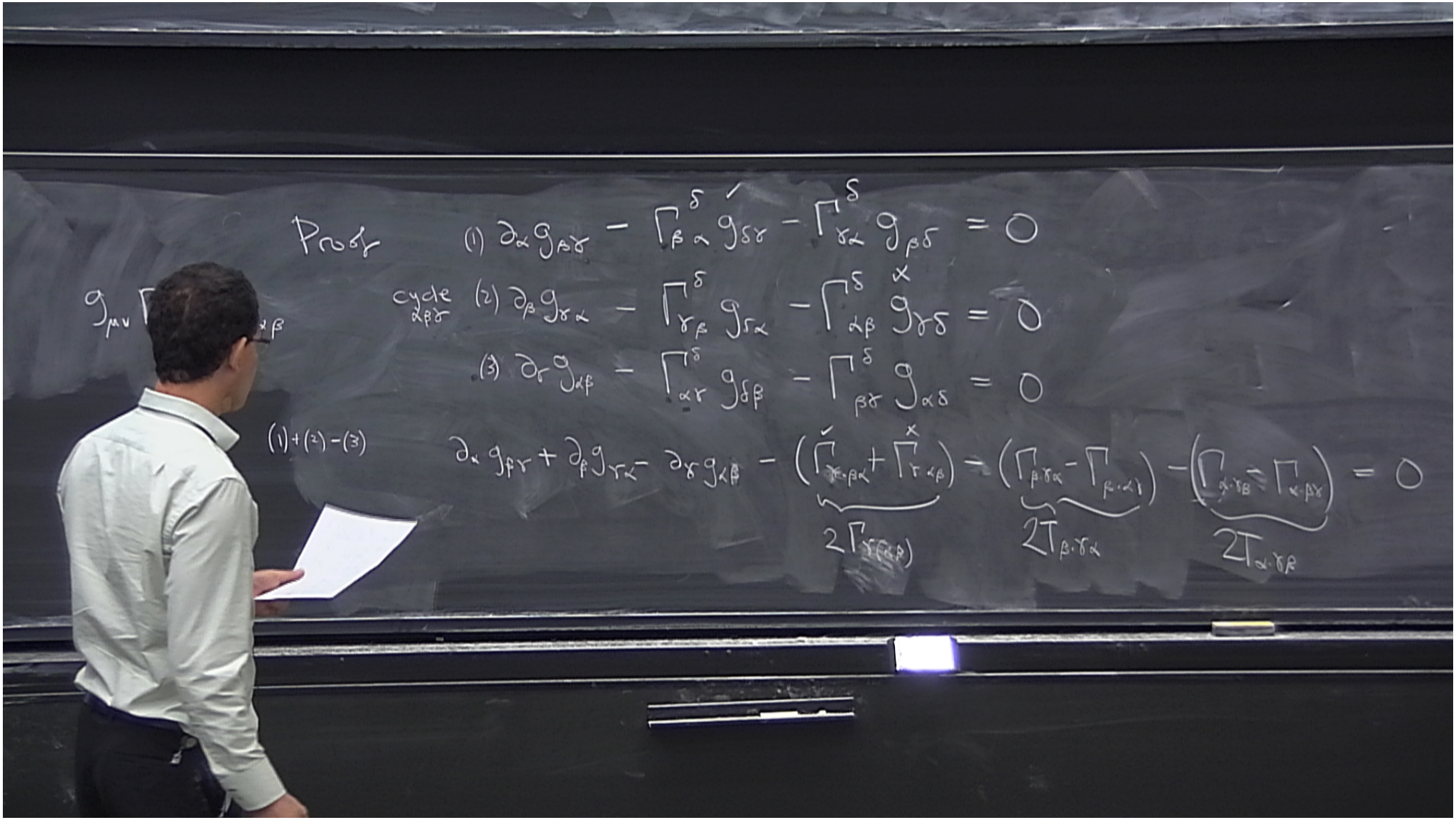

Theorem: if $\nabla_\alpha g_{\beta\gamma} = 0$
then $\Gamma^\alpha_{(\beta\gamma)} = \frac{1}{2} g^{\alpha\delta} (g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta})$

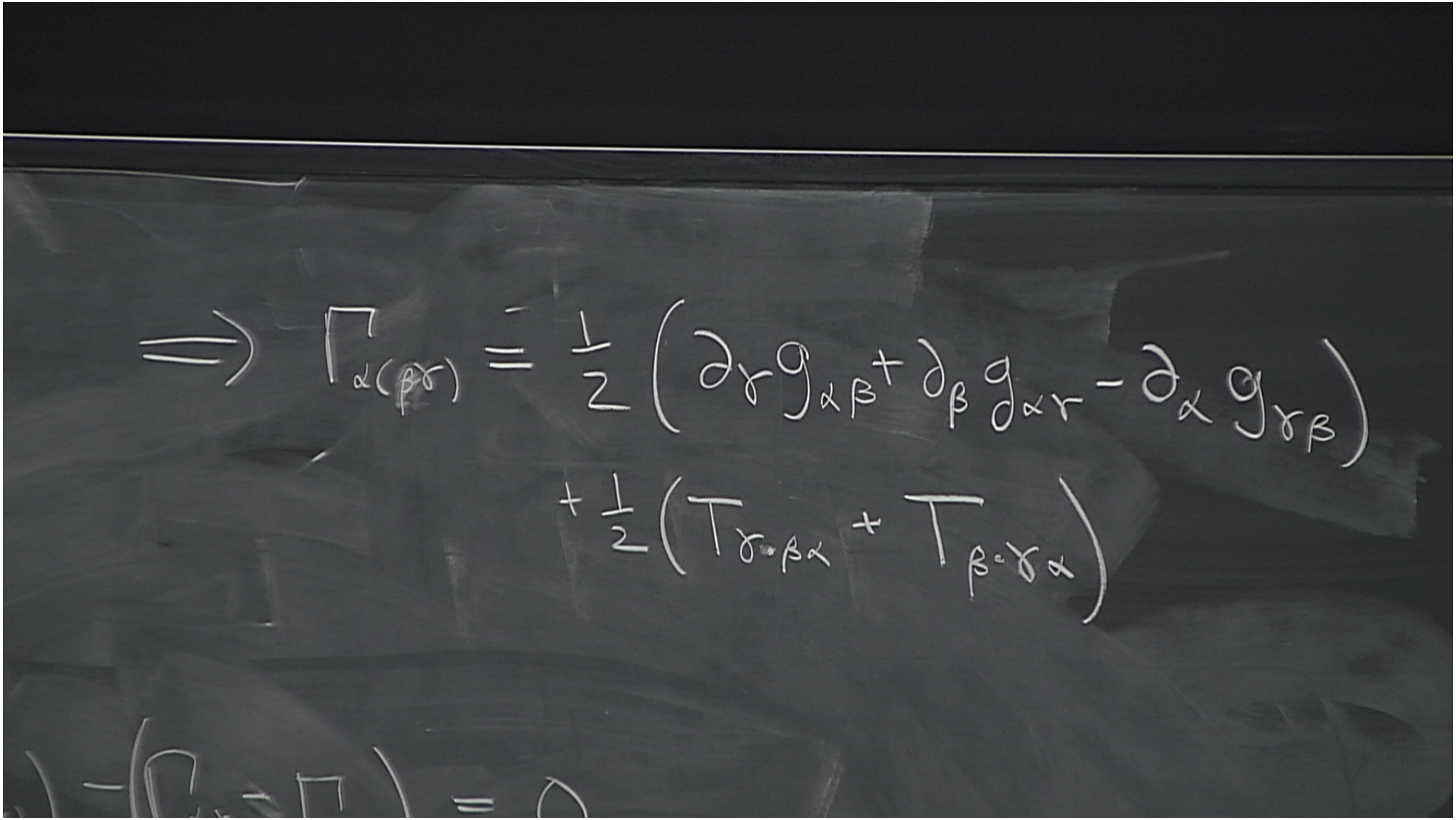

Theorem: if $\nabla_\alpha g_{\beta\gamma} = 0$
then $\Gamma^\alpha_{(\beta\gamma)} = \frac{1}{2} g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta})$
$\equiv \left\{ {\alpha \atop \beta\gamma} \right\}$ Christoffel symbol.


Prob 7
$\partial_\alpha g_{\beta\gamma} - \Gamma^{\delta}_{\beta\alpha} g_{\delta\gamma} - \Gamma^{\delta}_{\gamma\alpha} g_{\beta\delta} = 0$

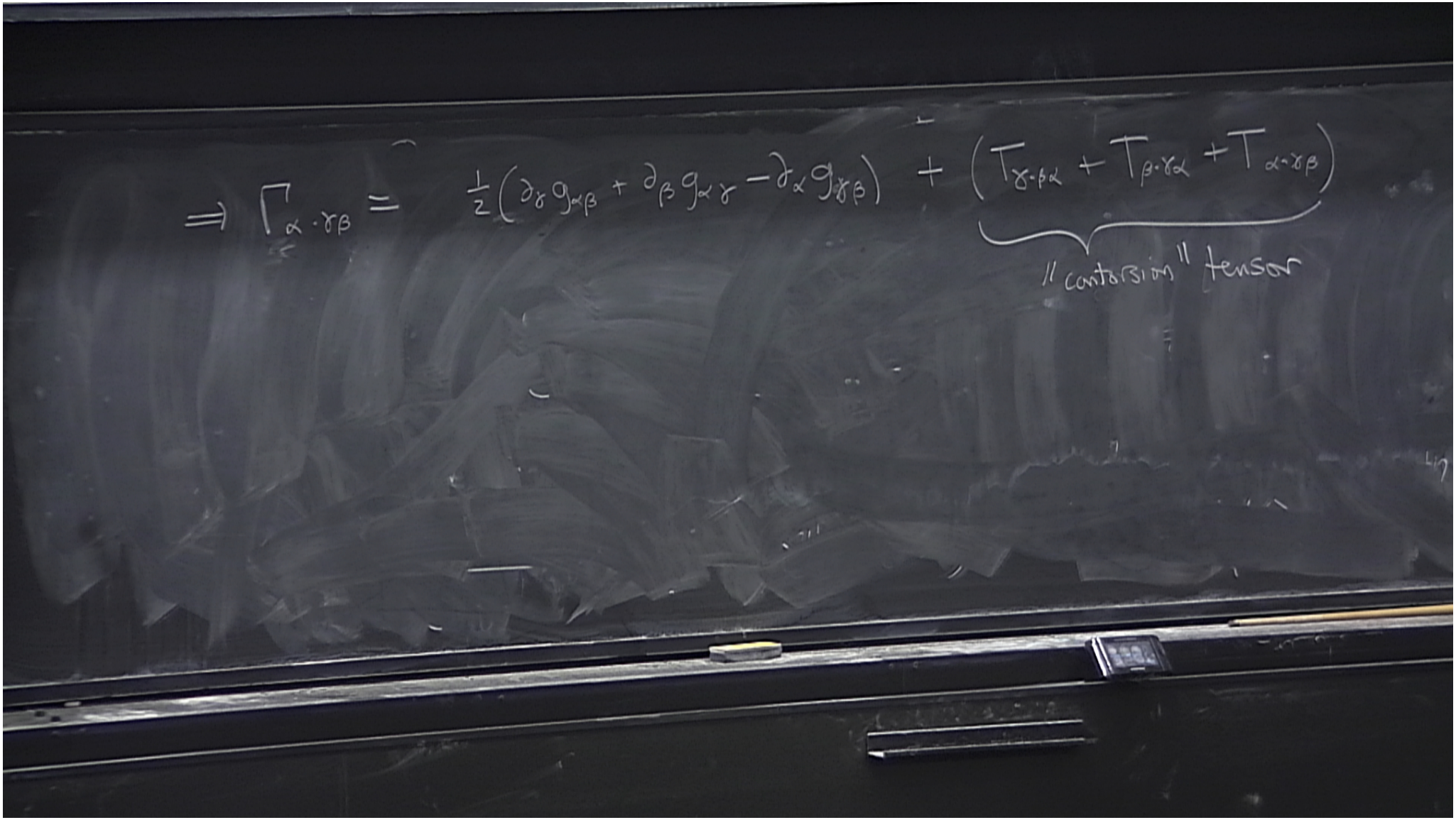

Proof
$\partial_\alpha g_{\beta\gamma} - \Gamma^\delta_{\beta\alpha} g_{\delta\gamma} - \Gamma^\delta_{\gamma\alpha} g_{\beta\delta} = 0$
cycle $\alpha\beta\gamma$
$\partial_\beta g_{\gamma\alpha} - \Gamma^\delta_{\gamma\beta} g_{\delta\alpha} - \Gamma^\delta_{\alpha\beta} g_{\gamma\delta} = 0$
$\partial_\gamma g_{\alpha\beta} - \Gamma^\delta_{\alpha\gamma} g_{\delta\beta} - \Gamma^\delta_{\beta\gamma} g_{\alpha\delta} = 0$


Proof
(1) $\partial_\alpha g_{\beta\gamma} - \Gamma^\delta_{\beta\alpha} g_{\delta\gamma} - \Gamma^\delta_{\gamma\alpha} g_{\beta\delta} = 0$
cycle $\alpha\beta\gamma$
(2) $\partial_\beta g_{\gamma\alpha} - \Gamma^\delta_{\gamma\beta} g_{\delta\alpha} - \Gamma^\delta_{\alpha\beta} g_{\gamma\delta} = 0$
(3) $\partial_\gamma g_{\alpha\beta} - \Gamma^\delta_{\alpha\gamma} g_{\delta\beta} - \Gamma^\delta_{\beta\gamma} g_{\alpha\delta} = 0$
(1)+(2)-(3)
$\partial_\alpha g_{\beta\gamma} + \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta} - (\Gamma_{\gamma,\beta\alpha} - \Gamma_{\gamma,\alpha\beta}) + (\Gamma_{\beta,\gamma\alpha} - \Gamma_{\beta,\alpha\gamma}) - (\Gamma_{\alpha,\gamma\beta} + \Gamma_{\alpha,\beta\gamma}) = 0$


Proof
(1) $\partial_\alpha g_{\beta\gamma} - \Gamma^{\delta}_{\beta\alpha} g_{\delta\gamma} - \Gamma^{\delta}_{\gamma\alpha} g_{\beta\delta} = 0$
cycle $\alpha\beta\gamma$ (2) $\partial_\beta g_{\gamma\alpha} - \Gamma^{\delta}_{\gamma\beta} g_{\delta\alpha} - \Gamma^{\delta}_{\alpha\beta} g_{\gamma\delta} = 0$
(3) $\partial_\gamma g_{\alpha\beta} - \Gamma^{\delta}_{\alpha\gamma} g_{\delta\beta} - \Gamma^{\delta}_{\beta\gamma} g_{\alpha\delta} = 0$
(1)+(2)-(3)
$\partial_\alpha g_{\beta\gamma} + \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta} - (\Gamma_{\gamma,\beta\alpha} + \Gamma_{\gamma,\alpha\beta}) - (\Gamma_{\beta,\gamma\alpha} - \Gamma_{\beta,\alpha\gamma}) - (\Gamma_{\alpha,\gamma\beta} - \Gamma_{\alpha,\beta\gamma}) = 0$
$2\Gamma_{\gamma,(\alpha\beta)}$
$2\Gamma_{\beta,\gamma\alpha}$
$2\Gamma_{\alpha,\gamma\beta}$


$\Rightarrow \Gamma_{\alpha(\beta\gamma)} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right)$
$+ \frac{1}{2}\left(T_{\gamma\cdot\beta\alpha} + T_{\beta\cdot\gamma\alpha}\right)$

$$\Rightarrow \Gamma^{\alpha}_{\gamma\beta} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right) + \left(T_{\gamma\cdot\beta\alpha} + T_{\beta\cdot\gamma\alpha} + T_{\alpha\cdot\gamma\beta}\right)$$


$\Rightarrow \Gamma^{\alpha}_{\gamma\beta} = \frac{1}{2}(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}) + (T_{\gamma\cdot\beta}^{\ \ \alpha} + T_{\beta\cdot\gamma}^{\ \ \alpha} + T_{\alpha\cdot\gamma\beta})$
"contorsion" tensor

if $\nabla_\alpha g_{\beta\gamma} = 0$ ($\nabla_\alpha g_{\beta\gamma}$ is a tensor, "metric

then $\Gamma^\alpha_{(\beta\gamma)} = \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta}\right)$ ... a theory in which ... cally determined.

$\equiv \left\{ {\alpha \atop \beta\gamma} \right\}$ Christoffel symbol.

$g_{\beta\sigma,\delta} \equiv \frac{\partial}{\partial x^\delta} g_{\beta\sigma}$

cycle $\alpha\beta\gamma$ (2) $\partial_\beta g_{\gamma\alpha} - \Gamma^\delta_{\gamma\beta} g_{\delta\alpha} - \Gamma^\delta_{\alpha\beta} g_{\gamma\delta} = 0$

(3) $\partial_\gamma g_{\alpha\beta} - \Gamma^\delta_{\alpha\gamma} g_{\delta\beta} - \Gamma^\delta_{\beta\gamma} g_{\alpha\delta} = 0$

$$\Rightarrow \Gamma_{\alpha\cdot\gamma\beta} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right) + \underbrace{\left(T_{\gamma\cdot\beta\alpha} + \right.}$$

// con

If torsion is set equal to zero, then

$$\Rightarrow \Gamma^{\delta}_{\beta\gamma} = \frac{1}{2} g^{\delta\alpha}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\gamma\alpha} - \partial_\alpha g_{\beta\gamma}\right)$$

$$\Rightarrow \Gamma_{\alpha\cdot\gamma\beta} = \frac{1}{2}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\alpha g_{\gamma\beta}\right) + \underbrace{\left(T_{\gamma\cdot\beta\alpha} + \right.}$$

// con

If torsion is set equal to zero, then

$$\Rightarrow \Gamma^{\delta}_{\beta\gamma} = \frac{1}{2} g^{\delta\alpha}\left(\partial_\gamma g_{\alpha\beta} + \partial_\beta g_{\gamma\alpha} - \partial_\alpha g_{\beta\gamma}\right)$$

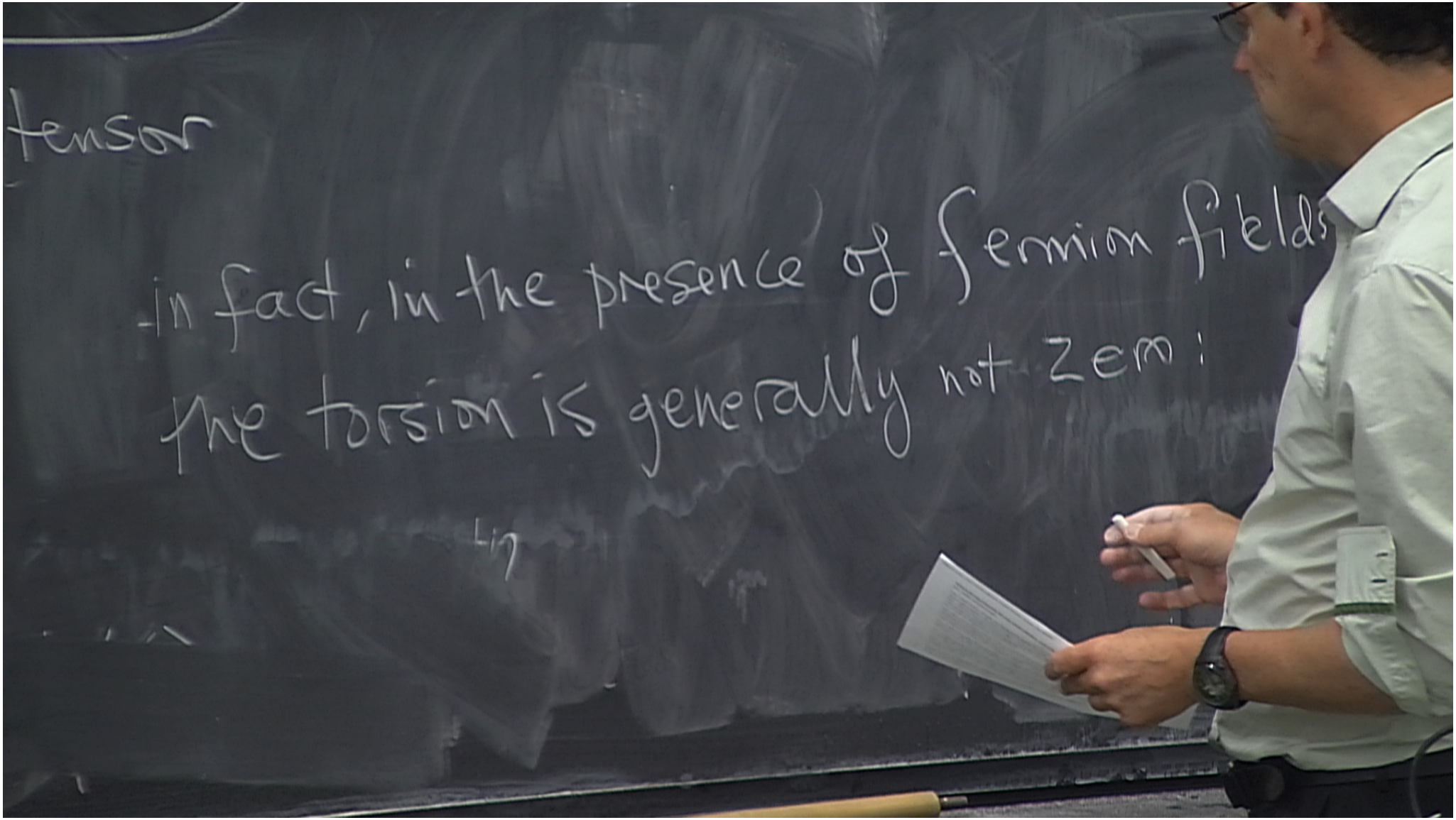
$$= \left\{ {\delta \atop \beta\gamma} \right\}$$

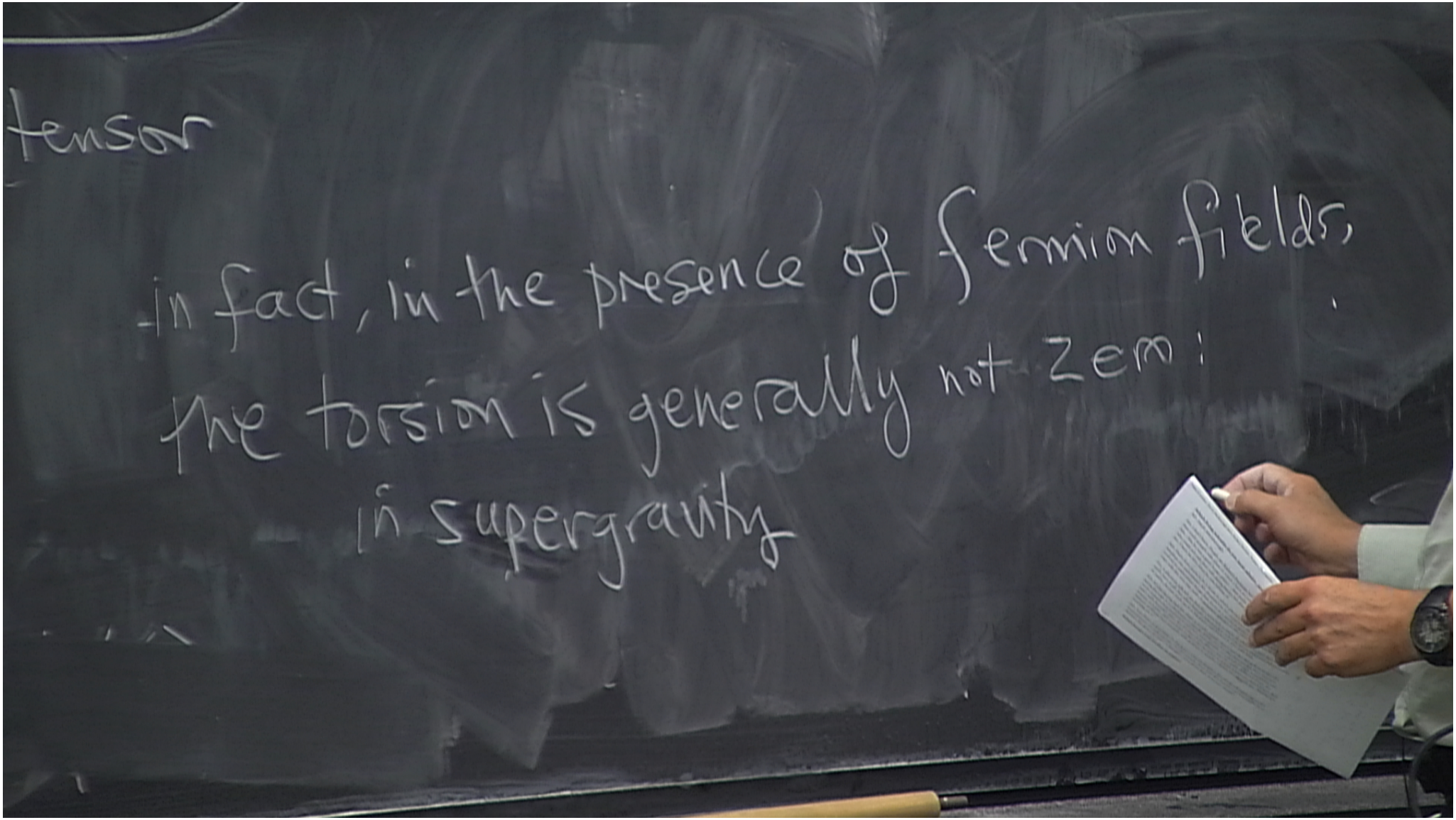
Theorem: if $\nabla_\alpha g_{\beta\gamma} = 0$ $\left(\nabla_\alpha g_{\beta}\right.$

and torsion is zero then $\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta}\right)$

$\equiv \left\{ {\alpha \atop \beta\gamma} \right\}$ Christoffel symbol.



