Title: Physics in Nature Presentation: Natural Chaos

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URL: http://pirsa.org/11080114

Abstract:

Natural Chaos

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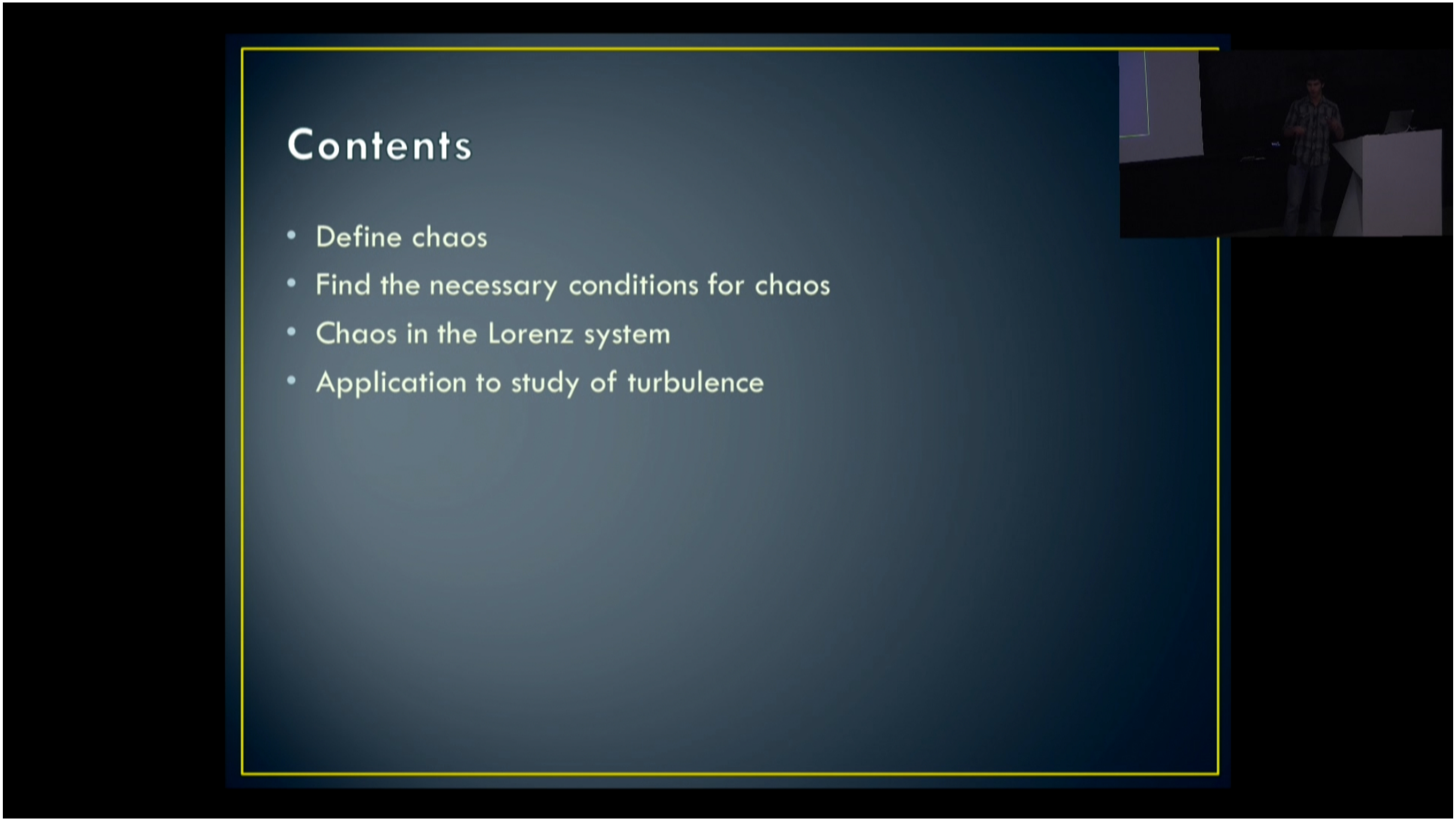
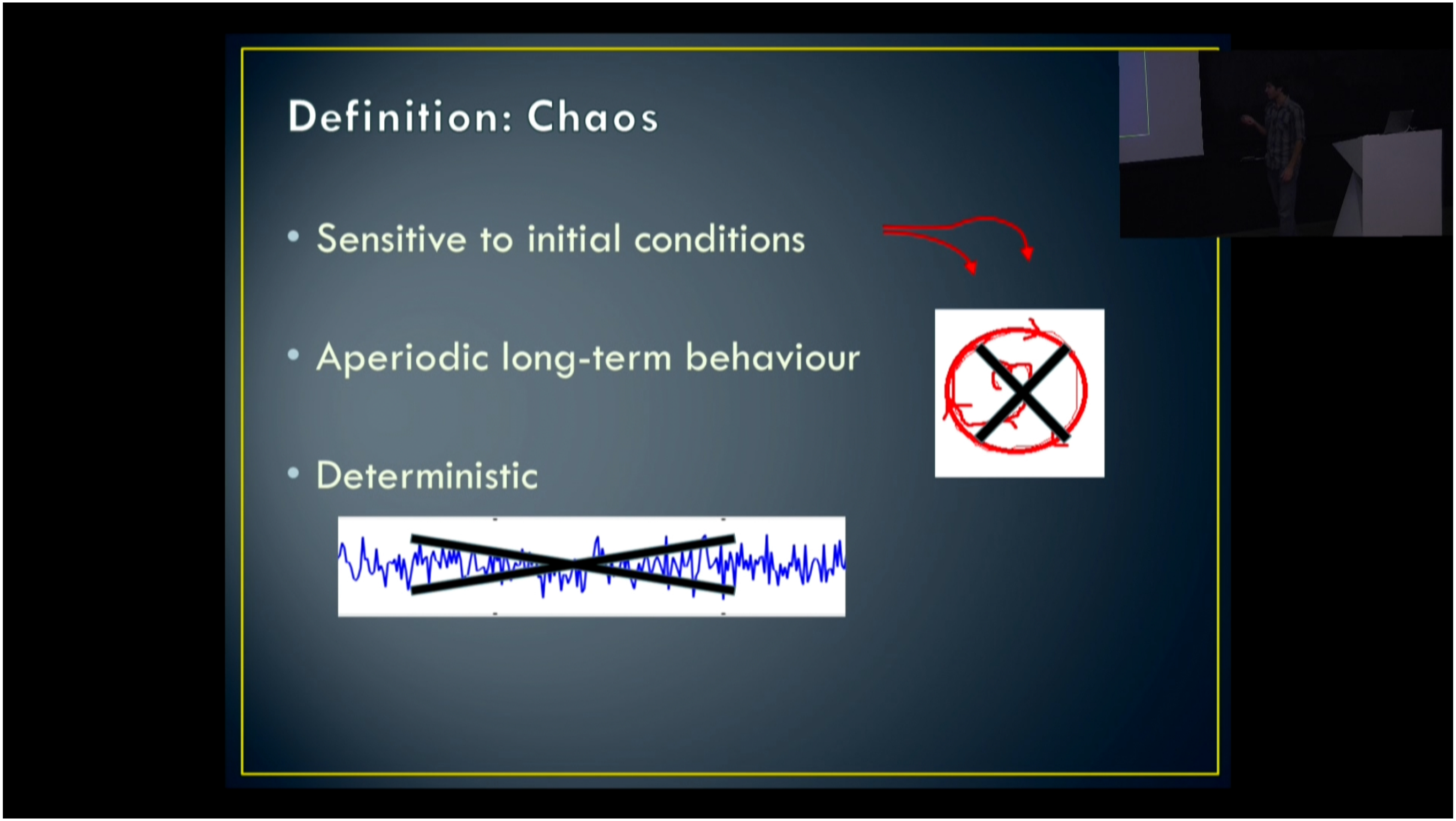
PSI
08/2011


Contents
Define chaos
Find the necessary conditions for chaos
Chaos in the Lorenz system
Application to study of turbulence


