Title: Physics in Nature Presentation: Physics Behind Rainbows

Date: Aug 19, 2011 02:00 PM

URL: http://pirsa.org/11080113

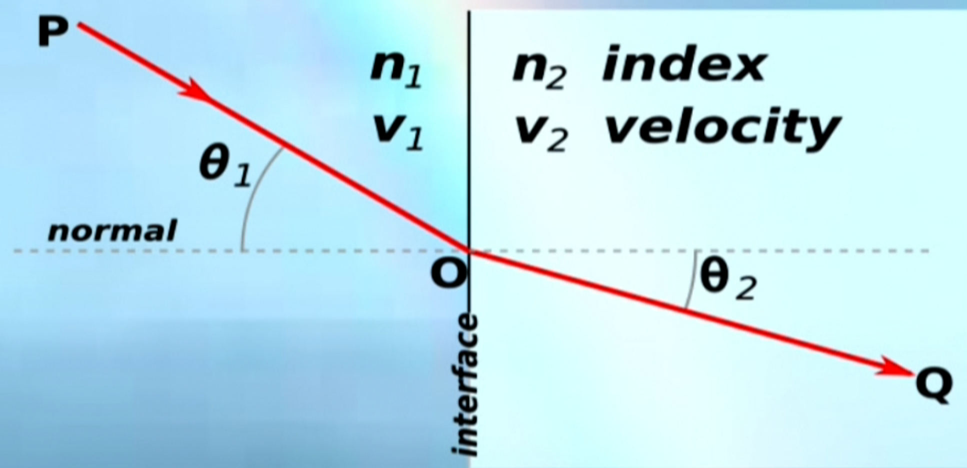
Abstract:

- A rainbow is an optical phenomenon that causes a spectrum of light to appear in the sky when the Sun shines on to droplets of moisture in the Earth's atmosphere.

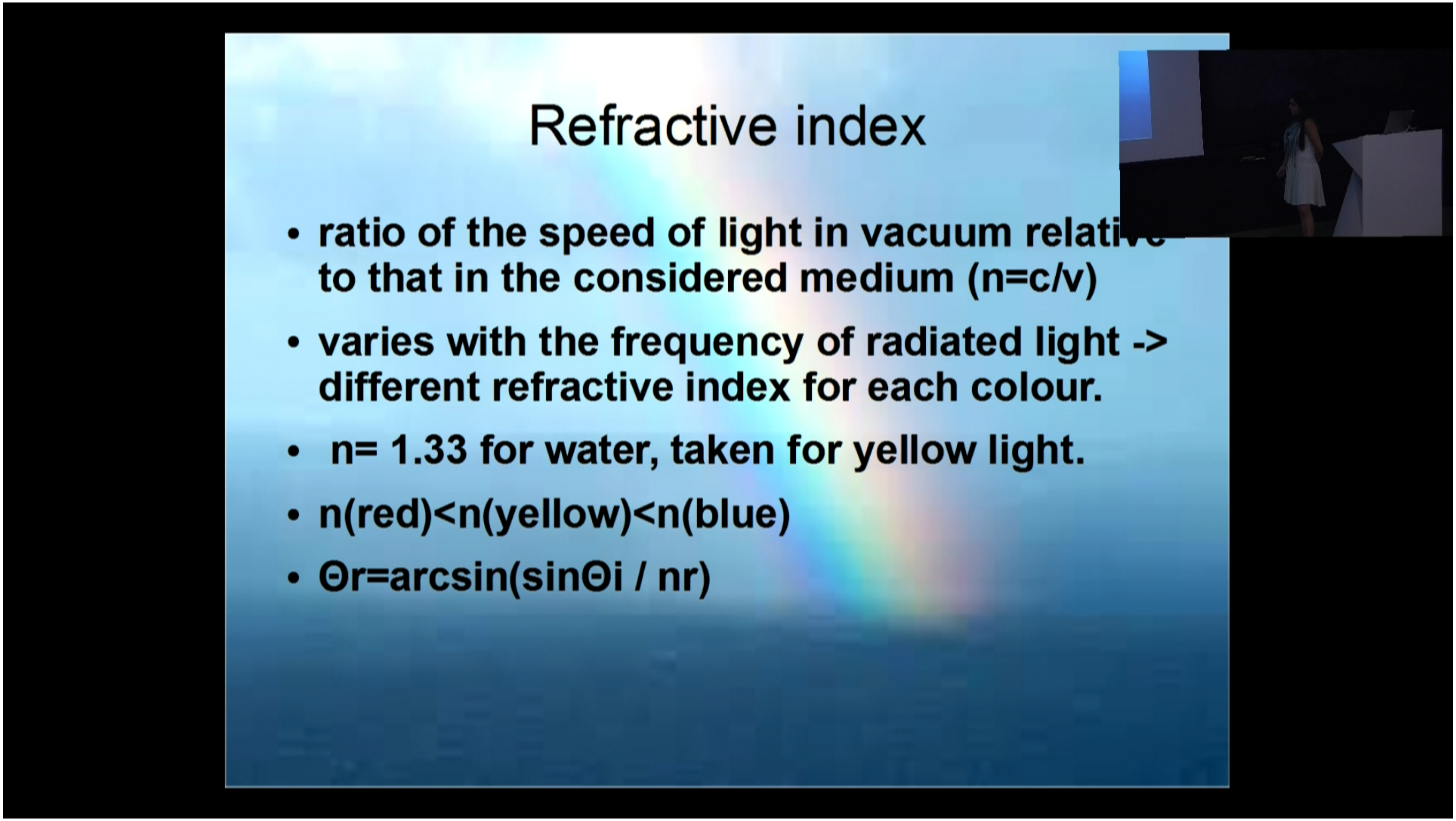
A little bit of history...

