Title: Physics in Nature Presentation: Soap Films and Minimal Surfaces

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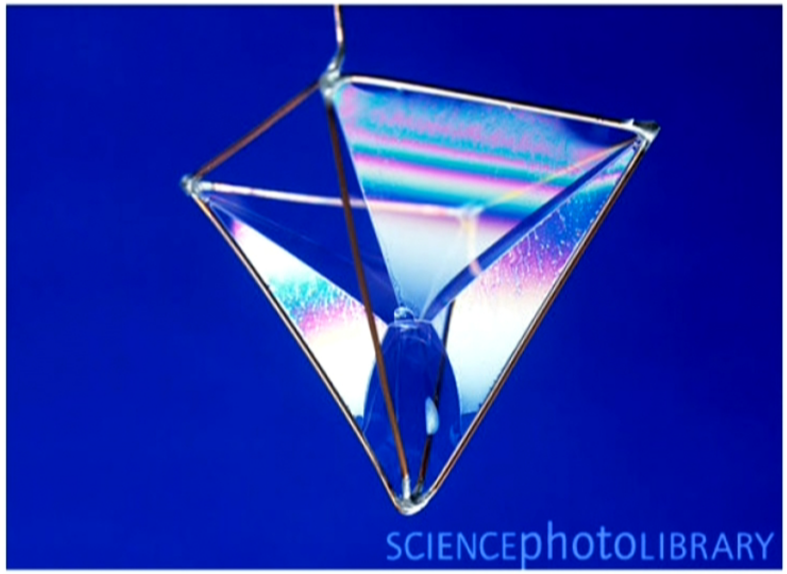
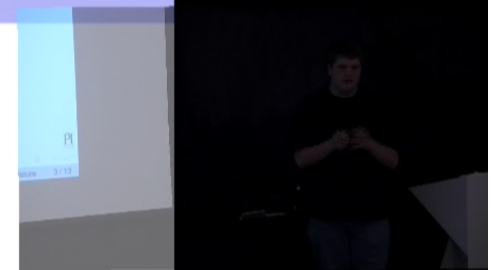
URL: http://pirsa.org/11080111

Abstract:

What are Soap Films?





- A thin film of soapy water stretching across a contour.

Marangoni Effect

- In the presence of surface tension gradient liquids move from low to high surface tension.
- Soap has lower surface tension than water.
- Soap stabilizes the film: As the film stretches surface tension increases.

Surface Tension and Potential Energy

- For equilibrium soap films potential energy is $E = \sigma A$.
- Soap films minimize surface area!

Minimizing Surface Area

Definition

$\Sigma \subset \mathbb{R}^3$ is a surface if there is a domain $\Omega \subset \mathbb{R}^2$ and a continuous function $u : \Omega \to \mathbb{R}$ such that

$$\Sigma = \{(x, y, u(x, y) \ : \ (x, y) \in \Omega\}$$

Definition

The area of a surface Σ is defined as:

$$A(\Sigma) = \int_\Omega \sqrt{1 + u_x^2 + u_y^2} d\mu$$

Minimizing Surface Area, Cont.

- Wire Frame introduced as boundary conditions of u on $\partial\Omega$
- Shape of soap film = Σ that minimizes $A(\Sigma)$ for fixed values of u on $\partial\Omega$
- Calculus of Variations gives the BVP

$$(1 + u_x^2)u_{yy} - 2u_x u_y u_{xy} + (1 + u_y)^2 u_{xx} = 0$$

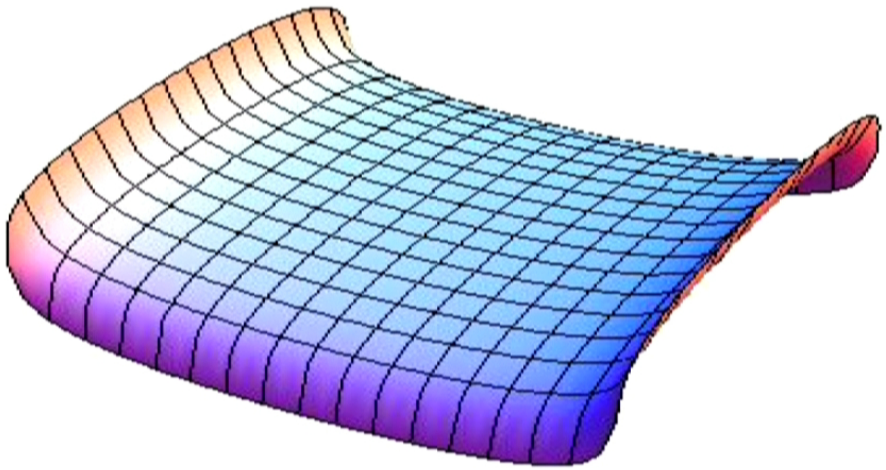
Minimal Surfaces

Definition

1. *The mean curvature H at a point $p \in \Sigma$ is the average of the principal curvatures.*
2. *A surface with $H = 0 \; \forall p \in \Sigma$ is called a minimal surface*