Definition: Chaos
Sensitive to initial conditions
Aperiodic long-term behaviour
Deterministic

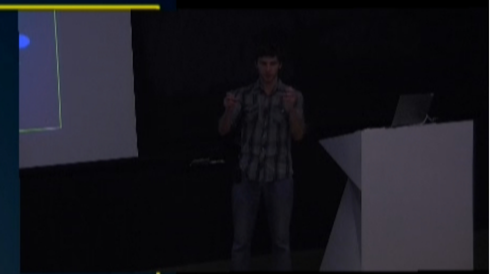
Definition: Chaos

- Sensitive to initial conditions

- Aperiodic long-term behaviour

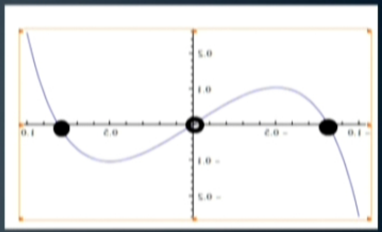
- Deterministic

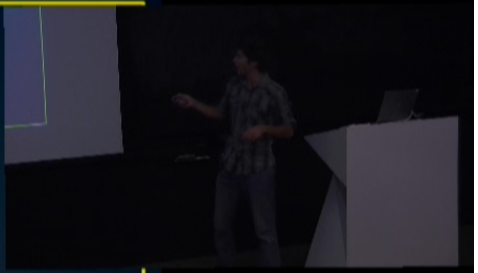
- Topological mixing

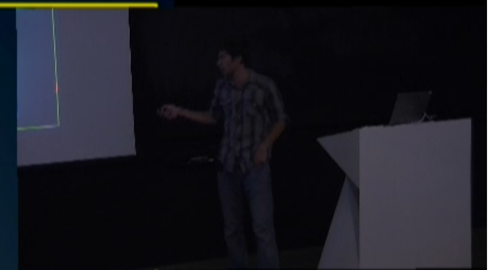


1D systems

- Stable/unstable fixed points
  - Example: $\frac{dx}{dt} = -\tan(x) + \beta x$







1D systems

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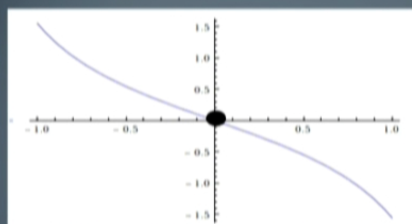
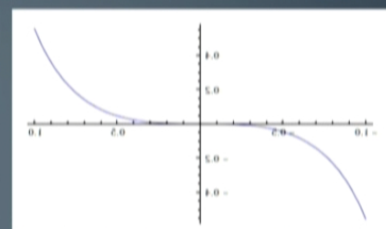
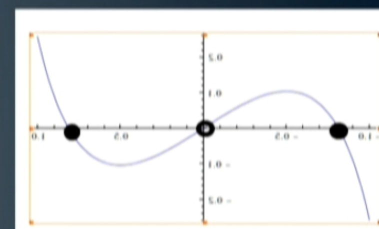




