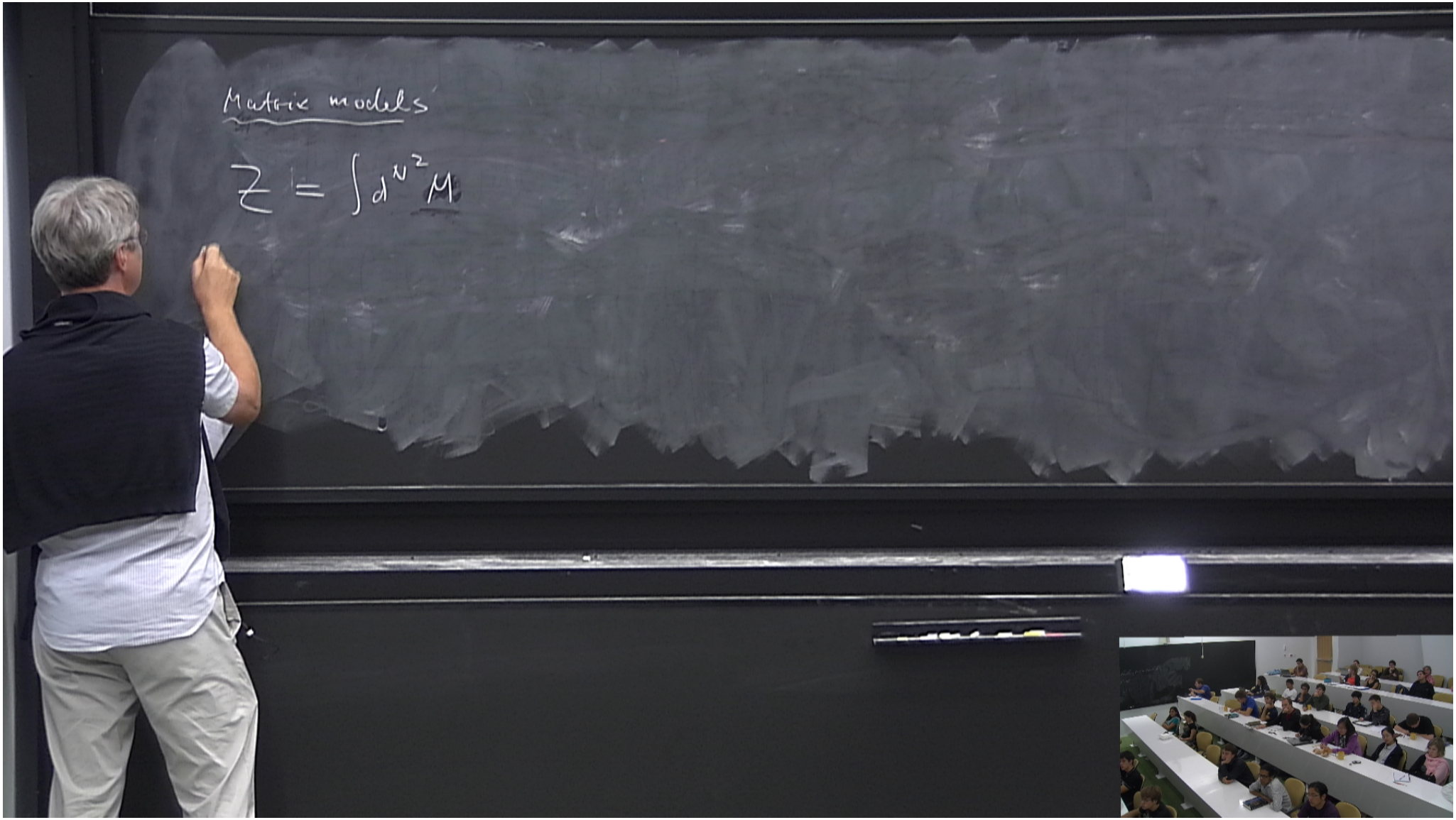
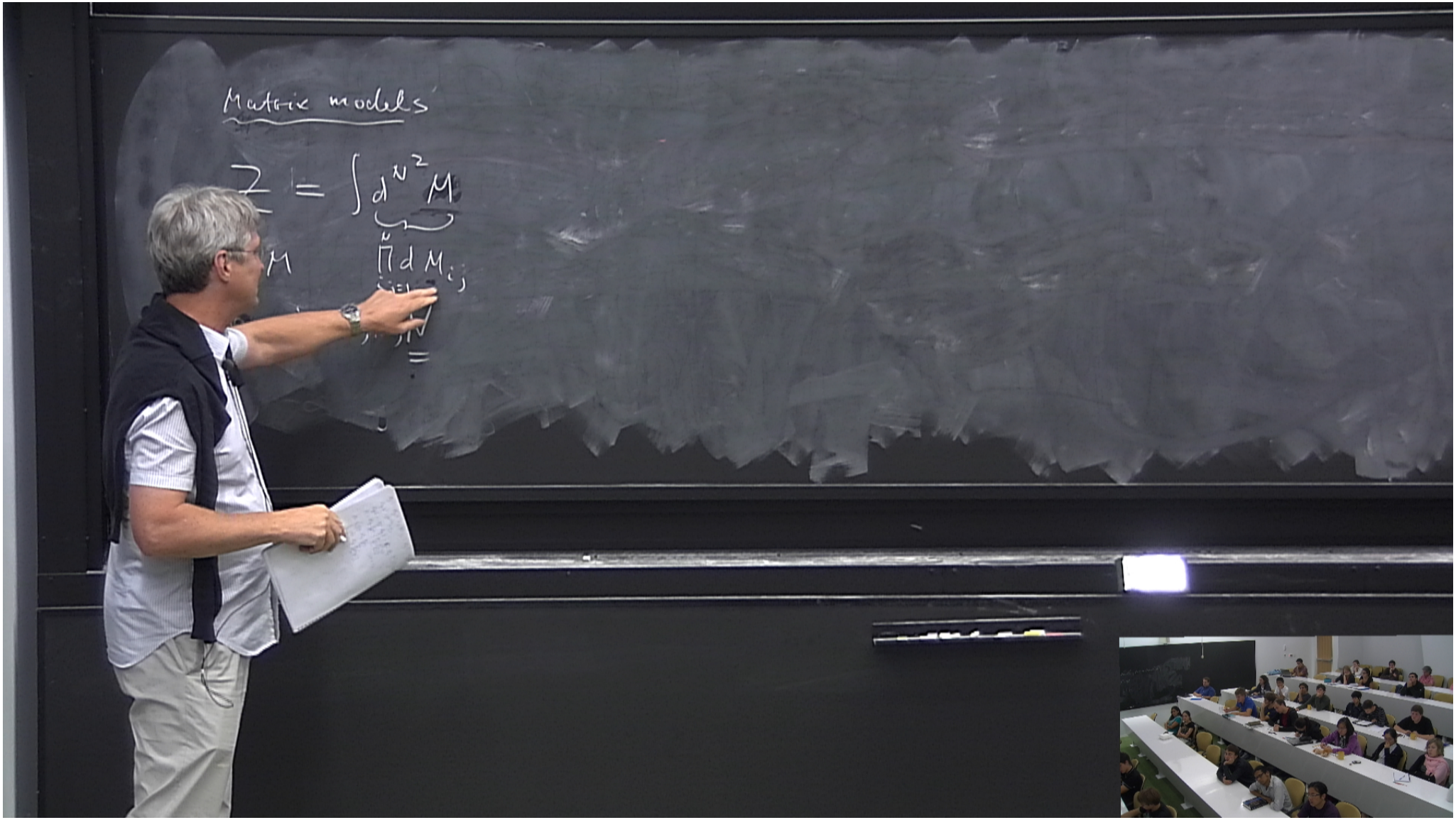
Title: Visiting Researcher Presentation: Vladimir Kazakov

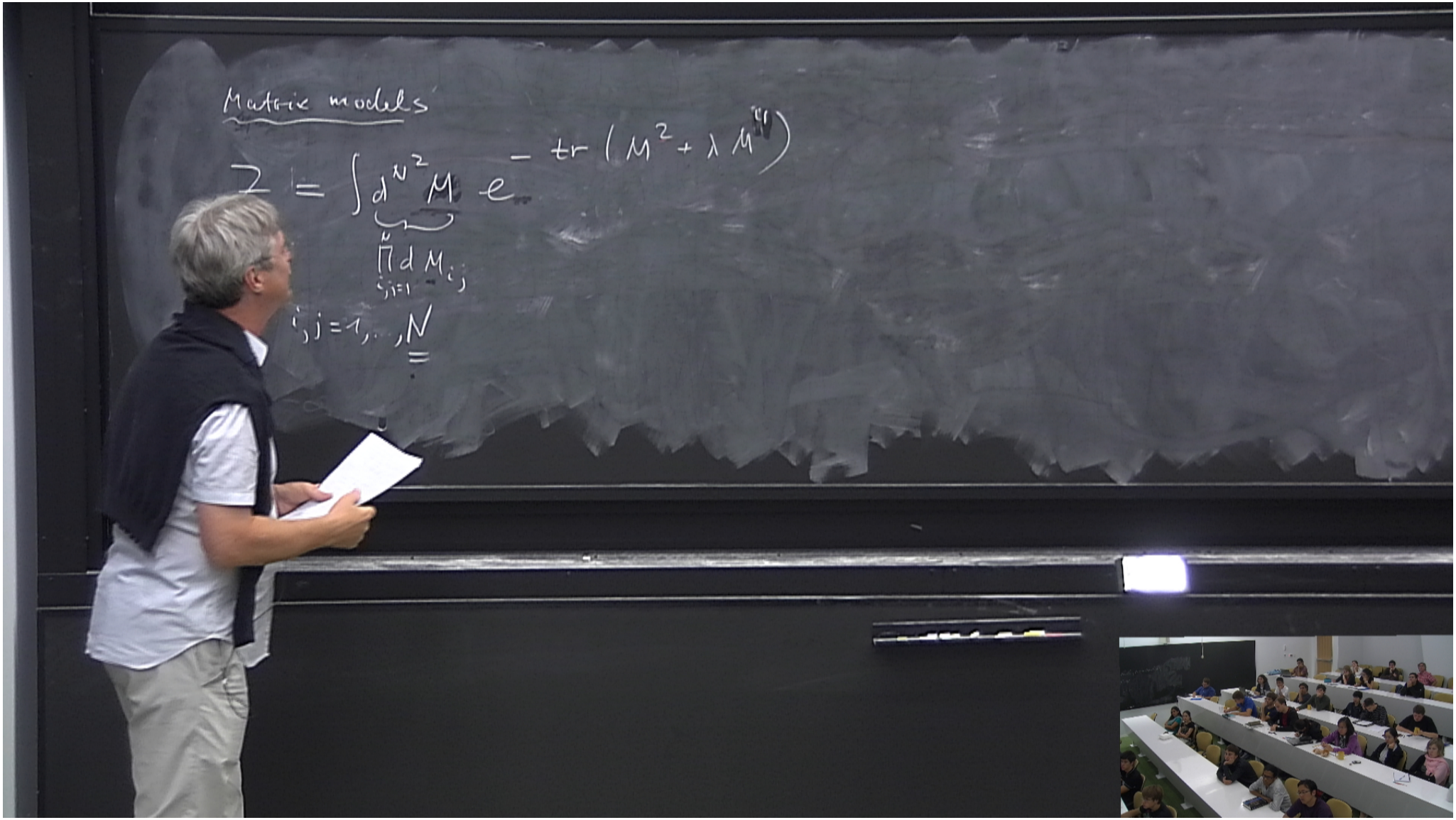
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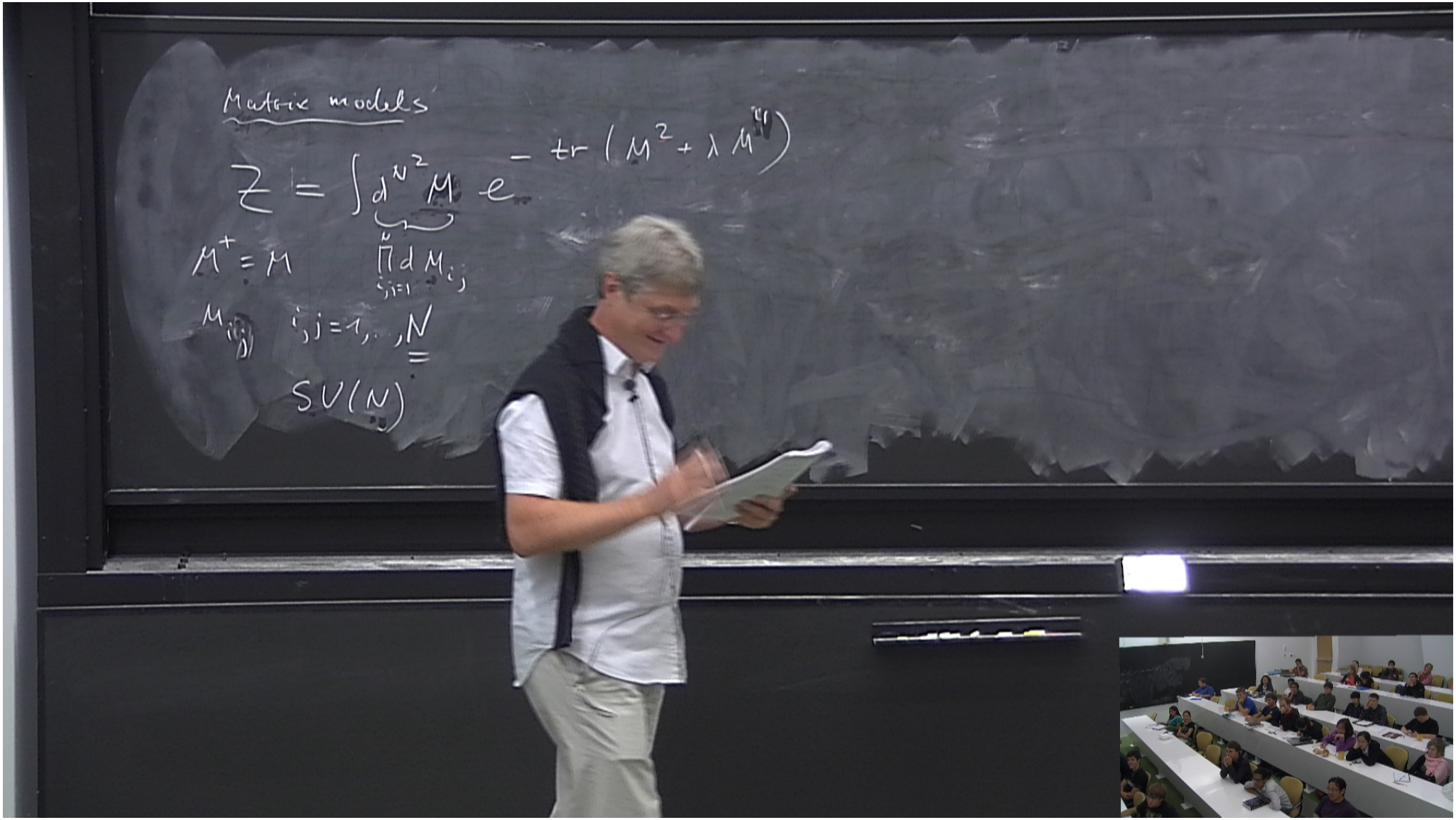
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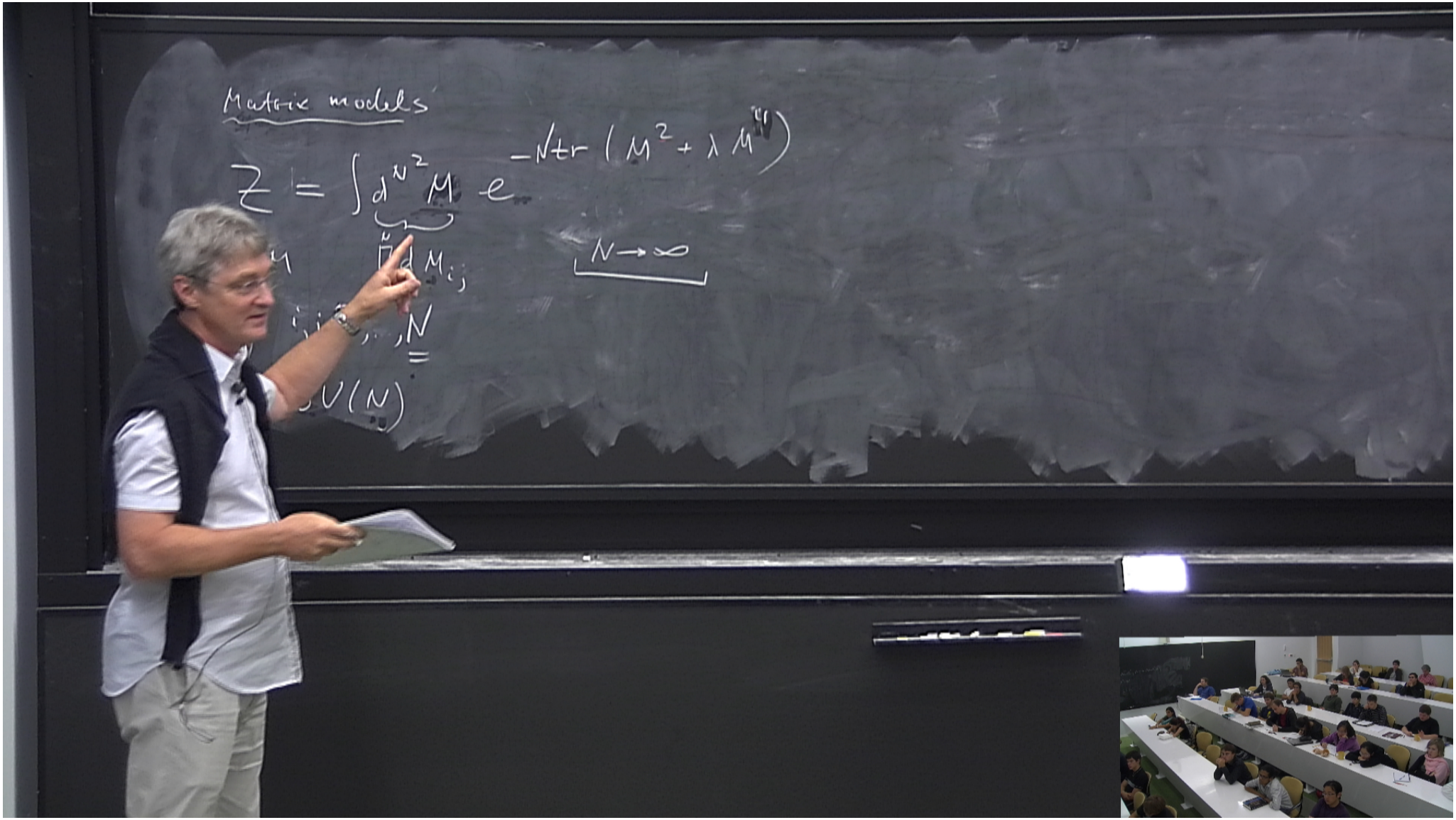
Abstract:

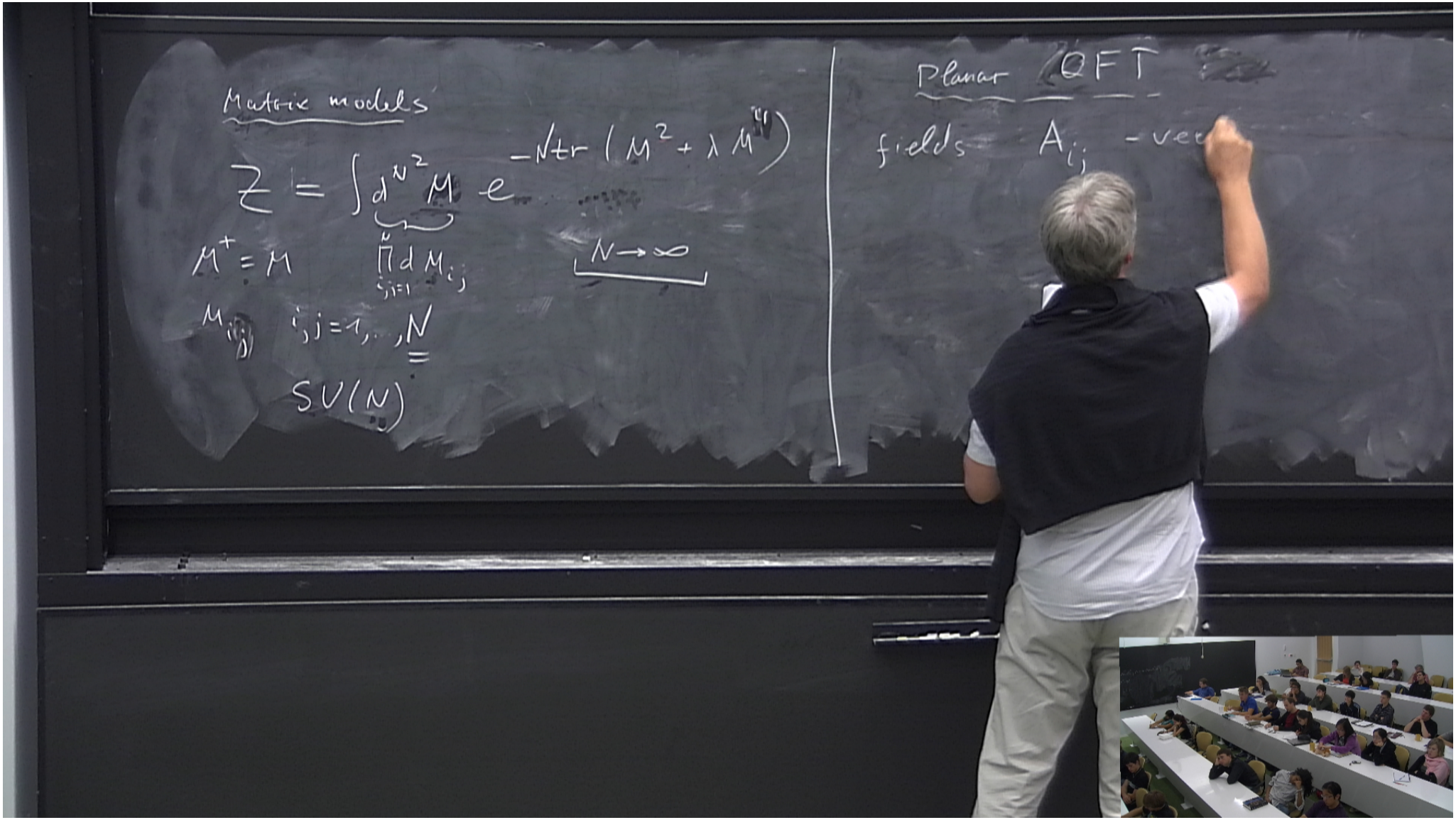

Matrix models
$Z = \int dM^{N^2} M$

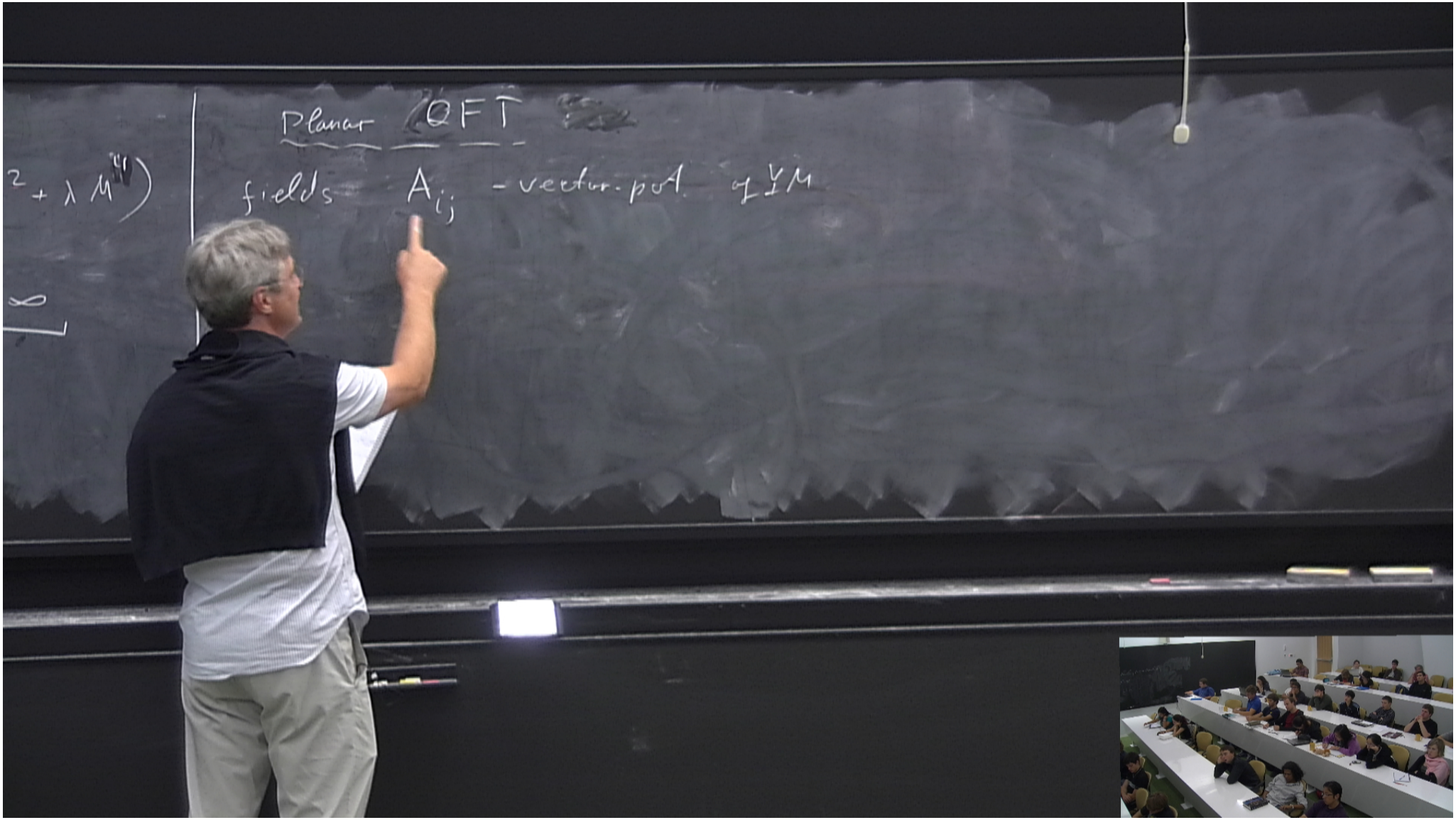

Matrix models
$Z = \int d^{N^2} M$
$\prod_{i,j=1}^{N} d M_{ij}$

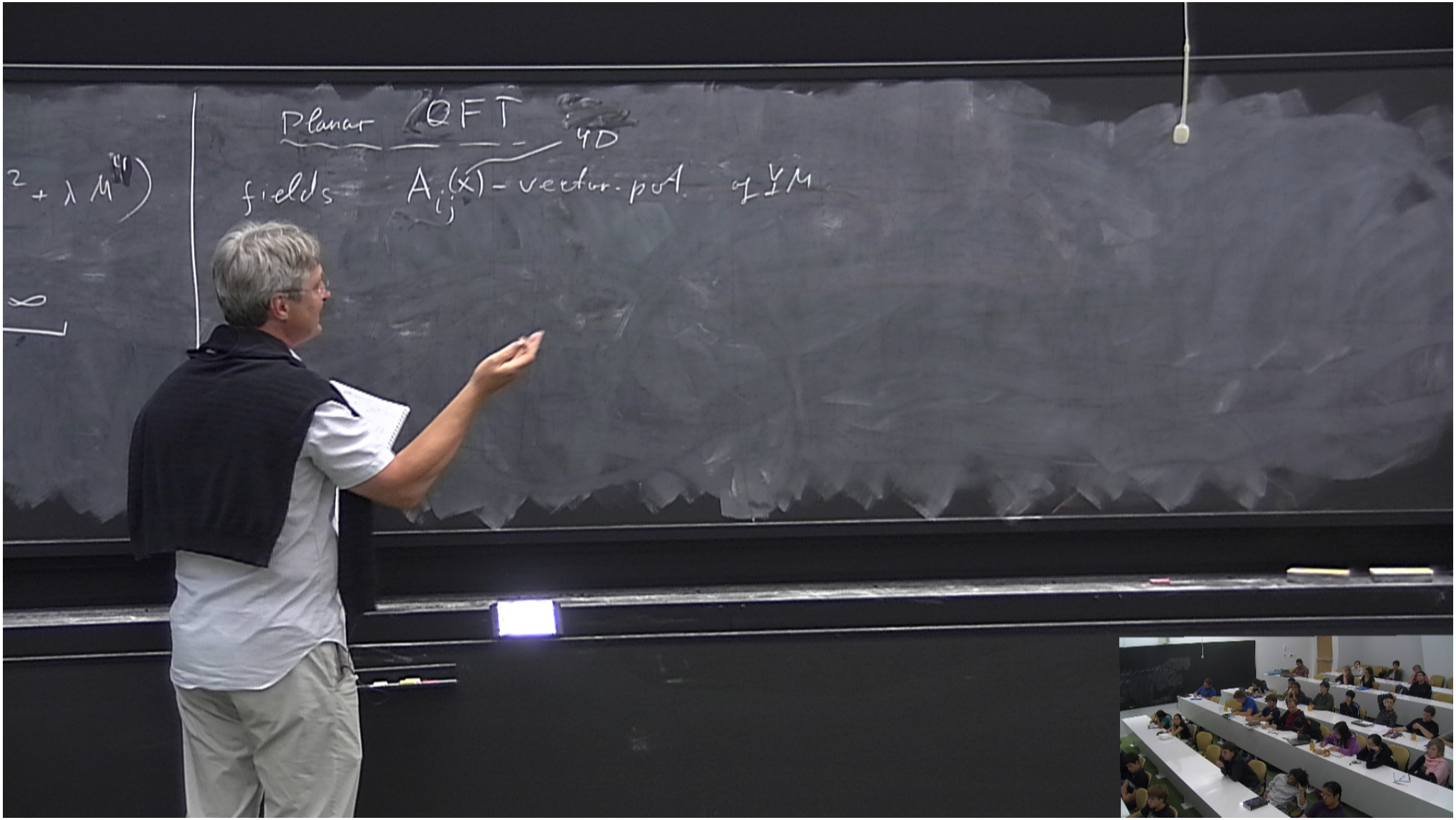

Matrix models
$Z = \int d^{N^2} M \; e^{-\mathrm{tr}\left(M^2 + \lambda M^4\right)}$
$\prod_{i,j=1} dM_{ij}$
$i,j = 1, \ldots, N$

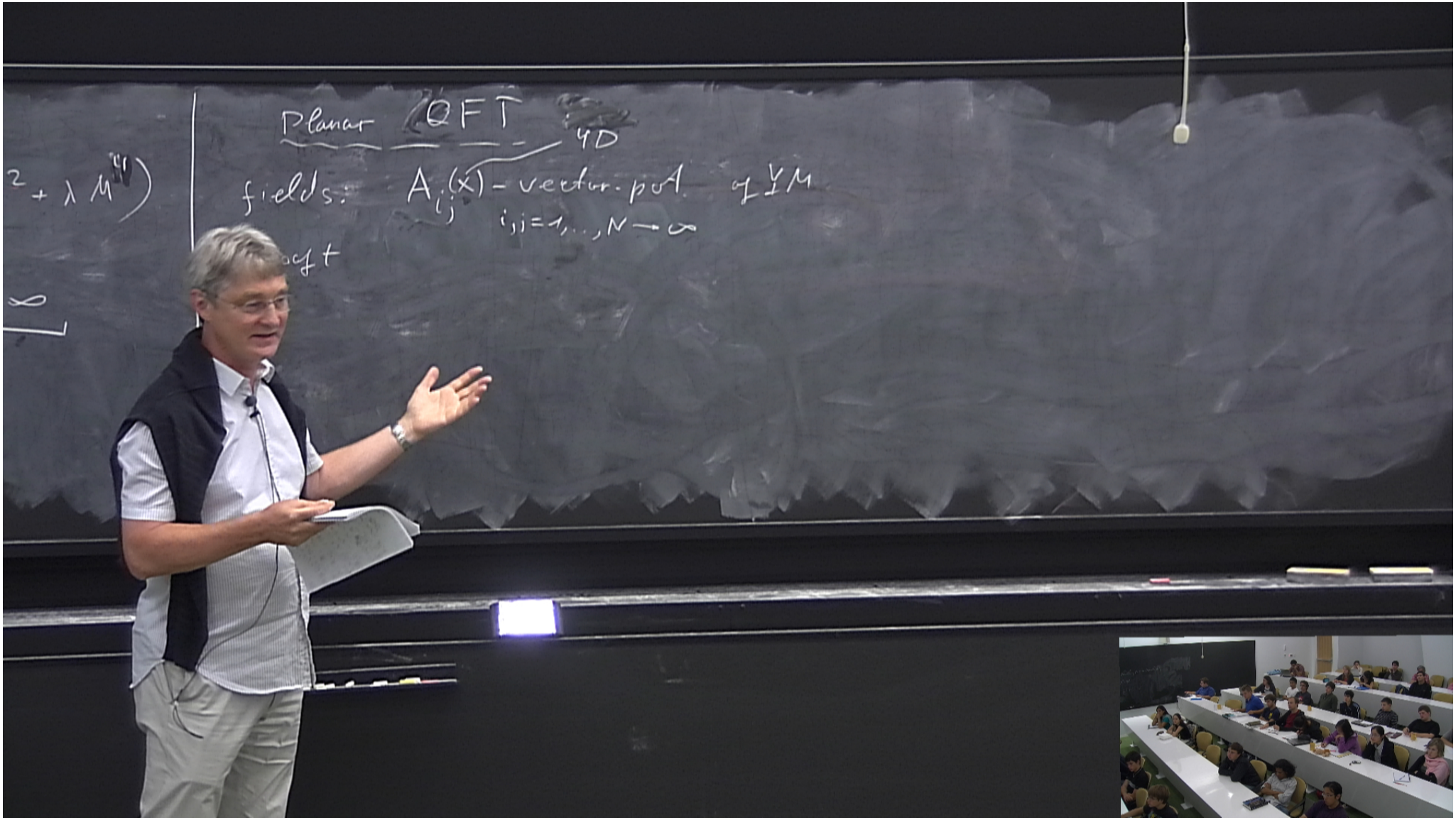

Matrix models
$Z = \int d^{N^2} M \; e^{-\mathrm{tr}\left(M^2 + \lambda M^4\right)}$
$M^+ = M$
$\prod_{i,j=1} dM_{ij}$
$M_{ij} \quad i,j = 1, \ldots, N$
$SU(N)$

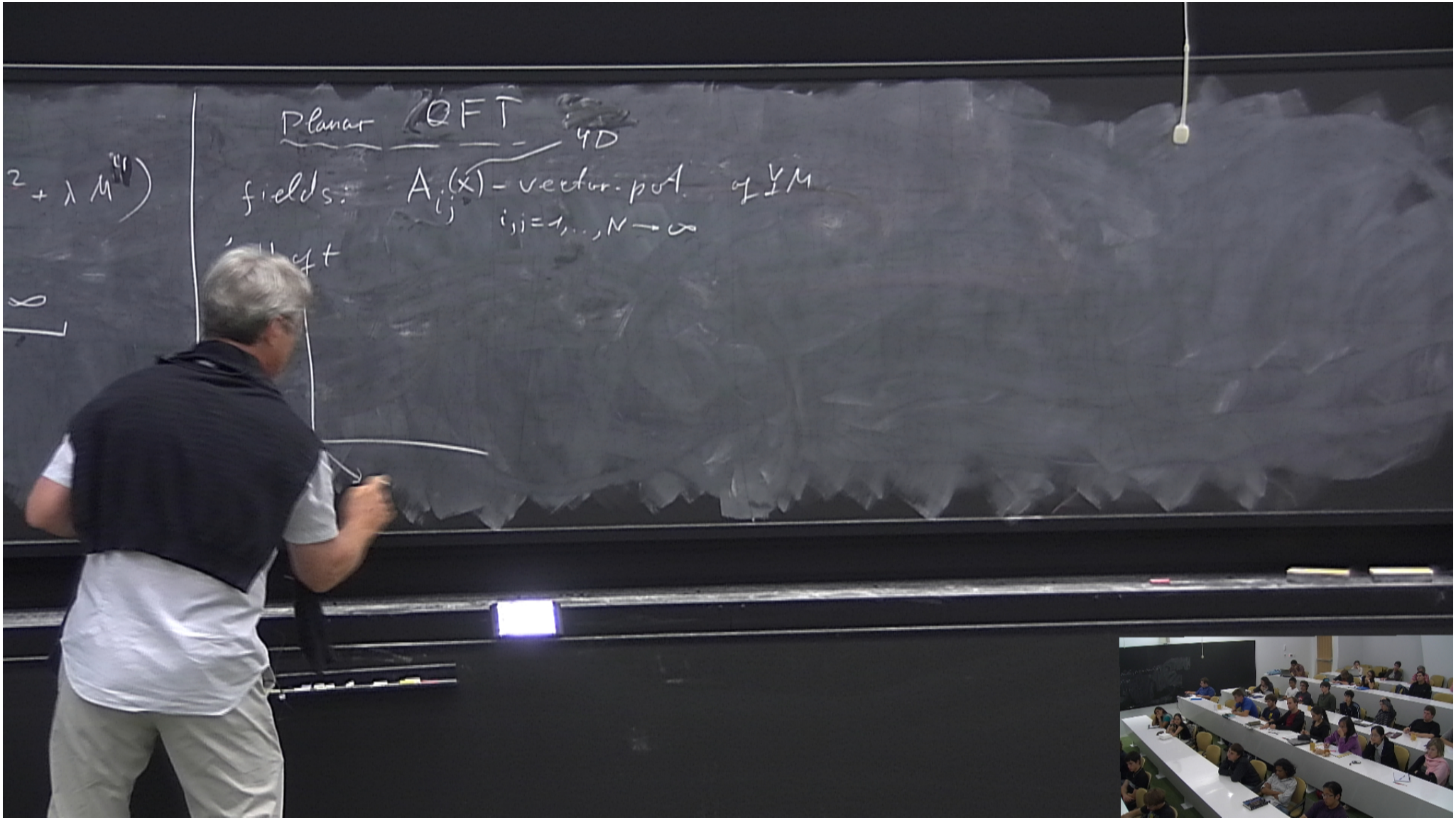

Matrix models
$Z = \int d^{N^2} M \; e^{-N \mathrm{tr}\left(M^2 + \lambda M^4\right)}$
$\prod_{i,j} dM_{ij}$
$N \to \infty$
$U(N)$

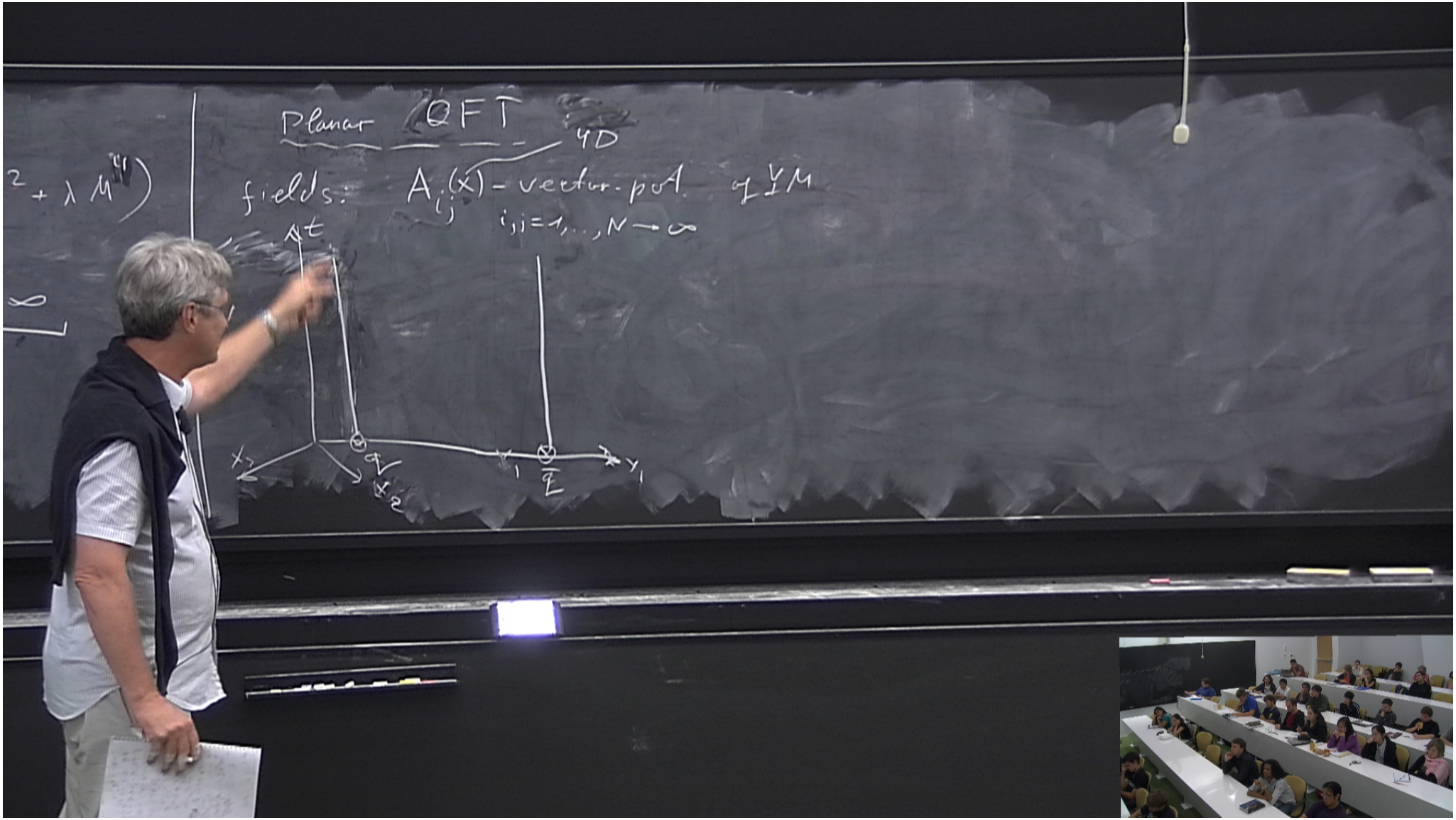

Matrix models
$Z = \int d^{N^2} M \; e^{-N \mathrm{tr}\,(M^2 + \lambda M^4)}$
$M^+ = M$
$\prod_{i,j=1} dM_{ij}$
$N \to \infty$
M_{ij} $i,j = 1, \ldots, N$
$SU(N)$
Planar QFT
fields A_{ij} - ver

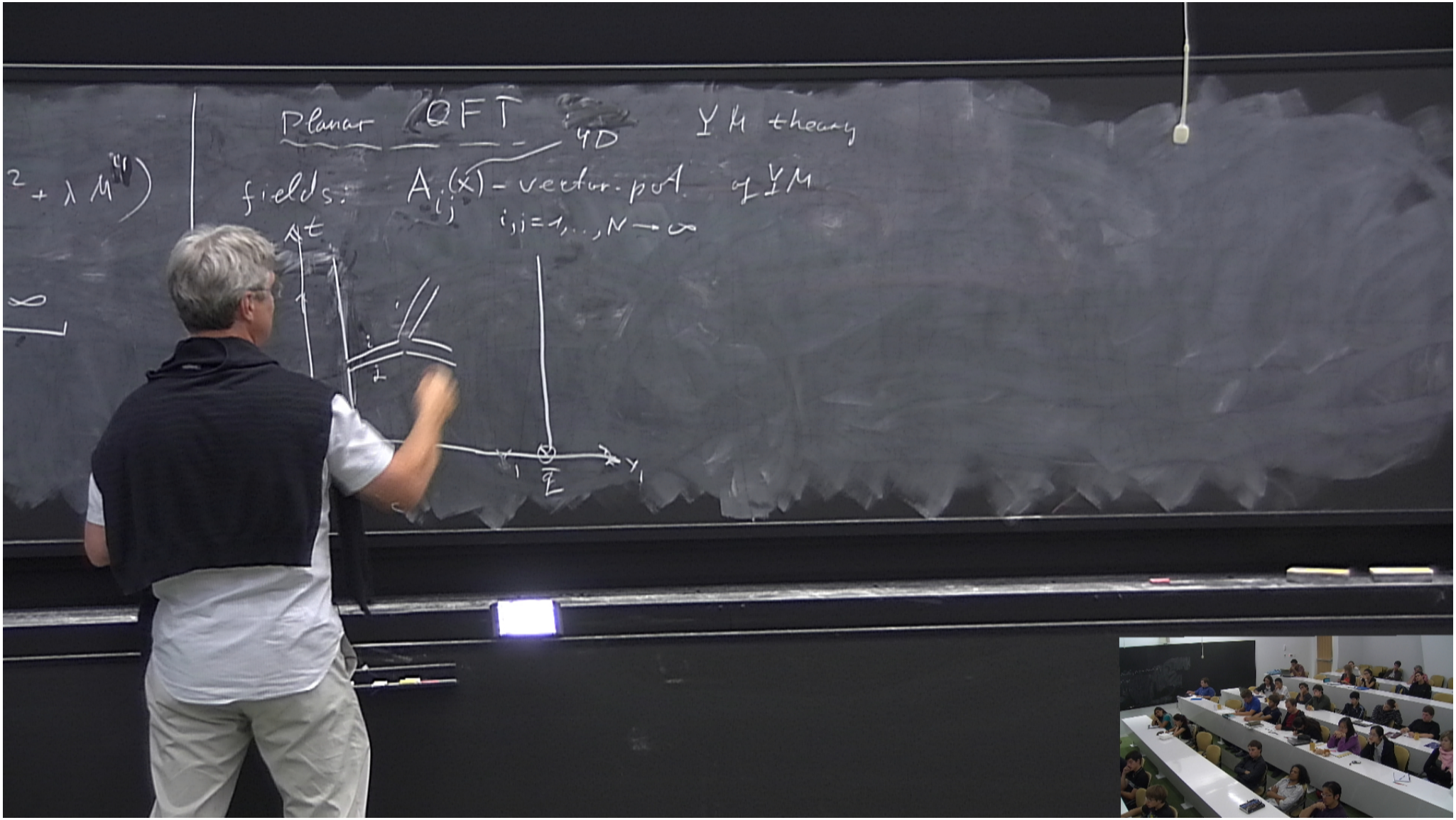

Planar QFT
fields A_{ij} - vector pot. of YM

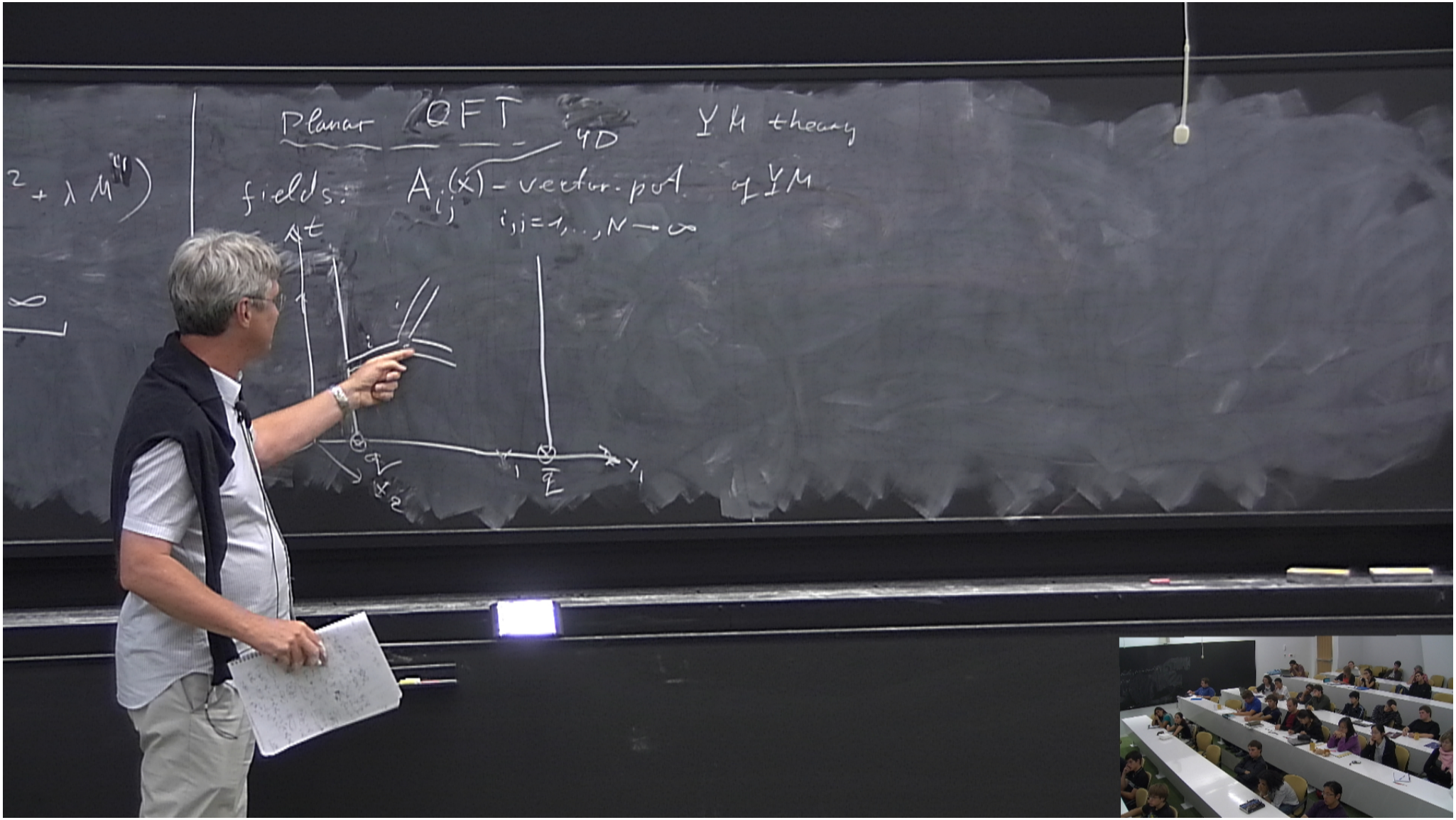

Planar QFT
4D
fields $A_{ij}(x)$ - vector pot.
$+\lambda M^4)$
∞

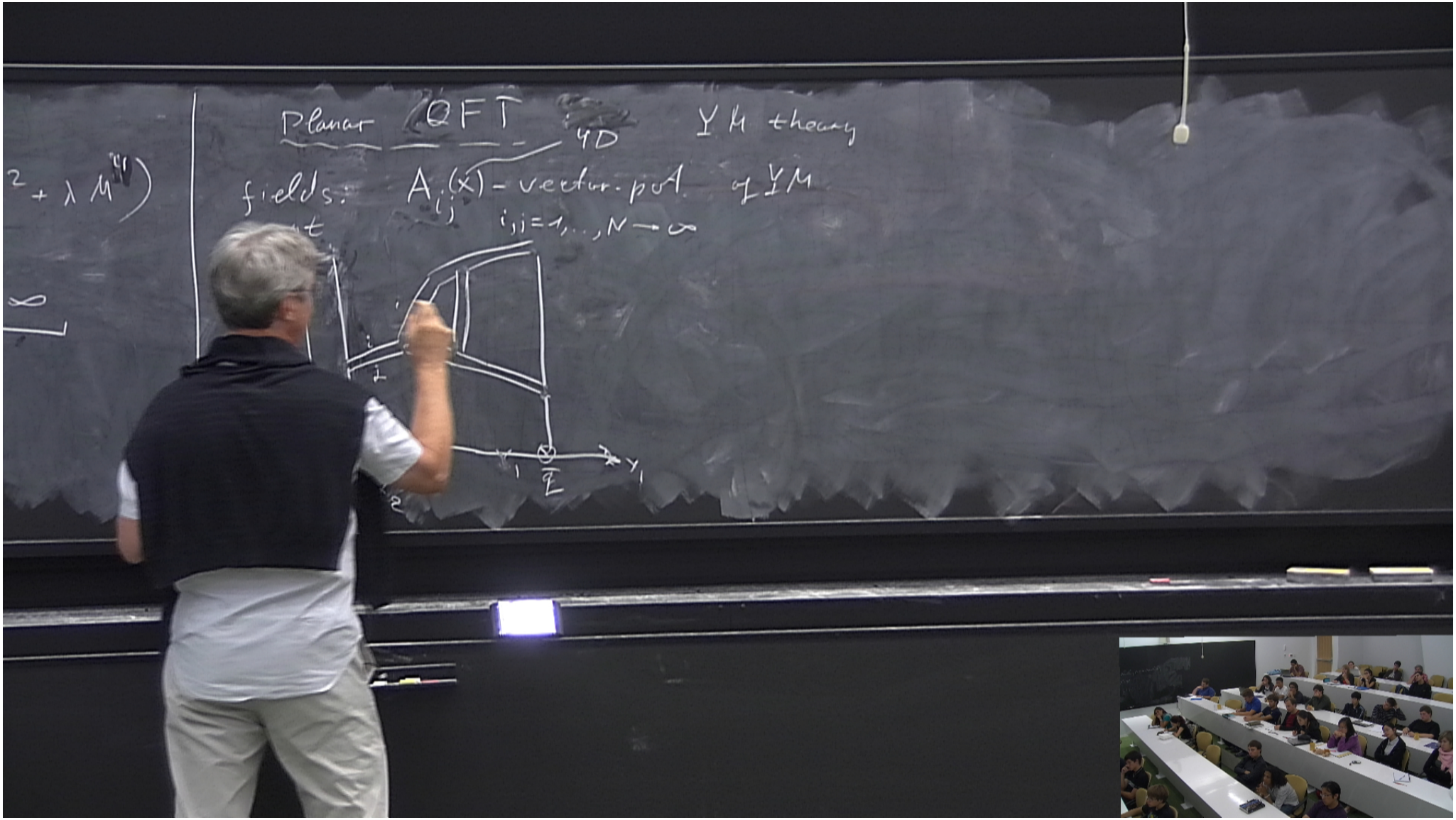

Planar QFT
4D
fields:
$A_{ij}(x)$ - vector pot.
$i,j=1,..,N\to\infty$

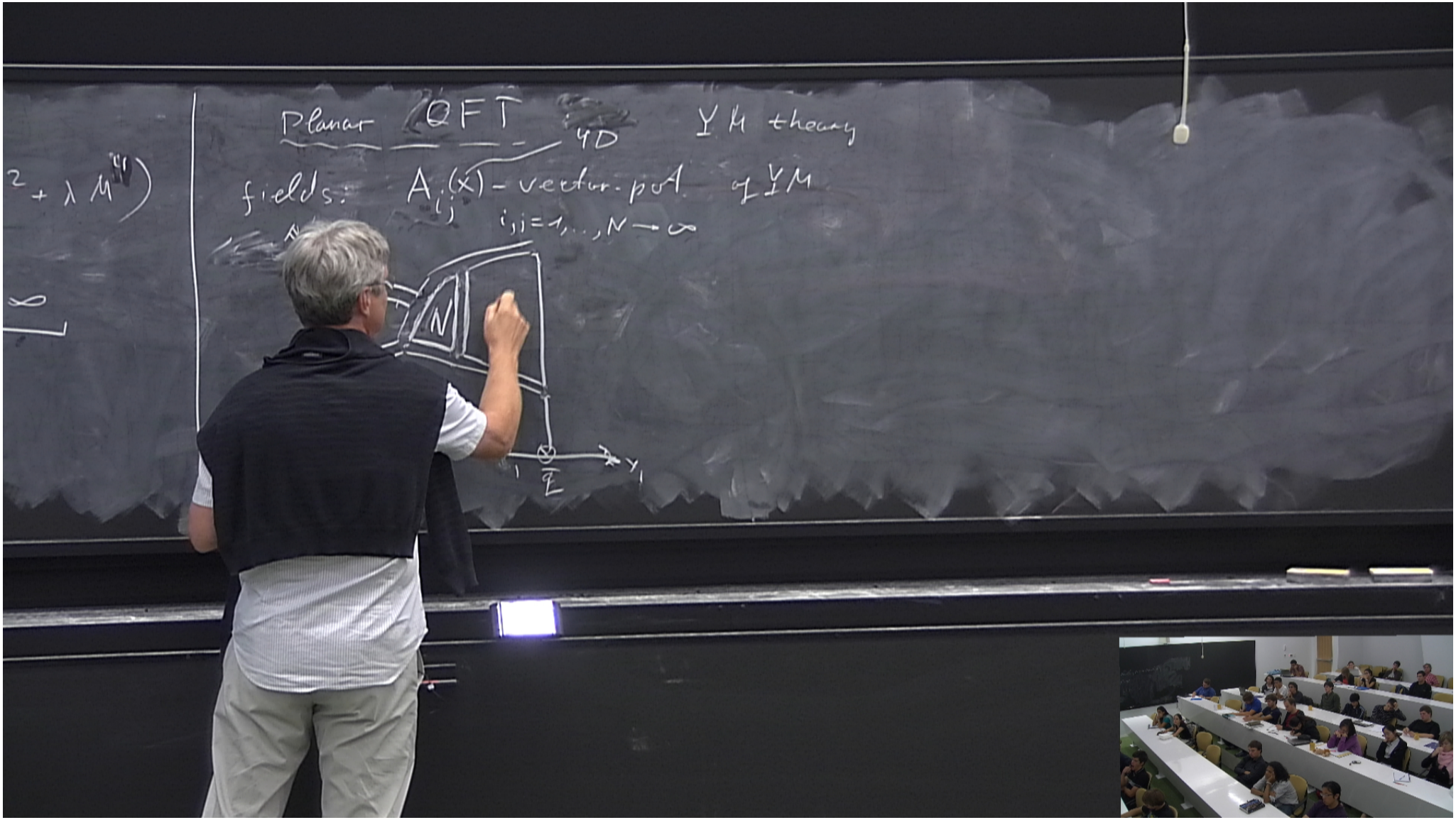

Planar QFT
4D
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j=1,..,N\to\infty$
$+\lambda M^{4}$)
∞

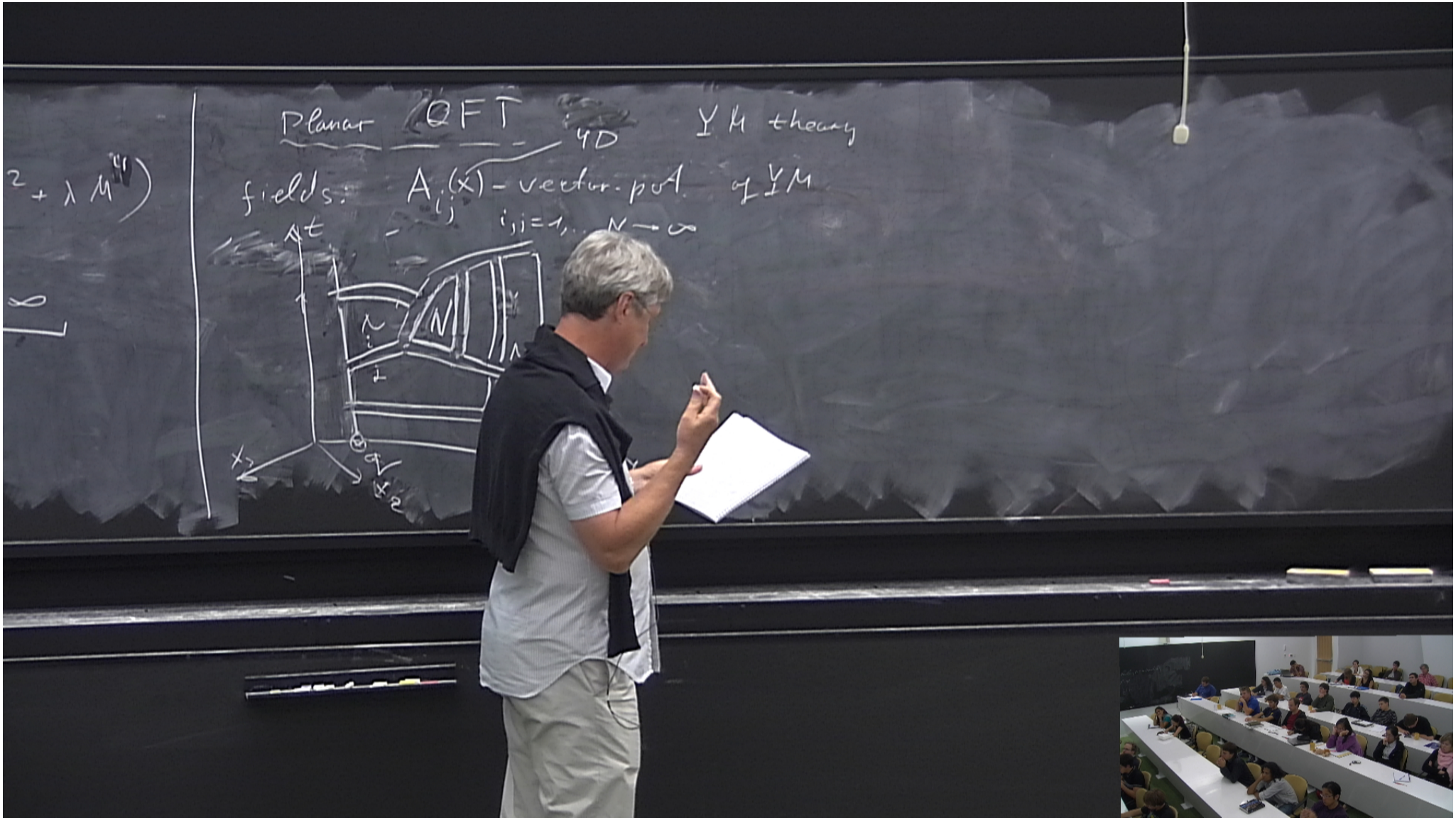

Planar QFT
4D
fields: $A_{ij}(x)$ - vector-pot of YM
$i,j=1,..,N\to\infty$
t
x_2
x_1
x_3
$\bar{q}$
q

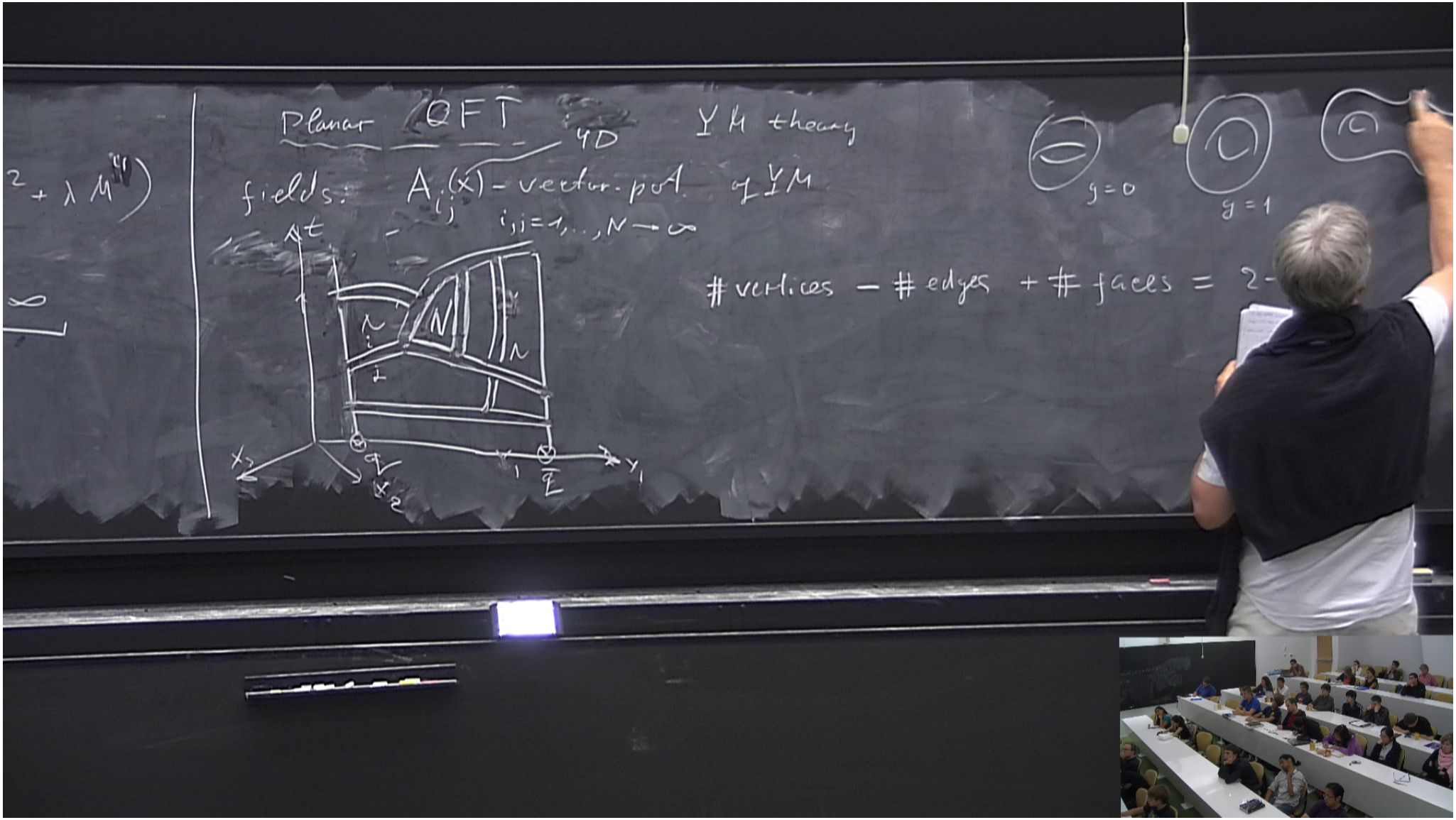

Planar QFT : 4D YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j = 1,\ldots,N \to \infty$
t
x_1

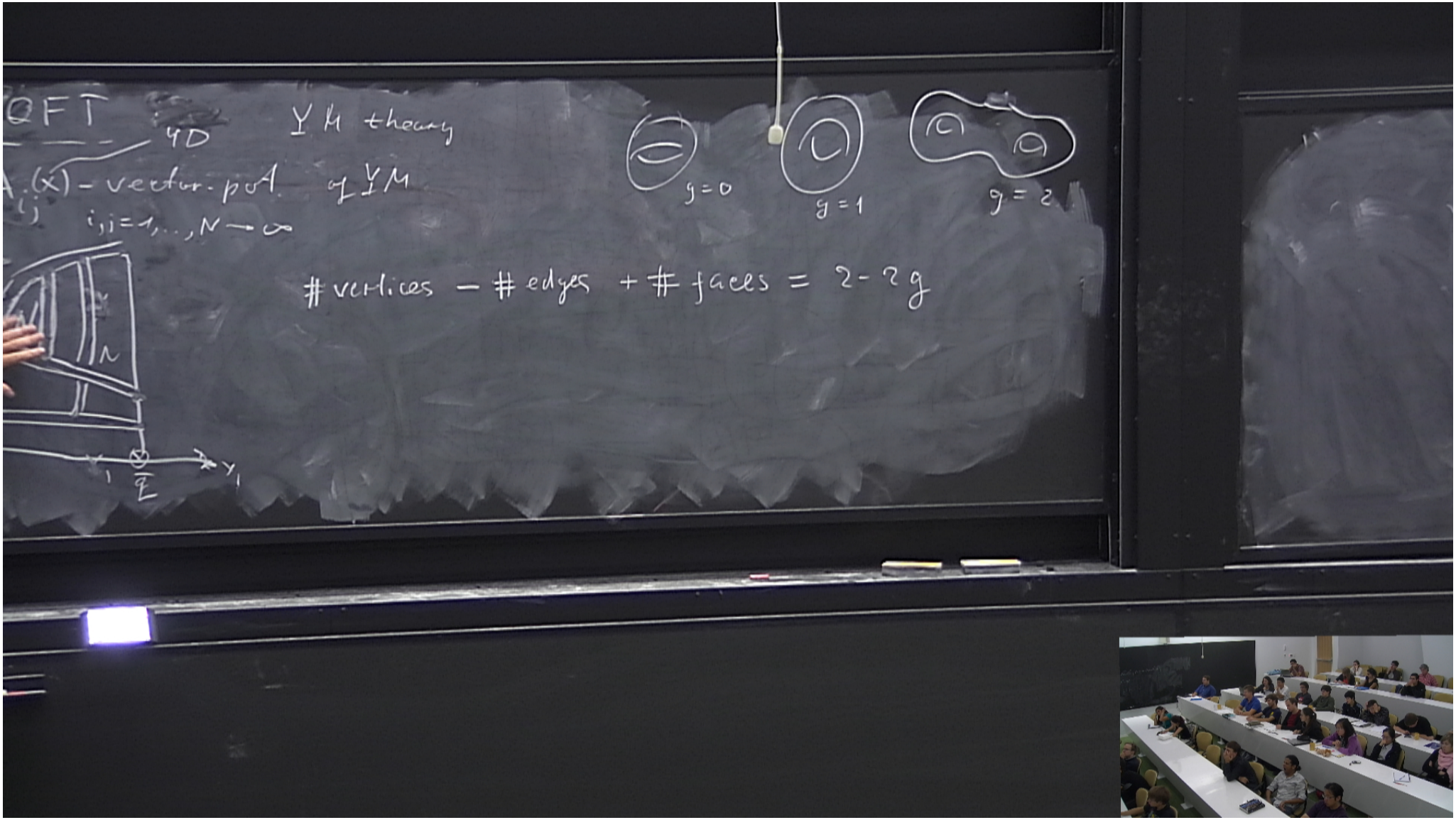

Planar QFT : 4D YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j=1,..,N\to\infty$
t
x_1
x_2
q
$\bar{q}$

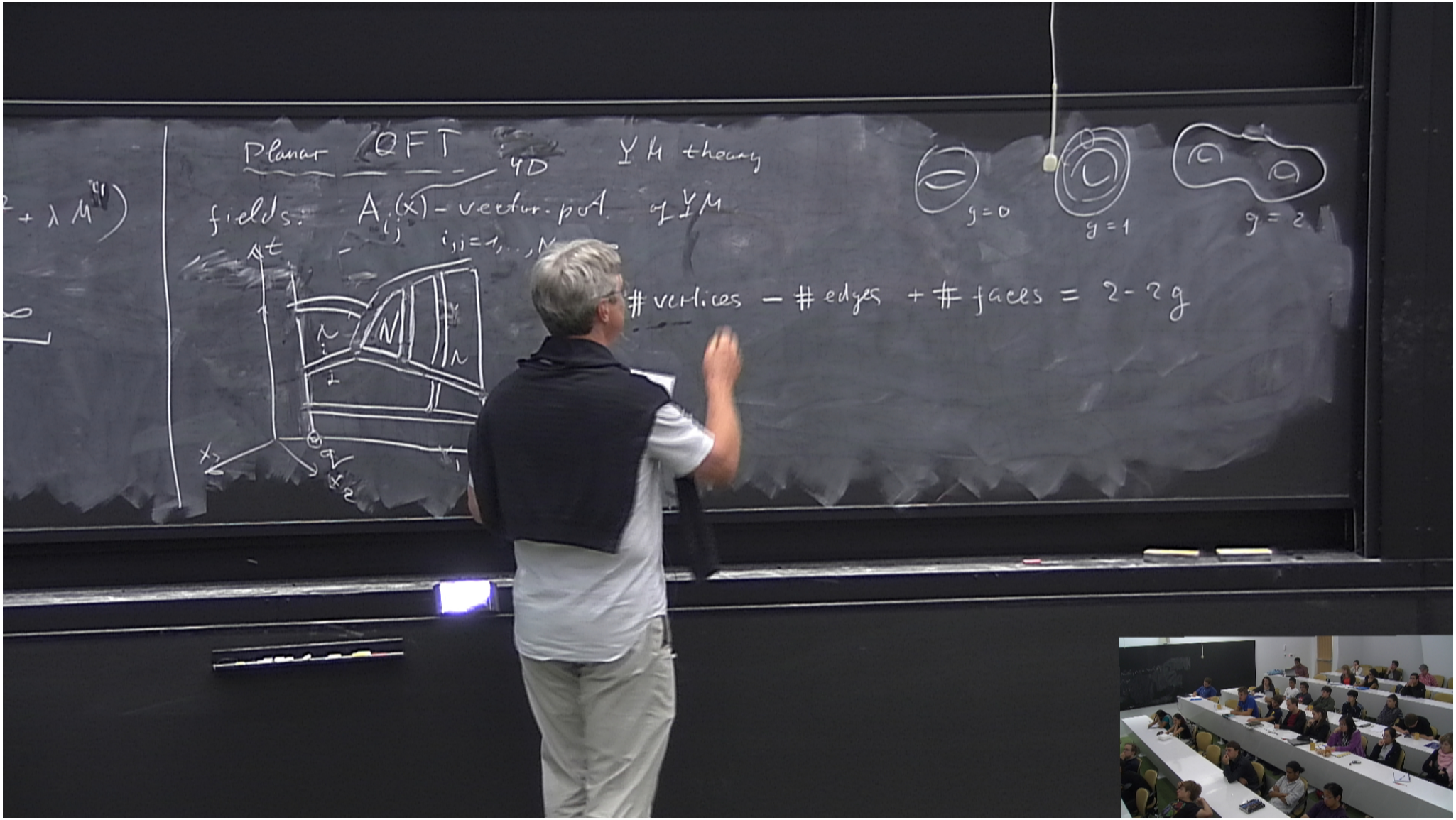

Planar QFT : 4D YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j=1,..,N\to\infty$

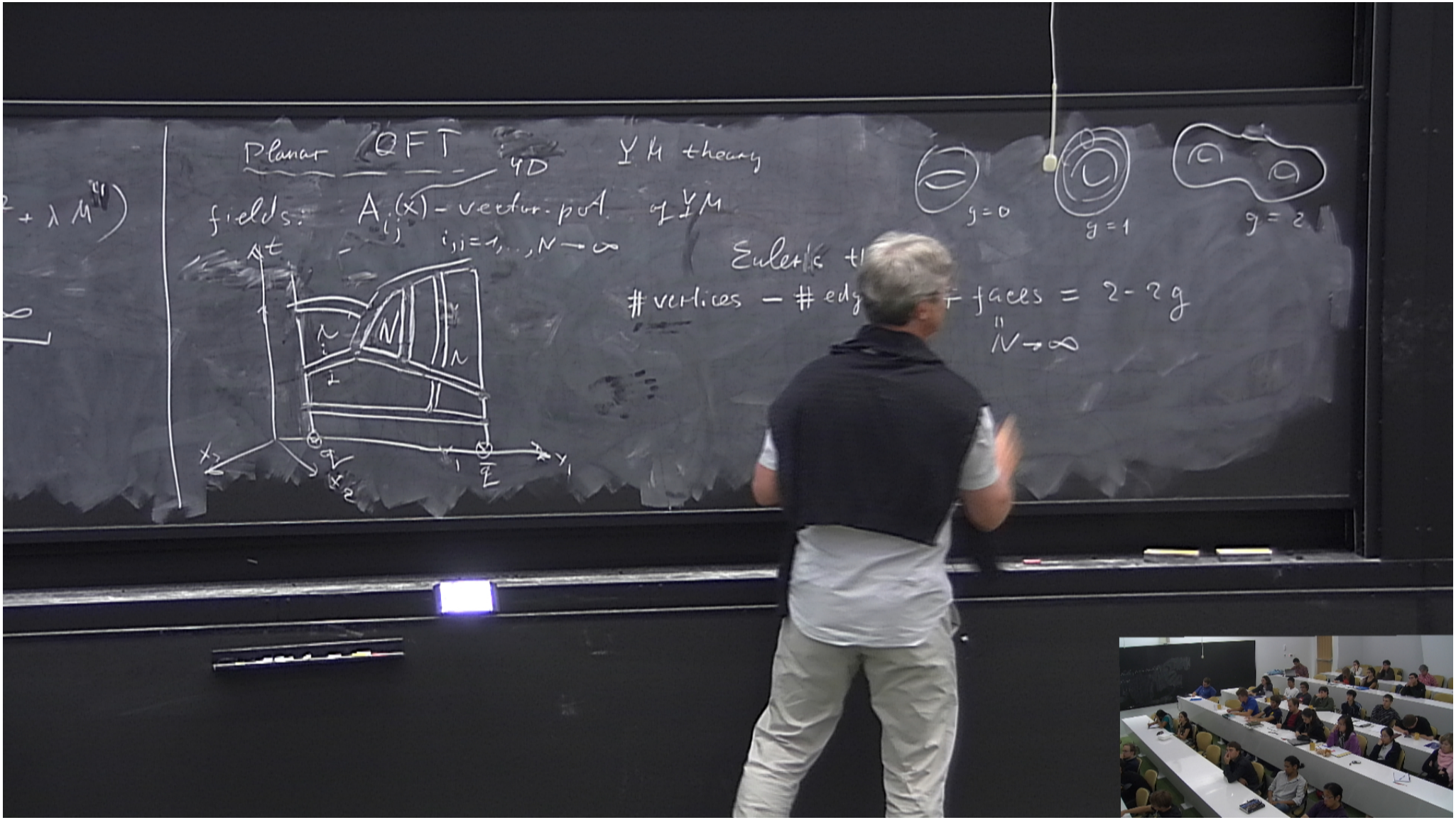

Planar QFT : 4D YM theory
fields: $A_{ij}(x)$ - vector-pot of YM
$i,j = 1,..,N \to \infty$
N
x_1

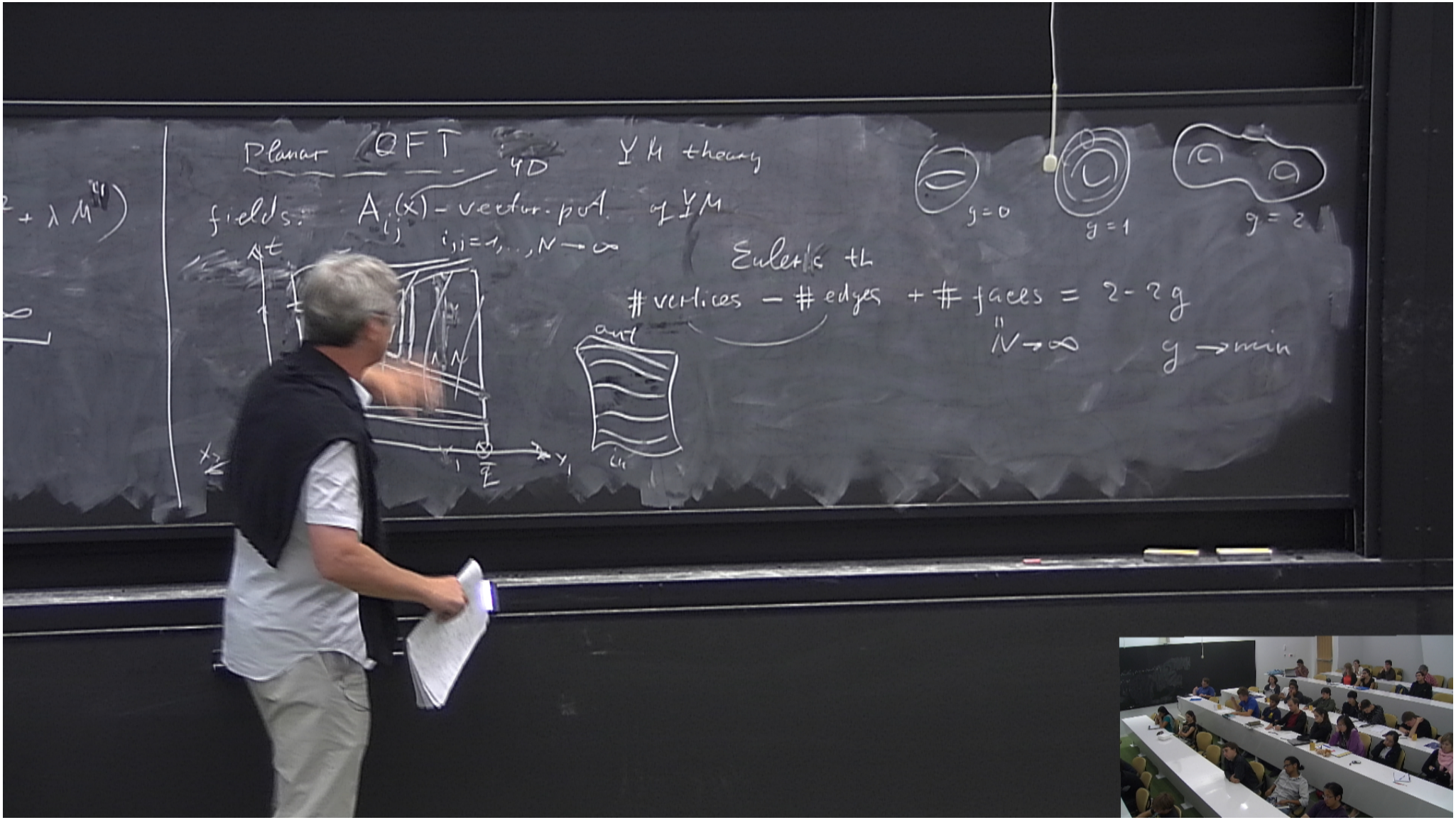

Planar QFT : 4D YM theory
fields: $A_{ij}(x)$ - vector pot of YM
$i,j=1,\dots N\to\infty$
Λt
x_1
x_2
N

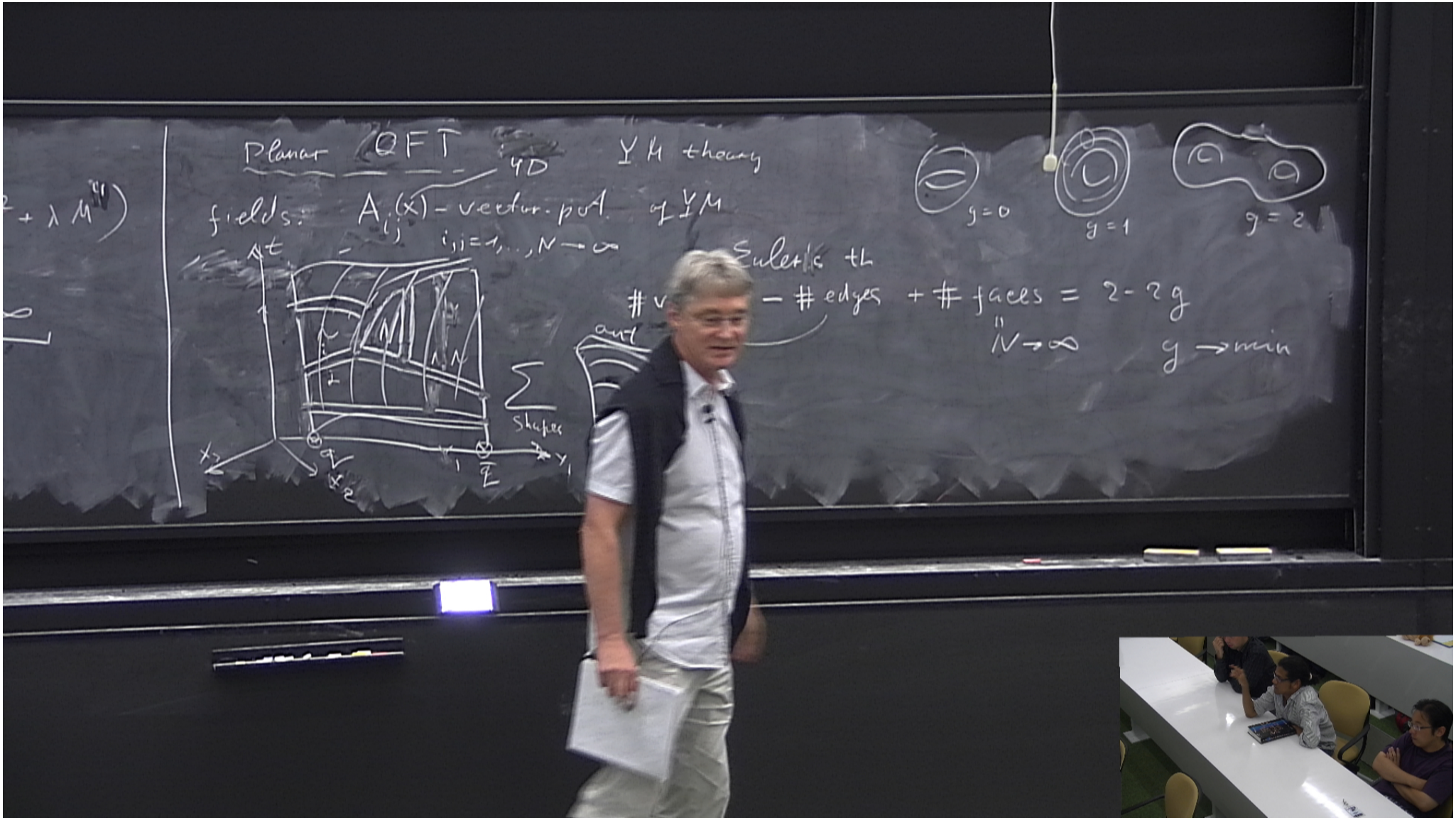

Planar QFT : 4D YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j = 1,..,N \to \infty$
t
x_3
x_2
x_1
N
N
$g=0$
$g=1$
vertices − # edges + # faces = 2-

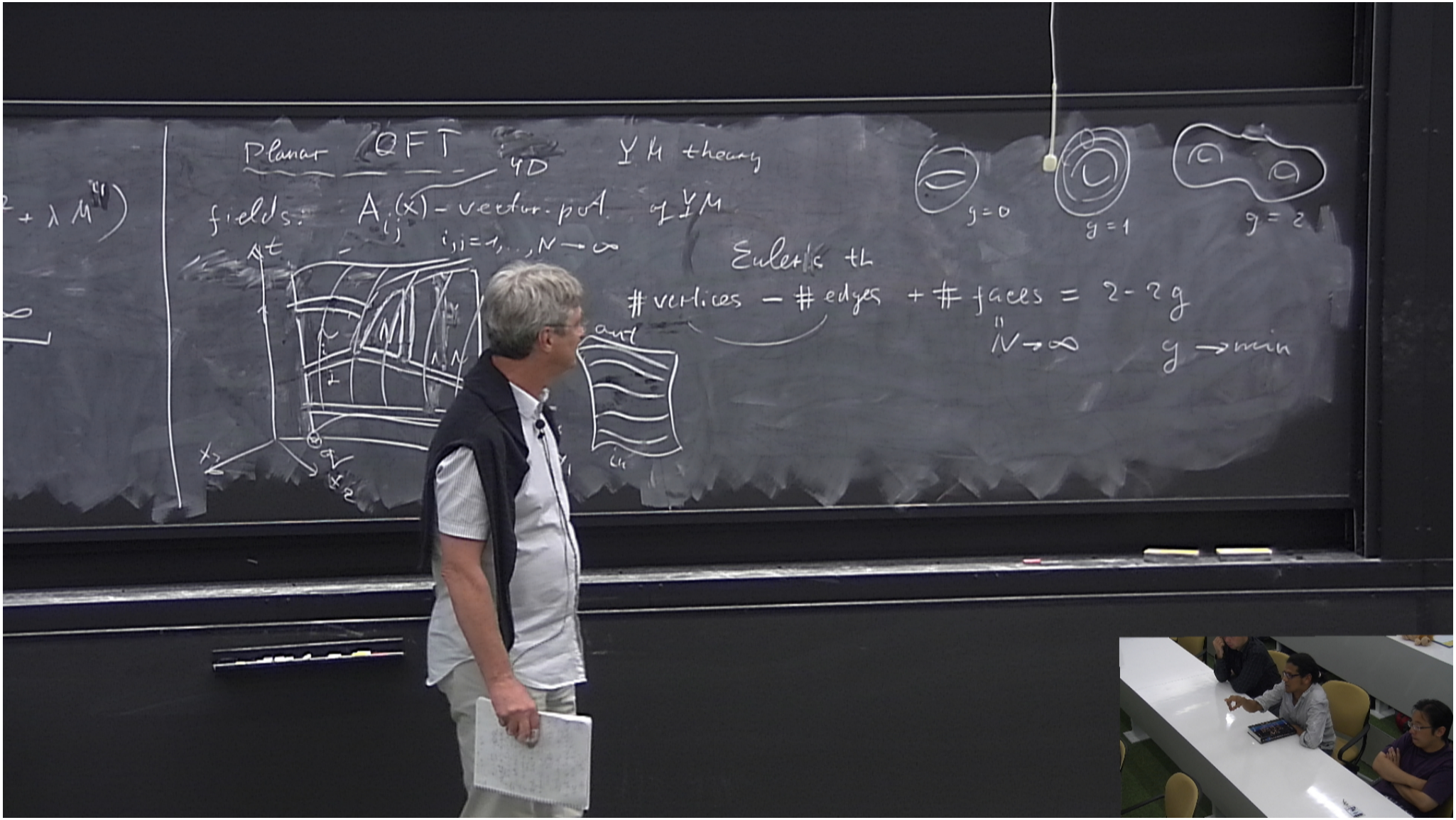

QFT
4D
YM theory
A_i(x) - vector pot.
i,j=1,..,N→∞
g=0
g=1
g=2
# vertices − # edges + # faces = 2 − 2g
N
1
2
y_1

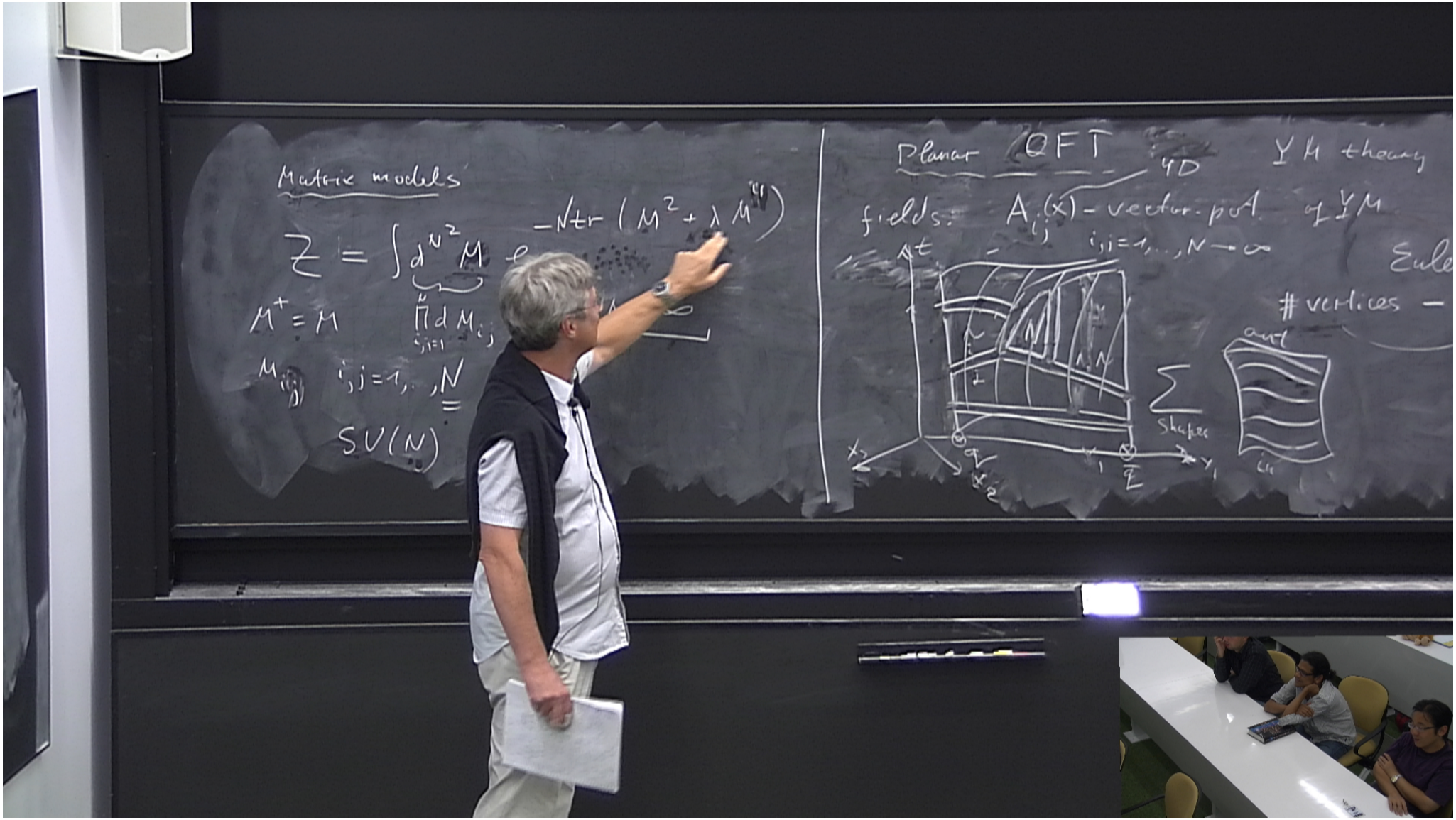

Planar QFT
4D
YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j=1,\dots,N$
# vertices − # edges + # faces = 2 − 2g
g=0
g=1
g=2

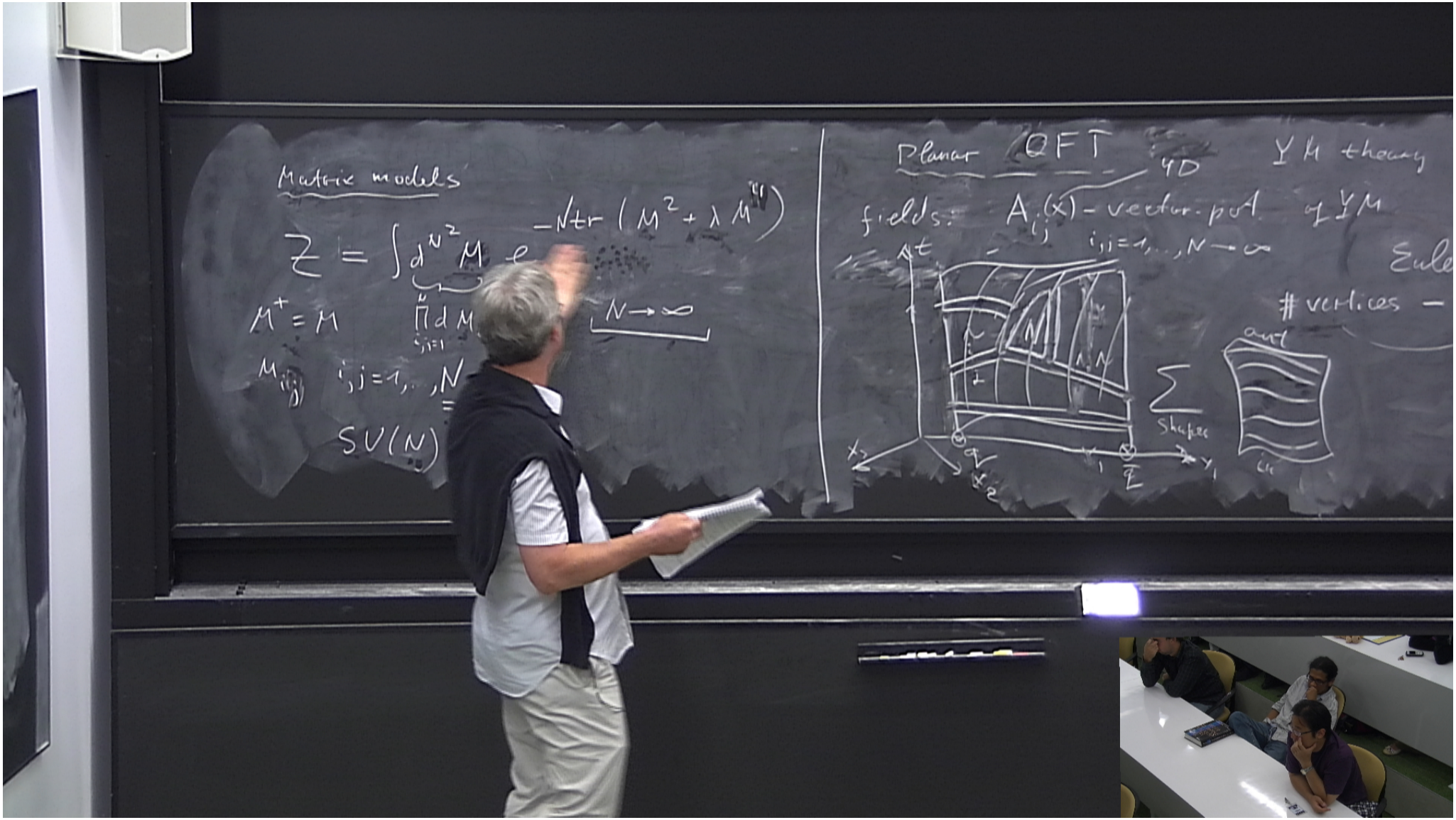

Planar QFT
4D
YM theory
fields: $A_{ij}(x)$ - vector pot.
$i,j=1,\dots,N\to\infty$
Euler's
#vertices − #edg + faces = 2 − 2g
$N\to\infty$
g=0
g=1
g=2

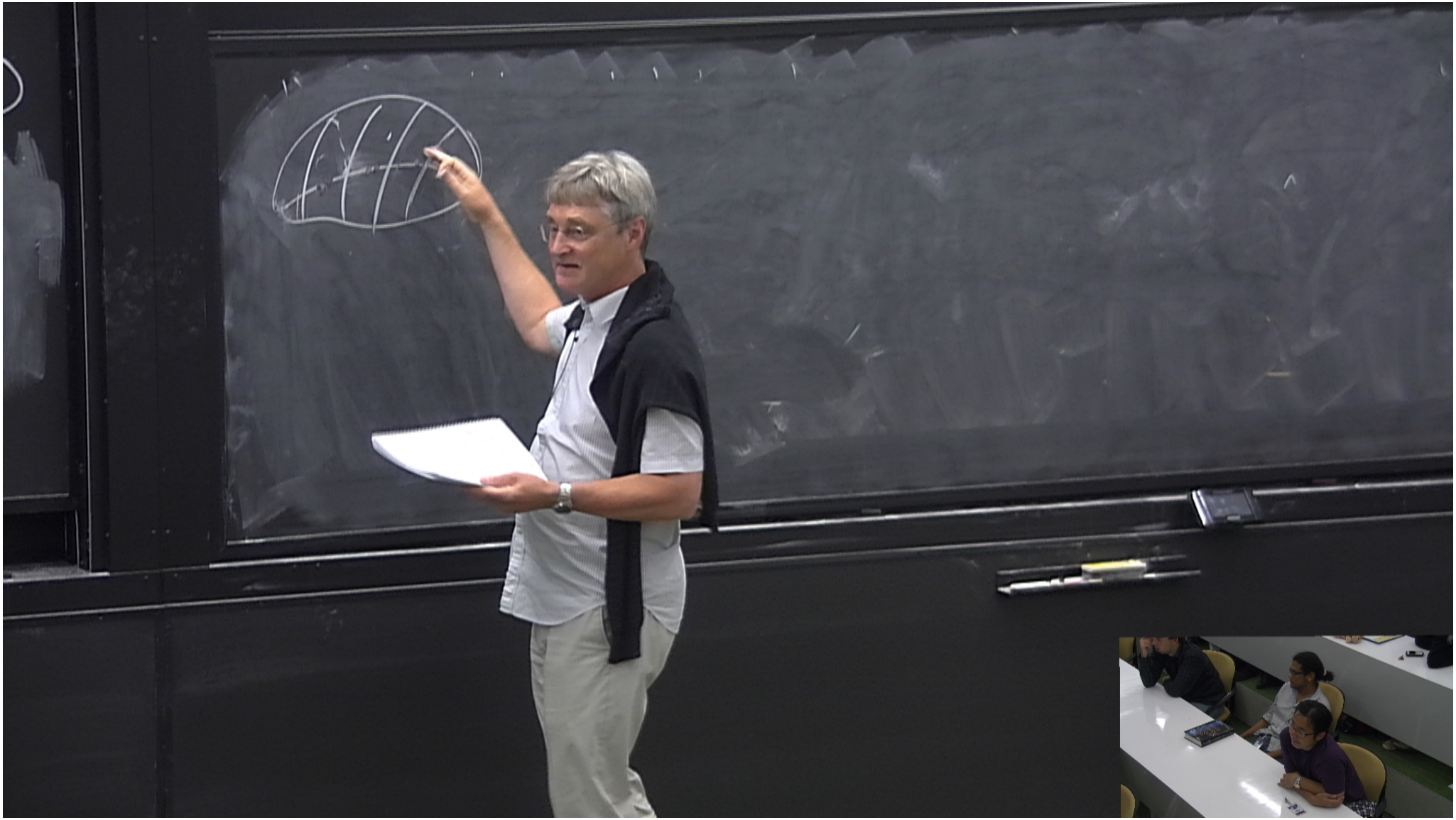

Planar QFT : 4D YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j=1,\dots,N\to\infty$
Euler's th
vertices − # edges + # faces = 2 − 2g
$N\to\infty$
$g\to\min$
g=0
g=1
g=2
cut

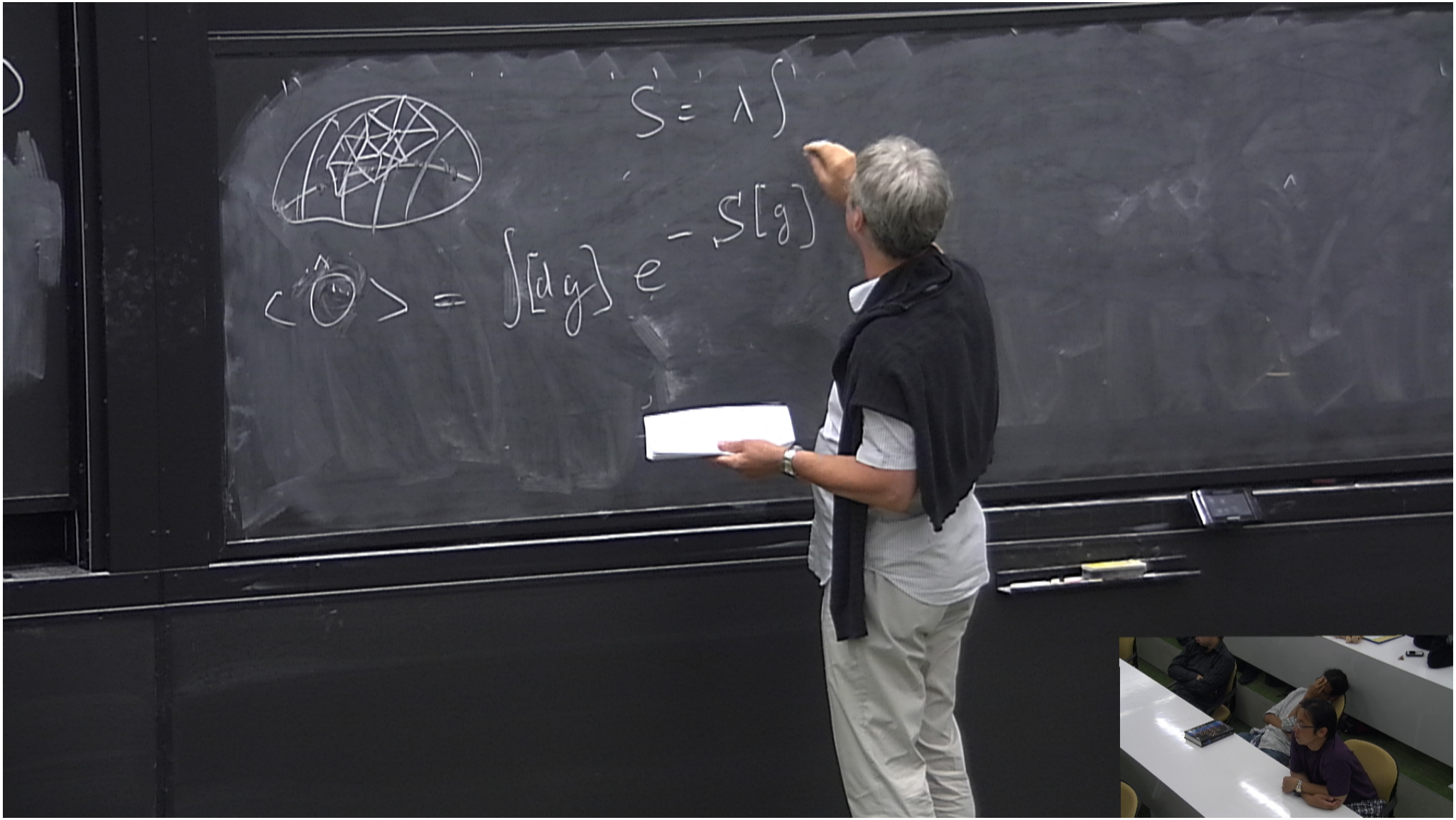

Planar QFT 4D YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j = 1,\dots,N \to \infty$
Euler's th
v - # edges + # faces = 2 - 2g
N → ∞
g → min
g = 0
g = 1
g = 2
Σ
Shapes

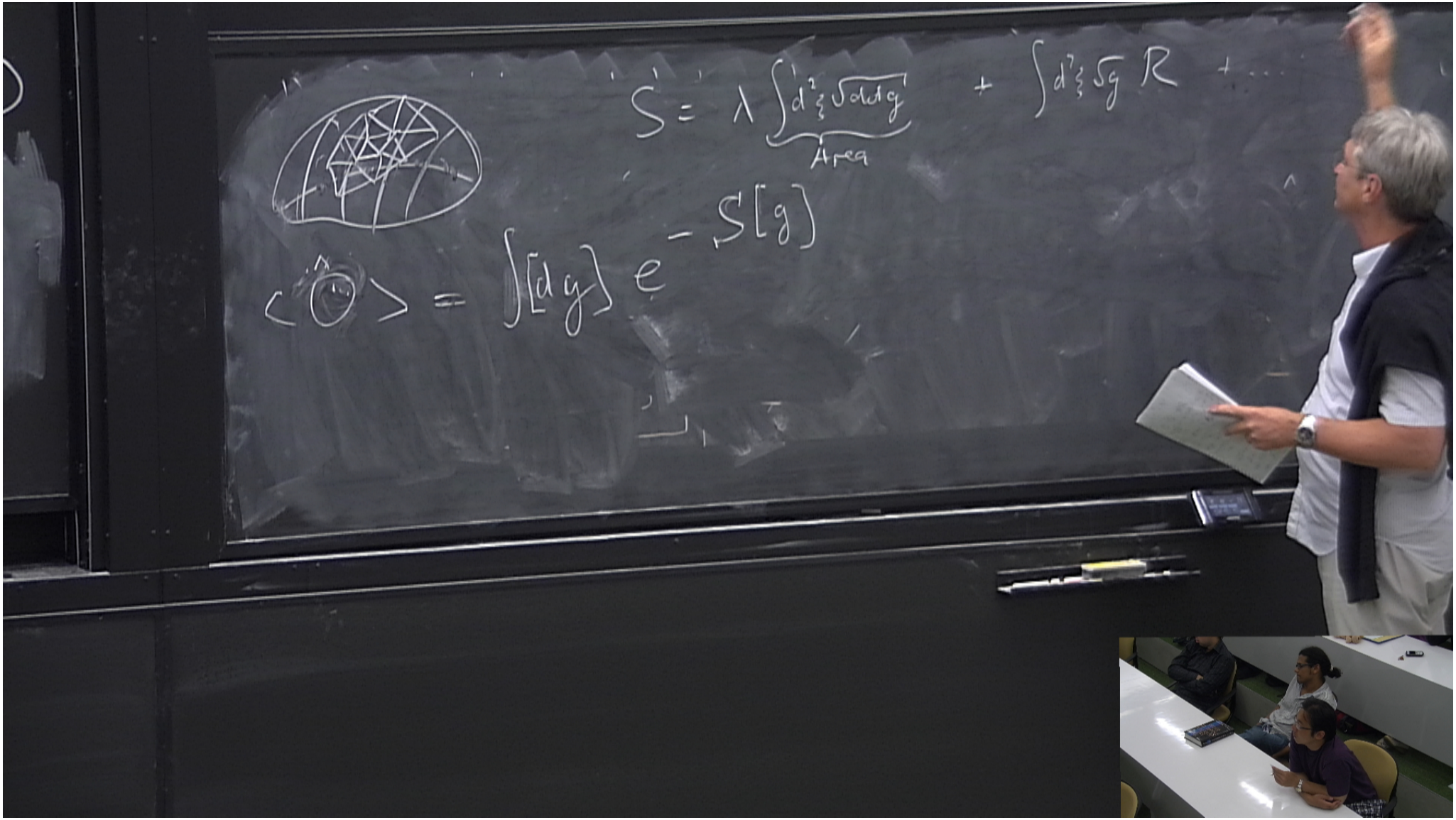

Planar QFT
4D
YM theory
fields: $A_{ij}(x)$ - vector pot. of YM
$i,j=1,\dots,N\to\infty$
Eulerian th
vertices − # edges + # faces = 2 − 2g
$N\to\infty$
$g\to\min$
g=0
g=1
g=2

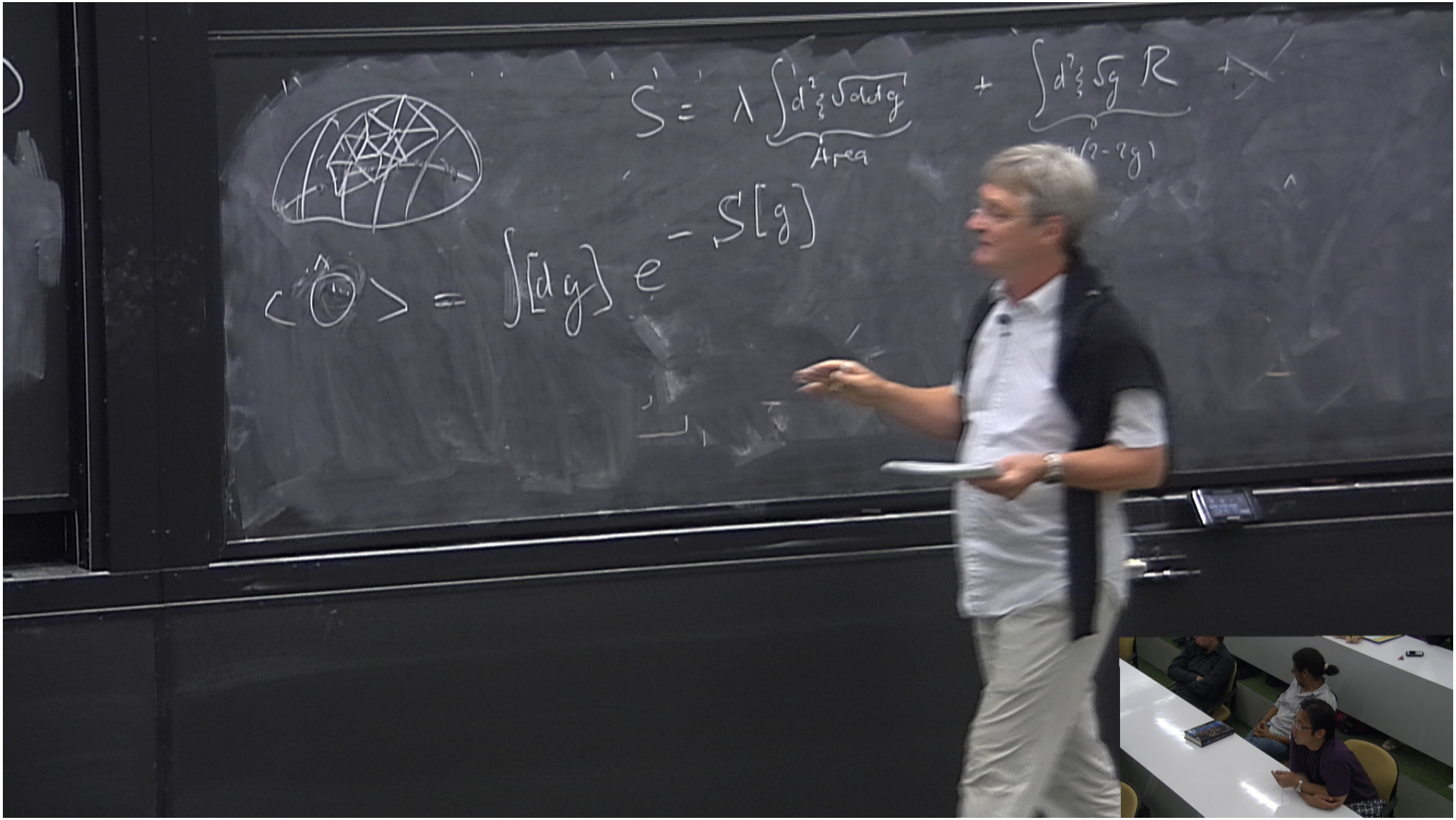

Matrix models
$Z = \int d^{N^2} M \, e^{-N \mathrm{tr}\,(M^2 + \lambda M^4)}$
$M^+ = M$
$\prod_{i,j=1} dM_{ij}$
M_{ij} $i,j = 1, \ldots, N$
$SU(N)$
Planar QFT
4D
YM theory
fields: $A_{ij}(x)$ - vector pot.
$i,j = 1, \ldots, N \to \infty$
of YM
Eule
vertices -
and
Sum
Shapes
χ_1
χ_2
χ_3
Li

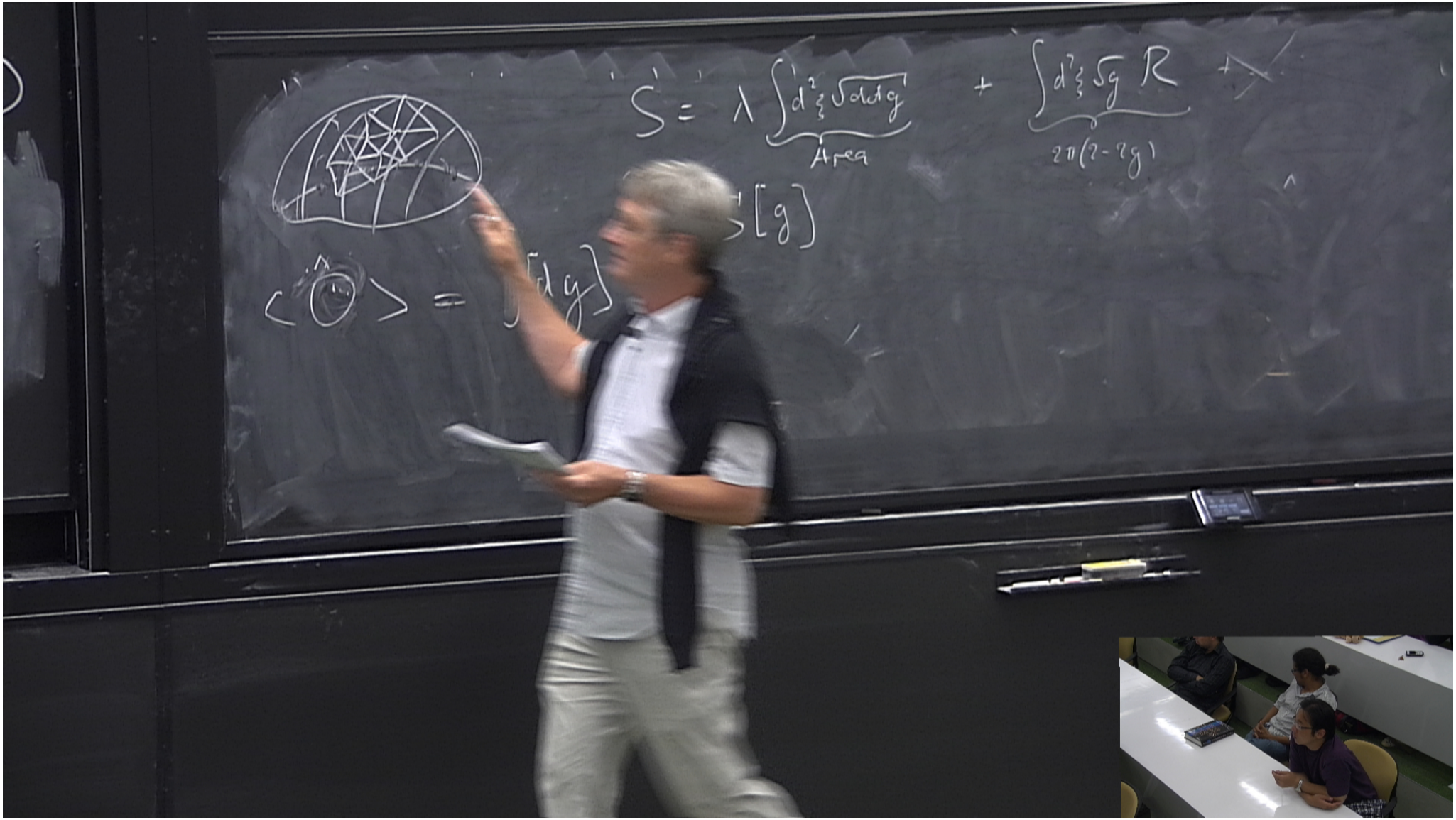

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$Z = \int d^{N^2} M \, e^{-N \mathrm{tr}\,(M^2 + \lambda M^4)}$
$M^+ = M$
$\prod_{i,j=1} dM$
$N \to \infty$
M_{ij} $i,j = 1, \ldots, N$
$SU(N)$
Planar QFT 4D YM theory
fields: $A_{ij}(x)$ - vector pot of YM
$i,j = 1, \ldots, N \to \infty$
Euler
vertices -
Sum
Shapes
Sum

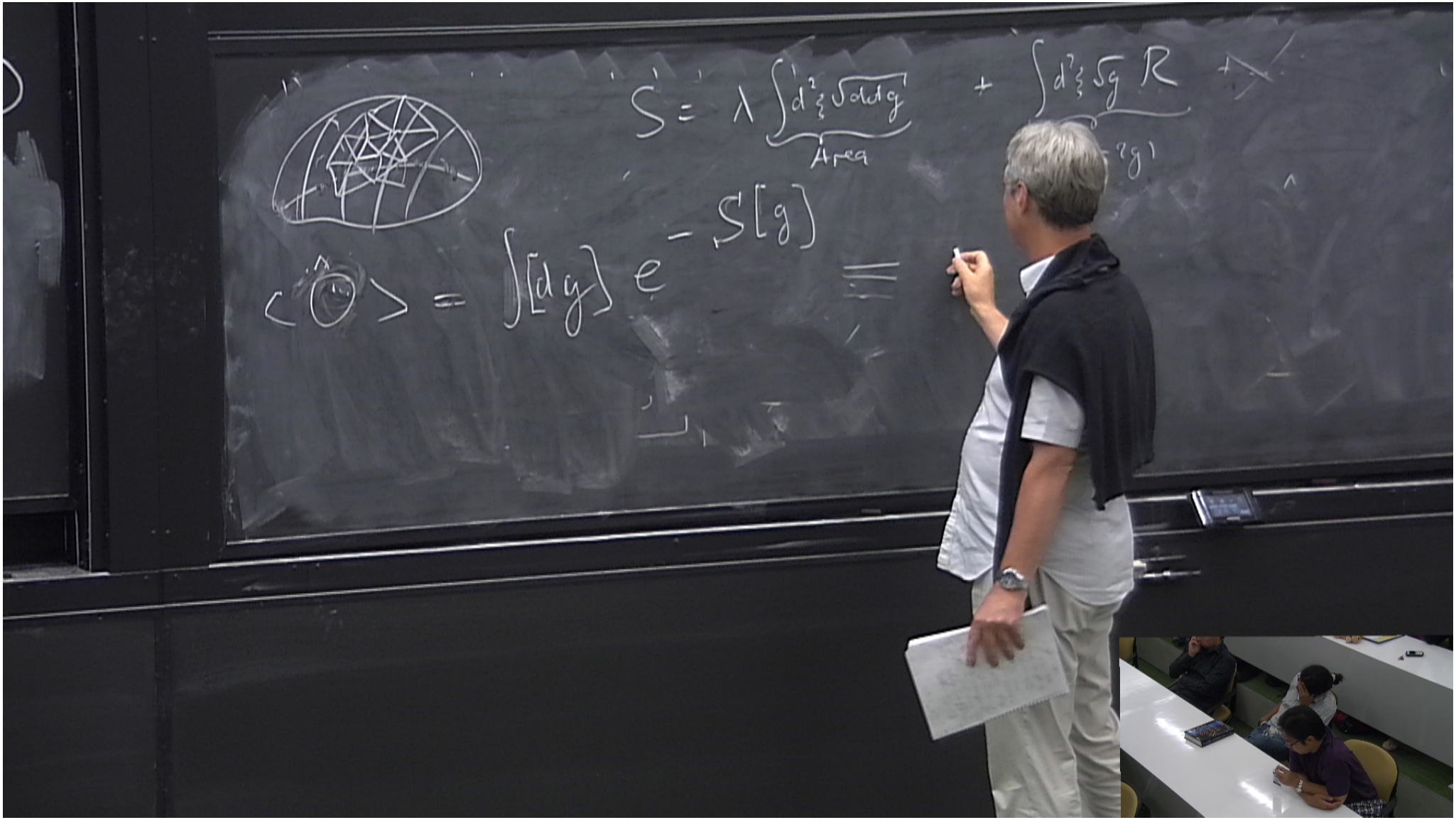


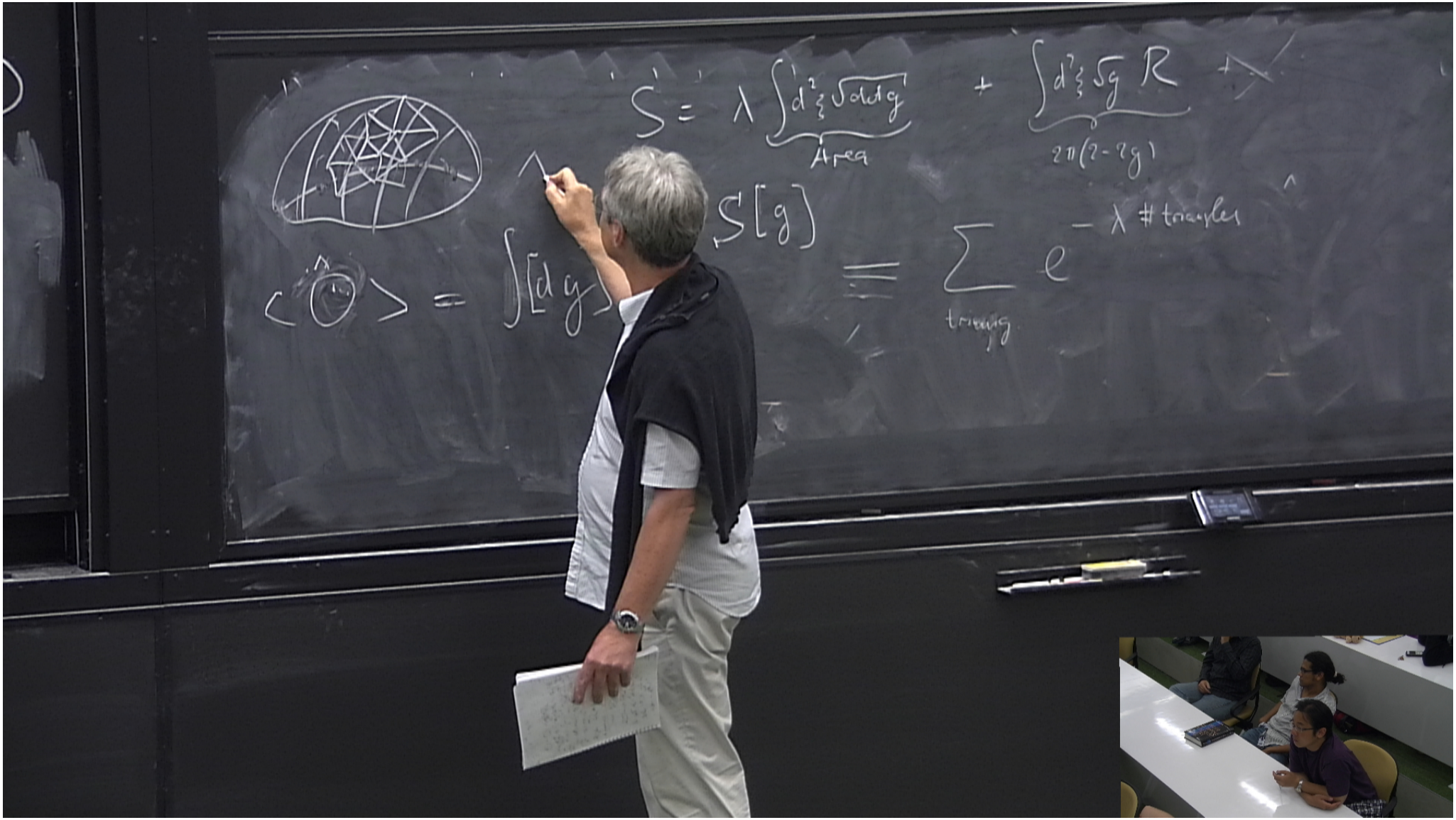

S = λ ∫
⟨Ô⟩ = ∫[dg] e^{-S[g]}

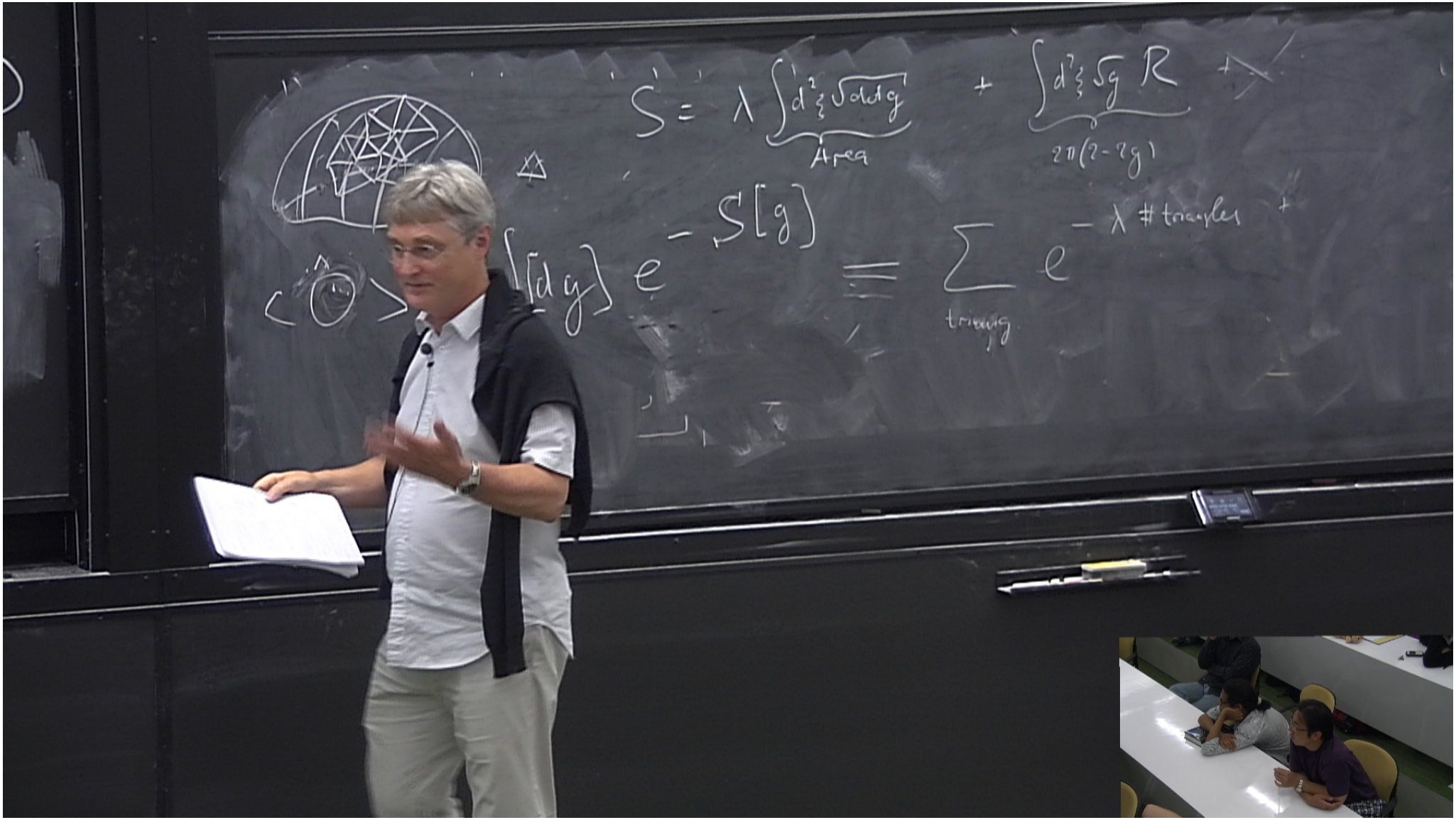

$S = \lambda \underbrace{\int d^2\xi \sqrt{\det g}}_{\text{Area}} + \int d^2\xi \sqrt{g} R + \dots$
$\langle \mathcal{O} \rangle = \int [dg] e^{-S[g]}$

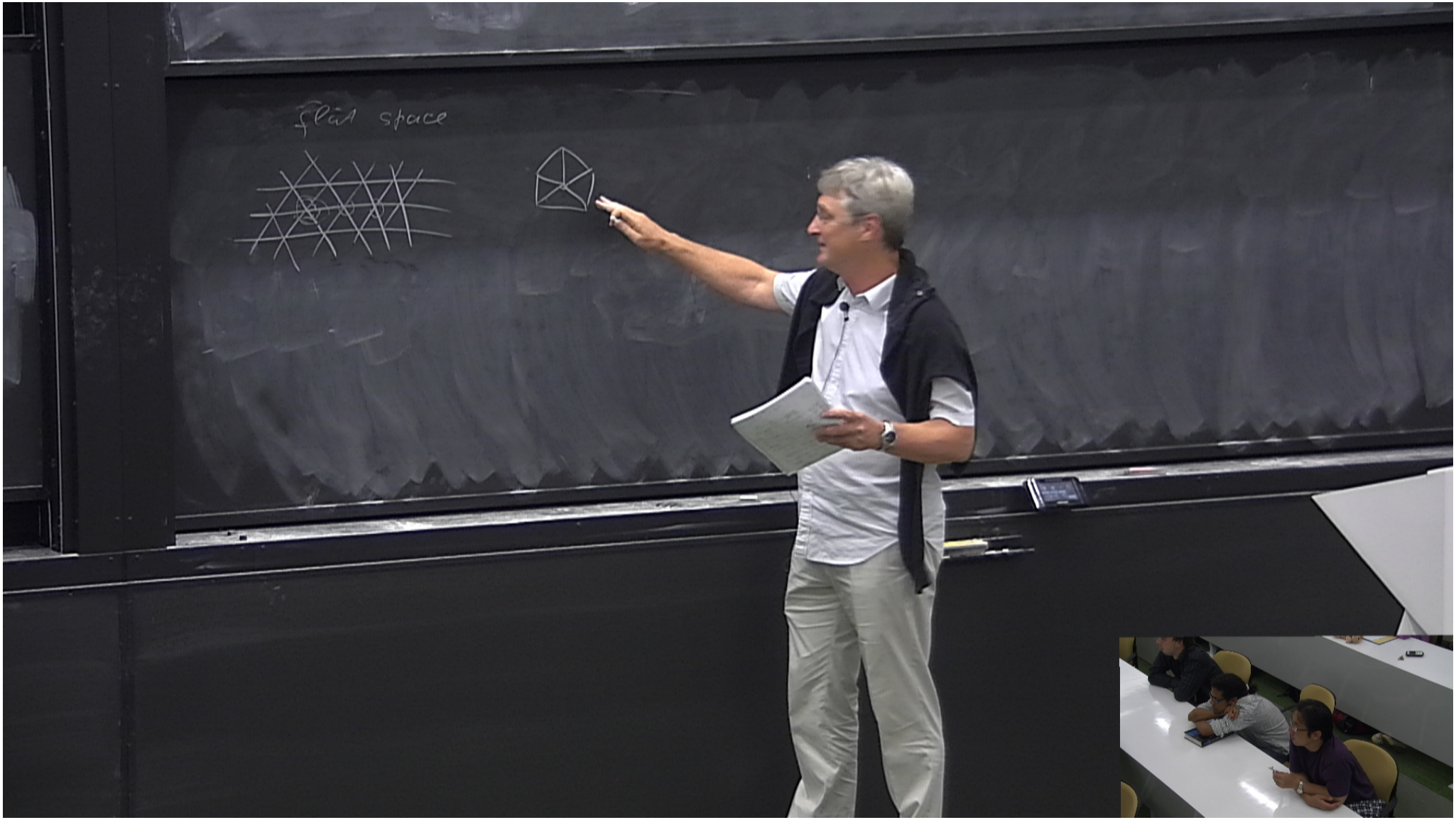

$S = \lambda \int d^2\xi \sqrt{\det g}$
Area
$+ \int d^2\xi \sqrt{g} R$
$\langle \mathcal{O} \rangle = \int [dg]\, e^{-S[g]}$

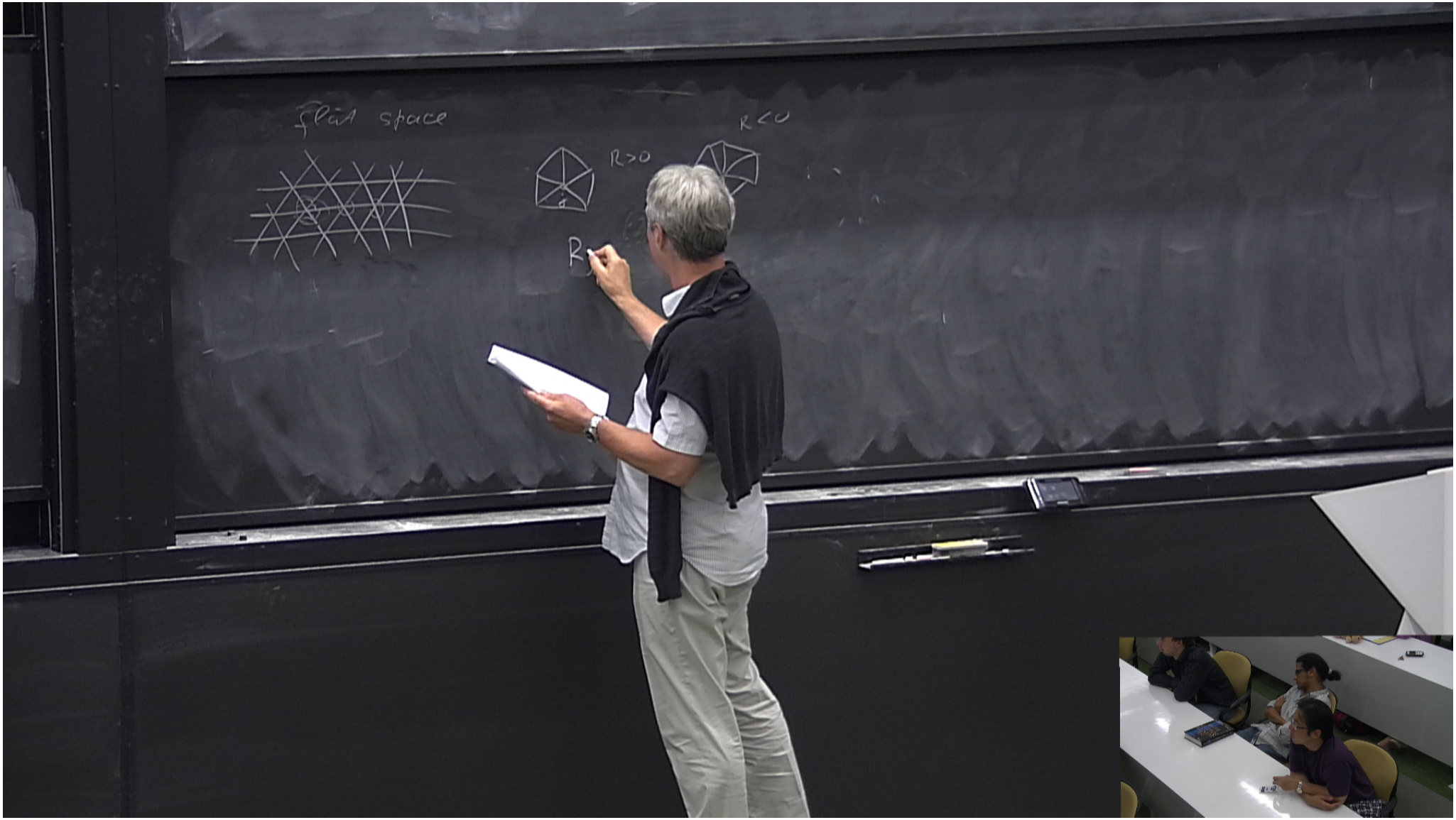

S = λ ∫ d²ξ √g dξ dξ + ∫ d²ξ √g R
Area
2π(2-2g)
[g]
⟨ ⟩ = ∫ [dg]

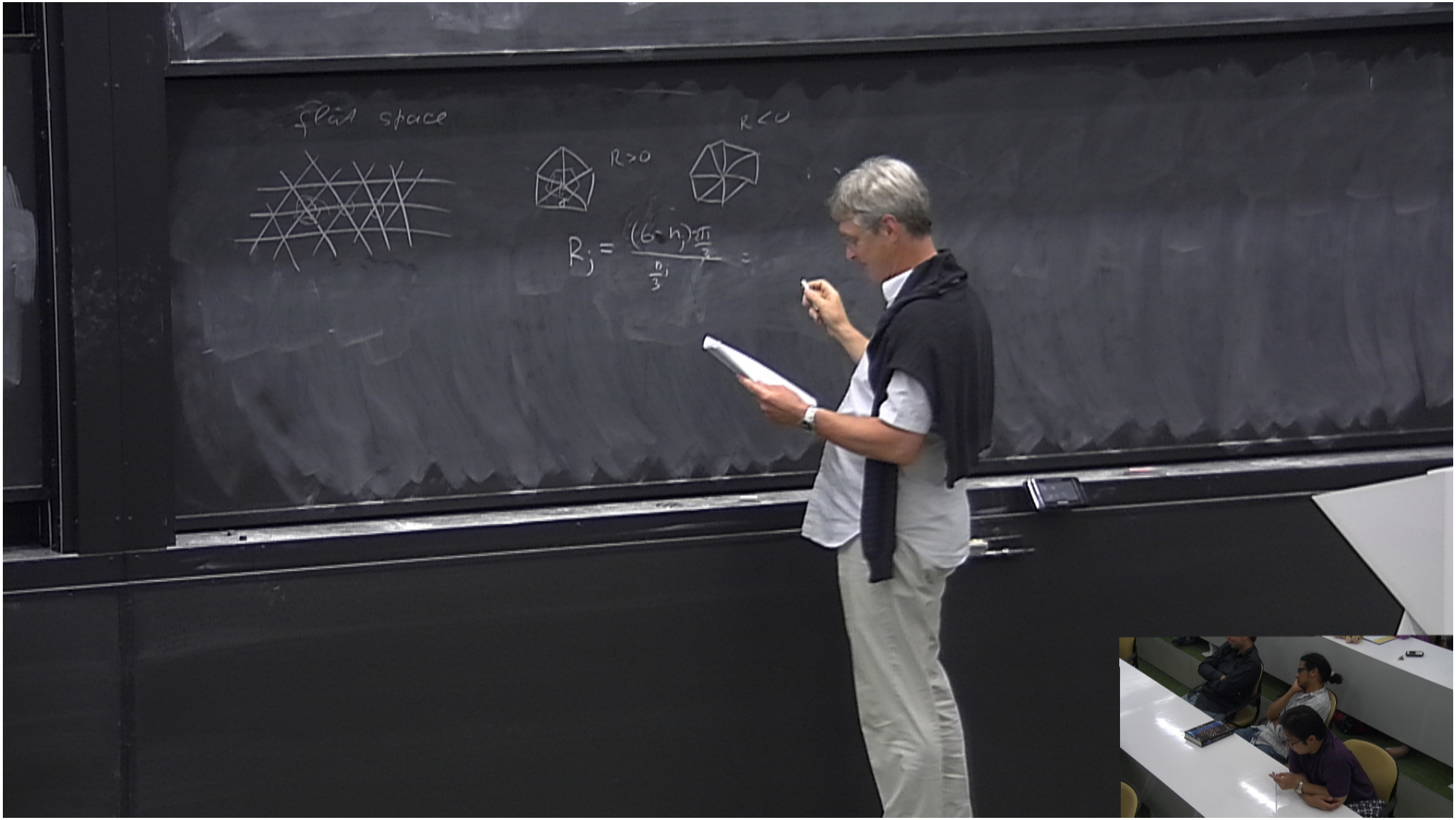

$S = \lambda \int d^2\xi \sqrt{g}\, d\, d\, g$
Area
$+ \int d^2\xi \sqrt{g}\, R$
$\langle \hat{O} \rangle = \int [dg]\, e^{-S[g]}$

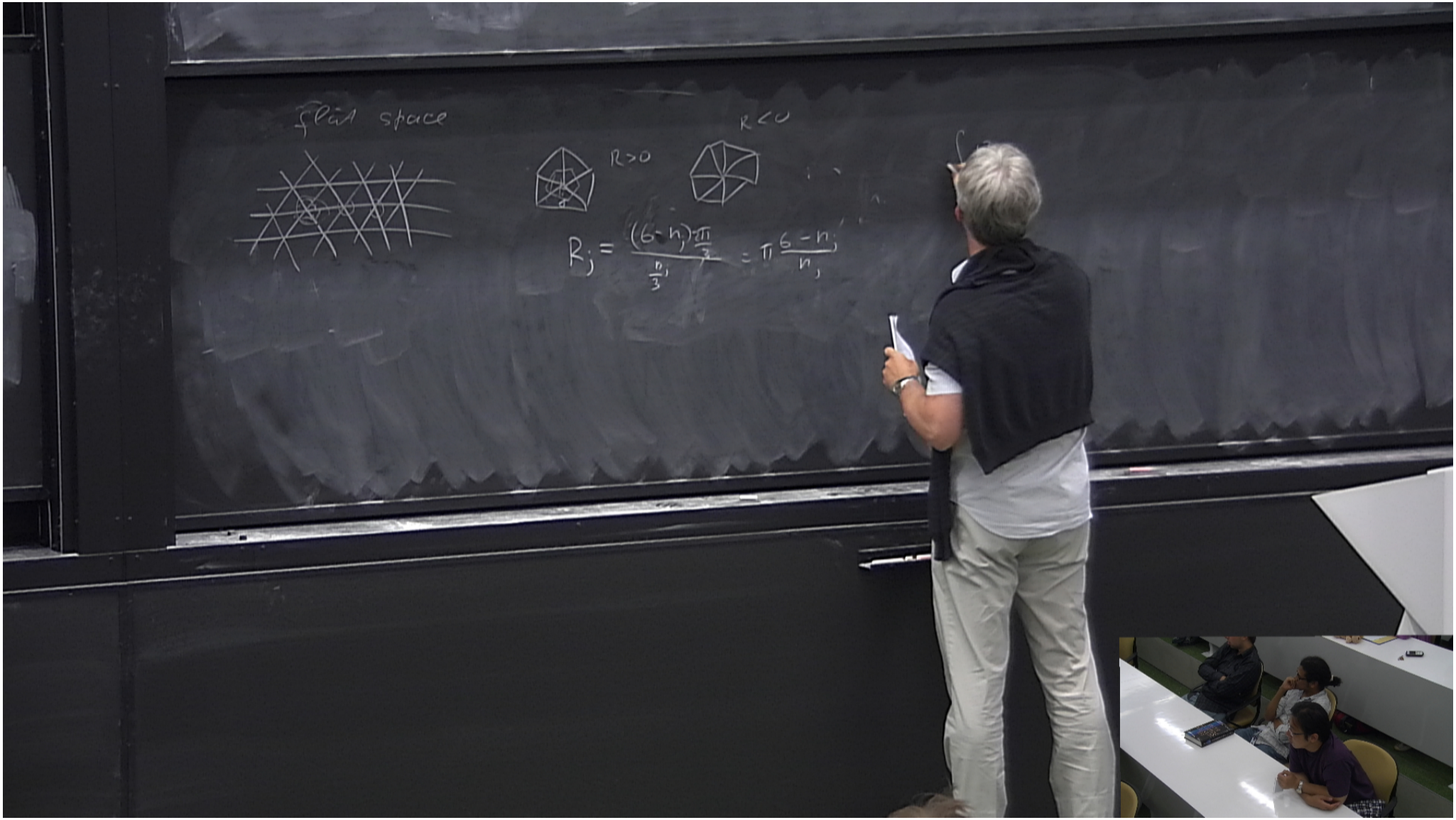

S = λ ∫ d²ξ √det g + ∫ d²ξ √g R + ...
Area
2π(2-2g)
⟨ O ⟩ = ∫ [dg] O S[g]
≡ Σ e^{-λ # triangles}
triang.

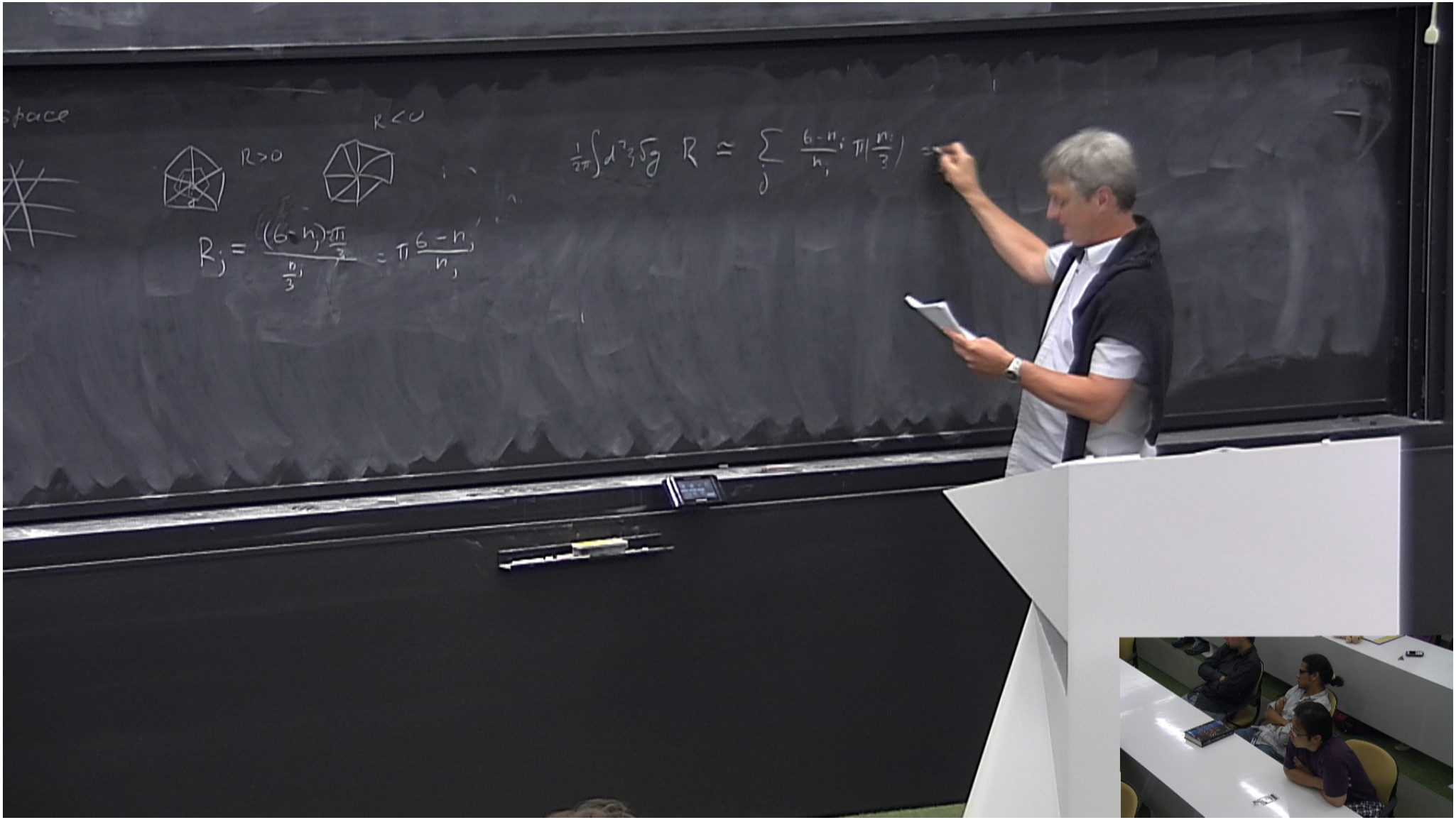

$S = \lambda \int d^2\xi \sqrt{\det g}$
Area
$+ \int d^2\xi \sqrt{g} R$
$2\pi(2-2g)$
$\int [dg] e^{-S[g]} = \sum_{\text{triang.}} e^{-\lambda \# \text{triangles}}$

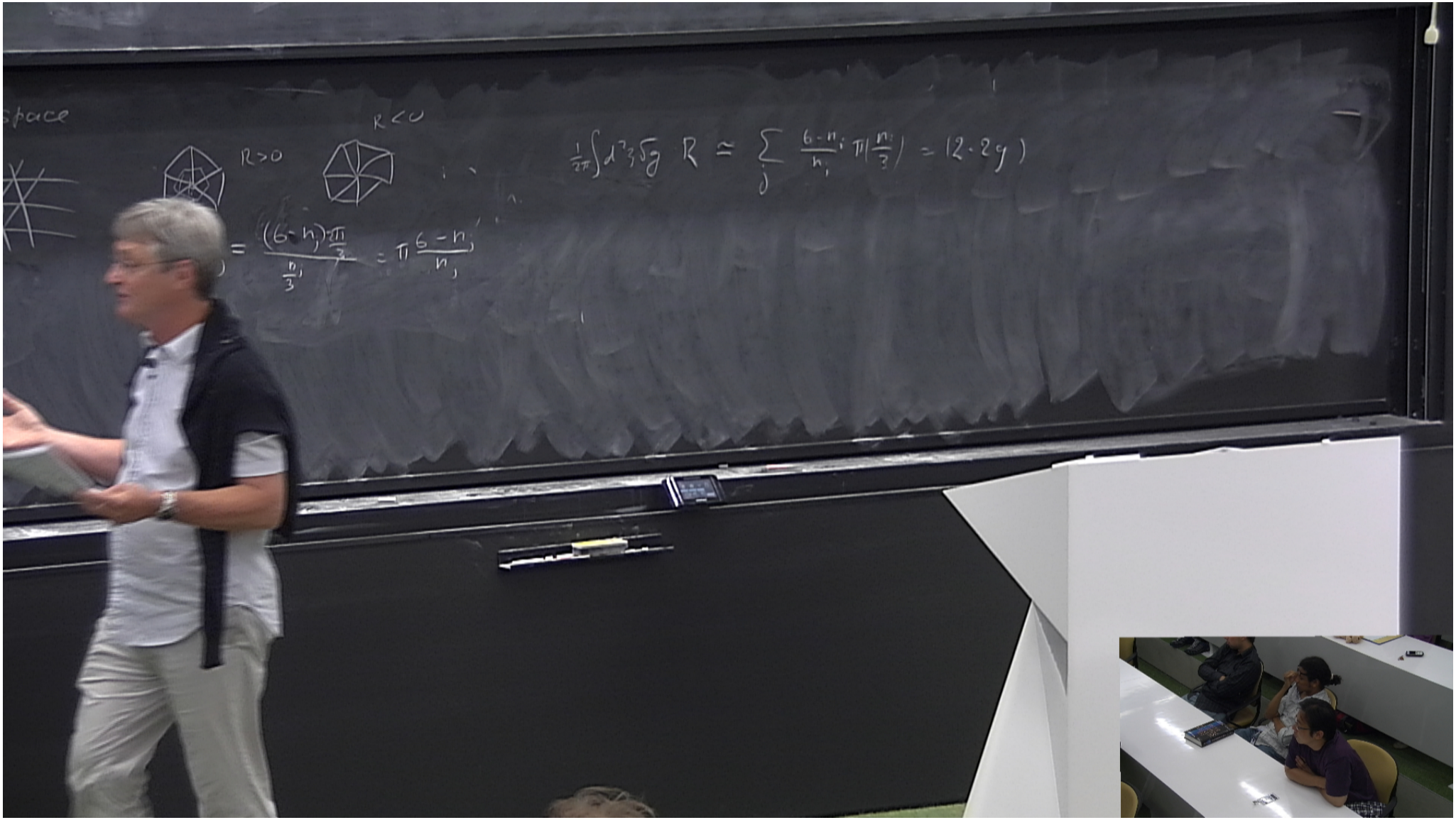

flat space

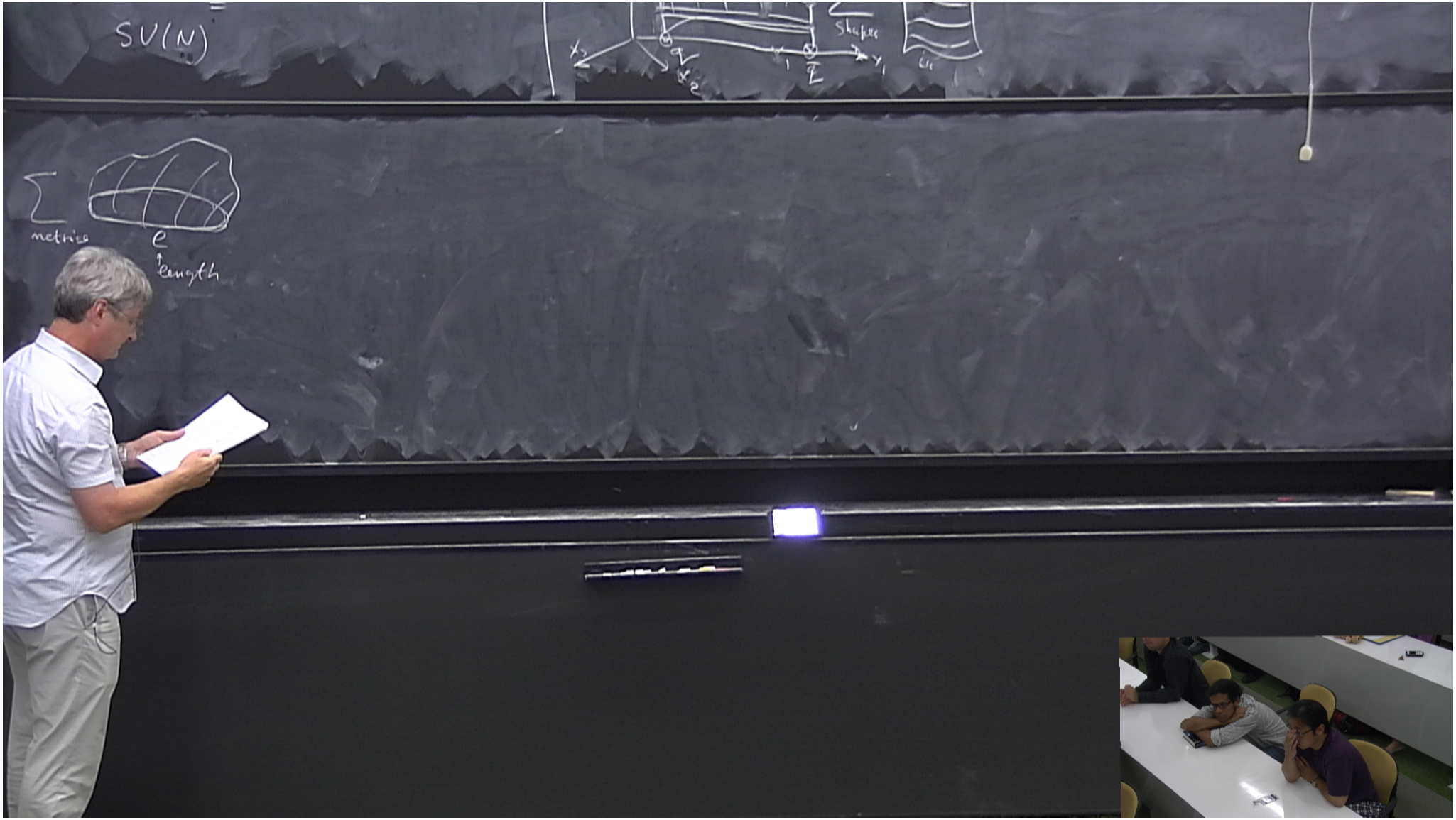

flat space
R>0
R<0

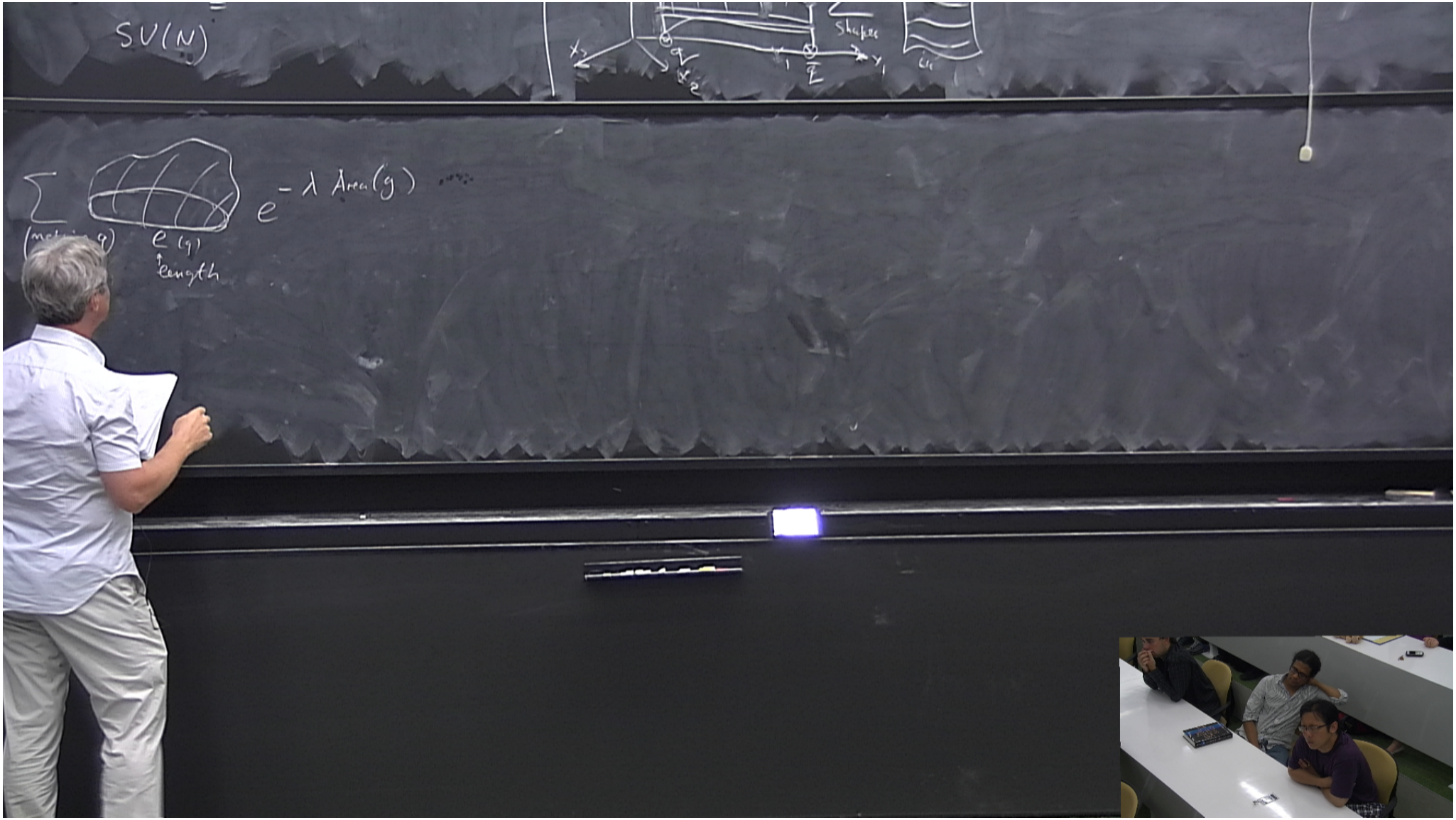

flat space
R>0
R<0
$R_j = \frac{(6-n_j)\cdot\frac{\pi}{2}}{\frac{n_j}{3}}$

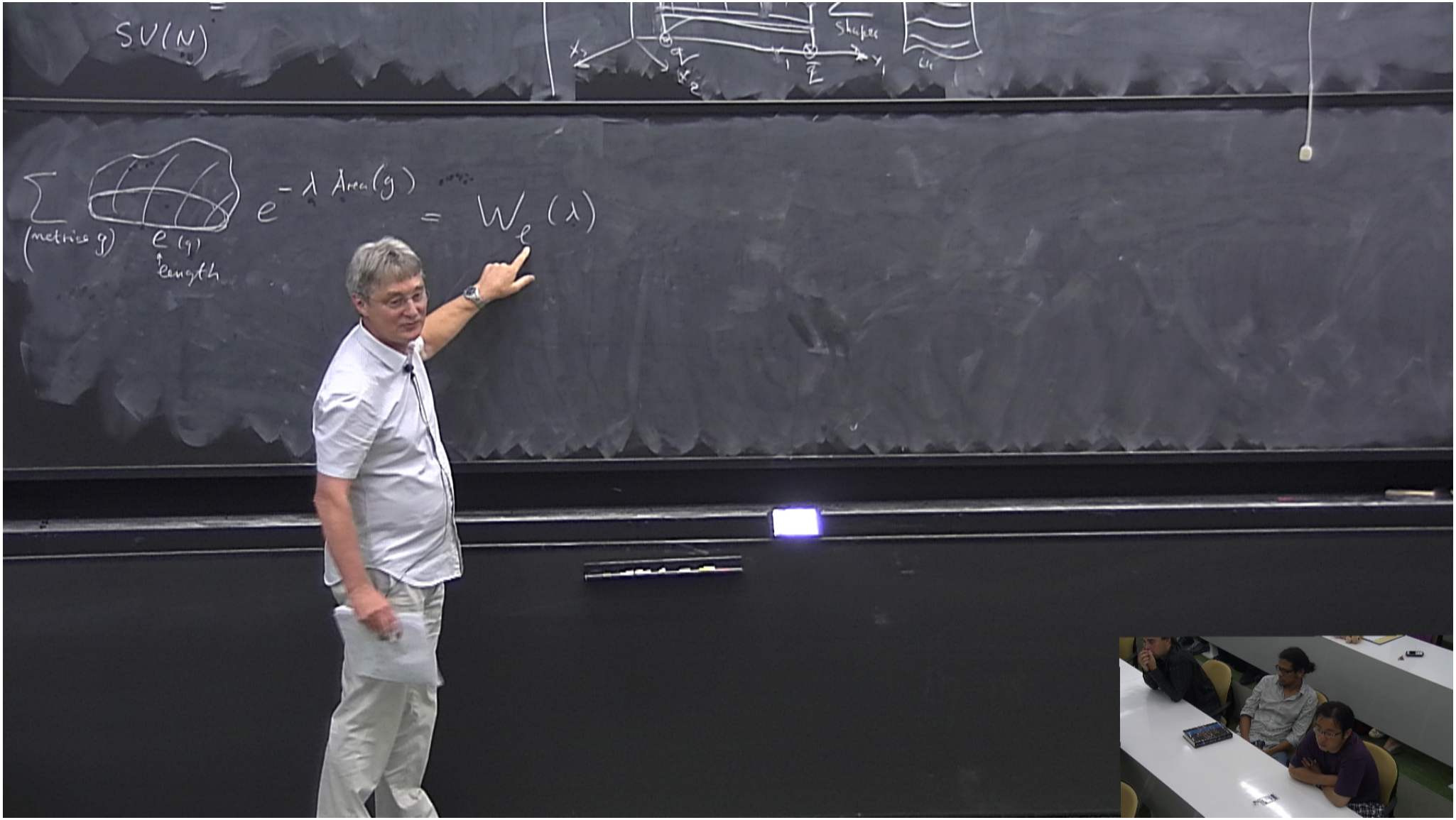

flat space
R>0
R<0
$R_j = \frac{(6-n_j)\cdot\frac{\pi}{3}}{\frac{n_j}{3}} = \pi\frac{6-n_j}{n_j}$

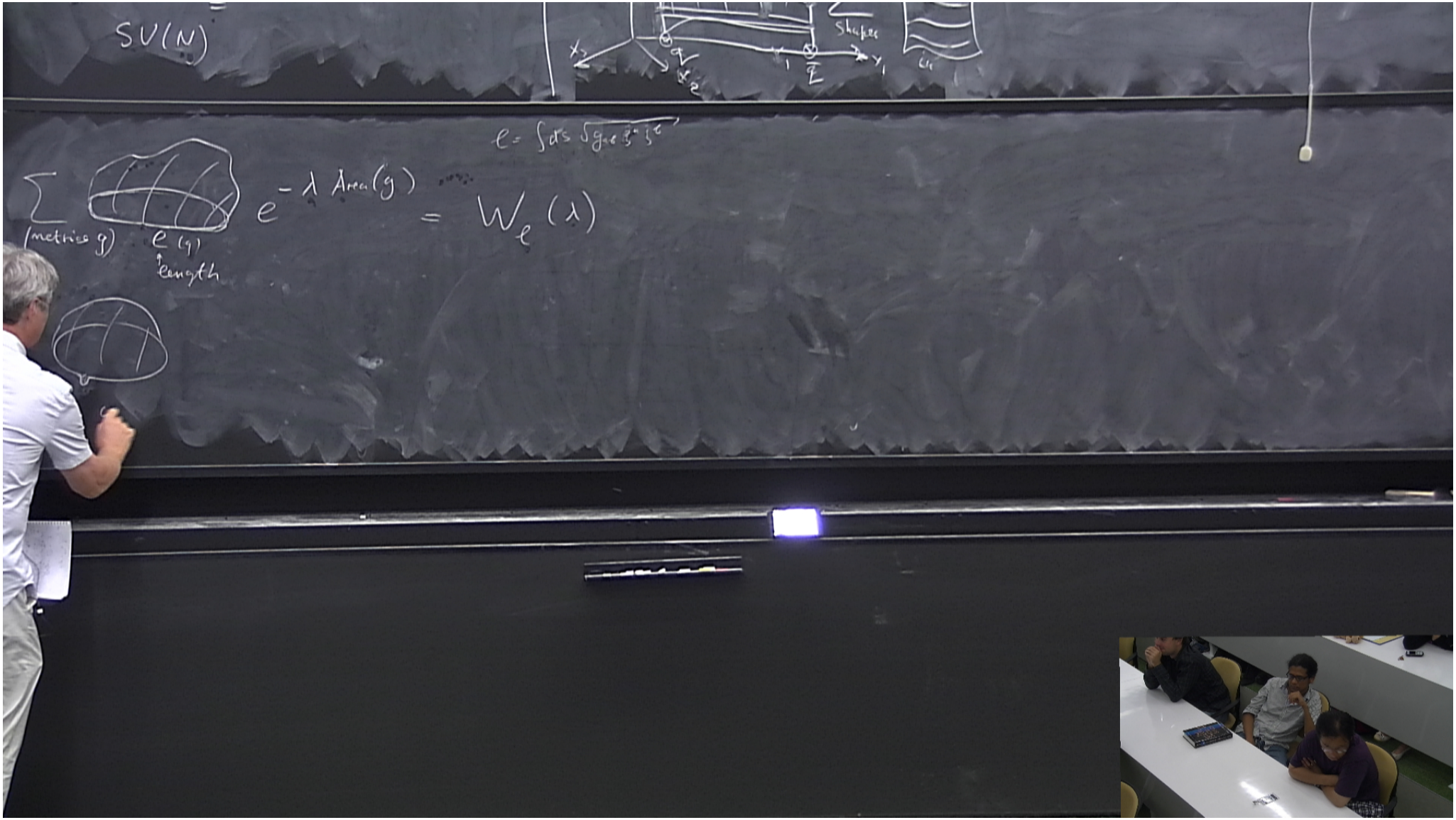

space
R>0
R<0
$R_j = \frac{(6-n_j)\cdot\frac{\pi}{3}}{\frac{n_j}{3}} = \pi\frac{6-n_j}{n_j}$
$\frac{1}{2\pi}\int d^2x\sqrt{g}\,R \simeq \sum_j \frac{6-n_j}{n_j}\cdot\pi\left(\frac{n_j}{3}\right)$

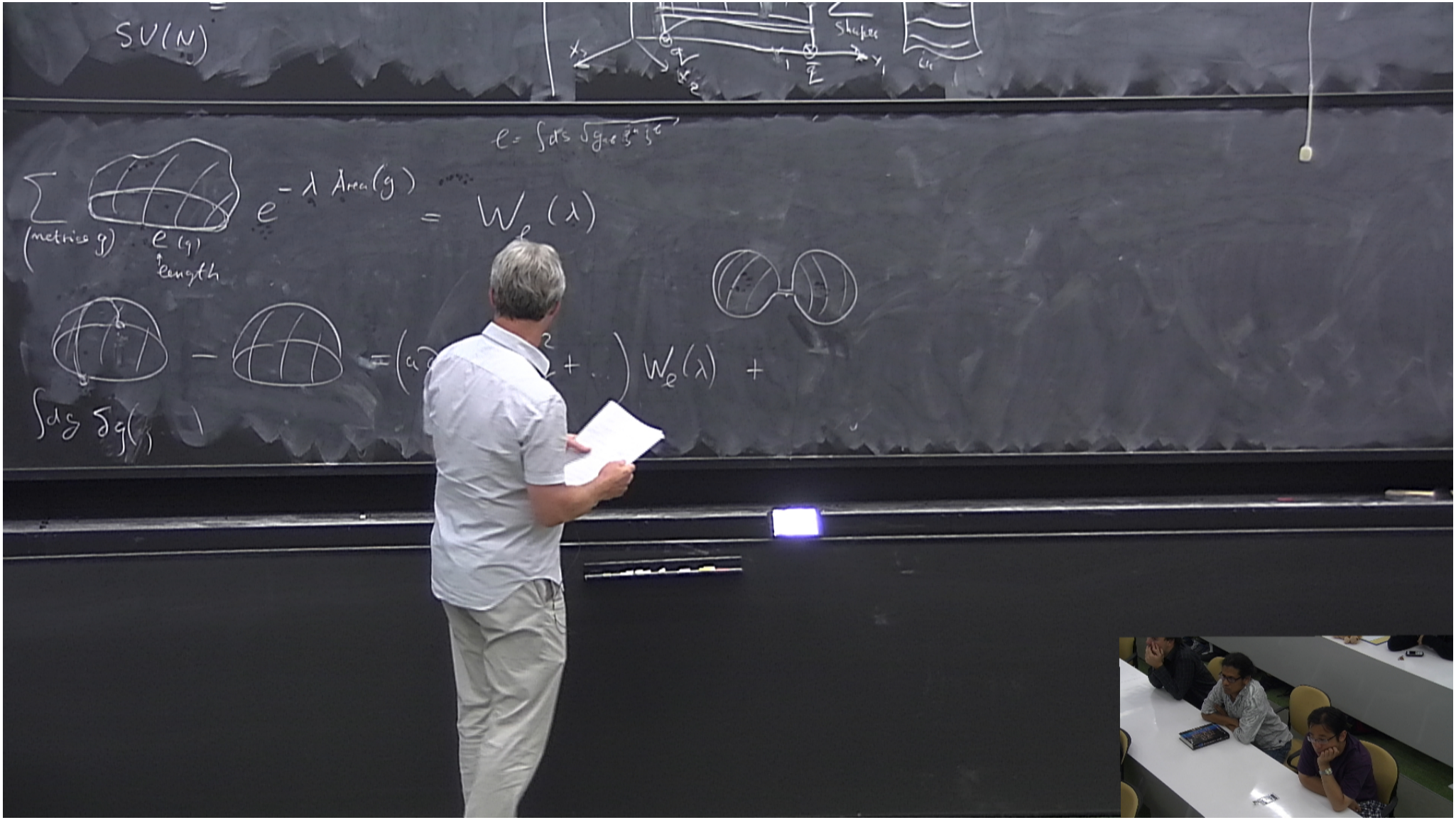

space
R>0
R<0

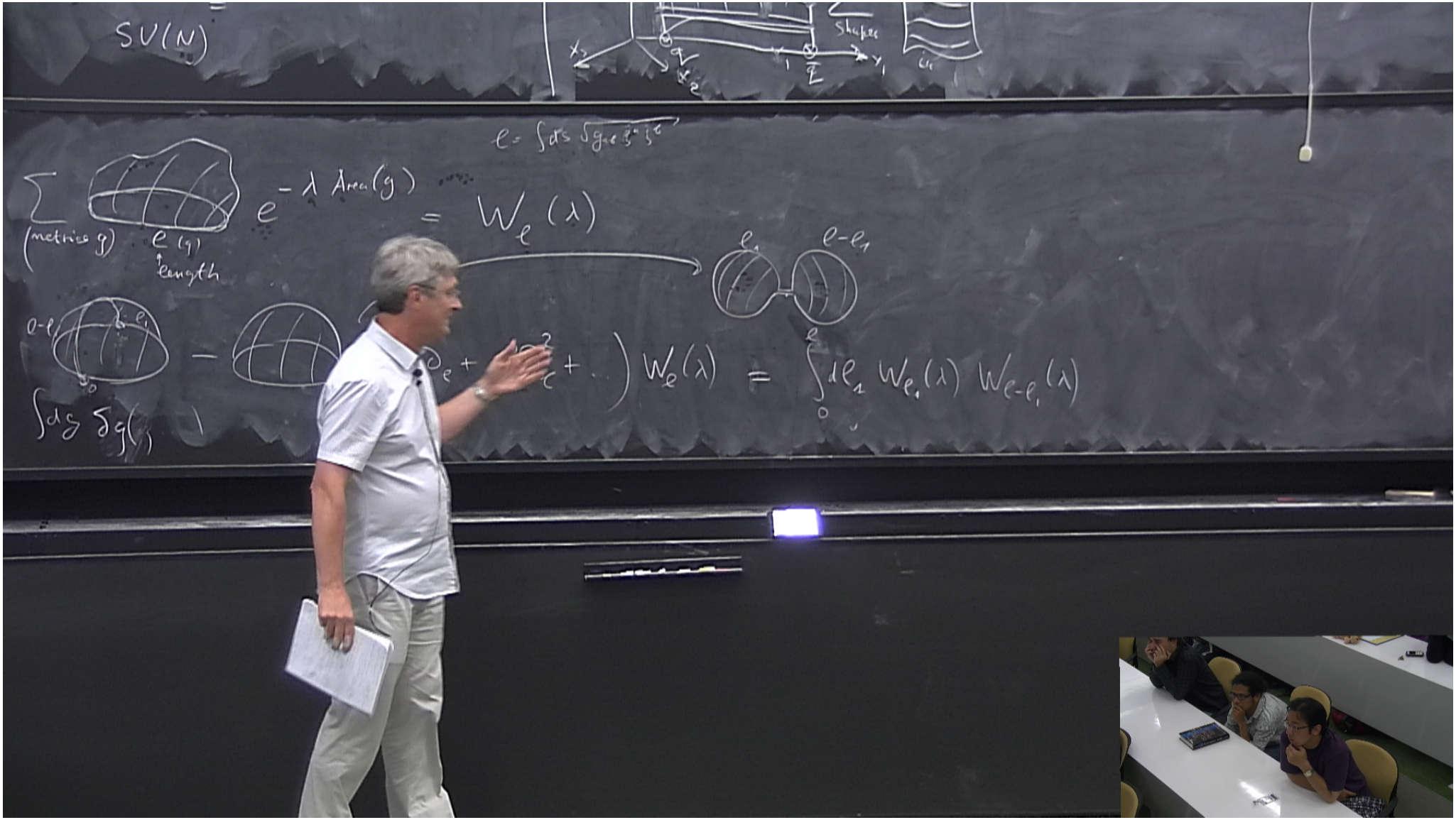

SU(N)
metrics
e
length

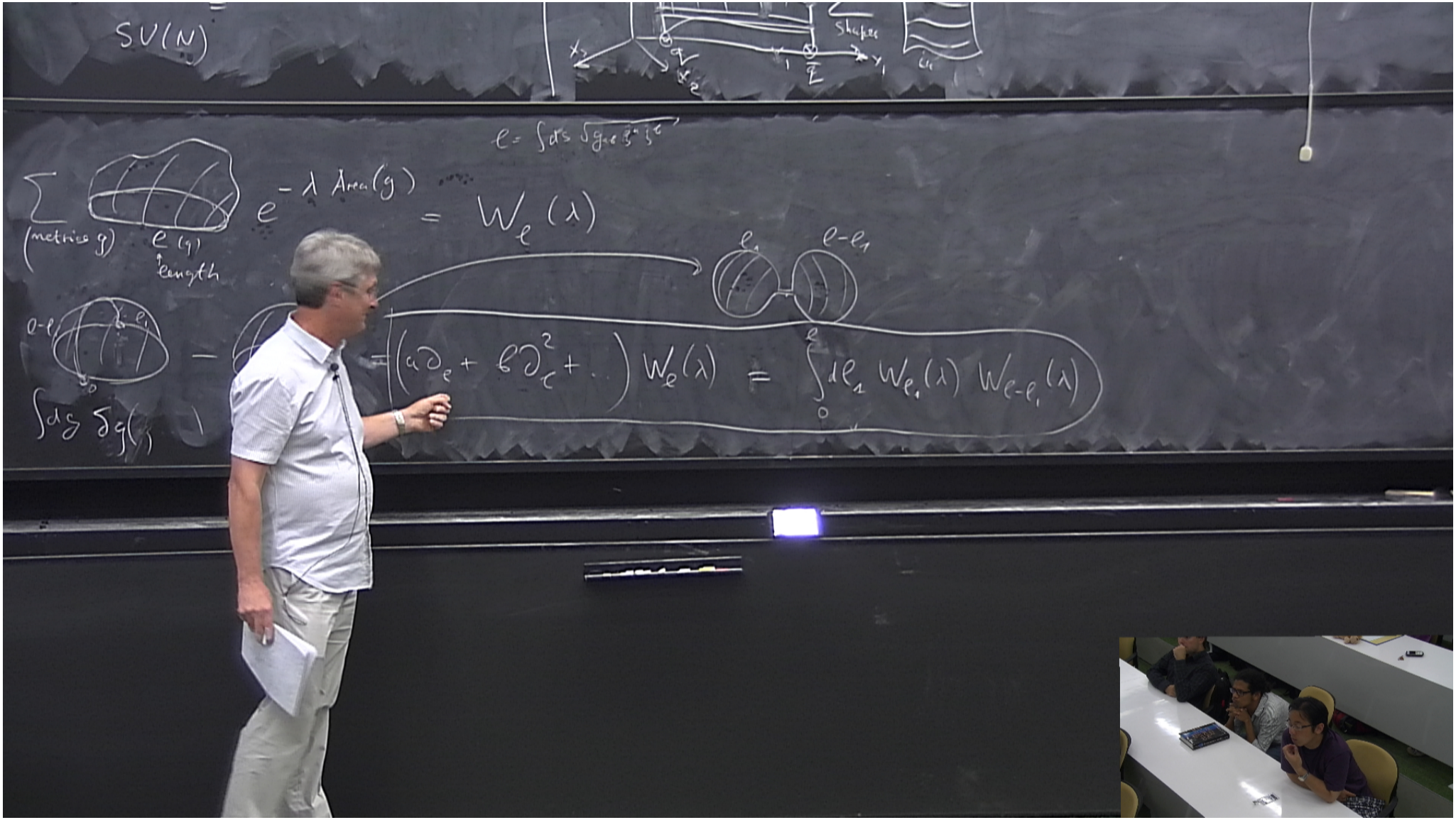

SU(N)
Shakes
$e^{-\lambda\,\mathrm{Area}(g)}$
$e(g)$
length

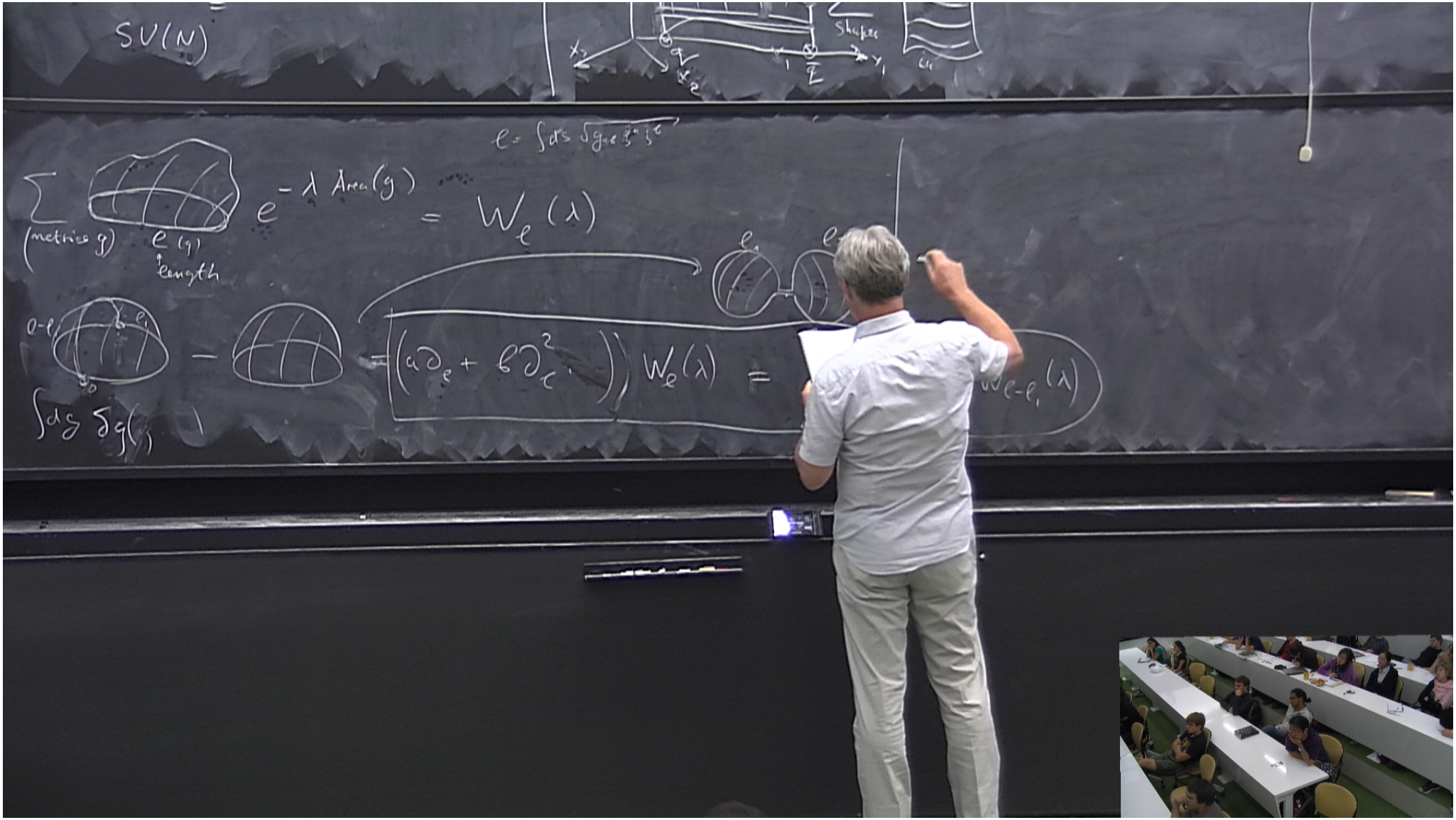

SU(N)
Shapes
$\sum_{\text{(metrics } g)} e^{-\lambda \text{Area}(g)} = W_\ell(\lambda)$
$\ell(g)$
length

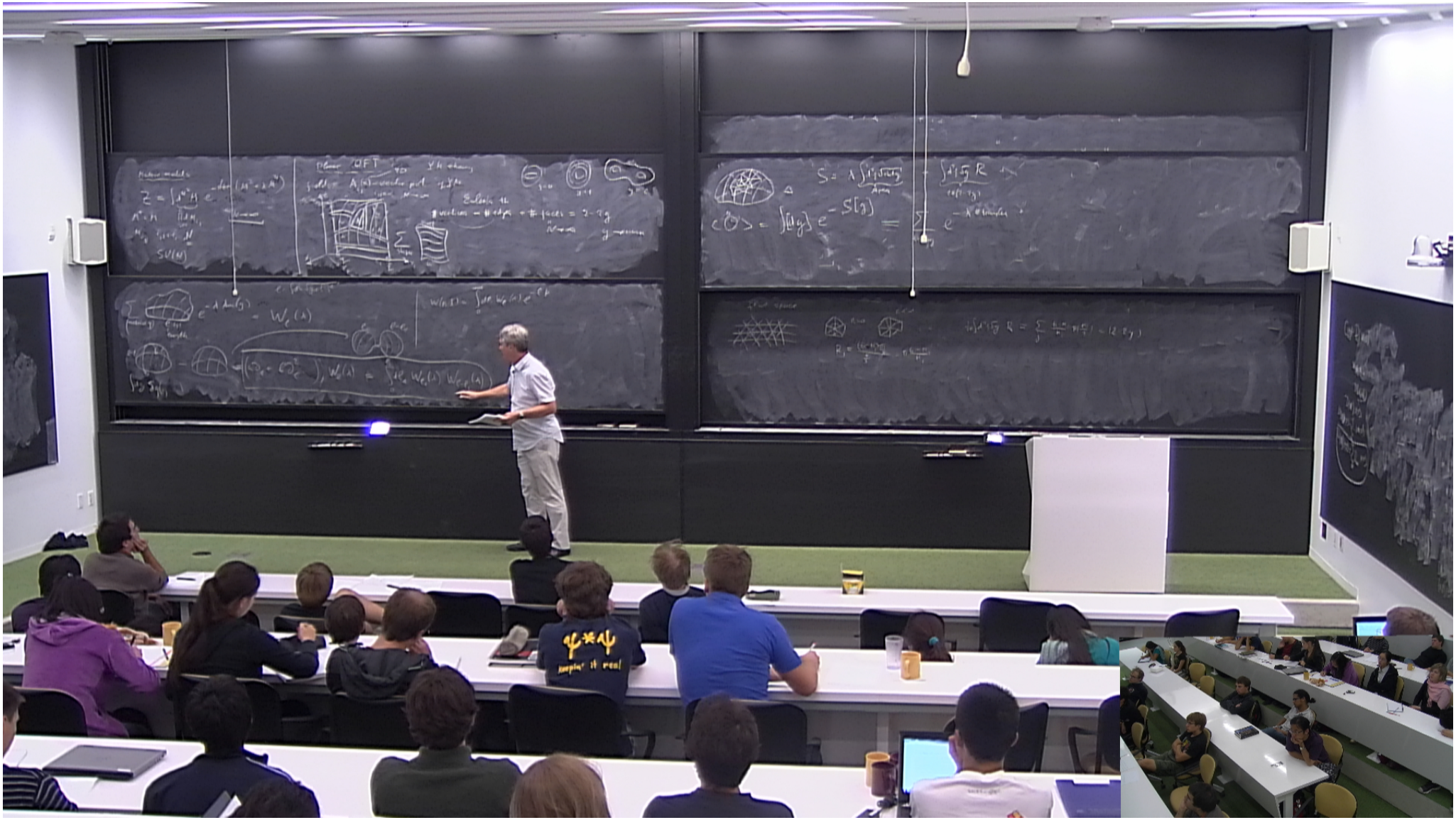

SU(N)
Shakes
$\ell = \int ds \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$
$\sum_{\text{(metrics } g)} e^{-\lambda \text{Area}(g)} = W_\ell(\lambda)$
$\ell(g)$
length

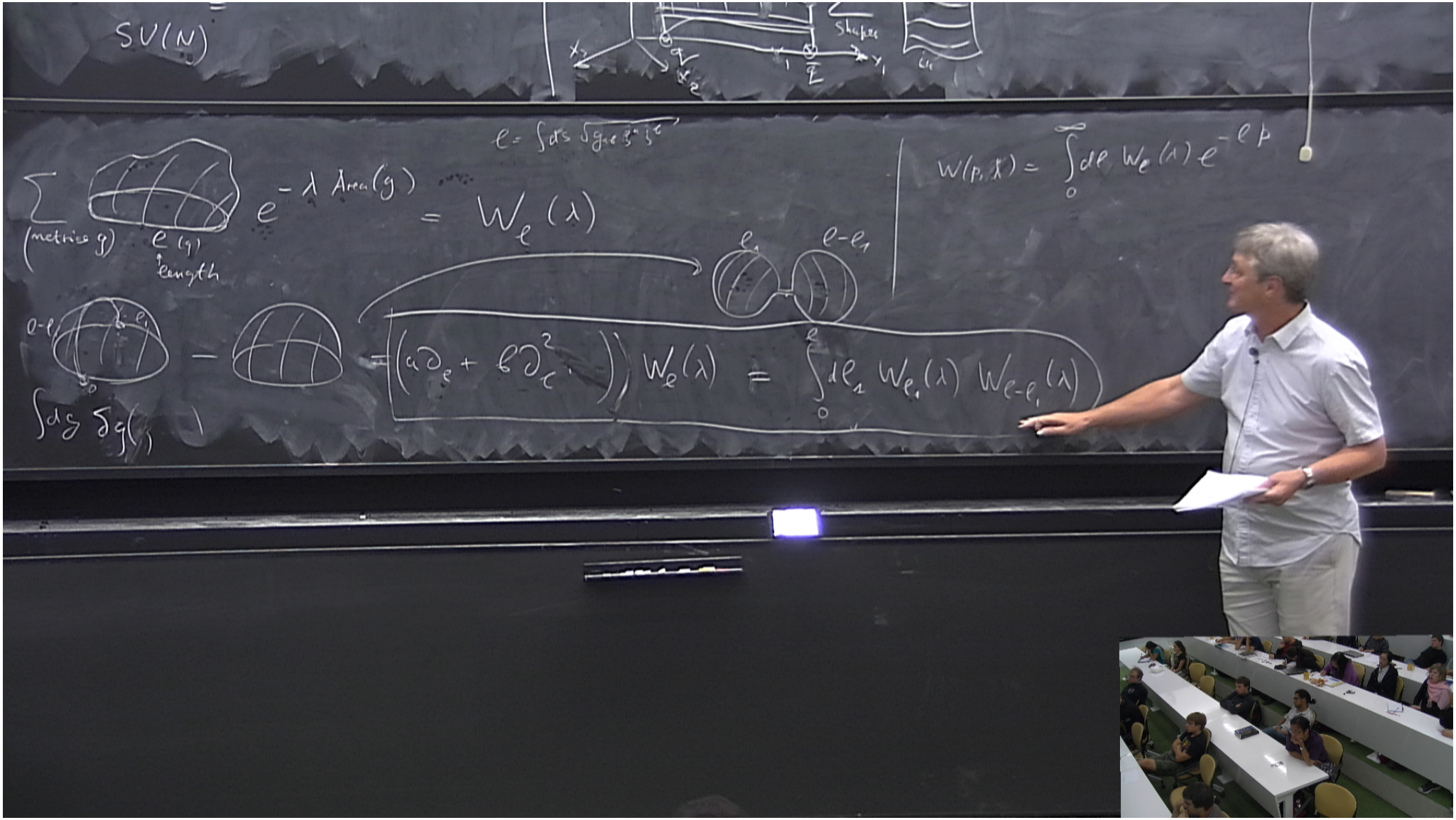

SU(N)
Shapes
$\ell = \int ds \sqrt{g_{\alpha\beta}\dot{\xi}^\alpha \dot{\xi}^\beta}$
$\sum_{(\text{metrics } g)} e^{-\lambda \text{Area}(g)} = W_e(\lambda)$
$e(g)$
length
$\int dg\, \delta g($
$W_e(\lambda)$ +

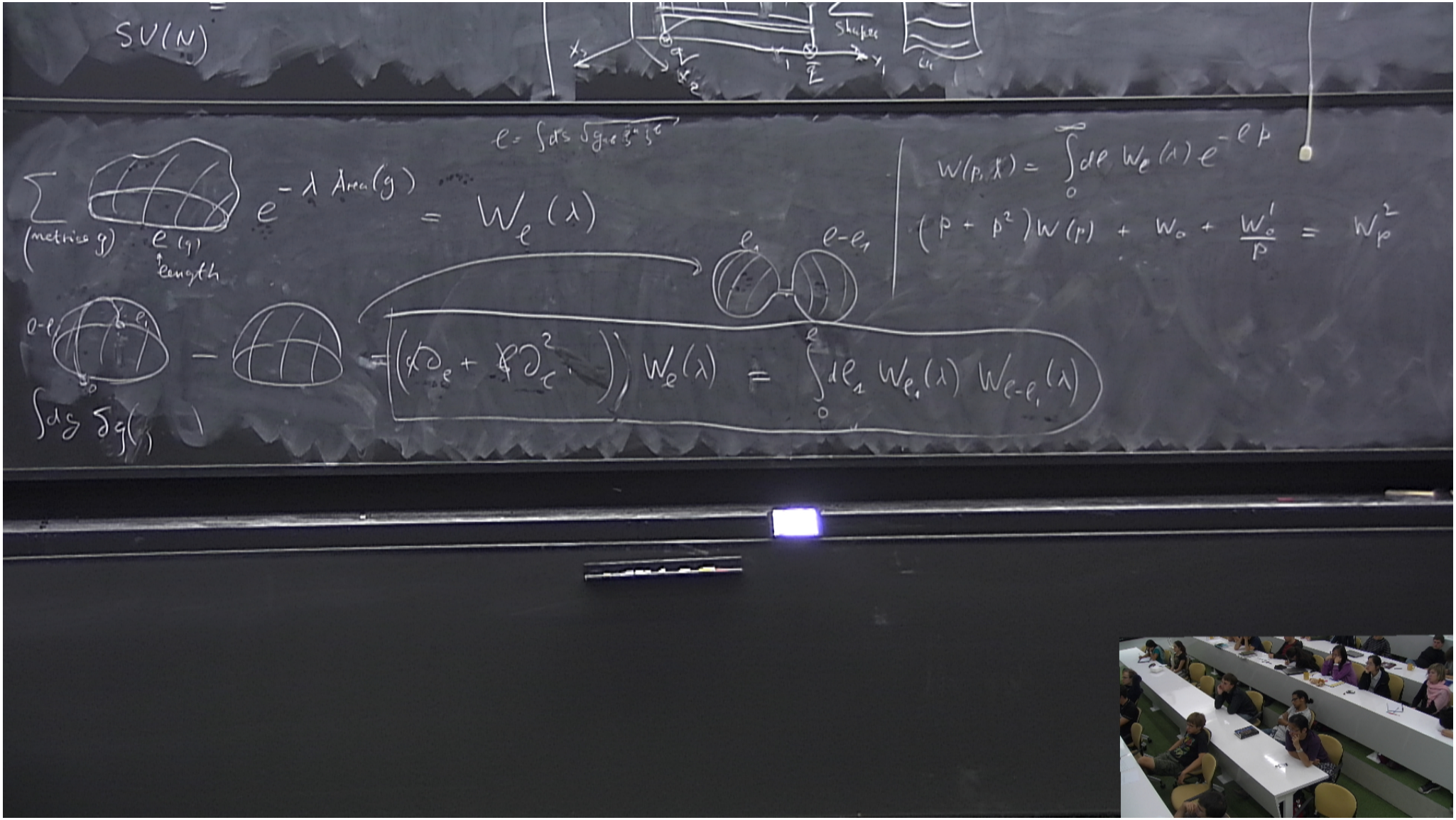

SU(N)
Shapes
$e = \int ds \sqrt{g_{ab} \dot{x}^a \dot{x}^b}$
$\sum_{\text{(metrics g)}} e^{-\lambda \text{Area}(g)} = W_\ell(\lambda)$
$\ell(g)$
length
ℓ_1
$\ell - \ell_1$
$\int dg\, \delta g(\,)$
$W_\ell(\lambda) = \int_0^\ell d\ell_1\, W_{\ell_1}(\lambda)\, W_{\ell-\ell_1}(\lambda)$

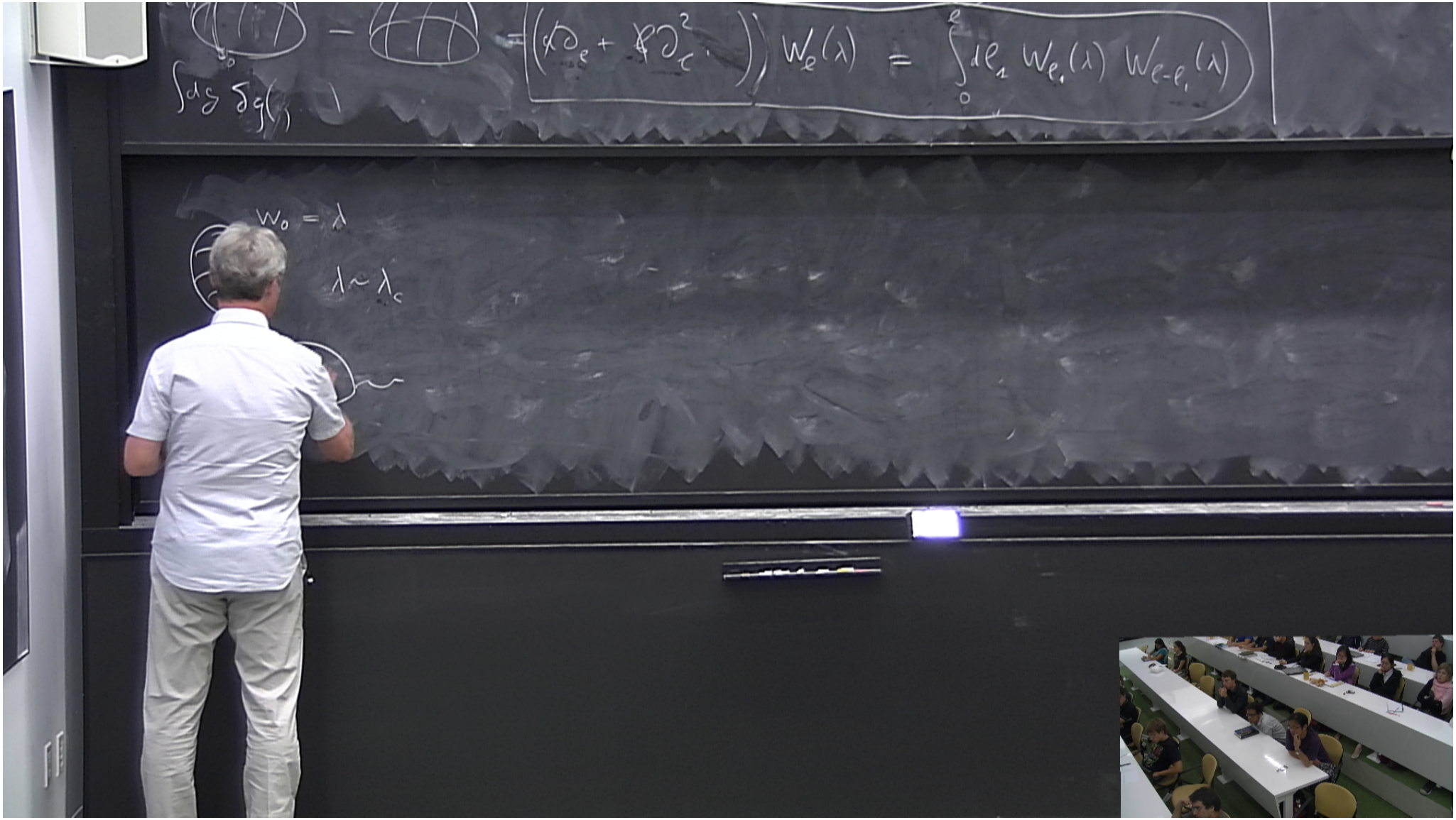

SU(N)
Shapes
$\epsilon = \int ds \sqrt{g_{ab} \dot{x}^a \dot{x}^b}$
$\sum_{\text{(metrics g)}} e^{-\lambda \text{Area}(g)} = W_\ell(\lambda)$
$\ell(g)$
length
ℓ_1
$\ell - \ell_1$
$\int dg\, \delta g(\ell)$
$= \left(a\partial_\ell + b\partial_\ell^2 + \ldots\right) W_\ell(\lambda) = \int_0^\ell d\ell_1\, W_{\ell_1}(\lambda)\, W_{\ell-\ell_1}(\lambda)$

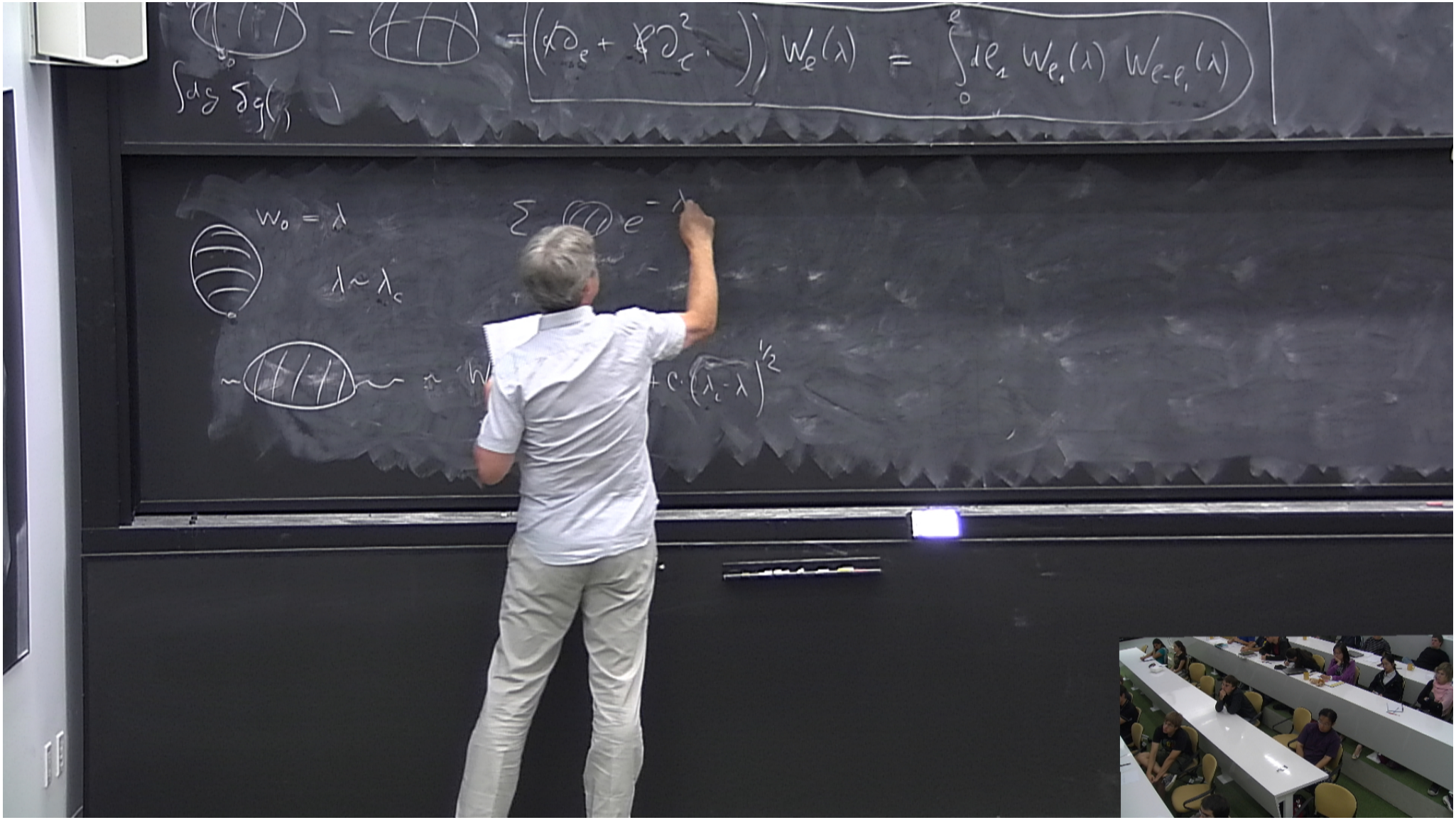

SU(N)
Shaker
$e = \int ds \sqrt{g_{ab} \dot{x}^a \dot{x}^b}$
$\sum_{\text{(metrics } g)} e^{-\lambda \text{Area}(g)} = W_e(\lambda)$
e (g)
length
$\int dg\, \delta g(\,)$
$= (a\partial_s + b\partial_s^2 \cdot) W_e(\lambda) =$
$W_{e-e_1}(\lambda)$