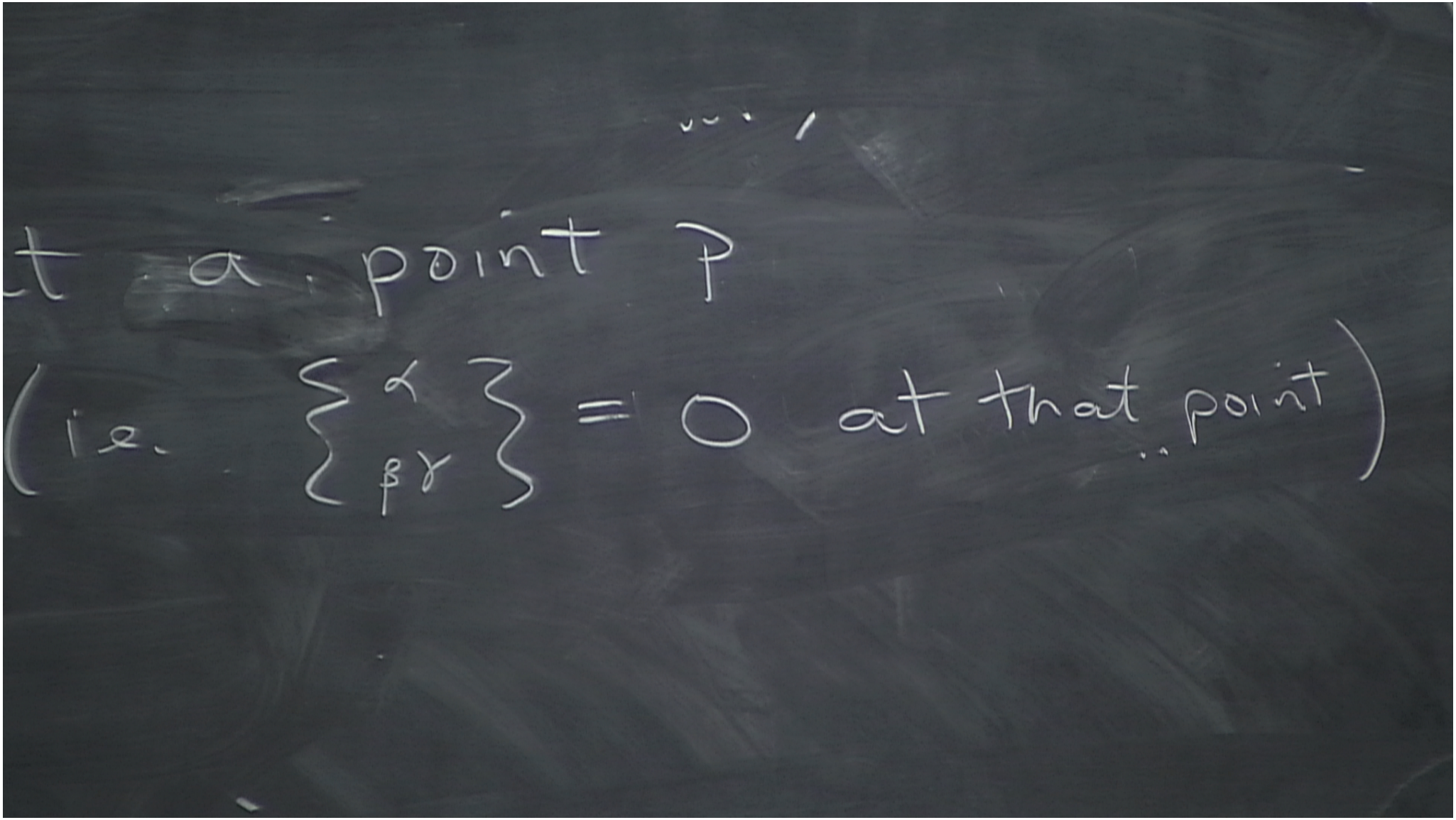

tensor
-in fact, in the presence of fermion fields
the torsion is generally not zero:

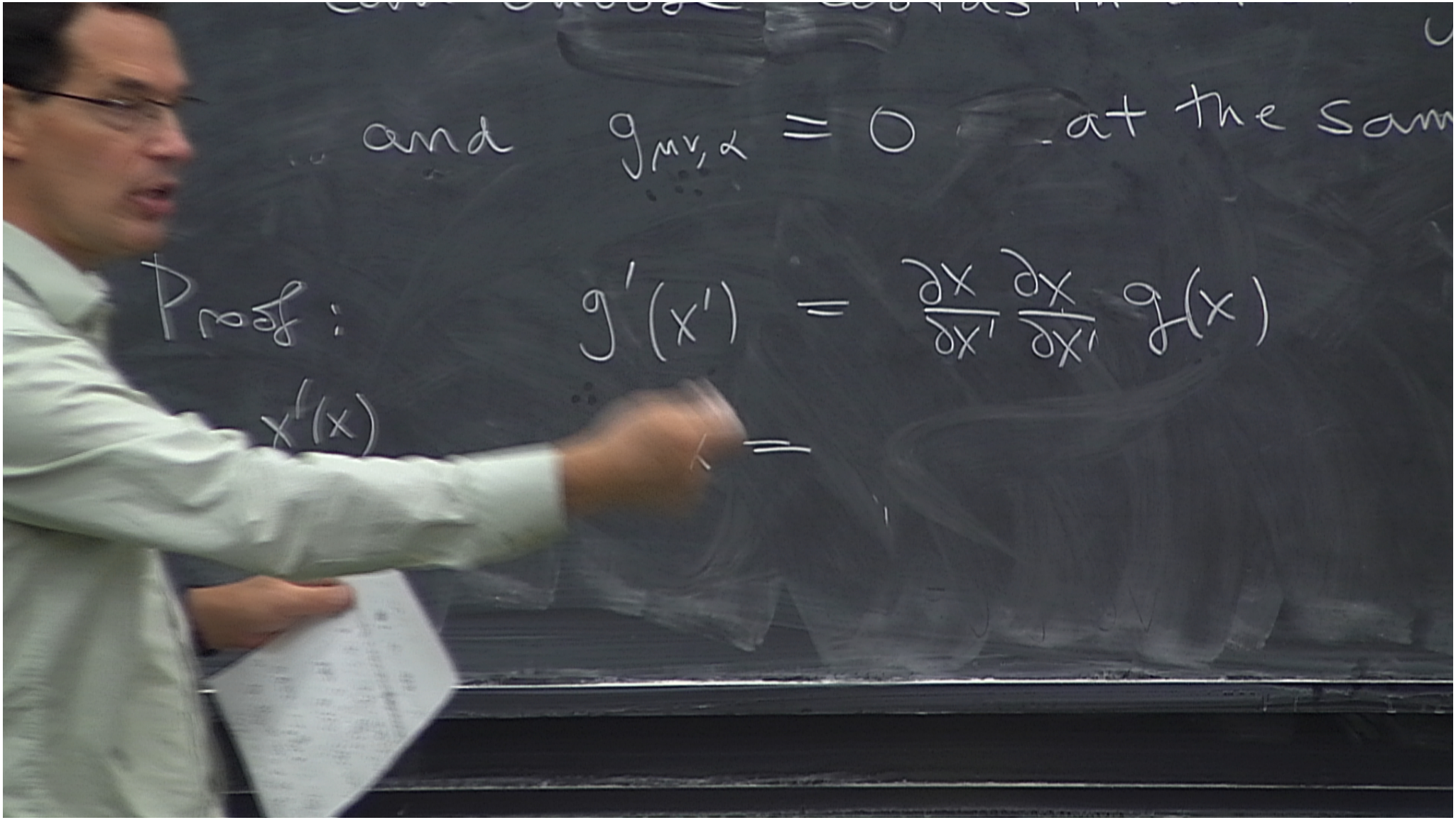

tensor
- in fact, in the presence of fermion fields,
the torsion is generally not zero:
in supergravity

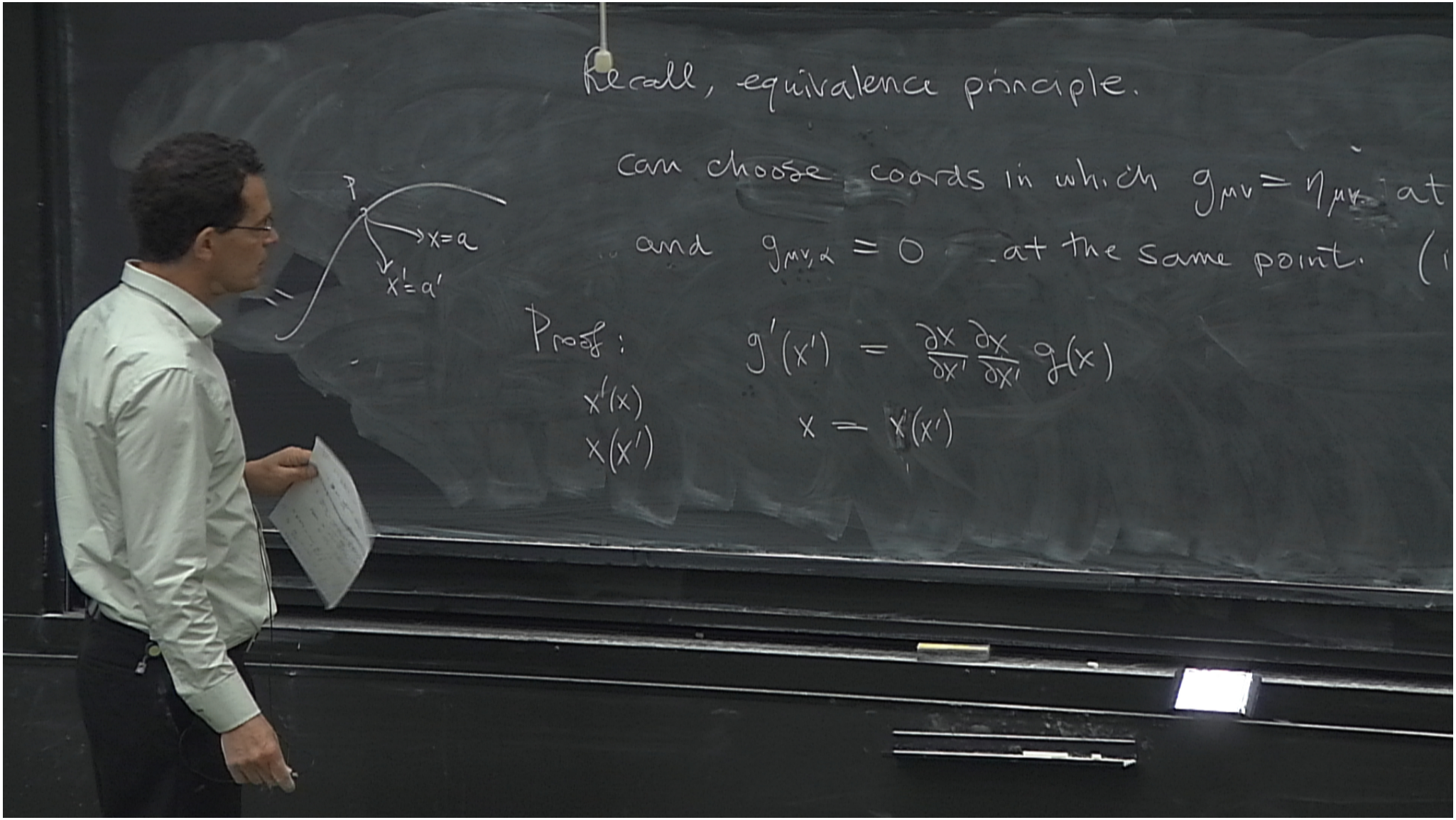
Recall, equivalence principle.

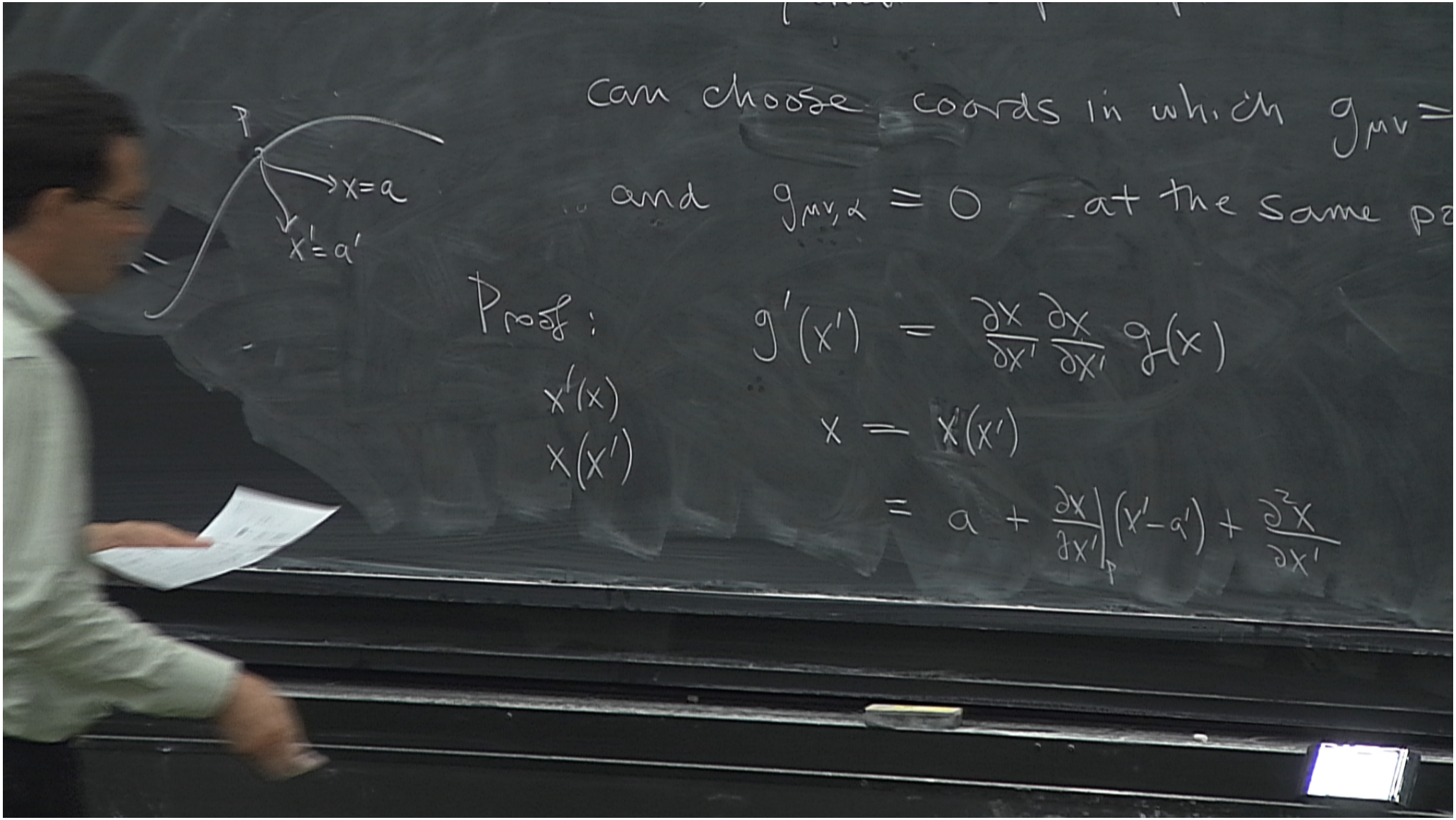
... can choose coords in which $g_{\mu\nu} = \eta_{\mu\nu}$ at a point P

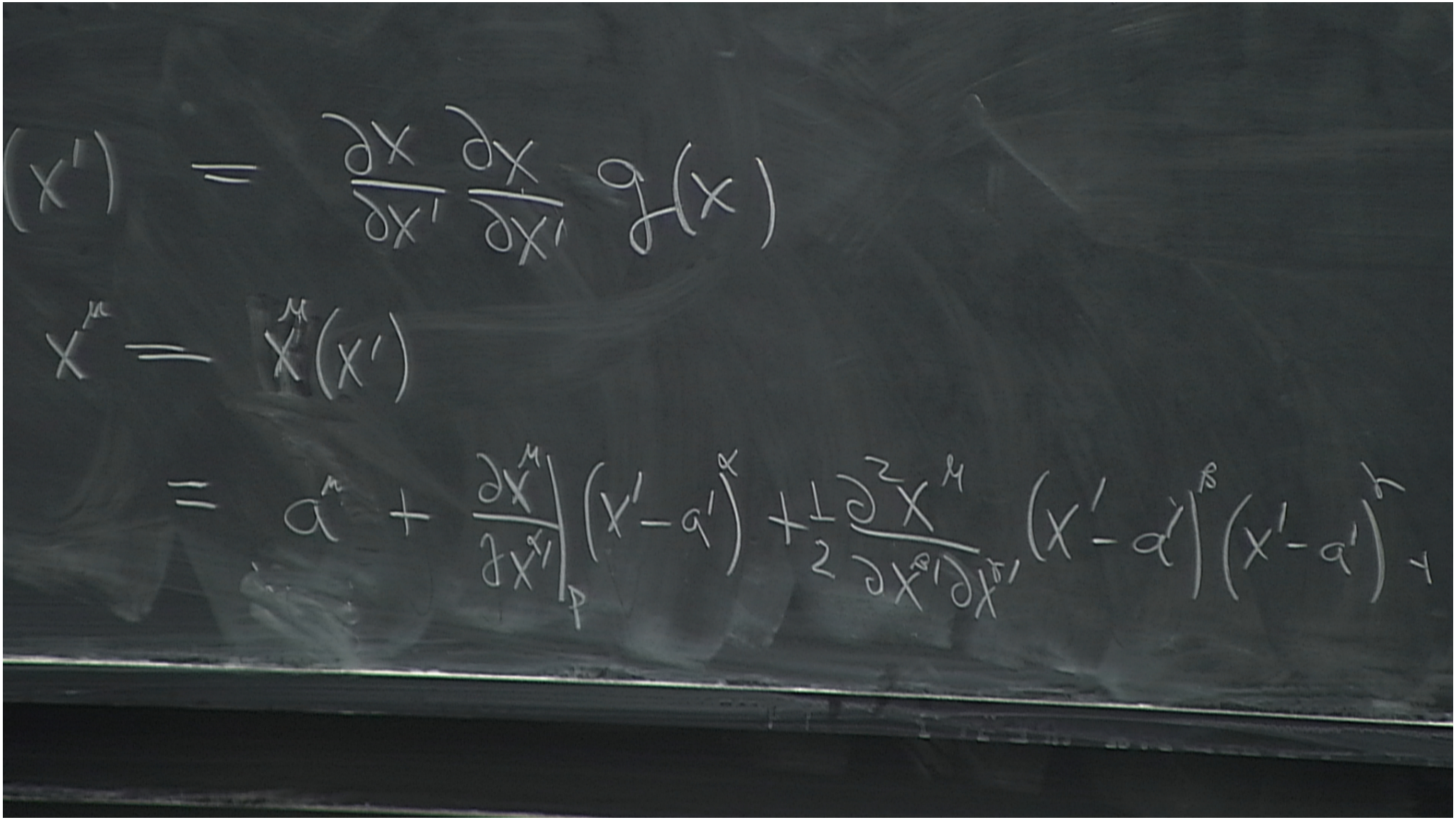
and $g_{\mu\nu,\alpha} = 0$ at the same point.


t a point P
(ie. $\left\{ {\alpha \atop \beta\gamma} \right\} = 0$ at that point)


and $g_{\mu\nu;\alpha} = 0$ at the sam
Proof:
$g'(x') = \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} g(x)$
$x'(x)$

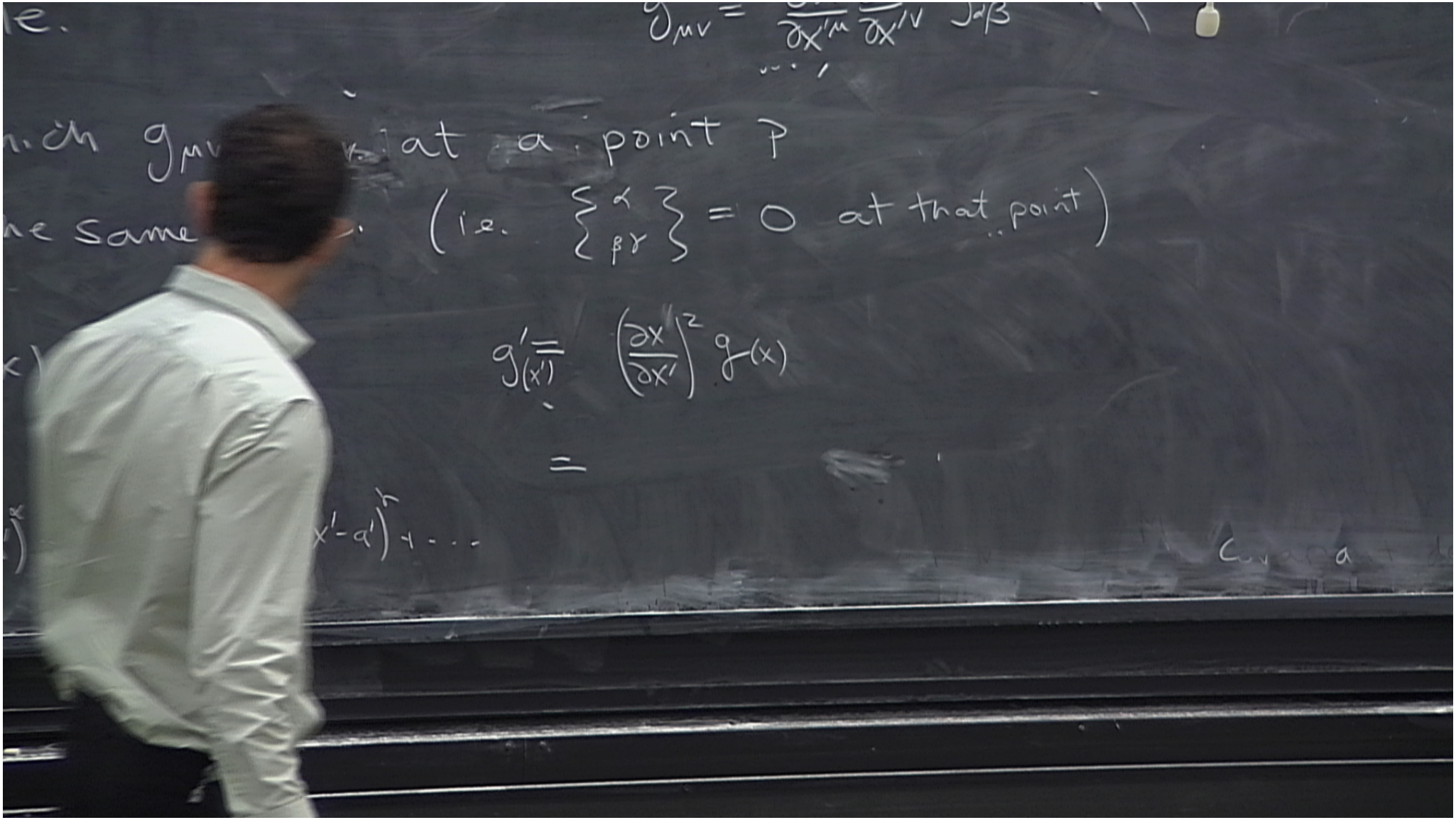

Recall, equivalence principle.
can choose coords in which $g_{\mu\nu} = \eta_{\mu\nu}$ at
and $g_{\mu\nu,\alpha} = 0$ at the same point.
P
x=a
x'=a'
Proof:
$x'(x)$
$x(x')$
$g'(x') = \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} g(x)$
$x = x'(x')$

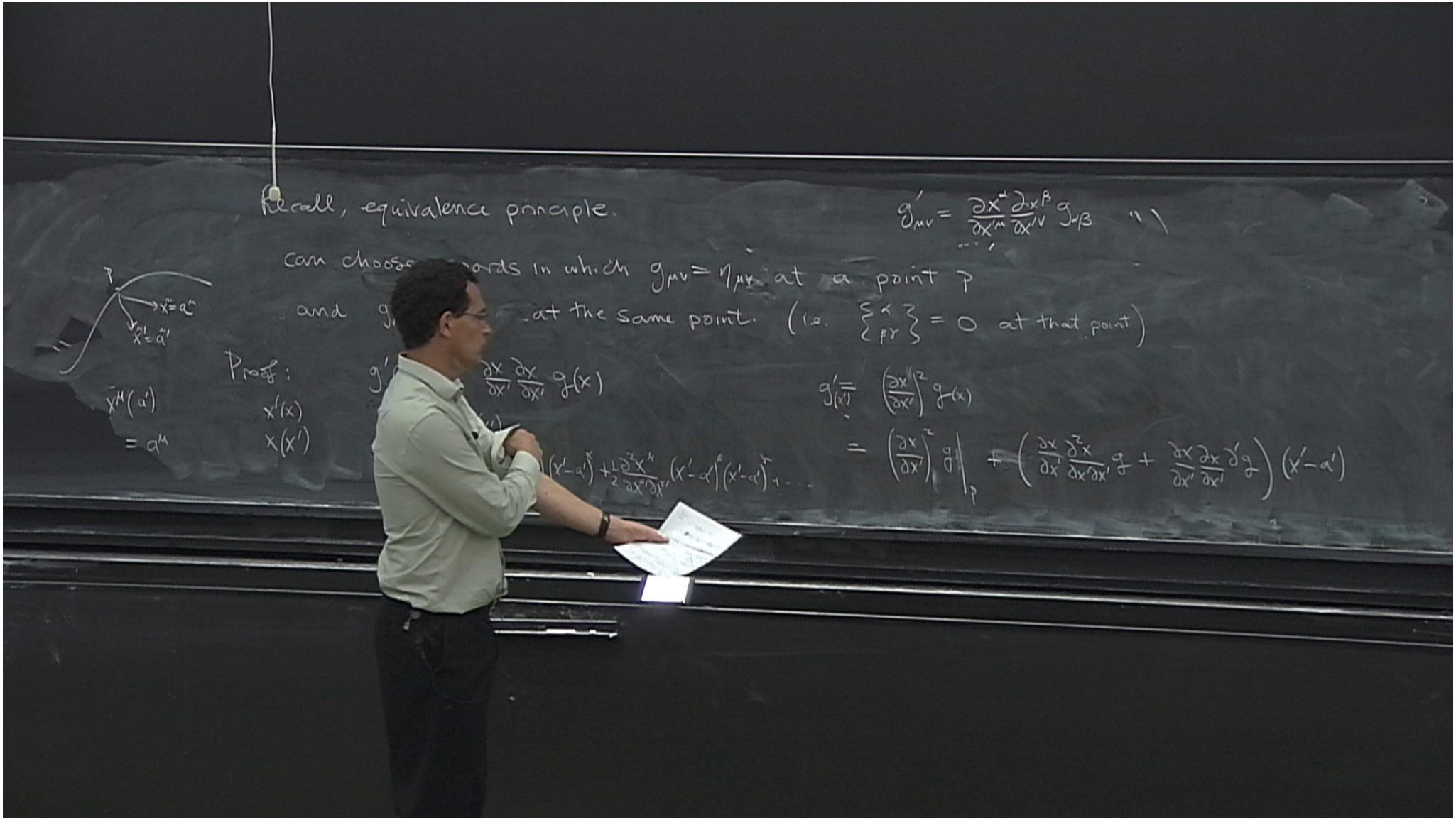

can choose coords in which $g_{\mu\nu} =$
and $g_{\mu\nu,\alpha} = 0$ at the same pa
$x = a$
$x' = a'$
Proof:
$x'(x)$
$x(x')$
$g'(x') = \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} g(x)$
$x = x(x')$
$= a + \frac{\partial x}{\partial x'}\Big|_{a'} (x' - a') + \frac{\partial^2 x}{\partial x'}$

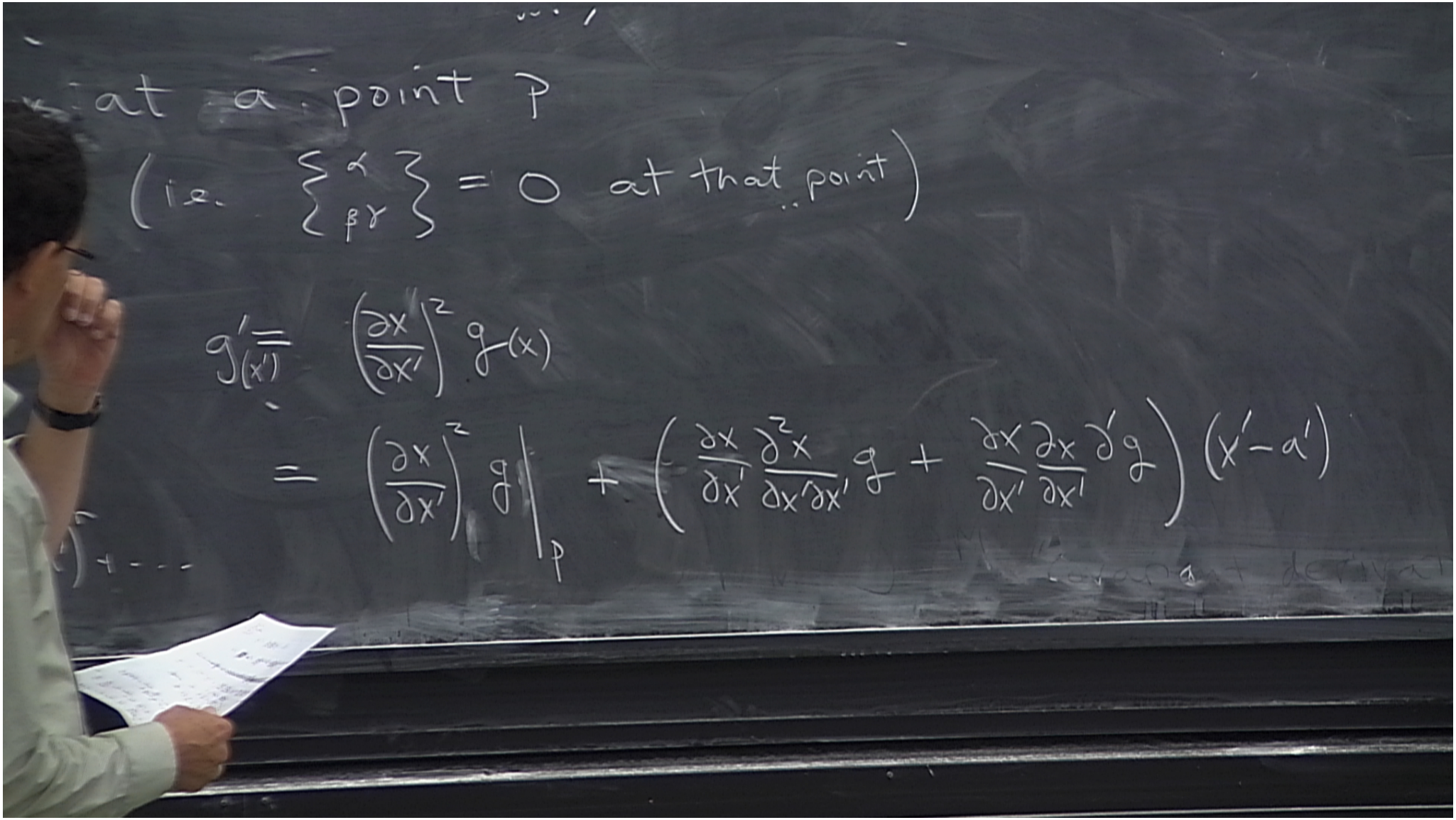

$g'(x') = \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} g(x)$
$x'^\mu = x'^\mu(x')$
$= a'^\mu + \frac{\partial x'^\mu}{\partial x^\rho}\Big|_p (x'-a')^\rho + \frac{1}{2}\frac{\partial^2 x'^\mu}{\partial x^\alpha \partial x^\beta}\Big|_p (x'-a')^\alpha (x'-a')^\beta +$

$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$$

point P


at a point P
(i.e. $\Gamma^{\alpha}_{\beta\gamma} = 0$ at that point)
$g'_{(x')} = \left(\frac{\partial x}{\partial x'}\right)^2 g_{(x)}$
=


Recall, equivalence principle.
$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$ "!"
can choose coords in which $g_{\mu\nu} = \eta_{\mu\nu}$ at a point P
and g ... at the same point. (i.e. $\left\{ {\kappa \atop \mu\nu} \right\} = 0$ at that point)
P
$x^\mu = a^\mu$
$x'^\mu = a'^\mu$
Proof:
$x'(x)$
$x(x')$
$x^\mu(a') = a^\mu$
$g'_{(x')} = \left(\frac{\partial x}{\partial x'}\right)^2 g(x)$
$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_P + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x' \partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'} \partial g\right)(x'-a')$


at a point P
(i.e. $\left\{ {\alpha \atop \beta\gamma} \right\} = 0$ at that point)
$g'_{(x')} = \left(\frac{\partial x}{\partial x'}\right)^2 g_{(x)}$
$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_p + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x' \partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial' g\right)(x'-a')$

at a point P

(i.e. $\left\{ {\alpha \atop \beta\gamma} \right\} = 0$ at that point)

$$g'_{(x')} = \left(\frac{\partial x}{\partial x'}\right)^2 \frac{\partial x}{\partial x'} \, g$$

$$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_p + \left(\frac{\partial x}{\partial x'} \frac{\partial^2 x}{\partial x \partial x'} g + \frac{\partial x}{\partial x'} \frac{\partial x}{\partial x'} \partial g \right) (x' - a')$$



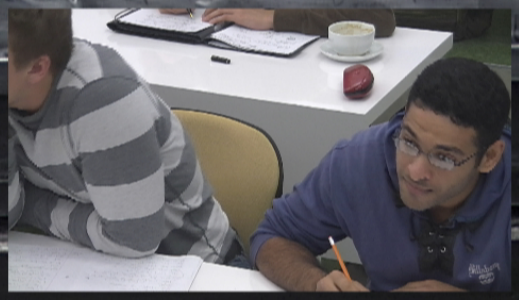
at a point P

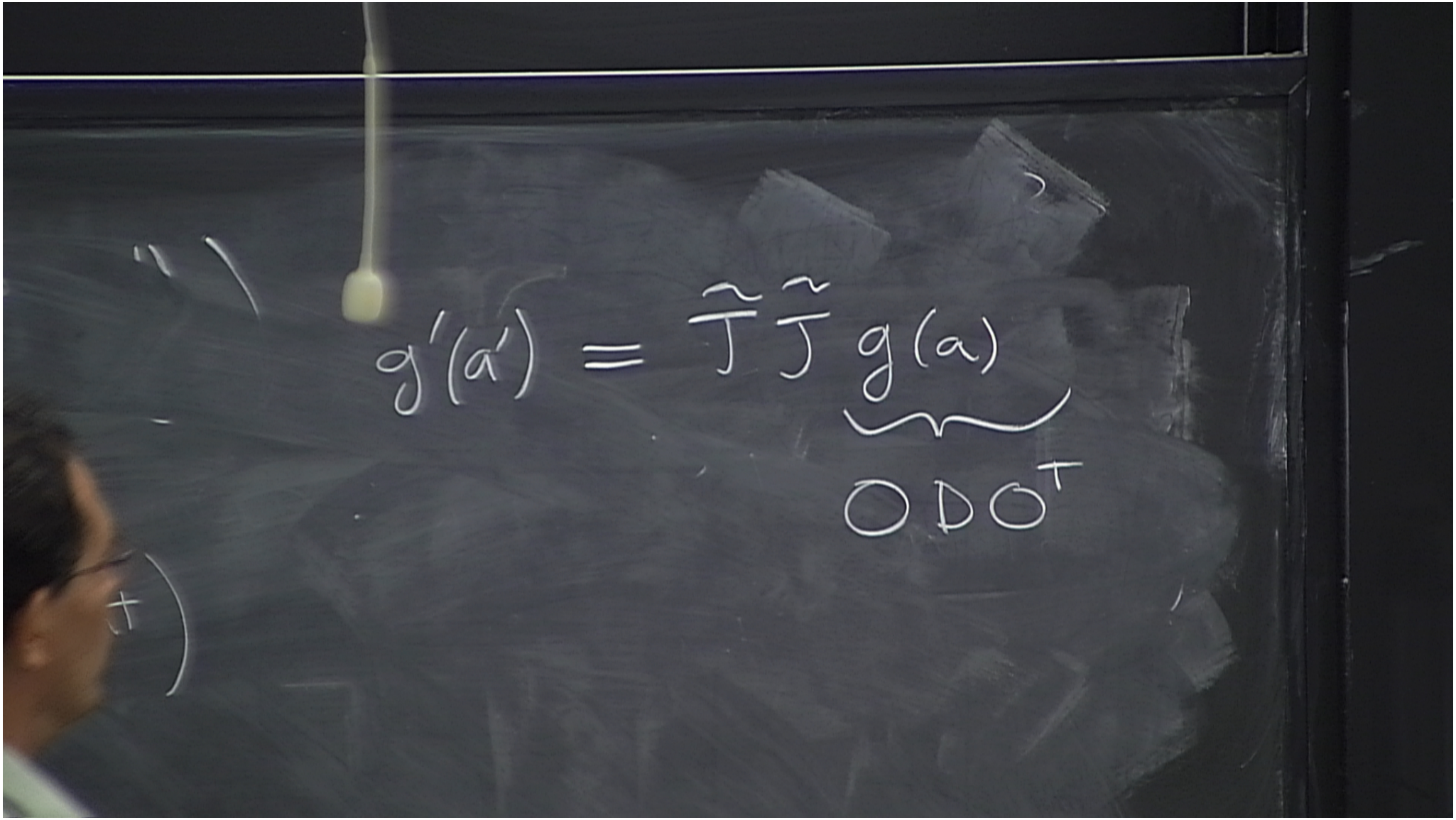
(i.e. $\left\{ {\alpha \atop \beta\gamma} \right\} = 0$ at that point)

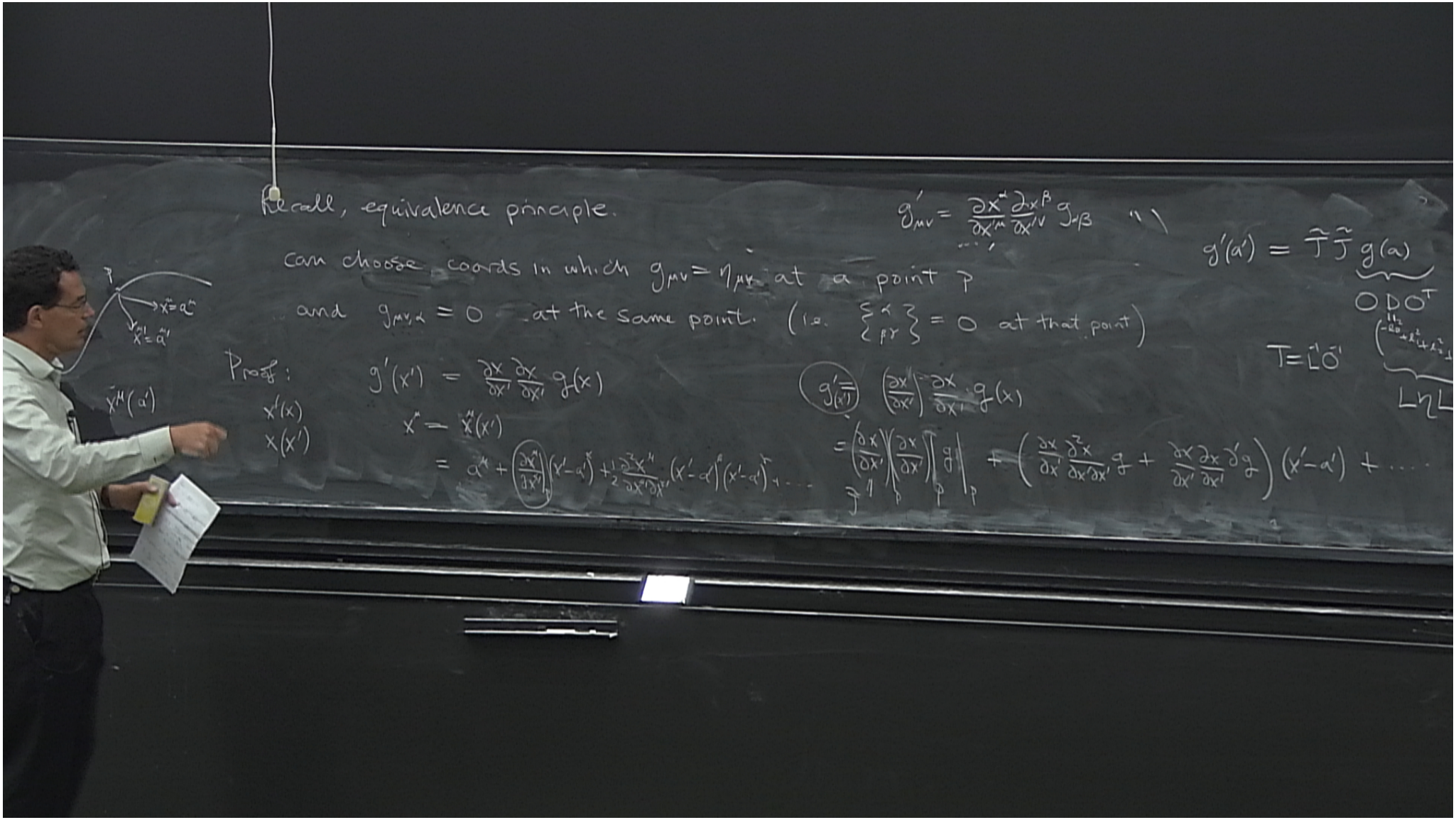
$$g'_{(x')} = \left(\frac{\partial x}{\partial x'}\right)^2 \frac{\partial x}{\partial x'} \cdot g(x)$$