- Bifurcation

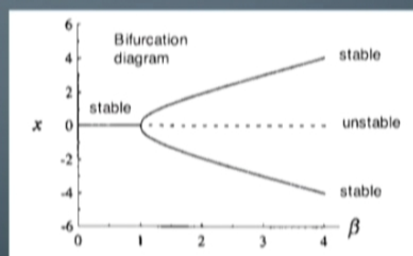
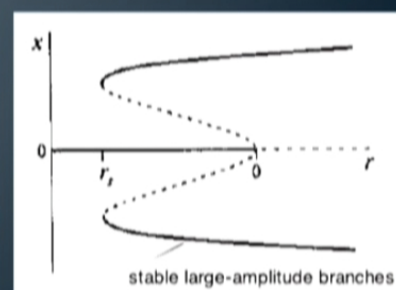
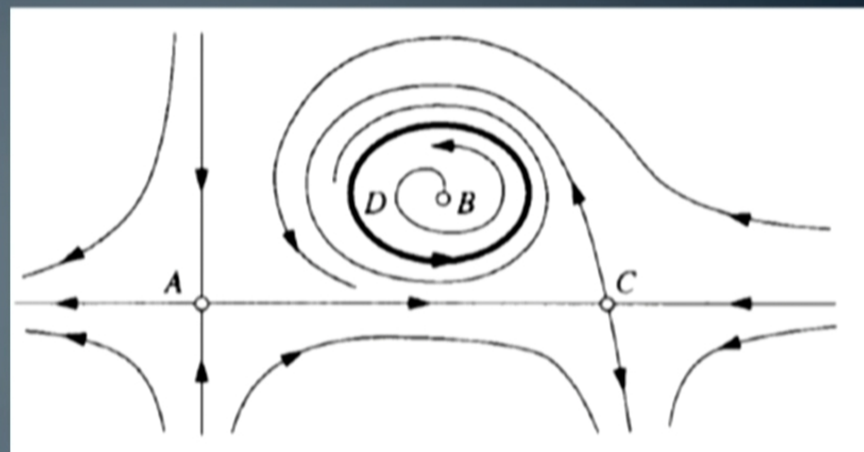


Fig. 3.4.4

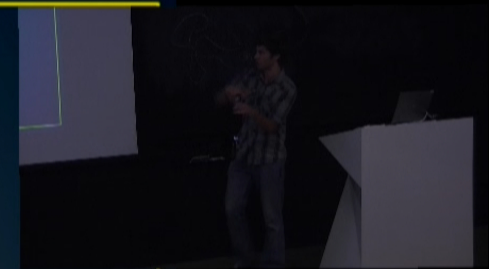


- No chaos

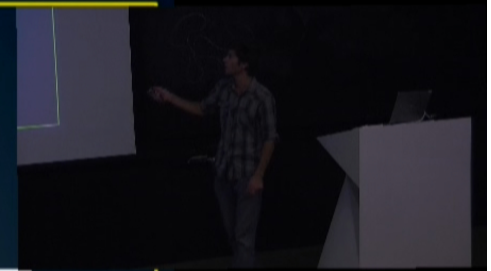
2D systems



- Still no chaos possible!



3D systems – Lorenz attractor



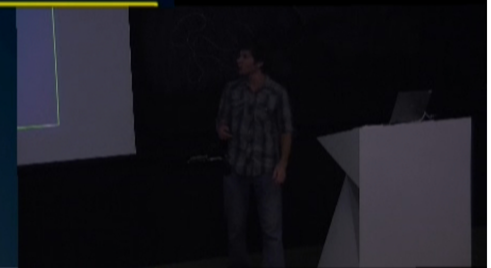
- Atmospheric convection flows:
  - x – fluid speed
  - y – relative temperature
  - z – non-linearity in temperature gradient
  - σ – Prandtl number (~viscosity)
  - ρ - Rayleigh number (~temperature difference)
  - β – relates to flow geometry

