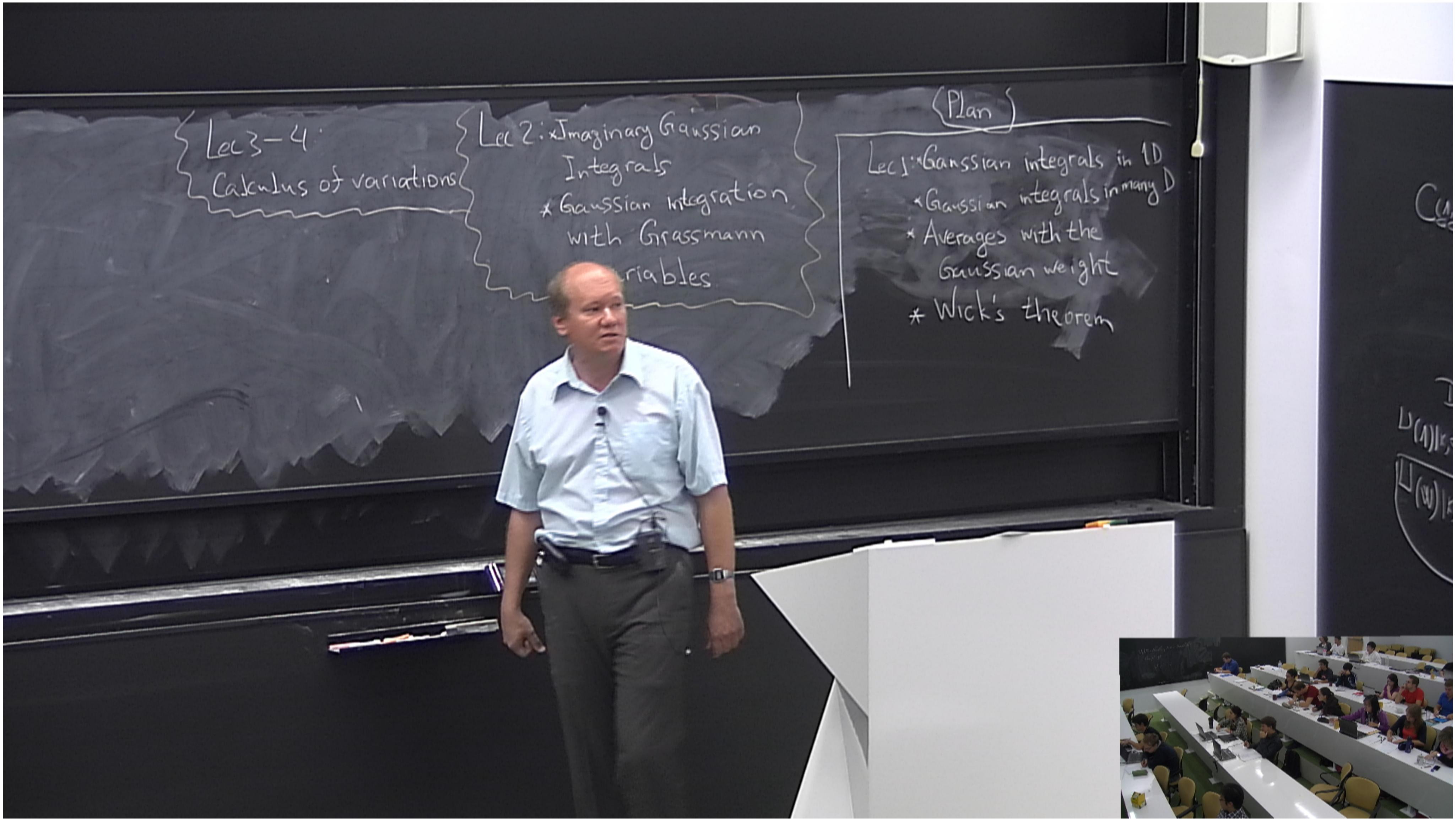
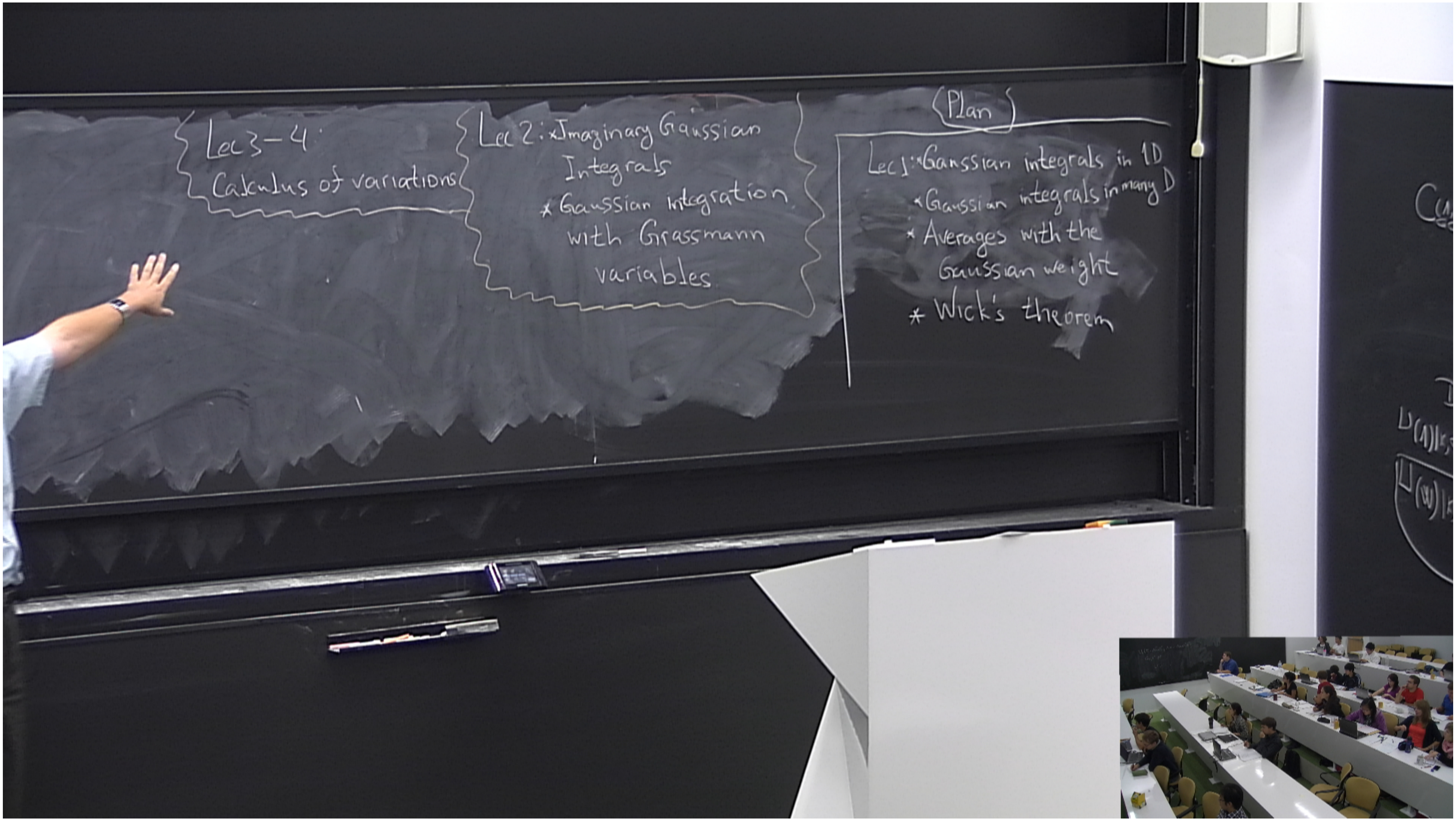
Title: Evaluation of Integrals and Calculus of Variations - Lecture 1

Date: Aug 24, 2011 10:45 AM

URL: http://pirsa.org/11080128

Abstract:


Lec 3-4:
Calculus of variations
Lec 2: * Imaginary Gaussian Integrals
* Gaussian integration with Grassmann variables
Plan
Lec 1: * Gaussian integrals in 1D
* Gaussian integrals in many D
* Averages with the Gaussian weight
* Wick's theorem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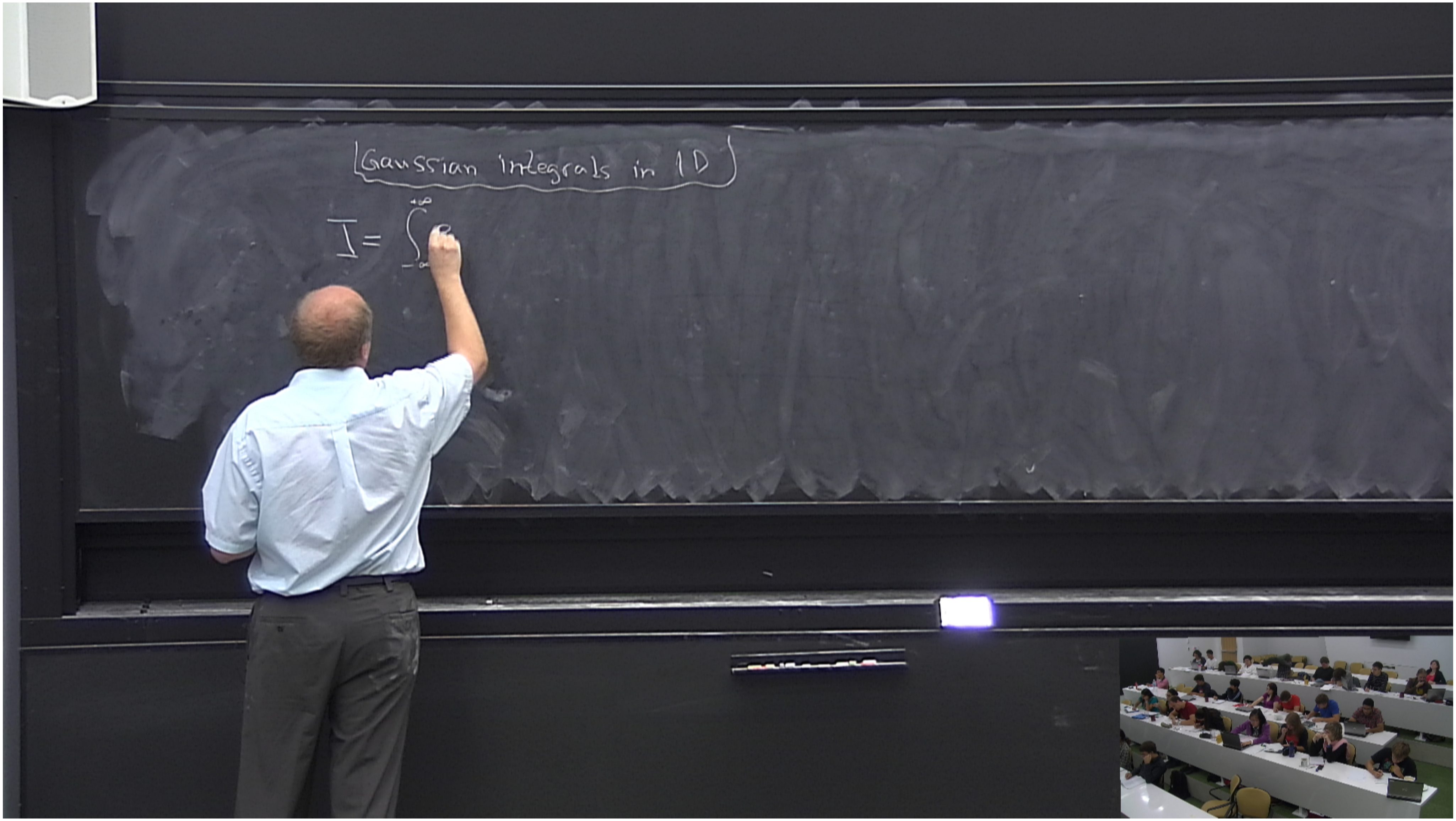
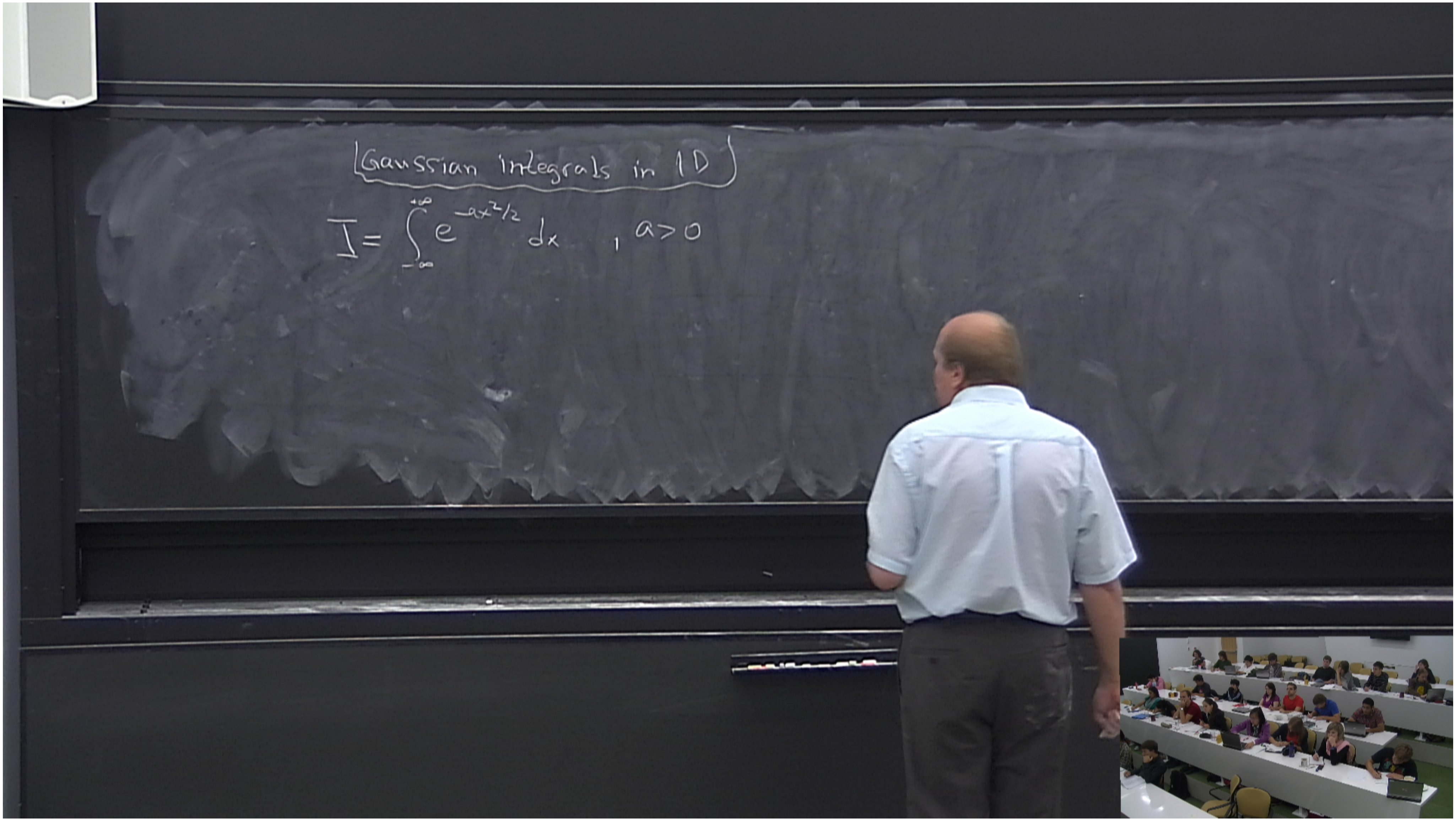
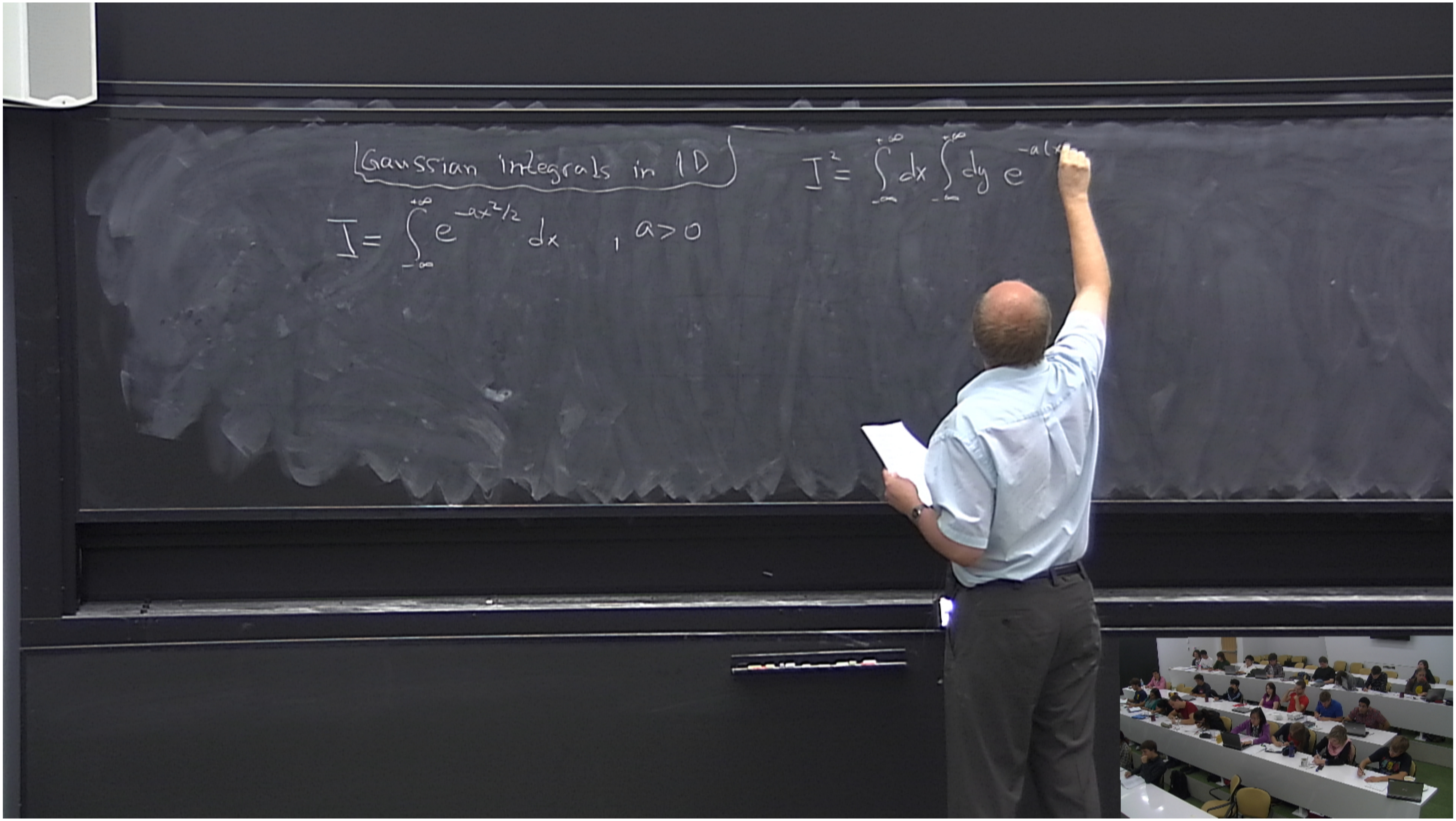
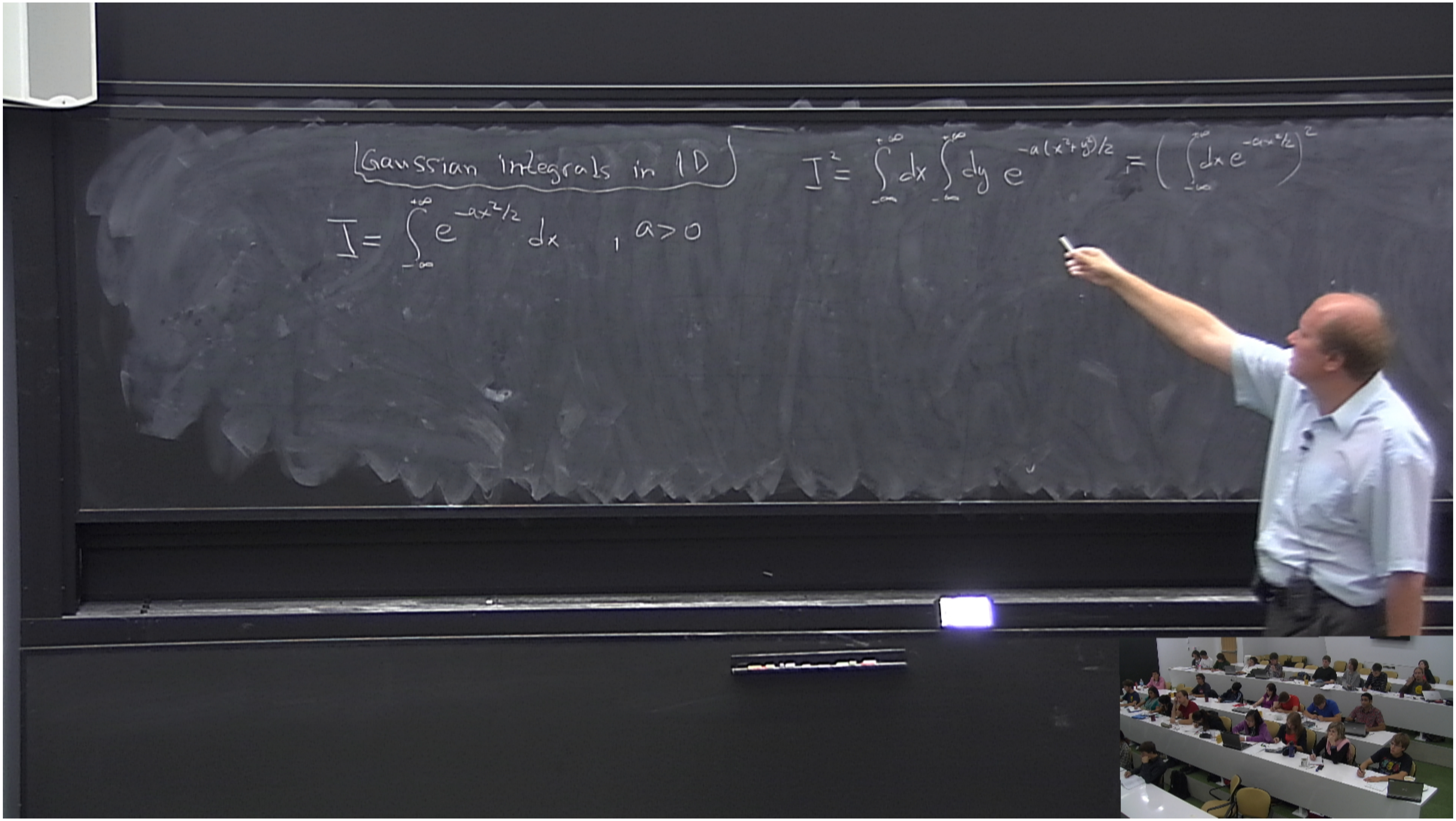
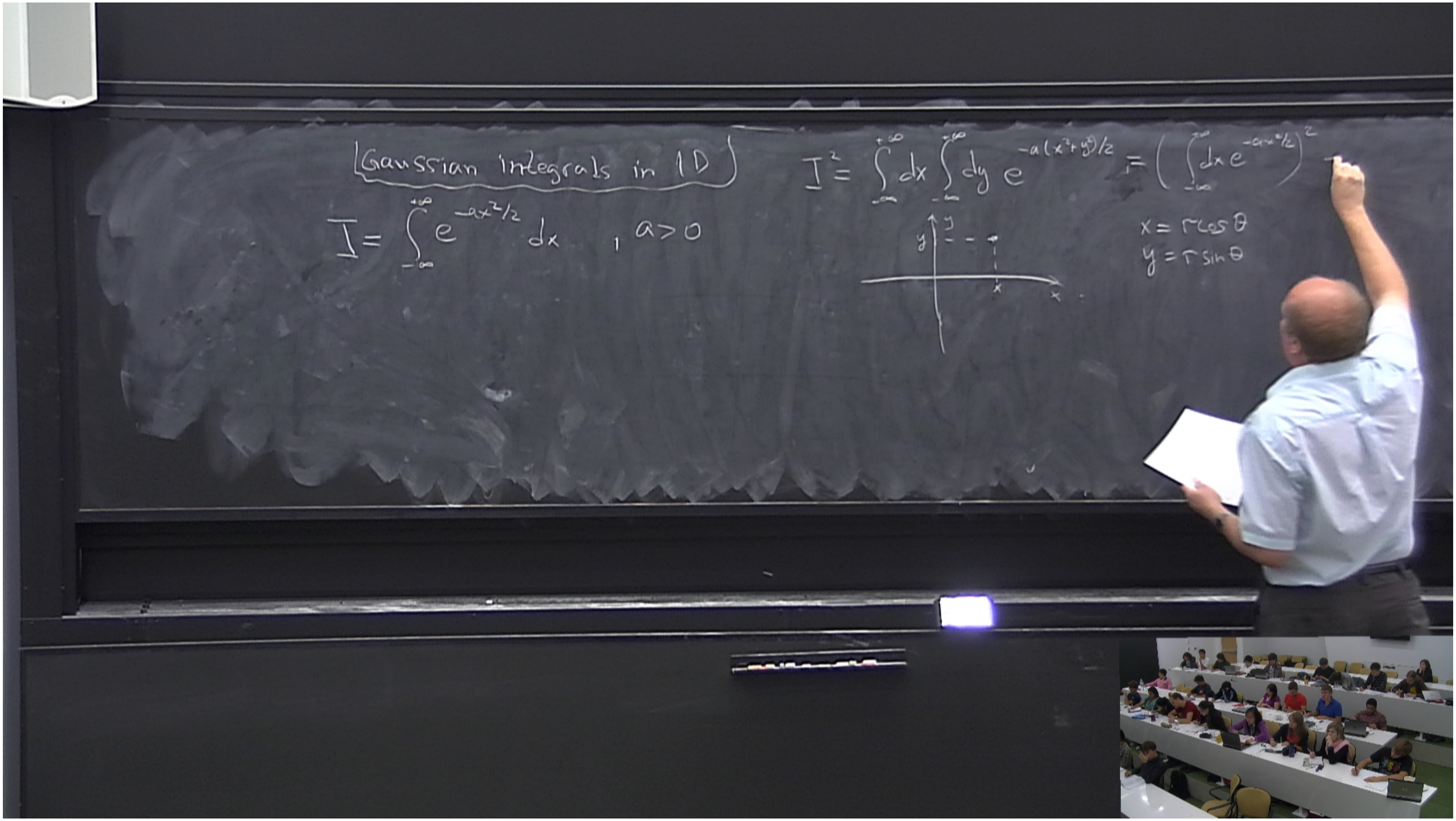
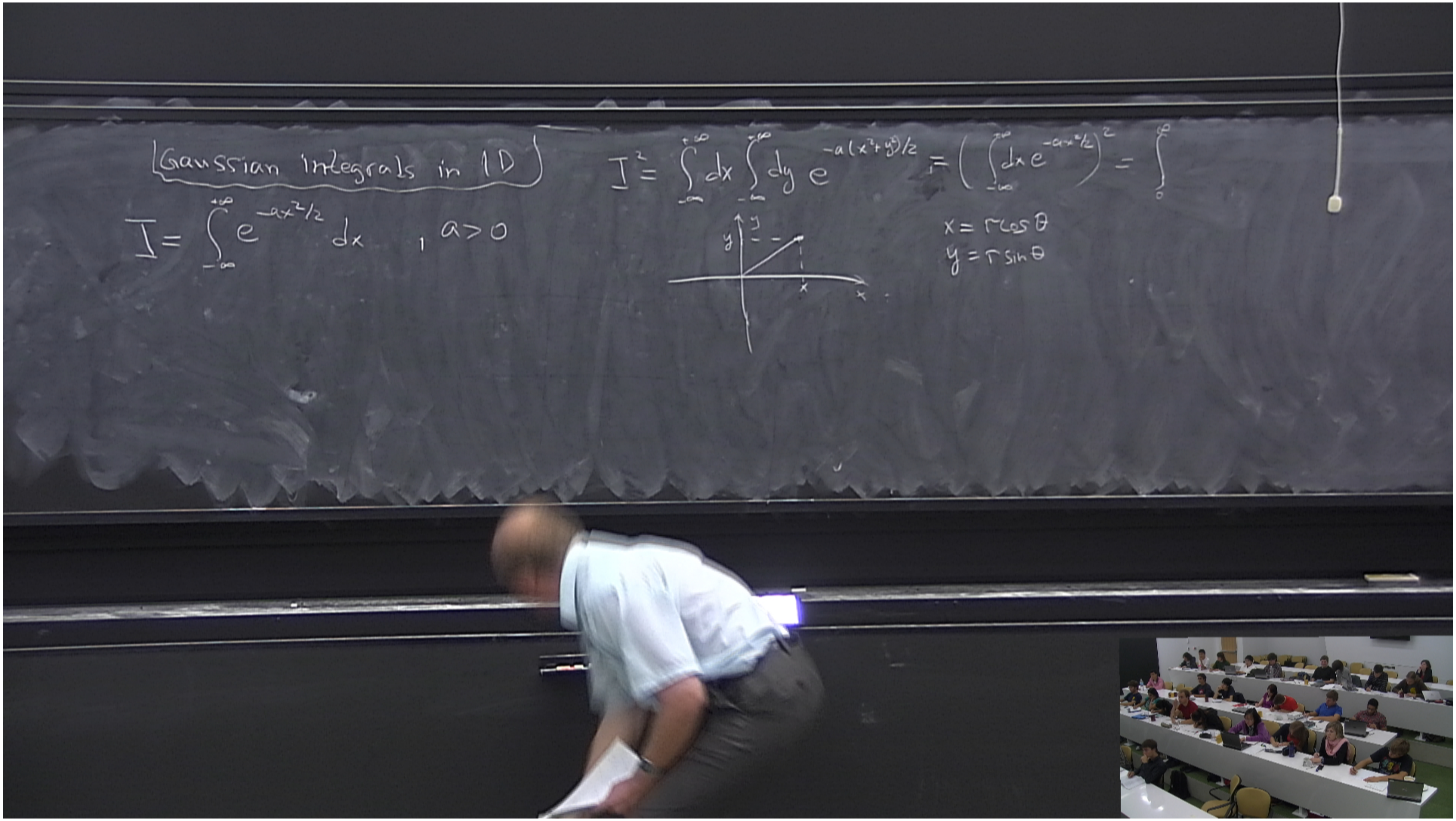

Gaussian integrals in 1D
I = ∫

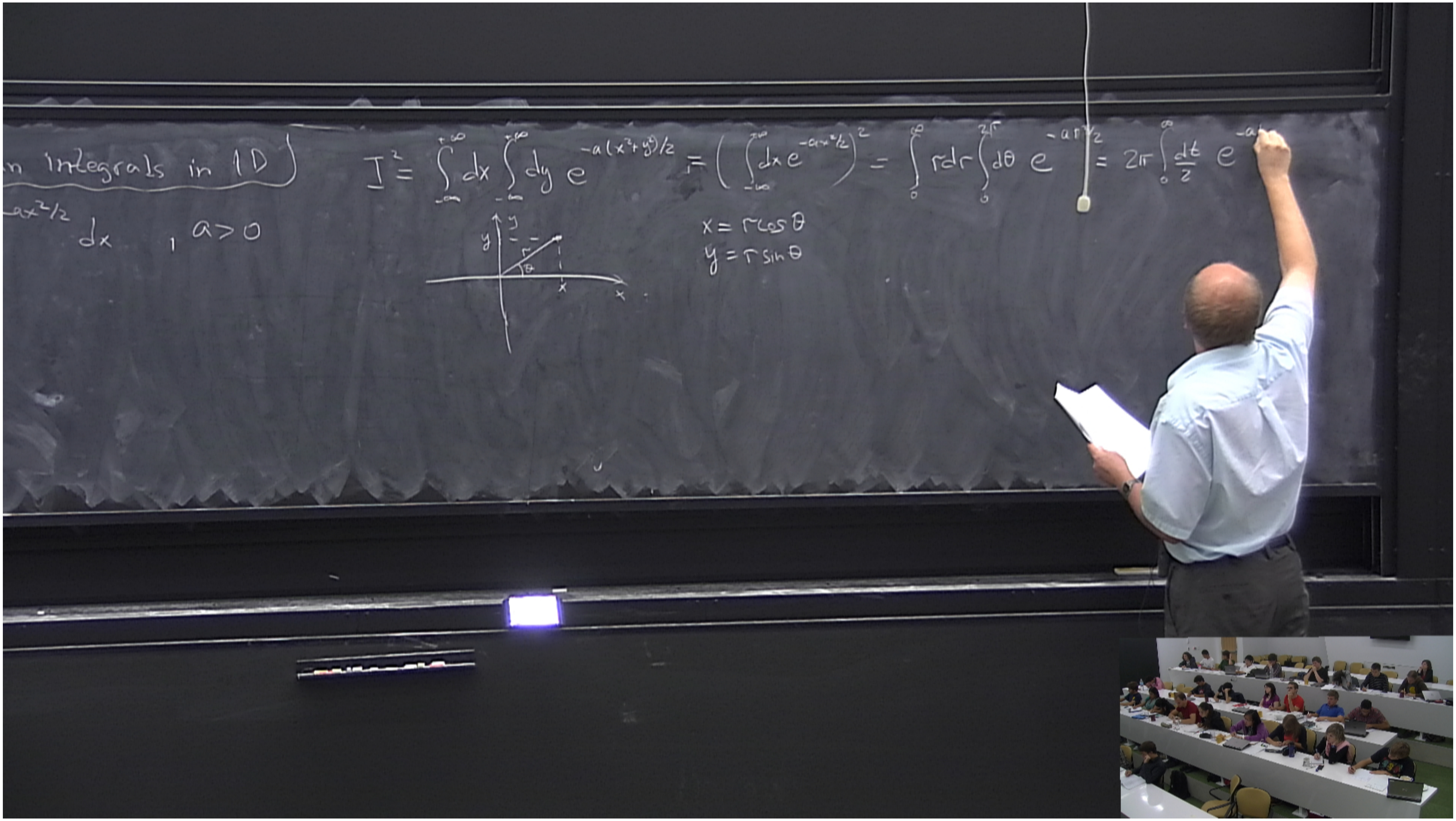

Gaussian integrals in 1D
$I = \int_{-\infty}^{+\infty} e^{-ax^2/2} dx \quad , a>0$

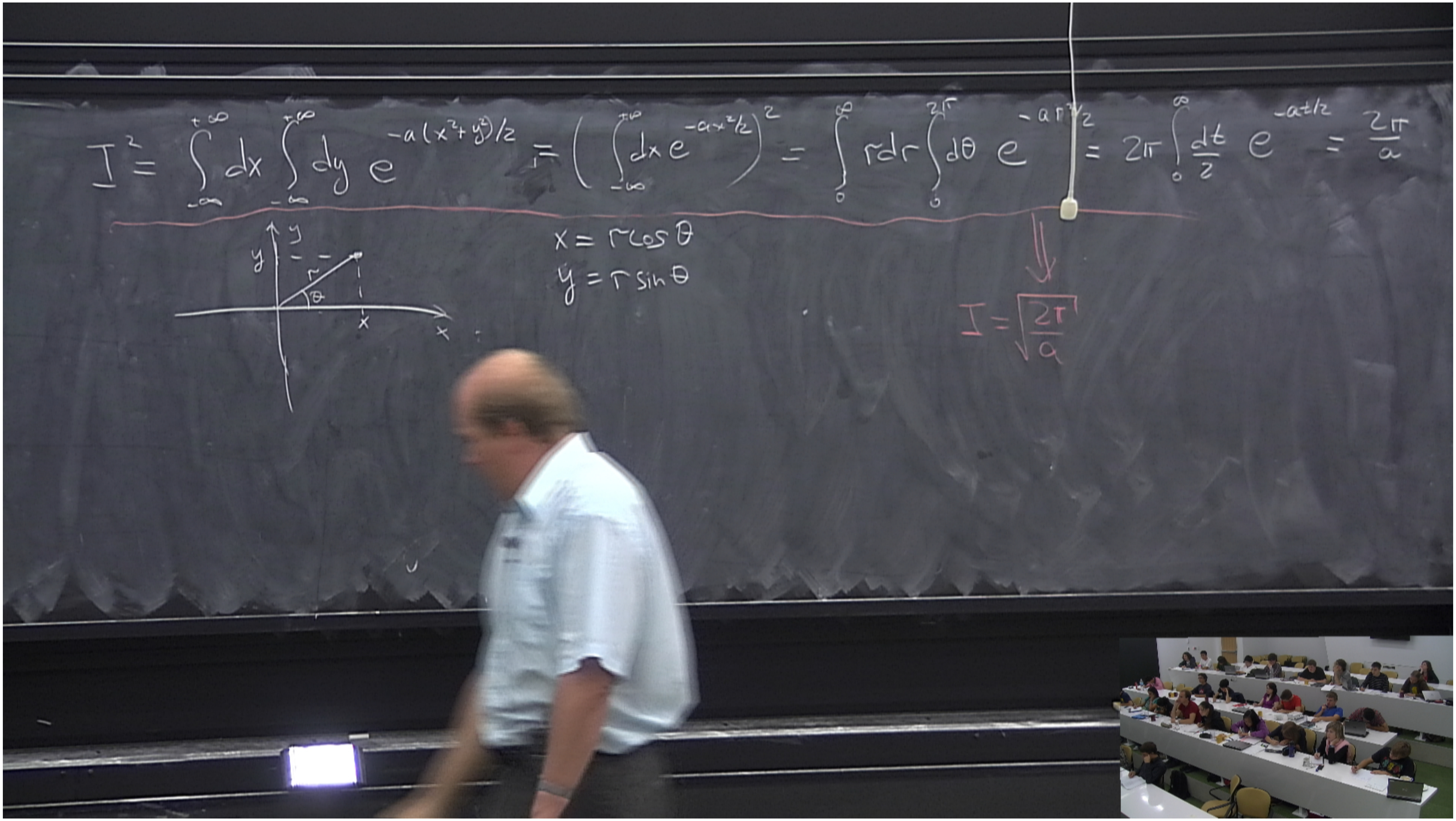

Gaussian integrals in 1D
I = ∫ e^{-ax²/2} dx , a > 0
I² = ∫ dx ∫ dy e^{-a(x


Gaussian integrals in 1D
$I = \int_{-\infty}^{+\infty} e^{-ax^2/2} dx \, , \, a>0$
$I^2 = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \, e^{-a(x^2+y^2)/2} = \left( \int_{-\infty}^{+\infty} dx \, e^{-ax^2/2} \right)^2$

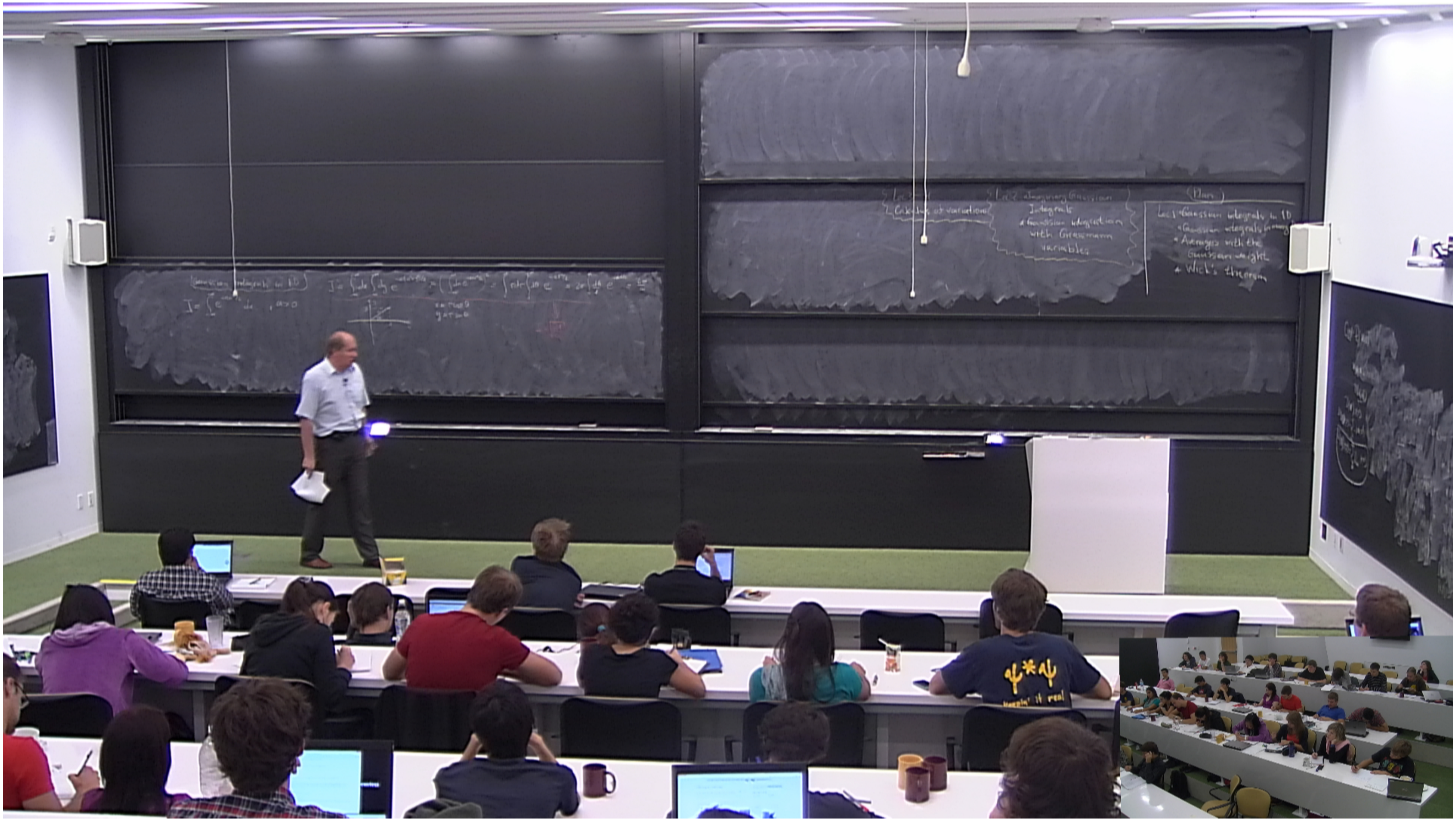

Gaussian integrals in 1D
I = ∫_{-∞}^{+∞} e^{-ax²/2} dx , a > 0
I² = ∫_{-∞}^{+∞} dx ∫_{-∞}^{+∞} dy e^{-a(x²+y²)/2} = ( ∫_{-∞}^{+∞} dx e^{-ax²/2} )²
x = r cos θ
y = r sin θ


Gaussian integrals in 1D
I = ∫ e^{-ax²/2} dx , a > 0
I² = ∫dx ∫dy e^{-a(x²+y²)/2} = (∫dx e^{-ax²/2})² =
x = r cos θ
y = r sin θ


...n integrals in 1D
-ax²/2 dx , a>0
I² = ∫dx ∫dy e^{-a(x²+y²)/2} = ( ∫dx e^{-ax²/2} )² = ∫ rdr ∫dθ e^{-ar²/2} = 2π ∫ dt/2 e^{-a...
x = r cos θ
y = r sin θ



$I^2 = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \, e^{-a(x^2+y^2)/2} = \left( \int_{-\infty}^{+\infty} dx \, e^{-ax^2/2} \right)^2 = \int_0^\infty r dr \int_0^{2\pi} d\theta \, e^{-ar^2/2} = 2\pi \int_0^\infty \frac{dt}{2} e^{-at/2} = \frac{2\pi}{a}$
$x = r\cos\theta$
$y = r\sin\theta$
$I = \sqrt{\frac{2\pi}{a}}$



Calculus of variations
Integrals
* Gaussian integration with Grassmann variables
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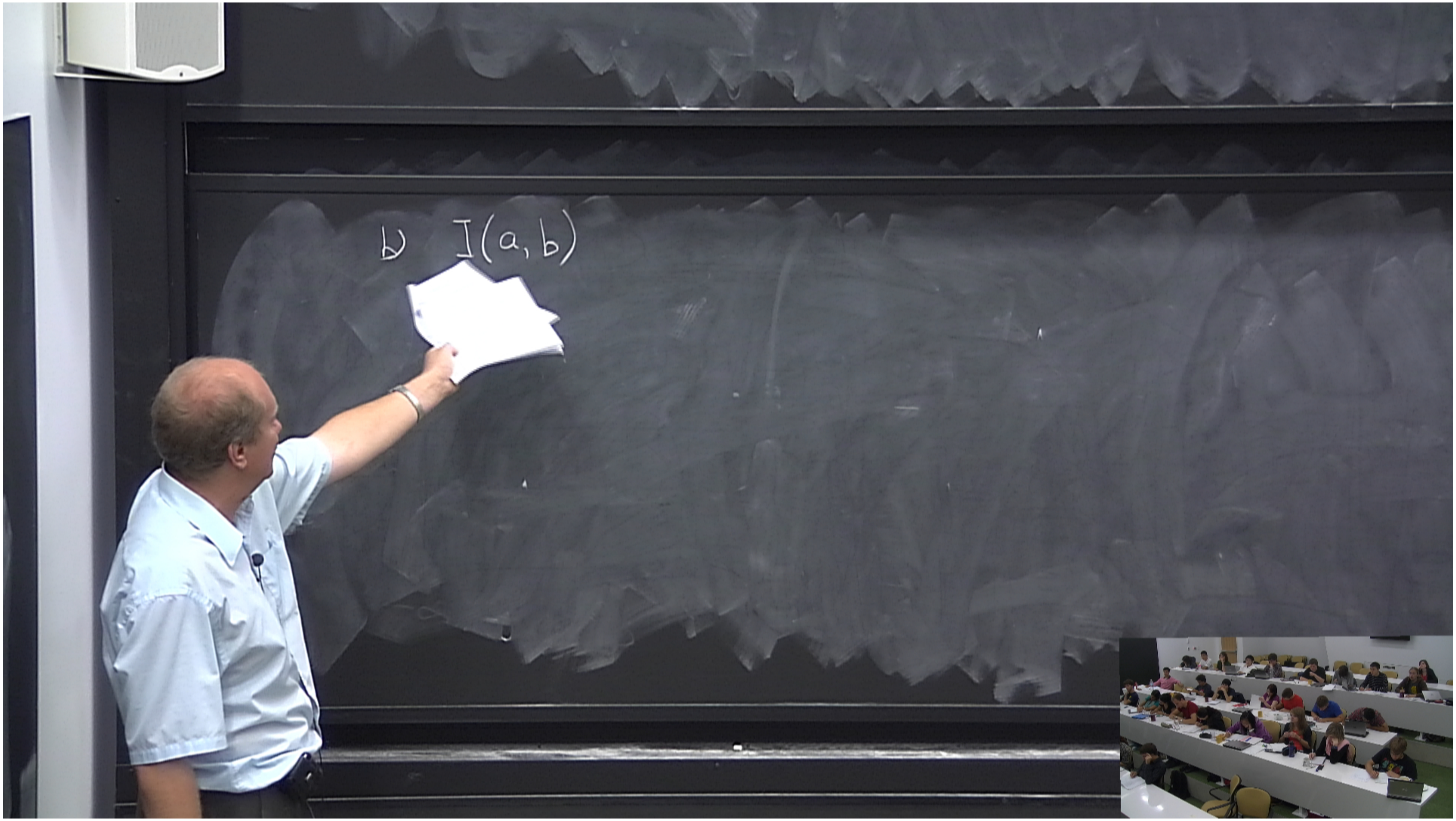
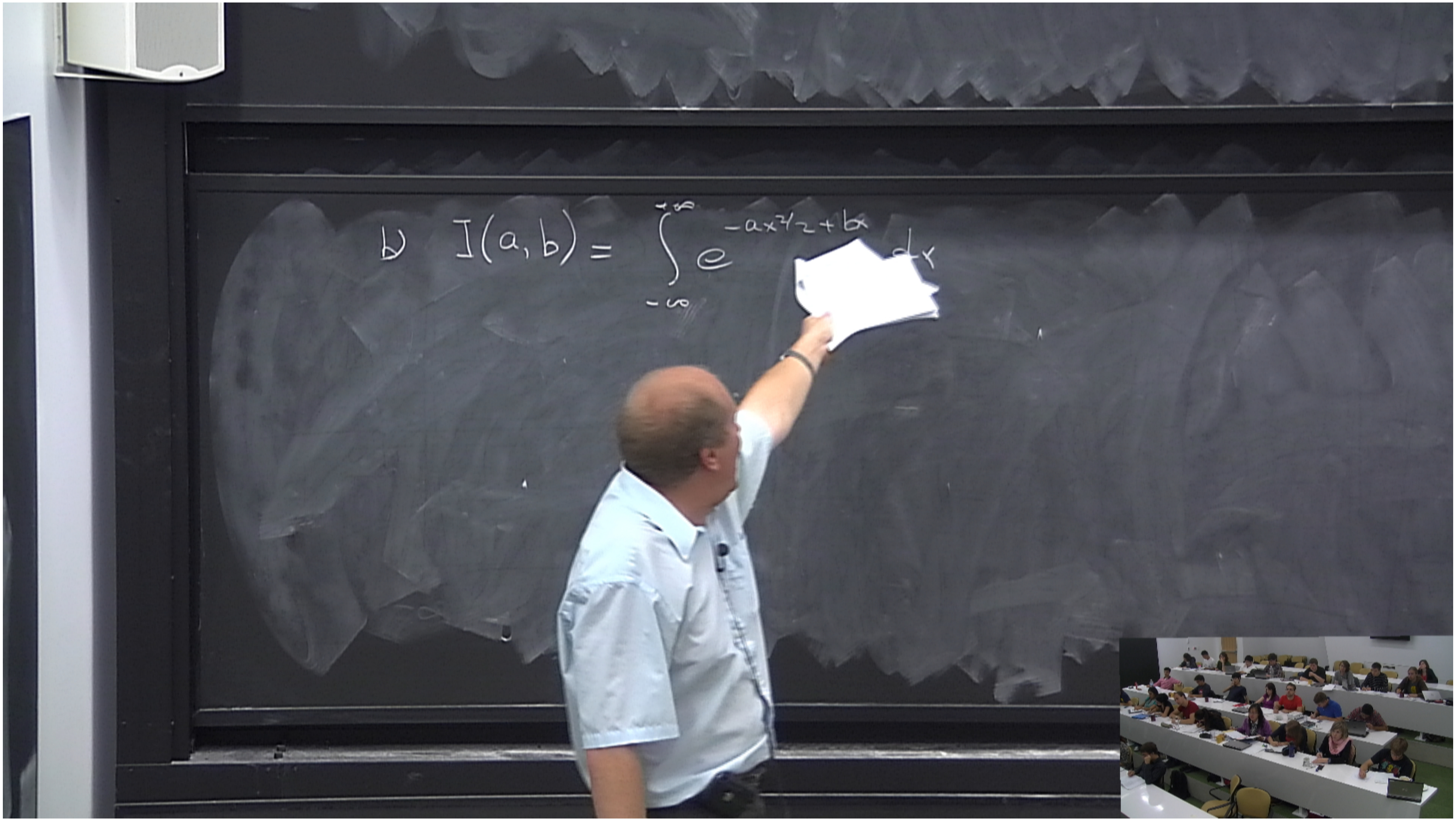
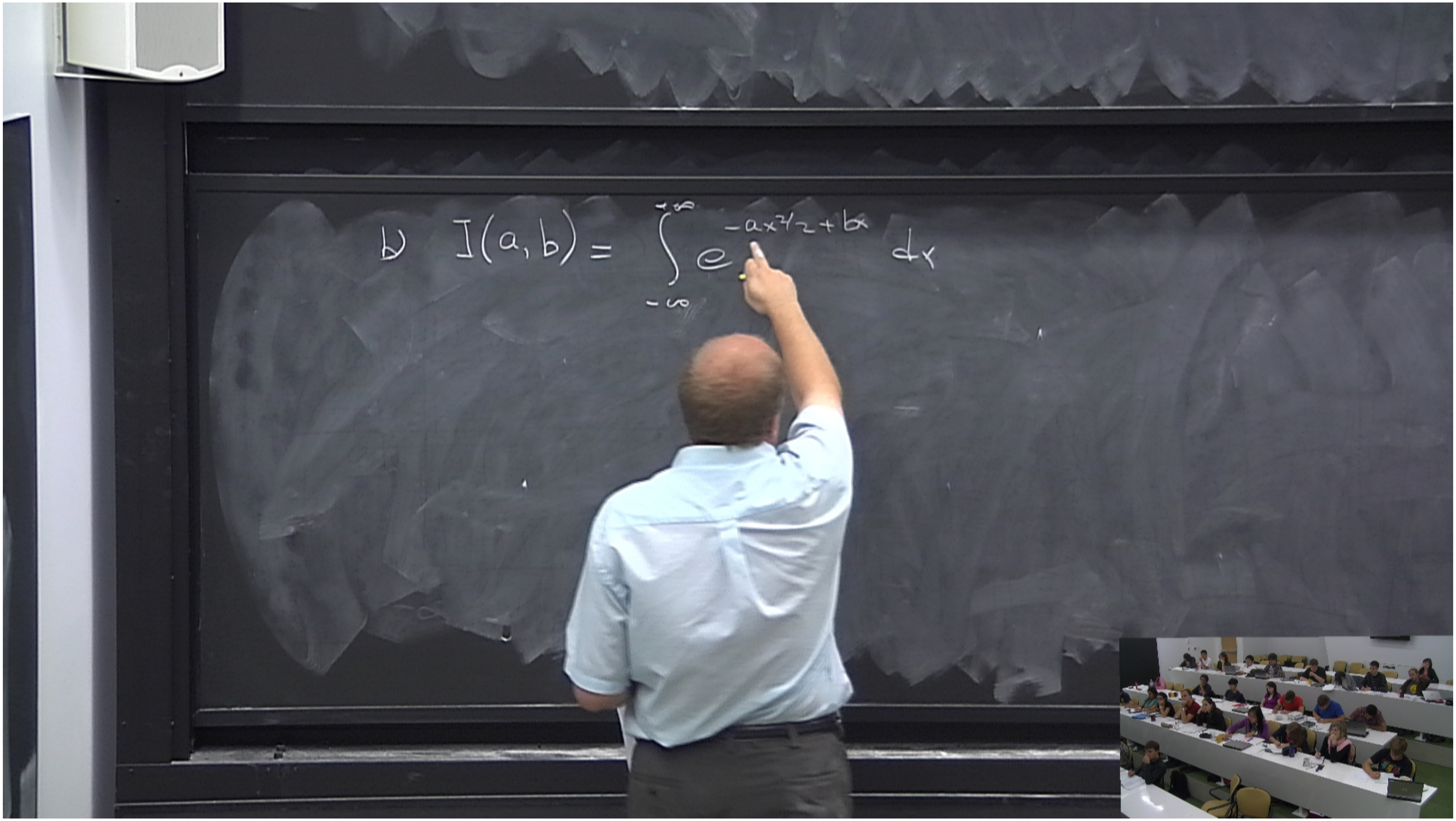
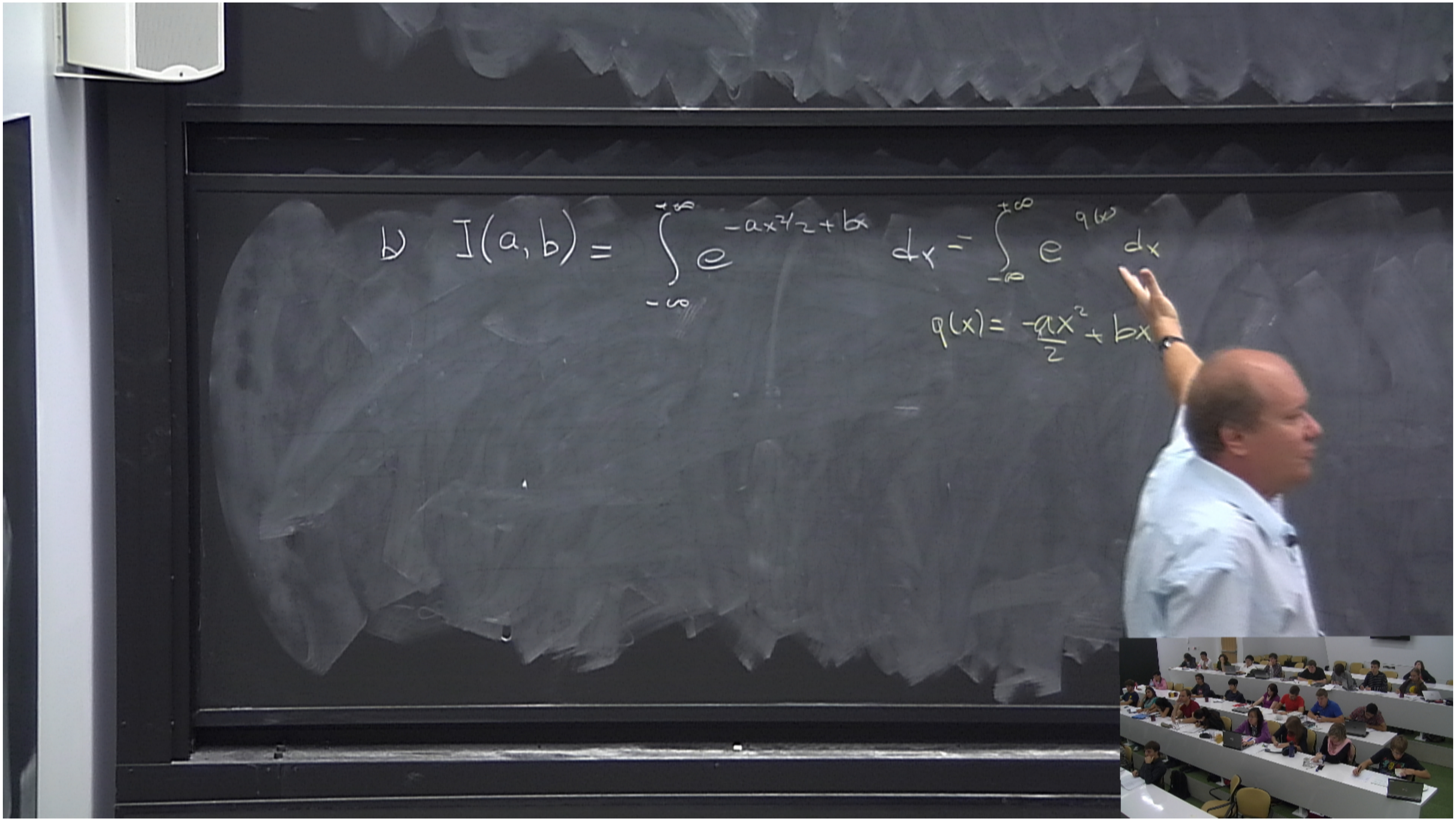
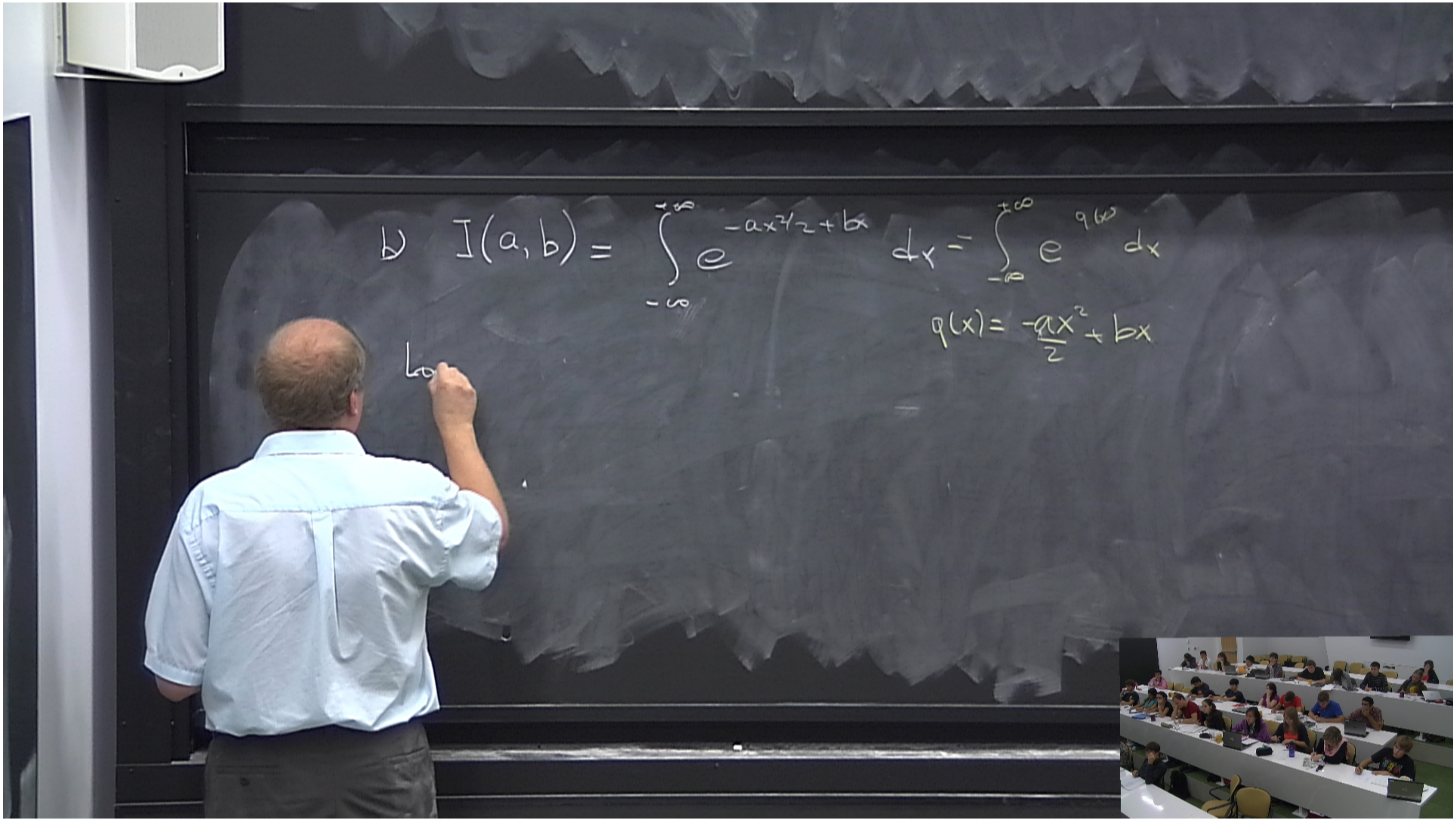

b) I(a, b)


b) I(a, b) = ∫ e^(−ax²/2 + bx) dx, integral from −∞ to +∞


b) $I(a,b) = \int_{-\infty}^{+\infty} e^{-ax^2/2+bx}\, dx$


b) $I(a,b) = \int_{-\infty}^{+\infty} e^{-ax^2/2+bx}\, dx = \int_{-\infty}^{+\infty} e^{q(x)}\, dx$
$q(x) = -\frac{a}{2}x^2 + bx$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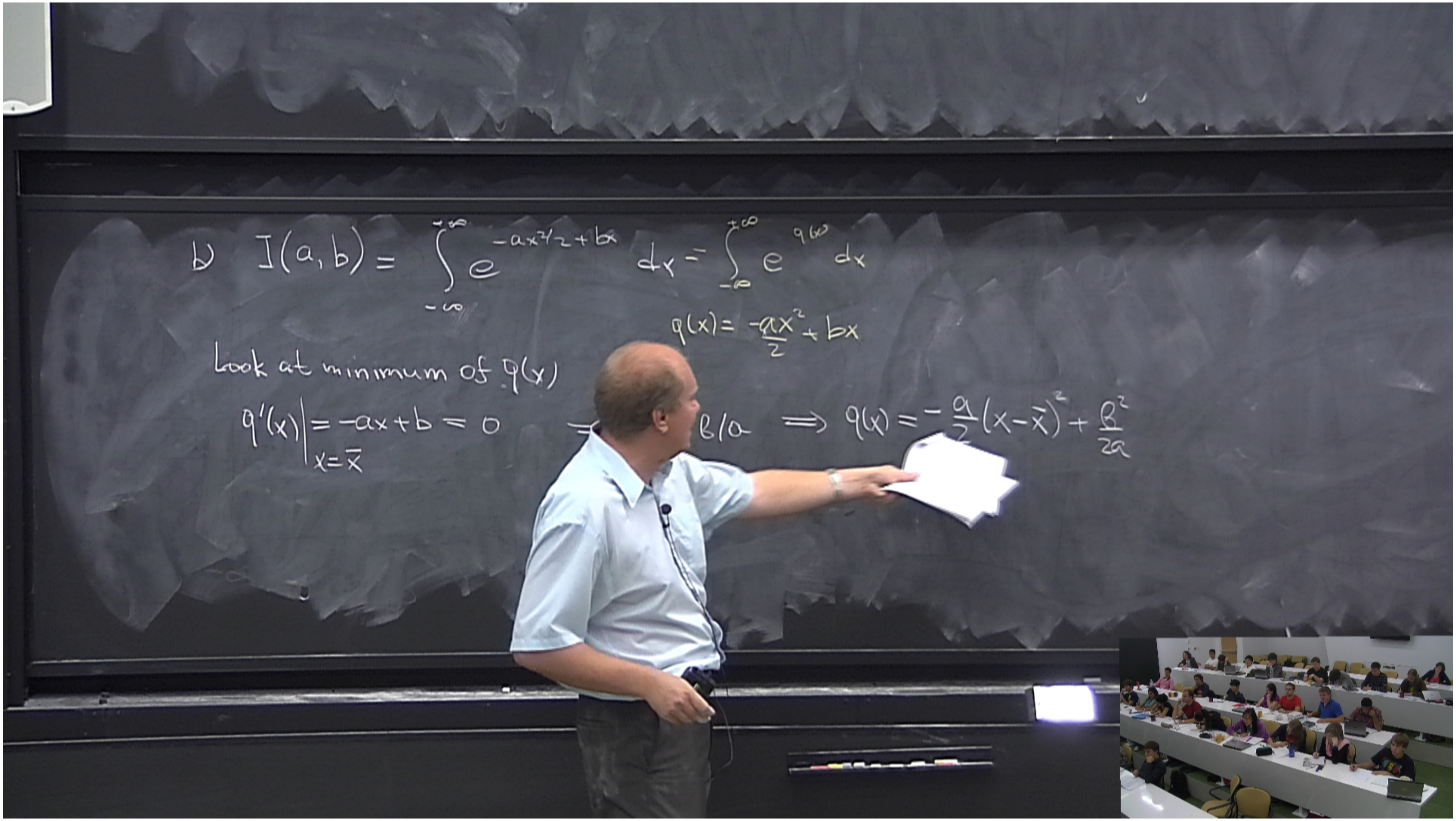
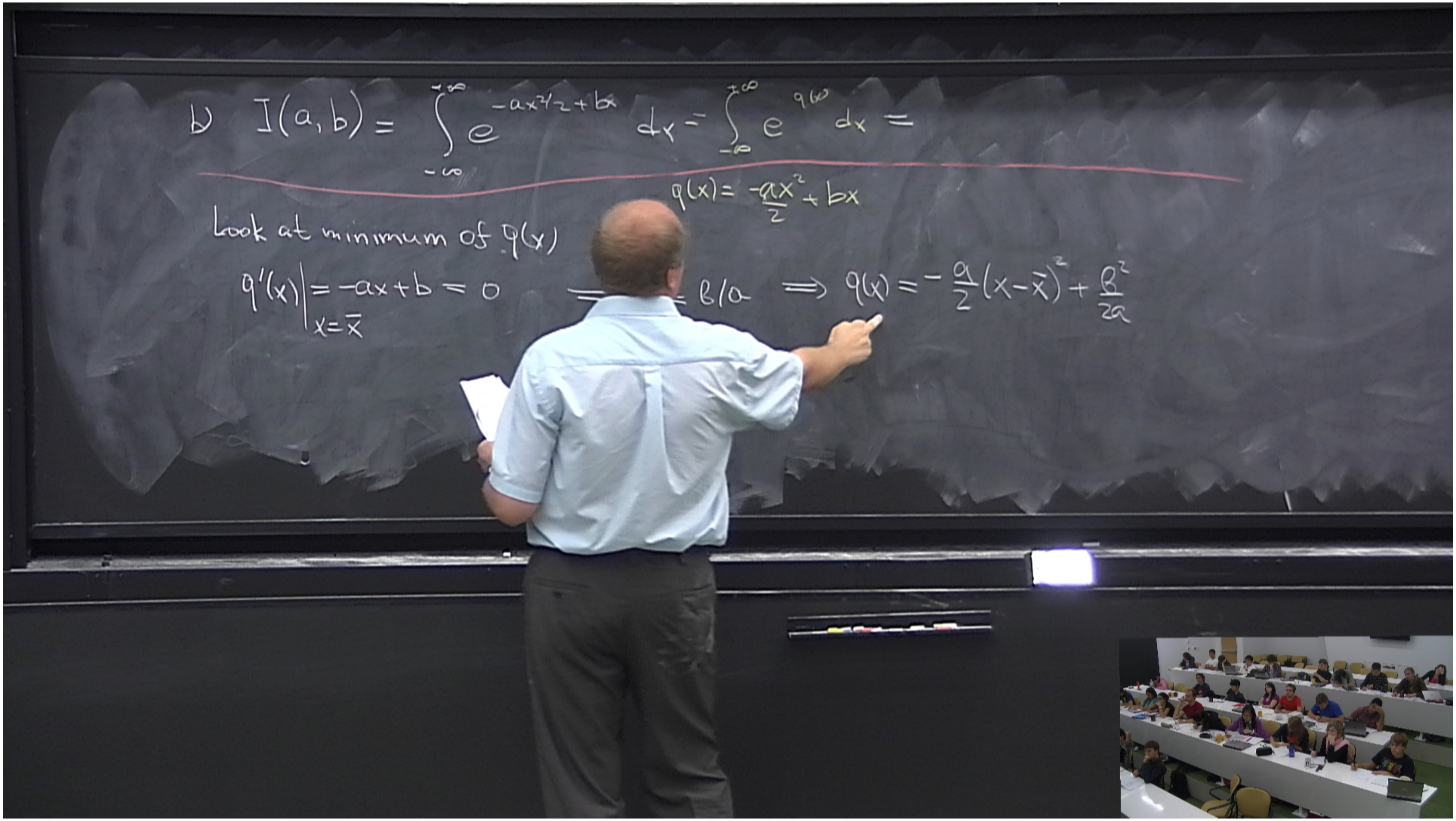
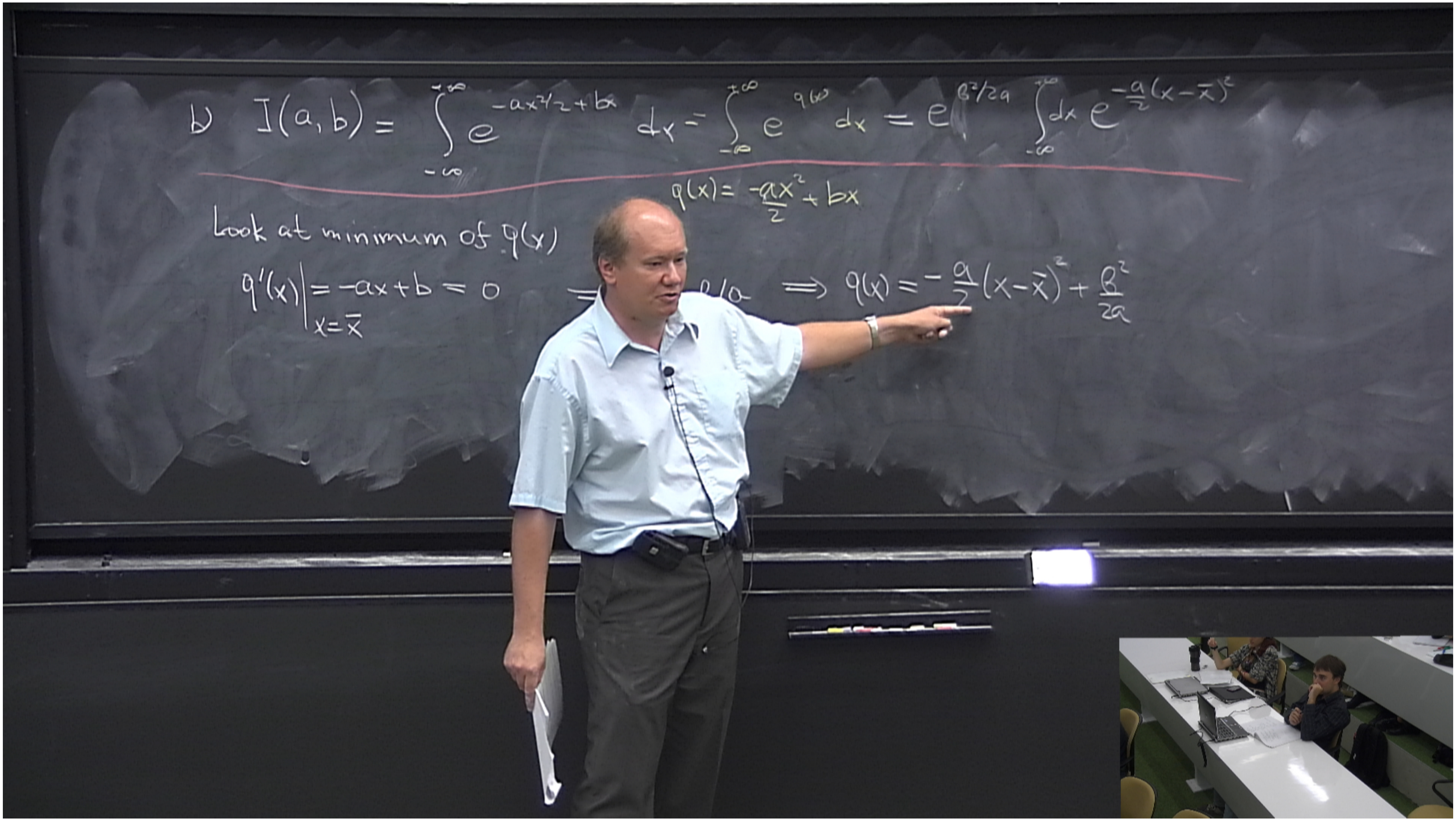
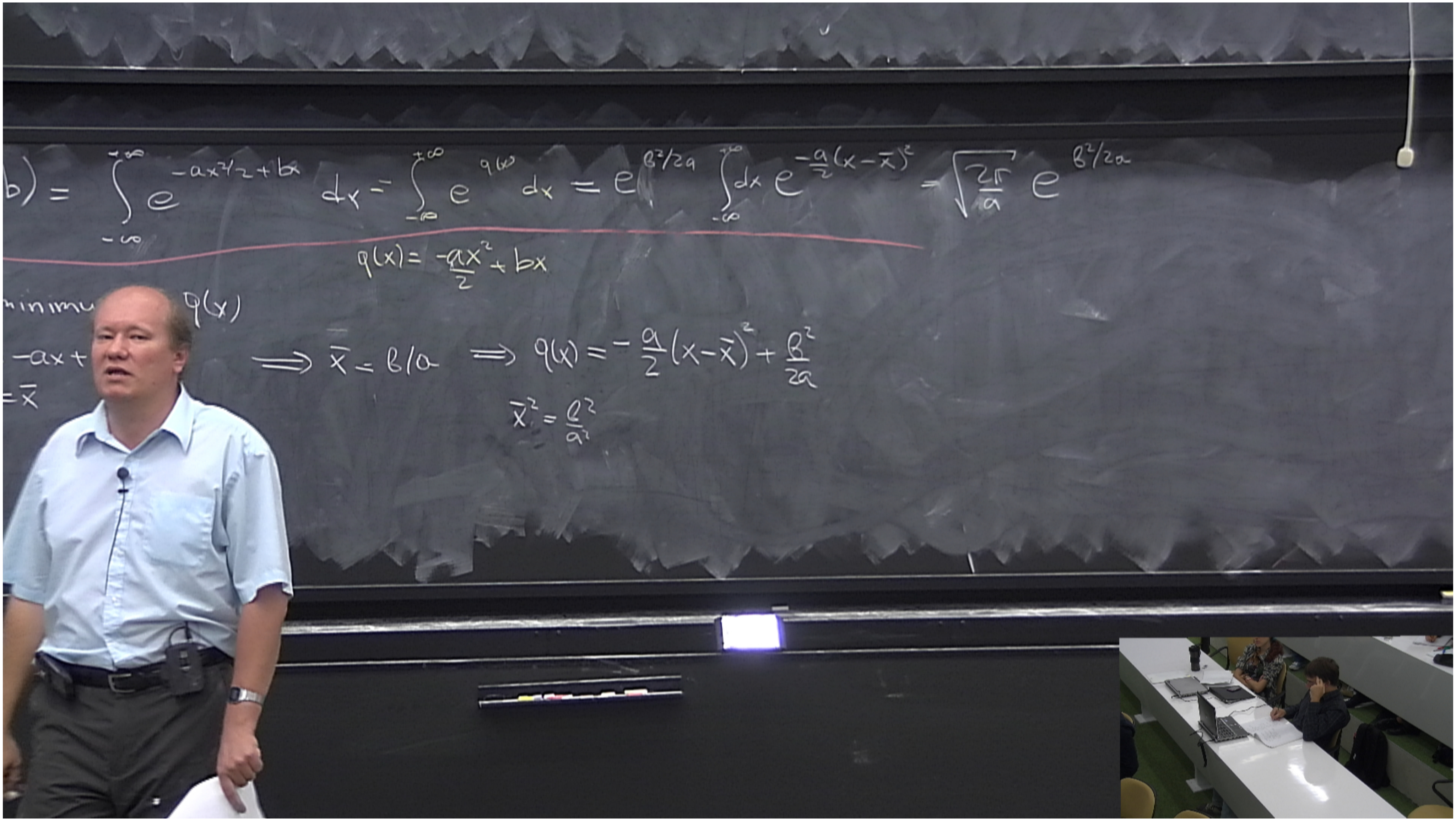
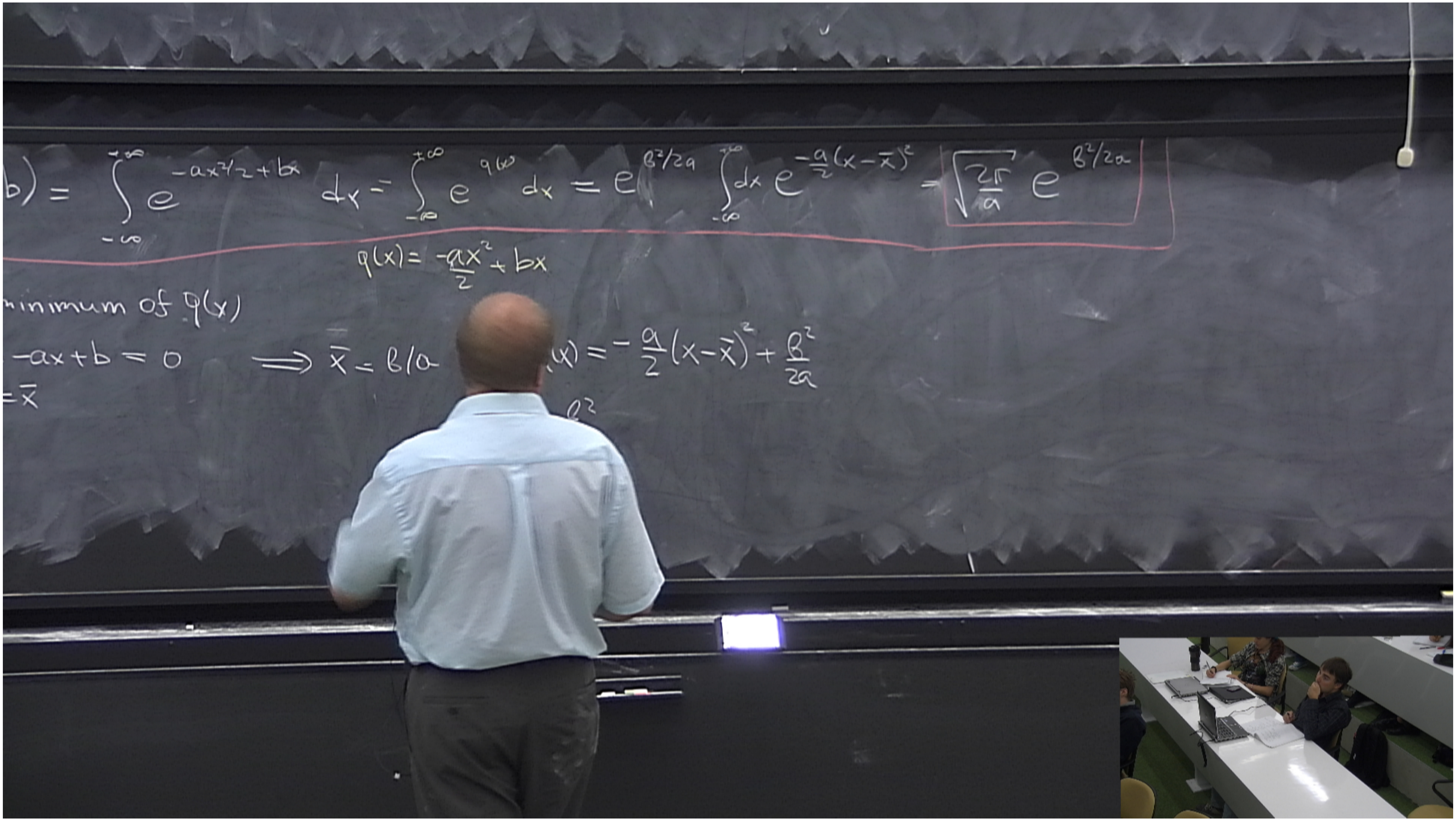
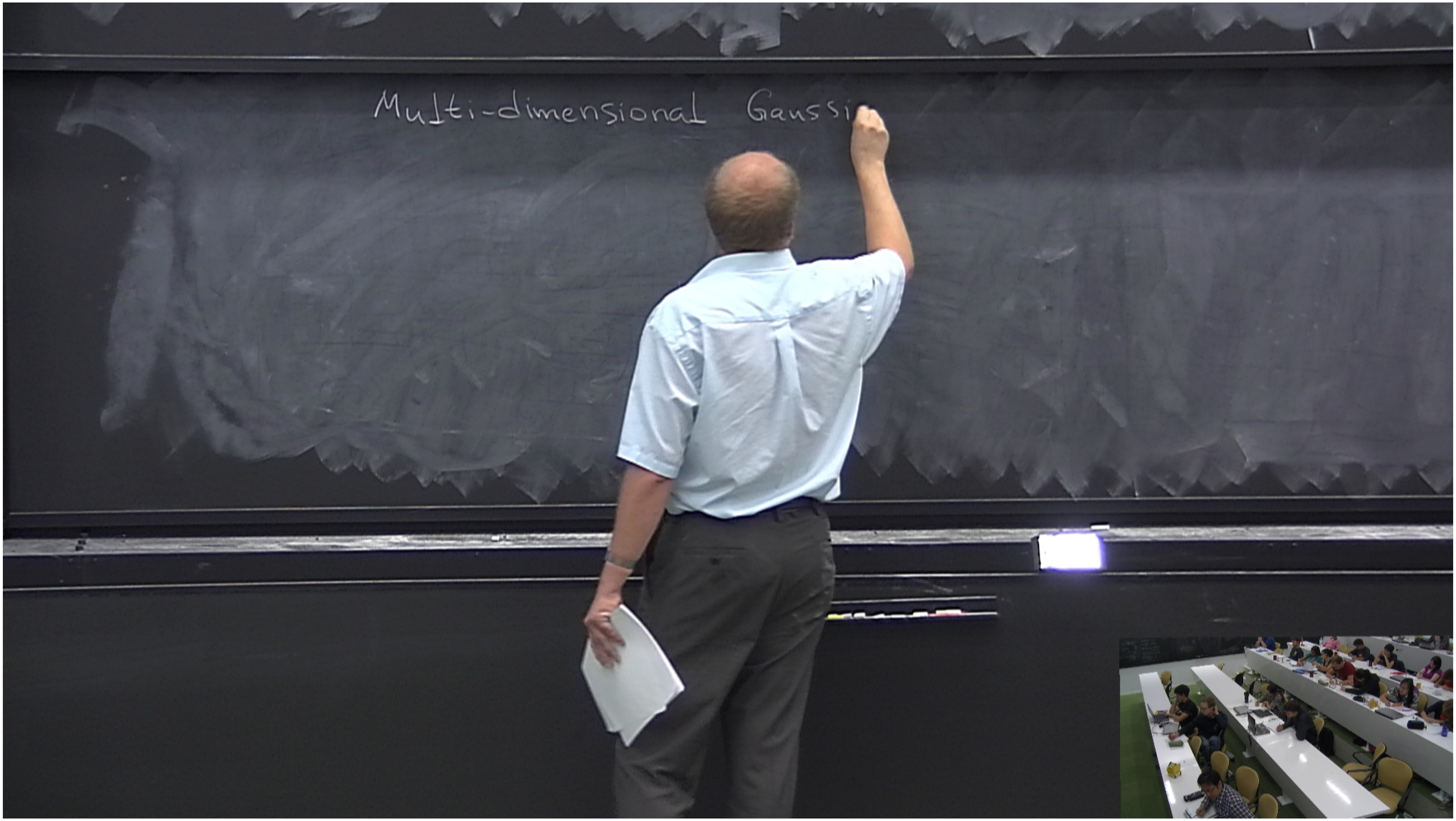

b) $I(a,b) = \int_{-\infty}^{+\infty} e^{-ax^2/2+bx}\,dx = \int_{-\infty}^{+\infty} e^{q(x)}\,dx$
$q(x) = -\frac{a}{2}x^2 + bx$
Look at minimum of q(x)
$q'(x)\big|_{x=\bar{x}} = -ax+b = 0$
$b/a \implies q(x) = -\frac{a}{2}(x-\bar{x})^2 + \frac{b^2}{2a}$

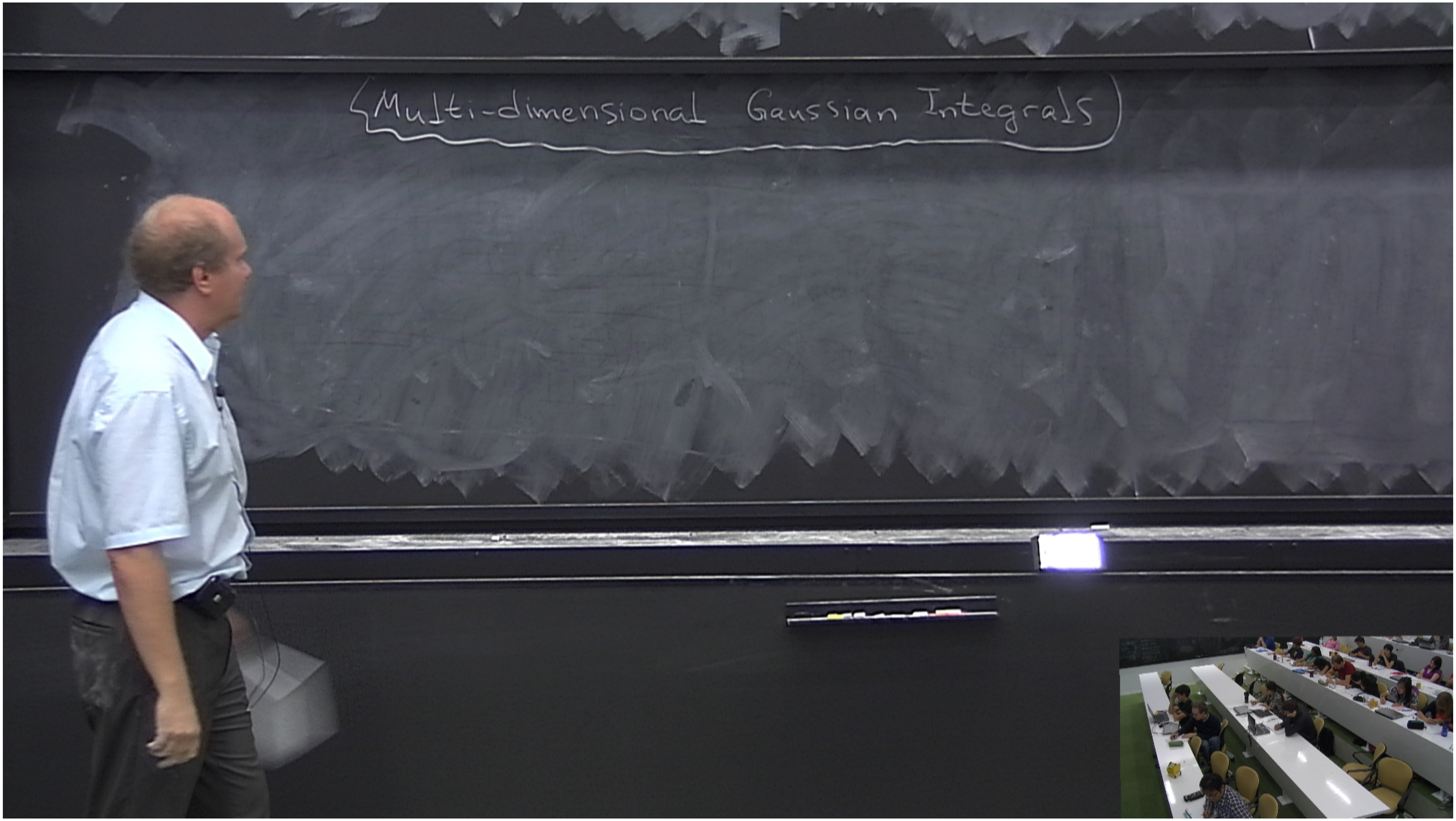

b) I(a,b) = ∫ e^(-ax²/2+bx) dx = ∫ e^(q(x)) dx =
q(x) = -a/2 x² + bx
Look at minimum of q(x)
q'(x)| x=x̄ = -ax+b = 0
= b/a ⟹ q(x) = - a/2 (x-x̄)² + b²/2a

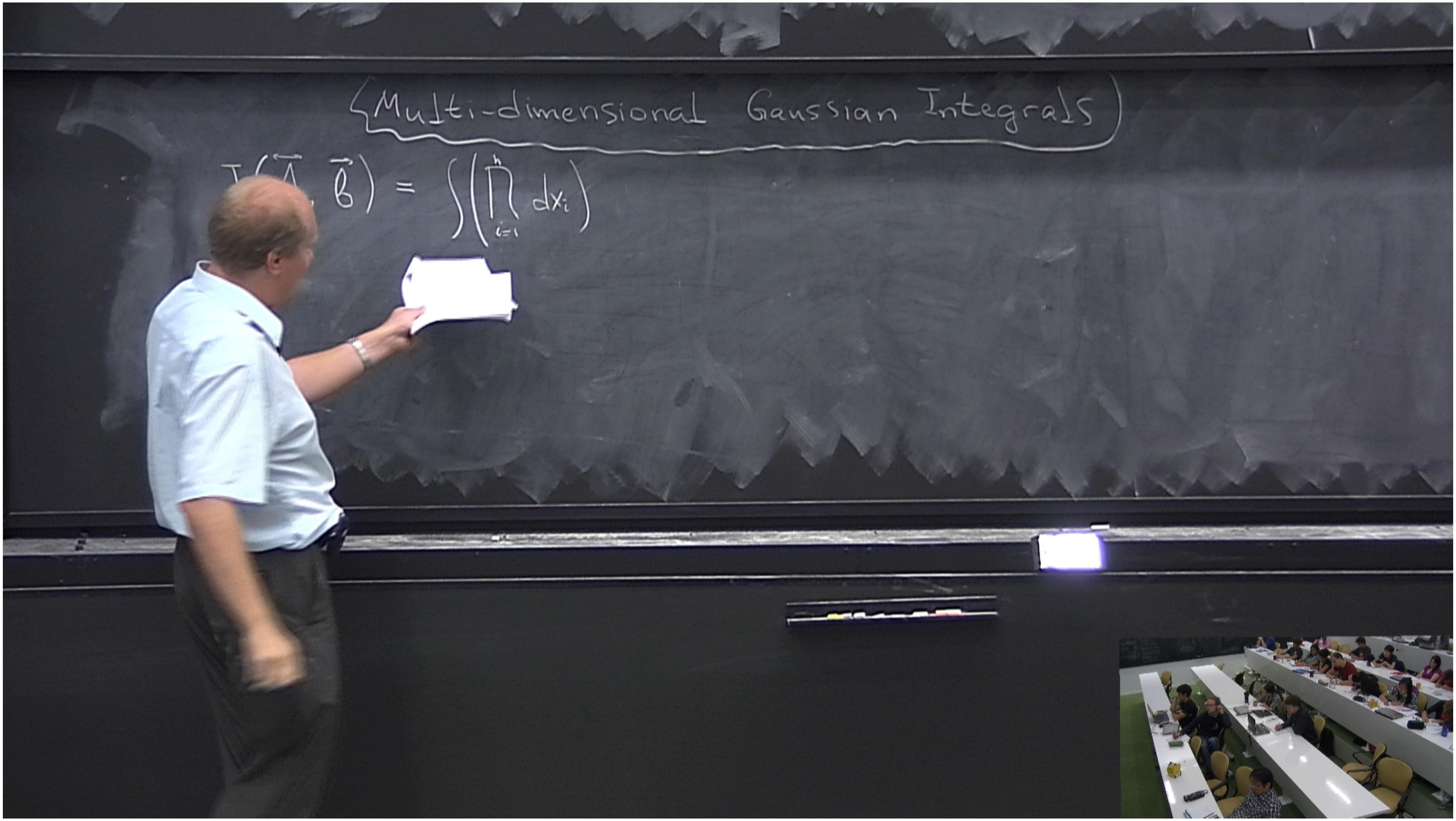

b) I(a, b) = ∫ e^{-ax²/2+bx} dx = ∫ e^{q(x)} dx = e^{β²/2a} ∫dx e^{-a/2(x-x̄)²}
q(x) = -a/2 x² + bx
Look at minimum of q(x)
q'(x)|x=x̄ = -ax + b = 0
⟹ q(x) = -a/2 (x-x̄)² + β²/2a

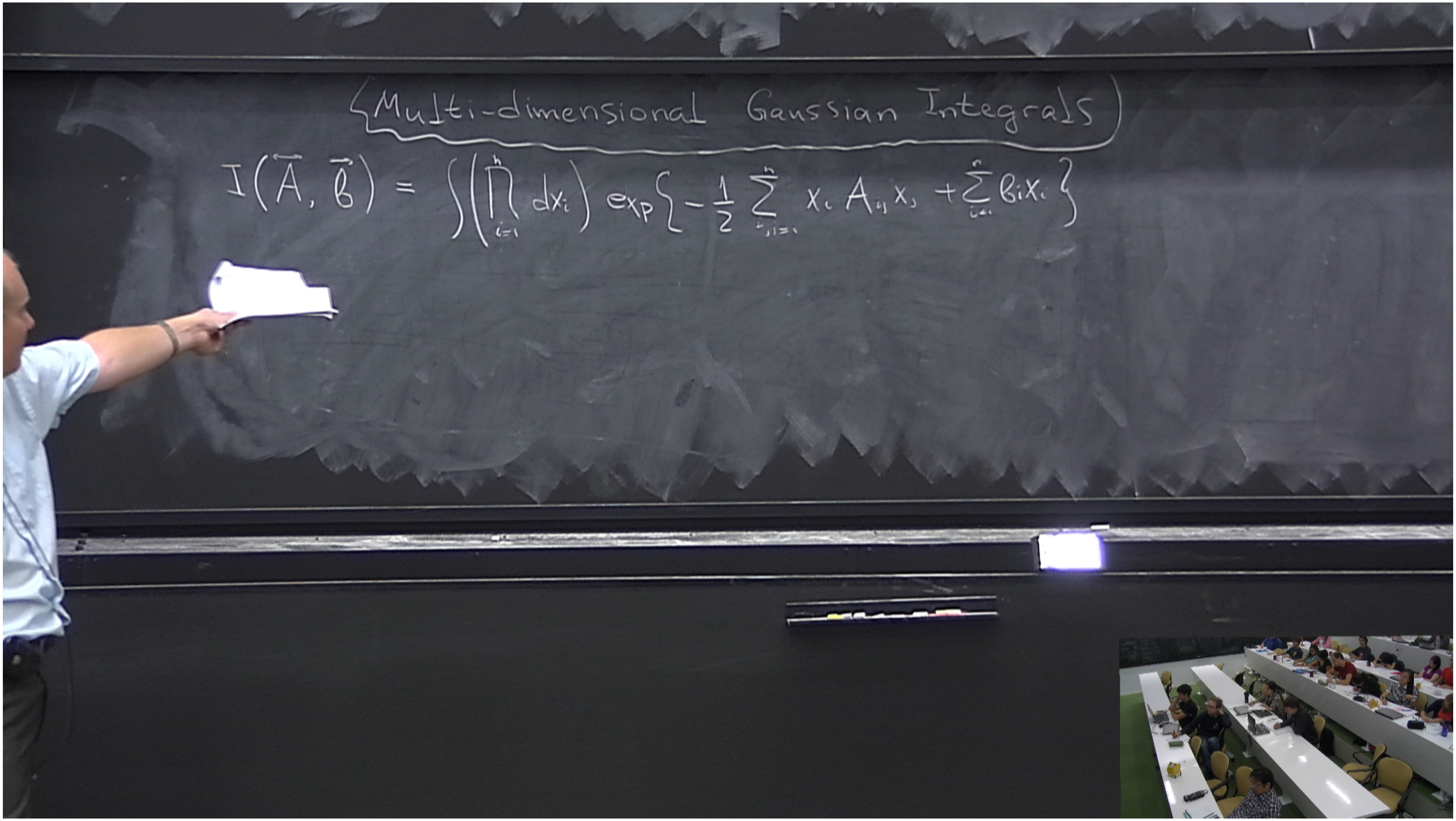

q(x) = -ax²/2 + bx
⟹ x̄ = b/a ⟹ q(x) = - a/2 (x-x̄)² + b²/2a
x̄² = b²/a²

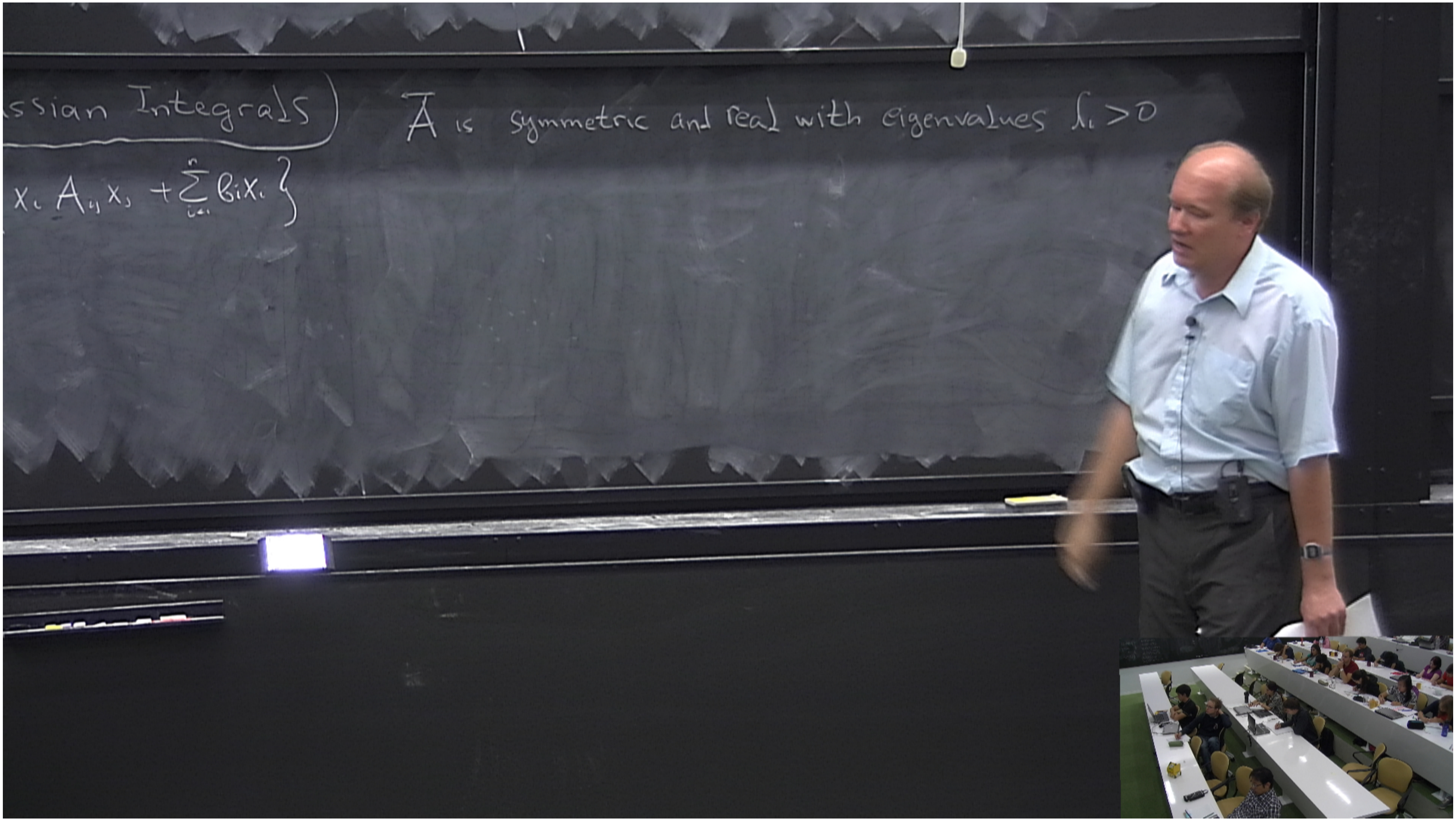

q(x) = -ax^2/2 + bx
minimum of q(x)
-ax + b = 0
x̄ = b/a

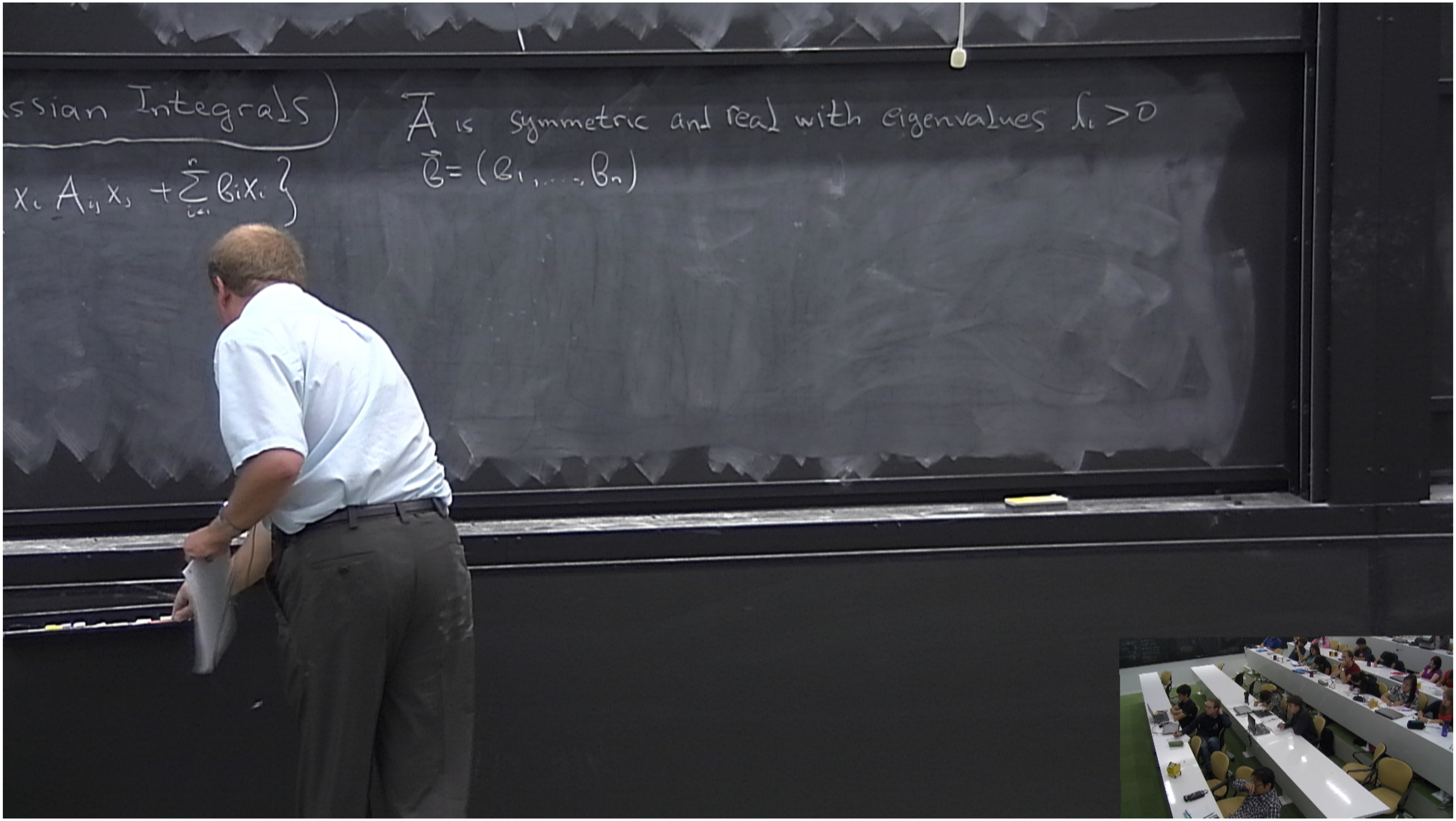

Multi-dimensional Gaussi


Multi-dimensional Gaussian Integrals


Multi-dimensional Gaussian Integrals


Multi-dimensional Gaussian Integrals
$I(\overleftrightarrow{A}, \vec{B}) = \int \left( \prod_{i=1}^{n} dx_i \right) \exp\left\{ -\frac{1}{2} \sum_{i,j=1}^{n} x_i A_{ij} x_j + \sum_{i=1}^{n} B_i x_i \right\}$


ssian Integrals
A is symmetric and real with eigenvalues λi > 0
xi Aij xj + Σ Bi xi }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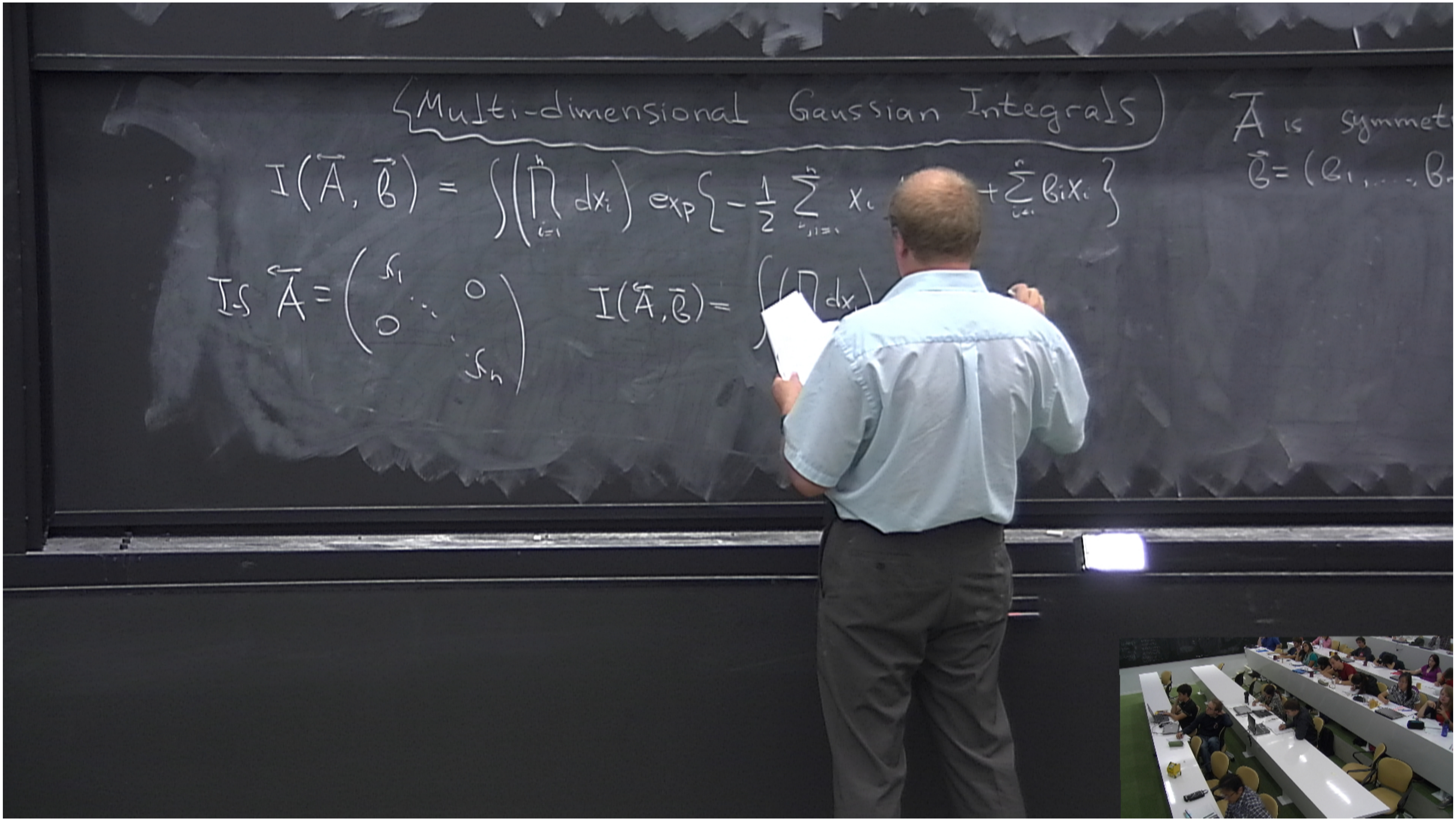
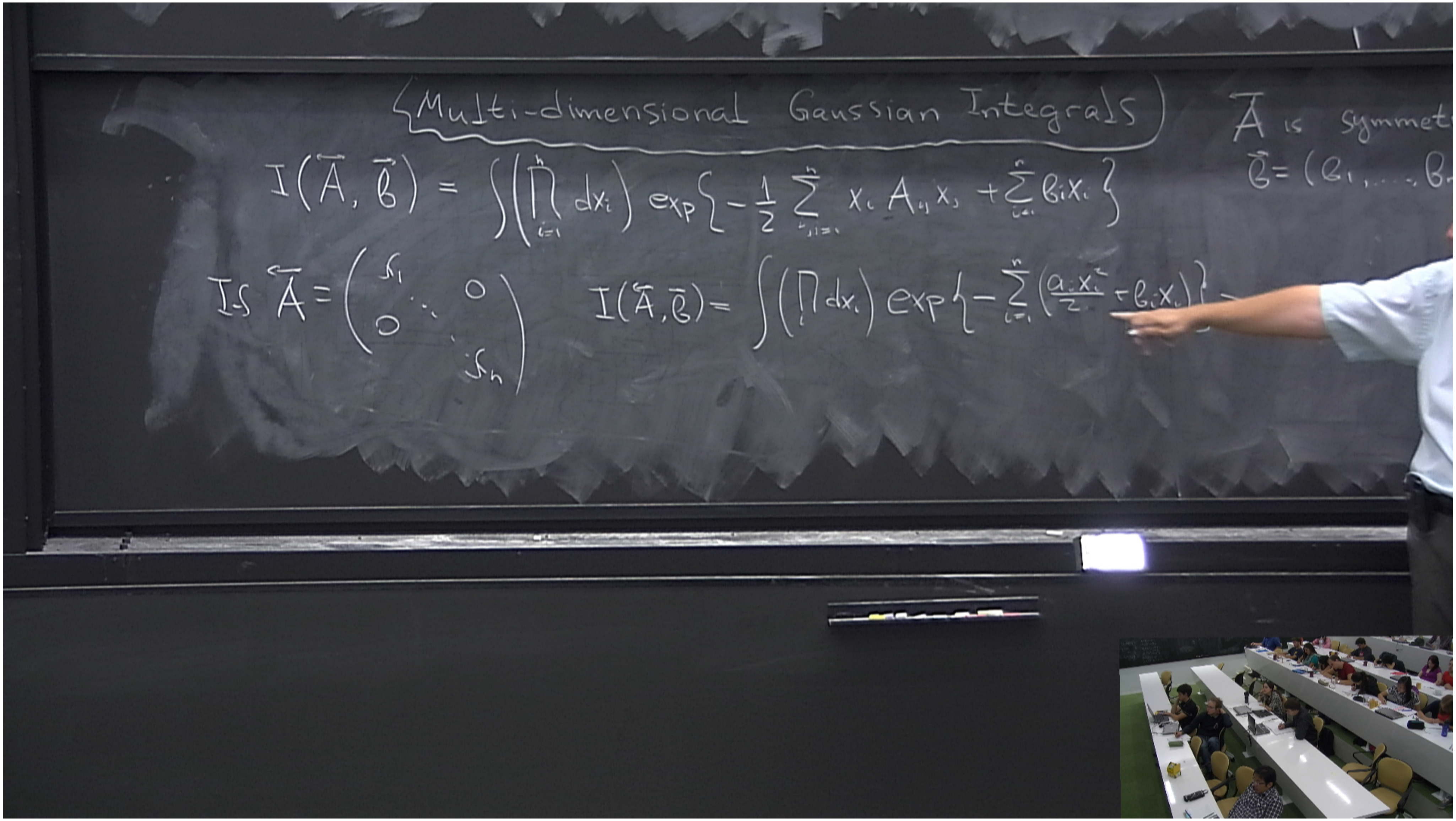
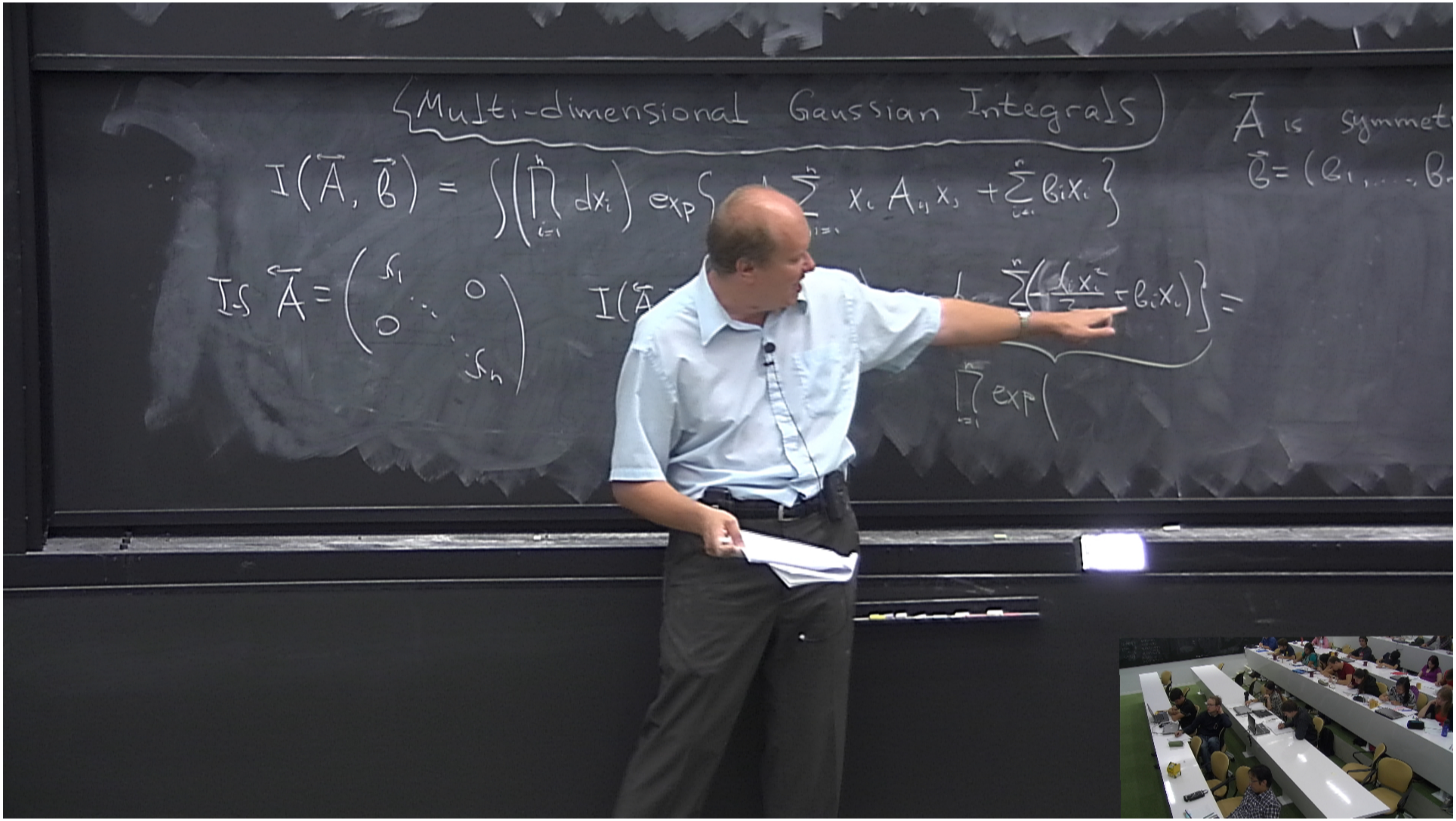
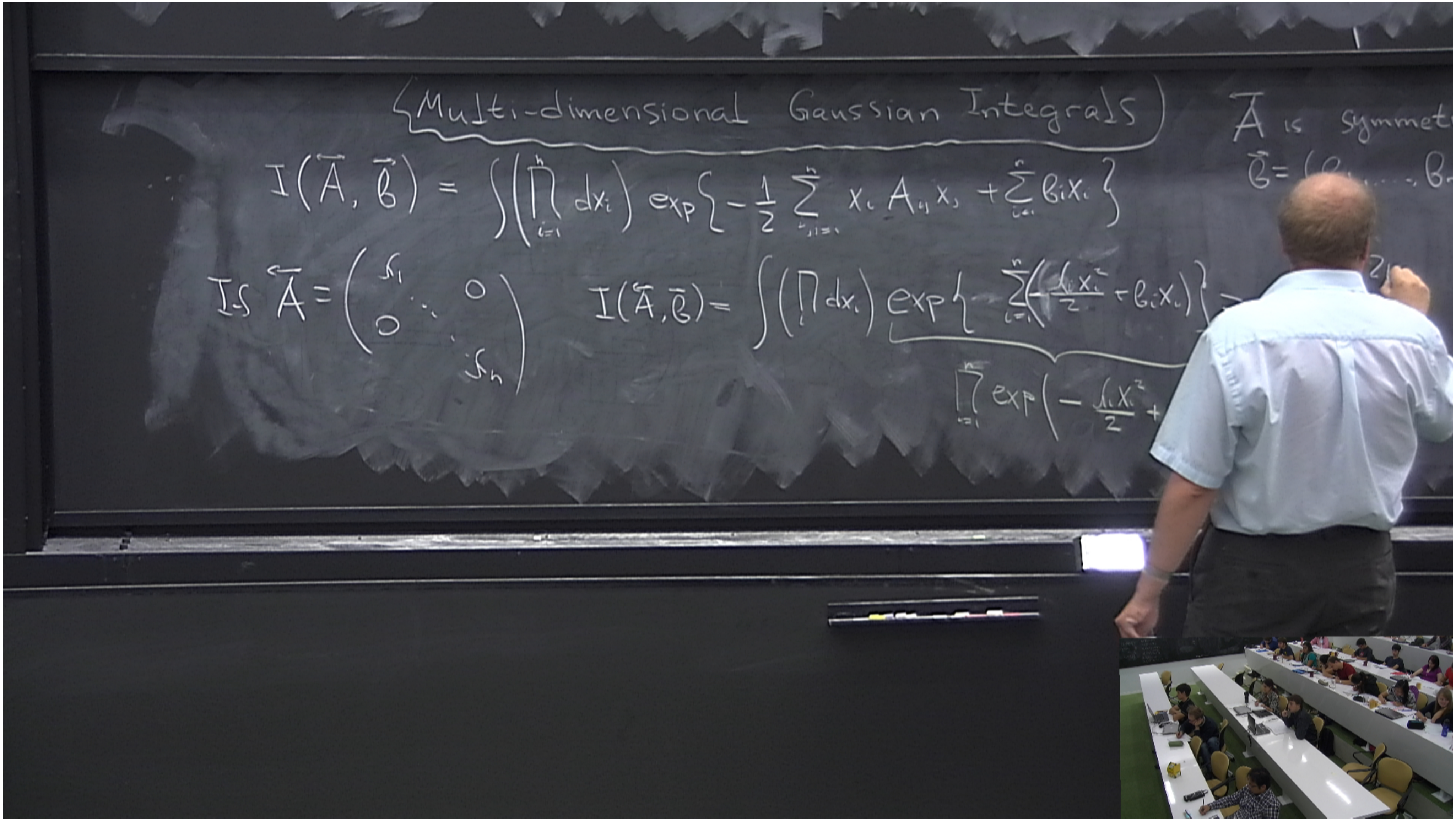
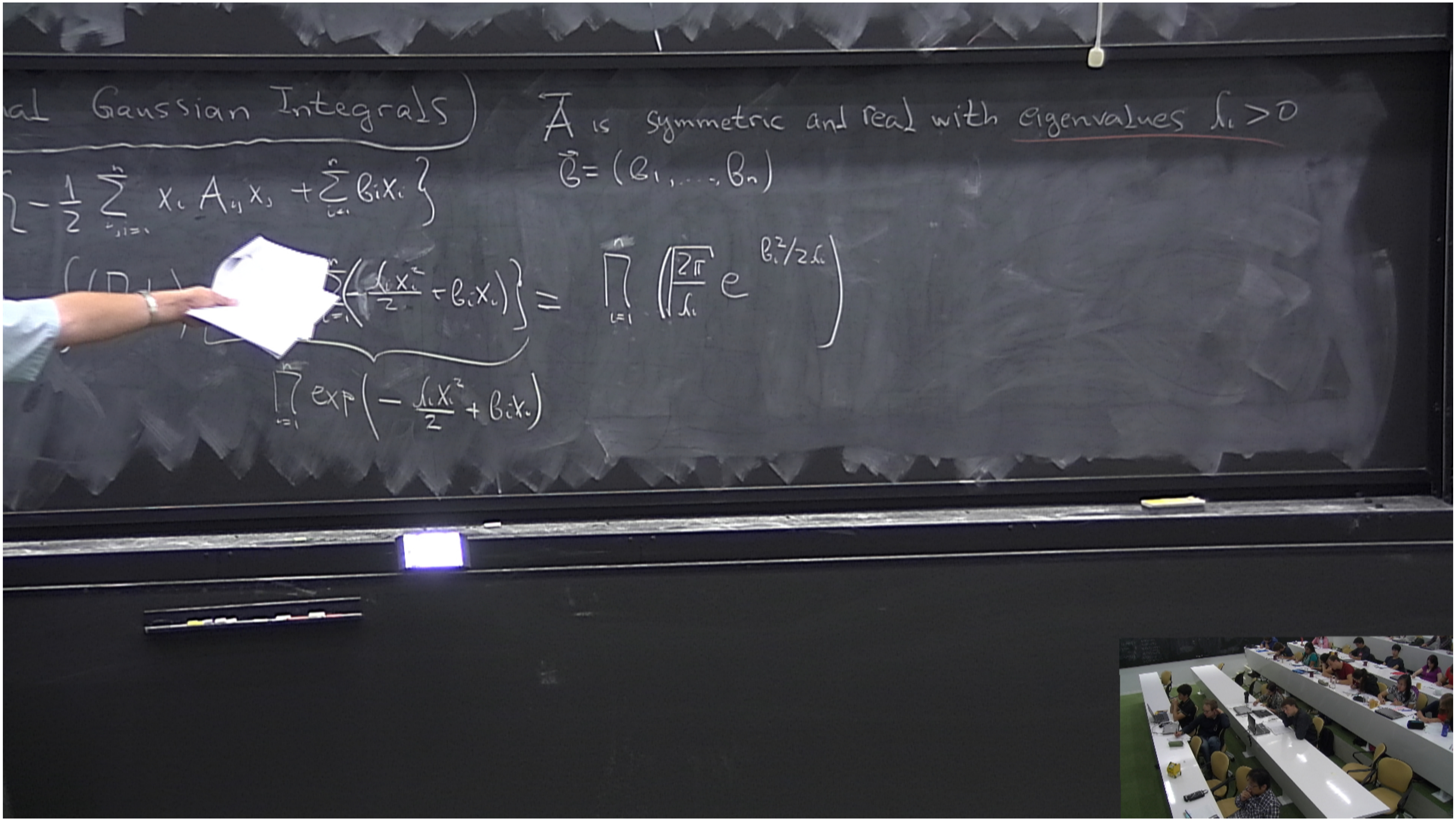
B = (B1, ..., Bn)
xi Aij xj + Σ Bi xi }

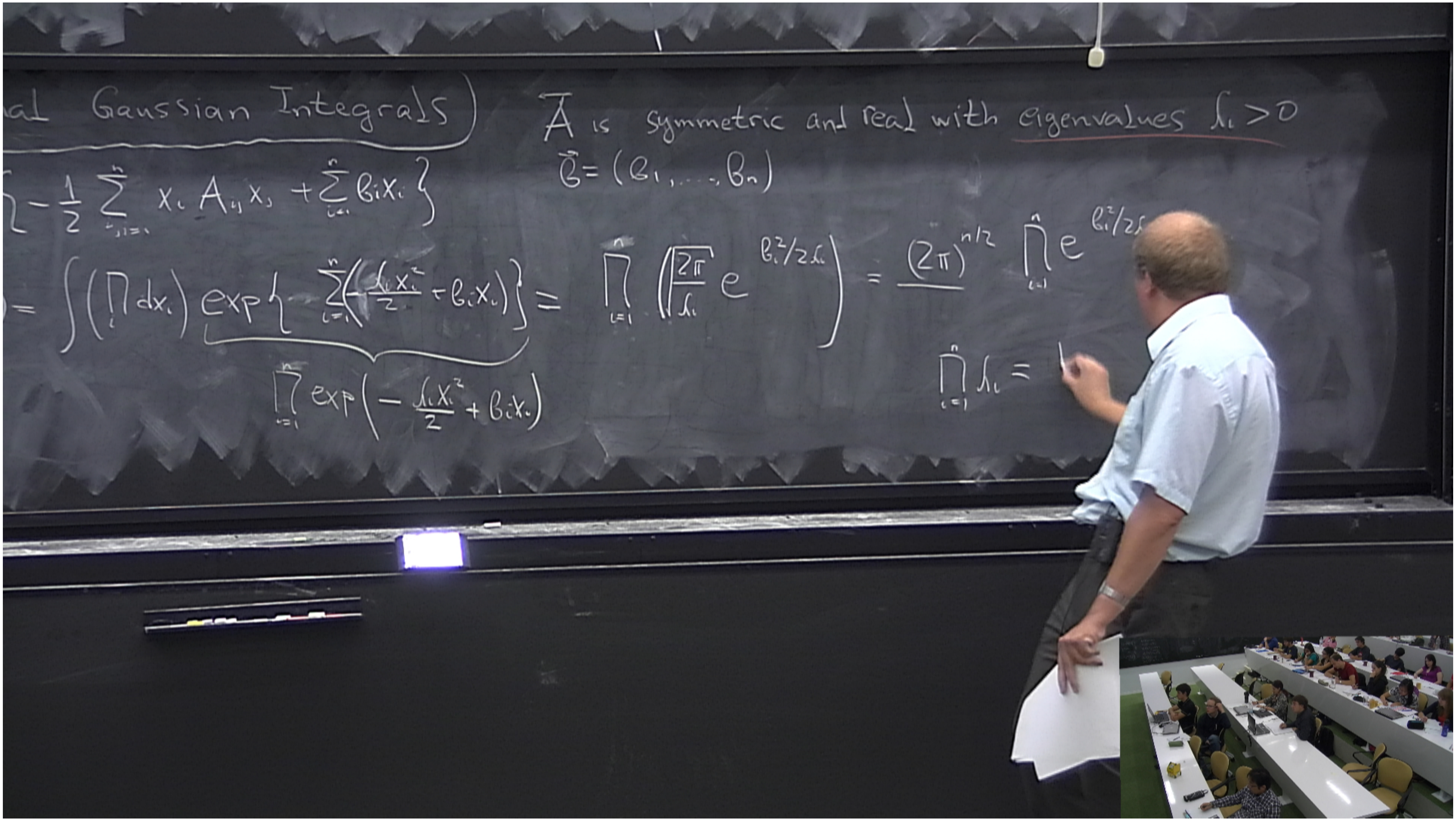

Multi-dimensional Gaussian Integrals
A is symmet
B = (B1, ..., B
I(A, B) =
exp
Is A =
I(A, B) =

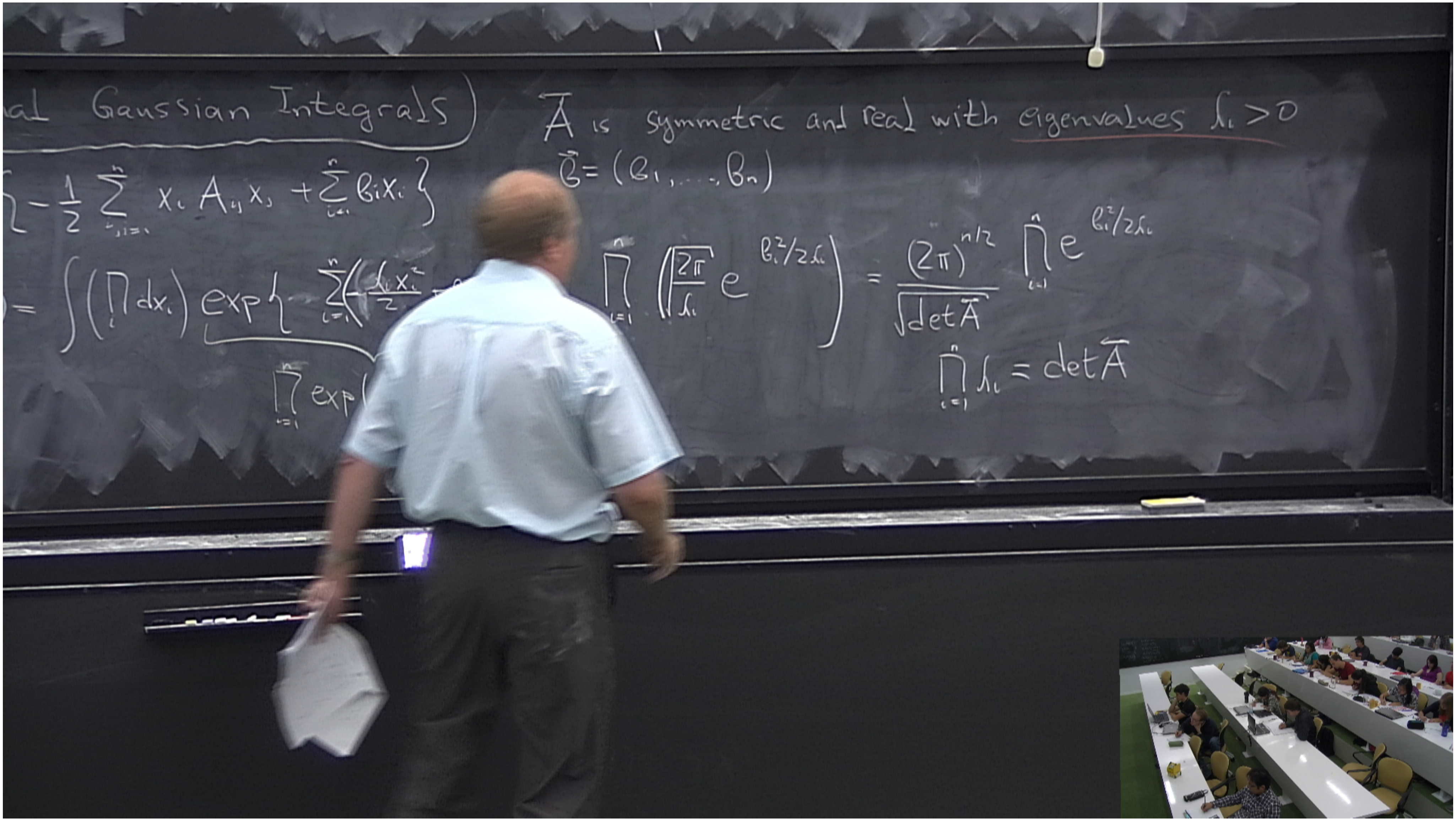

Multi-dimensional Gaussian Integrals
A is symmet
I(A, B) = ∫(∏ dxi) exp{−1/2 Σ xi Aij xj + Σ Bi xi}
If A = (λ1 ... 0; 0 ... λn)
I(A, B) = ∫(∏ dxi) exp{−Σ (ai xi²/2 + Bi xi)}

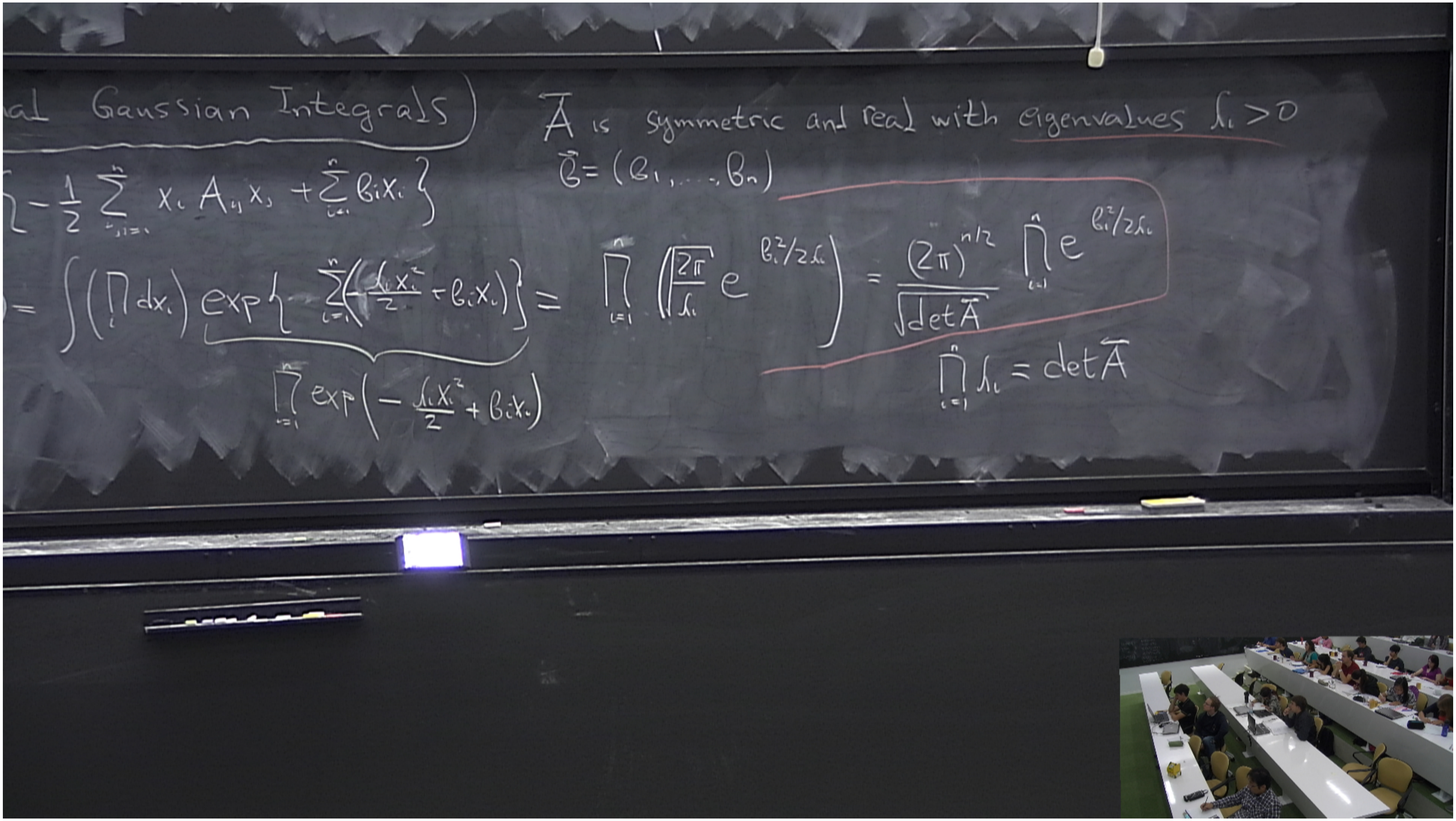

Multi-dimensional Gaussian Integrals
A is symmet
B = (B1, ..., B
I(A, B) = ∫(∏ dxi) exp{ ... Σ xi Aij xj + Σ Bi xi }
Is A = (λ1 ... 0 ; 0 ... λn)
I(A ... exp{ Σ(-λi xi²/2 + Bi xi)} =
∏ exp(

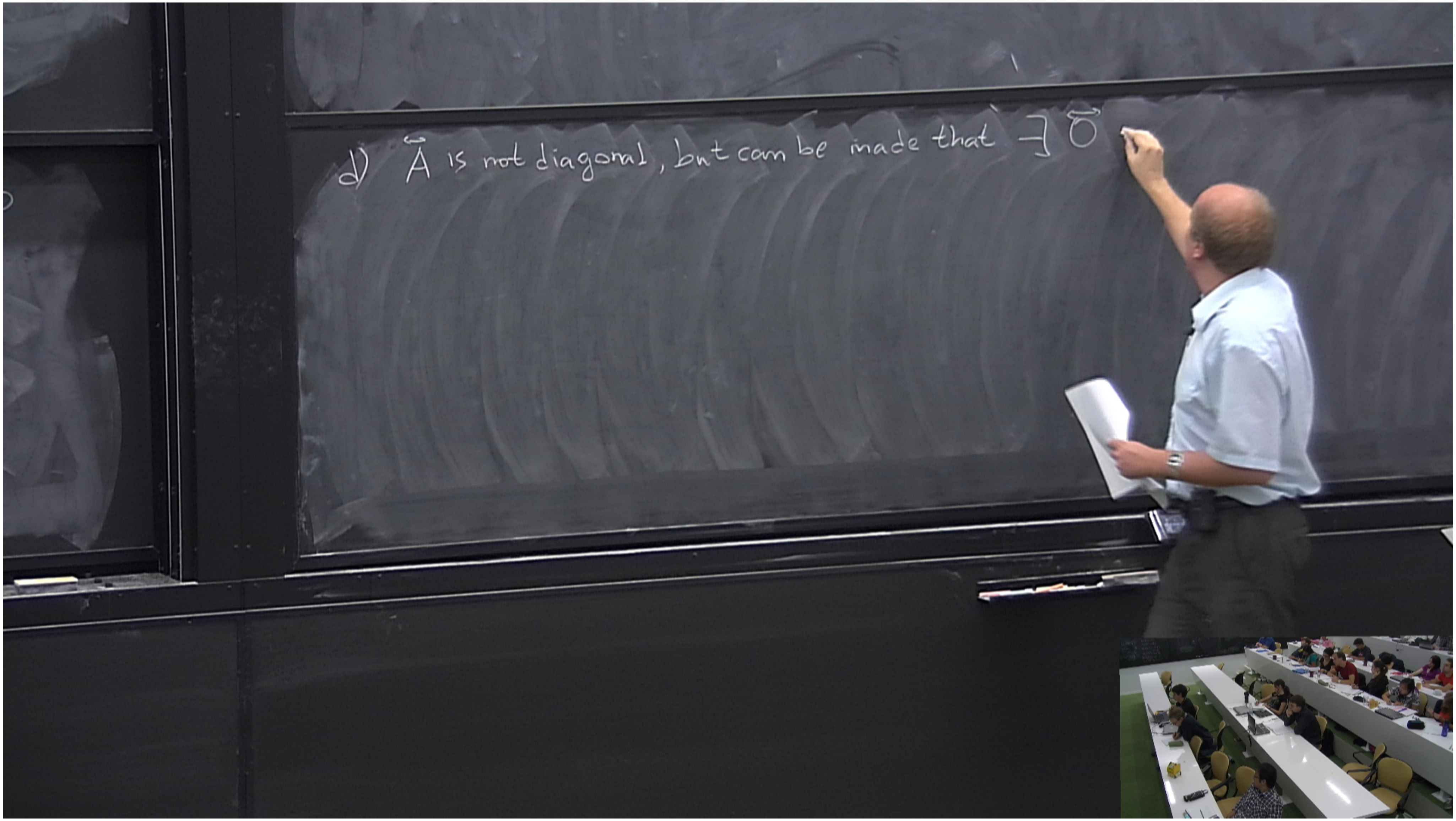

Multi-dimensional Gaussian Integrals
$I(\vec{A},\vec{B}) = \int \left(\prod_{i=1}^{n} dx_i\right) \exp\left\{-\frac{1}{2}\sum_{i,j=1}^{n} x_i A_{ij} x_j + \sum_{i=1}^{n} B_i x_i\right\}$
Is $\overleftarrow{A} = \begin{pmatrix} \lambda_1 & & 0 \\ 0 & \ddots & \\ & & \lambda_n \end{pmatrix}$
$I(\overleftarrow{A},\vec{B}) = \int \left(\prod dx_i\right) \exp\left\{\sum_{i=1}^{n}\left(-\frac{\lambda_i x_i^2}{2} + B_i x_i\right)\right\}$
$\prod_{i=1}^{n} \exp\left(-\frac{\lambda_i x_i^2}{2} + \right.$
$\overleftarrow{A}$ is

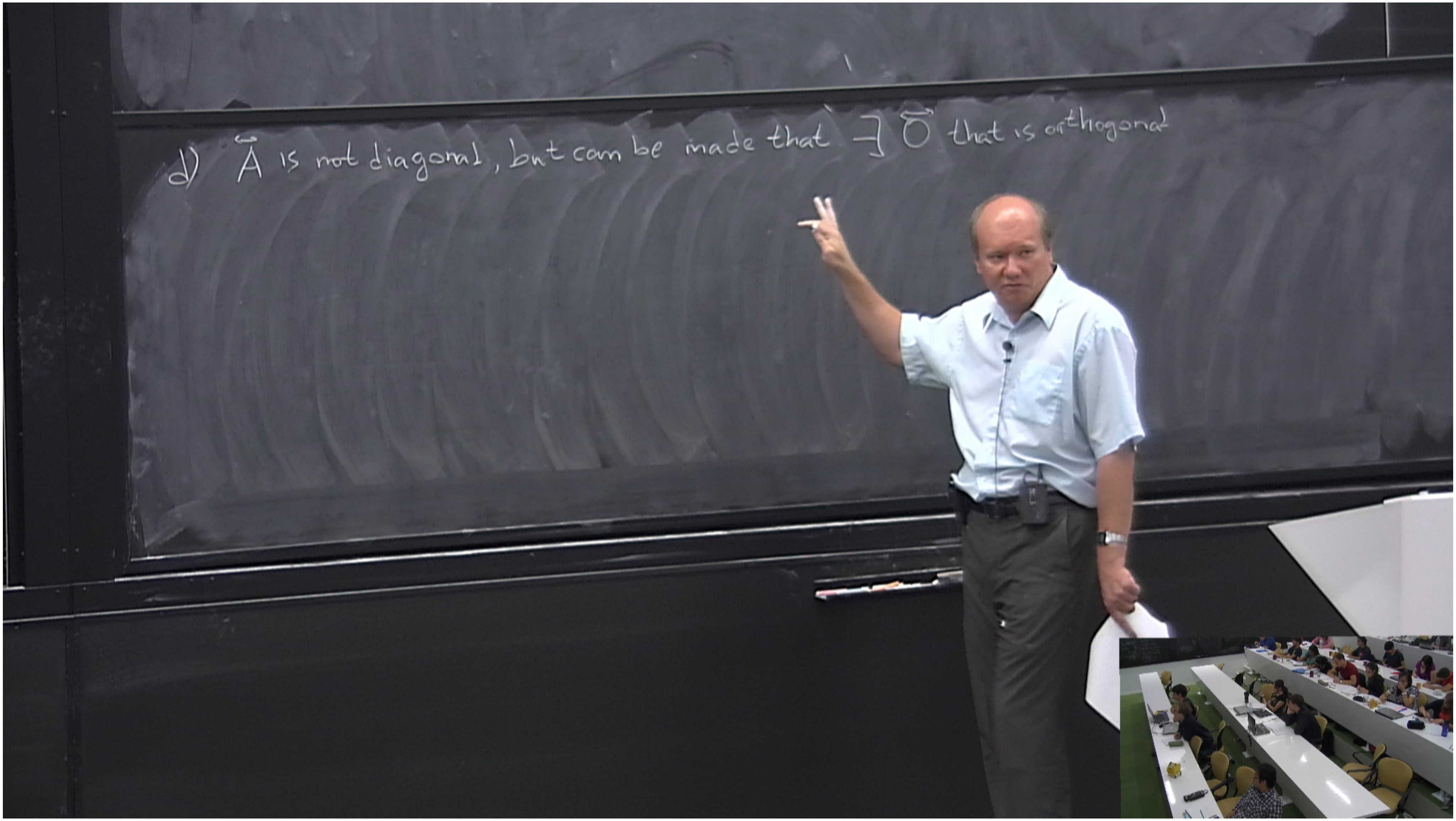

al Gaussian Integrals
A is symmetric and real with eigenvalues λi > 0
B = (B1, ..., Bn)
exp

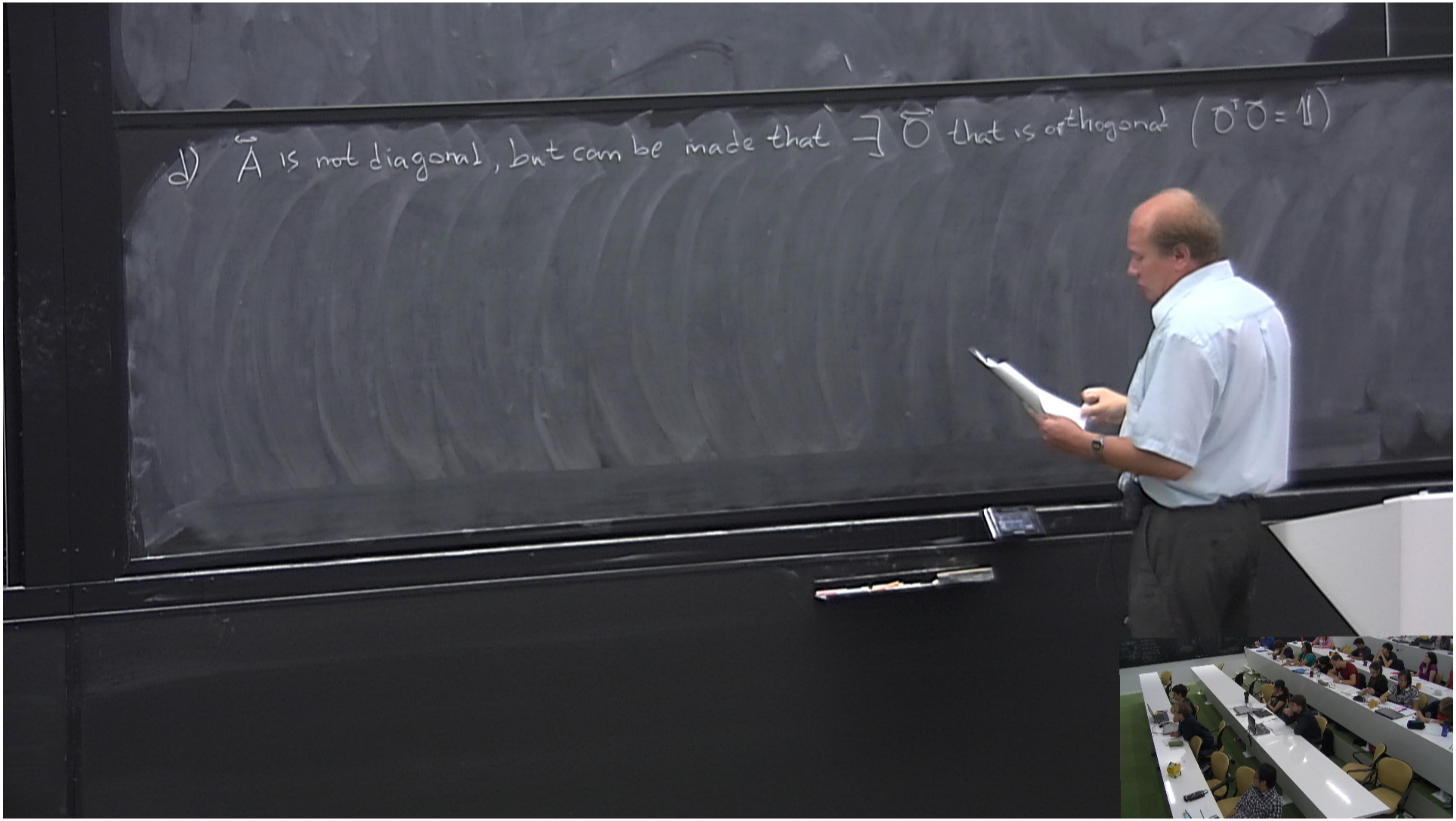

al Gaussian Integrals
$\bar{A}$ is symmetric and real with eigenvalues $\lambda_i > 0$
$\vec{B} = (B_1, \ldots, B_n)$
$\{-\frac{1}{2}\sum_{i,j=1}^{n} x_i A_{ij} x_j + \sum_{i=1}^{n} B_i x_i\}$
$= \int \left(\prod_i dx_i\right) \exp\{\sum_{i=1}^{n}\left(-\frac{\lambda_i x_i^2}{2} + B_i x_i\right)\} = \prod_{i=1}^{n}\left(\sqrt{\frac{2\pi}{\lambda_i}}\, e^{B_i^2/2\lambda_i}\right) = \frac{(2\pi)^{n/2}}{\quad} \prod_{i=1}^{n} e^{B_i^2/2\lambda_i}$
$\prod_{i=1}^{n} \exp\left(-\frac{\lambda_i x_i^2}{2} + B_i x_i\right)$
$\prod_{i=1}^{n} \lambda_i =$

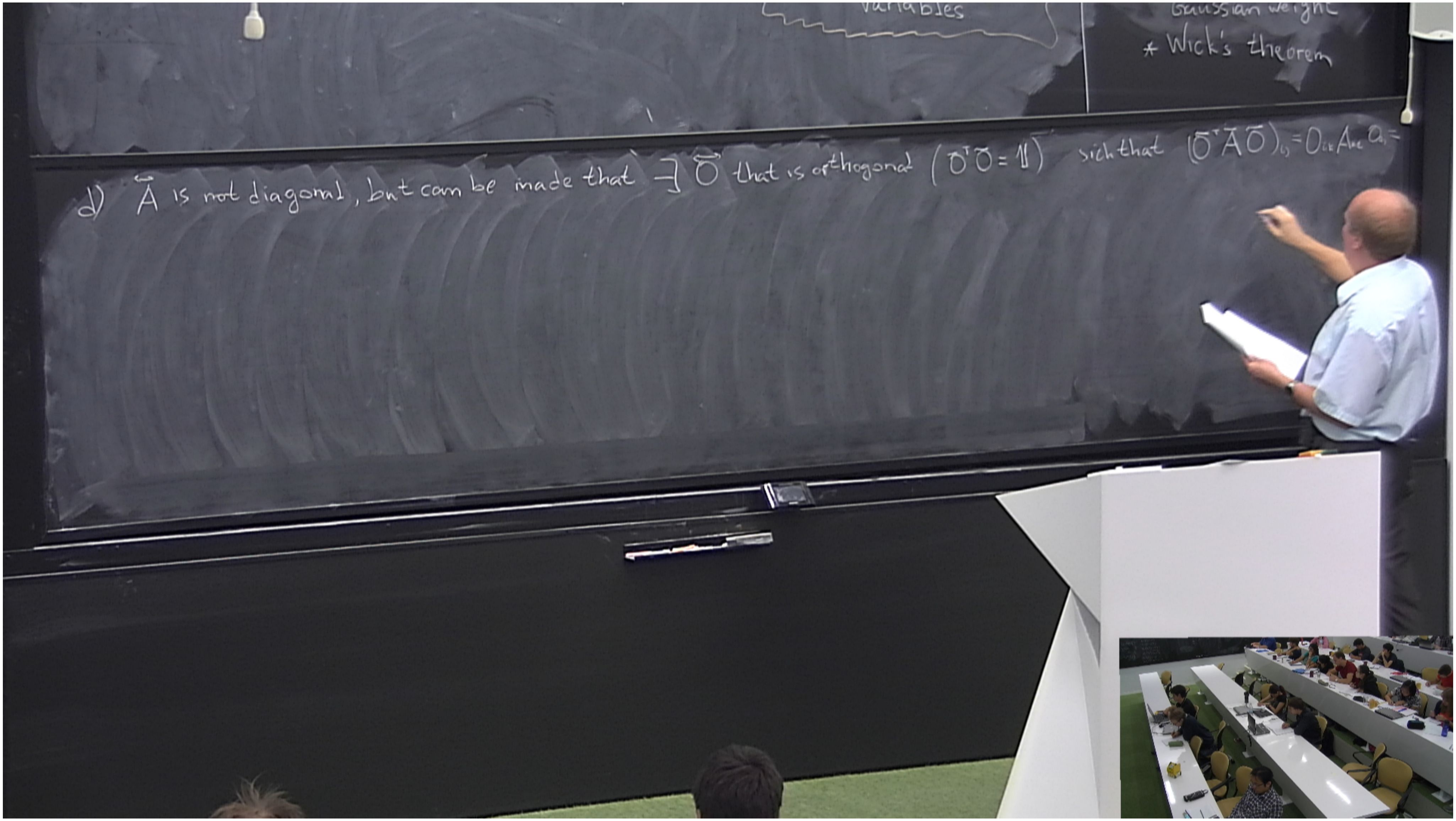

Gaussian Integrals
A is symmetric and real with eigenvalues λ_i > 0
B = (B_1, ..., B_n)
= (2π)^{n/2} / √det A ∏_{i=1}^{n} e^{B_i^2/2λ_i}
∏_{i=1}^{n} λ_i = det A

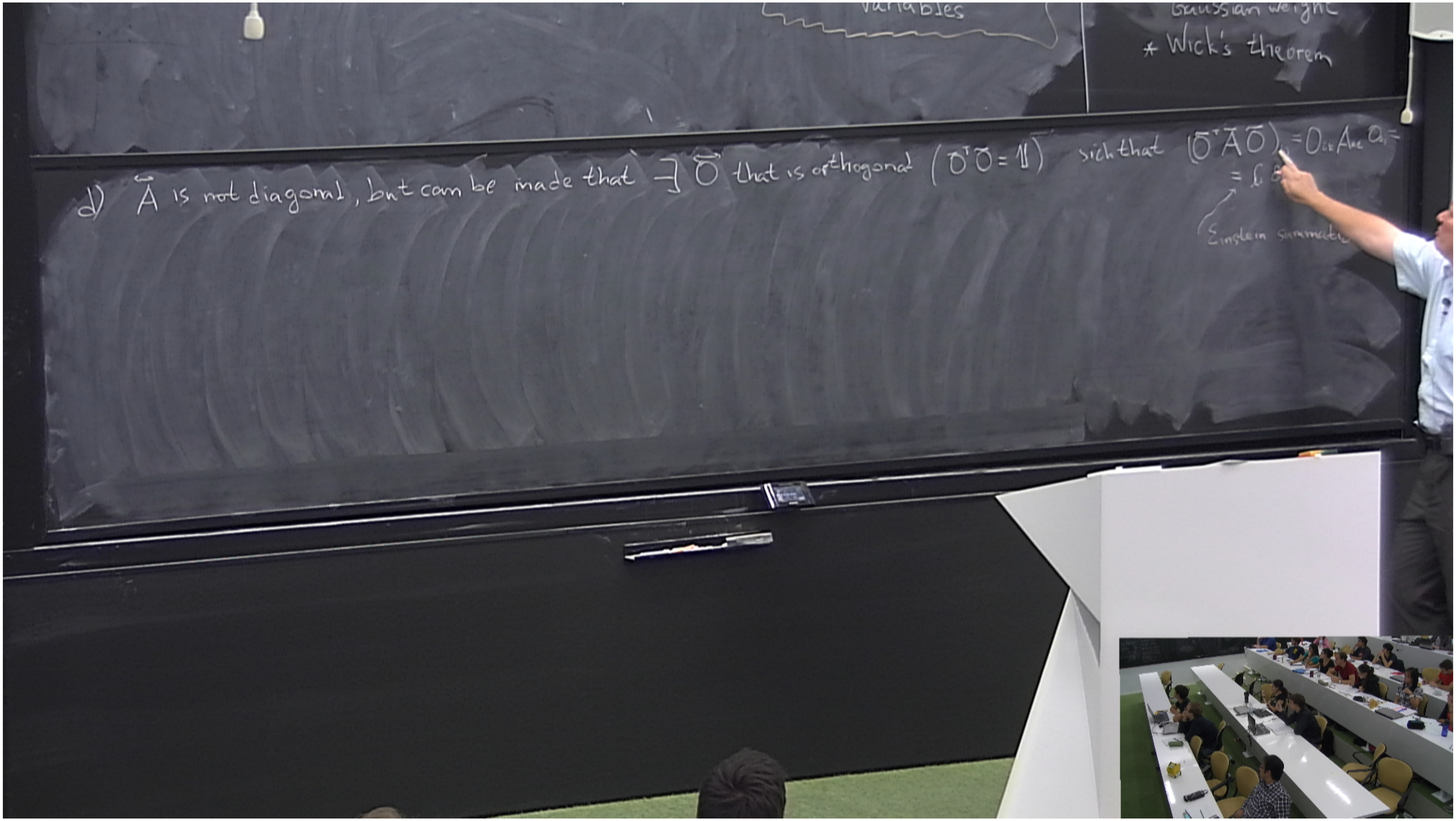

al Gaussian Integrals
A is symmetric and real with eigenvalues λi > 0
B = (B1, ..., Bn)
{ − 1/2 Σ xi Aij xj + Σ Bi xi }
= ∫ (Π dxi) exp{ Σ (− λi xi²/2 + Bi xi) } = Π ( √(2π/λi) e^(Bi²/2λi) ) = (2π)^(n/2) / √det A Π e^(Bi²/2λi)
Π exp(− λi xi²/2 + Bi xi)
Π λi = det A

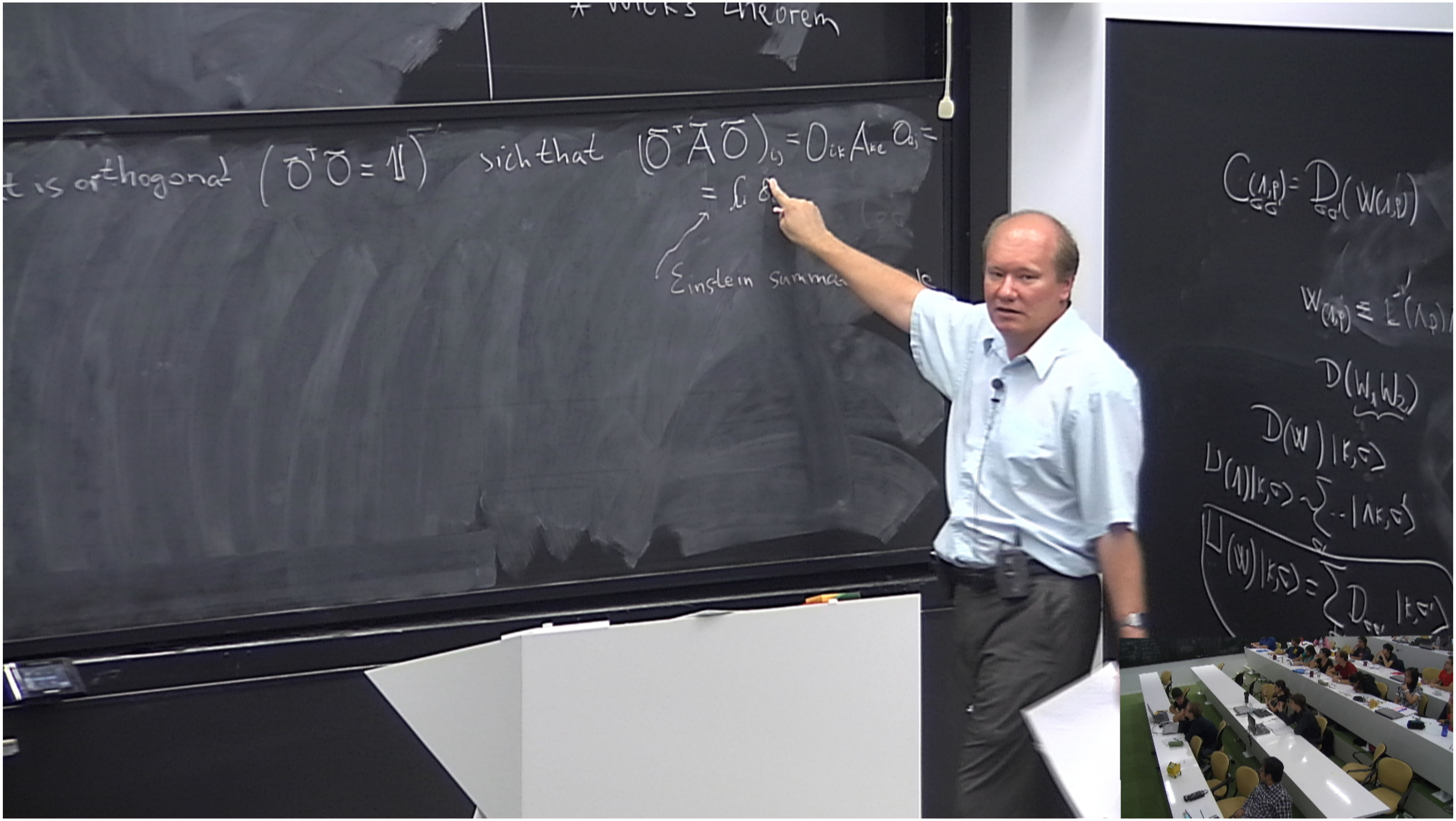

d) $\overleftrightarrow{A}$ is not diagonal, but can be made that $\exists$ $\overleftrightarrow{O}$


d) $\overleftrightarrow{A}$ is not diagonal, but can be made that ∃ $\overleftrightarrow{O}$ that is orthogonal


d) $\tilde{A}$ is not diagonal, but can be made that $\exists\ \tilde{O}$ that is orthogonal $(\tilde{O}^T\tilde{O} = \mathbb{1})$


* Wick's theorem
d) A is not diagonal, but can be made that ∃ O that is orthogonal (O^T O = 1) such that (O^T A O)_ij = O_ik A_kl O_lj =


variables
Gaussian weight
* Wick's theorem
d) A is not diagonal, but can be made that ∃ O that is orthogonal (OᵀO = 1) such that
Einstein summation


* Wick's theorem
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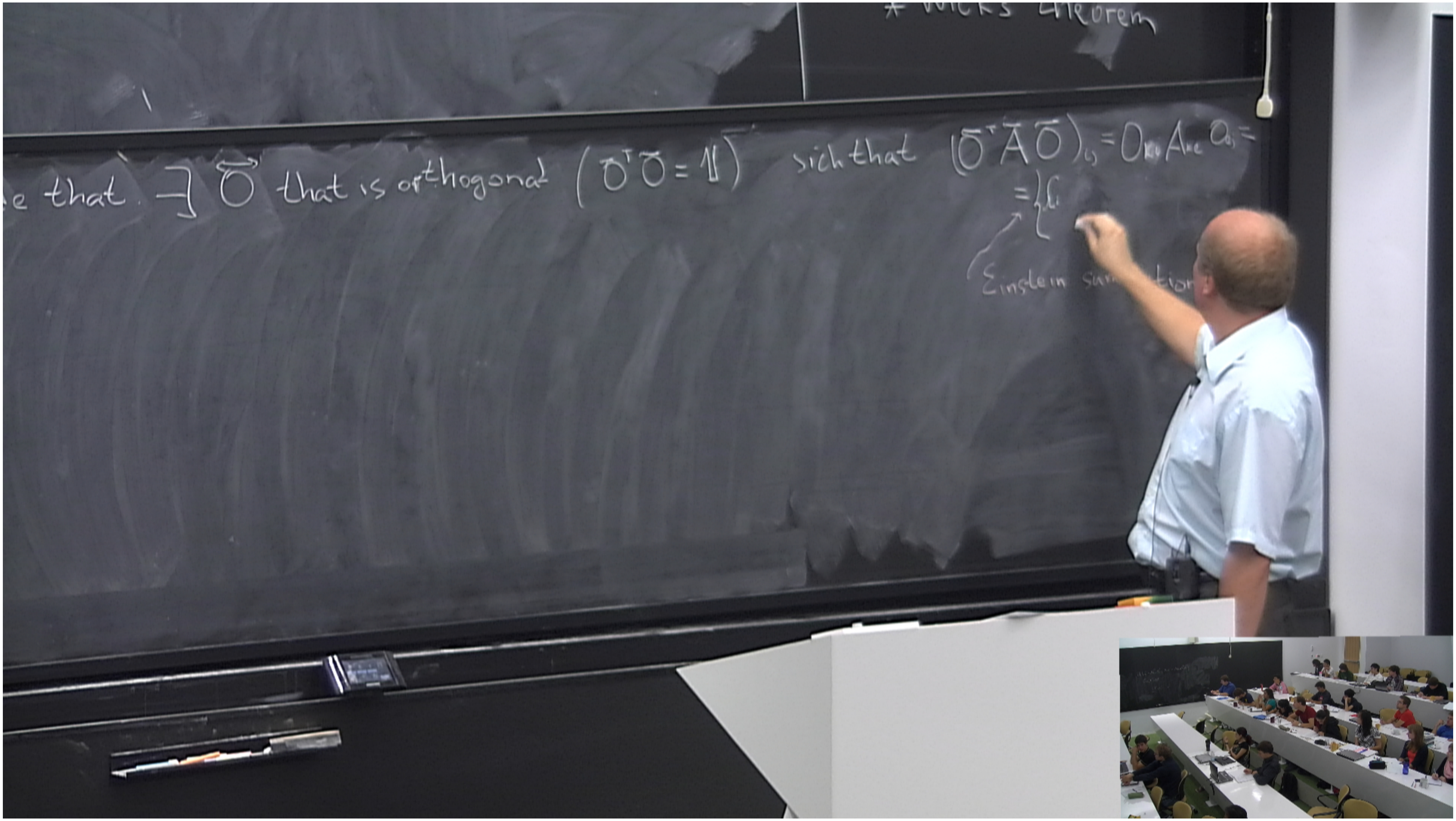
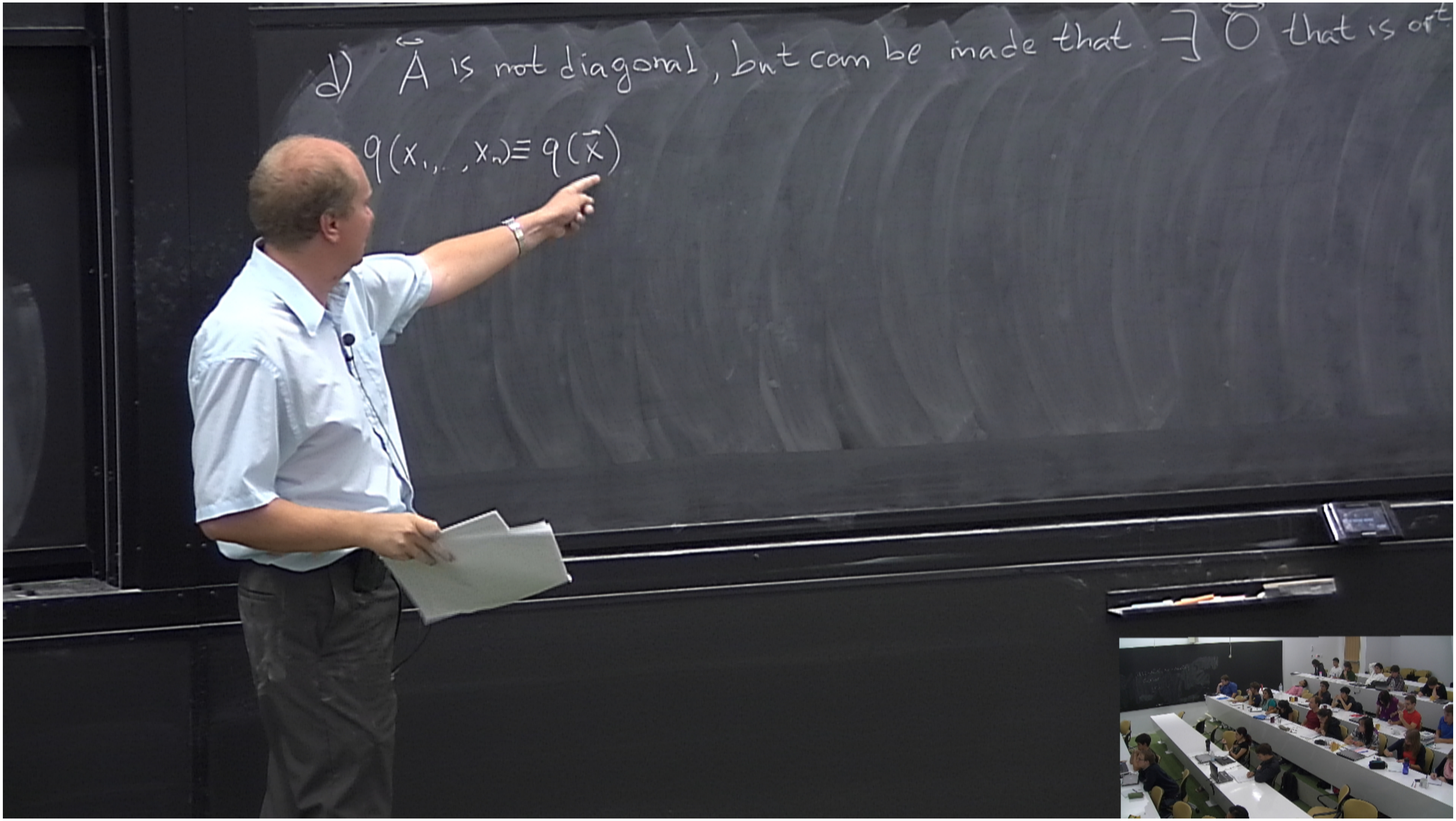
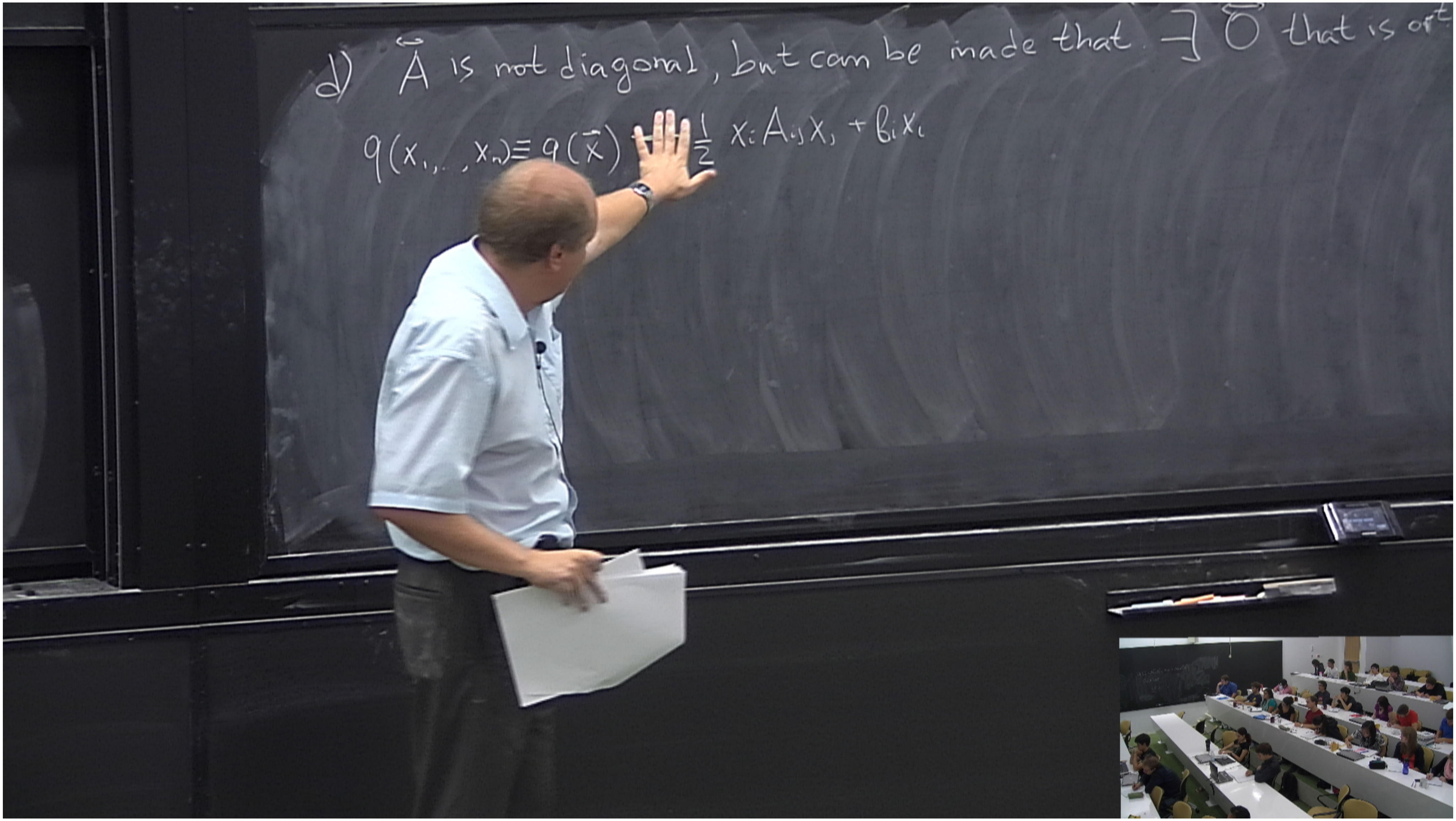
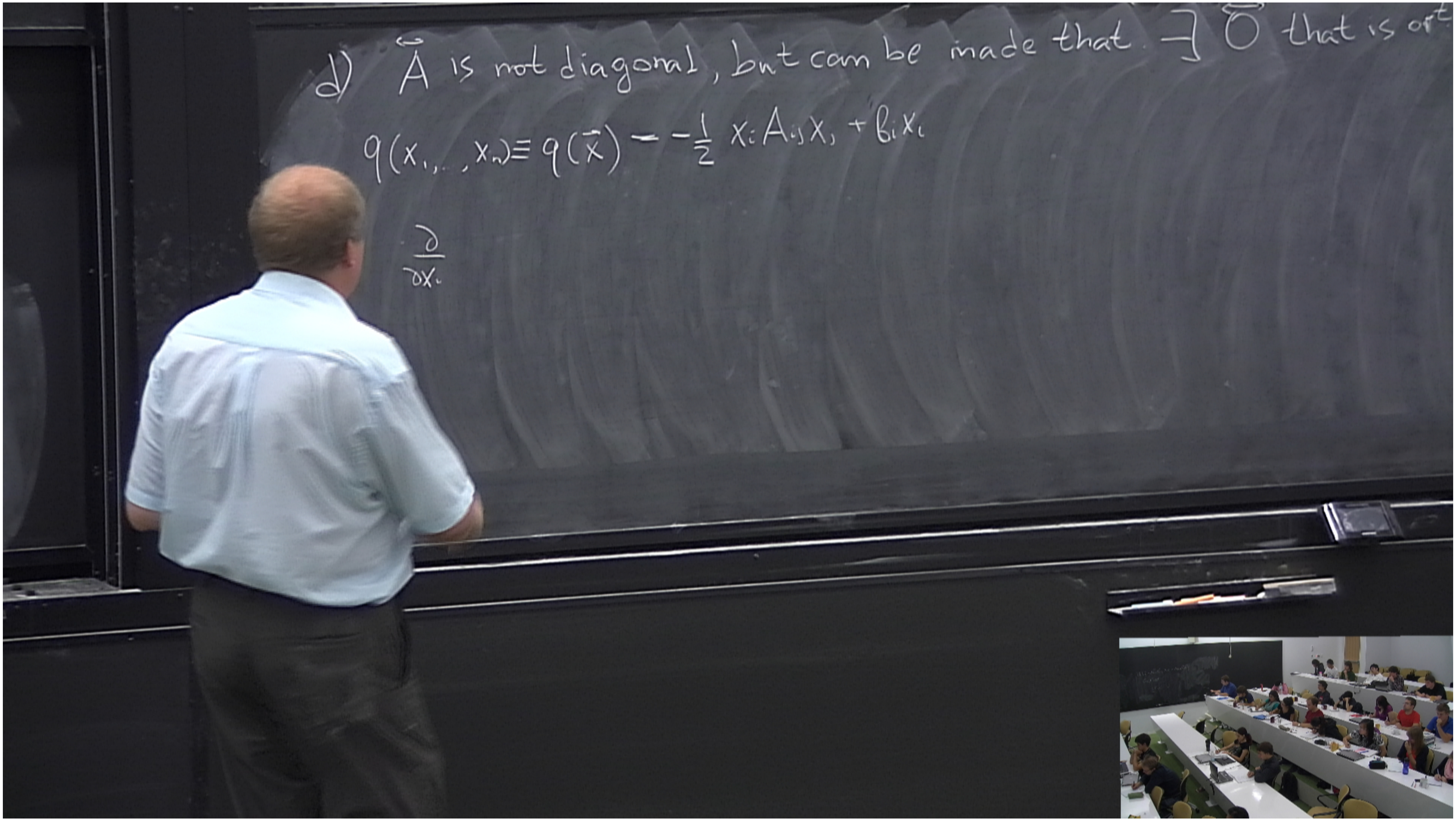
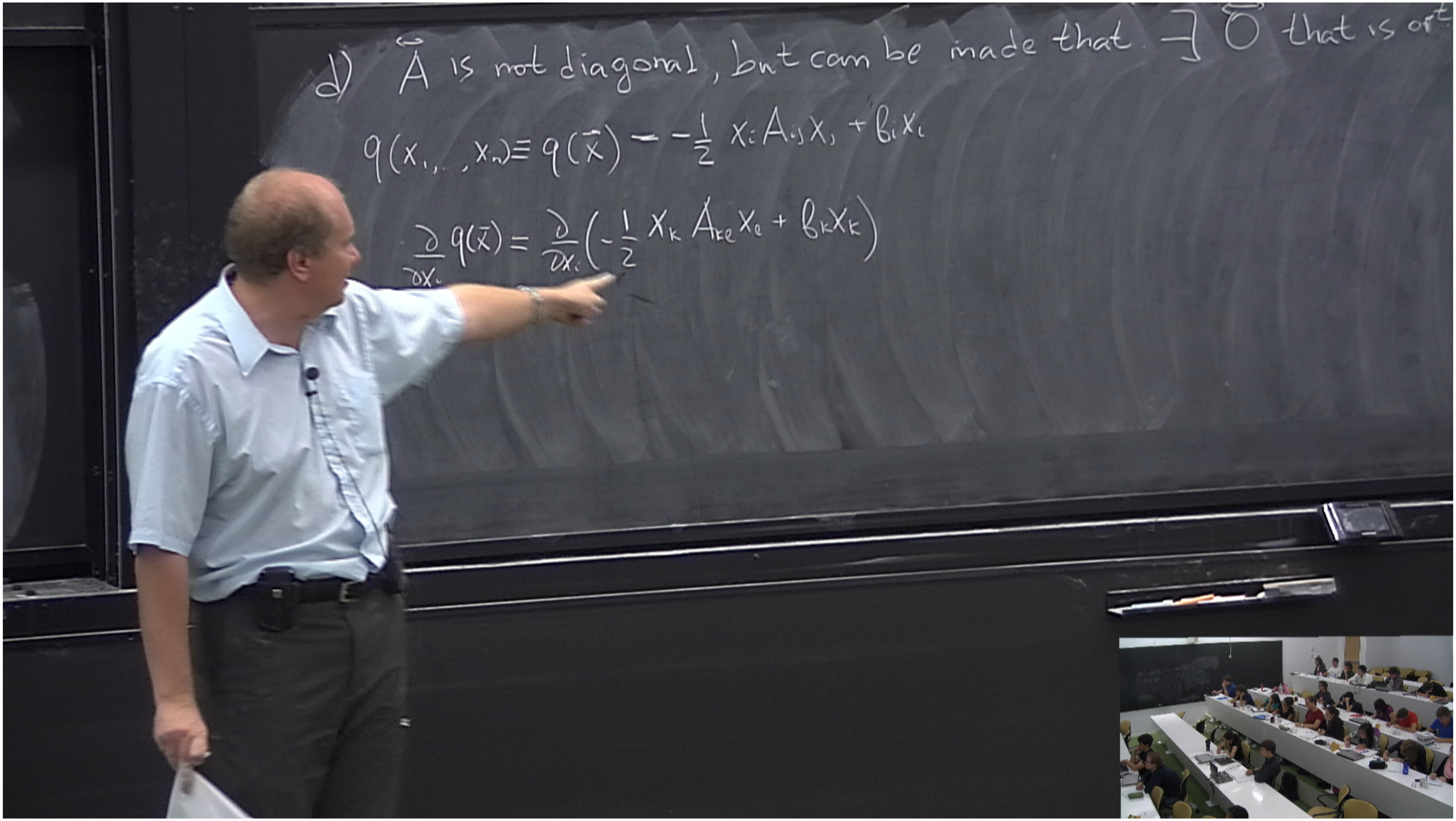
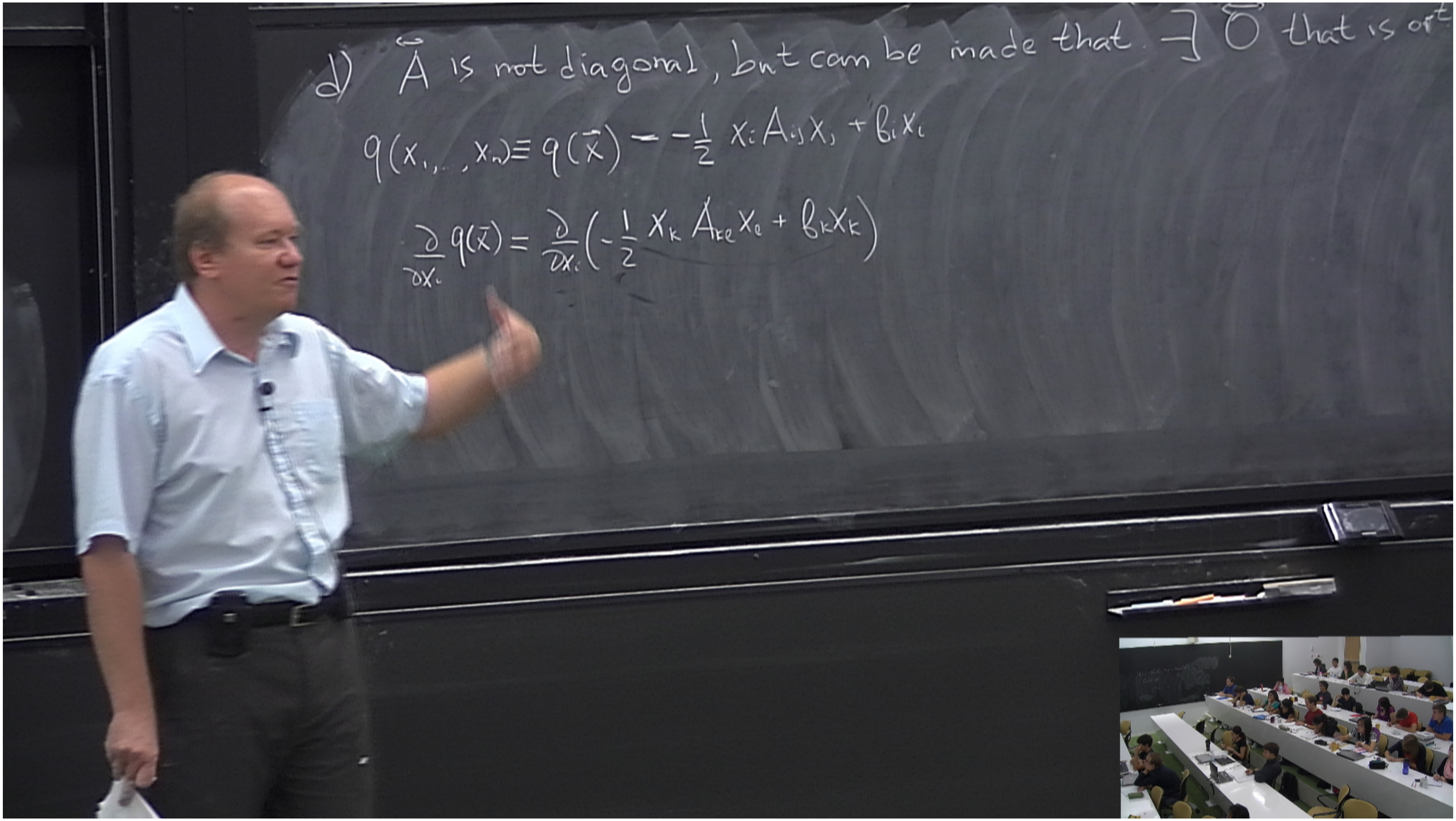

Wick's theorem
that ∃ O that is orthogonal (OᵀO = 1) such that (OᵀAO)ij = Oki Akl Olj =
Einstein summation

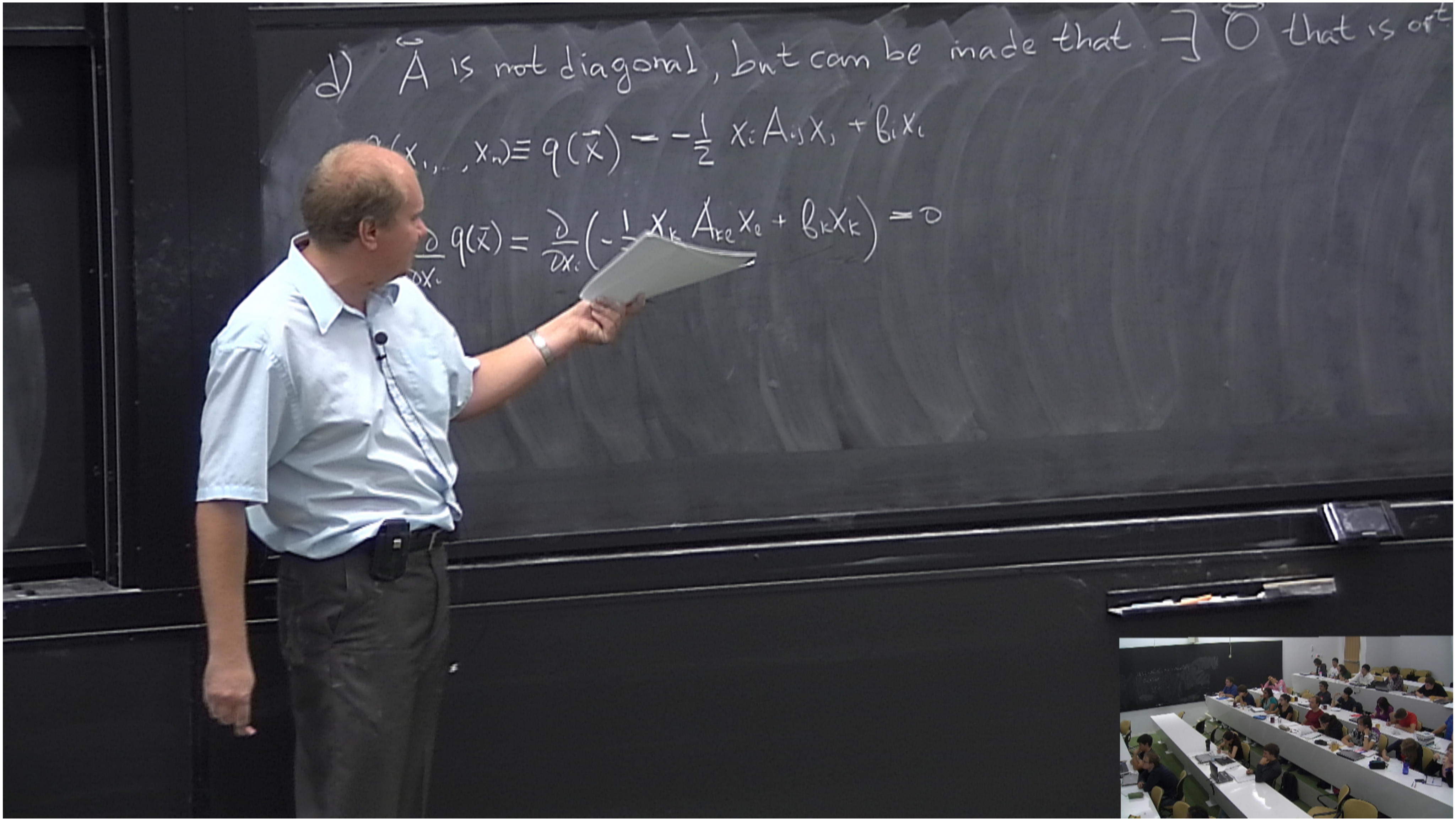

d) A is not diagonal, but can be made that ∃ O that is ort
q(x₁,…,xₙ) ≡ q(x)

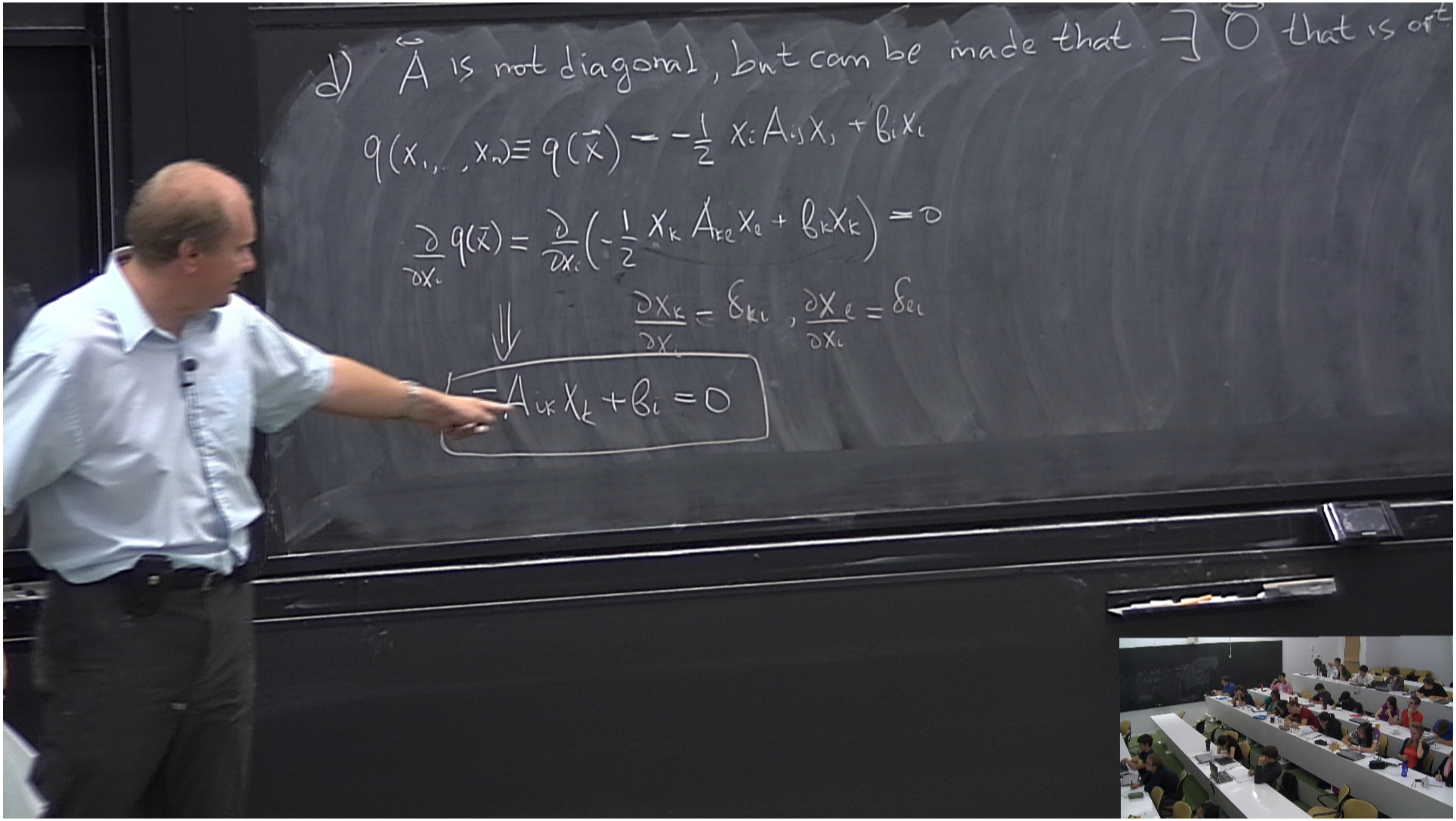

d) A is not diagonal, but can be made that ∃ O that is ort
q(x₁,..., xₙ) ≡ q(x) 1/2 xᵢ Aᵢⱼ xⱼ + bᵢ xᵢ

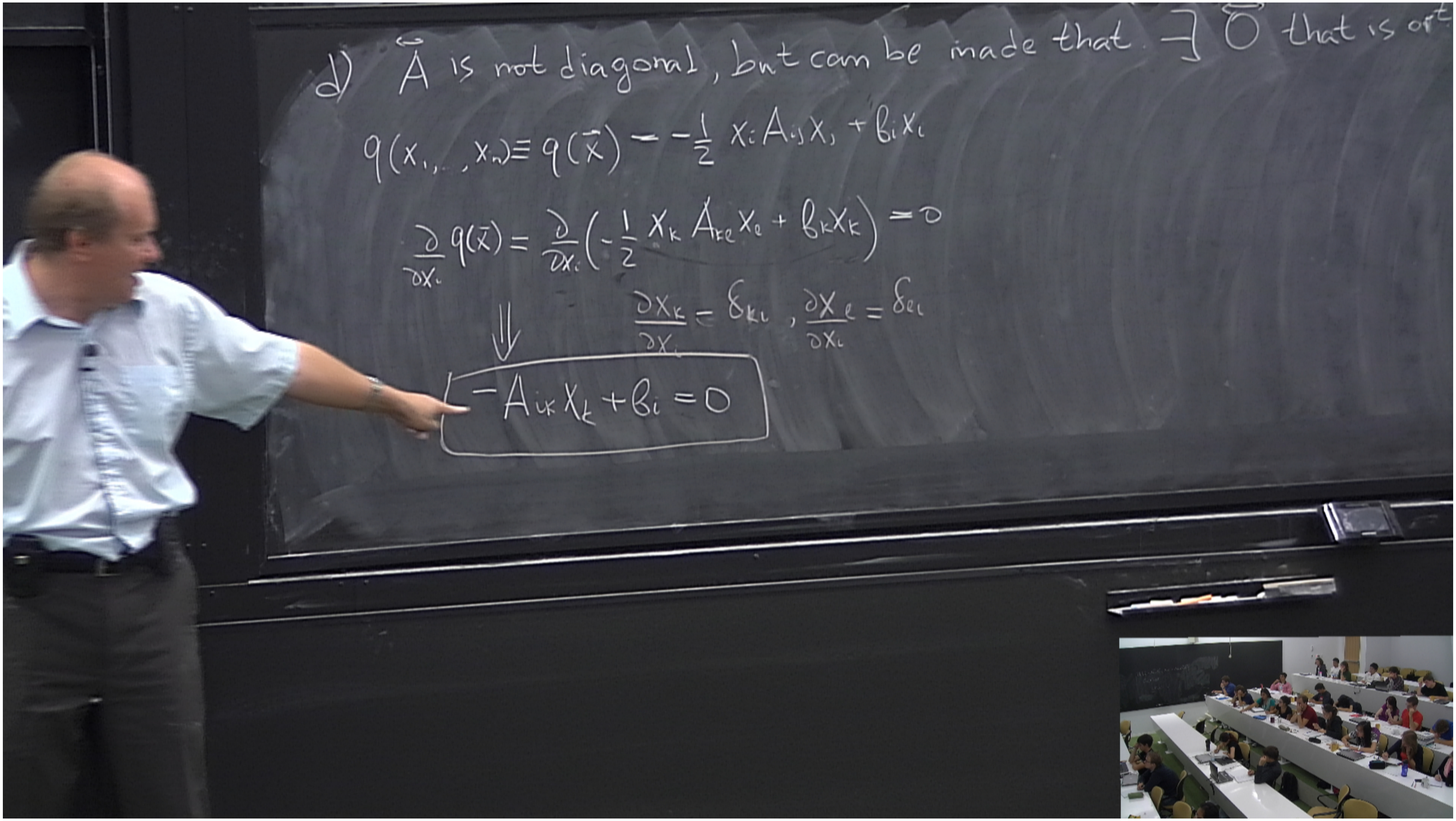

d) A is not diagonal, but can be made that ∃ O that is ort
q(x1,...,xn) ≡ q(x) = -1/2 xi Aij xj + bi xi
∂/∂xi

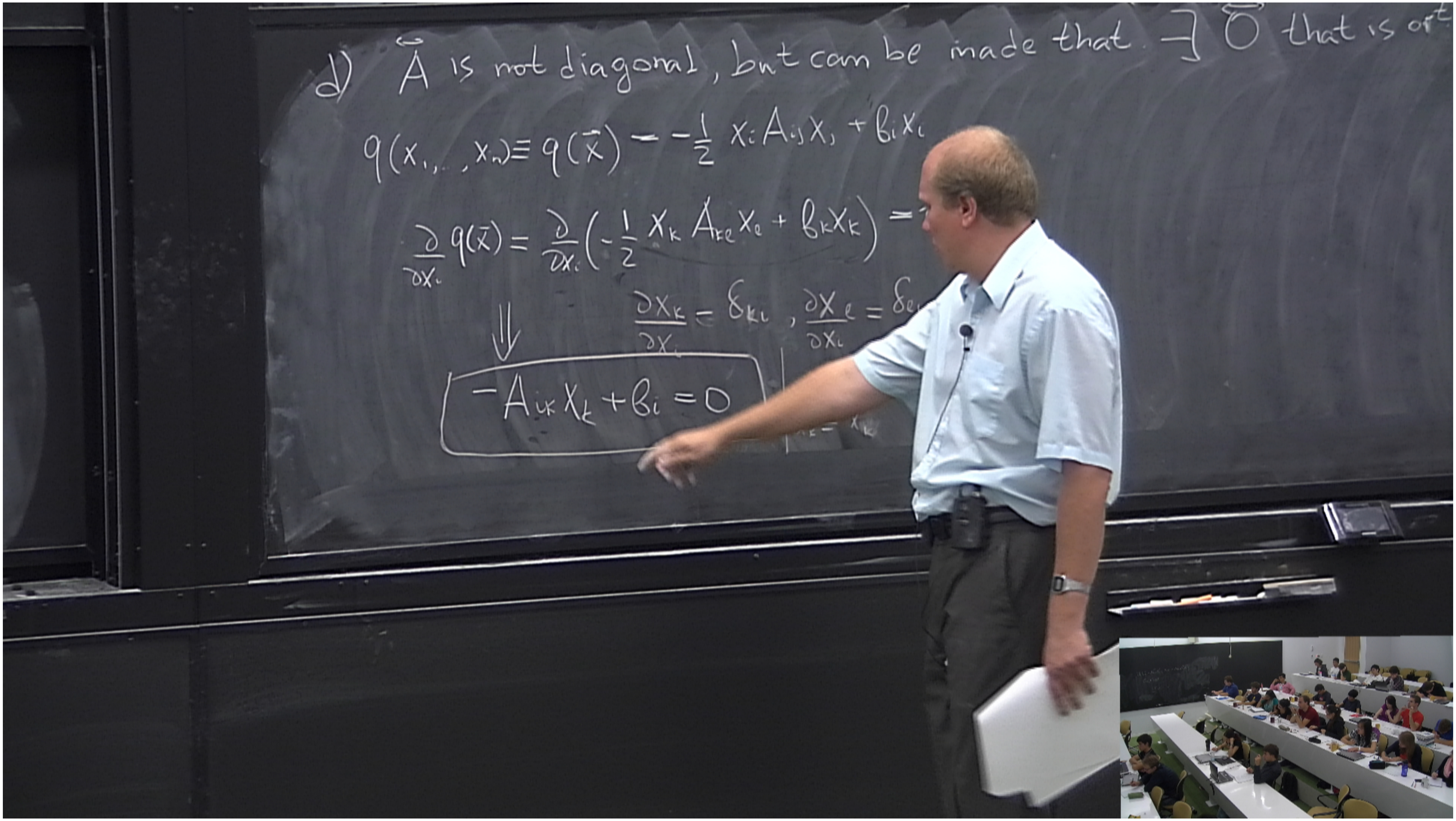

d) A is not diagonal, but can be made that ∃ O that is ort
q(x₁,…,xₙ) ≡ q(x̄) = −1/2 xᵢ Aᵢⱼ xⱼ + bᵢ xᵢ
∂/∂xᵢ q(x̄) = ∂/∂xᵢ (−1/2 xₖ Aₖₗ xₗ + bₖ xₖ)

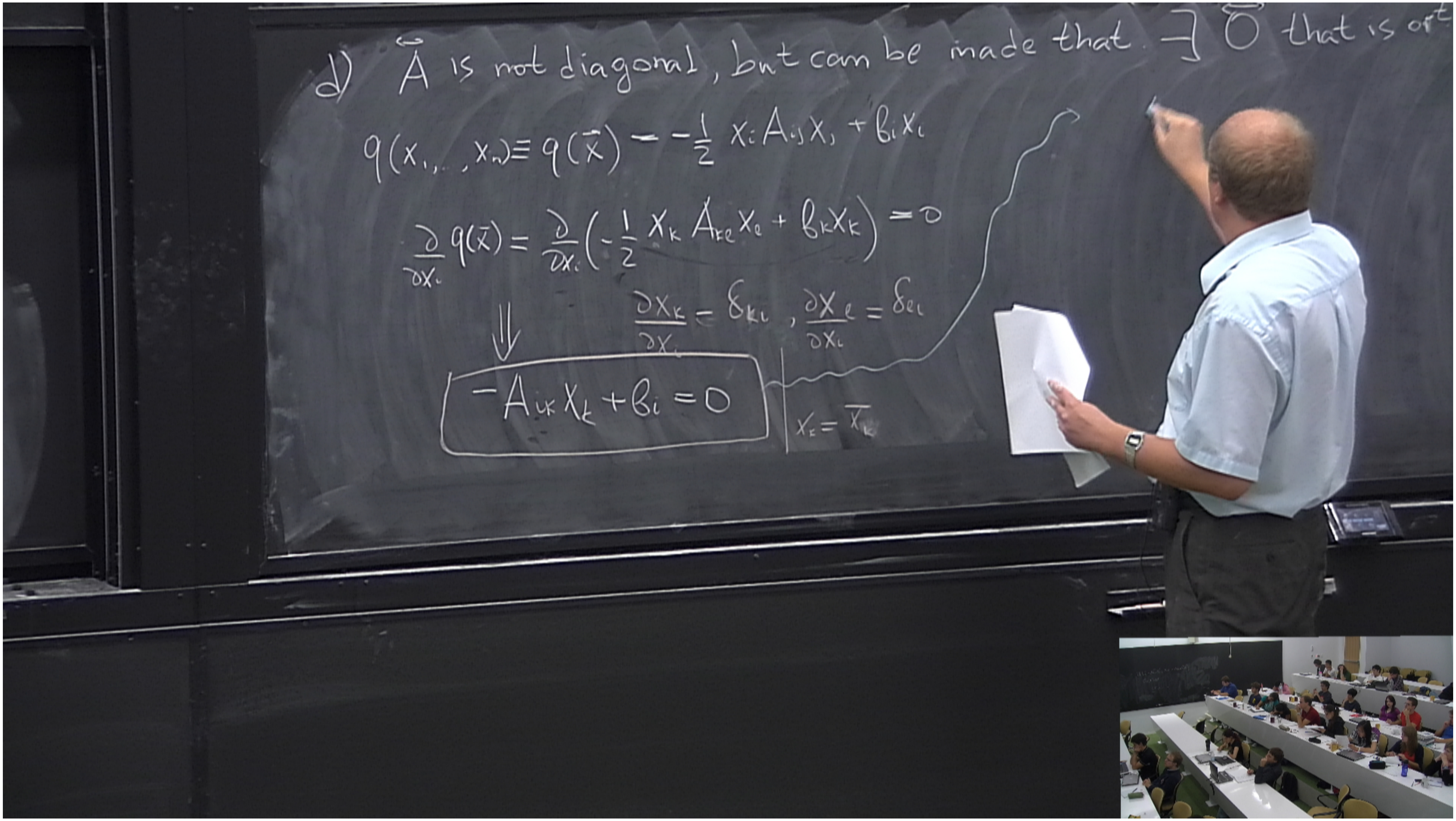

d) $\overleftrightarrow{A}$ is not diagonal, but can be made that ∃ $\overleftrightarrow{O}$ that is ort
$q(x_1, \ldots, x_n) \equiv q(\bar{x}) = -\frac{1}{2} x_i A_{ij} x_j + b_i x_i$
$\frac{\partial}{\partial x_i} q(\bar{x}) = \frac{\partial}{\partial x_i}\left(-\frac{1}{2} x_k A_{k\ell} x_\ell + b_k x_k\right)$

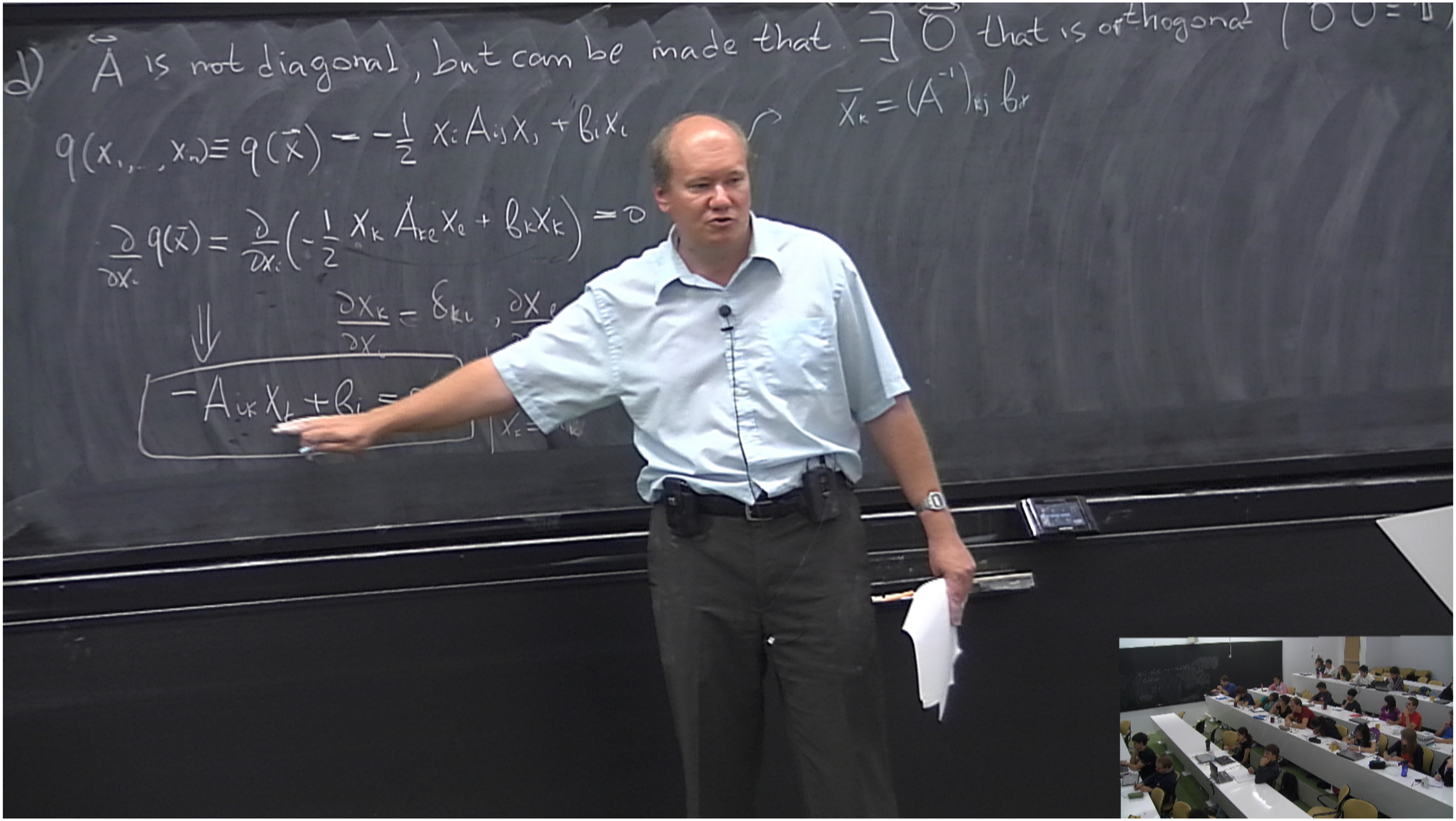

d) A is not diagonal, but can be made that ∃ O that is ort
(x1,..., xn) ≡ q(x) = −1/2 xi Aij xj + bi xi
q(x) = ∂/∂xi (−1/2 xk Akl xl + bk xk) = 0

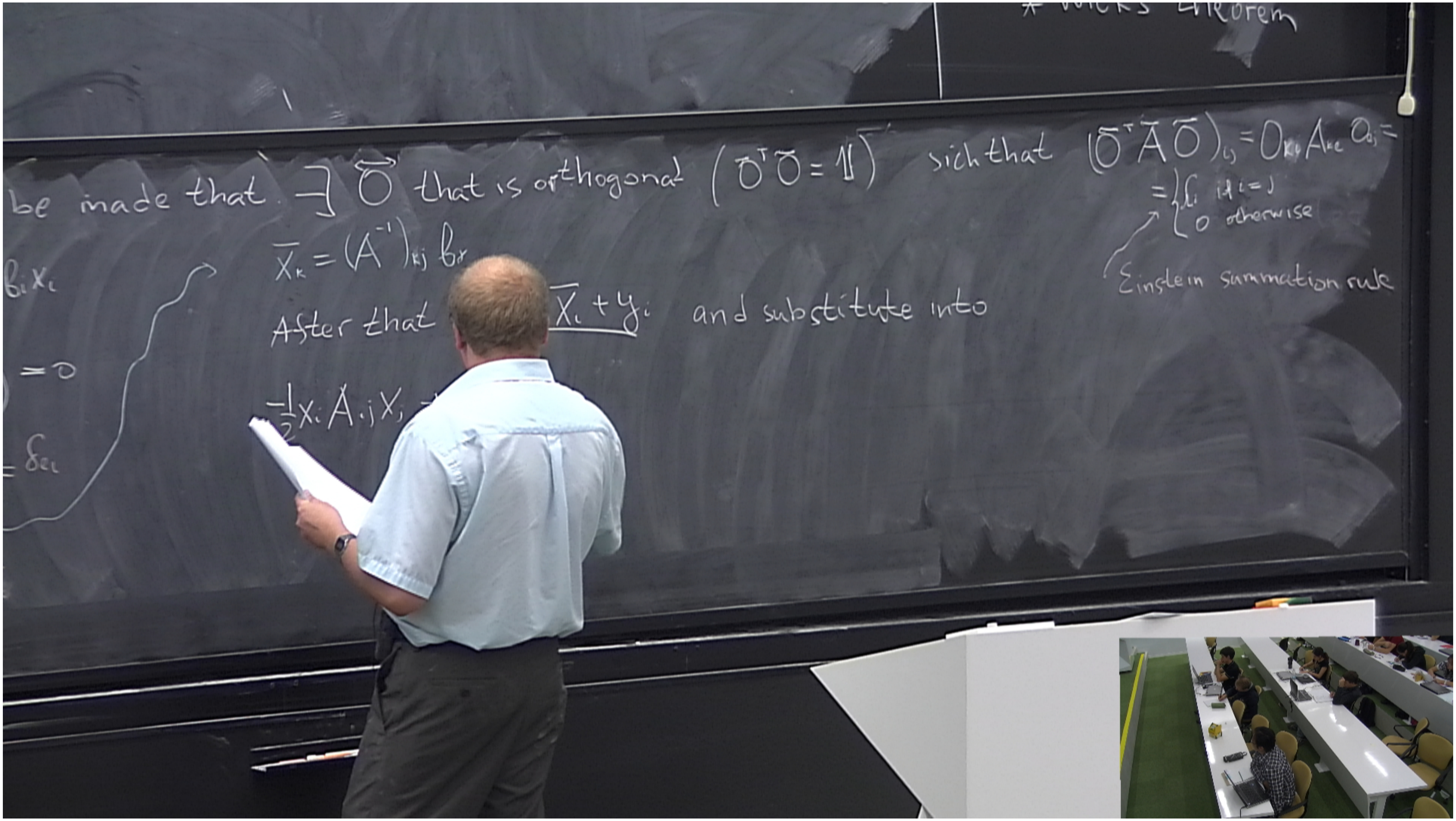

d) A is not diagonal, but can be made that ∃ O that is ort
$q(x_1, \ldots, x_n) \equiv q(\bar{x}) = -\frac{1}{2} x_i A_{ij} x_j + b_i x_i$
$\frac{\partial}{\partial x_i} q(\bar{x}) = \frac{\partial}{\partial x_i}\left(-\frac{1}{2} x_k A_{kl} x_l + b_k x_k\right) = 0$
$\frac{\partial x_k}{\partial x_i} = \delta_{ki}, \frac{\partial x_l}{\partial x_i} = \delta_{li}$
$-A_{ik} x_k + b_i = 0$

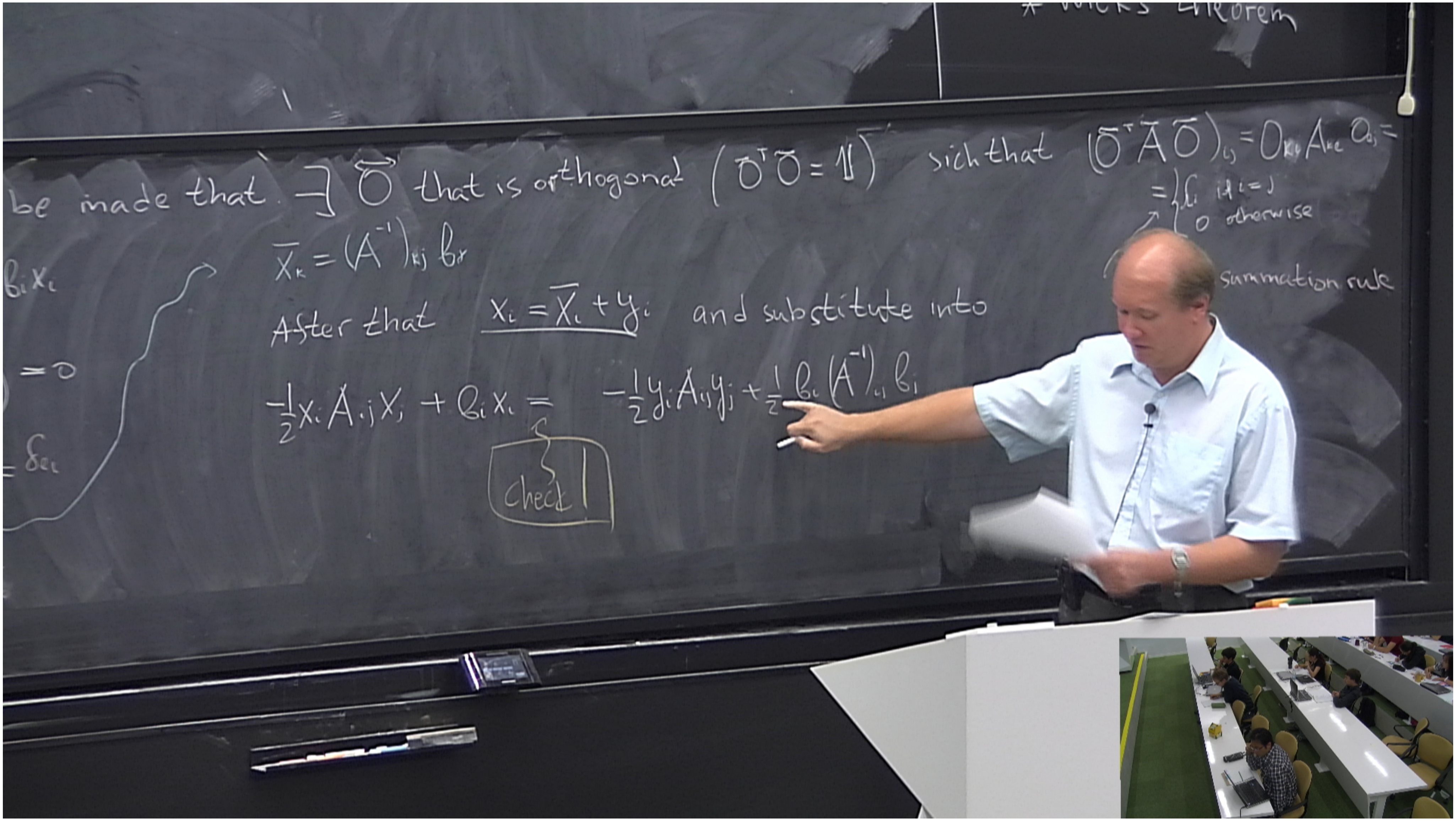

d) A is not diagonal, but can be made that ∃ O that is ort
q(x_1, ..., x_n) ≡ q(x) = -1/2 x_i A_ij x_j + b_i x_i
∂/∂x_i q(x) = ∂/∂x_i (-1/2 x_k A_kl x_l + b_k x_k) = 0
∂x_k/∂x_i = δ_ki , ∂x_l/∂x_i = δ_li
-A_ik x_k + b_i = 0

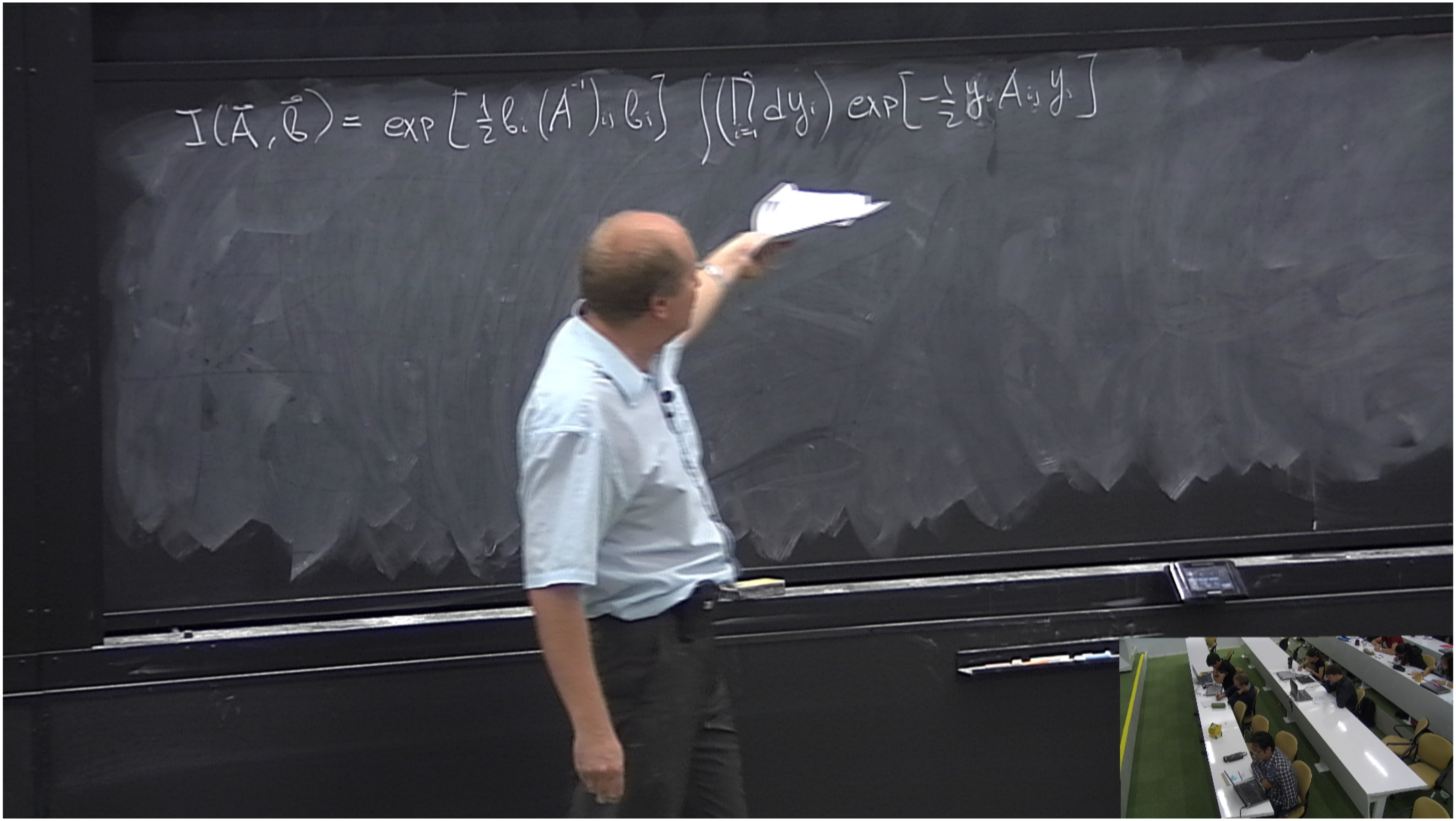

d) A is not diagonal, but can be made that ∃ O that is ort
q(x1,...,xn) ≡ q(x) = -1/2 xi Aij xj + bi xi
∂/∂xi q(x) = ∂/∂xi (-1/2 xk Akl xl + bk xk) =
∂xk/∂xi = δki , ∂xl/∂xi = δli
-Aik xk + bi = 0

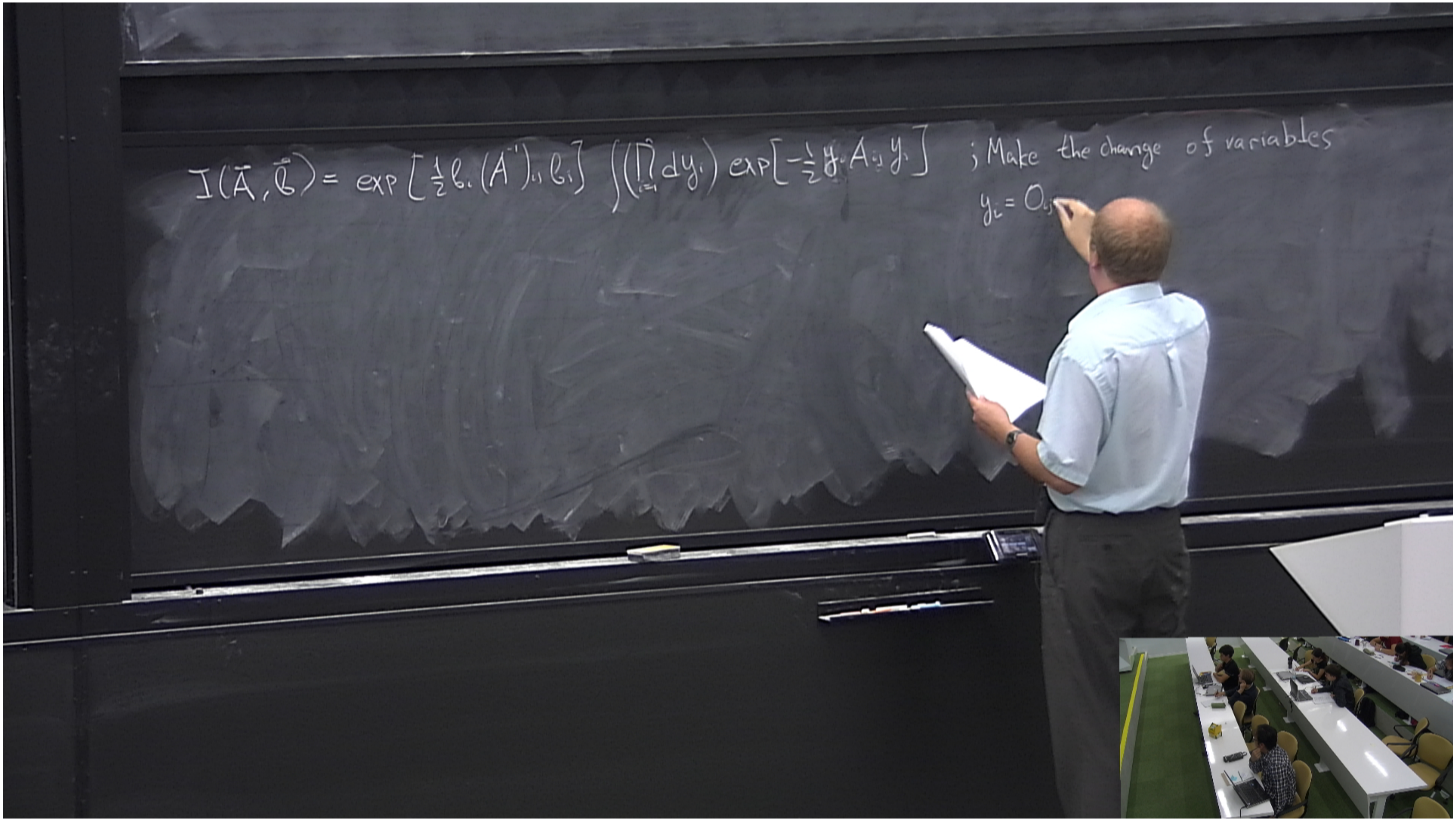

d) A is not diagonal, but can be made that ∃ O that is ort
q(x1,...,xn) ≡ q(x̄) = -1/2 xi Aij xj + bi xi
∂/∂xi q(x̄) = ∂/∂xi (-1/2 xk Akl xl + bk xk) = 0
∂xk/∂xi = δki , ∂xl/∂xi = δli
-Aik xk + bi = 0
xk = x̄k

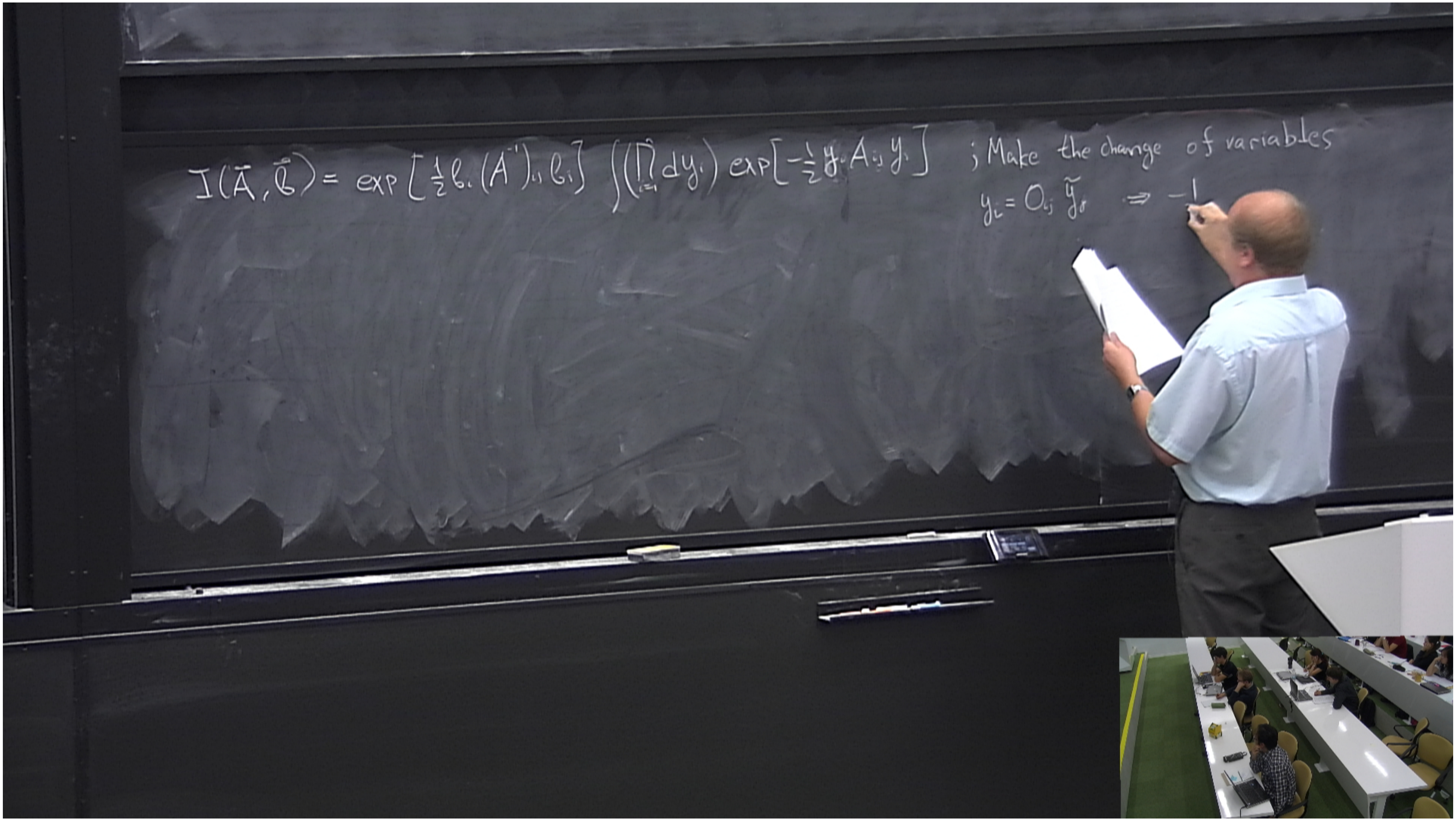

d) A is not diagonal, but can be made that ∃ O that is orthogonal
x̄_k = (A^-1)_kj b_j
q(x_1,..., x_n) ≡ q(x̄) = -1/2 x_i A_ij x_j + b_i x_i
∂/∂x_i q(x̄) = ∂/∂x_i (-1/2 x_k A_kl x_l + b_k x_k) = 0
∂x_k/∂x_i = δ_ki
-A_ik x_k + b_i =


Wick's theorem
be made that ∃ O that is orthogonal (OᵀO = 1) such that (OᵀAO)ᵢⱼ = Oₖᵢ Aₖₗ Oₗⱼ =
= { λᵢ if i = j ; 0 otherwise
Einstein summation rule
X̄ₖ = (A⁻¹)ₖⱼ bⱼ
After that X̄ᵢ + yᵢ and substitute into
−½ xᵢ Aᵢⱼ xⱼ

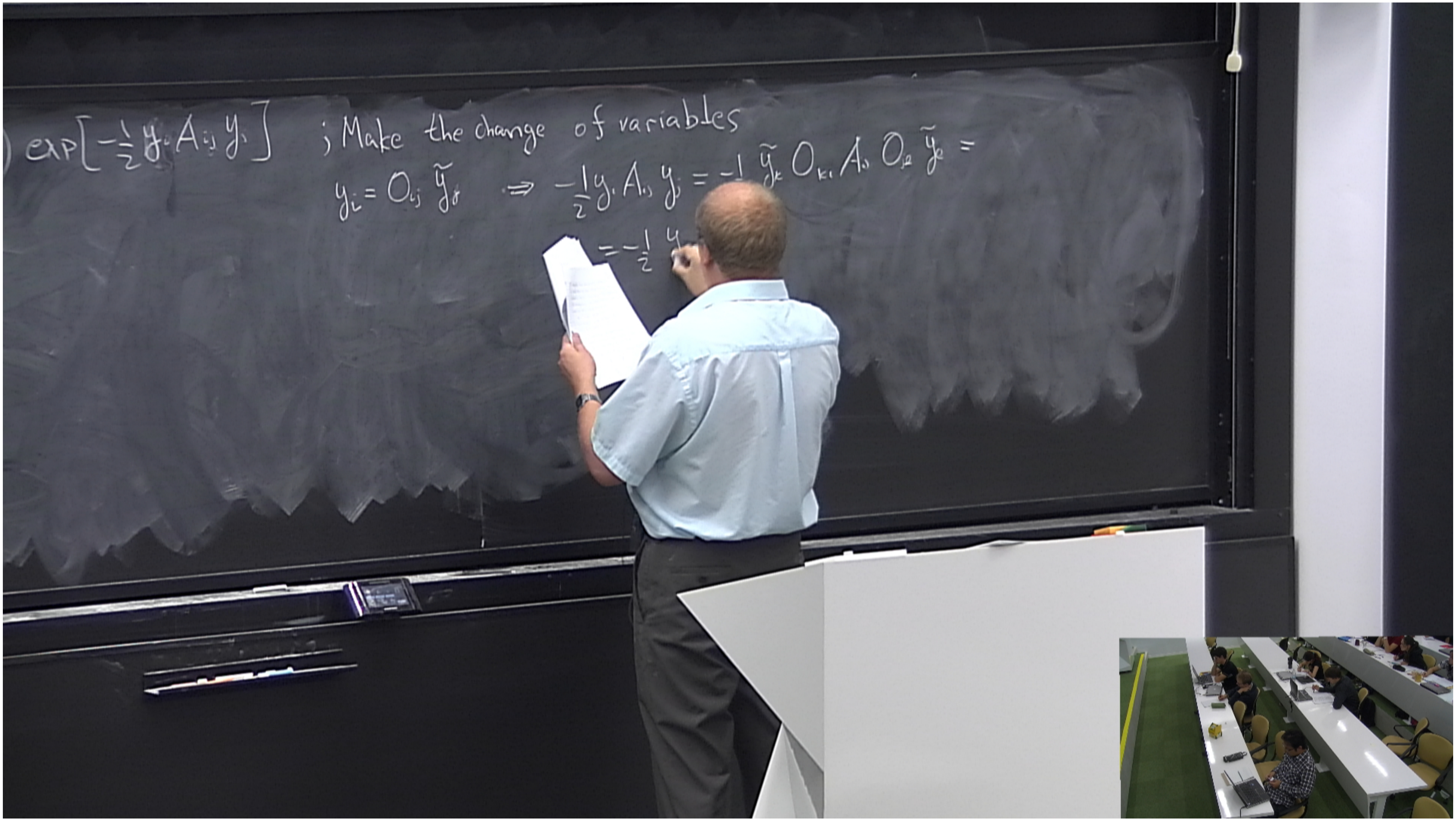

be made that ∃ Ō that is orthogonal (ŌᵀŌ = 1) sich that
otherwise
summation rule
After that x_i = x̄_i + y_i and substitute into
Check!

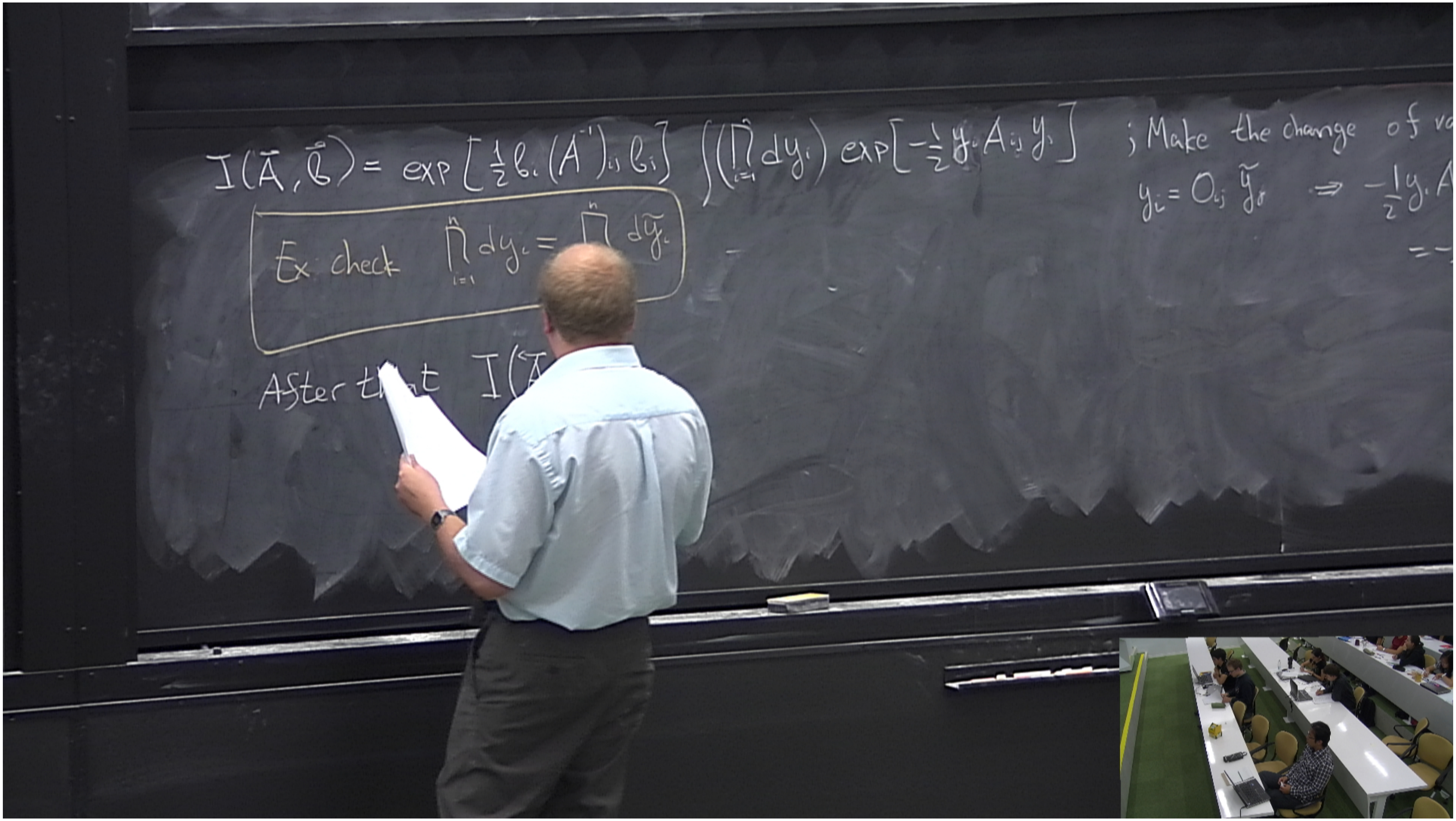

$I(\vec{A}, \vec{b}) = \exp\left[\frac{1}{2} b_i (A^{-1})_{ij} b_j\right] \int \left(\prod_{i=1}^{n} dy_i\right) \exp\left[-\frac{1}{2} y_i A_{ij} y_j\right]$

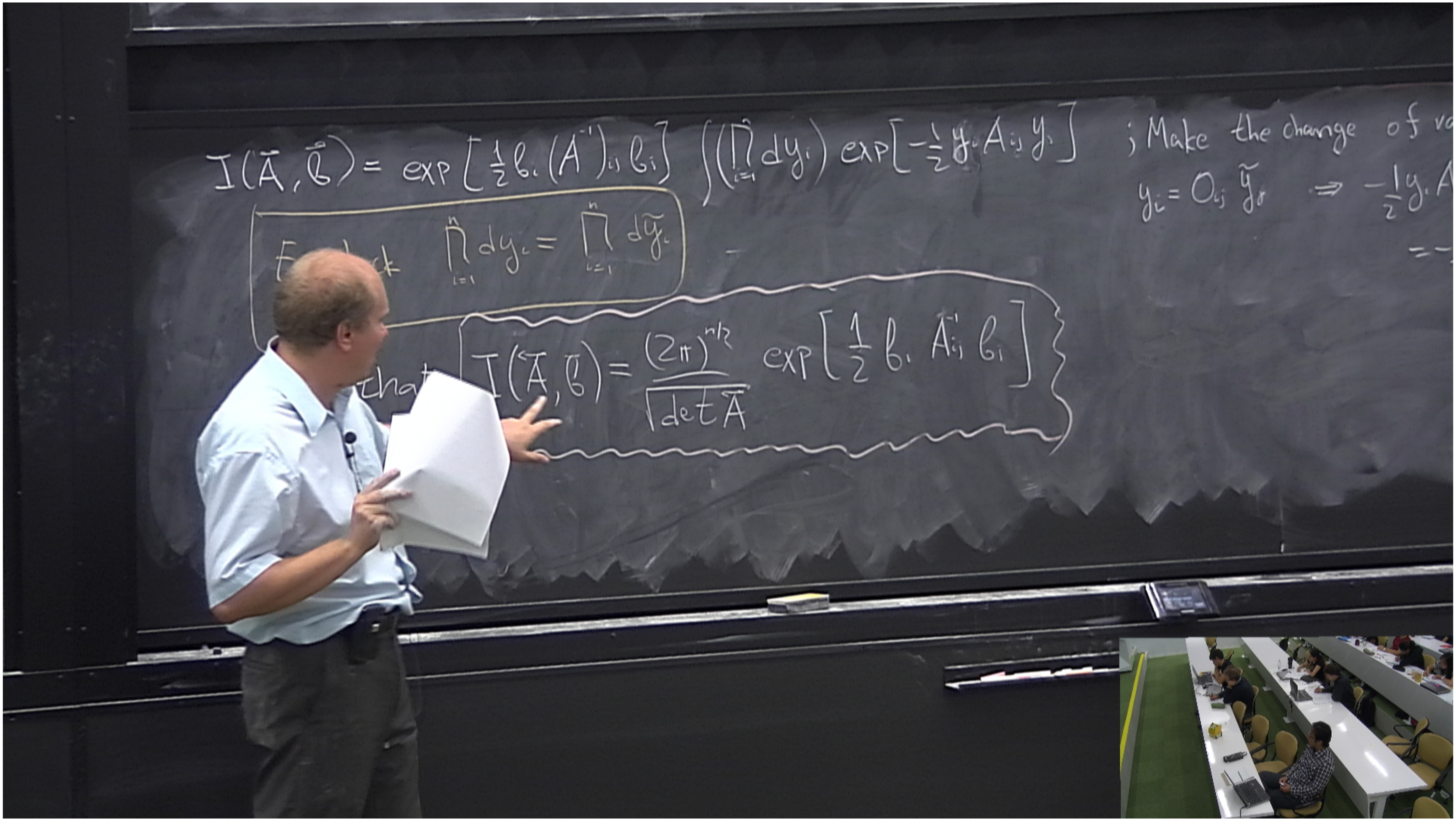

$I(\vec{A},\vec{b}) = \exp\left[\frac{1}{2} b_i (A^{-1})_{ij} b_j\right] \int \left(\prod_{i=1}^{n} dy_i\right) \exp\left[-\frac{1}{2} y_i A_{ij} y_j\right]$ ; Make the change of variables
$y_i = O_{ij}$

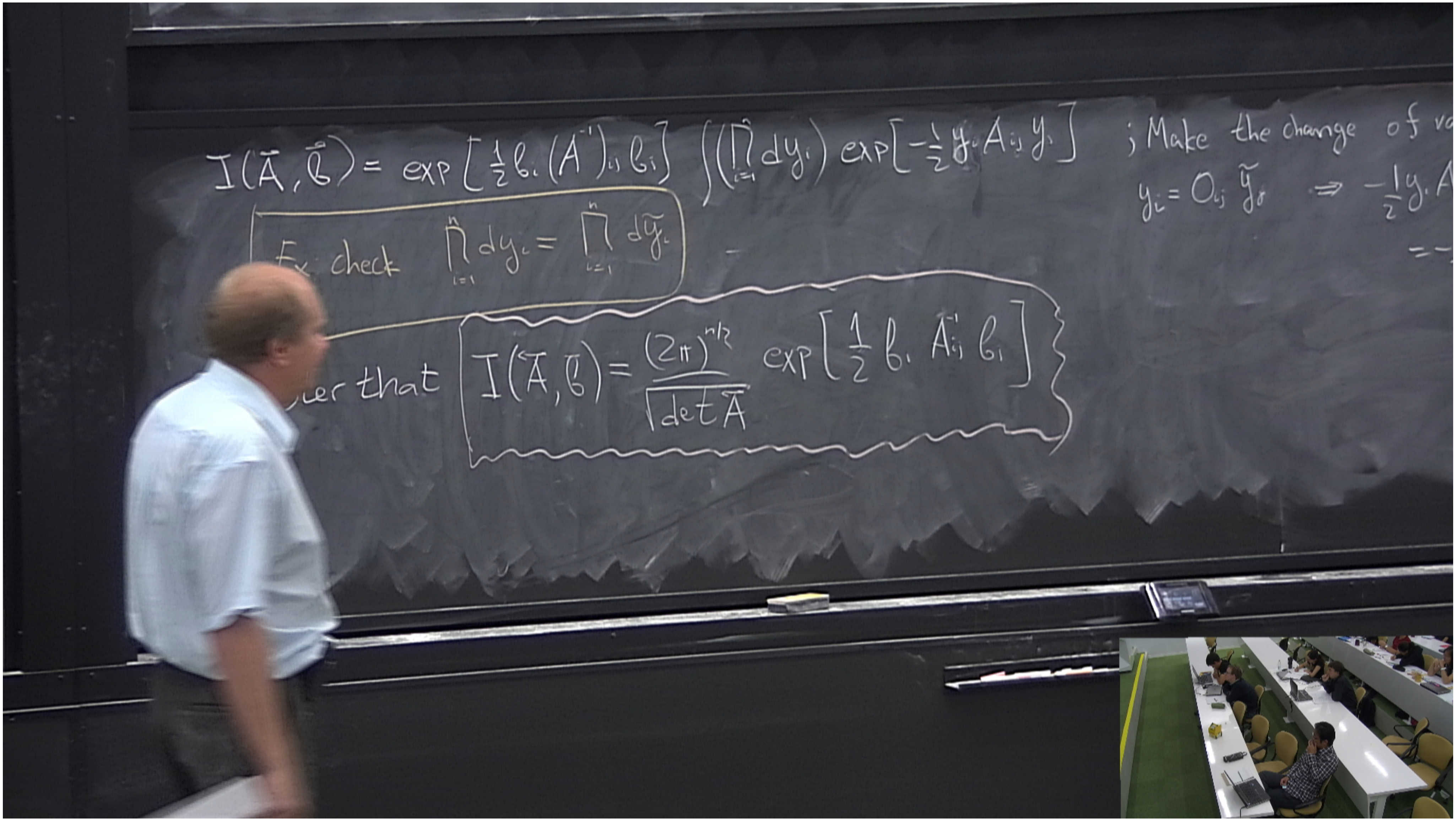


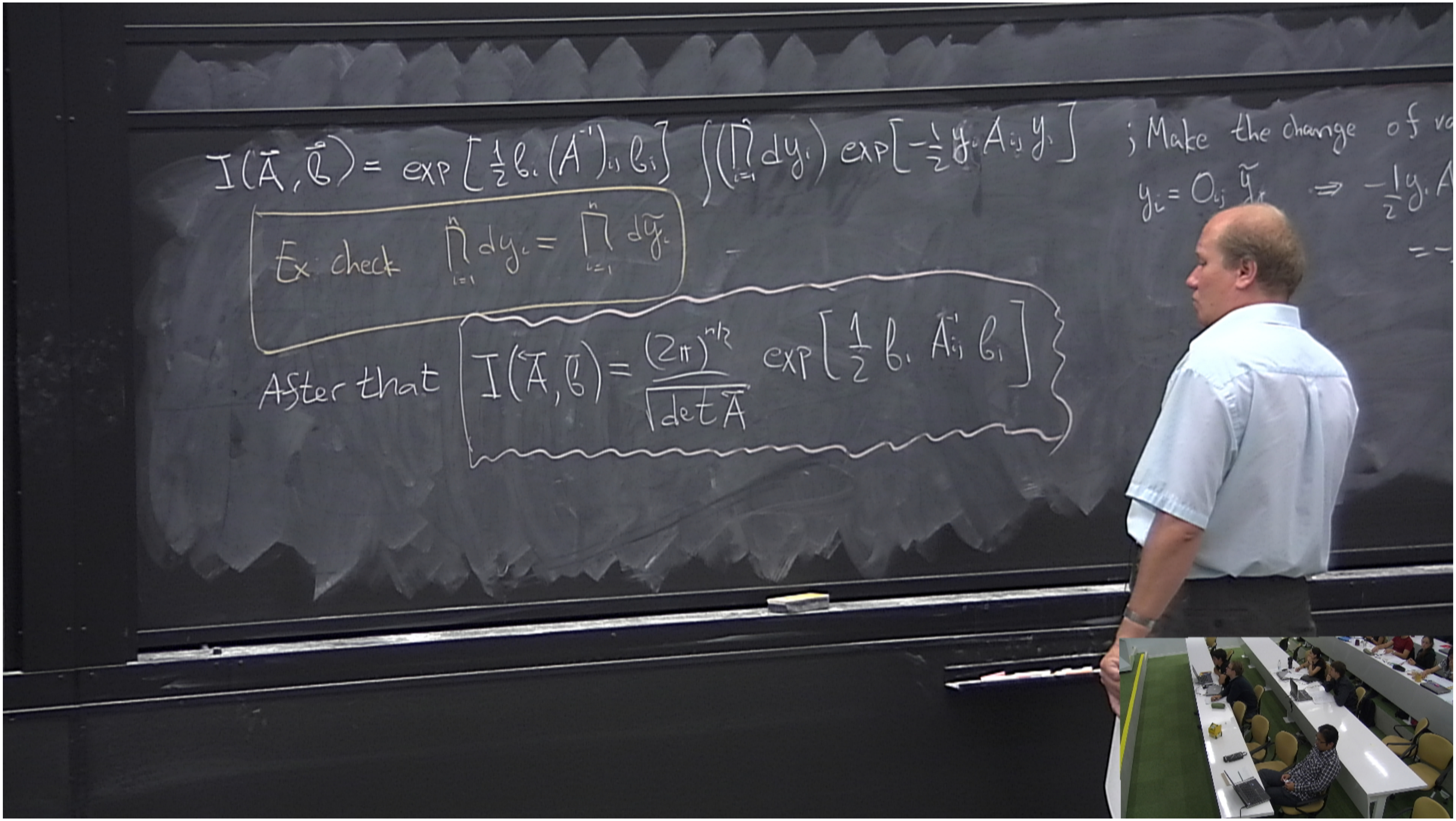
$I(\vec{A},\vec{b}) = \exp\left[\frac{1}{2} b_i (A^{-1})_{ij} b_j\right] \int \left(\prod_{i=1}^{n} dy_i\right) \exp\left[-\frac{1}{2} y_i A_{ij} y_j\right]$ ; Make the change of variables
$y_i = O_{ij} \tilde{y}_j \Rightarrow -1$


exp[-½ yᵢ Aᵢⱼ yⱼ] ; Make the change of variables
yᵢ = Oᵢⱼ ỹⱼ ⇒ -½ yᵢ Aᵢⱼ yⱼ = -½ ỹₖ Oₖᵢ Aᵢⱼ Oⱼₗ ỹₗ =
= -½ y

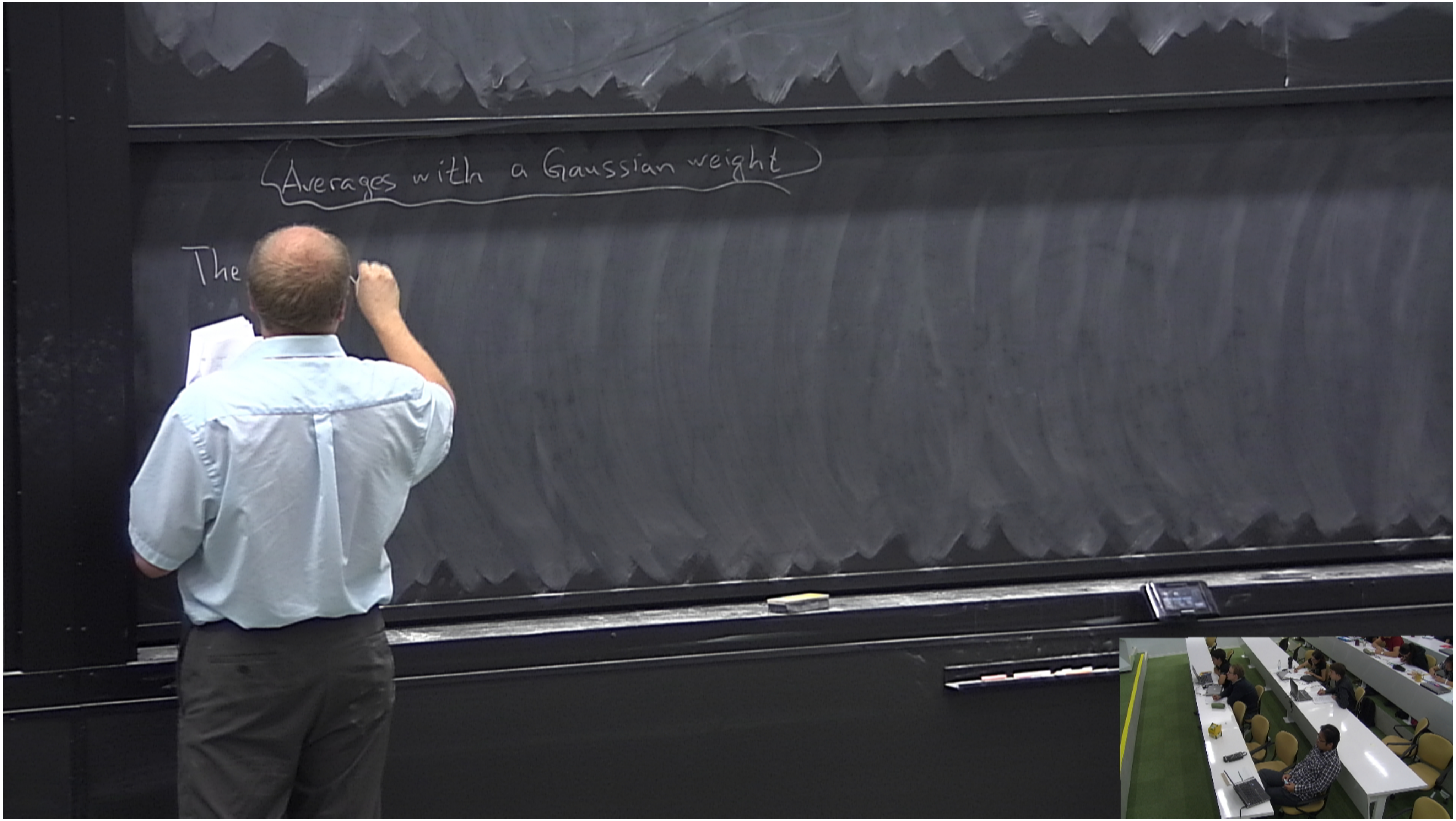

I(A, b) = exp [ ½ b_i (A^-1)_ij b_j ] ∫ ( ∏_{i=1}^n dy_i ) exp [ −½ y_i A_ij y_j ]
; Make the change of va
y_i = O_ij ỹ_j ⇒ −½ y_i A
Ex check ∏_{i=1}^n dy_i = ∏ dỹ_i
After that I(A

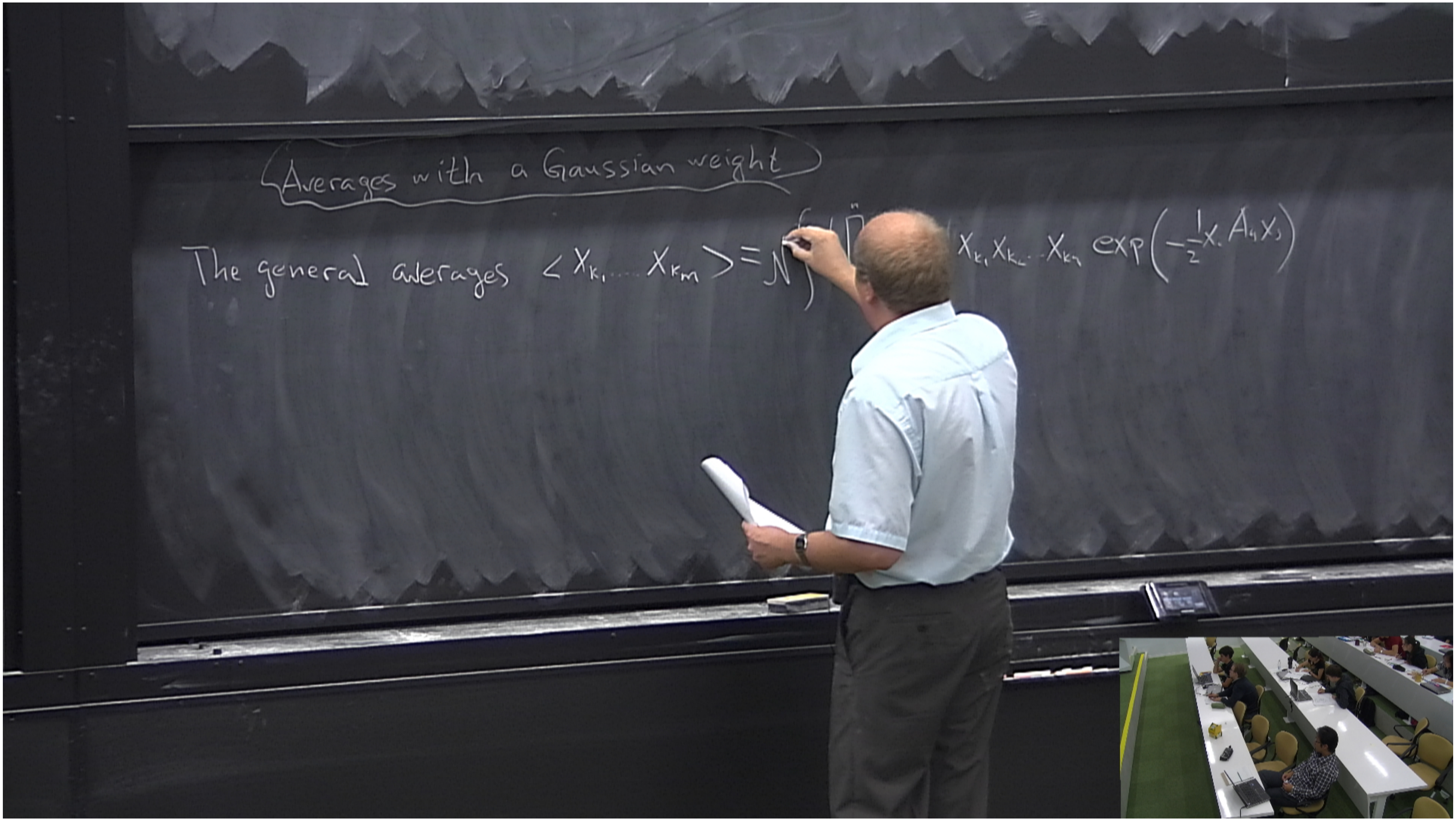

I(A, b) = exp[½ b_i (A^-1)_ij b_j] ∫(∏ dy_i) exp[−½ y_i A_ij y_j]
; Make the change of va
y_i = O_ij ỹ_j ⇒ −½ y_i A
∏ dy_i = ∏ dỹ_i
that
I(A, b) = (2π)^n/2 / √det A exp[½ b_i A^-1_ij b_j]

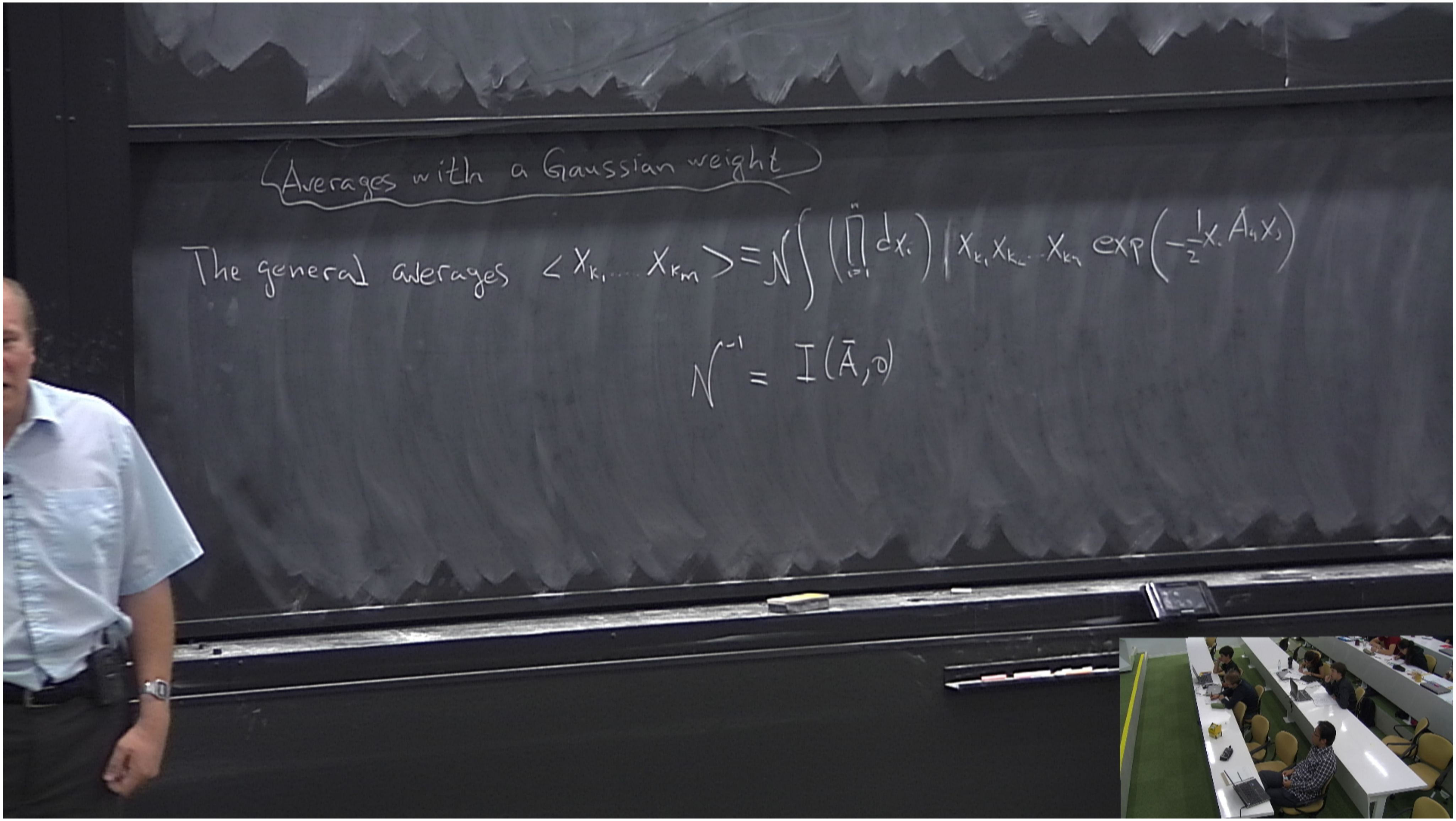

$I(\vec{A},\vec{b}) = \exp\left[\frac{1}{2} b_i (A^{-1})_{ij} b_j\right] \int \left(\prod_{i=1}^{n} dy_i\right) \exp\left[-\frac{1}{2} y_i A_{ij} y_j\right]$
; Make the change of va
$y_i = O_{ij} \tilde{y}_j \Rightarrow -\frac{1}{2} y_i A$
Ex: check $\prod_{i=1}^{n} dy_i = \prod_{i=1}^{n} d\tilde{y}_i$
er that
$I(\vec{A},\vec{b}) = \frac{(2\pi)^{n/2}}{\sqrt{\det \vec{A}}} \exp\left[\frac{1}{2} b_i A^{-1}_{ij} b_j\right]$

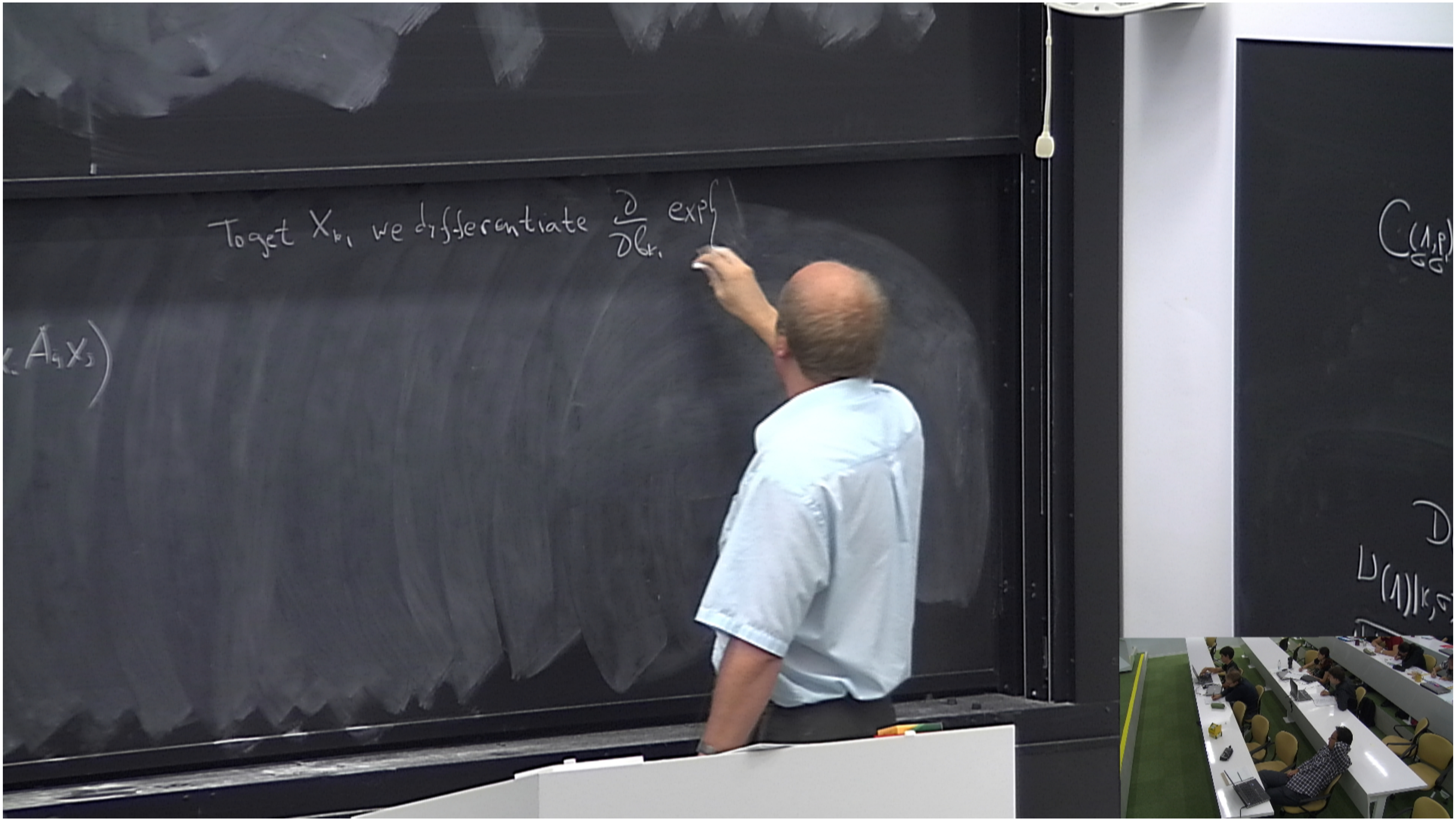


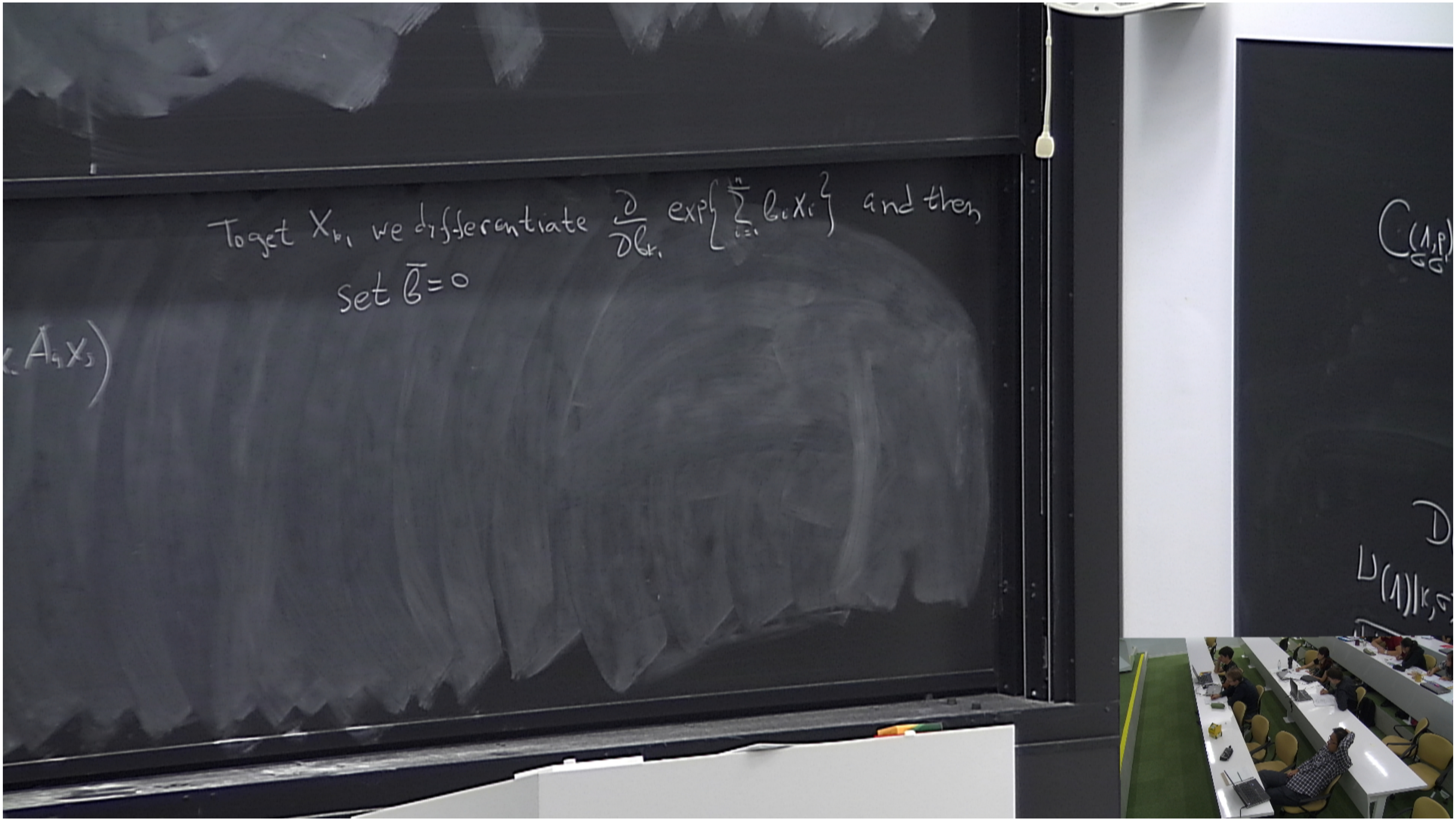
I(A, b) = exp [ ½ b_i (A^-1)_ij b_j ] ∫(∏ dy_i) exp[ -½ y_i A_ij y_j ]
; Make the change
y_i = O_ij ỹ_j ⇒ -½ y_i A
Ex: check ∏ dy_i = ∏ dỹ_i
After that
I(A, b) = (2π)^n/2 / √det A exp[ ½ b_i A^-1_ij b_j ]

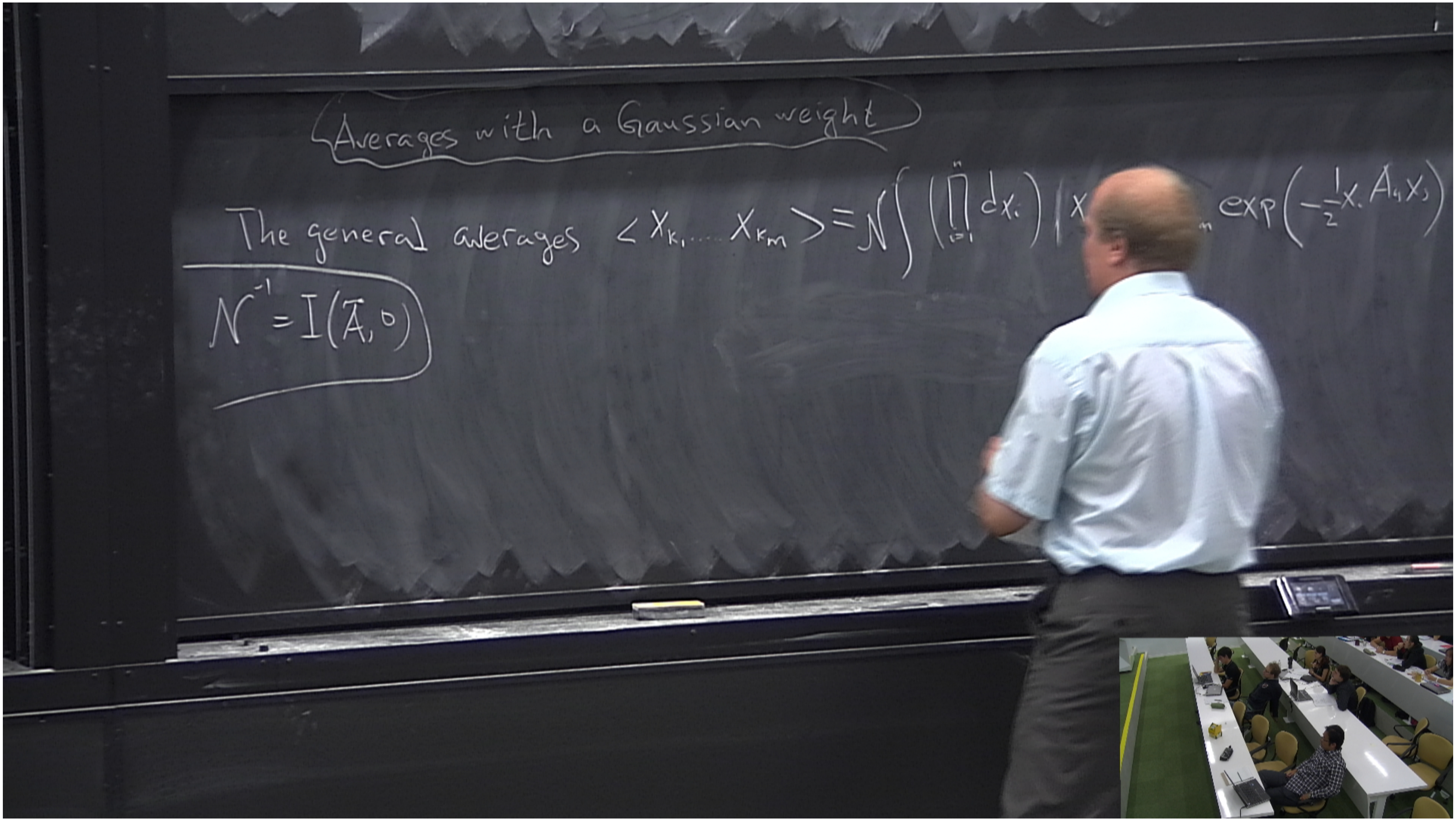

Averages with a Gaussian weight
The

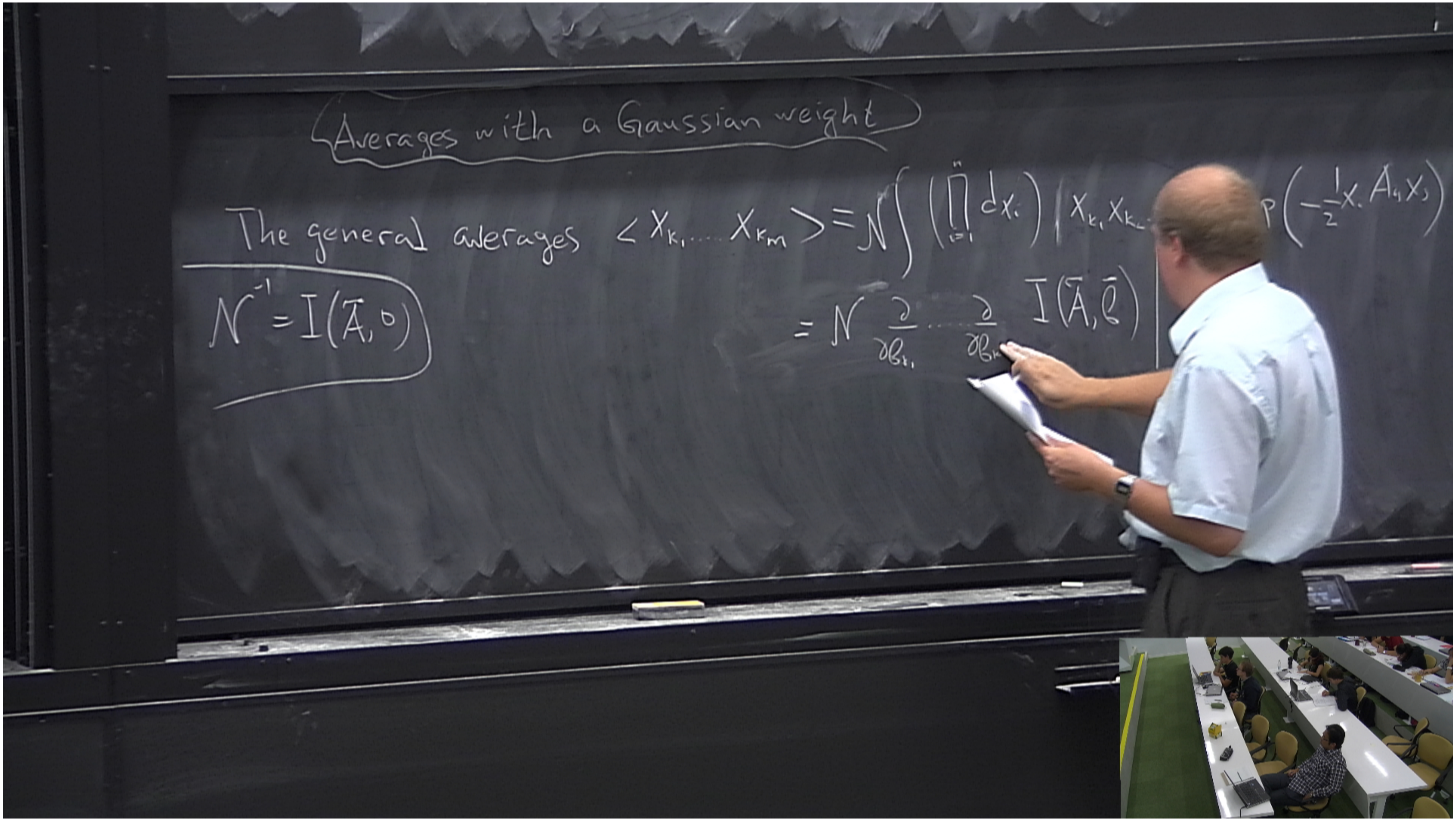

Averages with a Gaussian weight
The general averages

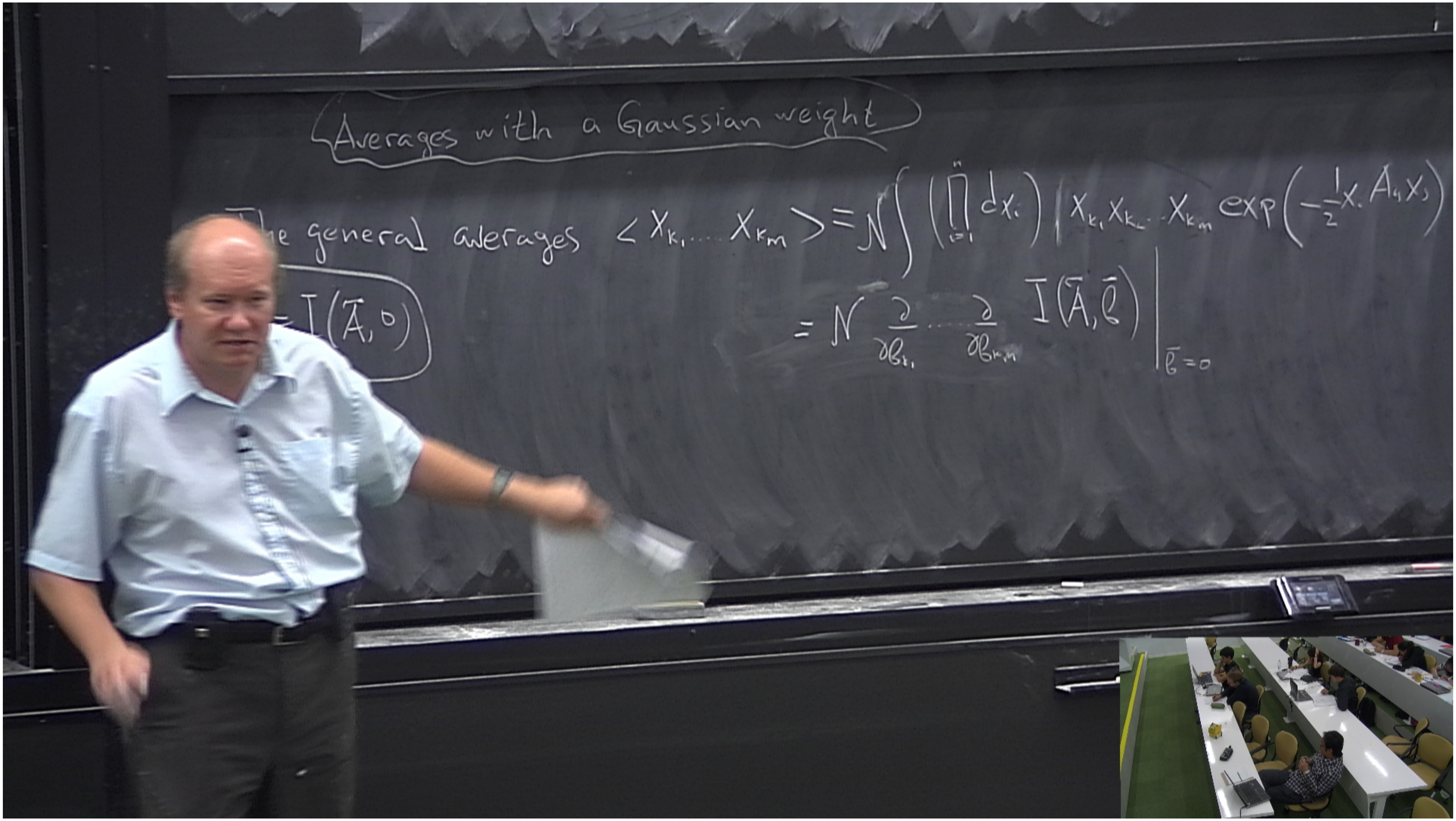

Averages with a Gaussian weight
The general averages $\langle x_{k_1} \ldots x_{k_m} \rangle = N \int \left( \prod_{i=1}^{n} dx_i \right) x_{k_1} x_{k_2} \ldots x_{k_m} \exp\left(-\frac{1}{2} x_i A_{ij} x_j\right)$
$N^{-1} = I(\bar{A}, 0)$

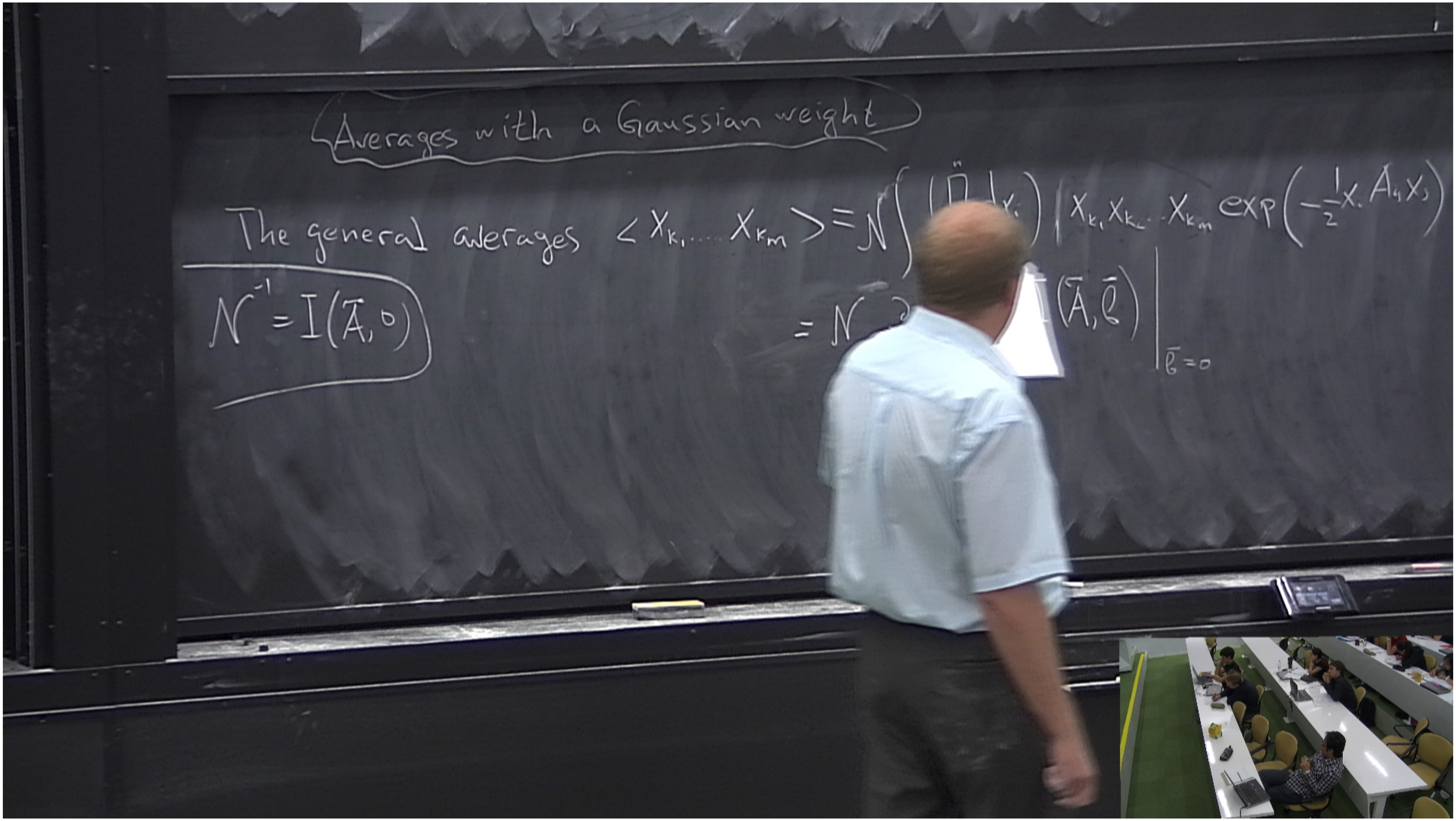

To get $X_{k_1}$ we differentiate $\frac{\partial}{\partial b_{k_1}} \exp\{$

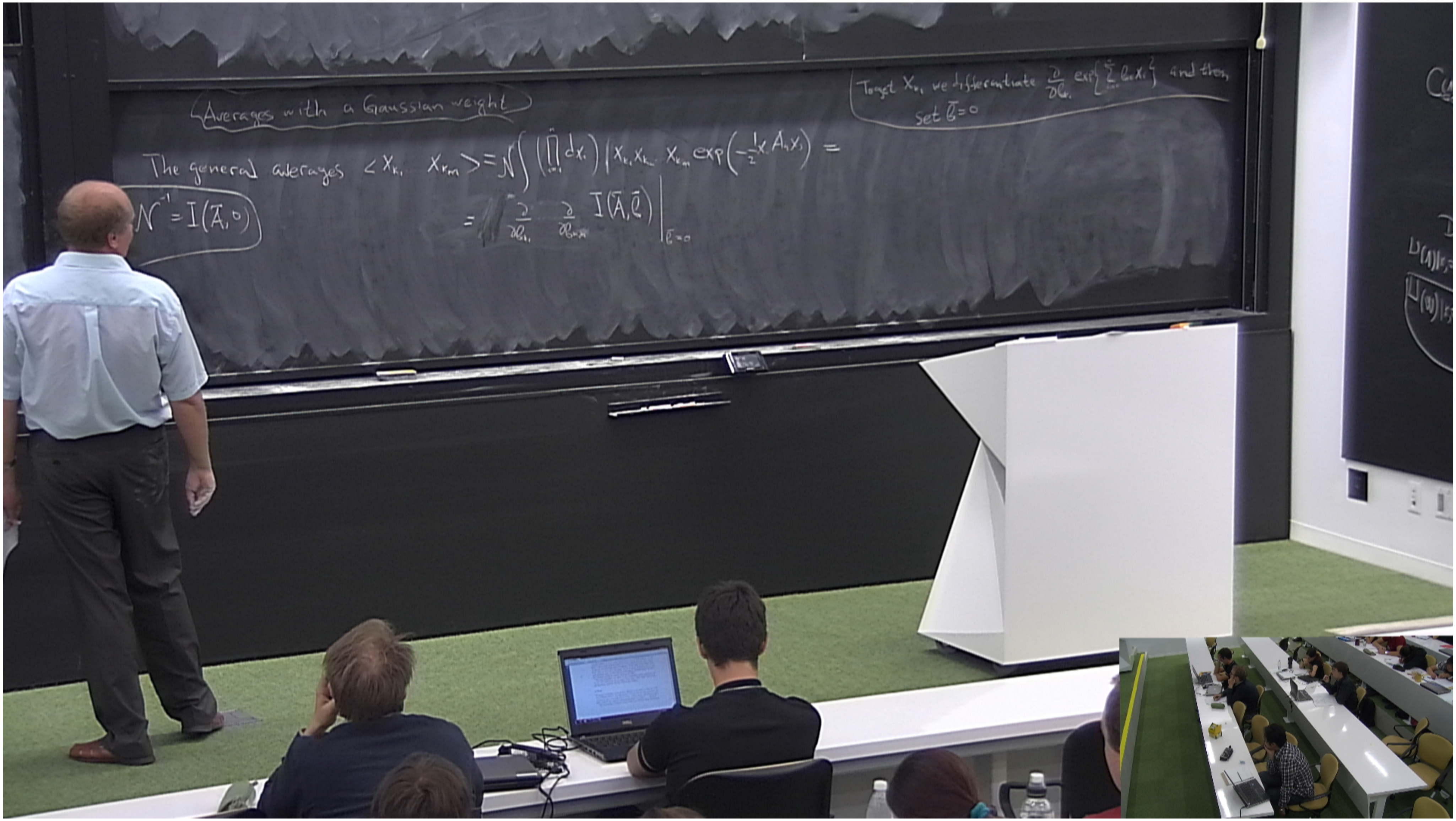

To get X_{k_1} we differentiate ∂/∂b_{k_1} exp{Σ_{i=1}^n b_i X_i} and then
set b̄ = 0

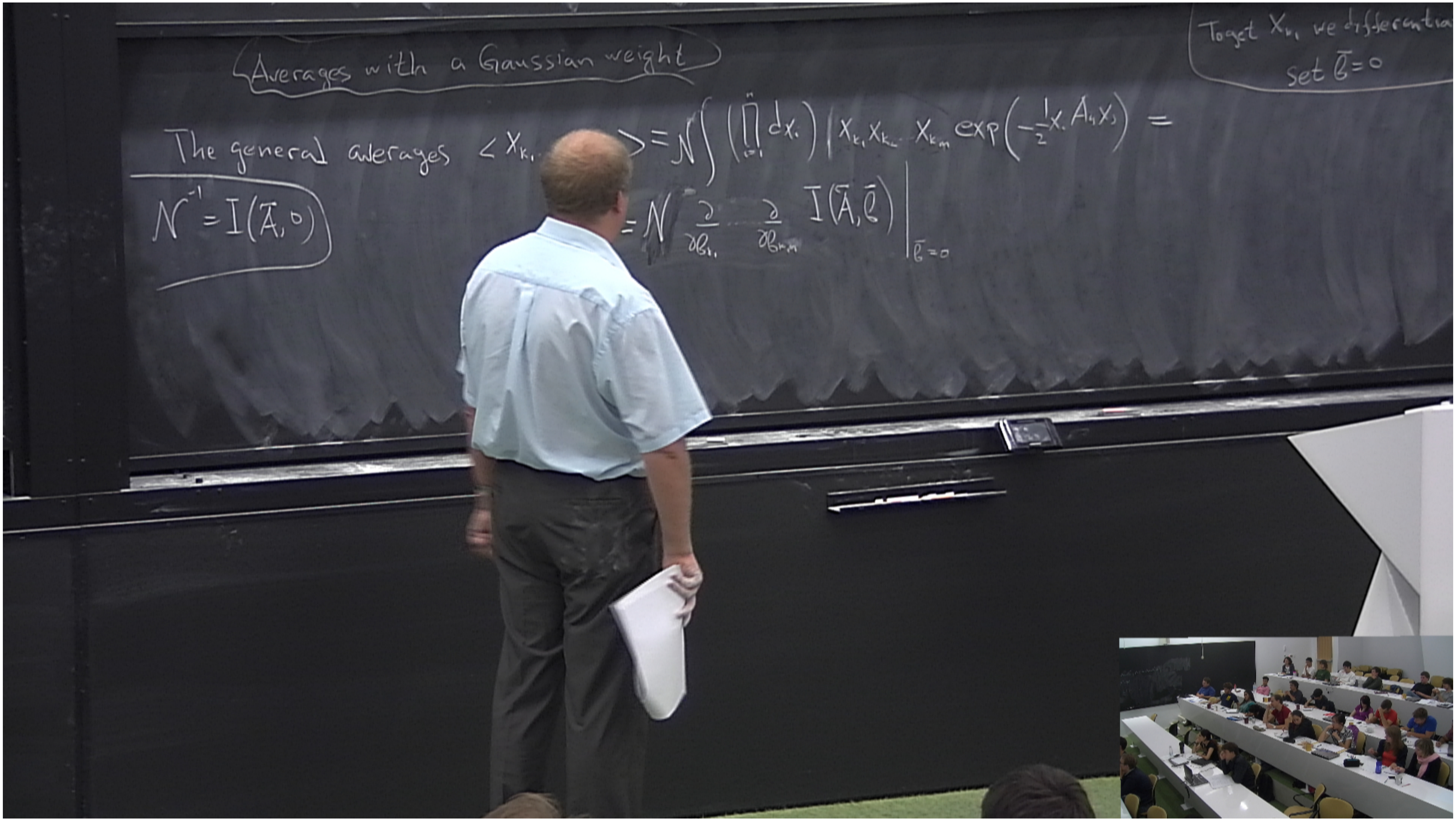

Averages with a Gaussian weight
The general averages $< x_{k_1} \ldots x_{k_m} > = N \int \left( \prod_{i=1}^{n} dx_i \right)$
$\exp\left(-\frac{1}{2} x_i A_{ij} x_j\right)$
$N^{-1} = I(A, 0)$

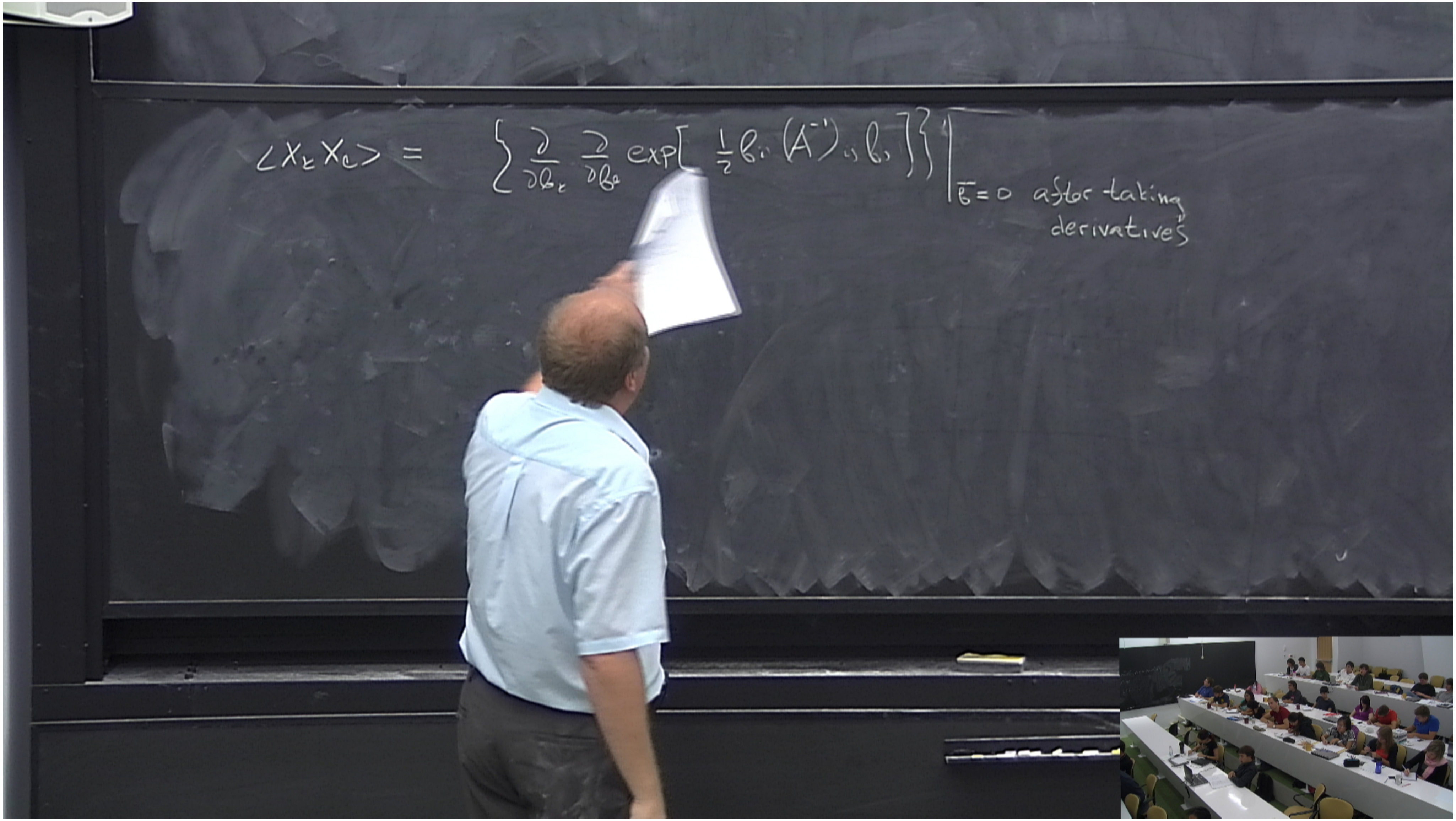

Averages with a Gaussian weight
The general averages < X_{k_1} ..... X_{k_m} > = N ∫ ( ∏_{i=1}^{n} dx_i ) X_{k_1} X_{k_2}
= N ∂/∂b_{k_1} ..... ∂/∂b_{k_m} I(Ā, b̄)
N^{-1} = I(Ā, 0)

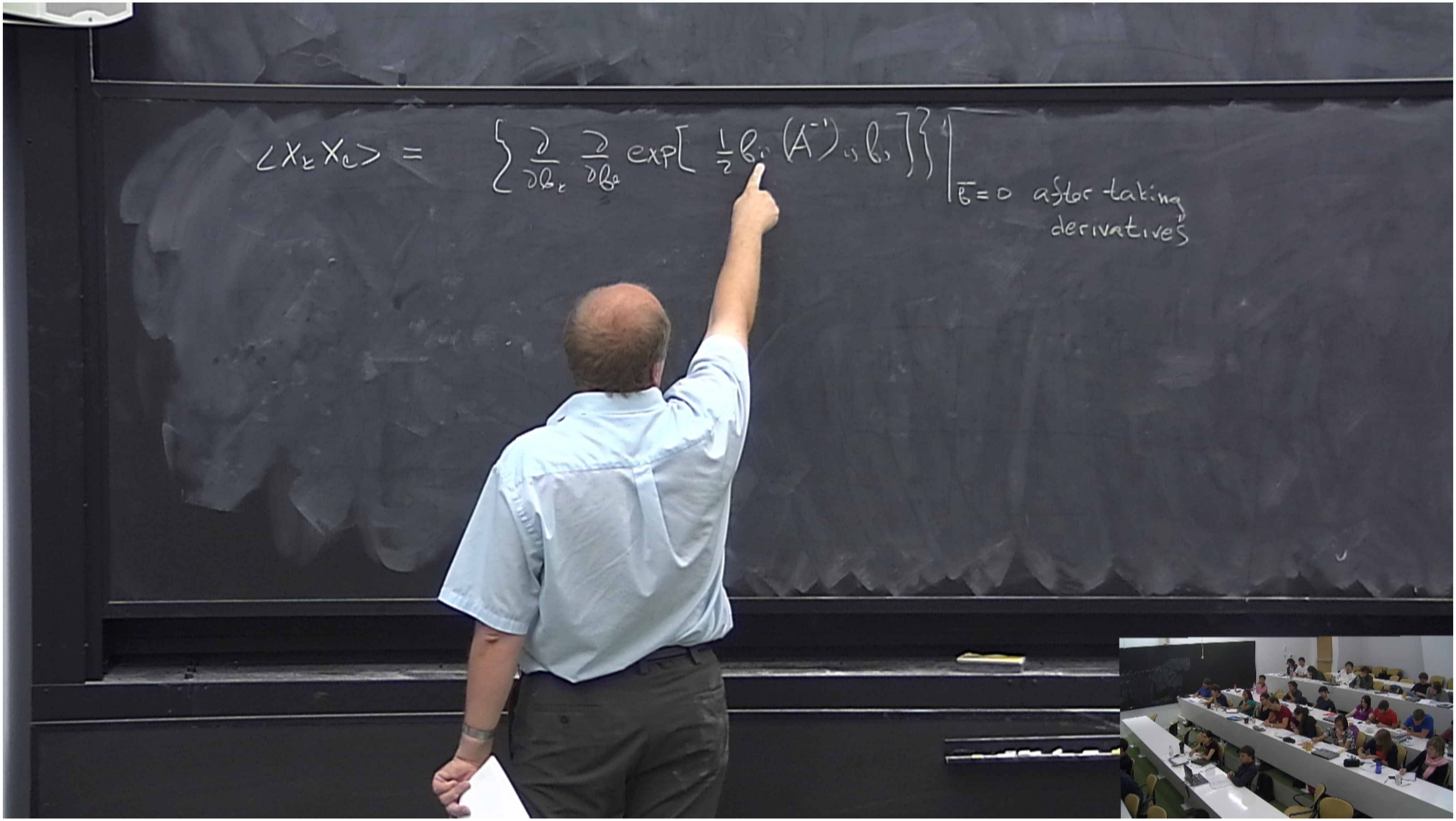

Averages with a Gaussian weight
general averages < X_{k_1} ..... X_{k_m} > = N ∫ ( ∏_{i=1}^{n} dx_i ) X_{k_1} X_{k_2} ... X_{k_m} exp( -1/2 x_i A_{ij} x_j )
I(Ā, 0)
= N ∂/∂b_{k_1} ..... ∂/∂b_{k_m} I(Ā, b̄) |_{b̄=0}

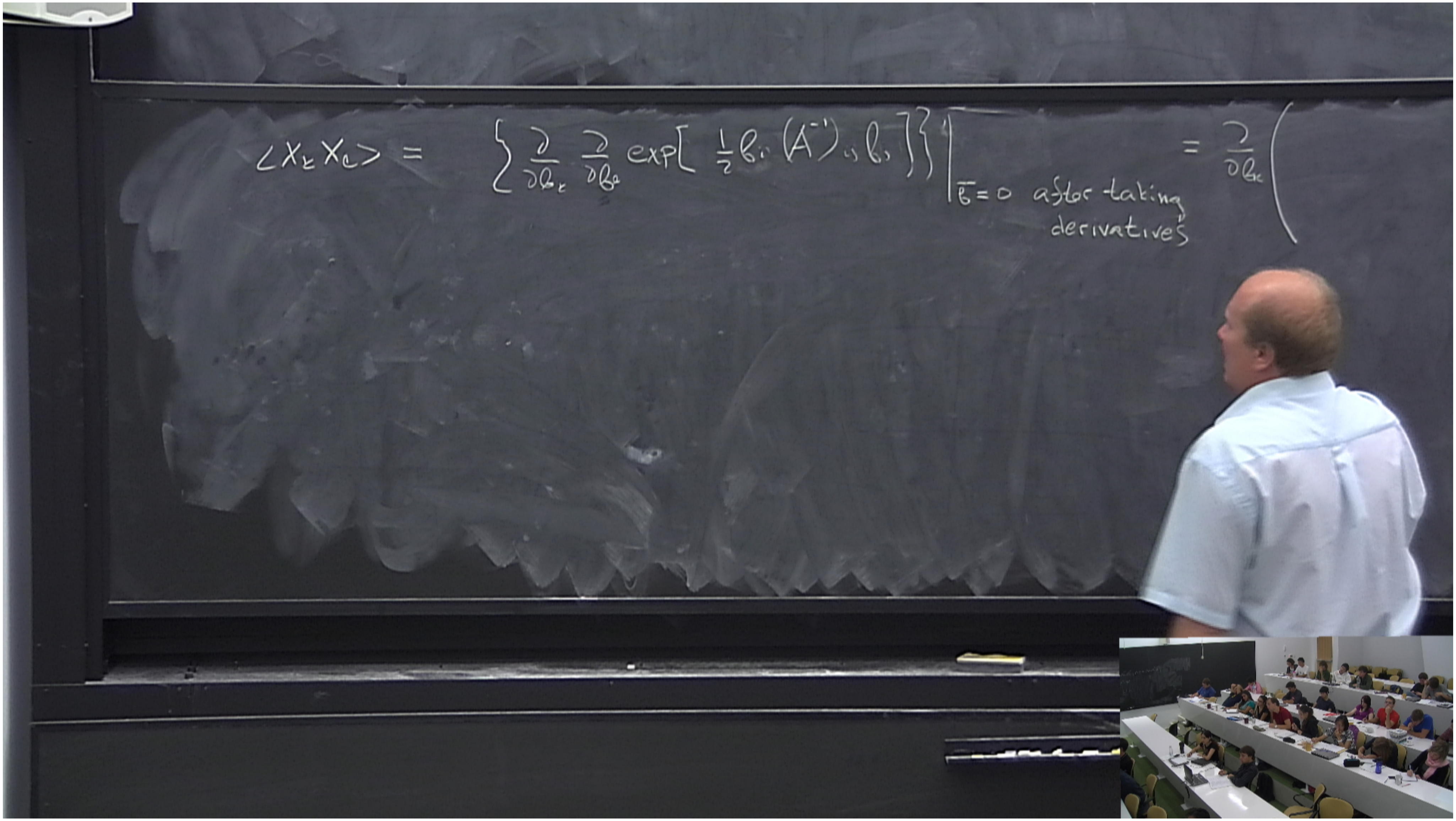

Averages with a Gaussian weight
The general averages $< X_{k_1} \dots X_{k_m} > = N \int \left(\prod dx_i\right) X_{k_1} X_{k_2} \dots X_{k_m} \exp\left(-\frac{1}{2} x_i A_{ij} x_j\right)$
$N^{-1} = I(\bar{A}, 0)$
$= N$
$I(\bar{A}, \bar{b}) \Big|_{\bar{b}=0}$

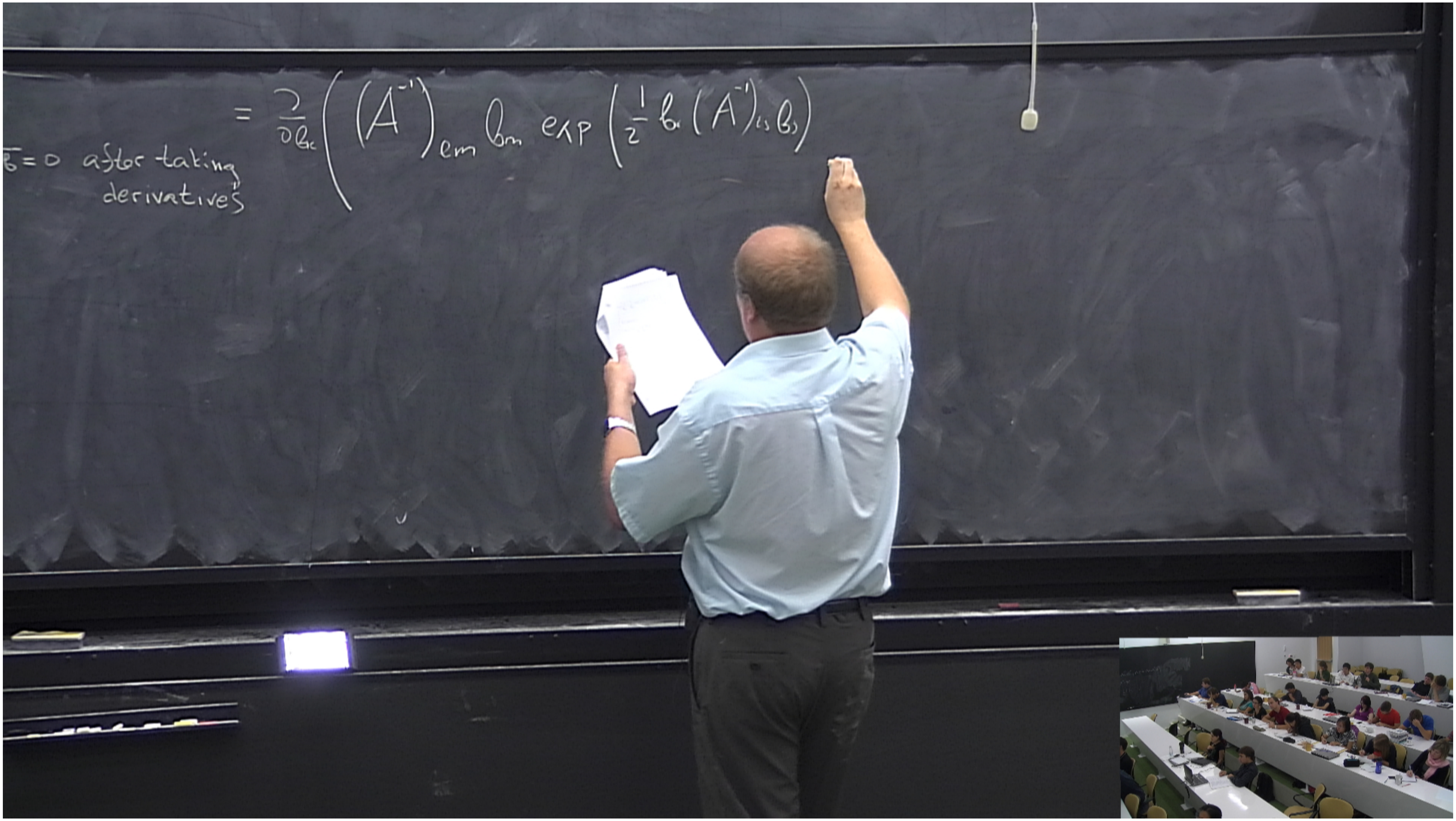

Averages with a Gaussian weight
The general averages
To get $X_{k_1}$ we differentiate
and then set $\vec{b}=0$

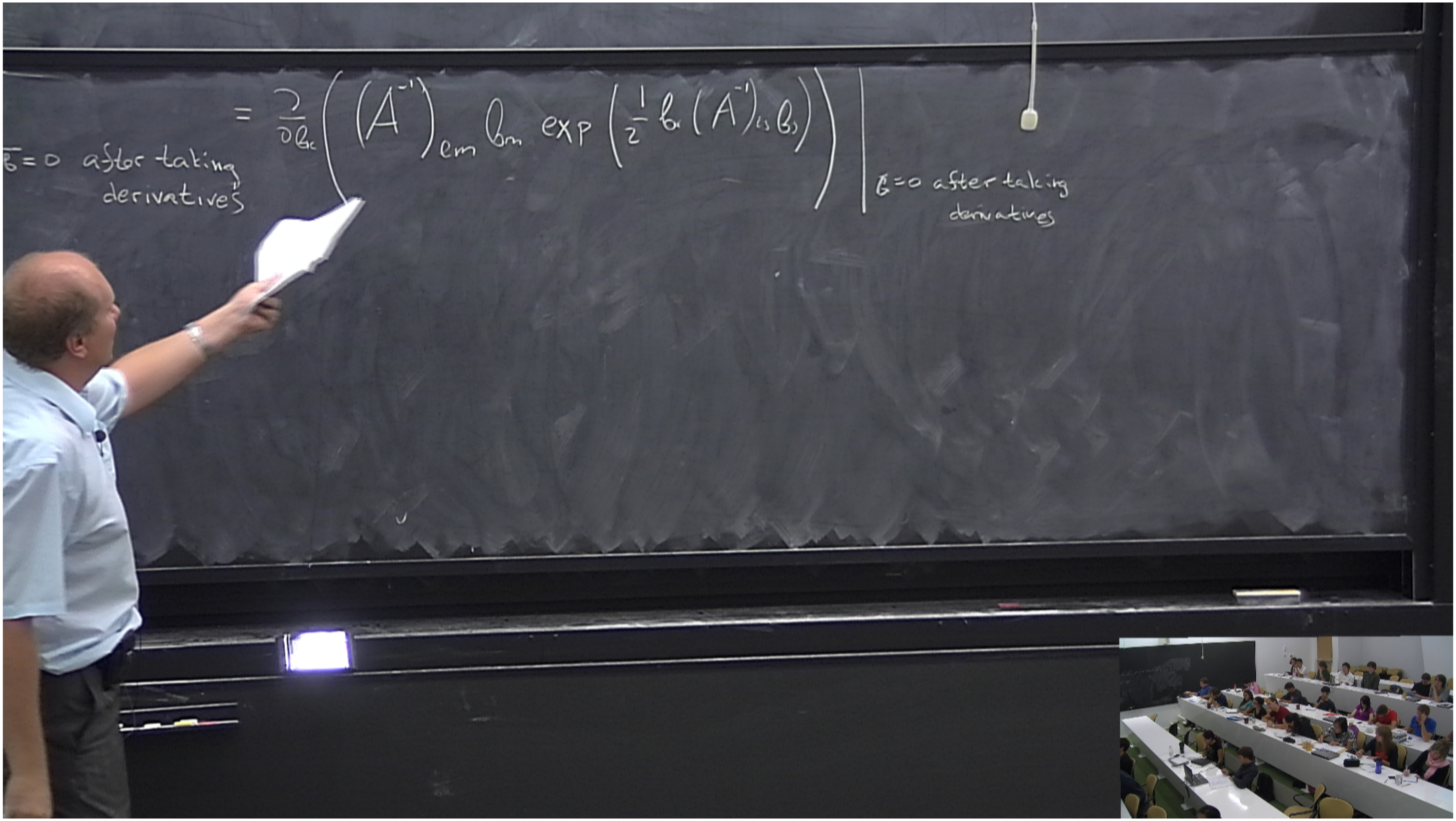

Averages with a Gaussian weight
The general averages
$N^{-1} = I(\vec{A}, 0)$
To get $X_{k_1}$ we differentiate
set $\vec{b} = 0$

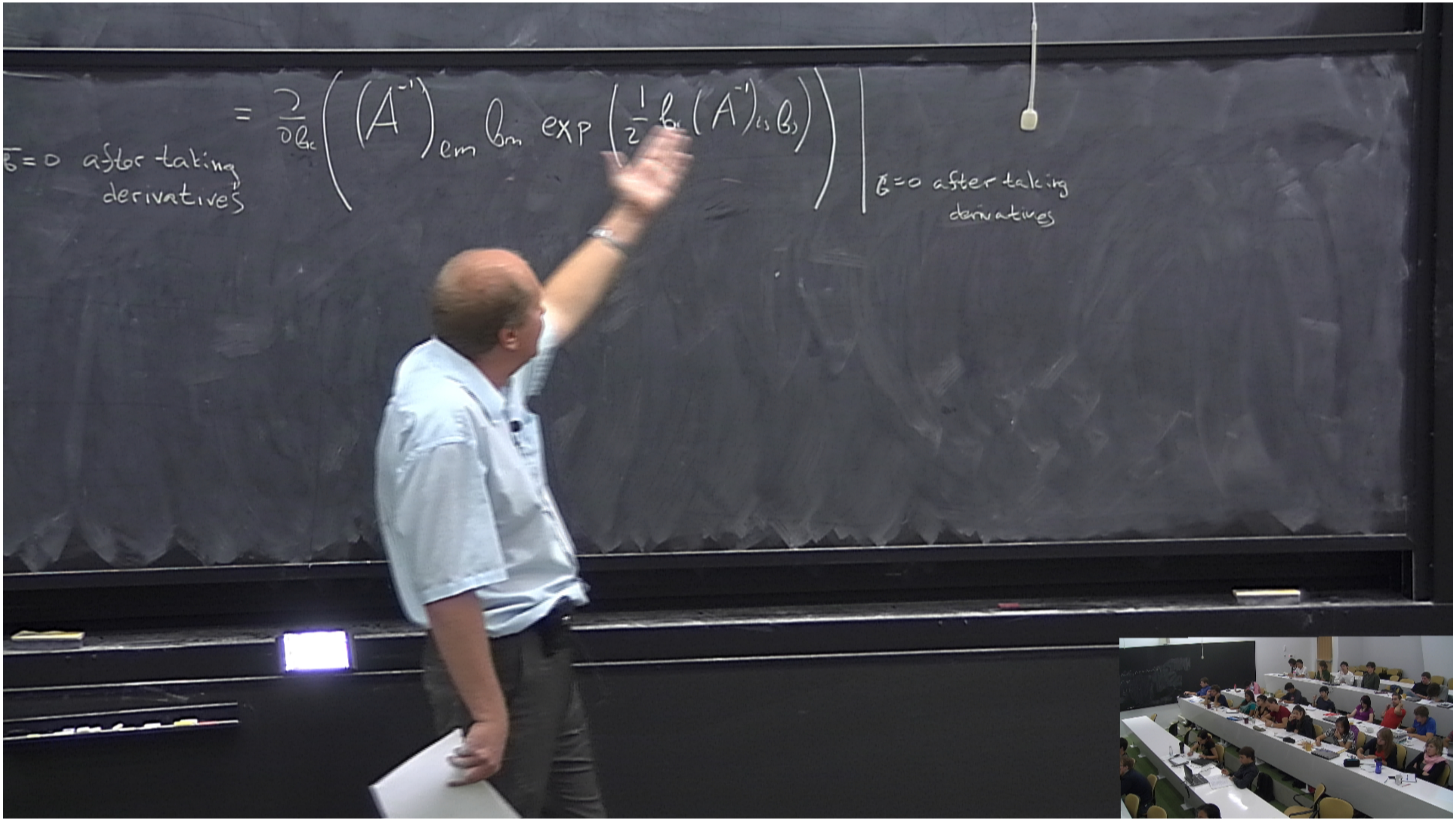

<X_t X_t'> = { ∂/∂b_t ∂/∂b_t' exp[ 1/2 b_i (A^-1)_ij b_j ] } |_b=0 after taking derivatives

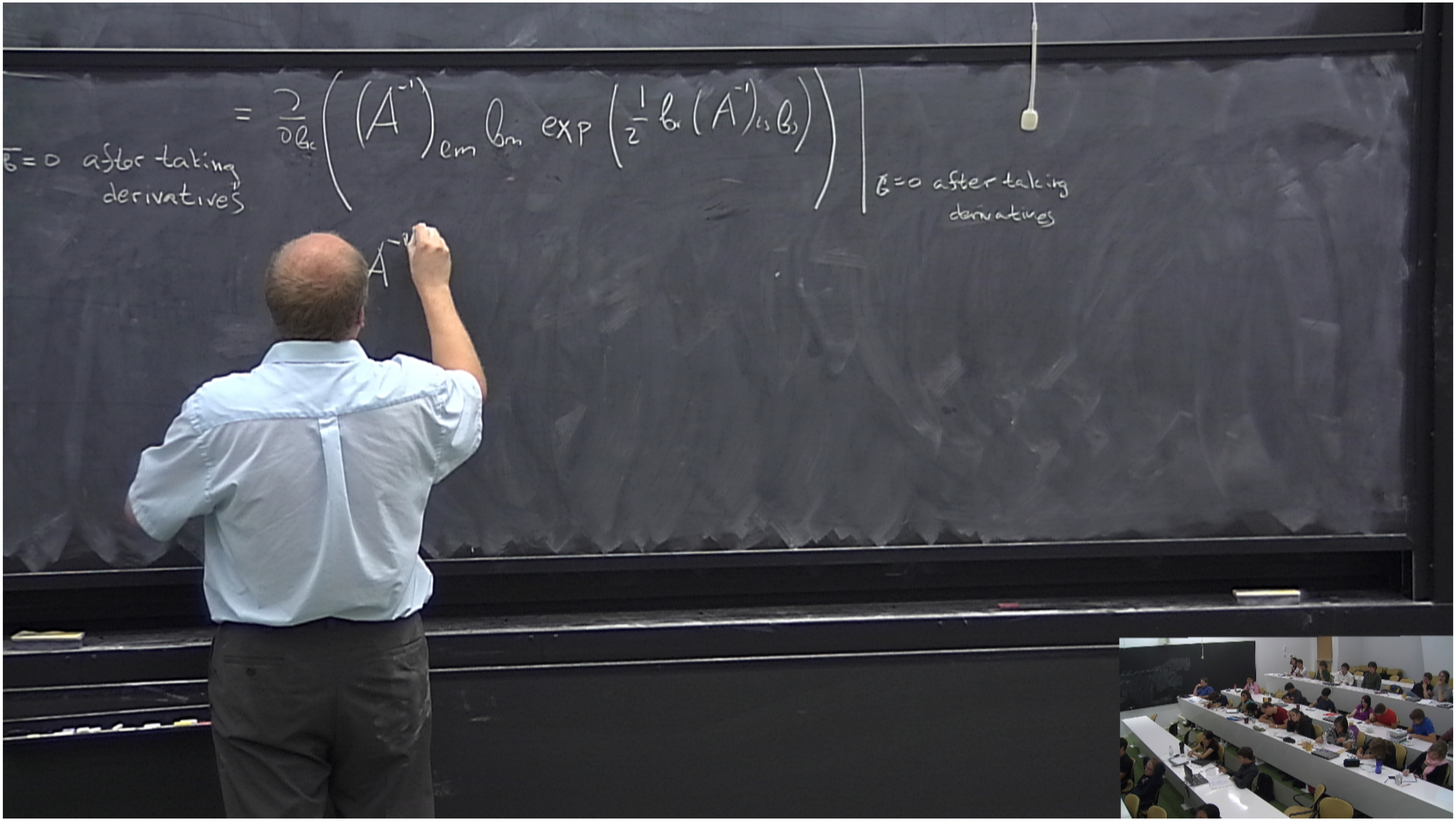

$\langle x_i x_k \rangle = \left\{ \frac{\partial}{\partial b_i} \frac{\partial}{\partial b_k} \exp\left[ \frac{1}{2} b_i (A^{-1})_{ij} b_j \right] \right\} \Big|_{\vec{b}=0}$
after taking derivatives

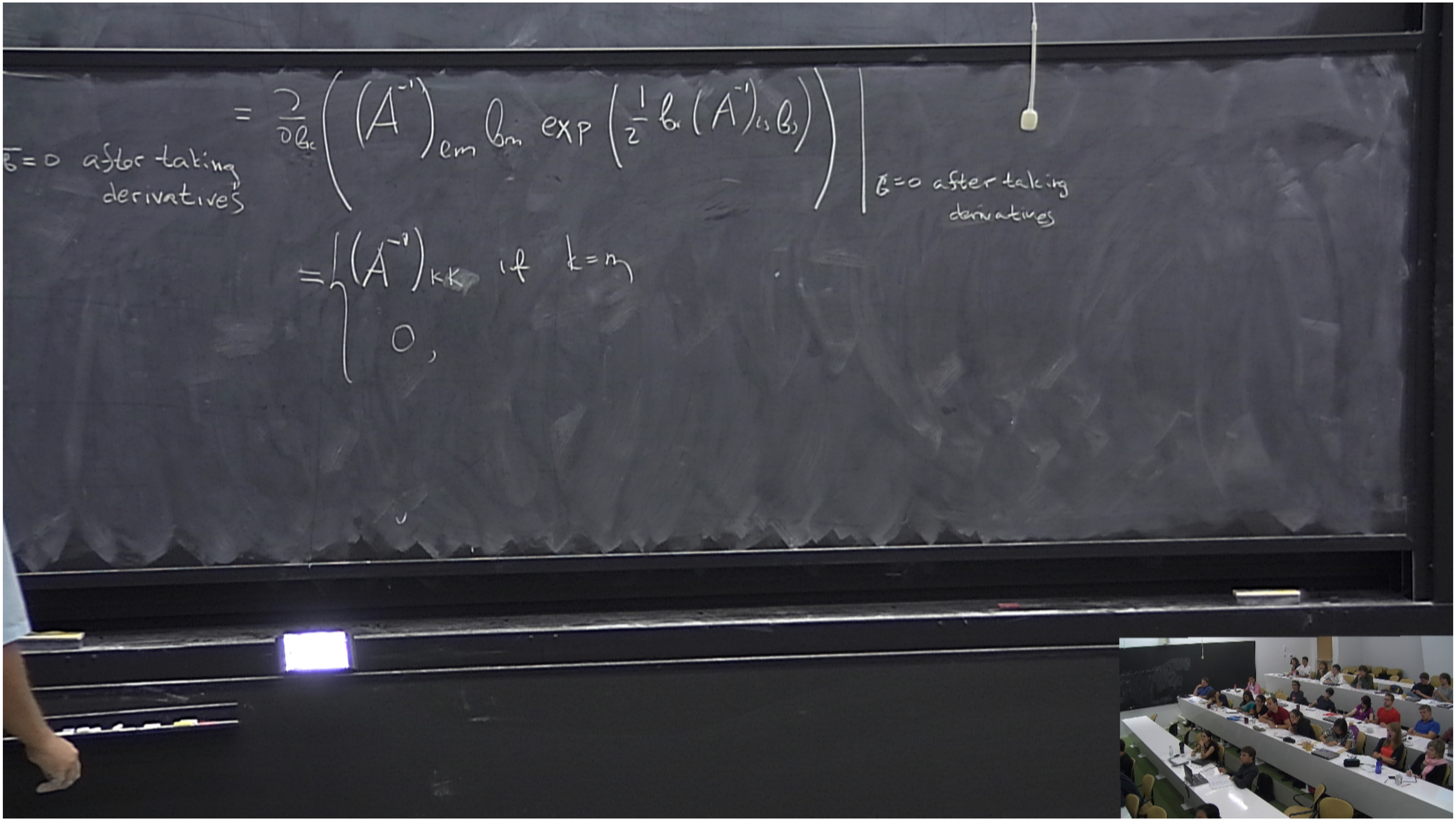

$\langle x_k x_l \rangle = \left\{ \frac{\partial}{\partial b_k} \frac{\partial}{\partial b_l} \exp\left[ \frac{1}{2} b_i (A^{-1})_{ij} b_j \right] \right\} \Big|_{\vec{b}=0}$ after taking derivatives
$= \frac{\partial}{\partial b_k} \Big($

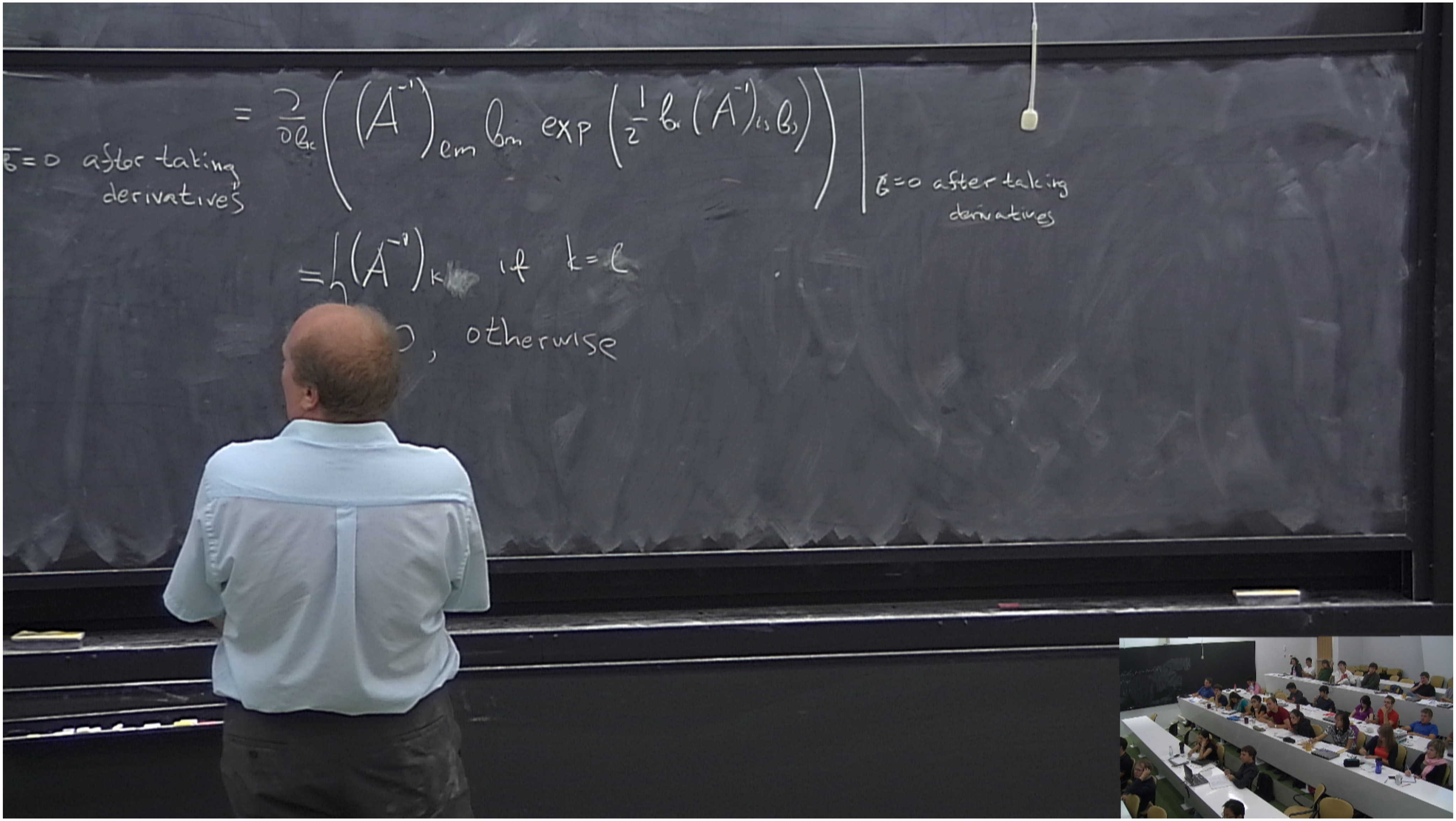

= ∂/∂b_c ((A^-1)_lm b_m exp(1/2 b_r (A^-1)_rs b_s))
b = 0 after taking
derivatives

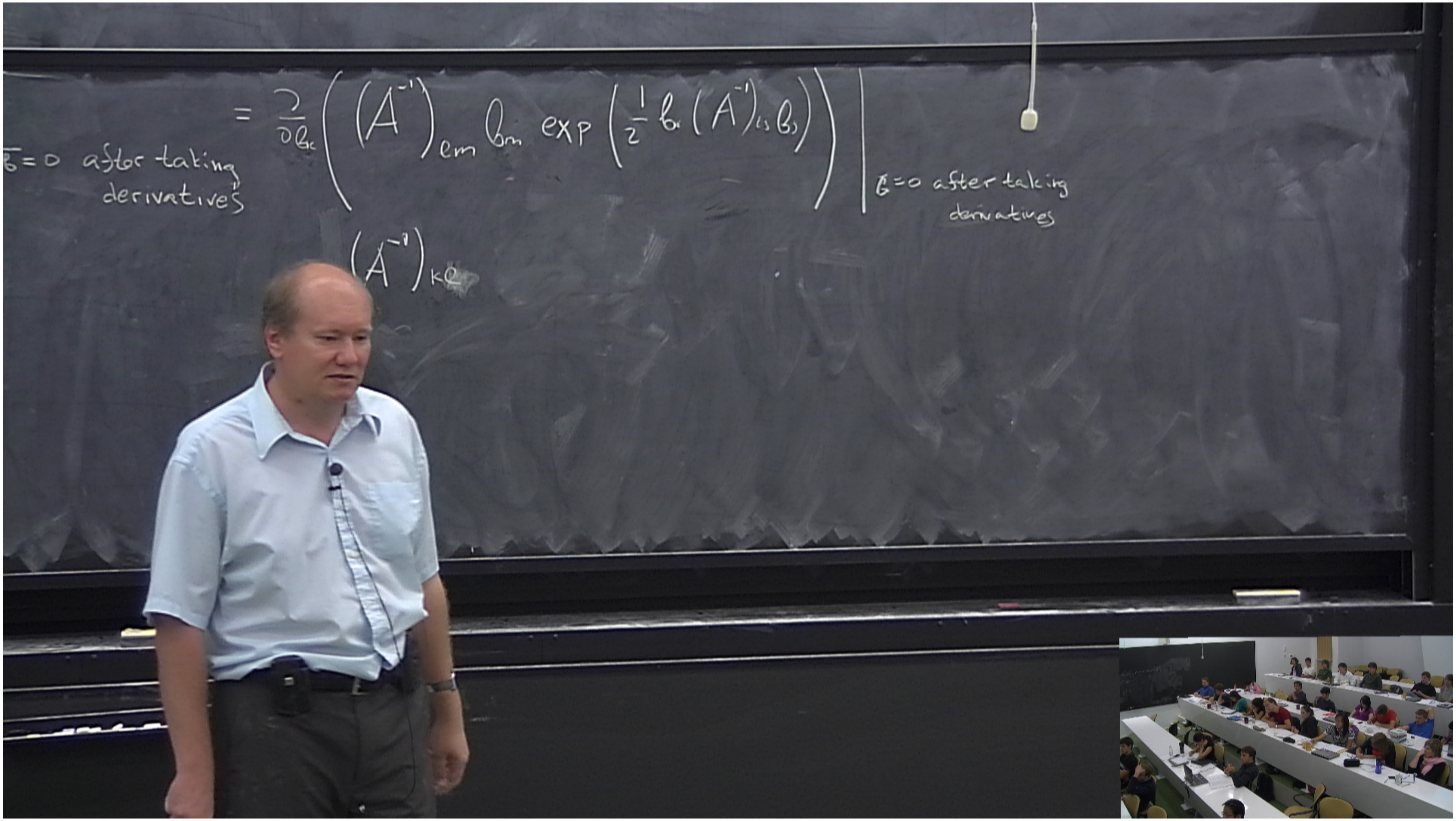

b = 0 after taking
derivatives
$= \frac{\partial}{\partial b_k}\left( (A^{-1})_{em} b_m \exp\left(\frac{1}{2} b_r (A^{-1})_{rs} b_s\right)\right)\Big|$
b = 0 after taking
derivatives


$= \frac{\partial}{\partial b_c}\left( (A^{-1})_{em} b_m \exp\left(\frac{1}{2} b_c (A^{-1})_{cs} b_s\right)\right)\Big|$
$b=0$ after taking derivatives
$b=0$ after taking derivatives


b = 0 after taking derivatives
= ∂/∂b_c ((A^-1)_em b_m exp(1/2 b_r (A^-1)_rs b_s))
b = 0 after taking derivatives
A^-1


b = 0 after taking derivatives
= ∂/∂b_k ( (A^-1)_lm b_m exp ( 1/2 b_r (A^-1)_rs b_s ) ) |
b = 0 after taking derivatives
= (A^-1)_kk if k = m
0,


b = 0 after taking derivatives
= ∂/∂b_k ( (A^-1)_lm b_m exp ( 1/2 b_r (A^-1)_rs b_s ) ) | b = 0 after taking derivatives
= (A^-1)_k if k = l
0, otherwise


= ∂/∂b_k ( (A^-1)_lm b_m exp ( 1/2 b_r (A^-1)_rs b_s ) )
b = 0 after taking derivatives
(A^-1)_kl
b = 0 after taking derivatives