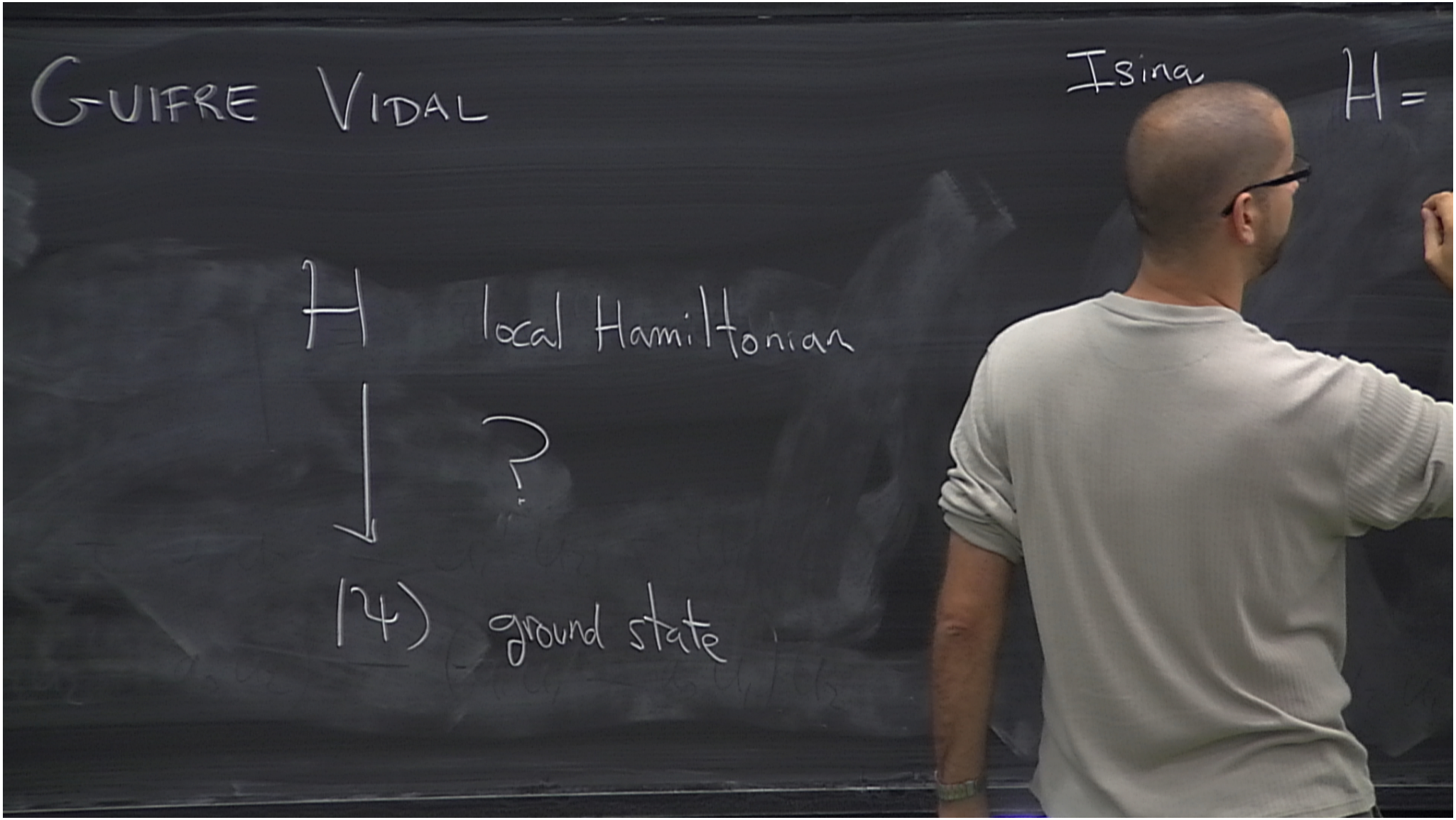
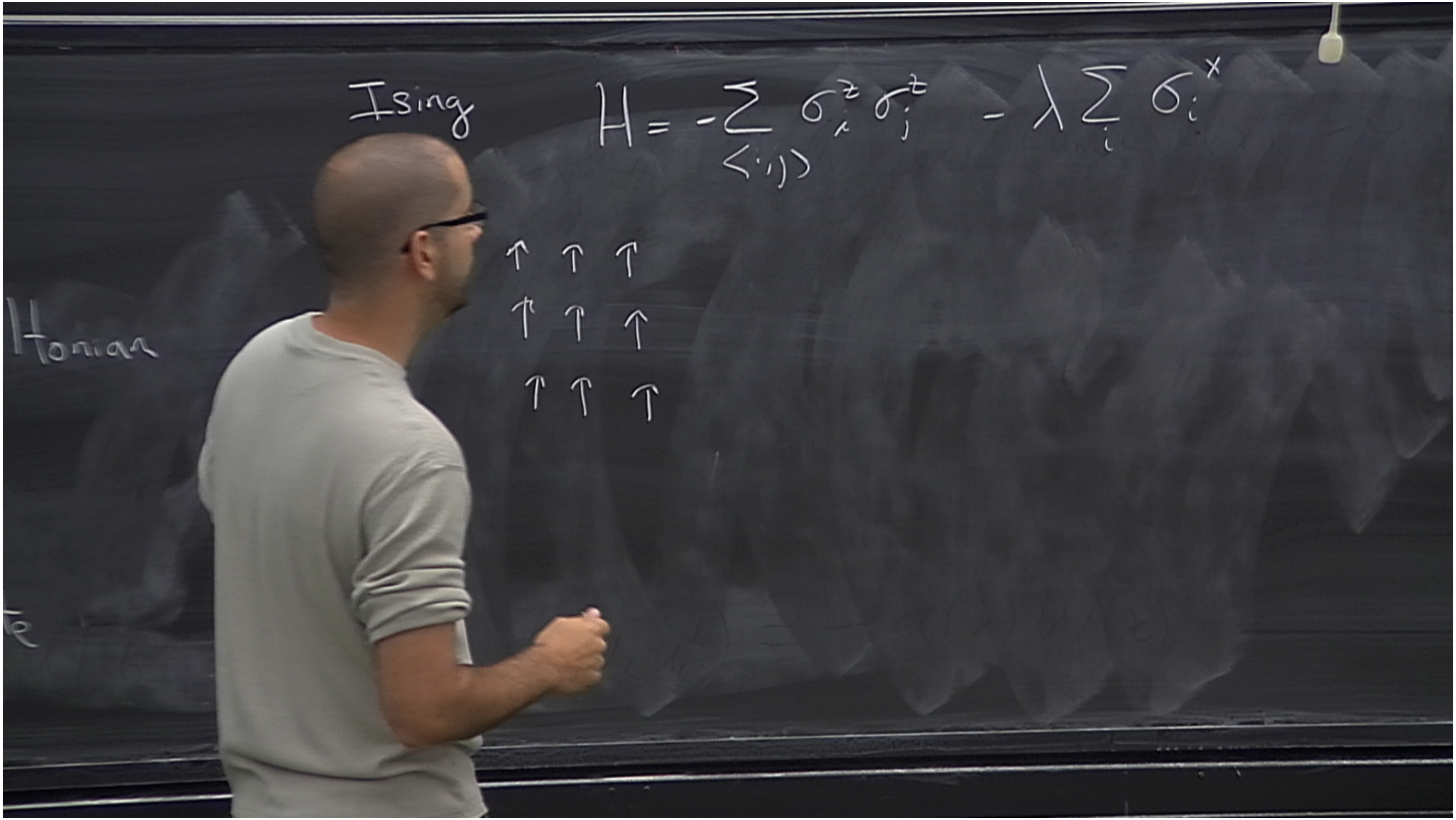
Title: Researcher Presentation: Guifre Vidal

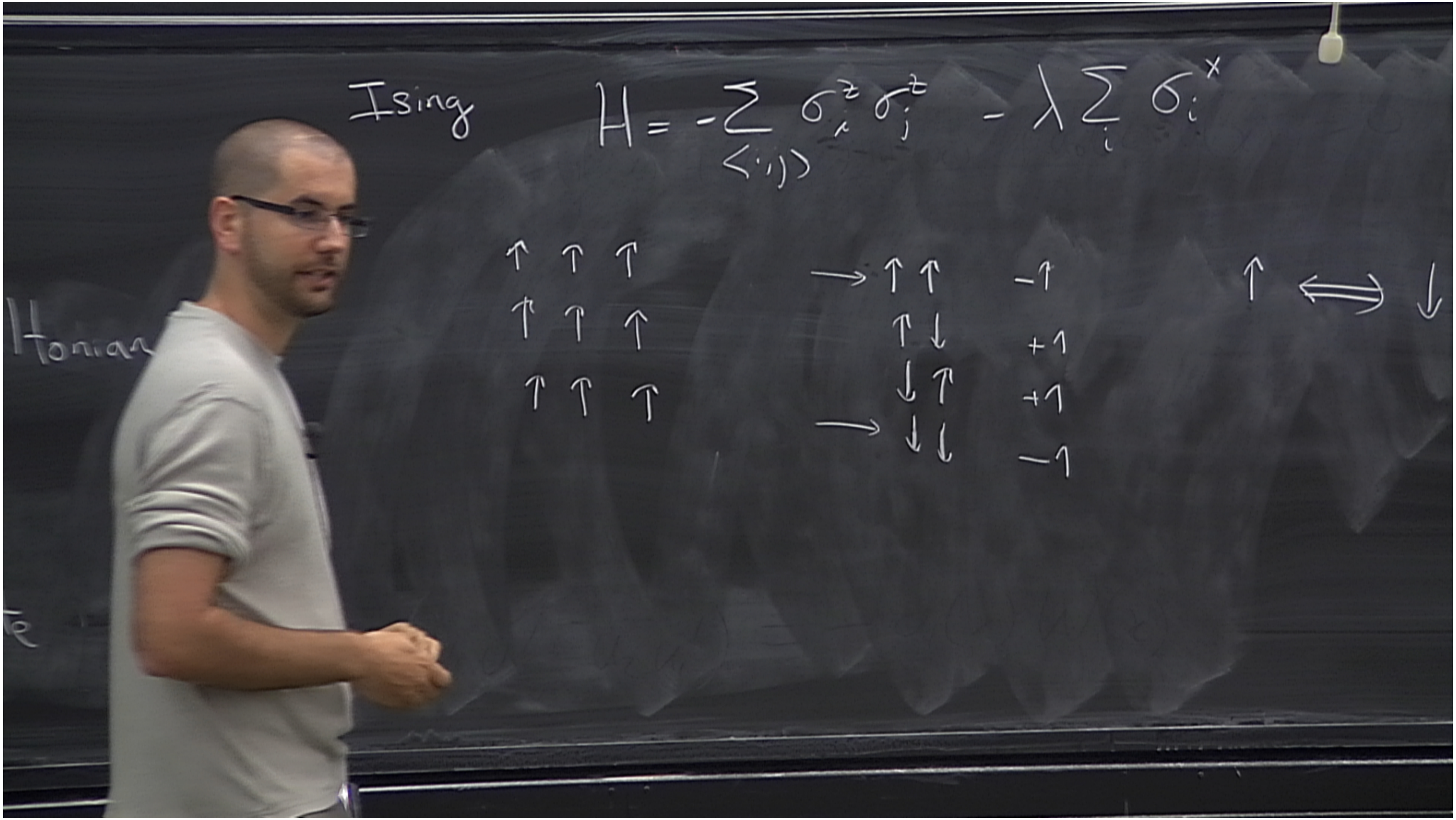
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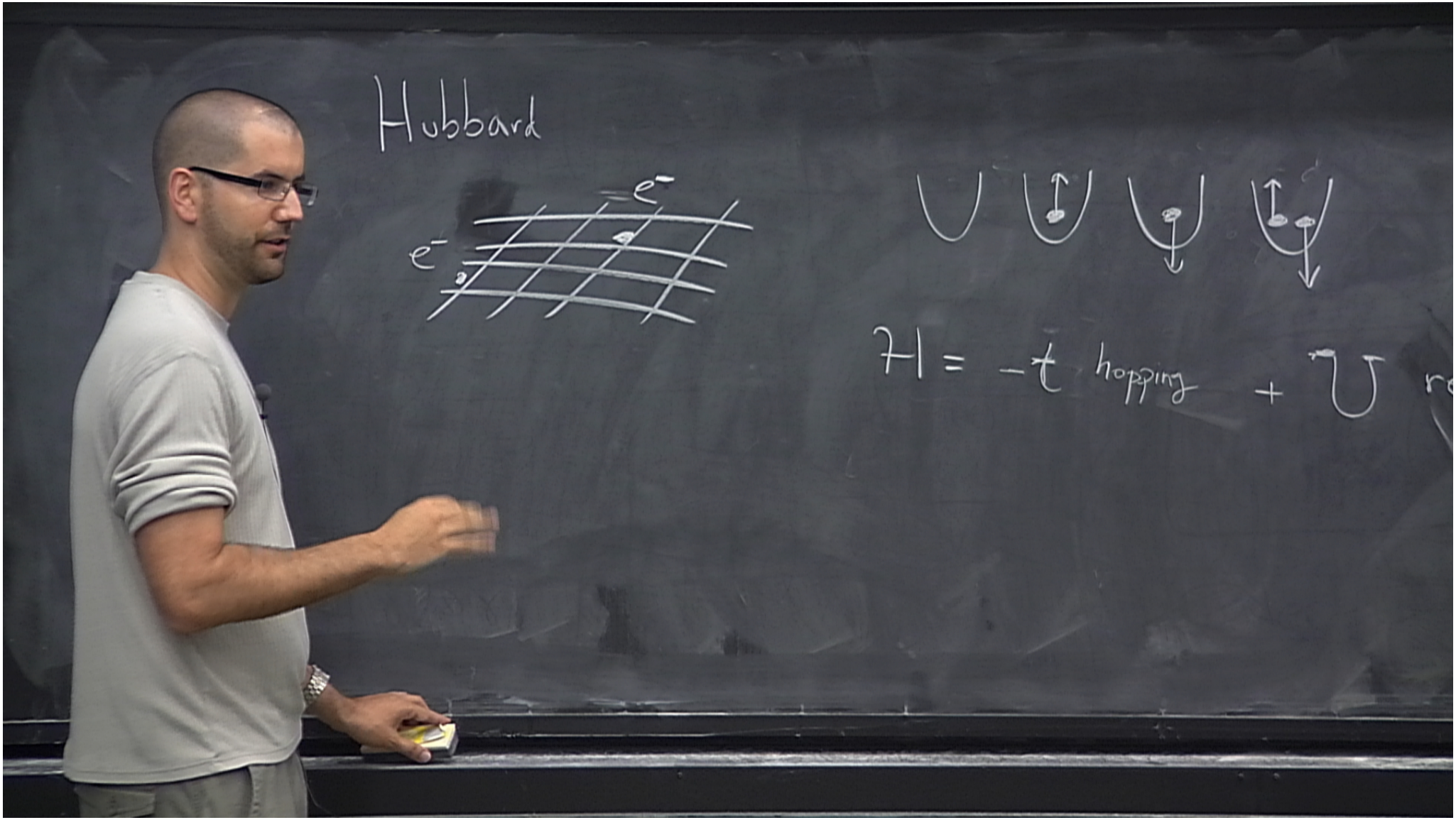
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Abstract:

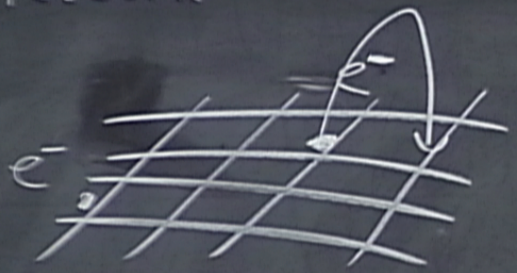

GUIFRE VIDAL
Ising
H =
H
local Hamiltonian
?
|4)
ground state

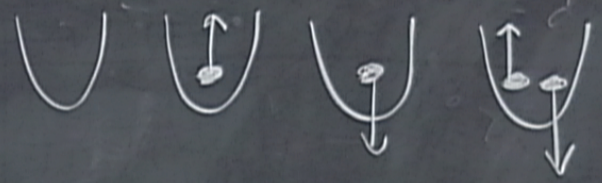

Ising
$H = -\sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \lambda \sum_i \sigma_i^x$
Honian


Ising
$H = -\sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - \lambda \sum_i \sigma_i^x$
Hamiltonian
$\uparrow\uparrow\uparrow$
$\uparrow\uparrow\uparrow$
$\uparrow\uparrow\uparrow$
$\uparrow\uparrow$ -1
$\uparrow\downarrow$ $+1$
$\downarrow\uparrow$ $+1$
$\downarrow\downarrow$ -1
$\uparrow \Longleftrightarrow \downarrow$


Hubbard
e⁻
e⁻
e⁻
H = -t hopping + U

Hubbard

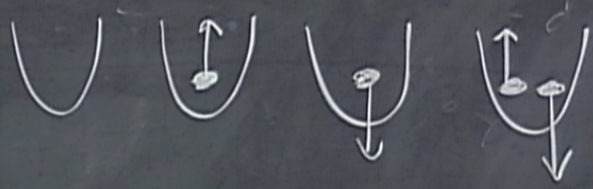




$$\mathcal{H} = -t \text{ hopping} + U \text{ repulsion}$$

Hubbard 1963

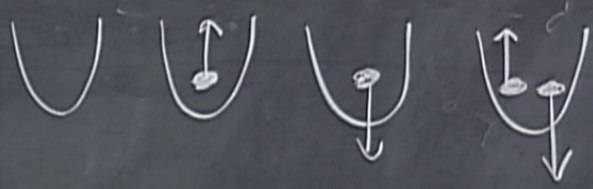




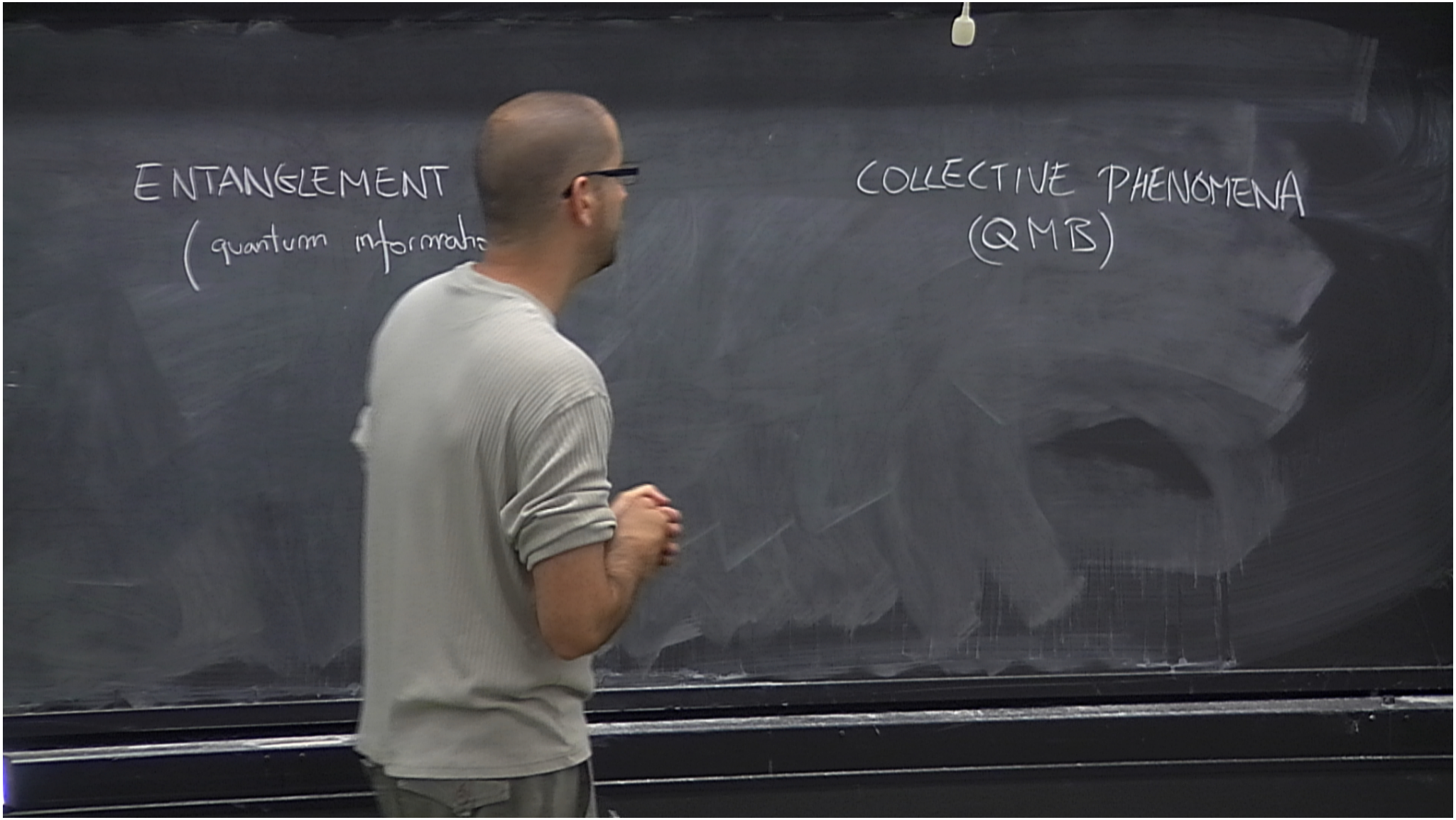
$\mathcal{H} = -t$ hopping $+ U$ repulsion