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

3D systems – Lorenz attractor



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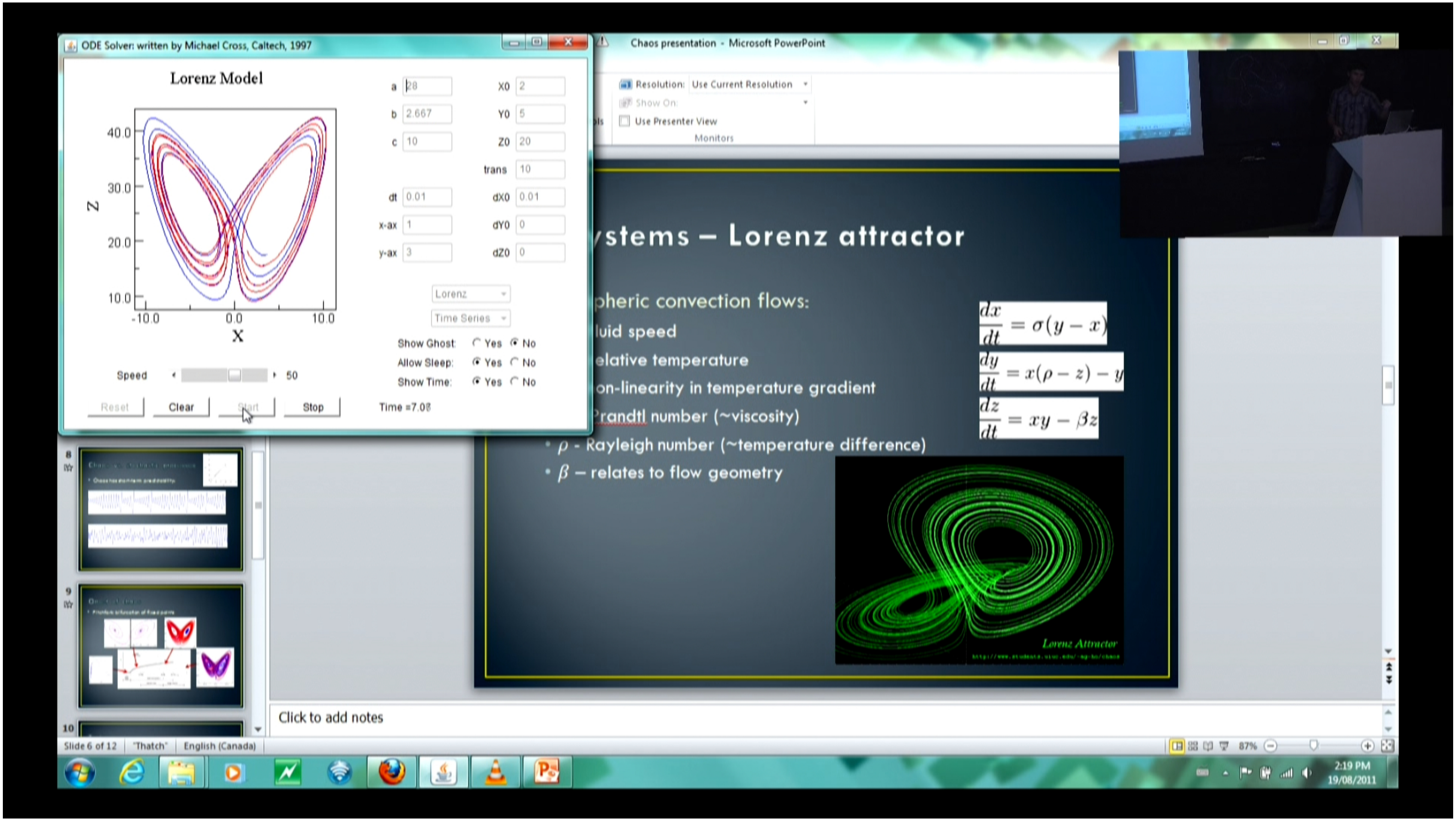
ODE Solver: written by Michael Cross, Caltech, 1997
Lorenz Model
Z
X
Speed
50
Reset
Clear
Start
Stop
Time =7.08
Show Ghost:
Allow Sleep:
Show Time:
Lorenz
Time Series
Chaos presentation - Microsoft PowerPoint
Resolution: Use Current Resolution
Use Presenter View
Monitors
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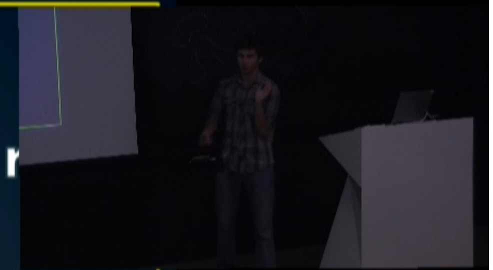
