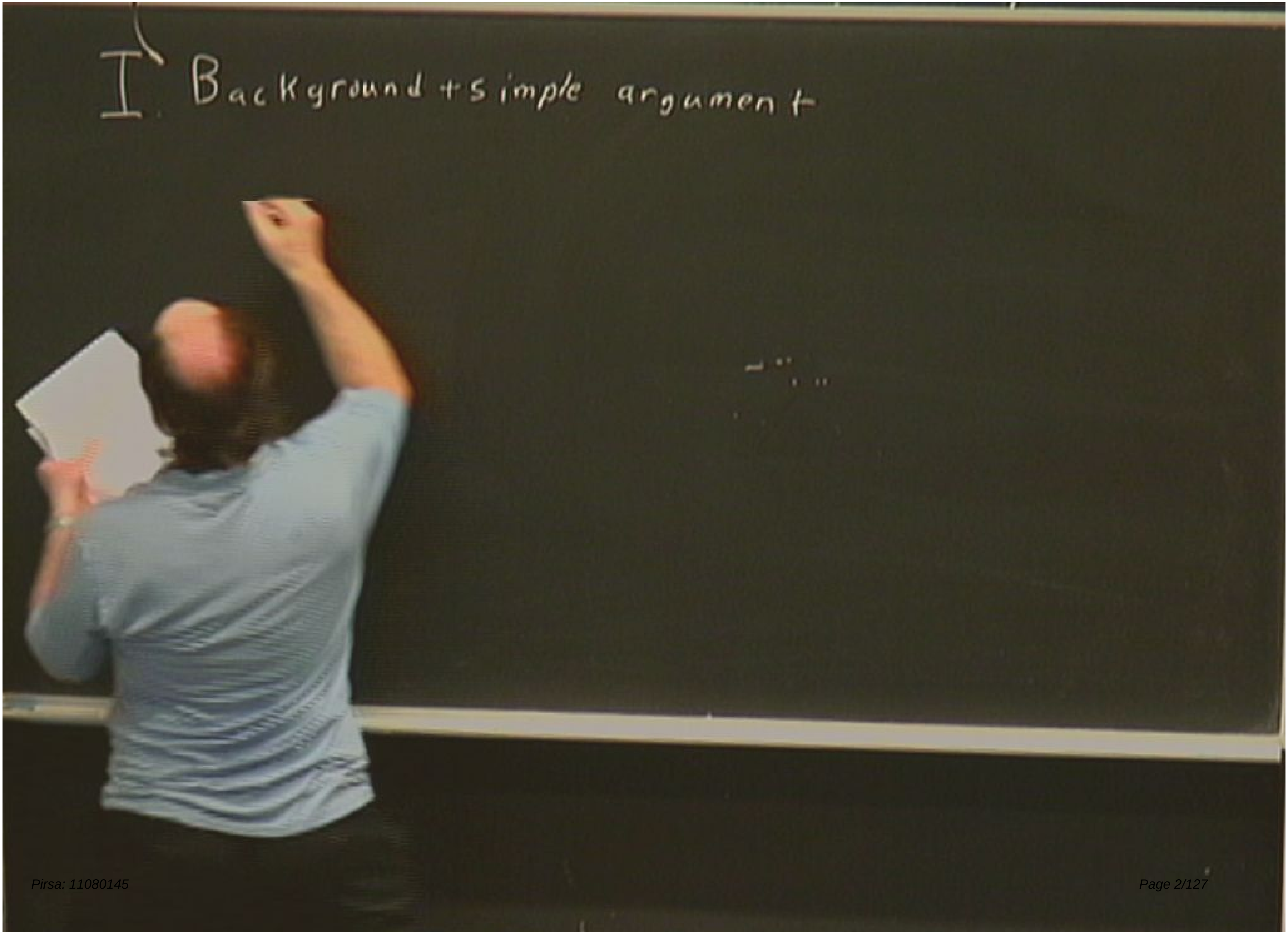
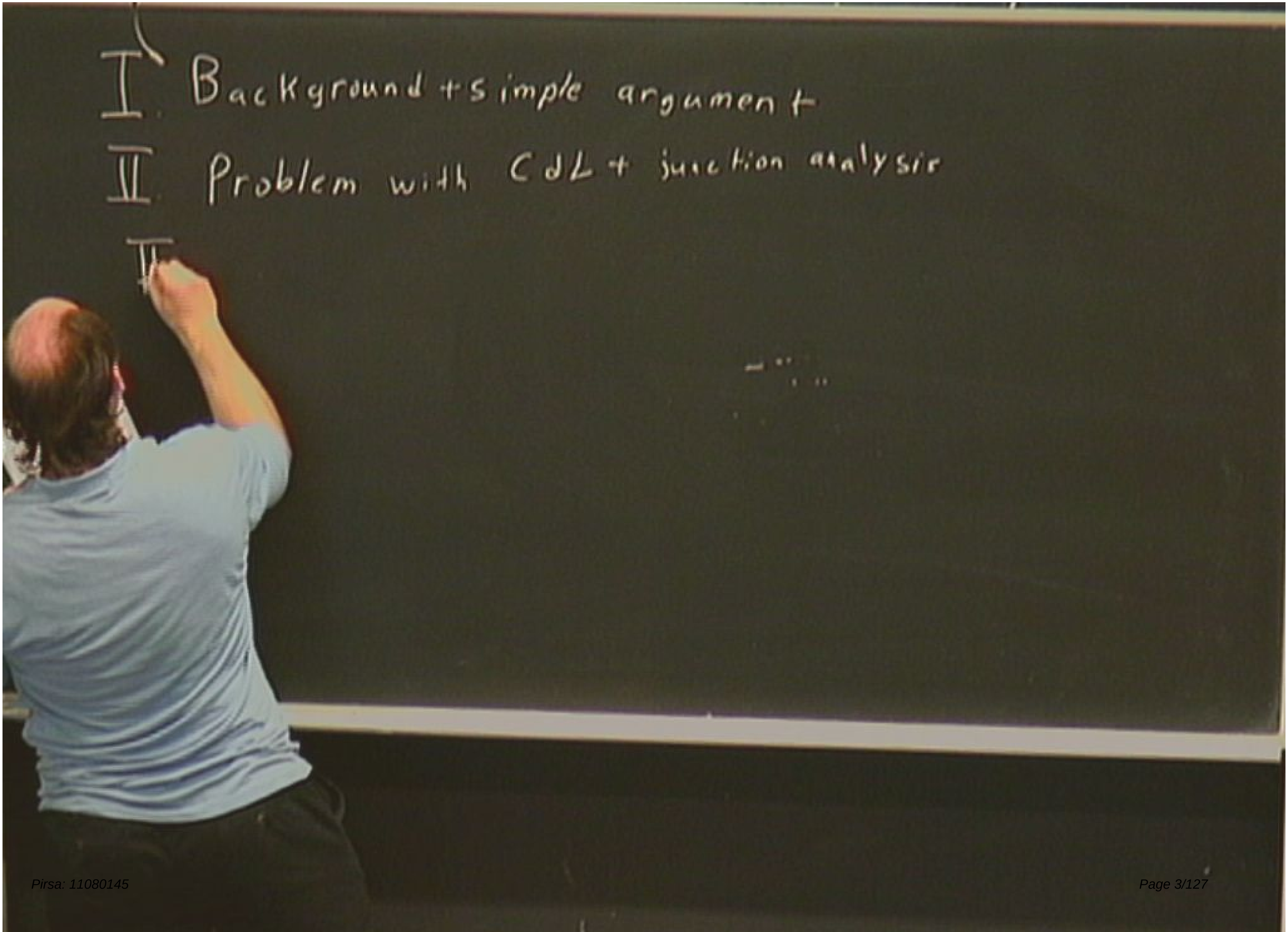
Title: Gravity and Rapid Tunnelling

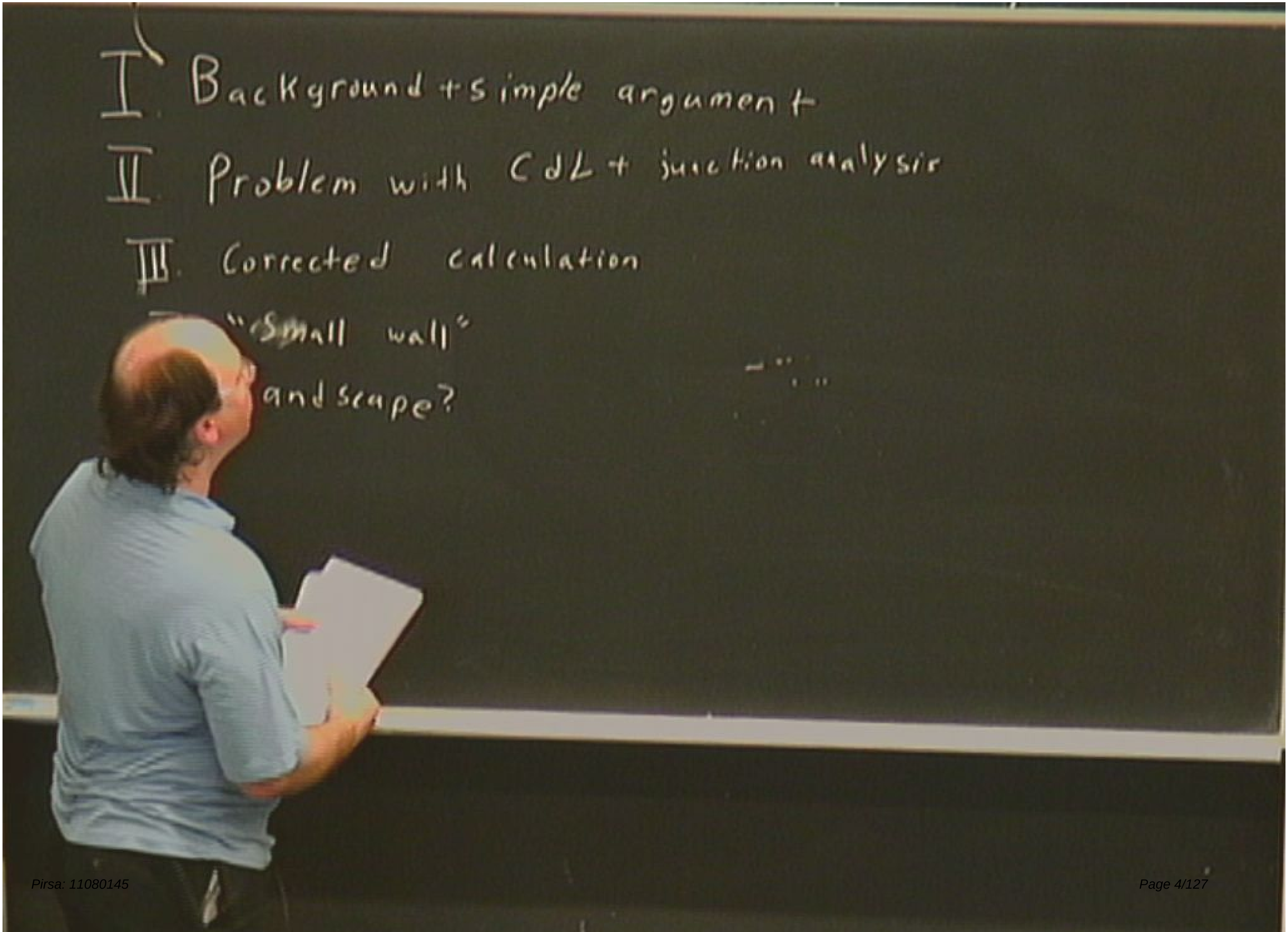
Date: Aug 24, 2011 11:30 AM

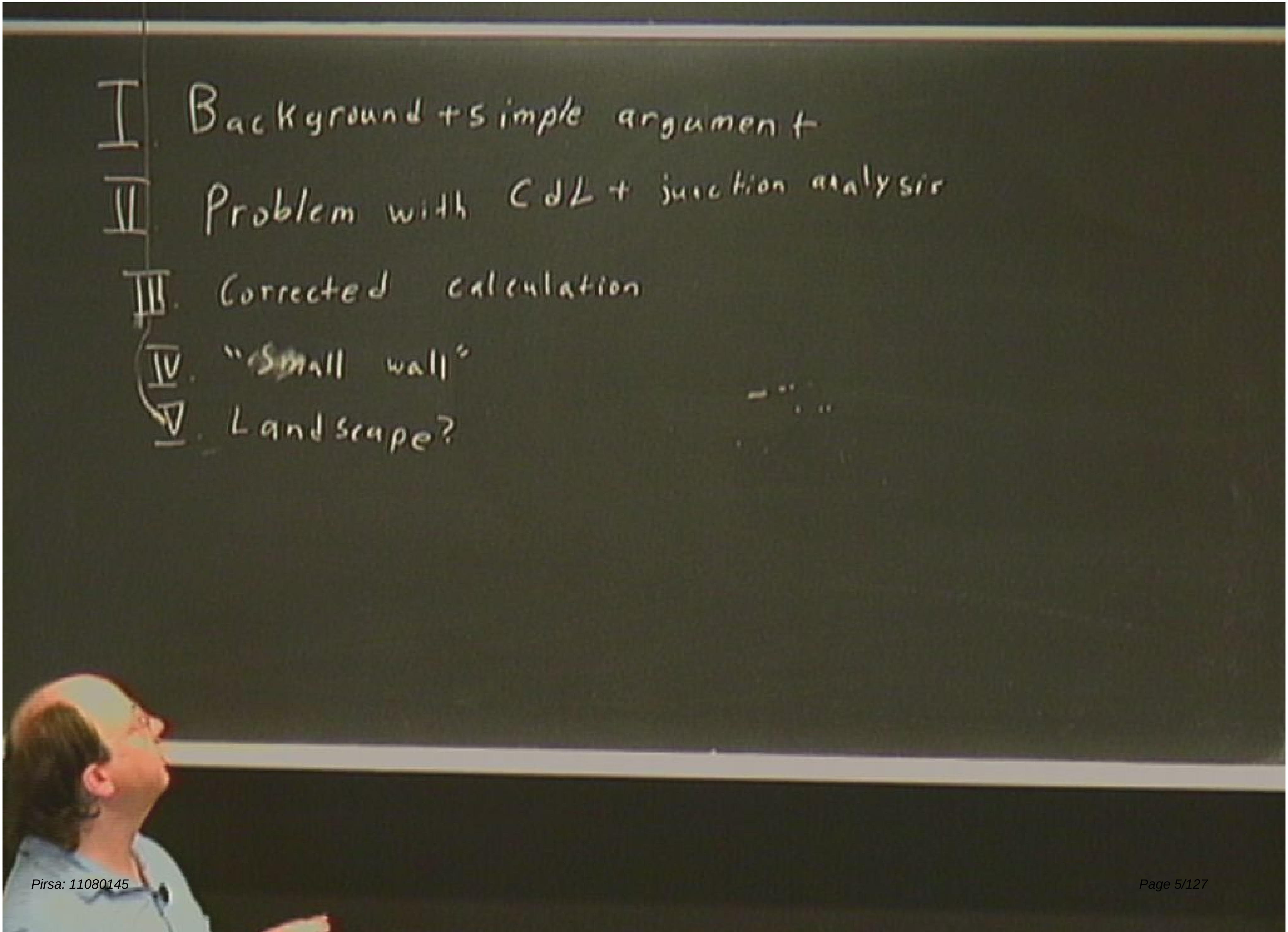
URL: http://pirsa.org/11080145

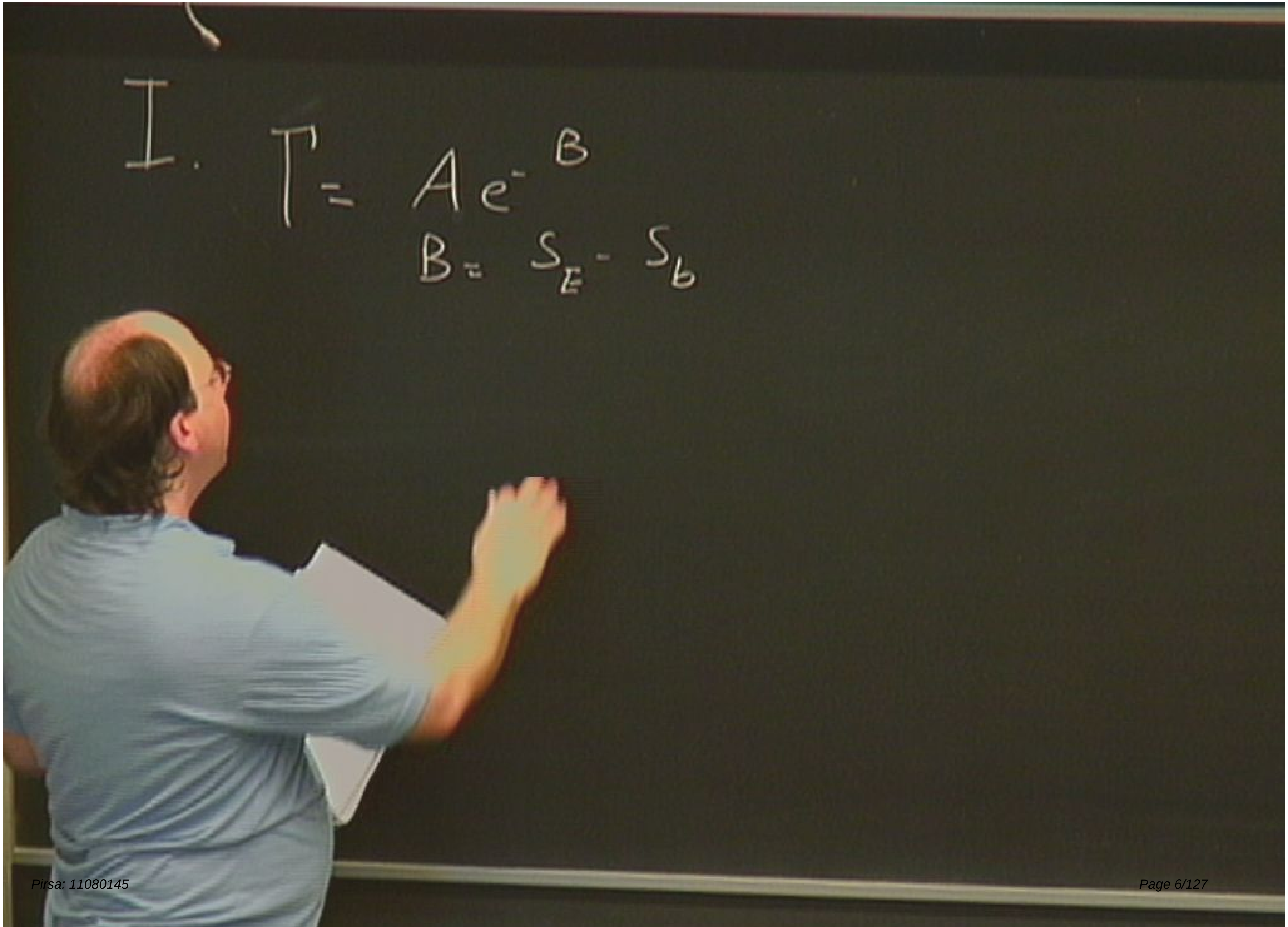
Abstract: I'll discuss my recent results (1108.2255) showing that, once gravity and some technical confusions are taken care of, two classes of potentials one expects to be generic in a landscape, namely ones resulting in thin-wall instantons or with small relative differences in potentials, result in instantons which typically decay rapidly, including exponentially enhanced rates for thin-wall instantons. I'll explain why this is true both generally and in detail and why the previous treatments have gone astray. This meeting will be designed to be a highly informal and interactive session to present these results in detail and address questions, confusions, and challenges of those, esp. cosmologists, with some level of background in tunnelling. I'll give a broader presentation of this material for a more general audience in the strings group meeting (Friday 11am, space room).

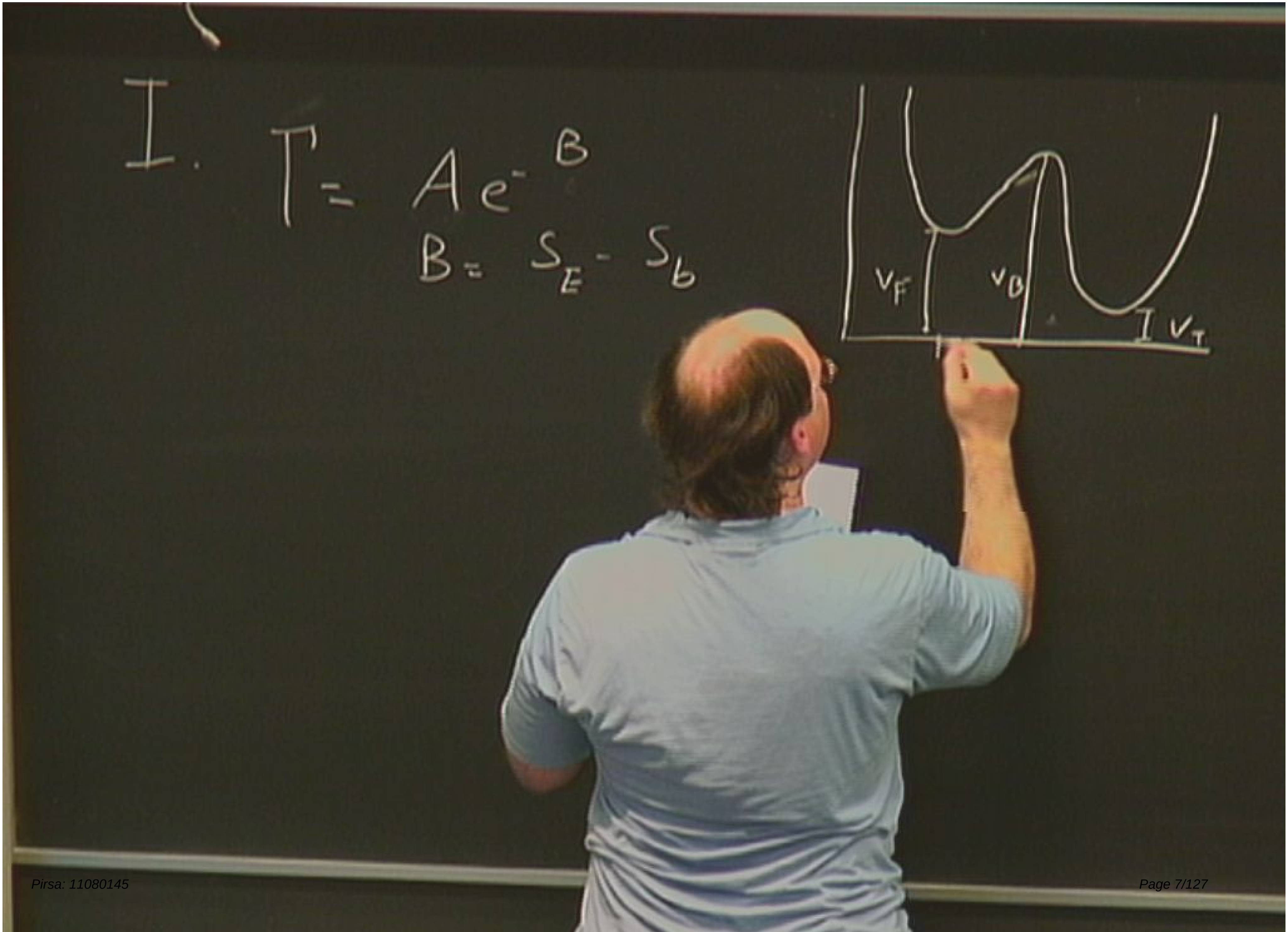

I. Background + simple argument


I. Background + simple argument
II. Problem with CdL + junction analysis
III


I. Background + simple argument
II. Problem with CdL + junction analysis
III. Corrected calculation
"Small wall"
andscape?


I. Background + simple argument
II. Problem with CdL + junction analysis
III. Corrected calculation
IV. "Small wall"
V. Landscape?


I. $\Gamma = A e^{-B}$
$B = S_E - S_b$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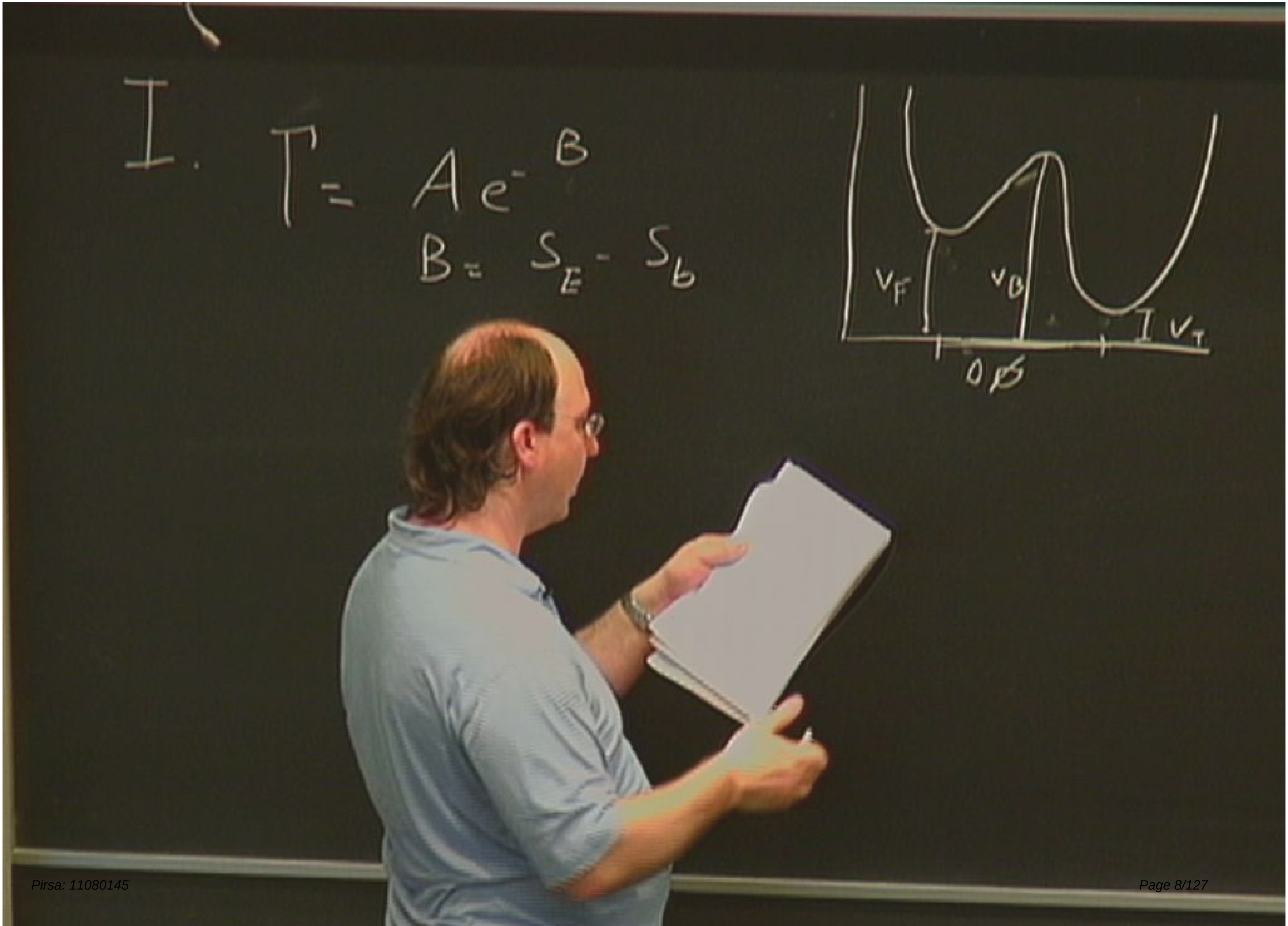
B = S_E - S_b
V_F
V_D
V_T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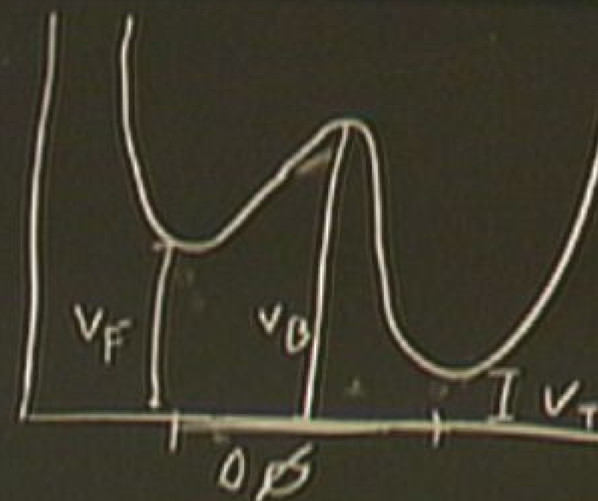
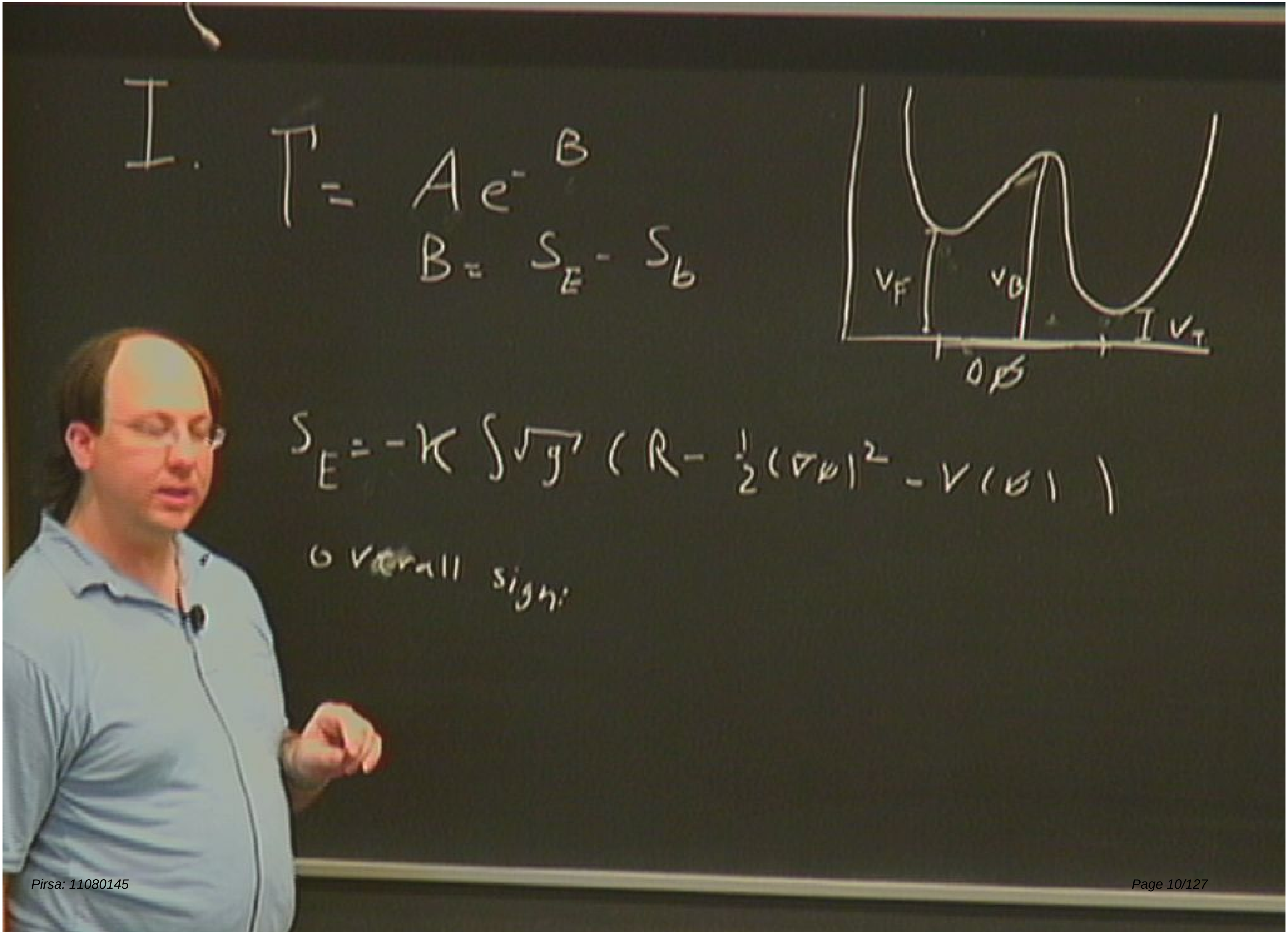


$$S_E = -\kappa \int \sqrt{g} \left(R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right)$$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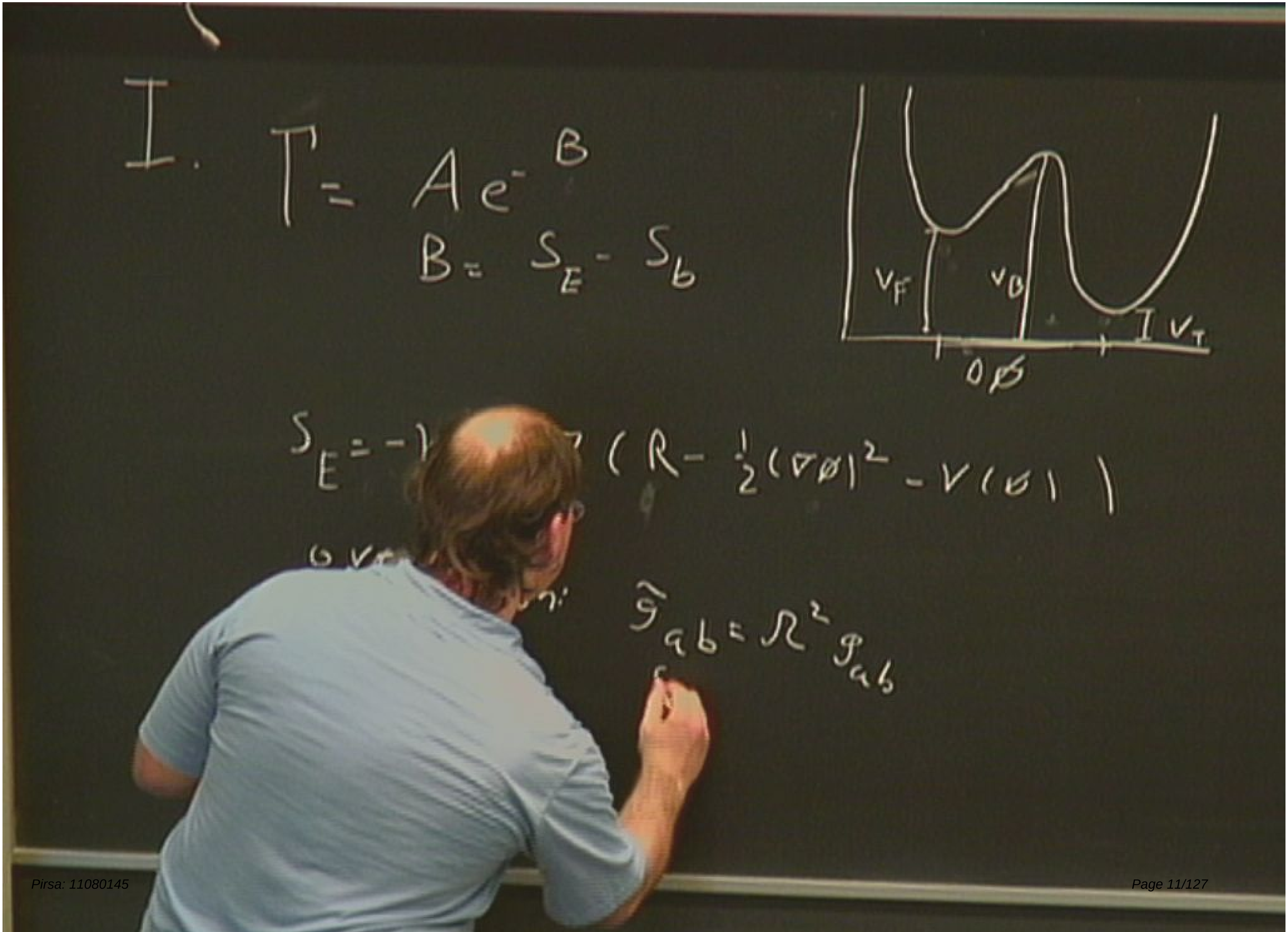
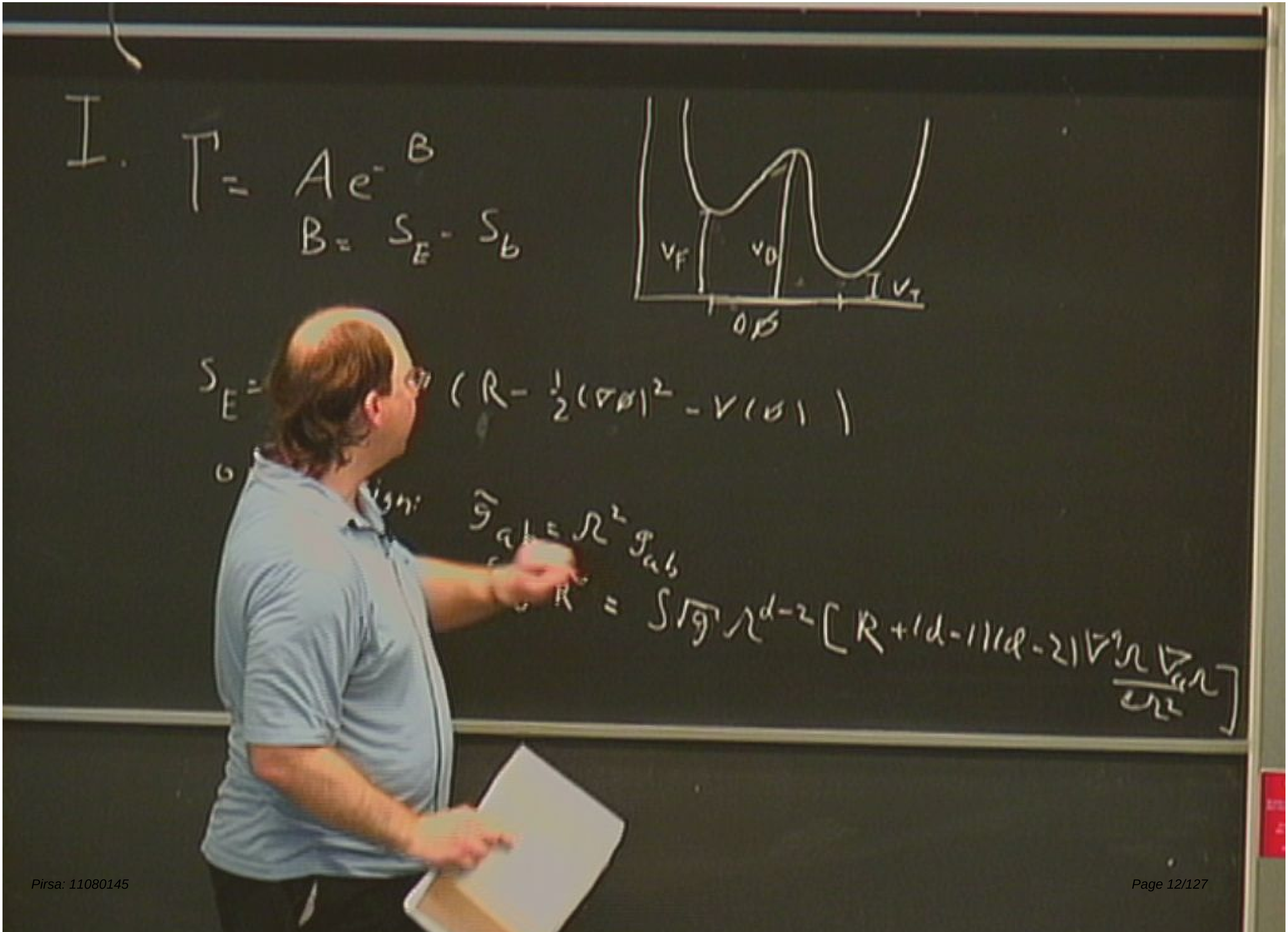
$S_E = -K \int \sqrt{g} \, (R - \frac{1}{2}(\nabla\phi)^2 - V(\phi))$
overall sign:



I.
Γ = Ae^-B
B = S_E - S_b
V_F
V_B
V_T
0
ø
(R - ½(∇ø)^2 - V(ø))
g̃_ab = Ω^2 g_ab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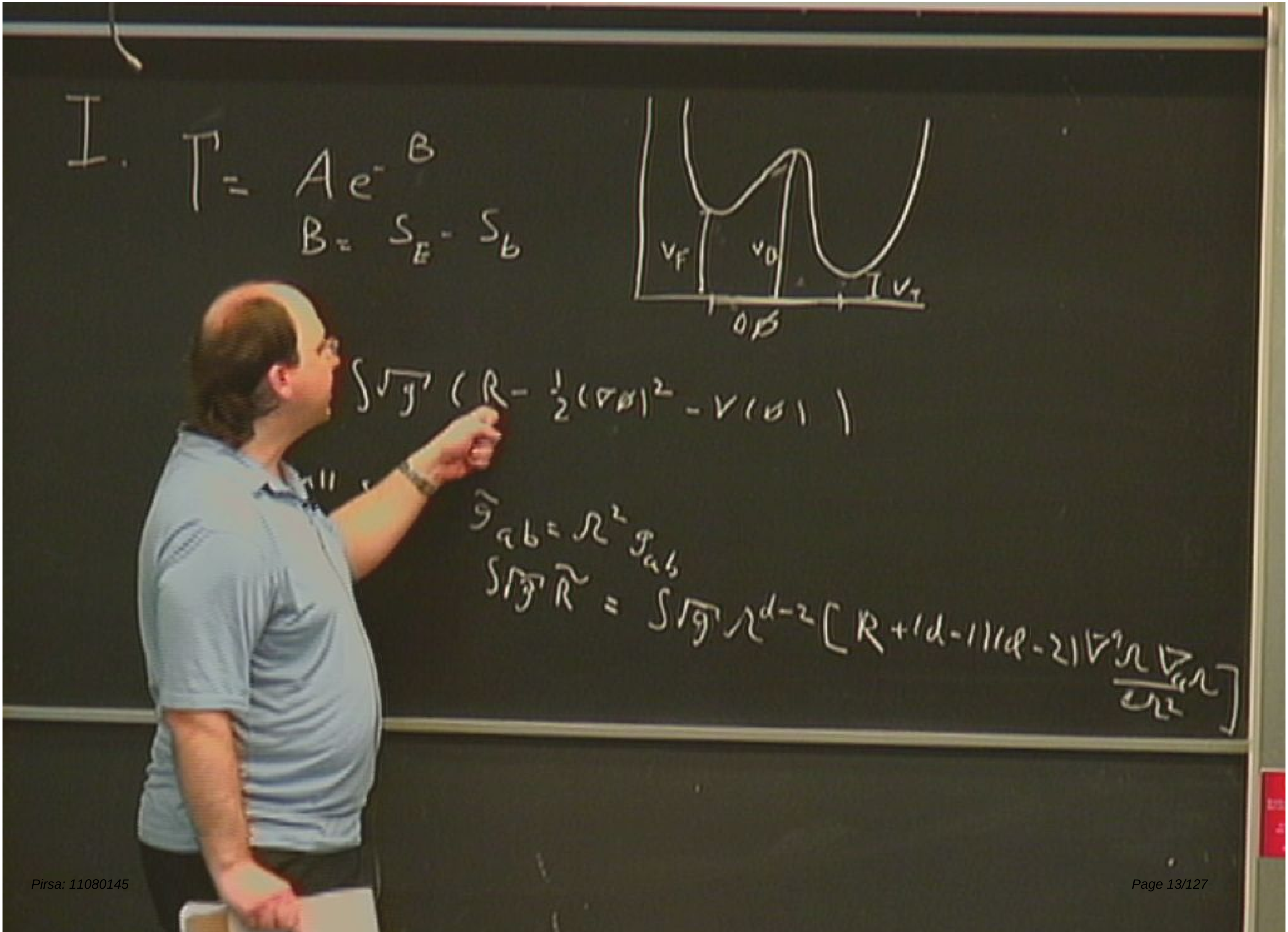
g̃_ab = Ω² g_ab
= ∫√g Ω^(d-2) [R + (d-1)(d-2) ∇^a Ω ∇_a Ω / Ω²]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I. $\Gamma = A e^{-B}$

