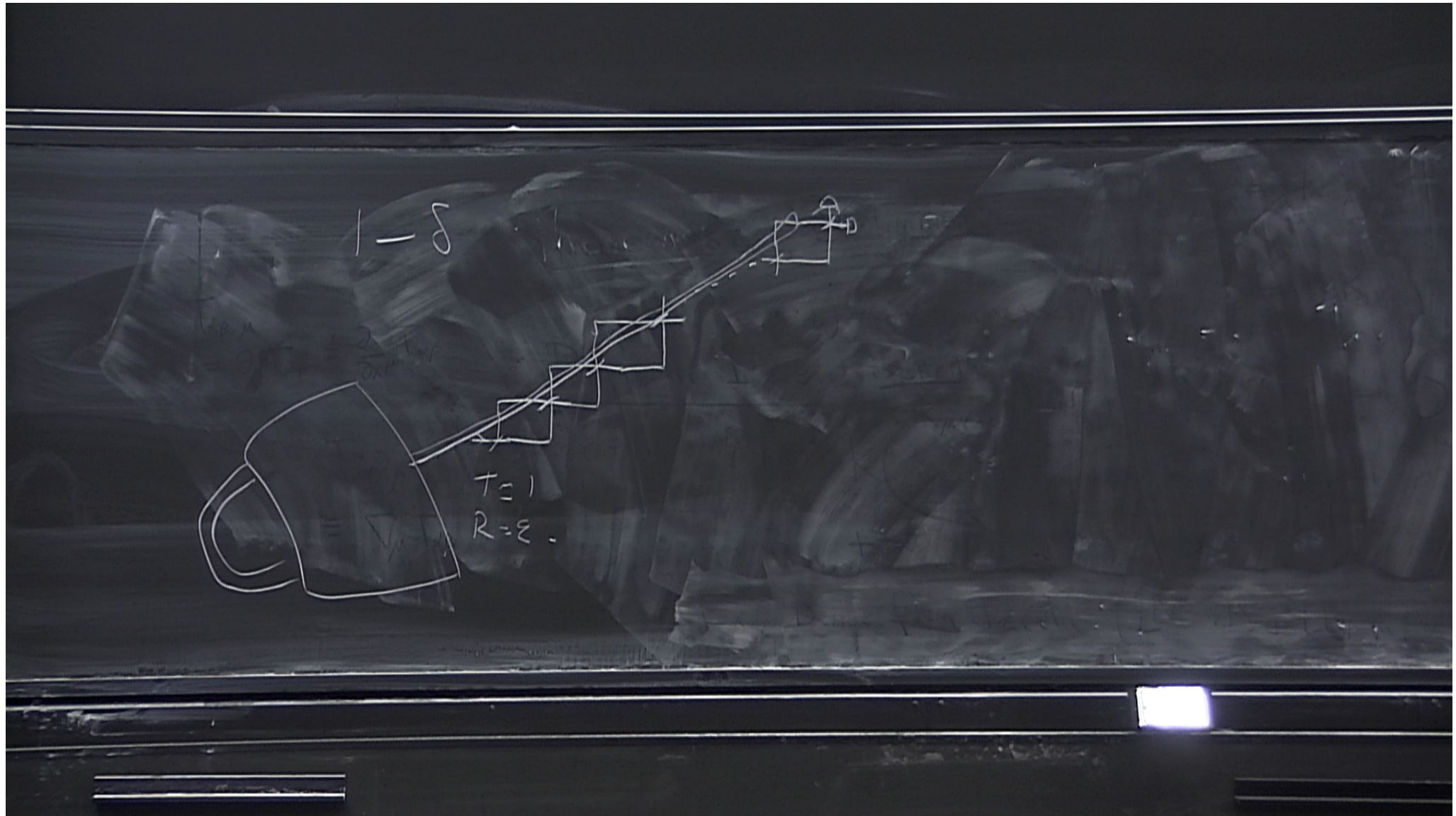


Title: Quantum Theory - Lecture 7

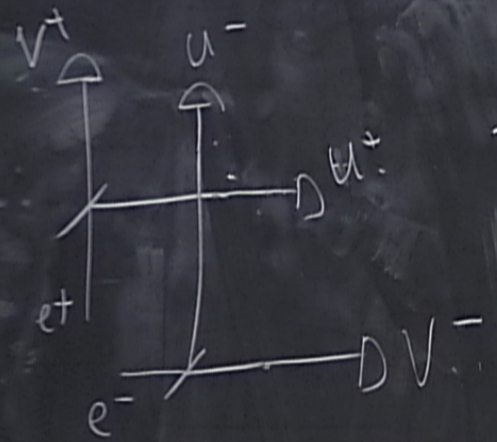
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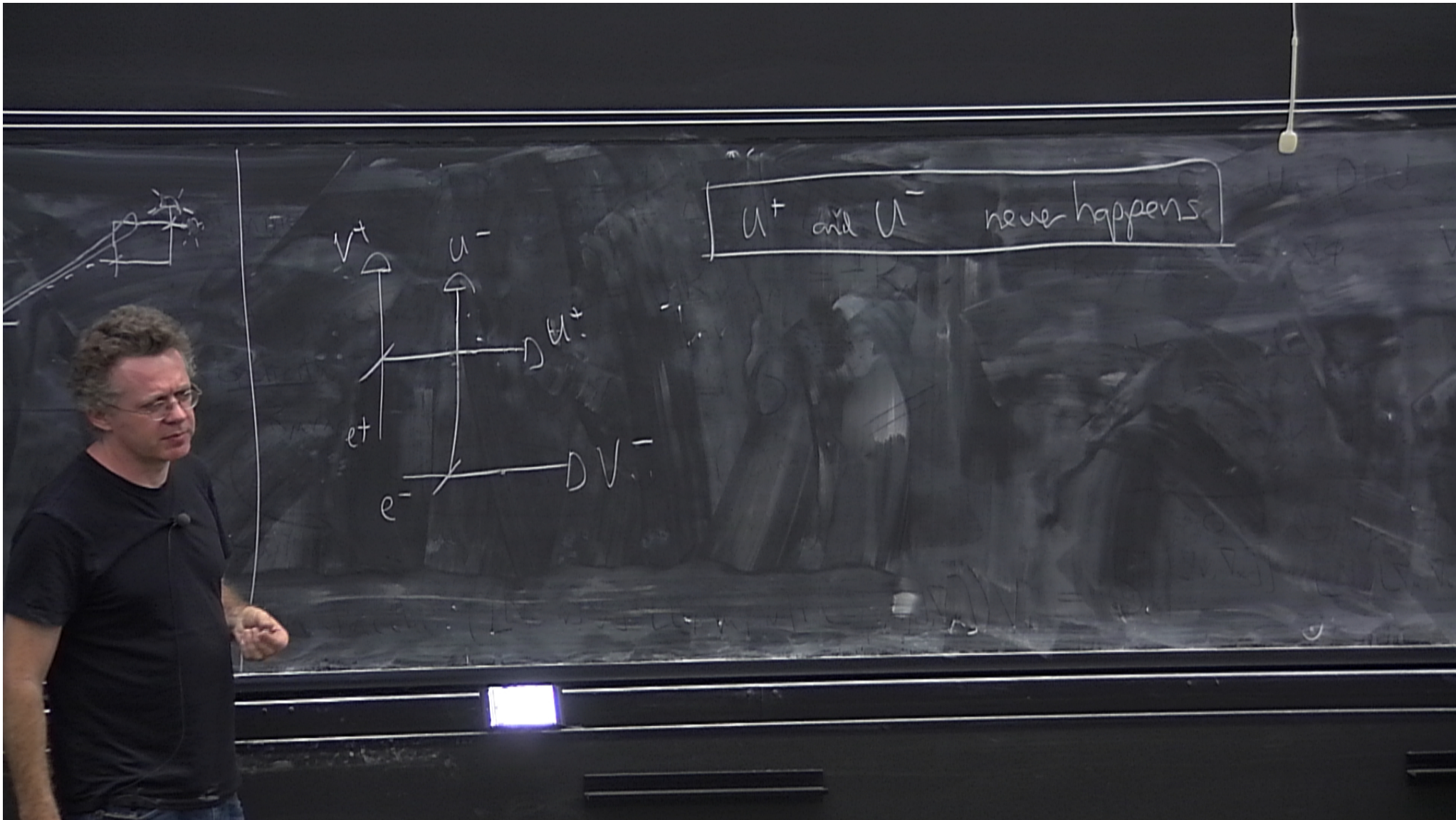
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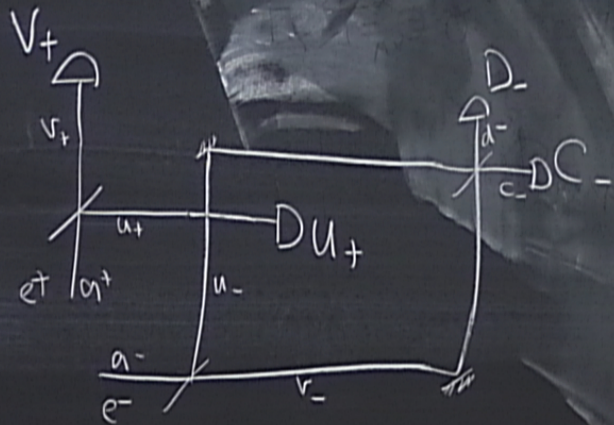
Abstract:



u^+ and u^- never happens





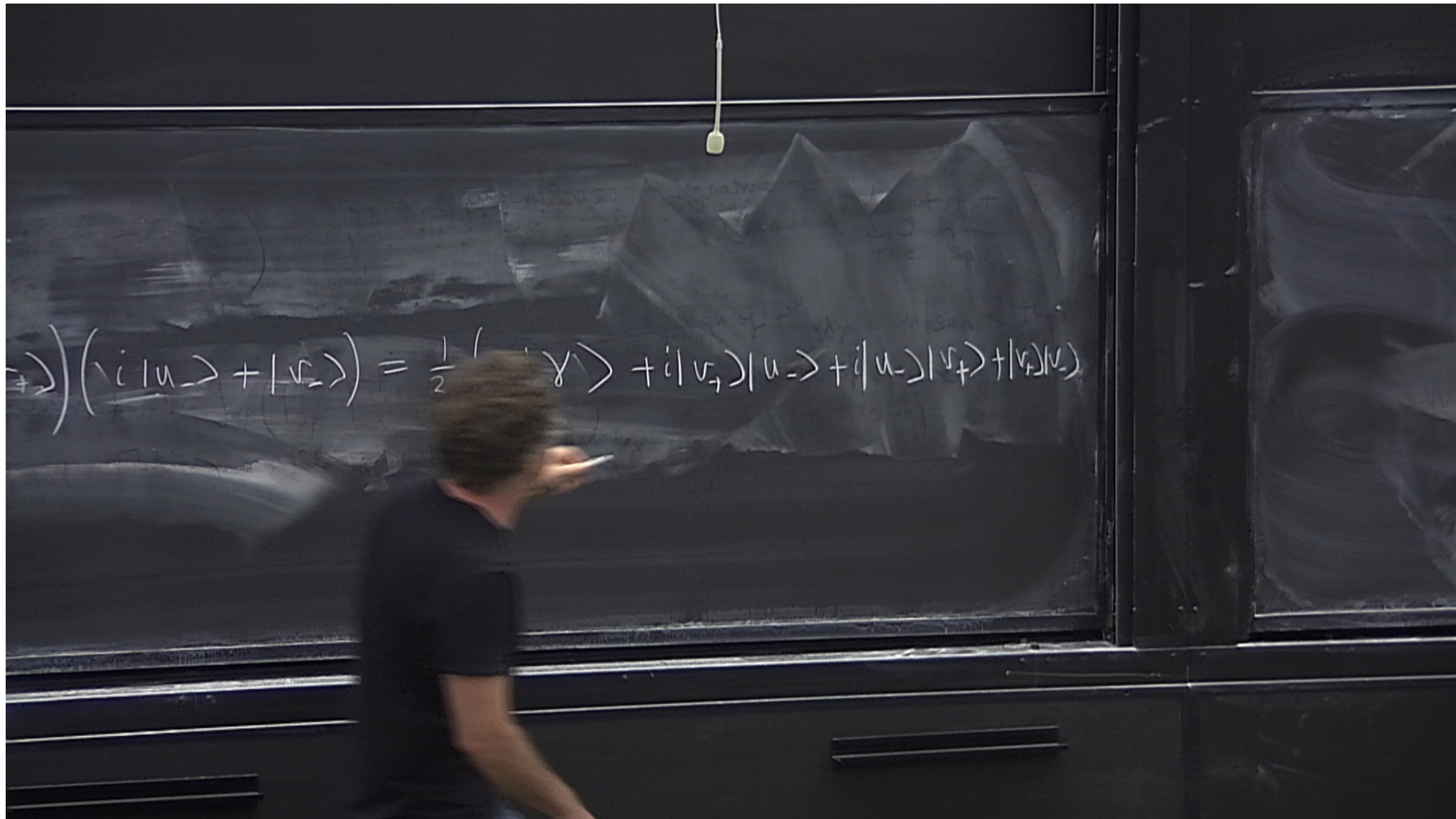


$$D_- \Rightarrow U_+$$

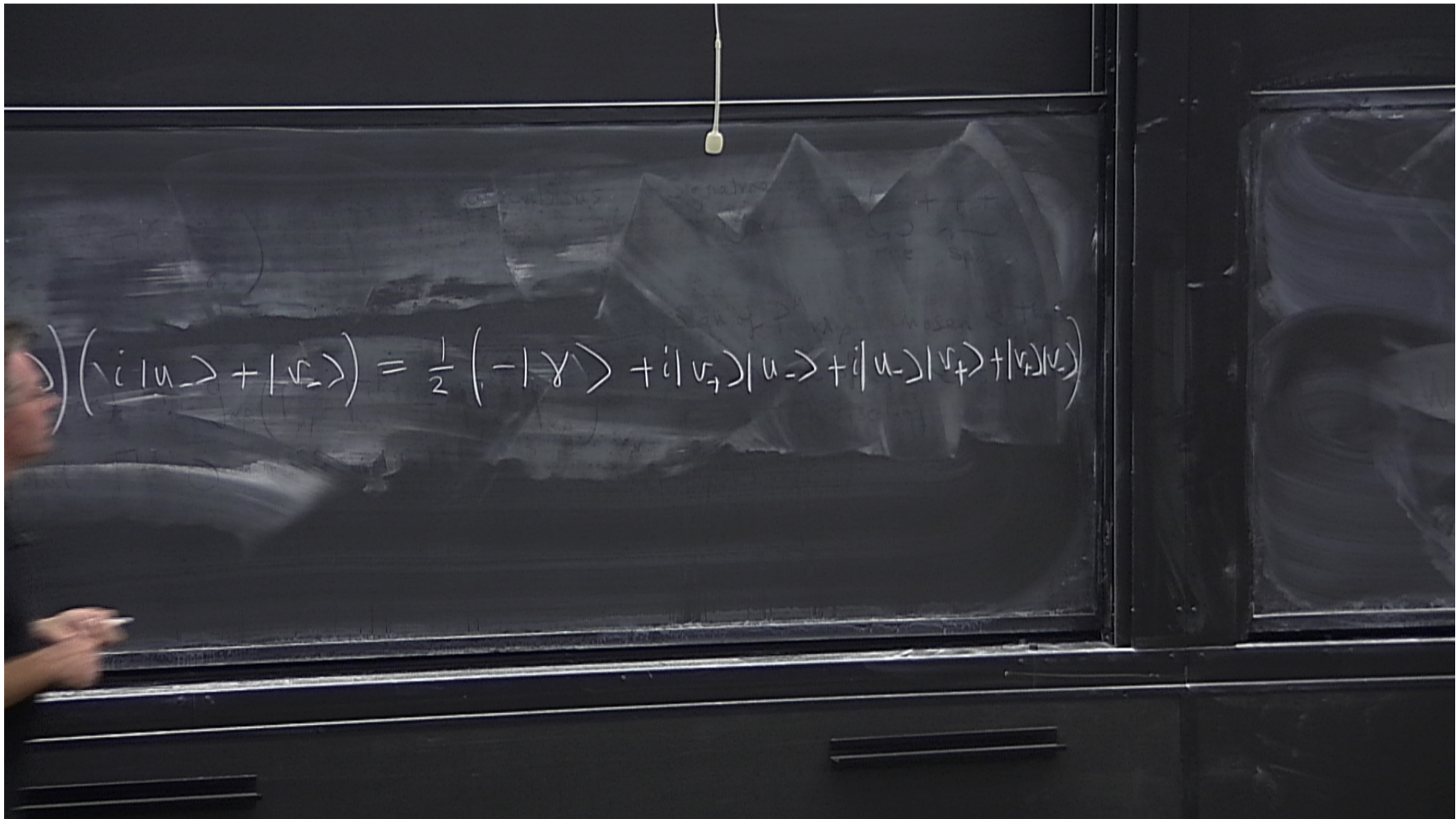
$$|a^+\rangle |a^-\rangle \rightarrow \frac{1}{\sqrt{2}} |c\rangle$$

$$D_- \Rightarrow U_+$$

$$|a^+\rangle |a^-\rangle \rightarrow \frac{1}{2} (i|u_+\rangle + |v_+\rangle) (i|u_-\rangle + |v_-\rangle) = \frac{1}{2} (-|x\rangle + i|y\rangle)$$



$$|+\rangle (i|u-\rangle + |v-\rangle) = \frac{1}{2} (|+\rangle + i|v_+\rangle|u-\rangle + i|u-\rangle|v_+\rangle + |v_+\rangle|v_-\rangle)$$



$$\rightarrow \frac{1}{2} (i|u_+\rangle + |v_+\rangle) (i|u_-\rangle + |v_-\rangle) = \frac{1}{2} (-|\gamma\rangle + i|v_+\rangle|u_-\rangle + i|u_+\rangle|v_-\rangle + |v_+\rangle|v_-\rangle)$$

$$> + \frac{i}{\sqrt{2}} |v_+\rangle (|c_-\rangle + i|d_-\rangle) + (i|u_+\rangle + |v_+\rangle) ($$

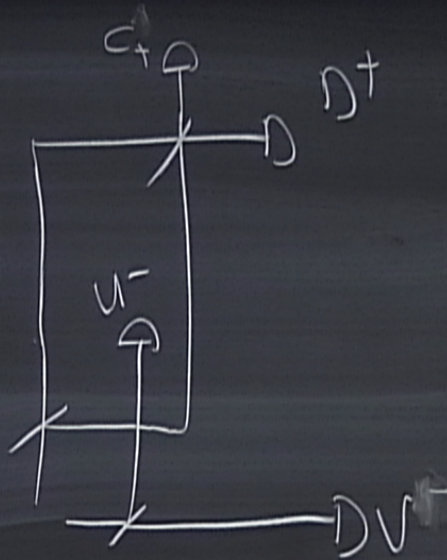
$\Rightarrow U_+$

$$|u_+\rangle |d_-\rangle \rightarrow \frac{1}{2} (i|u_+\rangle + |v_+\rangle) (i|u_-\rangle + |v_-\rangle) = \frac{1}{2} (-|x\rangle + i|v_+\rangle |u_-\rangle + i|u_+\rangle |v_-\rangle + |y\rangle)$$

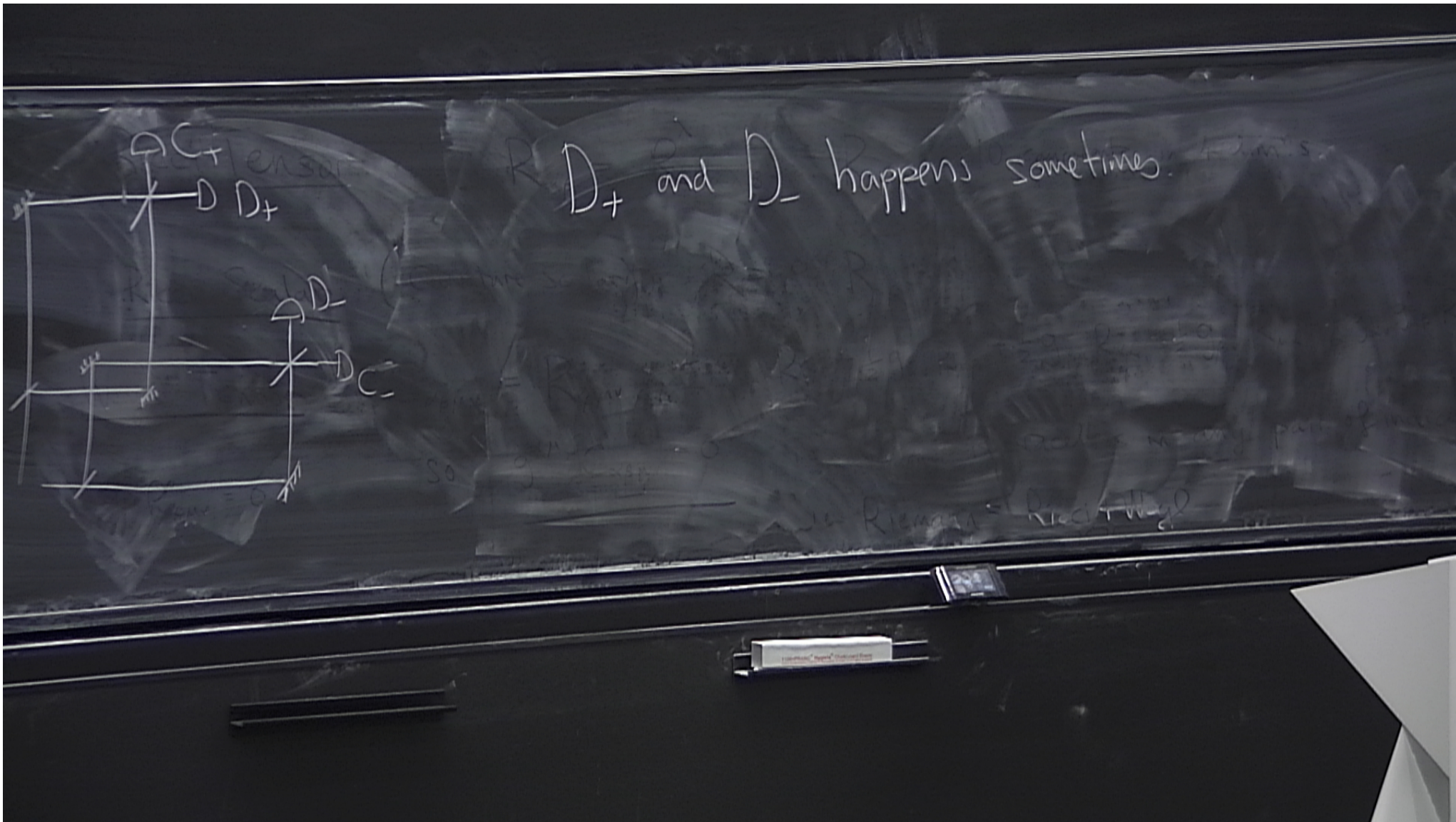
$$\rightarrow \frac{1}{2} \left(-|x\rangle + \frac{i}{\sqrt{2}} |v_+\rangle (|c_-\rangle + i|d_-\rangle) + (i|u_+\rangle + |v_+\rangle) (i|c_-\rangle + |d_-\rangle) \right) \frac{1}{\sqrt{2}}$$

