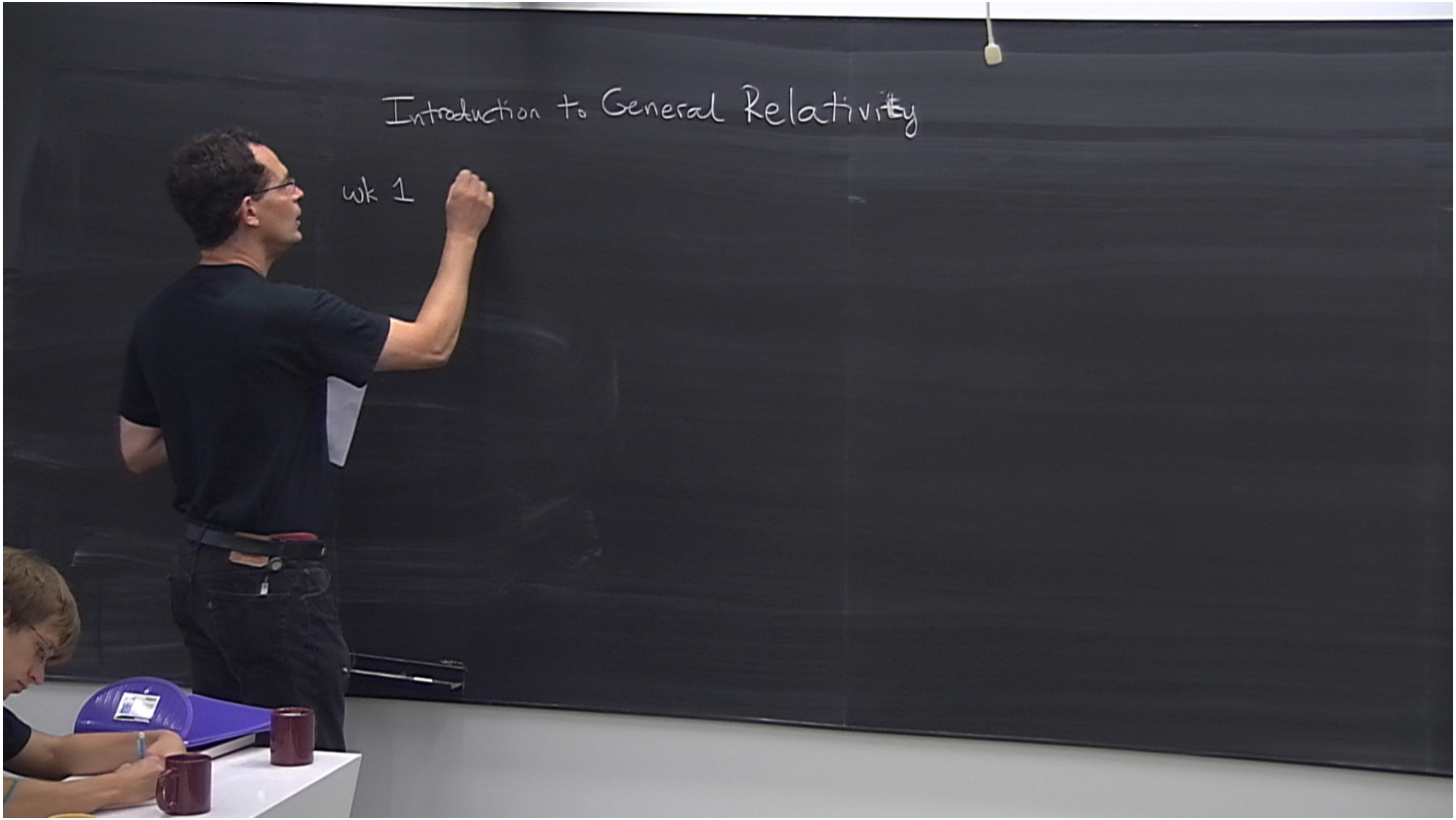
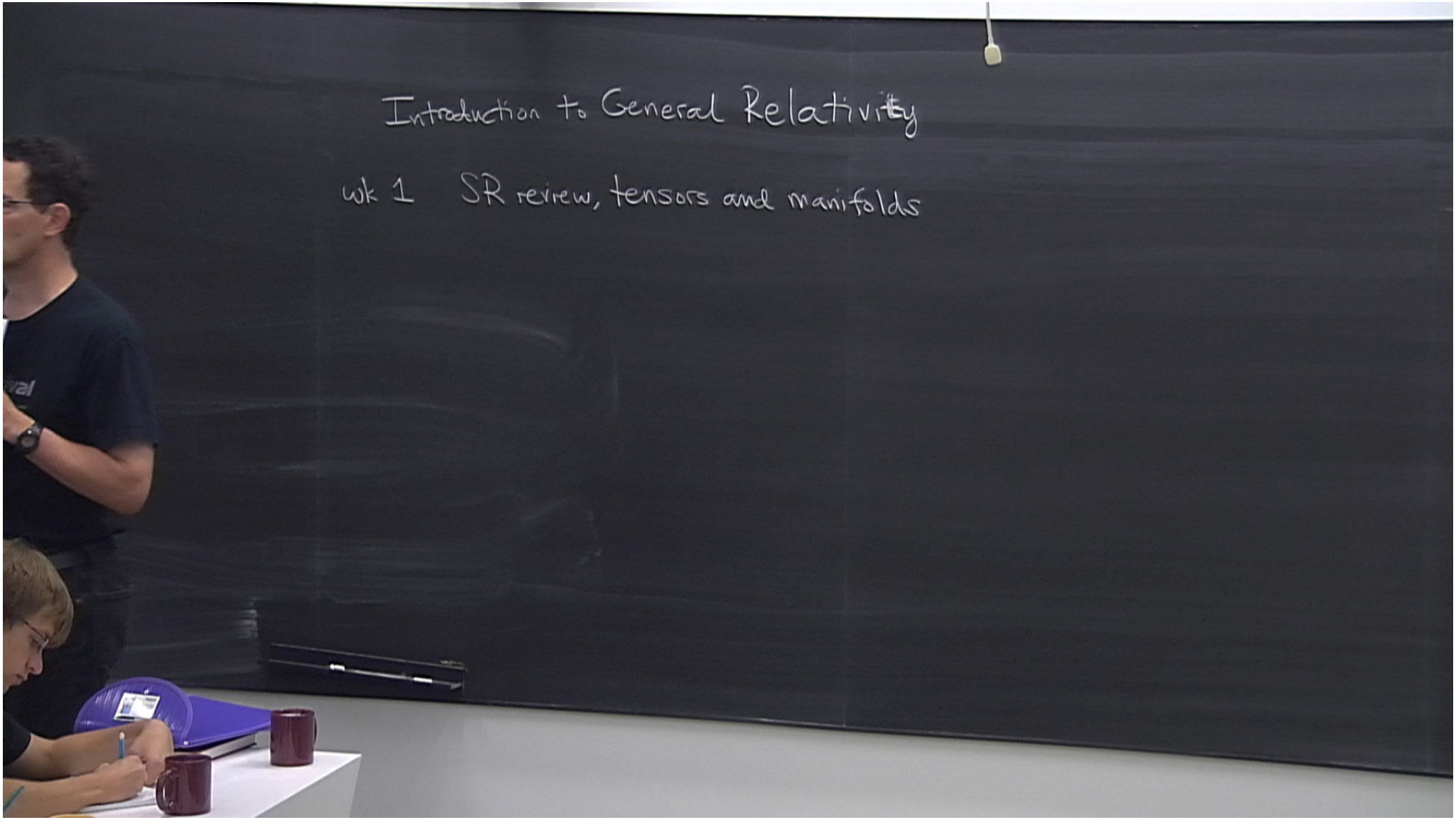
Title: Relativity - Lecture 1

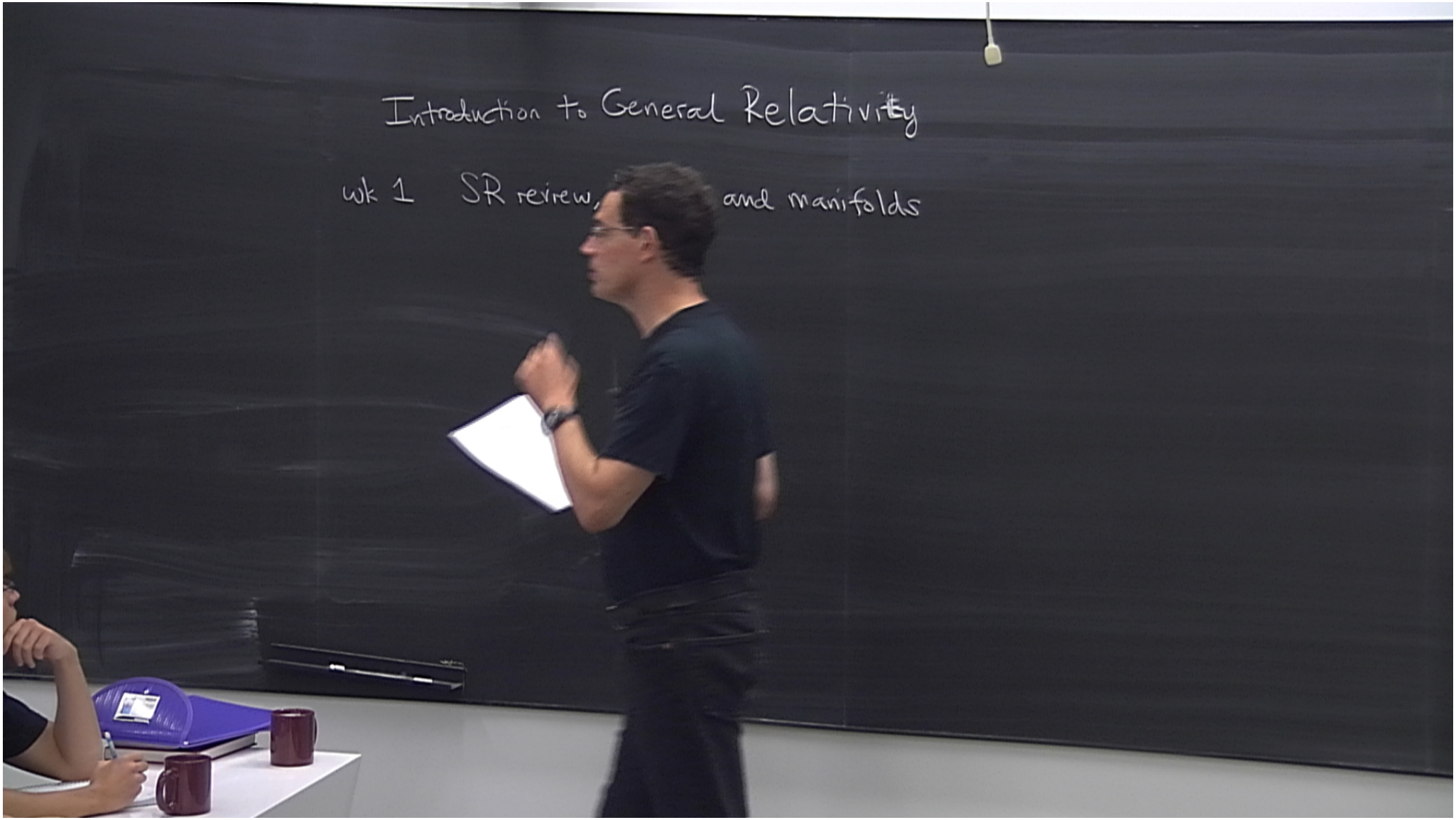
Date: Sep 12, 2011 10:30 AM

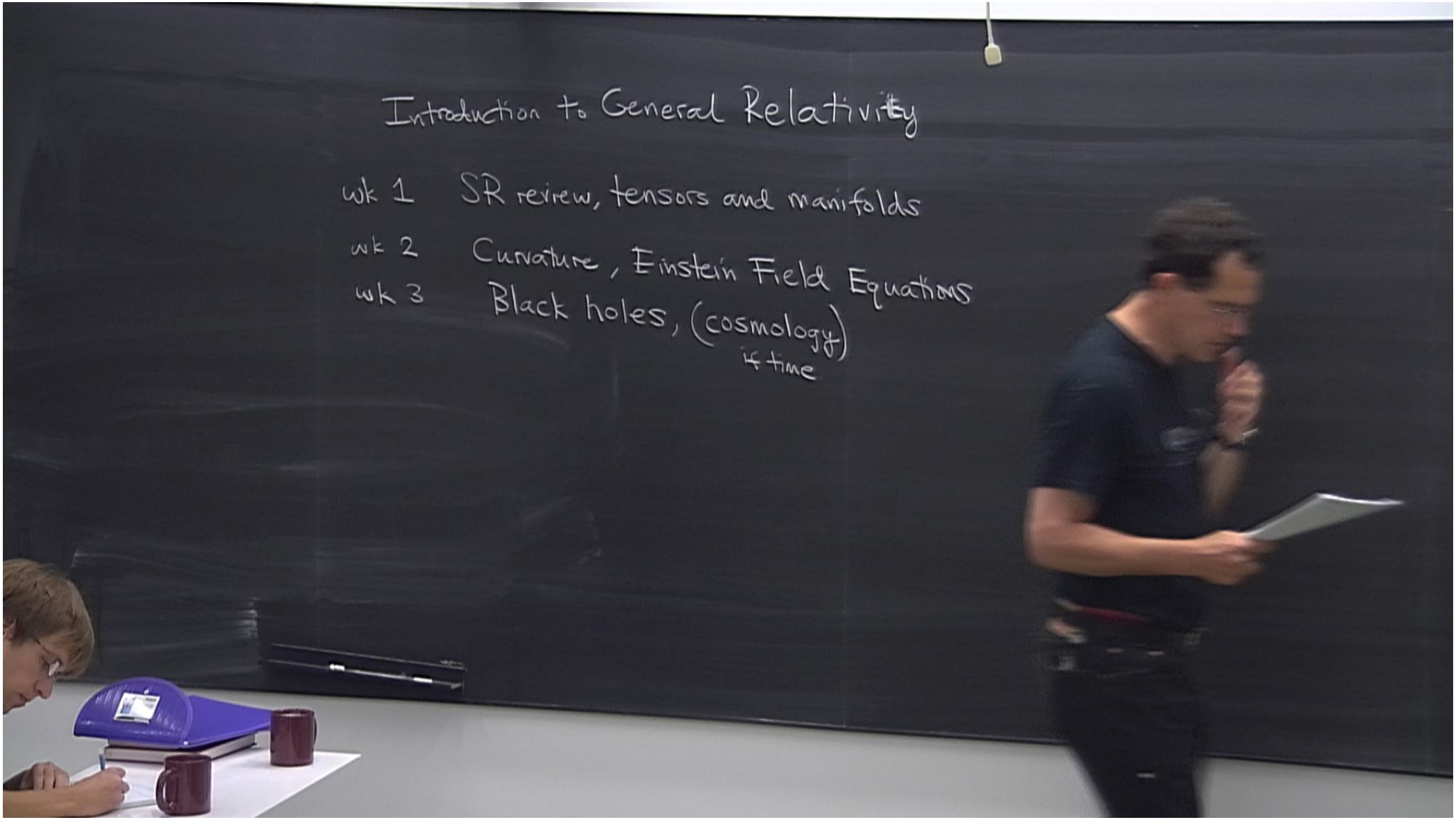
URL: http://pirsa.pi.local/11090024

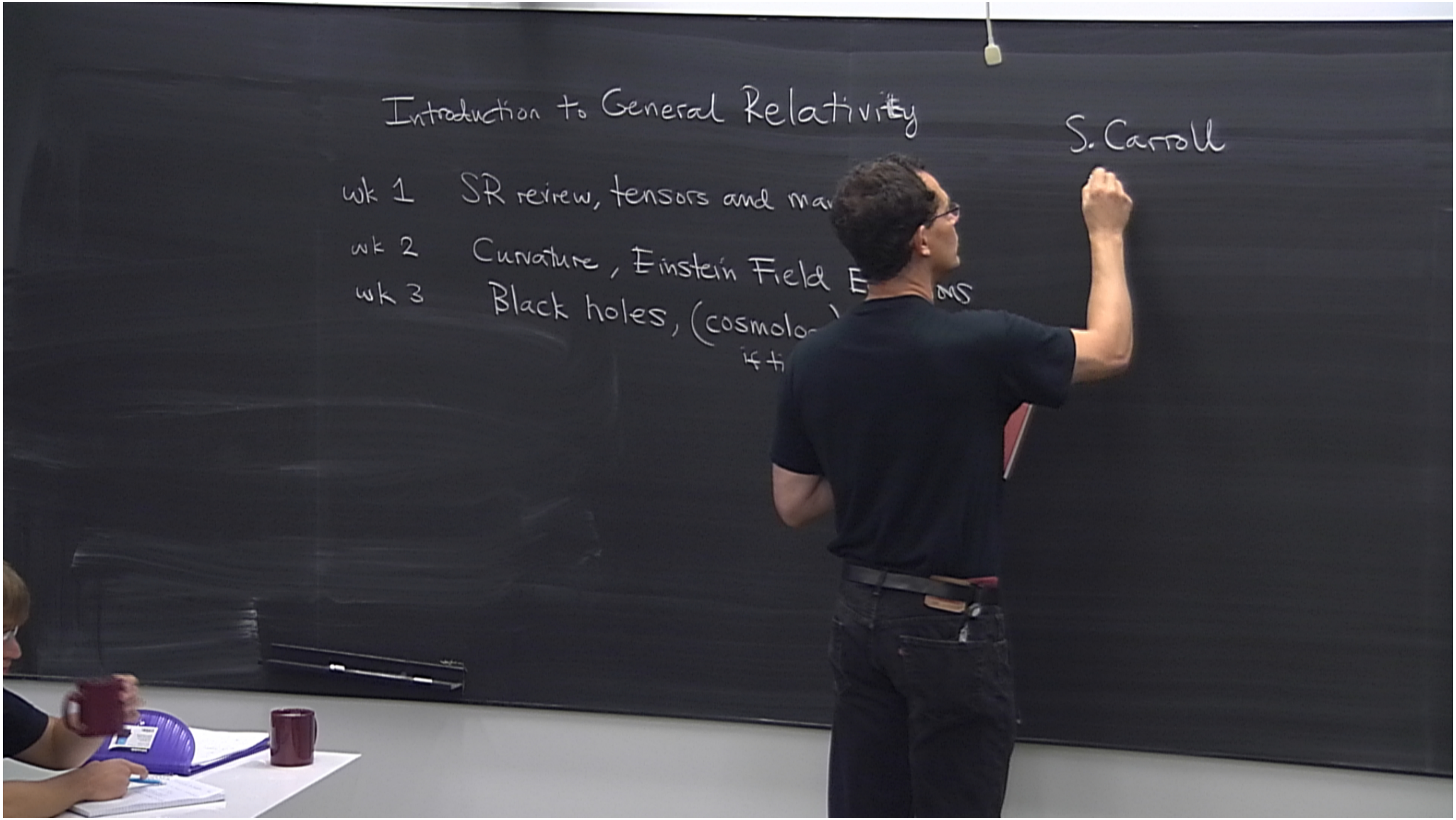
Abstract:

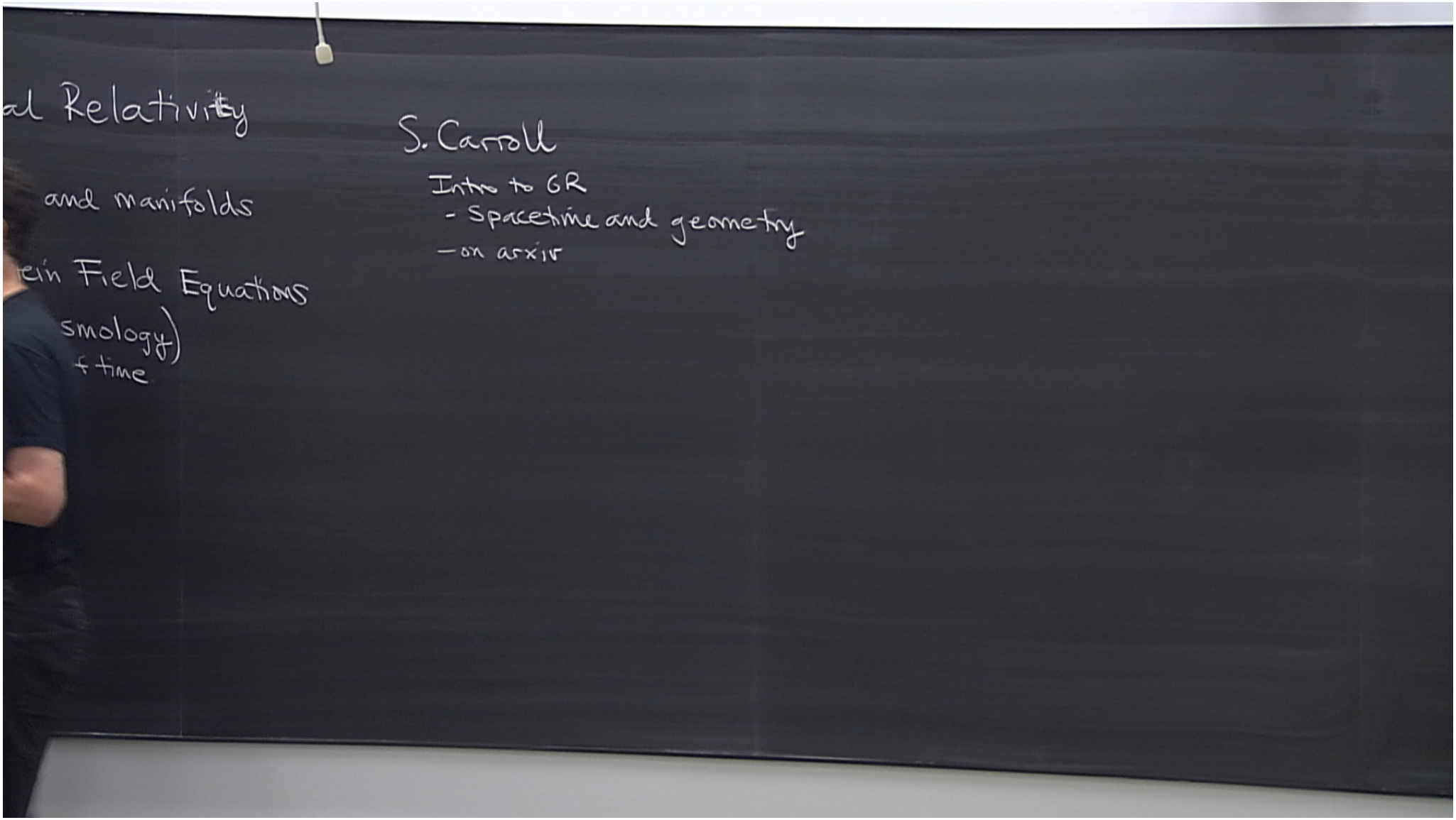

Introduction to General Relativity
wk 1


Introduction to General Relativity
wk 1 SR review, tensors and manifolds


Introduction to General Relativity
wk 1 SR review, and manifolds


Introduction to General Relativity
wk 1 SR review, tensors and manifolds
wk 2 Curvature, Einstein Field Equations
wk 3 Black holes, (cosmology)
if time


Introduction to General Relativity
S. Carroll
wk 1 SR review, tensors and ma
wk 2 Curvature, Einstein Field E
wk 3 Black holes, (cosmolo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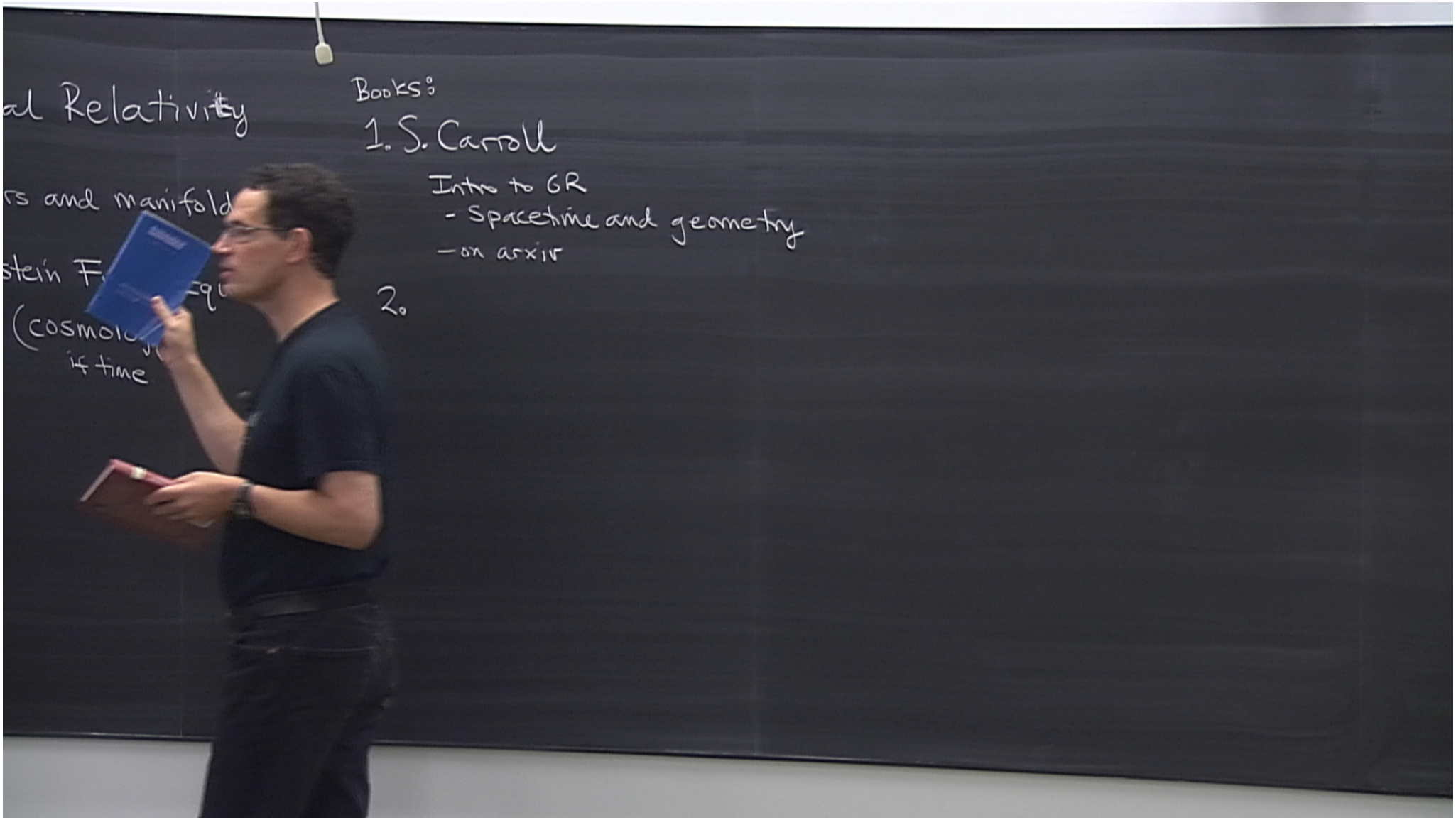
S. Carroll
Intro to GR
- Spacetime and geometry
- on arxiv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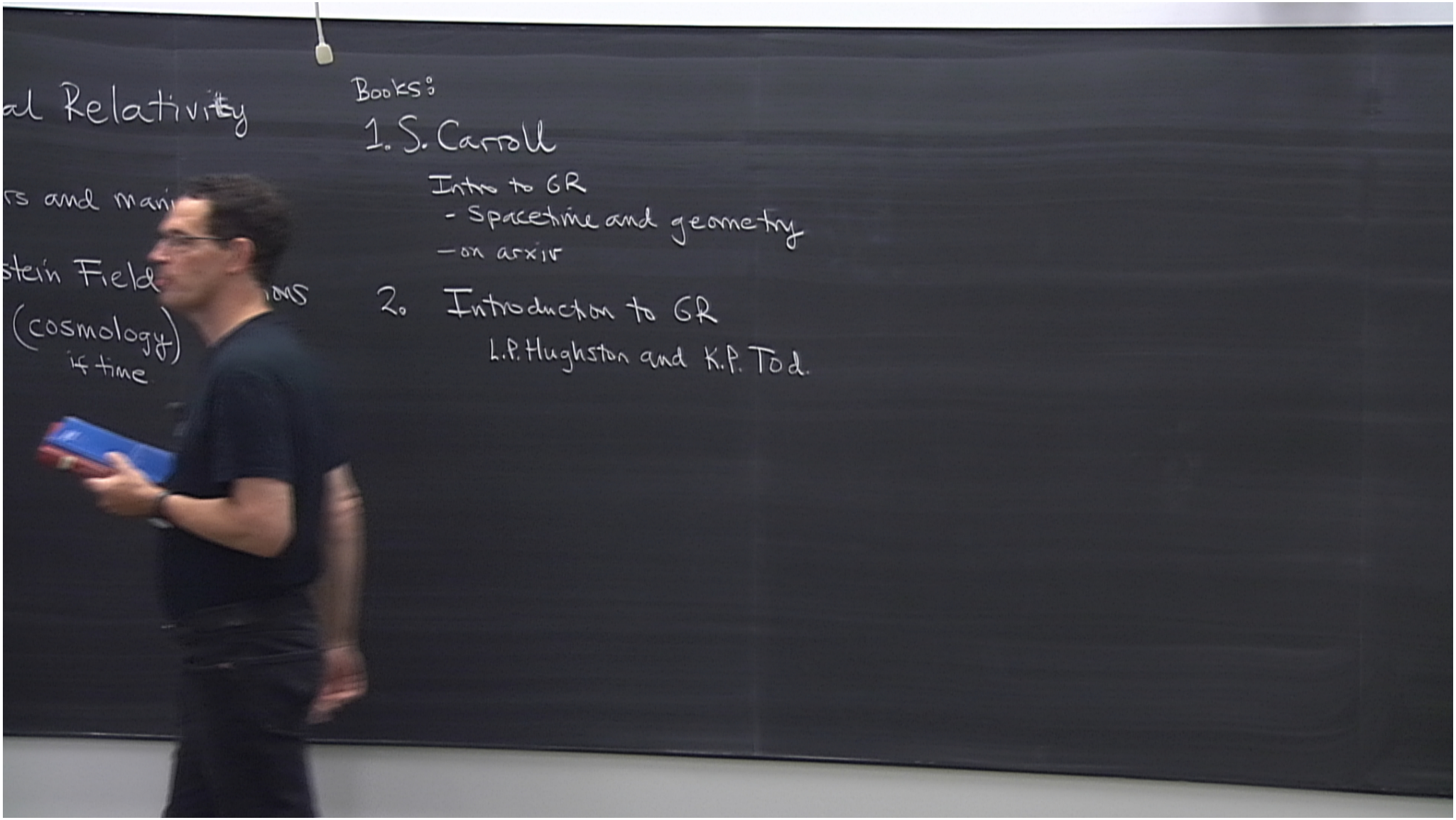
Books:
1. S. Carroll
Intro to GR
- Spacetime and geometry
- on arxiv
2.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(cosmology)
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Books:
1. S. Carroll
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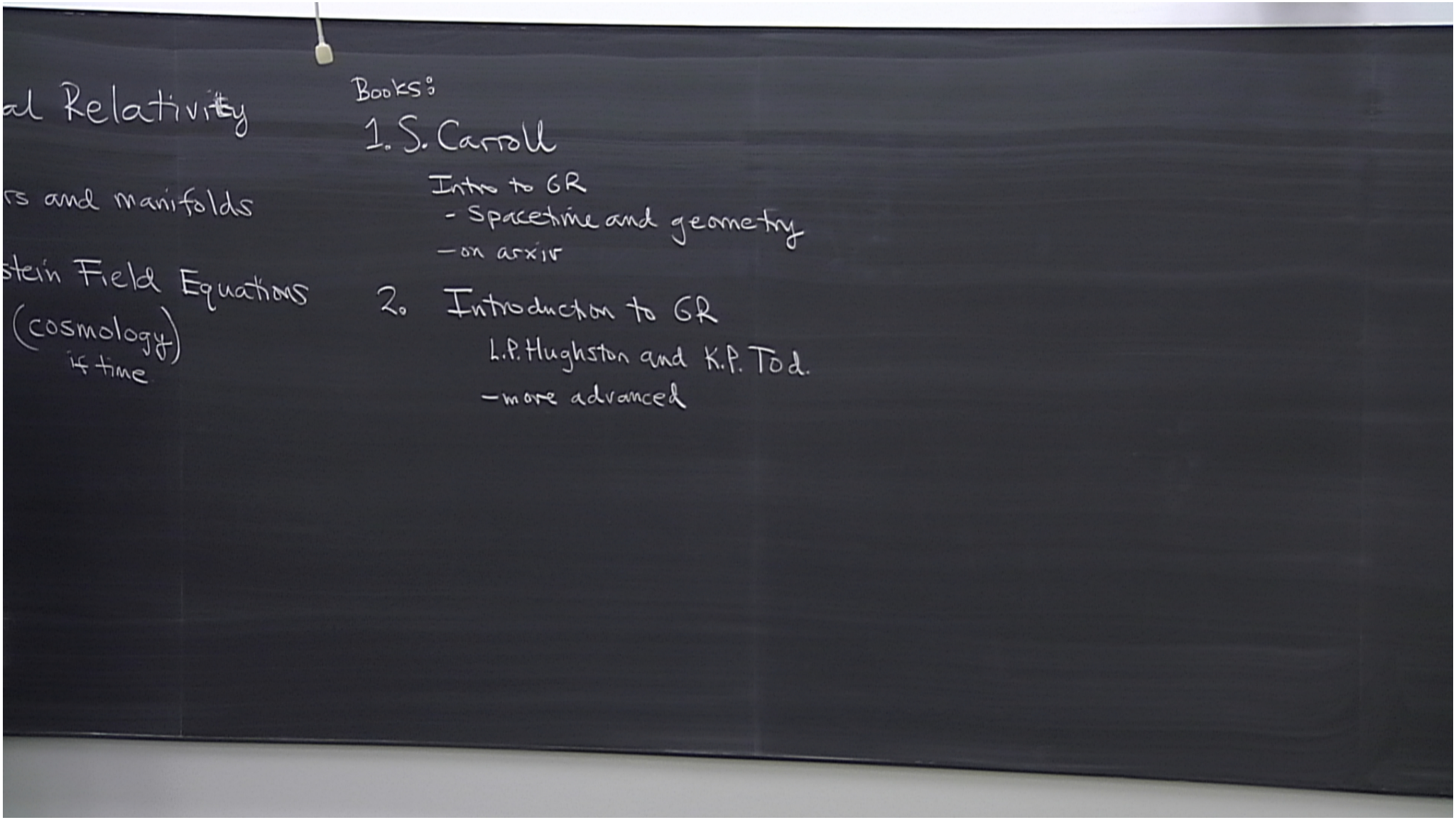
- Spacetime and geometry
- on arxiv
2. Introduction to GR
L.P. Hughston and K.P. Tod.


al Relativity
rs and manifolds
stein Field Equations
(cosmology)
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Books:
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Intro to GR
- Spacetime and geometry
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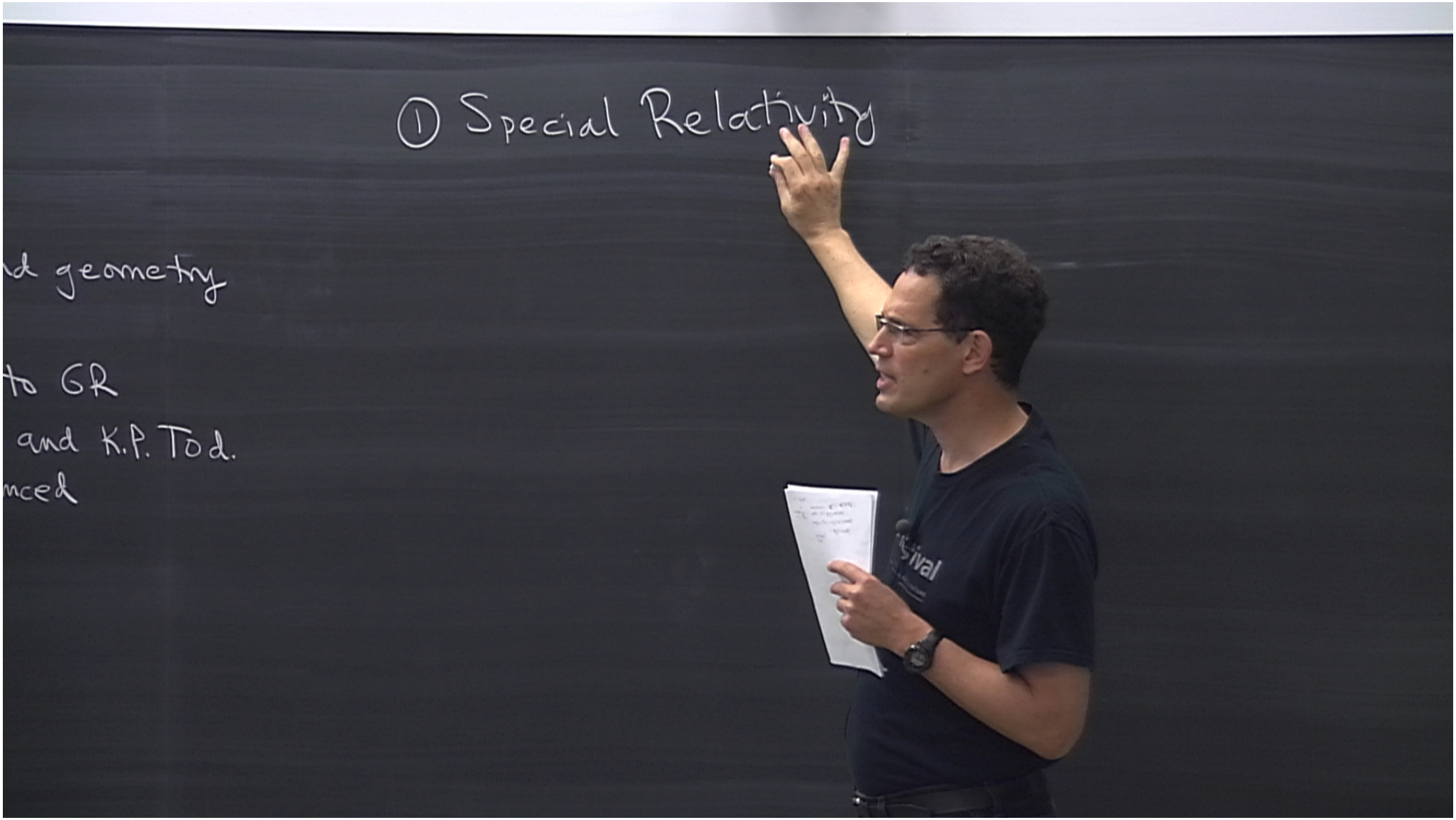
2. Introduction to GR
L.P. Hughston and K.P. Tod.
- more advanced


① Special Relativity
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to GR
and K.P. Tod.
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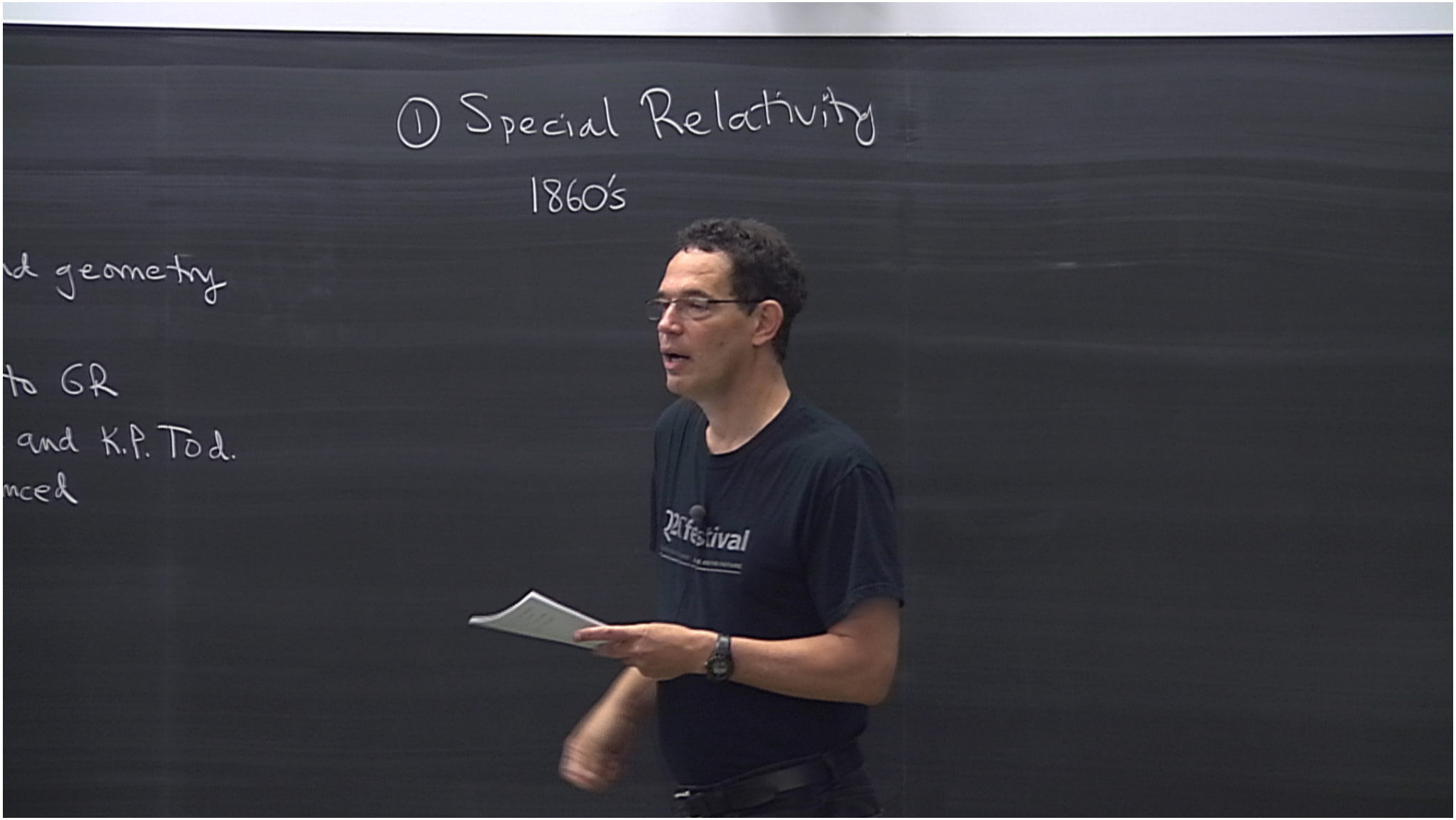
① Special Relativity
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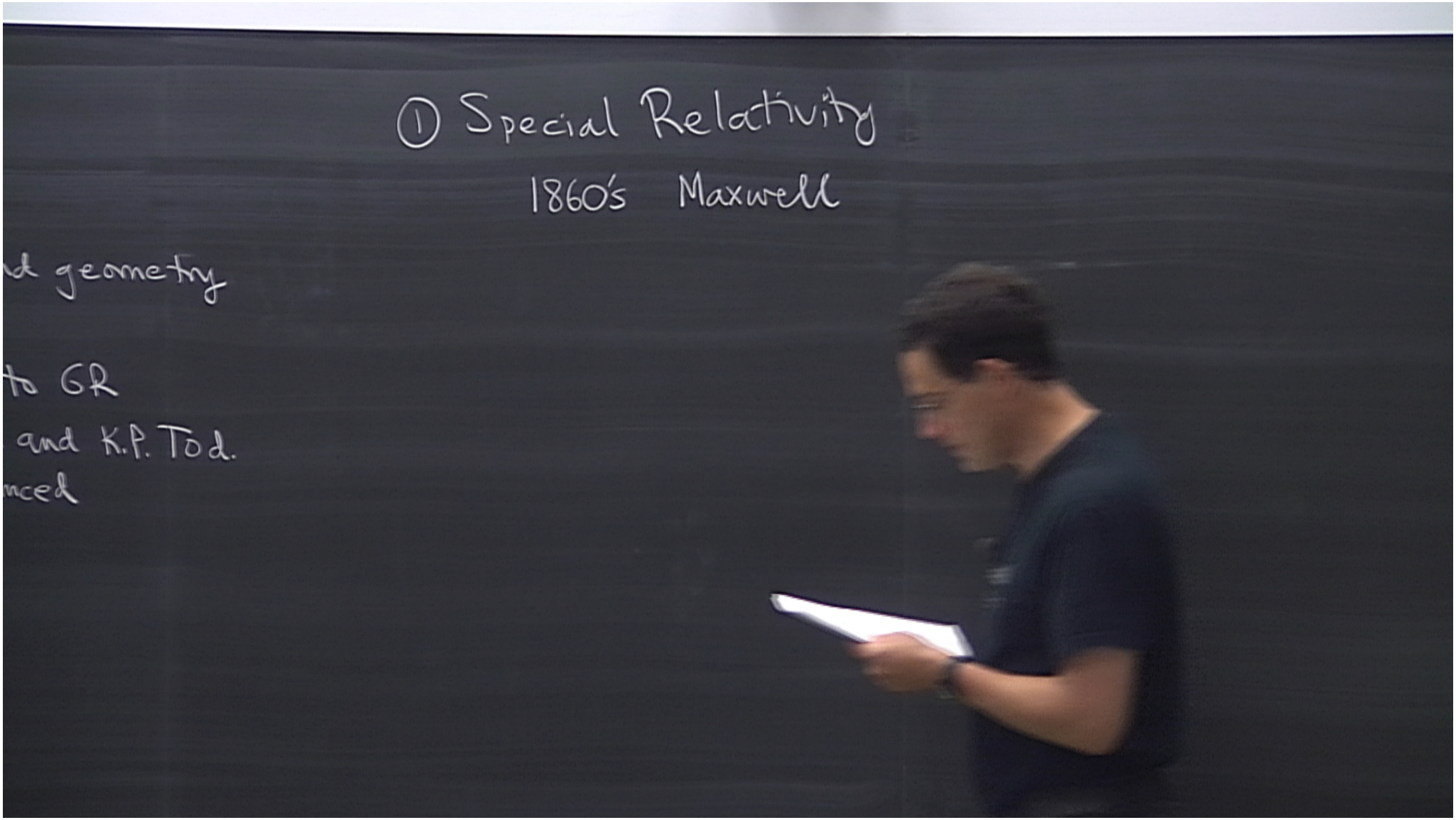
① Special Relativity
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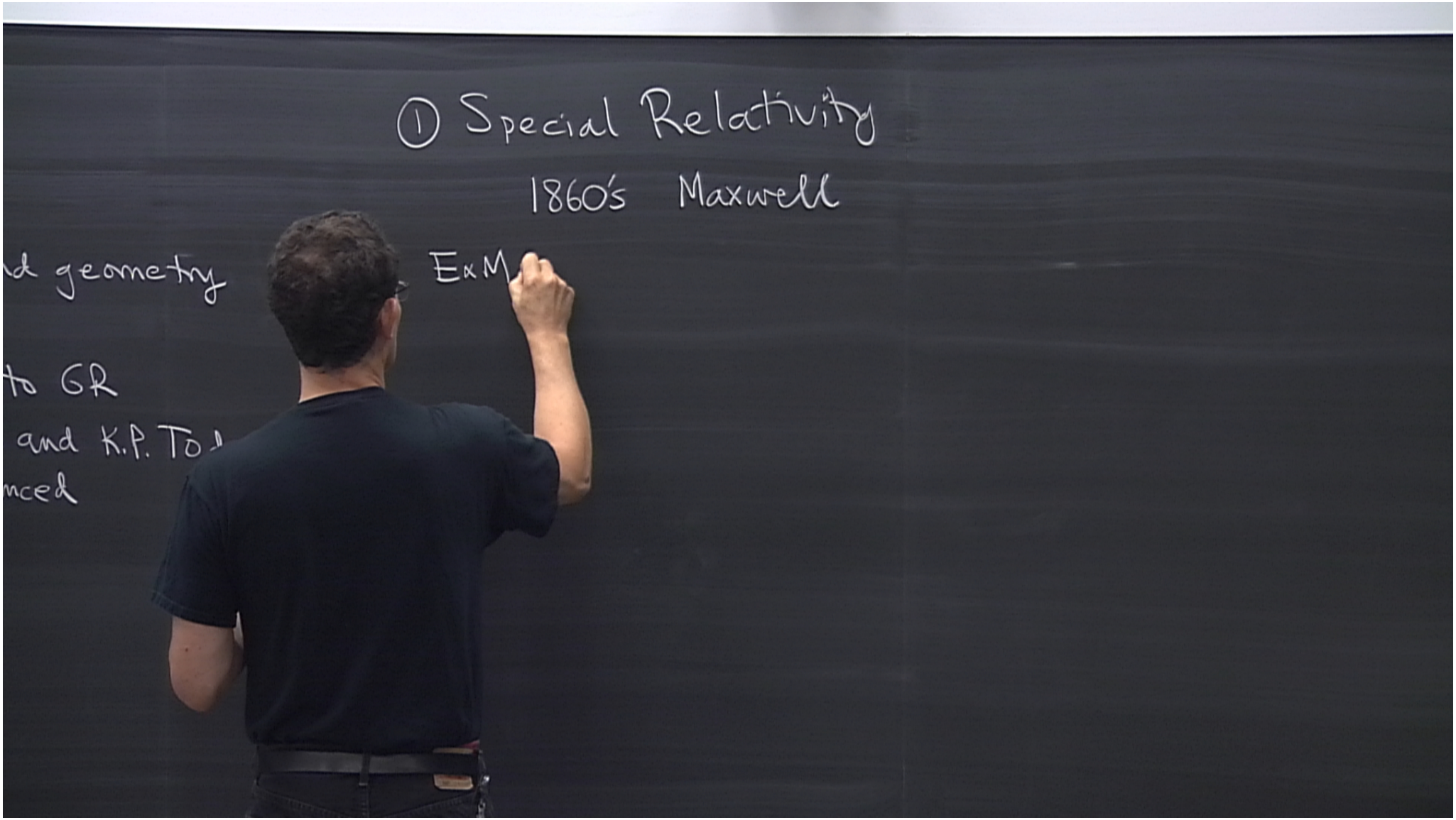
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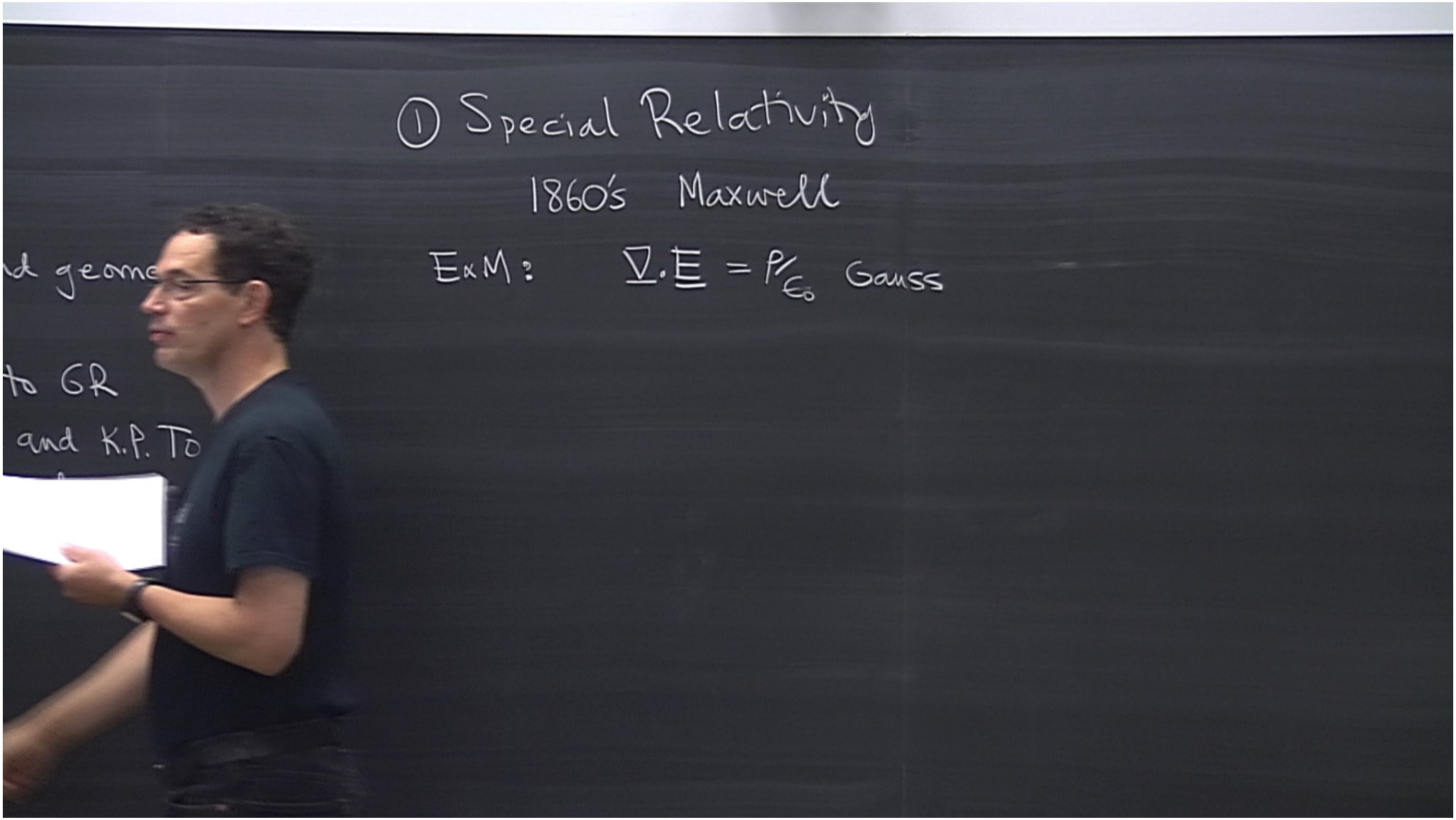
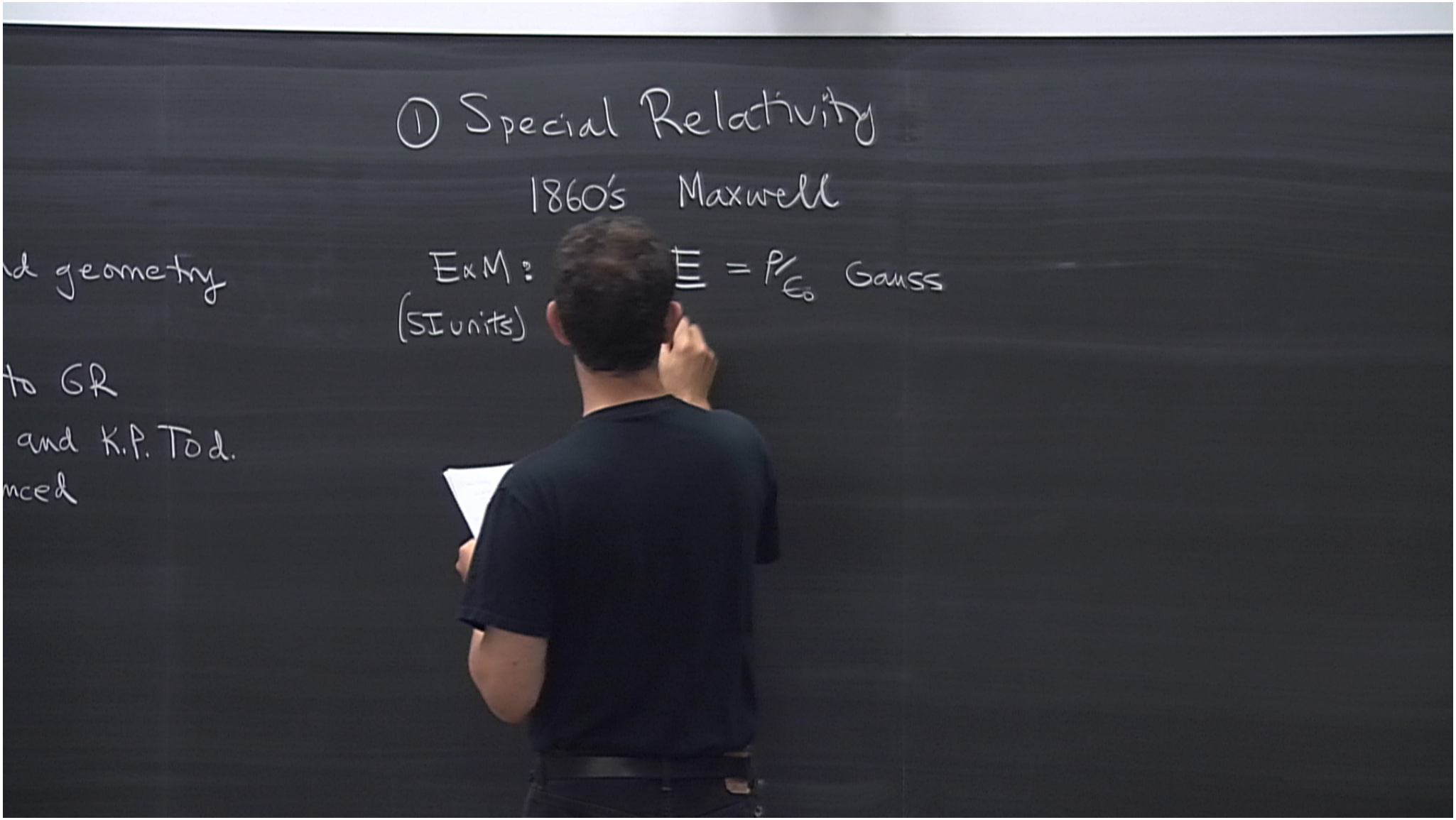

① Special Relativity
1860's Maxwell
E&M: ∇.E = ρ/ε₀ Gauss
to GR
and K.P. To


① Special Relativity
1860's Maxwell
E&M:
(SI units)
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to GR
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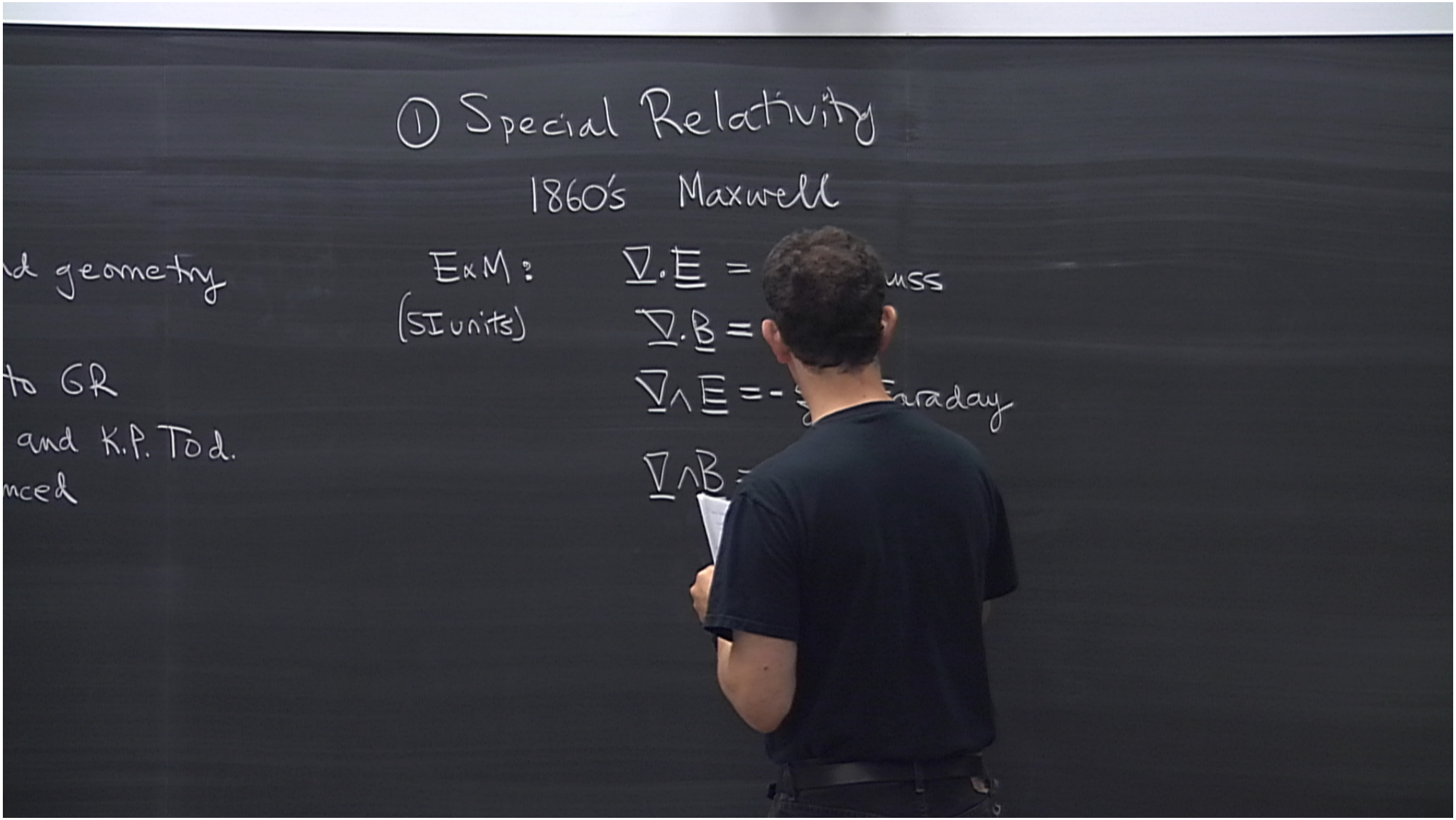
① Special Relativity
1860's Maxwell
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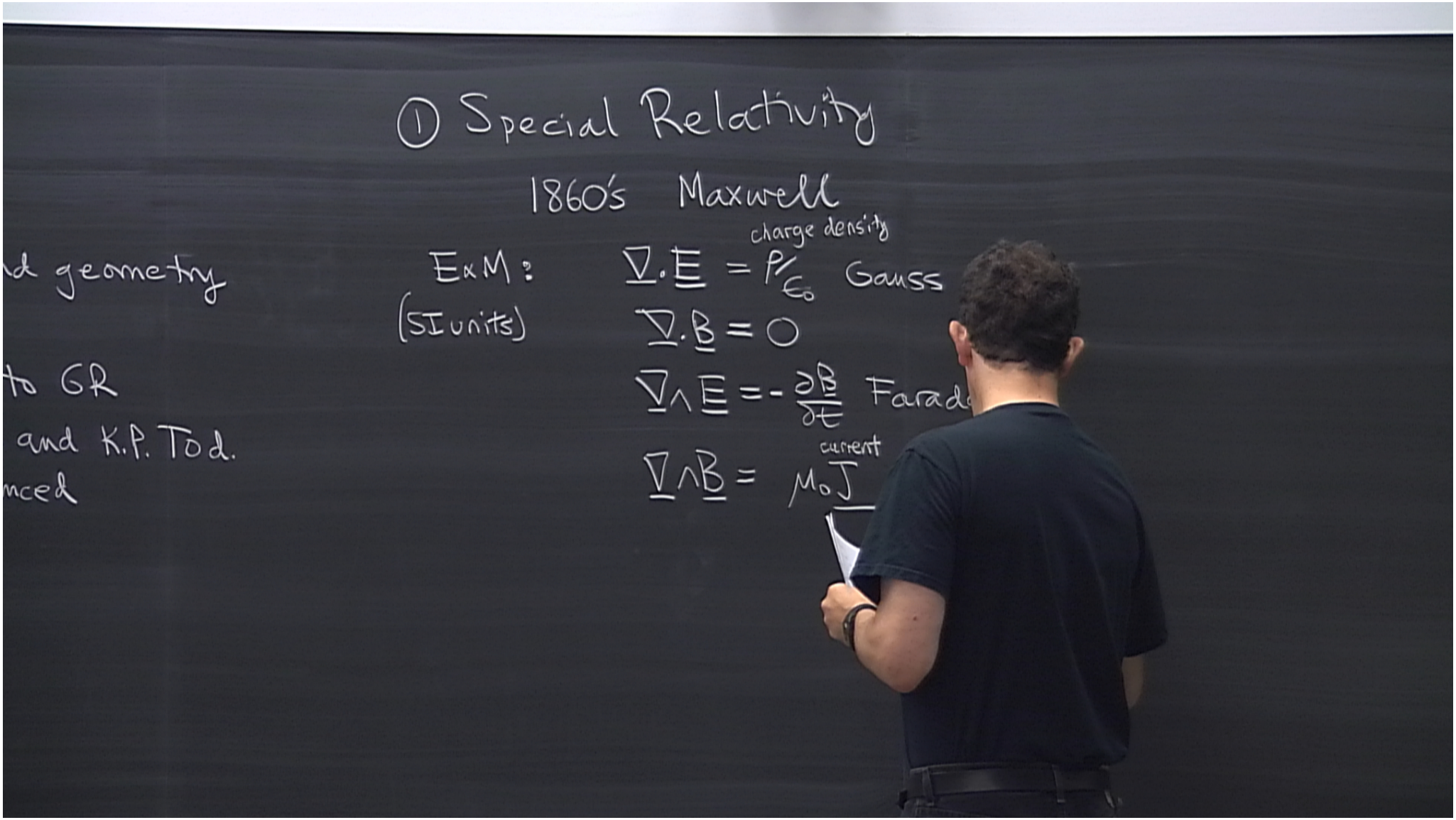
① Special Relativity
1860's Maxwell
charge density
E&M: $\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss
(SI units) $\nabla \cdot \underline{B} = 0$
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$
current
$\nabla \wedge \underline{B} = \mu_0 \underline{J}$
d geometry
to GR
and K.P. Tod.
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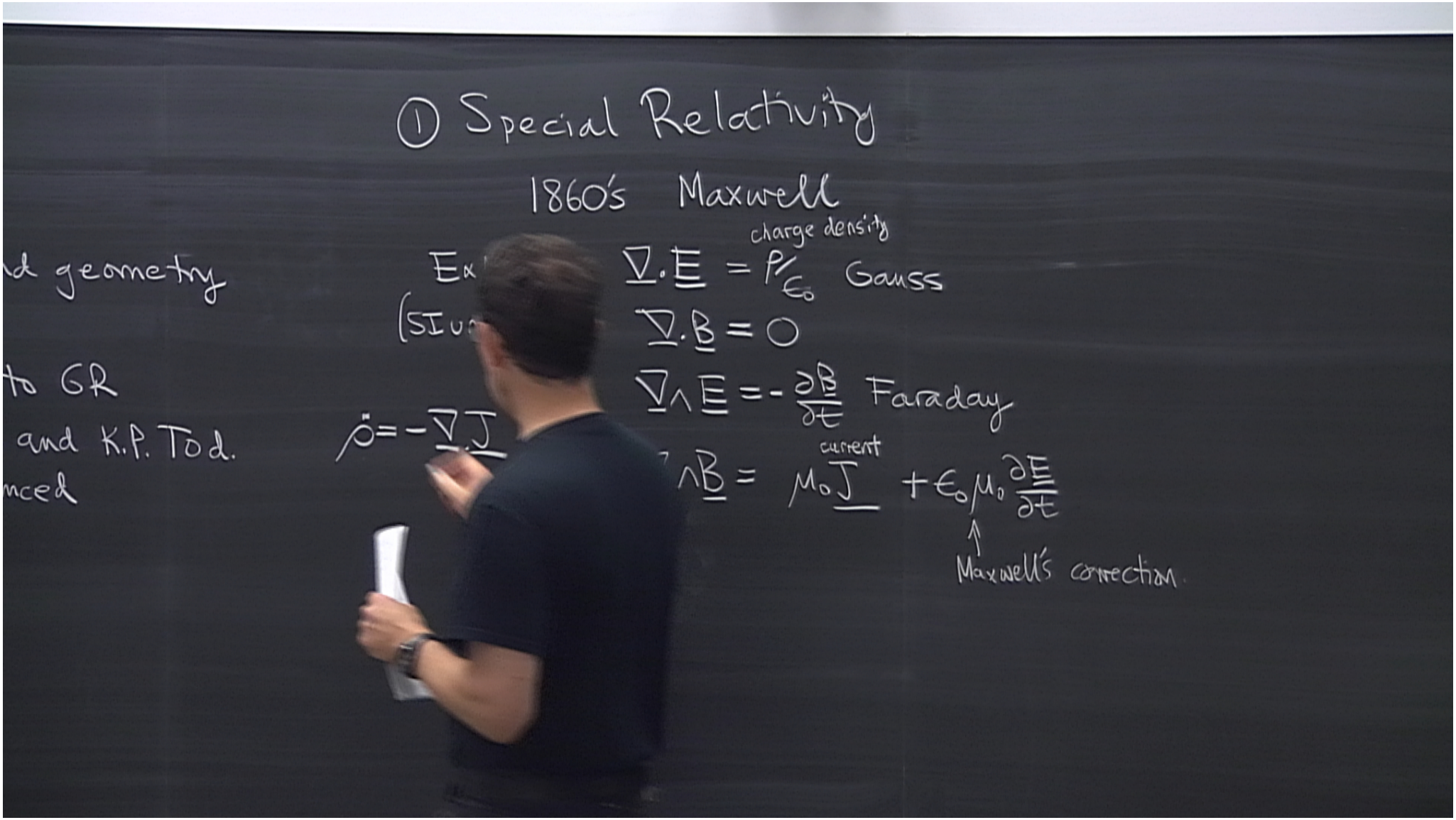

① Special Relativity
1860's Maxwell
charge density
$\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss
$\nabla \cdot \underline{B} = 0$
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday
current
$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$
Maxwell's correction
$\dot{\rho} = -\nabla \cdot \underline{J}$
d geometry
to GR
and K.P. Tod.
nced

① Special Relativity

1860's Maxwell

d geometry

to GR

and K.P. Tod.

nced

E×M: (SI units)

$\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss (charge density)

$\nabla \cdot \underline{B} = 0$

$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday

$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$ ($\underline{J}$: current)

↑ Maxwell's correction

charge conservation

$\dot{\rho} = -\nabla \cdot \underline{J}$

① Special Relativity

1860's Maxwell

d geometry

to GR

and K.P. Tod.

nced

E&M: (SI units)

charge density

$\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss

$\nabla \cdot \underline{B} = 0$

charge conservation

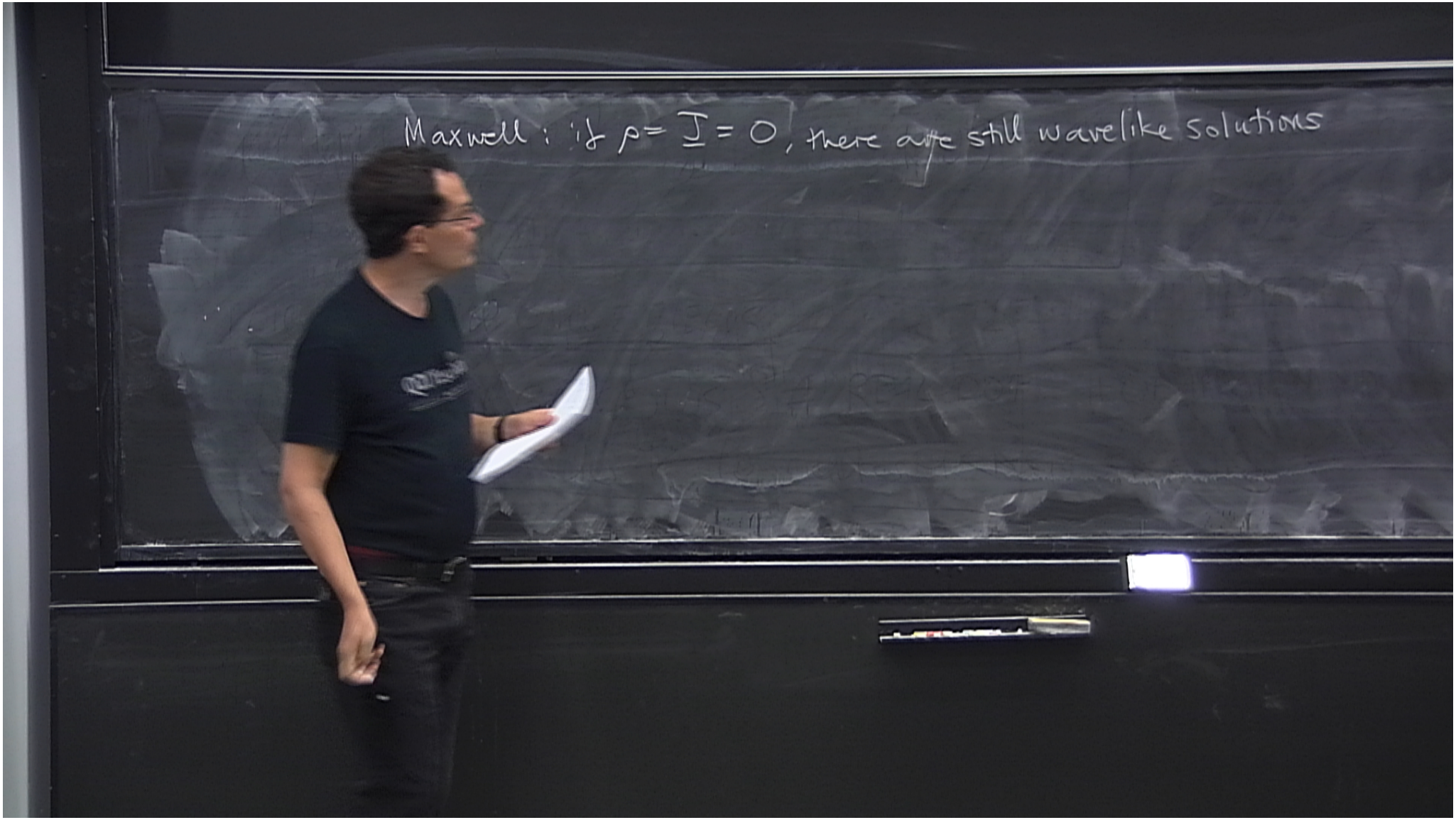
$\dot{\rho} = -\nabla \cdot \underline{J}$

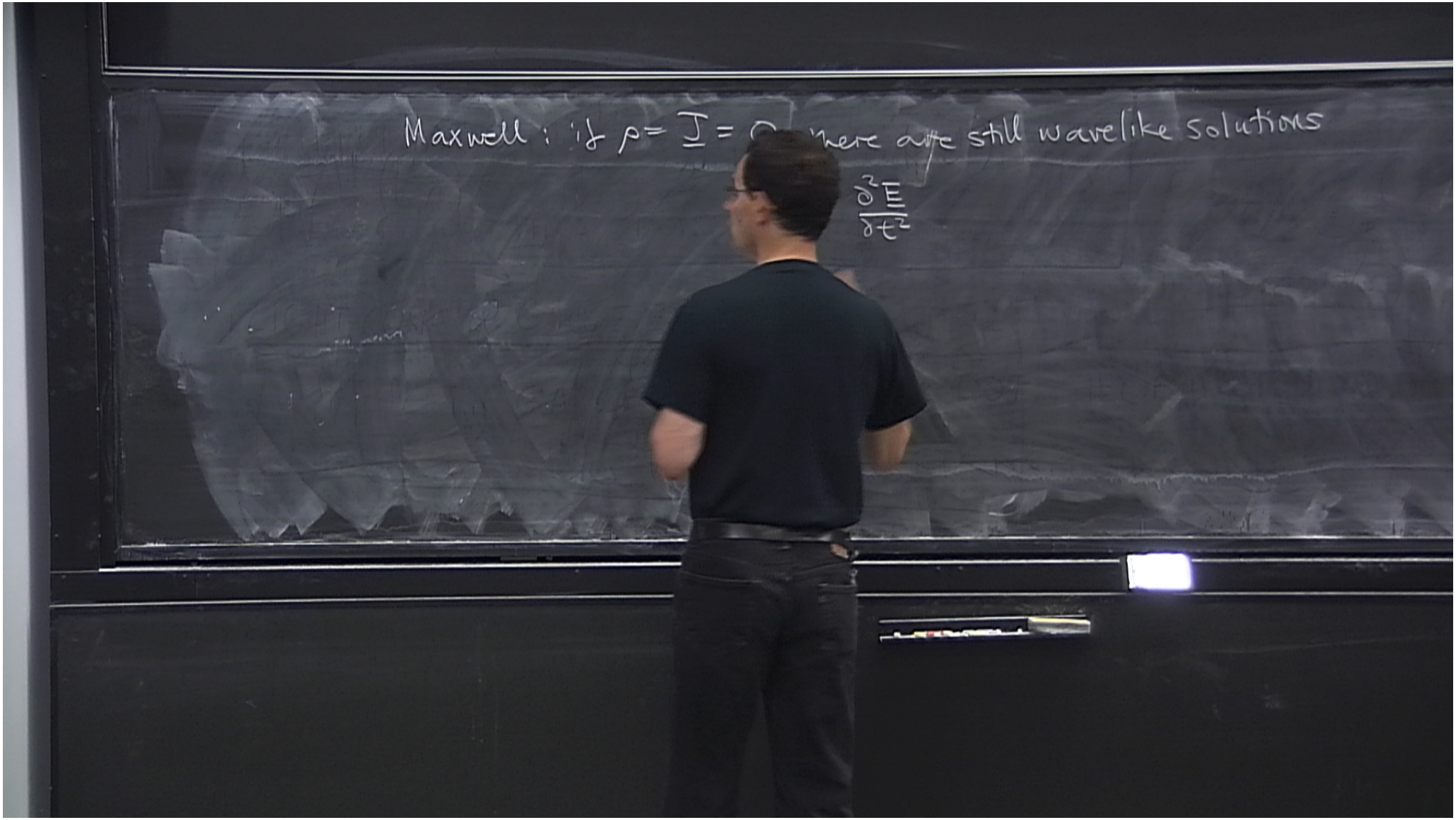
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday

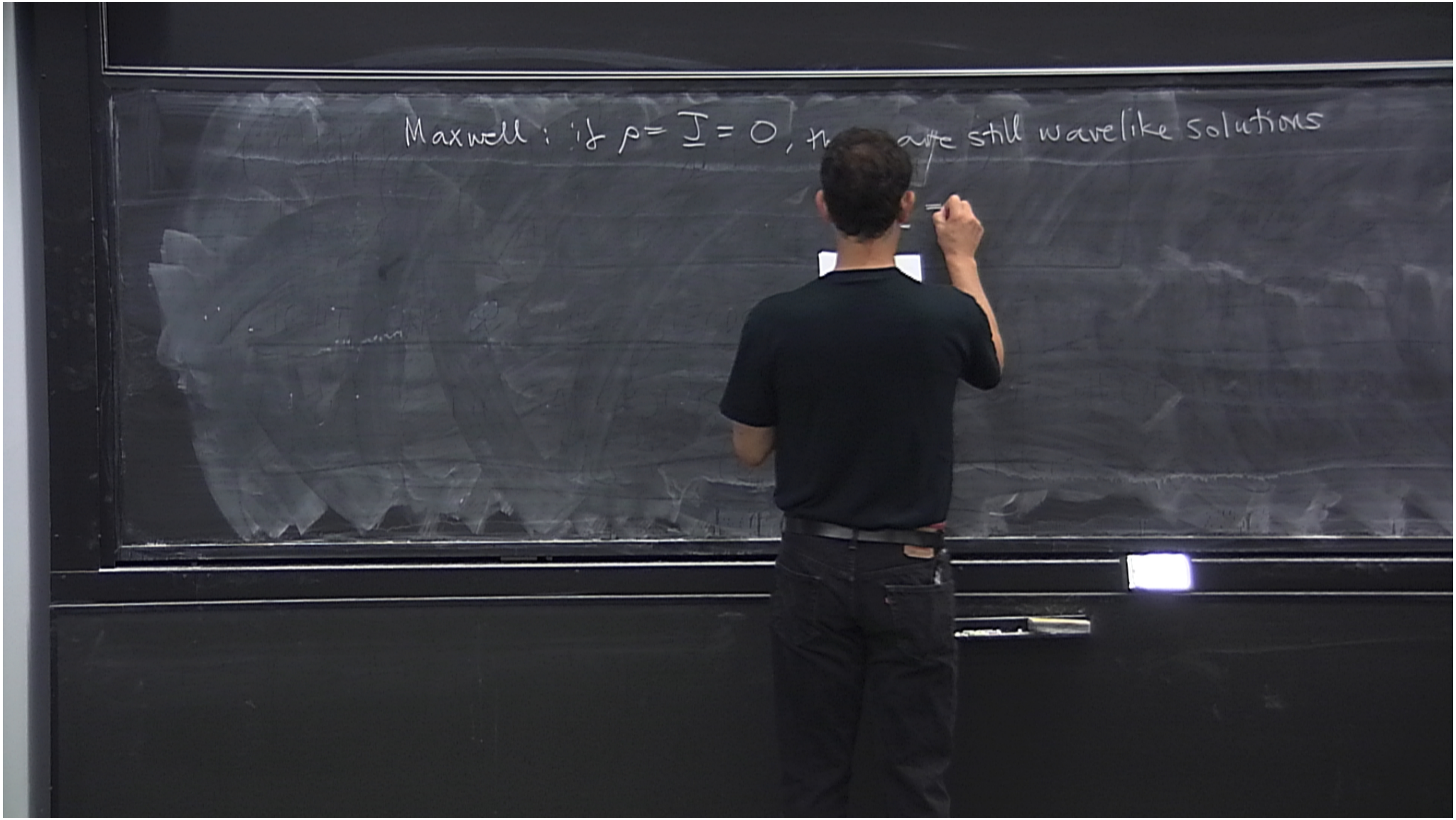
current

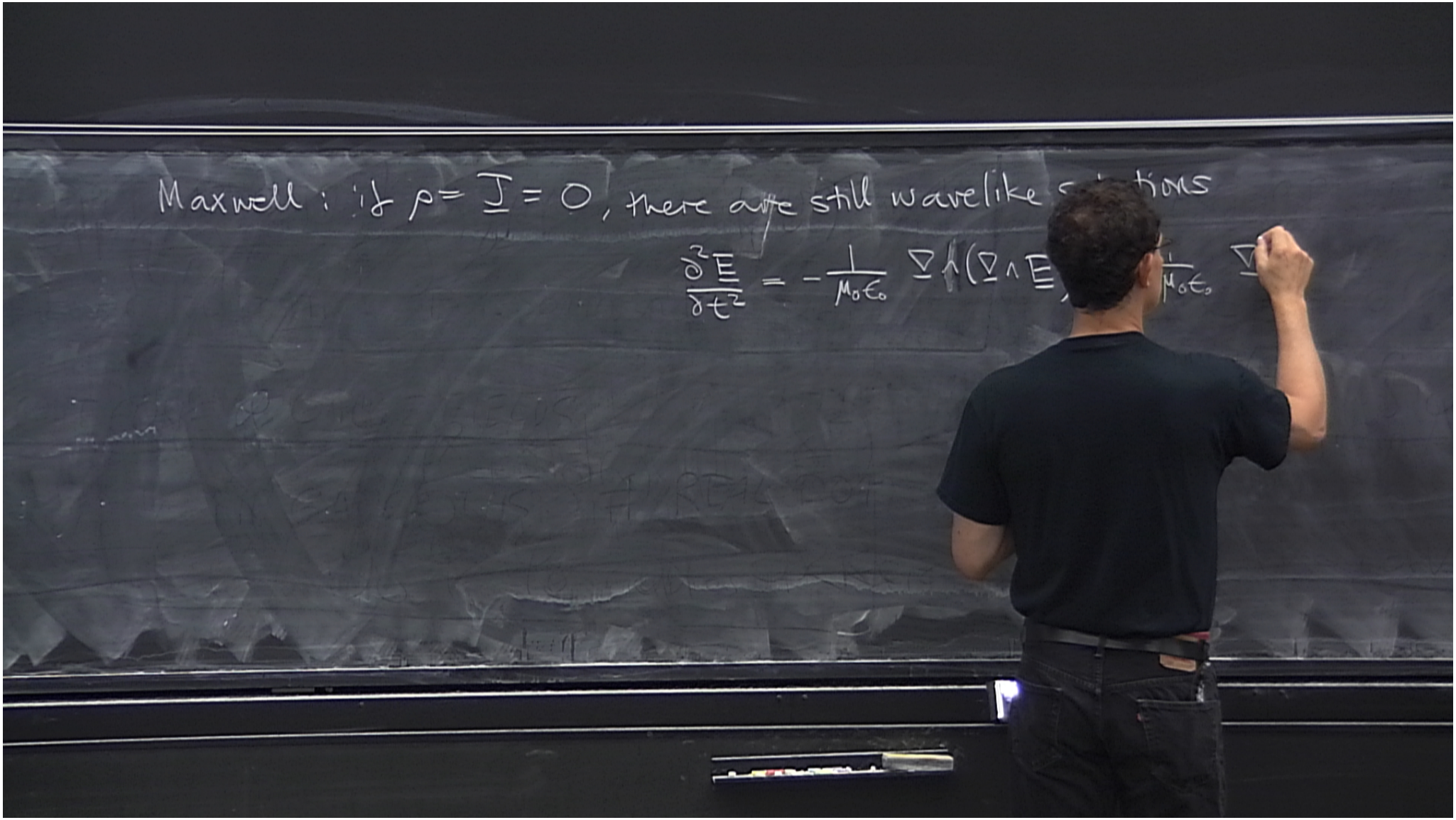
$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$ Ampere + Maxwell

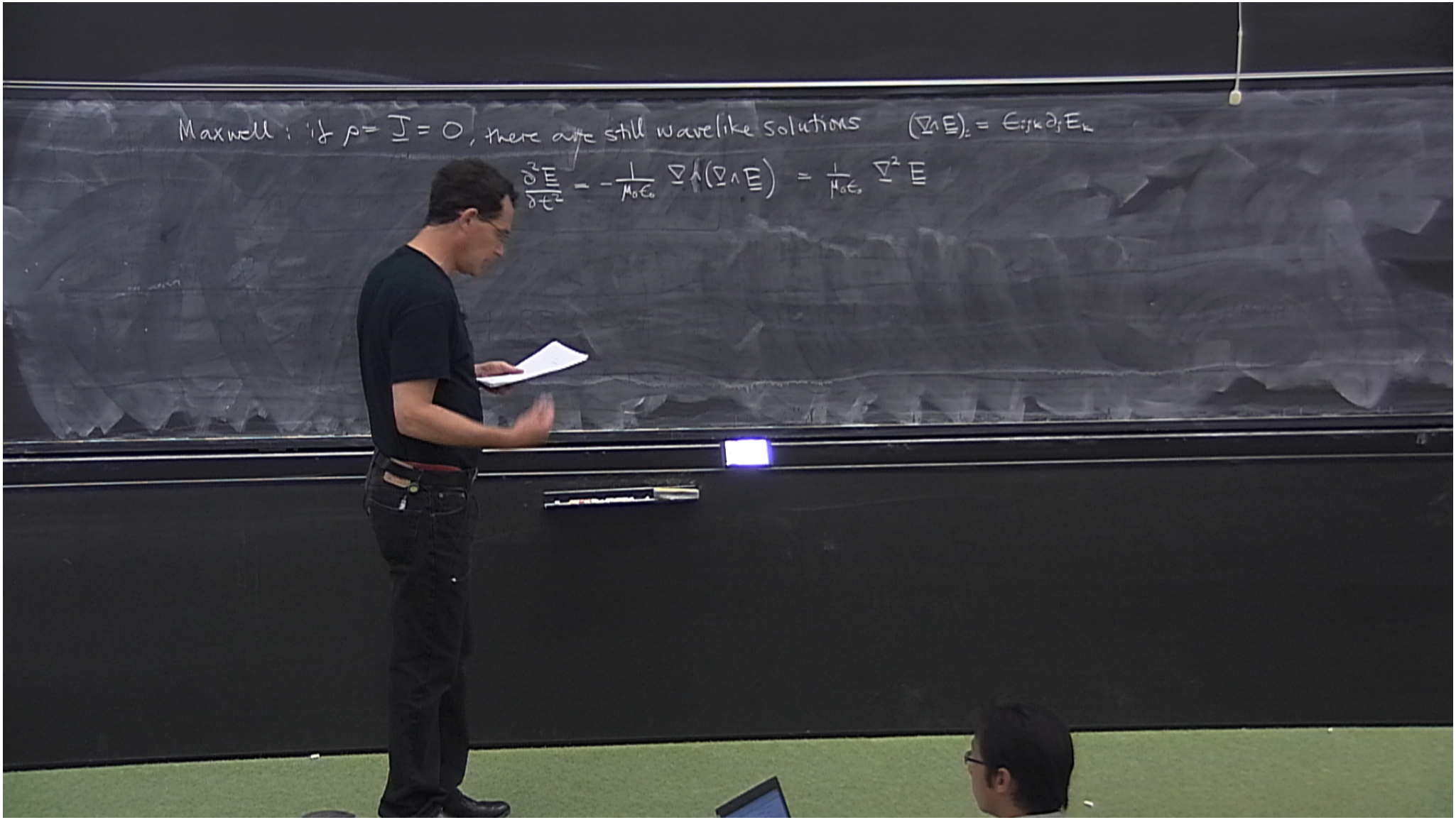
↑ Maxwell's correction

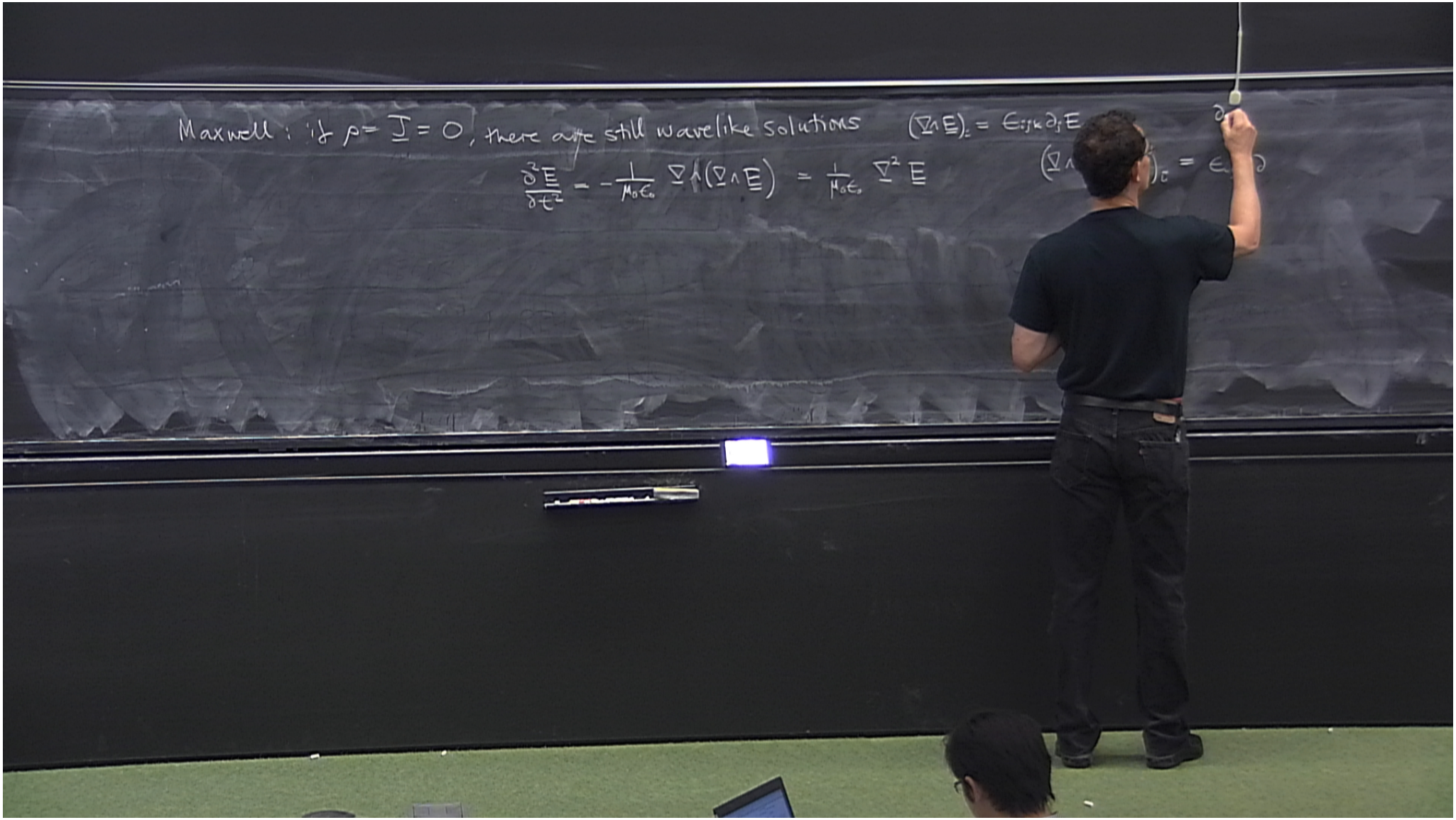

Maxwell: if ρ = J = 0, there are still wavelike solutions

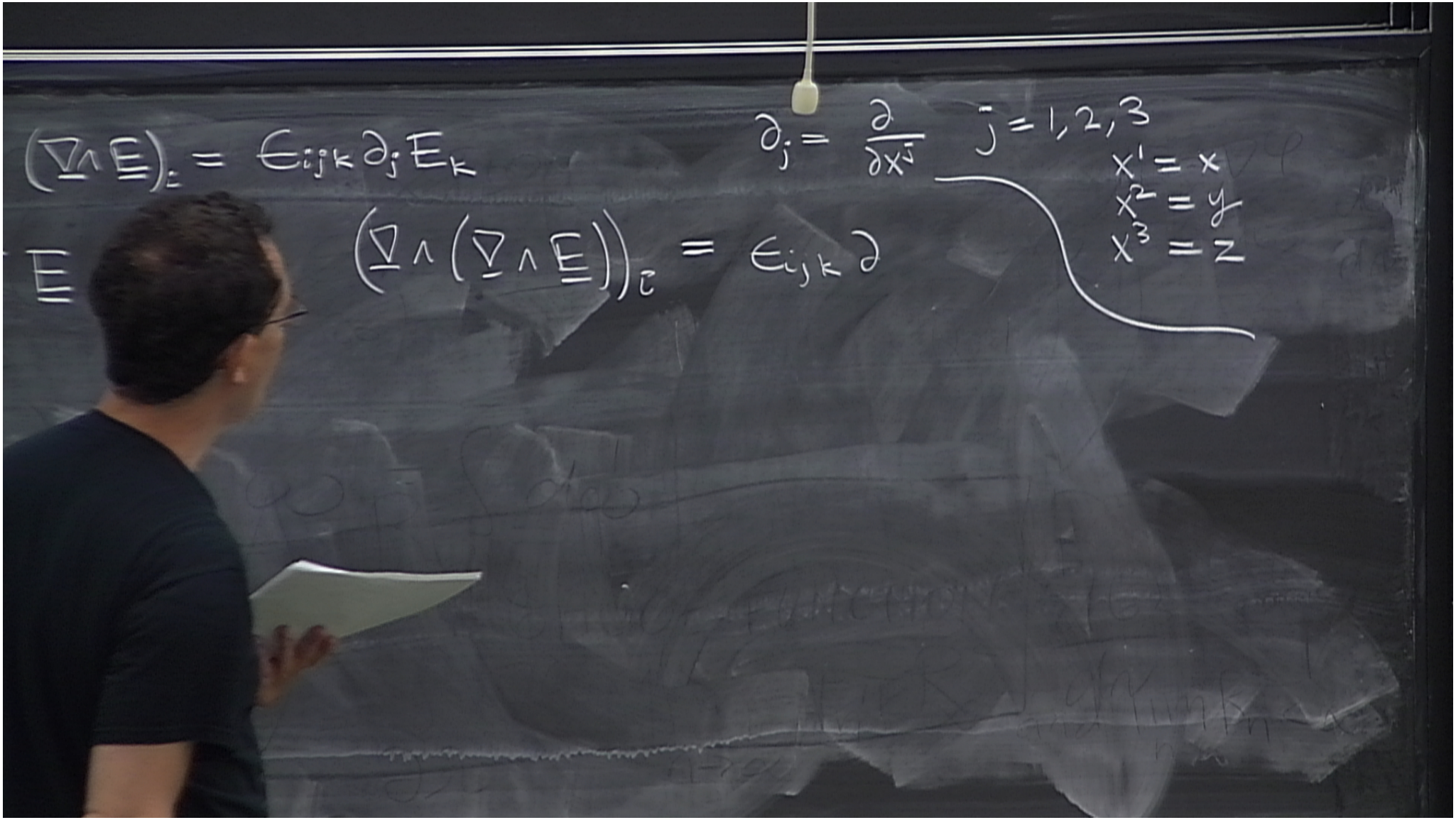

Maxwell: if ρ = J = 0 there are still wavelike solutions
∂²E/∂t²

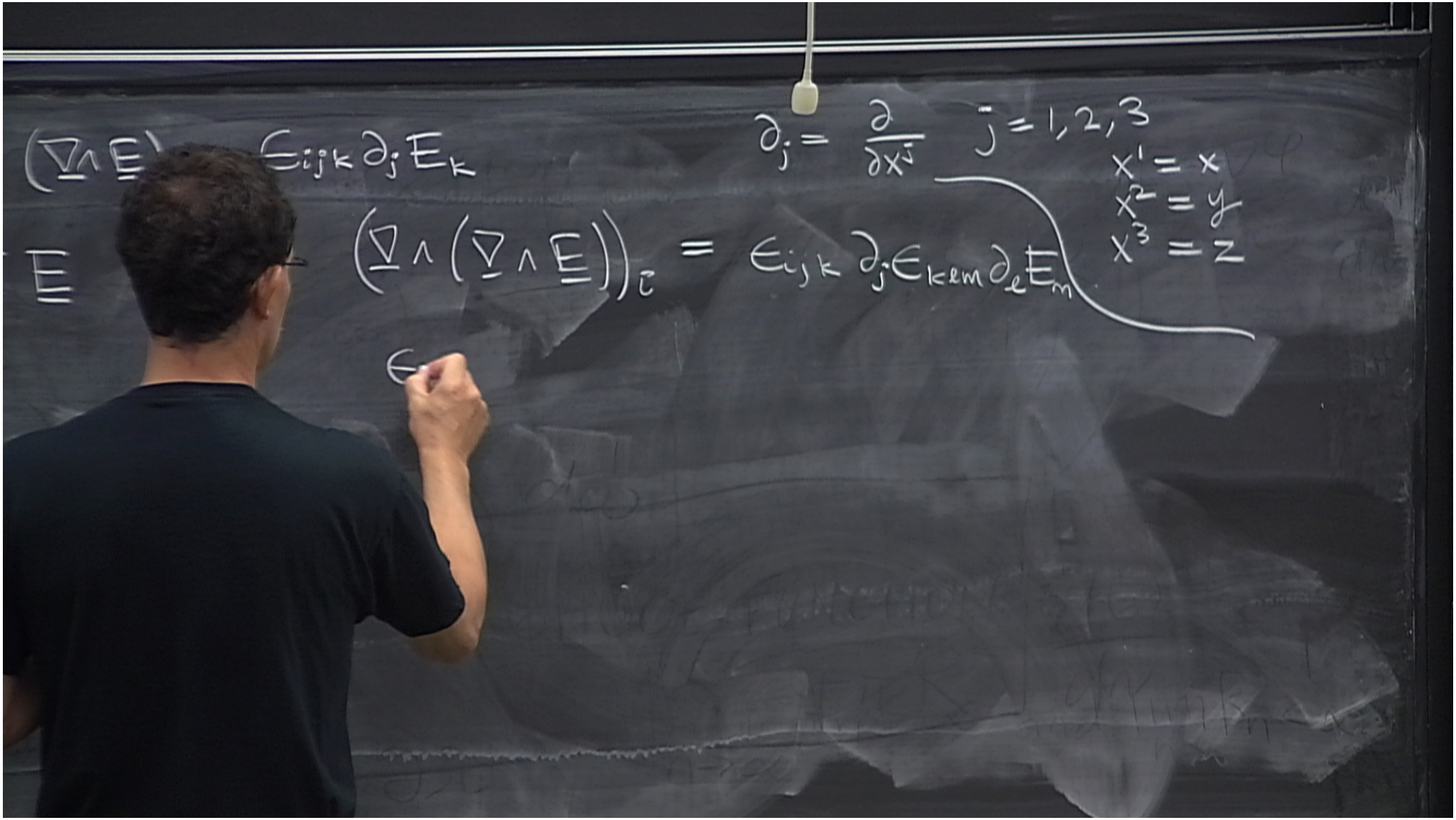

Maxwell: if ρ = J = 0, th
still wavelike solutions

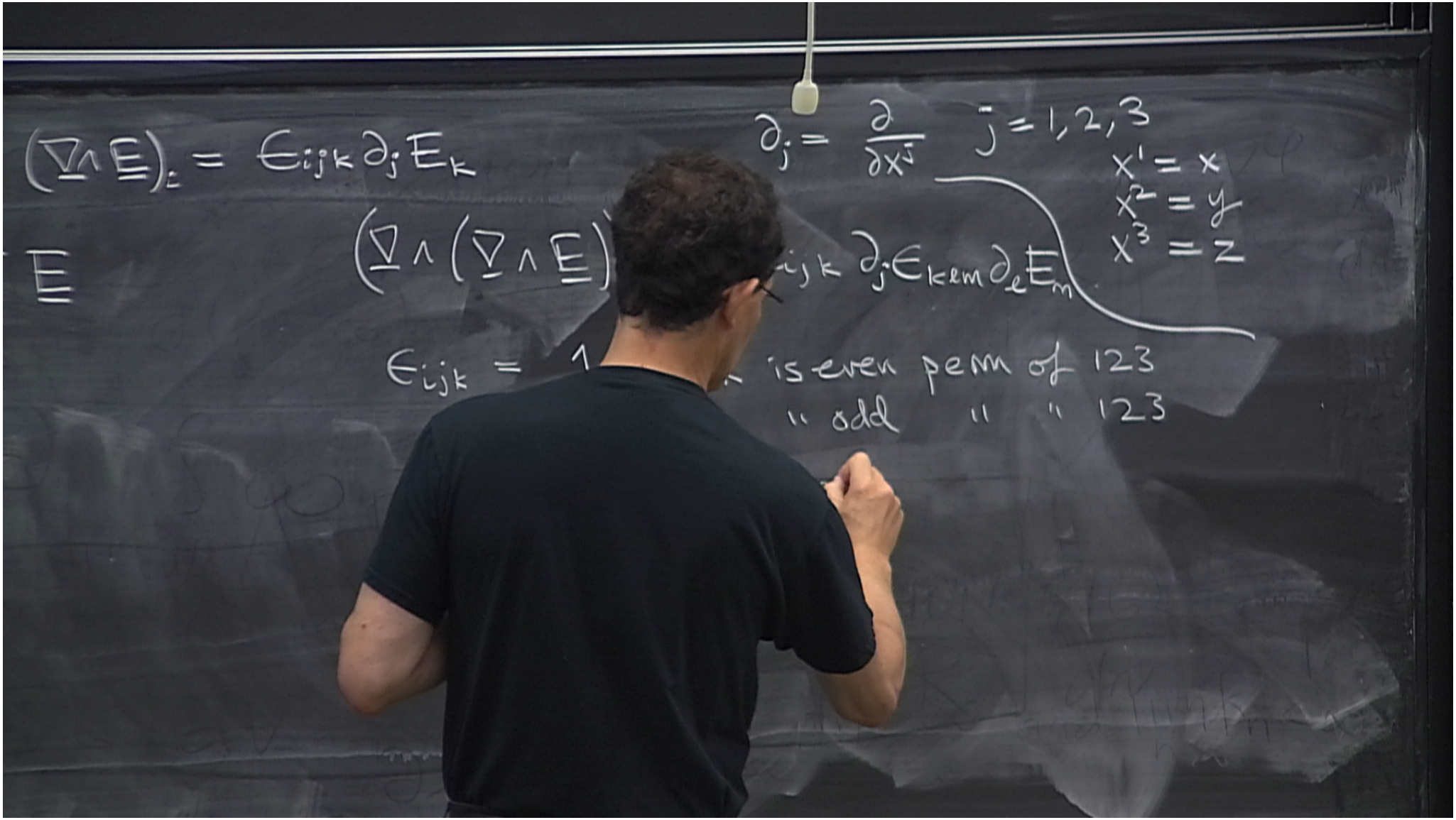

Maxwell: if $\rho = \underline{J} = 0$, there are still wavelike
$\frac{\partial^2 \underline{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \nabla \wedge (\nabla \wedge \underline{E})$,
$\frac{1}{\mu_0 \epsilon_0}$
∇


Maxwell: if $\rho = \underline{J} = 0$, there are still wavelike solutions $(\underline{\nabla} \wedge \underline{E})_i = \epsilon_{ijk} \partial_j E_k$
$\frac{\partial^2 \underline{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \underline{\nabla} \wedge (\underline{\nabla} \wedge \underline{E}) = \frac{1}{\mu_0 \epsilon_0} \underline{\nabla}^2 \underline{E}$


Maxwell: if $\rho = \underline{J} = 0$, there are still wavelike solutions
$\frac{\partial^2 \underline{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \underline{\nabla} \wedge (\underline{\nabla} \wedge \underline{E}) = \frac{1}{\mu_0 \epsilon_0} \underline{\nabla}^2 \underline{E}$
$(\underline{\nabla} \wedge \underline{E})_i = \epsilon_{ijk} \partial_j E$


$(\nabla \wedge \underline{E})_i = \epsilon_{ijk} \partial_j E_k$
$\partial_j = \frac{\partial}{\partial x^j}$
$j = 1, 2, 3$
$x^1 = x$
$x^2 = y$
$x^3 = z$
$\underline{E}$
$(\nabla \wedge (\nabla \wedge \underline{E}))_i = \epsilon_{ijk} \partial$


$(\nabla \wedge \underline{E})_i = \epsilon_{ijk} \partial_j E_k$
$\partial_j = \frac{\partial}{\partial x^j}$ $j = 1, 2, 3$
$x^1 = x$
$x^2 = y$
$x^3 = z$
$\underline{E}$
$(\underline{\nabla} \wedge (\underline{\nabla} \wedge \underline{E}))_i = \epsilon_{ijk} \partial_j \epsilon_{klm} \partial_l E_m$
ϵ


$(\nabla \wedge \mathbf{E})_i = \epsilon_{ijk} \partial_j E_k$
$\partial_j = \frac{\partial}{\partial x^j}$
$j = 1, 2, 3$
$x^1 = x$
$x^2 = y$
$x^3 = z$
$\epsilon_{ijk} = 1$
is even perm of 123
" odd " " 123

$(\nabla \wedge \mathbf{E})_i = \epsilon_{ijk} \partial_j E_k$

$\partial_j = \frac{\partial}{\partial x^j}$ $j = 1, 2, 3$

$x^1 = x$
$x^2 = y$
$x^3 = z$

$\mathbf{E}$

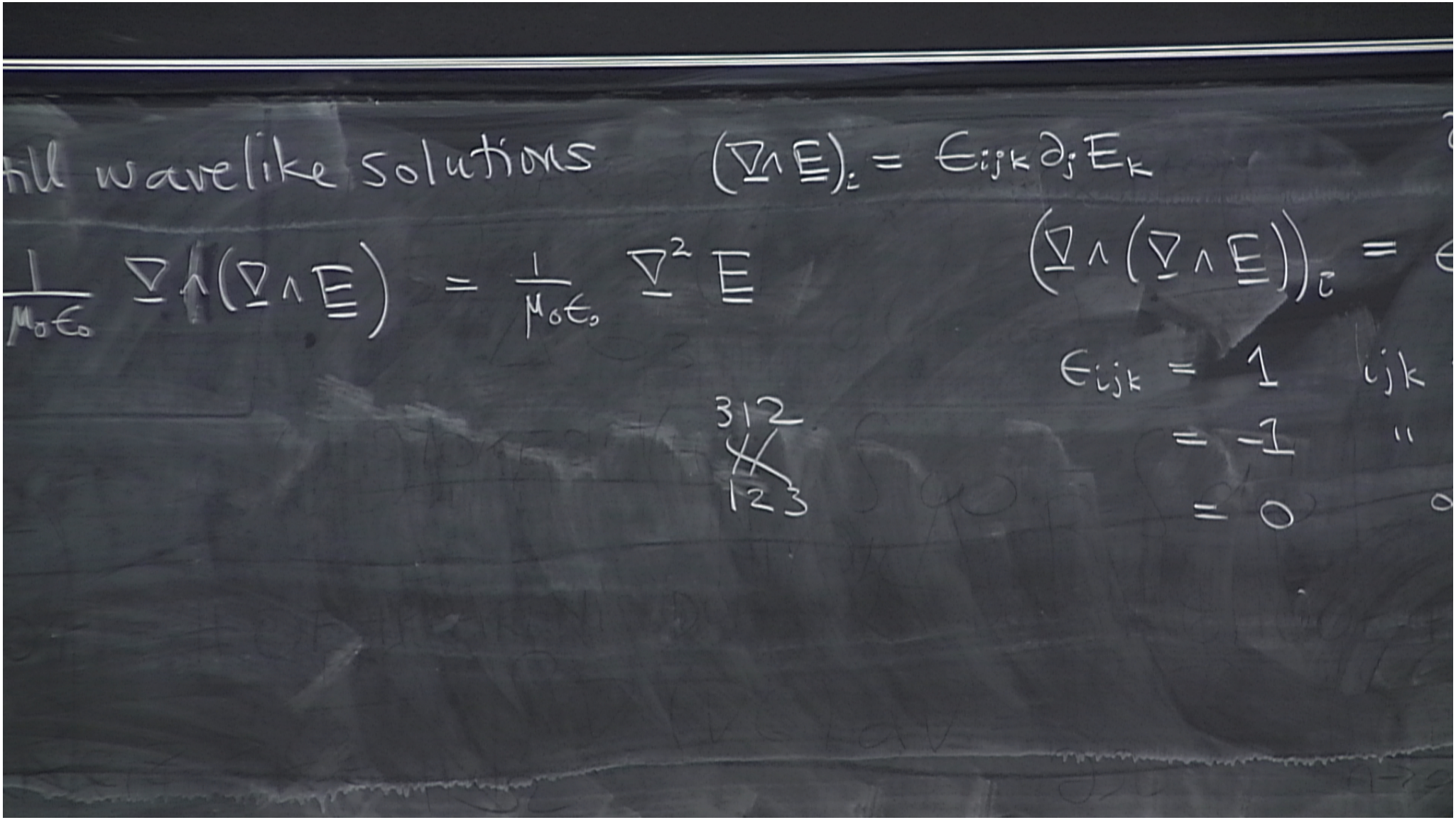
$\left(\nabla \wedge (\nabla \wedge \mathbf{E})\right)_i = \epsilon_{ijk} \partial_j \epsilon_{klm} \partial_l E_m$

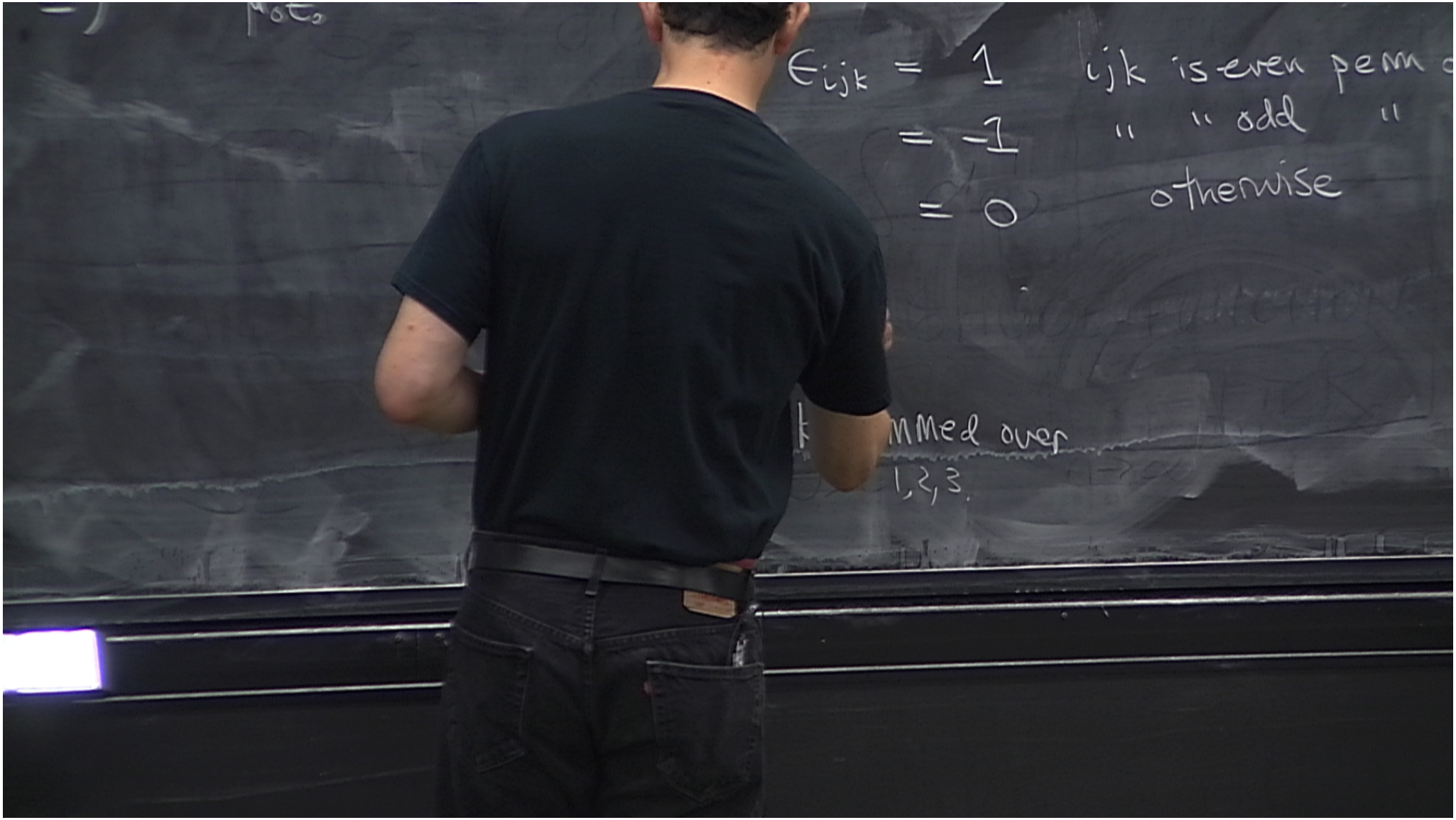
312

$\epsilon_{ijk} = 1$ ijk is even perm of 123

$= -1$ " " odd " " 123

$= 0$ otherwise


all wavelike solutions
$(\nabla \wedge \underline{E})_i = \epsilon_{ijk} \partial_j E_k$
$\frac{1}{\mu_0 \epsilon_0} \nabla \wedge (\nabla \wedge \underline{E}) = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underline{E}$
$(\nabla \wedge (\nabla \wedge \underline{E}))_i = \epsilon$
$\epsilon_{ijk} = 1$ ijk
$= -1$ "
$= 0$ o
3 1 2
1 2 3


$\epsilon_{ijk} = 1$ ijk is even perm
$= -1$ " " odd "
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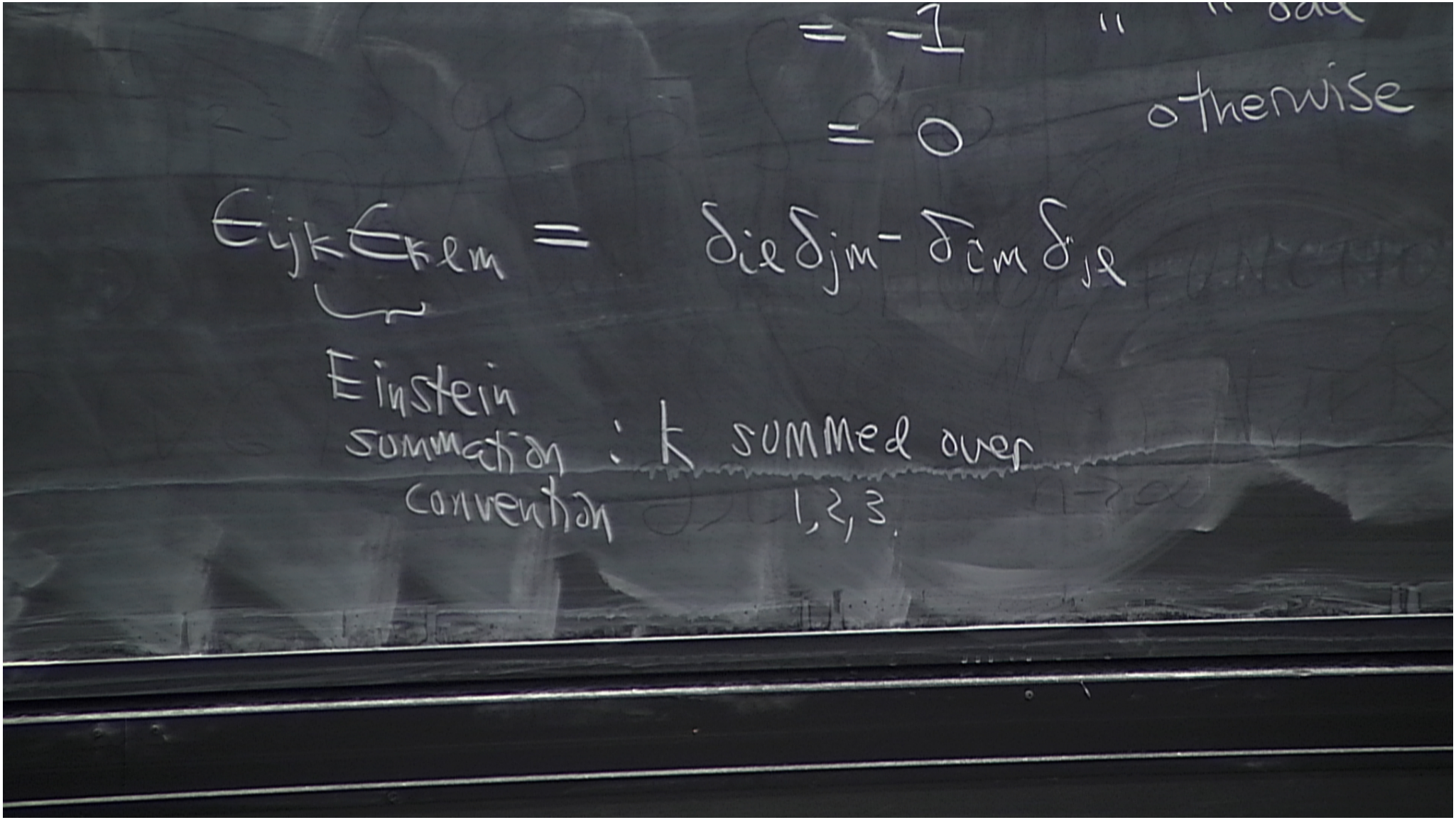
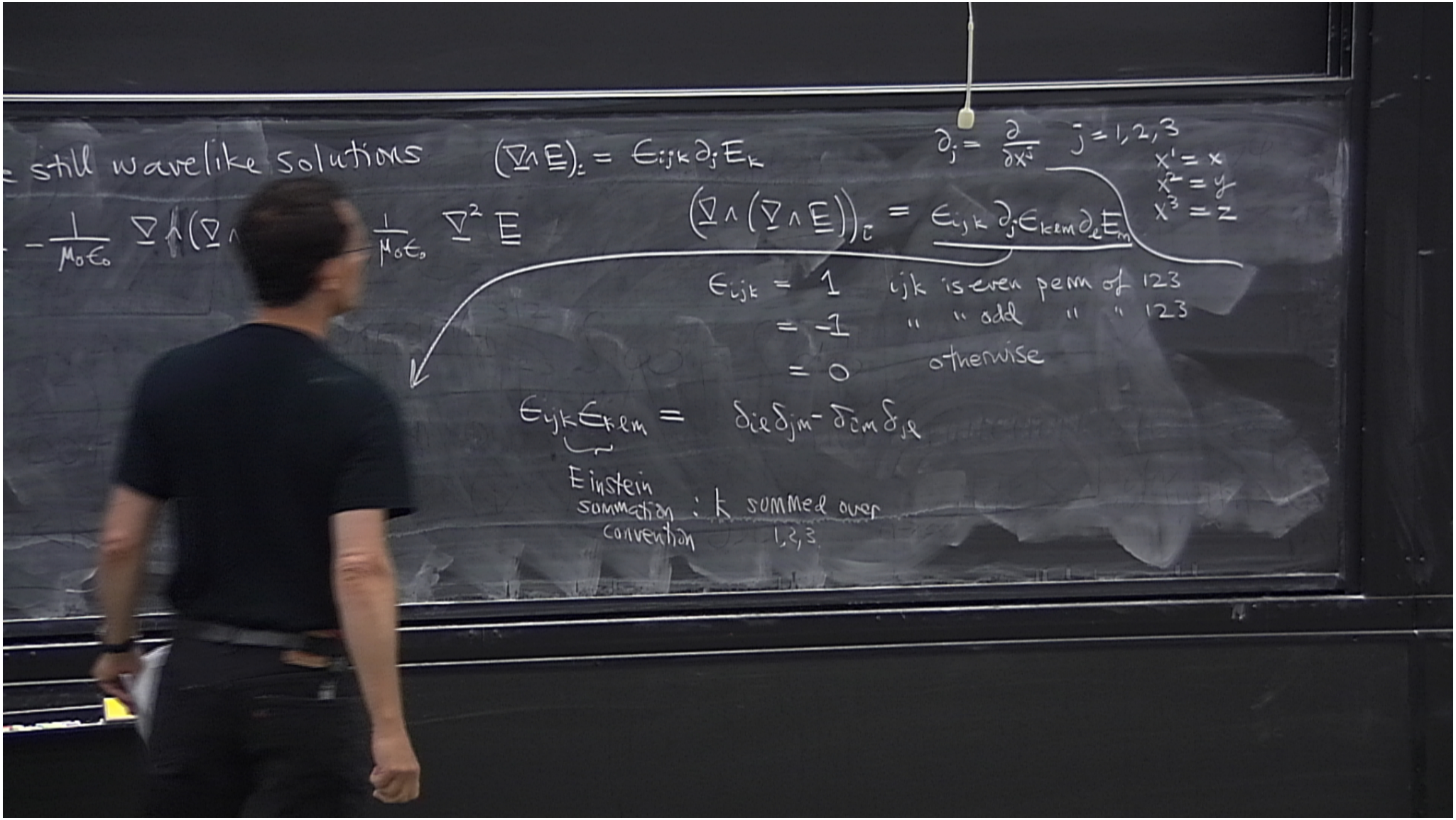
MMed over
1,2,3.


= -1 "
= 0 otherwise
$\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$
Einstein summation convention : k summed over 1,2,3.


still wavelike solutions
$(\nabla \times \mathbf{E})_i = \epsilon_{ijk} \partial_j E_k$
$\partial_j = \frac{\partial}{\partial x^j}$ $j = 1,2,3$
$x^1 = x$
$x^2 = y$
$x^3 = z$
$-\frac{1}{\mu_0 \epsilon_0} \nabla \times (\nabla \times \mathbf{E}) = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \mathbf{E}$
$(\nabla \times (\nabla \times \mathbf{E}))_i = \epsilon_{ijk} \partial_j \epsilon_{klm} \partial_l E_m$
$\epsilon_{ijk} = 1$ ijk is even perm of 123
$= -1$ " " odd " " 123
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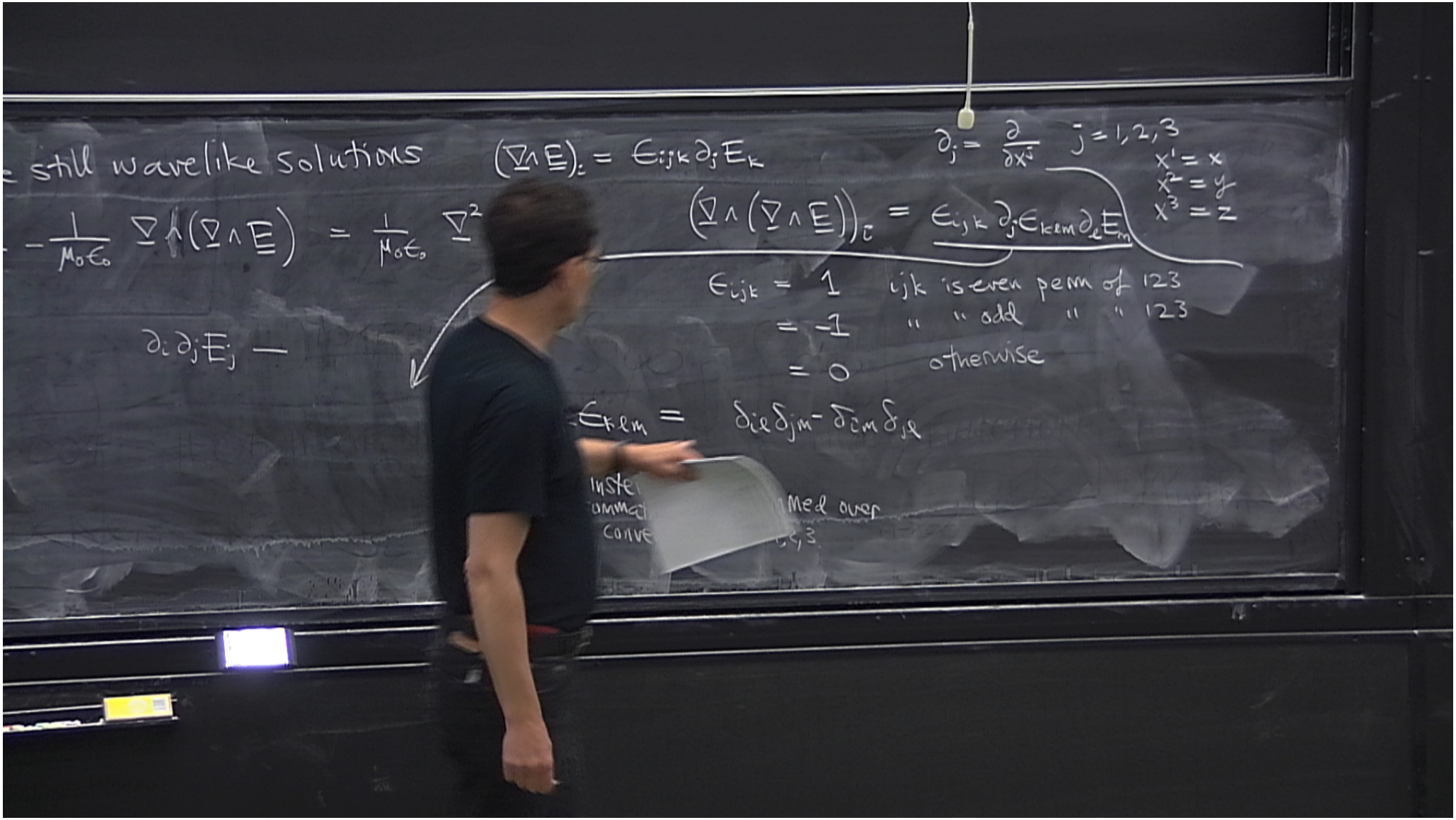
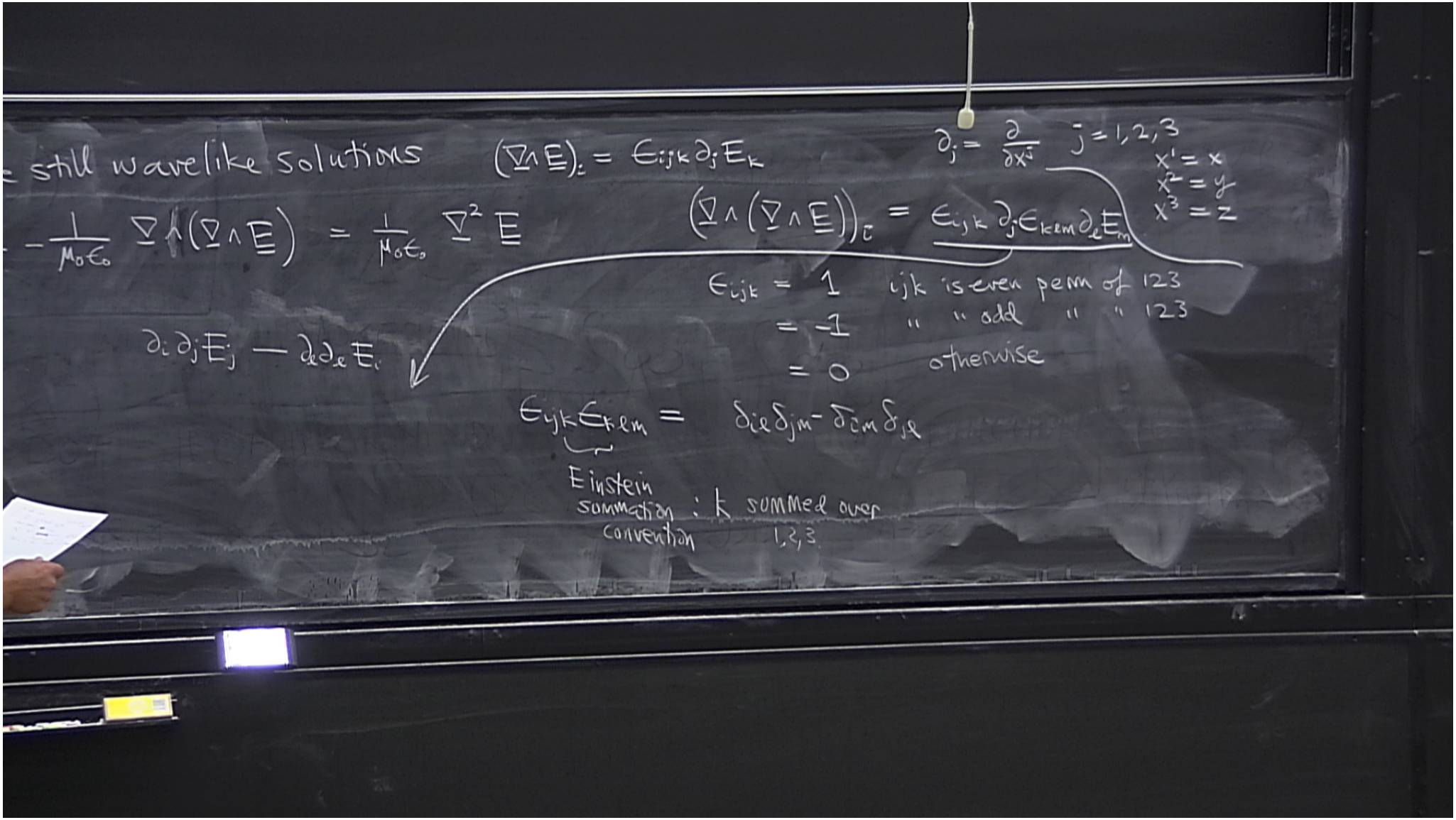
$\epsilon_{ijk} \epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$
Einstein summation convention : k summed over 1,2,3


still wavelike solutions
$(\nabla \times \underline{E})_i = \epsilon_{ijk} \partial_j E_k$
$\partial_j = \frac{\partial}{\partial x^j}$ $j = 1,2,3$
$x^1 = x$
$x^2 = y$
$x^3 = z$
$-\frac{1}{\mu_0 \epsilon_0} \nabla \times (\nabla \times \underline{E}) = \frac{1}{\mu_0 \epsilon_0} \nabla^2$
$(\nabla \times (\nabla \times \underline{E}))_i = \epsilon_{ijk} \partial_j \epsilon_{klm} \partial_l E_m$
$\epsilon_{ijk} = 1$ ijk is even perm of 123
$= -1$ " " odd " " 123
$= 0$ otherwise
$\partial_i \partial_j E_j$ —
$\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$
med over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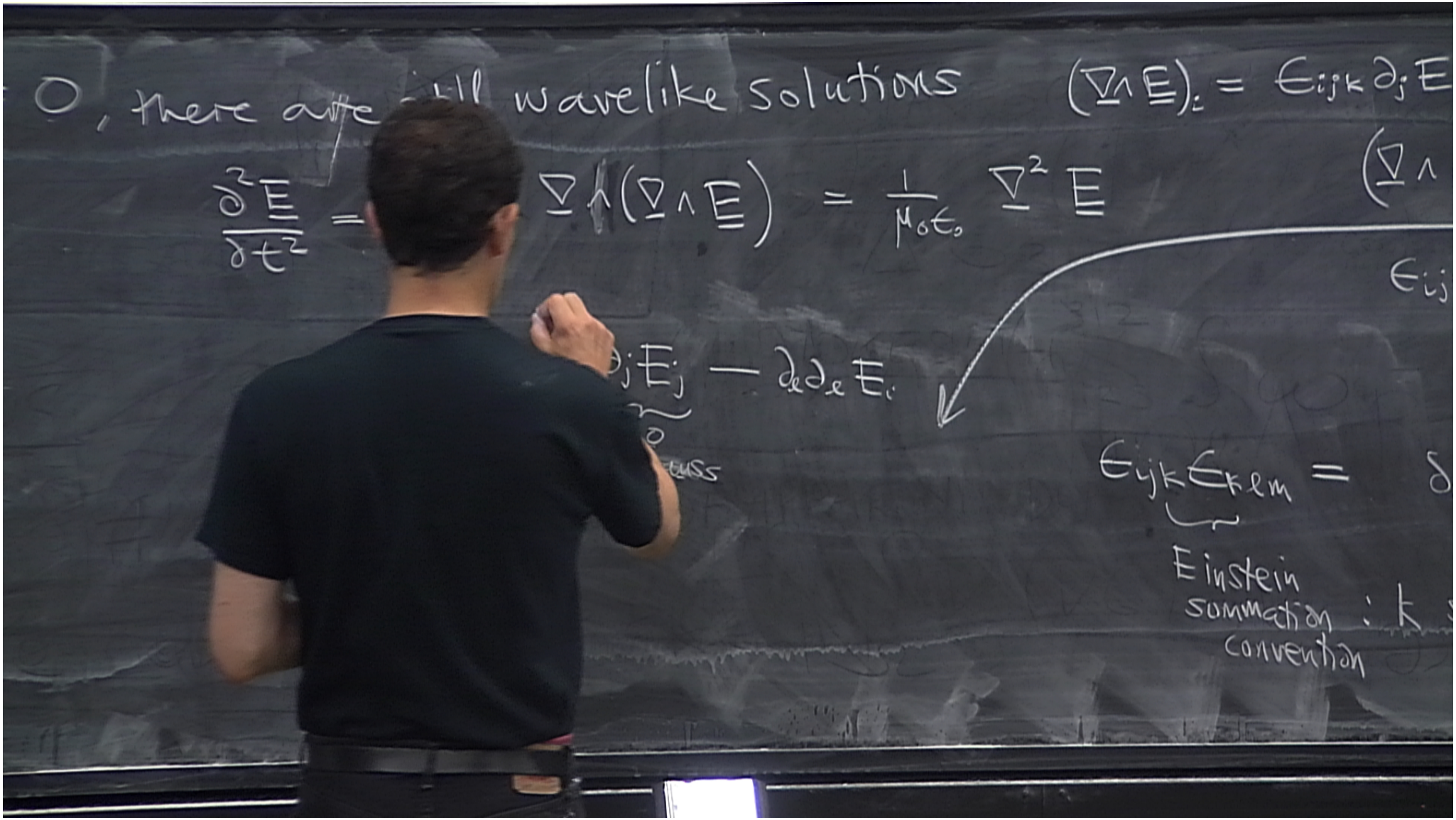
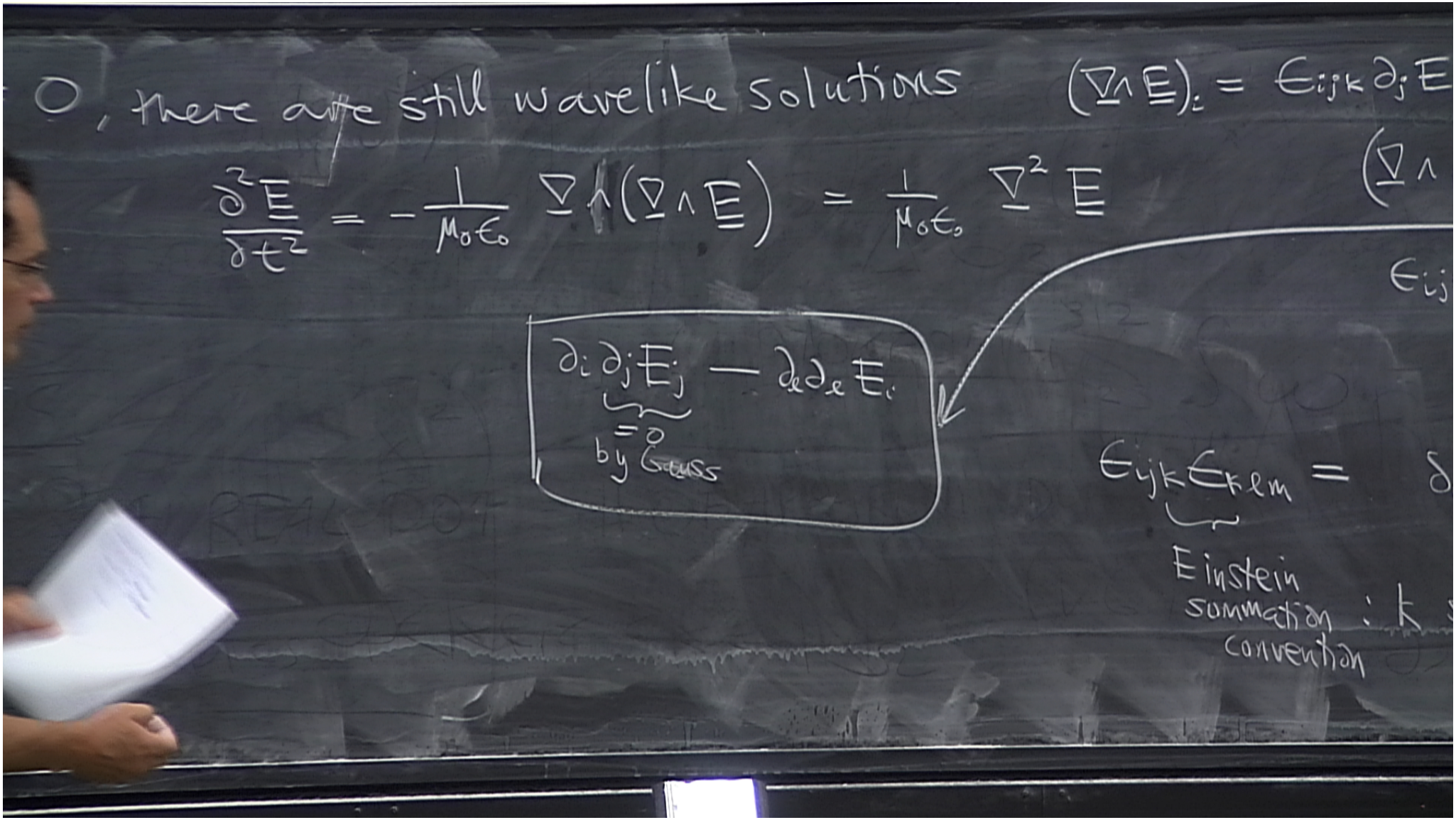
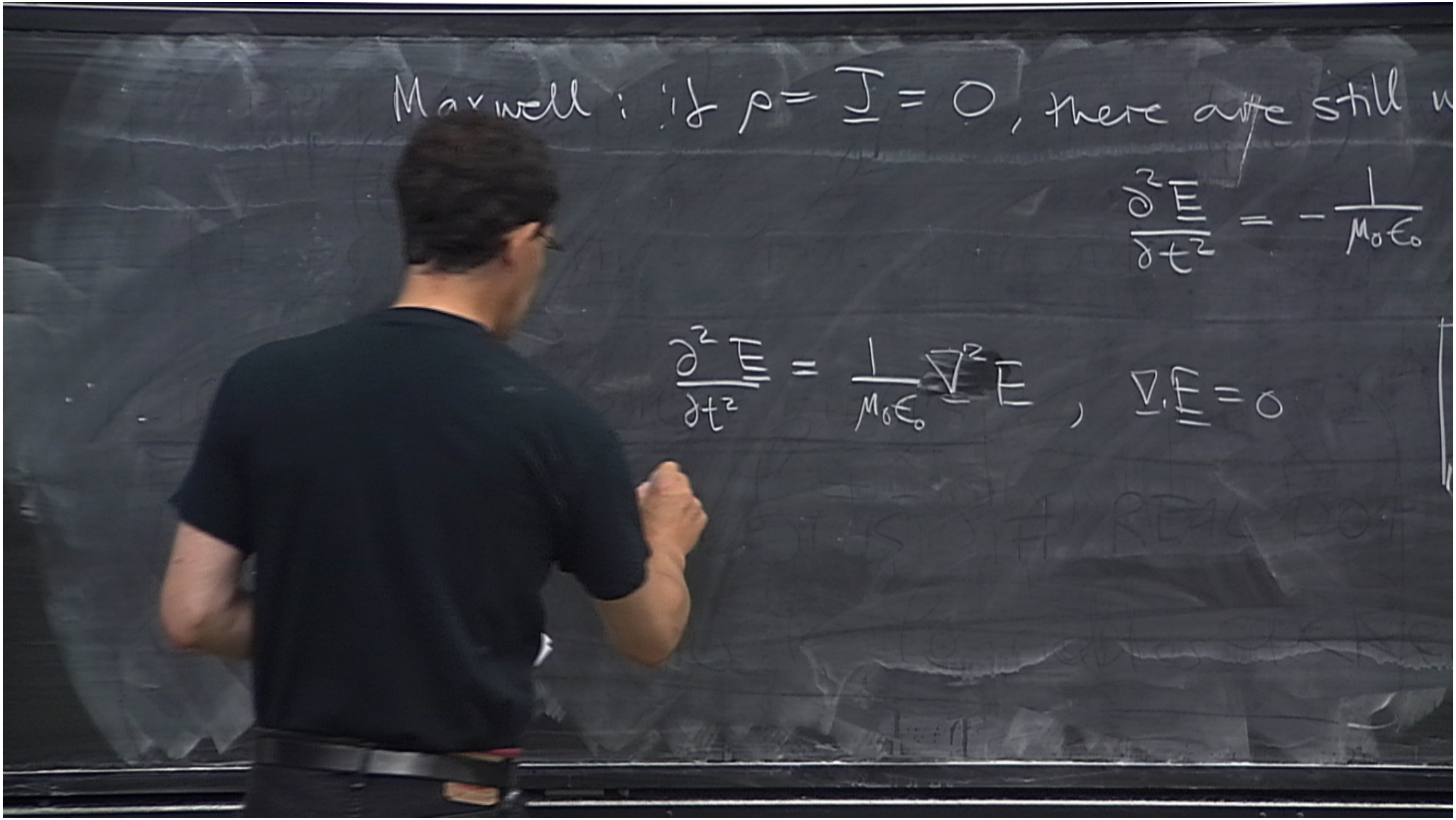
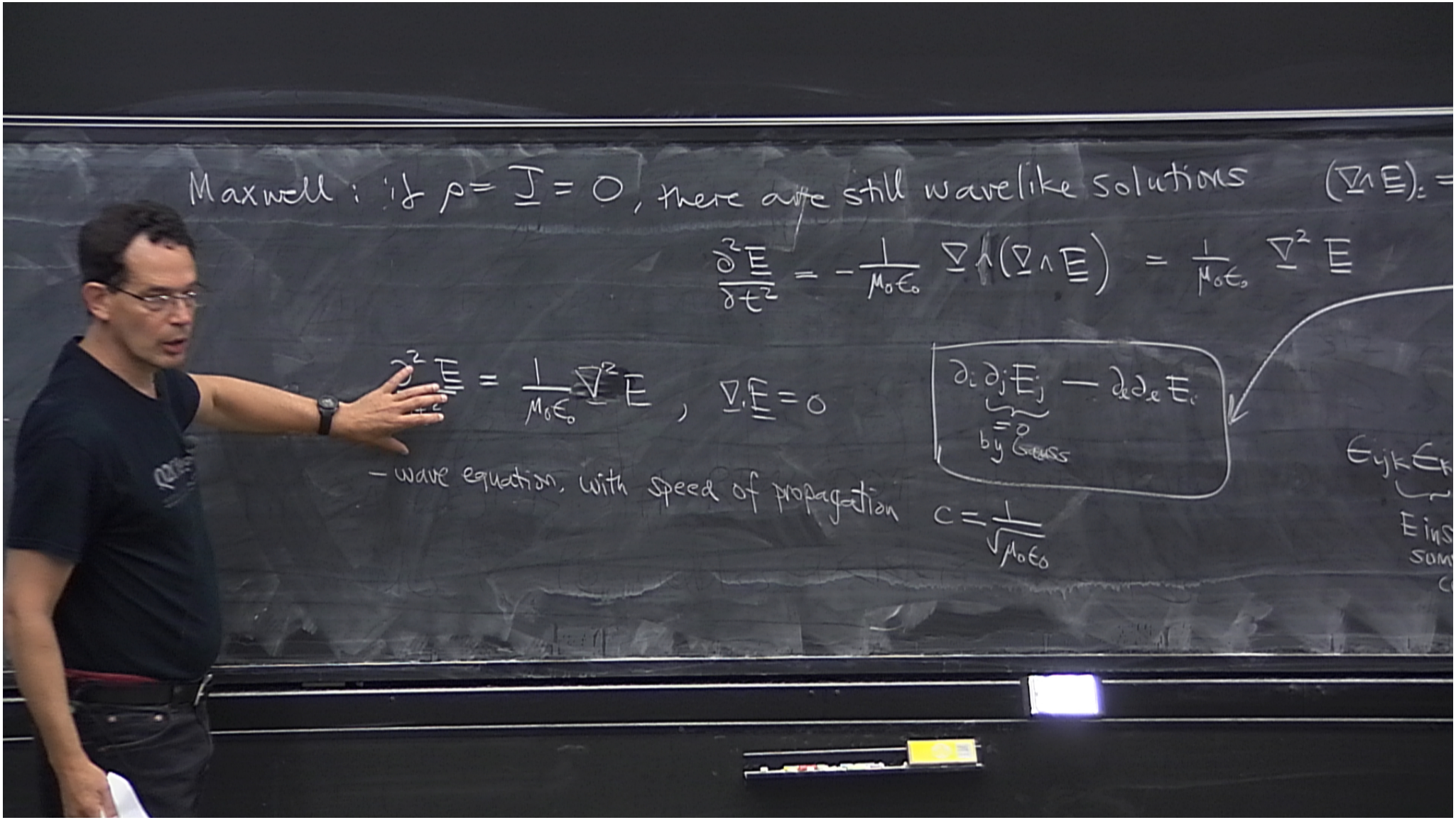
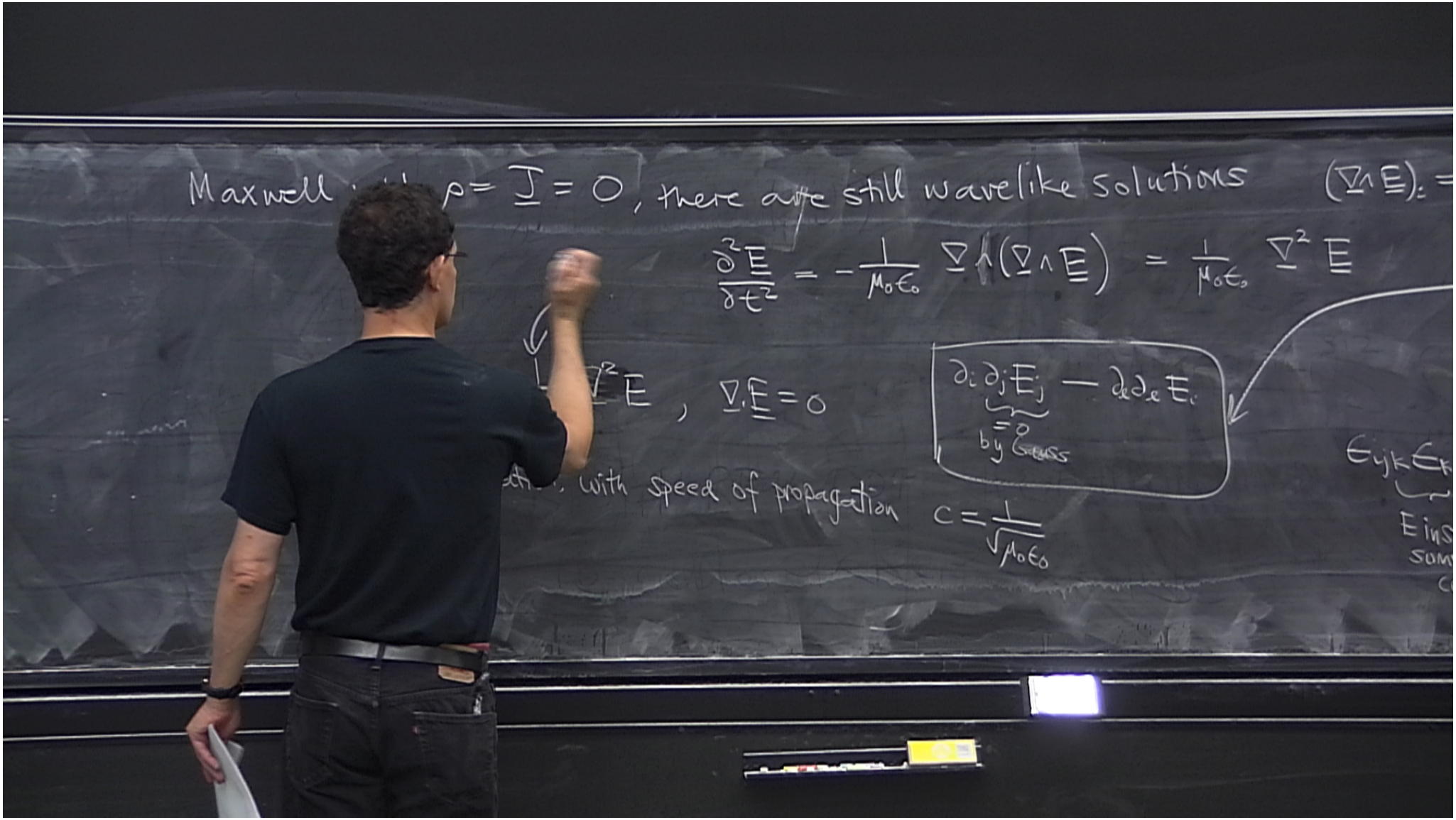
$(\nabla \wedge (\nabla \wedge \underline{E}))_i = \epsilon_{ijk} \partial_j \epsilon_{klm} \partial_l E_m$
$\epsilon_{ijk} = 1$ ijk is even perm of 123
$= -1$ " " odd " " 123
$= 0$ otherwise
$\partial_i \partial_j E_j - \partial_l \partial_l E_i$
$\epsilon_{ijk} \epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$
Einstein summation convention : k summed over 1,2,3

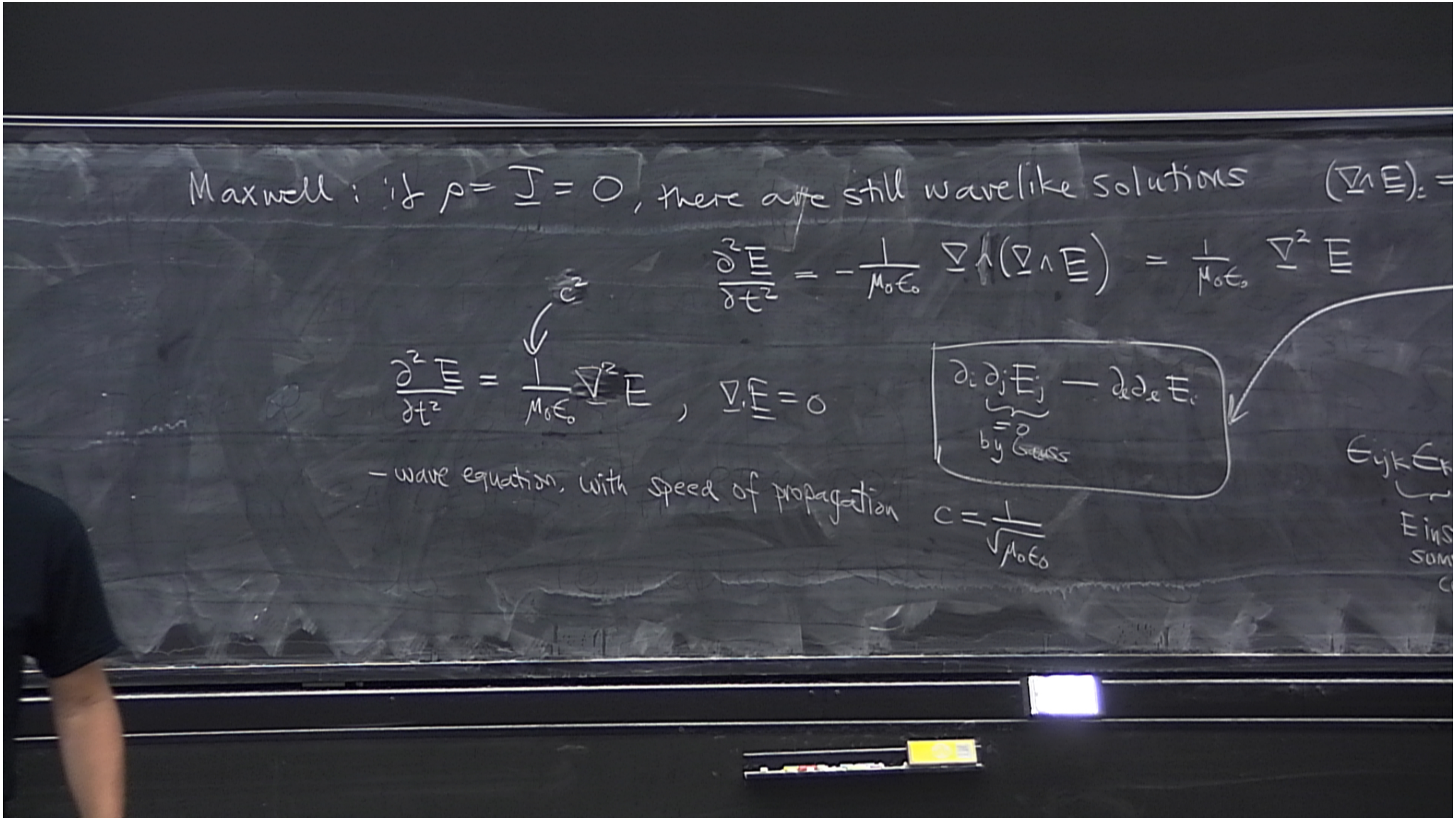

wavelike solutions
$(\nabla \times \mathbf{E})_i = \epsilon_{ijk} \partial_j E$
$\frac{\partial^2 \mathbf{E}}{\partial t^2} = \nabla \times (\nabla \times \mathbf{E}) = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \mathbf{E}$
$\partial_j E_j - \partial_k \partial_k E_i$
$\epsilon_{ijk} \epsilon_{klm} =$
Einstein summation convention


