

Title: Summer Undergraduate Presentation - John

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Abstract:

Non-Gaussian Probability Distribution and Primordial Black Holes

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Supervisor : PhD. Sarah Shandera



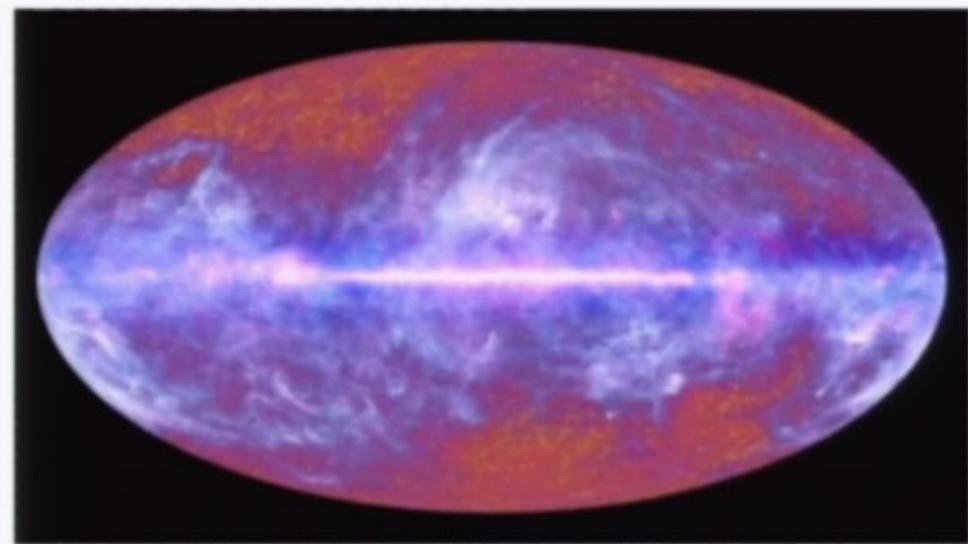
CONTENT

1. STANDARD MODEL OF THE UNIVERSE
2. INFLATION
3. SCALAR FIELD PERTURBATION
4. PRIMORDIAL BLACK HOLES
5. STANDARD CALCULATIONS
6. OUR CALCULATIONS
7. ADDING NON-GAUSSIAN
8. CONCLUTIONS



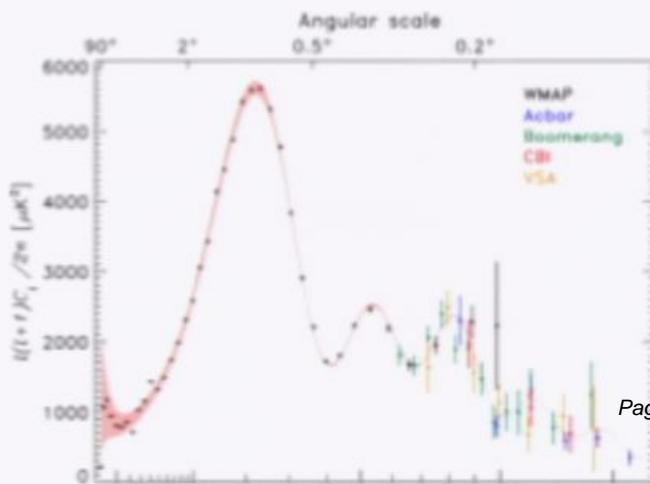
STANDARD MODEL OF THE UNIVERSE

The Big Bang



Causal contact?

- Horizon Problem



INFLATION

$$10^{-34} s \text{ to } 10^{-32} s$$

$$\ddot{a} > 0 \quad \rho + 3P < 0$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho_v}{3} \quad a_f = a_i e^{H(t_f - t_i)}$$

$$N = \ln\left(\frac{a_f}{a_i}\right) = \ln(10^{28}) \approx 60e - \text{folds}$$

- For the inflation we need to introduce a scalar field

$$\phi$$

$$L = \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \quad S = \int d^4x \sqrt{-g} L$$





$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$



$$P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

- The universo was dominated by the scalar field

- Klein-Gordon equation

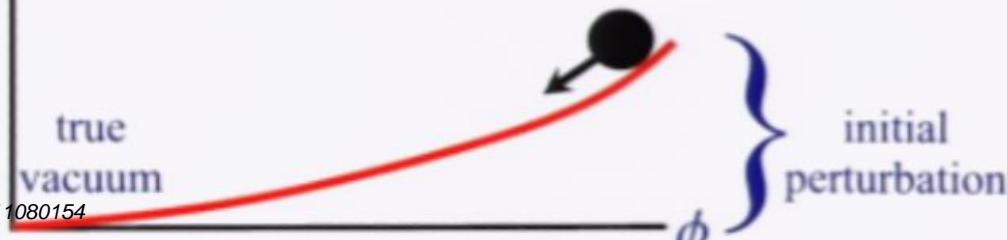


$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

The equation of motion of the homogeneous scalar field

$V(\phi)$

- SLOW ROLL describe the evolution of the scalar field in the potential

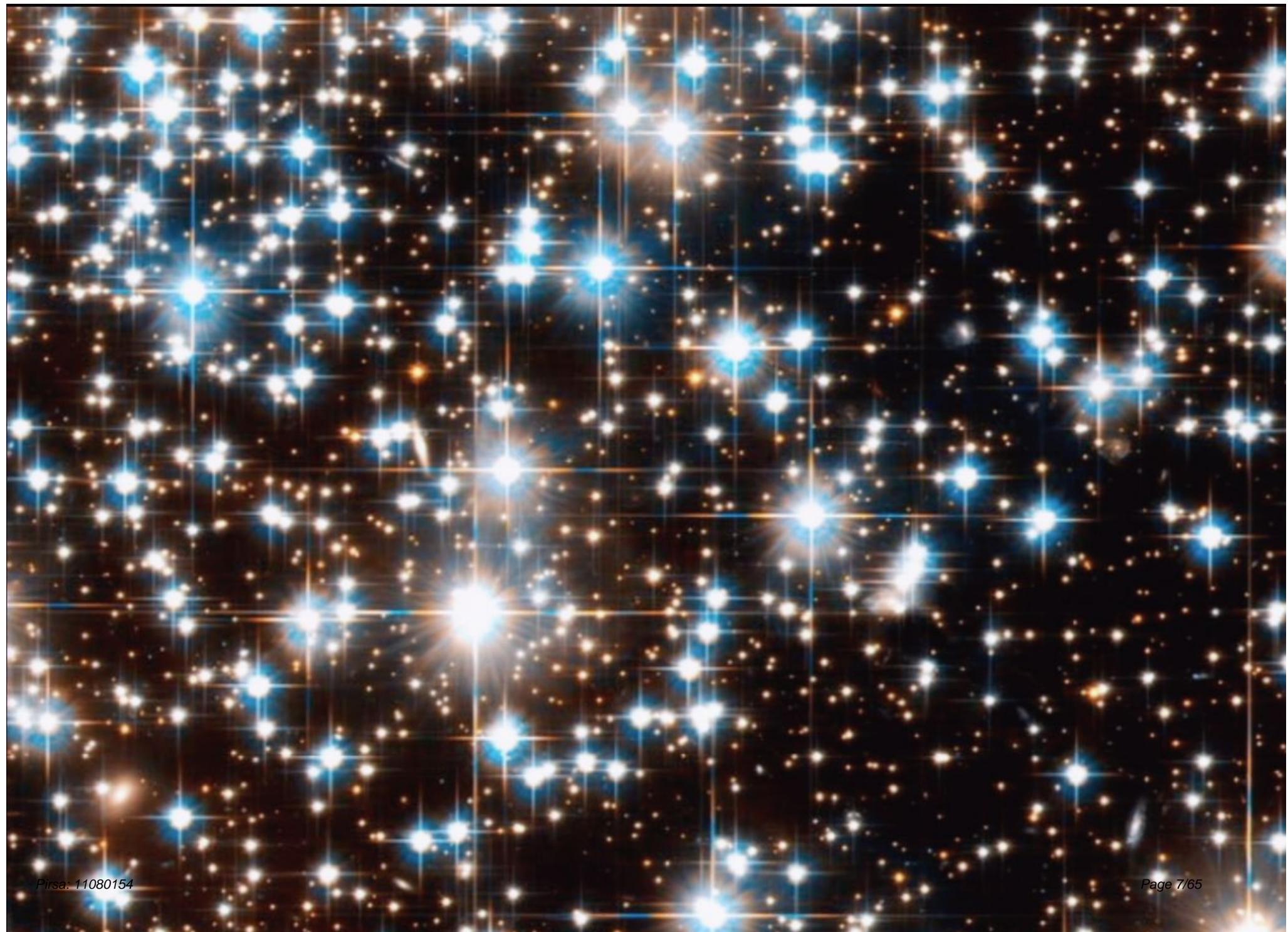


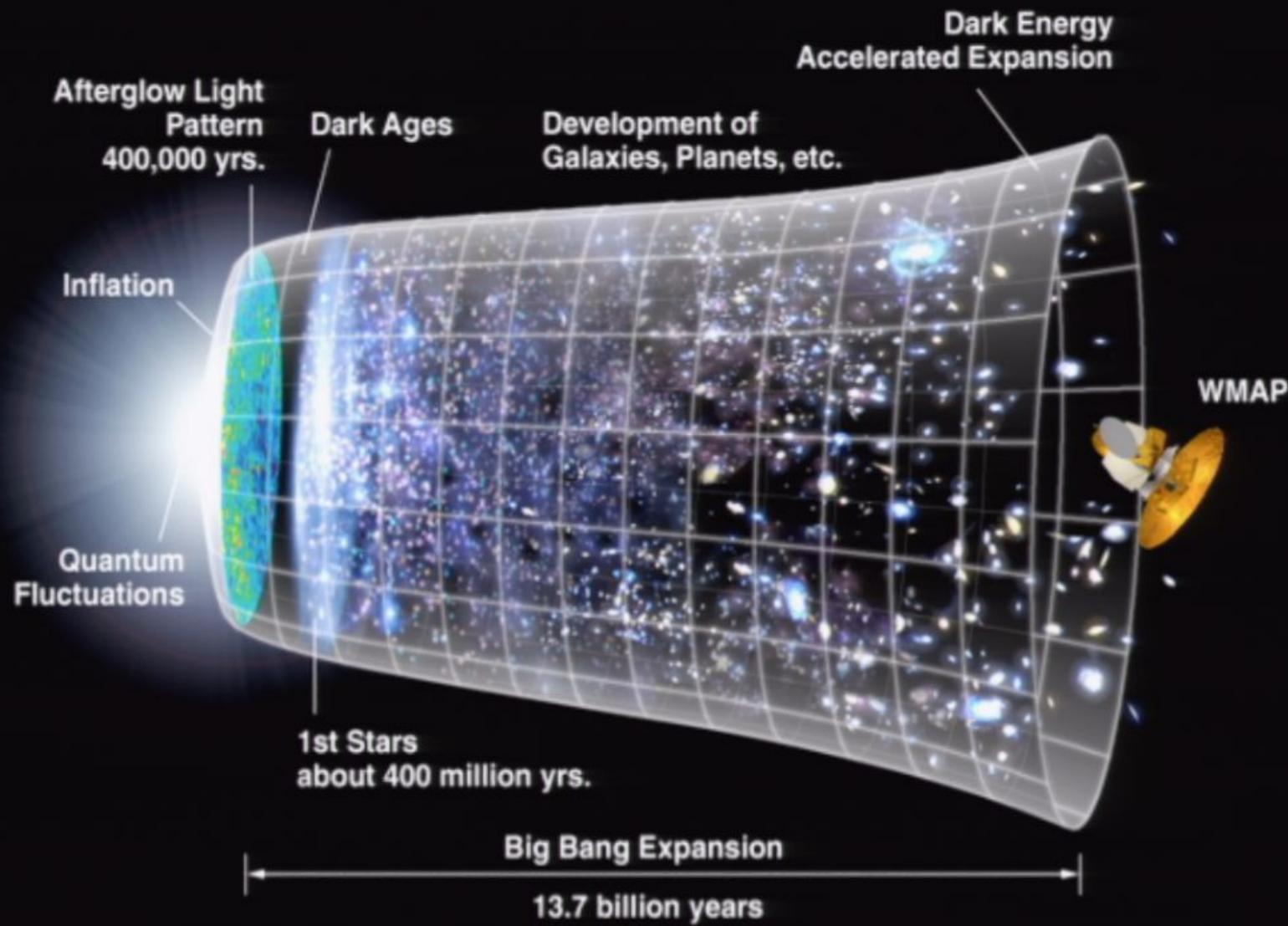
$$\dot{\phi} \ll V$$

$$\ddot{\phi} \ll 3H\dot{\phi}$$

$$\varepsilon = \frac{\dot{H}}{H^2} \ll 1$$

$$\eta = \frac{\dot{\varepsilon}}{H\varepsilon}$$





SCALAR FIELD PERTURBATIONS

$$\phi(\vec{x}, t) = \phi(t) + \delta\phi(\vec{x}, t)$$

- The perturbations are best described in terms of the Fourier modes

$$\delta\phi(x, t) = \int \frac{d^3 k}{(2\pi)^{3/2}} \text{Exp}(ik.x) \delta\phi_k(t)$$

- During the inflation, and before horizon exit, the fluctuations can still be regarded as quantum operators

$$\hat{\delta\phi}_k \equiv \omega_k(t) \hat{a}_k + \omega_k^*(t) \hat{a}_k^\dagger$$

- The commutation relations are

$$[a_k, a_{k'}^\dagger] = \delta^3(k - k')$$

$$[a_k, a_{k'}] = 0$$

- The statistical average is easy to calculate corresponding to the expectation value.

$$\langle \delta\phi_{k_1} \delta\phi_{k_2} \rangle \equiv \frac{(2\pi)^3}{k^3} P_{\delta\phi}(k) \delta^3(k_1 - k_2)$$


- Define the power spectrum for the scalar field perturbation
- To calculate $|\omega_k|^2$

$$P_{\delta\phi}(k) = \frac{k^3}{2\pi^2} |\omega_k|^2$$

$$\phi(\vec{x}, t) = \phi(t) + \delta\phi(\vec{x}, t)$$



$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$(\delta\phi)\ddot{+} 3H(\delta\phi)\dot{+}\left(\frac{k}{a}\right)^2 \delta\phi_k = 0$$



- The solution

$$|\omega_k|^2 \approx \frac{H}{\sqrt{2k^3}}$$

onsiderations

- The gauge-invariant curvature perturbation

$$\zeta = -\Psi - H \frac{\delta\phi}{\dot{\phi}}$$

- For super-horizon models $k < aH$

$$\zeta = -\Psi - H \frac{\delta\phi}{\dot{\phi}} \rightarrow \zeta = -\frac{3}{2}\Psi \rightarrow P_\zeta = \frac{9}{4}P_\Psi$$

- Spectral index

$$n(k) - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} \rightarrow n = 1 - 6\varepsilon + 2\eta$$

- We use this very common definition, for many inflationary models.

Gaussian Distribution

$$\langle \zeta_k \rangle = 0$$

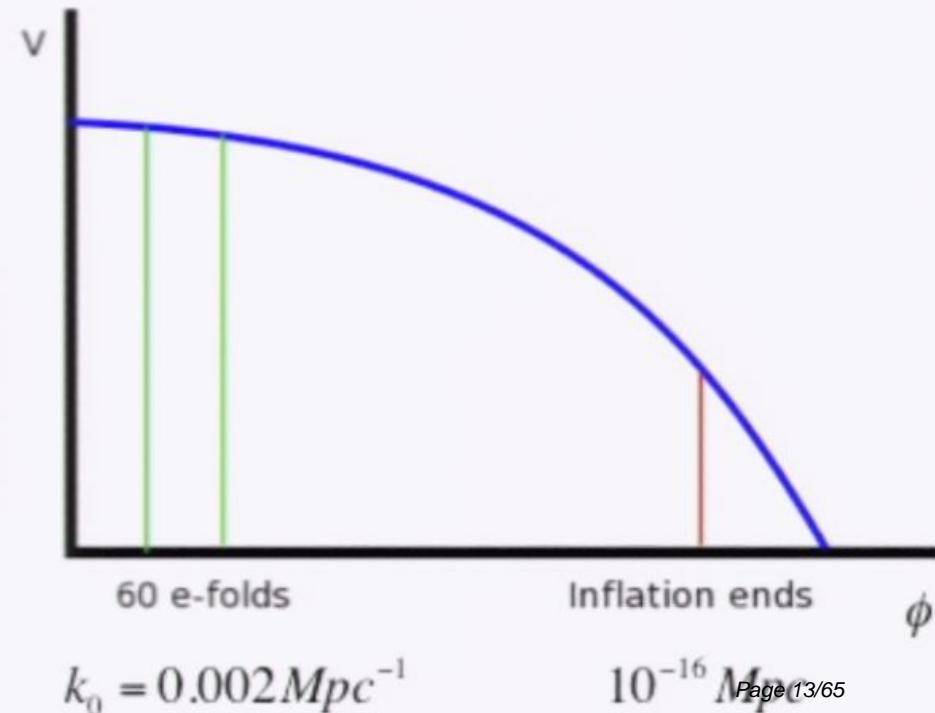
$$\langle \zeta_{k1} \zeta_{k2} \rangle = \sigma_\delta^2(R) = \frac{2\pi^2}{k^3} P_\zeta(k) \delta^3(k_1 + k_2)$$

- The vacuum fluctuations are gaussian
- The Fourier components are uncorrelated
- Delta Dirac enforcing the independence of the different modes

What happens if the fluctuations are Non-Gaussian?

PRIMORDIAL BLACK HOLES (PBHs)

- There are a few phenomenological probes of primordial fluctuations on very small scales.
- With PBHs we can constrain the density perturbation spectrum on extremely short scales.



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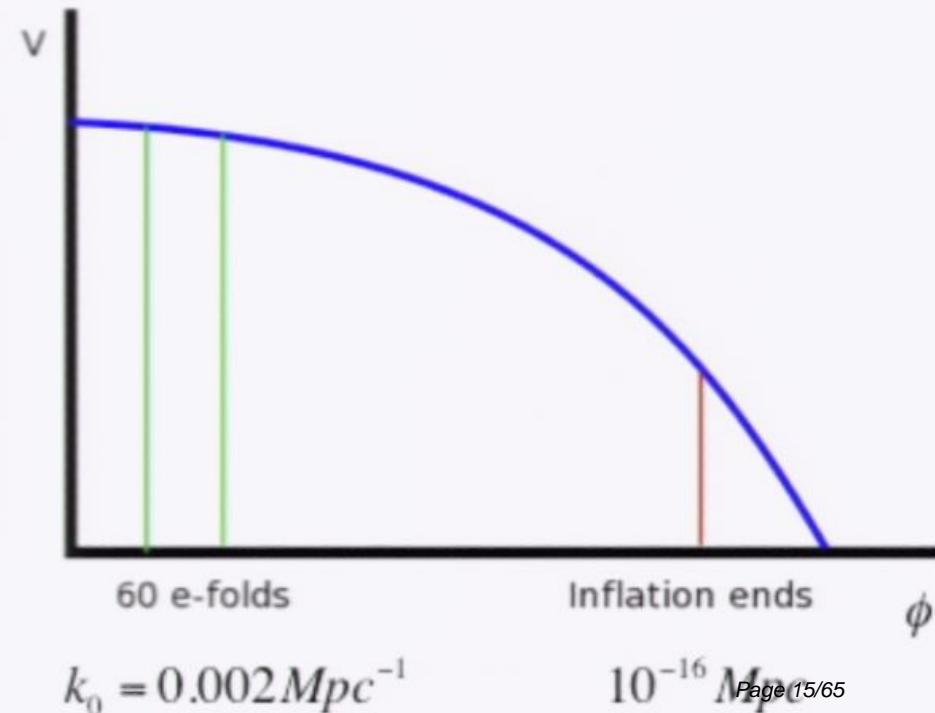
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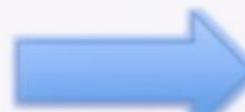
PBH FORMATION

Lifetime of an evaporating Black Hole

$$\frac{\tau}{10^{17} \text{ sec}} \approx \left(\frac{M}{10^{15} \text{ grams}} \right)^3$$

$$M < 10^{15} g$$

$$M > 10^{15} g$$



Evaporated today

$$\Omega_{PBH,0} < 1$$

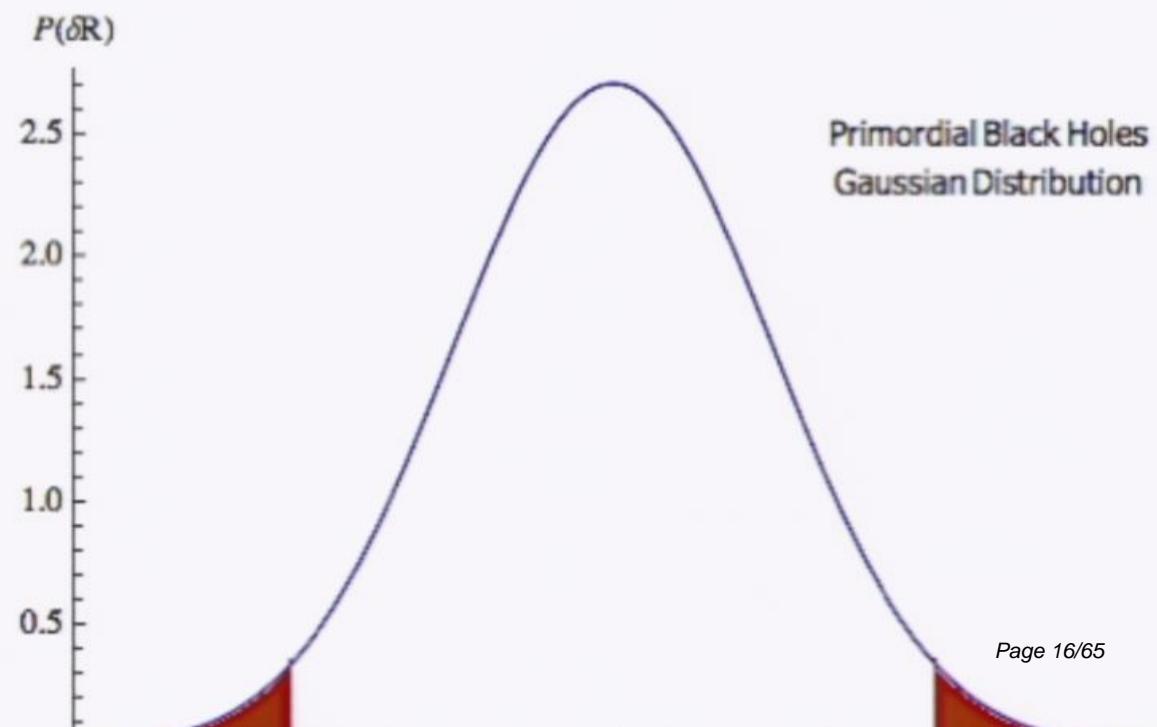
$$\Omega_{PBH,i} < 10^{-20}$$

For PBH formation we require the initial fluctuations to satisfy

$$\delta_i = \frac{(\tilde{\rho}_i - \rho_i)}{\rho_i}$$

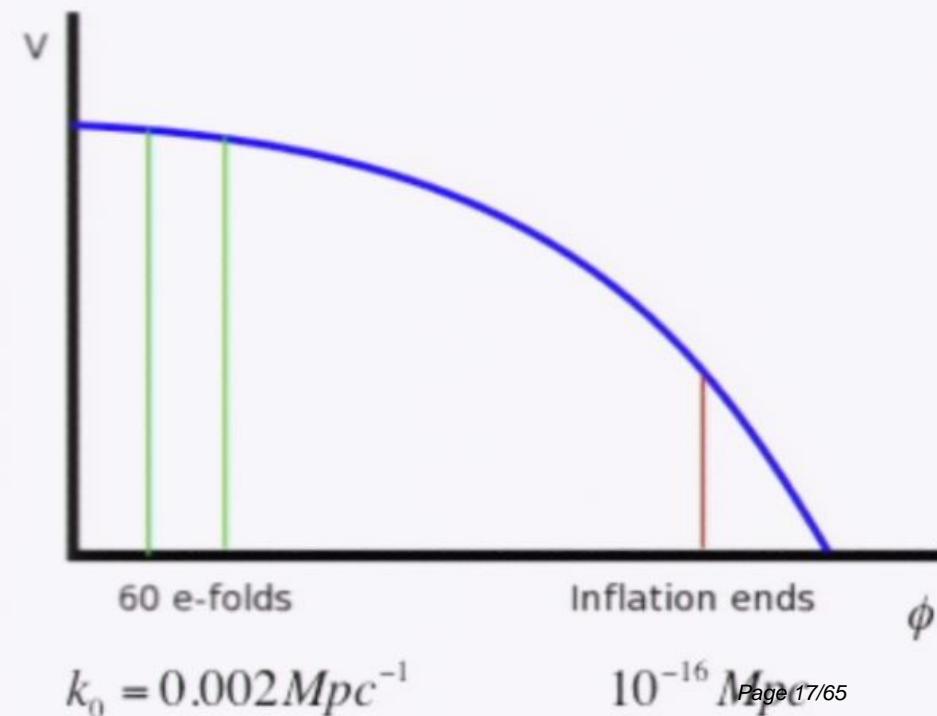
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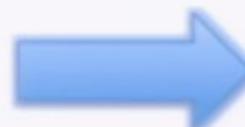


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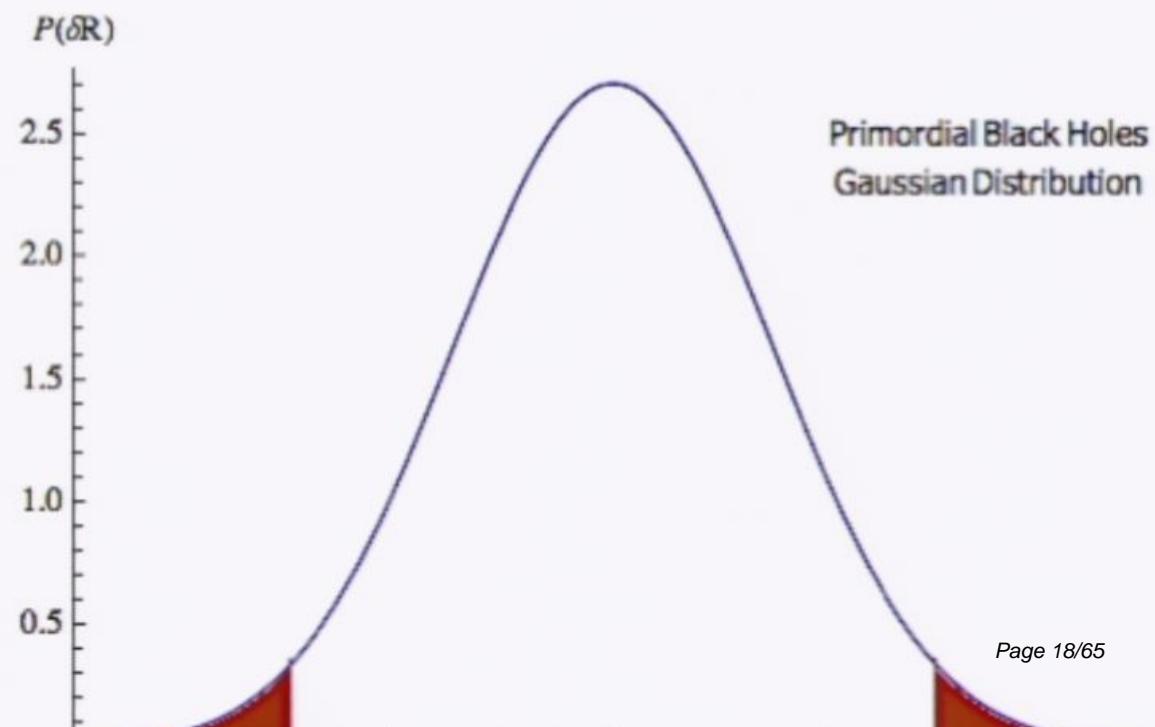
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ESTANDAR CACULATION

- Probability density

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- Dispersion (mass variance)

$$\sigma_\delta(R) = \frac{1}{2\pi^2} \int_0^\infty W_\delta^2(kR) P_\delta(k) \frac{dk}{k}$$

$$W_\delta(kR) = \exp\left(\frac{-k^2 R^2}{2}\right)$$

$$P_\delta(k) = \frac{4(1+w)^2}{(5+3w)^2} P_\zeta(k) \quad \rightarrow \quad w = \frac{1}{3}$$

• In the radiation epoch

- Press-Schechter Theory

$$\Omega_{PBH} = 2 \int_0^\infty p(\delta) d\delta = erfc\left(\frac{1/3}{\sqrt{2}\sigma_\delta(R)}\right)$$

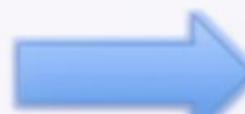
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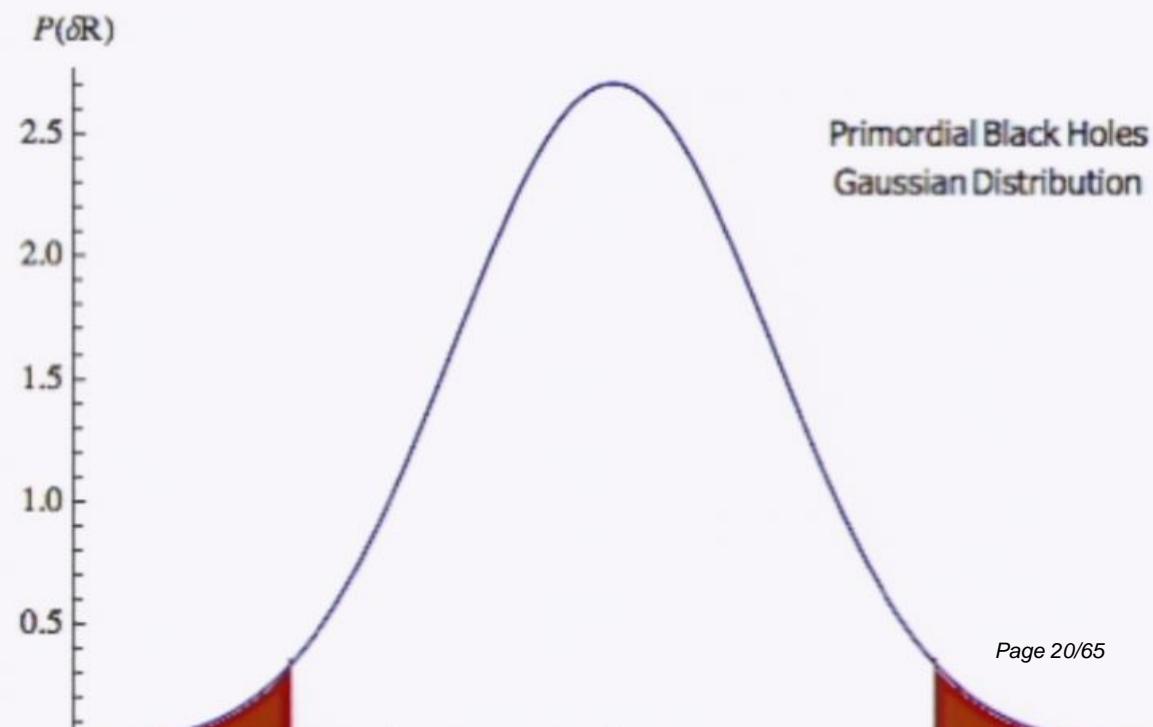
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- PBHs can only form in the early universe at the time when the horizon mass equals the black hole mass.

$$M_H = \frac{4\pi}{3} \rho (H^{-1})^3 \quad \longrightarrow \quad M_H = M_{H,eq} (k_{eq} R)^2$$

- The power spectrum for the primordial curvature perturbation.

$$P_\zeta = A \left(\frac{k}{k_0} \right)^{n_s - 1} \quad \longrightarrow \quad k_0 = 0.05 \text{Mpc}^{-1}$$

$$A = (0.8 \pm 0.1) 2.95 * 10^{-9}$$

WMAP-7 data(2011) $\longrightarrow n_s = 0.968 \pm 0.012$

$n_s > 1$ \longrightarrow

A significant number of PBHs can be produced for a “Blue Spectra”

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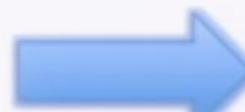
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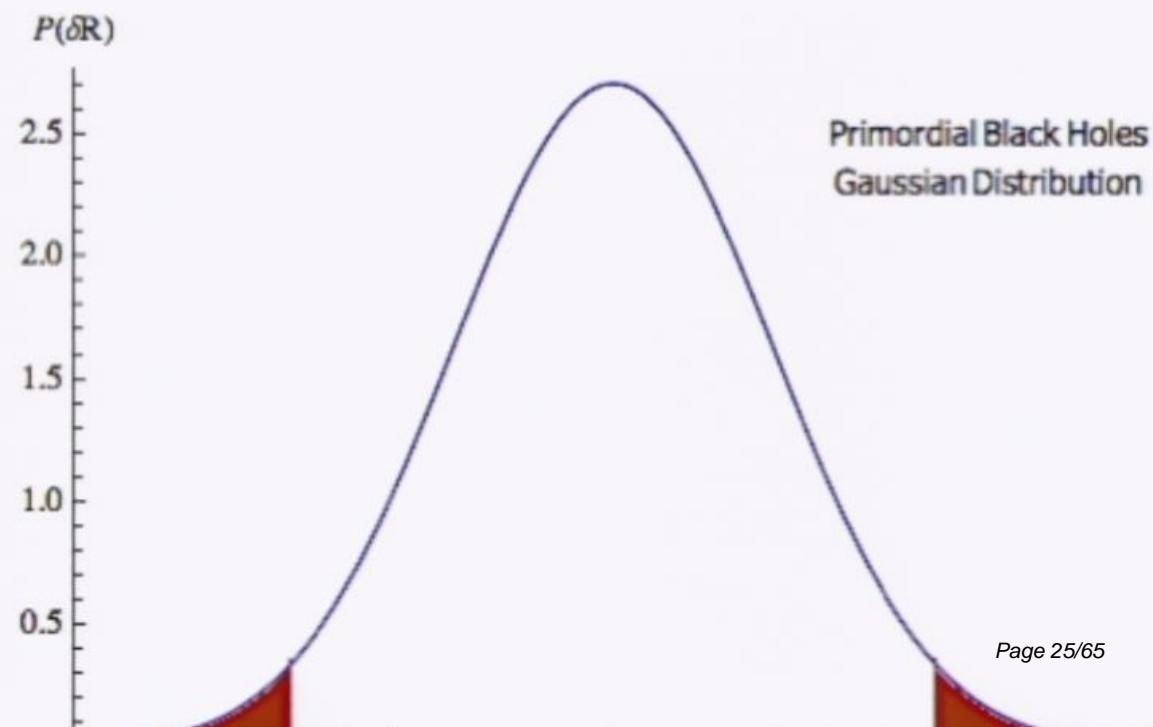
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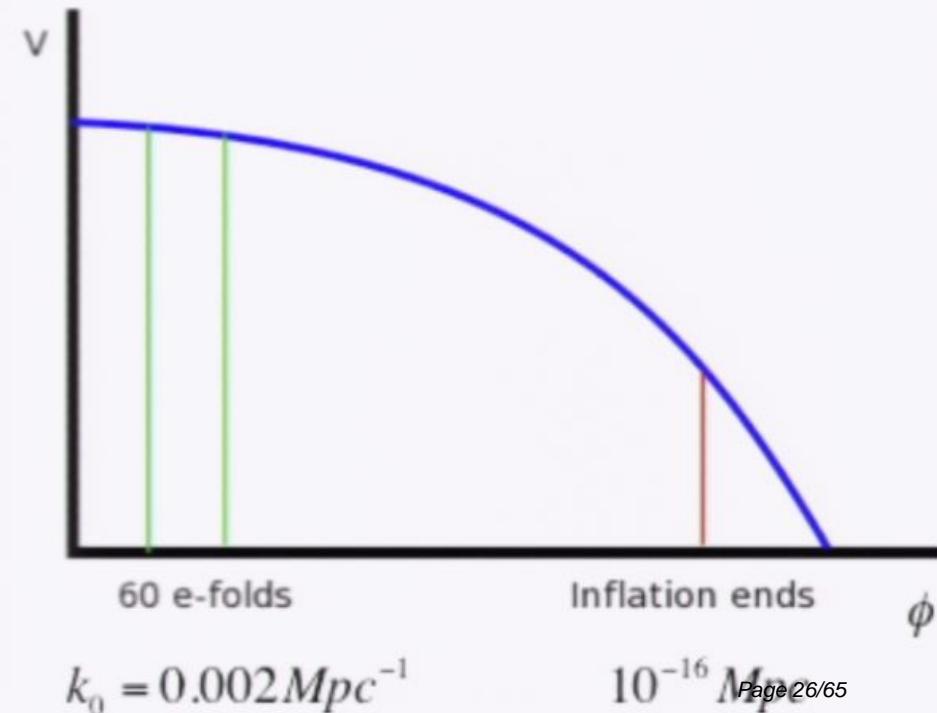
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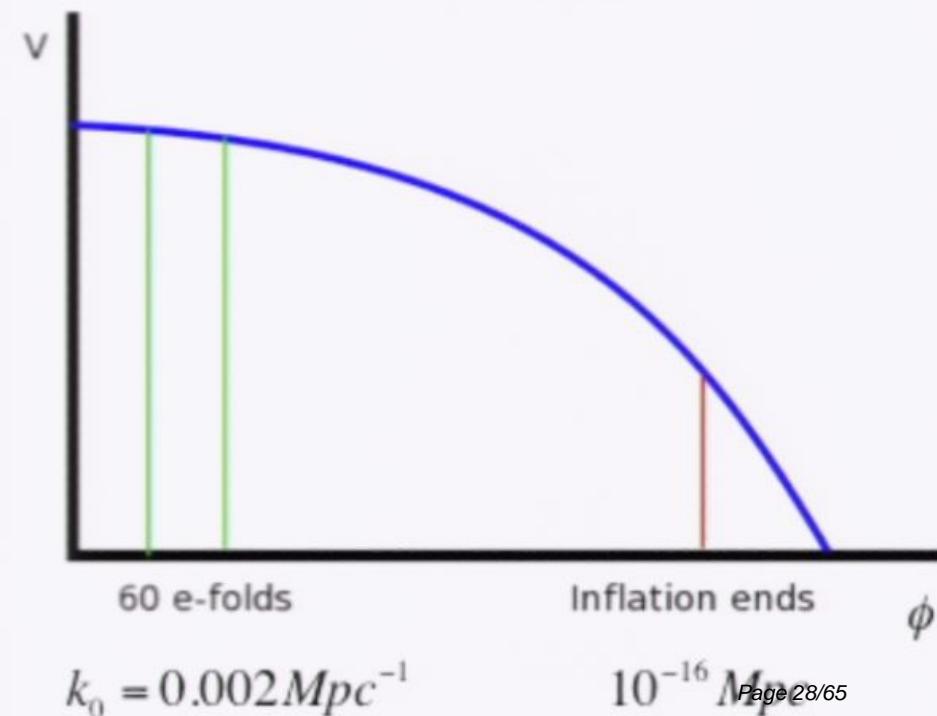
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OUR CALCULATIONS

Primordial Black Holes

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$$M = 10^{15} g$$

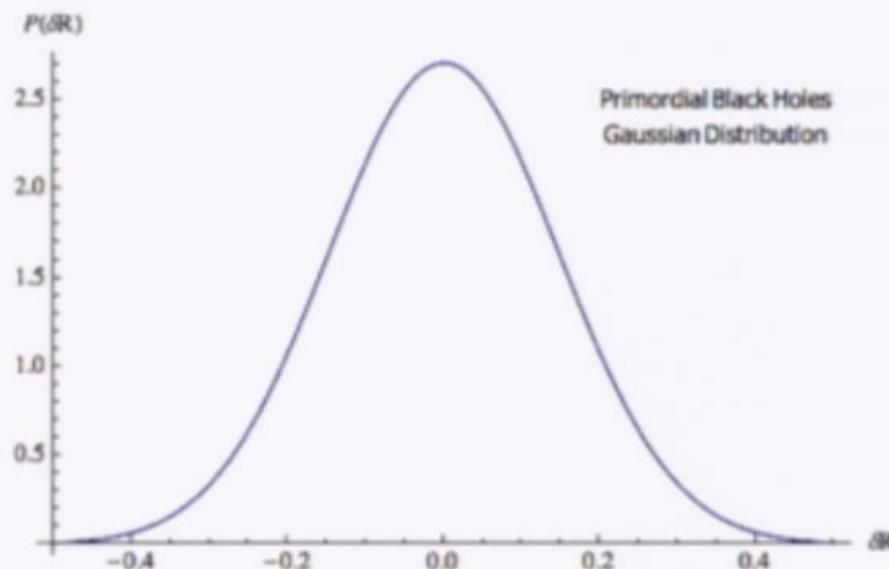
$$R = 2 * 10^{-16} Mpc$$

$$k_0 = 0.002$$

$$A = 2.32 * 10^{-9}$$

$$\sigma_R = 0.147288$$

$$\Omega_{PBH,o} = 0.023626$$



Constraint from CMB

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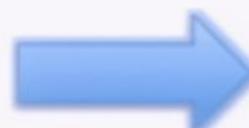
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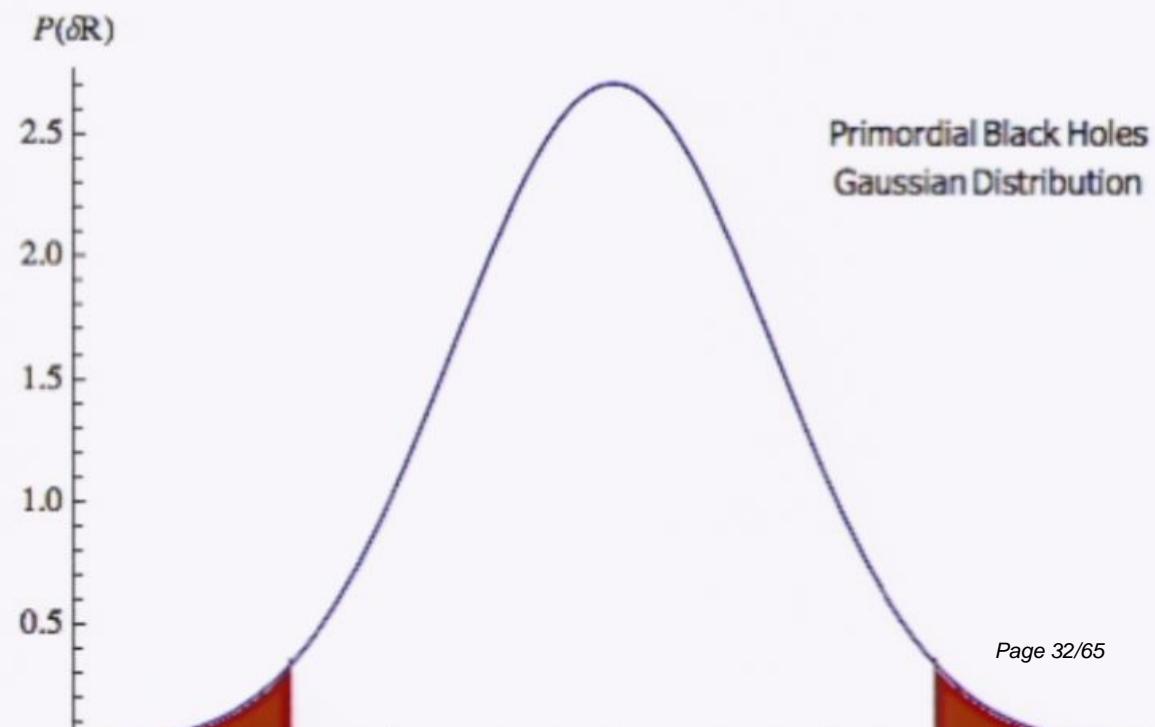
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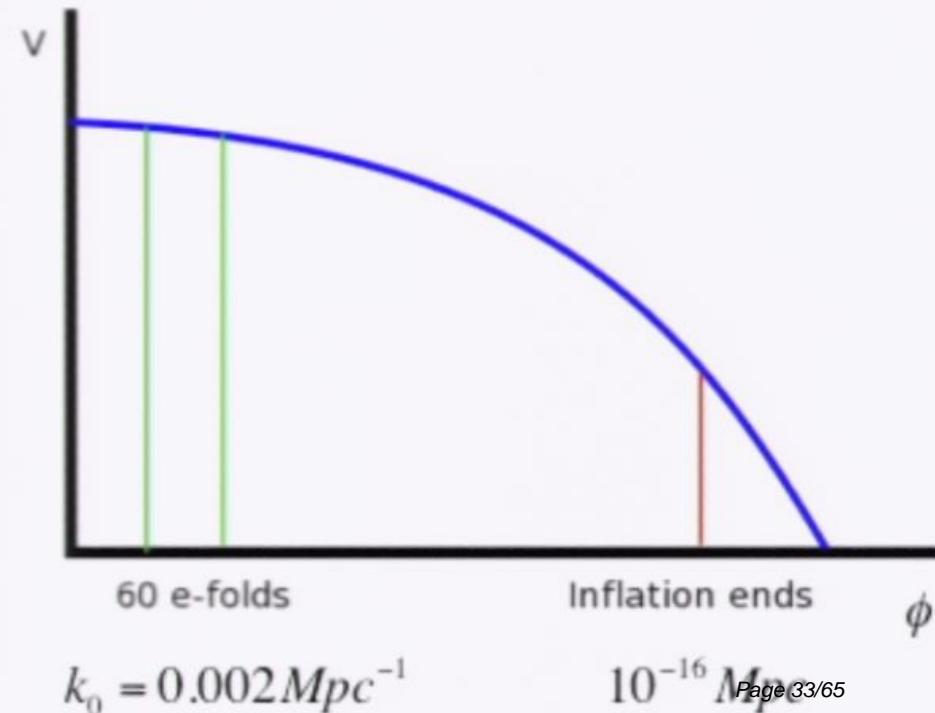
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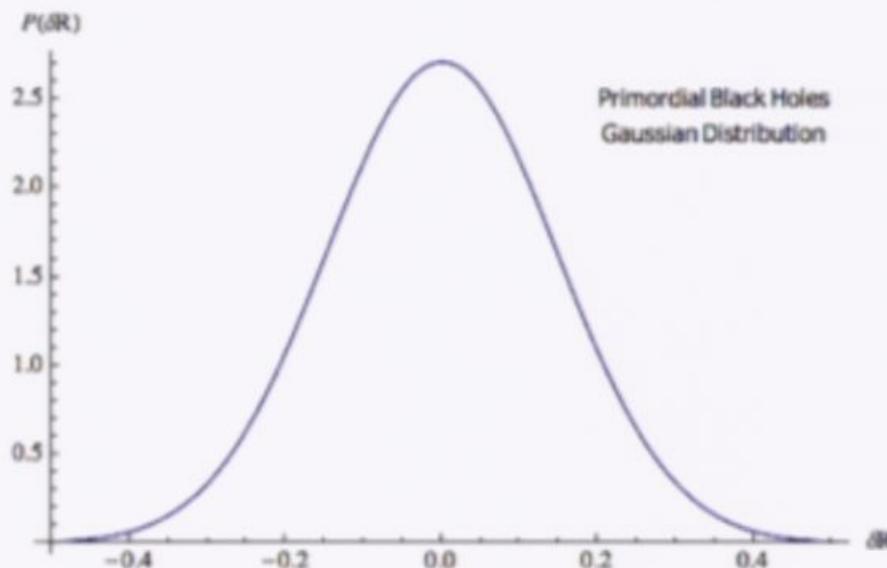
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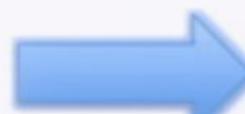
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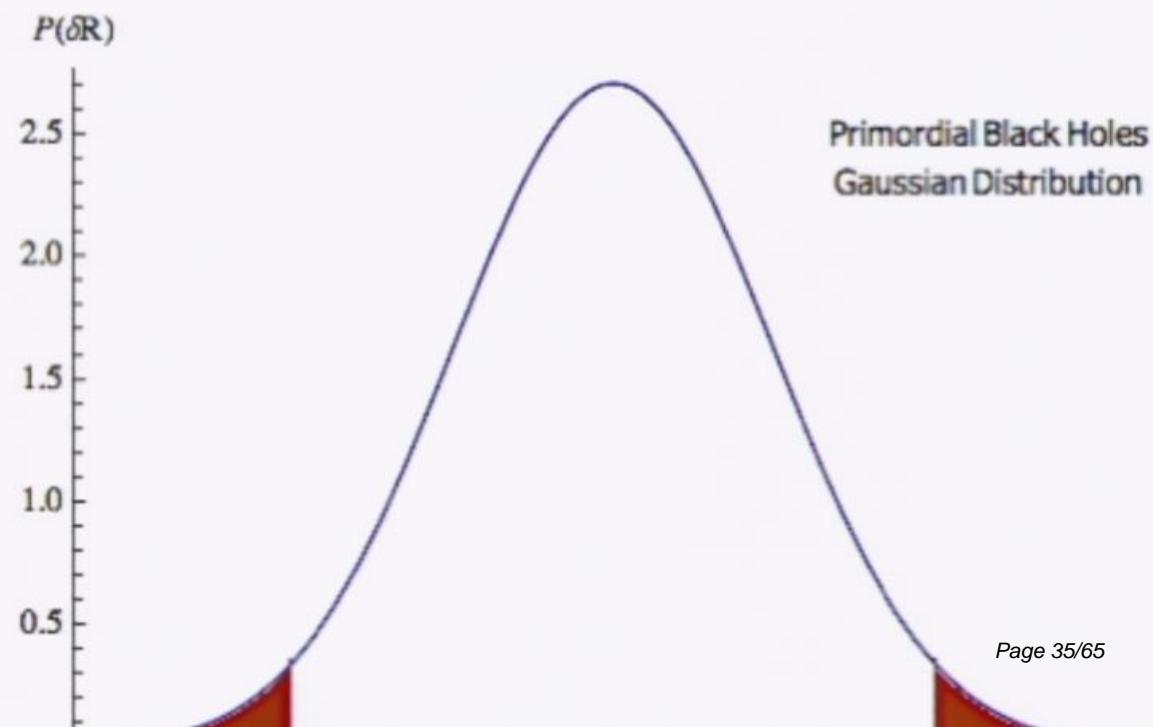
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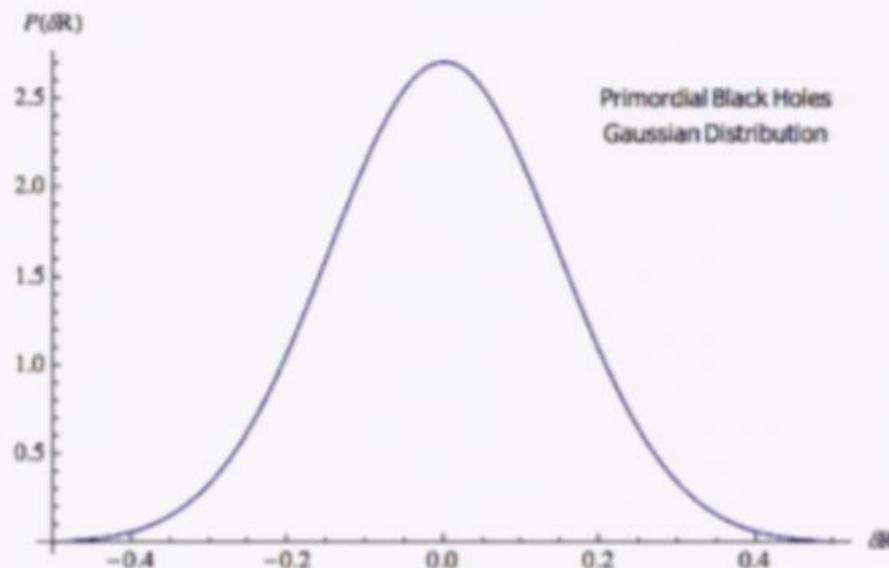
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ADDING NON-GAUSSIAN

Let's build the non gaussian for PBHs, but we need an expression for the PDF

- This technique leads to the well-known Edgeworth Expansion

$$\langle \delta_R^n \rangle \equiv \int_{-\infty}^{\infty} \delta_R^n p(\delta_R) d\delta_R$$

$$k_1 = \langle \delta_R \rangle_c$$

$$k_2 = \langle \delta_R^2 \rangle - \langle \delta_R \rangle_c^2$$

$$k_3 = \langle \delta_R^3 \rangle - 3\langle \delta_R^2 \rangle_c \langle \delta_R \rangle_c - \langle \delta_R \rangle_c^3$$

$$k_4 = \langle \delta_R^4 \rangle - 4\langle \delta_R^3 \rangle_c \langle \delta_R \rangle_c - 3\langle \delta_R^2 \rangle_c^3 - 6\langle \delta_R^2 \rangle_c \langle \delta_R \rangle_c^2 - \langle \delta_R \rangle_c^4$$

...

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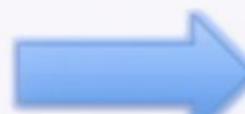
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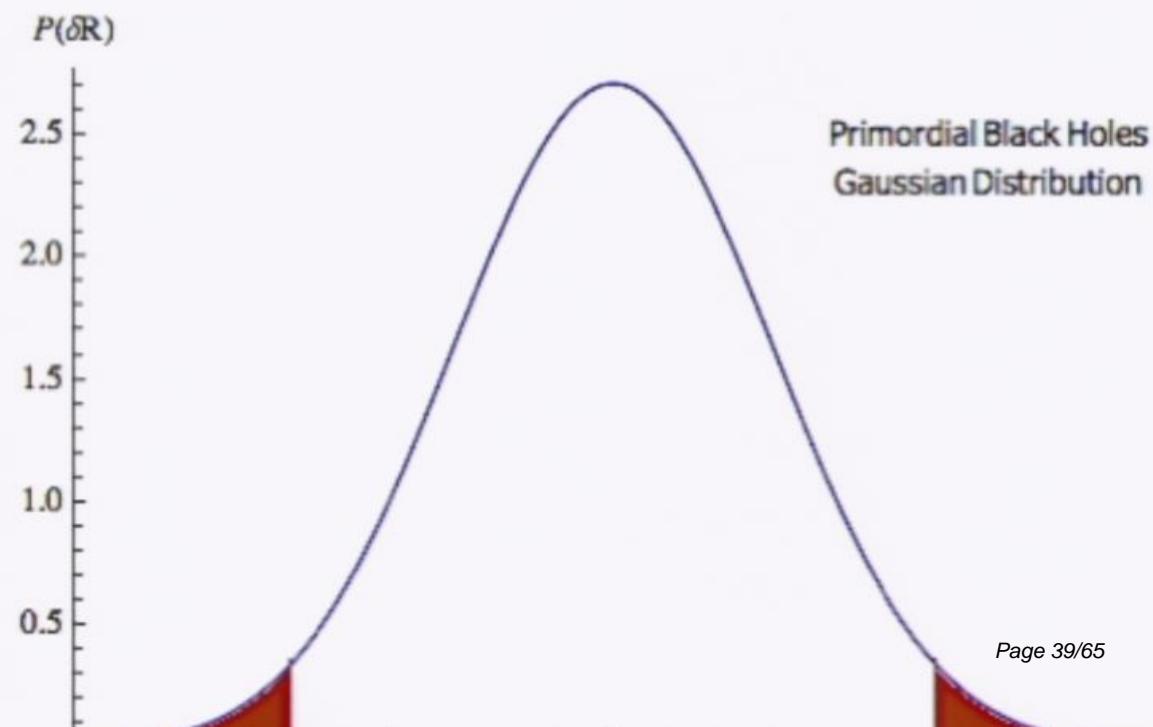
$$\Omega_{PBH,i} < 10^{-20}$$

For PBH formation we require the initial fluctuations to satisfy

$$\delta_i = \frac{(\tilde{\rho}_i - \rho_i)}{\rho_i}$$

$$\frac{1}{3} \leq \delta \leq 1$$

They are formed from fluctuations far out on the tail of the distribution



OUR CALCULATIONS

Primordial Black Holes

$$n_s = 1.4$$

$$M = 10^{15} g$$

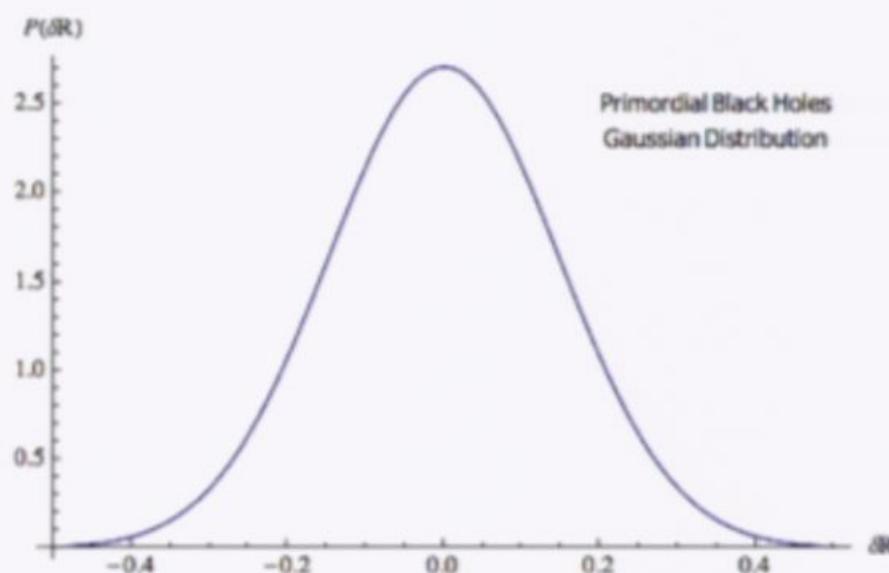
$$R = 2 * 10^{-16} Mpc$$

$$k_0 = 0.002$$

$$A = 2.32 * 10^{-9}$$

$$\sigma_R = 0.147288$$

$$\Omega_{PBH,o} = 0.023626$$



Constraint from CMB

$$n_s = 1$$

$$M = 10^{15} g$$

$$R = 2 * 10^{-16} Mpc$$

$$k_0 = 0.002$$

$$A = 2.32 * 10^{-9}$$

$$\sigma_p = 0.0001384$$



ADDING NON-GAUSSIAN

Let's build the non gaussian for PBHs, but we need an expression for the PDF

- This technique leads to the well-known Edgeworth Expansion

$$\langle \delta_R^n \rangle \equiv \int_{-\infty}^{\infty} \delta_R^n p(\delta_R) d\delta_R$$

$$k_1 = \langle \delta_R \rangle_c$$

$$k_2 = \langle \delta_R^2 \rangle - \langle \delta_R \rangle_c^2$$

$$k_3 = \langle \delta_R^3 \rangle - 3\langle \delta_R^2 \rangle_c \langle \delta_R \rangle_c - \langle \delta_R \rangle_c^3$$

$$k_4 = \langle \delta_R^4 \rangle - 4\langle \delta_R^3 \rangle_c \langle \delta_R \rangle_c - 3\langle \delta_R^2 \rangle_c^3 - 6\langle \delta_R^2 \rangle_c \langle \delta_R \rangle_c^2 - \langle \delta_R \rangle_c^4$$

...