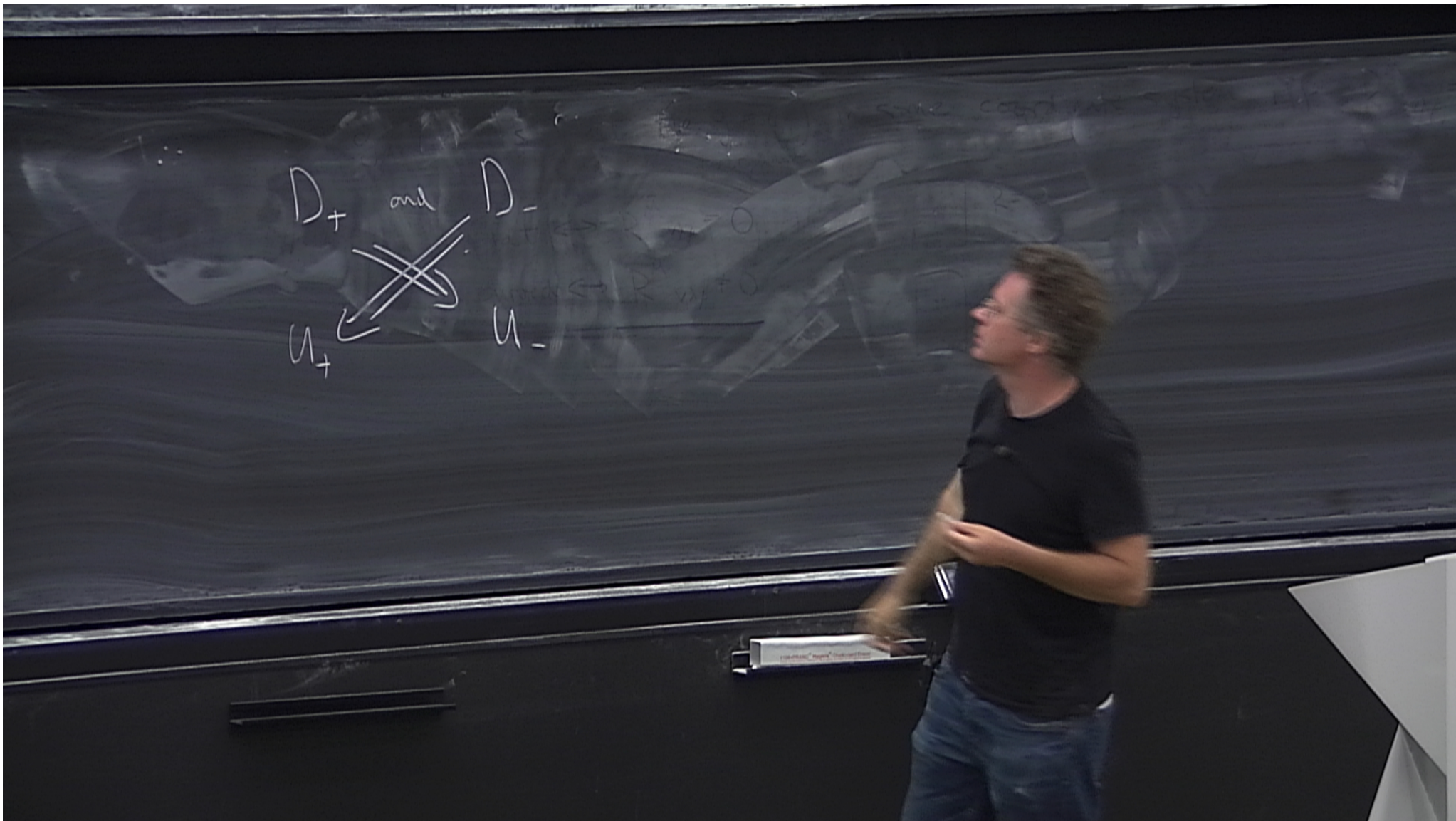
$$= \frac{1}{2\sqrt{2}} |u_+\rangle |d_-\rangle +$$

$$= \frac{1}{2\sqrt{2}} (|u_+\rangle |d_-\rangle +$$



$$D_+ \Rightarrow u^-$$

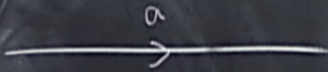




$$= \frac{1}{2\sqrt{2}} |u_+\rangle |d_-\rangle +$$

Quantum optical interferometry.

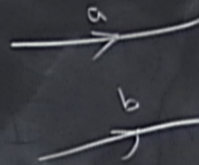
1)



$|n\rangle_a$

$$\alpha |0\rangle_a + \beta |1\rangle_a + \sqrt{2} |2\rangle_a + \dots$$

two modes.



$|n\rangle_a |m\rangle_b$

$$\alpha |0\rangle_a |0\rangle_b + \beta |0\rangle_a |1\rangle_b + \dots$$

a annihilation op.

a^+ creation op.

$$(|n-1\rangle\langle n|)|n\rangle = \langle n|n\rangle |n-1\rangle$$

$$a = \sum_{n=1}^{\infty} \sqrt{n} |n-1\rangle\langle n|$$

a annihilation op.

a^+ creation op.

$$(|n-1\rangle\langle n|)|n\rangle = \langle n|n\rangle |n-1\rangle$$

$$a = \sum_{n=1}^{\infty} v_n |n-1\rangle\langle n|$$

$$a|m\rangle = v_m |m-1\rangle$$

a annihilation op.

a^+ creation op.

$$(|n-1\rangle\langle n|)|n\rangle = \langle n|n\rangle |n-1\rangle$$

$$a = \sum_{n=1}^{\infty} v_n |n-1\rangle\langle n|$$

$$a|m\rangle = v_m |m-1\rangle$$

$$a|0\rangle = 0$$

a annihilation op.

a^+ creation op.

$$(|n-1\rangle\langle n|)|n\rangle = \langle n|n\rangle |n-1\rangle$$

$$a = \sum_{n=1}^{\infty} v_n |n-1\rangle\langle n|$$

$$a|m\rangle = v_m |m-1\rangle$$

$$a|0\rangle = 0$$



$$a^+ = \sum_{n=1}^{\infty} \sqrt{n} |n\rangle \langle n-1|$$

$$a^+ |m\rangle = \sqrt{m+1} |m+1\rangle$$

$$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1 = \sum_{n=1}^{\infty} |n\rangle\langle n|$$

Riemann-Roch-Weg

$$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1 = \sum_{n=0}^{\infty} |n\rangle\langle n|$$

$$V_n^* = V_n$$

$$V_m = \sqrt{m}$$

Riemann - Riemann-Weg



$$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1 = \sum_{n=0}^{\infty} |n\rangle \langle n|$$

$$V_n^* = V_n \geq 0$$

$$V_m = \sqrt{m}$$

$$a|n\rangle = \sqrt{n}|n-1\rangle$$

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

Riemann - Risch-Weyl

$$\begin{aligned}
 [a, a^\dagger] &= aa^\dagger - a^\dagger a = 1 = \sum_{n=0}^{\infty} |n\rangle \langle n| \\
 V_n^* &= V_n \geq 0 \\
 V_m &= \sqrt{m} \\
 a|n\rangle &= \sqrt{n}|n-1\rangle \\
 a^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle \\
 [a, a] &= 0 \\
 [a^\dagger, a^\dagger] &= 0
 \end{aligned}$$

$$[a, a^\dagger] = aa^\dagger - a^\dagger a = 1 = \sum_{n=0}^{\infty} |n\rangle \langle n|$$

$$V_n \geq 0$$

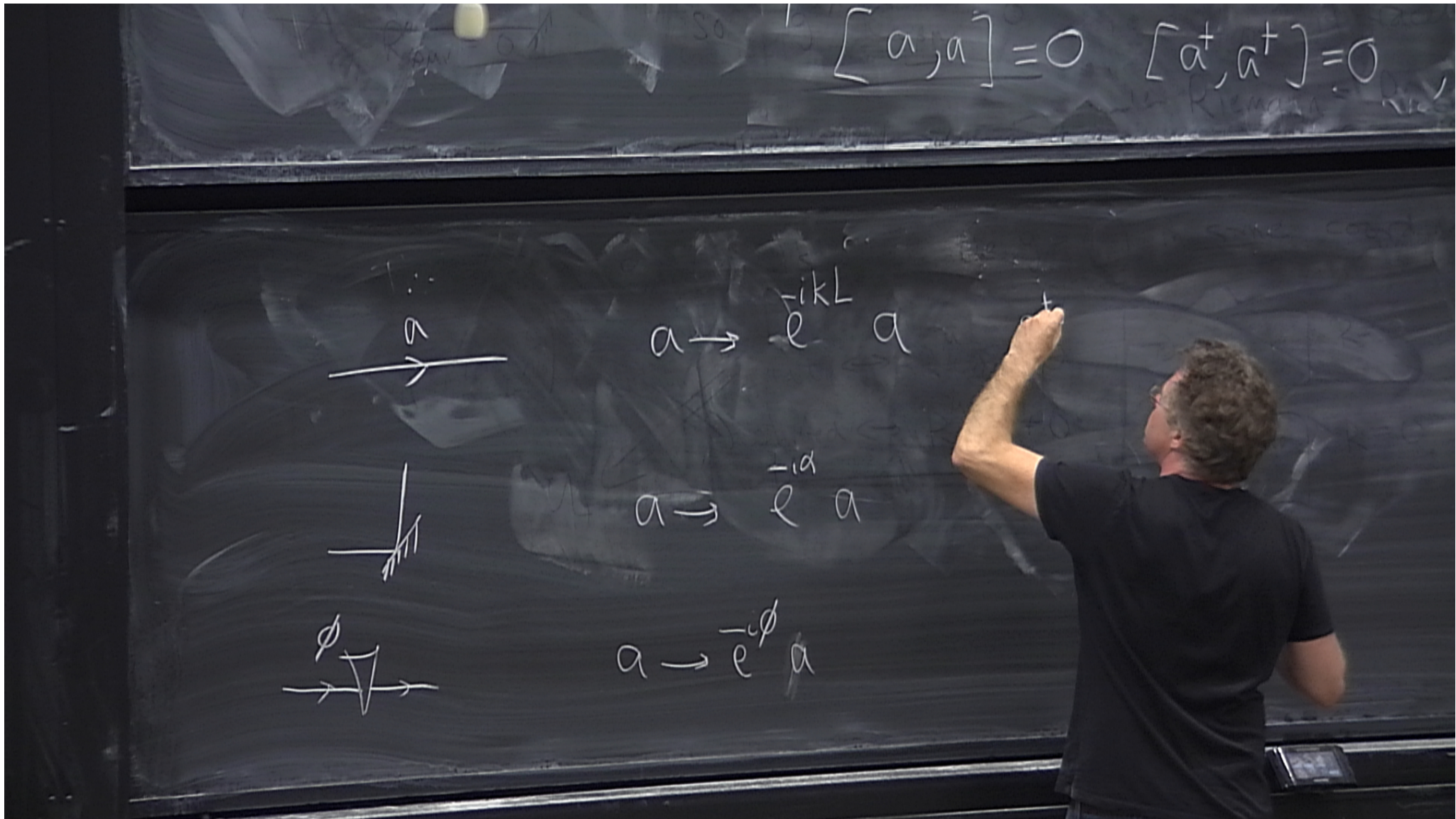
$$V_m = \sqrt{m}$$

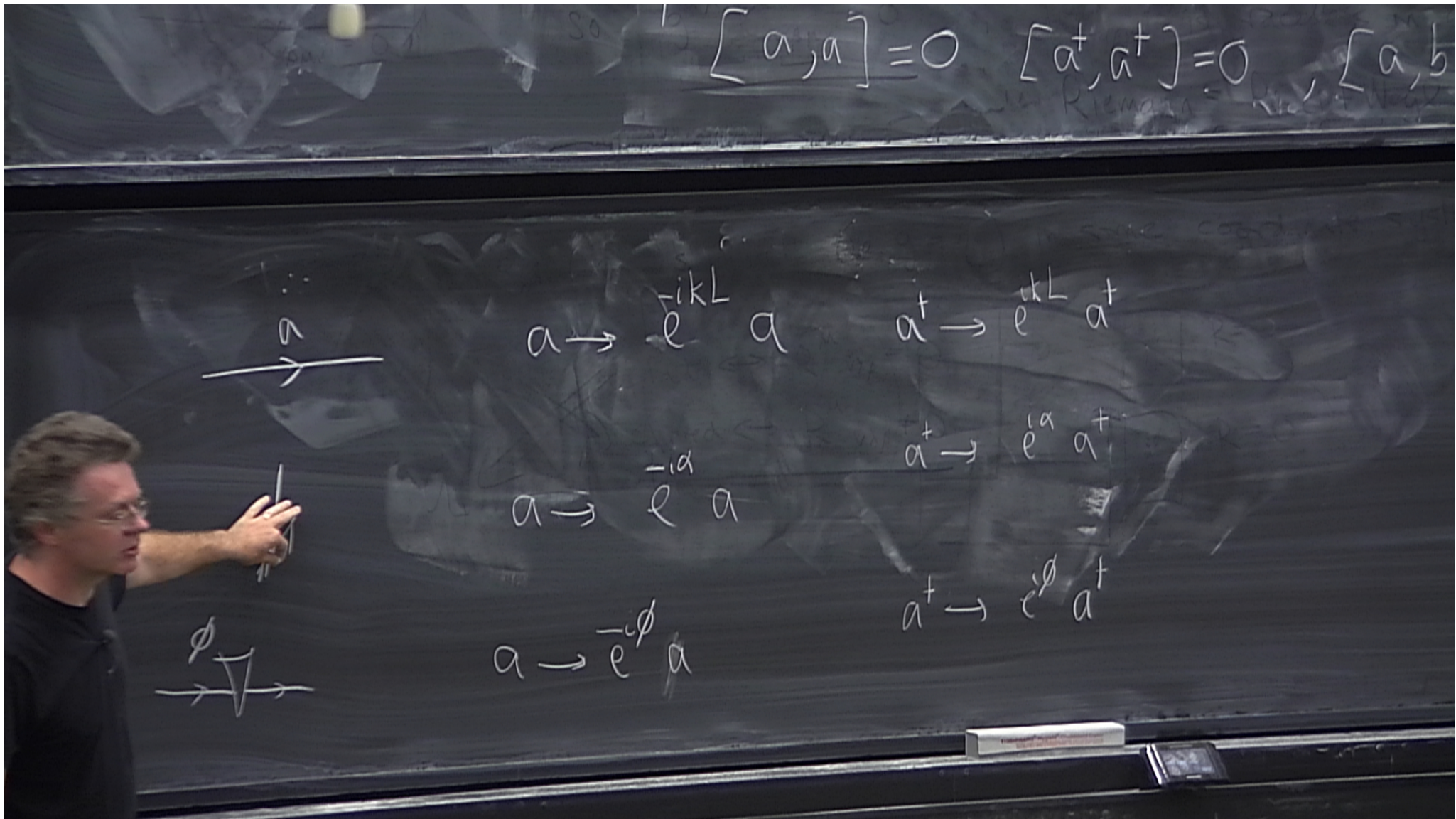
$$a|n\rangle = \sqrt{n}|n-1\rangle$$

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

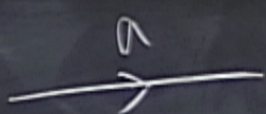
$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle$$

$$[a, a] = 0, [a^\dagger, a^\dagger] = 0, [a, b] = [a^\dagger, b^\dagger] = [a, b^\dagger] = 0$$





So $[a, a] = 0$ $[a^\dagger, a^\dagger] = 0$, $[a, b]$



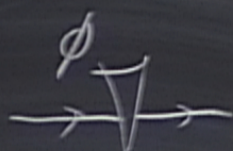
$$a \rightarrow e^{-ikL} a$$

$$a^\dagger \rightarrow e^{ikL} a^\dagger$$



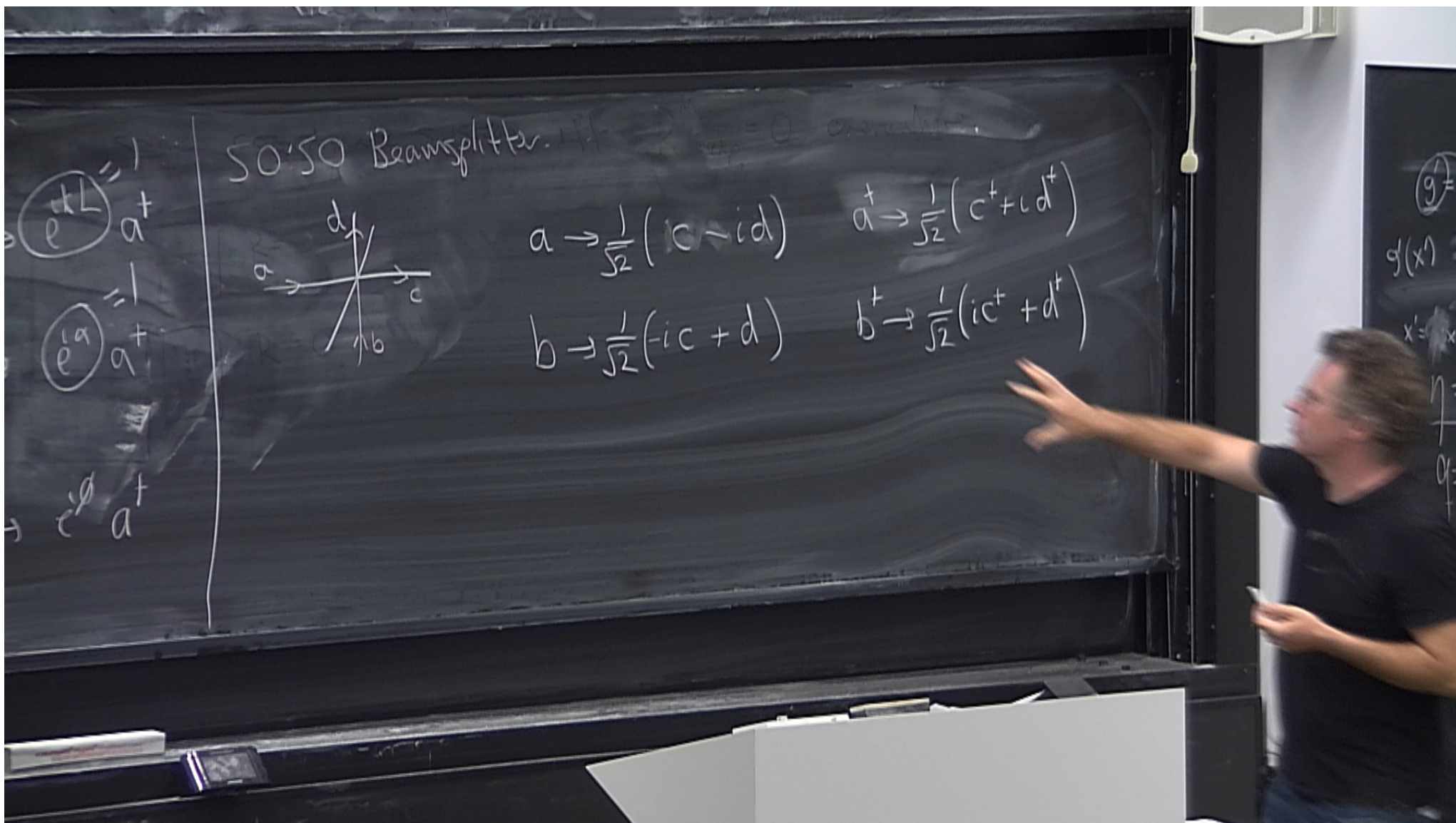
$$a \rightarrow e^{-i\alpha} a$$

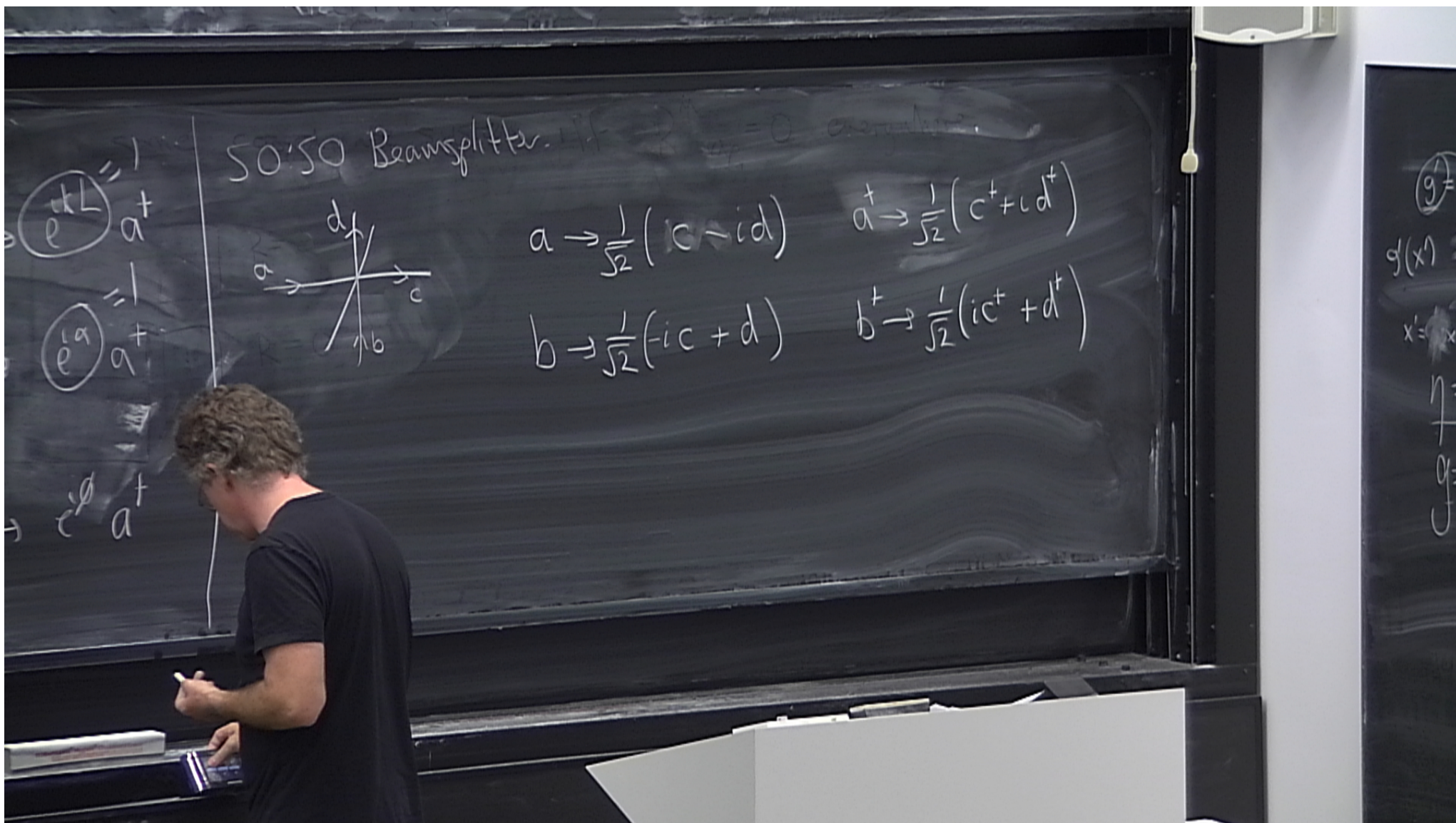
$$a^\dagger \rightarrow e^{i\alpha} a^\dagger$$



$$a \rightarrow e^{-i\phi} a$$

$$a^\dagger \rightarrow e^{i\phi} a^\dagger$$



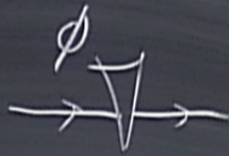




$$a \rightarrow e^{-i\phi} a$$

$$a^\dagger \rightarrow e^{i\phi} a^\dagger$$

$$|m\rangle_a |n\rangle_b = \frac{(a^\dagger)^m}{\sqrt{m!}} \frac{(b^\dagger)^n}{\sqrt{n!}} |0\rangle$$

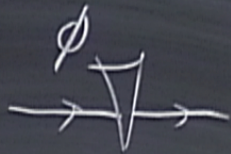


$$a \rightarrow e^{-i\phi} a$$

$$a^\dagger \rightarrow e^{i\phi} a^\dagger$$

$$|m\rangle_a |n\rangle_b = \frac{(a^\dagger)^m}{\sqrt{m!}} \frac{(b^\dagger)^n}{\sqrt{n!}} |0\rangle$$

$$\rightarrow \frac{\left(\frac{c + id^\dagger}{\sqrt{2}}\right)^m}{\sqrt{m!}} \frac{\left(\frac{ic^\dagger + d}{\sqrt{2}}\right)^n}{\sqrt{n!}} |0\rangle$$

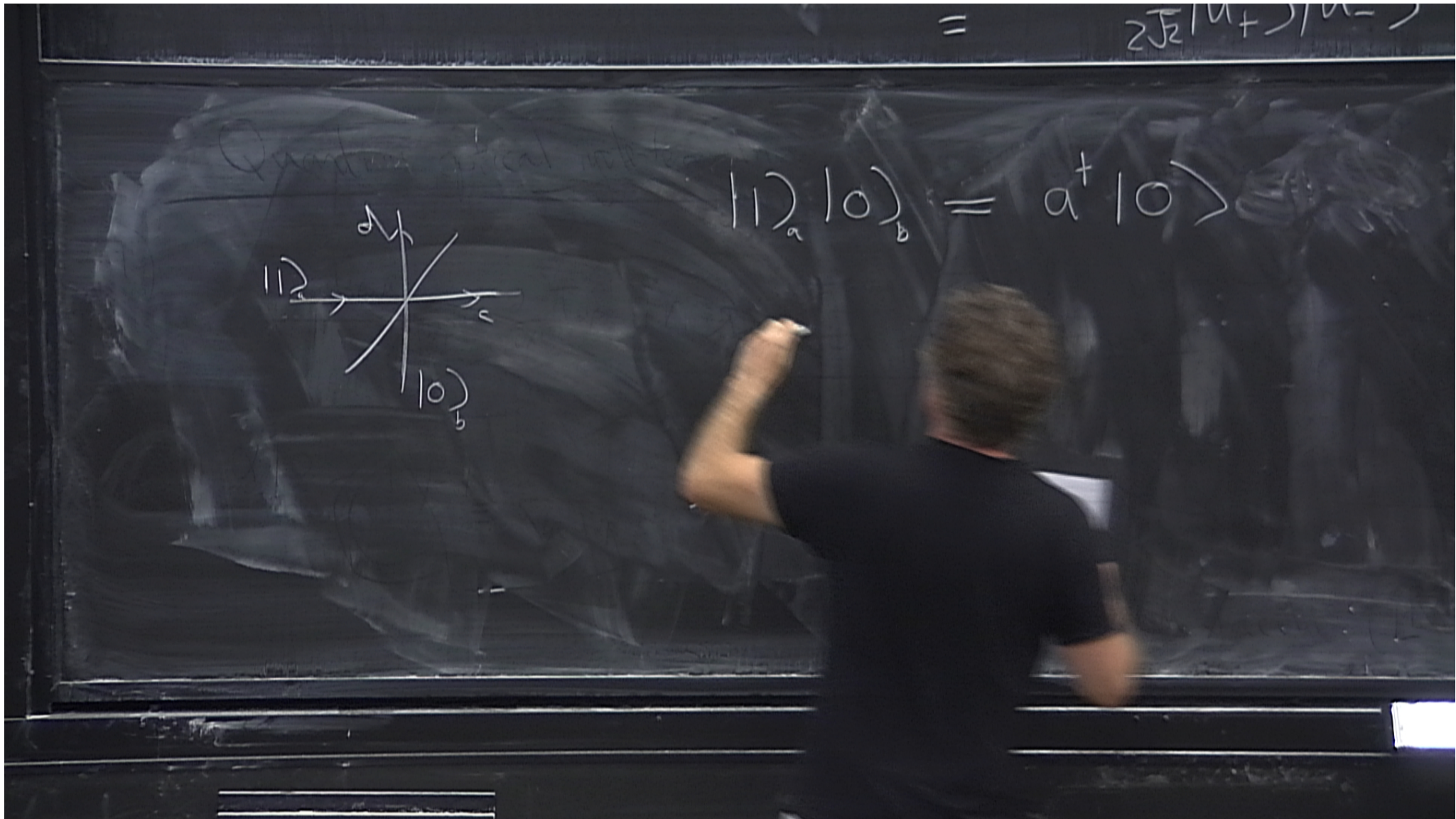


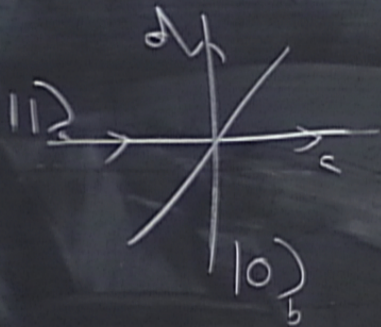
$$a \rightarrow e^{-i\phi} a$$

$$a^\dagger \rightarrow e^{i\phi} a^\dagger$$

$$|m\rangle_a |n\rangle_b = \frac{(a^\dagger)^m}{\sqrt{m!}} \frac{(b^\dagger)^n}{\sqrt{n!}} |0\rangle$$

$$\rightarrow \frac{\left(\frac{c + id^\dagger}{\sqrt{2}}\right)^m}{\sqrt{m!}} \frac{\left(\frac{ic^\dagger + d}{\sqrt{2}}\right)^n}{\sqrt{n!}} |0\rangle$$

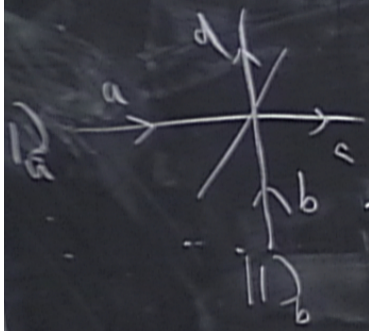




$$|1\rangle_a |0\rangle_b = a^\dagger |0\rangle$$

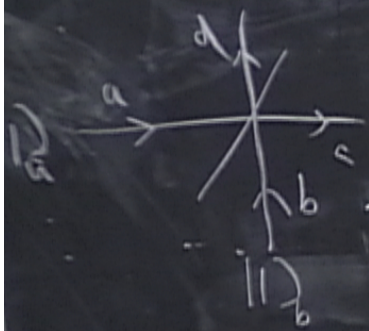
$$\rightarrow \frac{1}{\sqrt{2}} (c^\dagger + i d^\dagger) |0\rangle$$

$$= \frac{1}{\sqrt{2}} (|1\rangle_c |0\rangle_d + i |0\rangle_c |1\rangle_d)$$



$$|1\rangle_a |1\rangle_b = a^\dagger b^\dagger |0\rangle$$

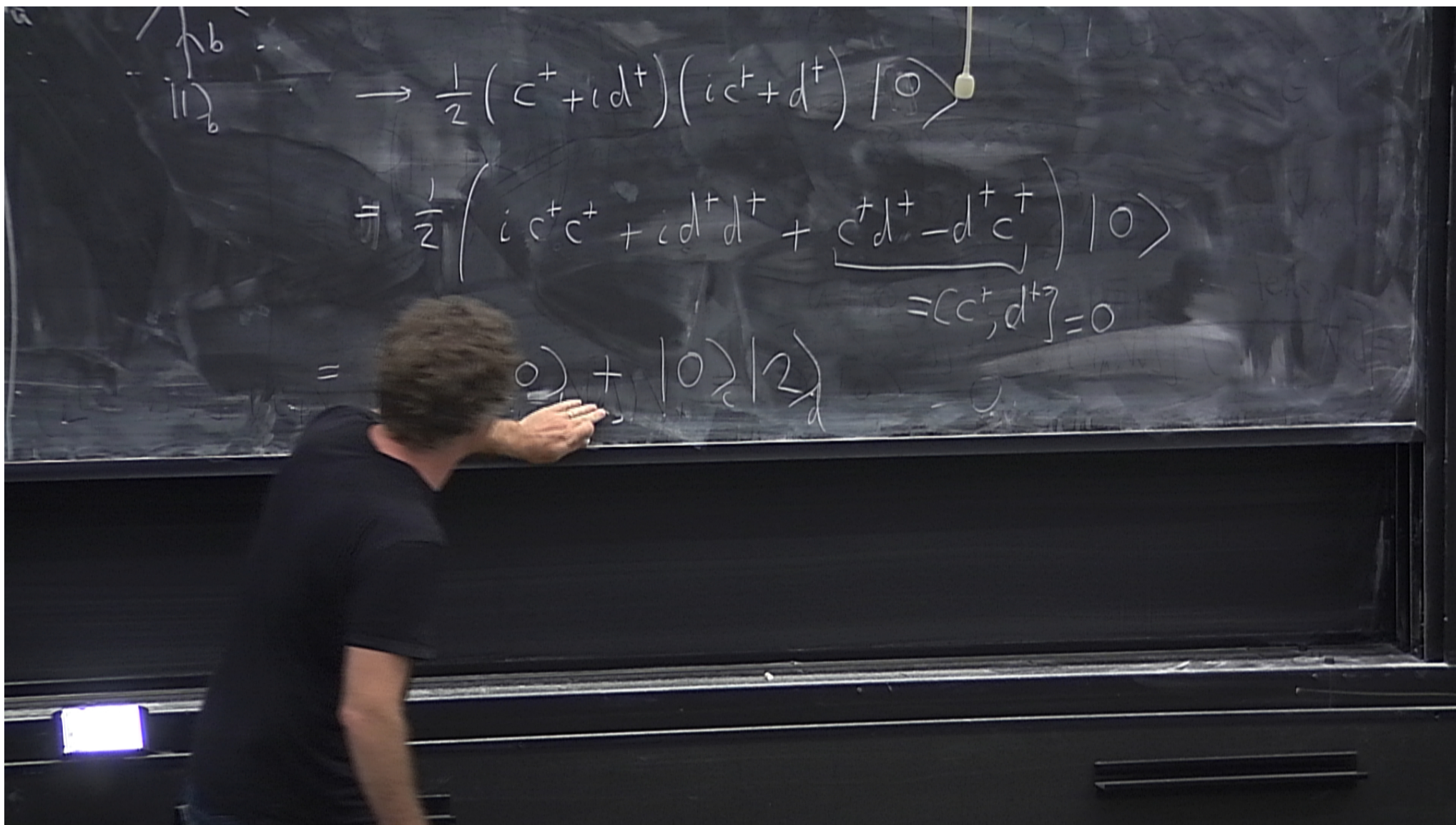
$$\rightarrow \frac{1}{2} (c^\dagger + i d^\dagger) (i c^\dagger + d^\dagger) |0\rangle$$

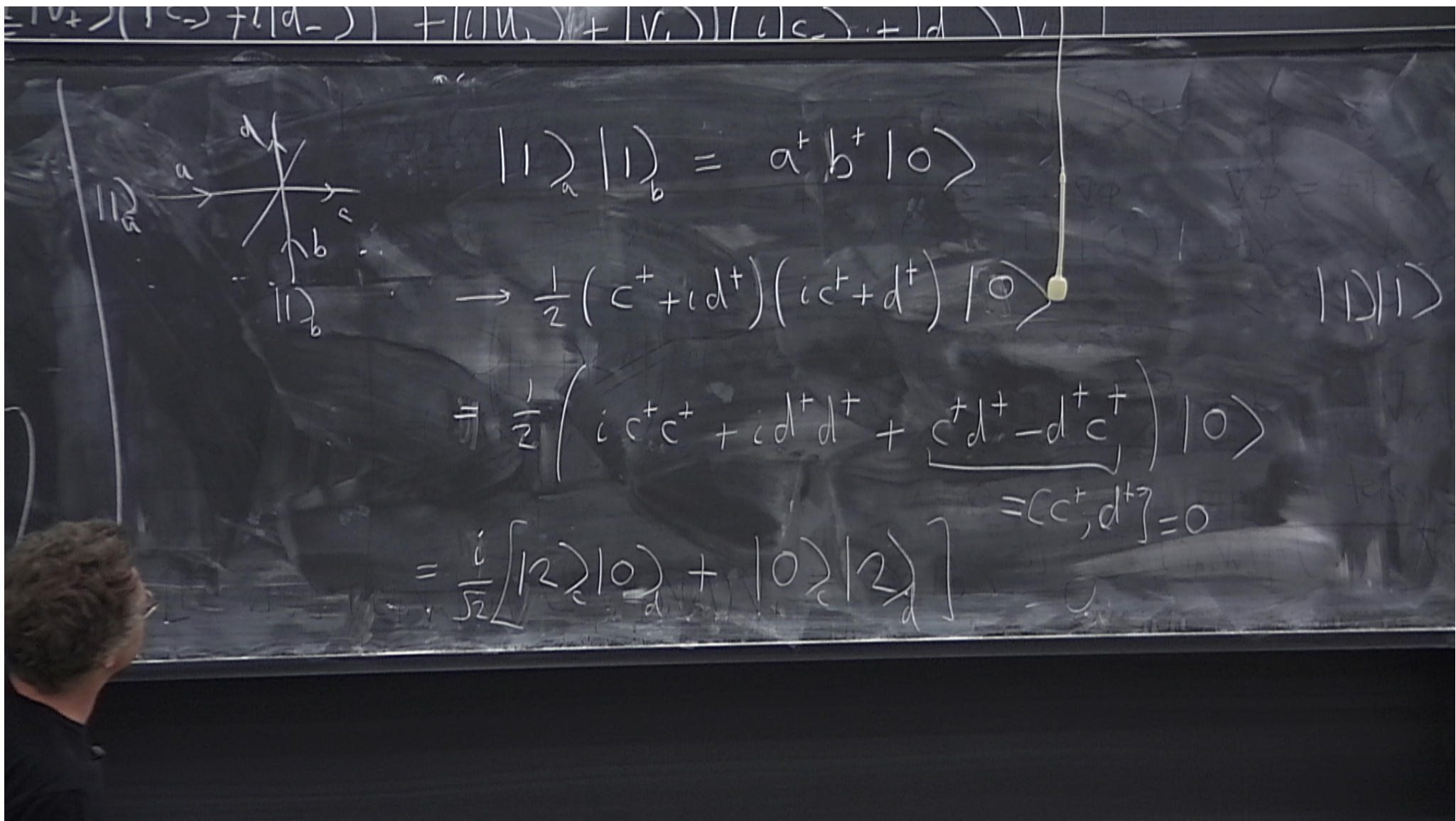


$$|1\rangle_a |1\rangle_b = a^\dagger b^\dagger |0\rangle$$

$$\rightarrow \frac{1}{2} (c^\dagger + i d^\dagger) (i c^\dagger + d^\dagger) |0\rangle$$

$$= \frac{1}{2} (i c^\dagger c^\dagger + i d^\dagger d^\dagger + c^\dagger d^\dagger - d^\dagger c^\dagger) |0\rangle$$

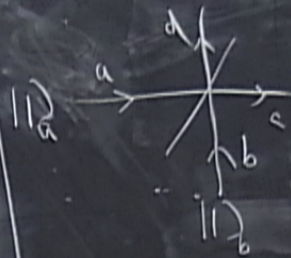




$$|1\rangle_a |0\rangle_b = a^\dagger |0\rangle$$

$$\rightarrow \frac{1}{\sqrt{2}} (c^\dagger + i d^\dagger) |0\rangle$$

$$= \frac{1}{\sqrt{2}} (|1\rangle_a |0\rangle_b + i |0\rangle_a |1\rangle_b)$$



$$|1\rangle_a |1\rangle_b = a^\dagger b^\dagger |0\rangle$$

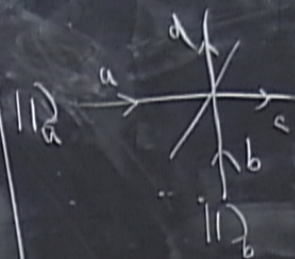
$$\rightarrow \frac{1}{2} (c^\dagger + i d^\dagger) (i c^\dagger + d^\dagger) |0\rangle$$

$$= \frac{1}{2} (i c^\dagger c^\dagger + i d^\dagger d^\dagger + \underbrace{c^\dagger d^\dagger - d^\dagger c^\dagger}_{= [c^\dagger, d^\dagger] = 0}) |0\rangle$$

$$= \frac{i}{\sqrt{2}} [|2\rangle_a |0\rangle_b + |0\rangle_a |2\rangle_b]$$

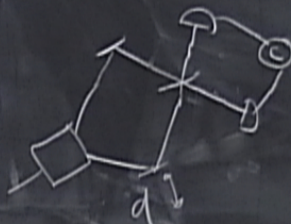
$$|1\rangle_a |0\rangle_b = a^\dagger |0\rangle$$

$$\rightarrow \frac{1}{\sqrt{2}}(c^\dagger + id^\dagger)|0\rangle$$



$$|1\rangle_a |1\rangle_b = a^\dagger b^\dagger |0\rangle$$

$$\rightarrow \frac{1}{2}(c^\dagger + id^\dagger)(ic^\dagger + d^\dagger)|0\rangle$$



$$= \frac{1}{2}(ic^\dagger c^\dagger + id^\dagger d^\dagger + \underbrace{c^\dagger d^\dagger - d^\dagger c^\dagger})|0\rangle$$

$$= [c^\dagger, d^\dagger] = 0$$

$$= \frac{i}{\sqrt{2}}[|2\rangle_a |0\rangle_b + |0\rangle_a |2\rangle_b]$$