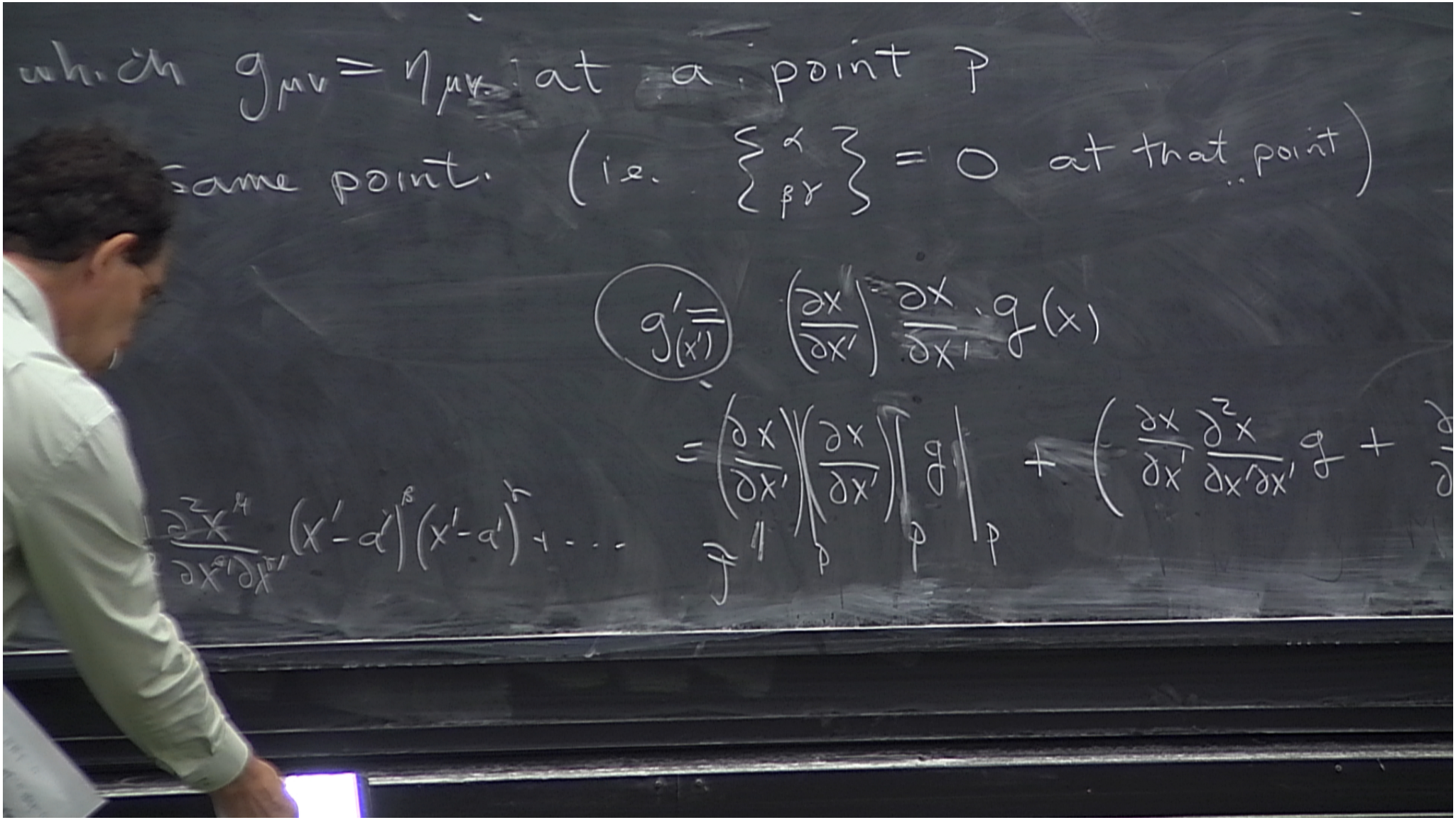
$$= \left(\frac{\partial x}{\partial x'}\right)^2 g\Big|_P + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x' \partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial' g\right)(x'-a')$$

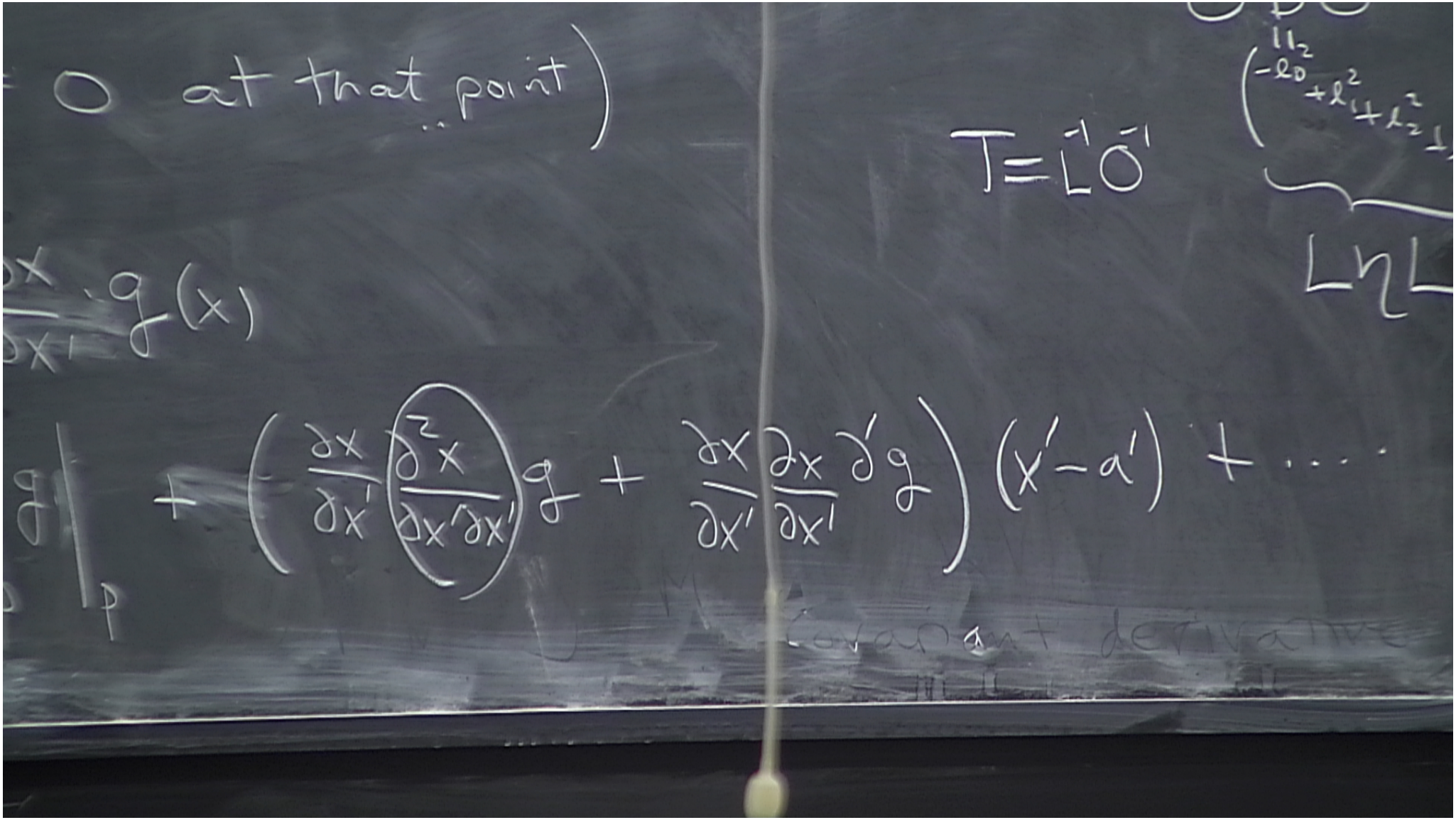
$(x'-a')^2 + \cdots$

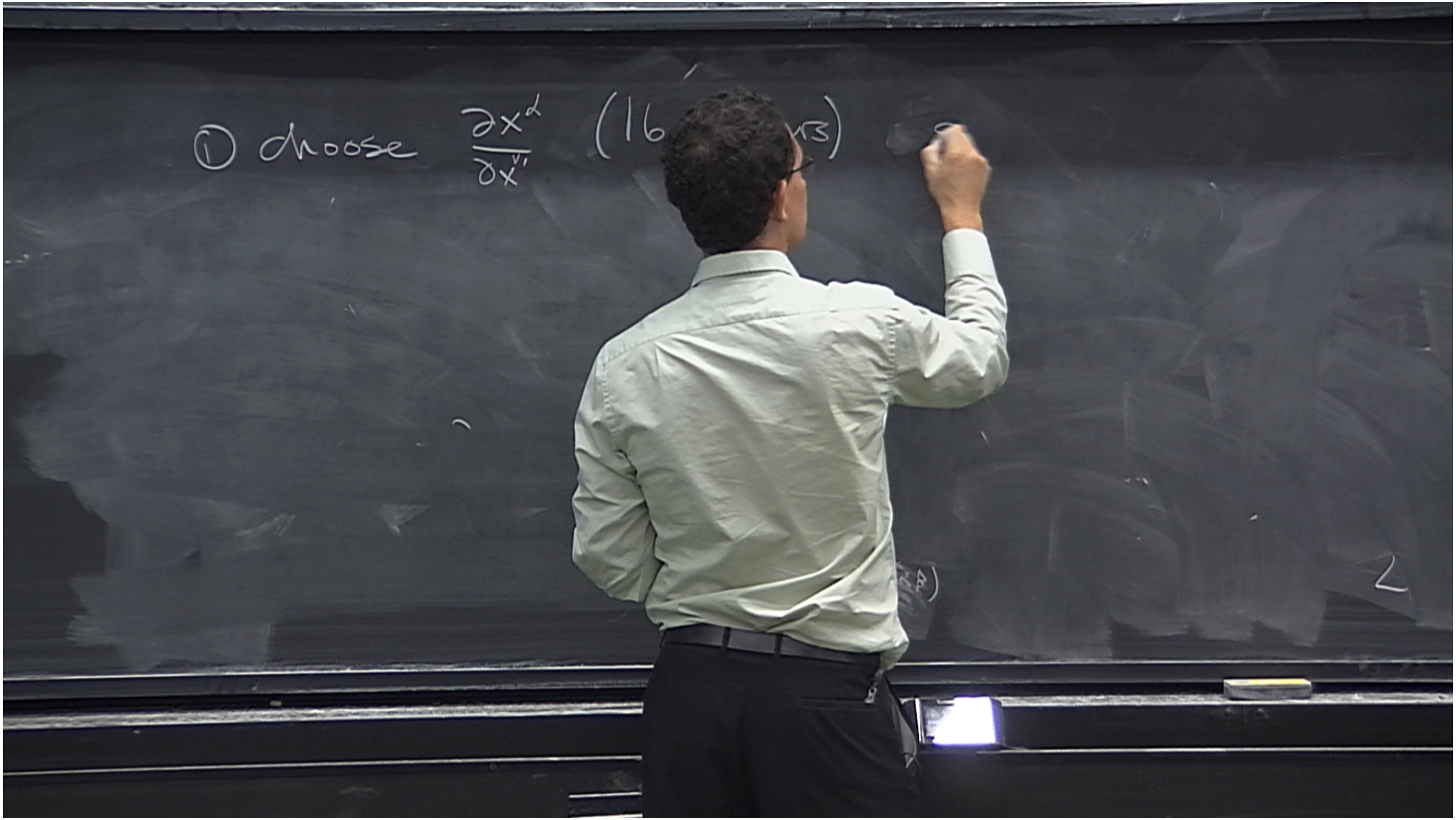


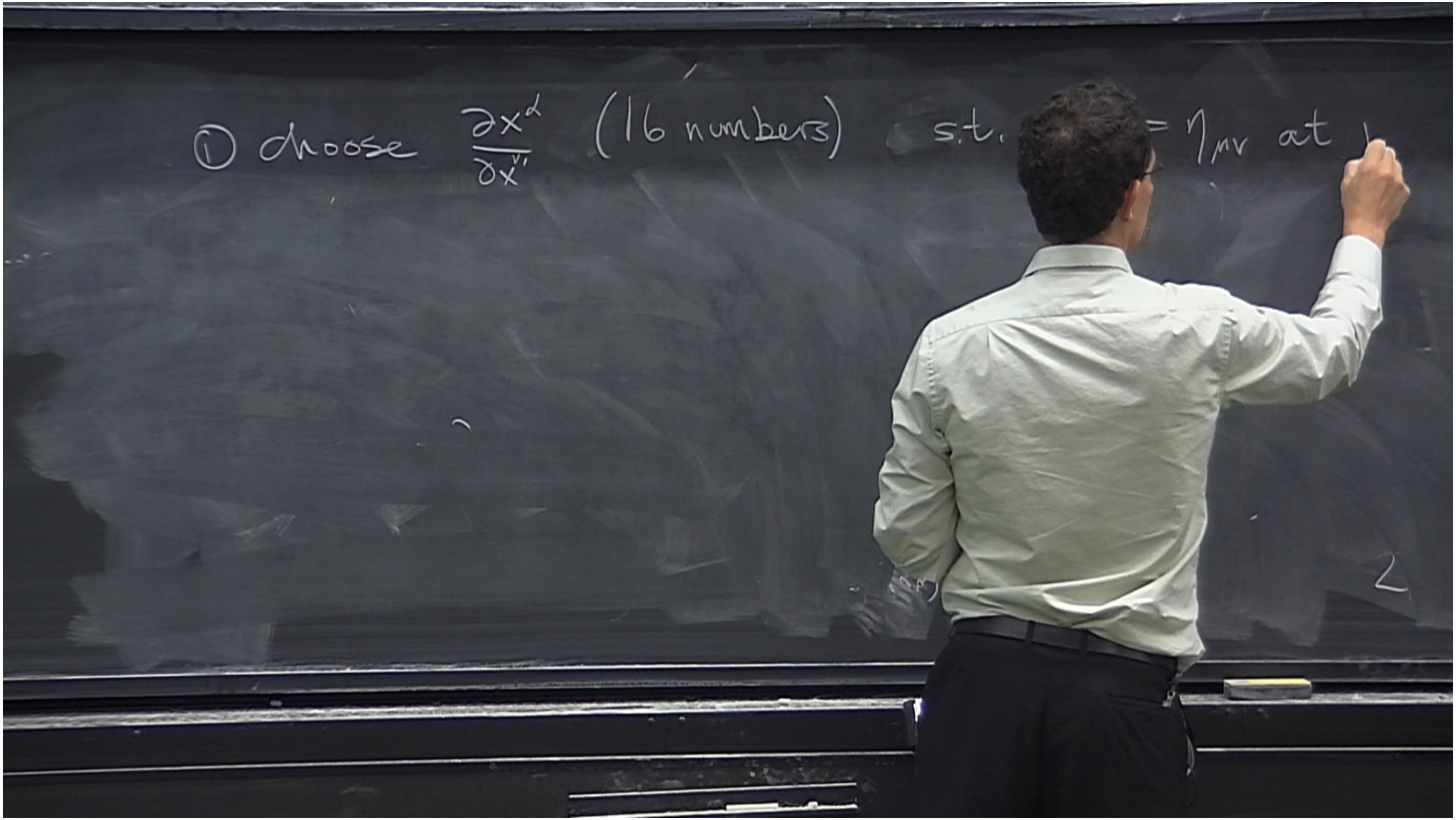

g'(a') = T̃ T̃ g(a)
O D O^T

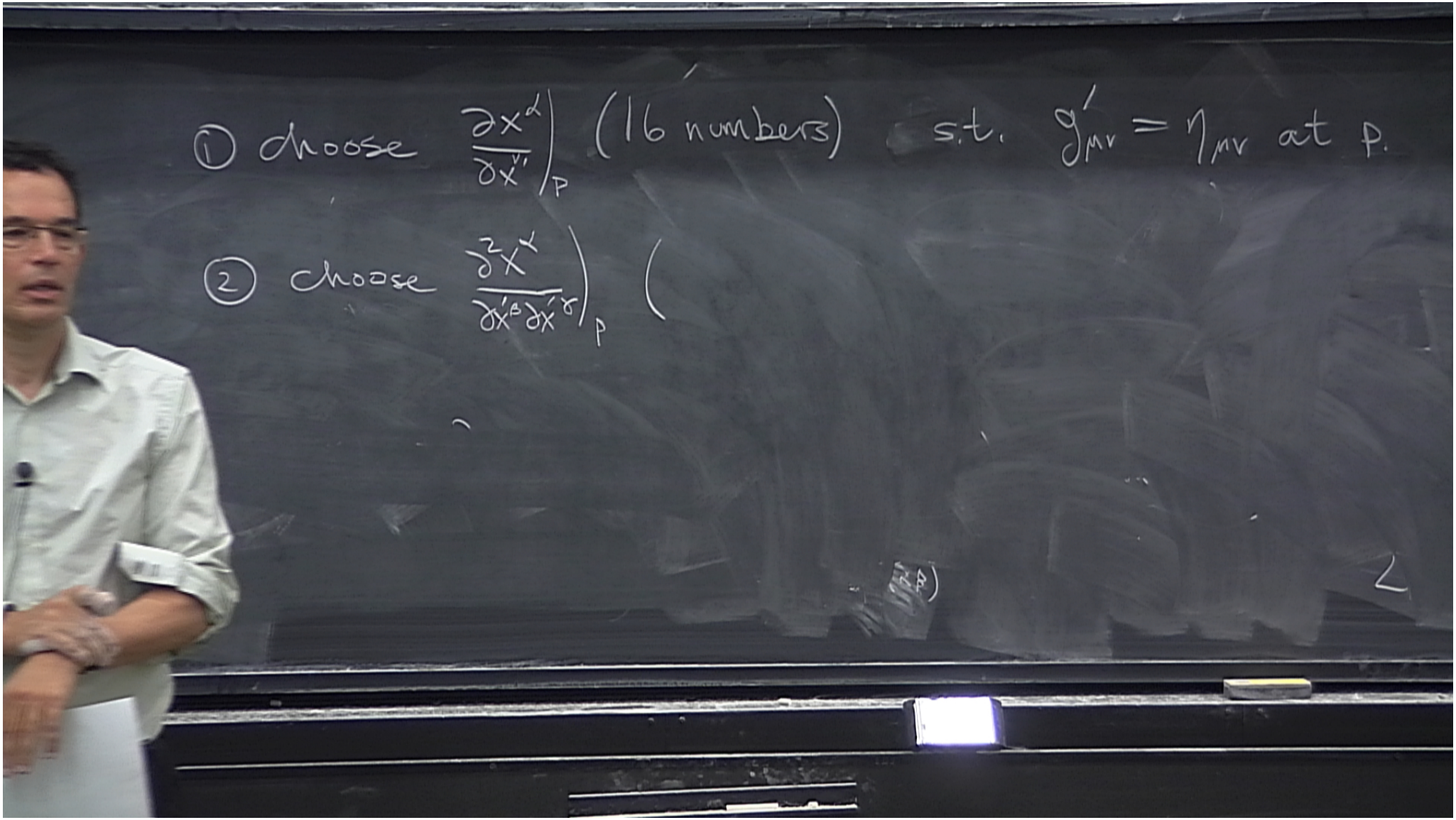

Recall, equivalence principle.
Can choose coords in which $g_{\mu\nu} = \eta_{\mu\nu}$ at a point P
and $g_{\mu\nu,\alpha} = 0$ at the same point. (i.e. $\Gamma^{\alpha}_{\beta\gamma} = 0$ at that point)
$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$ (1)
$g'(a') = \tilde{T}\tilde{T}\, g(a)$
$O D O^T$
$T = L\tilde{O}^{-1}$
$L\eta L$
P
$x = a$
$x' = a'$
$x^\mu(a')$
Proof:
$x'(x)$
$x(x')$
$g'(x') = \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'} g(x)$
$x^\mu = x^\mu(x')$
$= a^\mu + \left(\frac{\partial x^\mu}{\partial x'^\nu}\right)(x'-a')^\nu + \frac{1}{2}\frac{\partial^2 x^\mu}{\partial x'^\alpha \partial x'^\nu}(x'-a')^\alpha (x'-a')^\nu + \ldots$
$g'(x') = \left(\frac{\partial x}{\partial x'}\right)\left(\frac{\partial x}{\partial x'}\right) g(x)$
$= \left(\frac{\partial x}{\partial x'}\right)\left(\frac{\partial x}{\partial x'}\right) g \Big|_P + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x' \partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'} g'\right)(x'-a') + \ldots$
$\to 1$

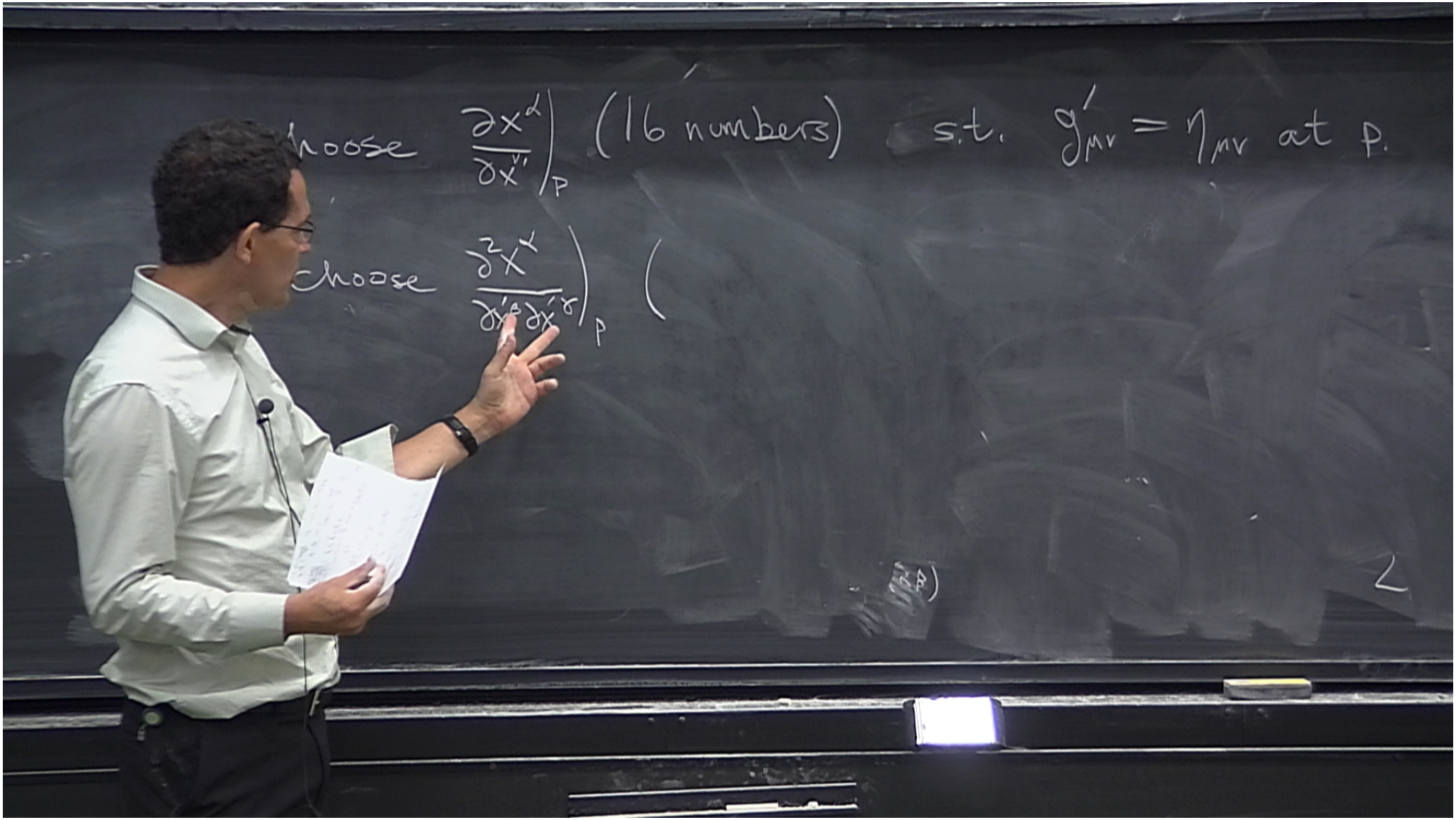

which $g_{\mu\nu} = \eta_{\mu\nu}$ at a point P
same point. (i.e. $\left\{ {\alpha \atop \beta\gamma} \right\} = 0$ at that point)
$g'(x') = \left(\frac{\partial x}{\partial x'}\right)\left(\frac{\partial x}{\partial x'}\right) g(x)$
$= \left(\frac{\partial x}{\partial x'}\right)\left(\frac{\partial x}{\partial x'}\right)\Big|_P g\Big|_P + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x' \partial x'} g + \right.$
$\frac{\partial^2 x^\mu}{\partial x'^\alpha \partial x'^\beta}(x'-a')^\beta (x'-a')^\gamma + \cdots$
$= 1$

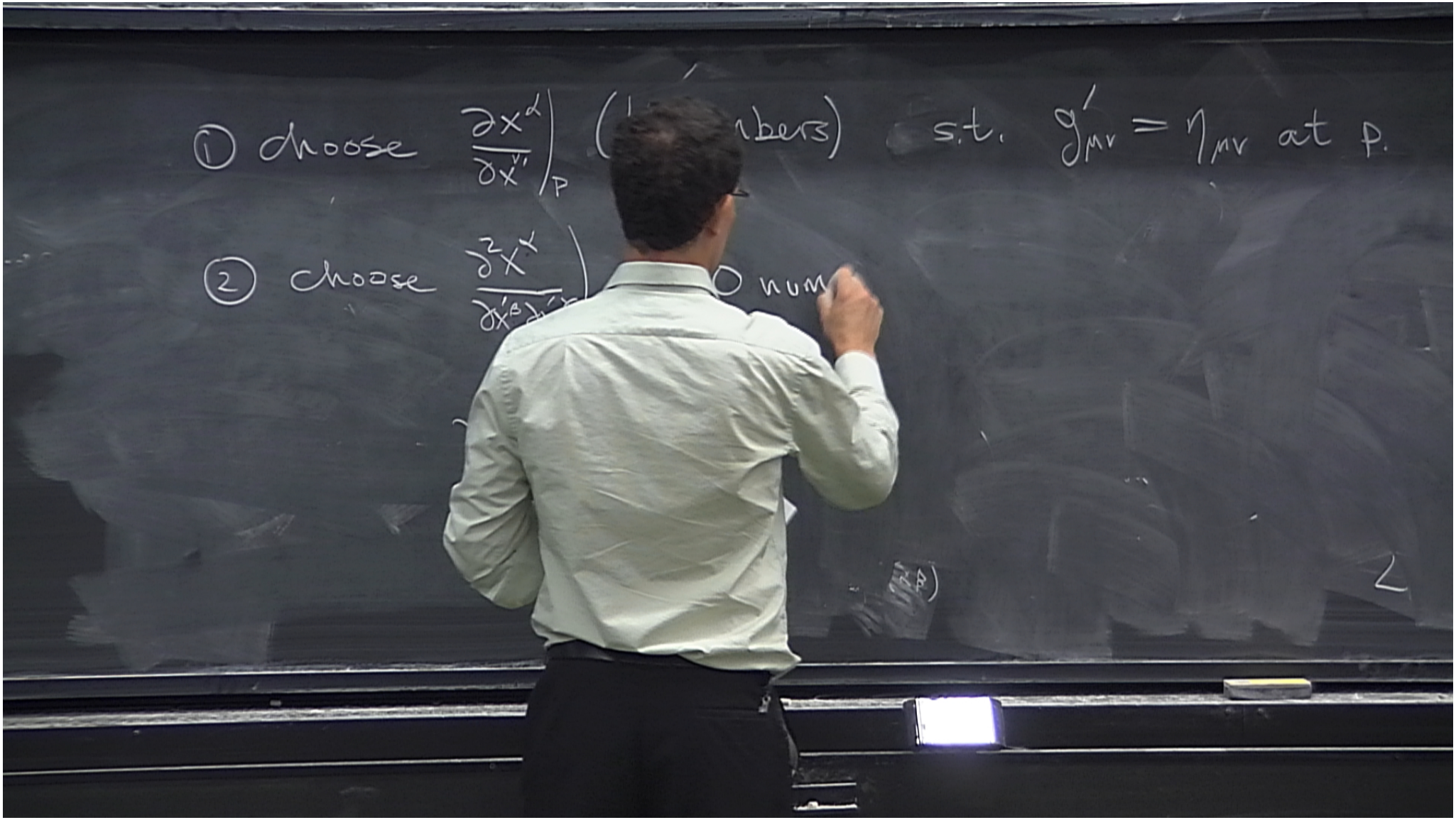

= 0 at that point)
T = L⁻¹O⁻¹
LηL
$\frac{\partial x}{\partial x'} \cdot g(x)$
$g\Big|_p + \left(\frac{\partial x}{\partial x'}\frac{\partial^2 x}{\partial x' \partial x'} g + \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'}\partial' g\right)(x'-a') + \ldots$

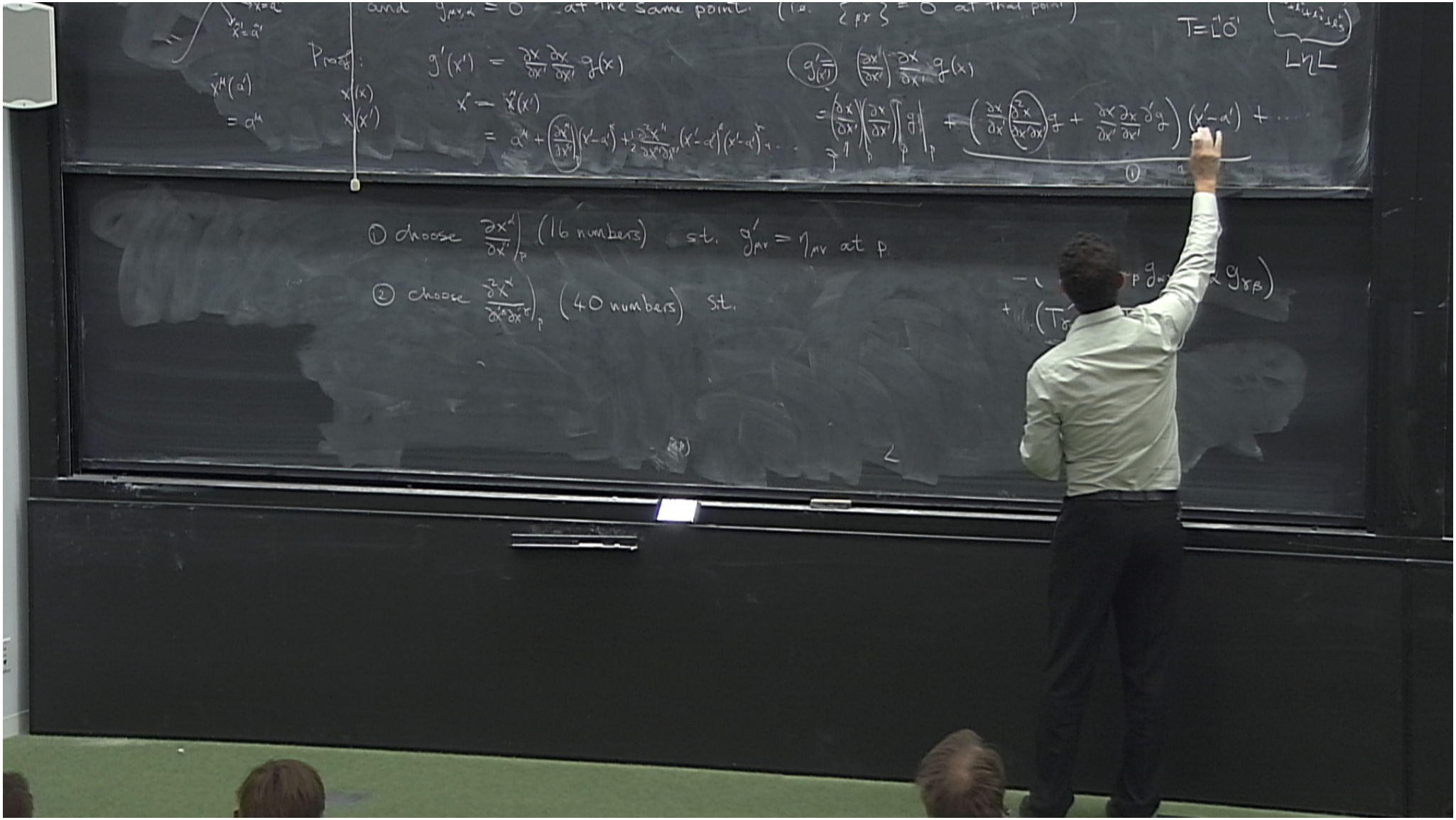

① choose

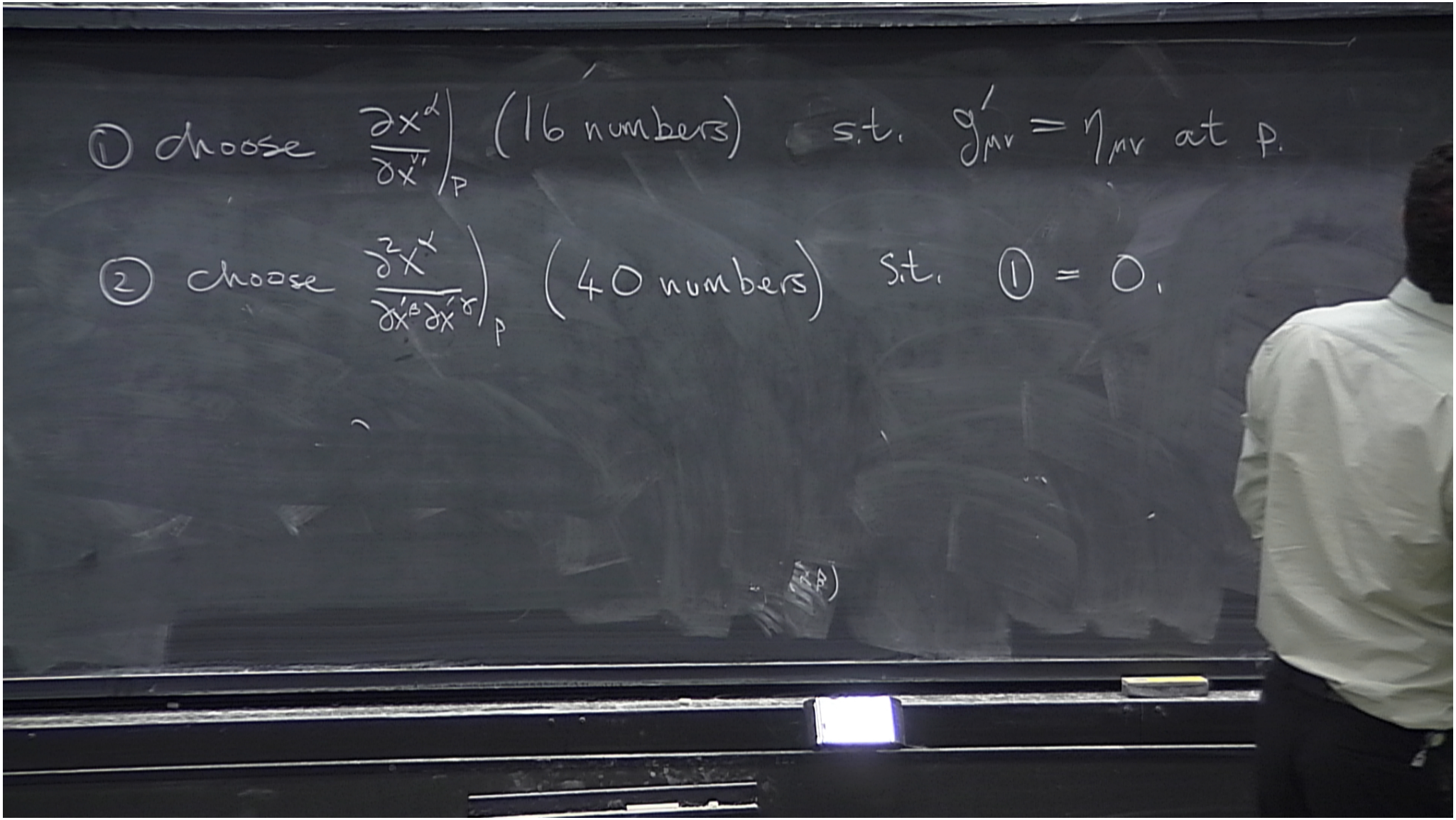

① choose $\frac{\partial x^{\alpha}}{\partial x'^{\mu}}$ (16 numbers) s.t. $= \eta_{\mu\nu}$ at

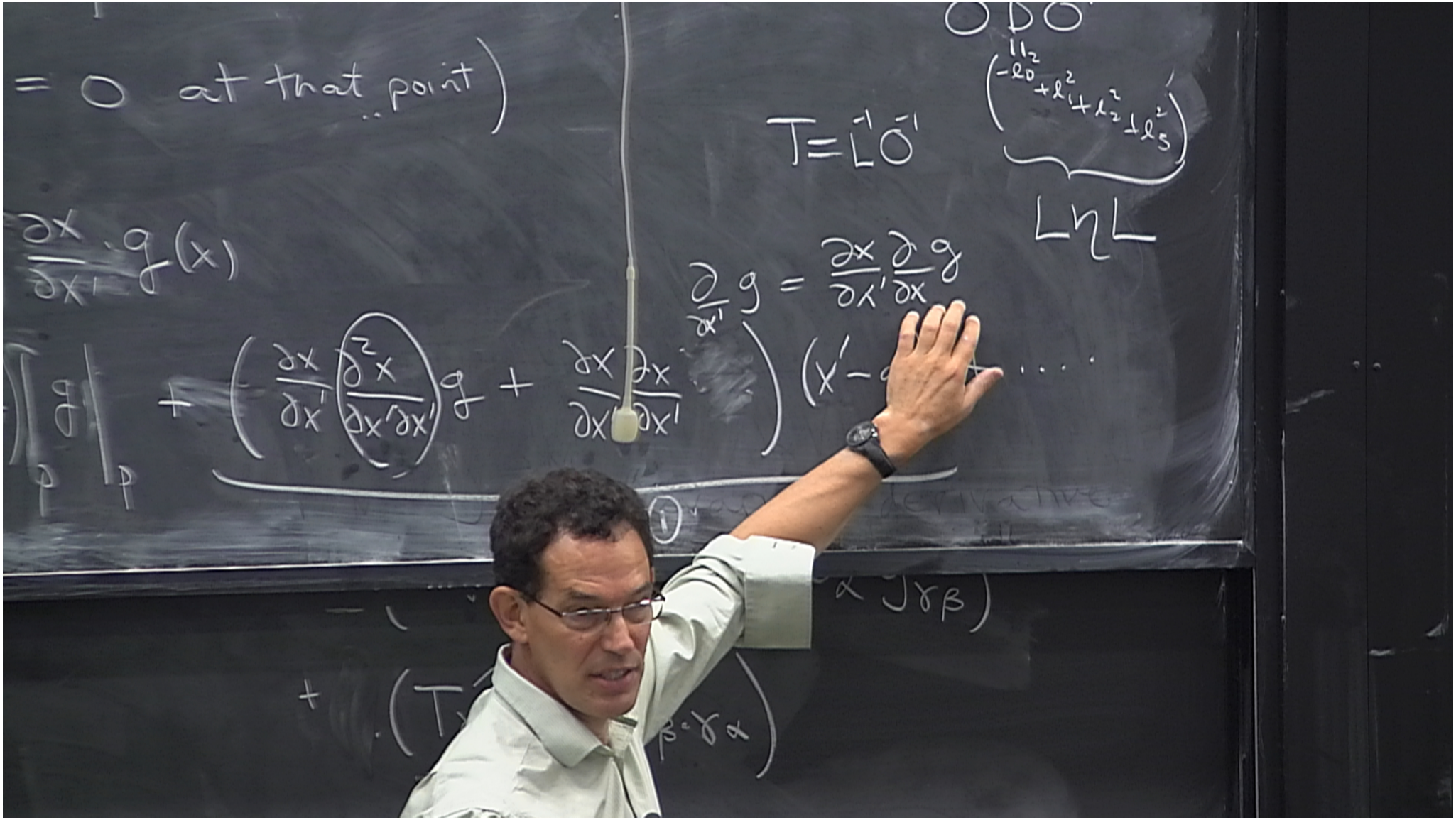

① choose $\left.\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\right|_P$ (16 numbers) s.t. $g'_{\mu\nu} = \eta_{\mu\nu}$ at P.
② choose $\left.\frac{\partial^2 x^{\gamma}}{\partial x'^{\beta} \partial x'^{\sigma}}\right)_P$ (

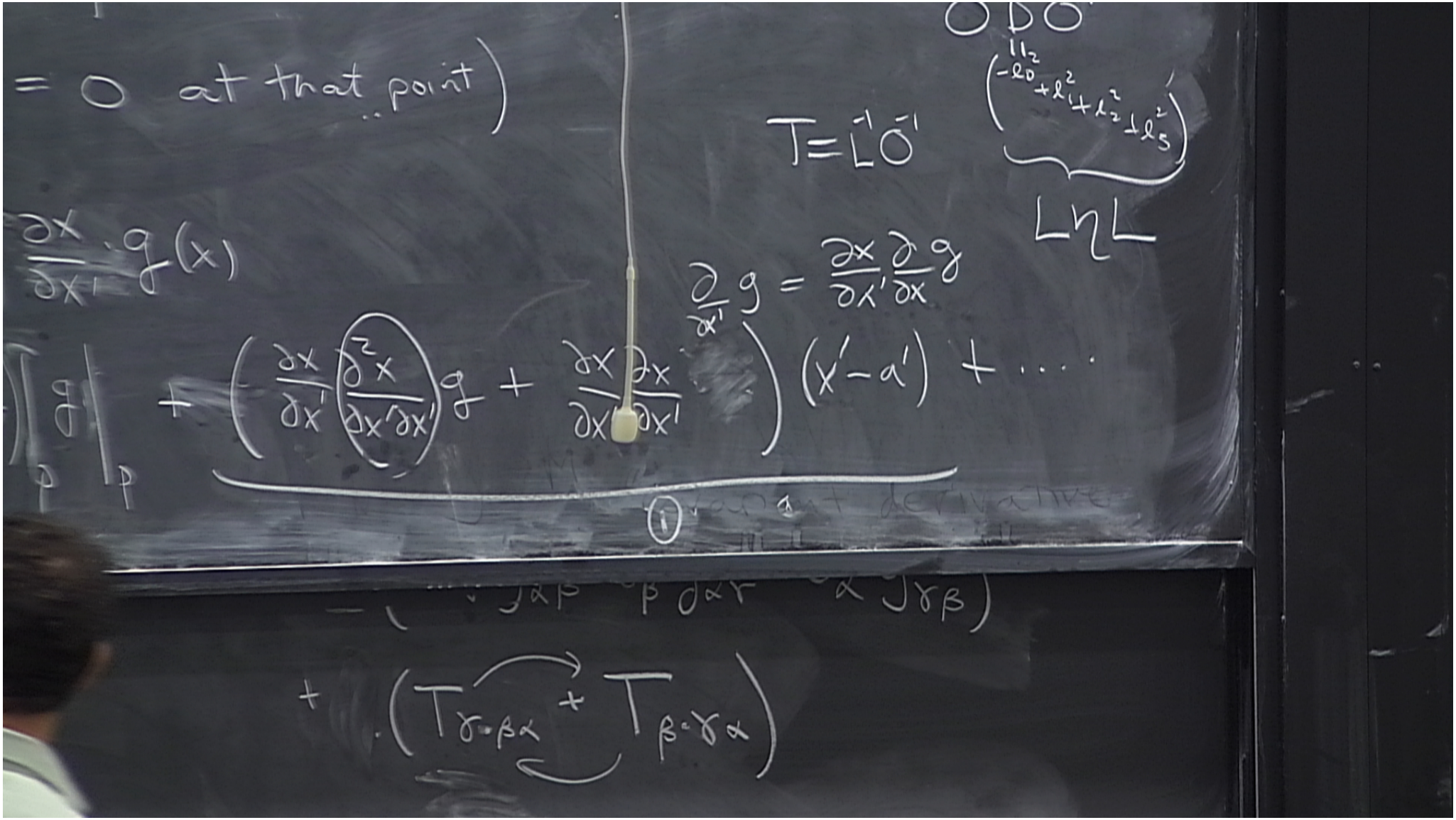

choose $\left.\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\right|_P$ (16 numbers) s.t. $g'_{\mu\nu} = \eta_{\mu\nu}$ at P.
choose $\left.\frac{\partial^2 x^{\gamma}}{\partial x'^{\beta} \partial x'^{\sigma}}\right)_P$ (

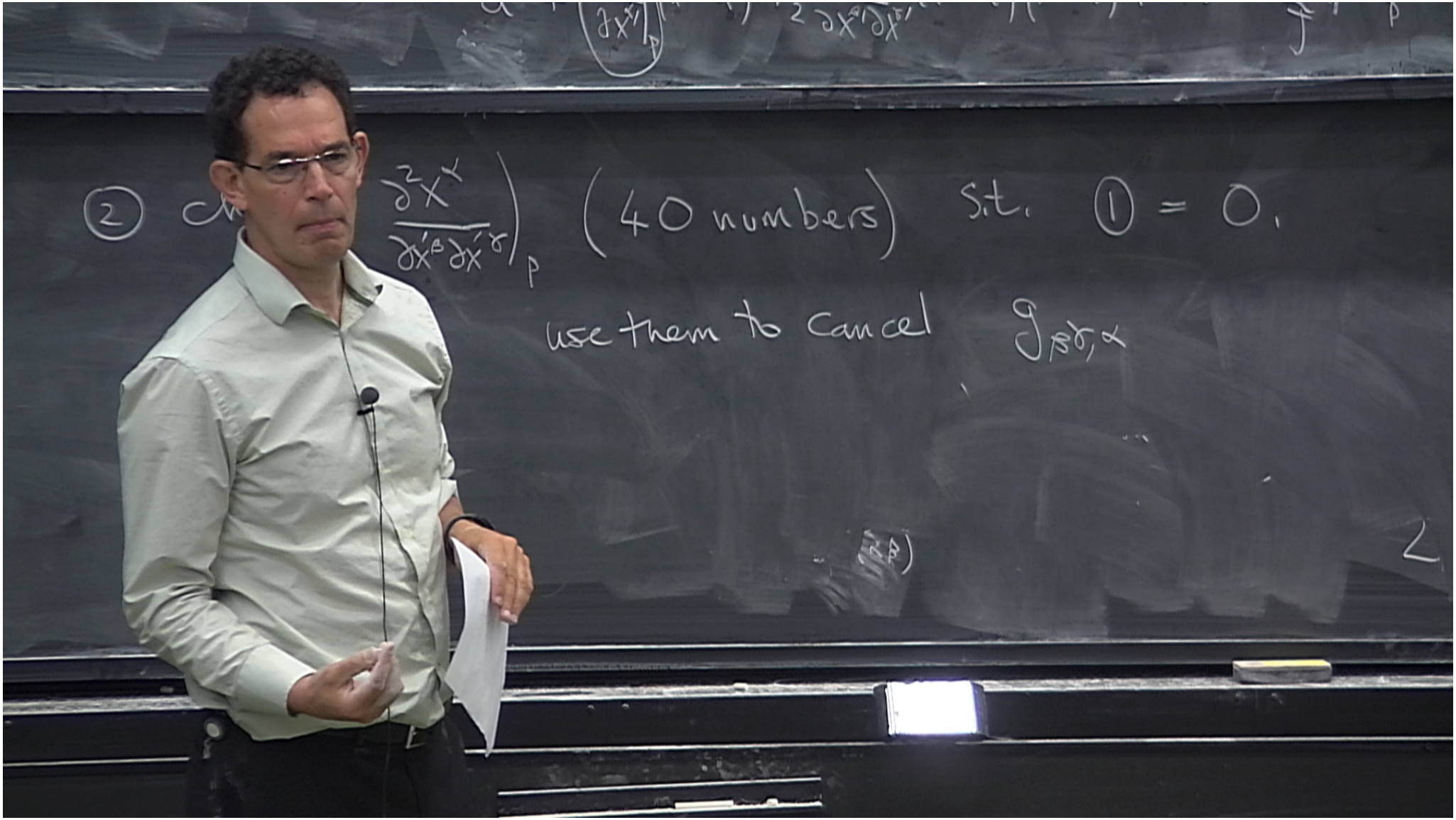

① choose
s.t.
② choose

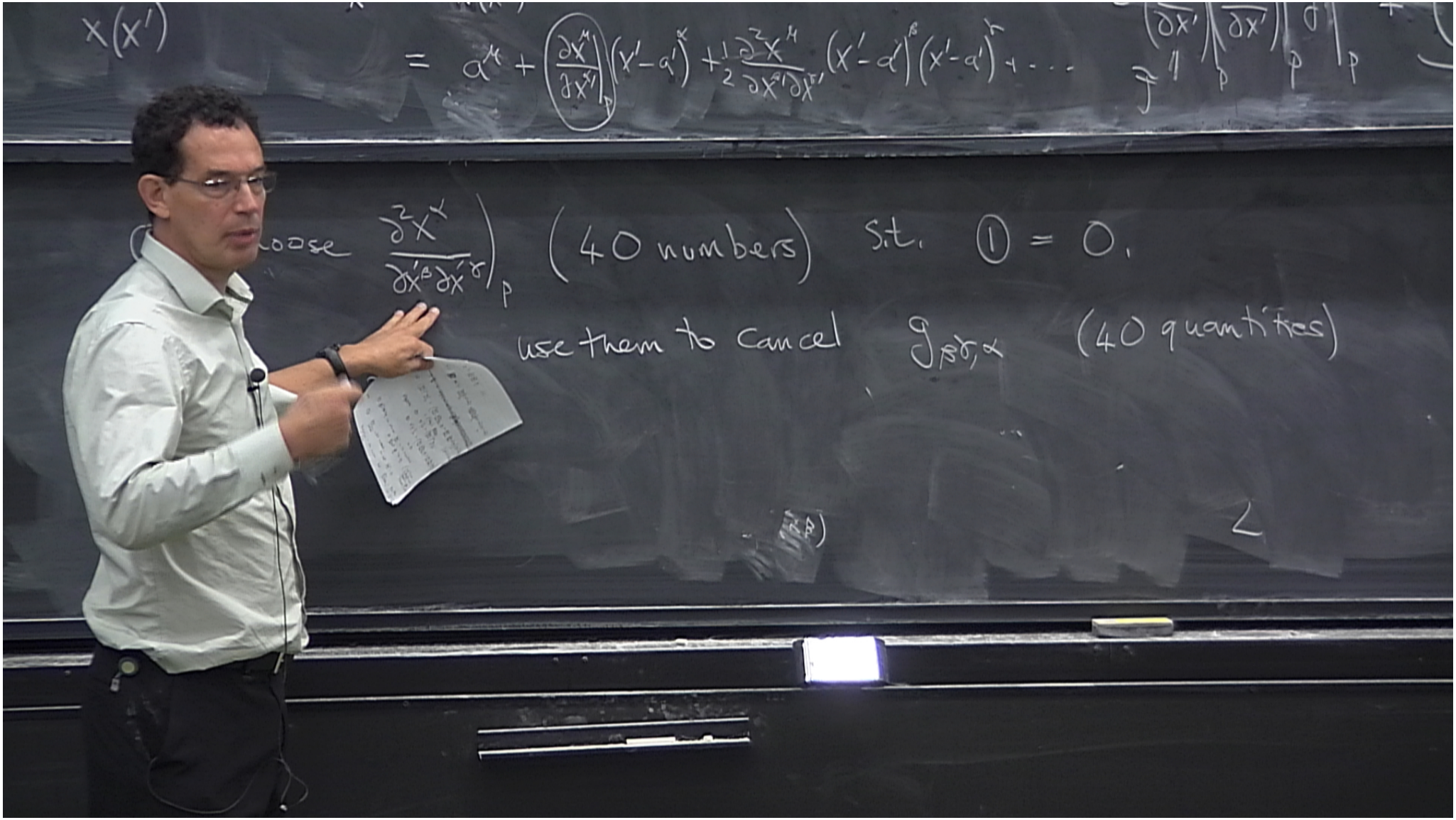

T=L0
LyL
① choose (16 numbers) s.t. at p
② choose (40 numbers) s.t.


① choose $\left.\frac{\partial x^{\alpha}}{\partial x'^{\mu'}}\right|_P$ (16 numbers) s.t. $g'_{\mu\nu} = \eta_{\mu\nu}$ at P.
② choose $\left.\frac{\partial^2 x^{\gamma}}{\partial x'^{\beta} \partial x'^{\sigma}}\right)_P$ (40 numbers) s.t. ① = 0.


= 0 at that point
T=L⁻¹O⁻¹
LηL


= 0 at that point)
T = L^{-1} O^{-1}
LηL
(1)


(2)
$\frac{\partial^2 x^\alpha}{\partial x'^\beta \partial x'^\gamma}\Big)_p$ (40 numbers) s.t. (1) = 0.
use them to cancel $g_{\beta\gamma,\alpha}$
2


x(x')
= a^μ + (∂x^μ/∂x^ν)_p (x'-a')^ν + ½ (∂²x^μ/∂x^ρ∂x^σ)_p (x'-a')^ρ (x'-a')^σ + ...
(40 numbers) s.t. ① = 0.
use them to cancel
(40 quantities)

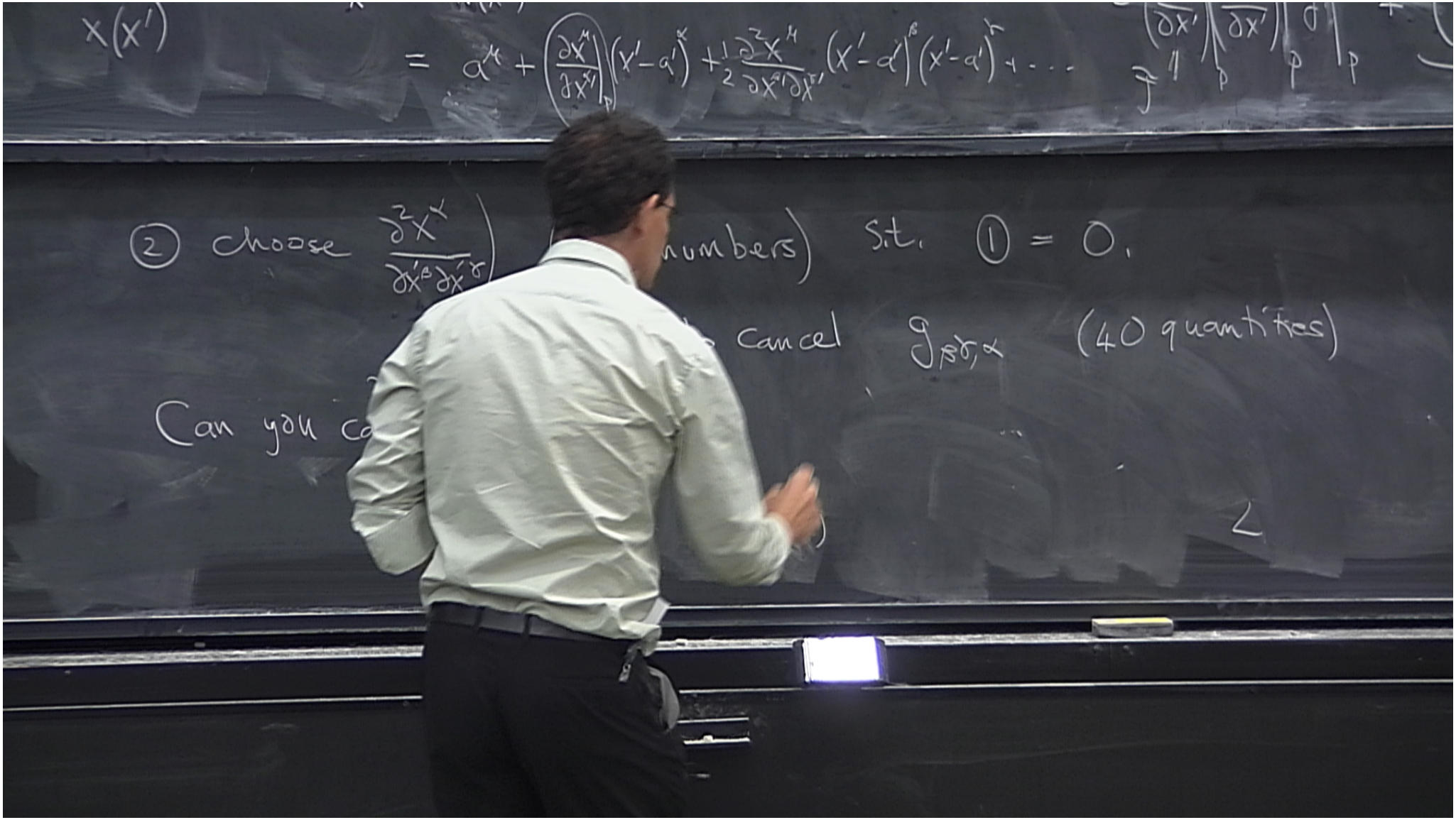
$x(x')$

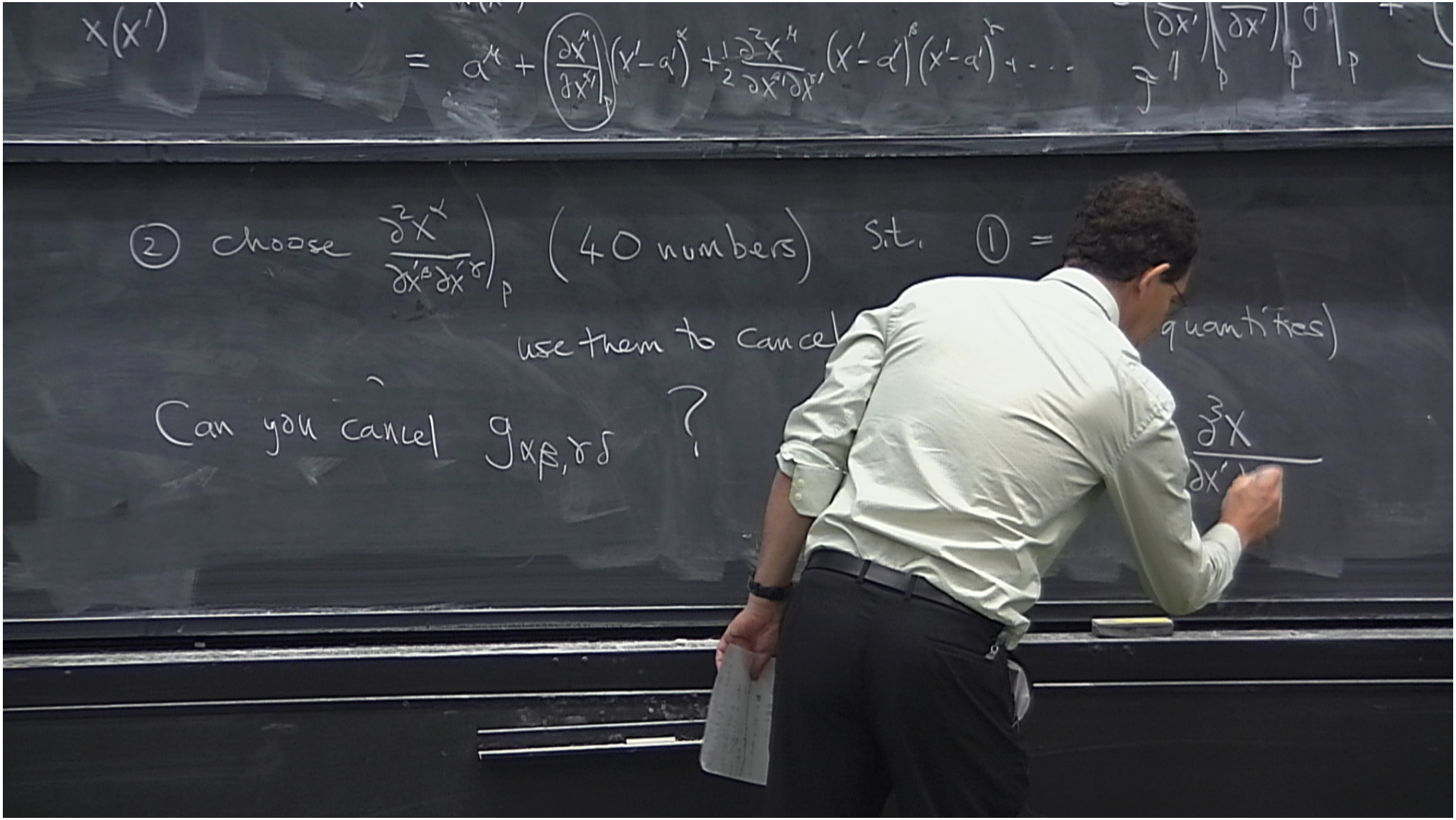
$= a^\mu + \left(\frac{\partial x^\mu}{\partial x'^\nu}\right)_p (x'-a')^\nu + \frac{1}{2}\frac{\partial^2 x^\mu}{\partial x'^\alpha \partial x'^\beta}(x'-a')^\alpha (x'-a')^\beta + \dots$

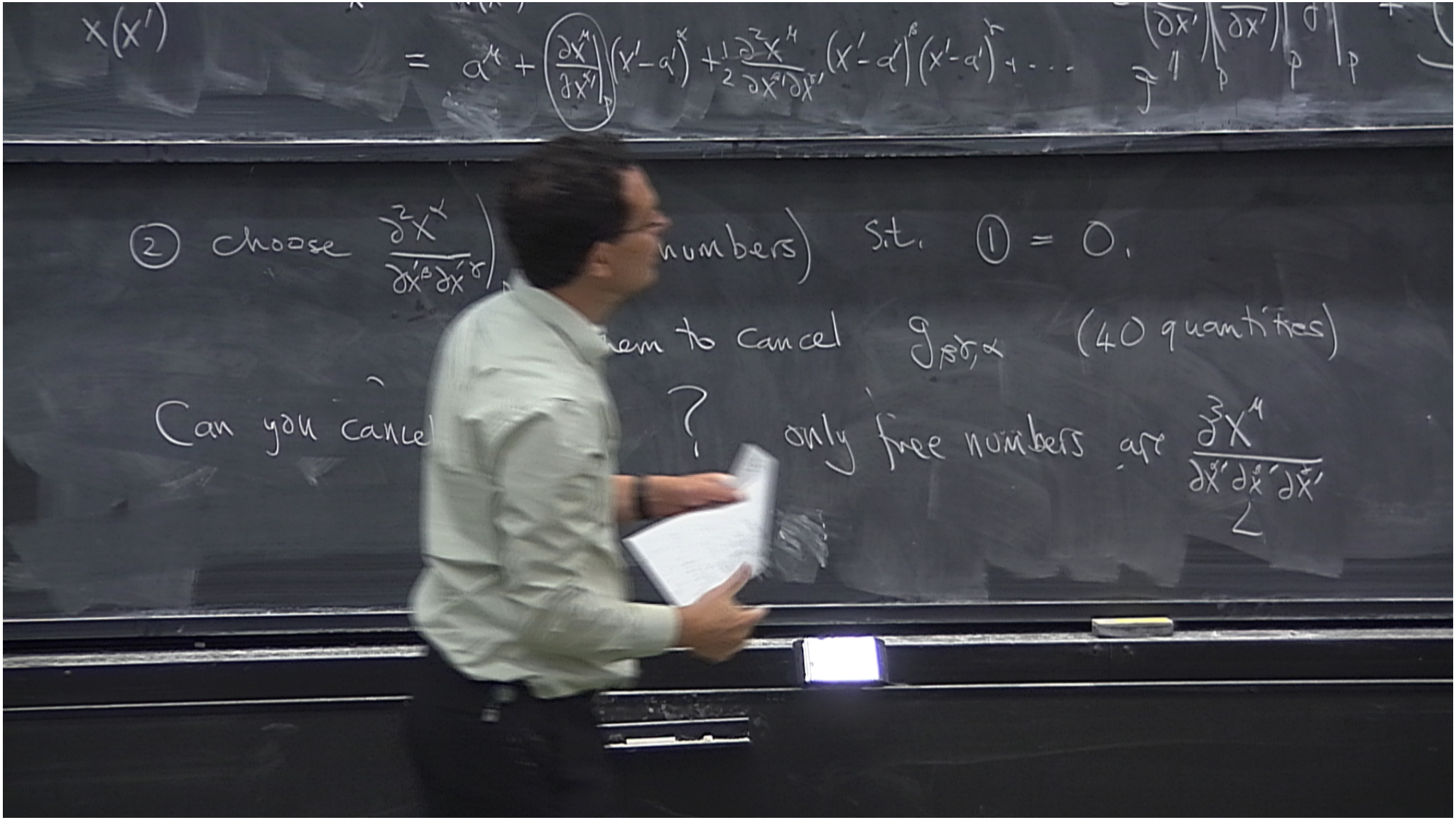
② choose $\left(\frac{\partial^2 x^\nu}{\partial x'^\beta \partial x'^\delta}\right)_p$ (40 numbers) s.t. ① = 0.

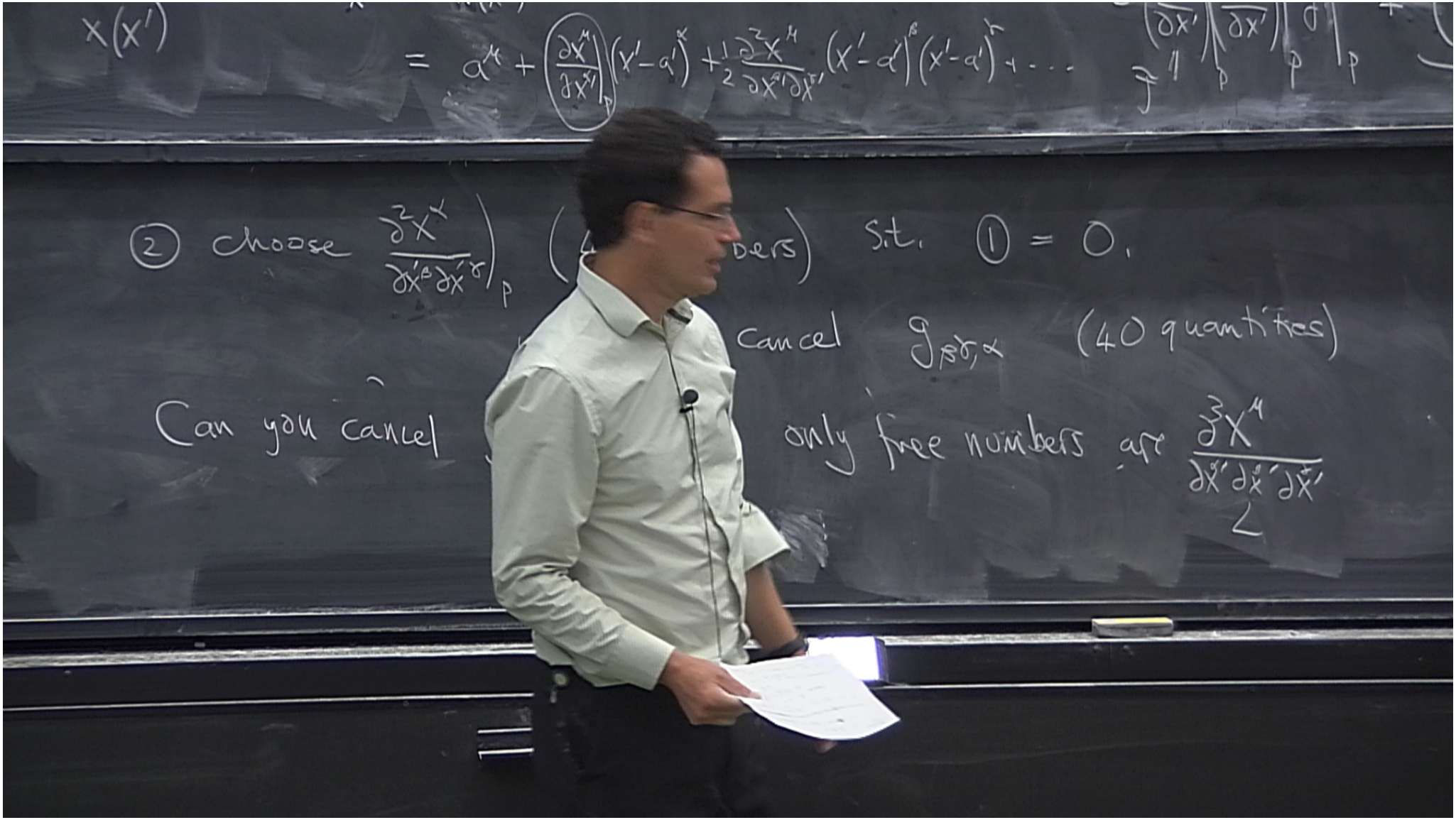
use them to cancel $g_{\beta\delta,\alpha}$ (40 quan