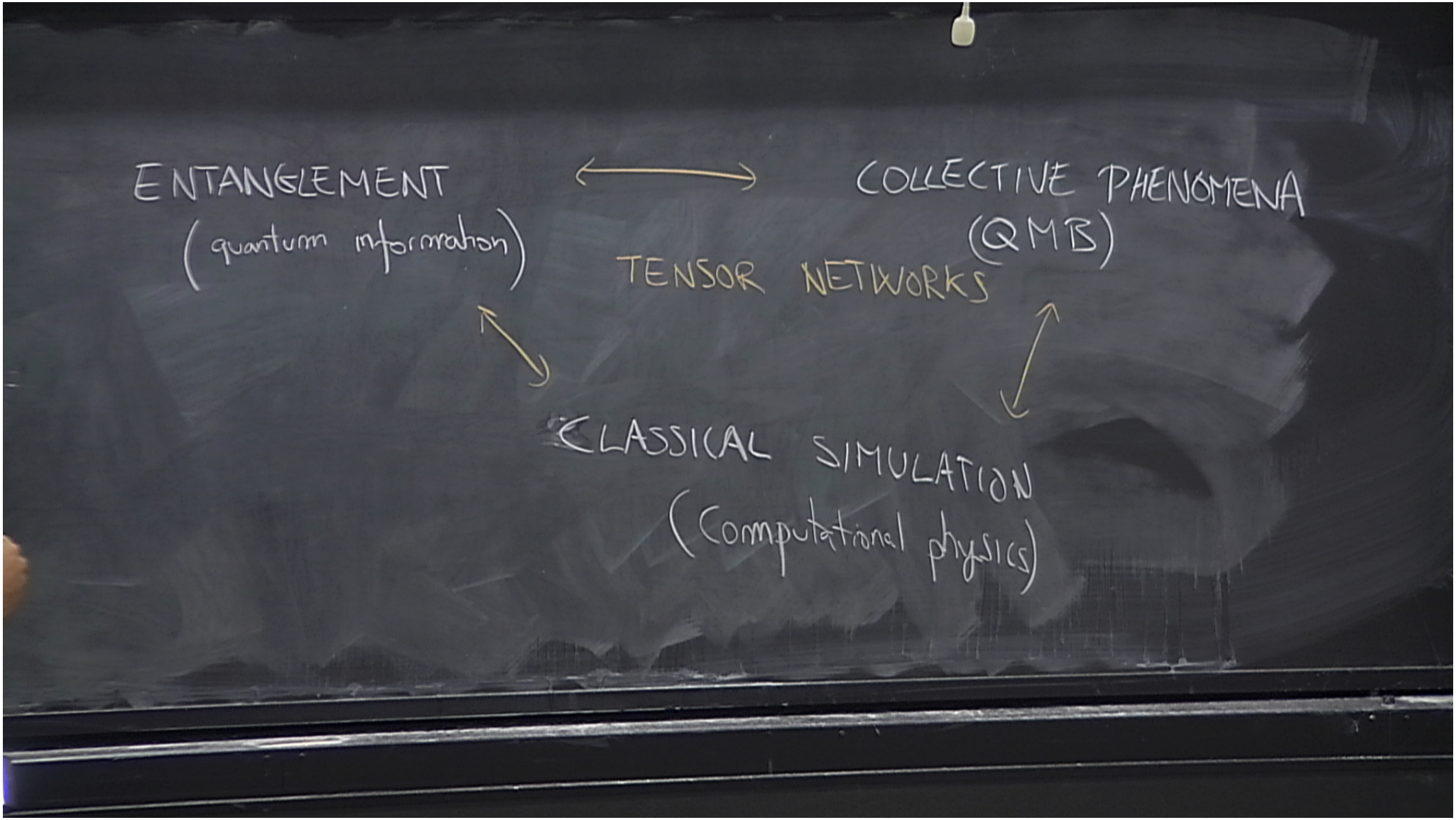
Hubbard 1963

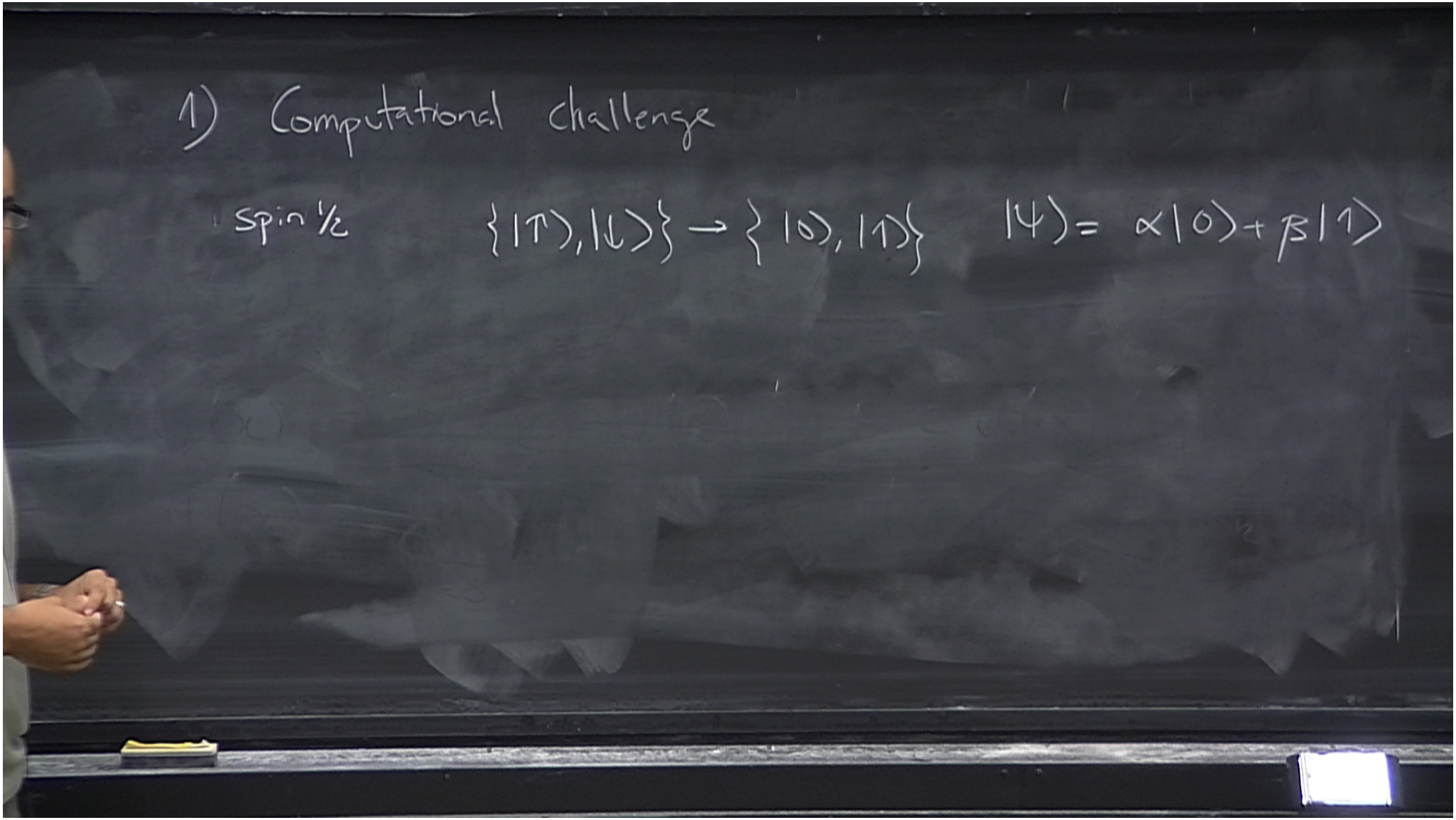


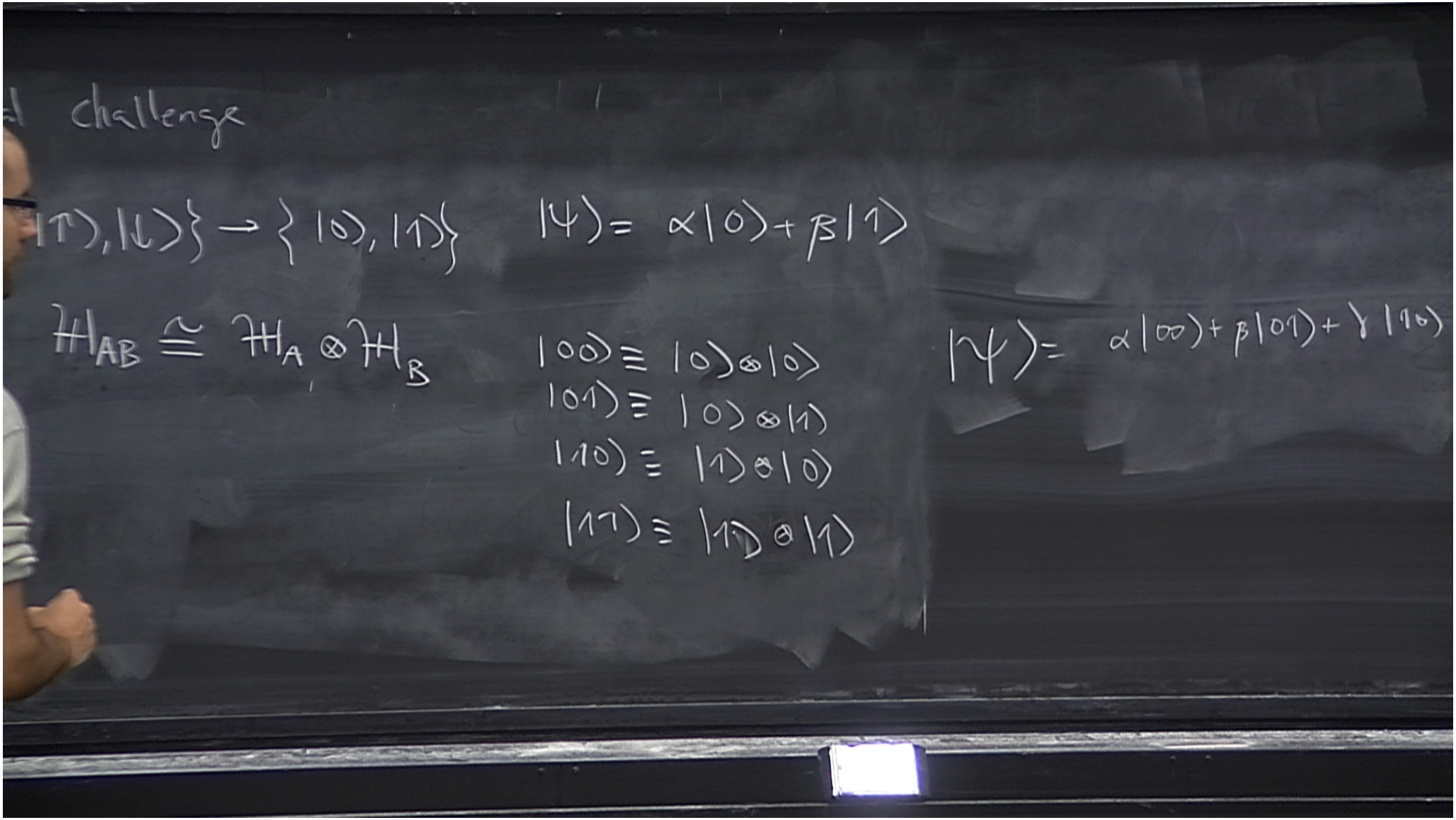


$\mathcal{H} = -t$ hopping $+ U$ repulsion


ENTANGLEMENT
COLLECTIVE PHENOMENA
(QMB)


ENTANGLEMENT
(quantum information)
COLLECTIVE PHENOMENA
(QMB)
TENSOR NETWORKS
CLASSICAL SIMULATION
(Computational physics)


1) Computational challenge
spin ½
$\{|\uparrow\rangle, |\downarrow\rangle\} \rightarrow \{|0\rangle, |1\rangle\}$
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$


challenge
$\{|T\rangle, |L\rangle\} \rightarrow \{|0\rangle, |1\rangle\}$
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$\mathcal{H}_{AB} \cong \mathcal{H}_A \otimes \mathcal{H}_B$
$|00\rangle \equiv |0\rangle \otimes |0\rangle$
$|01\rangle \equiv |0\rangle \otimes |1\rangle$
$|10\rangle \equiv |1\rangle \otimes |0\rangle$
$|11\rangle \equiv |1\rangle \otimes |1\rangle$
$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle$

1) Computational challenge

spin ½ — $d=2$

$N=1$ $\{|\uparrow\rangle, |\downarrow\rangle\} \to \{|0\rangle, |1\rangle\}$ $\quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$N=2$ $d=4$ $\quad \mathcal{H}_{AB} \cong \mathcal{H}_A \otimes \mathcal{H}_B$

$|00\rangle \equiv |0\rangle \otimes |0\rangle$ $\quad |\psi\rangle =$

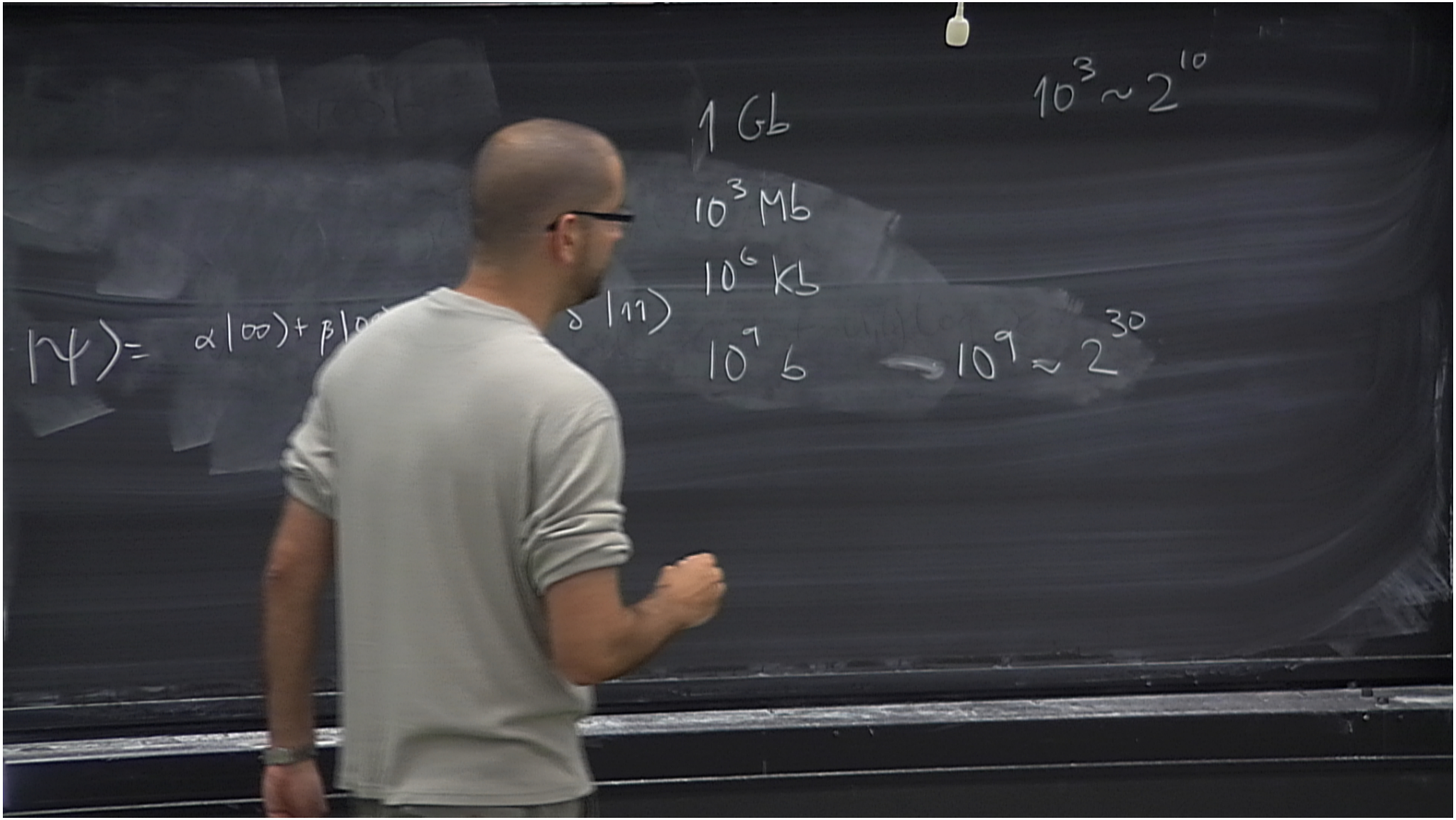
$|01\rangle \equiv |0\rangle \otimes |1\rangle$

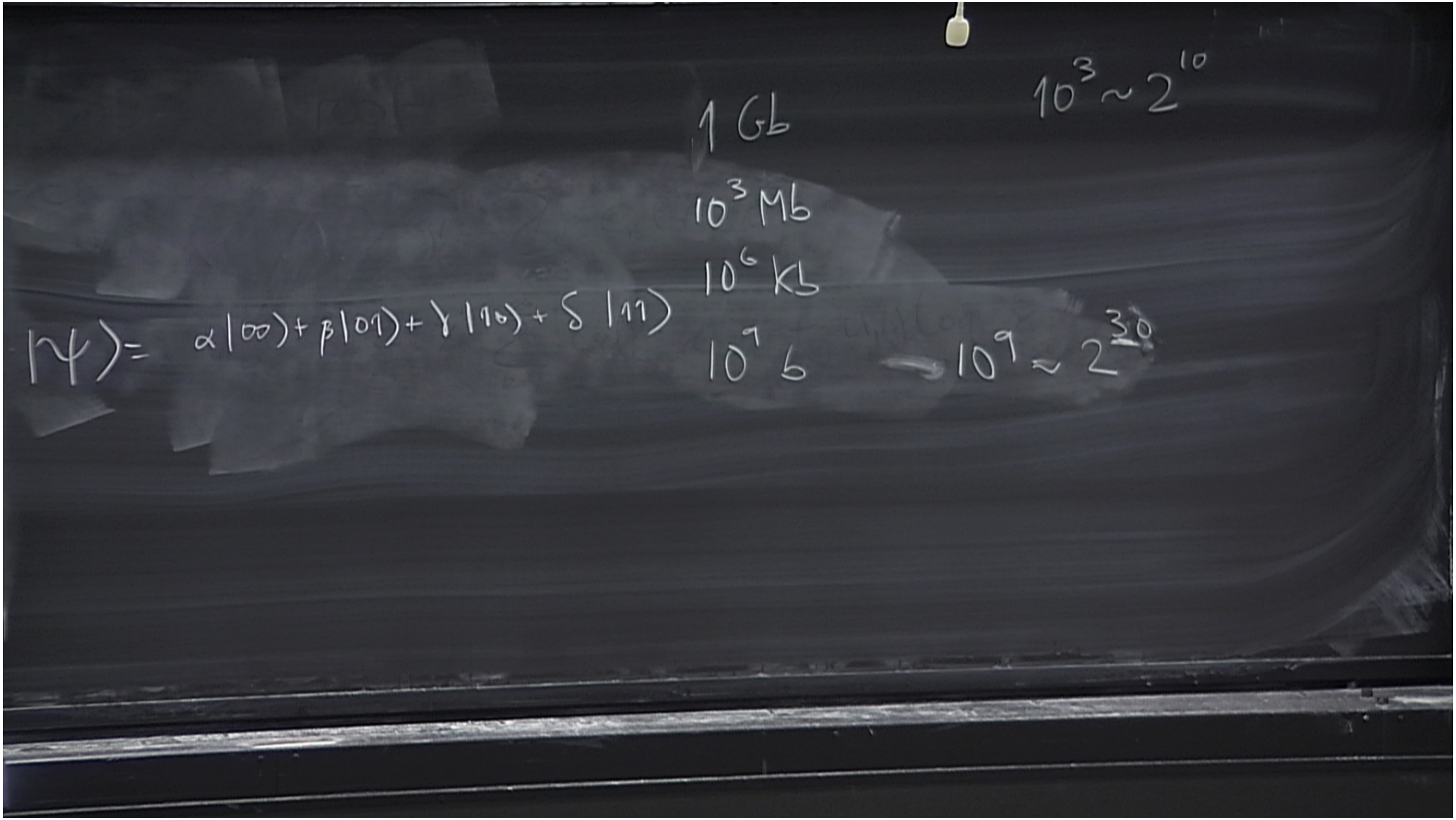
$|10\rangle \equiv |1\rangle \otimes |0\rangle$

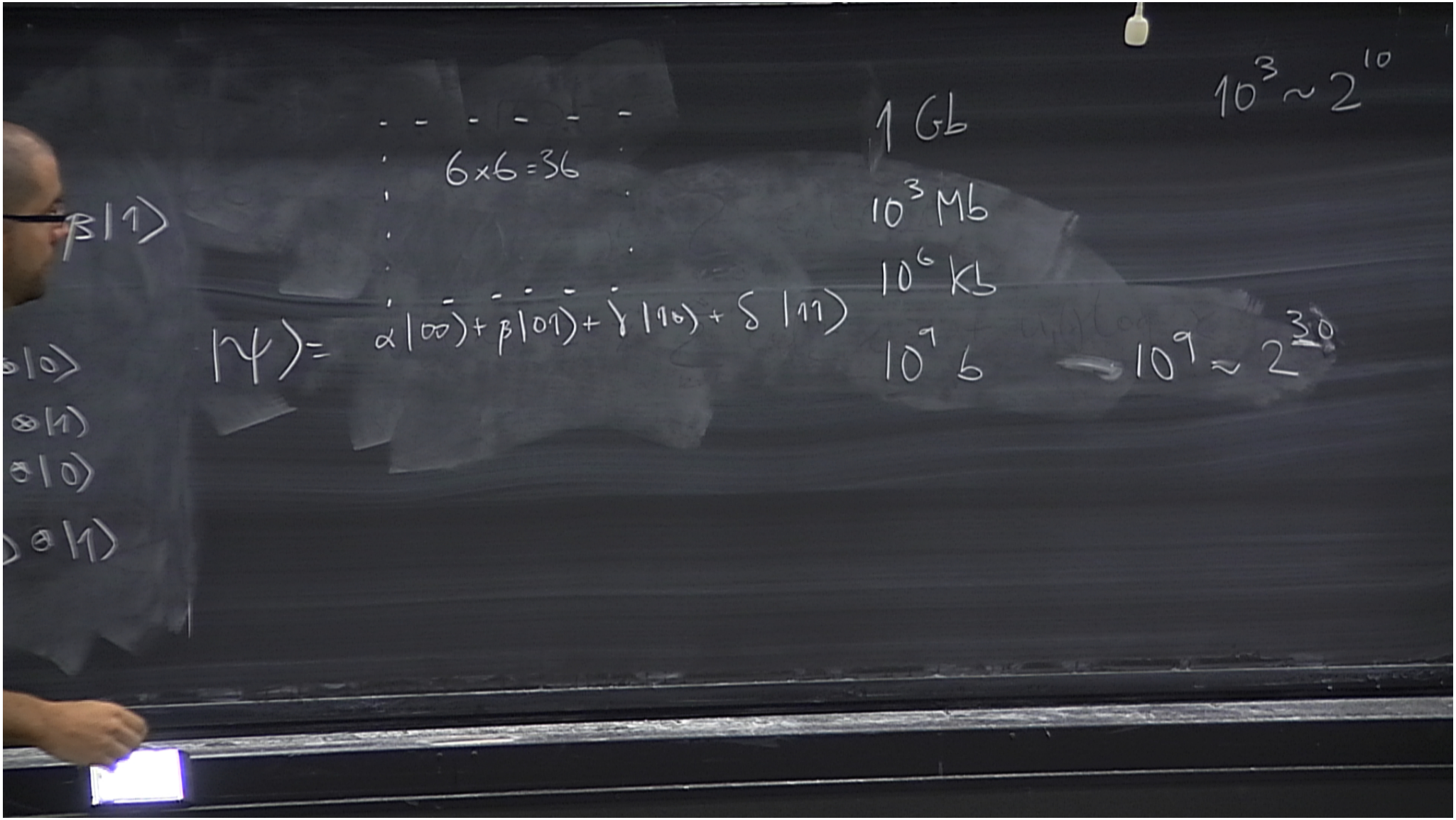
$|11\rangle \equiv |1\rangle \otimes |1\rangle$