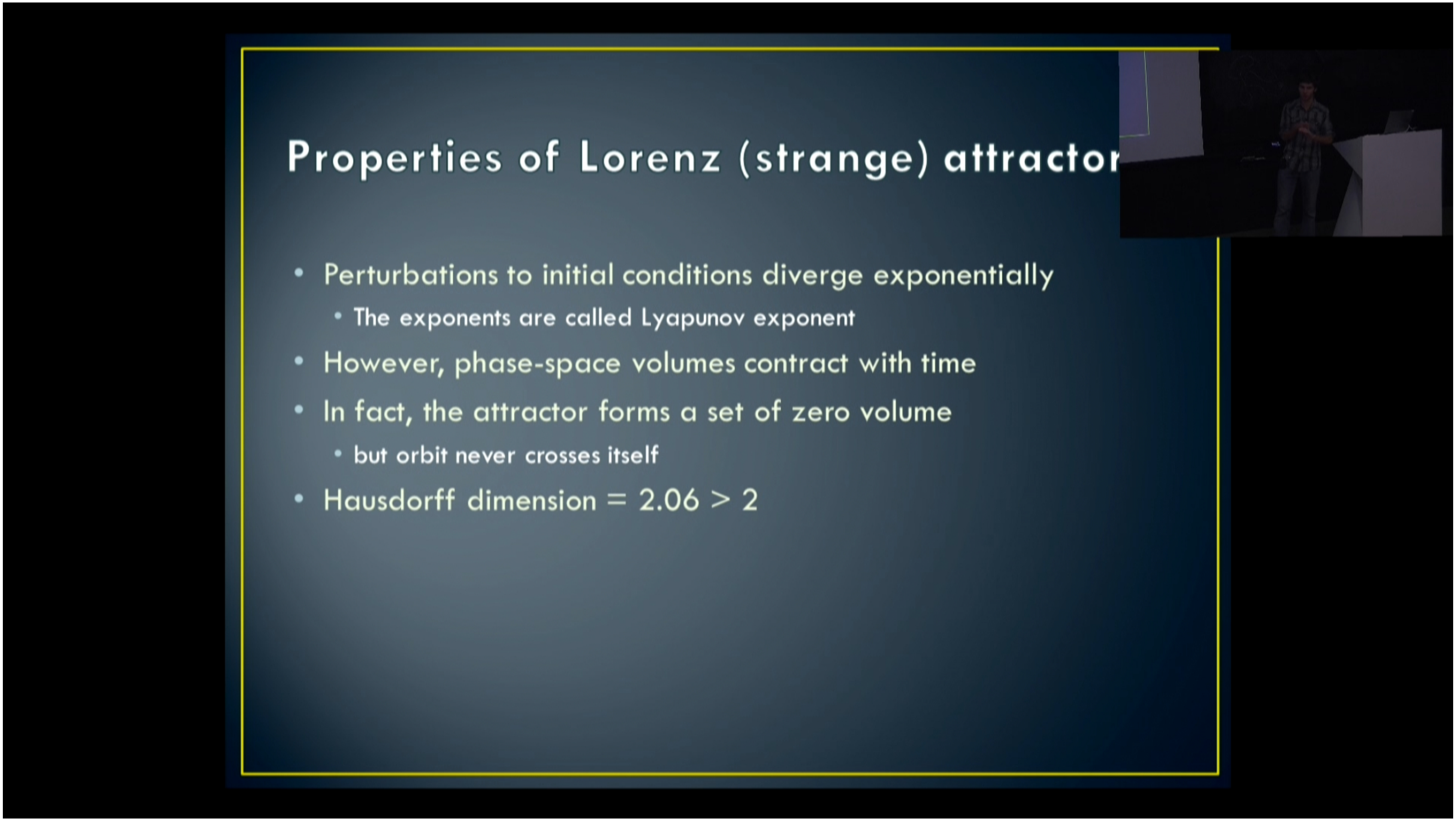
Lorenz Attractor
Click to add notes
Slide 6 of 12

Properties of Lorenz (strange) attractor



- Perturbations to initial conditions diverge exponentially
  - The exponents are called Lyapunov exponent
- However, phase-space volumes contract with time
- In fact, the attractor forms a set of zero volume
  - but orbit never crosses itself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Hausdorff dimension = 2.06 > 2

Chaos vs. stochastic processes

- Chaos has short-term predictability: