

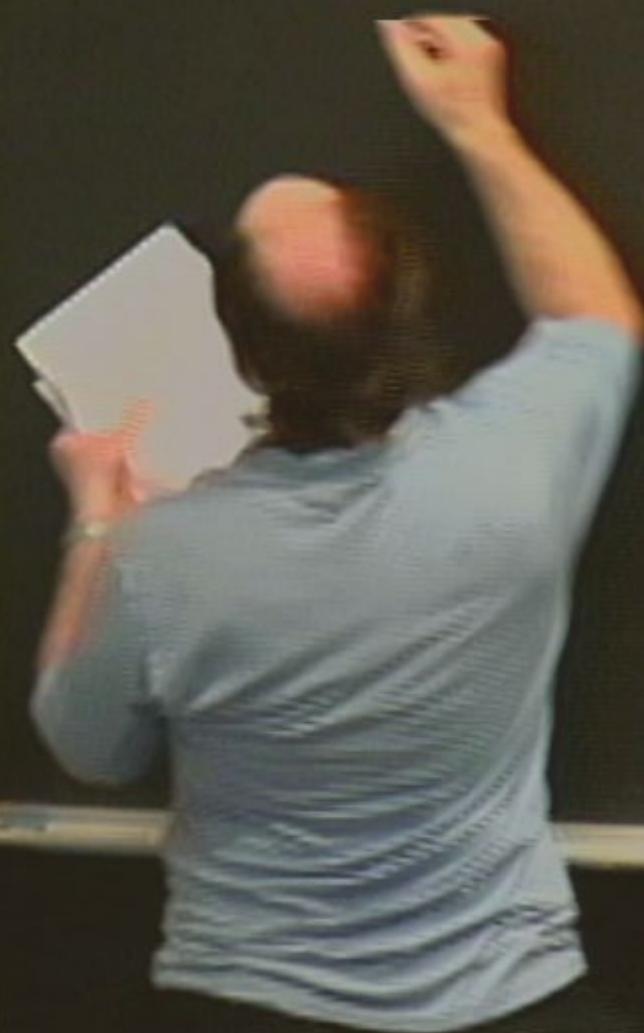
Title: Gravity and Rapid Tunnelling

Date: Aug 24, 2011 11:30 AM

URL: <http://pirsa.org/11080145>

Abstract: I'll discuss my recent results (1108.2255) showing that, once gravity and some technical confusions are taken care of, two classes of potentials one expects to be generic in a landscape, namely ones resulting in thin-wall instantons or with small relative differences in potentials, result in instantons which typically decay rapidly, including exponentially enhanced rates for thin-wall instantons. I'll explain why this is true both generally and in detail and why the previous treatments have gone astray. This meeting will be designed to be a highly informal and interactive session to present these results in detail and address questions, confusions, and challenges of those, esp. cosmologists, with some level of background in tunnelling. I'll give a broader presentation of this material for a more general audience in the strings group meeting (Friday 11am, space room).

I Background + simple argument



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II Problem with CdL + junction analysis

III



I Background + simple argument

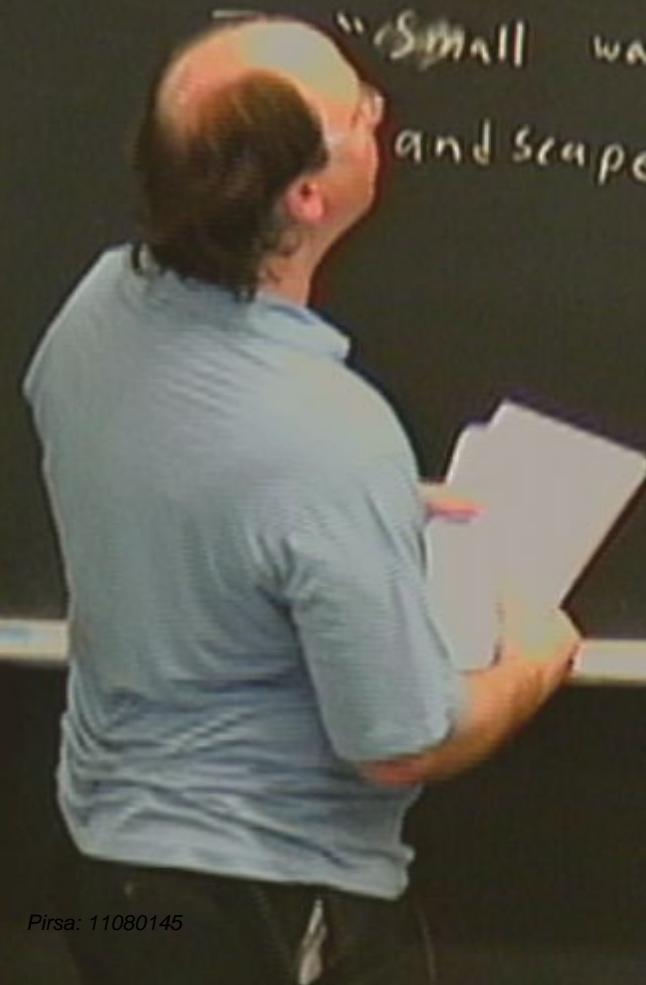
II Problem with CdL + junction analysis

III Corrected calculation

"Small wall"

and scape?

- - -



I Background + simple argument

II Problem with CDL + junction analysis

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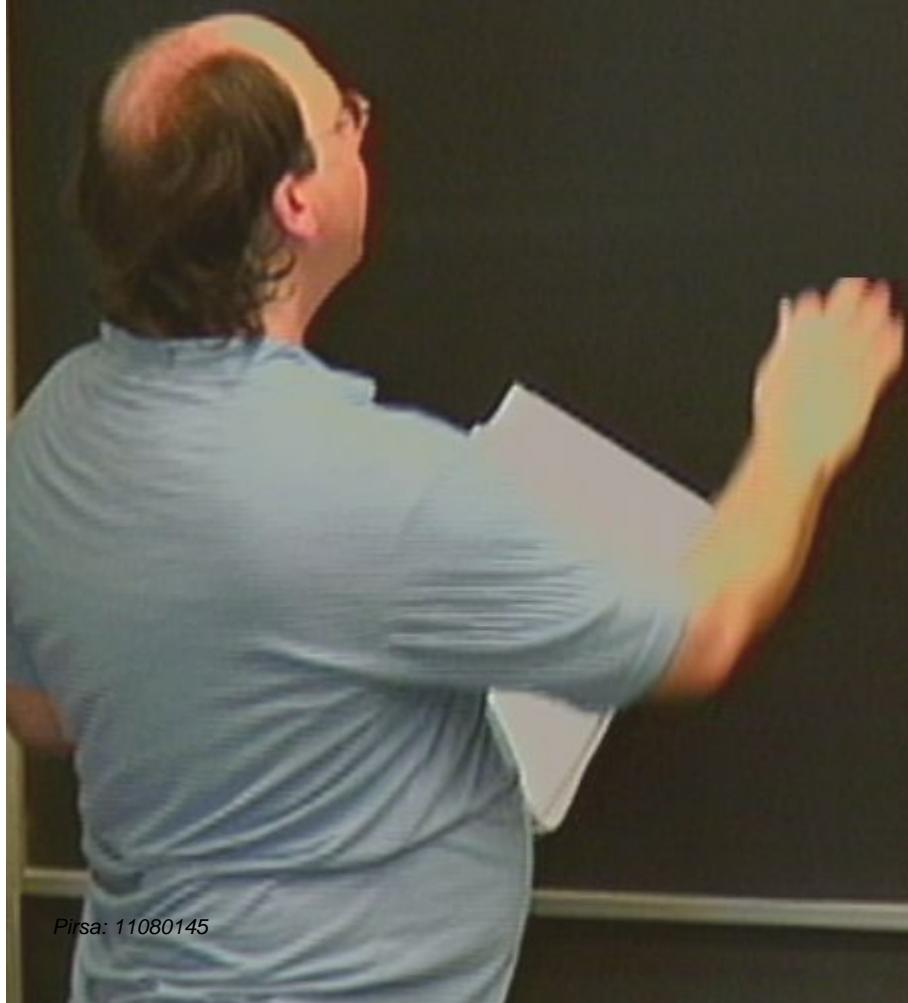
IV "Small wall"

V Landscape?

- - -

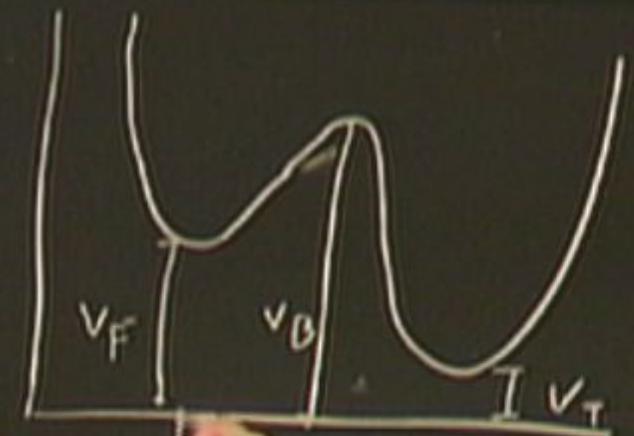
I. $T = A e^{-B}$

$$B = S_E - S_b$$



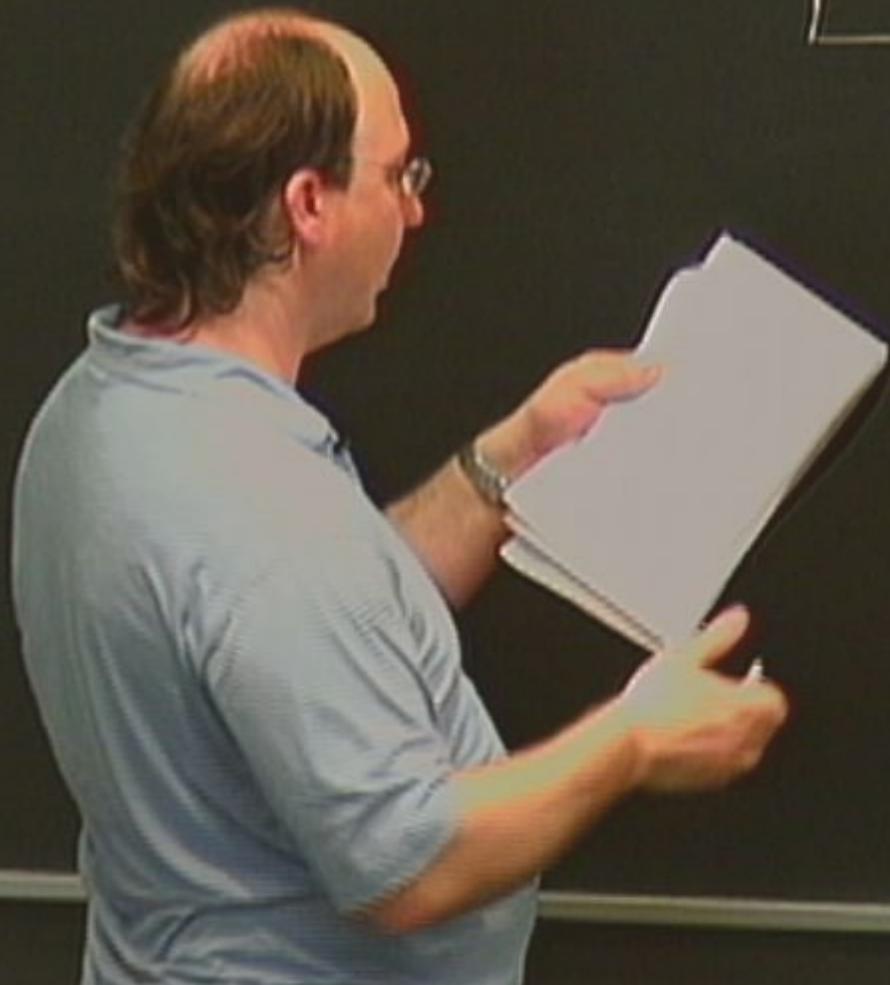
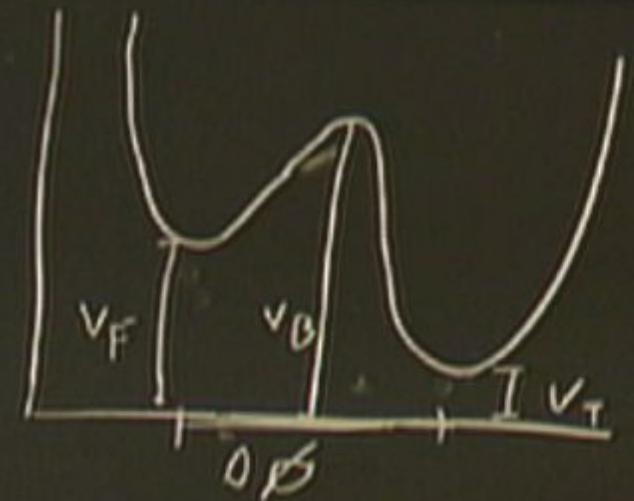
$$I. T = A e^{-\beta}$$

$$\beta = S_E - S_b$$



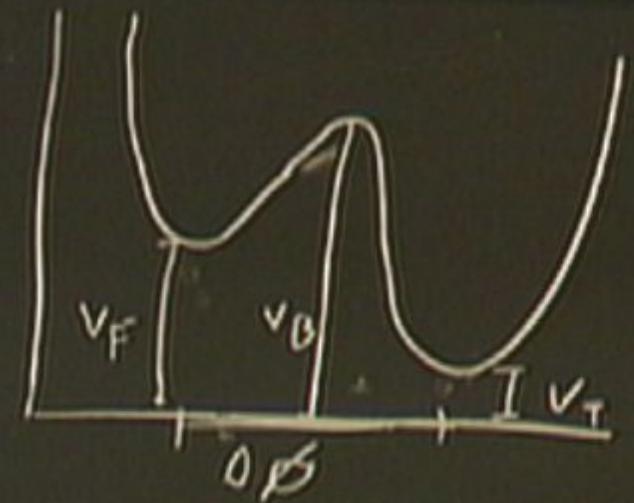
$$I. T = A e^{-B}$$

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$$I. T = A e^{-B}$$

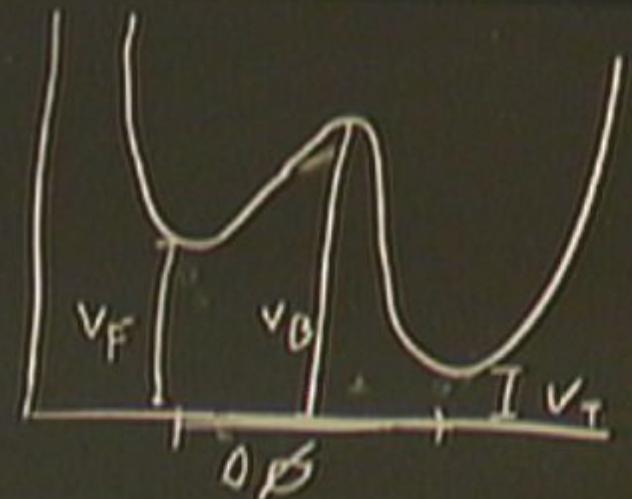
$$B = S_E - S_b$$



$$S_E = -k \int \sqrt{g'} (R - \frac{1}{2}(\nabla \phi)^2 - V(\phi))$$

$$I. \quad T = A e^{-B}$$

$$B = S_E - S_b$$

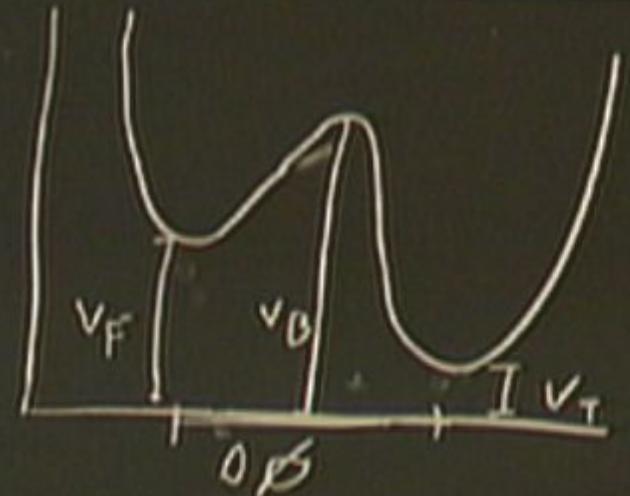


$$S_E = -k \int \sqrt{g'} (R - \frac{1}{2}(\nabla \phi)^2 - V(\phi))$$

Overall sign:

$$I. \quad T = A e^{-B}$$

$$B = S_E - S_b$$



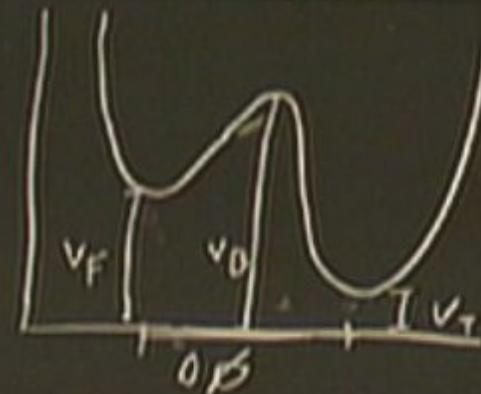
$$S_E = -\int d\mathbf{r} \left(R - \frac{1}{2}(\nabla \phi)^2 - V(\phi) \right)$$

$\propto V$

$$\hat{g}_{ab} = R^2 g_{ab}$$

$$I. \quad T = A e^{-B}$$

$$B = S_E - S_b$$



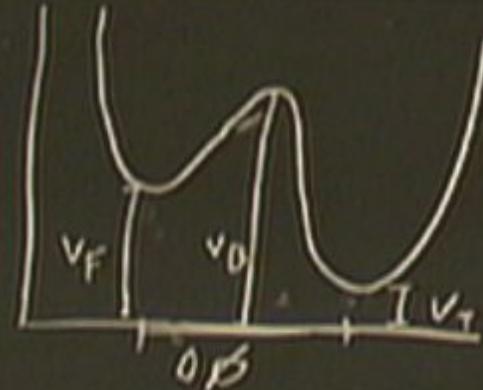
$$S_E = \frac{1}{2} (R - \frac{1}{2}(\sigma_0)^2 - V(\sigma_1))$$

sign: $\hat{\mathcal{G}}_{ab} = R^2 g_{ab}$

$$R = \sqrt{g} \sqrt{d-2} [R + (d-1)(d-2) V^2 \frac{\nabla^2 R}{R}]$$

$$I. \quad T = A e^{-B}$$

$$B = S_E - S_B$$



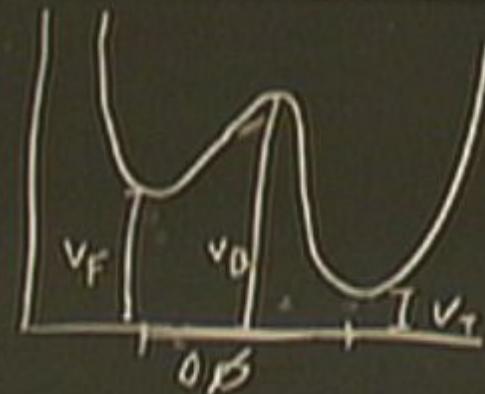
$$\int \sqrt{f} \left(R - \frac{1}{2}(\sigma_0)^2 - V(r) \right)$$

$$\hat{\mathcal{G}}_{ab} = R^2 g_{ab}$$

$$\int \sqrt{f} \hat{R} = \int \sqrt{g} R^{d-2} [R + (d-1)(d-2)V^2 \frac{\nabla^2 R}{R}]$$

$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



$$S_E = -k \int \sqrt{g} \left(R - \frac{1}{2} (\sigma_B)^2 - V(\sigma) \right)$$

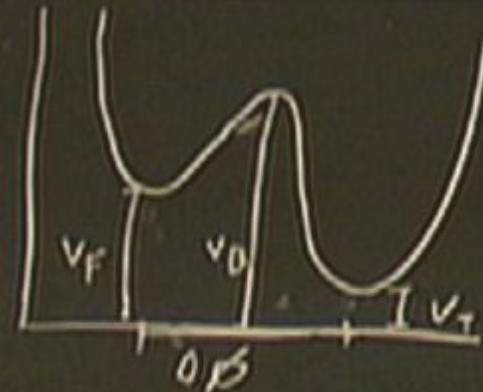
Overall sign:

$$\hat{\mathcal{G}}_{ab} = \mathcal{N}^2 g_{ab}$$

$$\int \sqrt{g} \hat{R} = \int \sqrt{g} \mathcal{N}^{d-2} [R + (d-1)(d-2)V^2 \mathcal{N} \frac{\nabla^2 \mathcal{N}}{\mathcal{N}^2}]$$

$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



$$S_E = -\mathcal{K} \int \sqrt{g} \left(R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right)$$

Overall sign:

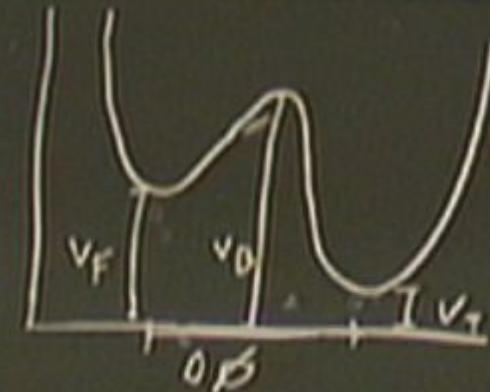
$$\hat{g}_{ab} = \mathcal{N}^2 g_{ab}$$

$$\int \sqrt{g} \hat{R} = \int \sqrt{g} \mathcal{N}^{d-2} [R + (d-1)(d-2)V \frac{\partial \phi}{\partial r}]$$

Bousso, Hawking, BH production

$$I. T = A e^{-B}$$

$$B = S_E - S_B$$



$$S_E = -K \int \sqrt{g} \left(R - \frac{1}{2} (\nabla \sigma)^2 - V(\sigma) \right)$$

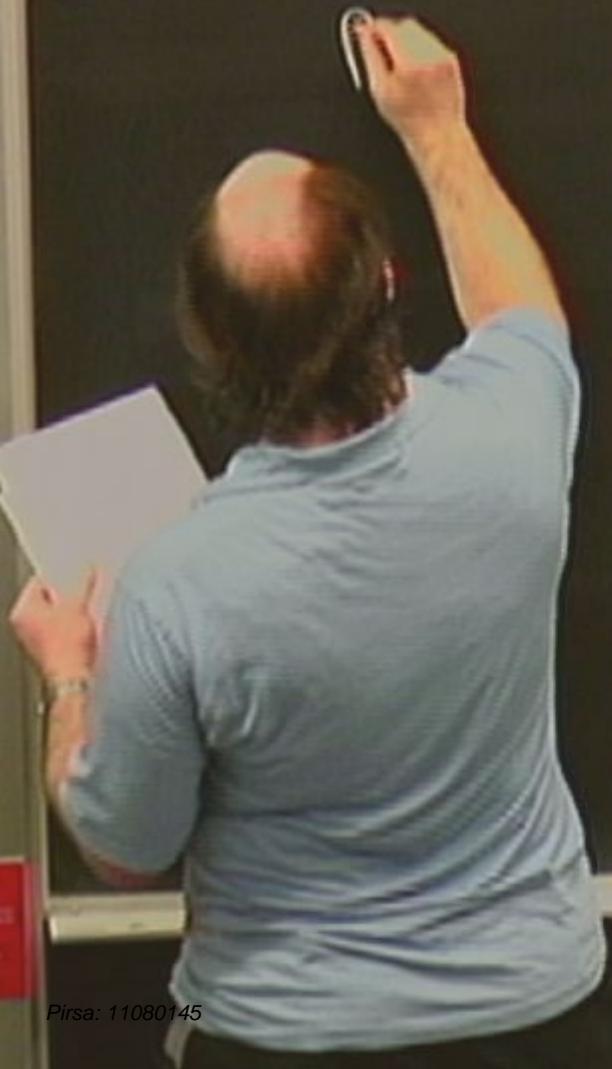
overall sign:

$$\hat{g}_{ab} = R^2 g_{ab}$$

$$\int \sqrt{g} R = \int \sqrt{g} R^{d-2} [R + (d-1)(d-2)V \frac{\nabla \sigma}{\sigma}]$$

Bousso, Hawking, BH production

$$R_{ab} - \frac{1}{2} \nabla_a \phi \nabla_b \phi - \frac{g_{ab}}{2} (R - \frac{1}{2} (\nabla \phi)^2 - V(\phi)) = 0$$



$$R_{ab} - \frac{1}{2} \nabla_a \phi \nabla_b \phi - \frac{g_{ab}}{2} (R - \frac{1}{2} (\nabla \phi)^2 - V(\phi)) = 0$$

$$R(1 - \frac{d}{2}) = \frac{1}{2}(1 - \frac{d}{2})(\nabla \phi)^2 - \frac{d}{2}V(\phi)$$



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$$R = \frac{1}{2} (\nabla \phi)^2 + \frac{d}{d-2} V(\phi)$$

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S

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$$R = \frac{1}{2}(\nabla \phi)^2 + \frac{d}{d-2}V(\phi)$$

$$S_E = -K \sqrt{g} \left(\frac{d}{d-2} \right)$$

$$R_{ab} - \frac{1}{2} \nabla_a \phi \nabla_b \phi - \frac{g_{ab}}{2} (R - \frac{1}{2} (\nabla \phi)^2 - V(\phi)) = 0$$

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$$R = \frac{1}{2}(\nabla \phi)^2 + \frac{d}{d-2}V(\phi)$$

$$S_F = -K \int \sqrt{g} \left(\frac{d}{d-2} - 1 \right) V(\phi) = -\frac{2K}{(d-2)} \int \sqrt{g} V(\phi)$$



$$R_{ab} - \frac{1}{2} \nabla_a \phi \nabla_b \phi - \frac{g_{ab}}{2} (R - \frac{1}{2} (\nabla \phi)^2 - V(\phi)) = 0$$

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$$R = \frac{1}{2}(\nabla \phi)^2 + \frac{d}{d-2}V(\phi)$$

$$S_E = -K \int \sqrt{g} \left(\frac{d}{d-2} - 1 \right) V(\phi) = -2K \underbrace{\int \sqrt{g} V(\phi)}_{(d-2)}$$

- no gradient terms

- potential (naively) comes in with opposite sign

$$R_{ab} - \frac{1}{2} \nabla_a \phi \nabla_b \phi - \frac{g_{ab}}{2} (R - \frac{1}{2} (\nabla \phi)^2 - V(\phi)) = 0$$

$$R(1 - \frac{d}{2}) = \frac{1}{2}(1 - \frac{d}{2})(\nabla \phi)^2 - \frac{d}{2} V(\phi)$$

$$R = \frac{1}{2} (\nabla \phi)^2 + \frac{d}{d-2} V(\phi)$$

$$S_E = -K \int \sqrt{g} \left(\frac{d}{d-2} - 1 \right) V(\phi) = -2K \frac{\int \sqrt{g} V(\phi)}{(d-2)}$$

- no gradient terms

- potential (naively) comes in with opposite sign

$$S_E^{(4)} = -\frac{4\delta K}{V_0}$$

$$R_{ab} - \frac{1}{2} \nabla_a \psi \nabla_b \psi - \frac{g_{ab}}{2} (R - \frac{1}{2} (\nabla \psi)^2 - V(\psi)) = 0$$

$$R(1 - \frac{d}{2}) = \frac{1}{2}(1 - \frac{d}{2})(\nabla \psi)^2 - \frac{d}{2}V(\psi)$$

$$R = \frac{1}{2}(\nabla \psi)^2 + \frac{d}{d-2}V(\psi)$$

$$S_E = -K \int \sqrt{g} \left(\frac{d}{d-2} - 1 \right) V(\psi) = -\frac{2K}{(d-2)} \int \sqrt{g} V(\psi)$$

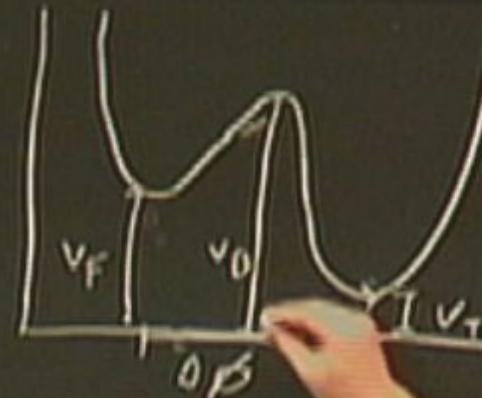
- no gradient terms

- potential (naively) comes in with opposite sign

$$S_E^{(4)} = -\frac{4\delta K}{V_0} \quad V(\psi) = V_0 > 0$$

$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



$$S_E = -K \int \sqrt{g} (R - \frac{1}{2}(\nabla \phi)^2 - V(\phi))$$

Overall sign:

$$\tilde{g}_{ab} = R^2 g_{ab}$$

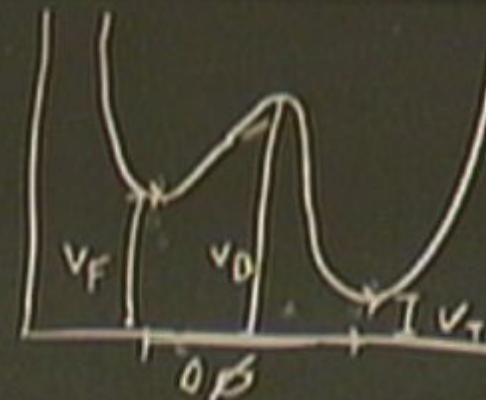
$$\int \sqrt{g} R = \int g \sqrt{R^{d-2}} [R + \dots]$$

Bousso, Hawking, BH production

$$\frac{\partial \Delta_{\text{BH}}}{\partial L}$$

$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



$$S_E = -k \int \sqrt{g} (R - \frac{1}{2}(\sigma \theta)^2 - V(r))$$

Overall sign:

$$\hat{g}_{ab} = R^2 g_{ab}$$

$$\int \sqrt{g} R = \int \sqrt{g} R^{d-2} [R + (d-1)(d-2)V^2 \frac{\nabla_a \nabla_b}{R}]$$

Bousso, Hawking, BH production

$$ds^2 = d\tau^2 + \rho^2(\tau) dR_{d-1}$$

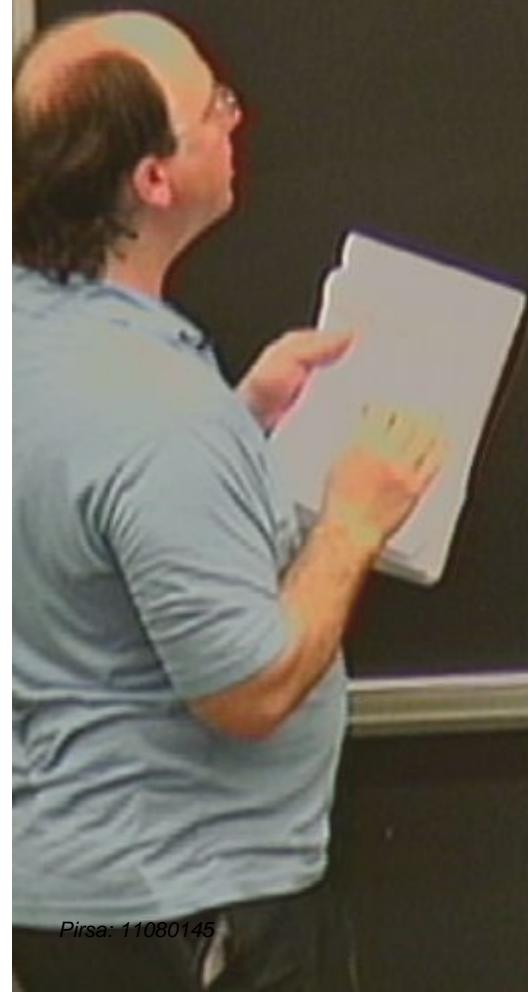


$$d\zeta^2 = d\tau^2 + \rho^2(\tau) dR_{d-1}^2, \rho(\tau)$$



$$\int \zeta^2 = \int \tau^2 + \rho^2(\tau) \int R_{d-1} \quad , \quad \mathcal{V}(\tau)$$

$$S_E = -\frac{2K R_{d-1}}{(d-2)} \int_0^{\tau_e} \int c \rho^{d-1} V(r)$$



$$ds^2 = d\tau^2 + \rho^2(\tau) d\mathcal{N}_{d-1}(\varphi(\tau))$$

$$S_E = -\frac{2K\mathcal{N}_{d-1}}{(d-2)} \int_0^{\tau_e} d\tau \rho^{d-1} V(\varphi)$$

$$\text{If } V(\varphi) = V_0$$

$$\rho = \frac{1}{\omega_0} \sin \omega_0 \tau \quad \omega_0^2 = \frac{V_0}{(d-1)(d-2)}$$

$$ds^2 = d\tau^2 + \rho^2(\tau) d\mathcal{N}_{d-1}(\varphi)$$

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$$S_E = -\frac{2K\mathcal{R}_{d-1}}{(d-2)} \int_0^\tau d\tau \rho^{d-1} V(\varphi)$$

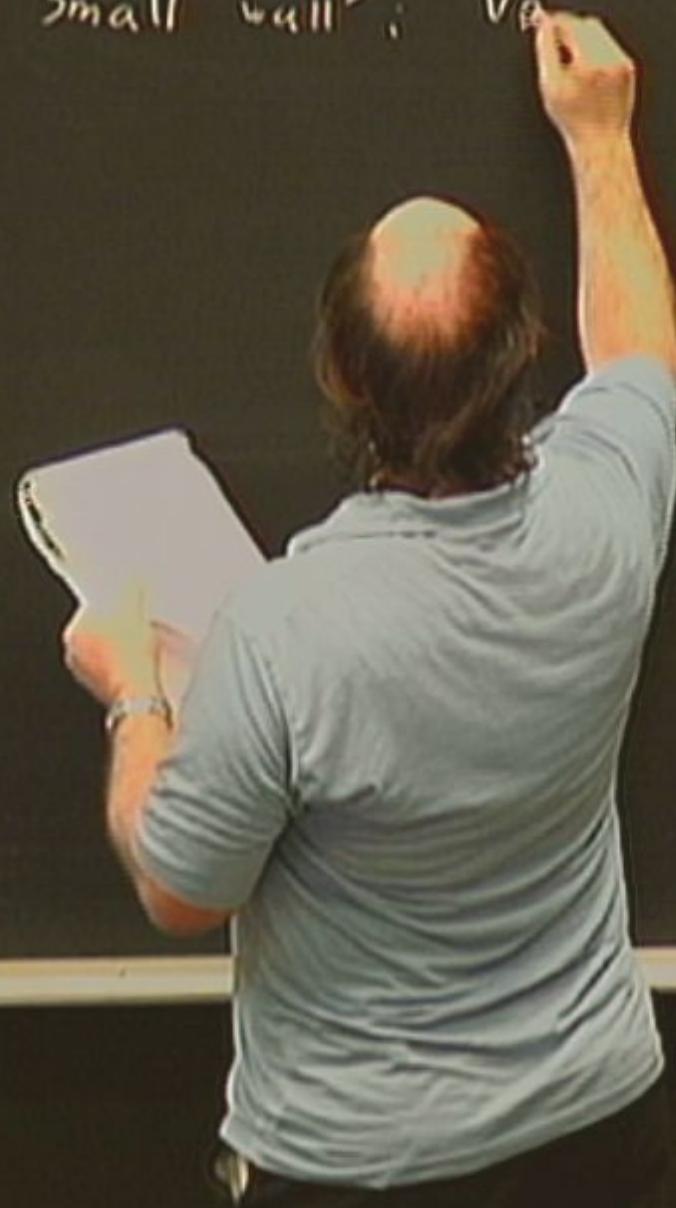
$$\text{If } V(v) = V_0$$

$$\rho = \frac{1}{\omega_0} \sin \omega_0 \tau \quad \omega_0^2 = \frac{V_0}{(d-1)(d-2)}$$



Two classes of potentials with rapid transitions:

i) "Small wall"; V_0



Two classes of potentials with rapid transitions

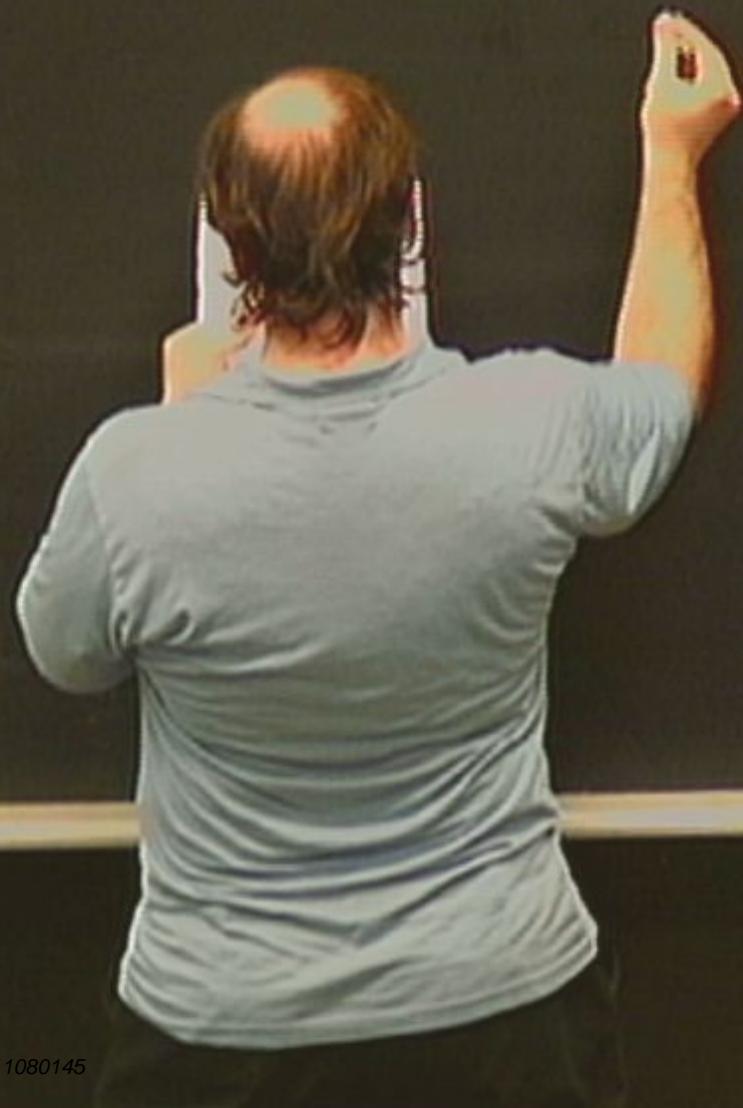
I) "Small wall": $V_B - V_T \ll V(r)$



Two classes of potentials with rapid transitions

i) "Small wall": $V_B - V_T \ll V(r)$

$$S_E - S_b$$



Two classes of potentials with rapid transitions

I) "Small wall": $V_B - V_T \ll V(\varphi)$

$$S_E - S_b \ll S_b$$

$$S_E - S_b \approx \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



Two classes of potentials with rapid transitions:

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1) "Small wall": $V_B - V_T \ll V(\varphi)$

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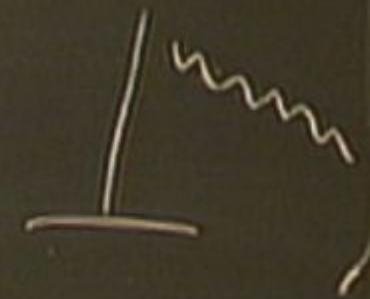
2) "thin wall": $\rho \approx \rho_0$ when φ not near a minimum

Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T \ll V(\varphi)$

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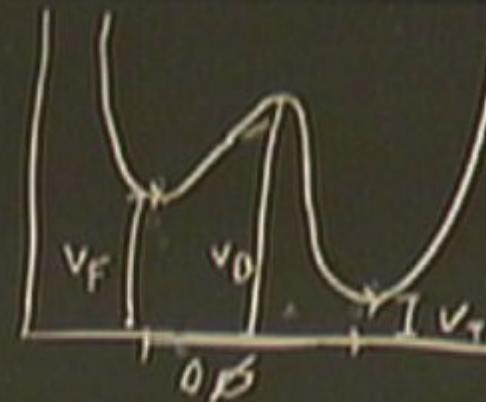
2) "thin wall": $\rho \approx \rho_0$ when φ not near a minimum

I.e. $\Delta\varphi \ll 1$



$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



$$[B] = L^0$$

$$[V] = \frac{1}{L^2}$$

$$\kappa = \frac{1}{16\pi G_d}$$

$$S_E = -\kappa \int \sqrt{g} (R - \frac{1}{2}(\sigma_\theta)^2 - V(r))$$

Overall sign:

$$\hat{g}_{ab} = R^2 g_{ab}$$

$$\int \sqrt{g} R = \int \hat{g} R^{d-2} [R + (d-1)(d-2)V_R \frac{\partial V}{\partial R}]$$

Bousso, Hawking, BH production

Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T \ll V(\varphi)$

$$S_E - S_b \ll S_b$$

$$S_E - S_b \approx \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



2) "thin wall": $\rho \approx \rho_0$ when φ not near a minimum

I.e. $\Delta\varphi \ll 1$ $V_B \gg V_F - V_T$

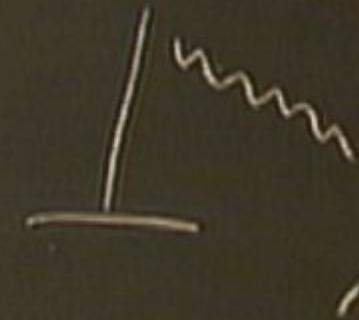


Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T \ll V(\varphi)$

$$S_E - S_b \ll S_b$$

$$S_E - S_b \approx \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



2) "thin wall": $\rho \neq \rho_0$ when φ not near a minimum

If $A\varphi \ll 1$ $V_B \gg V_F - V_T$

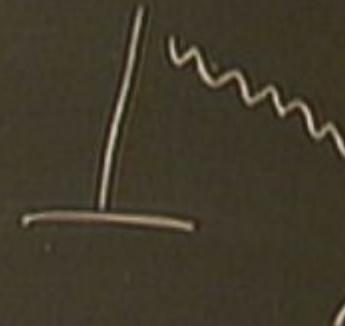
If $A\varphi = \mathcal{O}(1)$ $V_B \gg V_F$ + instanton starts close to the minimum

Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T \ll V(\varphi)$

$$S_E - S_b \ll S_b$$

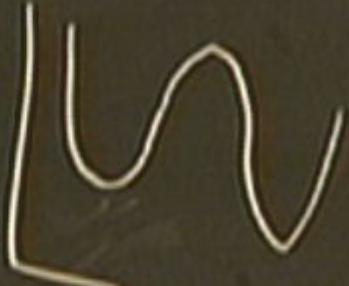
$$S_E - S_b \approx \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



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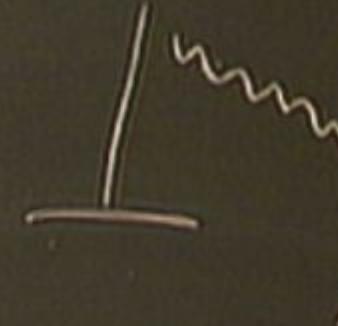


Two classes of potentials with rapid transitions:

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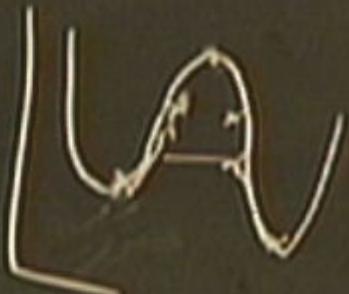
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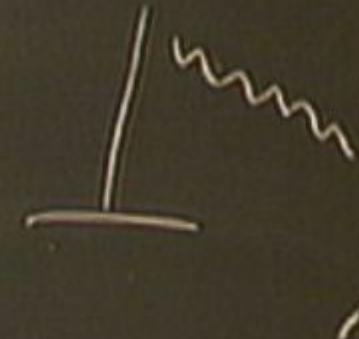


Two classes of potentials with rapid transitions:

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$$S_E - S_b \ll S_b$$

$$S_E - S_b \approx \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



2) "thin wall": $\rho \approx \rho_0$ when φ not near a minimum

If $A\mu \ll 1$ $V_B \gg V_F - V_T$

If $A\mu = \mathcal{O}(1)$ $V_B \gg V_F$ + instanton starts close to the minimum

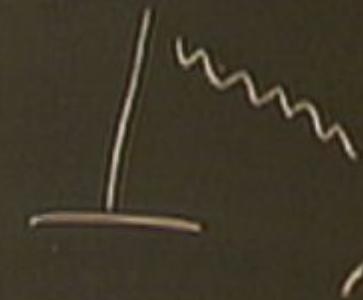


Two classes of potentials with rapid transitions

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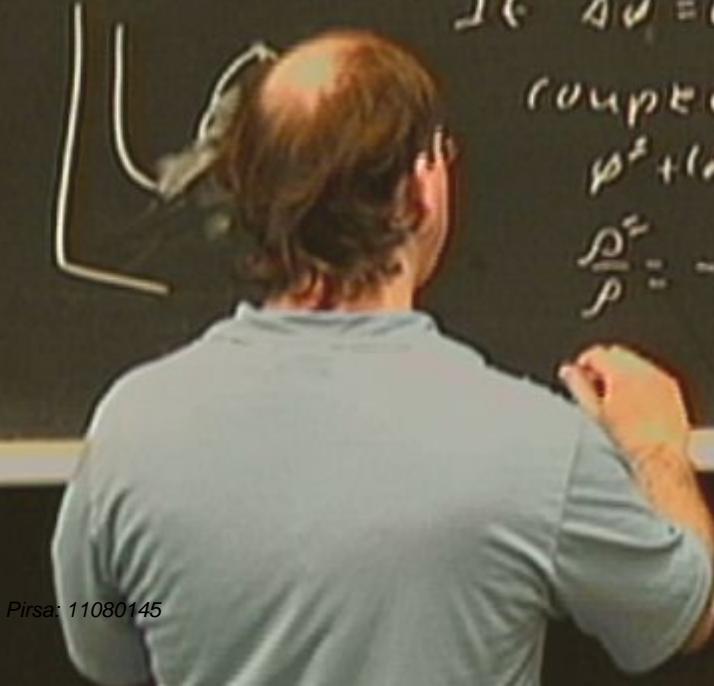
Ie $\Delta\varphi \ll 1$ $V_B \gg V_F - V_T$

Ie $\Delta\varphi = \mathcal{O}(1)$ $V_B \gg V_F$ + instanton starts close to the minimum

Coupled Einstein-Field equations

$$\rho^2 + (d-1)\rho' r' = V''(\varphi)$$

$$\frac{\rho''}{\rho} = -\frac{V''(\varphi)}{(d-1)(d-2)} - \frac{\rho' L}{2(d-1)}$$

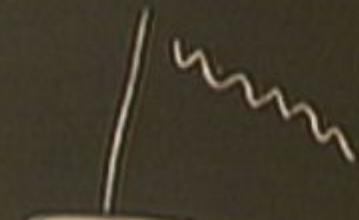


Two classes of potentials with rapid transitions:

1) "Small wall": $V_B - V_T \ll V(\varphi)$

$$S_E - S_b \ll S_b$$

$$S_E - S_b \approx \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



2) "thin wall": $\rho \approx \rho_0$ when φ not near a minimum

If $A\varphi \ll 1$ $V_B \gg V_F - V_T$

If $A\varphi = \mathcal{O}(1)$ $V_B \gg V_F$ + instanton starts close to the minimum

coupled Einstein-Field equations

$$\rho^2 + (d-1)\rho' \rho'' = V''(\varphi)$$

$$\frac{\rho'}{\rho} = -\frac{V''(\varphi)}{(d-1)Vd-2}$$

$$\rho'^2 = \frac{1}{1 + \frac{\rho^2}{Vd-2}} \left(\frac{\rho'^2}{2(d-1)} - V(\varphi) \right)$$



Ie $\Delta\mu \ll 1$ $V_B \gg V_F - V_T$

Ie $\Delta\mu = O(1)$ $V_B \gg V_F$ + instanton starts close to the minimum

Coupled Einstein Field equations

$$\varphi'' + (d-1)\rho' \varphi' = V'(v)$$

$$\rho' = -\frac{V'(v)}{\rho} = -\frac{\varphi' L}{\rho}$$

$$\rho' L = \frac{(d-1)(d-2)}{4\pi G_{d-4}} \left(\frac{\varphi' L}{\rho} - V(v) \right)$$

$$\rho(v) \approx 0$$

Ie $\alpha \ll 1$ $V_B \gg V_F - V_T$

Ie $\alpha = O(1)$ $V_B \gg V_F$ + instanton starts close to the minima
coupled Einstein Field equations

$$\rho' + (d-1)\frac{\rho'}{r} = V'(r)$$

$$\frac{\rho'}{\rho} = -\frac{V'(r)}{(d-1)r^{d-2}} - \frac{\rho' \lambda}{r^{d-1}}$$

$$\rho' \lambda = (d-1)r^{d-2} \left(\frac{\rho' \lambda}{r^2} - V(r) \right)$$

$$\rho(r) \approx 0$$



If $\alpha_s \ll 1$ $V_B \gg V_F - V_T$

If $\alpha_s = O(1)$ $V_B \gg V_F$ + instanton starts close to the minima

Coupled Einstein Field equations

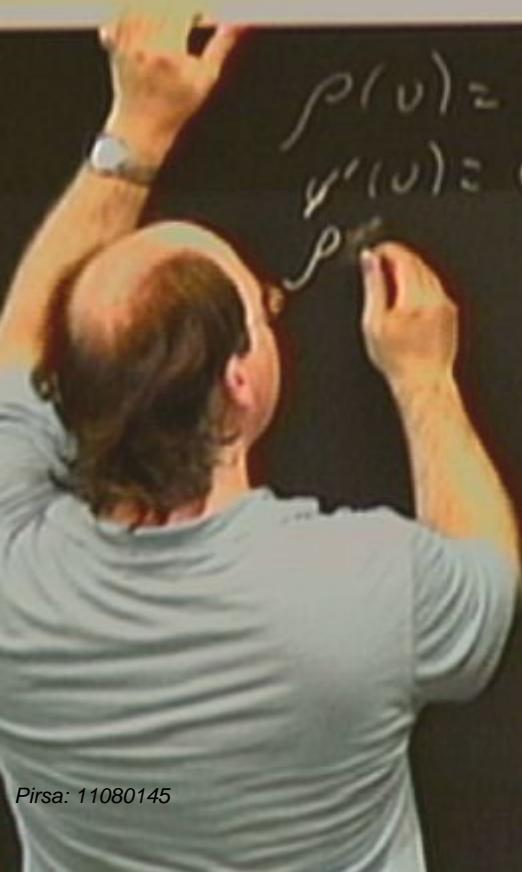
$$\varphi'' + (d-1)\rho' \varphi' = V'(v)$$

$$\rho' = -\frac{V'(v)}{(d-1)v^{d-2}}$$

$$\rho, \lambda = \frac{(d-1)v^{d-2}}{1 + \frac{\rho^L}{d-1} \left(\frac{\rho' \lambda}{2(d-1)} - V(v) \right)}$$

$$\rho(v) = 0$$

$$v'(v) = 0 \rightarrow v \sim v_0 + c_2 \tau^2$$



to the minimum

Coupled Einstein Field equations

$$\rho' + (d-1)\frac{\rho'}{\rho} = v'(v)$$

$$\frac{\rho''}{\rho} = -\frac{v''(v)}{(d-1)v^{d-2}}$$

$$\rho'/2 = \left(1 + \frac{\rho^d}{d-1}\right) \left(\frac{\rho'\lambda}{2(d-1)} - v(v)\right)$$

$$v(v) = 0$$

$$v'(v) = 0 \rightarrow v \sim v_0 + c_1 \tau^2, \dots$$

$$\rho \sim \tau + c_3 \tau^3, \dots$$

$$S_E - S_B \approx \mathcal{O}(\frac{\rho_0}{\sqrt{\tau}}) \rho_0$$

2) "thin wall": $\rho \approx \rho_0$ when ϕ not near a minimum

If $a\mu \ll 1$ $V_B \gg V_F - V_T$

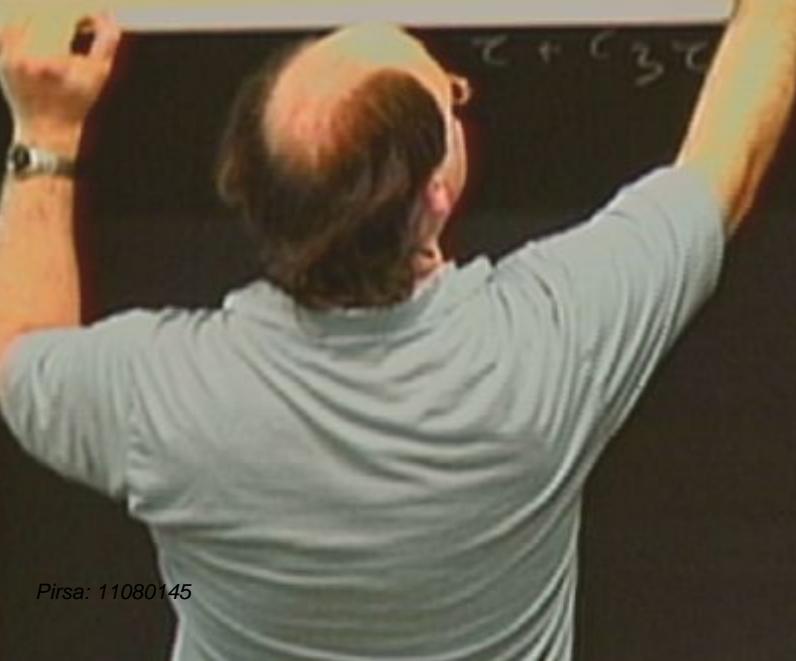
If $a\mu = \mathcal{O}(1)$ $V_B \gg V_F$ + inflation starts close to the minimum

Coupled Einstein Field equations

$$\phi'' + (d-1)\rho' \phi''' = V'(\phi)$$

$$\frac{\rho''}{\rho} = -\lambda \frac{\phi''}{\phi}$$

$$\rho''/L^2 = \frac{(d-2)}{d-1} \left(\frac{\rho''}{\rho} - \frac{\phi''}{\phi} \right)$$

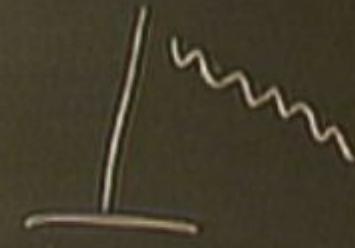


Two classes of potentials with rapid transitions:

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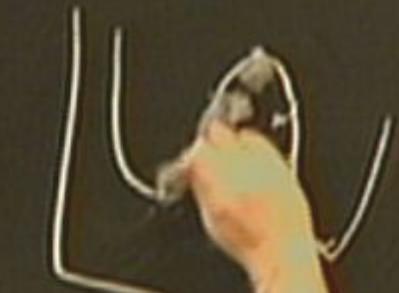
If $A\varphi = \mathcal{O}(1)$ $V_B \gg V_F$ + instanton starts close to the minimum

Coupled Einstein-Field equations

$$\rho^2 + (d-1)\rho' \rho'' = V'(\varphi)$$

$$\frac{\rho'}{\rho} = -\frac{V'(\varphi)}{(d-1)(d-2)}$$

$$\rho'^2 = \frac{1}{4} \frac{\rho'^2}{(d-1)} \left(\frac{\rho'^2}{(d-1)} - V(\varphi) \right)$$

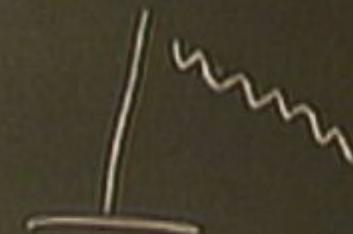


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$$\rho' + (d-1)\rho \frac{\varphi'}{\varphi} = V'(\varphi)$$

$$\frac{\rho''}{\rho} = -\frac{V''(\varphi)}{(d-1)(d-2)}$$

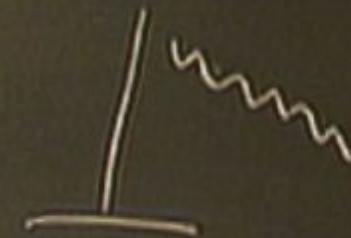
$$\rho' \varphi = 1 + \frac{\rho^2}{d-1} \left(\frac{\rho' \varphi}{\varphi} - V(\varphi) \right)$$

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Coupled Einstein-Field equations

$$\rho' + (d-1)\rho \frac{\rho'}{\rho} = V'(\varphi)$$

$$\frac{\rho'}{\rho} = -\frac{V(\varphi)}{(d-1)(d-2)} - \frac{\rho' L}{\rho^2(d-1)}$$

$$\rho' L = 1 + \frac{\rho^2}{L^2(d-1)} \left(\frac{\rho' L}{L} - V(\varphi) \right)$$

Two classes of potentials with rapid transitions:

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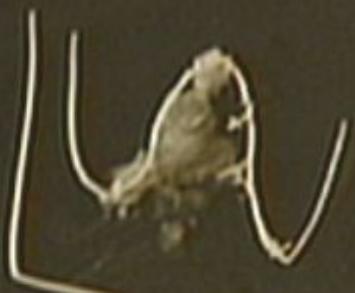
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$$\varphi'' + (d-1)\rho' \varphi''' = V''(\varphi)$$

$$\frac{\rho''}{\rho} = -\frac{V''(\varphi)}{(d-1)(d-2)}$$

$$\rho' \varphi''' = 1 + \frac{1}{(d-1)} \left(\frac{\rho'' \varphi''}{V''(\varphi)} - V(\varphi) \right)$$

Two classes of potentials with rapid transitions

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Coupled Einstein-Field equations

$$\varphi'' + (d-1)\frac{\rho'}{\rho} \varphi' = V'(\varphi)$$

$$\frac{\rho''}{\rho} = -\frac{V''(\varphi)}{(d-1)V(d-2)}$$

$$\rho'/2 = 14 \int_{\varphi=0}^{\varphi} \left(\frac{\rho'^2}{2} - V(\varphi) \right)$$

Two classes of potentials with rapid transitions

1) "Small wall": $V_B - V_T \ll V(\varphi)$

$$S_E - S_b \ll S_b$$

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Coupled Einstein-Field equations

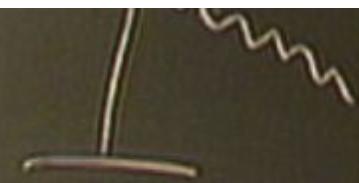
$$\rho' + (d-1)\rho' \frac{\varphi'}{\varphi} = V'(\varphi)$$

$$\frac{\rho''}{\rho} = -\frac{V''(\varphi)}{(d-1)V(d-2)}$$

$$\rho'/\varphi = 1 + \frac{B^k}{d-1} \left(\frac{\varphi'^2}{2(d-1)} - V(\varphi) \right)$$

$$S_E - S_b \ll S_b$$

$$S_E - S_b \approx \mathcal{O}\left(\frac{V_0 \cdot V_T}{V_F}\right) S_b$$



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Coupled Einstein-Field equations:

$$\rho' + (d-1)\rho'' = V'(v)$$

$$\frac{\rho''}{\rho} = -\frac{V(v)}{(d-1)V(d-2)}$$

$$\rho'' \lambda = (1 + \frac{\rho''}{\rho}) \left(\frac{\rho'' \lambda}{2} - V(v) \right),$$

$$\rho(v) = 0$$

$$V'(v) = 0 \rightarrow v \sim v_0 + c_2 \tau^2, \dots$$

$$\rho \sim \tau^{c_3 \tau^3}, \dots$$

$$\frac{d\tilde{\rho}}{\rho} = - \frac{V''(x)}{(d-1)(d-2)} - \frac{\rho' L}{(d-1)(\rho^L - V(x))}$$

$$\rho(v) \approx 0$$

$$V'(v) \approx 0 \rightarrow v \approx v_0 + c_2 \tau^2, \dots$$

$$\rho \approx \tau + c_3 \tau^3, \dots$$

$$S_E = \frac{T_{\text{thin wall}}}{\rho}$$

$$\frac{\rho'}{\rho} = -\frac{V(r)}{(d-1)\lambda^{d-2}} - \frac{\rho' L}{2(d-1)}$$

$$\rho' L = \frac{1}{(d-1)} \left(\frac{\rho' L}{2} - V(r) \right)$$

$$\rho(v) = 0$$

$$V(r) = v \rightarrow v \sim r_0 + c_2 r^2 + \dots$$

$$v \sim r + c_3 r^3 + \dots$$

Thin wall

$$S_E = -\frac{2K R_{d-1}}{(d-2)} \left[\int_{r_0}^{r_0} d\sigma \rho^{d-1} V(r) + \sum_{\Sigma_0} \int_{r_0}^{r_0} d\sigma \rho^{d-1} V(r) + \sum_{\Sigma_1} \int_{r_1}^{r_1} d\sigma \rho^{d-1} V(r) \right]$$

$$\frac{\rho}{\rho} = - \frac{V'(v)}{(d-1)\nabla^2 L} - \frac{\rho' L}{2(d-1)}$$

$$\rho' L = (1 + \frac{\rho^2}{d-1}) (\frac{\rho' L}{L} - V(v))$$

$$\rho(v) = 0$$

$$V(v) = 0 \rightarrow v = v_0 + c_2 \tau^2 + \dots$$

$$\rho = c_2 \tau + c_3 \tau^2 + \dots$$

Thin wall

$$S_E = - \frac{2K R_{d-1}}{(d-2)} \left[\underbrace{\int_0^{r_0} d\tau \rho^{d-1} |V(v)|}_{I_0} + \sum_{i=0}^{1/2} d\tau \rho^{d-1} |V(v)| + \sum_{i=1}^{d-1} d\tau \right]$$

I_0 : near true vac

$$\rho' = 1 + \frac{c}{\omega - \omega_0} (\omega^2 - V(\omega))$$

$$\rho(v) = 0$$

$$\rho'(v) = 0 \rightarrow v \sim v_0 + c_2 \tau^2, \dots$$

$$\rho''(v) \sim c_3 \tau^3, \dots$$

Thin wall

$$S_E = -\frac{2K\Omega_{d-1}}{(d-2)} \left[\underbrace{\int_0^\infty dz \rho^{d-1} |V(z)|}_{I_0: \text{near true}} + \underbrace{\int_0^\infty dz \rho^{d-1} |V(z)|}_{I_1: \text{wall}} \right] V(\infty)$$

calseva

$$\rho(v) = 0$$

$$v'(v) = v \rightarrow v \sim v_0 + c_1 v^1 + c_2 v^2 + \dots$$

$$\rho \sim c_1 v + c_2 v^2 + \dots$$

Thin wall

$$S_E = -\frac{2Kd-1}{(d-2)} \left[\underbrace{\int_{I_0}^{I_1} d\tau \rho^{d-1} |V(\tau)|}_{I_0: \text{near true vac}} + \underbrace{\int_{I_1}^{I_2} d\tau \rho^{d-1} |V(\tau)|}_{I_1: \text{"wall"} \text{ or } I_2: \text{near false vac}} + \underbrace{\int_{I_2}^{I_3} d\tau \rho^{d-1} |V(\tau)|}_{I_3: \text{far from walls}} \right]$$

$$\rho(v) = 0$$

$$v'(v) = 0 \rightarrow v \approx v_0 + c_1 v^1 + c_2 v^2 + \dots$$

$$\rho \approx c_1 v^1 + c_2 v^2 + \dots$$

Thin wall

$$S_E = -\frac{2K\sqrt{d-1}}{(d-2)} \left[\underbrace{\int_{I_0}^{I_1} d\tau \rho^{d-1} |V|^{1/d}}_{I_0: \text{near true vac}} + \underbrace{\int_{I_1}^{I_2} d\tau \rho^{d-1} |V|^{1/d}}_{I_1: \text{"wall": near false vac}} + \underbrace{\int_{I_2}^{I_3} d\tau \rho^{d-1} |V|^{1/d}}_{I_2: \text{near false vac}} \right]$$

If $\alpha \ll 1$ $V_B \gg V_F - V_T$

If $\alpha = O(1)$ $V_B \gg V_F$ + instanton starts close to the minimum
coupled Einstein Field equations

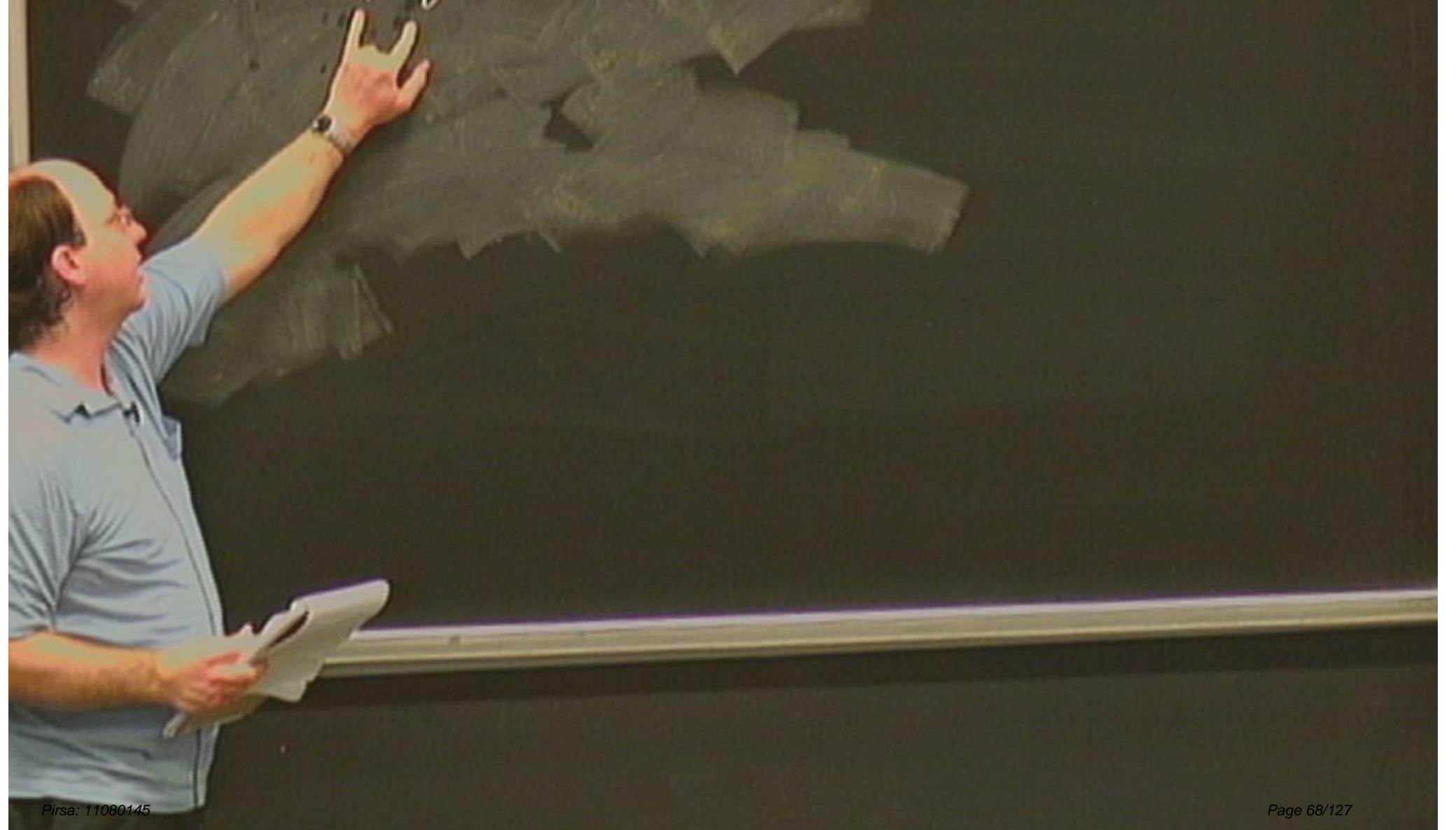
$$\varphi'' + (d-1)\rho' \varphi' = V'(\varphi)$$

$$\rho' = -\frac{V'(\varphi)}{(d-1)(d-2)}$$

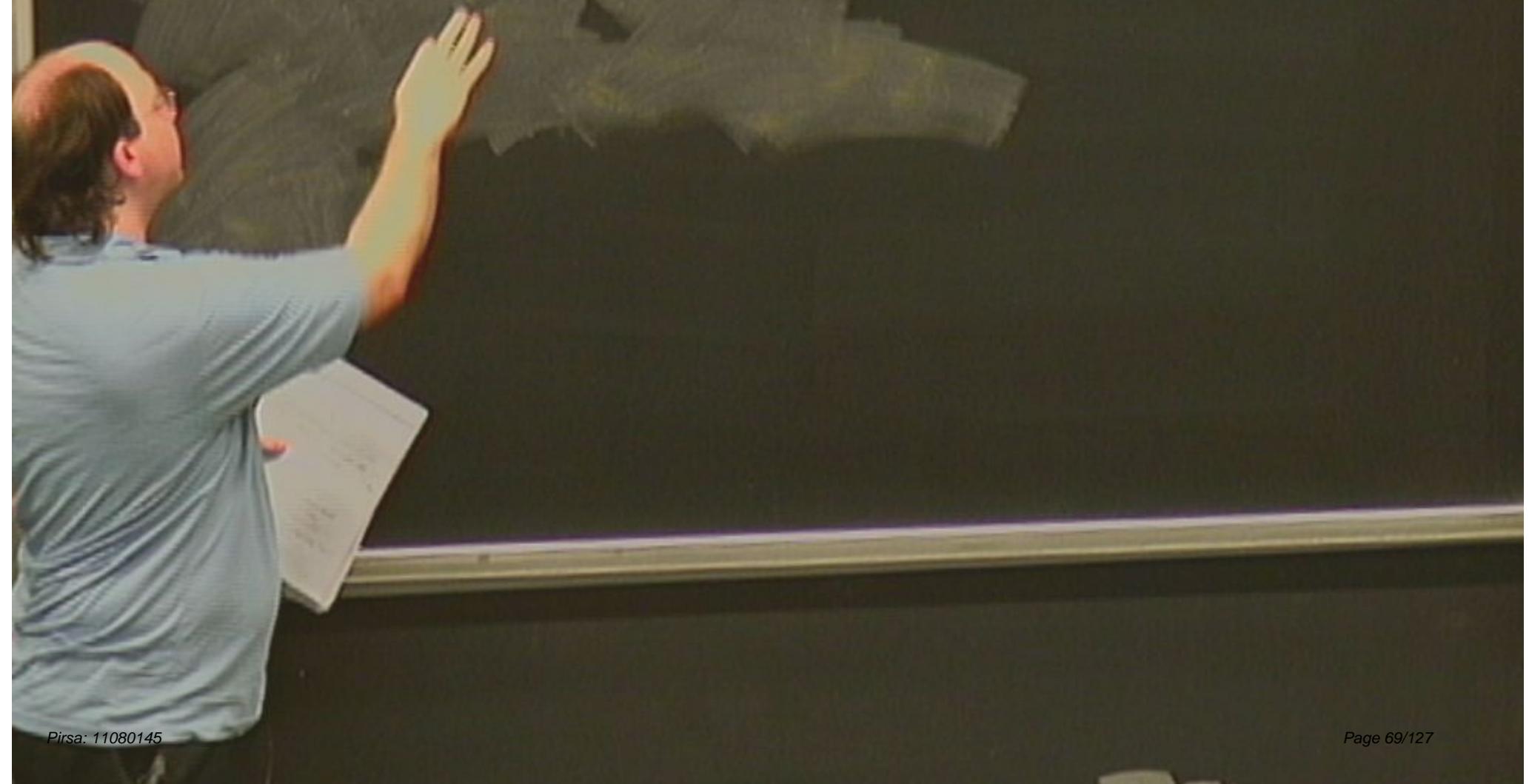
$$\rho'^2 = \frac{1}{1+\frac{\rho^2}{2}} \left(\frac{\rho'^2}{2(d-1)} - V(\varphi) \right)$$

$$\frac{2K\sqrt{d-1}}{(d-2)} \left[\underbrace{\int_0^r dz \rho^{d-1} |V'(z)|}_{I_0: \text{near true vac}} + \underbrace{\int_r^1 dz \rho^{d-1} |V(z)|}_{I_1: \text{"wall"}} + \underbrace{\int_1^\infty dz \rho^{d-1} |V(z)|}_{I_2: \text{near false vac}} \right]$$

$$- \Delta C \sim \frac{\rho \phi}{\sqrt{V_B}}$$



$$- \Delta C \sim \frac{\rho e}{\sqrt{V_B}} \sim \frac{1}{\sqrt{v_{ext}(B)}}$$



$$- D \sim \sqrt{\frac{P}{V_0}} \sim \sqrt{\frac{1}{V_0 - \rho_0}}$$

- I.

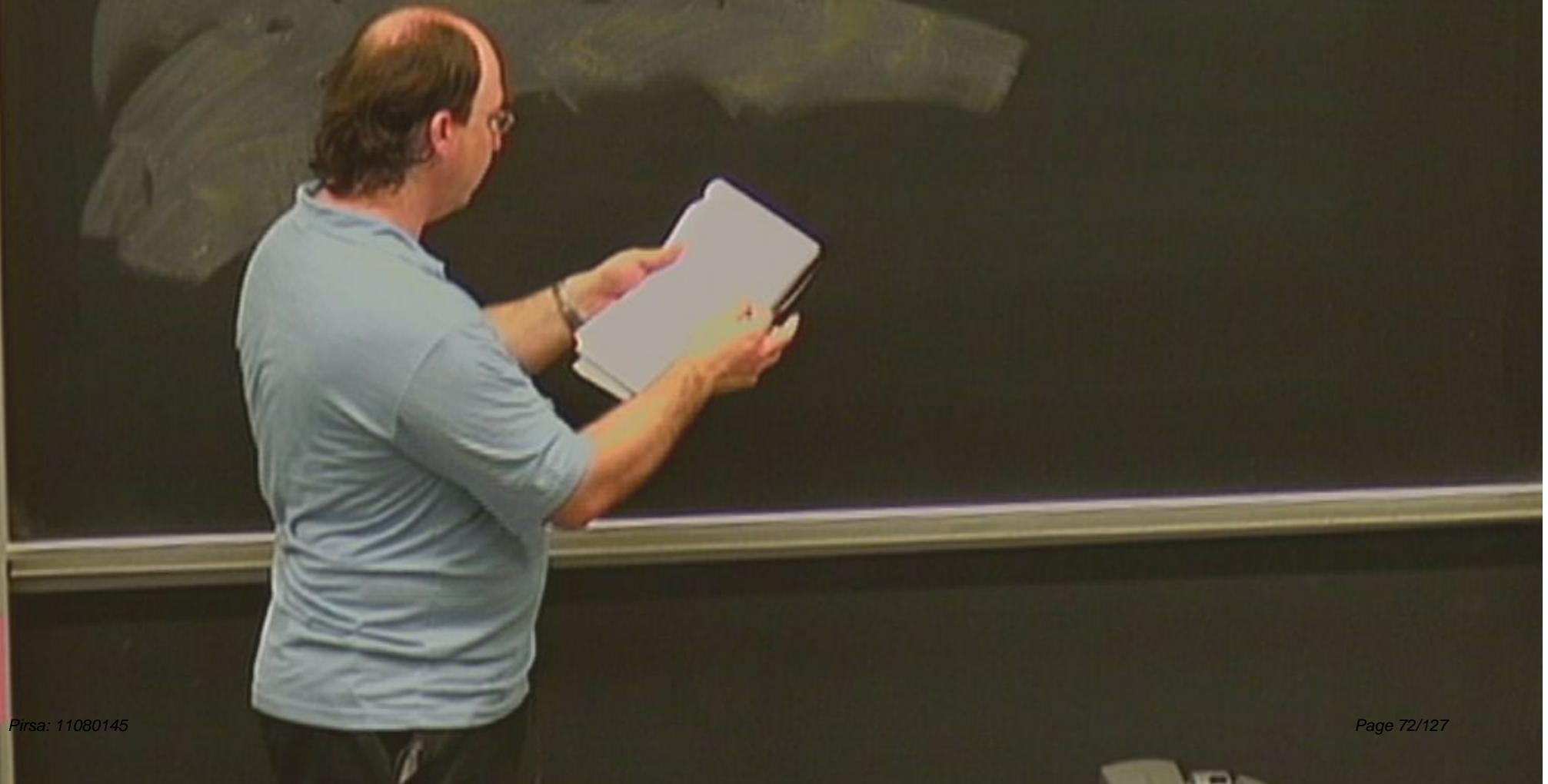
$$- \Delta C \sim \frac{\rho \epsilon}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V = \epsilon_{FS}}}$$