- First theory: Aristotle
- First measurement: Roger Bacon
- Teodorico of Freiberg (droplets) corroborated by Descartes

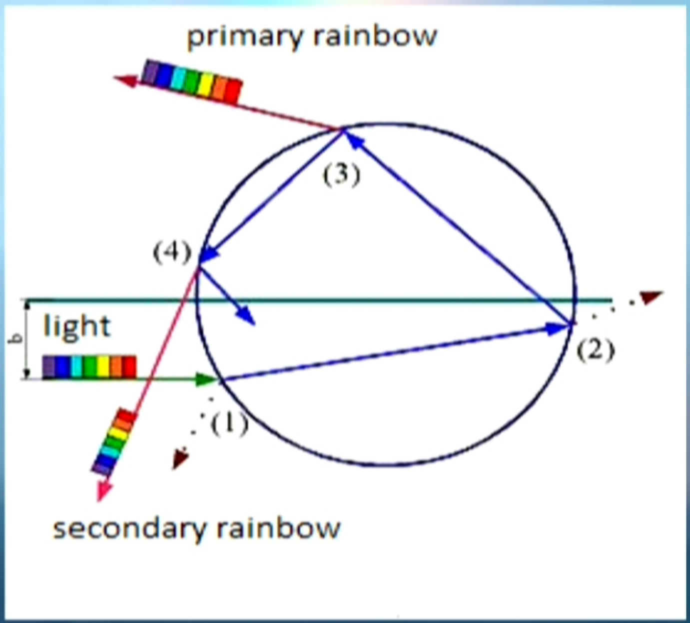
Snell's law

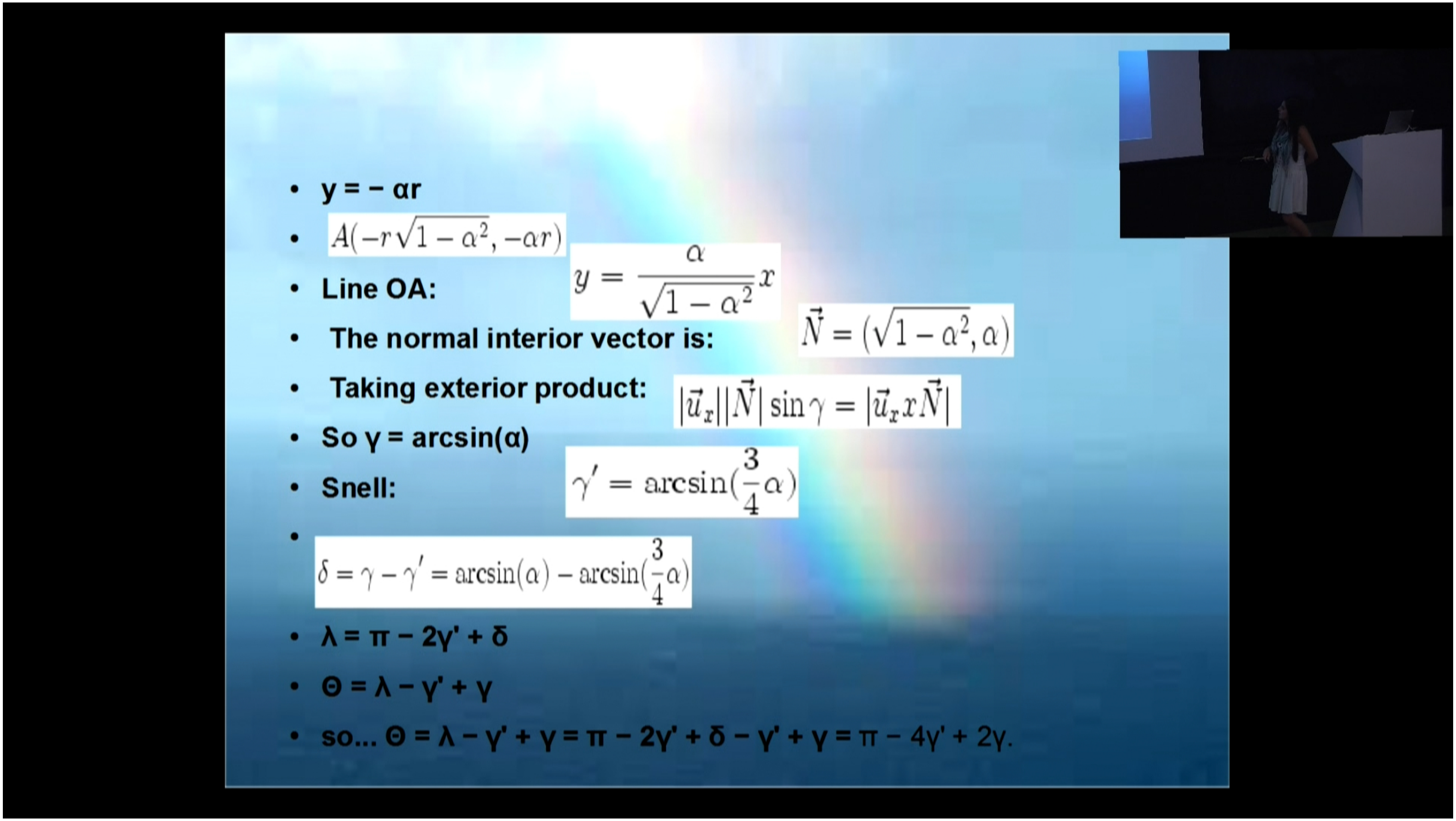


$$V2 \sin\Theta 1 = V1 \sin\Theta 2$$


Refractive index
• ratio of the speed of light in vacuum relati
to that in the considered medium (n=c/v)
• varies with the frequency of radiated light ->
different refractive index for each colour.
• n= 1.33 for water, taken for yellow light.
• n(red)<n(yellow)<n(blue)
• Θr=arcsin(sinΘi / nr)

- so...basic processes that form rainbows are reflection and refraction.





• y = − αr
• $A(-r\sqrt{1-\alpha^2}, -\alpha r)$
• Line OA: $y = \frac{\alpha}{\sqrt{1-\alpha^2}}x$
• The normal interior vector is: $\vec{N} = (\sqrt{1-\alpha^2}, \alpha)$
• Taking exterior product: $|\vec{u}_x||\vec{N}| \sin\gamma = |\vec{u}_x x \vec{N}|$
• So γ = arcsin(α)
• Snell: $\gamma' = \arcsin(\frac{3}{4}\alpha)$
• $\delta = \gamma - \gamma' = \arcsin(\alpha) - \arcsin(\frac{3}{4}\alpha)$
• λ = π − 2γ' + δ
• Θ = λ − γ' + γ
• so... Θ = λ − γ' + γ = π − 2γ' + δ − γ' + γ = π − 4γ' + 2γ.