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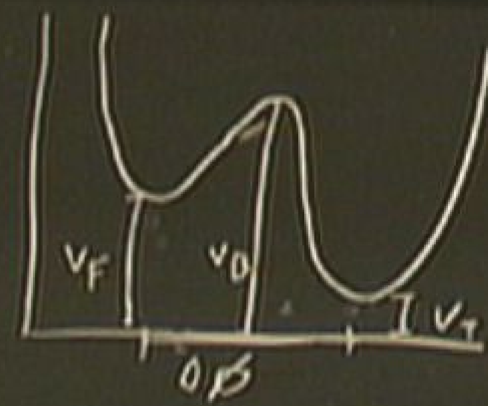
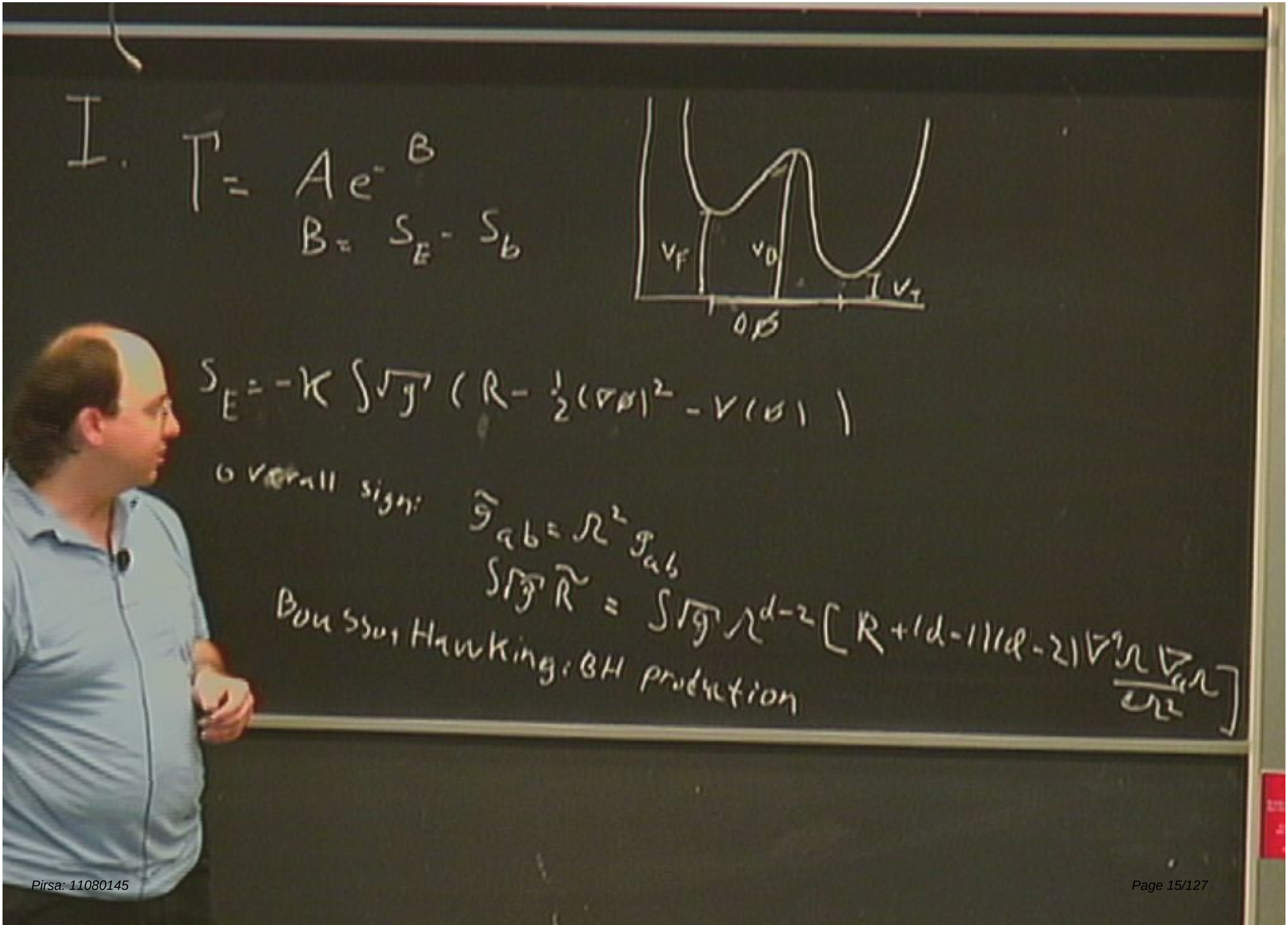


$$S_E = -K \int \sqrt{g}\,\left(R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right)$$

Overall sign: $\tilde{g}_{ab} = \Omega^2 g_{ab}$

$$\int \sqrt{\tilde{g}}\,\tilde{R} = \int \sqrt{g}\,\Omega^{d-2}\left[R + (d-1)(d-2)\frac{\nabla^a\Omega \nabla_a\Omega}{\Omega^2} \right]$$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Boussou Hawking: BH production

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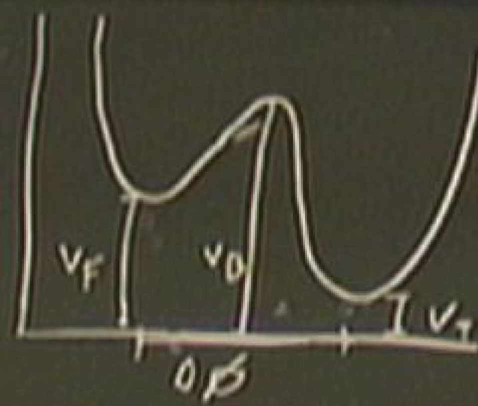

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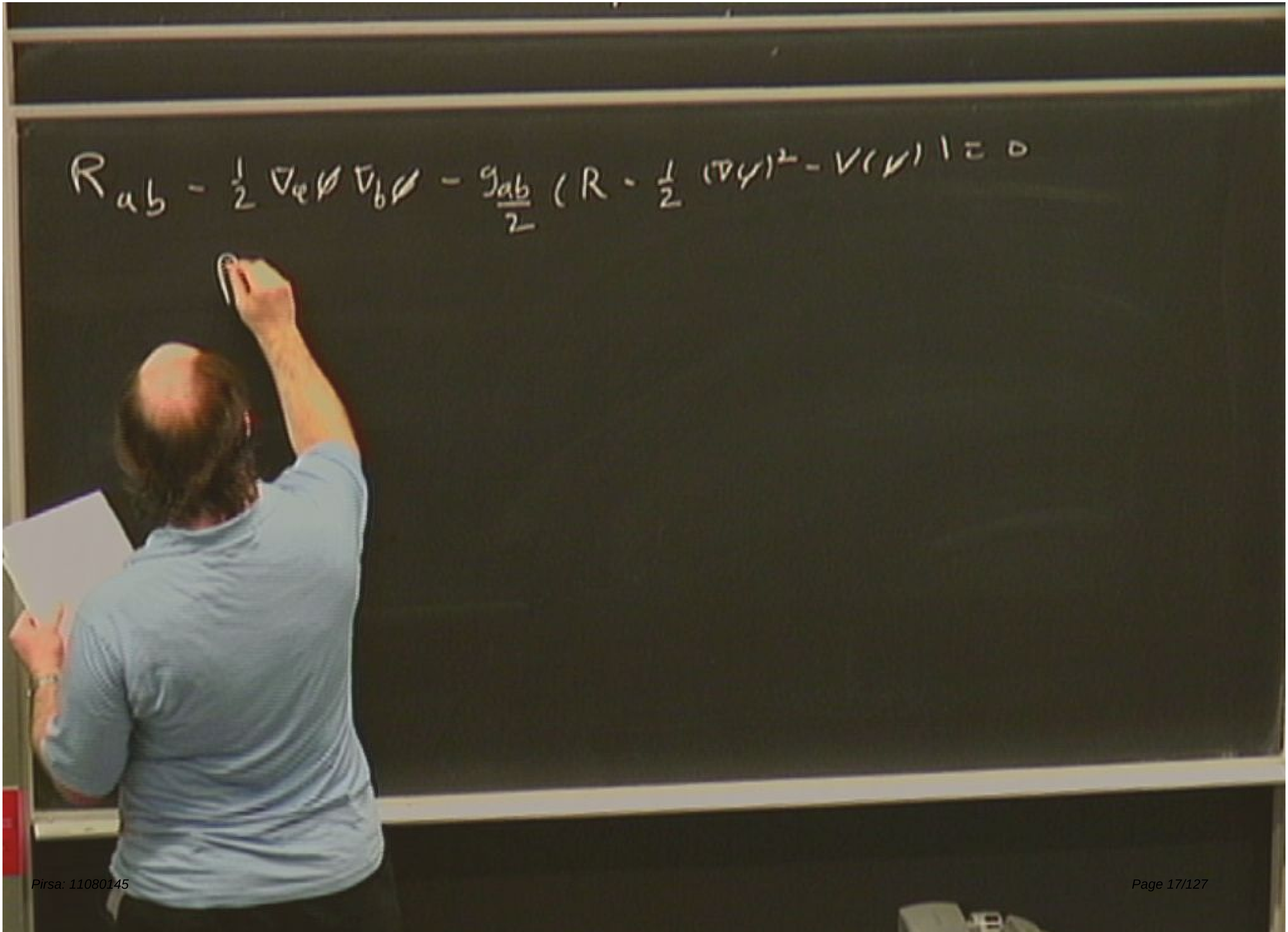
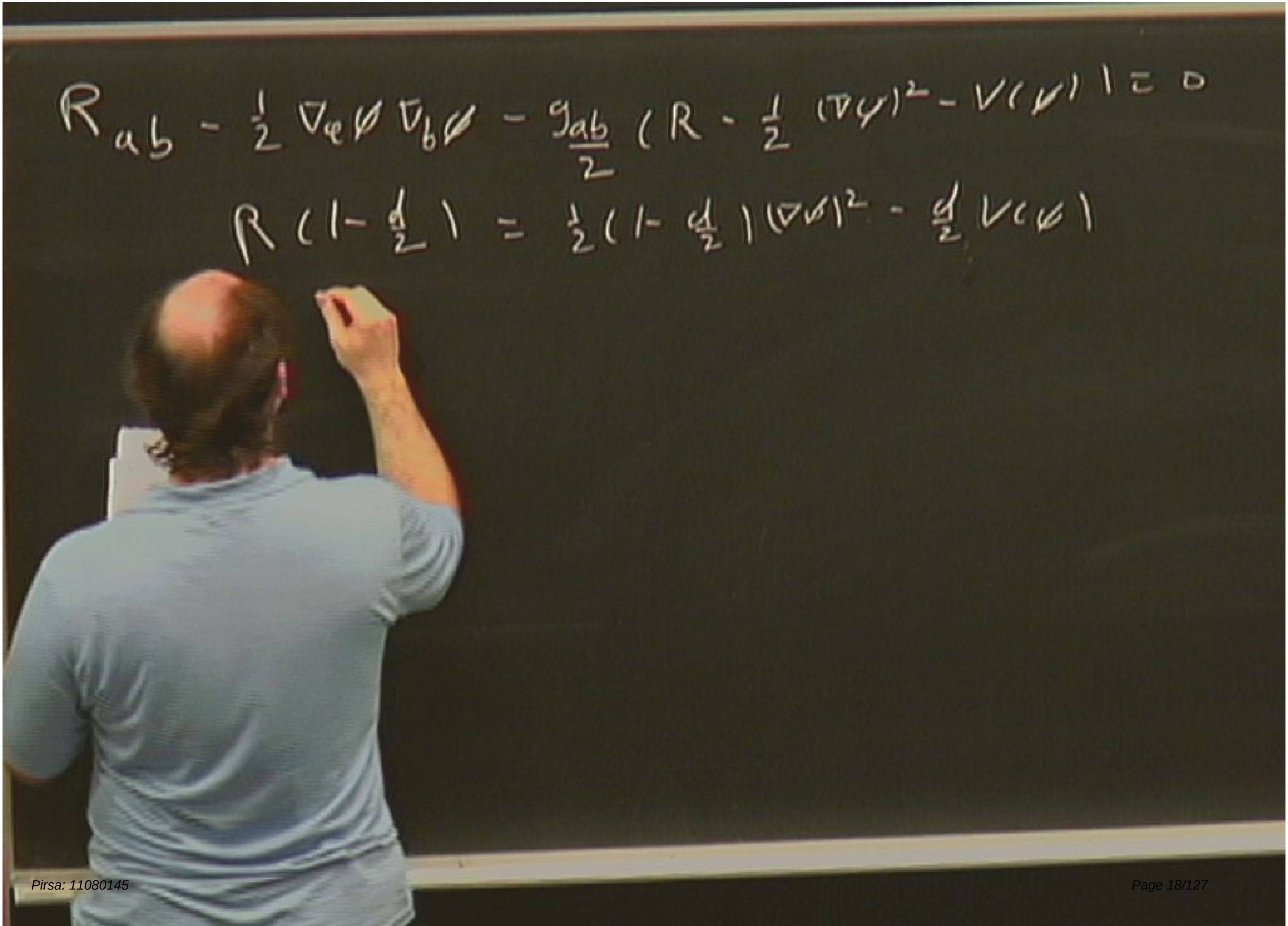
Bousso, Hawking: BH production


$R_{ab} - \frac{1}{2} \nabla_a \phi \nabla_b \phi - \frac{g_{ab}}{2} (R - \frac{1}{2} (\nabla \phi)^2 - V(\phi)) = 0$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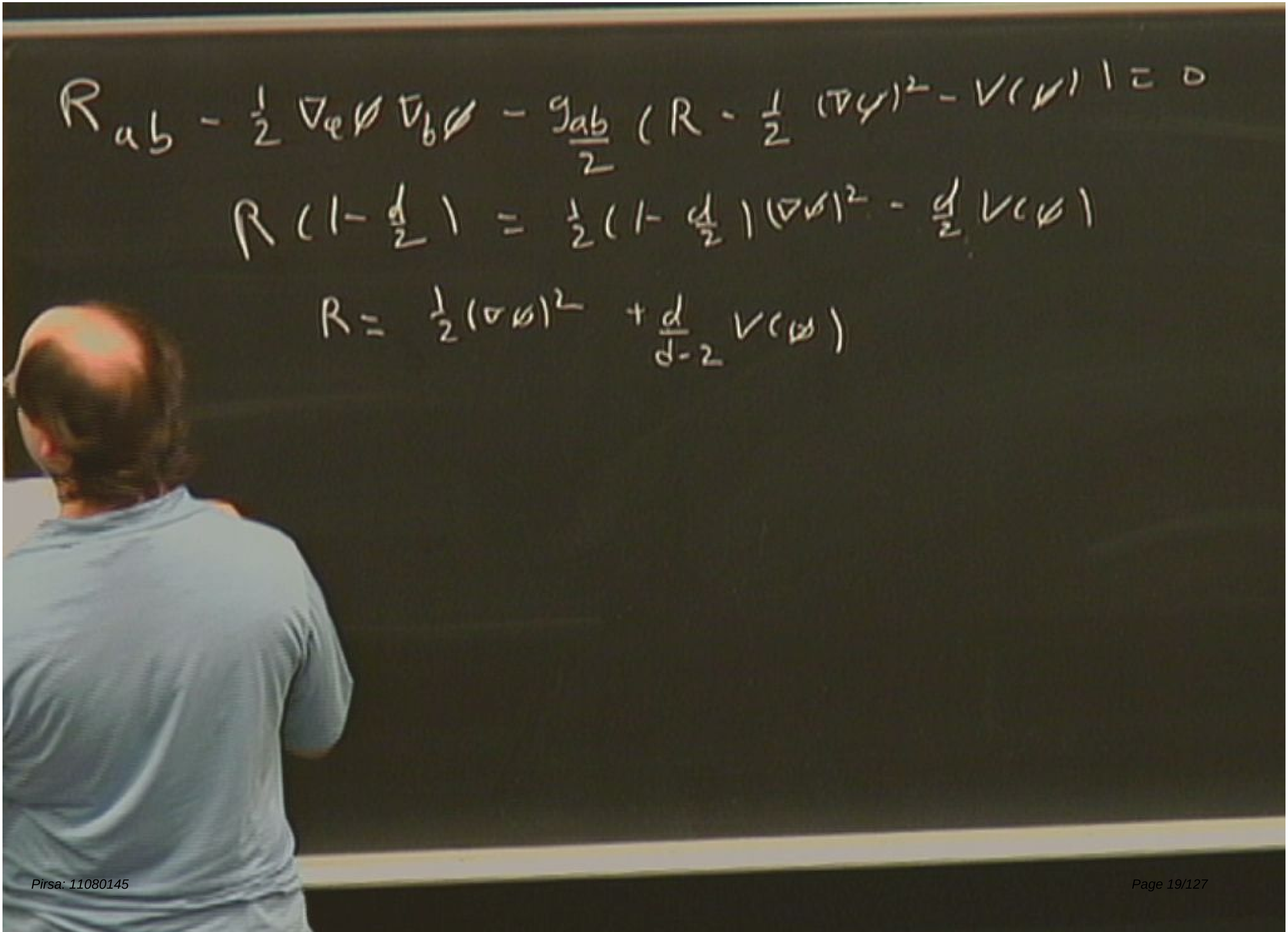
$R\left(1-\frac{d}{2}\right) = \frac{1}{2}\left(1-\frac{d}{2}\right)(\nabla\phi)^2 - \frac{d}{2}V(\phi)$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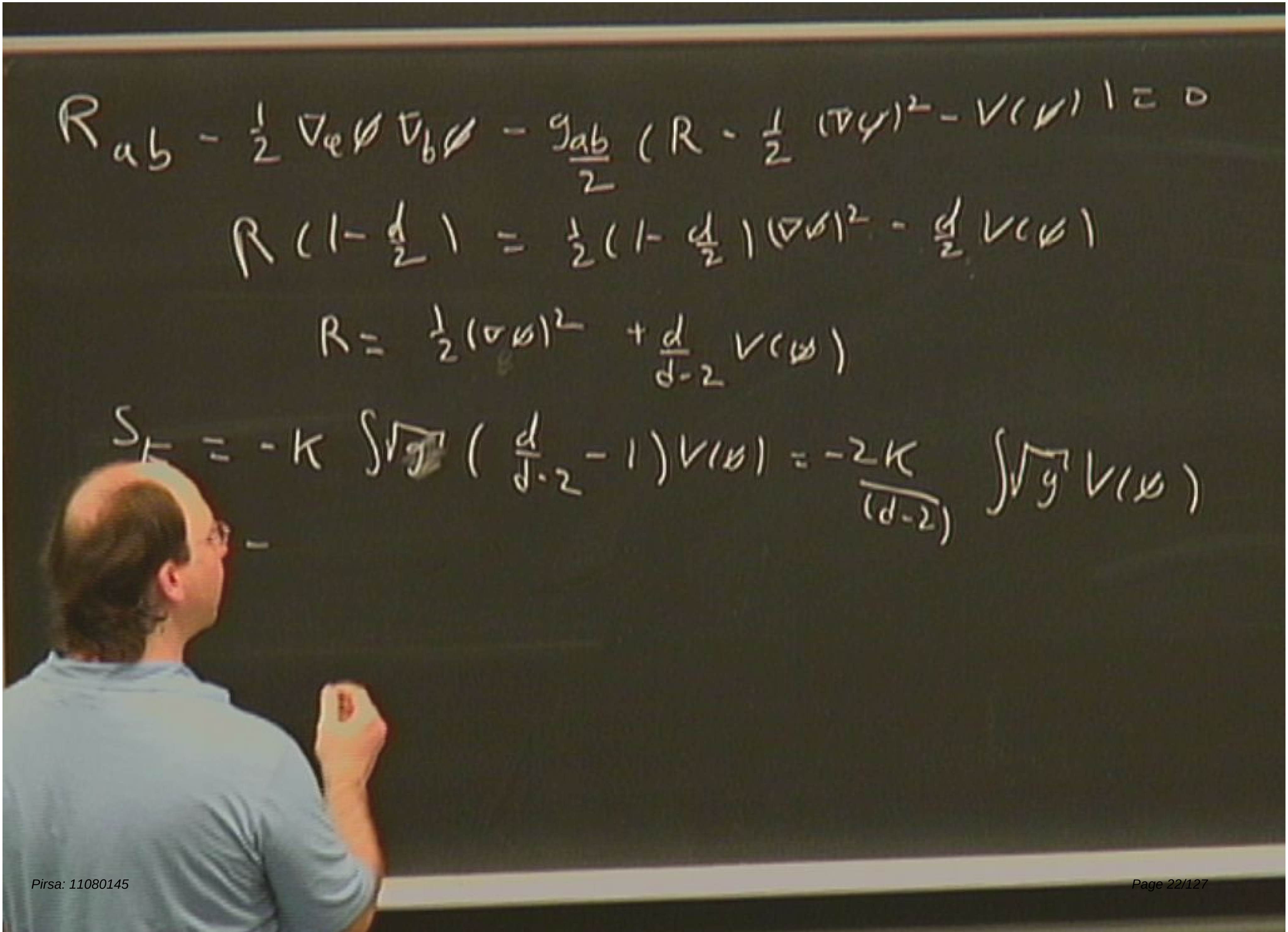
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$S_E = -K\int\sqrt{g}\left(\frac{d}{d-2} - 1\right)V(\phi) = \frac{-2K}{(d-2)}\int\sqrt{g}\,V(\phi)$

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- no gradient terms
- potential (naively) comes in with opposite sign

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$$S_E^{(4)} = -\frac{48K}{V_0}$$

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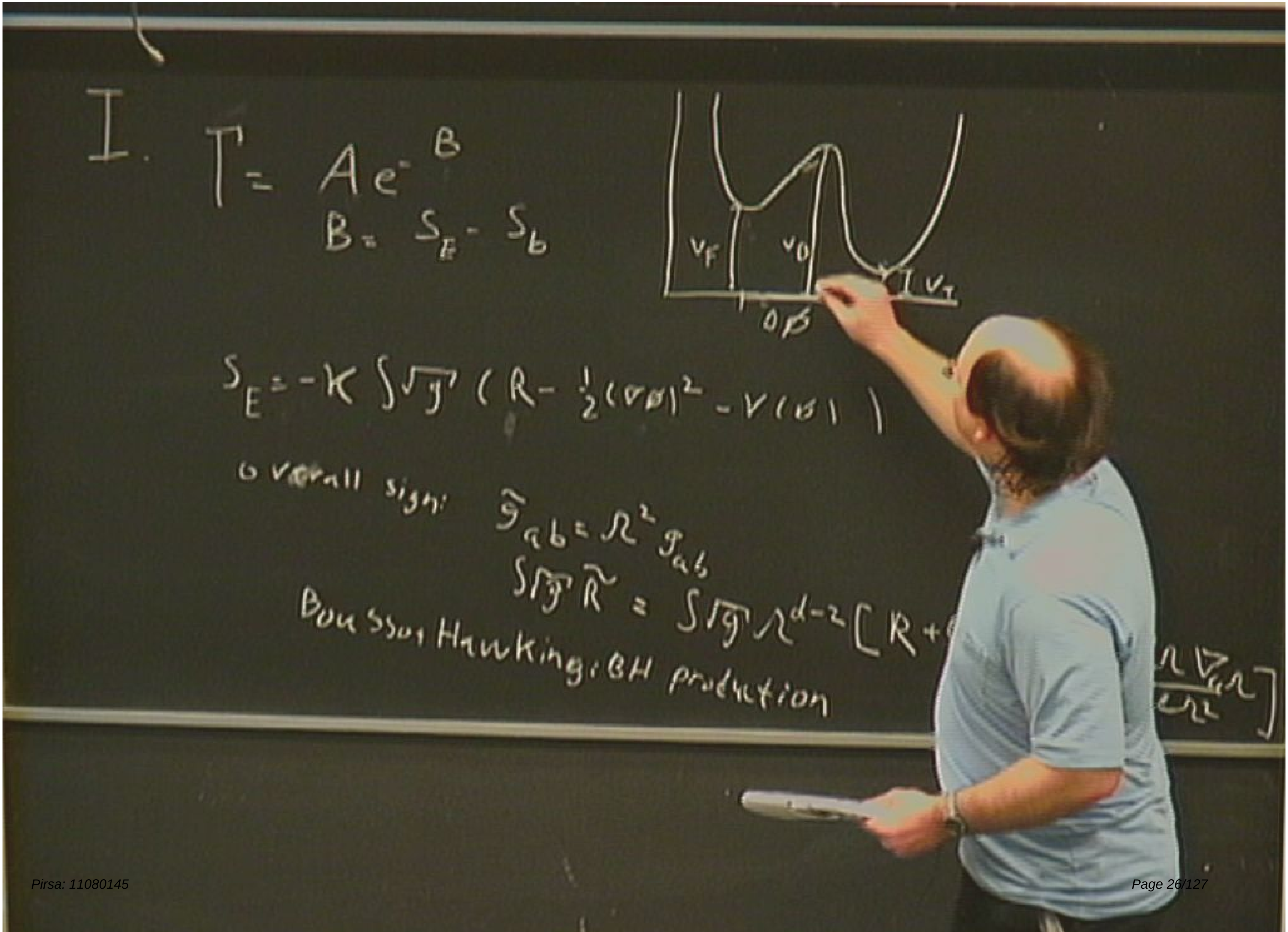
$$S_E^{(4)} = -\frac{48K}{V_0} \qquad V(\phi) = V_0 > 0$$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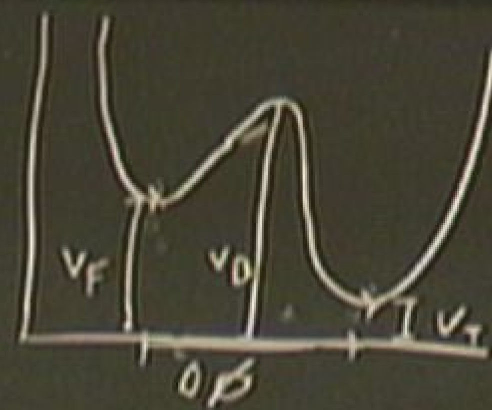

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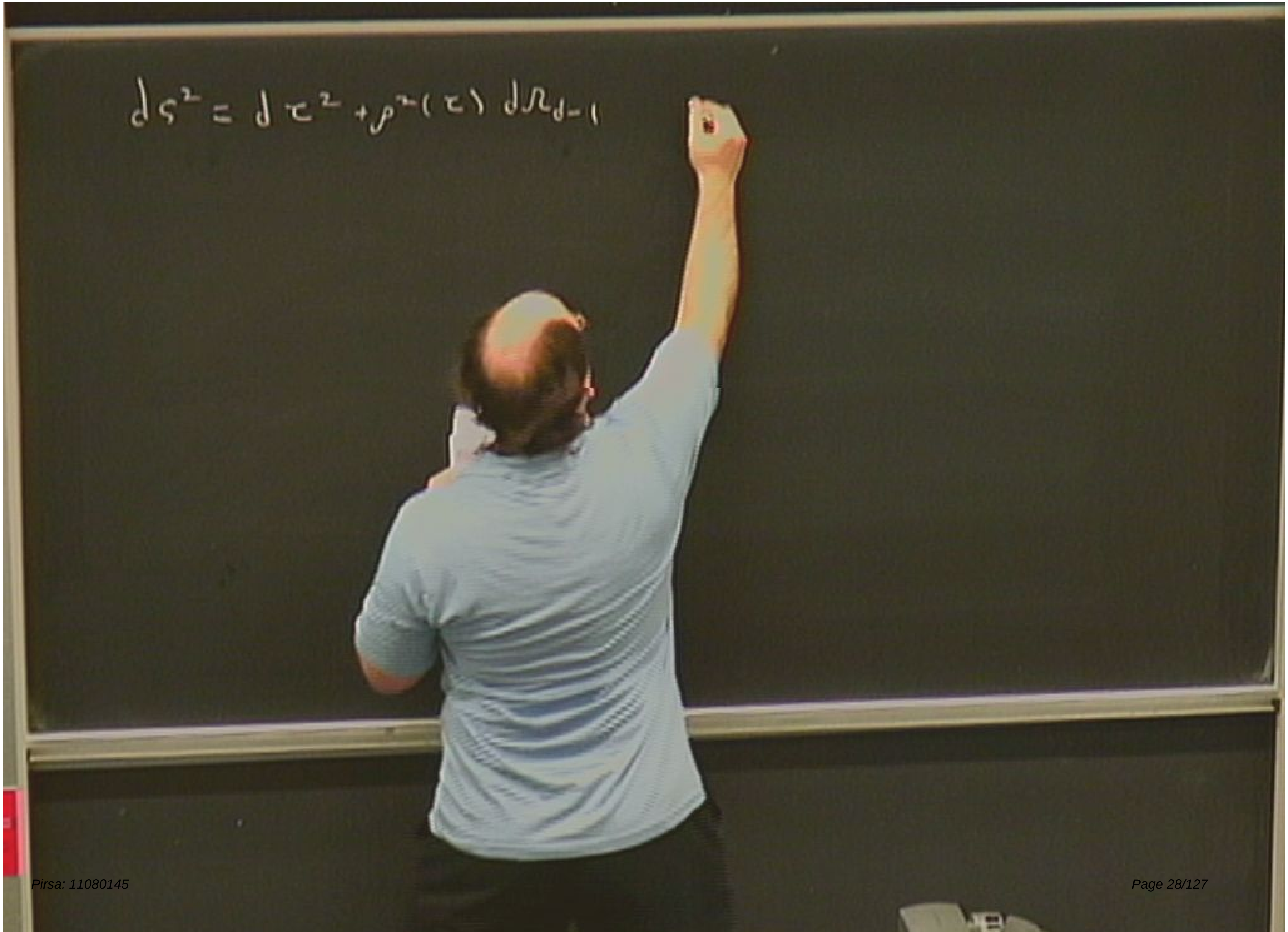
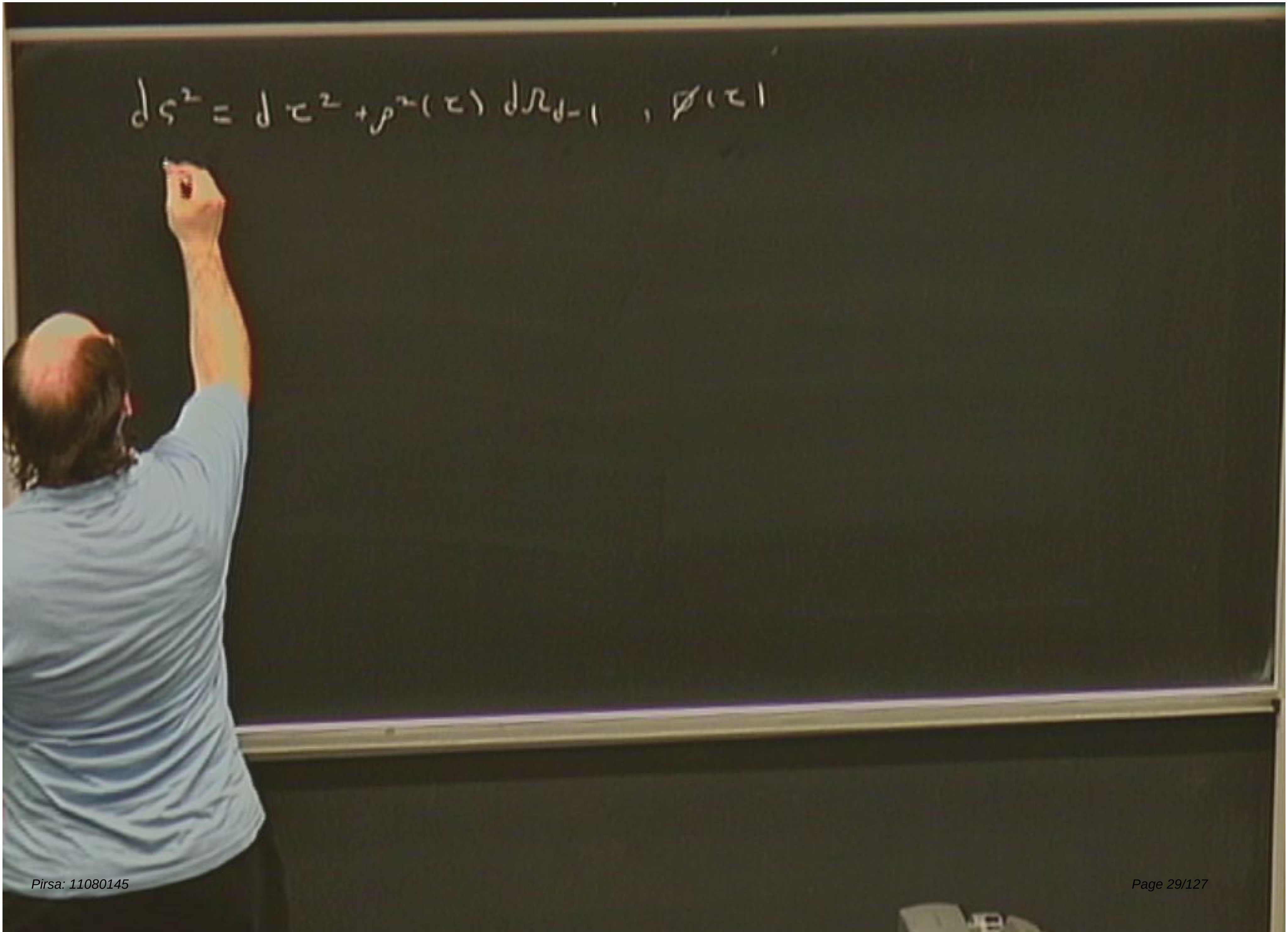
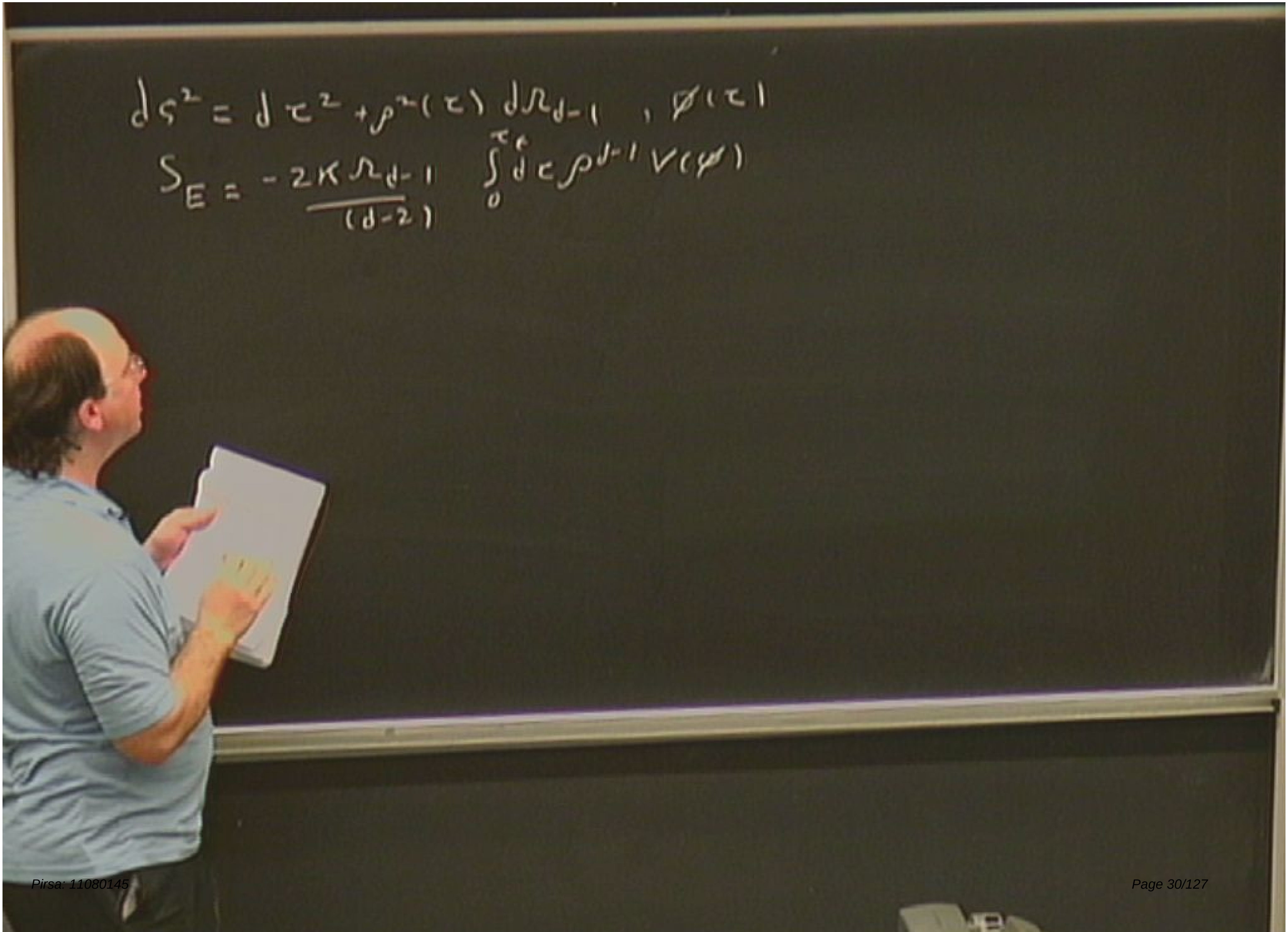

$ds^2 = d\tau^2 + \rho^2(\tau)\, d\Omega_{d-1}$


$ds^2 = d\tau^2 + \rho^2(\tau)\, d\Omega_{d-1}$, $\phi(\tau)$


$ds^2 = d\tau^2 + \rho^2(\tau)\, d\Omega_{d-1} \quad , \phi(\tau)$
$S_E = -\frac{2\kappa\Omega_{d-1}}{(d-2)} \int_0^{\tau_f} d\tau\, \rho^{d-1} V(\phi)$

$$ds^2 = d\tau^2 + \rho^2(\tau)\, d\Omega_{d-1} \quad , \quad \phi(\tau)$$

$$S_E = -\frac{2\kappa\, \Omega_{d-1}}{(d-2)} \int_0^{\tau_f} d\tau\, \rho^{d-1}\, V(\phi)$$

