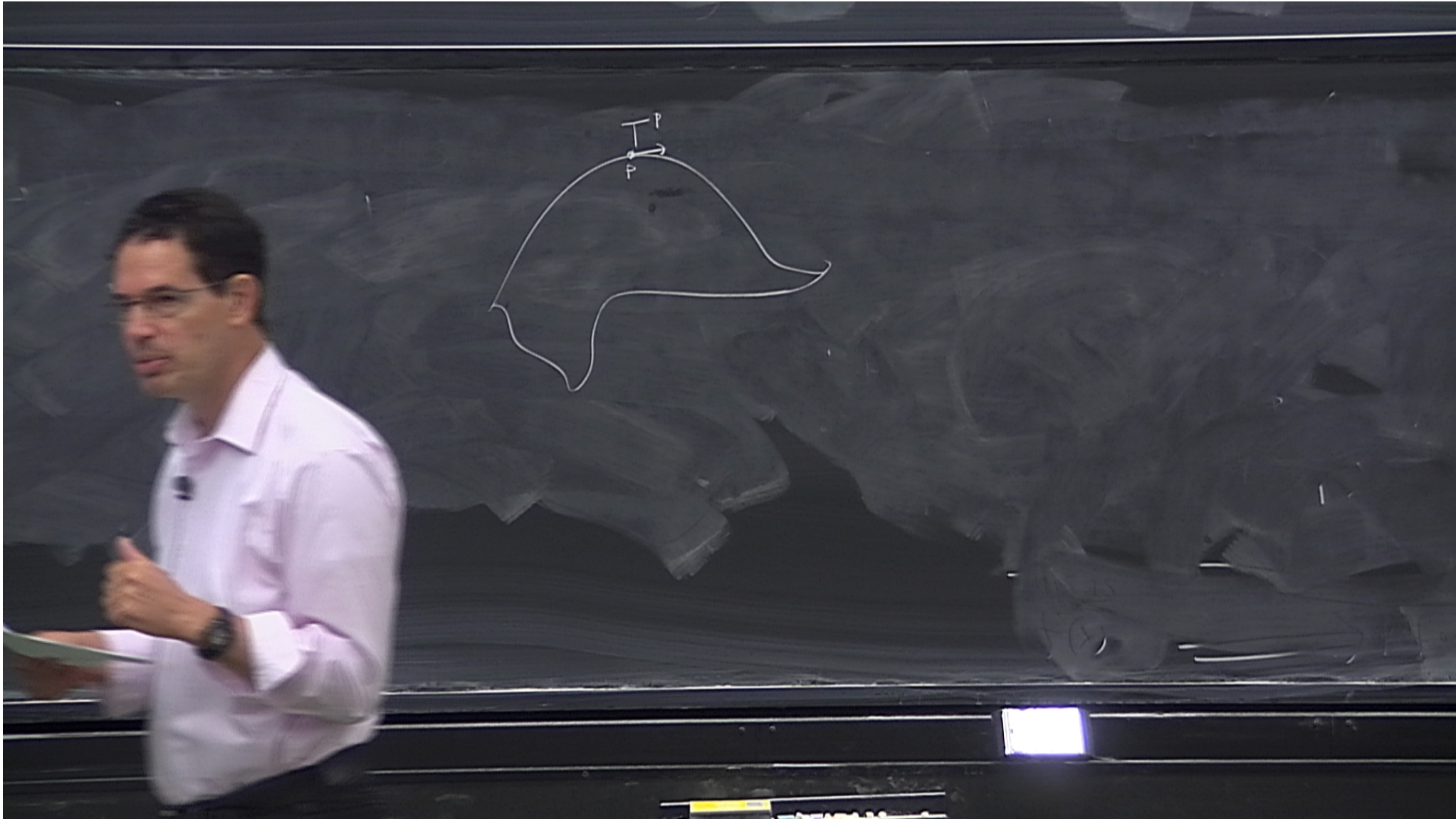


Title: Relativity - Lecture 4

Date: Sep 15, 2011 10:30 AM

URL: <http://pirsa.pi.local/11090027>

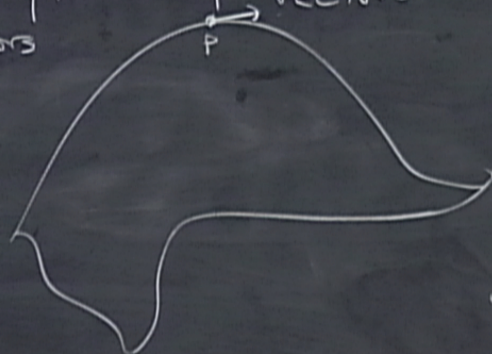
Abstract:



dual T_P^*
vectors

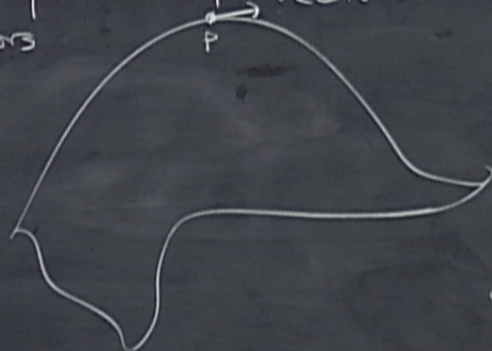
T_P vectors

Ten



dual T_P^*
vectors

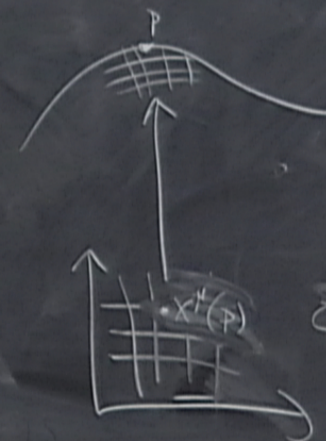
T_P vectors



Neel Carroll

Tensors on Manifolds

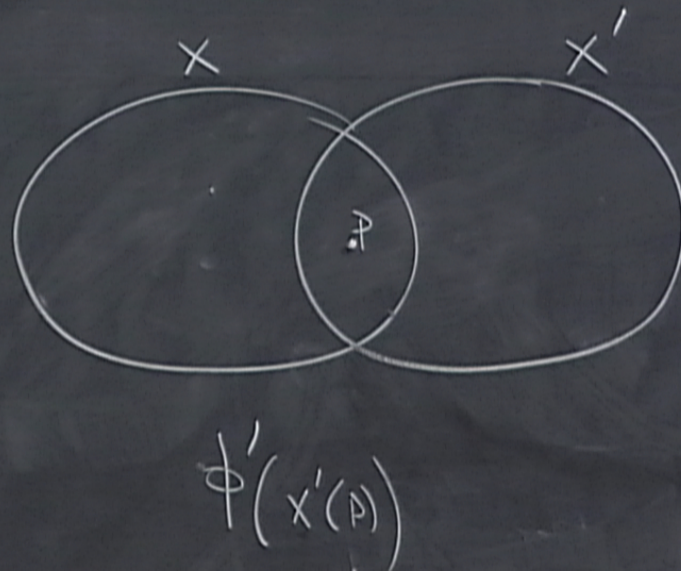
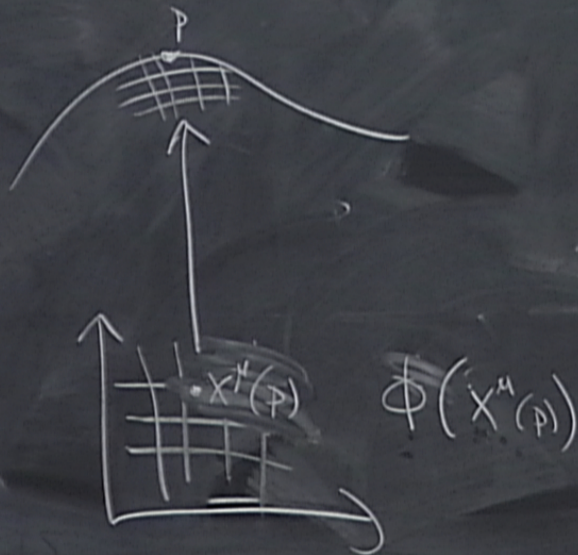
Scalar



$\phi(x^a(p))$

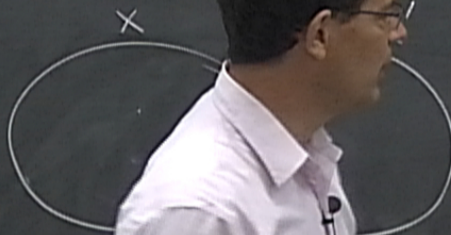
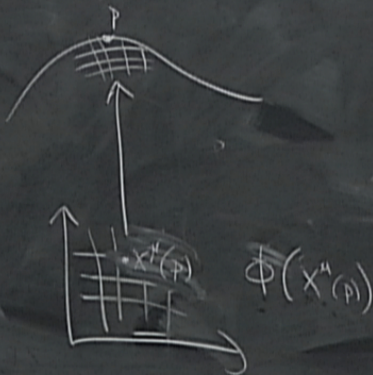
Tensors on Manifolds.

Scalar



Tensors on Manifolds

Scalar



$$\phi'(x') = \phi(x)$$

Contravariant vector

"Direction of a derivative at a point"

$$V = V^\mu(x) \frac{\partial}{\partial x^\mu}$$



$$V = V^\mu(x) \frac{\partial}{\partial x^\mu}$$
A hand-drawn diagram on a chalkboard showing a curved line with a small horizontal arrow pointing to the right, representing a direction of derivative at a point.

$$V = V^{\mu'}(x') \frac{\partial}{\partial x'^{\mu}} = V^{\mu}(x) \frac{\partial}{\partial x^{\mu}}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\alpha}}$$

$$x'(x)$$

$$x(x')$$

$$V^{n'}(x') = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}(x)$$

$$(x) \frac{\partial}{\partial x^\mu}$$

(general)

$$\Rightarrow V^{\mu'}(x') = \frac{\partial x'^{\mu}}{\partial x^\alpha} V^\alpha(x)$$

Covariant vector

example: gradient of a scalar.

$$\frac{\partial}{\partial x^\mu} \phi(x)$$

Covariant vector

example: gradient of a scalar.

$$W_\mu(x) = \frac{\partial}{\partial x^\mu} \phi(x) \quad ; \quad W'_\mu(x') = \frac{\partial}{\partial x'^\mu} \phi'(x')$$

$x(x')$

$\frac{\partial x^\alpha}{\partial x'^\mu}$

$$V(x) = \frac{\partial x^\alpha}{\partial x'^\mu} V_\alpha(x)$$

'vector'

Covariant vector

example: gradient of a scalar.

$$W_\mu(x) = \frac{\partial}{\partial x^\mu} \phi(x) \quad ; \quad W'_\mu(x') = \frac{\partial}{\partial x'^\mu} \phi'(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha} \phi(x) = \frac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)$$

Defⁿ of a covariant vector:

$$W'_\mu(x') = \frac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)$$

(dual vector
one form
covector ...)

a derivative at a point

$$(x') \frac{\partial}{\partial x'^{\mu}} = V^{\mu}(x) \frac{\partial}{\partial x^{\mu}}$$

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial}{\partial x^{\alpha}}$$

$\Rightarrow$

$$V^{\mu'}(x') = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} V^{\alpha}(x)$$

cf $\Lambda^{\mu'}$

(general)

'vector'

gradient of a scalar.