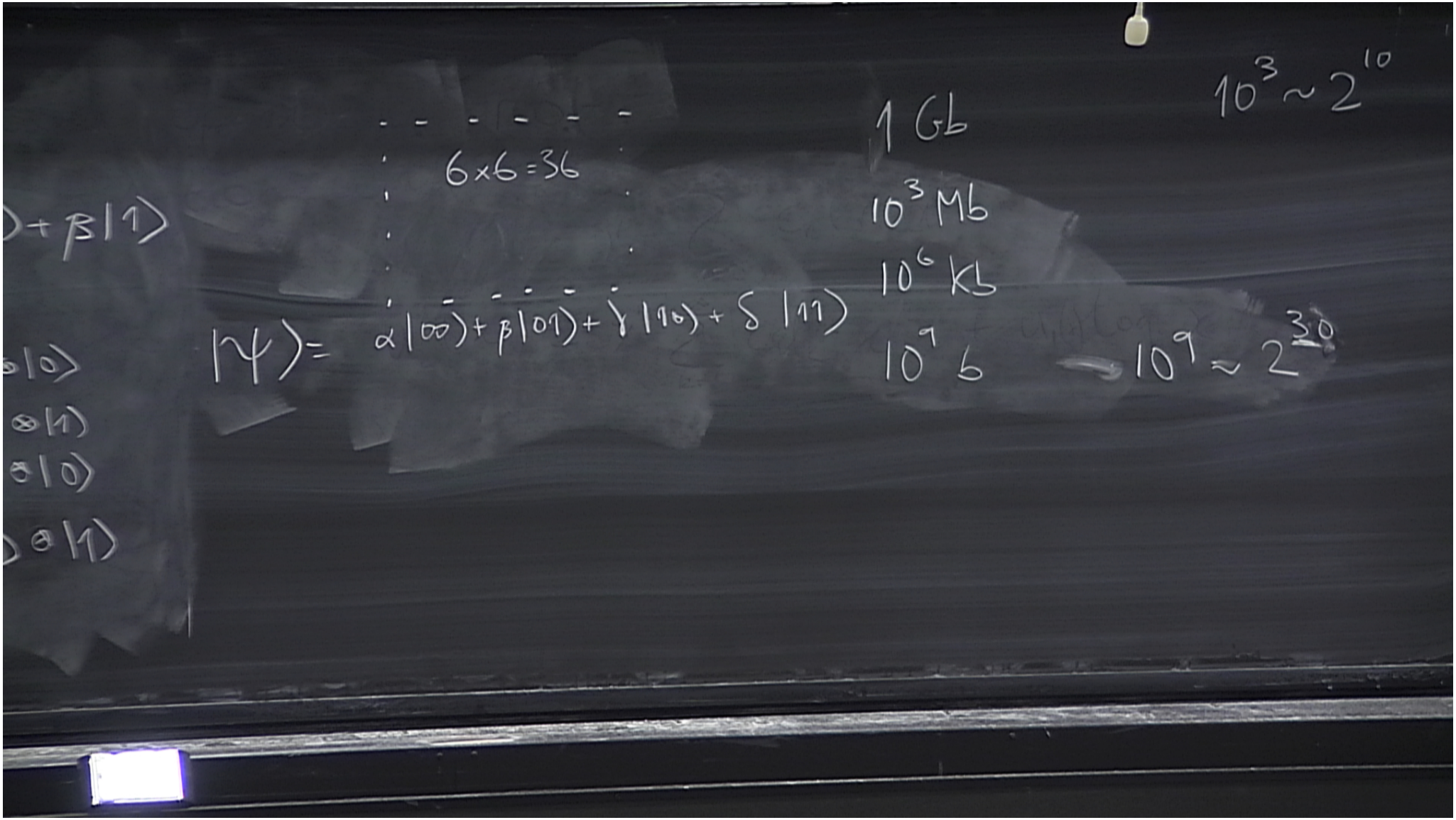
$N=3$ $d=8$

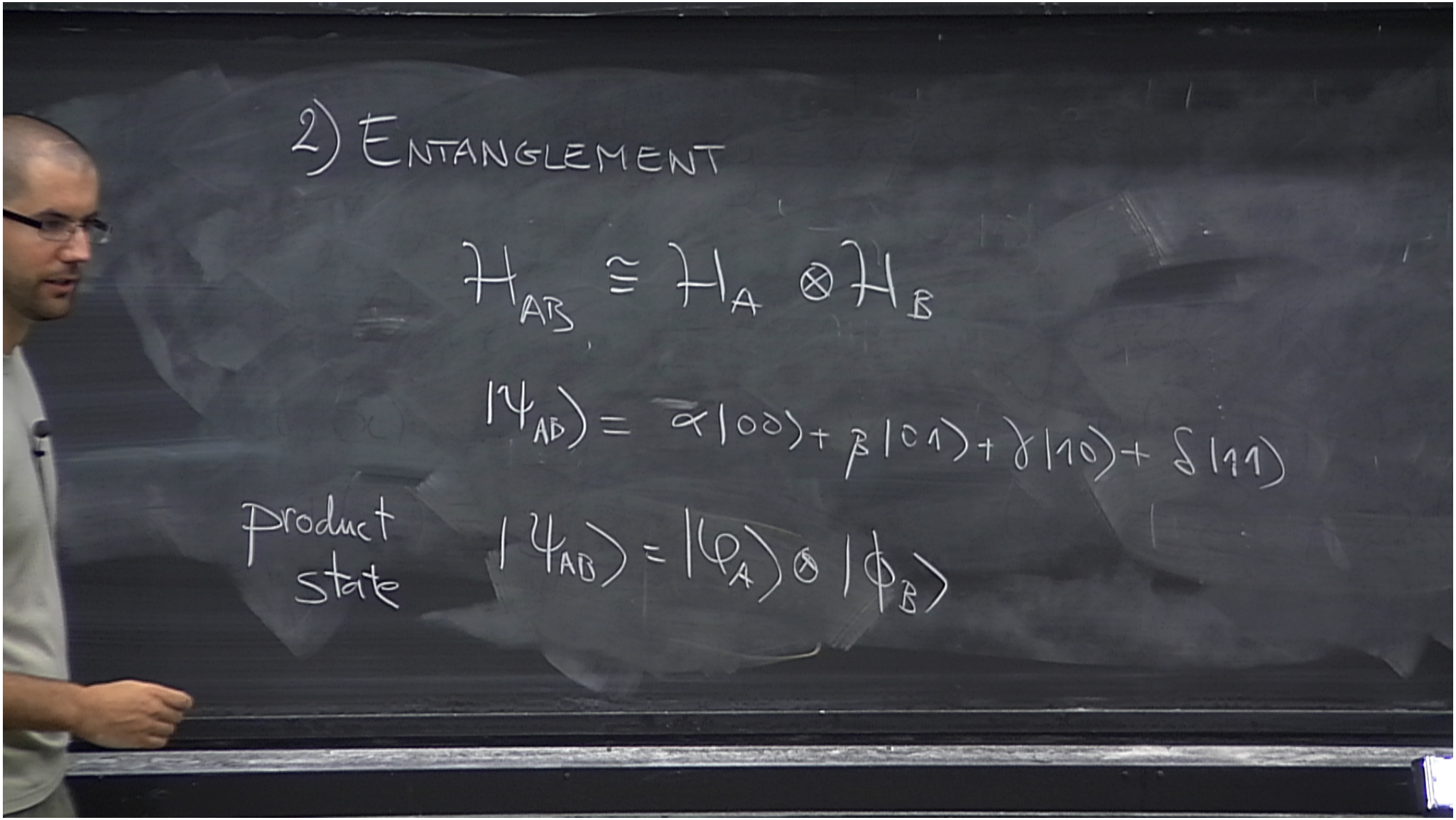
$N \qquad d=2^N$

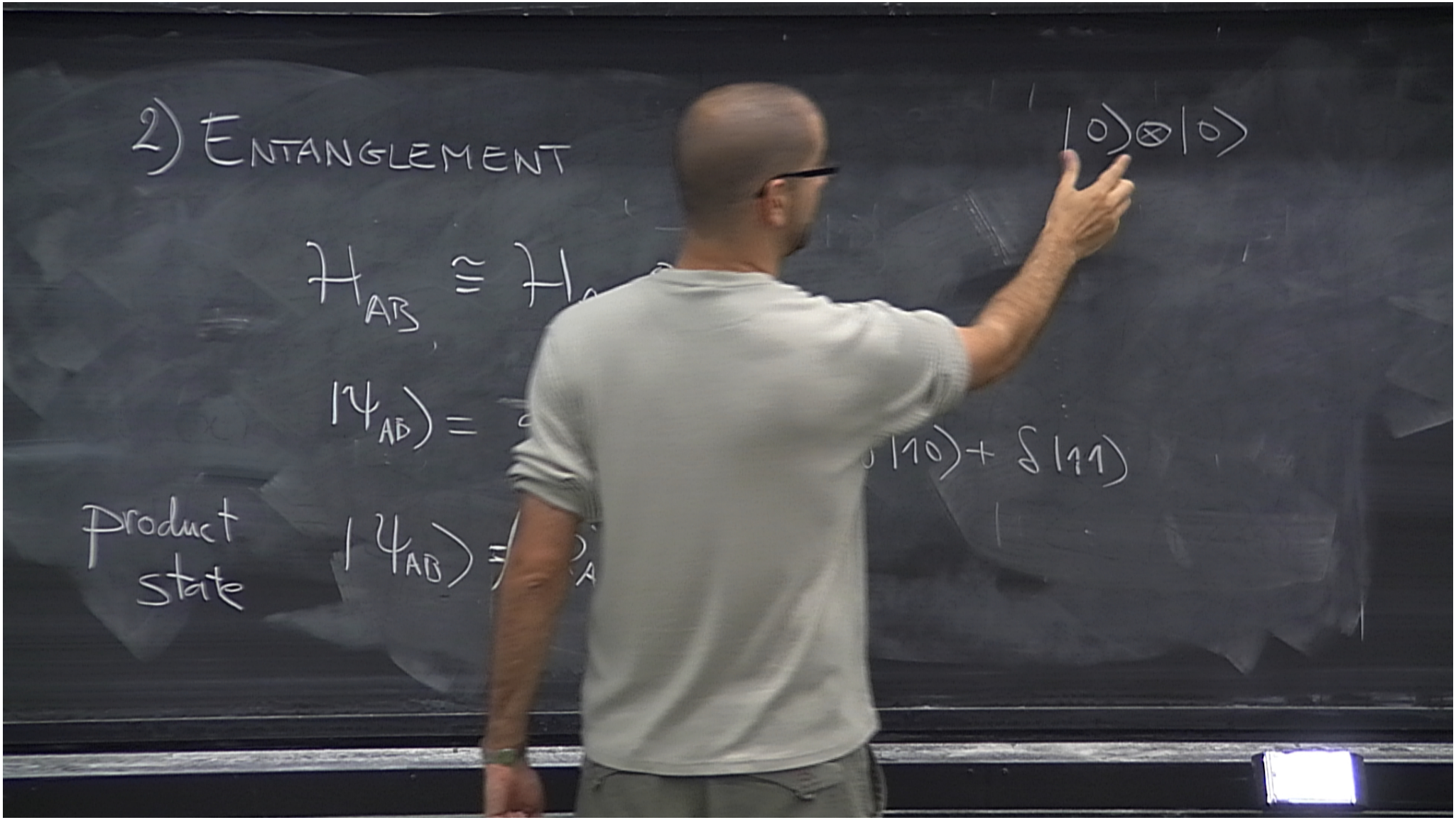

1 Gb
$10^3 \sim 2^{10}$
10^3 Mb
10^6 Kb
10^9 b
$\sim 10^9 \sim 2^{30}$
$|\psi\rangle = \alpha|00\rangle + \beta|$
$\delta|11\rangle$

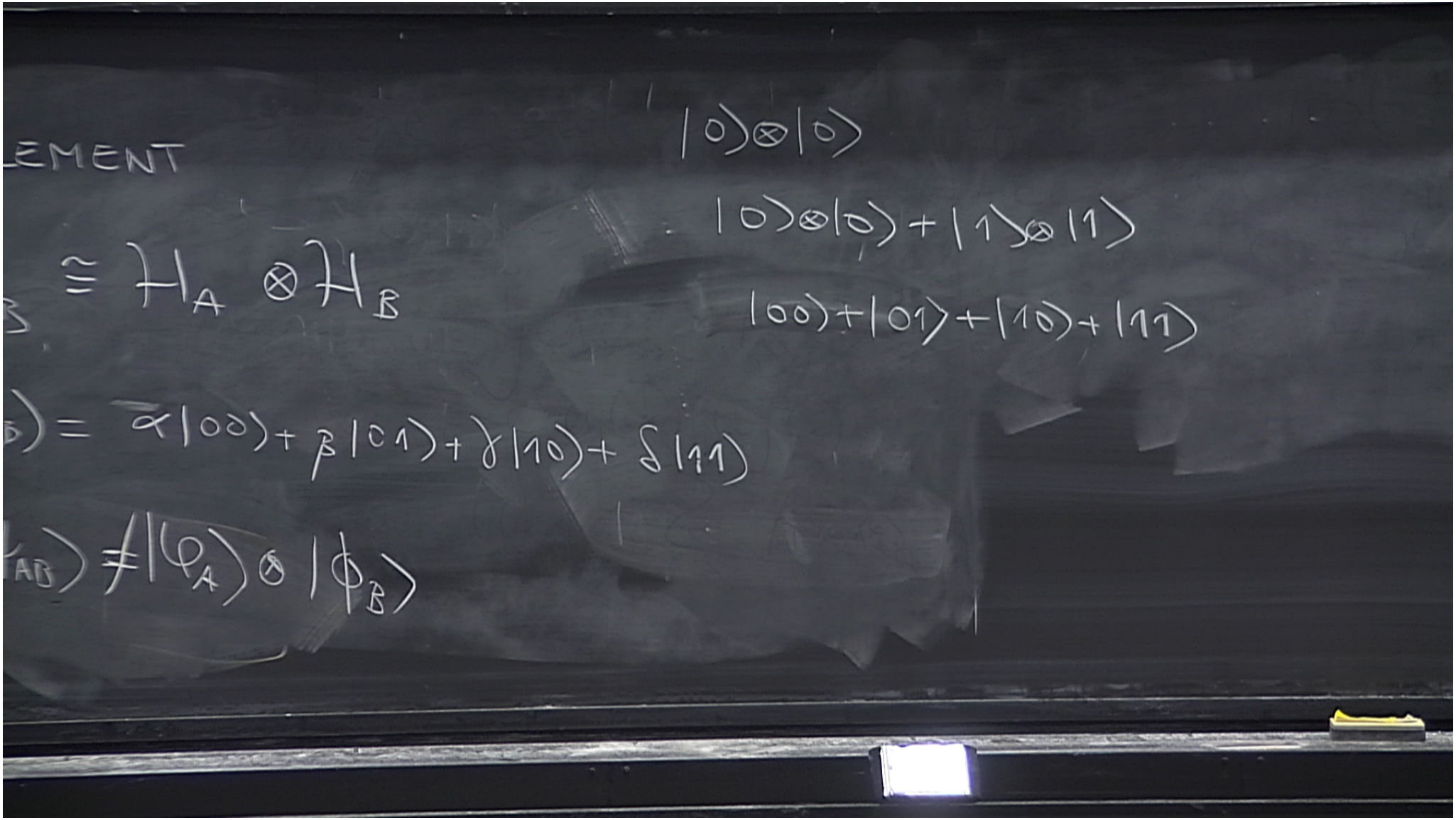

1 Gb
10^3 Mb
10^6 Kb
10^9 b
$10^3 \sim 2^{10}$
$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
$10^9 \sim 2^{30}$

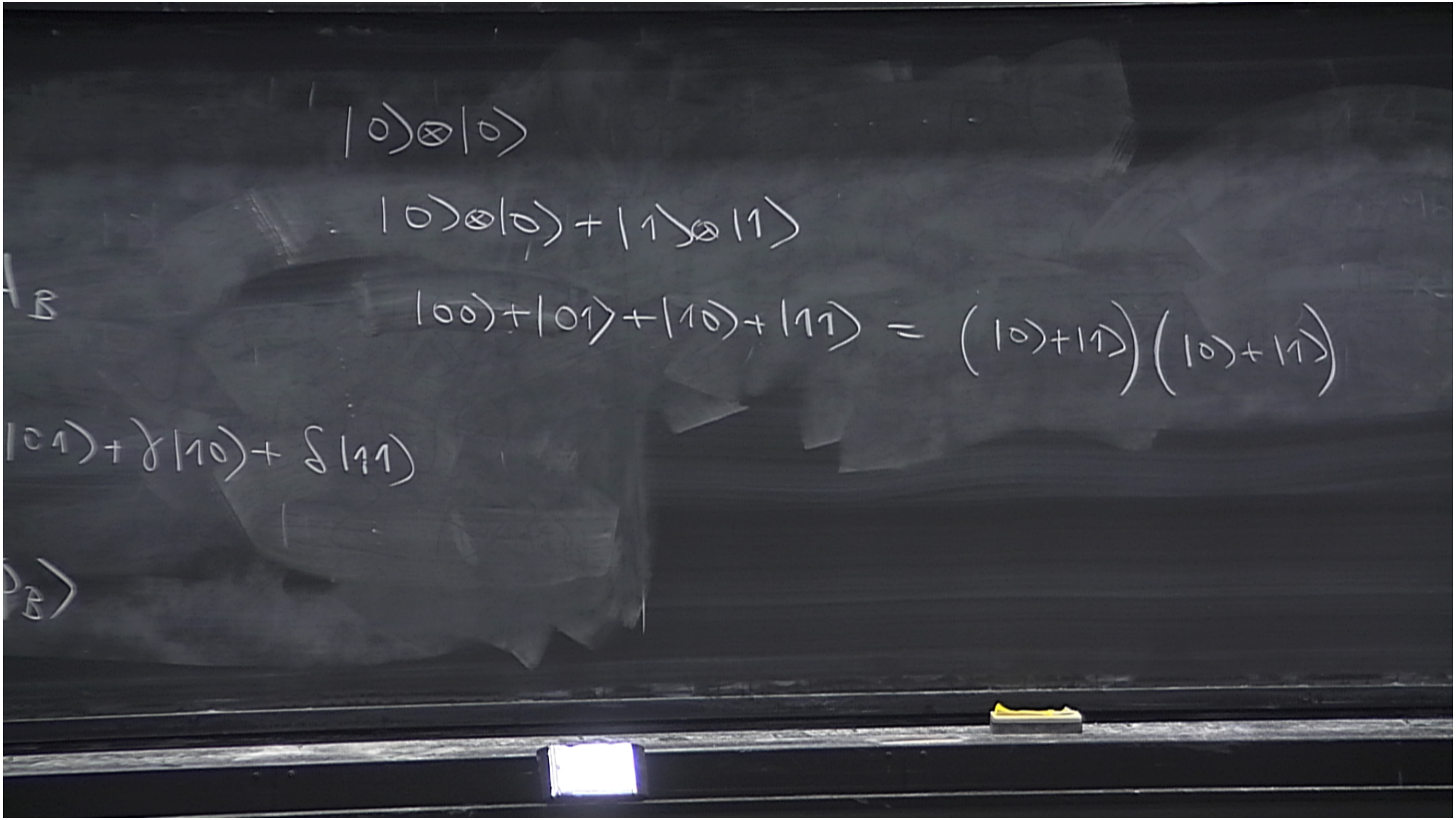

6×6=36
$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
1 Gb
10^3 Mb
10^6 Kb
10^9 b
$10^3 \sim 2^{10}$
$10^9 \sim 2^{30}$

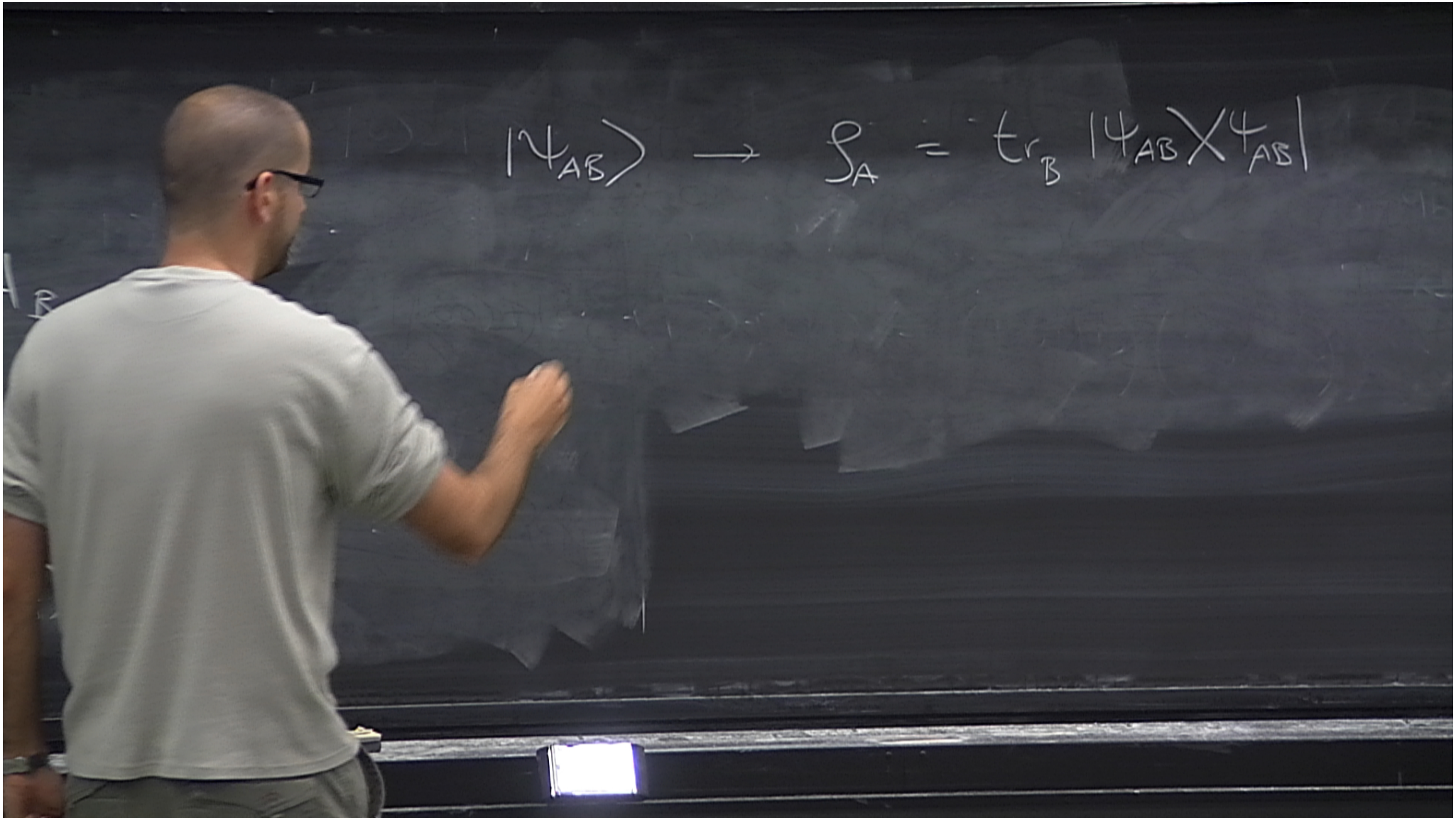

6×6=36
$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
1 Gb
10^3 Mb
10^6 Kb
10^9 b
$10^3 \sim 2^{10}$
$10^9 \sim 2^{30}$

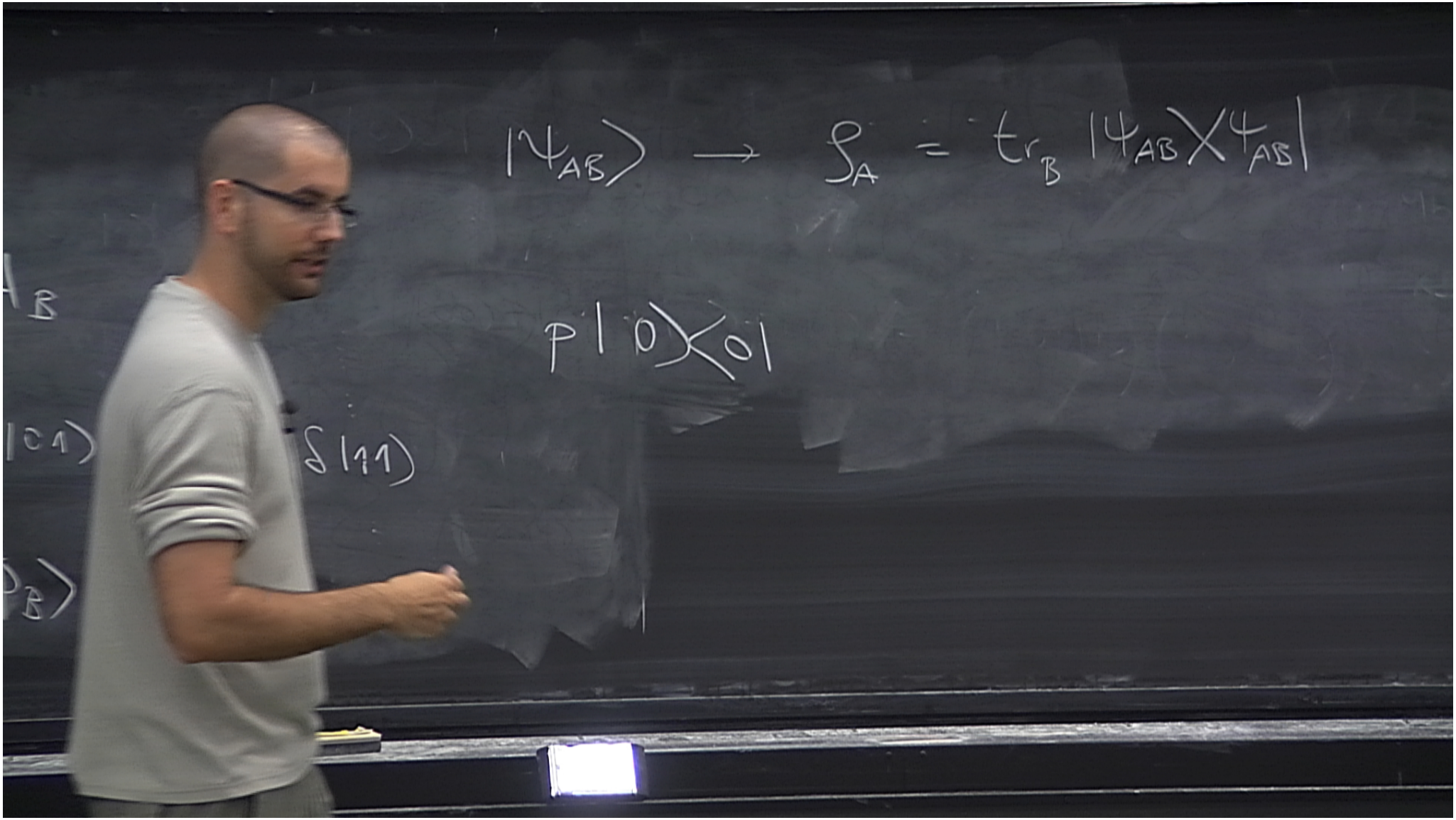

2) Entanglement
$\mathcal{H}_{AB} \cong \mathcal{H}_A \otimes \mathcal{H}_B$
$|\psi_{AB}\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
product state
$|\psi_{AB}\rangle = |\varphi_A\rangle \otimes |\phi_B\rangle$

