

Title: Physics in Nature Presentation: The Physics of Flight and Conformal Mappings

Date: Aug 19, 2011 03:00 PM

URL: <http://pirsa.org/11080106>

Abstract:

## The Airfoil

*Kutta condition* : fluid flowing over the upper and lower surfaces meet at the trailing edge of the airfoil.

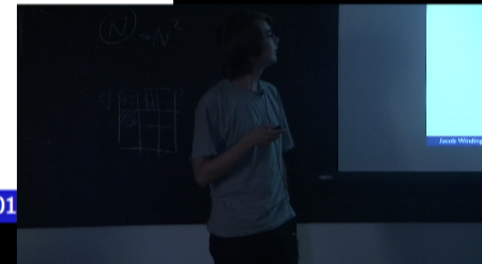
This accounts for the friction between the flowing air and the surface that is necessary for lift to be generated.



# Conformal mappings

A conformal map is a map that preserves angles.

On the complex plane, conformal = holomorphic.

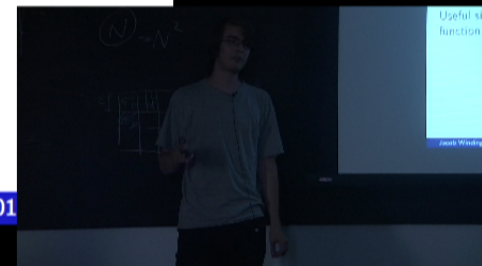


## Conformal mappings

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On the complex plane, conformal = holomorphic.

Useful since we can represent the information of the flow as a holomorphic function.

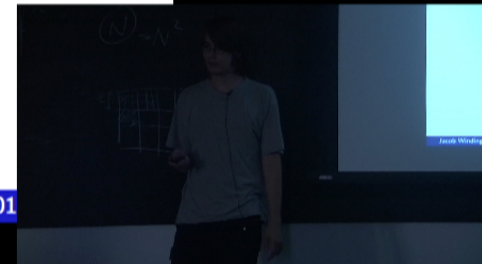


## The Joukowski transform

$$\omega(z) = z + \frac{\lambda^2}{z}$$

where  $\lambda$  is a parameter determining the shape.

The Joukowski transform maps a circle conformally to an airfoil-like shape.



## The Joukowski transform

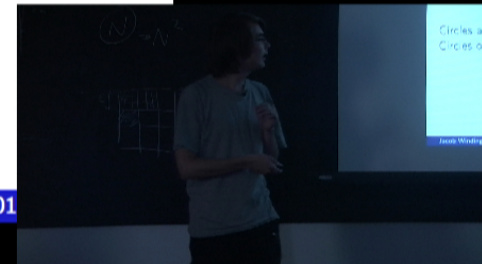
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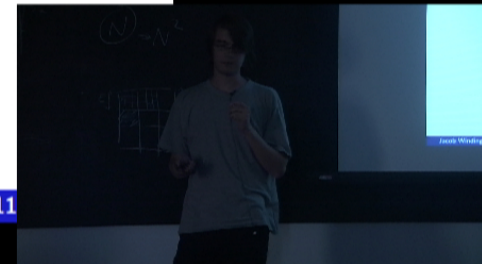
Circles around origin  $\mapsto$  ellipses or lines

Circles offset a little  $\mapsto$  airfoil-like shapes.



## The Joukowski transform

Also transforms the flow.  
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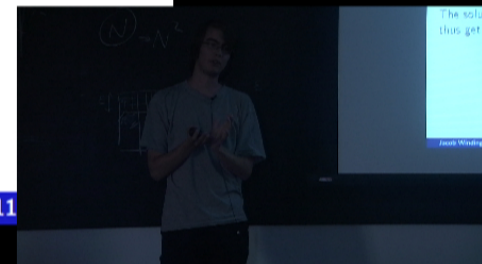