- The reduced cumulants are defined as

$$S_p(R) \equiv \frac{\langle \delta_R^p \rangle_c}{\langle \delta_R^2 \rangle_c^{p-1}}$$

- Edgeworth Expansion

$$p(v)dv = \frac{dv}{\sqrt{2\pi}} e^{\frac{-v^2}{2}} \left[1 + \sigma_\delta \frac{S_3(R)}{6} H_3(v) + \sigma_\delta^2 \left(\frac{S_4(R)}{24} H_4(v) + \frac{S_3(R)^2}{72} H_6(v) \right) + \dots \right]$$

- Hermite polynomials

$$\begin{aligned} H_3(v) &= v^3 - 3v \\ H_4(v) &= v^4 - 6v^2 + 3 \\ H_6(v) &= v^6 - 15v^4 + 45v^2 - 15 \end{aligned}$$

- An illustrative example is generated by the “local ansatz” for a non-gaussian curvature perturbation.

$$\Phi_{NG} = \Phi_G + f_{NL}(\Phi_G^2 - \langle \Phi_G^2 \rangle)$$

$$S_p(R) \equiv \frac{\langle \delta_R^p \rangle_c}{\langle \delta_R^2 \rangle_c^{p-1}} \quad \xrightarrow{\hspace{2cm}} \quad \langle \Phi_{NG}^3 \rangle = \langle \Phi_{NG} \Phi_{NG} \Phi_{NG} \rangle = 8f_{NL}^3 \Phi_G^6 + 6f_{NL} \Phi_G^4$$

$$\quad \xrightarrow{\hspace{2cm}} \quad \langle \Phi_{NG}^4 \rangle = \langle \Phi_{NG} \Phi_{NG} \Phi_{NG} \Phi_{NG} \rangle = 60f_{NL}^4 \Phi_G^8 + 60f_{NL}^2 \Phi_G^6$$

$$S_3(R) \equiv \frac{\langle \Phi_{NG}^3 \rangle_c}{\langle \Phi_{NG}^2 \rangle_c} = 6f_{NL} + 8f_{NL}^3 \sigma^2$$

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- The Edgeworth expansion is a good approximation to the true PDF if

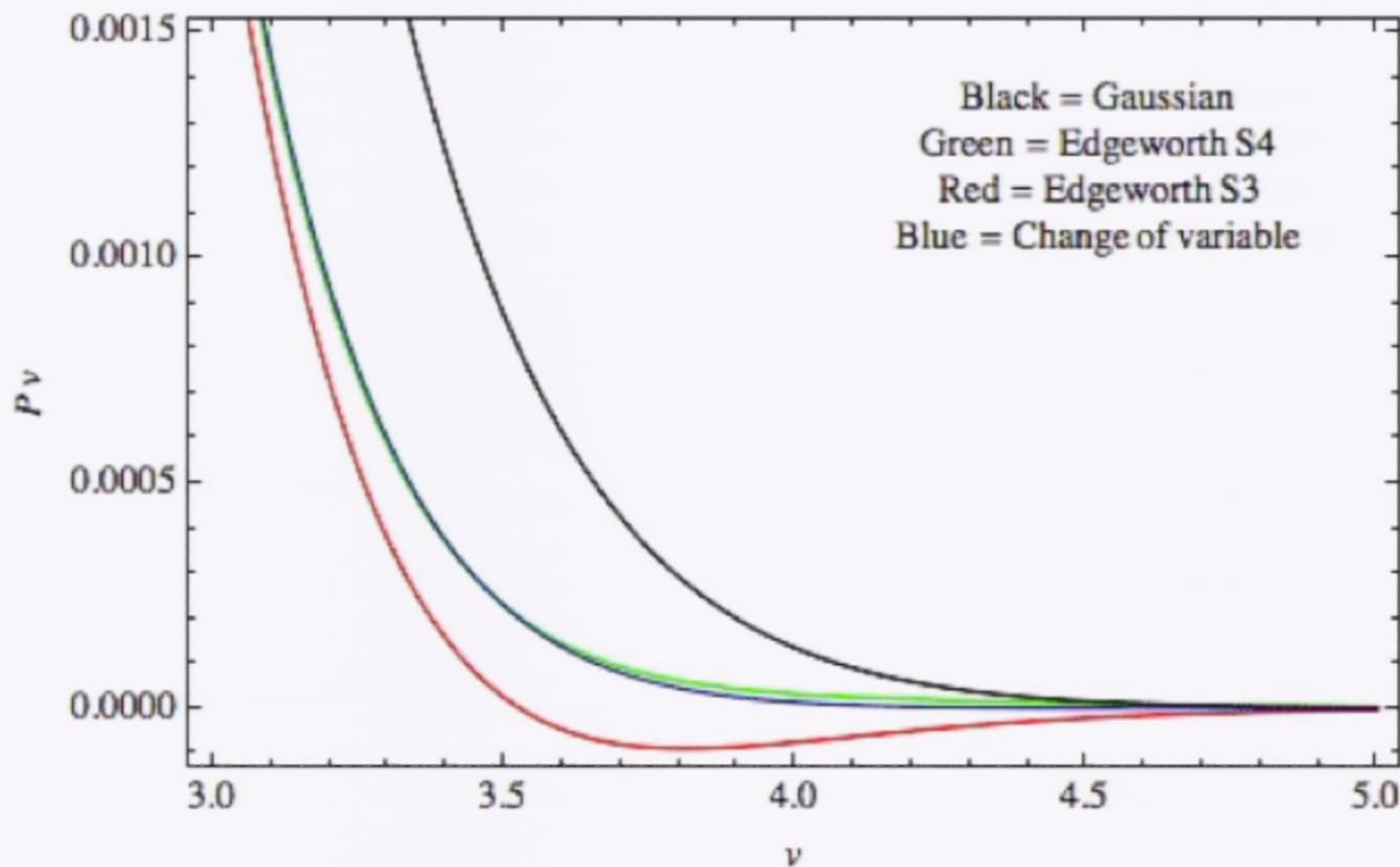
$$1 \gg S_3 \sigma \gg S_4 \sigma^2$$

- We can obtain the PDF by making a formal change of variable in the Gaussian Distribution

$$P(v)dv = \frac{dv}{\sqrt{1 + 4f_{NL}\sigma(v + f_{NL}\sigma)}} \frac{1}{\sqrt{2\pi}} (E_- + E_+)$$

$$E_\pm = \exp \left[\frac{-1}{4\sigma^2 f_{NL}^2} \left(1 + 2f_{NL}\sigma(v + f_{NL}\sigma) \mp \sqrt{1 + 4f_{NL}\sigma(v + f_{NL}\sigma)} \right) \right]$$

COMPARATION



The distributions are plotted for unsmoothed quantities.

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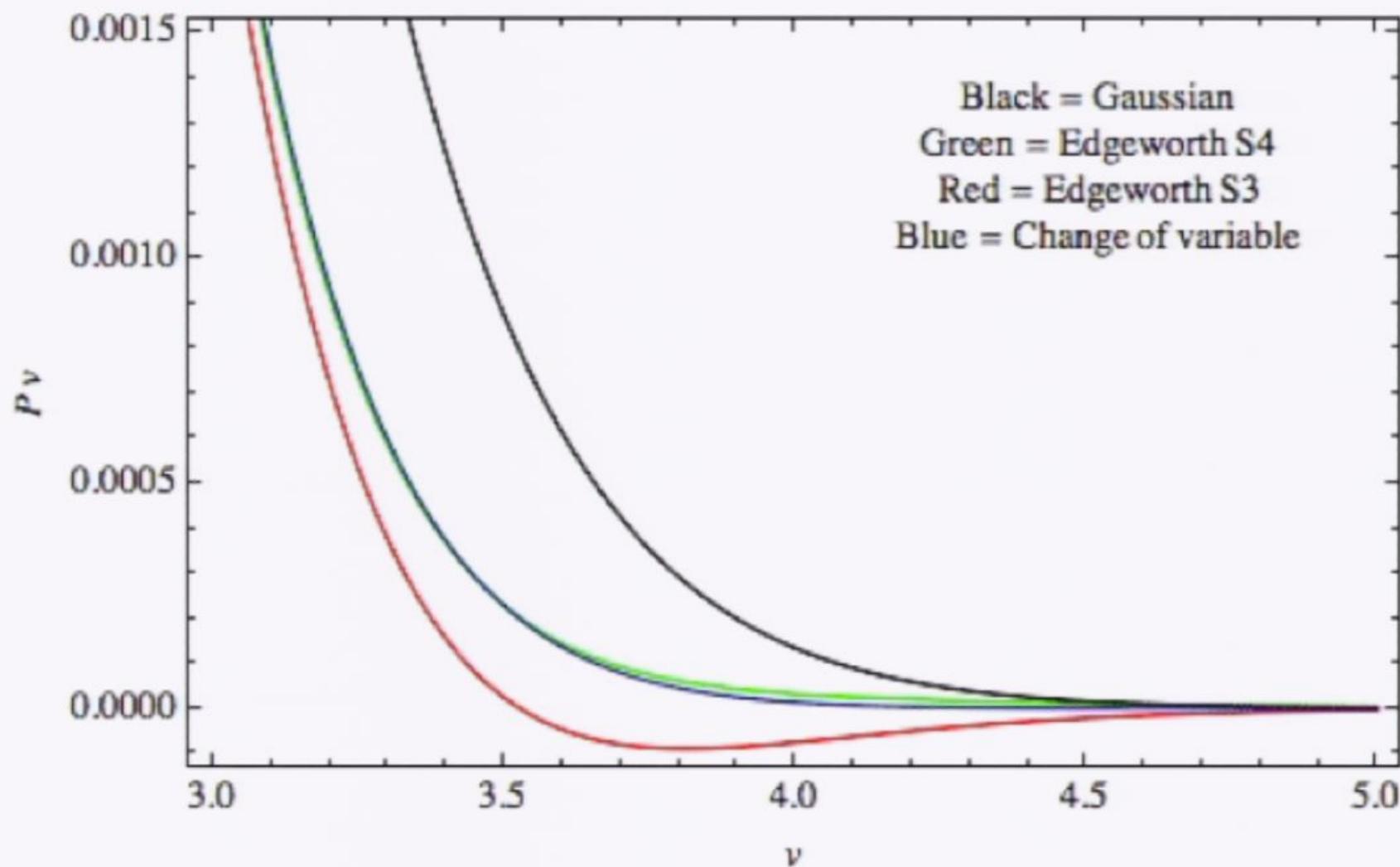
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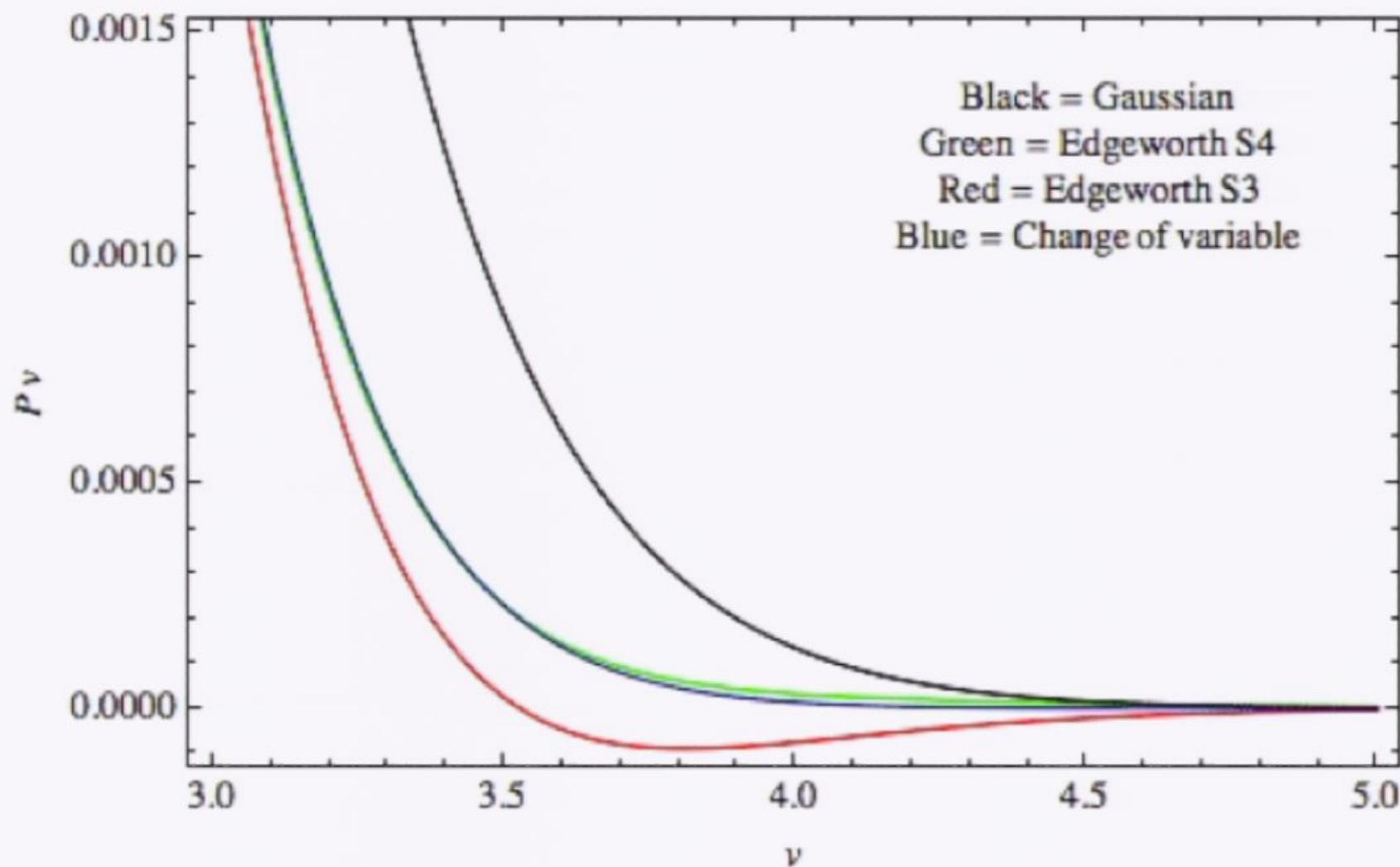
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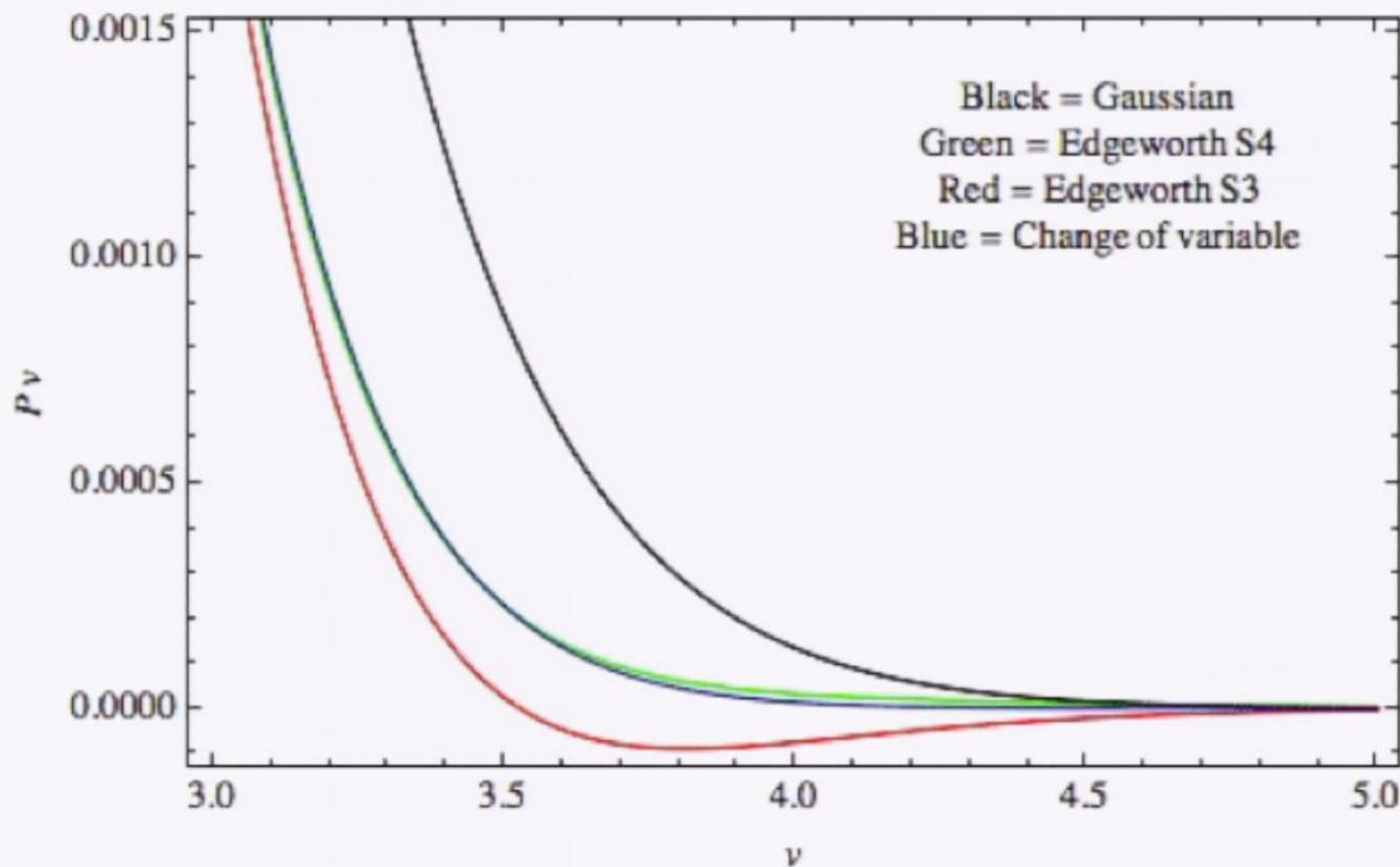
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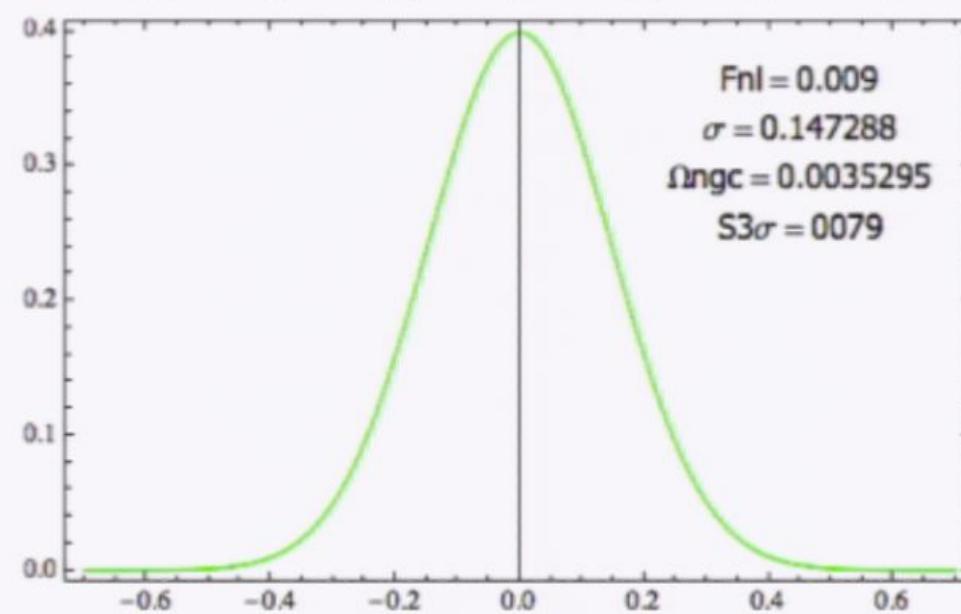
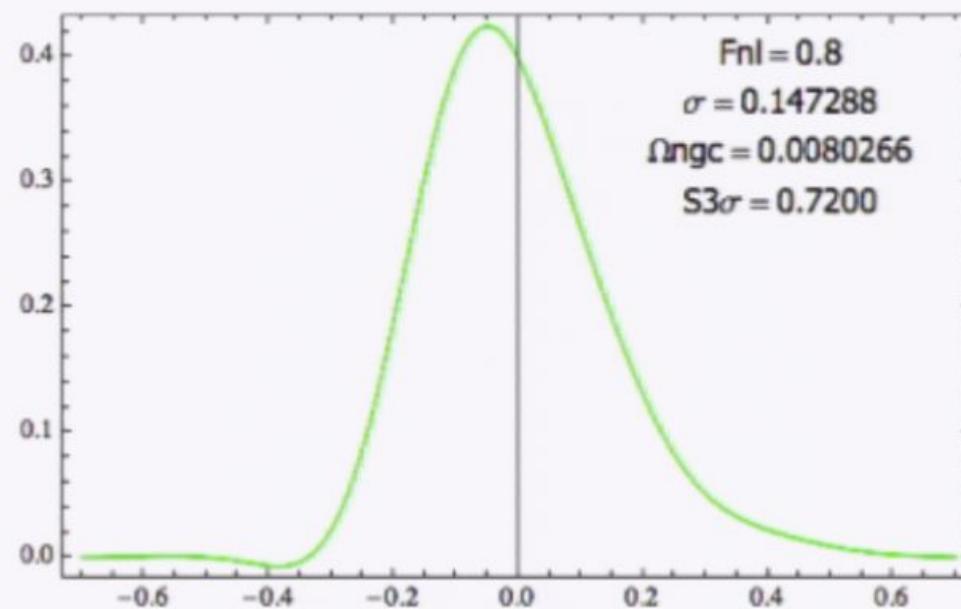
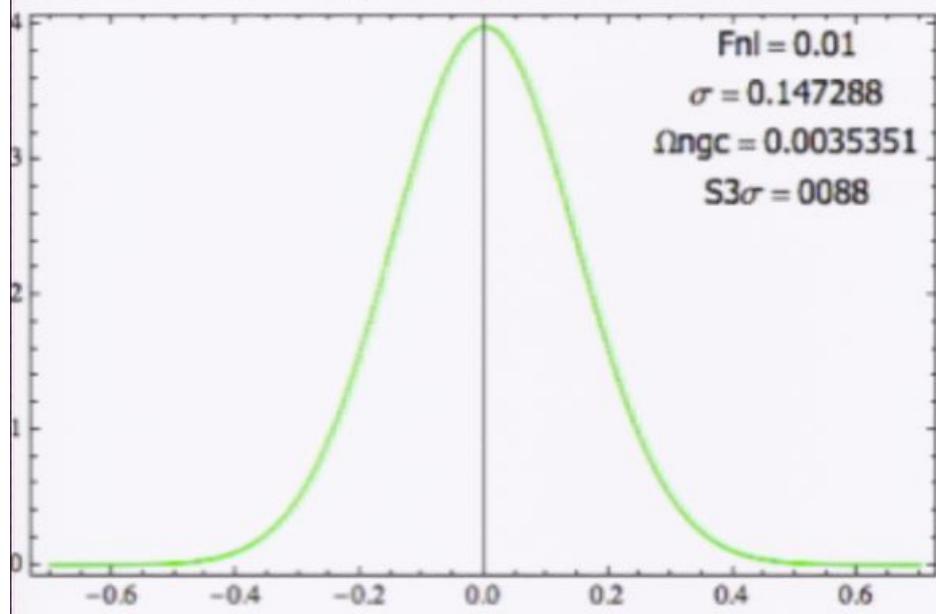
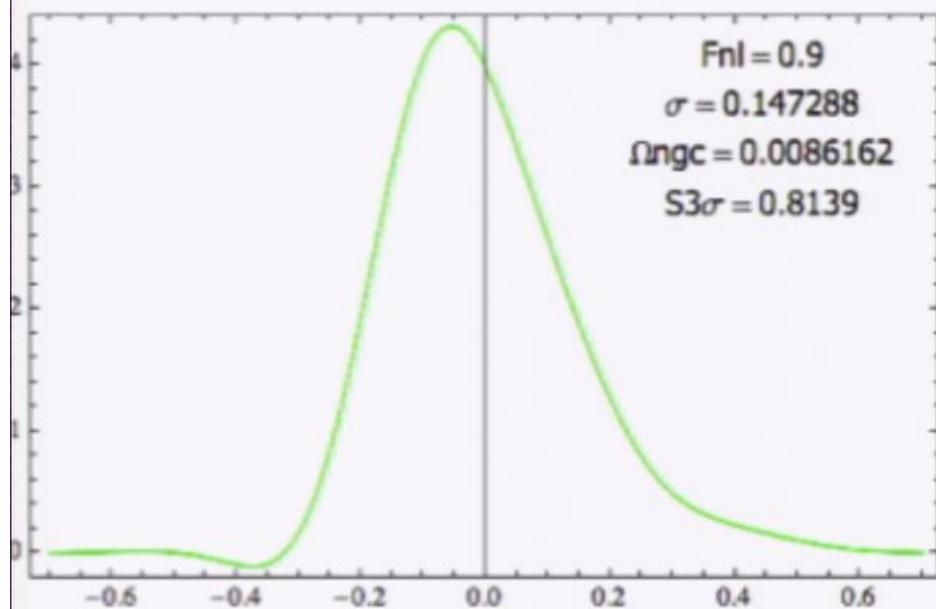
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COMPARATION