$$- I_s \sim \rho_0^{d-1} V_B \Delta C \sim \rho_0^{d-1} \rho \epsilon \sqrt{V_B}$$



$$- \Delta t \sim \frac{\rho \ell}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V_B} \cdot \rho \ell}$$

$$- I_s \sim \rho_0^{d-1} V_B \Delta t \sim \rho_0^{d-1} \rho \ell \sqrt{V_B}$$



$$- J_C \sim \frac{\rho \phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V = V_B}}$$

$$- I_s \sim \rho_0^{d-1} V_B J_C \sim \rho_0^{d-1} \rho \phi \sqrt{V_B}$$

As V unless

$$- J_C \sim \frac{\rho_0}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V_B} \cdot \rho_0}$$

$$- I_s \sim \rho_0^{d-1} V_B J_C \sim \rho_0^{d-1} D \propto \sqrt{V_B}$$

As $V_B \rightarrow \infty$, $\rho_0 \rightarrow 0$,

$$\rho(v) = 0$$

$$v'(v) = 0 \rightarrow v \sim v_0 + c_1 v^1 + \dots$$

$$\rho \sim c_1 v^1 + c_2 v^2 + \dots$$

$$S_E = -\frac{2K \pi d-1}{(d-2)} \left[\underbrace{\sum_0^d \int_{I_0} I^{d-1} |V|^{1/d}}_{I_0: \text{near true vac}} + \underbrace{\sum_1^d \int_{I_i} I^{d-1} V(I)}_{I_i: \text{"wall": near false vac}} + \underbrace{\sum_1^d \int_{I_2} I^{d-1} V(I)}_{I_2: \text{near false vac}} \right]$$

I_0 : near true vac

I_i : "wall":

I_2 : near false vac

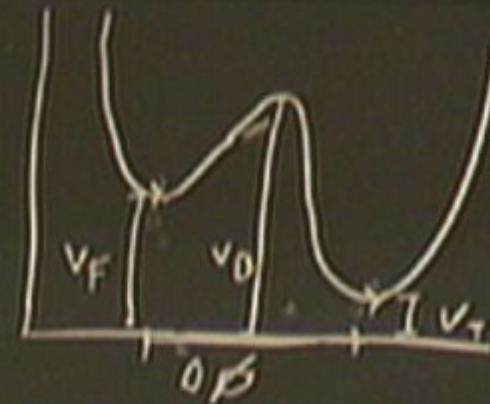
$$- \Delta t \sim \frac{\rho_0}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V_B + \rho_0 \beta}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta t \sim \rho_0^{d-1} \rho \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |x_{ij}|, |\gamma_{ij}|, |\sigma_b|$

$$I. \quad T = A e^{-B}$$

$$B = S_E - S_b$$



$$[B] = L^0$$

$$[V] = L^2$$

$$\kappa = \frac{1}{16\pi G}$$

$$S_E = -\kappa \int \sqrt{g} (R - \frac{1}{2}(\partial_\mu)^2 - V(r))$$

Overall sign:

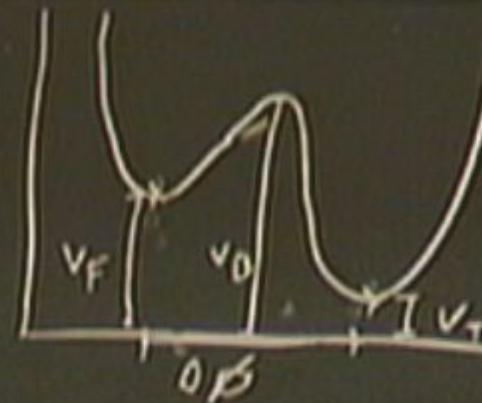
$$\hat{g}_{ab} = R^2 g_{ab}$$

$$\int \sqrt{g} R = \int \sqrt{g} R^{d-2} [R + (d-1)/d]$$

Bousso, Hawking, BH production

$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



$$S_E = -\kappa \int \sqrt{g} (R - \frac{1}{2}(\partial r)^2 - V(r))$$

overall sign:

$$\hat{g}_{ab} = \mathcal{R}^2 g_{ab}$$

$$\int \sqrt{g} R = \int \sqrt{g} \mathcal{R}^{d-2} [R + (d-1)\mathcal{R} - 2V] \mathcal{R} \frac{\partial \mathcal{R}}{\partial r}$$

$$Bousso, Hawking, BH production$$

$$[\rho] = L^0$$

$$[v] = \frac{1}{L^2}$$

$$\kappa = \frac{1}{16\pi G_d}$$

$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



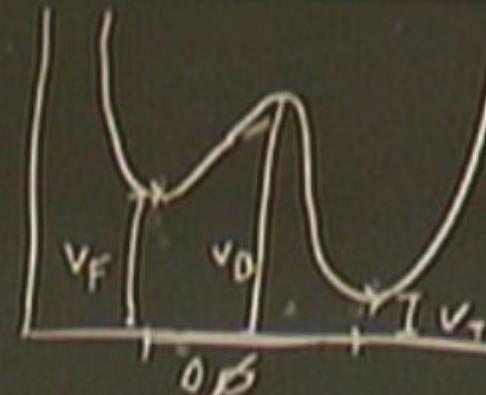
$$S_E = -\kappa \int \sqrt{g} (R - \frac{1}{2}(\partial \phi)^2 - V(\phi))$$

Overall sign:

$$\hat{g}_{ab} = \mathcal{N}^2 g_{ab}$$

$$\int \sqrt{g} R =$$

$$Bousso, Hawking, BH production$$



$$[B] = L^0$$

$$[V] = \frac{1}{L^2}$$

$$\kappa = \frac{1}{16\pi G_d}$$

$$- \Delta t \sim \frac{\rho \phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V = c \rho_B}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta t \sim \rho_0^{d-1} \rho \phi \sqrt{V_B}$$

As $V_B \gg 0$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |I_2|, |I_3|, |S_6|$

Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$- \Delta t \sim \frac{\rho_0}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V = c\rho_B}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta t \sim \rho_0^{d-1} \rho \propto \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $|I_1| \gg |I_0|, |I_2|, |S_b|$

$$- Suppose \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$- \Delta_C \sim \frac{\rho_F}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V = c \mu_B}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta_C \sim \rho_0^{d-1} \rho_F \sqrt{V_B}$$

As $V \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $|I_1| \gg |I_0|, |I_2|, |S_0|$

use $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$- \Delta \tau \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(V - c\rho_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} \rho \sqrt{V_B}$$

$V_B \gg 0$, unless $\rho_0 \rightarrow 0$, $|I_1| \gg |I_2|, |I_3|, |S_6|$

Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$- \Delta \tau \sim \frac{\rho \phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V_B (\rho_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} \rho \phi \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $|I_1| \gg |I_2|, |I_3|, |S_6|$

$$- Suppose \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\sim \frac{T}{(V_F - V_T)}$$

$$- \Delta \tau \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(\nu - \nu_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} \rho \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |I_2|, |I_3|, |S_b|$

$$- Suppose \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$- \Delta \tau \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(V = c \mu_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} \rho \propto \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |I_2|, |I_3|, |S_b|$

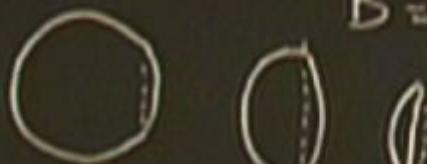
$$- Suppose \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$I. T = A e^{-B}$$

$$B = S_E - S_b$$



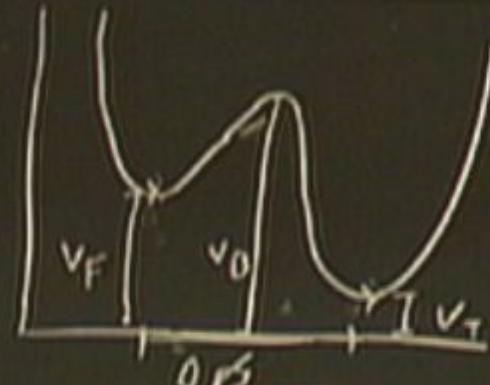
$$S_E = -\kappa \int \sqrt{g} (R - \frac{1}{2}(\nabla \theta)^2 - V)$$

Overall sign:

$$\hat{g}_{ab} = R^2 g_a$$

$$\int \hat{g} R =$$

Bousso, Hawking, BH



$$[\rho] = L^0$$

$$[v] = \frac{1}{L^2}$$

$$\kappa = \frac{1}{16 \pi G_d}$$

$$(d-2) \nabla^2 \ln \left[\frac{\nabla_{ab}}{C_{ab}} \right]$$

$$- D_C \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V_F(\rho_0)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B D_C \sim \rho_0^{d-1} D \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |J_{11}|, |J_{21}|, |S_b|$

$$- Suppose \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho^{d-1}(V_T) \sim T \rho^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$- \Delta \tau \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V - (\rho V_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} D_F \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $|I_1| \gg |I_2|, |I_3|, |S_b|$

$$- \text{Suppose } \rho_0 \ll \frac{1}{D_F}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$- \Delta \tau \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V - (\rho V_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} D_F \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |I_{-}|, |I_2|, |S_b|$

$$- \text{Suppose } \rho_0 \ll \frac{1}{D_F}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$- \Delta \tau \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V - (\rho V_B)}}$$

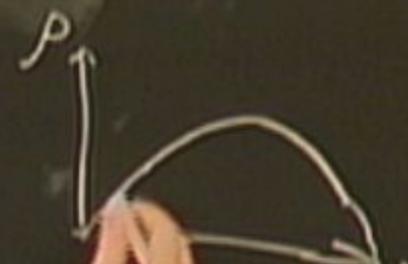
$$- I_s \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} D \phi \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_s \rightarrow 0$

- Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1} (V_F - V_T) \sim T$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$



$$- A_C \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V - (\rho_B)^d}}$$

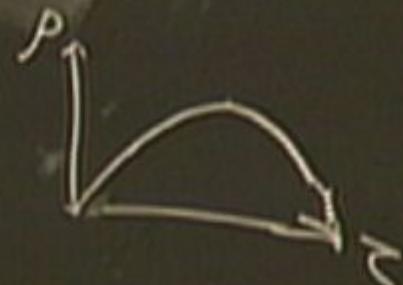
$$- I_1 \sim \rho_0^{d-1} V_B A_C \sim \rho_0^{d-1} \rho \sqrt{V_B}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |I_{-1}|, |J_1|, |S_b|$

- Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$



$$- A_C \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(V - \epsilon_{FB})}}$$

$$- I_1 \sim \rho_0^{d-1} V_B A_C \sim \rho_0^{d-1} D \sqrt{V_B}$$

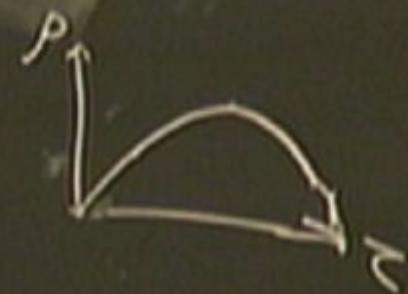
As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |I_{-1}|, |J_1|, |S_b|$

- Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$\phi = t(d-1) \frac{\rho}{\rho_0} = V/(t\epsilon)$$



$$R(1 - \frac{d}{2}) = \frac{1}{2}(1 - \frac{d}{2})(\nabla \phi)^2 - \frac{d}{2}V(\phi)$$

$$R = \frac{1}{2}(\nabla \phi)^2 + \frac{d}{d-2}V(\phi)$$

$$S_E = -K \int \sqrt{g} \left(\frac{d}{d-2} - 1 \right) V(\phi) = -\frac{2K}{(d-2)} \int \sqrt{g} V(\phi)$$

- no gradient terms

- potential (naively) comes in with opposite sign

$$S_E^{(4)} \approx -\frac{4\delta K}{V_0} \quad V(\phi) = V_0 > 0$$

$$- D_C \sim \frac{\rho_0}{\sqrt{V_0}} \sim \frac{1}{\sqrt{V_0}} \sim \frac{1}{\sqrt{V_0}}$$

$$- I_s \sim \rho_0^{d-1} V_B D_C \sim \rho_0^{d-1} D_C \sqrt{V_B}$$

$$- \Delta t \sim \frac{\rho_0}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V = \rho_0 V_B}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta t \sim \rho_0^{d-1} \rho \sqrt{V_B}$$

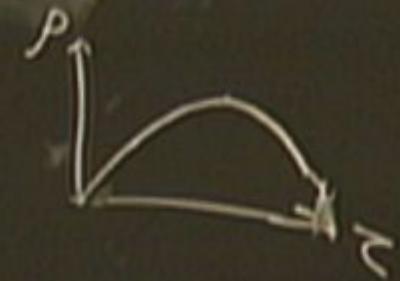
Assume $\rho_0 \neq 0$, unless $\rho_0 \rightarrow 0$, $T_1 \gg |T_{11}|, |T_{21}|, |S_b|$

Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$d^{-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

$$-1) \frac{(V_F - V_T)}{\rho_0^{d-2}} = V'(0)$$



$$S_E - S_b < 0$$

$$- \Delta t \sim \frac{\rho \phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{V - (\mu_B)^2}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta t \sim \rho_0^{d-1} \rho \phi \sqrt{V_B}$$

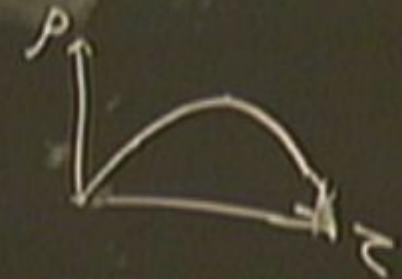
As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |x_a|, |x_b|, |S_b|$

- Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim T$$

$$- \rho = r(d-1) \frac{(V_F - V_T)}{\rho^d} = V \cdot |\theta|$$



$$S_E - S_b < 0$$

$$CDL: \beta = S_E \cdot S_b = \frac{96 \pi^2}{K^2 V_F}$$

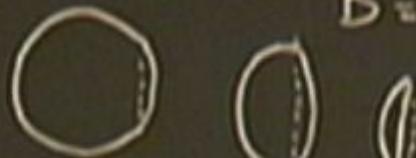
$$S_E = \frac{T \ln \text{wall}}{2 K n_{d-1} (d-2)} \left[\underbrace{\sum_0^d d \int \rho^{d-1} |V'| \rho |}_{I_0: \text{near true vac}} + \underbrace{\sum_1^d d \int \rho^{d-1} |V| \rho |}_{I_1: \text{"wall": near false vac}} + \underbrace{\sum_1^d d \int \rho^{d-1} |V| \rho |}_{I_2: \text{near false vac}} \right]$$

$$CDL: \beta = S_E \cdot S_b = \frac{96 \pi^2}{K^2 V_F}$$

$$S_E = \frac{\text{Thin wall}}{-2 K \eta_{d-1} / (d-2)} \left[\underbrace{\sum_0^d d \int \rho^{d-1} |V'| \nu_1}_{I_0: \text{neur true vac}} + \underbrace{\sum_1^d d \int \rho^{d-1} |V| \nu_1}_{I_1: \text{"wall": neur false vac}} + \underbrace{\sum_1^d d \int \rho^{d-1} |V| \nu_0}_{I_2: \text{heat false vac}} \right]$$

$$I. \quad T = A e^{-B}$$

$$B = S_E - S_b$$



$$S_E = -K \int \sqrt{g'} (R - \frac{1}{2}(\sigma\omega)^2 - V(\omega))$$

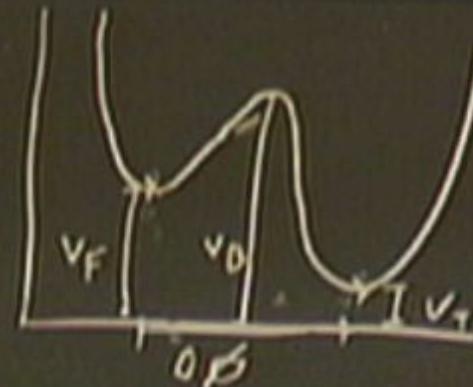
overall sign:

$$\hat{g}_{ab} = R^2 g_{ab}$$

$$\sqrt{g'} R =$$

$$Bousso, Hawking, BH production$$

$$\int g' R^{d-2} [R + (d-1)(d-2)V \ln \frac{\nabla_a \nabla^a}{R}]$$



$$[\rho] = L^0$$

$$[V] = \frac{1}{L^2}$$

$$K = \frac{1}{16 \pi G_d}$$

$$- A_C \sim \frac{\rho}{\sqrt{V_0}} \sim \frac{1}{\sqrt{(V_F - V_T)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B A_C \sim \rho_0^{d-1} D \sqrt{V_B}$$

$$S = \frac{C_0}{V_F^{d-2} - 1}$$

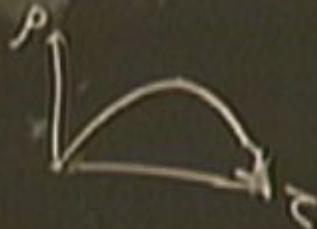
As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |x_a|, |x_b|, |S_b|$

$$- Suppose \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim T$$

$$- \rho = r(d-1) \frac{(V_F - V_T)}{\rho} \Rightarrow r = V^{-1}(G)$$



$$S_a \cdot S_b < 0$$

$$- \Delta \tau \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(V_F - V_T)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} D \sqrt{V_B} \quad \text{B. } S_L S_B = \frac{C_0}{V_F^{d/2-1}}$$

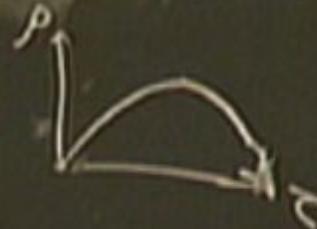
As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $|I_1| \gg |I_{1L}|, |I_{1R}|, |S_B|$

$$- \text{Suppose } \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim T$$

$$- \rho = r(d-1) \rho' \quad \rho' = \frac{(V_F - V_T)}{V'(0)}$$



$$S_L \cdot S_B < 0$$

$$- A_C \sim \frac{\rho}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(V_F - V_T)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B A_C \sim \rho_0^{d-1} D \sqrt{V_B} \quad \text{B. } S_L S_B = \frac{C_0}{V_F^{\frac{d}{2}-1}}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $|I_1| \gg |I_{1L}|, |I_{1R}|, |S_B|$

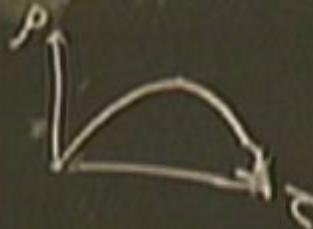
$$- \text{Suppose } \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim T$$

$$- \rho = r(d-1) \frac{(V_F - V_T)}{\rho} \Rightarrow r = V/(G)$$

$$S_L \cdot S_B < 0$$



$$R(1 - \frac{d}{2}) = \frac{1}{2}(1 - \frac{d}{2})(\nabla v)^2 - \frac{d}{2}V(v)$$

$$R = \frac{1}{2}(\nabla v)^2 + \frac{d}{d-2}V(v)$$

$$S_E = -K \int \sqrt{g} \left(\frac{d}{d-2} - 1 \right) V(v) = -2K \frac{\int \sqrt{g} V(v)}{(d-2)}$$

- no gradient terms

- potential (naively) comes in with opposite sign

$$S_E^{(4)} = -4\delta K \quad V(v) = V_0 > 0$$

$$-A \sim \frac{\rho g}{\sqrt{V_0}} \sim \frac{1}{\sqrt{V_0 + \mu_B}}$$

$$- A_C \sim \frac{\rho_0}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(V_F - V_B)}}$$

$$- I_1 \sim \rho_0^{d-1} V_B A_C \sim \rho_0^{d-1} D \sqrt{V_B}$$

$$B_s S_e S_b = \frac{C_0}{\sqrt{V_F - V_B}}$$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |I_2|, |I_3|, |S_b|$

$$- Suppose \rho_0 \ll \frac{1}{\sqrt{V_F}}$$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-1}$$

$$\rho_0 \sim \frac{T}{V_F - V_T}$$

$$- \phi = r/(d-1) \frac{(V_F - V_T)}{\rho_0} = V_T$$

- $\Delta \tau \sim \frac{\rho \phi}{\sqrt{V_B}} \sim \frac{1}{\sqrt{(V - V_F)T}}$

- $I_1 \sim \rho_0^{d-1} V_B \Delta \tau \sim \rho_0^{d-1} D \sqrt{V_B}$ $B_s S_a S_b = C_0 \frac{1}{\sqrt{V_F(d-1)}}$

As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $I_1 \gg |S_a|, |S_b|, |S_b|$

- Suppose $\rho_0 \ll \frac{1}{\sqrt{V_F}}$

$$\rho_0^{d-1} (V_F - V_T) \sim T \rho_0^{d-2}$$

$$\rho_0 \sim \frac{T}{(V_F - V_T)}$$

- $\phi = r/(d-1) \frac{\rho_0}{\rho_0'} = V/(r)$

$$- A \sim \frac{\rho_0}{\sqrt{V_B}} \sim \sqrt{\frac{1}{(V - V_F)}}$$

$$- I_s \sim \rho_0^{d-1} V_B \sim \rho_0^{d-1} D \sqrt{V_B}$$

$$B_s S_E S_B = \frac{C_0}{V_F^{\frac{d}{2}-1}}$$

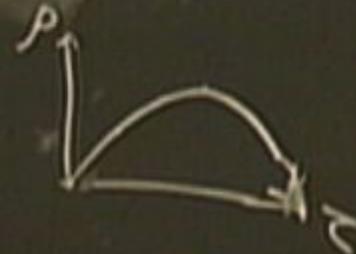
As $V_B \rightarrow \infty$, unless $\rho_0 \rightarrow 0$, $T_c \gg |J_{11}|, |J_{21}|, |S_b|$

$$- Suppose \rho_0 \ll \frac{1}{V_F}$$

$$\left(\frac{V_T}{V_F}\right)^{n_0} \rho_0^{d-1} (V_F - V_T) \sim$$

$$\rho_0 \sim T$$

$$- \phi = r/(d-1) \frac{(V_F - V_T)}{\rho} =$$



$$\varepsilon - S_b < 0$$

$$CdL: \beta = S_E \cdot S_b = \frac{96 \pi^2}{K^2 V_F}$$

(4 fm)

$$\frac{\text{Hil wall}}{-\frac{2K \eta_{d-1}}{(d-2)}} \left[\underbrace{\sum_0^2 d \int \rho^{d-1} |V'| \rho |}_{I_0: \text{near true vac}} + \underbrace{\sum_0^1 d \int \rho^{d-1} |V| \rho |}_{I_1: \text{"wall": near false vac}} + \underbrace{\sum_1^6 d \int \rho^{d-1} |V| \rho |}_{I_2: \text{near false vac}} \right]$$



$$CDL: \beta = S_E \cdot S_b = \frac{96 \pi^2}{K^2 V_F}$$

(4 fm)

CDL: i) Diff. form of action:

$$S_E = \frac{\text{Thin wall}}{-2 K \eta_{d-1} / (d-2)} \left[\underbrace{\sum_0^r d \int \rho^{d-1} |V| ds}_{I_0: \text{near true vac}} + \underbrace{\sum_0^1 d \int \rho^{d-1} |V| ds}_{I_1: \text{"wall": near false vac}} + \underbrace{\sum_1^r d \int \rho^{d-1} |V| ds}_{I_2: \text{near false vac}} \right]$$

$$CdL: \quad B = S_E \cdot S_b = \frac{96 \pi^2}{K^2 V_F} \quad (4 \text{ dm})$$

including backreaction

T_ϵ (instanton)

$I - T_\epsilon$ (background)

$$CdL: \quad \text{II) Diffr. form of action: } \checkmark$$

$$B = S_E - S_b$$

$$\text{Thin wall} \quad S_E = -\frac{2K \pi_{d-1}}{(d-2)} \left[\underbrace{\int_0^{r_o} dr \rho^{d-1} |V(r)|}_{I_0: \text{near true vac}} + \underbrace{\int_{r_o}^{\infty} dr \rho^{d-1} |V(r)|}_{I_1: \text{wall}} + \underbrace{\int_{r_o}^{\infty} dr \rho^{d-1} V(r)}_{I_2: \text{false vac}} \right]$$

$$B = K_1 \left[I_0 + T_1 + I_2 - (I_0 + I_1 + I_2) \right]$$

$\frac{I_0}{\text{true vac}}$ $\frac{T_1}{\text{false vac}}$ $\frac{I_2}{\text{false background vac}}$

Two classes of potentials with rapid transitions

1) "Small wall": $V_B - V_T \ll V(\varphi)$

$$S_E - S_b \ll S_b$$

$$S_E - S_b \approx \mathcal{O}\left(\frac{V_B - V_T}{V_T}\right) S_b$$



2) "thin wall": $\rho \approx \rho_0$ when φ not near a minima

If $A\varphi \ll 1$ $V_B \gg V_F - V_T$

If $A\varphi = \mathcal{O}(1)$ $V_B \gg V_F$ + instanton starts close to the minima

Coupled Einstein Field equations

$$\rho^2 + (d-1)\rho' \ddot{\varphi} = V'(\varphi)$$

$$\frac{\rho''}{\rho} = -\frac{V_{,L}}{(d-1)V_{,d-2}}$$

$$\rho'^2 = (1 + \frac{\rho^2}{d-1}) \left(\frac{V'^2}{V_{,d-2}} - \frac{V''^2}{V_{,d-2}^2} \right)$$



CDL: 1) Diff form of action

$$\beta = S_E - S_b$$

(instanton)

$\int \tau \epsilon$ (background)

$$S_E \cdot S_b \ll S_b$$

$$S_E \cdot S_b \approx \mathcal{O}(\frac{1}{\sqrt{\tau}})$$

2) "thin wall": $\rho \approx \rho_0$ when φ not near a minima

$$\text{Ic A} \quad V_B \gg V_F - V_T$$

$\text{Ic B} \quad \Delta\varphi = \mathcal{O}(1) \quad V_B \gg V_F$ + instanton starts close to the minimum

Coupled Einstein Field equations

$$\rho'' + (d-1)\rho' \varphi' = V''(\varphi)$$

$$\frac{\rho''}{\rho} = -\frac{V''}{V}$$

$$\rho'/\lambda = \frac{(d-1)(d-2)}{16(d-4)} \left(\frac{\varphi'^2}{2} - V(\varphi) \right)$$

CDL: $B: S_E \cdot S_b = \frac{96\pi^2}{K^2 V_F}$

(4 dm)

including backreaction

τ_ϵ (instanton)

$\neq \tau_\epsilon$ (background)

$$CDL: B = S_E \cdot S_b = \frac{96\pi^2}{K^2 V_F}$$

(4 km)

including backreaction

$$CDL: \text{II) Diff. form of action: } \checkmark$$

$$B = S_E \cdot S_b$$

τ_ϵ (instanton)
 $\neq \tau_\epsilon$ (background)

$$S_E = -\frac{2K\eta_{d-1}}{(d-2)} \left[\underbrace{\int_0^{\tau_0} d\tau \rho^{d-1} |V(\tau)|}_{I_0: \text{near true vac}} + \underbrace{\int_0^{\tau_1} d\tau \rho^{d-1} |V(\tau)|}_{I_1: \text{"wall"} \text{ near false vac}} + \underbrace{\int_{\tau_1}^{\tau_2} d\tau \rho^{d-1} |V(\tau)|}_{I_2: \text{near false vac}} \right]$$

$$B = K_1 [I_0 + I_1 + I_2 - \left. (I_0 + I_1 + I_2) \right|_{\substack{\text{false} \\ \text{vac}}}]$$

I_2 : false background

$$I. T = A e^{-B}$$

Infrared
Background

$$S_E = \sqrt{A} e^{-B}$$

Overall signal

Boussinesq

$$(S_E)^2 = V(\sigma_1)$$

$$\begin{aligned} D(\rho) &= L^0 \\ L(V) &= \frac{1}{L^2} \\ K &= \frac{1}{16\pi G_d} \end{aligned}$$

σ_{ab}

$$S \sqrt{\sigma^{d-2}} [R + (d-1)\sigma^{d-2} V^a \nabla_a \nabla_b]$$

production

Pirsa: 11080145

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Eq. 6.1

$K = V_F$

Including backreaction

CdL: 1) Diff. form of action, ✓

$$B = S_E - S_b$$

Thin wall

$$S_E = -\frac{2K\sqrt{d-1}}{(d-2)}$$

$$\left[\int_0^{\tau_0} d\tau \rho^{d-1} V(\tau) \right]$$

I_0 : near

$$B = K_1 [I_0 + T_1 + I_2] - (I_0 + T_1)_{\text{false}} + I_2 - (I_2)_{\text{true}}$$

CdL claim

τ_f (instanton)

$\int \tau_f$ (background)

$\rightarrow \rho(c_1)$: back

$\int \rho(c_1)$ instanton

$$\int_0^{\tau_0} d\tau \rho^{d-1} V(\tau) + \int_{\tau_0}^{\tau_f} d\tau \rho^{d-1} V(\tau)$$

I_2 : hear false

$$CdL: B = S_E \cdot S_b = \frac{q_6 \pi^2}{K^2 V_F} \quad (4 \text{ dm})$$

including backreaction

τ_ϵ (instanton)

$\not\tau_\epsilon$ (background)

$\rightarrow \rho(\epsilon_i)$: back

$\not\rho(\epsilon_i)$: instanton

CdL: 1) Diff form of action; ✓

$$B = S_E \cdot S_b$$

$$S = \frac{\kappa n_{d-1}}{(2)} \left[\int_0^{\tau_0} \int d\tau \rho^{d-1} |V''| + \underbrace{\int_0^{\tau_1} d\tau \rho^{d-1} |V'|}_{\text{wall}} + \underbrace{\int_{\tau_1}^{\tau_2} d\tau \rho^{d-1} |V'|}_{\text{near true vac}} + \underbrace{\int_{\tau_2}^{\tau_3} d\tau \rho^{d-1} |V'|}_{\text{near false vac}} \right]$$