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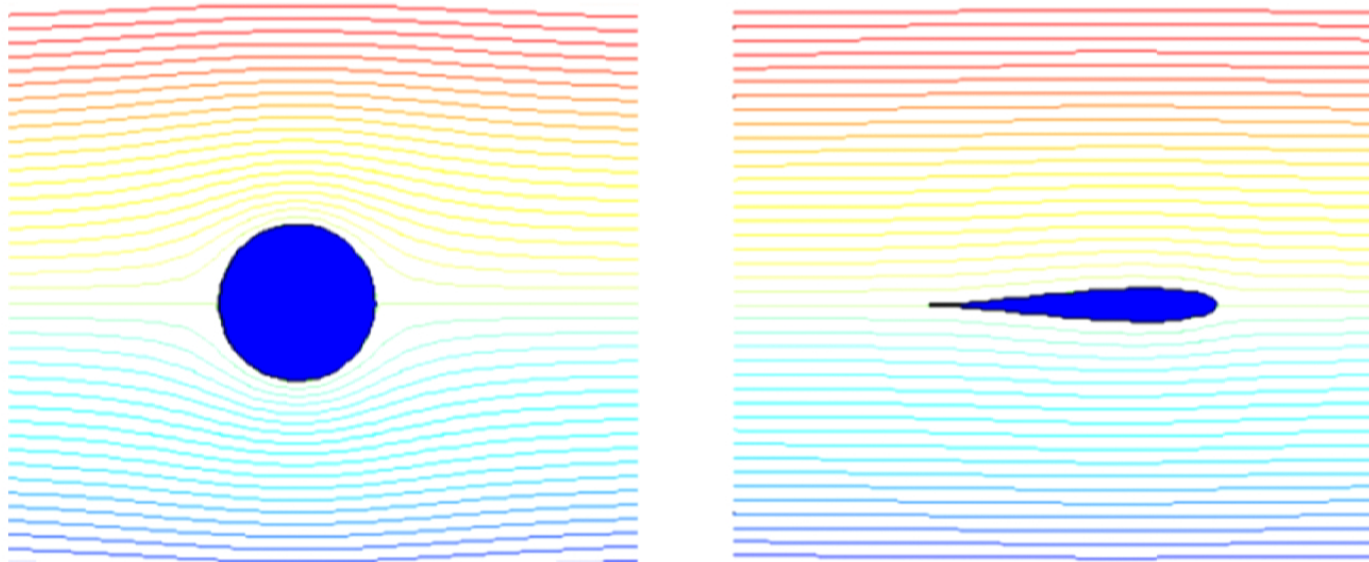
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For flow around the cylinder the symmetry greatly helps us solve the flow equation.

The solution can then be mapped by the Joukowski transform and we can thus get the flow around the airfoil-like shape.