$x(x')$

$\frac{\partial x^\mu}{\partial x'^\nu}$

$$V(x) = \frac{\partial x^\mu}{\partial x'^\nu} V(x)$$

vector

Covariant vector

example: gradient of a scalar.

$$W_\mu(x) = \frac{\partial}{\partial x^\mu} \phi(x) \quad ; \quad W'_\mu(x) = \frac{\partial}{\partial x'^\mu} \phi(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial}{\partial x^\alpha} \phi(x) = \frac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)$$

is a covariant vector:

$$W'_\mu(x') = \frac{\partial x^\alpha}{\partial x'^\mu} W_\alpha(x)$$

(dual vector
one form
covector ...)

Defⁿ of a covariant vector:

NB
$$\frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x'^\nu}{\partial x^\alpha} = \frac{\partial x'^\nu}{\partial x'^\mu} = \delta^\nu_\mu$$

Similarly, a tensor of valence $\begin{bmatrix} m \\ n \end{bmatrix}$

$$T^{\overbrace{\alpha\beta\gamma\dots}^m}_{\overbrace{\mu\nu\dots}^n}(x') = \left(\frac{\partial x'^{\alpha}}{\partial x^{\mu}} \dots \right) \left(\frac{\partial x^{\nu}}{\partial x'^{\mu}} \dots \right) T^{\rho\dots}_{\sigma\dots}$$

Tensor algebra:

If T, S are tensors of same valence, then

1) $T+S$ is a tensor

Tensor algebra:

If T, S are tensors of same valence, then

1) $T+S$ is a tensor

2) outer multiplication:

$$T^{\alpha\beta} \dots S_{\mu\nu} \dots$$

Similarly, a tensor of valence $\begin{bmatrix} m \\ n \end{bmatrix}$

$$T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} (x) = \left(\frac{\partial x^{\alpha_1}}{\partial x^{\alpha'}} \dots \frac{\partial x^{\alpha_m}}{\partial x^{\alpha'}} \right) \left(\frac{\partial x^{\beta_1}}{\partial x^{\beta'}} \dots \frac{\partial x^{\beta_n}}{\partial x^{\beta'}} \right) T^{\alpha' \dots \alpha'}_{\beta' \dots \beta'}$$

(rank $\equiv m+n$)

Tensor algebra:

If T, S are tensors

1) $T+S$ is a tensor (if T, S have same valence)

2) outer multiplication:

$$T^{\alpha\beta} \dots S_{\mu\nu} \dots$$

Similarly, a tensor of valence $\begin{bmatrix} m \\ n \end{bmatrix}$

$$T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n}(x) = \left(\frac{\partial x^{\alpha_1}}{\partial x^{\beta_1}} \dots \frac{\partial x^{\alpha_m}}{\partial x^{\beta_m}} \right) \left(\frac{\partial x^{\beta_1}}{\partial x^{\alpha_1}} \dots \frac{\partial x^{\beta_n}}{\partial x^{\alpha_n}} \right) T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n}$$

(rank $\equiv m+n$)

Tensor algebra:

T, S are tensors

1) $T+S$ is a tensor (if T, S have same valence)

2) outer multiplication: $T^{\alpha\beta\dots r}$ $S_{\mu\nu\dots s}$ is a tensor
rank $r+s$

3) contraction $T^{\alpha\beta\dots r} S_{\alpha\mu\nu\dots s}$ has rank $r+s-2$

Very useful fact:

if a tensor is zero in one coordinate system, then it is zero in all coordinate systems.

a tensor

The metric

Minkowski spacetime has line element ("Lorentz-invariant distance squared")

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = \underline{\underline{\eta_{\mu\nu} dx^\mu dx^\nu}}$$

The metric

Minkowski spacetime has line element ("Lorentz-invariant distance squared")

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

= Minkowski metric

Curved spacetime:

$$ds^2 = g_{\mu\nu}(x^\mu) dx^\mu dx^\nu$$

The metric

Minkowski spacetime has line element ("Lorentz-

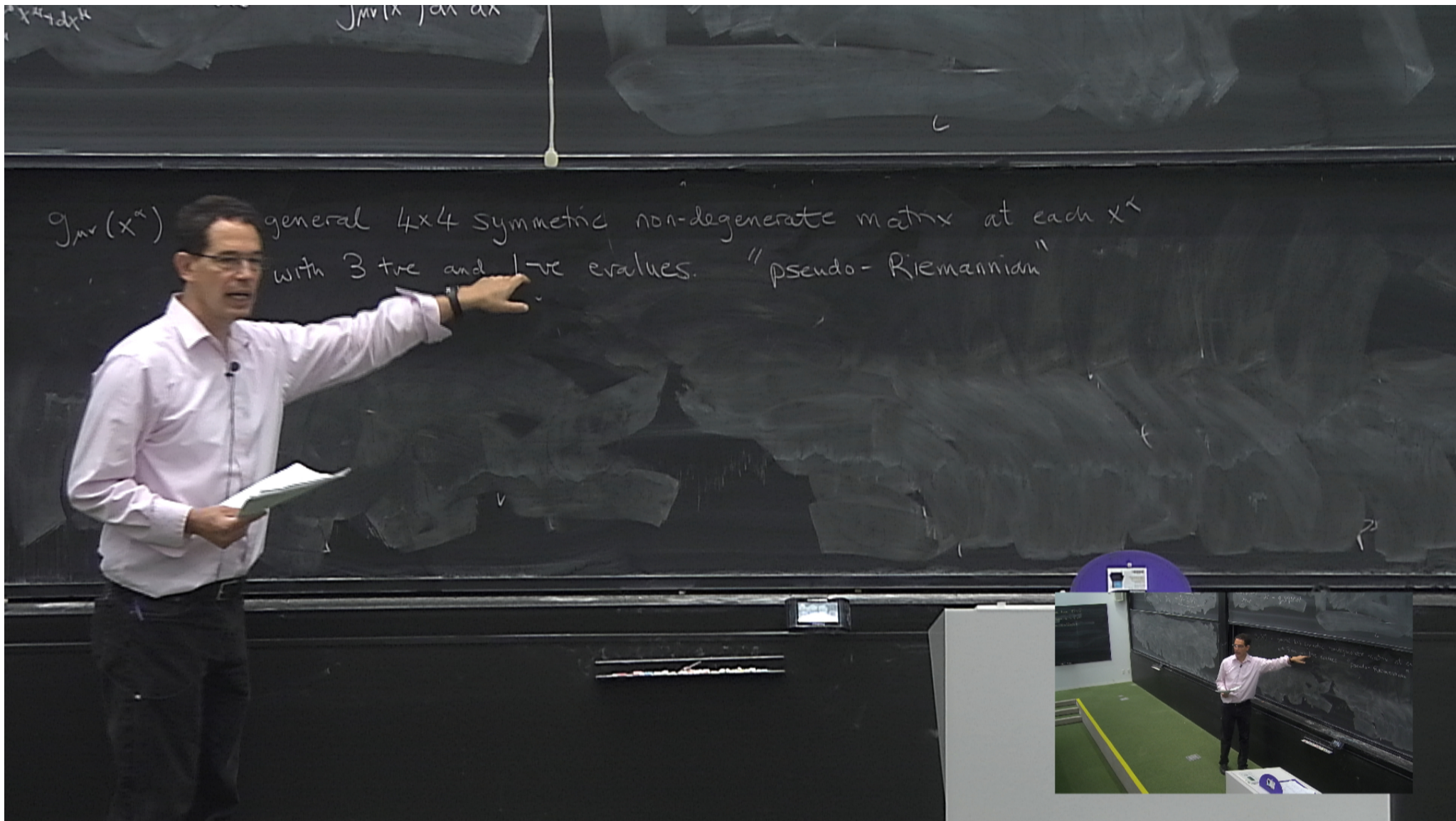
$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = \underline{\underline{\eta_{\mu\nu} dx^\mu dx^\nu}} \quad \text{Minko}$$

Curved spacetime:

$$ds^2 = g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu$$

$$\begin{matrix} \bullet & x^\mu + dx^\mu \\ & \nearrow \\ & x^\mu \end{matrix}$$

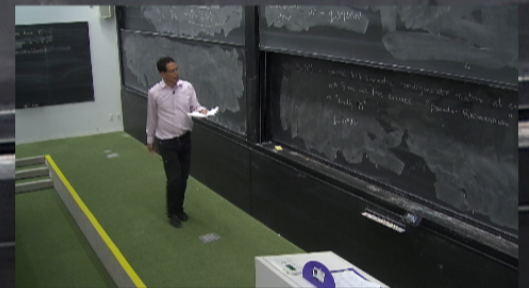
general 4×4 symmetric non-degenerate matrix at each x^i
with 3 +ve and 1 -ve evalues.



general 4×4 symmetric non-degenerate matrix at each x^μ
with 3 +ve and 1 -ve evalues. "pseudo-Riemannian"

$\Rightarrow$ "locally M^4 "

$$g_{\mu\nu} dx^\mu dx^\nu$$



general 4×4 symmetric non-degenerate matrix at each x^μ
with 3 +ve and 1 -ve evalues. "pseudo-Riemannian"

$\Rightarrow$ "locally $\mathbb{M}^4$ "

$$g_{\mu\nu} dx^\mu dx^\nu = dx^T O^T D O dx$$

general 4×4 symmetric non-degenerate matrix at each x^μ
 with 3 +ve and 1 -ve evalues. "pseudo-Riemannian"

$\Rightarrow$ "locally M^4 "

$$g_{\mu\nu} dx^\mu dx^\nu = dx^T O^T D O dx$$

$$\begin{pmatrix} -\lambda_0 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix} = \begin{pmatrix} -\lambda_0^2 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{pmatrix} = L^T \eta L$$

$\lambda_i > 0$

$$L = \begin{pmatrix} e^0 \\ e^1 \\ e^2 \\ e^3 \end{pmatrix}$$

$g_{\mu\nu}(x^\alpha)$: general 4×4 symmetric non-degenerate matrix at each x^α
 with 3 +ve and 1 -ve evalues. "pseudo-Riemannian"
 $\Rightarrow$ "locally M^4 "

$$dy = L \circ dx$$

$$g_{\mu\nu} dx^\mu dx^\nu = dx^T \circ^T D \circ dx$$

$$L^* = \begin{pmatrix} e^0 & e^1 & e^2 & e^3 \end{pmatrix}$$

$$\begin{pmatrix} -\lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix} = \begin{pmatrix} -\lambda_0^2 & +\lambda_1^2 & +\lambda_2^2 & +\lambda_3^2 \end{pmatrix} = L^T \eta L$$

$\lambda_i^2 > 0$

$$= dx^T \circ^T L^T \eta L \circ dx$$

$$= dy^T \eta dy$$

$J_{\mu\nu}(x)$

with 3 tree and 1-

$$y = L O x$$

$\Rightarrow$ "locally M^4 "

$$dy = L O dx$$

$$g_{\mu\nu} dx^\mu dx^\nu =$$

$$= dx^T O^T L^T \eta L O dx$$

$$= dy^T \eta dy$$



$$x'(x)$$

$$x(x')$$

$$\frac{\partial x^\mu}{\partial x'^\alpha} \frac{\partial}{\partial x^\mu}$$

$$\Rightarrow \boxed{V'^\alpha(x) = \frac{\partial x'^\alpha}{\partial x^\mu} V^\mu(x)}$$

$$c \Lambda^{\mu'}_{\mu}$$

(general)

'vector'

Principle of Equivalence

At every spacetime point, it is possible to choose a coordinate system (called a "free-fall" or "locally inertial" coordinate system) such that within a sufficiently small neighbourhood, the laws of physics take the same special relativistic form as they do in Minkowski spacetime.

$$V = V(x) \frac{\partial}{\partial x^\mu} ; \quad V = V(x') \frac{\partial}{\partial x'^\mu} = V^{\mu}(x) \frac{\partial}{\partial x^\mu}$$

$x'(x)$
 $x(x')$

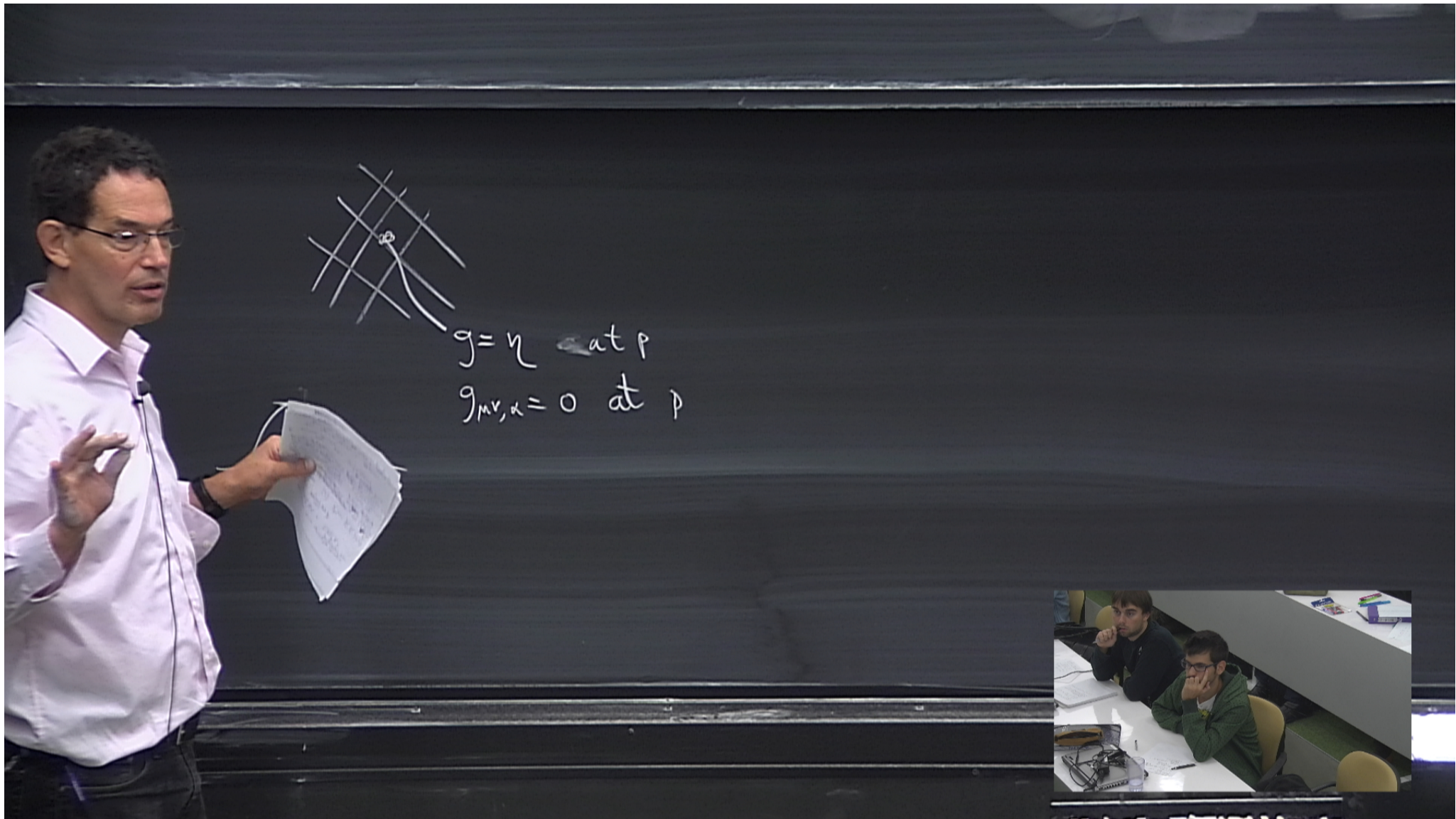
$$\Rightarrow \boxed{V^{\mu}(x) = \frac{\partial x'^\mu}{\partial x^\alpha} V^{\alpha}(x)}$$

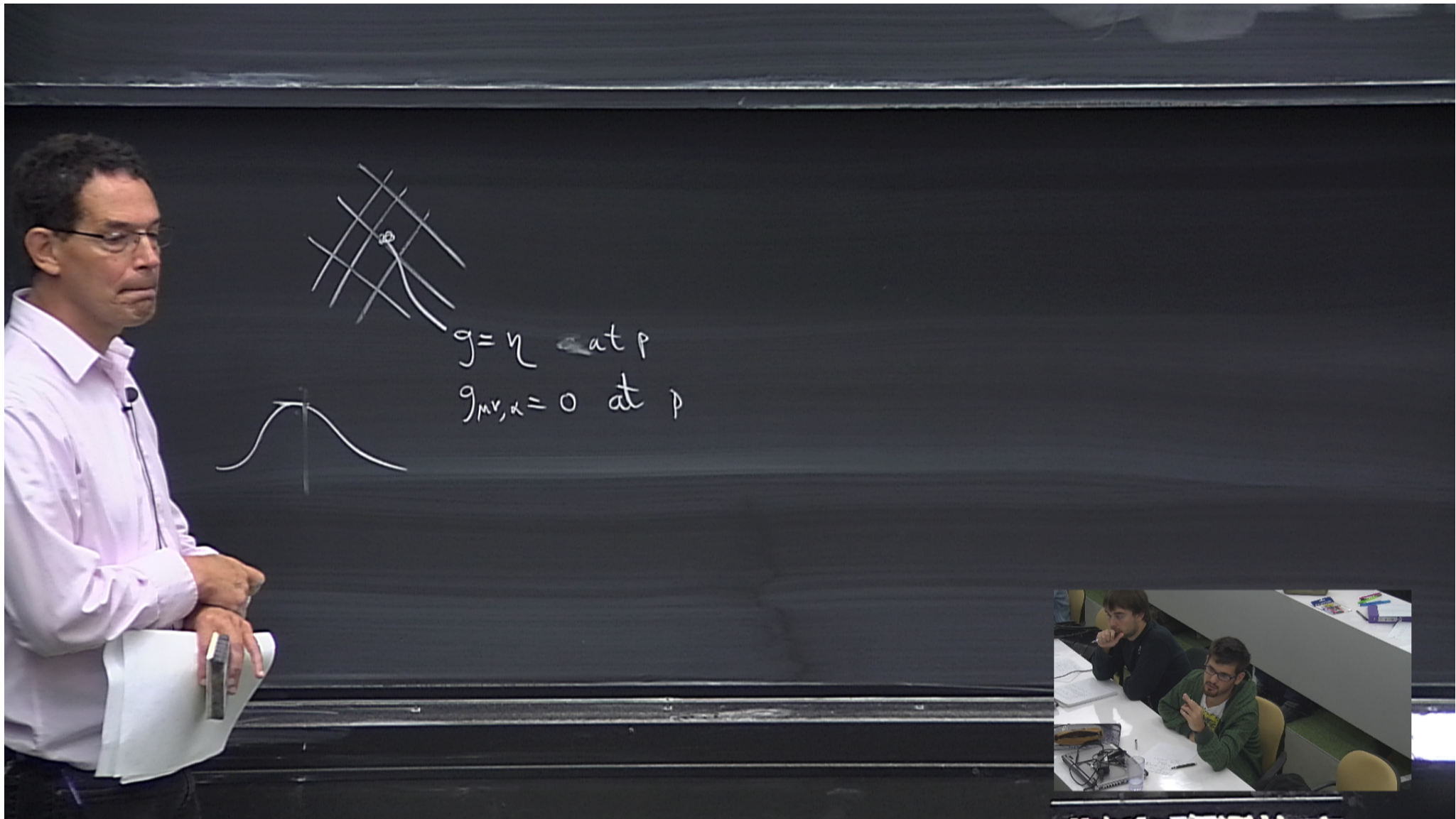
(general)
 'vector'

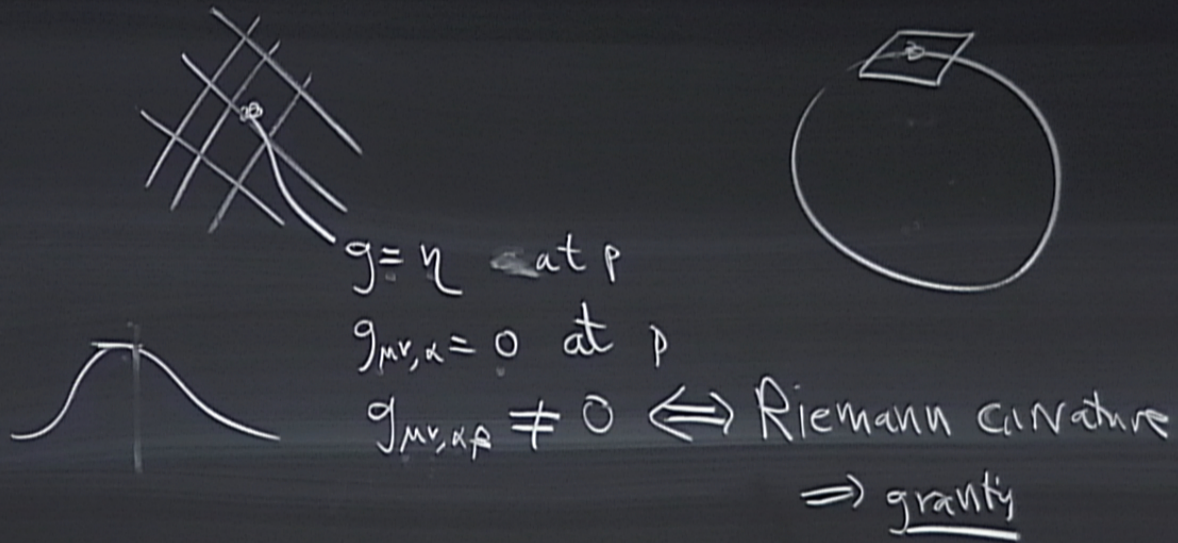
Principle of Equivalence

At every spacetime point, it is possible to choose a coordinate system (called a "free-fall" or "locally inertial" coordinate system) such that within a sufficiently small neighbourhood, the laws of physics take the same special relativistic form as they do in Minkowski spacetime.

Later, sharpen slightly: poss to choose coord sys. st $g_{\mu\nu} = \eta_{\mu\nu}$ and $g_{\mu\nu,\alpha} = 0$ at a point







Coordinate transformation
metric is $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

Coordinate transformation
metric is $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\det g' = \left(\det \frac{\partial x}{\partial x'} \right)^2 \det g$$

Coordinate transformation
metric is $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\det g' = \left(\det \frac{\partial x}{\partial x'} \right)^2 \det g$$

$$\sqrt{-\det g'} = \left| \frac{\partial x}{\partial x'} \right|$$

Coordinate transformation
metric is $\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\det g' = \left(\det \frac{\partial x}{\partial x'} \right)^2 \det g$$

$$\sqrt{-\det g'} = \left| \frac{\partial x}{\partial x'} \right| \sqrt{-\det g} \Rightarrow \sqrt{-\det g'} d^4 x' = \underbrace{\sqrt{-\det g} d^4 x}_{\text{"invariant volume element"}}$$

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}(x)$$

$$\det g' = \left(\det \frac{\partial x}{\partial x'} \right)^2 \det g$$

$$\sqrt{-\det g'} = \left| \frac{\partial x}{\partial x'} \right| \sqrt{-\det g} \Rightarrow \sqrt{-\det g'} d^4 x' = \underbrace{\sqrt{-\det g} d^4 x}_{\text{invariant volume}}$$

Inverse metric $g^{\mu\nu}$: $g^{\mu\nu} g_{\nu\alpha} = \delta^{\mu}_{\alpha}$

"invariant volume element"

Inverse metric g^{mv} : $g^{mv} g_{v\alpha} = \delta_{\alpha}^m$: $g'^{mv}(x') = \frac{\partial x'^m}{\partial x^\alpha} \frac{\partial x'^v}{\partial x^\beta} g^{\alpha\beta}(x)$

Can use metric to raise or lower indices : $T_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu}$ etc.

Examples: S^2

is phase on lower matrices

$$S^2 \in E^3$$

$$x^2 + y^2 + z^2 = 1$$

$$\begin{aligned}x &= \sin\theta \cos\phi \\y &= \sin\theta \sin\phi \\z &= \cos\theta\end{aligned}$$

metric to raise or lower indices :

$$T^{\alpha}_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu}$$

$$S^2 \in \mathbb{E}^3$$

$$x^2 + y^2 + z^2 = 1$$

metric :

$$\begin{aligned} x &= \sin\theta \cos\phi \\ y &= \sin\theta \sin\phi \\ z &= \cos\theta \end{aligned}$$

: embedding of S^2 in $\mathbb{E}^3$

"invariant volume element"

Inverse metric g^{mv} : $g^{mv} g_{vn} = \delta_n^m$: $g'^{mv}(x') = \frac{\partial x'^m}{\partial x^\alpha} \frac{\partial x'^v}{\partial x^\beta} g^{\alpha\beta}(x)$

Can use metric to raise or lower indices : $T^\alpha_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu}$ etc.

Examples : $S^2 \in E^3$
 $x^2 + y^2 + z^2 = 1$

$dx = \cos\theta \cos\phi d\theta$

$+ \sin\theta \sin\phi d\phi$ etc.

$x = \sin\theta \cos\phi$
 $y = \sin\theta \sin\phi$
 $z = \cos\theta$

: embedding of S^2 in E^3

metric : $dx^2 + dy^2 + dz^2$

raise or lower indices :

$$T^{\alpha}_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu} \quad \text{etc.}$$

$$S^2 \in E^3$$
$$x^2 + y^2 + z^2 = 1$$

$$x = \sin\theta \cos\phi$$
$$y = \sin\theta \sin\phi$$
$$z = \cos\theta$$

embedding of S^2 in E^3

metric: $dx^2 + dy^2 + dz^2 = d\theta^2 + \sin^2\theta d\phi^2$ i.e. $g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$

$$g_{\nu\alpha} = \delta_{\alpha\nu}$$

$$g^{\alpha\beta} = \frac{\partial x^\alpha}{\partial x'^\beta}$$

$$\frac{\partial x^\alpha}{\partial x'^\beta}$$

raise or lower indices:

$$T^{\alpha}_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu}$$

etc.

$$S^2 \in E^3$$

$$x^2 + y^2 + z^2 = 1$$

$$\begin{aligned} x &= \sin\theta \cos\phi \\ y &= \sin\theta \sin\phi \\ z &= \cos\theta \end{aligned}$$

: embedding of S^2 in E^3

$$\text{metric: } dx^2 + dy^2 + dz^2 = d\theta^2 + \sin^2\theta d\phi^2$$

$$\text{i.e. } g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}$$

etc.

Vol element is
 $\sin\theta d\theta d\phi$

$$\sqrt{g} = \sin\theta$$

$$\begin{pmatrix} \phi \\ \theta \\ \sin^2\theta \end{pmatrix}$$

Ex: stereographic coords

$$dx^2 = 4 \frac{dz_1^2 + dz_2^2}{(1 + z_1^2 + z_2^2)^2} = \frac{4}{(1 + R^2)^2} (dR^2 + R^2 d\phi^2)$$

$$R^2 = z_1^2 + z_2^2$$

azimuthal angle
in plane.

Ex:
de Sitter

$$-X^0{}^2 + X^1{}^2 + X^2{}^2 + X^3{}^2 + X^4{}^2 = 1 \text{ in } M^5$$

Ex:
de Sitter
 dS^4

$$-X^0^2 + X^1^2 + X^2^2 + X^3^2 + X^4^2 = 1 \text{ in } M^5$$

work out the metric.

$$\det \frac{\partial x'}{\partial x} > 0 \quad \sqrt{-\det g'} = \left| \frac{\partial x}{\partial x'} \right| \sqrt{-\det g} \Rightarrow \sqrt{-\det g'} d^4 x' = \underbrace{\sqrt{-\det g} d^4 x}_{\text{"invariant volume element"}}$$

The connection

We express laws of physics as differential equations. How can we take derivatives of tensors?

The connection

We express laws of physics as differential

$$\frac{\partial}{\partial x^\mu} \phi(x) = \text{tensor} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \checkmark$$

The connection

We express laws of physics as differential

$$\frac{\partial}{\partial x^\mu} \phi(x) = \text{tensor} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \checkmark$$

BUT

$$\frac{\partial}{\partial x'^\mu} V'^\alpha(x')$$

The connection

We express laws of physics as differential

$$\frac{\partial}{\partial x^\mu} \phi(x) = \text{tensor} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \checkmark$$

BUT

$$\frac{\partial}{\partial x'^\mu} V'^\alpha(x') = \frac{\partial x^\beta}{\partial x'^\mu} \frac{\partial}{\partial x^\beta} \left(\frac{\partial x'^\alpha}{\partial x^\nu} V^\nu(x) \right)$$

as differential equations. How can we take

✓

$$\left(\frac{\partial x'^{\alpha}}{\partial x^{\mu}} V^{\mu} \right) = \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\beta} \partial x^{\mu}} V^{\mu} + \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial x'^{\alpha}}{\partial x^{\mu}} \frac{\partial V^{\mu}}{\partial x^{\beta}}$$

al equations. How can we take derivati

✓ (for $[1]$ tensor)

$$\frac{\partial x^\beta}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\beta \partial x^\nu} V^\nu + \frac{\partial x^\beta}{\partial x'^\mu} \frac{\partial x'^\alpha}{\partial x^\nu} \frac{\partial V^\nu}{\partial x^\beta}$$

define the connection $\Gamma^M_{\nu\alpha}$, with the transformation law

$$\Gamma'^M_{\nu\rho} = \frac{\partial x'^M}{\partial x^\alpha} \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x'^\rho} \Gamma^\alpha_{\beta\gamma}$$

define the connection $\Gamma^M_{\nu\alpha}$, with the transformation law

$$\begin{aligned}\Gamma'^M_{\nu\rho} &= \frac{\partial x'^M}{\partial x^\alpha} \frac{\partial x^\beta}{\partial x'^\nu} \frac{\partial x^\gamma}{\partial x'^\rho} \Gamma^\alpha_{\beta\gamma} + \frac{\partial^2 x'^M}{\partial x^\lambda \partial x'^\nu} \frac{\partial^2 x^\lambda}{\partial x'^\rho \partial x'^\rho} \\ &= J^M_\alpha \tilde{J}^\beta_\nu \tilde{J}^\gamma_\rho \Gamma^\alpha_{\beta\gamma} + J^M_\lambda \partial'_\rho \tilde{J}^\lambda_\nu\end{aligned}$$

x^{μ}

$$\frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\nu}}{\partial x^{\mu}} = \delta^{\mu}_{\mu}$$

$$\frac{\partial x^{\mu}}{\partial x^{\nu}} \frac{\partial x^{\nu}}{\partial x^{\alpha}} = \frac{\partial x^{\mu}}{\partial x^{\alpha}} - \frac{\partial x^{\mu}}{\partial x^{\beta}} \frac{\partial x^{\beta}}{\partial x^{\alpha}}$$

define the connection $\Gamma_{\nu\alpha}^{\mu}$, with the transformation law

$$\Gamma_{\nu\rho}^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x^{\nu}} \frac{\partial x^{\gamma}}{\partial x^{\rho}} \Gamma_{\beta\gamma}^{\alpha} + \underbrace{\frac{\partial x^{\mu'}}{\partial x^{\lambda}} \frac{\partial^2 x^{\lambda}}{\partial x^{\nu} \partial x^{\rho}}}_{\frac{\partial x^{\mu'}}{\partial x^{\lambda}} \frac{\partial x^{\lambda}}{\partial x^{\nu}} \frac{\partial x^{\lambda}}{\partial x^{\rho}}}$$

$$= J_{\alpha}^{\mu'} \tilde{J}_{\nu}^{\beta} \tilde{J}_{\rho}^{\gamma} \Gamma_{\beta\gamma}^{\alpha} + J_{\lambda}^{\mu'} \partial_{\nu} \tilde{J}_{\rho}^{\lambda}$$

$$\frac{\partial x^{\mu'}}{\partial x^{\lambda}} \frac{\partial x^{\lambda}}{\partial x^{\nu}} \frac{\partial x^{\lambda}}{\partial x^{\rho}} = \delta_{\rho}^{\mu'}$$

$$\frac{\partial}{\partial x^{\nu}} \rightarrow \frac{\partial x^{\mu'}}{\partial x^{\lambda}} \frac{\partial^2 x^{\lambda}}{\partial x^{\nu} \partial x^{\rho}} = - \underbrace{\frac{\partial x^{\lambda}}{\partial x^{\nu}} \frac{\partial x^{\mu'}}{\partial x^{\lambda}} \frac{\partial^2 x^{\lambda}}{\partial x^{\nu} \partial x^{\rho}}}_{\frac{\partial x^{\mu'}}{\partial x^{\lambda}} \frac{\partial x^{\lambda}}{\partial x^{\nu}} \frac{\partial x^{\lambda}}{\partial x^{\rho}}}$$

define the connection $\Gamma_{\nu\alpha}^{\mu}$, with the tra

$$\Gamma_{\nu\rho}^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} \frac{\partial x^{\gamma}}{\partial x'^{\rho}} \Gamma_{\beta\gamma}^{\alpha} + \dots$$

$$= J_{\alpha}^{\mu} \tilde{J}_{\nu}^{\beta} \tilde{J}_{\rho}^{\gamma} \Gamma_{\beta\gamma}^{\alpha} + \dots$$

Ex: show that

$$\nabla_{\mu} V^{\nu} = \frac{\partial V^{\nu}}{\partial x^{\mu}} + \Gamma_{\alpha\mu}^{\nu} V^{\alpha}$$