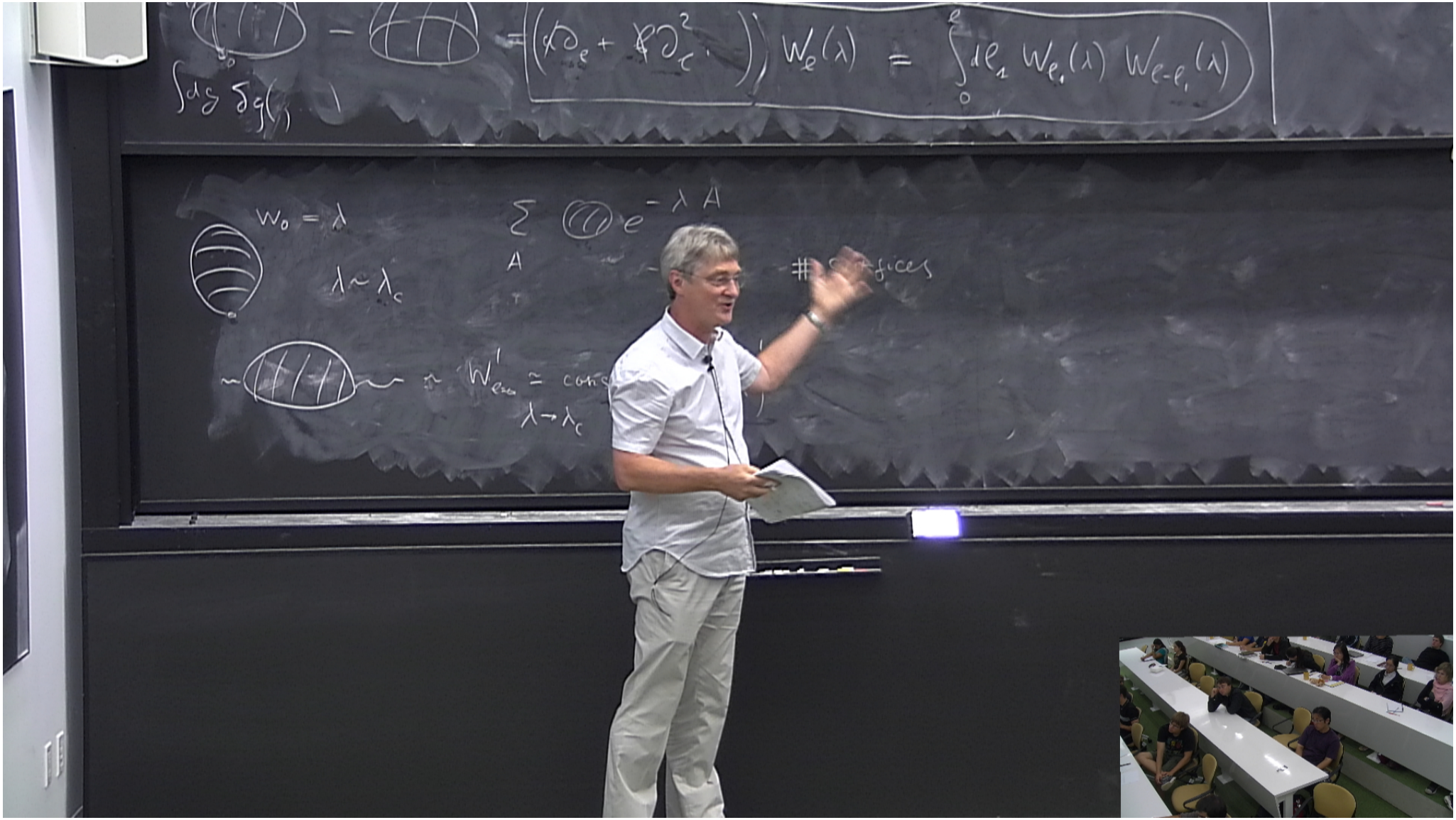


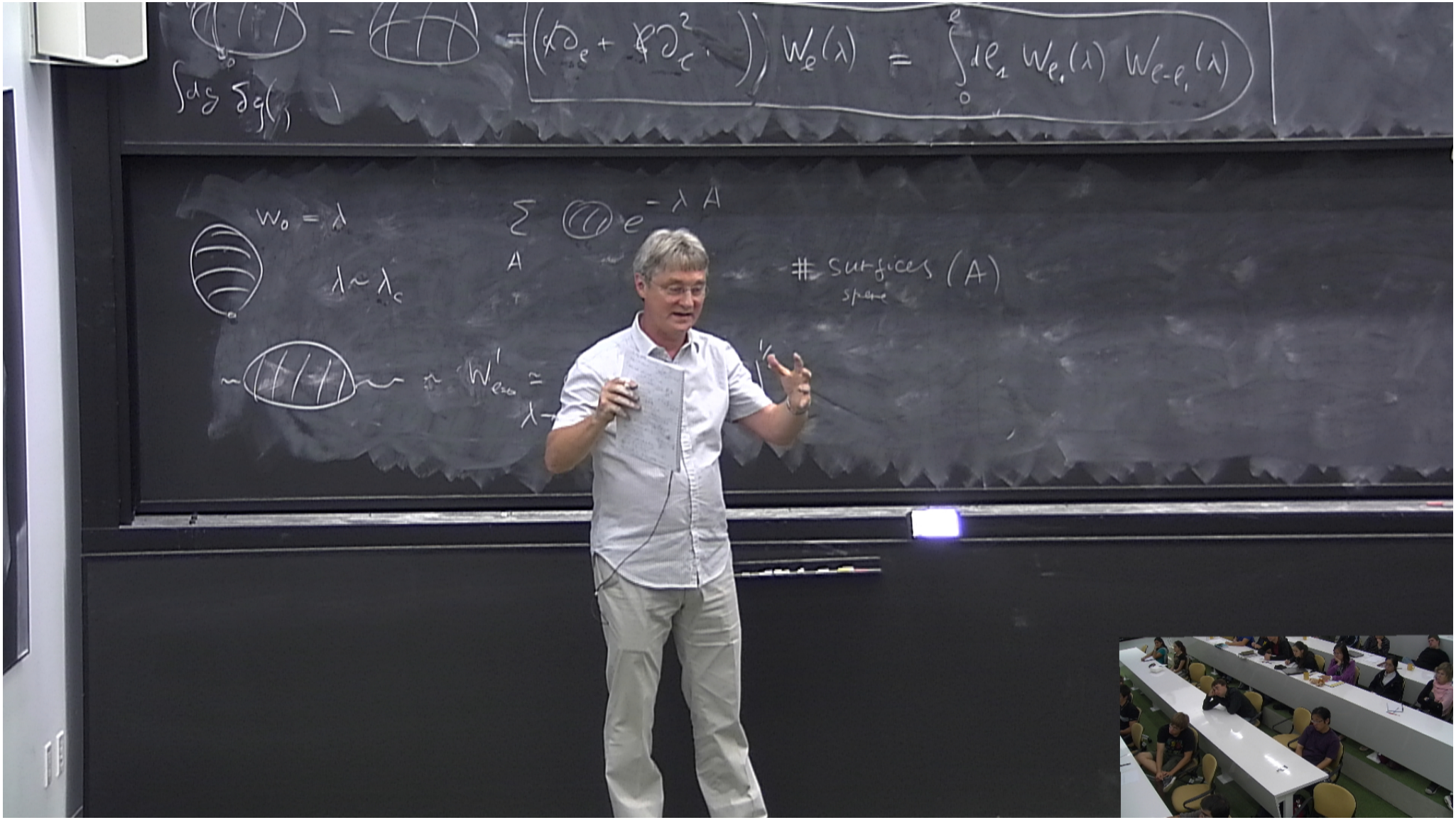

SU(N)
Shaker
$e = \int ds \sqrt{g_{ab} \dot{x}^a \dot{x}^b}$
$\sum_{\text{(metrics g)}} e^{-\lambda \, \text{Area}(g)} = W_\ell(\lambda)$
$\ell(g)$
length
$W(\ell, \lambda) = \int_0^\infty d\ell \, W_\ell(\lambda) \, e^{-\ell \mu}$
ℓ_1
$\ell - \ell_1$
ℓ
$0 - \ell$
$\int dg \, \delta g(\,)$
$= \left(a \partial_\ell + b \partial_\ell^2 \right) W_\ell(\lambda) = \int_0^\ell d\ell_1 \, W_{\ell_1}(\lambda) \, W_{\ell - \ell_1}(\lambda)$

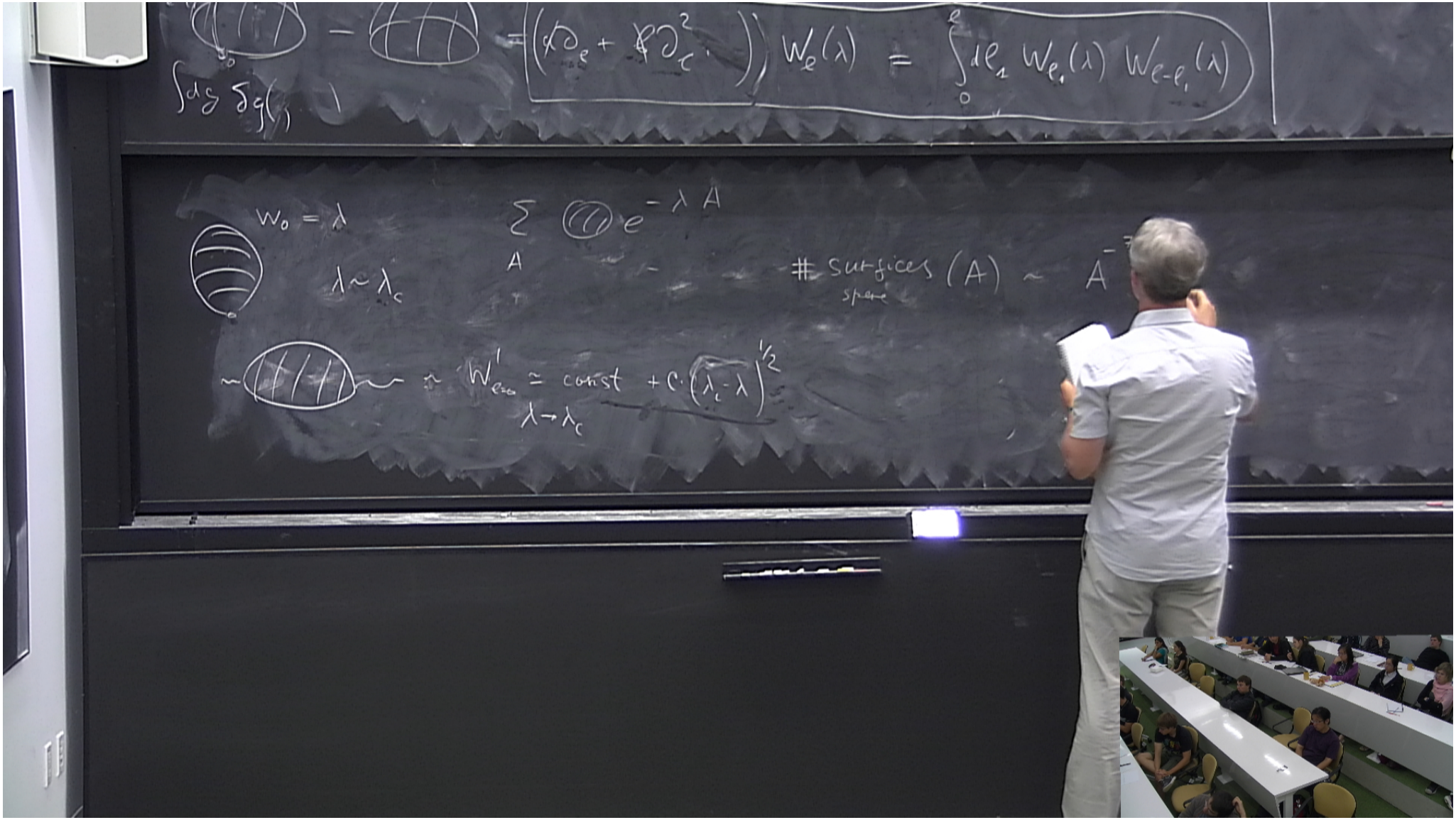

SU(N)
Shapes
(metrics g)
e (g)
length
$\sum_{(\text{metrics } g)} e^{-\lambda \text{Area}(g)} = W_\ell(\lambda)$
$\ell = \int ds \sqrt{g_{ab} \dot{x}^a \dot{x}^b}$
$W(p,\lambda) = \int_0^\infty d\ell\, W_\ell(\lambda)\, e^{-\ell p}$
$(p + p^2) W(p) + W_0 + \frac{W_0'}{p} = W_p^2$
ℓ_1
$\ell - \ell_1$
$0-\ell$
$\int dg\, \delta g(\,)$
$= \left(\alpha \partial_\ell + \beta \partial_\ell^2\right) W_\ell(\lambda) = \int_0^\ell d\ell_1\, W_{\ell_1}(\lambda)\, W_{\ell-\ell_1}(\lambda)$

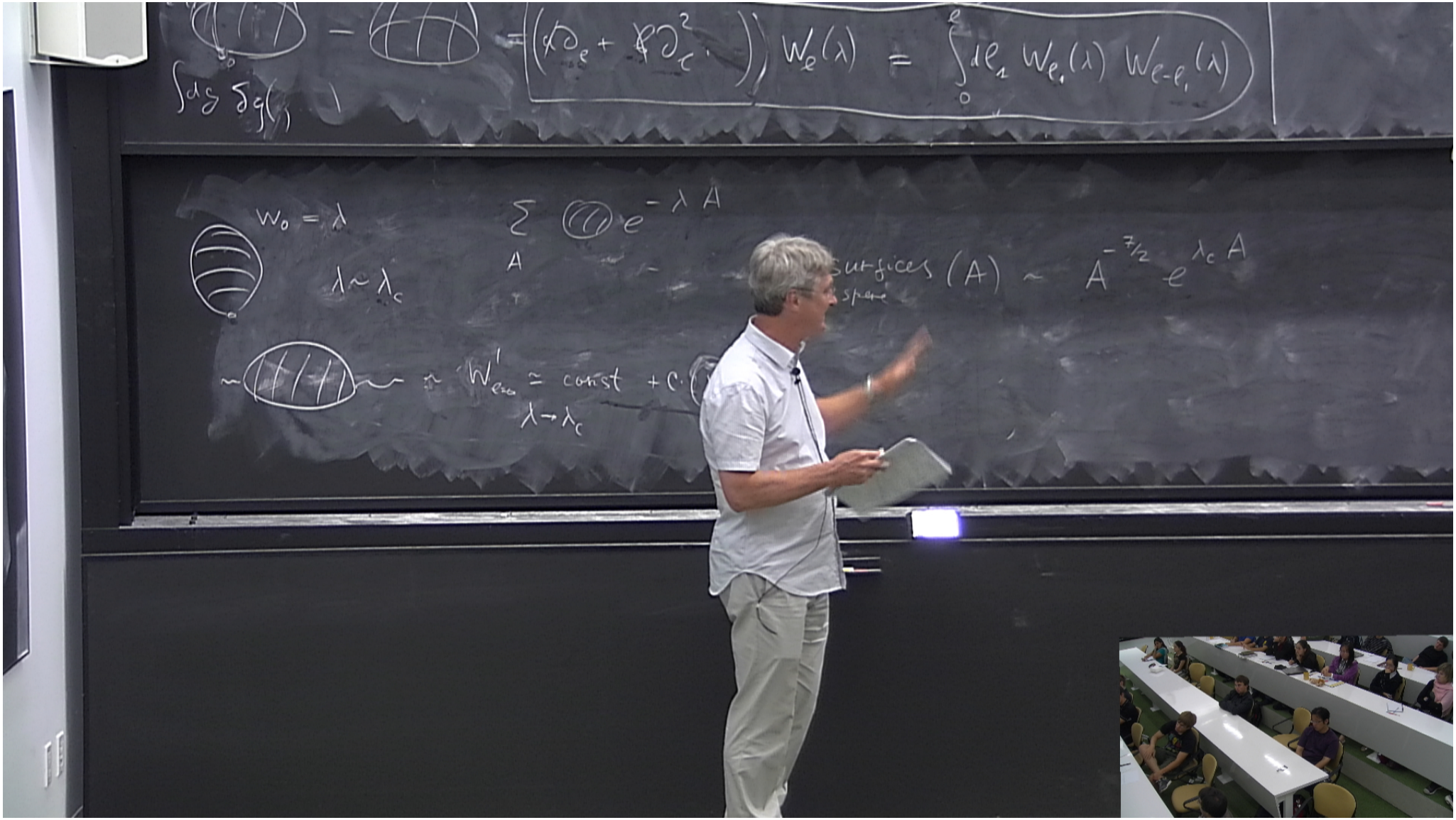

$W_0 = \lambda$
$\lambda \sim \lambda_c$

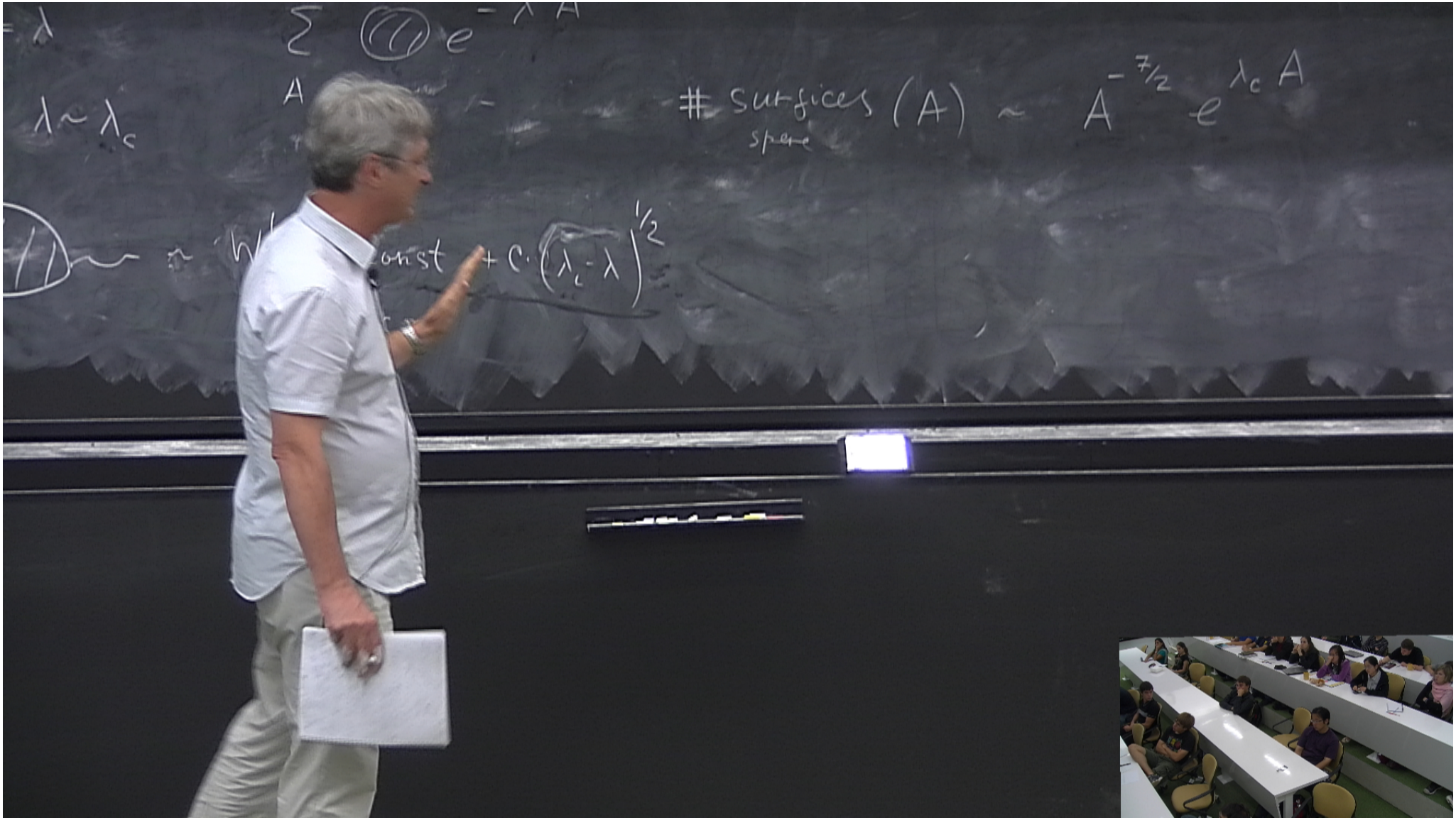

$W_0 = \lambda$
$\lambda \sim \lambda_c$
$W_\ell(\lambda) = \int_0^\ell d\ell_1 \, W_{\ell_1}(\lambda) \, W_{\ell-\ell_1}(\lambda)$
$+ C \cdot (\lambda_c - \lambda)^{1/2}$

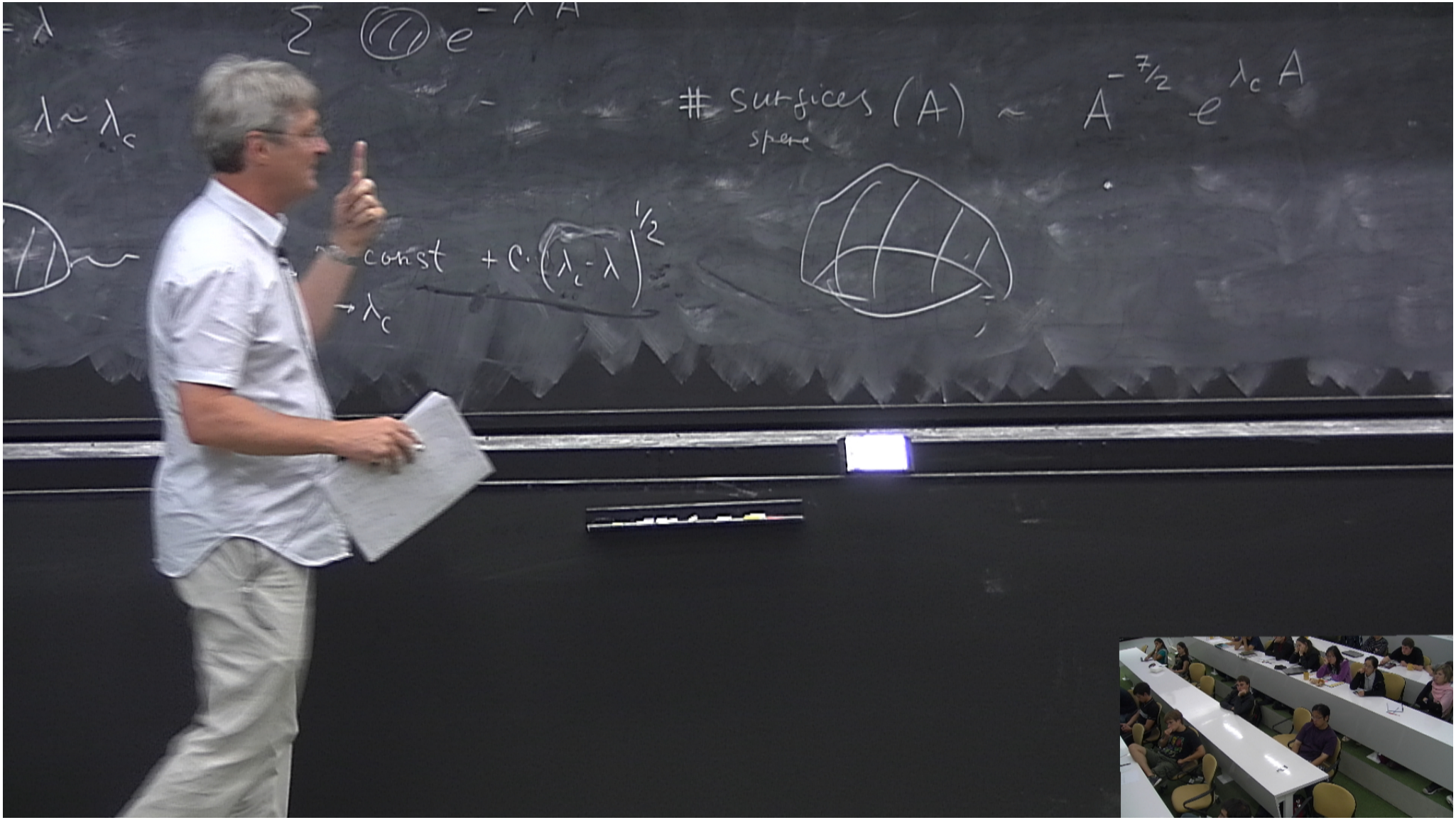

$W_0 = \lambda$
$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
$\lambda \to \lambda_c$

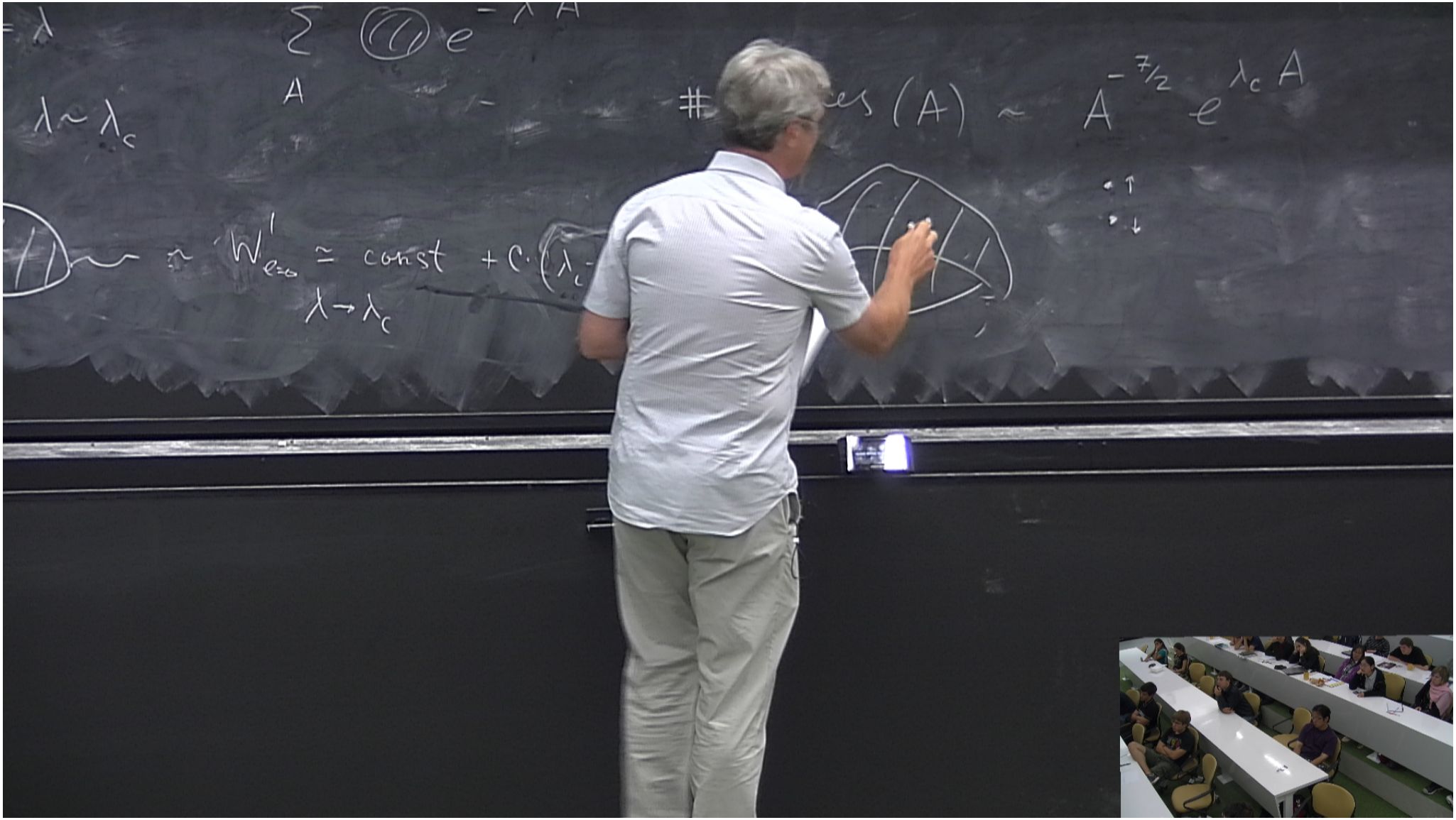

$W_0 = \lambda$
$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
surfaces (A)
space
$W_e(\lambda) = \int_0^e de_1 \, W_{e_1}(\lambda) \, W_{e-e_1}(\lambda)$

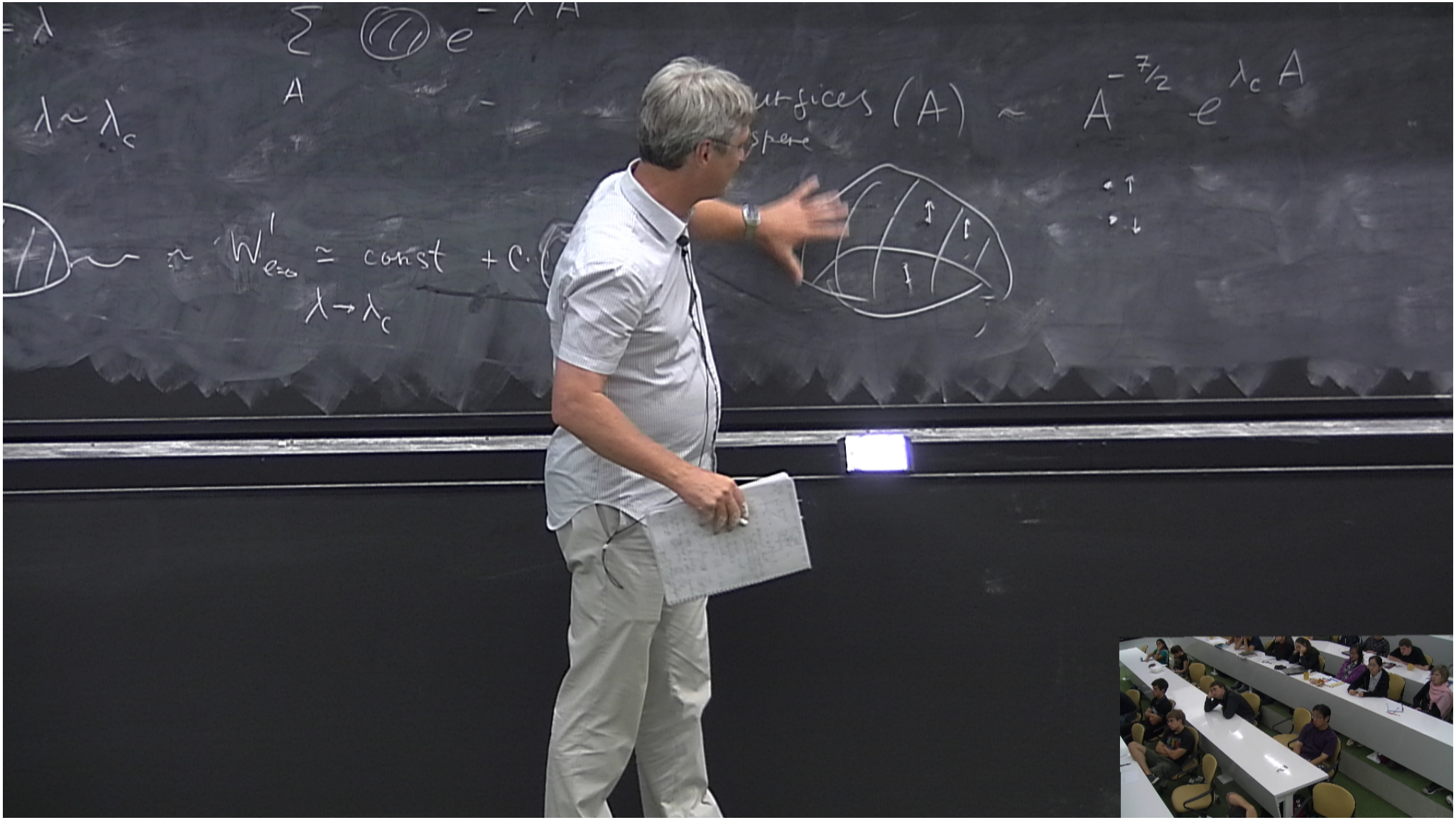

$W_e(\lambda) = \int_0^e de_1 \, W_{e_1}(\lambda) \, W_{e-e_1}(\lambda)$
$W_0 = \lambda$
$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
surfaces (A) ~ A^{-}
space
$W'_{\text{loop}} \simeq \text{const} + C \cdot (\lambda_c - \lambda)^{1/2}$
$\lambda \to \lambda_c$

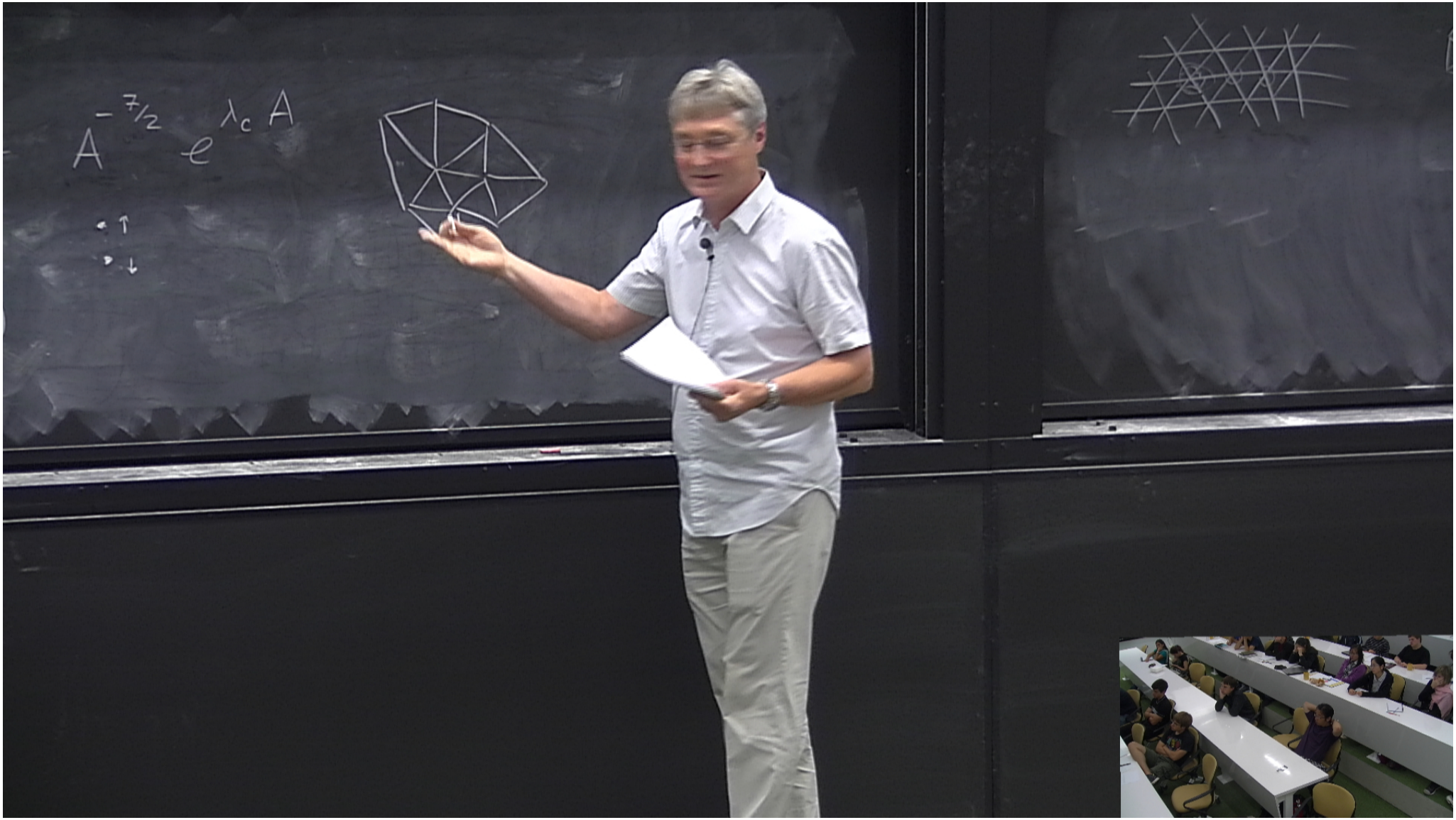

$W_0 = \lambda$
$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
surfaces $(A) \sim A^{-7/2} e^{\lambda_c A}$
sphere
$W'_{\text{conn}} \simeq \text{const} + C \cdot$
$\lambda \to \lambda_c$

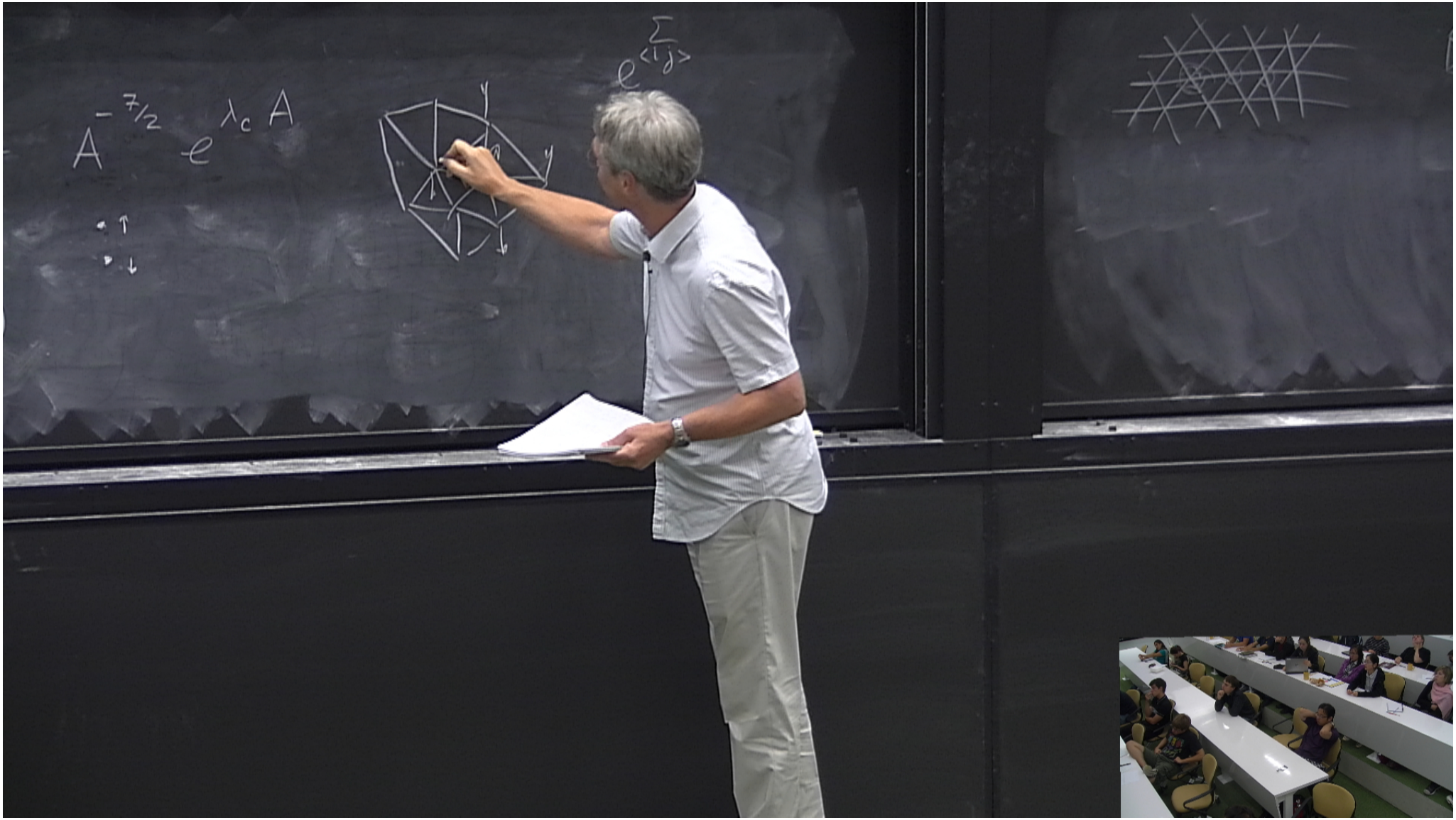

λ ~ λc
surfaces (A) ~ A^{-7/2} e^{λc A}
const + C · (λc - λ)^{1/2}

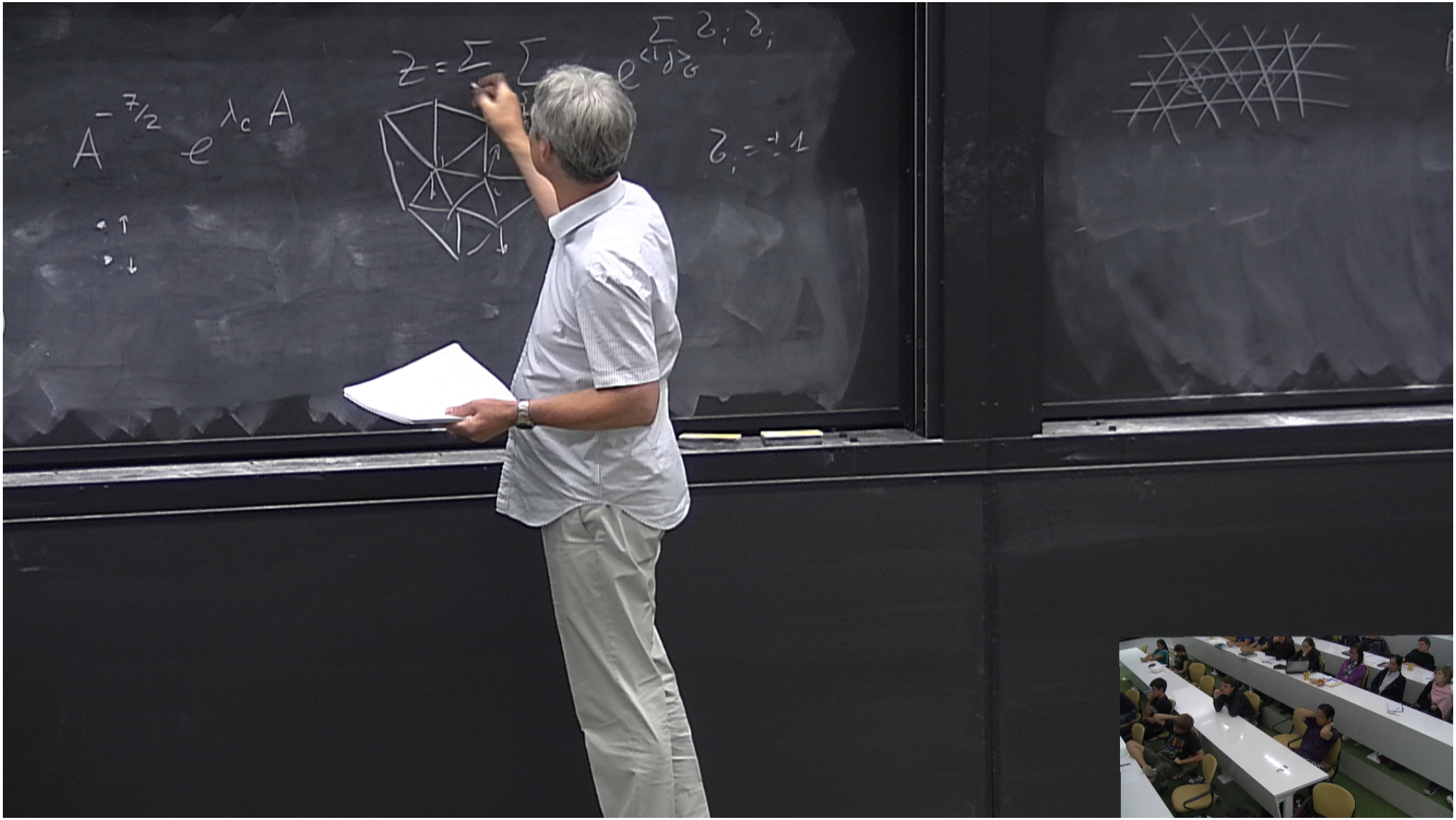

$\lambda \sim \lambda_c$
$\sum$
$e^{-\lambda A}$
surfaces (A) ~ $A^{-7/2}$ $e^{\lambda_c A}$
sphere
const + $C \cdot (\lambda_c - \lambda)^{1/2}$
$\to \lambda_c$

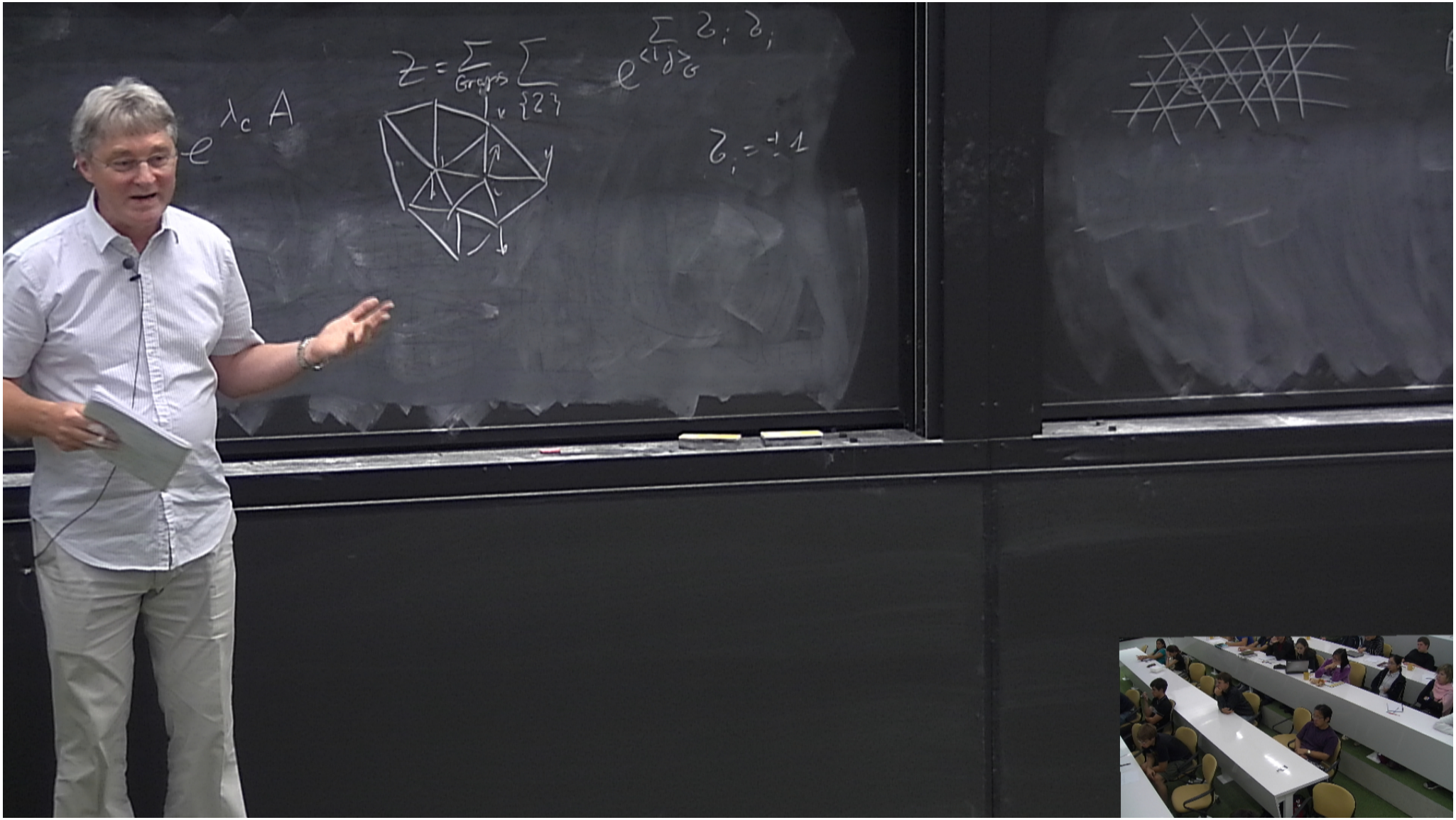

$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
$\#(A) \sim A^{-7/2} e^{\lambda_c A}$
$W'_{\ell=0} \simeq \text{const} + C \cdot (\lambda_c - \lambda)$
$\lambda \to \lambda_c$

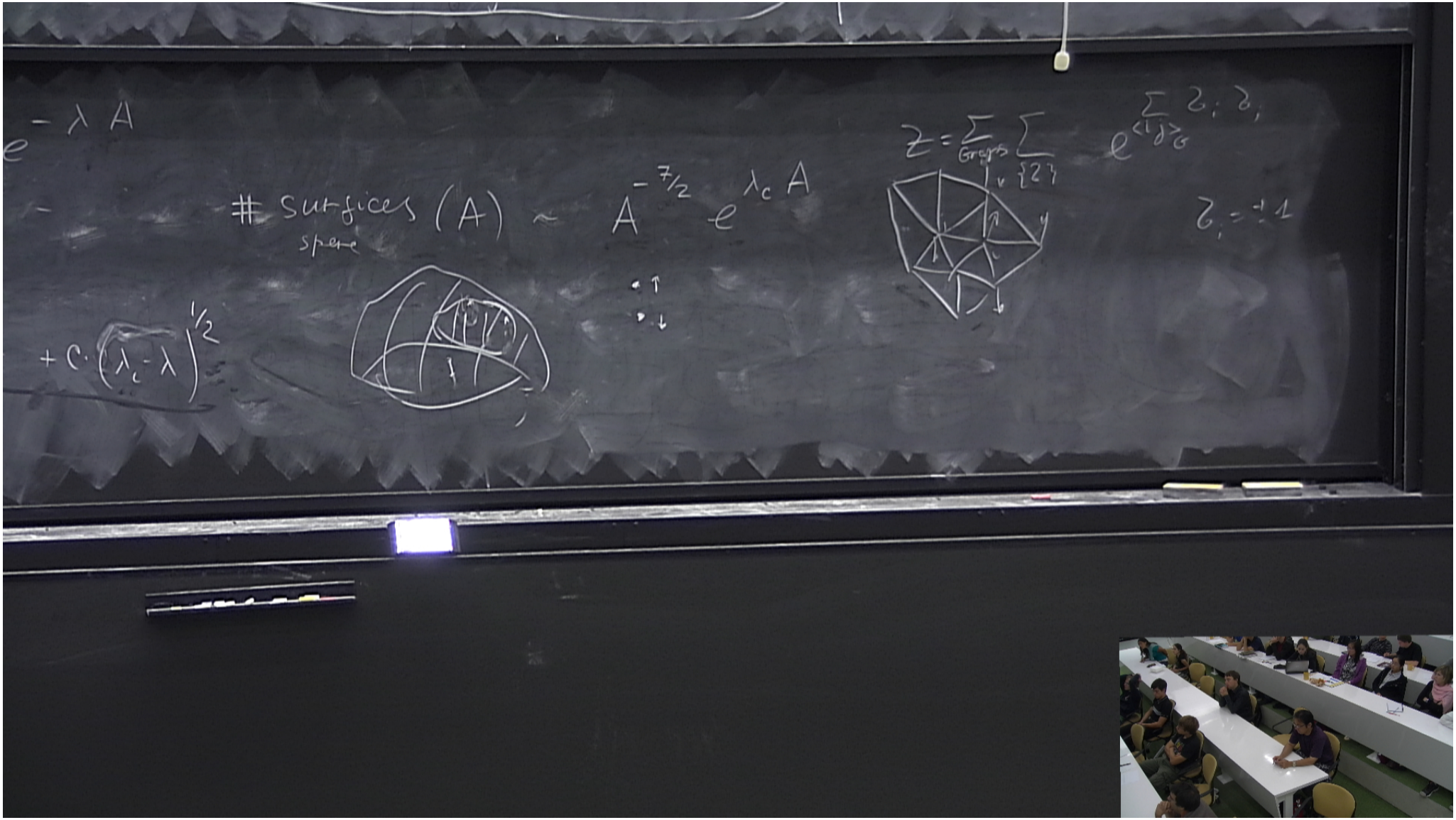

$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
surfaces $(A) \sim A^{-7/2} e^{\lambda_c A}$
sphere
$W'_{\ell=0} \simeq$ const $+ C \cdot$
$\lambda \to \lambda_c$

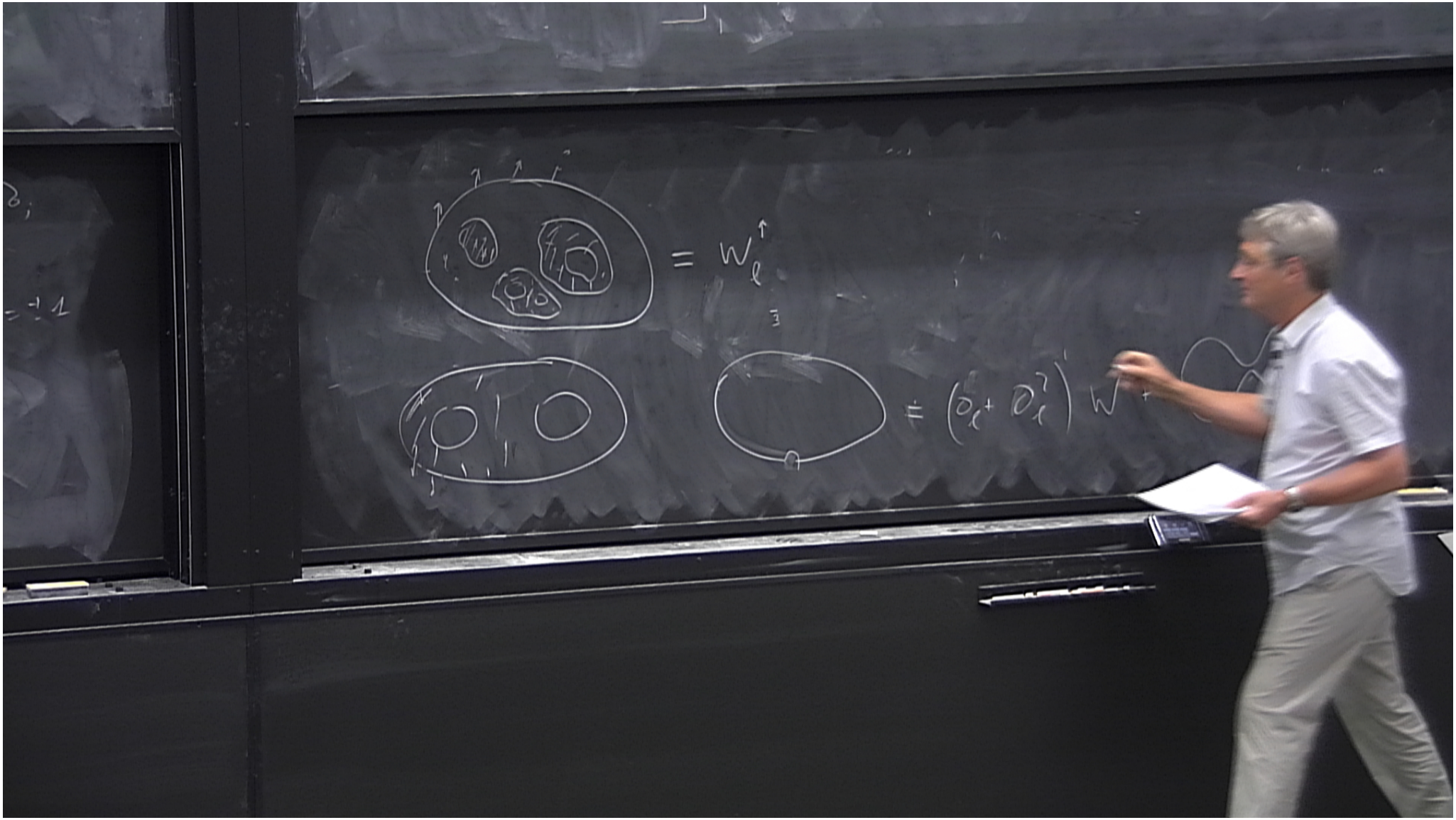

A^{-7/2} e^{λc A}

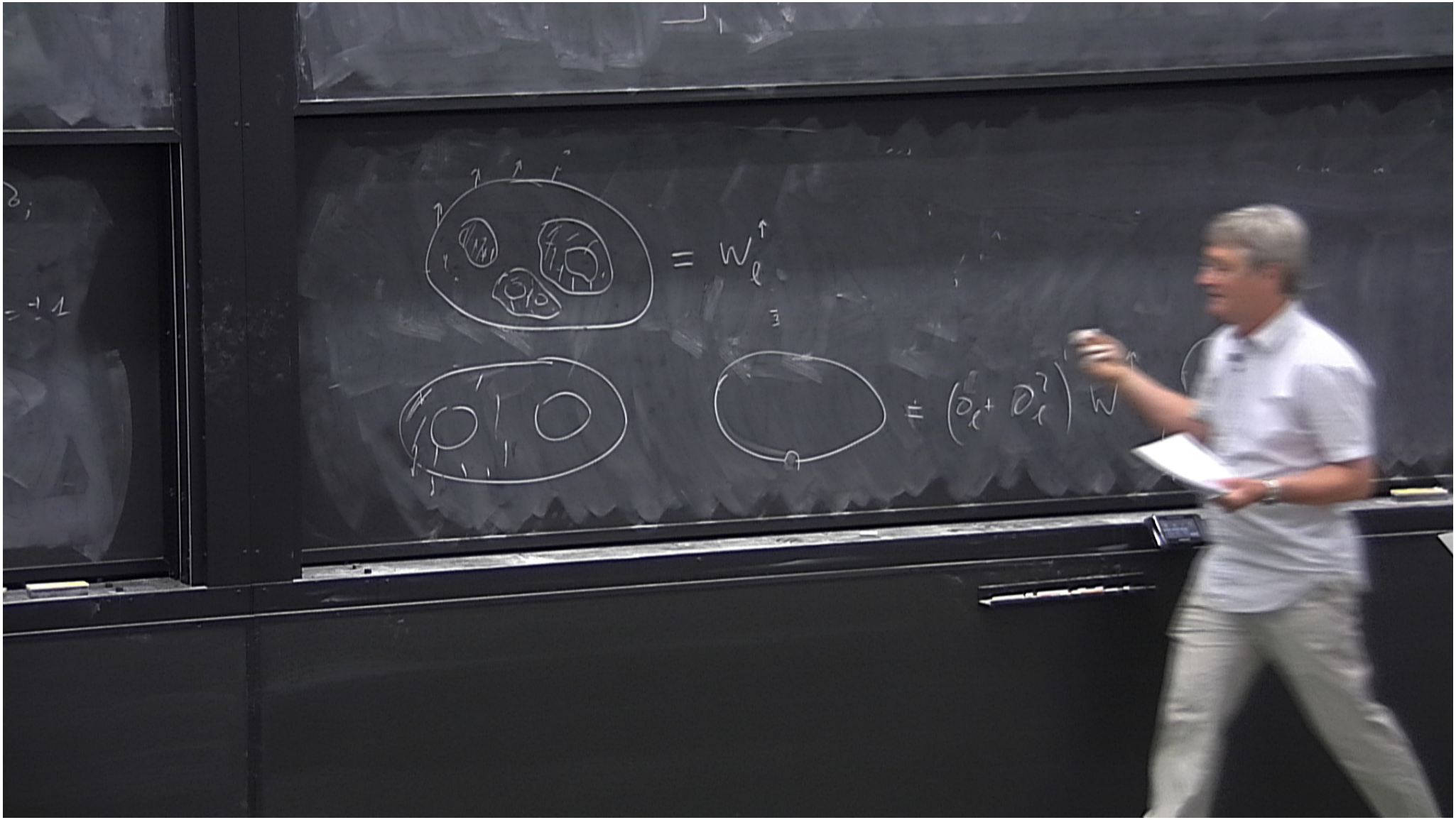

A^{-7/2} e^{λc A}
e^{Σ ⟨ij⟩}

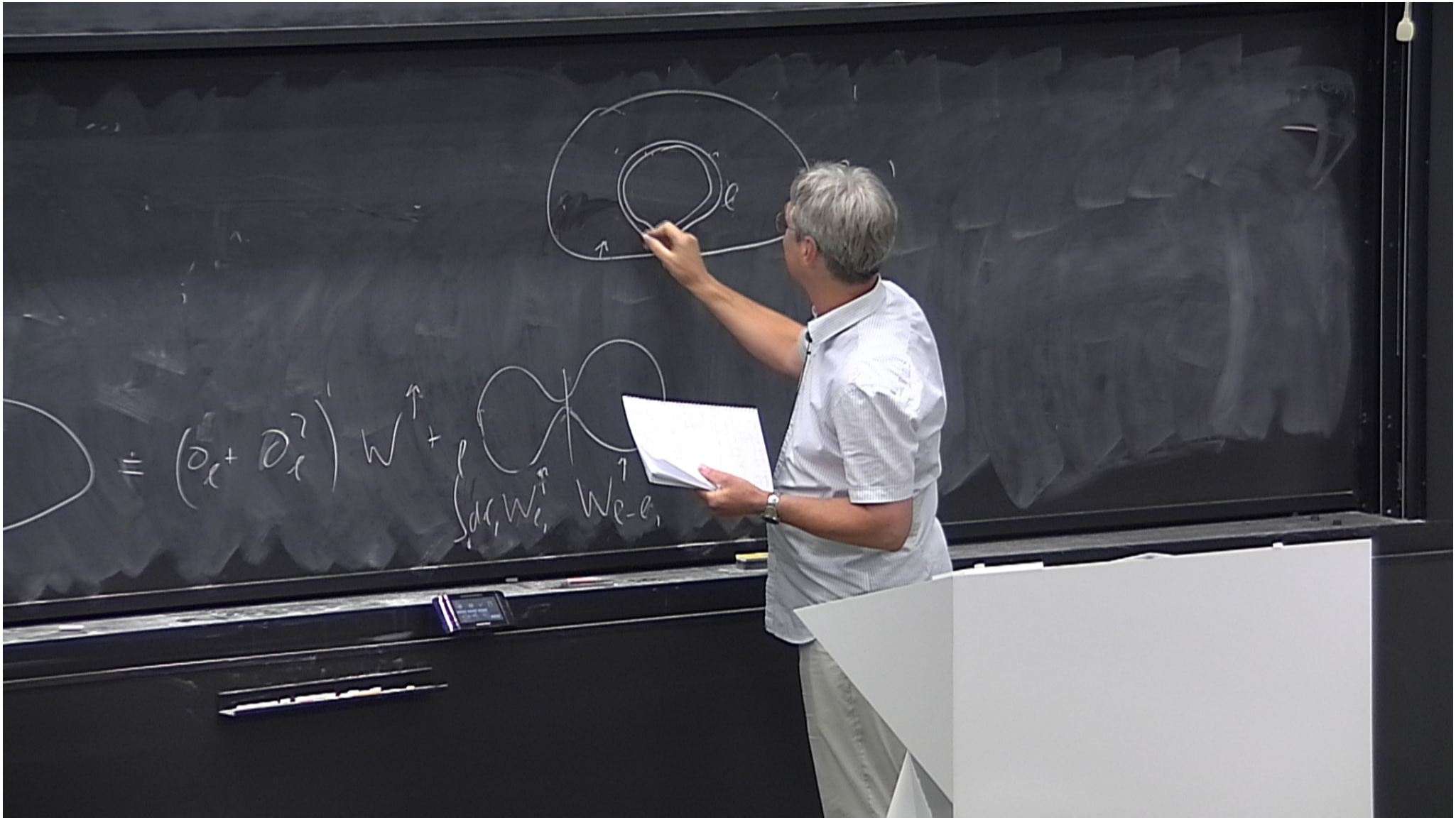

$A^{-z/2}$
$e^{\lambda_c A}$
$Z = \sum \sum$
$e^{\sum_{\langle ij \rangle} \sigma_i \sigma_j}$
$\sigma_i = \pm 1$

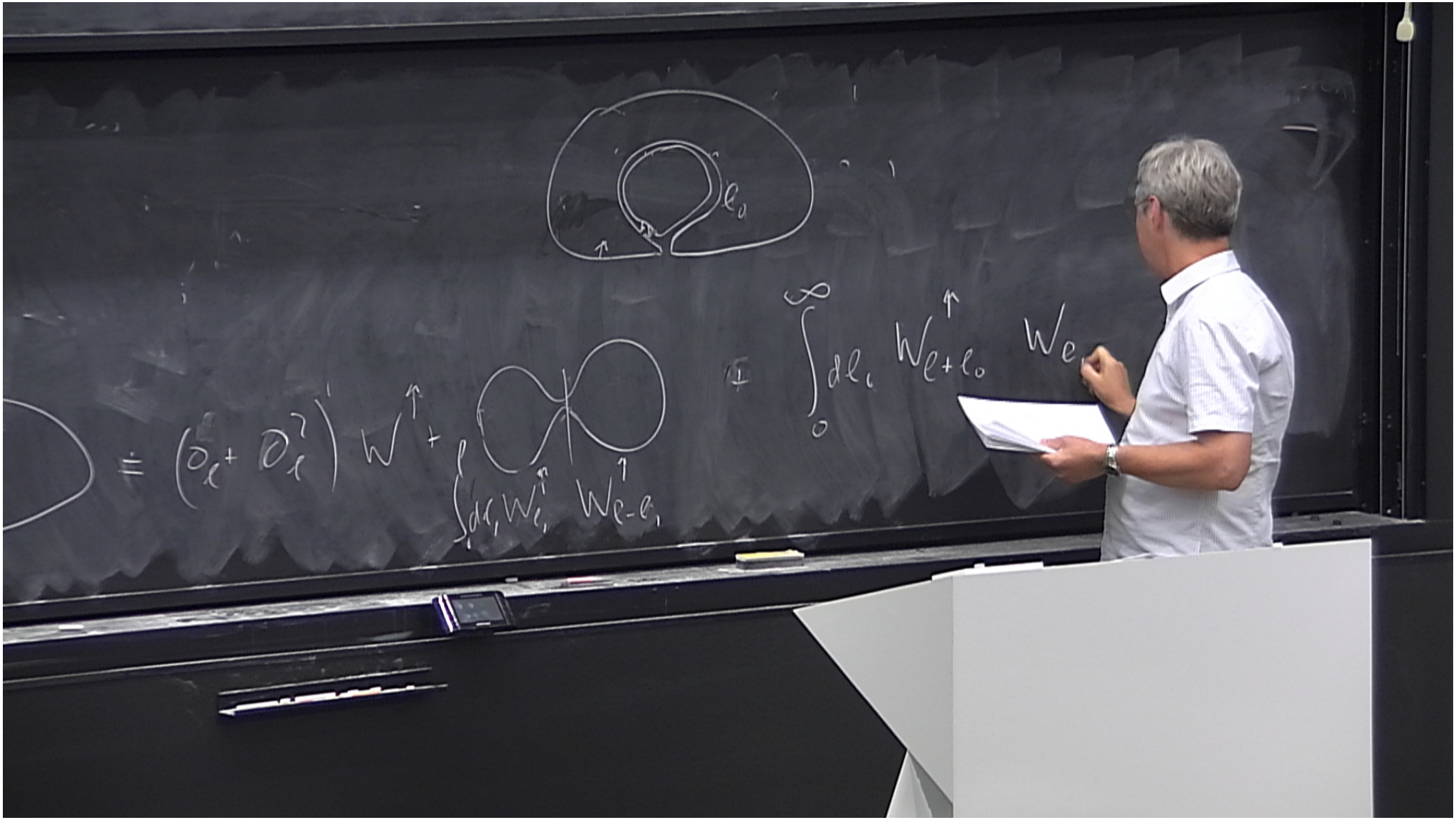

$e^{\lambda_c A}$
$Z = \sum_{\text{Graphs}} \sum_{\{z\}}$
$e^{\sum_{\langle ij \rangle_G} z_i z_j}$
$z_i = \pm 1$

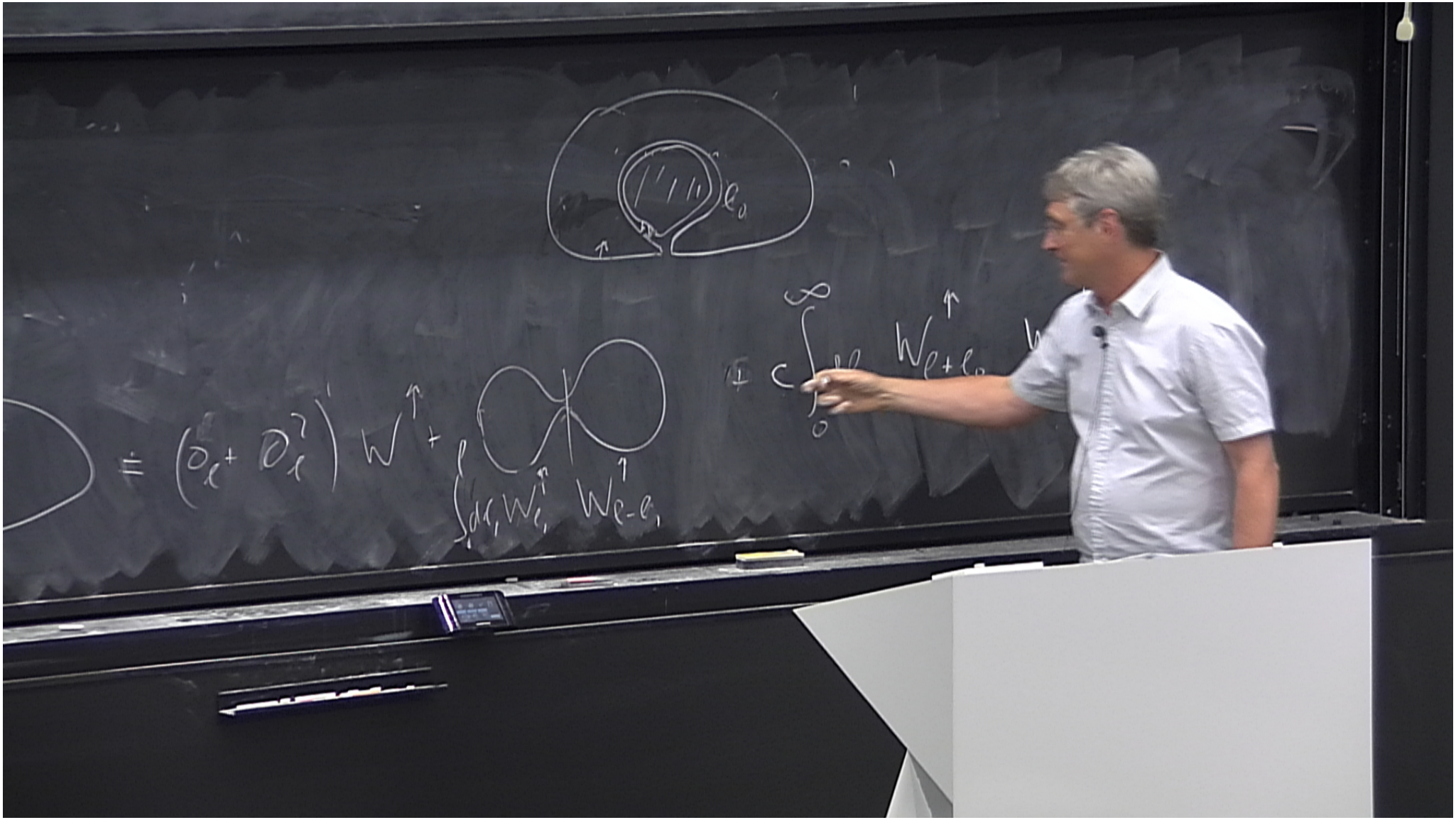

$e^{-\lambda A}$
surfaces (A) ~ $A^{-7/2} e^{\lambda_c A}$
sphere
$+C\cdot(\lambda_c-\lambda)^{1/2}$
$Z = \sum_{\text{Graphs}} \sum_{\{\sigma_i\}} e^{\sum_{\langle ij\rangle} \sigma_i \sigma_j}$
$\sigma_i = \pm 1$

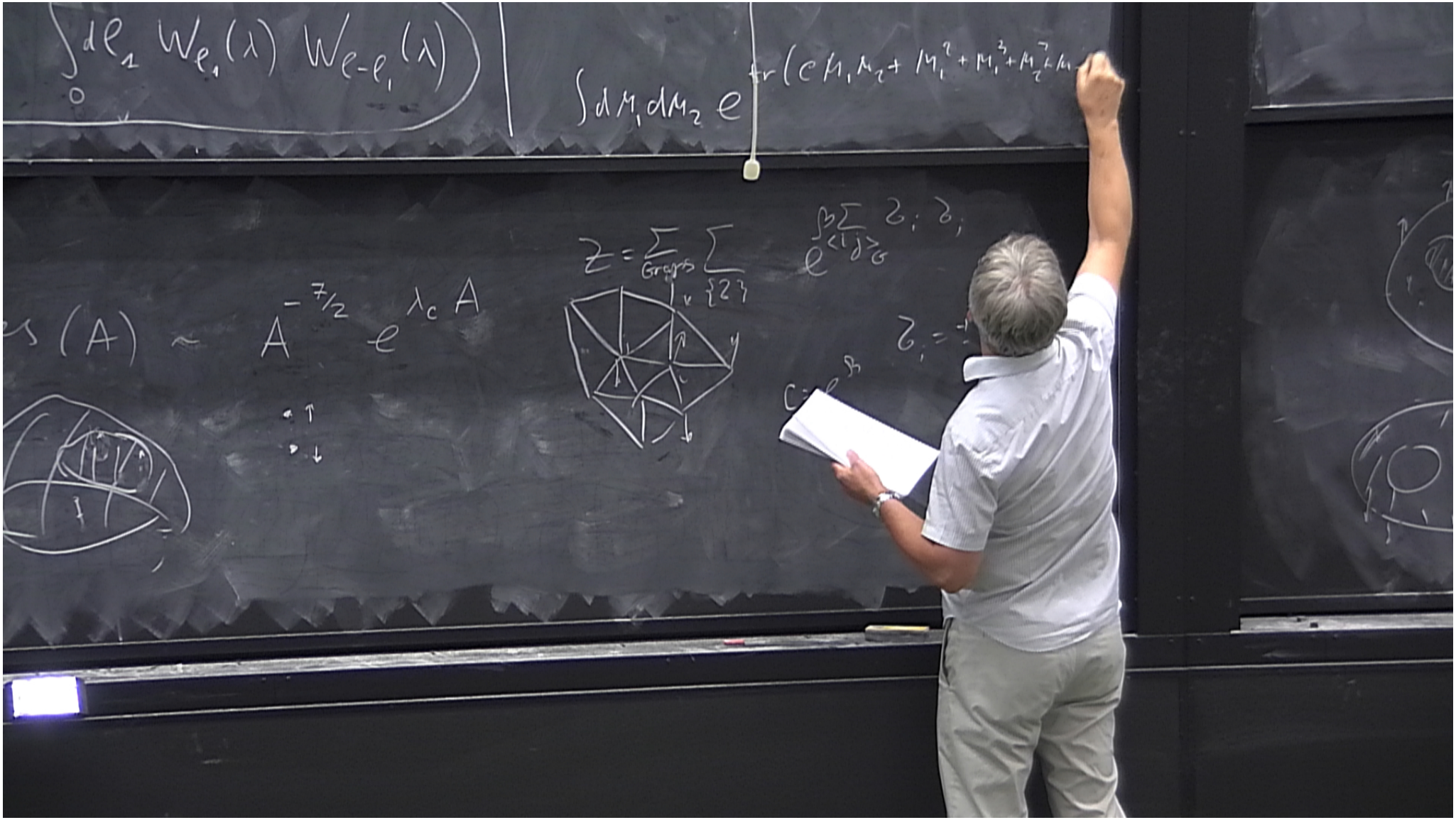

= W
= (O + O²)

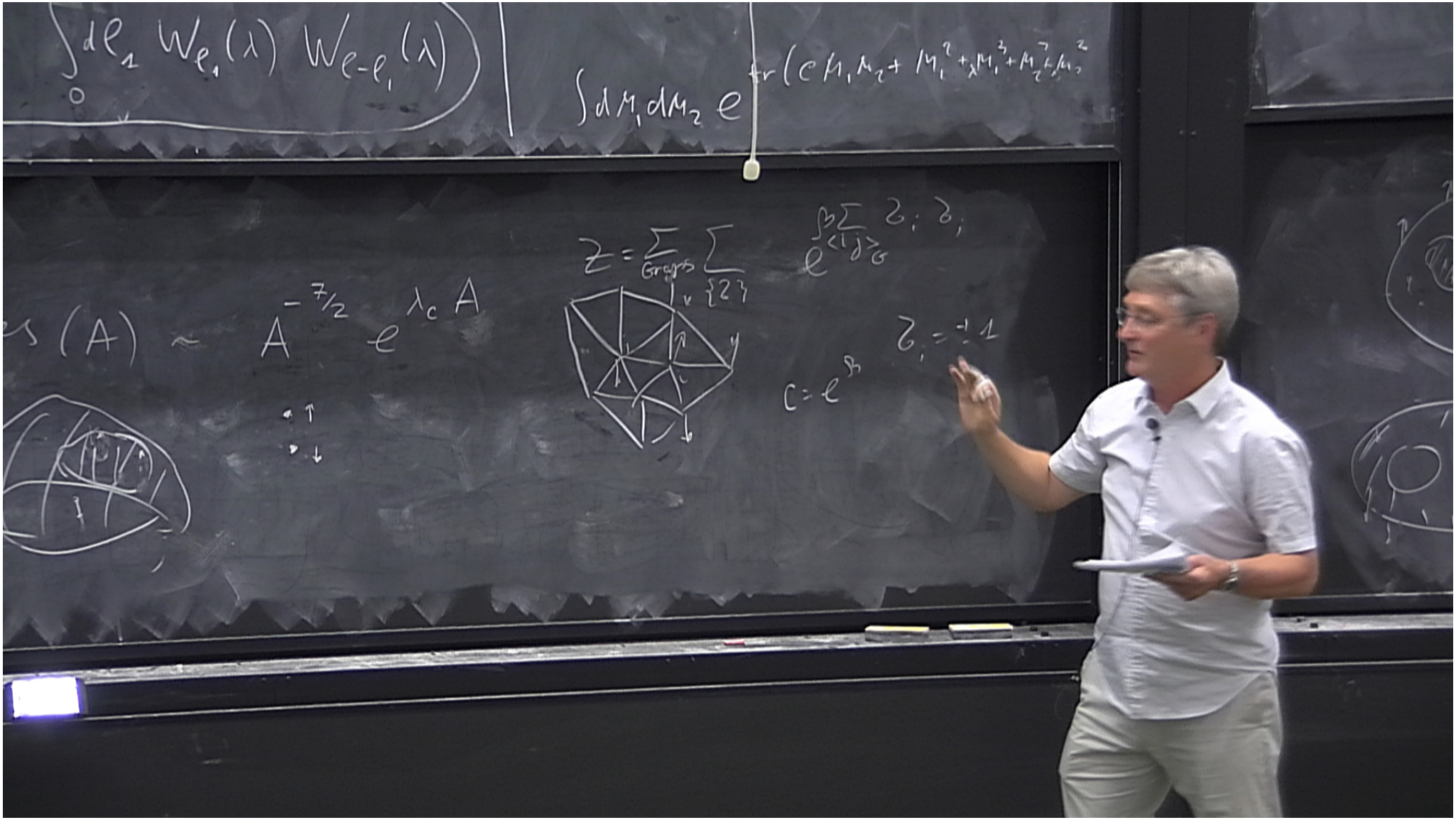

= W
= (O + O²) W

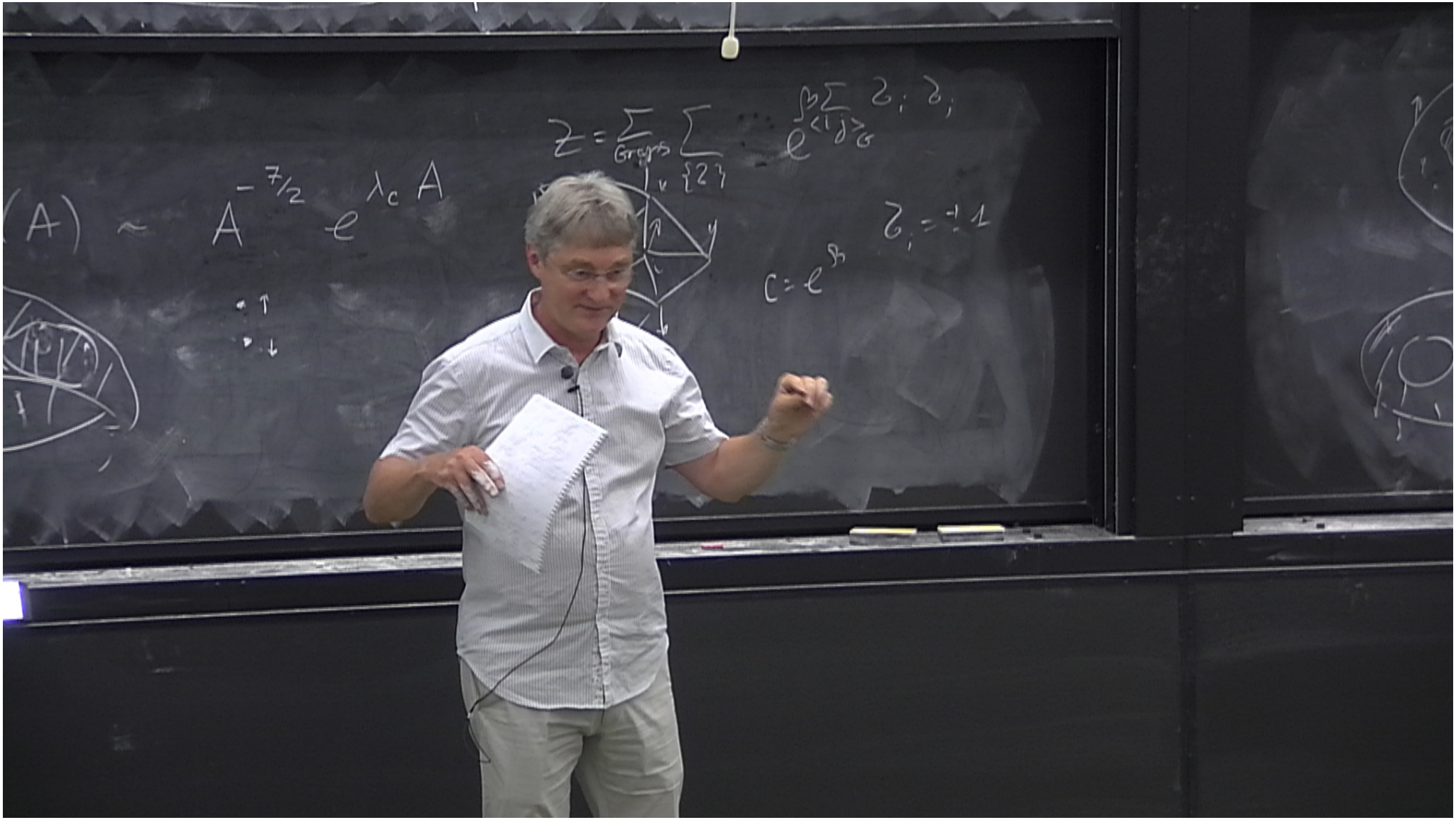


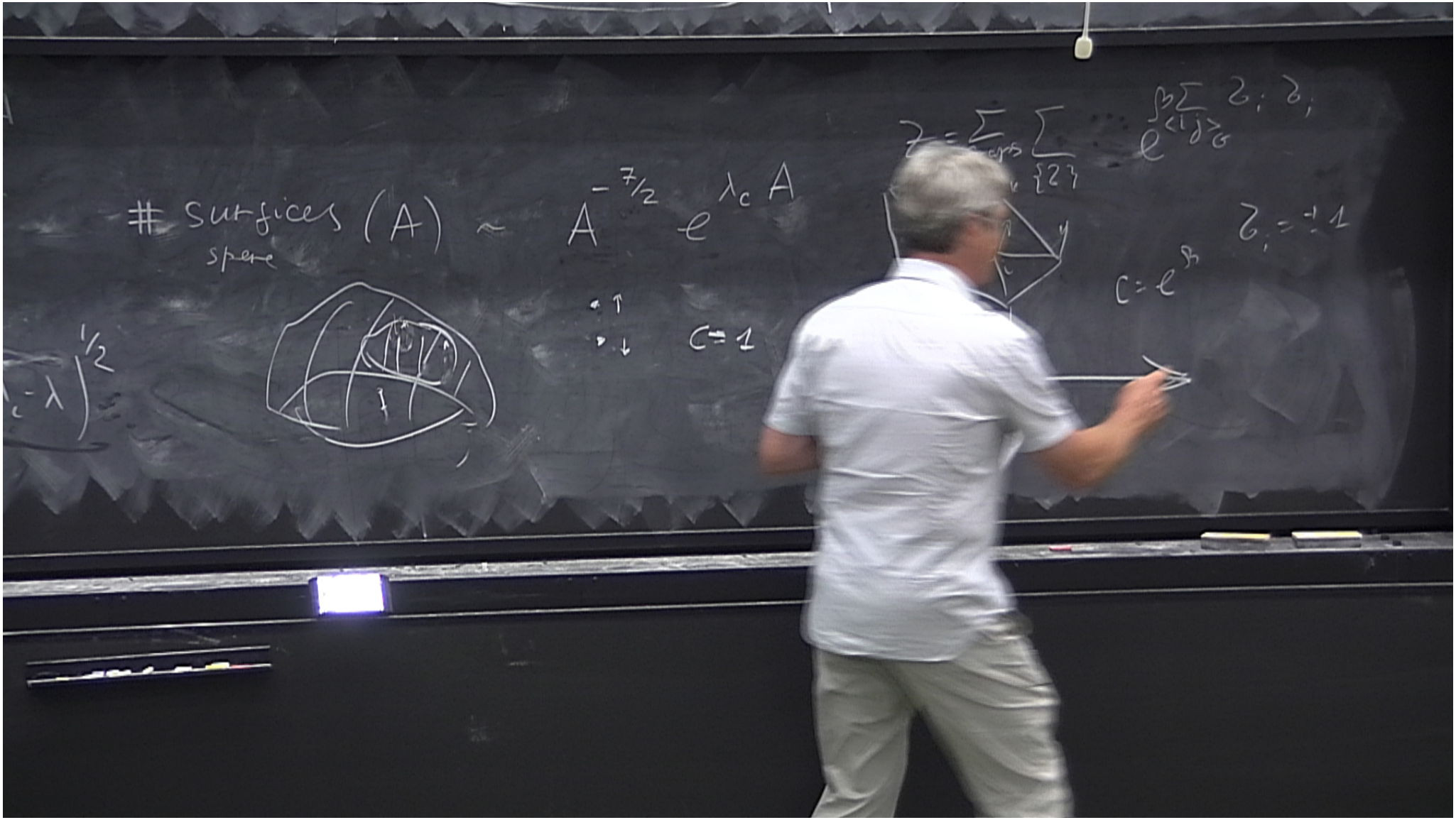

ℓ_0
$= (\partial_\ell + \partial_\ell^2)^{-1} W^{\uparrow}$
$+$
$\int_0^\infty d\ell_1 \, W^{\uparrow}_{\ell+\ell_0}$
$\int d\ell_1 W^{\uparrow}_{\ell_1} \, W^{\uparrow}_{\ell-\ell_1}$

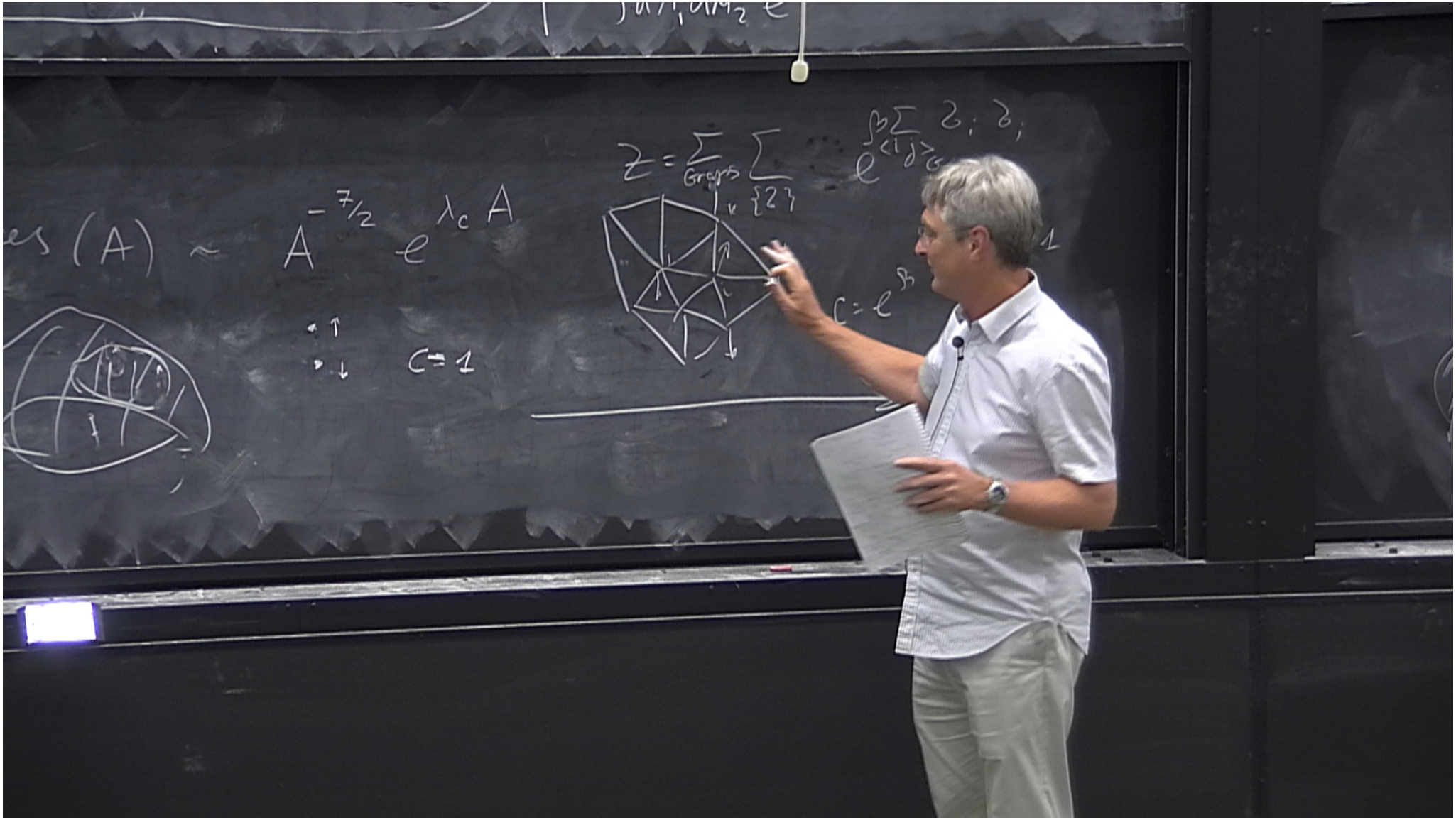


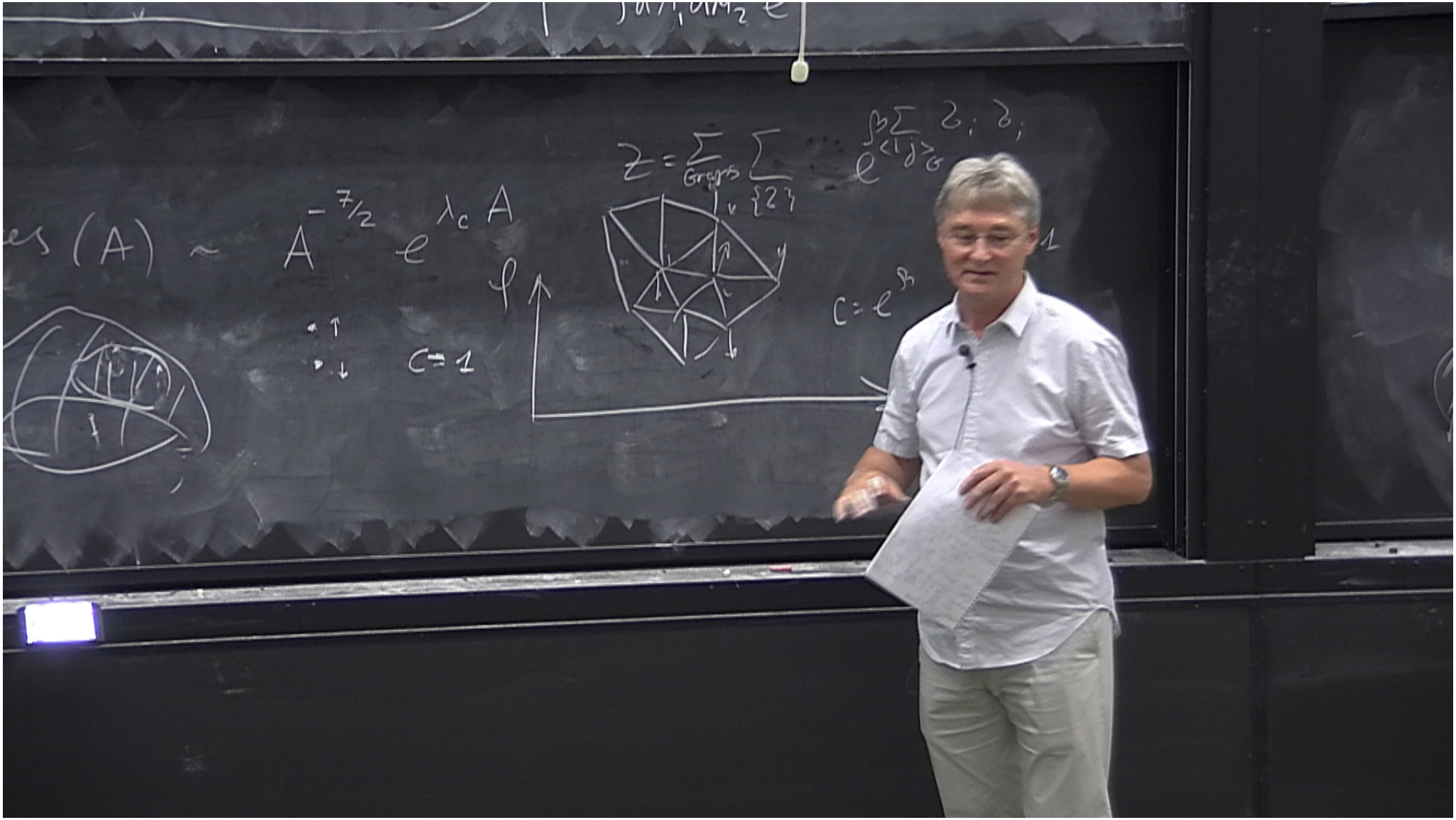

$\int_0^\infty d\ell_1\, W_{\ell_1}(\lambda)\, W_{\ell-\ell_1}(\lambda)$
$\int dM_1 dM_2\, e^{tr(cM_1M_2 + M_1^2 + M_1^3 + M_2^2 + M_1}$
$Z = \sum_{Graphs} \sum_{\{z\}}$
$e^{\beta \sum_{\langle ij \rangle} z_i z_j}$
$A^{-7/2}\, e^{\lambda_c A}$

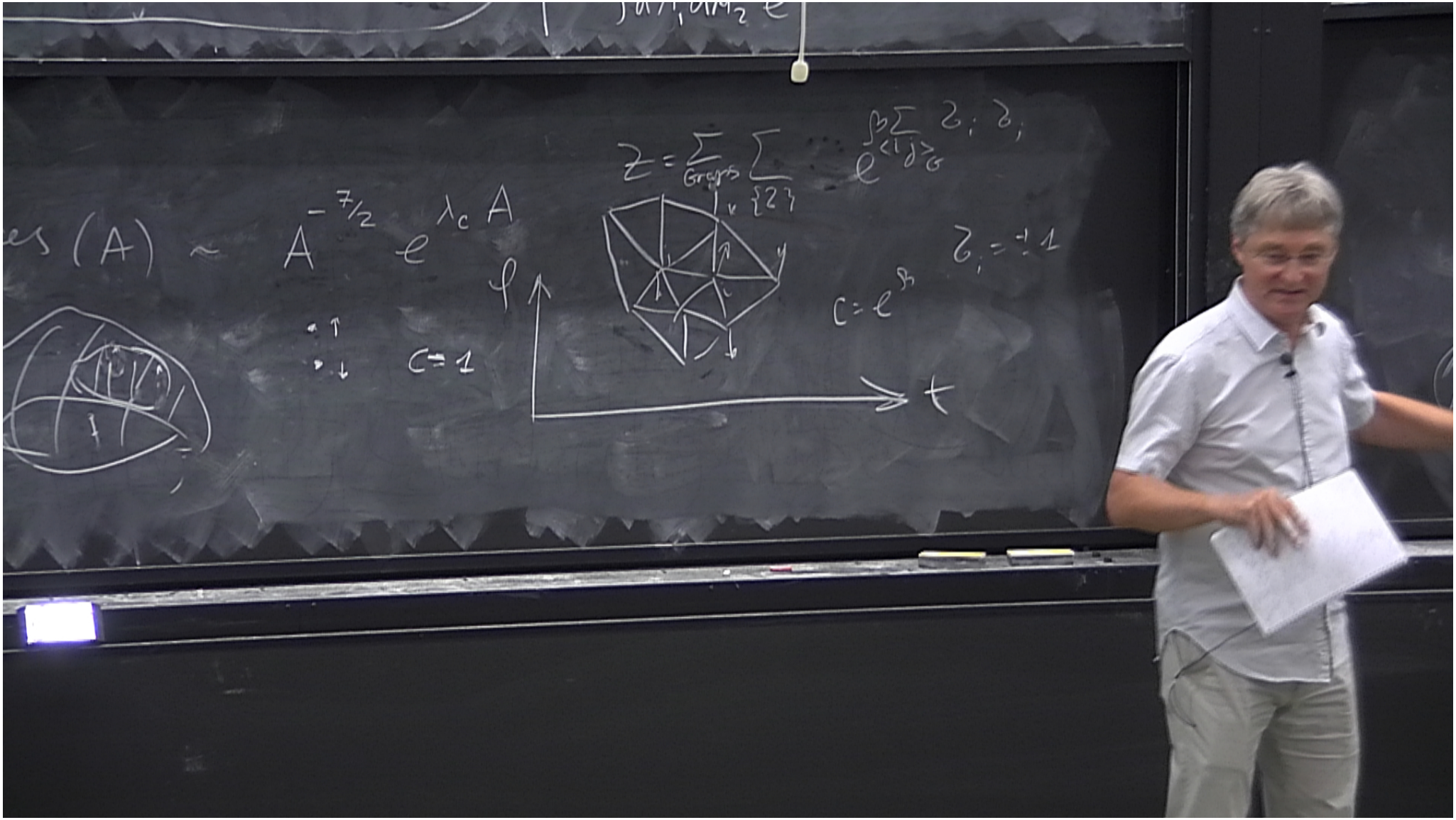

$\int_0 d\ell_1\, W_{\ell_1}(\lambda)\, W_{\ell-\ell_1}(\lambda)$
$\int dM_1 dM_2\, e^{\mathrm{tr}(cM_1M_2 + M_1^2 + \lambda M_1^3 + M_2^2 + \lambda M_2^3)}$
$Z = \sum_{\text{Graphs}} \sum_{\{\sigma\}}$
$e^{\beta \sum_{\langle ij \rangle} \sigma_i \sigma_j}$
$A^{-7/2}\, e^{\lambda_c A}$
$\sigma_i = \pm 1$
$c = e^{\beta}$

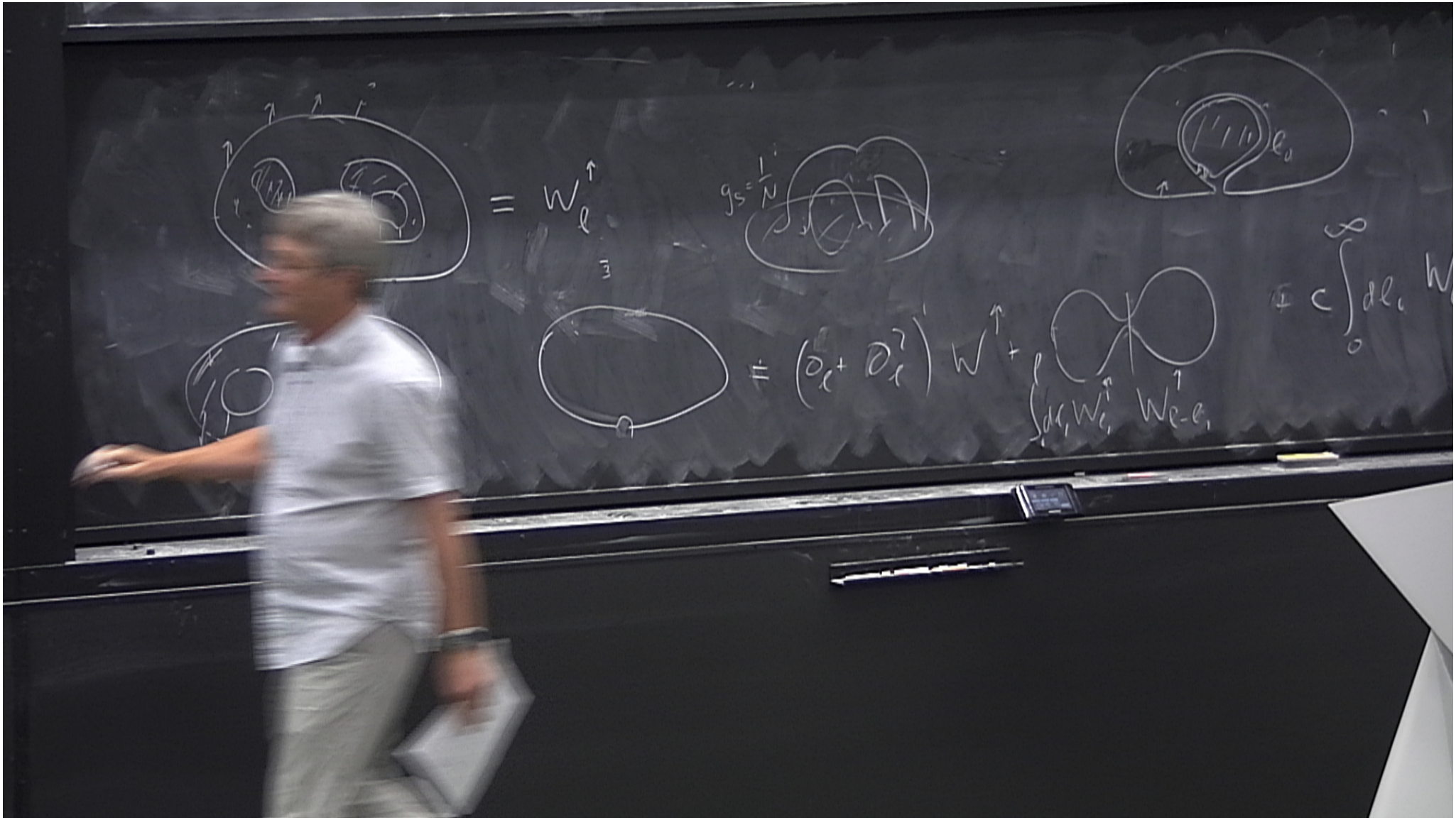

$Z = \sum_{G \text{ regular}} \sum_{\{z_v\}} e^{\beta \sum_{\langle i,j \rangle \in G} z_i z_j}$
$A^{-7/2} e^{\lambda_c A}$
$z_i = \pm 1$
$C = e^{\beta}$

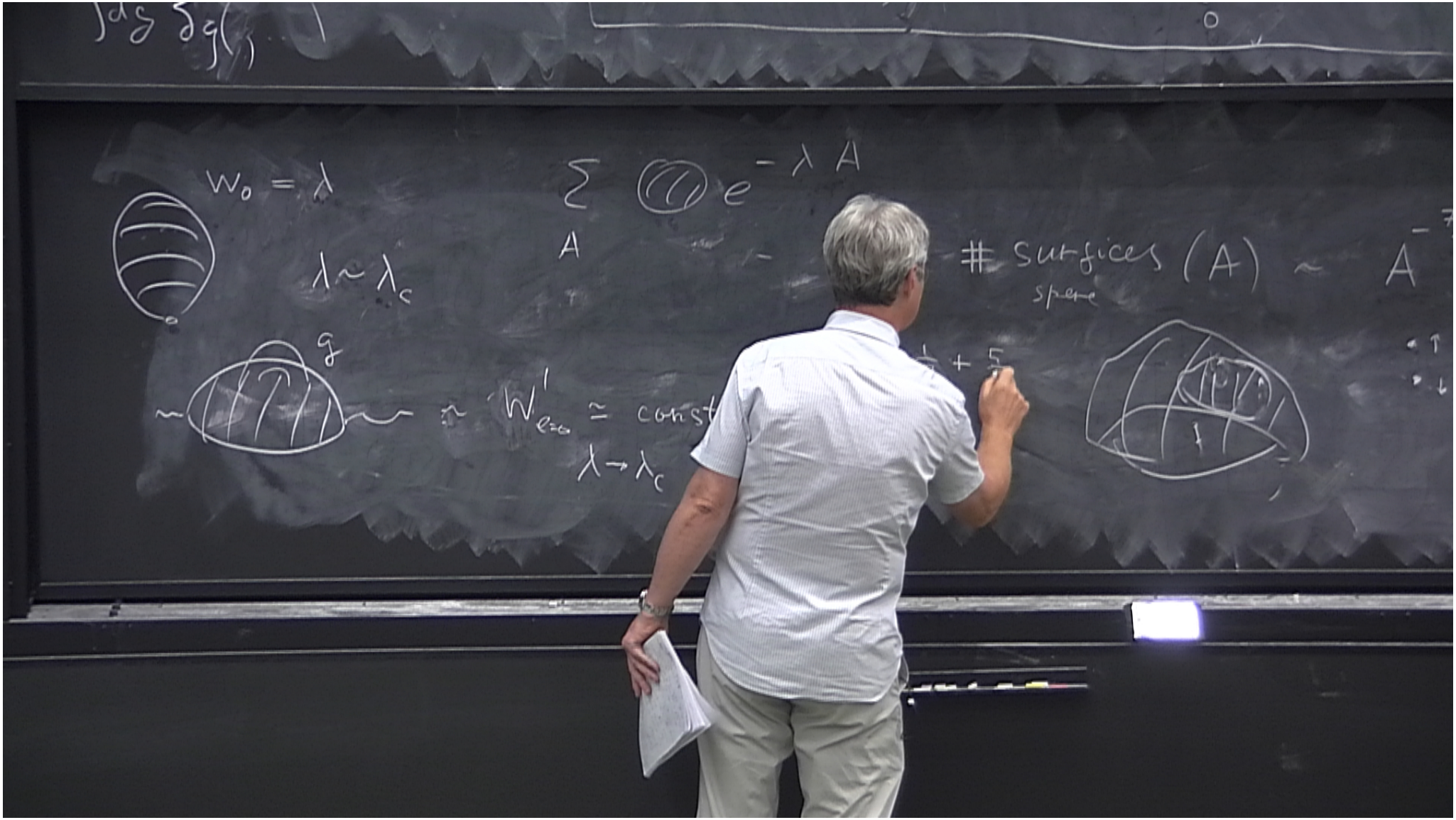

#surfaces (A) ~ A^{-7/2} e^{λc A}
sphere
C = 1
C = e^{β}
Z_i = ±1

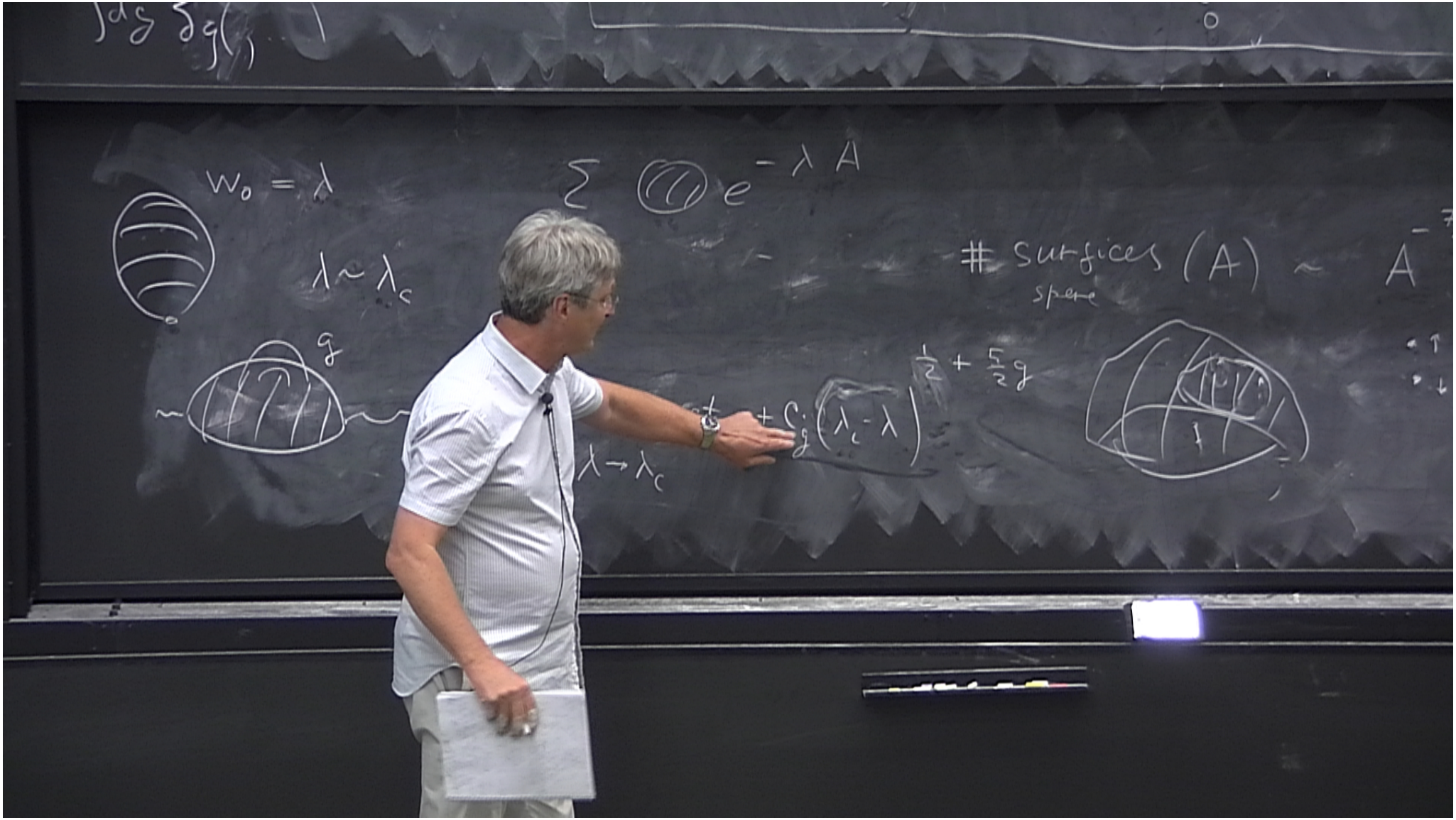

$Z = \sum_{\text{Graphs}} \sum_{\{z\}}$
$e^{\beta \sum_{\langle ij \rangle} z_i z_j}$
$A^{-7/2}$
$e^{\lambda_c A}$
$C = 1$
$C = e^{\beta}$

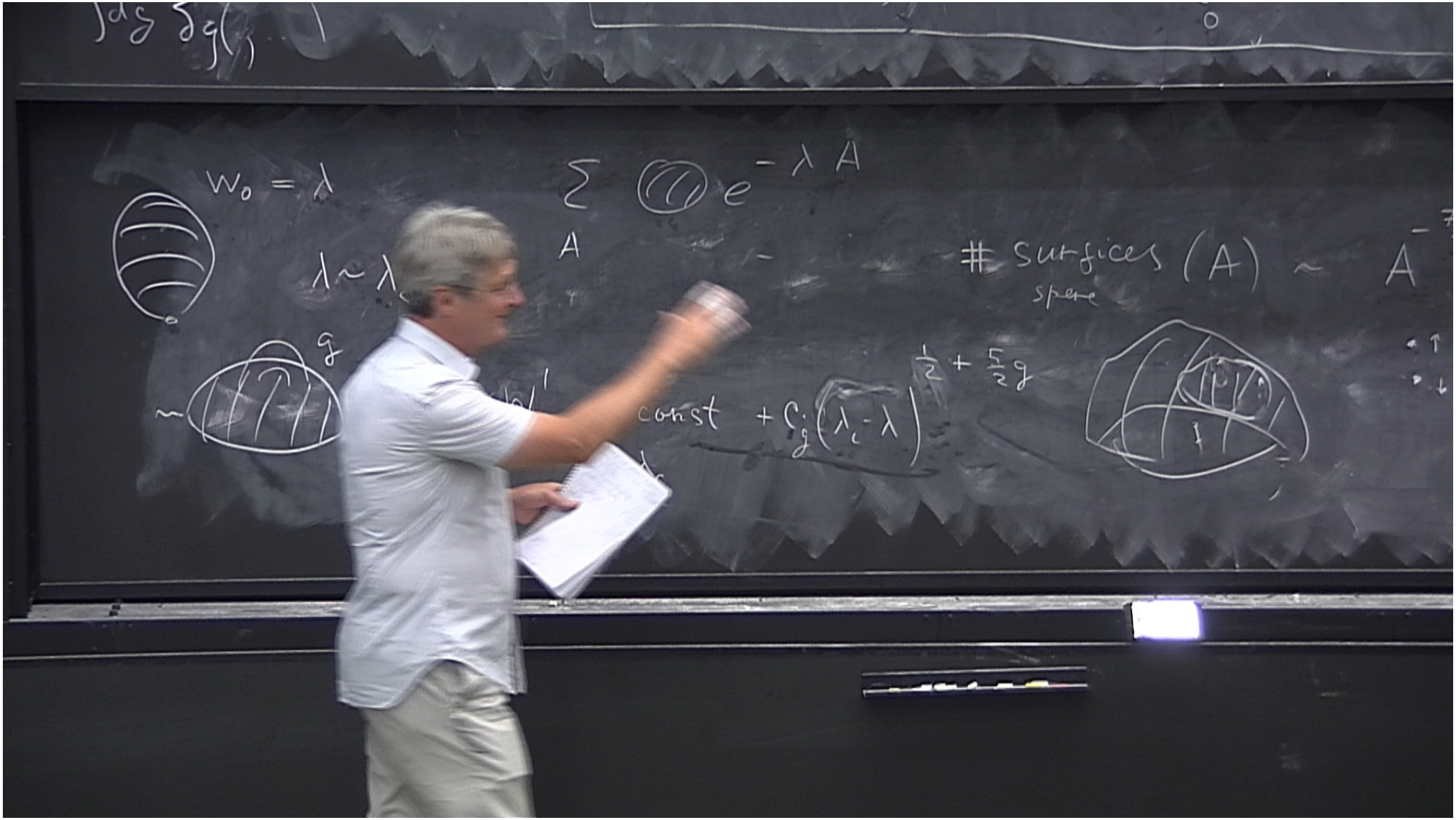

$A^{-7/2}$
$e^{\lambda_c A}$
$c = 1$
$Z = \sum_{\text{Graphs}} \sum_{\{z\}}$
$c = e^{\beta}$

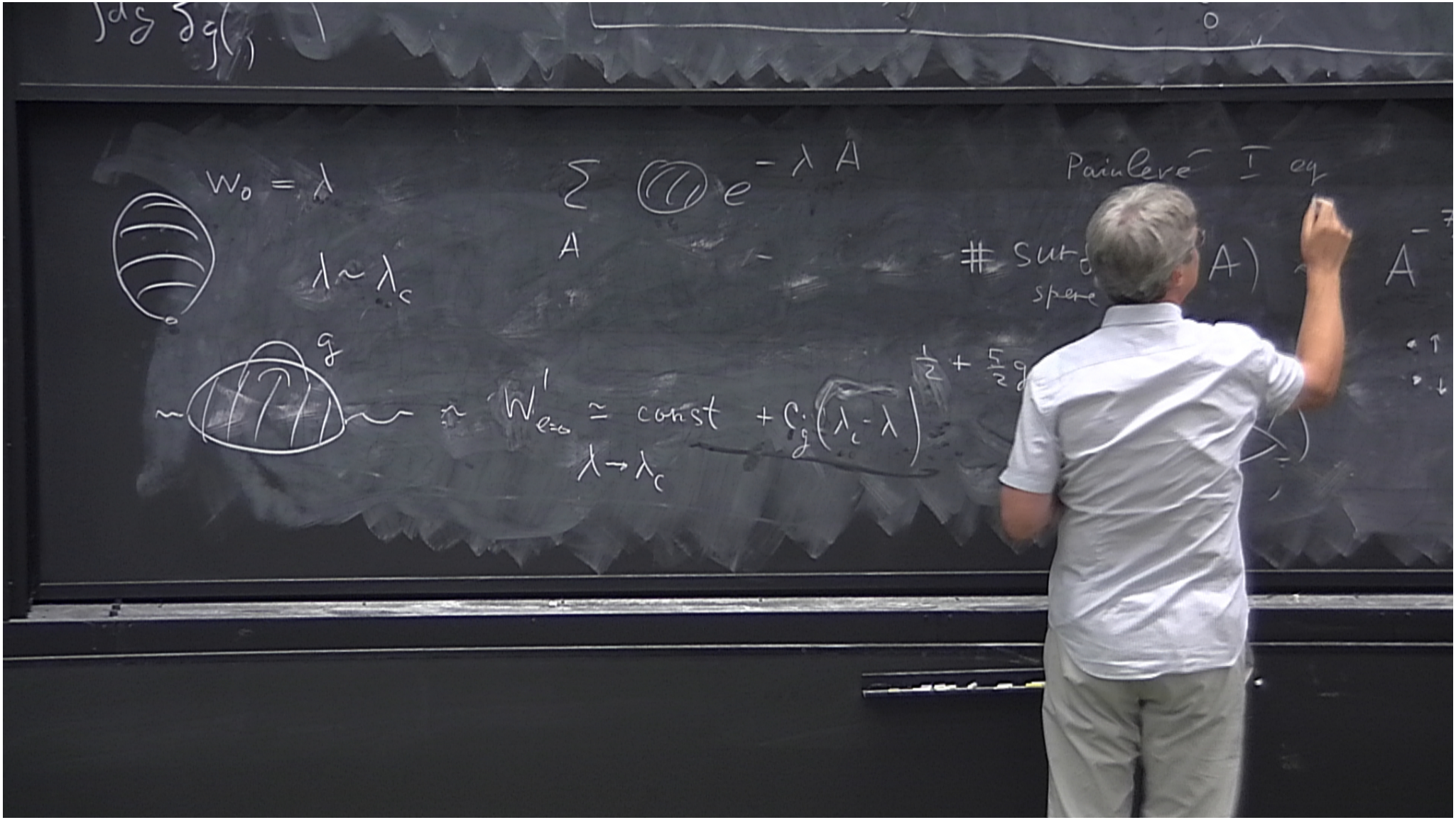

$Z = \sum_{\text{Graphs}} \sum_{\{z\}}$
$e^{\beta \sum_{\langle ij \rangle} z_i z_j}$
$A^{-7/2}$
$e^{\lambda_c A}$
$C = e^{\beta}$
$z_i = \pm 1$
$C = 1$
φ
t

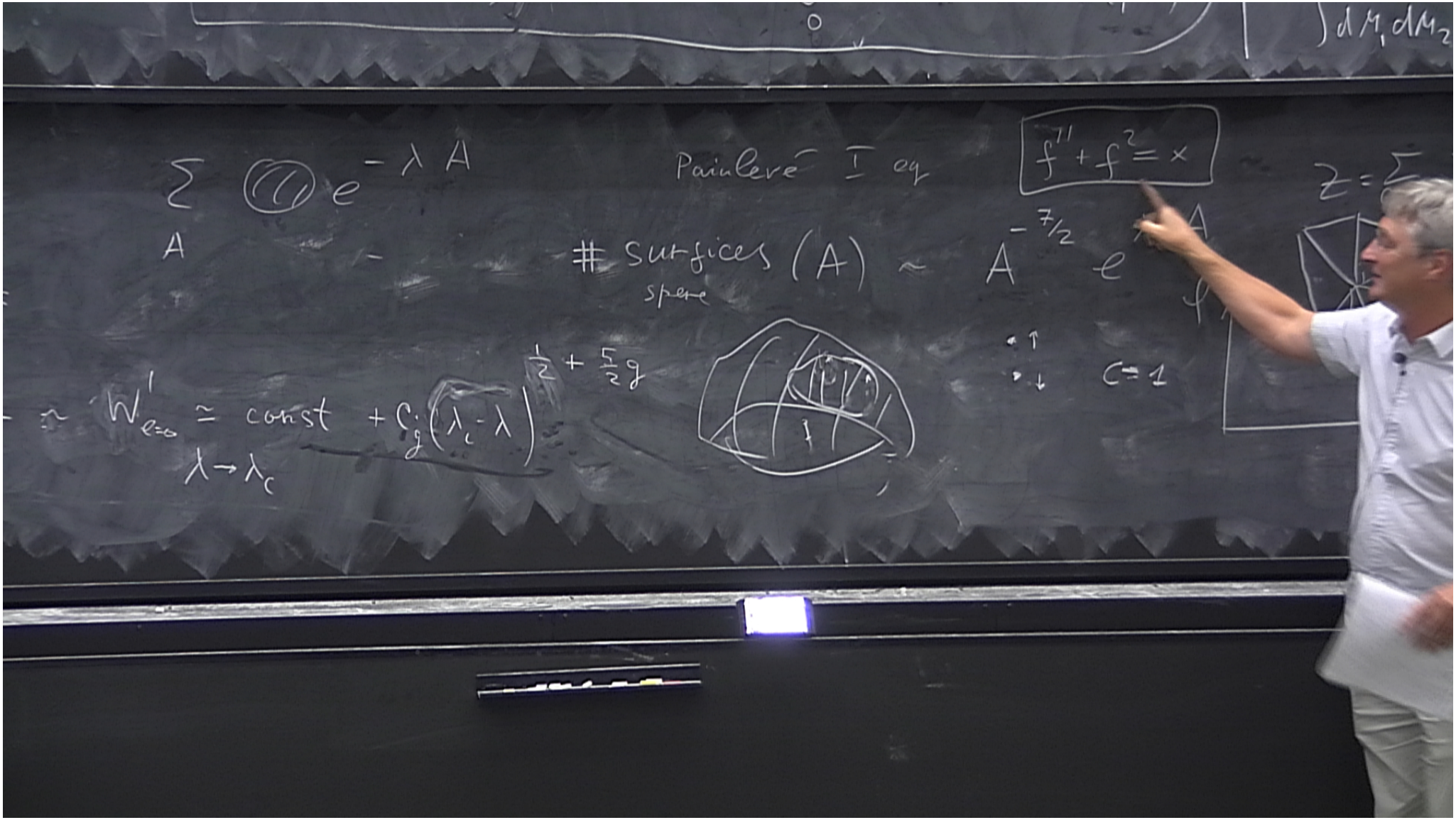

$= W_\ell$
$g_s = \frac{1}{N}$
$= (\partial_\ell + D_\ell^2)\, W^\ell +$
$\int d\alpha_i W_i^\ell \; W_{\ell-\ell_i}$
$= c \int_0^\infty d\ell_i$


$W_0 = \lambda$
$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
surfaces (A) ~ A^{-}
sphere
g
$W'_{e=0} \simeq$ const
$\lambda \to \lambda_c$


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$\lambda \sim \lambda$
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sphere
g
const $+ c_g (\lambda_c - \lambda)^{\frac{1}{2}} + \frac{5}{2} g$


$W_0 = \lambda$
$\lambda \sim \lambda_c$
$\sum_A e^{-\lambda A}$
Painlevé I eq
$W'_{e=0} \simeq \text{const} + c_g (\lambda_c - \lambda)^{\frac{1}{2} + \frac{5}{2}g}$
$\lambda \to \lambda_c$


$\sum_A$ $e^{-\lambda A}$
Painlevé I eq
$f'' + f^2 = x$
$Z = \sum$
surfaces (A) $\sim A^{-7/2} e^{\lambda A}$
sphere
$W'_{\ell=0} \simeq$ const $+ c_g (\lambda_c - \lambda)^{\frac{1}{2} + \frac{5}{2} g}$
$\lambda \to \lambda_c$
$C = 1$
$dM_1 dM_2$