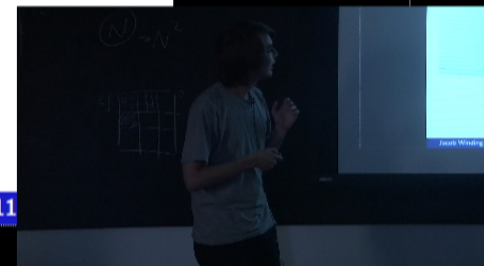
## A last example



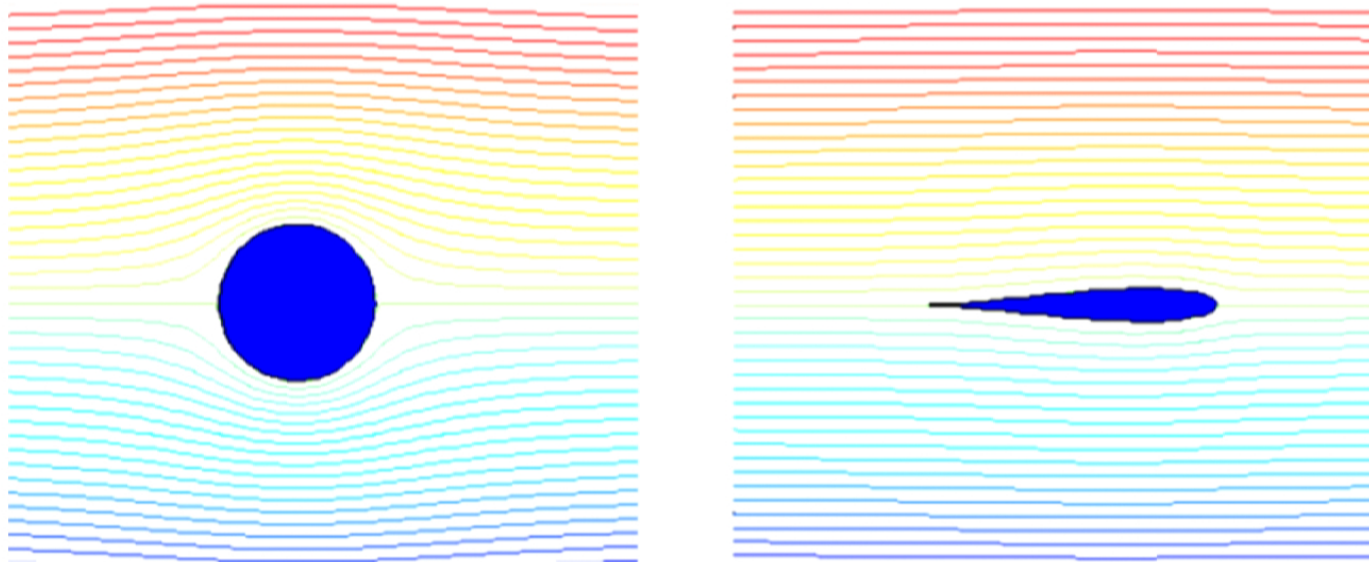
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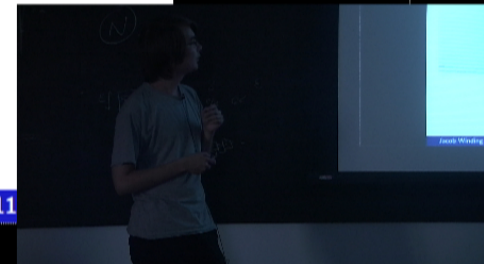
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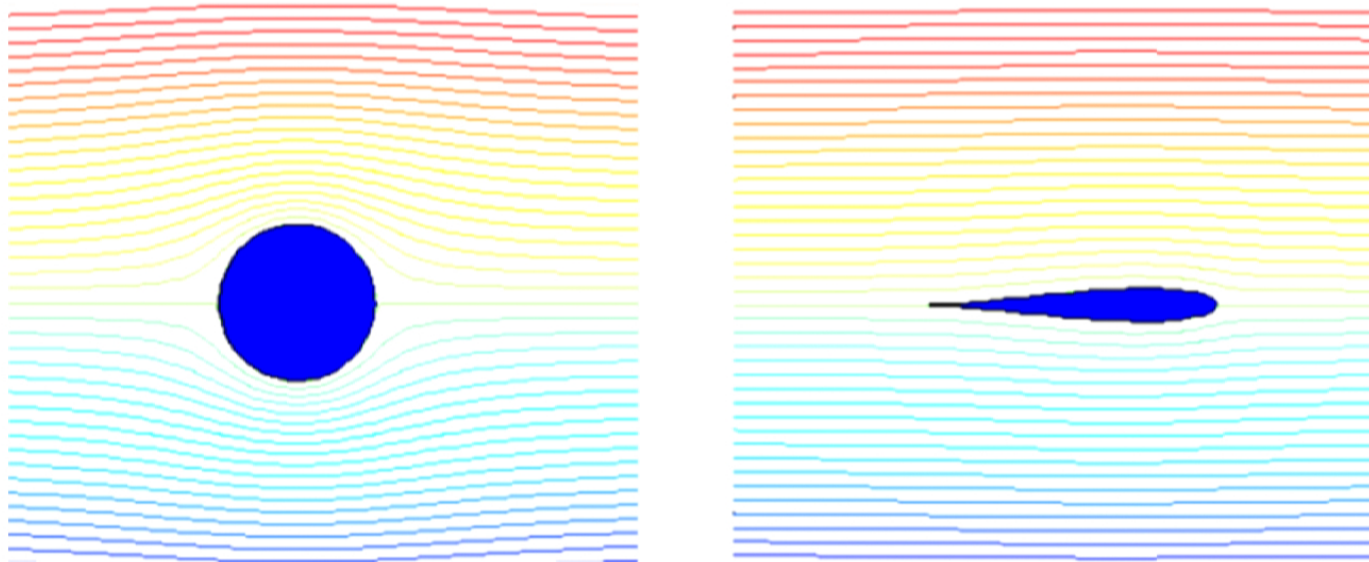
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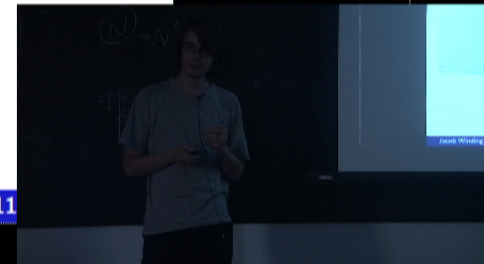
## A last example





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## References

-  Nitin R. Kapania, Katherine Terracciano, Shannon Taylor (2008)  
Modelling the Fluid Flow around Airfoils Using Conformal Mapping
-  NASA; <http://www.grc.nasa.gov/WWW/k-12/airplane/map.html>  
Conformal Mappings, Joukowski transform



# The Joukowski transform - an example