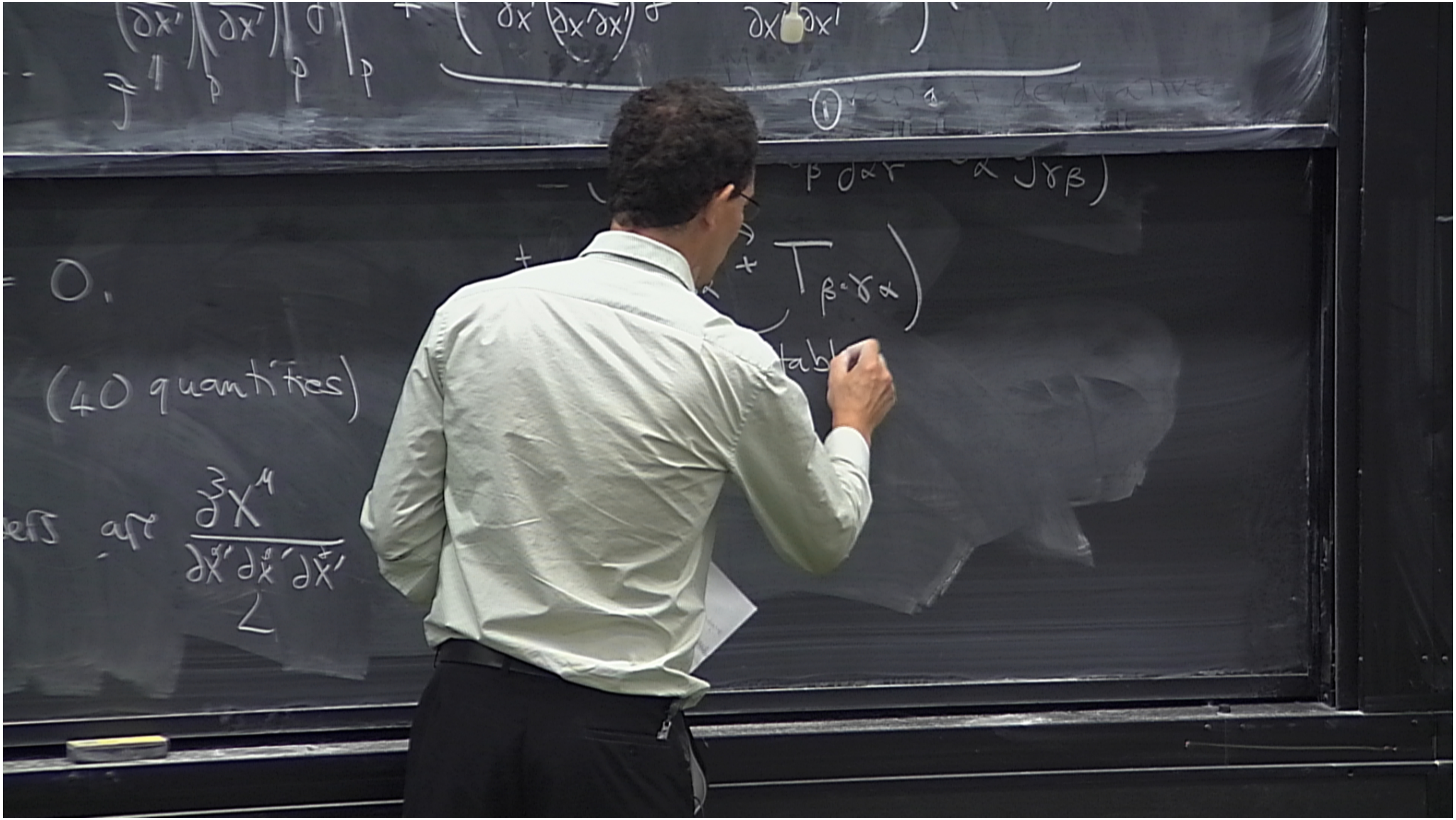
Can you cancel $g_{\alpha\beta,\gamma\delta}$

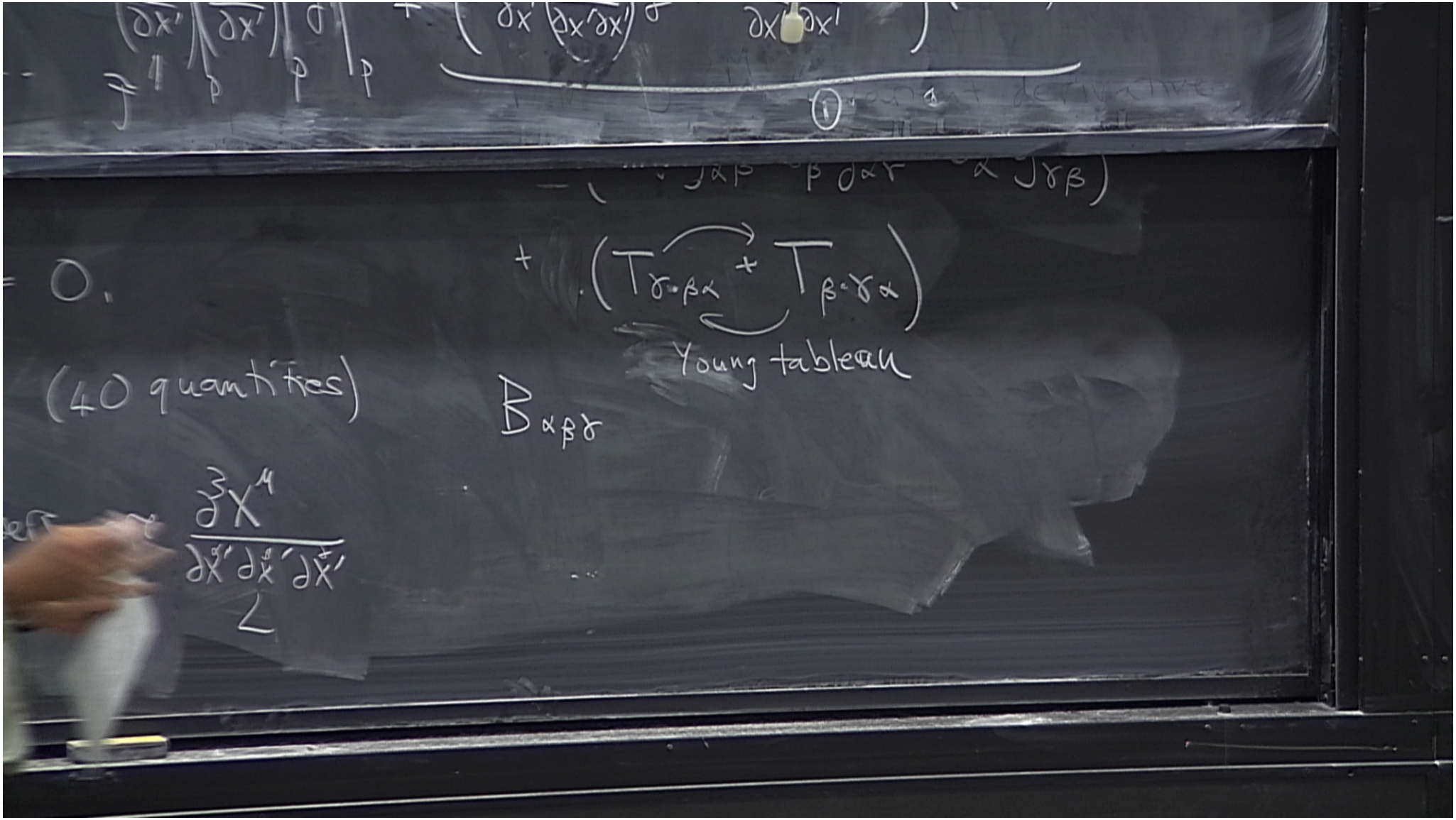

x(x')
= a^μ + (∂x^μ/∂x^ν)_p (x'-a')^ν + 1/2 (∂²x^μ/∂x^ρ∂x^σ)_p (x'-a')^ρ (x'-a')^σ + ...
② choose ∂²x^γ/∂x'^β∂x'^α
numbers) s.t. ① = 0.
cancel g_βδ,α (40 quantities)
Can you ca

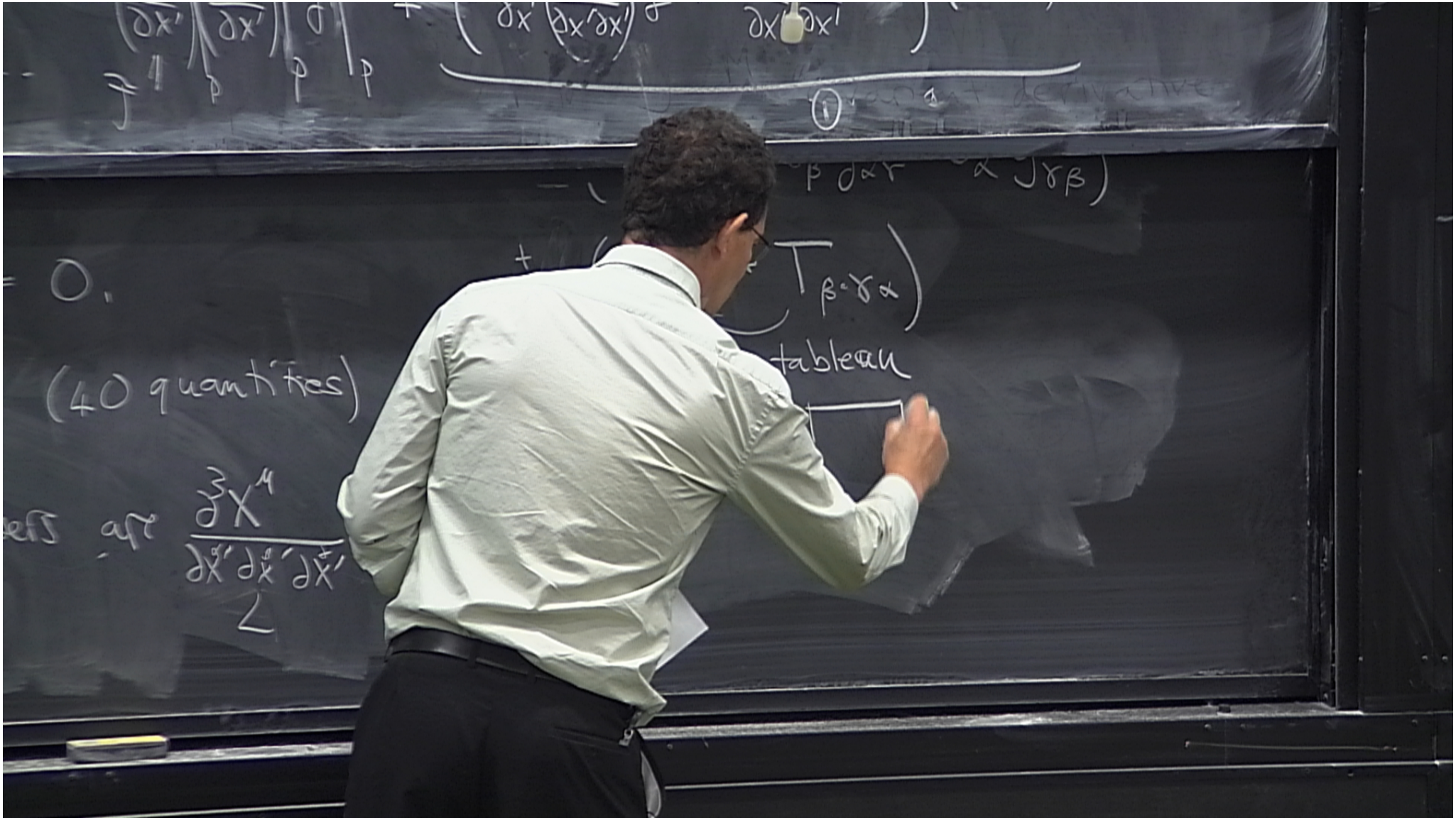

x(x')
= a^μ + (∂x^μ/∂x'^ν)_p (x'-a')^ν + ½ (∂²x^μ/∂x'^α∂x'^ν) (x'-a')^α (x'-a')^ν + ...
② choose (∂²x^γ/∂x'^β∂x'^δ)_p (40 numbers) s.t. ① =
use them to cancel
quantities)
Can you cancel g_αβ,γδ ?
∂³x

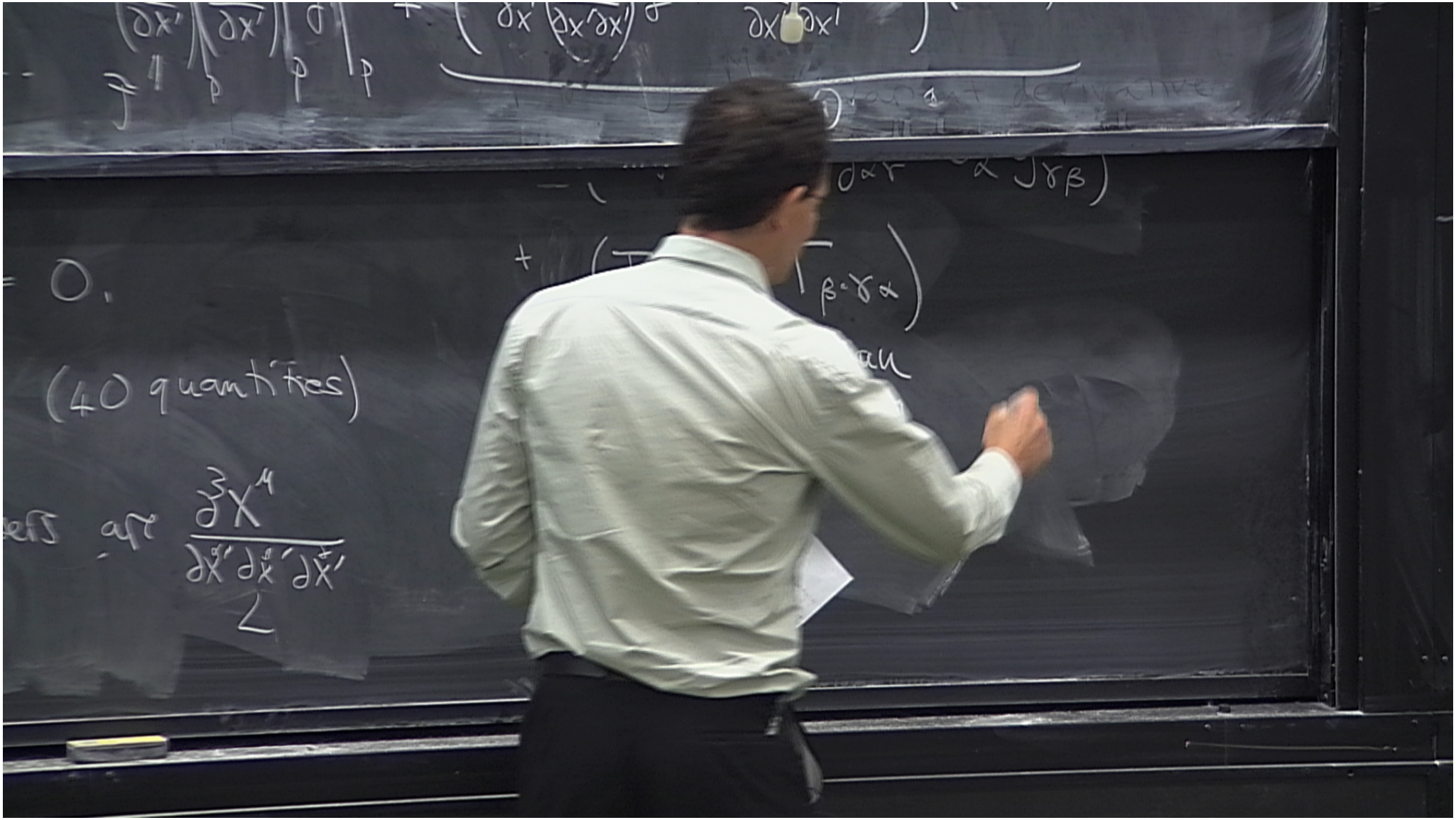

$x(x')$
$= a^\mu + \left(\frac{\partial x^\mu}{\partial x^\nu}\right)_p (x'-a')^\nu + \frac{1}{2}\frac{\partial^2 x^\mu}{\partial x^\rho \partial x^\nu}(x'-a')^\rho (x'-a')^\nu + \cdots$
② choose $\frac{\partial^2 x^\nu}{\partial x'^\beta \partial x'^\sigma}$ numbers) s.t. ① = 0.
em to cancel $g_{\beta\sigma,\alpha}$ (40 quantities)
Can you cancel ? only free numbers are $\frac{\partial^3 x^\mu}{\partial x'^\rho \partial x'^\sigma \partial x'^\tau}$

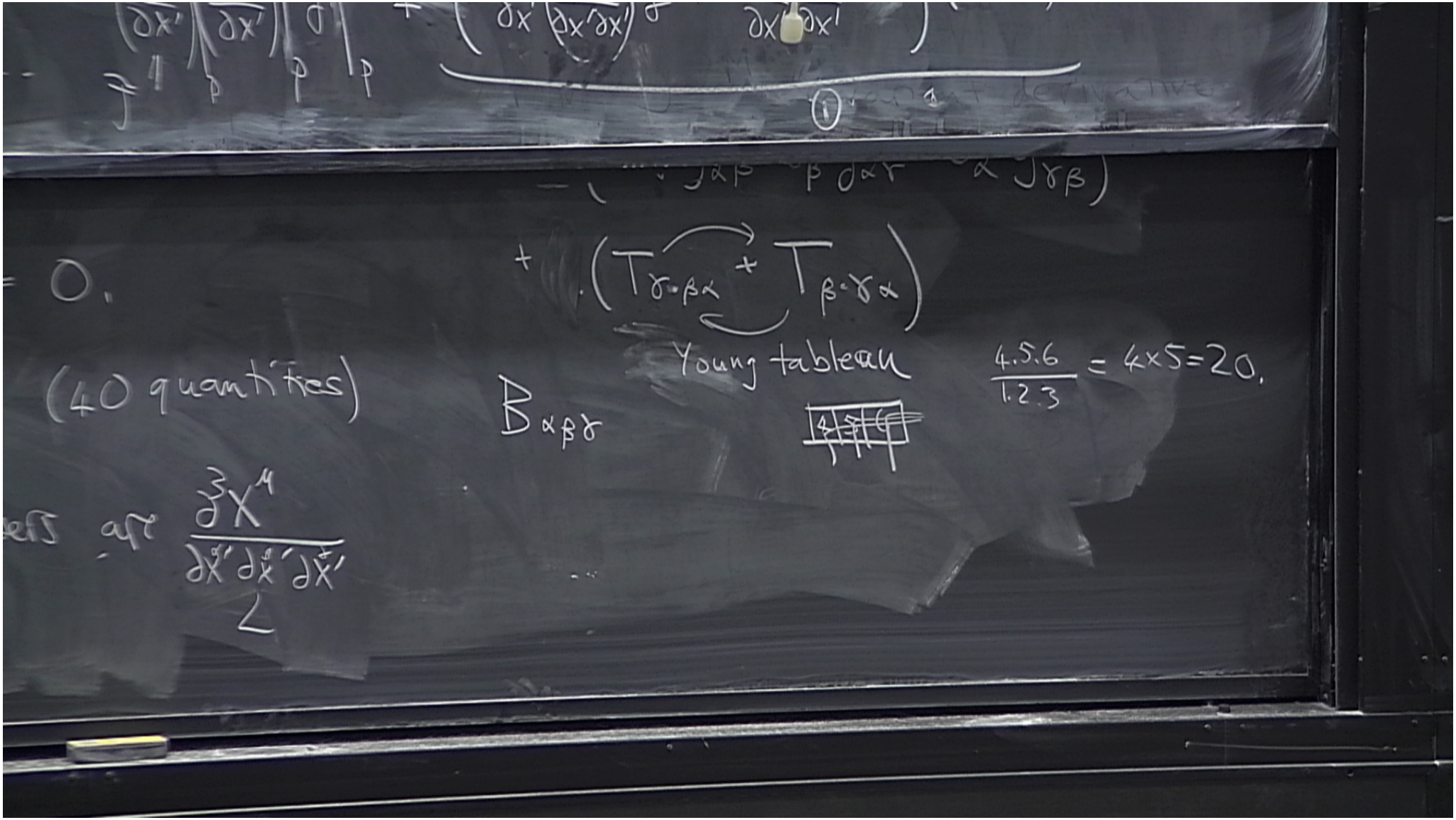

x(x')
= a^μ + (∂x^μ/∂x^ν)_p (x'-a')^ν + ½ (∂²x^μ/∂x^ρ∂x^σ)_p (x'-a')^ρ (x'-a')^σ + ...
② choose (∂²x^ν/∂x'^β∂x'^σ)_p
cancel g_βσ,α (40 quantities)
Can you cancel
only free numbers are ∂³x^μ/∂x'^ρ∂x'^σ∂x'^τ

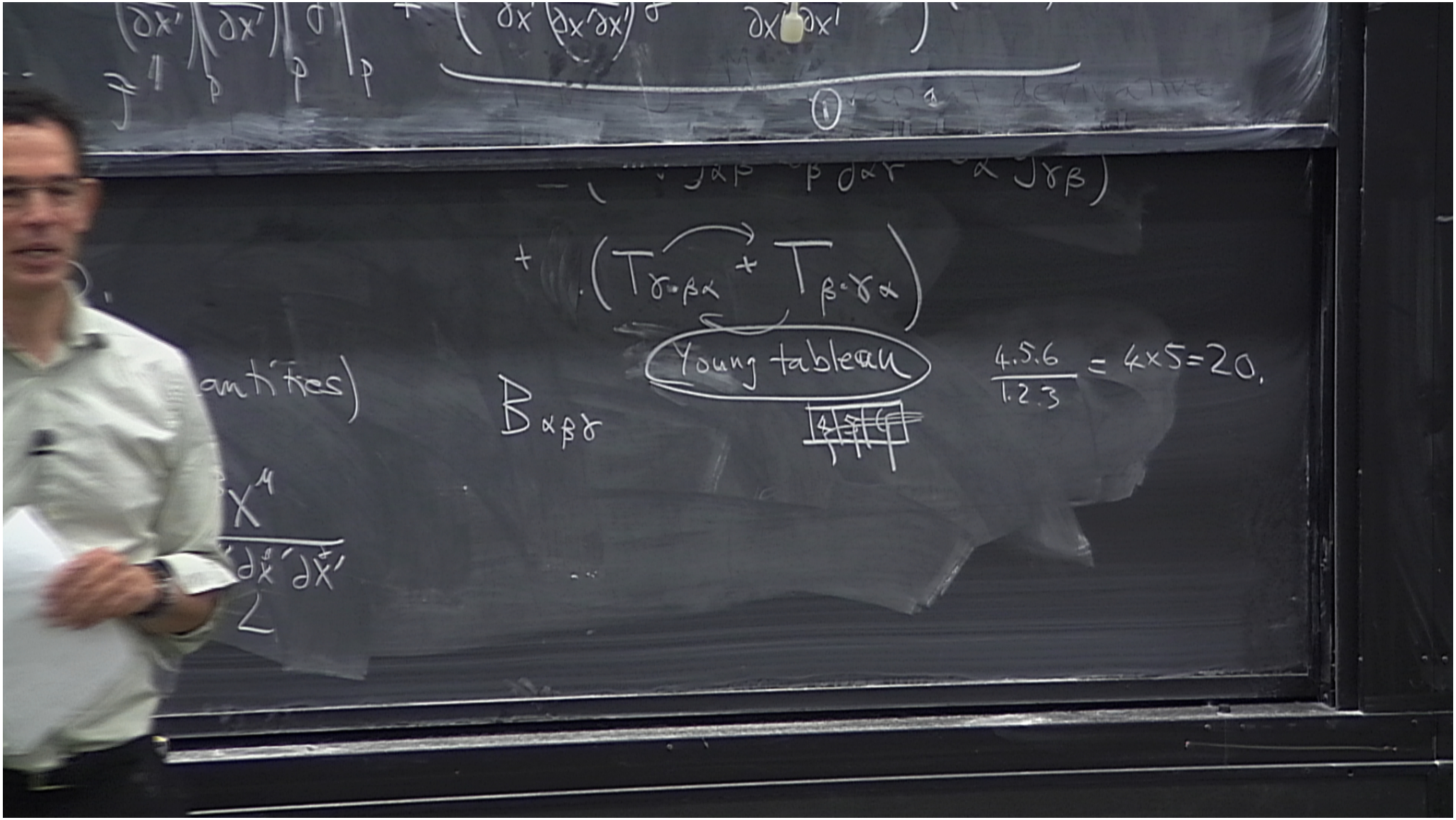

= 0.
(40 quantities)