τ_1 : "wall"; τ_2 : near false vac

$$\tau_1, \tau_2 = (\tau_0 + \tau_1 + \tau_2) \Big|_{\text{false background vac}}$$

initial
vac

CdL(Claim =)

Junction condition analysis

$$\varphi^+ + (d-1) \frac{\varphi'}{r} \varphi^- = V(r)$$

$$E = V(r) - \frac{\varphi^2}{2}$$



Junction condition analysis

$$\varphi'' + (d-1) \rho' \nu^{-1} = v'(r)$$

$$E = v(r) - \frac{\varphi'^2}{2}$$

$$E' = (d-1) \rho' (v')^2$$



Junction condition analysis

$$\varphi'' + (d-1)\rho' v^1 = v'(r)$$

$$E = v(r) - \frac{\rho'^2}{2}$$

$$E' = (d-1)\rho' (v')^2$$

$$V_F = -\frac{\rho'^2}{2} = V_T - \frac{v'^2}{2}$$



Junction condition analysis

$$\varphi^+ + (d-1) \rho' v^- = v'(x)$$

$$E = v(x) - \frac{\varphi'^2}{2}$$

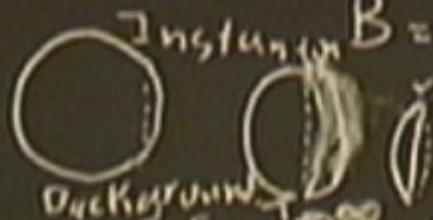
$$E' = (d-1) \rho' (v)^2$$

$$V_F - \frac{\varphi'^2}{2} = V_T - \frac{v'^2}{2}$$

$$V_F \neq V_T$$



$$I. T = A e^{-B}$$



$$B = S_E - S_b$$

$$S_E = -\kappa \int \sqrt{g} \left(R - \frac{1}{2}(\sigma \phi)^2 - V(\phi) \right)$$

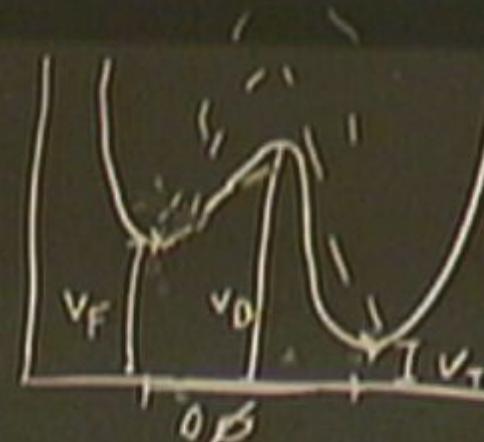
Overall sign:

$$\hat{g}_{ab} = \mathcal{R}^2 g_{ab}$$

$$\int \sqrt{g} R =$$

$$= \int \sqrt{g} \mathcal{R}^{d-2} [R + (d-1)(d-2)V \frac{\nabla^a \nabla_a}{\mathcal{R}}]$$

Bousso, Hawking, BH production



$$[B] = L^0$$

$$[V] = L^2$$

$$\kappa = \frac{1}{16\pi G_d}$$

Junction condition analysis

$$\phi' + (d-1)\rho' v' = v'(r)$$

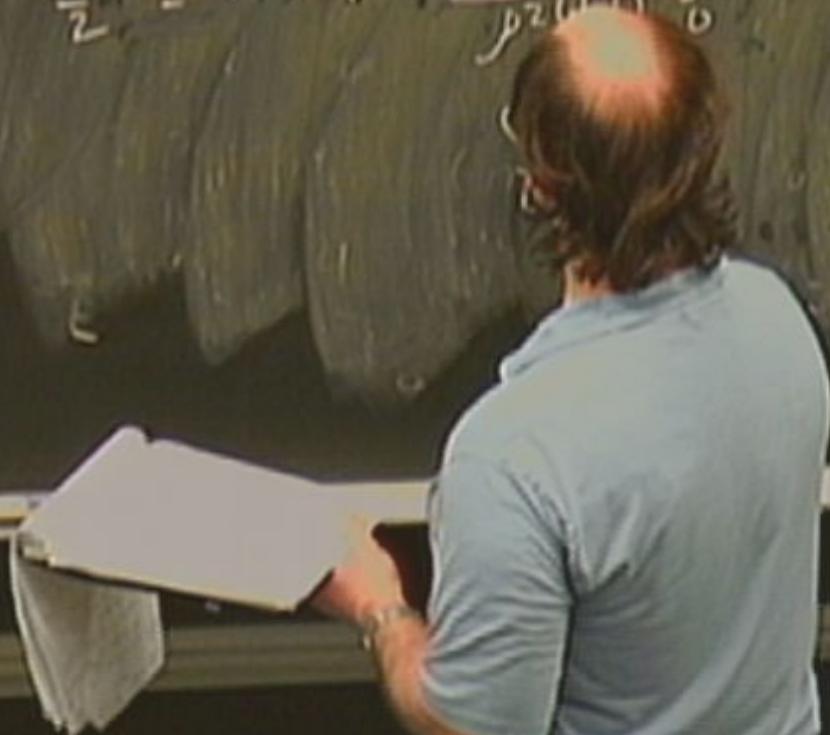
$$E = v(r) - \frac{\phi'^2}{2}$$

$$E' = (d-1)\rho' (v')^2$$

$$V_I = -\frac{\phi'^2}{2} \approx V_T - \frac{\phi'^2}{2}$$

$$\frac{\phi'^2}{2} = v(r) - \frac{2(d-1)}{\rho^{2(d-1)}} \sum_0^\infty \delta s V(r) \rho^{2d-3}$$

$$V_I = F V_T$$



Junction condition analysis

$$\psi' + (d-1)\rho' v' = v'(r)$$

$$E = v(r) - \frac{\psi'^2}{2}$$

$$E' = (d-1)\rho' (v')^2$$

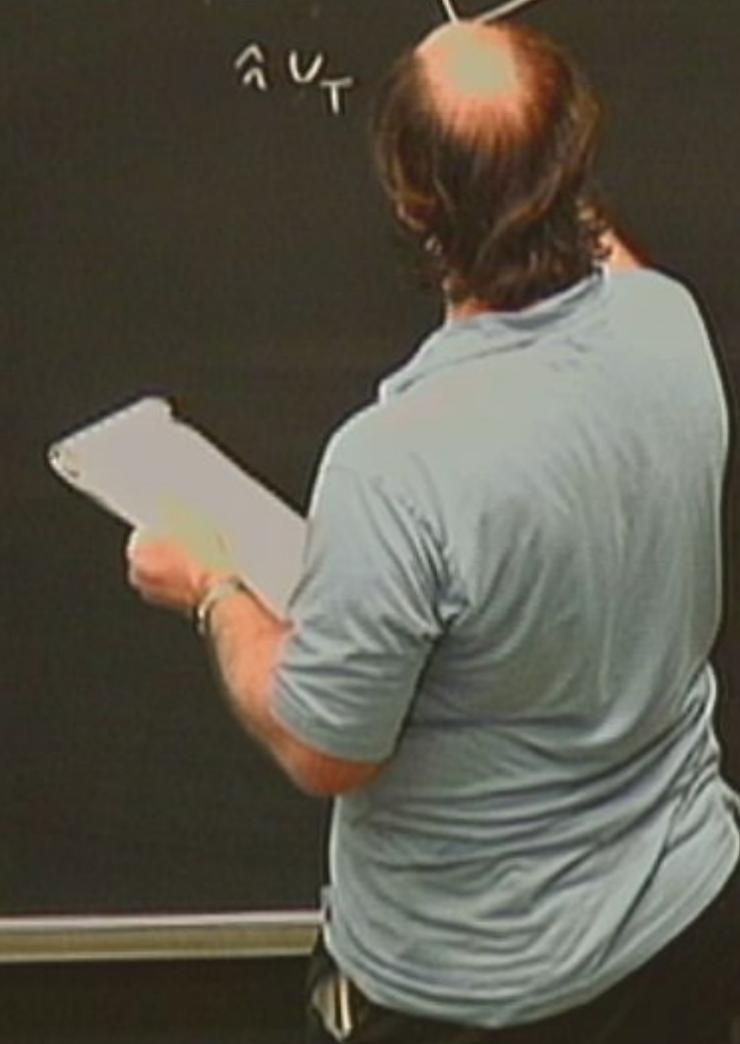
$$V_F - \frac{\psi'^2}{2} = V_T - \frac{v'^2}{2}$$

$$\frac{\psi'^2}{2} = v(r) - \frac{2(d-1)}{\rho^{2(d-1)}} \int_0^r d\zeta v(\zeta) \rho^{2d-3} \rho' \quad V_F = F V_T$$

$$\int_0^r d\zeta v(\zeta) \rho^{2d-3} \rho' = 0$$

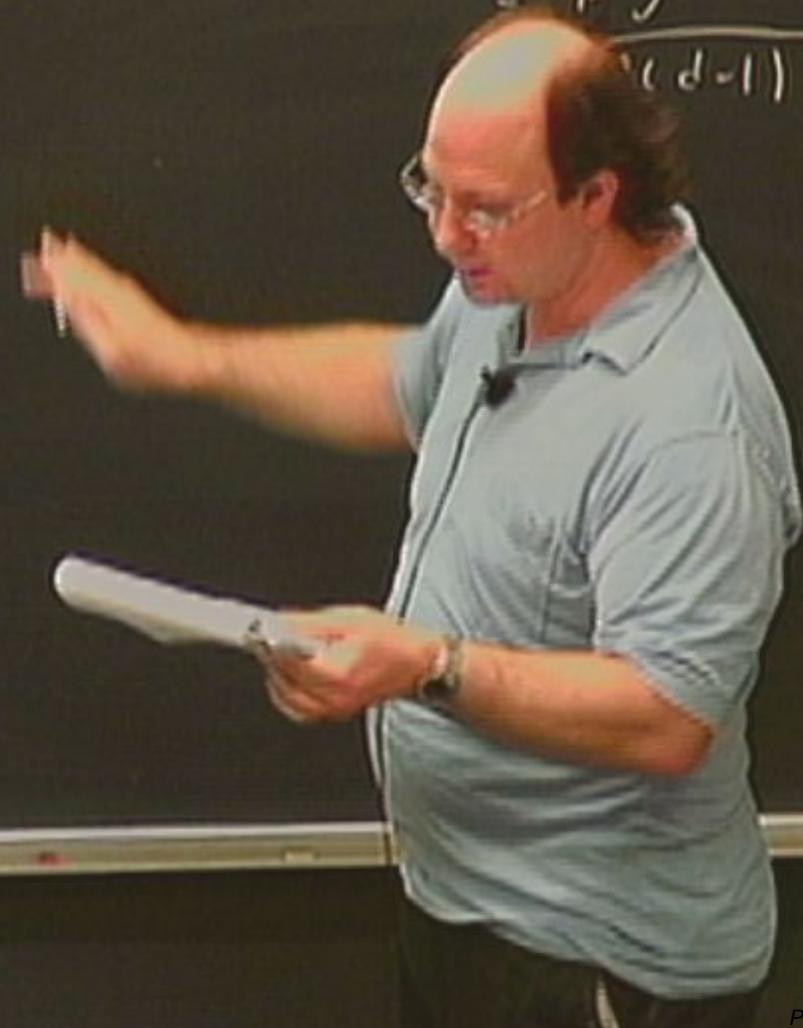
$$C_{EL} \cdot D \cdot S_E \cdot S_b = \frac{7670}{r^2 - v}$$

$$D = \underbrace{\int_0^{\tau_0} ds V(\varphi) \rho^2 d\cdot 3 \rho'}_{\hat{n} U_T} + \underbrace{\int_{\tau_0}^{\tau_1} ds V(\rho) \rho^2 d\cdot 3 \rho'} + \underbrace{\int_{\tau_1}^{\tau_2} ds V(v) \rho^2 d\cdot 3 \rho'}$$



$$\text{C.C.L.} \cdot D \cdot S_E \cdot S_b = \frac{7670}{\rho^2 \cdot V}$$

$$D = \underbrace{\int_0^{r_0} ds V(r) \rho^{2(d-3)} \rho'}_{\hat{v} v_T \frac{\rho^{2(d-1)}(r_0)}{2(d-1)}} + \underbrace{\int_{r_0}^{r_1} ds V(r) \rho^{2(d-3)} \rho'}_{-v_F \frac{\rho^{2(d-1)}(r_1)}{(d-1)}} + \underbrace{\int_{r_1}^{\infty} ds V(r) \rho^{2(d-3)} \rho'}$$



$$\text{Coul. } V - \mathcal{E} \cdot S_b = \frac{76\pi}{k^2 V}$$

(4 km) Measurement

$$D = \underbrace{\int_0^\infty ds v(\varphi) \rho^{2d-3} \rho'}_{\hat{v}_T \frac{\rho^{2(d-1)}(\tau_0)}{2(d-1)}} + \underbrace{\int_{\tau_0}^\infty ds v(\varphi) \rho^{2d-3} \rho'}_{v_F \frac{\rho^{2(d-1)}(\tau_1)}{2(d-1)}} + \underbrace{\int_{\tau_1}^\infty ds v(\varphi) \rho^{2d-3} \rho'}$$

$$D \approx -\frac{1}{2(d-1)} \int_{\tau_0}^{\tau_1} ds v''(\varphi) \varphi' \rho^{2d-2}$$



$$(4 \text{ km}) \cdot V \cdot E \cdot S_b = \frac{7670}{R^2 V_c}$$

$$\Omega = \underbrace{\int_0^{\tau_0} ds v(\varphi) \rho^{2d-3} \rho' + \int_{\tau_0}^{\tau_1} ds v(\varphi) \rho^{2d-3} \rho' + \int_{\tau_1}^{\tau_f} ds v(\varphi) \rho^{2d-3} \rho'}_{\hat{\Delta} V_T \frac{\rho^{2(d-1)}(\tau_0)}{2(d-1)}} - \underbrace{v_F \frac{\rho^{2(d-1)}(\tau_f)}{2(d-1)}}$$

$$\Omega \approx -\frac{1}{2(d-1)} \int_0^{\tau_f} ds v''(\varphi) \varphi' \rho^{2d-2}$$

$$\rightarrow \Omega = \sum_i \psi v'(\vartheta) = V_F - V_T$$

$$(4 \text{ km}) \quad D = D_E + S_b = \frac{76\pi}{R^2 V_c}$$

$$D = \underbrace{\int_0^{\tau_0} ds v(\varphi) \rho^{2d-3} \rho' + \int_{\tau_0}^{\tau_1} ds v(\varphi) \rho^{2d-3} \rho' + \int_{\tau_1}^{\tau_f} ds v(\varphi) \rho^{2d-3} \rho'}_{\hat{\Delta} V_T \frac{\rho^{2(d-1)}(\tau_0)}{2(d-1)}} - \frac{V_F \rho^{2(d-1)}(\tau_1)}{2(d-1)}$$

$$\boxed{D \approx -\frac{1}{2(d-1)} \int_0^{\tau_f} ds v''(\varphi) \varphi' \rho^{2d-2}}$$

$$\rightarrow D = \boxed{\int_{\tau_1}^{\tau_f} d\varphi v''(\varphi)} = V_F - V_T$$