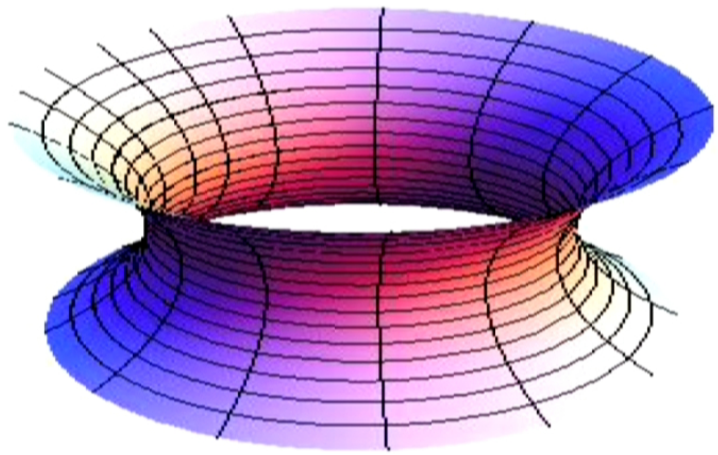
- It turns out $H = (1 + u_x^2)u_{yy} - 2u_x u_y u_{xy} + (1 + u_y)^2 u_{xx}$
- Σ is minimal if and only if Σ minimizes surface area!

Scherk Surface



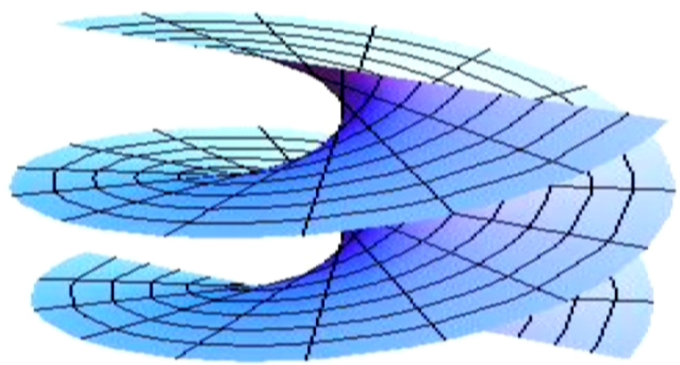
- Soap film with wire frame $u(x, -\frac{\pi}{2}) = u(x, \frac{\pi}{2}) = -1$, $u(-\frac{\pi}{2}, y) = u(\frac{\pi}{2}, y) = 1$

Catenoid



- Soap film formed by two parallel circles

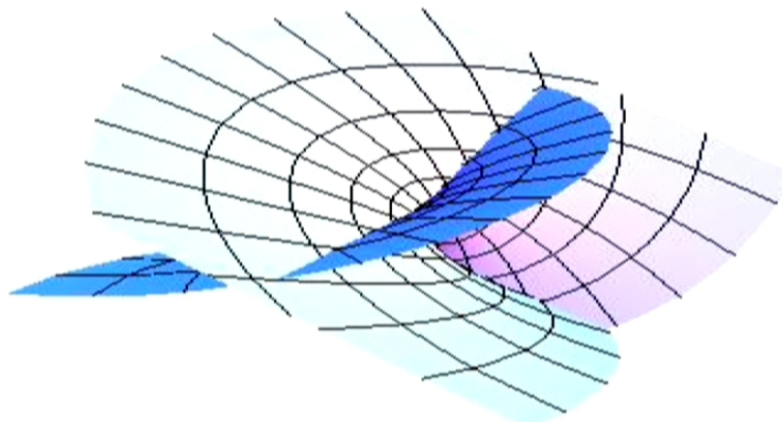
Helicoid



- Soap film formed by a helix.

Summary

- Soap acts as a stabilizer so that films may form
- Shape taken by film minimizes surface area
- Equivalent to having zero mean curvature
- $\Rightarrow$ Soap films are minimal surfaces



Thank You