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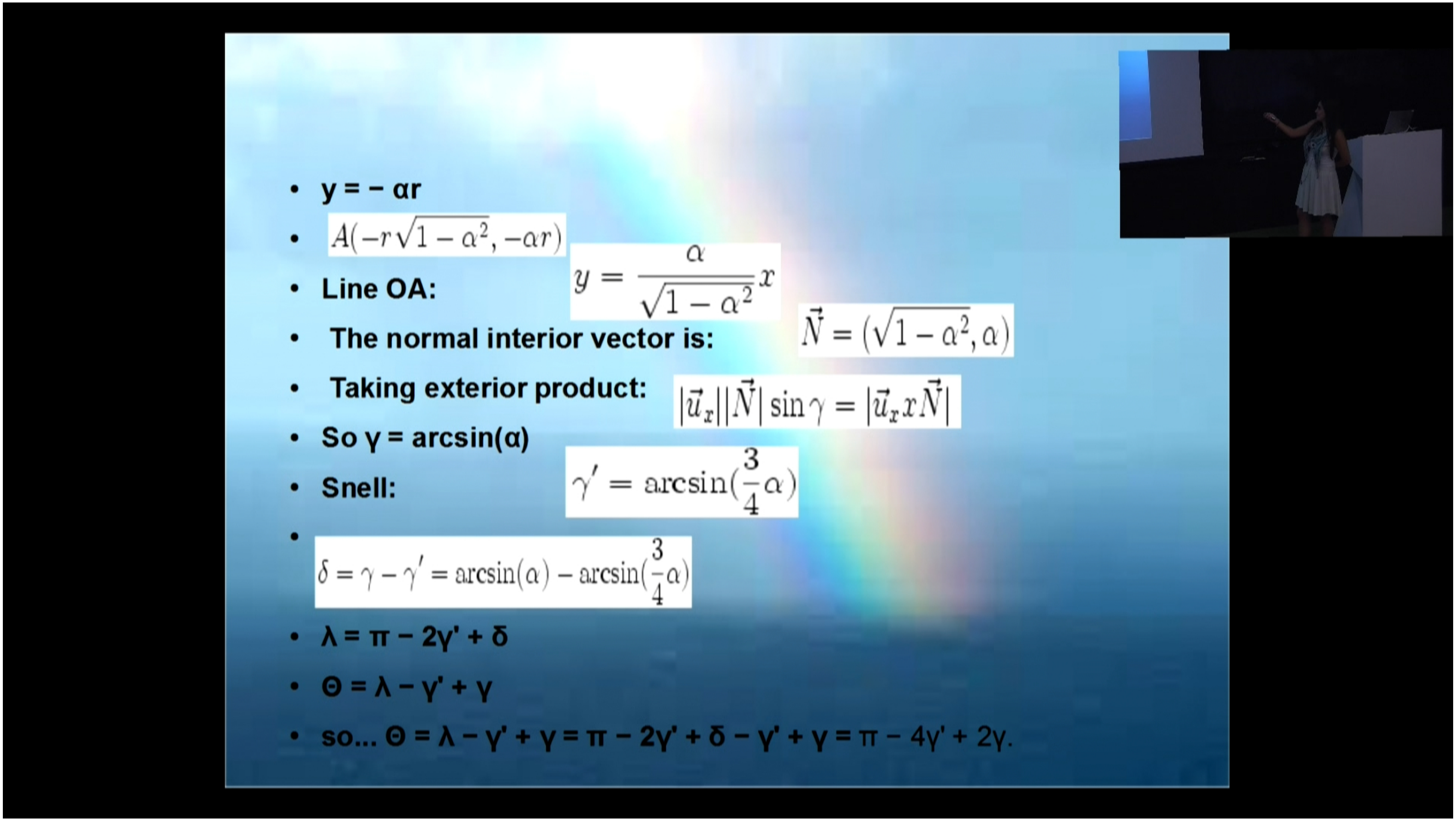
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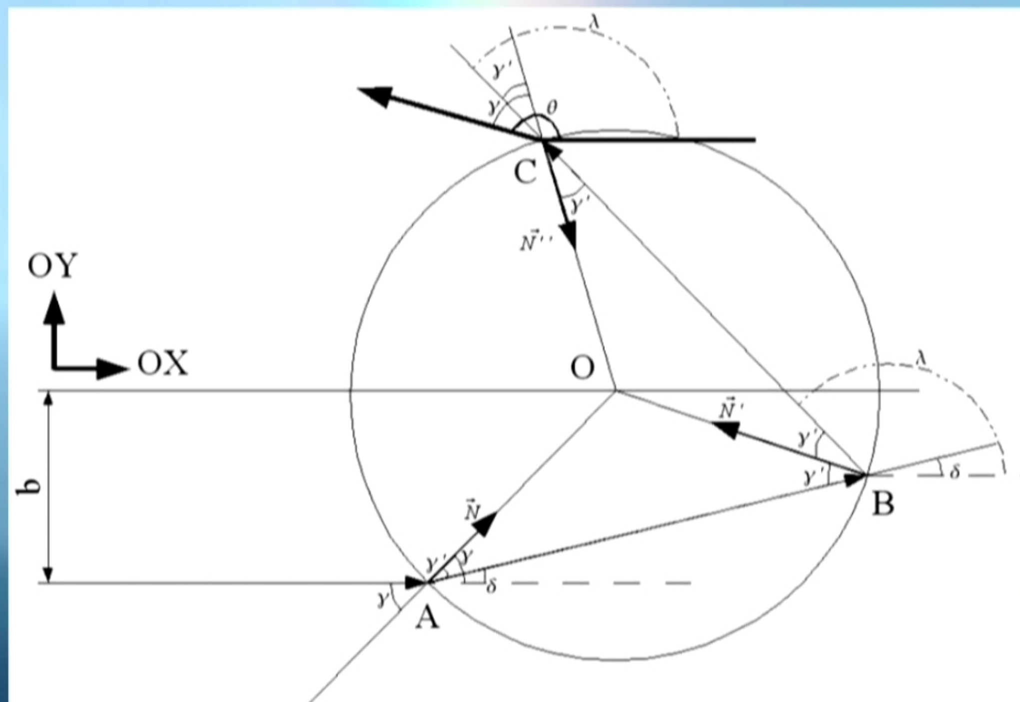