O, there are still wavelike solutions
$(\nabla \wedge \mathbf{E})_i = \epsilon_{ijk} \partial_j E$
$\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \nabla \wedge (\nabla \wedge \mathbf{E}) = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \mathbf{E}$
$\partial_i \partial_j E_j - \partial_k \partial_k E_i$
$= 0$ by Gauss
$\epsilon_{ijk} \epsilon_{klm} = \delta$
Einstein summation convention


Maxwell : if $\rho = \underline{J} = 0$, there are still
$\frac{\partial^2 \underline{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0}$
$\frac{\partial^2 \underline{E}}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underline{E}$, $\nabla \cdot \underline{E} = 0$


Maxwell: if $\rho = \underline{J} = 0$, there are still wavelike solutions
$\frac{\partial^2 \underline{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \nabla \wedge (\nabla \wedge \underline{E}) = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underline{E}$
$\frac{\partial^2 \underline{E}}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underline{E}$, $\nabla \cdot \underline{E} = 0$
– wave equation, with speed of propagation $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$
$\partial_i \partial_j E_j - \partial_k \partial_k E_i$
$= 0$ by Gauss


Maxwell
ρ = J = 0, there are still wavelike solutions
∂²E/∂t² = −1/μ₀ε₀ ∇×(∇×E) = 1/μ₀ε₀ ∇²E
∇²E , ∇·E = 0
with speed of propagation
c = 1/√(μ₀ε₀)
∂ᵢ∂ⱼEⱼ − ∂ₖ∂ₖEᵢ
by Gauss
εᵢⱼₖεₖ



Maxwell: if $\rho = \underline{J} = 0$, there are still wavelike solutions $(\underline{\nabla} \wedge \underline{E})_i =$

$$\frac{\partial^2 \underline{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \underline{\nabla} \wedge (\underline{\nabla} \wedge \underline{E}) = \frac{1}{\mu_0 \epsilon_0} \underline{\nabla}^2 \underline{E}$$

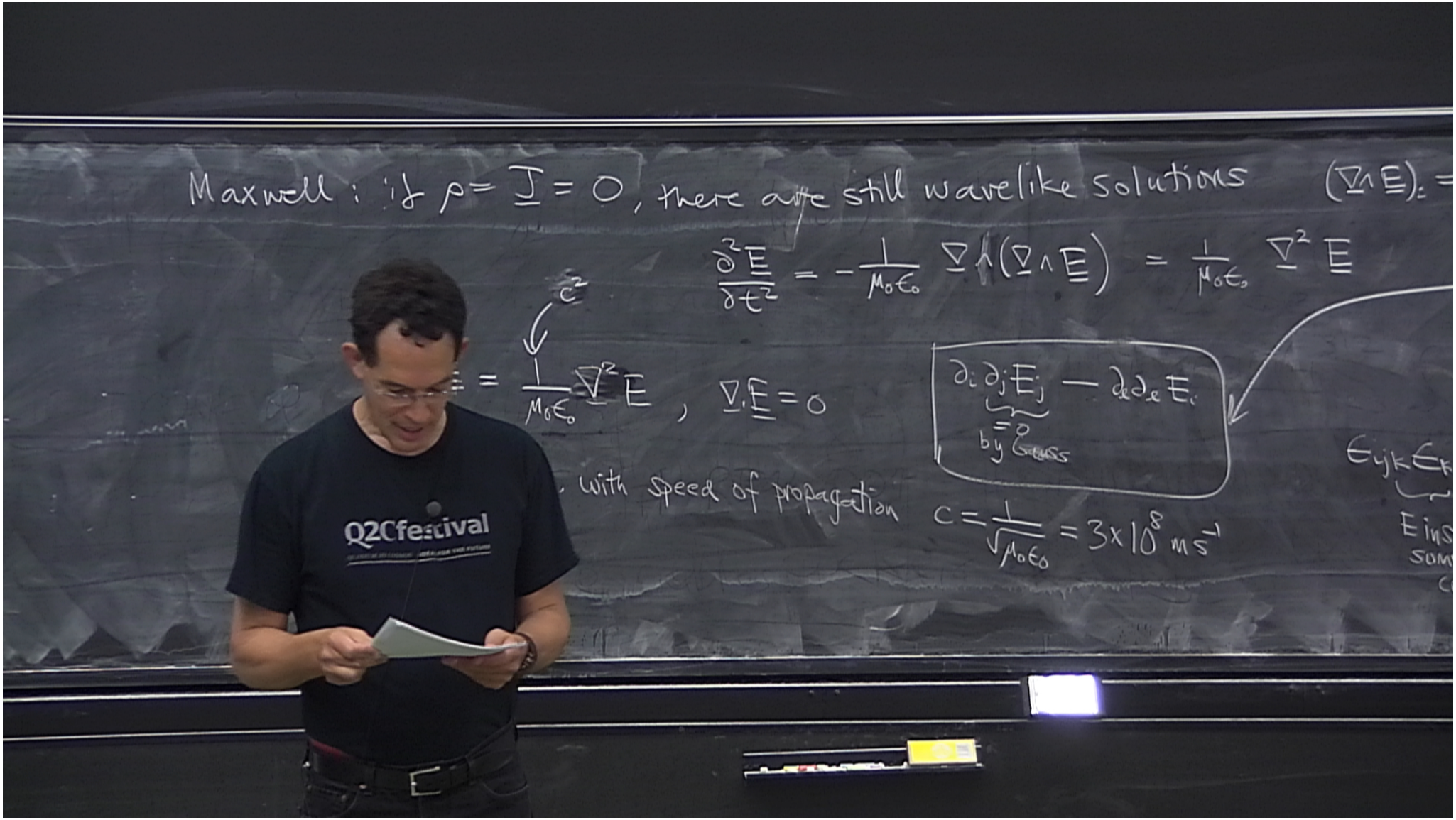
$$\frac{\partial^2 \underline{E}}{\partial t^2} = \overset{c^2}{\frac{1}{\mu_0 \epsilon_0}} \underline{\nabla}^2 \underline{E}, \quad \underline{\nabla} \cdot \underline{E} = 0$$

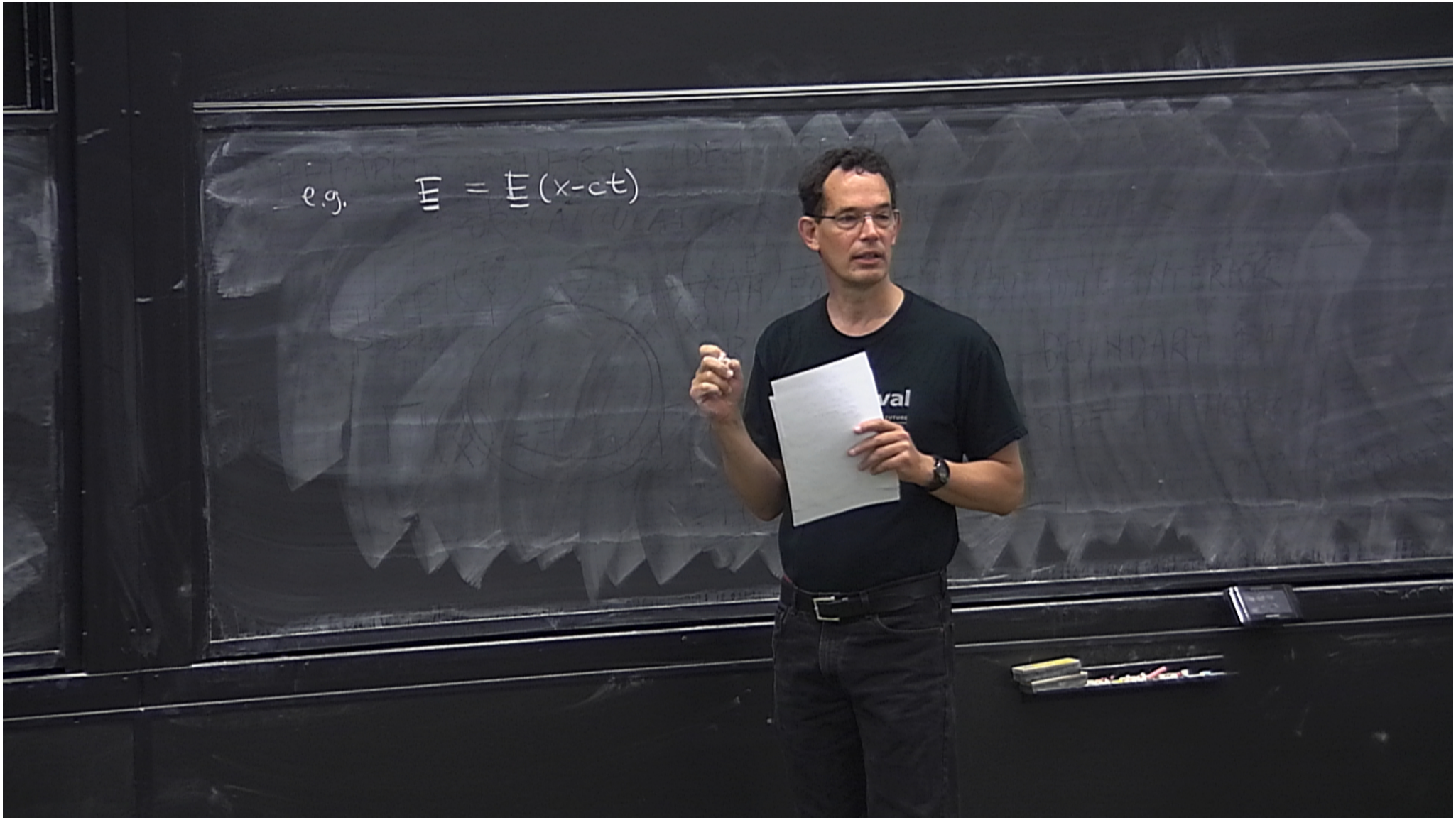
$$\boxed{\partial_i \underbrace{\partial_j E_j}_{\substack{=0 \\ \text{by Gauss}}} - \partial_\ell \partial_\ell E_i}$$

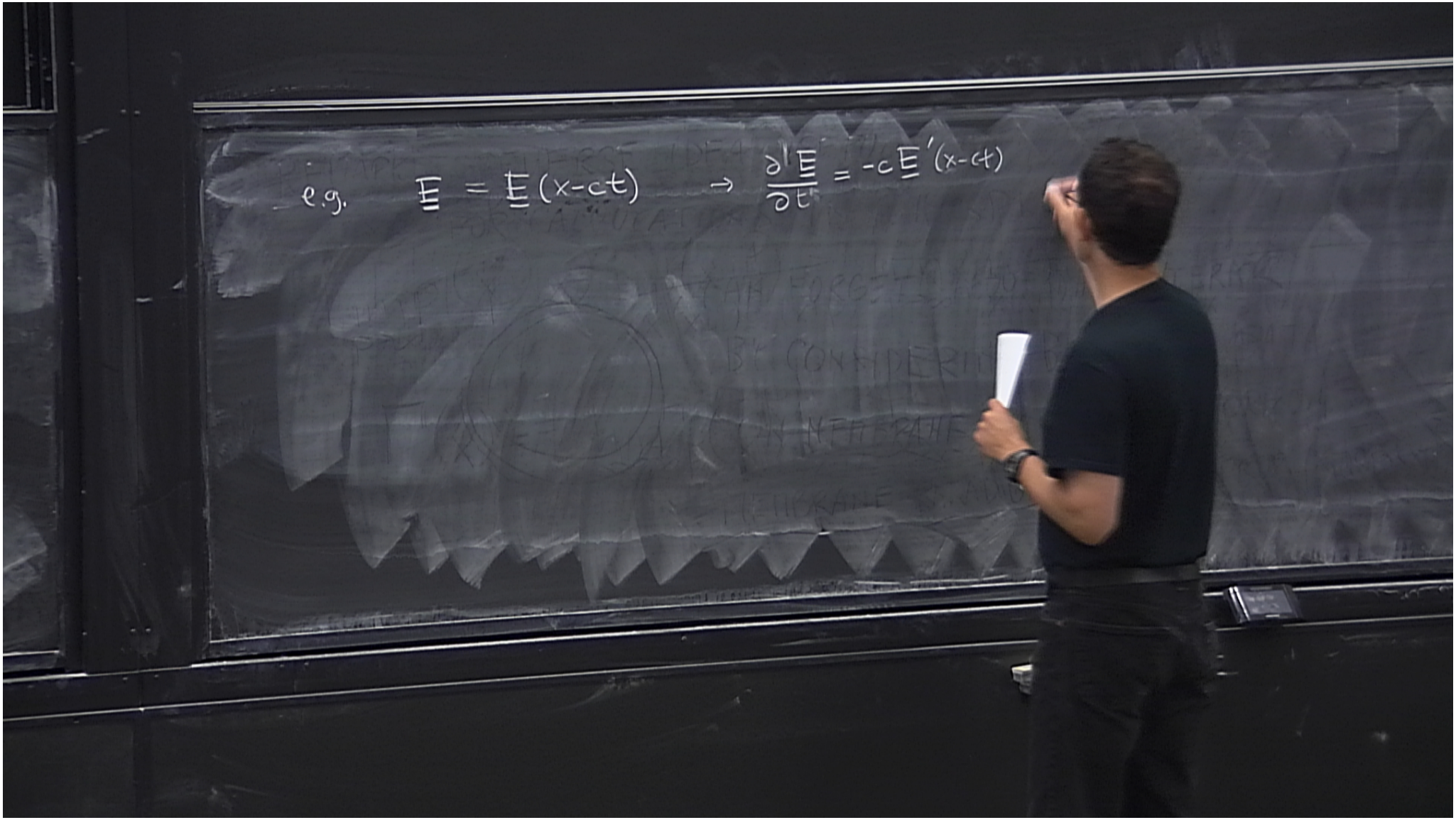
– wave equation, with speed of propagation $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

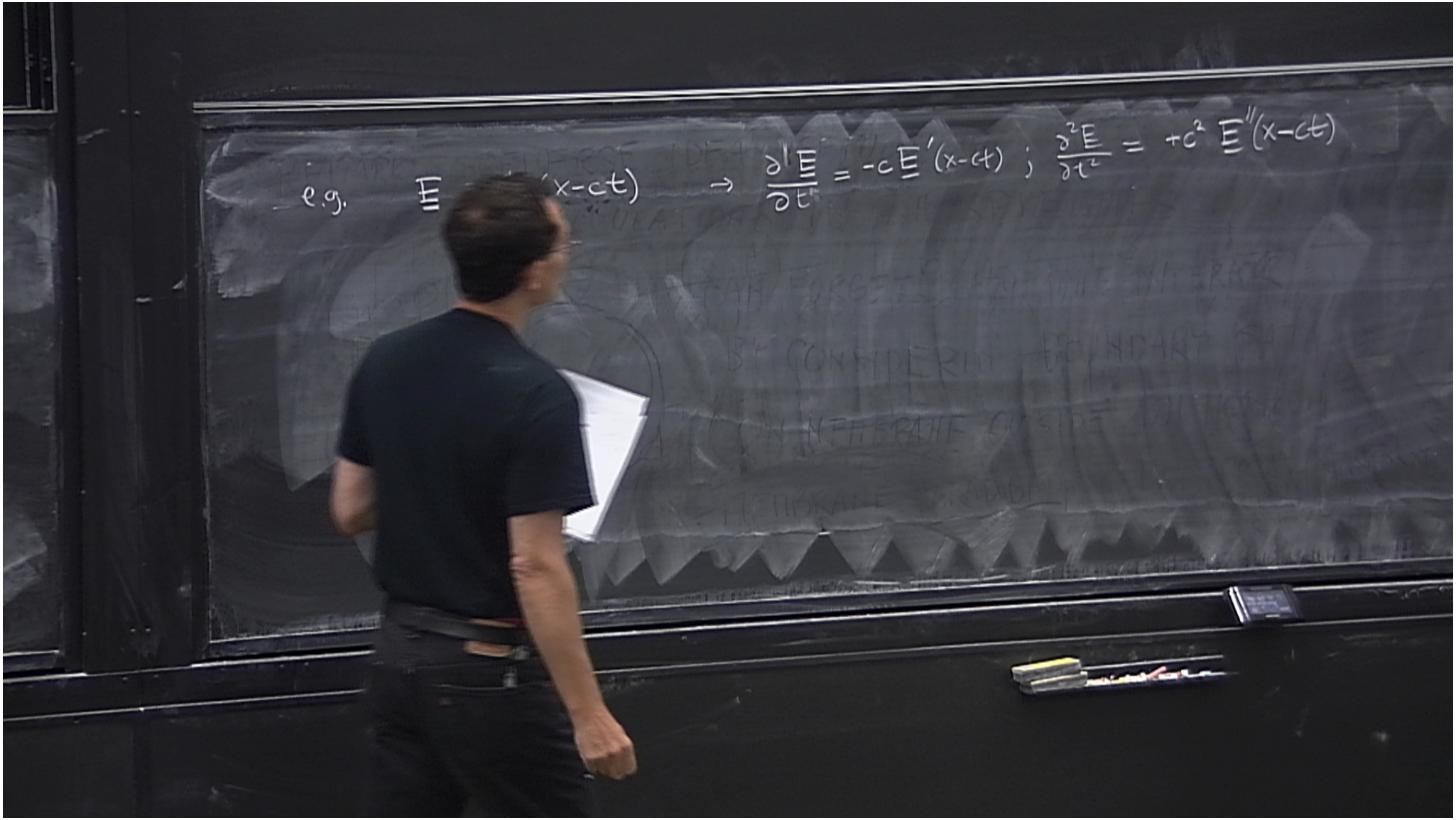
$\epsilon_{ijk} \epsilon_{k}$

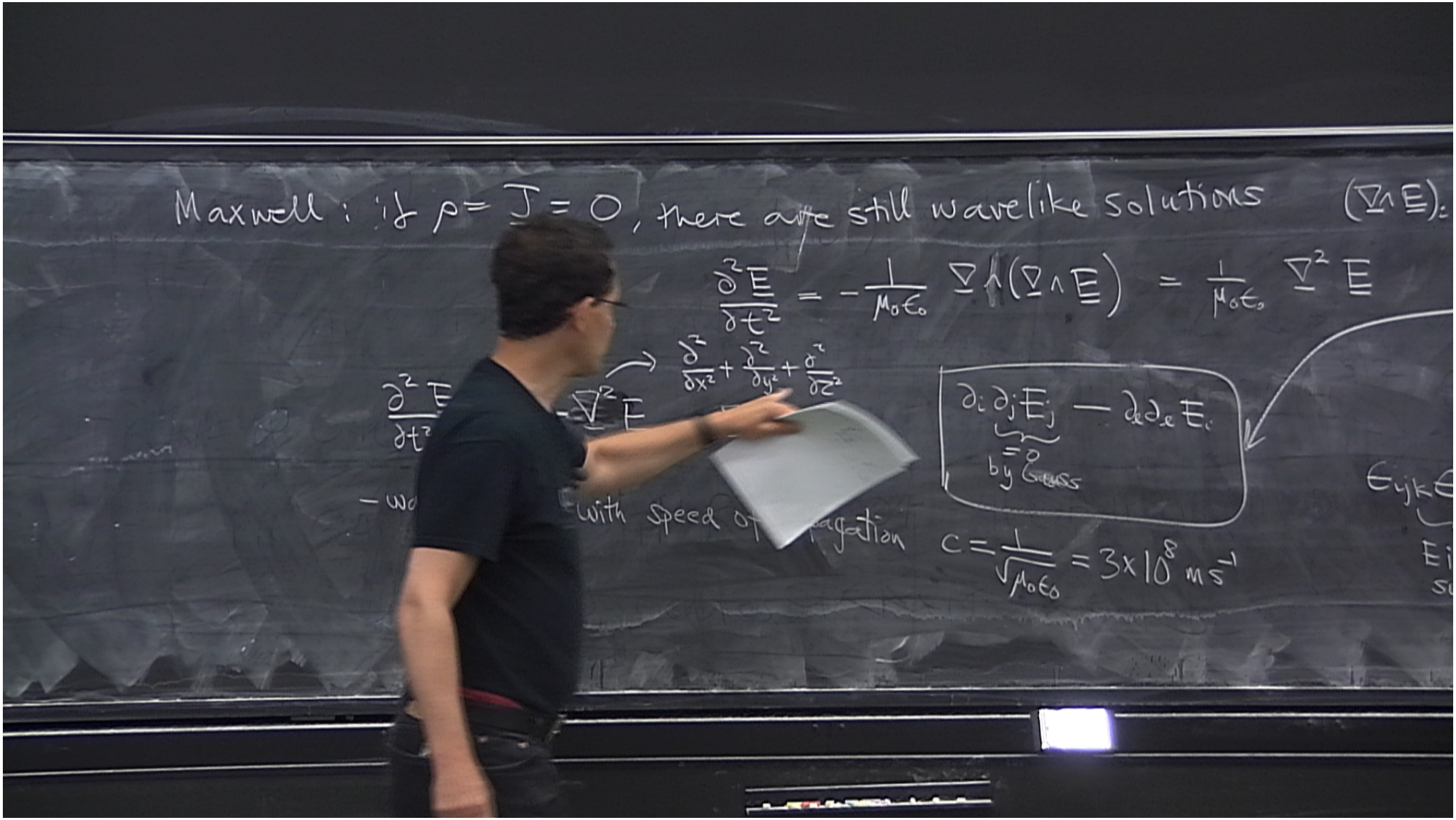
E ins
som
c

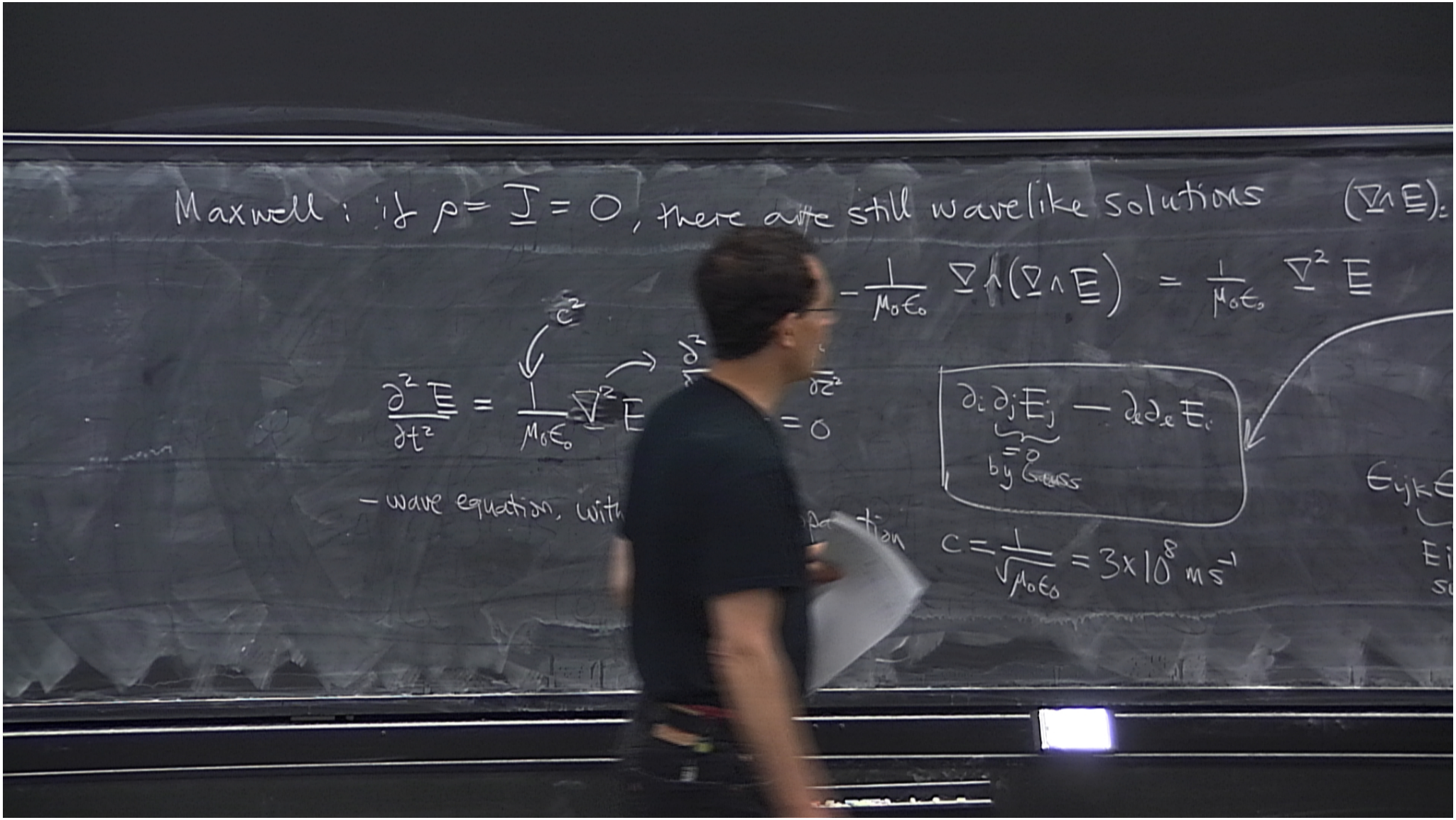

Maxwell: if ρ = J = 0, there are still wavelike solutions
$\frac{\partial^2 \underline{E}}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \nabla \wedge (\nabla \wedge \underline{E}) = \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underline{E}$
c^2
$= \frac{1}{\mu_0 \epsilon_0} \nabla^2 \underline{E}$, $\nabla \cdot \underline{E} = 0$
$\partial_i \partial_j E_j - \partial_k \partial_k E_i$
$= 0$ by Gauss
with speed of propagation
$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \, m s^{-1}$
$\epsilon_{ijk} \epsilon_{k}$
Q2Cfestival

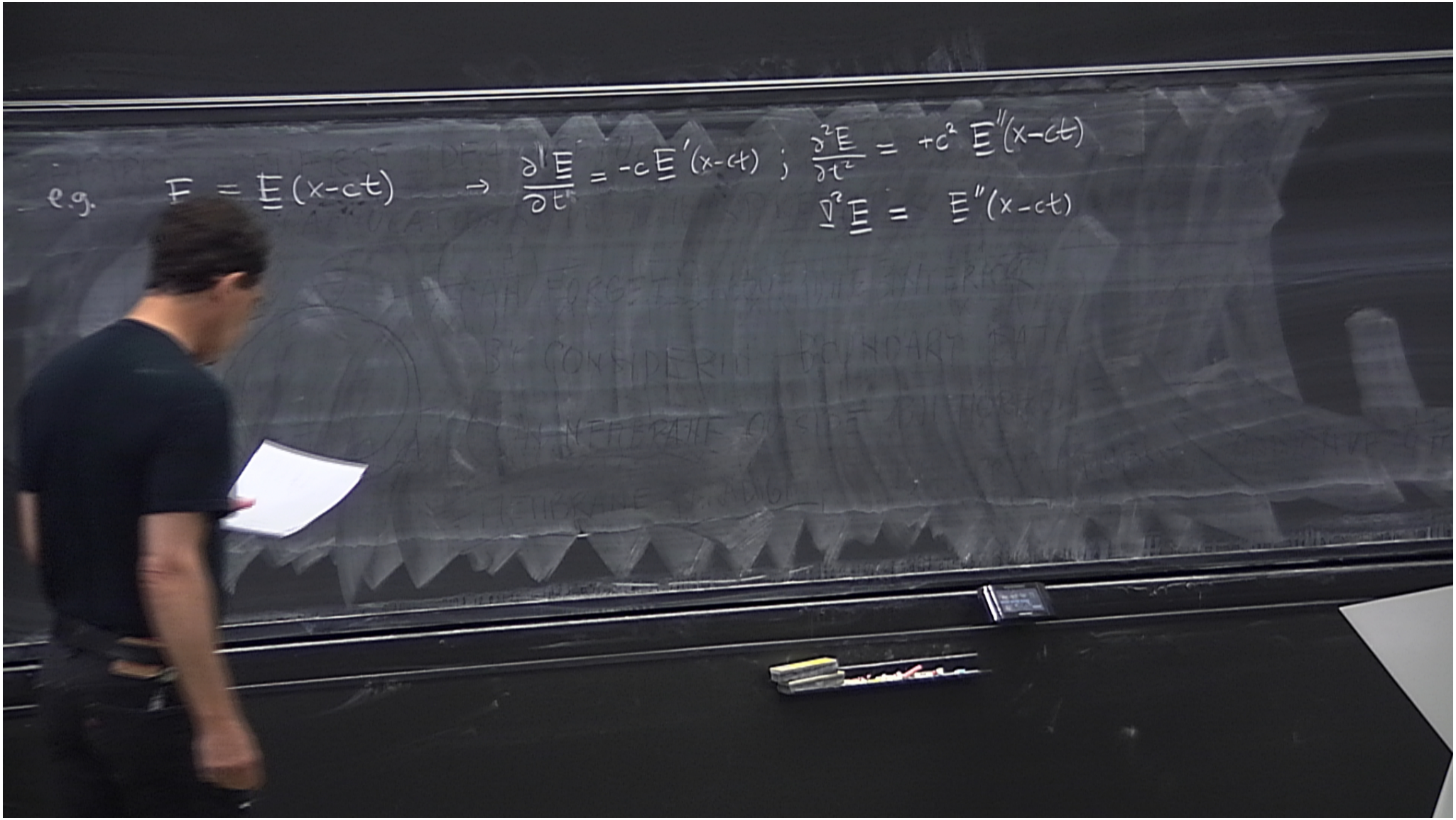

e.g. $\underline{\underline{E}} = \underline{\underline{E}}(x-ct)$

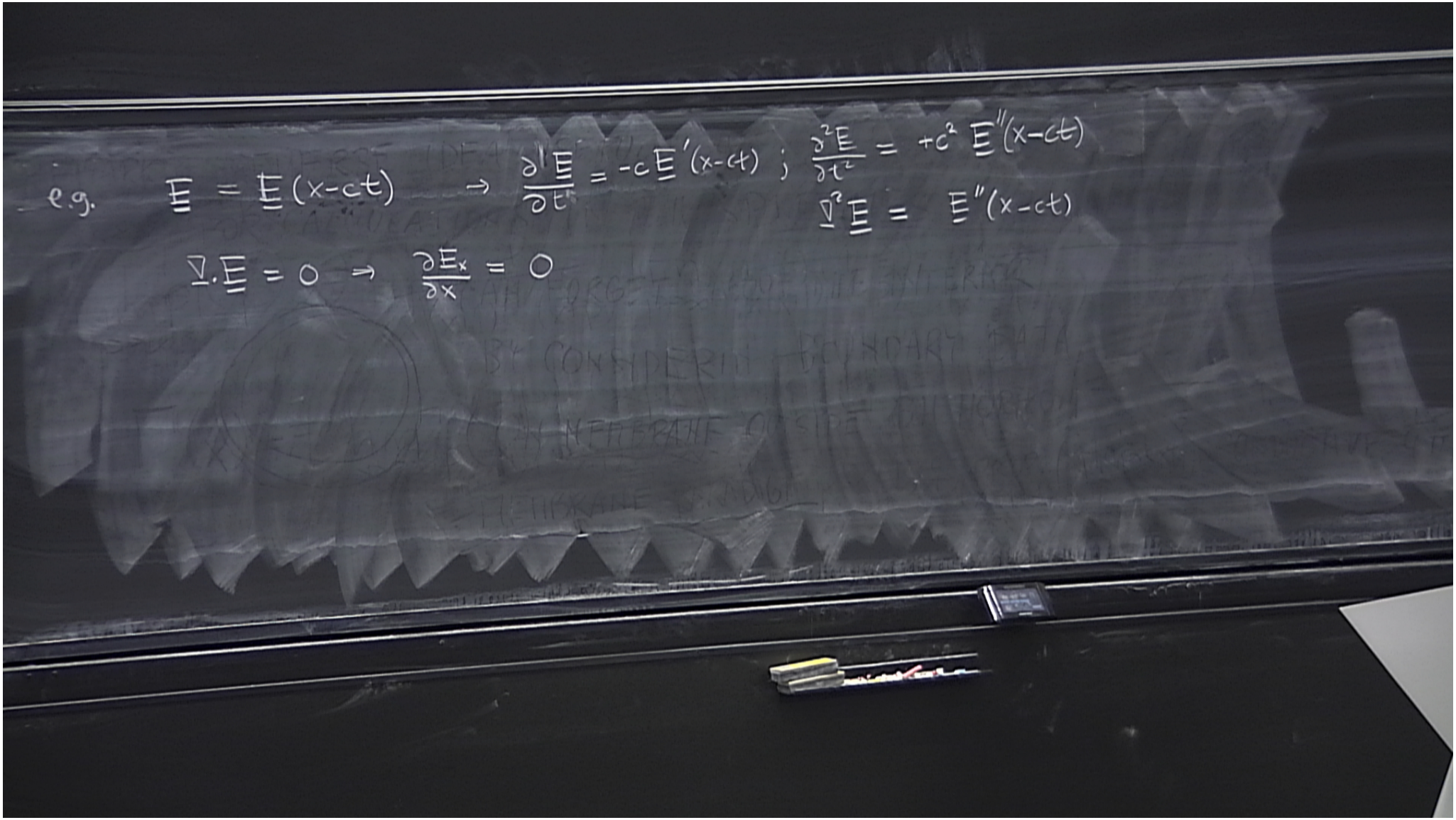

e.g. $\underline{E} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$

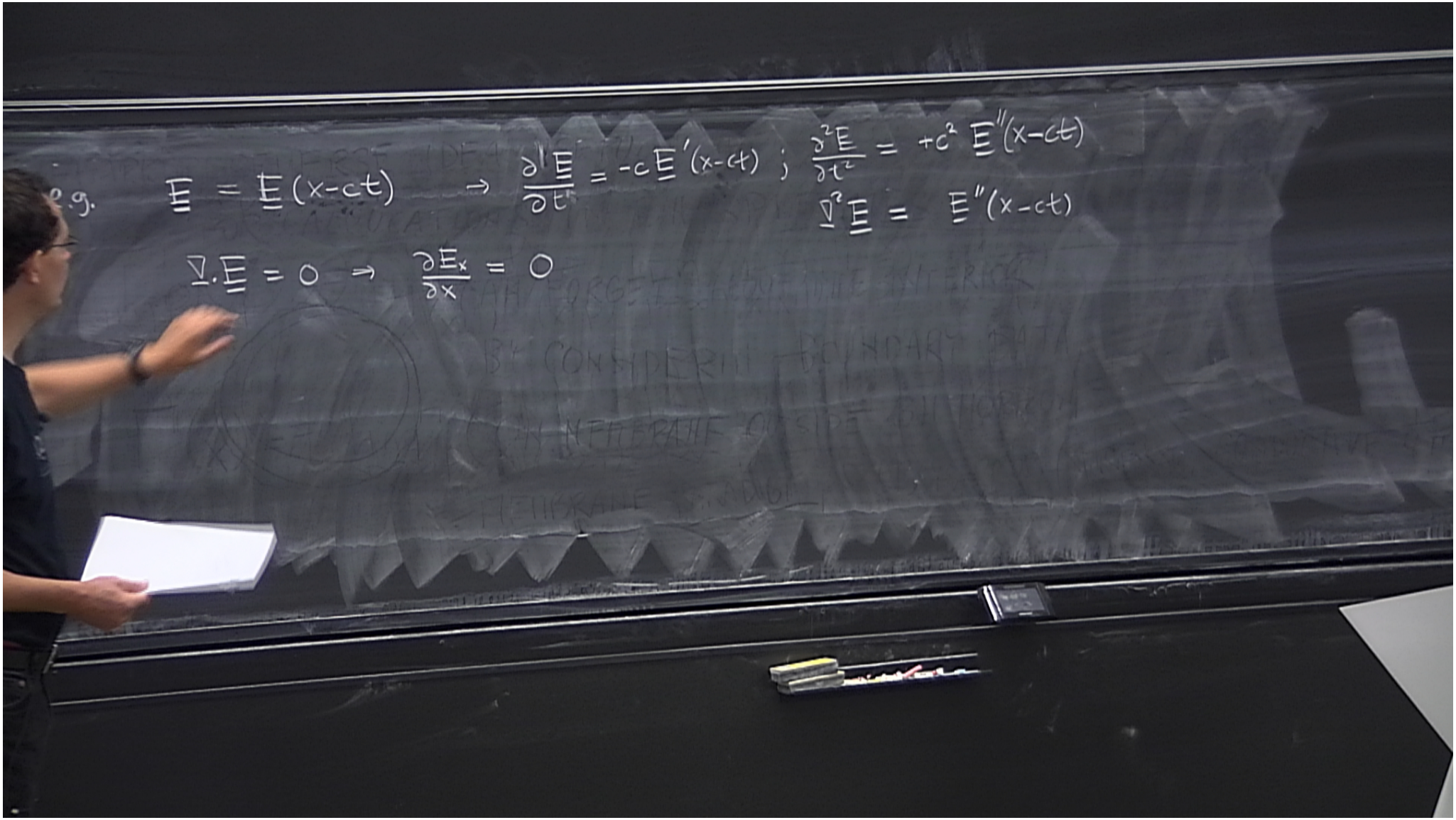

e.g. $\underline{E}$ $(x-ct)$ $\rightarrow$ $\frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2\,\underline{E}''(x-ct)$


Maxwell: if ρ = J = 0, there are still wavelike solutions
(∇ ∧ E):
∂²E/∂t² = −1/μ₀ε₀ ∇ ∧ (∇ ∧ E) = 1/μ₀ε₀ ∇² E
∂²E/∂t² = ∇² E
∂²/∂x² + ∂²/∂y² + ∂²/∂z²
∂ᵢ ∂ⱼ Eⱼ − ∂ₖ ∂ₖ Eᵢ
by Gauss
with speed of
c = 1/√(μ₀ε₀) = 3×10⁸ m s⁻¹
εijk


Maxwell: if ρ = J = 0, there are still wavelike solutions
$-\frac{1}{\mu_0\epsilon_0}\nabla\wedge(\nabla\wedge E) = \frac{1}{\mu_0\epsilon_0}\nabla^2 E$
$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0\epsilon_0}\nabla^2 E$
c^2
$\partial_i\partial_j E_j - \partial_k\partial_k E_i$
by Gauss
ϵ_{ijk}
$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8\ ms^{-1}$


e.g. $\underline{F} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$
$\nabla^2 \underline{E} = \underline{E}''(x-ct)$


e.g. $\underline{E} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$
$\nabla^2 \underline{E} = \underline{E}''(x-ct)$
$\nabla \cdot \underline{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} = 0$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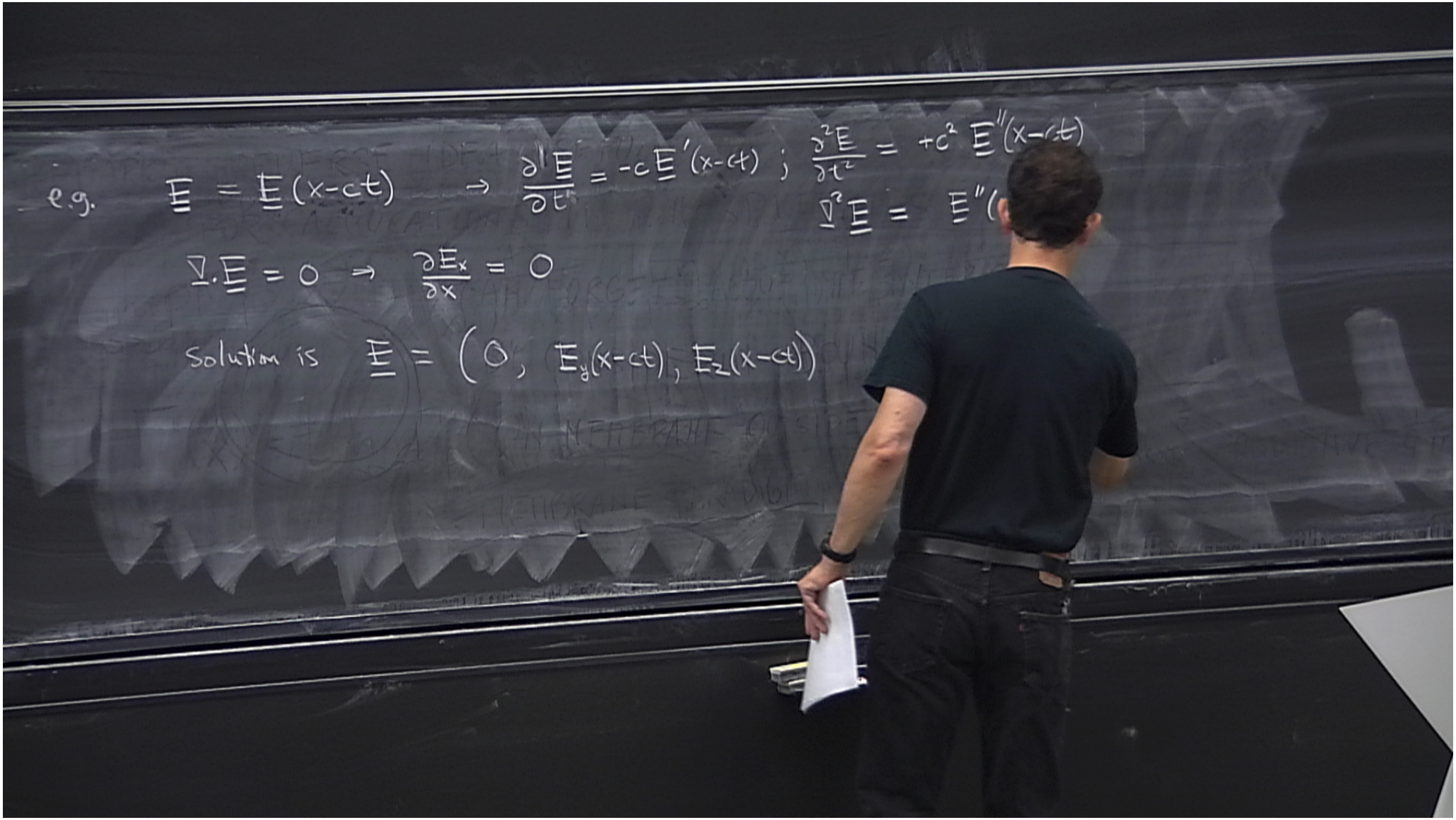
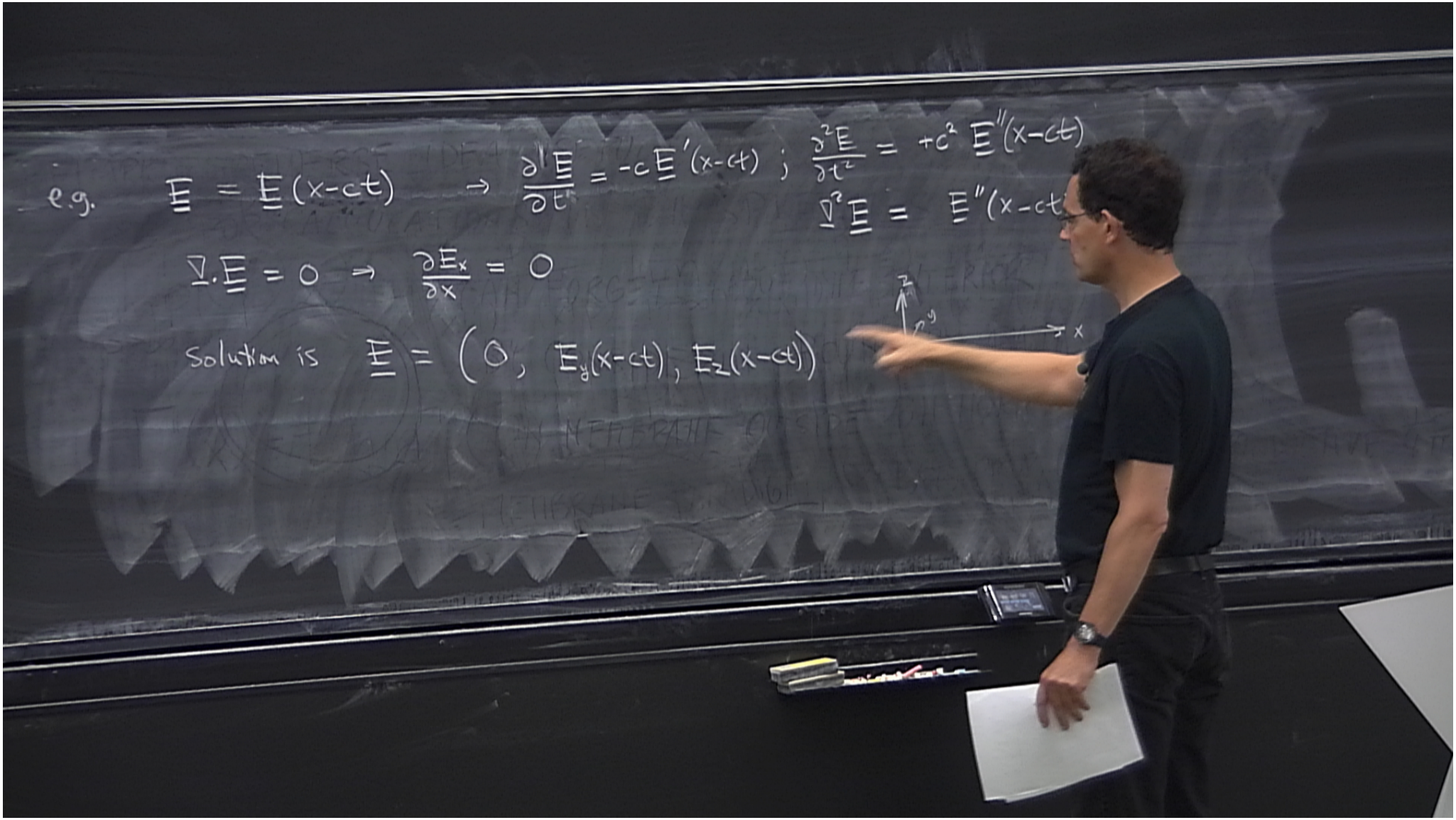
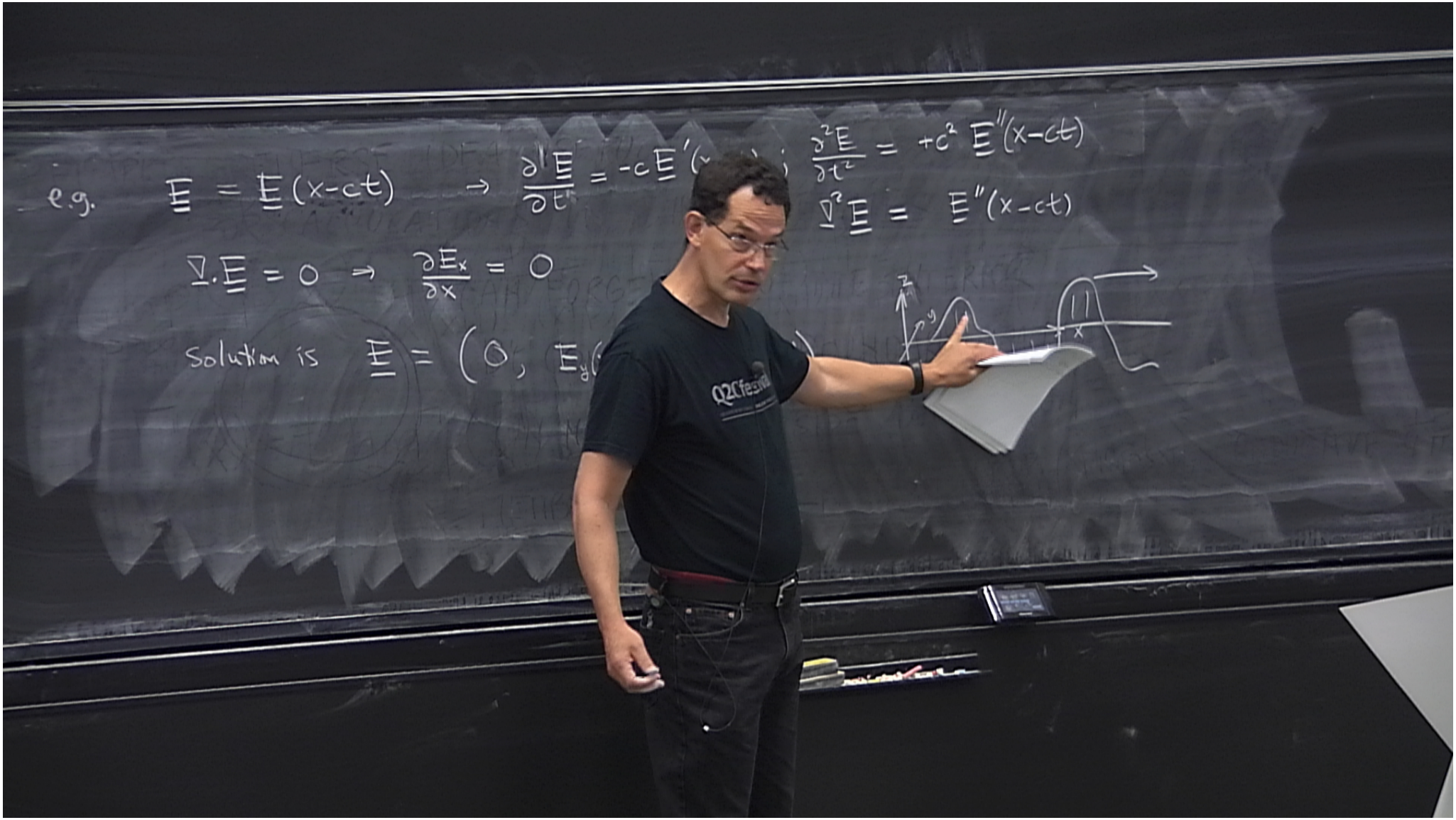
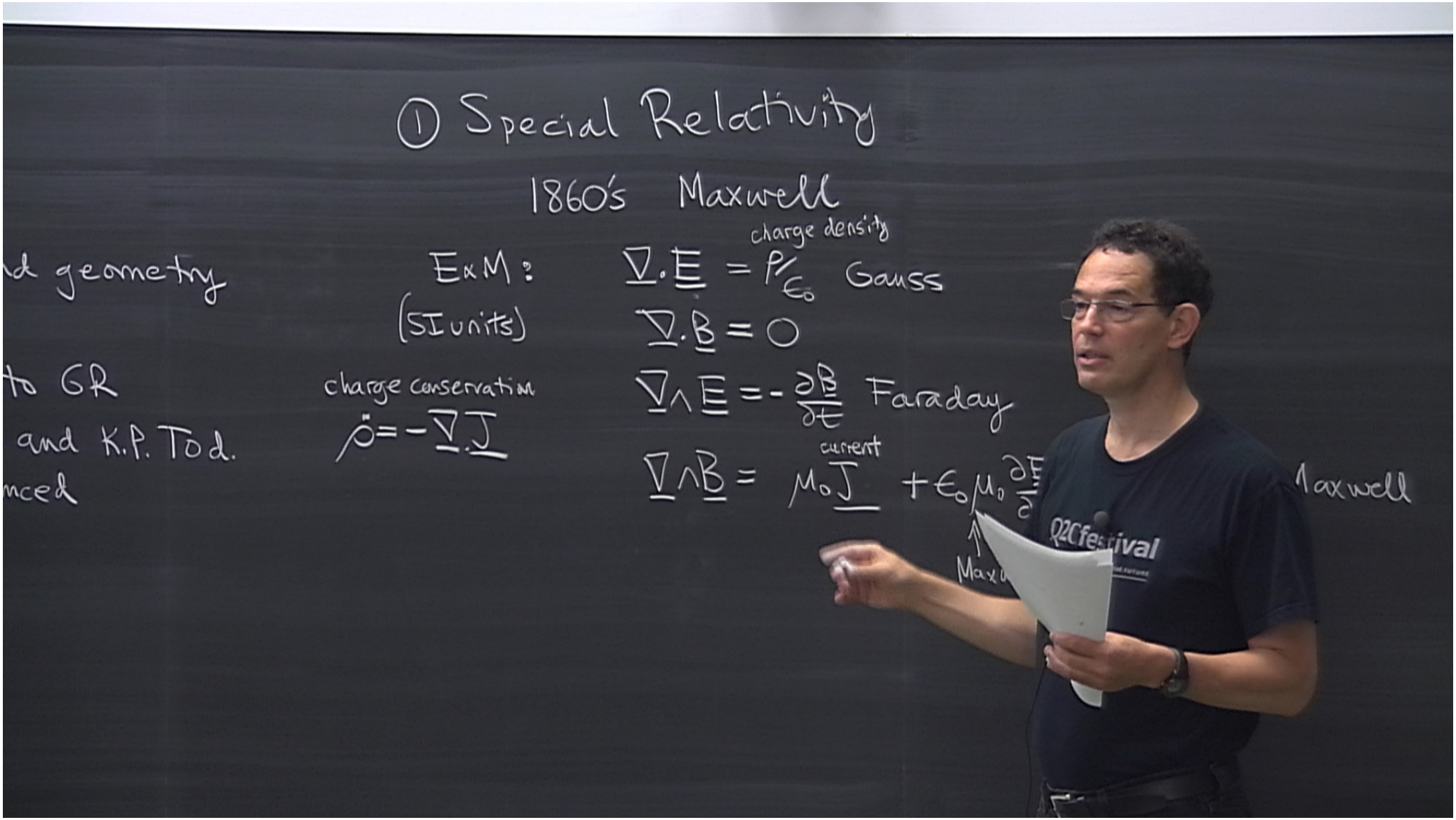

e.g. $\underline{E} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$
$\nabla^2 \underline{E} = \underline{E}''($
$\nabla . \underline{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} = 0$
Solution is $\underline{E} = (0, E_y(x-ct), E_z(x-ct))$


e.g. $\underline{E} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$
$\nabla^2 \underline{E} = \underline{E}''(x-ct)$
$\nabla . \underline{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} = 0$
Solution is $\underline{E} = (0, E_y(x-ct), E_z(x-ct))$
z
y
x


e.g. $\underline{E} = \underline{E}(x-ct)$ → $\frac{\partial^2 \underline{E}}{\partial t^2} = -c\underline{E}'($
$\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$
$\nabla^2 \underline{E} = \underline{E}''(x-ct)$
$\nabla \cdot \underline{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} = 0$
Solution is $\underline{E} = (0, E_y($
z
y
x


① Special Relativity
1860's Maxwell
d geometry
to GR
and K.P. Tod.
nced
E×M:
(SI units)
charge density
$\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss
$\nabla \cdot \underline{B} = 0$
charge conservation
$\dot{\rho} = -\nabla \cdot \underline{J}$
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday
current
$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial}$
Maxwell
Q2Cfestival

① Special Relativity

1860's Maxwell

d geometry

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E×M: (SI units)

charge density

$\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss

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current

$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$ Ampere + Maxwell

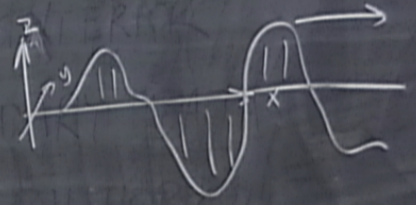
↑ Maxwell's correction.

e.g. $\underline{E} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$

$\nabla^2 \underline{E} = \underline{E}''(x-ct)$

$\nabla \cdot \underline{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} = 0$

Solution is $\underline{E} = (0, E_y(x-ct), E_z(x-ct))$

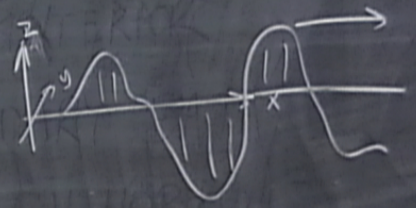


e.g. $\underline{E} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$

$\nabla^2 \underline{E} = \underline{E}''(x-ct)$

$\nabla \cdot \underline{E} = 0 \rightarrow \frac{\partial E_x}{\partial x} = 0$

Solution is $\underline{E} = (0, E_y(x-ct), E_z(x-ct))$



Ex: calculate $\underline{B}$.

e.g. $\underline{E} = \underline{E}(x-ct) \rightarrow \frac{\partial \underline{E}}{\partial t} = -c\underline{E}'(x-ct)$; $\frac{\partial^2 \underline{E}}{\partial t^2} = +c^2 \underline{E}''(x-ct)$

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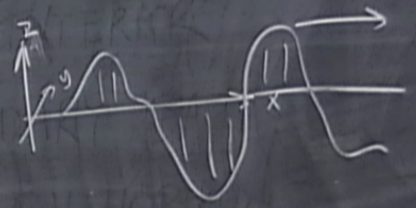
Solution is $\underline{E} = (0, E_y(x-ct), E_z(x-ct))$



Ex: calculate $\underline{B}$.

① Special Relativity

1860's Maxwell : Field theory

d geometry

to GR

and K.P. Tod.

nced

E×M: (SI units)

$\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss (charge density)

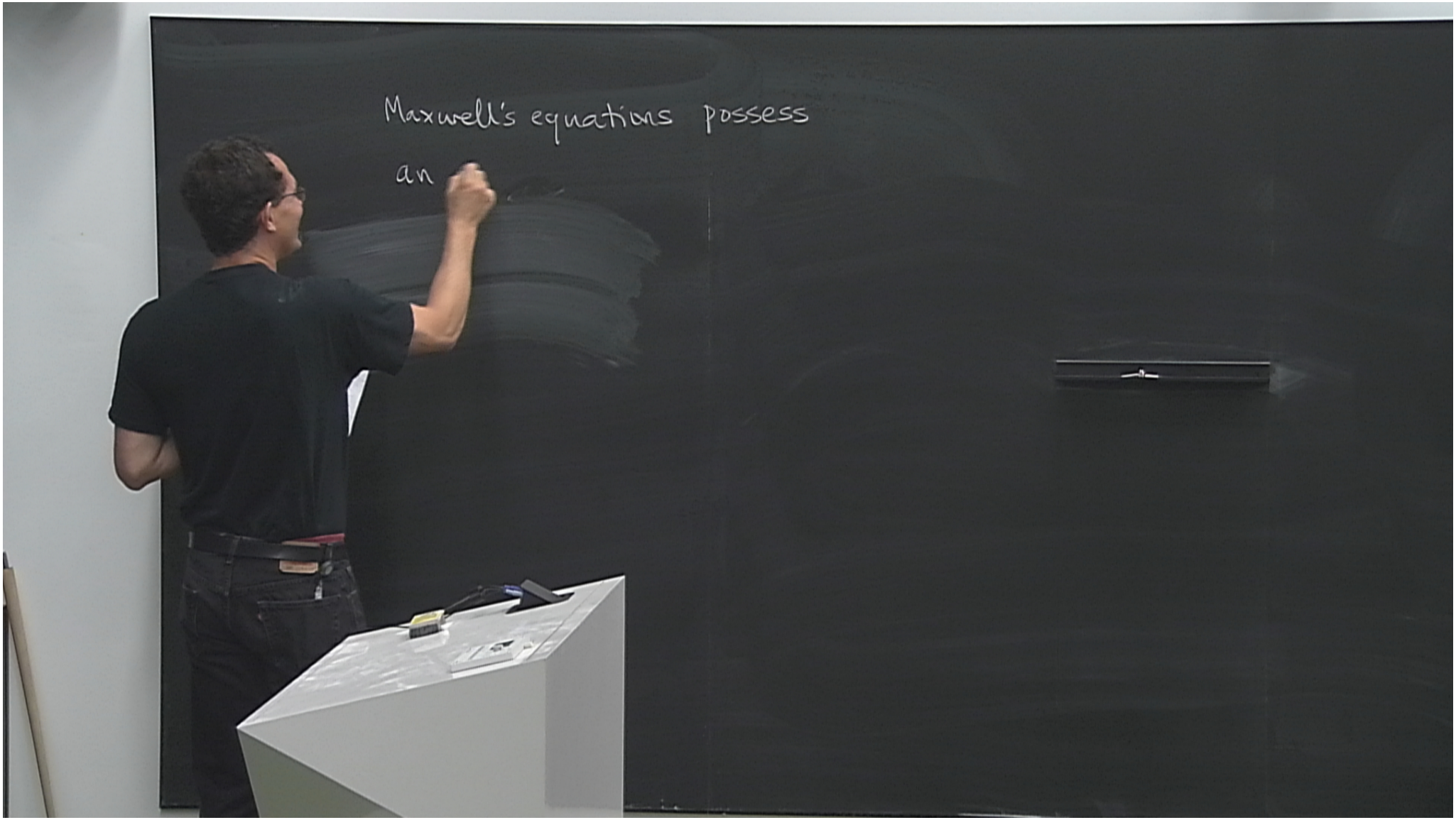
$\nabla \cdot \underline{B} = 0$

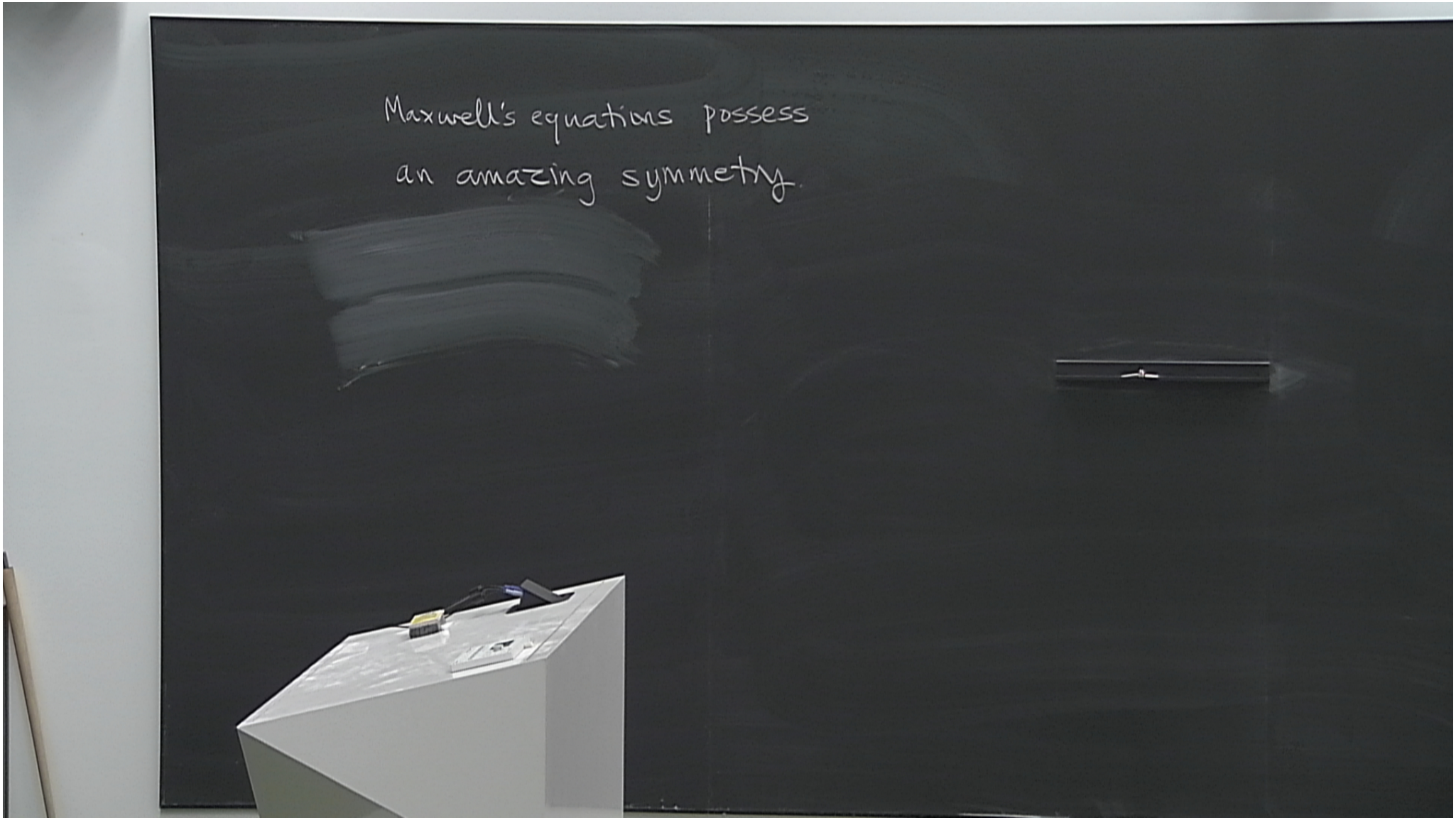
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday

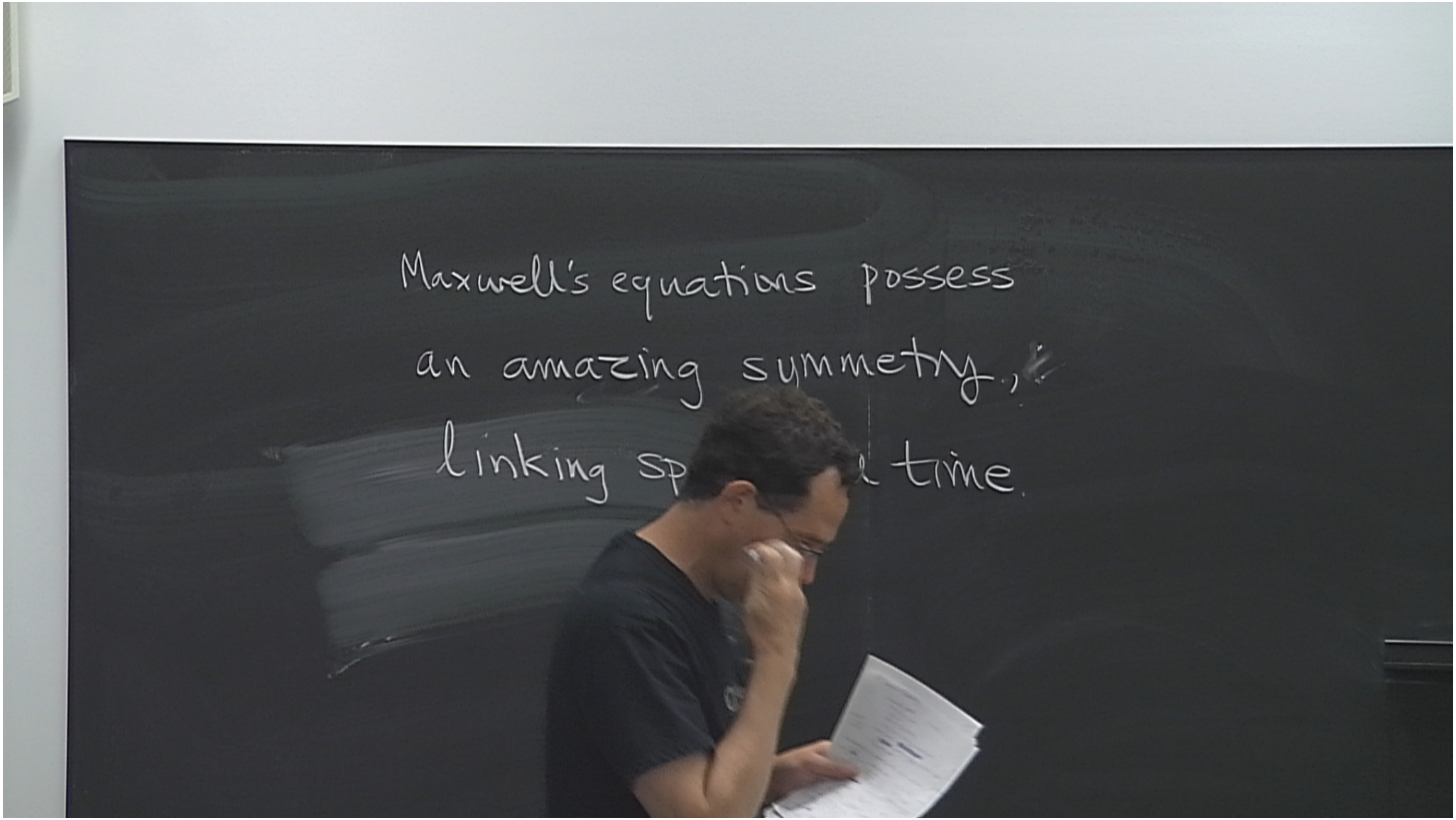
charge conservation $\dot{\rho} = -\nabla \cdot \underline{J}$

$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$ Ampere + Maxwell (current)