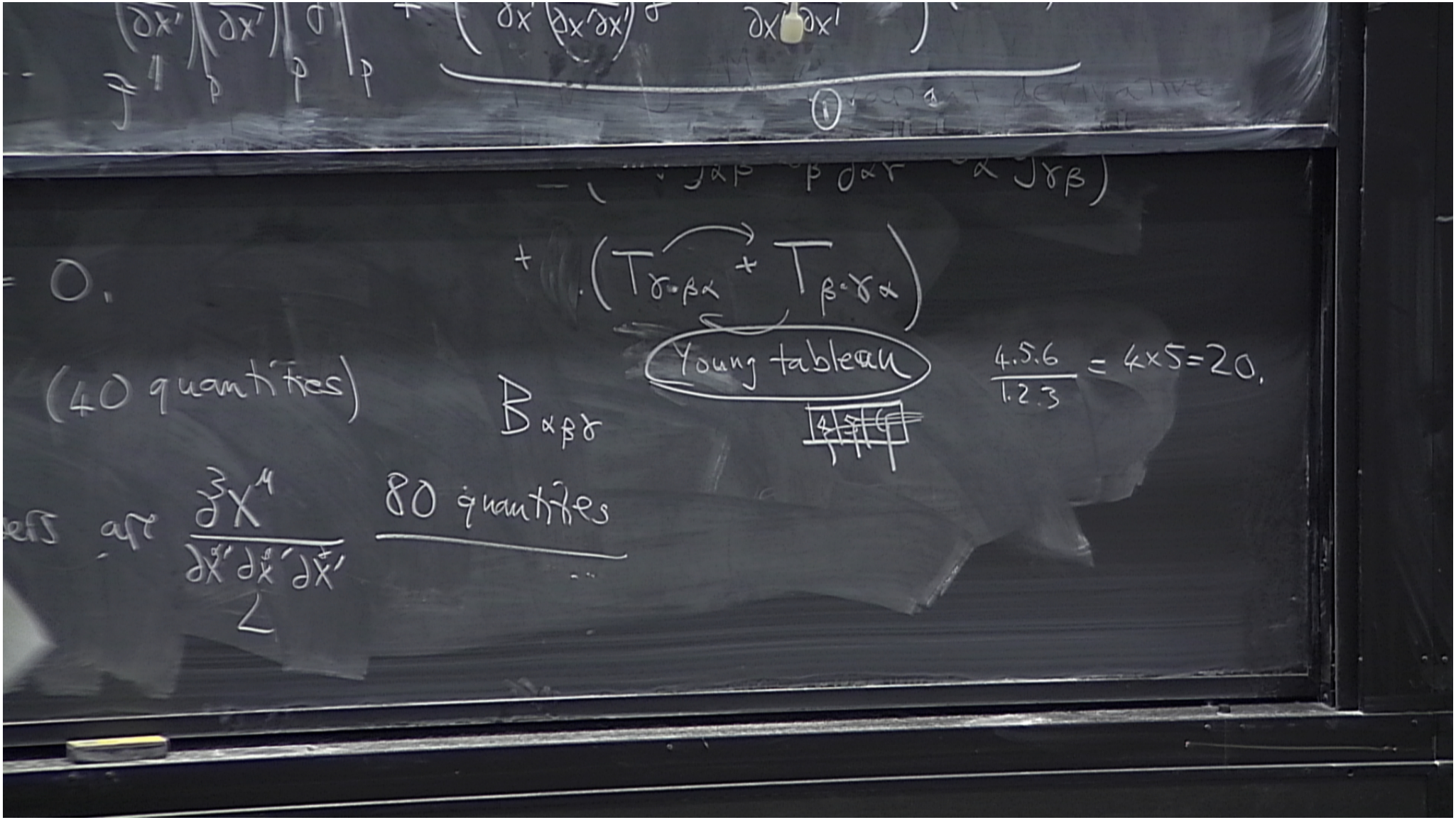

= 0.
(40 quantities)
Young tableau

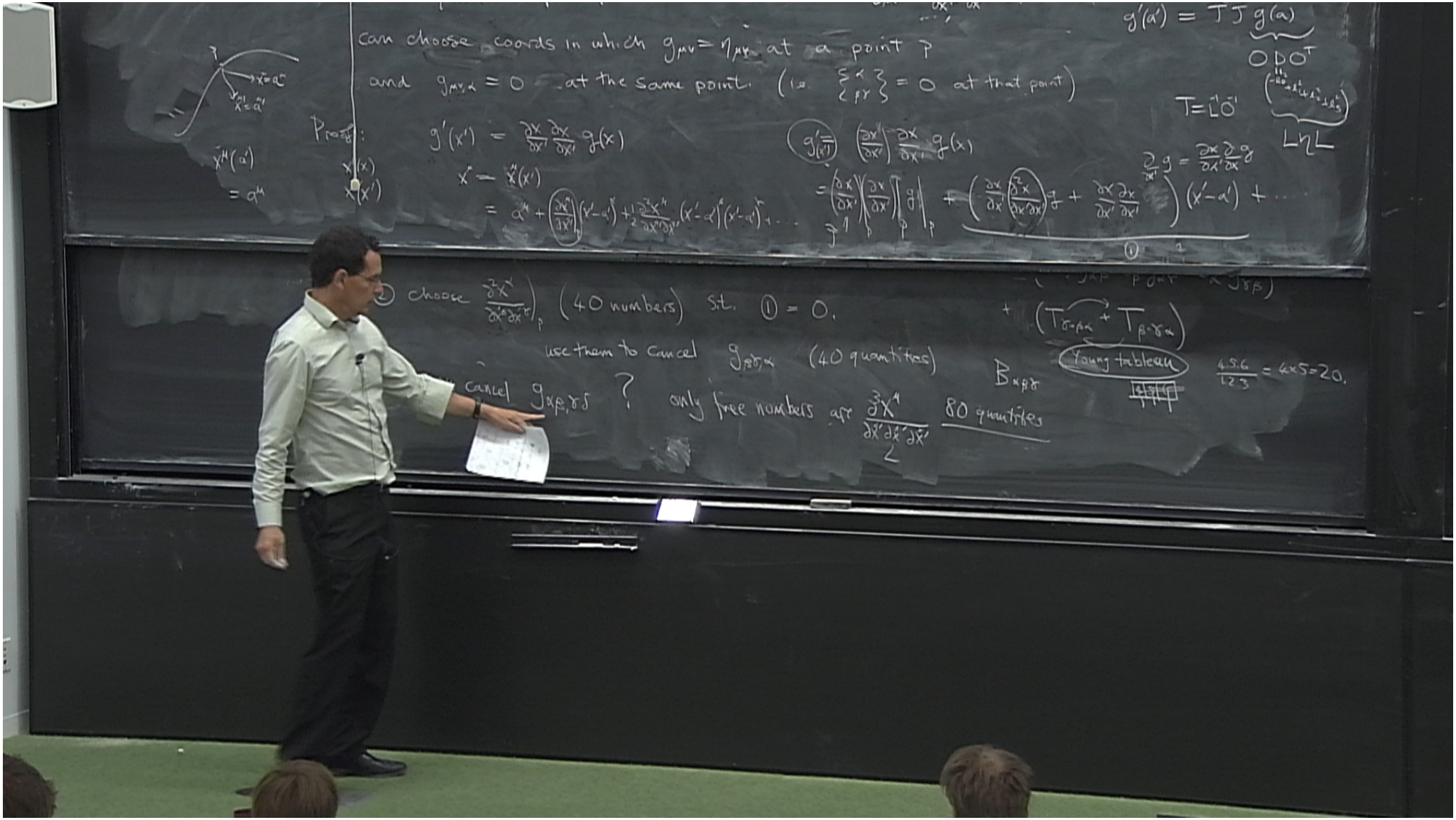

= 0.
(40 quantities)
tableau

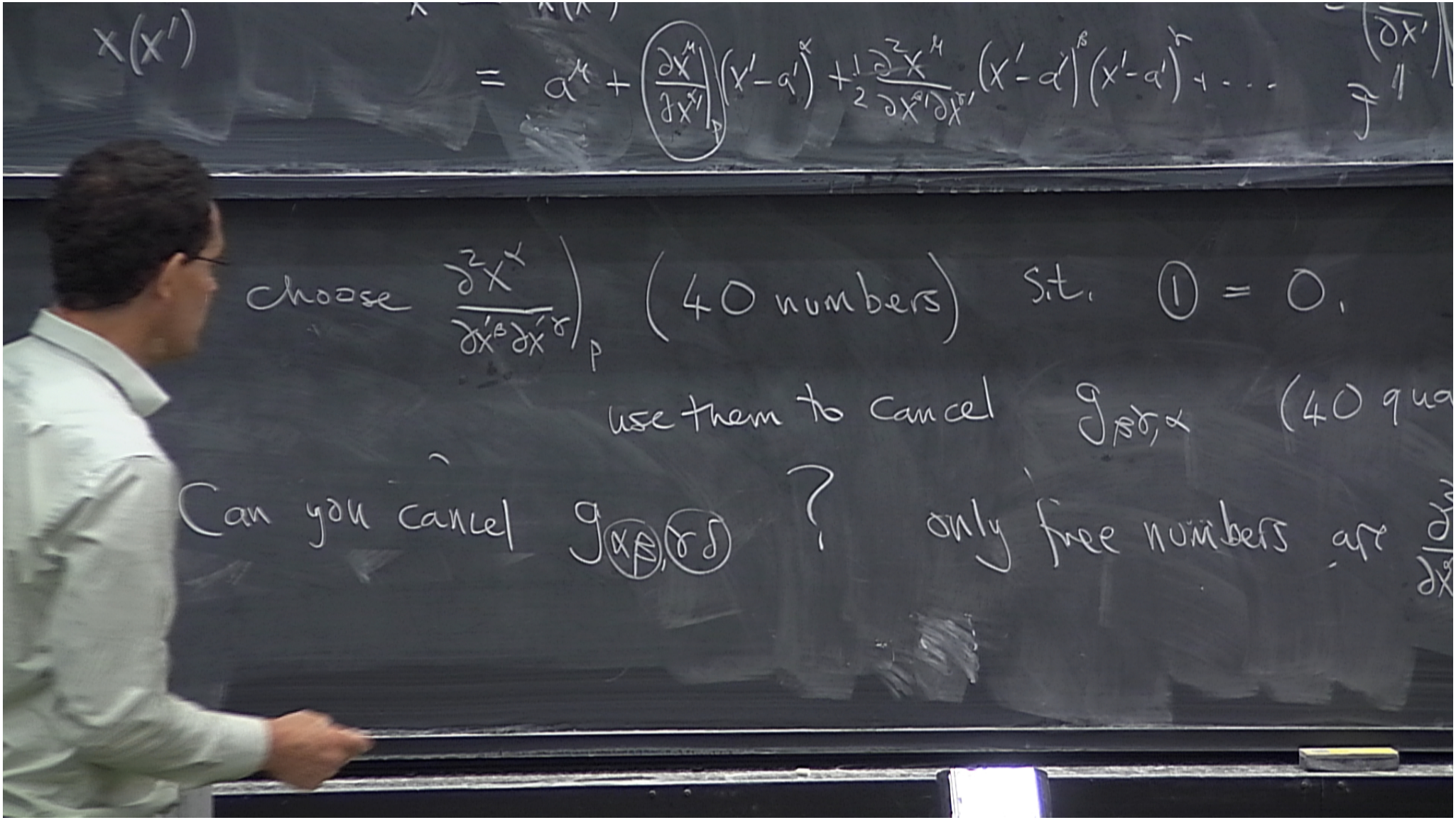

= 0.
(40 quantities)

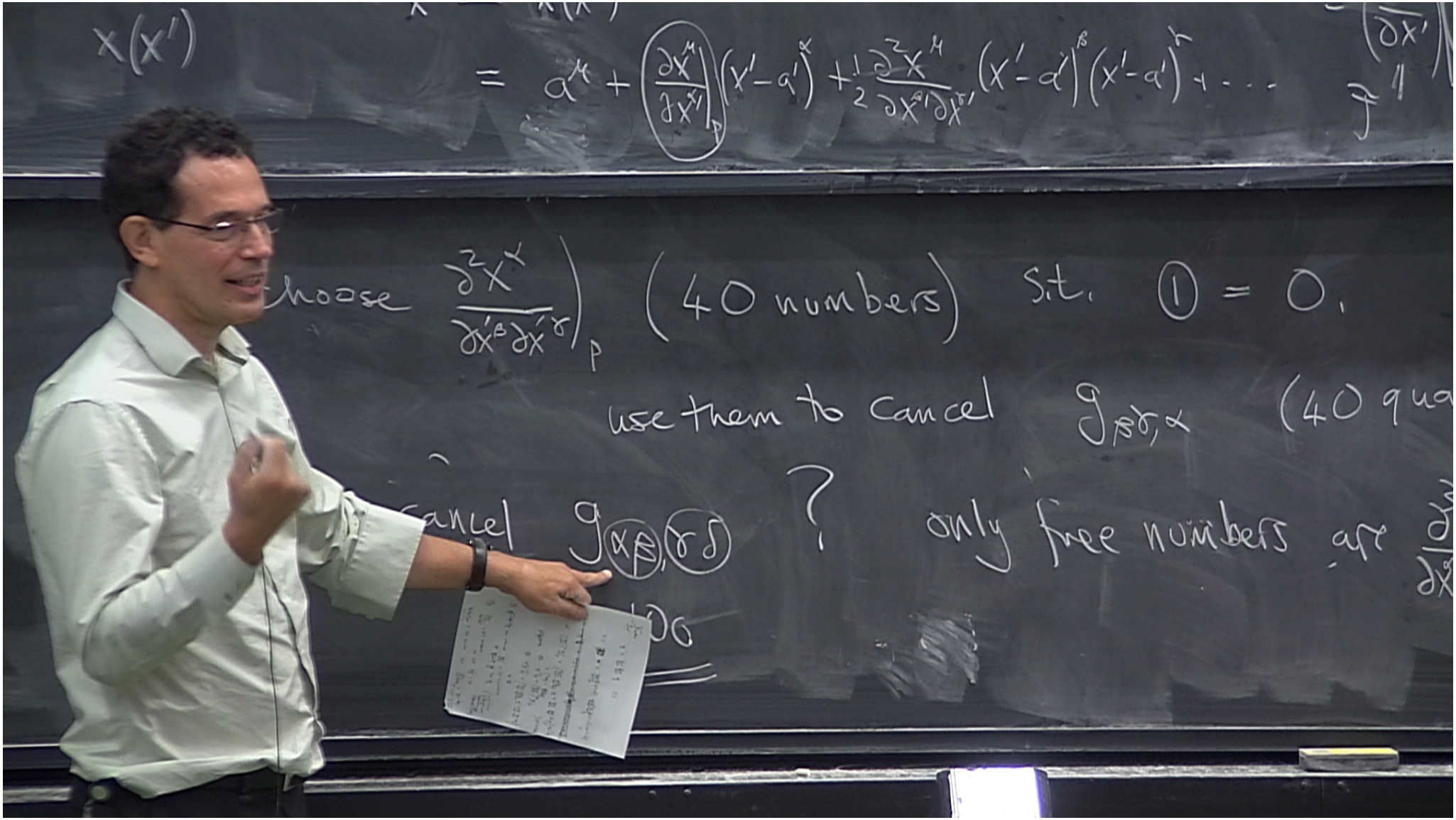

= 0.
(40 quantities)
Young tableau
4.5.6 / 1.2.3 = 4×5=20.

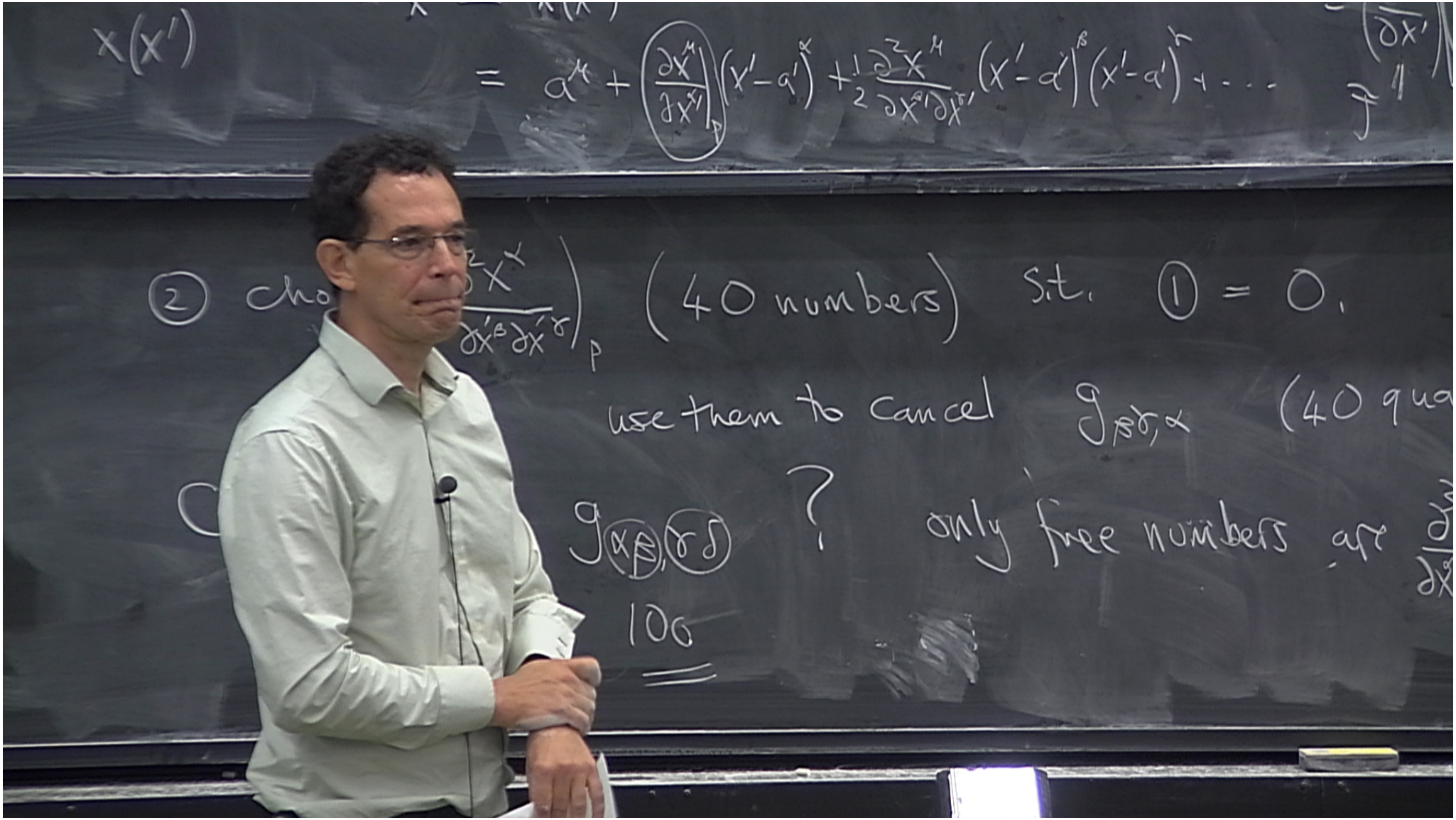

$T_{\gamma\cdot\beta\alpha} + T_{\beta\cdot\gamma\alpha}$
Young tableau
$B_{\alpha\beta\gamma}$
$\frac{4.5.6}{1.2.3} = 4\times5=20.$


= 0.
(40 quantities)
$B_{\alpha\beta\gamma}$
Young tableau
$\frac{4.5.6}{1.2.3} = 4\times5=20.$
80 quantities


can choose coords in which $g_{\mu\nu} = \eta_{\mu\nu}$ at a point P
and $g_{\mu\nu,\alpha} = 0$ at the same point. (i.e. $\Gamma^{\alpha}_{\beta\gamma} = 0$ at that point)
$g'(x') = \frac{\partial x}{\partial x'}\frac{\partial x}{\partial x'} g(x)$
$g'(a') = J J g(a)$
$O D O^T$
$T = L \tilde{O}$
LHL
choose $\frac{\partial^2 x^\mu}{\partial x'^\alpha \partial x'^\beta}\Big|_P$ (40 numbers) s.t. ① = 0.
use them to cancel $g_{\beta\gamma,\alpha}$ (40 quantities)
cancel $g_{\alpha\beta,\gamma}$? only free numbers are $\frac{\partial^3 x^\mu}{\partial x'^\alpha \partial x'^\beta \partial x'^\gamma}$ 80 quantities
$B_{\alpha\beta\gamma}$
Young tableau
$\frac{4 \cdot 5 \cdot 6}{1 \cdot 2 \cdot 3} = 4 \times 5 = 20.$


$x(x') = a^\mu + \left(\frac{\partial x^\mu}{\partial x'^\nu}\right)_p (x'-a')^\nu + \frac{1}{2}\frac{\partial^2 x^\mu}{\partial x'^\rho \partial x'^\sigma}(x'-a')^\rho (x'-a')^\sigma + \dots$
choose $\left(\frac{\partial^2 x^\nu}{\partial x'^\beta \partial x'^\gamma}\right)_p$ (40 numbers) s.t. ① = 0.
use them to cancel $g_{\beta\gamma,\alpha}$ (40 qu
Can you cancel $g_{\alpha\beta,\gamma\delta}$? only free numbers are


x(x')
= a^μ + (∂x^μ/∂x^ν)_p (x'-a')^ν + ½ ∂²x^μ/∂x'^σ∂x'^τ (x'-a')^σ (x'-a')^τ + ...
choose ∂²x^α/∂x'^β∂x'^γ)_p (40 numbers) s.t. ① = 0.
use them to cancel g_βγ,α (40 qu
cancel g_αβ,γδ ?
only free numbers


x(x')
= a^μ + (∂x^μ/∂x^ν)_p (x'-a')^ν + ½ (∂²x^μ/∂x'^σ ∂x'^τ)_p (x'-a')^σ (x'-a')^τ + ...
(40 numbers) s.t. ① = 0.
use them to cancel
(40 qu
only free numbers
100