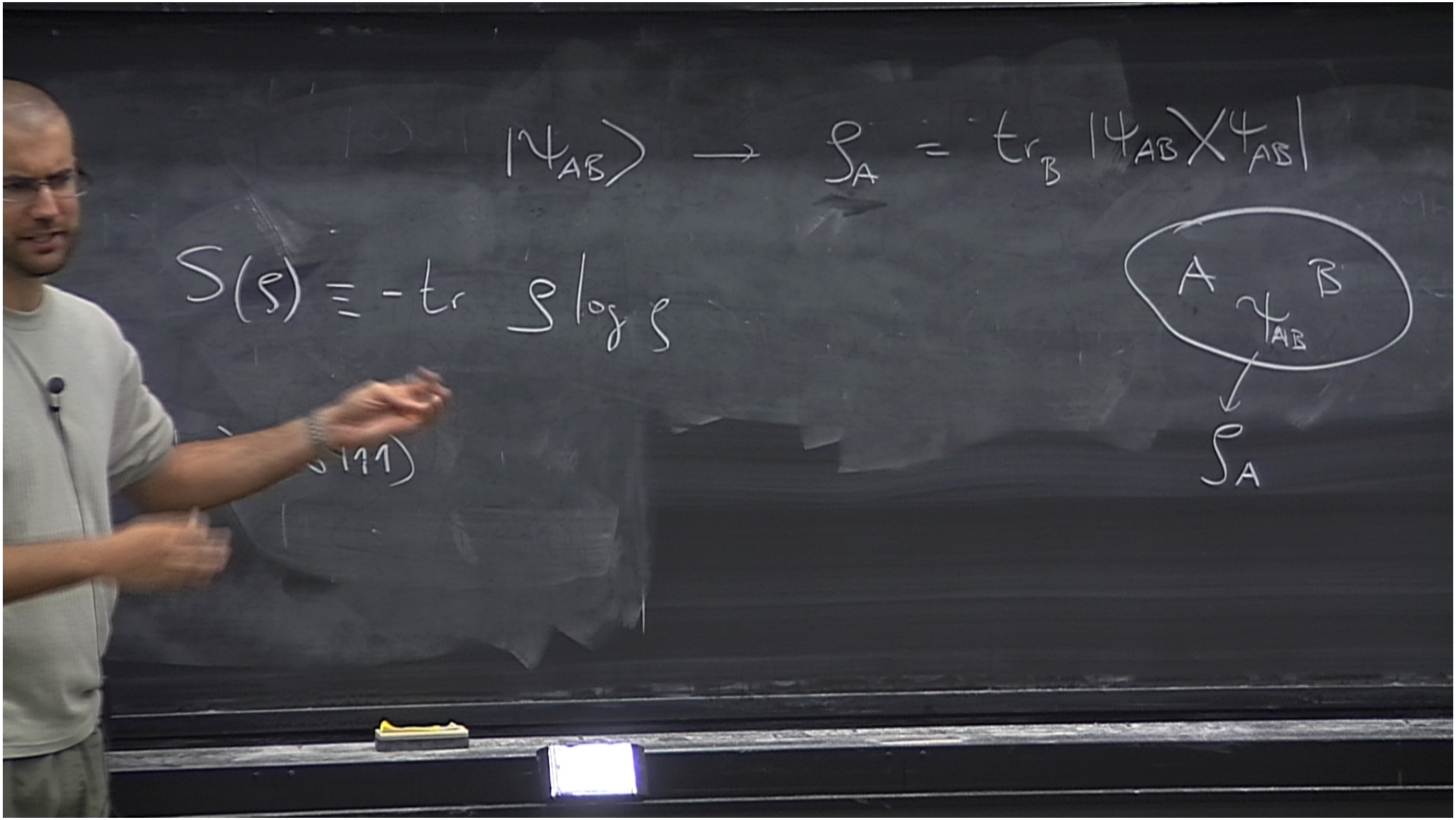

2) Entanglement
$|0\rangle \otimes |0\rangle$
$\mathcal{H}_{AB} \cong \mathcal{H}$
$|\psi_{AB}\rangle =$
$\beta|10\rangle + \delta|11\rangle$
Product state
$|\psi_{AB}\rangle =$

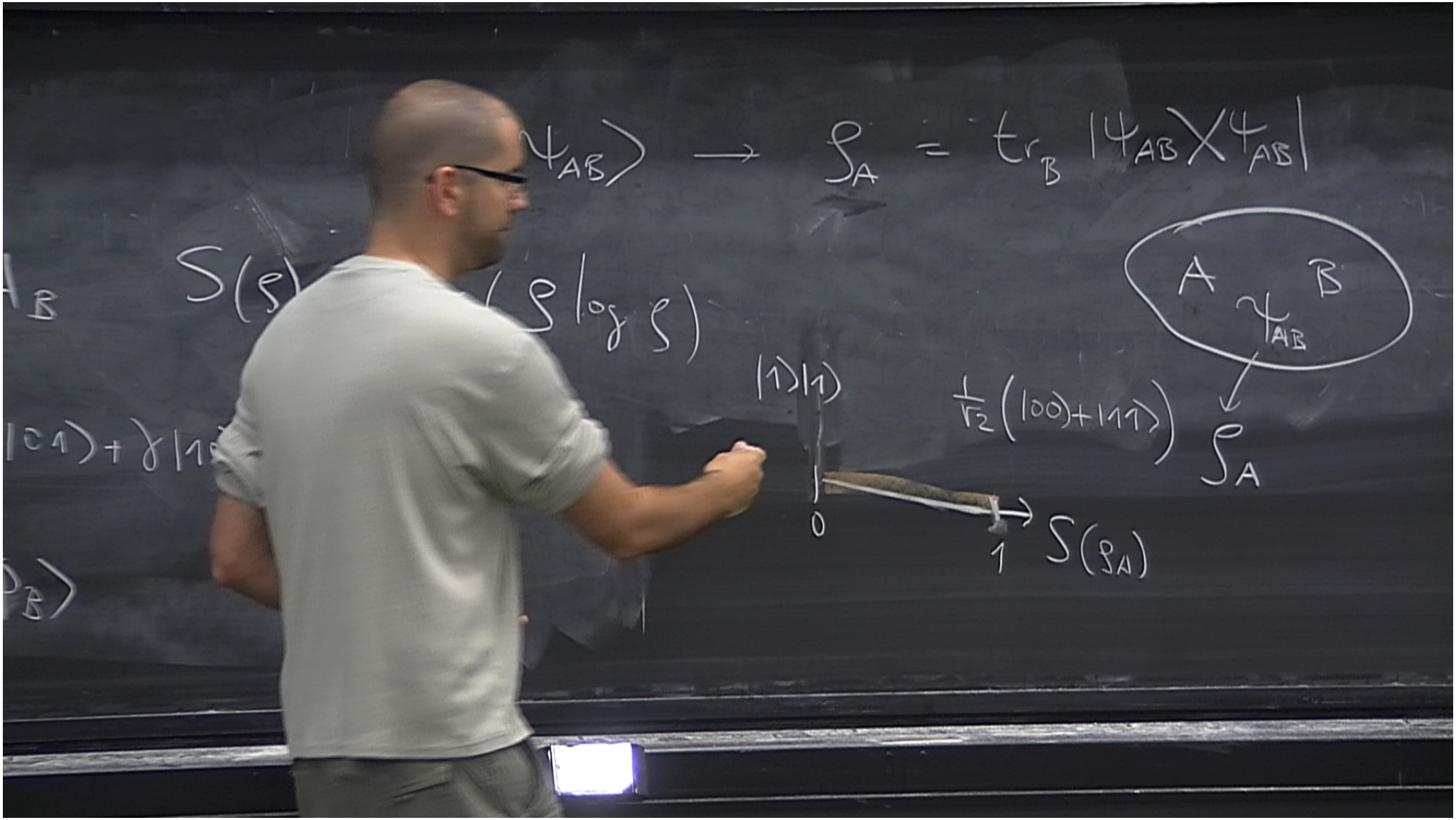

EMENT
$\cong \mathcal{H}_A \otimes \mathcal{H}_B$
$= \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$
$\neq |\phi_A\rangle \otimes |\phi_B\rangle$
$|0\rangle \otimes |0\rangle$
$|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle$
$|00\rangle + |01\rangle + |10\rangle + |11\rangle$

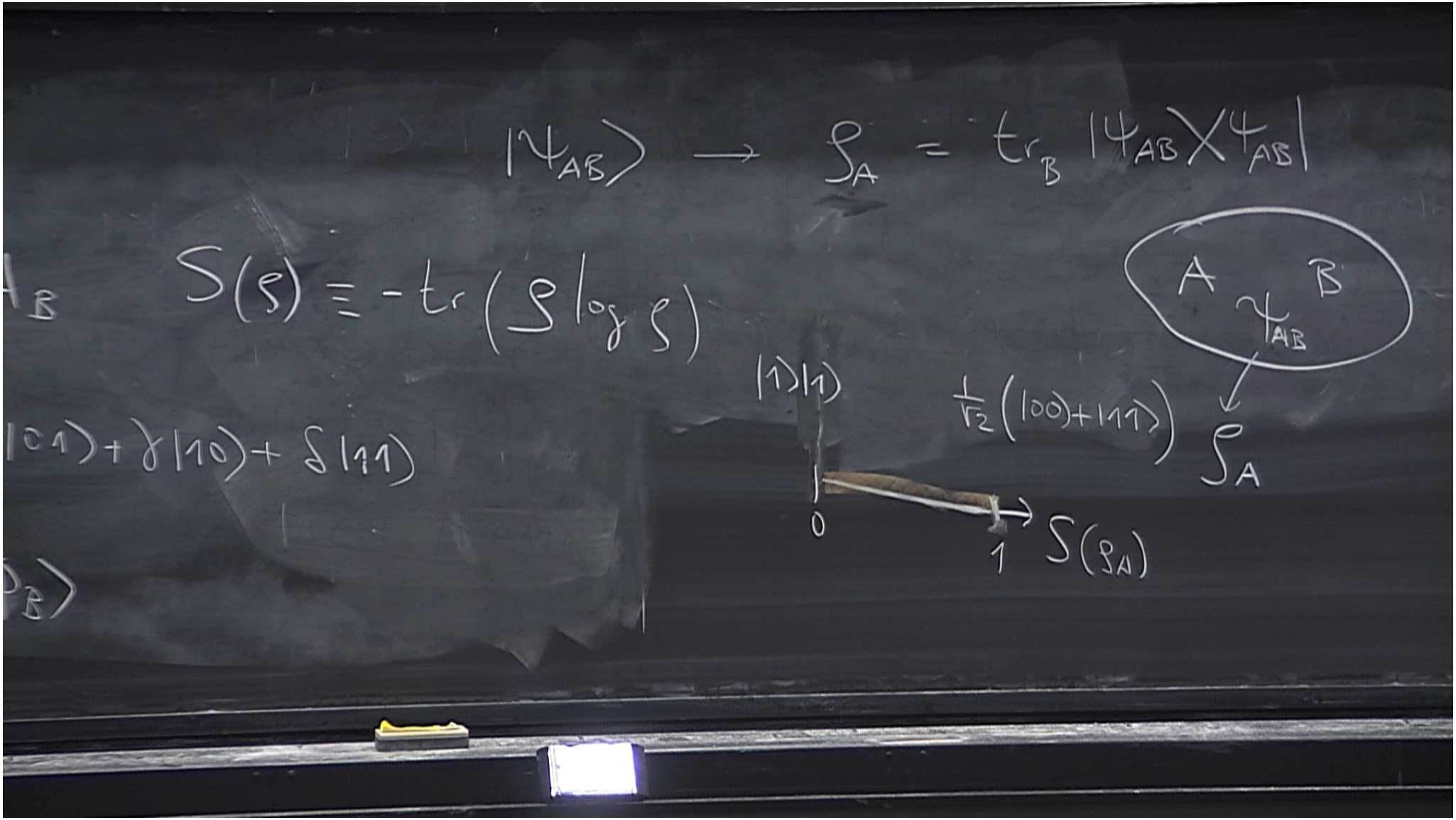

$|0\rangle\otimes|0\rangle$
$|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle$
$|00\rangle+|01\rangle+|10\rangle+|11\rangle = (|0\rangle+|1\rangle)(|0\rangle+|1\rangle)$
$\mathcal{H}_B$
$|c1\rangle+\gamma|10\rangle+\delta|11\rangle$
$\rho_B\rangle$

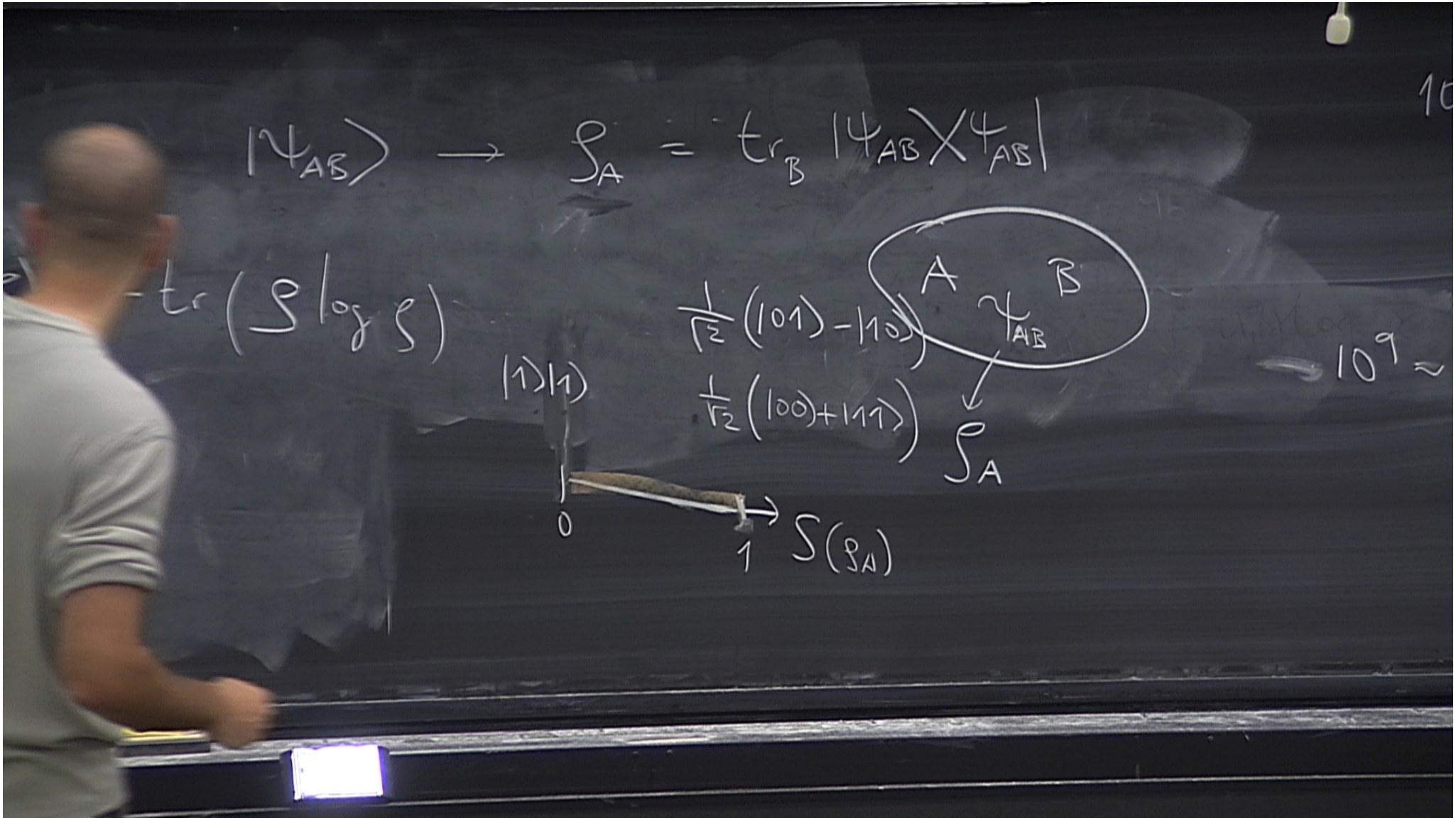

$|\psi_{AB}\rangle \rightarrow \rho_A = tr_B |\psi_{AB}\rangle\langle\psi_{AB}|$

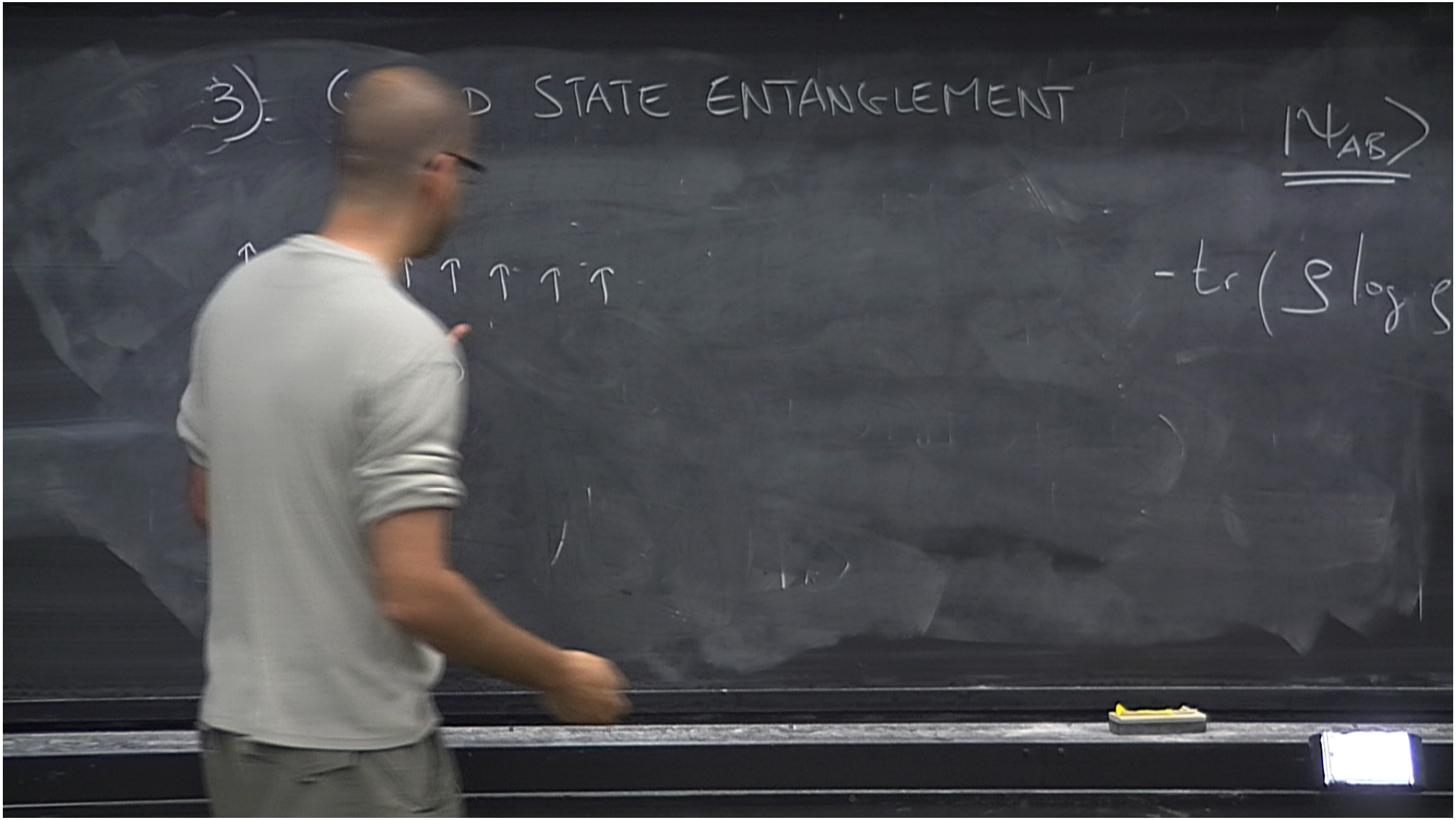

$|\psi_{AB}\rangle \rightarrow \rho_A = tr_B |\psi_{AB}\rangle\langle\psi_{AB}|$
$p|0\rangle\langle 0|$

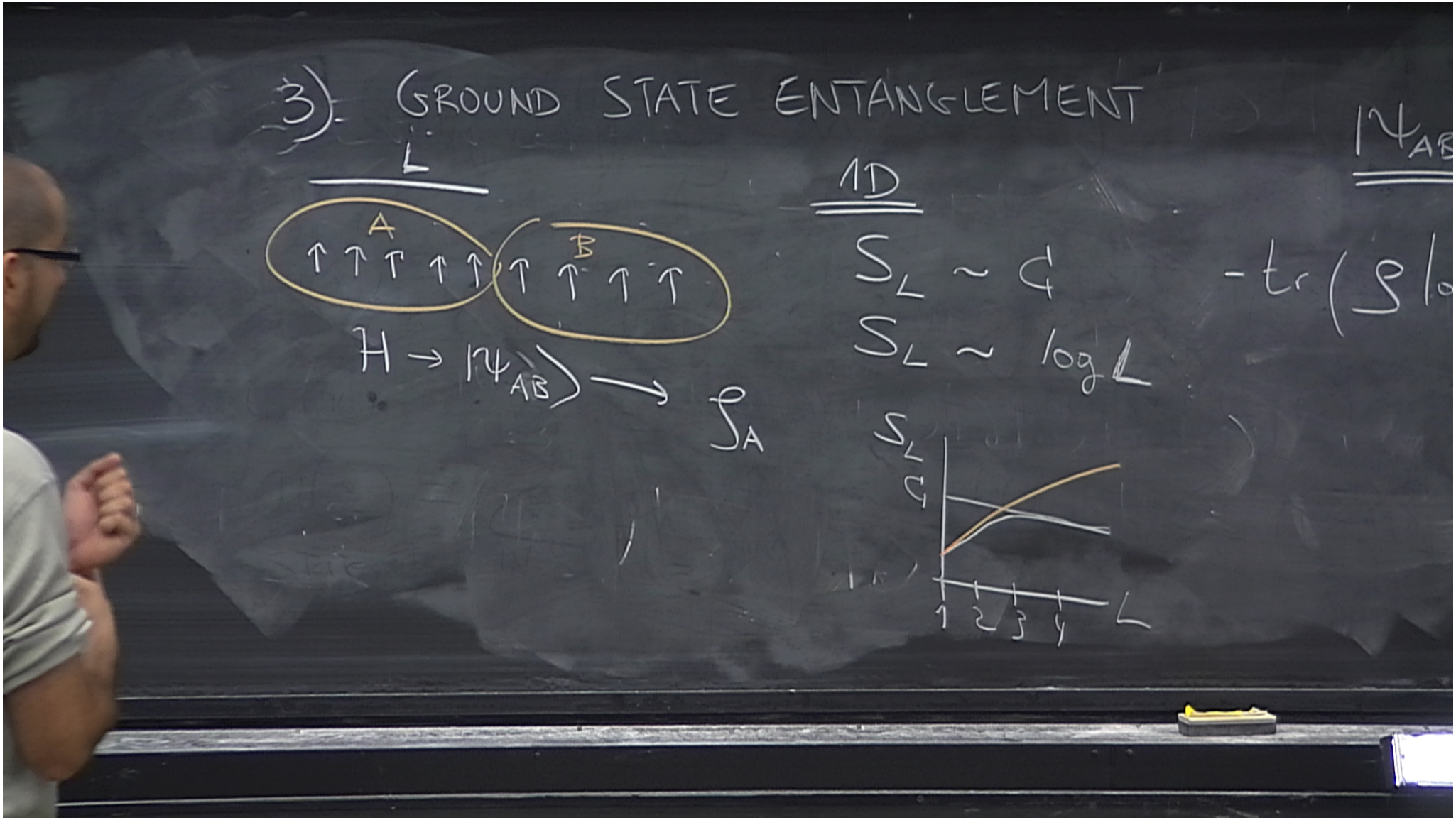

$|\psi_{AB}\rangle \rightarrow \rho_A = tr_B |\psi_{AB}\rangle\langle\psi_{AB}|$
$S(\rho) \equiv -tr\ \rho \log \rho$
A
B
ψ_{AB}
ρ_A

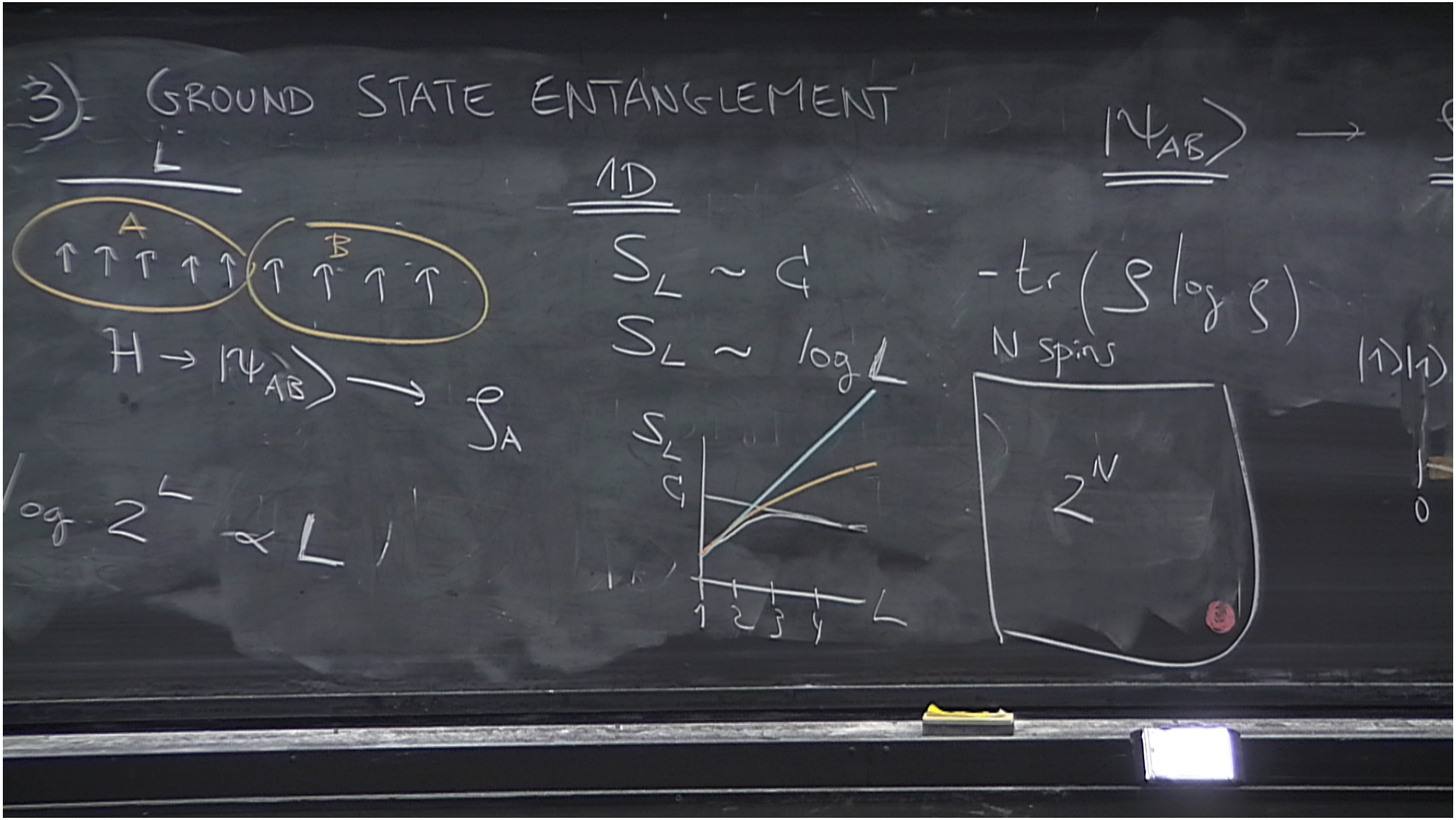

$\psi_{AB}\rangle \rightarrow \rho_A = tr_B |\psi_{AB}\rangle\langle\psi_{AB}|$
$S(\rho)$
$(\rho \log \rho)$
A
B
ψ_{AB}
$|1\rangle|1\rangle$
$\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$
ρ_A
0
1
$S(\rho_A)$

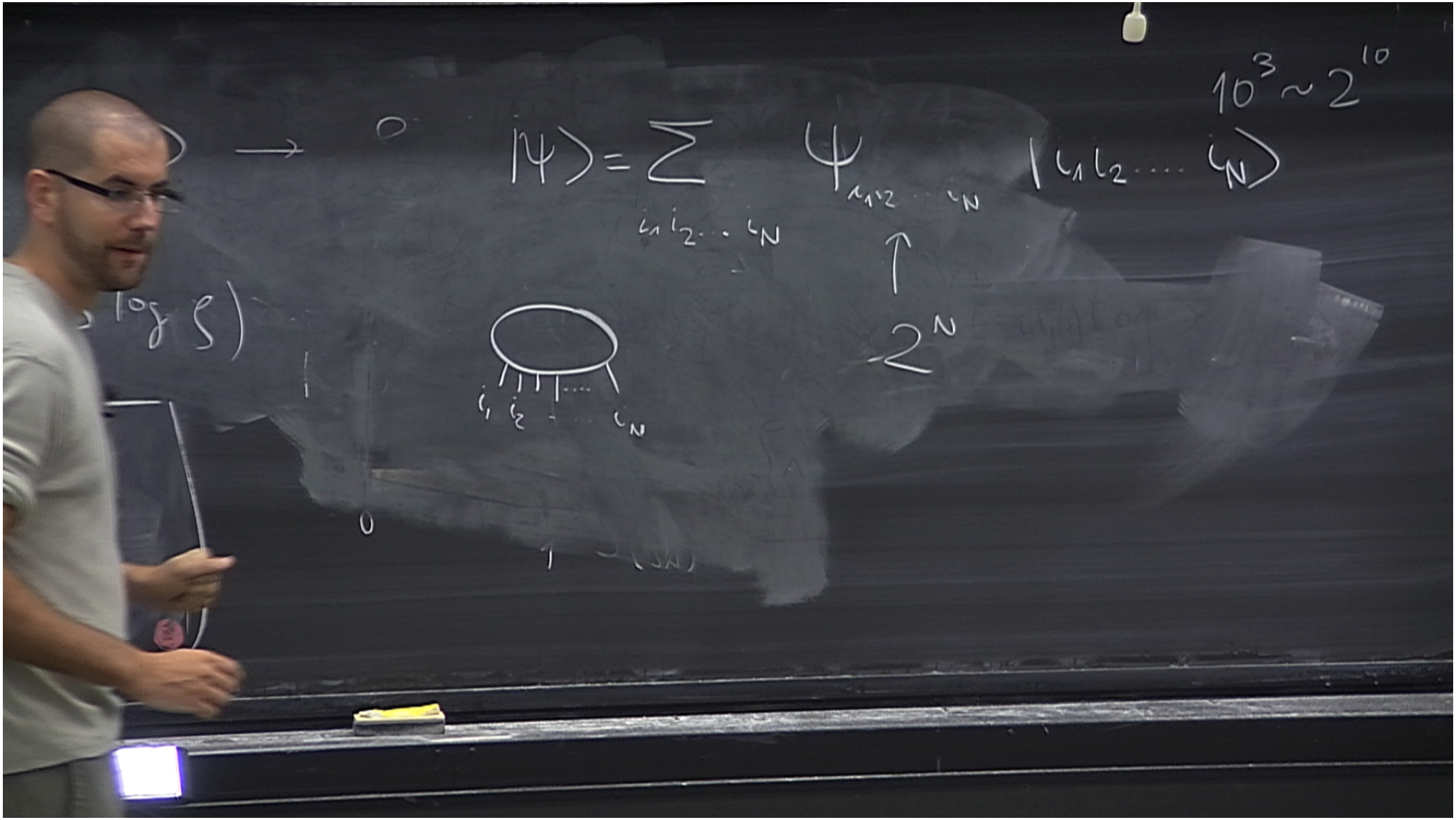

$|\psi_{AB}\rangle \rightarrow \rho_A = tr_B |\psi_{AB}\rangle\langle\psi_{AB}|$
$S(\rho) \equiv -tr(\rho \log \rho)$
$|11\rangle|11\rangle$
$\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$
A
B
ψ_{AB}
ρ_A
0
1
$S(\rho_A)$

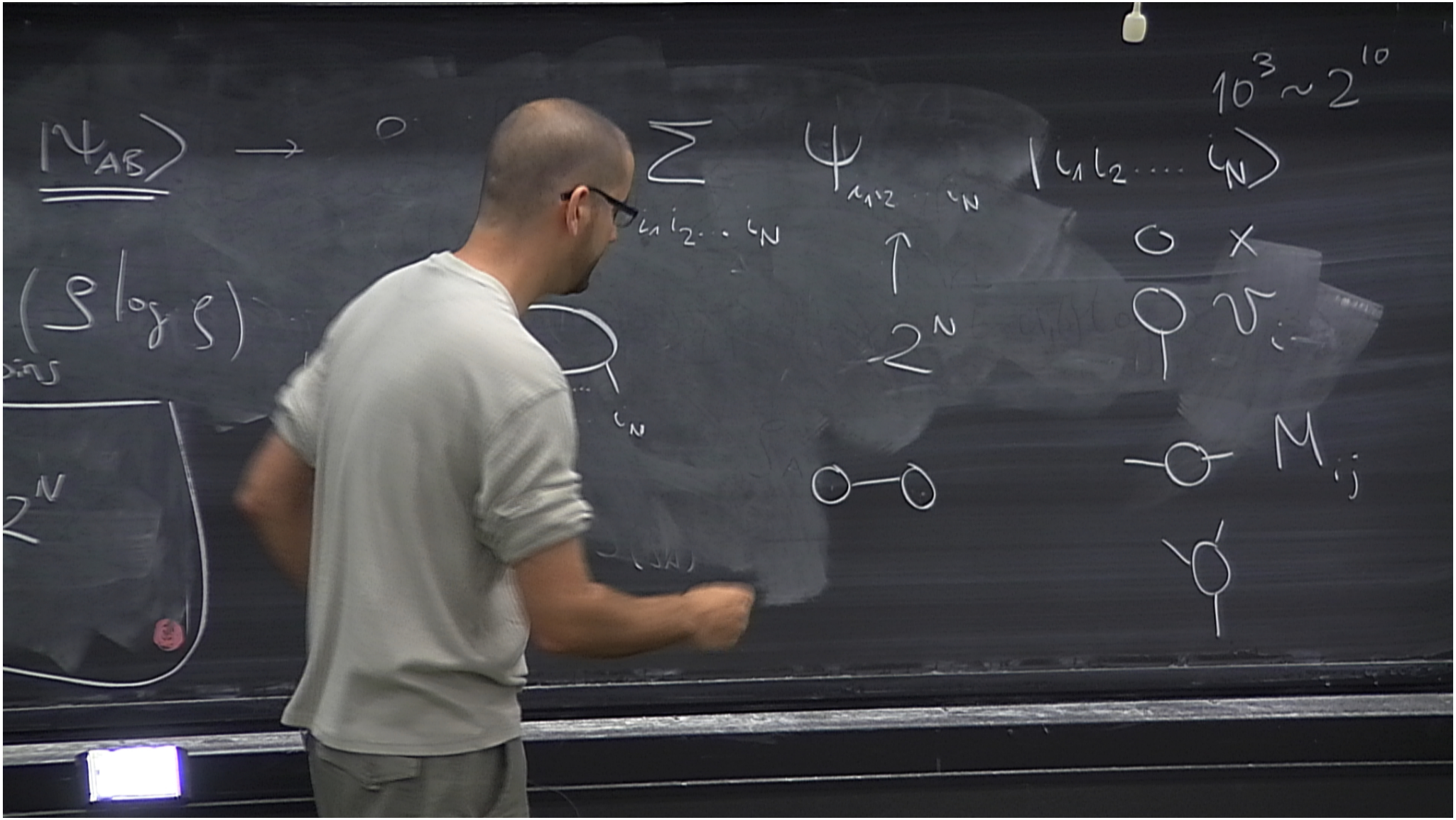

$|\psi_{AB}\rangle \rightarrow \rho_A = tr_B |\psi_{AB}\rangle\langle\psi_{AB}|$
$-tr(\rho \log \rho)$
$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
$|1\rangle|1\rangle$
A
B
ψ_{AB}
ρ_A
0
1
$S(\rho_A)$
$10^9 \approx$

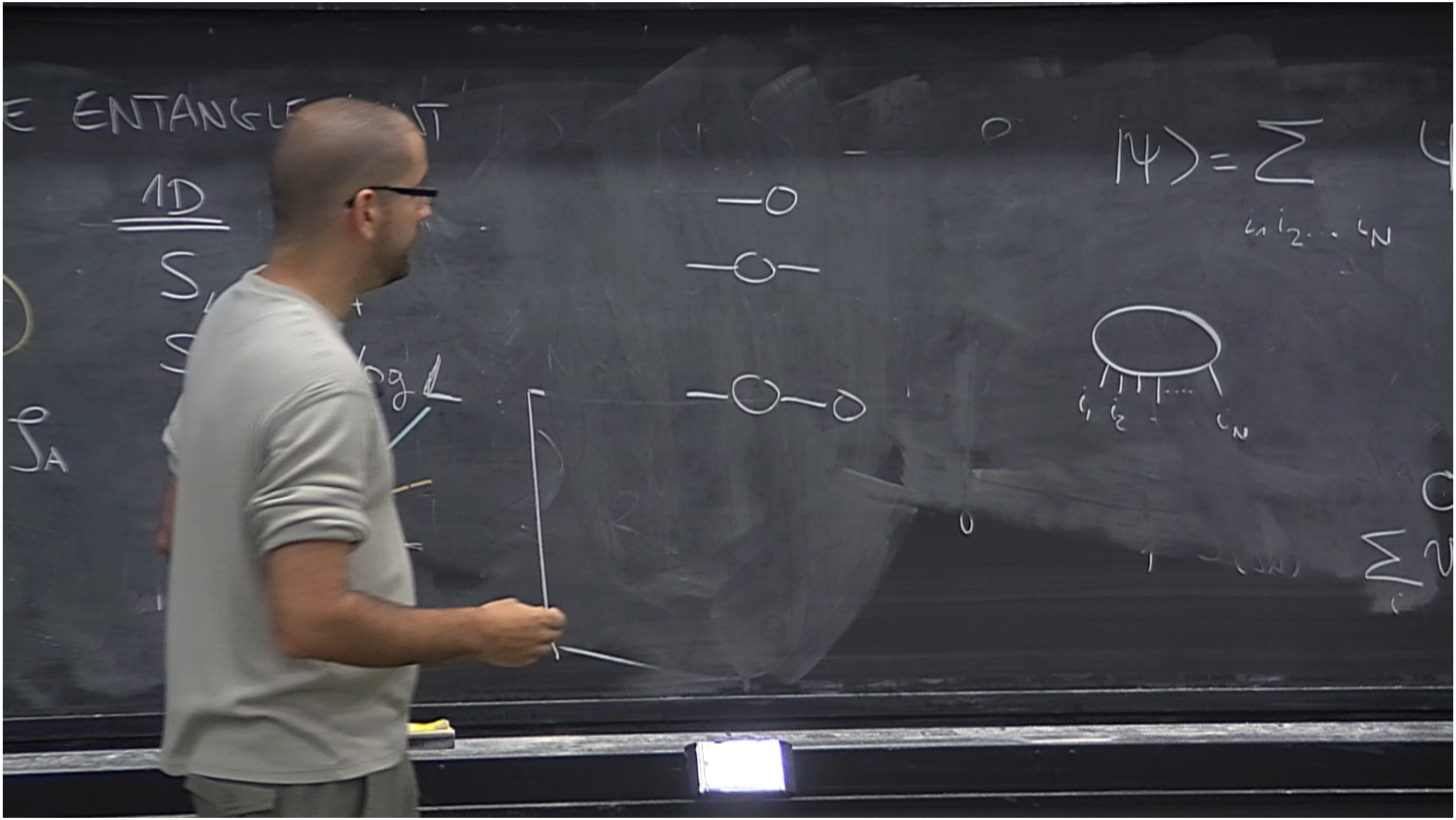

3).
STATE ENTANGLEMENT
$|\psi_{AB}\rangle$
$-tr(\rho \log \rho$

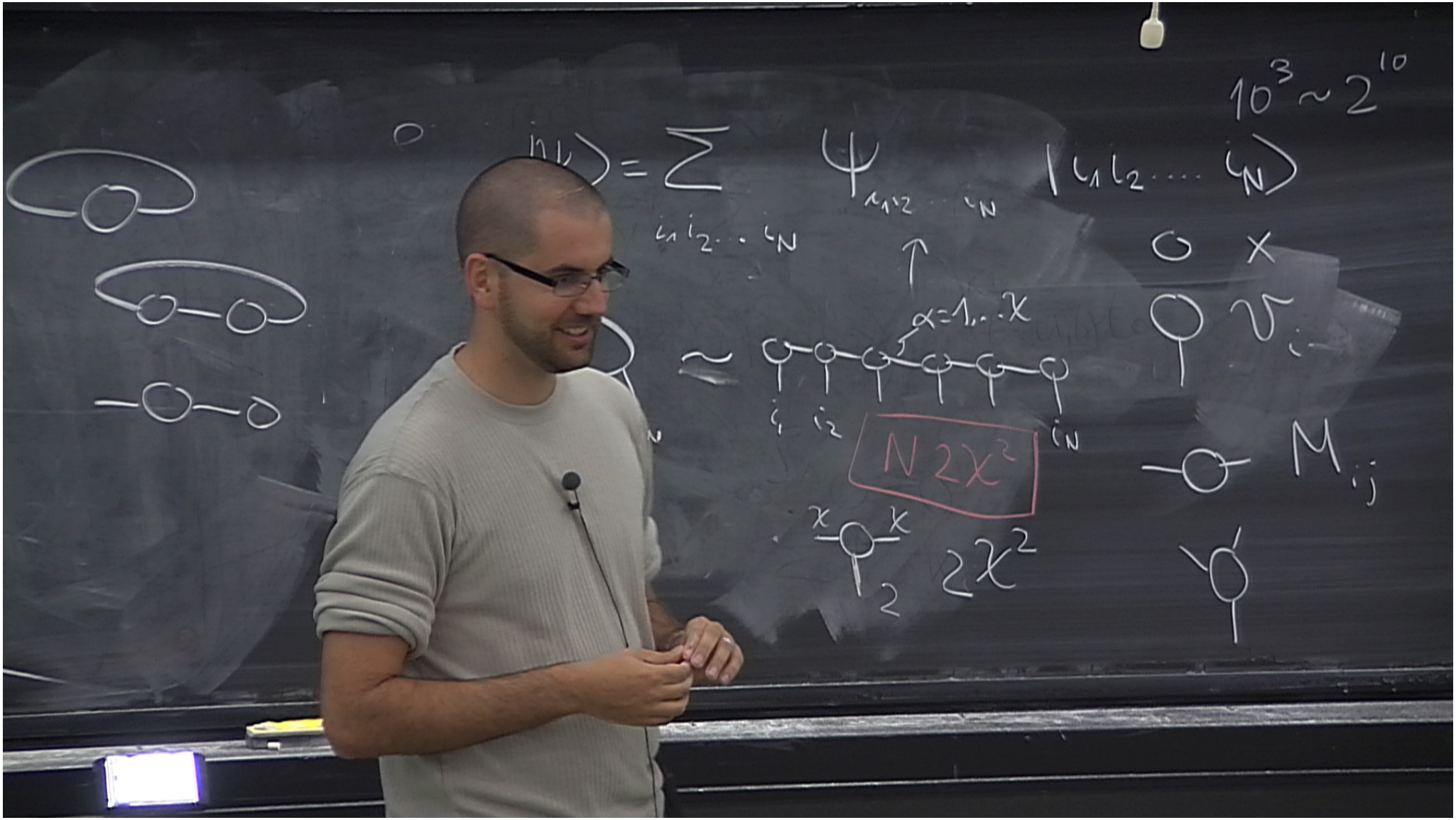

3). GROUND STATE ENTANGLEMENT
L
A
B
$H \to |\Psi_{AB}\rangle \to \rho_A$
1D
$S_L \sim C$
$S_L \sim \log L$
$-tr(\rho$
$|\Psi_{AB}$
S_L
C
1 2 3 4 L


3) GROUND STATE ENTANGLEMENT
L
A
B
H → |ψ_AB⟩ → ρ_A
log 2^L ∝ L
1D
S_L ~ cte
S_L ~ log L
S_L
cte
1 2 3 4 L
|ψ_AB⟩ →
-tr(ρ log ρ)
N spins
2^N
|1⟩|1⟩
1
0


$10^3 \sim 2^{10}$
$|\psi\rangle = \sum_{i_1 i_2 \cdots i_N} \psi_{i_1 i_2 \cdots i_N} |i_1 i_2 \cdots i_N\rangle$
2^N


$|\Psi_{AB}\rangle \rightarrow$
Σ
Ψ
$|i_1 i_2 \cdots i_N\rangle$
$10^3 \sim 2^{10}$
$(\rho \log \rho)$
2^N
M_{ij}


ENTANGL
1D
$|\psi\rangle = \Sigma$
$i_1 i_2 \ldots i_N$
ρ_A
$\log$


$10^3 \sim 2^{10}$
$\alpha = 1 \ldots \chi$
$N 2\chi^2$
$2\chi^2$