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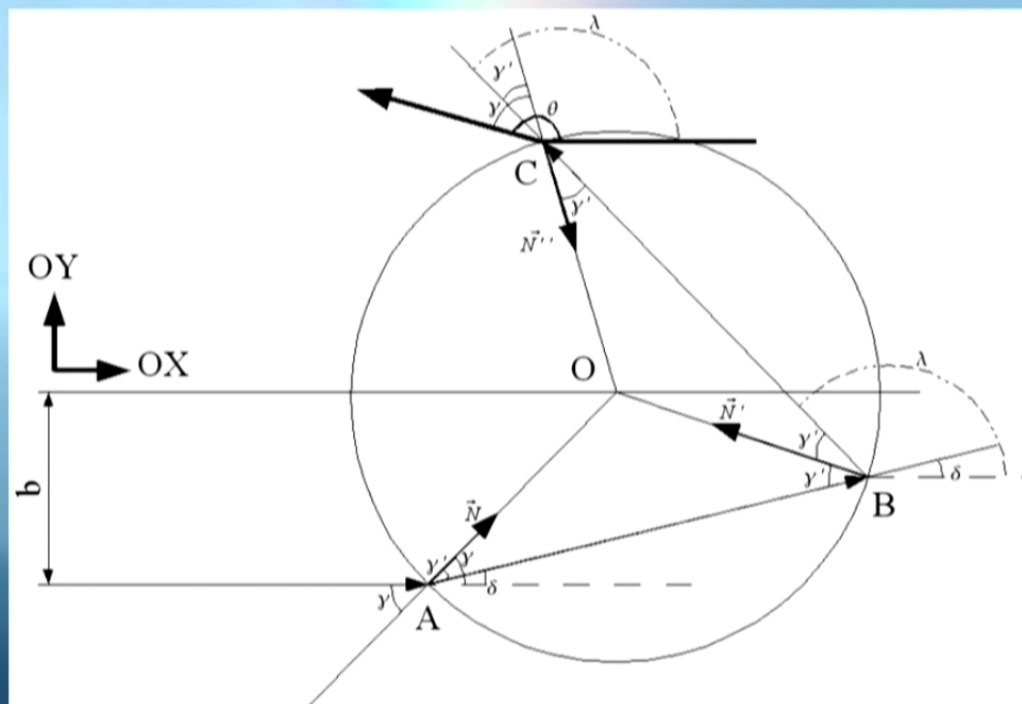