↑ Maxwell's correction


Maxwell's equations possess
an


Maxwell's equations possess
an amazing symmetry.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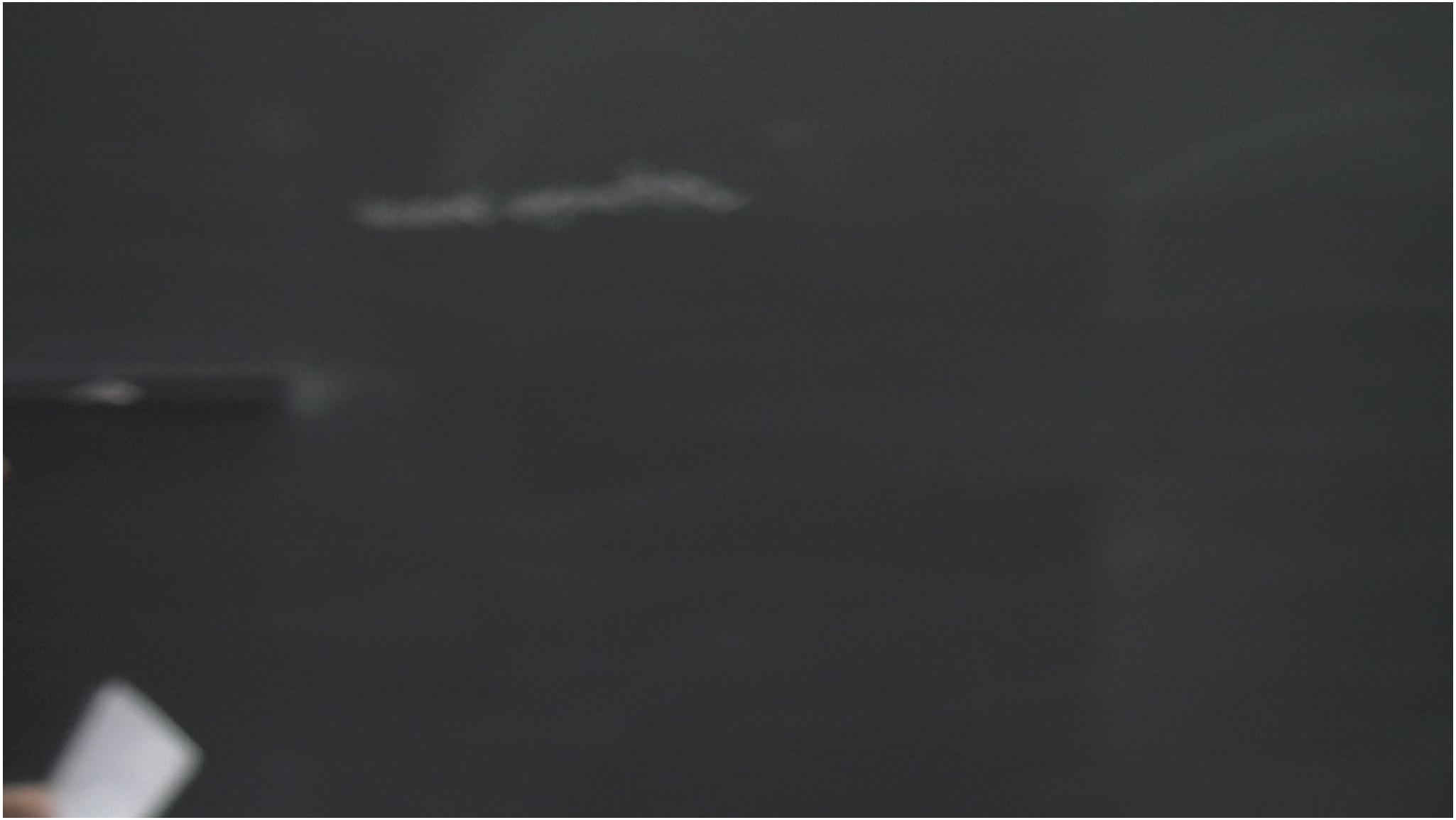
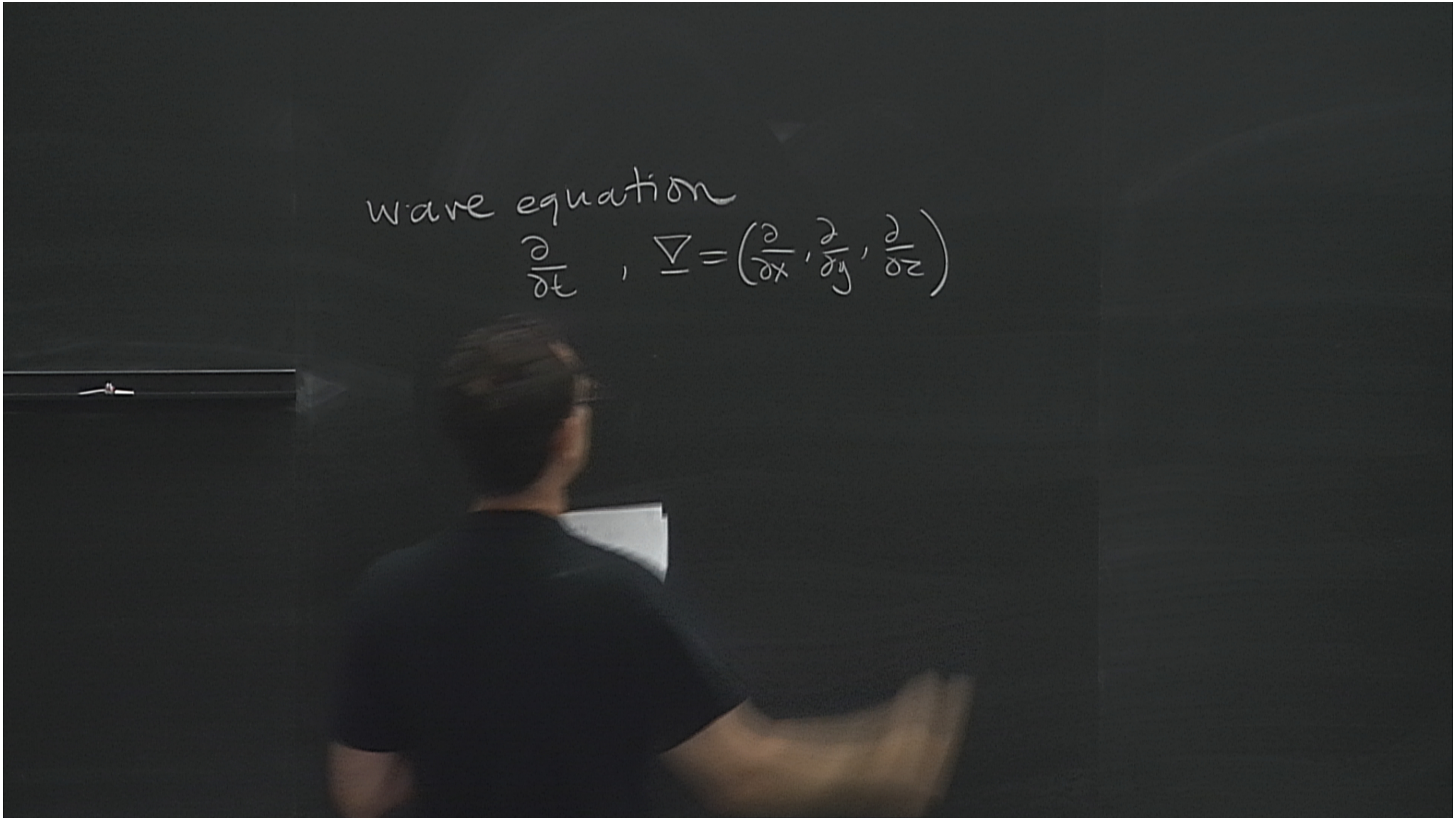
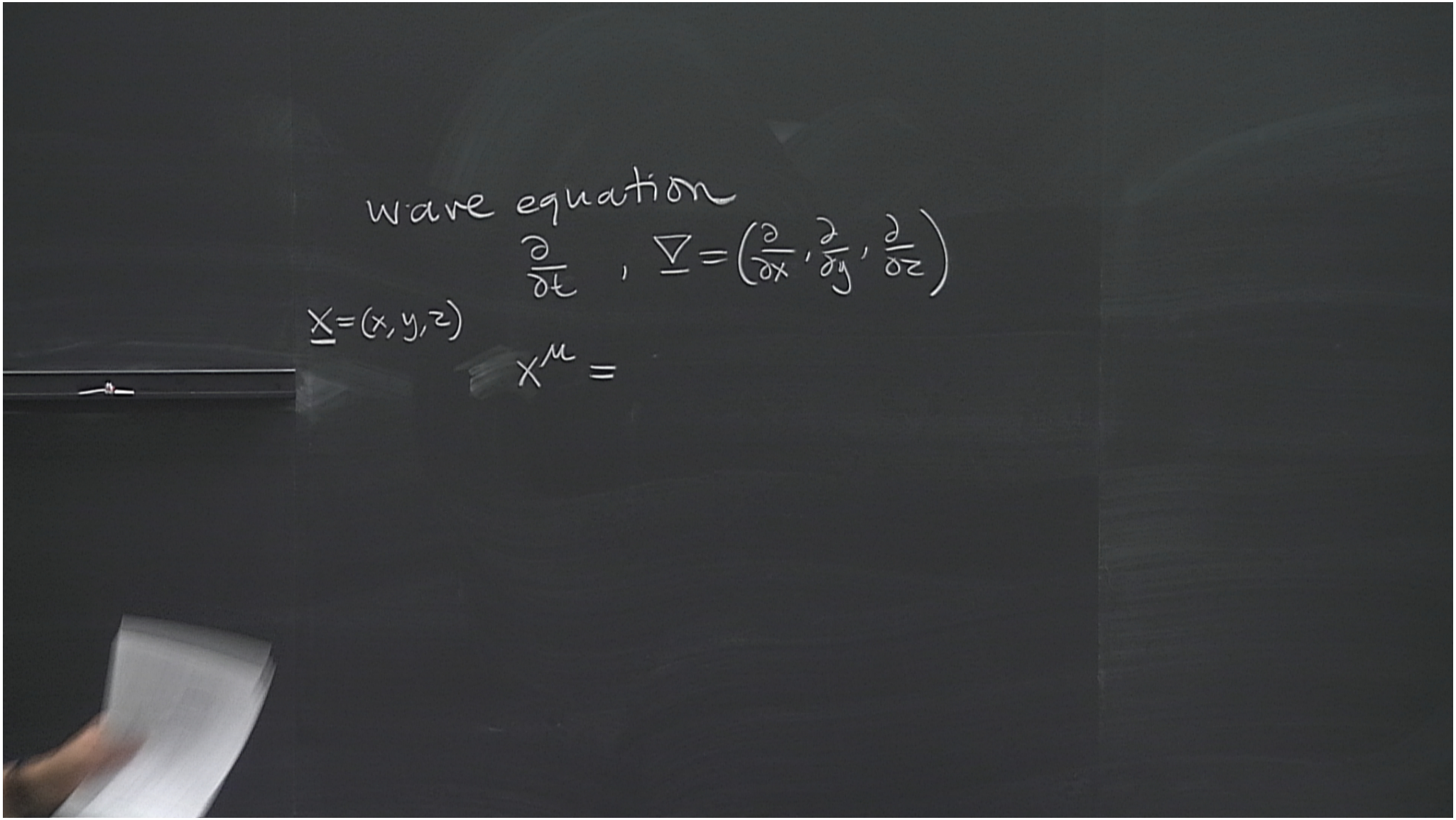
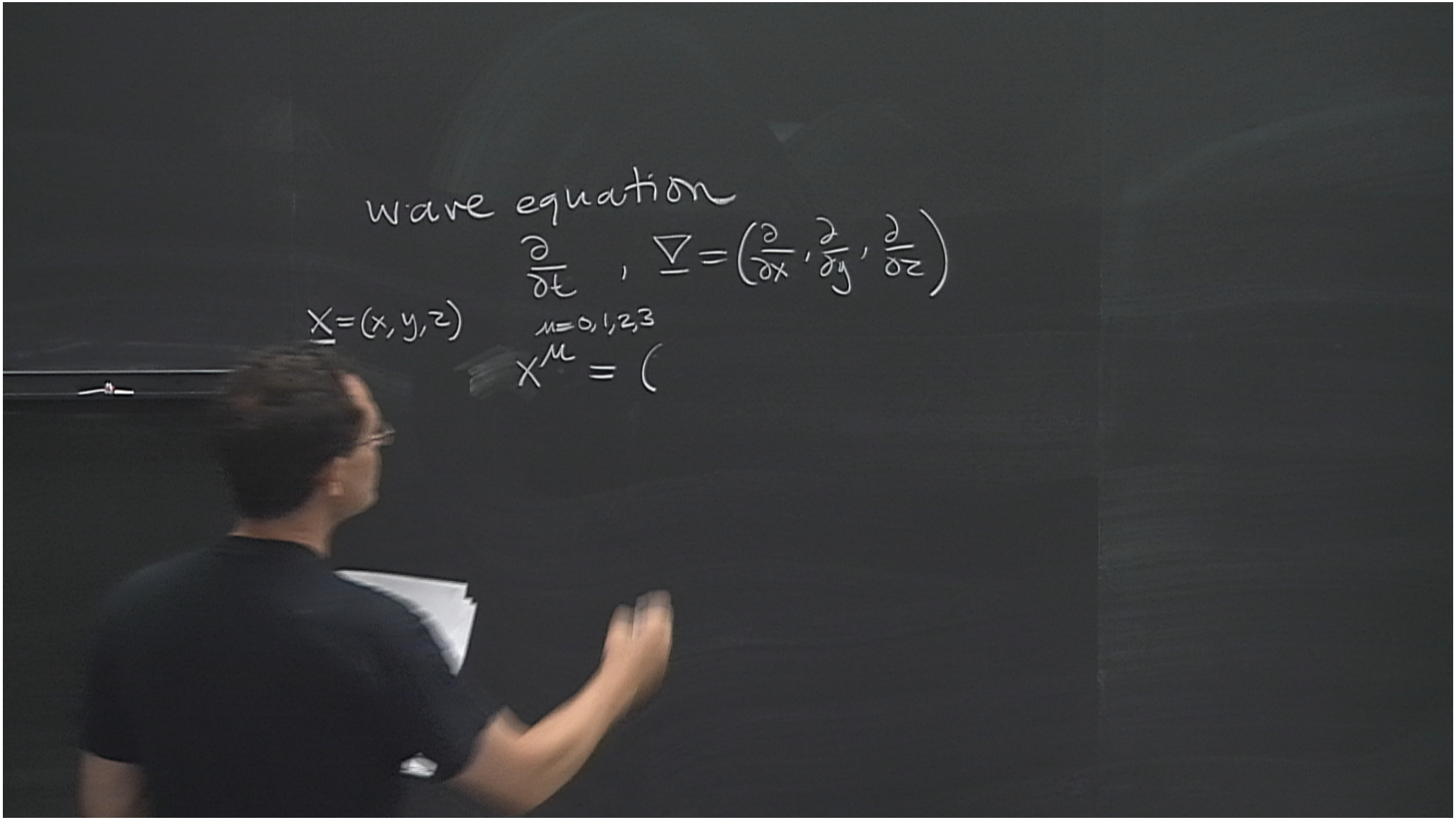
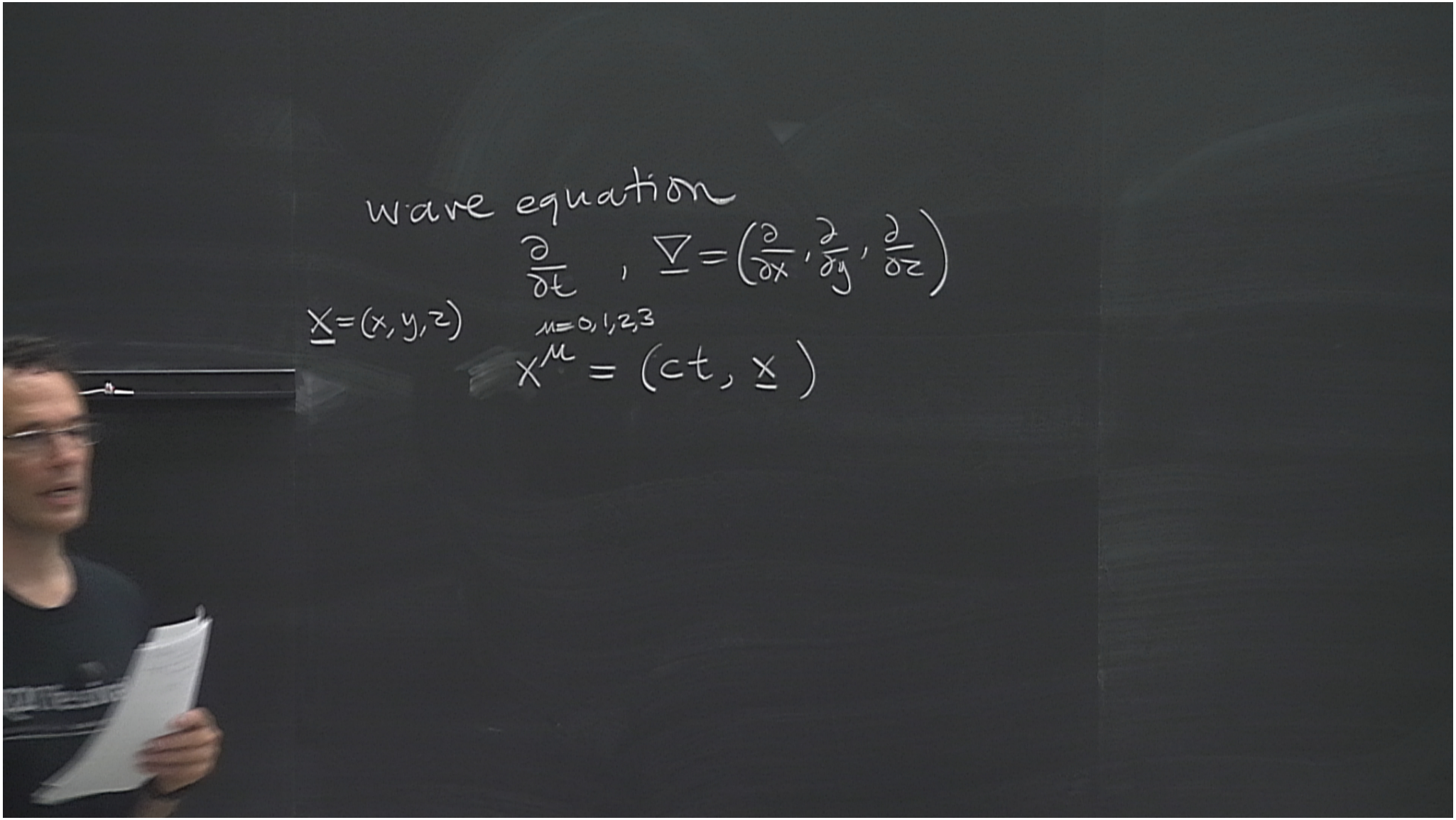
time.




wave equation
$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$


wave equation
$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
$\underline{x} = (x, y, z)$
$x^\mu =$


wave equation
$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
$\underline{x} = (x, y, z)$
$\mu = 0, 1, 2, 3$
$x^\mu = ($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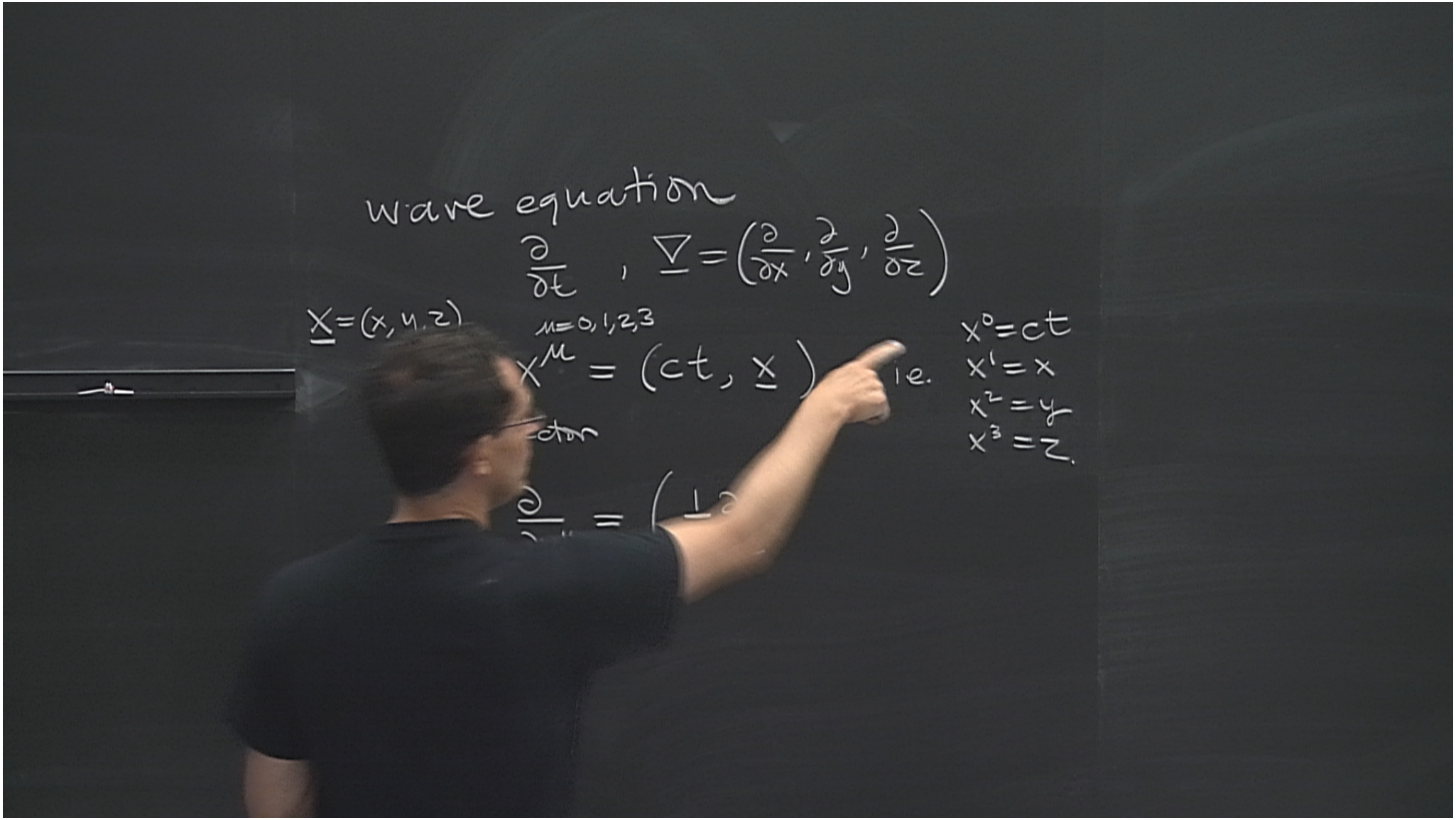
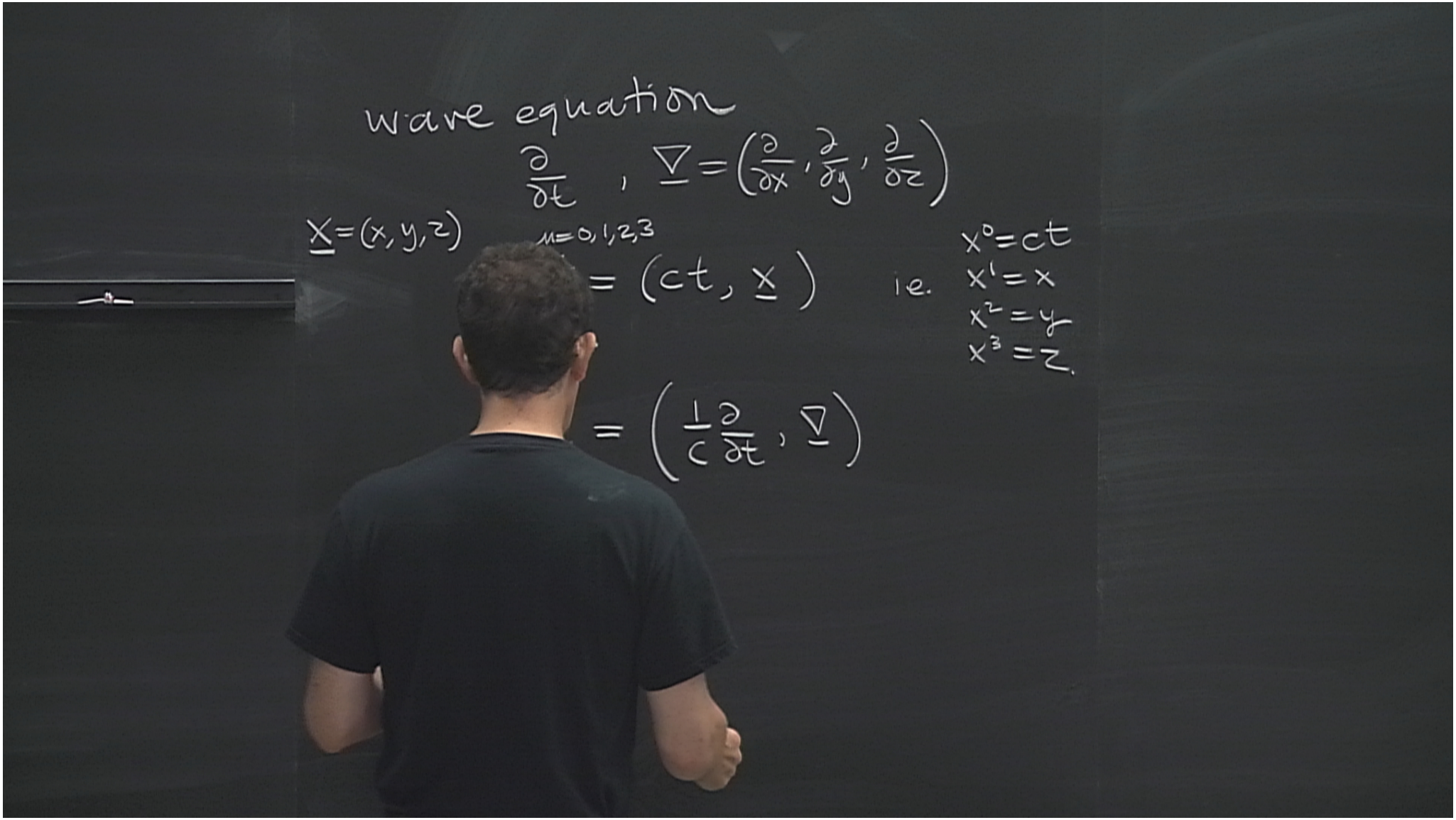
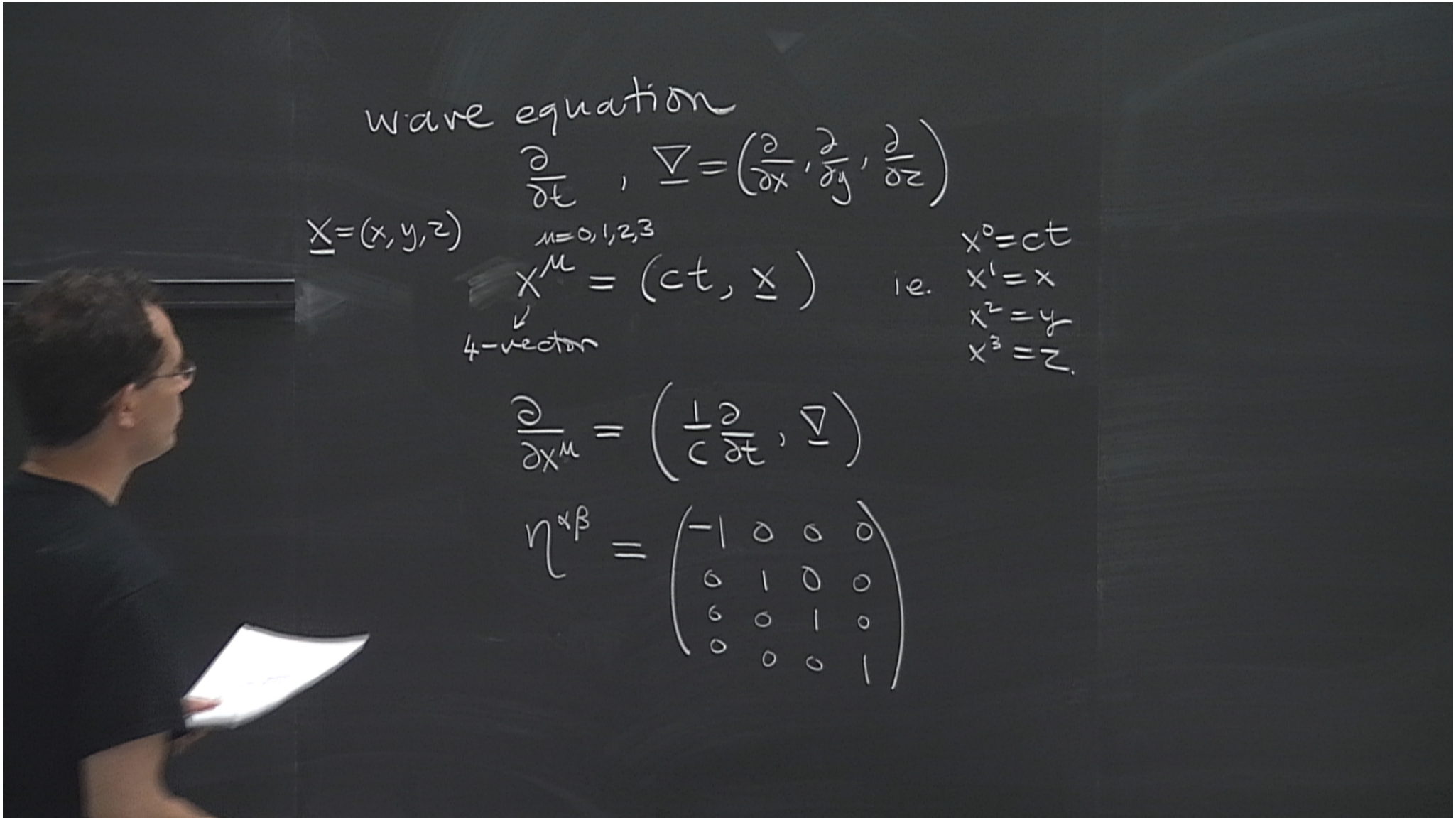
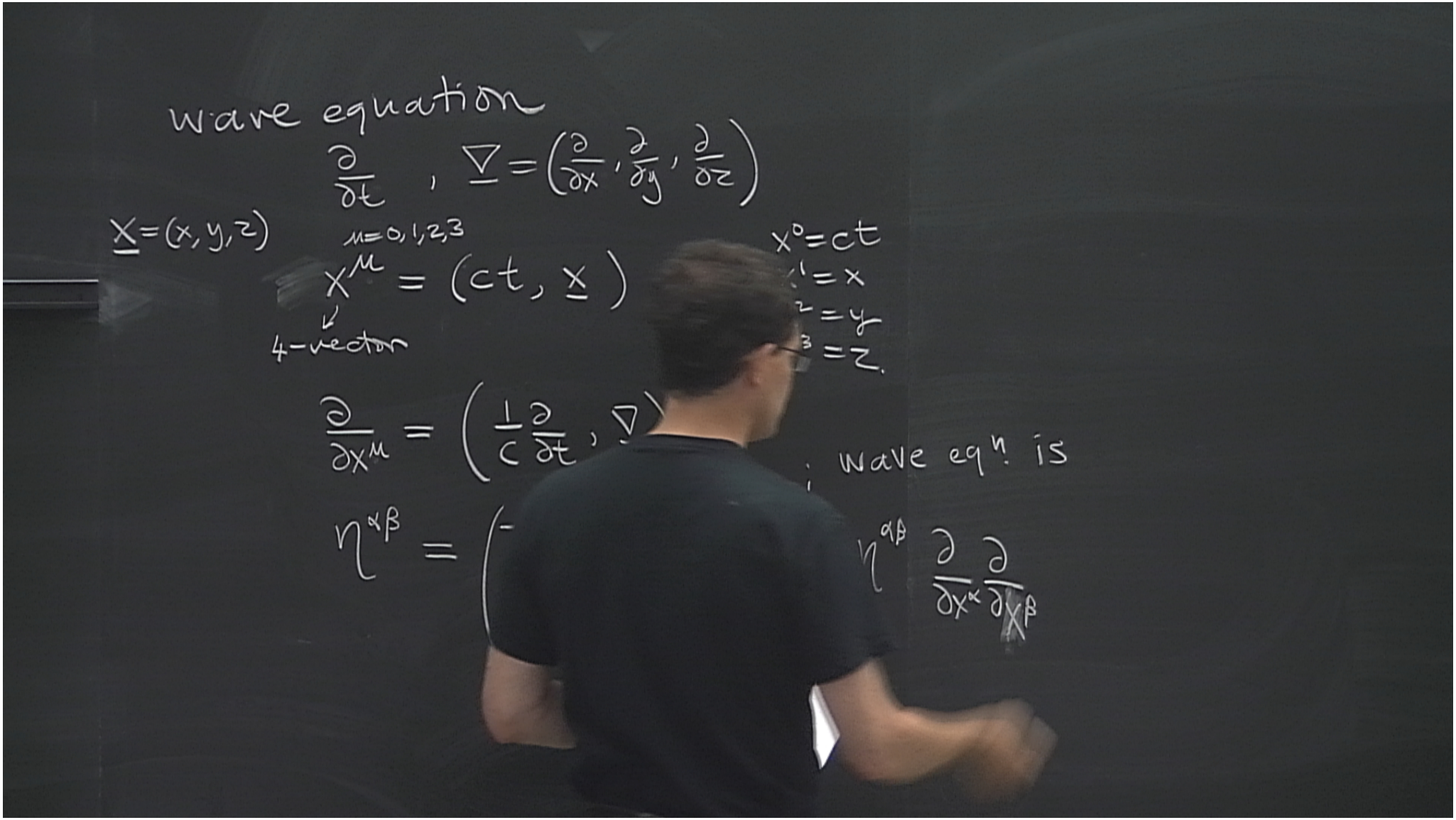
$\mu = 0, 1, 2, 3$
$x^\mu = (ct, \underline{x})$


wave equation
$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
$\underline{x} = (x, y, z)$
$\mu = 0, 1, 2, 3$
$x^\mu = (ct, \underline{x})$
i.e.
$x^0 = ct$
$x^1 = x$
$x^2 = y$
$x^3 = z$.


wave equation
$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
$\underline{x} = (x, y, z)$
$\mu = 0, 1, 2, 3$
$= (ct, \underline{x})$
ie.
$x^0 = ct$
$x^1 = x$
$x^2 = y$
$x^3 = z$.
$= \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right)$


wave equation
$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
$\underline{x} = (x, y, z)$
$\mu = 0, 1, 2, 3$
$x^\mu = (ct, \underline{x})$
4-vector
ie.
$x^0 = ct$
$x^1 = x$
$x^2 = y$
$x^3 = z$.
$\frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right)$
$\eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$


wave equation
$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
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$x^0 = ct$
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$\frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right)$
; wave eqn is
$\eta^{\alpha\beta} = \left($
$\eta^{\alpha\beta} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta}$

Wave equation

$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$\underline{x} = (x, y, z)$

$\mu = 0, 1, 2, 3$

$x^\mu = (ct, \underline{x})$ ie. $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$.

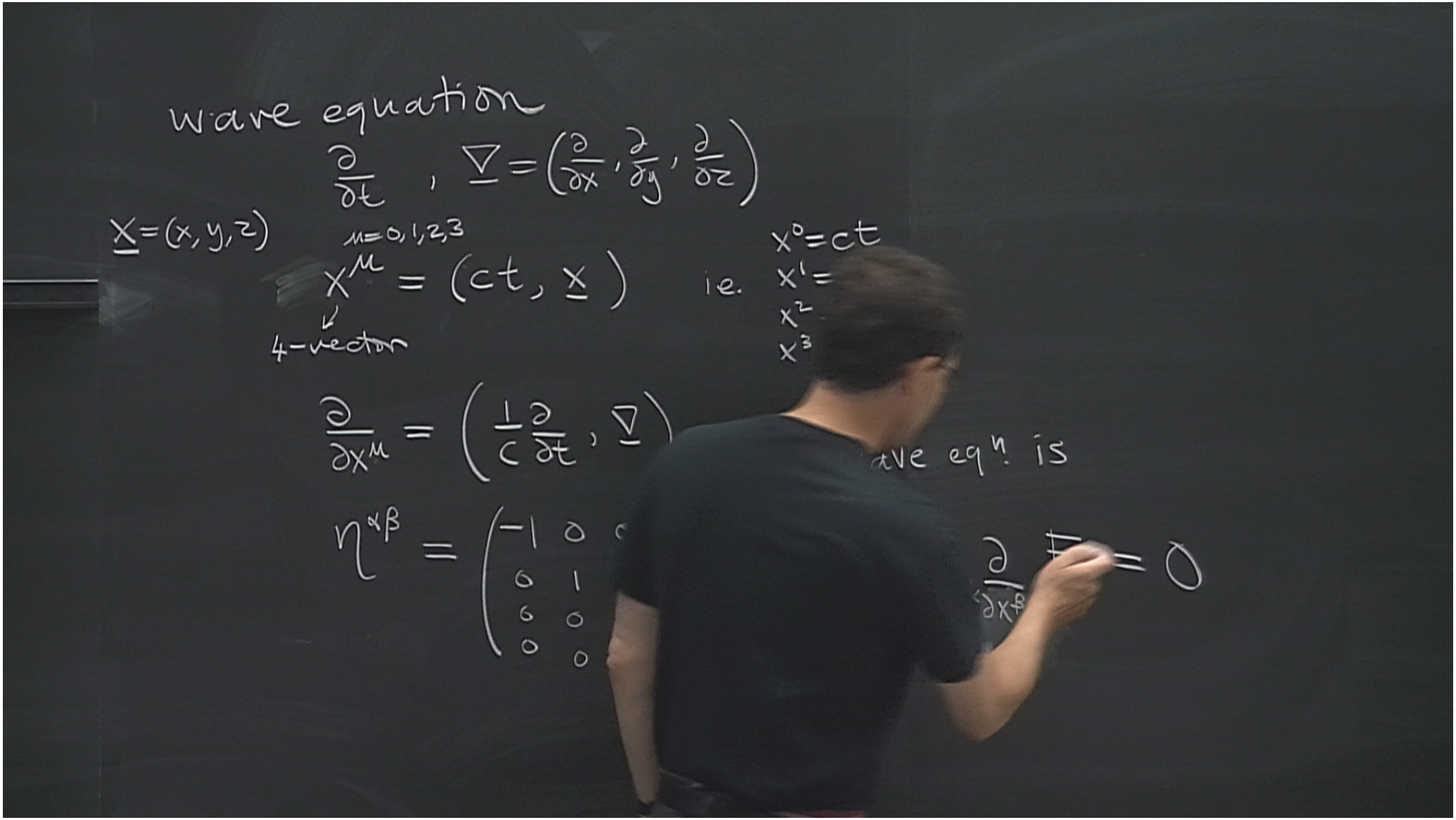
x^μ ↙ 4-vector

$\frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right)$

$\eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

; wave eqn is

$\eta^{\alpha\beta} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} F = 0$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$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$
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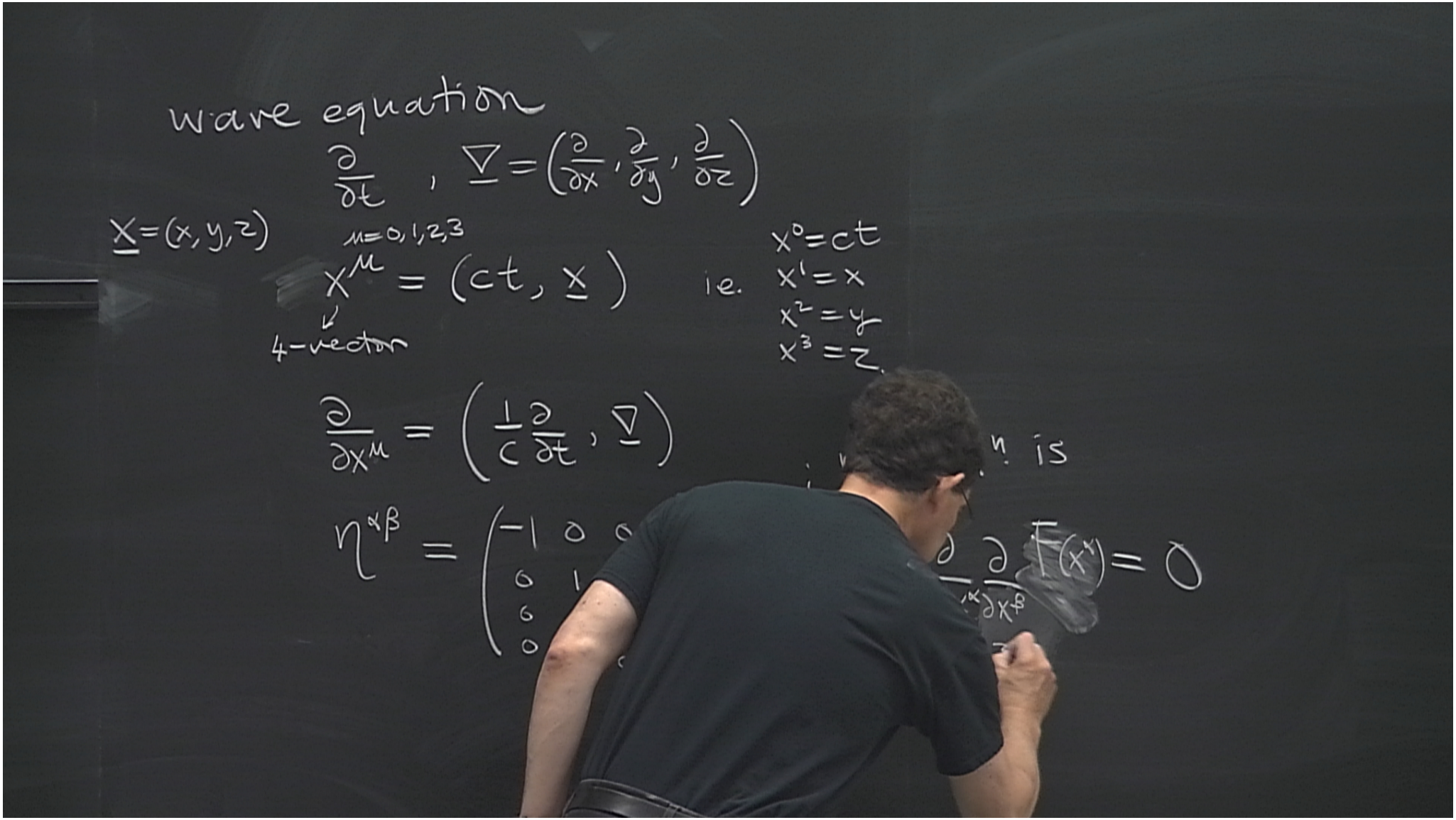
$\mu = 0, 1, 2, 3$
$x^\mu = (ct, \underline{x})$
4-vector
ie. $x^0 = ct$
$x^1 =$
x^2
x^3
$\frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right)$
ave eqn is
$\eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 & \\ 0 & 1 & \\ 0 & 0 & \\ 0 & 0 & \end{pmatrix}$
$= 0$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Wave equation

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4-vector

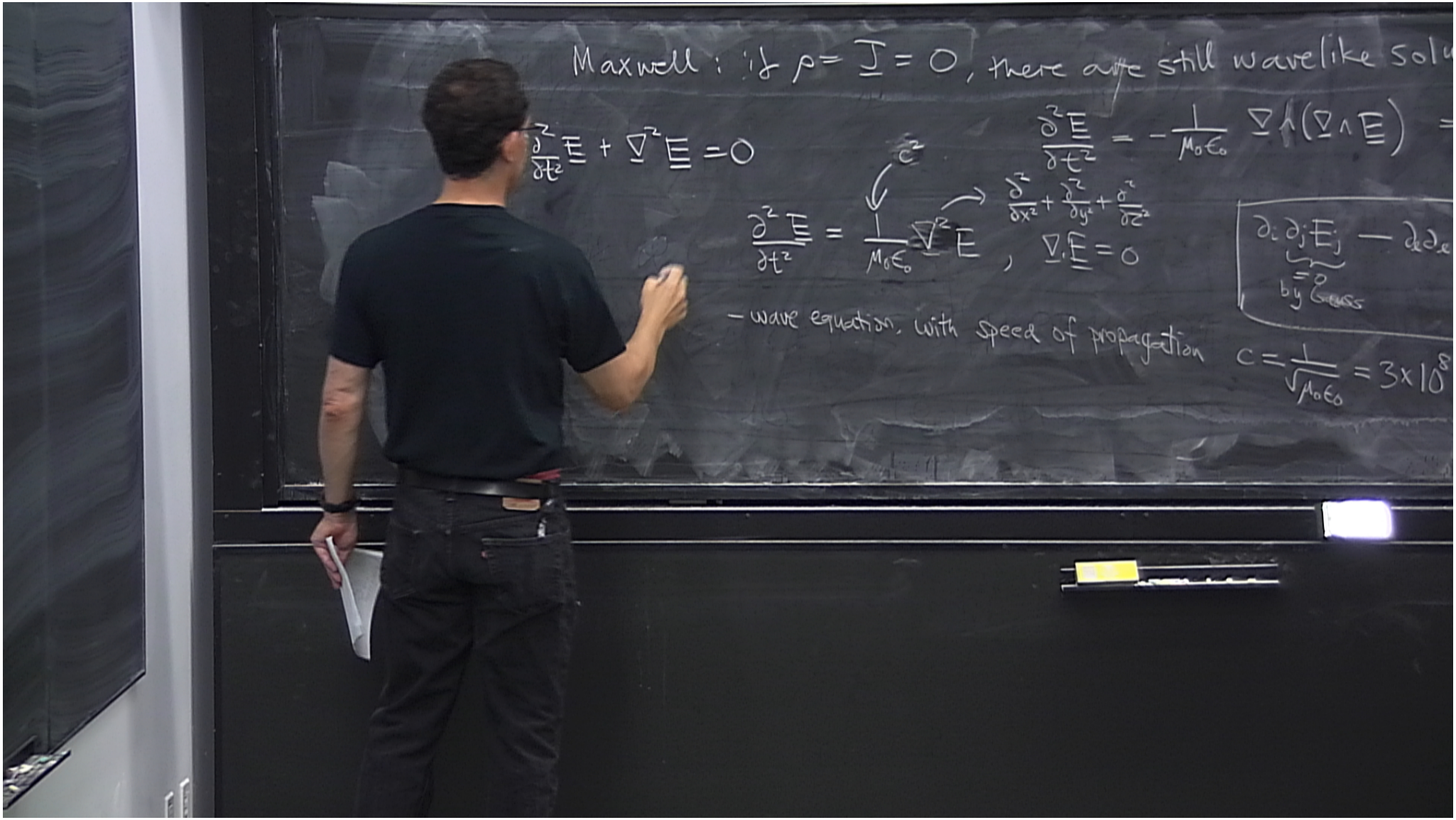
$\frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right)$

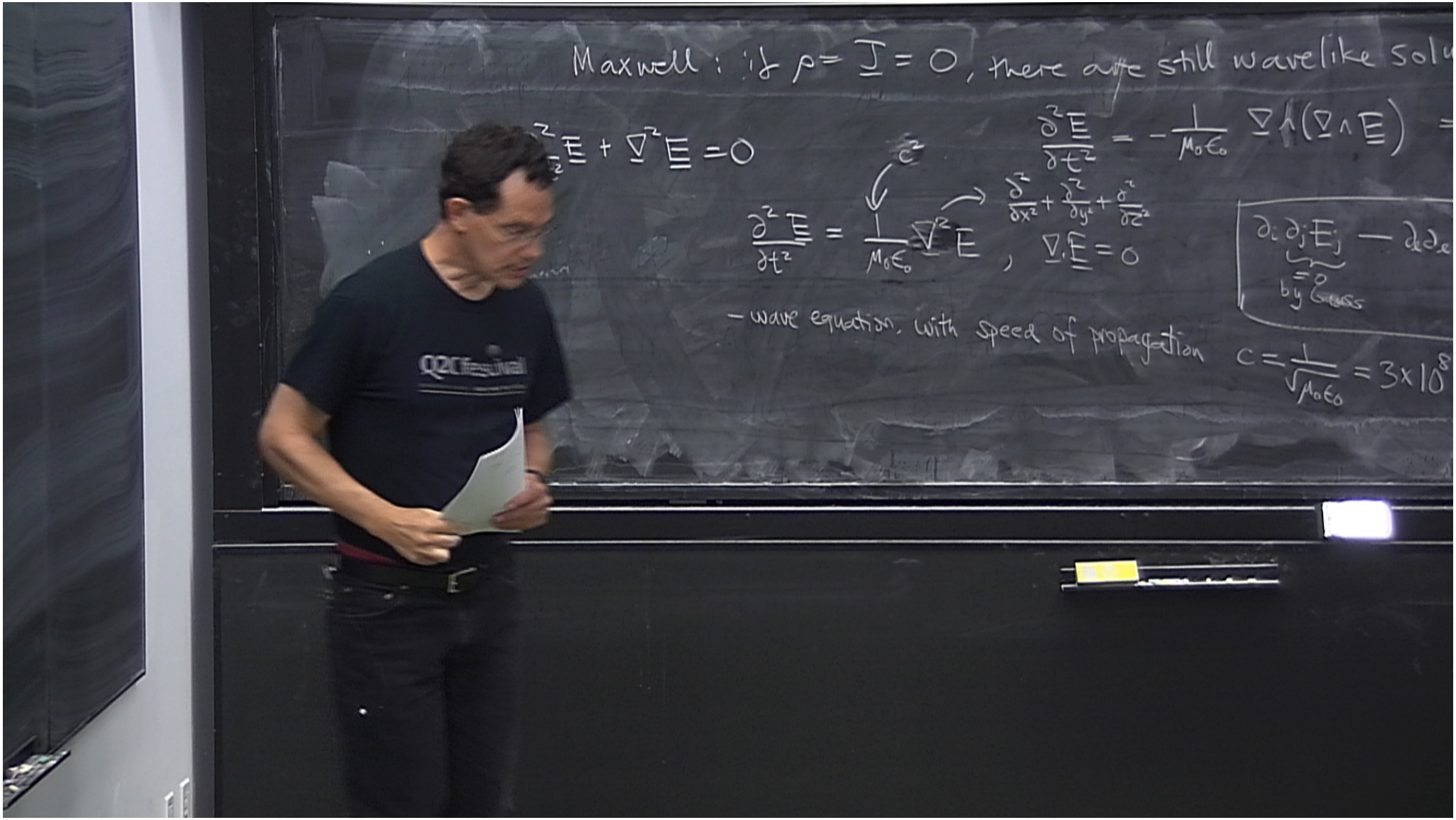
$\eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

∴ wave eqn is

$\eta^{\alpha\beta} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} f(x^\mu) = 0$

$-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$


Maxwell: if ρ = J = 0, there are still wavelike sol
$\frac{\partial^2 E}{\partial t^2} + \nabla^2 E = 0$
$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \nabla^2 E$, $\nabla \cdot E = 0$
c^2
$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
$\frac{\partial^2 E}{\partial t^2} = -\frac{1}{\mu_0 \epsilon_0} \nabla \times (\nabla \times E)$
$\partial_i \partial_j E_j - \partial_j \partial_j$
by Gauss
– wave equation, with speed of propagation $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$


Maxwell: if ρ = J = 0, there are still wavelike sol
∇²E + ∇²E = 0
∂²E/∂t² = −1/μ₀ε₀ ∇×(∇×E) =
c²
∂²E/∂t² = 1/μ₀ε₀ ∇²E
∂²/∂x² + ∂²/∂y² + ∂²/∂z²
∇·E = 0
∂ᵢ∂ⱼEⱼ − ∂ⱼ∂ⱼ
= 0 by Gauss
– wave equation, with speed of propagation c = 1/√(μ₀ε₀) = 3×10⁸

Wave equation

$\frac{\partial}{\partial t}$, $\underline{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$\underline{x} = (x, y, z)$

$\mu = 0, 1, 2, 3$

$x^\mu = (ct, \underline{x})$ ie. $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$.

4-vector

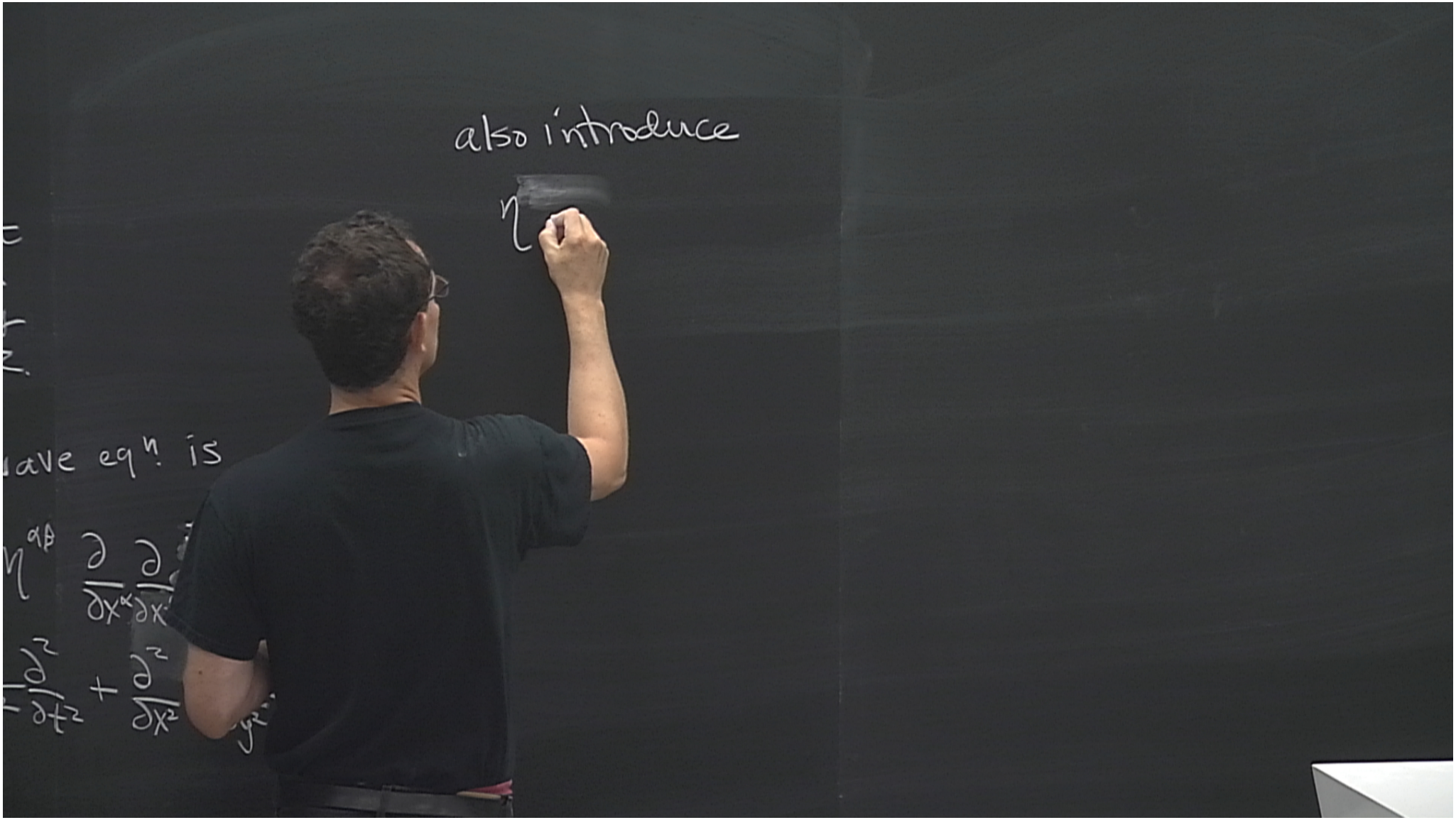
$\frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \underline{\nabla}\right)$

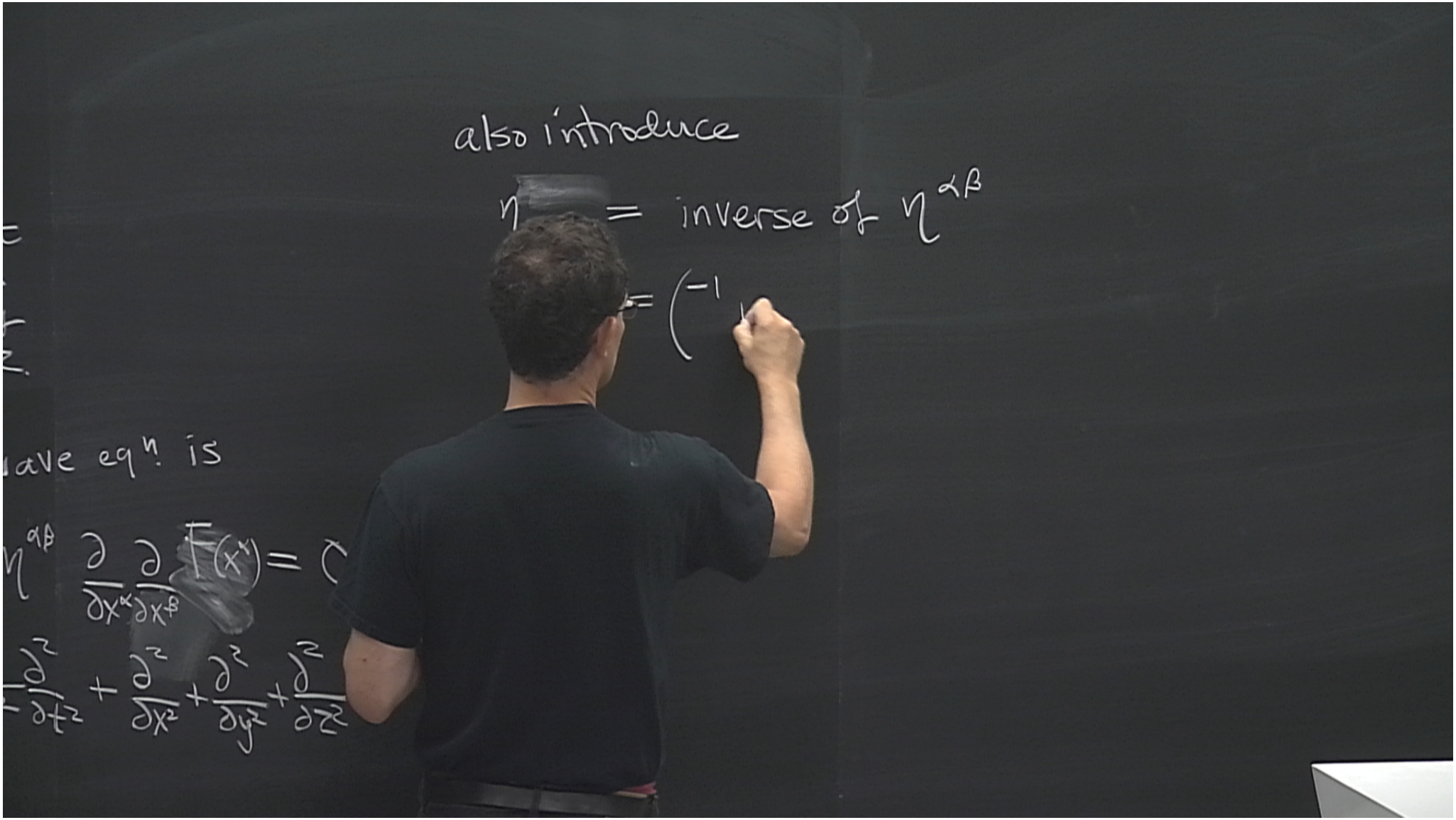
$\eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

; wave eqn is

$\eta^{\alpha\beta} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} \Gamma(x^\mu) = 0$

$-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$


also introduce
η
ave eqn is


also introduce
inverse of
ave eqn is

also introduce

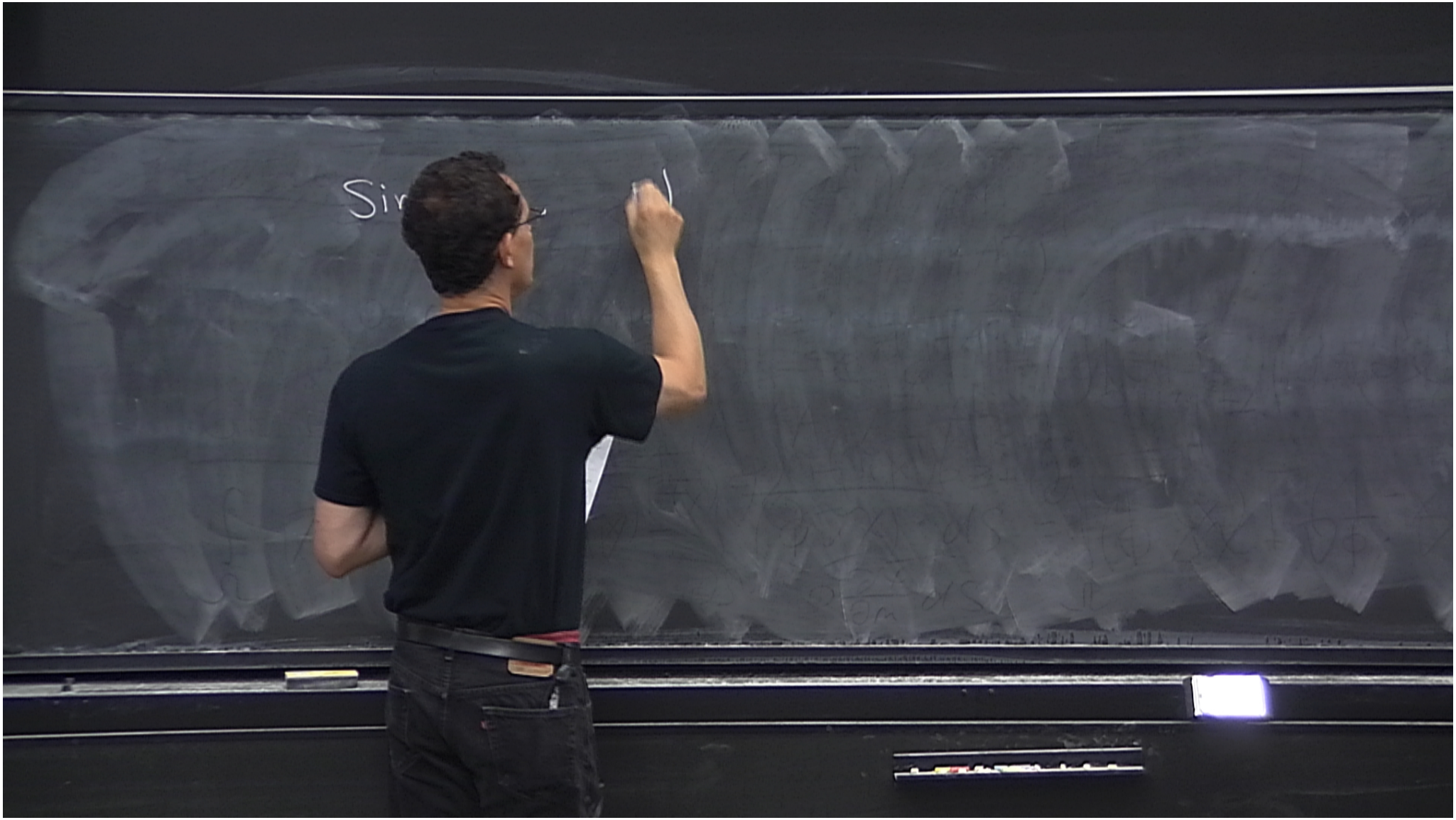
$\eta_{\alpha\beta}$ = inverse of $\eta^{\alpha\beta}$

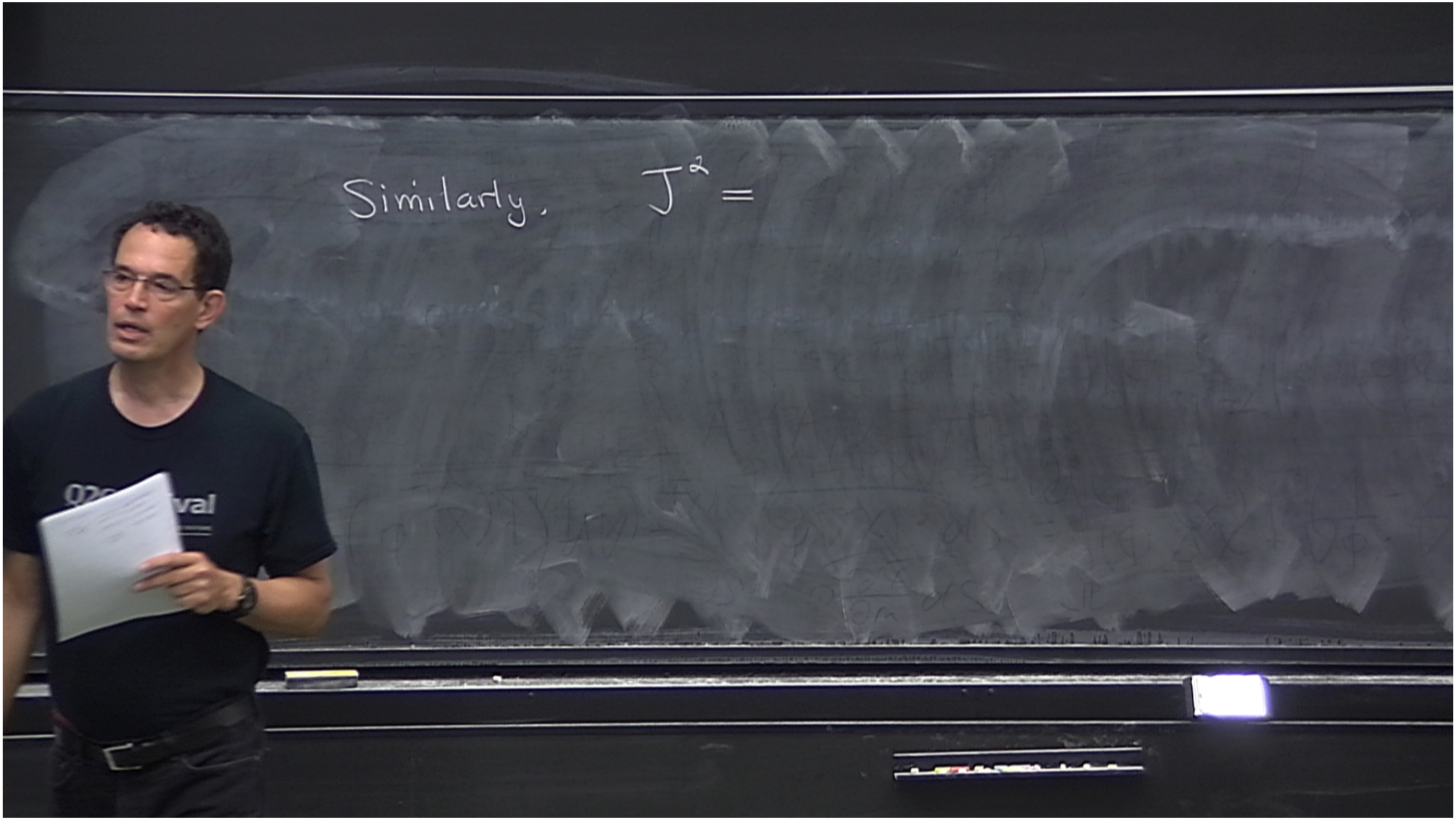
$$= \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

ave eqn is

$$\eta^{\alpha\beta} \frac{\partial}{\partial x^\alpha} \frac{\partial}{\partial x^\beta} F(x^\mu) = 0$$

$$\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$




Similarly. $J^2 =$

① Special Relativity

1860's Maxwell : Field theory

d geometry

to GR

and K.P. Tod.

nced

E×M: (SI units)

$\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss (charge density)

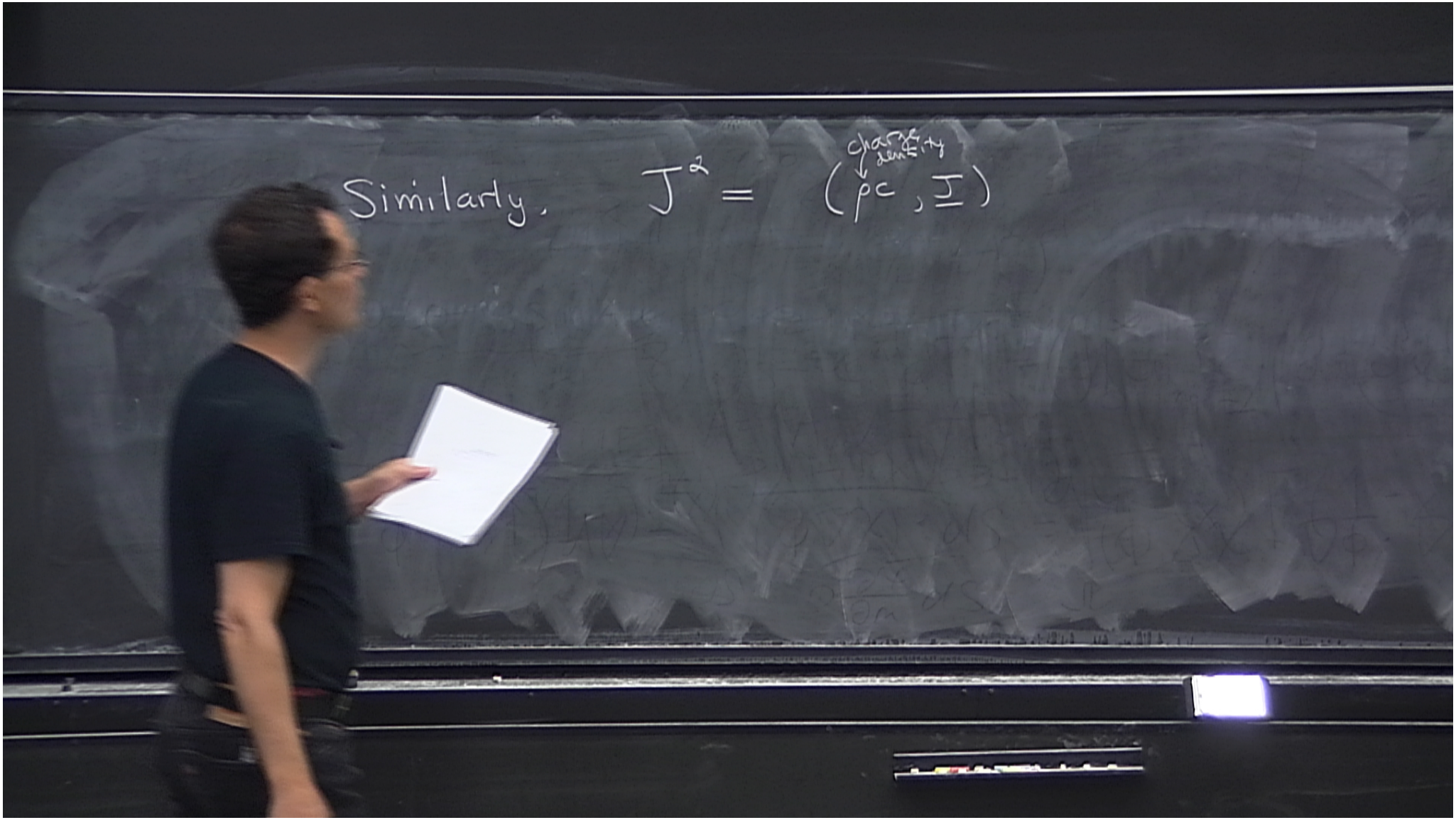
$\nabla \cdot \underline{B} = 0$

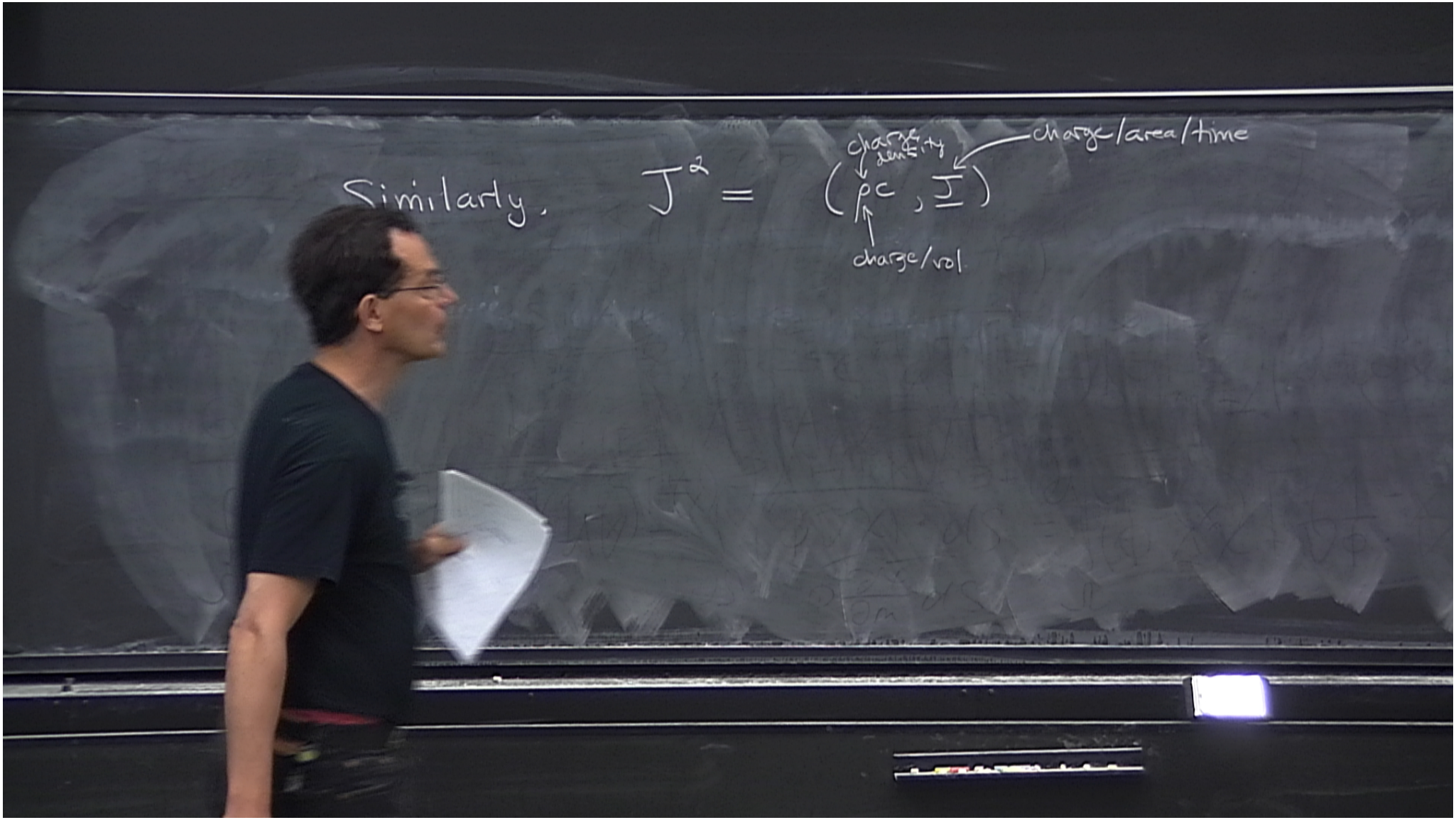
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday

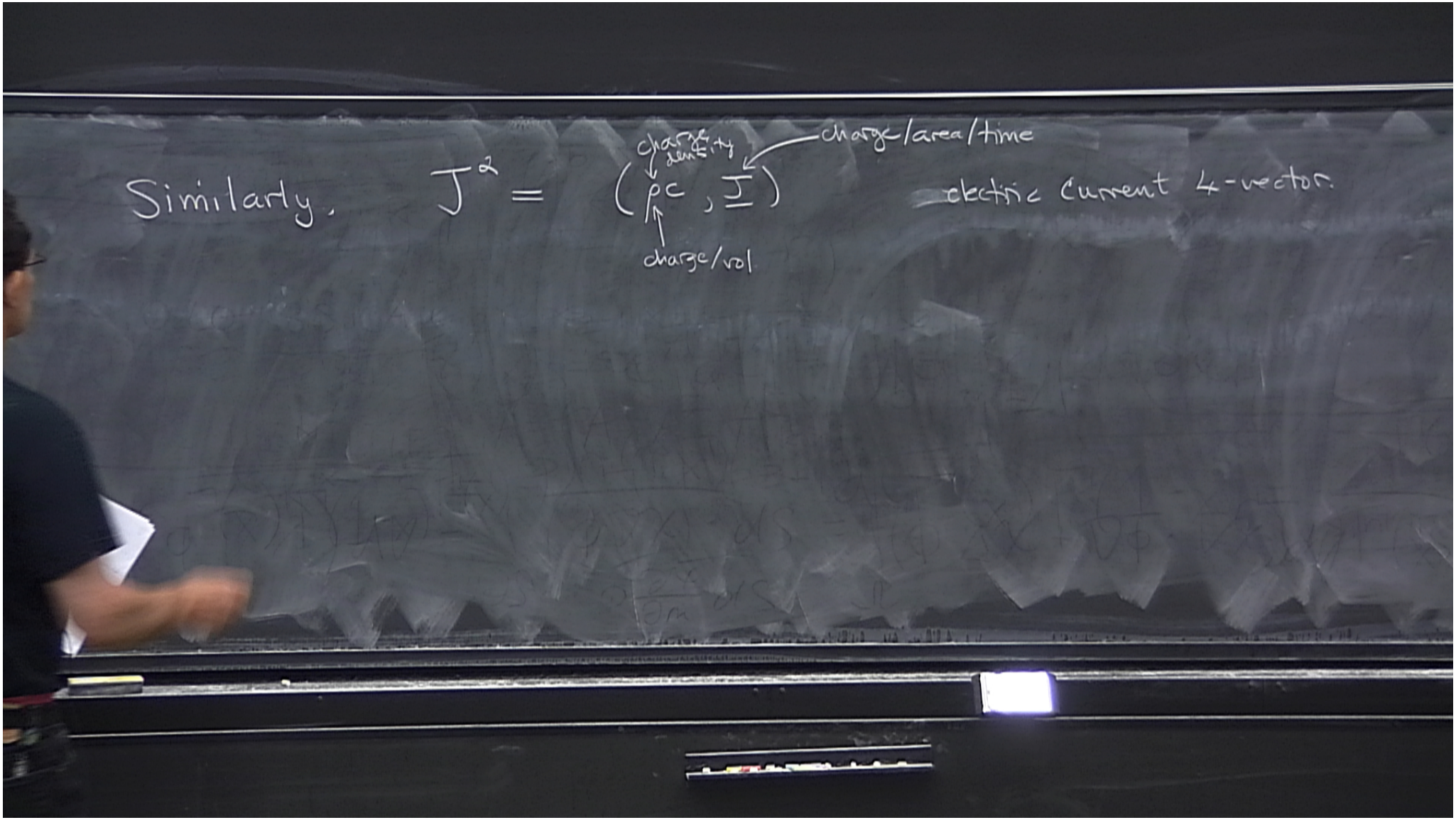
charge conservation $\dot{\rho} = -\nabla \cdot \underline{J}$

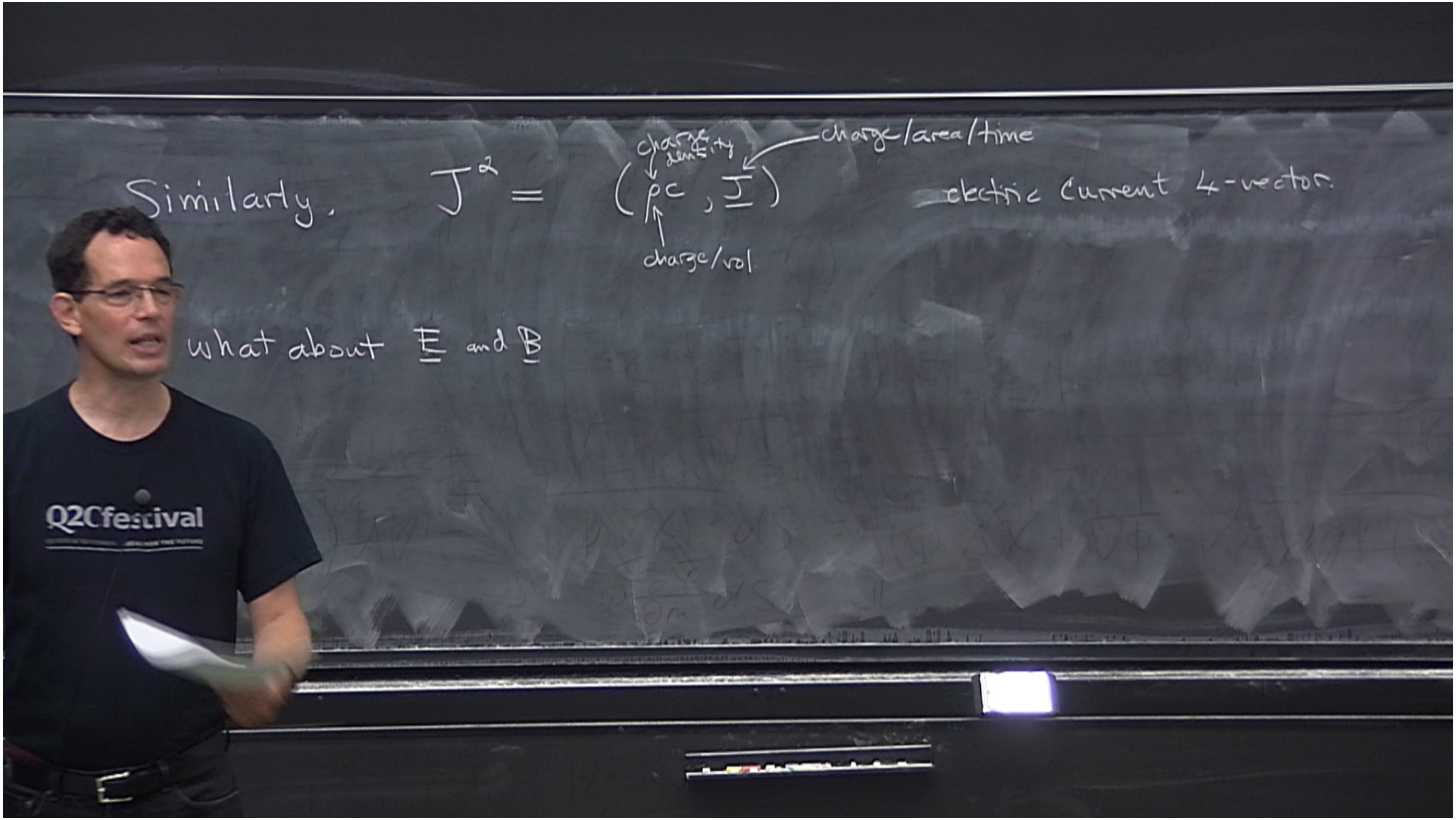
$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$ Ampere + Maxwell (current)

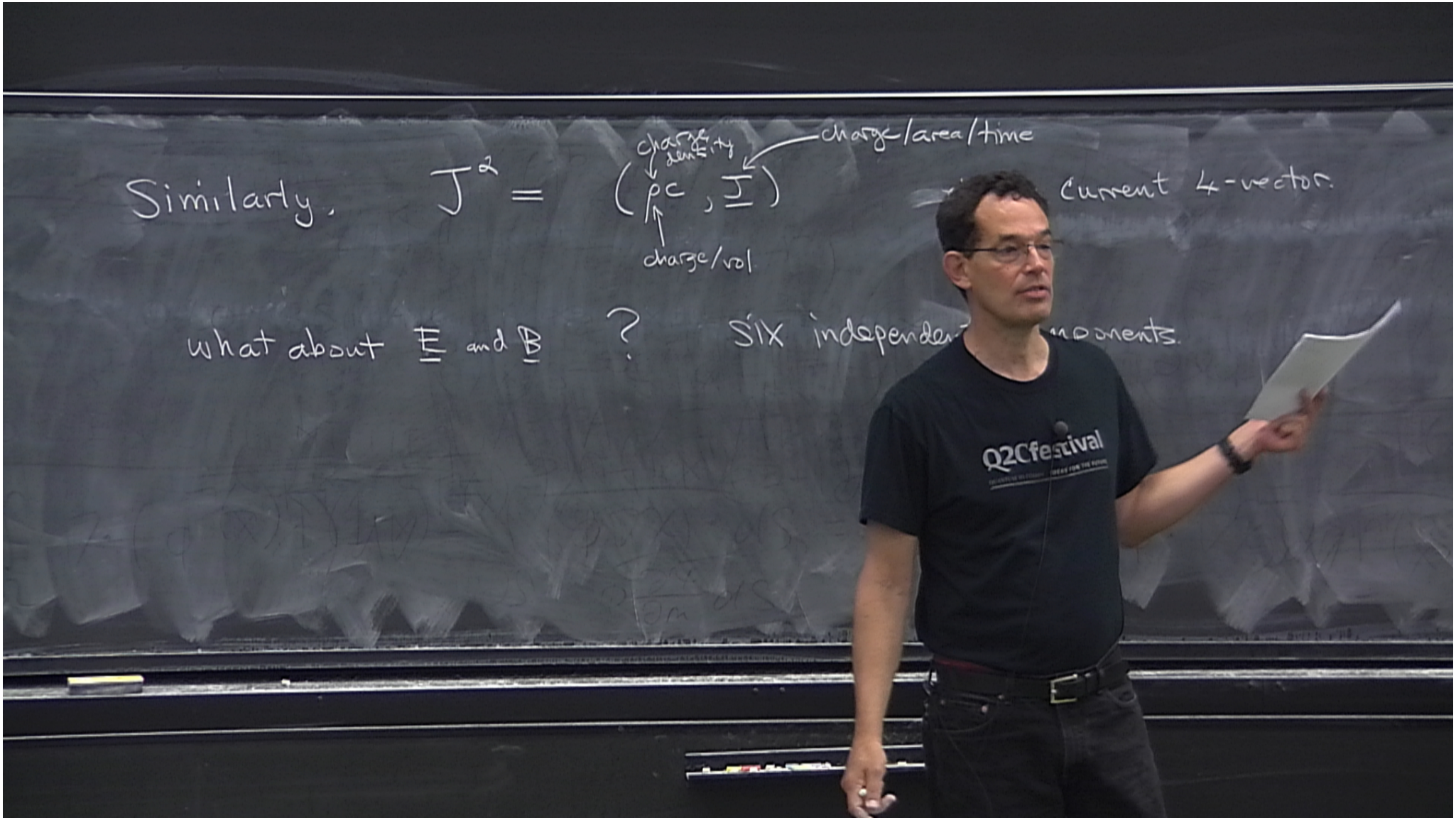
↑ Maxwell's correction.

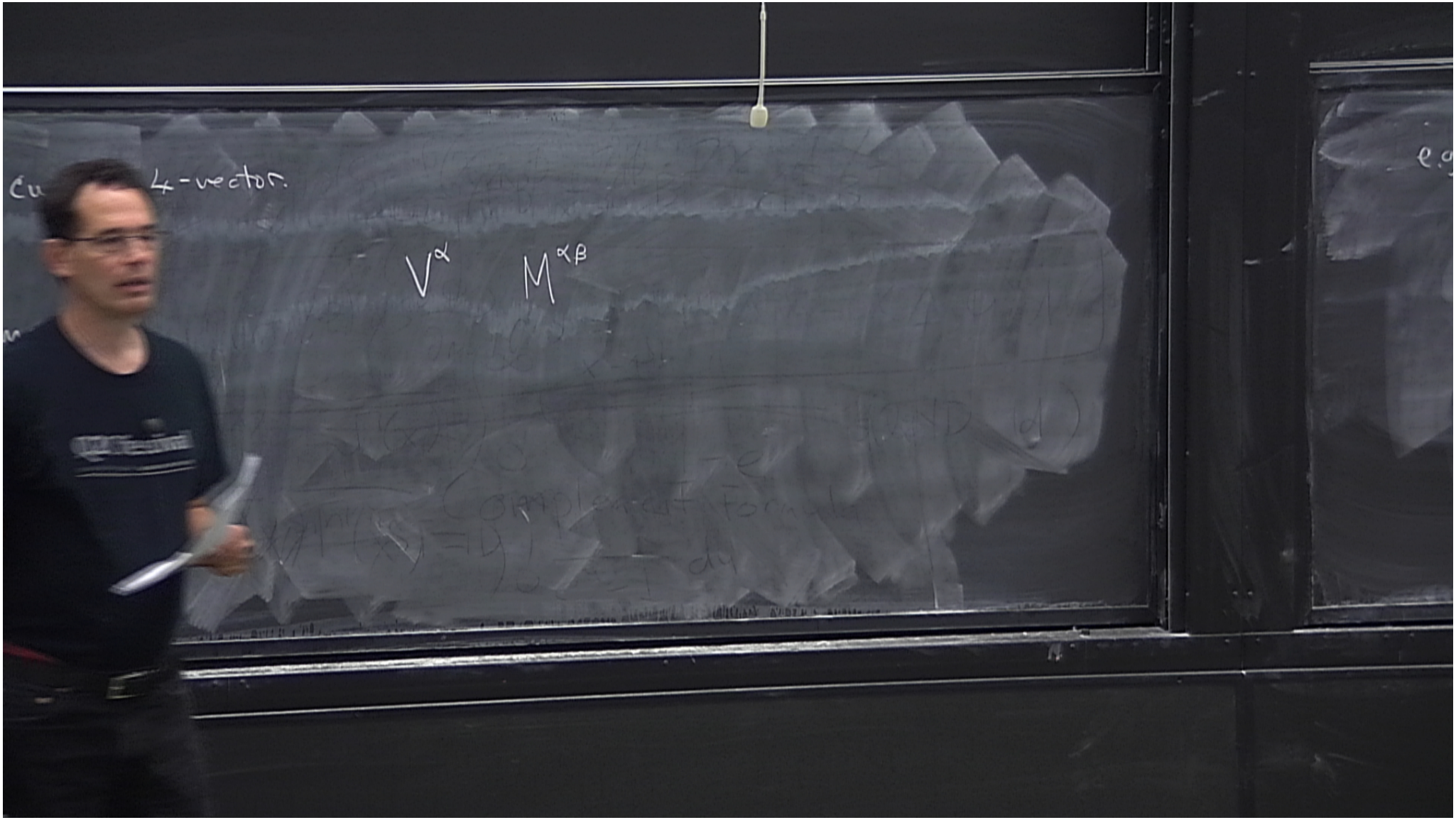

Similarly. $J^a = (\rho c, \underline{J})$
charge density

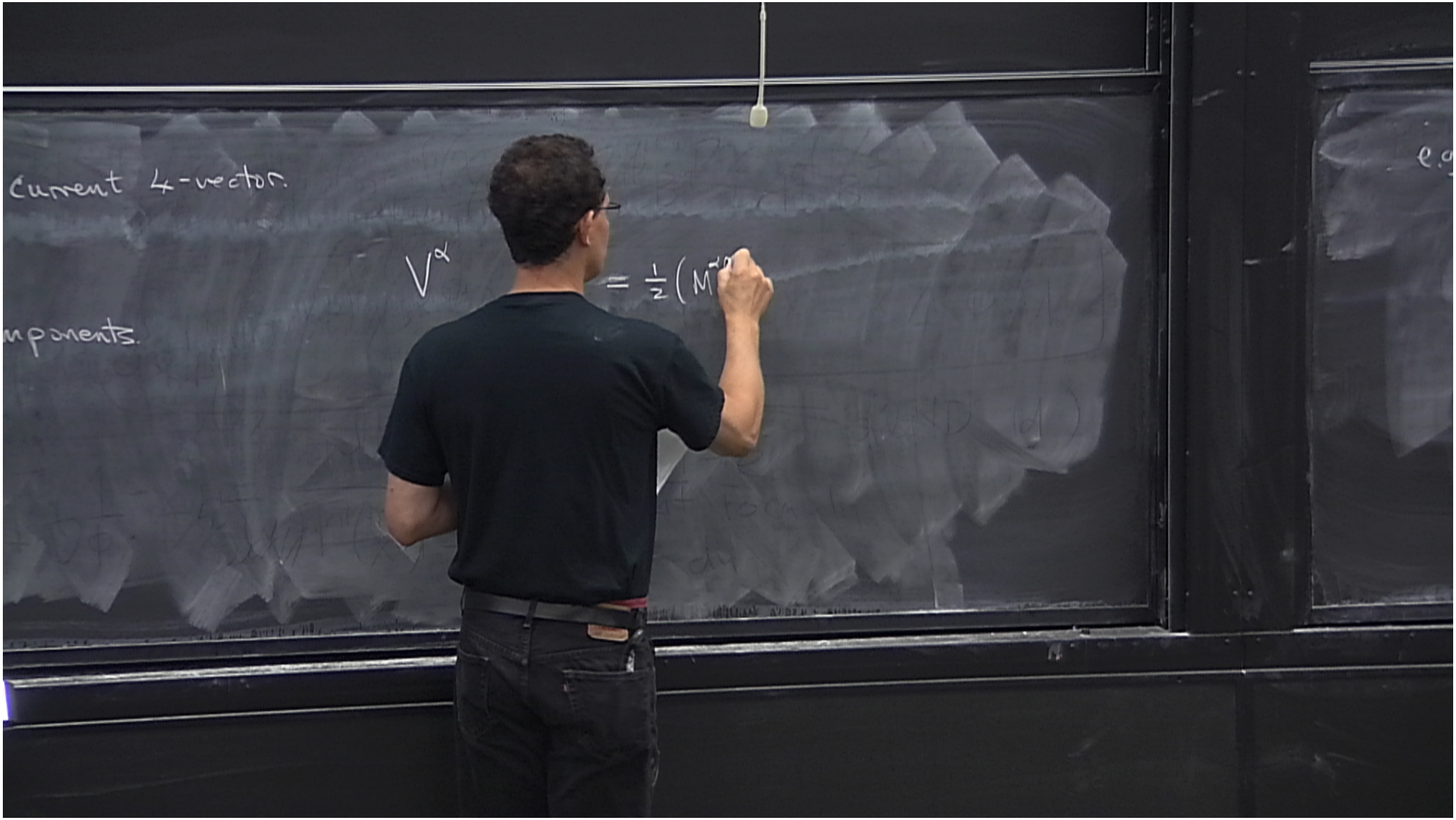

Similarly. $J^a = (\rho c, \underline{J})$
charge density
charge/area/time
charge/vol.

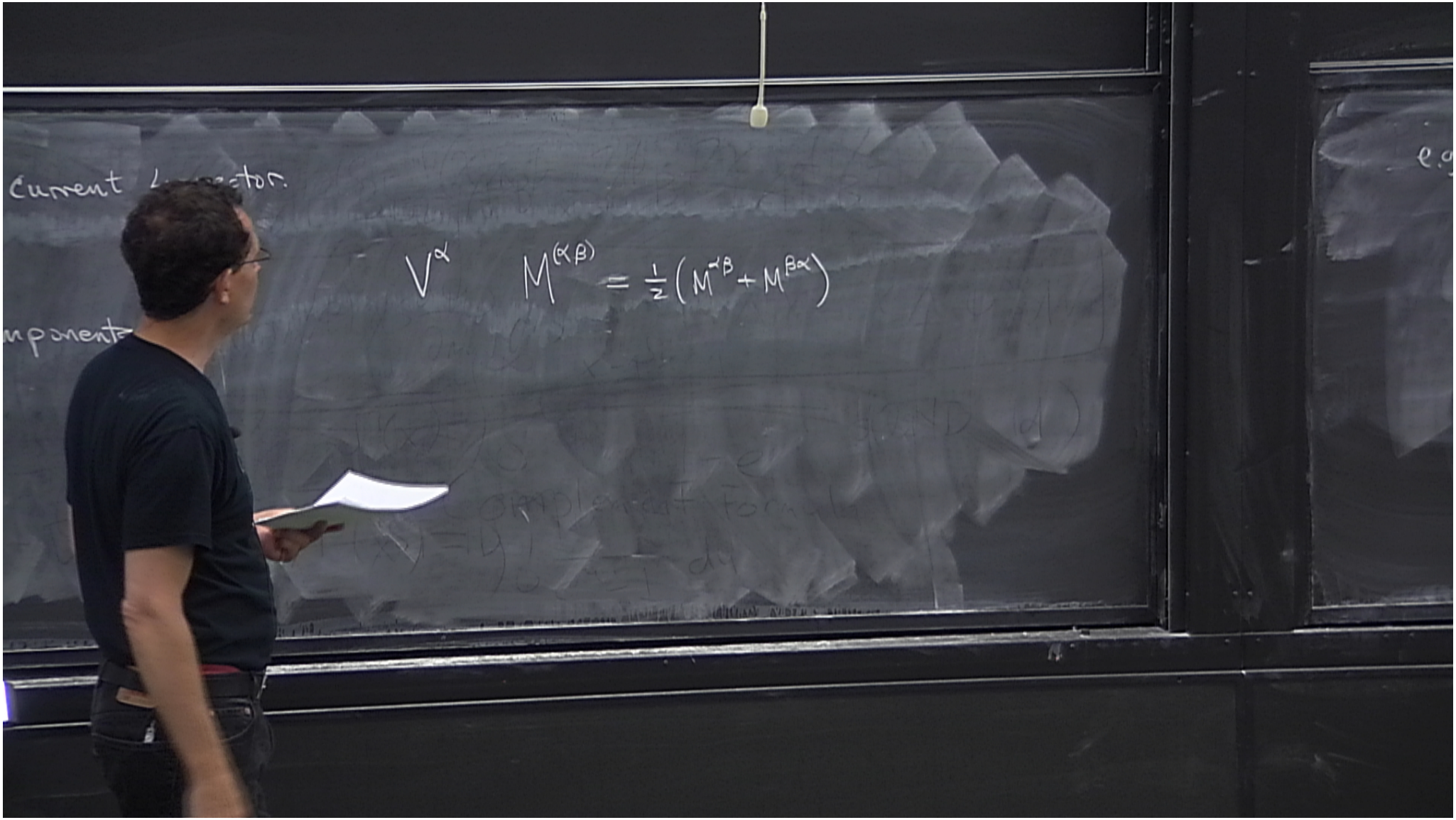

Similarly, $J^a = (\rho c, \underline{J})$
charge density
charge/area/time
charge/vol
electric current 4-vector.

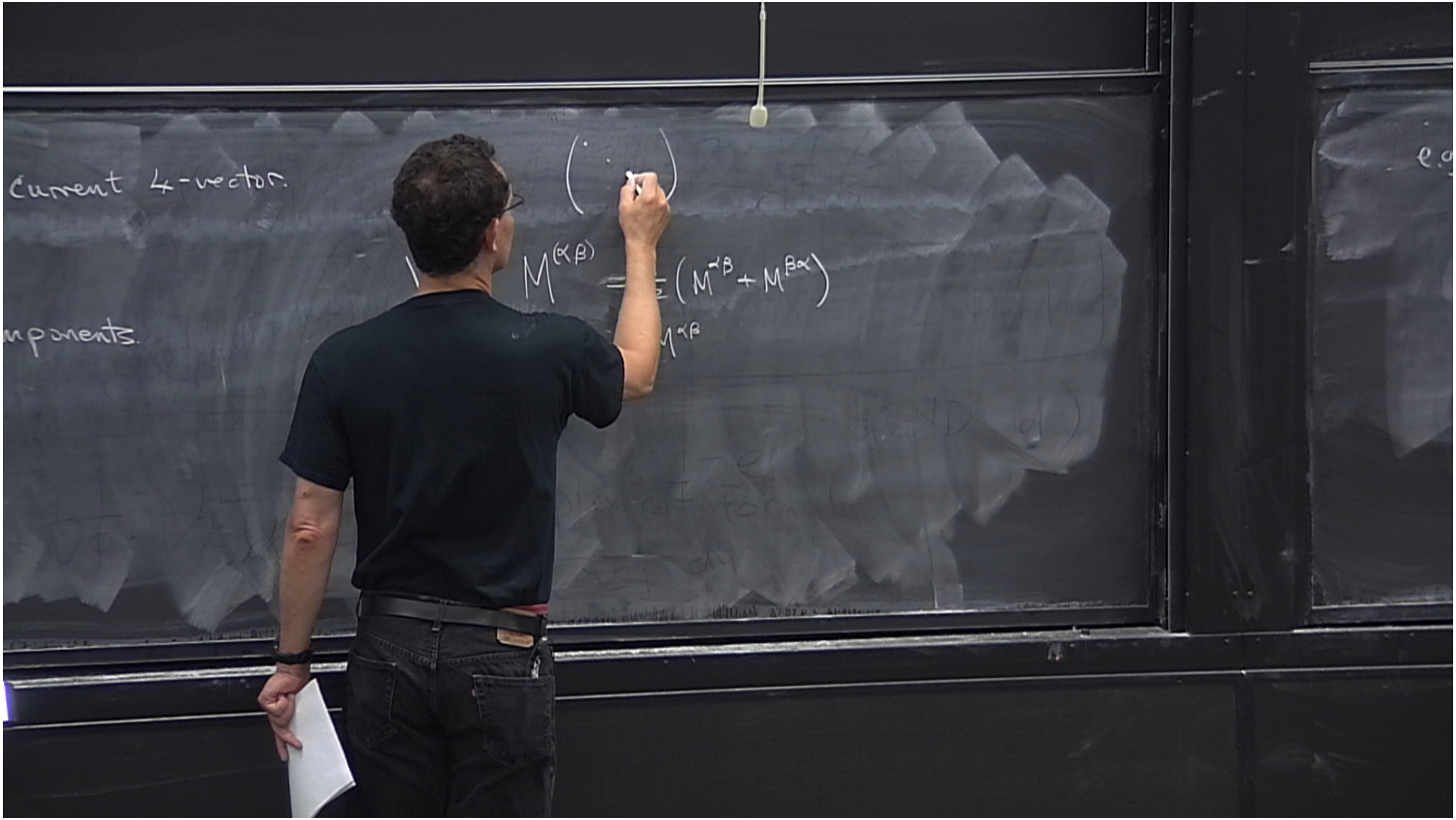

Similarly, $J^a = (\rho c, \underline{J})$
charge density
charge/area/time
electric current 4-vector.
charge/vol
what about $\underline{E}$ and $\underline{B}$
Q2Cfestival

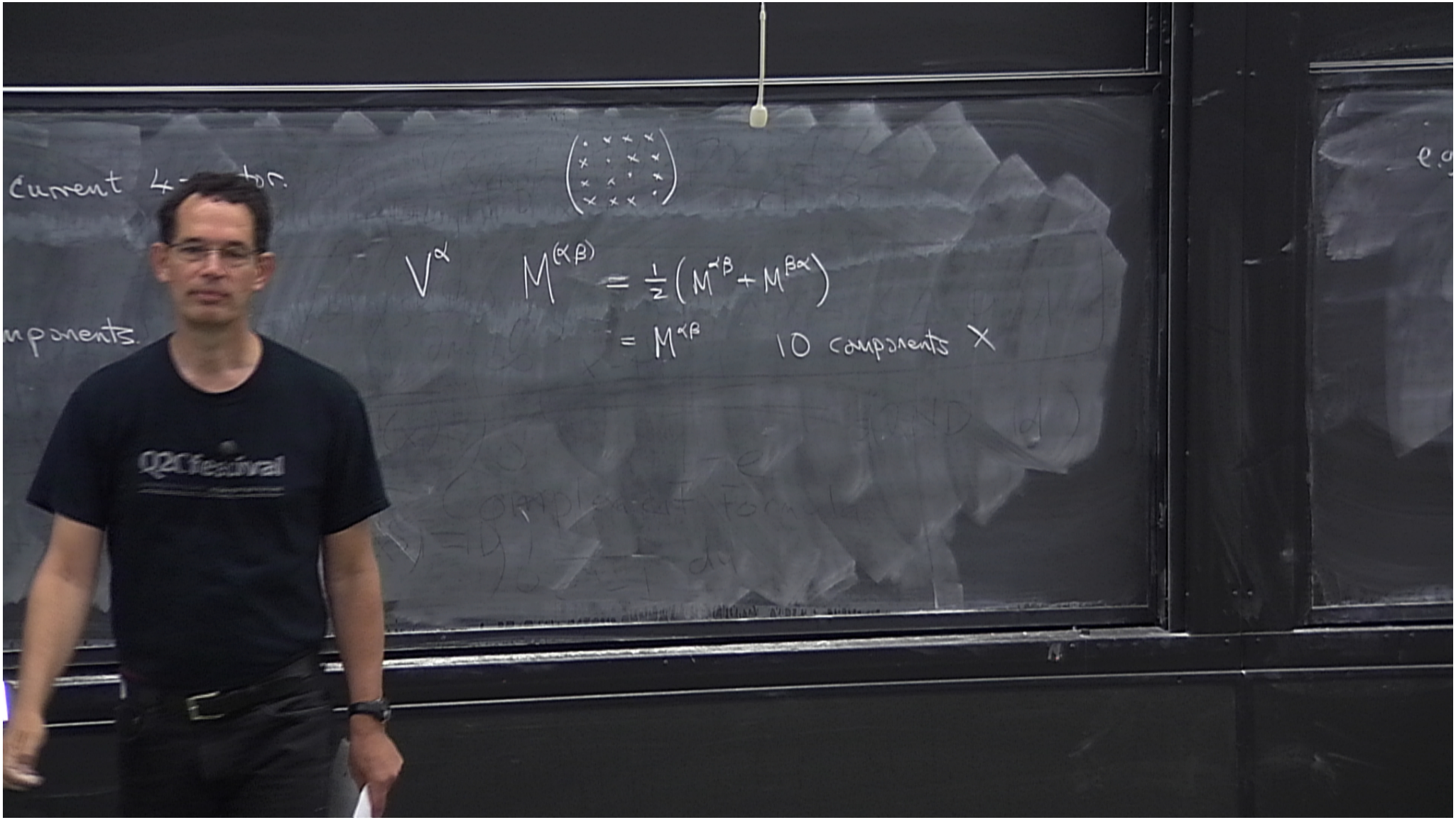

Similarly, $J^a = (\rho c, \underline{J})$
charge density
charge/area/time
charge/vol
current 4-vector.
what about $\underline{E}$ and $\underline{B}$?
six independent
components.
Q2Cfestival


4-vector.
V^α
$M^{\alpha\beta}$


Current 4-vector.
$V^\alpha = \frac{1}{2}(M$
mponents.


Current
$V^\alpha \quad M^{(\alpha\beta)} = \frac{1}{2}\left(M^{\alpha\beta} + M^{\beta\alpha}\right)$


Current 4-vector.
components.


Current 4-
components.
V^α
$M^{(\alpha\beta)} = \frac{1}{2}(M^{\alpha\beta} + M^{\beta\alpha})$
$= M^{\alpha\beta}$
10 components X

Current 4-vector.



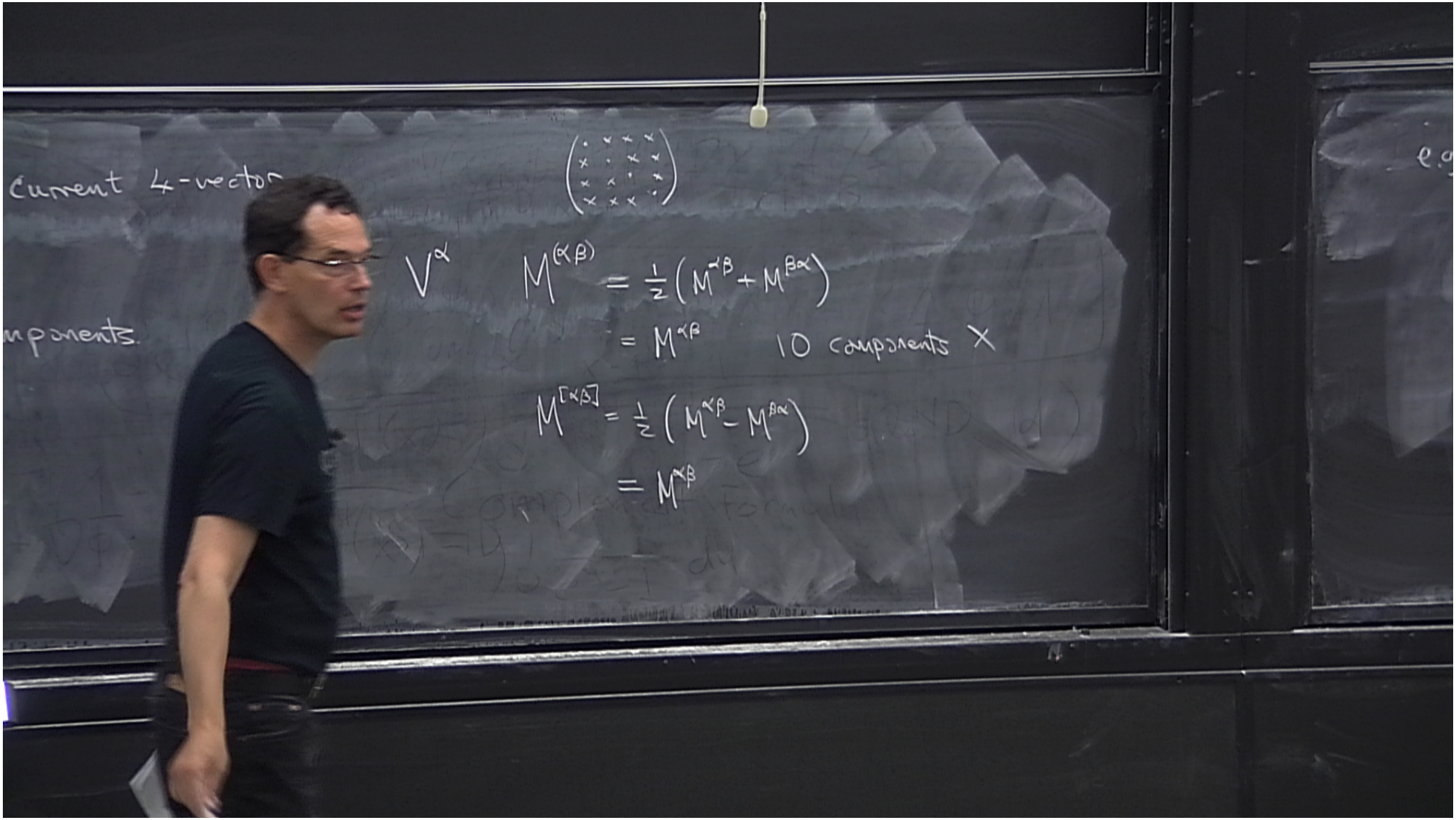
V^α $\quad M^{(\alpha\beta)} = \frac{1}{2}\left(M^{\alpha\beta} + M^{\beta\alpha}\right)$

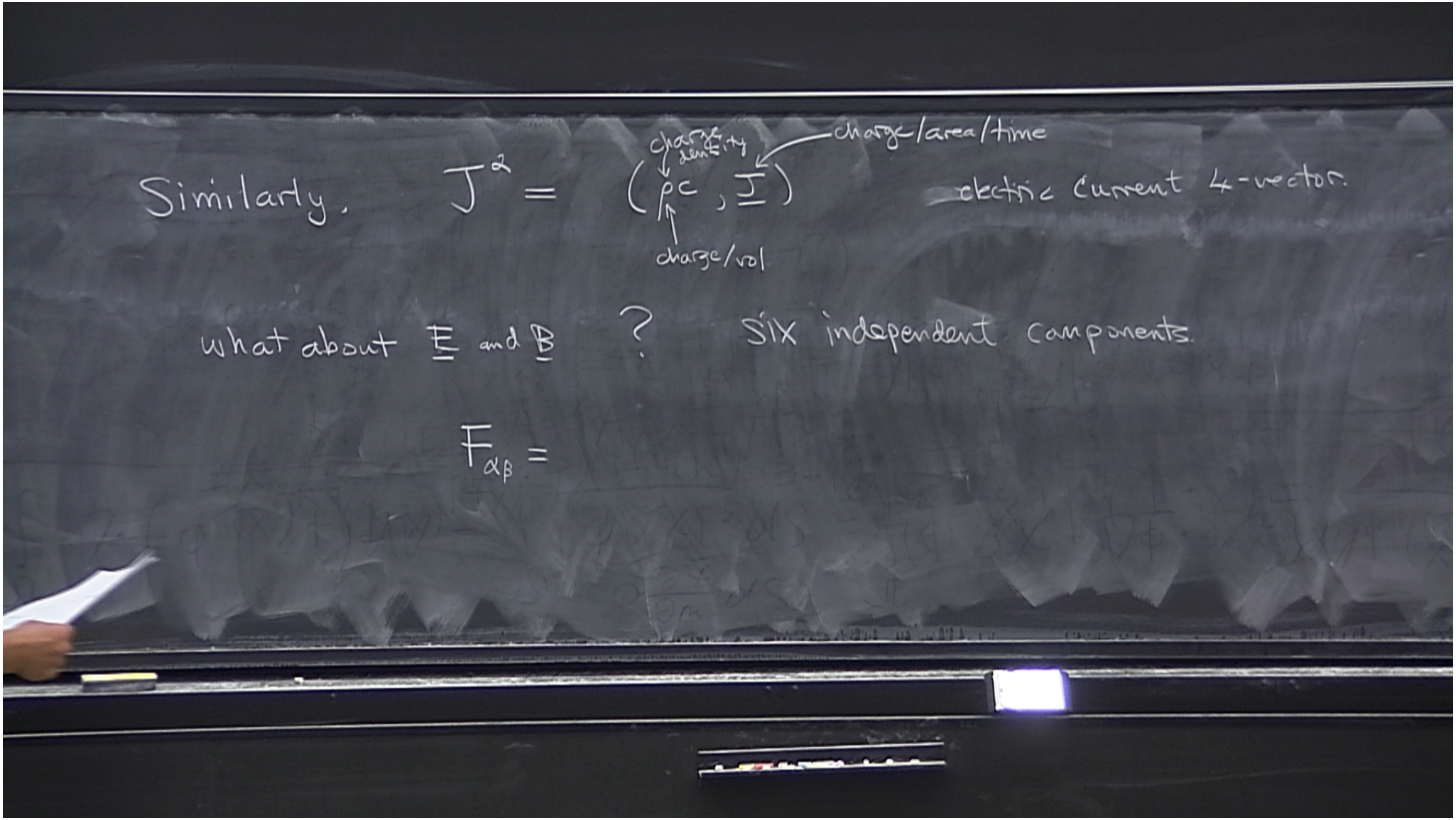
$= M^{\alpha\beta}$ $\quad$ 10 components ✓

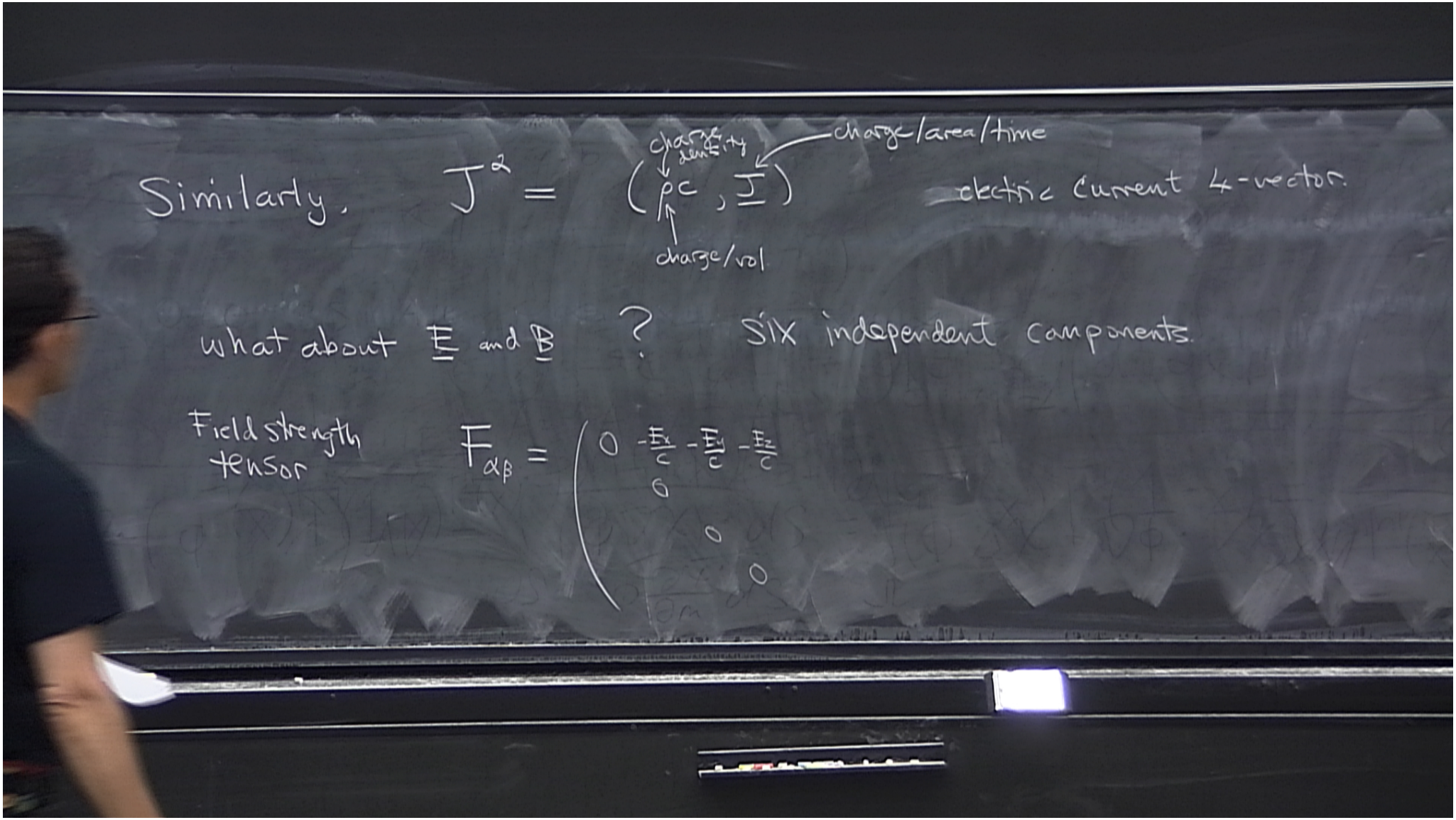
$M^{[\alpha\beta]} = \frac{1}{2}\left(M^{\alpha\beta} - M^{\beta\alpha}\right)$

$= M^{\alpha\beta}$

mponents.


current 4-vector
components.
V^α
$M^{(\alpha\beta)} = \frac{1}{2}(M^{\alpha\beta} + M^{\beta\alpha})$
$= M^{\alpha\beta}$ 10 components
$M^{[\alpha\beta]} = \frac{1}{2}(M^{\alpha\beta} - M^{\beta\alpha})$
$= M^{\alpha\beta}$


Similarly. $J^\alpha = (\rho c, \underline{J})$
charge density
charge/area/time
electric current 4-vector.
charge/vol
what about $\underline{E}$ and $\underline{B}$? six independent components.
$F_{\alpha\beta} =$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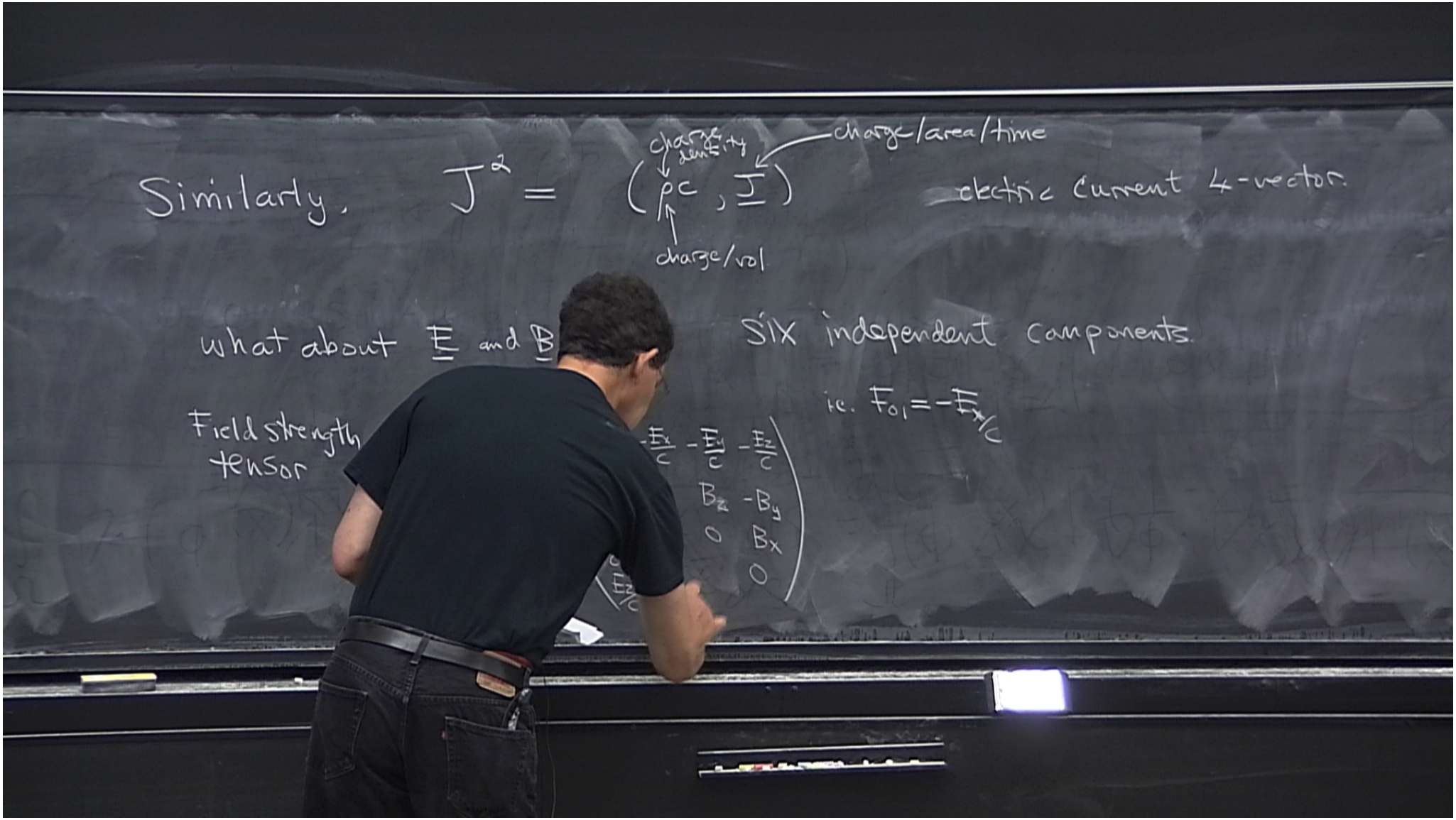
charge density
charge/area/time
charge/vol
electric current 4-vector.
what about $\underline{E}$ and $\underline{B}$?
six independent components.
Field strength tensor
$F_{\alpha\beta} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ 0 & & & \\ & 0 & & \\ & & 0 & \end{pmatrix}$


Similarly, $J^a = (\rho c, \underline{J})$
charge density
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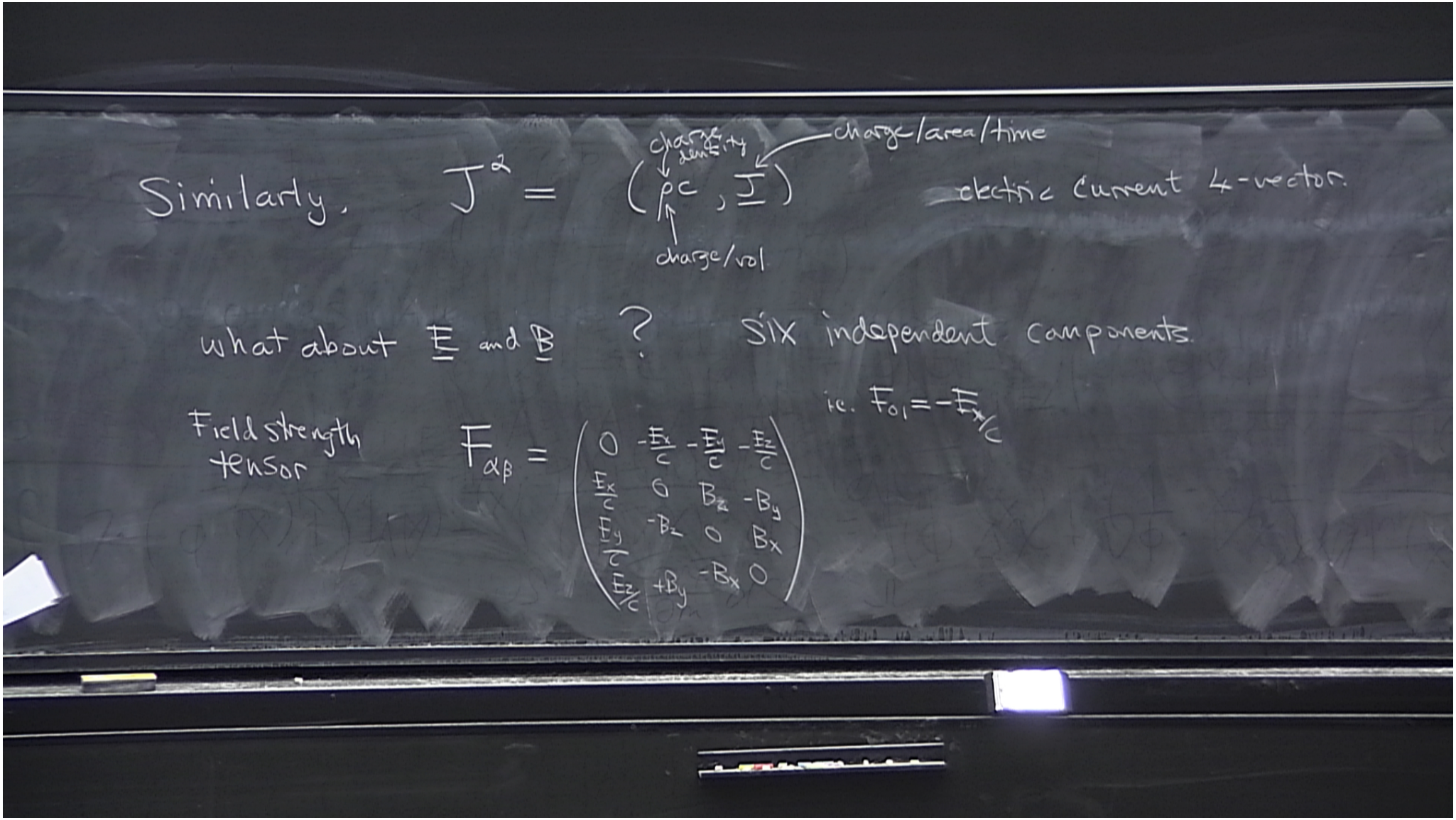
electric current 4-vector.
charge/vol
what about $\underline{E}$ and $\underline{B}$
Six independent components.
ie. $F_{01} = -E_x/c$
Field strength tensor
$-\frac{E_x}{c}$ $-\frac{E_y}{c}$ $-\frac{E_z}{c}$
B_z $-B_y$
0 B_x
0


Similarly, $J^\alpha = (\rho c, \underline{J})$
charge density
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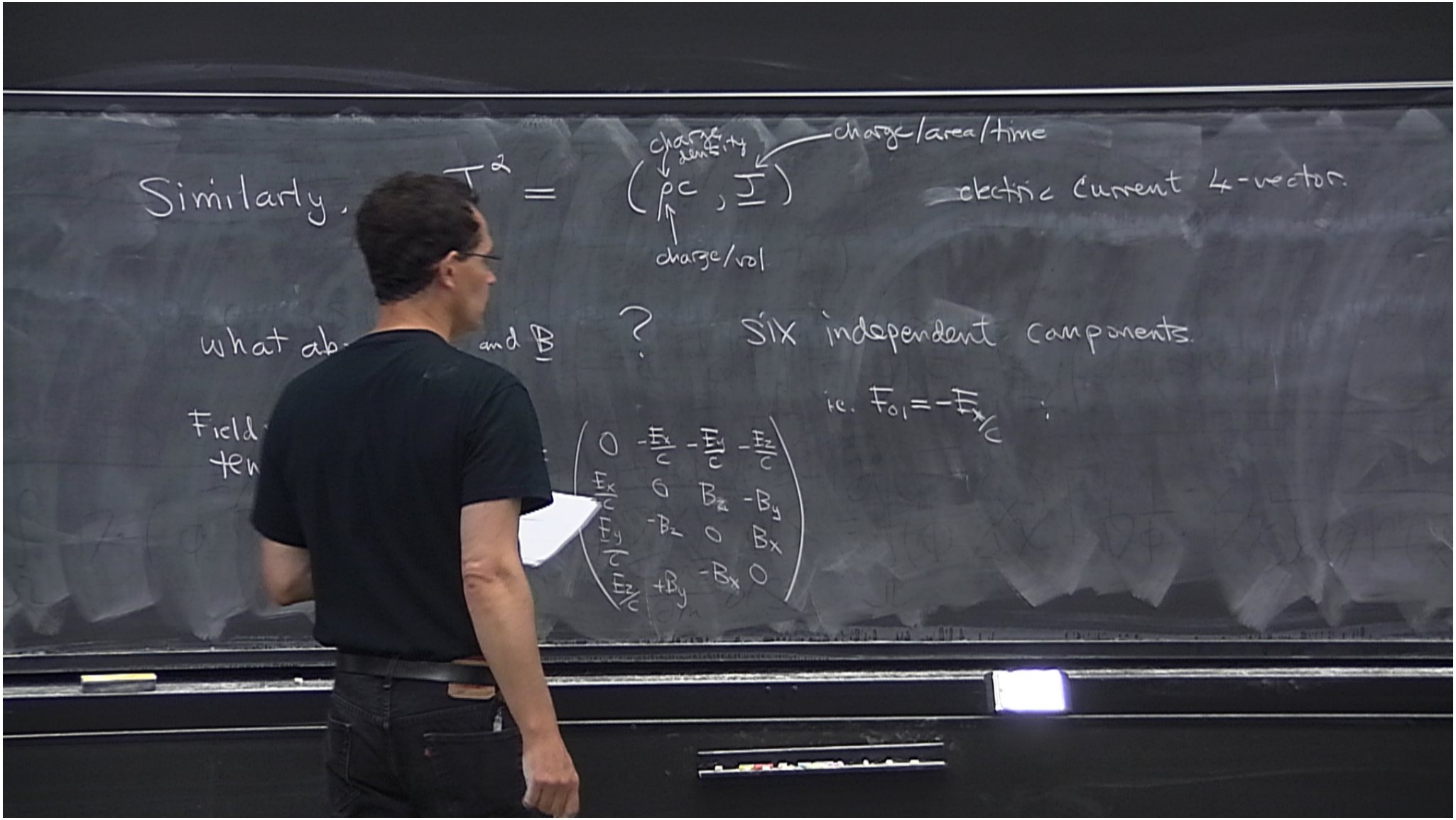
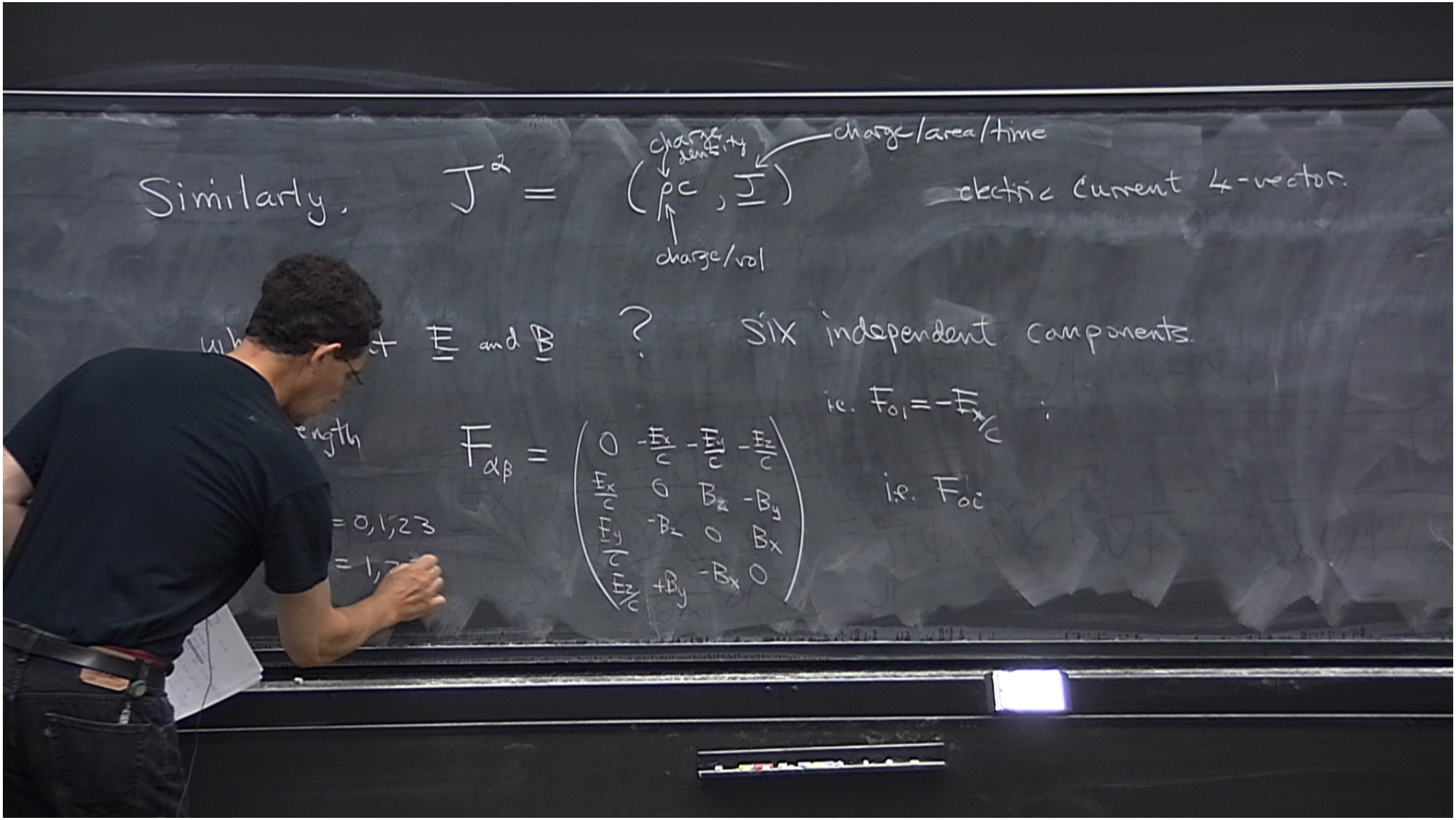
electric current 4-vector.
what about $\underline{E}$ and $\underline{B}$?
Six independent components.
ie. $F_{01} = -E_x/c$
Field strength tensor
$F_{\alpha\beta} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & B_z & -B_y \\ E_y/c & -B_z & 0 & B_x \\ E_z/c & +B_y & -B_x & 0 \end{pmatrix}$


Similarly, $T^\alpha = (\rho c, \underline{J})$
charge density
charge/area/time
charge/vol
electric current 4-vector.
what ab
and $\underline{B}$
?
Six independent components.
i.e. $F_{01} = -E_x/c$;
Field
ten
$\begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & B_z & -B_y \\ \frac{E_y}{c} & -B_z & 0 & B_x \\ \frac{E_z}{c} & +B_y & -B_x & 0 \end{pmatrix}$


Similarly, $J^\alpha = (\rho c, \underline{J})$
charge density
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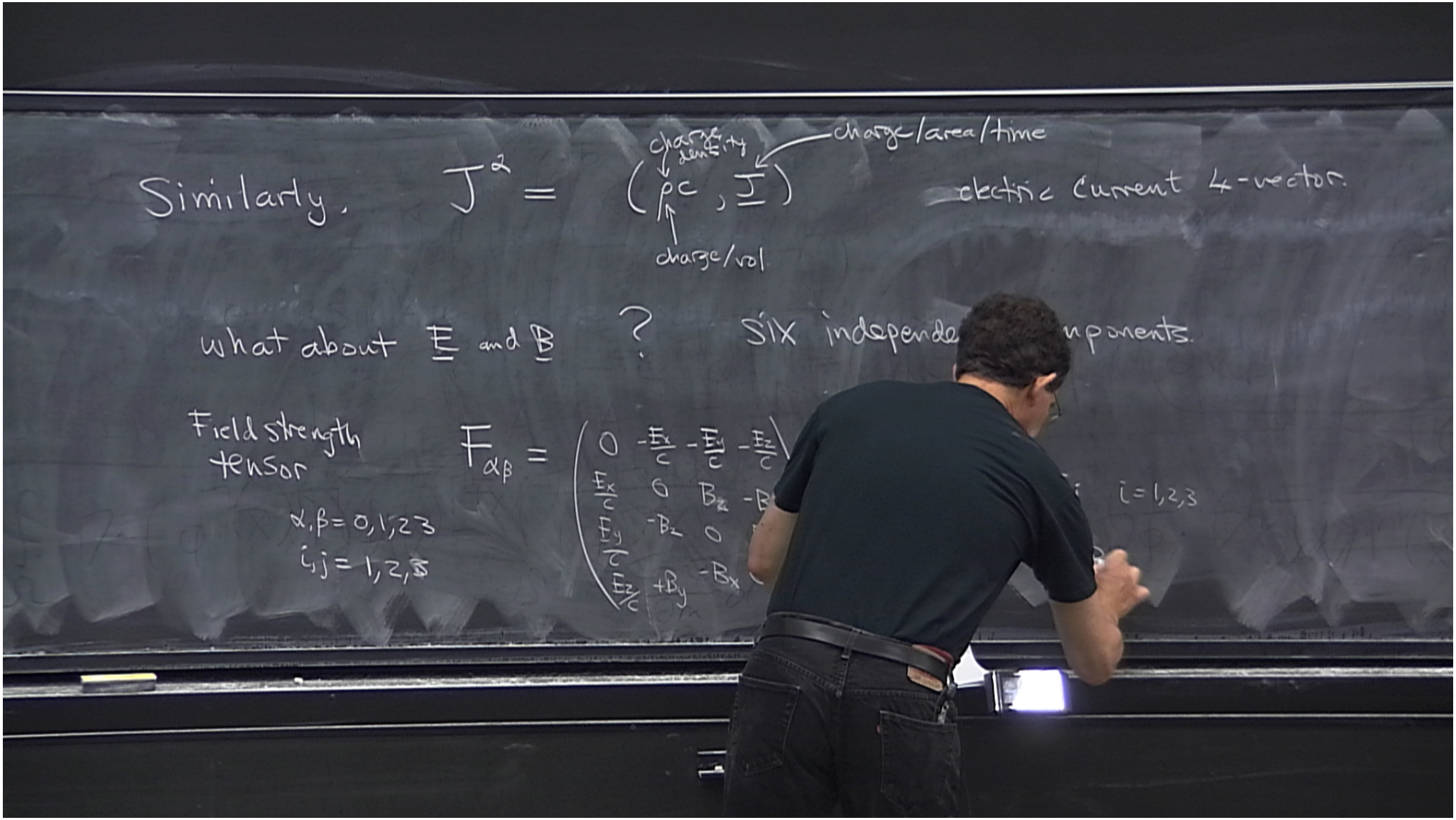
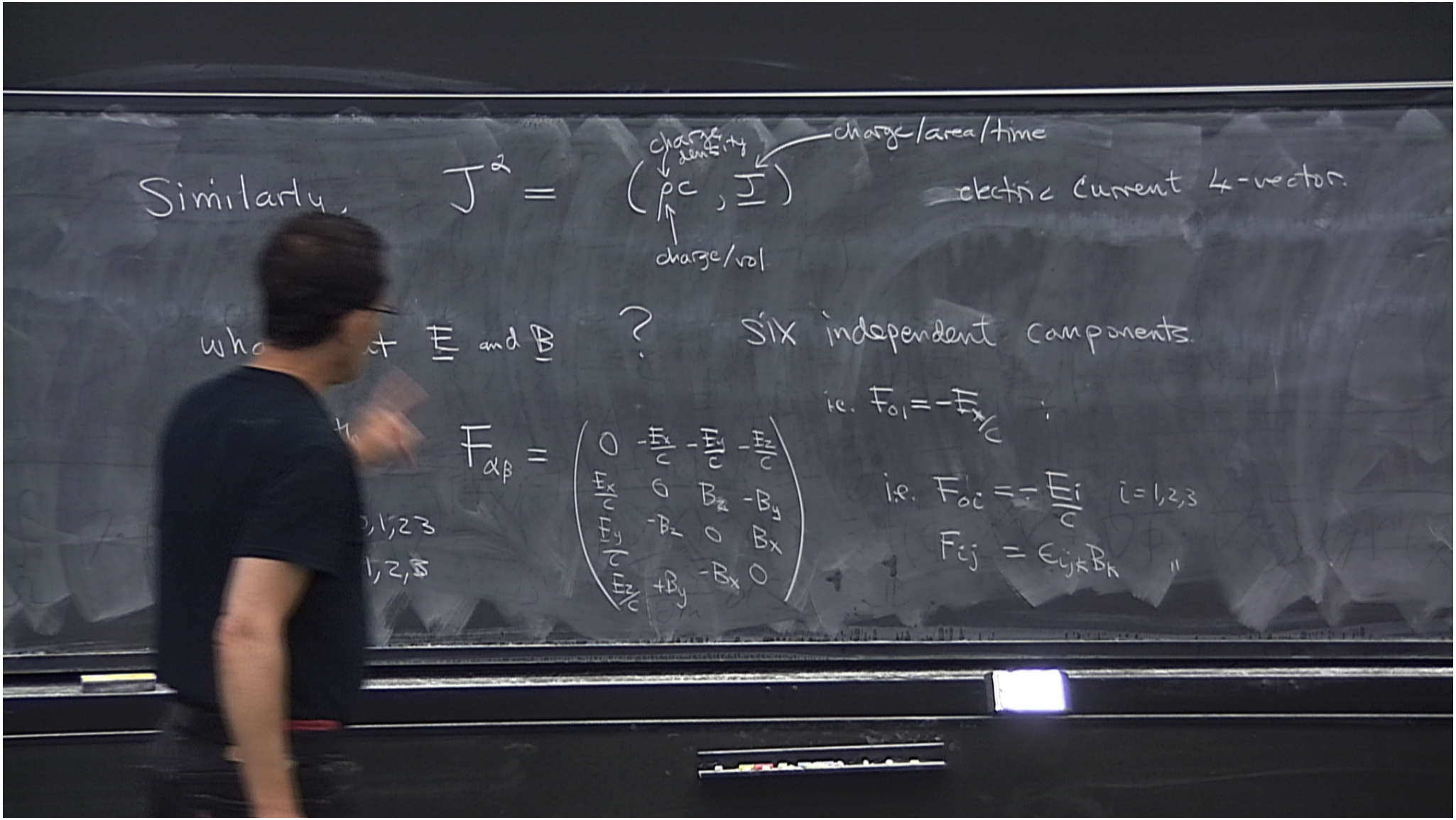
electric current 4-vector.
E and B ?
Six independent components.
i.e. $F_{01} = -E_x/c$;
i.e. F_{0i}
$F_{\alpha\beta} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & B_z & -B_y \\ \frac{E_y}{c} & -B_z & 0 & B_x \\ \frac{E_z}{c} & +B_y & -B_x & 0 \end{pmatrix}$
= 0,1,23


Similarly, $J^\alpha = (\rho c, \underline{J})$
charge density
charge/area/time
electric current 4-vector.
charge/vol
what about $\underline{E}$ and $\underline{B}$?
Six independe... components.
Field strength tensor
$F_{\alpha\beta} =$
$\alpha, \beta = 0,1,2,3$
$i, j = 1,2,3$
$i = 1,2,3$


Similarly, $J^\alpha = (\rho c, \underline{J})$
charge density
charge/area/time
electric current 4-vector.
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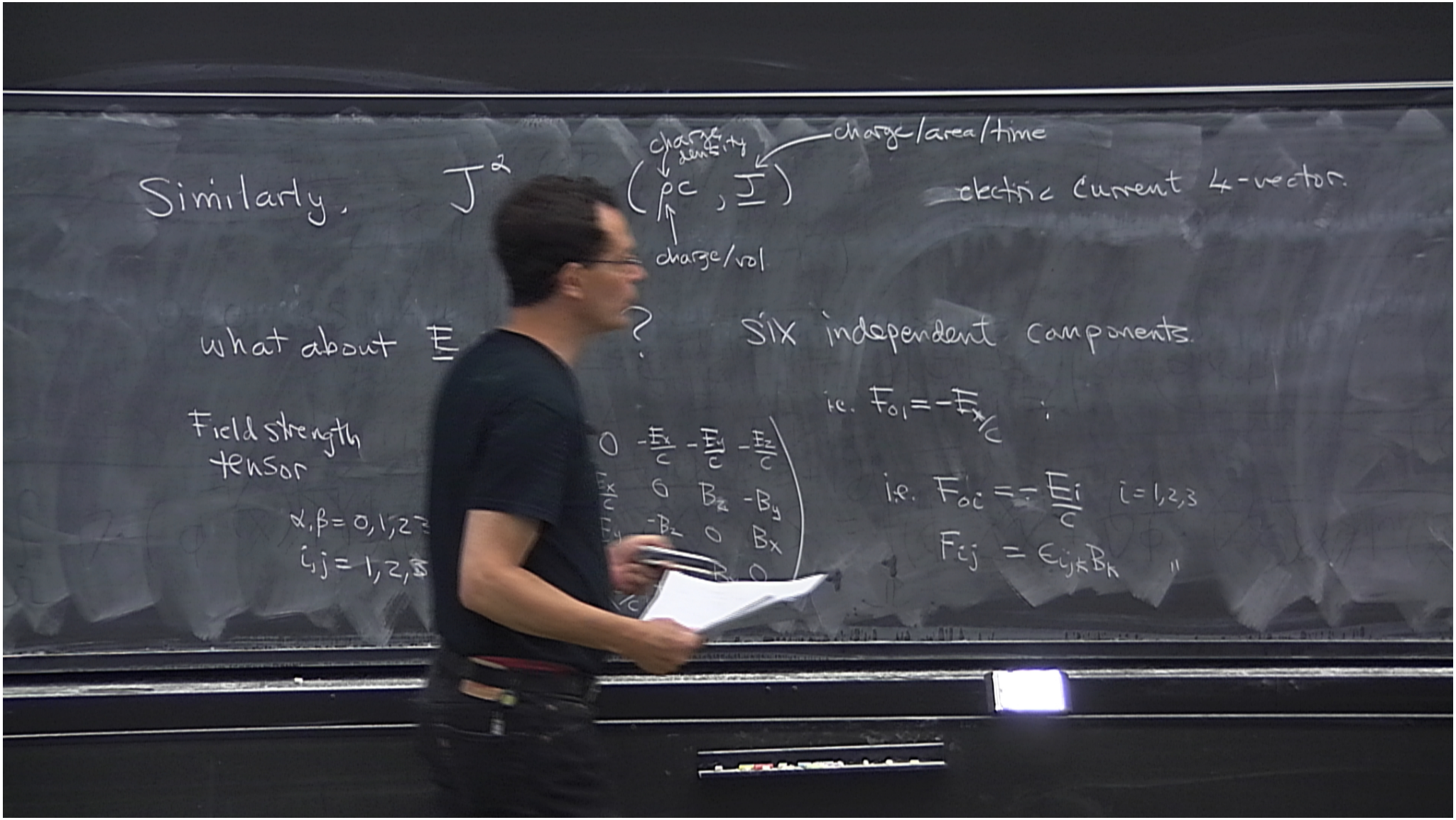
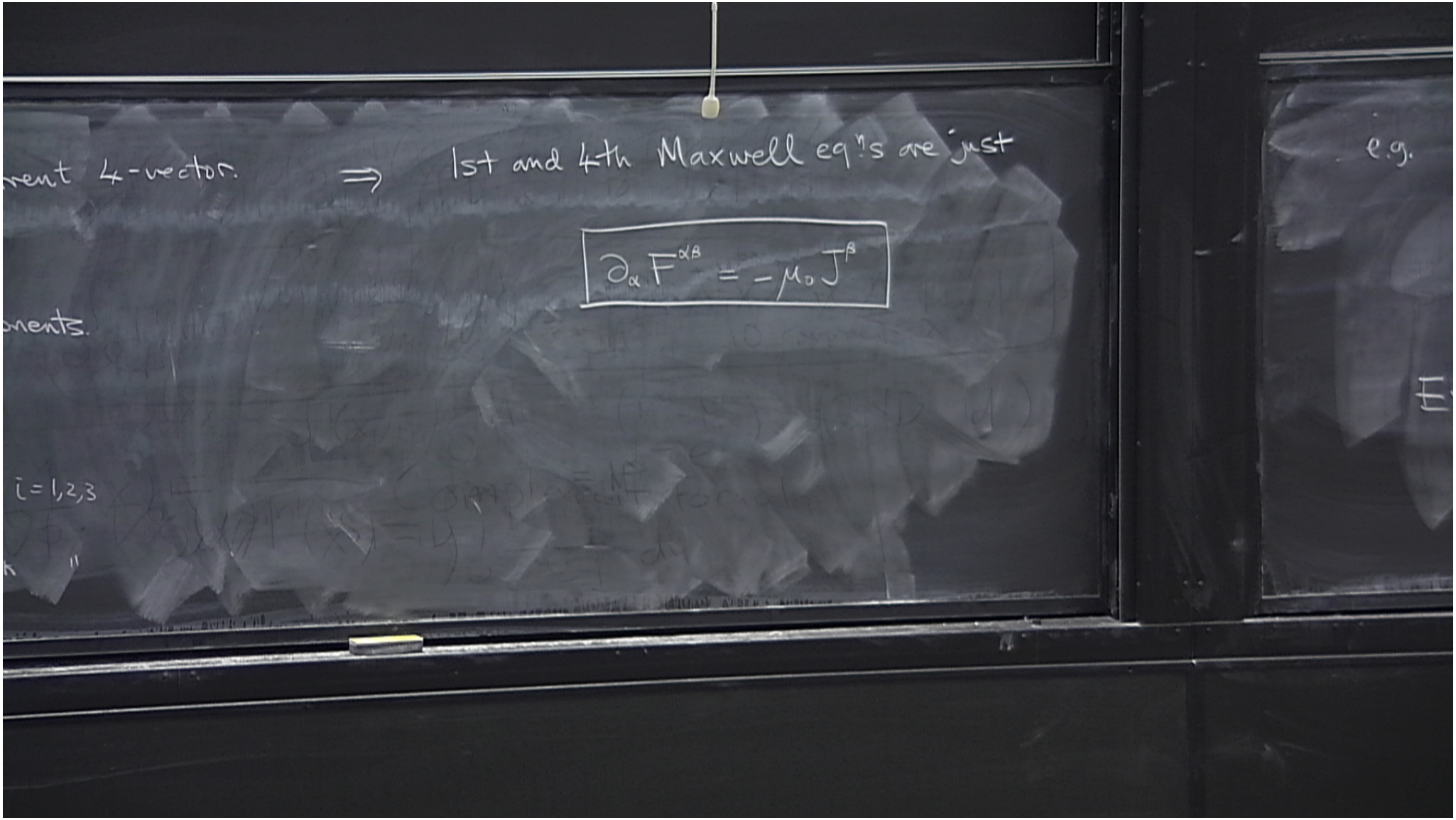
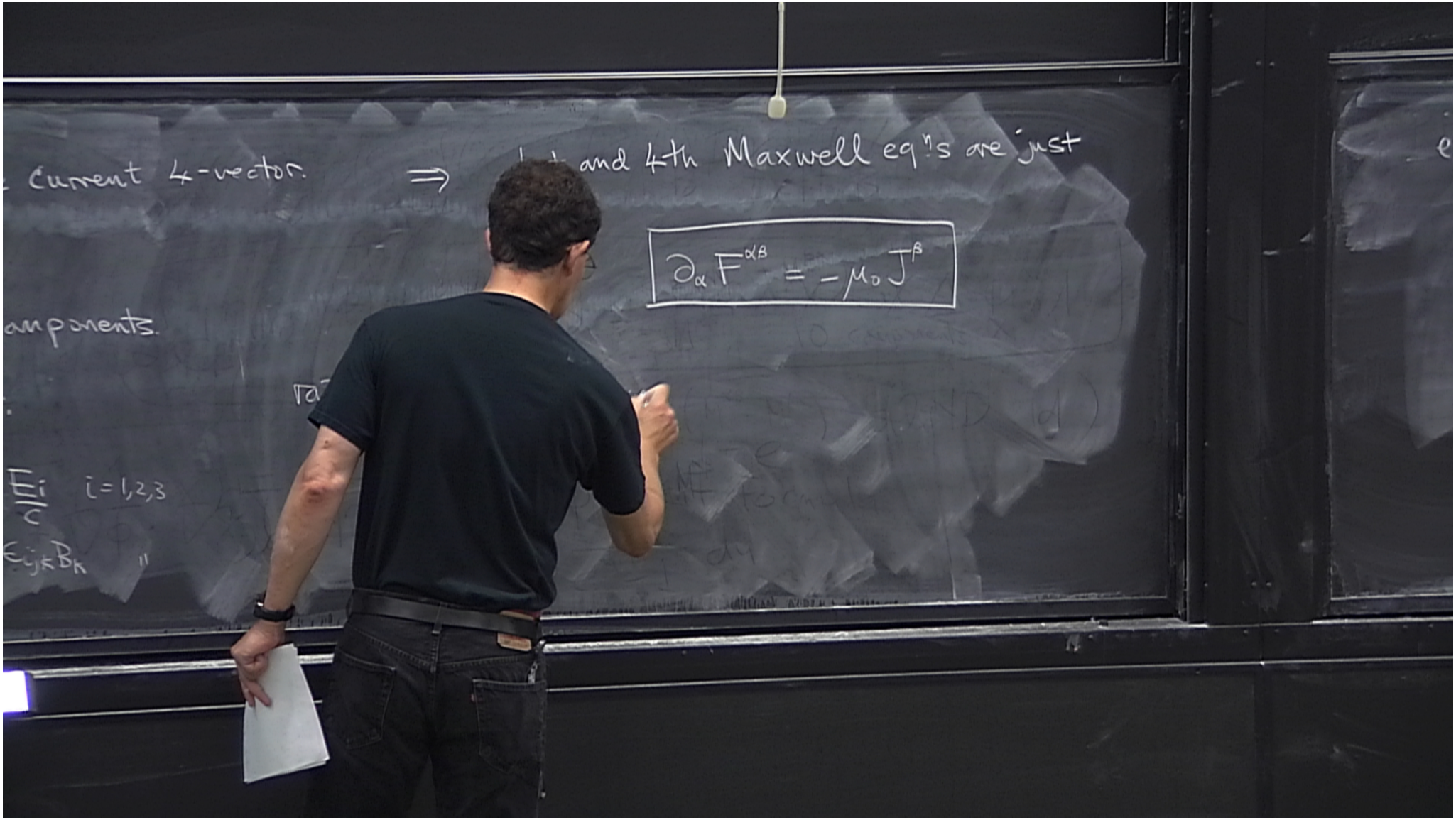
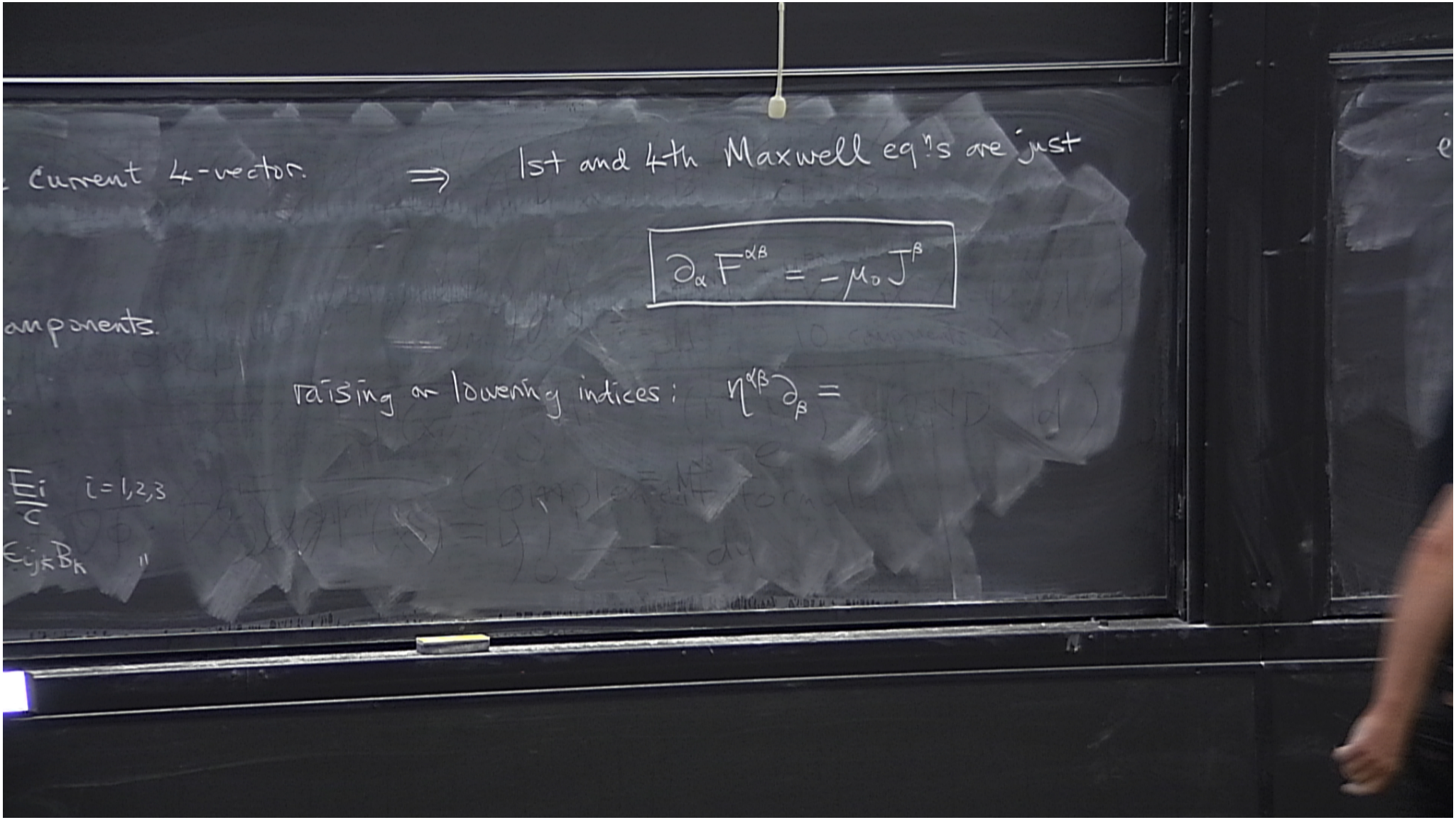
who[illegible]t $\underline{E}$ and $\underline{B}$? Six independent components.
$F_{\alpha\beta} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & B_z & -B_y \\ \frac{E_y}{c} & -B_z & 0 & B_x \\ \frac{E_z}{c} & +B_y & -B_x & 0 \end{pmatrix}$
i.e. $F_{0i} = -\frac{E_x}{c}$;
i.e. $F_{0i} = -\frac{E_i}{c}$ $i = 1,2,3$
$F_{ij} = \epsilon_{ijk} B_k$ "


Similarly, J^α $(\rho c, \underline{J})$
charge density
charge/area/time
charge/vol
electric current 4-vector.
what about $\underline{E}$?
Six independent components.
Field strength tensor
$\alpha, \beta = 0,1,2,3$
$i,j = 1,2,3$
i.e. $F_{01} = -E_x/c$;
i.e. $F_{0i} = -\frac{E_i}{c}$ $i = 1,2,3$
$F_{ij} = \epsilon_{ijk} B_k$ "


rent 4-vector. ⇒ 1st and 4th Maxwell eq'ns are just
$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$
e.g.
ments.
i = 1,2,3


Current 4-vector. ⇒ and 4th Maxwell eq'ns are just
$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$
components.
$\frac{E_i}{c}$ $i=1,2,3$


current 4-vector. ⇒ 1st and 4th Maxwell eq'n's are just
$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$
components.
raising or lowering indices: $\eta^{\alpha\beta}\partial_\beta =$
$\frac{E_i}{c}$ $i=1,2,3$
$\epsilon_{ijk}B_k$ "

current 4-vector. $\Rightarrow$ 1st and 4th Maxwell eq^n's are just

$$\boxed{\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta}$$

components.

raising or lowering indices: $\eta^{\alpha\beta}\partial_\beta = \left(-\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \equiv \partial^\alpha$

$\frac{E_i}{c}$ $i = 1,2,3$

$\epsilon_{ijk}B_k$ "

current 4-vector. $\Rightarrow$ 1st and 4th Maxwell eq^n's are just

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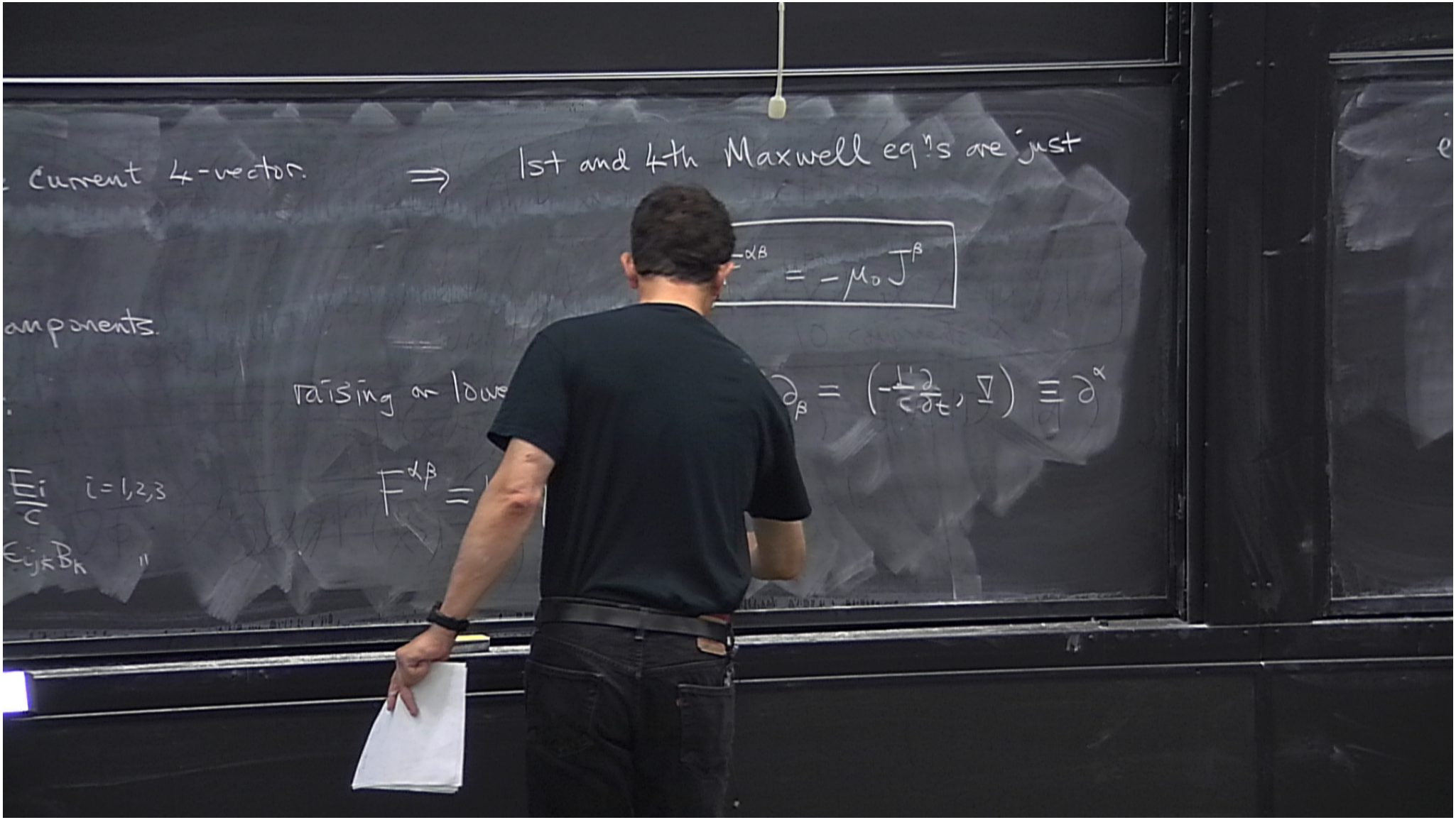
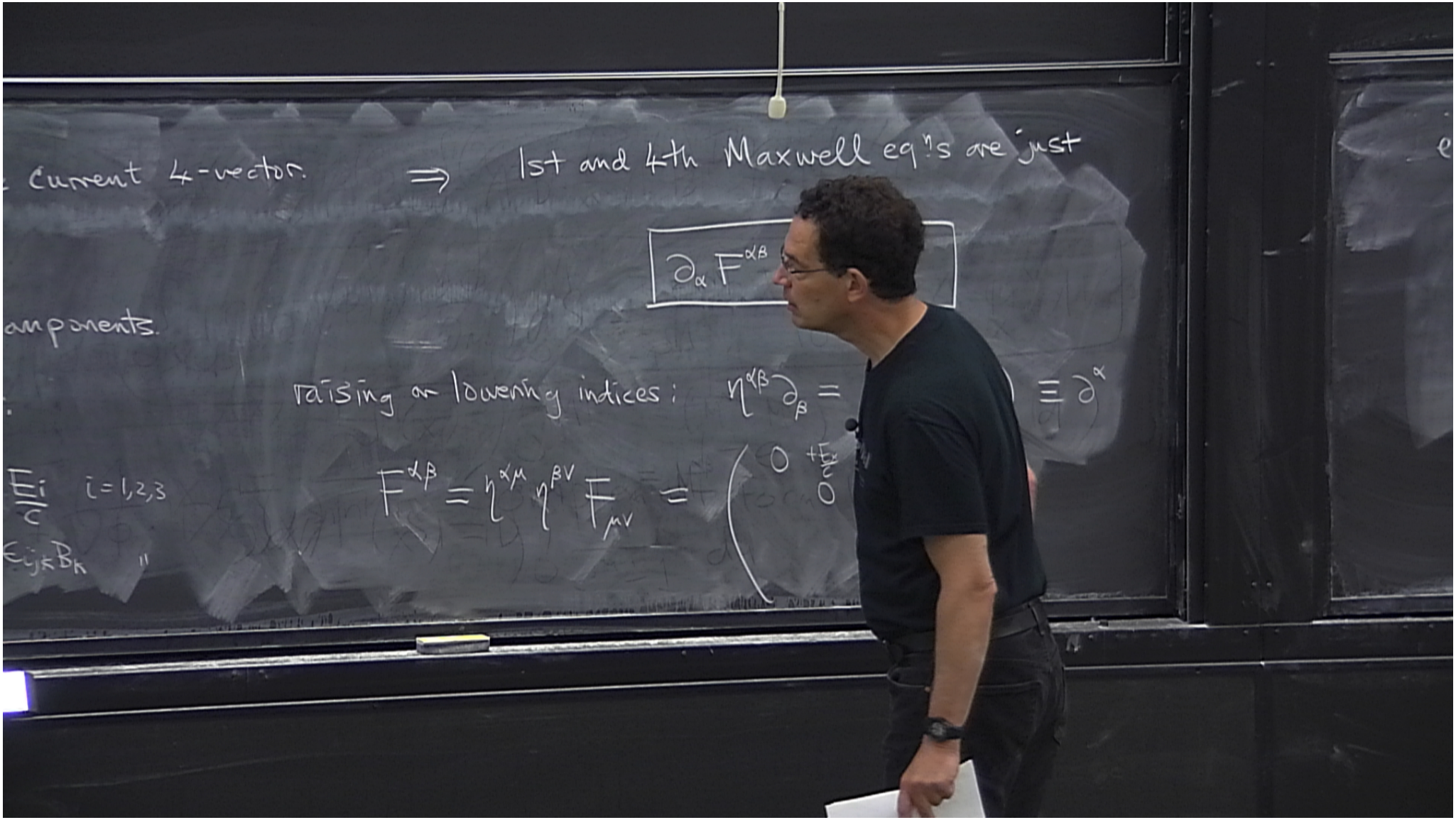
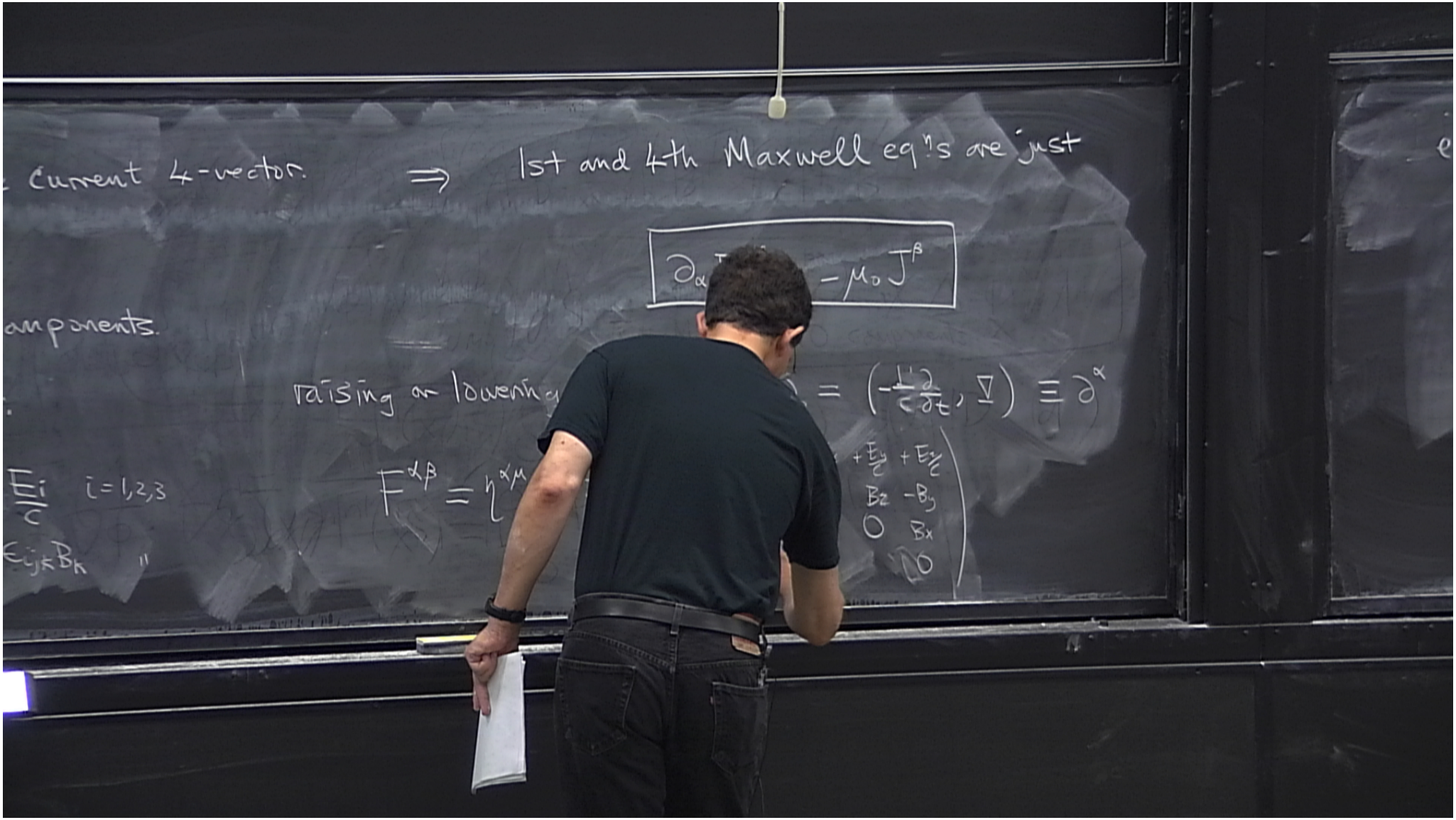
$\frac{E_i}{c}$ $i=1,2,3$

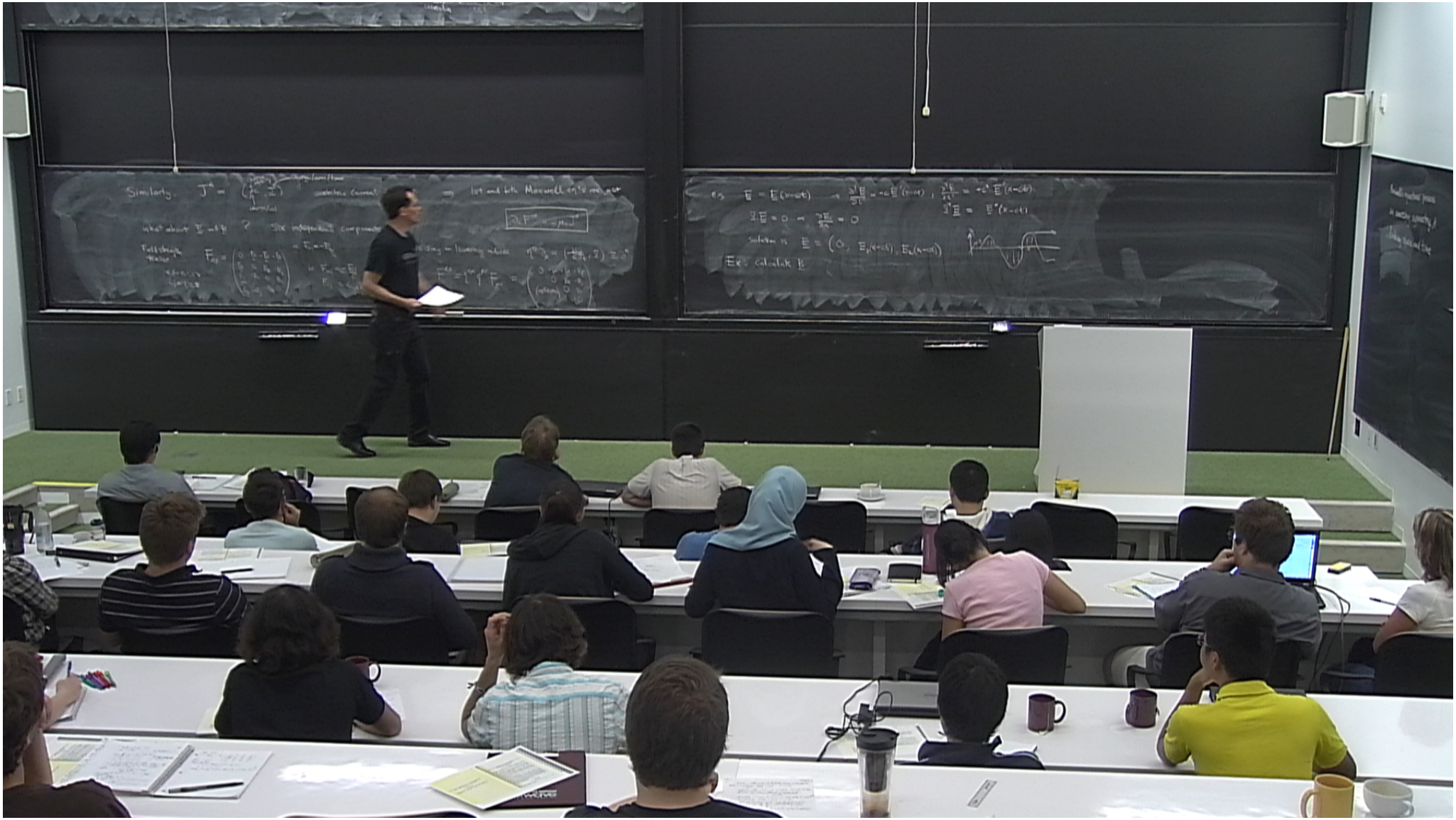
$\epsilon_{ijk}B_k$ "

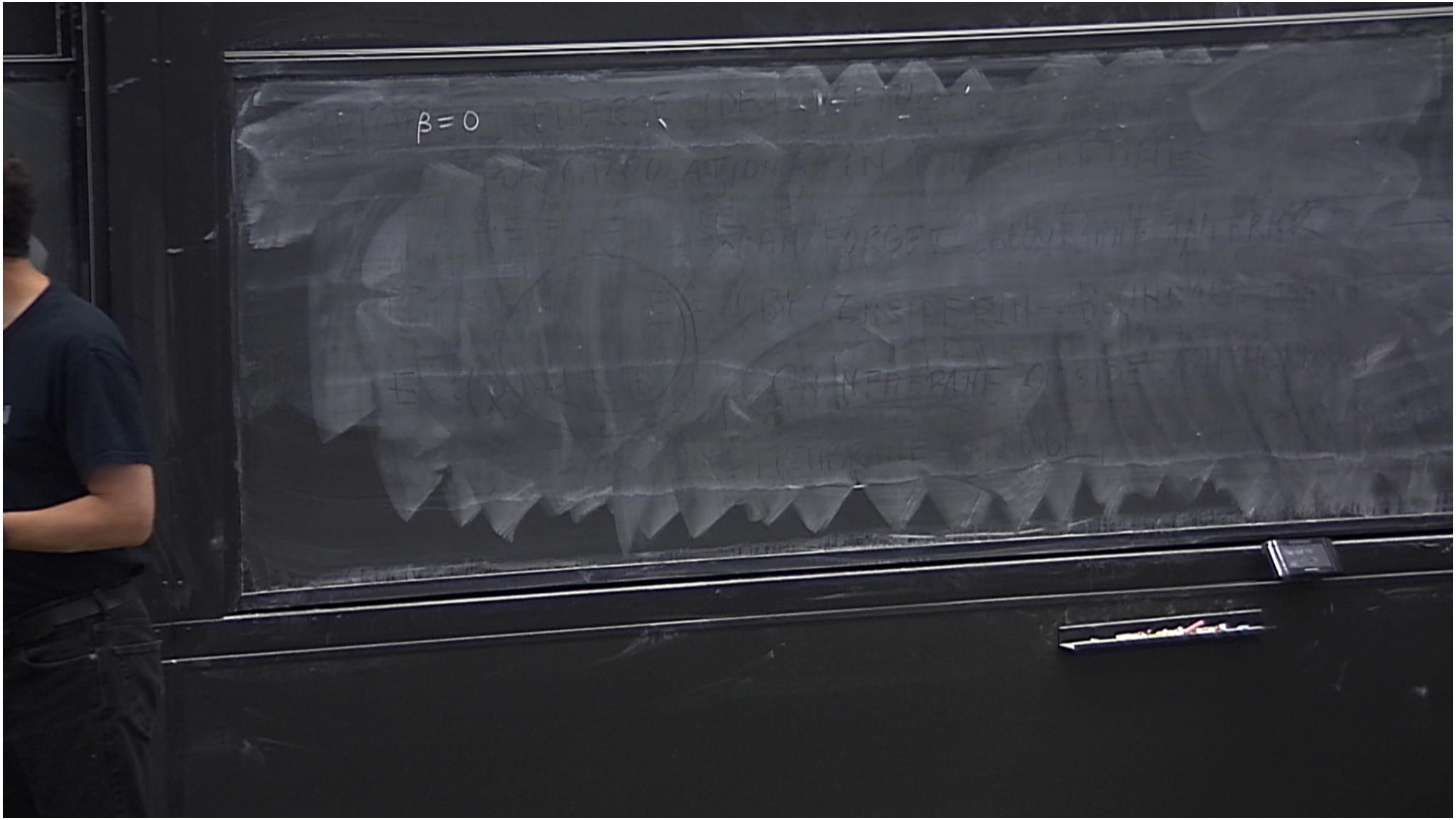
$$F^{\alpha\beta} = \eta^{\alpha\mu}\eta^{\beta\nu}F_{\mu\nu}$$

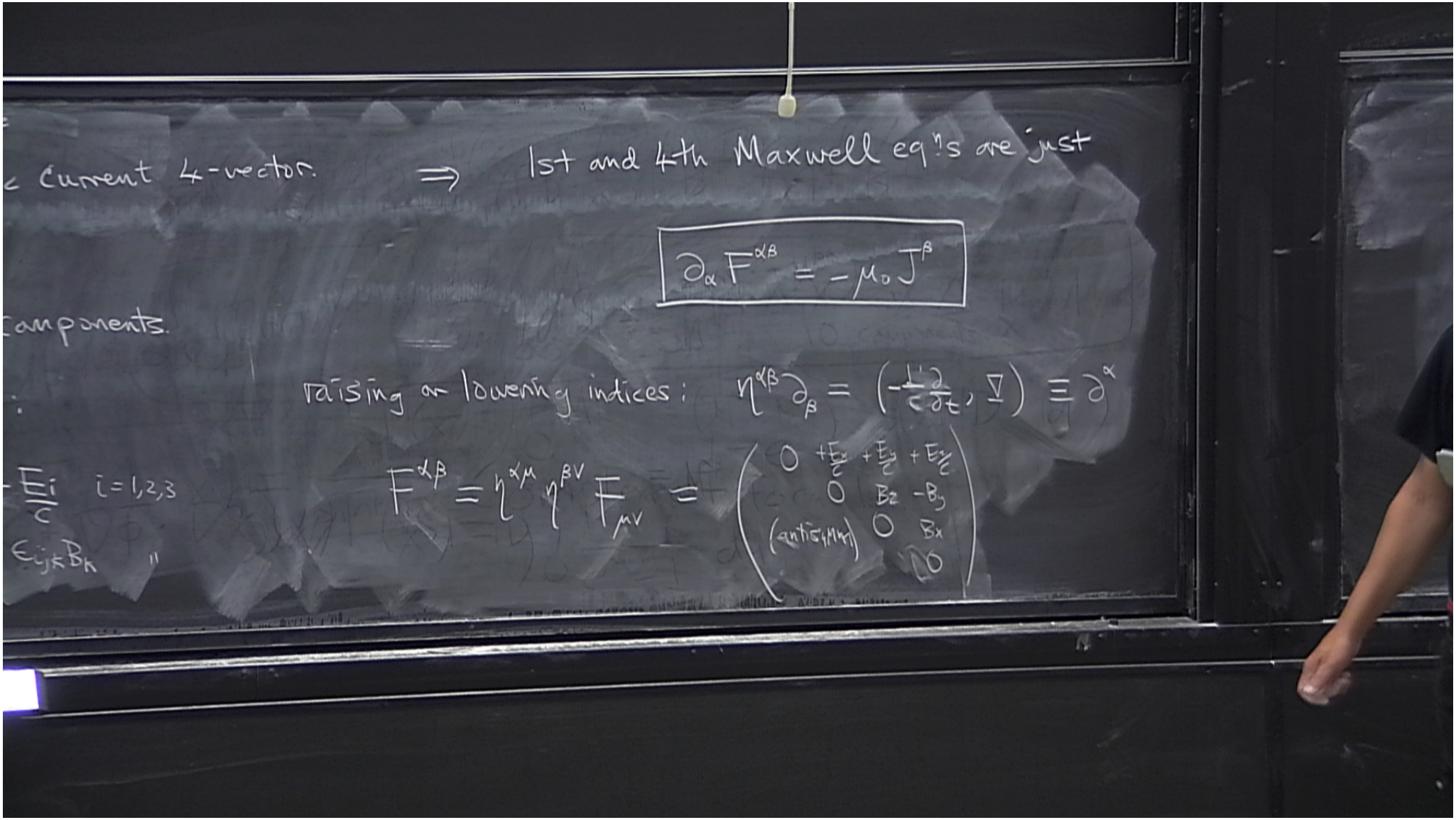

current 4-vector. ⇒ 1st and 4th Maxwell eq'n's are just
$= -\mu_0 J^\beta$
components.
raising or lowe
$\partial_\beta = \left(-\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \equiv \partial^\alpha$
$\frac{E_i}{c}$ $i = 1,2,3$
$F^{\alpha\beta} =$
$\epsilon_{ijk} B_k$ "

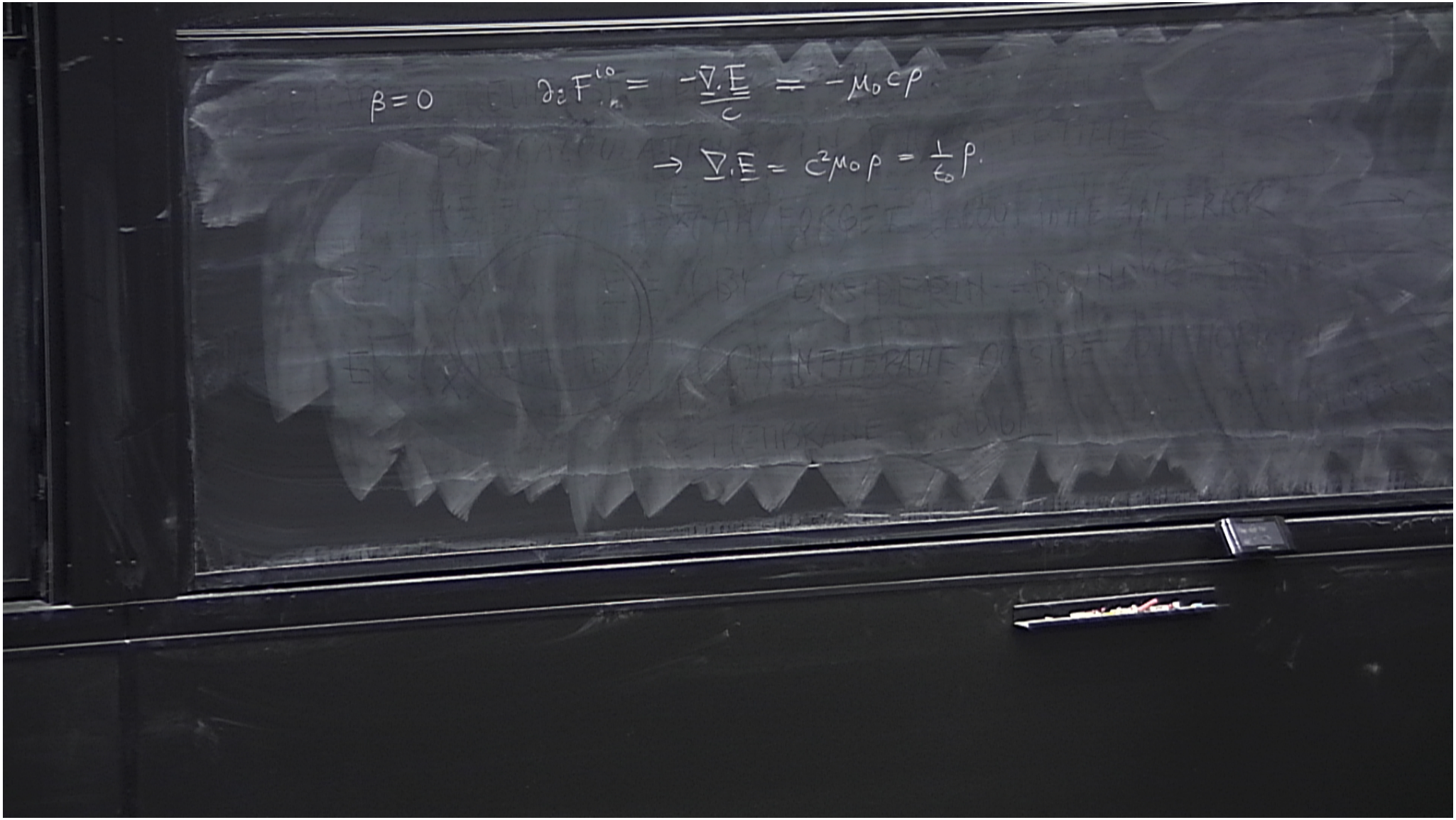

Current 4-vector. ⇒ 1st and 4th Maxwell eq'n's are just
components.
raising or lowering indices:
$\partial_\alpha F^{\alpha\beta}$
$\eta^{\alpha\beta}\partial_\beta =$
$\equiv \partial^\alpha$
$\frac{E_i}{c}$ $i=1,2,3$
$\epsilon_{ijk}B_k$ "
$F^{\alpha\beta} = \eta^{\alpha\mu}\eta^{\beta\nu}F_{\mu\nu} =$
$+\frac{E_x}{c}$
0

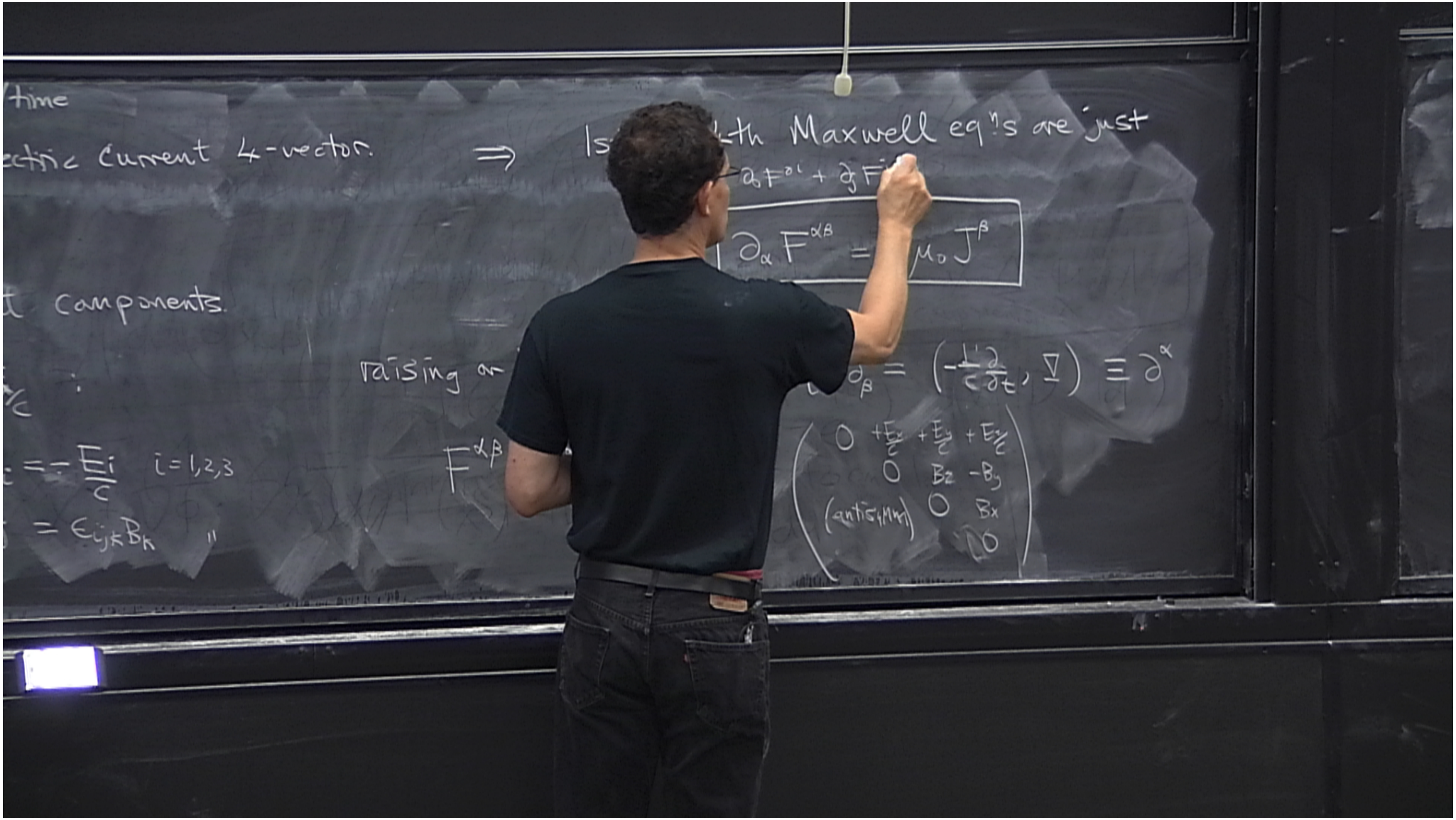

current 4-vector.
⇒ 1st and 4th Maxwell eq^ns are just
∂_α $-\mu_0 J^\beta$
amponents.
raising or lowering
$= \left(-\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \equiv \partial^\alpha$
$\frac{E_i}{c}$ $i=1,2,3$
$F^{\alpha\beta} = \eta^{\alpha\mu}$
$+\frac{E_y}{c}$ $+\frac{E_z}{c}$
B_z $-B_y$
0 B_x
0
$\epsilon_{ijk}B_k$ "

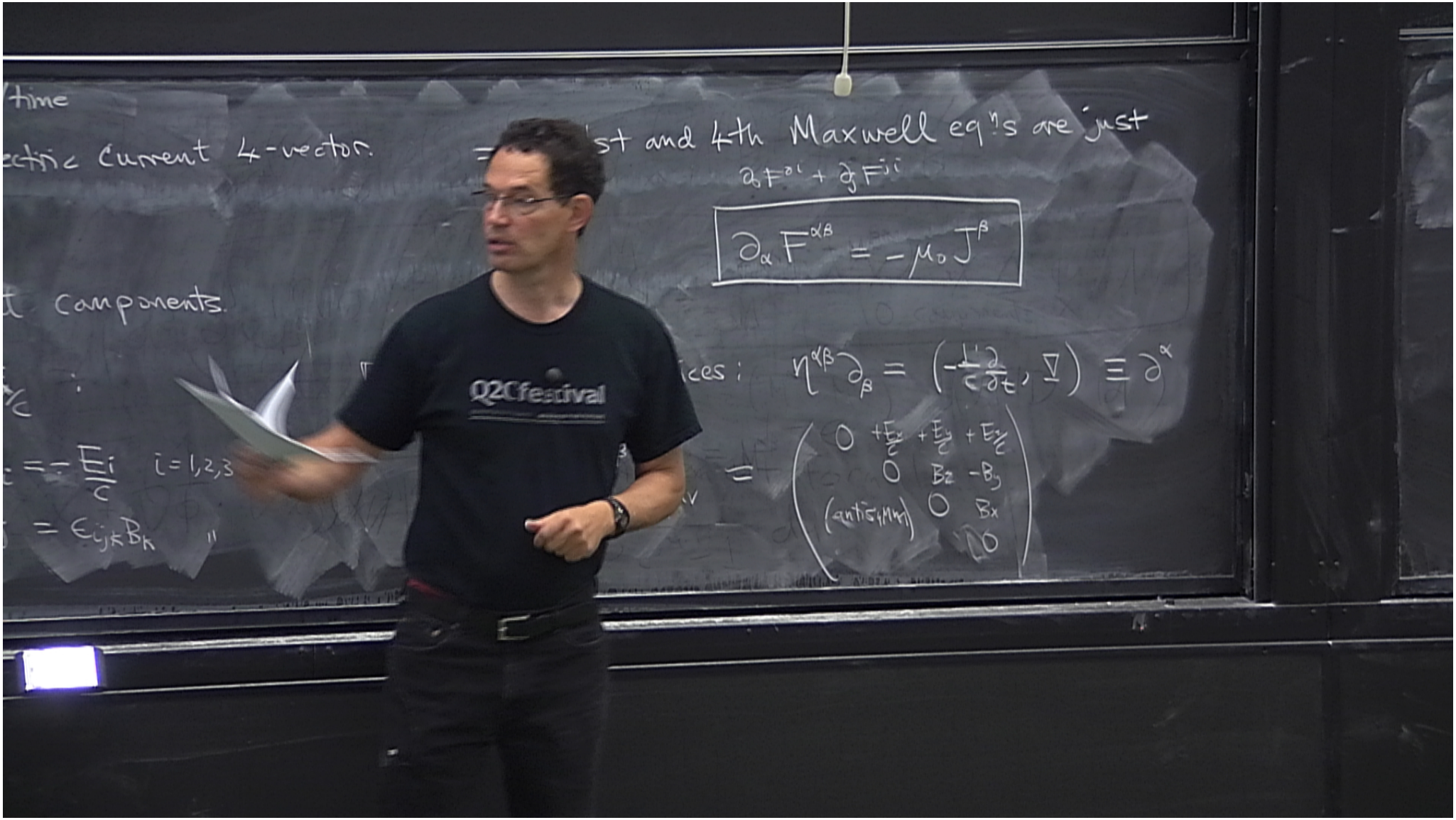


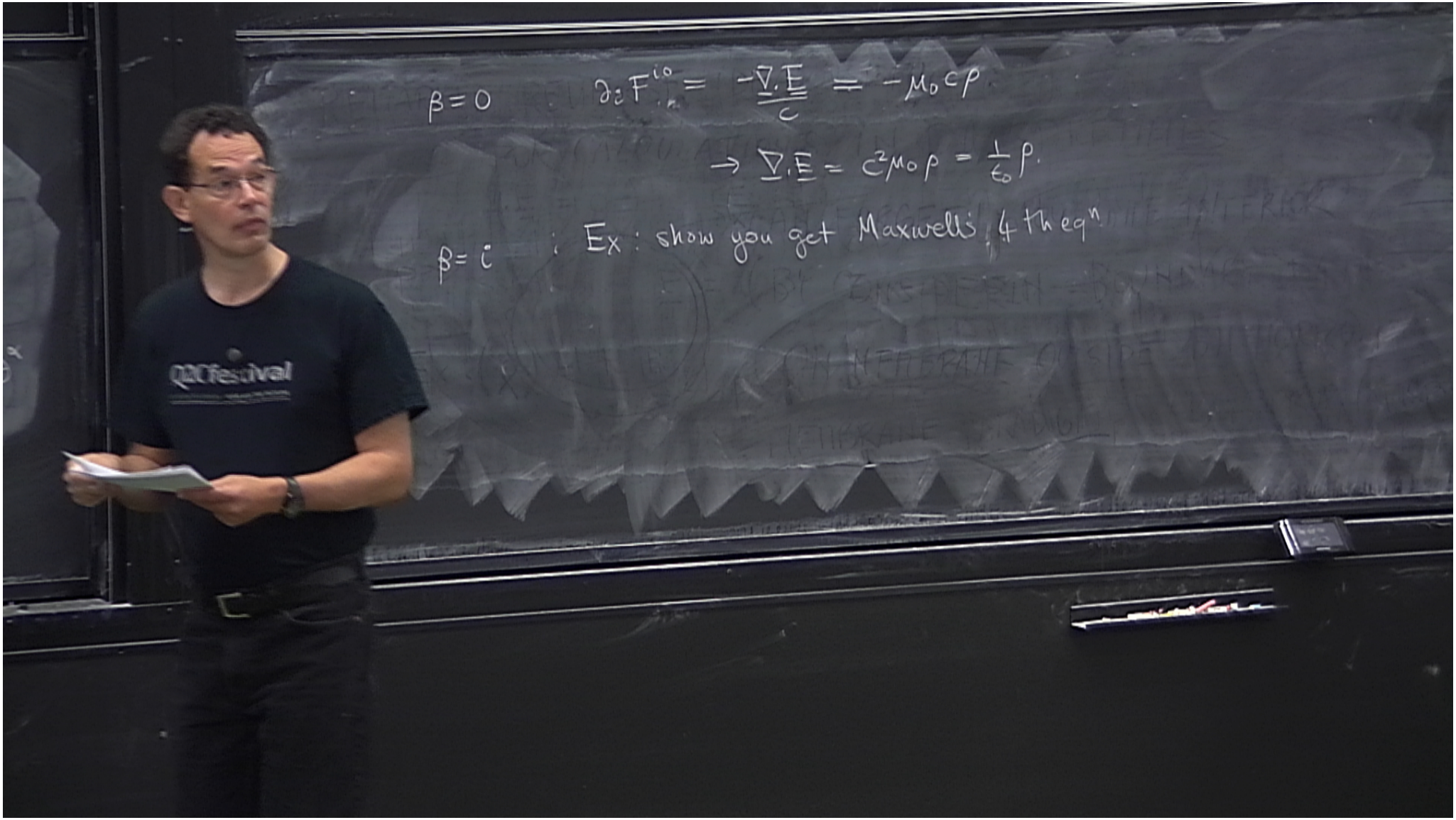

β=0

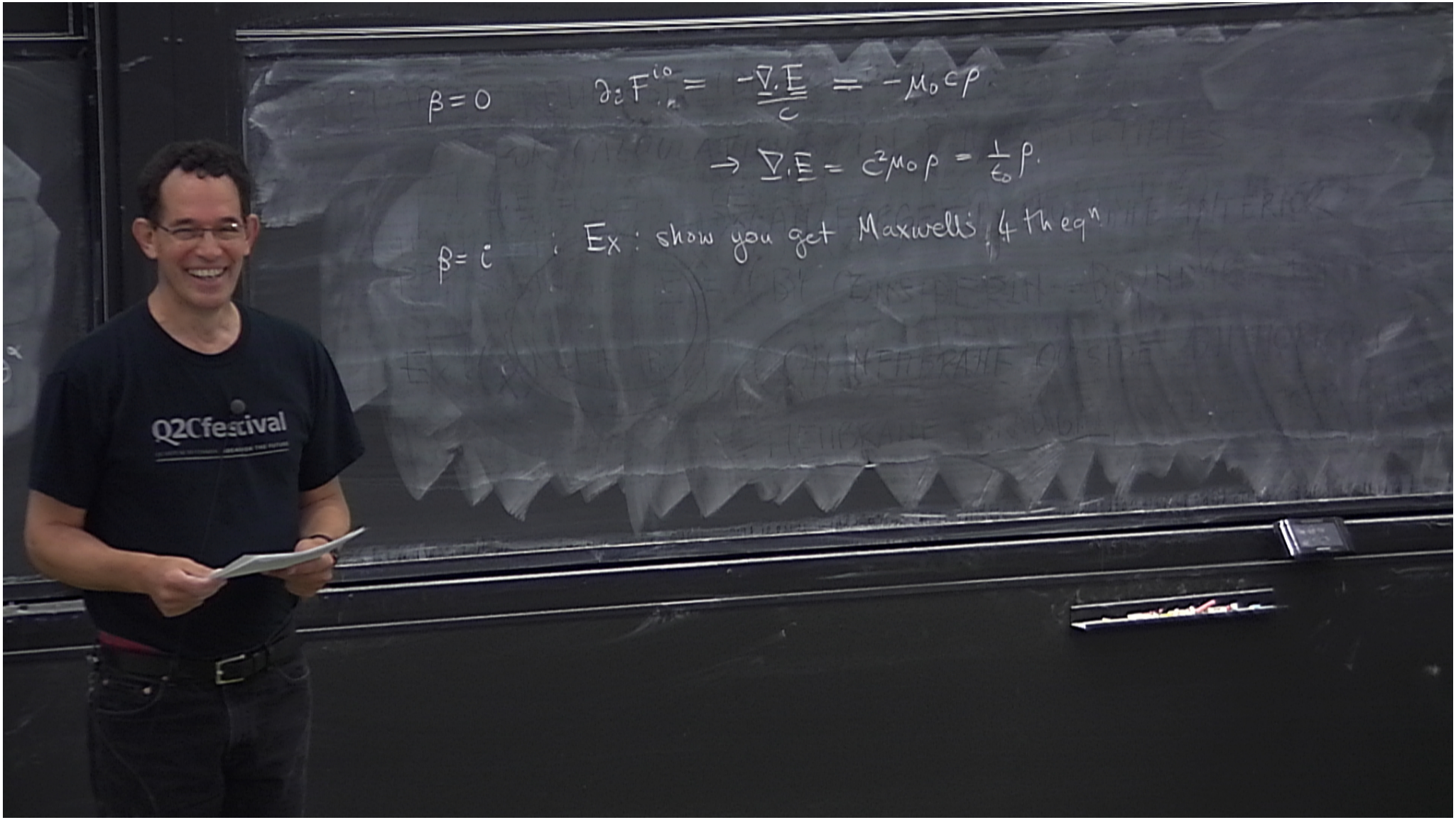

current 4-vector. ⇒ 1st and 4th Maxwell eq^n's are just
$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$
components.
raising or lowering indices: $\eta^{\alpha\beta}\partial_\beta = \left(-\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \equiv \partial^\alpha$
$\frac{E_i}{c}$ $i=1,2,3$
$\epsilon_{ijk}B_k$ "
$F^{\alpha\beta} = \eta^{\alpha\mu}\eta^{\beta\nu}F_{\mu\nu} =$
$\begin{pmatrix} 0 & +\frac{E_x}{c} & +\frac{E_y}{c} & +\frac{E_z}{c} \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ (\text{antisymm}) & & & 0 \end{pmatrix}$


$\beta = 0$
$\partial_2 F^{i0} = -\frac{\nabla \cdot \underline{E}}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot \underline{E} = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$


time
ectric current 4-vector. ⇒
components.
raising
$\partial_\alpha F^{\alpha\beta} = \mu_0 J^\beta$
$\partial_\beta = \left(-\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \equiv \partial^\alpha$
$= -\frac{E_i}{c}$ $i=1,2,3$
$= \epsilon_{ijk} B_k$
$F^{\alpha\beta}$
$\begin{pmatrix} 0 & +E_x/c & +E_y/c & +E_z/c \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$
(antisymm)


time
ectric current 4-vector.
st and 4th Maxwell eq^n's are just
$\partial_0 F^{0i} + \partial_j F^{ji}$
$\partial_\alpha F^{\alpha\beta} = -\mu_0 J^\beta$
t components.
ices:
$\eta^{\alpha\beta}\partial_\beta = \left(-\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) \equiv \partial^\alpha$
$= -\frac{E_i}{c}$ $i = 1,2,3$
$= \epsilon_{ijk} B_k$ "
$=$
$\begin{pmatrix} 0 & +\frac{E_x}{c} & +\frac{E_y}{c} & +\frac{E_z}{c} \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ \text{(antisymm)} & & & 0 \end{pmatrix}$
Q2Cfestival


$\beta = 0$ $\partial_i F^{i0} = -\frac{\underline{\nabla}.\underline{E}}{c} = -\mu_0 c \rho$
$\rightarrow \underline{\nabla}.\underline{E} = c^2 \mu_0 \rho = \frac{1}{\epsilon_0}\rho.$
$\beta = i$; Ex : show you get Maxwell's 4 th eq^n
Q2Cfestival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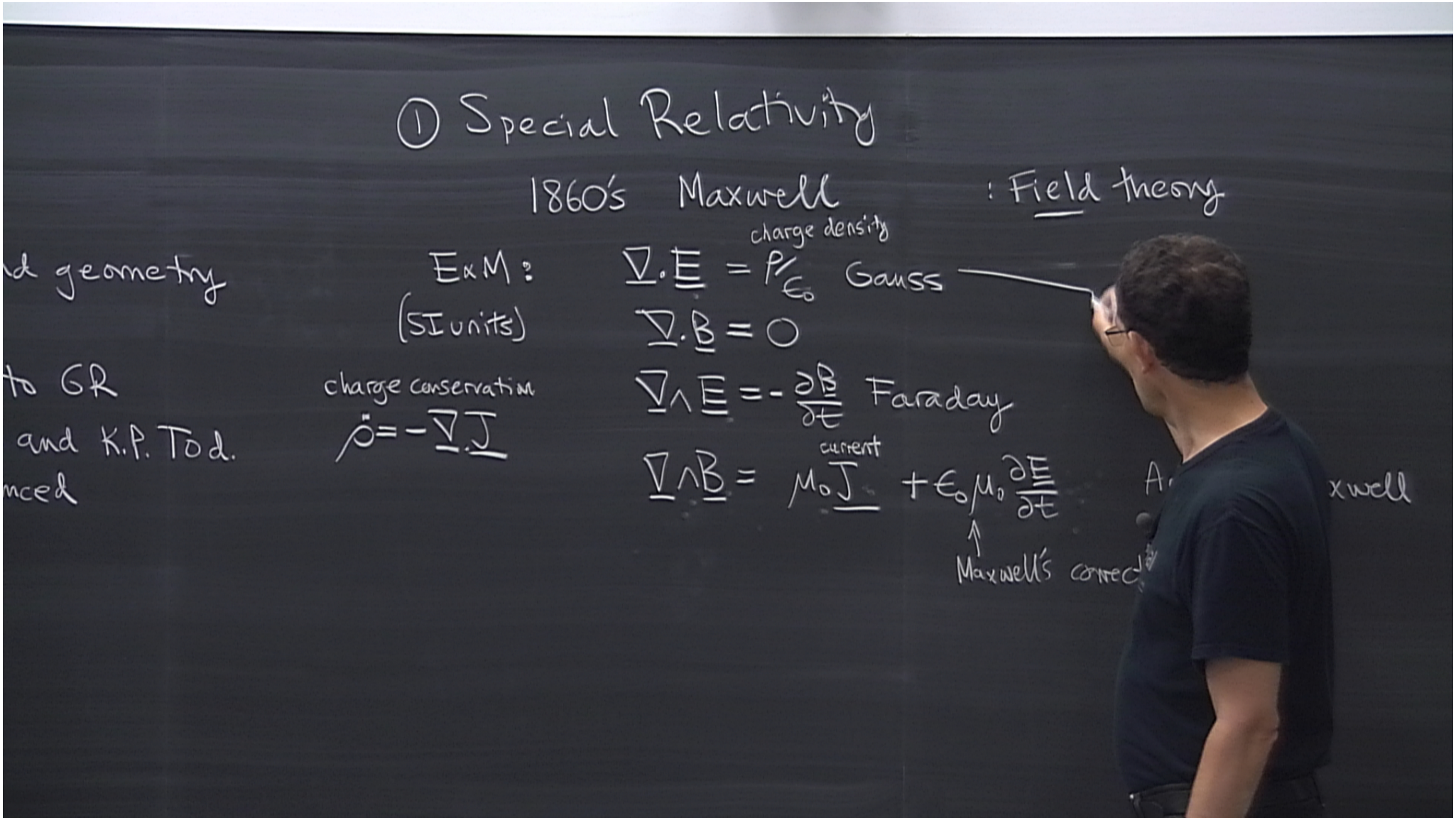

① Special Relativity
1860's Maxwell : Field theory
charge density
E×M: $\nabla \cdot \underline{E} = \rho/\epsilon_0$ Gauss
(SI units) $\nabla \cdot \underline{B} = 0$
charge conservation $\dot{\rho} = -\nabla \cdot \underline{J}$
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday
current
$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$
Maxwell's
d geometry
to GR
and K.P. Tod.
nced
xwell

① Special Relativity

1860's Maxwell : Field theory

d geometry

to GR

and K.P. Tod.

nced

E×M: (SI units)

$\nabla \cdot \underline{E} = \rho / \epsilon_0$ Gauss (ρ: charge density)

$\nabla \cdot \underline{B} = 0$

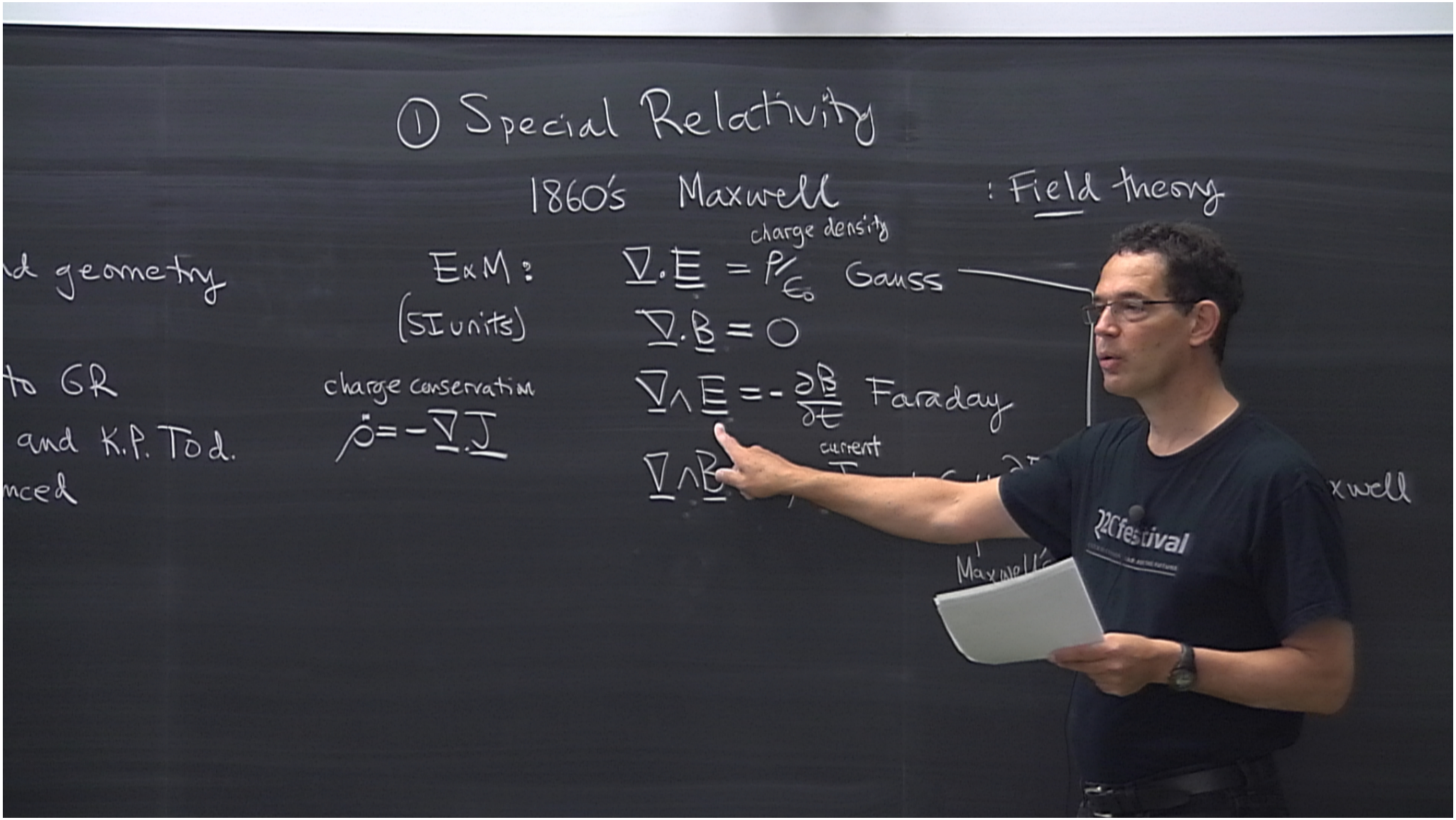
$\nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t}$ Faraday

$\nabla \wedge \underline{B} = \mu_0 \underline{J} + \epsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$ Ampere + Maxwell (J: current)

↑ Maxwell's correction

charge conservation

$\dot{\rho} = -\nabla \cdot \underline{J}$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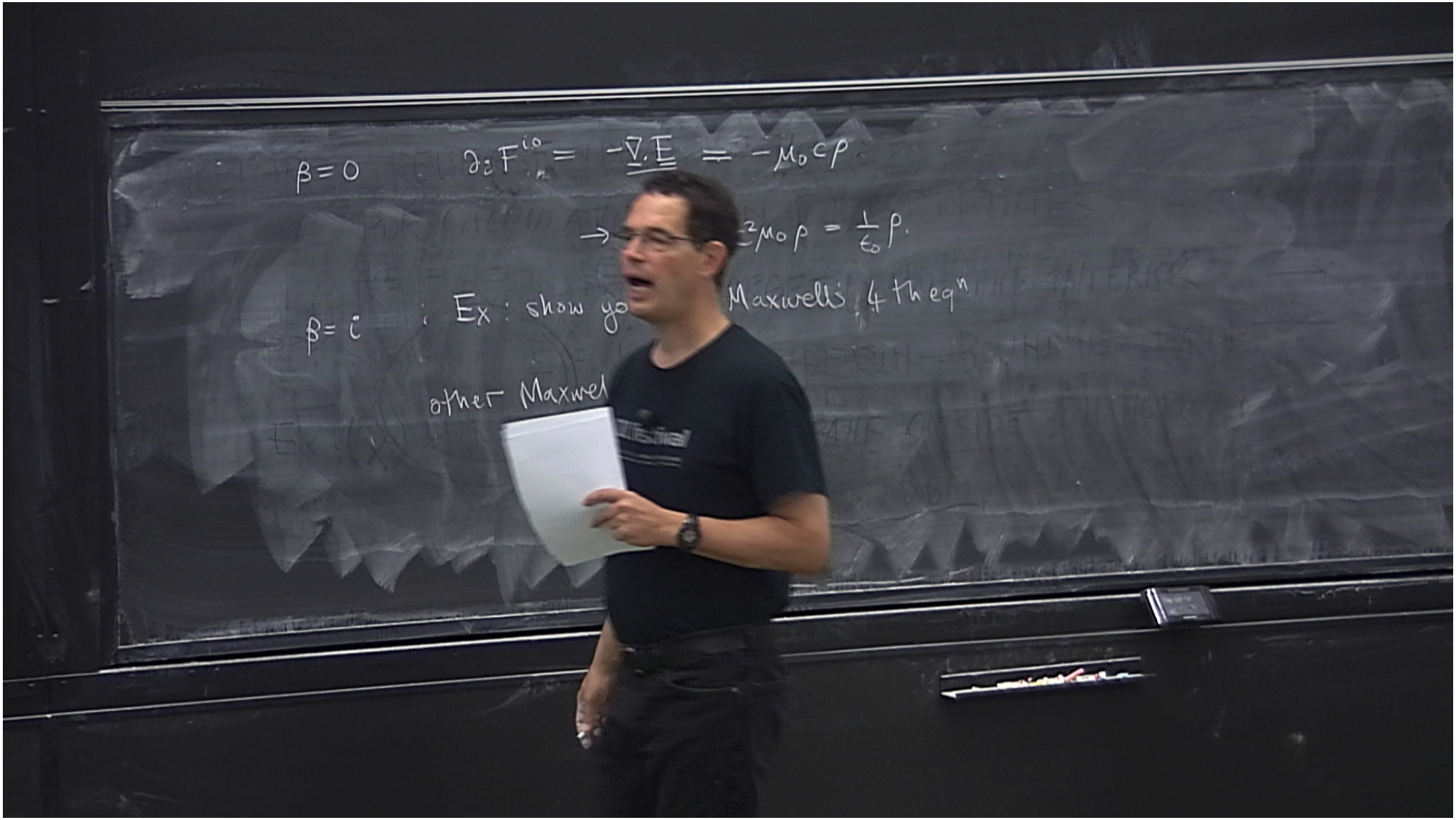
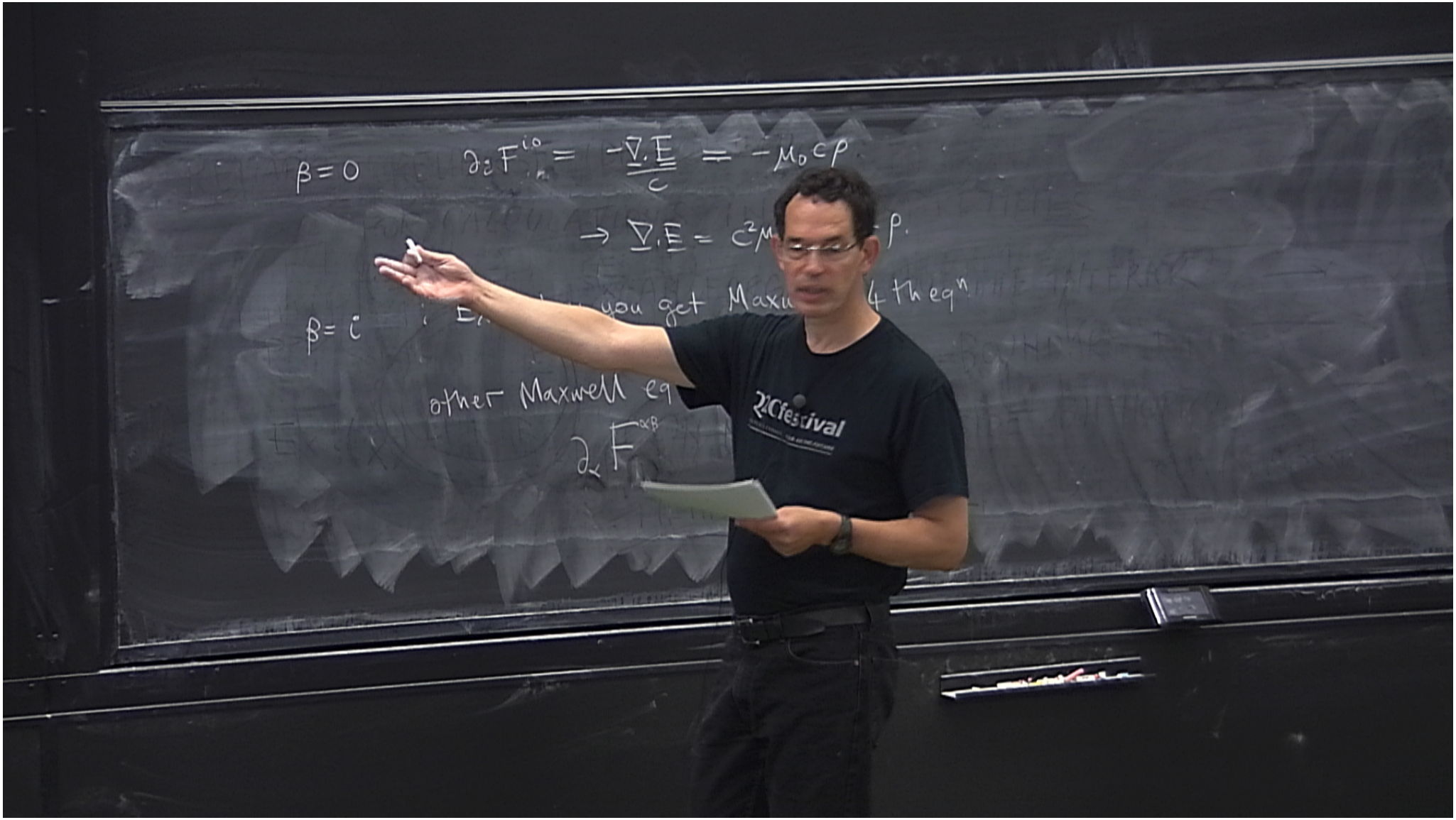
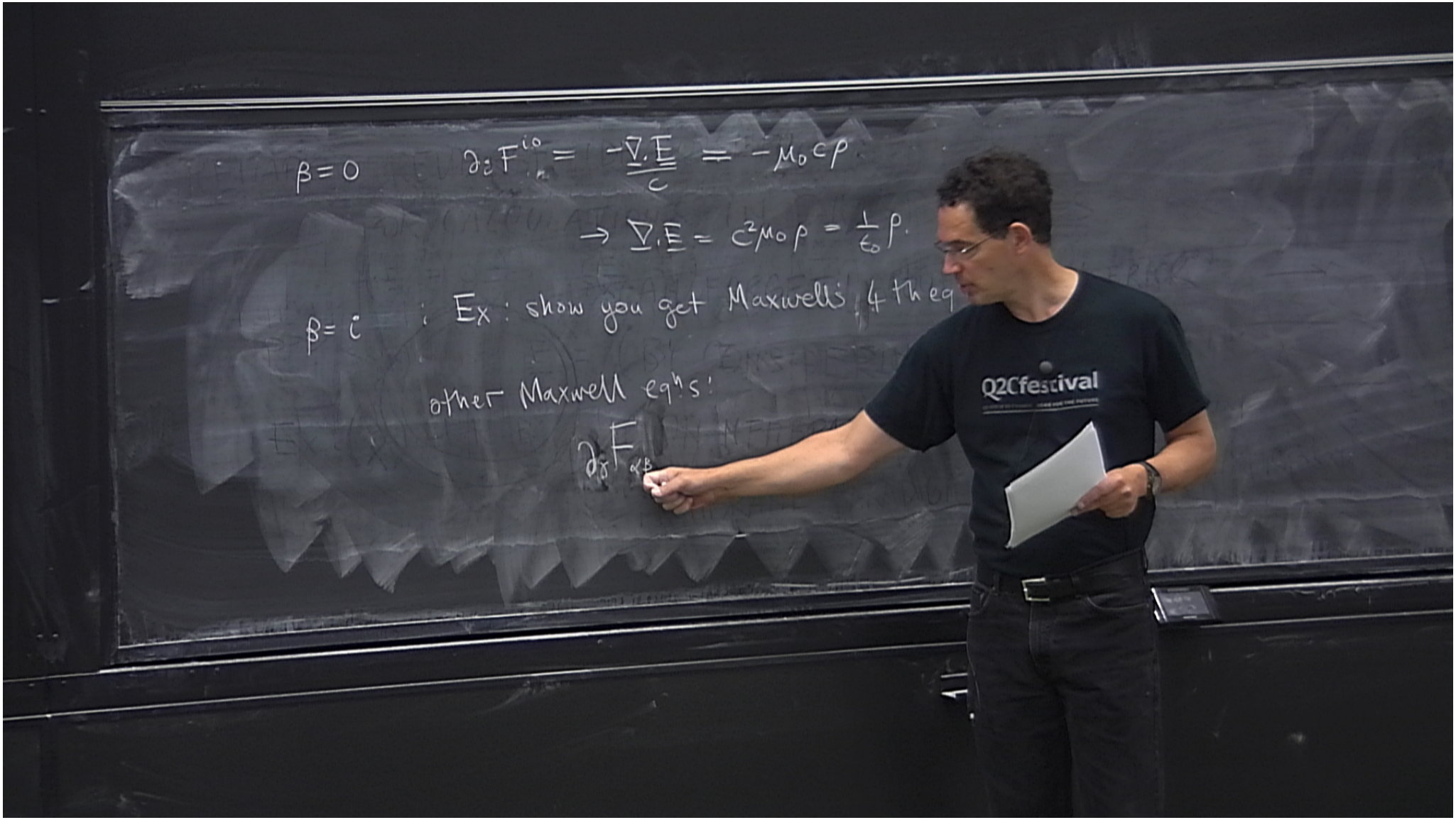
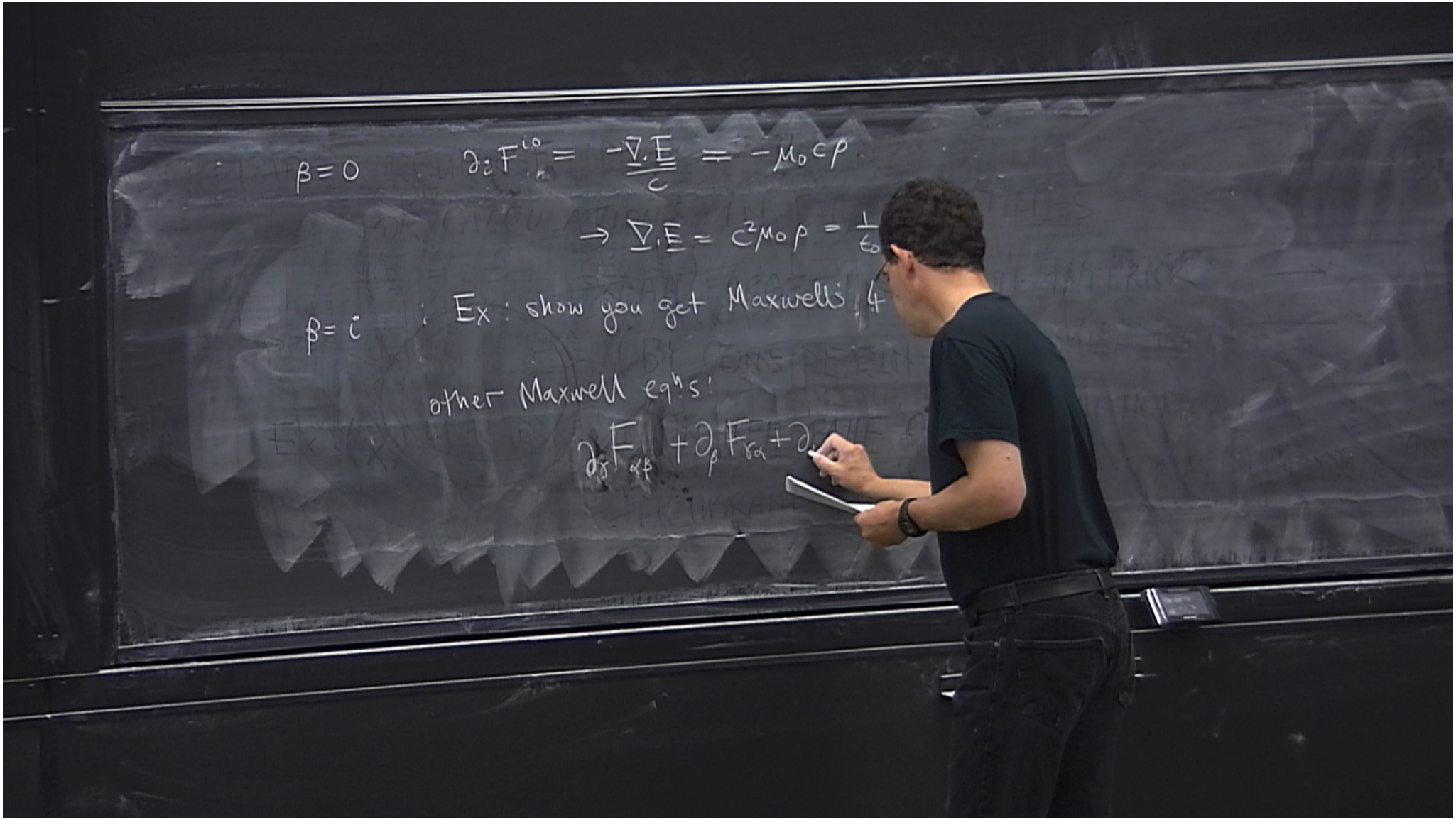
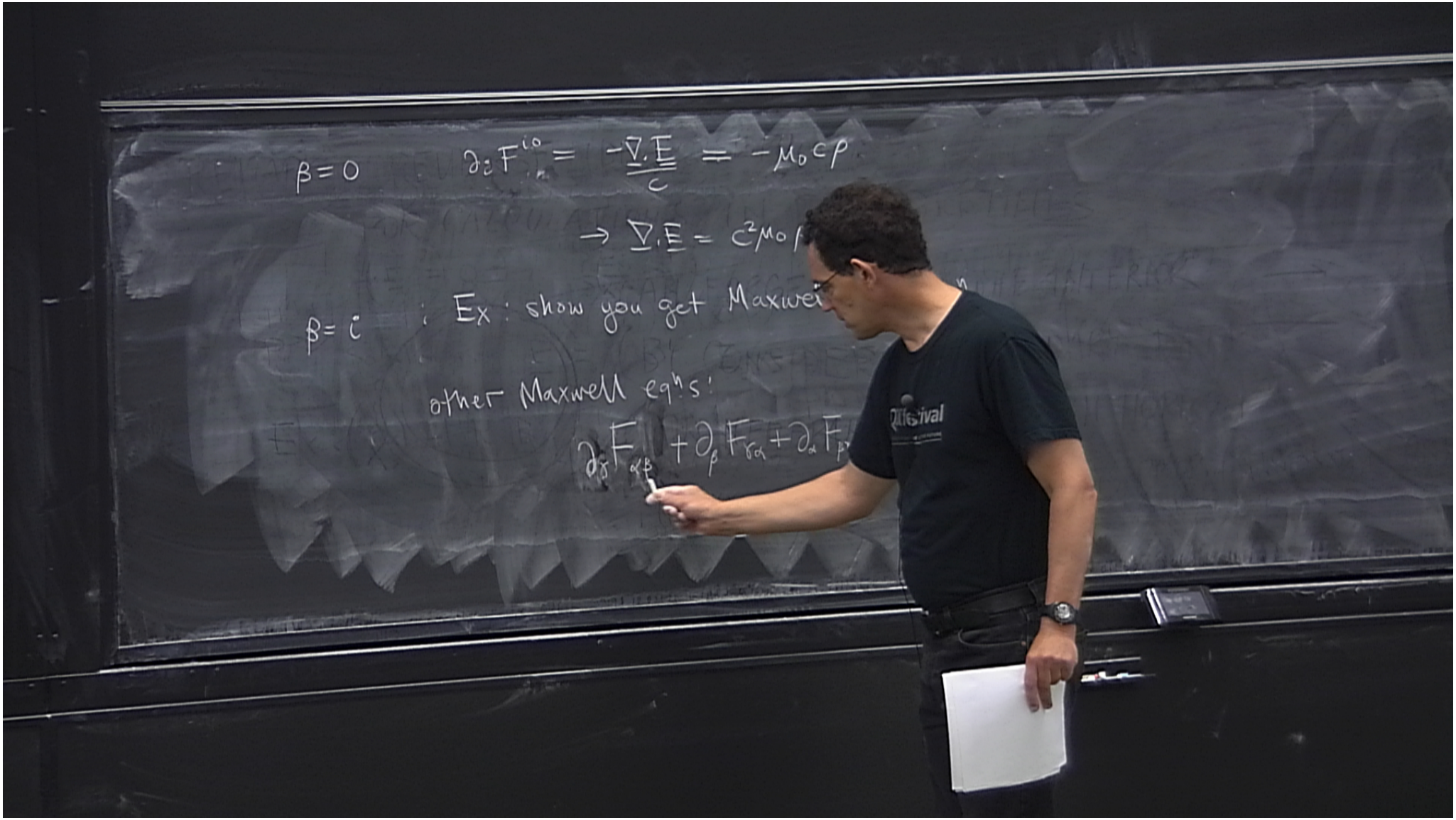
Maxwell's
22Cfestival

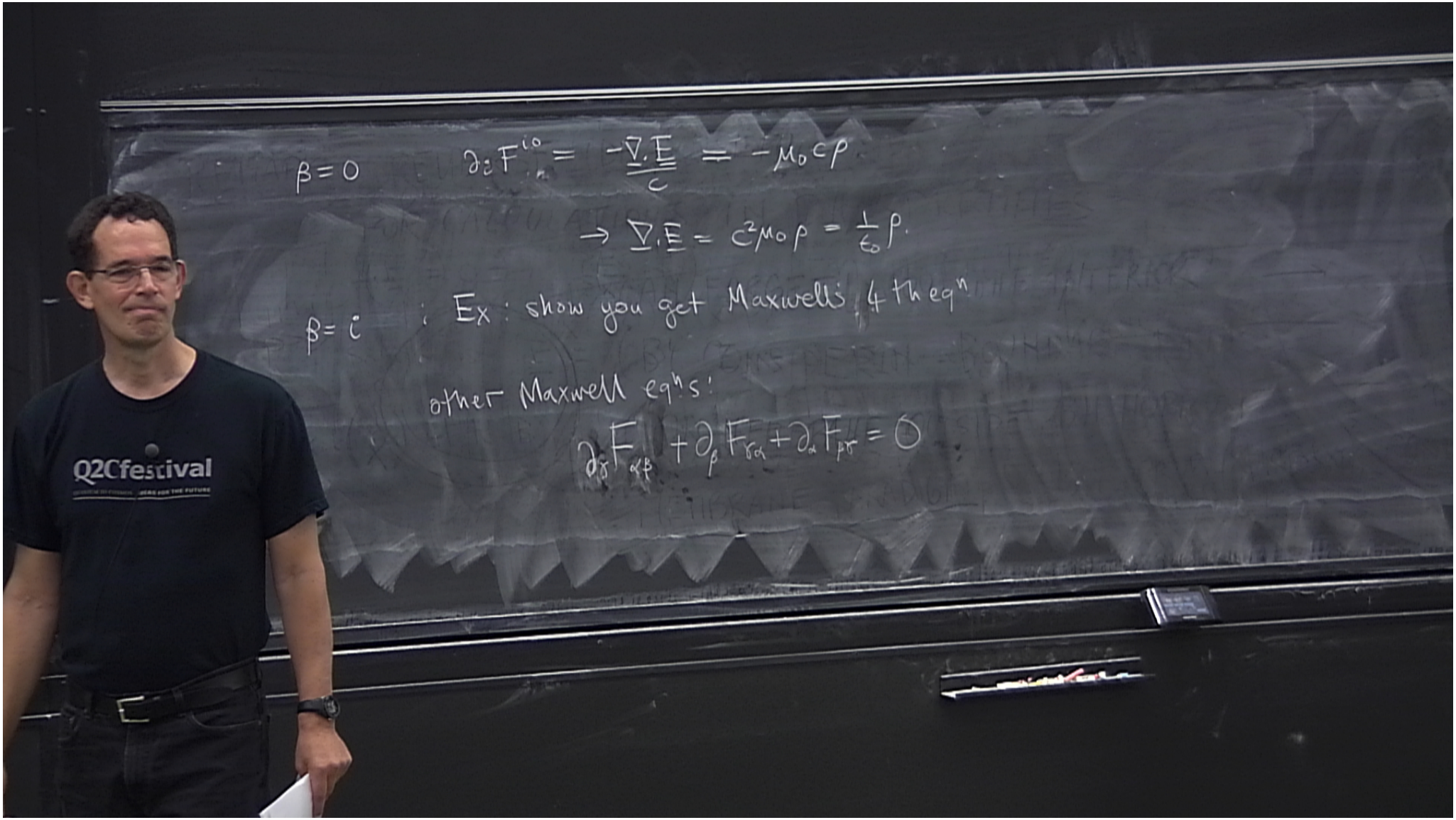

β=0
$\partial_i F^{i0} = -\nabla \cdot \underline{E} = -\mu_0 c \rho$
$\rightarrow$
$\mu_0 \rho = \frac{1}{\epsilon_0} \rho$.
β=i
; Ex: show yo
Maxwell's 4th eqn
other Maxwe


β=0
$\partial_i F^{i0} = -\frac{\nabla \cdot E}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot E = c^2 \mu$ ρ.
you get Maxu 4th eqn
β=i
other Maxwell eq
$\partial_\alpha F^{\alpha\beta}$


β = 0
$\partial_i F^{i0} = -\frac{\nabla \cdot E}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot E = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$
β = i ; Ex: show you get Maxwell's 4th eq
other Maxwell eq^n's:
$\partial_\gamma F_{\alpha\beta}$
Q2Cfestival


β = 0
$\partial_i F^{i0} = -\frac{\nabla \cdot E}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot E = c^2 \mu_0 \rho = \frac{1}{\epsilon_0}$
β = i ; Ex: show you get Maxwell's 4
other Maxwell eq^n's:
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha$


β = 0
$\partial_i F^{i0} = -\frac{\nabla \cdot E}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot E = c^2 \mu_0 \rho$
β = i ; Ex: show you get Maxwe
other Maxwell eq^n's:
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta}$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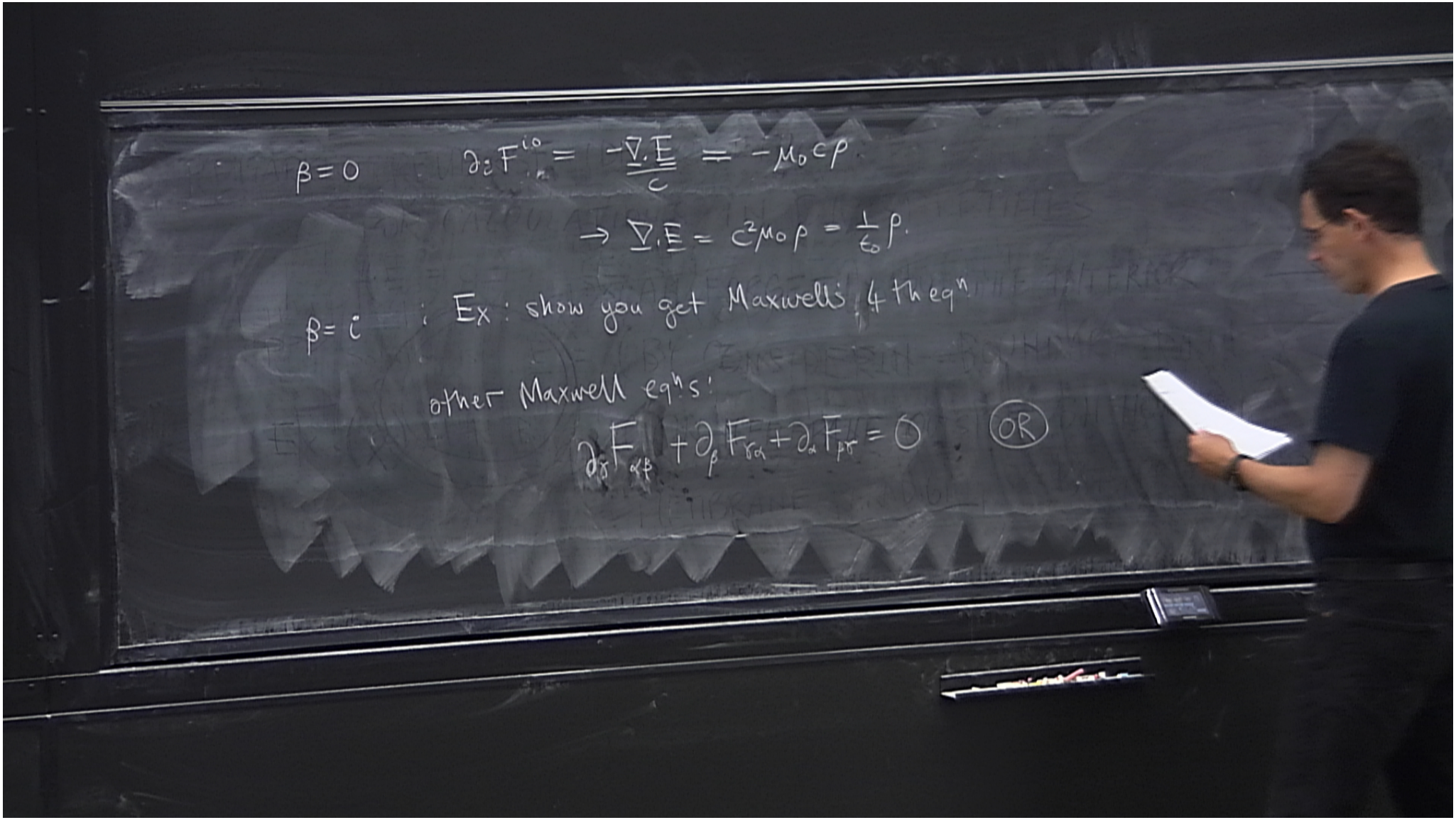
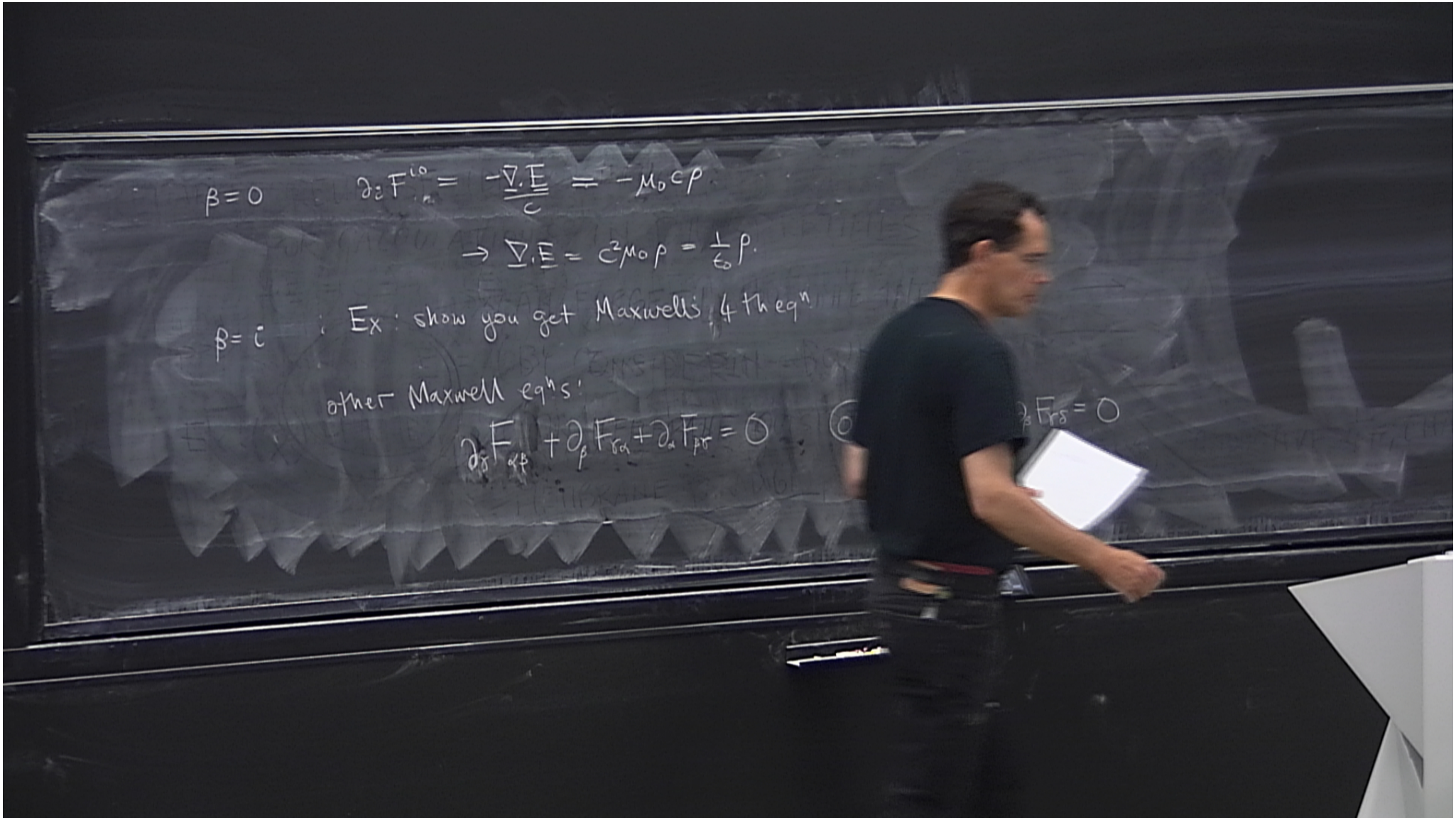
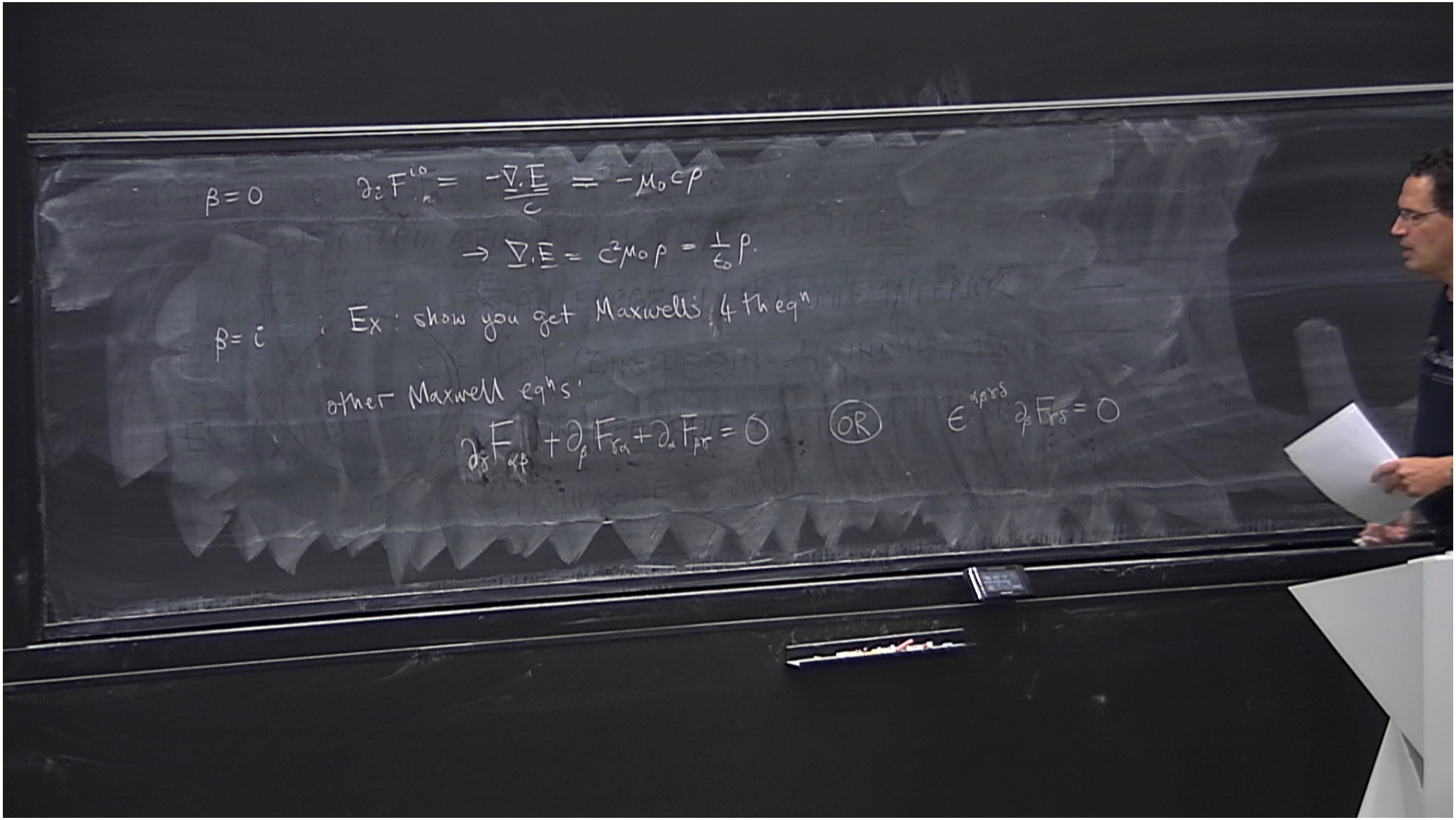
$\rightarrow \nabla \cdot E = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$
$\beta = i$; Ex: show you get Maxwell's 4th eqn
other Maxwell eqn's:
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$
Q2Cfestival


$\beta = 0$ $\partial_i F^{i0} = -\frac{\nabla \cdot E}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot E = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$
$\beta = i$; Ex: show you get Maxwell's 4th eqn
other Maxwell eqn's:
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$ (OR)


β = 0
$\partial_\alpha F^{\alpha\beta} = -\frac{\nabla \cdot \underline{E}}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot \underline{E} = c^2 \mu_0 \rho = \frac{1}{\epsilon_0}\rho$
β = i
Ex: show you get Maxwell's 4th eqn
other Maxwell eqn's:
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$
$\partial_\beta F_{\gamma\delta} = 0$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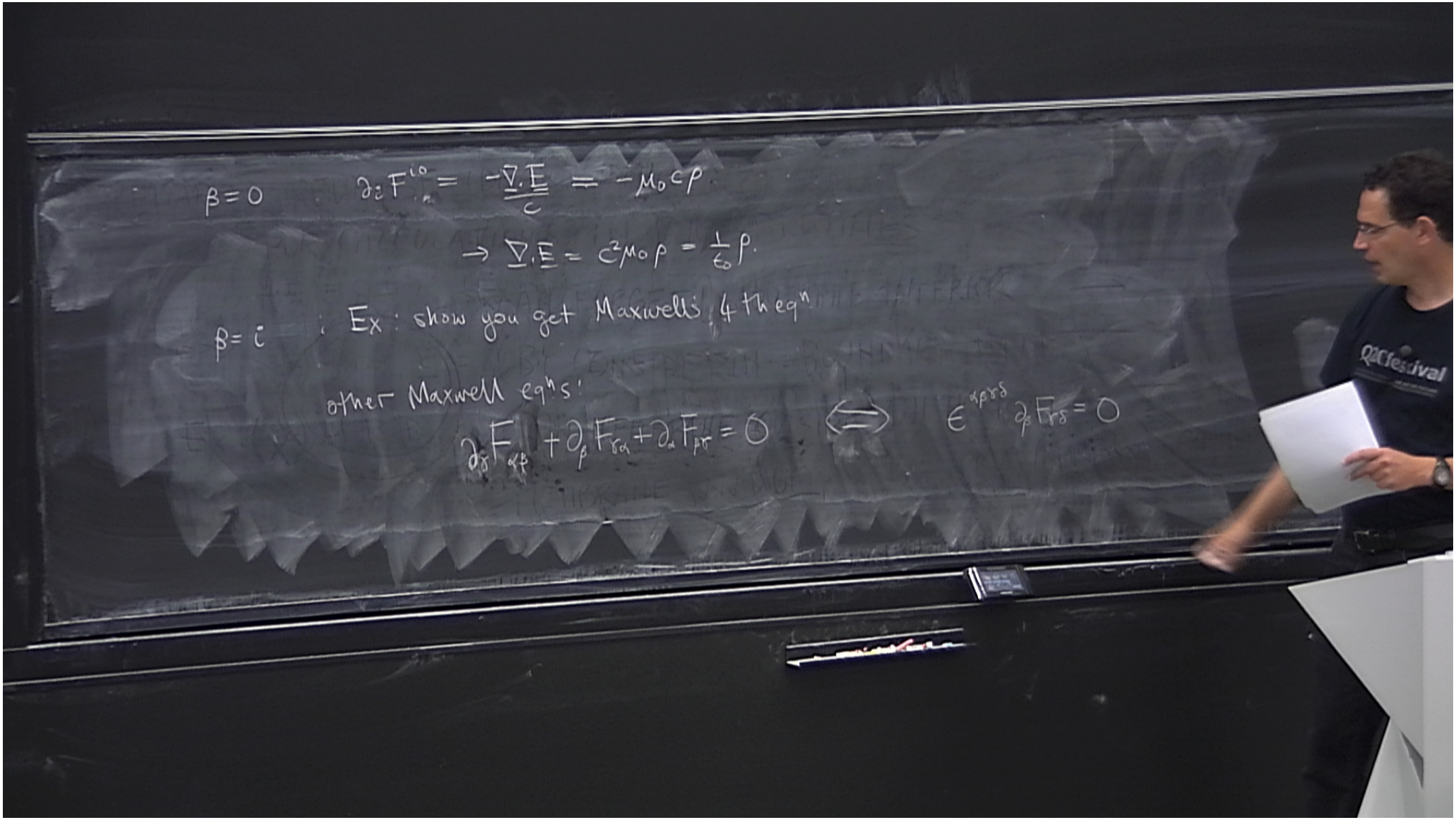
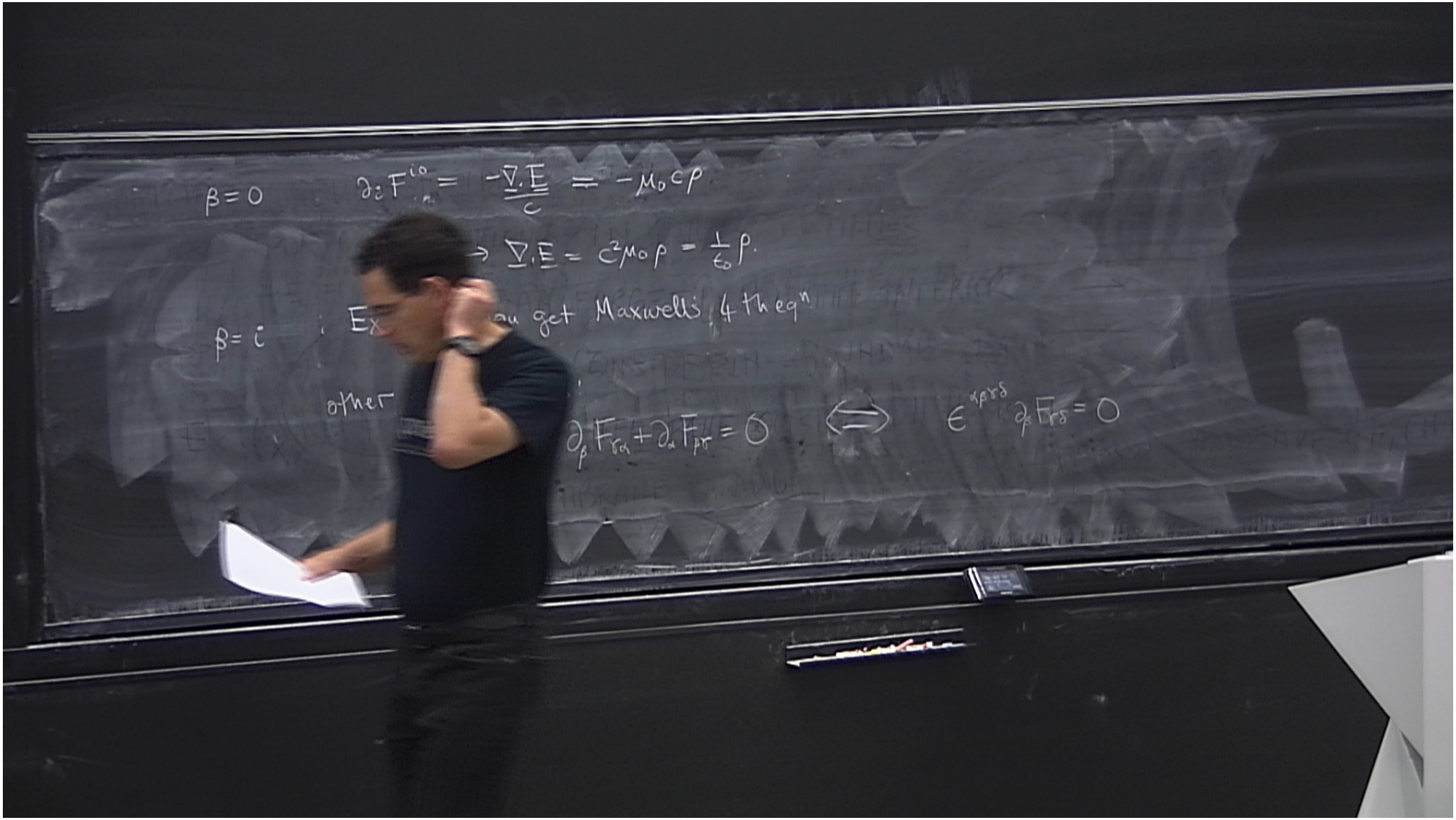
β = i ; Ex : show you get Maxwell's 4th eqn
other Maxwell eqn's:
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$
OR
$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$


$\beta = 0$
$\partial_i F^{i0} = -\frac{\nabla \cdot E}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot E = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$
$\beta = i$; Ex: show you get Maxwell's 4th eqn
other Maxwell eq^{n}s:
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0 \iff \epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$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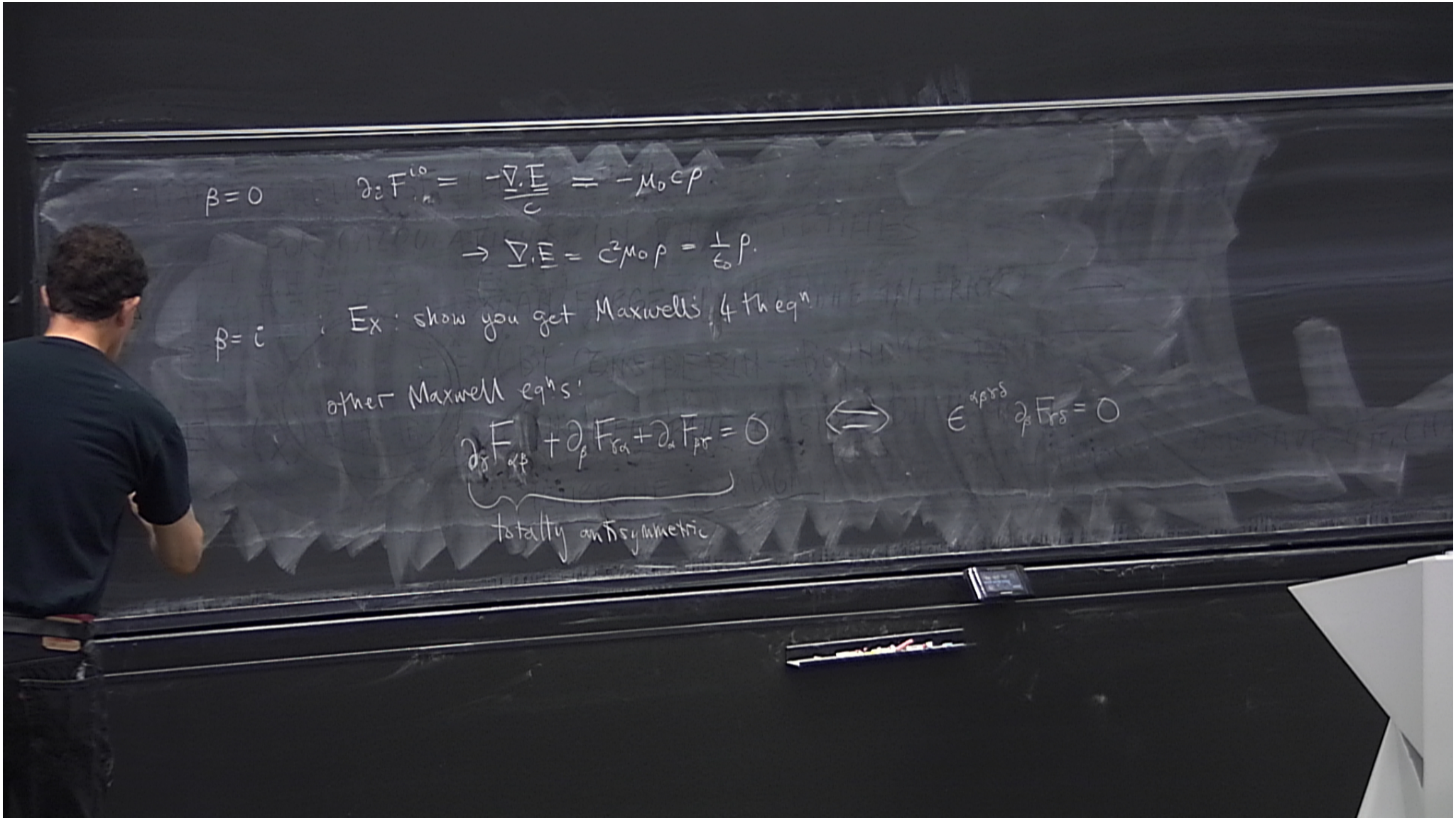
$\beta = i$
get Maxwell's 4 th eq^n
other
$\partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0 \iff \epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$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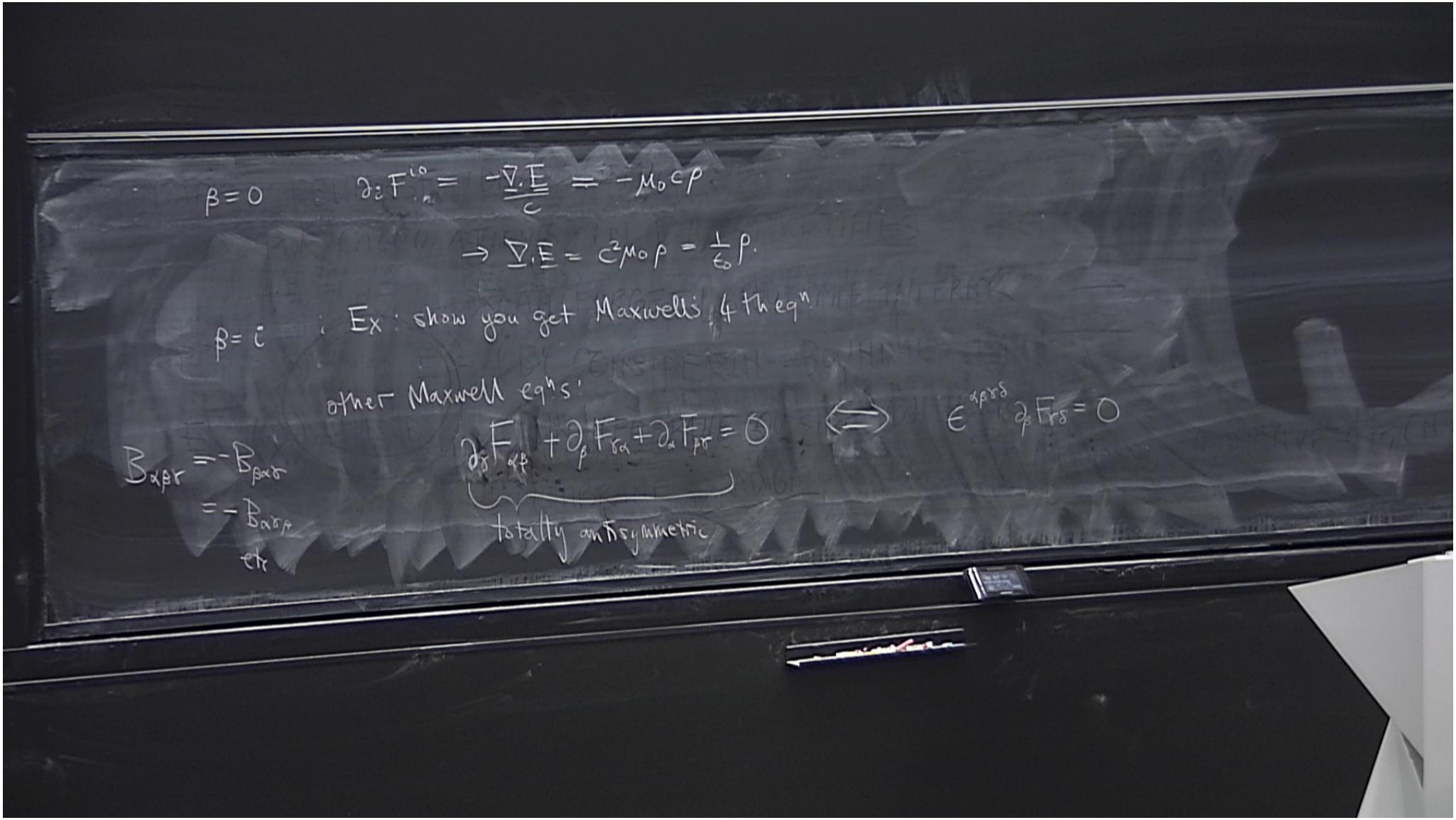
totally antisymmetric


$\beta = 0$
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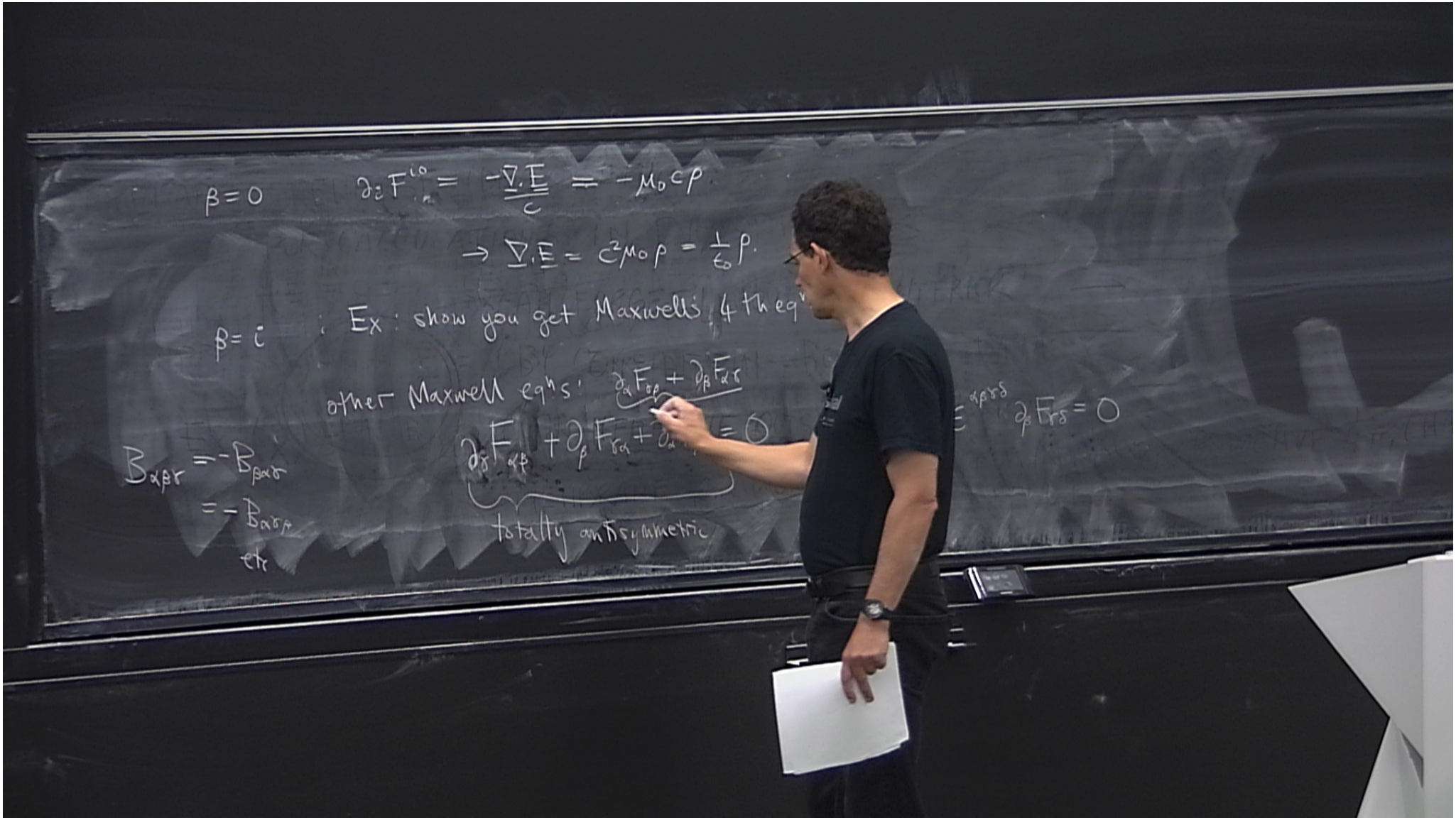
$\rightarrow \nabla \cdot \underline{E} = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$
$\beta = i$; Ex: show you get Maxwell's 4th eq^n
other Maxwell eq^{n}s:
$B_{\alpha\beta\gamma} = -B_{\beta\alpha\gamma}$
$= -B_{\alpha\gamma\beta}$
etc
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$
totally antisymmetric
$\Leftrightarrow$
$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$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$\partial_i F^{i0} = -\frac{\nabla \cdot E}{c} = -\mu_0 c \rho$
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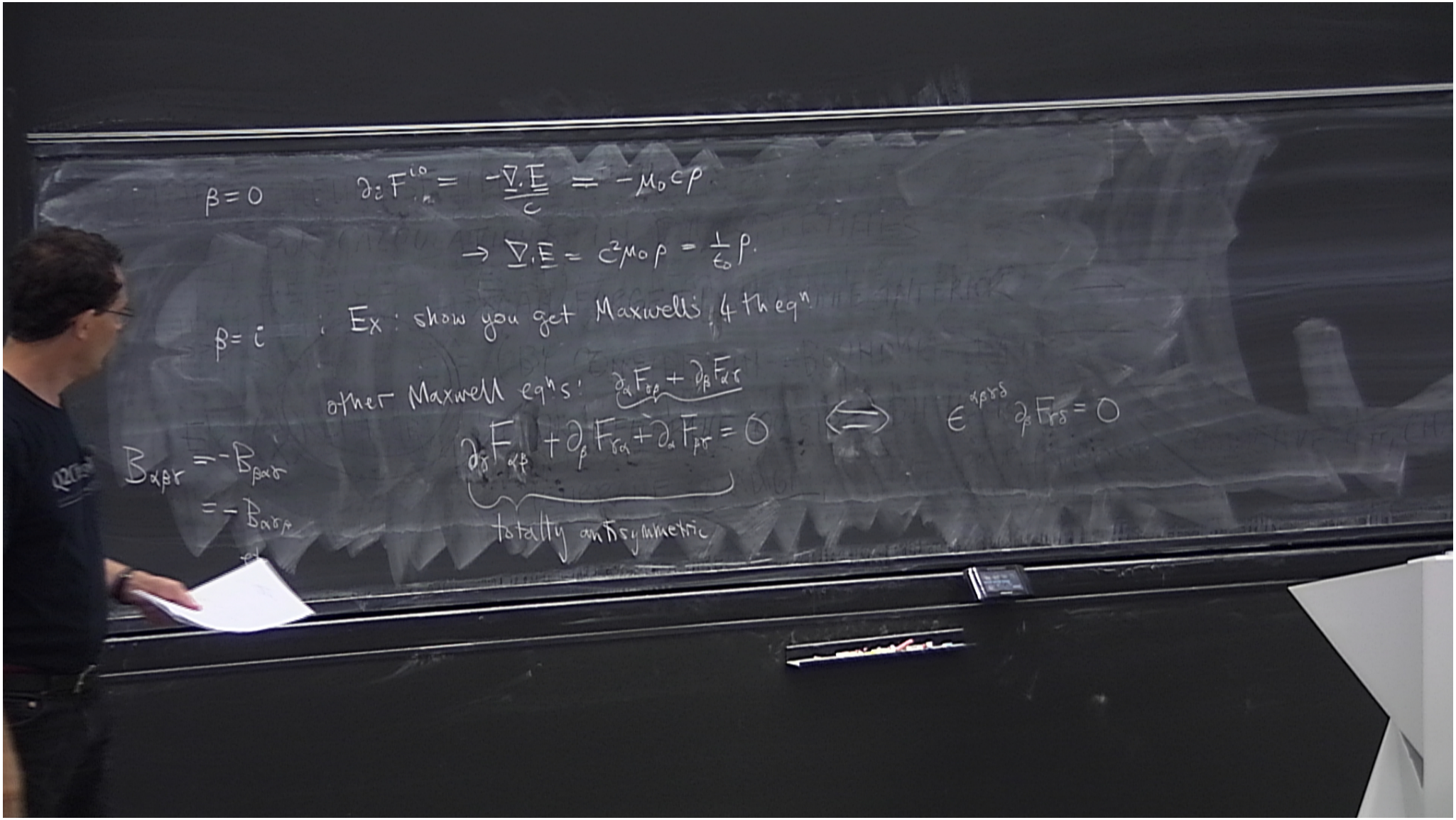
$\beta = i$; Ex: show you get Maxwell's 4th eq
other Maxwell eq^n's: $\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha}$
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha$ ... $= 0$
totally antisymmetric
$B_{\alpha\beta\gamma} = -B_{\beta\alpha\gamma}$
$= -B_{\alpha\gamma\beta}$
etc
$\partial_\beta F_{\gamma\delta} = 0$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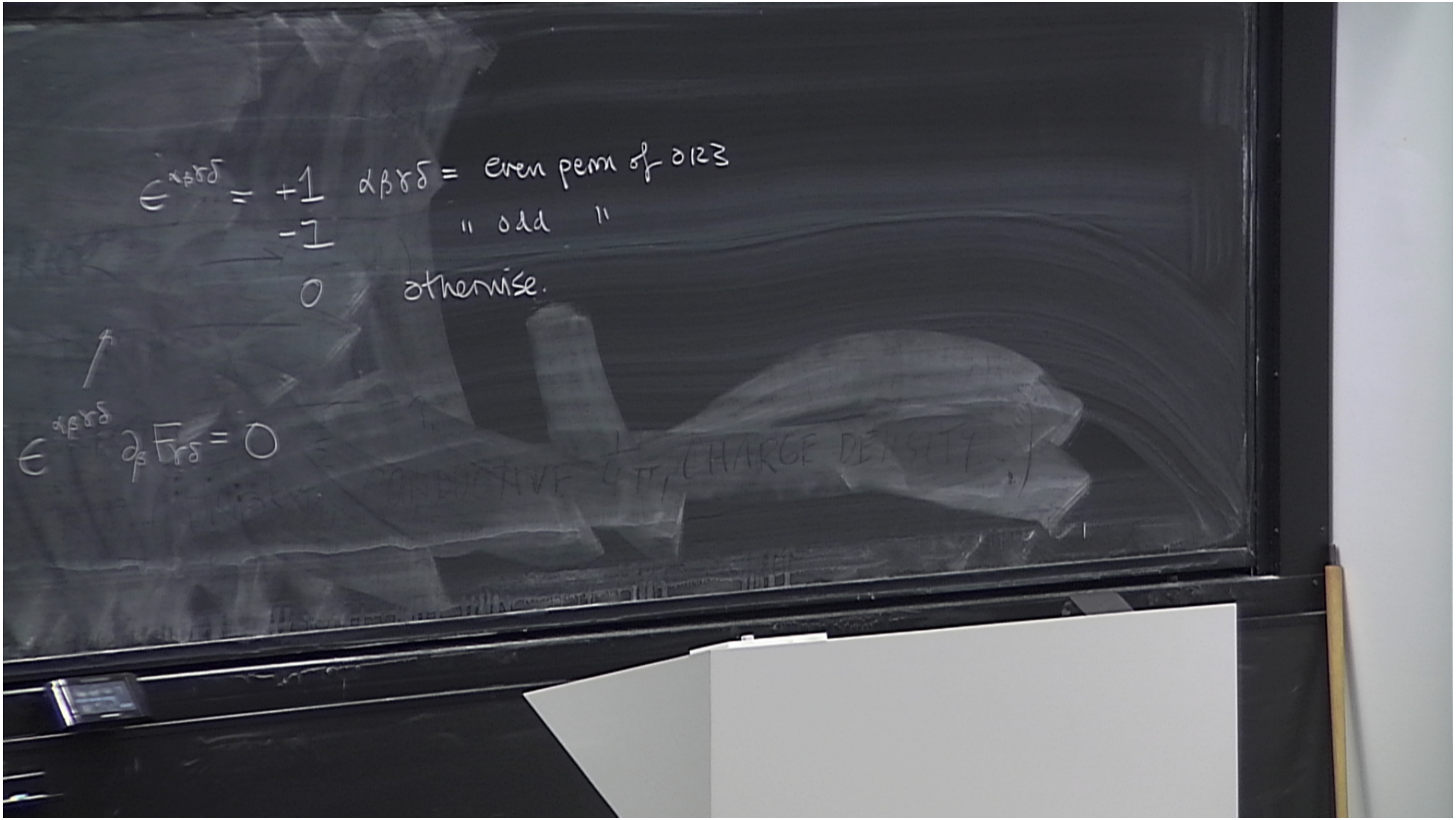
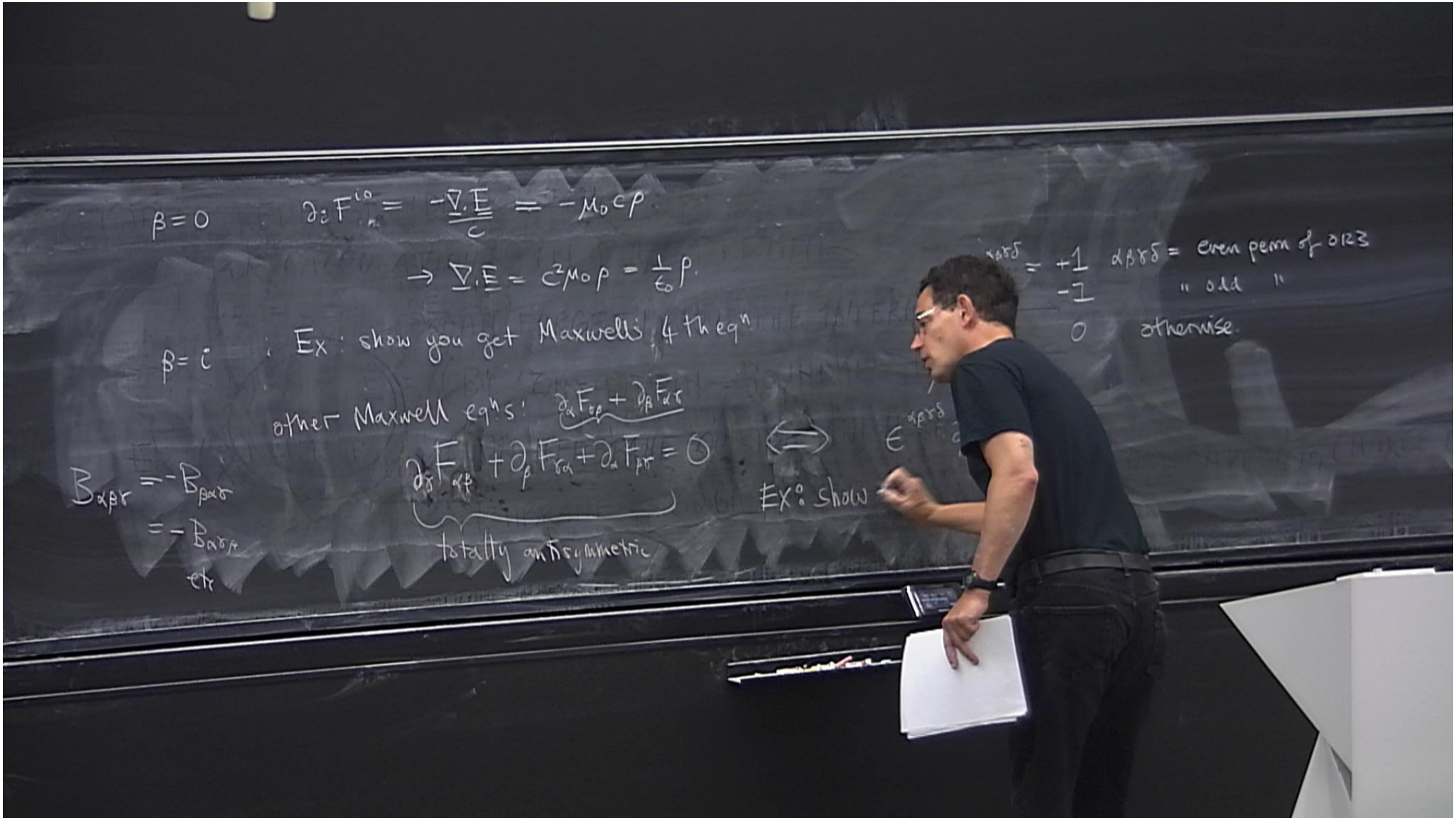
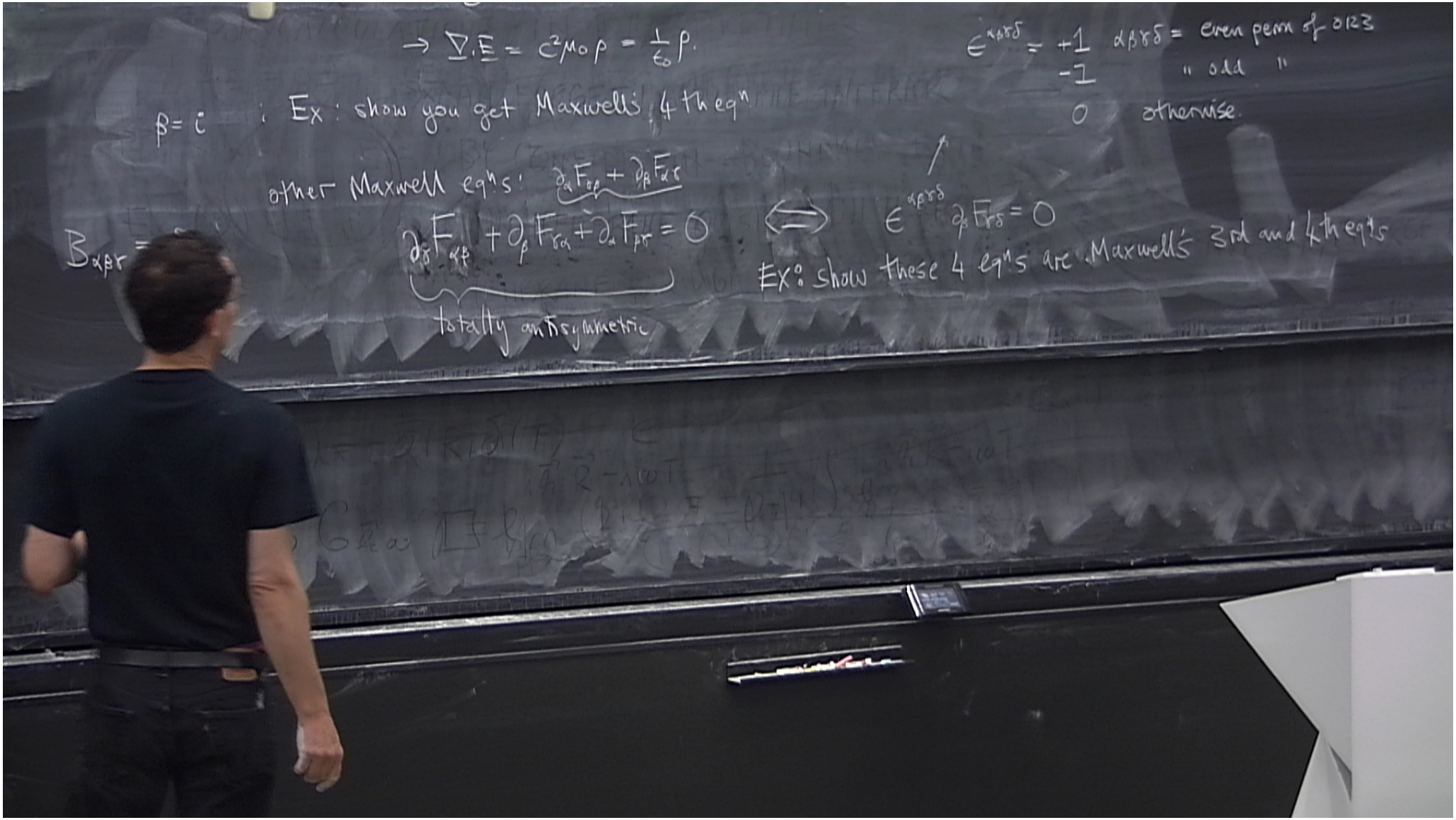
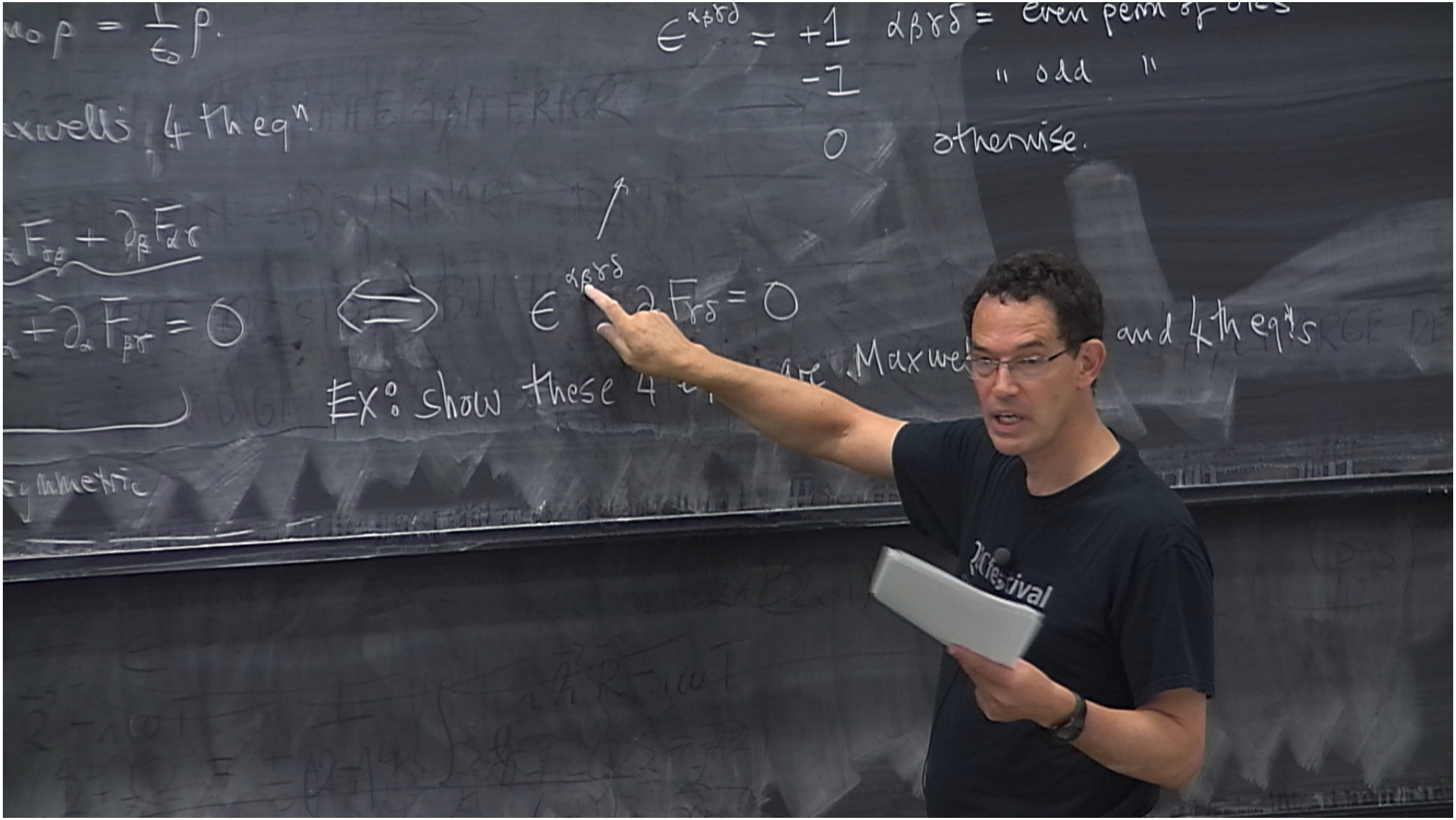
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$
totally antisymmetric
$\Leftrightarrow$
$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$
$B_{\alpha\beta\gamma} = -B_{\beta\alpha\gamma}$
$= -B_{\alpha\gamma\beta}$

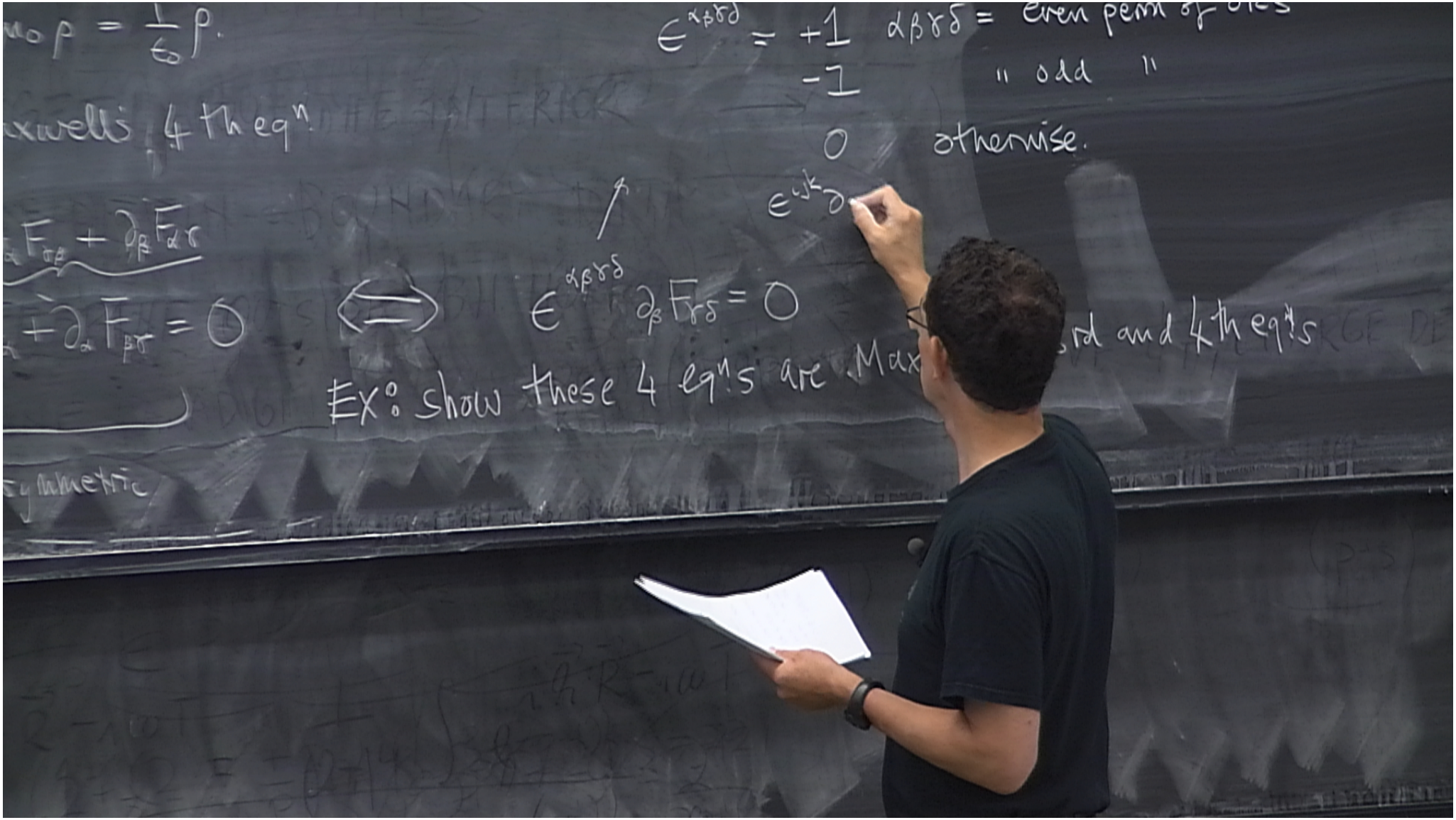

$\epsilon^{\alpha\beta\gamma\delta}$ = +1 $\alpha\beta\gamma\delta$ = even perm of 0123
-1 " odd "
0 otherwise.
$\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$

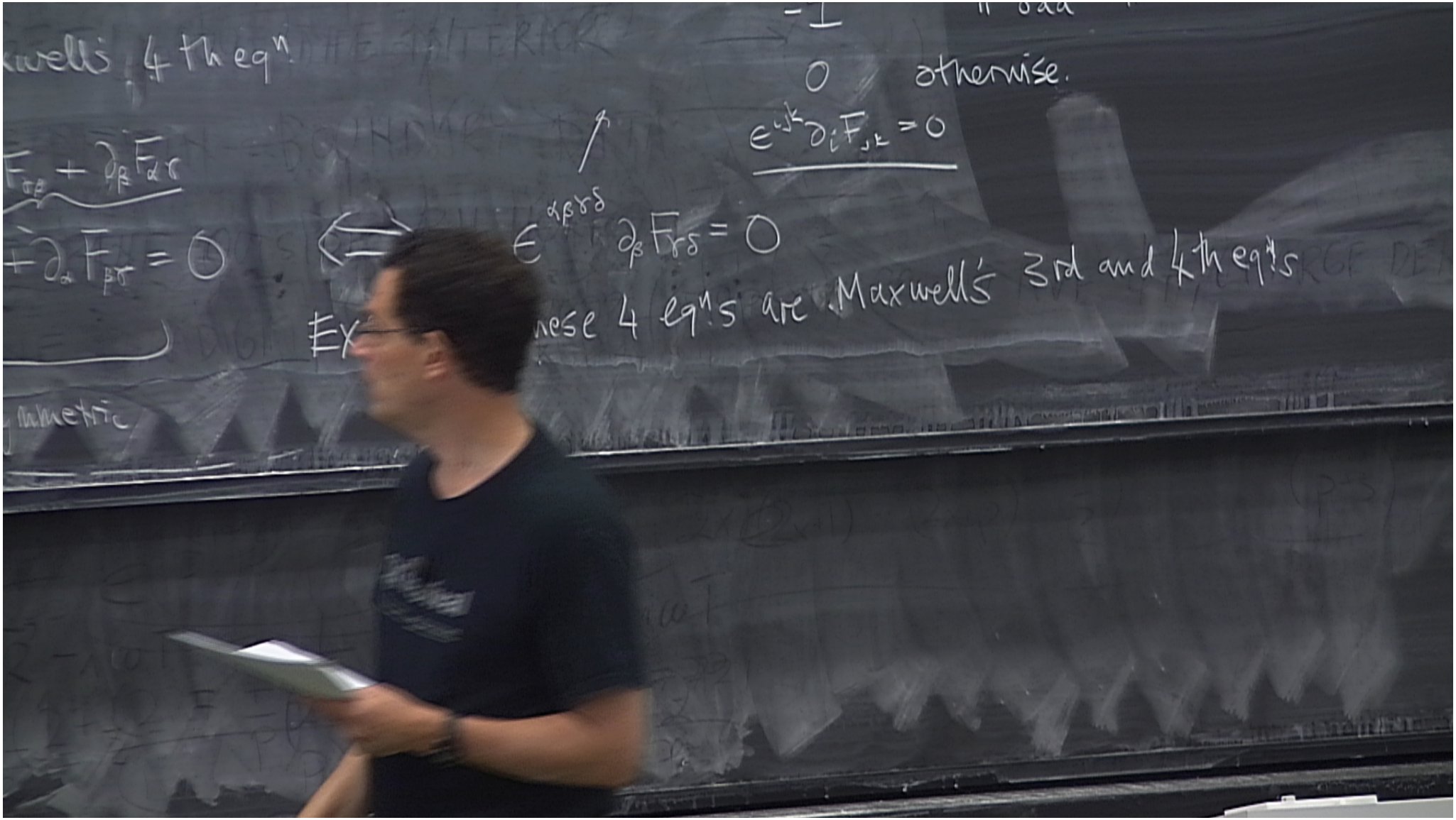

$\beta = 0$
$\partial_\alpha F^{i0} = -\frac{\nabla \cdot \underline{E}}{c} = -\mu_0 c \rho$
$\rightarrow \nabla \cdot \underline{E} = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$
$\beta = i$; Ex: show you get Maxwell's 4th eq^n
other Maxwell eq^{n}s: $\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha}$
$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$
totally antisymmetric
$B_{\alpha\beta\gamma} = -B_{\beta\alpha\gamma}$
$= -B_{\alpha\gamma\beta}$
etc
$\epsilon^{\alpha\beta\gamma\delta} = +1$
-1
0
$\alpha\beta\gamma\delta$ = even perm of 0123
" odd "
otherwise
Ex: show

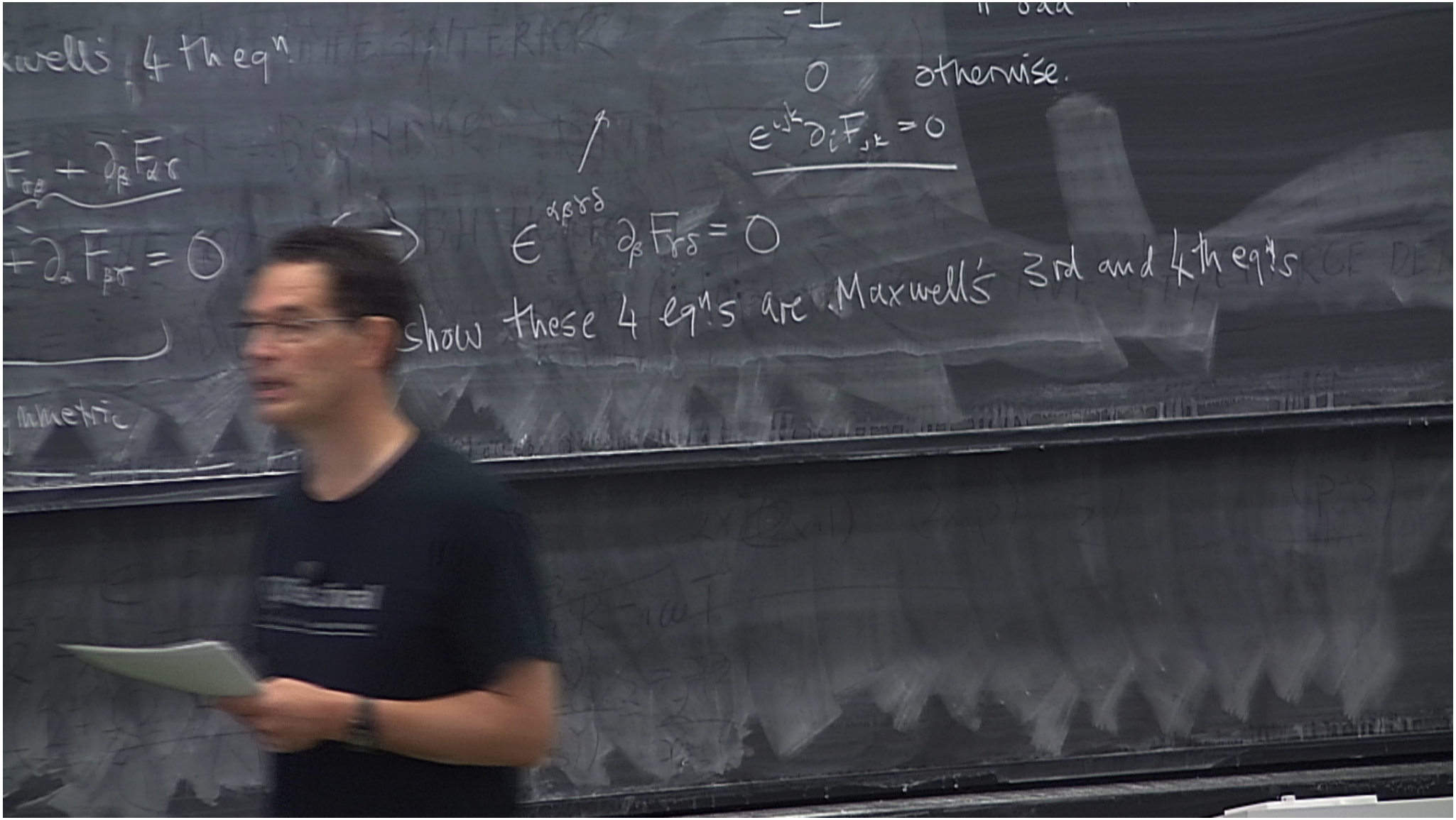

$\rightarrow \nabla \cdot \underline{E} = c^2 \mu_0 \rho = \frac{1}{\epsilon_0}\rho$.
$\epsilon^{\alpha\beta\gamma\delta}$ = +1 $\alpha\beta\gamma\delta$ = even perm of 0123
-1 " odd "
0 otherwise.
$\beta = i$; Ex: show you get Maxwell's 4th eqn
other Maxwell eqn's: $\partial_\alpha F_{\beta\rho} + \partial_\beta F_{\alpha\delta}$
$B_{\alpha\beta\gamma}$ =
$\partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} + \partial_\alpha F_{\beta\gamma} = 0$
totally antisymmetric
$\Leftrightarrow$
$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$
Ex: show these 4 eqn's are Maxwell's 3rd and 4th eqn's

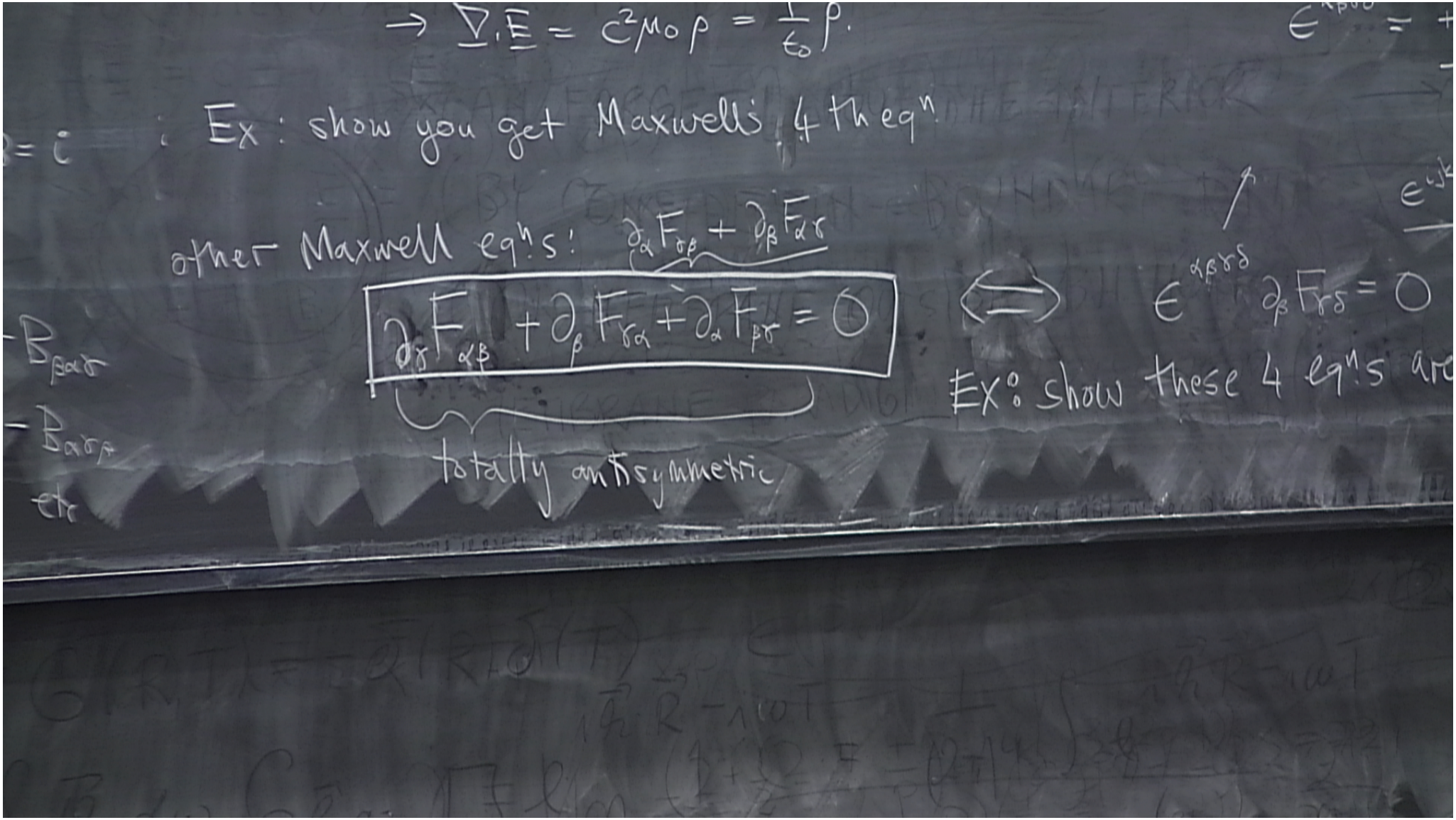


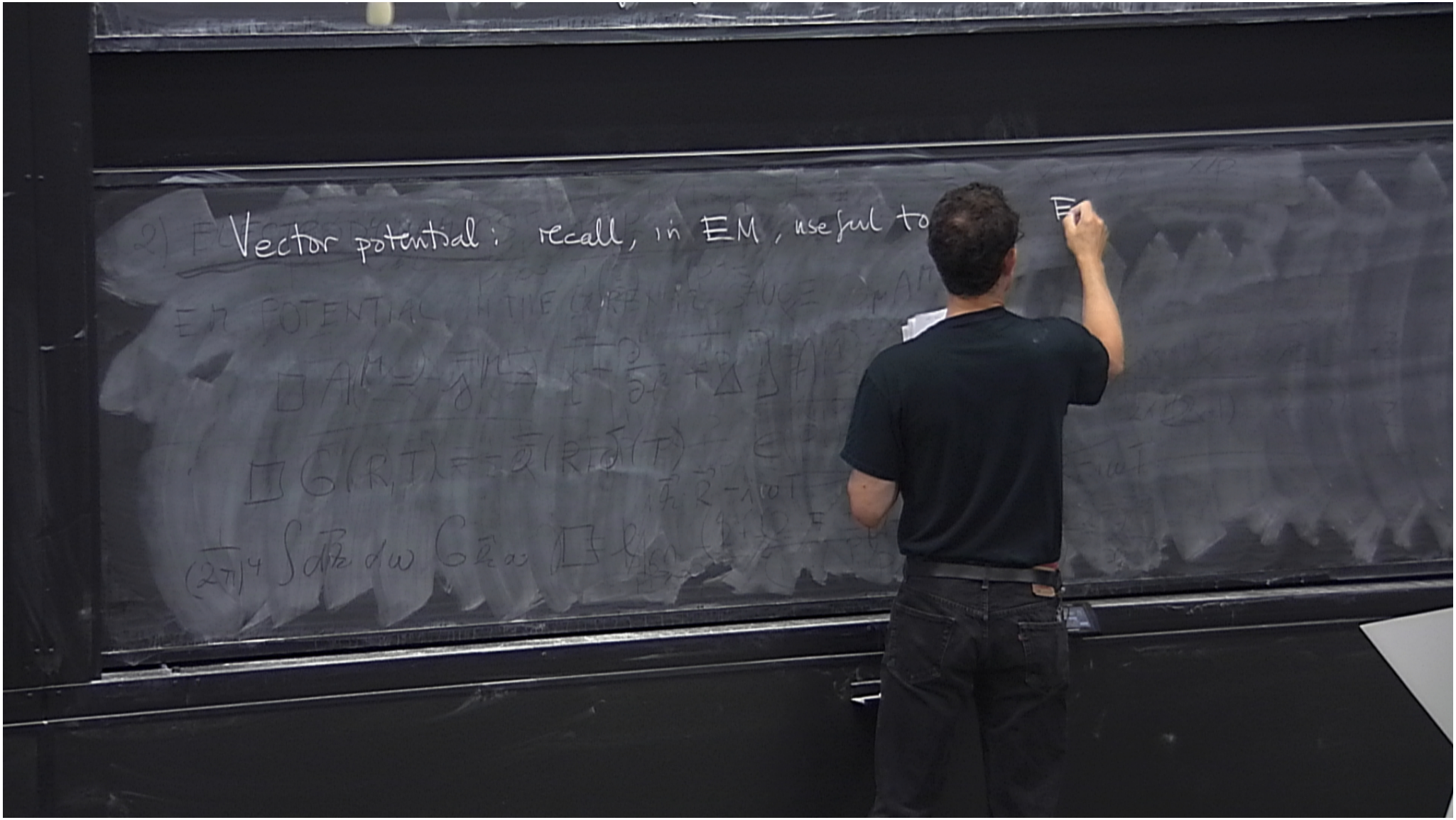
$\epsilon^{\alpha\beta\gamma\delta} = +1$ $\alpha\beta\gamma\delta$ = even perm of
-1 " odd "
0 otherwise.
xwell's 4 th eq^n
$\partial_\alpha F_{\gamma\beta} + \partial_\beta F_{\alpha\gamma}$
$+ \partial_\alpha F_{\beta\gamma} = 0$
$\Longleftrightarrow$
$\epsilon^{\alpha\beta\gamma\delta} \partial F_{\gamma\delta} = 0$
and 4 th eq^{n}s
Ex: show these 4
symmetric

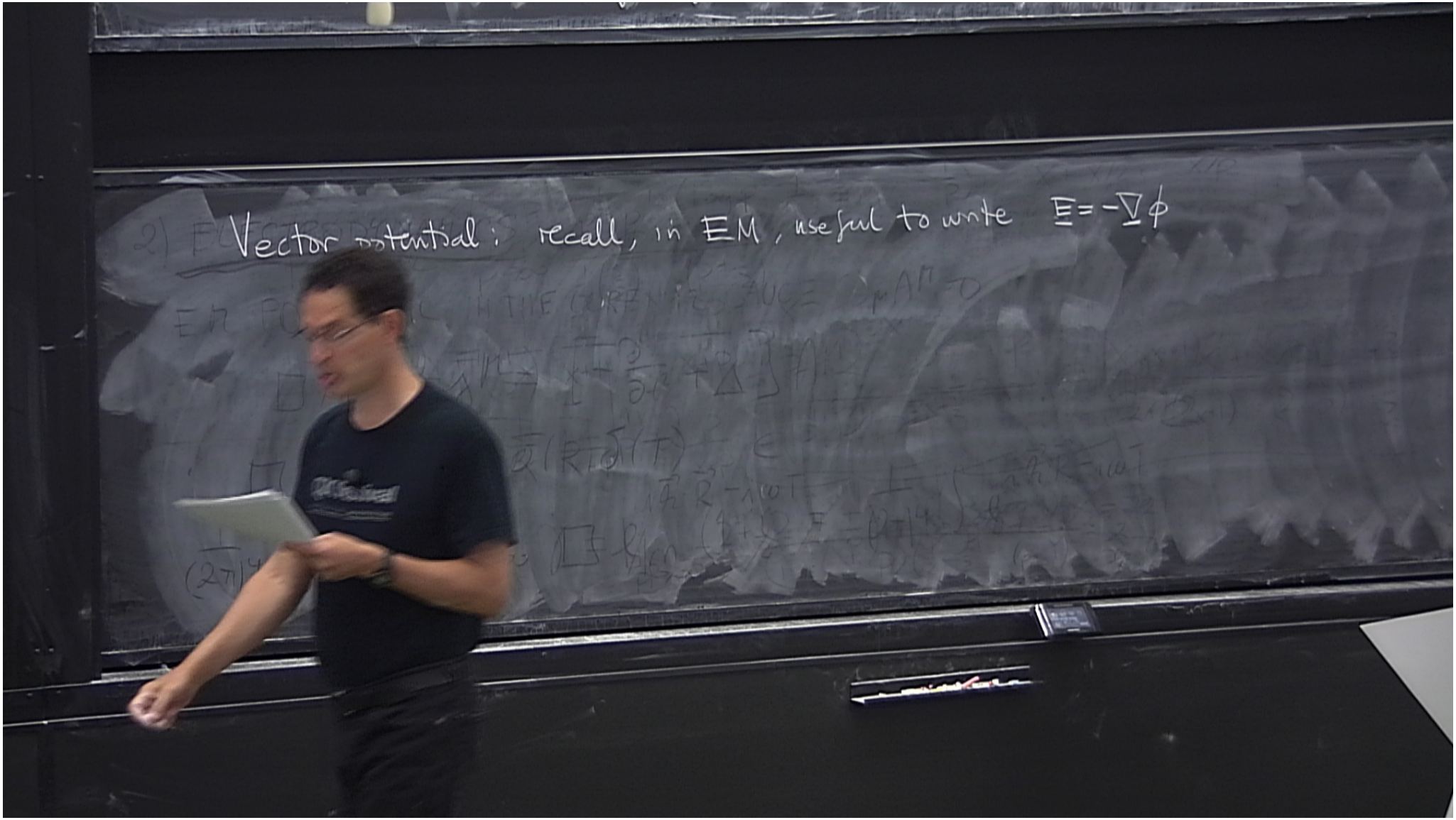

$\epsilon^{\alpha\beta\gamma\delta} = +1$ $\alpha\beta\gamma\delta =$ even perm of 0123
-1 " odd "
0 otherwise.
$\epsilon^{ijk}\partial$
$\epsilon^{\alpha\beta\gamma\delta}\,\partial_\beta F_{\gamma\delta} = 0$
Ex: show these 4 eq^{n}s are Max
rd and 4th eq^{n}s
ymmetric

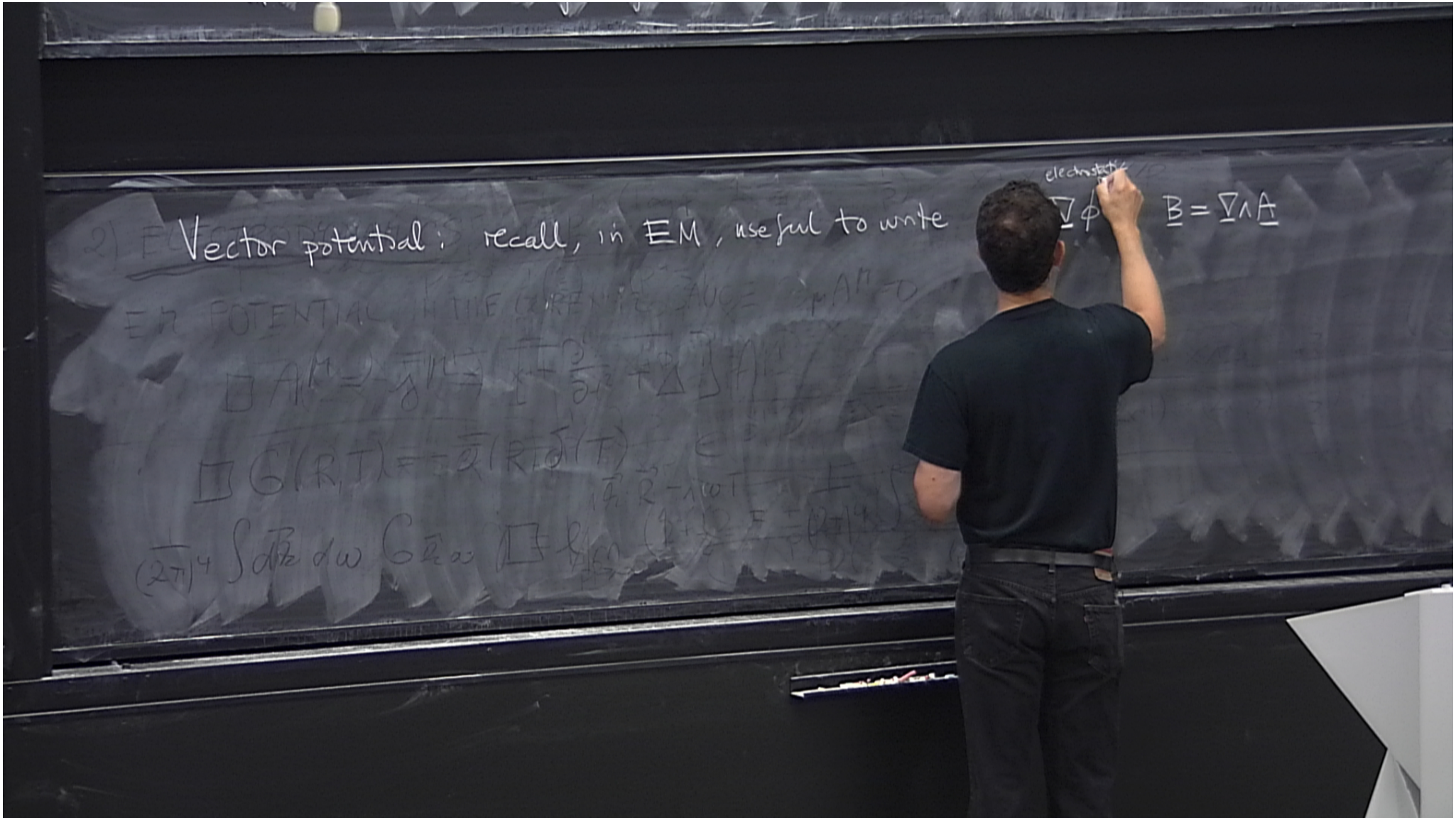

Maxwell's 4th eq^n
otherwise.
$\epsilon^{ijk}\partial_i F_{jk} = 0$
$\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$
$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha}$
$+\partial_\gamma F_{\alpha\beta} = 0$
these 4 eq^n's are Maxwell's 3rd and 4th eq^n's

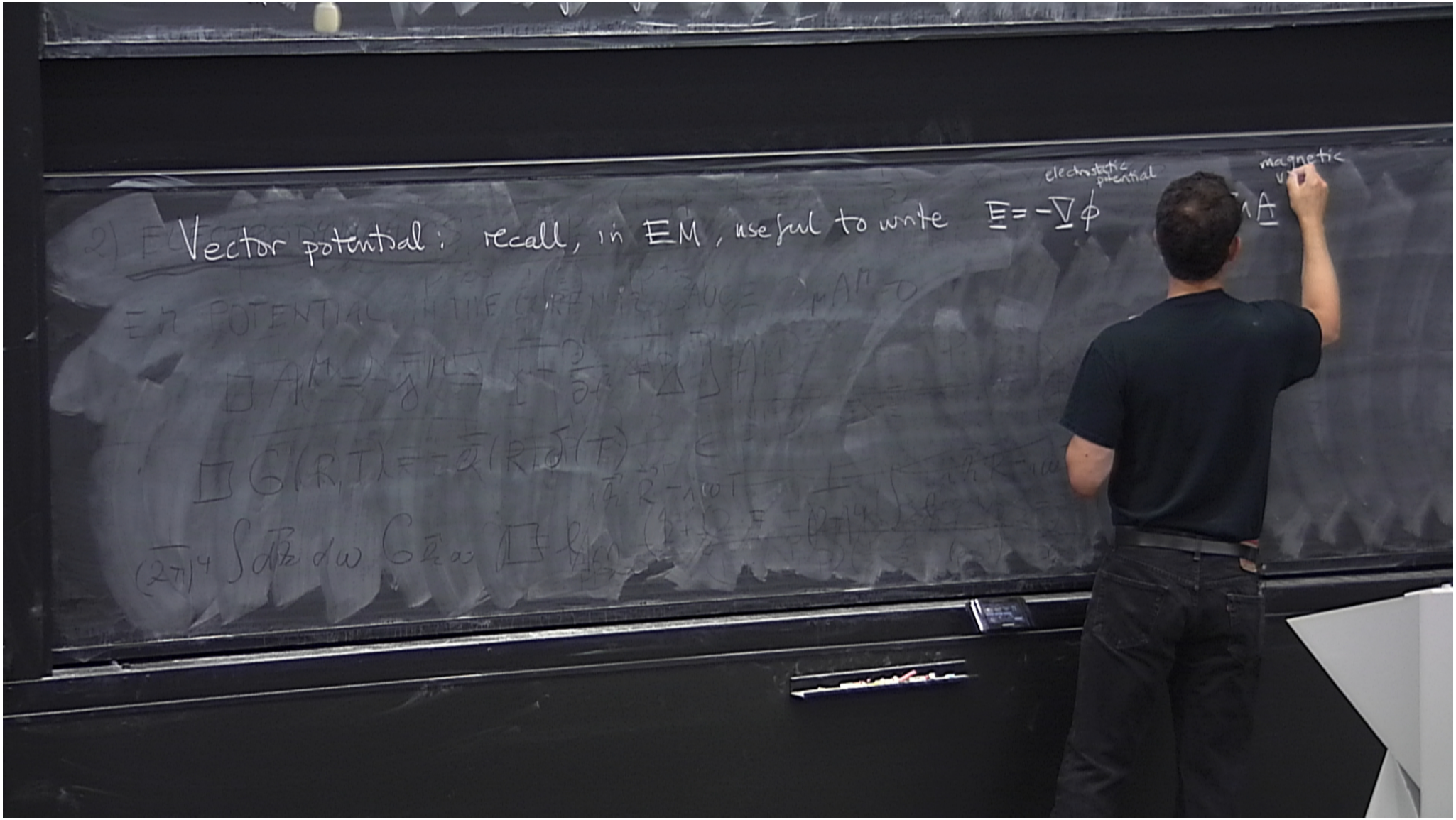

xwell's 4th eqn
$\epsilon^{ijk}\partial_i F_{jk} = 0$
otherwise.
$\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$
show these 4 eqn's are Maxwell's 3rd and 4th eq's

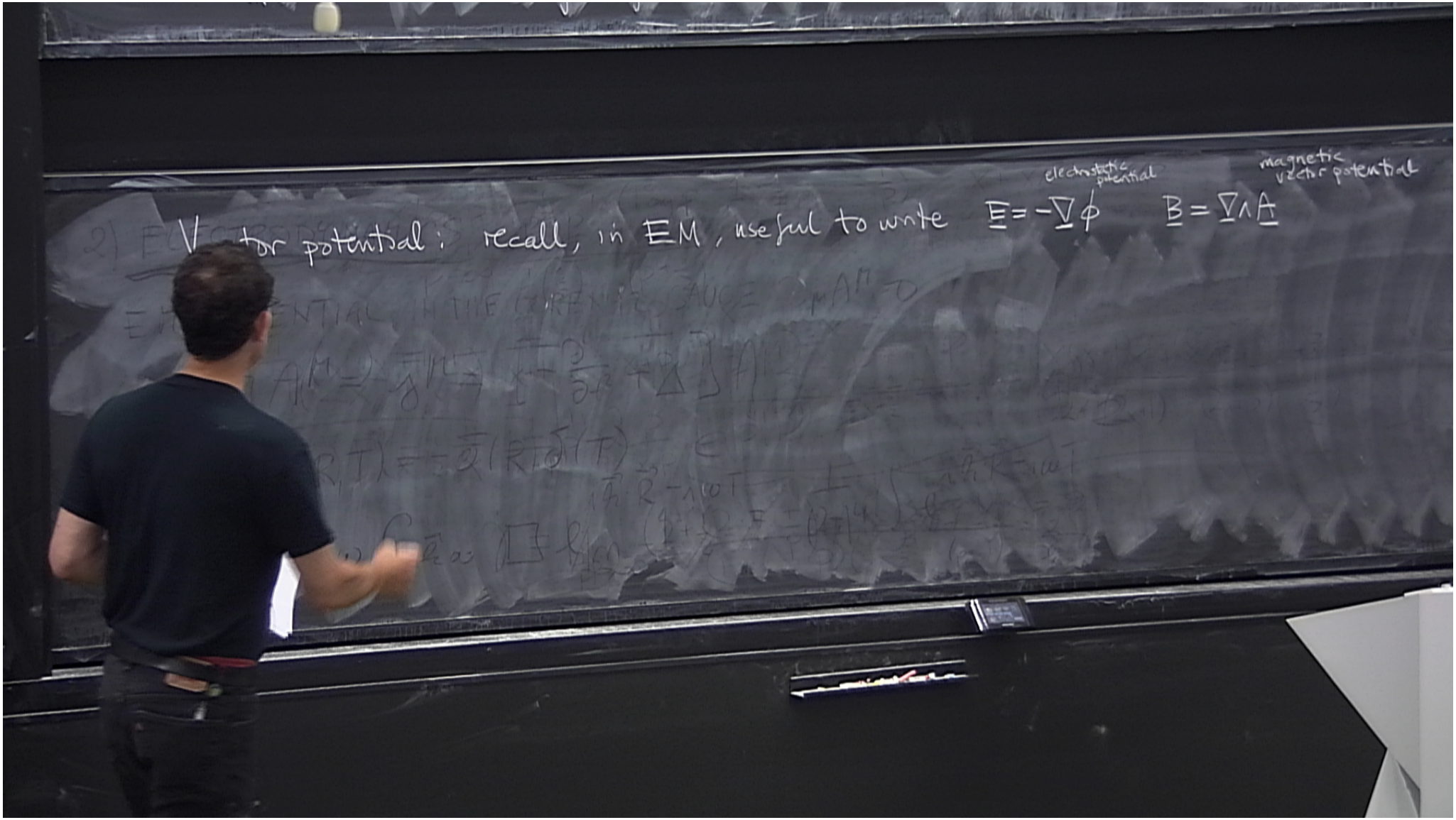

$\rightarrow \nabla \cdot \underline{E} = c^2 \mu_0 \rho = \frac{1}{\epsilon_0} \rho$.
Ex: show you get Maxwell's 4 th eqn
other Maxwell eq^{n}s: $\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha}$
$\partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} + \partial_\alpha F_{\beta\gamma} = 0$
totally antisymmetric
$\Leftrightarrow$
$\epsilon^{\alpha\beta\gamma\delta} \partial_\beta F_{\gamma\delta} = 0$
Ex: show these 4 eq^{n}s are
$-B_{\beta\alpha\delta}$
$-B_{\alpha\delta\beta}$
etc

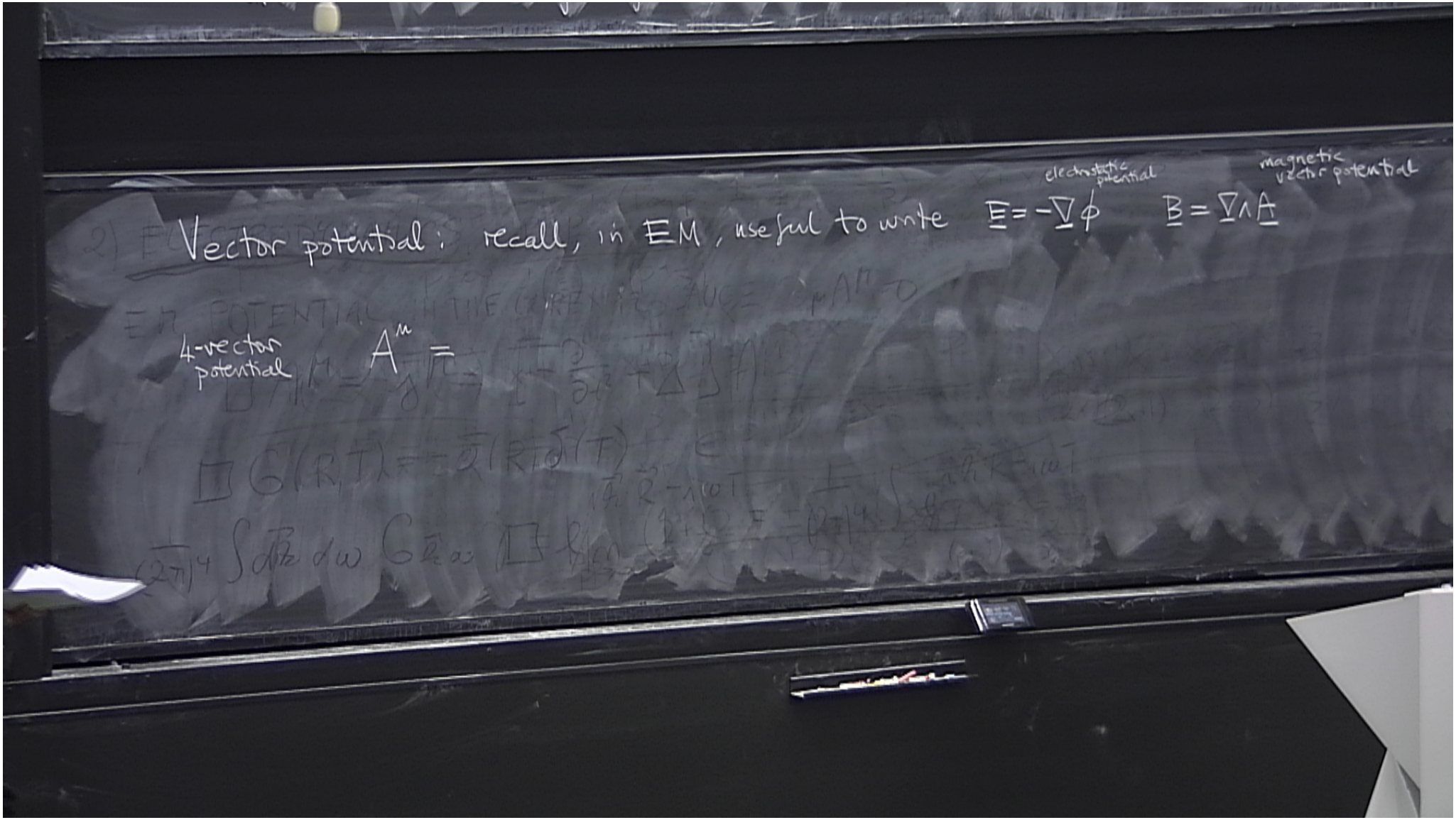

Vector potential: recall, in EM, useful to

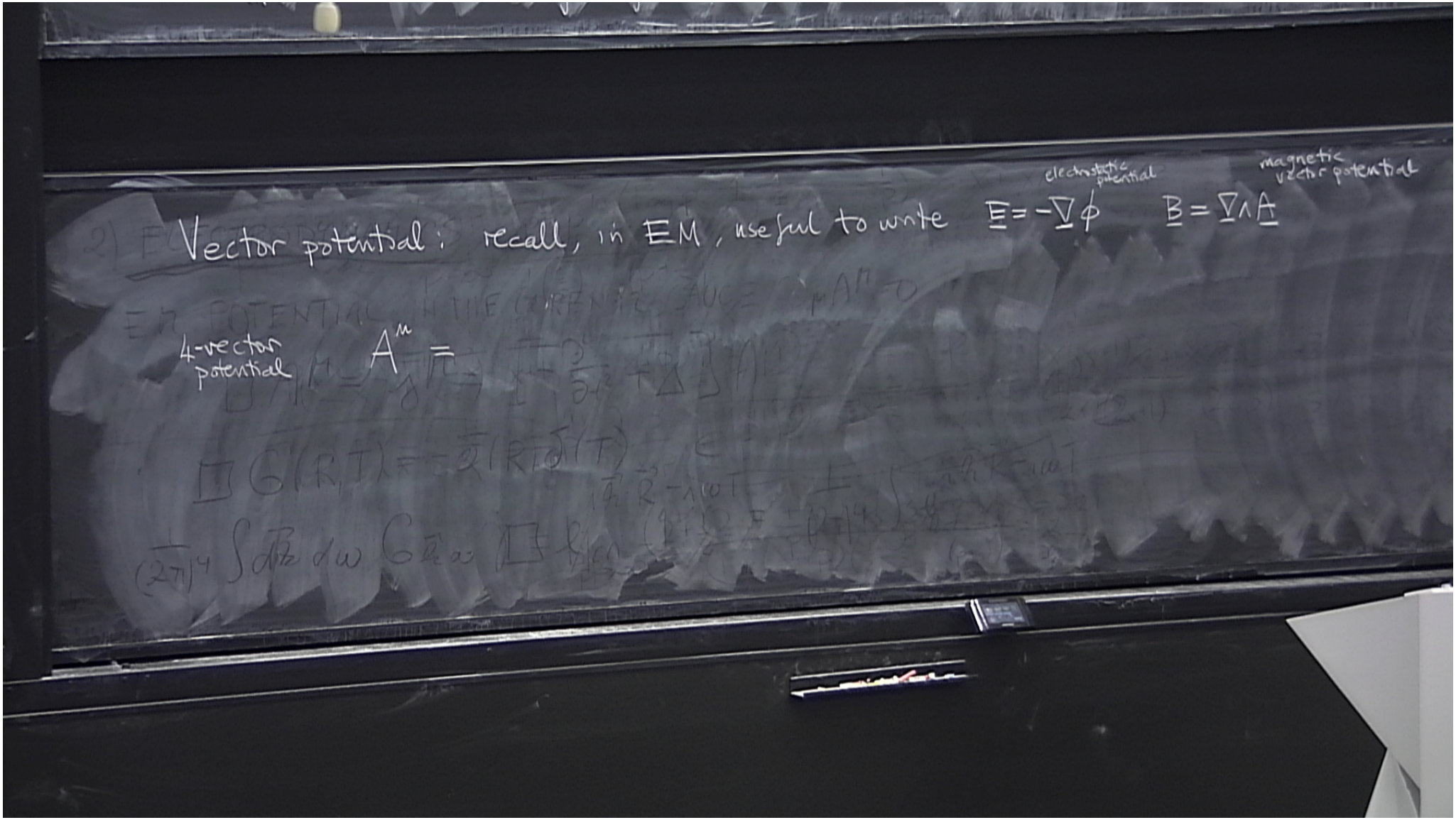

Vector potential: recall, in EM, useful to write $\underline{E} = -\underline{\nabla}\phi$

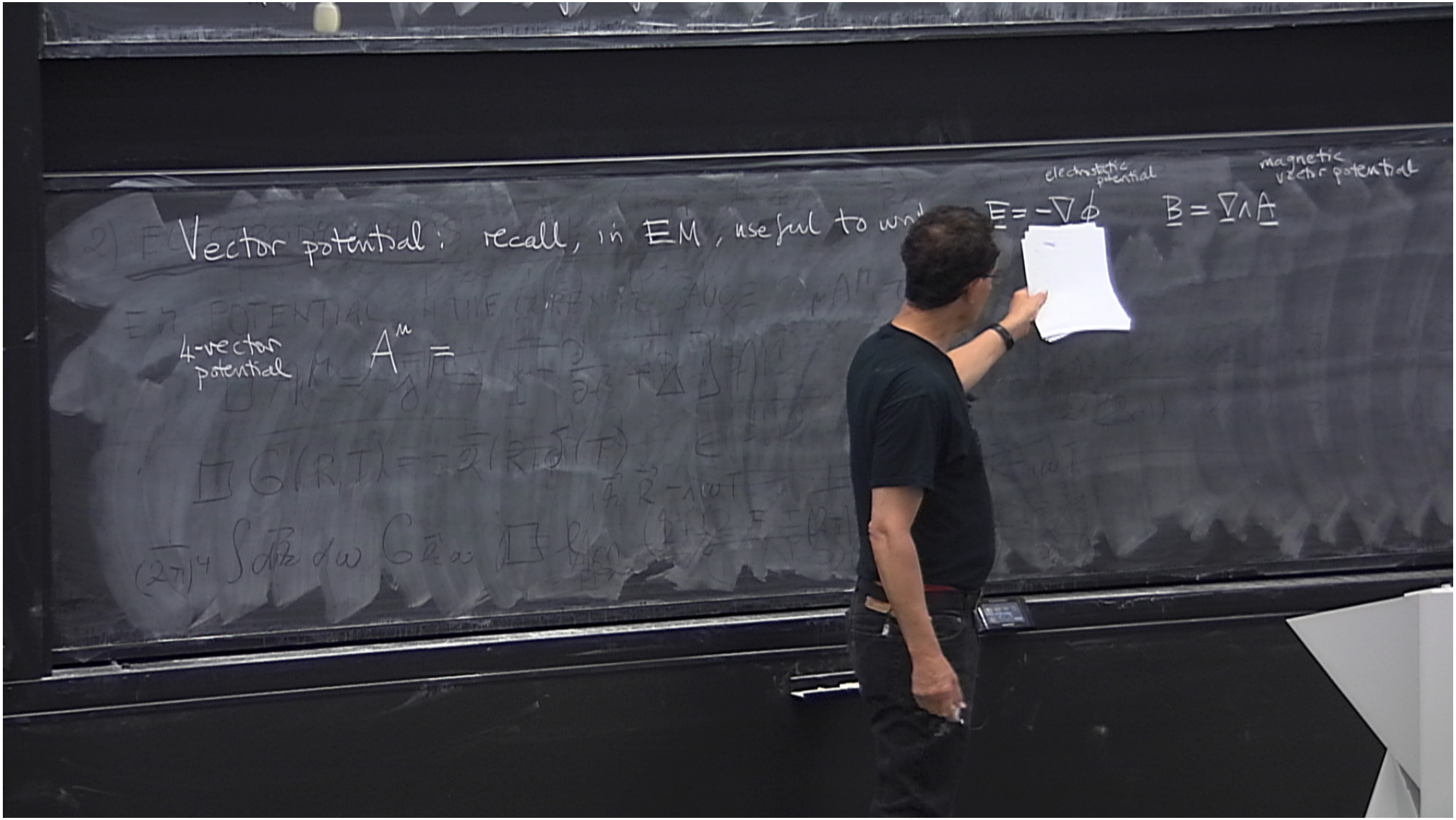

Vector potential: recall, in EM, useful to write
electrostatic
$\underline{B} = \underline{\nabla} \wedge \underline{A}$

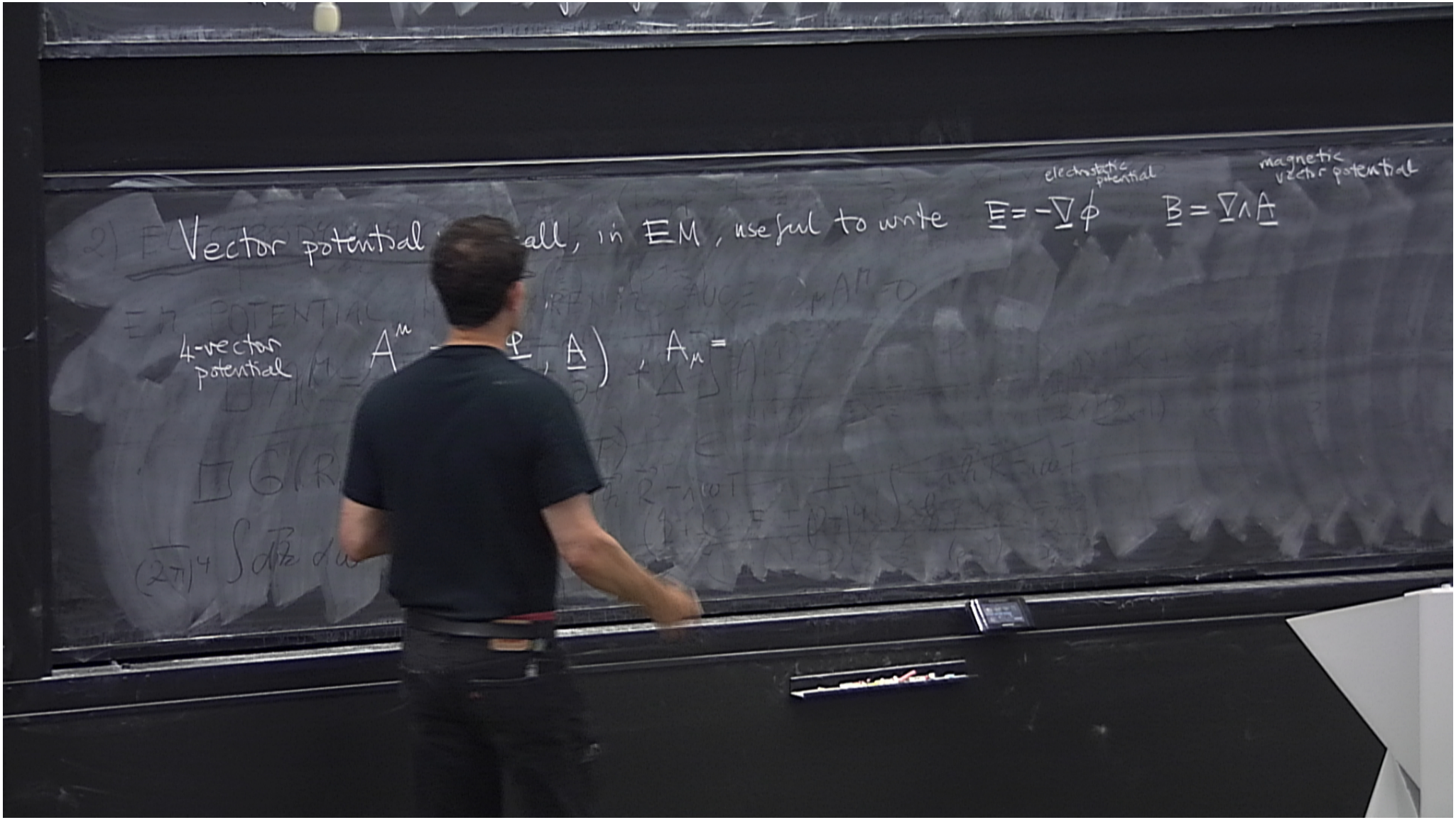

Vector potential: recall, in EM, useful to write
electrostatic potential
magnetic

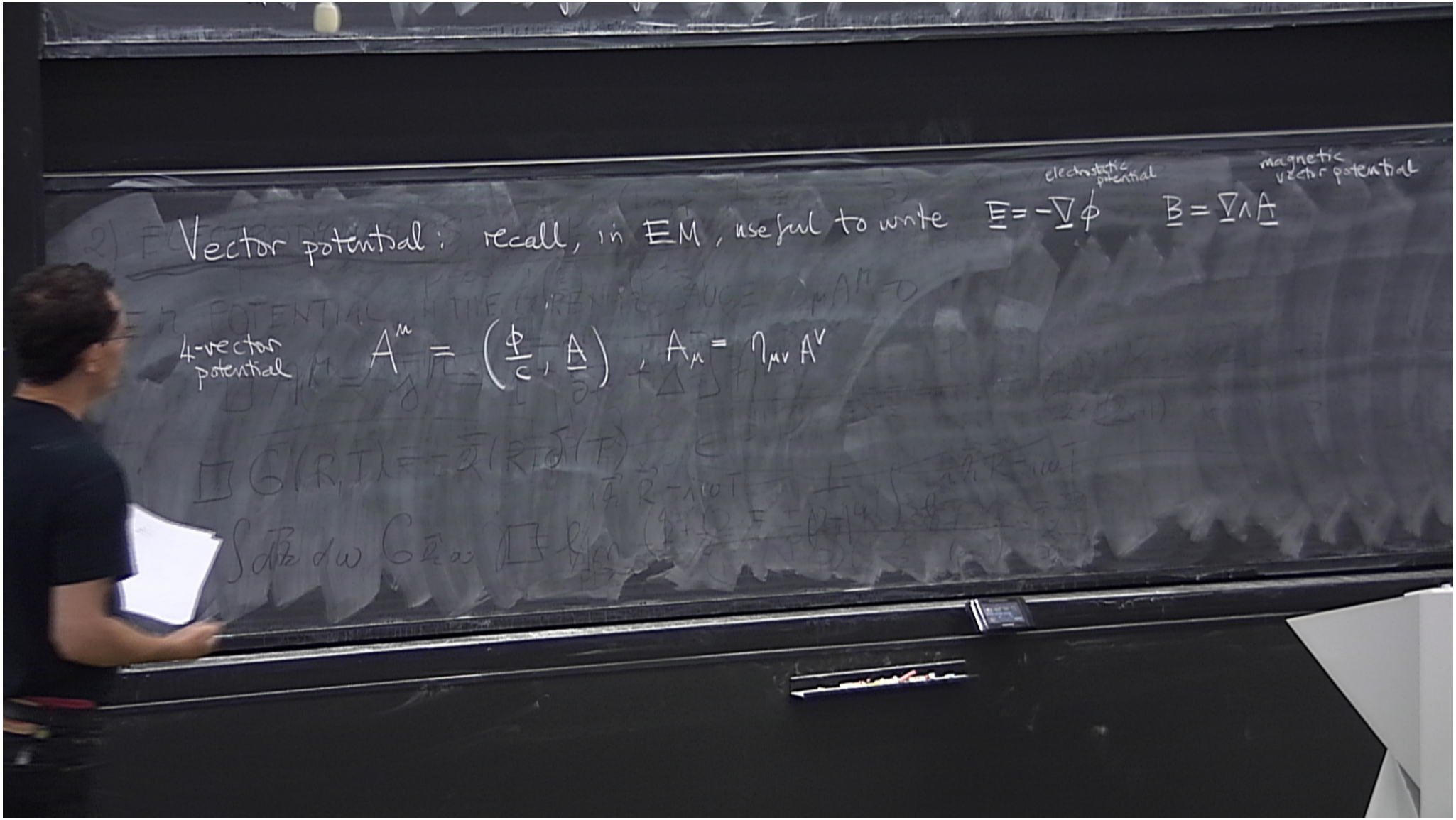

Vector potential: recall, in EM, useful to write
electrostatic potential
magnetic vector potential

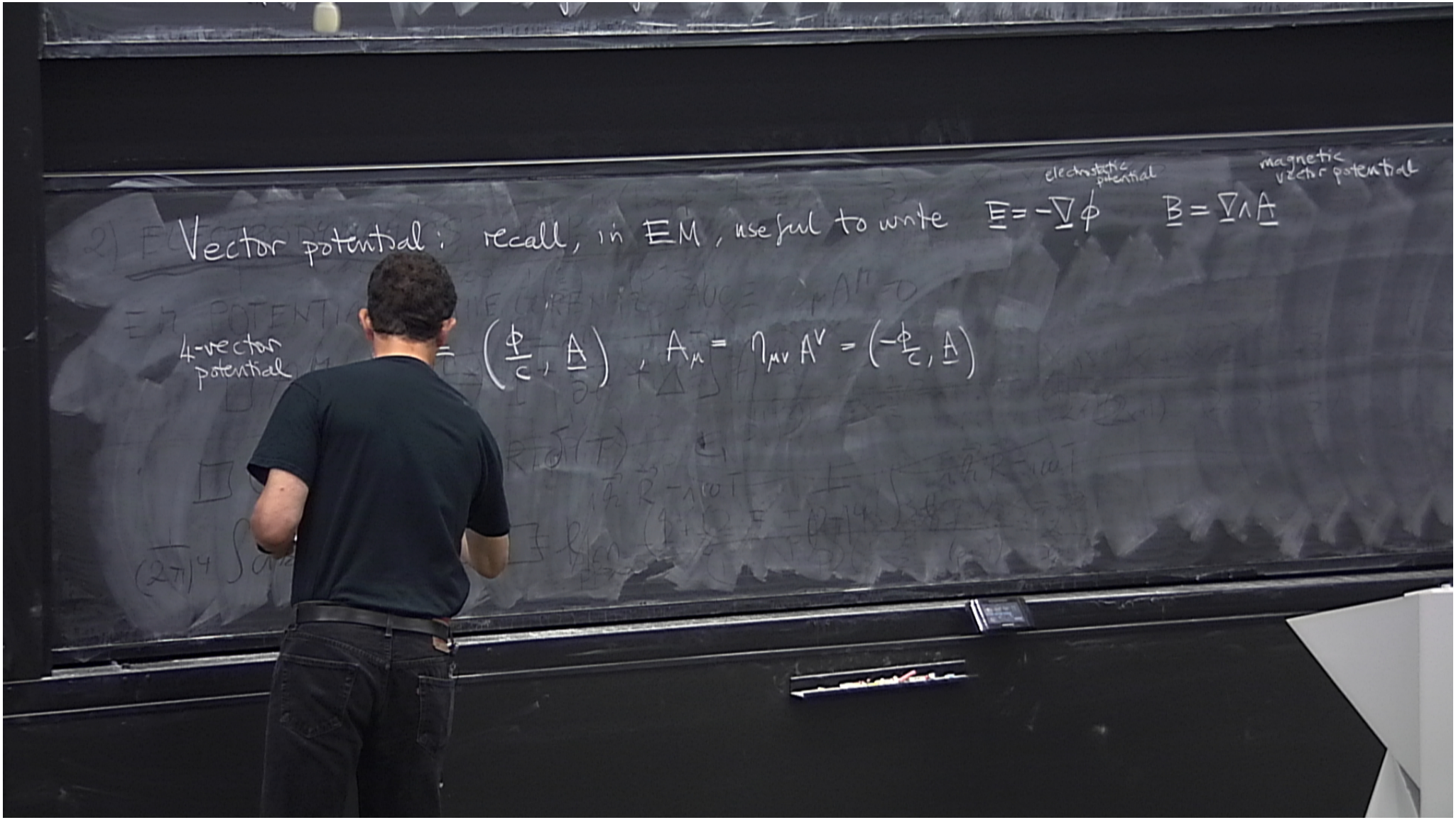

Vector potential: recall, in EM, useful to write
electrostatic potential
magnetic vector potential
4-vector potential


Vector potential: recall, in EM, useful to write $\underline{E} = -\underline{\nabla}\phi$ $\underline{B} = \underline{\nabla} \wedge \underline{A}$
electrostatic potential
magnetic vector potential
4-vector potential $A^\mu =$


Vector potential: recall, in EM, useful to
electrostatic potential
magnetic vector potential
$\underline{E} = -\nabla\phi$
$\underline{B} = \nabla \wedge \underline{A}$
4-vector potential
$A^\mu =$


Vector potential
all, in EM, useful to write
electrostatic potential
magnetic vector potential
4-vector potential


Vector potential: recall, in EM, useful to write
electrostatic potential
magnetic vector potential
$\underline{E} = -\underline{\nabla}\phi \qquad \underline{B} = \underline{\nabla} \wedge \underline{A}$
4-vector potential
$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right), \quad A_\mu = \eta_{\mu\nu} A^\nu$


Vector potential: recall, in EM, useful to write
electrostatic potential
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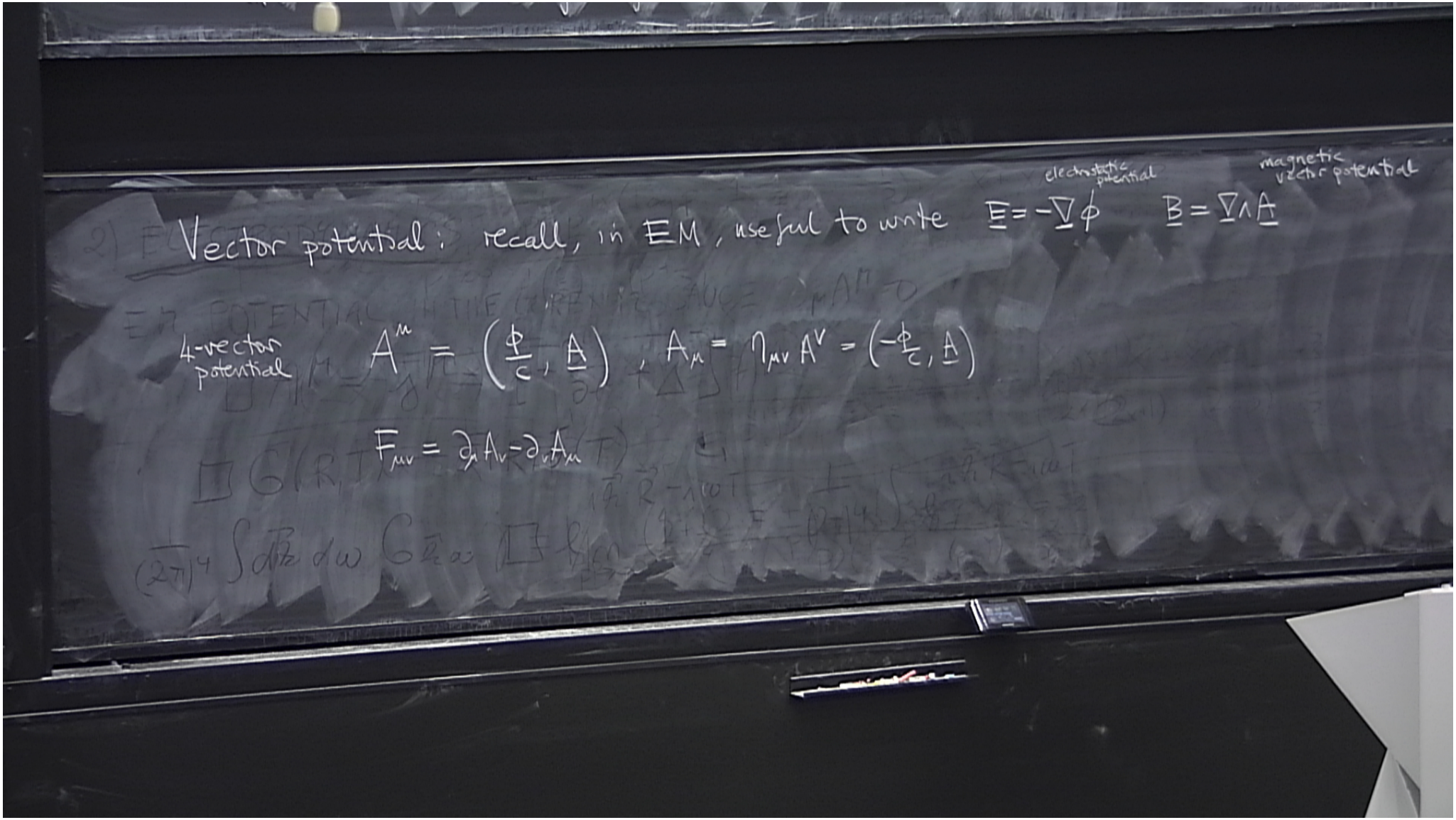
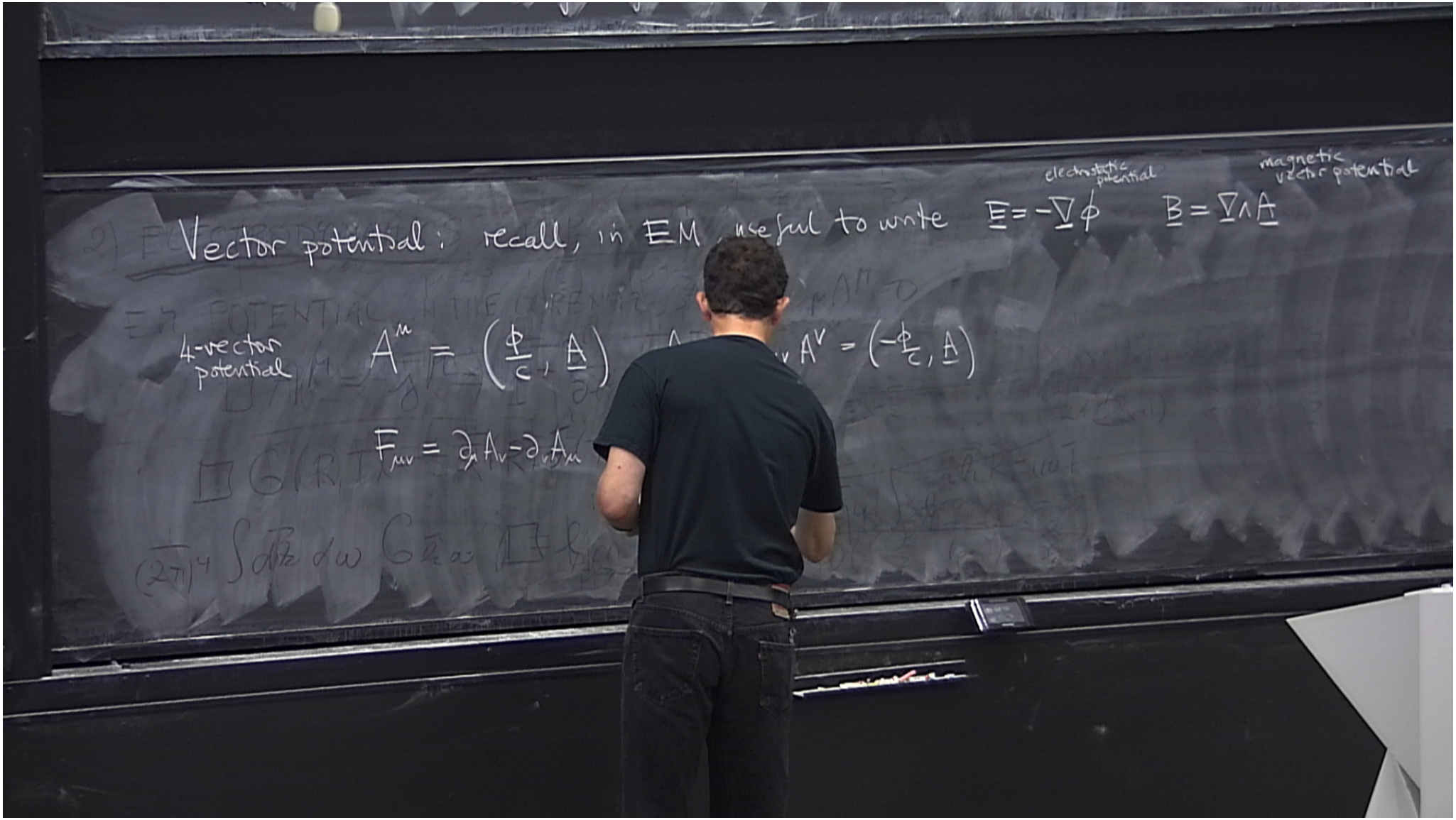
4-vector potential


Vector potential: recall, in EM, useful to write
electrostatic potential
magnetic vector potential
$\underline{E} = -\underline{\nabla}\phi \qquad \underline{B} = \underline{\nabla} \wedge \underline{A}$
4-vector potential
$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right), \quad A_\mu = \eta_{\mu\nu} A^\nu = \left(-\frac{\phi}{c}, \underline{A}\right)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$


Vector potential: recall, in EM useful to write
electrostatic potential
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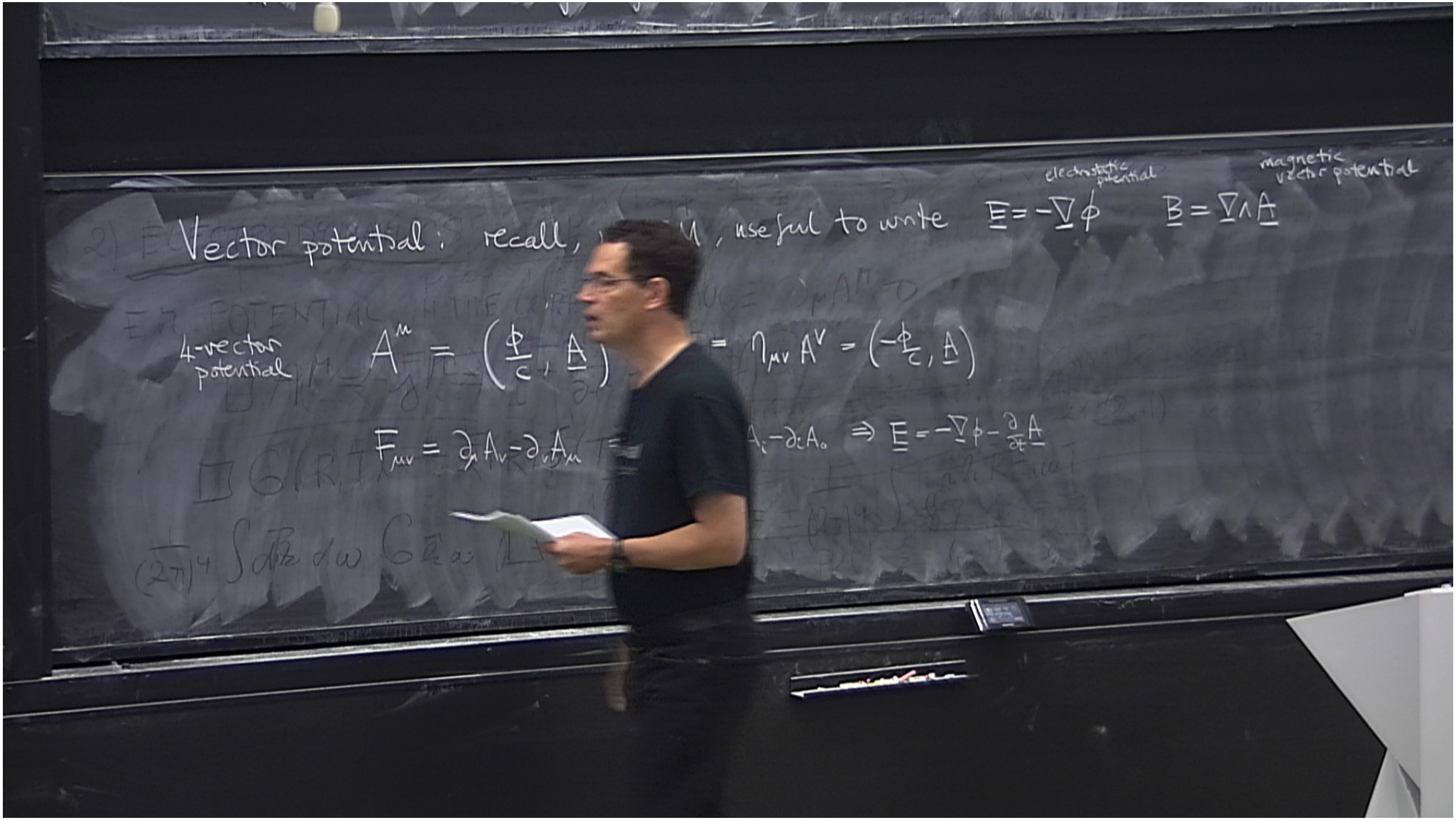
Vector potential: recall, useful to write
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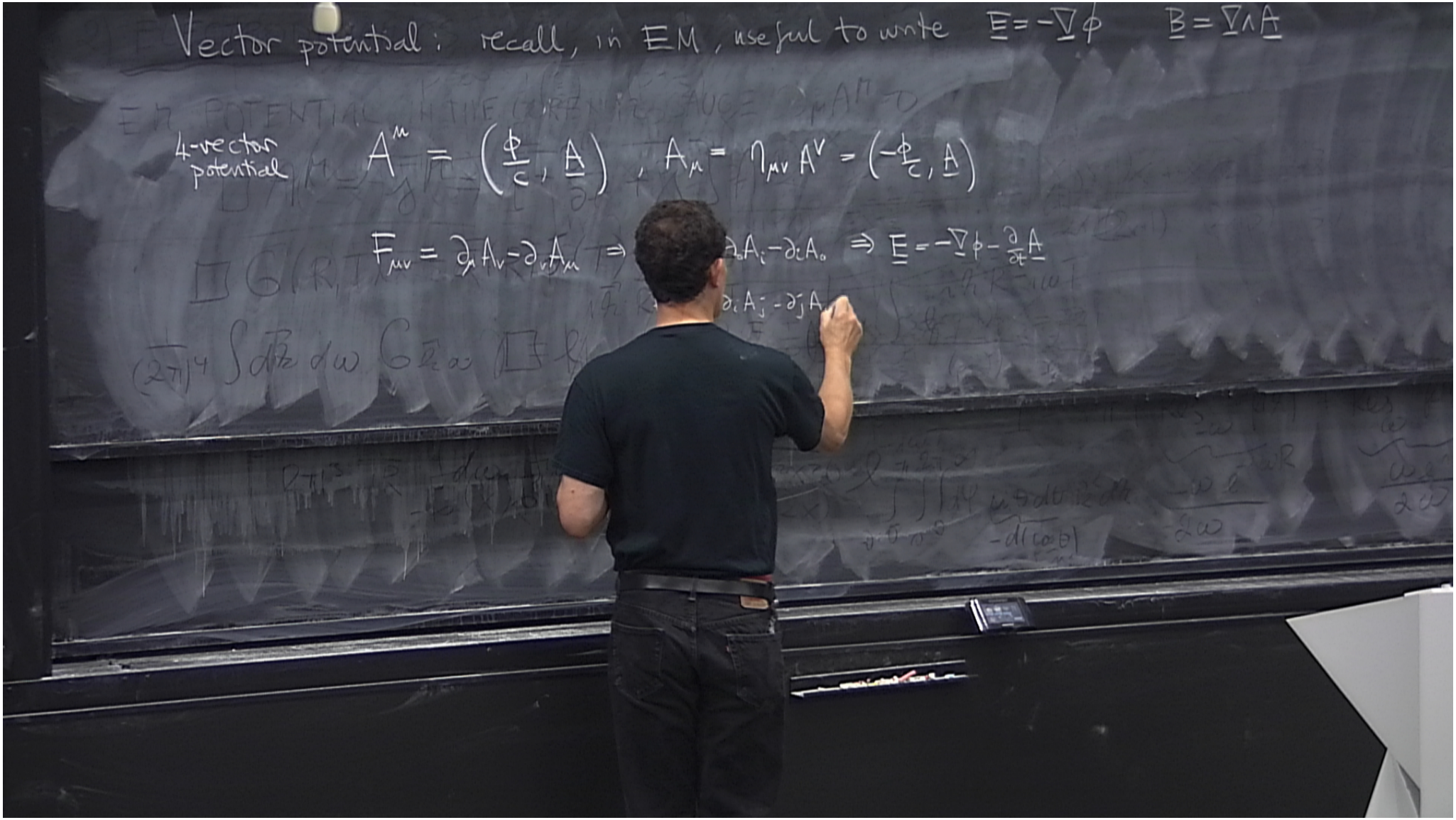
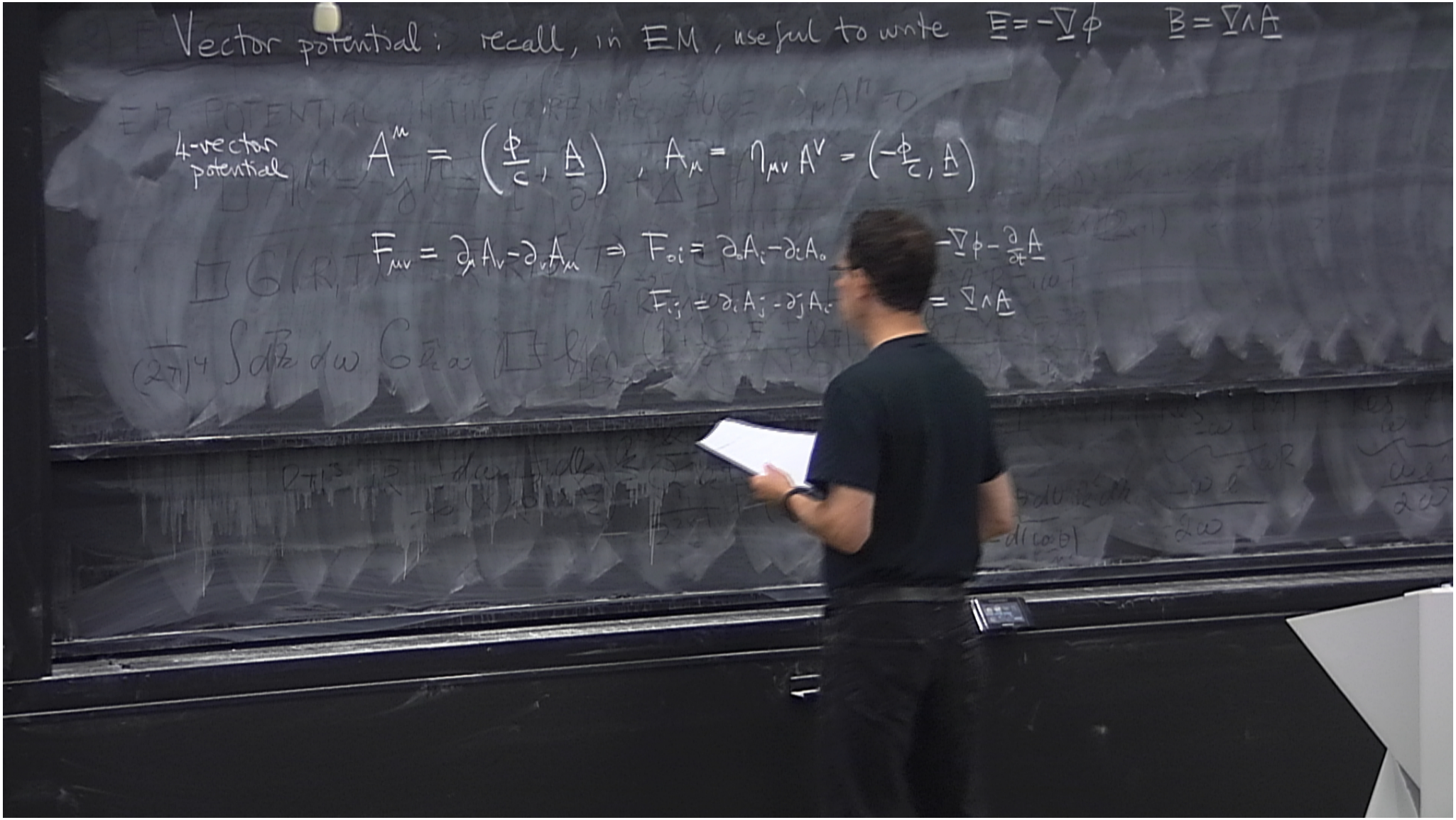
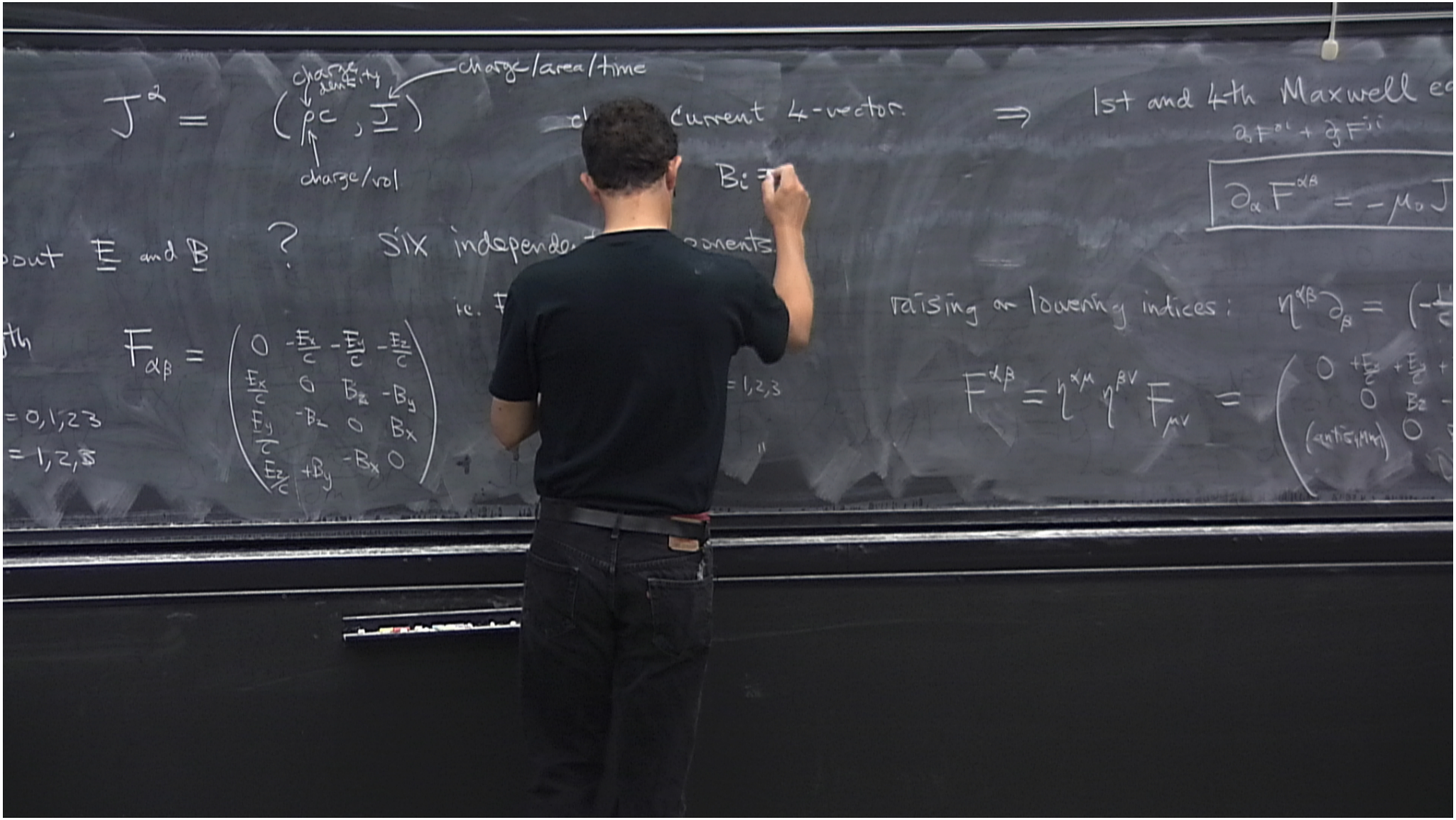
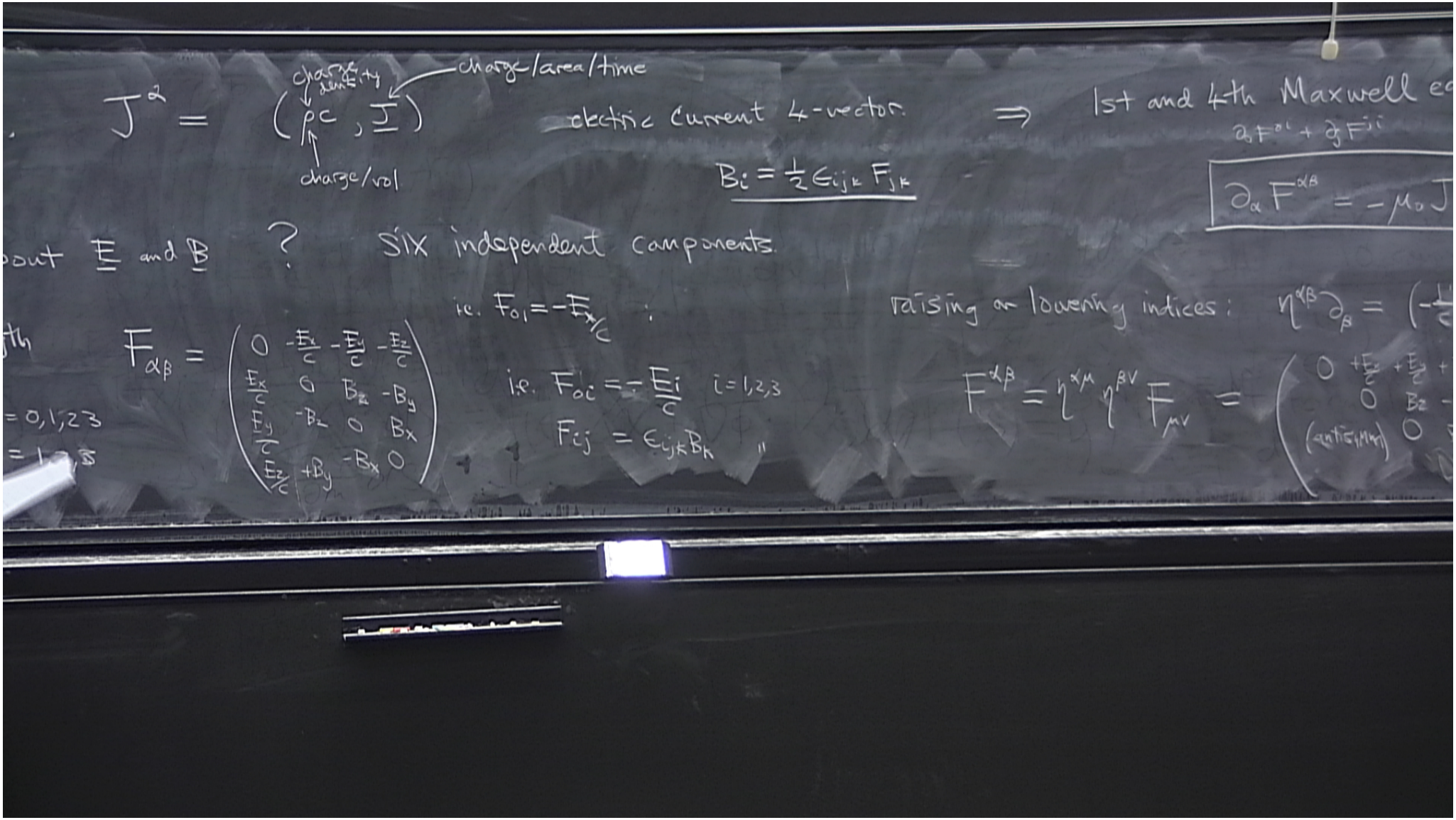
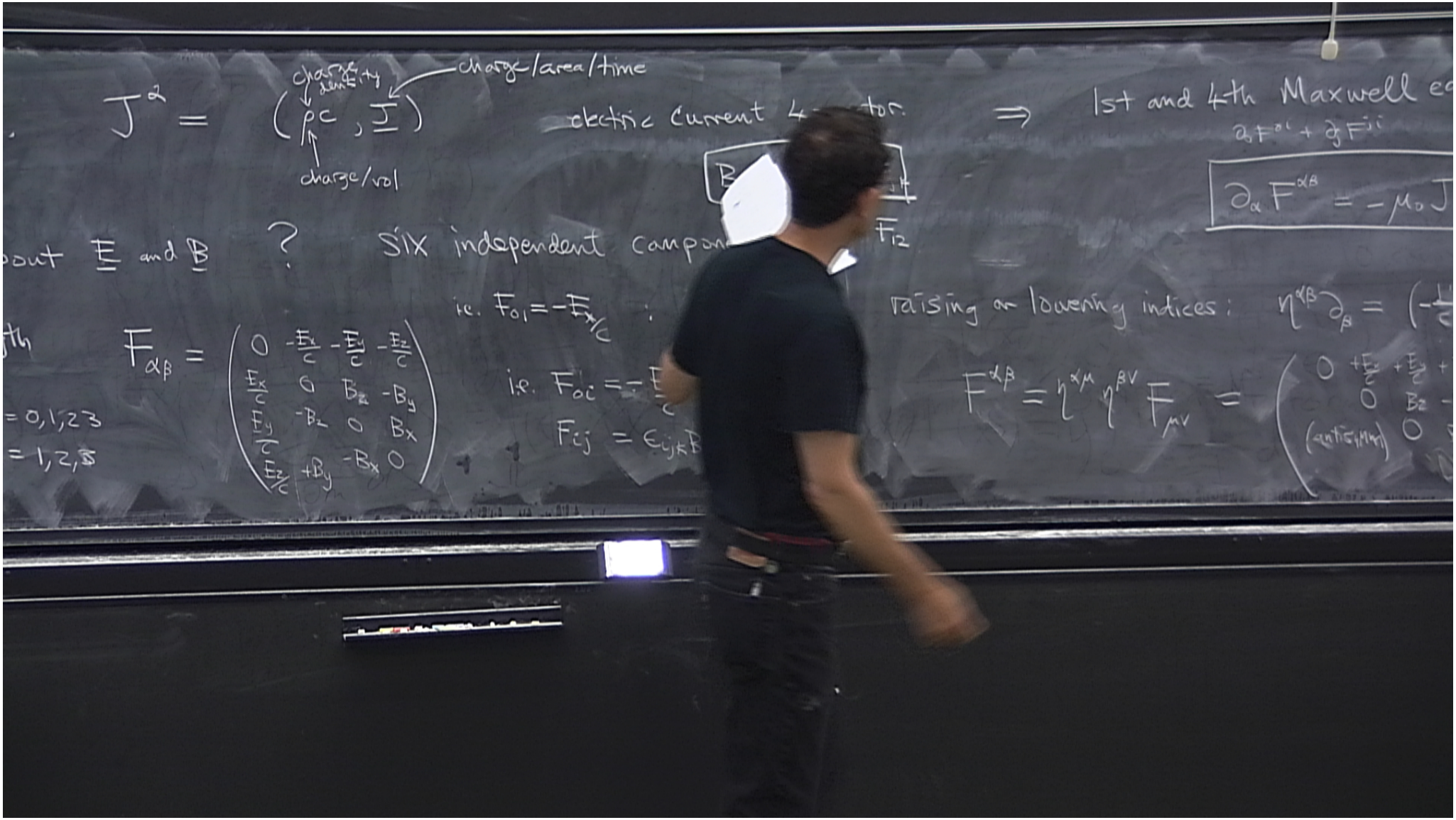
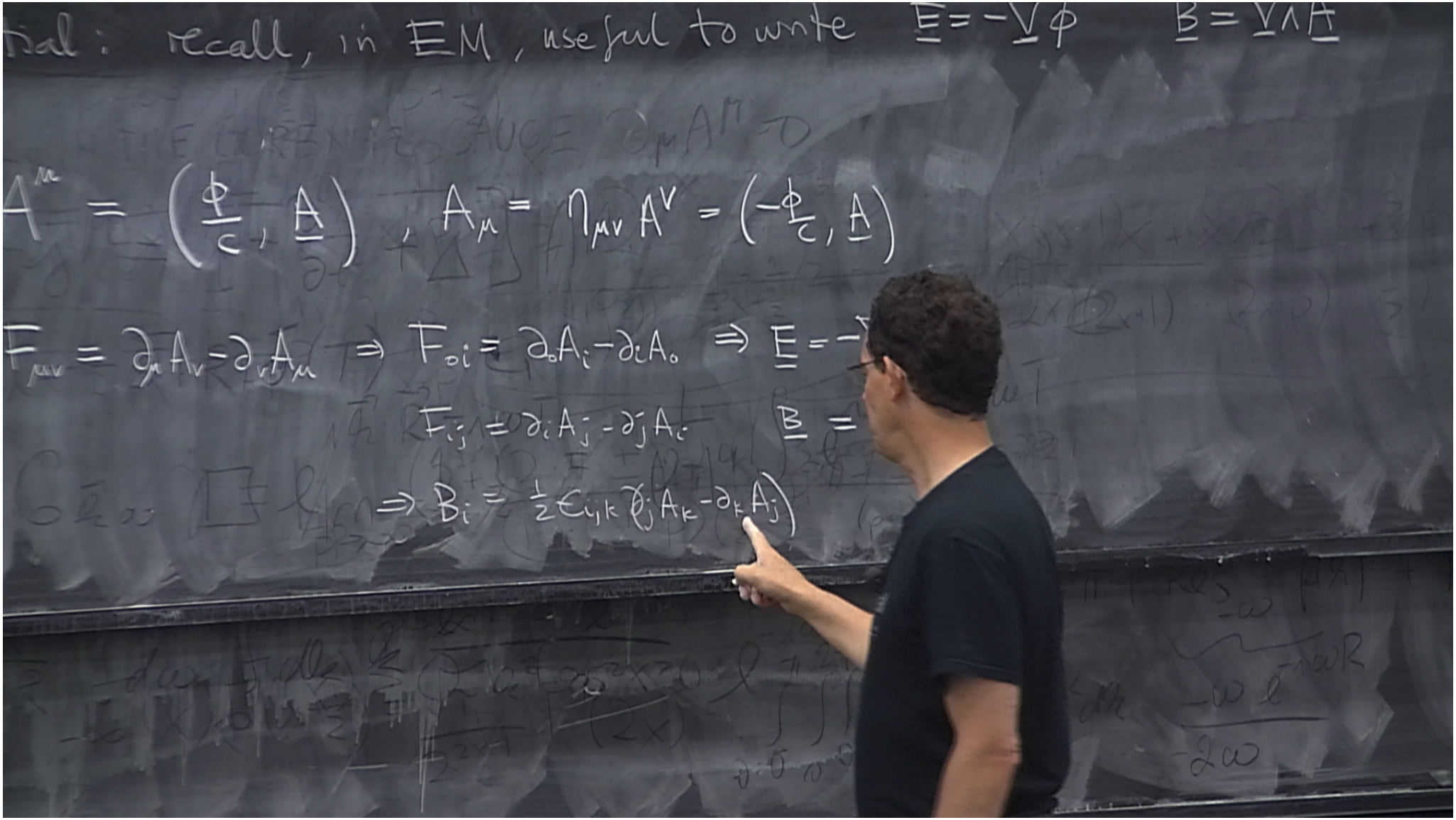

Vector potential: recall, in EM, useful to write $\underline{E} = -\underline{\nabla}\phi$ $\underline{B} = \underline{\nabla} \wedge \underline{A}$
4-vector potential
$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right)$, $A_\mu = \eta_{\mu\nu} A^\nu = \left(-\frac{\phi}{c}, \underline{A}\right)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Rightarrow$
$\Rightarrow \underline{E} = -\underline{\nabla}\phi - \frac{\partial}{\partial t}\underline{A}$

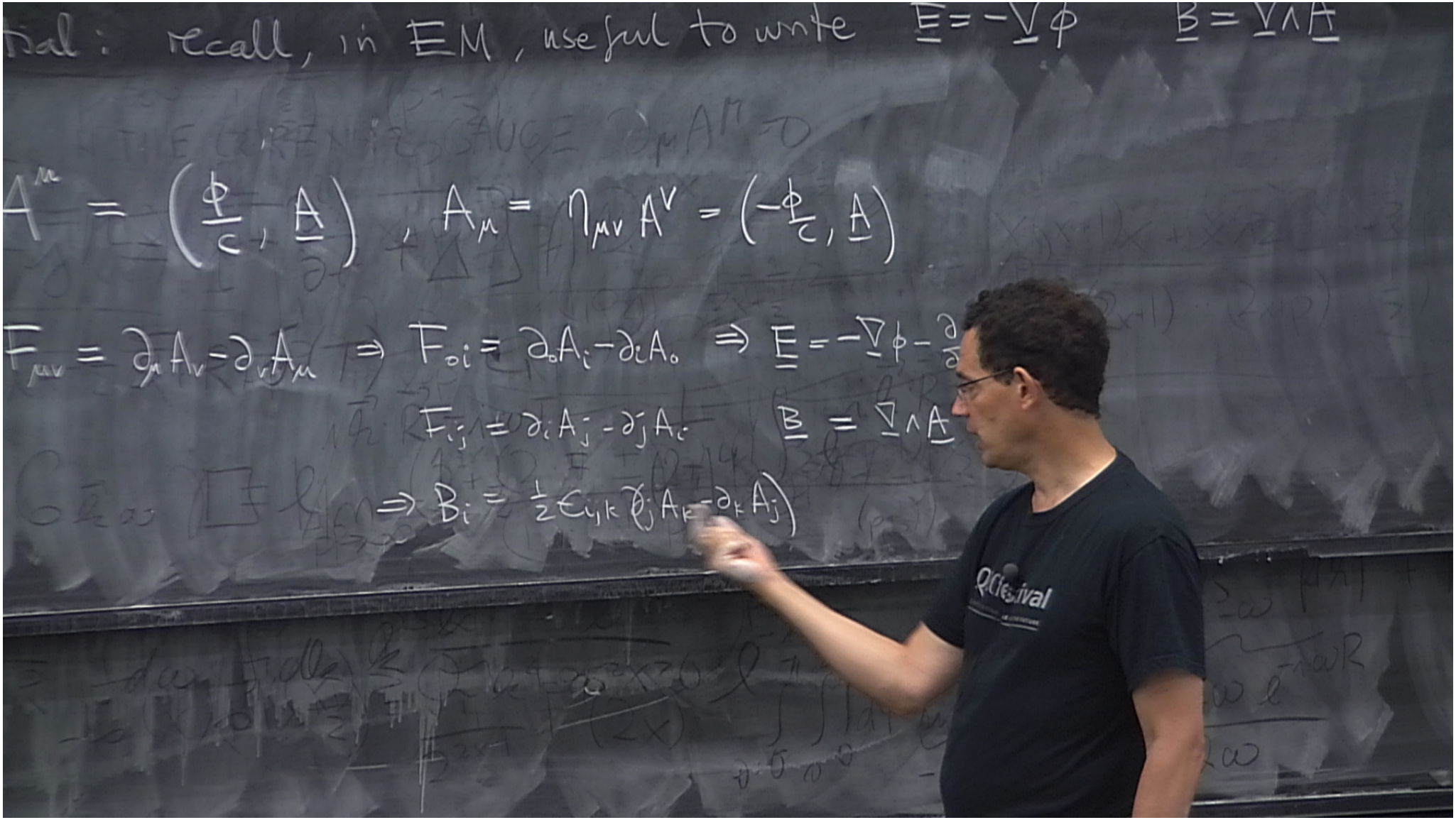

Vector potential: recall, in EM, useful to write $\underline{E} = -\underline{\nabla}\phi$ $\underline{B} = \underline{\nabla} \wedge \underline{A}$
4-vector potential
$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right)$, $A_\mu = \eta_{\mu\nu} A^\nu = \left(-\frac{\phi}{c}, \underline{A}\right)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Rightarrow F_{0i} = \partial_0 A_i - \partial_i A_0$ $-\underline{\nabla}\phi - \frac{\partial}{\partial t}\underline{A}$
$F_{ij} = \partial_i A_j - \partial_j A_i$ $= \underline{\nabla} \wedge \underline{A}$


charge density
charge/area/time
charge/vol
current 4-vector
1st and 4th Maxwell
six independe
raising or lowering indices:
= 0,1,2,3
= 1,2,3


$J^\alpha = (\rho c, \underline{J})$
charge density
charge/area/time
charge/vol
electric current 4-vector
$\Rightarrow$ 1st and 4th Maxwell eq
$B_i = \frac{1}{2}\epsilon_{ijk} F_{jk}$
$\partial_\alpha F^{\alpha\beta} = -\mu_0 J$
about $\underline{E}$ and $\underline{B}$? SIX independent components.
i.e. $F_{01} = -E_x/c$;
raising or lowering indices:
$F_{\alpha\beta} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & B_z & -B_y \\ E_y/c & -B_z & 0 & B_x \\ E_z/c & +B_y & -B_x & 0 \end{pmatrix}$
i.e. $F_{0i} = -\frac{E_i}{c}$ $i = 1,2,3$
$F_{ij} = \epsilon_{ijk} B_k$
$F^{\alpha\beta} = \eta^{\alpha\mu}\eta^{\beta\nu} F_{\mu\nu} =$


charge density
charge/area/time
charge/vol
electric current 4-vector
1st and 4th Maxwell
six independent
raising or lowering indices:
= 0,1,2,3
= 1,2,3


Recall, in EM, useful to write $\underline{E} = -\underline{\nabla}\phi$ $\underline{B} = \underline{\nabla}\wedge\underline{A}$
$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right)$, $A_\mu = \eta_{\mu\nu} A^\nu = \left(-\frac{\phi}{c}, \underline{A}\right)$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Rightarrow F_{0i} = \partial_0 A_i - \partial_i A_0 \Rightarrow \underline{E} = -$
$F_{ij} = \partial_i A_j - \partial_j A_i$ $\underline{B} =$
$\Rightarrow B_i = \frac{1}{2}\epsilon_{ijk}(\partial_j A_k - \partial_k A_j)$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$F_{ij} = \partial_i A_j - \partial_j A_i$ $\underline{B} = \underline{\nabla}\wedge\underline{A}$
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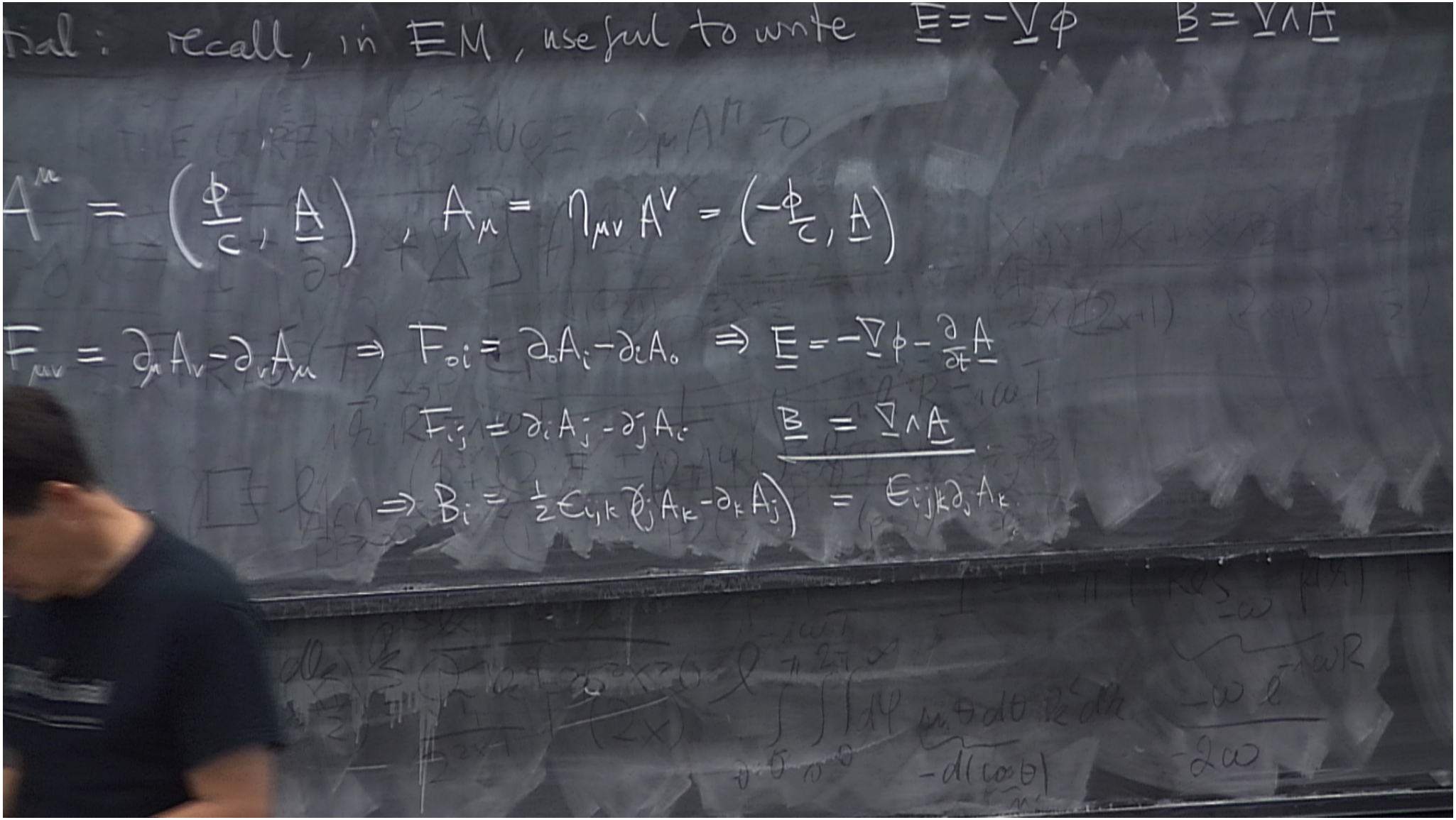
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$F_{ij} = \partial_i A_j - \partial_j A_i$ $\underline{B} = \underline{\nabla}\wedge\underline{A}$
$\Rightarrow B_i = \frac{1}{2}\epsilon_{ijk}(\partial_j A_k - \partial_k A_j) = \epsilon_{ijk}\partial_j A_k$