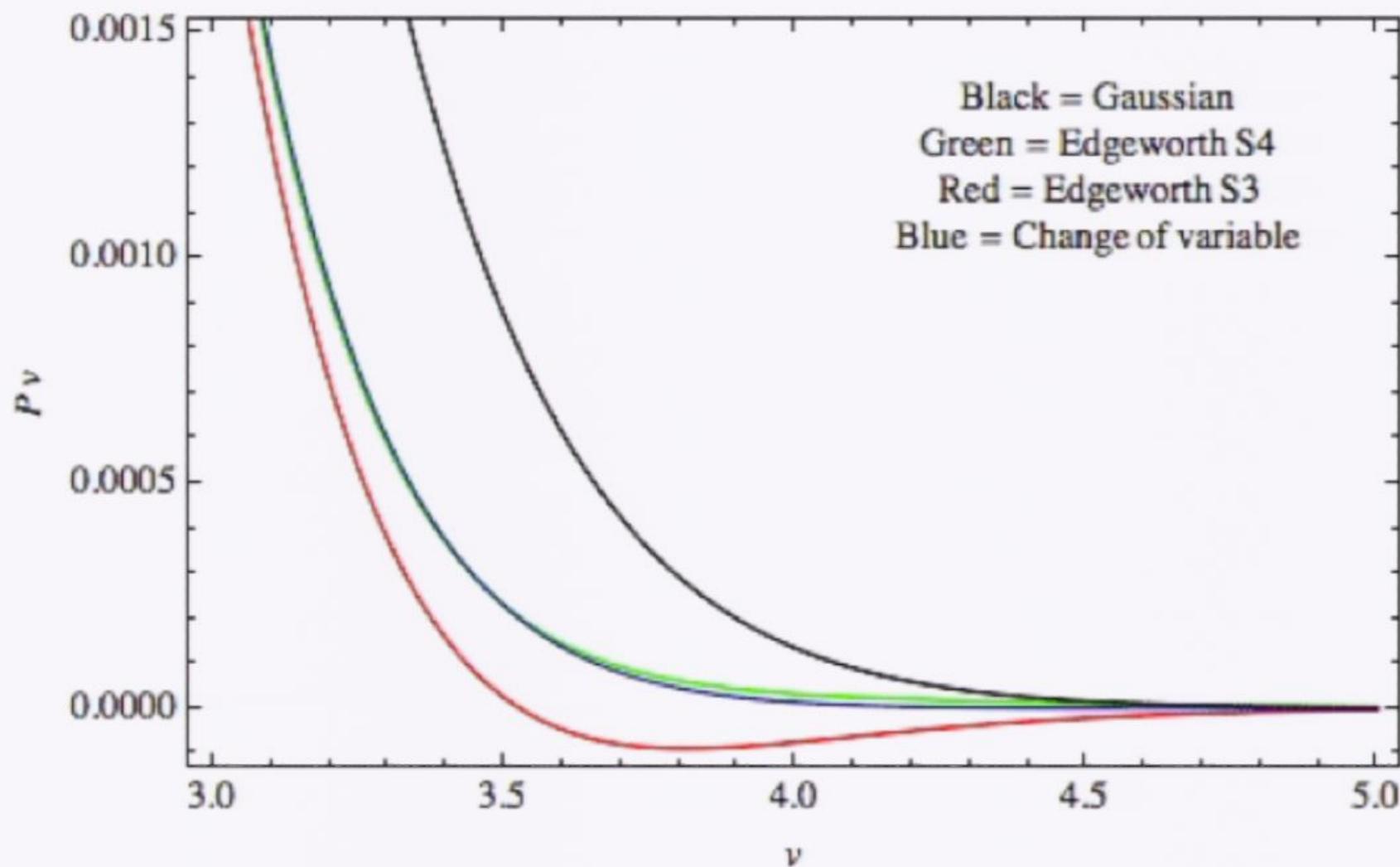


The distributions are plotted for unsmoothed quantities.

f_{NL} FROM EDGEWORTH EXPANSION



COMPARATION



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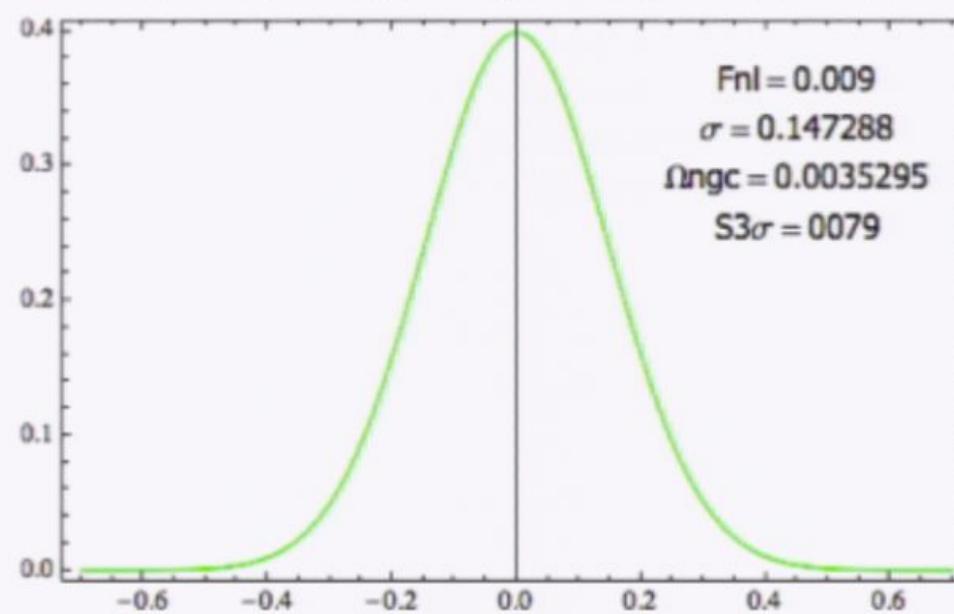
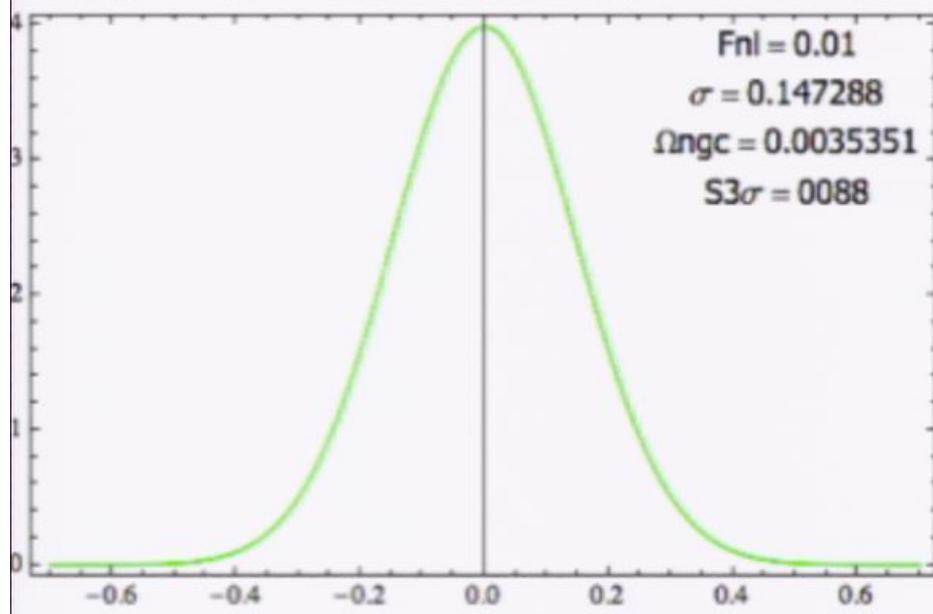
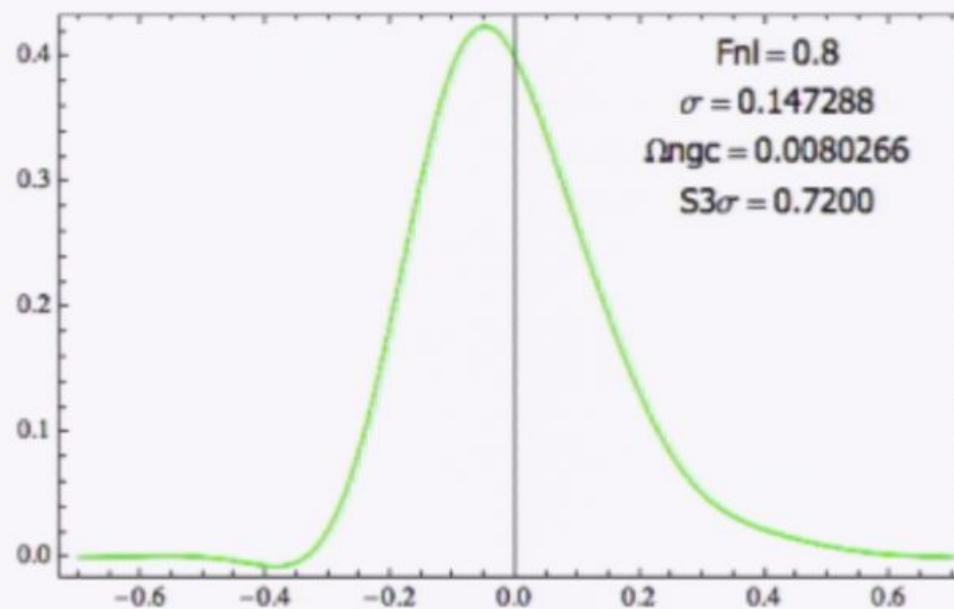
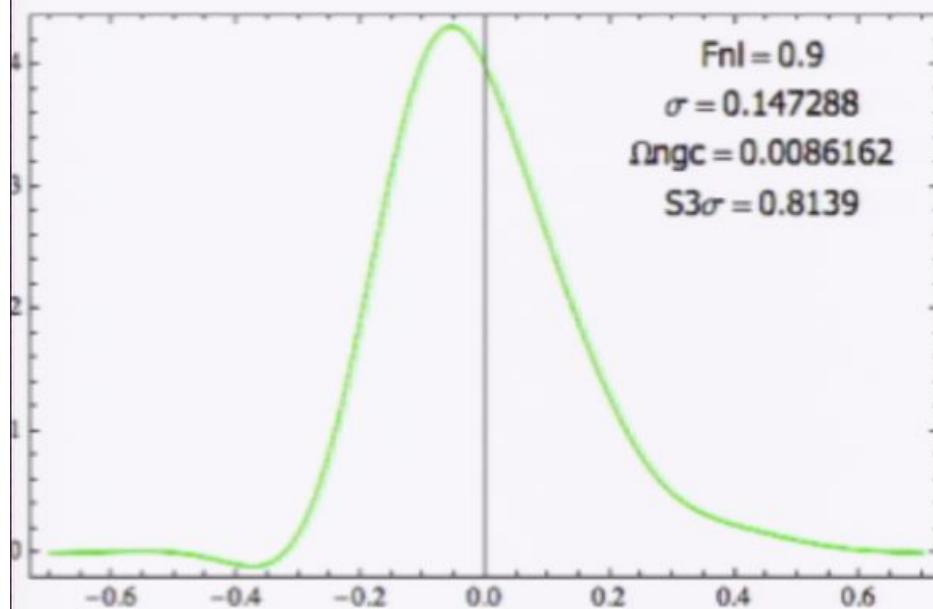
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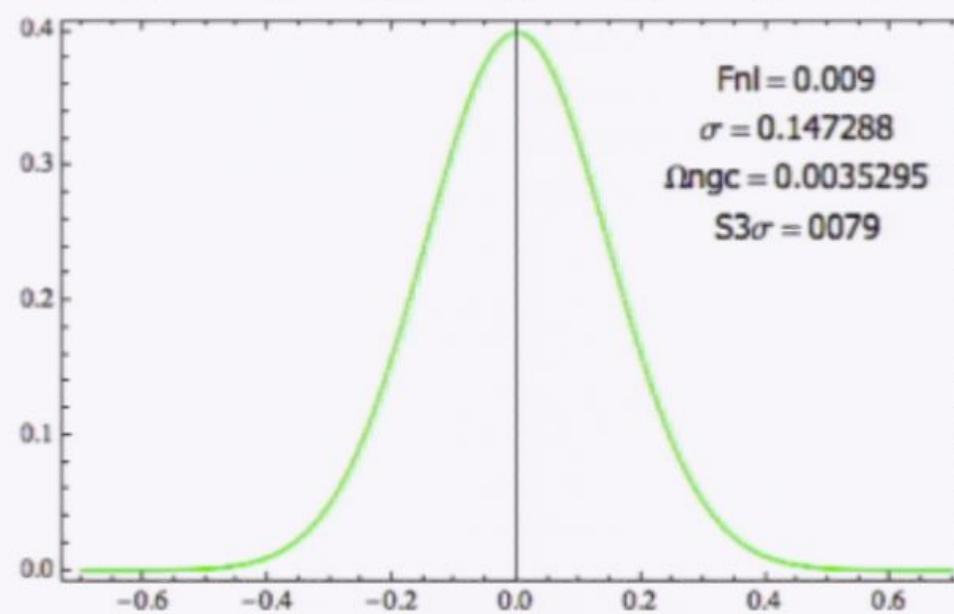
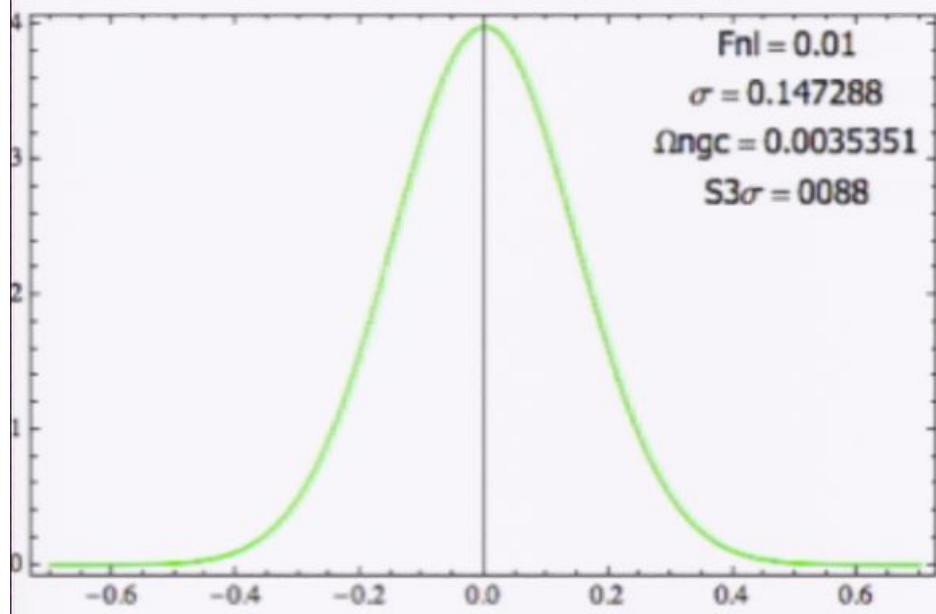
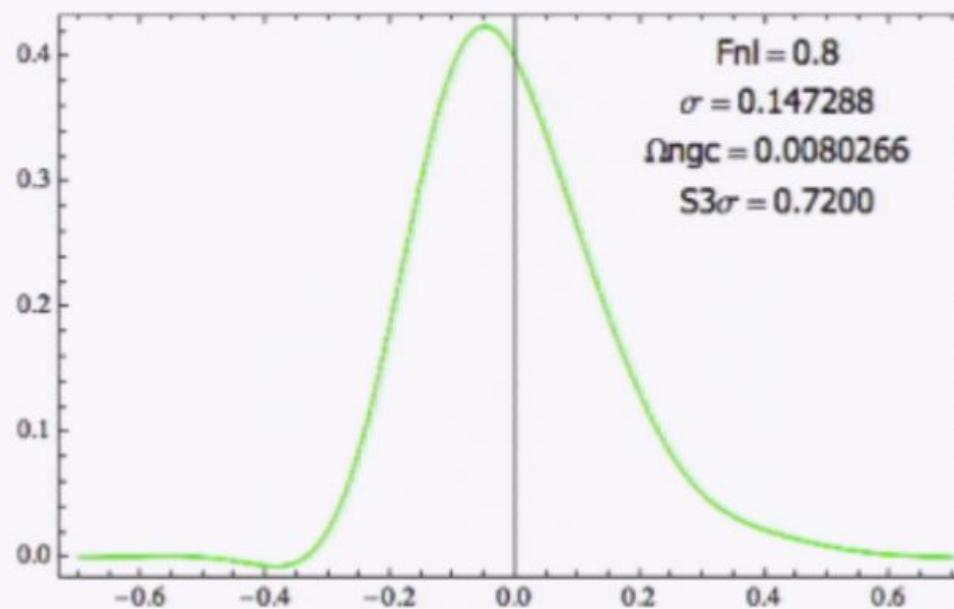
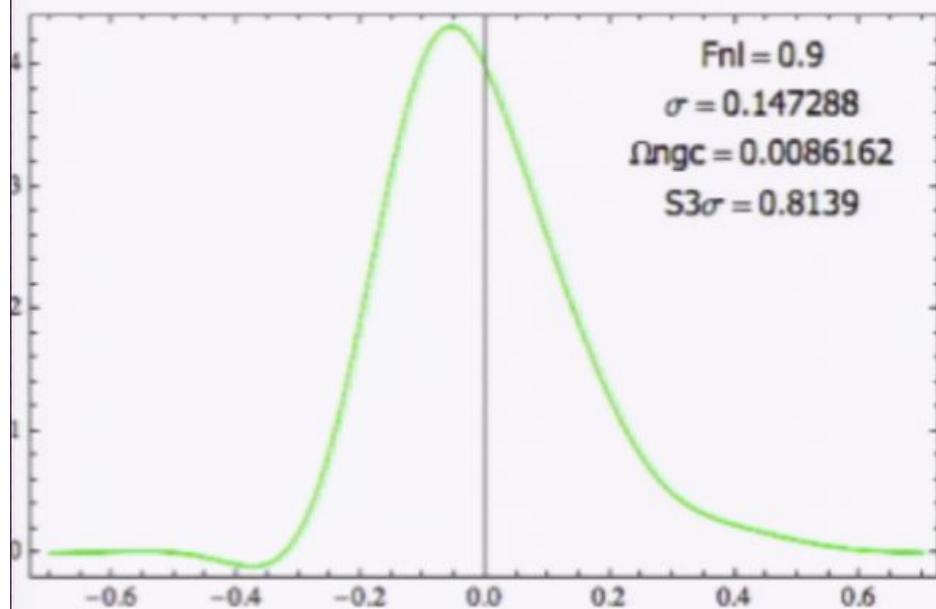
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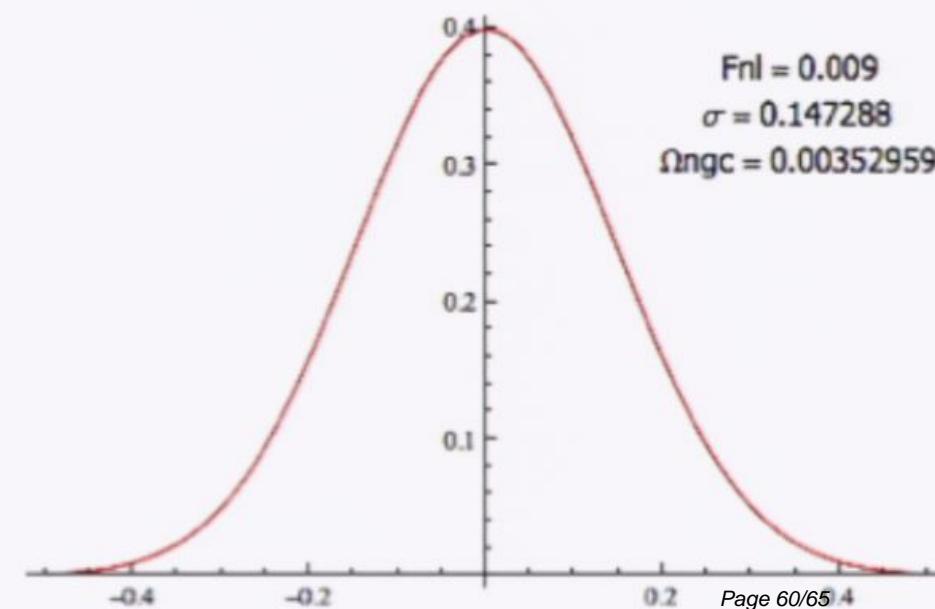
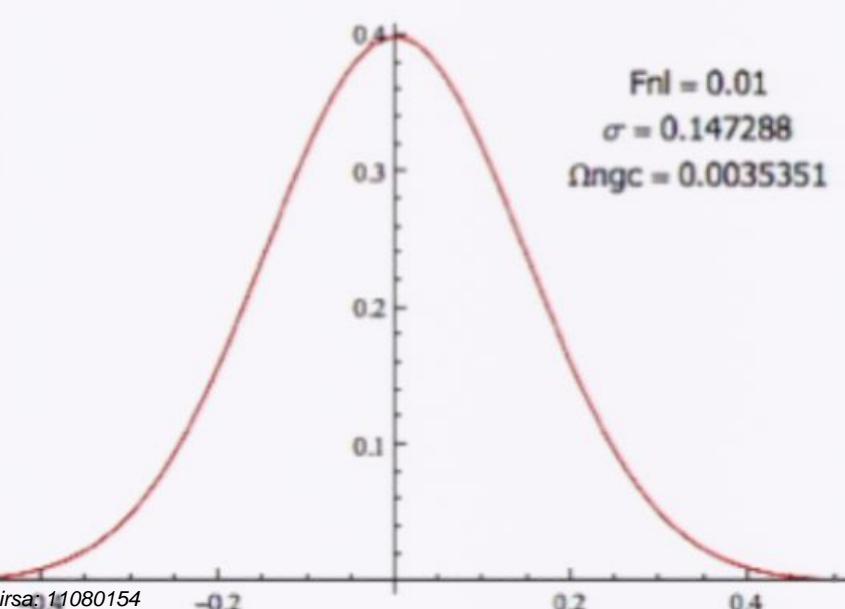
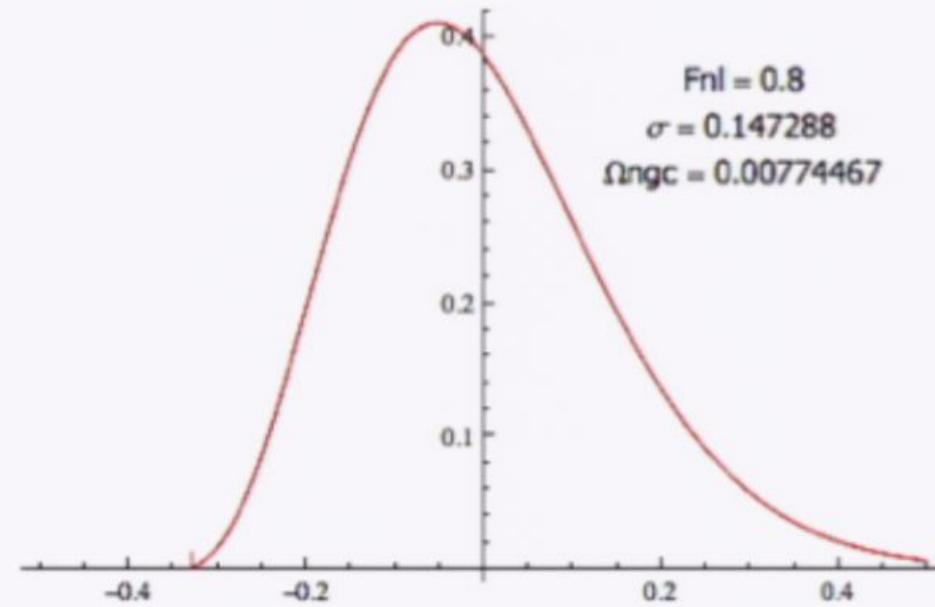
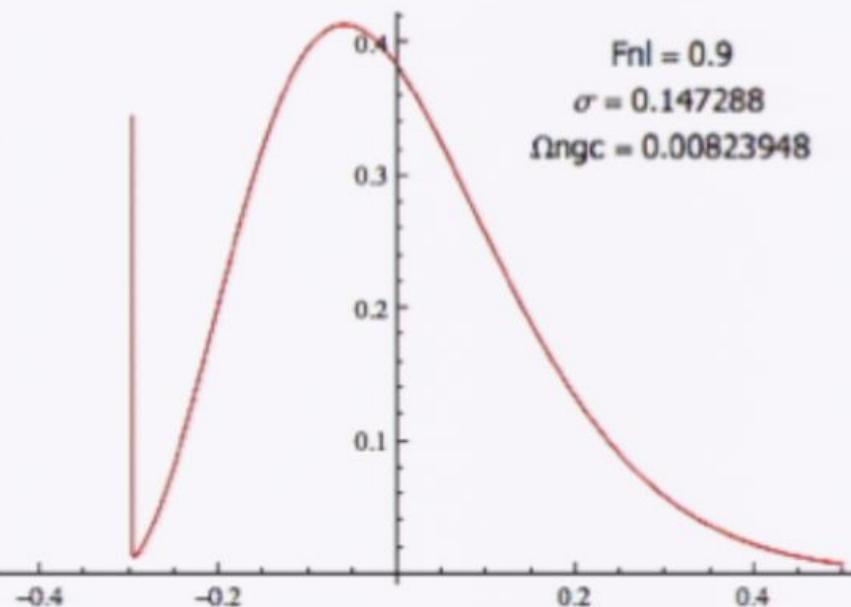
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f_{NL} FROM EDGEWORTH EXPANSION



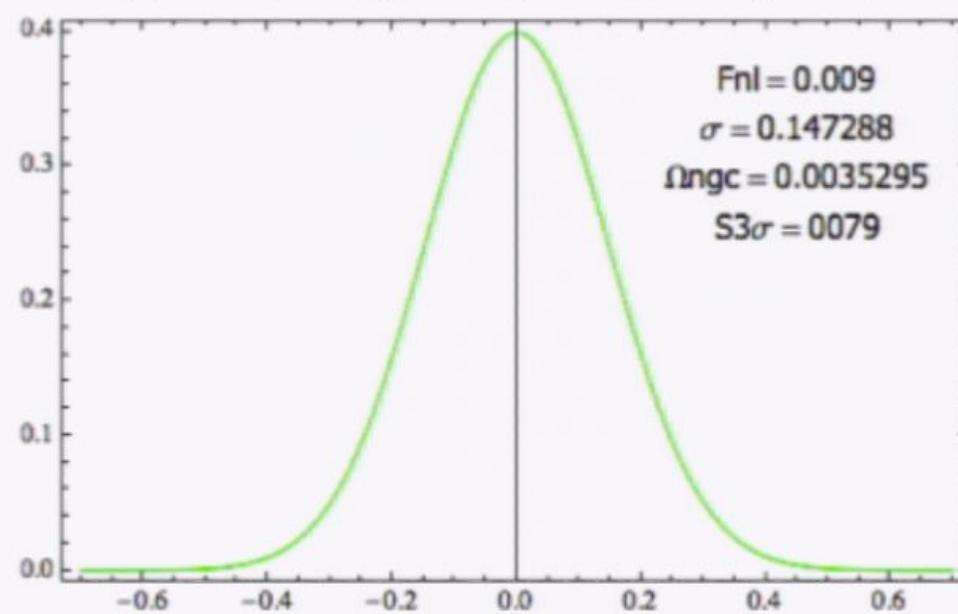
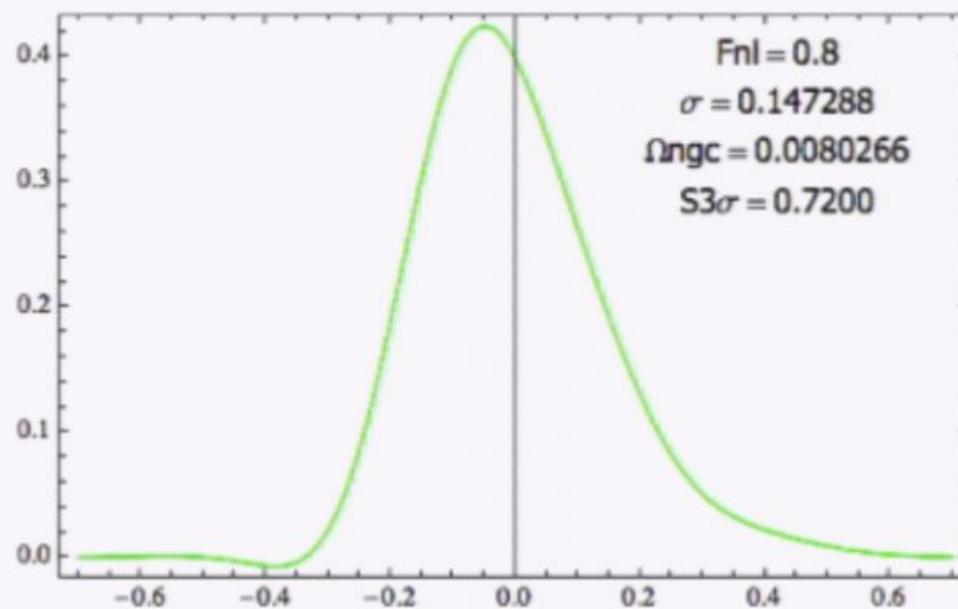
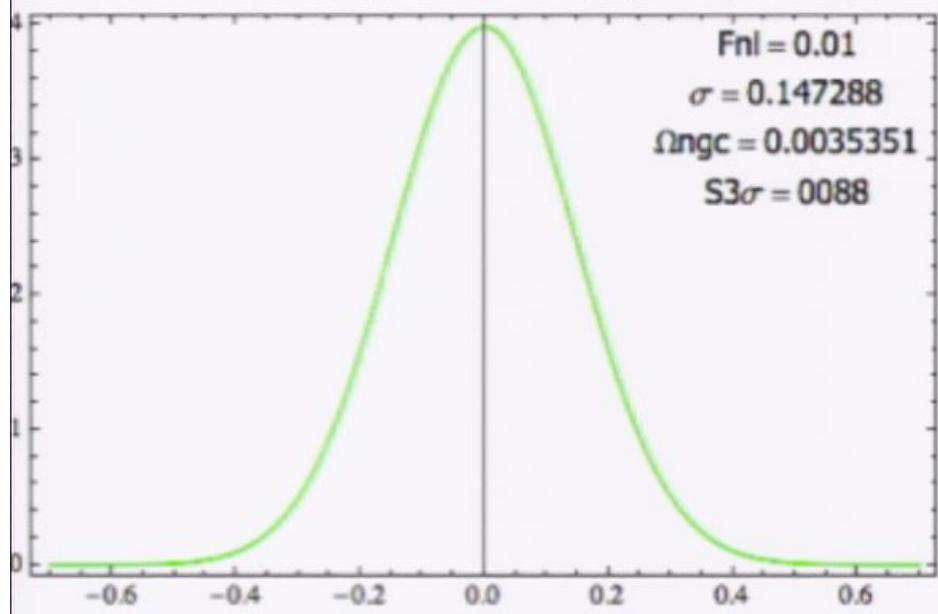
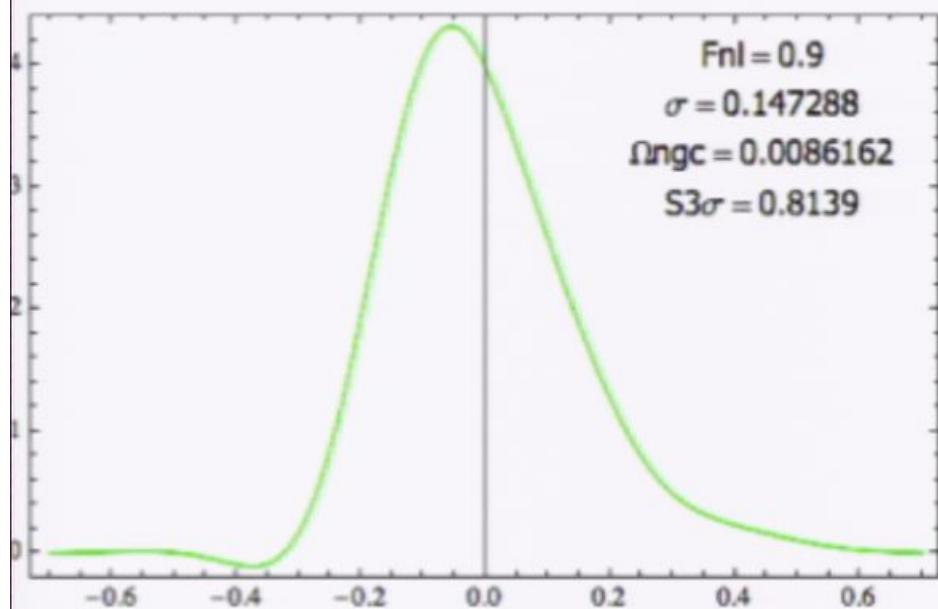
f_{NL} FROM CHANGE OF VARIABLE



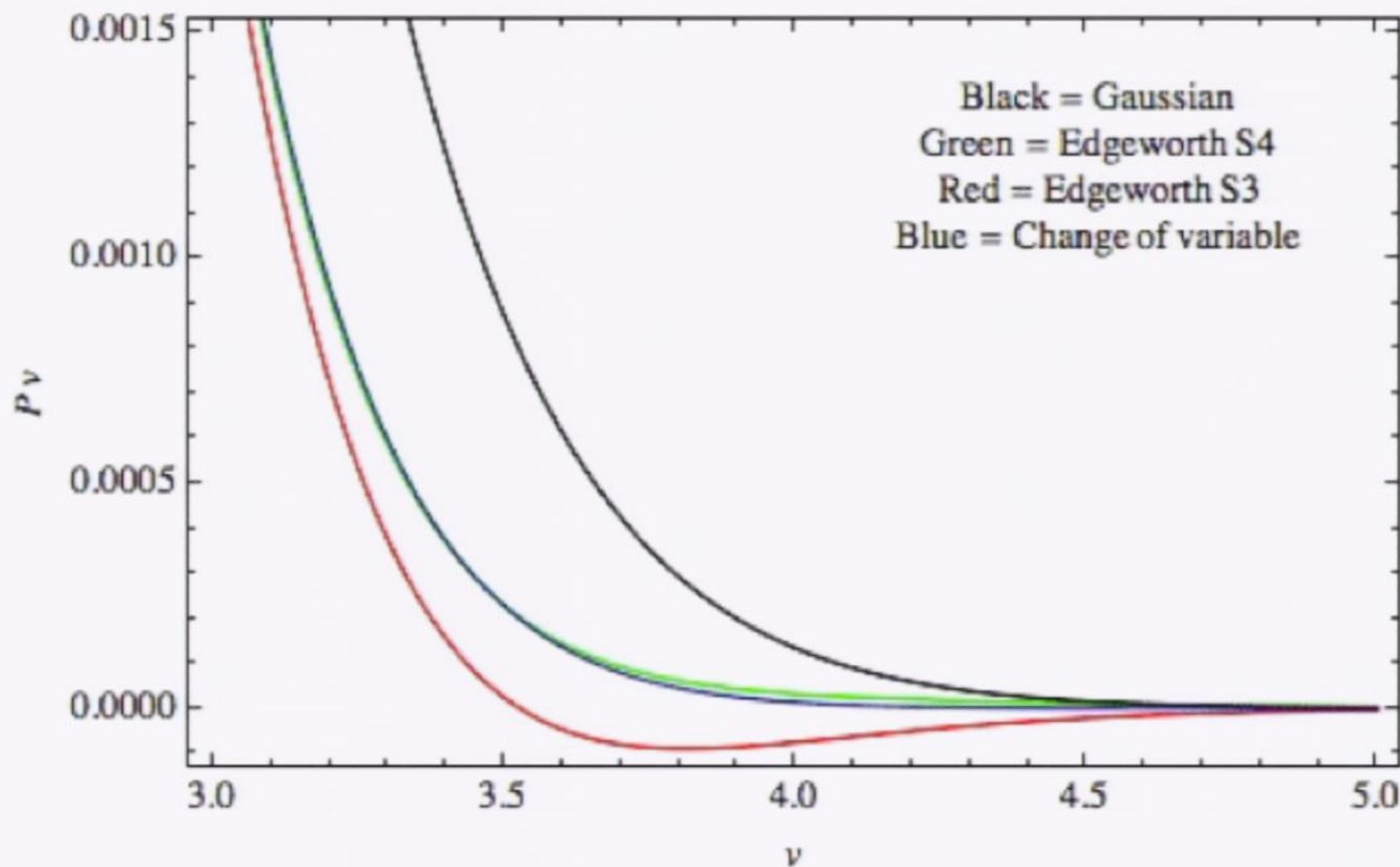
CONCLUSIONS

- With PBHs we can constrain the density perturbation spectrum on extremely short scales.
- The Edgeworth series is an exact expression for the PDF in terms of the cumulants.
- The non-gaussian effects are scale dependent.
- This scale dependent gives new importance to observational probes at multiple scales.

f_{NL} FROM EDGEWORTH EXPANSION



COMPARATION



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Thanks!!!