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$$\Theta(\alpha) = \pi - 4\arcsin(\frac{3}{4}\alpha) + 2\arcsin(\alpha)$$

Taking the derivative equal to zero:

$$\Theta'(\alpha) = \frac{-3}{\sqrt{1-\frac{9}{16}\alpha^2}} + \frac{2}{\sqrt{1-\alpha^2}}$$

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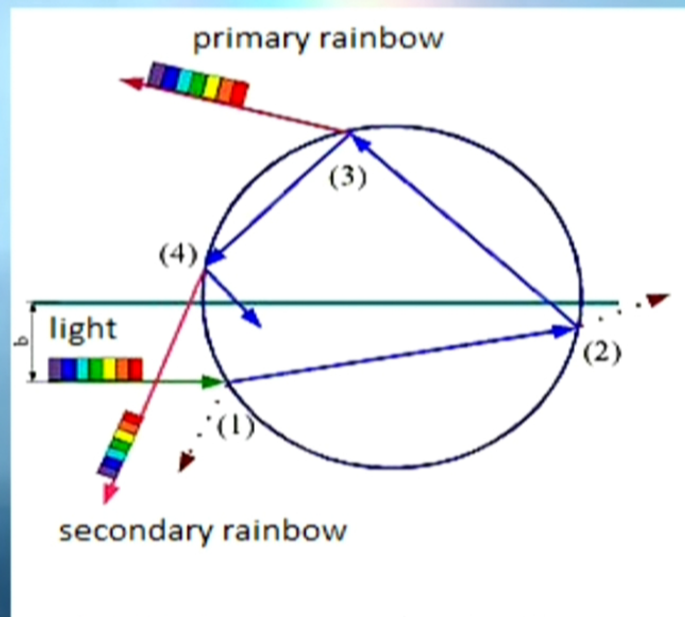
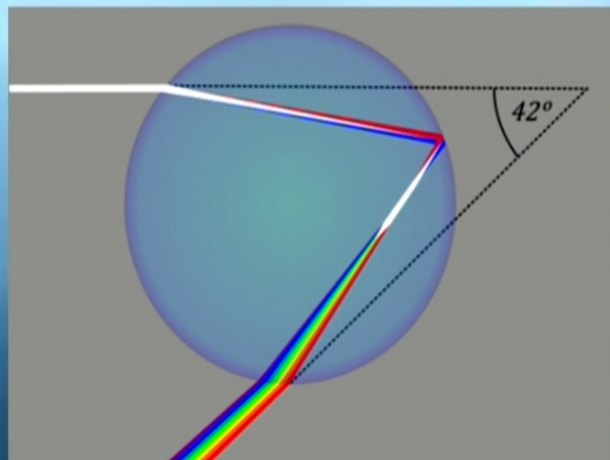
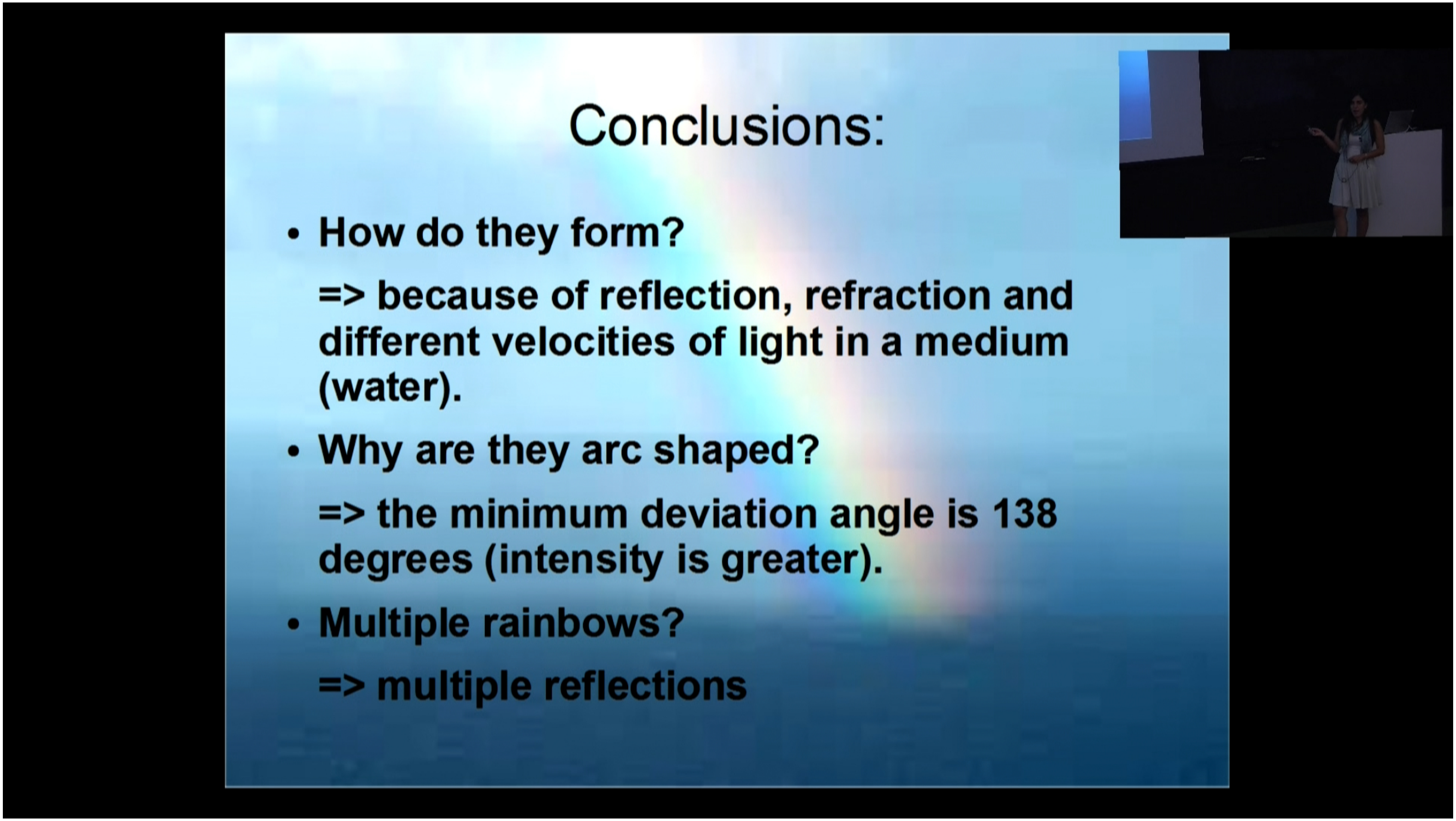


- Blue is refracted at a greater angle than red, but due to the reflection of rays from the back of the droplet, the blue light emerges at a smaller an to the original incident white light ray than the red light. Due to this angl blue is seen on the inside of the arc of the primary rainbow, and red on outside
- If the Sun is higher than 42°, then the rainbow is below the horizon and usually cannot be seen as there are not usually sufficient raindrops between the horizon and the ground, to contribute.




Conclusions:
• How do they form?
=> because of reflection, refraction and different velocities of light in a medium (water).
• Why are they arc shaped?
=> the minimum deviation angle is 138 degrees (intensity is greater).
• Multiple rainbows?
=> multiple reflections