If $\quad V(\phi) = V_0$

$$\rho = \frac{1}{\omega_0} \sin \omega_0 \tau \qquad \omega_0^2 = \frac{V_0}{(d-1)(d-2)}$$

$$ds^2 = d\tau^2 + \rho^2(\tau)\, d\Omega_{d-1} \quad , \quad \phi(\tau)$$

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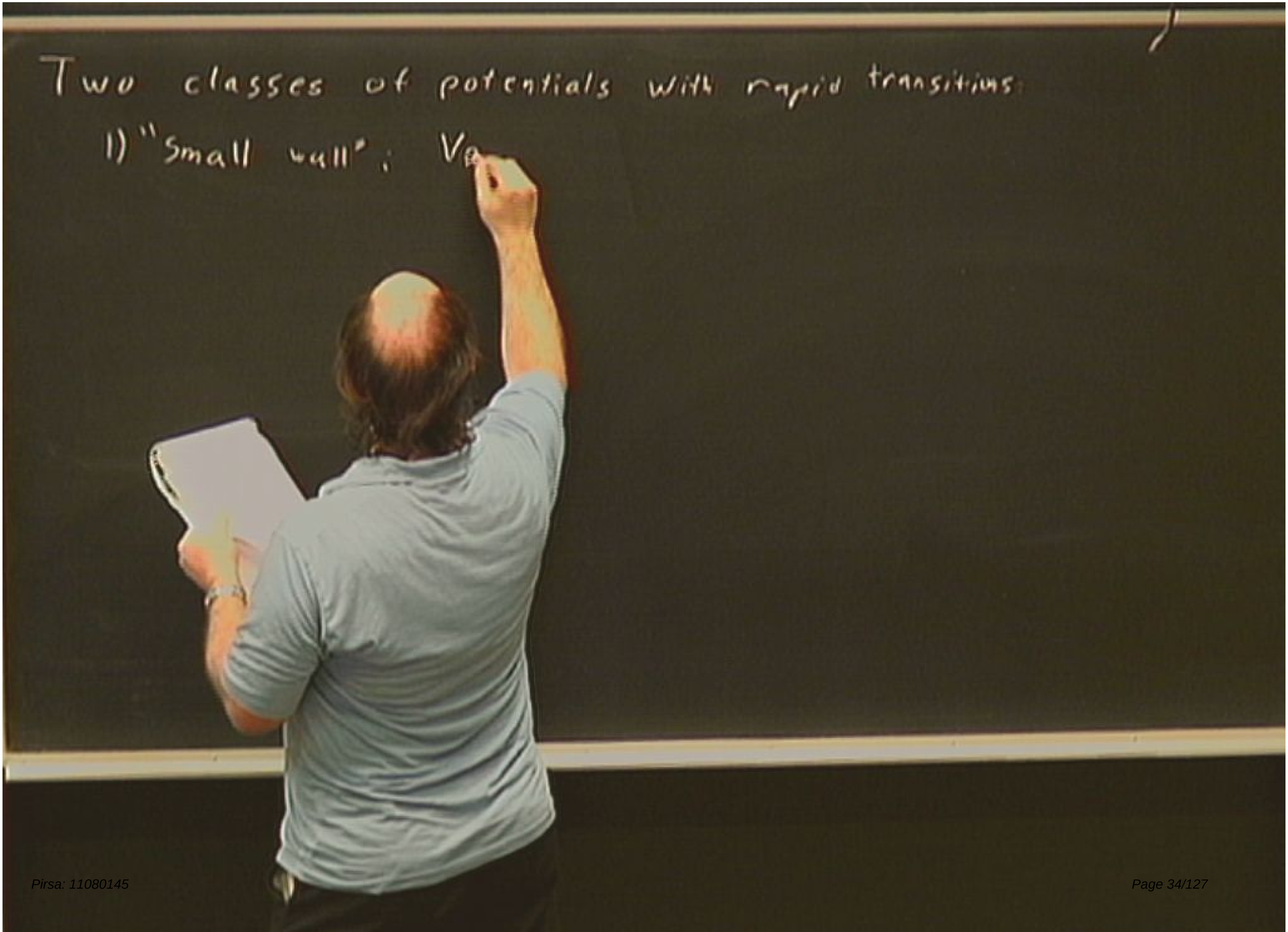
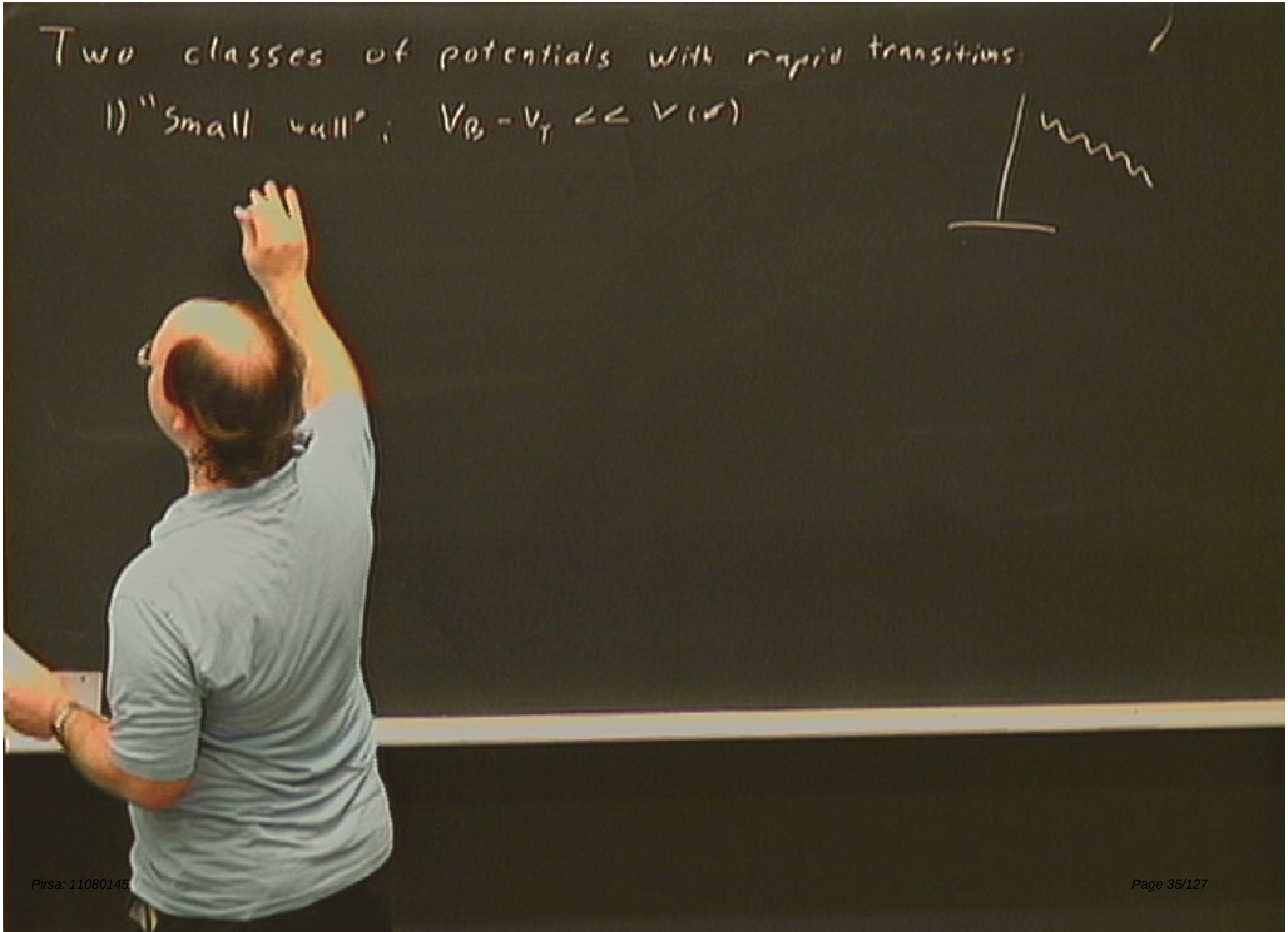
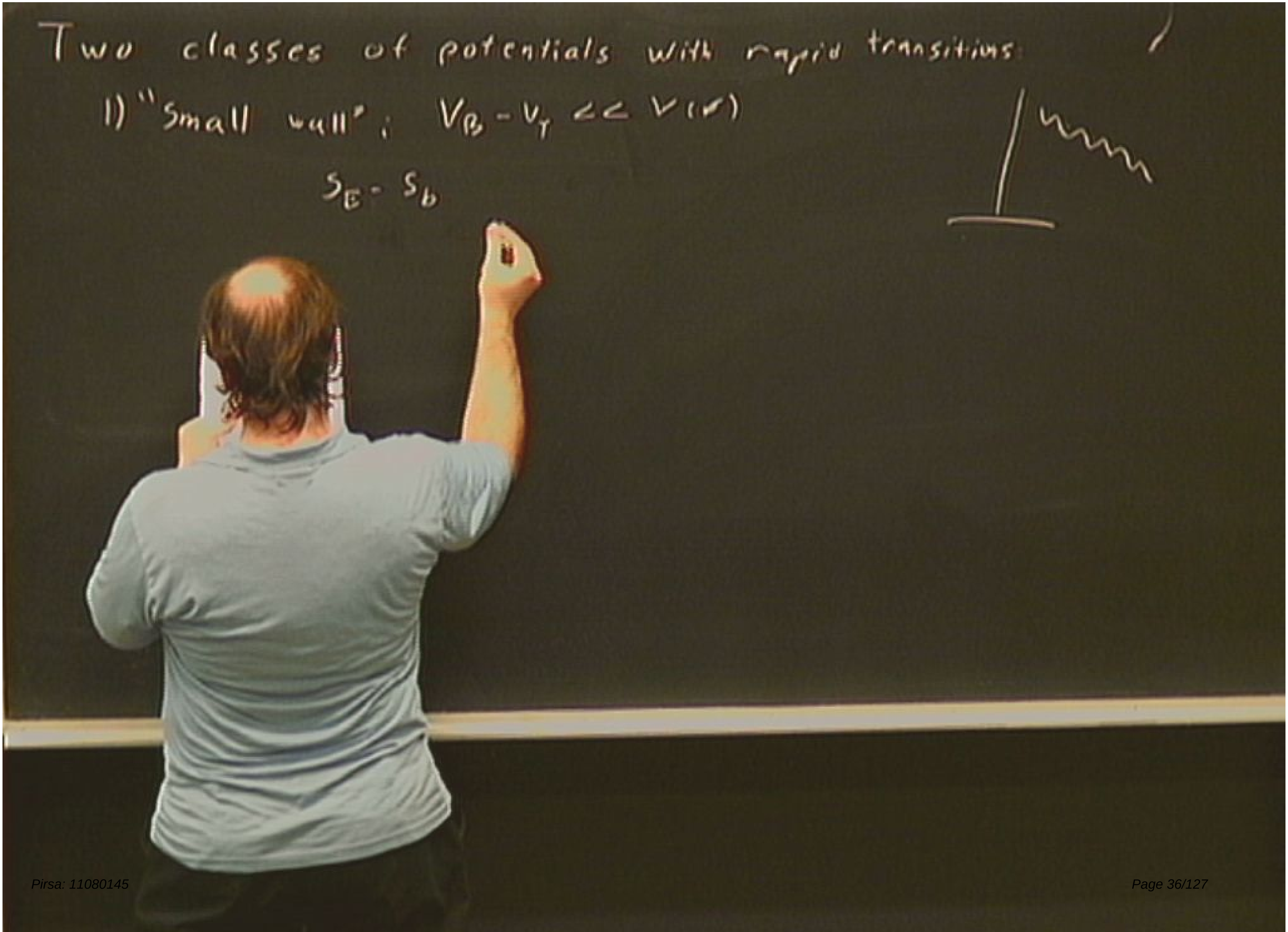
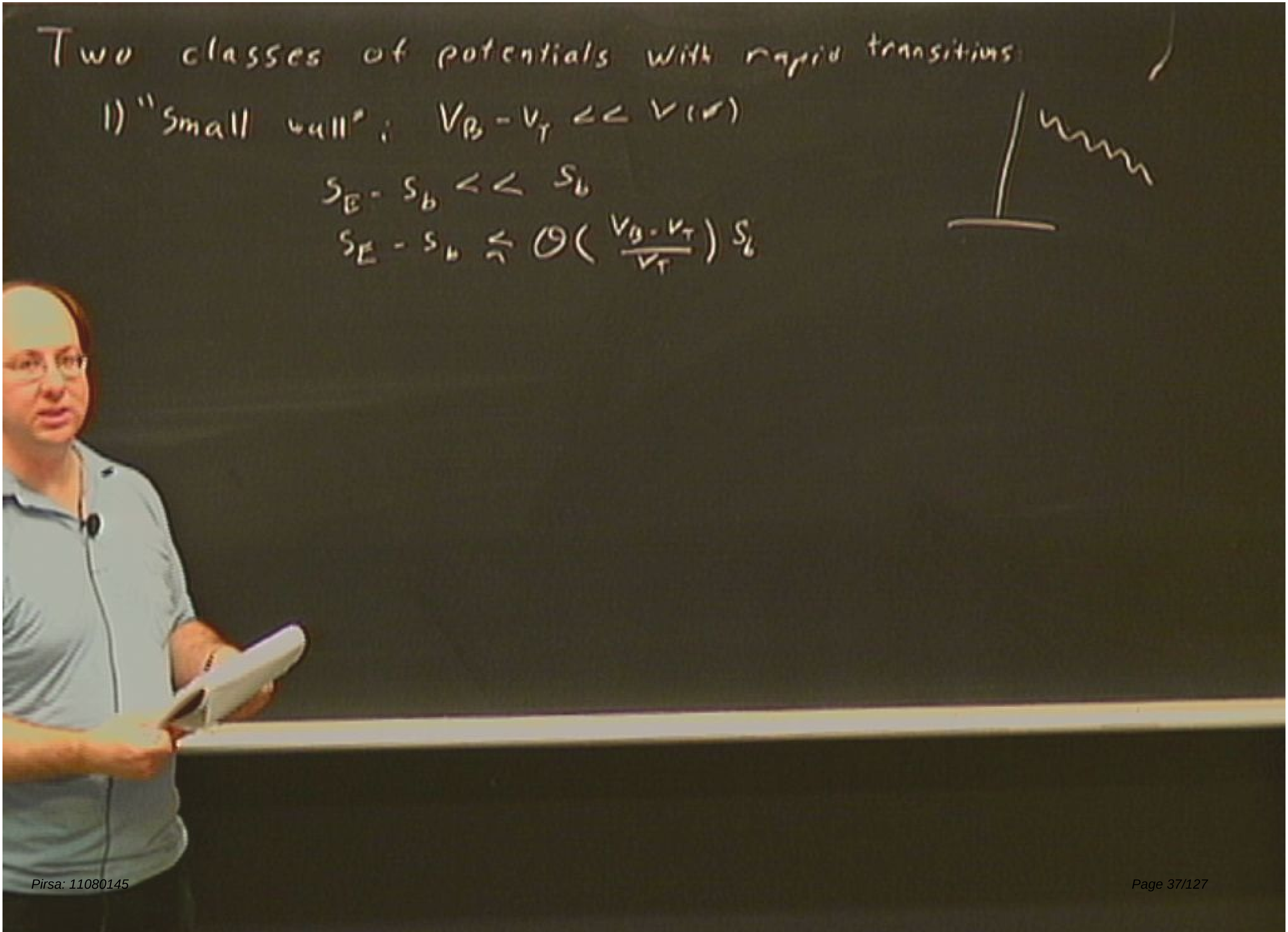

Two classes of potentials with rapid transitions:
1) "Small wall": V

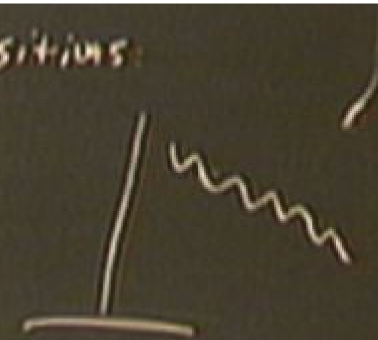

Two classes of potentials with rapid transitions
1) "Small wall": $V_B - V_T \ll V(\phi)$


Two classes of potentials with rapid transitions:
1) "Small wall": V_B - V_T << V(φ)
S_E - S_b


Two classes of potentials with rapid transitions:
1) "Small wall": V_B - V_T << V(φ)
S_E - S_b << S_b
S_E - S_b ≲ O((V_B - V_T)/V_T) S_b

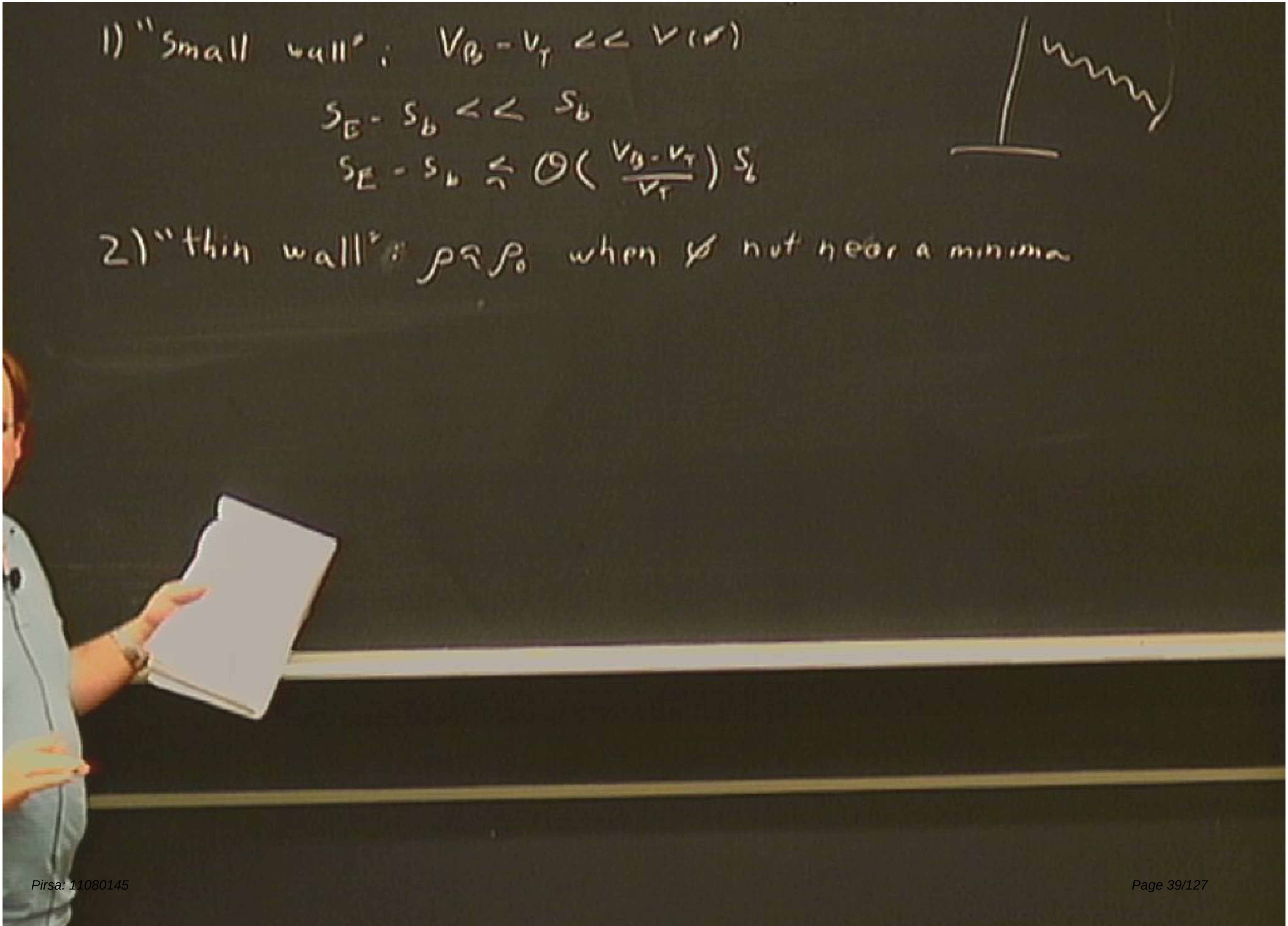
Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T << V(\phi)$



$$S_E - S_b << S_b$$

$$S_E - S_b \lesssim \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



1) "Small wall": $V_B - V_T << V(\phi)$
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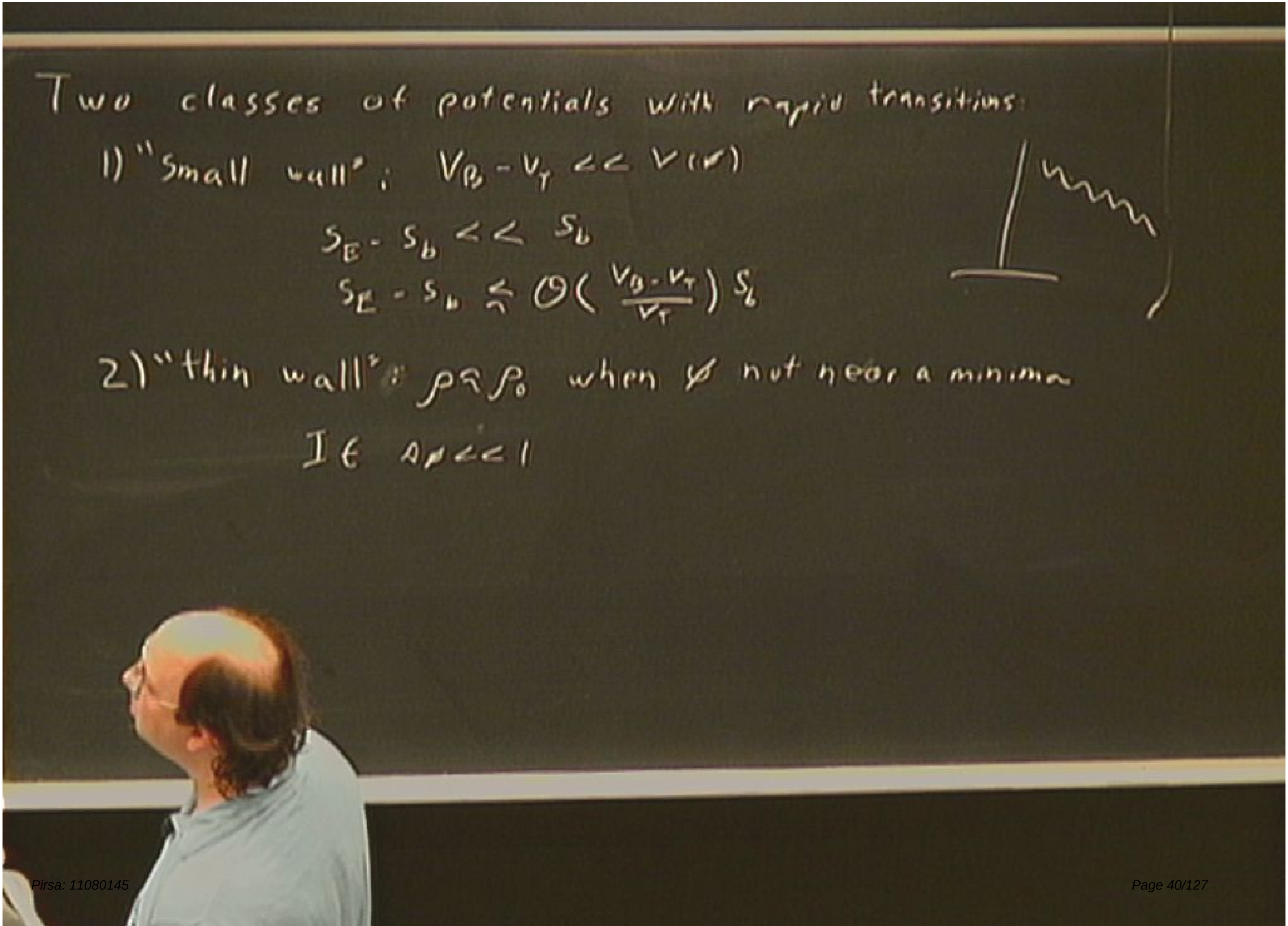
2) "thin wall": $\rho \approx \rho_0$ when ϕ not near a minima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If Δφ << 1

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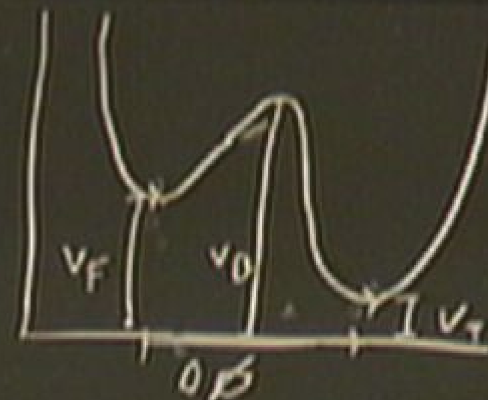

$[\phi] = L^0$

$[V] = \frac{1}{L^2}$

$\kappa = \frac{1}{16\pi G_d}$

$S_E = -\kappa \int \sqrt{g}\,\left(R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right)$

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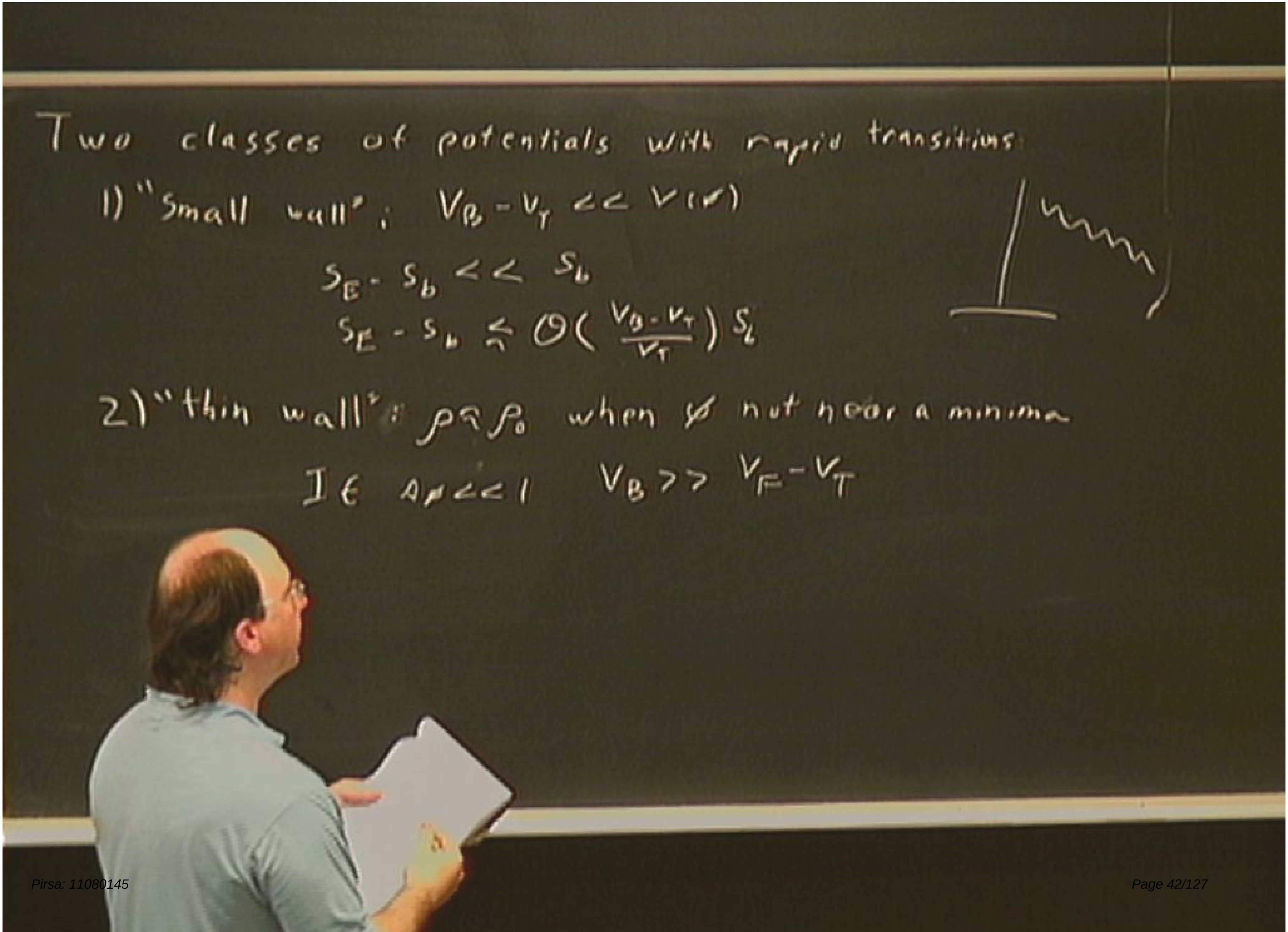
Bousso, Hawking: BH production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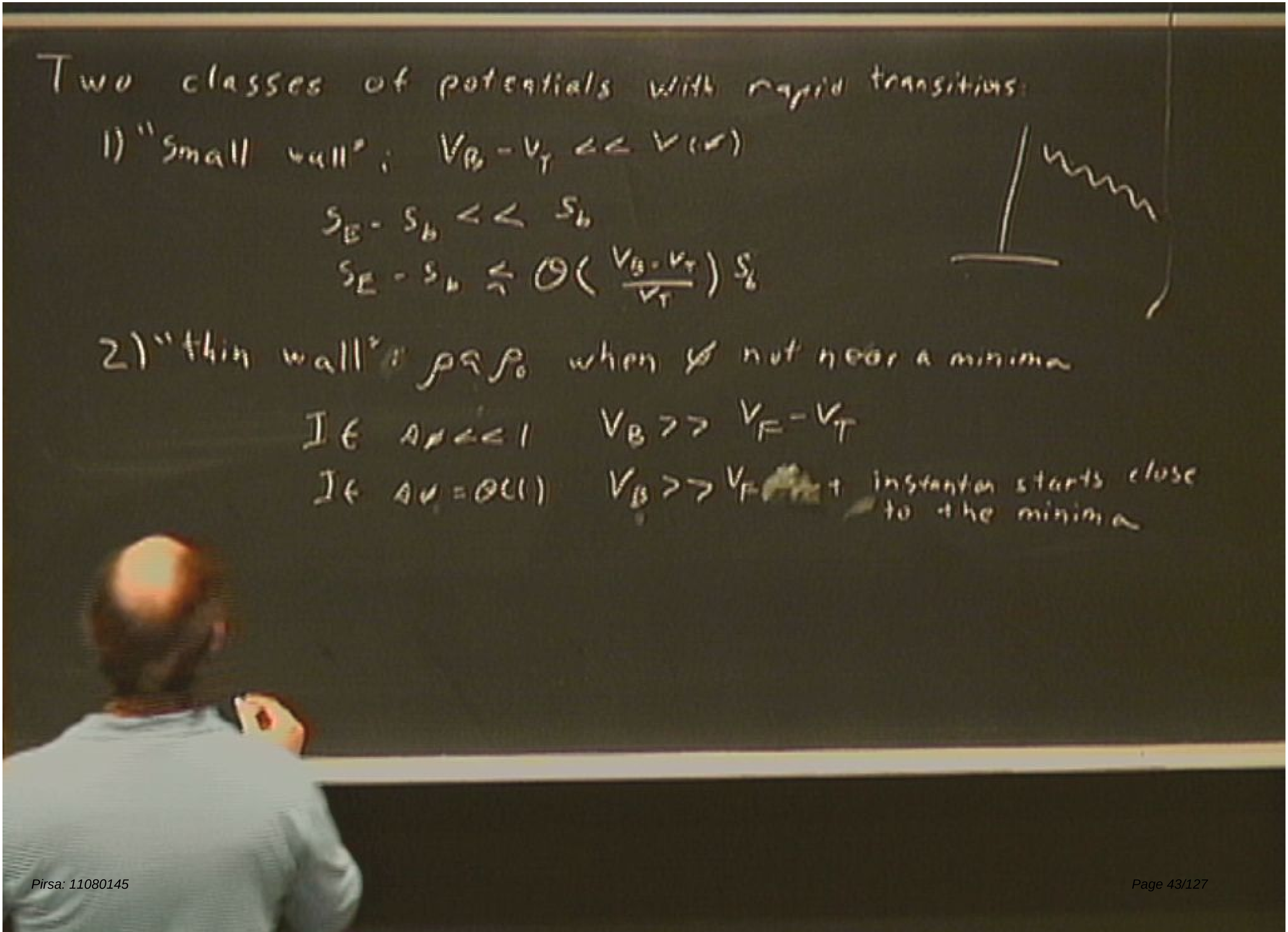
If Δφ << 1   V_B >> V_F - V_T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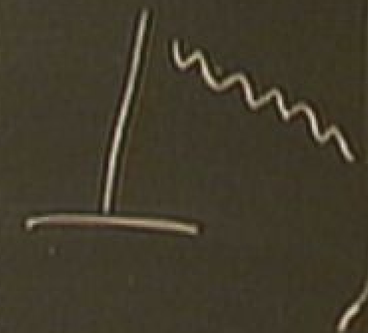
1) "small wall": V_B - V_T << V(φ)
S_E - S_b << S_b
S_E - S_b ≲ Θ((V_B - V_T)/V_T) S_b
2) "thin wall": ρ ≈ ρ_0 when φ not near a minima
If Δφ << 1   V_B >> V_F - V_T
If Δφ = Θ(1)   V_B >> V_F   + instanton starts close to the minima

Two classes of potentials with rapid transitions:

1) "small wall": $V_B - V_T \ll V(\phi)$



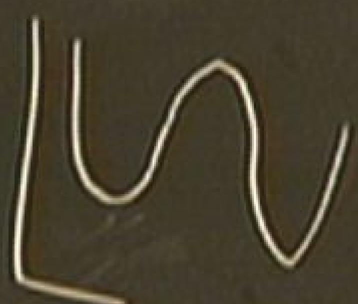
$$S_E - S_b \ll S_b$$

$$S_E - S_b \lesssim \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$

2) "thin wall": $\rho \approx \rho_0$ when ϕ not near a minima

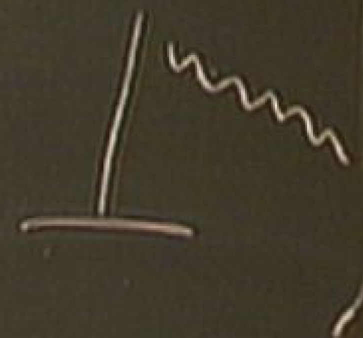
If $\Delta\phi \ll 1$ $\quad V_B \gg V_F - V_T$

If $\Delta\phi = \mathcal{O}(1)$ $\quad V_B \gg V_F$ + instanton starts close to the minima



Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T << V(\phi)$



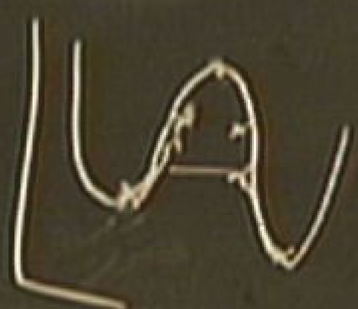
$$S_E - S_b << S_b$$

$$S_E - S_b \lesssim \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$

2) "thin wall": $\rho \approx \rho_0$ when ϕ not near a minima

If $\Delta\phi << 1$ $\quad V_B >> V_F - V_T$

If $\Delta\phi = \mathcal{O}(1)$ $\quad V_B >> V_F$ [illegible] + instanton starts close to the minima



Two classes of potentials with rapid transitions:

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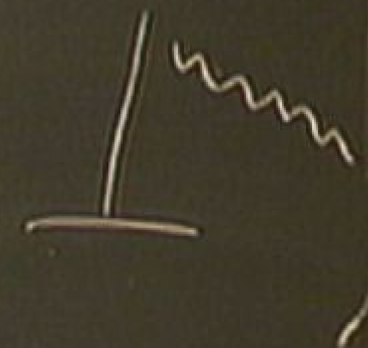

$$S_E - S_b << S_b$$

$$S_E - S_b \lesssim \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$

2) "thin wall": $\rho \simeq \rho_0$ when ϕ not near a minima

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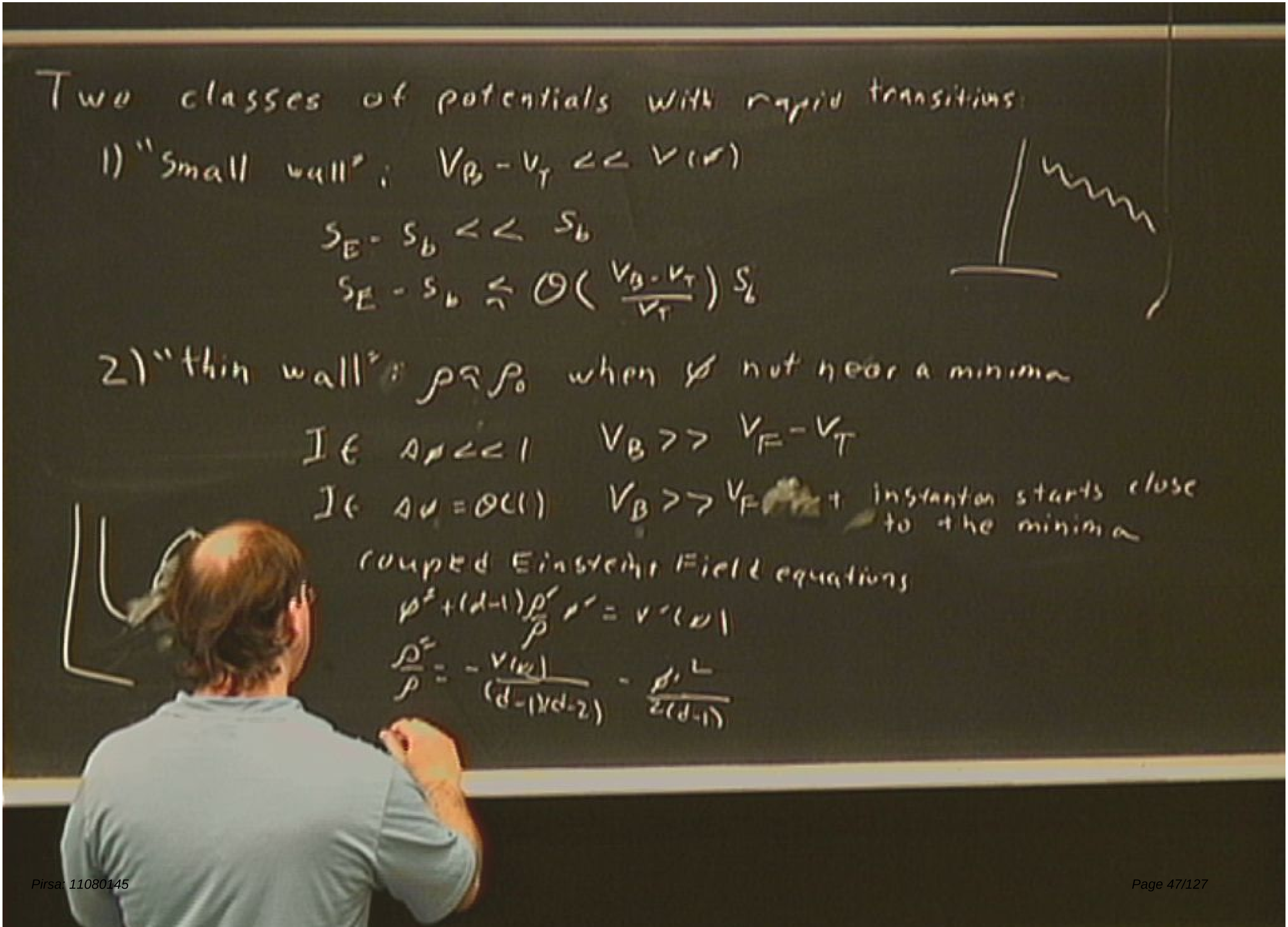

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If $\Delta\phi << 1$ $V_B >> V_F - V_T$
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coupled Einstein Field equations
$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$
$\frac{\rho''}{\rho} = -\frac{V(\phi)}{(d-1)(d-2)} - \frac{\phi'^2}{2(d-1)}$

Two classes of potentials with rapid transitions:

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coupled Einstein Field equations

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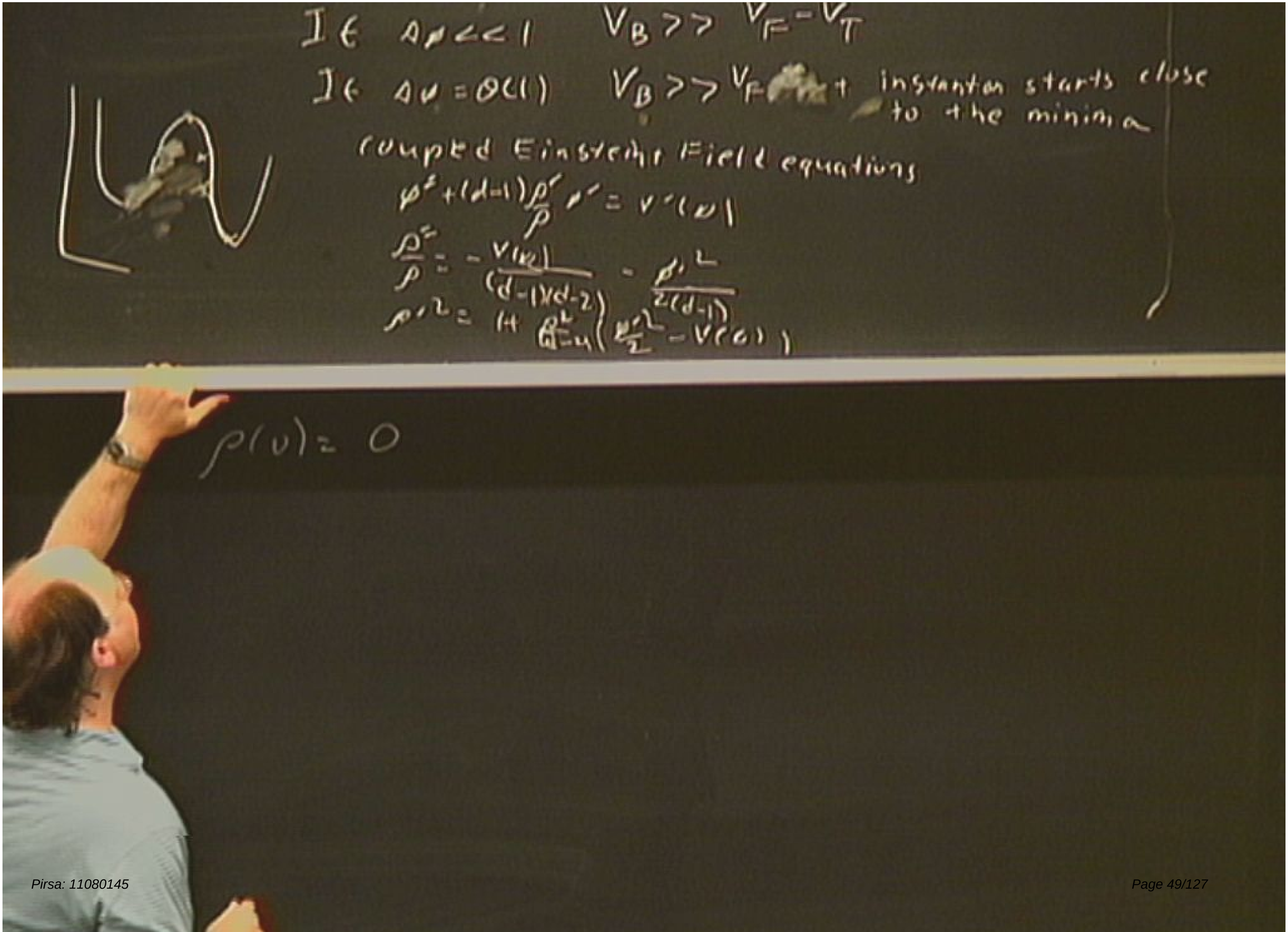
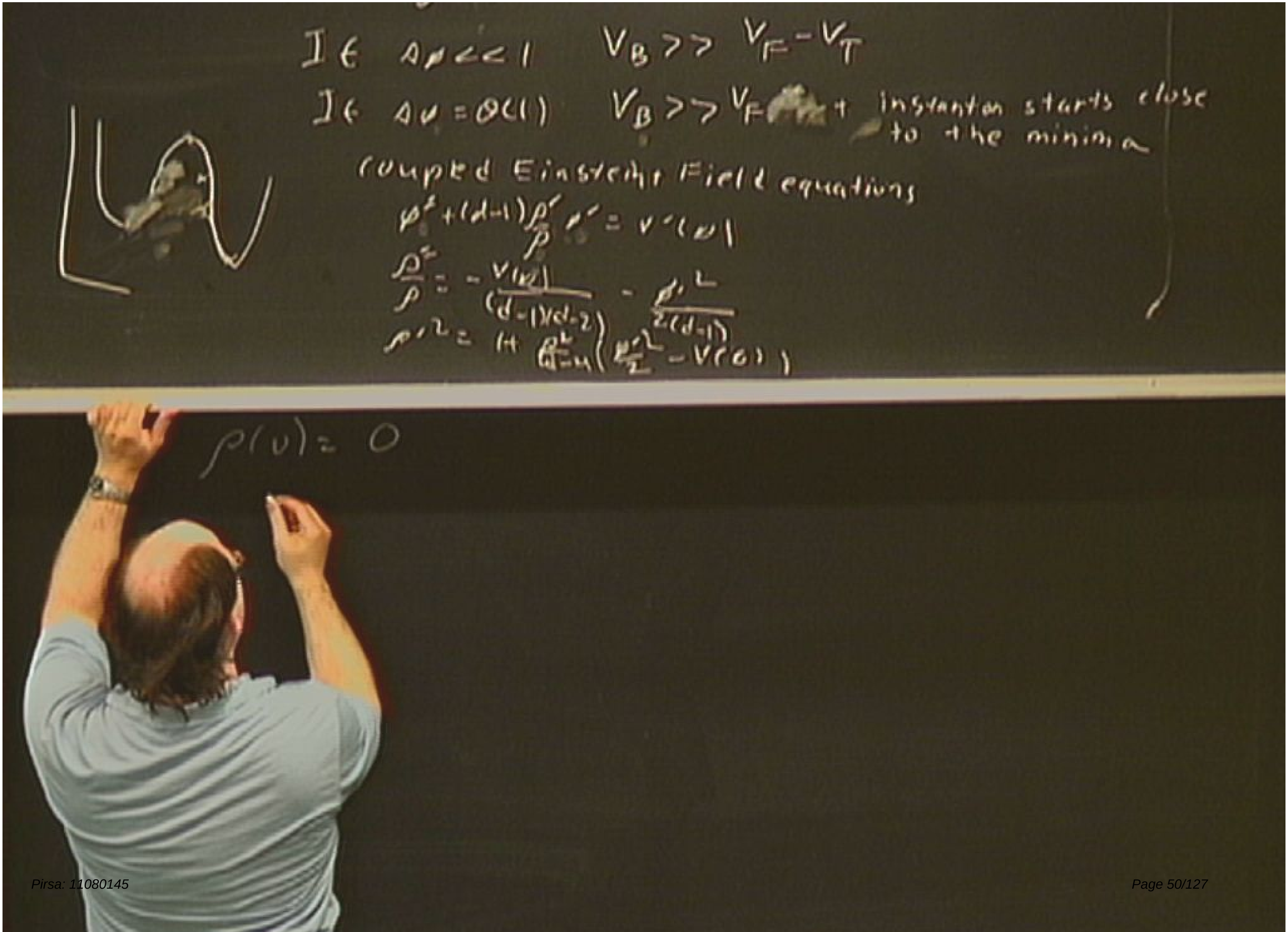
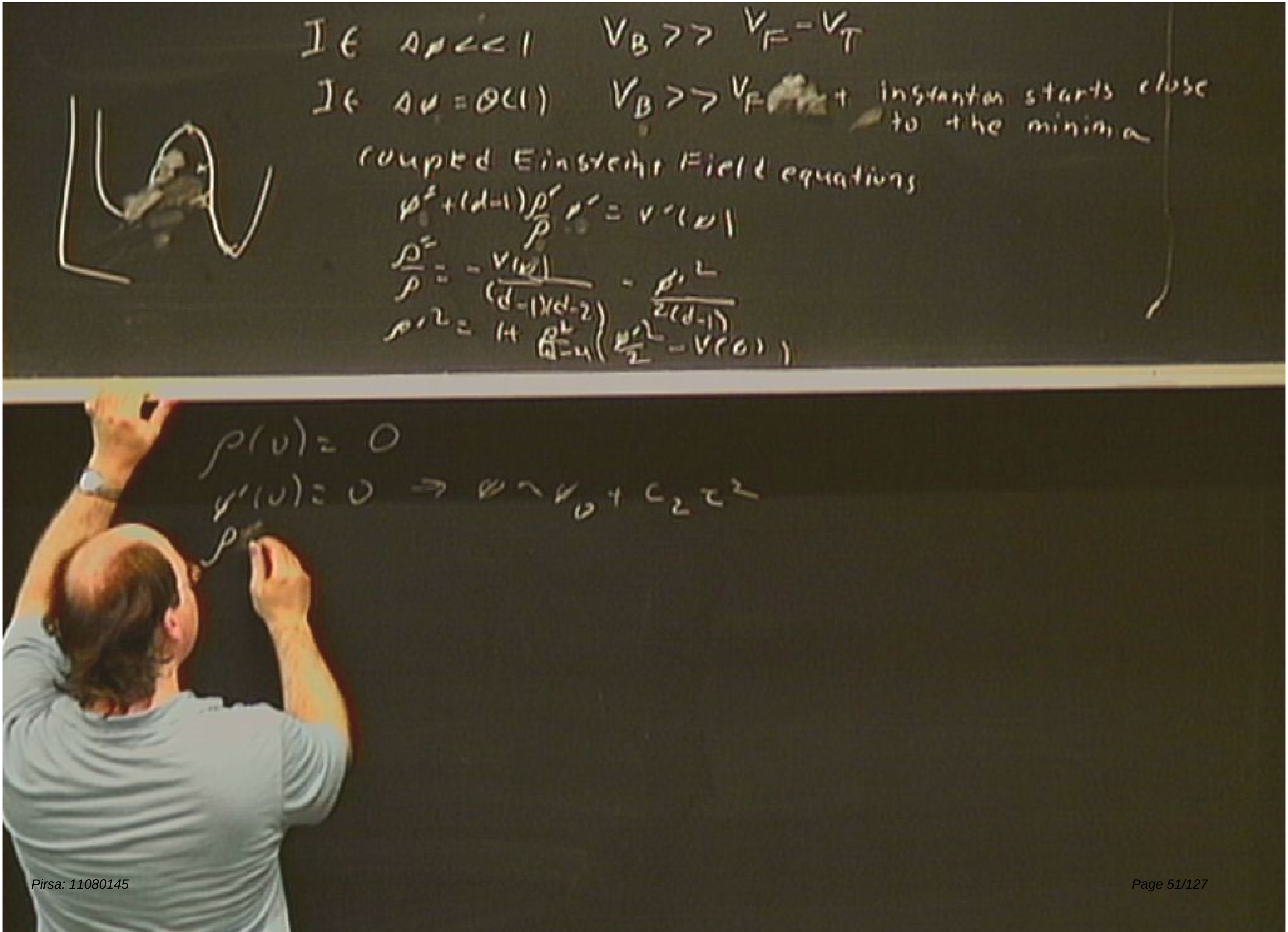
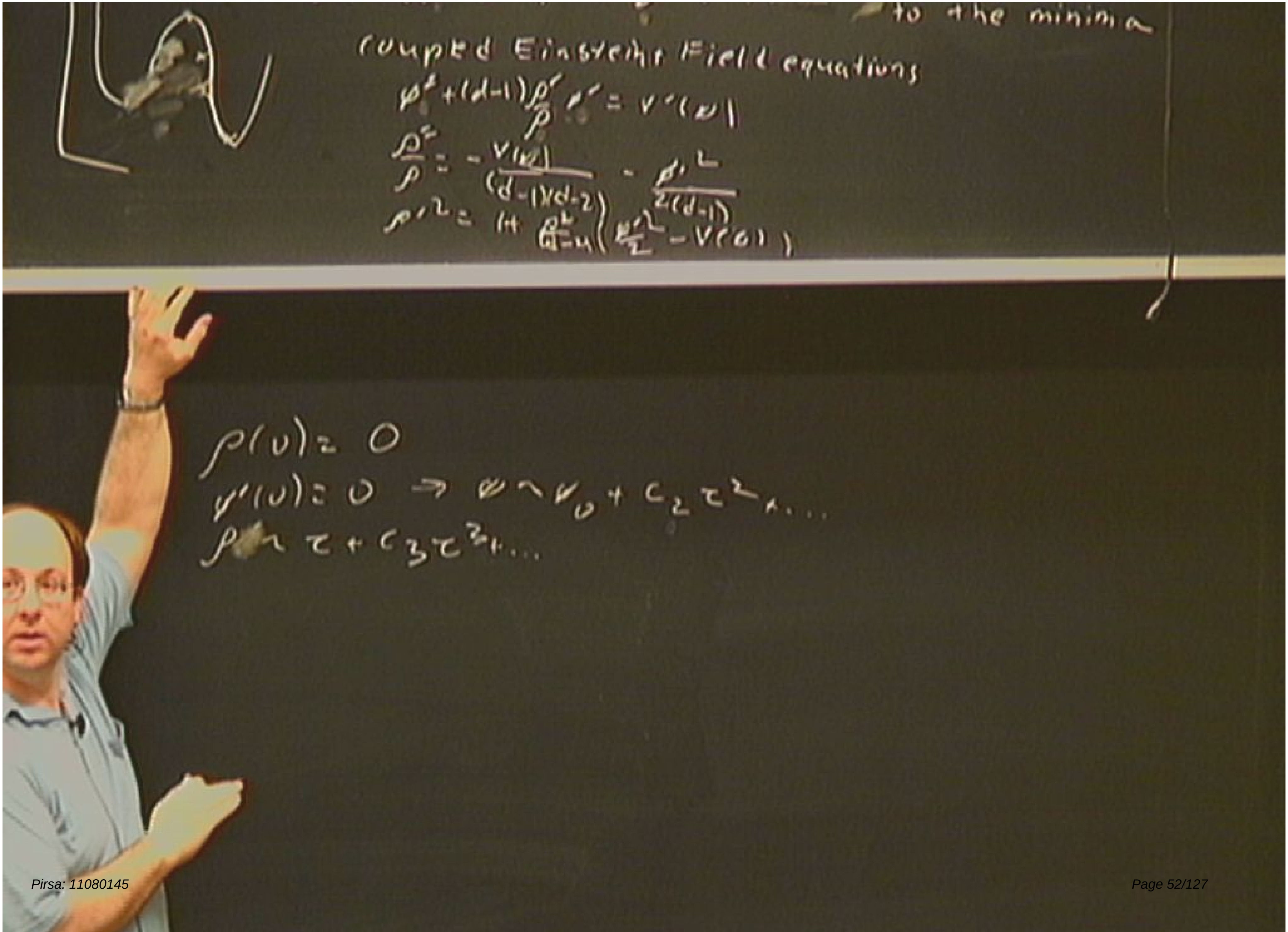
$\frac{\rho''}{\rho} = -\frac{V(\phi)}{(d-1)(d-2)} - \frac{\phi'^2}{2(d-1)}$

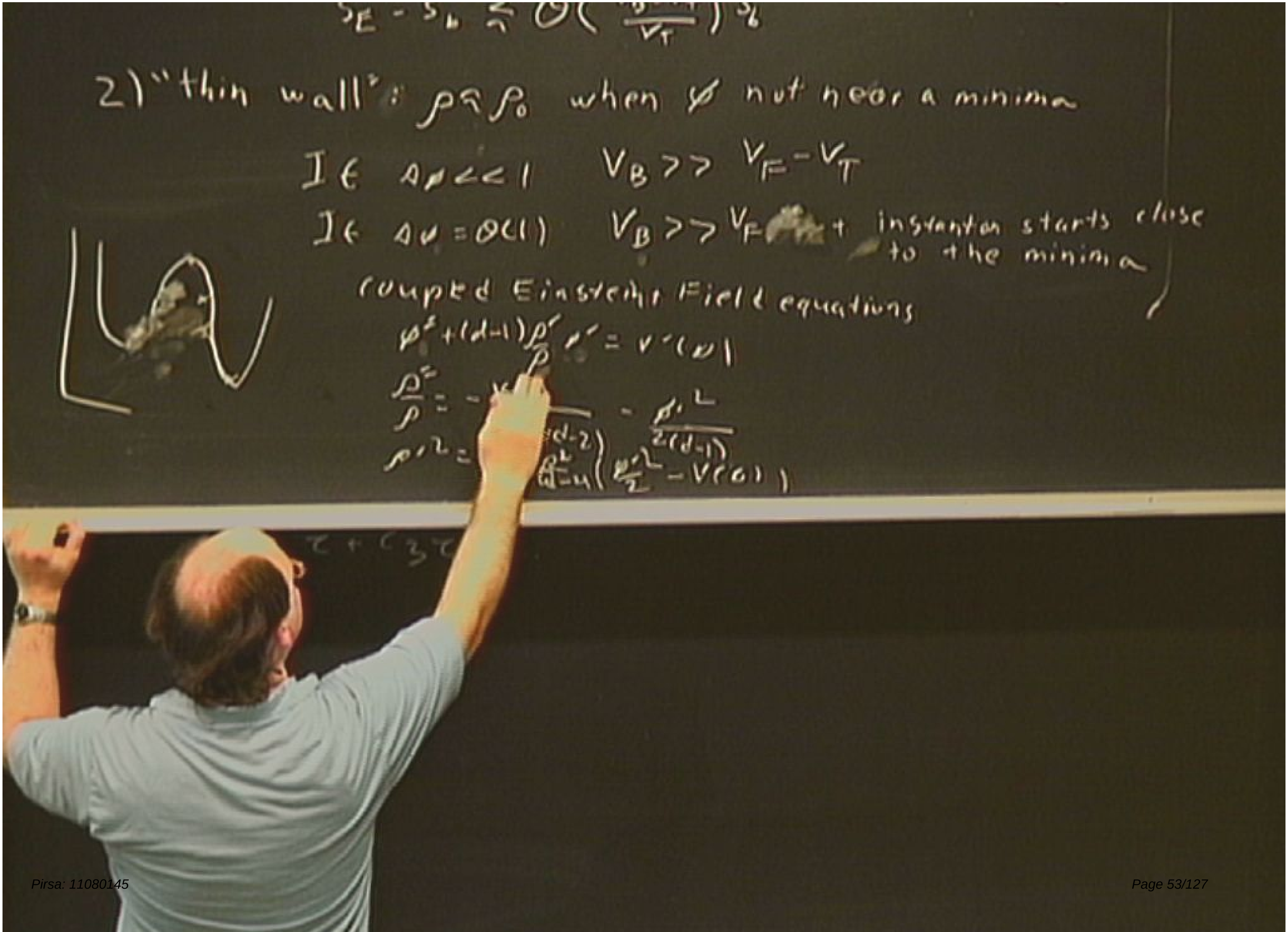
$\rho'^2 = 1 + \frac{\rho^2}{(d-1)(d-2)}\left(\frac{\phi'^2}{2} - V(\phi) \right)$

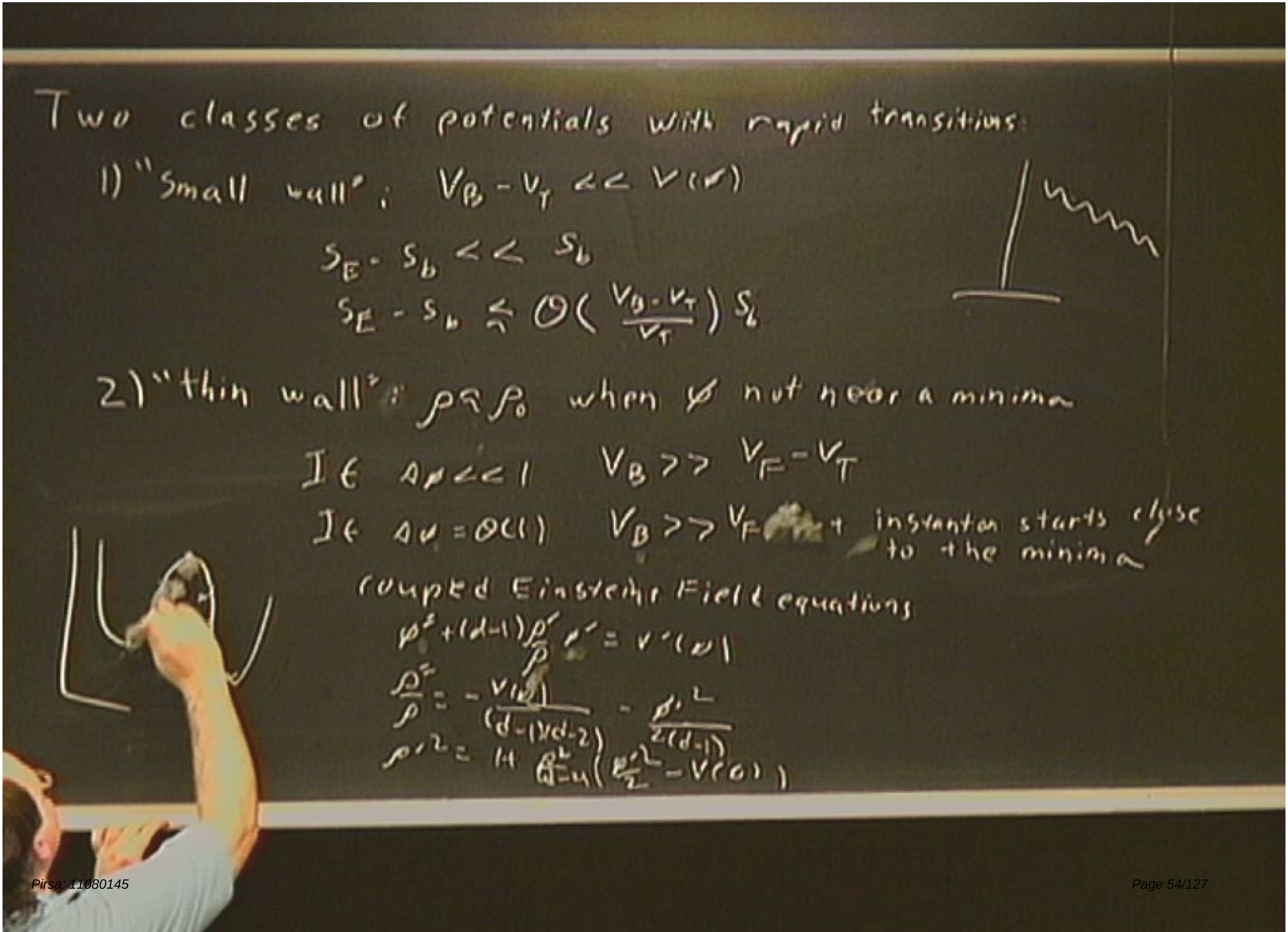

If Δφ << 1   V_B >> V_F − V_T
If Δφ = O(1)   V_B >> V_F + instanton starts close to the minima
coupled Einstein Field equations
φ'' + (d−1) ρ'/ρ φ' = V'(φ)
ρ''/ρ = − V(φ)/((d−1)(d−2)) − φ'^2/(2(d−1))
ρ(0) = 0


If Δφ << 1   V_B >> V_F − V_T
If Δφ = O(1)   V_B >> V_F
+ instanton starts close to the minima
Coupled Einstein Field equations
ρ(0) = 0


I ε Δφ << 1 V_B >> V_F - V_T
I ε Δφ = O(1) V_B >> V_F
+ instanton starts close to the minima
coupled Einstein Field equations
φ'' + (d-1) ρ'/ρ φ' = V'(φ)
ρ''/ρ = - V(φ)/((d-1)(d-2)) - φ'^2/(2(d-1))
ρ(0) = 0
φ'(0) = 0 → φ ~ φ_0 + c_2 τ^2


to the minima
coupled Einstein Field equations
$\rho(0) = 0$
$\varphi'(0) = 0 \Rightarrow \varphi \sim \varphi_0 + c_2 \tau^2 + \dots$
$\rho \sim \tau + c_3 \tau^3 + \dots$


2) "thin wall": ρ ≈ ρ₀ when φ not near a minima
If Δφ << 1 V_B >> V_F - V_T
If Δφ = O(1) V_B >> V_F
instanton starts close to the minima
coupled Einstein Field equations
φ'' + (d-1) ρ'/ρ φ' = V'(φ)



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Two classes of potentials with rapid transitions:

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If $\Delta\phi << 1$ $\quad V_B >> V_F - V_T$

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Coupled Einstein Field equations

$$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$$

$$\frac{\rho''}{\rho} = -\frac{V(\phi)}{(d-1)(d-2)} - \frac{\phi'^2}{2(d-1)}$$

$$\rho'^2 = 1 + \frac{\rho^2}{(d-1)}\left(\frac{\phi'^2}{2} - V(\phi)\right)$$

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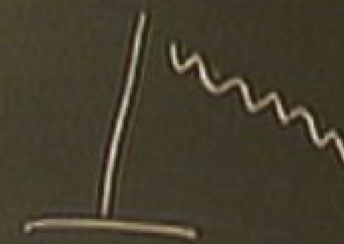

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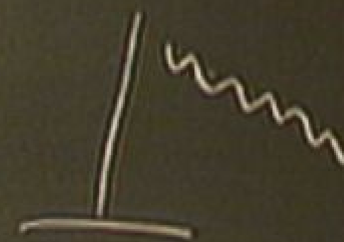

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coupled Einstein Field equations

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$$\rho'^2 = 1 + \frac{\rho^2}{(d-1)}\left(\frac{\phi'^2}{2} - V(\phi)\right)$$

Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T << V(\phi)$

$$S_E - S_b << S_b$$

$$S_E - S_b \lesssim \Theta\left(\frac{V_B - V_T}{V_T}\right) S_b$$

2) "thin wall": $\rho \approx \rho_0$ when ϕ not near a minima

If $\Delta\phi << 1$ $V_B >> V_F - V_T$

If $\Delta\phi = \Theta(1)$ $V_B >> V_F - V_T$ instanton starts close to the minima

coupled Einstein Field equations

$$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$$

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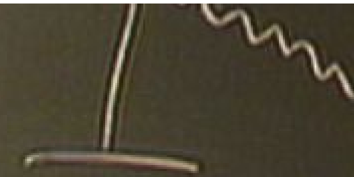

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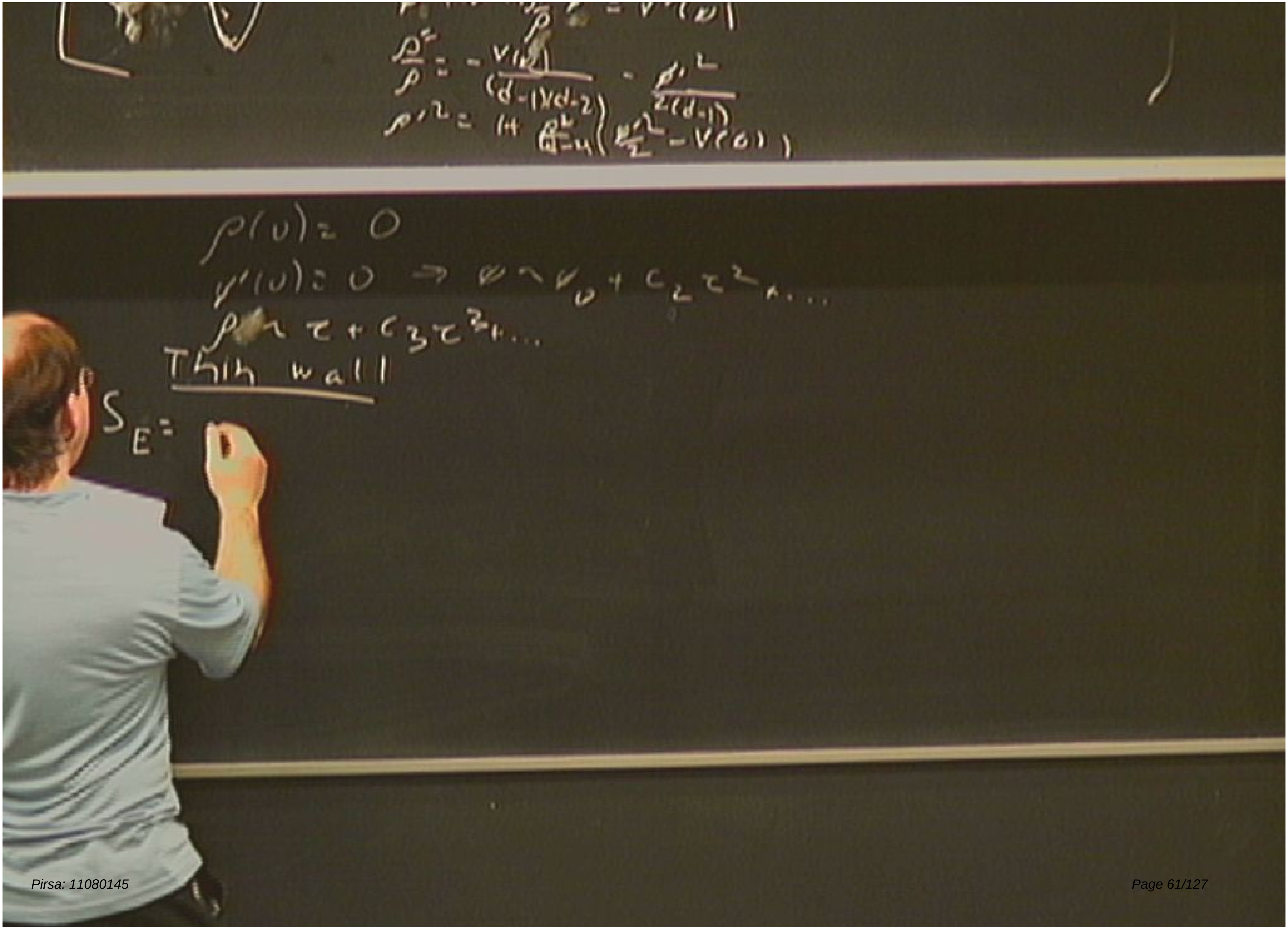
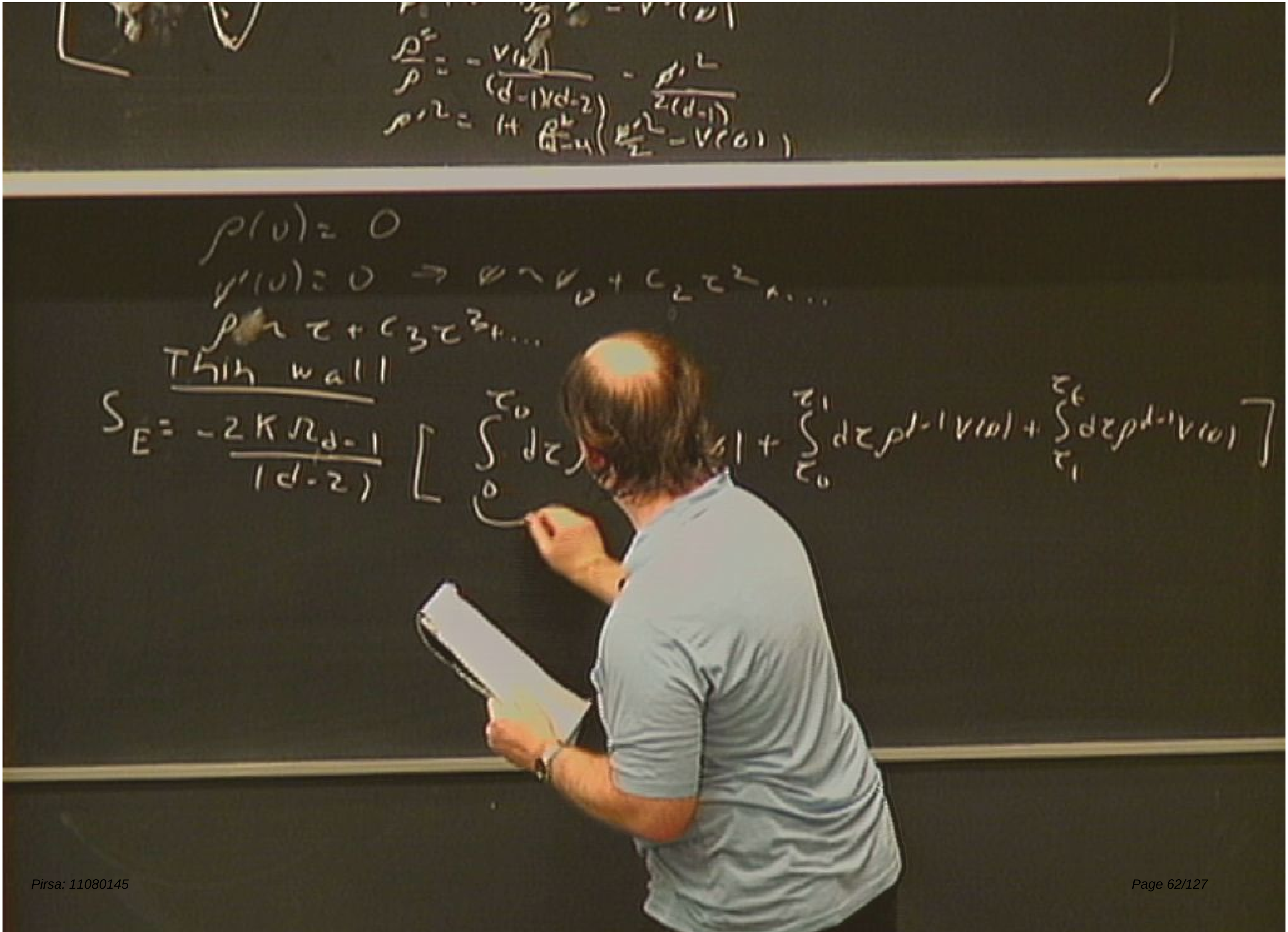
$\rho'^2 = 1 + \frac{\rho^2}{(d-1)}\left(\frac{\phi'^2}{2} - V(\phi)\right)$

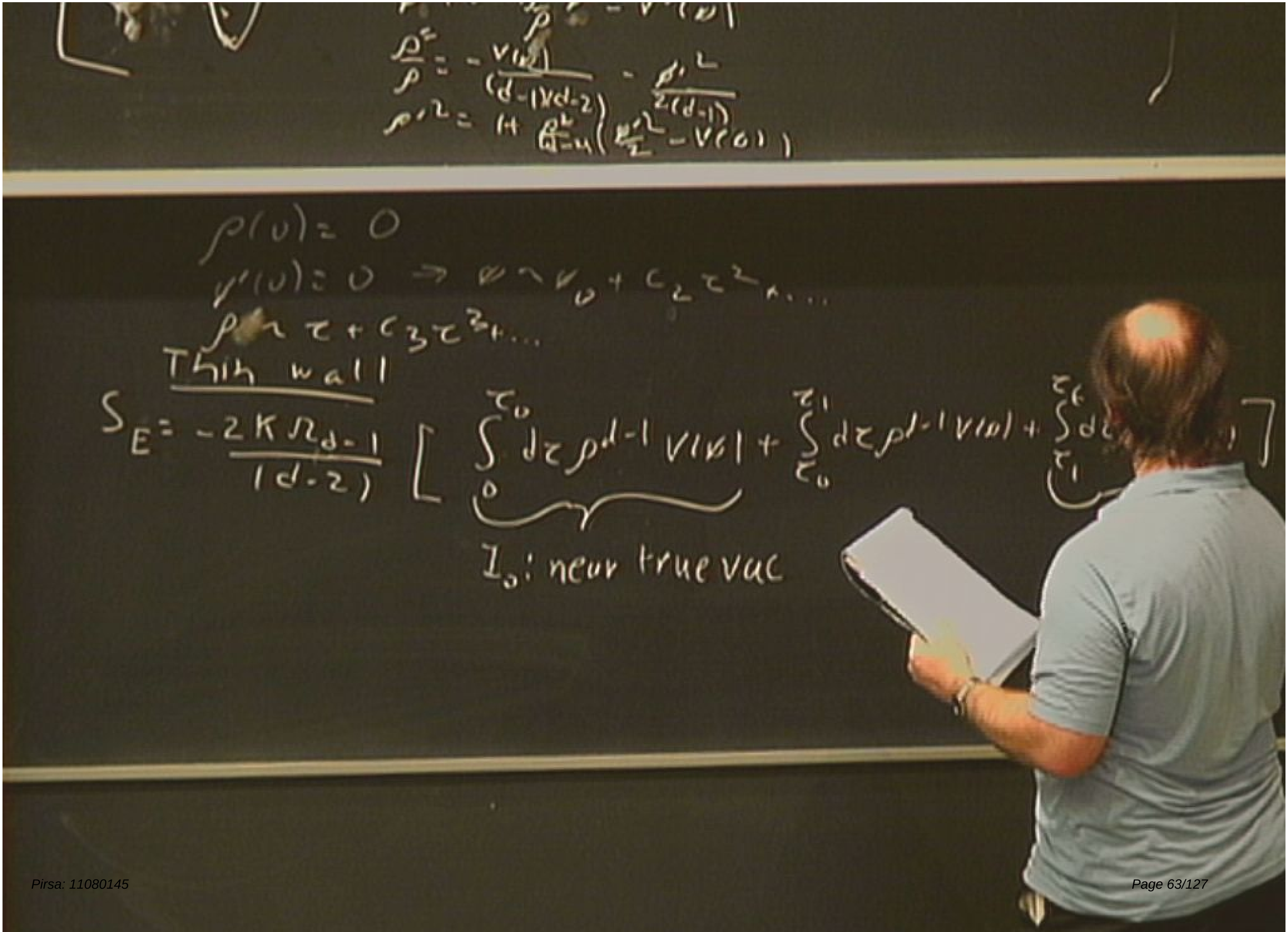
$\rho(0) = 0$

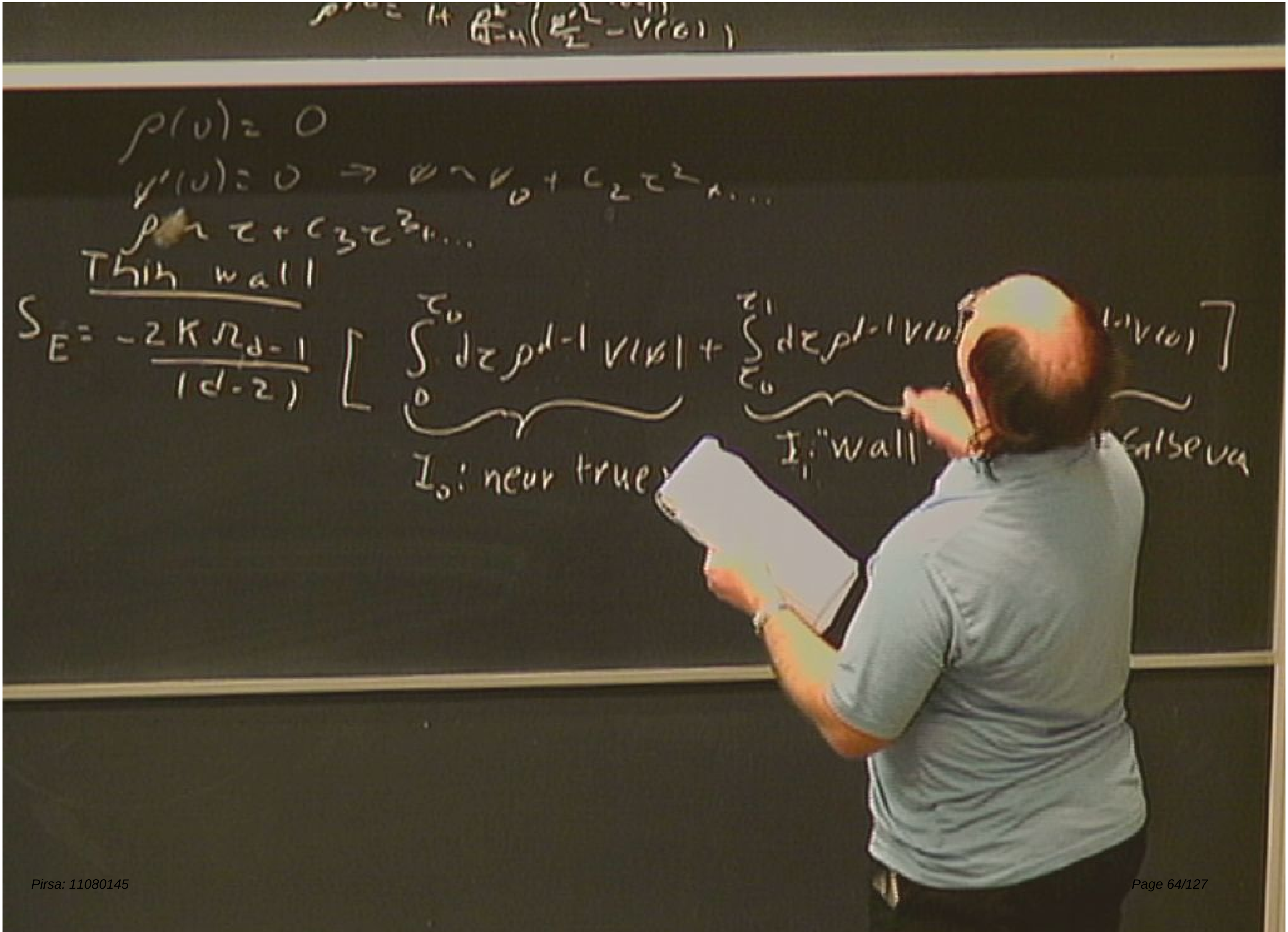
$\phi'(0) = 0 \Rightarrow \phi \sim \phi_0 + c_2\tau^2 + \ldots$

$\rho \sim \tau + c_3\tau^3 + \ldots$


ρ(0) = 0
φ'(0) = 0 → φ ~ φ0 + c2 τ2 + ...
ρ ~ τ + c3 τ3 + ...
Thin wall
SE =


ρ(0) = 0
Thin wall


ρ(0) = 0
ϕ'(0) = 0 → ϕ ~ ϕ_0 + c_2 τ^2 + ...
ρ ~ τ + c_3 τ^3 + ...
Thin wall
S_E = -2K Ω_{d-1} / (d-2) [∫_0^{τ_0} dτ ρ^{d-1} V(ϕ) + ∫_{τ_0}^{τ_1} dτ ρ^{d-1} V(ϕ) + ∫_{τ_1}^{τ_f} ...]
I_0: near true vac


$\rho(0) = 0$
$\phi'(0) = 0 \Rightarrow \phi \sim \phi_0 + c_2 \tau^2 + \dots$
$\rho \sim \tau + c_3 \tau^3 + \dots$
Thin wall
$S_E = -\frac{2K\Omega_{d-1}}{(d-2)} \left[\int_0^{\tau_0} d\tau \rho^{d-1} V(\phi) + \int_{\tau_0}^{\tau_1} d\tau \rho^{d-1} V(\phi) \dots V(\phi) \right]$
I_0: near true
I_1: "wall"

$\rho(0) = 0$

$\phi'(0) = 0 \Rightarrow \phi \sim \phi_0 + c_2 \tau^2 + \dots$

$\rho \sim \tau + c_3 \tau^3 + \dots$

Thin wall

$$S_E = -\frac{2K\Omega_{d-1}}{(d-2)} \left[\underbrace{\int_0^{\tau_0} d\tau\, \rho^{d-1} V(\phi)}_{I_0:\ \text{near true vac}} + \underbrace{\int_{\tau_0}^{\tau_1} d\tau\, \rho^{d-1} V(\phi)}_{I_1:\ \text{"wall"}} + \underbrace{\int_{\tau_1}^{\tau_f} d\tau\, \rho^{d-1} V(\phi)}_{I_2:\ \text{near false vac}} \right]$$

$\rho(0) = 0$

$\varphi'(0) = 0 \Rightarrow \varphi \sim \varphi_0 + c_2 \tau^2 + \ldots$

$\rho \sim \tau + c_3 \tau^3 + \ldots$

Thin wall

$$S_E = \frac{-2K\Omega_{d-1}}{(d-2)} \Big[\underbrace{\int_0^{\tau_0} d\tau\, \rho^{d-1} V(\varphi)}_{I_0:\ \text{near true vac}} + \underbrace{\int_{\tau_0}^{\tau_1} d\tau\, \rho^{d-1} V(\varphi)}_{I_1:\ \text{"wall"}} + \underbrace{\int_{\tau_1}^{\tau_f} d\tau\, \rho^{d-1} V(\varphi)}_{I_2:\ \text{near false vac}} \Big]$$

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If $\Delta\phi = O(1)$ $\quad V_B >> V_F$ [illegible] + instanton starts close to the minima

coupled Einstein Field equations

$$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$$

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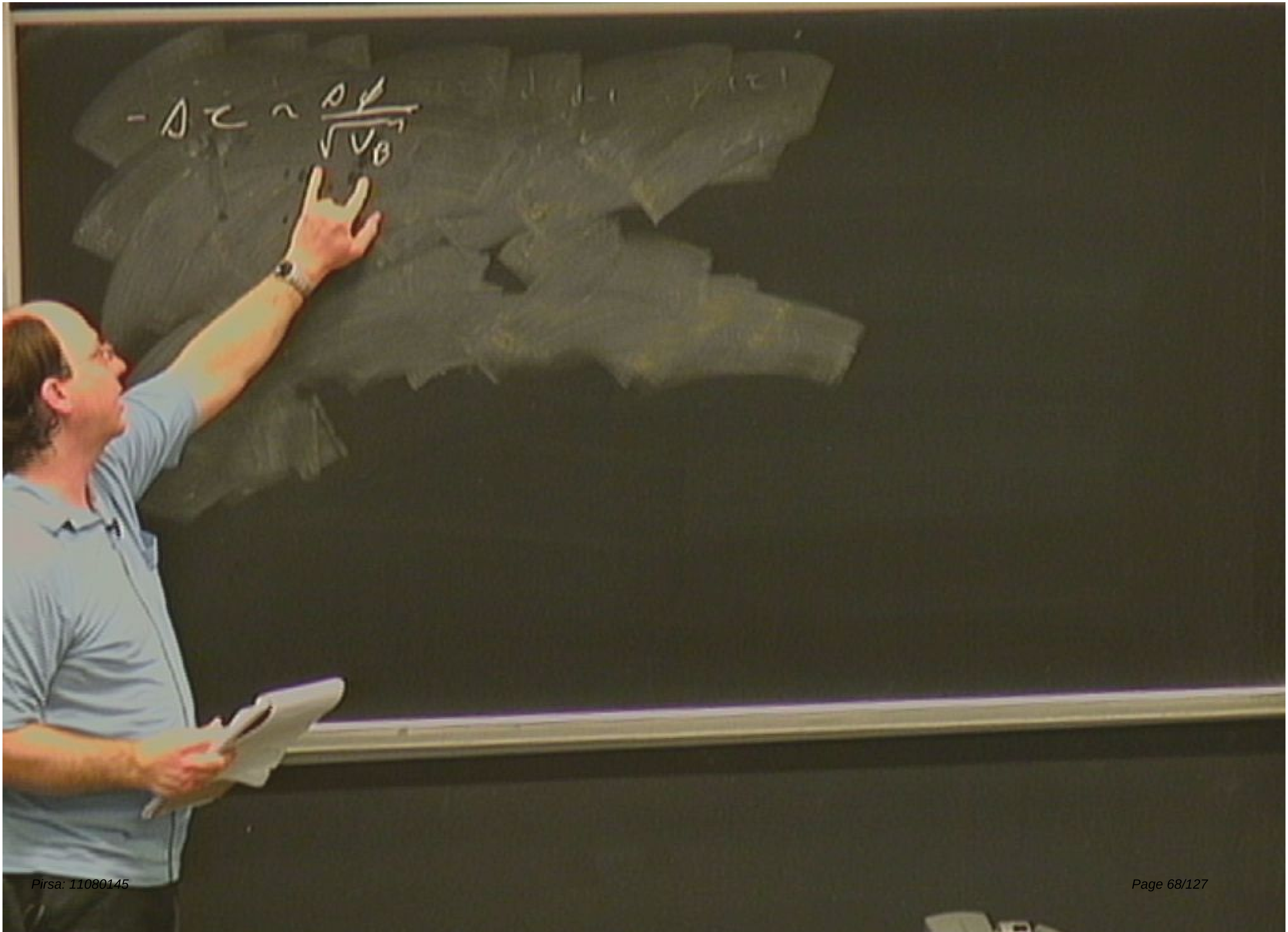
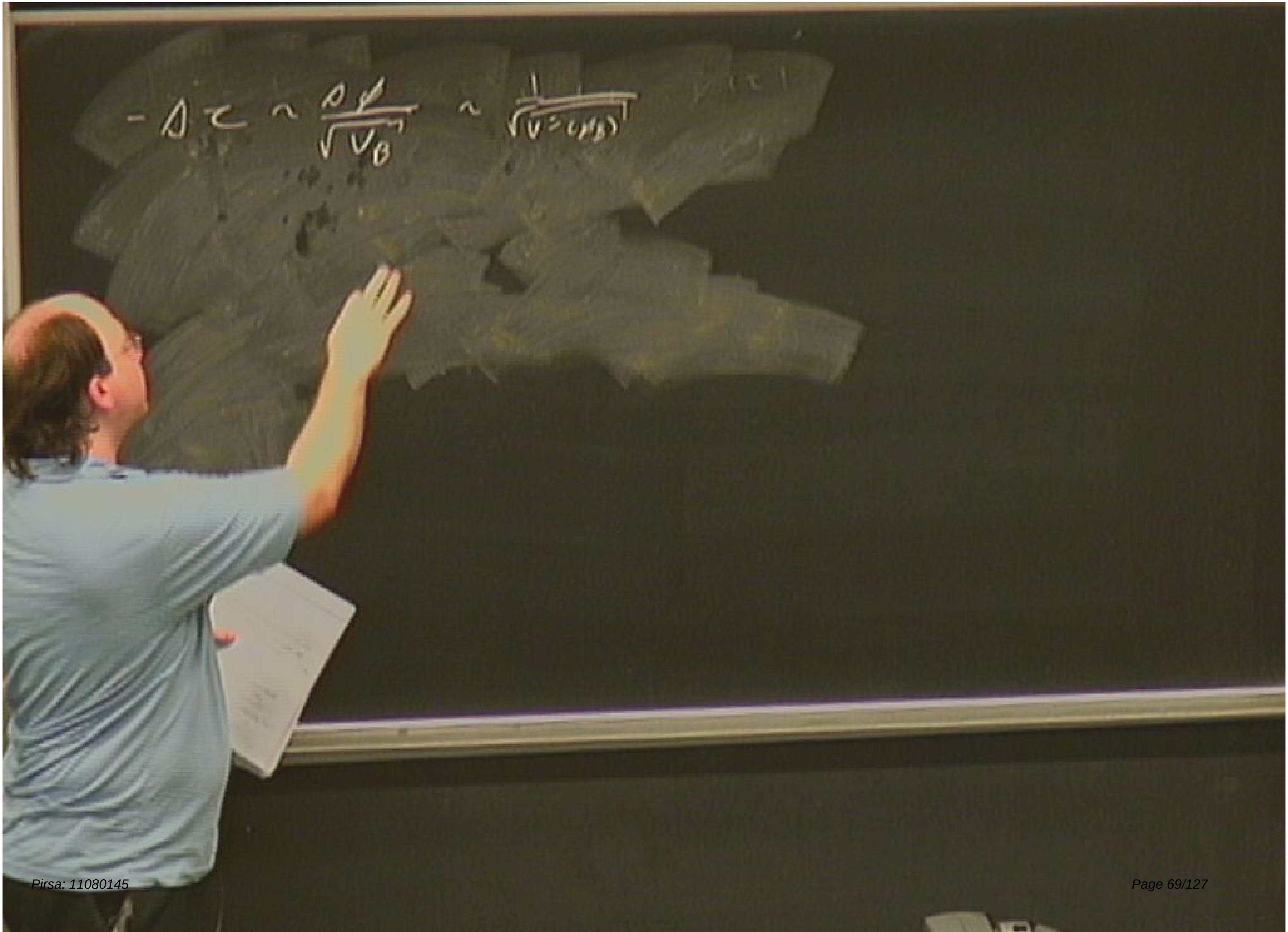
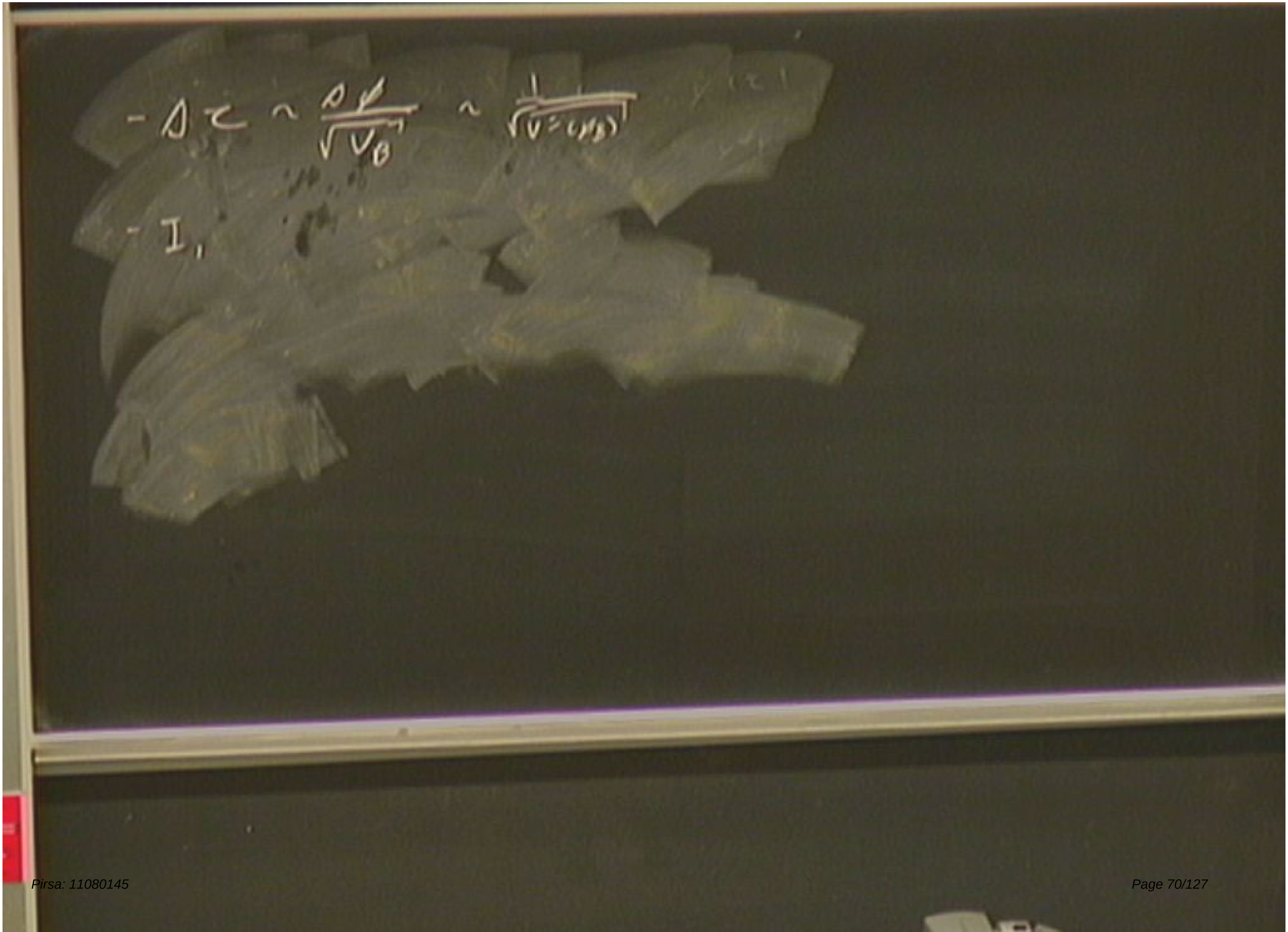
$$\rho'^2 = 1 + \frac{\rho^2}{d-4}\left(\frac{\phi'^2}{2} - V(\phi)\right)$$

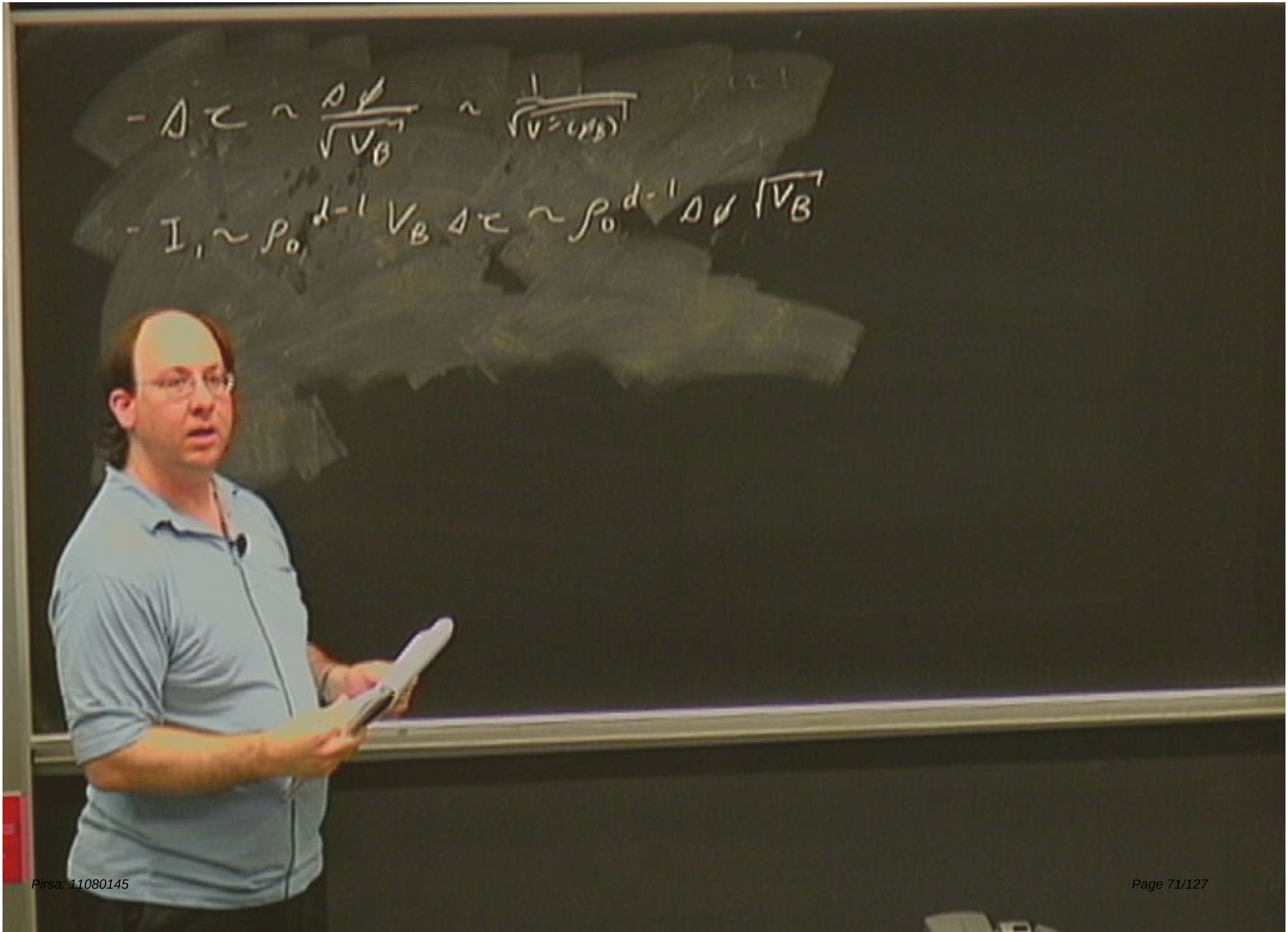
$$\frac{2K\Omega_{d-1}}{(d-2)}\left[\underbrace{\int_0^{\tau_0} d\tau\, \rho^{d-1} V(\phi)}_{I_0:\ \text{near true vac}} + \underbrace{\int_{\tau_0}^{\tau_1} d\tau\, \rho^{d-1} V(\phi)}_{I_1:\ \text{"wall"}} + \underbrace{\int_{\tau_1}^{\tau_f} d\tau\, \rho^{d-1} V(\phi)}_{I_2:\ \text{near false vac}}\right]$$

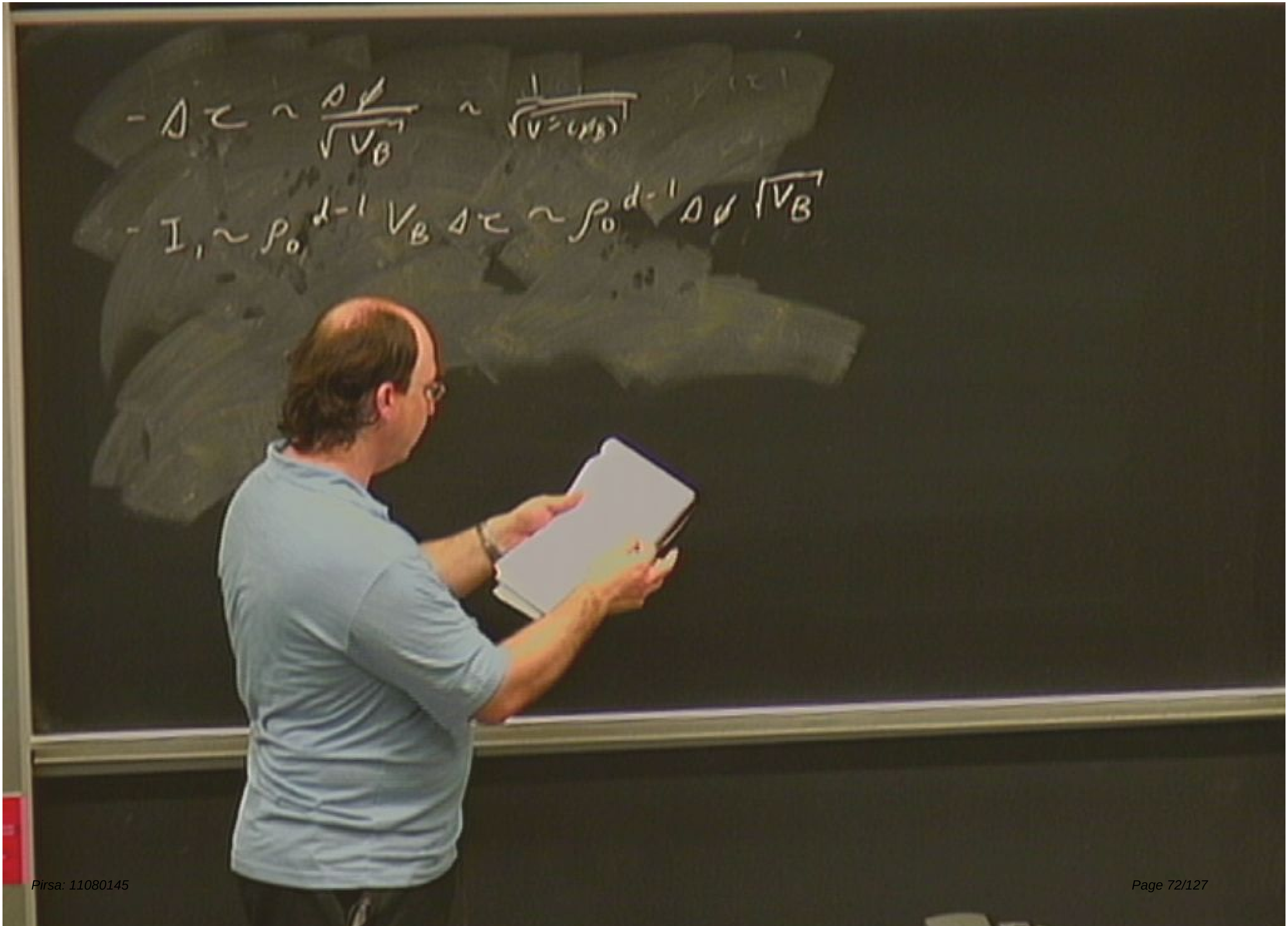

$-\Delta\tau \sim \frac{\Delta \ell}{\sqrt{V_B}}$



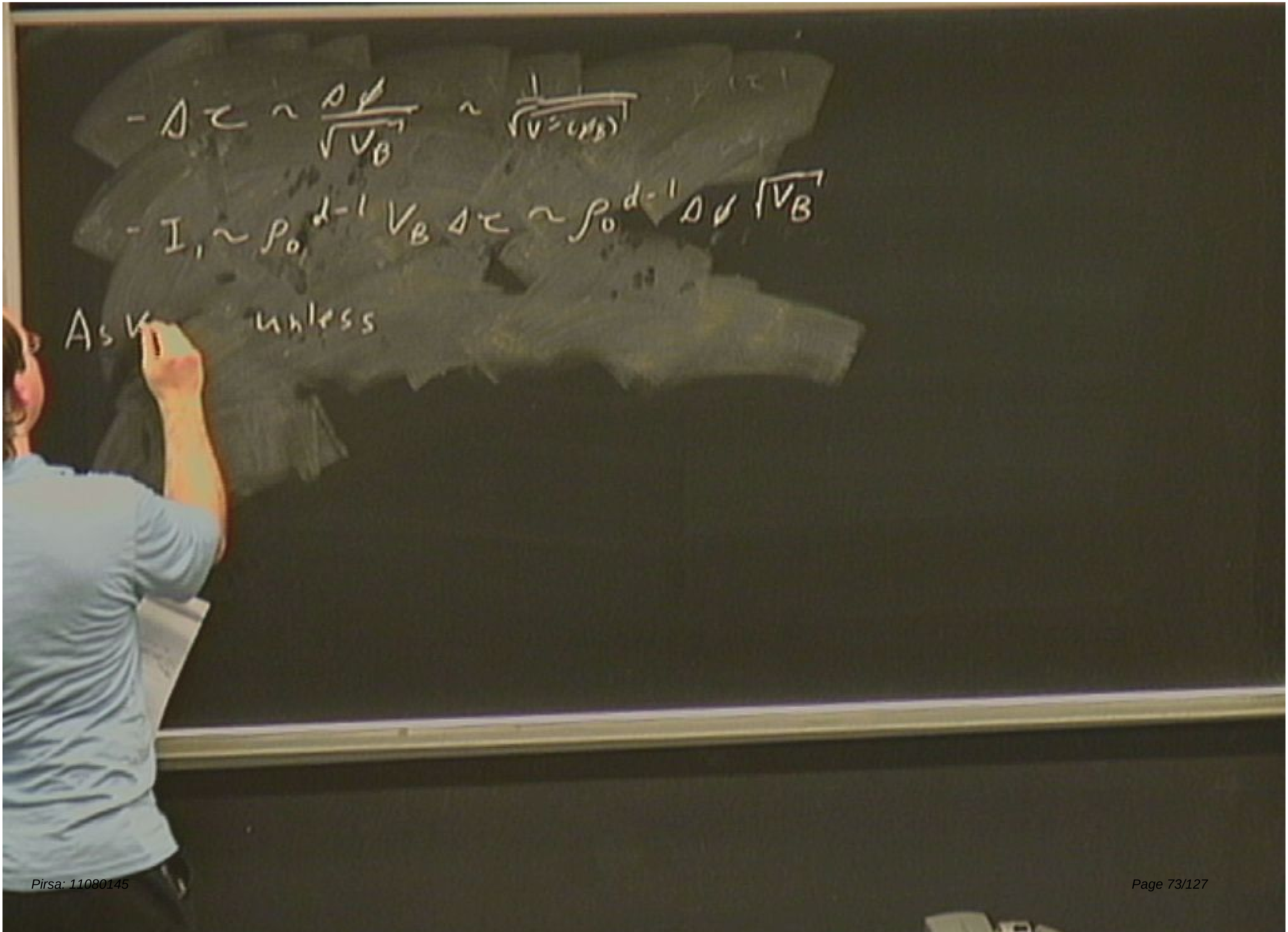


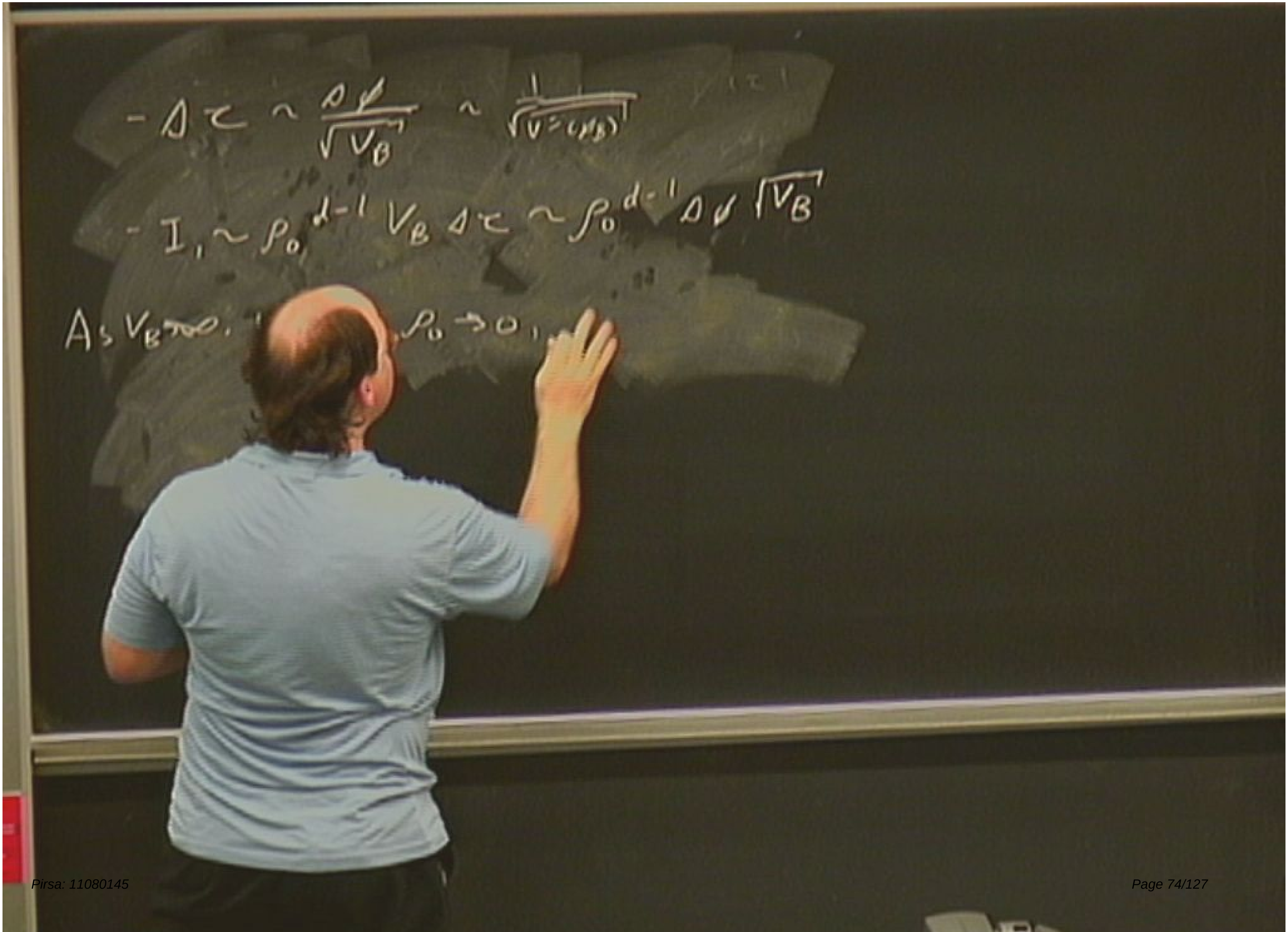
- $\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_0}} \sim \frac{1}{\sqrt{V=O(1)}}$
- I_1


$- \Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$
$- I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$



$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$
$-I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$


- Δτ ~ Δφ/√V_B ~ 1/√(V − V(φ_B))
- I_1 ~ ρ_0^(d-1) V_B Δτ ~ ρ_0^(d-1) Δφ √V_B
As V unless


$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V'(\phi_B)}}$
$-I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$
As $V_B \to 0$, $\rho_0 \to 0$,

$\rho(0) = 0$

$\phi'(0) = 0 \rightarrow \phi \sim \phi_0 + c_2 \tau^2 + \ldots$

$\rho \sim \tau + c_3 \tau^3 + \ldots$

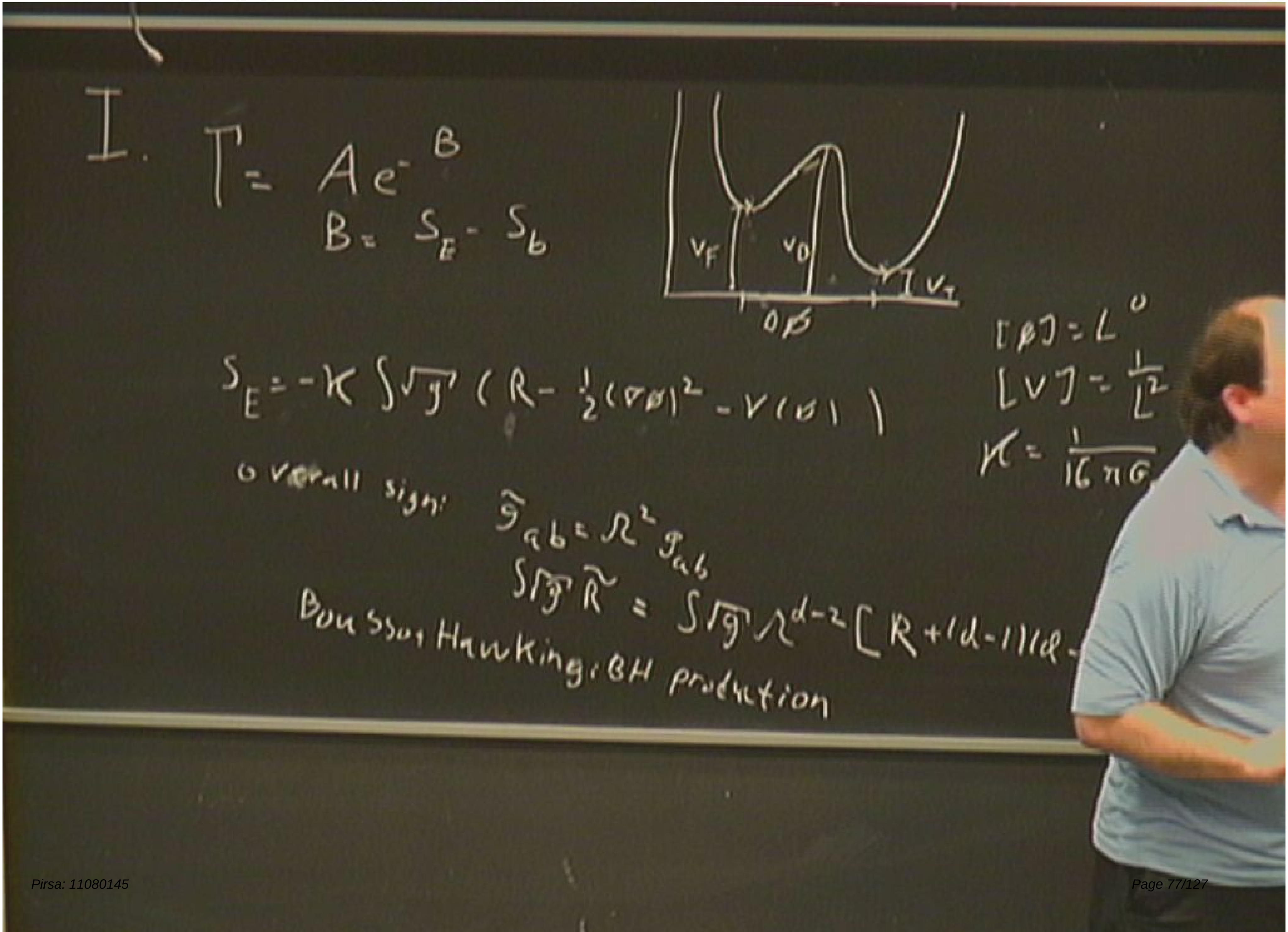
Thin wall

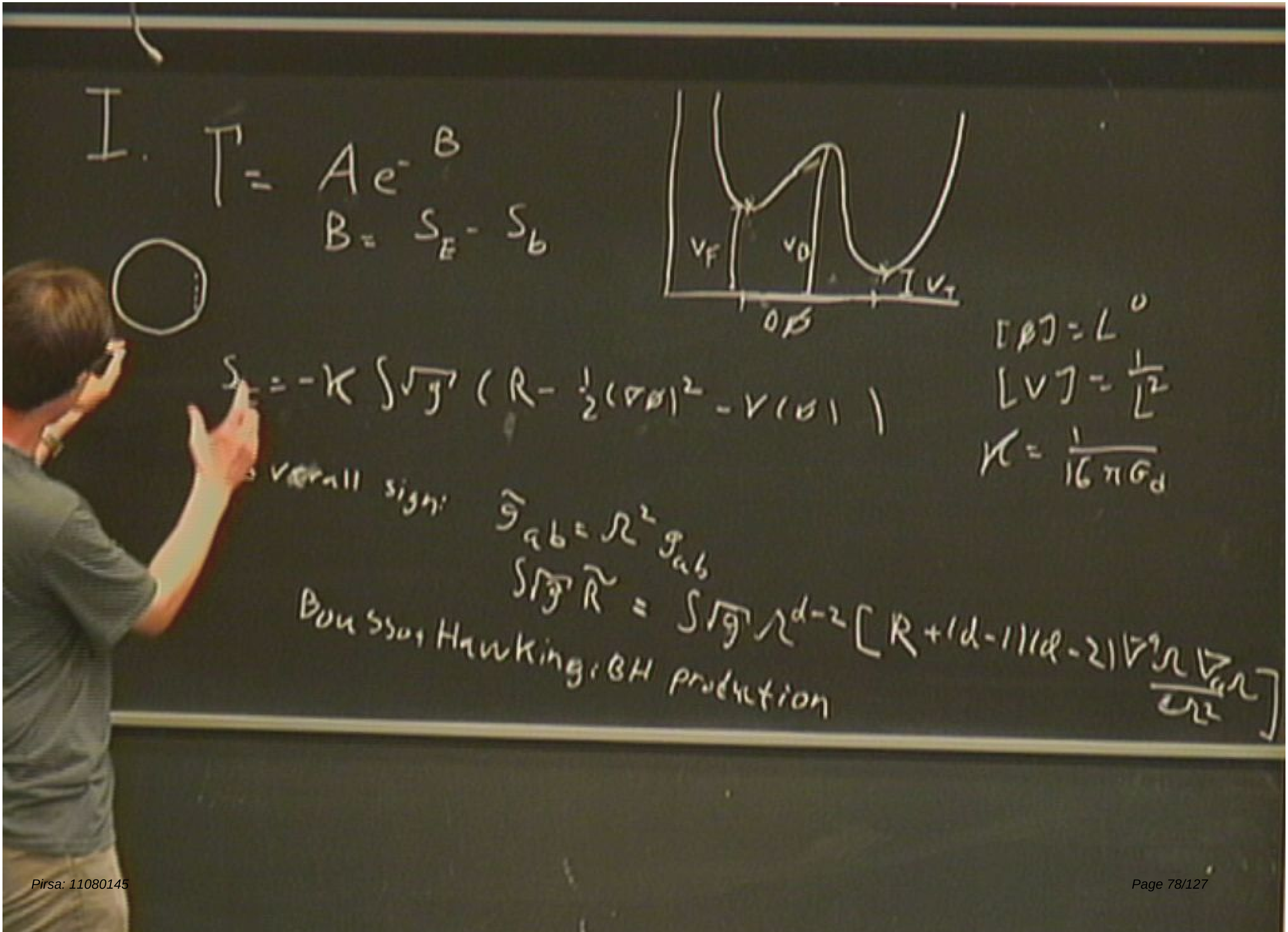
$$S_E = -\frac{2\kappa\,\Omega_{d-1}}{(d-2)} \Big[\underbrace{\int_0^{\tau_0} d\tau\, \rho^{d-1} V(\phi)}_{I_0:\ \text{near true vac}} + \underbrace{\int_{\tau_0}^{\tau_1} d\tau\, \rho^{d-1} V(\phi)}_{I_1:\ \text{"wall"}} + \underbrace{\int_{\tau_1}^{\tau_f} d\tau\, \rho^{d-1} V(\phi)}_{I_2:\ \text{near false va}} \Big]$$

$$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$$

$$-I_1 \sim \rho_0^{\,d-1} V_B \,\Delta\tau \sim \rho_0^{\,d-1} \Delta\phi \sqrt{V_B}$$

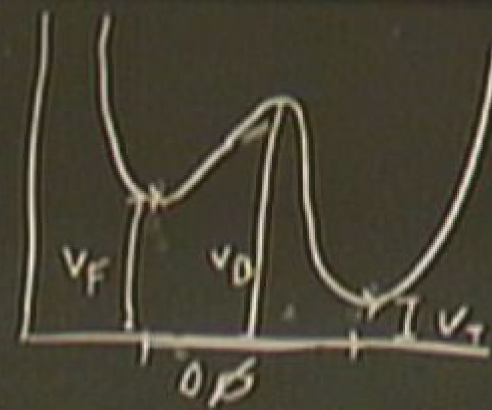
As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_w|, |I_2|, |S_b|$


I. Γ = A e^-B
B = S_E - S_b
V_F
V_0
V_T
0 φ
S_E = -K ∫√g (R - ½(∇φ)² - V(φ))
[φ] = L^0
[V] = 1/L²
K = 1/16πG
overall sign:
g̃_ab = Ω² g_ab
∫√g̃ R̃ = ∫√g Ω^(d-2) [R + (d-1)(d-
Bousso, Hawking: BH production


I. Γ = Ae^-B
B = S_E - S_b
V_F
V_D
V_T
0
[φ] = L^0
[V] = 1/L^2
K = 1/(16πG_d)
S = -K ∫√g (R - ½(∇φ)^2 - V(φ))
overall sign:
g̃_ab = Ω^2 g_ab
∫√g̃ R̃ = ∫√g Ω^(d-2) [R + (d-1)(d-2) ∇^a Ω ∇_a Ω / Ω^2]
Bousso, Hawking: BH production

I. $\Gamma = A e^{-B}$

$B = S_E - S_b$



$$S_E = -\kappa \int \sqrt{g} \left(R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right)$$

$[\phi] = L^0$

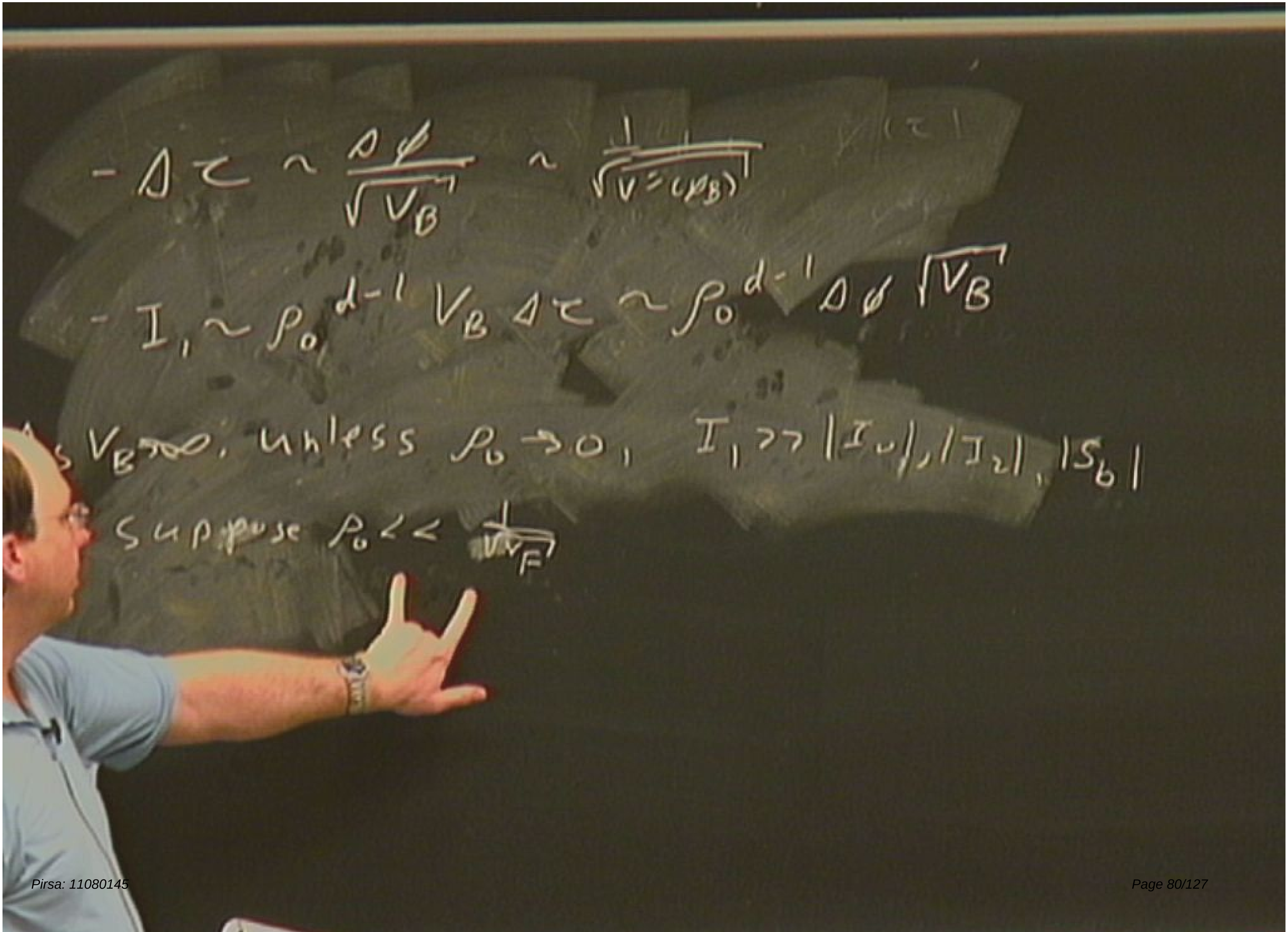
$[V] = \frac{1}{L^2}$

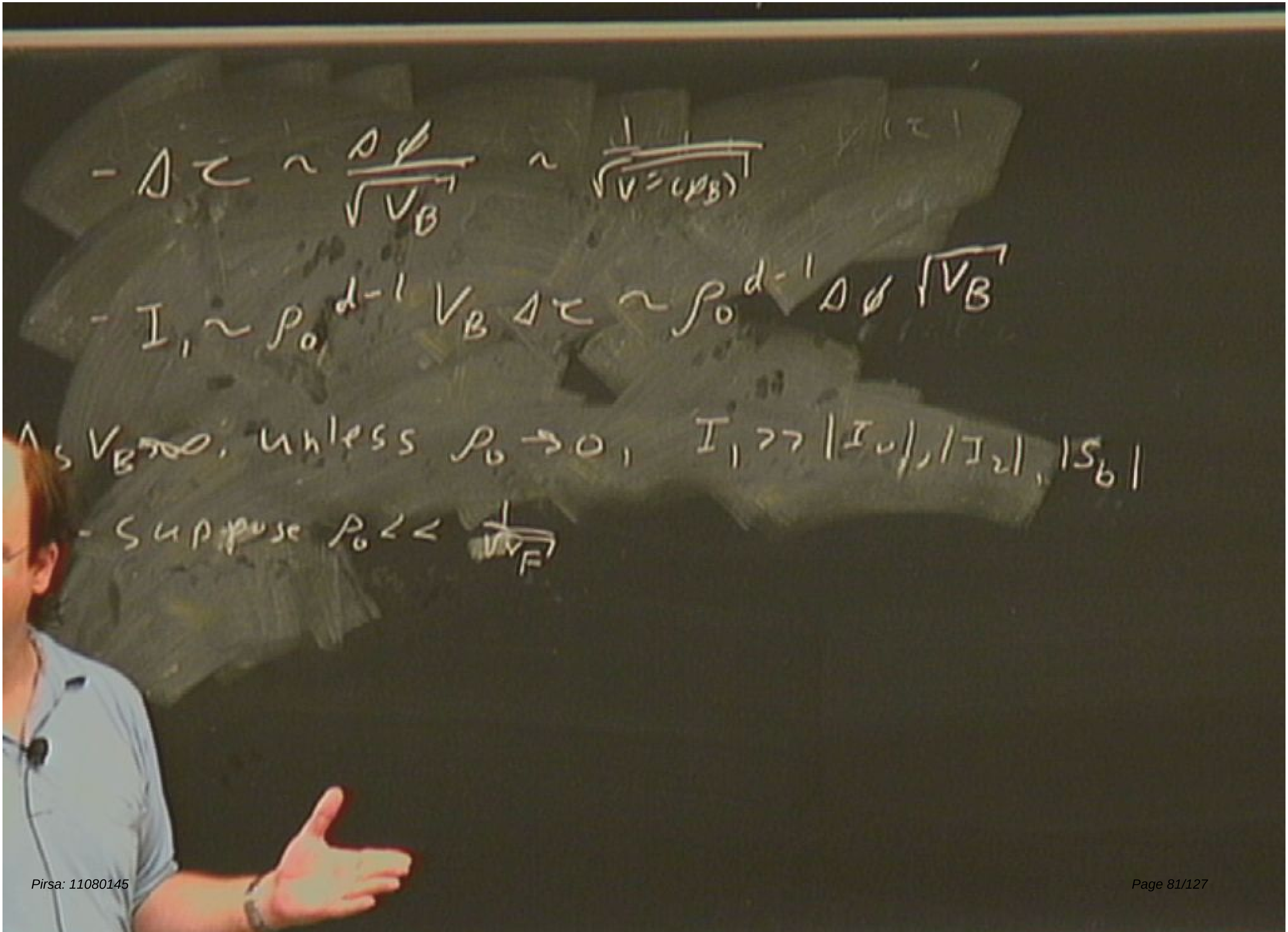
$\kappa = \frac{1}{16\pi G_d}$

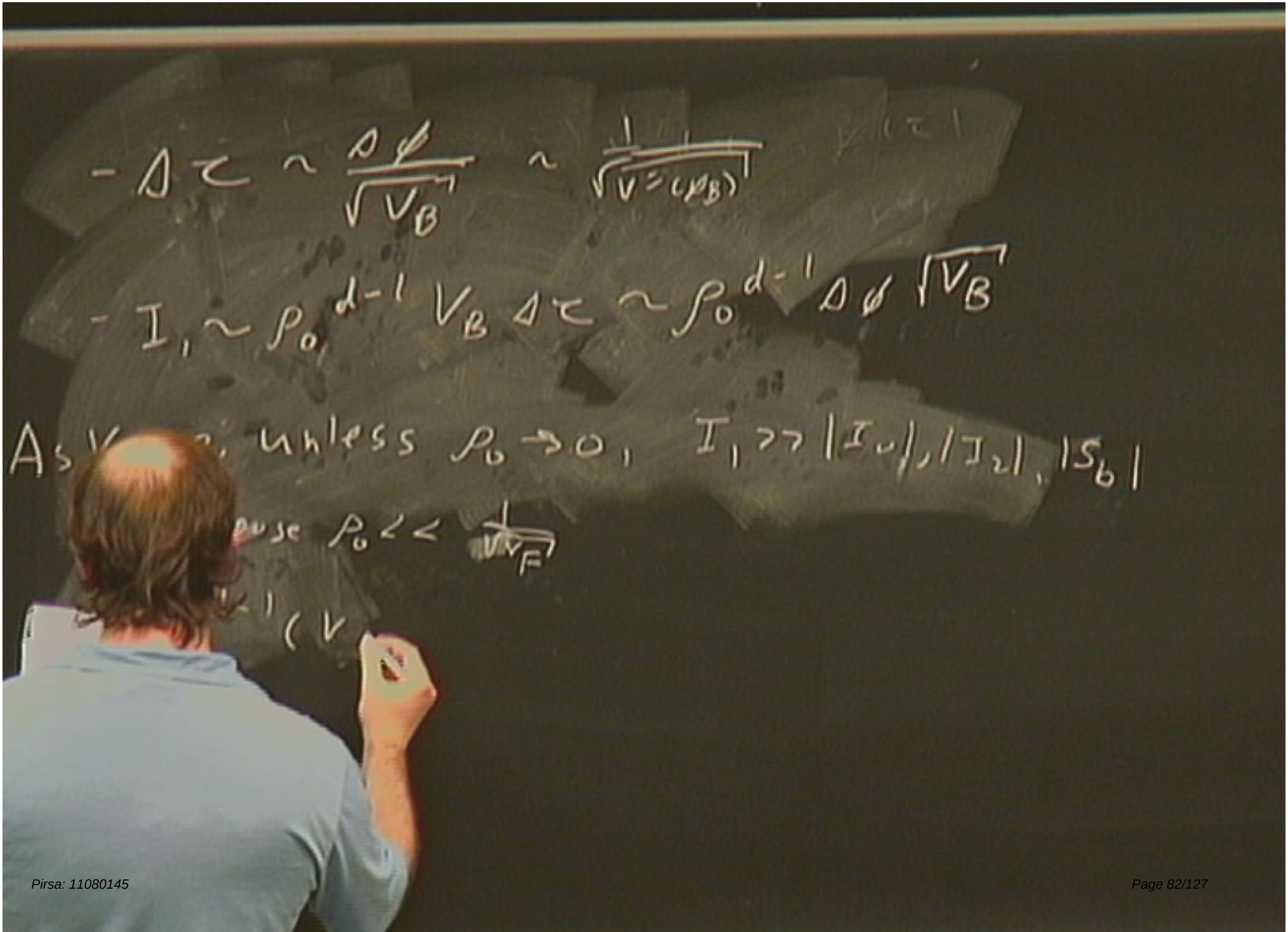
overall sign: $\tilde{g}_{ab} = \Omega^2 g_{ab}$

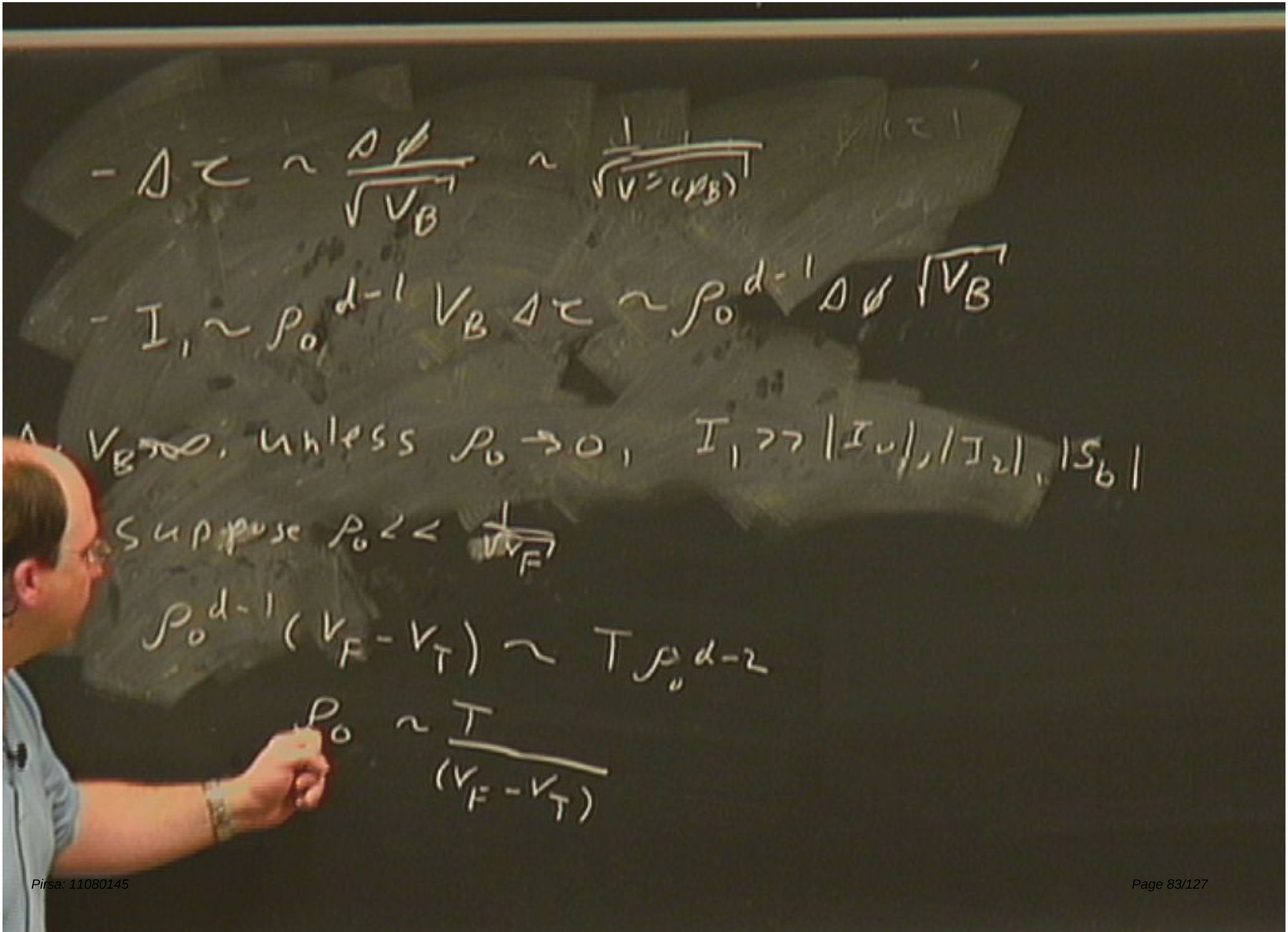
$$\int \sqrt{\tilde{g}}\, \tilde{R} = \int \sqrt{g}\, \Omega^{d-2} \left[R + (d-1)(d-2) \frac{\nabla^a \Omega \nabla_a \Omega}{\Omega^2} \right]$$

Bousso, Hawking: BH production


- Δτ ~ Δφ / √V_B ~ 1 / √V'(φ_B)
- I_1 ~ ρ_0^{d-1} V_B Δτ ~ ρ_0^{d-1} Δφ √V_B
V_B → ∞, unless ρ_0 → 0, I_1 >> |I_0|, |I_2|, |S_b|
suppose ρ_0 << 1/√V_F


- Δτ ~ Δφ / √V_B ~ 1 / √(V = (φ_B))
- I_1 ~ ρ_0^(d-1) V_B Δτ ~ ρ_0^(d-1) Δφ √V_B
As V_B → ∞, unless ρ_0 → 0, I_1 >> |I_0|, |I_2|, |S_0|
- Suppose ρ_0 << 1/√V_F


$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$
$-I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$
As … unless $\rho_0 \to 0$, $I_1 >> |I_0|, |I_2|, |S_b|$
…ause $\rho_0 << \frac{1}{\sqrt{V_F}}$



$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$
$-I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$
As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$
Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$
$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$
$\rho_0 \sim \frac{T}{(V_F - V_T)}$

- $\Delta\tau \sim \dfrac{\Delta\phi}{\sqrt{V_B}} \sim \dfrac{1}{\sqrt{V''(\rho_B)}}$

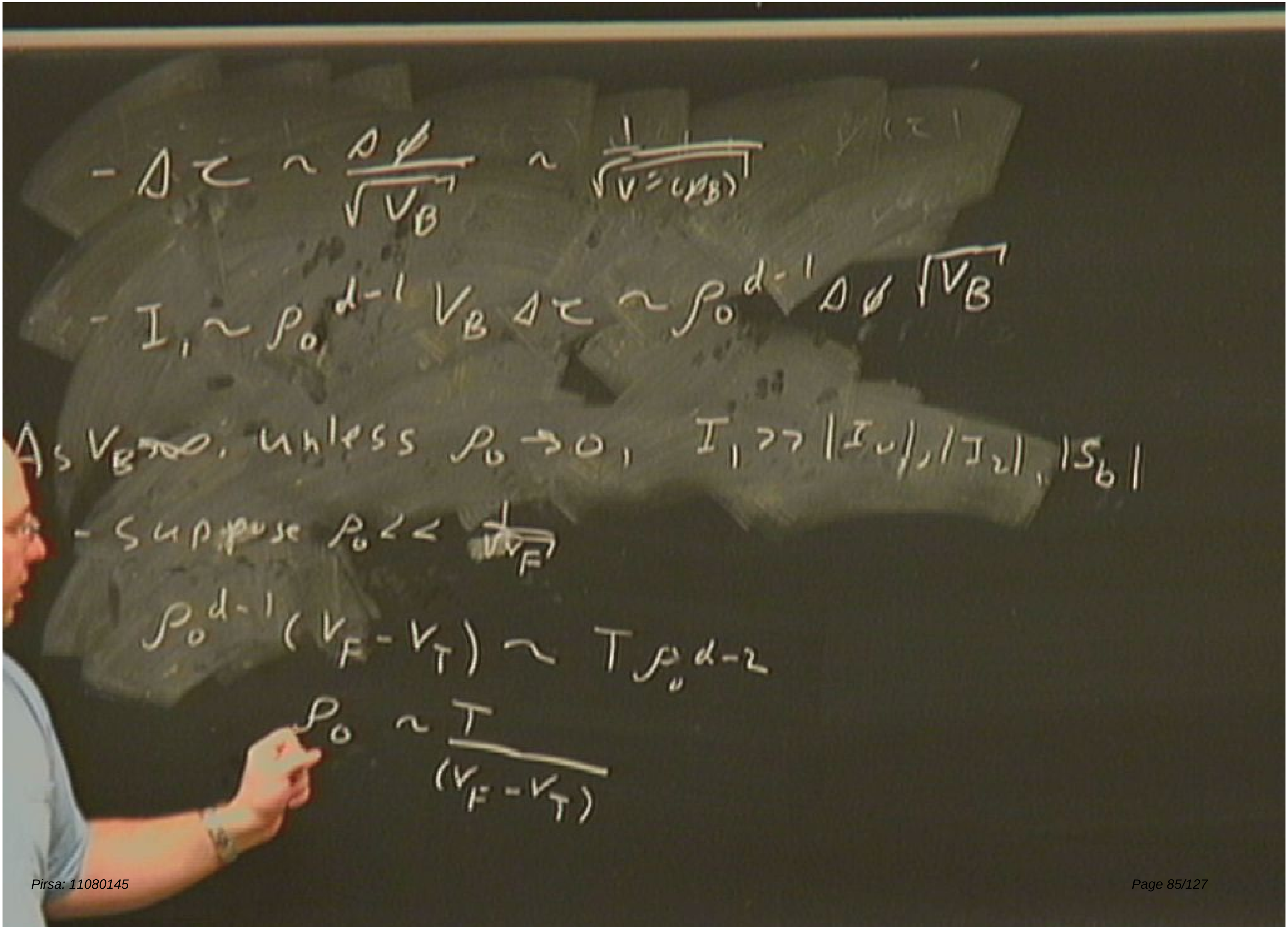
- $I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

- suppose $\rho_0 \ll \dfrac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$


$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$
$-I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$
As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_0|$
$-$ suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$
$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$
$\rho_0 \sim \frac{T}{(V_F - V_T)}$

$$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$$

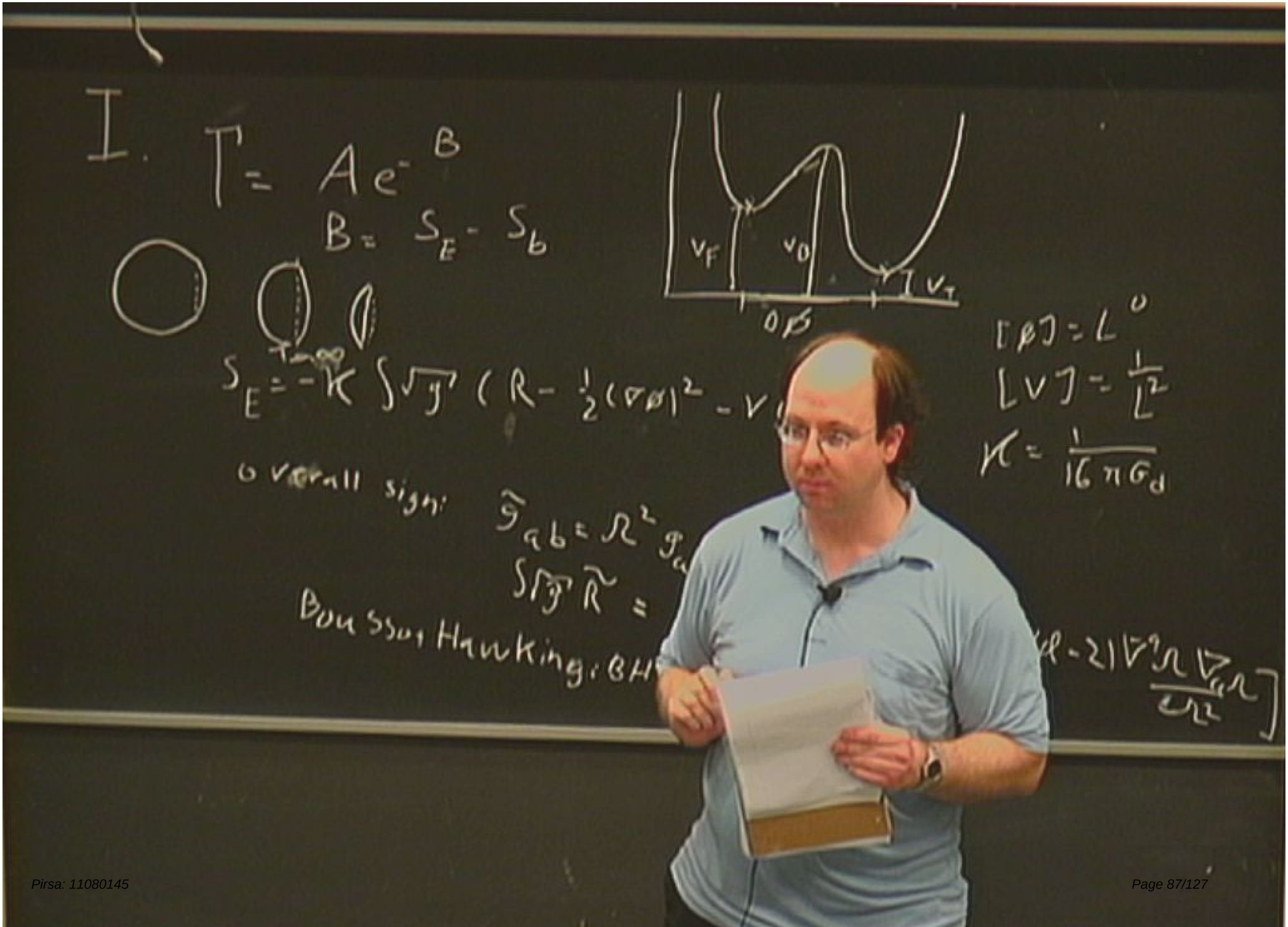
$$- I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$$

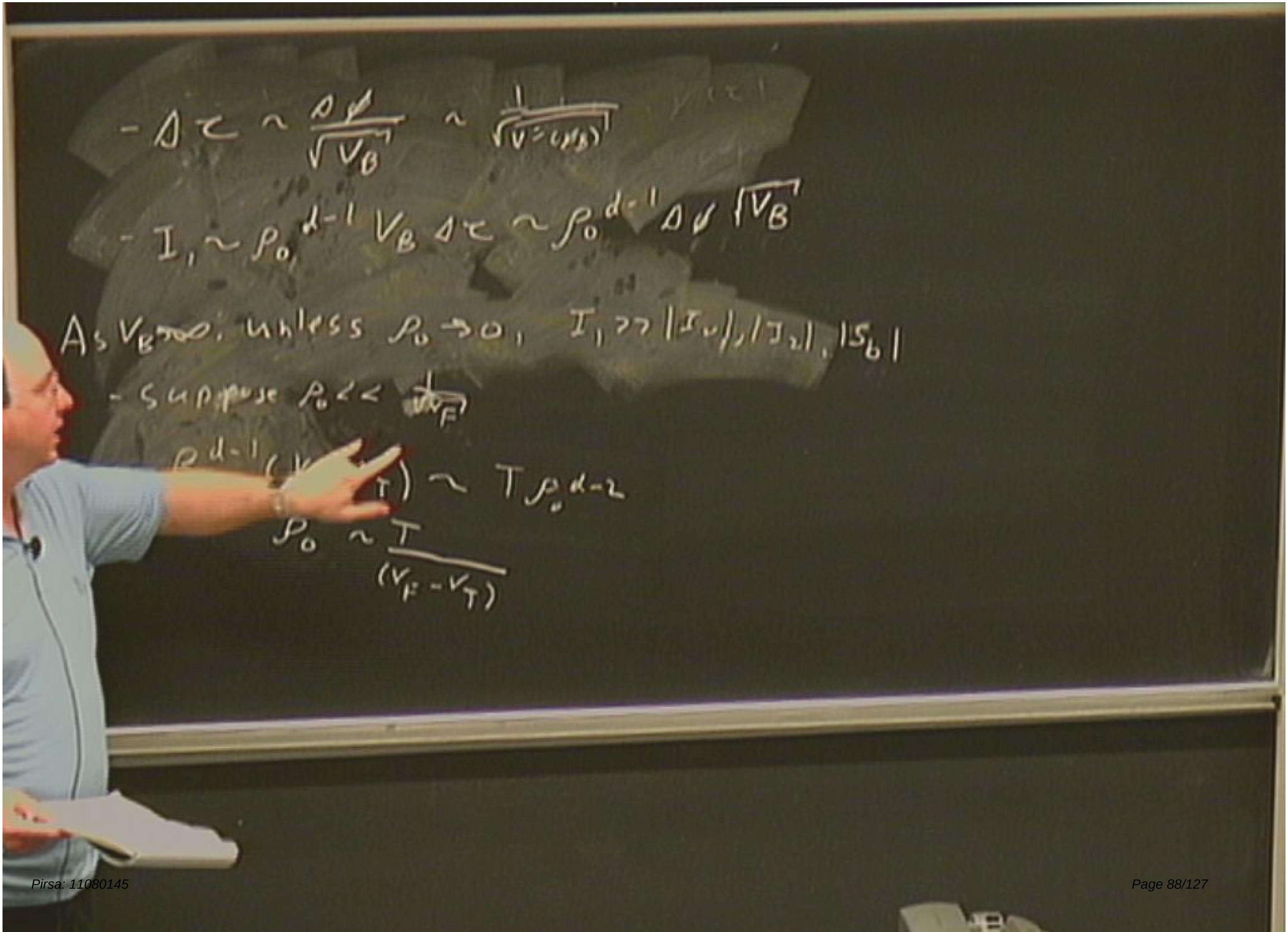
As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

- suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$


I. Γ = Ae^-B
B = S_E - S_b
V_F
V_0
V_T
S_E = -K ∫√g (R - ½(∇φ)² - V
overall sign:
[φ] = L^0
[V] = 1/L²
K = 1/16πG_d


- Δτ ~ Δφ / √V_B
- I_1 ~ ρ_0^(d-1) V_B Δτ ~ ρ_0^(d-1) Δφ √V_B
As V_B → ∞, unless ρ_0 → 0, I_1 >> |I_0|, |J_2|, |S_b|
- suppose ρ_0 <<
~ T ρ_0^(d-2)
ρ_0 ~ T / (v_F - v_T)

- $\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$

- $I_1 \sim \rho_0^{d-1} V_B \, \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

- suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

- $\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$

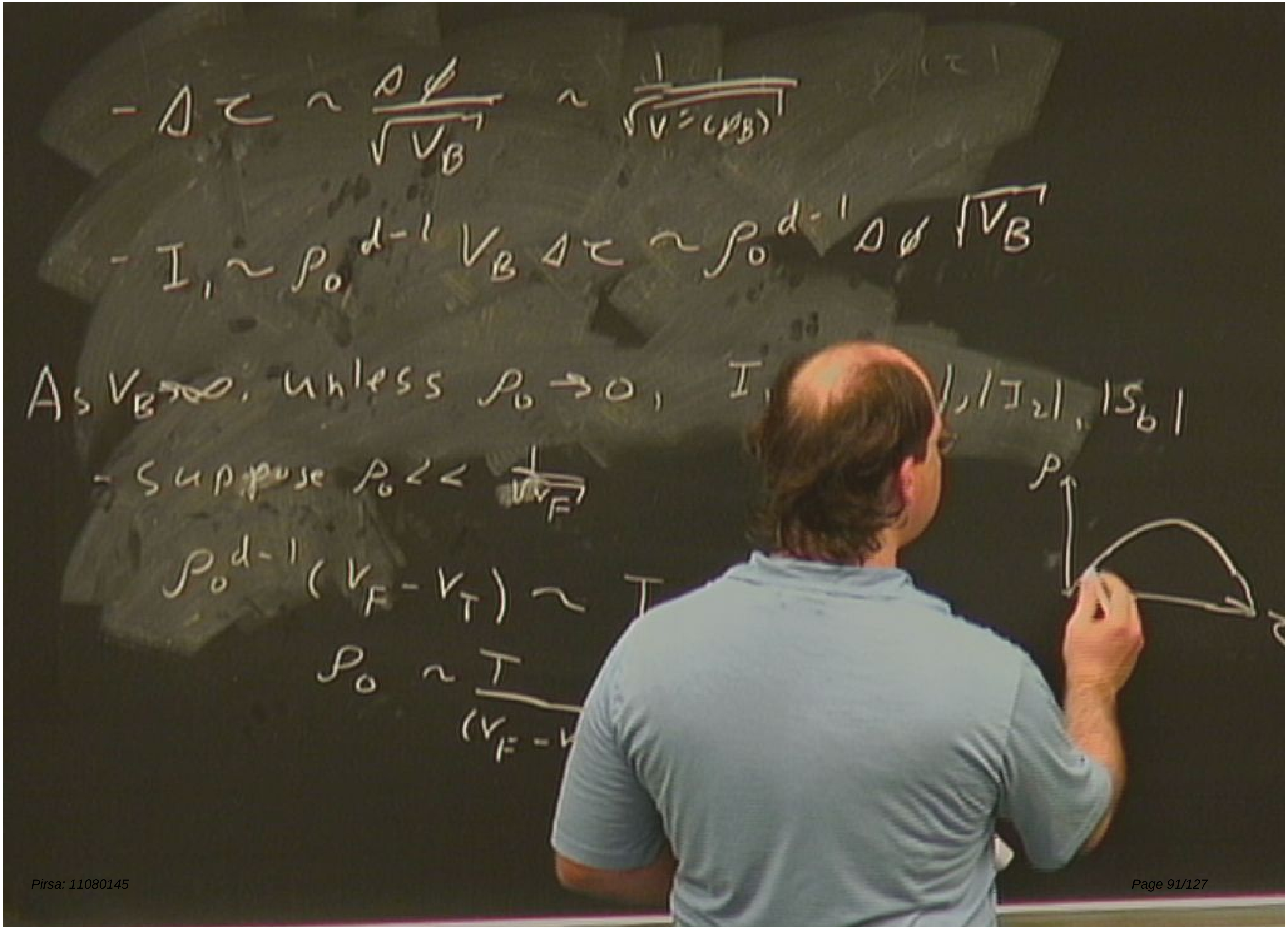
- $I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

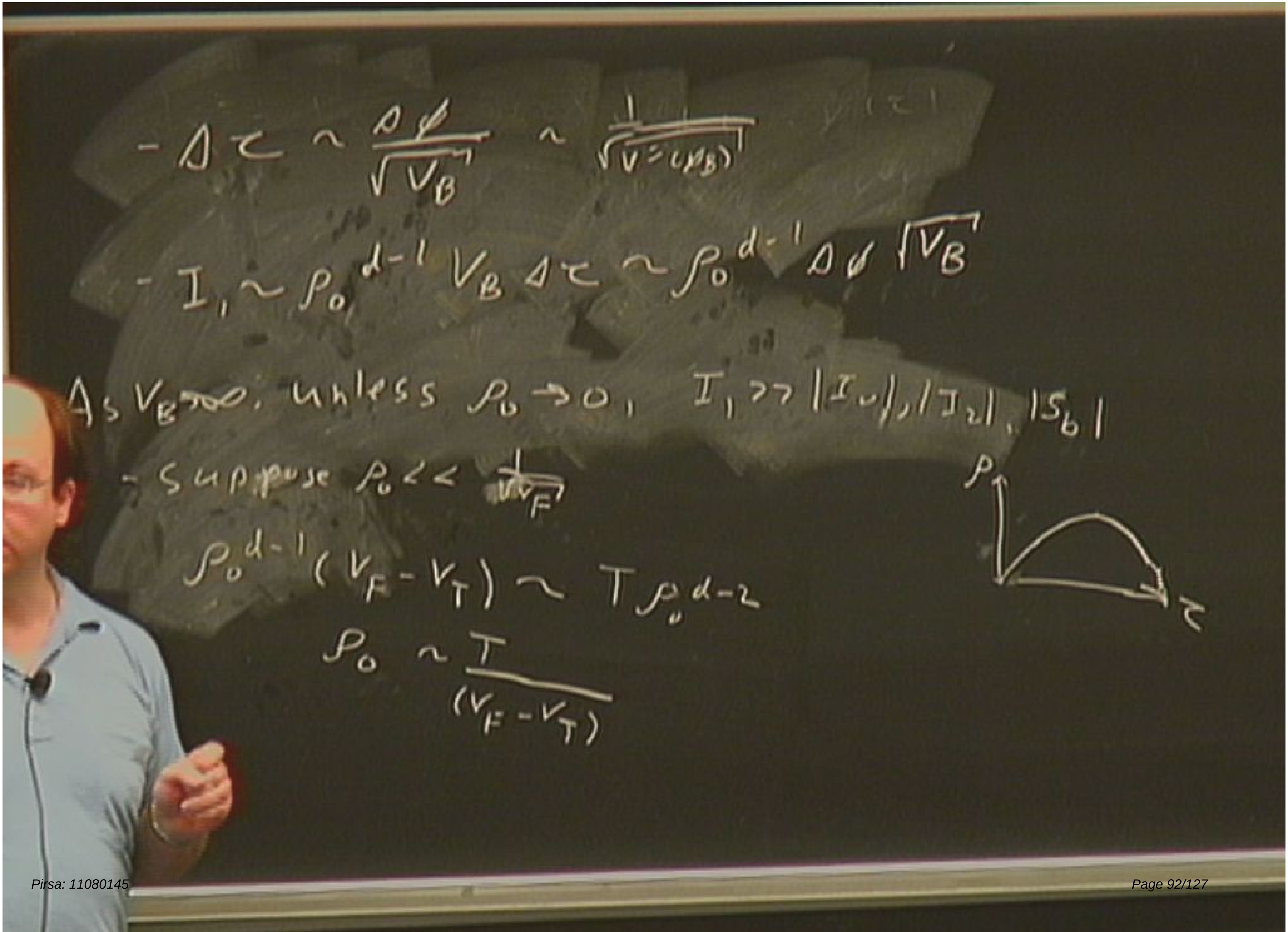
- suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$



$- \Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$
$- I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$
As $V_B \to \infty$, unless $\rho_0 \to 0$, I_1
$|, |I_2|, |S_b|$
$-$ suppose $\rho_0 << \frac{1}{\sqrt{V_F}}$
$\rho_0^{d-1}(V_F - V_T) \sim T$
$\rho_0 \sim \frac{T}{(V_F - V$
ρ

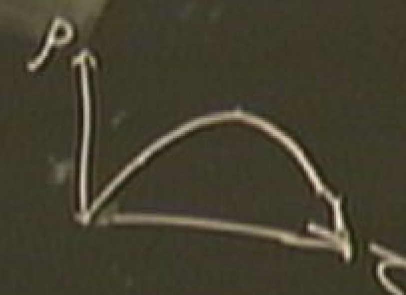

$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$
$-I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$
As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$
$-$ suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$
$\rho_0^{d-1}(V_F - V_T) \sim T\rho_0^{d-2}$
$\rho_0 \sim \frac{T}{(V_F - V_T)}$
ρ
τ

- $\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$

- $I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

- suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$



$$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

- $\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$

$$R\left(1-\frac{d}{2}\right) = \frac{1}{2}\left(1-\frac{d}{2}\right)(\nabla\phi)^2 - \frac{d}{2}V(\phi)$$

$$R = \frac{1}{2}(\nabla\phi)^2 + \frac{d}{d-2}V(\phi)$$

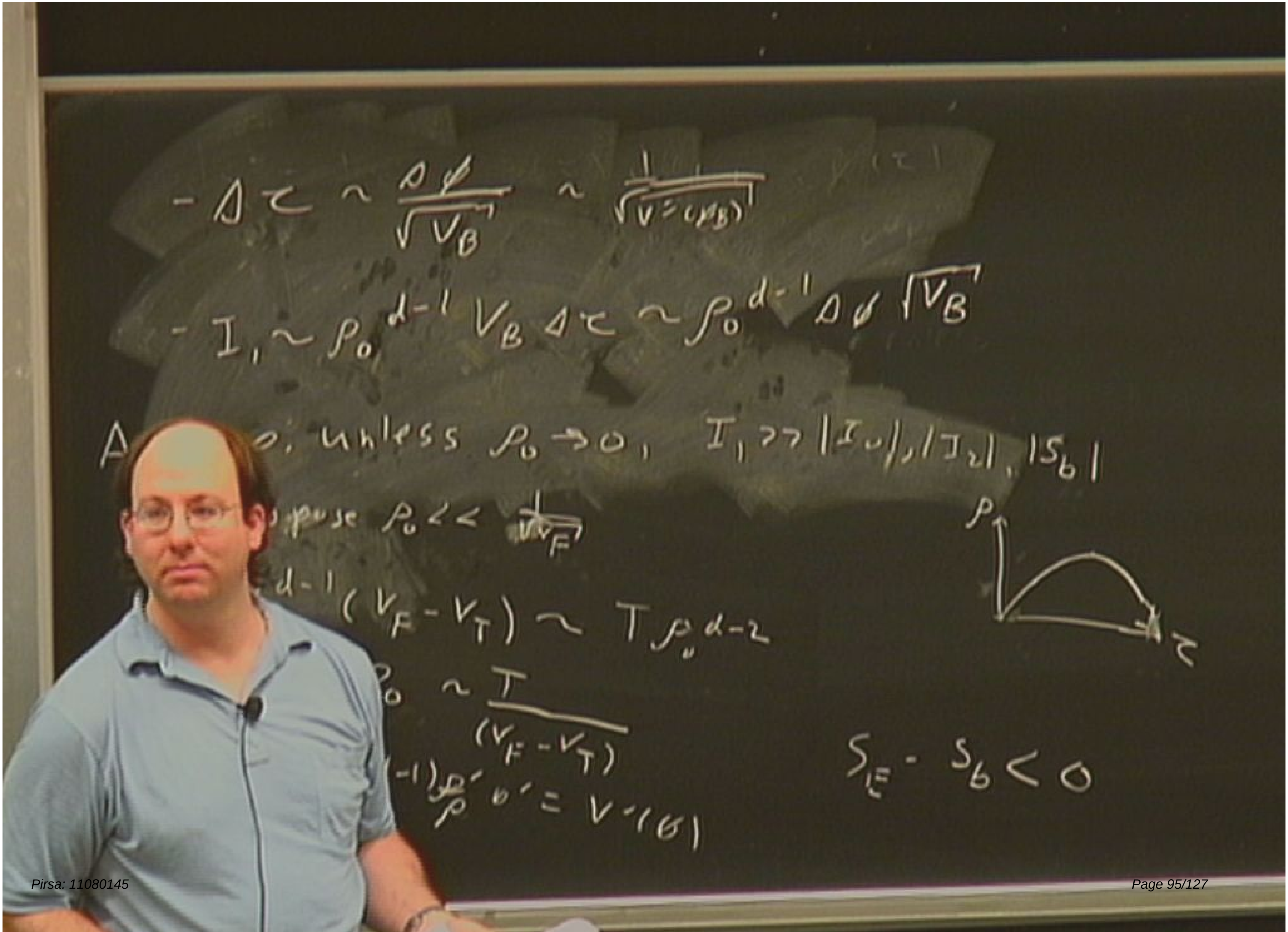
$$S_E = -K\int\sqrt{g}\left(\frac{d}{d-2}-1\right)V(\phi) = \frac{-2K}{(d-2)}\int\sqrt{g}\,V(\phi)$$

- no gradient terms
- potential (naively) comes in with opposite sign

$$S_E^{(4)} = -\frac{48K}{V_0} \qquad V(\phi) = V_0 > 0$$

$$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$$

$$-I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1}\Delta\phi\sqrt{V_B}$$



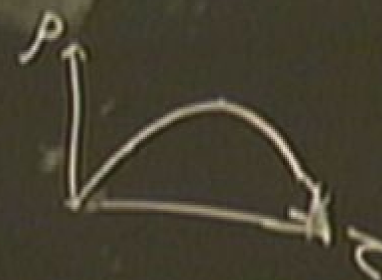
- Δτ ~ Δφ / √V_B ~ 1 / √V''(φ_B)
- I_1 ~ ρ_0^{d-1} V_B Δτ ~ ρ_0^{d-1} Δφ √V_B
unless ρ_0 → 0, I_1 >> |I_0|, |I_2|, |S_b|
ρ_0 << 1/√V_F
(V_F - V_T) ~ T ρ_0^{d-2}
ρ_0 ~ T / (V_F - V_T)
= V'(φ)
S_E - S_b < 0

$$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

- suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

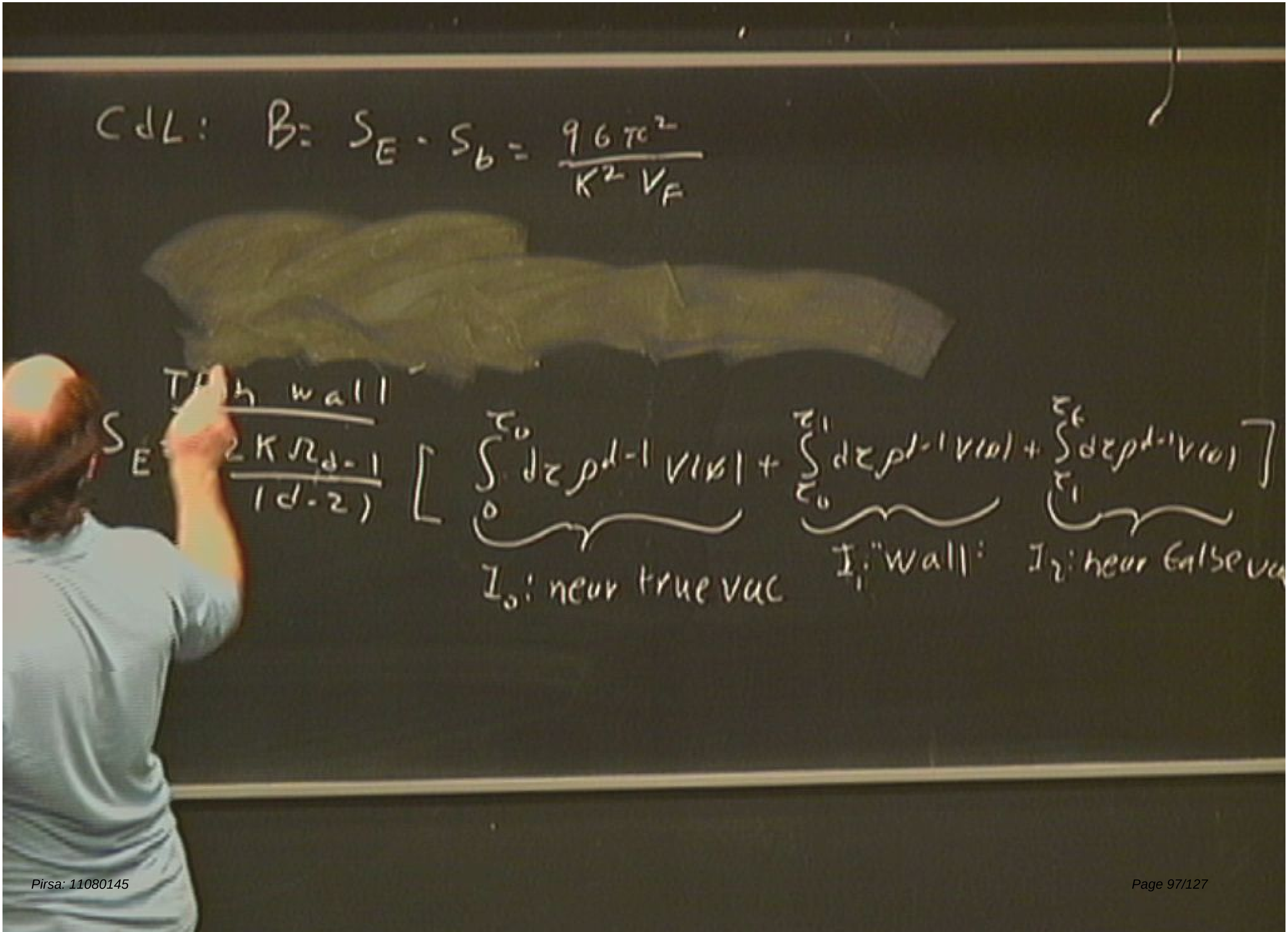


$$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$-\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$$

$$S_E - S_b < 0$$


CdL: B = S_E - S_b = 96π² / κ² V_F
wall
S_E = 2κΩ_{d-1} / (d-2) [∫_0^{τ_0} dτ ρ^{d-1} V(φ) + ∫_{τ_0}^{τ_1} dτ ρ^{d-1} V(φ) + ∫_{τ_1}^{τ_f} dτ ρ^{d-1} V(φ)]
I_0: near true vac
I_1: "wall"
I_2: near false va

CdL: $B = S_E - S_b = \frac{96\pi c^2}{K^2 V_F}$

Thin wall

$$S_E = -\frac{2K\Omega_{d-1}}{(d-2)}\left[\int_0^{\tau_0} d\tau\,\rho^{d-1}V(\phi) + \int_{\tau_0}^{\tau_1} d\tau\,\rho^{d-1}V(\phi) + \int_{\tau_1}^{\tau_f} d\tau\,\rho^{d-1}V(\phi)\right]$$

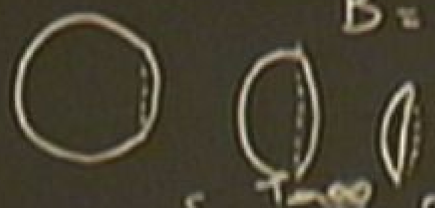
I_0: near true vac

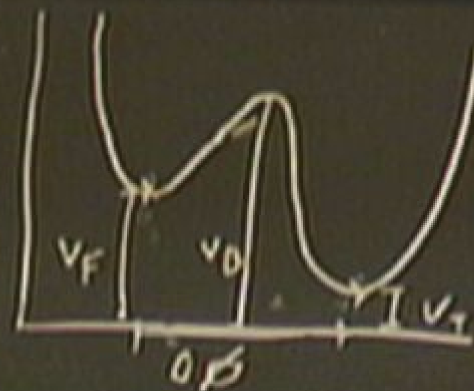
I_1: "wall"

I_2: near false vac

I. $\Gamma = A e^{-B}$

$B = S_E - S_b$





$$S_E = -\kappa \int \sqrt{g}\, \left(R - \tfrac{1}{2}(\nabla\phi)^2 - V(\phi) \right)$$

$[\phi] = L^0$

$[V] = \frac{1}{L^2}$

$\kappa = \frac{1}{16\pi G_d}$

overall sign: $\tilde{g}_{ab} = \Omega^2 g_{ab}$

$$\int \sqrt{\tilde{g}}\, \tilde{R} = \int \sqrt{g}\, \Omega^{d-2} \left[R + (d-1)(d-2) \frac{\nabla^a \Omega \nabla_a \Omega}{\Omega^2} \right]$$

Bousso, Hawking: BH production

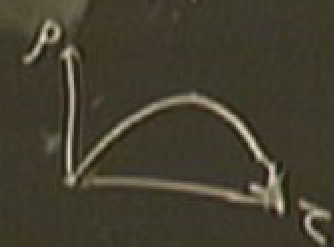
$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$

$- I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$

$S = \frac{C_0}{V_F^{\frac{d}{2}-1}}$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

- suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$



$\rho_0^{d-1}(V_F - V_T) \sim T\rho_0^{d-2}$

$\rho_0 \sim \frac{T}{(V_F - V_T)}$

$-\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$

$S_E \cdot S_b < 0$

$- \Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V''(\phi_B)}}$

$- I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$

$B \sim S_E \ldots = \frac{C_0}{V_F^{\frac{d}{2}-1}}$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |J_2|, |S_b|$

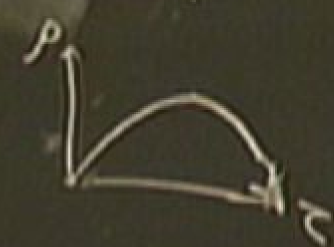
- suppose $\rho_0 \ll \frac{1}{V_F}$

$\rho_0^{d-1}(V_F - V_T) \sim T \rho_0^{d-2}$

$\rho_0 \sim \frac{T}{(V_F - V_T)}$

$- \phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$

$S_E \cdot S_b < 0$



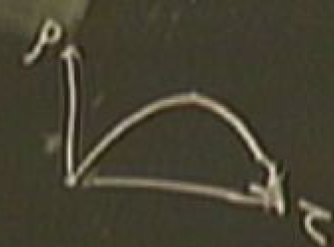
$- \Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V'(\phi_B)}}$

$- I_1 \sim \rho_0^{d-1} V_B \Delta\tau \sim \rho_0^{d-1} \Delta\phi \sqrt{V_B}$

$B \sim S_E \ldots = \frac{C_0}{V_F^{\frac{d}{2}-1}}$

As $V_B \to \infty$, unless $\rho_0 \to 0$, $I_1 \gg |I_0|, |I_2|, |S_b|$

$-$ suppose $\rho_0 \ll \frac{1}{V_F}$



$\rho_0^{d-1}(V_F - V_T) \sim T\rho_0^{d-2}$

$\rho_0 \sim \frac{T}{(V_F - V_T)}$

$- \phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$

$S_E \cdot S_b < 0$

$$R\left(1-\frac{d}{2}\right) = \frac{1}{2}\left(1-\frac{d}{2}\right)(\nabla\phi)^2 - \frac{d}{2}V(\phi)$$

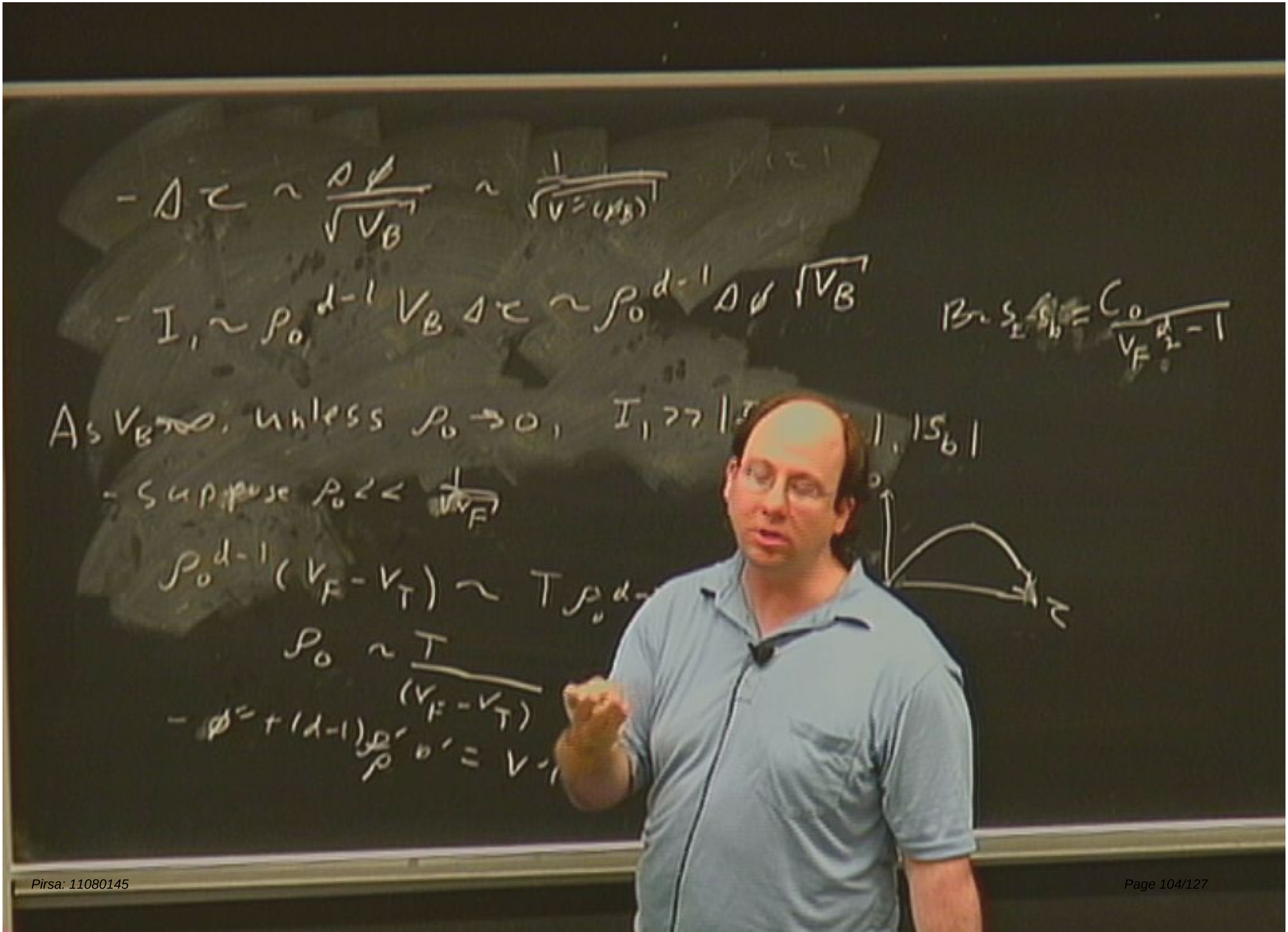
$$R = \frac{1}{2}(\nabla\phi)^2 + \frac{d}{d-2}V(\phi)$$

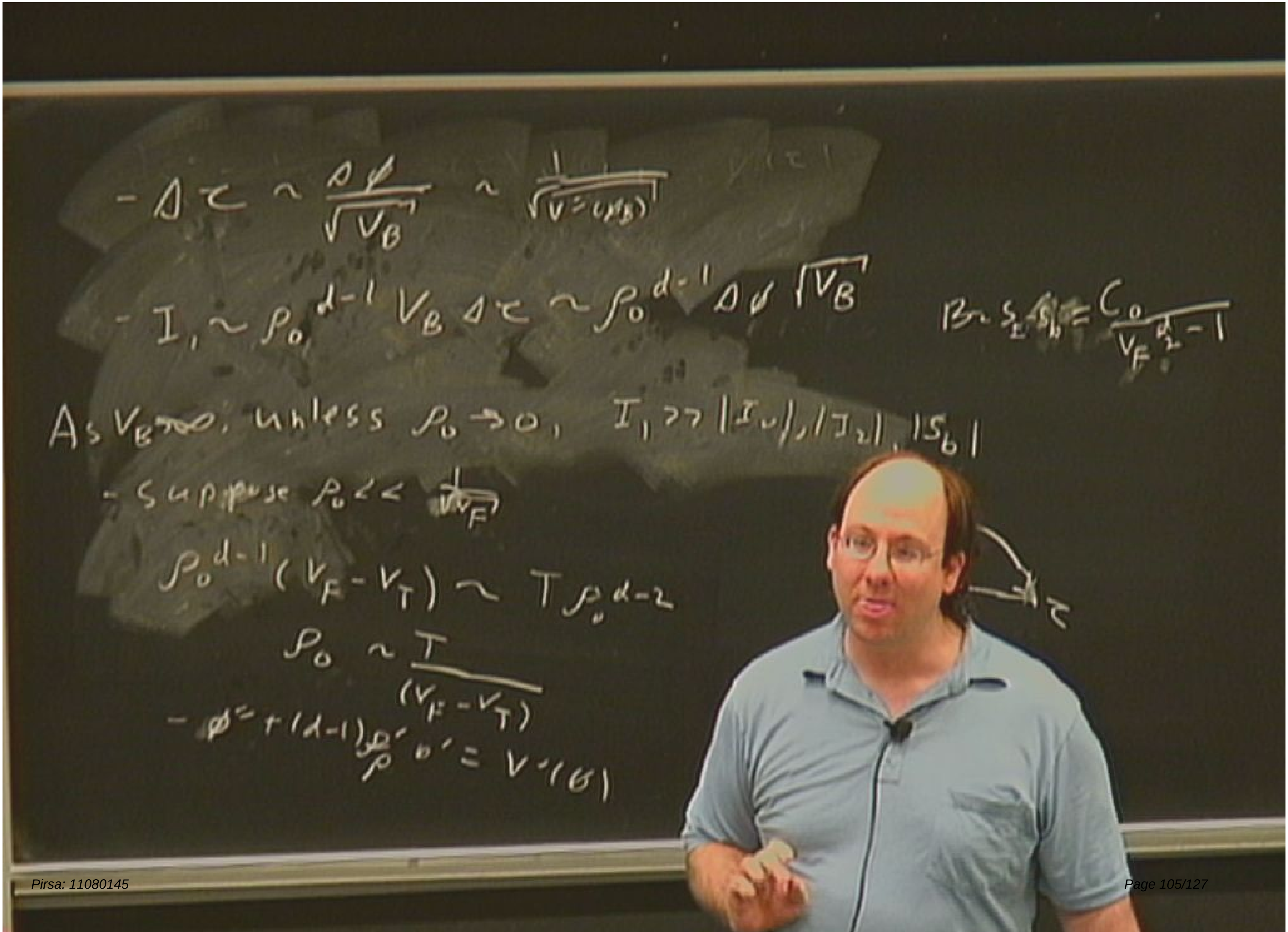
$$S_E = -K\int\sqrt{g}\left(\frac{d}{d-2}-1\right)V(\phi) = \frac{-2K}{(d-2)}\int\sqrt{g}\,V(\phi)$$

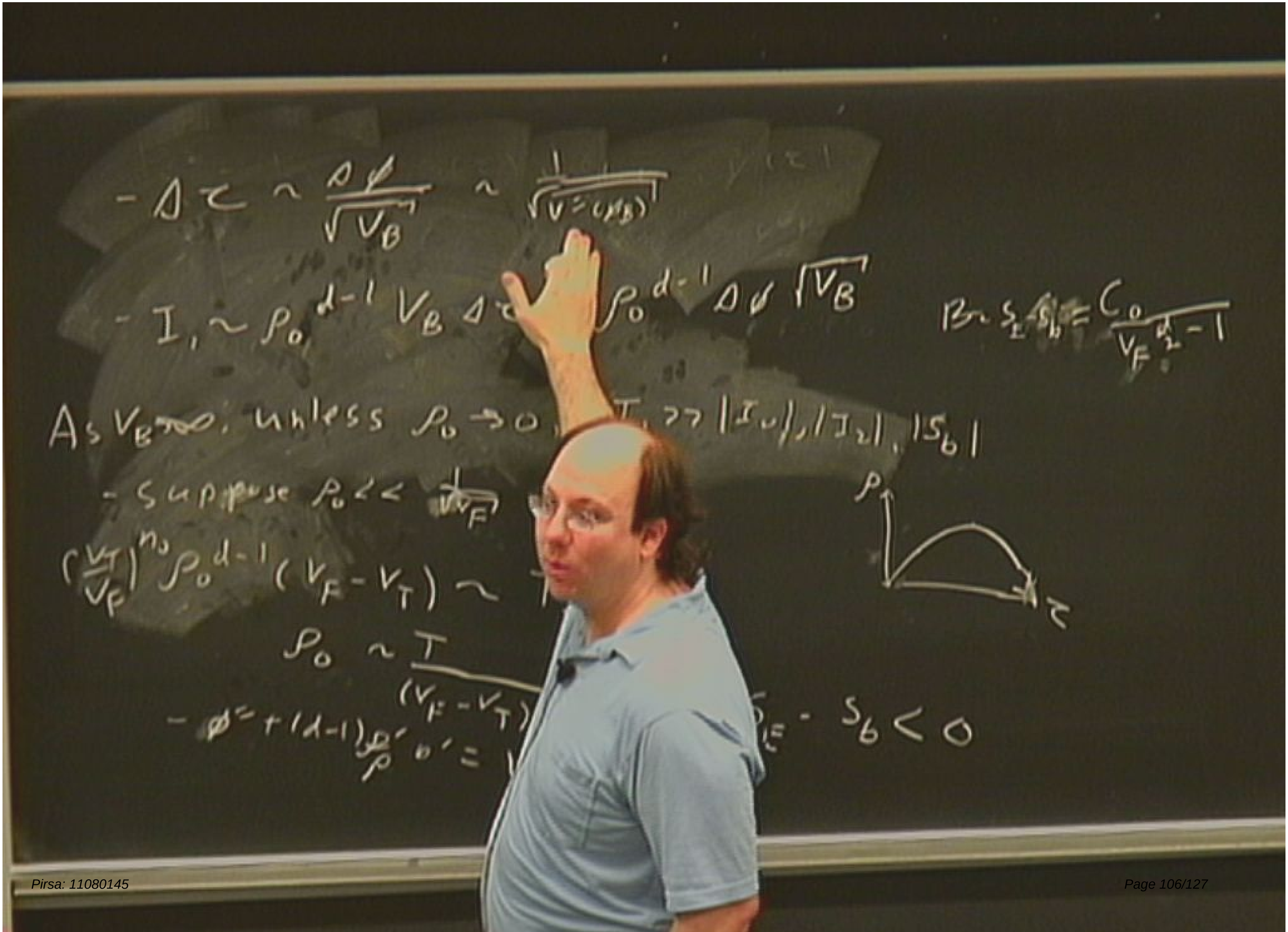
- no gradient terms
- potential (naively) comes in with opposite sign

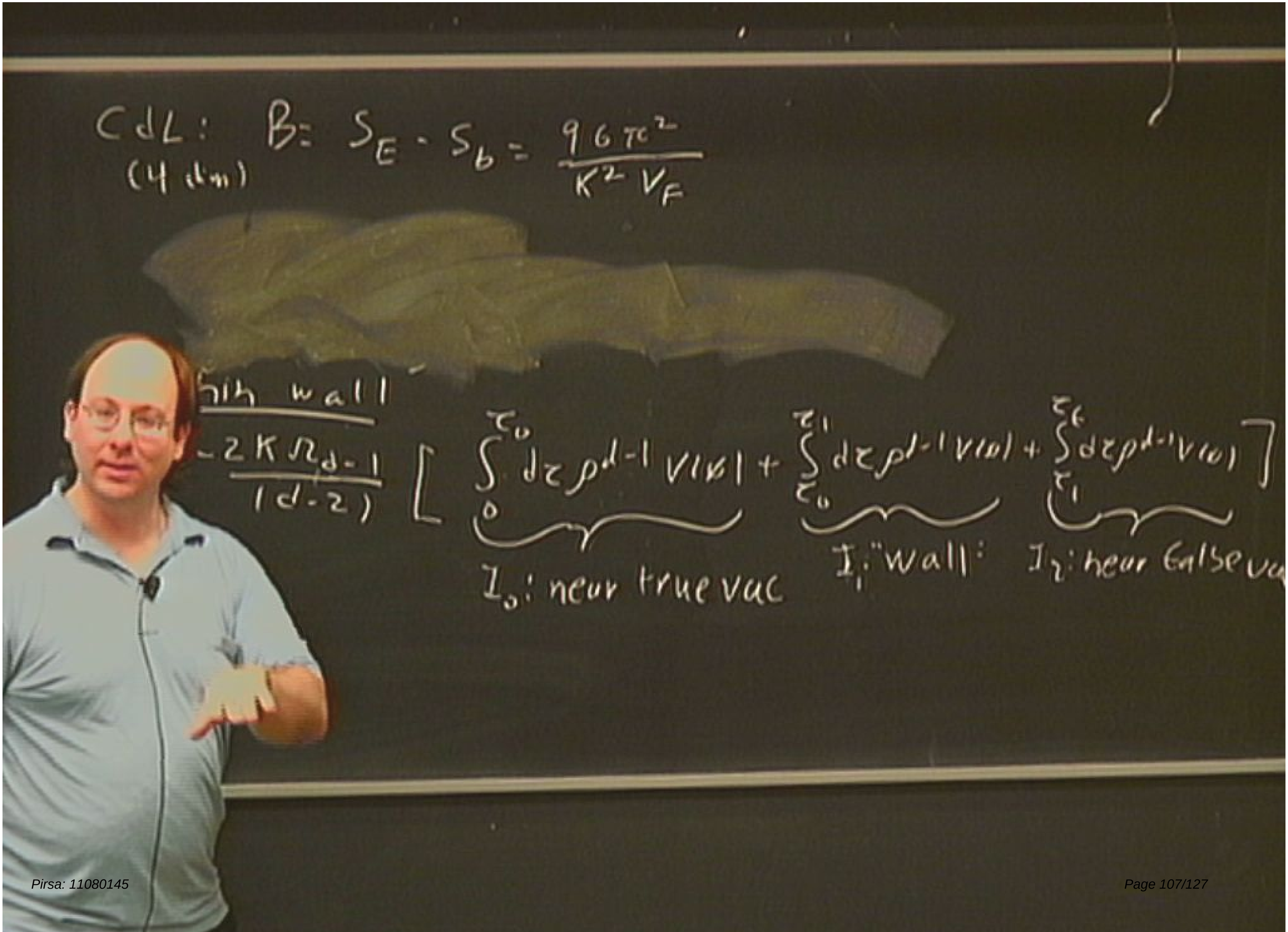
$$S_E^{(4)} = -\frac{48K}{V_0} \qquad V(\phi) = V_0 > 0$$

$$-\Delta\tau \sim \frac{\Delta\phi}{\sqrt{V_0}} \sim \frac{1}{\sqrt{V^2(\phi)}}$$


- Δτ ~ Δφ / √V_B ~ 1 / √(V - V(φ_B))
- I_1 ~ ρ_0^{d-1} V_B Δτ ~ ρ_0^{d-1} Δφ √V_B
As V_B → ∞, unless ρ_0 → 0, I_1 >> |S_b|
- suppose ρ_0 << 1/√V_F
ρ_0^{d-1} (V_F - V_T) ~ T ρ_0
ρ_0 ~ T / (V_F - V_T)
τ


- Δτ ~ Δφ / √V_B ~ 1 / √(V''(φ_B))
- I_1 ~ ρ_0^{d-1} V_B Δτ ~ ρ_0^{d-1} Δφ √V_B
B ~ S_E - S_b = C_0 / V_F^{d/2 - 1}
As V_B → ∞, unless ρ_0 → 0, I_1 >> |I_0|, |I_2|, |S_b|
- suppose ρ_0 << 1/√V_F
ρ_0^{d-1} (V_F - V_T) ~ T ρ_0^{d-2}
ρ_0 ~ T / (V_F - V_T)
- φ'' + (d-1) ρ'/ρ φ' = V'(φ)


As $V_B \to \infty$, unless $\rho_0 \to 0$
- suppose $\rho_0 \ll$
$S_b < 0$

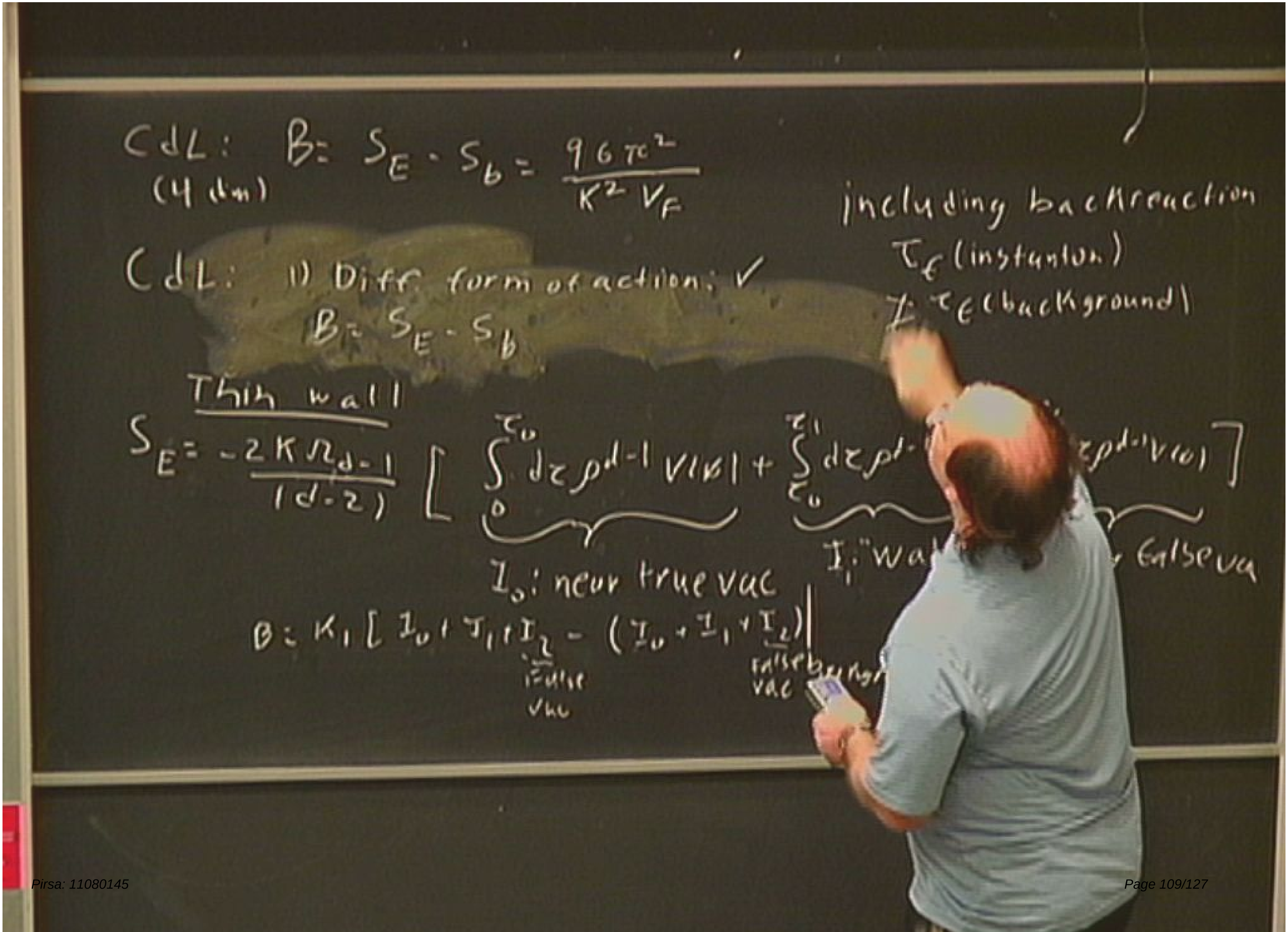

CdL: $B = S_E - S_b = \frac{96\pi^2}{\kappa^2 V_F}$
(4 dim)
thin wall
$= \frac{2\kappa\Omega_{d-1}}{(d-2)} \left[\int_0^{\tau_0} d\tau \rho^{d-1} V(\phi) + \int_{\tau_0}^{\tau_1} d\tau \rho^{d-1} V(\phi) + \int_{\tau_1}^{\tau_f} d\tau \rho^{d-1} V(\phi) \right]$
I_0: near true vac
I_1: "wall"
I_2: near false

CdL: $B = S_E - S_b = \frac{96\pi^2}{\kappa^2 V_F}$
(4 dim)

CdL: 1) Diff. form of action

<u>Thin wall</u>

$$S_E = -\frac{2\kappa\Omega_{d-1}}{(d-2)}\left[\underbrace{\int_0^{\tau_0} d\tau\,\rho^{d-1}V(\phi)}_{I_0:\text{ near true vac}} + \underbrace{\int_{\tau_0}^{\tau_1} d\tau\,\rho^{d-1}V(\phi)}_{I_1:\text{ "wall"}} + \underbrace{\int_{\tau_1}^{\tau_f} d\tau\,\rho^{d-1}V(\phi)}_{I_2:\text{ near false vac}}\right]$$


CdL: B = S_E - S_b = 96π²/(K² V_F)
(4 dim)
including backreaction
τ_E (instanton)
≠ τ_E (background)
CdL: 1) Diff. form of action: ✓
B = S_E - S_b
Thin wall
S_E = -2KΩ_{d-1}/(d-2) [∫_0^{τ_0} dτ ρ^{d-1} V(φ) + ∫_{τ_0}^{τ_1} dτ ρ^{d-1}
ρ^{d-1} V(φ)]
I_0: near true vac
I_1 "wal
false vac
B = K_1 [I_0 + I_1 + I_2 - (I_0 + I_1 + I_2)
false vac
false vac
backgr

Two classes of potentials with rapid transitions

1) "Small wall": $V_B - V_T << V(\phi)$

$S_E - S_b << S_b$

$S_E - S_b \lesssim \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$

2) "thin wall": $\rho \approx \rho_0$ when ϕ not near a minima

If $\Delta\phi << 1$ $V_B >> V_F - V_T$

If $\Delta\phi = \mathcal{O}(1)$ $V_B >> V_F$ + instanton starts close to the minima

coupled Einstein Field equations

$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$

$\frac{\rho''}{\rho} = -\frac{V(\phi)}{(d-1)(d-2)} - \frac{\phi'^2}{2(\ldots)}$

$\rho'^2 = 1 + \frac{\rho^2}{(d-1)}\left(\frac{\phi'^2}{2} - \ldots\right)$

CdL: 1) Diff. form of action

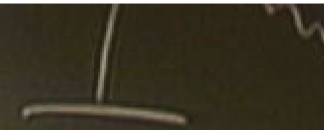
$B = S_E - S_b$

τ_E (instanton)

$\neq \tau_E$ (background)

$S_E - S_b << S_b$

$S_E - S_b \lesssim \mathcal{O}(\quad \frac{\quad}{V_T})$



2) "thin wall": $\rho \approx \rho_0$ when ϕ not near a minima

If $\Delta\phi << 1$ $\quad V_B >> V_F - V_T$

If $\Delta\phi = \mathcal{O}(1)$ $\quad V_B >> V_F$ + instanton starts close to the minima

coupled Einstein Field equations

$$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$$

$$\frac{\rho''}{\rho} = -\frac{V(\phi)}{(d-1)(d-2)} - \frac{\phi'^2}{2(d-1)}$$

$$\rho'^2 = 1 + \frac{\rho^2}{d-4}\left(\frac{\phi'^2}{2} - V(\phi)\right)$$

CdL: $B = S_E - S_b = \frac{96\pi^2}{K^2 V_F}$
(4 dim)

including backreaction
τ_E(instanton)
$\neq \tau_E$(background)

CdL: 1) Diff. form of action: ✓

$B = S_E - S_b$

CdL: $B = S_E - S_b = \frac{96\pi^2}{\kappa^2 V_F}$
(4 dim)

including backreaction
τ_E (instanton)
$\neq \tau_E$ (background)

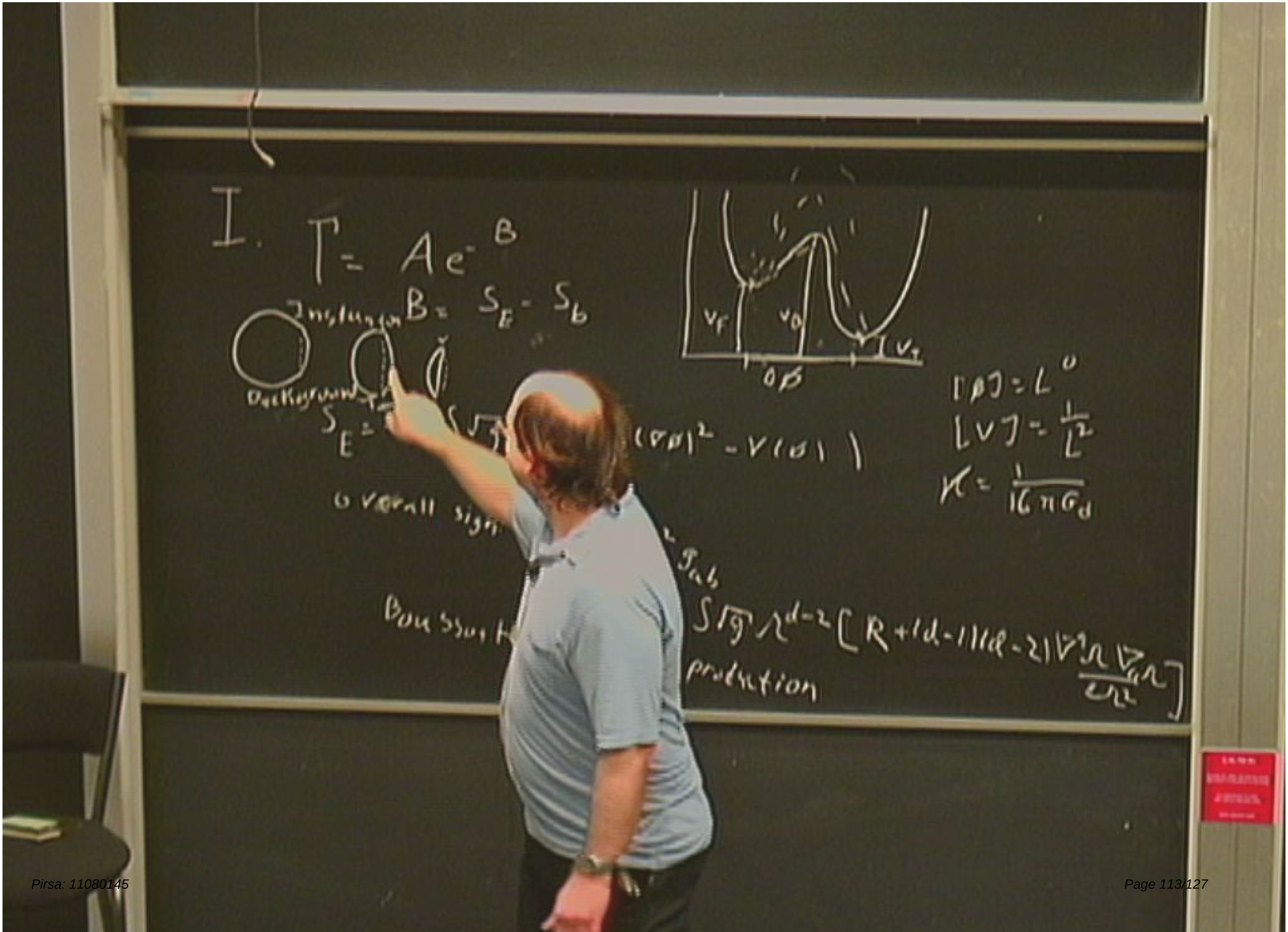
CdL: (1) Diff. form of action: ✓
$B = S_E - S_b$

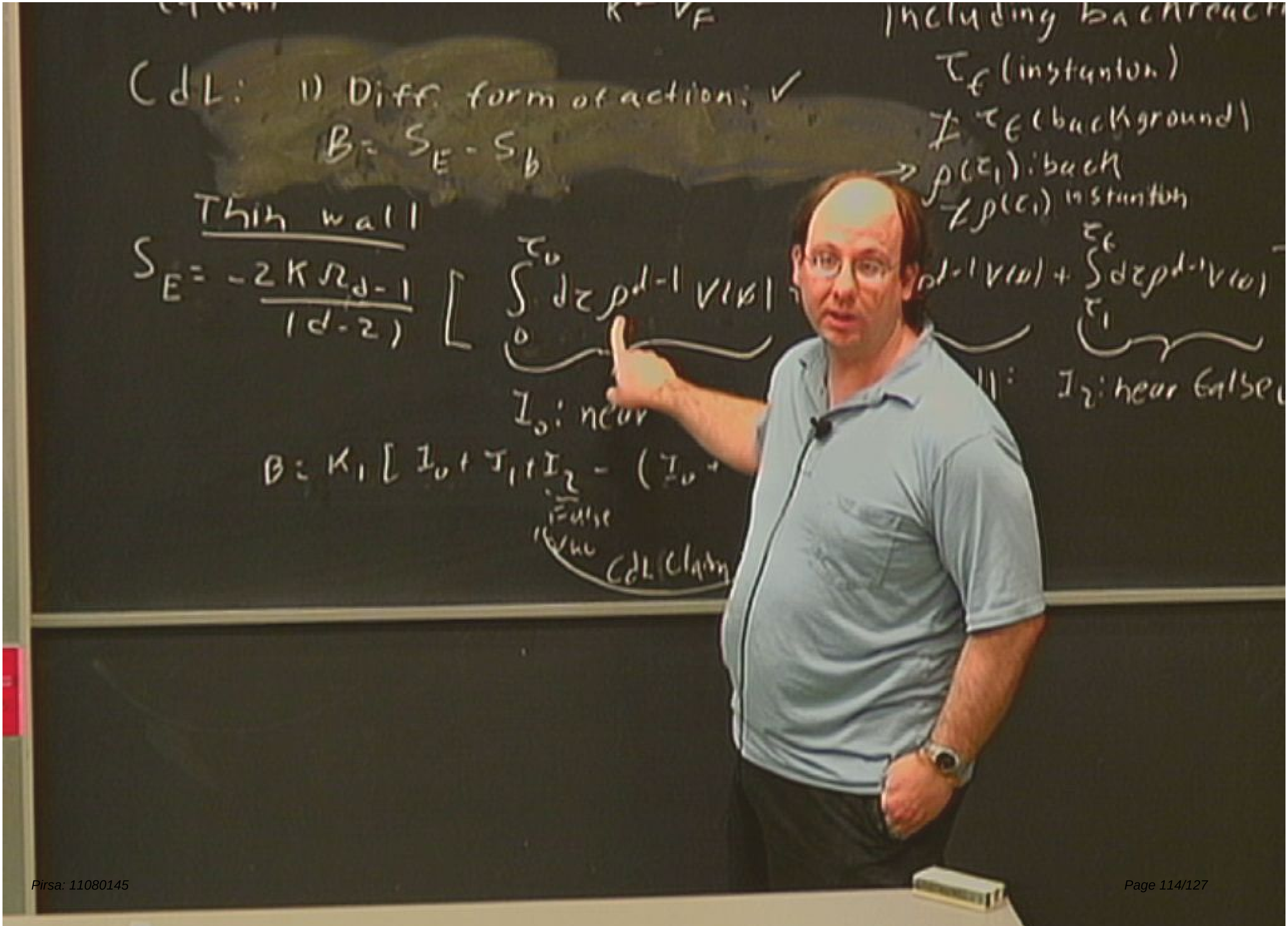
Thin wall

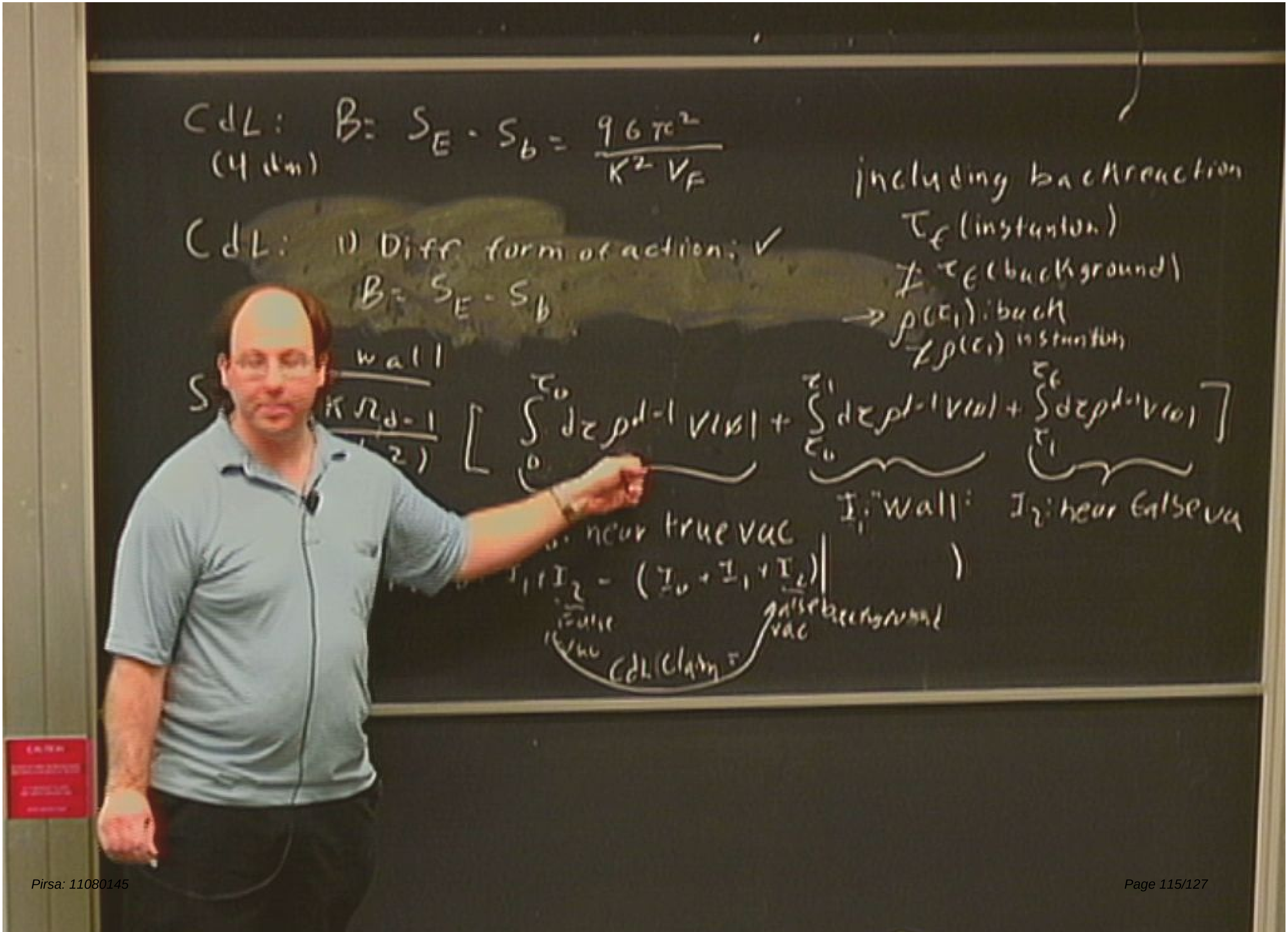
$$S_E = -\frac{2\kappa\Omega_{d-1}}{(d-2)}\left[\underbrace{\int_0^{\tau_0} d\tau\,\rho^{d-1} V(\phi)}_{I_0:\ \text{near true vac}} + \underbrace{\int_{\tau_0}^{\tau_1} d\tau\,\rho^{d-1} V(\phi)}_{I_1:\ \text{"wall"}} + \underbrace{\int_{\tau_1}^{\tau_f} d\tau\,\rho^{d-1} V(\phi)}_{I_2:\ \text{near false vac}}\right]$$

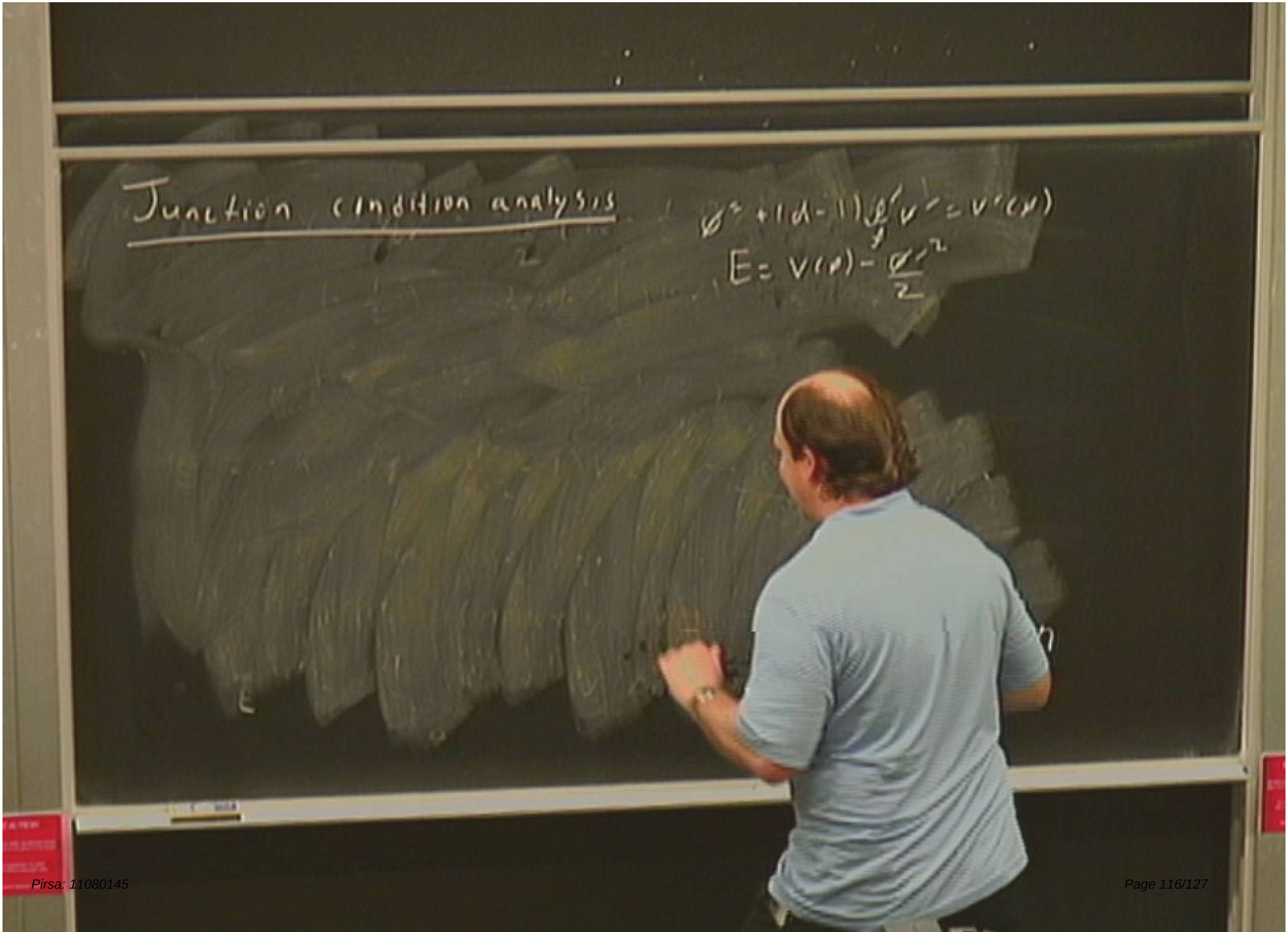
$$B = K_1\left[I_0 + I_1 + \underset{\text{false vac}}{I_2} - (I_0 + I_1 + \underset{\text{false vac}}{I_2})\right]$$

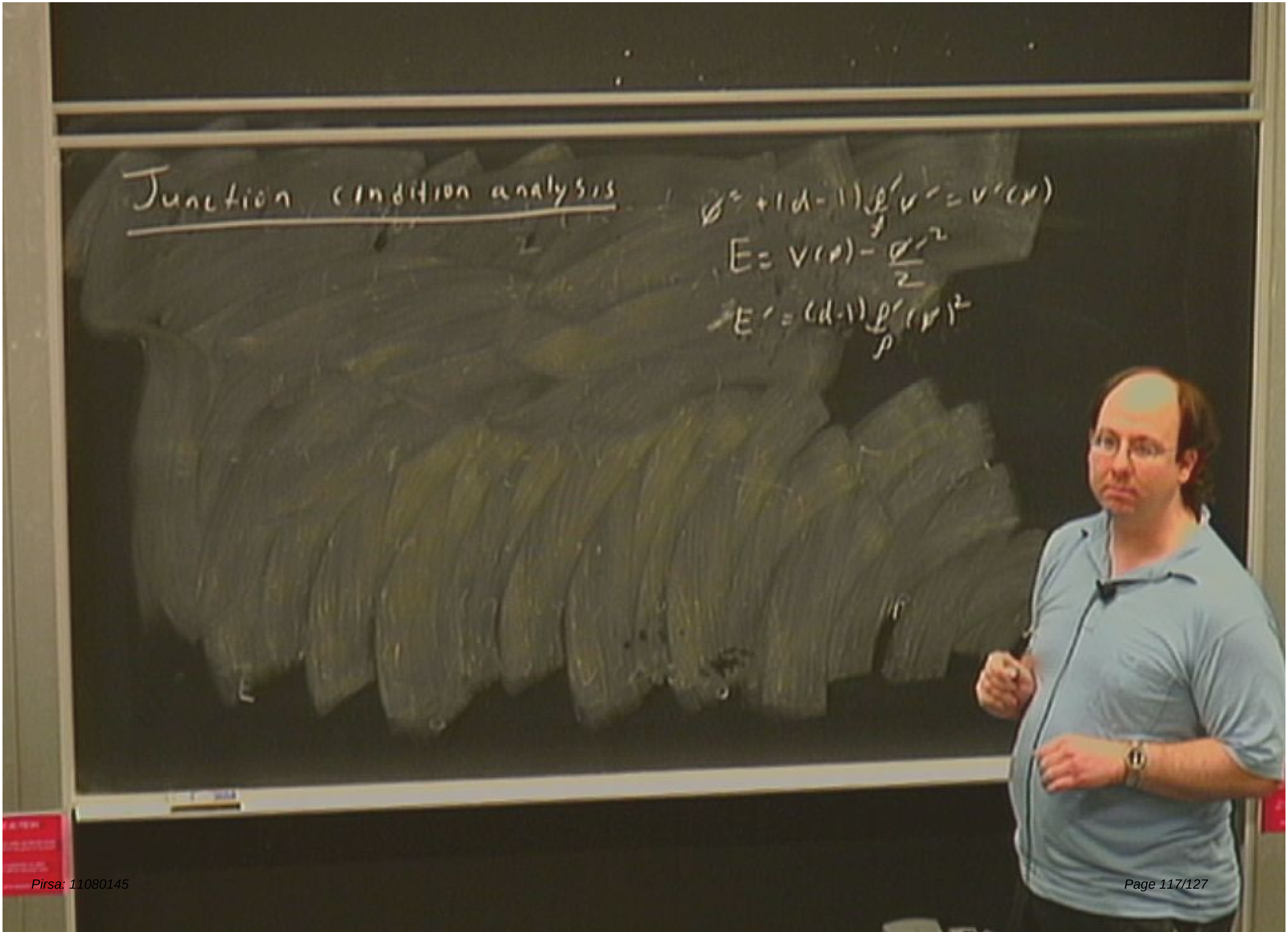
background

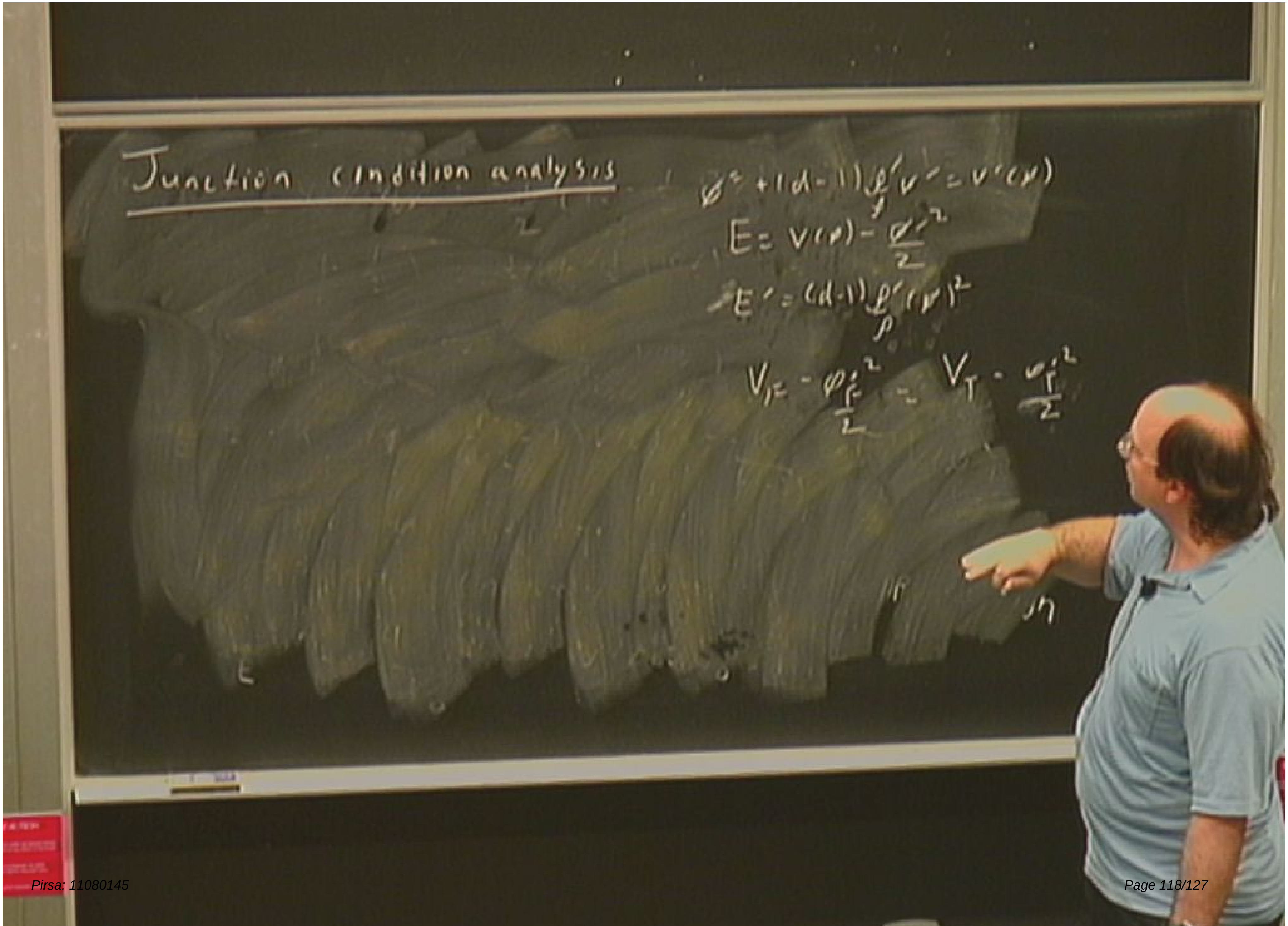

I.
Γ = A e^-B
Instanton B = S_E - S_b
background
overall
production
[φ] = L^0
[V] = 1/L^2
κ = 1/(16πG_d)

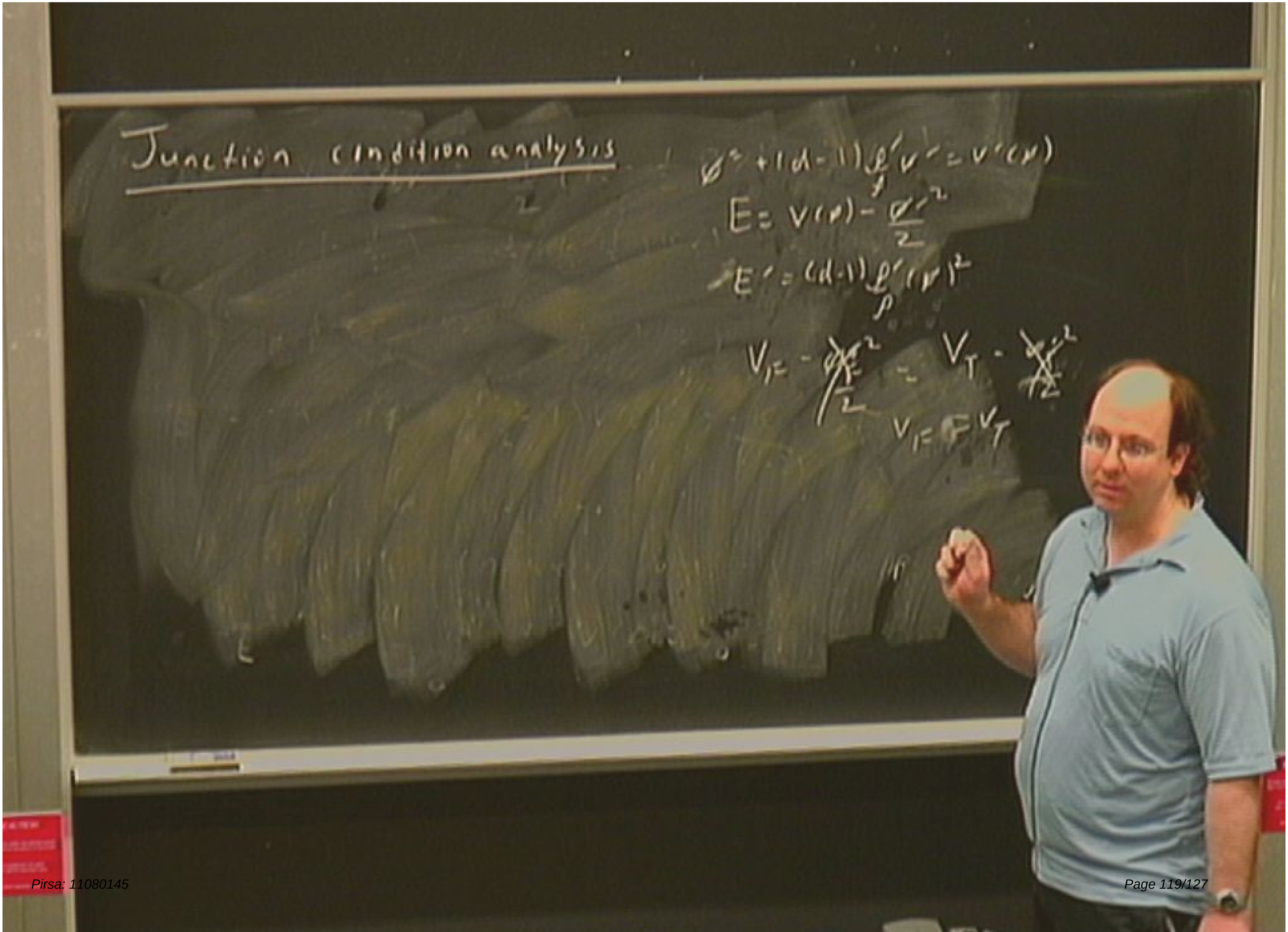

including backreaction
CdL: 1) Diff. form of action: ✓
B = S_E - S_b
Thin wall
τ_f (instanton)
≠ τ_f (background)
ρ(τ_1): back
≠ ρ(τ_1) instanton
S_E = -2KΩ_{d-1} / (d-2)
I_0: near
I_2: near false
B = K_1 [I_0 + I_1 + I_2 - (I_0 +
False
CdL Claim


CdL: B= S_E - S_b = 96π²/(κ² V_F)
(4 dim)
CdL: 1) Diff. form of action: ✓
B= S_E - S_b
including backreaction
τ_f (instanton)
≠ τ_f (background)
→ ρ(τ_1): back
≠ ρ(τ_1) instanton
wall
I_1: "wall"
I_2: near false vac
near true vac
CdL (Claim) =
false background vac


Junction condition analysis
φ'' + (d-1) φ'/r = V'(φ)
E = V(φ) - φ'²/2

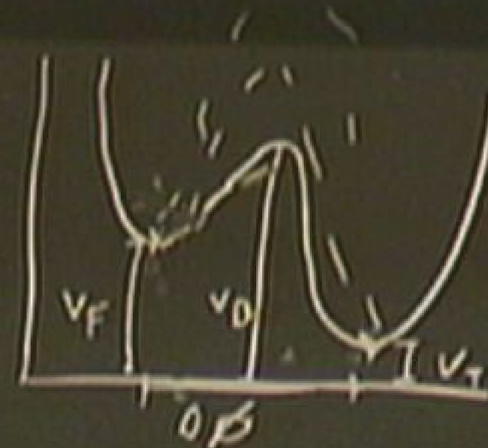

Junction condition analysis
φ'' + (d-1) ρ'/ρ φ' = V'(φ)
E = V(φ) - φ'²/2
E' = (d-1) ρ'/ρ (φ')²


Junction condition analysis
$\varphi'' + (d-1)\frac{\rho'}{\rho}\varphi' = V'(\varphi)$
$E = V(\varphi) - \frac{\varphi'^2}{2}$
$E' = (d-1)\frac{\rho'}{\rho}(\varphi')^2$
$V_F - \frac{\varphi_F'^2}{2} = V_T - \frac{\varphi_T'^2}{2}$


Junction condition analysis
E = V(φ) − φ'²/2
E' = (d−1) ρ'/ρ (φ')²
V_f − φ'²/2 = V_T − φ'²/2
V_f = V_T

I. $\Gamma = A e^{-B}$

Instanton $B = S_E - S_b$



$S_E = -\kappa \int \sqrt{g}\,\left(R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$

$[\phi] = L^0$

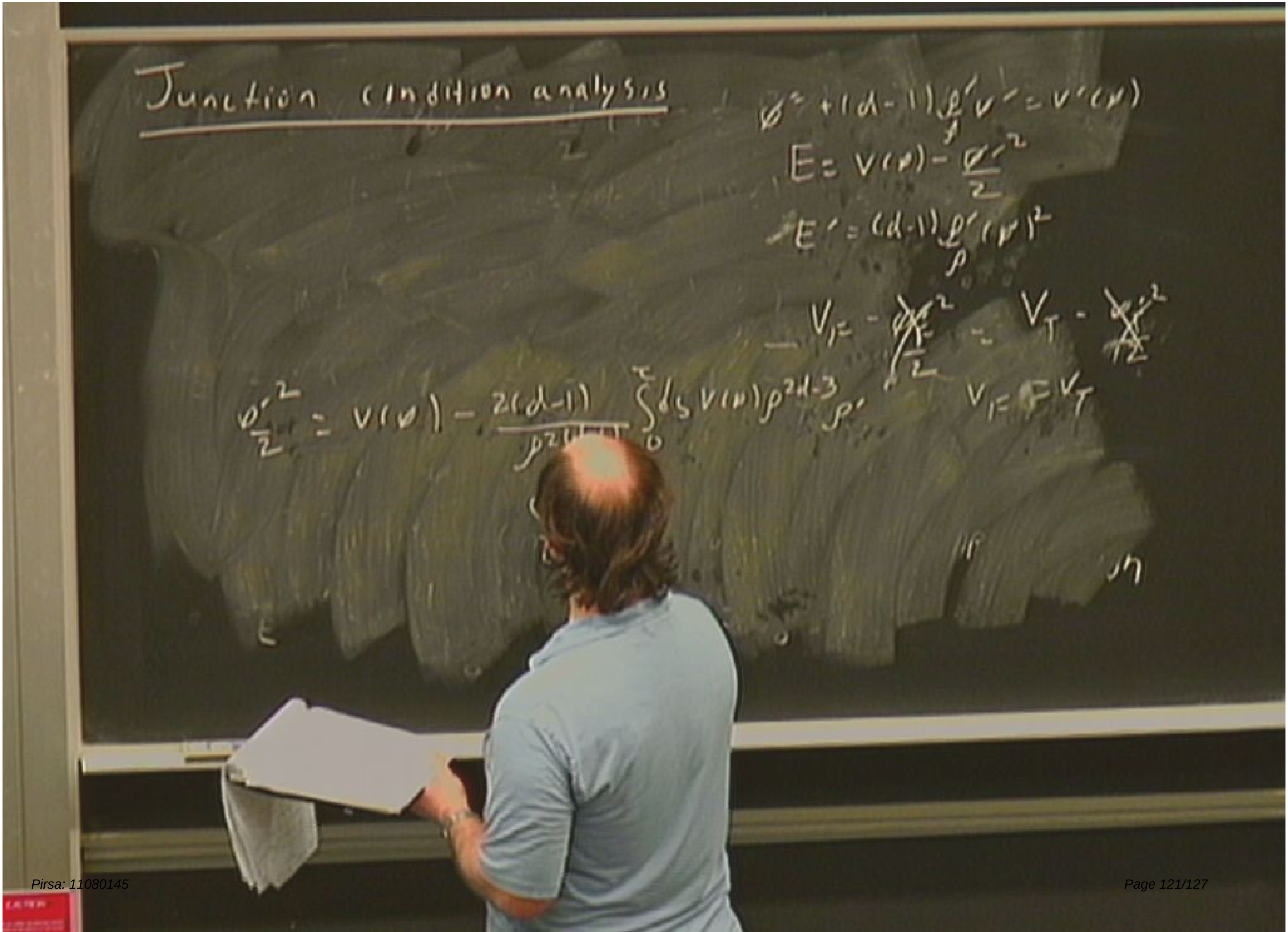
$[V] = \frac{1}{L^2}$

$\kappa = \frac{1}{16\pi G_d}$

overall sign: $\tilde{g}_{ab} = \Omega^2 g_{ab}$

$\int \sqrt{\tilde{g}}\,\tilde{R} = \int \sqrt{g}\,\Omega^{d-2}\left[R + (d-1)(d-2)\frac{\nabla^a\Omega\,\nabla_a\Omega}{\Omega^2}\right]$

Bousso Hawking: BH production


Junction condition analysis
$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$
$E = V(\phi) - \frac{\phi'^2}{2}$
$E' = (d-1)\frac{\rho'}{\rho}(\phi')^2$
$V_i - \frac{\phi'^2}{2} = V_T - \frac{\phi'^2}{2}$
$V_i = V_T$
$\frac{\phi'^2}{2} = V(\phi) - \frac{2(d-1)}{\rho^{2(d-1)}}\int_0^\tau ds\, V(\phi)\rho^{2d-3}\rho'$

Junction condition analysis

$$\phi'' + (d-1)\frac{\rho'}{\rho}\phi' = V'(\phi)$$

$$E = V(\phi) - \frac{\phi'^2}{2}$$

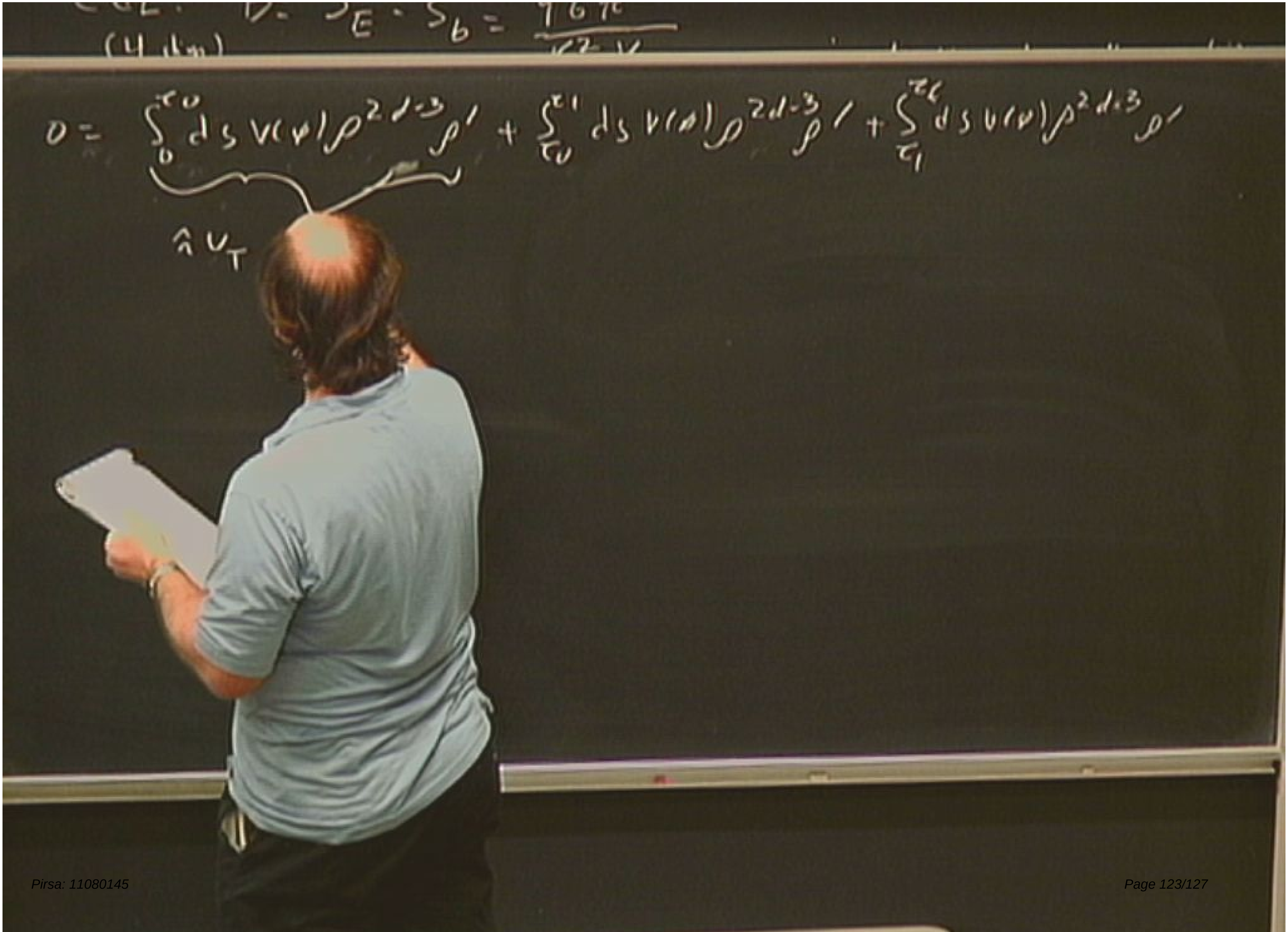
$$E' = (d-1)\frac{\rho'}{\rho}(\phi')^2$$

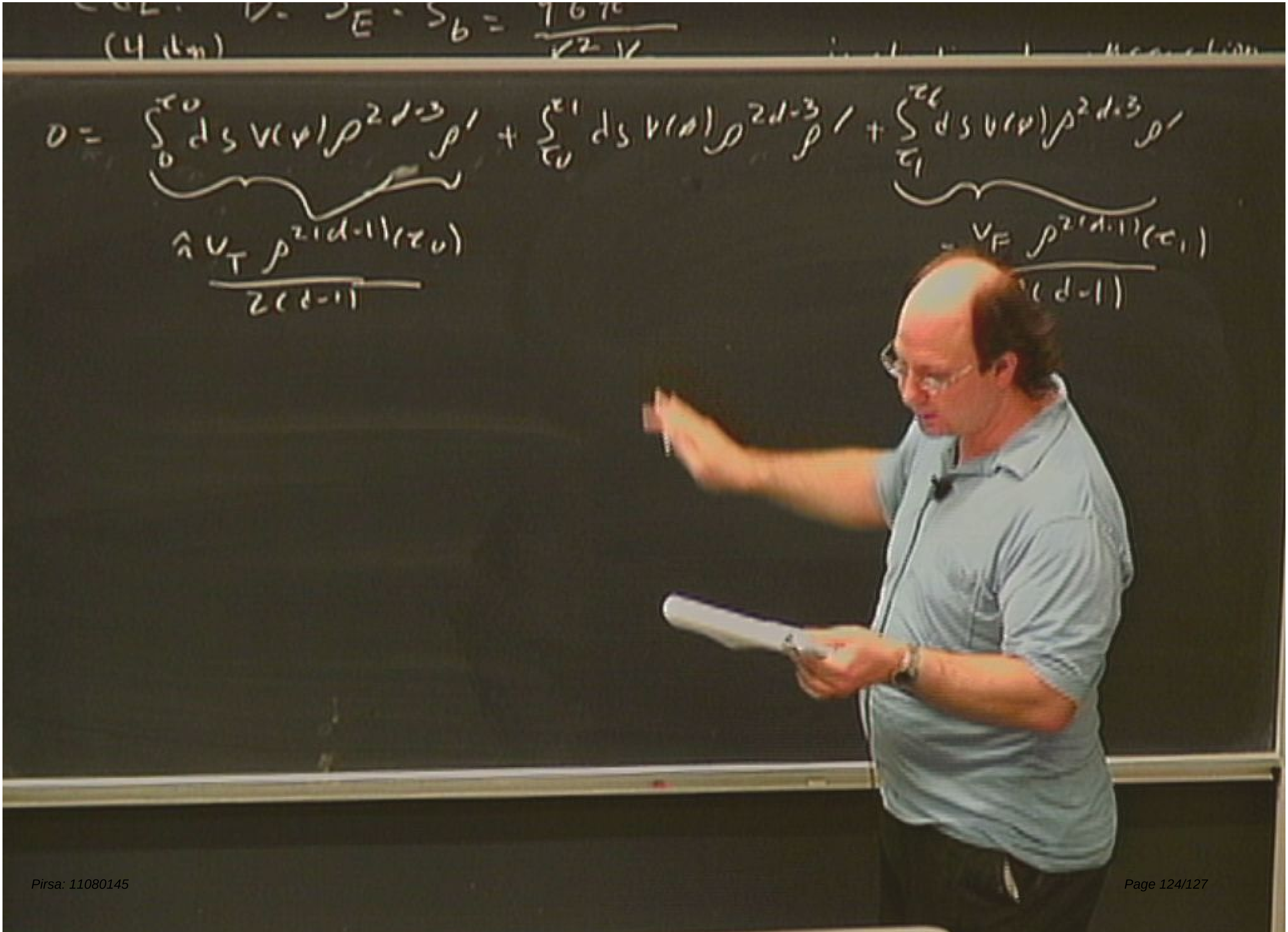
$$V_F - \frac{\phi'^2}{2} = V_T - \frac{\phi'^2}{2}$$

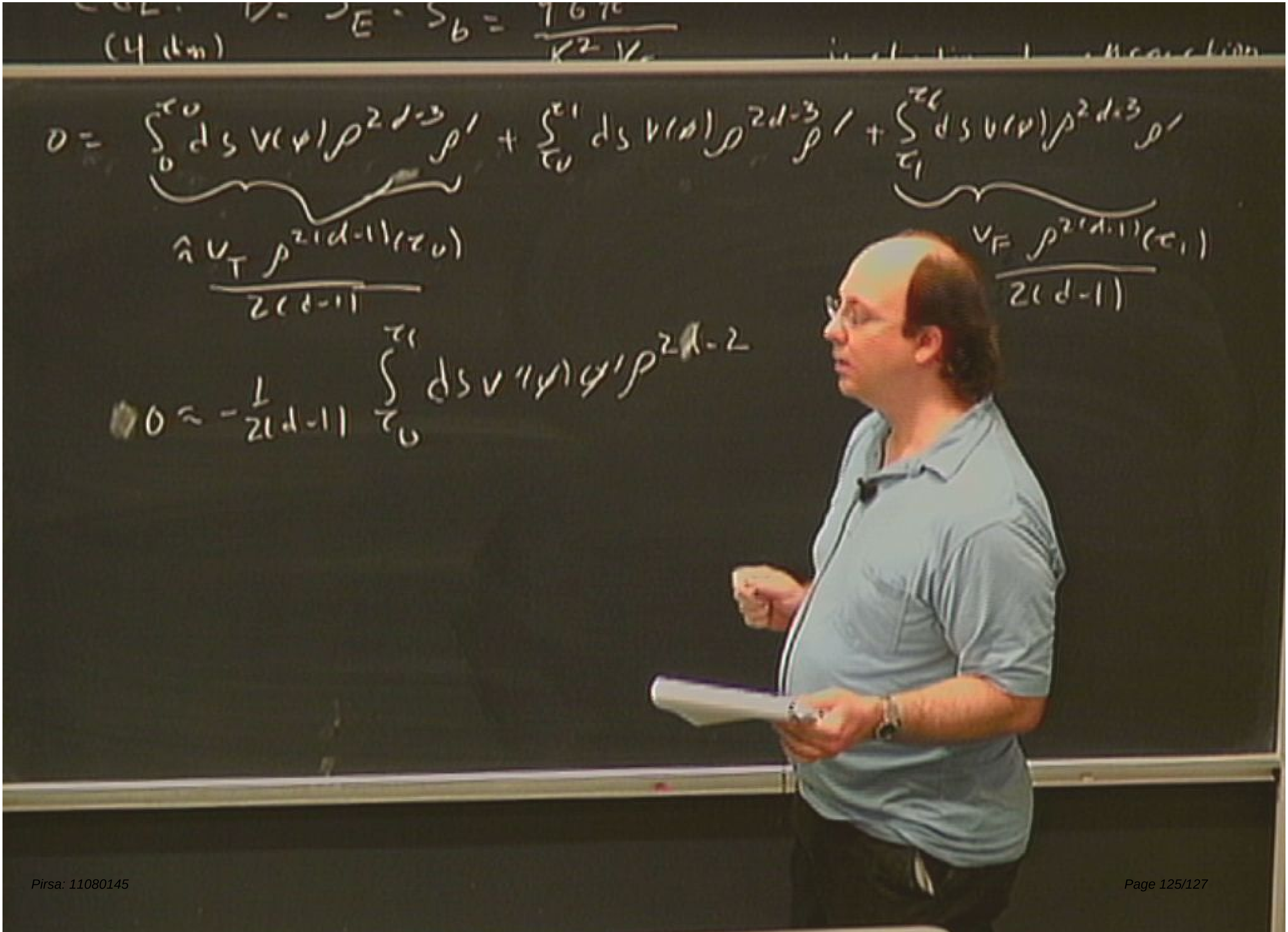
$$V_F = V_T$$

$$\frac{\phi'^2}{2} = V(\phi) - \frac{2(d-1)}{\rho^{2(d-1)}}\int_0^{r} ds\, V(\phi)\,\rho^{2d-3}\rho'$$

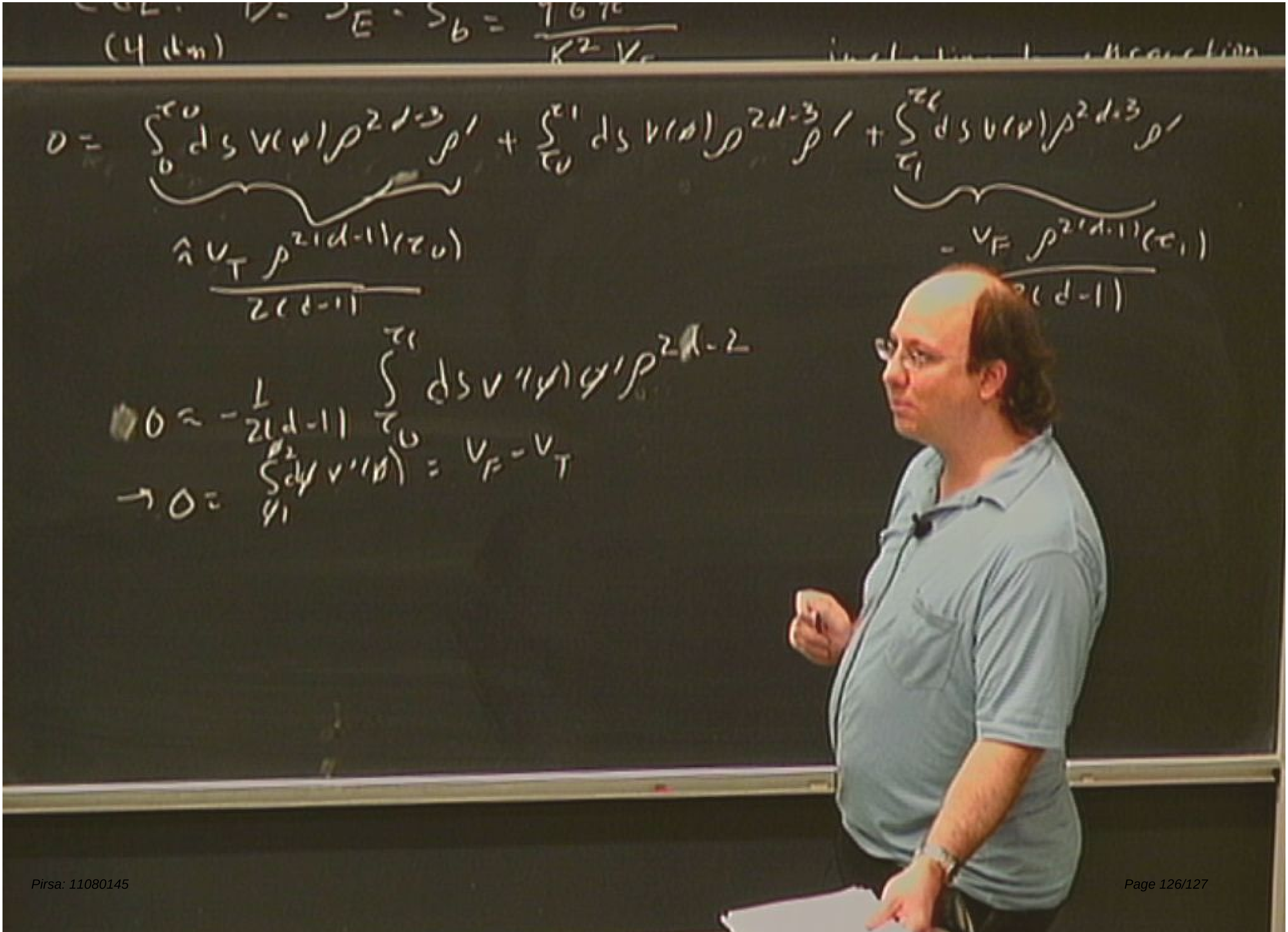
$$\int_0^{r_E} ds\, V(\phi)\,\rho^{2d-3}\rho' = 0$$

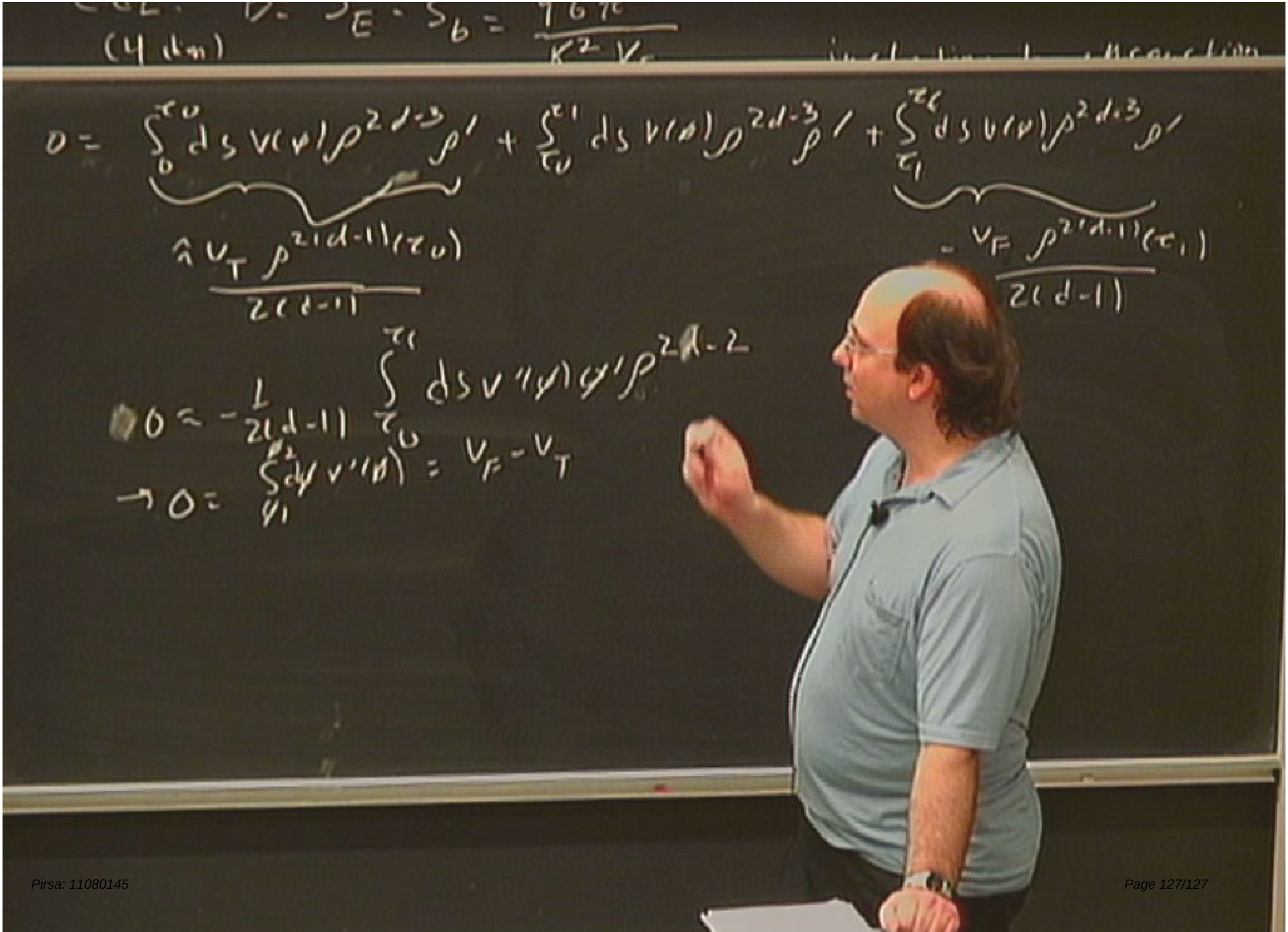

$0 = \int_0^{\tau_0} ds\, V(\varphi)\rho^{2d-3}\rho' + \int_{\tau_0}^{\tau_1} ds\, V(\varphi)\rho^{2d-3}\rho' + \int_{\tau_1}^{\tau_2} ds\, V(\varphi)\rho^{2d-3}\rho'$
$\hat{\approx} V_T$


(4 dim)
0 = ∫_0^τ0 ds V(φ) ρ^(2d-3) ρ' + ∫_τ0^τ1 ds V(φ) ρ^(2d-3) ρ' + ∫_τ1^τf ds V(φ) ρ^(2d-3) ρ'
≈ V_T ρ^(2(d-1))(τ0) / 2(d-1)
V_F ρ^(2(d-1))(τ1) / 2(d-1)



$0 = \int_0^{\tau_0} ds\, V(\varphi) \rho^{2d-3} \rho' + \int_{\tau_0}^{\tau_1} ds\, V(\varphi) \rho^{2d-3} \rho' + \int_{\tau_1}^{\tau_f} ds\, V(\varphi) \rho^{2d-3} \rho'$
$\hat{=} \frac{V_T \rho^{2(d-1)}(\tau_0)}{2(d-1)}$
$\frac{V_F \rho^{2(d-1)}(\tau_1)}{2(d-1)}$
$0 \approx -\frac{1}{2(d-1)} \int_{\tau_0}^{\tau_1} ds\, V'(\varphi) \varphi' \rho^{2d-2}$


(4 dim)
0 = ∫ ds v(φ) ρ^{2d-3} ρ' + ∫ ds v(φ) ρ^{2d-3} ρ' + ∫ ds v(φ) ρ^{2d-3} ρ'
v_T ρ^{2(d-1)}(τ_0) / 2(d-1)
- v_F ρ^{2(d-1)}(τ_1) / 2(d-1)
0 = - 1/2(d-1) ∫ ds v'(φ) φ' ρ^{2d-2}
→ 0 = ∫ dφ v'(φ) = v_F - v_T


(4 dim)
0 = ∫ ds v(φ) ρ^{2d-3} ρ' + ∫ ds v(φ) ρ^{2d-3} ρ' + ∫ ds v(φ) ρ^{2d-3} ρ'
2(d-1)
2(d-1)
0 = - 1/2(d-1) ∫ ds v'(φ) φ' ρ^{2d-2}
= V_F - V_T
→ 0 =